Computer algebra independent integration tests

1-Algebraic-functions/1.2-Trinomial-products/1.2.3-General/1.2.3.2-d-x-^m-a+b-x^n+c-x^-2-n-^p

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Contents

Chapter 1

Introduction

Local contents

This report gives the result of running the computer algebra independent integration test. Elementary Algebraic integrals version.

The download section below contains links to download the problems in plain text format used for all CAS systems.

The number of integrals in this report is [549]. This is test number [32].

1.1 Listing of CAS systems tested

The following systems were tested at this time.

1. Mathematica 12.3.1 (64 bit) on windows 10.
2. Rubi 4.16.1 in Mathematica 12.3.1 on windows 10.
3. Maple 2021.1 (64 bit) on windows 10.
4. Maxima 5.45 on Linux. (via sagemath 9.3)
5. Fricas 1.3.7 on Linux (via sagemath 9.3)
6. Giac/Xcas 1.7 on Linux. (via sagemath 9.3)
7. Sympy 1.8 under Python 3.8.8 using Anaconda distribution on Ubuntu.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 under windows 10 (64 bit)
9. `IntegrateAlgebraic` under Mathematica 12.3.1 on windows 10. `https://github.com/stblake/algebraic_integration`. September 15, 2021 version.

Maxima, Fricas and Giac/Xcas were called from inside SageMath. This was done using SageMath integrate command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly using Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	100.00 (549)	0.00 (0)
Mathematica	100.00 (549)	0.00 (0)
Fricas	97.45 (535)	2.55 (14)
Maple	90.35 (496)	9.65 (53)
Giac	75.05 (412)	24.95 (137)
Mupad	65.57 (360)	34.43 (189)
Maxima	55.19 (303)	44.81 (246)
Sympy	51.37 (282)	% 48.63 (267)
IntegrateAlgebraic	39.89 (219)	60.11 (330)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

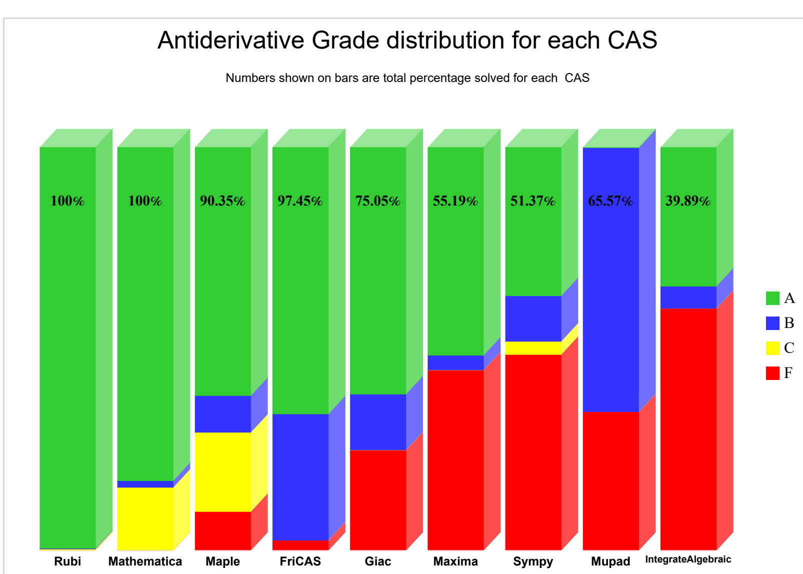
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

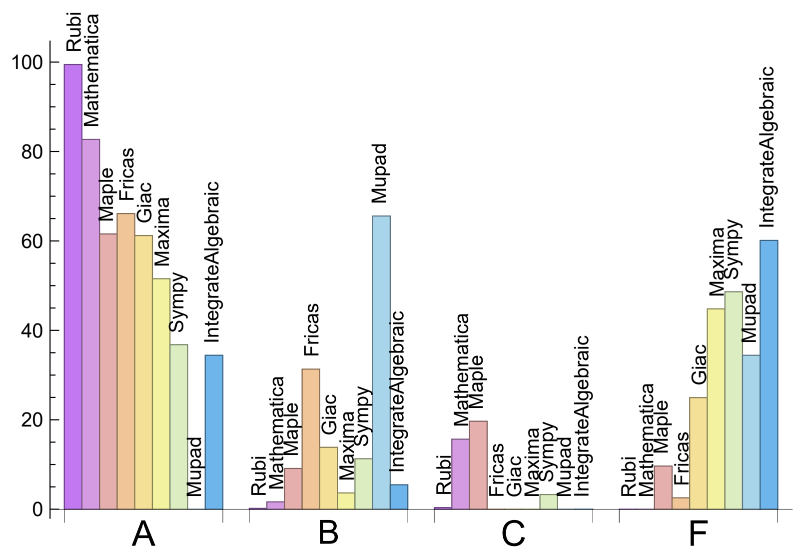
System	% A grade	% B grade	% C grade	% F grade
Rubi	99.45	0.18	0.36	0.00
Mathematica	82.70	1.64	15.66	0.00
Fricas	66.12	31.33	0.00	2.55
Maple	61.57	9.11	19.67	9.65
Giac	61.20	13.84	0.00	24.95
Maxima	51.55	3.64	0.00	44.81
Sympy	36.79	11.29	3.28	48.63
IntegrateAlgebraic	34.43	5.46	0.00	60.11
Mupad	N/A	65.57	0.00	34.43

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



The following table shows the distribution of the different types of failure for each CAS. There are 3 types of reasons why it can fail. The first is when CAS returns back the input within the time limit, which means it could not solve it. This the typical normal failure **F** .

The second is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned **F(-1)**.

The third is due to an exception generated. Assigned **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in CAS. This type of error requires more investigations to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	0	0.00 %	0.00 %	0.00 %
Mathematica	0	0.00 %	0.00 %	0.00 %
Maple	53	100.00 %	0.00 %	0.00 %
Fricas	14	0.00 %	100.00 %	0.00 %
IntegrateAlgebraic	330	100.00 %	0.00 %	0.00 %
Giac	137	88.32 %	2.92 %	8.76 %
Maxima	246	52.44 %	14.63 %	32.93 %
Sympy	267	74.91 %	23.97 %	1.12 %
Mupad	189	100.00 %	0.00 %	0.00 %

Table 1.4: Failure statistics for each CAS

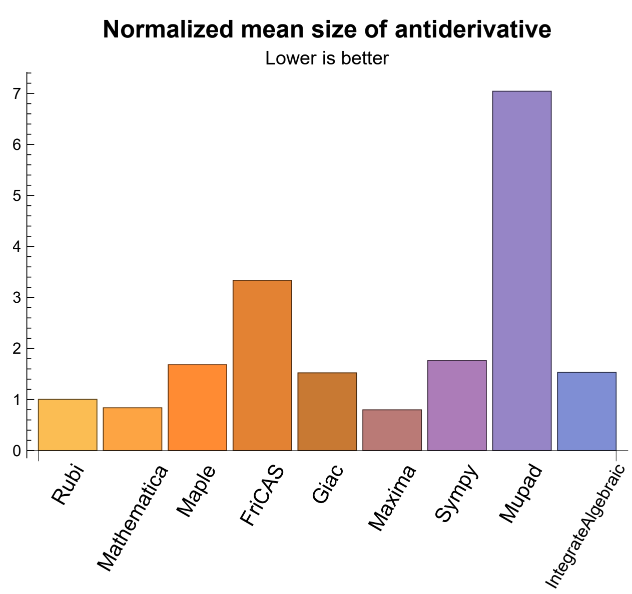
1.3 Performance

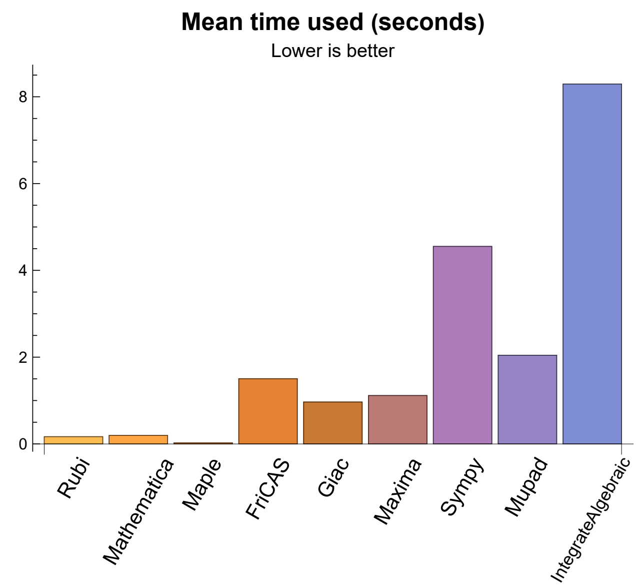
The table below summarizes the performance of each CAS system in terms of CPU time and leaf size of results.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.17	162.59	1.00	126.00	1.00
Mathematica	0.20	104.81	0.84	81.00	0.89
Maple	0.02	283.22	1.68	78.00	0.79
Maxima	1.12	80.96	0.80	55.00	0.76
Fricas	1.50	774.34	3.34	143.00	1.50
Sympy	4.55	183.60	1.76	59.00	0.94
Giac	0.97	256.17	1.52	91.00	0.85
Mupad	2.04	1652.88	7.04	119.00	0.99
IntegrateAlgebraic	8.29	229.20	1.53	101.00	0.88

Table 1.5: Time and leaf size performance for each CAS

The following are bar charts for the normalized leafsize and time used columns from the above table.




Mean time used (seconds)
Lower is better
0
2
4
6
8
Rubi
Mathematica
Maple
FriCAS
Giac
Maxima
Sympy
Mupad
IntegrateAlgebraic

1.4 list of integrals that has no closed form antiderivative

{}

1.5 list of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

IntegrateAlgebraic {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {}

Mathematica {271, 273, 275, 280, 282, 283, 285, 286, 288, 292, 294}

IntegrateAlgebraic {}

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

Mupad Verification phase not implemented yet.

1.7 Timing

The command `AboluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of _int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call has completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out is not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for Rubi and Mathematica. Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative produced was correct.

Verification phase has 3 minutes time out. An integral whose result was not verified could still be correct. Further investigation is needed on those integrals which failed verifications. Such integrals are marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since these integrals are run in a batch mode, using an automated script, and by using sagemath (SageMath uses Maxima), then any integral where Maxima needs an interactive response from the user to answer a question during evaluation of the integral in order to complete the integration, will fail and is counted as failed.

The exception raised is `ValueError`. Therefore Maxima result below is lower than what could result if Maxima was run directly and each question Maxima asks was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the Timofeev test file, there were about 14 such integrals out of total 705, or about 2 percent. This pecrentage can be higher or lower depending on the specific input test file.

Such integrals can be indentified by looking at the output of the integration in each section for Maxima. The exception message will indicate of the error is due to the interactive question being asked or not.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath loading of Maxima `abs_integrate` was found to cause some problem. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/ for reference.

1.9.2 Important note about FriCAS and Giac/XCAS results

There are Few integrals which failed due to SageMath not able to translate the result back to SageMath syntax and not because these CAS system were not able to do the integrations.

These will fail With error `Exception raised: NotImplementedError`

The number of such cases seems to be very small. About 1 or 2 percent of all integrals.

Hopefully the next version of SageMath will have complete translation of FriCAS and XCAS syntax and I will re-run all the tests again when this happens.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi and Maple, the buildin system function `LeafSize` is used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special buildin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy and Giac antiderivatives is determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which is called directly from Python, the following code is used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
         leafCount =1
```

1.9.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative, Maple was used to determine the leaf size of Mupad output by post processing.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

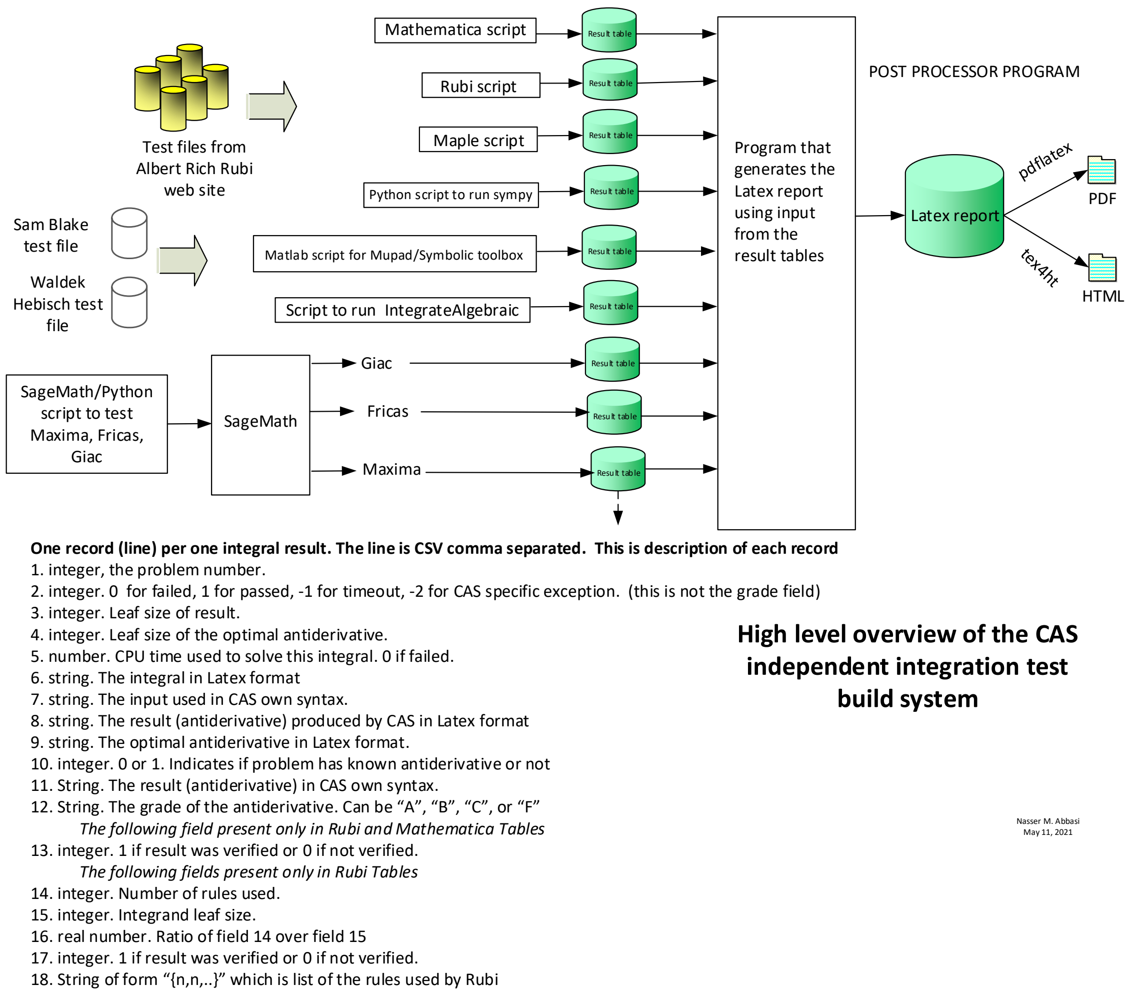
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



Chapter 2

detailed summary tables of results

Local contents

2.1 List of integrals sorted by grade for each CAS

Local contents

2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549 }

B grade: { 141 }

C grade: { 163, 414 }

F grade: { }

2.1.2 Mathematica

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 138, 139, 140, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 158, 161, 170, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 270, 272, 274, 281, 284, 287, 289, 290, 291, 293, 296, 308, 309, 310, 311, 312, 313, 314, 316, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 486, 487, 488, 489, 490, 491, 492, 493, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548 }

B grade: { 61, 141, 494, 495, 496, 497, 498, 499, 549 }

C grade: { 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 157, 159, 160, 162, 163, 164, 165, 166, 167, 168, 169, 171, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 271, 273, 275, 276, 277, 278, 279, 280, 282, 283, 285, 286, 288, 292, 294, 295, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 315, 317, 318, 319, 320, 321, 322, 323, 324, 348, 349, 350, 396, 397, 414, 436, 437, 438, 439, 440, 441, 470, 471, 483, 484, 485 }

F grade: { }

2.1.3 Maple

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 108, 111, 113, 114, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 138, 139, 140, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 158, 161, 164, 167, 170, 209, 210, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 255, 256, 257, 258, 259, 260, 261, 270, 271, 272, 273, 274, 275, 277, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 308, 310, 312, 313, 314, 316, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 349, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 363, 364, 365, 366, 375, 376, 379, 380, 381, 382, 383, 384, 385, 386, 387, 390, 391, 392, 393, 394, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 438, 439, 444, 445, 446, 447, 448, 449, 450, 452, 453, 454, 455, 456, 457, 459, 460, 461, 462, 463, 464, 465, 466, 468, 487, 488 }

B grade: { 61, 82, 107, 109, 110, 112, 115, 116, 141, 215, 254, 276, 278, 309, 311, 315, 317, 348, 350, 361, 362, 367, 368, 369, 370, 371, 372, 373, 374, 377, 378, 388, 389, 395, 473, 474, 475, 476, 477, 478, 479, 486, 494, 495, 496, 497, 498, 499, 548, 549 }

C grade: { 130, 131, 132, 133, 134, 135, 136, 137, 157, 159, 160, 162, 163, 165, 166, 168, 169, 171, 262, 263, 264, 265, 266, 267, 268, 269, 299, 300, 301, 302, 303, 304, 305, 306, 307, 318, 319, 320, 321, 322, 323, 324, 396, 397, 436, 437, 440, 441, 451, 458, 472, 480, 481, 482, 483, 484, 485, 491, 492, 493, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547 }

F grade: { 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 211, 212, 213, 214, 411, 412, 413, 414, 442, 443, 467, 469, 470, 471, 489, 490 }

2.1.4 Maxima

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 43, 44, 46, 47, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 80, 81, 83, 84, 86, 87, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 138, 139, 140, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 158, 161, 164, 167, 170, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 289, 291, 293, 295, 297, 308, 310, 312, 314, 316, 325, 326, 327, 329, 331, 332, 333, 334, 344, 345, 346, 347, 348, 349, 379, 380, 381, 382, 383, 384, 385, 386, 387, 394, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 415, 416, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 442, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 472, 491, 492, 493 }

B grade: { 9, 21, 42, 45, 48, 79, 82, 85, 88, 141, 328, 330, 494, 495, 496, 497, 498, 499, 548, 549 }

C grade: { }

F grade: { 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 157, 159, 160, 162, 163, 165, 166, 168, 169, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 280, 281, 282, 283, 284, 285, 286, 287, 288, 290, 292, 294, 296, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 309, 311, 313, 315, 317, 318, 319, 320, 321, 322, 323, 324, 335, 336, 337, 338, 339, 340, 341, 342, 343, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 388, 389, 390, 391, 392, 393, 395, 396, 397, 398, 414, 417, 436, 437, 438, 439, 440, 441, 443, 467, 468, 469, 470, 471, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547 }

2.1.5 FriCAS

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 138, 139, 140, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 158, 161, 164, 167, 170, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 212, 213, 214, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 238, 240, 242, 253, 255, 257, 259, 261, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 287, 289, 290, 291, 293, 295, 296, 297, 300, 303, 306, 308, 310, 314, 316, 325, 327, 331, 333, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 394, 399, 401, 402, 403, 404, 405, 406, 407, 408, 411, 412, 413, 414, 415, 427, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 477, 478, 479, 486, 487, 488, 489, 503, 504, 526, 528, 529 }

B grade: { 82, 130, 131, 132, 133, 134, 135, 136, 137, 141, 157, 159, 160, 162, 163, 165, 166, 168, 169, 171, 211, 215, 237, 239, 241, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 254, 256, 258, 260, 262, 263, 264, 265, 266, 267, 268, 269, 286, 288, 292, 294, 298, 299, 301, 302, 304, 305, 307, 309, 311, 312, 313, 315, 317, 318, 319, 320, 321, 322, 323, 324, 326, 328, 329, 330, 332, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 393, 395, 396, 397, 409, 410, 476, 480, 481, 482, 483, 484, 485, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 527, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549 }

C grade: { }

F grade: { 398, 400, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 428 }

2.1.6 Sympy

A grade: { 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 89, 90, 91, 92, 93, 94, 95, 96, 97, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 254, 256, 258, 260, 264, 265, 266, 267, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 281, 284, 287, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 327, 329, 331, 333, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 379, 380, 381, 382, 383, 384, 385, 386, 387, 395, 396, 399, 429, 430, 431, 432, 434, 435, 438, 439, 493, 500, 502, 505, 507, 525, 527, 530, 532 }

B grade: { 125, 126, 127, 128, 129, 253, 255, 257, 259, 326, 328, 330, 332, 334, 351, 352, 353, 354, 355, 356, 357, 361, 362, 363, 364, 365, 366, 370, 371, 372, 373, 374, 375, 376, 494, 495, 496, 497, 498, 499, 501, 503, 504, 506, 508, 509, 510, 511, 518, 520, 526, 528, 529, 531, 533, 534, 535, 536, 542, 544, 548, 549 }

C grade: { 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 280, 282, 283, 285, 286, 288

}

F grade: { 1, 2, 3, 4, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 261, 262, 263, 268, 269, 358, 359, 360, 367, 368, 369, 377, 378, 388, 389, 390, 391, 392, 393, 394, 397, 398, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 433, 436, 437, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 512, 513, 514, 515, 516, 517, 519, 521, 522, 523, 524, 537, 538, 539, 540, 541, 543, 545, 546, 547 }

2.1.7 Giac

A grade: { 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 120, 122, 123, 124, 125, 126, 127, 128, 129, 138, 139, 140, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 158, 161, 164, 167, 170, 175, 176, 186, 187, 199, 200, 209, 210, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 255, 257, 259, 261, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 291, 292, 293, 294, 295, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 329, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 394, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 413, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 432, 436, 437, 438, 451, 452, 453, 454, 459, 460, 461, 476, 501, 503, 506, 509, 511, 515, 524, 528, 548, 549 }

B grade: { 45, 61, 82, 118, 119, 121, 141, 157, 159, 160, 162, 163, 165, 166, 168, 215, 216, 254, 256, 258, 260, 290, 296, 328, 330, 395, 411, 412, 458, 482, 492, 493, 494, 495, 496, 497, 498, 499, 500, 502, 504, 507, 508, 510, 512, 513, 514, 516, 517, 518, 519, 520, 521, 522, 523, 525, 526, 527, 529, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547 }

C grade: { }

F grade: { 1, 2, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 130, 131, 132, 133, 134, 135, 136, 137, 169, 171, 172, 173, 174, 177, 178, 179, 180, 181, 182, 183, 184, 185, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 201, 202, 203, 204, 205, 206, 207, 208, 211, 212, 213, 214, 262, 263, 264, 265, 266, 267, 268, 269, 388, 389, 390, 391, 392, 393, 396, 397, 398, 414, 415, 429, 430, 431, 433, 434, 435, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 455, 456, 457, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 477, 478, 479, 480, 481, 483, 484, 485, 486, 487, 488, 489, 490, 491, 505, 530 }

2.1.8 Mupad

A grade: { }

B grade: { 1, 2, 3, 4, 5, 6, 9, 12, 15, 16, 17, 18, 19, 20, 21, 22, 27, 30, 43, 44, 45, 46, 47, 48, 49, 61, 80, 81, 82, 83, 84, 85, 86, 87, 88, 91, 94, 97, 100, 108, 111, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 186, 187, 199, 200, 201, 202, 208, 209, 210, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 390, 391, 394, 395, 396, 397, 399, 404, 407, 408, 409, 410, 411, 412, 413, 414, 421, 432, 476, 486, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549 }

C grade: { }

F grade: { 7, 8, 10, 11, 13, 14, 23, 24, 25, 26, 28, 29, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 89, 90, 92, 93, 95, 96, 98, 99, 101, 102, 103, 104, 105, 106, 107, 109, 110, 112, 113, 114, 115, 116, 117, 118, 119, 120, 179, 180, 181, 182, 183, 184, 185, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 203, 204, 205, 206, 207, 211, 212, 213, 214, 388, 389, 392, 393, 398, 400, 401, 402, 403, 405, 406, 415, 416, 417, 418, 419, 420, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 477, 478, 479, 480, 481, 482, 483, 484, 485, 487, 488, 489, 490 }

2.1.9 IntegrateAlgebraic

A grade: { 1, 2, 3, 4, 6, 7, 8, 9, 10, 11, 13, 14, 16, 17, 18, 19, 20, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 40, 41, 42, 43, 44, 46, 47, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 77, 78, 80, 81, 83, 84, 86, 87, 89, 90, 92, 93, 94, 95, 96, 98, 99, 101, 102, 104, 105, 107, 109, 110, 112, 113, 115, 116, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 388, 389, 390, 391, 392, 393, 394, 398, 399, 405, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 428, 429, 430, 431, 432, 433, 434, 435, 444, 445, 446, 447, 448, 449, 450, 455, 462, 465, 466, 473, 474, 475, 476, 477, 478, 479, 486, 487, 488, 489, 490 }

B grade: { 12, 15, 21, 39, 45, 48, 61, 76, 79, 82, 85, 88, 91, 97, 100, 103, 106, 108, 111, 114, 117, 400, 406, 407, 408, 409, 410, 415, 416, 427 }

C grade: { }

F grade: { 5, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 395, 396, 397, 401, 402, 403, 404, 411, 412, 413, 414, 436, 437, 438, 439, 440, 441, 442, 443, 451, 452, 453, 454, 456, 457, 458, 459, 460, 461, 463, 464, 467, 468, 469, 470, 471, 472, 480, 481, 482, 483, 484, 485, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column `N.S.` in the table below, which stands for **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To help make the table fit, `Mathematica` was abbreviated to `MMA` and `IntegrateAlgebraic` to `I.A.`

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	52	52	42	39	46	53	0	0	40	57
N.S.	1	1.00	0.81	0.75	0.88	1.02	0.00	0.00	0.77	1.10
time (sec)	N/A	0.048	0.024	0.006	0.443	1.973	0.000	0.000	1.226	0.279

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	25	25	25	29	14	28	0	0	29	25
N.S.	1	1.00	1.00	1.16	0.56	1.12	0.00	0.00	1.16	1.00
time (sec)	N/A	0.005	0.008	0.004	0.458	1.908	0.000	0.000	1.152	0.203

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	23	23	23	27	17	21	0	14	21	23
N.S.	1	1.00	1.00	1.17	0.74	0.91	0.00	0.61	0.91	1.00
time (sec)	N/A	0.005	0.008	0.004	0.449	1.167	0.000	24.996	1.151	0.407

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	77	77	46	46	38	54	0	52	51	46
N.S.	1	1.00	0.60	0.60	0.49	0.70	0.00	0.68	0.66	0.60
time (sec)	N/A	0.057	0.010	0.004	0.465	1.064	0.000	23.789	1.282	0.950

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	48	48	48	38	37	46	48	38	51	0
N.S.	1	1.00	1.00	0.79	0.77	0.96	1.00	0.79	1.06	0.00
time (sec)	N/A	0.023	0.017	0.007	0.946	1.546	0.147	0.316	0.095	0.000

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	79	79	39	36	83	13	12	23	59	39
N.S.	1	1.00	0.49	0.46	1.05	0.16	0.15	0.29	0.75	0.49
time (sec)	N/A	0.023	0.012	0.003	0.490	1.342	0.107	0.395	1.259	6.282

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	79	79	39	36	13	13	12	29	-1	39
N.S.	1	1.00	0.49	0.46	0.16	0.16	0.15	0.37	-0.01	0.49
time (sec)	N/A	0.023	0.008	0.001	0.466	1.328	0.109	0.371	0.000	6.451

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	79	79	39	36	13	13	12	29	-1	39
N.S.	1	1.00	0.49	0.46	0.16	0.16	0.15	0.37	-0.01	0.49
time (sec)	N/A	0.024	0.007	0.003	0.463	1.671	0.106	0.388	0.000	6.463

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	36	36	38	35	52	13	12	22	33	38
N.S.	1	1.00	1.06	0.97	1.44	0.36	0.33	0.61	0.92	1.06
time (sec)	N/A	0.028	0.009	0.003	0.462	1.667	0.105	0.362	1.229	6.097

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	79	79	39	36	13	13	12	29	-1	39
N.S.	1	1.00	0.49	0.46	0.16	0.16	0.15	0.37	-0.01	0.49
time (sec)	N/A	0.018	0.009	0.007	0.451	1.010	0.105	0.326	0.000	8.160

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	74	74	36	33	10	10	8	20	-1	36
N.S.	1	1.00	0.49	0.45	0.14	0.14	0.11	0.27	-0.01	0.49
time (sec)	N/A	0.012	0.007	0.001	0.440	1.062	0.103	0.379	0.000	11.163

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	75	75	37	34	96	11	10	28	109	197
N.S.	1	1.00	0.49	0.45	1.28	0.15	0.13	0.37	1.45	2.63
time (sec)	N/A	0.020	0.009	0.010	0.462	1.179	0.129	0.289	1.377	0.243

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	77	77	38	36	14	14	8	29	-1	38
N.S.	1	1.00	0.49	0.47	0.18	0.18	0.10	0.38	-0.01	0.49
time (sec)	N/A	0.021	0.009	0.001	0.463	1.072	0.125	0.323	0.000	16.226

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	74	74	37	34	15	15	8	26	-1	39
N.S.	1	1.00	0.50	0.46	0.20	0.20	0.11	0.35	-0.01	0.53
time (sec)	N/A	0.020	0.007	0.003	0.456	1.532	0.134	0.403	0.000	19.038

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	75	75	39	38	99	17	10	43	112	749
N.S.	1	1.00	0.52	0.51	1.32	0.23	0.13	0.57	1.49	9.99
time (sec)	N/A	0.021	0.009	0.009	0.465	0.885	0.162	0.346	1.384	0.615

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	77	77	37	34	13	13	14	30	33	39
N.S.	1	1.00	0.48	0.44	0.17	0.17	0.18	0.39	0.43	0.51
time (sec)	N/A	0.021	0.008	0.003	0.454	1.145	0.168	0.354	1.208	19.206

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	79	79	39	36	15	15	15	31	35	39
N.S.	1	1.00	0.49	0.46	0.19	0.19	0.19	0.39	0.44	0.49
time (sec)	N/A	0.022	0.008	0.003	0.456	0.953	0.179	0.348	1.176	18.832

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	79	79	37	34	86	13	14	30	33	118
N.S.	1	1.00	0.47	0.43	1.09	0.16	0.18	0.38	0.42	1.49
time (sec)	N/A	0.022	0.008	0.003	0.488	1.338	0.185	0.336	1.184	0.429

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	79	79	39	36	15	15	15	31	35	39
N.S.	1	1.00	0.49	0.46	0.19	0.19	0.19	0.39	0.44	0.49
time (sec)	N/A	0.022	0.008	0.003	0.457	1.909	0.193	0.339	1.267	19.284

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	79	79	39	36	15	15	15	31	35	39
N.S.	1	1.00	0.49	0.46	0.19	0.19	0.19	0.39	0.44	0.49
time (sec)	N/A	0.021	0.008	0.003	0.453	1.066	0.197	0.379	1.164	19.244

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	79	79	39	36	117	15	15	31	35	753
N.S.	1	1.00	0.49	0.46	1.48	0.19	0.19	0.39	0.44	9.53
time (sec)	N/A	0.022	0.008	0.004	0.473	1.370	0.209	0.357	1.151	3.559

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	79	79	39	36	15	15	15	31	35	39
N.S.	1	1.00	0.49	0.46	0.19	0.19	0.19	0.39	0.44	0.49
time (sec)	N/A	0.022	0.009	0.004	0.452	1.404	0.217	0.307	1.162	20.698

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	167	167	61	58	35	35	0	67	-1	61
N.S.	1	1.00	0.37	0.35	0.21	0.21	0.00	0.40	-0.01	0.37
time (sec)	N/A	0.043	0.020	0.007	0.472	1.220	0.000	0.385	0.000	11.614

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	119	167	61	58	114	35	0	67	-1	61
N.S.	1	1.40	0.51	0.49	0.96	0.29	0.00	0.56	-0.01	0.51
time (sec)	N/A	0.053	0.015	0.008	0.463	0.759	0.000	0.303	0.000	10.095

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	167	167	61	58	35	35	0	67	-1	61
N.S.	1	1.00	0.37	0.35	0.21	0.21	0.00	0.40	-0.01	0.37
time (sec)	N/A	0.043	0.016	0.007	0.448	1.320	0.000	0.295	0.000	9.620

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	167	167	61	58	35	35	0	67	-1	61
N.S.	1	1.00	0.37	0.35	0.21	0.21	0.00	0.40	-0.01	0.37
time (sec)	N/A	0.041	0.015	0.008	0.452	1.542	0.000	0.359	0.000	9.203

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	78	78	61	58	83	35	0	45	46	61
N.S.	1	1.00	0.78	0.74	1.06	0.45	0.00	0.58	0.59	0.78
time (sec)	N/A	0.050	0.015	0.007	0.487	1.482	0.000	0.437	1.249	8.621

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	167	167	61	58	35	35	0	67	-1	61
N.S.	1	1.00	0.37	0.35	0.21	0.21	0.00	0.40	-0.01	0.37
time (sec)	N/A	0.041	0.016	0.007	0.469	1.125	0.000	0.368	0.000	8.311

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	167	167	61	58	35	35	0	67	-1	61
N.S.	1	1.00	0.37	0.35	0.21	0.21	0.00	0.40	-0.01	0.37
time (sec)	N/A	0.041	0.015	0.007	0.480	1.338	0.000	0.447	0.000	8.242

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	36	36	60	57	52	35	0	44	36	60
N.S.	1	1.00	1.67	1.58	1.44	0.97	0.00	1.22	1.00	1.67
time (sec)	N/A	0.030	0.016	0.007	0.453	1.473	0.000	0.413	1.220	8.020

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	167	167	61	58	35	35	0	67	-1	61
N.S.	1	1.00	0.37	0.35	0.21	0.21	0.00	0.40	-0.01	0.37
time (sec)	N/A	0.038	0.015	0.005	0.443	1.515	0.000	0.374	0.000	10.189

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	162	162	59	56	32	32	0	64	-1	59
N.S.	1	1.00	0.36	0.35	0.20	0.20	0.00	0.40	-0.01	0.36
time (sec)	N/A	0.033	0.014	0.003	0.715	3.026	0.000	0.360	0.000	12.398

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	160	160	60	57	152	32	0	65	-1	256
N.S.	1	1.00	0.38	0.36	0.95	0.20	0.00	0.41	-0.01	1.60
time (sec)	N/A	0.048	0.020	0.010	0.517	1.095	0.000	0.426	0.000	0.442

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	165	165	61	58	37	37	0	67	-1	61
N.S.	1	1.00	0.37	0.35	0.22	0.22	0.00	0.41	-0.01	0.37
time (sec)	N/A	0.042	0.016	0.007	0.496	1.382	0.000	0.337	0.000	17.203

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	163	163	61	58	37	37	0	65	-1	61
N.S.	1	1.00	0.37	0.36	0.23	0.23	0.00	0.40	-0.01	0.37
time (sec)	N/A	0.042	0.018	0.006	0.498	1.469	0.000	0.384	0.000	19.881

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	161	161	62	59	156	38	0	85	-1	317
N.S.	1	1.00	0.39	0.37	0.97	0.24	0.00	0.53	-0.01	1.97
time (sec)	N/A	0.048	0.020	0.010	0.528	1.174	0.000	0.365	0.000	0.744

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	165	165	61	58	37	37	0	69	-1	61
N.S.	1	1.00	0.37	0.35	0.22	0.22	0.00	0.42	-0.01	0.37
time (sec)	N/A	0.042	0.015	0.006	0.525	1.152	0.000	0.362	0.000	20.334

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	163	163	61	58	37	37	0	68	-1	61
N.S.	1	1.00	0.37	0.36	0.23	0.23	0.00	0.42	-0.01	0.37
time (sec)	N/A	0.041	0.017	0.006	0.497	1.420	0.000	0.293	0.000	20.008

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	162	162	61	60	220	39	0	86	-1	1164
N.S.	1	1.00	0.38	0.37	1.36	0.24	0.00	0.53	-0.01	7.19
time (sec)	N/A	0.047	0.015	0.012	0.700	1.299	0.000	0.340	0.000	1.679

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	165	165	61	58	37	37	0	70	-1	61
N.S.	1	1.00	0.37	0.35	0.22	0.22	0.00	0.42	-0.01	0.37
time (sec)	N/A	0.041	0.014	0.006	0.738	1.917	0.000	0.470	0.000	20.101

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	162	162	61	58	37	37	0	67	-1	61
N.S.	1	1.00	0.38	0.36	0.23	0.23	0.00	0.41	-0.01	0.38
time (sec)	N/A	0.041	0.014	0.007	0.677	1.256	0.000	0.391	0.000	20.489

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	161	161	63	60	253	39	0	85	-1	293
N.S.	1	1.00	0.39	0.37	1.57	0.24	0.00	0.53	-0.01	1.82
time (sec)	N/A	0.046	0.021	0.013	0.724	1.311	0.000	0.377	0.000	1.166

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	165	165	61	58	37	37	0	69	151	61
N.S.	1	1.00	0.37	0.35	0.22	0.22	0.00	0.42	0.92	0.37
time (sec)	N/A	0.040	0.013	0.007	0.665	1.525	0.000	0.361	1.214	20.137

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	167	167	61	58	37	37	0	69	151	61
N.S.	1	1.00	0.37	0.35	0.22	0.22	0.00	0.41	0.90	0.37
time (sec)	N/A	0.041	0.016	0.007	0.718	1.191	0.000	0.419	1.224	20.142

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	A	F	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	41	41	59	56	148	35	0	68	151	310
N.S.	1	1.00	1.44	1.37	3.61	0.85	0.00	1.66	3.68	7.56
time (sec)	N/A	0.018	0.013	0.007	0.546	1.419	0.000	0.357	1.208	1.349

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	167	167	61	58	37	37	0	69	151	61
N.S.	1	1.00	0.37	0.35	0.22	0.22	0.00	0.41	0.90	0.37
time (sec)	N/A	0.041	0.013	0.010	0.689	1.365	0.000	0.358	1.191	18.763

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	167	167	61	58	37	37	0	69	151	61
N.S.	1	1.00	0.37	0.35	0.22	0.22	0.00	0.41	0.90	0.37
time (sec)	N/A	0.041	0.016	0.007	0.686	2.232	0.000	0.360	1.207	18.640

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	A	F	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	84	84	61	58	179	37	0	69	151	356
N.S.	1	1.00	0.73	0.69	2.13	0.44	0.00	0.82	1.80	4.24
time (sec)	N/A	0.040	0.014	0.012	0.522	0.850	0.000	0.414	1.209	1.016

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	167	167	61	58	37	37	0	69	151	61
N.S.	1	1.00	0.37	0.35	0.22	0.22	0.00	0.41	0.90	0.37
time (sec)	N/A	0.040	0.014	0.007	0.766	1.648	0.000	0.304	1.215	20.486

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	255	255	83	80	57	57	0	105	-1	83
N.S.	1	1.00	0.33	0.31	0.22	0.22	0.00	0.41	-0.00	0.33
time (sec)	N/A	0.064	0.025	0.013	0.830	0.976	0.000	0.326	0.000	27.589

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	255	255	83	80	57	57	0	105	-1	83
N.S.	1	1.00	0.33	0.31	0.22	0.22	0.00	0.41	-0.00	0.33
time (sec)	N/A	0.058	0.020	0.010	0.603	1.987	0.000	0.322	0.000	24.777

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	160	160	83	80	145	57	0	105	-1	83
N.S.	1	1.00	0.52	0.50	0.91	0.36	0.00	0.66	-0.01	0.52
time (sec)	N/A	0.120	0.021	0.012	0.725	1.450	0.000	0.351	0.000	21.673

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	255	255	83	80	57	57	0	105	-1	83
N.S.	1	1.00	0.33	0.31	0.22	0.22	0.00	0.41	-0.00	0.33
time (sec)	N/A	0.061	0.024	0.010	0.805	1.557	0.000	0.443	0.000	19.229

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	255	255	83	80	57	57	0	105	-1	83
N.S.	1	1.00	0.33	0.31	0.22	0.22	0.00	0.41	-0.00	0.33
time (sec)	N/A	0.057	0.020	0.008	0.674	1.140	0.000	0.401	0.000	18.134

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	119	119	83	80	114	57	0	105	-1	83
N.S.	1	1.00	0.70	0.67	0.96	0.48	0.00	0.88	-0.01	0.70
time (sec)	N/A	0.090	0.021	0.009	0.666	1.150	0.000	0.377	0.000	14.505

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	255	255	83	80	57	57	0	105	-1	83
N.S.	1	1.00	0.33	0.31	0.22	0.22	0.00	0.41	-0.00	0.33
time (sec)	N/A	0.060	0.021	0.008	0.774	1.141	0.000	0.339	0.000	13.722

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	255	255	83	80	57	57	0	105	-1	83
N.S.	1	1.00	0.33	0.31	0.22	0.22	0.00	0.41	-0.00	0.33
time (sec)	N/A	0.059	0.020	0.009	0.714	1.054	0.000	0.345	0.000	12.977

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	78	78	83	80	83	57	0	67	-1	83
N.S.	1	1.00	1.06	1.03	1.06	0.73	0.00	0.86	-0.01	1.06
time (sec)	N/A	0.056	0.021	0.009	0.625	1.126	0.000	0.291	0.000	11.576

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	255	255	83	80	57	57	0	105	-1	83
N.S.	1	1.00	0.33	0.31	0.22	0.22	0.00	0.41	-0.00	0.33
time (sec)	N/A	0.059	0.021	0.009	0.691	1.328	0.000	0.416	0.000	10.545

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	252	252	83	80	56	56	0	104	-1	83
N.S.	1	1.00	0.33	0.32	0.22	0.22	0.00	0.41	-0.00	0.33
time (sec)	N/A	0.057	0.020	0.008	0.511	1.158	0.000	0.317	0.000	10.380

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	B	A	A	F	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	36	36	82	79	52	57	0	66	36	82
N.S.	1	1.00	2.28	2.19	1.44	1.58	0.00	1.83	1.00	2.28
time (sec)	N/A	0.029	0.021	0.009	0.660	0.939	0.000	0.373	1.244	9.630

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	252	252	83	80	56	56	0	104	-1	83
N.S.	1	1.00	0.33	0.32	0.22	0.22	0.00	0.41	-0.00	0.33
time (sec)	N/A	0.054	0.020	0.005	1.101	0.814	0.000	0.368	0.000	11.176

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	247	247	81	78	53	53	0	101	-1	81
N.S.	1	1.00	0.33	0.32	0.21	0.21	0.00	0.41	-0.00	0.33
time (sec)	N/A	0.051	0.019	0.004	1.016	1.287	0.000	0.376	0.000	12.919

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	251	251	82	79	206	55	0	104	-1	314
N.S.	1	1.00	0.33	0.31	0.82	0.22	0.00	0.41	-0.00	1.25
time (sec)	N/A	0.069	0.025	0.008	1.156	1.170	0.000	0.352	0.000	0.580

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	251	251	83	80	59	59	0	105	-1	83
N.S.	1	1.00	0.33	0.32	0.24	0.24	0.00	0.42	-0.00	0.33
time (sec)	N/A	0.061	0.021	0.008	0.820	1.219	0.000	0.334	0.000	16.653

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	251	251	83	80	59	59	0	103	-1	83
N.S.	1	1.00	0.33	0.32	0.24	0.24	0.00	0.41	-0.00	0.33
time (sec)	N/A	0.059	0.021	0.008	1.231	1.217	0.000	0.362	0.000	18.808

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	252	252	85	82	214	61	0	124	-1	364
N.S.	1	1.00	0.34	0.33	0.85	0.24	0.00	0.49	-0.00	1.44
time (sec)	N/A	0.074	0.026	0.011	1.196	0.985	0.000	0.308	0.000	0.980

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	249	249	83	80	59	59	0	107	-1	83
N.S.	1	1.00	0.33	0.32	0.24	0.24	0.00	0.43	-0.00	0.33
time (sec)	N/A	0.059	0.023	0.007	0.972	1.343	0.000	0.351	0.000	18.555

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	251	251	83	80	59	59	0	106	-1	83
N.S.	1	1.00	0.33	0.32	0.24	0.24	0.00	0.42	-0.00	0.33
time (sec)	N/A	0.058	0.020	0.007	0.672	0.715	0.000	0.341	0.000	18.311

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	252	252	85	82	282	61	0	126	-1	368
N.S.	1	1.00	0.34	0.33	1.12	0.24	0.00	0.50	-0.00	1.46
time (sec)	N/A	0.073	0.024	0.013	1.245	1.309	0.000	0.367	0.000	1.164

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	248	248	83	80	59	59	0	107	-1	83
N.S.	1	1.00	0.33	0.32	0.24	0.24	0.00	0.43	-0.00	0.33
time (sec)	N/A	0.058	0.021	0.008	1.099	1.142	0.000	0.326	0.000	18.571

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	247	247	83	80	59	59	0	105	-1	83
N.S.	1	1.00	0.34	0.32	0.24	0.24	0.00	0.43	-0.00	0.34
time (sec)	N/A	0.059	0.021	0.008	1.326	1.285	0.000	0.336	0.000	18.466

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	252	252	85	82	313	61	0	127	-1	366
N.S.	1	1.00	0.34	0.33	1.24	0.24	0.00	0.50	-0.00	1.45
time (sec)	N/A	0.071	0.028	0.012	0.797	1.716	0.000	0.397	0.000	1.821

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	253	253	83	80	59	59	0	108	-1	83
N.S.	1	1.00	0.33	0.32	0.23	0.23	0.00	0.43	-0.00	0.33
time (sec)	N/A	0.062	0.017	0.006	1.013	1.197	0.000	0.397	0.000	18.356

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	247	247	83	80	59	59	0	106	-1	83
N.S.	1	1.00	0.34	0.32	0.24	0.24	0.00	0.43	-0.00	0.34
time (sec)	N/A	0.060	0.016	0.006	0.974	1.079	0.000	0.350	0.000	18.198

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	252	252	85	82	342	61	0	125	-1	2031
N.S.	1	1.00	0.34	0.33	1.36	0.24	0.00	0.50	-0.00	8.06
time (sec)	N/A	0.071	0.019	0.013	1.097	1.172	0.000	0.378	0.000	3.023

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	253	253	83	80	59	59	0	108	-1	83
N.S.	1	1.00	0.33	0.32	0.23	0.23	0.00	0.43	-0.00	0.33
time (sec)	N/A	0.059	0.017	0.006	0.986	1.179	0.000	0.335	0.000	19.396

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	248	248	83	80	59	59	0	105	-1	83
N.S.	1	1.00	0.33	0.32	0.24	0.24	0.00	0.42	-0.00	0.33
time (sec)	N/A	0.060	0.018	0.007	0.990	0.678	0.000	0.346	0.000	19.620

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	A	F	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	251	251	85	82	374	61	0	123	-1	2386
N.S.	1	1.00	0.34	0.33	1.49	0.24	0.00	0.49	-0.00	9.51
time (sec)	N/A	0.069	0.028	0.014	1.097	1.234	0.000	0.370	0.000	3.577

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	251	251	83	80	59	59	0	107	231	83
N.S.	1	1.00	0.33	0.32	0.24	0.24	0.00	0.43	0.92	0.33
time (sec)	N/A	0.058	0.017	0.009	0.985	1.156	0.000	0.340	1.260	20.035

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	253	253	83	80	59	59	0	107	231	83
N.S.	1	1.00	0.33	0.32	0.23	0.23	0.00	0.42	0.91	0.33
time (sec)	N/A	0.058	0.018	0.008	1.163	1.205	0.000	0.448	1.323	20.552

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	B	B	F	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	41	41	81	78	210	57	0	106	231	442
N.S.	1	1.00	1.98	1.90	5.12	1.39	0.00	2.59	5.63	10.78
time (sec)	N/A	0.018	0.017	0.007	1.097	1.141	0.000	0.291	1.222	2.887

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	253	253	83	80	59	59	0	107	231	83
N.S.	1	1.00	0.33	0.32	0.23	0.23	0.00	0.42	0.91	0.33
time (sec)	N/A	0.061	0.020	0.008	1.045	1.100	0.000	0.361	1.305	21.316

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	255	255	83	80	59	59	0	107	231	83
N.S.	1	1.00	0.33	0.31	0.23	0.23	0.00	0.42	0.91	0.33
time (sec)	N/A	0.056	0.017	0.009	0.983	1.111	0.000	0.318	1.241	21.387

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	A	F	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	84	84	83	80	241	59	0	107	231	488
N.S.	1	1.00	0.99	0.95	2.87	0.70	0.00	1.27	2.75	5.81
time (sec)	N/A	0.040	0.018	0.007	1.159	1.019	0.000	0.322	1.217	1.671

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	255	255	83	80	59	59	0	107	231	83
N.S.	1	1.00	0.33	0.31	0.23	0.23	0.00	0.42	0.91	0.33
time (sec)	N/A	0.058	0.018	0.010	0.984	1.107	0.000	0.330	1.220	23.704

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	255	255	83	80	59	59	0	107	231	83
N.S.	1	1.00	0.33	0.31	0.23	0.23	0.00	0.42	0.91	0.33
time (sec)	N/A	0.056	0.018	0.008	0.832	1.094	0.000	0.340	1.230	23.518

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	A	F	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	128	128	83	80	272	59	0	107	231	532
N.S.	1	1.00	0.65	0.62	2.12	0.46	0.00	0.84	1.80	4.16
time (sec)	N/A	0.057	0.020	0.010	1.021	1.275	0.000	0.395	1.222	1.537

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	240	240	131	113	109	123	32	146	-1	144
N.S.	1	1.00	0.55	0.47	0.45	0.51	0.13	0.61	-0.00	0.60
time (sec)	N/A	0.124	0.056	0.010	2.178	0.854	0.212	0.371	0.000	7.362

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	235	235	128	110	106	106	22	143	-1	139
N.S.	1	1.00	0.54	0.47	0.45	0.45	0.09	0.61	-0.00	0.59
time (sec)	N/A	0.114	0.032	0.007	1.365	1.174	0.203	0.409	0.000	6.524

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	44	44	35	32	15	13	10	22	33	149
N.S.	1	1.00	0.80	0.73	0.34	0.30	0.23	0.50	0.75	3.39
time (sec)	N/A	0.035	0.008	0.007	0.932	1.073	0.173	0.340	1.392	0.272

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	202	202	109	97	98	304	24	124	-1	135
N.S.	1	1.00	0.54	0.48	0.49	1.50	0.12	0.61	-0.00	0.67
time (sec)	N/A	0.085	0.025	0.004	1.277	1.755	0.179	0.417	0.000	8.355

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	202	202	109	97	98	299	20	122	-1	135
N.S.	1	1.00	0.54	0.48	0.49	1.48	0.10	0.60	-0.00	0.67
time (sec)	N/A	0.117	0.023	0.004	2.733	1.312	0.190	0.365	0.000	10.094

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	80	80	42	39	43	18	15	32	48	94
N.S.	1	1.00	0.52	0.49	0.54	0.22	0.19	0.40	0.60	1.18
time (sec)	N/A	0.034	0.012	0.008	1.146	1.118	0.276	0.343	1.387	0.201

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	238	238	133	111	106	103	29	131	-1	142
N.S.	1	1.00	0.56	0.47	0.45	0.43	0.12	0.55	-0.00	0.60
time (sec)	N/A	0.108	0.031	0.007	1.742	1.075	0.234	0.371	0.000	15.054

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	243	240	140	117	106	143	32	125	-1	144
N.S.	1	0.99	0.58	0.48	0.44	0.59	0.13	0.51	-0.00	0.59
time (sec)	N/A	0.107	0.034	0.013	2.211	1.298	0.261	0.334	0.000	17.846

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	125	122	54	51	73	33	31	50	75	383
N.S.	1	0.98	0.43	0.41	0.58	0.26	0.25	0.40	0.60	3.06
time (sec)	N/A	0.051	0.016	0.014	1.195	1.116	0.362	0.367	1.399	0.679

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	280	280	235	299	149	512	0	0	-1	169
N.S.	1	1.00	0.84	1.07	0.53	1.83	0.00	0.00	-0.00	0.60
time (sec)	N/A	0.135	0.075	0.025	1.810	1.379	0.000	0.000	0.000	11.029

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	276	276	235	299	146	503	0	0	-1	166
N.S.	1	1.00	0.85	1.08	0.53	1.82	0.00	0.00	-0.00	0.60
time (sec)	N/A	0.133	0.066	0.014	1.700	1.306	0.000	0.000	0.000	11.086

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	38	38	27	24	16	26	0	0	34	137
N.S.	1	1.00	0.71	0.63	0.42	0.68	0.00	0.00	0.89	3.61
time (sec)	N/A	0.029	0.011	0.006	0.888	1.410	0.000	0.000	1.193	0.565

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	277	277	237	301	147	514	0	0	-1	166
N.S.	1	1.00	0.86	1.09	0.53	1.86	0.00	0.00	-0.00	0.60
time (sec)	N/A	0.140	0.069	0.010	1.361	0.736	0.000	0.000	0.000	13.198

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	286	286	235	299	145	499	0	0	-1	164
N.S.	1	1.00	0.82	1.05	0.51	1.74	0.00	0.00	-0.00	0.57
time (sec)	N/A	0.147	0.068	0.010	1.916	1.343	0.000	0.000	0.000	15.693

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	147	147	74	107	88	90	0	0	-1	758
N.S.	1	1.00	0.50	0.73	0.60	0.61	0.00	0.00	-0.01	5.16
time (sec)	N/A	0.083	0.030	0.017	1.153	1.240	0.000	0.000	0.000	2.571

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	316	316	260	316	148	201	0	0	-1	177
N.S.	1	1.00	0.82	1.00	0.47	0.64	0.00	0.00	-0.00	0.56
time (sec)	N/A	0.160	0.084	0.020	1.949	1.397	0.000	0.000	0.000	21.141

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	316	316	266	322	150	242	0	0	-1	177
N.S.	1	1.00	0.84	1.02	0.47	0.77	0.00	0.00	-0.00	0.56
time (sec)	N/A	0.159	0.088	0.018	1.801	1.398	0.000	0.000	0.000	24.318

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	188	188	97	133	117	119	0	0	-1	796
N.S.	1	1.00	0.52	0.71	0.62	0.63	0.00	0.00	-0.01	4.23
time (sec)	N/A	0.097	0.033	0.024	0.994	1.267	0.000	0.000	0.000	3.838

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	A	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	359	359	218	519	195	723	0	0	-1	189
N.S.	1	1.00	0.61	1.45	0.54	2.01	0.00	0.00	-0.00	0.53
time (sec)	N/A	0.185	0.128	0.023	1.348	0.779	0.000	0.000	0.000	24.917

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	78	78	39	32	43	58	0	0	42	229
N.S.	1	1.00	0.50	0.41	0.55	0.74	0.00	0.00	0.54	2.94
time (sec)	N/A	0.053	0.016	0.009	0.857	1.181	0.000	0.000	1.284	0.765

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	A	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	368	368	229	521	195	734	0	0	-1	191
N.S.	1	1.00	0.62	1.42	0.53	1.99	0.00	0.00	-0.00	0.52
time (sec)	N/A	0.191	0.135	0.022	1.766	1.302	0.000	0.000	0.000	26.107

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	A	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	360	360	221	519	193	723	0	0	-1	189
N.S.	1	1.00	0.61	1.44	0.54	2.01	0.00	0.00	-0.00	0.52
time (sec)	N/A	0.180	0.132	0.019	2.284	1.230	0.000	0.000	0.000	26.811

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	38	38	27	24	16	48	0	0	34	209
N.S.	1	1.00	0.71	0.63	0.42	1.26	0.00	0.00	0.89	5.50
time (sec)	N/A	0.030	0.010	0.009	0.512	1.147	0.000	0.000	1.259	0.828

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	A	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	359	359	219	521	191	734	0	0	-1	188
N.S.	1	1.00	0.61	1.45	0.53	2.04	0.00	0.00	-0.00	0.52
time (sec)	N/A	0.193	0.120	0.011	2.184	1.270	0.000	0.000	0.000	28.089

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	364	364	211	519	189	719	0	0	-1	186
N.S.	1	1.00	0.58	1.43	0.52	1.98	0.00	0.00	-0.00	0.51
time (sec)	N/A	0.199	0.111	0.008	2.152	1.243	0.000	0.000	0.000	34.135

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	223	223	96	193	132	178	0	0	-1	3896
N.S.	1	1.00	0.43	0.87	0.59	0.80	0.00	0.00	-0.00	17.47
time (sec)	N/A	0.124	0.044	0.024	0.875	1.022	0.000	0.000	0.000	127.754

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	A	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	398	398	242	536	192	311	0	0	-1	199
N.S.	1	1.00	0.61	1.35	0.48	0.78	0.00	0.00	-0.00	0.50
time (sec)	N/A	0.216	0.129	0.025	2.333	0.966	0.000	0.000	0.000	35.250

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	A	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	398	398	234	542	194	352	0	0	-1	199
N.S.	1	1.00	0.59	1.36	0.49	0.88	0.00	0.00	-0.00	0.50
time (sec)	N/A	0.211	0.135	0.025	2.059	1.304	0.000	0.000	0.000	42.116

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	269	269	119	219	163	207	0	0	-1	2850
N.S.	1	1.00	0.44	0.81	0.61	0.77	0.00	0.00	-0.00	10.59
time (sec)	N/A	0.143	0.051	0.021	0.851	1.267	0.000	0.000	0.000	81.815

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	313	313	111	453	243	369	0	900	-1	0
N.S.	1	1.00	0.35	1.45	0.78	1.18	0.00	2.88	-0.00	0.00
time (sec)	N/A	0.138	0.091	0.008	0.952	1.906	0.000	0.673	0.000	3.285

Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	205	205	131	199	119	159	0	384	-1	0
N.S.	1	1.00	0.64	0.97	0.58	0.78	0.00	1.87	-0.00	0.00
time (sec)	N/A	0.085	0.065	0.015	1.037	1.197	0.000	0.524	0.000	1.804

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	97	97	53	56	35	35	0	83	-1	0
N.S.	1	1.00	0.55	0.58	0.36	0.36	0.00	0.86	-0.01	0.00
time (sec)	N/A	0.038	0.023	0.003	1.034	1.277	0.000	0.452	0.000	1.211

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	172	172	110	150	115	163	0	375	207	0
N.S.	1	1.00	0.64	0.87	0.67	0.95	0.00	2.18	1.20	0.00
time (sec)	N/A	0.113	0.056	0.014	0.808	1.168	0.000	0.530	1.310	0.527

Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	130	130	77	96	79	108	0	235	137	0
N.S.	1	1.00	0.59	0.74	0.61	0.83	0.00	1.81	1.05	0.00
time (sec)	N/A	0.079	0.033	0.007	1.222	1.129	0.000	0.439	1.219	0.492

Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	84	84	51	60	54	70	0	132	85	0
N.S.	1	1.00	0.61	0.71	0.64	0.83	0.00	1.57	1.01	0.00
time (sec)	N/A	0.055	0.020	0.007	0.628	1.116	0.000	0.479	1.193	0.288

Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	41	41	32	40	30	37	0	58	46	0
N.S.	1	1.00	0.78	0.98	0.73	0.90	0.00	1.41	1.12	0.00
time (sec)	N/A	0.028	0.006	0.007	0.947	1.389	0.000	0.482	1.155	0.171

Problem 125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F(-2)	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	81	81	78	111	0	254	316	75	1758	0
N.S.	1	1.00	0.96	1.37	0.00	3.14	3.90	0.93	21.70	0.00
time (sec)	N/A	0.083	0.052	0.006	0.000	1.290	2.695	1.138	1.981	0.001

Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F(-2)	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	63	63	62	60	0	197	223	59	1199	0
N.S.	1	1.00	0.98	0.95	0.00	3.13	3.54	0.94	19.03	0.00
time (sec)	N/A	0.058	0.023	0.003	0.000	0.833	1.369	1.186	1.800	0.001

Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F(-2)	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	38	38	42	37	0	129	131	36	174	0
N.S.	1	1.00	1.11	0.97	0.00	3.39	3.45	0.95	4.58	0.00
time (sec)	N/A	0.036	0.010	0.002	0.000	1.309	0.638	0.983	1.232	0.001

Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	F(-2)	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	69	69	66	66	0	223	253	66	1362	0
N.S.	1	1.00	0.96	0.96	0.00	3.23	3.67	0.96	19.74	0.00
time (sec)	N/A	0.070	0.023	0.007	0.000	1.407	6.731	1.003	1.922	0.001

Problem 129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	F(-2)	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	89	89	92	119	0	293	345	93	4281	0
N.S.	1	1.00	1.03	1.34	0.00	3.29	3.88	1.04	48.10	0.00
time (sec)	N/A	0.125	0.029	0.009	0.000	1.382	113.787	1.137	2.026	0.001

Problem 130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	B	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	636	636	70	61	0	5601	279	0	4069	0
N.S.	1	1.00	0.11	0.10	0.00	8.81	0.44	0.00	6.40	0.00
time (sec)	N/A	1.249	0.030	0.141	0.000	4.092	14.581	0.000	12.146	0.001

Problem 131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	B	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	631	631	70	59	0	5260	196	0	2280	0
N.S.	1	1.00	0.11	0.09	0.00	8.34	0.31	0.00	3.61	0.00
time (sec)	N/A	1.024	0.030	0.005	0.000	3.185	6.903	0.000	3.397	0.001

Problem 132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	B	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	558	558	44	43	0	3799	175	0	2695	0
N.S.	1	1.00	0.08	0.08	0.00	6.81	0.31	0.00	4.83	0.00
time (sec)	N/A	0.520	0.017	0.004	0.000	1.778	2.182	0.000	8.111	0.001

Problem 133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	B	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	558	558	42	43	0	2551	122	0	2129	0
N.S.	1	1.00	0.08	0.08	0.00	4.57	0.22	0.00	3.82	0.00
time (sec)	N/A	0.574	0.016	0.003	0.000	1.485	1.800	0.000	7.712	0.001

Problem 134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	B	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	558	558	43	41	0	2875	158	0	1543	0
N.S.	1	1.00	0.08	0.07	0.00	5.15	0.28	0.00	2.77	0.00
time (sec)	N/A	0.472	0.017	0.004	0.000	1.701	1.528	0.000	5.388	0.000

Problem 135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	B	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	558	558	45	40	0	3978	155	0	2597	0
N.S.	1	1.00	0.08	0.07	0.00	7.13	0.28	0.00	4.65	0.00
time (sec)	N/A	0.595	0.017	0.002	0.000	2.024	4.353	0.000	8.495	0.001

Problem 136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	B	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	610	610	71	61	0	5266	252	0	2978	0
N.S.	1	1.00	0.12	0.10	0.00	8.63	0.41	0.00	4.88	0.00
time (sec)	N/A	0.818	0.031	0.006	0.000	3.992	3.188	0.000	6.889	0.001

Problem 137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	B	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	612	612	75	62	0	5771	241	0	4063	0
N.S.	1	1.00	0.12	0.10	0.00	9.43	0.39	0.00	6.64	0.00
time (sec)	N/A	0.815	0.034	0.005	0.000	3.764	52.501	0.000	10.654	0.001

Problem 138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	35	35	35	28	27	27	29	29	27	0
N.S.	1	1.00	1.00	0.80	0.77	0.77	0.83	0.83	0.77	0.00
time (sec)	N/A	0.024	0.006	0.005	0.489	1.288	0.126	0.394	1.253	0.001

Problem 139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	28	28	28	23	22	22	22	24	22	0
N.S.	1	1.00	1.00	0.82	0.79	0.79	0.79	0.86	0.79	0.00
time (sec)	N/A	0.019	0.005	0.006	0.487	1.126	0.124	0.343	0.046	0.000

Problem 140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	21	21	21	18	17	17	15	19	17	0
N.S.	1	1.00	1.00	0.86	0.81	0.81	0.71	0.90	0.81	0.00
time (sec)	N/A	0.014	0.004	0.004	0.519	1.205	0.116	0.364	0.055	0.001

Problem 141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	B	B	B	B	B	A	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	10	21	21	18	17	17	15	19	16	0
N.S.	1	2.10	2.10	1.80	1.70	1.70	1.50	1.90	1.60	0.00
time (sec)	N/A	0.014	0.004	0.003	0.465	1.211	0.108	0.328	0.383	0.000

Problem 142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	27	27	27	31	23	21	20	24	21	0
N.S.	1	1.00	1.00	1.15	0.85	0.78	0.74	0.89	0.78	0.00
time (sec)	N/A	0.019	0.006	0.010	0.451	1.253	0.141	0.353	1.261	0.000

Problem 143	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	34	34	34	36	28	35	29	36	26	0
N.S.	1	1.00	1.00	1.06	0.82	1.03	0.85	1.06	0.76	0.00
time (sec)	N/A	0.032	0.006	0.010	0.682	1.271	0.170	0.301	1.233	0.001

Problem 144	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	41	41	41	41	35	40	34	41	32	0
N.S.	1	1.00	1.00	1.00	0.85	0.98	0.83	1.00	0.78	0.00
time (sec)	N/A	0.036	0.005	0.008	0.588	1.209	0.188	0.365	0.043	0.001

Problem 145	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	C	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	124	124	118	94	94	102	144	96	124	0
N.S.	1	1.00	0.95	0.76	0.76	0.82	1.16	0.77	1.00	0.00
time (sec)	N/A	0.114	0.054	0.010	1.230	1.299	0.619	0.365	0.244	0.000

Problem 146	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	C	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	122	122	114	92	92	90	129	94	119	0
N.S.	1	1.00	0.93	0.75	0.75	0.74	1.06	0.77	0.98	0.00
time (sec)	N/A	0.094	0.028	0.007	1.331	1.117	0.611	0.432	1.420	0.000

Problem 147	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	C	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	119	119	111	89	89	99	134	91	118	0
N.S.	1	1.00	0.93	0.75	0.75	0.83	1.13	0.76	0.99	0.00
time (sec)	N/A	0.082	0.027	0.007	1.305	1.180	0.607	0.336	0.186	0.000

Problem 148	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	C	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	113	113	111	85	85	88	126	87	104	0
N.S.	1	1.00	0.98	0.75	0.75	0.78	1.12	0.77	0.92	0.00
time (sec)	N/A	0.074	0.026	0.007	1.227	1.506	0.615	0.454	0.160	0.001

Problem 149	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	C	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	112	112	107	84	84	106	134	86	114	0
N.S.	1	1.00	0.96	0.75	0.75	0.95	1.20	0.77	1.02	0.00
time (sec)	N/A	0.068	0.024	0.006	1.607	1.468	0.600	0.303	1.371	0.001

Problem 150	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	C	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	112	112	106	84	84	102	110	86	113	0
N.S.	1	1.00	0.95	0.75	0.75	0.91	0.98	0.77	1.01	0.00
time (sec)	N/A	0.069	0.023	0.007	1.706	1.306	0.589	0.335	1.363	0.000

Problem 151	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	C	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	112	112	108	84	84	84	119	86	113	0
N.S.	1	1.00	0.96	0.75	0.75	0.75	1.06	0.77	1.01	0.00
time (sec)	N/A	0.069	0.025	0.005	1.597	1.018	1.857	0.363	1.364	0.000

Problem 152	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	C	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	112	112	107	84	84	124	124	86	110	0
N.S.	1	1.00	0.96	0.75	0.75	1.11	1.11	0.77	0.98	0.00
time (sec)	N/A	0.065	0.023	0.007	1.283	1.419	1.819	0.339	0.225	0.000

Problem 153	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	C	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	119	119	118	89	89	117	139	91	119	0
N.S.	1	1.00	0.99	0.75	0.75	0.98	1.17	0.76	1.00	0.00
time (sec)	N/A	0.082	0.041	0.007	1.085	1.182	1.830	0.414	1.380	0.000

Problem 154	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	C	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	119	119	113	89	89	126	128	91	118	0
N.S.	1	1.00	0.95	0.75	0.75	1.06	1.08	0.76	0.99	0.00
time (sec)	N/A	0.078	0.049	0.008	1.275	1.161	1.742	0.411	1.362	0.000

Problem 155	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	C	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	126	126	118	94	96	112	141	98	124	0
N.S.	1	1.00	0.94	0.75	0.76	0.89	1.12	0.78	0.98	0.00
time (sec)	N/A	0.103	0.048	0.011	1.198	1.160	1.836	0.363	0.189	0.001

Problem 156	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	C	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	126	126	118	94	96	153	136	98	121	0
N.S.	1	1.00	0.94	0.75	0.76	1.21	1.08	0.78	0.96	0.00
time (sec)	N/A	0.099	0.059	0.014	1.658	1.155	1.776	0.358	1.399	0.001

Problem 157	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	B	A	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	412	412	59	44	0	1028	26	638	320	0
N.S.	1	1.00	0.14	0.11	0.00	2.50	0.06	1.55	0.78	0.00
time (sec)	N/A	0.428	0.013	0.013	0.000	1.382	0.180	0.525	1.821	0.001

Problem 158	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	39	39	39	33	32	32	37	32	34	0
N.S.	1	1.00	1.00	0.85	0.82	0.82	0.95	0.82	0.87	0.00
time (sec)	N/A	0.035	0.011	0.005	1.041	1.180	0.128	0.433	1.214	0.002

Problem 159	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	B	A	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	411	411	41	40	0	1583	26	824	304	0
N.S.	1	1.00	0.10	0.10	0.00	3.85	0.06	2.00	0.74	0.00
time (sec)	N/A	0.281	0.009	0.007	0.000	1.730	0.181	0.688	1.717	0.001

Problem 160	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	B	A	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	411	411	39	40	0	1031	24	637	327	0
N.S.	1	1.00	0.09	0.10	0.00	2.51	0.06	1.55	0.80	0.00
time (sec)	N/A	0.278	0.009	0.007	0.000	1.425	0.179	0.574	1.840	0.000

Problem 161	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	23	23	23	19	18	18	27	18	20	0
N.S.	1	1.00	1.00	0.83	0.78	0.78	1.17	0.78	0.87	0.00
time (sec)	N/A	0.023	0.006	0.001	1.135	1.128	0.116	0.495	1.217	0.001

Problem 162	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	B	A	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	375	375	40	38	0	1583	26	812	304	0
N.S.	1	1.00	0.11	0.10	0.00	4.22	0.07	2.17	0.81	0.00
time (sec)	N/A	0.250	0.010	0.007	0.000	1.498	0.180	0.570	0.451	0.001

Problem 163	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	C	C	F	B	A	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	186	375	42	37	0	1027	20	629	327	0
N.S.	1	2.02	0.23	0.20	0.00	5.52	0.11	3.38	1.76	0.00
time (sec)	N/A	0.242	0.008	0.007	0.000	1.292	0.184	0.499	1.788	0.001

Problem 164	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	41	41	55	35	38	34	41	35	36	0
N.S.	1	1.00	1.34	0.85	0.93	0.83	1.00	0.85	0.88	0.00
time (sec)	N/A	0.039	0.012	0.006	1.447	1.230	0.148	0.352	1.231	0.000

Problem 165	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	B	A	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	416	416	61	50	0	1598	24	826	286	0
N.S.	1	1.00	0.15	0.12	0.00	3.84	0.06	1.99	0.69	0.00
time (sec)	N/A	0.300	0.013	0.009	0.000	1.650	0.198	0.550	1.656	0.001

Problem 166	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	B	A	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	418	418	65	50	0	1066	31	642	324	0
N.S.	1	1.00	0.16	0.12	0.00	2.55	0.07	1.54	0.78	0.00
time (sec)	N/A	0.342	0.013	0.009	0.000	1.446	0.206	0.551	1.717	0.000

Problem 167	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	48	48	51	40	43	51	48	45	41	0
N.S.	1	1.00	1.06	0.83	0.90	1.06	1.00	0.94	0.85	0.00
time (sec)	N/A	0.052	0.013	0.007	1.130	1.173	0.172	0.417	0.064	0.000

Problem 168	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	B	A	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	423	423	54	51	0	1623	39	836	318	0
N.S.	1	1.00	0.13	0.12	0.00	3.84	0.09	1.98	0.75	0.00
time (sec)	N/A	0.369	0.014	0.010	0.000	1.620	0.218	0.549	1.592	0.000

Problem 169	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	B	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	381	381	38	33	0	1996	24	0	513	0
N.S.	1	1.00	0.10	0.09	0.00	5.24	0.06	0.00	1.35	0.00
time (sec)	N/A	0.403	0.010	0.007	0.000	3.213	0.154	0.000	2.615	0.000

Problem 170	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	23	23	23	19	18	18	27	18	20	0
N.S.	1	1.00	1.00	0.83	0.78	0.78	1.17	0.78	0.87	0.00
time (sec)	N/A	0.024	0.008	0.003	1.360	1.188	0.113	0.504	0.046	0.000

Problem 171	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	B	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	399	399	37	36	0	1435	24	0	351	0
N.S.	1	1.00	0.09	0.09	0.00	3.60	0.06	0.00	0.88	0.00
time (sec)	N/A	0.305	0.009	0.006	0.000	1.506	0.148	0.000	2.614	0.000

Problem 172	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F(-2)	A	F	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	231	231	208	0	0	451	0	0	543	209
N.S.	1	1.00	0.90	0.00	0.00	1.95	0.00	0.00	2.35	0.90
time (sec)	N/A	0.300	0.160	0.053	0.000	1.353	0.000	0.000	2.939	0.800

Problem 173	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F(-2)	A	F	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	171	171	164	0	0	367	0	0	315	170
N.S.	1	1.00	0.96	0.00	0.00	2.15	0.00	0.00	1.84	0.99
time (sec)	N/A	0.152	0.134	0.046	0.000	1.750	0.000	0.000	1.866	0.582

Problem 174	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F(-2)	A	F	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	153	153	136	0	0	303	0	0	193	132
N.S.	1	1.00	0.89	0.00	0.00	1.98	0.00	0.00	1.26	0.86
time (sec)	N/A	0.136	0.069	0.032	0.000	1.220	0.000	0.000	1.591	0.424

Problem 175	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F(-2)	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	108	108	99	0	0	237	0	98	87	107
N.S.	1	1.00	0.92	0.00	0.00	2.19	0.00	0.91	0.81	0.99
time (sec)	N/A	0.085	0.074	0.028	0.000	1.219	0.000	0.491	1.390	0.323

Problem 176	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F(-2)	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	83	83	87	0	0	197	0	76	72	85
N.S.	1	1.00	1.05	0.00	0.00	2.37	0.00	0.92	0.87	1.02
time (sec)	N/A	0.060	0.010	0.026	0.000	1.340	0.000	0.556	1.390	0.245

Problem 177	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F(-2)	A	F	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	109	109	106	0	0	566	0	0	88	111
N.S.	1	1.00	0.97	0.00	0.00	5.19	0.00	0.00	0.81	1.02
time (sec)	N/A	0.111	0.042	0.036	0.000	1.367	0.000	0.000	1.362	0.290

Problem 178	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F(-2)	A	F	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	112	112	112	0	0	601	0	0	91	114
N.S.	1	1.00	1.00	0.00	0.00	5.37	0.00	0.00	0.81	1.02
time (sec)	N/A	0.114	0.048	0.043	0.000	1.568	0.000	0.000	1.553	0.300

Problem 179	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F(-2)	A	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	88	88	89	0	0	215	0	0	-1	91
N.S.	1	1.00	1.01	0.00	0.00	2.44	0.00	0.00	-0.01	1.03
time (sec)	N/A	0.073	0.043	0.039	0.000	1.366	0.000	0.000	0.000	0.384

Problem 180	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F(-2)	A	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	116	116	108	0	0	259	0	0	-1	108
N.S.	1	1.00	0.93	0.00	0.00	2.23	0.00	0.00	-0.01	0.93
time (sec)	N/A	0.095	0.074	0.050	0.000	1.363	0.000	0.000	0.000	0.654

Problem 181	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F(-2)	A	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	161	161	139	0	0	325	0	0	-1	141
N.S.	1	1.00	0.86	0.00	0.00	2.02	0.00	0.00	-0.01	0.88
time (sec)	N/A	0.147	0.088	0.060	0.000	1.400	0.000	0.000	0.000	0.837

Problem 182	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F(-2)	A	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	199	199	173	0	0	389	0	0	-1	176
N.S.	1	1.00	0.87	0.00	0.00	1.95	0.00	0.00	-0.01	0.88
time (sec)	N/A	0.227	0.116	0.061	0.000	1.522	0.000	0.000	0.000	1.197

Problem 183	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F(-2)	A	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	293	293	241	0	0	641	0	0	-1	314
N.S.	1	1.00	0.82	0.00	0.00	2.19	0.00	0.00	-0.00	1.07
time (sec)	N/A	0.402	0.349	0.049	0.000	1.555	0.000	0.000	0.000	1.398

Problem 184	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F(-2)	A	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	223	223	192	0	0	535	0	0	-1	255
N.S.	1	1.00	0.86	0.00	0.00	2.40	0.00	0.00	-0.00	1.14
time (sec)	N/A	0.206	0.192	0.032	0.000	1.245	0.000	0.000	0.000	1.134

Problem 185	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F(-2)	A	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	204	204	175	0	0	451	0	0	-1	209
N.S.	1	1.00	0.86	0.00	0.00	2.21	0.00	0.00	-0.00	1.02
time (sec)	N/A	0.188	0.159	0.031	0.000	1.296	0.000	0.000	0.000	0.883

Problem 186	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F(-2)	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	150	150	149	0	0	361	0	172	223	162
N.S.	1	1.00	0.99	0.00	0.00	2.41	0.00	1.15	1.49	1.08
time (sec)	N/A	0.116	0.146	0.036	0.000	1.212	0.000	0.561	1.575	0.661

Problem 187	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F(-2)	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	124	124	126	0	0	297	0	135	115	132
N.S.	1	1.00	1.02	0.00	0.00	2.40	0.00	1.09	0.93	1.06
time (sec)	N/A	0.086	0.085	0.033	0.000	1.383	0.000	0.611	1.444	0.495

Problem 188	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F(-2)	A	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	155	155	143	0	0	727	0	0	-1	151
N.S.	1	1.00	0.92	0.00	0.00	4.69	0.00	0.00	-0.01	0.97
time (sec)	N/A	0.179	0.143	0.042	0.000	1.821	0.000	0.000	0.000	0.703

Problem 189	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F(-2)	A	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	150	150	134	0	0	713	0	0	-1	137
N.S.	1	1.00	0.89	0.00	0.00	4.75	0.00	0.00	-0.01	0.91
time (sec)	N/A	0.170	0.111	0.065	0.000	1.554	0.000	0.000	0.000	0.727

Problem 190	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F(-2)	A	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	151	151	134	0	0	713	0	0	-1	134
N.S.	1	1.00	0.89	0.00	0.00	4.72	0.00	0.00	-0.01	0.89
time (sec)	N/A	0.162	0.172	0.056	0.000	1.457	0.000	0.000	0.000	0.796

Problem 191	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F(-2)	A	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	163	163	149	0	0	771	0	0	-1	148
N.S.	1	1.00	0.91	0.00	0.00	4.73	0.00	0.00	-0.01	0.91
time (sec)	N/A	0.176	0.206	0.056	0.000	1.596	0.000	0.000	0.000	0.991

Problem 192	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F(-2)	A	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	133	133	138	0	0	319	0	0	-1	139
N.S.	1	1.00	1.04	0.00	0.00	2.40	0.00	0.00	-0.01	1.05
time (sec)	N/A	0.108	0.164	0.051	0.000	1.404	0.000	0.000	0.000	1.109

Problem 193	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F(-2)	A	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	162	162	167	0	0	383	0	0	-1	176
N.S.	1	1.00	1.03	0.00	0.00	2.36	0.00	0.00	-0.01	1.09
time (sec)	N/A	0.146	0.140	0.065	0.000	1.673	0.000	0.000	0.000	1.551

Problem 194	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F(-2)	A	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	216	216	206	0	0	473	0	0	-1	221
N.S.	1	1.00	0.95	0.00	0.00	2.19	0.00	0.00	-0.00	1.02
time (sec)	N/A	0.207	0.210	0.078	0.000	2.090	0.000	0.000	0.000	2.272

Problem 195	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F(-2)	A	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	255	255	243	0	0	557	0	0	-1	270
N.S.	1	1.00	0.95	0.00	0.00	2.18	0.00	0.00	-0.00	1.06
time (sec)	N/A	0.312	0.486	0.093	0.000	2.626	0.000	0.000	0.000	2.484

Problem 196	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F(-2)	A	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	171	171	137	0	0	303	0	0	-1	138
N.S.	1	1.00	0.80	0.00	0.00	1.77	0.00	0.00	-0.01	0.81
time (sec)	N/A	0.216	0.111	0.036	0.000	0.641	0.000	0.000	0.000	0.415

Problem 197	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F(-2)	A	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	121	121	104	0	0	241	0	0	-1	101
N.S.	1	1.00	0.86	0.00	0.00	1.99	0.00	0.00	-0.01	0.83
time (sec)	N/A	0.105	0.050	0.040	0.000	1.357	0.000	0.000	0.000	0.315

Problem 198	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F(-2)	A	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	104	104	88	0	0	203	0	0	-1	91
N.S.	1	1.00	0.85	0.00	0.00	1.95	0.00	0.00	-0.01	0.88
time (sec)	N/A	0.089	0.033	0.027	0.000	1.456	0.000	0.000	0.000	0.288

Problem 199	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F(-2)	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	68	68	68	0	0	161	0	61	55	70
N.S.	1	1.00	1.00	0.00	0.00	2.37	0.00	0.90	0.81	1.03
time (sec)	N/A	0.057	0.016	0.024	0.000	1.347	0.000	0.674	1.492	0.188

Problem 200	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F(-2)	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	43	43	43	0	0	118	0	40	34	41
N.S.	1	1.00	1.00	0.00	0.00	2.74	0.00	0.93	0.79	0.95
time (sec)	N/A	0.034	0.006	0.020	0.000	1.214	0.000	0.676	1.575	0.140

Problem 201	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F(-2)	A	F	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	44	44	44	0	0	124	0	0	36	48
N.S.	1	1.00	1.00	0.00	0.00	2.82	0.00	0.00	0.82	1.09
time (sec)	N/A	0.040	0.006	0.020	0.000	1.101	0.000	0.000	1.571	0.126

Problem 202	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F(-2)	A	F	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	72	72	72	0	0	179	0	0	56	76
N.S.	1	1.00	1.00	0.00	0.00	2.49	0.00	0.00	0.78	1.06
time (sec)	N/A	0.060	0.024	0.033	0.000	1.360	0.000	0.000	1.560	0.215

Problem 203	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F(-2)	A	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	108	108	92	0	0	221	0	0	-1	91
N.S.	1	1.00	0.85	0.00	0.00	2.05	0.00	0.00	-0.01	0.84
time (sec)	N/A	0.101	0.059	0.037	0.000	1.414	0.000	0.000	0.000	0.343

Problem 204	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F(-2)	A	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	145	145	112	0	0	263	0	0	-1	110
N.S.	1	1.00	0.77	0.00	0.00	1.81	0.00	0.00	-0.01	0.76
time (sec)	N/A	0.158	0.078	0.042	0.000	1.275	0.000	0.000	0.000	0.531

Problem 205	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F(-2)	A	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	192	192	141	0	0	327	0	0	-1	141
N.S.	1	1.00	0.73	0.00	0.00	1.70	0.00	0.00	-0.01	0.73
time (sec)	N/A	0.233	0.098	0.046	0.000	1.776	0.000	0.000	0.000	0.785

Problem 206	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F(-2)	A	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	195	195	181	0	0	591	0	0	-1	175
N.S.	1	1.00	0.93	0.00	0.00	3.03	0.00	0.00	-0.01	0.90
time (sec)	N/A	0.229	0.181	0.065	0.000	1.906	0.000	0.000	0.000	0.958

Problem 207	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F(-2)	A	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	137	137	137	0	0	459	0	0	-1	131
N.S.	1	1.00	1.00	0.00	0.00	3.35	0.00	0.00	-0.01	0.96
time (sec)	N/A	0.111	0.119	0.055	0.000	1.371	0.000	0.000	0.000	0.667

Problem 208	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F(-2)	A	F	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	120	120	107	0	0	387	0	0	84	99
N.S.	1	1.00	0.89	0.00	0.00	3.22	0.00	0.00	0.70	0.82
time (sec)	N/A	0.090	0.098	0.028	0.000	1.338	0.000	0.000	1.658	0.520

Problem 209	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F(-2)	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	39	39	41	38	0	68	0	45	38	39
N.S.	1	1.00	1.05	0.97	0.00	1.74	0.00	1.15	0.97	1.00
time (sec)	N/A	0.029	0.099	0.006	0.000	0.945	0.000	1.321	1.429	0.394

Problem 210	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F(-2)	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	38	38	38	37	0	67	0	45	37	38
N.S.	1	1.00	1.00	0.97	0.00	1.76	0.00	1.18	0.97	1.00
time (sec)	N/A	0.025	0.024	0.006	0.000	1.198	0.000	1.407	1.370	0.325

Problem 211	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F(-2)	B	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	92	92	92	0	0	389	0	0	-1	101
N.S.	1	1.00	1.00	0.00	0.00	4.23	0.00	0.00	-0.01	1.10
time (sec)	N/A	0.078	0.123	0.062	0.000	1.666	0.000	0.000	0.000	0.540

Problem 212	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F(-2)	A	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	142	142	137	0	0	485	0	0	-1	132
N.S.	1	1.00	0.96	0.00	0.00	3.42	0.00	0.00	-0.01	0.93
time (sec)	N/A	0.124	0.083	0.087	0.000	1.527	0.000	0.000	0.000	0.682

Problem 213	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F(-2)	A	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	198	198	179	0	0	615	0	0	-1	175
N.S.	1	1.00	0.90	0.00	0.00	3.11	0.00	0.00	-0.01	0.88
time (sec)	N/A	0.204	0.126	0.098	0.000	1.846	0.000	0.000	0.000	1.015

Problem 214	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F(-2)	A	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	256	256	223	0	0	705	0	0	-1	224
N.S.	1	1.00	0.87	0.00	0.00	2.75	0.00	0.00	-0.00	0.88
time (sec)	N/A	0.286	0.177	0.133	0.000	2.103	0.000	0.000	0.000	1.354

Problem 215	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	B	A	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	101	101	70	301	110	241	1510	449	260	0
N.S.	1	1.00	0.69	2.98	1.09	2.39	14.95	4.45	2.57	0.00
time (sec)	N/A	0.061	0.071	0.008	1.102	1.078	5.824	0.465	1.524	1.170

Problem 216	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	52	52	35	78	50	71	314	119	89	0
N.S.	1	1.00	0.67	1.50	0.96	1.37	6.04	2.29	1.71	0.00
time (sec)	N/A	0.021	0.029	0.003	1.163	1.308	1.468	0.346	1.358	0.214

Problem 217	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	30	30	24	25	24	31	22	24	25	0
N.S.	1	1.00	0.80	0.83	0.80	1.03	0.73	0.80	0.83	0.00
time (sec)	N/A	0.012	0.014	0.009	2.156	0.931	0.126	0.340	0.047	0.000

Problem 218	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	22	22	18	19	18	23	15	18	18	0
N.S.	1	1.00	0.82	0.86	0.82	1.05	0.68	0.82	0.82	0.00
time (sec)	N/A	0.010	0.006	0.007	0.984	0.938	0.106	0.279	1.315	0.000

Problem 219	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	23	23	23	20	19	24	15	19	21	0
N.S.	1	1.00	1.00	0.87	0.83	1.04	0.65	0.83	0.91	0.00
time (sec)	N/A	0.009	0.009	0.006	2.015	0.942	0.118	0.372	1.365	0.000

Problem 220	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	11	11	11	10	9	9	8	9	11	0
N.S.	1	1.00	1.00	0.91	0.82	0.82	0.73	0.82	1.00	0.00
time (sec)	N/A	0.003	0.002	0.003	0.887	0.858	0.094	0.355	0.018	0.000

Problem 221	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	23	23	20	20	19	23	15	19	20	0
N.S.	1	1.00	0.87	0.87	0.83	1.00	0.65	0.83	0.87	0.00
time (sec)	N/A	0.007	0.005	0.004	2.173	1.119	0.116	0.392	0.026	0.000

Problem 222	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	24	24	24	21	24	32	19	29	20	0
N.S.	1	1.00	1.00	0.88	1.00	1.33	0.79	1.21	0.83	0.00
time (sec)	N/A	0.012	0.010	0.013	0.974	0.782	0.125	0.361	0.044	0.000

Problem 223	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	30	30	30	25	25	31	26	25	25	0
N.S.	1	1.00	1.00	0.83	0.83	1.03	0.87	0.83	0.83	0.00
time (sec)	N/A	0.012	0.011	0.010	1.976	1.021	0.147	0.335	0.040	0.001

Problem 224	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	33	33	33	28	33	44	31	33	31	0
N.S.	1	1.00	1.00	0.85	1.00	1.33	0.94	1.00	0.94	0.00
time (sec)	N/A	0.016	0.012	0.015	0.877	1.129	0.153	0.355	0.054	0.000

Problem 225	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	37	37	33	28	30	36	29	31	30	0
N.S.	1	1.00	0.89	0.76	0.81	0.97	0.78	0.84	0.81	0.00
time (sec)	N/A	0.016	0.011	0.012	2.487	1.097	0.170	0.296	0.046	0.001

Problem 226	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	104	104	94	69	83	132	90	83	45	0
N.S.	1	1.00	0.90	0.66	0.80	1.27	0.87	0.80	0.43	0.00
time (sec)	N/A	0.055	0.068	0.010	2.022	1.018	0.176	0.339	1.369	0.001

Problem 227	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	99	99	93	70	84	129	90	84	47	0
N.S.	1	1.00	0.94	0.71	0.85	1.30	0.91	0.85	0.47	0.00
time (sec)	N/A	0.051	0.061	0.008	2.075	1.366	0.179	0.293	1.326	0.001

Problem 228	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	97	97	90	68	82	126	82	82	45	0
N.S.	1	1.00	0.93	0.70	0.85	1.30	0.85	0.85	0.46	0.00
time (sec)	N/A	0.051	0.065	0.006	1.986	1.298	0.170	0.357	0.082	0.000

Problem 229	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	99	99	92	70	84	128	83	84	46	0
N.S.	1	1.00	0.93	0.71	0.85	1.29	0.84	0.85	0.46	0.00
time (sec)	N/A	0.050	0.049	0.007	2.065	1.108	0.174	0.345	0.045	0.001

Problem 230	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	97	97	91	68	82	127	88	82	44	0
N.S.	1	1.00	0.94	0.70	0.85	1.31	0.91	0.85	0.45	0.00
time (sec)	N/A	0.047	0.046	0.004	2.002	1.132	0.200	0.336	1.305	0.000

Problem 231	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	106	106	98	75	88	130	97	88	49	0
N.S.	1	1.00	0.92	0.71	0.83	1.23	0.92	0.83	0.46	0.00
time (sec)	N/A	0.052	0.069	0.009	2.025	1.275	0.207	0.358	1.314	0.000

Problem 232	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	106	106	96	73	90	140	99	87	51	0
N.S.	1	1.00	0.91	0.69	0.85	1.32	0.93	0.82	0.48	0.00
time (sec)	N/A	0.052	0.071	0.010	2.198	1.305	0.224	0.443	1.365	0.000

Problem 233	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	113	113	103	80	95	145	102	96	55	0
N.S.	1	1.00	0.91	0.71	0.84	1.28	0.90	0.85	0.49	0.00
time (sec)	N/A	0.054	0.076	0.012	2.012	1.279	0.232	0.297	0.093	0.001

Problem 234	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	113	113	101	78	95	145	102	94	55	0
N.S.	1	1.00	0.89	0.69	0.84	1.28	0.90	0.83	0.49	0.00
time (sec)	N/A	0.055	0.076	0.010	1.932	1.235	0.234	0.364	0.104	0.001

Problem 235	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	32	32	39	41	34	46	34	35	26	0
N.S.	1	1.00	1.22	1.28	1.06	1.44	1.06	1.09	0.81	0.00
time (sec)	N/A	0.013	0.022	0.009	1.032	1.141	0.122	0.326	0.047	0.000

Problem 236	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	26	26	22	19	18	23	15	19	20	0
N.S.	1	1.00	0.85	0.73	0.69	0.88	0.58	0.73	0.77	0.00
time (sec)	N/A	0.013	0.007	0.006	1.128	1.268	0.104	0.410	0.048	0.000

Problem 237	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	25	25	33	36	29	40	26	30	21	0
N.S.	1	1.00	1.32	1.44	1.16	1.60	1.04	1.20	0.84	0.00
time (sec)	N/A	0.010	0.011	0.009	0.914	1.168	0.118	0.496	1.269	0.000

Problem 238	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	13	13	11	10	9	9	8	9	11	0
N.S.	1	1.00	0.85	0.77	0.69	0.69	0.62	0.69	0.85	0.00
time (sec)	N/A	0.003	0.002	0.003	0.884	1.146	0.095	0.499	0.022	0.000

Problem 239	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	25	25	33	36	29	40	26	30	21	0
N.S.	1	1.00	1.32	1.44	1.16	1.60	1.04	1.20	0.84	0.00
time (sec)	N/A	0.008	0.007	0.010	0.918	1.260	0.119	0.471	0.035	0.000

Problem 240	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	28	28	26	47	24	32	19	30	22	0
N.S.	1	1.00	0.93	1.68	0.86	1.14	0.68	1.07	0.79	0.00
time (sec)	N/A	0.015	0.010	0.015	0.943	0.981	0.126	0.381	0.058	0.000

Problem 241	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	32	32	41	50	37	54	36	38	26	0
N.S.	1	1.00	1.28	1.56	1.16	1.69	1.12	1.19	0.81	0.00
time (sec)	N/A	0.013	0.017	0.017	0.953	1.162	0.150	0.414	0.044	0.000

Problem 242	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	37	37	35	54	35	50	29	36	32	0
N.S.	1	1.00	0.95	1.46	0.95	1.35	0.78	0.97	0.86	0.00
time (sec)	N/A	0.019	0.012	0.018	0.828	0.863	0.152	0.453	0.052	0.000

Problem 243	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	39	39	49	55	42	59	41	42	32	0
N.S.	1	1.00	1.26	1.41	1.08	1.51	1.05	1.08	0.82	0.00
time (sec)	N/A	0.017	0.014	0.020	0.937	1.227	0.176	0.328	0.051	0.000

Problem 244	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	34	34	38	43	28	49	32	30	26	0
N.S.	1	1.00	1.12	1.26	0.82	1.44	0.94	0.88	0.76	0.00
time (sec)	N/A	0.009	0.016	0.013	2.127	1.190	0.151	0.280	1.285	0.001

Problem 245	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	29	29	35	42	29	46	32	31	23	0
N.S.	1	1.00	1.21	1.45	1.00	1.59	1.10	1.07	0.79	0.00
time (sec)	N/A	0.008	0.014	0.015	2.027	1.232	0.153	0.400	0.034	0.001

Problem 246	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	27	27	31	42	27	43	26	29	21	0
N.S.	1	1.00	1.15	1.56	1.00	1.59	0.96	1.07	0.78	0.00
time (sec)	N/A	0.007	0.013	0.013	1.918	1.268	0.147	0.512	0.034	0.000

Problem 247	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	29	29	33	42	29	45	27	31	23	0
N.S.	1	1.00	1.14	1.45	1.00	1.55	0.93	1.07	0.79	0.00
time (sec)	N/A	0.008	0.012	0.010	1.711	1.217	0.149	0.335	0.032	0.000

Problem 248	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	27	27	33	42	27	44	31	29	21	0
N.S.	1	1.00	1.22	1.56	1.00	1.63	1.15	1.07	0.78	0.00
time (sec)	N/A	0.005	0.009	0.013	2.132	1.299	0.156	0.366	0.029	0.000

Problem 249	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	36	36	40	47	35	55	37	37	26	0
N.S.	1	1.00	1.11	1.31	0.97	1.53	1.03	1.03	0.72	0.00
time (sec)	N/A	0.010	0.016	0.014	2.086	1.255	0.179	0.375	0.042	0.000

Problem 250	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	36	36	38	47	37	63	39	34	28	0
N.S.	1	1.00	1.06	1.31	1.03	1.75	1.08	0.94	0.78	0.00
time (sec)	N/A	0.009	0.018	0.013	1.983	1.221	0.184	0.457	1.292	0.000

Problem 251	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	43	43	51	52	42	68	44	43	34	0
N.S.	1	1.00	1.19	1.21	0.98	1.58	1.02	1.00	0.79	0.00
time (sec)	N/A	0.012	0.020	0.020	2.434	1.358	0.201	0.451	0.044	0.001

Problem 252	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	43	43	43	52	42	68	44	41	34	0
N.S.	1	1.00	1.00	1.21	0.98	1.58	1.02	0.95	0.79	0.00
time (sec)	N/A	0.012	0.019	0.016	1.940	1.178	0.211	0.452	0.047	0.000

Problem 253	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F(-2)	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	81	81	78	111	0	254	316	75	3916	0
N.S.	1	1.00	0.96	1.37	0.00	3.14	3.90	0.93	48.35	0.00
time (sec)	N/A	0.084	0.051	0.006	0.000	1.365	4.113	17.069	2.687	0.001

Problem 254	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	B	A	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	192	192	210	360	0	1071	134	2043	5659	0
N.S.	1	1.00	1.09	1.88	0.00	5.58	0.70	10.64	29.47	0.00
time (sec)	N/A	0.339	0.123	0.030	0.000	1.439	4.419	18.166	3.071	0.001

Problem 255	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F(-2)	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	63	63	62	60	0	197	223	59	2654	0
N.S.	1	1.00	0.98	0.95	0.00	3.13	3.54	0.94	42.13	0.00
time (sec)	N/A	0.058	0.024	0.004	0.000	1.422	2.180	17.121	2.608	0.001

Problem 256	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	B	A	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	159	159	171	216	0	567	76	1036	1220	0
N.S.	1	1.00	1.08	1.36	0.00	3.57	0.48	6.52	7.67	0.00
time (sec)	N/A	0.126	0.086	0.018	0.000	1.249	2.509	18.741	2.810	0.001

Problem 257	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F(-2)	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	38	38	42	37	0	129	131	36	260	0
N.S.	1	1.00	1.11	0.97	0.00	3.39	3.45	0.95	6.84	0.00
time (sec)	N/A	0.034	0.009	0.003	0.000	1.295	0.770	17.345	1.370	0.001

Problem 258	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	B	A	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	154	154	133	120	0	619	88	1030	1105	0
N.S.	1	1.00	0.86	0.78	0.00	4.02	0.57	6.69	7.18	0.00
time (sec)	N/A	0.094	0.081	0.013	0.000	1.254	3.359	19.496	2.271	0.000

Problem 259	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	F(-2)	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	69	69	66	66	0	223	253	68	1690	0
N.S.	1	1.00	0.96	0.96	0.00	3.23	3.67	0.99	24.49	0.00
time (sec)	N/A	0.068	0.023	0.008	0.000	1.240	14.469	16.281	2.185	0.001

Problem 260	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	F	B	A	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	184	184	75	240	0	1134	153	2055	5451	0
N.S.	1	1.00	0.41	1.30	0.00	6.16	0.83	11.17	29.62	0.00
time (sec)	N/A	0.224	0.030	0.025	0.000	1.404	15.344	15.891	2.416	0.001

Problem 261	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	F(-2)	A	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	89	89	92	119	0	293	0	94	8817	0
N.S.	1	1.00	1.03	1.34	0.00	3.29	0.00	1.06	99.07	0.00
time (sec)	N/A	0.124	0.032	0.010	0.000	1.879	0.000	14.501	2.788	0.001

Problem 262	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F(-1)	B	F(-1)	F(-2)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	381	381	70	63	0	6296	0	0	12709	0
N.S.	1	1.00	0.18	0.17	0.00	16.52	0.00	0.00	33.36	0.00
time (sec)	N/A	0.647	0.040	0.038	0.000	5.663	0.000	0.000	3.491	0.001

Problem 263	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	B	F(-1)	F(-2)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	376	376	70	59	0	5082	0	0	10382	0
N.S.	1	1.00	0.19	0.16	0.00	13.52	0.00	0.00	27.61	0.00
time (sec)	N/A	0.574	0.037	0.007	0.000	2.612	0.000	0.000	3.969	0.001

Problem 264	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	B	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	325	325	44	43	0	3912	230	0	8033	0
N.S.	1	1.00	0.14	0.13	0.00	12.04	0.71	0.00	24.72	0.00
time (sec)	N/A	0.307	0.024	0.003	0.000	1.935	61.577	0.000	3.511	0.001

Problem 265	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	B	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	325	325	42	43	0	2479	126	0	8169	0
N.S.	1	1.00	0.13	0.13	0.00	7.63	0.39	0.00	25.14	0.00
time (sec)	N/A	0.296	0.021	0.003	0.000	1.531	2.917	0.000	3.633	0.001

Problem 266	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	B	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	315	315	43	43	0	2746	172	0	6067	0
N.S.	1	1.00	0.14	0.14	0.00	8.72	0.55	0.00	19.26	0.00
time (sec)	N/A	0.290	0.023	0.003	0.000	1.660	4.099	0.000	2.336	0.001

Problem 267	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	B	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	315	315	45	40	0	3929	177	0	10337	0
N.S.	1	1.00	0.14	0.13	0.00	12.47	0.56	0.00	32.82	0.00
time (sec)	N/A	0.304	0.025	0.003	0.000	1.868	19.792	0.000	3.416	0.000

Problem 268	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F(-1)	B	F(-1)	F(-2)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	363	363	71	63	0	5125	0	0	10509	0
N.S.	1	1.00	0.20	0.17	0.00	14.12	0.00	0.00	28.95	0.00
time (sec)	N/A	0.412	0.037	0.009	0.000	3.474	0.000	0.000	2.809	0.001

Problem 269	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F(-1)	B	F(-1)	F(-2)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	365	365	75	62	0	6324	0	0	16497	0
N.S.	1	1.00	0.21	0.17	0.00	17.33	0.00	0.00	45.20	0.00
time (sec)	N/A	0.399	0.042	0.012	0.000	4.075	0.000	0.000	5.568	0.001

Problem 270	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	44	44	44	36	35	35	42	35	37	0
N.S.	1	1.00	1.00	0.82	0.80	0.80	0.95	0.80	0.84	0.00
time (sec)	N/A	0.037	0.011	0.004	2.423	1.417	0.139	0.342	0.046	0.001

Problem 271	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	A	A	B	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	54	54	98	43	42	40	51	42	43	0
N.S.	1	1.00	1.81	0.80	0.78	0.74	0.94	0.78	0.80	0.00
time (sec)	N/A	0.058	0.173	0.006	2.394	1.226	0.140	0.315	0.042	0.000

Problem 272	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	37	37	37	31	30	30	37	30	32	0
N.S.	1	1.00	1.00	0.84	0.81	0.81	1.00	0.81	0.86	0.00
time (sec)	N/A	0.031	0.008	0.003	2.435	1.212	0.133	0.390	0.040	0.000

Problem 273	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	A	A	B	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	75	75	94	62	61	61	76	61	51	0
N.S.	1	1.00	1.25	0.83	0.81	0.81	1.01	0.81	0.68	0.00
time (sec)	N/A	0.077	0.119	0.004	2.490	0.973	0.206	0.389	0.093	0.000

Problem 274	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	23	23	23	19	18	18	26	18	17	0
N.S.	1	1.00	1.00	0.83	0.78	0.78	1.13	0.78	0.74	0.00
time (sec)	N/A	0.021	0.006	0.001	2.470	1.140	0.118	0.361	1.302	0.000

Problem 275	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	A	A	B	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	75	75	79	62	61	61	76	61	51	0
N.S.	1	1.00	1.05	0.83	0.81	0.81	1.01	0.81	0.68	0.00
time (sec)	N/A	0.064	0.048	0.004	2.589	1.501	0.202	0.309	1.275	0.000

Problem 276	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	B	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	39	39	138	87	36	32	41	36	34	0
N.S.	1	1.00	3.54	2.23	0.92	0.82	1.05	0.92	0.87	0.00
time (sec)	N/A	0.035	0.084	0.008	3.047	1.261	0.155	0.383	1.295	0.000

Problem 277	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	54	54	100	57	42	45	53	42	43	0
N.S.	1	1.00	1.85	1.06	0.78	0.83	0.98	0.78	0.80	0.00
time (sec)	N/A	0.052	0.049	0.006	2.392	1.084	0.164	0.378	0.036	0.000

Problem 278	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	B	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	48	48	141	94	41	49	48	46	41	0
N.S.	1	1.00	2.94	1.96	0.85	1.02	1.00	0.96	0.85	0.00
time (sec)	N/A	0.051	0.102	0.010	2.791	1.085	0.184	0.306	0.063	0.000

Problem 279	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	89	89	142	95	73	84	88	73	62	0
N.S.	1	1.00	1.60	1.07	0.82	0.94	0.99	0.82	0.70	0.00
time (sec)	N/A	0.096	0.110	0.009	2.372	1.458	0.249	0.266	0.039	0.001

Problem 280	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	F	A	C	A	B	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	141	141	139	110	0	212	192	109	100	0
N.S.	1	1.00	0.99	0.78	0.00	1.50	1.36	0.77	0.71	0.00
time (sec)	N/A	0.097	0.275	0.045	0.000	0.947	0.709	0.395	0.103	0.000

Problem 281	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	88	88	68	67	0	70	82	66	38	0
N.S.	1	1.00	0.77	0.76	0.00	0.80	0.93	0.75	0.43	0.00
time (sec)	N/A	0.058	0.020	0.013	0.000	1.211	0.179	0.326	1.310	0.000

Problem 282	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	F	A	C	A	B	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	140	140	135	109	0	211	197	108	99	0
N.S.	1	1.00	0.96	0.78	0.00	1.51	1.41	0.77	0.71	0.00
time (sec)	N/A	0.107	0.165	0.015	0.000	1.525	0.725	0.404	0.067	0.001

Problem 283	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	F	A	C	A	B	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	140	140	135	109	0	211	214	108	97	0
N.S.	1	1.00	0.96	0.78	0.00	1.51	1.53	0.77	0.69	0.00
time (sec)	N/A	0.083	0.151	0.010	0.000	1.447	0.713	0.335	1.311	0.000

Problem 284	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	88	88	68	67	0	70	82	66	40	0
N.S.	1	1.00	0.77	0.76	0.00	0.80	0.93	0.75	0.45	0.00
time (sec)	N/A	0.052	0.017	0.010	0.000	1.124	0.182	0.387	0.037	0.000

Problem 285	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	F	A	C	A	B	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	145	145	140	114	0	224	218	113	102	0
N.S.	1	1.00	0.97	0.79	0.00	1.54	1.50	0.78	0.70	0.00
time (sec)	N/A	0.112	0.208	0.013	0.000	1.318	0.738	0.330	0.048	0.001

Problem 286	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	F	B	C	A	B	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	147	147	148	114	0	240	197	113	104	0
N.S.	1	1.00	1.01	0.78	0.00	1.63	1.34	0.77	0.71	0.00
time (sec)	N/A	0.099	0.294	0.013	0.000	1.231	0.748	0.377	0.030	0.001

Problem 287	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	98	98	95	75	0	90	94	100	52	0
N.S.	1	1.00	0.97	0.77	0.00	0.92	0.96	1.02	0.53	0.00
time (sec)	N/A	0.084	0.034	0.013	0.000	1.196	0.217	0.404	0.038	0.001

Problem 288	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	F	B	C	A	B	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	154	154	171	119	0	246	209	120	110	0
N.S.	1	1.00	1.11	0.77	0.00	1.60	1.36	0.78	0.71	0.00
time (sec)	N/A	0.142	0.344	0.013	0.000	1.201	0.764	0.426	0.032	0.000

Problem 289	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	46	46	46	38	37	37	42	37	39	0
N.S.	1	1.00	1.00	0.83	0.80	0.80	0.91	0.80	0.85	0.00
time (sec)	N/A	0.040	0.012	0.006	2.112	1.135	0.140	0.342	0.047	0.000

Problem 290	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	A	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	57	57	55	44	0	47	48	99	29	0
N.S.	1	1.00	0.96	0.77	0.00	0.82	0.84	1.74	0.51	0.00
time (sec)	N/A	0.043	0.015	0.010	0.000	1.173	0.127	0.321	1.305	0.001

Problem 291	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	39	39	39	33	32	32	37	32	34	0
N.S.	1	1.00	1.00	0.85	0.82	0.82	0.95	0.82	0.87	0.00
time (sec)	N/A	0.033	0.008	0.004	1.968	1.163	0.137	0.423	1.281	0.000

Problem 292	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	F	B	A	A	B	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	82	82	98	65	0	171	70	76	53	0
N.S.	1	1.00	1.20	0.79	0.00	2.09	0.85	0.93	0.65	0.00
time (sec)	N/A	0.071	0.132	0.007	0.000	1.165	0.209	0.348	0.050	0.000

Problem 293	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	23	23	23	19	18	18	26	18	17	0
N.S.	1	1.00	1.00	0.83	0.78	0.78	1.13	0.78	0.74	0.00
time (sec)	N/A	0.022	0.007	0.001	1.937	1.702	0.122	0.410	1.285	0.000

Problem 294	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	F	B	A	A	B	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	82	82	83	65	0	171	70	64	53	0
N.S.	1	1.00	1.01	0.79	0.00	2.09	0.85	0.78	0.65	0.00
time (sec)	N/A	0.057	0.052	0.008	0.000	1.169	0.208	0.407	0.045	0.000

Problem 295	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	41	41	55	35	38	34	41	38	36	0
N.S.	1	1.00	1.34	0.85	0.93	0.83	1.00	0.93	0.88	0.00
time (sec)	N/A	0.038	0.013	0.006	1.947	1.115	0.157	0.346	1.292	0.000

Problem 296	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	A	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	57	57	55	44	0	50	49	99	29	0
N.S.	1	1.00	0.96	0.77	0.00	0.88	0.86	1.74	0.51	0.00
time (sec)	N/A	0.042	0.015	0.010	0.000	1.242	0.153	0.314	1.273	0.000

Problem 297	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	48	48	51	40	43	51	48	48	41	0
N.S.	1	1.00	1.06	0.83	0.90	1.06	1.00	1.00	0.85	0.00
time (sec)	N/A	0.054	0.014	0.007	1.968	1.224	0.190	0.404	0.066	0.000

Problem 298	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	F	B	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	96	96	56	75	0	193	83	56	63	0
N.S.	1	1.00	0.58	0.78	0.00	2.01	0.86	0.58	0.66	0.00
time (sec)	N/A	0.094	0.018	0.011	0.000	1.326	0.261	0.415	0.056	0.000

Problem 299	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	B	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	356	356	59	44	0	716	26	254	209	0
N.S.	1	1.00	0.17	0.12	0.00	2.01	0.07	0.71	0.59	0.00
time (sec)	N/A	0.333	0.015	0.016	0.000	1.430	3.202	0.347	0.157	0.000

Problem 300	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	275	275	41	32	0	215	165	205	53	0
N.S.	1	1.00	0.15	0.12	0.00	0.78	0.60	0.75	0.19	0.00
time (sec)	N/A	0.243	0.011	0.013	0.000	1.267	0.219	0.398	0.096	0.000

Problem 301	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	B	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	347	347	39	40	0	567	24	253	474	0
N.S.	1	1.00	0.11	0.12	0.00	1.63	0.07	0.73	1.37	0.00
time (sec)	N/A	0.209	0.011	0.012	0.000	1.324	3.187	0.480	1.328	0.000

Problem 302	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	B	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	355	355	40	40	0	567	26	253	286	0
N.S.	1	1.00	0.11	0.11	0.00	1.60	0.07	0.71	0.81	0.00
time (sec)	N/A	0.200	0.011	0.013	0.000	1.302	3.281	0.457	0.079	0.000

Problem 303	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	275	275	42	30	0	215	165	205	53	0
N.S.	1	1.00	0.15	0.11	0.00	0.78	0.60	0.75	0.19	0.00
time (sec)	N/A	0.211	0.011	0.006	0.000	1.461	0.220	0.400	0.037	0.000

Problem 304	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	B	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	360	360	61	52	0	732	29	258	253	0
N.S.	1	1.00	0.17	0.14	0.00	2.03	0.08	0.72	0.70	0.00
time (sec)	N/A	0.239	0.016	0.016	0.000	1.557	3.293	0.462	1.292	0.000

Problem 305	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	B	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	370	370	65	50	0	756	31	258	213	0
N.S.	1	1.00	0.18	0.14	0.00	2.04	0.08	0.70	0.58	0.00
time (sec)	N/A	0.236	0.014	0.014	0.000	0.964	3.252	0.353	1.288	0.000

Problem 306	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	287	287	54	43	0	238	182	217	63	0
N.S.	1	1.00	0.19	0.15	0.00	0.83	0.63	0.76	0.22	0.00
time (sec)	N/A	0.241	0.017	0.011	0.000	2.126	0.273	0.383	1.296	0.001

Problem 307	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	B	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	377	377	54	51	0	614	37	265	486	0
N.S.	1	1.00	0.14	0.14	0.00	1.63	0.10	0.70	1.29	0.00
time (sec)	N/A	0.286	0.015	0.019	0.000	1.731	3.366	0.381	0.064	0.001

Problem 308	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	62	62	57	38	50	62	60	50	64	0
N.S.	1	1.00	0.92	0.61	0.81	1.00	0.97	0.81	1.03	0.00
time (sec)	N/A	0.056	0.033	0.004	1.281	1.476	0.145	0.497	0.134	0.000

Problem 309	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	B	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	90	90	97	117	0	154	54	66	130	0
N.S.	1	1.00	1.08	1.30	0.00	1.71	0.60	0.73	1.44	0.00
time (sec)	N/A	0.142	0.152	0.050	0.000	1.290	0.207	0.561	1.343	0.000

Problem 310	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	55	55	53	33	45	56	53	45	59	0
N.S.	1	1.00	0.96	0.60	0.82	1.02	0.96	0.82	1.07	0.00
time (sec)	N/A	0.033	0.023	0.003	1.169	1.398	0.135	0.497	1.358	0.000

Problem 311	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	B	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	81	81	75	110	0	165	49	47	117	0
N.S.	1	1.00	0.93	1.36	0.00	2.04	0.60	0.58	1.44	0.00
time (sec)	N/A	0.084	0.048	0.024	0.000	1.447	0.201	0.551	0.119	0.000

Problem 312	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	23	23	38	19	31	43	42	31	30	0
N.S.	1	1.00	1.65	0.83	1.35	1.87	1.83	1.35	1.30	0.00
time (sec)	N/A	0.026	0.010	0.001	1.482	1.590	0.118	0.601	1.333	0.000

Problem 313	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	B	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	75	75	74	60	0	128	49	41	125	0
N.S.	1	1.00	0.99	0.80	0.00	1.71	0.65	0.55	1.67	0.00
time (sec)	N/A	0.058	0.037	0.016	0.000	1.364	0.204	0.419	0.054	0.000

Problem 314	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	57	57	55	35	51	58	58	51	42	0
N.S.	1	1.00	0.96	0.61	0.89	1.02	1.02	0.89	0.74	0.00
time (sec)	N/A	0.034	0.030	0.009	1.505	1.450	0.158	0.484	1.412	0.000

Problem 315	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	B	F	B	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	89	89	65	117	0	158	56	68	130	0
N.S.	1	1.00	0.73	1.31	0.00	1.78	0.63	0.76	1.46	0.00
time (sec)	N/A	0.072	0.017	0.028	0.000	0.955	0.238	0.456	1.304	0.000

Problem 316	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	66	66	60	42	56	76	65	63	49	0
N.S.	1	1.00	0.91	0.64	0.85	1.15	0.98	0.95	0.74	0.00
time (sec)	N/A	0.067	0.035	0.010	1.292	1.184	0.188	0.551	1.358	0.001

Problem 317	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	B	F	B	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	97	97	73	122	0	180	65	77	136	0
N.S.	1	1.00	0.75	1.26	0.00	1.86	0.67	0.79	1.40	0.00
time (sec)	N/A	0.134	0.018	0.027	0.000	0.786	0.271	0.509	0.124	0.001

Problem 318	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	B	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	460	440	58	46	0	1012	29	240	216	0
N.S.	1	0.96	0.13	0.10	0.00	2.20	0.06	0.52	0.47	0.00
time (sec)	N/A	0.416	0.015	0.013	0.000	1.678	1.591	0.725	1.438	0.001

Problem 319	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	B	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	431	431	41	40	0	725	26	239	149	0
N.S.	1	1.00	0.10	0.09	0.00	1.68	0.06	0.55	0.35	0.00
time (sec)	N/A	0.295	0.012	0.013	0.000	1.577	1.530	0.663	1.465	0.000

Problem 320	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	B	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	451	451	39	40	0	843	24	239	454	0
N.S.	1	1.00	0.09	0.09	0.00	1.87	0.05	0.53	1.01	0.00
time (sec)	N/A	0.280	0.011	0.016	0.000	1.104	1.482	0.765	0.196	0.000

Problem 321	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	B	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	427	431	40	40	0	955	26	239	275	0
N.S.	1	1.01	0.09	0.09	0.00	2.24	0.06	0.56	0.64	0.00
time (sec)	N/A	0.259	0.010	0.010	0.000	1.862	1.510	0.593	0.086	0.001

Problem 322	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	B	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	414	414	42	37	0	733	26	239	403	0
N.S.	1	1.00	0.10	0.09	0.00	1.77	0.06	0.58	0.97	0.00
time (sec)	N/A	0.257	0.010	0.010	0.000	1.763	1.519	0.528	0.083	0.001

Problem 323	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	B	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	416	416	61	52	0	1017	32	244	292	0
N.S.	1	1.00	0.15	0.12	0.00	2.44	0.08	0.59	0.70	0.00
time (sec)	N/A	0.285	0.016	0.013	0.000	1.500	1.609	0.603	1.291	0.000

Problem 324	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	B	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	466	466	65	50	0	1057	34	244	492	0
N.S.	1	1.00	0.14	0.11	0.00	2.27	0.07	0.52	1.06	0.00
time (sec)	N/A	0.367	0.016	0.012	0.000	1.795	1.624	0.702	0.184	0.001

Problem 325	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	62	62	56	38	50	62	58	53	64	0
N.S.	1	1.00	0.90	0.61	0.81	1.00	0.94	0.85	1.03	0.00
time (sec)	N/A	0.046	0.032	0.004	1.393	0.681	0.143	0.416	0.120	0.000

Problem 326	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	90	90	103	67	92	114	170	97	90	0
N.S.	1	1.00	1.14	0.74	1.02	1.27	1.89	1.08	1.00	0.00
time (sec)	N/A	0.072	0.054	0.006	1.359	1.162	0.380	0.463	1.328	0.000

Problem 327	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	55	55	53	33	45	57	53	48	59	0
N.S.	1	1.00	0.96	0.60	0.82	1.04	0.96	0.87	1.07	0.00
time (sec)	N/A	0.032	0.022	0.003	1.479	1.040	0.135	0.425	0.102	0.000

Problem 328	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	B	B	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	81	81	91	62	87	109	165	92	77	0
N.S.	1	1.00	1.12	0.77	1.07	1.35	2.04	1.14	0.95	0.00
time (sec)	N/A	0.053	0.034	0.003	1.917	1.168	0.371	0.445	1.381	0.001

Problem 329	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	23	23	38	19	31	43	42	33	30	0
N.S.	1	1.00	1.65	0.83	1.35	1.87	1.83	1.43	1.30	0.00
time (sec)	N/A	0.027	0.010	0.003	1.413	1.277	0.118	0.537	1.570	0.000

Problem 330	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	B	B	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	75	75	91	62	87	107	165	92	83	0
N.S.	1	1.00	1.21	0.83	1.16	1.43	2.20	1.23	1.11	0.00
time (sec)	N/A	0.038	0.029	0.004	1.450	1.228	0.366	0.433	1.302	0.000

Problem 331	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	57	57	55	64	51	59	58	54	42	0
N.S.	1	1.00	0.96	1.12	0.89	1.04	1.02	0.95	0.74	0.00
time (sec)	N/A	0.030	0.031	0.010	1.445	1.278	0.158	0.477	0.429	0.000

Problem 332	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	89	89	103	67	92	125	172	97	88	0
N.S.	1	1.00	1.16	0.75	1.03	1.40	1.93	1.09	0.99	0.00
time (sec)	N/A	0.058	0.057	0.007	1.462	1.109	0.406	0.384	0.064	0.001

Problem 333	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	66	66	61	71	56	76	66	66	49	0
N.S.	1	1.00	0.92	1.08	0.85	1.15	1.00	1.00	0.74	0.00
time (sec)	N/A	0.063	0.036	0.012	1.395	1.279	0.190	0.505	1.348	0.000

Problem 334	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	97	97	111	72	99	130	199	104	95	0
N.S.	1	1.00	1.14	0.74	1.02	1.34	2.05	1.07	0.98	0.00
time (sec)	N/A	0.090	0.071	0.009	1.325	1.200	0.439	0.409	1.385	0.000

Problem 335	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	B	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	170	170	160	205	0	304	58	148	246	0
N.S.	1	1.00	0.94	1.21	0.00	1.79	0.34	0.87	1.45	0.00
time (sec)	N/A	0.114	0.266	0.061	0.000	1.312	1.245	0.676	1.445	0.001

Problem 336	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	B	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	167	167	160	206	0	255	53	147	147	0
N.S.	1	1.00	0.96	1.23	0.00	1.53	0.32	0.88	0.88	0.00
time (sec)	N/A	0.078	0.150	0.030	0.000	1.268	1.222	0.748	0.194	0.000

Problem 337	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	B	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	173	173	132	206	0	271	49	147	269	0
N.S.	1	1.00	0.76	1.19	0.00	1.57	0.28	0.85	1.55	0.00
time (sec)	N/A	0.075	0.191	0.040	0.000	1.424	1.203	0.615	1.472	0.000

Problem 338	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	B	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	145	166	131	110	0	255	53	147	269	0
N.S.	1	1.14	0.90	0.76	0.00	1.76	0.37	1.01	1.86	0.00
time (sec)	N/A	0.060	0.044	0.029	0.000	1.080	1.185	0.624	0.081	0.000

Problem 339	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	B	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	169	169	160	206	0	251	53	147	245	0
N.S.	1	1.00	0.95	1.22	0.00	1.49	0.31	0.87	1.45	0.00
time (sec)	N/A	0.060	0.156	0.029	0.000	1.490	1.212	0.482	0.079	0.000

Problem 340	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	B	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	172	172	174	211	0	313	63	152	250	0
N.S.	1	1.00	1.01	1.23	0.00	1.82	0.37	0.88	1.45	0.00
time (sec)	N/A	0.089	0.267	0.029	0.000	1.312	1.250	0.542	1.339	0.000

Problem 341	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	B	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	182	182	166	209	0	327	63	152	268	0
N.S.	1	1.00	0.91	1.15	0.00	1.80	0.35	0.84	1.47	0.00
time (sec)	N/A	0.118	0.262	0.030	0.000	1.401	1.281	0.669	0.204	0.000

Problem 342	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	B	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	173	173	189	216	0	300	73	159	257	0
N.S.	1	1.00	1.09	1.25	0.00	1.73	0.42	0.92	1.49	0.00
time (sec)	N/A	0.140	0.274	0.039	0.000	1.193	1.290	0.539	1.487	0.001

Problem 343	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	B	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	189	189	189	216	0	332	70	159	291	0
N.S.	1	1.00	1.00	1.14	0.00	1.76	0.37	0.84	1.54	0.00
time (sec)	N/A	0.163	0.268	0.043	0.000	1.205	1.298	0.556	0.210	0.000

Problem 344	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	21	21	21	18	17	17	15	17	16	0
N.S.	1	1.00	1.00	0.86	0.81	0.81	0.71	0.81	0.76	0.00
time (sec)	N/A	0.013	0.004	0.005	0.587	1.276	0.116	0.282	0.063	0.000

Problem 345	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	26	26	26	23	22	22	19	22	22	0
N.S.	1	1.00	1.00	0.88	0.85	0.85	0.73	0.85	0.85	0.00
time (sec)	N/A	0.019	0.005	0.006	0.861	1.204	0.130	0.364	1.315	0.000

Problem 346	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	37	37	37	31	30	30	37	30	32	0
N.S.	1	1.00	1.00	0.84	0.81	0.81	1.00	0.81	0.86	0.00
time (sec)	N/A	0.034	0.013	0.003	2.046	1.199	0.143	2.721	1.346	0.000

Problem 347	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	23	23	23	19	18	18	27	18	20	0
N.S.	1	1.00	1.00	0.83	0.78	0.78	1.17	0.78	0.87	0.00
time (sec)	N/A	0.022	0.006	0.001	2.092	1.217	0.131	2.962	1.333	0.000

Problem 348	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	B	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	39	39	197	66	36	32	41	33	34	0
N.S.	1	1.00	5.05	1.69	0.92	0.82	1.05	0.85	0.87	0.00
time (sec)	N/A	0.035	0.037	0.039	2.033	1.164	0.165	0.408	0.060	0.000

Problem 349	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	48	48	208	73	41	49	48	45	41	0
N.S.	1	1.00	4.33	1.52	0.85	1.02	1.00	0.94	0.85	0.00
time (sec)	N/A	0.051	0.042	0.024	2.070	1.273	0.202	0.251	1.369	0.001

Problem 350	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	B	F	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	39	39	197	66	0	32	41	33	34	0
N.S.	1	1.00	5.05	1.69	0.00	0.82	1.05	0.85	0.87	0.00
time (sec)	N/A	0.036	0.020	0.022	0.000	1.061	0.166	0.350	0.033	0.000

Problem 351	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F(-2)	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	147	147	140	236	0	466	605	145	183	0
N.S.	1	1.00	0.95	1.61	0.00	3.17	4.12	0.99	1.24	0.00
time (sec)	N/A	0.137	0.122	0.006	0.000	1.303	1.249	0.291	0.159	0.001

Problem 352	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F(-2)	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	118	118	112	190	0	383	498	113	151	0
N.S.	1	1.00	0.95	1.61	0.00	3.25	4.22	0.96	1.28	0.00
time (sec)	N/A	0.105	0.085	0.003	0.000	1.337	1.012	0.392	1.396	0.001

Problem 353	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F(-2)	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	89	89	84	132	0	297	381	86	112	0
N.S.	1	1.00	0.94	1.48	0.00	3.34	4.28	0.97	1.26	0.00
time (sec)	N/A	0.086	0.104	0.005	0.000	1.413	0.835	0.257	0.131	0.001

Problem 354	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F(-2)	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	70	70	73	101	0	235	306	67	172	0
N.S.	1	1.00	1.04	1.44	0.00	3.36	4.37	0.96	2.46	0.00
time (sec)	N/A	0.049	0.064	0.003	0.000	1.434	0.597	0.274	1.418	0.001

Problem 355	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F(-2)	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	56	56	57	56	0	185	216	55	112	0
N.S.	1	1.00	1.02	1.00	0.00	3.30	3.86	0.98	2.00	0.00
time (sec)	N/A	0.036	0.031	0.003	0.000	1.349	0.318	0.291	0.168	0.001

Problem 356	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F(-2)	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	36	36	38	35	0	120	124	34	46	0
N.S.	1	1.00	1.06	0.97	0.00	3.33	3.44	0.94	1.28	0.00
time (sec)	N/A	0.032	0.007	0.001	0.000	1.243	0.218	0.345	0.046	0.001

Problem 357	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F(-2)	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	62	62	61	62	0	211	564	62	213	0
N.S.	1	1.00	0.98	1.00	0.00	3.40	9.10	1.00	3.44	0.00
time (sec)	N/A	0.048	0.067	0.006	0.000	1.406	4.080	0.297	1.715	0.001

Problem 358	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F(-2)	A	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	81	81	77	112	0	269	0	79	339	0
N.S.	1	1.00	0.95	1.38	0.00	3.32	0.00	0.98	4.19	0.00
time (sec)	N/A	0.102	0.079	0.006	0.000	0.889	0.000	0.386	1.814	0.001

Problem 359	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F(-2)	A	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	104	104	102	150	0	358	0	105	447	0
N.S.	1	1.00	0.98	1.44	0.00	3.44	0.00	1.01	4.30	0.00
time (sec)	N/A	0.153	0.136	0.007	0.000	1.361	0.000	0.314	1.866	0.001

Problem 360	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F(-2)	A	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	137	137	131	214	0	445	0	136	524	0
N.S.	1	1.00	0.96	1.56	0.00	3.25	0.00	0.99	3.82	0.00
time (sec)	N/A	0.193	0.099	0.010	0.000	1.471	0.000	0.382	1.923	0.001

Problem 361	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	B	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	196	196	163	434	0	1029	1012	188	382	0
N.S.	1	1.00	0.83	2.21	0.00	5.25	5.16	0.96	1.95	0.00
time (sec)	N/A	0.205	0.222	0.013	0.000	1.332	2.369	0.330	1.822	0.001

Problem 362	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	B	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	150	150	132	352	0	837	842	161	261	0
N.S.	1	1.00	0.88	2.35	0.00	5.58	5.61	1.07	1.74	0.00
time (sec)	N/A	0.148	0.177	0.012	0.000	1.620	1.727	0.369	1.798	0.001

Problem 363	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F(-2)	B	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	114	114	109	209	0	635	729	125	279	0
N.S.	1	1.00	0.96	1.83	0.00	5.57	6.39	1.10	2.45	0.00
time (sec)	N/A	0.100	0.140	0.010	0.000	1.284	1.323	0.362	1.859	0.001

Problem 364	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F(-2)	B	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	71	71	81	97	0	387	280	88	135	0
N.S.	1	1.00	1.14	1.37	0.00	5.45	3.94	1.24	1.90	0.00
time (sec)	N/A	0.042	0.092	0.007	0.000	1.200	0.597	0.379	1.370	0.001

Problem 365	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F(-2)	B	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	66	66	69	70	0	338	253	76	110	0
N.S.	1	1.00	1.05	1.06	0.00	5.12	3.83	1.15	1.67	0.00
time (sec)	N/A	0.034	0.064	0.003	0.000	1.171	0.562	0.371	1.369	0.001

Problem 366	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F(-2)	B	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	66	66	70	68	0	341	265	76	119	0
N.S.	1	1.00	1.06	1.03	0.00	5.17	4.02	1.15	1.80	0.00
time (sec)	N/A	0.033	0.077	0.003	0.000	1.264	0.579	0.395	0.083	0.001

Problem 367	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	B	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	108	108	107	237	0	781	0	126	620	0
N.S.	1	1.00	0.99	2.19	0.00	7.23	0.00	1.17	5.74	0.00
time (sec)	N/A	0.145	0.185	0.013	0.000	1.488	0.000	0.359	2.096	0.001

Problem 368	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	B	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	148	148	131	328	0	975	0	171	775	0
N.S.	1	1.00	0.89	2.22	0.00	6.59	0.00	1.16	5.24	0.00
time (sec)	N/A	0.182	0.266	0.014	0.000	1.787	0.000	0.423	2.134	0.001

Problem 369	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	B	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	202	202	175	418	0	1226	0	229	914	0
N.S.	1	1.00	0.87	2.07	0.00	6.07	0.00	1.13	4.52	0.00
time (sec)	N/A	0.239	0.343	0.017	0.000	2.004	0.000	0.330	2.298	0.001

Problem 370	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	B	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	238	238	260	1040	0	1926	1714	282	705	0
N.S.	1	1.00	1.09	4.37	0.00	8.09	7.20	1.18	2.96	0.00
time (sec)	N/A	0.291	0.370	0.017	0.000	1.006	4.196	0.379	2.000	0.001

Problem 371	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	B	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	190	190	221	530	0	1603	1510	245	620	0
N.S.	1	1.00	1.16	2.79	0.00	8.44	7.95	1.29	3.26	0.00
time (sec)	N/A	0.279	0.308	0.016	0.000	1.679	3.208	0.398	2.203	0.001

Problem 372	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	B	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	111	111	174	260	0	953	547	202	343	0
N.S.	1	1.00	1.57	2.34	0.00	8.59	4.93	1.82	3.09	0.00
time (sec)	N/A	0.067	0.174	0.010	0.000	0.755	1.411	0.416	0.195	0.001

Problem 373	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	B	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	107	107	126	223	0	872	513	163	271	0
N.S.	1	1.00	1.18	2.08	0.00	8.15	4.79	1.52	2.53	0.00
time (sec)	N/A	0.050	0.194	0.012	0.000	1.110	1.209	0.308	1.429	0.002

Problem 374	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	B	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	115	115	131	262	0	887	570	154	313	0
N.S.	1	1.00	1.14	2.28	0.00	7.71	4.96	1.34	2.72	0.00
time (sec)	N/A	0.068	0.140	0.010	0.000	1.103	1.282	0.436	1.500	0.001

Problem 375	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F(-2)	B	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	103	103	102	130	0	788	481	135	253	0
N.S.	1	1.00	0.99	1.26	0.00	7.65	4.67	1.31	2.46	0.00
time (sec)	N/A	0.041	0.096	0.005	0.000	1.186	1.092	0.409	1.430	0.001

Problem 376	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F(-2)	B	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	103	101	97	129	0	785	474	136	285	0
N.S.	1	0.98	0.94	1.25	0.00	7.62	4.60	1.32	2.77	0.00
time (sec)	N/A	0.041	0.096	0.004	0.000	1.696	1.111	0.337	1.424	0.001

Problem 377	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	B	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	185	185	178	781	0	1985	0	239	1089	0
N.S.	1	1.00	0.96	4.22	0.00	10.73	0.00	1.29	5.89	0.00
time (sec)	N/A	0.220	0.353	0.017	0.000	3.243	0.000	0.315	2.456	0.001

Problem 378	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	B	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	239	239	221	954	0	2280	0	309	1255	0
N.S.	1	1.00	0.92	3.99	0.00	9.54	0.00	1.29	5.25	0.00
time (sec)	N/A	0.277	0.437	0.023	0.000	3.188	0.000	0.468	2.554	0.001

Problem 379	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	40	40	40	31	30	30	34	32	26	0
N.S.	1	1.00	1.00	0.78	0.75	0.75	0.85	0.80	0.65	0.00
time (sec)	N/A	0.022	0.006	0.006	0.423	1.723	0.121	0.255	0.046	0.001

Problem 380	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	33	33	33	26	25	25	27	27	21	0
N.S.	1	1.00	1.00	0.79	0.76	0.76	0.82	0.82	0.64	0.00
time (sec)	N/A	0.020	0.004	0.004	0.418	0.759	0.119	0.280	1.310	0.000

Problem 381	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	26	26	26	21	20	20	20	22	16	0
N.S.	1	1.00	1.00	0.81	0.77	0.77	0.77	0.85	0.62	0.00
time (sec)	N/A	0.015	0.004	0.006	0.418	1.221	0.116	0.277	0.077	0.000

Problem 382	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	21	21	21	18	17	17	17	19	13	0
N.S.	1	1.00	1.00	0.86	0.81	0.81	0.81	0.90	0.62	0.00
time (sec)	N/A	0.011	0.003	0.006	0.420	1.403	0.111	0.275	0.066	0.001

Problem 383	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	23	23	21	18	17	17	15	19	8	0
N.S.	1	1.00	0.91	0.78	0.74	0.74	0.65	0.83	0.35	0.00
time (sec)	N/A	0.014	0.003	0.006	0.418	1.044	0.111	0.224	1.373	0.001

Problem 384	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	27	27	27	22	21	21	24	24	17	0
N.S.	1	1.00	1.00	0.81	0.78	0.78	0.89	0.89	0.63	0.00
time (sec)	N/A	0.017	0.004	0.007	0.421	1.302	0.149	0.250	1.385	0.000

Problem 385	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	34	34	34	27	26	30	31	29	22	0
N.S.	1	1.00	1.00	0.79	0.76	0.88	0.91	0.85	0.65	0.00
time (sec)	N/A	0.031	0.004	0.009	0.427	1.296	0.163	0.308	0.044	0.000

Problem 386	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	41	41	41	32	31	39	36	34	26	0
N.S.	1	1.00	1.00	0.78	0.76	0.95	0.88	0.83	0.63	0.00
time (sec)	N/A	0.035	0.005	0.007	0.421	1.301	0.173	0.298	1.312	0.001

Problem 387	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	48	48	48	37	36	44	41	39	32	0
N.S.	1	1.00	1.00	0.77	0.75	0.92	0.85	0.81	0.67	0.00
time (sec)	N/A	0.042	0.005	0.007	0.645	1.143	0.185	0.294	0.047	0.001

Problem 388	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	A	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	204	204	213	701	0	959	0	0	-1	209
N.S.	1	1.00	1.04	3.44	0.00	4.70	0.00	0.00	-0.00	1.02
time (sec)	N/A	0.231	0.522	0.019	0.000	1.592	0.000	0.000	0.000	8.555

Problem 389	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	A	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	145	145	163	334	0	709	0	0	-1	154
N.S.	1	1.00	1.12	2.30	0.00	4.89	0.00	0.00	-0.01	1.06
time (sec)	N/A	0.134	0.250	0.007	0.000	1.362	0.000	0.000	0.000	7.173

Problem 390	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	F(-2)	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	105	105	128	121	0	590	0	0	100	125
N.S.	1	1.00	1.22	1.15	0.00	5.62	0.00	0.00	0.95	1.19
time (sec)	N/A	0.083	0.081	0.007	0.000	1.253	0.000	0.000	0.126	5.604

Problem 391	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	F(-2)	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	67	67	89	88	0	171	0	0	53	93
N.S.	1	1.00	1.33	1.31	0.00	2.55	0.00	0.00	0.79	1.39
time (sec)	N/A	0.041	0.058	0.005	0.000	1.295	0.000	0.000	1.452	4.695

Problem 392	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	F(-2)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	133	133	138	197	0	465	0	0	-1	150
N.S.	1	1.00	1.04	1.48	0.00	3.50	0.00	0.00	-0.01	1.13
time (sec)	N/A	0.099	0.171	0.008	0.000	1.347	0.000	0.000	0.000	5.928

Problem 393	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	B	F	F(-2)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	220	220	256	376	0	1081	0	0	-1	273
N.S.	1	1.00	1.16	1.71	0.00	4.91	0.00	0.00	-0.00	1.24
time (sec)	N/A	0.194	0.378	0.012	0.000	1.863	0.000	0.000	0.000	7.914

Problem 394	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	73	73	32	40	8	8	0	29	134	32
N.S.	1	1.00	0.44	0.55	0.11	0.11	0.00	0.40	1.84	0.44
time (sec)	N/A	0.036	0.019	0.013	0.785	1.599	0.000	0.341	0.112	4.983

Problem 395	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	B	A	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	179	179	202	343	0	1059	129	2109	3026	0
N.S.	1	1.00	1.13	1.92	0.00	5.92	0.72	11.78	16.91	0.00
time (sec)	N/A	0.291	0.117	0.011	0.000	0.956	2.119	1.663	2.079	0.001

Problem 396	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	B	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	631	631	70	59	0	5260	196	0	2280	0
N.S.	1	1.00	0.11	0.09	0.00	8.34	0.31	0.00	3.61	0.00
time (sec)	N/A	1.170	0.036	0.006	0.000	3.179	6.965	0.000	4.540	0.001

Problem 397	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F(-1)	B	F(-1)	F(-2)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	376	376	70	59	0	5082	0	0	10382	0
N.S.	1	1.00	0.19	0.16	0.00	13.52	0.00	0.00	27.61	0.00
time (sec)	N/A	0.666	0.043	0.004	0.000	2.634	0.000	0.000	3.776	0.001

Problem 398	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	F(-1)	F	F(-2)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	106	106	106	84	0	0	0	0	-1	109
N.S.	1	1.00	1.00	0.79	0.00	0.00	0.00	0.00	-0.01	1.03
time (sec)	N/A	0.092	0.056	0.007	0.000	0.000	0.000	0.000	0.000	0.235

Problem 399	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	40	40	29	52	54	53	51	49	44	55
N.S.	1	1.00	0.72	1.30	1.35	1.32	1.28	1.22	1.10	1.38
time (sec)	N/A	0.019	0.025	0.001	0.869	1.068	0.330	0.332	0.038	0.027

Problem 400	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	F(-1)	F	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	75	75	50	50	23	0	0	45	-1	151
N.S.	1	1.00	0.67	0.67	0.31	0.00	0.00	0.60	-0.01	2.01
time (sec)	N/A	0.040	0.030	0.011	0.956	0.000	0.000	0.397	0.000	0.242

Problem 401	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	137	137	56	109	114	84	0	140	-1	0
N.S.	1	1.00	0.41	0.80	0.83	0.61	0.00	1.02	-0.01	0.00
time (sec)	N/A	0.081	0.049	0.016	0.906	1.467	0.000	0.511	0.000	2.556

Problem 402	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	137	137	56	87	114	61	0	102	-1	0
N.S.	1	1.00	0.41	0.64	0.83	0.45	0.00	0.74	-0.01	0.00
time (sec)	N/A	0.071	0.035	0.002	0.890	1.117	0.000	0.534	0.000	2.402

Problem 403	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	137	137	65	65	114	32	0	64	-1	0
N.S.	1	1.00	0.47	0.47	0.83	0.23	0.00	0.47	-0.01	0.00
time (sec)	N/A	0.056	0.032	0.003	0.914	1.395	0.000	0.368	0.000	1.683

Problem 404	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	88	88	43	43	114	10	0	26	71	0
N.S.	1	1.00	0.49	0.49	1.30	0.11	0.00	0.30	0.81	0.00
time (sec)	N/A	0.040	0.009	0.003	0.884	0.950	0.000	0.389	1.561	0.817

Problem 405	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	147	147	65	103	36	33	0	61	-1	204
N.S.	1	1.00	0.44	0.70	0.24	0.22	0.00	0.41	-0.01	1.39
time (sec)	N/A	0.070	0.036	0.018	0.876	1.371	0.000	0.489	0.000	0.359

Problem 406	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	130	130	72	92	55	113	0	64	-1	1439
N.S.	1	1.00	0.55	0.71	0.42	0.87	0.00	0.49	-0.01	11.07
time (sec)	N/A	0.072	0.045	0.008	0.617	1.231	0.000	0.532	0.000	1.315

Problem 407	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	135	135	58	54	53	136	0	43	53	284
N.S.	1	1.00	0.43	0.40	0.39	1.01	0.00	0.32	0.39	2.10
time (sec)	N/A	0.075	0.036	0.009	0.716	1.137	0.000	0.545	2.804	0.773

Problem 408	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	137	137	58	54	53	209	0	43	53	356
N.S.	1	1.00	0.42	0.39	0.39	1.53	0.00	0.31	0.39	2.60
time (sec)	N/A	0.078	0.035	0.007	0.538	1.309	0.000	0.589	3.228	0.894

Problem 409	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	F	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	137	137	58	54	53	275	0	43	53	434
N.S.	1	1.00	0.42	0.39	0.39	2.01	0.00	0.31	0.39	3.17
time (sec)	N/A	0.079	0.035	0.007	0.721	1.574	0.000	0.649	3.654	1.004

Problem 410	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	F	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	137	137	58	54	53	343	0	43	53	507
N.S.	1	1.00	0.42	0.39	0.39	2.50	0.00	0.31	0.39	3.70
time (sec)	N/A	0.078	0.037	0.007	0.543	1.412	0.000	0.743	4.344	1.220

Problem 411	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	A	A	F(-2)	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	468	468	207	0	362	579	0	1564	777	0
N.S.	1	1.00	0.44	0.00	0.77	1.24	0.00	3.34	1.66	0.00
time (sec)	N/A	0.224	0.216	0.009	0.704	1.657	0.000	0.627	3.515	0.544

Problem 412	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	A	A	F(-2)	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	315	315	143	0	198	297	0	745	390	0
N.S.	1	1.00	0.45	0.00	0.63	0.94	0.00	2.37	1.24	0.00
time (sec)	N/A	0.140	0.174	0.007	0.640	1.315	0.000	0.481	2.166	0.189

Problem 413	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	A	A	F	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	142	142	83	0	77	110	0	229	138	0
N.S.	1	1.00	0.58	0.00	0.54	0.77	0.00	1.61	0.97	0.00
time (sec)	N/A	0.068	0.049	0.006	0.798	1.498	0.000	0.483	1.542	0.157

Problem 414	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	C	F	F	A	F	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	146	162	101	0	0	82	0	0	69	0
N.S.	1	1.11	0.69	0.00	0.00	0.56	0.00	0.00	0.47	0.00
time (sec)	N/A	0.099	0.082	0.011	0.000	1.667	0.000	0.000	1.647	5.793

Problem 415	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	176	176	93	114	114	147	0	0	-1	1582
N.S.	1	1.00	0.53	0.65	0.65	0.84	0.00	0.00	-0.01	8.99
time (sec)	N/A	0.104	0.069	0.016	0.576	11.031	0.000	0.000	0.000	1.263

Problem 416	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	F(-1)	F(-1)	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	268	268	121	174	119	0	0	237	-1	2887
N.S.	1	1.00	0.45	0.65	0.44	0.00	0.00	0.88	-0.00	10.77
time (sec)	N/A	0.151	0.096	0.016	0.878	0.000	0.000	0.866	0.000	2.595

Problem 417	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	F(-1)	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	179	179	66	68	0	0	0	80	-1	77
N.S.	1	1.00	0.37	0.38	0.00	0.00	0.00	0.45	-0.01	0.43
time (sec)	N/A	0.092	0.034	0.030	0.000	0.000	0.000	0.390	0.000	5.928

Problem 418	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	F(-1)	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	391	391	125	115	79	0	0	173	-1	133
N.S.	1	1.00	0.32	0.29	0.20	0.00	0.00	0.44	-0.00	0.34
time (sec)	N/A	0.187	0.068	0.028	0.598	0.000	0.000	0.541	0.000	6.543

Problem 419	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	F(-1)	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	291	291	99	91	57	0	0	128	-1	109
N.S.	1	1.00	0.34	0.31	0.20	0.00	0.00	0.44	-0.00	0.37
time (sec)	N/A	0.137	0.057	0.010	0.635	0.000	0.000	0.505	0.000	6.281

Problem 420	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	F(-1)	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	189	189	77	69	30	0	0	79	-1	85
N.S.	1	1.00	0.41	0.37	0.16	0.00	0.00	0.42	-0.01	0.45
time (sec)	N/A	0.091	0.029	0.005	0.663	0.000	0.000	0.412	0.000	5.979

Problem 421	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	F(-1)	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	88	88	49	50	10	0	0	34	39	54
N.S.	1	1.00	0.56	0.57	0.11	0.00	0.00	0.39	0.44	0.61
time (sec)	N/A	0.053	0.014	0.005	0.733	0.000	0.000	0.397	1.426	5.303

Problem 422	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	F(-1)	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	190	190	86	78	44	0	0	77	-1	94
N.S.	1	1.00	0.45	0.41	0.23	0.00	0.00	0.41	-0.01	0.49
time (sec)	N/A	0.118	0.040	0.006	0.743	0.000	0.000	0.592	0.000	4.797

Problem 423	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	F(-1)	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	300	300	126	141	97	0	0	121	-1	135
N.S.	1	1.00	0.42	0.47	0.32	0.00	0.00	0.40	-0.00	0.45
time (sec)	N/A	0.187	0.086	0.011	1.068	0.000	0.000	0.660	0.000	6.436

Problem 424	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	F(-1)	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	410	410	152	199	139	0	0	141	-1	159
N.S.	1	1.00	0.37	0.49	0.34	0.00	0.00	0.34	-0.00	0.39
time (sec)	N/A	0.268	0.130	0.013	0.854	0.000	0.000	0.717	0.000	9.554

Problem 425	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	F(-1)	F(-1)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	289	289	98	94	57	0	0	126	-1	109
N.S.	1	1.00	0.34	0.33	0.20	0.00	0.00	0.44	-0.00	0.38
time (sec)	N/A	0.138	0.052	0.036	0.995	0.000	0.000	0.486	0.000	6.280

Problem 426	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	F(-1)	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	291	291	103	91	52	0	0	125	-1	109
N.S.	1	1.00	0.35	0.31	0.18	0.00	0.00	0.43	-0.00	0.37
time (sec)	N/A	0.137	0.048	0.026	0.940	0.000	0.000	0.481	0.000	6.305

Problem 427	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	222	222	98	152	99	302	0	84	-1	2765
N.S.	1	1.00	0.44	0.68	0.45	1.36	0.00	0.38	-0.00	12.45
time (sec)	N/A	0.126	0.087	0.012	1.022	0.906	0.000	0.590	0.000	2.806

Problem 428	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	F(-1)	F(-1)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	391	391	124	116	79	0	0	172	-1	135
N.S.	1	1.00	0.32	0.30	0.20	0.00	0.00	0.44	-0.00	0.35
time (sec)	N/A	0.180	0.066	0.032	0.947	0.000	0.000	0.461	0.000	6.779

Problem 429	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	46	46	38	62	45	38	42	0	-1	41
N.S.	1	1.00	0.83	1.35	0.98	0.83	0.91	0.00	-0.02	0.89
time (sec)	N/A	0.036	0.028	0.030	0.934	1.645	19.826	0.000	0.000	0.063

Problem 430	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	28	28	26	33	32	24	26	0	-1	34
N.S.	1	1.00	0.93	1.18	1.14	0.86	0.93	0.00	-0.04	1.21
time (sec)	N/A	0.026	0.014	0.024	0.872	2.059	13.890	0.000	0.000	0.070

Problem 431	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	15	15	15	18	19	15	37	0	-1	18
N.S.	1	1.00	1.00	1.20	1.27	1.00	2.47	0.00	-0.07	1.20
time (sec)	N/A	0.012	0.003	0.022	0.885	1.126	11.280	0.000	0.000	0.051

Problem 432	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	23	23	22	26	27	22	66	25	20	34
N.S.	1	1.00	0.96	1.13	1.17	0.96	2.87	1.09	0.87	1.48
time (sec)	N/A	0.019	0.007	0.020	0.896	1.463	16.485	0.251	1.369	0.054

Problem 433	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F(-2)	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	57	57	49	69	58	59	0	0	-1	59
N.S.	1	1.00	0.86	1.21	1.02	1.04	0.00	0.00	-0.02	1.04
time (sec)	N/A	0.042	0.066	0.024	0.911	1.386	0.000	0.000	0.000	0.074

Problem 434	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	76	76	62	88	71	72	73	0	-1	72
N.S.	1	1.00	0.82	1.16	0.93	0.95	0.96	0.00	-0.01	0.95
time (sec)	N/A	0.047	0.078	0.031	0.910	1.473	52.911	0.000	0.000	0.101

Problem 435	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	93	93	75	105	84	85	88	0	-1	85
N.S.	1	1.00	0.81	1.13	0.90	0.91	0.95	0.00	-0.01	0.91
time (sec)	N/A	0.054	0.095	0.032	0.921	1.224	105.734	0.000	0.000	0.117

Problem 436	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	A	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	236	236	34	54	0	272	0	203	-1	0
N.S.	1	1.00	0.14	0.23	0.00	1.15	0.00	0.86	-0.00	0.00
time (sec)	N/A	0.206	0.009	0.121	0.000	0.920	0.000	0.348	0.000	0.110

Problem 437	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	A	F(-1)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	160	160	34	54	0	212	0	136	-1	0
N.S.	1	1.00	0.21	0.34	0.00	1.32	0.00	0.85	-0.01	0.00
time (sec)	N/A	0.132	0.009	0.069	0.000	1.331	0.000	0.419	0.000	0.097

Problem 438	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	F	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	50	50	32	79	0	151	36	38	-1	0
N.S.	1	1.00	0.64	1.58	0.00	3.02	0.72	0.76	-0.02	0.00
time (sec)	N/A	0.035	0.007	0.073	0.000	1.283	12.999	0.375	0.000	0.086

Problem 439	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	F	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	68	68	34	97	0	161	58	0	-1	0
N.S.	1	1.00	0.50	1.43	0.00	2.37	0.85	0.00	-0.01	0.00
time (sec)	N/A	0.041	0.008	0.095	0.000	1.394	24.919	0.000	0.000	0.118

Problem 440	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	A	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	176	176	34	73	0	171	0	0	-1	0
N.S.	1	1.00	0.19	0.41	0.00	0.97	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.142	0.008	0.079	0.000	1.425	0.000	0.000	0.000	0.103

Problem 441	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	A	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	252	252	34	73	0	259	0	0	-1	0
N.S.	1	1.00	0.13	0.29	0.00	1.03	0.00	0.00	-0.00	0.00
time (sec)	N/A	0.225	0.008	0.092	0.000	1.385	0.000	0.000	0.000	0.122

Problem 442	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	A	A	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	37	37	38	0	43	59	0	0	-1	0
N.S.	1	1.00	1.03	0.00	1.16	1.59	0.00	0.00	-0.03	0.00
time (sec)	N/A	0.040	0.017	0.098	1.106	1.340	0.000	0.000	0.000	0.161

Problem 443	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F	A	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	38	38	43	0	0	59	0	0	-1	0
N.S.	1	1.00	1.13	0.00	0.00	1.55	0.00	0.00	-0.03	0.00
time (sec)	N/A	0.039	0.019	0.070	0.000	1.150	0.000	0.000	0.000	0.152

Problem 444	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F(-1)	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	112	112	40	208	74	74	0	0	-1	96
N.S.	1	1.00	0.36	1.86	0.66	0.66	0.00	0.00	-0.01	0.86
time (sec)	N/A	0.042	0.056	0.047	0.922	1.303	0.000	0.000	0.000	0.066

Problem 445	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F(-1)	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	112	112	40	135	48	48	0	0	-1	70
N.S.	1	1.00	0.36	1.21	0.43	0.43	0.00	0.00	-0.01	0.62
time (sec)	N/A	0.040	0.037	0.026	0.922	1.329	0.000	0.000	0.000	0.057

Problem 446	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	99	99	44	64	22	22	0	0	-1	44
N.S.	1	1.00	0.44	0.65	0.22	0.22	0.00	0.00	-0.01	0.44
time (sec)	N/A	0.031	0.023	0.023	0.890	1.250	0.000	0.000	0.000	0.051

Problem 447	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	90	90	46	71	32	24	0	0	-1	55
N.S.	1	1.00	0.51	0.79	0.36	0.27	0.00	0.00	-0.01	0.61
time (sec)	N/A	0.044	0.032	0.030	0.894	1.313	0.000	0.000	0.000	0.075

Problem 448	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	48	48	35	37	41	41	0	0	-1	42
N.S.	1	1.00	0.73	0.77	0.85	0.85	0.00	0.00	-0.02	0.88
time (sec)	N/A	0.027	0.016	0.026	0.915	1.304	0.000	0.000	0.000	0.057

Problem 449	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F(-1)	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	88	88	40	37	69	69	0	0	-1	42
N.S.	1	1.00	0.45	0.42	0.78	0.78	0.00	0.00	-0.01	0.48
time (sec)	N/A	0.052	0.032	0.030	0.947	1.183	0.000	0.000	0.000	0.141

Problem 450	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F(-1)	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	88	88	40	37	97	97	0	0	-1	42
N.S.	1	1.00	0.45	0.42	1.10	1.10	0.00	0.00	-0.01	0.48
time (sec)	N/A	0.051	0.034	0.032	0.975	1.210	0.000	0.000	0.000	0.148

Problem 451	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	A	A	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	108	108	55	132	47	57	0	173	-1	0
N.S.	1	1.00	0.51	1.22	0.44	0.53	0.00	1.60	-0.01	0.00
time (sec)	N/A	0.042	0.033	0.043	0.940	1.448	0.000	0.350	0.000	0.193

Problem 452	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	93	93	46	61	25	28	0	53	-1	0
N.S.	1	1.00	0.49	0.66	0.27	0.30	0.00	0.57	-0.01	0.00
time (sec)	N/A	0.028	0.022	0.016	0.949	1.105	0.000	0.258	0.000	0.177

Problem 453	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	93	93	46	61	25	28	0	53	-1	0
N.S.	1	1.00	0.49	0.66	0.27	0.30	0.00	0.57	-0.01	0.00
time (sec)	N/A	0.025	0.021	0.015	0.883	1.154	0.000	0.295	0.000	0.106

Problem 454	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	88	88	39	56	19	20	0	25	-1	0
N.S.	1	1.00	0.44	0.64	0.22	0.23	0.00	0.28	-0.01	0.00
time (sec)	N/A	0.019	0.014	0.016	1.043	1.323	0.000	0.219	0.000	0.087

Problem 455	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	85	85	37	54	13	15	0	0	-1	41
N.S.	1	1.00	0.44	0.64	0.15	0.18	0.00	0.00	-0.01	0.48
time (sec)	N/A	0.026	0.015	0.017	0.797	1.360	0.000	0.000	0.000	0.041

Problem 456	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	94	94	42	61	22	23	0	0	-1	0
N.S.	1	1.00	0.45	0.65	0.23	0.24	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.033	0.025	0.017	0.665	1.285	0.000	0.000	0.000	0.207

Problem 457	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	96	96	47	61	22	23	0	0	-1	0
N.S.	1	1.00	0.49	0.64	0.23	0.24	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.029	0.026	0.020	0.911	1.333	0.000	0.000	0.000	0.243

Problem 458	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	A	A	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	238	238	90	532	276	390	0	2719	-1	0
N.S.	1	1.00	0.38	2.24	1.16	1.64	0.00	11.42	-0.00	0.00
time (sec)	N/A	0.098	0.107	0.059	0.993	1.013	0.000	0.955	0.000	0.219

Problem 459	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	212	212	123	146	108	144	0	292	-1	0
N.S.	1	1.00	0.58	0.69	0.51	0.68	0.00	1.38	-0.00	0.00
time (sec)	N/A	0.062	0.078	0.017	0.928	1.101	0.000	0.434	0.000	0.148

Problem 460	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	211	211	124	145	109	145	0	292	-1	0
N.S.	1	1.00	0.59	0.69	0.52	0.69	0.00	1.38	-0.00	0.00
time (sec)	N/A	0.058	0.070	0.017	0.937	1.323	0.000	0.373	0.000	0.124

Problem 461	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	206	206	122	138	101	130	0	263	-1	0
N.S.	1	1.00	0.59	0.67	0.49	0.63	0.00	1.28	-0.00	0.00
time (sec)	N/A	0.052	0.075	0.016	0.945	1.361	0.000	0.489	0.000	0.085

Problem 462	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	196	196	68	127	43	44	0	0	-1	69
N.S.	1	1.00	0.35	0.65	0.22	0.22	0.00	0.00	-0.01	0.35
time (sec)	N/A	0.052	0.037	0.019	0.902	0.803	0.000	0.000	0.000	0.044

Problem 463	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	212	212	124	147	101	131	0	0	-1	0
N.S.	1	1.00	0.58	0.69	0.48	0.62	0.00	0.00	-0.00	0.00
time (sec)	N/A	0.070	0.091	0.025	0.945	1.343	0.000	0.000	0.000	0.192

Problem 464	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	218	218	124	145	101	134	0	0	-1	0
N.S.	1	1.00	0.57	0.67	0.46	0.61	0.00	0.00	-0.00	0.00
time (sec)	N/A	0.070	0.087	0.026	1.172	1.330	0.000	0.000	0.000	0.159

Problem 465	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	85	85	42	66	27	22	0	0	-1	55
N.S.	1	1.00	0.49	0.78	0.32	0.26	0.00	0.00	-0.01	0.65
time (sec)	N/A	0.036	0.015	0.020	1.066	1.217	0.000	0.000	0.000	0.043

Problem 466	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	159	159	76	104	70	106	0	0	-1	78
N.S.	1	1.00	0.48	0.65	0.44	0.67	0.00	0.00	-0.01	0.49
time (sec)	N/A	0.083	0.063	0.021	1.162	1.486	0.000	0.000	0.000	0.047

Problem 467	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F	A	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	52	52	58	0	0	79	0	0	-1	0
N.S.	1	1.00	1.12	0.00	0.00	1.52	0.00	0.00	-0.02	0.00
time (sec)	N/A	0.020	0.023	0.279	0.000	1.090	0.000	0.000	0.000	0.142

Problem 468	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	43	43	32	51	0	45	0	0	-1	0
N.S.	1	1.00	0.74	1.19	0.00	1.05	0.00	0.00	-0.02	0.00
time (sec)	N/A	0.013	0.062	0.043	0.000	1.279	0.000	0.000	0.000	0.090

Problem 469	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F	A	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	130	130	80	0	0	103	0	0	-1	0
N.S.	1	1.00	0.62	0.00	0.00	0.79	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.055	0.062	0.282	0.000	0.928	0.000	0.000	0.000	0.144

Problem 470	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	F	A	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	102	102	59	0	0	82	0	0	-1	0
N.S.	1	1.00	0.58	0.00	0.00	0.80	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.044	0.045	0.111	0.000	1.131	0.000	0.000	0.000	0.074

Problem 471	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	F	A	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	117	124	75	0	0	165	0	0	-1	0
N.S.	1	1.06	0.64	0.00	0.00	1.41	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.064	0.035	0.052	0.000	1.076	0.000	0.000	0.000	0.222

Problem 472	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	A	A	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	103	103	54	148	59	78	0	0	-1	0
N.S.	1	1.00	0.52	1.44	0.57	0.76	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.065	0.029	0.073	1.233	1.078	0.000	0.000	0.000	0.232

Problem 473	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	A	F(-1)	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	111	111	93	973	0	353	0	0	-1	126
N.S.	1	1.00	0.84	8.77	0.00	3.18	0.00	0.00	-0.01	1.14
time (sec)	N/A	0.122	0.194	0.164	0.000	1.183	0.000	0.000	0.000	0.248

Problem 474	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	A	F(-1)	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	87	87	80	664	0	285	0	0	-1	104
N.S.	1	1.00	0.92	7.63	0.00	3.28	0.00	0.00	-0.01	1.20
time (sec)	N/A	0.074	0.127	0.127	0.000	1.077	0.000	0.000	0.000	0.162

Problem 475	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	A	F(-1)	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	68	68	62	402	0	231	0	0	-1	87
N.S.	1	1.00	0.91	5.91	0.00	3.40	0.00	0.00	-0.01	1.28
time (sec)	N/A	0.051	0.066	0.101	0.000	1.579	0.000	0.000	0.000	0.123

Problem 476	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	B	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	39	39	39	113	0	159	0	39	39	57
N.S.	1	1.00	1.00	2.90	0.00	4.08	0.00	1.00	1.00	1.46
time (sec)	N/A	0.033	0.059	0.063	0.000	1.223	0.000	0.414	1.467	0.080

Problem 477	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	A	F(-1)	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	98	98	135	658	0	333	0	0	-1	120
N.S.	1	1.00	1.38	6.71	0.00	3.40	0.00	0.00	-0.01	1.22
time (sec)	N/A	0.126	0.665	0.148	0.000	1.408	0.000	0.000	0.000	0.191

Problem 478	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	A	F(-1)	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	126	126	112	958	0	429	0	0	-1	149
N.S.	1	1.00	0.89	7.60	0.00	3.40	0.00	0.00	-0.01	1.18
time (sec)	N/A	0.171	0.333	0.174	0.000	1.409	0.000	0.000	0.000	0.332

Problem 479	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	A	F(-1)	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	164	164	143	1300	0	522	0	0	-1	182
N.S.	1	1.00	0.87	7.93	0.00	3.18	0.00	0.00	-0.01	1.11
time (sec)	N/A	0.231	0.443	0.201	0.000	1.359	0.000	0.000	0.000	0.319

Problem 480	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	353	353	340	280	0	4426	0	0	-1	0
N.S.	1	1.00	0.96	0.79	0.00	12.54	0.00	0.00	-0.00	0.00
time (sec)	N/A	0.627	0.898	0.747	0.000	2.560	0.000	0.000	0.000	0.056

Problem 481	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F	B	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	610	610	526	260	0	4699	0	0	-1	0
N.S.	1	1.00	0.86	0.43	0.00	7.70	0.00	0.00	-0.00	0.00
time (sec)	N/A	1.152	0.763	0.461	0.000	1.984	0.000	0.000	0.000	0.056

Problem 482	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F	B	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	169	169	145	114	0	801	0	1035	-1	0
N.S.	1	1.00	0.86	0.67	0.00	4.74	0.00	6.12	-0.01	0.00
time (sec)	N/A	0.192	0.258	0.194	0.000	1.363	0.000	1.846	0.000	0.058

Problem 483	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	205	205	127	268	0	1229	0	0	-1	0
N.S.	1	1.00	0.62	1.31	0.00	6.00	0.00	0.00	-0.00	0.00
time (sec)	N/A	0.395	0.179	0.348	0.000	1.249	0.000	0.000	0.000	0.059

Problem 484	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	B	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	699	699	127	534	0	6279	0	0	-1	0
N.S.	1	1.00	0.18	0.76	0.00	8.98	0.00	0.00	-0.00	0.00
time (sec)	N/A	1.493	0.128	0.750	0.000	7.485	0.000	0.000	0.000	0.098

Problem 485	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	B	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	414	414	127	630	0	5712	0	0	-1	0
N.S.	1	1.00	0.31	1.52	0.00	13.80	0.00	0.00	-0.00	0.00
time (sec)	N/A	0.787	0.125	1.215	0.000	4.182	0.000	0.000	0.000	0.071

Problem 486	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	A	F(-1)	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	74	74	74	397	0	259	0	0	224	98
N.S.	1	1.00	1.00	5.36	0.00	3.50	0.00	0.00	3.03	1.32
time (sec)	N/A	0.066	0.139	0.096	0.000	1.005	0.000	0.000	1.608	0.148

Problem 487	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	119	119	110	125	0	658	0	0	-1	125
N.S.	1	1.00	0.92	1.05	0.00	5.53	0.00	0.00	-0.01	1.05
time (sec)	N/A	0.095	0.148	0.115	0.000	1.606	0.000	0.000	0.000	0.349

Problem 488	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	173	173	158	209	0	827	0	0	-1	169
N.S.	1	1.00	0.91	1.21	0.00	4.78	0.00	0.00	-0.01	0.98
time (sec)	N/A	0.159	0.282	0.047	0.000	1.803	0.000	0.000	0.000	0.892

Problem 489	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F	A	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	47	47	47	0	0	148	0	0	-1	51
N.S.	1	1.00	1.00	0.00	0.00	3.15	0.00	0.00	-0.02	1.09
time (sec)	N/A	0.033	0.063	0.036	0.000	1.336	0.000	0.000	0.000	0.171

Problem 490	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F	B	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	98	98	94	0	0	449	0	0	-1	107
N.S.	1	1.00	0.96	0.00	0.00	4.58	0.00	0.00	-0.01	1.09
time (sec)	N/A	0.073	0.310	0.013	0.000	1.703	0.000	0.000	0.000	0.592

Problem 491	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	A	B	F(-1)	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	182	182	137	3798	273	2303	0	0	1734	0
N.S.	1	1.00	0.75	20.87	1.50	12.65	0.00	0.00	9.53	0.00
time (sec)	N/A	0.156	0.355	0.113	1.299	1.391	0.000	0.000	2.156	0.177

Problem 492	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	A	B	F(-1)	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	117	117	86	1065	152	706	0	5454	543	0
N.S.	1	1.00	0.74	9.10	1.30	6.03	0.00	46.62	4.64	0.00
time (sec)	N/A	0.069	0.151	0.071	1.265	1.255	0.000	0.803	1.617	0.139

Problem 493	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	A	B	A	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	58	58	41	205	65	142	1239	557	83	0
N.S.	1	1.00	0.71	3.53	1.12	2.45	21.36	9.60	1.43	0.00
time (sec)	N/A	0.024	0.065	0.046	1.124	1.278	42.999	0.403	1.406	0.077

Problem 494	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	B	B	B	B	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	46	46	150	298	142	175	178	169	141	0
N.S.	1	1.00	3.26	6.48	3.09	3.80	3.87	3.67	3.07	0.00
time (sec)	N/A	0.054	0.039	0.002	1.018	1.156	0.106	0.408	0.076	0.000

Problem 495	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	B	B	B	B	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	89	89	401	1314	403	571	559	493	383	0
N.S.	1	1.00	4.51	14.76	4.53	6.42	6.28	5.54	4.30	0.00
time (sec)	N/A	0.187	0.114	0.003	1.061	1.145	0.194	0.410	1.479	0.000

Problem 496	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	B	B	B	B	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	138	138	797	7550	872	1335	1314	1109	777	0
N.S.	1	1.00	5.78	54.71	6.32	9.67	9.52	8.04	5.63	0.00
time (sec)	N/A	0.373	0.275	0.003	1.129	1.244	0.356	0.617	1.657	0.000

Problem 497	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	B	B	B	B	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	55	55	154	349	166	229	240	213	164	0
N.S.	1	1.00	2.80	6.35	3.02	4.16	4.36	3.87	2.98	0.00
time (sec)	N/A	0.053	0.010	0.001	1.077	1.089	0.112	0.300	0.077	0.000

Problem 498	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	B	B	B	B	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	104	104	405	1413	439	715	722	615	419	0
N.S.	1	1.00	3.89	13.59	4.22	6.88	6.94	5.91	4.03	0.00
time (sec)	N/A	0.164	0.074	0.000	1.034	1.016	0.212	0.433	1.472	0.000

Problem 499	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	B	B	B	B	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	159	159	801	7697	920	1635	1654	1360	825	0
N.S.	1	1.00	5.04	48.41	5.79	10.28	10.40	8.55	5.19	0.00
time (sec)	N/A	0.315	0.042	0.003	1.164	0.727	0.392	0.514	1.650	0.000

Problem 500	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F(-1)	B	A	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	193	193	219	158	0	1231	178	1194	3988	0
N.S.	1	1.00	1.13	0.82	0.00	6.38	0.92	6.19	20.66	0.00
time (sec)	N/A	0.438	0.138	0.069	0.000	2.179	3.065	0.431	2.324	0.001

Problem 501	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F	B	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	81	81	77	151	0	434	280	130	278	0
N.S.	1	1.00	0.95	1.86	0.00	5.36	3.46	1.60	3.43	0.00
time (sec)	N/A	0.129	0.044	0.003	0.000	1.008	1.638	0.405	1.764	0.001

Problem 502	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F	B	A	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	164	164	175	140	0	703	104	1285	590	0
N.S.	1	1.00	1.07	0.85	0.00	4.29	0.63	7.84	3.60	0.00
time (sec)	N/A	0.155	0.091	0.006	0.000	1.247	1.350	0.455	1.737	0.001

Problem 503	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	43	43	46	129	0	272	168	53	61	0
N.S.	1	1.00	1.07	3.00	0.00	6.33	3.91	1.23	1.42	0.00
time (sec)	N/A	0.061	0.016	0.007	0.000	1.302	1.036	0.452	0.094	0.001

Problem 504	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F(-1)	A	B	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	94	94	128	184	0	468	320	274	2173	0
N.S.	1	1.00	1.36	1.96	0.00	4.98	3.40	2.91	23.12	0.00
time (sec)	N/A	0.132	0.078	0.011	0.000	1.299	6.044	1.147	2.504	0.001

Problem 505	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F(-1)	B	A	F(-2)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	195	195	206	168	0	1339	211	0	3844	0
N.S.	1	1.00	1.06	0.86	0.00	6.87	1.08	0.00	19.71	0.00
time (sec)	N/A	0.286	0.358	0.010	0.000	1.361	4.269	0.000	2.389	0.001

Problem 506	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F(-1)	B	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	121	121	154	213	0	810	464	102	4950	0
N.S.	1	1.00	1.27	1.76	0.00	6.69	3.83	0.84	40.91	0.00
time (sec)	N/A	0.198	0.134	0.013	0.000	1.096	146.464	0.402	5.855	0.001

Problem 507	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F(-1)	B	A	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	224	224	235	188	0	2044	347	1243	5214	0
N.S.	1	1.00	1.05	0.84	0.00	9.12	1.55	5.55	23.28	0.00
time (sec)	N/A	0.497	0.209	0.010	0.000	1.224	12.774	0.495	2.833	0.001

Problem 508	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F	B	B	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	270	270	263	323	0	2454	573	1304	7327	0
N.S.	1	1.00	0.97	1.20	0.00	9.09	2.12	4.83	27.14	0.00
time (sec)	N/A	0.579	0.468	0.020	0.000	1.110	8.966	0.524	4.755	0.002

Problem 509	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F	B	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	97	97	100	276	0	1021	495	171	427	0
N.S.	1	1.00	1.03	2.85	0.00	10.53	5.10	1.76	4.40	0.00
time (sec)	N/A	0.135	0.131	0.021	0.000	1.026	4.875	0.565	1.770	0.001

Problem 510	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F	B	B	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	254	254	247	319	0	2474	578	1312	7200	0
N.S.	1	1.00	0.97	1.26	0.00	9.74	2.28	5.17	28.35	0.00
time (sec)	N/A	0.388	0.975	0.022	0.000	1.324	18.532	0.529	3.953	0.001

Problem 511	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F	B	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	98	96	98	270	0	1042	495	172	417	0
N.S.	1	0.98	1.00	2.76	0.00	10.63	5.05	1.76	4.26	0.00
time (sec)	N/A	0.122	0.122	0.023	0.000	0.872	4.675	0.462	1.720	0.001

Problem 512	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F	B	F(-1)	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	299	299	271	364	0	3228	0	1357	9056	0
N.S.	1	1.00	0.91	1.22	0.00	10.80	0.00	4.54	30.29	0.00
time (sec)	N/A	0.702	0.889	0.018	0.000	1.726	0.000	0.463	4.846	0.001

Problem 513	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F(-1)	B	F(-1)	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	162	162	235	693	0	2476	0	454	11072	0
N.S.	1	1.00	1.45	4.28	0.00	15.28	0.00	2.80	68.35	0.00
time (sec)	N/A	0.294	0.473	0.033	0.000	1.789	0.000	1.253	11.354	0.001

Problem 514	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F(-1)	B	F(-1)	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	348	348	339	1304	0	4330	0	847	10556	0
N.S.	1	1.00	0.97	3.75	0.00	12.44	0.00	2.43	30.33	0.00
time (sec)	N/A	1.653	1.632	0.032	0.000	1.861	0.000	0.787	6.384	0.001

Problem 515	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F(-1)	B	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	213	213	284	1014	0	4562	0	224	12436	0
N.S.	1	1.00	1.33	4.76	0.00	21.42	0.00	1.05	58.38	0.00
time (sec)	N/A	0.390	0.520	0.040	0.000	2.509	0.000	0.430	12.317	0.002

Problem 516	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F(-1)	B	F(-1)	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	408	408	384	1518	0	5734	0	1987	12239	0
N.S.	1	1.00	0.94	3.72	0.00	14.05	0.00	4.87	30.00	0.00
time (sec)	N/A	3.676	2.983	0.036	0.000	2.383	0.000	0.548	8.725	0.002

Problem 517	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F(-1)	B	F(-1)	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	341	341	328	704	0	6633	0	1688	12677	0
N.S.	1	1.00	0.96	2.06	0.00	19.45	0.00	4.95	37.18	0.00
time (sec)	N/A	0.951	4.434	0.050	0.000	2.043	0.000	0.746	7.019	0.002

Problem 518	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F(-1)	B	B	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	150	150	146	544	0	3739	1671	365	1182	0
N.S.	1	1.00	0.97	3.63	0.00	24.93	11.14	2.43	7.88	0.00
time (sec)	N/A	0.196	0.206	0.046	0.000	1.531	14.453	0.693	3.855	0.002

Problem 519	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F(-1)	B	F(-1)	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	363	363	382	885	0	7701	0	2295	14584	0
N.S.	1	1.00	1.05	2.44	0.00	21.21	0.00	6.32	40.18	0.00
time (sec)	N/A	1.039	4.861	0.051	0.000	2.258	0.000	0.866	7.429	0.002

Problem 520	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F(-1)	B	B	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	152	150	147	541	0	3708	1646	365	1157	0
N.S.	1	0.99	0.97	3.56	0.00	24.39	10.83	2.40	7.61	0.00
time (sec)	N/A	0.185	0.184	0.050	0.000	1.470	13.928	0.633	3.802	0.002

Problem 521	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F	B	F(-1)	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	437	437	463	1010	0	8554	0	2487	16086	0
N.S.	1	1.00	1.06	2.31	0.00	19.57	0.00	5.69	36.81	0.00
time (sec)	N/A	5.361	6.169	0.052	0.000	2.546	0.000	0.563	7.800	0.001

Problem 522	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F(-1)	B	F(-1)	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	255	255	391	4477	0	9908	0	1012	19440	0
N.S.	1	1.00	1.53	17.56	0.00	38.85	0.00	3.97	76.24	0.00
time (sec)	N/A	0.486	3.949	0.080	0.000	4.114	0.000	1.716	17.982	0.001

Problem 523	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F(-1)	B	F(-1)	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	484	484	560	6821	0	10260	0	1412	18112	0
N.S.	1	1.00	1.16	14.09	0.00	21.20	0.00	2.92	37.42	0.00
time (sec)	N/A	1.227	6.237	0.071	0.000	3.917	0.000	1.220	14.376	0.001

Problem 524	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F(-1)	B	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	325	325	491	5575	0	15165	0	377	21465	0
N.S.	1	1.00	1.51	17.15	0.00	46.66	0.00	1.16	66.05	0.00
time (sec)	N/A	0.585	6.179	0.086	0.000	7.534	0.000	0.639	22.450	0.001

Problem 525	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F(-1)	B	A	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	202	202	222	164	0	1346	219	1245	4605	0
N.S.	1	1.00	1.10	0.81	0.00	6.66	1.08	6.16	22.80	0.00
time (sec)	N/A	0.362	0.049	0.005	0.000	1.322	3.623	0.536	1.341	0.001

Problem 526	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F	A	B	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	87	87	80	154	0	446	332	162	287	0
N.S.	1	1.00	0.92	1.77	0.00	5.13	3.82	1.86	3.30	0.00
time (sec)	N/A	0.127	0.044	0.005	0.000	1.187	1.950	0.429	0.443	0.001

Problem 527	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F	B	A	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	170	170	178	143	0	799	124	1325	683	0
N.S.	1	1.00	1.05	0.84	0.00	4.70	0.73	7.79	4.02	0.00
time (sec)	N/A	0.153	0.101	0.002	0.000	1.103	1.595	0.466	1.792	0.001

Problem 528	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	44	44	47	130	0	274	189	62	477	0
N.S.	1	1.00	1.07	2.95	0.00	6.23	4.30	1.41	10.84	0.00
time (sec)	N/A	0.063	0.016	0.005	0.000	1.220	1.187	0.397	1.622	0.001

Problem 529	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F(-1)	A	B	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	103	103	131	190	0	474	348	285	2520	0
N.S.	1	1.00	1.27	1.84	0.00	4.60	3.38	2.77	24.47	0.00
time (sec)	N/A	0.138	0.069	0.007	0.000	1.184	7.157	1.193	3.462	0.002

Problem 530	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F(-1)	B	A	F(-2)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	204	204	209	174	0	1477	258	0	4339	0
N.S.	1	1.00	1.02	0.85	0.00	7.24	1.26	0.00	21.27	0.00
time (sec)	N/A	0.273	0.342	0.007	0.000	1.448	4.899	0.000	3.872	0.002

Problem 531	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F(-1)	B	B	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	133	133	157	222	0	828	532	348	5947	0
N.S.	1	1.00	1.18	1.67	0.00	6.23	4.00	2.62	44.71	0.00
time (sec)	N/A	0.195	0.136	0.008	0.000	1.935	156.628	1.106	6.981	0.002

Problem 532	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F(-1)	B	A	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	236	236	238	197	0	2212	411	1249	5771	0
N.S.	1	1.00	1.01	0.83	0.00	9.37	1.74	5.29	24.45	0.00
time (sec)	N/A	0.488	0.103	0.010	0.000	1.316	14.012	0.570	3.173	0.002

Problem 533	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F	B	B	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	279	279	266	695	0	2578	641	1370	8025	0
N.S.	1	1.00	0.95	2.49	0.00	9.24	2.30	4.91	28.76	0.00
time (sec)	N/A	0.539	0.461	0.018	0.000	1.256	9.821	0.603	5.092	0.002

Problem 534	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F	B	B	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	103	103	103	500	0	1077	556	211	460	0
N.S.	1	1.00	1.00	4.85	0.00	10.46	5.40	2.05	4.47	0.00
time (sec)	N/A	0.138	0.126	0.020	0.000	1.209	5.286	0.515	1.903	0.002

Problem 535	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F	B	B	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	263	263	250	693	0	2600	646	1378	7835	0
N.S.	1	1.00	0.95	2.63	0.00	9.89	2.46	5.24	29.79	0.00
time (sec)	N/A	0.368	0.939	0.017	0.000	1.316	19.757	0.675	4.398	0.002

Problem 536	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F	B	B	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	98	98	99	484	0	1066	525	211	442	0
N.S.	1	1.00	1.01	4.94	0.00	10.88	5.36	2.15	4.51	0.00
time (sec)	N/A	0.127	0.123	0.024	0.000	1.115	5.068	0.445	1.905	0.002

Problem 537	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F(-1)	B	F(-1)	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	174	174	238	714	0	2486	0	476	13434	0
N.S.	1	1.00	1.37	4.10	0.00	14.29	0.00	2.74	77.21	0.00
time (sec)	N/A	0.295	0.436	0.030	0.000	2.338	0.000	1.432	11.689	0.002

Problem 538	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F(-1)	B	F(-1)	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	360	360	342	1346	0	4520	0	999	12008	0
N.S.	1	1.00	0.95	3.74	0.00	12.56	0.00	2.78	33.36	0.00
time (sec)	N/A	1.599	1.471	0.028	0.000	1.763	0.000	1.021	7.289	0.001

Problem 539	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F(-1)	B	F(-1)	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	228	228	287	1047	0	4604	0	687	14830	0
N.S.	1	1.00	1.26	4.59	0.00	20.19	0.00	3.01	65.04	0.00
time (sec)	N/A	0.369	0.515	0.034	0.000	5.904	0.000	1.296	13.520	0.002

Problem 540	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F(-1)	B	F(-1)	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	423	423	387	1569	0	5954	0	2002	13781	0
N.S.	1	1.00	0.91	3.71	0.00	14.08	0.00	4.73	32.58	0.00
time (sec)	N/A	3.550	3.025	0.030	0.000	2.131	0.000	0.714	10.447	0.002

Problem 541	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F(-1)	B	F(-1)	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	353	353	331	3432	0	6770	0	1844	13840	0
N.S.	1	1.00	0.94	9.72	0.00	19.18	0.00	5.22	39.21	0.00
time (sec)	N/A	0.870	4.445	0.049	0.000	1.412	0.000	0.799	7.520	0.002

Problem 542	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F(-1)	B	B	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	159	159	149	2181	0	3843	1794	447	1267	0
N.S.	1	1.00	0.94	13.72	0.00	24.17	11.28	2.81	7.97	0.00
time (sec)	N/A	0.200	0.209	0.046	0.000	1.443	14.583	0.781	4.019	0.002

Problem 543	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F(-1)	B	F(-1)	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	375	375	385	4751	0	7838	0	2527	16025	0
N.S.	1	1.00	1.03	12.67	0.00	20.90	0.00	6.74	42.73	0.00
time (sec)	N/A	0.972	4.483	0.049	0.000	2.199	0.000	0.794	7.944	0.002

Problem 544	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F(-1)	B	B	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	153	153	148	2132	0	3748	1707	445	1199	0
N.S.	1	1.00	0.97	13.93	0.00	24.50	11.16	2.91	7.84	0.00
time (sec)	N/A	0.192	0.179	0.053	0.000	1.591	13.965	0.690	3.994	0.002

Problem 545	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F(-1)	B	F(-1)	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	270	270	394	4606	0	9926	0	1044	22621	0
N.S.	1	1.00	1.46	17.06	0.00	36.76	0.00	3.87	83.78	0.00
time (sec)	N/A	0.496	3.925	0.075	0.000	7.394	0.000	1.700	18.492	0.001

Problem 546	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F(-1)	B	F(-1)	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	499	499	575	7019	0	10518	0	1658	20580	0
N.S.	1	1.00	1.15	14.07	0.00	21.08	0.00	3.32	41.24	0.00
time (sec)	N/A	1.094	6.210	0.070	0.000	3.605	0.000	1.419	15.398	0.002

Problem 547	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F(-1)	B	F(-1)	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	343	343	509	5737	0	15231	0	1735	25334	0
N.S.	1	1.00	1.48	16.73	0.00	44.41	0.00	5.06	73.86	0.00
time (sec)	N/A	0.589	6.157	0.082	0.000	22.607	0.000	1.628	24.912	0.002

Problem 548	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	B	B	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	34	34	34	105	104	104	107	28	29	0
N.S.	1	1.00	1.00	3.09	3.06	3.06	3.15	0.82	0.85	0.00
time (sec)	N/A	0.036	0.011	0.003	0.543	1.171	0.101	0.349	1.584	0.000

Problem 549	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	B	B	B	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	56	56	188	175	174	174	187	46	46	0
N.S.	1	1.00	3.36	3.12	3.11	3.11	3.34	0.82	0.82	0.00
time (sec)	N/A	0.096	0.010	0.003	0.774	0.996	0.150	0.415	1.602	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [350] had the largest ratio of [.8000]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	2	2	1.00	15	0.133
2	A	1	1	1.00	15	0.067
3	A	1	1	1.00	15	0.067
4	A	3	3	1.00	15	0.200
5	A	8	8	1.00	11	0.727
6	A	3	2	1.00	26	0.077
7	A	3	2	1.00	26	0.077
8	A	3	2	1.00	26	0.077
9	A	2	2	1.00	26	0.077
10	A	3	2	1.00	24	0.083
11	A	2	1	1.00	22	0.045
12	A	3	2	1.00	26	0.077
13	A	3	2	1.00	26	0.077
14	A	3	2	1.00	26	0.077
15	A	3	2	1.00	26	0.077
16	A	3	2	1.00	26	0.077
17	A	3	2	1.00	26	0.077
18	A	3	2	1.00	26	0.077
19	A	3	2	1.00	26	0.077
20	A	3	2	1.00	26	0.077
21	A	3	2	1.00	26	0.077
22	A	3	2	1.00	26	0.077
23	A	3	2	1.00	26	0.077
24	A	4	3	1.40	26	0.115
25	A	3	2	1.00	26	0.077
26	A	3	2	1.00	26	0.077
27	A	4	3	1.00	26	0.115
28	A	3	2	1.00	26	0.077
29	A	3	2	1.00	26	0.077
30	A	2	2	1.00	26	0.077
31	A	3	2	1.00	24	0.083
32	A	3	2	1.00	22	0.091
33	A	4	3	1.00	26	0.115
34	A	3	2	1.00	26	0.077
35	A	3	2	1.00	26	0.077
36	A	4	3	1.00	26	0.115
37	A	3	2	1.00	26	0.077
38	A	3	2	1.00	26	0.077
39	A	4	3	1.00	26	0.115

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
40	A	3	2	1.00	26	0.077
41	A	3	2	1.00	26	0.077
42	A	4	3	1.00	26	0.115
43	A	3	2	1.00	26	0.077
44	A	3	2	1.00	26	0.077
45	A	2	2	1.00	26	0.077
46	A	3	2	1.00	26	0.077
47	A	3	2	1.00	26	0.077
48	A	4	4	1.00	26	0.154
49	A	3	2	1.00	26	0.077
50	A	3	2	1.00	26	0.077
51	A	3	2	1.00	26	0.077
52	A	4	3	1.00	26	0.115
53	A	3	2	1.00	26	0.077
54	A	3	2	1.00	26	0.077
55	A	4	3	1.00	26	0.115
56	A	3	2	1.00	26	0.077
57	A	3	2	1.00	26	0.077
58	A	4	3	1.00	26	0.115
59	A	3	2	1.00	26	0.077
60	A	3	2	1.00	26	0.077
61	A	2	2	1.00	26	0.077
62	A	3	2	1.00	24	0.083
63	A	3	2	1.00	22	0.091
64	A	4	3	1.00	26	0.115
65	A	3	2	1.00	26	0.077
66	A	3	2	1.00	26	0.077
67	A	4	3	1.00	26	0.115
68	A	3	2	1.00	26	0.077
69	A	3	2	1.00	26	0.077
70	A	4	3	1.00	26	0.115
71	A	3	2	1.00	26	0.077
72	A	3	2	1.00	26	0.077
73	A	4	3	1.00	26	0.115
74	A	3	2	1.00	26	0.077
75	A	3	2	1.00	26	0.077
76	A	4	3	1.00	26	0.115
77	A	3	2	1.00	26	0.077
78	A	3	2	1.00	26	0.077
79	A	4	3	1.00	26	0.115
80	A	3	2	1.00	26	0.077
81	A	3	2	1.00	26	0.077
82	A	2	2	1.00	26	0.077
83	A	3	2	1.00	26	0.077
84	A	3	2	1.00	26	0.077
85	A	4	4	1.00	26	0.154
86	A	3	2	1.00	26	0.077
87	A	3	2	1.00	26	0.077

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
88	A	5	4	1.00	26	0.154
89	A	8	8	1.00	26	0.308
90	A	8	8	1.00	26	0.308
91	A	3	3	1.00	26	0.115
92	A	7	7	1.00	24	0.292
93	A	7	7	1.00	22	0.318
94	A	5	5	1.00	26	0.192
95	A	8	8	1.00	26	0.308
96	A	8	8	0.99	26	0.308
97	A	4	3	0.98	26	0.115
98	A	9	9	1.00	26	0.346
99	A	9	9	1.00	26	0.346
100	A	2	2	1.00	26	0.077
101	A	9	8	1.00	24	0.333
102	A	9	8	1.00	22	0.364
103	A	4	3	1.00	26	0.115
104	A	10	9	1.00	26	0.346
105	A	10	9	1.00	26	0.346
106	A	4	3	1.00	26	0.115
107	A	11	9	1.00	26	0.346
108	A	4	3	1.00	26	0.115
109	A	11	9	1.00	26	0.346
110	A	11	9	1.00	26	0.346
111	A	2	2	1.00	26	0.077
112	A	11	8	1.00	24	0.333
113	A	11	8	1.00	22	0.364
114	A	4	3	1.00	26	0.115
115	A	12	9	1.00	26	0.346
116	A	12	9	1.00	26	0.346
117	A	4	3	1.00	26	0.115
118	A	3	2	1.00	28	0.071
119	A	3	2	1.00	28	0.071
120	A	3	2	1.00	28	0.071
121	A	4	3	1.00	24	0.125
122	A	4	3	1.00	24	0.125
123	A	4	3	1.00	24	0.125
124	A	2	2	1.00	24	0.083
125	A	6	6	1.00	18	0.333
126	A	5	5	1.00	18	0.278
127	A	3	3	1.00	18	0.167
128	A	7	7	1.00	18	0.389
129	A	8	7	1.00	18	0.389
130	A	14	8	1.00	18	0.444
131	A	14	8	1.00	18	0.444
132	A	13	7	1.00	18	0.389
133	A	13	7	1.00	18	0.389
134	A	13	7	1.00	16	0.438
135	A	13	7	1.00	14	0.500

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
136	A	14	8	1.00	18	0.444
137	A	14	8	1.00	18	0.444
138	A	6	4	1.00	16	0.250
139	A	5	4	1.00	16	0.250
140	A	4	3	1.00	16	0.188
141	B	4	3	2.10	16	0.188
142	A	6	5	1.00	16	0.312
143	A	4	3	1.00	16	0.188
144	A	4	3	1.00	16	0.188
145	A	15	10	1.00	16	0.625
146	A	15	10	1.00	16	0.625
147	A	14	9	1.00	16	0.562
148	A	14	9	1.00	16	0.562
149	A	13	8	1.00	16	0.500
150	A	13	8	1.00	16	0.500
151	A	13	8	1.00	14	0.571
152	A	13	8	1.00	12	0.667
153	A	14	9	1.00	16	0.562
154	A	14	9	1.00	16	0.562
155	A	15	10	1.00	16	0.625
156	A	15	10	1.00	16	0.625
157	A	14	8	1.00	16	0.500
158	A	5	5	1.00	16	0.312
159	A	13	7	1.00	16	0.438
160	A	13	7	1.00	16	0.438
161	A	3	3	1.00	16	0.188
162	A	13	7	1.00	14	0.500
163	C	13	7	2.02	12	0.583
164	A	7	7	1.00	16	0.438
165	A	14	8	1.00	16	0.500
166	A	14	8	1.00	16	0.500
167	A	8	7	1.00	16	0.438
168	A	16	10	1.00	16	0.625
169	A	13	7	1.00	10	0.700
170	A	3	3	1.00	14	0.214
171	A	13	7	1.00	14	0.500
172	A	7	7	1.00	20	0.350
173	A	6	6	1.00	20	0.300
174	A	6	6	1.00	20	0.300
175	A	5	5	1.00	20	0.250
176	A	4	4	1.00	20	0.200
177	A	7	6	1.00	20	0.300
178	A	7	6	1.00	20	0.300
179	A	4	4	1.00	20	0.200
180	A	5	5	1.00	20	0.250
181	A	6	6	1.00	20	0.300
182	A	7	7	1.00	20	0.350
183	A	8	7	1.00	20	0.350

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
184	A	7	6	1.00	20	0.300
185	A	7	6	1.00	20	0.300
186	A	6	5	1.00	20	0.250
187	A	5	4	1.00	20	0.200
188	A	8	7	1.00	20	0.350
189	A	8	7	1.00	20	0.350
190	A	8	7	1.00	20	0.350
191	A	8	7	1.00	20	0.350
192	A	5	4	1.00	20	0.200
193	A	6	5	1.00	20	0.250
194	A	7	6	1.00	20	0.300
195	A	8	7	1.00	20	0.350
196	A	6	6	1.00	20	0.300
197	A	5	5	1.00	20	0.250
198	A	5	5	1.00	20	0.250
199	A	4	4	1.00	20	0.200
200	A	3	3	1.00	20	0.150
201	A	3	3	1.00	20	0.150
202	A	4	4	1.00	20	0.200
203	A	5	5	1.00	20	0.250
204	A	6	6	1.00	20	0.300
205	A	7	6	1.00	20	0.300
206	A	6	6	1.00	20	0.300
207	A	5	5	1.00	20	0.250
208	A	5	5	1.00	20	0.250
209	A	2	2	1.00	20	0.100
210	A	2	2	1.00	20	0.100
211	A	5	5	1.00	20	0.250
212	A	5	5	1.00	20	0.250
213	A	6	6	1.00	20	0.300
214	A	7	6	1.00	20	0.300
215	A	2	1	1.00	20	0.050
216	A	2	1	1.00	18	0.056
217	A	5	5	1.00	16	0.312
218	A	4	3	1.00	16	0.188
219	A	4	4	1.00	16	0.250
220	A	2	2	1.00	16	0.125
221	A	4	4	1.00	14	0.286
222	A	4	3	1.00	16	0.188
223	A	5	5	1.00	16	0.312
224	A	4	3	1.00	16	0.188
225	A	6	5	1.00	16	0.312
226	A	12	9	1.00	16	0.562
227	A	11	8	1.00	16	0.500
228	A	11	8	1.00	16	0.500
229	A	11	8	1.00	16	0.500
230	A	11	8	1.00	12	0.667
231	A	12	9	1.00	16	0.562

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
232	A	12	9	1.00	16	0.562
233	A	13	9	1.00	16	0.562
234	A	13	9	1.00	16	0.562
235	A	5	5	1.00	16	0.312
236	A	4	3	1.00	16	0.188
237	A	4	4	1.00	16	0.250
238	A	2	2	1.00	16	0.125
239	A	4	4	1.00	14	0.286
240	A	4	3	1.00	16	0.188
241	A	5	5	1.00	16	0.312
242	A	4	3	1.00	16	0.188
243	A	6	5	1.00	16	0.312
244	A	6	6	1.00	16	0.375
245	A	5	5	1.00	16	0.312
246	A	5	5	1.00	16	0.312
247	A	5	5	1.00	16	0.312
248	A	5	5	1.00	12	0.417
249	A	6	6	1.00	16	0.375
250	A	6	6	1.00	16	0.375
251	A	7	6	1.00	16	0.375
252	A	7	6	1.00	16	0.375
253	A	6	6	1.00	18	0.333
254	A	5	4	1.00	18	0.222
255	A	5	5	1.00	18	0.278
256	A	4	3	1.00	18	0.167
257	A	3	3	1.00	18	0.167
258	A	4	3	1.00	16	0.188
259	A	7	7	1.00	18	0.389
260	A	5	4	1.00	18	0.222
261	A	8	7	1.00	18	0.389
262	A	8	5	1.00	18	0.278
263	A	8	5	1.00	18	0.278
264	A	7	4	1.00	18	0.222
265	A	7	4	1.00	18	0.222
266	A	7	4	1.00	18	0.222
267	A	7	4	1.00	14	0.286
268	A	8	5	1.00	18	0.278
269	A	8	5	1.00	18	0.278
270	A	6	6	1.00	14	0.429
271	A	7	5	1.00	14	0.357
272	A	5	5	1.00	14	0.357
273	A	10	7	1.00	14	0.500
274	A	3	3	1.00	14	0.214
275	A	10	6	1.00	12	0.500
276	A	7	7	1.00	14	0.500
277	A	7	5	1.00	14	0.357
278	A	8	7	1.00	14	0.500
279	A	13	10	1.00	14	0.714

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
280	A	20	7	1.00	14	0.500
281	A	9	6	1.00	14	0.429
282	A	19	7	1.00	14	0.500
283	A	19	6	1.00	14	0.429
284	A	9	6	1.00	10	0.600
285	A	20	8	1.00	14	0.571
286	A	20	7	1.00	14	0.500
287	A	12	9	1.00	14	0.643
288	A	22	10	1.00	14	0.714
289	A	6	6	1.00	16	0.375
290	A	5	4	1.00	16	0.250
291	A	5	5	1.00	16	0.312
292	A	10	7	1.00	16	0.438
293	A	3	3	1.00	16	0.188
294	A	10	6	1.00	14	0.429
295	A	7	7	1.00	16	0.438
296	A	5	4	1.00	16	0.250
297	A	8	7	1.00	16	0.438
298	A	13	10	1.00	16	0.625
299	A	20	7	1.00	16	0.438
300	A	19	6	1.00	16	0.375
301	A	19	7	1.00	16	0.438
302	A	19	6	1.00	16	0.375
303	A	19	6	1.00	12	0.500
304	A	22	8	1.00	16	0.500
305	A	20	7	1.00	16	0.438
306	A	22	9	1.00	16	0.562
307	A	22	10	1.00	16	0.625
308	A	5	4	1.00	16	0.250
309	A	5	4	1.00	16	0.250
310	A	4	3	1.00	16	0.188
311	A	4	3	1.00	16	0.188
312	A	3	3	1.00	16	0.188
313	A	4	3	1.00	14	0.214
314	A	6	5	1.00	16	0.312
315	A	5	4	1.00	16	0.250
316	A	7	5	1.00	16	0.312
317	A	6	5	1.00	16	0.312
318	A	20	8	0.96	16	0.500
319	A	19	7	1.00	16	0.438
320	A	19	7	1.00	16	0.438
321	A	19	7	1.01	16	0.438
322	A	19	7	1.00	12	0.583
323	A	20	8	1.00	16	0.500
324	A	20	8	1.00	16	0.500
325	A	5	4	1.00	16	0.250
326	A	5	4	1.00	16	0.250
327	A	4	3	1.00	16	0.188

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
328	A	4	3	1.00	16	0.188
329	A	3	3	1.00	16	0.188
330	A	4	3	1.00	14	0.214
331	A	6	5	1.00	16	0.312
332	A	5	4	1.00	16	0.250
333	A	7	5	1.00	16	0.312
334	A	6	5	1.00	16	0.312
335	A	8	5	1.00	16	0.312
336	A	7	4	1.00	16	0.250
337	A	7	4	1.00	16	0.250
338	A	7	4	1.14	16	0.250
339	A	7	4	1.00	12	0.333
340	A	8	5	1.00	16	0.312
341	A	8	5	1.00	16	0.312
342	A	9	6	1.00	16	0.375
343	A	9	6	1.00	16	0.375
344	A	4	3	1.00	16	0.188
345	A	5	4	1.00	16	0.250
346	A	5	5	1.00	14	0.357
347	A	3	3	1.00	14	0.214
348	A	7	7	1.00	14	0.500
349	A	8	7	1.00	14	0.500
350	A	8	8	1.00	10	0.800
351	A	7	6	1.00	18	0.333
352	A	7	6	1.00	18	0.333
353	A	7	6	1.00	16	0.375
354	A	6	6	1.00	14	0.429
355	A	5	5	1.00	18	0.278
356	A	3	3	1.00	18	0.167
357	A	7	7	1.00	18	0.389
358	A	8	7	1.00	18	0.389
359	A	8	7	1.00	18	0.389
360	A	8	7	1.00	18	0.389
361	A	8	7	1.00	16	0.438
362	A	8	7	1.00	14	0.500
363	A	7	7	1.00	18	0.389
364	A	4	4	1.00	18	0.222
365	A	4	4	1.00	18	0.222
366	A	4	4	1.00	18	0.222
367	A	8	7	1.00	18	0.389
368	A	8	7	1.00	18	0.389
369	A	8	7	1.00	18	0.389
370	A	9	8	1.00	14	0.571
371	A	8	8	1.00	18	0.444
372	A	5	4	1.00	18	0.222
373	A	5	5	1.00	18	0.278
374	A	5	5	1.00	18	0.278
375	A	5	5	1.00	18	0.278

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
376	A	5	4	0.98	18	0.222
377	A	9	8	1.00	18	0.444
378	A	9	8	1.00	18	0.444
379	A	6	4	1.00	18	0.222
380	A	6	4	1.00	16	0.250
381	A	5	4	1.00	14	0.286
382	A	4	3	1.00	18	0.167
383	A	4	3	1.00	18	0.167
384	A	6	5	1.00	18	0.278
385	A	4	3	1.00	18	0.167
386	A	4	3	1.00	18	0.167
387	A	4	3	1.00	18	0.167
388	A	9	7	1.00	16	0.438
389	A	8	7	1.00	16	0.438
390	A	7	6	1.00	16	0.375
391	A	4	4	1.00	16	0.250
392	A	5	5	1.00	16	0.312
393	A	6	6	1.00	16	0.375
394	A	4	3	1.00	22	0.136
395	A	5	4	1.00	14	0.286
396	A	15	9	1.00	14	0.643
397	A	9	6	1.00	14	0.429
398	A	7	6	1.00	20	0.300
399	A	4	3	1.00	23	0.130
400	A	4	4	1.00	22	0.182
401	A	4	3	1.00	26	0.115
402	A	4	3	1.00	26	0.115
403	A	3	2	1.00	26	0.077
404	A	4	3	1.00	26	0.115
405	A	4	3	1.00	26	0.115
406	A	4	3	1.00	26	0.115
407	A	4	3	1.00	26	0.115
408	A	4	3	1.00	26	0.115
409	A	4	3	1.00	26	0.115
410	A	4	3	1.00	26	0.115
411	A	4	3	1.00	28	0.107
412	A	4	3	1.00	26	0.115
413	A	4	3	1.00	24	0.125
414	C	7	3	1.11	77	0.039
415	A	4	3	1.00	26	0.115
416	A	4	3	1.00	26	0.115
417	A	5	4	1.00	24	0.167
418	A	5	4	1.00	26	0.154
419	A	5	4	1.00	26	0.154
420	A	5	4	1.00	26	0.154
421	A	4	3	1.00	26	0.115
422	A	5	4	1.00	26	0.154
423	A	5	4	1.00	26	0.154

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
424	A	5	4	1.00	26	0.154
425	A	5	4	1.00	26	0.154
426	A	5	4	1.00	26	0.154
427	A	4	3	1.00	26	0.115
428	A	5	4	1.00	26	0.154
429	A	4	3	1.00	23	0.130
430	A	4	3	1.00	23	0.130
431	A	2	2	1.00	23	0.087
432	A	5	5	1.00	21	0.238
433	A	4	3	1.00	23	0.130
434	A	4	3	1.00	23	0.130
435	A	4	3	1.00	23	0.130
436	A	12	9	1.00	25	0.360
437	A	9	9	1.00	25	0.360
438	A	5	5	1.00	25	0.200
439	A	6	6	1.00	25	0.240
440	A	11	11	1.00	25	0.440
441	A	14	11	1.00	25	0.440
442	A	1	1	1.00	26	0.038
443	A	1	1	1.00	28	0.036
444	A	4	3	1.00	32	0.094
445	A	4	3	1.00	32	0.094
446	A	3	2	1.00	32	0.062
447	A	4	3	1.00	32	0.094
448	A	2	2	1.00	32	0.062
449	A	4	3	1.00	32	0.094
450	A	4	3	1.00	32	0.094
451	A	5	4	1.00	30	0.133
452	A	3	2	1.00	28	0.071
453	A	3	2	1.00	26	0.077
454	A	2	1	1.00	24	0.042
455	A	3	2	1.00	28	0.071
456	A	3	2	1.00	28	0.071
457	A	3	2	1.00	28	0.071
458	A	9	4	1.00	30	0.133
459	A	3	2	1.00	28	0.071
460	A	3	2	1.00	26	0.077
461	A	3	2	1.00	24	0.083
462	A	4	3	1.00	28	0.107
463	A	3	2	1.00	28	0.071
464	A	3	2	1.00	28	0.071
465	A	5	5	1.00	28	0.179
466	A	4	3	1.00	28	0.107
467	A	2	2	1.00	36	0.056
468	A	2	2	1.00	31	0.065
469	A	3	3	1.00	34	0.088
470	A	3	3	1.00	33	0.091
471	A	3	3	1.06	35	0.086

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
472	A	4	3	1.00	30	0.100
473	A	7	6	1.00	24	0.250
474	A	6	6	1.00	24	0.250
475	A	5	5	1.00	24	0.208
476	A	3	3	1.00	22	0.136
477	A	8	7	1.00	24	0.292
478	A	8	7	1.00	24	0.292
479	A	8	7	1.00	24	0.292
480	A	8	5	1.00	26	0.192
481	A	14	8	1.00	26	0.308
482	A	4	3	1.00	26	0.115
483	A	6	5	1.00	26	0.192
484	A	16	10	1.00	26	0.385
485	A	10	7	1.00	26	0.269
486	A	7	7	1.00	20	0.350
487	A	7	6	1.00	22	0.273
488	A	8	7	1.00	22	0.318
489	A	3	3	1.00	22	0.136
490	A	5	5	1.00	22	0.227
491	A	14	3	1.00	22	0.136
492	A	10	3	1.00	22	0.136
493	A	6	3	1.00	20	0.150
494	A	3	2	1.00	28	0.071
495	A	4	3	1.00	30	0.100
496	A	4	3	1.00	30	0.100
497	A	3	2	1.00	31	0.065
498	A	4	3	1.00	33	0.091
499	A	4	3	1.00	33	0.091
500	A	5	4	1.00	30	0.133
501	A	6	6	1.00	30	0.200
502	A	4	3	1.00	30	0.100
503	A	4	4	1.00	28	0.143
504	A	8	8	1.00	30	0.267
505	A	5	4	1.00	30	0.133
506	A	9	8	1.00	30	0.267
507	A	6	5	1.00	30	0.167
508	A	5	4	1.00	30	0.133
509	A	5	5	1.00	30	0.167
510	A	5	4	1.00	30	0.133
511	A	5	5	0.98	28	0.179
512	A	5	4	1.00	22	0.182
513	A	9	8	1.00	30	0.267
514	A	6	5	1.00	30	0.167
515	A	9	8	1.00	30	0.267
516	A	7	5	1.00	30	0.167
517	A	6	5	1.00	30	0.167
518	A	6	6	1.00	30	0.200
519	A	6	5	1.00	30	0.167

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
520	A	6	5	0.99	28	0.179
521	A	6	5	1.00	22	0.227
522	A	10	9	1.00	30	0.300
523	A	7	6	1.00	30	0.200
524	A	10	9	1.00	30	0.300
525	A	5	4	1.00	33	0.121
526	A	6	6	1.00	33	0.182
527	A	4	3	1.00	33	0.091
528	A	4	4	1.00	31	0.129
529	A	8	8	1.00	33	0.242
530	A	5	4	1.00	33	0.121
531	A	9	8	1.00	33	0.242
532	A	6	5	1.00	33	0.152
533	A	5	4	1.00	33	0.121
534	A	5	5	1.00	33	0.152
535	A	5	4	1.00	33	0.121
536	A	5	5	1.00	31	0.161
537	A	9	8	1.00	33	0.242
538	A	6	5	1.00	33	0.152
539	A	9	8	1.00	33	0.242
540	A	7	5	1.00	33	0.152
541	A	6	5	1.00	33	0.152
542	A	6	6	1.00	33	0.182
543	A	6	5	1.00	33	0.152
544	A	6	5	1.00	31	0.161
545	A	10	9	1.00	33	0.273
546	A	7	6	1.00	33	0.182
547	A	10	9	1.00	33	0.273
548	A	3	2	1.00	24	0.083
549	A	4	3	1.00	26	0.115

Chapter 3

Listing of integrals

Local contents

3.1 $\int \left(ax^3+bx^6\right)^{5/3} dx$

Optimal. Leaf size=52

$$\frac{\left(ax^3+bx^6\right)^{8/3}}{11bx^5}-\frac{3a\left(ax^3+bx^6\right)^{8/3}}{88b^2x^8}$$

Rubi [A] time = 0.05, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2002, 2014}

$$\frac{\left(ax^3+bx^6\right)^{8/3}}{11bx^5}-\frac{3a\left(ax^3+bx^6\right)^{8/3}}{88b^2x^8}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x^3 + b*x^6)^(5/3),x]
```

```
[Out] (-3*a*(a*x^3 + b*x^6)^(8/3))/(88*b^2*x^8) + (a*x^3 + b*x^6)^(8/3)/(11*b*x^5
)
```

Rule 2002

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(a*(j*p + 1)*x^(j - 1)), x] - Dist[(b*(n*p + n - j + 1))/(a*
(j*p + 1)), Int[x^(n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, j, n, p
}, x] && !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(n*p + n - j + 1)/(n -
j)], 0] && NeQ[j*p + 1, 0]
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !IntegerQ[p] && NeQ[n,
j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\int \left(ax^3+bx^6\right)^{5/3} dx &= \frac{\left(ax^3+bx^6\right)^{8/3}}{11bx^5}-\frac{(3a)\int \frac{\left(ax^3+bx^6\right)^{5/3}}{x^3} dx}{11b}\\ &= -\frac{3a\left(ax^3+bx^6\right)^{8/3}}{88b^2x^8}+\frac{\left(ax^3+bx^6\right)^{8/3}}{11bx^5}\end{aligned}$$

Mathematica [A] time = 0.02, size = 42, normalized size = 0.81

$$\frac{x\left(a+bx^3\right)^3\left(8bx^3-3a\right)}{88b^2\sqrt[3]{x^3\left(a+bx^3\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x^3 + b*x^6)^(5/3),x]
```

```
[Out] (x*(a + b*x^3)^3*(-3*a + 8*b*x^3))/(88*b^2*(x^3*(a + b*x^3))^(1/3))
```

IntegrateAlgebraic [A] time = 0.28, size = 57, normalized size = 1.10

$$\frac{\left(ax^3+bx^6\right)^{2/3}\left(-3a^3+2a^2bx^3+13ab^2x^6+8b^3x^9\right)}{88b^2x^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a*x^3 + b*x^6)^(5/3),x]
[Out] ((a*x^3 + b*x^6)^(2/3)*(-3*a^3 + 2*a^2*b*x^3 + 13*a*b^2*x^6 + 8*b^3*x^9))/(
88*b^2*x^2)
```

fricas [A] time = 1.97, size = 53, normalized size = 1.02

$$\frac{\left(8\,b^3x^9+13\,ab^2x^6+2\,a^2bx^3-3\,a^3\right)\left(bx^6+ax^3\right)^{\frac{2}{3}}}{88\,b^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^6+a*x^3)^(5/3),x, algorithm="fricas")
[Out] 1/88*(8*b^3*x^9 + 13*a*b^2*x^6 + 2*a^2*b*x^3 - 3*a^3)*(b*x^6 + a*x^3)^(2/3)
/(b^2*x^2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(bx^6+ax^3\right)^{\frac{5}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^6+a*x^3)^(5/3),x, algorithm="giac")
[Out] integrate((b*x^6 + a*x^3)^(5/3), x)
```

maple [A] time = 0.01, size = 39, normalized size = 0.75

$$-\frac{\left(b\,x^3+a\right)\left(-8b\,x^3+3a\right)\left(b\,x^6+a\,x^3\right)^{\frac{5}{3}}}{88b^2x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^6+a*x^3)^(5/3),x)
[Out] -1/88*(b*x^3+a)*(-8*b*x^3+3*a)*(b*x^6+a*x^3)^(5/3)/b^2/x^5
```

maxima [A] time = 0.44, size = 46, normalized size = 0.88

$$\frac{\left(8\,b^3x^9+13\,ab^2x^6+2\,a^2bx^3-3\,a^3\right)\left(bx^3+a\right)^{\frac{2}{3}}}{88\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^6+a*x^3)^(5/3),x, algorithm="maxima")
[Out] 1/88*(8*b^3*x^9 + 13*a*b^2*x^6 + 2*a^2*b*x^3 - 3*a^3)*(b*x^3 + a)^(2/3)/b^2
```

mupad [B] time = 1.23, size = 40, normalized size = 0.77

$$-\frac{\left(b\,x^3+a\right)^2\left(b\,x^6+a\,x^3\right)^{2/3}\left(3\,a-8\,b\,x^3\right)}{88\,b^2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^3 + b*x^6)^(5/3),x)
```

```
[Out] -((a + b*x^3)^2*(a*x^3 + b*x^6)^(2/3)*(3*a - 8*b*x^3))/(88*b^2*x^2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(ax^3 + bx^6\right)^{\frac{5}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**6+a*x**3)**(5/3),x)
```

```
[Out] Integral((a*x**3 + b*x**6)**(5/3), x)
```

3.2 $\int \left(ax^3+bx^6\right)^{2/3} dx$

Optimal. Leaf size=25

$$\frac{\left(ax^3+bx^6\right)^{5/3}}{5bx^5}$$

Rubi [A] time = 0.00, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {2000}

$$\frac{\left(ax^3+bx^6\right)^{5/3}}{5bx^5}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x^3 + b*x^6)^(2/3),x]
```

```
[Out] (a*x^3 + b*x^6)^(5/3)/(5*b*x^5)
```

Rule 2000

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(b*(n - j)*(p + 1)*x^(n - 1)), x] /; FreeQ[{a, b, j, n, p},
x] &&  !IntegerQ[p] && NeQ[n, j] && EqQ[j*p - n + j + 1, 0]
```

Rubi steps

$$\int \left(ax^3+bx^6\right)^{2/3} dx = \frac{\left(ax^3+bx^6\right)^{5/3}}{5bx^5}$$

Mathematica [A] time = 0.01, size = 25, normalized size = 1.00

$$\frac{\left(x^3\left(a+bx^3\right)\right)^{5/3}}{5bx^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x^3 + b*x^6)^(2/3),x]
```

```
[Out] (x^3*(a + b*x^3))^(5/3)/(5*b*x^5)
```

IntegrateAlgebraic [A] time = 0.20, size = 25, normalized size = 1.00

$$\frac{\left(ax^3+bx^6\right)^{5/3}}{5bx^5}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a*x^3 + b*x^6)^(2/3),x]
```

```
[Out] (a*x^3 + b*x^6)^(5/3)/(5*b*x^5)
```

fricas [A] time = 1.91, size = 28, normalized size = 1.12

$$\frac{\left(bx^6+ax^3\right)^{\frac{2}{3}}\left(bx^3+a\right)}{5\,bx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^6+a*x^3)^(2/3),x, algorithm="fricas")
```

```
[Out] 1/5*(b*x^6 + a*x^3)^(2/3)*(b*x^3 + a)/(b*x^2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(bx^6 + ax^3\right)^{\frac{2}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^6+a*x^3)^(2/3),x, algorithm="giac")
```

```
[Out] integrate((b*x^6 + a*x^3)^(2/3), x)
```

maple [A] time = 0.00, size = 29, normalized size = 1.16

$$\frac{\left(b\,x^3 + a\right)\left(b\,x^6 + a\,x^3\right)^{\frac{2}{3}}}{5b\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^6+a*x^3)^(2/3),x)
```

```
[Out] 1/5*(b*x^3+a)/b/x^2*(b*x^6+a*x^3)^(2/3)
```

maxima [A] time = 0.46, size = 14, normalized size = 0.56

$$\frac{\left(bx^3 + a\right)^{\frac{5}{3}}}{5\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^6+a*x^3)^(2/3),x, algorithm="maxima")
```

```
[Out] 1/5*(b*x^3 + a)^(5/3)/b
```

mupad [B] time = 1.15, size = 29, normalized size = 1.16

$$\frac{\left(\frac{a}{5\,b} + \frac{x^3}{5}\right)\left(b\,x^6 + a\,x^3\right)^{2/3}}{x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^3 + b*x^6)^(2/3),x)
```

```
[Out] ((a/(5*b) + x^3/5)*(a*x^3 + b*x^6)^(2/3))/x^2
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(ax^3 + bx^6\right)^{\frac{2}{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**6+a*x**3)**(2/3),x)
```

```
[Out] Integral((a*x**3 + b*x**6)**(2/3), x)
```

3.3 $\int \frac{1}{\left(ax^3+bx^6\right)^{2/3}}\,dx$

Optimal. Leaf size=23

$$-\frac{\sqrt[3]{ax^3+bx^6}}{ax^2}$$

Rubi [A] time = 0.00, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {2000}

$$-\frac{\sqrt[3]{ax^3+bx^6}}{ax^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x^3 + b*x^6)^(-2/3),x]
```

```
[Out] -((a*x^3 + b*x^6)^(1/3)/(a*x^2))
```

Rule 2000

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j + b*x^n)^(p + 1)/(b*(n - j)*(p + 1)*x^(n - 1)), x] /; FreeQ[{a, b, j, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j] && EqQ[j*p - n + j + 1, 0]
```

Rubi steps

$$\int \frac{1}{\left(ax^3+bx^6\right)^{2/3}}\,dx=-\frac{\sqrt[3]{ax^3+bx^6}}{ax^2}$$

Mathematica [A] time = 0.01, size = 23, normalized size = 1.00

$$-\frac{\sqrt[3]{x^3\left(a+bx^3\right)}}{ax^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x^3 + b*x^6)^(-2/3),x]
```

```
[Out] -((x^3*(a + b*x^3))^(1/3)/(a*x^2))
```

IntegrateAlgebraic [A] time = 0.41, size = 23, normalized size = 1.00

$$-\frac{\sqrt[3]{ax^3+bx^6}}{ax^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a*x^3 + b*x^6)^(-2/3),x]
```

```
[Out] -((a*x^3 + b*x^6)^(1/3)/(a*x^2))
```

fricas [A] time = 1.17, size = 21, normalized size = 0.91

$$-\frac{\left(bx^6+ax^3\right)^{\frac{1}{3}}}{ax^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^6+a*x^3)^(2/3),x, algorithm="fricas")
[Out] -(b*x^6 + a*x^3)^(1/3)/(a*x^2)
```

giac [A] time = 25.00, size = 14, normalized size = 0.61

$$-\frac{\left(b+\frac{a}{x^3}\right)^{\frac{1}{3}}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^6+a*x^3)^(2/3),x, algorithm="giac")
[Out] -(b + a/x^3)^(1/3)/a
```

maple [A] time = 0.00, size = 27, normalized size = 1.17

$$-\frac{\left(b\,x^3+a\right)x}{\left(b\,x^6+a\,x^3\right)^{\frac{2}{3}}\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^6+a*x^3)^(2/3),x)
[Out] -x*(b*x^3+a)/a/(b*x^6+a*x^3)^(2/3)
```

maxima [A] time = 0.45, size = 17, normalized size = 0.74

$$-\frac{\left(bx^3+a\right)^{\frac{1}{3}}}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^6+a*x^3)^(2/3),x, algorithm="maxima")
[Out] -(b*x^3 + a)^(1/3)/(a*x)
```

mupad [B] time = 1.15, size = 21, normalized size = 0.91

$$-\frac{\left(b\,x^6+a\,x^3\right)^{1/3}}{a\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x^3 + b*x^6)^(2/3),x)
[Out] -(a*x^3 + b*x^6)^(1/3)/(a*x^2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\left(ax^3+bx^6\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**6+a*x**3)**(2/3),x)
[Out] Integral((a*x**3 + b*x**6)**(-2/3), x)
```

3.4 $\int \frac{1}{\left(ax^3+bx^6\right)^{5/3}}\,dx$

Optimal. Leaf size=77

$$\frac{9b\sqrt[3]{ax^3+bx^6}}{4a^3x^2}-\frac{3\sqrt[3]{ax^3+bx^6}}{4a^2x^5}+\frac{1}{2ax^2\left(ax^3+bx^6\right)^{2/3}}$$

Rubi [A] time = 0.06, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2001, 2016, 2000}

$$\frac{9b\sqrt[3]{ax^3+bx^6}}{4a^3x^2}-\frac{3\sqrt[3]{ax^3+bx^6}}{4a^2x^5}+\frac{1}{2ax^2\left(ax^3+bx^6\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x^3 + b*x^6)^(-5/3),x]
```

```
[Out] 1/(2*a*x^2*(a*x^3 + b*x^6)^(2/3)) - (3*(a*x^3 + b*x^6)^(1/3))/(4*a^2*x^5) +
 (9*b*(a*x^3 + b*x^6)^(1/3))/(4*a^3*x^2)
```

Rule 2000

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(b*(n - j)*(p + 1)*x^(n - 1)), x] /; FreeQ[{a, b, j, n, p},
x] &&  !IntegerQ[p] && NeQ[n, j] && EqQ[j*p - n + j + 1, 0]
```

Rule 2001

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> -Simp[(a*x^j +
 b*x^n)^(p + 1)/(a*(n - j)*(p + 1)*x^(j - 1)), x] + Dist[(n*p + n - j + 1)/
(a*(n - j)*(p + 1)), Int[(a*x^j + b*x^n)^(p + 1)/x^j, x], x] /; FreeQ[{a, b
, j, n}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(n*p + n - j + 1
)/(n - j)], 0] && LtQ[p, -1]
```

Rule 2016

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(ax^3+bx^6\right)^{5/3}}\,dx&=\frac{1}{2ax^2\left(ax^3+bx^6\right)^{2/3}}+\frac{3\int \frac{1}{x^3\left(ax^3+bx^6\right)^{2/3}}\,dx}{a}\\&=\frac{1}{2ax^2\left(ax^3+bx^6\right)^{2/3}}-\frac{3\sqrt[3]{ax^3+bx^6}}{4a^2x^5}-\frac{(9b)\int \frac{1}{\left(ax^3+bx^6\right)^{2/3}}\,dx}{4a^2}\\&=\frac{1}{2ax^2\left(ax^3+bx^6\right)^{2/3}}-\frac{3\sqrt[3]{ax^3+bx^6}}{4a^2x^5}+\frac{9b\sqrt[3]{ax^3+bx^6}}{4a^3x^2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 46, normalized size = 0.60

$$\frac{-a^2+6abx^3+9b^2x^6}{4a^3x^2\left(x^3\left(a+bx^3\right)\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x^3 + b*x^6)^(-5/3),x]
[Out] (-a^2 + 6*a*b*x^3 + 9*b^2*x^6)/(4*a^3*x^2*(x^3*(a + b*x^3))^(2/3))
```

IntegrateAlgebraic [A] time = 0.95, size = 46, normalized size = 0.60

$$\frac{-a^2+6abx^3+9b^2x^6}{4a^3x^2\left(ax^3+bx^6\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a*x^3 + b*x^6)^(-5/3),x]
[Out] (-a^2 + 6*a*b*x^3 + 9*b^2*x^6)/(4*a^3*x^2*(a*x^3 + b*x^6)^(2/3))
```

fricas [A] time = 1.06, size = 54, normalized size = 0.70

$$\frac{\left(9\,b^2x^6+6\,abx^3-a^2\right)\left(bx^6+ax^3\right)^{\frac{1}{3}}}{4\left(a^3bx^8+a^4x^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^6+a*x^3)^(5/3),x, algorithm="fricas")
[Out] 1/4*(9*b^2*x^6 + 6*a*b*x^3 - a^2)*(b*x^6 + a*x^3)^(1/3)/(a^3*b*x^8 + a^4*x^
5)
```

giac [A] time = 23.79, size = 52, normalized size = 0.68

$$\frac{b^2}{2\,a^3\left(b+\frac{a}{x^3}\right)^{\frac{2}{3}}}-\frac{a^9\left(b+\frac{a}{x^3}\right)^{\frac{4}{3}}-8\,a^9\left(b+\frac{a}{x^3}\right)^{\frac{1}{3}}b}{4\,a^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^6+a*x^3)^(5/3),x, algorithm="giac")
[Out] 1/2*b^2/(a^3*(b + a/x^3)^(2/3)) - 1/4*(a^9*(b + a/x^3)^(4/3) - 8*a^9*(b + a
/x^3)^(1/3)*b)/a^12
```

maple [A] time = 0.00, size = 46, normalized size = 0.60

$$-\frac{\left(b\,x^3+a\right)\left(-9b^2x^6-6b\,x^3a+a^2\right)x}{4\left(b\,x^6+a\,x^3\right)^{\frac{5}{3}}\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^6+a*x^3)^(5/3),x)
[Out] -1/4*x*(b*x^3+a)*(-9*b^2*x^6-6*a*b*x^3+a^2)/a^3/(b*x^6+a*x^3)^(5/3)
```

maxima [A] time = 0.47, size = 38, normalized size = 0.49

$$\frac{9\,b^2x^6+6\,abx^3-a^2}{4\,\left(bx^3+a\right)^{\frac{2}{3}}a^3x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^6+a*x^3)^(5/3),x, algorithm="maxima")
```

```
[Out] 1/4*(9*b^2*x^6 + 6*a*b*x^3 - a^2)/((b*x^3 + a)^(2/3)*a^3*x^4)
```

mupad [B] time = 1.28, size = 51, normalized size = 0.66

$$\frac{\left(b\,x^6+a\,x^3\right)^{1/3}\,\left(-a^2+6\,a\,b\,x^3+9\,b^2\,x^6\right)}{4\,a^3\,x^5\,\left(b\,x^3+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x^3 + b*x^6)^(5/3),x)
```

```
[Out] ((a*x^3 + b*x^6)^(1/3)*(9*b^2*x^6 - a^2 + 6*a*b*x^3))/(4*a^3*x^5*(a + b*x^3
))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\left(ax^3+bx^6\right)^{\frac{5}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**6+a*x**3)**(5/3),x)
```

```
[Out] Integral((a*x**3 + b*x**6)**(-5/3), x)
```

3.5 $\int \frac{1}{-x^3+x^6} dx$

Optimal. Leaf size=48

$$\frac{1}{2x^2} - \frac{1}{6}\log\left(x^2+x+1\right) + \frac{1}{3}\log(1-x) - \frac{\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}$$

Rubi [A] time = 0.02, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.727, Rules used = {1593, 325, 200, 31, 634, 618, 204, 628}

$$\frac{1}{2x^2} - \frac{1}{6}\log\left(x^2+x+1\right) + \frac{1}{3}\log(1-x) - \frac{\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(-x^3 + x^6)^(-1),x]
```

```
[Out] 1/(2*x^2) - ArcTan[(1 + 2*x)/Sqrt[3]]/Sqrt[3] + Log[1 - x]/3 - Log[1 + x +
x^2]/6
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{-x^3+x^6}\,dx &= \int \frac{1}{x^3\left(-1+x^3\right)}\,dx\\ &= \frac{1}{2x^2}+\int \frac{1}{-1+x^3}\,dx\\ &= \frac{1}{2x^2}+\frac{1}{3}\int \frac{1}{-1+x}\,dx+\frac{1}{3}\int \frac{-2-x}{1+x+x^2}\,dx\\ &= \frac{1}{2x^2}+\frac{1}{3}\log(1-x)-\frac{1}{6}\int \frac{1+2x}{1+x+x^2}\,dx-\frac{1}{2}\int \frac{1}{1+x+x^2}\,dx\\ &= \frac{1}{2x^2}+\frac{1}{3}\log(1-x)-\frac{1}{6}\log\left(1+x+x^2\right)+\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,1+2x\right)\\ &= \frac{1}{2x^2}-\frac{\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{1}{3}\log(1-x)-\frac{1}{6}\log\left(1+x+x^2\right)\end{aligned}$$

Mathematica [A] time = 0.02, size = 48, normalized size = 1.00

$$\frac{1}{2x^2}-\frac{1}{6}\log\left(x^2+x+1\right)+\frac{1}{3}\log(1-x)-\frac{\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-x^3 + x^6)^(-1),x]
[Out] 1/(2*x^2) - ArcTan[(1 + 2*x)/Sqrt[3]]/Sqrt[3] + Log[1 - x]/3 - Log[1 + x +
x^2]/6
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{-x^3+x^6}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(-x^3 + x^6)^(-1),x]
[Out] IntegrateAlgebraic[(-x^3 + x^6)^(-1), x]
```

fricas [A] time = 1.55, size = 46, normalized size = 0.96

$$-\frac{2\sqrt{3}\,x^2\arctan\left(\frac{1}{3}\sqrt{3}\,(2\,x+1)\right)+x^2\log\left(x^2+x+1\right)-2\,x^2\log\left(x-1\right)-3}{6\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^6-x^3),x, algorithm="fricas")
```

```
[Out] -1/6*(2*sqrt(3)*x^2*arctan(1/3*sqrt(3)*(2*x + 1)) + x^2*log(x^2 + x + 1) -
2*x^2*log(x - 1) - 3)/x^2
```

giac [A] time = 0.32, size = 38, normalized size = 0.79

$$-\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)+\frac{1}{2x^2}-\frac{1}{6}\log\left(x^2+x+1\right)+\frac{1}{3}\log\left(|x-1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^6-x^3),x, algorithm="giac")
```

```
[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/2/x^2 - 1/6*log(x^2 + x + 1)
 + 1/3*log(abs(x - 1))
```

maple [A] time = 0.01, size = 38, normalized size = 0.79

$$-\frac{\sqrt{3}\arctan\left(\frac{(2x+1)\sqrt{3}}{3}\right)}{3}+\frac{\ln(x-1)}{3}-\frac{\ln\left(x^2+x+1\right)}{6}+\frac{1}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^6-x^3),x)
```

```
[Out] -1/6*ln(x^2+x+1)-1/3*arctan(1/3*(2*x+1)*3^(1/2))*3^(1/2)+1/3*ln(x-1)+1/2/x^
2
```

maxima [A] time = 0.95, size = 37, normalized size = 0.77

$$-\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)+\frac{1}{2x^2}-\frac{1}{6}\log\left(x^2+x+1\right)+\frac{1}{3}\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^6-x^3),x, algorithm="maxima")
```

```
[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/2/x^2 - 1/6*log(x^2 + x + 1)
 + 1/3*log(x - 1)
```

mupad [B] time = 0.09, size = 51, normalized size = 1.06

$$\frac{\ln(x-1)}{3}+\ln\left(x+\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(-\frac{1}{6}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right)-\ln\left(x+\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{6}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right)+\frac{1}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/(x^3 - x^6),x)
```

```
[Out] log(x - 1)/3 + log(x - (3^(1/2)*1i)/2 + 1/2)*((3^(1/2)*1i)/6 - 1/6) - log(x
 + (3^(1/2)*1i)/2 + 1/2)*((3^(1/2)*1i)/6 + 1/6) + 1/(2*x^2)
```

sympy [A] time = 0.15, size = 48, normalized size = 1.00

$$\frac{\log(x-1)}{3}-\frac{\log\left(x^2+x+1\right)}{6}-\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x}{3}+\frac{\sqrt{3}}{3}\right)}{3}+\frac{1}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**6-x**3),x)
```

```
[Out] log(x - 1)/3 - log(x**2 + x + 1)/6 - sqrt(3)*atan(2*sqrt(3)*x/3 + sqrt(3)/3
)/3 + 1/(2*x**2)
```

3.6 $\int x^5\sqrt{a^2+2abx^3+b^2x^6}\,dx$

Optimal. Leaf size=79

$$\frac{ax^6\sqrt{a^2+2abx^3+b^2x^6}}{6\left(a+bx^3\right)}+\frac{bx^9\sqrt{a^2+2abx^3+b^2x^6}}{9\left(a+bx^3\right)}$$

Rubi [A] time = 0.02, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1355, 14}

$$\frac{bx^9\sqrt{a^2+2abx^3+b^2x^6}}{9\left(a+bx^3\right)}+\frac{ax^6\sqrt{a^2+2abx^3+b^2x^6}}{6\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6],x]
```

```
[Out] (a*x^6*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(6*(a + b*x^3)) + (b*x^9*Sqrt[a^2 +
 2*a*b*x^3 + b^2*x^6])/(9*(a + b*x^3))
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\begin{aligned}\int x^5\sqrt{a^2+2abx^3+b^2x^6}\,dx&=\frac{\sqrt{a^2+2abx^3+b^2x^6}\int x^5\left(ab+b^2x^3\right)dx}{ab+b^2x^3}\\&=\frac{\sqrt{a^2+2abx^3+b^2x^6}\int\left(abx^5+b^2x^8\right)dx}{ab+b^2x^3}\\&=\frac{ax^6\sqrt{a^2+2abx^3+b^2x^6}}{6\left(a+bx^3\right)}+\frac{bx^9\sqrt{a^2+2abx^3+b^2x^6}}{9\left(a+bx^3\right)}\end{aligned}$$

Mathematica [A] time = 0.01, size = 39, normalized size = 0.49

$$\frac{\sqrt{\left(a+bx^3\right)^2}\left(3ax^6+2bx^9\right)}{18\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6],x]
```

```
[Out] (Sqrt[(a + b*x^3)^2]*(3*a*x^6 + 2*b*x^9))/(18*(a + b*x^3))
```

IntegrateAlgebraic [A] time = 6.28, size = 39, normalized size = 0.49

$$\frac{\sqrt{\left(a+bx^3\right)^2}\left(3ax^6+2bx^9\right)}{18\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6],x]
[Out] (Sqrt[(a + b*x^3)^2]*(3*a*x^6 + 2*b*x^9))/(18*(a + b*x^3))
```

fricas [A] time = 1.34, size = 13, normalized size = 0.16

$$\frac{1}{9}bx^9+\frac{1}{6}ax^6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*((b*x^3+a)^2)^(1/2),x, algorithm="fricas")
[Out] 1/9*b*x^9 + 1/6*a*x^6
```

giac [A] time = 0.40, size = 23, normalized size = 0.29

$$\frac{1}{18}\left(2\,bx^9+3\,ax^6\right)\mathrm{sgn}\left(bx^3+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*((b*x^3+a)^2)^(1/2),x, algorithm="giac")
[Out] 1/18*(2*b*x^9 + 3*a*x^6)*sgn(b*x^3 + a)
```

maple [A] time = 0.00, size = 36, normalized size = 0.46

$$\frac{\left(2b\,x^3+3a\right)\sqrt{\left(b\,x^3+a\right)^2}\;x^6}{18b\,x^3+18a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*((b*x^3+a)^2)^(1/2),x)
[Out] 1/18*x^6*(2*b*x^3+3*a)*((b*x^3+a)^2)^(1/2)/(b*x^3+a)
```

maxima [A] time = 0.49, size = 83, normalized size = 1.05

$$-\frac{\sqrt{b^2x^6+2\,abx^3+a^2}\,ax^3}{6\,b}-\frac{\sqrt{b^2x^6+2\,abx^3+a^2}\,a^2}{6\,b^2}+\frac{\left(b^2x^6+2\,abx^3+a^2\right)^{\frac{3}{2}}}{9\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*((b*x^3+a)^2)^(1/2),x, algorithm="maxima")
[Out] -1/6*sqrt(b^2*x^6 + 2*a*b*x^3 + a^2)*a*x^3/b - 1/6*sqrt(b^2*x^6 + 2*a*b*x^3
 + a^2)*a^2/b^2 + 1/9*(b^2*x^6 + 2*a*b*x^3 + a^2)^(3/2)/b^2
```

mupad [B] time = 1.26, size = 59, normalized size = 0.75

$$\frac{\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}\;\left(8\,b^2\,\left(a^2+b^2\,x^6\right)-12\,a^2\,b^2+4\,a\,b^3\,x^3\right)}{72\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*((a + b*x^3)^2)^(1/2),x)
```

```
[Out] ((a^2 + b^2*x^6 + 2*a*b*x^3)^(1/2)*(8*b^2*(a^2 + b^2*x^6) - 12*a^2*b^2 + 4*
a*b^3*x^3))/(72*b^4)
```

sympy [A] time = 0.11, size = 12, normalized size = 0.15

$$\frac{ax^6}{6} + \frac{bx^9}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*((b*x**3+a)**2)**(1/2),x)
```

```
[Out] a*x**6/6 + b*x**9/9
```

3.7 $\int x^4\sqrt{a^2+2abx^3+b^2x^6}\,dx$

Optimal. Leaf size=79

$$\frac{bx^8\sqrt{a^2+2abx^3+b^2x^6}}{8\left(a+bx^3\right)}+\frac{ax^5\sqrt{a^2+2abx^3+b^2x^6}}{5\left(a+bx^3\right)}$$

Rubi [A] time = 0.02, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1355, 14}

$$\frac{bx^8\sqrt{a^2+2abx^3+b^2x^6}}{8\left(a+bx^3\right)}+\frac{ax^5\sqrt{a^2+2abx^3+b^2x^6}}{5\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6],x]
```

```
[Out] (a*x^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(5*(a + b*x^3)) + (b*x^8*Sqrt[a^2 +
 2*a*b*x^3 + b^2*x^6])/(8*(a + b*x^3))
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\begin{aligned}\int x^4\sqrt{a^2+2abx^3+b^2x^6}\,dx&=\frac{\sqrt{a^2+2abx^3+b^2x^6}\int x^4\left(ab+b^2x^3\right)\,dx}{ab+b^2x^3}\\&=\frac{\sqrt{a^2+2abx^3+b^2x^6}\int\left(abx^4+b^2x^7\right)\,dx}{ab+b^2x^3}\\&=\frac{ax^5\sqrt{a^2+2abx^3+b^2x^6}}{5\left(a+bx^3\right)}+\frac{bx^8\sqrt{a^2+2abx^3+b^2x^6}}{8\left(a+bx^3\right)}\end{aligned}$$

Mathematica [A] time = 0.01, size = 39, normalized size = 0.49

$$\frac{\sqrt{\left(a+bx^3\right)^2}\left(8ax^5+5bx^8\right)}{40\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6],x]
```

```
[Out] (Sqrt[(a + b*x^3)^2]*(8*a*x^5 + 5*b*x^8))/(40*(a + b*x^3))
```

IntegrateAlgebraic [A] time = 6.45, size = 39, normalized size = 0.49

$$\frac{\sqrt{\left(a+bx^3\right)^2}\,\left(8ax^5+5bx^8\right)}{40\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^4*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6],x]
[Out] (Sqrt[(a + b*x^3)^2]*(8*a*x^5 + 5*b*x^8))/(40*(a + b*x^3))
```

fricas [A] time = 1.33, size = 13, normalized size = 0.16

$$\frac{1}{8}\,bx^8+\frac{1}{5}\,ax^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*((b*x^3+a)^2)^(1/2),x, algorithm="fricas")
[Out] 1/8*b*x^8 + 1/5*a*x^5
```

giac [A] time = 0.37, size = 29, normalized size = 0.37

$$\frac{1}{8}\,bx^8\mathrm{sgn}\left(bx^3+a\right)+\frac{1}{5}\,ax^5\mathrm{sgn}\left(bx^3+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*((b*x^3+a)^2)^(1/2),x, algorithm="giac")
[Out] 1/8*b*x^8*sgn(b*x^3 + a) + 1/5*a*x^5*sgn(b*x^3 + a)
```

maple [A] time = 0.00, size = 36, normalized size = 0.46

$$\frac{\left(5b\,x^3+8a\right)\sqrt{\left(b\,x^3+a\right)^2}\;x^5}{40b\,x^3+40a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*((b*x^3+a)^2)^(1/2),x)
[Out] 1/40*x^5*(5*b*x^3+8*a)*((b*x^3+a)^2)^(1/2)/(b*x^3+a)
```

maxima [A] time = 0.47, size = 13, normalized size = 0.16

$$\frac{1}{8}\,bx^8+\frac{1}{5}\,ax^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*((b*x^3+a)^2)^(1/2),x, algorithm="maxima")
[Out] 1/8*b*x^8 + 1/5*a*x^5
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^4\,\sqrt{\left(b\,x^3+a\right)^2}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*((a + b*x^3)^2)^(1/2),x)
```

```
[Out] int(x^4*((a + b*x^3)^2)^(1/2), x)
```

sympy [A] time = 0.11, size = 12, normalized size = 0.15

$$\frac{ax^5}{5} + \frac{bx^8}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*((b*x**3+a)**2)**(1/2),x)
```

```
[Out] a*x**5/5 + b*x**8/8
```

3.8 $\int x^3\sqrt{a^2+2abx^3+b^2x^6}\,dx$

Optimal. Leaf size=79

$$\frac{bx^7\sqrt{a^2+2abx^3+b^2x^6}}{7\left(a+bx^3\right)}+\frac{ax^4\sqrt{a^2+2abx^3+b^2x^6}}{4\left(a+bx^3\right)}$$

Rubi [A] time = 0.02, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1355, 14}

$$\frac{bx^7\sqrt{a^2+2abx^3+b^2x^6}}{7\left(a+bx^3\right)}+\frac{ax^4\sqrt{a^2+2abx^3+b^2x^6}}{4\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6],x]
```

```
[Out] (a*x^4*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(4*(a + b*x^3)) + (b*x^7*Sqrt[a^2 +
 2*a*b*x^3 + b^2*x^6])/(7*(a + b*x^3))
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\begin{aligned}\int x^3\sqrt{a^2+2abx^3+b^2x^6}\,dx &= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int x^3\left(ab+b^2x^3\right)dx}{ab+b^2x^3}\\ &= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int\left(abx^3+b^2x^6\right)dx}{ab+b^2x^3}\\ &= \frac{ax^4\sqrt{a^2+2abx^3+b^2x^6}}{4\left(a+bx^3\right)}+\frac{bx^7\sqrt{a^2+2abx^3+b^2x^6}}{7\left(a+bx^3\right)}\end{aligned}$$

Mathematica [A] time = 0.01, size = 39, normalized size = 0.49

$$\frac{\sqrt{\left(a+bx^3\right)^2}\left(7ax^4+4bx^7\right)}{28\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6],x]
```

```
[Out] (Sqrt[(a + b*x^3)^2]*(7*a*x^4 + 4*b*x^7))/(28*(a + b*x^3))
```

IntegrateAlgebraic [A] time = 6.46, size = 39, normalized size = 0.49

$$\frac{\sqrt{\left(a+bx^3\right)^2}\left(7ax^4+4bx^7\right)}{28\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6],x]
[Out] (Sqrt[(a + b*x^3)^2]*(7*a*x^4 + 4*b*x^7))/(28*(a + b*x^3))
```

fricas [A] time = 1.67, size = 13, normalized size = 0.16

$$\frac{1}{7}bx^7+\frac{1}{4}ax^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*((b*x^3+a)^2)^(1/2),x, algorithm="fricas")
[Out] 1/7*b*x^7 + 1/4*a*x^4
```

giac [A] time = 0.39, size = 29, normalized size = 0.37

$$\frac{1}{7}bx^7\mathrm{sgn}\left(bx^3+a\right)+\frac{1}{4}ax^4\mathrm{sgn}\left(bx^3+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*((b*x^3+a)^2)^(1/2),x, algorithm="giac")
[Out] 1/7*b*x^7*sgn(b*x^3 + a) + 1/4*a*x^4*sgn(b*x^3 + a)
```

maple [A] time = 0.00, size = 36, normalized size = 0.46

$$\frac{\left(4b\,x^3+7a\right)\sqrt{\left(b\,x^3+a\right)^2}\;x^4}{28b\,x^3+28a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*((b*x^3+a)^2)^(1/2),x)
[Out] 1/28*x^4*(4*b*x^3+7*a)*((b*x^3+a)^2)^(1/2)/(b*x^3+a)
```

maxima [A] time = 0.46, size = 13, normalized size = 0.16

$$\frac{1}{7}bx^7+\frac{1}{4}ax^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*((b*x^3+a)^2)^(1/2),x, algorithm="maxima")
[Out] 1/7*b*x^7 + 1/4*a*x^4
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^3\sqrt{\left(b\,x^3+a\right)^2}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*((a + b*x^3)^2)^(1/2),x)
```

```
[Out] int(x^3*((a + b*x^3)^2)^(1/2), x)
```

sympy [**A**] time = 0.11, size = 12, normalized size = 0.15

$$\frac{ax^4}{4} + \frac{bx^7}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*((b*x**3+a)**2)**(1/2),x)
```

```
[Out] a*x**4/4 + b*x**7/7
```

3.9 $\int x^2\sqrt{a^2+2abx^3+b^2x^6}\,dx$

Optimal. Leaf size=36

$$\frac{(a+bx^3)\sqrt{a^2+2abx^3+b^2x^6}}{6b}$$

Rubi [A] time = 0.03, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {1352, 609}

$$\frac{(a+bx^3)\sqrt{a^2+2abx^3+b^2x^6}}{6b}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6],x]
```

```
[Out] ((a + b*x^3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(6*b)
```

Rule 609

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x)
*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] /; FreeQ[{a, b, c, p}, x] && EqQ[
b^2 - 4*a*c, 0] && NeQ[p, -2^(-1)]
```

Rule 1352

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a,
b, c, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

Rubi steps

$$\begin{aligned}\int x^2\sqrt{a^2+2abx^3+b^2x^6}\,dx &= \frac{1}{3}\text{Subst}\left(\int \sqrt{a^2+2abx+b^2x^2}\,dx, x, x^3\right)\\ &= \frac{(a+bx^3)\sqrt{a^2+2abx^3+b^2x^6}}{6b}\end{aligned}$$

Mathematica [A] time = 0.01, size = 38, normalized size = 1.06

$$\frac{\sqrt{(a+bx^3)^2}\,(2ax^3+bx^6)}{6(a+bx^3)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6],x]
```

```
[Out] (Sqrt[(a + b*x^3)^2]*(2*a*x^3 + b*x^6))/(6*(a + b*x^3))
```

IntegrateAlgebraic [A] time = 6.10, size = 38, normalized size = 1.06

$$\frac{\sqrt{(a+bx^3)^2}\,(2ax^3+bx^6)}{6(a+bx^3)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^2*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6],x]
[Out] (Sqrt[(a + b*x^3)^2]*(2*a*x^3 + b*x^6))/(6*(a + b*x^3))
```

fricas [A] time = 1.67, size = 13, normalized size = 0.36

$$\frac{1}{6}\,bx^6 + \frac{1}{3}\,ax^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*((b*x^3+a)^2)^(1/2),x, algorithm="fricas")
[Out] 1/6*b*x^6 + 1/3*a*x^3
```

giac [A] time = 0.36, size = 22, normalized size = 0.61

$$\frac{1}{6}\left(bx^6 + 2\,ax^3\right)\mathrm{sgn}\left(bx^3 + a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*((b*x^3+a)^2)^(1/2),x, algorithm="giac")
[Out] 1/6*(b*x^6 + 2*a*x^3)*sgn(b*x^3 + a)
```

maple [A] time = 0.00, size = 35, normalized size = 0.97

$$\frac{\left(b\,x^3 + 2a\right)\sqrt{\left(b\,x^3 + a\right)^2}\;x^3}{6b\,x^3 + 6a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*((b*x^3+a)^2)^(1/2),x)
[Out] 1/6*x^3*(b*x^3+2*a)*((b*x^3+a)^2)^(1/2)/(b*x^3+a)
```

maxima [B] time = 0.46, size = 52, normalized size = 1.44

$$\frac{1}{6}\,\sqrt{b^2x^6 + 2\,abx^3 + a^2}\,x^3 + \frac{\sqrt{b^2x^6 + 2\,abx^3 + a^2}\,a}{6\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*((b*x^3+a)^2)^(1/2),x, algorithm="maxima")
[Out] 1/6*sqrt(b^2*x^6 + 2*a*b*x^3 + a^2)*x^3 + 1/6*sqrt(b^2*x^6 + 2*a*b*x^3 + a^
2)*a/b
```

mupad [B] time = 1.23, size = 33, normalized size = 0.92

$$\left(\frac{a}{6\,b} + \frac{x^3}{6}\right)\,\sqrt{a^2 + 2\,a\,b\,x^3 + b^2\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*((a + b*x^3)^2)^(1/2),x)
[Out] (a/(6*b) + x^3/6)*(a^2 + b^2*x^6 + 2*a*b*x^3)^(1/2)
```

sympy [A] time = 0.11, size = 12, normalized size = 0.33

$$\frac{ax^3}{3} + \frac{bx^6}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*((b*x**3+a)**2)**(1/2),x)
[Out] a*x**3/3 + b*x**6/6
```

3.10 $\int x\sqrt{a^2+2abx^3+b^2x^6}\,dx$

Optimal. Leaf size=79

$$\frac{bx^5\sqrt{a^2+2abx^3+b^2x^6}}{5\left(a+bx^3\right)}+\frac{ax^2\sqrt{a^2+2abx^3+b^2x^6}}{2\left(a+bx^3\right)}$$

Rubi [A] time = 0.02, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1355, 14}

$$\frac{bx^5\sqrt{a^2+2abx^3+b^2x^6}}{5\left(a+bx^3\right)}+\frac{ax^2\sqrt{a^2+2abx^3+b^2x^6}}{2\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6],x]
```

```
[Out] (a*x^2*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(2*(a + b*x^3)) + (b*x^5*Sqrt[a^2 +
 2*a*b*x^3 + b^2*x^6])/(5*(a + b*x^3))
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\begin{aligned}\int x\sqrt{a^2+2abx^3+b^2x^6}\,dx &= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int x\left(ab+b^2x^3\right)dx}{ab+b^2x^3}\\ &= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int\left(abx+b^2x^4\right)dx}{ab+b^2x^3}\\ &= \frac{ax^2\sqrt{a^2+2abx^3+b^2x^6}}{2\left(a+bx^3\right)}+\frac{bx^5\sqrt{a^2+2abx^3+b^2x^6}}{5\left(a+bx^3\right)}\end{aligned}$$

Mathematica [A] time = 0.01, size = 39, normalized size = 0.49

$$\frac{\sqrt{\left(a+bx^3\right)^2}\left(5ax^2+2bx^5\right)}{10\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6],x]
```

```
[Out] (Sqrt[(a + b*x^3)^2]*(5*a*x^2 + 2*b*x^5))/(10*(a + b*x^3))
```

IntegrateAlgebraic [A] time = 8.16, size = 39, normalized size = 0.49

$$\frac{\sqrt{\left(a+bx^3\right)^2}\left(5ax^2+2bx^5\right)}{10\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6],x]
[Out] (Sqrt[(a + b*x^3)^2]*(5*a*x^2 + 2*b*x^5))/(10*(a + b*x^3))
```

fricas [A] time = 1.01, size = 13, normalized size = 0.16

$$\frac{1}{5}bx^5+\frac{1}{2}ax^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*((b*x^3+a)^2)^(1/2),x, algorithm="fricas")
[Out] 1/5*b*x^5 + 1/2*a*x^2
```

giac [A] time = 0.33, size = 29, normalized size = 0.37

$$\frac{1}{5}bx^5\operatorname{sgn}\left(bx^3+a\right)+\frac{1}{2}ax^2\operatorname{sgn}\left(bx^3+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*((b*x^3+a)^2)^(1/2),x, algorithm="giac")
[Out] 1/5*b*x^5*sgn(b*x^3 + a) + 1/2*a*x^2*sgn(b*x^3 + a)
```

maple [A] time = 0.01, size = 36, normalized size = 0.46

$$\frac{\left(2bx^3+5a\right)\sqrt{\left(bx^3+a\right)^2}\,x^2}{10bx^3+10a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*((b*x^3+a)^2)^(1/2),x)
[Out] 1/10*x^2*(2*b*x^3+5*a)*((b*x^3+a)^2)^(1/2)/(b*x^3+a)
```

maxima [A] time = 0.45, size = 13, normalized size = 0.16

$$\frac{1}{5}bx^5+\frac{1}{2}ax^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*((b*x^3+a)^2)^(1/2),x, algorithm="maxima")
[Out] 1/5*b*x^5 + 1/2*a*x^2
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x\sqrt{\left(bx^3+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*((a + b*x^3)^2)^(1/2),x)
```

```
[Out] int(x*((a + b*x^3)^2)^(1/2), x)
```

sympy [A] time = 0.10, size = 12, normalized size = 0.15

$$\frac{ax^2}{2} + \frac{bx^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*((b*x**3+a)**2)**(1/2),x)
```

```
[Out] a*x**2/2 + b*x**5/5
```

3.11 $\int \sqrt{a^2+2abx^3+b^2x^6}\,dx$

Optimal. Leaf size=74

$$\frac{ax\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}+\frac{bx^4\sqrt{a^2+2abx^3+b^2x^6}}{4\left(a+bx^3\right)}$$

Rubi [A] time = 0.01, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {1343}

$$\frac{bx^4\sqrt{a^2+2abx^3+b^2x^6}}{4\left(a+bx^3\right)}+\frac{ax\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6],x]
```

```
[Out] (a*x*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(a + b*x^3) + (b*x^4*Sqrt[a^2 + 2*a*b
*x^3 + b^2*x^6])/(4*(a + b*x^3))
```

Rule 1343

```
Int[((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_), x_Symbol] :> Dist[(
a + b*x^n + c*x^(2*n))^p/(b + 2*c*x^n)^(2*p), Int[(b + 2*c*x^n)^(2*p), x],
x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}\int \sqrt{a^2+2abx^3+b^2x^6}\,dx&=\frac{\sqrt{a^2+2abx^3+b^2x^6}\int \left(2ab+2b^2x^3\right)\,dx}{2ab+2b^2x^3}\\&=\frac{ax\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}+\frac{bx^4\sqrt{a^2+2abx^3+b^2x^6}}{4\left(a+bx^3\right)}\end{aligned}$$

Mathematica [A] time = 0.01, size = 36, normalized size = 0.49

$$\frac{\sqrt{\left(a+bx^3\right)^2}\left(4ax+bx^4\right)}{4\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6],x]
```

```
[Out] (Sqrt[(a + b*x^3)^2]*(4*a*x + b*x^4))/(4*(a + b*x^3))
```

IntegrateAlgebraic [A] time = 11.16, size = 36, normalized size = 0.49

$$\frac{\sqrt{\left(a+bx^3\right)^2}\left(4ax+bx^4\right)}{4\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6],x]
```

[Out] (Sqrt[(a + b*x^3)^2]*(4*a*x + b*x^4))/(4*(a + b*x^3))

fricas [A] time = 1.06, size = 10, normalized size = 0.14

$$\frac{1}{4}bx^4 + ax$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(((b*x^3+a)^2)^(1/2),x, algorithm="fricas")

[Out] 1/4*b*x^4 + a*x

giac [A] time = 0.38, size = 20, normalized size = 0.27

$$\frac{1}{4}\left(bx^4 + 4\,ax\right)\mathrm{sgn}\left(bx^3 + a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(((b*x^3+a)^2)^(1/2),x, algorithm="giac")

[Out] 1/4*(b*x^4 + 4*a*x)*sgn(b*x^3 + a)

maple [A] time = 0.00, size = 33, normalized size = 0.45

$$\frac{\left(b\,x^3 + 4a\right)\sqrt{\left(b\,x^3 + a\right)^2}\;x}{4b\,x^3 + 4a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((b*x^3+a)^2)^(1/2),x)

[Out] 1/4*x*(b*x^3+4*a)*((b*x^3+a)^2)^(1/2)/(b*x^3+a)

maxima [A] time = 0.44, size = 10, normalized size = 0.14

$$\frac{1}{4}bx^4 + ax$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(((b*x^3+a)^2)^(1/2),x, algorithm="maxima")

[Out] 1/4*b*x^4 + a*x

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{\left(b\,x^3 + a\right)^2}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((a + b*x^3)^2)^(1/2),x)

[Out] int(((a + b*x^3)^2)^(1/2), x)

sympy [A] time = 0.10, size = 8, normalized size = 0.11

$$ax + \frac{bx^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(((b*x**3+a)**2)**(1/2),x)

[Out] a*x + b*x**4/4

3.12 $\int \frac{\sqrt{a^2+2abx^3+b^2x^6}}{x} \, dx$

Optimal. Leaf size=75

$$\frac{bx^3\sqrt{a^2+2abx^3+b^2x^6}}{3\left(a+bx^3\right)}+\frac{a\log(x)\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}$$

Rubi [A] time = 0.02, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1355, 14}

$$\frac{bx^3\sqrt{a^2+2abx^3+b^2x^6}}{3\left(a+bx^3\right)}+\frac{a\log(x)\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]/x,x]
```

```
[Out] (b*x^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(3*(a + b*x^3)) + (a*Sqrt[a^2 + 2*a
*b*x^3 + b^2*x^6]*Log[x])/(a + b*x^3)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{a^2+2abx^3+b^2x^6}}{x} \, dx &= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{ab+b^2x^3}{x} \, dx}{ab+b^2x^3}\\ &= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \left(\frac{ab}{x}+b^2x^2\right) \, dx}{ab+b^2x^3}\\ &= \frac{bx^3\sqrt{a^2+2abx^3+b^2x^6}}{3\left(a+bx^3\right)}+\frac{a\sqrt{a^2+2abx^3+b^2x^6}\log(x)}{a+bx^3}\end{aligned}$$

Mathematica [A] time = 0.01, size = 37, normalized size = 0.49

$$\frac{\sqrt{\left(a+bx^3\right)^2}\left(3a\log(x)+bx^3\right)}{3\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]/x,x]
```

```
[Out] (Sqrt[(a + b*x^3)^2]*(b*x^3 + 3*a*Log[x]))/(3*(a + b*x^3))
```

IntegrateAlgebraic [B] time = 0.24, size = 197, normalized size = 2.63

$$\frac{1}{6}\sqrt{a^2+2abx^3+b^2x^6}+\frac{1}{6}a\log\left(\sqrt{a^2+2abx^3+b^2x^6}-a-\sqrt{b^2}\,x^3\right)-\frac{a\left(\sqrt{b^2}+b\right)\log\left(\sqrt{a^2+2abx^3+b^2x^6}+a-\sqrt{b^2}\,x^3\right)}{6b}-\frac{a\sqrt{b^2}\,\log\left(b\sqrt{a^2+2abx^3+b^2x^6}-ab-b\sqrt{b^2}\,x^3\right)}{6b}-\frac{1}{6}\sqrt{b^2}\,x^3$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]/x,x]
```

```
[Out] -1/6*(Sqrt[b^2]*x^3) + Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]/6 + (a*Log[-a - Sqrt
[b^2]*x^3 + Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]])/6 - (a*(b + Sqrt[b^2])*Log[a
- Sqrt[b^2]*x^3 + Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]])/(6*b) - (a*Sqrt[b^2]*Lo
g[-(a*b) - b*Sqrt[b^2]*x^3 + b*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]])/(6*b)
```

fricas [A] time = 1.18, size = 11, normalized size = 0.15

$$\frac{1}{3}\,bx^3+a\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^3+a)^2)^(1/2)/x,x, algorithm="fricas")
```

```
[Out] 1/3*b*x^3 + a*log(x)
```

giac [A] time = 0.29, size = 28, normalized size = 0.37

$$\frac{1}{3}\,bx^3\mathrm{sgn}\left(bx^3+a\right)+a\log\left(|x|\right)\mathrm{sgn}\left(bx^3+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^3+a)^2)^(1/2)/x,x, algorithm="giac")
```

```
[Out] 1/3*b*x^3*sgn(b*x^3 + a) + a*log(abs(x))*sgn(b*x^3 + a)
```

maple [A] time = 0.01, size = 34, normalized size = 0.45

$$\frac{\sqrt{\left(b\,x^3+a\right)^2}\,\left(b\,x^3+3a\ln(x)\right)}{3b\,x^3+3a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*x^3+a)^2)^(1/2)/x,x)
```

```
[Out] 1/3*((b*x^3+a)^2)^(1/2)*(b*x^3+3*a*ln(x))/(b*x^3+a)
```

maxima [A] time = 0.46, size = 96, normalized size = 1.28

$$\frac{1}{3}\,(-1)^{2\,b^2x^3+2\,ab}\,a\log\left(2\,b^2x^3+2\,ab\right)-\frac{1}{3}\,(-1)^{2\,abx^3+2\,a^2}\,a\log\left(\frac{2\,abx}{|x|}+\frac{2\,a^2}{x^2|x|}\right)+\frac{1}{3}\,\sqrt{b^2x^6+2\,abx^3+a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^3+a)^2)^(1/2)/x,x, algorithm="maxima")
```

```
[Out] 1/3*(-1)^(2*b^2*x^3 + 2*a*b)*a*log(2*b^2*x^3 + 2*a*b) - 1/3*(-1)^(2*a*b*x^3
 + 2*a^2)*a*log(2*a*b*x/abs(x) + 2*a^2/(x^2*abs(x))) + 1/3*sqrt(b^2*x^6 + 2
*a*b*x^3 + a^2)
```

mupad [B] time = 1.38, size = 109, normalized size = 1.45

$$\frac{\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{3}-\frac{\ln\left(a\,b+\frac{a^2}{x^3}+\frac{\sqrt{a^2}\,\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{x^3}\right)\,\sqrt{a^2}}{3}+\frac{a\,b\,\ln\left(a\,b+\sqrt{\left(b\,x^3+a\right)^2}\,\sqrt{b^2}+b^2\,x^3\right)}{3\,\sqrt{b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^3)^2)^(1/2)/x,x)
```

```
[Out] (a^2 + b^2*x^6 + 2*a*b*x^3)^(1/2)/3 - (log(a*b + a^2/x^3 + ((a^2)^(1/2)*(a^
2 + b^2*x^6 + 2*a*b*x^3)^(1/2))/x^3)*(a^2)^(1/2))/3 + (a*b*log(a*b + ((a +
b*x^3)^2)^(1/2)*(b^2)^(1/2) + b^2*x^3))/(3*(b^2)^(1/2))
```

sympy [A] time = 0.13, size = 10, normalized size = 0.13

$$a\log(x) + \frac{bx^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x**3+a)**2)**(1/2)/x,x)
```

```
[Out] a*log(x) + b*x**3/3
```

3.13 $\int \frac{\sqrt{a^2+2abx^3+b^2x^6}}{x^2}\,dx$

Optimal. Leaf size=77

$$\frac{bx^2\sqrt{a^2+2abx^3+b^2x^6}}{2\left(a+bx^3\right)}-\frac{a\sqrt{a^2+2abx^3+b^2x^6}}{x\left(a+bx^3\right)}$$

Rubi [A] time = 0.02, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1355, 14}

$$\frac{bx^2\sqrt{a^2+2abx^3+b^2x^6}}{2\left(a+bx^3\right)}-\frac{a\sqrt{a^2+2abx^3+b^2x^6}}{x\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]/x^2,x]
```

```
[Out] -((a*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(x*(a + b*x^3))) + (b*x^2*Sqrt[a^2 +
2*a*b*x^3 + b^2*x^6])/(2*(a + b*x^3))
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{a^2+2abx^3+b^2x^6}}{x^2}\,dx &= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{ab+b^2x^3}{x^2}\,dx}{ab+b^2x^3}\\ &= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \left(\frac{ab}{x^2}+b^2x\right)dx}{ab+b^2x^3}\\ &= -\frac{a\sqrt{a^2+2abx^3+b^2x^6}}{x\left(a+bx^3\right)}+\frac{bx^2\sqrt{a^2+2abx^3+b^2x^6}}{2\left(a+bx^3\right)}\end{aligned}$$

Mathematica [A] time = 0.01, size = 38, normalized size = 0.49

$$\frac{\left(bx^3-2a\right)\sqrt{\left(a+bx^3\right)^2}}{2x\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]/x^2,x]
```

```
[Out] ((-2*a + b*x^3)*Sqrt[(a + b*x^3)^2])/(2*x*(a + b*x^3))
```

IntegrateAlgebraic [A] time = 16.23, size = 38, normalized size = 0.49

$$\frac{\left(bx^3-2a\right)\sqrt{\left(a+bx^3\right)^2}}{2x\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]/x^2,x]
[Out] ((-2*a + b*x^3)*Sqrt[(a + b*x^3)^2])/(2*x*(a + b*x^3))
```

fricas [A] time = 1.07, size = 14, normalized size = 0.18

$$\frac{bx^3-2\,a}{2\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^3+a)^2)^(1/2)/x^2,x, algorithm="fricas")
[Out] 1/2*(b*x^3 - 2*a)/x
```

giac [A] time = 0.32, size = 29, normalized size = 0.38

$$\frac{1}{2}\,bx^2\mathrm{sgn}\left(bx^3+a\right)-\frac{a\mathrm{sgn}\left(bx^3+a\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^3+a)^2)^(1/2)/x^2,x, algorithm="giac")
[Out] 1/2*b*x^2*sgn(b*x^3 + a) - a*sgn(b*x^3 + a)/x
```

maple [A] time = 0.00, size = 36, normalized size = 0.47

$$-\frac{\left(-b\,x^3+2a\right)\sqrt{\left(b\,x^3+a\right)^2}}{2\left(b\,x^3+a\right)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*x^3+a)^2)^(1/2)/x^2,x)
[Out] -1/2*(-b*x^3+2*a)*((b*x^3+a)^2)^(1/2)/x/(b*x^3+a)
```

maxima [A] time = 0.46, size = 14, normalized size = 0.18

$$\frac{bx^3-2\,a}{2\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^3+a)^2)^(1/2)/x^2,x, algorithm="maxima")
[Out] 1/2*(b*x^3 - 2*a)/x
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{\left(b\,x^3+a\right)^2}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^3)^2)^(1/2)/x^2,x)
```

```
[Out] int(((a + b*x^3)^2)^(1/2)/x^2, x)
```

sympy [A] time = 0.13, size = 8, normalized size = 0.10

$$-\frac{a}{x}+\frac{bx^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x**3+a)**2)**(1/2)/x**2,x)
```

```
[Out] -a/x + b*x**2/2
```

3.14 $\int \frac{\sqrt{a^2+2abx^3+b^2x^6}}{x^3}\,dx$

Optimal. Leaf size=74

$$\frac{bx\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}-\frac{a\sqrt{a^2+2abx^3+b^2x^6}}{2x^2\left(a+bx^3\right)}$$

Rubi [A] time = 0.02, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1355, 14}

$$\frac{bx\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}-\frac{a\sqrt{a^2+2abx^3+b^2x^6}}{2x^2\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]/x^3,x]
```

```
[Out] -(a*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(2*x^2*(a + b*x^3)) + (b*x*Sqrt[a^2 +
2*a*b*x^3 + b^2*x^6])/(a + b*x^3)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{a^2+2abx^3+b^2x^6}}{x^3}\,dx &= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{ab+b^2x^3}{x^3}\,dx}{ab+b^2x^3}\\ &= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \left(b^2+\frac{ab}{x^3}\right)dx}{ab+b^2x^3}\\ &= -\frac{a\sqrt{a^2+2abx^3+b^2x^6}}{2x^2\left(a+bx^3\right)}+\frac{bx\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}\end{aligned}$$

Mathematica [A] time = 0.01, size = 37, normalized size = 0.50

$$-\frac{\left(a-2bx^3\right)\sqrt{\left(a+bx^3\right)^2}}{2x^2\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]/x^3,x]
```

```
[Out] -1/2*((a - 2*b*x^3)*Sqrt[(a + b*x^3)^2])/(x^2*(a + b*x^3))
```

IntegrateAlgebraic [A] time = 19.04, size = 39, normalized size = 0.53

$$\frac{\sqrt{\left(a+bx^3\right)^2}\left(2bx^3-a\right)}{2x^2\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]/x^3,x]
[Out] (Sqrt[(a + b*x^3)^2]*(-a + 2*b*x^3))/(2*x^2*(a + b*x^3))
```

fricas [A] time = 1.53, size = 15, normalized size = 0.20

$$\frac{2\,bx^3-a}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^3+a)^2)^(1/2)/x^3,x, algorithm="fricas")
[Out] 1/2*(2*b*x^3 - a)/x^2
```

giac [A] time = 0.40, size = 26, normalized size = 0.35

$$bx\mathrm{sgn}\left(bx^3+a\right)-\frac{a\mathrm{sgn}\left(bx^3+a\right)}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^3+a)^2)^(1/2)/x^3,x, algorithm="giac")
[Out] b*x*sgn(b*x^3 + a) - 1/2*a*sgn(b*x^3 + a)/x^2
```

maple [A] time = 0.00, size = 34, normalized size = 0.46

$$-\frac{\left(-2b\,x^3+a\right)\sqrt{\left(b\,x^3+a\right)^2}}{2\left(b\,x^3+a\right)x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*x^3+a)^2)^(1/2)/x^3,x)
[Out] -1/2*(-2*b*x^3+a)*((b*x^3+a)^2)^(1/2)/x^2/(b*x^3+a)
```

maxima [A] time = 0.46, size = 15, normalized size = 0.20

$$\frac{2\,bx^3-a}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^3+a)^2)^(1/2)/x^3,x, algorithm="maxima")
[Out] 1/2*(2*b*x^3 - a)/x^2
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{\left(b\,x^3+a\right)^2}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^3)^2)^(1/2)/x^3,x)
```

```
[Out] int(((a + b*x^3)^2)^(1/2)/x^3, x)
```

sympy [**A**] time = 0.13, size = 8, normalized size = 0.11

$$-\frac{a}{2x^2}+bx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x**3+a)**2)**(1/2)/x**3,x)
```

```
[Out] -a/(2*x**2) + b*x
```

3.15 $\int \frac{\sqrt{a^2+2abx^3+b^2x^6}}{x^4}\,dx$

Optimal. Leaf size=75

$$\frac{b\log(x)\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}-\frac{a\sqrt{a^2+2abx^3+b^2x^6}}{3x^3\left(a+bx^3\right)}$$

Rubi [A] time = 0.02, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1355, 14}

$$\frac{b\log(x)\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}-\frac{a\sqrt{a^2+2abx^3+b^2x^6}}{3x^3\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]/x^4,x]
```

```
[Out] -(a*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(3*x^3*(a + b*x^3)) + (b*Sqrt[a^2 + 2*
a*b*x^3 + b^2*x^6]*Log[x])/(a + b*x^3)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{a^2+2abx^3+b^2x^6}}{x^4}\,dx &= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{ab+b^2x^3}{x^4}\,dx}{ab+b^2x^3}\\ &= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \left(\frac{ab}{x^4}+\frac{b^2}{x}\right)dx}{ab+b^2x^3}\\ &= -\frac{a\sqrt{a^2+2abx^3+b^2x^6}}{3x^3\left(a+bx^3\right)}+\frac{b\sqrt{a^2+2abx^3+b^2x^6}\log(x)}{a+bx^3}\end{aligned}$$

Mathematica [A] time = 0.01, size = 39, normalized size = 0.52

$$-\frac{\sqrt{\left(a+bx^3\right)^2}\left(a-3bx^3\log(x)\right)}{3x^3\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]/x^4,x]
```

```
[Out] -1/3*(Sqrt[(a + b*x^3)^2]*(a - 3*b*x^3*Log[x]))/(x^3*(a + b*x^3))
```

IntegrateAlgebraic [B] time = 0.61, size = 749, normalized size = 9.99

$$\frac{-\frac{1}{3}\left(b^2\right)^{3/2}x^6\log\left(\sqrt{a^2+2abx^3+b^2x^6}-a-\sqrt{b^2}\,x^3\right)-\frac{1}{3}\left(b^2\right)^{3/2}x^6\log\left(\sqrt{a^2+2abx^3+b^2x^6}+a-\sqrt{b^2}\,x^3\right)-\frac{1}{3}ab\sqrt{b^2}\,x^3\log\left(\sqrt{a^2+2abx^3+b^2x^6}-a-\sqrt{b^2}\,x^3\right)+\frac{1}{3}b^2x^3\sqrt{a^2+2abx^3+b^2x^6}\,\log\left(\sqrt{a^2+2abx^3+b^2x^6}-a-\sqrt{b^2}\,x^3\right)-\frac{1}{3}ab\sqrt{b^2}\,x^3\log\left(\sqrt{a^2+2abx^3+b^2x^6}+a-\sqrt{b^2}\,x^3\right)+\frac{1}{3}b^2x^3\sqrt{a^2+2abx^3+b^2x^6}\,\log\left(\sqrt{a^2+2abx^3+b^2x^6}+a-\sqrt{b^2}\,x^3\right)+\frac{2}{3}a^2\sqrt{b^2}}{\left(\sqrt{a^2+2abx^3+b^2x^6}-a-\sqrt{b^2}\,x^3\right)\left(\sqrt{a^2+2abx^3+b^2x^6}+a-\sqrt{b^2}\,x^3\right)}+\frac{-\frac{2}{3}ab\sqrt{a^2+2abx^3+b^2x^6}-\frac{2}{3}ab^2x^3\tanh^{-1}\left(\frac{\sqrt{a^2+2abx^3+b^2x^6}-\sqrt{b^2}\,x^3}{a}\right)+\frac{2}{3}b\sqrt{b^2}\,x^3\sqrt{a^2+2abx^3+b^2x^6}\,\tanh^{-1}\left(\frac{\sqrt{a^2+2abx^3+b^2x^6}-\sqrt{b^2}\,x^3}{a}\right)-\frac{2}{3}b^3x^6\tanh^{-1}\left(\frac{\sqrt{a^2+2abx^3+b^2x^6}-\sqrt{b^2}\,x^3}{a}\right)+\frac{2}{3}ab\sqrt{b^2}\,x^3}{\left(\sqrt{a^2+2abx^3+b^2x^6}-a-\sqrt{b^2}\,x^3\right)\left(\sqrt{a^2+2abx^3+b^2x^6}+a-\sqrt{b^2}\,x^3\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]/x^4,x]
```

```
[Out] ((2*a*b*Sqrt[b^2]*x^3)/3 - (2*a*b*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/3 - (2*a
*b^2*x^3*ArcTanh[(-(Sqrt[b^2]*x^3) + Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/a])/3
 - (2*b^3*x^6*ArcTanh[(-(Sqrt[b^2]*x^3) + Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/
a])/3 + (2*b*Sqrt[b^2]*x^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]*ArcTanh[(-(Sqrt[
b^2]*x^3) + Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/a])/3)/((-a - Sqrt[b^2]*x^3 +
Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])*(a - Sqrt[b^2]*x^3 + Sqrt[a^2 + 2*a*b*x^3
+ b^2*x^6])) + ((2*a^2*Sqrt[b^2])/3 - (a*b*Sqrt[b^2]*x^3*Log[-a - Sqrt[b^2]
*x^3 + Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]])/3 - ((b^2)^(3/2)*x^6*Log[-a - Sqrt
[b^2]*x^3 + Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]])/3 + (b^2*x^3*Sqrt[a^2 + 2*a*b
*x^3 + b^2*x^6]*Log[-a - Sqrt[b^2]*x^3 + Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]])/
3 - (a*b*Sqrt[b^2]*x^3*Log[a - Sqrt[b^2]*x^3 + Sqrt[a^2 + 2*a*b*x^3 + b^2*x
^6]])/3 - ((b^2)^(3/2)*x^6*Log[a - Sqrt[b^2]*x^3 + Sqrt[a^2 + 2*a*b*x^3 + b
^2*x^6]])/3 + (b^2*x^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]*Log[a - Sqrt[b^2]*x^
3 + Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]])/3)/((-a - Sqrt[b^2]*x^3 + Sqrt[a^2 +
2*a*b*x^3 + b^2*x^6])*(a - Sqrt[b^2]*x^3 + Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])
)
```

fricas [A] time = 0.89, size = 17, normalized size = 0.23

$$\frac{3\,bx^3\log(x)-a}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^3+a)^2)^(1/2)/x^4,x, algorithm="fricas")
```

```
[Out] 1/3*(3*b*x^3*log(x) - a)/x^3
```

giac [A] time = 0.35, size = 43, normalized size = 0.57

$$b\log(|x|)\operatorname{sgn}\left(bx^3+a\right)-\frac{bx^3\operatorname{sgn}\left(bx^3+a\right)+a\operatorname{sgn}\left(bx^3+a\right)}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^3+a)^2)^(1/2)/x^4,x, algorithm="giac")
```

```
[Out] b*log(abs(x))*sgn(b*x^3 + a) - 1/3*(b*x^3*sgn(b*x^3 + a) + a*sgn(b*x^3 + a)
)/x^3
```

maple [A] time = 0.01, size = 38, normalized size = 0.51

$$\frac{\sqrt{\left(b\,x^3+a\right)^2}\;\left(3b\,x^3\ln(x)-a\right)}{3\left(b\,x^3+a\right)x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*x^3+a)^2)^(1/2)/x^4,x)
```

```
[Out] 1/3*((b*x^3+a)^2)^(1/2)*(3*b*ln(x)*x^3-a)/(b*x^3+a)/x^3
```

maxima [A] time = 0.46, size = 99, normalized size = 1.32

$$\frac{1}{3}\left(-1\right)^{2\,b^{2}x^{3}+2\,ab}b\log\left(2\,b^{2}x^{3}+2\,ab\right)-\frac{1}{3}\left(-1\right)^{2\,abx^{3}+2\,a^{2}}b\log\left(\frac{2\,abx}{|x|}+\frac{2\,a^{2}}{x^{2}|x|}\right)-\frac{\sqrt{b^{2}x^{6}+2\,abx^{3}+a^{2}}}{3\,x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^3+a)^2)^(1/2)/x^4,x, algorithm="maxima")
```

```
[Out] 1/3*(-1)^(2*b^2*x^3 + 2*a*b)*b*log(2*b^2*x^3 + 2*a*b) - 1/3*(-1)^(2*a*b*x^3
 + 2*a^2)*b*log(2*a*b*x/abs(x) + 2*a^2/(x^2*abs(x))) - 1/3*sqrt(b^2*x^6 + 2
*a*b*x^3 + a^2)/x^3
```

mupad [B] time = 1.38, size = 112, normalized size = 1.49

$$\frac{\ln\left(a\,b+\sqrt{\left(b\,x^{3}+a\right)^{2}}\,\sqrt{b^{2}}+b^{2}\,x^{3}\right)\sqrt{b^{2}}}{3}-\frac{\sqrt{a^{2}+2\,a\,b\,x^{3}+b^{2}\,x^{6}}}{3\,x^{3}}-\frac{a\,b\,\ln\left(a\,b+\frac{a^{2}}{x^{3}}+\frac{\sqrt{a^{2}}\,\sqrt{a^{2}+2\,a\,b\,x^{3}+b^{2}\,x^{6}}}{x^{3}}\right)}{3\,\sqrt{a^{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^3)^2)^(1/2)/x^4,x)
```

```
[Out] (log(a*b + ((a + b*x^3)^2)^(1/2)*(b^2)^(1/2) + b^2*x^3)*(b^2)^(1/2))/3 - (a
^2 + b^2*x^6 + 2*a*b*x^3)^(1/2)/(3*x^3) - (a*b*log(a*b + a^2/x^3 + ((a^2)^(
1/2)*(a^2 + b^2*x^6 + 2*a*b*x^3)^(1/2))/x^3))/(3*(a^2)^(1/2))
```

sympy [A] time = 0.16, size = 10, normalized size = 0.13

$$-\frac{a}{3x^{3}}+b\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x**3+a)**2)**(1/2)/x**4,x)
```

```
[Out] -a/(3*x**3) + b*log(x)
```

3.16 $\int \frac{\sqrt{a^2+2abx^3+b^2x^6}}{x^5}\,dx$

Optimal. Leaf size=77

$$-\frac{b\sqrt{a^2+2abx^3+b^2x^6}}{x\left(a+bx^3\right)}-\frac{a\sqrt{a^2+2abx^3+b^2x^6}}{4x^4\left(a+bx^3\right)}$$

Rubi [A] time = 0.02, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1355, 14}

$$-\frac{a\sqrt{a^2+2abx^3+b^2x^6}}{4x^4\left(a+bx^3\right)}-\frac{b\sqrt{a^2+2abx^3+b^2x^6}}{x\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]/x^5,x]
```

```
[Out] -(a*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(4*x^4*(a + b*x^3)) - (b*Sqrt[a^2 + 2*
a*b*x^3 + b^2*x^6])/(x*(a + b*x^3))
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{a^2+2abx^3+b^2x^6}}{x^5}\,dx &= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{ab+b^2x^3}{x^5}\,dx}{ab+b^2x^3}\\
&= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \left(\frac{ab}{x^5}+\frac{b^2}{x^2}\right)dx}{ab+b^2x^3}\\
&= -\frac{a\sqrt{a^2+2abx^3+b^2x^6}}{4x^4\left(a+bx^3\right)}-\frac{b\sqrt{a^2+2abx^3+b^2x^6}}{x\left(a+bx^3\right)}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 37, normalized size = 0.48

$$-\frac{\sqrt{\left(a+bx^3\right)^2}\left(a+4bx^3\right)}{4x^4\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]/x^5,x]
```

```
[Out] -1/4*(Sqrt[(a + b*x^3)^2]*(a + 4*b*x^3))/(x^4*(a + b*x^3))
```

IntegrateAlgebraic [A] time = 19.21, size = 39, normalized size = 0.51

$$\frac{\left(-a-4bx^3\right)\sqrt{\left(a+bx^3\right)^2}}{4x^4\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]/x^5,x]
[Out] ((-a - 4*b*x^3)*Sqrt[(a + b*x^3)^2])/(4*x^4*(a + b*x^3))
```

fricas [A] time = 1.15, size = 13, normalized size = 0.17

$$-\frac{4\,bx^3+a}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^3+a)^2)^(1/2)/x^5,x, algorithm="fricas")
[Out] -1/4*(4*b*x^3 + a)/x^4
```

giac [A] time = 0.35, size = 30, normalized size = 0.39

$$-\frac{4\,bx^3\mathrm{sgn}\left(bx^3+a\right)+a\mathrm{sgn}\left(bx^3+a\right)}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^3+a)^2)^(1/2)/x^5,x, algorithm="giac")
[Out] -1/4*(4*b*x^3*sgn(b*x^3 + a) + a*sgn(b*x^3 + a))/x^4
```

maple [A] time = 0.00, size = 34, normalized size = 0.44

$$-\frac{\left(4b\,x^3+a\right)\sqrt{\left(b\,x^3+a\right)^2}}{4\left(b\,x^3+a\right)x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*x^3+a)^2)^(1/2)/x^5,x)
[Out] -1/4*(4*b*x^3+a)*((b*x^3+a)^2)^(1/2)/x^4/(b*x^3+a)
```

maxima [A] time = 0.45, size = 13, normalized size = 0.17

$$-\frac{4\,bx^3+a}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^3+a)^2)^(1/2)/x^5,x, algorithm="maxima")
[Out] -1/4*(4*b*x^3 + a)/x^4
```

mupad [B] time = 1.21, size = 33, normalized size = 0.43

$$-\frac{\left(4\,b\,x^3+a\right)\sqrt{\left(b\,x^3+a\right)^2}}{4\,x^4\,\left(b\,x^3+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^3)^2)^(1/2)/x^5,x)
```

```
[Out] -((a + 4*b*x^3)*((a + b*x^3)^2)^(1/2))/(4*x^4*(a + b*x^3))
```

sympy [A] time = 0.17, size = 14, normalized size = 0.18

$$\frac{-a-4bx^3}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x**3+a)**2)**(1/2)/x**5,x)
```

```
[Out] (-a - 4*b*x**3)/(4*x**4)
```

3.17 $\int \frac{\sqrt{a^2+2abx^3+b^2x^6}}{x^6}\,dx$

Optimal. Leaf size=79

$$-\frac{a\sqrt{a^2+2abx^3+b^2x^6}}{5x^5\left(a+bx^3\right)}-\frac{b\sqrt{a^2+2abx^3+b^2x^6}}{2x^2\left(a+bx^3\right)}$$

Rubi [A] time = 0.02, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1355, 14}

$$-\frac{a\sqrt{a^2+2abx^3+b^2x^6}}{5x^5\left(a+bx^3\right)}-\frac{b\sqrt{a^2+2abx^3+b^2x^6}}{2x^2\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]/x^6,x]
```

```
[Out] -(a*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(5*x^5*(a + b*x^3)) - (b*Sqrt[a^2 + 2*
a*b*x^3 + b^2*x^6])/(2*x^2*(a + b*x^3))
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{a^2+2abx^3+b^2x^6}}{x^6}\,dx &= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{ab+b^2x^3}{x^6}\,dx}{ab+b^2x^3}\\ &= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \left(\frac{ab}{x^6}+\frac{b^2}{x^3}\right)dx}{ab+b^2x^3}\\ &= -\frac{a\sqrt{a^2+2abx^3+b^2x^6}}{5x^5\left(a+bx^3\right)}-\frac{b\sqrt{a^2+2abx^3+b^2x^6}}{2x^2\left(a+bx^3\right)}\end{aligned}$$

Mathematica [A] time = 0.01, size = 39, normalized size = 0.49

$$-\frac{\sqrt{\left(a+bx^3\right)^2}\left(2a+5bx^3\right)}{10x^5\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]/x^6,x]
```

```
[Out] -1/10*(Sqrt[(a + b*x^3)^2]*(2*a + 5*b*x^3))/(x^5*(a + b*x^3))
```

IntegrateAlgebraic [A] time = 18.83, size = 39, normalized size = 0.49

$$\frac{\left(-2a-5bx^3\right)\sqrt{\left(a+bx^3\right)^2}}{10x^5\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]/x^6,x]
[Out] ((-2*a - 5*b*x^3)*Sqrt[(a + b*x^3)^2])/(10*x^5*(a + b*x^3))
```

fricas [A] time = 0.95, size = 15, normalized size = 0.19

$$-\frac{5\,bx^3+2\,a}{10\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^3+a)^2)^(1/2)/x^6,x, algorithm="fricas")
[Out] -1/10*(5*b*x^3 + 2*a)/x^5
```

giac [A] time = 0.35, size = 31, normalized size = 0.39

$$-\frac{5\,bx^3\mathrm{sgn}\left(bx^3+a\right)+2\,a\mathrm{sgn}\left(bx^3+a\right)}{10\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^3+a)^2)^(1/2)/x^6,x, algorithm="giac")
[Out] -1/10*(5*b*x^3*sgn(b*x^3 + a) + 2*a*sgn(b*x^3 + a))/x^5
```

maple [A] time = 0.00, size = 36, normalized size = 0.46

$$-\frac{\left(5b\,x^3+2a\right)\sqrt{\left(b\,x^3+a\right)^2}}{10\left(b\,x^3+a\right)x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*x^3+a)^2)^(1/2)/x^6,x)
[Out] -1/10*(5*b*x^3+2*a)*((b*x^3+a)^2)^(1/2)/x^5/(b*x^3+a)
```

maxima [A] time = 0.46, size = 15, normalized size = 0.19

$$-\frac{5\,bx^3+2\,a}{10\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^3+a)^2)^(1/2)/x^6,x, algorithm="maxima")
[Out] -1/10*(5*b*x^3 + 2*a)/x^5
```

mupad [B] time = 1.18, size = 35, normalized size = 0.44

$$-\frac{\left(5\,b\,x^3+2\,a\right)\sqrt{\left(b\,x^3+a\right)^2}}{10\,x^5\,\left(b\,x^3+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^3)^2)^(1/2)/x^6,x)
```

```
[Out] -((2*a + 5*b*x^3)*((a + b*x^3)^2)^(1/2))/(10*x^5*(a + b*x^3))
```

sympy [A] time = 0.18, size = 15, normalized size = 0.19

$$\frac{-2a - 5bx^3}{10x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x**3+a)**2)**(1/2)/x**6,x)
```

```
[Out] (-2*a - 5*b*x**3)/(10*x**5)
```

3.18 $\int \frac{\sqrt{a^2+2abx^3+b^2x^6}}{x^7}\,dx$

Optimal. Leaf size=79

$$-\frac{a\sqrt{a^2+2abx^3+b^2x^6}}{6x^6\left(a+bx^3\right)}-\frac{b\sqrt{a^2+2abx^3+b^2x^6}}{3x^3\left(a+bx^3\right)}$$

Rubi [A] time = 0.02, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1355, 14}

$$-\frac{a\sqrt{a^2+2abx^3+b^2x^6}}{6x^6\left(a+bx^3\right)}-\frac{b\sqrt{a^2+2abx^3+b^2x^6}}{3x^3\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]/x^7,x]
```

```
[Out] -(a*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(6*x^6*(a + b*x^3)) - (b*Sqrt[a^2 + 2*
a*b*x^3 + b^2*x^6])/(3*x^3*(a + b*x^3))
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{a^2+2abx^3+b^2x^6}}{x^7}\,dx &= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{ab+b^2x^3}{x^7}\,dx}{ab+b^2x^3}\\ &= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \left(\frac{ab}{x^7}+\frac{b^2}{x^4}\right)dx}{ab+b^2x^3}\\ &= -\frac{a\sqrt{a^2+2abx^3+b^2x^6}}{6x^6\left(a+bx^3\right)}-\frac{b\sqrt{a^2+2abx^3+b^2x^6}}{3x^3\left(a+bx^3\right)}\end{aligned}$$

Mathematica [A] time = 0.01, size = 37, normalized size = 0.47

$$-\frac{\sqrt{\left(a+bx^3\right)^2}\left(a+2bx^3\right)}{6x^6\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]/x^7,x]
```

```
[Out] -1/6*(Sqrt[(a + b*x^3)^2]*(a + 2*b*x^3))/(x^6*(a + b*x^3))
```

IntegrateAlgebraic [A] time = 0.43, size = 118, normalized size = 1.49

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\left(-ab-2b^2x^3\right)+\sqrt{b^2}\left(a^2+3abx^3+2b^2x^6\right)}{6x^6\left(ab+b^2x^3\right)-6\sqrt{b^2}\,x^6\sqrt{a^2+2abx^3+b^2x^6}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]/x^7,x]
```

```
[Out] ((-(a*b) - 2*b^2*x^3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6] + Sqrt[b^2]*(a^2 + 3*
a*b*x^3 + 2*b^2*x^6))/(6*x^6*(a*b + b^2*x^3) - 6*Sqrt[b^2]*x^6*Sqrt[a^2 + 2
*a*b*x^3 + b^2*x^6])
```

fricas [A] time = 1.34, size = 13, normalized size = 0.16

$$-\frac{2\,bx^3+a}{6\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^3+a)^2)^(1/2)/x^7,x, algorithm="fricas")
```

```
[Out] -1/6*(2*b*x^3 + a)/x^6
```

giac [A] time = 0.34, size = 30, normalized size = 0.38

$$-\frac{2\,bx^3\mathrm{sgn}\left(bx^3+a\right)+a\mathrm{sgn}\left(bx^3+a\right)}{6\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^3+a)^2)^(1/2)/x^7,x, algorithm="giac")
```

```
[Out] -1/6*(2*b*x^3*sgn(b*x^3 + a) + a*sgn(b*x^3 + a))/x^6
```

maple [A] time = 0.00, size = 34, normalized size = 0.43

$$-\frac{\left(2b\,x^3+a\right)\sqrt{\left(b\,x^3+a\right)^2}}{6\left(b\,x^3+a\right)x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*x^3+a)^2)^(1/2)/x^7,x)
```

```
[Out] -1/6*(2*b*x^3+a)*((b*x^3+a)^2)^(1/2)/x^6/(b*x^3+a)
```

maxima [A] time = 0.49, size = 86, normalized size = 1.09

$$\frac{\sqrt{b^2x^6+2\,abx^3+a^2}\,b^2}{6\,a^2}+\frac{\sqrt{b^2x^6+2\,abx^3+a^2}\,b}{6\,ax^3}-\frac{\left(b^2x^6+2\,abx^3+a^2\right)^{\frac{3}{2}}}{6\,a^2x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^3+a)^2)^(1/2)/x^7,x, algorithm="maxima")
```

```
[Out] 1/6*sqrt(b^2*x^6 + 2*a*b*x^3 + a^2)*b^2/a^2 + 1/6*sqrt(b^2*x^6 + 2*a*b*x^3
+ a^2)*b/(a*x^3) - 1/6*(b^2*x^6 + 2*a*b*x^3 + a^2)^(3/2)/(a^2*x^6)
```

mupad [B] time = 1.18, size = 33, normalized size = 0.42

$$-\frac{\left(2\,b\,x^3+a\right)\,\sqrt{{\left(b\,x^3+a\right)}^2}}{6\,x^6\,\left(b\,x^3+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^3)^2)^(1/2)/x^7,x)
[Out] -((a + 2*b*x^3)*((a + b*x^3)^2)^(1/2))/(6*x^6*(a + b*x^3))
```

sympy [A] time = 0.19, size = 14, normalized size = 0.18

$$\frac{-a-2bx^3}{6x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x**3+a)**2)**(1/2)/x**7,x)
[Out] (-a - 2*b*x**3)/(6*x**6)
```

3.19 $\int \frac{\sqrt{a^2+2abx^3+b^2x^6}}{x^8}\,dx$

Optimal. Leaf size=79

$$-\frac{a\sqrt{a^2+2abx^3+b^2x^6}}{7x^7\left(a+bx^3\right)}-\frac{b\sqrt{a^2+2abx^3+b^2x^6}}{4x^4\left(a+bx^3\right)}$$

Rubi [A] time = 0.02, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1355, 14}

$$-\frac{a\sqrt{a^2+2abx^3+b^2x^6}}{7x^7\left(a+bx^3\right)}-\frac{b\sqrt{a^2+2abx^3+b^2x^6}}{4x^4\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]/x^8,x]
```

```
[Out] -(a*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(7*x^7*(a + b*x^3)) - (b*Sqrt[a^2 + 2*
a*b*x^3 + b^2*x^6])/(4*x^4*(a + b*x^3))
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{a^2+2abx^3+b^2x^6}}{x^8}\,dx &= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{ab+b^2x^3}{x^8}\,dx}{ab+b^2x^3}\\ &= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \left(\frac{ab}{x^8}+\frac{b^2}{x^5}\right)dx}{ab+b^2x^3}\\ &= -\frac{a\sqrt{a^2+2abx^3+b^2x^6}}{7x^7\left(a+bx^3\right)}-\frac{b\sqrt{a^2+2abx^3+b^2x^6}}{4x^4\left(a+bx^3\right)}\end{aligned}$$

Mathematica [A] time = 0.01, size = 39, normalized size = 0.49

$$-\frac{\sqrt{\left(a+bx^3\right)^2}\left(4a+7bx^3\right)}{28x^7\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]/x^8,x]
```

```
[Out] -1/28*(Sqrt[(a + b*x^3)^2]*(4*a + 7*b*x^3))/(x^7*(a + b*x^3))
```

IntegrateAlgebraic [A]   time = 19.28, size = 39, normalized size = 0.49

$$\frac{\left(-4a-7bx^3\right)\sqrt{\left(a+bx^3\right)^2}}{28x^7\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]/x^8,x]
[Out] ((-4*a - 7*b*x^3)*Sqrt[(a + b*x^3)^2])/(28*x^7*(a + b*x^3))
```

fricas [A]   time = 1.91, size = 15, normalized size = 0.19

$$-\frac{7\,bx^3+4\,a}{28\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^3+a)^2)^(1/2)/x^8,x, algorithm="fricas")
[Out] -1/28*(7*b*x^3 + 4*a)/x^7
```

giac [A]   time = 0.34, size = 31, normalized size = 0.39

$$-\frac{7\,bx^3\mathrm{sgn}\left(bx^3+a\right)+4\,a\mathrm{sgn}\left(bx^3+a\right)}{28\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^3+a)^2)^(1/2)/x^8,x, algorithm="giac")
[Out] -1/28*(7*b*x^3*sgn(b*x^3 + a) + 4*a*sgn(b*x^3 + a))/x^7
```

maple [A]   time = 0.00, size = 36, normalized size = 0.46

$$-\frac{\left(7b\,x^3+4a\right)\sqrt{\left(b\,x^3+a\right)^2}}{28\left(b\,x^3+a\right)x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*x^3+a)^2)^(1/2)/x^8,x)
[Out] -1/28*(7*b*x^3+4*a)*((b*x^3+a)^2)^(1/2)/x^7/(b*x^3+a)
```

maxima [A]   time = 0.46, size = 15, normalized size = 0.19

$$-\frac{7\,bx^3+4\,a}{28\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^3+a)^2)^(1/2)/x^8,x, algorithm="maxima")
[Out] -1/28*(7*b*x^3 + 4*a)/x^7
```

mupad [B]   time = 1.27, size = 35, normalized size = 0.44

$$-\frac{\left(7\,b\,x^3+4\,a\right)\,\sqrt{\left(b\,x^3+a\right)^2}}{28\,x^7\,\left(b\,x^3+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^3)^2)^(1/2)/x^8,x)
```

```
[Out] -((4*a + 7*b*x^3)*((a + b*x^3)^2)^(1/2))/(28*x^7*(a + b*x^3))
```

sympy [A] time = 0.19, size = 15, normalized size = 0.19

$$\frac{-4a-7bx^3}{28x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x**3+a)**2)**(1/2)/x**8,x)
```

```
[Out] (-4*a - 7*b*x**3)/(28*x**7)
```

3.20 $\int \frac{\sqrt{a^2+2abx^3+b^2x^6}}{x^9}\,dx$

Optimal. Leaf size=79

$$-\frac{a\sqrt{a^2+2abx^3+b^2x^6}}{8x^8\left(a+bx^3\right)}-\frac{b\sqrt{a^2+2abx^3+b^2x^6}}{5x^5\left(a+bx^3\right)}$$

Rubi [A] time = 0.02, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1355, 14}

$$-\frac{a\sqrt{a^2+2abx^3+b^2x^6}}{8x^8\left(a+bx^3\right)}-\frac{b\sqrt{a^2+2abx^3+b^2x^6}}{5x^5\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]/x^9,x]
```

```
[Out] -(a*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(8*x^8*(a + b*x^3)) - (b*Sqrt[a^2 + 2*
a*b*x^3 + b^2*x^6])/(5*x^5*(a + b*x^3))
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{a^2+2abx^3+b^2x^6}}{x^9}\,dx &= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{ab+b^2x^3}{x^9}\,dx}{ab+b^2x^3}\\
&= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \left(\frac{ab}{x^9}+\frac{b^2}{x^6}\right)dx}{ab+b^2x^3}\\
&= -\frac{a\sqrt{a^2+2abx^3+b^2x^6}}{8x^8\left(a+bx^3\right)}-\frac{b\sqrt{a^2+2abx^3+b^2x^6}}{5x^5\left(a+bx^3\right)}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 39, normalized size = 0.49

$$-\frac{\sqrt{\left(a+bx^3\right)^2}\left(5a+8bx^3\right)}{40x^8\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]/x^9,x]
```

```
[Out] -1/40*(Sqrt[(a + b*x^3)^2]*(5*a + 8*b*x^3))/(x^8*(a + b*x^3))
```

IntegrateAlgebraic [A] time = 19.24, size = 39, normalized size = 0.49

$$\frac{\left(-5a-8bx^3\right)\sqrt{\left(a+bx^3\right)^2}}{40x^8\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]/x^9,x]
[Out] ((-5*a - 8*b*x^3)*Sqrt[(a + b*x^3)^2])/(40*x^8*(a + b*x^3))
```

fricas [A] time = 1.07, size = 15, normalized size = 0.19

$$-\frac{8\,bx^3+5\,a}{40\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^3+a)^2)^(1/2)/x^9,x, algorithm="fricas")
[Out] -1/40*(8*b*x^3 + 5*a)/x^8
```

giac [A] time = 0.38, size = 31, normalized size = 0.39

$$-\frac{8\,bx^3\mathrm{sgn}\left(bx^3+a\right)+5\,a\mathrm{sgn}\left(bx^3+a\right)}{40\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^3+a)^2)^(1/2)/x^9,x, algorithm="giac")
[Out] -1/40*(8*b*x^3*sgn(b*x^3 + a) + 5*a*sgn(b*x^3 + a))/x^8
```

maple [A] time = 0.00, size = 36, normalized size = 0.46

$$-\frac{\left(8b\,x^3+5a\right)\sqrt{\left(b\,x^3+a\right)^2}}{40\left(b\,x^3+a\right)x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*x^3+a)^2)^(1/2)/x^9,x)
[Out] -1/40*(8*b*x^3+5*a)*((b*x^3+a)^2)^(1/2)/x^8/(b*x^3+a)
```

maxima [A] time = 0.45, size = 15, normalized size = 0.19

$$-\frac{8\,bx^3+5\,a}{40\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^3+a)^2)^(1/2)/x^9,x, algorithm="maxima")
[Out] -1/40*(8*b*x^3 + 5*a)/x^8
```

mupad [B] time = 1.16, size = 35, normalized size = 0.44

$$-\frac{\left(8\,b\,x^3+5\,a\right)\sqrt{\left(b\,x^3+a\right)^2}}{40\,x^8\,\left(b\,x^3+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^3)^2)^(1/2)/x^9,x)
```

```
[Out] -((5*a + 8*b*x^3)*((a + b*x^3)^2)^(1/2))/(40*x^8*(a + b*x^3))
```

sympy [A] time = 0.20, size = 15, normalized size = 0.19

$$\frac{-5a - 8bx^3}{40x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x**3+a)**2)**(1/2)/x**9,x)
```

```
[Out] (-5*a - 8*b*x**3)/(40*x**8)
```

3.21 $\int \frac{\sqrt{a^2+2abx^3+b^2x^6}}{x^{10}}\,dx$

Optimal. Leaf size=79

$$-\frac{b\sqrt{a^2+2abx^3+b^2x^6}}{6x^6\left(a+bx^3\right)}-\frac{a\sqrt{a^2+2abx^3+b^2x^6}}{9x^9\left(a+bx^3\right)}$$

Rubi [A] time = 0.02, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1355, 14}

$$-\frac{a\sqrt{a^2+2abx^3+b^2x^6}}{9x^9\left(a+bx^3\right)}-\frac{b\sqrt{a^2+2abx^3+b^2x^6}}{6x^6\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]/x^10,x]
```

```
[Out] -(a*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(9*x^9*(a + b*x^3)) - (b*Sqrt[a^2 + 2*
a*b*x^3 + b^2*x^6])/(6*x^6*(a + b*x^3))
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{a^2+2abx^3+b^2x^6}}{x^{10}}\,dx &= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{ab+b^2x^3}{x^{10}}\,dx}{ab+b^2x^3}\\ &= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \left(\frac{ab}{x^{10}}+\frac{b^2}{x^7}\right)dx}{ab+b^2x^3}\\ &= -\frac{a\sqrt{a^2+2abx^3+b^2x^6}}{9x^9\left(a+bx^3\right)}-\frac{b\sqrt{a^2+2abx^3+b^2x^6}}{6x^6\left(a+bx^3\right)}\end{aligned}$$

Mathematica [A] time = 0.01, size = 39, normalized size = 0.49

$$-\frac{\sqrt{\left(a+bx^3\right)^2}\left(2a+3bx^3\right)}{18x^9\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]/x^10,x]
```

```
[Out] -1/18*(Sqrt[(a + b*x^3)^2]*(2*a + 3*b*x^3))/(x^9*(a + b*x^3))
```

IntegrateAlgebraic [B] time = 3.56, size = 753, normalized size = 9.53

$$\frac{2\sqrt{a^2+2abx^3+b^2x^6}\left(-2a^{15}b^3-55a^{14}b^4x^3-704a^{13}b^5x^6-5563a^{12}b^6x^9-30344a^{11}b^7x^{12}-121000a^{10}b^8x^{15}-364320a^9b^9x^{18}-843216a^8b^{10}x^{21}-1512192a^7b^{11}x^{24}-2100736a^6b^{12}x^{27}-2241536a^5b^{13}x^{30}-1803520a^4b^{14}x^{33}-1058816a^3b^{15}x^{36}-428032a^2b^{16}x^{39}-106496ab^{17}x^{42}-12288b^{18}x^{45}\right)+2\sqrt{b^2}\left(2a^{16}b^2+57a^{15}b^3x^3+759a^{14}b^4x^6+6267a^{13}b^5x^9+35907a^{12}b^6x^{12}+151344a^{11}b^7x^{15}+485320a^{10}b^8x^{18}+1207536a^9b^9x^{21}+2355408a^8b^{10}x^{24}+3612928a^7b^{11}x^{27}+4342272a^6b^{12}x^{30}+4045056a^5b^{13}x^{33}+2862336a^4b^{14}x^{36}+1486848a^3b^{15}x^{39}+534528a^2b^{16}x^{42}+118784ab^{17}x^{45}+12288b^{18}x^{48}\right)}{9\sqrt{b^2}\,x^9\sqrt{a^2+2abx^3+b^2x^6}\left(-4a^{14}b^2-104a^{13}b^3x^3-1252a^{12}b^4x^6-9248a^{11}b^5x^9-46816a^{10}b^6x^{12}-171776a^9b^7x^{15}-470976a^8b^8x^{18}-979968a^7b^9x^{21}-1554432a^6b^{10}x^{24}-1869824a^5b^{11}x^{27}-1678336a^4b^{12}x^{30}-1089536a^3b^{13}x^{33}-483328a^2b^{14}x^{36}-131072ab^{15}x^{39}-16384b^{16}x^{42}\right)+9x^9\left(4a^{15}b^3+108a^{14}b^4x^3+1356a^{13}b^5x^6+10500a^{12}b^6x^9+56064a^{11}b^7x^{12}+218592a^{10}b^8x^{15}+642752a^9b^9x^{18}+1450944a^8b^{10}x^{21}+2534400a^7b^{11}x^{24}+3424256a^6b^{12}x^{27}+3548160a^5b^{13}x^{30}+2767872a^4b^{14}x^{33}+1572864a^3b^{15}x^{36}+614400a^2b^{16}x^{39}+147456ab^{17}x^{42}+16384b^{18}x^{45}\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]/x^10,x]
```

```
[Out] (2*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]*(-2*a^15*b^3 - 55*a^14*b^4*x^3 - 704*a^1
3*b^5*x^6 - 5563*a^12*b^6*x^9 - 30344*a^11*b^7*x^12 - 121000*a^10*b^8*x^15
- 364320*a^9*b^9*x^18 - 843216*a^8*b^10*x^21 - 1512192*a^7*b^11*x^24 - 2100
736*a^6*b^12*x^27 - 2241536*a^5*b^13*x^30 - 1803520*a^4*b^14*x^33 - 1058816
*a^3*b^15*x^36 - 428032*a^2*b^16*x^39 - 106496*a*b^17*x^42 - 12288*b^18*x^4
5) + 2*Sqrt[b^2]*(2*a^16*b^2 + 57*a^15*b^3*x^3 + 759*a^14*b^4*x^6 + 6267*a^
13*b^5*x^9 + 35907*a^12*b^6*x^12 + 151344*a^11*b^7*x^15 + 485320*a^10*b^8*x
^18 + 1207536*a^9*b^9*x^21 + 2355408*a^8*b^10*x^24 + 3612928*a^7*b^11*x^27
+ 4342272*a^6*b^12*x^30 + 4045056*a^5*b^13*x^33 + 2862336*a^4*b^14*x^36 + 1
486848*a^3*b^15*x^39 + 534528*a^2*b^16*x^42 + 118784*a*b^17*x^45 + 12288*b^
18*x^48))/(9*Sqrt[b^2]*x^9*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]*(-4*a^14*b^2 - 1
04*a^13*b^3*x^3 - 1252*a^12*b^4*x^6 - 9248*a^11*b^5*x^9 - 46816*a^10*b^6*x^
12 - 171776*a^9*b^7*x^15 - 470976*a^8*b^8*x^18 - 979968*a^7*b^9*x^21 - 1554
432*a^6*b^10*x^24 - 1869824*a^5*b^11*x^27 - 1678336*a^4*b^12*x^30 - 1089536
*a^3*b^13*x^33 - 483328*a^2*b^14*x^36 - 131072*a*b^15*x^39 - 16384*b^16*x^4
2) + 9*x^9*(4*a^15*b^3 + 108*a^14*b^4*x^3 + 1356*a^13*b^5*x^6 + 10500*a^12*
b^6*x^9 + 56064*a^11*b^7*x^12 + 218592*a^10*b^8*x^15 + 642752*a^9*b^9*x^18
+ 1450944*a^8*b^10*x^21 + 2534400*a^7*b^11*x^24 + 3424256*a^6*b^12*x^27 + 3
548160*a^5*b^13*x^30 + 2767872*a^4*b^14*x^33 + 1572864*a^3*b^15*x^36 + 6144
00*a^2*b^16*x^39 + 147456*a*b^17*x^42 + 16384*b^18*x^45))
```

fricas [A] time = 1.37, size = 15, normalized size = 0.19

$$-\frac{3\,bx^3+2\,a}{18\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^3+a)^2)^(1/2)/x^10,x, algorithm="fricas")
```

```
[Out] -1/18*(3*b*x^3 + 2*a)/x^9
```

giac [A] time = 0.36, size = 31, normalized size = 0.39

$$-\frac{3\,bx^3\mathrm{sgn}\left(bx^3+a\right)+2\,a\mathrm{sgn}\left(bx^3+a\right)}{18\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^3+a)^2)^(1/2)/x^10,x, algorithm="giac")
```

```
[Out] -1/18*(3*b*x^3*sgn(b*x^3 + a) + 2*a*sgn(b*x^3 + a))/x^9
```

maple [A] time = 0.00, size = 36, normalized size = 0.46

$$-\frac{\left(3b\,x^3+2a\right)\sqrt{\left(b\,x^3+a\right)^2}}{18\left(b\,x^3+a\right)x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*x^3+a)^2)^(1/2)/x^10,x)
```

```
[Out] -1/18*(3*b*x^3+2*a)*((b*x^3+a)^2)^(1/2)/x^9/(b*x^3+a)
```

maxima [B] time = 0.47, size = 117, normalized size = 1.48

$$-\frac{\sqrt{b^2x^6 + 2\,abx^3 + a^2}\,b^3}{6\,a^3} - \frac{\sqrt{b^2x^6 + 2\,abx^3 + a^2}\,b^2}{6\,a^2x^3} + \frac{\left(b^2x^6 + 2\,abx^3 + a^2\right)^{\frac{3}{2}}b}{6\,a^3x^6} - \frac{\left(b^2x^6 + 2\,abx^3 + a^2\right)^{\frac{3}{2}}}{9\,a^2x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^3+a)^2)^(1/2)/x^10,x, algorithm="maxima")
```

```
[Out] -1/6*sqrt(b^2*x^6 + 2*a*b*x^3 + a^2)*b^3/a^3 - 1/6*sqrt(b^2*x^6 + 2*a*b*x^3
 + a^2)*b^2/(a^2*x^3) + 1/6*(b^2*x^6 + 2*a*b*x^3 + a^2)^(3/2)*b/(a^3*x^6) -
 1/9*(b^2*x^6 + 2*a*b*x^3 + a^2)^(3/2)/(a^2*x^9)
```

mupad [B] time = 1.15, size = 35, normalized size = 0.44

$$-\frac{\left(3\,b\,x^3 + 2\,a\right)\,\sqrt{{\left(b\,x^3 + a\right)}^2}}{18\,x^9\,\left(b\,x^3 + a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^3)^2)^(1/2)/x^10,x)
```

```
[Out] -((2*a + 3*b*x^3)*((a + b*x^3)^2)^(1/2))/(18*x^9*(a + b*x^3))
```

sympy [A] time = 0.21, size = 15, normalized size = 0.19

$$\frac{-2a - 3bx^3}{18x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x**3+a)**2)**(1/2)/x**10,x)
```

```
[Out] (-2*a - 3*b*x**3)/(18*x**9)
```

3.22 $\int \frac{\sqrt{a^2+2abx^3+b^2x^6}}{x^{11}}\,dx$

Optimal. Leaf size=79

$$-\frac{a\sqrt{a^2+2abx^3+b^2x^6}}{10x^{10}\left(a+bx^3\right)}-\frac{b\sqrt{a^2+2abx^3+b^2x^6}}{7x^7\left(a+bx^3\right)}$$

Rubi [A] time = 0.02, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1355, 14}

$$-\frac{a\sqrt{a^2+2abx^3+b^2x^6}}{10x^{10}\left(a+bx^3\right)}-\frac{b\sqrt{a^2+2abx^3+b^2x^6}}{7x^7\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]/x^11,x]
```

```
[Out] -(a*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(10*x^10*(a + b*x^3)) - (b*Sqrt[a^2 +
2*a*b*x^3 + b^2*x^6])/(7*x^7*(a + b*x^3))
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{a^2+2abx^3+b^2x^6}}{x^{11}}\,dx &= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{ab+b^2x^3}{x^{11}}\,dx}{ab+b^2x^3}\\
&= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \left(\frac{ab}{x^{11}}+\frac{b^2}{x^8}\right)dx}{ab+b^2x^3}\\
&= -\frac{a\sqrt{a^2+2abx^3+b^2x^6}}{10x^{10}\left(a+bx^3\right)}-\frac{b\sqrt{a^2+2abx^3+b^2x^6}}{7x^7\left(a+bx^3\right)}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 39, normalized size = 0.49

$$-\frac{\sqrt{\left(a+bx^3\right)^2}\left(7a+10bx^3\right)}{70x^{10}\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]/x^11,x]
```

```
[Out] -1/70*(Sqrt[(a + b*x^3)^2]*(7*a + 10*b*x^3))/(x^10*(a + b*x^3))
```

IntegrateAlgebraic [A] time = 20.70, size = 39, normalized size = 0.49

$$\frac{\left(-7a-10bx^3\right)\sqrt{\left(a+bx^3\right)^2}}{70x^{10}\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]/x^11,x]
[Out] ((-7*a - 10*b*x^3)*Sqrt[(a + b*x^3)^2])/(70*x^10*(a + b*x^3))
```

fricas [A] time = 1.40, size = 15, normalized size = 0.19

$$-\frac{10\,bx^3+7\,a}{70\,x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^3+a)^2)^(1/2)/x^11,x, algorithm="fricas")
[Out] -1/70*(10*b*x^3 + 7*a)/x^10
```

giac [A] time = 0.31, size = 31, normalized size = 0.39

$$-\frac{10\,bx^3\mathrm{sgn}\left(bx^3+a\right)+7\,a\mathrm{sgn}\left(bx^3+a\right)}{70\,x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^3+a)^2)^(1/2)/x^11,x, algorithm="giac")
[Out] -1/70*(10*b*x^3*sgn(b*x^3 + a) + 7*a*sgn(b*x^3 + a))/x^10
```

maple [A] time = 0.00, size = 36, normalized size = 0.46

$$-\frac{\left(10b\,x^3+7a\right)\sqrt{\left(b\,x^3+a\right)^2}}{70\left(b\,x^3+a\right)x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*x^3+a)^2)^(1/2)/x^11,x)
[Out] -1/70*(10*b*x^3+7*a)*((b*x^3+a)^2)^(1/2)/x^10/(b*x^3+a)
```

maxima [A] time = 0.45, size = 15, normalized size = 0.19

$$-\frac{10\,bx^3+7\,a}{70\,x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^3+a)^2)^(1/2)/x^11,x, algorithm="maxima")
[Out] -1/70*(10*b*x^3 + 7*a)/x^10
```

mupad [B] time = 1.16, size = 35, normalized size = 0.44

$$-\frac{\left(10\,b\,x^3+7\,a\right)\,\sqrt{\left(b\,x^3+a\right)^2}}{70\,x^{10}\,\left(b\,x^3+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^3)^2)^(1/2)/x^11,x)
```

```
[Out] -((7*a + 10*b*x^3)*((a + b*x^3)^2)^(1/2))/(70*x^10*(a + b*x^3))
```

sympy [A] time = 0.22, size = 15, normalized size = 0.19

$$\frac{-7a - 10bx^3}{70x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x**3+a)**2)**(1/2)/x**11,x)
```

```
[Out] (-7*a - 10*b*x**3)/(70*x**10)
```

3.23 $\int x^9 \left(a^2+2abx^3+b^2x^6\right)^{3/2} dx$

Optimal. Leaf size=167

$$\frac{3ab^2x^{16}\sqrt{a^2+2abx^3+b^2x^6}}{16\left(a+bx^3\right)}+\frac{3a^2bx^{13}\sqrt{a^2+2abx^3+b^2x^6}}{13\left(a+bx^3\right)}+\frac{b^3x^{19}\sqrt{a^2+2abx^3+b^2x^6}}{19\left(a+bx^3\right)}+\frac{a^3x^{10}\sqrt{a^2+2abx^3+b^2x}}{10\left(a+bx^3\right)}$$

Rubi [A] time = 0.04, antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1355, 270}

$$\frac{b^3x^{19}\sqrt{a^2+2abx^3+b^2x^6}}{19\left(a+bx^3\right)}+\frac{3ab^2x^{16}\sqrt{a^2+2abx^3+b^2x^6}}{16\left(a+bx^3\right)}+\frac{3a^2bx^{13}\sqrt{a^2+2abx^3+b^2x^6}}{13\left(a+bx^3\right)}+\frac{a^3x^{10}\sqrt{a^2+2abx^3+b^2x^6}}{10\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x^9*(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2),x]
```

```
[Out] (a^3*x^10*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(10*(a + b*x^3)) + (3*a^2*b*x^13
*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(13*(a + b*x^3)) + (3*a*b^2*x^16*Sqrt[a^2
 + 2*a*b*x^3 + b^2*x^6])/(16*(a + b*x^3)) + (b^3*x^19*Sqrt[a^2 + 2*a*b*x^3
+ b^2*x^6])/(19*(a + b*x^3))
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\begin{aligned}
\int x^9\left(a^2+2abx^3+b^2x^6\right)^{3/2} dx &= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int x^9\left(ab+b^2x^3\right)^3 dx}{b^2\left(ab+b^2x^3\right)}\\
&= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int\left(a^3b^3x^9+3a^2b^4x^{12}+3ab^5x^{15}+b^6x^{18}\right)dx}{b^2\left(ab+b^2x^3\right)}\\
&= \frac{a^3x^{10}\sqrt{a^2+2abx^3+b^2x^6}}{10\left(a+bx^3\right)}+\frac{3a^2bx^{13}\sqrt{a^2+2abx^3+b^2x^6}}{13\left(a+bx^3\right)}+\frac{3ab^2x^{16}\sqrt{a^2+2a}}{16\left(a+bx\right.}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 61, normalized size = 0.37

$$\frac{x^{10}\sqrt{\left(a+bx^3\right)^2}\left(1976a^3+4560a^2bx^3+3705ab^2x^6+1040b^3x^9\right)}{19760\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^9*(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2),x]
```

```
[Out] (x^10*Sqrt[(a + b*x^3)^2]*(1976*a^3 + 4560*a^2*b*x^3 + 3705*a*b^2*x^6 + 104
0*b^3*x^9))/(19760*(a + b*x^3))
```

IntegrateAlgebraic [A] time = 11.61, size = 61, normalized size = 0.37

$$\frac{\sqrt{\left(a+bx^3\right)^2}\left(1976a^3x^{10}+4560a^2bx^{13}+3705ab^2x^{16}+1040b^3x^{19}\right)}{19760\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^9*(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2),x]
```

```
[Out] (Sqrt[(a + b*x^3)^2]*(1976*a^3*x^10 + 4560*a^2*b*x^13 + 3705*a*b^2*x^16 + 1
040*b^3*x^19))/(19760*(a + b*x^3))
```

fricas [A] time = 1.22, size = 35, normalized size = 0.21

$$\frac{1}{19}b^3x^{19}+\frac{3}{16}ab^2x^{16}+\frac{3}{13}a^2bx^{13}+\frac{1}{10}a^3x^{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9*(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/19*b^3*x^19 + 3/16*a*b^2*x^16 + 3/13*a^2*b*x^13 + 1/10*a^3*x^10
```

giac [A] time = 0.39, size = 67, normalized size = 0.40

$$\frac{1}{19}b^3x^{19}\mathrm{sgn}\left(bx^3+a\right)+\frac{3}{16}ab^2x^{16}\mathrm{sgn}\left(bx^3+a\right)+\frac{3}{13}a^2bx^{13}\mathrm{sgn}\left(bx^3+a\right)+\frac{1}{10}a^3x^{10}\mathrm{sgn}\left(bx^3+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9*(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/19*b^3*x^19*sgn(b*x^3 + a) + 3/16*a*b^2*x^16*sgn(b*x^3 + a) + 3/13*a^2*b*
x^13*sgn(b*x^3 + a) + 1/10*a^3*x^10*sgn(b*x^3 + a)
```

maple [A] time = 0.01, size = 58, normalized size = 0.35

$$\frac{\left(1040b^3x^9+3705a\,b^2x^6+4560a^2b\,x^3+1976a^3\right)\left(\left(b\,x^3+a\right)^2\right)^{\frac{3}{2}}x^{10}}{19760\left(b\,x^3+a\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9*(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x)
```

```
[Out] 1/19760*x^10*(1040*b^3*x^9+3705*a*b^2*x^6+4560*a^2*b*x^3+1976*a^3)*((b*x^3+
a)^2)^(3/2)/(b*x^3+a)^3
```

maxima [A] time = 0.47, size = 35, normalized size = 0.21

$$\frac{1}{19}b^3x^{19}+\frac{3}{16}ab^2x^{16}+\frac{3}{13}a^2bx^{13}+\frac{1}{10}a^3x^{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9*(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/19*b^3*x^19 + 3/16*a*b^2*x^16 + 3/13*a^2*b*x^13 + 1/10*a^3*x^10
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^9 \left(a^2 + 2\,a\,b\,x^3 + b^2\,x^6\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9*(a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2),x)
[Out] int(x^9*(a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^9 \left(\left(a + bx^3\right)^2\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**9*(b**2*x**6+2*a*b*x**3+a**2)**(3/2),x)
[Out] Integral(x**9*((a + b*x**3)**2)**(3/2), x)
```

3.24 $\int x^8 \left(a^2+2abx^3+b^2x^6\right)^{3/2}\,dx$

Optimal. Leaf size=119

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\left(a+bx^3\right)^5}{18b^3}-\frac{2a\sqrt{a^2+2abx^3+b^2x^6}\left(a+bx^3\right)^4}{15b^3}+\frac{a^2\sqrt{a^2+2abx^3+b^2x^6}\left(a+bx^3\right)^3}{12b^3}$$

Rubi [A] time = 0.05, antiderivative size = 167, normalized size of antiderivative = 1.40, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1355, 266, 43}

$$\frac{b^3x^{18}\sqrt{a^2+2abx^3+b^2x^6}}{18\left(a+bx^3\right)}+\frac{ab^2x^{15}\sqrt{a^2+2abx^3+b^2x^6}}{5\left(a+bx^3\right)}+\frac{a^2bx^{12}\sqrt{a^2+2abx^3+b^2x^6}}{4\left(a+bx^3\right)}+\frac{a^3x^9\sqrt{a^2+2abx^3+b^2x^6}}{9\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x^8*(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2),x]
```

```
[Out] (a^3*x^9*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(9*(a + b*x^3)) + (a^2*b*x^12*Sqr
t[a^2 + 2*a*b*x^3 + b^2*x^6])/(4*(a + b*x^3)) + (a*b^2*x^15*Sqrt[a^2 + 2*a*
b*x^3 + b^2*x^6])/(5*(a + b*x^3)) + (b^3*x^18*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^
6])/(18*(a + b*x^3))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\begin{aligned}\int x^8\left(a^2+2abx^3+b^2x^6\right)^{3/2}\,dx &= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int x^8\left(ab+b^2x^3\right)^3\,dx}{b^2\left(ab+b^2x^3\right)}\\ &= \frac{\sqrt{a^2+2abx^3+b^2x^6}\,\text{Subst}\left(\int x^2\left(ab+b^2x\right)^3\,dx,x,x^3\right)}{3b^2\left(ab+b^2x^3\right)}\\ &= \frac{\sqrt{a^2+2abx^3+b^2x^6}\,\text{Subst}\left(\int\left(a^3b^3x^2+3a^2b^4x^3+3ab^5x^4+b^6x^5\right)dx,x,x^3\right)}{3b^2\left(ab+b^2x^3\right)}\\ &= \frac{a^3x^9\sqrt{a^2+2abx^3+b^2x^6}}{9\left(a+bx^3\right)}+\frac{a^2bx^{12}\sqrt{a^2+2abx^3+b^2x^6}}{4\left(a+bx^3\right)}+\frac{ab^2x^{15}\sqrt{a^2+2ab}}{5\left(a+bx\right.}\end{aligned}$$

Mathematica [A] time = 0.02, size = 61, normalized size = 0.51

$$\frac{x^9\sqrt{\left(a+bx^3\right)^2}\left(20a^3+45a^2bx^3+36ab^2x^6+10b^3x^9\right)}{180\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8*(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2),x]
```

```
[Out] (x^9*Sqrt[(a + b*x^3)^2]*(20*a^3 + 45*a^2*b*x^3 + 36*a*b^2*x^6 + 10*b^3*x^9
))/(180*(a + b*x^3))
```

IntegrateAlgebraic [A] time = 10.09, size = 61, normalized size = 0.51

$$\frac{x^9\sqrt{\left(a+bx^3\right)^2}\left(20a^3+45a^2bx^3+36ab^2x^6+10b^3x^9\right)}{180\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^8*(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2),x]
```

```
[Out] (x^9*Sqrt[(a + b*x^3)^2]*(20*a^3 + 45*a^2*b*x^3 + 36*a*b^2*x^6 + 10*b^3*x^9
))/(180*(a + b*x^3))
```

fricas [A] time = 0.76, size = 35, normalized size = 0.29

$$\frac{1}{18}\,b^3x^{18}+\frac{1}{5}\,ab^2x^{15}+\frac{1}{4}\,a^2bx^{12}+\frac{1}{9}\,a^3x^9$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/18*b^3*x^18 + 1/5*a*b^2*x^15 + 1/4*a^2*b*x^12 + 1/9*a^3*x^9
```

giac [A] time = 0.30, size = 67, normalized size = 0.56

$$\frac{1}{18}\,b^3x^{18}\mathrm{sgn}\left(bx^3+a\right)+\frac{1}{5}\,ab^2x^{15}\mathrm{sgn}\left(bx^3+a\right)+\frac{1}{4}\,a^2bx^{12}\mathrm{sgn}\left(bx^3+a\right)+\frac{1}{9}\,a^3x^9\mathrm{sgn}\left(bx^3+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/18*b^3*x^18*sgn(b*x^3 + a) + 1/5*a*b^2*x^15*sgn(b*x^3 + a) + 1/4*a^2*b*x^
12*sgn(b*x^3 + a) + 1/9*a^3*x^9*sgn(b*x^3 + a)
```

maple [A] time = 0.01, size = 58, normalized size = 0.49

$$\frac{\left(10b^3x^9+36a\,b^2x^6+45a^2b\,x^3+20a^3\right)\left(\left(b\,x^3+a\right)^2\right)^{\frac{3}{2}}x^9}{180\left(b\,x^3+a\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x)
```

```
[Out] 1/180*x^9*(10*b^3*x^9+36*a*b^2*x^6+45*a^2*b*x^3+20*a^3)*((b*x^3+a)^2)^(3/2)
/(b*x^3+a)^3
```

maxima [A] time = 0.46, size = 114, normalized size = 0.96

$$\frac{\left(b^2x^6+2\,abx^3+a^2\right)^{\frac{3}{2}}a^2x^3}{12\,b^2}+\frac{\left(b^2x^6+2\,abx^3+a^2\right)^{\frac{5}{2}}x^3}{18\,b^2}+\frac{\left(b^2x^6+2\,abx^3+a^2\right)^{\frac{3}{2}}a^3}{12\,b^3}-\frac{7\left(b^2x^6+2\,abx^3+a^2\right)^{\frac{5}{2}}a}{90\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/12*(b^2*x^6 + 2*a*b*x^3 + a^2)^(3/2)*a^2*x^3/b^2 + 1/18*(b^2*x^6 + 2*a*b*
x^3 + a^2)^(5/2)*x^3/b^2 + 1/12*(b^2*x^6 + 2*a*b*x^3 + a^2)^(3/2)*a^3/b^3 -
 7/90*(b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2)*a/b^3
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^8\left(a^2+2\,a\,b\,x^3+b^2\,x^6\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*(a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2),x)
```

```
[Out] int(x^8*(a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^8\left(\left(a+bx^3\right)^2\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8*(b**2*x**6+2*a*b*x**3+a**2)**(3/2),x)
```

```
[Out] Integral(x**8*((a + b*x**3)**2)**(3/2), x)
```

3.25 $\int x^7 \left(a^2+2abx^3+b^2x^6\right)^{3/2} \, dx$

Optimal. Leaf size=167

$$\frac{3ab^2x^{14}\sqrt{a^2+2abx^3+b^2x^6}}{14\left(a+bx^3\right)}+\frac{3a^2bx^{11}\sqrt{a^2+2abx^3+b^2x^6}}{11\left(a+bx^3\right)}+\frac{b^3x^{17}\sqrt{a^2+2abx^3+b^2x^6}}{17\left(a+bx^3\right)}+\frac{a^3x^8\sqrt{a^2+2abx^3+b^2x^6}}{8\left(a+bx^3\right)}$$

Rubi [A] time = 0.04, antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {1355, 270}

$$\frac{b^3x^{17}\sqrt{a^2+2abx^3+b^2x^6}}{17\left(a+bx^3\right)}+\frac{3ab^2x^{14}\sqrt{a^2+2abx^3+b^2x^6}}{14\left(a+bx^3\right)}+\frac{3a^2bx^{11}\sqrt{a^2+2abx^3+b^2x^6}}{11\left(a+bx^3\right)}+\frac{a^3x^8\sqrt{a^2+2abx^3+b^2x^6}}{8\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x^7*(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2),x]
```

```
[Out] (a^3*x^8*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(8*(a + b*x^3)) + (3*a^2*b*x^11*S
qrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(11*(a + b*x^3)) + (3*a*b^2*x^14*Sqrt[a^2 +
 2*a*b*x^3 + b^2*x^6])/(14*(a + b*x^3)) + (b^3*x^17*Sqrt[a^2 + 2*a*b*x^3 +
b^2*x^6])/(17*(a + b*x^3))
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\begin{aligned}\int x^7\left(a^2+2abx^3+b^2x^6\right)^{3/2}\,dx&=\frac{\sqrt{a^2+2abx^3+b^2x^6}\int x^7\left(ab+b^2x^3\right)^3\,dx}{b^2\left(ab+b^2x^3\right)}\\&=\frac{\sqrt{a^2+2abx^3+b^2x^6}\int\left(a^3b^3x^7+3a^2b^4x^{10}+3ab^5x^{13}+b^6x^{16}\right)dx}{b^2\left(ab+b^2x^3\right)}\\&=\frac{a^3x^8\sqrt{a^2+2abx^3+b^2x^6}}{8\left(a+bx^3\right)}+\frac{3a^2bx^{11}\sqrt{a^2+2abx^3+b^2x^6}}{11\left(a+bx^3\right)}+\frac{3ab^2x^{14}\sqrt{a^2+2ab}}{14\left(a+bx\right.}\end{aligned}$$

Mathematica [A] time = 0.02, size = 61, normalized size = 0.37

$$\frac{x^8\sqrt{\left(a+bx^3\right)^2}\left(1309a^3+2856a^2bx^3+2244ab^2x^6+616b^3x^9\right)}{10472\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7*(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2),x]
```

```
[Out] (x^8*Sqrt[(a + b*x^3)^2]*(1309*a^3 + 2856*a^2*b*x^3 + 2244*a*b^2*x^6 + 616*
b^3*x^9))/(10472*(a + b*x^3))
```

IntegrateAlgebraic [A] time = 9.62, size = 61, normalized size = 0.37

$$\frac{\sqrt{\left(a+bx^3\right)^2}\left(1309a^3x^8+2856a^2bx^{11}+2244ab^2x^{14}+616b^3x^{17}\right)}{10472\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^7*(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2),x]
```

```
[Out] (Sqrt[(a + b*x^3)^2]*(1309*a^3*x^8 + 2856*a^2*b*x^11 + 2244*a*b^2*x^14 + 61
6*b^3*x^17))/(10472*(a + b*x^3))
```

fricas [A] time = 1.32, size = 35, normalized size = 0.21

$$\frac{1}{17}b^3x^{17}+\frac{3}{14}ab^2x^{14}+\frac{3}{11}a^2bx^{11}+\frac{1}{8}a^3x^8$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/17*b^3*x^17 + 3/14*a*b^2*x^14 + 3/11*a^2*b*x^11 + 1/8*a^3*x^8
```

giac [A] time = 0.29, size = 67, normalized size = 0.40

$$\frac{1}{17}b^3x^{17}\mathrm{sgn}\left(bx^3+a\right)+\frac{3}{14}ab^2x^{14}\mathrm{sgn}\left(bx^3+a\right)+\frac{3}{11}a^2bx^{11}\mathrm{sgn}\left(bx^3+a\right)+\frac{1}{8}a^3x^8\mathrm{sgn}\left(bx^3+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/17*b^3*x^17*sgn(b*x^3 + a) + 3/14*a*b^2*x^14*sgn(b*x^3 + a) + 3/11*a^2*b*
x^11*sgn(b*x^3 + a) + 1/8*a^3*x^8*sgn(b*x^3 + a)
```

maple [A] time = 0.01, size = 58, normalized size = 0.35

$$\frac{\left(616b^3x^9+2244a\,b^2x^6+2856a^2b\,x^3+1309a^3\right)\left(\left(b\,x^3+a\right)^2\right)^{\frac{3}{2}}x^8}{10472\left(b\,x^3+a\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x)
```

```
[Out] 1/10472*x^8*(616*b^3*x^9+2244*a*b^2*x^6+2856*a^2*b*x^3+1309*a^3)*((b*x^3+a)
^2)^(3/2)/(b*x^3+a)^3
```

maxima [A] time = 0.45, size = 35, normalized size = 0.21

$$\frac{1}{17}b^3x^{17}+\frac{3}{14}ab^2x^{14}+\frac{3}{11}a^2bx^{11}+\frac{1}{8}a^3x^8$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/17*b^3*x^17 + 3/14*a*b^2*x^14 + 3/11*a^2*b*x^11 + 1/8*a^3*x^8
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^7 \left(a^2 + 2\,a\,b\,x^3 + b^2\,x^6\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2),x)
[Out] int(x^7*(a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^7 \left(\left(a + bx^3\right)^2\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7*(b**2*x**6+2*a*b*x**3+a**2)**(3/2),x)
[Out] Integral(x**7*((a + b*x**3)**2)**(3/2), x)
```

3.26 $\int x^6 \left(a^2+2abx^3+b^2x^6\right)^{3/2} \, dx$

Optimal. Leaf size=167

$$\frac{3ab^2x^{13}\sqrt{a^2+2abx^3+b^2x^6}}{13\left(a+bx^3\right)}+\frac{3a^2bx^{10}\sqrt{a^2+2abx^3+b^2x^6}}{10\left(a+bx^3\right)}+\frac{b^3x^{16}\sqrt{a^2+2abx^3+b^2x^6}}{16\left(a+bx^3\right)}+\frac{a^3x^7\sqrt{a^2+2abx^3+b}}{7\left(a+bx^3\right)}$$

Rubi [A] time = 0.04, antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1355, 270}

$$\frac{b^3x^{16}\sqrt{a^2+2abx^3+b^2x^6}}{16\left(a+bx^3\right)}+\frac{3ab^2x^{13}\sqrt{a^2+2abx^3+b^2x^6}}{13\left(a+bx^3\right)}+\frac{3a^2bx^{10}\sqrt{a^2+2abx^3+b^2x^6}}{10\left(a+bx^3\right)}+\frac{a^3x^7\sqrt{a^2+2abx^3+b^2x^6}}{7\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x^6*(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2),x]
```

```
[Out] (a^3*x^7*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(7*(a + b*x^3)) + (3*a^2*b*x^10*S
qrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(10*(a + b*x^3)) + (3*a*b^2*x^13*Sqrt[a^2 +
 2*a*b*x^3 + b^2*x^6])/(13*(a + b*x^3)) + (b^3*x^16*Sqrt[a^2 + 2*a*b*x^3 +
b^2*x^6])/(16*(a + b*x^3))
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\begin{aligned}
\int x^6 \left(a^2+2abx^3+b^2x^6\right)^{3/2} \, dx &= \frac{\sqrt{a^2+2abx^3+b^2x^6} \int x^6 \left(ab+b^2x^3\right)^3 \, dx}{b^2\left(ab+b^2x^3\right)}\\
&= \frac{\sqrt{a^2+2abx^3+b^2x^6} \int \left(a^3b^3x^6+3a^2b^4x^9+3ab^5x^{12}+b^6x^{15}\right) \, dx}{b^2\left(ab+b^2x^3\right)}\\
&= \frac{a^3x^7\sqrt{a^2+2abx^3+b^2x^6}}{7\left(a+bx^3\right)}+\frac{3a^2bx^{10}\sqrt{a^2+2abx^3+b^2x^6}}{10\left(a+bx^3\right)}+\frac{3ab^2x^{13}\sqrt{a^2+2}}{13\left(a+\right.}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 61, normalized size = 0.37

$$\frac{x^7\sqrt{\left(a+bx^3\right)^2}\left(1040a^3+2184a^2bx^3+1680ab^2x^6+455b^3x^9\right)}{7280\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6*(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2),x]
```

```
[Out] (x^7*Sqrt[(a + b*x^3)^2]*(1040*a^3 + 2184*a^2*b*x^3 + 1680*a*b^2*x^6 + 455*
b^3*x^9))/(7280*(a + b*x^3))
```

IntegrateAlgebraic [A] time = 9.20, size = 61, normalized size = 0.37

$$\frac{\sqrt{\left(a+bx^3\right)^2}\left(1040a^3x^7+2184a^2bx^{10}+1680ab^2x^{13}+455b^3x^{16}\right)}{7280\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^6*(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2),x]
```

```
[Out] (Sqrt[(a + b*x^3)^2]*(1040*a^3*x^7 + 2184*a^2*b*x^10 + 1680*a*b^2*x^13 + 45
5*b^3*x^16))/(7280*(a + b*x^3))
```

fricas [A] time = 1.54, size = 35, normalized size = 0.21

$$\frac{1}{16}b^3x^{16}+\frac{3}{13}ab^2x^{13}+\frac{3}{10}a^2bx^{10}+\frac{1}{7}a^3x^7$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/16*b^3*x^16 + 3/13*a*b^2*x^13 + 3/10*a^2*b*x^10 + 1/7*a^3*x^7
```

giac [A] time = 0.36, size = 67, normalized size = 0.40

$$\frac{1}{16}b^3x^{16}\mathrm{sgn}\left(bx^3+a\right)+\frac{3}{13}ab^2x^{13}\mathrm{sgn}\left(bx^3+a\right)+\frac{3}{10}a^2bx^{10}\mathrm{sgn}\left(bx^3+a\right)+\frac{1}{7}a^3x^7\mathrm{sgn}\left(bx^3+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/16*b^3*x^16*sgn(b*x^3 + a) + 3/13*a*b^2*x^13*sgn(b*x^3 + a) + 3/10*a^2*b*
x^10*sgn(b*x^3 + a) + 1/7*a^3*x^7*sgn(b*x^3 + a)
```

maple [A] time = 0.01, size = 58, normalized size = 0.35

$$\frac{\left(455b^3x^9+1680a\,b^2x^6+2184a^2b\,x^3+1040a^3\right)\left(\left(b\,x^3+a\right)^2\right)^{\frac{3}{2}}x^7}{7280\left(b\,x^3+a\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x)
```

```
[Out] 1/7280*x^7*(455*b^3*x^9+1680*a*b^2*x^6+2184*a^2*b*x^3+1040*a^3)*((b*x^3+a)^
2)^(3/2)/(b*x^3+a)^3
```

maxima [A] time = 0.45, size = 35, normalized size = 0.21

$$\frac{1}{16}b^3x^{16}+\frac{3}{13}ab^2x^{13}+\frac{3}{10}a^2bx^{10}+\frac{1}{7}a^3x^7$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/16*b^3*x^16 + 3/13*a*b^2*x^13 + 3/10*a^2*b*x^10 + 1/7*a^3*x^7
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^6 \left(a^2 + 2\,a\,b\,x^3 + b^2\,x^6\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2),x)
[Out] int(x^6*(a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^6 \left(\left(a + bx^3\right)^2\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6*(b**2*x**6+2*a*b*x**3+a**2)**(3/2),x)
[Out] Integral(x**6*((a + b*x**3)**2)**(3/2), x)
```

3.27 $\int x^5 \left(a^2+2abx^3+b^2x^6\right)^{3/2} dx$

Optimal. Leaf size=78

$$\frac{\left(a+bx^3\right)^4 \sqrt{a^2+2abx^3+b^2x^6}}{15b^2}-\frac{a\left(a+bx^3\right)^3 \sqrt{a^2+2abx^3+b^2x^6}}{12b^2}$$

Rubi [A] time = 0.05, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1355, 266, 43}

$$\frac{\left(a+bx^3\right)^4 \sqrt{a^2+2abx^3+b^2x^6}}{15b^2}-\frac{a\left(a+bx^3\right)^3 \sqrt{a^2+2abx^3+b^2x^6}}{12b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2),x]
```

```
[Out] -(a*(a + b*x^3)^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(12*b^2) + ((a + b*x^3)^
4*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(15*b^2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\begin{aligned}
\int x^5 \left(a^2+2abx^3+b^2x^6\right)^{3/2} dx &= \frac{\sqrt{a^2+2abx^3+b^2x^6} \int x^5 \left(ab+b^2x^3\right)^3 dx}{b^2\left(ab+b^2x^3\right)}\\
&= \frac{\sqrt{a^2+2abx^3+b^2x^6}\, \text{Subst}\left(\int x\left(ab+b^2x\right)^3 dx, x, x^3\right)}{3b^2\left(ab+b^2x^3\right)}\\
&= \frac{\sqrt{a^2+2abx^3+b^2x^6}\, \text{Subst}\left(\int \left(-\frac{a\left(ab+b^2x\right)^3}{b}+\frac{\left(ab+b^2x\right)^4}{b^2}\right) dx, x, x^3\right)}{3b^2\left(ab+b^2x^3\right)}\\
&= -\frac{a\left(a+bx^3\right)^3 \sqrt{a^2+2abx^3+b^2x^6}}{12b^2}+\frac{\left(a+bx^3\right)^4 \sqrt{a^2+2abx^3+b^2x^6}}{15b^2}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 61, normalized size = 0.78

$$\frac{x^6\sqrt{(a+bx^3)^2}\,(10a^3+20a^2bx^3+15ab^2x^6+4b^3x^9)}{60(a+bx^3)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2),x]
```

```
[Out] (x^6*Sqrt[(a + b*x^3)^2]*(10*a^3 + 20*a^2*b*x^3 + 15*a*b^2*x^6 + 4*b^3*x^9)
)/(60*(a + b*x^3))
```

IntegrateAlgebraic [A] time = 8.62, size = 61, normalized size = 0.78

$$\frac{x^6\sqrt{(a+bx^3)^2}\,(10a^3+20a^2bx^3+15ab^2x^6+4b^3x^9)}{60(a+bx^3)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^5*(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2),x]
```

```
[Out] (x^6*Sqrt[(a + b*x^3)^2]*(10*a^3 + 20*a^2*b*x^3 + 15*a*b^2*x^6 + 4*b^3*x^9)
)/(60*(a + b*x^3))
```

fricas [A] time = 1.48, size = 35, normalized size = 0.45

$$\frac{1}{15}\,b^3x^{15}+\frac{1}{4}\,ab^2x^{12}+\frac{1}{3}\,a^2bx^9+\frac{1}{6}\,a^3x^6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/15*b^3*x^15 + 1/4*a*b^2*x^12 + 1/3*a^2*b*x^9 + 1/6*a^3*x^6
```

giac [A] time = 0.44, size = 45, normalized size = 0.58

$$\frac{1}{60}\,(4\,b^3x^{15}+15\,ab^2x^{12}+20\,a^2bx^9+10\,a^3x^6)\mathrm{sgn}\,(bx^3+a)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/60*(4*b^3*x^15 + 15*a*b^2*x^12 + 20*a^2*b*x^9 + 10*a^3*x^6)*sgn(b*x^3 + a
)
```

maple [A] time = 0.01, size = 58, normalized size = 0.74

$$\frac{(4b^3x^9+15a\,b^2x^6+20a^2b\,x^3+10a^3)\left((b\,x^3+a)^2\right)^{\frac{3}{2}}x^6}{60\,(b\,x^3+a)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x)
```

```
[Out] 1/60*x^6*(4*b^3*x^9+15*a*b^2*x^6+20*a^2*b*x^3+10*a^3)*((b*x^3+a)^2)^(3/2)/(
b*x^3+a)^3
```

maxima [A] time = 0.49, size = 83, normalized size = 1.06

$$-\frac{\left(b^2x^6+2\,abx^3+a^2\right)^{\frac{3}{2}}ax^3}{12\,b}-\frac{\left(b^2x^6+2\,abx^3+a^2\right)^{\frac{3}{2}}a^2}{12\,b^2}+\frac{\left(b^2x^6+2\,abx^3+a^2\right)^{\frac{5}{2}}}{15\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/12*(b^2*x^6 + 2*a*b*x^3 + a^2)^(3/2)*a*x^3/b - 1/12*(b^2*x^6 + 2*a*b*x^3
 + a^2)^(3/2)*a^2/b^2 + 1/15*(b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2)/b^2
```

mupad [B] time = 1.25, size = 46, normalized size = 0.59

$$\frac{\left(a^2+2\,a\,b\,x^3+b^2\,x^6\right)^{3/2}\,\left(-a^2+3\,a\,b\,x^3+4\,b^2\,x^6\right)}{60\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2),x)
```

```
[Out] ((a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2)*(4*b^2*x^6 - a^2 + 3*a*b*x^3))/(60*b^2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^5\left(\left(a+bx^3\right)^2\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(b**2*x**6+2*a*b*x**3+a**2)**(3/2),x)
```

```
[Out] Integral(x**5*((a + b*x**3)**2)**(3/2), x)
```

3.28 $\int x^4 \left(a^2+2abx^3+b^2x^6\right)^{3/2} dx$

Optimal. Leaf size=167

$$\frac{3ab^2x^{11}\sqrt{a^2+2abx^3+b^2x^6}}{11\left(a+bx^3\right)}+\frac{3a^2bx^8\sqrt{a^2+2abx^3+b^2x^6}}{8\left(a+bx^3\right)}+\frac{b^3x^{14}\sqrt{a^2+2abx^3+b^2x^6}}{14\left(a+bx^3\right)}+\frac{a^3x^5\sqrt{a^2+2abx^3+b^2}}{5\left(a+bx^3\right)}$$

Rubi [A] time = 0.04, antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1355, 270}

$$\frac{b^3x^{14}\sqrt{a^2+2abx^3+b^2x^6}}{14\left(a+bx^3\right)}+\frac{3ab^2x^{11}\sqrt{a^2+2abx^3+b^2x^6}}{11\left(a+bx^3\right)}+\frac{3a^2bx^8\sqrt{a^2+2abx^3+b^2x^6}}{8\left(a+bx^3\right)}+\frac{a^3x^5\sqrt{a^2+2abx^3+b^2x^6}}{5\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2),x]
```

```
[Out] (a^3*x^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(5*(a + b*x^3)) + (3*a^2*b*x^8*Sq
rt[a^2 + 2*a*b*x^3 + b^2*x^6])/(8*(a + b*x^3)) + (3*a*b^2*x^11*Sqrt[a^2 + 2
*a*b*x^3 + b^2*x^6])/(11*(a + b*x^3)) + (b^3*x^14*Sqrt[a^2 + 2*a*b*x^3 + b^
2*x^6])/(14*(a + b*x^3))
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\begin{aligned}
\int x^4\left(a^2+2abx^3+b^2x^6\right)^{3/2} dx &= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int x^4\left(ab+b^2x^3\right)^3 dx}{b^2\left(ab+b^2x^3\right)}\\
&= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \left(a^3b^3x^4+3a^2b^4x^7+3ab^5x^{10}+b^6x^{13}\right) dx}{b^2\left(ab+b^2x^3\right)}\\
&= \frac{a^3x^5\sqrt{a^2+2abx^3+b^2x^6}}{5\left(a+bx^3\right)}+\frac{3a^2bx^8\sqrt{a^2+2abx^3+b^2x^6}}{8\left(a+bx^3\right)}+\frac{3ab^2x^{11}\sqrt{a^2+2a}}{11\left(a+b\right.}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 61, normalized size = 0.37

$$\frac{x^5\sqrt{\left(a+bx^3\right)^2}\left(616a^3+1155a^2bx^3+840ab^2x^6+220b^3x^9\right)}{3080\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2),x]
```

```
[Out] (x^5*Sqrt[(a + b*x^3)^2]*(616*a^3 + 1155*a^2*b*x^3 + 840*a*b^2*x^6 + 220*b^
3*x^9))/(3080*(a + b*x^3))
```

IntegrateAlgebraic [A] time = 8.31, size = 61, normalized size = 0.37

$$\frac{\sqrt{\left(a+bx^3\right)^2}\left(616a^3x^5+1155a^2bx^8+840ab^2x^{11}+220b^3x^{14}\right)}{3080\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^4*(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2),x]
```

```
[Out] (Sqrt[(a + b*x^3)^2]*(616*a^3*x^5 + 1155*a^2*b*x^8 + 840*a*b^2*x^11 + 220*b
^3*x^14))/(3080*(a + b*x^3))
```

fricas [A] time = 1.13, size = 35, normalized size = 0.21

$$\frac{1}{14}b^3x^{14}+\frac{3}{11}ab^2x^{11}+\frac{3}{8}a^2bx^8+\frac{1}{5}a^3x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/14*b^3*x^14 + 3/11*a*b^2*x^11 + 3/8*a^2*b*x^8 + 1/5*a^3*x^5
```

giac [A] time = 0.37, size = 67, normalized size = 0.40

$$\frac{1}{14}b^3x^{14}\mathrm{sgn}\left(bx^3+a\right)+\frac{3}{11}ab^2x^{11}\mathrm{sgn}\left(bx^3+a\right)+\frac{3}{8}a^2bx^8\mathrm{sgn}\left(bx^3+a\right)+\frac{1}{5}a^3x^5\mathrm{sgn}\left(bx^3+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/14*b^3*x^14*sgn(b*x^3 + a) + 3/11*a*b^2*x^11*sgn(b*x^3 + a) + 3/8*a^2*b*x
^8*sgn(b*x^3 + a) + 1/5*a^3*x^5*sgn(b*x^3 + a)
```

maple [A] time = 0.01, size = 58, normalized size = 0.35

$$\frac{\left(220b^3x^9+840a\,b^2x^6+1155a^2b\,x^3+616a^3\right)\left(\left(b\,x^3+a\right)^2\right)^{\frac{3}{2}}x^5}{3080\left(b\,x^3+a\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x)
```

```
[Out] 1/3080*x^5*(220*b^3*x^9+840*a*b^2*x^6+1155*a^2*b*x^3+616*a^3)*((b*x^3+a)^2)
^(3/2)/(b*x^3+a)^3
```

maxima [A] time = 0.47, size = 35, normalized size = 0.21

$$\frac{1}{14}b^3x^{14}+\frac{3}{11}ab^2x^{11}+\frac{3}{8}a^2bx^8+\frac{1}{5}a^3x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/14*b^3*x^14 + 3/11*a*b^2*x^11 + 3/8*a^2*b*x^8 + 1/5*a^3*x^5
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^4 \left(a^2 + 2\,a\,b\,x^3 + b^2\,x^6\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2),x)
[Out] int(x^4*(a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^4 \left(\left(a + bx^3\right)^2\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(b**2*x**6+2*a*b*x**3+a**2)**(3/2),x)
[Out] Integral(x**4*((a + b*x**3)**2)**(3/2), x)
```

3.29 $\int x^3 \left(a^2+2abx^3+b^2x^6\right)^{3/2} \, dx$

Optimal. Leaf size=167

$$\frac{3ab^2x^{10}\sqrt{a^2+2abx^3+b^2x^6}}{10\left(a+bx^3\right)}+\frac{3a^2bx^7\sqrt{a^2+2abx^3+b^2x^6}}{7\left(a+bx^3\right)}+\frac{b^3x^{13}\sqrt{a^2+2abx^3+b^2x^6}}{13\left(a+bx^3\right)}+\frac{a^3x^4\sqrt{a^2+2abx^3+b^2x^6}}{4\left(a+bx^3\right)}$$

Rubi [A] time = 0.04, antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1355, 270}

$$\frac{b^3x^{13}\sqrt{a^2+2abx^3+b^2x^6}}{13\left(a+bx^3\right)}+\frac{3ab^2x^{10}\sqrt{a^2+2abx^3+b^2x^6}}{10\left(a+bx^3\right)}+\frac{3a^2bx^7\sqrt{a^2+2abx^3+b^2x^6}}{7\left(a+bx^3\right)}+\frac{a^3x^4\sqrt{a^2+2abx^3+b^2x^6}}{4\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2),x]
```

```
[Out] (a^3*x^4*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(4*(a + b*x^3)) + (3*a^2*b*x^7*Sq
rt[a^2 + 2*a*b*x^3 + b^2*x^6])/(7*(a + b*x^3)) + (3*a*b^2*x^10*Sqrt[a^2 + 2
*a*b*x^3 + b^2*x^6])/(10*(a + b*x^3)) + (b^3*x^13*Sqrt[a^2 + 2*a*b*x^3 + b^
2*x^6])/(13*(a + b*x^3))
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\begin{aligned}
\int x^3 \left(a^2+2abx^3+b^2x^6\right)^{3/2} \, dx &= \frac{\sqrt{a^2+2abx^3+b^2x^6} \int x^3 \left(ab+b^2x^3\right)^3 \, dx}{b^2\left(ab+b^2x^3\right)}\\
&= \frac{\sqrt{a^2+2abx^3+b^2x^6} \int \left(a^3b^3x^3+3a^2b^4x^6+3ab^5x^9+b^6x^{12}\right) dx}{b^2\left(ab+b^2x^3\right)}\\
&= \frac{a^3x^4\sqrt{a^2+2abx^3+b^2x^6}}{4\left(a+bx^3\right)}+\frac{3a^2bx^7\sqrt{a^2+2abx^3+b^2x^6}}{7\left(a+bx^3\right)}+\frac{3ab^2x^{10}\sqrt{a^2+2abx}}{10\left(a+bx^3\right)}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 61, normalized size = 0.37

$$\frac{x^4\sqrt{\left(a+bx^3\right)^2}\left(455a^3+780a^2bx^3+546ab^2x^6+140b^3x^9\right)}{1820\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2),x]
```

```
[Out] (x^4*Sqrt[(a + b*x^3)^2]*(455*a^3 + 780*a^2*b*x^3 + 546*a*b^2*x^6 + 140*b^3
*x^9))/(1820*(a + b*x^3))
```

IntegrateAlgebraic [A] time = 8.24, size = 61, normalized size = 0.37

$$\frac{\sqrt{\left(a+bx^3\right)^2}\left(455a^3x^4+780a^2bx^7+546ab^2x^{10}+140b^3x^{13}\right)}{1820\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^3*(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2),x]
```

```
[Out] (Sqrt[(a + b*x^3)^2]*(455*a^3*x^4 + 780*a^2*b*x^7 + 546*a*b^2*x^10 + 140*b^
3*x^13))/(1820*(a + b*x^3))
```

fricas [A] time = 1.34, size = 35, normalized size = 0.21

$$\frac{1}{13}b^3x^{13}+\frac{3}{10}ab^2x^{10}+\frac{3}{7}a^2bx^7+\frac{1}{4}a^3x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/13*b^3*x^13 + 3/10*a*b^2*x^10 + 3/7*a^2*b*x^7 + 1/4*a^3*x^4
```

giac [A] time = 0.45, size = 67, normalized size = 0.40

$$\frac{1}{13}b^3x^{13}\mathrm{sgn}\left(bx^3+a\right)+\frac{3}{10}ab^2x^{10}\mathrm{sgn}\left(bx^3+a\right)+\frac{3}{7}a^2bx^7\mathrm{sgn}\left(bx^3+a\right)+\frac{1}{4}a^3x^4\mathrm{sgn}\left(bx^3+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/13*b^3*x^13*sgn(b*x^3 + a) + 3/10*a*b^2*x^10*sgn(b*x^3 + a) + 3/7*a^2*b*x
^7*sgn(b*x^3 + a) + 1/4*a^3*x^4*sgn(b*x^3 + a)
```

maple [A] time = 0.01, size = 58, normalized size = 0.35

$$\frac{\left(140b^3x^9+546a\,b^2x^6+780a^2b\,x^3+455a^3\right)\left(\left(b\,x^3+a\right)^2\right)^{\frac{3}{2}}x^4}{1820\left(b\,x^3+a\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x)
```

```
[Out] 1/1820*x^4*(140*b^3*x^9+546*a*b^2*x^6+780*a^2*b*x^3+455*a^3)*((b*x^3+a)^2)^
(3/2)/(b*x^3+a)^3
```

maxima [A] time = 0.48, size = 35, normalized size = 0.21

$$\frac{1}{13}b^3x^{13}+\frac{3}{10}ab^2x^{10}+\frac{3}{7}a^2bx^7+\frac{1}{4}a^3x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/13*b^3*x^13 + 3/10*a*b^2*x^10 + 3/7*a^2*b*x^7 + 1/4*a^3*x^4
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^3 \left(a^2 + 2\,a\,b\,x^3 + b^2\,x^6\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2),x)
[Out] int(x^3*(a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^3 \left(\left(a + bx^3\right)^2\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b**2*x**6+2*a*b*x**3+a**2)**(3/2),x)
[Out] Integral(x**3*((a + b*x**3)**2)**(3/2), x)
```

3.30 $\int x^2 \left(a^2+2abx^3+b^2x^6\right)^{3/2} \, dx$

Optimal. Leaf size=36

$$\frac{\left(a+bx^3\right)\left(a^2+2abx^3+b^2x^6\right)^{3/2}}{12b}$$

Rubi [A] time = 0.03, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1352, 609}

$$\frac{\left(a+bx^3\right)\left(a^2+2abx^3+b^2x^6\right)^{3/2}}{12b}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2),x]
```

```
[Out] ((a + b*x^3)*(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2))/(12*b)
```

Rule 609

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x)
*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] /; FreeQ[{a, b, c, p}, x] && EqQ[
b^2 - 4*a*c, 0] && NeQ[p, -2^(-1)]
```

Rule 1352

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a,
b, c, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

Rubi steps

$$\begin{aligned}
\int x^2 \left(a^2+2abx^3+b^2x^6\right)^{3/2} \, dx &= \frac{1}{3}\text{Subst}\left(\int \left(a^2+2abx+b^2x^2\right)^{3/2} \, dx,x,x^3\right)\\
&= \frac{\left(a+bx^3\right)\left(a^2+2abx^3+b^2x^6\right)^{3/2}}{12b}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 60, normalized size = 1.67

$$\frac{x^3\sqrt{\left(a+bx^3\right)^2}\left(4a^3+6a^2bx^3+4ab^2x^6+b^3x^9\right)}{12\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2),x]
```

```
[Out] (x^3*Sqrt[(a + b*x^3)^2]*(4*a^3 + 6*a^2*b*x^3 + 4*a*b^2*x^6 + b^3*x^9))/(12
*(a + b*x^3))
```

IntegrateAlgebraic [A] time = 8.02, size = 60, normalized size = 1.67

$$\frac{x^3\sqrt{\left(a+bx^3\right)^2}\left(4a^3+6a^2bx^3+4ab^2x^6+b^3x^9\right)}{12\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^2*(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2),x]
```

```
[Out] (x^3*Sqrt[(a + b*x^3)^2]*(4*a^3 + 6*a^2*b*x^3 + 4*a*b^2*x^6 + b^3*x^9))/(12
*(a + b*x^3))
```

fricas [A] time = 1.47, size = 35, normalized size = 0.97

$$\frac{1}{12}\,b^3x^{12} + \frac{1}{3}\,ab^2x^9 + \frac{1}{2}\,a^2bx^6 + \frac{1}{3}\,a^3x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/12*b^3*x^12 + 1/3*a*b^2*x^9 + 1/2*a^2*b*x^6 + 1/3*a^3*x^3
```

giac [A] time = 0.41, size = 44, normalized size = 1.22

$$\frac{1}{12}\left(2\left(bx^6 + 2\,ax^3\right)a^2 + \left(bx^6 + 2\,ax^3\right)^2b\right)\mathrm{sgn}\left(bx^3 + a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/12*(2*(b*x^6 + 2*a*x^3)*a^2 + (b*x^6 + 2*a*x^3)^2*b)*sgn(b*x^3 + a)
```

maple [A] time = 0.01, size = 57, normalized size = 1.58

$$\frac{\left(b^3x^9 + 4a\,b^2x^6 + 6a^2b\,x^3 + 4a^3\right)\left(\left(b\,x^3 + a\right)^2\right)^{\frac{3}{2}}\,x^3}{12\left(b\,x^3 + a\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x)
```

```
[Out] 1/12*x^3*(b^3*x^9+4*a*b^2*x^6+6*a^2*b*x^3+4*a^3)*((b*x^3+a)^2)^(3/2)/(b*x^3
+a)^3
```

maxima [A] time = 0.45, size = 52, normalized size = 1.44

$$\frac{1}{12}\left(b^2x^6 + 2\,abx^3 + a^2\right)^{\frac{3}{2}}x^3 + \frac{\left(b^2x^6 + 2\,abx^3 + a^2\right)^{\frac{3}{2}}a}{12\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/12*(b^2*x^6 + 2*a*b*x^3 + a^2)^(3/2)*x^3 + 1/12*(b^2*x^6 + 2*a*b*x^3 + a^
2)^(3/2)*a/b
```

mupad [B] time = 1.22, size = 36, normalized size = 1.00

$$\frac{\left(b^2\,x^3 + a\,b\right)\,\left(a^2 + 2\,a\,b\,x^3 + b^2\,x^6\right)^{3/2}}{12\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2),x)
```

```
[Out] ((a*b + b^2*x^3)*(a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2))/(12*b^2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \left(\left(a + bx^3\right)^2\right)^{\frac{3}{2}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b**2*x**6+2*a*b*x**3+a**2)**(3/2),x)
[Out] Integral(x**2*((a + b*x**3)**2)**(3/2), x)
```

3.31 $\int x\left(a^2+2abx^3+b^2x^6\right)^{3/2}\,dx$

Optimal. Leaf size=167

$$\frac{3ab^2x^8\sqrt{a^2+2abx^3+b^2x^6}}{8\left(a+bx^3\right)}+\frac{3a^2bx^5\sqrt{a^2+2abx^3+b^2x^6}}{5\left(a+bx^3\right)}+\frac{b^3x^{11}\sqrt{a^2+2abx^3+b^2x^6}}{11\left(a+bx^3\right)}+\frac{a^3x^2\sqrt{a^2+2abx^3+b^2x^6}}{2\left(a+bx^3\right)}$$

Rubi [A] time = 0.04, antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1355, 270}

$$\frac{b^3x^{11}\sqrt{a^2+2abx^3+b^2x^6}}{11\left(a+bx^3\right)}+\frac{3ab^2x^8\sqrt{a^2+2abx^3+b^2x^6}}{8\left(a+bx^3\right)}+\frac{3a^2bx^5\sqrt{a^2+2abx^3+b^2x^6}}{5\left(a+bx^3\right)}+\frac{a^3x^2\sqrt{a^2+2abx^3+b^2x^6}}{2\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2),x]
```

```
[Out] (a^3*x^2*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(2*(a + b*x^3)) + (3*a^2*b*x^5*Sq
rt[a^2 + 2*a*b*x^3 + b^2*x^6])/(5*(a + b*x^3)) + (3*a*b^2*x^8*Sqrt[a^2 + 2*
a*b*x^3 + b^2*x^6])/(8*(a + b*x^3)) + (b^3*x^11*Sqrt[a^2 + 2*a*b*x^3 + b^2*
x^6])/(11*(a + b*x^3))
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\begin{aligned}\int x\left(a^2+2abx^3+b^2x^6\right)^{3/2}\,dx&=\frac{\sqrt{a^2+2abx^3+b^2x^6}\int x\left(ab+b^2x^3\right)^3\,dx}{b^2\left(ab+b^2x^3\right)}\\&=\frac{\sqrt{a^2+2abx^3+b^2x^6}\int\left(a^3b^3x+3a^2b^4x^4+3ab^5x^7+b^6x^{10}\right)\,dx}{b^2\left(ab+b^2x^3\right)}\\&=\frac{a^3x^2\sqrt{a^2+2abx^3+b^2x^6}}{2\left(a+bx^3\right)}+\frac{3a^2bx^5\sqrt{a^2+2abx^3+b^2x^6}}{5\left(a+bx^3\right)}+\frac{3ab^2x^8\sqrt{a^2+2abx^3}}{8\left(a+bx^3\right)}\end{aligned}$$

Mathematica [A] time = 0.02, size = 61, normalized size = 0.37

$$\frac{x^2\sqrt{\left(a+bx^3\right)^2}\left(220a^3+264a^2bx^3+165ab^2x^6+40b^3x^9\right)}{440\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2),x]
```

```
[Out] (x^2*Sqrt[(a + b*x^3)^2]*(220*a^3 + 264*a^2*b*x^3 + 165*a*b^2*x^6 + 40*b^3*
x^9))/(440*(a + b*x^3))
```

IntegrateAlgebraic [A] time = 10.19, size = 61, normalized size = 0.37

$$\frac{\sqrt{\left(a+bx^3\right)^2}\left(220a^3x^2+264a^2bx^5+165ab^2x^8+40b^3x^{11}\right)}{440\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x*(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2),x]
```

```
[Out] (Sqrt[(a + b*x^3)^2]*(220*a^3*x^2 + 264*a^2*b*x^5 + 165*a*b^2*x^8 + 40*b^3*
x^11))/(440*(a + b*x^3))
```

fricas [A] time = 1.51, size = 35, normalized size = 0.21

$$\frac{1}{11}\,b^3x^{11}+\frac{3}{8}\,ab^2x^8+\frac{3}{5}\,a^2bx^5+\frac{1}{2}\,a^3x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/11*b^3*x^11 + 3/8*a*b^2*x^8 + 3/5*a^2*b*x^5 + 1/2*a^3*x^2
```

giac [A] time = 0.37, size = 67, normalized size = 0.40

$$\frac{1}{11}\,b^3x^{11}\mathrm{sgn}\left(bx^3+a\right)+\frac{3}{8}\,ab^2x^8\mathrm{sgn}\left(bx^3+a\right)+\frac{3}{5}\,a^2bx^5\mathrm{sgn}\left(bx^3+a\right)+\frac{1}{2}\,a^3x^2\mathrm{sgn}\left(bx^3+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/11*b^3*x^11*sgn(b*x^3 + a) + 3/8*a*b^2*x^8*sgn(b*x^3 + a) + 3/5*a^2*b*x^5
*sgn(b*x^3 + a) + 1/2*a^3*x^2*sgn(b*x^3 + a)
```

maple [A] time = 0.00, size = 58, normalized size = 0.35

$$\frac{\left(40b^3x^9+165a\,b^2x^6+264a^2b\,x^3+220a^3\right)\left(\left(b\,x^3+a\right)^2\right)^{\frac{3}{2}}x^2}{440\left(b\,x^3+a\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x)
```

```
[Out] 1/440*x^2*(40*b^3*x^9+165*a*b^2*x^6+264*a^2*b*x^3+220*a^3)*((b*x^3+a)^2)^(3
/2)/(b*x^3+a)^3
```

maxima [A] time = 0.44, size = 35, normalized size = 0.21

$$\frac{1}{11}\,b^3x^{11}+\frac{3}{8}\,ab^2x^8+\frac{3}{5}\,a^2bx^5+\frac{1}{2}\,a^3x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/11*b^3*x^11 + 3/8*a*b^2*x^8 + 3/5*a^2*b*x^5 + 1/2*a^3*x^2
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x \left(a^2 + 2\,a\,b\,x^3 + b^2\,x^6\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2),x)
[Out] int(x*(a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x \left(\left(a + bx^3\right)^2\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b**2*x**6+2*a*b*x**3+a**2)**(3/2),x)
[Out] Integral(x*((a + b*x**3)**2)**(3/2), x)
```

3.32 $\int \left(a^2+2abx^3+b^2x^6\right)^{3/2} dx$

Optimal. Leaf size=162

$$\frac{3ab^2x^7\left(a^2+2abx^3+b^2x^6\right)^{3/2}}{7\left(a+bx^3\right)^3}+\frac{3a^2bx^4\left(a^2+2abx^3+b^2x^6\right)^{3/2}}{4\left(a+bx^3\right)^3}+\frac{b^3x^{10}\left(a^2+2abx^3+b^2x^6\right)^{3/2}}{10\left(a+bx^3\right)^3}+\frac{a^3x\left(a^2+2abx^3\right.}{\left(a+bx\right.}$$

Rubi [A] time = 0.03, antiderivative size = 162, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1343, 194}

$$\frac{b^3x^{10}\left(a^2+2abx^3+b^2x^6\right)^{3/2}}{10\left(a+bx^3\right)^3}+\frac{3ab^2x^7\left(a^2+2abx^3+b^2x^6\right)^{3/2}}{7\left(a+bx^3\right)^3}+\frac{3a^2bx^4\left(a^2+2abx^3+b^2x^6\right)^{3/2}}{4\left(a+bx^3\right)^3}+\frac{a^3x\left(a^2+2abx^3+b^2x^6\right)^{3/2}}{\left(a+bx^3\right)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2),x]
```

```
[Out] (a^3*x*(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2))/(a + b*x^3)^3 + (3*a^2*b*x^4*(a^2
 + 2*a*b*x^3 + b^2*x^6)^(3/2))/(4*(a + b*x^3)^3) + (3*a*b^2*x^7*(a^2 + 2*a*
b*x^3 + b^2*x^6)^(3/2))/(7*(a + b*x^3)^3) + (b^3*x^10*(a^2 + 2*a*b*x^3 + b^
2*x^6)^(3/2))/(10*(a + b*x^3)^3)
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 1343

```
Int[((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_), x_Symbol] :> Dist[(
a + b*x^n + c*x^(2*n))^p/(b + 2*c*x^n)^(2*p), Int[(b + 2*c*x^n)^(2*p), x],
x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}\int \left(a^2+2abx^3+b^2x^6\right)^{3/2} dx &= \frac{\left(a^2+2abx^3+b^2x^6\right)^{3/2}\int \left(2ab+2b^2x^3\right)^3 dx}{\left(2ab+2b^2x^3\right)^3}\\ &= \frac{\left(a^2+2abx^3+b^2x^6\right)^{3/2}\int \left(8a^3b^3+24a^2b^4x^3+24ab^5x^6+8b^6x^9\right) dx}{\left(2ab+2b^2x^3\right)^3}\\ &= \frac{a^3x\left(a^2+2abx^3+b^2x^6\right)^{3/2}}{\left(a+bx^3\right)^3}+\frac{3a^2bx^4\left(a^2+2abx^3+b^2x^6\right)^{3/2}}{4\left(a+bx^3\right)^3}+\frac{3ab^2x^7\left(a^2+2a\right.}{7\left(a+\right.}\end{aligned}$$

Mathematica [A] time = 0.01, size = 59, normalized size = 0.36

$$\frac{x\sqrt{\left(a+bx^3\right)^2}\left(140a^3+105a^2bx^3+60ab^2x^6+14b^3x^9\right)}{140\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2),x]
```

```
[Out] (x*Sqrt[(a + b*x^3)^2]*(140*a^3 + 105*a^2*b*x^3 + 60*a*b^2*x^6 + 14*b^3*x^9
))/(140*(a + b*x^3))
```

IntegrateAlgebraic [A] time = 12.40, size = 59, normalized size = 0.36

$$\frac{\sqrt{\left(a+bx^3\right)^2}\left(140a^3x+105a^2bx^4+60ab^2x^7+14b^3x^{10}\right)}{140\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2),x]
```

```
[Out] (Sqrt[(a + b*x^3)^2]*(140*a^3*x + 105*a^2*b*x^4 + 60*a*b^2*x^7 + 14*b^3*x^1
0))/(140*(a + b*x^3))
```

fricas [A] time = 3.03, size = 32, normalized size = 0.20

$$\frac{1}{10}b^3x^{10}+\frac{3}{7}ab^2x^7+\frac{3}{4}a^2bx^4+a^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/10*b^3*x^10 + 3/7*a*b^2*x^7 + 3/4*a^2*b*x^4 + a^3*x
```

giac [A] time = 0.36, size = 64, normalized size = 0.40

$$\frac{1}{10}b^3x^{10}\mathrm{sgn}\left(bx^3+a\right)+\frac{3}{7}ab^2x^7\mathrm{sgn}\left(bx^3+a\right)+\frac{3}{4}a^2bx^4\mathrm{sgn}\left(bx^3+a\right)+a^3x\mathrm{sgn}\left(bx^3+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/10*b^3*x^10*sgn(b*x^3 + a) + 3/7*a*b^2*x^7*sgn(b*x^3 + a) + 3/4*a^2*b*x^4
*sgn(b*x^3 + a) + a^3*x*sgn(b*x^3 + a)
```

maple [A] time = 0.00, size = 56, normalized size = 0.35

$$\frac{\left(14b^3x^9+60a\,b^2x^6+105a^2b\,x^3+140a^3\right)\left(\left(b\,x^3+a\right)^2\right)^{\frac{3}{2}}x}{140\left(b\,x^3+a\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^6+2*a*b*x^3+a^2)^(3/2),x)
```

```
[Out] 1/140*x*(14*b^3*x^9+60*a*b^2*x^6+105*a^2*b*x^3+140*a^3)*((b*x^3+a)^2)^(3/2)
/(b*x^3+a)^3
```

maxima [A] time = 0.71, size = 32, normalized size = 0.20

$$\frac{1}{10}b^3x^{10}+\frac{3}{7}ab^2x^7+\frac{3}{4}a^2bx^4+a^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/10*b^3*x^10 + 3/7*a*b^2*x^7 + 3/4*a^2*b*x^4 + a^3*x
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(a^2 + 2\,a\,b\,x^3 + b^2\,x^6\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2),x)
```

```
[Out] int((a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a^2 + 2abx^3 + b^2x^6\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**6+2*a*b*x**3+a**2)**(3/2),x)
```

```
[Out] Integral((a**2 + 2*a*b*x**3 + b**2*x**6)**(3/2), x)
```

3.33 $\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{3/2}}{x}\,dx$

Optimal. Leaf size=160

$$\frac{ab^2x^6\sqrt{a^2+2abx^3+b^2x^6}}{2\left(a+bx^3\right)}+\frac{a^2bx^3\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}+\frac{b^3x^9\sqrt{a^2+2abx^3+b^2x^6}}{9\left(a+bx^3\right)}+\frac{a^3\log(x)\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}$$

Rubi [A] time = 0.05, antiderivative size = 160, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1355, 266, 43}

$$\frac{b^3x^9\sqrt{a^2+2abx^3+b^2x^6}}{9\left(a+bx^3\right)}+\frac{ab^2x^6\sqrt{a^2+2abx^3+b^2x^6}}{2\left(a+bx^3\right)}+\frac{a^2bx^3\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}+\frac{a^3\log(x)\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)/x,x]
```

```
[Out] (a^2*b*x^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(a + b*x^3) + (a*b^2*x^6*Sqrt[a
^2 + 2*a*b*x^3 + b^2*x^6])/(2*(a + b*x^3)) + (b^3*x^9*Sqrt[a^2 + 2*a*b*x^3
+ b^2*x^6])/(9*(a + b*x^3)) + (a^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]*Log[x])/
(a + b*x^3)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{3/2}}{x}\,dx = \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{\left(ab+b^2x^3\right)^3}{x}\,dx}{b^2\left(ab+b^2x^3\right)}$$

$$= \frac{\sqrt{a^2+2abx^3+b^2x^6}\ \mathrm{Subst}\left(\int \frac{\left(ab+b^2x\right)^3}{x}\,dx, x, x^3\right)}{3b^2\left(ab+b^2x^3\right)}$$

$$= \frac{\sqrt{a^2+2abx^3+b^2x^6}\ \mathrm{Subst}\left(\int \left(3a^2b^4+\frac{a^3b^3}{x}+3ab^5x+b^6x^2\right)dx, x, x^3\right)}{3b^2\left(ab+b^2x^3\right)}$$

$$= \frac{a^2bx^3\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3} + \frac{ab^2x^6\sqrt{a^2+2abx^3+b^2x^6}}{2\left(a+bx^3\right)} + \frac{b^3x^9\sqrt{a^2+2abx^3+}}{9\left(a+bx^3\right)}$$

Mathematica [**A**] time = 0.02, size = 60, normalized size = 0.38

$$\frac{\sqrt{\left(a+bx^3\right)^2}\left(18a^3\log(x)+bx^3\left(18a^2+9abx^3+2b^2x^6\right)\right)}{18\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)/x,x]
```

```
[Out] (Sqrt[(a + b*x^3)^2]*(b*x^3*(18*a^2 + 9*a*b*x^3 + 2*b^2*x^6) + 18*a^3*Log[x
]))/(18*(a + b*x^3))
```

IntegrateAlgebraic [**A**] time = 0.44, size = 256, normalized size = 1.60

$$\frac{1}{36}\sqrt{a^2+2abx^3+b^2x^6}\left(11a^2+7abx^3+2b^2x^6\right)+\frac{1}{36}\left(-18a^2\sqrt{b^2}\,x^3-9ab\sqrt{b^2}\,x^6-2\left(b^2\right)^{3/2}x^9\right)+\frac{1}{6}a^3\log\left(\sqrt{a^2+2abx^3+b^2x^6}-a-\sqrt{b^2}\,x^3\right)-\frac{a^3\left(\sqrt{b^2}+b\right)\log\left(\sqrt{a^2+2abx^3+b^2x^6}+a-\sqrt{b^2}\,x^3\right)}{6b}-\frac{a^3\sqrt{b^2}\log\left(b\sqrt{a^2+2abx^3+b^2x^6}-ab-b\sqrt{b^2}\,x^3\right)}{6b}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)/x,x]
```

```
[Out] (Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]*(11*a^2 + 7*a*b*x^3 + 2*b^2*x^6))/36 + (-1
8*a^2*Sqrt[b^2]*x^3 - 9*a*b*Sqrt[b^2]*x^6 - 2*(b^2)^(3/2)*x^9)/36 + (a^3*Lo
g[-a - Sqrt[b^2]*x^3 + Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]])/6 - (a^3*(b + Sqrt
[b^2])*Log[a - Sqrt[b^2]*x^3 + Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]])/(6*b) - (a
^3*Sqrt[b^2]*Log[-(a*b) - b*Sqrt[b^2]*x^3 + b*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^
6]])/(6*b)
```

fricas [**A**] time = 1.09, size = 32, normalized size = 0.20

$$\frac{1}{9}\,b^3x^9+\frac{1}{2}\,ab^2x^6+a^2bx^3+a^3\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x,x, algorithm="fricas")
```

```
[Out] 1/9*b^3*x^9 + 1/2*a*b^2*x^6 + a^2*b*x^3 + a^3*log(x)
```

giac [**A**] time = 0.43, size = 65, normalized size = 0.41

$$\frac{1}{9}\,b^3x^9\mathrm{sgn}\left(bx^3+a\right)+\frac{1}{2}\,ab^2x^6\mathrm{sgn}\left(bx^3+a\right)+a^2bx^3\mathrm{sgn}\left(bx^3+a\right)+a^3\log\left(|x|\right)\mathrm{sgn}\left(bx^3+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x,x, algorithm="giac")
```

```
[Out] 1/9*b^3*x^9*sgn(b*x^3 + a) + 1/2*a*b^2*x^6*sgn(b*x^3 + a) + a^2*b*x^3*sgn(b
*x^3 + a) + a^3*log(abs(x))*sgn(b*x^3 + a)
```

maple [A] time = 0.01, size = 57, normalized size = 0.36

$$\frac{\left(\left(b\,x^{3}+a\right)^{2}\right)^{\frac{3}{2}}\left(2b^{3}x^{9}+9a\,b^{2}x^{6}+18a^{2}b\,x^{3}+18a^{3}\ln\left(x\right)\right)}{18\left(b\,x^{3}+a\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x,x)
```

```
[Out] 1/18*((b*x^3+a)^2)^(3/2)*(2*b^3*x^9+9*a*b^2*x^6+18*a^2*b*x^3+18*a^3*ln(x))/
(b*x^3+a)^3
```

maxima [A] time = 0.52, size = 152, normalized size = 0.95

$$\frac{1}{6}\sqrt{b^{2}x^{6}+2\,abx^{3}+a^{2}}\,abx^{3}+\frac{1}{3}\left(-1\right)^{2\,b^{2}x^{3}+2\,ab}a^{3}\log\left(2\,b^{2}x^{3}+2\,ab\right)-\frac{1}{3}\left(-1\right)^{2\,abx^{3}+2\,a^{2}}a^{3}\log\left(\frac{2\,abx}{|x|}+\frac{2\,a^{2}}{x^{2}|x|}\right)+\frac{1}{2}\sqrt{b^{2}x^{6}+2\,abx^{3}+a^{2}}\,a^{2}+\frac{1}{9}\left(b^{2}x^{6}+2\,abx^{3}+a^{2}\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x,x, algorithm="maxima")
```

```
[Out] 1/6*sqrt(b^2*x^6 + 2*a*b*x^3 + a^2)*a*b*x^3 + 1/3*(-1)^(2*b^2*x^3 + 2*a*b)*
a^3*log(2*b^2*x^3 + 2*a*b) - 1/3*(-1)^(2*a*b*x^3 + 2*a^2)*a^3*log(2*a*b*x/a
bs(x) + 2*a^2/(x^2*abs(x))) + 1/2*sqrt(b^2*x^6 + 2*a*b*x^3 + a^2)*a^2 + 1/9
*(b^2*x^6 + 2*a*b*x^3 + a^2)^(3/2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(a^2+2\,a\,b\,x^3+b^2\,x^6\right)}^{3/2}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2)/x,x)
```

```
[Out] int((a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2)/x, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\left(a+bx^{3}\right)^{2}\right)^{\frac{3}{2}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**6+2*a*b*x**3+a**2)**(3/2)/x,x)
```

```
[Out] Integral(((a + b*x**3)**2)**(3/2)/x, x)
```

3.34 $\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{3/2}}{x^2}\,dx$

Optimal. Leaf size=165

$$\frac{3ab^2x^5\sqrt{a^2+2abx^3+b^2x^6}}{5\left(a+bx^3\right)}+\frac{3a^2bx^2\sqrt{a^2+2abx^3+b^2x^6}}{2\left(a+bx^3\right)}+\frac{b^3x^8\sqrt{a^2+2abx^3+b^2x^6}}{8\left(a+bx^3\right)}-\frac{a^3\sqrt{a^2+2abx^3+b^2x^6}}{x\left(a+bx^3\right)}$$

Rubi [A] time = 0.04, antiderivative size = 165, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1355, 270}

$$\frac{b^3x^8\sqrt{a^2+2abx^3+b^2x^6}}{8\left(a+bx^3\right)}+\frac{3ab^2x^5\sqrt{a^2+2abx^3+b^2x^6}}{5\left(a+bx^3\right)}+\frac{3a^2bx^2\sqrt{a^2+2abx^3+b^2x^6}}{2\left(a+bx^3\right)}-\frac{a^3\sqrt{a^2+2abx^3+b^2x^6}}{x\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)/x^2,x]
```

```
[Out] -((a^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(x*(a + b*x^3))) + (3*a^2*b*x^2*Sqr
t[a^2 + 2*a*b*x^3 + b^2*x^6])/(2*(a + b*x^3)) + (3*a*b^2*x^5*Sqrt[a^2 + 2*a
*b*x^3 + b^2*x^6])/(5*(a + b*x^3)) + (b^3*x^8*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^
6])/(8*(a + b*x^3))
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{3/2}}{x^2}\,dx &= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{\left(ab+b^2x^3\right)^3}{x^2}\,dx}{b^2\left(ab+b^2x^3\right)}\\ &= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \left(\frac{a^3b^3}{x^2}+3a^2b^4x+3ab^5x^4+b^6x^7\right)dx}{b^2\left(ab+b^2x^3\right)}\\ &= -\frac{a^3\sqrt{a^2+2abx^3+b^2x^6}}{x\left(a+bx^3\right)}+\frac{3a^2bx^2\sqrt{a^2+2abx^3+b^2x^6}}{2\left(a+bx^3\right)}+\frac{3ab^2x^5\sqrt{a^2+2abx^3}}{5\left(a+bx^3\right)}\end{aligned}$$

Mathematica [A] time = 0.02, size = 61, normalized size = 0.37

$$\frac{\sqrt{\left(a+bx^3\right)^2}\left(-40a^3+60a^2bx^3+24ab^2x^6+5b^3x^9\right)}{40x\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)/x^2,x]
```

```
[Out] (Sqrt[(a + b*x^3)^2]*(-40*a^3 + 60*a^2*b*x^3 + 24*a*b^2*x^6 + 5*b^3*x^9))/(
40*x*(a + b*x^3))
```

IntegrateAlgebraic [A] time = 17.20, size = 61, normalized size = 0.37

$$\frac{\sqrt{\left(a+bx^3\right)^2}\left(-40a^3+60a^2bx^3+24ab^2x^6+5b^3x^9\right)}{40x\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)/x^2,x]
```

```
[Out] (Sqrt[(a + b*x^3)^2]*(-40*a^3 + 60*a^2*b*x^3 + 24*a*b^2*x^6 + 5*b^3*x^9))/(
40*x*(a + b*x^3))
```

fricas [A] time = 1.38, size = 37, normalized size = 0.22

$$\frac{5\,b^3x^9+24\,ab^2x^6+60\,a^2bx^3-40\,a^3}{40\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^2,x, algorithm="fricas")
```

```
[Out] 1/40*(5*b^3*x^9 + 24*a*b^2*x^6 + 60*a^2*b*x^3 - 40*a^3)/x
```

giac [A] time = 0.34, size = 67, normalized size = 0.41

$$\frac{1}{8}\,b^3x^8\mathrm{sgn}\left(bx^3+a\right)+\frac{3}{5}\,ab^2x^5\mathrm{sgn}\left(bx^3+a\right)+\frac{3}{2}\,a^2bx^2\mathrm{sgn}\left(bx^3+a\right)-\frac{a^3\mathrm{sgn}\left(bx^3+a\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^2,x, algorithm="giac")
```

```
[Out] 1/8*b^3*x^8*sgn(b*x^3 + a) + 3/5*a*b^2*x^5*sgn(b*x^3 + a) + 3/2*a^2*b*x^2*s
gn(b*x^3 + a) - a^3*sgn(b*x^3 + a)/x
```

maple [A] time = 0.01, size = 58, normalized size = 0.35

$$-\frac{\left(-5b^3x^9-24a\,b^2x^6-60a^2b\,x^3+40a^3\right)\left(\left(b\,x^3+a\right)^2\right)^{\frac{3}{2}}}{40\left(b\,x^3+a\right)^3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^2,x)
```

```
[Out] -1/40*(-5*b^3*x^9-24*a*b^2*x^6-60*a^2*b*x^3+40*a^3)*((b*x^3+a)^2)^(3/2)/x/(
b*x^3+a)^3
```

maxima [A] time = 0.50, size = 37, normalized size = 0.22

$$\frac{5\,b^3x^9+24\,ab^2x^6+60\,a^2bx^3-40\,a^3}{40\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^2,x, algorithm="maxima")
[Out] 1/40*(5*b^3*x^9 + 24*a*b^2*x^6 + 60*a^2*b*x^3 - 40*a^3)/x
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a^2 + 2\,a\,b\,x^3 + b^2\,x^6\right)^{3/2}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2)/x^2,x)
[Out] int((a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2)/x^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\left(a + bx^3\right)^2\right)^{\frac{3}{2}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**6+2*a*b*x**3+a**2)**(3/2)/x**2,x)
[Out] Integral(((a + b*x**3)**2)**(3/2)/x**2, x)
```

3.35 $\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{3/2}}{x^3}\,dx$

Optimal. Leaf size=163

$$\frac{3a^2bx\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}+\frac{3ab^2x^4\sqrt{a^2+2abx^3+b^2x^6}}{4\left(a+bx^3\right)}+\frac{b^3x^7\sqrt{a^2+2abx^3+b^2x^6}}{7\left(a+bx^3\right)}-\frac{a^3\sqrt{a^2+2abx^3+b^2x^6}}{2x^2\left(a+bx^3\right)}$$

Rubi [A] time = 0.04, antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1355, 270}

$$\frac{b^3x^7\sqrt{a^2+2abx^3+b^2x^6}}{7\left(a+bx^3\right)}+\frac{3ab^2x^4\sqrt{a^2+2abx^3+b^2x^6}}{4\left(a+bx^3\right)}+\frac{3a^2bx\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}-\frac{a^3\sqrt{a^2+2abx^3+b^2x^6}}{2x^2\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)/x^3,x]
```

```
[Out] -(a^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(2*x^2*(a + b*x^3)) + (3*a^2*b*x*Sqr
t[a^2 + 2*a*b*x^3 + b^2*x^6])/(a + b*x^3) + (3*a*b^2*x^4*Sqrt[a^2 + 2*a*b*x
^3 + b^2*x^6])/(4*(a + b*x^3)) + (b^3*x^7*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/
(7*(a + b*x^3))
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{3/2}}{x^3}\,dx &= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{\left(ab+b^2x^3\right)^3}{x^3}\,dx}{b^2\left(ab+b^2x^3\right)}\\ &= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \left(3a^2b^4+\frac{a^3b^3}{x^3}+3ab^5x^3+b^6x^6\right)dx}{b^2\left(ab+b^2x^3\right)}\\ &= -\frac{a^3\sqrt{a^2+2abx^3+b^2x^6}}{2x^2\left(a+bx^3\right)}+\frac{3a^2bx\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}+\frac{3ab^2x^4\sqrt{a^2+2abx^3+b^2}}{4\left(a+bx^3\right)}\end{aligned}$$

Mathematica [A] time = 0.02, size = 61, normalized size = 0.37

$$\frac{\sqrt{\left(a+bx^3\right)^2}\left(-14a^3+84a^2bx^3+21ab^2x^6+4b^3x^9\right)}{28x^2\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)/x^3,x]
```

```
[Out] (Sqrt[(a + b*x^3)^2]*(-14*a^3 + 84*a^2*b*x^3 + 21*a*b^2*x^6 + 4*b^3*x^9))/(
28*x^2*(a + b*x^3))
```

IntegrateAlgebraic [A] time = 19.88, size = 61, normalized size = 0.37

$$\frac{\sqrt{\left(a+bx^3\right)^2}\left(-14a^3+84a^2bx^3+21ab^2x^6+4b^3x^9\right)}{28x^2\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)/x^3,x]
```

```
[Out] (Sqrt[(a + b*x^3)^2]*(-14*a^3 + 84*a^2*b*x^3 + 21*a*b^2*x^6 + 4*b^3*x^9))/(
28*x^2*(a + b*x^3))
```

fricas [A] time = 1.47, size = 37, normalized size = 0.23

$$\frac{4\,b^3x^9+21\,ab^2x^6+84\,a^2bx^3-14\,a^3}{28\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^3,x, algorithm="fricas")
```

```
[Out] 1/28*(4*b^3*x^9 + 21*a*b^2*x^6 + 84*a^2*b*x^3 - 14*a^3)/x^2
```

giac [A] time = 0.38, size = 65, normalized size = 0.40

$$\frac{1}{7}\,b^3x^7\mathrm{sgn}\left(bx^3+a\right)+\frac{3}{4}\,ab^2x^4\mathrm{sgn}\left(bx^3+a\right)+3\,a^2bx\mathrm{sgn}\left(bx^3+a\right)-\frac{a^3\mathrm{sgn}\left(bx^3+a\right)}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^3,x, algorithm="giac")
```

```
[Out] 1/7*b^3*x^7*sgn(b*x^3 + a) + 3/4*a*b^2*x^4*sgn(b*x^3 + a) + 3*a^2*b*x*sgn(b
*x^3 + a) - 1/2*a^3*sgn(b*x^3 + a)/x^2
```

maple [A] time = 0.01, size = 58, normalized size = 0.36

$$-\frac{\left(-4b^3x^9-21a\,b^2x^6-84a^2b\,x^3+14a^3\right)\left(\left(b\,x^3+a\right)^2\right)^{\frac{3}{2}}}{28\left(b\,x^3+a\right)^3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^3,x)
```

```
[Out] -1/28*(-4*b^3*x^9-21*a*b^2*x^6-84*a^2*b*x^3+14*a^3)*((b*x^3+a)^2)^(3/2)/x^2
/(b*x^3+a)^3
```

maxima [A] time = 0.50, size = 37, normalized size = 0.23

$$\frac{4\,b^3x^9+21\,ab^2x^6+84\,a^2bx^3-14\,a^3}{28\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^3,x, algorithm="maxima")
[Out] 1/28*(4*b^3*x^9 + 21*a*b^2*x^6 + 84*a^2*b*x^3 - 14*a^3)/x^2
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a^2+2\,a\,b\,x^3+b^2\,x^6\right)^{3/2}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2)/x^3,x)
[Out] int((a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2)/x^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\left(a+bx^3\right)^2\right)^{\frac{3}{2}}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**6+2*a*b*x**3+a**2)**(3/2)/x**3,x)
[Out] Integral(((a + b*x**3)**2)**(3/2)/x**3, x)
```

3.36 $\int \frac{(a^2+2abx^3+b^2x^6)^{3/2}}{x^4}\,dx$

Optimal. Leaf size=161

$$\frac{ab^2x^3\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}+\frac{3a^2b\log(x)\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}+\frac{b^3x^6\sqrt{a^2+2abx^3+b^2x^6}}{6\left(a+bx^3\right)}-\frac{a^3\sqrt{a^2+2abx^3+b^2x^6}}{3x^3\left(a+bx^3\right)}$$

Rubi [A] time = 0.05, antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1355, 266, 43}

$$\frac{b^3x^6\sqrt{a^2+2abx^3+b^2x^6}}{6\left(a+bx^3\right)}+\frac{ab^2x^3\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}-\frac{a^3\sqrt{a^2+2abx^3+b^2x^6}}{3x^3\left(a+bx^3\right)}+\frac{3a^2b\log(x)\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)/x^4,x]
```

```
[Out] -(a^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(3*x^3*(a + b*x^3)) + (a*b^2*x^3*Sqr
t[a^2 + 2*a*b*x^3 + b^2*x^6])/(a + b*x^3) + (b^3*x^6*Sqrt[a^2 + 2*a*b*x^3 +
 b^2*x^6])/(6*(a + b*x^3)) + (3*a^2*b*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]*Log[x
])/(a + b*x^3)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{3/2}}{x^4}\,dx = \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{\left(ab+b^2x^3\right)^3}{x^4}\,dx}{b^2\left(ab+b^2x^3\right)}$$

$$= \frac{\sqrt{a^2+2abx^3+b^2x^6}\ \text{Subst}\left(\int \frac{\left(ab+b^2x\right)^3}{x^2}\,dx, x, x^3\right)}{3b^2\left(ab+b^2x^3\right)}$$

$$= \frac{\sqrt{a^2+2abx^3+b^2x^6}\ \text{Subst}\left(\int \left(3ab^5+\frac{a^3b^3}{x^2}+\frac{3a^2b^4}{x}+b^6x\right)dx, x, x^3\right)}{3b^2\left(ab+b^2x^3\right)}$$

$$= -\frac{a^3\sqrt{a^2+2abx^3+b^2x^6}}{3x^3\left(a+bx^3\right)} + \frac{ab^2x^3\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3} + \frac{b^3x^6\sqrt{a^2+2abx^3+b^2x^6}}{6\left(a+bx^3\right)}$$

Mathematica [A] time = 0.02, size = 62, normalized size = 0.39

$$\frac{\sqrt{\left(a+bx^3\right)^2}\left(-2a^3+18a^2bx^3\log(x)+6ab^2x^6+b^3x^9\right)}{6x^3\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)/x^4,x]
```

```
[Out] (Sqrt[(a + b*x^3)^2]*(-2*a^3 + 6*a*b^2*x^6 + b^3*x^9 + 18*a^2*b*x^3*Log[x])
)/(6*x^3*(a + b*x^3))
```

IntegrateAlgebraic [A] time = 0.74, size = 317, normalized size = 1.97

$$-\frac{1}{2}a^2\sqrt{b^2}\log\left(\sqrt{a^2+2abx^3+b^2x^6}-a-\sqrt{b^2}x^3\right)-\frac{1}{2}a^2\sqrt{b^2}\log\left(\sqrt{a^2+2abx^3+b^2x^6}+a-\sqrt{b^2}x^3\right)+a^2b\tanh^{-1}\left(\frac{\sqrt{b^2}x^3}{a}-\frac{\sqrt{a^2+2abx^3+b^2x^6}}{a}\right)+\frac{\sqrt{a^2+2abx^3+b^2x^6}\left(-8a^3b-21a^2b^2x^3+24ab^3x^6+4b^4x^9\right)+\sqrt{b^2}\left(8a^4+29a^3bx^3-3a^2b^2x^6-28ab^3x^9-4b^4x^{12}\right)}{24x^3\left(ab+b^2x^3\right)-24\sqrt{b^2}x^3\sqrt{a^2+2abx^3+b^2x^6}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)/x^4,x]
```

```
[Out] (Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]*(-8*a^3*b - 21*a^2*b^2*x^3 + 24*a*b^3*x^6
+ 4*b^4*x^9) + Sqrt[b^2]*(8*a^4 + 29*a^3*b*x^3 - 3*a^2*b^2*x^6 - 28*a*b^3*x
^9 - 4*b^4*x^12))/(24*x^3*(a*b + b^2*x^3) - 24*Sqrt[b^2]*x^3*Sqrt[a^2 + 2*a
*b*x^3 + b^2*x^6]) + a^2*b*ArcTanh[(Sqrt[b^2]*x^3)/a - Sqrt[a^2 + 2*a*b*x^3
 + b^2*x^6]/a] - (a^2*Sqrt[b^2]*Log[-a - Sqrt[b^2]*x^3 + Sqrt[a^2 + 2*a*b*x
^3 + b^2*x^6]])/2 - (a^2*Sqrt[b^2]*Log[a - Sqrt[b^2]*x^3 + Sqrt[a^2 + 2*a*b
*x^3 + b^2*x^6]])/2
```

fricas [A] time = 1.17, size = 38, normalized size = 0.24

$$\frac{b^3x^9+6\,ab^2x^6+18\,a^2bx^3\log(x)-2\,a^3}{6\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^4,x, algorithm="fricas")
```

```
[Out] 1/6*(b^3*x^9 + 6*a*b^2*x^6 + 18*a^2*b*x^3*log(x) - 2*a^3)/x^3
```

giac [A] time = 0.37, size = 85, normalized size = 0.53

$$\frac{1}{6}\,b^3x^6\text{sgn}\left(bx^3+a\right)+ab^2x^3\text{sgn}\left(bx^3+a\right)+3\,a^2b\log\left(|x|\right)\text{sgn}\left(bx^3+a\right)-\frac{3\,a^2bx^3\text{sgn}\left(bx^3+a\right)+a^3\text{sgn}\left(bx^3+a\right)}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^4,x, algorithm="giac")
```

```
[Out] 1/6*b^3*x^6*sgn(b*x^3 + a) + a*b^2*x^3*sgn(b*x^3 + a) + 3*a^2*b*log(abs(x))
*sgn(b*x^3 + a) - 1/3*(3*a^2*b*x^3*sgn(b*x^3 + a) + a^3*sgn(b*x^3 + a))/x^3
```

maple [A] time = 0.01, size = 59, normalized size = 0.37

$$\frac{\left(\left(b\,x^{3}+a\right)^{2}\right)^{\frac{3}{2}}\left(b^{3}x^{9}+6a\,b^{2}x^{6}+18a^{2}b\,x^{3}\ln(x)-2a^{3}\right)}{6\left(b\,x^{3}+a\right)^{3}x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^4,x)
```

```
[Out] 1/6*((b*x^3+a)^2)^(3/2)*(b^3*x^9+6*a*b^2*x^6+18*a^2*b*ln(x)*x^3-2*a^3)/(b*x
^3+a)^3/x^3
```

maxima [A] time = 0.53, size = 156, normalized size = 0.97

$$\frac{1}{2}\sqrt{b^{2}x^{6}+2\,abx^{3}+a^{2}}\,b^{2}x^{3}+(-1)^{2\,b^{2}x^{3}+2\,ab}\,a^{2}b\log\left(2\,b^{2}x^{3}+2\,ab\right)-(-1)^{2\,abx^{3}+2\,a^{2}}\,a^{2}b\log\left(\frac{2\,abx}{|x|}+\frac{2\,a^{2}}{x^{2}|x|}\right)+\frac{3}{2}\sqrt{b^{2}x^{6}+2\,abx^{3}+a^{2}}\,ab-\frac{\left(b^{2}x^{6}+2\,abx^{3}+a^{2}\right)^{\frac{3}{2}}}{3\,x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^4,x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(b^2*x^6 + 2*a*b*x^3 + a^2)*b^2*x^3 + (-1)^(2*b^2*x^3 + 2*a*b)*a^2*
b*log(2*b^2*x^3 + 2*a*b) - (-1)^(2*a*b*x^3 + 2*a^2)*a^2*b*log(2*a*b*x/abs(x
) + 2*a^2/(x^2*abs(x))) + 3/2*sqrt(b^2*x^6 + 2*a*b*x^3 + a^2)*a*b - 1/3*(b^
2*x^6 + 2*a*b*x^3 + a^2)^(3/2)/x^3
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a^{2}+2\,a\,b\,x^{3}+b^{2}\,x^{6}\right)^{3/2}}{x^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2)/x^4,x)
```

```
[Out] int((a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2)/x^4, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\left(a+bx^{3}\right)^{2}\right)^{\frac{3}{2}}}{x^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**6+2*a*b*x**3+a**2)**(3/2)/x**4,x)
```

```
[Out] Integral(((a + b*x**3)**2)**(3/2)/x**4, x)
```

3.37 $\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{3/2}}{x^5}\,dx$

Optimal. Leaf size=165

$$-\frac{3a^2b\sqrt{a^2+2abx^3+b^2x^6}}{x\left(a+bx^3\right)}+\frac{3ab^2x^2\sqrt{a^2+2abx^3+b^2x^6}}{2\left(a+bx^3\right)}+\frac{b^3x^5\sqrt{a^2+2abx^3+b^2x^6}}{5\left(a+bx^3\right)}-\frac{a^3\sqrt{a^2+2abx^3+b^2x^6}}{4x^4\left(a+bx^3\right)}$$

Rubi [A] time = 0.04, antiderivative size = 165, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {1355, 270}

$$\frac{b^3x^5\sqrt{a^2+2abx^3+b^2x^6}}{5\left(a+bx^3\right)}+\frac{3ab^2x^2\sqrt{a^2+2abx^3+b^2x^6}}{2\left(a+bx^3\right)}-\frac{3a^2b\sqrt{a^2+2abx^3+b^2x^6}}{x\left(a+bx^3\right)}-\frac{a^3\sqrt{a^2+2abx^3+b^2x^6}}{4x^4\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)/x^5,x]
```

```
[Out] -(a^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(4*x^4*(a + b*x^3)) - (3*a^2*b*Sqrt[
a^2 + 2*a*b*x^3 + b^2*x^6])/(x*(a + b*x^3)) + (3*a*b^2*x^2*Sqrt[a^2 + 2*a*b
*x^3 + b^2*x^6])/(2*(a + b*x^3)) + (b^3*x^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]
)/(5*(a + b*x^3))
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{3/2}}{x^5}\,dx &= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{\left(ab+b^2x^3\right)^3}{x^5}\,dx}{b^2\left(ab+b^2x^3\right)}\\
&= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \left(\frac{a^3b^3}{x^5}+\frac{3a^2b^4}{x^2}+3ab^5x+b^6x^4\right)dx}{b^2\left(ab+b^2x^3\right)}\\
&= -\frac{a^3\sqrt{a^2+2abx^3+b^2x^6}}{4x^4\left(a+bx^3\right)}-\frac{3a^2b\sqrt{a^2+2abx^3+b^2x^6}}{x\left(a+bx^3\right)}+\frac{3ab^2x^2\sqrt{a^2+2abx^3+b^2x}}{2\left(a+bx^3\right)}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 61, normalized size = 0.37

$$\frac{\sqrt{\left(a+bx^3\right)^2}\left(-5a^3-60a^2bx^3+30ab^2x^6+4b^3x^9\right)}{20x^4\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)/x^5,x]
```

```
[Out] (Sqrt[(a + b*x^3)^2]*(-5*a^3 - 60*a^2*b*x^3 + 30*a*b^2*x^6 + 4*b^3*x^9))/(2
0*x^4*(a + b*x^3))
```

IntegrateAlgebraic [A] time = 20.33, size = 61, normalized size = 0.37

$$\frac{\sqrt{\left(a+bx^3\right)^2}\left(-5a^3-60a^2bx^3+30ab^2x^6+4b^3x^9\right)}{20x^4\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)/x^5,x]
```

```
[Out] (Sqrt[(a + b*x^3)^2]*(-5*a^3 - 60*a^2*b*x^3 + 30*a*b^2*x^6 + 4*b^3*x^9))/(2
0*x^4*(a + b*x^3))
```

fricas [A] time = 1.15, size = 37, normalized size = 0.22

$$\frac{4\,b^3x^9+30\,ab^2x^6-60\,a^2bx^3-5\,a^3}{20\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^5,x, algorithm="fricas")
```

```
[Out] 1/20*(4*b^3*x^9 + 30*a*b^2*x^6 - 60*a^2*b*x^3 - 5*a^3)/x^4
```

giac [A] time = 0.36, size = 69, normalized size = 0.42

$$\frac{1}{5}\,b^3x^5\mathrm{sgn}\left(bx^3+a\right)+\frac{3}{2}\,ab^2x^2\mathrm{sgn}\left(bx^3+a\right)-\frac{12\,a^2bx^3\mathrm{sgn}\left(bx^3+a\right)+a^3\mathrm{sgn}\left(bx^3+a\right)}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^5,x, algorithm="giac")
```

```
[Out] 1/5*b^3*x^5*sgn(b*x^3 + a) + 3/2*a*b^2*x^2*sgn(b*x^3 + a) - 1/4*(12*a^2*b*x
^3*sgn(b*x^3 + a) + a^3*sgn(b*x^3 + a))/x^4
```

maple [A] time = 0.01, size = 58, normalized size = 0.35

$$-\frac{\left(-4b^3x^9-30a\,b^2x^6+60a^2b\,x^3+5a^3\right)\left(\left(b\,x^3+a\right)^2\right)^{\frac{3}{2}}}{20\left(b\,x^3+a\right)^3x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^5,x)
```

```
[Out] -1/20*(-4*b^3*x^9-30*a*b^2*x^6+60*a^2*b*x^3+5*a^3)*((b*x^3+a)^2)^(3/2)/x^4/
(b*x^3+a)^3
```

maxima [A] time = 0.53, size = 37, normalized size = 0.22

$$\frac{4\,b^3x^9+30\,ab^2x^6-60\,a^2bx^3-5\,a^3}{20\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^5,x, algorithm="maxima")
```

```
[Out] 1/20*(4*b^3*x^9 + 30*a*b^2*x^6 - 60*a^2*b*x^3 - 5*a^3)/x^4
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a^2+2\,a\,b\,x^3+b^2\,x^6\right)^{3/2}}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2)/x^5,x)
```

```
[Out] int((a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2)/x^5, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\left(a+bx^3\right)^2\right)^{\frac{3}{2}}}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**6+2*a*b*x**3+a**2)**(3/2)/x**5,x)
```

```
[Out] Integral(((a + b*x**3)**2)**(3/2)/x**5, x)
```

3.38 $\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{3/2}}{x^6}\,dx$

Optimal. Leaf size=163

$$\frac{3ab^2x\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}-\frac{3a^2b\sqrt{a^2+2abx^3+b^2x^6}}{2x^2\left(a+bx^3\right)}+\frac{b^3x^4\sqrt{a^2+2abx^3+b^2x^6}}{4\left(a+bx^3\right)}-\frac{a^3\sqrt{a^2+2abx^3+b^2x^6}}{5x^5\left(a+bx^3\right)}$$

Rubi [A] time = 0.04, antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1355, 270}

$$\frac{b^3x^4\sqrt{a^2+2abx^3+b^2x^6}}{4\left(a+bx^3\right)}+\frac{3ab^2x\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}-\frac{3a^2b\sqrt{a^2+2abx^3+b^2x^6}}{2x^2\left(a+bx^3\right)}-\frac{a^3\sqrt{a^2+2abx^3+b^2x^6}}{5x^5\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)/x^6,x]
```

```
[Out] -(a^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(5*x^5*(a + b*x^3)) - (3*a^2*b*Sqrt[
a^2 + 2*a*b*x^3 + b^2*x^6])/(2*x^2*(a + b*x^3)) + (3*a*b^2*x*Sqrt[a^2 + 2*a
*b*x^3 + b^2*x^6])/(a + b*x^3) + (b^3*x^4*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/
(4*(a + b*x^3))
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{3/2}}{x^6}\,dx &= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{\left(ab+b^2x^3\right)^3}{x^6}\,dx}{b^2\left(ab+b^2x^3\right)}\\ &= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \left(3ab^5+\frac{a^3b^3}{x^6}+\frac{3a^2b^4}{x^3}+b^6x^3\right)dx}{b^2\left(ab+b^2x^3\right)}\\ &= -\frac{a^3\sqrt{a^2+2abx^3+b^2x^6}}{5x^5\left(a+bx^3\right)}-\frac{3a^2b\sqrt{a^2+2abx^3+b^2x^6}}{2x^2\left(a+bx^3\right)}+\frac{3ab^2x\sqrt{a^2+2abx^3+b}}{a+bx^3}\end{aligned}$$

Mathematica [A] time = 0.02, size = 61, normalized size = 0.37

$$\frac{\sqrt{\left(a+bx^3\right)^2}\left(-4a^3-30a^2bx^3+60ab^2x^6+5b^3x^9\right)}{20x^5\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)/x^6,x]
```

```
[Out] (Sqrt[(a + b*x^3)^2]*(-4*a^3 - 30*a^2*b*x^3 + 60*a*b^2*x^6 + 5*b^3*x^9))/(2
0*x^5*(a + b*x^3))
```

IntegrateAlgebraic [A] time = 20.01, size = 61, normalized size = 0.37

$$\frac{\sqrt{\left(a+bx^3\right)^2}\left(-4a^3-30a^2bx^3+60ab^2x^6+5b^3x^9\right)}{20x^5\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)/x^6,x]
```

```
[Out] (Sqrt[(a + b*x^3)^2]*(-4*a^3 - 30*a^2*b*x^3 + 60*a*b^2*x^6 + 5*b^3*x^9))/(2
0*x^5*(a + b*x^3))
```

fricas [A] time = 1.42, size = 37, normalized size = 0.23

$$\frac{5\,b^3x^9+60\,ab^2x^6-30\,a^2bx^3-4\,a^3}{20\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^6,x, algorithm="fricas")
```

```
[Out] 1/20*(5*b^3*x^9 + 60*a*b^2*x^6 - 30*a^2*b*x^3 - 4*a^3)/x^5
```

giac [A] time = 0.29, size = 68, normalized size = 0.42

$$\frac{1}{4}\,b^3x^4\mathrm{sgn}\left(bx^3+a\right)+3\,ab^2x\mathrm{sgn}\left(bx^3+a\right)-\frac{15\,a^2bx^3\mathrm{sgn}\left(bx^3+a\right)+2\,a^3\mathrm{sgn}\left(bx^3+a\right)}{10\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^6,x, algorithm="giac")
```

```
[Out] 1/4*b^3*x^4*sgn(b*x^3 + a) + 3*a*b^2*x*sgn(b*x^3 + a) - 1/10*(15*a^2*b*x^3*
sgn(b*x^3 + a) + 2*a^3*sgn(b*x^3 + a))/x^5
```

maple [A] time = 0.01, size = 58, normalized size = 0.36

$$-\frac{\left(-5b^3x^9-60a\,b^2x^6+30a^2b\,x^3+4a^3\right)\left(\left(b\,x^3+a\right)^2\right)^{\frac{3}{2}}}{20\left(b\,x^3+a\right)^3x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^6,x)
```

```
[Out] -1/20*(-5*b^3*x^9-60*a*b^2*x^6+30*a^2*b*x^3+4*a^3)*((b*x^3+a)^2)^(3/2)/x^5/
(b*x^3+a)^3
```

maxima [A] time = 0.50, size = 37, normalized size = 0.23

$$\frac{5\,b^3x^9+60\,ab^2x^6-30\,a^2bx^3-4\,a^3}{20\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^6,x, algorithm="maxima")
```

```
[Out] 1/20*(5*b^3*x^9 + 60*a*b^2*x^6 - 30*a^2*b*x^3 - 4*a^3)/x^5
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a^2+2\,a\,b\,x^3+b^2\,x^6\right)^{3/2}}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2)/x^6,x)
```

```
[Out] int((a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2)/x^6, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\left(a+bx^3\right)^2\right)^{\frac{3}{2}}}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**6+2*a*b*x**3+a**2)**(3/2)/x**6,x)
```

```
[Out] Integral(((a + b*x**3)**2)**(3/2)/x**6, x)
```

3.39 $\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{3/2}}{x^7}\,dx$

Optimal. Leaf size=162

$$-\frac{a^2b\sqrt{a^2+2abx^3+b^2x^6}}{x^3\left(a+bx^3\right)}+\frac{3ab^2\log(x)\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}+\frac{b^3x^3\sqrt{a^2+2abx^3+b^2x^6}}{3\left(a+bx^3\right)}-\frac{a^3\sqrt{a^2+2abx^3+b^2x^6}}{6x^6\left(a+bx^3\right)}$$

Rubi [A] time = 0.05, antiderivative size = 162, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1355, 266, 43}

$$-\frac{a^3\sqrt{a^2+2abx^3+b^2x^6}}{6x^6\left(a+bx^3\right)}-\frac{a^2b\sqrt{a^2+2abx^3+b^2x^6}}{x^3\left(a+bx^3\right)}+\frac{b^3x^3\sqrt{a^2+2abx^3+b^2x^6}}{3\left(a+bx^3\right)}+\frac{3ab^2\log(x)\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)/x^7,x]
```

```
[Out] -(a^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(6*x^6*(a + b*x^3)) - (a^2*b*Sqrt[a^
2 + 2*a*b*x^3 + b^2*x^6])/(x^3*(a + b*x^3)) + (b^3*x^3*Sqrt[a^2 + 2*a*b*x^3
 + b^2*x^6])/(3*(a + b*x^3)) + (3*a*b^2*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]*Log
[x])/(a + b*x^3)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{3/2}}{x^7}\,dx = \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{\left(ab+b^2x^3\right)^3}{x^7}\,dx}{b^2\left(ab+b^2x^3\right)}$$

$$= \frac{\sqrt{a^2+2abx^3+b^2x^6}\ \mathrm{Subst}\left(\int \frac{\left(ab+b^2x\right)^3}{x^3}\,dx, x, x^3\right)}{3b^2\left(ab+b^2x^3\right)}$$

$$= \frac{\sqrt{a^2+2abx^3+b^2x^6}\ \mathrm{Subst}\left(\int \left(b^6+\frac{a^3b^3}{x^3}+\frac{3a^2b^4}{x^2}+\frac{3ab^5}{x}\right)dx, x, x^3\right)}{3b^2\left(ab+b^2x^3\right)}$$

$$= -\frac{a^3\sqrt{a^2+2abx^3+b^2x^6}}{6x^6\left(a+bx^3\right)} - \frac{a^2b\sqrt{a^2+2abx^3+b^2x^6}}{x^3\left(a+bx^3\right)} + \frac{b^3x^3\sqrt{a^2+2abx^3+b^2x^6}}{3\left(a+bx^3\right)}$$

Mathematica [A] time = 0.02, size = 61, normalized size = 0.38

$$-\frac{\sqrt{\left(a+bx^3\right)^2}\left(a^3+6a^2bx^3-18ab^2x^6\log(x)-2b^3x^9\right)}{6x^6\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)/x^7,x]
```

```
[Out] -1/6*(Sqrt[(a + b*x^3)^2]*(a^3 + 6*a^2*b*x^3 - 2*b^3*x^9 - 18*a*b^2*x^6*Log
[x]))/(x^6*(a + b*x^3))
```

IntegrateAlgebraic [B] time = 1.68, size = 1164, normalized size = 7.19

[illegible]

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)/x^7,x]
```

```
[Out] (Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]*(-(a^3*b) - 6*a^2*b^2*x^3 + a*b^3*x^6 + 2*
b^4*x^9) + Sqrt[b^2]*(a^4 + 7*a^3*b*x^3 + 5*a^2*b^2*x^6 - 3*a*b^3*x^9 - 2*b
^4*x^12))/(6*x^6*(a*b + b^2*x^3) - 6*Sqrt[b^2]*x^6*Sqrt[a^2 + 2*a*b*x^3 + b
^2*x^6]) + (a*b^2*Log[-a - Sqrt[b^2]*x^3 + Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]]
)/2 - (a*b*Sqrt[b^2]*Log[-a - Sqrt[b^2]*x^3 + Sqrt[a^2 + 2*a*b*x^3 + b^2*x^
6]])/2 - (a^5*b^2*Log[a - Sqrt[b^2]*x^3 + Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]])
/(2*(-a - Sqrt[b^2]*x^3 + Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])^2*(a - Sqrt[b^2]
*x^3 + Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])^2) - (a^5*b*Sqrt[b^2]*Log[a - Sqrt[
b^2]*x^3 + Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]])/(2*(-a - Sqrt[b^2]*x^3 + Sqrt[
a^2 + 2*a*b*x^3 + b^2*x^6])^2*(a - Sqrt[b^2]*x^3 + Sqrt[a^2 + 2*a*b*x^3 + b
^2*x^6])^2) + (a^3*b^2*(-(Sqrt[b^2]*x^3) + Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])
^2*Log[a - Sqrt[b^2]*x^3 + Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]])/((-a - Sqrt[b^
2]*x^3 + Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])^2*(a - Sqrt[b^2]*x^3 + Sqrt[a^2 +
 2*a*b*x^3 + b^2*x^6])^2) + (a^3*b*Sqrt[b^2]*(-(Sqrt[b^2]*x^3) + Sqrt[a^2 +
 2*a*b*x^3 + b^2*x^6])^2*Log[a - Sqrt[b^2]*x^3 + Sqrt[a^2 + 2*a*b*x^3 + b^2
*x^6]])/((-a - Sqrt[b^2]*x^3 + Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])^2*(a - Sqrt
[b^2]*x^3 + Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])^2) - (a*b^2*(-(Sqrt[b^2]*x^3)
+ Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])^4*Log[a - Sqrt[b^2]*x^3 + Sqrt[a^2 + 2*a
*b*x^3 + b^2*x^6]])/(2*(-a - Sqrt[b^2]*x^3 + Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6
])^2*(a - Sqrt[b^2]*x^3 + Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])^2) - (a*b*Sqrt[b
^2]*(-(Sqrt[b^2]*x^3) + Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])^4*Log[a - Sqrt[b^2
]*x^3 + Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]])/(2*(-a - Sqrt[b^2]*x^3 + Sqrt[a^2
```

```
 + 2*a*b*x^3 + b^2*x^6])^2*(a - Sqrt[b^2]*x^3 + Sqrt[a^2 + 2*a*b*x^3 + b^2*
x^6])^2)
```

fricas [A] time = 1.30, size = 39, normalized size = 0.24

$$\frac{2\,b^3x^9+18\,ab^2x^6\log(x)-6\,a^2bx^3-a^3}{6\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^7,x, algorithm="fricas")
```

```
[Out] 1/6*(2*b^3*x^9 + 18*a*b^2*x^6*log(x) - 6*a^2*b*x^3 - a^3)/x^6
```

giac [A] time = 0.34, size = 86, normalized size = 0.53

$$\frac{1}{3}\,b^3x^3\mathrm{sgn}\left(bx^3+a\right)+3\,ab^2\log\left(|x|\right)\mathrm{sgn}\left(bx^3+a\right)-\frac{9\,ab^2x^6\mathrm{sgn}\left(bx^3+a\right)+6\,a^2bx^3\mathrm{sgn}\left(bx^3+a\right)+a^3\mathrm{sgn}\left(bx^3+a\right)}{6\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^7,x, algorithm="giac")
```

```
[Out] 1/3*b^3*x^3*sgn(b*x^3 + a) + 3*a*b^2*log(abs(x))*sgn(b*x^3 + a) - 1/6*(9*a*
b^2*x^6*sgn(b*x^3 + a) + 6*a^2*b*x^3*sgn(b*x^3 + a) + a^3*sgn(b*x^3 + a))/x
^6
```

maple [A] time = 0.01, size = 60, normalized size = 0.37

$$\frac{\left(\left(b\,x^3+a\right)^2\right)^{\frac{3}{2}}\left(2b^3x^9+18a\,b^2x^6\ln(x)-6a^2b\,x^3-a^3\right)}{6\left(b\,x^3+a\right)^3x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^7,x)
```

```
[Out] 1/6*((b*x^3+a)^2)^(3/2)*(2*b^3*x^9+18*a*b^2*ln(x)*x^6-6*a^2*b*x^3-a^3)/(b*x
^3+a)^3/x^6
```

maxima [A] time = 0.70, size = 220, normalized size = 1.36

$$\frac{\sqrt{b^2x^6+2\,abx^3+a^2}\,b^3x^3}{2\,a}+(-1)^{2\,b^2x^3+2\,ab}ab^2\log\left(2\,b^2x^3+2\,ab\right)-(-1)^{2\,abx^3+2\,a^2}ab^2\log\left(\frac{2\,abx}{|x|}+\frac{2\,a^2}{x^2|x|}\right)+\frac{3}{2}\sqrt{b^2x^6+2\,abx^3+a^2}\,b^2+\frac{\left(b^2x^6+2\,abx^3+a^2\right)^{\frac{3}{2}}b^2}{6\,a^2}-\frac{\left(b^2x^6+2\,abx^3+a^2\right)^{\frac{3}{2}}b}{6\,ax^3}-\frac{\left(b^2x^6+2\,abx^3+a^2\right)^{\frac{5}{2}}}{6\,a^2x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^7,x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(b^2*x^6 + 2*a*b*x^3 + a^2)*b^3*x^3/a + (-1)^(2*b^2*x^3 + 2*a*b)*a*
b^2*log(2*b^2*x^3 + 2*a*b) - (-1)^(2*a*b*x^3 + 2*a^2)*a*b^2*log(2*a*b*x/abs
(x) + 2*a^2/(x^2*abs(x))) + 3/2*sqrt(b^2*x^6 + 2*a*b*x^3 + a^2)*b^2 + 1/6*(
b^2*x^6 + 2*a*b*x^3 + a^2)^(3/2)*b^2/a^2 - 1/6*(b^2*x^6 + 2*a*b*x^3 + a^2)^
(3/2)*b/(a*x^3) - 1/6*(b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2)/(a^2*x^6)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a^2+2\,a\,b\,x^3+b^2\,x^6\right)^{3/2}}{x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2)/x^7,x)
```

```
[Out] int((a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2)/x^7, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\left(a + bx^3\right)^2\right)^{\frac{3}{2}}}{x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**6+2*a*b*x**3+a**2)**(3/2)/x**7,x)
[Out] Integral(((a + b*x**3)**2)**(3/2)/x**7, x)
```

3.40 $\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{3/2}}{x^8}\,dx$

Optimal. Leaf size=165

$$-\frac{3ab^2\sqrt{a^2+2abx^3+b^2x^6}}{x\left(a+bx^3\right)}-\frac{3a^2b\sqrt{a^2+2abx^3+b^2x^6}}{4x^4\left(a+bx^3\right)}+\frac{b^3x^2\sqrt{a^2+2abx^3+b^2x^6}}{2\left(a+bx^3\right)}-\frac{a^3\sqrt{a^2+2abx^3+b^2x^6}}{7x^7\left(a+bx^3\right)}$$

Rubi [A] time = 0.04, antiderivative size = 165, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1355, 270}

$$-\frac{a^3\sqrt{a^2+2abx^3+b^2x^6}}{7x^7\left(a+bx^3\right)}-\frac{3a^2b\sqrt{a^2+2abx^3+b^2x^6}}{4x^4\left(a+bx^3\right)}-\frac{3ab^2\sqrt{a^2+2abx^3+b^2x^6}}{x\left(a+bx^3\right)}+\frac{b^3x^2\sqrt{a^2+2abx^3+b^2x^6}}{2\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)/x^8,x]
```

```
[Out] -(a^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(7*x^7*(a + b*x^3)) - (3*a^2*b*Sqrt[
a^2 + 2*a*b*x^3 + b^2*x^6])/(4*x^4*(a + b*x^3)) - (3*a*b^2*Sqrt[a^2 + 2*a*b
*x^3 + b^2*x^6])/(x*(a + b*x^3)) + (b^3*x^2*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]
)/(2*(a + b*x^3))
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{3/2}}{x^8}\,dx &= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{\left(ab+b^2x^3\right)^3}{x^8}\,dx}{b^2\left(ab+b^2x^3\right)}\\
&= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \left(\frac{a^3b^3}{x^8}+\frac{3a^2b^4}{x^5}+\frac{3ab^5}{x^2}+b^6x\right)dx}{b^2\left(ab+b^2x^3\right)}\\
&= -\frac{a^3\sqrt{a^2+2abx^3+b^2x^6}}{7x^7\left(a+bx^3\right)}-\frac{3a^2b\sqrt{a^2+2abx^3+b^2x^6}}{4x^4\left(a+bx^3\right)}-\frac{3ab^2\sqrt{a^2+2abx^3+b^2x^6}}{x\left(a+bx^3\right)}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 61, normalized size = 0.37

$$-\frac{\sqrt{\left(a+bx^3\right)^2}\left(4a^3+21a^2bx^3+84ab^2x^6-14b^3x^9\right)}{28x^7\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)/x^8,x]
```

```
[Out] -1/28*(Sqrt[(a + b*x^3)^2]*(4*a^3 + 21*a^2*b*x^3 + 84*a*b^2*x^6 - 14*b^3*x^
9))/(x^7*(a + b*x^3))
```

IntegrateAlgebraic [A] time = 20.10, size = 61, normalized size = 0.37

$$\frac{\sqrt{\left(a+bx^3\right)^2}\left(-4a^3-21a^2bx^3-84ab^2x^6+14b^3x^9\right)}{28x^7\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)/x^8,x]
```

```
[Out] (Sqrt[(a + b*x^3)^2]*(-4*a^3 - 21*a^2*b*x^3 - 84*a*b^2*x^6 + 14*b^3*x^9))/(
28*x^7*(a + b*x^3))
```

fricas [A] time = 1.92, size = 37, normalized size = 0.22

$$\frac{14\,b^3x^9-84\,ab^2x^6-21\,a^2bx^3-4\,a^3}{28\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^8,x, algorithm="fricas")
```

```
[Out] 1/28*(14*b^3*x^9 - 84*a*b^2*x^6 - 21*a^2*b*x^3 - 4*a^3)/x^7
```

giac [A] time = 0.47, size = 70, normalized size = 0.42

$$\frac{1}{2}\,b^3x^2\mathrm{sgn}\left(bx^3+a\right)-\frac{84\,ab^2x^6\mathrm{sgn}\left(bx^3+a\right)+21\,a^2bx^3\mathrm{sgn}\left(bx^3+a\right)+4\,a^3\mathrm{sgn}\left(bx^3+a\right)}{28\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^8,x, algorithm="giac")
```

```
[Out] 1/2*b^3*x^2*sgn(b*x^3 + a) - 1/28*(84*a*b^2*x^6*sgn(b*x^3 + a) + 21*a^2*b*x
^3*sgn(b*x^3 + a) + 4*a^3*sgn(b*x^3 + a))/x^7
```

maple [A] time = 0.01, size = 58, normalized size = 0.35

$$-\frac{\left(-14b^3x^9+84a\,b^2x^6+21a^2b\,x^3+4a^3\right)\left(\left(b\,x^3+a\right)^2\right)^{\frac{3}{2}}}{28\left(b\,x^3+a\right)^3x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^8,x)
```

```
[Out] -1/28*(-14*b^3*x^9+84*a*b^2*x^6+21*a^2*b*x^3+4*a^3)*((b*x^3+a)^2)^(3/2)/x^7
/(b*x^3+a)^3
```

maxima [A] time = 0.74, size = 37, normalized size = 0.22

$$\frac{14\,b^3x^9-84\,ab^2x^6-21\,a^2bx^3-4\,a^3}{28\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^8,x, algorithm="maxima")
[Out] 1/28*(14*b^3*x^9 - 84*a*b^2*x^6 - 21*a^2*b*x^3 - 4*a^3)/x^7
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a^2 + 2\,a\,b\,x^3 + b^2\,x^6\right)^{3/2}}{x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2)/x^8,x)
[Out] int((a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2)/x^8, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\left(a + bx^3\right)^2\right)^{\frac{3}{2}}}{x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**6+2*a*b*x**3+a**2)**(3/2)/x**8,x)
[Out] Integral(((a + b*x**3)**2)**(3/2)/x**8, x)
```

3.41 $\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{3/2}}{x^9}\,dx$

Optimal. Leaf size=162

$$-\frac{3a^2b\sqrt{a^2+2abx^3+b^2x^6}}{5x^5\left(a+bx^3\right)}-\frac{3ab^2\sqrt{a^2+2abx^3+b^2x^6}}{2x^2\left(a+bx^3\right)}+\frac{b^3x\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}-\frac{a^3\sqrt{a^2+2abx^3+b^2x^6}}{8x^8\left(a+bx^3\right)}$$

Rubi [A] time = 0.04, antiderivative size = 162, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1355, 270}

$$-\frac{a^3\sqrt{a^2+2abx^3+b^2x^6}}{8x^8\left(a+bx^3\right)}-\frac{3a^2b\sqrt{a^2+2abx^3+b^2x^6}}{5x^5\left(a+bx^3\right)}-\frac{3ab^2\sqrt{a^2+2abx^3+b^2x^6}}{2x^2\left(a+bx^3\right)}+\frac{b^3x\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)/x^9,x]
```

```
[Out] -(a^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(8*x^8*(a + b*x^3)) - (3*a^2*b*Sqrt[
a^2 + 2*a*b*x^3 + b^2*x^6])/(5*x^5*(a + b*x^3)) - (3*a*b^2*Sqrt[a^2 + 2*a*b
*x^3 + b^2*x^6])/(2*x^2*(a + b*x^3)) + (b^3*x*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^
6])/(a + b*x^3)
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{3/2}}{x^9}\,dx &= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{\left(ab+b^2x^3\right)^3}{x^9}\,dx}{b^2\left(ab+b^2x^3\right)}\\
&= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \left(b^6+\frac{a^3b^3}{x^9}+\frac{3a^2b^4}{x^6}+\frac{3ab^5}{x^3}\right)dx}{b^2\left(ab+b^2x^3\right)}\\
&= -\frac{a^3\sqrt{a^2+2abx^3+b^2x^6}}{8x^8\left(a+bx^3\right)}-\frac{3a^2b\sqrt{a^2+2abx^3+b^2x^6}}{5x^5\left(a+bx^3\right)}-\frac{3ab^2\sqrt{a^2+2abx^3+b^2}}{2x^2\left(a+bx^3\right)}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 61, normalized size = 0.38

$$-\frac{\sqrt{\left(a+bx^3\right)^2}\left(5a^3+24a^2bx^3+60ab^2x^6-40b^3x^9\right)}{40x^8\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)/x^9,x]
```

```
[Out] -1/40*(Sqrt[(a + b*x^3)^2]*(5*a^3 + 24*a^2*b*x^3 + 60*a*b^2*x^6 - 40*b^3*x^
9))/(x^8*(a + b*x^3))
```

IntegrateAlgebraic [A] time = 20.49, size = 61, normalized size = 0.38

$$\frac{\sqrt{\left(a+bx^3\right)^2}\,\left(-5a^3-24a^2bx^3-60ab^2x^6+40b^3x^9\right)}{40x^8\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)/x^9,x]
```

```
[Out] (Sqrt[(a + b*x^3)^2]*(-5*a^3 - 24*a^2*b*x^3 - 60*a*b^2*x^6 + 40*b^3*x^9))/(
40*x^8*(a + b*x^3))
```

fricas [A] time = 1.26, size = 37, normalized size = 0.23

$$\frac{40\,b^3x^9-60\,ab^2x^6-24\,a^2bx^3-5\,a^3}{40\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^9,x, algorithm="fricas")
```

```
[Out] 1/40*(40*b^3*x^9 - 60*a*b^2*x^6 - 24*a^2*b*x^3 - 5*a^3)/x^8
```

giac [A] time = 0.39, size = 67, normalized size = 0.41

$$b^3x\mathrm{sgn}\left(bx^3+a\right)-\frac{60\,ab^2x^6\mathrm{sgn}\left(bx^3+a\right)+24\,a^2bx^3\mathrm{sgn}\left(bx^3+a\right)+5\,a^3\mathrm{sgn}\left(bx^3+a\right)}{40\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^9,x, algorithm="giac")
```

```
[Out] b^3*x*sgn(b*x^3 + a) - 1/40*(60*a*b^2*x^6*sgn(b*x^3 + a) + 24*a^2*b*x^3*sgn
(b*x^3 + a) + 5*a^3*sgn(b*x^3 + a))/x^8
```

maple [A] time = 0.01, size = 58, normalized size = 0.36

$$-\frac{\left(-40b^3x^9+60a\,b^2x^6+24a^2b\,x^3+5a^3\right)\left(\left(b\,x^3+a\right)^2\right)^{\frac{3}{2}}}{40\left(b\,x^3+a\right)^3x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^9,x)
```

```
[Out] -1/40*(-40*b^3*x^9+60*a*b^2*x^6+24*a^2*b*x^3+5*a^3)*((b*x^3+a)^2)^(3/2)/x^8
/(b*x^3+a)^3
```

maxima [A] time = 0.68, size = 37, normalized size = 0.23

$$\frac{40\,b^3x^9-60\,ab^2x^6-24\,a^2bx^3-5\,a^3}{40\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^9,x, algorithm="maxima")
```

```
[Out] 1/40*(40*b^3*x^9 - 60*a*b^2*x^6 - 24*a^2*b*x^3 - 5*a^3)/x^8
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a^2 + 2\,a\,b\,x^3 + b^2\,x^6\right)^{3/2}}{x^9}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2)/x^9,x)
```

```
[Out] int((a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2)/x^9, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\left(a + bx^3\right)^2\right)^{\frac{3}{2}}}{x^9}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**6+2*a*b*x**3+a**2)**(3/2)/x**9,x)
```

```
[Out] Integral(((a + b*x**3)**2)**(3/2)/x**9, x)
```

3.42 $\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{3/2}}{x^{10}} dx$

Optimal. Leaf size=161

$$-\frac{a^2b\sqrt{a^2+2abx^3+b^2x^6}}{2x^6\left(a+bx^3\right)}-\frac{ab^2\sqrt{a^2+2abx^3+b^2x^6}}{x^3\left(a+bx^3\right)}+\frac{b^3\log(x)\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}-\frac{a^3\sqrt{a^2+2abx^3+b^2x^6}}{9x^9\left(a+bx^3\right)}$$

Rubi [A] time = 0.05, antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {1355, 266, 43}

$$-\frac{a^3\sqrt{a^2+2abx^3+b^2x^6}}{9x^9\left(a+bx^3\right)}-\frac{a^2b\sqrt{a^2+2abx^3+b^2x^6}}{2x^6\left(a+bx^3\right)}-\frac{ab^2\sqrt{a^2+2abx^3+b^2x^6}}{x^3\left(a+bx^3\right)}+\frac{b^3\log(x)\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)/x^10,x]
```

```
[Out] -(a^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(9*x^9*(a + b*x^3)) - (a^2*b*Sqrt[a^
2 + 2*a*b*x^3 + b^2*x^6])/(2*x^6*(a + b*x^3)) - (a*b^2*Sqrt[a^2 + 2*a*b*x^3
 + b^2*x^6])/(x^3*(a + b*x^3)) + (b^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]*Log[x
])/(a + b*x^3)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{3/2}}{x^{10}}\,dx = \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{\left(ab+b^2x^3\right)^3}{x^{10}}\,dx}{b^2\left(ab+b^2x^3\right)}$$
$$= \frac{\sqrt{a^2+2abx^3+b^2x^6}\ \text{Subst}\left(\int \frac{\left(ab+b^2x\right)^3}{x^4}\,dx,x,x^3\right)}{3b^2\left(ab+b^2x^3\right)}$$
$$= \frac{\sqrt{a^2+2abx^3+b^2x^6}\ \text{Subst}\left(\int \left(\frac{a^3b^3}{x^4}+\frac{3a^2b^4}{x^3}+\frac{3ab^5}{x^2}+\frac{b^6}{x}\right)dx,x,x^3\right)}{3b^2\left(ab+b^2x^3\right)}$$
$$= -\frac{a^3\sqrt{a^2+2abx^3+b^2x^6}}{9x^9\left(a+bx^3\right)}-\frac{a^2b\sqrt{a^2+2abx^3+b^2x^6}}{2x^6\left(a+bx^3\right)}-\frac{ab^2\sqrt{a^2+2abx^3+b^2x^6}}{x^3\left(a+bx^3\right)}$$

Mathematica [A] time = 0.02, size = 63, normalized size = 0.39

$$-\frac{\sqrt{\left(a+bx^3\right)^2}\left(a\left(2a^2+9abx^3+18b^2x^6\right)-18b^3x^9\log(x)\right)}{18x^9\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)/x^10,x]
```

```
[Out] -1/18*(Sqrt[(a + b*x^3)^2]*(a*(2*a^2 + 9*a*b*x^3 + 18*b^2*x^6) - 18*b^3*x^9
*Log[x]))/(x^9*(a + b*x^3))
```

IntegrateAlgebraic [A] time = 1.17, size = 293, normalized size = 1.82

$$-\frac{1}{6}\left(b^2\right)^{3/2}\log\left(\sqrt{a^2+2abx^3+b^2x^6}-a-\sqrt{b^2}\,x^3\right)-\frac{1}{6}\left(b^2\right)^{3/2}\log\left(\sqrt{a^2+2abx^3+b^2x^6}+a-\sqrt{b^2}\,x^3\right)+\frac{1}{3}b^3\tanh^{-1}\left(\frac{\sqrt{b^2}\,x^3}{a}-\frac{\sqrt{a^2+2abx^3+b^2x^6}}{a}\right)+\frac{a\sqrt{a^2+2abx^3+b^2x^6}\left(-2a^2b-9ab^2x^3-18b^3x^6\right)+a\sqrt{b^2}\left(2a^3+11a^2bx^3+27ab^2x^6+18b^3x^9\right)}{18x^9\left(ab+b^2x^3\right)-18\sqrt{b^2}\,x^9\sqrt{a^2+2abx^3+b^2x^6}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)/x^10,x]
```

```
[Out] (a*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]*(-2*a^2*b - 9*a*b^2*x^3 - 18*b^3*x^6) +
a*Sqrt[b^2]*(2*a^3 + 11*a^2*b*x^3 + 27*a*b^2*x^6 + 18*b^3*x^9))/(18*x^9*(a*
b + b^2*x^3) - 18*Sqrt[b^2]*x^9*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) + (b^3*Arc
Tanh[(Sqrt[b^2]*x^3)/a - Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]/a])/3 - ((b^2)^(3/
2)*Log[-a - Sqrt[b^2]*x^3 + Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]])/6 - ((b^2)^(3
/2)*Log[a - Sqrt[b^2]*x^3 + Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]])/6
```

fricas [A] time = 1.31, size = 39, normalized size = 0.24

$$\frac{18\,b^3x^9\log(x)-18\,ab^2x^6-9\,a^2bx^3-2\,a^3}{18\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^10,x, algorithm="fricas")
```

```
[Out] 1/18*(18*b^3*x^9*log(x) - 18*a*b^2*x^6 - 9*a^2*b*x^3 - 2*a^3)/x^9
```

giac [A] time = 0.38, size = 85, normalized size = 0.53

$$b^3\log\left(|x|\right)\mathrm{sgn}\left(bx^3+a\right)-\frac{11\,b^3x^9\mathrm{sgn}\left(bx^3+a\right)+18\,ab^2x^6\mathrm{sgn}\left(bx^3+a\right)+9\,a^2bx^3\mathrm{sgn}\left(bx^3+a\right)+2\,a^3\mathrm{sgn}\left(bx^3+a\right)}{18\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^10,x, algorithm="giac")
```

```
[Out] b^3*log(abs(x))*sgn(b*x^3 + a) - 1/18*(11*b^3*x^9*sgn(b*x^3 + a) + 18*a*b^2
*x^6*sgn(b*x^3 + a) + 9*a^2*b*x^3*sgn(b*x^3 + a) + 2*a^3*sgn(b*x^3 + a))/x^
9
```

maple [A] time = 0.01, size = 60, normalized size = 0.37

$$\frac{\left(\left(b\,x^{3}+a\right)^{2}\right)^{\frac{3}{2}}\left(18b^{3}x^{9}\ln(x)-18a\,b^{2}x^{6}-9a^{2}b\,x^{3}-2a^{3}\right)}{18\left(b\,x^{3}+a\right)^{3}x^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^10,x)
```

```
[Out] 1/18*((b*x^3+a)^2)^(3/2)*(18*b^3*ln(x)*x^9-18*a*b^2*x^6-9*a^2*b*x^3-2*a^3)/
(b*x^3+a)^3/x^9
```

maxima [B] time = 0.72, size = 253, normalized size = 1.57

$$\frac{\sqrt{b^{2}x^{6}+2\,abx^{3}+a^{2}}\,b^{4}x^{3}}{6\,a^{2}}+\frac{1}{3}\,(-1)^{2\,b^{2}x^{3}+2\,ab}\,b^{3}\log\left(2\,b^{2}x^{3}+2\,ab\right)-\frac{1}{3}\,(-1)^{2\,abx^{3}+2\,a^{2}}\,b^{3}\log\left(\frac{2\,abx}{|x|}+\frac{2\,a^{2}}{x^{2}|x|}\right)+\frac{\sqrt{b^{2}x^{6}+2\,abx^{3}+a^{2}}\,b^{3}}{2\,a}-\frac{\left(b^{2}x^{6}+2\,abx^{3}+a^{2}\right)^{\frac{3}{2}}b^{3}}{18\,a^{3}}-\frac{\left(b^{2}x^{6}+2\,abx^{3}+a^{2}\right)^{\frac{3}{2}}b^{2}}{6\,a^{2}x^{3}}+\frac{\left(b^{2}x^{6}+2\,abx^{3}+a^{2}\right)^{\frac{5}{2}}b}{18\,a^{3}x^{6}}-\frac{\left(b^{2}x^{6}+2\,abx^{3}+a^{2}\right)^{\frac{5}{2}}}{9\,a^{2}x^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^10,x, algorithm="maxima")
```

```
[Out] 1/6*sqrt(b^2*x^6 + 2*a*b*x^3 + a^2)*b^4*x^3/a^2 + 1/3*(-1)^(2*b^2*x^3 + 2*a
*b)*b^3*log(2*b^2*x^3 + 2*a*b) - 1/3*(-1)^(2*a*b*x^3 + 2*a^2)*b^3*log(2*a*b
*x/abs(x) + 2*a^2/(x^2*abs(x))) + 1/2*sqrt(b^2*x^6 + 2*a*b*x^3 + a^2)*b^3/a
 - 1/18*(b^2*x^6 + 2*a*b*x^3 + a^2)^(3/2)*b^3/a^3 - 1/6*(b^2*x^6 + 2*a*b*x^
3 + a^2)^(3/2)*b^2/(a^2*x^3) + 1/18*(b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2)*b/(a^
3*x^6) - 1/9*(b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2)/(a^2*x^9)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(a^2+2\,a\,b\,x^3+b^2\,x^6\right)}^{3/2}}{x^{10}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2)/x^10,x)
```

```
[Out] int((a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2)/x^10, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\left(a+bx^{3}\right)^{2}\right)^{\frac{3}{2}}}{x^{10}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**6+2*a*b*x**3+a**2)**(3/2)/x**10,x)
```

```
[Out] Integral(((a + b*x**3)**2)**(3/2)/x**10, x)
```

3.43 $\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{3/2}}{x^{11}}\,dx$

Optimal. Leaf size=165

$$-\frac{3a^2b\sqrt{a^2+2abx^3+b^2x^6}}{7x^7\left(a+bx^3\right)}-\frac{3ab^2\sqrt{a^2+2abx^3+b^2x^6}}{4x^4\left(a+bx^3\right)}-\frac{b^3\sqrt{a^2+2abx^3+b^2x^6}}{x\left(a+bx^3\right)}-\frac{a^3\sqrt{a^2+2abx^3+b^2x^6}}{10x^{10}\left(a+bx^3\right)}$$

Rubi [A] time = 0.04, antiderivative size = 165, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1355, 270}

$$-\frac{a^3\sqrt{a^2+2abx^3+b^2x^6}}{10x^{10}\left(a+bx^3\right)}-\frac{3a^2b\sqrt{a^2+2abx^3+b^2x^6}}{7x^7\left(a+bx^3\right)}-\frac{3ab^2\sqrt{a^2+2abx^3+b^2x^6}}{4x^4\left(a+bx^3\right)}-\frac{b^3\sqrt{a^2+2abx^3+b^2x^6}}{x\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)/x^11,x]
```

```
[Out] -(a^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(10*x^10*(a + b*x^3)) - (3*a^2*b*Sqr
t[a^2 + 2*a*b*x^3 + b^2*x^6])/(7*x^7*(a + b*x^3)) - (3*a*b^2*Sqrt[a^2 + 2*a
*b*x^3 + b^2*x^6])/(4*x^4*(a + b*x^3)) - (b^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^
6])/(x*(a + b*x^3))
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{3/2}}{x^{11}}\,dx &= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{\left(ab+b^2x^3\right)^3}{x^{11}}\,dx}{b^2\left(ab+b^2x^3\right)}\\ &= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \left(\frac{a^3b^3}{x^{11}}+\frac{3a^2b^4}{x^8}+\frac{3ab^5}{x^5}+\frac{b^6}{x^2}\right)dx}{b^2\left(ab+b^2x^3\right)}\\ &= -\frac{a^3\sqrt{a^2+2abx^3+b^2x^6}}{10x^{10}\left(a+bx^3\right)}-\frac{3a^2b\sqrt{a^2+2abx^3+b^2x^6}}{7x^7\left(a+bx^3\right)}-\frac{3ab^2\sqrt{a^2+2abx^3+b^2\ldots}}{4x^4\left(a+bx^3\right)}\end{aligned}$$

Mathematica [A] time = 0.01, size = 61, normalized size = 0.37

$$-\frac{\sqrt{\left(a+bx^3\right)^2}\left(14a^3+60a^2bx^3+105ab^2x^6+140b^3x^9\right)}{140x^{10}\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)/x^11,x]
```

```
[Out] -1/140*(Sqrt[(a + b*x^3)^2]*(14*a^3 + 60*a^2*b*x^3 + 105*a*b^2*x^6 + 140*b^
3*x^9))/(x^10*(a + b*x^3))
```

IntegrateAlgebraic [A] time = 20.14, size = 61, normalized size = 0.37

$$\frac{\sqrt{\left(a+bx^3\right)^2}\left(-14a^3-60a^2bx^3-105ab^2x^6-140b^3x^9\right)}{140x^{10}\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)/x^11,x]
```

```
[Out] (Sqrt[(a + b*x^3)^2]*(-14*a^3 - 60*a^2*b*x^3 - 105*a*b^2*x^6 - 140*b^3*x^9)
)/(140*x^10*(a + b*x^3))
```

fricas [A] time = 1.52, size = 37, normalized size = 0.22

$$-\frac{140\,b^3x^9+105\,ab^2x^6+60\,a^2bx^3+14\,a^3}{140\,x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^11,x, algorithm="fricas")
```

```
[Out] -1/140*(140*b^3*x^9 + 105*a*b^2*x^6 + 60*a^2*b*x^3 + 14*a^3)/x^10
```

giac [A] time = 0.36, size = 69, normalized size = 0.42

$$-\frac{140\,b^3x^9\mathrm{sgn}\left(bx^3+a\right)+105\,ab^2x^6\mathrm{sgn}\left(bx^3+a\right)+60\,a^2bx^3\mathrm{sgn}\left(bx^3+a\right)+14\,a^3\mathrm{sgn}\left(bx^3+a\right)}{140\,x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^11,x, algorithm="giac")
```

```
[Out] -1/140*(140*b^3*x^9*sgn(b*x^3 + a) + 105*a*b^2*x^6*sgn(b*x^3 + a) + 60*a^2*
b*x^3*sgn(b*x^3 + a) + 14*a^3*sgn(b*x^3 + a))/x^10
```

maple [A] time = 0.01, size = 58, normalized size = 0.35

$$-\frac{\left(140b^3x^9+105a\,b^2x^6+60a^2b\,x^3+14a^3\right)\left(\left(b\,x^3+a\right)^2\right)^{\frac{3}{2}}}{140\left(b\,x^3+a\right)^3x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^11,x)
```

```
[Out] -1/140*(140*b^3*x^9+105*a*b^2*x^6+60*a^2*b*x^3+14*a^3)*((b*x^3+a)^2)^(3/2)/
x^10/(b*x^3+a)^3
```

maxima [A] time = 0.66, size = 37, normalized size = 0.22

$$-\frac{140\,b^3x^9+105\,ab^2x^6+60\,a^2bx^3+14\,a^3}{140\,x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^11,x, algorithm="maxima")
```

```
[Out] -1/140*(140*b^3*x^9 + 105*a*b^2*x^6 + 60*a^2*b*x^3 + 14*a^3)/x^10
```

mupad [B] time = 1.21, size = 151, normalized size = 0.92

$$-\frac{a^3\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{10\,x^{10}\,\left(b\,x^3+a\right)}-\frac{b^3\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{x\,\left(b\,x^3+a\right)}-\frac{3\,a\,b^2\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{4\,x^4\,\left(b\,x^3+a\right)}-\frac{3\,a^2\,b\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{7\,x^7\,\left(b\,x^3+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2)/x^11,x)
```

```
[Out] - (a^3*(a^2 + b^2*x^6 + 2*a*b*x^3)^(1/2))/(10*x^10*(a + b*x^3)) - (b^3*(a^2
 + b^2*x^6 + 2*a*b*x^3)^(1/2))/(x*(a + b*x^3)) - (3*a*b^2*(a^2 + b^2*x^6 +
2*a*b*x^3)^(1/2))/(4*x^4*(a + b*x^3)) - (3*a^2*b*(a^2 + b^2*x^6 + 2*a*b*x^3
)^(1/2))/(7*x^7*(a + b*x^3))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\left(a + bx^{3}\right)^{2}\right)^{\frac{3}{2}}}{x^{11}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**6+2*a*b*x**3+a**2)**(3/2)/x**11,x)
```

```
[Out] Integral(((a + b*x**3)**2)**(3/2)/x**11, x)
```

3.44 $\int \frac{(a^2+2abx^3+b^2x^6)^{3/2}}{x^{12}} dx$

Optimal. Leaf size=167

$$-\frac{3a^2b\sqrt{a^2+2abx^3+b^2x^6}}{8x^8(a+bx^3)}-\frac{3ab^2\sqrt{a^2+2abx^3+b^2x^6}}{5x^5(a+bx^3)}-\frac{b^3\sqrt{a^2+2abx^3+b^2x^6}}{2x^2(a+bx^3)}-\frac{a^3\sqrt{a^2+2abx^3+b^2x^6}}{11x^{11}(a+bx^3)}$$

Rubi [A] time = 0.04, antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {1355, 270}

$$-\frac{a^3\sqrt{a^2+2abx^3+b^2x^6}}{11x^{11}(a+bx^3)}-\frac{3a^2b\sqrt{a^2+2abx^3+b^2x^6}}{8x^8(a+bx^3)}-\frac{3ab^2\sqrt{a^2+2abx^3+b^2x^6}}{5x^5(a+bx^3)}-\frac{b^3\sqrt{a^2+2abx^3+b^2x^6}}{2x^2(a+bx^3)}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)/x^12,x]
```

```
[Out] -(a^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(11*x^11*(a + b*x^3)) - (3*a^2*b*Sqr
t[a^2 + 2*a*b*x^3 + b^2*x^6])/(8*x^8*(a + b*x^3)) - (3*a*b^2*Sqrt[a^2 + 2*a
*b*x^3 + b^2*x^6])/(5*x^5*(a + b*x^3)) - (b^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^
6])/(2*x^2*(a + b*x^3))
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a^2+2abx^3+b^2x^6)^{3/2}}{x^{12}} dx &= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{(ab+b^2x^3)^3}{x^{12}} dx}{b^2(ab+b^2x^3)} \\
&= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \left(\frac{a^3b^3}{x^{12}}+\frac{3a^2b^4}{x^9}+\frac{3ab^5}{x^6}+\frac{b^6}{x^3}\right) dx}{b^2(ab+b^2x^3)} \\
&= -\frac{a^3\sqrt{a^2+2abx^3+b^2x^6}}{11x^{11}(a+bx^3)}-\frac{3a^2b\sqrt{a^2+2abx^3+b^2x^6}}{8x^8(a+bx^3)}-\frac{3ab^2\sqrt{a^2+2abx^3+b^2x^6}}{5x^5(a+bx^3)}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 61, normalized size = 0.37

$$-\frac{\sqrt{(a+bx^3)^2}\left(40a^3+165a^2bx^3+264ab^2x^6+220b^3x^9\right)}{440x^{11}(a+bx^3)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)/x^12,x]
```

```
[Out] -1/440*(Sqrt[(a + b*x^3)^2]*(40*a^3 + 165*a^2*b*x^3 + 264*a*b^2*x^6 + 220*b
^3*x^9))/(x^11*(a + b*x^3))
```

IntegrateAlgebraic [A] time = 20.14, size = 61, normalized size = 0.37

$$\frac{\sqrt{\left(a+bx^3\right)^2}\left(-40a^3-165a^2bx^3-264ab^2x^6-220b^3x^9\right)}{440x^{11}\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)/x^12,x]
```

```
[Out] (Sqrt[(a + b*x^3)^2]*(-40*a^3 - 165*a^2*b*x^3 - 264*a*b^2*x^6 - 220*b^3*x^9
))/(440*x^11*(a + b*x^3))
```

fricas [A] time = 1.19, size = 37, normalized size = 0.22

$$-\frac{220\,b^3x^9+264\,ab^2x^6+165\,a^2bx^3+40\,a^3}{440\,x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^12,x, algorithm="fricas")
```

```
[Out] -1/440*(220*b^3*x^9 + 264*a*b^2*x^6 + 165*a^2*b*x^3 + 40*a^3)/x^11
```

giac [A] time = 0.42, size = 69, normalized size = 0.41

$$-\frac{220\,b^3x^9\mathrm{sgn}\left(bx^3+a\right)+264\,ab^2x^6\mathrm{sgn}\left(bx^3+a\right)+165\,a^2bx^3\mathrm{sgn}\left(bx^3+a\right)+40\,a^3\mathrm{sgn}\left(bx^3+a\right)}{440\,x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^12,x, algorithm="giac")
```

```
[Out] -1/440*(220*b^3*x^9*sgn(b*x^3 + a) + 264*a*b^2*x^6*sgn(b*x^3 + a) + 165*a^2
*b*x^3*sgn(b*x^3 + a) + 40*a^3*sgn(b*x^3 + a))/x^11
```

maple [A] time = 0.01, size = 58, normalized size = 0.35

$$-\frac{\left(220b^3x^9+264a\,b^2x^6+165a^2b\,x^3+40a^3\right)\left(\left(b\,x^3+a\right)^2\right)^{\frac{3}{2}}}{440\left(b\,x^3+a\right)^3x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^12,x)
```

```
[Out] -1/440*(220*b^3*x^9+264*a*b^2*x^6+165*a^2*b*x^3+40*a^3)*((b*x^3+a)^2)^(3/2)
/x^11/(b*x^3+a)^3
```

maxima [A] time = 0.72, size = 37, normalized size = 0.22

$$-\frac{220\,b^3x^9+264\,ab^2x^6+165\,a^2bx^3+40\,a^3}{440\,x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^12,x, algorithm="maxima")
```

```
[Out] -1/440*(220*b^3*x^9 + 264*a*b^2*x^6 + 165*a^2*b*x^3 + 40*a^3)/x^11
```

mupad [B] time = 1.22, size = 151, normalized size = 0.90

$$-\frac{a^3\,\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{11\,x^{11}\,\left(b\,x^3+a\right)}-\frac{b^3\,\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{2\,x^2\,\left(b\,x^3+a\right)}-\frac{3\,a\,b^2\,\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{5\,x^5\,\left(b\,x^3+a\right)}-\frac{3\,a^2\,b\,\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{8\,x^8\,\left(b\,x^3+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2)/x^12,x)
```

```
[Out] - (a^3*(a^2 + b^2*x^6 + 2*a*b*x^3)^(1/2))/(11*x^11*(a + b*x^3)) - (b^3*(a^2
 + b^2*x^6 + 2*a*b*x^3)^(1/2))/(2*x^2*(a + b*x^3)) - (3*a*b^2*(a^2 + b^2*x^
6 + 2*a*b*x^3)^(1/2))/(5*x^5*(a + b*x^3)) - (3*a^2*b*(a^2 + b^2*x^6 + 2*a*b
*x^3)^(1/2))/(8*x^8*(a + b*x^3))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\left(a + bx^{3}\right)^{2}\right)^{\frac{3}{2}}}{x^{12}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**6+2*a*b*x**3+a**2)**(3/2)/x**12,x)
```

```
[Out] Integral(((a + b*x**3)**2)**(3/2)/x**12, x)
```

3.45 $\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{3/2}}{x^{13}}\,dx$

Optimal. Leaf size=41

$$-\frac{\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}}{12ax^{12}}$$

Rubi [A] time = 0.02, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1355, 264}

$$-\frac{\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}}{12ax^{12}}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)/x^13,x]
```

```
[Out] -((a + b*x^3)^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(12*a*x^12)
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{3/2}}{x^{13}}\,dx &= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{\left(ab+b^2x^3\right)^3}{x^{13}}\,dx}{b^2\left(ab+b^2x^3\right)}\\ &= -\frac{\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}}{12ax^{12}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 59, normalized size = 1.44

$$-\frac{\sqrt{\left(a+bx^3\right)^2}\left(a^3+4a^2bx^3+6ab^2x^6+4b^3x^9\right)}{12x^{12}\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)/x^13,x]
```

```
[Out] -1/12*(Sqrt[(a + b*x^3)^2]*(a^3 + 4*a^2*b*x^3 + 6*a*b^2*x^6 + 4*b^3*x^9))/(
x^12*(a + b*x^3))
```

IntegrateAlgebraic [B] time = 1.35, size = 310, normalized size = 7.56

$$\frac{2b^3\sqrt{a^2+2abx^3+b^2x^6}\left(-a^6b-7a^5b^2x^3-21a^4b^3x^6-35a^3b^4x^9-34a^2b^5x^{12}-18ab^6x^{15}-4b^7x^{18}\right)+2\sqrt{b^2}\,b^3\left(a^7+8a^6bx^3+28a^5b^2x^6+56a^4b^3x^9+69a^3b^4x^{12}+52a^2b^5x^{15}+22ab^6x^{18}+4b^7x^{21}\right)}{3\sqrt{b^2}\,x^{12}\sqrt{a^2+2abx^3+b^2x^6}\left(-8a^3b^3-24a^2b^4x^3-24ab^5x^6-8b^6x^9\right)+3x^{12}\left(8a^4b^4+32a^3b^5x^3+48a^2b^6x^6+32ab^7x^9+8b^8x^{12}\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)/x^13,x]
```

```
[Out] (2*b^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]*(-(a^6*b) - 7*a^5*b^2*x^3 - 21*a^4*b
^3*x^6 - 35*a^3*b^4*x^9 - 34*a^2*b^5*x^12 - 18*a*b^6*x^15 - 4*b^7*x^18) + 2
*b^3*Sqrt[b^2]*(a^7 + 8*a^6*b*x^3 + 28*a^5*b^2*x^6 + 56*a^4*b^3*x^9 + 69*a^
3*b^4*x^12 + 52*a^2*b^5*x^15 + 22*a*b^6*x^18 + 4*b^7*x^21))/(3*Sqrt[b^2]*x^
12*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]*(-8*a^3*b^3 - 24*a^2*b^4*x^3 - 24*a*b^5*
x^6 - 8*b^6*x^9) + 3*x^12*(8*a^4*b^4 + 32*a^3*b^5*x^3 + 48*a^2*b^6*x^6 + 32
*a*b^7*x^9 + 8*b^8*x^12))
```

fricas [A] time = 1.42, size = 35, normalized size = 0.85

$$-\frac{4\,b^3x^9+6\,ab^2x^6+4\,a^2bx^3+a^3}{12\,x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^13,x, algorithm="fricas")
```

```
[Out] -1/12*(4*b^3*x^9 + 6*a*b^2*x^6 + 4*a^2*b*x^3 + a^3)/x^12
```

giac [B] time = 0.36, size = 68, normalized size = 1.66

$$-\frac{4\,b^3x^9\mathrm{sgn}\left(bx^3+a\right)+6\,ab^2x^6\mathrm{sgn}\left(bx^3+a\right)+4\,a^2bx^3\mathrm{sgn}\left(bx^3+a\right)+a^3\mathrm{sgn}\left(bx^3+a\right)}{12\,x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^13,x, algorithm="giac")
```

```
[Out] -1/12*(4*b^3*x^9*sgn(b*x^3 + a) + 6*a*b^2*x^6*sgn(b*x^3 + a) + 4*a^2*b*x^3*
sgn(b*x^3 + a) + a^3*sgn(b*x^3 + a))/x^12
```

maple [A] time = 0.01, size = 56, normalized size = 1.37

$$-\frac{\left(4b^3x^9+6a\,b^2x^6+4a^2b\,x^3+a^3\right)\left(\left(b\,x^3+a\right)^2\right)^{\frac{3}{2}}}{12\left(b\,x^3+a\right)^3x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^13,x)
```

```
[Out] -1/12*(4*b^3*x^9+6*a*b^2*x^6+4*a^2*b*x^3+a^3)*((b*x^3+a)^2)^(3/2)/x^12/(b*x
^3+a)^3
```

maxima [B] time = 0.55, size = 148, normalized size = 3.61

$$\frac{\left(b^2x^6+2\,abx^3+a^2\right)^{\frac{3}{2}}b^4}{12\,a^4}+\frac{\left(b^2x^6+2\,abx^3+a^2\right)^{\frac{3}{2}}b^3}{12\,a^3x^3}-\frac{\left(b^2x^6+2\,abx^3+a^2\right)^{\frac{5}{2}}b^2}{12\,a^4x^6}+\frac{\left(b^2x^6+2\,abx^3+a^2\right)^{\frac{5}{2}}b}{12\,a^3x^9}-\frac{\left(b^2x^6+2\,abx^3+a^2\right)^{\frac{5}{2}}}{12\,a^2x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^13,x, algorithm="maxima")
```

```
[Out] 1/12*(b^2*x^6 + 2*a*b*x^3 + a^2)^(3/2)*b^4/a^4 + 1/12*(b^2*x^6 + 2*a*b*x^3
+ a^2)^(3/2)*b^3/(a^3*x^3) - 1/12*(b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2)*b^2/(a^
```

```
4*x^6) + 1/12*(b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2)*b/(a^3*x^9) - 1/12*(b^2*x^6
 + 2*a*b*x^3 + a^2)^(5/2)/(a^2*x^12)
```

mupad [B] time = 1.21, size = 151, normalized size = 3.68

$$-\frac{a^3\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{12\,x^{12}\,\left(b\,x^3+a\right)}-\frac{b^3\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{3\,x^3\,\left(b\,x^3+a\right)}-\frac{a\,b^2\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{2\,x^6\,\left(b\,x^3+a\right)}-\frac{a^2\,b\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{3\,x^9\,\left(b\,x^3+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2)/x^13,x)
```

```
[Out] - (a^3*(a^2 + b^2*x^6 + 2*a*b*x^3)^(1/2))/(12*x^12*(a + b*x^3)) - (b^3*(a^2
 + b^2*x^6 + 2*a*b*x^3)^(1/2))/(3*x^3*(a + b*x^3)) - (a*b^2*(a^2 + b^2*x^6
+ 2*a*b*x^3)^(1/2))/(2*x^6*(a + b*x^3)) - (a^2*b*(a^2 + b^2*x^6 + 2*a*b*x^3
)^(1/2))/(3*x^9*(a + b*x^3))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\left(a+bx^3\right)^2\right)^{\frac{3}{2}}}{x^{13}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**6+2*a*b*x**3+a**2)**(3/2)/x**13,x)
```

```
[Out] Integral(((a + b*x**3)**2)**(3/2)/x**13, x)
```

3.46 $\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{3/2}}{x^{14}}\,dx$

Optimal. Leaf size=167

$$-\frac{3a^2b\sqrt{a^2+2abx^3+b^2x^6}}{10x^{10}\left(a+bx^3\right)}-\frac{3ab^2\sqrt{a^2+2abx^3+b^2x^6}}{7x^7\left(a+bx^3\right)}-\frac{b^3\sqrt{a^2+2abx^3+b^2x^6}}{4x^4\left(a+bx^3\right)}-\frac{a^3\sqrt{a^2+2abx^3+b^2x^6}}{13x^{13}\left(a+bx^3\right)}$$

Rubi [A] time = 0.04, antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1355, 270}

$$-\frac{a^3\sqrt{a^2+2abx^3+b^2x^6}}{13x^{13}\left(a+bx^3\right)}-\frac{3a^2b\sqrt{a^2+2abx^3+b^2x^6}}{10x^{10}\left(a+bx^3\right)}-\frac{3ab^2\sqrt{a^2+2abx^3+b^2x^6}}{7x^7\left(a+bx^3\right)}-\frac{b^3\sqrt{a^2+2abx^3+b^2x^6}}{4x^4\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)/x^14,x]
```

```
[Out] -(a^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(13*x^13*(a + b*x^3)) - (3*a^2*b*Sqr
t[a^2 + 2*a*b*x^3 + b^2*x^6])/(10*x^10*(a + b*x^3)) - (3*a*b^2*Sqrt[a^2 + 2
*a*b*x^3 + b^2*x^6])/(7*x^7*(a + b*x^3)) - (b^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*
x^6])/(4*x^4*(a + b*x^3))
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{3/2}}{x^{14}}\,dx &= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{\left(ab+b^2x^3\right)^3}{x^{14}}\,dx}{b^2\left(ab+b^2x^3\right)}\\
&= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \left(\frac{a^3b^3}{x^{14}}+\frac{3a^2b^4}{x^{11}}+\frac{3ab^5}{x^8}+\frac{b^6}{x^5}\right)dx}{b^2\left(ab+b^2x^3\right)}\\
&= -\frac{a^3\sqrt{a^2+2abx^3+b^2x^6}}{13x^{13}\left(a+bx^3\right)}-\frac{3a^2b\sqrt{a^2+2abx^3+b^2x^6}}{10x^{10}\left(a+bx^3\right)}-\frac{3ab^2\sqrt{a^2+2abx^3+b^2x^6}}{7x^7\left(a+bx^3\right)}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 61, normalized size = 0.37

$$-\frac{\sqrt{\left(a+bx^3\right)^2}\left(140a^3+546a^2bx^3+780ab^2x^6+455b^3x^9\right)}{1820x^{13}\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)/x^14,x]
```

```
[Out] -1/1820*(Sqrt[(a + b*x^3)^2]*(140*a^3 + 546*a^2*b*x^3 + 780*a*b^2*x^6 + 455
*b^3*x^9))/(x^13*(a + b*x^3))
```

IntegrateAlgebraic [A] time = 18.76, size = 61, normalized size = 0.37

$$\frac{\sqrt{\left(a+bx^3\right)^2}\left(-140a^3-546a^2bx^3-780ab^2x^6-455b^3x^9\right)}{1820x^{13}\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)/x^14,x]
```

```
[Out] (Sqrt[(a + b*x^3)^2]*(-140*a^3 - 546*a^2*b*x^3 - 780*a*b^2*x^6 - 455*b^3*x^
9))/(1820*x^13*(a + b*x^3))
```

fricas [A] time = 1.36, size = 37, normalized size = 0.22

$$-\frac{455\,b^3x^9+780\,ab^2x^6+546\,a^2bx^3+140\,a^3}{1820\,x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^14,x, algorithm="fricas")
```

```
[Out] -1/1820*(455*b^3*x^9 + 780*a*b^2*x^6 + 546*a^2*b*x^3 + 140*a^3)/x^13
```

giac [A] time = 0.36, size = 69, normalized size = 0.41

$$-\frac{455\,b^3x^9\mathrm{sgn}\left(bx^3+a\right)+780\,ab^2x^6\mathrm{sgn}\left(bx^3+a\right)+546\,a^2bx^3\mathrm{sgn}\left(bx^3+a\right)+140\,a^3\mathrm{sgn}\left(bx^3+a\right)}{1820\,x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^14,x, algorithm="giac")
```

```
[Out] -1/1820*(455*b^3*x^9*sgn(b*x^3 + a) + 780*a*b^2*x^6*sgn(b*x^3 + a) + 546*a^
2*b*x^3*sgn(b*x^3 + a) + 140*a^3*sgn(b*x^3 + a))/x^13
```

maple [A] time = 0.01, size = 58, normalized size = 0.35

$$-\frac{\left(455b^3x^9+780a\,b^2x^6+546a^2b\,x^3+140a^3\right)\left(\left(b\,x^3+a\right)^2\right)^{\frac{3}{2}}}{1820\left(b\,x^3+a\right)^3x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^14,x)
```

```
[Out] -1/1820*(455*b^3*x^9+780*a*b^2*x^6+546*a^2*b*x^3+140*a^3)*((b*x^3+a)^2)^(3/
2)/x^13/(b*x^3+a)^3
```

maxima [A] time = 0.69, size = 37, normalized size = 0.22

$$-\frac{455\,b^3x^9+780\,ab^2x^6+546\,a^2bx^3+140\,a^3}{1820\,x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^14,x, algorithm="maxima")
[Out] -1/1820*(455*b^3*x^9 + 780*a*b^2*x^6 + 546*a^2*b*x^3 + 140*a^3)/x^13
```

mupad [B] time = 1.19, size = 151, normalized size = 0.90

$$-\frac{a^3\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{13\,x^{13}\,\left(b\,x^3+a\right)}-\frac{b^3\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{4\,x^4\,\left(b\,x^3+a\right)}-\frac{3\,a\,b^2\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{7\,x^7\,\left(b\,x^3+a\right)}-\frac{3\,a^2\,b\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{10\,x^{10}\,\left(b\,x^3+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2)/x^14,x)
[Out] - (a^3*(a^2 + b^2*x^6 + 2*a*b*x^3)^(1/2))/(13*x^13*(a + b*x^3)) - (b^3*(a^2
 + b^2*x^6 + 2*a*b*x^3)^(1/2))/(4*x^4*(a + b*x^3)) - (3*a*b^2*(a^2 + b^2*x^
6 + 2*a*b*x^3)^(1/2))/(7*x^7*(a + b*x^3)) - (3*a^2*b*(a^2 + b^2*x^6 + 2*a*b
*x^3)^(1/2))/(10*x^10*(a + b*x^3))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(\left(a+bx^3\right)^2\right)^{\frac{3}{2}}}{x^{14}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**6+2*a*b*x**3+a**2)**(3/2)/x**14,x)
[Out] Integral(((a + b*x**3)**2)**(3/2)/x**14, x)
```

3.47 $\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{3/2}}{x^{15}}\,dx$

Optimal. Leaf size=167

$$-\frac{3a^2b\sqrt{a^2+2abx^3+b^2x^6}}{11x^{11}\left(a+bx^3\right)}-\frac{3ab^2\sqrt{a^2+2abx^3+b^2x^6}}{8x^8\left(a+bx^3\right)}-\frac{b^3\sqrt{a^2+2abx^3+b^2x^6}}{5x^5\left(a+bx^3\right)}-\frac{a^3\sqrt{a^2+2abx^3+b^2x^6}}{14x^{14}\left(a+bx^3\right)}$$

Rubi [A] time = 0.04, antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1355, 270}

$$-\frac{a^3\sqrt{a^2+2abx^3+b^2x^6}}{14x^{14}\left(a+bx^3\right)}-\frac{3a^2b\sqrt{a^2+2abx^3+b^2x^6}}{11x^{11}\left(a+bx^3\right)}-\frac{3ab^2\sqrt{a^2+2abx^3+b^2x^6}}{8x^8\left(a+bx^3\right)}-\frac{b^3\sqrt{a^2+2abx^3+b^2x^6}}{5x^5\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)/x^15,x]
```

```
[Out] -(a^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(14*x^14*(a + b*x^3)) - (3*a^2*b*Sqr
t[a^2 + 2*a*b*x^3 + b^2*x^6])/(11*x^11*(a + b*x^3)) - (3*a*b^2*Sqrt[a^2 + 2
*a*b*x^3 + b^2*x^6])/(8*x^8*(a + b*x^3)) - (b^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*
x^6])/(5*x^5*(a + b*x^3))
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{3/2}}{x^{15}}\,dx &= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{\left(ab+b^2x^3\right)^3}{x^{15}}\,dx}{b^2\left(ab+b^2x^3\right)}\\
&= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \left(\frac{a^3b^3}{x^{15}}+\frac{3a^2b^4}{x^{12}}+\frac{3ab^5}{x^9}+\frac{b^6}{x^6}\right)dx}{b^2\left(ab+b^2x^3\right)}\\
&= -\frac{a^3\sqrt{a^2+2abx^3+b^2x^6}}{14x^{14}\left(a+bx^3\right)}-\frac{3a^2b\sqrt{a^2+2abx^3+b^2x^6}}{11x^{11}\left(a+bx^3\right)}-\frac{3ab^2\sqrt{a^2+2abx^3+b^2}}{8x^8\left(a+bx^3\right)}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 61, normalized size = 0.37

$$-\frac{\sqrt{\left(a+bx^3\right)^2}\left(220a^3+840a^2bx^3+1155ab^2x^6+616b^3x^9\right)}{3080x^{14}\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)/x^15,x]
```

```
[Out] -1/3080*(Sqrt[(a + b*x^3)^2]*(220*a^3 + 840*a^2*b*x^3 + 1155*a*b^2*x^6 + 61
6*b^3*x^9))/(x^14*(a + b*x^3))
```

IntegrateAlgebraic [A] time = 18.64, size = 61, normalized size = 0.37

$$\frac{\sqrt{\left(a+bx^3\right)^2}\left(-220a^3-840a^2bx^3-1155ab^2x^6-616b^3x^9\right)}{3080x^{14}\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)/x^15,x]
```

```
[Out] (Sqrt[(a + b*x^3)^2]*(-220*a^3 - 840*a^2*b*x^3 - 1155*a*b^2*x^6 - 616*b^3*x
^9))/(3080*x^14*(a + b*x^3))
```

fricas [A] time = 2.23, size = 37, normalized size = 0.22

$$-\frac{616\,b^3x^9+1155\,ab^2x^6+840\,a^2bx^3+220\,a^3}{3080\,x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^15,x, algorithm="fricas")
```

```
[Out] -1/3080*(616*b^3*x^9 + 1155*a*b^2*x^6 + 840*a^2*b*x^3 + 220*a^3)/x^14
```

giac [A] time = 0.36, size = 69, normalized size = 0.41

$$-\frac{616\,b^3x^9\mathrm{sgn}\left(bx^3+a\right)+1155\,ab^2x^6\mathrm{sgn}\left(bx^3+a\right)+840\,a^2bx^3\mathrm{sgn}\left(bx^3+a\right)+220\,a^3\mathrm{sgn}\left(bx^3+a\right)}{3080\,x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^15,x, algorithm="giac")
```

```
[Out] -1/3080*(616*b^3*x^9*sgn(b*x^3 + a) + 1155*a*b^2*x^6*sgn(b*x^3 + a) + 840*a
^2*b*x^3*sgn(b*x^3 + a) + 220*a^3*sgn(b*x^3 + a))/x^14
```

maple [A] time = 0.01, size = 58, normalized size = 0.35

$$-\frac{\left(616b^3x^9+1155a\,b^2x^6+840a^2b\,x^3+220a^3\right)\left(\left(b\,x^3+a\right)^2\right)^{\frac{3}{2}}}{3080\left(b\,x^3+a\right)^3x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^15,x)
```

```
[Out] -1/3080*(616*b^3*x^9+1155*a*b^2*x^6+840*a^2*b*x^3+220*a^3)*((b*x^3+a)^2)^(3
/2)/x^14/(b*x^3+a)^3
```

maxima [A] time = 0.69, size = 37, normalized size = 0.22

$$-\frac{616\,b^3x^9+1155\,ab^2x^6+840\,a^2bx^3+220\,a^3}{3080\,x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^15,x, algorithm="maxima")
```

```
[Out] -1/3080*(616*b^3*x^9 + 1155*a*b^2*x^6 + 840*a^2*b*x^3 + 220*a^3)/x^14
```

mupad [B] time = 1.21, size = 151, normalized size = 0.90

$$-\frac{a^3\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{14\,x^{14}\,\left(b\,x^3+a\right)}-\frac{b^3\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{5\,x^5\,\left(b\,x^3+a\right)}-\frac{3\,a\,b^2\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{8\,x^8\,\left(b\,x^3+a\right)}-\frac{3\,a^2\,b\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{11\,x^{11}\,\left(b\,x^3+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2)/x^15,x)
```

```
[Out] - (a^3*(a^2 + b^2*x^6 + 2*a*b*x^3)^(1/2))/(14*x^14*(a + b*x^3)) - (b^3*(a^2
 + b^2*x^6 + 2*a*b*x^3)^(1/2))/(5*x^5*(a + b*x^3)) - (3*a*b^2*(a^2 + b^2*x^
6 + 2*a*b*x^3)^(1/2))/(8*x^8*(a + b*x^3)) - (3*a^2*b*(a^2 + b^2*x^6 + 2*a*b
*x^3)^(1/2))/(11*x^11*(a + b*x^3))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\left(a+bx^3\right)^2\right)^{\frac{3}{2}}}{x^{15}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**6+2*a*b*x**3+a**2)**(3/2)/x**15,x)
```

```
[Out] Integral(((a + b*x**3)**2)**(3/2)/x**15, x)
```

3.48 $\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{3/2}}{x^{16}} dx$

Optimal. Leaf size=84

$$\frac{b\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}}{60a^2x^{12}}-\frac{\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}}{15ax^{15}}$$

Rubi [A] time = 0.04, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {1355, 266, 45, 37}

$$\frac{b\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}}{60a^2x^{12}}-\frac{\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}}{15ax^{15}}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)/x^16,x]
```

```
[Out] -((a + b*x^3)^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(15*a*x^15) + (b*(a + b*x^
3)^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(60*a^2*x^12)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{3/2}}{x^{16}}\,dx = \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{\left(ab+b^2x^3\right)^3}{x^{16}}\,dx}{b^2\left(ab+b^2x^3\right)}$$

$$= \frac{\sqrt{a^2+2abx^3+b^2x^6}\ \text{Subst}\left(\int \frac{\left(ab+b^2x\right)^3}{x^6}\,dx, x, x^3\right)}{3b^2\left(ab+b^2x^3\right)}$$

$$= -\frac{\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}}{15ax^{15}} - \frac{\sqrt{a^2+2abx^3+b^2x^6}\ \text{Subst}\left(\int \frac{\left(ab+b^2x\right)^3}{x^5}\,dx,\right.}{15ab\left(ab+b^2x^3\right)}$$

$$= -\frac{\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}}{15ax^{15}} + \frac{b\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}}{60a^2x^{12}}$$

Mathematica [A] time = 0.01, size = 61, normalized size = 0.73

$$-\frac{\sqrt{\left(a+bx^3\right)^2}\left(4a^3+15a^2bx^3+20ab^2x^6+10b^3x^9\right)}{60x^{15}\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)/x^16,x]
```

```
[Out] -1/60*(Sqrt[(a + b*x^3)^2]*(4*a^3 + 15*a^2*b*x^3 + 20*a*b^2*x^6 + 10*b^3*x^
9))/(x^15*(a + b*x^3))
```

IntegrateAlgebraic [B] time = 1.02, size = 356, normalized size = 4.24

$$\frac{4b^4\sqrt{a^2+2abx^3+b^2x^6}\left(-4a^7b-31a^6b^2x^3-104a^5b^3x^6-196a^4b^4x^9-224a^3b^5x^{12}-155a^2b^6x^{15}-60ab^7x^{18}-10b^8x^{21}\right)+4\sqrt{b^2}\,b^4\left(4a^8+35a^7bx^3+135a^6b^2x^6+300a^5b^3x^9+420a^4b^4x^{12}+379a^3b^5x^{15}+215a^2b^6x^{18}+70ab^7x^{21}+10b^8x^{24}\right)}{15\sqrt{b^2}\,x^{15}\sqrt{a^2+2abx^3+b^2x^6}\left(-16a^4b^4-64a^3b^5x^3-96a^2b^6x^6-64ab^7x^9-16b^8x^{12}\right)+15x^{15}\left(16a^5b^5+80a^4b^6x^3+160a^3b^7x^6+160a^2b^8x^9+80ab^9x^{12}+16b^{10}x^{15}\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)/x^16,x]
```

```
[Out] (4*b^4*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]*(-4*a^7*b - 31*a^6*b^2*x^3 - 104*a^5
*b^3*x^6 - 196*a^4*b^4*x^9 - 224*a^3*b^5*x^12 - 155*a^2*b^6*x^15 - 60*a*b^7
*x^18 - 10*b^8*x^21) + 4*b^4*Sqrt[b^2]*(4*a^8 + 35*a^7*b*x^3 + 135*a^6*b^2*
x^6 + 300*a^5*b^3*x^9 + 420*a^4*b^4*x^12 + 379*a^3*b^5*x^15 + 215*a^2*b^6*x
^18 + 70*a*b^7*x^21 + 10*b^8*x^24))/(15*Sqrt[b^2]*x^15*Sqrt[a^2 + 2*a*b*x^3
 + b^2*x^6]*(-16*a^4*b^4 - 64*a^3*b^5*x^3 - 96*a^2*b^6*x^6 - 64*a*b^7*x^9 -
 16*b^8*x^12) + 15*x^15*(16*a^5*b^5 + 80*a^4*b^6*x^3 + 160*a^3*b^7*x^6 + 16
0*a^2*b^8*x^9 + 80*a*b^9*x^12 + 16*b^10*x^15))
```

fricas [A] time = 0.85, size = 37, normalized size = 0.44

$$-\frac{10\,b^3x^9+20\,ab^2x^6+15\,a^2bx^3+4\,a^3}{60\,x^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^16,x, algorithm="fricas")
```

```
[Out] -1/60*(10*b^3*x^9 + 20*a*b^2*x^6 + 15*a^2*b*x^3 + 4*a^3)/x^15
```

giac [A] time = 0.41, size = 69, normalized size = 0.82

$$-\frac{10\,b^3x^9\text{sgn}\left(bx^3+a\right)+20\,ab^2x^6\text{sgn}\left(bx^3+a\right)+15\,a^2bx^3\text{sgn}\left(bx^3+a\right)+4\,a^3\text{sgn}\left(bx^3+a\right)}{60\,x^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^16,x, algorithm="giac")
```

```
[Out] -1/60*(10*b^3*x^9*sgn(b*x^3 + a) + 20*a*b^2*x^6*sgn(b*x^3 + a) + 15*a^2*b*x
^3*sgn(b*x^3 + a) + 4*a^3*sgn(b*x^3 + a))/x^15
```

maple [A] time = 0.01, size = 58, normalized size = 0.69

$$-\frac{\left(10b^{3}x^{9}+20a\,b^{2}x^{6}+15a^{2}b\,x^{3}+4a^{3}\right)\left(\left(b\,x^{3}+a\right)^{2}\right)^{\frac{3}{2}}}{60\left(b\,x^{3}+a\right)^{3}x^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^16,x)
```

```
[Out] -1/60*(10*b^3*x^9+20*a*b^2*x^6+15*a^2*b*x^3+4*a^3)*((b*x^3+a)^2)^(3/2)/x^15
/(b*x^3+a)^3
```

maxima [B] time = 0.52, size = 179, normalized size = 2.13

$$-\frac{\left(b^{2}x^{6}+2\,abx^{3}+a^{2}\right)^{\frac{3}{2}}b^{5}}{12\,a^{5}}-\frac{\left(b^{2}x^{6}+2\,abx^{3}+a^{2}\right)^{\frac{3}{2}}b^{4}}{12\,a^{4}x^{3}}+\frac{\left(b^{2}x^{6}+2\,abx^{3}+a^{2}\right)^{\frac{5}{2}}b^{3}}{12\,a^{5}x^{6}}-\frac{\left(b^{2}x^{6}+2\,abx^{3}+a^{2}\right)^{\frac{5}{2}}b^{2}}{12\,a^{4}x^{9}}+\frac{\left(b^{2}x^{6}+2\,abx^{3}+a^{2}\right)^{\frac{5}{2}}b}{12\,a^{3}x^{12}}-\frac{\left(b^{2}x^{6}+2\,abx^{3}+a^{2}\right)^{\frac{5}{2}}}{15\,a^{2}x^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^16,x, algorithm="maxima")
```

```
[Out] -1/12*(b^2*x^6 + 2*a*b*x^3 + a^2)^(3/2)*b^5/a^5 - 1/12*(b^2*x^6 + 2*a*b*x^3
 + a^2)^(3/2)*b^4/(a^4*x^3) + 1/12*(b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2)*b^3/(a
^5*x^6) - 1/12*(b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2)*b^2/(a^4*x^9) + 1/12*(b^2*
x^6 + 2*a*b*x^3 + a^2)^(5/2)*b/(a^3*x^12) - 1/15*(b^2*x^6 + 2*a*b*x^3 + a^2
)^(5/2)/(a^2*x^15)
```

mupad [B] time = 1.21, size = 151, normalized size = 1.80

$$-\frac{a^{3}\sqrt{a^{2}+2\,a\,b\,x^{3}+b^{2}\,x^{6}}}{15\,x^{15}\left(b\,x^{3}+a\right)}-\frac{b^{3}\sqrt{a^{2}+2\,a\,b\,x^{3}+b^{2}\,x^{6}}}{6\,x^{6}\left(b\,x^{3}+a\right)}-\frac{a\,b^{2}\sqrt{a^{2}+2\,a\,b\,x^{3}+b^{2}\,x^{6}}}{3\,x^{9}\left(b\,x^{3}+a\right)}-\frac{a^{2}\,b\sqrt{a^{2}+2\,a\,b\,x^{3}+b^{2}\,x^{6}}}{4\,x^{12}\left(b\,x^{3}+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2)/x^16,x)
```

```
[Out] - (a^3*(a^2 + b^2*x^6 + 2*a*b*x^3)^(1/2))/(15*x^15*(a + b*x^3)) - (b^3*(a^2
 + b^2*x^6 + 2*a*b*x^3)^(1/2))/(6*x^6*(a + b*x^3)) - (a*b^2*(a^2 + b^2*x^6
+ 2*a*b*x^3)^(1/2))/(3*x^9*(a + b*x^3)) - (a^2*b*(a^2 + b^2*x^6 + 2*a*b*x^3
)^(1/2))/(4*x^12*(a + b*x^3))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(\left(a+bx^{3}\right)^{2}\right)^{\frac{3}{2}}}{x^{16}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**6+2*a*b*x**3+a**2)**(3/2)/x**16,x)
```

```
[Out] Integral(((a + b*x**3)**2)**(3/2)/x**16, x)
```

3.49 $\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{3/2}}{x^{17}}\,dx$

Optimal. Leaf size=167

$$-\frac{3a^2b\sqrt{a^2+2abx^3+b^2x^6}}{13x^{13}\left(a+bx^3\right)}-\frac{3ab^2\sqrt{a^2+2abx^3+b^2x^6}}{10x^{10}\left(a+bx^3\right)}-\frac{b^3\sqrt{a^2+2abx^3+b^2x^6}}{7x^7\left(a+bx^3\right)}-\frac{a^3\sqrt{a^2+2abx^3+b^2x^6}}{16x^{16}\left(a+bx^3\right)}$$

Rubi [A] time = 0.04, antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1355, 270}

$$-\frac{a^3\sqrt{a^2+2abx^3+b^2x^6}}{16x^{16}\left(a+bx^3\right)}-\frac{3a^2b\sqrt{a^2+2abx^3+b^2x^6}}{13x^{13}\left(a+bx^3\right)}-\frac{3ab^2\sqrt{a^2+2abx^3+b^2x^6}}{10x^{10}\left(a+bx^3\right)}-\frac{b^3\sqrt{a^2+2abx^3+b^2x^6}}{7x^7\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)/x^17,x]
```

```
[Out] -(a^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(16*x^16*(a + b*x^3)) - (3*a^2*b*Sqr
t[a^2 + 2*a*b*x^3 + b^2*x^6])/(13*x^13*(a + b*x^3)) - (3*a*b^2*Sqrt[a^2 + 2
*a*b*x^3 + b^2*x^6])/(10*x^10*(a + b*x^3)) - (b^3*Sqrt[a^2 + 2*a*b*x^3 + b^
2*x^6])/(7*x^7*(a + b*x^3))
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{3/2}}{x^{17}}\,dx &= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{\left(ab+b^2x^3\right)^3}{x^{17}}\,dx}{b^2\left(ab+b^2x^3\right)}\\
&= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \left(\frac{a^3b^3}{x^{17}}+\frac{3a^2b^4}{x^{14}}+\frac{3ab^5}{x^{11}}+\frac{b^6}{x^8}\right)dx}{b^2\left(ab+b^2x^3\right)}\\
&= -\frac{a^3\sqrt{a^2+2abx^3+b^2x^6}}{16x^{16}\left(a+bx^3\right)}-\frac{3a^2b\sqrt{a^2+2abx^3+b^2x^6}}{13x^{13}\left(a+bx^3\right)}-\frac{3ab^2\sqrt{a^2+2abx^3+b^2}}{10x^{10}\left(a+bx^3\right)}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 61, normalized size = 0.37

$$-\frac{\sqrt{\left(a+bx^3\right)^2}\left(455a^3+1680a^2bx^3+2184ab^2x^6+1040b^3x^9\right)}{7280x^{16}\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)/x^17,x]
```

```
[Out] -1/7280*(Sqrt[(a + b*x^3)^2]*(455*a^3 + 1680*a^2*b*x^3 + 2184*a*b^2*x^6 + 1
040*b^3*x^9))/(x^16*(a + b*x^3))
```

IntegrateAlgebraic [A] time = 20.49, size = 61, normalized size = 0.37

$$\frac{\sqrt{\left(a+bx^3\right)^2}\left(-455a^3-1680a^2bx^3-2184ab^2x^6-1040b^3x^9\right)}{7280x^{16}\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)/x^17,x]
```

```
[Out] (Sqrt[(a + b*x^3)^2]*(-455*a^3 - 1680*a^2*b*x^3 - 2184*a*b^2*x^6 - 1040*b^3
*x^9))/(7280*x^16*(a + b*x^3))
```

fricas [A] time = 1.65, size = 37, normalized size = 0.22

$$-\frac{1040\,b^3x^9+2184\,ab^2x^6+1680\,a^2bx^3+455\,a^3}{7280\,x^{16}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^17,x, algorithm="fricas")
```

```
[Out] -1/7280*(1040*b^3*x^9 + 2184*a*b^2*x^6 + 1680*a^2*b*x^3 + 455*a^3)/x^16
```

giac [A] time = 0.30, size = 69, normalized size = 0.41

$$-\frac{1040\,b^3x^9\mathrm{sgn}\left(bx^3+a\right)+2184\,ab^2x^6\mathrm{sgn}\left(bx^3+a\right)+1680\,a^2bx^3\mathrm{sgn}\left(bx^3+a\right)+455\,a^3\mathrm{sgn}\left(bx^3+a\right)}{7280\,x^{16}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^17,x, algorithm="giac")
```

```
[Out] -1/7280*(1040*b^3*x^9*sgn(b*x^3 + a) + 2184*a*b^2*x^6*sgn(b*x^3 + a) + 1680
*a^2*b*x^3*sgn(b*x^3 + a) + 455*a^3*sgn(b*x^3 + a))/x^16
```

maple [A] time = 0.01, size = 58, normalized size = 0.35

$$-\frac{\left(1040b^3x^9+2184a\,b^2x^6+1680a^2b\,x^3+455a^3\right)\left(\left(b\,x^3+a\right)^2\right)^{\frac{3}{2}}}{7280\left(b\,x^3+a\right)^3x^{16}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^17,x)
```

```
[Out] -1/7280*(1040*b^3*x^9+2184*a*b^2*x^6+1680*a^2*b*x^3+455*a^3)*((b*x^3+a)^2)^
(3/2)/x^16/(b*x^3+a)^3
```

maxima [A] time = 0.77, size = 37, normalized size = 0.22

$$-\frac{1040\,b^3x^9+2184\,ab^2x^6+1680\,a^2bx^3+455\,a^3}{7280\,x^{16}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^17,x, algorithm="maxima")
```

```
[Out] -1/7280*(1040*b^3*x^9 + 2184*a*b^2*x^6 + 1680*a^2*b*x^3 + 455*a^3)/x^16
```

mupad [B] time = 1.22, size = 151, normalized size = 0.90

$$-\frac{a^3\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{16\,x^{16}\,\left(b\,x^3+a\right)}-\frac{b^3\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{7\,x^7\,\left(b\,x^3+a\right)}-\frac{3\,a\,b^2\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{10\,x^{10}\,\left(b\,x^3+a\right)}-\frac{3\,a^2\,b\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{13\,x^{13}\,\left(b\,x^3+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2)/x^17,x)
```

```
[Out] - (a^3*(a^2 + b^2*x^6 + 2*a*b*x^3)^(1/2))/(16*x^16*(a + b*x^3)) - (b^3*(a^2
 + b^2*x^6 + 2*a*b*x^3)^(1/2))/(7*x^7*(a + b*x^3)) - (3*a*b^2*(a^2 + b^2*x^
6 + 2*a*b*x^3)^(1/2))/(10*x^10*(a + b*x^3)) - (3*a^2*b*(a^2 + b^2*x^6 + 2*a
*b*x^3)^(1/2))/(13*x^13*(a + b*x^3))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\left(a+bx^3\right)^2\right)^{\frac{3}{2}}}{x^{17}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**6+2*a*b*x**3+a**2)**(3/2)/x**17,x)
```

```
[Out] Integral(((a + b*x**3)**2)**(3/2)/x**17, x)
```

3.50 $\int x^{13} \left(a^2+2abx^3+b^2x^6\right)^{5/2} \, dx$

Optimal. Leaf size=255

$$\frac{b^5x^{29}\sqrt{a^2+2abx^3+b^2x^6}}{29\left(a+bx^3\right)}+\frac{5ab^4x^{26}\sqrt{a^2+2abx^3+b^2x^6}}{26\left(a+bx^3\right)}+\frac{10a^2b^3x^{23}\sqrt{a^2+2abx^3+b^2x^6}}{23\left(a+bx^3\right)}+\frac{a^5x^{14}\sqrt{a^2+2abx^3+b}}{14\left(a+bx^3\right)}$$

Rubi [A] time = 0.06, antiderivative size = 255, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1355, 270}

$$\frac{b^5x^{29}\sqrt{a^2+2abx^3+b^2x^6}}{29\left(a+bx^3\right)}+\frac{5ab^4x^{26}\sqrt{a^2+2abx^3+b^2x^6}}{26\left(a+bx^3\right)}+\frac{10a^2b^3x^{23}\sqrt{a^2+2abx^3+b^2x^6}}{23\left(a+bx^3\right)}+\frac{a^3b^2x^{20}\sqrt{a^2+2abx^3+b^2x^6}}{2\left(a+bx^3\right)}+\frac{5a^4bx^{17}\sqrt{a^2+2abx^3+b^2x^6}}{17\left(a+bx^3\right)}+\frac{a^5x^{14}\sqrt{a^2+2abx^3+b^2x^6}}{14\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x^13*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
```

```
[Out] (a^5*x^14*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(14*(a + b*x^3)) + (5*a^4*b*x^17
*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(17*(a + b*x^3)) + (a^3*b^2*x^20*Sqrt[a^2
 + 2*a*b*x^3 + b^2*x^6])/(2*(a + b*x^3)) + (10*a^2*b^3*x^23*Sqrt[a^2 + 2*a*
b*x^3 + b^2*x^6])/(23*(a + b*x^3)) + (5*a*b^4*x^26*Sqrt[a^2 + 2*a*b*x^3 + b
^2*x^6])/(26*(a + b*x^3)) + (b^5*x^29*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(29*
(a + b*x^3))
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\begin{aligned}
\int x^{13}\left(a^2+2abx^3+b^2x^6\right)^{5/2}\,dx &= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int x^{13}\left(ab+b^2x^3\right)^5\,dx}{b^4\left(ab+b^2x^3\right)}\\
&= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int\left(a^5b^5x^{13}+5a^4b^6x^{16}+10a^3b^7x^{19}+10a^2b^8x^{22}+5ab^9x\right.}{b^4\left(ab+b^2x^3\right)}\\
&= \frac{a^5x^{14}\sqrt{a^2+2abx^3+b^2x^6}}{14\left(a+bx^3\right)}+\frac{5a^4bx^{17}\sqrt{a^2+2abx^3+b^2x^6}}{17\left(a+bx^3\right)}+\frac{a^3b^2x^{20}\sqrt{a^2+2a}}{2\left(a+bx\right.}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 83, normalized size = 0.33

$$\frac{x^{14}\sqrt{\left(a+bx^3\right)^2}\left(147407a^5+606970a^4bx^3+1031849a^3b^2x^6+897260a^2b^3x^9+396865ab^4x^{12}+71162b^5x^{15}\right)}{2063698\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^13*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
```

```
[Out] (x^14*Sqrt[(a + b*x^3)^2]*(147407*a^5 + 606970*a^4*b*x^3 + 1031849*a^3*b^2*
x^6 + 897260*a^2*b^3*x^9 + 396865*a*b^4*x^12 + 71162*b^5*x^15))/(2063698*(a
 + b*x^3))
```

IntegrateAlgebraic [A] time = 27.59, size = 83, normalized size = 0.33

$$\frac{\sqrt{\left(a+bx^3\right)^2}\left(147407a^5x^{14}+606970a^4bx^{17}+1031849a^3b^2x^{20}+897260a^2b^3x^{23}+396865ab^4x^{26}+71162b^5x^{29}\right)}{2063698\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^13*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
```

```
[Out] (Sqrt[(a + b*x^3)^2]*(147407*a^5*x^14 + 606970*a^4*b*x^17 + 1031849*a^3*b^2
*x^20 + 897260*a^2*b^3*x^23 + 396865*a*b^4*x^26 + 71162*b^5*x^29))/(2063698
*(a + b*x^3))
```

fricas [A] time = 0.98, size = 57, normalized size = 0.22

$$\frac{1}{29}\,b^5x^{29}+\frac{5}{26}\,ab^4x^{26}+\frac{10}{23}\,a^2b^3x^{23}+\frac{1}{2}\,a^3b^2x^{20}+\frac{5}{17}\,a^4bx^{17}+\frac{1}{14}\,a^5x^{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^13*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/29*b^5*x^29 + 5/26*a*b^4*x^26 + 10/23*a^2*b^3*x^23 + 1/2*a^3*b^2*x^20 + 5
/17*a^4*b*x^17 + 1/14*a^5*x^14
```

giac [A] time = 0.33, size = 105, normalized size = 0.41

$$\frac{1}{29}\,b^5x^{29}\mathrm{sgn}\left(bx^3+a\right)+\frac{5}{26}\,ab^4x^{26}\mathrm{sgn}\left(bx^3+a\right)+\frac{10}{23}\,a^2b^3x^{23}\mathrm{sgn}\left(bx^3+a\right)+\frac{1}{2}\,a^3b^2x^{20}\mathrm{sgn}\left(bx^3+a\right)+\frac{5}{17}\,a^4bx^{17}\mathrm{sgn}\left(bx^3+a\right)+\frac{1}{14}\,a^5x^{14}\mathrm{sgn}\left(bx^3+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^13*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="giac")
```

```
[Out] 1/29*b^5*x^29*sgn(b*x^3 + a) + 5/26*a*b^4*x^26*sgn(b*x^3 + a) + 10/23*a^2*b
^3*x^23*sgn(b*x^3 + a) + 1/2*a^3*b^2*x^20*sgn(b*x^3 + a) + 5/17*a^4*b*x^17*
sgn(b*x^3 + a) + 1/14*a^5*x^14*sgn(b*x^3 + a)
```

maple [A] time = 0.01, size = 80, normalized size = 0.31

$$\frac{\left(71162b^5x^{15}+396865a\,b^4x^{12}+897260a^2b^3x^9+1031849a^3b^2x^6+606970a^4b\,x^3+147407a^5\right)\left(\left(b\,x^3+a\right)^2\right)^{\frac{5}{2}}x^{14}}{2063698\left(b\,x^3+a\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^13*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x)
```

```
[Out] 1/2063698*x^14*(71162*b^5*x^15+396865*a*b^4*x^12+897260*a^2*b^3*x^9+1031849
*a^3*b^2*x^6+606970*a^4*b*x^3+147407*a^5)*((b*x^3+a)^2)^(5/2)/(b*x^3+a)^5
```

maxima [A] time = 0.83, size = 57, normalized size = 0.22

$$\frac{1}{29}\,b^5x^{29}+\frac{5}{26}\,ab^4x^{26}+\frac{10}{23}\,a^2b^3x^{23}+\frac{1}{2}\,a^3b^2x^{20}+\frac{5}{17}\,a^4bx^{17}+\frac{1}{14}\,a^5x^{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^13*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/29*b^5*x^29 + 5/26*a*b^4*x^26 + 10/23*a^2*b^3*x^23 + 1/2*a^3*b^2*x^20 + 5
/17*a^4*b*x^17 + 1/14*a^5*x^14
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^{13} \left(a^2 + 2\,a\,b\,x^3 + b^2\,x^6\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^13*(a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2),x)
```

```
[Out] int(x^13*(a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^{13} \left(\left(a + bx^3\right)^2\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**13*(b**2*x**6+2*a*b*x**3+a**2)**(5/2),x)
```

```
[Out] Integral(x**13*((a + b*x**3)**2)**(5/2), x)
```

3.51 $\int x^{12}\left(a^2+2abx^3+b^2x^6\right)^{5/2}\,dx$

Optimal. Leaf size=255

$$\frac{b^5x^{28}\sqrt{a^2+2abx^3+b^2x^6}}{28\left(a+bx^3\right)}+\frac{ab^4x^{25}\sqrt{a^2+2abx^3+b^2x^6}}{5\left(a+bx^3\right)}+\frac{5a^2b^3x^{22}\sqrt{a^2+2abx^3+b^2x^6}}{11\left(a+bx^3\right)}+\frac{a^5x^{13}\sqrt{a^2+2abx^3+b}}{13\left(a+bx^3\right)}$$

Rubi [A] time = 0.06, antiderivative size = 255, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1355, 270}

$$\frac{b^5x^{28}\sqrt{a^2+2abx^3+b^2x^6}}{28\left(a+bx^3\right)}+\frac{ab^4x^{25}\sqrt{a^2+2abx^3+b^2x^6}}{5\left(a+bx^3\right)}+\frac{5a^2b^3x^{22}\sqrt{a^2+2abx^3+b^2x^6}}{11\left(a+bx^3\right)}+\frac{10a^3b^2x^{19}\sqrt{a^2+2abx^3+b^2x^6}}{19\left(a+bx^3\right)}+\frac{5a^4bx^{16}\sqrt{a^2+2abx^3+b^2x^6}}{16\left(a+bx^3\right)}+\frac{a^5x^{13}\sqrt{a^2+2abx^3+b^2x^6}}{13\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x^12*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
```

```
[Out] (a^5*x^13*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(13*(a + b*x^3)) + (5*a^4*b*x^16
*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(16*(a + b*x^3)) + (10*a^3*b^2*x^19*Sqrt[
a^2 + 2*a*b*x^3 + b^2*x^6])/(19*(a + b*x^3)) + (5*a^2*b^3*x^22*Sqrt[a^2 + 2
*a*b*x^3 + b^2*x^6])/(11*(a + b*x^3)) + (a*b^4*x^25*Sqrt[a^2 + 2*a*b*x^3 +
b^2*x^6])/(5*(a + b*x^3)) + (b^5*x^28*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(28*
(a + b*x^3))
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\begin{aligned}\int x^{12}\left(a^2+2abx^3+b^2x^6\right)^{5/2}\,dx&=\frac{\sqrt{a^2+2abx^3+b^2x^6}\int x^{12}\left(ab+b^2x^3\right)^5\,dx}{b^4\left(ab+b^2x^3\right)}\\&=\frac{\sqrt{a^2+2abx^3+b^2x^6}\int\left(a^5b^5x^{12}+5a^4b^6x^{15}+10a^3b^7x^{18}+10a^2b^8x^{21}+5ab\right.}{b^4\left(ab+b^2x^3\right)}\\&=\frac{a^5x^{13}\sqrt{a^2+2abx^3+b^2x^6}}{13\left(a+bx^3\right)}+\frac{5a^4bx^{16}\sqrt{a^2+2abx^3+b^2x^6}}{16\left(a+bx^3\right)}+\frac{10a^3b^2x^{19}\sqrt{a^2}}{19\left(a\right.}\end{aligned}$$

Mathematica [A] time = 0.02, size = 83, normalized size = 0.33

$$\frac{x^{13}\sqrt{\left(a+bx^3\right)^2}\left(117040a^5+475475a^4bx^3+800800a^3b^2x^6+691600a^2b^3x^9+304304ab^4x^{12}+54340b^5x^{15}\right)}{1521520\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^12*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
```

```
[Out] (x^13*Sqrt[(a + b*x^3)^2]*(117040*a^5 + 475475*a^4*b*x^3 + 800800*a^3*b^2*x
^6 + 691600*a^2*b^3*x^9 + 304304*a*b^4*x^12 + 54340*b^5*x^15))/(1521520*(a
+ b*x^3))
```

IntegrateAlgebraic [A] time = 24.78, size = 83, normalized size = 0.33

$$\frac{\sqrt{(a+bx^3)^2}\left(117040a^5x^{13}+475475a^4bx^{16}+800800a^3b^2x^{19}+691600a^2b^3x^{22}+304304ab^4x^{25}+54340b^5x^{28}\right)}{1521520\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^12*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
```

```
[Out] (Sqrt[(a + b*x^3)^2]*(117040*a^5*x^13 + 475475*a^4*b*x^16 + 800800*a^3*b^2*
x^19 + 691600*a^2*b^3*x^22 + 304304*a*b^4*x^25 + 54340*b^5*x^28))/(1521520*
(a + b*x^3))
```

fricas [A] time = 1.99, size = 57, normalized size = 0.22

$$\frac{1}{28}b^5x^{28}+\frac{1}{5}ab^4x^{25}+\frac{5}{11}a^2b^3x^{22}+\frac{10}{19}a^3b^2x^{19}+\frac{5}{16}a^4bx^{16}+\frac{1}{13}a^5x^{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^12*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/28*b^5*x^28 + 1/5*a*b^4*x^25 + 5/11*a^2*b^3*x^22 + 10/19*a^3*b^2*x^19 + 5
/16*a^4*b*x^16 + 1/13*a^5*x^13
```

giac [A] time = 0.32, size = 105, normalized size = 0.41

$$\frac{1}{28}b^5x^{28}\mathrm{sgn}\left(bx^3+a\right)+\frac{1}{5}ab^4x^{25}\mathrm{sgn}\left(bx^3+a\right)+\frac{5}{11}a^2b^3x^{22}\mathrm{sgn}\left(bx^3+a\right)+\frac{10}{19}a^3b^2x^{19}\mathrm{sgn}\left(bx^3+a\right)+\frac{5}{16}a^4bx^{16}\mathrm{sgn}\left(bx^3+a\right)+\frac{1}{13}a^5x^{13}\mathrm{sgn}\left(bx^3+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^12*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="giac")
```

```
[Out] 1/28*b^5*x^28*sgn(b*x^3 + a) + 1/5*a*b^4*x^25*sgn(b*x^3 + a) + 5/11*a^2*b^3
*x^22*sgn(b*x^3 + a) + 10/19*a^3*b^2*x^19*sgn(b*x^3 + a) + 5/16*a^4*b*x^16*
sgn(b*x^3 + a) + 1/13*a^5*x^13*sgn(b*x^3 + a)
```

maple [A] time = 0.01, size = 80, normalized size = 0.31

$$\frac{\left(54340b^5x^{15}+304304a\,b^4x^{12}+691600a^2b^3x^9+800800a^3b^2x^6+475475a^4b\,x^3+117040a^5\right)\left(\left(b\,x^3+a\right)^2\right)^{\frac{5}{2}}x^{13}}{1521520\left(b\,x^3+a\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^12*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x)
```

```
[Out] 1/1521520*x^13*(54340*b^5*x^15+304304*a*b^4*x^12+691600*a^2*b^3*x^9+800800*
a^3*b^2*x^6+475475*a^4*b*x^3+117040*a^5)*((b*x^3+a)^2)^(5/2)/(b*x^3+a)^5
```

maxima [A] time = 0.60, size = 57, normalized size = 0.22

$$\frac{1}{28}b^5x^{28}+\frac{1}{5}ab^4x^{25}+\frac{5}{11}a^2b^3x^{22}+\frac{10}{19}a^3b^2x^{19}+\frac{5}{16}a^4bx^{16}+\frac{1}{13}a^5x^{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^12*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/28*b^5*x^28 + 1/5*a*b^4*x^25 + 5/11*a^2*b^3*x^22 + 10/19*a^3*b^2*x^19 + 5
/16*a^4*b*x^16 + 1/13*a^5*x^13
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^{12}\left(a^2+2\,a\,b\,x^3+b^2\,x^6\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^12*(a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2),x)
[Out] int(x^12*(a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^{12}\left(\left(a+bx^3\right)^2\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**12*(b**2*x**6+2*a*b*x**3+a**2)**(5/2),x)
[Out] Integral(x**12*((a + b*x**3)**2)**(5/2), x)
```

3.52 $\int x^{11} \left(a^2+2abx^3+b^2x^6\right)^{5/2} dx$

Optimal. Leaf size=160

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\left(a+bx^3\right)^8}{27b^4}-\frac{a\sqrt{a^2+2abx^3+b^2x^6}\left(a+bx^3\right)^7}{8b^4}+\frac{a^2\sqrt{a^2+2abx^3+b^2x^6}\left(a+bx^3\right)^6}{7b^4}-\frac{a^3\sqrt{a^2+2}}{}$$

Rubi [A] time = 0.12, antiderivative size = 160, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1355, 266, 43}

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\left(a+bx^3\right)^8}{27b^4}-\frac{a\sqrt{a^2+2abx^3+b^2x^6}\left(a+bx^3\right)^7}{8b^4}+\frac{a^2\sqrt{a^2+2abx^3+b^2x^6}\left(a+bx^3\right)^6}{7b^4}-\frac{a^3\sqrt{a^2+2abx^3+b^2x^6}\left(a+bx^3\right)^5}{18b^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^11*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
```

```
[Out] -(a^3*(a + b*x^3)^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(18*b^4) + (a^2*(a + b
*x^3)^6*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(7*b^4) - (a*(a + b*x^3)^7*Sqrt[a^
2 + 2*a*b*x^3 + b^2*x^6])/(8*b^4) + ((a + b*x^3)^8*Sqrt[a^2 + 2*a*b*x^3 + b
^2*x^6])/(27*b^4)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\int x^{11}\left(a^2+2abx^3+b^2x^6\right)^{5/2} dx = \frac{\sqrt{a^2+2abx^3+b^2x^6}\int x^{11}\left(ab+b^2x^3\right)^5 dx}{b^4\left(ab+b^2x^3\right)}$$

$$= \frac{\sqrt{a^2+2abx^3+b^2x^6}\ \text{Subst}\left(\int x^3\left(ab+b^2x\right)^5 dx, x, x^3\right)}{3b^4\left(ab+b^2x^3\right)}$$

$$= \frac{\sqrt{a^2+2abx^3+b^2x^6}\ \text{Subst}\left(\int\left(-\frac{a^3\left(ab+b^2x\right)^5}{b^3}+\frac{3a^2\left(ab+b^2x\right)^6}{b^4}-\frac{3a\left(ab+b^2x\right)^7}{b^5}+\frac{\left(ab+b^2\right.}{b^6}\right.\right.}{3b^4\left(ab+b^2x^3\right)}$$

$$= -\frac{a^3\left(a+bx^3\right)^5\sqrt{a^2+2abx^3+b^2x^6}}{18b^4}+\frac{a^2\left(a+bx^3\right)^6\sqrt{a^2+2abx^3+b^2x^6}}{7b^4}-\frac{a}{}$$

Mathematica [A] time = 0.02, size = 83, normalized size = 0.52

$$\frac{x^{12}\sqrt{\left(a+bx^3\right)^2}\left(126a^5+504a^4bx^3+840a^3b^2x^6+720a^2b^3x^9+315ab^4x^{12}+56b^5x^{15}\right)}{1512\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^11*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
```

```
[Out] (x^12*Sqrt[(a + b*x^3)^2]*(126*a^5 + 504*a^4*b*x^3 + 840*a^3*b^2*x^6 + 720*
a^2*b^3*x^9 + 315*a*b^4*x^12 + 56*b^5*x^15))/(1512*(a + b*x^3))
```

IntegrateAlgebraic [A] time = 21.67, size = 83, normalized size = 0.52

$$\frac{x^{12}\sqrt{\left(a+bx^3\right)^2}\left(126a^5+504a^4bx^3+840a^3b^2x^6+720a^2b^3x^9+315ab^4x^{12}+56b^5x^{15}\right)}{1512\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^11*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
```

```
[Out] (x^12*Sqrt[(a + b*x^3)^2]*(126*a^5 + 504*a^4*b*x^3 + 840*a^3*b^2*x^6 + 720*
a^2*b^3*x^9 + 315*a*b^4*x^12 + 56*b^5*x^15))/(1512*(a + b*x^3))
```

fricas [A] time = 1.45, size = 57, normalized size = 0.36

$$\frac{1}{27}b^5x^{27}+\frac{5}{24}ab^4x^{24}+\frac{10}{21}a^2b^3x^{21}+\frac{5}{9}a^3b^2x^{18}+\frac{1}{3}a^4bx^{15}+\frac{1}{12}a^5x^{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/27*b^5*x^27 + 5/24*a*b^4*x^24 + 10/21*a^2*b^3*x^21 + 5/9*a^3*b^2*x^18 + 1
/3*a^4*b*x^15 + 1/12*a^5*x^12
```

giac [A] time = 0.35, size = 105, normalized size = 0.66

$$\frac{1}{27}b^5x^{27}\mathrm{sgn}\left(bx^3+a\right)+\frac{5}{24}ab^4x^{24}\mathrm{sgn}\left(bx^3+a\right)+\frac{10}{21}a^2b^3x^{21}\mathrm{sgn}\left(bx^3+a\right)+\frac{5}{9}a^3b^2x^{18}\mathrm{sgn}\left(bx^3+a\right)+\frac{1}{3}a^4bx^{15}\mathrm{sgn}\left(bx^3+a\right)+\frac{1}{12}a^5x^{12}\mathrm{sgn}\left(bx^3+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="giac")
```

```
[Out] 1/27*b^5*x^27*sgn(b*x^3 + a) + 5/24*a*b^4*x^24*sgn(b*x^3 + a) + 10/21*a^2*b
^3*x^21*sgn(b*x^3 + a) + 5/9*a^3*b^2*x^18*sgn(b*x^3 + a) + 1/3*a^4*b*x^15*s
gn(b*x^3 + a) + 1/12*a^5*x^12*sgn(b*x^3 + a)
```

maple [A] time = 0.01, size = 80, normalized size = 0.50

$$\frac{\left(56b^5x^{15}+315a\,b^4x^{12}+720a^2b^3x^9+840a^3b^2x^6+504a^4b\,x^3+126a^5\right)\left(\left(b\,x^3+a\right)^2\right)^{\frac{5}{2}}x^{12}}{1512\left(b\,x^3+a\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x)
```

```
[Out] 1/1512*x^12*(56*b^5*x^15+315*a*b^4*x^12+720*a^2*b^3*x^9+840*a^3*b^2*x^6+504
*a^4*b*x^3+126*a^5)*((b*x^3+a)^2)^(5/2)/(b*x^3+a)^5
```

maxima [A] time = 0.72, size = 145, normalized size = 0.91

$$\frac{\left(b^2x^6+2\,abx^3+a^2\right)^{\frac{7}{2}}x^6}{27\,b^2}-\frac{\left(b^2x^6+2\,abx^3+a^2\right)^{\frac{5}{2}}a^3x^3}{18\,b^3}-\frac{11\left(b^2x^6+2\,abx^3+a^2\right)^{\frac{7}{2}}ax^3}{216\,b^3}-\frac{\left(b^2x^6+2\,abx^3+a^2\right)^{\frac{5}{2}}a^4}{18\,b^4}+\frac{83\left(b^2x^6+2\,abx^3+a^2\right)^{\frac{7}{2}}a^2}{1512\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/27*(b^2*x^6 + 2*a*b*x^3 + a^2)^(7/2)*x^6/b^2 - 1/18*(b^2*x^6 + 2*a*b*x^3
+ a^2)^(5/2)*a^3*x^3/b^3 - 11/216*(b^2*x^6 + 2*a*b*x^3 + a^2)^(7/2)*a*x^3/b
^3 - 1/18*(b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2)*a^4/b^4 + 83/1512*(b^2*x^6 + 2*
a*b*x^3 + a^2)^(7/2)*a^2/b^4
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^{11}\left(a^2+2\,a\,b\,x^3+b^2\,x^6\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11*(a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2),x)
```

```
[Out] int(x^11*(a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^{11}\left(\left(a+bx^3\right)^2\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**11*(b**2*x**6+2*a*b*x**3+a**2)**(5/2),x)
```

```
[Out] Integral(x**11*((a + b*x**3)**2)**(5/2), x)
```

3.53 $\int x^{10} \left(a^2+2abx^3+b^2x^6\right)^{5/2} \, dx$

Optimal. Leaf size=255

$$\frac{b^5x^{26}\sqrt{a^2+2abx^3+b^2x^6}}{26\left(a+bx^3\right)}+\frac{5ab^4x^{23}\sqrt{a^2+2abx^3+b^2x^6}}{23\left(a+bx^3\right)}+\frac{a^2b^3x^{20}\sqrt{a^2+2abx^3+b^2x^6}}{2\left(a+bx^3\right)}+\frac{a^5x^{11}\sqrt{a^2+2abx^3+b}}{11\left(a+bx^3\right)}$$

Rubi [A] time = 0.06, antiderivative size = 255, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1355, 270}

$$\frac{b^5x^{26}\sqrt{a^2+2abx^3+b^2x^6}}{26\left(a+bx^3\right)}+\frac{5ab^4x^{23}\sqrt{a^2+2abx^3+b^2x^6}}{23\left(a+bx^3\right)}+\frac{a^2b^3x^{20}\sqrt{a^2+2abx^3+b^2x^6}}{2\left(a+bx^3\right)}+\frac{10a^3b^2x^{17}\sqrt{a^2+2abx^3+b^2x^6}}{17\left(a+bx^3\right)}+\frac{5a^4bx^{14}\sqrt{a^2+2abx^3+b^2x^6}}{14\left(a+bx^3\right)}+\frac{a^5x^{11}\sqrt{a^2+2abx^3+b^2x^6}}{11\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x^10*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
```

```
[Out] (a^5*x^11*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(11*(a + b*x^3)) + (5*a^4*b*x^14
*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(14*(a + b*x^3)) + (10*a^3*b^2*x^17*Sqrt[
a^2 + 2*a*b*x^3 + b^2*x^6])/(17*(a + b*x^3)) + (a^2*b^3*x^20*Sqrt[a^2 + 2*a
*b*x^3 + b^2*x^6])/(2*(a + b*x^3)) + (5*a*b^4*x^23*Sqrt[a^2 + 2*a*b*x^3 + b
^2*x^6])/(23*(a + b*x^3)) + (b^5*x^26*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(26*
(a + b*x^3))
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\begin{aligned}\int x^{10}\left(a^2+2abx^3+b^2x^6\right)^{5/2}\,dx &= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int x^{10}\left(ab+b^2x^3\right)^5\,dx}{b^4\left(ab+b^2x^3\right)}\\ &= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int\left(a^5b^5x^{10}+5a^4b^6x^{13}+10a^3b^7x^{16}+10a^2b^8x^{19}+5a\right.}{b^4\left(ab+b^2x^3\right)}\\ &= \frac{a^5x^{11}\sqrt{a^2+2abx^3+b^2x^6}}{11\left(a+bx^3\right)}+\frac{5a^4bx^{14}\sqrt{a^2+2abx^3+b^2x^6}}{14\left(a+bx^3\right)}+\frac{10a^3b^2x^{17}\sqrt{a^2}}{17\left(a\right.}\end{aligned}$$

Mathematica [A] time = 0.02, size = 83, normalized size = 0.33

$$\frac{x^{11}\sqrt{\left(a+bx^3\right)^2}\left(71162a^5+279565a^4bx^3+460460a^3b^2x^6+391391a^2b^3x^9+170170ab^4x^{12}+30107b^5x^{15}\right)}{782782\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^10*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
```

```
[Out] (x^11*Sqrt[(a + b*x^3)^2]*(71162*a^5 + 279565*a^4*b*x^3 + 460460*a^3*b^2*x^
6 + 391391*a^2*b^3*x^9 + 170170*a*b^4*x^12 + 30107*b^5*x^15))/(782782*(a +
b*x^3))
```

IntegrateAlgebraic [A] time = 19.23, size = 83, normalized size = 0.33

$$\frac{\sqrt{\left(a+bx^3\right)^2}\left(71162a^5x^{11}+279565a^4bx^{14}+460460a^3b^2x^{17}+391391a^2b^3x^{20}+170170ab^4x^{23}+30107b^5x^{26}\right)}{782782\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^10*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
```

```
[Out] (Sqrt[(a + b*x^3)^2]*(71162*a^5*x^11 + 279565*a^4*b*x^14 + 460460*a^3*b^2*x
^17 + 391391*a^2*b^3*x^20 + 170170*a*b^4*x^23 + 30107*b^5*x^26))/(782782*(a
 + b*x^3))
```

fricas [A] time = 1.56, size = 57, normalized size = 0.22

$$\frac{1}{26}\,b^5x^{26}+\frac{5}{23}\,ab^4x^{23}+\frac{1}{2}\,a^2b^3x^{20}+\frac{10}{17}\,a^3b^2x^{17}+\frac{5}{14}\,a^4bx^{14}+\frac{1}{11}\,a^5x^{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/26*b^5*x^26 + 5/23*a*b^4*x^23 + 1/2*a^2*b^3*x^20 + 10/17*a^3*b^2*x^17 + 5
/14*a^4*b*x^14 + 1/11*a^5*x^11
```

giac [A] time = 0.44, size = 105, normalized size = 0.41

$$\frac{1}{26}\,b^5x^{26}\mathrm{sgn}\left(bx^3+a\right)+\frac{5}{23}\,ab^4x^{23}\mathrm{sgn}\left(bx^3+a\right)+\frac{1}{2}\,a^2b^3x^{20}\mathrm{sgn}\left(bx^3+a\right)+\frac{10}{17}\,a^3b^2x^{17}\mathrm{sgn}\left(bx^3+a\right)+\frac{5}{14}\,a^4bx^{14}\mathrm{sgn}\left(bx^3+a\right)+\frac{1}{11}\,a^5x^{11}\mathrm{sgn}\left(bx^3+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="giac")
```

```
[Out] 1/26*b^5*x^26*sgn(b*x^3 + a) + 5/23*a*b^4*x^23*sgn(b*x^3 + a) + 1/2*a^2*b^3
*x^20*sgn(b*x^3 + a) + 10/17*a^3*b^2*x^17*sgn(b*x^3 + a) + 5/14*a^4*b*x^14*
sgn(b*x^3 + a) + 1/11*a^5*x^11*sgn(b*x^3 + a)
```

maple [A] time = 0.01, size = 80, normalized size = 0.31

$$\frac{\left(30107b^5x^{15}+170170a\,b^4x^{12}+391391a^2b^3x^9+460460a^3b^2x^6+279565a^4b\,x^3+71162a^5\right)\left(\left(b\,x^3+a\right)^2\right)^{\frac{5}{2}}x^{11}}{782782\left(b\,x^3+a\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^10*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x)
```

```
[Out] 1/782782*x^11*(30107*b^5*x^15+170170*a*b^4*x^12+391391*a^2*b^3*x^9+460460*a
^3*b^2*x^6+279565*a^4*b*x^3+71162*a^5)*((b*x^3+a)^2)^(5/2)/(b*x^3+a)^5
```

maxima [A] time = 0.81, size = 57, normalized size = 0.22

$$\frac{1}{26}\,b^5x^{26}+\frac{5}{23}\,ab^4x^{23}+\frac{1}{2}\,a^2b^3x^{20}+\frac{10}{17}\,a^3b^2x^{17}+\frac{5}{14}\,a^4bx^{14}+\frac{1}{11}\,a^5x^{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/26*b^5*x^26 + 5/23*a*b^4*x^23 + 1/2*a^2*b^3*x^20 + 10/17*a^3*b^2*x^17 + 5
/14*a^4*b*x^14 + 1/11*a^5*x^11
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^{10} \left(a^2 + 2\,a\,b\,x^3 + b^2\,x^6\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^10*(a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2),x)
[Out] int(x^10*(a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^{10} \left(\left(a + bx^3\right)^2\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**10*(b**2*x**6+2*a*b*x**3+a**2)**(5/2),x)
[Out] Integral(x**10*((a + b*x**3)**2)**(5/2), x)
```

3.54 $\int x^9 \left(a^2+2abx^3+b^2x^6\right)^{5/2}\,dx$

Optimal. Leaf size=255

$$\frac{b^5x^{25}\sqrt{a^2+2abx^3+b^2x^6}}{25\left(a+bx^3\right)}+\frac{5ab^4x^{22}\sqrt{a^2+2abx^3+b^2x^6}}{22\left(a+bx^3\right)}+\frac{10a^2b^3x^{19}\sqrt{a^2+2abx^3+b^2x^6}}{19\left(a+bx^3\right)}+\frac{a^5x^{10}\sqrt{a^2+2abx^3+b}}{10\left(a+bx^3\right)}$$

Rubi [A] time = 0.06, antiderivative size = 255, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1355, 270}

$$\frac{b^5x^{25}\sqrt{a^2+2abx^3+b^2x^6}}{25\left(a+bx^3\right)}+\frac{5ab^4x^{22}\sqrt{a^2+2abx^3+b^2x^6}}{22\left(a+bx^3\right)}+\frac{10a^2b^3x^{19}\sqrt{a^2+2abx^3+b^2x^6}}{19\left(a+bx^3\right)}+\frac{5a^3b^2x^{16}\sqrt{a^2+2abx^3+b^2x^6}}{8\left(a+bx^3\right)}+\frac{5a^4bx^{13}\sqrt{a^2+2abx^3+b^2x^6}}{13\left(a+bx^3\right)}+\frac{a^5x^{10}\sqrt{a^2+2abx^3+b^2x^6}}{10\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x^9*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
```

```
[Out] (a^5*x^10*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(10*(a + b*x^3)) + (5*a^4*b*x^13
*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(13*(a + b*x^3)) + (5*a^3*b^2*x^16*Sqrt[a
^2 + 2*a*b*x^3 + b^2*x^6])/(8*(a + b*x^3)) + (10*a^2*b^3*x^19*Sqrt[a^2 + 2*
a*b*x^3 + b^2*x^6])/(19*(a + b*x^3)) + (5*a*b^4*x^22*Sqrt[a^2 + 2*a*b*x^3 +
 b^2*x^6])/(22*(a + b*x^3)) + (b^5*x^25*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(2
5*(a + b*x^3))
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\begin{aligned}\int x^9\left(a^2+2abx^3+b^2x^6\right)^{5/2}\,dx &= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int x^9\left(ab+b^2x^3\right)^5\,dx}{b^4\left(ab+b^2x^3\right)}\\ &= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int\left(a^5b^5x^9+5a^4b^6x^{12}+10a^3b^7x^{15}+10a^2b^8x^{18}+5ab^9x^{21}\right.}{b^4\left(ab+b^2x^3\right)}\\ &= \frac{a^5x^{10}\sqrt{a^2+2abx^3+b^2x^6}}{10\left(a+bx^3\right)}+\frac{5a^4bx^{13}\sqrt{a^2+2abx^3+b^2x^6}}{13\left(a+bx^3\right)}+\frac{5a^3b^2x^{16}\sqrt{a^2+2a}}{8\left(a+bx\right.}\end{aligned}$$

Mathematica [A] time = 0.02, size = 83, normalized size = 0.33

$$\frac{x^{10}\sqrt{\left(a+bx^3\right)^2}\left(54340a^5+209000a^4bx^3+339625a^3b^2x^6+286000a^2b^3x^9+123500ab^4x^{12}+21736b^5x^{15}\right)}{543400\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^9*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
```

```
[Out] (x^10*Sqrt[(a + b*x^3)^2]*(54340*a^5 + 209000*a^4*b*x^3 + 339625*a^3*b^2*x^
6 + 286000*a^2*b^3*x^9 + 123500*a*b^4*x^12 + 21736*b^5*x^15))/(543400*(a +
b*x^3))
```

IntegrateAlgebraic [A] time = 18.13, size = 83, normalized size = 0.33

$$\frac{\sqrt{\left(a+bx^3\right)^2}\left(54340a^5x^{10}+209000a^4bx^{13}+339625a^3b^2x^{16}+286000a^2b^3x^{19}+123500ab^4x^{22}+21736b^5x^{25}\right)}{543400\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^9*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
```

```
[Out] (Sqrt[(a + b*x^3)^2]*(54340*a^5*x^10 + 209000*a^4*b*x^13 + 339625*a^3*b^2*x
^16 + 286000*a^2*b^3*x^19 + 123500*a*b^4*x^22 + 21736*b^5*x^25))/(543400*(a
 + b*x^3))
```

fricas [A] time = 1.14, size = 57, normalized size = 0.22

$$\frac{1}{25}b^5x^{25}+\frac{5}{22}ab^4x^{22}+\frac{10}{19}a^2b^3x^{19}+\frac{5}{8}a^3b^2x^{16}+\frac{5}{13}a^4bx^{13}+\frac{1}{10}a^5x^{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/25*b^5*x^25 + 5/22*a*b^4*x^22 + 10/19*a^2*b^3*x^19 + 5/8*a^3*b^2*x^16 + 5
/13*a^4*b*x^13 + 1/10*a^5*x^10
```

giac [A] time = 0.40, size = 105, normalized size = 0.41

$$\frac{1}{25}b^5x^{25}\mathrm{sgn}\left(bx^3+a\right)+\frac{5}{22}ab^4x^{22}\mathrm{sgn}\left(bx^3+a\right)+\frac{10}{19}a^2b^3x^{19}\mathrm{sgn}\left(bx^3+a\right)+\frac{5}{8}a^3b^2x^{16}\mathrm{sgn}\left(bx^3+a\right)+\frac{5}{13}a^4bx^{13}\mathrm{sgn}\left(bx^3+a\right)+\frac{1}{10}a^5x^{10}\mathrm{sgn}\left(bx^3+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="giac")
```

```
[Out] 1/25*b^5*x^25*sgn(b*x^3 + a) + 5/22*a*b^4*x^22*sgn(b*x^3 + a) + 10/19*a^2*b
^3*x^19*sgn(b*x^3 + a) + 5/8*a^3*b^2*x^16*sgn(b*x^3 + a) + 5/13*a^4*b*x^13*
sgn(b*x^3 + a) + 1/10*a^5*x^10*sgn(b*x^3 + a)
```

maple [A] time = 0.01, size = 80, normalized size = 0.31

$$\frac{\left(21736b^5x^{15}+123500a\,b^4x^{12}+286000a^2b^3x^9+339625a^3b^2x^6+209000a^4b\,x^3+54340a^5\right)\left(\left(b\,x^3+a\right)^2\right)^{\frac{5}{2}}x^{10}}{543400\left(b\,x^3+a\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x)
```

```
[Out] 1/543400*x^10*(21736*b^5*x^15+123500*a*b^4*x^12+286000*a^2*b^3*x^9+339625*a
^3*b^2*x^6+209000*a^4*b*x^3+54340*a^5)*((b*x^3+a)^2)^(5/2)/(b*x^3+a)^5
```

maxima [A] time = 0.67, size = 57, normalized size = 0.22

$$\frac{1}{25}b^5x^{25}+\frac{5}{22}ab^4x^{22}+\frac{10}{19}a^2b^3x^{19}+\frac{5}{8}a^3b^2x^{16}+\frac{5}{13}a^4bx^{13}+\frac{1}{10}a^5x^{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/25*b^5*x^25 + 5/22*a*b^4*x^22 + 10/19*a^2*b^3*x^19 + 5/8*a^3*b^2*x^16 + 5
/13*a^4*b*x^13 + 1/10*a^5*x^10
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^9 \left(a^2 + 2\,a\,b\,x^3 + b^2\,x^6\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9*(a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2),x)
[Out] int(x^9*(a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^9 \left(\left(a + bx^3\right)^2\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**9*(b**2*x**6+2*a*b*x**3+a**2)**(5/2),x)
[Out] Integral(x**9*((a + b*x**3)**2)**(5/2), x)
```

3.55 $\int x^8 \left(a^2+2abx^3+b^2x^6\right)^{5/2} dx$

Optimal. Leaf size=119

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\left(a+bx^3\right)^7}{24b^3}-\frac{2a\sqrt{a^2+2abx^3+b^2x^6}\left(a+bx^3\right)^6}{21b^3}+\frac{a^2\sqrt{a^2+2abx^3+b^2x^6}\left(a+bx^3\right)^5}{18b^3}$$

Rubi [A] time = 0.09, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1355, 266, 43}

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\left(a+bx^3\right)^7}{24b^3}-\frac{2a\sqrt{a^2+2abx^3+b^2x^6}\left(a+bx^3\right)^6}{21b^3}+\frac{a^2\sqrt{a^2+2abx^3+b^2x^6}\left(a+bx^3\right)^5}{18b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^8*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
```

```
[Out] (a^2*(a + b*x^3)^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(18*b^3) - (2*a*(a + b*
x^3)^6*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(21*b^3) + ((a + b*x^3)^7*Sqrt[a^2
+ 2*a*b*x^3 + b^2*x^6])/(24*b^3)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\int x^8\left(a^2+2abx^3+b^2x^6\right)^{5/2} dx=\frac{\sqrt{a^2+2abx^3+b^2x^6}\int x^8\left(ab+b^2x^3\right)^5 dx}{b^4\left(ab+b^2x^3\right)}$$

$$=\frac{\sqrt{a^2+2abx^3+b^2x^6}\,\text{Subst}\left(\int x^2\left(ab+b^2x\right)^5 dx,x,x^3\right)}{3b^4\left(ab+b^2x^3\right)}$$

$$=\frac{\sqrt{a^2+2abx^3+b^2x^6}\,\text{Subst}\left(\int\left(\frac{a^2\left(ab+b^2x\right)^5}{b^2}-\frac{2a\left(ab+b^2x\right)^6}{b^3}+\frac{\left(ab+b^2x\right)^7}{b^4}\right)dx,x,x^3\right)}{3b^4\left(ab+b^2x^3\right)}$$

$$=\frac{a^2\left(a+bx^3\right)^5\sqrt{a^2+2abx^3+b^2x^6}}{18b^3}-\frac{2a\left(a+bx^3\right)^6\sqrt{a^2+2abx^3+b^2x^6}}{21b^3}+$$

Mathematica [A] time = 0.02, size = 83, normalized size = 0.70

$$\frac{x^9\sqrt{\left(a+bx^3\right)^2}\left(56a^5+210a^4bx^3+336a^3b^2x^6+280a^2b^3x^9+120ab^4x^{12}+21b^5x^{15}\right)}{504\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
```

```
[Out] (x^9*Sqrt[(a + b*x^3)^2]*(56*a^5 + 210*a^4*b*x^3 + 336*a^3*b^2*x^6 + 280*a^
2*b^3*x^9 + 120*a*b^4*x^12 + 21*b^5*x^15))/(504*(a + b*x^3))
```

IntegrateAlgebraic [A] time = 14.51, size = 83, normalized size = 0.70

$$\frac{x^9\sqrt{\left(a+bx^3\right)^2}\left(56a^5+210a^4bx^3+336a^3b^2x^6+280a^2b^3x^9+120ab^4x^{12}+21b^5x^{15}\right)}{504\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^8*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
```

```
[Out] (x^9*Sqrt[(a + b*x^3)^2]*(56*a^5 + 210*a^4*b*x^3 + 336*a^3*b^2*x^6 + 280*a^
2*b^3*x^9 + 120*a*b^4*x^12 + 21*b^5*x^15))/(504*(a + b*x^3))
```

fricas [A] time = 1.15, size = 57, normalized size = 0.48

$$\frac{1}{24}b^5x^{24}+\frac{5}{21}ab^4x^{21}+\frac{5}{9}a^2b^3x^{18}+\frac{2}{3}a^3b^2x^{15}+\frac{5}{12}a^4bx^{12}+\frac{1}{9}a^5x^9$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/24*b^5*x^24 + 5/21*a*b^4*x^21 + 5/9*a^2*b^3*x^18 + 2/3*a^3*b^2*x^15 + 5/1
2*a^4*b*x^12 + 1/9*a^5*x^9
```

giac [A] time = 0.38, size = 105, normalized size = 0.88

$$\frac{1}{24}b^5x^{24}\mathrm{sgn}\left(bx^3+a\right)+\frac{5}{21}ab^4x^{21}\mathrm{sgn}\left(bx^3+a\right)+\frac{5}{9}a^2b^3x^{18}\mathrm{sgn}\left(bx^3+a\right)+\frac{2}{3}a^3b^2x^{15}\mathrm{sgn}\left(bx^3+a\right)+\frac{5}{12}a^4bx^{12}\mathrm{sgn}\left(bx^3+a\right)+\frac{1}{9}a^5x^9\mathrm{sgn}\left(bx^3+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="giac")
```

```
[Out] 1/24*b^5*x^24*sgn(b*x^3 + a) + 5/21*a*b^4*x^21*sgn(b*x^3 + a) + 5/9*a^2*b^3
*x^18*sgn(b*x^3 + a) + 2/3*a^3*b^2*x^15*sgn(b*x^3 + a) + 5/12*a^4*b*x^12*sg
n(b*x^3 + a) + 1/9*a^5*x^9*sgn(b*x^3 + a)
```

maple [A] time = 0.01, size = 80, normalized size = 0.67

$$\frac{\left(21b^5x^{15}+120a\,b^4x^{12}+280a^2b^3x^9+336a^3b^2x^6+210a^4b\,x^3+56a^5\right)\left(\left(b\,x^3+a\right)^2\right)^{\frac{5}{2}}x^9}{504\left(b\,x^3+a\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x)
```

```
[Out] 1/504*x^9*(21*b^5*x^15+120*a*b^4*x^12+280*a^2*b^3*x^9+336*a^3*b^2*x^6+210*a
^4*b*x^3+56*a^5)*((b*x^3+a)^2)^(5/2)/(b*x^3+a)^5
```

maxima [A] time = 0.67, size = 114, normalized size = 0.96

$$\frac{\left(b^2x^6+2\,abx^3+a^2\right)^{\frac{5}{2}}a^2x^3}{18\,b^2}+\frac{\left(b^2x^6+2\,abx^3+a^2\right)^{\frac{7}{2}}x^3}{24\,b^2}+\frac{\left(b^2x^6+2\,abx^3+a^2\right)^{\frac{5}{2}}a^3}{18\,b^3}-\frac{3\left(b^2x^6+2\,abx^3+a^2\right)^{\frac{7}{2}}a}{56\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/18*(b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2)*a^2*x^3/b^2 + 1/24*(b^2*x^6 + 2*a*b*
x^3 + a^2)^(7/2)*x^3/b^2 + 1/18*(b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2)*a^3/b^3 -
 3/56*(b^2*x^6 + 2*a*b*x^3 + a^2)^(7/2)*a/b^3
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^8\left(a^2+2\,a\,b\,x^3+b^2\,x^6\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*(a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2),x)
```

```
[Out] int(x^8*(a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^8\left(\left(a+bx^3\right)^2\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8*(b**2*x**6+2*a*b*x**3+a**2)**(5/2),x)
```

```
[Out] Integral(x**8*((a + b*x**3)**2)**(5/2), x)
```

3.56 $\int x^7 \left(a^2+2abx^3+b^2x^6\right)^{5/2} dx$

Optimal. Leaf size=255

$$\frac{b^5x^{23}\sqrt{a^2+2abx^3+b^2x^6}}{23\left(a+bx^3\right)}+\frac{ab^4x^{20}\sqrt{a^2+2abx^3+b^2x^6}}{4\left(a+bx^3\right)}+\frac{10a^2b^3x^{17}\sqrt{a^2+2abx^3+b^2x^6}}{17\left(a+bx^3\right)}+\frac{a^5x^8\sqrt{a^2+2abx^3+b^2x^6}}{8\left(a+bx^3\right)}$$

Rubi [A] time = 0.06, antiderivative size = 255, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1355, 270}

$$\frac{b^5x^{23}\sqrt{a^2+2abx^3+b^2x^6}}{23\left(a+bx^3\right)}+\frac{ab^4x^{20}\sqrt{a^2+2abx^3+b^2x^6}}{4\left(a+bx^3\right)}+\frac{10a^2b^3x^{17}\sqrt{a^2+2abx^3+b^2x^6}}{17\left(a+bx^3\right)}+\frac{5a^3b^2x^{14}\sqrt{a^2+2abx^3+b^2x^6}}{7\left(a+bx^3\right)}+\frac{5a^4bx^{11}\sqrt{a^2+2abx^3+b^2x^6}}{11\left(a+bx^3\right)}+\frac{a^5x^8\sqrt{a^2+2abx^3+b^2x^6}}{8\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x^7*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
```

```
[Out] (a^5*x^8*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(8*(a + b*x^3)) + (5*a^4*b*x^11*S
qrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(11*(a + b*x^3)) + (5*a^3*b^2*x^14*Sqrt[a^2
 + 2*a*b*x^3 + b^2*x^6])/(7*(a + b*x^3)) + (10*a^2*b^3*x^17*Sqrt[a^2 + 2*a*
b*x^3 + b^2*x^6])/(17*(a + b*x^3)) + (a*b^4*x^20*Sqrt[a^2 + 2*a*b*x^3 + b^2
*x^6])/(4*(a + b*x^3)) + (b^5*x^23*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(23*(a
+ b*x^3))
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\begin{aligned}\int x^7\left(a^2+2abx^3+b^2x^6\right)^{5/2}dx &= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int x^7\left(ab+b^2x^3\right)^5 dx}{b^4\left(ab+b^2x^3\right)}\\ &= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int\left(a^5b^5x^7+5a^4b^6x^{10}+10a^3b^7x^{13}+10a^2b^8x^{16}+5ab^9x^{19}\right.}{b^4\left(ab+b^2x^3\right)}\\ &= \frac{a^5x^8\sqrt{a^2+2abx^3+b^2x^6}}{8\left(a+bx^3\right)}+\frac{5a^4bx^{11}\sqrt{a^2+2abx^3+b^2x^6}}{11\left(a+bx^3\right)}+\frac{5a^3b^2x^{14}\sqrt{a^2+2ab}}{7\left(a+bx^3\right.}\end{aligned}$$

Mathematica [A] time = 0.02, size = 83, normalized size = 0.33

$$\frac{x^8\sqrt{\left(a+bx^3\right)^2}\left(30107a^5+109480a^4bx^3+172040a^3b^2x^6+141680a^2b^3x^9+60214ab^4x^{12}+10472b^5x^{15}\right)}{240856\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
```

```
[Out] (x^8*Sqrt[(a + b*x^3)^2]*(30107*a^5 + 109480*a^4*b*x^3 + 172040*a^3*b^2*x^6
 + 141680*a^2*b^3*x^9 + 60214*a*b^4*x^12 + 10472*b^5*x^15))/(240856*(a + b*
x^3))
```

IntegrateAlgebraic [A] time = 13.72, size = 83, normalized size = 0.33

$$\frac{\sqrt{\left(a+bx^3\right)^2}\left(30107a^5x^8+109480a^4bx^{11}+172040a^3b^2x^{14}+141680a^2b^3x^{17}+60214ab^4x^{20}+10472b^5x^{23}\right)}{240856\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^7*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
```

```
[Out] (Sqrt[(a + b*x^3)^2]*(30107*a^5*x^8 + 109480*a^4*b*x^11 + 172040*a^3*b^2*x^
14 + 141680*a^2*b^3*x^17 + 60214*a*b^4*x^20 + 10472*b^5*x^23))/(240856*(a +
 b*x^3))
```

fricas [A] time = 1.14, size = 57, normalized size = 0.22

$$\frac{1}{23}\,b^5x^{23}+\frac{1}{4}\,ab^4x^{20}+\frac{10}{17}\,a^2b^3x^{17}+\frac{5}{7}\,a^3b^2x^{14}+\frac{5}{11}\,a^4bx^{11}+\frac{1}{8}\,a^5x^8$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/23*b^5*x^23 + 1/4*a*b^4*x^20 + 10/17*a^2*b^3*x^17 + 5/7*a^3*b^2*x^14 + 5/
11*a^4*b*x^11 + 1/8*a^5*x^8
```

giac [A] time = 0.34, size = 105, normalized size = 0.41

$$\frac{1}{23}\,b^5x^{23}\mathrm{sgn}\left(bx^3+a\right)+\frac{1}{4}\,ab^4x^{20}\mathrm{sgn}\left(bx^3+a\right)+\frac{10}{17}\,a^2b^3x^{17}\mathrm{sgn}\left(bx^3+a\right)+\frac{5}{7}\,a^3b^2x^{14}\mathrm{sgn}\left(bx^3+a\right)+\frac{5}{11}\,a^4bx^{11}\mathrm{sgn}\left(bx^3+a\right)+\frac{1}{8}\,a^5x^8\mathrm{sgn}\left(bx^3+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="giac")
```

```
[Out] 1/23*b^5*x^23*sgn(b*x^3 + a) + 1/4*a*b^4*x^20*sgn(b*x^3 + a) + 10/17*a^2*b^
3*x^17*sgn(b*x^3 + a) + 5/7*a^3*b^2*x^14*sgn(b*x^3 + a) + 5/11*a^4*b*x^11*s
gn(b*x^3 + a) + 1/8*a^5*x^8*sgn(b*x^3 + a)
```

maple [A] time = 0.01, size = 80, normalized size = 0.31

$$\frac{\left(10472b^5x^{15}+60214a\,b^4x^{12}+141680a^2b^3x^9+172040a^3b^2x^6+109480a^4b\,x^3+30107a^5\right)\left(\left(b\,x^3+a\right)^2\right)^{\frac{5}{2}}x^8}{240856\left(b\,x^3+a\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x)
```

```
[Out] 1/240856*x^8*(10472*b^5*x^15+60214*a*b^4*x^12+141680*a^2*b^3*x^9+172040*a^3
*b^2*x^6+109480*a^4*b*x^3+30107*a^5)*((b*x^3+a)^2)^(5/2)/(b*x^3+a)^5
```

maxima [A] time = 0.77, size = 57, normalized size = 0.22

$$\frac{1}{23}\,b^5x^{23}+\frac{1}{4}\,ab^4x^{20}+\frac{10}{17}\,a^2b^3x^{17}+\frac{5}{7}\,a^3b^2x^{14}+\frac{5}{11}\,a^4bx^{11}+\frac{1}{8}\,a^5x^8$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/23*b^5*x^23 + 1/4*a*b^4*x^20 + 10/17*a^2*b^3*x^17 + 5/7*a^3*b^2*x^14 + 5/
11*a^4*b*x^11 + 1/8*a^5*x^8
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^7\left(a^2+2\,a\,b\,x^3+b^2\,x^6\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2),x)
```

```
[Out] int(x^7*(a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^7\left(\left(a+bx^3\right)^2\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7*(b**2*x**6+2*a*b*x**3+a**2)**(5/2),x)
```

```
[Out] Integral(x**7*((a + b*x**3)**2)**(5/2), x)
```

3.57 $\int x^6 \left(a^2+2abx^3+b^2x^6\right)^{5/2} dx$

Optimal. Leaf size=255

$$\frac{b^5x^{22}\sqrt{a^2+2abx^3+b^2x^6}}{22\left(a+bx^3\right)}+\frac{5ab^4x^{19}\sqrt{a^2+2abx^3+b^2x^6}}{19\left(a+bx^3\right)}+\frac{5a^2b^3x^{16}\sqrt{a^2+2abx^3+b^2x^6}}{8\left(a+bx^3\right)}+\frac{a^5x^7\sqrt{a^2+2abx^3+}}{7\left(a+bx^3\right)}$$

Rubi [A] time = 0.06, antiderivative size = 255, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1355, 270}

$$\frac{b^5x^{22}\sqrt{a^2+2abx^3+b^2x^6}}{22\left(a+bx^3\right)}+\frac{5ab^4x^{19}\sqrt{a^2+2abx^3+b^2x^6}}{19\left(a+bx^3\right)}+\frac{5a^2b^3x^{16}\sqrt{a^2+2abx^3+b^2x^6}}{8\left(a+bx^3\right)}+\frac{10a^3b^2x^{13}\sqrt{a^2+2abx^3+b^2x^6}}{13\left(a+bx^3\right)}+\frac{a^4bx^{10}\sqrt{a^2+2abx^3+b^2x^6}}{2\left(a+bx^3\right)}+\frac{a^5x^7\sqrt{a^2+2abx^3+b^2x^6}}{7\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x^6*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
```

```
[Out] (a^5*x^7*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(7*(a + b*x^3)) + (a^4*b*x^10*Sqr
t[a^2 + 2*a*b*x^3 + b^2*x^6])/(2*(a + b*x^3)) + (10*a^3*b^2*x^13*Sqrt[a^2 +
 2*a*b*x^3 + b^2*x^6])/(13*(a + b*x^3)) + (5*a^2*b^3*x^16*Sqrt[a^2 + 2*a*b*
x^3 + b^2*x^6])/(8*(a + b*x^3)) + (5*a*b^4*x^19*Sqrt[a^2 + 2*a*b*x^3 + b^2*
x^6])/(19*(a + b*x^3)) + (b^5*x^22*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(22*(a
+ b*x^3))
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\begin{aligned}
\int x^6\left(a^2+2abx^3+b^2x^6\right)^{5/2} dx &= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int x^6\left(ab+b^2x^3\right)^5 dx}{b^4\left(ab+b^2x^3\right)}\\
&= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int\left(a^5b^5x^6+5a^4b^6x^9+10a^3b^7x^{12}+10a^2b^8x^{15}+5ab^9x\right.}{b^4\left(ab+b^2x^3\right)}\\
&= \frac{a^5x^7\sqrt{a^2+2abx^3+b^2x^6}}{7\left(a+bx^3\right)}+\frac{a^4bx^{10}\sqrt{a^2+2abx^3+b^2x^6}}{2\left(a+bx^3\right)}+\frac{10a^3b^2x^{13}\sqrt{a^2+}}{13\left(a+\right.}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 83, normalized size = 0.33

$$\frac{x^7\sqrt{\left(a+bx^3\right)^2}\left(21736a^5+76076a^4bx^3+117040a^3b^2x^6+95095a^2b^3x^9+40040ab^4x^{12}+6916b^5x^{15}\right)}{152152\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
```

```
[Out] (x^7*Sqrt[(a + b*x^3)^2]*(21736*a^5 + 76076*a^4*b*x^3 + 117040*a^3*b^2*x^6
+ 95095*a^2*b^3*x^9 + 40040*a*b^4*x^12 + 6916*b^5*x^15))/(152152*(a + b*x^3
))
```

IntegrateAlgebraic [A] time = 12.98, size = 83, normalized size = 0.33

$$\frac{\sqrt{\left(a+bx^3\right)^2}\left(21736a^5x^7+76076a^4bx^{10}+117040a^3b^2x^{13}+95095a^2b^3x^{16}+40040ab^4x^{19}+6916b^5x^{22}\right)}{152152\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^6*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
```

```
[Out] (Sqrt[(a + b*x^3)^2]*(21736*a^5*x^7 + 76076*a^4*b*x^10 + 117040*a^3*b^2*x^1
3 + 95095*a^2*b^3*x^16 + 40040*a*b^4*x^19 + 6916*b^5*x^22))/(152152*(a + b*
x^3))
```

fricas [A] time = 1.05, size = 57, normalized size = 0.22

$$\frac{1}{22}\,b^5x^{22}+\frac{5}{19}\,ab^4x^{19}+\frac{5}{8}\,a^2b^3x^{16}+\frac{10}{13}\,a^3b^2x^{13}+\frac{1}{2}\,a^4bx^{10}+\frac{1}{7}\,a^5x^7$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/22*b^5*x^22 + 5/19*a*b^4*x^19 + 5/8*a^2*b^3*x^16 + 10/13*a^3*b^2*x^13 + 1
/2*a^4*b*x^10 + 1/7*a^5*x^7
```

giac [A] time = 0.34, size = 105, normalized size = 0.41

$$\frac{1}{22}\,b^5x^{22}\mathrm{sgn}\left(bx^3+a\right)+\frac{5}{19}\,ab^4x^{19}\mathrm{sgn}\left(bx^3+a\right)+\frac{5}{8}\,a^2b^3x^{16}\mathrm{sgn}\left(bx^3+a\right)+\frac{10}{13}\,a^3b^2x^{13}\mathrm{sgn}\left(bx^3+a\right)+\frac{1}{2}\,a^4bx^{10}\mathrm{sgn}\left(bx^3+a\right)+\frac{1}{7}\,a^5x^7\mathrm{sgn}\left(bx^3+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="giac")
```

```
[Out] 1/22*b^5*x^22*sgn(b*x^3 + a) + 5/19*a*b^4*x^19*sgn(b*x^3 + a) + 5/8*a^2*b^3
*x^16*sgn(b*x^3 + a) + 10/13*a^3*b^2*x^13*sgn(b*x^3 + a) + 1/2*a^4*b*x^10*s
gn(b*x^3 + a) + 1/7*a^5*x^7*sgn(b*x^3 + a)
```

maple [A] time = 0.01, size = 80, normalized size = 0.31

$$\frac{\left(6916b^5x^{15}+40040a\,b^4x^{12}+95095a^2b^3x^9+117040a^3b^2x^6+76076a^4b\,x^3+21736a^5\right)\left(\left(b\,x^3+a\right)^2\right)^{\frac{5}{2}}x^7}{152152\left(b\,x^3+a\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x)
```

```
[Out] 1/152152*x^7*(6916*b^5*x^15+40040*a*b^4*x^12+95095*a^2*b^3*x^9+117040*a^3*b
^2*x^6+76076*a^4*b*x^3+21736*a^5)*((b*x^3+a)^2)^(5/2)/(b*x^3+a)^5
```

maxima [A] time = 0.71, size = 57, normalized size = 0.22

$$\frac{1}{22}\,b^5x^{22}+\frac{5}{19}\,ab^4x^{19}+\frac{5}{8}\,a^2b^3x^{16}+\frac{10}{13}\,a^3b^2x^{13}+\frac{1}{2}\,a^4bx^{10}+\frac{1}{7}\,a^5x^7$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/22*b^5*x^22 + 5/19*a*b^4*x^19 + 5/8*a^2*b^3*x^16 + 10/13*a^3*b^2*x^13 + 1
/2*a^4*b*x^10 + 1/7*a^5*x^7
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^6 \left(a^2 + 2\,a\,b\,x^3 + b^2\,x^6\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2),x)
```

```
[Out] int(x^6*(a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^6 \left(\left(a + bx^3\right)^2\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6*(b**2*x**6+2*a*b*x**3+a**2)**(5/2),x)
```

```
[Out] Integral(x**6*((a + b*x**3)**2)**(5/2), x)
```

3.58 $\int x^5 \left(a^2+2abx^3+b^2x^6\right)^{5/2} dx$

Optimal. Leaf size=78

$$\frac{\left(a+bx^3\right)^6 \sqrt{a^2+2abx^3+b^2x^6}}{21b^2}-\frac{a\left(a+bx^3\right)^5 \sqrt{a^2+2abx^3+b^2x^6}}{18b^2}$$

Rubi [A] time = 0.06, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1355, 266, 43}

$$\frac{\left(a+bx^3\right)^6 \sqrt{a^2+2abx^3+b^2x^6}}{21b^2}-\frac{a\left(a+bx^3\right)^5 \sqrt{a^2+2abx^3+b^2x^6}}{18b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
```

```
[Out] -(a*(a + b*x^3)^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(18*b^2) + ((a + b*x^3)^
6*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(21*b^2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\begin{aligned}
\int x^5 \left(a^2+2abx^3+b^2x^6\right)^{5/2} dx &= \frac{\sqrt{a^2+2abx^3+b^2x^6} \int x^5 \left(ab+b^2x^3\right)^5 dx}{b^4\left(ab+b^2x^3\right)}\\
&= \frac{\sqrt{a^2+2abx^3+b^2x^6}\ \text{Subst}\left(\int x\left(ab+b^2x\right)^5 dx, x, x^3\right)}{3b^4\left(ab+b^2x^3\right)}\\
&= \frac{\sqrt{a^2+2abx^3+b^2x^6}\ \text{Subst}\left(\int \left(-\frac{a\left(ab+b^2x\right)^5}{b}+\frac{\left(ab+b^2x\right)^6}{b^2}\right) dx, x, x^3\right)}{3b^4\left(ab+b^2x^3\right)}\\
&= -\frac{a\left(a+bx^3\right)^5 \sqrt{a^2+2abx^3+b^2x^6}}{18b^2}+\frac{\left(a+bx^3\right)^6 \sqrt{a^2+2abx^3+b^2x^6}}{21b^2}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 83, normalized size = 1.06

$$\frac{x^6\sqrt{(a+bx^3)^2}\left(21a^5+70a^4bx^3+105a^3b^2x^6+84a^2b^3x^9+35ab^4x^{12}+6b^5x^{15}\right)}{126(a+bx^3)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
```

```
[Out] (x^6*Sqrt[(a + b*x^3)^2]*(21*a^5 + 70*a^4*b*x^3 + 105*a^3*b^2*x^6 + 84*a^2*
b^3*x^9 + 35*a*b^4*x^12 + 6*b^5*x^15))/(126*(a + b*x^3))
```

IntegrateAlgebraic [A] time = 11.58, size = 83, normalized size = 1.06

$$\frac{x^6\sqrt{(a+bx^3)^2}\left(21a^5+70a^4bx^3+105a^3b^2x^6+84a^2b^3x^9+35ab^4x^{12}+6b^5x^{15}\right)}{126(a+bx^3)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^5*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
```

```
[Out] (x^6*Sqrt[(a + b*x^3)^2]*(21*a^5 + 70*a^4*b*x^3 + 105*a^3*b^2*x^6 + 84*a^2*
b^3*x^9 + 35*a*b^4*x^12 + 6*b^5*x^15))/(126*(a + b*x^3))
```

fricas [A] time = 1.13, size = 57, normalized size = 0.73

$$\frac{1}{21}b^5x^{21}+\frac{5}{18}ab^4x^{18}+\frac{2}{3}a^2b^3x^{15}+\frac{5}{6}a^3b^2x^{12}+\frac{5}{9}a^4bx^9+\frac{1}{6}a^5x^6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/21*b^5*x^21 + 5/18*a*b^4*x^18 + 2/3*a^2*b^3*x^15 + 5/6*a^3*b^2*x^12 + 5/9
*a^4*b*x^9 + 1/6*a^5*x^6
```

giac [A] time = 0.29, size = 67, normalized size = 0.86

$$\frac{1}{126}\left(6\,b^5x^{21}+35\,ab^4x^{18}+84\,a^2b^3x^{15}+105\,a^3b^2x^{12}+70\,a^4bx^9+21\,a^5x^6\right)\mathrm{sgn}\left(bx^3+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="giac")
```

```
[Out] 1/126*(6*b^5*x^21 + 35*a*b^4*x^18 + 84*a^2*b^3*x^15 + 105*a^3*b^2*x^12 + 70
*a^4*b*x^9 + 21*a^5*x^6)*sgn(b*x^3 + a)
```

maple [A] time = 0.01, size = 80, normalized size = 1.03

$$\frac{\left(6b^5x^{15}+35a\,b^4x^{12}+84a^2b^3x^9+105a^3b^2x^6+70a^4b\,x^3+21a^5\right)\left(\left(b\,x^3+a\right)^2\right)^{\frac{5}{2}}x^6}{126\left(b\,x^3+a\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x)
```

```
[Out] 1/126*x^6*(6*b^5*x^15+35*a*b^4*x^12+84*a^2*b^3*x^9+105*a^3*b^2*x^6+70*a^4*b
*x^3+21*a^5)*((b*x^3+a)^2)^(5/2)/(b*x^3+a)^5
```

maxima [A] time = 0.63, size = 83, normalized size = 1.06

$$-\frac{\left(b^{2}x^{6}+2\,abx^{3}+a^{2}\right)^{\frac{5}{2}}ax^{3}}{18\,b}-\frac{\left(b^{2}x^{6}+2\,abx^{3}+a^{2}\right)^{\frac{5}{2}}a^{2}}{18\,b^{2}}+\frac{\left(b^{2}x^{6}+2\,abx^{3}+a^{2}\right)^{\frac{7}{2}}}{21\,b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/18*(b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2)*a*x^3/b - 1/18*(b^2*x^6 + 2*a*b*x^3
 + a^2)^(5/2)*a^2/b^2 + 1/21*(b^2*x^6 + 2*a*b*x^3 + a^2)^(7/2)/b^2
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^5\,{\left(a^2+2\,a\,b\,x^3+b^2\,x^6\right)}^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2),x)
```

```
[Out] int(x^5*(a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^{5}\left(\left(a+bx^{3}\right)^{2}\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(b**2*x**6+2*a*b*x**3+a**2)**(5/2),x)
```

```
[Out] Integral(x**5*((a + b*x**3)**2)**(5/2), x)
```

3.59 $\int x^4 \left(a^2+2abx^3+b^2x^6\right)^{5/2}\,dx$

Optimal. Leaf size=255

$$\frac{b^5x^{20}\sqrt{a^2+2abx^3+b^2x^6}}{20\left(a+bx^3\right)}+\frac{5ab^4x^{17}\sqrt{a^2+2abx^3+b^2x^6}}{17\left(a+bx^3\right)}+\frac{5a^2b^3x^{14}\sqrt{a^2+2abx^3+b^2x^6}}{7\left(a+bx^3\right)}+\frac{a^5x^5\sqrt{a^2+2abx^3+}}{5\left(a+bx^3\right)}$$

Rubi [A] time = 0.06, antiderivative size = 255, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}=0.077$, Rules used = {1355, 270}

$$\frac{b^5x^{20}\sqrt{a^2+2abx^3+b^2x^6}}{20\left(a+bx^3\right)}+\frac{5ab^4x^{17}\sqrt{a^2+2abx^3+b^2x^6}}{17\left(a+bx^3\right)}+\frac{5a^2b^3x^{14}\sqrt{a^2+2abx^3+b^2x^6}}{7\left(a+bx^3\right)}+\frac{10a^3b^2x^{11}\sqrt{a^2+2abx^3+b^2x^6}}{11\left(a+bx^3\right)}+\frac{5a^4bx^8\sqrt{a^2+2abx^3+b^2x^6}}{8\left(a+bx^3\right)}+\frac{a^5x^5\sqrt{a^2+2abx^3+b^2x^6}}{5\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
```

```
[Out] (a^5*x^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(5*(a + b*x^3)) + (5*a^4*b*x^8*Sq
rt[a^2 + 2*a*b*x^3 + b^2*x^6])/(8*(a + b*x^3)) + (10*a^3*b^2*x^11*Sqrt[a^2
+ 2*a*b*x^3 + b^2*x^6])/(11*(a + b*x^3)) + (5*a^2*b^3*x^14*Sqrt[a^2 + 2*a*b
*x^3 + b^2*x^6])/(7*(a + b*x^3)) + (5*a*b^4*x^17*Sqrt[a^2 + 2*a*b*x^3 + b^2
*x^6])/(17*(a + b*x^3)) + (b^5*x^20*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(20*(a
 + b*x^3))
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\begin{aligned}\int x^4\left(a^2+2abx^3+b^2x^6\right)^{5/2}\,dx&=\frac{\sqrt{a^2+2abx^3+b^2x^6}\int x^4\left(ab+b^2x^3\right)^5\,dx}{b^4\left(ab+b^2x^3\right)}\\&=\frac{\sqrt{a^2+2abx^3+b^2x^6}\int\left(a^5b^5x^4+5a^4b^6x^7+10a^3b^7x^{10}+10a^2b^8x^{13}+5ab^9x\right.}{b^4\left(ab+b^2x^3\right)}\\&=\frac{a^5x^5\sqrt{a^2+2abx^3+b^2x^6}}{5\left(a+bx^3\right)}+\frac{5a^4bx^8\sqrt{a^2+2abx^3+b^2x^6}}{8\left(a+bx^3\right)}+\frac{10a^3b^2x^{11}\sqrt{a^2+}}{11\left(a+\right.}\end{aligned}$$

Mathematica [A] time = 0.02, size = 83, normalized size = 0.33

$$\frac{x^5\sqrt{\left(a+bx^3\right)^2}\left(10472a^5+32725a^4bx^3+47600a^3b^2x^6+37400a^2b^3x^9+15400ab^4x^{12}+2618b^5x^{15}\right)}{52360\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
```

```
[Out] (x^5*Sqrt[(a + b*x^3)^2]*(10472*a^5 + 32725*a^4*b*x^3 + 47600*a^3*b^2*x^6 +
 37400*a^2*b^3*x^9 + 15400*a*b^4*x^12 + 2618*b^5*x^15))/(52360*(a + b*x^3))
```

IntegrateAlgebraic [A] time = 10.54, size = 83, normalized size = 0.33

$$\frac{\sqrt{\left(a+bx^3\right)^2}\left(10472a^5x^5+32725a^4bx^8+47600a^3b^2x^{11}+37400a^2b^3x^{14}+15400ab^4x^{17}+2618b^5x^{20}\right)}{52360\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^4*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
```

```
[Out] (Sqrt[(a + b*x^3)^2]*(10472*a^5*x^5 + 32725*a^4*b*x^8 + 47600*a^3*b^2*x^11
+ 37400*a^2*b^3*x^14 + 15400*a*b^4*x^17 + 2618*b^5*x^20))/(52360*(a + b*x^3
))
```

fricas [A] time = 1.33, size = 57, normalized size = 0.22

$$\frac{1}{20}\,b^5x^{20}+\frac{5}{17}\,ab^4x^{17}+\frac{5}{7}\,a^2b^3x^{14}+\frac{10}{11}\,a^3b^2x^{11}+\frac{5}{8}\,a^4bx^8+\frac{1}{5}\,a^5x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/20*b^5*x^20 + 5/17*a*b^4*x^17 + 5/7*a^2*b^3*x^14 + 10/11*a^3*b^2*x^11 + 5
/8*a^4*b*x^8 + 1/5*a^5*x^5
```

giac [A] time = 0.42, size = 105, normalized size = 0.41

$$\frac{1}{20}\,b^5x^{20}\mathrm{sgn}\left(bx^3+a\right)+\frac{5}{17}\,ab^4x^{17}\mathrm{sgn}\left(bx^3+a\right)+\frac{5}{7}\,a^2b^3x^{14}\mathrm{sgn}\left(bx^3+a\right)+\frac{10}{11}\,a^3b^2x^{11}\mathrm{sgn}\left(bx^3+a\right)+\frac{5}{8}\,a^4bx^8\mathrm{sgn}\left(bx^3+a\right)+\frac{1}{5}\,a^5x^5\mathrm{sgn}\left(bx^3+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="giac")
```

```
[Out] 1/20*b^5*x^20*sgn(b*x^3 + a) + 5/17*a*b^4*x^17*sgn(b*x^3 + a) + 5/7*a^2*b^3
*x^14*sgn(b*x^3 + a) + 10/11*a^3*b^2*x^11*sgn(b*x^3 + a) + 5/8*a^4*b*x^8*sg
n(b*x^3 + a) + 1/5*a^5*x^5*sgn(b*x^3 + a)
```

maple [A] time = 0.01, size = 80, normalized size = 0.31

$$\frac{\left(2618b^5x^{15}+15400a\,b^4x^{12}+37400a^2b^3x^9+47600a^3b^2x^6+32725a^4b\,x^3+10472a^5\right)\left(\left(b\,x^3+a\right)^2\right)^{\frac{5}{2}}x^5}{52360\left(b\,x^3+a\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x)
```

```
[Out] 1/52360*x^5*(2618*b^5*x^15+15400*a*b^4*x^12+37400*a^2*b^3*x^9+47600*a^3*b^2
*x^6+32725*a^4*b*x^3+10472*a^5)*((b*x^3+a)^2)^(5/2)/(b*x^3+a)^5
```

maxima [A] time = 0.69, size = 57, normalized size = 0.22

$$\frac{1}{20}\,b^5x^{20}+\frac{5}{17}\,ab^4x^{17}+\frac{5}{7}\,a^2b^3x^{14}+\frac{10}{11}\,a^3b^2x^{11}+\frac{5}{8}\,a^4bx^8+\frac{1}{5}\,a^5x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/20*b^5*x^20 + 5/17*a*b^4*x^17 + 5/7*a^2*b^3*x^14 + 10/11*a^3*b^2*x^11 + 5
/8*a^4*b*x^8 + 1/5*a^5*x^5
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^4 \left(a^2 + 2\,a\,b\,x^3 + b^2\,x^6\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2),x)
```

```
[Out] int(x^4*(a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^4 \left(\left(a + bx^3\right)^2\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(b**2*x**6+2*a*b*x**3+a**2)**(5/2),x)
```

```
[Out] Integral(x**4*((a + b*x**3)**2)**(5/2), x)
```

3.60 $\int x^3 \left(a^2+2abx^3+b^2x^6\right)^{5/2}\,dx$

Optimal. Leaf size=252

$$\frac{b^5x^{19}\sqrt{a^2+2abx^3+b^2x^6}}{19\left(a+bx^3\right)}+\frac{5ab^4x^{16}\sqrt{a^2+2abx^3+b^2x^6}}{16\left(a+bx^3\right)}+\frac{10a^2b^3x^{13}\sqrt{a^2+2abx^3+b^2x^6}}{13\left(a+bx^3\right)}+\frac{a^5x^4\sqrt{a^2+2abx^3+b^2}}{4\left(a+bx^3\right)}$$

Rubi [A] time = 0.06, antiderivative size = 252, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1355, 270}

$$\frac{b^5x^{19}\sqrt{a^2+2abx^3+b^2x^6}}{19\left(a+bx^3\right)}+\frac{5ab^4x^{16}\sqrt{a^2+2abx^3+b^2x^6}}{16\left(a+bx^3\right)}+\frac{10a^2b^3x^{13}\sqrt{a^2+2abx^3+b^2x^6}}{13\left(a+bx^3\right)}+\frac{a^3b^2x^{10}\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}+\frac{5a^4bx^7\sqrt{a^2+2abx^3+b^2x^6}}{7\left(a+bx^3\right)}+\frac{a^5x^4\sqrt{a^2+2abx^3+b^2x^6}}{4\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
```

```
[Out] (a^5*x^4*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(4*(a + b*x^3)) + (5*a^4*b*x^7*Sq
rt[a^2 + 2*a*b*x^3 + b^2*x^6])/(7*(a + b*x^3)) + (a^3*b^2*x^10*Sqrt[a^2 + 2
*a*b*x^3 + b^2*x^6])/(a + b*x^3) + (10*a^2*b^3*x^13*Sqrt[a^2 + 2*a*b*x^3 +
b^2*x^6])/(13*(a + b*x^3)) + (5*a*b^4*x^16*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])
/(16*(a + b*x^3)) + (b^5*x^19*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(19*(a + b*x
^3))
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\begin{aligned}
\int x^3\left(a^2+2abx^3+b^2x^6\right)^{5/2}\,dx &= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int x^3\left(ab+b^2x^3\right)^5\,dx}{b^4\left(ab+b^2x^3\right)}\\
&= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int\left(a^5b^5x^3+5a^4b^6x^6+10a^3b^7x^9+10a^2b^8x^{12}+5ab^9x^{15}+\right.}{b^4\left(ab+b^2x^3\right)}\\
&= \frac{a^5x^4\sqrt{a^2+2abx^3+b^2x^6}}{4\left(a+bx^3\right)}+\frac{5a^4bx^7\sqrt{a^2+2abx^3+b^2x^6}}{7\left(a+bx^3\right)}+\frac{a^3b^2x^{10}\sqrt{a^2+2abx}}{a+bx^3}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 83, normalized size = 0.33

$$\frac{x^4\sqrt{\left(a+bx^3\right)^2}\left(6916a^5+19760a^4bx^3+27664a^3b^2x^6+21280a^2b^3x^9+8645ab^4x^{12}+1456b^5x^{15}\right)}{27664\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
```

```
[Out] (x^4*Sqrt[(a + b*x^3)^2]*(6916*a^5 + 19760*a^4*b*x^3 + 27664*a^3*b^2*x^6 +
21280*a^2*b^3*x^9 + 8645*a*b^4*x^12 + 1456*b^5*x^15))/(27664*(a + b*x^3))
```

IntegrateAlgebraic [A] time = 10.38, size = 83, normalized size = 0.33

$$\frac{\sqrt{\left(a+bx^3\right)^2}\left(6916a^5x^4+19760a^4bx^7+27664a^3b^2x^{10}+21280a^2b^3x^{13}+8645ab^4x^{16}+1456b^5x^{19}\right)}{27664\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^3*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
```

```
[Out] (Sqrt[(a + b*x^3)^2]*(6916*a^5*x^4 + 19760*a^4*b*x^7 + 27664*a^3*b^2*x^10 +
 21280*a^2*b^3*x^13 + 8645*a*b^4*x^16 + 1456*b^5*x^19))/(27664*(a + b*x^3))
```

fricas [A] time = 1.16, size = 56, normalized size = 0.22

$$\frac{1}{19}\,b^5x^{19}+\frac{5}{16}\,ab^4x^{16}+\frac{10}{13}\,a^2b^3x^{13}+a^3b^2x^{10}+\frac{5}{7}\,a^4bx^7+\frac{1}{4}\,a^5x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/19*b^5*x^19 + 5/16*a*b^4*x^16 + 10/13*a^2*b^3*x^13 + a^3*b^2*x^10 + 5/7*a
^4*b*x^7 + 1/4*a^5*x^4
```

giac [A] time = 0.32, size = 104, normalized size = 0.41

$$\frac{1}{19}\,b^5x^{19}\mathrm{sgn}\left(bx^3+a\right)+\frac{5}{16}\,ab^4x^{16}\mathrm{sgn}\left(bx^3+a\right)+\frac{10}{13}\,a^2b^3x^{13}\mathrm{sgn}\left(bx^3+a\right)+a^3b^2x^{10}\mathrm{sgn}\left(bx^3+a\right)+\frac{5}{7}\,a^4bx^7\mathrm{sgn}\left(bx^3+a\right)+\frac{1}{4}\,a^5x^4\mathrm{sgn}\left(bx^3+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="giac")
```

```
[Out] 1/19*b^5*x^19*sgn(b*x^3 + a) + 5/16*a*b^4*x^16*sgn(b*x^3 + a) + 10/13*a^2*b
^3*x^13*sgn(b*x^3 + a) + a^3*b^2*x^10*sgn(b*x^3 + a) + 5/7*a^4*b*x^7*sgn(b*
x^3 + a) + 1/4*a^5*x^4*sgn(b*x^3 + a)
```

maple [A] time = 0.01, size = 80, normalized size = 0.32

$$\frac{\left(1456b^5x^{15}+8645a\,b^4x^{12}+21280a^2b^3x^9+27664a^3b^2x^6+19760a^4b\,x^3+6916a^5\right)\left(\left(b\,x^3+a\right)^2\right)^{\frac{5}{2}}x^4}{27664\left(b\,x^3+a\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x)
```

```
[Out] 1/27664*x^4*(1456*b^5*x^15+8645*a*b^4*x^12+21280*a^2*b^3*x^9+27664*a^3*b^2*
x^6+19760*a^4*b*x^3+6916*a^5)*((b*x^3+a)^2)^(5/2)/(b*x^3+a)^5
```

maxima [A] time = 0.51, size = 56, normalized size = 0.22

$$\frac{1}{19}\,b^5x^{19}+\frac{5}{16}\,ab^4x^{16}+\frac{10}{13}\,a^2b^3x^{13}+a^3b^2x^{10}+\frac{5}{7}\,a^4bx^7+\frac{1}{4}\,a^5x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/19*b^5*x^19 + 5/16*a*b^4*x^16 + 10/13*a^2*b^3*x^13 + a^3*b^2*x^10 + 5/7*a
^4*b*x^7 + 1/4*a^5*x^4
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^3 \left(a^2 + 2\,a\,b\,x^3 + b^2\,x^6\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2),x)
[Out] int(x^3*(a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^3 \left(\left(a + bx^3\right)^2\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b**2*x**6+2*a*b*x**3+a**2)**(5/2),x)
[Out] Integral(x**3*((a + b*x**3)**2)**(5/2), x)
```

3.61 $\int x^2 \left(a^2+2abx^3+b^2x^6\right)^{5/2} dx$

Optimal. Leaf size=36

$$\frac{\left(a+bx^3\right)\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{18b}$$

Rubi [A] time = 0.03, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1352, 609}

$$\frac{\left(a+bx^3\right)\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{18b}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
```

```
[Out] ((a + b*x^3)*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2))/(18*b)
```

Rule 609

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x)
*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] /; FreeQ[{a, b, c, p}, x] && EqQ[
b^2 - 4*a*c, 0] && NeQ[p, -2^(-1)]
```

Rule 1352

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a,
b, c, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

Rubi steps

$$\begin{aligned}\int x^2 \left(a^2+2abx^3+b^2x^6\right)^{5/2} dx &= \frac{1}{3}\operatorname{Subst}\left(\int \left(a^2+2abx+b^2x^2\right)^{5/2} dx, x, x^3\right)\\ &= \frac{\left(a+bx^3\right)\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{18b}\end{aligned}$$

Mathematica [B] time = 0.02, size = 82, normalized size = 2.28

$$\frac{x^3\sqrt{\left(a+bx^3\right)^2}\left(6a^5+15a^4bx^3+20a^3b^2x^6+15a^2b^3x^9+6ab^4x^{12}+b^5x^{15}\right)}{18\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
```

```
[Out] (x^3*Sqrt[(a + b*x^3)^2]*(6*a^5 + 15*a^4*b*x^3 + 20*a^3*b^2*x^6 + 15*a^2*b^
3*x^9 + 6*a*b^4*x^12 + b^5*x^15))/(18*(a + b*x^3))
```

IntegrateAlgebraic [B] time = 9.63, size = 82, normalized size = 2.28

$$\frac{x^3\sqrt{\left(a+bx^3\right)^2}\left(6a^5+15a^4bx^3+20a^3b^2x^6+15a^2b^3x^9+6ab^4x^{12}+b^5x^{15}\right)}{18\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^2*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
```

```
[Out] (x^3*Sqrt[(a + b*x^3)^2]*(6*a^5 + 15*a^4*b*x^3 + 20*a^3*b^2*x^6 + 15*a^2*b^
3*x^9 + 6*a*b^4*x^12 + b^5*x^15))/(18*(a + b*x^3))
```

fricas [A] time = 0.94, size = 57, normalized size = 1.58

$$\frac{1}{18}b^5x^{18} + \frac{1}{3}ab^4x^{15} + \frac{5}{6}a^2b^3x^{12} + \frac{10}{9}a^3b^2x^9 + \frac{5}{6}a^4bx^6 + \frac{1}{3}a^5x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/18*b^5*x^18 + 1/3*a*b^4*x^15 + 5/6*a^2*b^3*x^12 + 10/9*a^3*b^2*x^9 + 5/6*
a^4*b*x^6 + 1/3*a^5*x^3
```

giac [B] time = 0.37, size = 66, normalized size = 1.83

$$\frac{1}{18}\left(3\left(bx^6 + 2ax^3\right)a^4 + 3\left(bx^6 + 2ax^3\right)^2a^2b + \left(bx^6 + 2ax^3\right)^3b^2\right)\mathrm{sgn}\left(bx^3 + a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="giac")
```

```
[Out] 1/18*(3*(b*x^6 + 2*a*x^3)*a^4 + 3*(b*x^6 + 2*a*x^3)^2*a^2*b + (b*x^6 + 2*a*
x^3)^3*b^2)*sgn(b*x^3 + a)
```

maple [B] time = 0.01, size = 79, normalized size = 2.19

$$\frac{\left(b^5x^{15} + 6ab^4x^{12} + 15a^2b^3x^9 + 20a^3b^2x^6 + 15a^4bx^3 + 6a^5\right)\left(\left(bx^3 + a\right)^2\right)^{\frac{5}{2}}x^3}{18\left(bx^3 + a\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x)
```

```
[Out] 1/18*x^3*(b^5*x^15+6*a*b^4*x^12+15*a^2*b^3*x^9+20*a^3*b^2*x^6+15*a^4*b*x^3+
6*a^5)*((b*x^3+a)^2)^(5/2)/(b*x^3+a)^5
```

maxima [A] time = 0.66, size = 52, normalized size = 1.44

$$\frac{1}{18}\left(b^2x^6 + 2abx^3 + a^2\right)^{\frac{5}{2}}x^3 + \frac{\left(b^2x^6 + 2abx^3 + a^2\right)^{\frac{5}{2}}a}{18b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/18*(b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2)*x^3 + 1/18*(b^2*x^6 + 2*a*b*x^3 + a^
2)^(5/2)*a/b
```

mupad [B] time = 1.24, size = 36, normalized size = 1.00

$$\frac{\left(b^2x^3 + ab\right)\left(a^2 + 2abx^3 + b^2x^6\right)^{5/2}}{18b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2),x)
[Out] ((a*b + b^2*x^3)*(a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2))/(18*b^2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \left(\left(a + bx^3\right)^2\right)^{\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b**2*x**6+2*a*b*x**3+a**2)**(5/2),x)
[Out] Integral(x**2*((a + b*x**3)**2)**(5/2), x)
```

3.62 $\int x\left(a^2+2abx^3+b^2x^6\right)^{5/2}\,dx$

Optimal. Leaf size=252

$$\frac{b^5x^{17}\sqrt{a^2+2abx^3+b^2x^6}}{17\left(a+bx^3\right)}+\frac{5ab^4x^{14}\sqrt{a^2+2abx^3+b^2x^6}}{14\left(a+bx^3\right)}+\frac{10a^2b^3x^{11}\sqrt{a^2+2abx^3+b^2x^6}}{11\left(a+bx^3\right)}+\frac{a^5x^2\sqrt{a^2+2abx^3+b^2}}{2\left(a+bx^3\right)}$$

Rubi [A] time = 0.05, antiderivative size = 252, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1355, 270}

$$\frac{b^5x^{17}\sqrt{a^2+2abx^3+b^2x^6}}{17\left(a+bx^3\right)}+\frac{5ab^4x^{14}\sqrt{a^2+2abx^3+b^2x^6}}{14\left(a+bx^3\right)}+\frac{10a^2b^3x^{11}\sqrt{a^2+2abx^3+b^2x^6}}{11\left(a+bx^3\right)}+\frac{5a^3b^2x^8\sqrt{a^2+2abx^3+b^2x^6}}{4\left(a+bx^3\right)}+\frac{a^4bx^5\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}+\frac{a^5x^2\sqrt{a^2+2abx^3+b^2x^6}}{2\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
```

```
[Out] (a^5*x^2*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(2*(a + b*x^3)) + (a^4*b*x^5*Sqrt
[a^2 + 2*a*b*x^3 + b^2*x^6])/(a + b*x^3) + (5*a^3*b^2*x^8*Sqrt[a^2 + 2*a*b*
x^3 + b^2*x^6])/(4*(a + b*x^3)) + (10*a^2*b^3*x^11*Sqrt[a^2 + 2*a*b*x^3 + b
^2*x^6])/(11*(a + b*x^3)) + (5*a*b^4*x^14*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/
(14*(a + b*x^3)) + (b^5*x^17*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(17*(a + b*x^
3))
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\begin{aligned}
\int x\left(a^2+2abx^3+b^2x^6\right)^{5/2}\,dx &= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int x\left(ab+b^2x^3\right)^5\,dx}{b^4\left(ab+b^2x^3\right)}\\
&= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int\left(a^5b^5x+5a^4b^6x^4+10a^3b^7x^7+10a^2b^8x^{10}+5ab^9x^{13}+b\right.}{b^4\left(ab+b^2x^3\right)}\\
&= \frac{a^5x^2\sqrt{a^2+2abx^3+b^2x^6}}{2\left(a+bx^3\right)}+\frac{a^4bx^5\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}+\frac{5a^3b^2x^8\sqrt{a^2+2abx^3}}{4\left(a+bx^3\right)}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 83, normalized size = 0.33

$$\frac{x^2\sqrt{\left(a+bx^3\right)^2}\left(2618a^5+5236a^4bx^3+6545a^3b^2x^6+4760a^2b^3x^9+1870ab^4x^{12}+308b^5x^{15}\right)}{5236\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
```

```
[Out] (x^2*Sqrt[(a + b*x^3)^2]*(2618*a^5 + 5236*a^4*b*x^3 + 6545*a^3*b^2*x^6 + 47
60*a^2*b^3*x^9 + 1870*a*b^4*x^12 + 308*b^5*x^15))/(5236*(a + b*x^3))
```

IntegrateAlgebraic [A] time = 11.18, size = 83, normalized size = 0.33

$$\frac{\sqrt{\left(a+bx^3\right)^2}\left(2618a^5x^2+5236a^4bx^5+6545a^3b^2x^8+4760a^2b^3x^{11}+1870ab^4x^{14}+308b^5x^{17}\right)}{5236\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
```

```
[Out] (Sqrt[(a + b*x^3)^2]*(2618*a^5*x^2 + 5236*a^4*b*x^5 + 6545*a^3*b^2*x^8 + 47
60*a^2*b^3*x^11 + 1870*a*b^4*x^14 + 308*b^5*x^17))/(5236*(a + b*x^3))
```

fricas [A] time = 0.81, size = 56, normalized size = 0.22

$$\frac{1}{17}\,b^5x^{17}+\frac{5}{14}\,ab^4x^{14}+\frac{10}{11}\,a^2b^3x^{11}+\frac{5}{4}\,a^3b^2x^8+a^4bx^5+\frac{1}{2}\,a^5x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/17*b^5*x^17 + 5/14*a*b^4*x^14 + 10/11*a^2*b^3*x^11 + 5/4*a^3*b^2*x^8 + a^
4*b*x^5 + 1/2*a^5*x^2
```

giac [A] time = 0.37, size = 104, normalized size = 0.41

$$\frac{1}{17}\,b^5x^{17}\mathrm{sgn}\left(bx^3+a\right)+\frac{5}{14}\,ab^4x^{14}\mathrm{sgn}\left(bx^3+a\right)+\frac{10}{11}\,a^2b^3x^{11}\mathrm{sgn}\left(bx^3+a\right)+\frac{5}{4}\,a^3b^2x^8\mathrm{sgn}\left(bx^3+a\right)+a^4bx^5\mathrm{sgn}\left(bx^3+a\right)+\frac{1}{2}\,a^5x^2\mathrm{sgn}\left(bx^3+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="giac")
```

```
[Out] 1/17*b^5*x^17*sgn(b*x^3 + a) + 5/14*a*b^4*x^14*sgn(b*x^3 + a) + 10/11*a^2*b
^3*x^11*sgn(b*x^3 + a) + 5/4*a^3*b^2*x^8*sgn(b*x^3 + a) + a^4*b*x^5*sgn(b*x
^3 + a) + 1/2*a^5*x^2*sgn(b*x^3 + a)
```

maple [A] time = 0.00, size = 80, normalized size = 0.32

$$\frac{\left(308b^5x^{15}+1870a\,b^4x^{12}+4760a^2b^3x^9+6545a^3b^2x^6+5236a^4b\,x^3+2618a^5\right)\left(\left(b\,x^3+a\right)^2\right)^{\frac{5}{2}}x^2}{5236\left(b\,x^3+a\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x)
```

```
[Out] 1/5236*x^2*(308*b^5*x^15+1870*a*b^4*x^12+4760*a^2*b^3*x^9+6545*a^3*b^2*x^6+
5236*a^4*b*x^3+2618*a^5)*((b*x^3+a)^2)^(5/2)/(b*x^3+a)^5
```

maxima [A] time = 1.10, size = 56, normalized size = 0.22

$$\frac{1}{17}\,b^5x^{17}+\frac{5}{14}\,ab^4x^{14}+\frac{10}{11}\,a^2b^3x^{11}+\frac{5}{4}\,a^3b^2x^8+a^4bx^5+\frac{1}{2}\,a^5x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/17*b^5*x^17 + 5/14*a*b^4*x^14 + 10/11*a^2*b^3*x^11 + 5/4*a^3*b^2*x^8 + a^
4*b*x^5 + 1/2*a^5*x^2
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x\left(a^2 + 2\,a\,b\,x^3 + b^2\,x^6\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2),x)
```

```
[Out] int(x*(a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\left(\left(a + bx^3\right)^2\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b**2*x**6+2*a*b*x**3+a**2)**(5/2),x)
```

```
[Out] Integral(x*((a + b*x**3)**2)**(5/2), x)
```

3.63 $\int \left(a^2+2abx^3+b^2x^6\right)^{5/2}\,dx$

Optimal. Leaf size=247

$$\frac{b^5x^{16}\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{16\left(a+bx^3\right)^5}+\frac{5ab^4x^{13}\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{13\left(a+bx^3\right)^5}+\frac{a^2b^3x^{10}\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{\left(a+bx^3\right)^5}+\frac{a^5x\left(a^2+2ab\cdots\right.}{\left(a+b\cdots\right.}$$

Rubi [A] time = 0.05, antiderivative size = 247, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1343, 194}

$$\frac{b^5x^{16}\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{16\left(a+bx^3\right)^5}+\frac{5ab^4x^{13}\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{13\left(a+bx^3\right)^5}+\frac{a^2b^3x^{10}\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{\left(a+bx^3\right)^5}+\frac{10a^3b^2x^7\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{7\left(a+bx^3\right)^5}+\frac{5a^4bx^4\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{4\left(a+bx^3\right)^5}+\frac{a^5x\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{\left(a+bx^3\right)^5}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
```

```
[Out] (a^5*x*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2))/(a + b*x^3)^5 + (5*a^4*b*x^4*(a^2
 + 2*a*b*x^3 + b^2*x^6)^(5/2))/(4*(a + b*x^3)^5) + (10*a^3*b^2*x^7*(a^2 + 2
*a*b*x^3 + b^2*x^6)^(5/2))/(7*(a + b*x^3)^5) + (a^2*b^3*x^10*(a^2 + 2*a*b*x
^3 + b^2*x^6)^(5/2))/(a + b*x^3)^5 + (5*a*b^4*x^13*(a^2 + 2*a*b*x^3 + b^2*x
^6)^(5/2))/(13*(a + b*x^3)^5) + (b^5*x^16*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)
)/(16*(a + b*x^3)^5)
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 1343

```
Int[((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_), x_Symbol] :> Dist[(
a + b*x^n + c*x^(2*n))^p/(b + 2*c*x^n)^(2*p), Int[(b + 2*c*x^n)^(2*p), x],
x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}
\int \left(a^2+2abx^3+b^2x^6\right)^{5/2}\,dx &= \frac{\left(a^2+2abx^3+b^2x^6\right)^{5/2}\int \left(2ab+2b^2x^3\right)^5\,dx}{\left(2ab+2b^2x^3\right)^5}\\
&= \frac{\left(a^2+2abx^3+b^2x^6\right)^{5/2}\int \left(32a^5b^5+160a^4b^6x^3+320a^3b^7x^6+320a^2b^8x^9+16\cdots\right.}{\left(2ab+2b^2x^3\right)^5\cdots}\\
&= \frac{a^5x\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{\left(a+bx^3\right)^5}+\frac{5a^4bx^4\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{4\left(a+bx^3\right)^5}+\frac{10a^3b^2x^7\left(a^2+\cdots\right.}{7\left(a+\cdots\right.}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 81, normalized size = 0.33

$$\frac{x\sqrt{\left(a+bx^3\right)^2}\left(1456a^5+1820a^4bx^3+2080a^3b^2x^6+1456a^2b^3x^9+560ab^4x^{12}+91b^5x^{15}\right)}{1456\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
```

```
[Out] (x*Sqrt[(a + b*x^3)^2]*(1456*a^5 + 1820*a^4*b*x^3 + 2080*a^3*b^2*x^6 + 1456
*a^2*b^3*x^9 + 560*a*b^4*x^12 + 91*b^5*x^15))/(1456*(a + b*x^3))
```

IntegrateAlgebraic [A] time = 12.92, size = 81, normalized size = 0.33

$$\frac{\sqrt{\left(a+bx^3\right)^2}\,\left(1456a^5x+1820a^4bx^4+2080a^3b^2x^7+1456a^2b^3x^{10}+560ab^4x^{13}+91b^5x^{16}\right)}{1456\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
```

```
[Out] (Sqrt[(a + b*x^3)^2]*(1456*a^5*x + 1820*a^4*b*x^4 + 2080*a^3*b^2*x^7 + 1456
*a^2*b^3*x^10 + 560*a*b^4*x^13 + 91*b^5*x^16))/(1456*(a + b*x^3))
```

fricas [A] time = 1.29, size = 53, normalized size = 0.21

$$\frac{1}{16}\,b^5x^{16}+\frac{5}{13}\,ab^4x^{13}+a^2b^3x^{10}+\frac{10}{7}\,a^3b^2x^7+\frac{5}{4}\,a^4bx^4+a^5x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/16*b^5*x^16 + 5/13*a*b^4*x^13 + a^2*b^3*x^10 + 10/7*a^3*b^2*x^7 + 5/4*a^4
*b*x^4 + a^5*x
```

giac [A] time = 0.38, size = 101, normalized size = 0.41

$$\frac{1}{16}\,b^5x^{16}\mathrm{sgn}\left(bx^3+a\right)+\frac{5}{13}\,ab^4x^{13}\mathrm{sgn}\left(bx^3+a\right)+a^2b^3x^{10}\mathrm{sgn}\left(bx^3+a\right)+\frac{10}{7}\,a^3b^2x^7\mathrm{sgn}\left(bx^3+a\right)+\frac{5}{4}\,a^4bx^4\mathrm{sgn}\left(bx^3+a\right)+a^5x\mathrm{sgn}\left(bx^3+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="giac")
```

```
[Out] 1/16*b^5*x^16*sgn(b*x^3 + a) + 5/13*a*b^4*x^13*sgn(b*x^3 + a) + a^2*b^3*x^1
0*sgn(b*x^3 + a) + 10/7*a^3*b^2*x^7*sgn(b*x^3 + a) + 5/4*a^4*b*x^4*sgn(b*x^
3 + a) + a^5*x*sgn(b*x^3 + a)
```

maple [A] time = 0.00, size = 78, normalized size = 0.32

$$\frac{\left(91b^5x^{15}+560a\,b^4x^{12}+1456a^2b^3x^9+2080a^3b^2x^6+1820a^4b\,x^3+1456a^5\right)\left(\left(b\,x^3+a\right)^2\right)^{\frac{5}{2}}x}{1456\left(b\,x^3+a\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^6+2*a*b*x^3+a^2)^(5/2),x)
```

```
[Out] 1/1456*x*(91*b^5*x^15+560*a*b^4*x^12+1456*a^2*b^3*x^9+2080*a^3*b^2*x^6+1820
*a^4*b*x^3+1456*a^5)*((b*x^3+a)^2)^(5/2)/(b*x^3+a)^5
```

maxima [A] time = 1.02, size = 53, normalized size = 0.21

$$\frac{1}{16}\,b^5x^{16}+\frac{5}{13}\,ab^4x^{13}+a^2b^3x^{10}+\frac{10}{7}\,a^3b^2x^7+\frac{5}{4}\,a^4bx^4+a^5x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/16*b^5*x^16 + 5/13*a*b^4*x^13 + a^2*b^3*x^10 + 10/7*a^3*b^2*x^7 + 5/4*a^4
*b*x^4 + a^5*x
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \left(a^2 + 2\,a\,b\,x^3 + b^2\,x^6\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2),x)
[Out] int((a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a^2 + 2abx^3 + b^2x^6\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**6+2*a*b*x**3+a**2)**(5/2),x)
[Out] Integral((a**2 + 2*a*b*x**3 + b**2*x**6)**(5/2), x)
```

3.64 $\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{x}\, dx$

Optimal. Leaf size=251

$$\frac{b^5x^{15}\sqrt{a^2+2abx^3+b^2x^6}}{15\left(a+bx^3\right)}+\frac{5ab^4x^{12}\sqrt{a^2+2abx^3+b^2x^6}}{12\left(a+bx^3\right)}+\frac{10a^2b^3x^9\sqrt{a^2+2abx^3+b^2x^6}}{9\left(a+bx^3\right)}+\frac{a^5\log(x)\sqrt{a^2+2abx^3-}}{a+bx^3}$$

Rubi [A] time = 0.07, antiderivative size = 251, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1355, 266, 43}

$$\frac{b^5x^{15}\sqrt{a^2+2abx^3+b^2x^6}}{15\left(a+bx^3\right)}+\frac{5ab^4x^{12}\sqrt{a^2+2abx^3+b^2x^6}}{12\left(a+bx^3\right)}+\frac{10a^2b^3x^9\sqrt{a^2+2abx^3+b^2x^6}}{9\left(a+bx^3\right)}+\frac{5a^3b^2x^6\sqrt{a^2+2abx^3+b^2x^6}}{3\left(a+bx^3\right)}+\frac{5a^4bx^3\sqrt{a^2+2abx^3+b^2x^6}}{3\left(a+bx^3\right)}+\frac{a^5\log(x)\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x,x]
```

```
[Out] (5*a^4*b*x^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(3*(a + b*x^3)) + (5*a^3*b^2*
x^6*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(3*(a + b*x^3)) + (10*a^2*b^3*x^9*Sqrt
[a^2 + 2*a*b*x^3 + b^2*x^6])/(9*(a + b*x^3)) + (5*a*b^4*x^12*Sqrt[a^2 + 2*a
*b*x^3 + b^2*x^6])/(12*(a + b*x^3)) + (b^5*x^15*Sqrt[a^2 + 2*a*b*x^3 + b^2*
x^6])/(15*(a + b*x^3)) + (a^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]*Log[x])/(a +
b*x^3)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{x}\,dx = \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{\left(ab+b^2x^3\right)^5}{x}\,dx}{b^4\left(ab+b^2x^3\right)}$$

$$= \frac{\sqrt{a^2+2abx^3+b^2x^6}\ \mathrm{Subst}\left(\int \frac{\left(ab+b^2x\right)^5}{x}\,dx, x, x^3\right)}{3b^4\left(ab+b^2x^3\right)}$$

$$= \frac{\sqrt{a^2+2abx^3+b^2x^6}\ \mathrm{Subst}\left(\int \left(5a^4b^6+\frac{a^5b^5}{x}+10a^3b^7x+10a^2b^8x^2+5ab^9x^3+\right.\right.}{3b^4\left(ab+b^2x^3\right)}$$

$$= \frac{5a^4bx^3\sqrt{a^2+2abx^3+b^2x^6}}{3\left(a+bx^3\right)}+\frac{5a^3b^2x^6\sqrt{a^2+2abx^3+b^2x^6}}{3\left(a+bx^3\right)}+\frac{10a^2b^3x^9\sqrt{a^2+}}{9\left(a+\right.}$$

Mathematica [A] time = 0.02, size = 82, normalized size = 0.33

$$\frac{\sqrt{\left(a+bx^3\right)^2}\left(180a^5\log(x)+bx^3\left(300a^4+300a^3bx^3+200a^2b^2x^6+75ab^3x^9+12b^4x^{12}\right)\right)}{180\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x,x]
```

```
[Out] (Sqrt[(a + b*x^3)^2]*(b*x^3*(300*a^4 + 300*a^3*b*x^3 + 200*a^2*b^2*x^6 + 75
*a*b^3*x^9 + 12*b^4*x^12) + 180*a^5*Log[x]))/(180*(a + b*x^3))
```

IntegrateAlgebraic [A] time = 0.58, size = 314, normalized size = 1.25

$$\frac{1}{6}a^5\log\left(\sqrt{a^2+2abx^3+b^2x^6}-a-\sqrt{b^2}\,x^3\right)-\frac{a^5\left(\sqrt{b^2}+b\right)\log\left(\sqrt{a^2+2abx^3+b^2x^6}+a-\sqrt{b^2}\,x^3\right)}{6b}-\frac{a^5\sqrt{b^2}\log\left(b\sqrt{a^2+2abx^3+b^2x^6}-ab-b\sqrt{b^2}\,x^3\right)}{6b}+\frac{1}{360}\sqrt{a^2+2abx^3+b^2x^6}\left(137a^4+163a^3bx^3+137a^2b^2x^6+63ab^3x^9+12b^4x^{12}\right)+\frac{1}{360}\left(-300a^4\sqrt{b^2}\,x^3-300a^3b\sqrt{b^2}\,x^6-200a^2\left(b^2\right)^{3/2}x^9-75ab^3\sqrt{b^2}\,x^{12}-12b^4\sqrt{b^2}\,x^{15}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x,x]
```

```
[Out] (Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]*(137*a^4 + 163*a^3*b*x^3 + 137*a^2*b^2*x^6
 + 63*a*b^3*x^9 + 12*b^4*x^12))/360 + (-300*a^4*Sqrt[b^2]*x^3 - 300*a^3*b*S
qrt[b^2]*x^6 - 200*a^2*(b^2)^(3/2)*x^9 - 75*a*b^3*Sqrt[b^2]*x^12 - 12*b^4*S
qrt[b^2]*x^15)/360 + (a^5*Log[-a - Sqrt[b^2]*x^3 + Sqrt[a^2 + 2*a*b*x^3 + b
^2*x^6]])/6 - (a^5*(b + Sqrt[b^2])*Log[a - Sqrt[b^2]*x^3 + Sqrt[a^2 + 2*a*b
*x^3 + b^2*x^6]])/(6*b) - (a^5*Sqrt[b^2]*Log[-(a*b) - b*Sqrt[b^2]*x^3 + b*S
qrt[a^2 + 2*a*b*x^3 + b^2*x^6]])/(6*b)
```

fricas [A] time = 1.17, size = 55, normalized size = 0.22

$$\frac{1}{15}\,b^5x^{15}+\frac{5}{12}\,ab^4x^{12}+\frac{10}{9}\,a^2b^3x^9+\frac{5}{3}\,a^3b^2x^6+\frac{5}{3}\,a^4bx^3+a^5\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x,x, algorithm="fricas")
```

```
[Out] 1/15*b^5*x^15 + 5/12*a*b^4*x^12 + 10/9*a^2*b^3*x^9 + 5/3*a^3*b^2*x^6 + 5/3*
a^4*b*x^3 + a^5*log(x)
```

giac [A] time = 0.35, size = 104, normalized size = 0.41

$$\frac{1}{15}\,b^5x^{15}\mathrm{sgn}\left(bx^3+a\right)+\frac{5}{12}\,ab^4x^{12}\mathrm{sgn}\left(bx^3+a\right)+\frac{10}{9}\,a^2b^3x^9\mathrm{sgn}\left(bx^3+a\right)+\frac{5}{3}\,a^3b^2x^6\mathrm{sgn}\left(bx^3+a\right)+\frac{5}{3}\,a^4bx^3\mathrm{sgn}\left(bx^3+a\right)+a^5\log\left(|x|\right)\mathrm{sgn}\left(bx^3+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x,x, algorithm="giac")
```

```
[Out] 1/15*b^5*x^15*sgn(b*x^3 + a) + 5/12*a*b^4*x^12*sgn(b*x^3 + a) + 10/9*a^2*b^
3*x^9*sgn(b*x^3 + a) + 5/3*a^3*b^2*x^6*sgn(b*x^3 + a) + 5/3*a^4*b*x^3*sgn(b
*x^3 + a) + a^5*log(abs(x))*sgn(b*x^3 + a)
```

maple [A] time = 0.01, size = 79, normalized size = 0.31

$$\frac{\left(\left(b\,x^{3}+a\right)^{2}\right)^{\frac{5}{2}}\left(12b^{5}x^{15}+75a\,b^{4}x^{12}+200a^{2}b^{3}x^{9}+300a^{3}b^{2}x^{6}+300a^{4}b\,x^{3}+180a^{5}\ln(x)\right)}{180\left(b\,x^{3}+a\right)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x,x)
```

```
[Out] 1/180*((b*x^3+a)^2)^(5/2)*(12*b^5*x^15+75*a*b^4*x^12+200*a^2*b^3*x^9+300*a^
3*b^2*x^6+300*a^4*b*x^3+180*a^5*ln(x))/(b*x^3+a)^5
```

maxima [A] time = 1.16, size = 206, normalized size = 0.82

$$\frac{1}{6}\sqrt{b^{2}x^{6}+2\,abx^{3}+a^{2}}\,a^{3}bx^{3}+\frac{1}{3}\left(-1\right)^{2\,b^{2}x^{3}+2\,ab}a^{5}\log\left(2\,b^{2}x^{3}+2\,ab\right)-\frac{1}{3}\left(-1\right)^{2\,abx^{3}+2\,a^{2}}a^{5}\log\left(\frac{2\,abx}{|x|}+\frac{2\,a^{2}}{x^{2}|x|}\right)+\frac{1}{12}\left(b^{2}x^{6}+2\,abx^{3}+a^{2}\right)^{\frac{3}{2}}abx^{3}+\frac{1}{2}\sqrt{b^{2}x^{6}+2\,abx^{3}+a^{2}}\,a^{4}+\frac{7}{36}\left(b^{2}x^{6}+2\,abx^{3}+a^{2}\right)^{\frac{3}{2}}a^{2}+\frac{1}{15}\left(b^{2}x^{6}+2\,abx^{3}+a^{2}\right)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x,x, algorithm="maxima")
```

```
[Out] 1/6*sqrt(b^2*x^6 + 2*a*b*x^3 + a^2)*a^3*b*x^3 + 1/3*(-1)^(2*b^2*x^3 + 2*a*b
)*a^5*log(2*b^2*x^3 + 2*a*b) - 1/3*(-1)^(2*a*b*x^3 + 2*a^2)*a^5*log(2*a*b*x
/abs(x) + 2*a^2/(x^2*abs(x))) + 1/12*(b^2*x^6 + 2*a*b*x^3 + a^2)^(3/2)*a*b*
x^3 + 1/2*sqrt(b^2*x^6 + 2*a*b*x^3 + a^2)*a^4 + 7/36*(b^2*x^6 + 2*a*b*x^3 +
 a^2)^(3/2)*a^2 + 1/15*(b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a^2+2\,a\,b\,x^3+b^2\,x^6\right)^{5/2}}{x} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2)/x,x)
```

```
[Out] int((a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2)/x, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\left(a + b x^{3}\right)^{2}\right)^{\frac{5}{2}}}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**6+2*a*b*x**3+a**2)**(5/2)/x,x)
```

```
[Out] Integral(((a + b*x**3)**2)**(5/2)/x, x)
```

3.65 $\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{x^2}\,dx$

Optimal. Leaf size=251

$$\frac{b^5x^{14}\sqrt{a^2+2abx^3+b^2x^6}}{14\left(a+bx^3\right)}+\frac{5ab^4x^{11}\sqrt{a^2+2abx^3+b^2x^6}}{11\left(a+bx^3\right)}+\frac{5a^2b^3x^8\sqrt{a^2+2abx^3+b^2x^6}}{4\left(a+bx^3\right)}-\frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{x\left(a+bx^3\right)}$$

Rubi [A] time = 0.06, antiderivative size = 251, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1355, 270}

$$\frac{b^5x^{14}\sqrt{a^2+2abx^3+b^2x^6}}{14\left(a+bx^3\right)}+\frac{5ab^4x^{11}\sqrt{a^2+2abx^3+b^2x^6}}{11\left(a+bx^3\right)}+\frac{5a^2b^3x^8\sqrt{a^2+2abx^3+b^2x^6}}{4\left(a+bx^3\right)}+\frac{2a^3b^2x^5\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}+\frac{5a^4bx^2\sqrt{a^2+2abx^3+b^2x^6}}{2\left(a+bx^3\right)}-\frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{x\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^2,x]
```

```
[Out] -((a^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(x*(a + b*x^3))) + (5*a^4*b*x^2*Sqr
t[a^2 + 2*a*b*x^3 + b^2*x^6])/(2*(a + b*x^3)) + (2*a^3*b^2*x^5*Sqrt[a^2 + 2
*a*b*x^3 + b^2*x^6])/(a + b*x^3) + (5*a^2*b^3*x^8*Sqrt[a^2 + 2*a*b*x^3 + b^
2*x^6])/(4*(a + b*x^3)) + (5*a*b^4*x^11*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(1
1*(a + b*x^3)) + (b^5*x^14*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(14*(a + b*x^3)
)
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{x^2}\,dx &= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{\left(ab+b^2x^3\right)^5}{x^2}\,dx}{b^4\left(ab+b^2x^3\right)}\\ &= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \left(\frac{a^5b^5}{x^2}+5a^4b^6x+10a^3b^7x^4+10a^2b^8x^7+5ab^9x^{10}+b^{10}\right.}{b^4\left(ab+b^2x^3\right)}\\ &= -\frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{x\left(a+bx^3\right)}+\frac{5a^4bx^2\sqrt{a^2+2abx^3+b^2x^6}}{2\left(a+bx^3\right)}+\frac{2a^3b^2x^5\sqrt{a^2+2abx^3}}{a+bx^3}\end{aligned}$$

Mathematica [A] time = 0.02, size = 83, normalized size = 0.33

$$\frac{\sqrt{\left(a+bx^3\right)^2}\left(-308a^5+770a^4bx^3+616a^3b^2x^6+385a^2b^3x^9+140ab^4x^{12}+22b^5x^{15}\right)}{308x\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^2,x]
```

```
[Out] (Sqrt[(a + b*x^3)^2]*(-308*a^5 + 770*a^4*b*x^3 + 616*a^3*b^2*x^6 + 385*a^2*
b^3*x^9 + 140*a*b^4*x^12 + 22*b^5*x^15))/(308*x*(a + b*x^3))
```

IntegrateAlgebraic [A] time = 16.65, size = 83, normalized size = 0.33

$$\frac{\sqrt{\left(a+bx^3\right)^2}\left(-308a^5+770a^4bx^3+616a^3b^2x^6+385a^2b^3x^9+140ab^4x^{12}+22b^5x^{15}\right)}{308x\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^2,x]
```

```
[Out] (Sqrt[(a + b*x^3)^2]*(-308*a^5 + 770*a^4*b*x^3 + 616*a^3*b^2*x^6 + 385*a^2*
b^3*x^9 + 140*a*b^4*x^12 + 22*b^5*x^15))/(308*x*(a + b*x^3))
```

fricas [A] time = 1.22, size = 59, normalized size = 0.24

$$\frac{22\,b^5x^{15}+140\,ab^4x^{12}+385\,a^2b^3x^9+616\,a^3b^2x^6+770\,a^4bx^3-308\,a^5}{308\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^2,x, algorithm="fricas")
```

```
[Out] 1/308*(22*b^5*x^15 + 140*a*b^4*x^12 + 385*a^2*b^3*x^9 + 616*a^3*b^2*x^6 + 7
70*a^4*b*x^3 - 308*a^5)/x
```

giac [A] time = 0.33, size = 105, normalized size = 0.42

$$\frac{1}{14}\,b^5x^{14}\mathrm{sgn}\left(bx^3+a\right)+\frac{5}{11}\,ab^4x^{11}\mathrm{sgn}\left(bx^3+a\right)+\frac{5}{4}\,a^2b^3x^8\mathrm{sgn}\left(bx^3+a\right)+2\,a^3b^2x^5\mathrm{sgn}\left(bx^3+a\right)+\frac{5}{2}\,a^4bx^2\mathrm{sgn}\left(bx^3+a\right)-\frac{a^5\mathrm{sgn}\left(bx^3+a\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^2,x, algorithm="giac")
```

```
[Out] 1/14*b^5*x^14*sgn(b*x^3 + a) + 5/11*a*b^4*x^11*sgn(b*x^3 + a) + 5/4*a^2*b^3
*x^8*sgn(b*x^3 + a) + 2*a^3*b^2*x^5*sgn(b*x^3 + a) + 5/2*a^4*b*x^2*sgn(b*x^
3 + a) - a^5*sgn(b*x^3 + a)/x
```

maple [A] time = 0.01, size = 80, normalized size = 0.32

$$-\frac{\left(-22b^5x^{15}-140a\,b^4x^{12}-385a^2b^3x^9-616a^3b^2x^6-770a^4b\,x^3+308a^5\right)\left(\left(b\,x^3+a\right)^2\right)^{\frac{5}{2}}}{308\left(b\,x^3+a\right)^5x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^2,x)
```

```
[Out] -1/308*(-22*b^5*x^15-140*a*b^4*x^12-385*a^2*b^3*x^9-616*a^3*b^2*x^6-770*a^4
*b*x^3+308*a^5)*((b*x^3+a)^2)^(5/2)/x/(b*x^3+a)^5
```

maxima [A] time = 0.82, size = 59, normalized size = 0.24

$$\frac{22\,b^5x^{15}+140\,ab^4x^{12}+385\,a^2b^3x^9+616\,a^3b^2x^6+770\,a^4bx^3-308\,a^5}{308\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^2,x, algorithm="maxima")
```

```
[Out] 1/308*(22*b^5*x^15 + 140*a*b^4*x^12 + 385*a^2*b^3*x^9 + 616*a^3*b^2*x^6 + 7
70*a^4*b*x^3 - 308*a^5)/x
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a^2+2\,a\,b\,x^3+b^2\,x^6\right)^{5/2}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2)/x^2,x)
```

```
[Out] int((a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2)/x^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\left(a+bx^3\right)^2\right)^{\frac{5}{2}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**6+2*a*b*x**3+a**2)**(5/2)/x**2,x)
```

```
[Out] Integral(((a + b*x**3)**2)**(5/2)/x**2, x)
```

3.66 $\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{x^3}\,dx$

Optimal. Leaf size=251

$$\frac{b^5x^{13}\sqrt{a^2+2abx^3+b^2x^6}}{13\left(a+bx^3\right)}+\frac{ab^4x^{10}\sqrt{a^2+2abx^3+b^2x^6}}{2\left(a+bx^3\right)}+\frac{10a^2b^3x^7\sqrt{a^2+2abx^3+b^2x^6}}{7\left(a+bx^3\right)}-\frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{2x^2\left(a+bx^3\right)}+$$

Rubi [A] time = 0.06, antiderivative size = 251, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {1355, 270}

$$\frac{b^5x^{13}\sqrt{a^2+2abx^3+b^2x^6}}{13\left(a+bx^3\right)}+\frac{ab^4x^{10}\sqrt{a^2+2abx^3+b^2x^6}}{2\left(a+bx^3\right)}+\frac{10a^2b^3x^7\sqrt{a^2+2abx^3+b^2x^6}}{7\left(a+bx^3\right)}+\frac{5a^3b^2x^4\sqrt{a^2+2abx^3+b^2x^6}}{2\left(a+bx^3\right)}+\frac{5a^4bx\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}-\frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{2x^2\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^3,x]
```

```
[Out] -(a^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(2*x^2*(a + b*x^3)) + (5*a^4*b*x*Sqr
t[a^2 + 2*a*b*x^3 + b^2*x^6])/(a + b*x^3) + (5*a^3*b^2*x^4*Sqrt[a^2 + 2*a*b
*x^3 + b^2*x^6])/(2*(a + b*x^3)) + (10*a^2*b^3*x^7*Sqrt[a^2 + 2*a*b*x^3 + b
^2*x^6])/(7*(a + b*x^3)) + (a*b^4*x^10*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(2*
(a + b*x^3)) + (b^5*x^13*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(13*(a + b*x^3))
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{x^3}\,dx&=\frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{\left(ab+b^2x^3\right)^5}{x^3}\,dx}{b^4\left(ab+b^2x^3\right)}\\&=\frac{\sqrt{a^2+2abx^3+b^2x^6}\int \left(5a^4b^6+\frac{a^5b^5}{x^3}+10a^3b^7x^3+10a^2b^8x^6+5ab^9x^9+b^{10}x^{12}\right)}{b^4\left(ab+b^2x^3\right)}\\&=-\frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{2x^2\left(a+bx^3\right)}+\frac{5a^4bx\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}+\frac{5a^3b^2x^4\sqrt{a^2+2abx^3+b}}{2\left(a+bx^3\right)}\end{aligned}$$

Mathematica [A] time = 0.02, size = 83, normalized size = 0.33

$$\frac{\sqrt{\left(a+bx^3\right)^2}\left(-91a^5+910a^4bx^3+455a^3b^2x^6+260a^2b^3x^9+91ab^4x^{12}+14b^5x^{15}\right)}{182x^2\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^3,x]
```

```
[Out] (Sqrt[(a + b*x^3)^2]*(-91*a^5 + 910*a^4*b*x^3 + 455*a^3*b^2*x^6 + 260*a^2*b
^3*x^9 + 91*a*b^4*x^12 + 14*b^5*x^15))/(182*x^2*(a + b*x^3))
```

IntegrateAlgebraic [A] time = 18.81, size = 83, normalized size = 0.33

$$\frac{\sqrt{\left(a+bx^3\right)^2}\left(-91a^5+910a^4bx^3+455a^3b^2x^6+260a^2b^3x^9+91ab^4x^{12}+14b^5x^{15}\right)}{182x^2\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^3,x]
```

```
[Out] (Sqrt[(a + b*x^3)^2]*(-91*a^5 + 910*a^4*b*x^3 + 455*a^3*b^2*x^6 + 260*a^2*b
^3*x^9 + 91*a*b^4*x^12 + 14*b^5*x^15))/(182*x^2*(a + b*x^3))
```

fricas [A] time = 1.22, size = 59, normalized size = 0.24

$$\frac{14\,b^5x^{15}+91\,ab^4x^{12}+260\,a^2b^3x^9+455\,a^3b^2x^6+910\,a^4bx^3-91\,a^5}{182\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^3,x, algorithm="fricas")
```

```
[Out] 1/182*(14*b^5*x^15 + 91*a*b^4*x^12 + 260*a^2*b^3*x^9 + 455*a^3*b^2*x^6 + 91
0*a^4*b*x^3 - 91*a^5)/x^2
```

giac [A] time = 0.36, size = 103, normalized size = 0.41

$$\frac{1}{13}\,b^5x^{13}\mathrm{sgn}\left(bx^3+a\right)+\frac{1}{2}\,ab^4x^{10}\mathrm{sgn}\left(bx^3+a\right)+\frac{10}{7}\,a^2b^3x^7\mathrm{sgn}\left(bx^3+a\right)+\frac{5}{2}\,a^3b^2x^4\mathrm{sgn}\left(bx^3+a\right)+5\,a^4bx\mathrm{sgn}\left(bx^3+a\right)-\frac{a^5\mathrm{sgn}\left(bx^3+a\right)}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^3,x, algorithm="giac")
```

```
[Out] 1/13*b^5*x^13*sgn(b*x^3 + a) + 1/2*a*b^4*x^10*sgn(b*x^3 + a) + 10/7*a^2*b^3
*x^7*sgn(b*x^3 + a) + 5/2*a^3*b^2*x^4*sgn(b*x^3 + a) + 5*a^4*b*x*sgn(b*x^3
+ a) - 1/2*a^5*sgn(b*x^3 + a)/x^2
```

maple [A] time = 0.01, size = 80, normalized size = 0.32

$$-\frac{\left(-14b^5x^{15}-91a\,b^4x^{12}-260a^2b^3x^9-455a^3b^2x^6-910a^4b\,x^3+91a^5\right)\left(\left(b\,x^3+a\right)^2\right)^{\frac{5}{2}}}{182\left(b\,x^3+a\right)^5x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^3,x)
```

```
[Out] -1/182*(-14*b^5*x^15-91*a*b^4*x^12-260*a^2*b^3*x^9-455*a^3*b^2*x^6-910*a^4*
b*x^3+91*a^5)*((b*x^3+a)^2)^(5/2)/x^2/(b*x^3+a)^5
```

maxima [A] time = 1.23, size = 59, normalized size = 0.24

$$\frac{14\,b^5x^{15}+91\,ab^4x^{12}+260\,a^2b^3x^9+455\,a^3b^2x^6+910\,a^4bx^3-91\,a^5}{182\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^3,x, algorithm="maxima")
```

```
[Out] 1/182*(14*b^5*x^15 + 91*a*b^4*x^12 + 260*a^2*b^3*x^9 + 455*a^3*b^2*x^6 + 91
0*a^4*b*x^3 - 91*a^5)/x^2
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a^2+2\,a\,b\,x^3+b^2\,x^6\right)^{5/2}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2)/x^3,x)
```

```
[Out] int((a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2)/x^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\left(a+bx^3\right)^2\right)^{\frac{5}{2}}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**6+2*a*b*x**3+a**2)**(5/2)/x**3,x)
```

```
[Out] Integral(((a + b*x**3)**2)**(5/2)/x**3, x)
```

3.67 $\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{x^4}\,dx$

Optimal. Leaf size=252

$$\frac{b^5x^{12}\sqrt{a^2+2abx^3+b^2x^6}}{12\left(a+bx^3\right)}+\frac{5ab^4x^9\sqrt{a^2+2abx^3+b^2x^6}}{9\left(a+bx^3\right)}+\frac{5a^2b^3x^6\sqrt{a^2+2abx^3+b^2x^6}}{3\left(a+bx^3\right)}-\frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{3x^3\left(a+bx^3\right)}$$

Rubi [A] time = 0.07, antiderivative size = 252, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1355, 266, 43}

$$\frac{b^5x^{12}\sqrt{a^2+2abx^3+b^2x^6}}{12\left(a+bx^3\right)}+\frac{5ab^4x^9\sqrt{a^2+2abx^3+b^2x^6}}{9\left(a+bx^3\right)}+\frac{5a^2b^3x^6\sqrt{a^2+2abx^3+b^2x^6}}{3\left(a+bx^3\right)}+\frac{10a^3b^2x^3\sqrt{a^2+2abx^3+b^2x^6}}{3\left(a+bx^3\right)}-\frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{3x^3\left(a+bx^3\right)}+\frac{5a^4b\log(x)\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^4,x]
```

```
[Out] -(a^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(3*x^3*(a + b*x^3)) + (10*a^3*b^2*x^
3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(3*(a + b*x^3)) + (5*a^2*b^3*x^6*Sqrt[a^
2 + 2*a*b*x^3 + b^2*x^6])/(3*(a + b*x^3)) + (5*a*b^4*x^9*Sqrt[a^2 + 2*a*b*x
^3 + b^2*x^6])/(9*(a + b*x^3)) + (b^5*x^12*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])
/(12*(a + b*x^3)) + (5*a^4*b*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]*Log[x])/(a + b
*x^3)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{x^4}\,dx = \frac{\sqrt{a^2+2abx^3+b^2x^6}\int\frac{\left(ab+b^2x^3\right)^5}{x^4}\,dx}{b^4\left(ab+b^2x^3\right)}$$

$$= \frac{\sqrt{a^2+2abx^3+b^2x^6}\,\text{Subst}\left(\int\frac{\left(ab+b^2x\right)^5}{x^2}\,dx,x,x^3\right)}{3b^4\left(ab+b^2x^3\right)}$$

$$= \frac{\sqrt{a^2+2abx^3+b^2x^6}\,\text{Subst}\left(\int\left(10a^3b^7+\frac{a^5b^5}{x^2}+\frac{5a^4b^6}{x}+10a^2b^8x+5ab^9x^2+b^{10}x^3\right.\right.}{3b^4\left(ab+b^2x^3\right)}$$

$$= -\frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{3x^3\left(a+bx^3\right)}+\frac{10a^3b^2x^3\sqrt{a^2+2abx^3+b^2x^6}}{3\left(a+bx^3\right)}+\frac{5a^2b^3x^6\sqrt{a^2+2abx^3}}{3\left(a+bx^3\right)}$$

Mathematica [A]   time = 0.03, size = 85, normalized size = 0.34

$$\frac{\sqrt{\left(a+bx^3\right)^2}\left(-12a^5+180a^4bx^3\log(x)+120a^3b^2x^6+60a^2b^3x^9+20ab^4x^{12}+3b^5x^{15}\right)}{36x^3\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^4,x]
```

```
[Out] (Sqrt[(a + b*x^3)^2]*(-12*a^5 + 120*a^3*b^2*x^6 + 60*a^2*b^3*x^9 + 20*a*b^4
*x^12 + 3*b^5*x^15 + 180*a^4*b*x^3*Log[x]))/(36*x^3*(a + b*x^3))
```

IntegrateAlgebraic [A]   time = 0.98, size = 364, normalized size = 1.44

$$-\frac{5}{6}a^4\sqrt{b^2}\log\left(\sqrt{a^2+2abx^3+b^2x^6}-a-\sqrt{b^2}x^3\right)-\frac{5}{6}a^4\sqrt{b^2}\log\left(\sqrt{a^2+2abx^3+b^2x^6}+a-\sqrt{b^2}x^3\right)+\frac{5}{3}a^4b\tanh^{-1}\left(\frac{\sqrt{b^2}x^3}{a}-\frac{\sqrt{a^2+2abx^3+b^2x^6}}{a}\right)+\frac{\sqrt{a^2+2abx^3+b^2x^6}\left(-192a^5b-395a^4b^2x^3+1920a^3b^3x^6+960a^2b^4x^9+320ab^5x^{12}+48b^6x^{15}\right)+\sqrt{b^2}\left(192a^6+587a^5bx^3-1525a^4b^2x^6-2880a^3b^3x^9-1280a^2b^4x^{12}-368ab^5x^{15}-48b^6x^{18}\right)}{576x^3\left(ab+b^2x^3\right)-576\sqrt{b^2}x^3\sqrt{a^2+2abx^3+b^2x^6}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^4,x]
```

```
[Out] (Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]*(-192*a^5*b - 395*a^4*b^2*x^3 + 1920*a^3*b
^3*x^6 + 960*a^2*b^4*x^9 + 320*a*b^5*x^12 + 48*b^6*x^15) + Sqrt[b^2]*(192*a
^6 + 587*a^5*b*x^3 - 1525*a^4*b^2*x^6 - 2880*a^3*b^3*x^9 - 1280*a^2*b^4*x^1
2 - 368*a*b^5*x^15 - 48*b^6*x^18))/(576*x^3*(a*b + b^2*x^3) - 576*Sqrt[b^2]
*x^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) + (5*a^4*b*ArcTanh[(Sqrt[b^2]*x^3)/a
- Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]/a])/3 - (5*a^4*Sqrt[b^2]*Log[-a - Sqrt[b^
2]*x^3 + Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]])/6 - (5*a^4*Sqrt[b^2]*Log[a - Sqr
t[b^2]*x^3 + Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]])/6
```

fricas [A]   time = 0.98, size = 61, normalized size = 0.24

$$\frac{3\,b^5x^{15}+20\,ab^4x^{12}+60\,a^2b^3x^9+120\,a^3b^2x^6+180\,a^4bx^3\log(x)-12\,a^5}{36\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^4,x, algorithm="fricas")
```

```
[Out] 1/36*(3*b^5*x^15 + 20*a*b^4*x^12 + 60*a^2*b^3*x^9 + 120*a^3*b^2*x^6 + 180*a
^4*b*x^3*log(x) - 12*a^5)/x^3
```

giac [A]   time = 0.31, size = 124, normalized size = 0.49

$$\frac{1}{12}b^5x^{12}\text{sgn}\left(bx^3+a\right)+\frac{5}{9}ab^4x^9\text{sgn}\left(bx^3+a\right)+\frac{5}{3}a^2b^3x^6\text{sgn}\left(bx^3+a\right)+\frac{10}{3}a^3b^2x^3\text{sgn}\left(bx^3+a\right)+5\,a^4b\log\left(|x|\right)\text{sgn}\left(bx^3+a\right)-\frac{5\,a^4bx^3\text{sgn}\left(bx^3+a\right)+a^5\text{sgn}\left(bx^3+a\right)}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^4,x, algorithm="giac")
```

```
[Out] 1/12*b^5*x^12*sgn(b*x^3 + a) + 5/9*a*b^4*x^9*sgn(b*x^3 + a) + 5/3*a^2*b^3*x
^6*sgn(b*x^3 + a) + 10/3*a^3*b^2*x^3*sgn(b*x^3 + a) + 5*a^4*b*log(abs(x))*s
gn(b*x^3 + a) - 1/3*(5*a^4*b*x^3*sgn(b*x^3 + a) + a^5*sgn(b*x^3 + a))/x^3
```

maple [A] time = 0.01, size = 82, normalized size = 0.33

$$\frac{\left(\left(b\,x^{3}+a\right)^{2}\right)^{\frac{5}{2}}\left(3b^{5}x^{15}+20a\,b^{4}x^{12}+60a^{2}b^{3}x^{9}+120a^{3}b^{2}x^{6}+180a^{4}b\,x^{3}\ln(x)-12a^{5}\right)}{36\left(b\,x^{3}+a\right)^{5}x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^4,x)
```

```
[Out] 1/36*((b*x^3+a)^2)^(5/2)*(3*b^5*x^15+20*a*b^4*x^12+60*a^2*b^3*x^9+120*a^3*b
^2*x^6+180*a^4*b*ln(x)*x^3-12*a^5)/(b*x^3+a)^5/x^3
```

maxima [A] time = 1.20, size = 214, normalized size = 0.85

$$\frac{5}{6}\sqrt{b^{2}x^{6}+2\,abx^{3}+a^{2}}\,a^{2}b^{2}x^{3}+\frac{5}{3}\,(-1)^{2\,b^{2}x^{3}+2\,ab}\,a^{4}b\log\left(2\,b^{2}x^{3}+2\,ab\right)-\frac{5}{3}\,(-1)^{2\,abx^{3}+2\,a^{2}}\,a^{4}b\log\left(\frac{2\,abx}{|x|}+\frac{2\,a^{2}}{x^{2}|x|}\right)+\frac{5}{12}\left(b^{2}x^{6}+2\,abx^{3}+a^{2}\right)^{\frac{3}{2}}b^{2}x^{3}+\frac{5}{2}\sqrt{b^{2}x^{6}+2\,abx^{3}+a^{2}}\,a^{3}b+\frac{35}{36}\left(b^{2}x^{6}+2\,abx^{3}+a^{2}\right)^{\frac{3}{2}}ab-\frac{\left(b^{2}x^{6}+2\,abx^{3}+a^{2}\right)^{\frac{5}{2}}}{3\,x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^4,x, algorithm="maxima")
```

```
[Out] 5/6*sqrt(b^2*x^6 + 2*a*b*x^3 + a^2)*a^2*b^2*x^3 + 5/3*(-1)^(2*b^2*x^3 + 2*a
*b)*a^4*b*log(2*b^2*x^3 + 2*a*b) - 5/3*(-1)^(2*a*b*x^3 + 2*a^2)*a^4*b*log(2
*a*b*x/abs(x) + 2*a^2/(x^2*abs(x))) + 5/12*(b^2*x^6 + 2*a*b*x^3 + a^2)^(3/2
)*b^2*x^3 + 5/2*sqrt(b^2*x^6 + 2*a*b*x^3 + a^2)*a^3*b + 35/36*(b^2*x^6 + 2*
a*b*x^3 + a^2)^(3/2)*a*b - 1/3*(b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2)/x^3
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a^{2}+2\,a\,b\,x^{3}+b^{2}\,x^{6}\right)^{5/2}}{x^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2)/x^4,x)
```

```
[Out] int((a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2)/x^4, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\left(a+bx^{3}\right)^{2}\right)^{\frac{5}{2}}}{x^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**6+2*a*b*x**3+a**2)**(5/2)/x**4,x)
```

```
[Out] Integral(((a + b*x**3)**2)**(5/2)/x**4, x)
```

3.68 $\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^5}\,dx$

Optimal. Leaf size=249

$$\frac{b^5x^{11}\sqrt{a^2+2abx^3+b^2x^6}}{11(a+bx^3)}+\frac{5ab^4x^8\sqrt{a^2+2abx^3+b^2x^6}}{8(a+bx^3)}+\frac{2a^2b^3x^5\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}-\frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{4x^4(a+bx^3)}-\frac{5}{}$$

Rubi [A] time = 0.06, antiderivative size = 249, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {1355, 270}

$$\frac{b^5x^{11}\sqrt{a^2+2abx^3+b^2x^6}}{11(a+bx^3)}+\frac{5ab^4x^8\sqrt{a^2+2abx^3+b^2x^6}}{8(a+bx^3)}+\frac{2a^2b^3x^5\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}+\frac{5a^3b^2x^2\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}-\frac{5a^4b\sqrt{a^2+2abx^3+b^2x^6}}{x(a+bx^3)}-\frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{4x^4(a+bx^3)}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^5,x]
```

```
[Out] -(a^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(4*x^4*(a + b*x^3)) - (5*a^4*b*Sqrt[
a^2 + 2*a*b*x^3 + b^2*x^6])/(x*(a + b*x^3)) + (5*a^3*b^2*x^2*Sqrt[a^2 + 2*a
*b*x^3 + b^2*x^6])/(a + b*x^3) + (2*a^2*b^3*x^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*
x^6])/(a + b*x^3) + (5*a*b^4*x^8*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(8*(a + b
*x^3)) + (b^5*x^11*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(11*(a + b*x^3))
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\begin{aligned}\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^5}\,dx &= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{(ab+b^2x^3)^5}{x^5}\,dx}{b^4(ab+b^2x^3)}\\ &= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \left(\frac{a^5b^5}{x^5}+\frac{5a^4b^6}{x^2}+10a^3b^7x+10a^2b^8x^4+5ab^9x^7+b^{10}x^{10}\right)dx}{b^4(ab+b^2x^3)}\\ &= -\frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{4x^4(a+bx^3)}-\frac{5a^4b\sqrt{a^2+2abx^3+b^2x^6}}{x(a+bx^3)}+\frac{5a^3b^2x^2\sqrt{a^2+2abx^3+b^2}}{a+bx^3}\end{aligned}$$

Mathematica [A] time = 0.02, size = 83, normalized size = 0.33

$$\frac{\sqrt{(a+bx^3)^2}\left(-22a^5-440a^4bx^3+440a^3b^2x^6+176a^2b^3x^9+55ab^4x^{12}+8b^5x^{15}\right)}{88x^4(a+bx^3)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^5,x]
```

```
[Out] (Sqrt[(a + b*x^3)^2]*(-22*a^5 - 440*a^4*b*x^3 + 440*a^3*b^2*x^6 + 176*a^2*b
^3*x^9 + 55*a*b^4*x^12 + 8*b^5*x^15))/(88*x^4*(a + b*x^3))
```

IntegrateAlgebraic [A] time = 18.55, size = 83, normalized size = 0.33

$$\frac{\sqrt{\left(a+bx^3\right)^2}\left(-22a^5-440a^4bx^3+440a^3b^2x^6+176a^2b^3x^9+55ab^4x^{12}+8b^5x^{15}\right)}{88x^4\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^5,x]
```

```
[Out] (Sqrt[(a + b*x^3)^2]*(-22*a^5 - 440*a^4*b*x^3 + 440*a^3*b^2*x^6 + 176*a^2*b
^3*x^9 + 55*a*b^4*x^12 + 8*b^5*x^15))/(88*x^4*(a + b*x^3))
```

fricas [A] time = 1.34, size = 59, normalized size = 0.24

$$\frac{8\,b^5x^{15}+55\,ab^4x^{12}+176\,a^2b^3x^9+440\,a^3b^2x^6-440\,a^4bx^3-22\,a^5}{88\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^5,x, algorithm="fricas")
```

```
[Out] 1/88*(8*b^5*x^15 + 55*a*b^4*x^12 + 176*a^2*b^3*x^9 + 440*a^3*b^2*x^6 - 440*
a^4*b*x^3 - 22*a^5)/x^4
```

giac [A] time = 0.35, size = 107, normalized size = 0.43

$$\frac{1}{11}\,b^5x^{11}\mathrm{sgn}\left(bx^3+a\right)+\frac{5}{8}\,ab^4x^8\mathrm{sgn}\left(bx^3+a\right)+2\,a^2b^3x^5\mathrm{sgn}\left(bx^3+a\right)+5\,a^3b^2x^2\mathrm{sgn}\left(bx^3+a\right)-\frac{20\,a^4bx^3\mathrm{sgn}\left(bx^3+a\right)+a^5\mathrm{sgn}\left(bx^3+a\right)}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^5,x, algorithm="giac")
```

```
[Out] 1/11*b^5*x^11*sgn(b*x^3 + a) + 5/8*a*b^4*x^8*sgn(b*x^3 + a) + 2*a^2*b^3*x^5
*sgn(b*x^3 + a) + 5*a^3*b^2*x^2*sgn(b*x^3 + a) - 1/4*(20*a^4*b*x^3*sgn(b*x^
3 + a) + a^5*sgn(b*x^3 + a))/x^4
```

maple [A] time = 0.01, size = 80, normalized size = 0.32

$$-\frac{\left(-8b^5x^{15}-55a\,b^4x^{12}-176a^2b^3x^9-440a^3b^2x^6+440a^4b\,x^3+22a^5\right)\left(\left(b\,x^3+a\right)^2\right)^{\frac{5}{2}}}{88\left(b\,x^3+a\right)^5x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^5,x)
```

```
[Out] -1/88*(-8*b^5*x^15-55*a*b^4*x^12-176*a^2*b^3*x^9-440*a^3*b^2*x^6+440*a^4*b*
x^3+22*a^5)*((b*x^3+a)^2)^(5/2)/x^4/(b*x^3+a)^5
```

maxima [A] time = 0.97, size = 59, normalized size = 0.24

$$\frac{8\,b^5x^{15}+55\,ab^4x^{12}+176\,a^2b^3x^9+440\,a^3b^2x^6-440\,a^4bx^3-22\,a^5}{88\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^5,x, algorithm="maxima")
```

```
[Out] 1/88*(8*b^5*x^15 + 55*a*b^4*x^12 + 176*a^2*b^3*x^9 + 440*a^3*b^2*x^6 - 440*
a^4*b*x^3 - 22*a^5)/x^4
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a^2+2\,a\,b\,x^3+b^2\,x^6\right)^{5/2}}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2)/x^5,x)
```

```
[Out] int((a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2)/x^5, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\left(a+bx^3\right)^2\right)^{\frac{5}{2}}}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**6+2*a*b*x**3+a**2)**(5/2)/x**5,x)
```

```
[Out] Integral(((a + b*x**3)**2)**(5/2)/x**5, x)
```

3.69 $\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{x^6}\,dx$

Optimal. Leaf size=251

$$\frac{b^5x^{10}\sqrt{a^2+2abx^3+b^2x^6}}{10\left(a+bx^3\right)}+\frac{5ab^4x^7\sqrt{a^2+2abx^3+b^2x^6}}{7\left(a+bx^3\right)}+\frac{5a^2b^3x^4\sqrt{a^2+2abx^3+b^2x^6}}{2\left(a+bx^3\right)}-\frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{5x^5\left(a+bx^3\right)}$$

Rubi [A] time = 0.06, antiderivative size = 251, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1355, 270}

$$\frac{b^5x^{10}\sqrt{a^2+2abx^3+b^2x^6}}{10\left(a+bx^3\right)}+\frac{5ab^4x^7\sqrt{a^2+2abx^3+b^2x^6}}{7\left(a+bx^3\right)}+\frac{5a^2b^3x^4\sqrt{a^2+2abx^3+b^2x^6}}{2\left(a+bx^3\right)}+\frac{10a^3b^2x\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}-\frac{5a^4b\sqrt{a^2+2abx^3+b^2x^6}}{2x^2\left(a+bx^3\right)}-\frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{5x^5\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^6,x]
```

```
[Out] -(a^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(5*x^5*(a + b*x^3)) - (5*a^4*b*Sqrt[
a^2 + 2*a*b*x^3 + b^2*x^6])/(2*x^2*(a + b*x^3)) + (10*a^3*b^2*x*Sqrt[a^2 +
2*a*b*x^3 + b^2*x^6])/(a + b*x^3) + (5*a^2*b^3*x^4*Sqrt[a^2 + 2*a*b*x^3 + b
^2*x^6])/(2*(a + b*x^3)) + (5*a*b^4*x^7*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(7
*(a + b*x^3)) + (b^5*x^10*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(10*(a + b*x^3))
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{x^6}\,dx &= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{\left(ab+b^2x^3\right)^5}{x^6}\,dx}{b^4\left(ab+b^2x^3\right)}\\ &= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \left(10a^3b^7+\frac{a^5b^5}{x^6}+\frac{5a^4b^6}{x^3}+10a^2b^8x^3+5ab^9x^6+b^{10}x^9\right)\,d}{b^4\left(ab+b^2x^3\right)}\\ &= -\frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{5x^5\left(a+bx^3\right)}-\frac{5a^4b\sqrt{a^2+2abx^3+b^2x^6}}{2x^2\left(a+bx^3\right)}+\frac{10a^3b^2x\sqrt{a^2+2abx^3+}}{a+bx^3}\end{aligned}$$

Mathematica [A] time = 0.02, size = 83, normalized size = 0.33

$$\frac{\sqrt{\left(a+bx^3\right)^2}\left(-14a^5-175a^4bx^3+700a^3b^2x^6+175a^2b^3x^9+50ab^4x^{12}+7b^5x^{15}\right)}{70x^5\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^6,x]
```

```
[Out] (Sqrt[(a + b*x^3)^2]*(-14*a^5 - 175*a^4*b*x^3 + 700*a^3*b^2*x^6 + 175*a^2*b
^3*x^9 + 50*a*b^4*x^12 + 7*b^5*x^15))/(70*x^5*(a + b*x^3))
```

IntegrateAlgebraic [A] time = 18.31, size = 83, normalized size = 0.33

$$\frac{\sqrt{\left(a+bx^3\right)^2}\,\left(-14a^5-175a^4bx^3+700a^3b^2x^6+175a^2b^3x^9+50ab^4x^{12}+7b^5x^{15}\right)}{70x^5\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^6,x]
```

```
[Out] (Sqrt[(a + b*x^3)^2]*(-14*a^5 - 175*a^4*b*x^3 + 700*a^3*b^2*x^6 + 175*a^2*b
^3*x^9 + 50*a*b^4*x^12 + 7*b^5*x^15))/(70*x^5*(a + b*x^3))
```

fricas [A] time = 0.71, size = 59, normalized size = 0.24

$$\frac{7\,b^5x^{15}+50\,ab^4x^{12}+175\,a^2b^3x^9+700\,a^3b^2x^6-175\,a^4bx^3-14\,a^5}{70\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^6,x, algorithm="fricas")
```

```
[Out] 1/70*(7*b^5*x^15 + 50*a*b^4*x^12 + 175*a^2*b^3*x^9 + 700*a^3*b^2*x^6 - 175*
a^4*b*x^3 - 14*a^5)/x^5
```

giac [A] time = 0.34, size = 106, normalized size = 0.42

$$\frac{1}{10}\,b^5x^{10}\mathrm{sgn}\left(bx^3+a\right)+\frac{5}{7}\,ab^4x^7\mathrm{sgn}\left(bx^3+a\right)+\frac{5}{2}\,a^2b^3x^4\mathrm{sgn}\left(bx^3+a\right)+10\,a^3b^2x\mathrm{sgn}\left(bx^3+a\right)-\frac{25\,a^4bx^3\mathrm{sgn}\left(bx^3+a\right)+2\,a^5\mathrm{sgn}\left(bx^3+a\right)}{10\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^6,x, algorithm="giac")
```

```
[Out] 1/10*b^5*x^10*sgn(b*x^3 + a) + 5/7*a*b^4*x^7*sgn(b*x^3 + a) + 5/2*a^2*b^3*x
^4*sgn(b*x^3 + a) + 10*a^3*b^2*x*sgn(b*x^3 + a) - 1/10*(25*a^4*b*x^3*sgn(b*
x^3 + a) + 2*a^5*sgn(b*x^3 + a))/x^5
```

maple [A] time = 0.01, size = 80, normalized size = 0.32

$$-\frac{\left(-7b^5x^{15}-50a\,b^4x^{12}-175a^2b^3x^9-700a^3b^2x^6+175a^4b\,x^3+14a^5\right)\left(\left(b\,x^3+a\right)^2\right)^{\frac{5}{2}}}{70\left(b\,x^3+a\right)^5x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^6,x)
```

```
[Out] -1/70*(-7*b^5*x^15-50*a*b^4*x^12-175*a^2*b^3*x^9-700*a^3*b^2*x^6+175*a^4*b*
x^3+14*a^5)*((b*x^3+a)^2)^(5/2)/x^5/(b*x^3+a)^5
```

maxima [A] time = 0.67, size = 59, normalized size = 0.24

$$\frac{7\,b^5x^{15}+50\,ab^4x^{12}+175\,a^2b^3x^9+700\,a^3b^2x^6-175\,a^4bx^3-14\,a^5}{70\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^6,x, algorithm="maxima")
```

```
[Out] 1/70*(7*b^5*x^15 + 50*a*b^4*x^12 + 175*a^2*b^3*x^9 + 700*a^3*b^2*x^6 - 175*
a^4*b*x^3 - 14*a^5)/x^5
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a^2+2\,a\,b\,x^3+b^2\,x^6\right)^{5/2}}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2)/x^6,x)
```

```
[Out] int((a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2)/x^6, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\left(a+bx^3\right)^2\right)^{\frac{5}{2}}}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**6+2*a*b*x**3+a**2)**(5/2)/x**6,x)
```

```
[Out] Integral(((a + b*x**3)**2)**(5/2)/x**6, x)
```

3.70 $\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^7}\,dx$

Optimal. Leaf size=252

$$\frac{b^5x^9\sqrt{a^2+2abx^3+b^2x^6}}{9\left(a+bx^3\right)}+\frac{5ab^4x^6\sqrt{a^2+2abx^3+b^2x^6}}{6\left(a+bx^3\right)}+\frac{10a^2b^3x^3\sqrt{a^2+2abx^3+b^2x^6}}{3\left(a+bx^3\right)}-\frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{6x^6\left(a+bx^3\right)}-5$$

Rubi [A] time = 0.07, antiderivative size = 252, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1355, 266, 43}

$$\frac{b^5x^9\sqrt{a^2+2abx^3+b^2x^6}}{9\left(a+bx^3\right)}+\frac{5ab^4x^6\sqrt{a^2+2abx^3+b^2x^6}}{6\left(a+bx^3\right)}+\frac{10a^2b^3x^3\sqrt{a^2+2abx^3+b^2x^6}}{3\left(a+bx^3\right)}-\frac{5a^4b\sqrt{a^2+2abx^3+b^2x^6}}{3x^3\left(a+bx^3\right)}-\frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{6x^6\left(a+bx^3\right)}+\frac{10a^3b^2\log(x)\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^7,x]
```

```
[Out] -(a^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(6*x^6*(a + b*x^3)) - (5*a^4*b*Sqrt[
a^2 + 2*a*b*x^3 + b^2*x^6])/(3*x^3*(a + b*x^3)) + (10*a^2*b^3*x^3*Sqrt[a^2
+ 2*a*b*x^3 + b^2*x^6])/(3*(a + b*x^3)) + (5*a*b^4*x^6*Sqrt[a^2 + 2*a*b*x^3
 + b^2*x^6])/(6*(a + b*x^3)) + (b^5*x^9*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(9
*(a + b*x^3)) + (10*a^3*b^2*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]*Log[x])/(a + b*
x^3)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{x^7}\,dx &= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{\left(ab+b^2x^3\right)^5}{x^7}dx}{b^4\left(ab+b^2x^3\right)} \\
&= \frac{\sqrt{a^2+2abx^3+b^2x^6}\ \mathrm{Subst}\left(\int \frac{\left(ab+b^2x\right)^5}{x^3}dx, x, x^3\right)}{3b^4\left(ab+b^2x^3\right)} \\
&= \frac{\sqrt{a^2+2abx^3+b^2x^6}\ \mathrm{Subst}\left(\int \left(10a^2b^8+\frac{a^5b^5}{x^3}+\frac{5a^4b^6}{x^2}+\frac{10a^3b^7}{x}+5ab^9x+b^{10}x^2\right.\right.}{3b^4\left(ab+b^2x^3\right)} \\
&= -\frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{6x^6\left(a+bx^3\right)}-\frac{5a^4b\sqrt{a^2+2abx^3+b^2x^6}}{3x^3\left(a+bx^3\right)}+\frac{10a^2b^3x^3\sqrt{a^2+2abx^3}}{3\left(a+bx^3\right)}
\end{aligned}
$$

Mathematica [A] time = 0.02, size = 85, normalized size = 0.34

$$
\frac{\sqrt{\left(a+bx^3\right)^2}\left(-3a^5-30a^4bx^3+180a^3b^2x^6\log(x)+60a^2b^3x^9+15ab^4x^{12}+2b^5x^{15}\right)}{18x^6\left(a+bx^3\right)}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^7,x]
```

```
[Out] (Sqrt[(a + b*x^3)^2]*(-3*a^5 - 30*a^4*b*x^3 + 60*a^2*b^3*x^9 + 15*a*b^4*x^1
2 + 2*b^5*x^15 + 180*a^3*b^2*x^6*Log[x]))/(18*x^6*(a + b*x^3))
```

IntegrateAlgebraic [A] time = 1.16, size = 368, normalized size = 1.46

$$
-\frac{5}{3}a^3b\sqrt{b^2}\log\left(\sqrt{a^2+2abx^3+b^2x^6}-a-\sqrt{b^2}x^3\right)-\frac{5}{3}a^3b\sqrt{b^2}\log\left(\sqrt{a^2+2abx^3+b^2x^6}+a-\sqrt{b^2}x^3\right)+\frac{10}{3}a^3b^2\tanh^{-1}\left(\frac{\sqrt{b^2}x^3}{a}-\frac{\sqrt{a^2+2abx^3+b^2x^6}}{a}\right)+\frac{\sqrt{a^2+2abx^3+b^2x^6}\left(-6a^5b-60a^4b^2x^3+53a^3b^3x^6+120a^2b^4x^9+30ab^5x^{12}+4b^6x^{15}\right)+\sqrt{b^2}\left(6a^6+66a^5bx^3+7a^4b^2x^6-173a^3b^3x^9-150a^2b^4x^{12}-34ab^5x^{15}-4b^6x^{18}\right)}{36x^6\left(ab+b^2x^3\right)-36\sqrt{b^2}x^6\sqrt{a^2+2abx^3+b^2x^6}}
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^7,x]
```

```
[Out] (Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]*(-6*a^5*b - 60*a^4*b^2*x^3 + 53*a^3*b^3*x^
6 + 120*a^2*b^4*x^9 + 30*a*b^5*x^12 + 4*b^6*x^15) + Sqrt[b^2]*(6*a^6 + 66*a
^5*b*x^3 + 7*a^4*b^2*x^6 - 173*a^3*b^3*x^9 - 150*a^2*b^4*x^12 - 34*a*b^5*x^
15 - 4*b^6*x^18))/(36*x^6*(a*b + b^2*x^3) - 36*Sqrt[b^2]*x^6*Sqrt[a^2 + 2*a
*b*x^3 + b^2*x^6]) + (10*a^3*b^2*ArcTanh[(Sqrt[b^2]*x^3)/a - Sqrt[a^2 + 2*a
*b*x^3 + b^2*x^6]/a])/3 - (5*a^3*b*Sqrt[b^2]*Log[-a - Sqrt[b^2]*x^3 + Sqrt[
a^2 + 2*a*b*x^3 + b^2*x^6]])/3 - (5*a^3*b*Sqrt[b^2]*Log[a - Sqrt[b^2]*x^3 +
 Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]])/3
```

fricas [A] time = 1.31, size = 61, normalized size = 0.24

$$
\frac{2\,b^5x^{15}+15\,ab^4x^{12}+60\,a^2b^3x^9+180\,a^3b^2x^6\log(x)-30\,a^4bx^3-3\,a^5}{18\,x^6}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^7,x, algorithm="fricas")
```

```
[Out] 1/18*(2*b^5*x^15 + 15*a*b^4*x^12 + 60*a^2*b^3*x^9 + 180*a^3*b^2*x^6*log(x)
- 30*a^4*b*x^3 - 3*a^5)/x^6
```

giac [A] time = 0.37, size = 126, normalized size = 0.50

$$
\frac{1}{9}\,b^5x^9\mathrm{sgn}\left(bx^3+a\right)+\frac{5}{6}\,ab^4x^6\mathrm{sgn}\left(bx^3+a\right)+\frac{10}{3}\,a^2b^3x^3\mathrm{sgn}\left(bx^3+a\right)+10\,a^3b^2\log\left(|x|\right)\mathrm{sgn}\left(bx^3+a\right)-\frac{30\,a^3b^2x^6\mathrm{sgn}\left(bx^3+a\right)+10\,a^4bx^3\mathrm{sgn}\left(bx^3+a\right)+a^5\mathrm{sgn}\left(bx^3+a\right)}{6\,x^6}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^7,x, algorithm="giac")
```

```
[Out] 1/9*b^5*x^9*sgn(b*x^3 + a) + 5/6*a*b^4*x^6*sgn(b*x^3 + a) + 10/3*a^2*b^3*x^
3*sgn(b*x^3 + a) + 10*a^3*b^2*log(abs(x))*sgn(b*x^3 + a) - 1/6*(30*a^3*b^2*
x^6*sgn(b*x^3 + a) + 10*a^4*b*x^3*sgn(b*x^3 + a) + a^5*sgn(b*x^3 + a))/x^6
```

maple [A] time = 0.01, size = 82, normalized size = 0.33

$$\frac{\left(\left(b\,x^{3}+a\right)^{2}\right)^{\frac{5}{2}}\left(2b^{5}x^{15}+15a\,b^{4}x^{12}+60a^{2}b^{3}x^{9}+180a^{3}b^{2}x^{6}\ln(x)-30a^{4}b\,x^{3}-3a^{5}\right)}{18\left(b\,x^{3}+a\right)^{5}x^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^7,x)
```

```
[Out] 1/18*((b*x^3+a)^2)^(5/2)*(2*b^5*x^15+15*a*b^4*x^12+60*a^2*b^3*x^9+180*a^3*b
^2*ln(x)*x^6-30*a^4*b*x^3-3*a^5)/(b*x^3+a)^5/x^6
```

maxima [A] time = 1.24, size = 282, normalized size = 1.12

$$\frac{5}{3}\sqrt{b^{2}x^{6}+2\,abx^{3}+a^{2}}\,ab^{3}x^{3}+\frac{10}{3}\,(-1)^{2\,b^{2}x^{3}+2\,ab}\,a^{3}b^{2}\log\left(2\,b^{2}x^{3}+2\,ab\right)-\frac{10}{3}\,(-1)^{2\,abx^{3}+2\,a^{2}}\,a^{3}b^{2}\log\left(\frac{2\,abx}{|x|}+\frac{2\,a^{2}}{x^{2}|x|}\right)+\frac{5\left(b^{2}x^{6}+2\,abx^{3}+a^{2}\right)^{\frac{3}{2}}b^{3}x^{3}}{6\,a}+5\sqrt{b^{2}x^{6}+2\,abx^{3}+a^{2}}\,a^{2}b^{2}+\frac{35}{18}\left(b^{2}x^{6}+2\,abx^{3}+a^{2}\right)^{\frac{3}{2}}b^{2}+\frac{\left(b^{2}x^{6}+2\,abx^{3}+a^{2}\right)^{\frac{5}{2}}b^{2}}{6\,a^{2}}-\frac{\left(b^{2}x^{6}+2\,abx^{3}+a^{2}\right)^{\frac{5}{2}}b}{2\,ax^{3}}-\frac{\left(b^{2}x^{6}+2\,abx^{3}+a^{2}\right)^{\frac{7}{2}}}{6\,a^{2}x^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^7,x, algorithm="maxima")
```

```
[Out] 5/3*sqrt(b^2*x^6 + 2*a*b*x^3 + a^2)*a*b^3*x^3 + 10/3*(-1)^(2*b^2*x^3 + 2*a*
b)*a^3*b^2*log(2*b^2*x^3 + 2*a*b) - 10/3*(-1)^(2*a*b*x^3 + 2*a^2)*a^3*b^2*l
og(2*a*b*x/abs(x) + 2*a^2/(x^2*abs(x))) + 5/6*(b^2*x^6 + 2*a*b*x^3 + a^2)^(
3/2)*b^3*x^3/a + 5*sqrt(b^2*x^6 + 2*a*b*x^3 + a^2)*a^2*b^2 + 35/18*(b^2*x^6
 + 2*a*b*x^3 + a^2)^(3/2)*b^2 + 1/6*(b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2)*b^2/a
^2 - 1/2*(b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2)*b/(a*x^3) - 1/6*(b^2*x^6 + 2*a*b
*x^3 + a^2)^(7/2)/(a^2*x^6)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a^{2}+2\,a\,b\,x^{3}+b^{2}\,x^{6}\right)^{5/2}}{x^{7}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2)/x^7,x)
```

```
[Out] int((a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2)/x^7, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\left(a+bx^{3}\right)^{2}\right)^{\frac{5}{2}}}{x^{7}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**6+2*a*b*x**3+a**2)**(5/2)/x**7,x)
```

```
[Out] Integral(((a + b*x**3)**2)**(5/2)/x**7, x)
```

3.71 $\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{x^8}\,dx$

Optimal. Leaf size=248

$$\frac{b^5x^8\sqrt{a^2+2abx^3+b^2x^6}}{8\left(a+bx^3\right)}+\frac{ab^4x^5\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}+\frac{5a^2b^3x^2\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}-\frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{7x^7\left(a+bx^3\right)}-$$

Rubi [A] time = 0.06, antiderivative size = 248, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {1355, 270}

$$\frac{b^5x^8\sqrt{a^2+2abx^3+b^2x^6}}{8\left(a+bx^3\right)}+\frac{ab^4x^5\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}+\frac{5a^2b^3x^2\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}-\frac{10a^3b^2\sqrt{a^2+2abx^3+b^2x^6}}{x\left(a+bx^3\right)}-\frac{5a^4b\sqrt{a^2+2abx^3+b^2x^6}}{4x^4\left(a+bx^3\right)}-\frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{7x^7\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^8,x]
```

```
[Out] -(a^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(7*x^7*(a + b*x^3)) - (5*a^4*b*Sqrt[
a^2 + 2*a*b*x^3 + b^2*x^6])/(4*x^4*(a + b*x^3)) - (10*a^3*b^2*Sqrt[a^2 + 2*
a*b*x^3 + b^2*x^6])/(x*(a + b*x^3)) + (5*a^2*b^3*x^2*Sqrt[a^2 + 2*a*b*x^3 +
 b^2*x^6])/(a + b*x^3) + (a*b^4*x^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(a + b
*x^3) + (b^5*x^8*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(8*(a + b*x^3))
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{x^8}\,dx &= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{\left(ab+b^2x^3\right)^5}{x^8}\,dx}{b^4\left(ab+b^2x^3\right)}\\
&= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \left(\frac{a^5b^5}{x^8}+\frac{5a^4b^6}{x^5}+\frac{10a^3b^7}{x^2}+10a^2b^8x+5ab^9x^4+b^{10}x^7\right)dx}{b^4\left(ab+b^2x^3\right)}\\
&= -\frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{7x^7\left(a+bx^3\right)}-\frac{5a^4b\sqrt{a^2+2abx^3+b^2x^6}}{4x^4\left(a+bx^3\right)}-\frac{10a^3b^2\sqrt{a^2+2abx^3+}}{x\left(a+bx^3\right)}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 83, normalized size = 0.33

$$\frac{\sqrt{\left(a+bx^3\right)^2}\left(-8a^5-70a^4bx^3-560a^3b^2x^6+280a^2b^3x^9+56ab^4x^{12}+7b^5x^{15}\right)}{56x^7\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^8,x]
```

```
[Out] (Sqrt[(a + b*x^3)^2]*(-8*a^5 - 70*a^4*b*x^3 - 560*a^3*b^2*x^6 + 280*a^2*b^3
*x^9 + 56*a*b^4*x^12 + 7*b^5*x^15))/(56*x^7*(a + b*x^3))
```

IntegrateAlgebraic [A] time = 18.57, size = 83, normalized size = 0.33

$$\frac{\sqrt{\left(a+bx^3\right)^2}\left(-8a^5-70a^4bx^3-560a^3b^2x^6+280a^2b^3x^9+56ab^4x^{12}+7b^5x^{15}\right)}{56x^7\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^8,x]
```

```
[Out] (Sqrt[(a + b*x^3)^2]*(-8*a^5 - 70*a^4*b*x^3 - 560*a^3*b^2*x^6 + 280*a^2*b^3
*x^9 + 56*a*b^4*x^12 + 7*b^5*x^15))/(56*x^7*(a + b*x^3))
```

fricas [A] time = 1.14, size = 59, normalized size = 0.24

$$\frac{7\,b^5x^{15}+56\,ab^4x^{12}+280\,a^2b^3x^9-560\,a^3b^2x^6-70\,a^4bx^3-8\,a^5}{56\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^8,x, algorithm="fricas")
```

```
[Out] 1/56*(7*b^5*x^15 + 56*a*b^4*x^12 + 280*a^2*b^3*x^9 - 560*a^3*b^2*x^6 - 70*a
^4*b*x^3 - 8*a^5)/x^7
```

giac [A] time = 0.33, size = 107, normalized size = 0.43

$$\frac{1}{8}\,b^5x^8\mathrm{sgn}\left(bx^3+a\right)+ab^4x^5\mathrm{sgn}\left(bx^3+a\right)+5\,a^2b^3x^2\mathrm{sgn}\left(bx^3+a\right)-\frac{280\,a^3b^2x^6\mathrm{sgn}\left(bx^3+a\right)+35\,a^4bx^3\mathrm{sgn}\left(bx^3+a\right)+4\,a^5\mathrm{sgn}\left(bx^3+a\right)}{28\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^8,x, algorithm="giac")
```

```
[Out] 1/8*b^5*x^8*sgn(b*x^3 + a) + a*b^4*x^5*sgn(b*x^3 + a) + 5*a^2*b^3*x^2*sgn(b
*x^3 + a) - 1/28*(280*a^3*b^2*x^6*sgn(b*x^3 + a) + 35*a^4*b*x^3*sgn(b*x^3 +
 a) + 4*a^5*sgn(b*x^3 + a))/x^7
```

maple [A] time = 0.01, size = 80, normalized size = 0.32

$$-\frac{\left(-7b^5x^{15}-56a\,b^4x^{12}-280a^2b^3x^9+560a^3b^2x^6+70a^4b\,x^3+8a^5\right)\left(\left(b\,x^3+a\right)^2\right)^{\frac{5}{2}}}{56\left(b\,x^3+a\right)^5x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^8,x)
```

```
[Out] -1/56*(-7*b^5*x^15-56*a*b^4*x^12-280*a^2*b^3*x^9+560*a^3*b^2*x^6+70*a^4*b*x
^3+8*a^5)*((b*x^3+a)^2)^(5/2)/x^7/(b*x^3+a)^5
```

maxima [A] time = 1.10, size = 59, normalized size = 0.24

$$\frac{7\,b^5x^{15}+56\,ab^4x^{12}+280\,a^2b^3x^9-560\,a^3b^2x^6-70\,a^4bx^3-8\,a^5}{56\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^8,x, algorithm="maxima")
```

```
[Out] 1/56*(7*b^5*x^15 + 56*a*b^4*x^12 + 280*a^2*b^3*x^9 - 560*a^3*b^2*x^6 - 70*a
^4*b*x^3 - 8*a^5)/x^7
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a^2+2\,a\,b\,x^3+b^2\,x^6\right)^{5/2}}{x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2)/x^8,x)
```

```
[Out] int((a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2)/x^8, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\left(a+bx^3\right)^2\right)^{\frac{5}{2}}}{x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**6+2*a*b*x**3+a**2)**(5/2)/x**8,x)
```

```
[Out] Integral(((a + b*x**3)**2)**(5/2)/x**8, x)
```

3.72 $\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{x^9}\,dx$

Optimal. Leaf size=247

$$\frac{b^5x^7\sqrt{a^2+2abx^3+b^2x^6}}{7\left(a+bx^3\right)}+\frac{5ab^4x^4\sqrt{a^2+2abx^3+b^2x^6}}{4\left(a+bx^3\right)}+\frac{10a^2b^3x\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}-\frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{8x^8\left(a+bx^3\right)}-\cdots$$

Rubi [A] time = 0.06, antiderivative size = 247, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1355, 270}

$$\frac{b^5x^7\sqrt{a^2+2abx^3+b^2x^6}}{7\left(a+bx^3\right)}+\frac{5ab^4x^4\sqrt{a^2+2abx^3+b^2x^6}}{4\left(a+bx^3\right)}+\frac{10a^2b^3x\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}-\frac{5a^3b^2\sqrt{a^2+2abx^3+b^2x^6}}{x^2\left(a+bx^3\right)}-\frac{a^4b\sqrt{a^2+2abx^3+b^2x^6}}{x^5\left(a+bx^3\right)}-\frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{8x^8\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^9,x]
```

```
[Out] -(a^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(8*x^8*(a + b*x^3)) - (a^4*b*Sqrt[a^
2 + 2*a*b*x^3 + b^2*x^6])/(x^5*(a + b*x^3)) - (5*a^3*b^2*Sqrt[a^2 + 2*a*b*x
^3 + b^2*x^6])/(x^2*(a + b*x^3)) + (10*a^2*b^3*x*Sqrt[a^2 + 2*a*b*x^3 + b^2
*x^6])/(a + b*x^3) + (5*a*b^4*x^4*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(4*(a +
b*x^3)) + (b^5*x^7*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(7*(a + b*x^3))
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{x^9}\,dx &= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{\left(ab+b^2x^3\right)^5}{x^9}\,dx}{b^4\left(ab+b^2x^3\right)}\\
&= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \left(10a^2b^8+\frac{a^5b^5}{x^9}+\frac{5a^4b^6}{x^6}+\frac{10a^3b^7}{x^3}+5ab^9x^3+b^{10}x^6\right)dx}{b^4\left(ab+b^2x^3\right)}\\
&= -\frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{8x^8\left(a+bx^3\right)}-\frac{a^4b\sqrt{a^2+2abx^3+b^2x^6}}{x^5\left(a+bx^3\right)}-\frac{5a^3b^2\sqrt{a^2+2abx^3+b^2x^6}}{x^2\left(a+bx^3\right)}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 83, normalized size = 0.34

$$\frac{\sqrt{\left(a+bx^3\right)^2}\left(-7a^5-56a^4bx^3-280a^3b^2x^6+560a^2b^3x^9+70ab^4x^{12}+8b^5x^{15}\right)}{56x^8\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^9,x]
```

```
[Out] (Sqrt[(a + b*x^3)^2]*(-7*a^5 - 56*a^4*b*x^3 - 280*a^3*b^2*x^6 + 560*a^2*b^3
*x^9 + 70*a*b^4*x^12 + 8*b^5*x^15))/(56*x^8*(a + b*x^3))
```

IntegrateAlgebraic [A] time = 18.47, size = 83, normalized size = 0.34

$$\frac{\sqrt{\left(a+bx^3\right)^2}\left(-7a^5-56a^4bx^3-280a^3b^2x^6+560a^2b^3x^9+70ab^4x^{12}+8b^5x^{15}\right)}{56x^8\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^9,x]
```

```
[Out] (Sqrt[(a + b*x^3)^2]*(-7*a^5 - 56*a^4*b*x^3 - 280*a^3*b^2*x^6 + 560*a^2*b^3
*x^9 + 70*a*b^4*x^12 + 8*b^5*x^15))/(56*x^8*(a + b*x^3))
```

fricas [A] time = 1.29, size = 59, normalized size = 0.24

$$\frac{8\,b^5x^{15}+70\,ab^4x^{12}+560\,a^2b^3x^9-280\,a^3b^2x^6-56\,a^4bx^3-7\,a^5}{56\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^9,x, algorithm="fricas")
```

```
[Out] 1/56*(8*b^5*x^15 + 70*a*b^4*x^12 + 560*a^2*b^3*x^9 - 280*a^3*b^2*x^6 - 56*a
^4*b*x^3 - 7*a^5)/x^8
```

giac [A] time = 0.34, size = 105, normalized size = 0.43

$$\frac{1}{7}\,b^5x^7\mathrm{sgn}\left(bx^3+a\right)+\frac{5}{4}\,ab^4x^4\mathrm{sgn}\left(bx^3+a\right)+10\,a^2b^3x\mathrm{sgn}\left(bx^3+a\right)-\frac{40\,a^3b^2x^6\mathrm{sgn}\left(bx^3+a\right)+8\,a^4bx^3\mathrm{sgn}\left(bx^3+a\right)+a^5\mathrm{sgn}\left(bx^3+a\right)}{8\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^9,x, algorithm="giac")
```

```
[Out] 1/7*b^5*x^7*sgn(b*x^3 + a) + 5/4*a*b^4*x^4*sgn(b*x^3 + a) + 10*a^2*b^3*x*sg
n(b*x^3 + a) - 1/8*(40*a^3*b^2*x^6*sgn(b*x^3 + a) + 8*a^4*b*x^3*sgn(b*x^3 +
 a) + a^5*sgn(b*x^3 + a))/x^8
```

maple [A] time = 0.01, size = 80, normalized size = 0.32

$$-\frac{\left(-8b^5x^{15}-70a\,b^4x^{12}-560a^2b^3x^9+280a^3b^2x^6+56a^4b\,x^3+7a^5\right)\left(\left(b\,x^3+a\right)^2\right)^{\frac{5}{2}}}{56\left(b\,x^3+a\right)^5x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^9,x)
```

```
[Out] -1/56*(-8*b^5*x^15-70*a*b^4*x^12-560*a^2*b^3*x^9+280*a^3*b^2*x^6+56*a^4*b*x
^3+7*a^5)*((b*x^3+a)^2)^(5/2)/x^8/(b*x^3+a)^5
```

maxima [A] time = 1.33, size = 59, normalized size = 0.24

$$\frac{8\,b^5x^{15}+70\,ab^4x^{12}+560\,a^2b^3x^9-280\,a^3b^2x^6-56\,a^4bx^3-7\,a^5}{56\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^9,x, algorithm="maxima")
```

```
[Out] 1/56*(8*b^5*x^15 + 70*a*b^4*x^12 + 560*a^2*b^3*x^9 - 280*a^3*b^2*x^6 - 56*a
^4*b*x^3 - 7*a^5)/x^8
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a^2 + 2\,a\,b\,x^3 + b^2\,x^6\right)^{5/2}}{x^9}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2)/x^9,x)
```

```
[Out] int((a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2)/x^9, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\left(a + bx^3\right)^2\right)^{\frac{5}{2}}}{x^9}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**6+2*a*b*x**3+a**2)**(5/2)/x**9,x)
```

```
[Out] Integral(((a + b*x**3)**2)**(5/2)/x**9, x)
```

3.73 $\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{x^{10}}\,dx$

Optimal. Leaf size=252

$$\frac{b^5x^6\sqrt{a^2+2abx^3+b^2x^6}}{6\left(a+bx^3\right)}+\frac{5ab^4x^3\sqrt{a^2+2abx^3+b^2x^6}}{3\left(a+bx^3\right)}+\frac{10a^2b^3\log(x)\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}-\frac{a^5\sqrt{a^2+2abx^3+}}{9x^9\left(a+bx^3\right.}$$

Rubi [A] time = 0.07, antiderivative size = 252, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {1355, 266, 43}

$$\frac{b^5x^6\sqrt{a^2+2abx^3+b^2x^6}}{6\left(a+bx^3\right)}+\frac{5ab^4x^3\sqrt{a^2+2abx^3+b^2x^6}}{3\left(a+bx^3\right)}-\frac{10a^3b^2\sqrt{a^2+2abx^3+b^2x^6}}{3x^3\left(a+bx^3\right)}-\frac{5a^4b\sqrt{a^2+2abx^3+b^2x^6}}{6x^6\left(a+bx^3\right)}-\frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{9x^9\left(a+bx^3\right)}+\frac{10a^2b^3\log(x)\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^10,x]
```

```
[Out] -(a^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(9*x^9*(a + b*x^3)) - (5*a^4*b*Sqrt[
a^2 + 2*a*b*x^3 + b^2*x^6])/(6*x^6*(a + b*x^3)) - (10*a^3*b^2*Sqrt[a^2 + 2*
a*b*x^3 + b^2*x^6])/(3*x^3*(a + b*x^3)) + (5*a*b^4*x^3*Sqrt[a^2 + 2*a*b*x^3
 + b^2*x^6])/(3*(a + b*x^3)) + (b^5*x^6*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(6
*(a + b*x^3)) + (10*a^2*b^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]*Log[x])/(a + b*
x^3)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{x^{10}}\,dx = \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{\left(ab+b^2x^3\right)^5}{x^{10}}dx}{b^4\left(ab+b^2x^3\right)}$$

$$= \frac{\sqrt{a^2+2abx^3+b^2x^6}\ \text{Subst}\left(\int \frac{\left(ab+b^2x\right)^5}{x^4}dx, x, x^3\right)}{3b^4\left(ab+b^2x^3\right)}$$

$$= \frac{\sqrt{a^2+2abx^3+b^2x^6}\ \text{Subst}\left(\int \left(5ab^9+\frac{a^5b^5}{x^4}+\frac{5a^4b^6}{x^3}+\frac{10a^3b^7}{x^2}+\frac{10a^2b^8}{x}+b^{10}x\right)dx, x\right.}{3b^4\left(ab+b^2x^3\right)}$$

$$= -\frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{9x^9\left(a+bx^3\right)} - \frac{5a^4b\sqrt{a^2+2abx^3+b^2x^6}}{6x^6\left(a+bx^3\right)} - \frac{10a^3b^2\sqrt{a^2+2abx^3+b^2x}}{3x^3\left(a+bx^3\right)}$$

Mathematica [A] time = 0.03, size = 85, normalized size = 0.34

$$\frac{\sqrt{\left(a+bx^3\right)^2}\left(-2a^5-15a^4bx^3-60a^3b^2x^6+180a^2b^3x^9\log(x)+30ab^4x^{12}+3b^5x^{15}\right)}{18x^9\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^10,x]
```

```
[Out] (Sqrt[(a + b*x^3)^2]*(-2*a^5 - 15*a^4*b*x^3 - 60*a^3*b^2*x^6 + 30*a*b^4*x^1
2 + 3*b^5*x^15 + 180*a^2*b^3*x^9*Log[x]))/(18*x^9*(a + b*x^3))
```

IntegrateAlgebraic [A] time = 1.82, size = 366, normalized size = 1.45

$$-\frac{5}{3}a^2\left(b^2\right)^{3/2}\log\left(\sqrt{a^2+2abx^3+b^2x^6}-a-\sqrt{b^2}x^3\right)-\frac{5}{3}a^2\left(b^2\right)^{3/2}\log\left(\sqrt{a^2+2abx^3+b^2x^6}+a-\sqrt{b^2}x^3\right)+\frac{10}{3}a^2b^3\tanh^{-1}\left(\frac{\sqrt{b^2}x^3}{a}-\frac{\sqrt{a^2+2abx^3+b^2x^6}}{a}\right)+\frac{\sqrt{a^2+2abx^3+b^2x^6}\left(-8a^5b-60a^4b^2x^3-240a^3b^3x^6-391a^2b^4x^9+120ab^5x^{12}+12b^6x^{15}\right)+\sqrt{b^2}\left(8a^6+68a^5bx^3+300a^4b^2x^6+631a^3b^3x^9+271a^2b^4x^{12}-132ab^5x^{15}-12b^6x^{18}\right)}{72x^9\left(ab+b^2x^3\right)-72\sqrt{b^2}x^9\sqrt{a^2+2abx^3+b^2x^6}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^10,x]
```

```
[Out] (Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]*(-8*a^5*b - 60*a^4*b^2*x^3 - 240*a^3*b^3*x
^6 - 391*a^2*b^4*x^9 + 120*a*b^5*x^12 + 12*b^6*x^15) + Sqrt[b^2]*(8*a^6 + 6
8*a^5*b*x^3 + 300*a^4*b^2*x^6 + 631*a^3*b^3*x^9 + 271*a^2*b^4*x^12 - 132*a*
b^5*x^15 - 12*b^6*x^18))/(72*x^9*(a*b + b^2*x^3) - 72*Sqrt[b^2]*x^9*Sqrt[a^
2 + 2*a*b*x^3 + b^2*x^6]) + (10*a^2*b^3*ArcTanh[(Sqrt[b^2]*x^3)/a - Sqrt[a^
2 + 2*a*b*x^3 + b^2*x^6]/a])/3 - (5*a^2*(b^2)^(3/2)*Log[-a - Sqrt[b^2]*x^3
+ Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]])/3 - (5*a^2*(b^2)^(3/2)*Log[a - Sqrt[b^2
]*x^3 + Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]])/3
```

fricas [A] time = 1.72, size = 61, normalized size = 0.24

$$\frac{3\,b^5x^{15}+30\,ab^4x^{12}+180\,a^2b^3x^9\log(x)-60\,a^3b^2x^6-15\,a^4bx^3-2\,a^5}{18\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^10,x, algorithm="fricas")
```

```
[Out] 1/18*(3*b^5*x^15 + 30*a*b^4*x^12 + 180*a^2*b^3*x^9*log(x) - 60*a^3*b^2*x^6
- 15*a^4*b*x^3 - 2*a^5)/x^9
```

giac [A] time = 0.40, size = 127, normalized size = 0.50

$$\frac{1}{6}\,b^5x^6\mathrm{sgn}\left(bx^3+a\right)+\frac{5}{3}\,ab^4x^3\mathrm{sgn}\left(bx^3+a\right)+10\,a^2b^3\log\left(|x|\right)\mathrm{sgn}\left(bx^3+a\right)-\frac{110\,a^2b^3x^9\mathrm{sgn}\left(bx^3+a\right)+60\,a^3b^2x^6\mathrm{sgn}\left(bx^3+a\right)+15\,a^4bx^3\mathrm{sgn}\left(bx^3+a\right)+2\,a^5\mathrm{sgn}\left(bx^3+a\right)}{18\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^10,x, algorithm="giac")
```

```
[Out] 1/6*b^5*x^6*sgn(b*x^3 + a) + 5/3*a*b^4*x^3*sgn(b*x^3 + a) + 10*a^2*b^3*log(
abs(x))*sgn(b*x^3 + a) - 1/18*(110*a^2*b^3*x^9*sgn(b*x^3 + a) + 60*a^3*b^2*
x^6*sgn(b*x^3 + a) + 15*a^4*b*x^3*sgn(b*x^3 + a) + 2*a^5*sgn(b*x^3 + a))/x^
9
```

maple [A] time = 0.01, size = 82, normalized size = 0.33

$$\frac{\left(\left(b\,x^{3}+a\right)^{2}\right)^{\frac{5}{2}}\left(3b^{5}x^{15}+30a\,b^{4}x^{12}+180a^{2}b^{3}x^{9}\ln\left(x\right)-60a^{3}b^{2}x^{6}-15a^{4}b\,x^{3}-2a^{5}\right)}{18\left(b\,x^{3}+a\right)^{5}x^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^10,x)
```

```
[Out] 1/18*((b*x^3+a)^2)^(5/2)*(3*b^5*x^15+30*a*b^4*x^12+180*a^2*b^3*ln(x)*x^9-60
*a^3*b^2*x^6-15*a^4*b*x^3-2*a^5)/(b*x^3+a)^5/x^9
```

maxima [A] time = 0.80, size = 313, normalized size = 1.24

$$\frac{5}{3}\sqrt{b^{2}x^{6}+2\,abx^{3}+a^{2}}\,b^{4}x^{3}+\frac{10}{3}\left(-1\right)^{2\,b^{2}x^{3}+2\,ab}a^{2}b^{3}\log\left(2\,b^{2}x^{3}+2\,ab\right)-\frac{10}{3}\left(-1\right)^{2\,abx^{3}+2\,a^{2}}a^{2}b^{3}\log\left(\frac{2\,abx}{|x|}+\frac{2\,a^{2}}{x^{2}|x|}\right)+\frac{5\left(b^{2}x^{6}+2\,abx^{3}+a^{2}\right)^{\frac{3}{2}}b^{4}x^{3}}{6\,a^{2}}+5\sqrt{b^{2}x^{6}+2\,abx^{3}+a^{2}}\,ab^{3}+\frac{35\left(b^{2}x^{6}+2\,abx^{3}+a^{2}\right)^{\frac{3}{2}}b^{3}}{18\,a}+\frac{\left(b^{2}x^{6}+2\,abx^{3}+a^{2}\right)^{\frac{5}{2}}b^{3}}{18\,a^{3}}-\frac{11\left(b^{2}x^{6}+2\,abx^{3}+a^{2}\right)^{\frac{5}{2}}b^{2}}{18\,a^{2}x^{3}}-\frac{\left(b^{2}x^{6}+2\,abx^{3}+a^{2}\right)^{\frac{7}{2}}b}{18\,a^{3}x^{6}}-\frac{\left(b^{2}x^{6}+2\,abx^{3}+a^{2}\right)^{\frac{7}{2}}}{9\,a^{2}x^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^10,x, algorithm="maxima")
```

```
[Out] 5/3*sqrt(b^2*x^6 + 2*a*b*x^3 + a^2)*b^4*x^3 + 10/3*(-1)^(2*b^2*x^3 + 2*a*b)
*a^2*b^3*log(2*b^2*x^3 + 2*a*b) - 10/3*(-1)^(2*a*b*x^3 + 2*a^2)*a^2*b^3*log
(2*a*b*x/abs(x) + 2*a^2/(x^2*abs(x))) + 5/6*(b^2*x^6 + 2*a*b*x^3 + a^2)^(3/
2)*b^4*x^3/a^2 + 5*sqrt(b^2*x^6 + 2*a*b*x^3 + a^2)*a*b^3 + 35/18*(b^2*x^6 +
 2*a*b*x^3 + a^2)^(3/2)*b^3/a + 1/18*(b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2)*b^3/
a^3 - 11/18*(b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2)*b^2/(a^2*x^3) - 1/18*(b^2*x^6
 + 2*a*b*x^3 + a^2)^(7/2)*b/(a^3*x^6) - 1/9*(b^2*x^6 + 2*a*b*x^3 + a^2)^(7/
2)/(a^2*x^9)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(a^2+2\,a\,b\,x^3+b^2\,x^6\right)}^{5/2}}{x^{10}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2)/x^10,x)
```

```
[Out] int((a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2)/x^10, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\left(a + bx^{3}\right)^{2}\right)^{\frac{5}{2}}}{x^{10}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**6+2*a*b*x**3+a**2)**(5/2)/x**10,x)
```

```
[Out] Integral(((a + b*x**3)**2)**(5/2)/x**10, x)
```

3.74 $\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{x^{11}}\,dx$

Optimal. Leaf size=253

$$\frac{b^5x^5\sqrt{a^2+2abx^3+b^2x^6}}{5\left(a+bx^3\right)}+\frac{5ab^4x^2\sqrt{a^2+2abx^3+b^2x^6}}{2\left(a+bx^3\right)}-\frac{10a^2b^3\sqrt{a^2+2abx^3+b^2x^6}}{x\left(a+bx^3\right)}-\frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{10x^{10}\left(a+bx^3\right)}-5a$$

Rubi [A] time = 0.06, antiderivative size = 253, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {1355, 270}

$$-\frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{10x^{10}\left(a+bx^3\right)}-\frac{5a^4b\sqrt{a^2+2abx^3+b^2x^6}}{7x^7\left(a+bx^3\right)}-\frac{5a^3b^2\sqrt{a^2+2abx^3+b^2x^6}}{2x^4\left(a+bx^3\right)}-\frac{10a^2b^3\sqrt{a^2+2abx^3+b^2x^6}}{x\left(a+bx^3\right)}+\frac{5ab^4x^2\sqrt{a^2+2abx^3+b^2x^6}}{2\left(a+bx^3\right)}+\frac{b^5x^5\sqrt{a^2+2abx^3+b^2x^6}}{5\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^11,x]
```

```
[Out] -(a^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(10*x^10*(a + b*x^3)) - (5*a^4*b*Sqr
t[a^2 + 2*a*b*x^3 + b^2*x^6])/(7*x^7*(a + b*x^3)) - (5*a^3*b^2*Sqrt[a^2 + 2
*a*b*x^3 + b^2*x^6])/(2*x^4*(a + b*x^3)) - (10*a^2*b^3*Sqrt[a^2 + 2*a*b*x^3
 + b^2*x^6])/(x*(a + b*x^3)) + (5*a*b^4*x^2*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]
)/(2*(a + b*x^3)) + (b^5*x^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(5*(a + b*x^3
))
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{x^{11}}\,dx &= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{\left(ab+b^2x^3\right)^5}{x^{11}}\,dx}{b^4\left(ab+b^2x^3\right)}\\ &= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \left(\frac{a^5b^5}{x^{11}}+\frac{5a^4b^6}{x^8}+\frac{10a^3b^7}{x^5}+\frac{10a^2b^8}{x^2}+5ab^9x+b^{10}x^4\right)dx}{b^4\left(ab+b^2x^3\right)}\\ &= -\frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{10x^{10}\left(a+bx^3\right)}-\frac{5a^4b\sqrt{a^2+2abx^3+b^2x^6}}{7x^7\left(a+bx^3\right)}-\frac{5a^3b^2\sqrt{a^2+2abx^3+b^2x^6}}{2x^4\left(a+bx^3\right)}\end{aligned}$$

Mathematica [A] time = 0.02, size = 83, normalized size = 0.33

$$-\frac{\sqrt{\left(a+bx^3\right)^2}\left(7a^5+50a^4bx^3+175a^3b^2x^6+700a^2b^3x^9-175ab^4x^{12}-14b^5x^{15}\right)}{70x^{10}\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^11,x]
```

```
[Out] -1/70*(Sqrt[(a + b*x^3)^2]*(7*a^5 + 50*a^4*b*x^3 + 175*a^3*b^2*x^6 + 700*a^
2*b^3*x^9 - 175*a*b^4*x^12 - 14*b^5*x^15))/(x^10*(a + b*x^3))
```

IntegrateAlgebraic [A] time = 18.36, size = 83, normalized size = 0.33

$$\frac{\sqrt{\left(a+bx^3\right)^2}\left(-7a^5-50a^4bx^3-175a^3b^2x^6-700a^2b^3x^9+175ab^4x^{12}+14b^5x^{15}\right)}{70x^{10}\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^11,x]
```

```
[Out] (Sqrt[(a + b*x^3)^2]*(-7*a^5 - 50*a^4*b*x^3 - 175*a^3*b^2*x^6 - 700*a^2*b^3
*x^9 + 175*a*b^4*x^12 + 14*b^5*x^15))/(70*x^10*(a + b*x^3))
```

fricas [A] time = 1.20, size = 59, normalized size = 0.23

$$\frac{14\,b^5x^{15}+175\,ab^4x^{12}-700\,a^2b^3x^9-175\,a^3b^2x^6-50\,a^4bx^3-7\,a^5}{70\,x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^11,x, algorithm="fricas")
```

```
[Out] 1/70*(14*b^5*x^15 + 175*a*b^4*x^12 - 700*a^2*b^3*x^9 - 175*a^3*b^2*x^6 - 50
*a^4*b*x^3 - 7*a^5)/x^10
```

giac [A] time = 0.40, size = 108, normalized size = 0.43

$$\frac{1}{5}\,b^5x^5\mathrm{sgn}\left(bx^3+a\right)+\frac{5}{2}\,ab^4x^2\mathrm{sgn}\left(bx^3+a\right)-\frac{700\,a^2b^3x^9\mathrm{sgn}\left(bx^3+a\right)+175\,a^3b^2x^6\mathrm{sgn}\left(bx^3+a\right)+50\,a^4bx^3\mathrm{sgn}\left(bx^3+a\right)+7\,a^5\mathrm{sgn}\left(bx^3+a\right)}{70\,x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^11,x, algorithm="giac")
```

```
[Out] 1/5*b^5*x^5*sgn(b*x^3 + a) + 5/2*a*b^4*x^2*sgn(b*x^3 + a) - 1/70*(700*a^2*b
^3*x^9*sgn(b*x^3 + a) + 175*a^3*b^2*x^6*sgn(b*x^3 + a) + 50*a^4*b*x^3*sgn(b
*x^3 + a) + 7*a^5*sgn(b*x^3 + a))/x^10
```

maple [A] time = 0.01, size = 80, normalized size = 0.32

$$-\frac{\left(-14b^5x^{15}-175a\,b^4x^{12}+700a^2b^3x^9+175a^3b^2x^6+50a^4b\,x^3+7a^5\right)\left(\left(b\,x^3+a\right)^2\right)^{\frac{5}{2}}}{70\left(b\,x^3+a\right)^5x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^11,x)
```

```
[Out] -1/70*(-14*b^5*x^15-175*a*b^4*x^12+700*a^2*b^3*x^9+175*a^3*b^2*x^6+50*a^4*b
*x^3+7*a^5)*((b*x^3+a)^2)^(5/2)/x^10/(b*x^3+a)^5
```

maxima [A] time = 1.01, size = 59, normalized size = 0.23

$$\frac{14\,b^5x^{15}+175\,ab^4x^{12}-700\,a^2b^3x^9-175\,a^3b^2x^6-50\,a^4bx^3-7\,a^5}{70\,x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^11,x, algorithm="maxima")
```

```
[Out] 1/70*(14*b^5*x^15 + 175*a*b^4*x^12 - 700*a^2*b^3*x^9 - 175*a^3*b^2*x^6 - 50
*a^4*b*x^3 - 7*a^5)/x^10
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a^2+2\,a\,b\,x^3+b^2\,x^6\right)^{5/2}}{x^{11}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2)/x^11,x)
```

```
[Out] int((a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2)/x^11, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\left(a+bx^3\right)^2\right)^{\frac{5}{2}}}{x^{11}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**6+2*a*b*x**3+a**2)**(5/2)/x**11,x)
```

```
[Out] Integral(((a + b*x**3)**2)**(5/2)/x**11, x)
```

3.75 $\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{x^{12}}\,dx$

Optimal. Leaf size=247

$$\frac{b^5x^4\sqrt{a^2+2abx^3+b^2x^6}}{4\left(a+bx^3\right)}+\frac{5ab^4x\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}-\frac{5a^2b^3\sqrt{a^2+2abx^3+b^2x^6}}{x^2\left(a+bx^3\right)}-\frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{11x^{11}\left(a+bx^3\right)}-\frac{5a}{}$$

Rubi [A] time = 0.06, antiderivative size = 247, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1355, 270}

$$-\frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{11x^{11}\left(a+bx^3\right)}-\frac{5a^4b\sqrt{a^2+2abx^3+b^2x^6}}{8x^8\left(a+bx^3\right)}-\frac{2a^3b^2\sqrt{a^2+2abx^3+b^2x^6}}{x^5\left(a+bx^3\right)}-\frac{5a^2b^3\sqrt{a^2+2abx^3+b^2x^6}}{x^2\left(a+bx^3\right)}+\frac{5ab^4x\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}+\frac{b^5x^4\sqrt{a^2+2abx^3+b^2x^6}}{4\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^12,x]
```

```
[Out] -(a^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(11*x^11*(a + b*x^3)) - (5*a^4*b*Sqr
t[a^2 + 2*a*b*x^3 + b^2*x^6])/(8*x^8*(a + b*x^3)) - (2*a^3*b^2*Sqrt[a^2 + 2
*a*b*x^3 + b^2*x^6])/(x^5*(a + b*x^3)) - (5*a^2*b^3*Sqrt[a^2 + 2*a*b*x^3 +
b^2*x^6])/(x^2*(a + b*x^3)) + (5*a*b^4*x*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(
a + b*x^3) + (b^5*x^4*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(4*(a + b*x^3))
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{x^{12}}\,dx &= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{\left(ab+b^2x^3\right)^5}{x^{12}}\,dx}{b^4\left(ab+b^2x^3\right)}\\ &= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \left(5ab^9+\frac{a^5b^5}{x^{12}}+\frac{5a^4b^6}{x^9}+\frac{10a^3b^7}{x^6}+\frac{10a^2b^8}{x^3}+b^{10}x^3\right)dx}{b^4\left(ab+b^2x^3\right)}\\ &= -\frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{11x^{11}\left(a+bx^3\right)}-\frac{5a^4b\sqrt{a^2+2abx^3+b^2x^6}}{8x^8\left(a+bx^3\right)}-\frac{2a^3b^2\sqrt{a^2+2abx^3+b}}{x^5\left(a+bx^3\right)}\end{aligned}$$

Mathematica [A] time = 0.02, size = 83, normalized size = 0.34

$$-\frac{\sqrt{\left(a+bx^3\right)^2}\left(8a^5+55a^4bx^3+176a^3b^2x^6+440a^2b^3x^9-440ab^4x^{12}-22b^5x^{15}\right)}{88x^{11}\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^12,x]
```

```
[Out] -1/88*(Sqrt[(a + b*x^3)^2]*(8*a^5 + 55*a^4*b*x^3 + 176*a^3*b^2*x^6 + 440*a^
2*b^3*x^9 - 440*a*b^4*x^12 - 22*b^5*x^15))/(x^11*(a + b*x^3))
```

IntegrateAlgebraic [A] time = 18.20, size = 83, normalized size = 0.34

$$\frac{\sqrt{\left(a+bx^3\right)^2}\left(-8a^5-55a^4bx^3-176a^3b^2x^6-440a^2b^3x^9+440ab^4x^{12}+22b^5x^{15}\right)}{88x^{11}\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^12,x]
```

```
[Out] (Sqrt[(a + b*x^3)^2]*(-8*a^5 - 55*a^4*b*x^3 - 176*a^3*b^2*x^6 - 440*a^2*b^3
*x^9 + 440*a*b^4*x^12 + 22*b^5*x^15))/(88*x^11*(a + b*x^3))
```

fricas [A] time = 1.08, size = 59, normalized size = 0.24

$$\frac{22\,b^5x^{15}+440\,ab^4x^{12}-440\,a^2b^3x^9-176\,a^3b^2x^6-55\,a^4bx^3-8\,a^5}{88\,x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^12,x, algorithm="fricas")
```

```
[Out] 1/88*(22*b^5*x^15 + 440*a*b^4*x^12 - 440*a^2*b^3*x^9 - 176*a^3*b^2*x^6 - 55
*a^4*b*x^3 - 8*a^5)/x^11
```

giac [A] time = 0.35, size = 106, normalized size = 0.43

$$\frac{1}{4}\,b^5x^4\mathrm{sgn}\left(bx^3+a\right)+5\,ab^4x\mathrm{sgn}\left(bx^3+a\right)-\frac{440\,a^2b^3x^9\mathrm{sgn}\left(bx^3+a\right)+176\,a^3b^2x^6\mathrm{sgn}\left(bx^3+a\right)+55\,a^4bx^3\mathrm{sgn}\left(bx^3+a\right)+8\,a^5\mathrm{sgn}\left(bx^3+a\right)}{88\,x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^12,x, algorithm="giac")
```

```
[Out] 1/4*b^5*x^4*sgn(b*x^3 + a) + 5*a*b^4*x*sgn(b*x^3 + a) - 1/88*(440*a^2*b^3*x
^9*sgn(b*x^3 + a) + 176*a^3*b^2*x^6*sgn(b*x^3 + a) + 55*a^4*b*x^3*sgn(b*x^3
 + a) + 8*a^5*sgn(b*x^3 + a))/x^11
```

maple [A] time = 0.01, size = 80, normalized size = 0.32

$$-\frac{\left(-22b^5x^{15}-440a\,b^4x^{12}+440a^2b^3x^9+176a^3b^2x^6+55a^4b\,x^3+8a^5\right)\left(\left(b\,x^3+a\right)^2\right)^{\frac{5}{2}}}{88\left(b\,x^3+a\right)^5x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^12,x)
```

```
[Out] -1/88*(-22*b^5*x^15-440*a*b^4*x^12+440*a^2*b^3*x^9+176*a^3*b^2*x^6+55*a^4*b
*x^3+8*a^5)*((b*x^3+a)^2)^(5/2)/x^11/(b*x^3+a)^5
```

maxima [A] time = 0.97, size = 59, normalized size = 0.24

$$\frac{22\,b^5x^{15}+440\,ab^4x^{12}-440\,a^2b^3x^9-176\,a^3b^2x^6-55\,a^4bx^3-8\,a^5}{88\,x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^12,x, algorithm="maxima")
```

```
[Out] 1/88*(22*b^5*x^15 + 440*a*b^4*x^12 - 440*a^2*b^3*x^9 - 176*a^3*b^2*x^6 - 55
*a^4*b*x^3 - 8*a^5)/x^11
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a^2 + 2\,a\,b\,x^3 + b^2\,x^6\right)^{5/2}}{x^{12}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2)/x^12,x)
```

```
[Out] int((a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2)/x^12, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\left(a + bx^3\right)^2\right)^{\frac{5}{2}}}{x^{12}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**6+2*a*b*x**3+a**2)**(5/2)/x**12,x)
```

```
[Out] Integral(((a + b*x**3)**2)**(5/2)/x**12, x)
```

3.76 $\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{x^{13}}\,dx$

Optimal. Leaf size=252

$$\frac{b^5x^3\sqrt{a^2+2abx^3+b^2x^6}}{3\left(a+bx^3\right)}+\frac{5ab^4\log(x)\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}-\frac{10a^2b^3\sqrt{a^2+2abx^3+b^2x^6}}{3x^3\left(a+bx^3\right)}-\frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{12x^{12}\left(a+bx^3\right)}$$

Rubi [A] time = 0.07, antiderivative size = 252, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1355, 266, 43}

$$-\frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{12x^{12}\left(a+bx^3\right)}-\frac{5a^4b\sqrt{a^2+2abx^3+b^2x^6}}{9x^9\left(a+bx^3\right)}-\frac{5a^3b^2\sqrt{a^2+2abx^3+b^2x^6}}{3x^6\left(a+bx^3\right)}-\frac{10a^2b^3\sqrt{a^2+2abx^3+b^2x^6}}{3x^3\left(a+bx^3\right)}+\frac{b^5x^3\sqrt{a^2+2abx^3+b^2x^6}}{3\left(a+bx^3\right)}+\frac{5ab^4\log(x)\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^13,x]
```

```
[Out] -(a^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(12*x^12*(a + b*x^3)) - (5*a^4*b*Sqr
t[a^2 + 2*a*b*x^3 + b^2*x^6])/(9*x^9*(a + b*x^3)) - (5*a^3*b^2*Sqrt[a^2 + 2
*a*b*x^3 + b^2*x^6])/(3*x^6*(a + b*x^3)) - (10*a^2*b^3*Sqrt[a^2 + 2*a*b*x^3
 + b^2*x^6])/(3*x^3*(a + b*x^3)) + (b^5*x^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]
)/(3*(a + b*x^3)) + (5*a*b^4*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]*Log[x])/(a + b
*x^3)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^{13}}\,dx = \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{(ab+b^2x^3)^5}{x^{13}}\,dx}{b^4(ab+b^2x^3)}$$
$$= \frac{\sqrt{a^2+2abx^3+b^2x^6}\,\text{Subst}\left(\int \frac{(ab+b^2x)^5}{x^5}\,dx, x, x^3\right)}{3b^4(ab+b^2x^3)}$$
$$= \frac{\sqrt{a^2+2abx^3+b^2x^6}\,\text{Subst}\left(\int \left(b^{10}+\frac{a^5b^5}{x^5}+\frac{5a^4b^6}{x^4}+\frac{10a^3b^7}{x^3}+\frac{10a^2b^8}{x^2}+\frac{5ab^9}{x}\right)dx,\right.}{3b^4(ab+b^2x^3)}$$
$$= -\frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{12x^{12}(a+bx^3)} - \frac{5a^4b\sqrt{a^2+2abx^3+b^2x^6}}{9x^9(a+bx^3)} - \frac{5a^3b^2\sqrt{a^2+2abx^3+b^2}}{3x^6(a+bx^3)}$$

Mathematica [A] time = 0.02, size = 85, normalized size = 0.34

$$-\frac{\sqrt{(a+bx^3)^2}\left(3a^5+20a^4bx^3+60a^3b^2x^6+120a^2b^3x^9-180ab^4x^{12}\log(x)-12b^5x^{15}\right)}{36x^{12}(a+bx^3)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^13,x]
```

```
[Out] -1/36*(Sqrt[(a + b*x^3)^2]*(3*a^5 + 20*a^4*b*x^3 + 60*a^3*b^2*x^6 + 120*a^2
*b^3*x^9 - 12*b^5*x^15 - 180*a*b^4*x^12*Log[x]))/(x^12*(a + b*x^3))
```

IntegrateAlgebraic [B] time = 3.02, size = 2031, normalized size = 8.06

Result too large to show

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^13,x]
```

```
[Out] (-2*b^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]*(3*a^8*b + 29*a^7*b^2*x^3 + 129*a^6
*b^3*x^6 + 363*a^5*b^4*x^9 + 554*a^4*b^5*x^12 + 390*a^3*b^6*x^15 + 66*a^2*b
^7*x^18 - 42*a*b^8*x^21 - 12*b^9*x^24) - 2*b^3*Sqrt[b^2]*(-3*a^9 - 32*a^8*b
*x^3 - 158*a^7*b^2*x^6 - 492*a^6*b^3*x^9 - 917*a^5*b^4*x^12 - 944*a^4*b^5*x
^15 - 456*a^3*b^6*x^18 - 24*a^2*b^7*x^21 + 54*a*b^8*x^24 + 12*b^9*x^27))/(9
*Sqrt[b^2]*x^12*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]*(-8*a^3*b^3 - 24*a^2*b^4*x^
3 - 24*a*b^5*x^6 - 8*b^6*x^9) + 9*x^12*(8*a^4*b^4 + 32*a^3*b^5*x^3 + 48*a^2
*b^6*x^6 + 32*a*b^7*x^9 + 8*b^8*x^12)) + (5*a*b^4*Log[-a - Sqrt[b^2]*x^3 +
Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]])/6 - (5*a*b^3*Sqrt[b^2]*Log[-a - Sqrt[b^2]
*x^3 + Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]])/6 - (5*a^9*b^4*Log[a - Sqrt[b^2]*x
^3 + Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]])/(6*(-a - Sqrt[b^2]*x^3 + Sqrt[a^2 +
2*a*b*x^3 + b^2*x^6])^4*(a - Sqrt[b^2]*x^3 + Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6
])^4) - (5*a^9*b^3*Sqrt[b^2]*Log[a - Sqrt[b^2]*x^3 + Sqrt[a^2 + 2*a*b*x^3 +
 b^2*x^6]])/(6*(-a - Sqrt[b^2]*x^3 + Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])^4*(a
- Sqrt[b^2]*x^3 + Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])^4) + (10*a^7*b^4*(-(Sqrt
[b^2]*x^3) + Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])^2*Log[a - Sqrt[b^2]*x^3 + Sqr
t[a^2 + 2*a*b*x^3 + b^2*x^6]])/(3*(-a - Sqrt[b^2]*x^3 + Sqrt[a^2 + 2*a*b*x^
3 + b^2*x^6])^4*(a - Sqrt[b^2]*x^3 + Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])^4) +
(10*a^7*b^3*Sqrt[b^2]*(-(Sqrt[b^2]*x^3) + Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])^
2*Log[a - Sqrt[b^2]*x^3 + Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]])/(3*(-a - Sqrt[b
^2]*x^3 + Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])^4*(a - Sqrt[b^2]*x^3 + Sqrt[a^2
+ 2*a*b*x^3 + b^2*x^6])^4) - (5*a^5*b^4*(-(Sqrt[b^2]*x^3) + Sqrt[a^2 + 2*a*
```

```
b*x^3 + b^2*x^6])^4*Log[a - Sqrt[b^2]*x^3 + Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]
])/((-a - Sqrt[b^2]*x^3 + Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])^4*(a - Sqrt[b^2]
*x^3 + Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])^4) - (5*a^5*b^3*Sqrt[b^2]*(-(Sqrt[b
^2]*x^3) + Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])^4*Log[a - Sqrt[b^2]*x^3 + Sqrt[
a^2 + 2*a*b*x^3 + b^2*x^6]])/((-a - Sqrt[b^2]*x^3 + Sqrt[a^2 + 2*a*b*x^3 +
b^2*x^6])^4*(a - Sqrt[b^2]*x^3 + Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])^4) + (10*
a^3*b^4*(-(Sqrt[b^2]*x^3) + Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])^6*Log[a - Sqrt
[b^2]*x^3 + Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]])/(3*(-a - Sqrt[b^2]*x^3 + Sqrt
[a^2 + 2*a*b*x^3 + b^2*x^6])^4*(a - Sqrt[b^2]*x^3 + Sqrt[a^2 + 2*a*b*x^3 +
b^2*x^6])^4) + (10*a^3*b^3*Sqrt[b^2]*(-(Sqrt[b^2]*x^3) + Sqrt[a^2 + 2*a*b*x
^3 + b^2*x^6])^6*Log[a - Sqrt[b^2]*x^3 + Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]])/
(3*(-a - Sqrt[b^2]*x^3 + Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])^4*(a - Sqrt[b^2]*
x^3 + Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])^4) - (5*a*b^4*(-(Sqrt[b^2]*x^3) + Sq
rt[a^2 + 2*a*b*x^3 + b^2*x^6])^8*Log[a - Sqrt[b^2]*x^3 + Sqrt[a^2 + 2*a*b*x
^3 + b^2*x^6]])/(6*(-a - Sqrt[b^2]*x^3 + Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])^4
*(a - Sqrt[b^2]*x^3 + Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])^4) - (5*a*b^3*Sqrt[b
^2]*(-(Sqrt[b^2]*x^3) + Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])^8*Log[a - Sqrt[b^2
]*x^3 + Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]])/(6*(-a - Sqrt[b^2]*x^3 + Sqrt[a^2
 + 2*a*b*x^3 + b^2*x^6])^4*(a - Sqrt[b^2]*x^3 + Sqrt[a^2 + 2*a*b*x^3 + b^2*
x^6])^4)
```

fricas [A] time = 1.17, size = 61, normalized size = 0.24

$$\frac{12\,b^{5}x^{15}+180\,ab^{4}x^{12}\log\left(x\right)-120\,a^{2}b^{3}x^{9}-60\,a^{3}b^{2}x^{6}-20\,a^{4}bx^{3}-3\,a^{5}}{36\,x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^13,x, algorithm="fricas")
```

```
[Out] 1/36*(12*b^5*x^15 + 180*a*b^4*x^12*log(x) - 120*a^2*b^3*x^9 - 60*a^3*b^2*x^
6 - 20*a^4*b*x^3 - 3*a^5)/x^12
```

giac [A] time = 0.38, size = 125, normalized size = 0.50

$$\frac{1}{3}\,b^{5}x^{3}\mathrm{sgn}\left(bx^{3}+a\right)+5\,ab^{4}\log\left(|x|\right)\mathrm{sgn}\left(bx^{3}+a\right)-\frac{125\,ab^{4}x^{12}\mathrm{sgn}\left(bx^{3}+a\right)+120\,a^{2}b^{3}x^{9}\mathrm{sgn}\left(bx^{3}+a\right)+60\,a^{3}b^{2}x^{6}\mathrm{sgn}\left(bx^{3}+a\right)+20\,a^{4}bx^{3}\mathrm{sgn}\left(bx^{3}+a\right)+3\,a^{5}\mathrm{sgn}\left(bx^{3}+a\right)}{36\,x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^13,x, algorithm="giac")
```

```
[Out] 1/3*b^5*x^3*sgn(b*x^3 + a) + 5*a*b^4*log(abs(x))*sgn(b*x^3 + a) - 1/36*(125
*a*b^4*x^12*sgn(b*x^3 + a) + 120*a^2*b^3*x^9*sgn(b*x^3 + a) + 60*a^3*b^2*x^
6*sgn(b*x^3 + a) + 20*a^4*b*x^3*sgn(b*x^3 + a) + 3*a^5*sgn(b*x^3 + a))/x^12
```

maple [A] time = 0.01, size = 82, normalized size = 0.33

$$\frac{\left(\left(b\,x^{3}+a\right)^{2}\right)^{\frac{5}{2}}\left(12b^{5}x^{15}+180a\,b^{4}x^{12}\ln\left(x\right)-120a^{2}b^{3}x^{9}-60a^{3}b^{2}x^{6}-20a^{4}b\,x^{3}-3a^{5}\right)}{36\left(b\,x^{3}+a\right)^{5}x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^13,x)
```

```
[Out] 1/36*((b*x^3+a)^2)^(5/2)*(12*b^5*x^15+180*a*b^4*ln(x)*x^12-120*a^2*b^3*x^9-
60*a^3*b^2*x^6-20*a^4*b*x^3-3*a^5)/(b*x^3+a)^5/x^12
```

maxima [A] time = 1.10, size = 342, normalized size = 1.36

$$\frac{5\,\sqrt{b^{2}x^{6}+2\,abx^{3}+a^{2}}\,b^{5}x^{3}}{6\,a}+\frac{5}{3}\,\left(-1\right)^{2\,b^{2}x^{3}+2\,ab}\,ab^{4}\log\left(2\,b^{2}x^{3}+2\,ab\right)-\frac{5}{3}\,\left(-1\right)^{2\,abx^{3}+2\,a^{2}}\,ab^{4}\log\left(\frac{2\,abx}{|x|}+\frac{2\,a^{2}}{x^{2}|x|}\right)+\frac{5\,\left(b^{2}x^{6}+2\,abx^{3}+a^{2}\right)^{\frac{3}{2}}b^{5}x^{3}}{12\,a^{3}}+\frac{5}{2}\,\sqrt{b^{2}x^{6}+2\,abx^{3}+a^{2}}\,b^{4}+\frac{35\,\left(b^{2}x^{6}+2\,abx^{3}+a^{2}\right)^{\frac{3}{2}}b^{4}}{36\,a^{2}}+\frac{\left(b^{2}x^{6}+2\,abx^{3}+a^{2}\right)^{\frac{5}{2}}b^{4}}{9\,a^{4}}-\frac{2\,\left(b^{2}x^{6}+2\,abx^{3}+a^{2}\right)^{\frac{5}{2}}b^{3}}{9\,a^{3}x^{3}}-\frac{\left(b^{2}x^{6}+2\,abx^{3}+a^{2}\right)^{\frac{7}{2}}b^{2}}{9\,a^{4}x^{6}}+\frac{\left(b^{2}x^{6}+2\,abx^{3}+a^{2}\right)^{\frac{7}{2}}b}{36\,a^{3}x^{9}}-\frac{\left(b^{2}x^{6}+2\,abx^{3}+a^{2}\right)^{\frac{7}{2}}}{12\,a^{2}x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^13,x, algorithm="maxima")
```

```
[Out] 5/6*sqrt(b^2*x^6 + 2*a*b*x^3 + a^2)*b^5*x^3/a + 5/3*(-1)^(2*b^2*x^3 + 2*a*b
)*a*b^4*log(2*b^2*x^3 + 2*a*b) - 5/3*(-1)^(2*a*b*x^3 + 2*a^2)*a*b^4*log(2*a
*b*x/abs(x) + 2*a^2/(x^2*abs(x))) + 5/12*(b^2*x^6 + 2*a*b*x^3 + a^2)^(3/2)*
b^5*x^3/a^3 + 5/2*sqrt(b^2*x^6 + 2*a*b*x^3 + a^2)*b^4 + 35/36*(b^2*x^6 + 2*
a*b*x^3 + a^2)^(3/2)*b^4/a^2 + 1/9*(b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2)*b^4/a^
4 - 2/9*(b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2)*b^3/(a^3*x^3) - 1/9*(b^2*x^6 + 2*
a*b*x^3 + a^2)^(7/2)*b^2/(a^4*x^6) + 1/36*(b^2*x^6 + 2*a*b*x^3 + a^2)^(7/2)
*b/(a^3*x^9) - 1/12*(b^2*x^6 + 2*a*b*x^3 + a^2)^(7/2)/(a^2*x^12)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a^2+2\,a\,b\,x^3+b^2\,x^6\right)^{5/2}}{x^{13}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2)/x^13,x)
```

```
[Out] int((a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2)/x^13, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\left(a+bx^3\right)^2\right)^{\frac{5}{2}}}{x^{13}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**6+2*a*b*x**3+a**2)**(5/2)/x**13,x)
```

```
[Out] Integral(((a + b*x**3)**2)**(5/2)/x**13, x)
```

3.77 $\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{x^{14}}\,dx$

Optimal. Leaf size=253

$$\frac{b^5x^2\sqrt{a^2+2abx^3+b^2x^6}}{2\left(a+bx^3\right)}-\frac{5ab^4\sqrt{a^2+2abx^3+b^2x^6}}{x\left(a+bx^3\right)}-\frac{5a^2b^3\sqrt{a^2+2abx^3+b^2x^6}}{2x^4\left(a+bx^3\right)}-\frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{13x^{13}\left(a+bx^3\right)}-\frac{a^4b\sqrt{a}}{2}$$

Rubi [A] time = 0.06, antiderivative size = 253, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1355, 270}

$$-\frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{13x^{13}\left(a+bx^3\right)}-\frac{a^4b\sqrt{a^2+2abx^3+b^2x^6}}{2x^{10}\left(a+bx^3\right)}-\frac{10a^3b^2\sqrt{a^2+2abx^3+b^2x^6}}{7x^7\left(a+bx^3\right)}-\frac{5a^2b^3\sqrt{a^2+2abx^3+b^2x^6}}{2x^4\left(a+bx^3\right)}-\frac{5ab^4\sqrt{a^2+2abx^3+b^2x^6}}{x\left(a+bx^3\right)}+\frac{b^5x^2\sqrt{a^2+2abx^3+b^2x^6}}{2\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^14,x]
```

```
[Out] -(a^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(13*x^13*(a + b*x^3)) - (a^4*b*Sqrt[
a^2 + 2*a*b*x^3 + b^2*x^6])/(2*x^10*(a + b*x^3)) - (10*a^3*b^2*Sqrt[a^2 + 2
*a*b*x^3 + b^2*x^6])/(7*x^7*(a + b*x^3)) - (5*a^2*b^3*Sqrt[a^2 + 2*a*b*x^3
+ b^2*x^6])/(2*x^4*(a + b*x^3)) - (5*a*b^4*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])
/(x*(a + b*x^3)) + (b^5*x^2*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(2*(a + b*x^3)
)
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{x^{14}}\,dx &= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{\left(ab+b^2x^3\right)^5}{x^{14}}\,dx}{b^4\left(ab+b^2x^3\right)}\\
&= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \left(\frac{a^5b^5}{x^{14}}+\frac{5a^4b^6}{x^{11}}+\frac{10a^3b^7}{x^8}+\frac{10a^2b^8}{x^5}+\frac{5ab^9}{x^2}+b^{10}x\right)dx}{b^4\left(ab+b^2x^3\right)}\\
&= -\frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{13x^{13}\left(a+bx^3\right)}-\frac{a^4b\sqrt{a^2+2abx^3+b^2x^6}}{2x^{10}\left(a+bx^3\right)}-\frac{10a^3b^2\sqrt{a^2+2abx^3+b^2x^6}}{7x^7\left(a+bx^3\right)}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 83, normalized size = 0.33

$$-\frac{\sqrt{\left(a+bx^3\right)^2}\left(14a^5+91a^4bx^3+260a^3b^2x^6+455a^2b^3x^9+910ab^4x^{12}-91b^5x^{15}\right)}{182x^{13}\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^14,x]
```

```
[Out] -1/182*(Sqrt[(a + b*x^3)^2]*(14*a^5 + 91*a^4*b*x^3 + 260*a^3*b^2*x^6 + 455*
a^2*b^3*x^9 + 910*a*b^4*x^12 - 91*b^5*x^15))/(x^13*(a + b*x^3))
```

IntegrateAlgebraic [A] time = 19.40, size = 83, normalized size = 0.33

$$\frac{\sqrt{\left(a+bx^3\right)^2}\left(-14a^5-91a^4bx^3-260a^3b^2x^6-455a^2b^3x^9-910ab^4x^{12}+91b^5x^{15}\right)}{182x^{13}\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^14,x]
```

```
[Out] (Sqrt[(a + b*x^3)^2]*(-14*a^5 - 91*a^4*b*x^3 - 260*a^3*b^2*x^6 - 455*a^2*b^
3*x^9 - 910*a*b^4*x^12 + 91*b^5*x^15))/(182*x^13*(a + b*x^3))
```

fricas [A] time = 1.18, size = 59, normalized size = 0.23

$$\frac{91\,b^5x^{15}-910\,ab^4x^{12}-455\,a^2b^3x^9-260\,a^3b^2x^6-91\,a^4bx^3-14\,a^5}{182\,x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^14,x, algorithm="fricas")
```

```
[Out] 1/182*(91*b^5*x^15 - 910*a*b^4*x^12 - 455*a^2*b^3*x^9 - 260*a^3*b^2*x^6 - 9
1*a^4*b*x^3 - 14*a^5)/x^13
```

giac [A] time = 0.33, size = 108, normalized size = 0.43

$$\frac{1}{2}\,b^5x^2\mathrm{sgn}\left(bx^3+a\right)-\frac{910\,ab^4x^{12}\mathrm{sgn}\left(bx^3+a\right)+455\,a^2b^3x^9\mathrm{sgn}\left(bx^3+a\right)+260\,a^3b^2x^6\mathrm{sgn}\left(bx^3+a\right)+91\,a^4bx^3\mathrm{sgn}\left(bx^3+a\right)+14\,a^5\mathrm{sgn}\left(bx^3+a\right)}{182\,x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^14,x, algorithm="giac")
```

```
[Out] 1/2*b^5*x^2*sgn(b*x^3 + a) - 1/182*(910*a*b^4*x^12*sgn(b*x^3 + a) + 455*a^2
*b^3*x^9*sgn(b*x^3 + a) + 260*a^3*b^2*x^6*sgn(b*x^3 + a) + 91*a^4*b*x^3*sgn
(b*x^3 + a) + 14*a^5*sgn(b*x^3 + a))/x^13
```

maple [A] time = 0.01, size = 80, normalized size = 0.32

$$-\frac{\left(-91b^5x^{15}+910a\,b^4x^{12}+455a^2b^3x^9+260a^3b^2x^6+91a^4b\,x^3+14a^5\right)\left(\left(b\,x^3+a\right)^2\right)^{\frac{5}{2}}}{182\left(b\,x^3+a\right)^5x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^14,x)
```

```
[Out] -1/182*(-91*b^5*x^15+910*a*b^4*x^12+455*a^2*b^3*x^9+260*a^3*b^2*x^6+91*a^4*
b*x^3+14*a^5)*((b*x^3+a)^2)^(5/2)/x^13/(b*x^3+a)^5
```

maxima [A] time = 0.99, size = 59, normalized size = 0.23

$$\frac{91\,b^5x^{15}-910\,ab^4x^{12}-455\,a^2b^3x^9-260\,a^3b^2x^6-91\,a^4bx^3-14\,a^5}{182\,x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^14,x, algorithm="maxima")
```

```
[Out] 1/182*(91*b^5*x^15 - 910*a*b^4*x^12 - 455*a^2*b^3*x^9 - 260*a^3*b^2*x^6 - 9
1*a^4*b*x^3 - 14*a^5)/x^13
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a^2+2\,a\,b\,x^3+b^2\,x^6\right)^{5/2}}{x^{14}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2)/x^14,x)
```

```
[Out] int((a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2)/x^14, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\left(a+bx^3\right)^2\right)^{\frac{5}{2}}}{x^{14}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**6+2*a*b*x**3+a**2)**(5/2)/x**14,x)
```

```
[Out] Integral(((a + b*x**3)**2)**(5/2)/x**14, x)
```

3.78 $\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{x^{15}}\,dx$

Optimal. Leaf size=248

$$\frac{b^5x\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}-\frac{5ab^4\sqrt{a^2+2abx^3+b^2x^6}}{2x^2\left(a+bx^3\right)}-\frac{2a^2b^3\sqrt{a^2+2abx^3+b^2x^6}}{x^5\left(a+bx^3\right)}-\frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{14x^{14}\left(a+bx^3\right)}-\frac{5a^4b}{}$$

Rubi [A] time = 0.06, antiderivative size = 248, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1355, 270}

$$-\frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{14x^{14}\left(a+bx^3\right)}-\frac{5a^4b\sqrt{a^2+2abx^3+b^2x^6}}{11x^{11}\left(a+bx^3\right)}-\frac{5a^3b^2\sqrt{a^2+2abx^3+b^2x^6}}{4x^8\left(a+bx^3\right)}-\frac{2a^2b^3\sqrt{a^2+2abx^3+b^2x^6}}{x^5\left(a+bx^3\right)}-\frac{5ab^4\sqrt{a^2+2abx^3+b^2x^6}}{2x^2\left(a+bx^3\right)}+\frac{b^5x\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^15,x]
```

```
[Out] -(a^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(14*x^14*(a + b*x^3)) - (5*a^4*b*Sqr
t[a^2 + 2*a*b*x^3 + b^2*x^6])/(11*x^11*(a + b*x^3)) - (5*a^3*b^2*Sqrt[a^2 +
 2*a*b*x^3 + b^2*x^6])/(4*x^8*(a + b*x^3)) - (2*a^2*b^3*Sqrt[a^2 + 2*a*b*x^
3 + b^2*x^6])/(x^5*(a + b*x^3)) - (5*a*b^4*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])
/(2*x^2*(a + b*x^3)) + (b^5*x*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(a + b*x^3)
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{x^{15}}\,dx &= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{\left(ab+b^2x^3\right)^5}{x^{15}}\,dx}{b^4\left(ab+b^2x^3\right)}\\ &= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \left(b^{10}+\frac{a^5b^5}{x^{15}}+\frac{5a^4b^6}{x^{12}}+\frac{10a^3b^7}{x^9}+\frac{10a^2b^8}{x^6}+\frac{5ab^9}{x^3}\right)dx}{b^4\left(ab+b^2x^3\right)}\\ &= -\frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{14x^{14}\left(a+bx^3\right)}-\frac{5a^4b\sqrt{a^2+2abx^3+b^2x^6}}{11x^{11}\left(a+bx^3\right)}-\frac{5a^3b^2\sqrt{a^2+2abx^3+b^}}{4x^8\left(a+bx^3\right)}\end{aligned}$$

Mathematica [A] time = 0.02, size = 83, normalized size = 0.33

$$-\frac{\sqrt{\left(a+bx^3\right)^2}\left(22a^5+140a^4bx^3+385a^3b^2x^6+616a^2b^3x^9+770ab^4x^{12}-308b^5x^{15}\right)}{308x^{14}\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^15,x]
```

```
[Out] -1/308*(Sqrt[(a + b*x^3)^2]*(22*a^5 + 140*a^4*b*x^3 + 385*a^3*b^2*x^6 + 616
*a^2*b^3*x^9 + 770*a*b^4*x^12 - 308*b^5*x^15))/(x^14*(a + b*x^3))
```

IntegrateAlgebraic [A] time = 19.62, size = 83, normalized size = 0.33

$$\frac{\sqrt{\left(a+bx^3\right)^2}\left(-22a^5-140a^4bx^3-385a^3b^2x^6-616a^2b^3x^9-770ab^4x^{12}+308b^5x^{15}\right)}{308x^{14}\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^15,x]
```

```
[Out] (Sqrt[(a + b*x^3)^2]*(-22*a^5 - 140*a^4*b*x^3 - 385*a^3*b^2*x^6 - 616*a^2*b
^3*x^9 - 770*a*b^4*x^12 + 308*b^5*x^15))/(308*x^14*(a + b*x^3))
```

fricas [A] time = 0.68, size = 59, normalized size = 0.24

$$\frac{308\,b^5x^{15}-770\,ab^4x^{12}-616\,a^2b^3x^9-385\,a^3b^2x^6-140\,a^4bx^3-22\,a^5}{308\,x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^15,x, algorithm="fricas")
```

```
[Out] 1/308*(308*b^5*x^15 - 770*a*b^4*x^12 - 616*a^2*b^3*x^9 - 385*a^3*b^2*x^6 -
140*a^4*b*x^3 - 22*a^5)/x^14
```

giac [A] time = 0.35, size = 105, normalized size = 0.42

$$b^5x\mathrm{sgn}\left(bx^3+a\right)-\frac{770\,ab^4x^{12}\mathrm{sgn}\left(bx^3+a\right)+616\,a^2b^3x^9\mathrm{sgn}\left(bx^3+a\right)+385\,a^3b^2x^6\mathrm{sgn}\left(bx^3+a\right)+140\,a^4bx^3\mathrm{sgn}\left(bx^3+a\right)+22\,a^5\mathrm{sgn}\left(bx^3+a\right)}{308\,x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^15,x, algorithm="giac")
```

```
[Out] b^5*x*sgn(b*x^3 + a) - 1/308*(770*a*b^4*x^12*sgn(b*x^3 + a) + 616*a^2*b^3*x
^9*sgn(b*x^3 + a) + 385*a^3*b^2*x^6*sgn(b*x^3 + a) + 140*a^4*b*x^3*sgn(b*x^
3 + a) + 22*a^5*sgn(b*x^3 + a))/x^14
```

maple [A] time = 0.01, size = 80, normalized size = 0.32

$$-\frac{\left(-308b^5x^{15}+770a\,b^4x^{12}+616a^2b^3x^9+385a^3b^2x^6+140a^4b\,x^3+22a^5\right)\left(\left(b\,x^3+a\right)^2\right)^{\frac{5}{2}}}{308\left(b\,x^3+a\right)^5x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^15,x)
```

```
[Out] -1/308*(-308*b^5*x^15+770*a*b^4*x^12+616*a^2*b^3*x^9+385*a^3*b^2*x^6+140*a^
4*b*x^3+22*a^5)*((b*x^3+a)^2)^(5/2)/x^14/(b*x^3+a)^5
```

maxima [A] time = 0.99, size = 59, normalized size = 0.24

$$\frac{308\,b^5x^{15}-770\,ab^4x^{12}-616\,a^2b^3x^9-385\,a^3b^2x^6-140\,a^4bx^3-22\,a^5}{308\,x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^15,x, algorithm="maxima")
```

```
[Out] 1/308*(308*b^5*x^15 - 770*a*b^4*x^12 - 616*a^2*b^3*x^9 - 385*a^3*b^2*x^6 -
140*a^4*b*x^3 - 22*a^5)/x^14
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a^2 + 2\,a\,b\,x^3 + b^2\,x^6\right)^{5/2}}{x^{15}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2)/x^15,x)
```

```
[Out] int((a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2)/x^15, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\left(a + bx^3\right)^2\right)^{\frac{5}{2}}}{x^{15}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**6+2*a*b*x**3+a**2)**(5/2)/x**15,x)
```

```
[Out] Integral(((a + b*x**3)**2)**(5/2)/x**15, x)
```

3.79 $\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{x^{16}}\,dx$

Optimal. Leaf size=251

$$\frac{b^5\log(x)\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}-\frac{5ab^4\sqrt{a^2+2abx^3+b^2x^6}}{3x^3\left(a+bx^3\right)}-\frac{5a^2b^3\sqrt{a^2+2abx^3+b^2x^6}}{3x^6\left(a+bx^3\right)}-\frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{15x^{15}\left(a+bx^3\right)}-5\ldots$$

Rubi [A] time = 0.07, antiderivative size = 251, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1355, 266, 43}

$$-\frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{15x^{15}\left(a+bx^3\right)}-\frac{5a^4b\sqrt{a^2+2abx^3+b^2x^6}}{12x^{12}\left(a+bx^3\right)}-\frac{10a^3b^2\sqrt{a^2+2abx^3+b^2x^6}}{9x^9\left(a+bx^3\right)}-\frac{5a^2b^3\sqrt{a^2+2abx^3+b^2x^6}}{3x^6\left(a+bx^3\right)}-\frac{5ab^4\sqrt{a^2+2abx^3+b^2x^6}}{3x^3\left(a+bx^3\right)}+\frac{b^5\log(x)\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^16,x]
```

```
[Out] -(a^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(15*x^15*(a + b*x^3)) - (5*a^4*b*Sqr
t[a^2 + 2*a*b*x^3 + b^2*x^6])/(12*x^12*(a + b*x^3)) - (10*a^3*b^2*Sqrt[a^2
+ 2*a*b*x^3 + b^2*x^6])/(9*x^9*(a + b*x^3)) - (5*a^2*b^3*Sqrt[a^2 + 2*a*b*x
^3 + b^2*x^6])/(3*x^6*(a + b*x^3)) - (5*a*b^4*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^
6])/(3*x^3*(a + b*x^3)) + (b^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]*Log[x])/(a +
 b*x^3)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{x^{16}}\,dx = \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{\left(ab+b^2x^3\right)^5}{x^{16}}\,dx}{b^4\left(ab+b^2x^3\right)}$$

$$= \frac{\sqrt{a^2+2abx^3+b^2x^6}\,\text{Subst}\left(\int \frac{\left(ab+b^2x\right)^5}{x^6}\,dx,x,x^3\right)}{3b^4\left(ab+b^2x^3\right)}$$

$$= \frac{\sqrt{a^2+2abx^3+b^2x^6}\,\text{Subst}\left(\int \left(\frac{a^5b^5}{x^6}+\frac{5a^4b^6}{x^5}+\frac{10a^3b^7}{x^4}+\frac{10a^2b^8}{x^3}+\frac{5ab^9}{x^2}+\frac{b^{10}}{x}\right)dx,\right.}{3b^4\left(ab+b^2x^3\right)}$$

$$= -\frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{15x^{15}\left(a+bx^3\right)} - \frac{5a^4b\sqrt{a^2+2abx^3+b^2x^6}}{12x^{12}\left(a+bx^3\right)} - \frac{10a^3b^2\sqrt{a^2+2abx^3+}}{9x^9\left(a+bx^3\right)}$$

Mathematica [A] time = 0.03, size = 85, normalized size = 0.34

$$-\frac{\sqrt{\left(a+bx^3\right)^2}\left(a\left(12a^4+75a^3bx^3+200a^2b^2x^6+300ab^3x^9+300b^4x^{12}\right)-180b^5x^{15}\log(x)\right)}{180x^{15}\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^16,x]
```

```
[Out] -1/180*(Sqrt[(a + b*x^3)^2]*(a*(12*a^4 + 75*a^3*b*x^3 + 200*a^2*b^2*x^6 + 3
00*a*b^3*x^9 + 300*b^4*x^12) - 180*b^5*x^15*Log[x]))/(x^15*(a + b*x^3))
```

IntegrateAlgebraic [B] time = 3.58, size = 2386, normalized size = 9.51

Result too large to show

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^16,x]
```

```
[Out] (4*a*b^4*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]*(-12*a^8*b - 123*a^7*b^2*x^3 - 572
*a^6*b^3*x^6 - 1598*a^5*b^4*x^9 - 3012*a^4*b^5*x^12 - 3875*a^3*b^6*x^15 - 3
200*a^2*b^7*x^18 - 1500*a*b^8*x^21 - 300*b^9*x^24) + 4*a*b^4*Sqrt[b^2]*(12*
a^9 + 135*a^8*b*x^3 + 695*a^7*b^2*x^6 + 2170*a^6*b^3*x^9 + 4610*a^5*b^4*x^1
2 + 6887*a^4*b^5*x^15 + 7075*a^3*b^6*x^18 + 4700*a^2*b^7*x^21 + 1800*a*b^8*
x^24 + 300*b^9*x^27))/(45*Sqrt[b^2]*x^15*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]*(-
16*a^4*b^4 - 64*a^3*b^5*x^3 - 96*a^2*b^6*x^6 - 64*a*b^7*x^9 - 16*b^8*x^12)
+ 45*x^15*(16*a^5*b^5 + 80*a^4*b^6*x^3 + 160*a^3*b^7*x^6 + 160*a^2*b^8*x^9
+ 80*a*b^9*x^12 + 16*b^10*x^15)) + (b^5*Log[-a - Sqrt[b^2]*x^3 + Sqrt[a^2 +
 2*a*b*x^3 + b^2*x^6]])/6 - (b^4*Sqrt[b^2]*Log[-a - Sqrt[b^2]*x^3 + Sqrt[a^
2 + 2*a*b*x^3 + b^2*x^6]])/6 + (a^10*b^5*Log[a - Sqrt[b^2]*x^3 + Sqrt[a^2 +
 2*a*b*x^3 + b^2*x^6]])/(6*(-a - Sqrt[b^2]*x^3 + Sqrt[a^2 + 2*a*b*x^3 + b^2
*x^6])^5*(a - Sqrt[b^2]*x^3 + Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])^5) + (a^10*b
^4*Sqrt[b^2]*Log[a - Sqrt[b^2]*x^3 + Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]])/(6*(
-a - Sqrt[b^2]*x^3 + Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])^5*(a - Sqrt[b^2]*x^3
+ Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])^5) - (5*a^8*b^5*(-(Sqrt[b^2]*x^3) + Sqrt
[a^2 + 2*a*b*x^3 + b^2*x^6])^2*Log[a - Sqrt[b^2]*x^3 + Sqrt[a^2 + 2*a*b*x^3
 + b^2*x^6]])/(6*(-a - Sqrt[b^2]*x^3 + Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])^5*(
a - Sqrt[b^2]*x^3 + Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])^5) - (5*a^8*b^4*Sqrt[b
^2]*(-(Sqrt[b^2]*x^3) + Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])^2*Log[a - Sqrt[b^2
]*x^3 + Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]])/(6*(-a - Sqrt[b^2]*x^3 + Sqrt[a^2
 + 2*a*b*x^3 + b^2*x^6])^5*(a - Sqrt[b^2]*x^3 + Sqrt[a^2 + 2*a*b*x^3 + b^2*
```

```
x^6])^5) + (5*a^6*b^5*(-(Sqrt[b^2]*x^3) + Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])^
4*Log[a - Sqrt[b^2]*x^3 + Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]])/(3*(-a - Sqrt[b
^2]*x^3 + Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])^5*(a - Sqrt[b^2]*x^3 + Sqrt[a^2
+ 2*a*b*x^3 + b^2*x^6])^5) + (5*a^6*b^4*Sqrt[b^2]*(-(Sqrt[b^2]*x^3) + Sqrt[
a^2 + 2*a*b*x^3 + b^2*x^6])^4*Log[a - Sqrt[b^2]*x^3 + Sqrt[a^2 + 2*a*b*x^3
+ b^2*x^6]])/(3*(-a - Sqrt[b^2]*x^3 + Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])^5*(a
 - Sqrt[b^2]*x^3 + Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])^5) - (5*a^4*b^5*(-(Sqrt
[b^2]*x^3) + Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])^6*Log[a - Sqrt[b^2]*x^3 + Sqr
t[a^2 + 2*a*b*x^3 + b^2*x^6]])/(3*(-a - Sqrt[b^2]*x^3 + Sqrt[a^2 + 2*a*b*x^
3 + b^2*x^6])^5*(a - Sqrt[b^2]*x^3 + Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])^5) -
(5*a^4*b^4*Sqrt[b^2]*(-(Sqrt[b^2]*x^3) + Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])^6
*Log[a - Sqrt[b^2]*x^3 + Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]])/(3*(-a - Sqrt[b^
2]*x^3 + Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])^5*(a - Sqrt[b^2]*x^3 + Sqrt[a^2 +
 2*a*b*x^3 + b^2*x^6])^5) + (5*a^2*b^5*(-(Sqrt[b^2]*x^3) + Sqrt[a^2 + 2*a*b
*x^3 + b^2*x^6])^8*Log[a - Sqrt[b^2]*x^3 + Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]]
)/(6*(-a - Sqrt[b^2]*x^3 + Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])^5*(a - Sqrt[b^2
]*x^3 + Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])^5) + (5*a^2*b^4*Sqrt[b^2]*(-(Sqrt[
b^2]*x^3) + Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])^8*Log[a - Sqrt[b^2]*x^3 + Sqrt
[a^2 + 2*a*b*x^3 + b^2*x^6]])/(6*(-a - Sqrt[b^2]*x^3 + Sqrt[a^2 + 2*a*b*x^3
 + b^2*x^6])^5*(a - Sqrt[b^2]*x^3 + Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])^5) - (
b^5*(-(Sqrt[b^2]*x^3) + Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])^10*Log[a - Sqrt[b^
2]*x^3 + Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]])/(6*(-a - Sqrt[b^2]*x^3 + Sqrt[a^
2 + 2*a*b*x^3 + b^2*x^6])^5*(a - Sqrt[b^2]*x^3 + Sqrt[a^2 + 2*a*b*x^3 + b^2
*x^6])^5) - (b^4*Sqrt[b^2]*(-(Sqrt[b^2]*x^3) + Sqrt[a^2 + 2*a*b*x^3 + b^2*x
^6])^10*Log[a - Sqrt[b^2]*x^3 + Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]])/(6*(-a -
Sqrt[b^2]*x^3 + Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])^5*(a - Sqrt[b^2]*x^3 + Sqr
t[a^2 + 2*a*b*x^3 + b^2*x^6])^5)
```

fricas [A] time = 1.23, size = 61, normalized size = 0.24

$$\frac{180\,b^5x^{15}\log(x) - 300\,ab^4x^{12} - 300\,a^2b^3x^9 - 200\,a^3b^2x^6 - 75\,a^4bx^3 - 12\,a^5}{180\,x^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^16,x, algorithm="fricas")
```

```
[Out] 1/180*(180*b^5*x^15*log(x) - 300*a*b^4*x^12 - 300*a^2*b^3*x^9 - 200*a^3*b^2
*x^6 - 75*a^4*b*x^3 - 12*a^5)/x^15
```

giac [A] time = 0.37, size = 123, normalized size = 0.49

$$b^5\log(|x|)\,\mathrm{sgn}\left(bx^3+a\right) - \frac{137\,b^5x^{15}\mathrm{sgn}\left(bx^3+a\right) + 300\,ab^4x^{12}\mathrm{sgn}\left(bx^3+a\right) + 300\,a^2b^3x^9\mathrm{sgn}\left(bx^3+a\right) + 200\,a^3b^2x^6\mathrm{sgn}\left(bx^3+a\right) + 75\,a^4bx^3\mathrm{sgn}\left(bx^3+a\right) + 12\,a^5\mathrm{sgn}\left(bx^3+a\right)}{180\,x^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^16,x, algorithm="giac")
```

```
[Out] b^5*log(abs(x))*sgn(b*x^3 + a) - 1/180*(137*b^5*x^15*sgn(b*x^3 + a) + 300*a
*b^4*x^12*sgn(b*x^3 + a) + 300*a^2*b^3*x^9*sgn(b*x^3 + a) + 200*a^3*b^2*x^6
*sgn(b*x^3 + a) + 75*a^4*b*x^3*sgn(b*x^3 + a) + 12*a^5*sgn(b*x^3 + a))/x^15
```

maple [A] time = 0.01, size = 82, normalized size = 0.33

$$\frac{\left(\left(b\,x^3+a\right)^2\right)^{\frac{5}{2}}\left(180b^5x^{15}\ln(x) - 300a\,b^4x^{12} - 300a^2b^3x^9 - 200a^3b^2x^6 - 75a^4b\,x^3 - 12a^5\right)}{180\left(b\,x^3+a\right)^5 x^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^16,x)
```

```
[Out] 1/180*((b*x^3+a)^2)^(5/2)*(180*b^5*ln(x)*x^15-300*a*b^4*x^12-300*a^2*b^3*x^
9-200*a^3*b^2*x^6-75*a^4*b*x^3-12*a^5)/(b*x^3+a)^5/x^15
```

maxima [B] time = 1.10, size = 374, normalized size = 1.49

$$\frac{\sqrt{b^2x^6+2\,abx^3+a^2}\,b^6x^3}{6\,a^2}+\frac{1}{3}\,(-1)^{2\,b^2x^3+2\,ab}\,b^5\log\left(2\,b^2x^3+2\,ab\right)-\frac{1}{3}\,(-1)^{2\,abx^3+2\,a^2}\,b^5\log\left(\frac{2\,abx}{|x|}+\frac{2\,a^2}{x^2|x|}\right)+\frac{\left(b^2x^6+2\,abx^3+a^2\right)^{\frac{3}{2}}b^6x^3}{12\,a^4}+\frac{\sqrt{b^2x^6+2\,abx^3+a^2}\,b^5}{2\,a}+\frac{7\left(b^2x^6+2\,abx^3+a^2\right)^{\frac{3}{2}}b^5}{36\,a^3}-\frac{2\left(b^2x^6+2\,abx^3+a^2\right)^{\frac{5}{2}}b^5}{45\,a^5}-\frac{\left(b^2x^6+2\,abx^3+a^2\right)^{\frac{5}{2}}b^4}{9\,a^4x^3}+\frac{2\left(b^2x^6+2\,abx^3+a^2\right)^{\frac{7}{2}}b^3}{45\,a^5x^6}-\frac{11\left(b^2x^6+2\,abx^3+a^2\right)^{\frac{7}{2}}b^2}{180\,a^4x^9}+\frac{\left(b^2x^6+2\,abx^3+a^2\right)^{\frac{7}{2}}b}{20\,a^3x^{12}}-\frac{\left(b^2x^6+2\,abx^3+a^2\right)^{\frac{7}{2}}}{15\,a^2x^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^16,x, algorithm="maxima")
```

```
[Out] 1/6*sqrt(b^2*x^6 + 2*a*b*x^3 + a^2)*b^6*x^3/a^2 + 1/3*(-1)^(2*b^2*x^3 + 2*a
*b)*b^5*log(2*b^2*x^3 + 2*a*b) - 1/3*(-1)^(2*a*b*x^3 + 2*a^2)*b^5*log(2*a*b
*x/abs(x) + 2*a^2/(x^2*abs(x))) + 1/12*(b^2*x^6 + 2*a*b*x^3 + a^2)^(3/2)*b^
6*x^3/a^4 + 1/2*sqrt(b^2*x^6 + 2*a*b*x^3 + a^2)*b^5/a + 7/36*(b^2*x^6 + 2*a
*b*x^3 + a^2)^(3/2)*b^5/a^3 - 2/45*(b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2)*b^5/a^
5 - 1/9*(b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2)*b^4/(a^4*x^3) + 2/45*(b^2*x^6 + 2
*a*b*x^3 + a^2)^(7/2)*b^3/(a^5*x^6) - 11/180*(b^2*x^6 + 2*a*b*x^3 + a^2)^(7
/2)*b^2/(a^4*x^9) + 1/20*(b^2*x^6 + 2*a*b*x^3 + a^2)^(7/2)*b/(a^3*x^12) - 1
/15*(b^2*x^6 + 2*a*b*x^3 + a^2)^(7/2)/(a^2*x^15)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(a^2+2\,a\,b\,x^3+b^2\,x^6\right)}^{5/2}}{x^{16}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2)/x^16,x)
```

```
[Out] int((a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2)/x^16, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\left(a + bx^{3}\right)^{2}\right)^{\frac{5}{2}}}{x^{16}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**6+2*a*b*x**3+a**2)**(5/2)/x**16,x)
```

```
[Out] Integral(((a + b*x**3)**2)**(5/2)/x**16, x)
```

3.80 $\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^{17}} dx$

Optimal. Leaf size=251

$$-\frac{b^5\sqrt{a^2+2abx^3+b^2x^6}}{x(a+bx^3)}-\frac{5ab^4\sqrt{a^2+2abx^3+b^2x^6}}{4x^4(a+bx^3)}-\frac{10a^2b^3\sqrt{a^2+2abx^3+b^2x^6}}{7x^7(a+bx^3)}-\frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{16x^{16}(a+bx^3)}-\frac{5a^4b\sqrt{}}{1}$$

Rubi [A] time = 0.06, antiderivative size = 251, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {1355, 270}

$$-\frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{16x^{16}(a+bx^3)}-\frac{5a^4b\sqrt{a^2+2abx^3+b^2x^6}}{13x^{13}(a+bx^3)}-\frac{a^3b^2\sqrt{a^2+2abx^3+b^2x^6}}{x^{10}(a+bx^3)}-\frac{10a^2b^3\sqrt{a^2+2abx^3+b^2x^6}}{7x^7(a+bx^3)}-\frac{5ab^4\sqrt{a^2+2abx^3+b^2x^6}}{4x^4(a+bx^3)}-\frac{b^5\sqrt{a^2+2abx^3+b^2x^6}}{x(a+bx^3)}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^17,x]
```

```
[Out] -(a^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(16*x^16*(a + b*x^3)) - (5*a^4*b*Sqr
t[a^2 + 2*a*b*x^3 + b^2*x^6])/(13*x^13*(a + b*x^3)) - (a^3*b^2*Sqrt[a^2 + 2
*a*b*x^3 + b^2*x^6])/(x^10*(a + b*x^3)) - (10*a^2*b^3*Sqrt[a^2 + 2*a*b*x^3
+ b^2*x^6])/(7*x^7*(a + b*x^3)) - (5*a*b^4*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])
/(4*x^4*(a + b*x^3)) - (b^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(x*(a + b*x^3)
)
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^{17}}\,dx &= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{(ab+b^2x^3)^5}{x^{17}}\,dx}{b^4(ab+b^2x^3)}\\
&= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \left(\frac{a^5b^5}{x^{17}}+\frac{5a^4b^6}{x^{14}}+\frac{10a^3b^7}{x^{11}}+\frac{10a^2b^8}{x^8}+\frac{5ab^9}{x^5}+\frac{b^{10}}{x^2}\right)dx}{b^4(ab+b^2x^3)}\\
&= -\frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{16x^{16}(a+bx^3)}-\frac{5a^4b\sqrt{a^2+2abx^3+b^2x^6}}{13x^{13}(a+bx^3)}-\frac{a^3b^2\sqrt{a^2+2abx^3+b^2x^6}}{x^{10}(a+bx^3)}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 83, normalized size = 0.33

$$-\frac{\sqrt{(a+bx^3)^2}\left(91a^5+560a^4bx^3+1456a^3b^2x^6+2080a^2b^3x^9+1820ab^4x^{12}+1456b^5x^{15}\right)}{1456x^{16}(a+bx^3)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^17,x]
```

```
[Out] -1/1456*(Sqrt[(a + b*x^3)^2]*(91*a^5 + 560*a^4*b*x^3 + 1456*a^3*b^2*x^6 + 2
080*a^2*b^3*x^9 + 1820*a*b^4*x^12 + 1456*b^5*x^15))/(x^16*(a + b*x^3))
```

IntegrateAlgebraic [A] time = 20.03, size = 83, normalized size = 0.33

$$\frac{\sqrt{\left(a+bx^3\right)^2}\left(-91a^5-560a^4bx^3-1456a^3b^2x^6-2080a^2b^3x^9-1820ab^4x^{12}-1456b^5x^{15}\right)}{1456x^{16}\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^17,x]
```

```
[Out] (Sqrt[(a + b*x^3)^2]*(-91*a^5 - 560*a^4*b*x^3 - 1456*a^3*b^2*x^6 - 2080*a^2
*b^3*x^9 - 1820*a*b^4*x^12 - 1456*b^5*x^15))/(1456*x^16*(a + b*x^3))
```

fricas [A] time = 1.16, size = 59, normalized size = 0.24

$$-\frac{1456\,b^5x^{15}+1820\,ab^4x^{12}+2080\,a^2b^3x^9+1456\,a^3b^2x^6+560\,a^4bx^3+91\,a^5}{1456\,x^{16}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^17,x, algorithm="fricas")
```

```
[Out] -1/1456*(1456*b^5*x^15 + 1820*a*b^4*x^12 + 2080*a^2*b^3*x^9 + 1456*a^3*b^2*
x^6 + 560*a^4*b*x^3 + 91*a^5)/x^16
```

giac [A] time = 0.34, size = 107, normalized size = 0.43

$$-\frac{1456\,b^5x^{15}\mathrm{sgn}\left(bx^3+a\right)+1820\,ab^4x^{12}\mathrm{sgn}\left(bx^3+a\right)+2080\,a^2b^3x^9\mathrm{sgn}\left(bx^3+a\right)+1456\,a^3b^2x^6\mathrm{sgn}\left(bx^3+a\right)+560\,a^4bx^3\mathrm{sgn}\left(bx^3+a\right)+91\,a^5\mathrm{sgn}\left(bx^3+a\right)}{1456\,x^{16}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^17,x, algorithm="giac")
```

```
[Out] -1/1456*(1456*b^5*x^15*sgn(b*x^3 + a) + 1820*a*b^4*x^12*sgn(b*x^3 + a) + 20
80*a^2*b^3*x^9*sgn(b*x^3 + a) + 1456*a^3*b^2*x^6*sgn(b*x^3 + a) + 560*a^4*b
*x^3*sgn(b*x^3 + a) + 91*a^5*sgn(b*x^3 + a))/x^16
```

maple [A] time = 0.01, size = 80, normalized size = 0.32

$$-\frac{\left(1456b^5x^{15}+1820a\,b^4x^{12}+2080a^2b^3x^9+1456a^3b^2x^6+560a^4b\,x^3+91a^5\right)\left(\left(b\,x^3+a\right)^2\right)^{\frac{5}{2}}}{1456\left(b\,x^3+a\right)^5x^{16}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^17,x)
```

```
[Out] -1/1456*(1456*b^5*x^15+1820*a*b^4*x^12+2080*a^2*b^3*x^9+1456*a^3*b^2*x^6+56
0*a^4*b*x^3+91*a^5)*((b*x^3+a)^2)^(5/2)/x^16/(b*x^3+a)^5
```

maxima [A] time = 0.99, size = 59, normalized size = 0.24

$$-\frac{1456\,b^5x^{15}+1820\,ab^4x^{12}+2080\,a^2b^3x^9+1456\,a^3b^2x^6+560\,a^4bx^3+91\,a^5}{1456\,x^{16}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^17,x, algorithm="maxima")
```

```
[Out] -1/1456*(1456*b^5*x^15 + 1820*a*b^4*x^12 + 2080*a^2*b^3*x^9 + 1456*a^3*b^2*
x^6 + 560*a^4*b*x^3 + 91*a^5)/x^16
```

mupad [B] time = 1.26, size = 231, normalized size = 0.92

$$-\frac{a^5\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{16\,x^{16}\,\left(b\,x^3+a\right)}-\frac{b^5\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{x\,\left(b\,x^3+a\right)}-\frac{5\,a\,b^4\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{4\,x^4\,\left(b\,x^3+a\right)}-\frac{5\,a^4\,b\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{13\,x^{13}\,\left(b\,x^3+a\right)}-\frac{10\,a^2\,b^3\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{7\,x^7\,\left(b\,x^3+a\right)}-\frac{a^3\,b^2\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{x^{10}\,\left(b\,x^3+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2)/x^17,x)
```

```
[Out] - (a^5*(a^2 + b^2*x^6 + 2*a*b*x^3)^(1/2))/(16*x^16*(a + b*x^3)) - (b^5*(a^2
 + b^2*x^6 + 2*a*b*x^3)^(1/2))/(x*(a + b*x^3)) - (5*a*b^4*(a^2 + b^2*x^6 +
2*a*b*x^3)^(1/2))/(4*x^4*(a + b*x^3)) - (5*a^4*b*(a^2 + b^2*x^6 + 2*a*b*x^3
)^(1/2))/(13*x^13*(a + b*x^3)) - (10*a^2*b^3*(a^2 + b^2*x^6 + 2*a*b*x^3)^(1
/2))/(7*x^7*(a + b*x^3)) - (a^3*b^2*(a^2 + b^2*x^6 + 2*a*b*x^3)^(1/2))/(x^1
0*(a + b*x^3))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\left(a + bx^{3}\right)^{2}\right)^{\frac{5}{2}}}{x^{17}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**6+2*a*b*x**3+a**2)**(5/2)/x**17,x)
```

```
[Out] Integral(((a + b*x**3)**2)**(5/2)/x**17, x)
```

3.81 $\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{x^{18}} \, dx$

Optimal. Leaf size=253

$$-\frac{b^5\sqrt{a^2+2abx^3+b^2x^6}}{2x^2\left(a+bx^3\right)}-\frac{ab^4\sqrt{a^2+2abx^3+b^2x^6}}{x^5\left(a+bx^3\right)}-\frac{5a^2b^3\sqrt{a^2+2abx^3+b^2x^6}}{4x^8\left(a+bx^3\right)}-\frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{17x^{17}\left(a+bx^3\right)}-\frac{5a^4b}{1}$$

Rubi [A] time = 0.06, antiderivative size = 253, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {1355, 270}

$$-\frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{17x^{17}\left(a+bx^3\right)}-\frac{5a^4b\sqrt{a^2+2abx^3+b^2x^6}}{14x^{14}\left(a+bx^3\right)}-\frac{10a^3b^2\sqrt{a^2+2abx^3+b^2x^6}}{11x^{11}\left(a+bx^3\right)}-\frac{5a^2b^3\sqrt{a^2+2abx^3+b^2x^6}}{4x^8\left(a+bx^3\right)}-\frac{ab^4\sqrt{a^2+2abx^3+b^2x^6}}{x^5\left(a+bx^3\right)}-\frac{b^5\sqrt{a^2+2abx^3+b^2x^6}}{2x^2\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^18,x]
```

```
[Out] -(a^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(17*x^17*(a + b*x^3)) - (5*a^4*b*Sqr
t[a^2 + 2*a*b*x^3 + b^2*x^6])/(14*x^14*(a + b*x^3)) - (10*a^3*b^2*Sqrt[a^2
+ 2*a*b*x^3 + b^2*x^6])/(11*x^11*(a + b*x^3)) - (5*a^2*b^3*Sqrt[a^2 + 2*a*b
*x^3 + b^2*x^6])/(4*x^8*(a + b*x^3)) - (a*b^4*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^
6])/(x^5*(a + b*x^3)) - (b^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(2*x^2*(a + b
*x^3))
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{x^{18}} \, dx &= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{\left(ab+b^2x^3\right)^5}{x^{18}} \, dx}{b^4\left(ab+b^2x^3\right)}\\ &= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \left(\frac{a^5b^5}{x^{18}}+\frac{5a^4b^6}{x^{15}}+\frac{10a^3b^7}{x^{12}}+\frac{10a^2b^8}{x^9}+\frac{5ab^9}{x^6}+\frac{b^{10}}{x^3}\right) dx}{b^4\left(ab+b^2x^3\right)}\\ &= -\frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{17x^{17}\left(a+bx^3\right)}-\frac{5a^4b\sqrt{a^2+2abx^3+b^2x^6}}{14x^{14}\left(a+bx^3\right)}-\frac{10a^3b^2\sqrt{a^2+2abx^3+}}{11x^{11}\left(a+bx^3\right)}\end{aligned}$$

Mathematica [A] time = 0.02, size = 83, normalized size = 0.33

$$-\frac{\sqrt{\left(a+bx^3\right)^2}\left(308a^5+1870a^4bx^3+4760a^3b^2x^6+6545a^2b^3x^9+5236ab^4x^{12}+2618b^5x^{15}\right)}{5236x^{17}\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^18,x]
```

```
[Out] -1/5236*(Sqrt[(a + b*x^3)^2]*(308*a^5 + 1870*a^4*b*x^3 + 4760*a^3*b^2*x^6 +
 6545*a^2*b^3*x^9 + 5236*a*b^4*x^12 + 2618*b^5*x^15))/(x^17*(a + b*x^3))
```

IntegrateAlgebraic [A] time = 20.55, size = 83, normalized size = 0.33

$$\frac{\sqrt{\left(a+bx^3\right)^2}\left(-308a^5-1870a^4bx^3-4760a^3b^2x^6-6545a^2b^3x^9-5236ab^4x^{12}-2618b^5x^{15}\right)}{5236x^{17}\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^18,x]
```

```
[Out] (Sqrt[(a + b*x^3)^2]*(-308*a^5 - 1870*a^4*b*x^3 - 4760*a^3*b^2*x^6 - 6545*a
^2*b^3*x^9 - 5236*a*b^4*x^12 - 2618*b^5*x^15))/(5236*x^17*(a + b*x^3))
```

fricas [A] time = 1.21, size = 59, normalized size = 0.23

$$-\frac{2618\,b^5x^{15}+5236\,ab^4x^{12}+6545\,a^2b^3x^9+4760\,a^3b^2x^6+1870\,a^4bx^3+308\,a^5}{5236\,x^{17}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^18,x, algorithm="fricas")
```

```
[Out] -1/5236*(2618*b^5*x^15 + 5236*a*b^4*x^12 + 6545*a^2*b^3*x^9 + 4760*a^3*b^2*
x^6 + 1870*a^4*b*x^3 + 308*a^5)/x^17
```

giac [A] time = 0.45, size = 107, normalized size = 0.42

$$-\frac{2618\,b^5x^{15}\mathrm{sgn}\left(bx^3+a\right)+5236\,ab^4x^{12}\mathrm{sgn}\left(bx^3+a\right)+6545\,a^2b^3x^9\mathrm{sgn}\left(bx^3+a\right)+4760\,a^3b^2x^6\mathrm{sgn}\left(bx^3+a\right)+1870\,a^4bx^3\mathrm{sgn}\left(bx^3+a\right)+308\,a^5\mathrm{sgn}\left(bx^3+a\right)}{5236\,x^{17}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^18,x, algorithm="giac")
```

```
[Out] -1/5236*(2618*b^5*x^15*sgn(b*x^3 + a) + 5236*a*b^4*x^12*sgn(b*x^3 + a) + 65
45*a^2*b^3*x^9*sgn(b*x^3 + a) + 4760*a^3*b^2*x^6*sgn(b*x^3 + a) + 1870*a^4*
b*x^3*sgn(b*x^3 + a) + 308*a^5*sgn(b*x^3 + a))/x^17
```

maple [A] time = 0.01, size = 80, normalized size = 0.32

$$-\frac{\left(2618b^5x^{15}+5236a\,b^4x^{12}+6545a^2b^3x^9+4760a^3b^2x^6+1870a^4b\,x^3+308a^5\right)\left(\left(b\,x^3+a\right)^2\right)^{\frac{5}{2}}}{5236\left(b\,x^3+a\right)^5x^{17}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^18,x)
```

```
[Out] -1/5236*(2618*b^5*x^15+5236*a*b^4*x^12+6545*a^2*b^3*x^9+4760*a^3*b^2*x^6+18
70*a^4*b*x^3+308*a^5)*((b*x^3+a)^2)^(5/2)/x^17/(b*x^3+a)^5
```

maxima [A] time = 1.16, size = 59, normalized size = 0.23

$$-\frac{2618\,b^5x^{15}+5236\,ab^4x^{12}+6545\,a^2b^3x^9+4760\,a^3b^2x^6+1870\,a^4bx^3+308\,a^5}{5236\,x^{17}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^18,x, algorithm="maxima")
```

```
[Out] -1/5236*(2618*b^5*x^15 + 5236*a*b^4*x^12 + 6545*a^2*b^3*x^9 + 4760*a^3*b^2*
x^6 + 1870*a^4*b*x^3 + 308*a^5)/x^17
```

mupad [B] time = 1.32, size = 231, normalized size = 0.91

$$-\frac{a^5\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{17\,x^{17}\,\left(b\,x^3+a\right)}-\frac{b^5\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{2\,x^2\,\left(b\,x^3+a\right)}-\frac{a\,b^4\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{x^5\,\left(b\,x^3+a\right)}-\frac{5\,a^4\,b\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{14\,x^{14}\,\left(b\,x^3+a\right)}-\frac{5\,a^2\,b^3\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{4\,x^8\,\left(b\,x^3+a\right)}-\frac{10\,a^3\,b^2\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{11\,x^{11}\,\left(b\,x^3+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2)/x^18,x)
```

```
[Out] - (a^5*(a^2 + b^2*x^6 + 2*a*b*x^3)^(1/2))/(17*x^17*(a + b*x^3)) - (b^5*(a^2
 + b^2*x^6 + 2*a*b*x^3)^(1/2))/(2*x^2*(a + b*x^3)) - (a*b^4*(a^2 + b^2*x^6
+ 2*a*b*x^3)^(1/2))/(x^5*(a + b*x^3)) - (5*a^4*b*(a^2 + b^2*x^6 + 2*a*b*x^3
)^(1/2))/(14*x^14*(a + b*x^3)) - (5*a^2*b^3*(a^2 + b^2*x^6 + 2*a*b*x^3)^(1/
2))/(4*x^8*(a + b*x^3)) - (10*a^3*b^2*(a^2 + b^2*x^6 + 2*a*b*x^3)^(1/2))/(1
1*x^11*(a + b*x^3))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\left(a + bx^3\right)^2\right)^{\frac{5}{2}}}{x^{18}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**6+2*a*b*x**3+a**2)**(5/2)/x**18,x)
```

```
[Out] Integral(((a + b*x**3)**2)**(5/2)/x**18, x)
```

3.82 $\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^{19}}\,dx$

Optimal. Leaf size=41

$$-\frac{(a+bx^3)^5\sqrt{a^2+2abx^3+b^2x^6}}{18ax^{18}}$$

Rubi [A] time = 0.02, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1355, 264}

$$-\frac{(a+bx^3)^5\sqrt{a^2+2abx^3+b^2x^6}}{18ax^{18}}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^19,x]
```

```
[Out] -((a + b*x^3)^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(18*a*x^18)
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^{19}}\,dx &= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{(ab+b^2x^3)^5}{x^{19}}\,dx}{b^4(ab+b^2x^3)}\\
&= -\frac{(a+bx^3)^5\sqrt{a^2+2abx^3+b^2x^6}}{18ax^{18}}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 81, normalized size = 1.98

$$-\frac{\sqrt{(a+bx^3)^2}\left(a^5+6a^4bx^3+15a^3b^2x^6+20a^2b^3x^9+15ab^4x^{12}+6b^5x^{15}\right)}{18x^{18}(a+bx^3)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^19,x]
```

```
[Out] -1/18*(Sqrt[(a + b*x^3)^2]*(a^5 + 6*a^4*b*x^3 + 15*a^3*b^2*x^6 + 20*a^2*b^3
*x^9 + 15*a*b^4*x^12 + 6*b^5*x^15))/(x^18*(a + b*x^3))
```

IntegrateAlgebraic [B] time = 2.89, size = 442, normalized size = 10.78

$$\frac{16b^5\sqrt{a^2+2abx^3+b^2x^6}\left(-a^{10}b-11a^9b^2x^3-55a^8b^3x^6-165a^7b^4x^9-330a^6b^5x^{12}-462a^5b^6x^{15}-461a^4b^7x^{18}-325a^3b^8x^{21}-155a^2b^9x^{24}-45ab^{10}x^{27}-6b^{11}x^{30}\right)+16\sqrt{b^2}\,b^5\left(a^{11}+12a^{10}bx^3+66a^9b^2x^6+220a^8b^3x^9+495a^7b^4x^{12}+792a^6b^5x^{15}+923a^5b^6x^{18}+786a^4b^7x^{21}+480a^3b^8x^{24}+200a^2b^9x^{27}+51ab^{10}x^{30}+6b^{11}x^{33}\right)}{9\sqrt{b^2}\,x^{18}\sqrt{a^2+2abx^3+b^2x^6}\left(-32a^5b^5-160a^4b^6x^3-320a^3b^7x^6-320a^2b^8x^9-160ab^9x^{12}-32b^{10}x^{15}\right)+9x^{18}\left(32a^6b^6+192a^5b^7x^3+480a^4b^8x^6+640a^3b^9x^9+480a^2b^{10}x^{12}+192ab^{11}x^{15}+32b^{12}x^{18}\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^19,x]
```

```
[Out] (16*b^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]*(-(a^10*b) - 11*a^9*b^2*x^3 - 55*a^
8*b^3*x^6 - 165*a^7*b^4*x^9 - 330*a^6*b^5*x^12 - 462*a^5*b^6*x^15 - 461*a^4
*b^7*x^18 - 325*a^3*b^8*x^21 - 155*a^2*b^9*x^24 - 45*a*b^10*x^27 - 6*b^11*x
^30) + 16*b^5*Sqrt[b^2]*(a^11 + 12*a^10*b*x^3 + 66*a^9*b^2*x^6 + 220*a^8*b^
3*x^9 + 495*a^7*b^4*x^12 + 792*a^6*b^5*x^15 + 923*a^5*b^6*x^18 + 786*a^4*b^
7*x^21 + 480*a^3*b^8*x^24 + 200*a^2*b^9*x^27 + 51*a*b^10*x^30 + 6*b^11*x^33
))/(9*Sqrt[b^2]*x^18*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]*(-32*a^5*b^5 - 160*a^4
*b^6*x^3 - 320*a^3*b^7*x^6 - 320*a^2*b^8*x^9 - 160*a*b^9*x^12 - 32*b^10*x^1
5) + 9*x^18*(32*a^6*b^6 + 192*a^5*b^7*x^3 + 480*a^4*b^8*x^6 + 640*a^3*b^9*x
^9 + 480*a^2*b^10*x^12 + 192*a*b^11*x^15 + 32*b^12*x^18))
```

fricas [B] time = 1.14, size = 57, normalized size = 1.39

$$-\frac{6\,b^5x^{15}+15\,ab^4x^{12}+20\,a^2b^3x^9+15\,a^3b^2x^6+6\,a^4bx^3+a^5}{18\,x^{18}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^19,x, algorithm="fricas")
```

```
[Out] -1/18*(6*b^5*x^15 + 15*a*b^4*x^12 + 20*a^2*b^3*x^9 + 15*a^3*b^2*x^6 + 6*a^4
*b*x^3 + a^5)/x^18
```

giac [B] time = 0.29, size = 106, normalized size = 2.59

$$-\frac{6\,b^5x^{15}\mathrm{sgn}\left(bx^3+a\right)+15\,ab^4x^{12}\mathrm{sgn}\left(bx^3+a\right)+20\,a^2b^3x^9\mathrm{sgn}\left(bx^3+a\right)+15\,a^3b^2x^6\mathrm{sgn}\left(bx^3+a\right)+6\,a^4bx^3\mathrm{sgn}\left(bx^3+a\right)+a^5\mathrm{sgn}\left(bx^3+a\right)}{18\,x^{18}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^19,x, algorithm="giac")
```

```
[Out] -1/18*(6*b^5*x^15*sgn(b*x^3 + a) + 15*a*b^4*x^12*sgn(b*x^3 + a) + 20*a^2*b^
3*x^9*sgn(b*x^3 + a) + 15*a^3*b^2*x^6*sgn(b*x^3 + a) + 6*a^4*b*x^3*sgn(b*x^
3 + a) + a^5*sgn(b*x^3 + a))/x^18
```

maple [B] time = 0.01, size = 78, normalized size = 1.90

$$-\frac{\left(6b^5x^{15}+15a\,b^4x^{12}+20a^2b^3x^9+15a^3b^2x^6+6a^4b\,x^3+a^5\right)\left(\left(b\,x^3+a\right)^2\right)^{\frac{5}{2}}}{18\left(b\,x^3+a\right)^5x^{18}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^19,x)
```

```
[Out] -1/18*(6*b^5*x^15+15*a*b^4*x^12+20*a^2*b^3*x^9+15*a^3*b^2*x^6+6*a^4*b*x^3+a
^5)*((b*x^3+a)^2)^(5/2)/x^18/(b*x^3+a)^5
```

maxima [B] time = 1.10, size = 210, normalized size = 5.12

$$\frac{\left(b^2x^6+2\,abx^3+a^2\right)^{\frac{5}{2}}b^6}{18\,a^6}+\frac{\left(b^2x^6+2\,abx^3+a^2\right)^{\frac{5}{2}}b^5}{18\,a^5x^3}-\frac{\left(b^2x^6+2\,abx^3+a^2\right)^{\frac{7}{2}}b^4}{18\,a^6x^6}+\frac{\left(b^2x^6+2\,abx^3+a^2\right)^{\frac{7}{2}}b^3}{18\,a^5x^9}-\frac{\left(b^2x^6+2\,abx^3+a^2\right)^{\frac{7}{2}}b^2}{18\,a^4x^{12}}+\frac{\left(b^2x^6+2\,abx^3+a^2\right)^{\frac{7}{2}}b}{18\,a^3x^{15}}-\frac{\left(b^2x^6+2\,abx^3+a^2\right)^{\frac{7}{2}}}{18\,a^2x^{18}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^19,x, algorithm="maxima")
```

```
[Out] 1/18*(b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2)*b^6/a^6 + 1/18*(b^2*x^6 + 2*a*b*x^3
+ a^2)^(5/2)*b^5/(a^5*x^3) - 1/18*(b^2*x^6 + 2*a*b*x^3 + a^2)^(7/2)*b^4/(a^
```

```
6*x^6) + 1/18*(b^2*x^6 + 2*a*b*x^3 + a^2)^(7/2)*b^3/(a^5*x^9) - 1/18*(b^2*x
^6 + 2*a*b*x^3 + a^2)^(7/2)*b^2/(a^4*x^12) + 1/18*(b^2*x^6 + 2*a*b*x^3 + a^
2)^(7/2)*b/(a^3*x^15) - 1/18*(b^2*x^6 + 2*a*b*x^3 + a^2)^(7/2)/(a^2*x^18)
```

mupad [B] time = 1.22, size = 231, normalized size = 5.63

$$-\frac{a^5\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{18\,x^{18}\,\left(b\,x^3+a\right)}-\frac{b^5\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{3\,x^3\,\left(b\,x^3+a\right)}-\frac{5\,a\,b^4\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{6\,x^6\,\left(b\,x^3+a\right)}-\frac{a^4\,b\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{3\,x^{15}\,\left(b\,x^3+a\right)}-\frac{10\,a^2\,b^3\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{9\,x^9\,\left(b\,x^3+a\right)}-\frac{5\,a^3\,b^2\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{6\,x^{12}\,\left(b\,x^3+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2)/x^19,x)
```

```
[Out] - (a^5*(a^2 + b^2*x^6 + 2*a*b*x^3)^(1/2))/(18*x^18*(a + b*x^3)) - (b^5*(a^2
 + b^2*x^6 + 2*a*b*x^3)^(1/2))/(3*x^3*(a + b*x^3)) - (5*a*b^4*(a^2 + b^2*x^
6 + 2*a*b*x^3)^(1/2))/(6*x^6*(a + b*x^3)) - (a^4*b*(a^2 + b^2*x^6 + 2*a*b*x
^3)^(1/2))/(3*x^15*(a + b*x^3)) - (10*a^2*b^3*(a^2 + b^2*x^6 + 2*a*b*x^3)^(
1/2))/(9*x^9*(a + b*x^3)) - (5*a^3*b^2*(a^2 + b^2*x^6 + 2*a*b*x^3)^(1/2))/(
6*x^12*(a + b*x^3))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\left(a + bx^3\right)^2\right)^{\frac{5}{2}}}{x^{19}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**6+2*a*b*x**3+a**2)**(5/2)/x**19,x)
```

```
[Out] Integral(((a + b*x**3)**2)**(5/2)/x**19, x)
```

3.83 $\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{x^{20}}\,dx$

Optimal. Leaf size=253

$$-\frac{b^5\sqrt{a^2+2abx^3+b^2x^6}}{4x^4\left(a+bx^3\right)}-\frac{5ab^4\sqrt{a^2+2abx^3+b^2x^6}}{7x^7\left(a+bx^3\right)}-\frac{a^2b^3\sqrt{a^2+2abx^3+b^2x^6}}{x^{10}\left(a+bx^3\right)}-\frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{19x^{19}\left(a+bx^3\right)}-\frac{5a^4b\cdots}{1\cdots}$$

Rubi [A] time = 0.06, antiderivative size = 253, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1355, 270}

$$-\frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{19x^{19}\left(a+bx^3\right)}-\frac{5a^4b\sqrt{a^2+2abx^3+b^2x^6}}{16x^{16}\left(a+bx^3\right)}-\frac{10a^3b^2\sqrt{a^2+2abx^3+b^2x^6}}{13x^{13}\left(a+bx^3\right)}-\frac{a^2b^3\sqrt{a^2+2abx^3+b^2x^6}}{x^{10}\left(a+bx^3\right)}-\frac{5ab^4\sqrt{a^2+2abx^3+b^2x^6}}{7x^7\left(a+bx^3\right)}-\frac{b^5\sqrt{a^2+2abx^3+b^2x^6}}{4x^4\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^20,x]
```

```
[Out] -(a^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(19*x^19*(a + b*x^3)) - (5*a^4*b*Sqr
t[a^2 + 2*a*b*x^3 + b^2*x^6])/(16*x^16*(a + b*x^3)) - (10*a^3*b^2*Sqrt[a^2
+ 2*a*b*x^3 + b^2*x^6])/(13*x^13*(a + b*x^3)) - (a^2*b^3*Sqrt[a^2 + 2*a*b*x
^3 + b^2*x^6])/(x^10*(a + b*x^3)) - (5*a*b^4*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6
])/(7*x^7*(a + b*x^3)) - (b^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(4*x^4*(a +
b*x^3))
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{x^{20}}\,dx &= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{\left(ab+b^2x^3\right)^5}{x^{20}}\,dx}{b^4\left(ab+b^2x^3\right)}\\ &= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \left(\frac{a^5b^5}{x^{20}}+\frac{5a^4b^6}{x^{17}}+\frac{10a^3b^7}{x^{14}}+\frac{10a^2b^8}{x^{11}}+\frac{5ab^9}{x^8}+\frac{b^{10}}{x^5}\right)dx}{b^4\left(ab+b^2x^3\right)}\\ &= -\frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{19x^{19}\left(a+bx^3\right)}-\frac{5a^4b\sqrt{a^2+2abx^3+b^2x^6}}{16x^{16}\left(a+bx^3\right)}-\frac{10a^3b^2\sqrt{a^2+2abx^3+\cdots}}{13x^{13}\left(a+bx^3\right)}\end{aligned}$$

Mathematica [A] time = 0.02, size = 83, normalized size = 0.33

$$-\frac{\sqrt{\left(a+bx^3\right)^2}\left(1456a^5+8645a^4bx^3+21280a^3b^2x^6+27664a^2b^3x^9+19760ab^4x^{12}+6916b^5x^{15}\right)}{27664x^{19}\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^20,x]
```

```
[Out] -1/27664*(Sqrt[(a + b*x^3)^2]*(1456*a^5 + 8645*a^4*b*x^3 + 21280*a^3*b^2*x^
6 + 27664*a^2*b^3*x^9 + 19760*a*b^4*x^12 + 6916*b^5*x^15))/(x^19*(a + b*x^3
))
```

IntegrateAlgebraic [A] time = 21.32, size = 83, normalized size = 0.33

$$\frac{\sqrt{\left(a+bx^3\right)^2}\left(-1456a^5-8645a^4bx^3-21280a^3b^2x^6-27664a^2b^3x^9-19760ab^4x^{12}-6916b^5x^{15}\right)}{27664x^{19}\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^20,x]
```

```
[Out] (Sqrt[(a + b*x^3)^2]*(-1456*a^5 - 8645*a^4*b*x^3 - 21280*a^3*b^2*x^6 - 2766
4*a^2*b^3*x^9 - 19760*a*b^4*x^12 - 6916*b^5*x^15))/(27664*x^19*(a + b*x^3))
```

fricas [A] time = 1.10, size = 59, normalized size = 0.23

$$-\frac{6916\,b^5x^{15}+19760\,ab^4x^{12}+27664\,a^2b^3x^9+21280\,a^3b^2x^6+8645\,a^4bx^3+1456\,a^5}{27664\,x^{19}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^20,x, algorithm="fricas")
```

```
[Out] -1/27664*(6916*b^5*x^15 + 19760*a*b^4*x^12 + 27664*a^2*b^3*x^9 + 21280*a^3*
b^2*x^6 + 8645*a^4*b*x^3 + 1456*a^5)/x^19
```

giac [A] time = 0.36, size = 107, normalized size = 0.42

$$-\frac{6916\,b^5x^{15}\mathrm{sgn}\left(bx^3+a\right)+19760\,ab^4x^{12}\mathrm{sgn}\left(bx^3+a\right)+27664\,a^2b^3x^9\mathrm{sgn}\left(bx^3+a\right)+21280\,a^3b^2x^6\mathrm{sgn}\left(bx^3+a\right)+8645\,a^4bx^3\mathrm{sgn}\left(bx^3+a\right)+1456\,a^5\mathrm{sgn}\left(bx^3+a\right)}{27664\,x^{19}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^20,x, algorithm="giac")
```

```
[Out] -1/27664*(6916*b^5*x^15*sgn(b*x^3 + a) + 19760*a*b^4*x^12*sgn(b*x^3 + a) +
27664*a^2*b^3*x^9*sgn(b*x^3 + a) + 21280*a^3*b^2*x^6*sgn(b*x^3 + a) + 8645*
a^4*b*x^3*sgn(b*x^3 + a) + 1456*a^5*sgn(b*x^3 + a))/x^19
```

maple [A] time = 0.01, size = 80, normalized size = 0.32

$$-\frac{\left(6916b^5x^{15}+19760a\,b^4x^{12}+27664a^2b^3x^9+21280a^3b^2x^6+8645a^4b\,x^3+1456a^5\right)\left(\left(b\,x^3+a\right)^2\right)^{\frac{5}{2}}}{27664\left(b\,x^3+a\right)^5x^{19}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^20,x)
```

```
[Out] -1/27664*(6916*b^5*x^15+19760*a*b^4*x^12+27664*a^2*b^3*x^9+21280*a^3*b^2*x^
6+8645*a^4*b*x^3+1456*a^5)*((b*x^3+a)^2)^(5/2)/x^19/(b*x^3+a)^5
```

maxima [A] time = 1.04, size = 59, normalized size = 0.23

$$-\frac{6916\,b^5x^{15}+19760\,ab^4x^{12}+27664\,a^2b^3x^9+21280\,a^3b^2x^6+8645\,a^4bx^3+1456\,a^5}{27664\,x^{19}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^20,x, algorithm="maxima")
```

```
[Out] -1/27664*(6916*b^5*x^15 + 19760*a*b^4*x^12 + 27664*a^2*b^3*x^9 + 21280*a^3*
b^2*x^6 + 8645*a^4*b*x^3 + 1456*a^5)/x^19
```

mupad [B] time = 1.31, size = 231, normalized size = 0.91

$$-\frac{a^5\,\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{19\,x^{19}\,\left(b\,x^3+a\right)}-\frac{b^5\,\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{4\,x^4\,\left(b\,x^3+a\right)}-\frac{5\,a\,b^4\,\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{7\,x^7\,\left(b\,x^3+a\right)}-\frac{5\,a^4\,b\,\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{16\,x^{16}\,\left(b\,x^3+a\right)}-\frac{a^2\,b^3\,\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{x^{10}\,\left(b\,x^3+a\right)}-\frac{10\,a^3\,b^2\,\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{13\,x^{13}\,\left(b\,x^3+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2)/x^20,x)
```

```
[Out] - (a^5*(a^2 + b^2*x^6 + 2*a*b*x^3)^(1/2))/(19*x^19*(a + b*x^3)) - (b^5*(a^2
 + b^2*x^6 + 2*a*b*x^3)^(1/2))/(4*x^4*(a + b*x^3)) - (5*a*b^4*(a^2 + b^2*x^
6 + 2*a*b*x^3)^(1/2))/(7*x^7*(a + b*x^3)) - (5*a^4*b*(a^2 + b^2*x^6 + 2*a*b
*x^3)^(1/2))/(16*x^16*(a + b*x^3)) - (a^2*b^3*(a^2 + b^2*x^6 + 2*a*b*x^3)^(
1/2))/(x^10*(a + b*x^3)) - (10*a^3*b^2*(a^2 + b^2*x^6 + 2*a*b*x^3)^(1/2))/(
13*x^13*(a + b*x^3))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\left(a + bx^3\right)^2\right)^{\frac{5}{2}}}{x^{20}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**6+2*a*b*x**3+a**2)**(5/2)/x**20,x)
```

```
[Out] Integral(((a + b*x**3)**2)**(5/2)/x**20, x)
```

3.84 $\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{x^{21}}\,dx$

Optimal. Leaf size=255

$$-\frac{b^5\sqrt{a^2+2abx^3+b^2x^6}}{5x^5\left(a+bx^3\right)}-\frac{5ab^4\sqrt{a^2+2abx^3+b^2x^6}}{8x^8\left(a+bx^3\right)}-\frac{10a^2b^3\sqrt{a^2+2abx^3+b^2x^6}}{11x^{11}\left(a+bx^3\right)}-\frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{20x^{20}\left(a+bx^3\right)}-\frac{5a^4b\sqrt{\ }}{1}$$

Rubi [A] time = 0.06, antiderivative size = 255, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {1355, 270}

$$-\frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{20x^{20}\left(a+bx^3\right)}-\frac{5a^4b\sqrt{a^2+2abx^3+b^2x^6}}{17x^{17}\left(a+bx^3\right)}-\frac{5a^3b^2\sqrt{a^2+2abx^3+b^2x^6}}{7x^{14}\left(a+bx^3\right)}-\frac{10a^2b^3\sqrt{a^2+2abx^3+b^2x^6}}{11x^{11}\left(a+bx^3\right)}-\frac{5ab^4\sqrt{a^2+2abx^3+b^2x^6}}{8x^8\left(a+bx^3\right)}-\frac{b^5\sqrt{a^2+2abx^3+b^2x^6}}{5x^5\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^21,x]
```

```
[Out] -(a^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(20*x^20*(a + b*x^3)) - (5*a^4*b*Sqr
t[a^2 + 2*a*b*x^3 + b^2*x^6])/(17*x^17*(a + b*x^3)) - (5*a^3*b^2*Sqrt[a^2 +
 2*a*b*x^3 + b^2*x^6])/(7*x^14*(a + b*x^3)) - (10*a^2*b^3*Sqrt[a^2 + 2*a*b*
x^3 + b^2*x^6])/(11*x^11*(a + b*x^3)) - (5*a*b^4*Sqrt[a^2 + 2*a*b*x^3 + b^2
*x^6])/(8*x^8*(a + b*x^3)) - (b^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(5*x^5*(
a + b*x^3))
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{x^{21}}\,dx &= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{\left(ab+b^2x^3\right)^5}{x^{21}}\,dx}{b^4\left(ab+b^2x^3\right)}\\
&= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \left(\frac{a^5b^5}{x^{21}}+\frac{5a^4b^6}{x^{18}}+\frac{10a^3b^7}{x^{15}}+\frac{10a^2b^8}{x^{12}}+\frac{5ab^9}{x^9}+\frac{b^{10}}{x^6}\right)dx}{b^4\left(ab+b^2x^3\right)}\\
&= -\frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{20x^{20}\left(a+bx^3\right)}-\frac{5a^4b\sqrt{a^2+2abx^3+b^2x^6}}{17x^{17}\left(a+bx^3\right)}-\frac{5a^3b^2\sqrt{a^2+2abx^3+b^2x^6}}{7x^{14}\left(a+bx^3\right)}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 83, normalized size = 0.33

$$-\frac{\sqrt{\left(a+bx^3\right)^2}\left(2618a^5+15400a^4bx^3+37400a^3b^2x^6+47600a^2b^3x^9+32725ab^4x^{12}+10472b^5x^{15}\right)}{52360x^{20}\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^21,x]
```

```
[Out] -1/52360*(Sqrt[(a + b*x^3)^2]*(2618*a^5 + 15400*a^4*b*x^3 + 37400*a^3*b^2*x
^6 + 47600*a^2*b^3*x^9 + 32725*a*b^4*x^12 + 10472*b^5*x^15))/(x^20*(a + b*x
^3))
```

IntegrateAlgebraic [A] time = 21.39, size = 83, normalized size = 0.33

$$\frac{\sqrt{\left(a+bx^3\right)^2}\left(-2618a^5-15400a^4bx^3-37400a^3b^2x^6-47600a^2b^3x^9-32725ab^4x^{12}-10472b^5x^{15}\right)}{52360x^{20}\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^21,x]
```

```
[Out] (Sqrt[(a + b*x^3)^2]*(-2618*a^5 - 15400*a^4*b*x^3 - 37400*a^3*b^2*x^6 - 476
00*a^2*b^3*x^9 - 32725*a*b^4*x^12 - 10472*b^5*x^15))/(52360*x^20*(a + b*x^3
))
```

fricas [A] time = 1.11, size = 59, normalized size = 0.23

$$-\frac{10472\,b^5x^{15}+32725\,ab^4x^{12}+47600\,a^2b^3x^9+37400\,a^3b^2x^6+15400\,a^4bx^3+2618\,a^5}{52360\,x^{20}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^21,x, algorithm="fricas")
```

```
[Out] -1/52360*(10472*b^5*x^15 + 32725*a*b^4*x^12 + 47600*a^2*b^3*x^9 + 37400*a^3
*b^2*x^6 + 15400*a^4*b*x^3 + 2618*a^5)/x^20
```

giac [A] time = 0.32, size = 107, normalized size = 0.42

$$-\frac{10472\,b^5x^{15}\mathrm{sgn}\left(bx^3+a\right)+32725\,ab^4x^{12}\mathrm{sgn}\left(bx^3+a\right)+47600\,a^2b^3x^9\mathrm{sgn}\left(bx^3+a\right)+37400\,a^3b^2x^6\mathrm{sgn}\left(bx^3+a\right)+15400\,a^4bx^3\mathrm{sgn}\left(bx^3+a\right)+2618\,a^5\mathrm{sgn}\left(bx^3+a\right)}{52360\,x^{20}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^21,x, algorithm="giac")
```

```
[Out] -1/52360*(10472*b^5*x^15*sgn(b*x^3 + a) + 32725*a*b^4*x^12*sgn(b*x^3 + a) +
 47600*a^2*b^3*x^9*sgn(b*x^3 + a) + 37400*a^3*b^2*x^6*sgn(b*x^3 + a) + 1540
0*a^4*b*x^3*sgn(b*x^3 + a) + 2618*a^5*sgn(b*x^3 + a))/x^20
```

maple [A] time = 0.01, size = 80, normalized size = 0.31

$$-\frac{\left(10472b^5x^{15}+32725a\,b^4x^{12}+47600a^2b^3x^9+37400a^3b^2x^6+15400a^4b\,x^3+2618a^5\right)\left(\left(b\,x^3+a\right)^2\right)^{\frac{5}{2}}}{52360\left(b\,x^3+a\right)^5x^{20}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^21,x)
```

```
[Out] -1/52360*(10472*b^5*x^15+32725*a*b^4*x^12+47600*a^2*b^3*x^9+37400*a^3*b^2*x
^6+15400*a^4*b*x^3+2618*a^5)*((b*x^3+a)^2)^(5/2)/x^20/(b*x^3+a)^5
```

maxima [A] time = 0.98, size = 59, normalized size = 0.23

$$-\frac{10472\,b^5x^{15}+32725\,ab^4x^{12}+47600\,a^2b^3x^9+37400\,a^3b^2x^6+15400\,a^4bx^3+2618\,a^5}{52360\,x^{20}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^21,x, algorithm="maxima")

[Out] -1/52360*(10472*b^5*x^15 + 32725*a*b^4*x^12 + 47600*a^2*b^3*x^9 + 37400*a^3*b^2*x^6 + 15400*a^4*b*x^3 + 2618*a^5)/x^20

mupad [B] time = 1.24, size = 231, normalized size = 0.91

$$-\frac{a^5\,\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{20\,x^{20}\,\left(b\,x^3+a\right)}-\frac{b^5\,\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{5\,x^5\,\left(b\,x^3+a\right)}-\frac{5\,a\,b^4\,\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{8\,x^8\,\left(b\,x^3+a\right)}-\frac{5\,a^4\,b\,\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{17\,x^{17}\,\left(b\,x^3+a\right)}-\frac{10\,a^2\,b^3\,\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{11\,x^{11}\,\left(b\,x^3+a\right)}-\frac{5\,a^3\,b^2\,\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{7\,x^{14}\,\left(b\,x^3+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2)/x^21,x)

[Out] - (a^5*(a^2 + b^2*x^6 + 2*a*b*x^3)^(1/2))/(20*x^20*(a + b*x^3)) - (b^5*(a^2 + b^2*x^6 + 2*a*b*x^3)^(1/2))/(5*x^5*(a + b*x^3)) - (5*a*b^4*(a^2 + b^2*x^6 + 2*a*b*x^3)^(1/2))/(8*x^8*(a + b*x^3)) - (5*a^4*b*(a^2 + b^2*x^6 + 2*a*b*x^3)^(1/2))/(17*x^17*(a + b*x^3)) - (10*a^2*b^3*(a^2 + b^2*x^6 + 2*a*b*x^3)^(1/2))/(11*x^11*(a + b*x^3)) - (5*a^3*b^2*(a^2 + b^2*x^6 + 2*a*b*x^3)^(1/2))/(7*x^14*(a + b*x^3))

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\left(a + bx^3\right)^2\right)^{\frac{5}{2}}}{x^{21}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b**2*x**6+2*a*b*x**3+a**2)**(5/2)/x**21,x)

[Out] Integral(((a + b*x**3)**2)**(5/2)/x**21, x)

3.85 $\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{x^{22}}\,dx$

Optimal. Leaf size=84

$$\frac{b\left(a+bx^3\right)^5\sqrt{a^2+2abx^3+b^2x^6}}{126a^2x^{18}}-\frac{\left(a+bx^3\right)^5\sqrt{a^2+2abx^3+b^2x^6}}{21ax^{21}}$$

Rubi [A] time = 0.04, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {1355, 266, 45, 37}

$$\frac{b\left(a+bx^3\right)^5\sqrt{a^2+2abx^3+b^2x^6}}{126a^2x^{18}}-\frac{\left(a+bx^3\right)^5\sqrt{a^2+2abx^3+b^2x^6}}{21ax^{21}}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^22,x]
```

```
[Out] -((a + b*x^3)^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(21*a*x^21) + (b*(a + b*x^
3)^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(126*a^2*x^18)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{x^{22}}\,dx = \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{\left(ab+b^2x^3\right)^5}{x^{22}}\,dx}{b^4\left(ab+b^2x^3\right)}$$

$$= \frac{\sqrt{a^2+2abx^3+b^2x^6}\,\text{Subst}\left(\int \frac{\left(ab+b^2x\right)^5}{x^8}\,dx, x, x^3\right)}{3b^4\left(ab+b^2x^3\right)}$$

$$= -\frac{\left(a+bx^3\right)^5\sqrt{a^2+2abx^3+b^2x^6}}{21ax^{21}} - \frac{\sqrt{a^2+2abx^3+b^2x^6}\,\text{Subst}\left(\int \frac{\left(ab+b^2x\right)^5}{x^7}\,dx, x,\right.}{21ab^3\left(ab+b^2x^3\right)}$$

$$= -\frac{\left(a+bx^3\right)^5\sqrt{a^2+2abx^3+b^2x^6}}{21ax^{21}} + \frac{b\left(a+bx^3\right)^5\sqrt{a^2+2abx^3+b^2x^6}}{126a^2x^{18}}$$

Mathematica [A] time = 0.02, size = 83, normalized size = 0.99

$$-\frac{\sqrt{\left(a+bx^3\right)^2}\left(6a^5+35a^4bx^3+84a^3b^2x^6+105a^2b^3x^9+70ab^4x^{12}+21b^5x^{15}\right)}{126x^{21}\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^22,x]
```

```
[Out] -1/126*(Sqrt[(a + b*x^3)^2]*(6*a^5 + 35*a^4*b*x^3 + 84*a^3*b^2*x^6 + 105*a^
2*b^3*x^9 + 70*a*b^4*x^12 + 21*b^5*x^15))/(x^21*(a + b*x^3))
```

IntegrateAlgebraic [B] time = 1.67, size = 488, normalized size = 5.81

$$\frac{32b^6\sqrt{a^2+2abx^3+b^2x^6}\left(-6a^{11}b-71a^{10}b^2x^3-384a^9b^3x^6-1254a^8b^4x^9-2750a^7b^5x^{12}-4257a^6b^6x^{15}-4752a^5b^7x^{18}-3829a^4b^8x^{21}-2184a^3b^9x^{24}-840a^2b^{10}x^{27}-196ab^{11}x^{30}-21b^{12}x^{33}\right)+32\sqrt{b^2}\,b^6\left(6a^{12}+77a^{11}bx^3+455a^{10}b^2x^6+1638a^9b^3x^9+4004a^8b^4x^{12}+7007a^7b^5x^{15}+9009a^6b^6x^{18}+8581a^5b^7x^{21}+6013a^4b^8x^{24}+3024a^3b^9x^{27}+1036a^2b^{10}x^{30}+217ab^{11}x^{33}+21b^{12}x^{36}\right)}{63\sqrt{b^2}\,x^{21}\sqrt{a^2+2abx^3+b^2x^6}\left(-64a^6b^6-384a^5b^7x^3-960a^4b^8x^6-1280a^3b^9x^9-960a^2b^{10}x^{12}-384ab^{11}x^{15}-64b^{12}x^{18}\right)+63x^{21}\left(64a^7b^7+448a^6b^8x^3+1344a^5b^9x^6+2240a^4b^{10}x^9+2240a^3b^{11}x^{12}+1344a^2b^{12}x^{15}+448ab^{13}x^{18}+64b^{14}x^{21}\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^22,x]
```

```
[Out] (32*b^6*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]*(-6*a^11*b - 71*a^10*b^2*x^3 - 384*
a^9*b^3*x^6 - 1254*a^8*b^4*x^9 - 2750*a^7*b^5*x^12 - 4257*a^6*b^6*x^15 - 47
52*a^5*b^7*x^18 - 3829*a^4*b^8*x^21 - 2184*a^3*b^9*x^24 - 840*a^2*b^10*x^27
 - 196*a*b^11*x^30 - 21*b^12*x^33) + 32*b^6*Sqrt[b^2]*(6*a^12 + 77*a^11*b*x
^3 + 455*a^10*b^2*x^6 + 1638*a^9*b^3*x^9 + 4004*a^8*b^4*x^12 + 7007*a^7*b^5
*x^15 + 9009*a^6*b^6*x^18 + 8581*a^5*b^7*x^21 + 6013*a^4*b^8*x^24 + 3024*a^
3*b^9*x^27 + 1036*a^2*b^10*x^30 + 217*a*b^11*x^33 + 21*b^12*x^36))/(63*Sqrt
[b^2]*x^21*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]*(-64*a^6*b^6 - 384*a^5*b^7*x^3 -
 960*a^4*b^8*x^6 - 1280*a^3*b^9*x^9 - 960*a^2*b^10*x^12 - 384*a*b^11*x^15 -
 64*b^12*x^18) + 63*x^21*(64*a^7*b^7 + 448*a^6*b^8*x^3 + 1344*a^5*b^9*x^6 +
 2240*a^4*b^10*x^9 + 2240*a^3*b^11*x^12 + 1344*a^2*b^12*x^15 + 448*a*b^13*x
^18 + 64*b^14*x^21))
```

fricas [A] time = 1.02, size = 59, normalized size = 0.70

$$-\frac{21\,b^5x^{15}+70\,ab^4x^{12}+105\,a^2b^3x^9+84\,a^3b^2x^6+35\,a^4bx^3+6\,a^5}{126\,x^{21}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^22,x, algorithm="fricas")
```

```
[Out] -1/126*(21*b^5*x^15 + 70*a*b^4*x^12 + 105*a^2*b^3*x^9 + 84*a^3*b^2*x^6 + 35
*a^4*b*x^3 + 6*a^5)/x^21
```

giac [A] time = 0.32, size = 107, normalized size = 1.27

$$-\frac{21\,b^5x^{15}\mathrm{sgn}\left(bx^3+a\right)+70\,ab^4x^{12}\mathrm{sgn}\left(bx^3+a\right)+105\,a^2b^3x^9\mathrm{sgn}\left(bx^3+a\right)+84\,a^3b^2x^6\mathrm{sgn}\left(bx^3+a\right)+35\,a^4bx^3\mathrm{sgn}\left(bx^3+a\right)+6\,a^5\mathrm{sgn}\left(bx^3+a\right)}{126\,x^{21}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^22,x, algorithm="giac")
```

```
[Out] -1/126*(21*b^5*x^15*sgn(b*x^3 + a) + 70*a*b^4*x^12*sgn(b*x^3 + a) + 105*a^2
*b^3*x^9*sgn(b*x^3 + a) + 84*a^3*b^2*x^6*sgn(b*x^3 + a) + 35*a^4*b*x^3*sgn(
b*x^3 + a) + 6*a^5*sgn(b*x^3 + a))/x^21
```

maple [A] time = 0.01, size = 80, normalized size = 0.95

$$-\frac{\left(21b^5x^{15}+70a\,b^4x^{12}+105a^2b^3x^9+84a^3b^2x^6+35a^4b\,x^3+6a^5\right)\left(\left(b\,x^3+a\right)^2\right)^{\frac{5}{2}}}{126\left(b\,x^3+a\right)^5x^{21}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^22,x)
```

```
[Out] -1/126*(21*b^5*x^15+70*a*b^4*x^12+105*a^2*b^3*x^9+84*a^3*b^2*x^6+35*a^4*b*x
^3+6*a^5)*((b*x^3+a)^2)^(5/2)/x^21/(b*x^3+a)^5
```

maxima [B] time = 1.16, size = 241, normalized size = 2.87

$$-\frac{\left(b^2x^6+2\,abx^3+a^2\right)^{\frac{5}{2}}b^7}{18\,a^7}-\frac{\left(b^2x^6+2\,abx^3+a^2\right)^{\frac{5}{2}}b^6}{18\,a^6x^3}+\frac{\left(b^2x^6+2\,abx^3+a^2\right)^{\frac{7}{2}}b^5}{18\,a^7x^6}-\frac{\left(b^2x^6+2\,abx^3+a^2\right)^{\frac{7}{2}}b^4}{18\,a^6x^9}+\frac{\left(b^2x^6+2\,abx^3+a^2\right)^{\frac{7}{2}}b^3}{18\,a^5x^{12}}-\frac{\left(b^2x^6+2\,abx^3+a^2\right)^{\frac{7}{2}}b^2}{18\,a^4x^{15}}+\frac{\left(b^2x^6+2\,abx^3+a^2\right)^{\frac{7}{2}}b}{18\,a^3x^{18}}-\frac{\left(b^2x^6+2\,abx^3+a^2\right)^{\frac{7}{2}}}{21\,a^2x^{21}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^22,x, algorithm="maxima")
```

```
[Out] -1/18*(b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2)*b^7/a^7 - 1/18*(b^2*x^6 + 2*a*b*x^3
 + a^2)^(5/2)*b^6/(a^6*x^3) + 1/18*(b^2*x^6 + 2*a*b*x^3 + a^2)^(7/2)*b^5/(a
^7*x^6) - 1/18*(b^2*x^6 + 2*a*b*x^3 + a^2)^(7/2)*b^4/(a^6*x^9) + 1/18*(b^2*
x^6 + 2*a*b*x^3 + a^2)^(7/2)*b^3/(a^5*x^12) - 1/18*(b^2*x^6 + 2*a*b*x^3 + a
^2)^(7/2)*b^2/(a^4*x^15) + 1/18*(b^2*x^6 + 2*a*b*x^3 + a^2)^(7/2)*b/(a^3*x^
18) - 1/21*(b^2*x^6 + 2*a*b*x^3 + a^2)^(7/2)/(a^2*x^21)
```

mupad [B] time = 1.22, size = 231, normalized size = 2.75

$$-\frac{a^5\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{21\,x^{21}\left(b\,x^3+a\right)}-\frac{b^5\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{6\,x^6\left(b\,x^3+a\right)}-\frac{5\,a\,b^4\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{9\,x^9\left(b\,x^3+a\right)}-\frac{5\,a^4\,b\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{18\,x^{18}\left(b\,x^3+a\right)}-\frac{5\,a^2\,b^3\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{6\,x^{12}\left(b\,x^3+a\right)}-\frac{2\,a^3\,b^2\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{3\,x^{15}\left(b\,x^3+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2)/x^22,x)
```

```
[Out] - (a^5*(a^2 + b^2*x^6 + 2*a*b*x^3)^(1/2))/(21*x^21*(a + b*x^3)) - (b^5*(a^2
 + b^2*x^6 + 2*a*b*x^3)^(1/2))/(6*x^6*(a + b*x^3)) - (5*a*b^4*(a^2 + b^2*x^
6 + 2*a*b*x^3)^(1/2))/(9*x^9*(a + b*x^3)) - (5*a^4*b*(a^2 + b^2*x^6 + 2*a*b
*x^3)^(1/2))/(18*x^18*(a + b*x^3)) - (5*a^2*b^3*(a^2 + b^2*x^6 + 2*a*b*x^3)
^(1/2))/(6*x^12*(a + b*x^3)) - (2*a^3*b^2*(a^2 + b^2*x^6 + 2*a*b*x^3)^(1/2)
)/(3*x^15*(a + b*x^3))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(\left(a+bx^3\right)^2\right)^{\frac{5}{2}}}{x^{22}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**6+2*a*b*x**3+a**2)**(5/2)/x**22,x)
```

```
[Out] Integral(((a + b*x**3)**2)**(5/2)/x**22, x)
```

3.86 $\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{x^{23}}\,dx$

Optimal. Leaf size=255

$$-\frac{b^5\sqrt{a^2+2abx^3+b^2x^6}}{7x^7\left(a+bx^3\right)}-\frac{ab^4\sqrt{a^2+2abx^3+b^2x^6}}{2x^{10}\left(a+bx^3\right)}-\frac{10a^2b^3\sqrt{a^2+2abx^3+b^2x^6}}{13x^{13}\left(a+bx^3\right)}-\frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{22x^{22}\left(a+bx^3\right)}-\frac{5a^4b}{}$$

Rubi [A] time = 0.06, antiderivative size = 255, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1355, 270}

$$-\frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{22x^{22}\left(a+bx^3\right)}-\frac{5a^4b\sqrt{a^2+2abx^3+b^2x^6}}{19x^{19}\left(a+bx^3\right)}-\frac{5a^3b^2\sqrt{a^2+2abx^3+b^2x^6}}{8x^{16}\left(a+bx^3\right)}-\frac{10a^2b^3\sqrt{a^2+2abx^3+b^2x^6}}{13x^{13}\left(a+bx^3\right)}-\frac{ab^4\sqrt{a^2+2abx^3+b^2x^6}}{2x^{10}\left(a+bx^3\right)}-\frac{b^5\sqrt{a^2+2abx^3+b^2x^6}}{7x^7\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^23,x]
```

```
[Out] -(a^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(22*x^22*(a + b*x^3)) - (5*a^4*b*Sqr
t[a^2 + 2*a*b*x^3 + b^2*x^6])/(19*x^19*(a + b*x^3)) - (5*a^3*b^2*Sqrt[a^2 +
 2*a*b*x^3 + b^2*x^6])/(8*x^16*(a + b*x^3)) - (10*a^2*b^3*Sqrt[a^2 + 2*a*b*
x^3 + b^2*x^6])/(13*x^13*(a + b*x^3)) - (a*b^4*Sqrt[a^2 + 2*a*b*x^3 + b^2*x
^6])/(2*x^10*(a + b*x^3)) - (b^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(7*x^7*(a
 + b*x^3))
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{x^{23}}\,dx &= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{\left(ab+b^2x^3\right)^5}{x^{23}}\,dx}{b^4\left(ab+b^2x^3\right)}\\
&= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \left(\frac{a^5b^5}{x^{23}}+\frac{5a^4b^6}{x^{20}}+\frac{10a^3b^7}{x^{17}}+\frac{10a^2b^8}{x^{14}}+\frac{5ab^9}{x^{11}}+\frac{b^{10}}{x^8}\right)dx}{b^4\left(ab+b^2x^3\right)}\\
&= -\frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{22x^{22}\left(a+bx^3\right)}-\frac{5a^4b\sqrt{a^2+2abx^3+b^2x^6}}{19x^{19}\left(a+bx^3\right)}-\frac{5a^3b^2\sqrt{a^2+2abx^3+b^2}}{8x^{16}\left(a+bx^3\right)}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 83, normalized size = 0.33

$$-\frac{\sqrt{\left(a+bx^3\right)^2}\left(6916a^5+40040a^4bx^3+95095a^3b^2x^6+117040a^2b^3x^9+76076ab^4x^{12}+21736b^5x^{15}\right)}{152152x^{22}\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^23,x]
```

```
[Out] -1/152152*(Sqrt[(a + b*x^3)^2]*(6916*a^5 + 40040*a^4*b*x^3 + 95095*a^3*b^2*
x^6 + 117040*a^2*b^3*x^9 + 76076*a*b^4*x^12 + 21736*b^5*x^15))/(x^22*(a + b
*x^3))
```

IntegrateAlgebraic [A] time = 23.70, size = 83, normalized size = 0.33

$$\frac{\sqrt{\left(a+bx^3\right)^2}\left(-6916a^5-40040a^4bx^3-95095a^3b^2x^6-117040a^2b^3x^9-76076ab^4x^{12}-21736b^5x^{15}\right)}{152152x^{22}\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^23,x]
```

```
[Out] (Sqrt[(a + b*x^3)^2]*(-6916*a^5 - 40040*a^4*b*x^3 - 95095*a^3*b^2*x^6 - 117
040*a^2*b^3*x^9 - 76076*a*b^4*x^12 - 21736*b^5*x^15))/(152152*x^22*(a + b*x
^3))
```

fricas [A] time = 1.11, size = 59, normalized size = 0.23

$$-\frac{21736\,b^5x^{15}+76076\,ab^4x^{12}+117040\,a^2b^3x^9+95095\,a^3b^2x^6+40040\,a^4bx^3+6916\,a^5}{152152\,x^{22}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^23,x, algorithm="fricas")
```

```
[Out] -1/152152*(21736*b^5*x^15 + 76076*a*b^4*x^12 + 117040*a^2*b^3*x^9 + 95095*a
^3*b^2*x^6 + 40040*a^4*b*x^3 + 6916*a^5)/x^22
```

giac [A] time = 0.33, size = 107, normalized size = 0.42

$$-\frac{21736\,b^5x^{15}\mathrm{sgn}\left(bx^3+a\right)+76076\,ab^4x^{12}\mathrm{sgn}\left(bx^3+a\right)+117040\,a^2b^3x^9\mathrm{sgn}\left(bx^3+a\right)+95095\,a^3b^2x^6\mathrm{sgn}\left(bx^3+a\right)+40040\,a^4bx^3\mathrm{sgn}\left(bx^3+a\right)+6916\,a^5\mathrm{sgn}\left(bx^3+a\right)}{152152\,x^{22}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^23,x, algorithm="giac")
```

```
[Out] -1/152152*(21736*b^5*x^15*sgn(b*x^3 + a) + 76076*a*b^4*x^12*sgn(b*x^3 + a)
+ 117040*a^2*b^3*x^9*sgn(b*x^3 + a) + 95095*a^3*b^2*x^6*sgn(b*x^3 + a) + 40
040*a^4*b*x^3*sgn(b*x^3 + a) + 6916*a^5*sgn(b*x^3 + a))/x^22
```

maple [A] time = 0.01, size = 80, normalized size = 0.31

$$-\frac{\left(21736b^5x^{15}+76076a\,b^4x^{12}+117040a^2b^3x^9+95095a^3b^2x^6+40040a^4b\,x^3+6916a^5\right)\left(\left(b\,x^3+a\right)^2\right)^{\frac{5}{2}}}{152152\left(b\,x^3+a\right)^5x^{22}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^23,x)
```

```
[Out] -1/152152*(21736*b^5*x^15+76076*a*b^4*x^12+117040*a^2*b^3*x^9+95095*a^3*b^2
*x^6+40040*a^4*b*x^3+6916*a^5)*((b*x^3+a)^2)^(5/2)/x^22/(b*x^3+a)^5
```

maxima [A] time = 0.98, size = 59, normalized size = 0.23

$$-\frac{21736\,b^5x^{15}+76076\,ab^4x^{12}+117040\,a^2b^3x^9+95095\,a^3b^2x^6+40040\,a^4bx^3+6916\,a^5}{152152\,x^{22}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^23,x, algorithm="maxima")
```

```
[Out] -1/152152*(21736*b^5*x^15 + 76076*a*b^4*x^12 + 117040*a^2*b^3*x^9 + 95095*a
^3*b^2*x^6 + 40040*a^4*b*x^3 + 6916*a^5)/x^22
```

mupad [B] time = 1.22, size = 231, normalized size = 0.91

$$-\frac{a^5\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{22\,x^{22}\,\left(b\,x^3+a\right)}-\frac{b^5\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{7\,x^7\,\left(b\,x^3+a\right)}-\frac{a\,b^4\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{2\,x^{10}\,\left(b\,x^3+a\right)}-\frac{5\,a^4\,b\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{19\,x^{19}\,\left(b\,x^3+a\right)}-\frac{10\,a^2\,b^3\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{13\,x^{13}\,\left(b\,x^3+a\right)}-\frac{5\,a^3\,b^2\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{8\,x^{16}\,\left(b\,x^3+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2)/x^23,x)
```

```
[Out] - (a^5*(a^2 + b^2*x^6 + 2*a*b*x^3)^(1/2))/(22*x^22*(a + b*x^3)) - (b^5*(a^2
 + b^2*x^6 + 2*a*b*x^3)^(1/2))/(7*x^7*(a + b*x^3)) - (a*b^4*(a^2 + b^2*x^6
+ 2*a*b*x^3)^(1/2))/(2*x^10*(a + b*x^3)) - (5*a^4*b*(a^2 + b^2*x^6 + 2*a*b*
x^3)^(1/2))/(19*x^19*(a + b*x^3)) - (10*a^2*b^3*(a^2 + b^2*x^6 + 2*a*b*x^3)
^(1/2))/(13*x^13*(a + b*x^3)) - (5*a^3*b^2*(a^2 + b^2*x^6 + 2*a*b*x^3)^(1/2
))/(8*x^16*(a + b*x^3))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\left(a + bx^3\right)^2\right)^{\frac{5}{2}}}{x^{23}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**6+2*a*b*x**3+a**2)**(5/2)/x**23,x)
```

```
[Out] Integral(((a + b*x**3)**2)**(5/2)/x**23, x)
```

3.87 $\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{x^{24}}\,dx$

Optimal. Leaf size=255

$$-\frac{b^5\sqrt{a^2+2abx^3+b^2x^6}}{8x^8\left(a+bx^3\right)}-\frac{5ab^4\sqrt{a^2+2abx^3+b^2x^6}}{11x^{11}\left(a+bx^3\right)}-\frac{5a^2b^3\sqrt{a^2+2abx^3+b^2x^6}}{7x^{14}\left(a+bx^3\right)}-\frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{23x^{23}\left(a+bx^3\right)}-\frac{a^4b\sqrt{a^2}}{4x}$$

Rubi [A] time = 0.06, antiderivative size = 255, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1355, 270}

$$-\frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{23x^{23}\left(a+bx^3\right)}-\frac{a^4b\sqrt{a^2+2abx^3+b^2x^6}}{4x^{20}\left(a+bx^3\right)}-\frac{10a^3b^2\sqrt{a^2+2abx^3+b^2x^6}}{17x^{17}\left(a+bx^3\right)}-\frac{5a^2b^3\sqrt{a^2+2abx^3+b^2x^6}}{7x^{14}\left(a+bx^3\right)}-\frac{5ab^4\sqrt{a^2+2abx^3+b^2x^6}}{11x^{11}\left(a+bx^3\right)}-\frac{b^5\sqrt{a^2+2abx^3+b^2x^6}}{8x^8\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^24,x]
```

```
[Out] -(a^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(23*x^23*(a + b*x^3)) - (a^4*b*Sqrt[
a^2 + 2*a*b*x^3 + b^2*x^6])/(4*x^20*(a + b*x^3)) - (10*a^3*b^2*Sqrt[a^2 + 2
*a*b*x^3 + b^2*x^6])/(17*x^17*(a + b*x^3)) - (5*a^2*b^3*Sqrt[a^2 + 2*a*b*x^
3 + b^2*x^6])/(7*x^14*(a + b*x^3)) - (5*a*b^4*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^
6])/(11*x^11*(a + b*x^3)) - (b^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(8*x^8*(a
 + b*x^3))
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{x^{24}}\,dx &= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{\left(ab+b^2x^3\right)^5}{x^{24}}\,dx}{b^4\left(ab+b^2x^3\right)}\\ &= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \left(\frac{a^5b^5}{x^{24}}+\frac{5a^4b^6}{x^{21}}+\frac{10a^3b^7}{x^{18}}+\frac{10a^2b^8}{x^{15}}+\frac{5ab^9}{x^{12}}+\frac{b^{10}}{x^9}\right)dx}{b^4\left(ab+b^2x^3\right)}\\ &= -\frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{23x^{23}\left(a+bx^3\right)}-\frac{a^4b\sqrt{a^2+2abx^3+b^2x^6}}{4x^{20}\left(a+bx^3\right)}-\frac{10a^3b^2\sqrt{a^2+2abx^3+b^2x^6}}{17x^{17}\left(a+bx^3\right)}\end{aligned}$$

Mathematica [A] time = 0.02, size = 83, normalized size = 0.33

$$-\frac{\sqrt{\left(a+bx^3\right)^2}\left(10472a^5+60214a^4bx^3+141680a^3b^2x^6+172040a^2b^3x^9+109480ab^4x^{12}+30107b^5x^{15}\right)}{240856x^{23}\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^24,x]
```

```
[Out] -1/240856*(Sqrt[(a + b*x^3)^2]*(10472*a^5 + 60214*a^4*b*x^3 + 141680*a^3*b^
2*x^6 + 172040*a^2*b^3*x^9 + 109480*a*b^4*x^12 + 30107*b^5*x^15))/(x^23*(a
+ b*x^3))
```

IntegrateAlgebraic [A] time = 23.52, size = 83, normalized size = 0.33

$$\frac{\sqrt{\left(a+bx^3\right)^2}\left(-10472a^5-60214a^4bx^3-141680a^3b^2x^6-172040a^2b^3x^9-109480ab^4x^{12}-30107b^5x^{15}\right)}{240856x^{23}\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^24,x]
```

```
[Out] (Sqrt[(a + b*x^3)^2]*(-10472*a^5 - 60214*a^4*b*x^3 - 141680*a^3*b^2*x^6 - 1
72040*a^2*b^3*x^9 - 109480*a*b^4*x^12 - 30107*b^5*x^15))/(240856*x^23*(a +
b*x^3))
```

fricas [A] time = 1.09, size = 59, normalized size = 0.23

$$-\frac{30107\,b^5x^{15}+109480\,ab^4x^{12}+172040\,a^2b^3x^9+141680\,a^3b^2x^6+60214\,a^4bx^3+10472\,a^5}{240856\,x^{23}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^24,x, algorithm="fricas")
```

```
[Out] -1/240856*(30107*b^5*x^15 + 109480*a*b^4*x^12 + 172040*a^2*b^3*x^9 + 141680
*a^3*b^2*x^6 + 60214*a^4*b*x^3 + 10472*a^5)/x^23
```

giac [A] time = 0.34, size = 107, normalized size = 0.42

$$-\frac{30107\,b^5x^{15}\mathrm{sgn}\left(bx^3+a\right)+109480\,ab^4x^{12}\mathrm{sgn}\left(bx^3+a\right)+172040\,a^2b^3x^9\mathrm{sgn}\left(bx^3+a\right)+141680\,a^3b^2x^6\mathrm{sgn}\left(bx^3+a\right)+60214\,a^4bx^3\mathrm{sgn}\left(bx^3+a\right)+10472\,a^5\mathrm{sgn}\left(bx^3+a\right)}{240856\,x^{23}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^24,x, algorithm="giac")
```

```
[Out] -1/240856*(30107*b^5*x^15*sgn(b*x^3 + a) + 109480*a*b^4*x^12*sgn(b*x^3 + a)
 + 172040*a^2*b^3*x^9*sgn(b*x^3 + a) + 141680*a^3*b^2*x^6*sgn(b*x^3 + a) +
60214*a^4*b*x^3*sgn(b*x^3 + a) + 10472*a^5*sgn(b*x^3 + a))/x^23
```

maple [A] time = 0.01, size = 80, normalized size = 0.31

$$-\frac{\left(30107b^5x^{15}+109480a\,b^4x^{12}+172040a^2b^3x^9+141680a^3b^2x^6+60214a^4b\,x^3+10472a^5\right)\left(\left(b\,x^3+a\right)^2\right)^{\frac{5}{2}}}{240856\left(b\,x^3+a\right)^5x^{23}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^24,x)
```

```
[Out] -1/240856*(30107*b^5*x^15+109480*a*b^4*x^12+172040*a^2*b^3*x^9+141680*a^3*b
^2*x^6+60214*a^4*b*x^3+10472*a^5)*((b*x^3+a)^2)^(5/2)/x^23/(b*x^3+a)^5
```

maxima [A] time = 0.83, size = 59, normalized size = 0.23

$$-\frac{30107\,b^5x^{15}+109480\,ab^4x^{12}+172040\,a^2b^3x^9+141680\,a^3b^2x^6+60214\,a^4bx^3+10472\,a^5}{240856\,x^{23}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^24,x, algorithm="maxima")
```

```
[Out] -1/240856*(30107*b^5*x^15 + 109480*a*b^4*x^12 + 172040*a^2*b^3*x^9 + 141680
*a^3*b^2*x^6 + 60214*a^4*b*x^3 + 10472*a^5)/x^23
```

mupad [B] time = 1.23, size = 231, normalized size = 0.91

$$-\frac{a^5\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{23\,x^{23}\,\left(b\,x^3+a\right)}-\frac{b^5\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{8\,x^8\,\left(b\,x^3+a\right)}-\frac{5\,a\,b^4\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{11\,x^{11}\,\left(b\,x^3+a\right)}-\frac{a^4\,b\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{4\,x^{20}\,\left(b\,x^3+a\right)}-\frac{5\,a^2\,b^3\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{7\,x^{14}\,\left(b\,x^3+a\right)}-\frac{10\,a^3\,b^2\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{17\,x^{17}\,\left(b\,x^3+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2)/x^24,x)
```

```
[Out] - (a^5*(a^2 + b^2*x^6 + 2*a*b*x^3)^(1/2))/(23*x^23*(a + b*x^3)) - (b^5*(a^2
 + b^2*x^6 + 2*a*b*x^3)^(1/2))/(8*x^8*(a + b*x^3)) - (5*a*b^4*(a^2 + b^2*x^
6 + 2*a*b*x^3)^(1/2))/(11*x^11*(a + b*x^3)) - (a^4*b*(a^2 + b^2*x^6 + 2*a*b
*x^3)^(1/2))/(4*x^20*(a + b*x^3)) - (5*a^2*b^3*(a^2 + b^2*x^6 + 2*a*b*x^3)^
(1/2))/(7*x^14*(a + b*x^3)) - (10*a^3*b^2*(a^2 + b^2*x^6 + 2*a*b*x^3)^(1/2)
)/(17*x^17*(a + b*x^3))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\left(a + bx^3\right)^2\right)^{\frac{5}{2}}}{x^{24}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**6+2*a*b*x**3+a**2)**(5/2)/x**24,x)
```

```
[Out] Integral(((a + b*x**3)**2)**(5/2)/x**24, x)
```

3.88 $\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{x^{25}}\,dx$

Optimal. Leaf size=128

$$-\frac{\sqrt{a^2+2abx^3+b^2x^6}\left(a+bx^3\right)^5}{24ax^{24}}+\frac{b\sqrt{a^2+2abx^3+b^2x^6}\left(a+bx^3\right)^5}{84a^2x^{21}}-\frac{b^2\sqrt{a^2+2abx^3+b^2x^6}\left(a+bx^3\right)^5}{504a^3x^{18}}$$

Rubi [A] time = 0.06, antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {1355, 266, 45, 37}

$$-\frac{b^2\sqrt{a^2+2abx^3+b^2x^6}\left(a+bx^3\right)^5}{504a^3x^{18}}+\frac{b\sqrt{a^2+2abx^3+b^2x^6}\left(a+bx^3\right)^5}{84a^2x^{21}}-\frac{\sqrt{a^2+2abx^3+b^2x^6}\left(a+bx^3\right)^5}{24ax^{24}}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^25,x]
```

```
[Out] -((a + b*x^3)^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(24*a*x^24) + (b*(a + b*x^
3)^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(84*a^2*x^21) - (b^2*(a + b*x^3)^5*Sq
rt[a^2 + 2*a*b*x^3 + b^2*x^6])/(504*a^3*x^18)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{x^{25}}\,dx = \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{\left(ab+b^2x^3\right)^5}{x^{25}}\,dx}{b^4\left(ab+b^2x^3\right)}$$

$$= \frac{\sqrt{a^2+2abx^3+b^2x^6}\ \text{Subst}\left(\int \frac{\left(ab+b^2x\right)^5}{x^9}\,dx, x, x^3\right)}{3b^4\left(ab+b^2x^3\right)}$$

$$= -\frac{\left(a+bx^3\right)^5\sqrt{a^2+2abx^3+b^2x^6}}{24ax^{24}} - \frac{\sqrt{a^2+2abx^3+b^2x^6}\ \text{Subst}\left(\int \frac{\left(ab+b^2x\right)^5}{x^8}\,dx, x,\right.}{12ab^3\left(ab+b^2x^3\right)}$$

$$= -\frac{\left(a+bx^3\right)^5\sqrt{a^2+2abx^3+b^2x^6}}{24ax^{24}} + \frac{b\left(a+bx^3\right)^5\sqrt{a^2+2abx^3+b^2x^6}}{84a^2x^{21}} + \frac{\sqrt{a^2+2a}}{}$$

$$= -\frac{\left(a+bx^3\right)^5\sqrt{a^2+2abx^3+b^2x^6}}{24ax^{24}} + \frac{b\left(a+bx^3\right)^5\sqrt{a^2+2abx^3+b^2x^6}}{84a^2x^{21}} - \frac{b^2\left(a+b\right.}{}$$

Mathematica [A] time = 0.02, size = 83, normalized size = 0.65

$$-\frac{\sqrt{\left(a+bx^3\right)^2}\left(21a^5+120a^4bx^3+280a^3b^2x^6+336a^2b^3x^9+210ab^4x^{12}+56b^5x^{15}\right)}{504x^{24}\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^25,x]
```

```
[Out] -1/504*(Sqrt[(a + b*x^3)^2]*(21*a^5 + 120*a^4*b*x^3 + 280*a^3*b^2*x^6 + 336
*a^2*b^3*x^9 + 210*a*b^4*x^12 + 56*b^5*x^15))/(x^24*(a + b*x^3))
```

IntegrateAlgebraic [B] time = 1.54, size = 532, normalized size = 4.16

$$\frac{16b^7\sqrt{a^2+2abx^3+b^2x^6}\left(-21a^{12}b-267a^{11}b^2x^3-1561a^{10}b^3x^6-5551a^9b^4x^9-13377a^8b^5x^{12}-23023a^7b^6x^{15}-29029a^6b^7x^{18}-27027a^5b^8x^{21}-18446a^4b^9x^{24}-9002a^3b^{10}x^{27}-2982a^2b^{11}x^{30}-602ab^{12}x^{33}-56b^{13}x^{36}\right)+16\sqrt{b^2}b^7\left(21a^{13}+288a^{12}bx^3+1828a^{11}b^2x^6+7112a^{10}b^3x^9+18928a^9b^4x^{12}+36400a^8b^5x^{15}+52052a^7b^6x^{18}+56056a^6b^7x^{21}+45473a^5b^8x^{24}+27448a^4b^9x^{27}+11984a^3b^{10}x^{30}+3584a^2b^{11}x^{33}+658ab^{12}x^{36}+56b^{13}x^{39}\right)}{63\sqrt{b^2}x^{24}\sqrt{a^2+2abx^3+b^2x^6}\left(-128a^7b^7-896a^6b^8x^3-2688a^5b^9x^6-4480a^4b^{10}x^9-4480a^3b^{11}x^{12}-2688a^2b^{12}x^{15}-896ab^{13}x^{18}-128b^{14}x^{21}\right)+63x^{24}\left(128a^8b^8+1024a^7b^9x^3+3584a^6b^{10}x^6+7168a^5b^{11}x^9+8960a^4b^{12}x^{12}+7168a^3b^{13}x^{15}+3584a^2b^{14}x^{18}+1024ab^{15}x^{21}+128b^{16}x^{24}\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^25,x]
```

```
[Out] (16*b^7*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]*(-21*a^12*b - 267*a^11*b^2*x^3 - 15
61*a^10*b^3*x^6 - 5551*a^9*b^4*x^9 - 13377*a^8*b^5*x^12 - 23023*a^7*b^6*x^1
5 - 29029*a^6*b^7*x^18 - 27027*a^5*b^8*x^21 - 18446*a^4*b^9*x^24 - 9002*a^3
*b^10*x^27 - 2982*a^2*b^11*x^30 - 602*a*b^12*x^33 - 56*b^13*x^36) + 16*b^7*
Sqrt[b^2]*(21*a^13 + 288*a^12*b*x^3 + 1828*a^11*b^2*x^6 + 7112*a^10*b^3*x^9
 + 18928*a^9*b^4*x^12 + 36400*a^8*b^5*x^15 + 52052*a^7*b^6*x^18 + 56056*a^6
*b^7*x^21 + 45473*a^5*b^8*x^24 + 27448*a^4*b^9*x^27 + 11984*a^3*b^10*x^30 +
 3584*a^2*b^11*x^33 + 658*a*b^12*x^36 + 56*b^13*x^39))/(63*Sqrt[b^2]*x^24*S
qrt[a^2 + 2*a*b*x^3 + b^2*x^6]*(-128*a^7*b^7 - 896*a^6*b^8*x^3 - 2688*a^5*b
^9*x^6 - 4480*a^4*b^10*x^9 - 4480*a^3*b^11*x^12 - 2688*a^2*b^12*x^15 - 896*
a*b^13*x^18 - 128*b^14*x^21) + 63*x^24*(128*a^8*b^8 + 1024*a^7*b^9*x^3 + 35
84*a^6*b^10*x^6 + 7168*a^5*b^11*x^9 + 8960*a^4*b^12*x^12 + 7168*a^3*b^13*x^
15 + 3584*a^2*b^14*x^18 + 1024*a*b^15*x^21 + 128*b^16*x^24))
```

fricas [A] time = 1.27, size = 59, normalized size = 0.46

$$-\frac{56\,b^5x^{15}+210\,ab^4x^{12}+336\,a^2b^3x^9+280\,a^3b^2x^6+120\,a^4bx^3+21\,a^5}{504\,x^{24}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^25,x, algorithm="fricas")
```

```
[Out] -1/504*(56*b^5*x^15 + 210*a*b^4*x^12 + 336*a^2*b^3*x^9 + 280*a^3*b^2*x^6 +
120*a^4*b*x^3 + 21*a^5)/x^24
```

giac [A] time = 0.39, size = 107, normalized size = 0.84

$$-\frac{56\,b^{5}x^{15}\mathrm{sgn}\left(bx^{3}+a\right)+210\,ab^{4}x^{12}\mathrm{sgn}\left(bx^{3}+a\right)+336\,a^{2}b^{3}x^{9}\mathrm{sgn}\left(bx^{3}+a\right)+280\,a^{3}b^{2}x^{6}\mathrm{sgn}\left(bx^{3}+a\right)+120\,a^{4}bx^{3}\mathrm{sgn}\left(bx^{3}+a\right)+21\,a^{5}\mathrm{sgn}\left(bx^{3}+a\right)}{504\,x^{24}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^25,x, algorithm="giac")
```

```
[Out] -1/504*(56*b^5*x^15*sgn(b*x^3 + a) + 210*a*b^4*x^12*sgn(b*x^3 + a) + 336*a^
2*b^3*x^9*sgn(b*x^3 + a) + 280*a^3*b^2*x^6*sgn(b*x^3 + a) + 120*a^4*b*x^3*s
gn(b*x^3 + a) + 21*a^5*sgn(b*x^3 + a))/x^24
```

maple [A] time = 0.01, size = 80, normalized size = 0.62

$$-\frac{\left(56b^{5}x^{15}+210a\,b^{4}x^{12}+336a^{2}b^{3}x^{9}+280a^{3}b^{2}x^{6}+120a^{4}b\,x^{3}+21a^{5}\right)\left(\left(b\,x^{3}+a\right)^{2}\right)^{\frac{5}{2}}}{504\left(b\,x^{3}+a\right)^{5}x^{24}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^25,x)
```

```
[Out] -1/504*(56*b^5*x^15+210*a*b^4*x^12+336*a^2*b^3*x^9+280*a^3*b^2*x^6+120*a^4*
b*x^3+21*a^5)*((b*x^3+a)^2)^(5/2)/x^24/(b*x^3+a)^5
```

maxima [B] time = 1.02, size = 272, normalized size = 2.12

$$\frac{\left(b^{2}x^{6}+2\,abx^{3}+a^{2}\right)^{\frac{5}{2}}b^{8}}{18\,a^{8}}+\frac{\left(b^{2}x^{6}+2\,abx^{3}+a^{2}\right)^{\frac{5}{2}}b^{7}}{18\,a^{7}x^{3}}-\frac{\left(b^{2}x^{6}+2\,abx^{3}+a^{2}\right)^{\frac{7}{2}}b^{6}}{18\,a^{8}x^{6}}+\frac{\left(b^{2}x^{6}+2\,abx^{3}+a^{2}\right)^{\frac{7}{2}}b^{5}}{18\,a^{7}x^{9}}-\frac{\left(b^{2}x^{6}+2\,abx^{3}+a^{2}\right)^{\frac{7}{2}}b^{4}}{18\,a^{6}x^{12}}+\frac{\left(b^{2}x^{6}+2\,abx^{3}+a^{2}\right)^{\frac{7}{2}}b^{3}}{18\,a^{5}x^{15}}-\frac{\left(b^{2}x^{6}+2\,abx^{3}+a^{2}\right)^{\frac{7}{2}}b^{2}}{18\,a^{4}x^{18}}+\frac{3\left(b^{2}x^{6}+2\,abx^{3}+a^{2}\right)^{\frac{7}{2}}b}{56\,a^{3}x^{21}}-\frac{\left(b^{2}x^{6}+2\,abx^{3}+a^{2}\right)^{\frac{7}{2}}}{24\,a^{2}x^{24}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^25,x, algorithm="maxima")
```

```
[Out] 1/18*(b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2)*b^8/a^8 + 1/18*(b^2*x^6 + 2*a*b*x^3
+ a^2)^(5/2)*b^7/(a^7*x^3) - 1/18*(b^2*x^6 + 2*a*b*x^3 + a^2)^(7/2)*b^6/(a^
8*x^6) + 1/18*(b^2*x^6 + 2*a*b*x^3 + a^2)^(7/2)*b^5/(a^7*x^9) - 1/18*(b^2*x
^6 + 2*a*b*x^3 + a^2)^(7/2)*b^4/(a^6*x^12) + 1/18*(b^2*x^6 + 2*a*b*x^3 + a^
2)^(7/2)*b^3/(a^5*x^15) - 1/18*(b^2*x^6 + 2*a*b*x^3 + a^2)^(7/2)*b^2/(a^4*x
^18) + 3/56*(b^2*x^6 + 2*a*b*x^3 + a^2)^(7/2)*b/(a^3*x^21) - 1/24*(b^2*x^6
+ 2*a*b*x^3 + a^2)^(7/2)/(a^2*x^24)
```

mupad [B] time = 1.22, size = 231, normalized size = 1.80

$$-\frac{a^{5}\,\sqrt{a^{2}+2\,a\,b\,x^{3}+b^{2}\,x^{6}}}{24\,x^{24}\,\left(b\,x^{3}+a\right)}-\frac{b^{5}\,\sqrt{a^{2}+2\,a\,b\,x^{3}+b^{2}\,x^{6}}}{9\,x^{9}\,\left(b\,x^{3}+a\right)}-\frac{5\,a\,b^{4}\,\sqrt{a^{2}+2\,a\,b\,x^{3}+b^{2}\,x^{6}}}{12\,x^{12}\,\left(b\,x^{3}+a\right)}-\frac{5\,a^{4}\,b\,\sqrt{a^{2}+2\,a\,b\,x^{3}+b^{2}\,x^{6}}}{21\,x^{21}\,\left(b\,x^{3}+a\right)}-\frac{2\,a^{2}\,b^{3}\,\sqrt{a^{2}+2\,a\,b\,x^{3}+b^{2}\,x^{6}}}{3\,x^{15}\,\left(b\,x^{3}+a\right)}-\frac{5\,a^{3}\,b^{2}\,\sqrt{a^{2}+2\,a\,b\,x^{3}+b^{2}\,x^{6}}}{9\,x^{18}\,\left(b\,x^{3}+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2)/x^25,x)
```

```
[Out] - (a^5*(a^2 + b^2*x^6 + 2*a*b*x^3)^(1/2))/(24*x^24*(a + b*x^3)) - (b^5*(a^2
 + b^2*x^6 + 2*a*b*x^3)^(1/2))/(9*x^9*(a + b*x^3)) - (5*a*b^4*(a^2 + b^2*x^
6 + 2*a*b*x^3)^(1/2))/(12*x^12*(a + b*x^3)) - (5*a^4*b*(a^2 + b^2*x^6 + 2*a
*b*x^3)^(1/2))/(21*x^21*(a + b*x^3)) - (2*a^2*b^3*(a^2 + b^2*x^6 + 2*a*b*x^
3)^(1/2))/(3*x^15*(a + b*x^3)) - (5*a^3*b^2*(a^2 + b^2*x^6 + 2*a*b*x^3)^(1/
2))/(9*x^18*(a + b*x^3))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\left(a+bx^3\right)^2\right)^{\frac{5}{2}}}{x^{25}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**6+2*a*b*x**3+a**2)**(5/2)/x**25,x)
```

```
[Out] Integral(((a + b*x**3)**2)**(5/2)/x**25, x)
```

3.89 $\int \frac{x^4}{\sqrt{a^2+2abx^3+b^2x^6}} \, dx$

Optimal. Leaf size=240

$$\frac{x^2\left(a+bx^3\right)}{2b\sqrt{a^2+2abx^3+b^2x^6}}+\frac{a^{2/3}\left(a+bx^3\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3b^{5/3}\sqrt{a^2+2abx^3+b^2x^6}}+\frac{a^{2/3}\left(a+bx^3\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,b^{5/3}\sqrt{a^2+2abx^3+b^2x^6}}-\frac{a^{2/3}\left(a+bx^3\right)\log\left(\right.}{6b^{5/3}\sqrt{a^2}}$$

Rubi [A] time = 0.12, antiderivative size = 240, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {1355, 321, 292, 31, 634, 617, 204, 628}

$$\frac{x^2\left(a+bx^3\right)}{2b\sqrt{a^2+2abx^3+b^2x^6}}+\frac{a^{2/3}\left(a+bx^3\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3b^{5/3}\sqrt{a^2+2abx^3+b^2x^6}}-\frac{a^{2/3}\left(a+bx^3\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{6b^{5/3}\sqrt{a^2+2abx^3+b^2x^6}}+\frac{a^{2/3}\left(a+bx^3\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,b^{5/3}\sqrt{a^2+2abx^3+b^2x^6}}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6],x]
```

```
[Out] (x^2*(a + b*x^3))/(2*b*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) + (a^(2/3)*(a + b*x
^3)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*b^(5/3)*Sqr
t[a^2 + 2*a*b*x^3 + b^2*x^6]) + (a^(2/3)*(a + b*x^3)*Log[a^(1/3) + b^(1/3)*
x])/(3*b^(5/3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) - (a^(2/3)*(a + b*x^3)*Log[
a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*b^(5/3)*Sqrt[a^2 + 2*a*b*x^3
 + b^2*x^6])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\int \frac{x^4}{\sqrt{a^2+2abx^3+b^2x^6}}\,dx = \frac{\left(ab+b^2x^3\right)\int \frac{x^4}{ab+b^2x^3}\,dx}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$= \frac{x^2\left(a+bx^3\right)}{2b\sqrt{a^2+2abx^3+b^2x^6}} - \frac{\left(a\left(ab+b^2x^3\right)\right)\int \frac{x}{ab+b^2x^3}\,dx}{b\sqrt{a^2+2abx^3+b^2x^6}}$$

$$= \frac{x^2\left(a+bx^3\right)}{2b\sqrt{a^2+2abx^3+b^2x^6}} + \frac{\left(a^{2/3}\left(ab+b^2x^3\right)\right)\int \frac{1}{\sqrt[3]{a}\sqrt[3]{b}+b^{2/3}x}\,dx}{3b^2\sqrt{a^2+2abx^3+b^2x^6}} - \frac{\left(a^{2/3}\left(ab+b^2x^3\right)\right)\int}{3b^2\sqrt{a^2+2}}$$

$$= \frac{x^2\left(a+bx^3\right)}{2b\sqrt{a^2+2abx^3+b^2x^6}} + \frac{a^{2/3}\left(a+bx^3\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3b^{5/3}\sqrt{a^2+2abx^3+b^2x^6}} - \frac{\left(a^{2/3}\left(ab+b^2x^3\right)\right)\int \frac{}{a^{2/}}}{6b^{8/3}\sqrt{a^2+2ab}}$$

$$= \frac{x^2\left(a+bx^3\right)}{2b\sqrt{a^2+2abx^3+b^2x^6}} + \frac{a^{2/3}\left(a+bx^3\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3b^{5/3}\sqrt{a^2+2abx^3+b^2x^6}} - \frac{a^{2/3}\left(a+bx^3\right)\log\left(a^{2/3}\right.}{6b^{5/3}\sqrt{a^2+2a}}$$

$$= \frac{x^2\left(a+bx^3\right)}{2b\sqrt{a^2+2abx^3+b^2x^6}} + \frac{a^{2/3}\left(a+bx^3\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,b^{5/3}\sqrt{a^2+2abx^3+b^2x^6}} + \frac{a^{2/3}\left(a+bx^3\right)\log\left(\sqrt[3]{a}\right.}{3b^{5/3}\sqrt{a^2+2abx^3+}}$$

Mathematica [A] time = 0.06, size = 131, normalized size = 0.55

$$\frac{\left(a+bx^3\right)\left(-a^{2/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2\right)+2a^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)+2\sqrt{3}\,a^{2/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a}}}{\sqrt{3}}\right)+3b^{2/3}x^2\right)}{6b^{5/3}\sqrt{\left(a+bx^3\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6],x]
```

```
[Out] ((a + b*x^3)*(3*b^(2/3)*x^2 + 2*Sqrt[3]*a^(2/3)*ArcTan[(1 - (2*b^(1/3)*x)/a
^(1/3))/Sqrt[3]] + 2*a^(2/3)*Log[a^(1/3) + b^(1/3)*x] - a^(2/3)*Log[a^(2/3)
 - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]))/(6*b^(5/3)*Sqrt[(a + b*x^3)^2])
```

IntegrateAlgebraic [A] time = 7.36, size = 144, normalized size = 0.60

$$\frac{\left(a+bx^3\right)\left(-\frac{a^{2/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6b^{5/3}}+\frac{a^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3b^{5/3}}+\frac{a^{2/3}\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}b^{5/3}}+\frac{x^2}{2b}\right)}{\sqrt{\left(a+bx^3\right)^2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^4/Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6],x]
```

```
[Out] ((a + b*x^3)*(x^2/(2*b) + (a^(2/3)*ArcTan[1/Sqrt[3] - (2*b^(1/3)*x)/(Sqrt[3
]*a^(1/3))])/(Sqrt[3]*b^(5/3)) + (a^(2/3)*Log[a^(1/3) + b^(1/3)*x])/(3*b^(5
/3)) - (a^(2/3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*b^(5/3))
))/Sqrt[(a + b*x^3)^2]
```

fricas [A] time = 0.85, size = 123, normalized size = 0.51

$$\frac{3x^2-2\sqrt{3}\left(\frac{a^2}{b^2}\right)^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}bx\left(\frac{a^2}{b^2}\right)^{\frac{1}{3}}-\sqrt{3}a}{3a}\right)-\left(\frac{a^2}{b^2}\right)^{\frac{1}{3}}\log\left(ax^2-bx\left(\frac{a^2}{b^2}\right)^{\frac{2}{3}}+a\left(\frac{a^2}{b^2}\right)^{\frac{1}{3}}\right)+2\left(\frac{a^2}{b^2}\right)^{\frac{1}{3}}\log\left(ax+b\left(\frac{a^2}{b^2}\right)^{\frac{2}{3}}\right)}{6b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/((b*x^3+a)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/6*(3*x^2 - 2*sqrt(3)*(a^2/b^2)^(1/3)*arctan(1/3*(2*sqrt(3)*b*x*(a^2/b^2)^
(1/3) - sqrt(3)*a)/a) - (a^2/b^2)^(1/3)*log(a*x^2 - b*x*(a^2/b^2)^(2/3) + a
*(a^2/b^2)^(1/3)) + 2*(a^2/b^2)^(1/3)*log(a*x + b*(a^2/b^2)^(2/3)))/b
```

giac [A] time = 0.37, size = 146, normalized size = 0.61

$$\frac{x^2\operatorname{sgn}\left(bx^3+a\right)}{2b}+\frac{\left(-\frac{a}{b}\right)^{\frac{2}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)\operatorname{sgn}\left(bx^3+a\right)}{3b}+\frac{\sqrt{3}\left(-ab^2\right)^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)\operatorname{sgn}\left(bx^3+a\right)}{3b^3}-\frac{\left(-ab^2\right)^{\frac{2}{3}}\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)\operatorname{sgn}\left(bx^3+a\right)}{6b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/((b*x^3+a)^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*x^2*sgn(b*x^3 + a)/b + 1/3*(-a/b)^(2/3)*log(abs(x - (-a/b)^(1/3)))*sgn(
b*x^3 + a)/b + 1/3*sqrt(3)*(-a*b^2)^(2/3)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^
(1/3))/(-a/b)^(1/3))*sgn(b*x^3 + a)/b^3 - 1/6*(-a*b^2)^(2/3)*log(x^2 + x*(-
a/b)^(1/3) + (-a/b)^(2/3))*sgn(b*x^3 + a)/b^3
```

maple [A] time = 0.01, size = 113, normalized size = 0.47

$$\frac{\left(bx^3+a\right)\left(3\left(\frac{a}{b}\right)^{\frac{1}{3}}bx^2+2\sqrt{3}\,a\arctan\left(\frac{\sqrt{3}\left(-2x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)+2a\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)-a\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)\right)}{6\sqrt{\left(bx^3+a\right)^2}\left(\frac{a}{b}\right)^{\frac{1}{3}}b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/((b*x^3+a)^2)^(1/2),x)
```

```
[Out] 1/6*(b*x^3+a)*(3*x^2*b*(a/b)^(1/3)+2*arctan(1/3*3^(1/2)*(-2*x+(a/b)^(1/3))/
(a/b)^(1/3))*3^(1/2)*a+2*ln(x+(a/b)^(1/3))*a-ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/
3))*a)/((b*x^3+a)^2)^(1/2)/b^2/(a/b)^(1/3)
```

maxima [A] time = 2.18, size = 109, normalized size = 0.45

$$\frac{x^2}{2\,b} - \frac{\sqrt{3}\,a\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,b^2\left(\frac{a}{b}\right)^{\frac{1}{3}}} - \frac{a\log\left(x^2 - x\left(\frac{a}{b}\right)^{\frac{1}{3}} + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,b^2\left(\frac{a}{b}\right)^{\frac{1}{3}}} + \frac{a\log\left(x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,b^2\left(\frac{a}{b}\right)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/((b*x^3+a)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*x^2/b - 1/3*sqrt(3)*a*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3
))/(b^2*(a/b)^(1/3)) - 1/6*a*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(b^2*(a
/b)^(1/3)) + 1/3*a*log(x + (a/b)^(1/3))/(b^2*(a/b)^(1/3))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^4}{\sqrt{\left(b\,x^3 + a\right)^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/((a + b*x^3)^2)^(1/2),x)
```

```
[Out] int(x^4/((a + b*x^3)^2)^(1/2), x)
```

sympy [A] time = 0.21, size = 32, normalized size = 0.13

$$\operatorname{RootSum}\left(27t^3b^5 - a^2, \left(t \mapsto t\log\left(\frac{9t^2b^3}{a} + x\right)\right)\right) + \frac{x^2}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/((b*x**3+a)**2)**(1/2),x)
```

```
[Out] RootSum(27*_t**3*b**5 - a**2, Lambda(_t, _t*log(9*_t**2*b**3/a + x))) + x**
2/(2*b)
```

3.90 $\int \frac{x^3}{\sqrt{a^2+2abx^3+b^2x^6}}\,dx$

Optimal. Leaf size=235

$$\frac{x\left(a+bx^3\right)}{b\sqrt{a^2+2abx^3+b^2x^6}}-\frac{\sqrt[3]{a}\left(a+bx^3\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3b^{4/3}\sqrt{a^2+2abx^3+b^2x^6}}+\frac{\sqrt[3]{a}\left(a+bx^3\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}b^{4/3}\sqrt{a^2+2abx^3+b^2x^6}}+\frac{\sqrt[3]{a}\left(a+bx^3\right)\log\left(a\right.}{6b^{4/3}\sqrt{a^2+\cdots}}$$

Rubi [A] time = 0.11, antiderivative size = 235, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {1355, 321, 200, 31, 634, 617, 204, 628}

$$\frac{x\left(a+bx^3\right)}{b\sqrt{a^2+2abx^3+b^2x^6}}-\frac{\sqrt[3]{a}\left(a+bx^3\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3b^{4/3}\sqrt{a^2+2abx^3+b^2x^6}}+\frac{\sqrt[3]{a}\left(a+bx^3\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6b^{4/3}\sqrt{a^2+2abx^3+b^2x^6}}+\frac{\sqrt[3]{a}\left(a+bx^3\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}b^{4/3}\sqrt{a^2+2abx^3+b^2x^6}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6],x]
```

```
[Out] (x*(a + b*x^3))/(b*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) + (a^(1/3)*(a + b*x^3)*
ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*b^(4/3)*Sqrt[a^
2 + 2*a*b*x^3 + b^2*x^6]) - (a^(1/3)*(a + b*x^3)*Log[a^(1/3) + b^(1/3)*x])/
(3*b^(4/3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) + (a^(1/3)*(a + b*x^3)*Log[a^(2
/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*b^(4/3)*Sqrt[a^2 + 2*a*b*x^3 + b
^2*x^6])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\int \frac{x^3}{\sqrt{a^2+2abx^3+b^2x^6}}\,dx = \frac{\left(ab+b^2x^3\right)\int \frac{x^3}{ab+b^2x^3}\,dx}{\sqrt{a^2+2abx^3+b^2x^6}}$$
$$= \frac{x\left(a+bx^3\right)}{b\sqrt{a^2+2abx^3+b^2x^6}} - \frac{\left(a\left(ab+b^2x^3\right)\right)\int \frac{1}{ab+b^2x^3}\,dx}{b\sqrt{a^2+2abx^3+b^2x^6}}$$
$$= \frac{x\left(a+bx^3\right)}{b\sqrt{a^2+2abx^3+b^2x^6}} - \frac{\left(\sqrt[3]{a}\left(ab+b^2x^3\right)\right)\int \frac{1}{\sqrt[3]{a}\sqrt[3]{b}+b^{2/3}x}\,dx}{3b^{5/3}\sqrt{a^2+2abx^3+b^2x^6}} - \frac{\left(\sqrt[3]{a}\left(ab+b^2x^3\right)\right)\int \frac{\,}{a}}{3b^{5/3}\sqrt{a^2+2a}}$$
$$= \frac{x\left(a+bx^3\right)}{b\sqrt{a^2+2abx^3+b^2x^6}} - \frac{\sqrt[3]{a}\left(a+bx^3\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3b^{4/3}\sqrt{a^2+2abx^3+b^2x^6}} + \frac{\left(\sqrt[3]{a}\left(ab+b^2x^3\right)\right)\int \frac{\,}{a^{2/3}b}}{6b^{7/3}\sqrt{a^2+2abx}}$$
$$= \frac{x\left(a+bx^3\right)}{b\sqrt{a^2+2abx^3+b^2x^6}} - \frac{\sqrt[3]{a}\left(a+bx^3\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3b^{4/3}\sqrt{a^2+2abx^3+b^2x^6}} + \frac{\sqrt[3]{a}\left(a+bx^3\right)\log\left(a^{2/3}-\right.}{6b^{4/3}\sqrt{a^2+2ab}}$$
$$= \frac{x\left(a+bx^3\right)}{b\sqrt{a^2+2abx^3+b^2x^6}} + \frac{\sqrt[3]{a}\left(a+bx^3\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,b^{4/3}\sqrt{a^2+2abx^3+b^2x^6}} - \frac{\sqrt[3]{a}\left(a+bx^3\right)\log\left(\sqrt[3]{a}+\right.}{3b^{4/3}\sqrt{a^2+2abx^3+b}}$$

Mathematica [A] time = 0.03, size = 128, normalized size = 0.54

$$\frac{\left(a+bx^3\right)\left(\sqrt[3]{a}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)-2\sqrt[3]{a}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)+2\sqrt{3}\sqrt[3]{a}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)+6\sqrt[3]{b}x\right)}{6b^{4/3}\sqrt{\left(a+bx^3\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6],x]
```

```
[Out] ((a + b*x^3)*(6*b^(1/3)*x + 2*Sqrt[3]*a^(1/3)*ArcTan[(1 - (2*b^(1/3)*x)/a^(
1/3))/Sqrt[3]] - 2*a^(1/3)*Log[a^(1/3) + b^(1/3)*x] + a^(1/3)*Log[a^(2/3) -
 a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]))/(6*b^(4/3)*Sqrt[(a + b*x^3)^2])
```

IntegrateAlgebraic [A] time = 6.52, size = 139, normalized size = 0.59

$$\frac{\left(a+bx^3\right)\left(\frac{\sqrt[3]{a}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6b^{4/3}}-\frac{\sqrt[3]{a}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3b^{4/3}}+\frac{\sqrt[3]{a}\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}b^{4/3}}+\frac{x}{b}\right)}{\sqrt{\left(a+bx^3\right)^2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^3/Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6],x]
```

```
[Out] ((a + b*x^3)*(x/b + (a^(1/3)*ArcTan[1/Sqrt[3] - (2*b^(1/3)*x)/(Sqrt[3]*a^(1
/3))])/(Sqrt[3]*b^(4/3)) - (a^(1/3)*Log[a^(1/3) + b^(1/3)*x])/(3*b^(4/3)) +
 (a^(1/3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*b^(4/3))))/Sqr
t[(a + b*x^3)^2]
```

fricas [A] time = 1.17, size = 106, normalized size = 0.45

$$\frac{2\sqrt{3}\left(-\frac{a}{b}\right)^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}bx\left(-\frac{a}{b}\right)^{\frac{2}{3}}-\sqrt{3}a}{3a}\right)-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)+2\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)+6x}{6b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/((b*x^3+a)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/6*(2*sqrt(3)*(-a/b)^(1/3)*arctan(1/3*(2*sqrt(3)*b*x*(-a/b)^(2/3) - sqrt(3
)*a)/a) - (-a/b)^(1/3)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3)) + 2*(-a/b)^
(1/3)*log(x - (-a/b)^(1/3)) + 6*x)/b
```

giac [A] time = 0.41, size = 143, normalized size = 0.61

$$\frac{\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)\mathrm{sgn}\left(bx^3+a\right)}{3b}+\frac{x\mathrm{sgn}\left(bx^3+a\right)}{b}-\frac{\sqrt{3}\left(-ab^2\right)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)\mathrm{sgn}\left(bx^3+a\right)}{3b^2}-\frac{\left(-ab^2\right)^{\frac{1}{3}}\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)\mathrm{sgn}\left(bx^3+a\right)}{6b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/((b*x^3+a)^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/3*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))*sgn(b*x^3 + a)/b + x*sgn(b*x^3
+ a)/b - 1/3*sqrt(3)*(-a*b^2)^(1/3)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))
/(-a/b)^(1/3))*sgn(b*x^3 + a)/b^2 - 1/6*(-a*b^2)^(1/3)*log(x^2 + x*(-a/b)^(
1/3) + (-a/b)^(2/3))*sgn(b*x^3 + a)/b^2
```

maple [A] time = 0.01, size = 110, normalized size = 0.47

$$\frac{\left(bx^3+a\right)\left(2\sqrt{3}\,a\arctan\left(\frac{\sqrt{3}\left(-2x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)-2a\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)+a\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)+6\left(\frac{a}{b}\right)^{\frac{2}{3}}bx\right)}{6\sqrt{\left(bx^3+a\right)^2}\left(\frac{a}{b}\right)^{\frac{2}{3}}b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/((b*x^3+a)^2)^(1/2),x)
```

```
[Out] 1/6*(b*x^3+a)*(6*x*b*(a/b)^(2/3)+2*3^(1/2)*a*arctan(1/3*3^(1/2)*(-2*x+(a/b)
^(1/3))/(a/b)^(1/3))-2*a*ln(x+(a/b)^(1/3))+a*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/
3)))/((b*x^3+a)^2)^(1/2)/b^2/(a/b)^(2/3)
```

maxima [A]   time = 1.36, size = 106, normalized size = 0.45

$$\frac{x}{b} - \frac{\sqrt{3}\, a \arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\, b^{2} \left(\frac{a}{b}\right)^{\frac{2}{3}}} + \frac{a \log\left(x^{2} - x \left(\frac{a}{b}\right)^{\frac{1}{3}} + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\, b^{2} \left(\frac{a}{b}\right)^{\frac{2}{3}}} - \frac{a \log\left(x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\, b^{2} \left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/((b*x^3+a)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] x/b - 1/3*sqrt(3)*a*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(b^
2*(a/b)^(2/3)) + 1/6*a*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(b^2*(a/b)^(2
/3)) - 1/3*a*log(x + (a/b)^(1/3))/(b^2*(a/b)^(2/3))
```

mupad [F]   time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3}{\sqrt{\left(b\, x^3 + a\right)^2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/((a + b*x^3)^2)^(1/2),x)
```

```
[Out] int(x^3/((a + b*x^3)^2)^(1/2), x)
```

sympy [A]   time = 0.20, size = 22, normalized size = 0.09

$$\operatorname{RootSum}\left(27 t^{3} b^{4} + a, \left(t \mapsto t \log\left(-3 t b + x\right) \right)\right) + \frac{x}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/((b*x**3+a)**2)**(1/2),x)
```

```
[Out] RootSum(27*_t**3*b**4 + a, Lambda(_t, _t*log(-3*_t*b + x))) + x/b
```

3.91 $\int \frac{x^2}{\sqrt{a^2+2abx^3+b^2x^6}}\,dx$

Optimal. Leaf size=44

$$\frac{(a+bx^3)\log(a+bx^3)}{3b\sqrt{a^2+2abx^3+b^2x^6}}$$

Rubi [A] time = 0.03, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1352, 608, 31}

$$\frac{(a+bx^3)\log(a+bx^3)}{3b\sqrt{a^2+2abx^3+b^2x^6}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6],x]
```

```
[Out] ((a + b*x^3)*Log[a + b*x^3])/(3*b*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 608

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[(b/2 + c*x)
/Sqrt[a + b*x + c*x^2], Int[1/(b/2 + c*x), x], x] /; FreeQ[{a, b, c}, x] &&
 EqQ[b^2 - 4*a*c, 0]
```

Rule 1352

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a,
b, c, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2}{\sqrt{a^2+2abx^3+b^2x^6}}\,dx &= \frac{1}{3}\operatorname{Subst}\left(\int \frac{1}{\sqrt{a^2+2abx+b^2x^2}}\,dx, x, x^3\right)\\
&= \frac{(ab+b^2x^3)\operatorname{Subst}\left(\int \frac{1}{ab+b^2x}\,dx, x, x^3\right)}{3\sqrt{a^2+2abx^3+b^2x^6}}\\
&= \frac{(a+bx^3)\log(a+bx^3)}{3b\sqrt{a^2+2abx^3+b^2x^6}}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 35, normalized size = 0.80

$$\frac{(a+bx^3)\log(a+bx^3)}{3b\sqrt{(a+bx^3)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6],x]
```

```
[Out] ((a + b*x^3)*Log[a + b*x^3])/(3*b*Sqrt[(a + b*x^3)^2])
```

IntegrateAlgebraic [B] time = 0.27, size = 149, normalized size = 3.39

$$-\frac{\log\left(\sqrt{a^2+2abx^3+b^2x^6}-a-\sqrt{b^2}\,x^3\right)}{6\sqrt{b^2}}-\frac{\log\left(\sqrt{a^2+2abx^3+b^2x^6}+a-\sqrt{b^2}\,x^3\right)}{6\sqrt{b^2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{b^2}\,x^3}{a}-\frac{\sqrt{a^2+2abx^3+b^2x^6}}{a}\right)}{3b}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^2/Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6],x]
```

```
[Out] -1/3*ArcTanh[(Sqrt[b^2]*x^3)/a - Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]/a]/b - Log
[-a - Sqrt[b^2]*x^3 + Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]]/(6*Sqrt[b^2]) - Log[
a - Sqrt[b^2]*x^3 + Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]]/(6*Sqrt[b^2])
```

fricas [A] time = 1.07, size = 13, normalized size = 0.30

$$\frac{\log\left(bx^3+a\right)}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/((b*x^3+a)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/3*log(b*x^3 + a)/b
```

giac [A] time = 0.34, size = 22, normalized size = 0.50

$$\frac{\log\left(\left|bx^3+a\right|\right)\operatorname{sgn}\left(bx^3+a\right)}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/((b*x^3+a)^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/3*log(abs(b*x^3 + a))*sgn(b*x^3 + a)/b
```

maple [A] time = 0.01, size = 32, normalized size = 0.73

$$\frac{\left(b\,x^3+a\right)\ln\left(b\,x^3+a\right)}{3\sqrt{\left(b\,x^3+a\right)^2}\;b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((b*x^3+a)^2)^(1/2),x)
```

```
[Out] 1/3*(b*x^3+a)*ln(b*x^3+a)/b/((b*x^3+a)^2)^(1/2)
```

maxima [A] time = 0.93, size = 15, normalized size = 0.34

$$\frac{\log\left(x^3+\frac{a}{b}\right)}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/((b*x^3+a)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/3*log(x^3 + a/b)/b
```

mupad [B] time = 1.39, size = 33, normalized size = 0.75

$$\frac{\ln\left(b^2\,x^3+a\,b\right)\,\mathrm{sign}\left(2\,b^2\,x^3+2\,a\,b\right)}{3\,\sqrt{b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((a + b*x^3)^2)^(1/2),x)
```

```
[Out] (log(a*b + b^2*x^3)*sign(2*a*b + 2*b^2*x^3))/(3*(b^2)^(1/2))
```

sympy [A] time = 0.17, size = 10, normalized size = 0.23

$$\frac{\log\left(a+bx^3\right)}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/((b*x**3+a)**2)**(1/2),x)
```

```
[Out] log(a + b*x**3)/(3*b)
```

3.92 $\int \frac{x}{\sqrt{a^2+2abx^3+b^2x^6}}\,dx$

Optimal. Leaf size=202

$$-\frac{(a+bx^3)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}\,b^{2/3}\sqrt{a^2+2abx^3+b^2x^6}}-\frac{(a+bx^3)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\sqrt[3]{a}\,b^{2/3}\sqrt{a^2+2abx^3+b^2x^6}}+\frac{(a+bx^3)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6\sqrt[3]{a}\,b^{2/3}\sqrt{a^2+2abx^3+b^2x^6}}$$

Rubi [A] time = 0.08, antiderivative size = 202, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {1355, 292, 31, 634, 617, 204, 628}

$$-\frac{(a+bx^3)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}\,b^{2/3}\sqrt{a^2+2abx^3+b^2x^6}}+\frac{(a+bx^3)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6\sqrt[3]{a}\,b^{2/3}\sqrt{a^2+2abx^3+b^2x^6}}-\frac{(a+bx^3)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\sqrt[3]{a}\,b^{2/3}\sqrt{a^2+2abx^3+b^2x^6}}$$

Antiderivative was successfully verified.

```
[In] Int[x/Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6],x]
```

```
[Out] -(((a + b*x^3)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*
a^(1/3)*b^(2/3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])) - ((a + b*x^3)*Log[a^(1/3
) + b^(1/3)*x])/(3*a^(1/3)*b^(2/3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) + ((a +
 b*x^3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(1/3)*b^(2/3)*
Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\int \frac{x}{\sqrt{a^2+2abx^3+b^2x^6}}\,dx = \frac{\left(ab+b^2x^3\right)\int \frac{x}{ab+b^2x^3}\,dx}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$= -\frac{\left(ab+b^2x^3\right)\int \frac{1}{\sqrt[3]{a}\sqrt[3]{b}+b^{2/3}x}\,dx}{3\sqrt[3]{a}\,b\sqrt{a^2+2abx^3+b^2x^6}} + \frac{\left(ab+b^2x^3\right)\int \frac{\sqrt[3]{a}\sqrt[3]{b}+b^{2/3}x}{a^{2/3}b^{2/3}-\sqrt[3]{a}\,bx+b^{4/3}x^2}\,dx}{3\sqrt[3]{a}\,b\sqrt{a^2+2abx^3+b^2x^6}}$$

$$= -\frac{\left(a+bx^3\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3\sqrt[3]{a}\,b^{2/3}\sqrt{a^2+2abx^3+b^2x^6}} + \frac{\left(ab+b^2x^3\right)\int \frac{-\sqrt[3]{a}\,b+2b^{4/3}x}{a^{2/3}b^{2/3}-\sqrt[3]{a}\,bx+b^{4/3}x^2}\,dx}{6\sqrt[3]{a}\,b^{5/3}\sqrt{a^2+2abx^3+b^2x^6}} + \frac{\left(ab+b^2\cdots\right.}{2b^{2/\cdots}}$$

$$= -\frac{\left(a+bx^3\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3\sqrt[3]{a}\,b^{2/3}\sqrt{a^2+2abx^3+b^2x^6}} + \frac{\left(a+bx^3\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{6\sqrt[3]{a}\,b^{2/3}\sqrt{a^2+2abx^3+b^2x^6}} + \frac{\left(ab\cdots\right.}{\cdots}$$

$$= -\frac{\left(a+bx^3\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\sqrt[3]{a}\,b^{2/3}\sqrt{a^2+2abx^3+b^2x^6}} - \frac{\left(a+bx^3\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3\sqrt[3]{a}\,b^{2/3}\sqrt{a^2+2abx^3+b^2x^6}} + \frac{\left(a+bx^3\right)\log\cdots}{6\sqrt[3]{a}\,b^{2/\cdots}}$$

Mathematica [A] time = 0.02, size = 109, normalized size = 0.54

$$\frac{\left(a+bx^3\right)\left(\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2\right)-2\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)-2\sqrt{3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a}}}{\sqrt{3}}\right)\right)}{6\sqrt[3]{a}\,b^{2/3}\sqrt{\left(a+bx^3\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6],x]
```

```
[Out] ((a + b*x^3)*(-2*Sqrt[3]*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] - 2*Lo
g[a^(1/3) + b^(1/3)*x] + Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]))/(
6*a^(1/3)*b^(2/3)*Sqrt[(a + b*x^3)^2])
```

IntegrateAlgebraic [A] time = 8.36, size = 135, normalized size = 0.67

$$\frac{\left(a+bx^3\right)\left(\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{6\sqrt[3]{a}\,b^{2/3}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3\sqrt[3]{a}\,b^{2/3}}-\frac{\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{b}\,x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\sqrt[3]{a}\,b^{2/3}}\right)}{\sqrt{\left(a+bx^3\right)^2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x/Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6],x]
```

```
[Out] ((a + b*x^3)*(-(ArcTan[1/Sqrt[3] - (2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))]/(Sqrt[3
]*a^(1/3)*b^(2/3))) - Log[a^(1/3) + b^(1/3)*x]/(3*a^(1/3)*b^(2/3)) + Log[a^
(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]/(6*a^(1/3)*b^(2/3))))/Sqrt[(a + b*
x^3)^2]
```

fricas [A] time = 1.76, size = 304, normalized size = 1.50

$$\left[\frac{3\sqrt{\frac{1}{3}}ab\sqrt{\frac{\left(-ab^2\right)^{\frac{1}{3}}}{a}}\log\left(\frac{2b^2x^3-ab+3\sqrt{\frac{1}{3}}\left(abx+2\left(-ab^2\right)^{\frac{2}{3}}x^2+\left(-ab^2\right)^{\frac{1}{3}}a\right)\sqrt{\frac{\left(-ab^2\right)^{\frac{1}{3}}}{a}}-3\left(-ab^2\right)^{\frac{2}{3}}x}{bx^3+a}\right)+\left(-ab^2\right)^{\frac{2}{3}}\log\left(b^2x^2+\left(-ab^2\right)^{\frac{1}{3}}bx+\left(-ab^2\right)^{\frac{2}{3}}\right)-2\left(-ab^2\right)^{\frac{2}{3}}\log\left(bx-\left(-ab^2\right)^{\frac{1}{3}}\right)}{6ab^2}, \frac{6\sqrt{\frac{1}{3}}ab\sqrt{-\frac{\left(-ab^2\right)^{\frac{1}{3}}}{a}}\arctan\left(\frac{\sqrt{\frac{1}{3}}\left(2bx+\left(-ab^2\right)^{\frac{1}{3}}\right)\sqrt{-\frac{\left(-ab^2\right)^{\frac{1}{3}}}{a}}}{b}\right)+\left(-ab^2\right)^{\frac{2}{3}}\log\left(b^2x^2+\left(-ab^2\right)^{\frac{1}{3}}bx+\left(-ab^2\right)^{\frac{2}{3}}\right)-2\left(-ab^2\right)^{\frac{2}{3}}\log\left(bx-\left(-ab^2\right)^{\frac{1}{3}}\right)}{6ab^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/((b*x^3+a)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/6*(3*sqrt(1/3)*a*b*sqrt((-a*b^2)^(1/3)/a)*log((2*b^2*x^3 - a*b + 3*sqrt(
1/3)*(a*b*x + 2*(-a*b^2)^(2/3)*x^2 + (-a*b^2)^(1/3)*a)*sqrt((-a*b^2)^(1/3)/
a) - 3*(-a*b^2)^(2/3)*x)/(b*x^3 + a)) + (-a*b^2)^(2/3)*log(b^2*x^2 + (-a*b^
2)^(1/3)*b*x + (-a*b^2)^(2/3)) - 2*(-a*b^2)^(2/3)*log(b*x - (-a*b^2)^(1/3))
)/(a*b^2), 1/6*(6*sqrt(1/3)*a*b*sqrt(-(-a*b^2)^(1/3)/a)*arctan(sqrt(1/3)*(2
*b*x + (-a*b^2)^(1/3))*sqrt(-(-a*b^2)^(1/3)/a)/b) + (-a*b^2)^(2/3)*log(b^2*
x^2 + (-a*b^2)^(1/3)*b*x + (-a*b^2)^(2/3)) - 2*(-a*b^2)^(2/3)*log(b*x - (-a
*b^2)^(1/3)))/(a*b^2)]
```

giac [A] time = 0.42, size = 124, normalized size = 0.61

$$\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)\mathrm{sgn}\left(bx^3+a\right)}{3\left(-ab^2\right)^{\frac{1}{3}}}-\frac{\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)\mathrm{sgn}\left(bx^3+a\right)}{6\left(-ab^2\right)^{\frac{1}{3}}}-\frac{\left(-\frac{a}{b}\right)^{\frac{2}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)\mathrm{sgn}\left(bx^3+a\right)}{3a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/((b*x^3+a)^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))*sgn(b*x^3
 + a)/(-a*b^2)^(1/3) - 1/6*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))*sgn(b*x
^3 + a)/(-a*b^2)^(1/3) - 1/3*(-a/b)^(2/3)*log(abs(x - (-a/b)^(1/3)))*sgn(b*
x^3 + a)/a
```

maple [A] time = 0.00, size = 97, normalized size = 0.48

$$-\frac{\left(bx^3+a\right)\left(2\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(-2x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)+2\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)-\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)\right)}{6\sqrt{\left(bx^3+a\right)^2}\,\left(\frac{a}{b}\right)^{\frac{1}{3}}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((b*x^3+a)^2)^(1/2),x)
```

```
[Out] -1/6*(b*x^3+a)*(2*3^(1/2)*arctan(1/3*3^(1/2)*(-2*x+(a/b)^(1/3))/(a/b)^(1/3)
)+2*ln(x+(a/b)^(1/3))-ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3)))/((b*x^3+a)^2)^(1/2
)/b/(a/b)^(1/3)
```

maxima [A] time = 1.28, size = 98, normalized size = 0.49

$$\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{1}{3}}}-\frac{\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/((b*x^3+a)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(b*(a/b)^(1
/3)) + 1/6*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(b*(a/b)^(1/3)) - 1/3*log
(x + (a/b)^(1/3))/(b*(a/b)^(1/3))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x}{\sqrt{\left(b\,x^3+a\right)^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((a + b*x^3)^2)^(1/2),x)
```

```
[Out] int(x/((a + b*x^3)^2)^(1/2), x)
```

sympy [A] time = 0.18, size = 24, normalized size = 0.12

$$\operatorname{RootSum}\left(27t^3ab^2+1,\left(t\mapsto t\log\left(9t^2ab+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/((b*x**3+a)**2)**(1/2),x)
```

```
[Out] RootSum(27*_t**3*a*b**2 + 1, Lambda(_t, _t*log(9*_t**2*a*b + x)))
```

3.93 $\int \frac{1}{\sqrt{a^2+2abx^3+b^2x^6}} \, dx$

Optimal. Leaf size=202

$$\frac{(a+bx^3)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3a^{2/3}\sqrt[3]{b}\,\sqrt{a^2+2abx^3+b^2x^6}}-\frac{(a+bx^3)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{2/3}\sqrt[3]{b}\,\sqrt{a^2+2abx^3+b^2x^6}}-\frac{(a+bx^3)\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{6a^{2/3}\sqrt[3]{b}\,\sqrt{a^2+2abx^3+b^2x^6}}$$

Rubi [A] time = 0.12, antiderivative size = 202, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.318$, Rules used = {1343, 200, 31, 634, 617, 204, 628}

$$\frac{(a+bx^3)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3a^{2/3}\sqrt[3]{b}\,\sqrt{a^2+2abx^3+b^2x^6}}-\frac{(a+bx^3)\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{6a^{2/3}\sqrt[3]{b}\,\sqrt{a^2+2abx^3+b^2x^6}}-\frac{(a+bx^3)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{2/3}\sqrt[3]{b}\,\sqrt{a^2+2abx^3+b^2x^6}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6],x]
```

```
[Out] -(((a + b*x^3)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*
a^(2/3)*b^(1/3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])) + ((a + b*x^3)*Log[a^(1/3
) + b^(1/3)*x])/(3*a^(2/3)*b^(1/3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) - ((a +
 b*x^3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(2/3)*b^(1/3)*
Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1343

```
Int[((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_), x_Symbol] :> Dist[(
a + b*x^n + c*x^(2*n))^p/(b + 2*c*x^n)^(2*p), Int[(b + 2*c*x^n)^(2*p), x],
x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{a^2+2abx^3+b^2x^6}}\,dx = \frac{\left(2ab+2b^2x^3\right)\int \frac{1}{2ab+2b^2x^3}\,dx}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$= \frac{\left(2ab+2b^2x^3\right)\int \frac{1}{\sqrt[3]{2}\,\sqrt[3]{a}\,\sqrt[3]{b}+\sqrt[3]{2}\,b^{2/3}x}\,dx}{3\,2^{2/3}a^{2/3}b^{2/3}\sqrt{a^2+2abx^3+b^2x^6}} + \frac{\left(2ab+2b^2x^3\right)\int \frac{2\sqrt[3]{2}\,\sqrt[3]{a}\,\sqrt[3]{b}-\sqrt[3]{2}\,b^{2/3}x}{2^{2/3}a^{2/3}b^{2/3}-2^{2/3}\sqrt[3]{a}\,bx+2^{2/3}b^{4/3}\ldots}}{3\,2^{2/3}a^{2/3}b^{2/3}\sqrt{a^2+2abx^3+b^2x^6}}$$

$$= \frac{\left(a+bx^3\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3a^{2/3}\sqrt[3]{b}\,\sqrt{a^2+2abx^3+b^2x^6}} - \frac{\left(2ab+2b^2x^3\right)\int \frac{-2^{2/3}\sqrt[3]{a}\,b+2\,2^{2/3}b^{4/3}x}{2^{2/3}a^{2/3}b^{2/3}-2^{2/3}\sqrt[3]{a}\,bx+2^{2/3}b^{4/3}x^2}\,dx}{12a^{2/3}b^{4/3}\sqrt{a^2+2abx^3+b^2x^6}} + \ldots$$

$$= \frac{\left(a+bx^3\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3a^{2/3}\sqrt[3]{b}\,\sqrt{a^2+2abx^3+b^2x^6}} - \frac{\left(a+bx^3\right)\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{6a^{2/3}\sqrt[3]{b}\,\sqrt{a^2+2abx^3+b^2x^6}} + \frac{\left(2ab\ldots\right.}{\ldots}$$

$$= -\frac{\left(a+bx^3\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{2/3}\sqrt[3]{b}\,\sqrt{a^2+2abx^3+b^2x^6}} + \frac{\left(a+bx^3\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3a^{2/3}\sqrt[3]{b}\,\sqrt{a^2+2abx^3+b^2x^6}} - \frac{\left(a+bx^3\right)\log\ldots}{6a^{2/3}\sqrt[3]{\ldots}}$$

Mathematica [A] time = 0.02, size = 109, normalized size = 0.54

$$-\frac{\left(a+bx^3\right)\left(\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)-2\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)+2\sqrt{3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a}}}{\sqrt{3}}\right)\right)}{6a^{2/3}\sqrt[3]{b}\,\sqrt{\left(a+bx^3\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6],x]
```

```
[Out] -1/6*((a + b*x^3)*(2*Sqrt[3]*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] -
2*Log[a^(1/3) + b^(1/3)*x] + Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]
))/(a^(2/3)*b^(1/3)*Sqrt[(a + b*x^3)^2])
```

IntegrateAlgebraic [A] time = 10.09, size = 135, normalized size = 0.67

$$\frac{\left(a+bx^3\right)\left(-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{6a^{2/3}\sqrt[3]{b}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3a^{2/3}\sqrt[3]{b}}-\frac{\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{2/3}\sqrt[3]{b}}\right)}{\sqrt{\left(a+bx^3\right)^2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6],x]
```

```
[Out] ((a + b*x^3)*(-(ArcTan[1/Sqrt[3] - (2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))]/(Sqrt[3
]*a^(2/3)*b^(1/3))) + Log[a^(1/3) + b^(1/3)*x]/(3*a^(2/3)*b^(1/3)) - Log[a^
(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]/(6*a^(2/3)*b^(1/3))))/Sqrt[(a + b*
x^3)^2]
```

fricas [A] time = 1.31, size = 299, normalized size = 1.48

$$\left[\frac{3 \sqrt{\frac{1}{3}}\, ab \sqrt{-\frac{\left(a^2 b\right)^{\frac{1}{3}}}{b}} \log\left(\frac{2\, abx^3 - 3\left(a^2 b\right)^{\frac{1}{3}} ax - a^2 + 3\sqrt{\frac{1}{3}}\left(2\, abx^2 + \left(a^2 b\right)^{\frac{2}{3}} x - \left(a^2 b\right)^{\frac{1}{3}} a\right)\sqrt{-\frac{\left(a^2 b\right)^{\frac{1}{3}}}{b}}}{bx^3 + a}\right) - \left(a^2 b\right)^{\frac{2}{3}} \log\left(abx^2 - \left(a^2 b\right)^{\frac{2}{3}} x + \left(a^2 b\right)^{\frac{1}{3}} a\right) + 2\left(a^2 b\right)^{\frac{2}{3}} \log\left(abx + \left(a^2 b\right)^{\frac{2}{3}}\right)}{6\, a^2 b}, \frac{6\sqrt{\frac{1}{3}}\, ab\sqrt{\frac{\left(a^2 b\right)^{\frac{1}{3}}}{b}} \arctan\left(\frac{\sqrt{\frac{1}{3}}\left(2\left(a^2 b\right)^{\frac{2}{3}} x - \left(a^2 b\right)^{\frac{1}{3}} a\right)\sqrt{\frac{\left(a^2 b\right)^{\frac{1}{3}}}{b}}}{a^2}\right) - \left(a^2 b\right)^{\frac{2}{3}} \log\left(abx^2 - \left(a^2 b\right)^{\frac{2}{3}} x + \left(a^2 b\right)^{\frac{1}{3}} a\right) + 2\left(a^2 b\right)^{\frac{2}{3}} \log\left(abx + \left(a^2 b\right)^{\frac{2}{3}}\right)}{6\, a^2 b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x^3+a)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/6*(3*sqrt(1/3)*a*b*sqrt(-(a^2*b)^(1/3)/b)*log((2*a*b*x^3 - 3*(a^2*b)^(1/
3)*a*x - a^2 + 3*sqrt(1/3)*(2*a*b*x^2 + (a^2*b)^(2/3)*x - (a^2*b)^(1/3)*a)*
sqrt(-(a^2*b)^(1/3)/b))/(b*x^3 + a)) - (a^2*b)^(2/3)*log(a*b*x^2 - (a^2*b)^
(2/3)*x + (a^2*b)^(1/3)*a) + 2*(a^2*b)^(2/3)*log(a*b*x + (a^2*b)^(2/3)))/(a
^2*b), 1/6*(6*sqrt(1/3)*a*b*sqrt((a^2*b)^(1/3)/b)*arctan(sqrt(1/3)*(2*(a^2*
b)^(2/3)*x - (a^2*b)^(1/3)*a)*sqrt((a^2*b)^(1/3)/b)/a^2) - (a^2*b)^(2/3)*lo
g(a*b*x^2 - (a^2*b)^(2/3)*x + (a^2*b)^(1/3)*a) + 2*(a^2*b)^(2/3)*log(a*b*x
+ (a^2*b)^(2/3)))/(a^2*b)]
```

giac [A] time = 0.36, size = 122, normalized size = 0.60

$$-\frac{1}{6}\left(\frac{2\left(-\frac{a}{b}\right)^{\frac{1}{3}} \log\left(\left|x - \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{a} - \frac{2\sqrt{3}\left(-ab^2\right)^{\frac{1}{3}} \arctan\left(\frac{\sqrt{3}\left(2x + \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{ab} - \frac{\left(-ab^2\right)^{\frac{1}{3}} \log\left(x^2 + x\left(-\frac{a}{b}\right)^{\frac{1}{3}} + \left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{ab}\right) \operatorname{sgn}\left(bx^3 + a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x^3+a)^2)^(1/2),x, algorithm="giac")
```

```
[Out] -1/6*(2*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/a - 2*sqrt(3)*(-a*b^2)^(1/3
)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(a*b) - (-a*b^2)^(1
/3)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a*b))*sgn(b*x^3 + a)
```

maple [A] time = 0.00, size = 97, normalized size = 0.48

$$\frac{\left(b\,x^3 + a\right)\left(-2\sqrt{3}\ \arctan\left(\frac{\sqrt{3}\left(-2x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right) + 2\ln\left(x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right) - \ln\left(x^2 - \left(\frac{a}{b}\right)^{\frac{1}{3}} x + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)\right)}{6\sqrt{\left(b\,x^3 + a\right)^2}\ \left(\frac{a}{b}\right)^{\frac{2}{3}} b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((b*x^3+a)^2)^(1/2),x)
```

```
[Out] 1/6*(b*x^3+a)*(-2*3^(1/2)*arctan(1/3*3^(1/2)*(-2*x+(a/b)^(1/3))/(a/b)^(1/3)
)+2*ln(x+(a/b)^(1/3))-ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3)))/((b*x^3+a)^2)^(1/2
)/b/(a/b)^(2/3)
```

maxima [A] time = 2.73, size = 98, normalized size = 0.49

$$\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x^3+a)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(b*(a/b)^(2
/3)) - 1/6*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(b*(a/b)^(2/3)) + 1/3*log
(x + (a/b)^(1/3))/(b*(a/b)^(2/3))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\sqrt{\left(b\,x^3+a\right)^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x^3)^2)^(1/2),x)
```

```
[Out] int(1/((a + b*x^3)^2)^(1/2), x)
```

sympy [A] time = 0.19, size = 20, normalized size = 0.10

$$\operatorname{RootSum}\left(27t^3a^2b-1,\left(t\mapsto t\log\left(3ta+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x**3+a)**2)**(1/2),x)
```

```
[Out] RootSum(27*_t**3*a**2*b - 1, Lambda(_t, _t*log(3*_t*a + x)))
```

3.94 $\int \frac{1}{x\sqrt{a^2+2abx^3+b^2x^6}}\,dx$

Optimal. Leaf size=80

$$\frac{\log(x)\left(a+bx^3\right)}{a\sqrt{a^2+2abx^3+b^2x^6}}-\frac{\left(a+bx^3\right)\log\left(a+bx^3\right)}{3a\sqrt{a^2+2abx^3+b^2x^6}}$$

Rubi [A] time = 0.03, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {1355, 266, 36, 29, 31}

$$\frac{\log(x)\left(a+bx^3\right)}{a\sqrt{a^2+2abx^3+b^2x^6}}-\frac{\left(a+bx^3\right)\log\left(a+bx^3\right)}{3a\sqrt{a^2+2abx^3+b^2x^6}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]),x]
```

```
[Out] ((a + b*x^3)*Log[x])/(a*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) - ((a + b*x^3)*Log
[a + b*x^3])/(3*a*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\int \frac{1}{x\sqrt{a^2+2abx^3+b^2x^6}}\,dx = \frac{(ab+b^2x^3)\int \frac{1}{x(ab+b^2x^3)}\,dx}{\sqrt{a^2+2abx^3+b^2x^6}}$$
$$= \frac{(ab+b^2x^3)\,\text{Subst}\left(\int \frac{1}{x(ab+b^2x)}\,dx, x, x^3\right)}{3\sqrt{a^2+2abx^3+b^2x^6}}$$
$$= \frac{(ab+b^2x^3)\,\text{Subst}\left(\int \frac{1}{x}\,dx, x, x^3\right)}{3ab\sqrt{a^2+2abx^3+b^2x^6}} - \frac{(b(ab+b^2x^3))\,\text{Subst}\left(\int \frac{1}{ab+b^2x}\,dx, x, x^3\right)}{3a\sqrt{a^2+2abx^3+b^2x^6}}$$
$$= \frac{(a+bx^3)\log(x)}{a\sqrt{a^2+2abx^3+b^2x^6}} - \frac{(a+bx^3)\log(a+bx^3)}{3a\sqrt{a^2+2abx^3+b^2x^6}}$$

Mathematica [A] time = 0.01, size = 42, normalized size = 0.52

$$\frac{(a+bx^3)(3\log(x)-\log(a+bx^3))}{3a\sqrt{(a+bx^3)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]),x]
[Out] ((a + b*x^3)*(3*Log[x] - Log[a + b*x^3]))/(3*a*Sqrt[(a + b*x^3)^2])
```

IntegrateAlgebraic [A] time = 0.20, size = 94, normalized size = 1.18

$$\frac{\log\left(-a\sqrt{a^2+2abx^3+b^2x^6}+a^2+a\sqrt{b^2}\,x^3\right)}{3a} - \frac{\log\left(\sqrt{a^2+2abx^3+b^2x^6}+a-\sqrt{b^2}\,x^3\right)}{3a}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]),x]
[Out] -1/3*Log[a - Sqrt[b^2]*x^3 + Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]]/a + Log[a^2 + 
 a*Sqrt[b^2]*x^3 - a*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]]/(3*a)
```

fricas [A] time = 1.12, size = 18, normalized size = 0.22

$$-\frac{\log(bx^3+a)-3\,\log(x)}{3\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/((b*x^3+a)^2)^(1/2),x, algorithm="fricas")
[Out] -1/3*(log(b*x^3 + a) - 3*log(x))/a
```

giac [A] time = 0.34, size = 32, normalized size = 0.40

$$-\frac{1}{3}\left(\frac{\log(|bx^3+a|)}{a} - \frac{3\,\log(|x|)}{a}\right)\text{sgn}(bx^3+a)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/((b*x^3+a)^2)^(1/2),x, algorithm="giac")
[Out] -1/3*(log(abs(b*x^3 + a))/a - 3*log(abs(x))/a)*sgn(b*x^3 + a)
```

maple [A] time = 0.01, size = 39, normalized size = 0.49

$$\frac{\left(b\,x^{3}+a\right)\left(3\ln(x)-\ln\left(b\,x^{3}+a\right)\right)}{3\sqrt{\left(b\,x^{3}+a\right)^{2}}\;a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/((b*x^3+a)^2)^(1/2),x)
```

```
[Out] 1/3*(b*x^3+a)*(3*ln(x)-ln(b*x^3+a))/((b*x^3+a)^2)^(1/2)/a
```

maxima [A] time = 1.15, size = 43, normalized size = 0.54

$$-\frac{(-1)^{2\,abx^{3}+2\,a^{2}}\log\left(\frac{2\,abx}{|x|}+\frac{2\,a^{2}}{x^{2}|x|}\right)}{3\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/((b*x^3+a)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/3*(-1)^(2*a*b*x^3 + 2*a^2)*log(2*a*b*x/abs(x) + 2*a^2/(x^2*abs(x)))/a
```

mupad [B] time = 1.39, size = 48, normalized size = 0.60

$$-\frac{\ln\left(a\,b+\frac{a^{2}}{x^{3}}+\frac{\sqrt{a^{2}}\,\sqrt{a^{2}+2\,a\,b\,x^{3}+b^{2}\,x^{6}}}{x^{3}}\right)}{3\,\sqrt{a^{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*((a + b*x^3)^2)^(1/2)),x)
```

```
[Out] -log(a*b + a^2/x^3 + ((a^2)^(1/2)*(a^2 + b^2*x^6 + 2*a*b*x^3)^(1/2))/x^3)/(
3*(a^2)^(1/2))
```

sympy [A] time = 0.28, size = 15, normalized size = 0.19

$$\frac{\log\left(x\right)}{a}-\frac{\log\left(\frac{a}{b}+x^{3}\right)}{3a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/((b*x**3+a)**2)**(1/2),x)
```

```
[Out] log(x)/a - log(a/b + x**3)/(3*a)
```

3.95 $\int \frac{1}{x^2\sqrt{a^2+2abx^3+b^2x^6}}\,dx$

Optimal. Leaf size=238

$$-\frac{a+bx^3}{ax\sqrt{a^2+2abx^3+b^2x^6}}+\frac{\sqrt[3]{b}\left(a+bx^3\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{4/3}\sqrt{a^2+2abx^3+b^2x^6}}+\frac{\sqrt[3]{b}\left(a+bx^3\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{4/3}\sqrt{a^2+2abx^3+b^2x^6}}-\frac{\sqrt[3]{b}\left(a+bx^3\right)\log}{6a^{4/3}\sqrt{a^2}}$$

Rubi [A] time = 0.11, antiderivative size = 238, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {1355, 325, 292, 31, 634, 617, 204, 628}

$$-\frac{a+bx^3}{ax\sqrt{a^2+2abx^3+b^2x^6}}+\frac{\sqrt[3]{b}\left(a+bx^3\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{4/3}\sqrt{a^2+2abx^3+b^2x^6}}-\frac{\sqrt[3]{b}\left(a+bx^3\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{4/3}\sqrt{a^2+2abx^3+b^2x^6}}+\frac{\sqrt[3]{b}\left(a+bx^3\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{4/3}\sqrt{a^2+2abx^3+b^2x^6}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]),x]
```

```
[Out] -((a + b*x^3)/(a*x*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])) + (b^(1/3)*(a + b*x^3)
*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*a^(4/3)*Sqrt[a
^2 + 2*a*b*x^3 + b^2*x^6]) + (b^(1/3)*(a + b*x^3)*Log[a^(1/3) + b^(1/3)*x])
/(3*a^(4/3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) - (b^(1/3)*(a + b*x^3)*Log[a^(
2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(4/3)*Sqrt[a^2 + 2*a*b*x^3 +
b^2*x^6])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x^2\sqrt{a^2+2abx^3+b^2x^6}}\,dx &= \frac{\left(ab+b^2x^3\right)\int \frac{1}{x^2\left(ab+b^2x^3\right)}\,dx}{\sqrt{a^2+2abx^3+b^2x^6}}\\
&= -\frac{a+bx^3}{ax\sqrt{a^2+2abx^3+b^2x^6}}-\frac{\left(b\left(ab+b^2x^3\right)\right)\int \frac{x}{ab+b^2x^3}\,dx}{a\sqrt{a^2+2abx^3+b^2x^6}}\\
&= -\frac{a+bx^3}{ax\sqrt{a^2+2abx^3+b^2x^6}}+\frac{\left(ab+b^2x^3\right)\int \frac{1}{\sqrt[3]{a}\sqrt[3]{b}+b^{2/3}x}\,dx}{3a^{4/3}\sqrt{a^2+2abx^3+b^2x^6}}-\frac{\left(ab+b^2x^3\right)\int \frac{\sqrt[3]{a}\sqrt[3]{\ldots}}{a^{2/3}b^{2/3}-\ldots}}{3a^{4/3}\sqrt{a^2+2abx^{3}\ldots}}\\
&= -\frac{a+bx^3}{ax\sqrt{a^2+2abx^3+b^2x^6}}+\frac{\sqrt[3]{b}\left(a+bx^3\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{4/3}\sqrt{a^2+2abx^3+b^2x^6}}-\frac{\left(ab+b^2x^3\right)\int \frac{-\ldots}{a^{2/3}b^{2/\ldots}}}{6a^{4/3}b^{2/3}\sqrt{a^2+2\ldots}}\\
&= -\frac{a+bx^3}{ax\sqrt{a^2+2abx^3+b^2x^6}}+\frac{\sqrt[3]{b}\left(a+bx^3\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{4/3}\sqrt{a^2+2abx^3+b^2x^6}}-\frac{\sqrt[3]{b}\left(a+bx^3\right)\log\left(\ldots\right.}{6a^{4/3}\sqrt{a^2+\ldots}}\\
&= -\frac{a+bx^3}{ax\sqrt{a^2+2abx^3+b^2x^6}}+\frac{\sqrt[3]{b}\left(a+bx^3\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{4/3}\sqrt{a^2+2abx^3+b^2x^6}}+\frac{\sqrt[3]{b}\left(a+bx^3\right)\log\left(\ldots\right.}{3a^{4/3}\sqrt{a^2+2abx\ldots}}
\end{aligned}
$$

Mathematica [A] time = 0.03, size = 133, normalized size = 0.56

$$
-\frac{\left(a+bx^3\right)\left(\sqrt[3]{b}x\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)-2\sqrt[3]{b}x\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)-2\sqrt{3}\sqrt[3]{b}x\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)+6\sqrt[3]{a}\right)}{6a^{4/3}x\sqrt{\left(a+bx^3\right)^2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]),x]
```

```
[Out] -1/6*((a + b*x^3)*(6*a^(1/3) - 2*Sqrt[3]*b^(1/3)*x*ArcTan[(1 - (2*b^(1/3)*x
)/a^(1/3))/Sqrt[3]] - 2*b^(1/3)*x*Log[a^(1/3) + b^(1/3)*x] + b^(1/3)*x*Log[
```

```
a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]))/(a^(4/3)*x*Sqrt[(a + b*x^3)^2]
)
```

IntegrateAlgebraic [A] time = 15.05, size = 142, normalized size = 0.60

$$\frac{\left(a+bx^3\right)\left(-\frac{\sqrt[3]{b}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{4/3}}+\frac{\sqrt[3]{b}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{4/3}}+\frac{\sqrt[3]{b}\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{4/3}}-\frac{1}{ax}\right)}{\sqrt{\left(a+bx^3\right)^2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^2*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]),x]
```

```
[Out] ((a + b*x^3)*(-(1/(a*x)) + (b^(1/3)*ArcTan[1/Sqrt[3] - (2*b^(1/3)*x)/(Sqrt[
3]*a^(1/3))])/(Sqrt[3]*a^(4/3)) + (b^(1/3)*Log[a^(1/3) + b^(1/3)*x])/(3*a^(
4/3)) - (b^(1/3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(4/3)
)))/Sqrt[(a + b*x^3)^2]
```

fricas [A] time = 1.08, size = 103, normalized size = 0.43

$$-\frac{2\sqrt{3}x\left(\frac{b}{a}\right)^{\frac{1}{3}}\arctan\left(\frac{2}{3}\sqrt{3}x\left(\frac{b}{a}\right)^{\frac{1}{3}}-\frac{1}{3}\sqrt{3}\right)+x\left(\frac{b}{a}\right)^{\frac{1}{3}}\log\left(bx^2-ax\left(\frac{b}{a}\right)^{\frac{2}{3}}+a\left(\frac{b}{a}\right)^{\frac{1}{3}}\right)-2x\left(\frac{b}{a}\right)^{\frac{1}{3}}\log\left(bx+a\left(\frac{b}{a}\right)^{\frac{2}{3}}\right)+6}{6ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/((b*x^3+a)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/6*(2*sqrt(3)*x*(b/a)^(1/3)*arctan(2/3*sqrt(3)*x*(b/a)^(1/3) - 1/3*sqrt(3
)) + x*(b/a)^(1/3)*log(b*x^2 - a*x*(b/a)^(2/3) + a*(b/a)^(1/3)) - 2*x*(b/a)
^(1/3)*log(b*x + a*(b/a)^(2/3)) + 6)/(a*x)
```

giac [A] time = 0.37, size = 131, normalized size = 0.55

$$\frac{1}{6}\left(\frac{2b\left(-\frac{a}{b}\right)^{\frac{2}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{a^2}+\frac{2\sqrt{3}\left(-ab^2\right)^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{a^2b}-\frac{\left(-ab^2\right)^{\frac{2}{3}}\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{a^2b}-\frac{6}{ax}\right)\mathrm{sgn}\left(bx^3+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/((b*x^3+a)^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/6*(2*b*(-a/b)^(2/3)*log(abs(x - (-a/b)^(1/3)))/a^2 + 2*sqrt(3)*(-a*b^2)^(
2/3)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(a^2*b) - (-a*b^
2)^(2/3)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a^2*b) - 6/(a*x))*sgn(b*
x^3 + a)
```

maple [A] time = 0.01, size = 111, normalized size = 0.47

$$-\frac{\left(bx^3+a\right)\left(-2\sqrt{3}\,x\arctan\left(\frac{\sqrt{3}\left(-2x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)-2x\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)+x\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)+6\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{6\sqrt{\left(bx^3+a\right)^2}\left(\frac{a}{b}\right)^{\frac{1}{3}}ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/((b*x^3+a)^2)^(1/2),x)
```

```
[Out] -1/6*(b*x^3+a)*(-2*3^(1/2)*arctan(1/3*3^(1/2)*(-2*x+(a/b)^(1/3))/(a/b)^(1/3
))*x+ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))*x-2*ln(x+(a/b)^(1/3))*x+6*(a/b)^(1/3
))/((b*x^3+a)^2)^(1/2)/a/x/(a/b)^(1/3)
```

maxima [A] time = 1.74, size = 106, normalized size = 0.45

$$-\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3a\left(\frac{a}{b}\right)^{\frac{1}{3}}}-\frac{\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6a\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3a\left(\frac{a}{b}\right)^{\frac{1}{3}}}-\frac{1}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/((b*x^3+a)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a*(a/b)^(
1/3)) - 1/6*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a*(a/b)^(1/3)) + 1/3*lo
g(x + (a/b)^(1/3))/(a*(a/b)^(1/3)) - 1/(a*x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^2\sqrt{\left(b\,x^3+a\right)^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*((a + b*x^3)^2)^(1/2)),x)
```

```
[Out] int(1/(x^2*((a + b*x^3)^2)^(1/2)), x)
```

sympy [A] time = 0.23, size = 29, normalized size = 0.12

$$\operatorname{RootSum}\left(27t^3a^4-b,\left(t\mapsto t\log\left(\frac{9t^2a^3}{b}+x\right)\right)\right)-\frac{1}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/((b*x**3+a)**2)**(1/2),x)
```

```
[Out] RootSum(27*_t**3*a**4 - b, Lambda(_t, _t*log(9*_t**2*a**3/b + x))) - 1/(a*x
)
```

3.96 $\int \frac{1}{x^3\sqrt{a^2+2abx^3+b^2x^6}}\,dx$

Optimal. Leaf size=243

$$\frac{-a-bx^3}{2ax^2\sqrt{a^2+2abx^3+b^2x^6}}-\frac{b^{2/3}\left(a+bx^3\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3a^{5/3}\sqrt{a^2+2abx^3+b^2x^6}}+\frac{b^{2/3}\left(a+bx^3\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{5/3}\sqrt{a^2+2abx^3+b^2x^6}}+\frac{b^{2/3}\left(a+bx^3\right)\log}{6a^{5/3}\sqrt{a}}$$

Rubi [A] time = 0.11, antiderivative size = 240, normalized size of antiderivative = 0.99, number of steps used = 8, number of rules used = 8, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {1355, 325, 200, 31, 634, 617, 204, 628}

$$-\frac{a+bx^3}{2ax^2\sqrt{a^2+2abx^3+b^2x^6}}-\frac{b^{2/3}\left(a+bx^3\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3a^{5/3}\sqrt{a^2+2abx^3+b^2x^6}}+\frac{b^{2/3}\left(a+bx^3\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{6a^{5/3}\sqrt{a^2+2abx^3+b^2x^6}}+\frac{b^{2/3}\left(a+bx^3\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{5/3}\sqrt{a^2+2abx^3+b^2x^6}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]),x]
```

```
[Out] -(a + b*x^3)/(2*a*x^2*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) + (b^(2/3)*(a + b*x^
3)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*a^(5/3)*Sqrt
[a^2 + 2*a*b*x^3 + b^2*x^6]) - (b^(2/3)*(a + b*x^3)*Log[a^(1/3) + b^(1/3)*x
])/(3*a^(5/3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) + (b^(2/3)*(a + b*x^3)*Log[a
^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(5/3)*Sqrt[a^2 + 2*a*b*x^3
+ b^2*x^6])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\int \frac{1}{x^3\sqrt{a^2+2abx^3+b^2x^6}}\,dx = \frac{\left(ab+b^2x^3\right)\int \frac{1}{x^3\left(ab+b^2x^3\right)}\,dx}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$= -\frac{a+bx^3}{2ax^2\sqrt{a^2+2abx^3+b^2x^6}} - \frac{\left(b\left(ab+b^2x^3\right)\right)\int \frac{1}{ab+b^2x^3}\,dx}{a\sqrt{a^2+2abx^3+b^2x^6}}$$

$$= -\frac{a+bx^3}{2ax^2\sqrt{a^2+2abx^3+b^2x^6}} - \frac{\left(\sqrt[3]{b}\left(ab+b^2x^3\right)\right)\int \frac{1}{\sqrt[3]{a}\sqrt[3]{b}+b^{2/3}x}\,dx}{3a^{5/3}\sqrt{a^2+2abx^3+b^2x^6}} - \frac{\left(\sqrt[3]{b}\left(ab+b^{2}x\right.\right.}{3a^{5/3}\sqrt{}}$$

$$= -\frac{a+bx^3}{2ax^2\sqrt{a^2+2abx^3+b^2x^6}} - \frac{b^{2/3}\left(a+bx^3\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{5/3}\sqrt{a^2+2abx^3+b^2x^6}} + \frac{\left(ab+b^2x^3\right)\int \frac{}{a^{2/3}}}{6a^{5/3}\sqrt[3]{b}\sqrt{a^2+}}$$

$$= -\frac{a+bx^3}{2ax^2\sqrt{a^2+2abx^3+b^2x^6}} - \frac{b^{2/3}\left(a+bx^3\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{5/3}\sqrt{a^2+2abx^3+b^2x^6}} + \frac{b^{2/3}\left(a+bx^3\right)\log}{6a^{5/3}\sqrt{a}}$$

$$= -\frac{a+bx^3}{2ax^2\sqrt{a^2+2abx^3+b^2x^6}} + \frac{b^{2/3}\left(a+bx^3\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{5/3}\sqrt{a^2+2abx^3+b^2x^6}} - \frac{b^{2/3}\left(a+bx^3\right)\log}{3a^{5/3}\sqrt{a^2+2a}}$$

Mathematica [A] time = 0.03, size = 140, normalized size = 0.58

$$-\frac{\left(a+bx^3\right)\left(-b^{2/3}x^2\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)+3a^{2/3}+2b^{2/3}x^2\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)-2\sqrt{3}b^{2/3}x^2\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)\right)}{6a^{5/3}x^2\sqrt{\left(a+bx^3\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]),x]
```

```
[Out] -1/6*((a + b*x^3)*(3*a^(2/3) - 2*Sqrt[3]*b^(2/3)*x^2*ArcTan[(1 - (2*b^(1/3)
*x)/a^(1/3))/Sqrt[3]] + 2*b^(2/3)*x^2*Log[a^(1/3) + b^(1/3)*x] - b^(2/3)*x^
```

```
2*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]))/(a^(5/3)*x^2*Sqrt[(a + b
*x^3)^2])
```

IntegrateAlgebraic [A] time = 17.85, size = 144, normalized size = 0.59

$$\frac{\left(a+bx^3\right)\left(\frac{b^{2/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{5/3}}-\frac{b^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{5/3}}+\frac{b^{2/3}\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{5/3}}-\frac{1}{2ax^2}\right)}{\sqrt{\left(a+bx^3\right)^2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]),x]
```

```
[Out] ((a + b*x^3)*(-1/2*1/(a*x^2) + (b^(2/3)*ArcTan[1/Sqrt[3] - (2*b^(1/3)*x)/(S
qrt[3]*a^(1/3))])/(Sqrt[3]*a^(5/3)) - (b^(2/3)*Log[a^(1/3) + b^(1/3)*x])/(3
*a^(5/3)) + (b^(2/3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(
5/3))))/Sqrt[(a + b*x^3)^2]
```

fricas [A] time = 1.30, size = 143, normalized size = 0.59

$$\frac{2\sqrt{3}x^2\left(-\frac{b^2}{a^2}\right)^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}ax\left(-\frac{b^2}{a^2}\right)^{\frac{2}{3}}-\sqrt{3}b}{3b}\right)-x^2\left(-\frac{b^2}{a^2}\right)^{\frac{1}{3}}\log\left(b^2x^2+abx\left(-\frac{b^2}{a^2}\right)^{\frac{1}{3}}+a^2\left(-\frac{b^2}{a^2}\right)^{\frac{2}{3}}\right)+2x^2\left(-\frac{b^2}{a^2}\right)^{\frac{1}{3}}\log\left(bx-a\left(-\frac{b^2}{a^2}\right)^{\frac{1}{3}}\right)-3}{6ax^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/((b*x^3+a)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/6*(2*sqrt(3)*x^2*(-b^2/a^2)^(1/3)*arctan(1/3*(2*sqrt(3)*a*x*(-b^2/a^2)^(2
/3) - sqrt(3)*b)/b) - x^2*(-b^2/a^2)^(1/3)*log(b^2*x^2 + a*b*x*(-b^2/a^2)^(
1/3) + a^2*(-b^2/a^2)^(2/3)) + 2*x^2*(-b^2/a^2)^(1/3)*log(b*x - a*(-b^2/a^2
)^(1/3)) - 3)/(a*x^2)
```

giac [A] time = 0.33, size = 125, normalized size = 0.51

$$\frac{1}{6}\left(\frac{2b\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{a^2}-\frac{2\sqrt{3}\left(-ab^2\right)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{a^2}-\frac{\left(-ab^2\right)^{\frac{1}{3}}\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{a^2}-\frac{3}{ax^2}\right)\operatorname{sgn}\left(bx^3+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/((b*x^3+a)^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/6*(2*b*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/a^2 - 2*sqrt(3)*(-a*b^2)^(
1/3)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/a^2 - (-a*b^2)^(
1/3)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/a^2 - 3/(a*x^2))*sgn(b*x^3 +
a)
```

maple [A] time = 0.01, size = 117, normalized size = 0.48

$$\frac{\left(bx^3+a\right)\left(2\sqrt{3}\,x^2\arctan\left(\frac{\sqrt{3}\left(-2x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)-2x^2\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)+x^2\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)-3\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\sqrt{\left(bx^3+a\right)^2}\,\left(\frac{a}{b}\right)^{\frac{2}{3}}ax^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/((b*x^3+a)^2)^(1/2),x)
```

```
[Out] 1/6*(b*x^3+a)*(2*3^(1/2)*arctan(1/3*3^(1/2)*(-2*x+(a/b)^(1/3))/(a/b)^(1/3))
*x^2+ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))*x^2-2*ln(x+(a/b)^(1/3))*x^2-3*(a/b)^
(2/3))/((b*x^3+a)^2)^(1/2)/x^2/a/(a/b)^(2/3)
```

maxima [A] time = 2.21, size = 106, normalized size = 0.44

$$-\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3a\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\log\left(x^{2}-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6a\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3a\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{1}{2ax^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/((b*x^3+a)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a*(a/b)^(
2/3)) + 1/6*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a*(a/b)^(2/3)) - 1/3*lo
g(x + (a/b)^(1/3))/(a*(a/b)^(2/3)) - 1/2/(a*x^2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^{3}\sqrt{\left(b\,x^{3}+a\right)^{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*((a + b*x^3)^2)^(1/2)),x)
```

```
[Out] int(1/(x^3*((a + b*x^3)^2)^(1/2)), x)
```

sympy [A] time = 0.26, size = 32, normalized size = 0.13

$$\operatorname{RootSum}\left(27t^{3}a^{5}+b^{2},\left(t\mapsto t\log\left(-\frac{3ta^{2}}{b}+x\right)\right)\right)-\frac{1}{2ax^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/((b*x**3+a)**2)**(1/2),x)
```

```
[Out] RootSum(27*_t**3*a**5 + b**2, Lambda(_t, _t*log(-3*_t*a**2/b + x))) - 1/(2*
a*x**2)
```

3.97 $\int \frac{1}{x^4 \sqrt{a^2+2abx^3+b^2x^6}} \, dx$

Optimal. Leaf size=125

$$\frac{-a-bx^3}{3ax^3\sqrt{a^2+2abx^3+b^2x^6}}-\frac{b\log(x)\left(a+bx^3\right)}{a^2\sqrt{a^2+2abx^3+b^2x^6}}+\frac{b\left(a+bx^3\right)\log\left(a+bx^3\right)}{3a^2\sqrt{a^2+2abx^3+b^2x^6}}$$

Rubi [A] time = 0.05, antiderivative size = 122, normalized size of antiderivative = 0.98, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1355, 266, 44}

$$-\frac{a+bx^3}{3ax^3\sqrt{a^2+2abx^3+b^2x^6}}-\frac{b\log(x)\left(a+bx^3\right)}{a^2\sqrt{a^2+2abx^3+b^2x^6}}+\frac{b\left(a+bx^3\right)\log\left(a+bx^3\right)}{3a^2\sqrt{a^2+2abx^3+b^2x^6}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]),x]
```

```
[Out] -(a + b*x^3)/(3*a*x^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) - (b*(a + b*x^3)*Log
[x])/(a^2*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) + (b*(a + b*x^3)*Log[a + b*x^3])
/(3*a^2*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^4\sqrt{a^2+2abx^3+b^2x^6}}\,dx &= \frac{\left(ab+b^2x^3\right)\int \frac{1}{x^4\left(ab+b^2x^3\right)}\,dx}{\sqrt{a^2+2abx^3+b^2x^6}}\\
&= \frac{\left(ab+b^2x^3\right)\text{Subst}\left(\int \frac{1}{x^2\left(ab+b^2x\right)}\,dx,x,x^3\right)}{3\sqrt{a^2+2abx^3+b^2x^6}}\\
&= \frac{\left(ab+b^2x^3\right)\text{Subst}\left(\int \left(\frac{1}{abx^2}-\frac{1}{a^2x}+\frac{b}{a^2(a+bx)}\right)dx,x,x^3\right)}{3\sqrt{a^2+2abx^3+b^2x^6}}\\
&= -\frac{a+bx^3}{3ax^3\sqrt{a^2+2abx^3+b^2x^6}}-\frac{b\left(a+bx^3\right)\log(x)}{a^2\sqrt{a^2+2abx^3+b^2x^6}}+\frac{b\left(a+bx^3\right)\log\left(a+bx^3\right)}{3a^2\sqrt{a^2+2abx^3+b^2x^6}}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 54, normalized size = 0.43

$$-\frac{\left(a+bx^3\right)\left(-bx^3\log\left(a+bx^3\right)+a+3bx^3\log(x)\right)}{3a^2x^3\sqrt{\left(a+bx^3\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]),x]
```

```
[Out] -1/3*((a + b*x^3)*(a + 3*b*x^3*Log[x] - b*x^3*Log[a + b*x^3]))/(a^2*x^3*Sqr
t[(a + b*x^3)^2])
```

IntegrateAlgebraic [B] time = 0.68, size = 383, normalized size = 3.06

$$\frac{\left(\sqrt{a^2+2abx^3+b^2x^6}-\sqrt{b^2}\,x^3\right)^2\left(\frac{b\log\left(\sqrt{a^2+2abx^3+b^2x^6}+a-\sqrt{b^2}\,x^3\right)}{3a^2}-\frac{b\log\left(a^3+a^2\sqrt{b^2}\,x^3-a^2\sqrt{a^2+2abx^3+b^2x^6}\right)}{3a^2}\right)}{-2\sqrt{b^2}\,x^3\sqrt{a^2+2abx^3+b^2x^6}+a^2+2abx^3+2b^2x^6}+\frac{-2\sqrt{a^2+2abx^3+b^2x^6}\left(-a^2b-4ab^2x^3-4b^3x^6\right)-2\sqrt{b^2}\left(a^3+5a^2bx^3+8ab^2x^6+4b^3x^9\right)}{3a\sqrt{b^2}\sqrt{a^2+2abx^3+b^2x^6}\left(2a^2x^3+8abx^6+8b^2x^9\right)+3a\left(-2a^3bx^3-10a^2b^2x^6-16ab^3x^9-8b^4x^{12}\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^4*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]),x]
```

```
[Out] (-2*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]*(-(a^2*b) - 4*a*b^2*x^3 - 4*b^3*x^6) -
2*Sqrt[b^2]*(a^3 + 5*a^2*b*x^3 + 8*a*b^2*x^6 + 4*b^3*x^9))/(3*a*Sqrt[b^2]*S
qrt[a^2 + 2*a*b*x^3 + b^2*x^6]*(2*a^2*x^3 + 8*a*b*x^6 + 8*b^2*x^9) + 3*a*(-
2*a^3*b*x^3 - 10*a^2*b^2*x^6 - 16*a*b^3*x^9 - 8*b^4*x^12)) + ((-(Sqrt[b^2]*
x^3) + Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])^2*((b*Log[a - Sqrt[b^2]*x^3 + Sqrt[
a^2 + 2*a*b*x^3 + b^2*x^6]])/(3*a^2) - (b*Log[a^3 + a^2*Sqrt[b^2]*x^3 - a^2
*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]])/(3*a^2)))/(a^2 + 2*a*b*x^3 + 2*b^2*x^6 -
 2*Sqrt[b^2]*x^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])
```

fricas [A] time = 1.12, size = 33, normalized size = 0.26

$$\frac{bx^3\log\left(bx^3+a\right)-3\,bx^3\log(x)-a}{3\,a^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/((b*x^3+a)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/3*(b*x^3*log(b*x^3 + a) - 3*b*x^3*log(x) - a)/(a^2*x^3)
```

giac [A] time = 0.37, size = 50, normalized size = 0.40

$$\frac{1}{3}\left(\frac{b\log\left(\left|bx^3+a\right|\right)}{a^2}-\frac{3\,b\log\left(|x|\right)}{a^2}+\frac{bx^3-a}{a^2x^3}\right)\mathrm{sgn}\left(bx^3+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/((b*x^3+a)^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/3*(b*log(abs(b*x^3 + a))/a^2 - 3*b*log(abs(x))/a^2 + (b*x^3 - a)/(a^2*x^3
))*sgn(b*x^3 + a)
```

maple [A] time = 0.01, size = 51, normalized size = 0.41

$$-\frac{\left(b\,x^3+a\right)\left(3b\,x^3\ln(x)-b\,x^3\ln\left(b\,x^3+a\right)+a\right)}{3\sqrt{\left(b\,x^3+a\right)^2}\;a^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/((b*x^3+a)^2)^(1/2),x)
```

```
[Out] -1/3*(b*x^3+a)*(3*b*x^3*ln(x)-b*ln(b*x^3+a)*x^3+a)/((b*x^3+a)^2)^(1/2)/a^2/
x^3
```

maxima [A] time = 1.20, size = 73, normalized size = 0.58

$$\frac{(-1)^{2\,abx^3+2\,a^2}\,b\log\left(\frac{2\,abx}{|x|}+\frac{2\,a^2}{x^2|x|}\right)}{3\,a^2}-\frac{\sqrt{b^2x^6+2\,abx^3+a^2}}{3\,a^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/((b*x^3+a)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/3*(-1)^(2*a*b*x^3 + 2*a^2)*b*log(2*a*b*x/abs(x) + 2*a^2/(x^2*abs(x)))/a^2
 - 1/3*sqrt(b^2*x^6 + 2*a*b*x^3 + a^2)/(a^2*x^3)
```

mupad [B] time = 1.40, size = 75, normalized size = 0.60

$$\frac{a\,b\,\mathrm{atanh}\left(\frac{a^2+b\,a\,x^3}{\sqrt{a^2}\,\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}\right)}{3\,\left(a^2\right)^{3/2}}-\frac{\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{3\,a^2\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*((a + b*x^3)^2)^(1/2)),x)
```

```
[Out] (a*b*atanh((a^2 + a*b*x^3)/((a^2)^(1/2)*(a^2 + b^2*x^6 + 2*a*b*x^3)^(1/2)))
)/(3*(a^2)^(3/2)) - (a^2 + b^2*x^6 + 2*a*b*x^3)^(1/2)/(3*a^2*x^3)
```

sympy [A] time = 0.36, size = 31, normalized size = 0.25

$$-\frac{1}{3ax^3}-\frac{b\log\left(x\right)}{a^2}+\frac{b\log\left(\frac{a}{b}+x^3\right)}{3a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/((b*x**3+a)**2)**(1/2),x)
```

```
[Out] -1/(3*a*x**3) - b*log(x)/a**2 + b*log(a/b + x**3)/(3*a**2)
```

3.98 $\int \frac{x^4}{\left(a^2+2abx^3+b^2x^6\right)^{3/2}}\,dx$

Optimal. Leaf size=280

$$\frac{x^2}{9ab\sqrt{a^2+2abx^3+b^2x^6}}-\frac{x^2}{6b\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}}-\frac{\left(a+bx^3\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{27a^{4/3}b^{5/3}\sqrt{a^2+2abx^3+b^2x^6}}-\frac{\left(a+bx^3\right)\tan^{-1}\left(\right.}{9\sqrt{3}\,a^{4/3}b^{5/3}\sqrt{a^2+2}}$$

Rubi [A] time = 0.14, antiderivative size = 280, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.346, Rules used = {1355, 288, 290, 292, 31, 634, 617, 204, 628}

$$\frac{x^2}{9ab\sqrt{a^2+2abx^3+b^2x^6}}-\frac{x^2}{6b\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}}-\frac{\left(a+bx^3\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{27a^{4/3}b^{5/3}\sqrt{a^2+2abx^3+b^2x^6}}+\frac{\left(a+bx^3\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{54a^{4/3}b^{5/3}\sqrt{a^2+2abx^3+b^2x^6}}-\frac{\left(a+bx^3\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}\,a^{4/3}b^{5/3}\sqrt{a^2+2abx^3+b^2x^6}}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2),x]
```

```
[Out] x^2/(9*a*b*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) - x^2/(6*b*(a + b*x^3)*Sqrt[a^2
 + 2*a*b*x^3 + b^2*x^6]) - ((a + b*x^3)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqr
t[3]*a^(1/3))])/(9*Sqrt[3]*a^(4/3)*b^(5/3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])
 - ((a + b*x^3)*Log[a^(1/3) + b^(1/3)*x])/(27*a^(4/3)*b^(5/3)*Sqrt[a^2 + 2*
a*b*x^3 + b^2*x^6]) + ((a + b*x^3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3
)*x^2])/(54*a^(4/3)*b^(5/3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
```

```
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\int \frac{x^4}{\left(a^2+2abx^3+b^2x^6\right)^{3/2}}\,dx = \frac{\left(b^2\left(ab+b^2x^3\right)\right)\int \frac{x^4}{\left(ab+b^2x^3\right)^3}\,dx}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$= -\frac{x^2}{6b\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}} + \frac{\left(ab+b^2x^3\right)\int \frac{x}{\left(ab+b^2x^3\right)^2}\,dx}{3\sqrt{a^2+2abx^3+b^2x^6}}$$

$$= \frac{x^2}{9ab\sqrt{a^2+2abx^3+b^2x^6}} - \frac{x^2}{6b\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}} + \frac{\left(ab+b^2x^3\right)\int \frac{\cdots}{ab+\cdots}}{9ab\sqrt{a^2+2abx^3}\cdots}$$

$$= \frac{x^2}{9ab\sqrt{a^2+2abx^3+b^2x^6}} - \frac{x^2}{6b\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}} - \frac{\left(ab+b^2x^3\right)\int \frac{\cdots}{\sqrt[3]{a}\cdots}}{27a^{4/3}b^2\sqrt{a^2+2a\cdots}}$$

$$= \frac{x^2}{9ab\sqrt{a^2+2abx^3+b^2x^6}} - \frac{x^2}{6b\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}} - \frac{\left(a+bx^3\right)\log\left(\sqrt[3]{\cdots}\right.}{27a^{4/3}b^{5/3}\sqrt{a^2+2\cdots}}$$

$$= \frac{x^2}{9ab\sqrt{a^2+2abx^3+b^2x^6}} - \frac{x^2}{6b\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}} - \frac{\left(a+bx^3\right)\log\left(\sqrt[3]{\cdots}\right.}{27a^{4/3}b^{5/3}\sqrt{a^2+2\cdots}}$$

$$= \frac{x^2}{9ab\sqrt{a^2+2abx^3+b^2x^6}} - \frac{x^2}{6b\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}} - \frac{\left(a+bx^3\right)\tan^{-1}\cdots}{9\sqrt{3}\,a^{4/3}b^{5/3}\sqrt{a^2+\cdots}}$$

Mathematica [A] time = 0.08, size = 235, normalized size = 0.84

$$\frac{-3a^{4/3}b^{2/3}x^2+2abx^3\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)+b^2x^6\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)+a^2\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)+6\sqrt[3]{a}b^{5/3}x^5-2\left(a+bx^3\right)^2\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)-2\sqrt{3}\left(a+bx^3\right)^2\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{54a^{4/3}b^{5/3}\left(a+bx^3\right)\sqrt{\left(a+bx^3\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2),x]
```

```
[Out] (-3*a^(4/3)*b^(2/3)*x^2 + 6*a^(1/3)*b^(5/3)*x^5 - 2*Sqrt[3]*(a + b*x^3)^2*A
rcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] - 2*(a + b*x^3)^2*Log[a^(1/3) +
b^(1/3)*x] + a^2*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2] + 2*a*b*x^3
*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2] + b^2*x^6*Log[a^(2/3) - a^(
1/3)*b^(1/3)*x + b^(2/3)*x^2])/(54*a^(4/3)*b^(5/3)*(a + b*x^3)*Sqrt[(a + b*
x^3)^2])
```

IntegrateAlgebraic [A] time = 11.03, size = 169, normalized size = 0.60

$$\frac{\left(a+bx^3\right)\left(\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{54a^{4/3}b^{5/3}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{4/3}b^{5/3}}-\frac{\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}\,a^{4/3}b^{5/3}}+\frac{2bx^5-ax^2}{18ab\left(a+bx^3\right)^2}\right)}{\sqrt{\left(a+bx^3\right)^2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^4/(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2),x]
```

```
[Out] ((a + b*x^3)*((-(a*x^2) + 2*b*x^5)/(18*a*b*(a + b*x^3)^2) - ArcTan[1/Sqrt[3
] - (2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))]/(9*Sqrt[3]*a^(4/3)*b^(5/3)) - Log[a^(1
```

```
/3) + b^(1/3)*x]/(27*a^(4/3)*b^(5/3)) + Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b
^(2/3)*x^2]/(54*a^(4/3)*b^(5/3))))/Sqrt[(a + b*x^3)^2]
```

fricas [A] time = 1.38, size = 512, normalized size = 1.83

$$\left[\frac{6\,ab^{3}x^{5} - 3\,a^{2}b^{2}x^{2} + 3\,\sqrt{\frac{1}{3}}\left(ab^{3}x^{6} + 2\,a^{2}b^{2}x^{3} + a^{3}b\right)\sqrt{\frac{\left(-ab^{2}\right)^{\frac{1}{3}}}{a}}\log\left(\frac{2\,b^{2}x^{3} - ab + 3\,\sqrt{\frac{1}{3}}\left(abx + 2\left(-ab^{2}\right)^{\frac{2}{3}}x^{2} + \left(-ab^{2}\right)^{\frac{1}{3}}a\right)\sqrt{\frac{\left(-ab^{2}\right)^{\frac{1}{3}}}{a}} - 3\left(-ab^{2}\right)^{\frac{2}{3}}x}{bx^{3} + a}\right) + \left(b^{2}x^{6} + 2\,abx^{3} + a^{2}\right)\left(-ab^{2}\right)^{\frac{2}{3}}\log\left(b^{2}x^{2} + \left(-ab^{2}\right)^{\frac{1}{3}}bx + \left(-ab^{2}\right)^{\frac{2}{3}}\right) - 2\left(b^{2}x^{6} + 2\,abx^{3} + a^{2}\right)\left(-ab^{2}\right)^{\frac{2}{3}}\log\left(bx - \left(-ab^{2}\right)^{\frac{1}{3}}\right)}{54\left(a^{2}b^{5}x^{6} + 2\,a^{3}b^{4}x^{3} + a^{4}b^{3}\right)}, \frac{6\,ab^{3}x^{5} - 3\,a^{2}b^{2}x^{2} + 6\,\sqrt{\frac{1}{3}}\left(ab^{3}x^{6} + 2\,a^{2}b^{2}x^{3} + a^{3}b\right)\sqrt{-\frac{\left(-ab^{2}\right)^{\frac{1}{3}}}{a}}\arctan\left(\frac{\sqrt{\frac{1}{3}}\left(2\,bx + \left(-ab^{2}\right)^{\frac{1}{3}}\right)\sqrt{-\frac{\left(-ab^{2}\right)^{\frac{1}{3}}}{a}}}{b}\right) + \left(b^{2}x^{6} + 2\,abx^{3} + a^{2}\right)\left(-ab^{2}\right)^{\frac{2}{3}}\log\left(b^{2}x^{2} + \left(-ab^{2}\right)^{\frac{1}{3}}bx + \left(-ab^{2}\right)^{\frac{2}{3}}\right) - 2\left(b^{2}x^{6} + 2\,abx^{3} + a^{2}\right)\left(-ab^{2}\right)^{\frac{2}{3}}\log\left(bx - \left(-ab^{2}\right)^{\frac{1}{3}}\right)}{54\left(a^{2}b^{5}x^{6} + 2\,a^{3}b^{4}x^{3} + a^{4}b^{3}\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/54*(6*a*b^3*x^5 - 3*a^2*b^2*x^2 + 3*sqrt(1/3)*(a*b^3*x^6 + 2*a^2*b^2*x^3
 + a^3*b)*sqrt((-a*b^2)^(1/3)/a)*log((2*b^2*x^3 - a*b + 3*sqrt(1/3)*(a*b*x
+ 2*(-a*b^2)^(2/3)*x^2 + (-a*b^2)^(1/3)*a)*sqrt((-a*b^2)^(1/3)/a) - 3*(-a*b
^2)^(2/3)*x)/(b*x^3 + a)) + (b^2*x^6 + 2*a*b*x^3 + a^2)*(-a*b^2)^(2/3)*log(
b^2*x^2 + (-a*b^2)^(1/3)*b*x + (-a*b^2)^(2/3)) - 2*(b^2*x^6 + 2*a*b*x^3 + a
^2)*(-a*b^2)^(2/3)*log(b*x - (-a*b^2)^(1/3)))/(a^2*b^5*x^6 + 2*a^3*b^4*x^3
+ a^4*b^3), 1/54*(6*a*b^3*x^5 - 3*a^2*b^2*x^2 + 6*sqrt(1/3)*(a*b^3*x^6 + 2*
a^2*b^2*x^3 + a^3*b)*sqrt(-(-a*b^2)^(1/3)/a)*arctan(sqrt(1/3)*(2*b*x + (-a*
b^2)^(1/3))*sqrt(-(-a*b^2)^(1/3)/a)/b) + (b^2*x^6 + 2*a*b*x^3 + a^2)*(-a*b^
2)^(2/3)*log(b^2*x^2 + (-a*b^2)^(1/3)*b*x + (-a*b^2)^(2/3)) - 2*(b^2*x^6 +
2*a*b*x^3 + a^2)*(-a*b^2)^(2/3)*log(b*x - (-a*b^2)^(1/3)))/(a^2*b^5*x^6 + 2
*a^3*b^4*x^3 + a^4*b^3)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_{0}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

maple [A] time = 0.02, size = 299, normalized size = 1.07

$$\frac{\left(-2\sqrt{3}\,b^{2}x^{6}\arctan\left(\frac{\sqrt{3}\left(-2x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right) - 2b^{2}x^{6}\ln\left(x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right) + b^{2}x^{6}\ln\left(x^{2} - \left(\frac{a}{b}\right)^{\frac{1}{3}}x + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right) + 6\left(\frac{a}{b}\right)^{\frac{1}{3}}b^{2}x^{5} - 4\sqrt{3}\,ab\,x^{3}\arctan\left(\frac{\sqrt{3}\left(-2x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right) - 4ab\,x^{3}\ln\left(x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right) + 2ab\,x^{3}\ln\left(x^{2} - \left(\frac{a}{b}\right)^{\frac{1}{3}}x + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right) - 3\left(\frac{a}{b}\right)^{\frac{1}{3}}ab\,x^{2} - 2\sqrt{3}\,a^{2}\arctan\left(\frac{\sqrt{3}\left(-2x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right) - 2a^{2}\ln\left(x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right) + a^{2}\ln\left(x^{2} - \left(\frac{a}{b}\right)^{\frac{1}{3}}x + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)\right)\left(b\,x^{3} + a\right)}{54\left(\frac{a}{b}\right)^{\frac{1}{3}}\left(\left(b\,x^{3} + a\right)^{2}\right)^{\frac{3}{2}}a\,b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x)
```

```
[Out] 1/54*(-2*3^(1/2)*arctan(1/3*3^(1/2)*(-2*x+(a/b)^(1/3))/(a/b)^(1/3))*x^6*b^2
-2*ln(x+(a/b)^(1/3))*x^6*b^2+ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))*x^6*b^2+6*(a
/b)^(1/3)*x^5*b^2-4*3^(1/2)*arctan(1/3*3^(1/2)*(-2*x+(a/b)^(1/3))/(a/b)^(1/
3))*x^3*a*b-4*ln(x+(a/b)^(1/3))*x^3*a*b+2*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))
*x^3*a*b-3*(a/b)^(1/3)*x^2*a*b-2*3^(1/2)*arctan(1/3*3^(1/2)*(-2*x+(a/b)^(1/
3))/(a/b)^(1/3))*a^2-2*ln(x+(a/b)^(1/3))*a^2+ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/
3))*a^2)*(b*x^3+a)/(a/b)^(1/3)/b^2/a/((b*x^3+a)^2)^(3/2)
```

maxima [A] time = 1.81, size = 149, normalized size = 0.53

$$\frac{2\,bx^{5} - ax^{2}}{18\left(ab^{3}x^{6} + 2\,a^{2}b^{2}x^{3} + a^{3}b\right)} + \frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2\,x - \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,ab^{2}\left(\frac{a}{b}\right)^{\frac{1}{3}}} + \frac{\log\left(x^{2} - x\left(\frac{a}{b}\right)^{\frac{1}{3}} + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\,ab^{2}\left(\frac{a}{b}\right)^{\frac{1}{3}}} - \frac{\log\left(x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\,ab^{2}\left(\frac{a}{b}\right)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/18*(2*b*x^5 - a*x^2)/(a*b^3*x^6 + 2*a^2*b^2*x^3 + a^3*b) + 1/27*sqrt(3)*a
rctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a*b^2*(a/b)^(1/3)) + 1/
54*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a*b^2*(a/b)^(1/3)) - 1/27*log(x
+ (a/b)^(1/3))/(a*b^2*(a/b)^(1/3))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^4}{\left(a^2+2\,a\,b\,x^3+b^2\,x^6\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2),x)
```

```
[Out] int(x^4/(a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\left(\left(a+bx^3\right)^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(b**2*x**6+2*a*b*x**3+a**2)**(3/2),x)
```

```
[Out] Integral(x**4/((a + b*x**3)**2)**(3/2), x)
```

3.99 $\int \frac{x^3}{\left(a^2+2abx^3+b^2x^6\right)^{3/2}}\,dx$

Optimal. Leaf size=276

$$\frac{x}{18ab\sqrt{a^2+2abx^3+b^2x^6}}-\frac{x}{6b\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}}+\frac{\left(a+bx^3\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{27a^{5/3}b^{4/3}\sqrt{a^2+2abx^3+b^2x^6}}-\frac{\left(a+bx^3\right)\tan}{9\sqrt{3}\,a^{5/3}b^{4/3}\sqrt{a}}$$

Rubi [A] time = 0.13, antiderivative size = 276, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.346, Rules used = {1355, 288, 199, 200, 31, 634, 617, 204, 628}

$$\frac{x}{18ab\sqrt{a^2+2abx^3+b^2x^6}}-\frac{x}{6b\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}}+\frac{\left(a+bx^3\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{27a^{5/3}b^{4/3}\sqrt{a^2+2abx^3+b^2x^6}}-\frac{\left(a+bx^3\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{54a^{5/3}b^{4/3}\sqrt{a^2+2abx^3+b^2x^6}}-\frac{\left(a+bx^3\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}\,a^{5/3}b^{4/3}\sqrt{a^2+2abx^3+b^2x^6}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2),x]
```

```
[Out] x/(18*a*b*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) - x/(6*b*(a + b*x^3)*Sqrt[a^2 +
2*a*b*x^3 + b^2*x^6]) - ((a + b*x^3)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3
]*a^(1/3))])/(9*Sqrt[3]*a^(5/3)*b^(4/3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) +
((a + b*x^3)*Log[a^(1/3) + b^(1/3)*x])/(27*a^(5/3)*b^(4/3)*Sqrt[a^2 + 2*a*b
*x^3 + b^2*x^6]) - ((a + b*x^3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x
^2])/(54*a^(5/3)*b^(4/3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] && !I
```

```
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\int \frac{x^3}{\left(a^2+2abx^3+b^2x^6\right)^{3/2}}\,dx = \frac{\left(b^2\left(ab+b^2x^3\right)\right)\int \frac{x^3}{\left(ab+b^2x^3\right)^3}\,dx}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$= -\frac{x}{6b\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}} + \frac{\left(ab+b^2x^3\right)\int \frac{1}{\left(ab+b^2x^3\right)^2}\,dx}{6\sqrt{a^2+2abx^3+b^2x^6}}$$

$$= \frac{x}{18ab\sqrt{a^2+2abx^3+b^2x^6}} - \frac{x}{6b\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}} + \frac{\left(ab+b^2x^3\right)\int \cdots}{9ab\sqrt{a^2+2ab\cdots}}$$

$$= \frac{x}{18ab\sqrt{a^2+2abx^3+b^2x^6}} - \frac{x}{6b\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}} + \frac{\left(ab+b^2x^3\right)\cdots}{27a^{5/3}b^{5/3}\sqrt{a^2\cdots}}$$

$$= \frac{x}{18ab\sqrt{a^2+2abx^3+b^2x^6}} - \frac{x}{6b\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}} + \frac{\left(a+bx^3\right)\log\cdots}{27a^{5/3}b^{4/3}\sqrt{a^2\cdots}}$$

$$= \frac{x}{18ab\sqrt{a^2+2abx^3+b^2x^6}} - \frac{x}{6b\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}} + \frac{\left(a+bx^3\right)\log\cdots}{27a^{5/3}b^{4/3}\sqrt{a^2\cdots}}$$

$$= \frac{x}{18ab\sqrt{a^2+2abx^3+b^2x^6}} - \frac{x}{6b\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}} - \frac{\left(a+bx^3\right)\tan\cdots}{9\sqrt{3}\,a^{5/3}b^{4/3}\sqrt{\cdots}}$$

Mathematica [A] time = 0.07, size = 235, normalized size = 0.85

$$\frac{3a^{2/3}b^{4/3}x^4-2abx^3\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)-b^2x^6\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)-6a^{5/3}\sqrt[3]{b}x-a^2\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)+2\left(a+bx^3\right)^2\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)-2\sqrt{3}\left(a+bx^3\right)^2\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{54a^{5/3}b^{4/3}\left(a+bx^3\right)\sqrt{\left(a+bx^3\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2),x]
```

```
[Out] (-6*a^(5/3)*b^(1/3)*x + 3*a^(2/3)*b^(4/3)*x^4 - 2*Sqrt[3]*(a + b*x^3)^2*Arc
Tan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] + 2*(a + b*x^3)^2*Log[a^(1/3) + b^
(1/3)*x] - a^2*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2] - 2*a*b*x^3*L
og[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2] - b^2*x^6*Log[a^(2/3) - a^(1/
3)*b^(1/3)*x + b^(2/3)*x^2])/(54*a^(5/3)*b^(4/3)*(a + b*x^3)*Sqrt[(a + b*x^
3)^2])
```

IntegrateAlgebraic [A] time = 11.09, size = 166, normalized size = 0.60

$$\frac{\left(a+bx^3\right)\left(-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{54a^{5/3}b^{4/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{5/3}b^{4/3}}-\frac{\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}\,a^{5/3}b^{4/3}}+\frac{bx^4-2ax}{18ab\left(a+bx^3\right)^2}\right)}{\sqrt{\left(a+bx^3\right)^2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^3/(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2),x]
```

```
[Out] ((a + b*x^3)*((-2*a*x + b*x^4)/(18*a*b*(a + b*x^3)^2) - ArcTan[1/Sqrt[3] -
(2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))]/(9*Sqrt[3]*a^(5/3)*b^(4/3)) + Log[a^(1/3)
```

```
+ b^(1/3)*x]/(27*a^(5/3)*b^(4/3)) - Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/
3)*x^2]/(54*a^(5/3)*b^(4/3))))/Sqrt[(a + b*x^3)^2]
```

fricas [A] time = 1.31, size = 503, normalized size = 1.82

$$\left[\frac{3\,a^{2}b^{2}x^{4} - 6\,a^{3}bx + 3\,\sqrt{\frac{1}{3}}\left(ab^{3}x^{6} + 2\,a^{2}b^{2}x^{3} + a^{3}b\right)\sqrt{-\frac{\left(a^{2}b\right)^{\frac{1}{3}}}{b}}\log\left(\frac{2\,abx^{3} - 3\left(a^{2}b\right)^{\frac{1}{3}}ax - a^{2} + 3\,\sqrt{\frac{1}{3}}\left(2\,abx^{2} + \left(a^{2}b\right)^{\frac{2}{3}}x - \left(a^{2}b\right)^{\frac{1}{3}}a\right)\sqrt{-\frac{\left(a^{2}b\right)^{\frac{1}{3}}}{b}}}{bx^{3}+a}\right) - \left(b^{2}x^{6} + 2\,abx^{3} + a^{2}\right)\left(a^{2}b\right)^{\frac{2}{3}}\log\left(abx^{2} - \left(a^{2}b\right)^{\frac{2}{3}}x + \left(a^{2}b\right)^{\frac{1}{3}}a\right) + 2\left(b^{2}x^{6} + 2\,abx^{3} + a^{2}\right)\left(a^{2}b\right)^{\frac{2}{3}}\log\left(abx + \left(a^{2}b\right)^{\frac{2}{3}}\right)}{54\left(a^{3}b^{4}x^{6} + 2\,a^{4}b^{3}x^{3} + a^{5}b^{2}\right)}, \frac{3\,a^{2}b^{2}x^{4} - 6\,a^{3}bx + 6\,\sqrt{\frac{1}{3}}\left(ab^{3}x^{6} + 2\,a^{2}b^{2}x^{3} + a^{3}b\right)\sqrt{\frac{\left(a^{2}b\right)^{\frac{1}{3}}}{b}}\arctan\left(\frac{\sqrt{\frac{1}{3}}\left(2\left(a^{2}b\right)^{\frac{2}{3}}x - \left(a^{2}b\right)^{\frac{1}{3}}a\right)\sqrt{\frac{\left(a^{2}b\right)^{\frac{1}{3}}}{b}}}{a^{2}}\right) - \left(b^{2}x^{6} + 2\,abx^{3} + a^{2}\right)\left(a^{2}b\right)^{\frac{2}{3}}\log\left(abx^{2} - \left(a^{2}b\right)^{\frac{2}{3}}x + \left(a^{2}b\right)^{\frac{1}{3}}a\right) + 2\left(b^{2}x^{6} + 2\,abx^{3} + a^{2}\right)\left(a^{2}b\right)^{\frac{2}{3}}\log\left(abx + \left(a^{2}b\right)^{\frac{2}{3}}\right)}{54\left(a^{3}b^{4}x^{6} + 2\,a^{4}b^{3}x^{3} + a^{5}b^{2}\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/54*(3*a^2*b^2*x^4 - 6*a^3*b*x + 3*sqrt(1/3)*(a*b^3*x^6 + 2*a^2*b^2*x^3 +
 a^3*b)*sqrt(-(a^2*b)^(1/3)/b)*log((2*a*b*x^3 - 3*(a^2*b)^(1/3)*a*x - a^2 +
 3*sqrt(1/3)*(2*a*b*x^2 + (a^2*b)^(2/3)*x - (a^2*b)^(1/3)*a)*sqrt(-(a^2*b)^
(1/3)/b))/(b*x^3 + a)) - (b^2*x^6 + 2*a*b*x^3 + a^2)*(a^2*b)^(2/3)*log(a*b*
x^2 - (a^2*b)^(2/3)*x + (a^2*b)^(1/3)*a) + 2*(b^2*x^6 + 2*a*b*x^3 + a^2)*(a
^2*b)^(2/3)*log(a*b*x + (a^2*b)^(2/3)))/(a^3*b^4*x^6 + 2*a^4*b^3*x^3 + a^5*
b^2), 1/54*(3*a^2*b^2*x^4 - 6*a^3*b*x + 6*sqrt(1/3)*(a*b^3*x^6 + 2*a^2*b^2*
x^3 + a^3*b)*sqrt((a^2*b)^(1/3)/b)*arctan(sqrt(1/3)*(2*(a^2*b)^(2/3)*x - (a
^2*b)^(1/3)*a)*sqrt((a^2*b)^(1/3)/b)/a^2) - (b^2*x^6 + 2*a*b*x^3 + a^2)*(a^
2*b)^(2/3)*log(a*b*x^2 - (a^2*b)^(2/3)*x + (a^2*b)^(1/3)*a) + 2*(b^2*x^6 +
2*a*b*x^3 + a^2)*(a^2*b)^(2/3)*log(a*b*x + (a^2*b)^(2/3)))/(a^3*b^4*x^6 + 2
*a^4*b^3*x^3 + a^5*b^2)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

maple [A] time = 0.01, size = 299, normalized size = 1.08

$$\frac{\left(-2\sqrt{3}\,b^{2}x^{6}\arctan\left(\frac{\sqrt{3}\left(-2x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right) + 2b^{2}x^{6}\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right) - b^{2}x^{6}\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right) - 4\sqrt{3}\,ab\,x^{3}\arctan\left(\frac{\sqrt{3}\left(-2x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right) + 4ab\,x^{3}\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right) - 2ab\,x^{3}\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right) + 3\left(\frac{a}{b}\right)^{\frac{2}{3}}b^{2}x^{4} - 2\sqrt{3}\,a^{2}\arctan\left(\frac{\sqrt{3}\left(-2x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right) + 2a^{2}\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right) - a^{2}\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right) - 6\left(\frac{a}{b}\right)^{\frac{2}{3}}abx\right)\left(b\,x^{3}+a\right)}{54\left(\frac{a}{b}\right)^{\frac{2}{3}}\left(\left(b\,x^{3}+a\right)^{2}\right)^{\frac{3}{2}}a\,b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x)
```

```
[Out] 1/54*(-2*3^(1/2)*b^2*x^6*arctan(1/3*3^(1/2)*(-2*x+(a/b)^(1/3))/(a/b)^(1/3))
+2*b^2*x^6*ln(x+(a/b)^(1/3))-b^2*x^6*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+3*(a
/b)^(2/3)*x^4*b^2-4*3^(1/2)*a*b*x^3*arctan(1/3*3^(1/2)*(-2*x+(a/b)^(1/3))/(
a/b)^(1/3))+4*a*b*x^3*ln(x+(a/b)^(1/3))-2*a*b*x^3*ln(x^2-(a/b)^(1/3)*x+(a/b
)^(2/3))-6*(a/b)^(2/3)*x*a*b-2*3^(1/2)*a^2*arctan(1/3*3^(1/2)*(-2*x+(a/b)^(
1/3))/(a/b)^(1/3))+2*a^2*ln(x+(a/b)^(1/3))-a^2*ln(x^2-(a/b)^(1/3)*x+(a/b)^(
2/3)))*(b*x^3+a)/(a/b)^(2/3)/b^2/a/((b*x^3+a)^2)^(3/2)
```

maxima [A] time = 1.70, size = 146, normalized size = 0.53

$$\frac{bx^{4} - 2\,ax}{18\left(ab^{3}x^{6} + 2\,a^{2}b^{2}x^{3} + a^{3}b\right)} + \frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,ab^{2}\left(\frac{a}{b}\right)^{\frac{2}{3}}} - \frac{\log\left(x^{2} - x\left(\frac{a}{b}\right)^{\frac{1}{3}} + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\,ab^{2}\left(\frac{a}{b}\right)^{\frac{2}{3}}} + \frac{\log\left(x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\,ab^{2}\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/18*(b*x^4 - 2*a*x)/(a*b^3*x^6 + 2*a^2*b^2*x^3 + a^3*b) + 1/27*sqrt(3)*arc
tan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a*b^2*(a/b)^(2/3)) - 1/54
*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a*b^2*(a/b)^(2/3)) + 1/27*log(x +
(a/b)^(1/3))/(a*b^2*(a/b)^(2/3))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3}{\left(a^2+2\,a\,b\,x^3+b^2\,x^6\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2),x)
```

```
[Out] int(x^3/(a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\left(\left(a+bx^3\right)^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(b**2*x**6+2*a*b*x**3+a**2)**(3/2),x)
```

```
[Out] Integral(x**3/((a + b*x**3)**2)**(3/2), x)
```

3.100 $\int \frac{x^2}{\left(a^2+2abx^3+b^2x^6\right)^{3/2}} dx$

Optimal. Leaf size=38

$$-\frac{1}{6b\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}}$$

Rubi [A] time = 0.03, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1352, 607}

$$-\frac{1}{6b\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2),x]
```

```
[Out] -1/(6*b*(a + b*x^3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])
```

Rule 607

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(2*(a + b*x
+ c*x^2)^(p + 1))/((2*p + 1)*(b + 2*c*x)), x] /; FreeQ[{a, b, c, p}, x] &&
EqQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 1352

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a,
b, c, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2}{\left(a^2+2abx^3+b^2x^6\right)^{3/2}} dx &= \frac{1}{3}\text{Subst}\left(\int \frac{1}{\left(a^2+2abx+b^2x^2\right)^{3/2}} dx, x, x^3\right)\\ &= -\frac{1}{6b\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 27, normalized size = 0.71

$$-\frac{a+bx^3}{6b\left(\left(a+bx^3\right)^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2),x]
```

```
[Out] -1/6*(a + b*x^3)/(b*((a + b*x^3)^2)^(3/2))
```

IntegrateAlgebraic [B] time = 0.57, size = 137, normalized size = 3.61

$$\frac{\sqrt{b^2}\left(a-bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}+a^2b+b^3x^6}{3b\sqrt{b^2}\,x^6\left(2a^2b^2+4ab^3x^3+2b^4x^6\right)+3bx^6\left(-2ab^3-2b^4x^3\right)\sqrt{a^2+2abx^3+b^2x^6}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^2/(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2),x]
```

```
[Out] (a^2*b + b^3*x^6 + Sqrt[b^2]*(a - b*x^3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(
3*b*x^6*(-2*a*b^3 - 2*b^4*x^3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6] + 3*b*Sqrt[b
^2]*x^6*(2*a^2*b^2 + 4*a*b^3*x^3 + 2*b^4*x^6))
```

fricas [A] time = 1.41, size = 26, normalized size = 0.68

$$-\frac{1}{6\left(b^3x^6 + 2\,ab^2x^3 + a^2b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/6/(b^3*x^6 + 2*a*b^2*x^3 + a^2*b)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

maple [A] time = 0.01, size = 24, normalized size = 0.63

$$-\frac{b\,x^3 + a}{6\left(\left(b\,x^3 + a\right)^2\right)^{\frac{3}{2}} b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x)
```

```
[Out] -1/6*(b*x^3+a)/b/((b*x^3+a)^2)^(3/2)
```

maxima [A] time = 0.89, size = 16, normalized size = 0.42

$$-\frac{1}{6\left(x^3 + \frac{a}{b}\right)^2 b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/6/((x^3 + a/b)^2*b^3)
```

mupad [B] time = 1.19, size = 34, normalized size = 0.89

$$-\frac{\sqrt{a^2 + 2\,a\,b\,x^3 + b^2\,x^6}}{6\,b\left(b\,x^3 + a\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2),x)
```

```
[Out] -(a^2 + b^2*x^6 + 2*a*b*x^3)^(1/2)/(6*b*(a + b*x^3)^3)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(\left(a+bx^3\right)^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b**2*x**6+2*a*b*x**3+a**2)**(3/2),x)
```

```
[Out] Integral(x**2/((a + b*x**3)**2)**(3/2), x)
```

3.101 $\int \frac{x}{\left(a^2+2abx^3+b^2x^6\right)^{3/2}}\,dx$

Optimal. Leaf size=277

$$\frac{x^2}{6a\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}}+\frac{2x^2}{9a^2\sqrt{a^2+2abx^3+b^2x^6}}-\frac{2\left(a+bx^3\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{27a^{7/3}b^{2/3}\sqrt{a^2+2abx^3+b^2x^6}}-\frac{2\left(a+bx^3\right)\tan}{9\sqrt{3}\,a^{7/3}b^{2/3}\sqrt{a^2}}$$

Rubi [A] time = 0.14, antiderivative size = 277, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1355, 290, 292, 31, 634, 617, 204, 628}

$$\frac{x^2}{6a\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}}+\frac{2x^2}{9a^2\sqrt{a^2+2abx^3+b^2x^6}}-\frac{2\left(a+bx^3\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{27a^{7/3}b^{2/3}\sqrt{a^2+2abx^3+b^2x^6}}+\frac{\left(a+bx^3\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{27a^{7/3}b^{2/3}\sqrt{a^2+2abx^3+b^2x^6}}-\frac{2\left(a+bx^3\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}\,a^{7/3}b^{2/3}\sqrt{a^2+2abx^3+b^2x^6}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2),x]
```

```
[Out] (2*x^2)/(9*a^2*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) + x^2/(6*a*(a + b*x^3)*Sqrt
[a^2 + 2*a*b*x^3 + b^2*x^6]) - (2*(a + b*x^3)*ArcTan[(a^(1/3) - 2*b^(1/3)*x
)/(Sqrt[3]*a^(1/3))])/(9*Sqrt[3]*a^(7/3)*b^(2/3)*Sqrt[a^2 + 2*a*b*x^3 + b^2
*x^6]) - (2*(a + b*x^3)*Log[a^(1/3) + b^(1/3)*x])/(27*a^(7/3)*b^(2/3)*Sqrt[
a^2 + 2*a*b*x^3 + b^2*x^6]) + ((a + b*x^3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x
+ b^(2/3)*x^2])/(27*a^(7/3)*b^(2/3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\int \frac{x}{\left(a^2+2abx^3+b^2x^6\right)^{3/2}}\,dx = \frac{\left(b^2\left(ab+b^2x^3\right)\right)\int \frac{x}{\left(ab+b^2x^3\right)^3}\,dx}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$= \frac{x^2}{6a\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}} + \frac{\left(2b\left(ab+b^2x^3\right)\right)\int \frac{x}{\left(ab+b^2x^3\right)^2}\,dx}{3a\sqrt{a^2+2abx^3+b^2x^6}}$$

$$= \frac{2x^2}{9a^2\sqrt{a^2+2abx^3+b^2x^6}} + \frac{x^2}{6a\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}} + \frac{\left(2\left(ab+b^2x^3\right)\right)\int}{9a^2\sqrt{a^2+2abx^3}}$$

$$= \frac{2x^2}{9a^2\sqrt{a^2+2abx^3+b^2x^6}} + \frac{x^2}{6a\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}} - \frac{\left(2\left(ab+b^2x^3\right)\right)\int}{27a^{7/3}b\sqrt{a^2+2}}$$

$$= \frac{2x^2}{9a^2\sqrt{a^2+2abx^3+b^2x^6}} + \frac{x^2}{6a\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}} - \frac{2\left(a+bx^3\right)\log(}{27a^{7/3}b^{2/3}\sqrt{a^2+2}}$$

$$= \frac{2x^2}{9a^2\sqrt{a^2+2abx^3+b^2x^6}} + \frac{x^2}{6a\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}} - \frac{2\left(a+bx^3\right)\log(}{27a^{7/3}b^{2/3}\sqrt{a^2+2}}$$

$$= \frac{2x^2}{9a^2\sqrt{a^2+2abx^3+b^2x^6}} + \frac{x^2}{6a\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}} - \frac{2\left(a+bx^3\right)\tan^{-1}}{9\sqrt{3}\,a^{7/3}b^{2/3}\sqrt{a^2+}}$$

Mathematica [A] time = 0.07, size = 237, normalized size = 0.86

$$\frac{21a^{4/3}b^{2/3}x^2+4abx^3\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)+2b^2x^6\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)+2a^2\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)+12\sqrt[3]{a}b^{5/3}x^5-4\left(a+bx^3\right)^2\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)-4\sqrt{3}\left(a+bx^3\right)^2\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{54a^{7/3}b^{2/3}\left(a+bx^3\right)\sqrt{\left(a+bx^3\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2),x]
```

```
[Out] (21*a^(4/3)*b^(2/3)*x^2 + 12*a^(1/3)*b^(5/3)*x^5 - 4*Sqrt[3]*(a + b*x^3)^2*
ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] - 4*(a + b*x^3)^2*Log[a^(1/3) +
 b^(1/3)*x] + 2*a^2*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2] + 4*a*b*
x^3*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2] + 2*b^2*x^6*Log[a^(2/3)
- a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(54*a^(7/3)*b^(2/3)*(a + b*x^3)*Sqrt[(a
 + b*x^3)^2])
```

IntegrateAlgebraic [A] time = 13.20, size = 166, normalized size = 0.60

$$\frac{\left(a+bx^3\right)\left(\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{27a^{7/3}b^{2/3}}-\frac{2\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{7/3}b^{2/3}}-\frac{2\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{7/3}b^{2/3}}+\frac{x^2\left(7a+4bx^3\right)}{18a^2\left(a+bx^3\right)^2}\right)}{\sqrt{\left(a+bx^3\right)^2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x/(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2),x]
```

```
[Out] ((a + b*x^3)*((x^2*(7*a + 4*b*x^3))/(18*a^2*(a + b*x^3)^2) - (2*ArcTan[1/Sq
rt[3] - (2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(9*Sqrt[3]*a^(7/3)*b^(2/3)) - (2*
Log[a^(1/3) + b^(1/3)*x])/(27*a^(7/3)*b^(2/3)) + Log[a^(2/3) - a^(1/3)*b^(1
/3)*x + b^(2/3)*x^2]/(27*a^(7/3)*b^(2/3))))/Sqrt[(a + b*x^3)^2]
```

fricas [A] time = 0.74, size = 514, normalized size = 1.86

$$\left[\frac{12ab^3x^5+21a^2b^2x^2+6\sqrt{\frac{1}{3}}\left(ab^3x^6+2a^2b^2x^3+a^3b\right)\sqrt{\frac{\left(-ab^2\right)^{\frac{1}{3}}}{a}}\log\left(\frac{2b^2x^3-ab+3\sqrt{\frac{1}{3}}\left(abx+2\left(-ab^2\right)^{\frac{2}{3}}x^2+\left(-ab^2\right)^{\frac{1}{3}}a\right)\sqrt{\frac{\left(-ab^2\right)^{\frac{1}{3}}}{a}}-3\left(-ab^2\right)^{\frac{2}{3}}x}{bx^3+a}\right)+2\left(b^2x^6+2abx^3+a^2\right)\left(-ab^2\right)^{\frac{2}{3}}\log\left(b^2x^2+\left(-ab^2\right)^{\frac{1}{3}}bx+\left(-ab^2\right)^{\frac{2}{3}}\right)-4\left(b^2x^6+2abx^3+a^2\right)\left(-ab^2\right)^{\frac{2}{3}}\log\left(bx-\left(-ab^2\right)^{\frac{1}{3}}\right)}{54\left(a^3b^4x^6+2a^4b^3x^3+a^5b^2\right)}, \frac{12ab^3x^5+21a^2b^2x^2+12\sqrt{\frac{1}{3}}\left(ab^3x^6+2a^2b^2x^3+a^3b\right)\sqrt{-\frac{\left(-ab^2\right)^{\frac{1}{3}}}{a}}\arctan\left(\frac{\sqrt{\frac{1}{3}}\left(2bx+\left(-ab^2\right)^{\frac{1}{3}}\right)\sqrt{-\frac{\left(-ab^2\right)^{\frac{1}{3}}}{a}}}{b}\right)+2\left(b^2x^6+2abx^3+a^2\right)\left(-ab^2\right)^{\frac{2}{3}}\log\left(b^2x^2+\left(-ab^2\right)^{\frac{1}{3}}bx+\left(-ab^2\right)^{\frac{2}{3}}\right)-4\left(b^2x^6+2abx^3+a^2\right)\left(-ab^2\right)^{\frac{2}{3}}\log\left(bx-\left(-ab^2\right)^{\frac{1}{3}}\right)}{54\left(a^3b^4x^6+2a^4b^3x^3+a^5b^2\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/54*(12*a*b^3*x^5 + 21*a^2*b^2*x^2 + 6*sqrt(1/3)*(a*b^3*x^6 + 2*a^2*b^2*x
^3 + a^3*b)*sqrt((-a*b^2)^(1/3)/a)*log((2*b^2*x^3 - a*b + 3*sqrt(1/3)*(a*b*
x + 2*(-a*b^2)^(2/3)*x^2 + (-a*b^2)^(1/3)*a)*sqrt((-a*b^2)^(1/3)/a) - 3*(-a
*b^2)^(2/3)*x)/(b*x^3 + a)) + 2*(b^2*x^6 + 2*a*b*x^3 + a^2)*(-a*b^2)^(2/3)*
log(b^2*x^2 + (-a*b^2)^(1/3)*b*x + (-a*b^2)^(2/3)) - 4*(b^2*x^6 + 2*a*b*x^3
 + a^2)*(-a*b^2)^(2/3)*log(b*x - (-a*b^2)^(1/3)))/(a^3*b^4*x^6 + 2*a^4*b^3*
x^3 + a^5*b^2), 1/54*(12*a*b^3*x^5 + 21*a^2*b^2*x^2 + 12*sqrt(1/3)*(a*b^3*x
^6 + 2*a^2*b^2*x^3 + a^3*b)*sqrt(-(-a*b^2)^(1/3)/a)*arctan(sqrt(1/3)*(2*b*x
 + (-a*b^2)^(1/3))*sqrt(-(-a*b^2)^(1/3)/a)/b) + 2*(b^2*x^6 + 2*a*b*x^3 + a^
2)*(-a*b^2)^(2/3)*log(b^2*x^2 + (-a*b^2)^(1/3)*b*x + (-a*b^2)^(2/3)) - 4*(b
^2*x^6 + 2*a*b*x^3 + a^2)*(-a*b^2)^(2/3)*log(b*x - (-a*b^2)^(1/3)))/(a^3*b^
4*x^6 + 2*a^4*b^3*x^3 + a^5*b^2)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

maple [A] time = 0.01, size = 301, normalized size = 1.09

$$\frac{\left(-4\sqrt{3}\,b^2x^6\arctan\left(\frac{\sqrt{3}\left(-2x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)-4b^2x^6\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)+2b^2x^6\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)+12\left(\frac{a}{b}\right)^{\frac{1}{3}}b^2x^5-8\sqrt{3}\,ab\,x^3\arctan\left(\frac{\sqrt{3}\left(-2x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)-8ab\,x^3\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)+4ab\,x^3\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)+21\left(\frac{a}{b}\right)^{\frac{1}{3}}ab\,x^2-4\sqrt{3}\,a^2\arctan\left(\frac{\sqrt{3}\left(-2x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)-4a^2\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)+2a^2\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)\right)\left(b\,x^3+a\right)}{54\left(\frac{a}{b}\right)^{\frac{1}{3}}\left(\left(b\,x^3+a\right)^2\right)^{\frac{3}{2}}a^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x)
```

```
[Out] 1/54*(-4*3^(1/2)*b^2*x^6*arctan(1/3*3^(1/2)*(-2*x+(a/b)^(1/3))/(a/b)^(1/3))
-4*b^2*x^6*ln(x+(a/b)^(1/3))+2*b^2*x^6*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+12
*(a/b)^(1/3)*b^2*x^5-8*3^(1/2)*a*b*x^3*arctan(1/3*3^(1/2)*(-2*x+(a/b)^(1/3)
)/(a/b)^(1/3))-8*a*b*x^3*ln(x+(a/b)^(1/3))+4*a*b*x^3*ln(x^2-(a/b)^(1/3)*x+(
a/b)^(2/3))+21*(a/b)^(1/3)*a*b*x^2-4*3^(1/2)*a^2*arctan(1/3*3^(1/2)*(-2*x+(
a/b)^(1/3))/(a/b)^(1/3))-4*a^2*ln(x+(a/b)^(1/3))+2*a^2*ln(x^2-(a/b)^(1/3)*x
+(a/b)^(2/3)))*(b*x^3+a)/(a/b)^(1/3)/b/a^2/((b*x^3+a)^2)^(3/2)
```

maxima [A] time = 1.36, size = 147, normalized size = 0.53

$$\frac{4\,bx^5+7\,ax^2}{18\left(a^2b^2x^6+2\,a^3bx^3+a^4\right)}+\frac{2\,\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,a^2b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{27\,a^2b\left(\frac{a}{b}\right)^{\frac{1}{3}}}-\frac{2\,\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\,a^2b\left(\frac{a}{b}\right)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/18*(4*b*x^5 + 7*a*x^2)/(a^2*b^2*x^6 + 2*a^3*b*x^3 + a^4) + 2/27*sqrt(3)*a
rctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a^2*b*(a/b)^(1/3)) + 1/
27*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a^2*b*(a/b)^(1/3)) - 2/27*log(x
+ (a/b)^(1/3))/(a^2*b*(a/b)^(1/3))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x}{\left(a^2+2\,a\,b\,x^3+b^2\,x^6\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2),x)
```

```
[Out] int(x/(a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(\left(a+bx^3\right)^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b**2*x**6+2*a*b*x**3+a**2)**(3/2),x)
```

```
[Out] Integral(x/((a + b*x**3)**2)**(3/2), x)
```

3.102 $\int \frac{1}{\left(a^2+2abx^3+b^2x^6\right)^{3/2}}\,dx$

Optimal. Leaf size=286

$$\frac{5x\left(a+bx^3\right)^2}{18a^2\left(a^2+2abx^3+b^2x^6\right)^{3/2}}+\frac{x\left(a+bx^3\right)}{6a\left(a^2+2abx^3+b^2x^6\right)^{3/2}}+\frac{5\left(a+bx^3\right)^3\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{27a^{8/3}\sqrt[3]{b}\left(a^2+2abx^3+b^2x^6\right)^{3/2}}-\frac{5\left(a+bx^3\right)^3\tan}{9\sqrt{3}\,a^{8/3}\sqrt[3]{b}\left(a^2+\right.}$$

Rubi [A] time = 0.15, antiderivative size = 286, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {1343, 199, 200, 31, 634, 617, 204, 628}

$$\frac{5x\left(a+bx^3\right)^2}{18a^2\left(a^2+2abx^3+b^2x^6\right)^{3/2}}+\frac{x\left(a+bx^3\right)}{6a\left(a^2+2abx^3+b^2x^6\right)^{3/2}}+\frac{5\left(a+bx^3\right)^3\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{27a^{8/3}\sqrt[3]{b}\left(a^2+2abx^3+b^2x^6\right)^{3/2}}-\frac{5\left(a+bx^3\right)^3\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{54a^{8/3}\sqrt[3]{b}\left(a^2+2abx^3+b^2x^6\right)^{3/2}}-\frac{5\left(a+bx^3\right)^3\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}\,a^{8/3}\sqrt[3]{b}\left(a^2+2abx^3+b^2x^6\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(-3/2),x]
```

```
[Out] (x*(a + b*x^3))/(6*a*(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)) + (5*x*(a + b*x^3)^
2)/(18*a^2*(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)) - (5*(a + b*x^3)^3*ArcTan[(a^
(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(9*Sqrt[3]*a^(8/3)*b^(1/3)*(a^2 +
2*a*b*x^3 + b^2*x^6)^(3/2)) + (5*(a + b*x^3)^3*Log[a^(1/3) + b^(1/3)*x])/(2
7*a^(8/3)*b^(1/3)*(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)) - (5*(a + b*x^3)^3*Log
[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(54*a^(8/3)*b^(1/3)*(a^2 + 2*a
*b*x^3 + b^2*x^6)^(3/2))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
```

```
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1343

```
Int[((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_), x_Symbol] :> Dist[(
a + b*x^n + c*x^(2*n))^p/(b + 2*c*x^n)^(2*p), Int[(b + 2*c*x^n)^(2*p), x],
x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{\left(a^2+2abx^3+b^2x^6\right)^{3/2}}\,dx &= \frac{\left(2ab+2b^2x^3\right)^3 \int \frac{1}{\left(2ab+2b^2x^3\right)^3}\,dx}{\left(a^2+2abx^3+b^2x^6\right)^{3/2}} \\
&= \frac{x\left(a+bx^3\right)}{6a\left(a^2+2abx^3+b^2x^6\right)^{3/2}}+\frac{\left(5\left(2ab+2b^2x^3\right)^3\right)\int \frac{1}{\left(2ab+2b^2x^3\right)^2}\,dx}{12ab\left(a^2+2abx^3+b^2x^6\right)^{3/2}} \\
&= \frac{x\left(a+bx^3\right)}{6a\left(a^2+2abx^3+b^2x^6\right)^{3/2}}+\frac{5x\left(a+bx^3\right)^2}{18a^2\left(a^2+2abx^3+b^2x^6\right)^{3/2}}+\frac{\left(5\left(2ab+2b^2x^3\right)^3\right)\int \cdots}{36a^2b^2\left(a^2+2abx^3 \cdots\right.} \\
&= \frac{x\left(a+bx^3\right)}{6a\left(a^2+2abx^3+b^2x^6\right)^{3/2}}+\frac{5x\left(a+bx^3\right)^2}{18a^2\left(a^2+2abx^3+b^2x^6\right)^{3/2}}+\frac{\left(5\left(2ab+2b^2x^3\right)^3\right)\int \cdots}{108\,2^{2/3}a^{8/3}b^{8/3}\left(a^2 \cdots\right.} \\
&= \frac{x\left(a+bx^3\right)}{6a\left(a^2+2abx^3+b^2x^6\right)^{3/2}}+\frac{5x\left(a+bx^3\right)^2}{18a^2\left(a^2+2abx^3+b^2x^6\right)^{3/2}}+\frac{5\left(a+bx^3\right)^3\log\left(\sqrt[3]{a} \cdots\right.}{27a^{8/3}\sqrt[3]{b}\left(a^2+2abx^3 \cdots\right.} \\
&= \frac{x\left(a+bx^3\right)}{6a\left(a^2+2abx^3+b^2x^6\right)^{3/2}}+\frac{5x\left(a+bx^3\right)^2}{18a^2\left(a^2+2abx^3+b^2x^6\right)^{3/2}}+\frac{5\left(a+bx^3\right)^3\log\left(\sqrt[3]{a} \cdots\right.}{27a^{8/3}\sqrt[3]{b}\left(a^2+2abx^3 \cdots\right.} \\
&= \frac{x\left(a+bx^3\right)}{6a\left(a^2+2abx^3+b^2x^6\right)^{3/2}}+\frac{5x\left(a+bx^3\right)^2}{18a^2\left(a^2+2abx^3+b^2x^6\right)^{3/2}}-\frac{5\left(a+bx^3\right)^3\tan^{-1}\left(\cdots\right.}{9\sqrt{3}\,a^{8/3}\sqrt[3]{b}\left(a^2+2ab \cdots\right.}
\end{aligned}
$$

Mathematica [A] time = 0.07, size = 235, normalized size = 0.82

$$
\frac{15a^{2/3}b^{4/3}x^4-10abx^3\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)-5b^2x^6\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)+24a^{5/3}\sqrt[3]{b}x-5a^2\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)+10\left(a+bx^3\right)^2\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)-10\sqrt{3}\left(a+bx^3\right)^2\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{54a^{8/3}\sqrt[3]{b}\left(a+bx^3\right)\sqrt{\left(a+bx^3\right)^2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(-3/2),x]
```

```
[Out] (24*a^(5/3)*b^(1/3)*x + 15*a^(2/3)*b^(4/3)*x^4 - 10*Sqrt[3]*(a + b*x^3)^2*A
rcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] + 10*(a + b*x^3)^2*Log[a^(1/3) +
 b^(1/3)*x] - 5*a^2*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2] - 10*a*b
*x^3*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2] - 5*b^2*x^6*Log[a^(2/3)
 - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(54*a^(8/3)*b^(1/3)*(a + b*x^3)*Sqrt[(
a + b*x^3)^2])
```

IntegrateAlgebraic [A] time = 15.69, size = 164, normalized size = 0.57

$$\frac{\left(a+bx^3\right)\left(-\frac{5\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{54a^{8/3}\sqrt[3]{b}}+\frac{5\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{8/3}\sqrt[3]{b}}-\frac{5\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{8/3}\sqrt[3]{b}}+\frac{x\left(8a+5bx^3\right)}{18a^2\left(a+bx^3\right)^2}\right)}{\sqrt{\left(a+bx^3\right)^2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a^2 + 2*a*b*x^3 + b^2*x^6)^(-3/2),x]
```

```
[Out] ((a + b*x^3)*((x*(8*a + 5*b*x^3))/(18*a^2*(a + b*x^3)^2) - (5*ArcTan[1/Sqrt
[3] - (2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(9*Sqrt[3]*a^(8/3)*b^(1/3)) + (5*Lo
g[a^(1/3) + b^(1/3)*x])/(27*a^(8/3)*b^(1/3)) - (5*Log[a^(2/3) - a^(1/3)*b^(
1/3)*x + b^(2/3)*x^2])/(54*a^(8/3)*b^(1/3))))/Sqrt[(a + b*x^3)^2]
```

fricas [A] time = 1.34, size = 499, normalized size = 1.74

$$\left[\frac{15a^2b^2x^4+24a^3bx+15\sqrt{\frac{1}{3}}\left(ab^3x^6+2a^2b^2x^3+a^3b\right)\sqrt{-\frac{\left(a^2b\right)^{\frac{1}{3}}}{b}}\log\left(\frac{2abx^3-3\left(a^2b\right)^{\frac{1}{3}}ax-a^2+3\sqrt{\frac{1}{3}}\left(2abx^2+\left(a^2b\right)^{\frac{2}{3}}x-\left(a^2b\right)^{\frac{1}{3}}a\right)\sqrt{-\frac{\left(a^2b\right)^{\frac{1}{3}}}{b}}}{bx^3+a}\right)-5\left(b^2x^6+2abx^3+a^2\right)\left(a^2b\right)^{\frac{2}{3}}\log\left(abx^2-\left(a^2b\right)^{\frac{2}{3}}x+\left(a^2b\right)^{\frac{1}{3}}a\right)+10\left(b^2x^6+2abx^3+a^2\right)\left(a^2b\right)^{\frac{2}{3}}\log\left(abx+\left(a^2b\right)^{\frac{2}{3}}\right)}{54\left(a^4b^3x^6+2a^5b^2x^3+a^6b\right)}, \frac{15a^2b^2x^4+24a^3bx+30\sqrt{\frac{1}{3}}\left(ab^3x^6+2a^2b^2x^3+a^3b\right)\sqrt{\frac{\left(a^2b\right)^{\frac{1}{3}}}{b}}\arctan\left(\frac{\sqrt{\frac{1}{3}}\left(2\left(a^2b\right)^{\frac{2}{3}}x-\left(a^2b\right)^{\frac{1}{3}}a\right)\sqrt{\frac{\left(a^2b\right)^{\frac{1}{3}}}{b}}}{a^2}\right)-5\left(b^2x^6+2abx^3+a^2\right)\left(a^2b\right)^{\frac{2}{3}}\log\left(abx^2-\left(a^2b\right)^{\frac{2}{3}}x+\left(a^2b\right)^{\frac{1}{3}}a\right)+10\left(b^2x^6+2abx^3+a^2\right)\left(a^2b\right)^{\frac{2}{3}}\log\left(abx+\left(a^2b\right)^{\frac{2}{3}}\right)}{54\left(a^4b^3x^6+2a^5b^2x^3+a^6b\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/54*(15*a^2*b^2*x^4 + 24*a^3*b*x + 15*sqrt(1/3)*(a*b^3*x^6 + 2*a^2*b^2*x^
3 + a^3*b)*sqrt(-(a^2*b)^(1/3)/b)*log((2*a*b*x^3 - 3*(a^2*b)^(1/3)*a*x - a^
2 + 3*sqrt(1/3)*(2*a*b*x^2 + (a^2*b)^(2/3)*x - (a^2*b)^(1/3)*a)*sqrt(-(a^2*
b)^(1/3)/b))/(b*x^3 + a)) - 5*(b^2*x^6 + 2*a*b*x^3 + a^2)*(a^2*b)^(2/3)*log
(a*b*x^2 - (a^2*b)^(2/3)*x + (a^2*b)^(1/3)*a) + 10*(b^2*x^6 + 2*a*b*x^3 + a
^2)*(a^2*b)^(2/3)*log(a*b*x + (a^2*b)^(2/3)))/(a^4*b^3*x^6 + 2*a^5*b^2*x^3
+ a^6*b), 1/54*(15*a^2*b^2*x^4 + 24*a^3*b*x + 30*sqrt(1/3)*(a*b^3*x^6 + 2*a
^2*b^2*x^3 + a^3*b)*sqrt((a^2*b)^(1/3)/b)*arctan(sqrt(1/3)*(2*(a^2*b)^(2/3)
*x - (a^2*b)^(1/3)*a)*sqrt((a^2*b)^(1/3)/b)/a^2) - 5*(b^2*x^6 + 2*a*b*x^3 +
 a^2)*(a^2*b)^(2/3)*log(a*b*x^2 - (a^2*b)^(2/3)*x + (a^2*b)^(1/3)*a) + 10*(
b^2*x^6 + 2*a*b*x^3 + a^2)*(a^2*b)^(2/3)*log(a*b*x + (a^2*b)^(2/3)))/(a^4*b
^3*x^6 + 2*a^5*b^2*x^3 + a^6*b)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

maple [A] time = 0.01, size = 299, normalized size = 1.05

$$\frac{\left(-10\sqrt{3}\,b^2x^6\arctan\left(\frac{\sqrt{3}\left(-2x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)+10b^2x^6\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)-5b^2x^6\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)-20\sqrt{3}\,abx^3\arctan\left(\frac{\sqrt{3}\left(-2x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)+20abx^3\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)-10abx^3\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)+15\left(\frac{a}{b}\right)^{\frac{2}{3}}b^2x^4-10\sqrt{3}\,a^2\arctan\left(\frac{\sqrt{3}\left(-2x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)+10a^2\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)-5a^2\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)+24\left(\frac{a}{b}\right)^{\frac{2}{3}}abx\right)\left(bx^3+a\right)}{54\left(\frac{a}{b}\right)^{\frac{2}{3}}\left(\left(bx^3+a\right)^2\right)^{\frac{3}{2}}a^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x)
```

```
[Out] 1/54*(-10*3^(1/2)*b^2*x^6*arctan(1/3*3^(1/2)*(-2*x+(a/b)^(1/3))/(a/b)^(1/3)
)+10*b^2*x^6*ln(x+(a/b)^(1/3))-5*b^2*x^6*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+
15*(a/b)^(2/3)*b^2*x^4-20*3^(1/2)*a*b*x^3*arctan(1/3*3^(1/2)*(-2*x+(a/b)^(1
/3))/(a/b)^(1/3))+20*a*b*x^3*ln(x+(a/b)^(1/3))-10*a*b*x^3*ln(x^2-(a/b)^(1/3
)*x+(a/b)^(2/3))+24*(a/b)^(2/3)*a*b*x-10*3^(1/2)*a^2*arctan(1/3*3^(1/2)*(-2
*x+(a/b)^(1/3))/(a/b)^(1/3))+10*a^2*ln(x+(a/b)^(1/3))-5*a^2*ln(x^2-(a/b)^(1
/3)*x+(a/b)^(2/3)))*(b*x^3+a)/(a/b)^(2/3)/b/a^2/((b*x^3+a)^2)^(3/2)
```

maxima [A] time = 1.92, size = 145, normalized size = 0.51

$$\frac{5\,bx^4+8\,ax}{18\left(a^2b^2x^6+2\,a^3bx^3+a^4\right)}+\frac{5\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,a^2b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{5\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\,a^2b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{5\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\,a^2b\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/18*(5*b*x^4 + 8*a*x)/(a^2*b^2*x^6 + 2*a^3*b*x^3 + a^4) + 5/27*sqrt(3)*arc
tan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a^2*b*(a/b)^(2/3)) - 5/54
*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a^2*b*(a/b)^(2/3)) + 5/27*log(x +
(a/b)^(1/3))/(a^2*b*(a/b)^(2/3))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{{\left(a^2+2\,a\,b\,x^3+b^2\,x^6\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2),x)
```

```
[Out] int(1/(a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a^{2}+2abx^{3}+b^{2}x^{6}\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b**2*x**6+2*a*b*x**3+a**2)**(3/2),x)
```

```
[Out] Integral((a**2 + 2*a*b*x**3 + b**2*x**6)**(-3/2), x)
```

3.103 $\int \frac{1}{x\left(a^2+2abx^3+b^2x^6\right)^{3/2}}\,dx$

Optimal. Leaf size=147

$$\frac{1}{6a\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}}+\frac{1}{3a^2\sqrt{a^2+2abx^3+b^2x^6}}+\frac{\log(x)\left(a+bx^3\right)}{a^3\sqrt{a^2+2abx^3+b^2x^6}}-\frac{\left(a+bx^3\right)\log\left(a+bx^3\right)}{3a^3\sqrt{a^2+2abx^3+b^2x^6}}$$

Rubi [A] time = 0.08, antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1355, 266, 44}

$$\frac{1}{6a\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}}+\frac{1}{3a^2\sqrt{a^2+2abx^3+b^2x^6}}+\frac{\log(x)\left(a+bx^3\right)}{a^3\sqrt{a^2+2abx^3+b^2x^6}}-\frac{\left(a+bx^3\right)\log\left(a+bx^3\right)}{3a^3\sqrt{a^2+2abx^3+b^2x^6}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)),x]
```

```
[Out] 1/(3*a^2*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) + 1/(6*a*(a + b*x^3)*Sqrt[a^2 + 2
*a*b*x^3 + b^2*x^6]) + ((a + b*x^3)*Log[x])/(a^3*Sqrt[a^2 + 2*a*b*x^3 + b^2
*x^6]) - ((a + b*x^3)*Log[a + b*x^3])/(3*a^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6
])
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\int \frac{1}{x\left(a^2+2abx^3+b^2x^6\right)^{3/2}}\,dx = \frac{\left(b^2\left(ab+b^2x^3\right)\right)\int \frac{1}{x\left(ab+b^2x^3\right)^3}\,dx}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$= \frac{\left(b^2\left(ab+b^2x^3\right)\right)\text{Subst}\left(\int \frac{1}{x\left(ab+b^2x\right)^3}\,dx, x, x^3\right)}{3\sqrt{a^2+2abx^3+b^2x^6}}$$

$$= \frac{\left(b^2\left(ab+b^2x^3\right)\right)\text{Subst}\left(\int \left(\frac{1}{a^3b^3x}-\frac{1}{ab^2(a+bx)^3}-\frac{1}{a^2b^2(a+bx)^2}-\frac{1}{a^3b^2(a+bx)}\right)dx, x, x^3\right)}{3\sqrt{a^2+2abx^3+b^2x^6}}$$

$$= \frac{1}{3a^2\sqrt{a^2+2abx^3+b^2x^6}} + \frac{1}{6a\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}} + \frac{\left(a+bx^3\right)\log}{a^3\sqrt{a^2+2abx^3}}$$

Mathematica [A] time = 0.03, size = 74, normalized size = 0.50

$$\frac{a\left(3a+2bx^3\right)+6\log(x)\left(a+bx^3\right)^2-2\left(a+bx^3\right)^2\log\left(a+bx^3\right)}{6a^3\left(a+bx^3\right)\sqrt{\left(a+bx^3\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)),x]
```

```
[Out] (a*(3*a + 2*b*x^3) + 6*(a + b*x^3)^2*Log[x] - 2*(a + b*x^3)^2*Log[a + b*x^3
])/(6*a^3*(a + b*x^3)*Sqrt[(a + b*x^3)^2])
```

IntegrateAlgebraic [B] time = 2.57, size = 758, normalized size = 5.16

$$\frac{2\left(\sqrt{a^2+2abx^3+b^2x^6}-\sqrt{b^2}x^3\right)^4\tanh^{-1}\left(\frac{\sqrt{b^2}x^3}{a}-\frac{\sqrt{a^2+2abx^3+b^2x^6}}{a}\right)}{3a^3\left(a^4+4a^3bx^3+12a^2b^2x^6-8ab\sqrt{b^2}x^6\sqrt{a^2+2abx^3+b^2x^6}-4a^2\sqrt{b^2}x^3\sqrt{a^2+2abx^3+b^2x^6}-8\left(b^2\right)^{3/2}x^9\sqrt{a^2+2abx^3+b^2x^6}+16ab^3x^9+8b^4x^{12}\right)} + \frac{\sqrt{b^2}\left(-a^{11}-3a^9b^2x^6-50a^8b^3x^9-368a^7b^4x^{12}-1568a^6b^5x^{15}-4256a^5b^6x^{18}-7616a^4b^7x^{21}-8960a^3b^8x^{24}-6656a^2b^9x^{27}-2816ab^{10}x^{30}-512b^{11}x^{33}\right)+\sqrt{a^2+2abx^3+b^2x^6}\left(-a^{10}b+a^9b^2x^3+2a^8b^3x^6+48a^7b^4x^9+320a^6b^5x^{12}+1248a^5b^6x^{15}+3008a^4b^7x^{18}+4608a^3b^8x^{21}+4352a^2b^9x^{24}+2304ab^{10}x^{27}+512b^{11}x^{30}\right)}{3a^2b\sqrt{b^2}x^6\sqrt{a^2+2abx^3+b^2x^6}\left(-2a^9b-34a^8b^2x^3-256a^7b^3x^6-1120a^6b^4x^9-3136a^5b^5x^{12}-5824a^4b^6x^{15}-7168a^3b^7x^{18}-5632a^2b^8x^{21}-2560ab^9x^{24}-512b^{10}x^{27}\right)+3a^2bx^6\left(2a^{10}b^2+36a^9b^3x^3+290a^8b^4x^6+1376a^7b^5x^9+4256a^6b^6x^{12}+8960a^5b^7x^{15}+12992a^4b^8x^{18}+12800a^3b^9x^{21}+8192a^2b^{10}x^{24}+3072ab^{11}x^{27}+512b^{12}x^{30}\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x*(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)),x]
```

```
[Out] (Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]*(-(a^10*b) + a^9*b^2*x^3 + 2*a^8*b^3*x^6 +
 48*a^7*b^4*x^9 + 320*a^6*b^5*x^12 + 1248*a^5*b^6*x^15 + 3008*a^4*b^7*x^18
+ 4608*a^3*b^8*x^21 + 4352*a^2*b^9*x^24 + 2304*a*b^10*x^27 + 512*b^11*x^30)
 + Sqrt[b^2]*(-a^11 - 3*a^9*b^2*x^6 - 50*a^8*b^3*x^9 - 368*a^7*b^4*x^12 - 1
568*a^6*b^5*x^15 - 4256*a^5*b^6*x^18 - 7616*a^4*b^7*x^21 - 8960*a^3*b^8*x^2
4 - 6656*a^2*b^9*x^27 - 2816*a*b^10*x^30 - 512*b^11*x^33))/(3*a^2*b*Sqrt[b^
2]*x^6*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]*(-2*a^9*b - 34*a^8*b^2*x^3 - 256*a^7
*b^3*x^6 - 1120*a^6*b^4*x^9 - 3136*a^5*b^5*x^12 - 5824*a^4*b^6*x^15 - 7168*
a^3*b^7*x^18 - 5632*a^2*b^8*x^21 - 2560*a*b^9*x^24 - 512*b^10*x^27) + 3*a^2
*b*x^6*(2*a^10*b^2 + 36*a^9*b^3*x^3 + 290*a^8*b^4*x^6 + 1376*a^7*b^5*x^9 +
4256*a^6*b^6*x^12 + 8960*a^5*b^7*x^15 + 12992*a^4*b^8*x^18 + 12800*a^3*b^9*
x^21 + 8192*a^2*b^10*x^24 + 3072*a*b^11*x^27 + 512*b^12*x^30)) + (2*(-(Sqrt
[b^2]*x^3) + Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])^4*ArcTanh[(Sqrt[b^2]*x^3)/a -
 Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]/a])/(3*a^3*(a^4 + 4*a^3*b*x^3 + 12*a^2*b^2
*x^6 + 16*a*b^3*x^9 + 8*b^4*x^12 - 4*a^2*Sqrt[b^2]*x^3*Sqrt[a^2 + 2*a*b*x^3
 + b^2*x^6] - 8*a*b*Sqrt[b^2]*x^6*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6] - 8*(b^2)
^(3/2)*x^9*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]))
```

fricas [A] time = 1.24, size = 90, normalized size = 0.61

$$\frac{2\,abx^3+3\,a^2-2\left(b^2x^6+2\,abx^3+a^2\right)\log\left(bx^3+a\right)+6\left(b^2x^6+2\,abx^3+a^2\right)\log(x)}{6\left(a^3b^2x^6+2\,a^4bx^3+a^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/6*(2*a*b*x^3 + 3*a^2 - 2*(b^2*x^6 + 2*a*b*x^3 + a^2)*log(b*x^3 + a) + 6*(
b^2*x^6 + 2*a*b*x^3 + a^2)*log(x))/(a^3*b^2*x^6 + 2*a^4*b*x^3 + a^5)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

maple [A] time = 0.02, size = 107, normalized size = 0.73

$$\frac{\left(6b^2x^6\ln(x)-2b^2x^6\ln\left(b\,x^3+a\right)+12ab\,x^3\ln(x)-4ab\,x^3\ln\left(b\,x^3+a\right)+2ab\,x^3+6a^2\ln(x)-2a^2\ln\left(b\,x^3+a\right)+3a^2\right)\left(b\,x^3+a\right)}{6\left(\left(b\,x^3+a\right)^2\right)^{\frac{3}{2}}a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x)
```

```
[Out] 1/6*(6*ln(x)*x^6*b^2-2*ln(b*x^3+a)*x^6*b^2+12*ln(x)*x^3*a*b-4*ln(b*x^3+a)*x
^3*a*b+2*a*b*x^3+6*ln(x)*a^2-2*ln(b*x^3+a)*a^2+3*a^2)*(b*x^3+a)/a^3/((b*x^3
+a)^2)^(3/2)
```

maxima [A] time = 1.15, size = 88, normalized size = 0.60

$$-\frac{(-1)^{2\,abx^3+2\,a^2}\log\left(\frac{2\,abx}{|x|}+\frac{2\,a^2}{x^2|x|}\right)}{3\,a^3}+\frac{1}{3\,\sqrt{b^2x^6+2\,abx^3+a^2}\,a^2}+\frac{1}{6\left(x^3+\frac{a}{b}\right)^2ab^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/3*(-1)^(2*a*b*x^3 + 2*a^2)*log(2*a*b*x/abs(x) + 2*a^2/(x^2*abs(x)))/a^3
+ 1/3/(sqrt(b^2*x^6 + 2*a*b*x^3 + a^2)*a^2) + 1/6/((x^3 + a/b)^2*a*b^2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x\left(a^2+2\,a\,b\,x^3+b^2\,x^6\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2)),x)
```

```
[Out] int(1/(x*(a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(\left(a+bx^3\right)^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b**2*x**6+2*a*b*x**3+a**2)**(3/2),x)
```

```
[Out] Integral(1/(x*((a + b*x**3)**2)**(3/2)), x)
```

3.104 $\int \frac{1}{x^2\left(a^2+2abx^3+b^2x^6\right)^{3/2}}\,dx$

Optimal. Leaf size=316

$$\frac{7}{18a^2x\sqrt{a^2+2abx^3+b^2x^6}}+\frac{1}{6ax\sqrt{a^2+2abx^3+b^2x^6}\,\left(a+bx^3\right)}+\frac{14\sqrt[3]{b}\,\left(a+bx^3\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{27a^{10/3}\sqrt{a^2+2abx^3+b^2x^6}}+\frac{14\sqrt[3]{b}\,\left(a+bx^3\right.}{9\sqrt{3}\,a^{10/3}\sqrt{}}$$

Rubi [A] time = 0.16, antiderivative size = 316, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.346, Rules used = {1355, 290, 325, 292, 31, 634, 617, 204, 628}

$$-\frac{14\left(a+bx^3\right)}{9a^3x\sqrt{a^2+2abx^3+b^2x^6}}+\frac{7}{18a^2x\sqrt{a^2+2abx^3+b^2x^6}}+\frac{1}{6ax\sqrt{a^2+2abx^3+b^2x^6}\,\left(a+bx^3\right)}+\frac{14\sqrt[3]{b}\,\left(a+bx^3\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{27a^{10/3}\sqrt{a^2+2abx^3+b^2x^6}}-\frac{7\sqrt[3]{b}\,\left(a+bx^3\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{27a^{10/3}\sqrt{a^2+2abx^3+b^2x^6}}+\frac{14\sqrt[3]{b}\,\left(a+bx^3\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}\,a^{10/3}\sqrt{a^2+2abx^3+b^2x^6}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)),x]
```

```
[Out] 7/(18*a^2*x*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) + 1/(6*a*x*(a + b*x^3)*Sqrt[a^
2 + 2*a*b*x^3 + b^2*x^6]) - (14*(a + b*x^3))/(9*a^3*x*Sqrt[a^2 + 2*a*b*x^3
+ b^2*x^6]) + (14*b^(1/3)*(a + b*x^3)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[
3]*a^(1/3))])/(9*Sqrt[3]*a^(10/3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) + (14*b^
(1/3)*(a + b*x^3)*Log[a^(1/3) + b^(1/3)*x])/(27*a^(10/3)*Sqrt[a^2 + 2*a*b*x
^3 + b^2*x^6]) - (7*b^(1/3)*(a + b*x^3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b
^(2/3)*x^2])/(27*a^(10/3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
```

```
x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\int \frac{1}{x^2\left(a^2+2abx^3+b^2x^6\right)^{3/2}}\,dx = \frac{\left(b^2\left(ab+b^2x^3\right)\right)\int \frac{1}{x^2\left(ab+b^2x^3\right)^3}\,dx}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$= \frac{1}{6ax\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}} + \frac{\left(7b\left(ab+b^2x^3\right)\right)\int \frac{1}{x^2\left(ab+b^2x^3\right)^2}\,dx}{6a\sqrt{a^2+2abx^3+b^2x^6}}$$

$$= \frac{7}{18a^2x\sqrt{a^2+2abx^3+b^2x^6}} + \frac{1}{6ax\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}} + \frac{\left(14\left(ab+b\right.\right.}{9a^2\sqrt{a^2}}$$

$$= \frac{7}{18a^2x\sqrt{a^2+2abx^3+b^2x^6}} + \frac{1}{6ax\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}} - \frac{14\left(a\right.}{9a^3x\sqrt{a^2+}}$$

$$= \frac{7}{18a^2x\sqrt{a^2+2abx^3+b^2x^6}} + \frac{1}{6ax\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}} - \frac{14\left(a\right.}{9a^3x\sqrt{a^2+}}$$

$$= \frac{7}{18a^2x\sqrt{a^2+2abx^3+b^2x^6}} + \frac{1}{6ax\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}} - \frac{14\left(a\right.}{9a^3x\sqrt{a^2+}}$$

$$= \frac{7}{18a^2x\sqrt{a^2+2abx^3+b^2x^6}} + \frac{1}{6ax\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}} - \frac{14\left(a\right.}{9a^3x\sqrt{a^2+}}$$

$$= \frac{7}{18a^2x\sqrt{a^2+2abx^3+b^2x^6}} + \frac{1}{6ax\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}} - \frac{14\left(a\right.}{9a^3x\sqrt{a^2+}}$$

Mathematica [A] time = 0.08, size = 260, normalized size = 0.82

$$\frac{-14b^{7/3}x^7\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)-28ab^{4/3}x^4\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)-147a^{4/3}bx^3-54a^{7/3}-14a^2\sqrt[3]{b}x\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)-84\sqrt[3]{a}b^2x^6+28\sqrt[3]{b}x\left(a+bx^3\right)^2\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)+28\sqrt{3}\sqrt[3]{b}x\left(a+bx^3\right)^2\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{54a^{10/3}x\left(a+bx^3\right)\sqrt{\left(a+bx^3\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)),x]
```

```
[Out] (-54*a^(7/3) - 147*a^(4/3)*b*x^3 - 84*a^(1/3)*b^2*x^6 + 28*Sqrt[3]*b^(1/3)*
x*(a + b*x^3)^2*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] + 28*b^(1/3)*x*
(a + b*x^3)^2*Log[a^(1/3) + b^(1/3)*x] - 14*a^2*b^(1/3)*x*Log[a^(2/3) - a^(
1/3)*b^(1/3)*x + b^(2/3)*x^2] - 28*a*b^(4/3)*x^4*Log[a^(2/3) - a^(1/3)*b^(1
/3)*x + b^(2/3)*x^2] - 14*b^(7/3)*x^7*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(
2/3)*x^2])/(54*a^(10/3)*x*(a + b*x^3)*Sqrt[(a + b*x^3)^2])
```

IntegrateAlgebraic [A] time = 21.14, size = 177, normalized size = 0.56

$$\frac{\left(a+bx^3\right)\left(-\frac{7\sqrt[3]{b}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{27a^{10/3}}+\frac{14\sqrt[3]{b}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{10/3}}+\frac{14\sqrt[3]{b}\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{10/3}}+\frac{-18a^2-49abx^3-28b^2x^6}{18a^3x\left(a+bx^3\right)^2}\right)}{\sqrt{\left(a+bx^3\right)^2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^2*(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)),x]
```

```
[Out] ((a + b*x^3)*((-18*a^2 - 49*a*b*x^3 - 28*b^2*x^6)/(18*a^3*x*(a + b*x^3)^2)
+ (14*b^(1/3)*ArcTan[1/Sqrt[3] - (2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(9*Sqrt[
3]*a^(10/3)) + (14*b^(1/3)*Log[a^(1/3) + b^(1/3)*x])/(27*a^(10/3)) - (7*b^(
1/3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(27*a^(10/3))))/Sqrt[(
a + b*x^3)^2]
```

fricas [A] time = 1.40, size = 201, normalized size = 0.64

$$-\frac{84\,b^{2}x^{6}+147\,abx^{3}+28\,\sqrt{3}\left(b^{2}x^{7}+2\,abx^{4}+a^{2}x\right)\left(\frac{b}{a}\right)^{\frac{1}{3}}\arctan\left(\frac{2}{3}\,\sqrt{3}x\left(\frac{b}{a}\right)^{\frac{1}{3}}-\frac{1}{3}\,\sqrt{3}\right)+14\left(b^{2}x^{7}+2\,abx^{4}+a^{2}x\right)\left(\frac{b}{a}\right)^{\frac{1}{3}}\log\left(bx^{2}-ax\left(\frac{b}{a}\right)^{\frac{2}{3}}+a\left(\frac{b}{a}\right)^{\frac{1}{3}}\right)-28\left(b^{2}x^{7}+2\,abx^{4}+a^{2}x\right)\left(\frac{b}{a}\right)^{\frac{1}{3}}\log\left(bx+a\left(\frac{b}{a}\right)^{\frac{2}{3}}\right)+54\,a^{2}}{54\left(a^{3}b^{2}x^{7}+2\,a^{4}bx^{4}+a^{5}x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/54*(84*b^2*x^6 + 147*a*b*x^3 + 28*sqrt(3)*(b^2*x^7 + 2*a*b*x^4 + a^2*x)*
(b/a)^(1/3)*arctan(2/3*sqrt(3)*x*(b/a)^(1/3) - 1/3*sqrt(3)) + 14*(b^2*x^7 +
 2*a*b*x^4 + a^2*x)*(b/a)^(1/3)*log(b*x^2 - a*x*(b/a)^(2/3) + a*(b/a)^(1/3)
) - 28*(b^2*x^7 + 2*a*b*x^4 + a^2*x)*(b/a)^(1/3)*log(b*x + a*(b/a)^(2/3)) +
 54*a^2)/(a^3*b^2*x^7 + 2*a^4*b*x^4 + a^5*x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_{0}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

maple [A] time = 0.02, size = 316, normalized size = 1.00

$$-\frac{\left(-28\sqrt{3}\,b^{2}x^{7}\arctan\left(\frac{\sqrt{3}\left(-2x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)-28b^{2}x^{7}\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)+14b^{2}x^{7}\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)+84\left(\frac{a}{b}\right)^{\frac{1}{3}}b^{2}x^{6}-56\sqrt{3}\,ab\,x^{4}\arctan\left(\frac{\sqrt{3}\left(-2x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)-56ab\,x^{4}\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)+28ab\,x^{4}\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)+147\left(\frac{a}{b}\right)^{\frac{1}{3}}ab\,x^{3}-28\sqrt{3}\,a^{2}x\arctan\left(\frac{\sqrt{3}\left(-2x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)-28a^{2}x\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)+14a^{2}x\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)+54\left(\frac{a}{b}\right)^{\frac{1}{3}}a^{2}\right)\left(b\,x^{3}+a\right)}{54\left(\frac{a}{b}\right)^{\frac{1}{3}}\left(\left(b\,x^{3}+a\right)^{2}\right)^{\frac{3}{2}}a^{3}x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x)
```

```
[Out] -1/54*(-28*3^(1/2)*arctan(1/3*3^(1/2)*(-2*x+(a/b)^(1/3))/(a/b)^(1/3))*x^7*b
^2-28*ln(x+(a/b)^(1/3))*x^7*b^2+14*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))*x^7*b^
2+84*(a/b)^(1/3)*x^6*b^2-56*3^(1/2)*arctan(1/3*3^(1/2)*(-2*x+(a/b)^(1/3))/(
a/b)^(1/3))*x^4*a*b-56*ln(x+(a/b)^(1/3))*x^4*a*b+28*ln(x^2-(a/b)^(1/3)*x+(a
/b)^(2/3))*x^4*a*b+147*(a/b)^(1/3)*x^3*a*b-28*3^(1/2)*arctan(1/3*3^(1/2)*(-
2*x+(a/b)^(1/3))/(a/b)^(1/3))*x*a^2-28*ln(x+(a/b)^(1/3))*x*a^2+14*ln(x^2-(a
/b)^(1/3)*x+(a/b)^(2/3))*x*a^2+54*(a/b)^(1/3)*a^2)*(b*x^3+a)/(a/b)^(1/3)/x/
a^3/((b*x^3+a)^2)^(3/2)
```

maxima [A] time = 1.95, size = 148, normalized size = 0.47

$$-\frac{28\,b^{2}x^{6}+49\,abx^{3}+18\,a^{2}}{18\left(a^{3}b^{2}x^{7}+2\,a^{4}bx^{4}+a^{5}x\right)}-\frac{14\,\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,a^{3}\left(\frac{a}{b}\right)^{\frac{1}{3}}}-\frac{7\,\log\left(x^{2}-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{27\,a^{3}\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{14\,\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\,a^{3}\left(\frac{a}{b}\right)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/18*(28*b^2*x^6 + 49*a*b*x^3 + 18*a^2)/(a^3*b^2*x^7 + 2*a^4*b*x^4 + a^5*x
) - 14/27*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a^3*
```

```
(a/b)^(1/3)) - 7/27*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a^3*(a/b)^(1/3)
) + 14/27*log(x + (a/b)^(1/3))/(a^3*(a/b)^(1/3))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^2 \left(a^2 + 2\,a\,b\,x^3 + b^2\,x^6\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2)),x)
[Out] int(1/(x^2*(a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2 \left(\left(a + bx^3\right)^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b**2*x**6+2*a*b*x**3+a**2)**(3/2),x)
[Out] Integral(1/(x**2*((a + b*x**3)**2)**(3/2)), x)
```

3.105 $\int \frac{1}{x^3\left(a^2+2abx^3+b^2x^6\right)^{3/2}}\,dx$

Optimal. Leaf size=316

$$\frac{4}{9a^2x^2\sqrt{a^2+2abx^3+b^2x^6}}+\frac{1}{6ax^2\sqrt{a^2+2abx^3+b^2x^6}\left(a+bx^3\right)}-\frac{20b^{2/3}\left(a+bx^3\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{11/3}\sqrt{a^2+2abx^3+b^2x^6}}+\frac{20b^{2/3}\left(a\right.}{9\sqrt{3}\,a^{1}}$$

Rubi [A] time = 0.16, antiderivative size = 316, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.346, Rules used = {1355, 290, 325, 200, 31, 634, 617, 204, 628}

$$-\frac{10\left(a+bx^3\right)}{9a^3x^2\sqrt{a^2+2abx^3+b^2x^6}}+\frac{4}{9a^2x^2\sqrt{a^2+2abx^3+b^2x^6}}+\frac{1}{6ax^2\sqrt{a^2+2abx^3+b^2x^6}\left(a+bx^3\right)}-\frac{20b^{2/3}\left(a+bx^3\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{11/3}\sqrt{a^2+2abx^3+b^2x^6}}+\frac{10b^{2/3}\left(a+bx^3\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{27a^{11/3}\sqrt{a^2+2abx^3+b^2x^6}}+\frac{20b^{2/3}\left(a+bx^3\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}\,a^{11/3}\sqrt{a^2+2abx^3+b^2x^6}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)),x]
```

```
[Out] 4/(9*a^2*x^2*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) + 1/(6*a*x^2*(a + b*x^3)*Sqrt
[a^2 + 2*a*b*x^3 + b^2*x^6]) - (10*(a + b*x^3))/(9*a^3*x^2*Sqrt[a^2 + 2*a*b
*x^3 + b^2*x^6]) + (20*b^(2/3)*(a + b*x^3)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(
Sqrt[3]*a^(1/3))])/(9*Sqrt[3]*a^(11/3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) - (
20*b^(2/3)*(a + b*x^3)*Log[a^(1/3) + b^(1/3)*x])/(27*a^(11/3)*Sqrt[a^2 + 2*
a*b*x^3 + b^2*x^6]) + (10*b^(2/3)*(a + b*x^3)*Log[a^(2/3) - a^(1/3)*b^(1/3)
*x + b^(2/3)*x^2])/(27*a^(11/3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
```

```
x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\int \frac{1}{x^3\left(a^2+2abx^3+b^2x^6\right)^{3/2}}\,dx = \frac{\left(b^2\left(ab+b^2x^3\right)\right)\int \frac{1}{x^3\left(ab+b^2x^3\right)^3}\,dx}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$= \frac{1}{6ax^2\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}} + \frac{\left(4b\left(ab+b^2x^3\right)\right)\int \frac{1}{x^3\left(ab+b^2x^3\right)^2}\,dx}{3a\sqrt{a^2+2abx^3+b^2x^6}}$$

$$= \frac{4}{9a^2x^2\sqrt{a^2+2abx^3+b^2x^6}} + \frac{1}{6ax^2\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}} + \frac{\left(20\left(ab\right.\right.}{9a^2}$$

$$= \frac{4}{9a^2x^2\sqrt{a^2+2abx^3+b^2x^6}} + \frac{1}{6ax^2\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}} - \frac{1}{9a^3x^2\sqrt{}}$$

$$= \frac{4}{9a^2x^2\sqrt{a^2+2abx^3+b^2x^6}} + \frac{1}{6ax^2\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}} - \frac{1}{9a^3x^2\sqrt{}}$$

$$= \frac{4}{9a^2x^2\sqrt{a^2+2abx^3+b^2x^6}} + \frac{1}{6ax^2\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}} - \frac{1}{9a^3x^2\sqrt{}}$$

$$= \frac{4}{9a^2x^2\sqrt{a^2+2abx^3+b^2x^6}} + \frac{1}{6ax^2\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}} - \frac{1}{9a^3x^2\sqrt{}}$$

$$= \frac{4}{9a^2x^2\sqrt{a^2+2abx^3+b^2x^6}} + \frac{1}{6ax^2\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}} - \frac{1}{9a^3x^2\sqrt{}}$$

Mathematica [A] time = 0.09, size = 266, normalized size = 0.84

$$\frac{20b^{8/3}x^8\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)+40ab^{5/3}x^5\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)-60a^{2/3}b^2x^6-96a^{5/3}bx^3-27a^{8/3}+20a^2b^{2/3}x^2\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)-40b^{2/3}x^2\left(a+bx^3\right)^2\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)+40\sqrt{3}b^{2/3}x^2\left(a+bx^3\right)^2\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{54a^{11/3}x^2\left(a+bx^3\right)\sqrt{\left(a+bx^3\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)),x]
```

```
[Out] (-27*a^(8/3) - 96*a^(5/3)*b*x^3 - 60*a^(2/3)*b^2*x^6 + 40*Sqrt[3]*b^(2/3)*x
^2*(a + b*x^3)^2*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] - 40*b^(2/3)*x
^2*(a + b*x^3)^2*Log[a^(1/3) + b^(1/3)*x] + 20*a^2*b^(2/3)*x^2*Log[a^(2/3)
- a^(1/3)*b^(1/3)*x + b^(2/3)*x^2] + 40*a*b^(5/3)*x^5*Log[a^(2/3) - a^(1/3)
*b^(1/3)*x + b^(2/3)*x^2] + 20*b^(8/3)*x^8*Log[a^(2/3) - a^(1/3)*b^(1/3)*x
+ b^(2/3)*x^2])/(54*a^(11/3)*x^2*(a + b*x^3)*Sqrt[(a + b*x^3)^2])
```

IntegrateAlgebraic [A] time = 24.32, size = 177, normalized size = 0.56

$$\frac{\left(a+bx^3\right)\left(\frac{10b^{2/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{27a^{11/3}}-\frac{20b^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{11/3}}+\frac{20b^{2/3}\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{11/3}}+\frac{-9a^2-32abx^3-20b^2x^6}{18a^3x^2\left(a+bx^3\right)^2}\right)}{\sqrt{\left(a+bx^3\right)^2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^3*(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)),x]
```

```
[Out] ((a + b*x^3)*((-9*a^2 - 32*a*b*x^3 - 20*b^2*x^6)/(18*a^3*x^2*(a + b*x^3)^2)
 + (20*b^(2/3)*ArcTan[1/Sqrt[3] - (2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(9*Sqrt
[3]*a^(11/3)) - (20*b^(2/3)*Log[a^(1/3) + b^(1/3)*x])/(27*a^(11/3)) + (10*b
^(2/3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(27*a^(11/3))))/Sqrt
[(a + b*x^3)^2]
```

fricas [A] time = 1.40, size = 242, normalized size = 0.77

$$-\frac{60\,b^{2}x^{6}+96\,abx^{3}-40\,\sqrt{3}\left(b^{2}x^{8}+2\,abx^{5}+a^{2}x^{2}\right)\left(-\frac{b^{2}}{a^{2}}\right)^{\frac{1}{3}}\arctan\left(\frac{2\,\sqrt{3}\,ax\left(-\frac{b^{2}}{a^{2}}\right)^{\frac{2}{3}}-\sqrt{3}\,b}{3\,b}\right)+20\left(b^{2}x^{8}+2\,abx^{5}+a^{2}x^{2}\right)\left(-\frac{b^{2}}{a^{2}}\right)^{\frac{1}{3}}\log\left(b^{2}x^{2}+abx\left(-\frac{b^{2}}{a^{2}}\right)^{\frac{1}{3}}+a^{2}\left(-\frac{b^{2}}{a^{2}}\right)^{\frac{2}{3}}\right)-40\left(b^{2}x^{8}+2\,abx^{5}+a^{2}x^{2}\right)\left(-\frac{b^{2}}{a^{2}}\right)^{\frac{1}{3}}\log\left(bx-a\left(-\frac{b^{2}}{a^{2}}\right)^{\frac{1}{3}}\right)+27\,a^{2}}{54\left(a^{3}b^{2}x^{8}+2\,a^{4}bx^{5}+a^{5}x^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/54*(60*b^2*x^6 + 96*a*b*x^3 - 40*sqrt(3)*(b^2*x^8 + 2*a*b*x^5 + a^2*x^2)
*(-b^2/a^2)^(1/3)*arctan(1/3*(2*sqrt(3)*a*x*(-b^2/a^2)^(2/3) - sqrt(3)*b)/b
) + 20*(b^2*x^8 + 2*a*b*x^5 + a^2*x^2)*(-b^2/a^2)^(1/3)*log(b^2*x^2 + a*b*x
*(-b^2/a^2)^(1/3) + a^2*(-b^2/a^2)^(2/3)) - 40*(b^2*x^8 + 2*a*b*x^5 + a^2*x
^2)*(-b^2/a^2)^(1/3)*log(b*x - a*(-b^2/a^2)^(1/3)) + 27*a^2)/(a^3*b^2*x^8 +
 2*a^4*b*x^5 + a^5*x^2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_{0}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

maple [A] time = 0.02, size = 322, normalized size = 1.02

$$-\frac{\left(-40\sqrt{3}\,b^{2}x^{8}\arctan\left(\frac{\sqrt{3}\left(-2x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)+40b^{2}x^{8}\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)-20b^{2}x^{8}\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)-80\sqrt{3}\,ab\,x^{5}\arctan\left(\frac{\sqrt{3}\left(-2x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)+80ab\,x^{5}\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)-40ab\,x^{5}\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)+60\left(\frac{a}{b}\right)^{\frac{2}{3}}b^{2}x^{6}-40\sqrt{3}\,a^{2}x^{2}\arctan\left(\frac{\sqrt{3}\left(-2x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)+40a^{2}x^{2}\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)-20a^{2}x^{2}\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)+96\left(\frac{a}{b}\right)^{\frac{2}{3}}ab\,x^{3}+27\left(\frac{a}{b}\right)^{\frac{2}{3}}a^{2}\right)\left(b\,x^{3}+a\right)}{54\left(\frac{a}{b}\right)^{\frac{2}{3}}\left(\left(b\,x^{3}+a\right)^{2}\right)^{\frac{3}{2}}a^{3}x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x)
```

```
[Out] -1/54*(-40*3^(1/2)*arctan(1/3*3^(1/2)*(-2*x+(a/b)^(1/3))/(a/b)^(1/3))*x^8*b
^2+40*ln(x+(a/b)^(1/3))*x^8*b^2-20*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))*x^8*b^
2+60*(a/b)^(2/3)*x^6*b^2-80*3^(1/2)*arctan(1/3*3^(1/2)*(-2*x+(a/b)^(1/3))/(
a/b)^(1/3))*x^5*a*b+80*ln(x+(a/b)^(1/3))*x^5*a*b-40*ln(x^2-(a/b)^(1/3)*x+(a
/b)^(2/3))*x^5*a*b+96*(a/b)^(2/3)*x^3*a*b-40*3^(1/2)*arctan(1/3*3^(1/2)*(-2
*x+(a/b)^(1/3))/(a/b)^(1/3))*x^2*a^2+40*ln(x+(a/b)^(1/3))*x^2*a^2-20*ln(x^2
-(a/b)^(1/3)*x+(a/b)^(2/3))*x^2*a^2+27*(a/b)^(2/3)*a^2)*(b*x^3+a)/(a/b)^(2/
3)/x^2/a^3/((b*x^3+a)^2)^(3/2)
```

maxima [A] time = 1.80, size = 150, normalized size = 0.47

$$-\frac{20\,b^{2}x^{6}+32\,abx^{3}+9\,a^{2}}{18\left(a^{3}b^{2}x^{8}+2\,a^{4}bx^{5}+a^{5}x^{2}\right)}-\frac{20\,\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,a^{3}\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{10\,\log\left(x^{2}-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{27\,a^{3}\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{20\,\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\,a^{3}\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/18*(20*b^2*x^6 + 32*a*b*x^3 + 9*a^2)/(a^3*b^2*x^8 + 2*a^4*b*x^5 + a^5*x^
2) - 20/27*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a^3
```

```
*(a/b)^(2/3)) + 10/27*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a^3*(a/b)^(2/
3)) - 20/27*log(x + (a/b)^(1/3))/(a^3*(a/b)^(2/3))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^3 \left(a^2 + 2\,a\,b\,x^3 + b^2\,x^6\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2)),x)
```

```
[Out] int(1/(x^3*(a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3 \left(\left(a + bx^3\right)^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(b**2*x**6+2*a*b*x**3+a**2)**(3/2),x)
```

```
[Out] Integral(1/(x**3*((a + b*x**3)**2)**(3/2)), x)
```

3.106 $\int \frac{1}{x^4\left(a^2+2abx^3+b^2x^6\right)^{3/2}}\,dx$

Optimal. Leaf size=188

$$-\frac{b}{6a^2\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}}-\frac{3b\log(x)\left(a+bx^3\right)}{a^4\sqrt{a^2+2abx^3+b^2x^6}}+\frac{b\left(a+bx^3\right)\log\left(a+bx^3\right)}{a^4\sqrt{a^2+2abx^3+b^2x^6}}-\frac{2b}{3a^3\sqrt{a^2+2abx^3+b^2x^6}}$$

Rubi [A] time = 0.10, antiderivative size = 188, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {1355, 266, 44}

$$-\frac{b}{6a^2\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}}-\frac{2b}{3a^3\sqrt{a^2+2abx^3+b^2x^6}}-\frac{a+bx^3}{3a^3x^3\sqrt{a^2+2abx^3+b^2x^6}}-\frac{3b\log(x)\left(a+bx^3\right)}{a^4\sqrt{a^2+2abx^3+b^2x^6}}+\frac{b\left(a+bx^3\right)\log\left(a+bx^3\right)}{a^4\sqrt{a^2+2abx^3+b^2x^6}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)),x]
```

```
[Out] (-2*b)/(3*a^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) - b/(6*a^2*(a + b*x^3)*Sqrt[
a^2 + 2*a*b*x^3 + b^2*x^6]) - (a + b*x^3)/(3*a^3*x^3*Sqrt[a^2 + 2*a*b*x^3 +
 b^2*x^6]) - (3*b*(a + b*x^3)*Log[x])/(a^4*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])
 + (b*(a + b*x^3)*Log[a + b*x^3])/(a^4*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\int \frac{1}{x^4\left(a^2+2abx^3+b^2x^6\right)^{3/2}}\,dx = \frac{\left(b^2\left(ab+b^2x^3\right)\right)\int \frac{1}{x^4\left(ab+b^2x^3\right)^3}\,dx}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$= \frac{\left(b^2\left(ab+b^2x^3\right)\right)\text{Subst}\left(\int \frac{1}{x^2\left(ab+b^2x\right)^3}\,dx, x, x^3\right)}{3\sqrt{a^2+2abx^3+b^2x^6}}$$

$$= \frac{\left(b^2\left(ab+b^2x^3\right)\right)\text{Subst}\left(\int \left(\frac{1}{a^3b^3x^2}-\frac{3}{a^4b^2x}+\frac{1}{a^2b(a+bx)^3}+\frac{2}{a^3b(a+bx)^2}+\frac{3}{a^4b(a+bx)}\right.\right.}{3\sqrt{a^2+2abx^3+b^2x^6}}$$

$$= -\frac{2b}{3a^3\sqrt{a^2+2abx^3+b^2x^6}} - \frac{b}{6a^2\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}} - \frac{}{3a^3x^3\sqrt{a^2}}$$

Mathematica [A] time = 0.03, size = 97, normalized size = 0.52

$$\frac{-a\left(2a^2+9abx^3+6b^2x^6\right)-18bx^3\log(x)\left(a+bx^3\right)^2+6bx^3\left(a+bx^3\right)^2\log\left(a+bx^3\right)}{6a^4x^3\left(a+bx^3\right)\sqrt{\left(a+bx^3\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)),x]
```

```
[Out] (-(a*(2*a^2 + 9*a*b*x^3 + 6*b^2*x^6)) - 18*b*x^3*(a + b*x^3)^2*Log[x] + 6*b
*x^3*(a + b*x^3)^2*Log[a + b*x^3])/(6*a^4*x^3*(a + b*x^3)*Sqrt[(a + b*x^3)^
2])
```

IntegrateAlgebraic [B] time = 3.84, size = 796, normalized size = 4.23

[illegible]

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^4*(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)),x]
```

```
[Out] (-(a^16*b) + 2*a^15*b^2*x^3 + 61*a^14*b^3*x^6 + 864*a^13*b^4*x^9 + 7540*a^1
2*b^5*x^12 + 45344*a^11*b^6*x^15 + 199056*a^10*b^7*x^18 + 658944*a^9*b^8*x^
21 + 1674816*a^8*b^9*x^24 + 3294720*a^7*b^10*x^27 + 5015296*a^6*b^11*x^30 +
 5857280*a^5*b^12*x^33 + 5151744*a^4*b^13*x^36 + 3301376*a^3*b^14*x^39 + 14
54080*a^2*b^15*x^42 + 393216*a*b^16*x^45 + 49152*b^17*x^48 + Sqrt[b^2]*Sqrt
[a^2 + 2*a*b*x^3 + b^2*x^6]*(-a^15 - a^14*b*x^3 - 60*a^13*b^2*x^6 - 804*a^1
2*b^3*x^9 - 6736*a^11*b^4*x^12 - 38608*a^10*b^5*x^15 - 160448*a^9*b^6*x^18
- 498496*a^8*b^7*x^21 - 1176320*a^7*b^8*x^24 - 2118400*a^6*b^9*x^27 - 28968
96*a^5*b^10*x^30 - 2960384*a^4*b^11*x^33 - 2191360*a^3*b^12*x^36 - 1110016*
a^2*b^13*x^39 - 344064*a*b^14*x^42 - 49152*b^15*x^45))/(3*a^3*x^6*Sqrt[a^2
+ 2*a*b*x^3 + b^2*x^6]*(-2*a^14*b^2 - 54*a^13*b^3*x^3 - 676*a^12*b^4*x^6 -
5200*a^11*b^5*x^9 - 27456*a^10*b^6*x^12 - 105248*a^9*b^7*x^15 - 302016*a^8*
b^8*x^18 - 658944*a^7*b^9*x^21 - 1098240*a^6*b^10*x^24 - 1391104*a^5*b^11*x
^27 - 1317888*a^4*b^12*x^30 - 905216*a^3*b^13*x^33 - 425984*a^2*b^14*x^36 -
 122880*a*b^15*x^39 - 16384*b^16*x^42) + 3*a^3*Sqrt[b^2]*x^6*(2*a^15*b + 56
*a^14*b^2*x^3 + 730*a^13*b^3*x^6 + 5876*a^12*b^4*x^9 + 32656*a^11*b^5*x^12
+ 132704*a^10*b^6*x^15 + 407264*a^9*b^7*x^18 + 960960*a^8*b^8*x^21 + 175718
4*a^7*b^9*x^24 + 2489344*a^6*b^10*x^27 + 2708992*a^5*b^11*x^30 + 2223104*a^
4*b^12*x^33 + 1331200*a^3*b^13*x^36 + 548864*a^2*b^14*x^39 + 139264*a*b^15*
x^42 + 16384*b^16*x^45)) - (2*b*ArcTanh[(Sqrt[b^2]*x^3)/a - Sqrt[a^2 + 2*a*
b*x^3 + b^2*x^6]/a])/a^4
```

fricas [A] time = 1.27, size = 119, normalized size = 0.63

$$-\frac{6\,ab^2x^6+9\,a^2bx^3+2\,a^3-6\left(b^3x^9+2\,ab^2x^6+a^2bx^3\right)\log\left(bx^3+a\right)+18\left(b^3x^9+2\,ab^2x^6+a^2bx^3\right)\log(x)}{6\left(a^4b^2x^9+2\,a^5bx^6+a^6x^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/6*(6*a*b^2*x^6 + 9*a^2*b*x^3 + 2*a^3 - 6*(b^3*x^9 + 2*a*b^2*x^6 + a^2*b*
x^3)*log(b*x^3 + a) + 18*(b^3*x^9 + 2*a*b^2*x^6 + a^2*b*x^3)*log(x))/(a^4*b
^2*x^9 + 2*a^5*b*x^6 + a^6*x^3)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

maple [A] time = 0.02, size = 133, normalized size = 0.71

$$-\frac{\left(18b^3x^9\ln(x)-6b^3x^9\ln\left(bx^3+a\right)+36a\,b^2x^6\ln(x)-12a\,b^2x^6\ln\left(bx^3+a\right)+6a\,b^2x^6+18a^2bx^3\ln(x)-6a^2bx^3\ln\left(bx^3+a\right)+9a^2bx^3+2a^3\right)\left(bx^3+a\right)}{6\left(\left(bx^3+a\right)^2\right)^{\frac{3}{2}}a^4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x)
```

```
[Out] -1/6*(18*b^3*x^9*ln(x)-6*ln(b*x^3+a)*x^9*b^3+36*a*b^2*x^6*ln(x)-12*ln(b*x^3
+a)*x^6*a*b^2+6*a*b^2*x^6+18*a^2*b*x^3*ln(x)-6*ln(b*x^3+a)*x^3*a^2*b+9*a^2*
b*x^3+2*a^3)*(b*x^3+a)/x^3/a^4/((b*x^3+a)^2)^(3/2)
```

maxima [A] time = 0.99, size = 117, normalized size = 0.62

$$\frac{(-1)^{2\,abx^3+2\,a^2}b\log\left(\frac{2\,abx}{|x|}+\frac{2\,a^2}{x^2|x|}\right)}{a^4}-\frac{b}{\sqrt{b^2x^6+2\,abx^3+a^2}\,a^3}-\frac{1}{6\left(x^3+\frac{a}{b}\right)^2a^2b}-\frac{1}{3\sqrt{b^2x^6+2\,abx^3+a^2}\,a^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="maxima")
```

```
[Out] (-1)^(2*a*b*x^3 + 2*a^2)*b*log(2*a*b*x/abs(x) + 2*a^2/(x^2*abs(x)))/a^4 - b
/(sqrt(b^2*x^6 + 2*a*b*x^3 + a^2)*a^3) - 1/6/((x^3 + a/b)^2*a^2*b) - 1/3/(s
qrt(b^2*x^6 + 2*a*b*x^3 + a^2)*a^2*x^3)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^4\left(a^2+2\,a\,b\,x^3+b^2\,x^6\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2)),x)
```

```
[Out] int(1/(x^4*(a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^4\left(\left(a+bx^3\right)^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(b**2*x**6+2*a*b*x**3+a**2)**(3/2),x)
```

```
[Out] Integral(1/(x**4*((a + b*x**3)**2)**(3/2)), x)
```

3.107 $\int \frac{x^6}{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}\,dx$

Optimal. Leaf size=359

$$\frac{x}{162ab^2\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}}+\frac{5x}{486a^2b^2\sqrt{a^2+2abx^3+b^2x^6}}-\frac{x}{27b^2\left(a+bx^3\right)^2\sqrt{a^2+2abx^3+b^2x^6}}-\frac{}{12b}$$

Rubi [A] time = 0.19, antiderivative size = 359, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 9, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.346, Rules used = {1355, 288, 199, 200, 31, 634, 617, 204, 628}

$$-\frac{x^4}{12b\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}}+\frac{x}{162ab^2\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}}+\frac{5x}{486a^2b^2\sqrt{a^2+2abx^3+b^2x^6}}-\frac{x}{27b^2\left(a+bx^3\right)^2\sqrt{a^2+2abx^3+b^2x^6}}+\frac{5\left(a+bx^3\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{729a^{8/3}b^{7/3}\sqrt{a^2+2abx^3+b^2x^6}}-\frac{5\left(a+bx^3\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{1458a^{8/3}b^{7/3}\sqrt{a^2+2abx^3+b^2x^6}}-\frac{5\left(a+bx^3\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{243\sqrt{3}a^{8/3}b^{7/3}\sqrt{a^2+2abx^3+b^2x^6}}$$

Antiderivative was successfully verified.

```
[In] Int[x^6/(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
```

```
[Out] (5*x)/(486*a^2*b^2*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) - x^4/(12*b*(a + b*x^3)
^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) - x/(27*b^2*(a + b*x^3)^2*Sqrt[a^2 + 2*
a*b*x^3 + b^2*x^6]) + x/(162*a*b^2*(a + b*x^3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x
^6]) - (5*(a + b*x^3)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(2
43*Sqrt[3]*a^(8/3)*b^(7/3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) + (5*(a + b*x^3
)*Log[a^(1/3) + b^(1/3)*x])/(729*a^(8/3)*b^(7/3)*Sqrt[a^2 + 2*a*b*x^3 + b^2
*x^6]) - (5*(a + b*x^3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(14
58*a^(8/3)*b^(7/3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
```

```
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\int \frac{x^6}{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}\,dx = \frac{\left(b^4\left(ab+b^2x^3\right)\right)\int \frac{x^6}{\left(ab+b^2x^3\right)^5}\,dx}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$= -\frac{x^4}{12b\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}} + \frac{\left(b^2\left(ab+b^2x^3\right)\right)\int \frac{x^3}{\left(ab+b^2x^3\right)^4}\,dx}{3\sqrt{a^2+2abx^3+b^2x^6}}$$

$$= -\frac{x^4}{12b\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}} - \frac{x}{27b^2\left(a+bx^3\right)^2\sqrt{a^2+2abx^3+b^2x^6}} + \cdots$$

$$= -\frac{x^4}{12b\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}} - \frac{x}{27b^2\left(a+bx^3\right)^2\sqrt{a^2+2abx^3+b^2x^6}} + \cdots$$

$$= \frac{5x}{486a^2b^2\sqrt{a^2+2abx^3+b^2x^6}} - \frac{x^4}{12b\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}} - \cdots$$

$$= \frac{5x}{486a^2b^2\sqrt{a^2+2abx^3+b^2x^6}} - \frac{x^4}{12b\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}} - \cdots$$

$$= \frac{5x}{486a^2b^2\sqrt{a^2+2abx^3+b^2x^6}} - \frac{x^4}{12b\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}} - \cdots$$

$$= \frac{5x}{486a^2b^2\sqrt{a^2+2abx^3+b^2x^6}} - \frac{x^4}{12b\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}} - \cdots$$

$$= \frac{5x}{486a^2b^2\sqrt{a^2+2abx^3+b^2x^6}} - \frac{x^4}{12b\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}} - \cdots$$

Mathematica [A]   time = 0.13, size = 218, normalized size = 0.61

$$\frac{\left(a+bx^3\right)\left(-\frac{10\left(a+bx^3\right)^4\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{a^{8/3}}+\frac{20\left(a+bx^3\right)^4\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{a^{8/3}}+\frac{20\sqrt{3}\left(a+bx^3\right)^4\tan^{-1}\left(\frac{2\sqrt[3]{b}x-\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{a^{8/3}}+\frac{30\sqrt[3]{b}x\left(a+bx^3\right)^3}{a^2}+\frac{18\sqrt[3]{b}x\left(a+bx^3\right)^2}{a}-351\sqrt[3]{b}x\left(a+bx^3\right)+243a\sqrt[3]{b}x\right)}{2916b^{7/3}\left(\left(a+bx^3\right)^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6/(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
```

```
[Out] ((a + b*x^3)*(243*a*b^(1/3)*x - 351*b^(1/3)*x*(a + b*x^3) + (18*b^(1/3)*x*(
a + b*x^3)^2)/a + (30*b^(1/3)*x*(a + b*x^3)^3)/a^2 + (20*Sqrt[3]*(a + b*x^3
)^4*ArcTan[(-a^(1/3) + 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/a^(8/3) + (20*(a +
b*x^3)^4*Log[a^(1/3) + b^(1/3)*x])/a^(8/3) - (10*(a + b*x^3)^4*Log[a^(2/3)
- a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/a^(8/3)))/(2916*b^(7/3)*((a + b*x^3)^2)
^(5/2))
```

IntegrateAlgebraic [A] time = 24.92, size = 189, normalized size = 0.53

$$\frac{\left(a+bx^3\right)\left(-\frac{5\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{1458a^{8/3}b^{7/3}}+\frac{5\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{729a^{8/3}b^{7/3}}-\frac{5\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{243\sqrt{3}a^{8/3}b^{7/3}}+\frac{-20a^3x-75a^2bx^4+36ab^2x^7+10b^3x^{10}}{972a^2b^2\left(a+bx^3\right)^4}\right)}{\sqrt{\left(a+bx^3\right)^2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^6/(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
```

```
[Out] ((a + b*x^3)*((-20*a^3*x - 75*a^2*b*x^4 + 36*a*b^2*x^7 + 10*b^3*x^10)/(972*
a^2*b^2*(a + b*x^3)^4) - (5*ArcTan[1/Sqrt[3] - (2*b^(1/3)*x)/(Sqrt[3]*a^(1/
3))])/(243*Sqrt[3]*a^(8/3)*b^(7/3)) + (5*Log[a^(1/3) + b^(1/3)*x])/(729*a^(
8/3)*b^(7/3)) - (5*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(1458*a^
(8/3)*b^(7/3))))/Sqrt[(a + b*x^3)^2]
```

fricas [A] time = 0.78, size = 723, normalized size = 2.01

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/2916*(30*a^2*b^4*x^10 + 108*a^3*b^3*x^7 - 225*a^4*b^2*x^4 - 60*a^5*b*x +
 30*sqrt(1/3)*(a*b^5*x^12 + 4*a^2*b^4*x^9 + 6*a^3*b^3*x^6 + 4*a^4*b^2*x^3 +
 a^5*b)*sqrt(-(a^2*b)^(1/3)/b)*log((2*a*b*x^3 - 3*(a^2*b)^(1/3)*a*x - a^2 +
 3*sqrt(1/3)*(2*a*b*x^2 + (a^2*b)^(2/3)*x - (a^2*b)^(1/3)*a)*sqrt(-(a^2*b)^
(1/3)/b))/(b*x^3 + a)) - 10*(b^4*x^12 + 4*a*b^3*x^9 + 6*a^2*b^2*x^6 + 4*a^3
*b*x^3 + a^4)*(a^2*b)^(2/3)*log(a*b*x^2 - (a^2*b)^(2/3)*x + (a^2*b)^(1/3)*a
) + 20*(b^4*x^12 + 4*a*b^3*x^9 + 6*a^2*b^2*x^6 + 4*a^3*b*x^3 + a^4)*(a^2*b)
^(2/3)*log(a*b*x + (a^2*b)^(2/3)))/(a^4*b^7*x^12 + 4*a^5*b^6*x^9 + 6*a^6*b^
5*x^6 + 4*a^7*b^4*x^3 + a^8*b^3), 1/2916*(30*a^2*b^4*x^10 + 108*a^3*b^3*x^7
 - 225*a^4*b^2*x^4 - 60*a^5*b*x + 60*sqrt(1/3)*(a*b^5*x^12 + 4*a^2*b^4*x^9
+ 6*a^3*b^3*x^6 + 4*a^4*b^2*x^3 + a^5*b)*sqrt((a^2*b)^(1/3)/b)*arctan(sqrt(
1/3)*(2*(a^2*b)^(2/3)*x - (a^2*b)^(1/3)*a)*sqrt((a^2*b)^(1/3)/b)/a^2) - 10*
(b^4*x^12 + 4*a*b^3*x^9 + 6*a^2*b^2*x^6 + 4*a^3*b*x^3 + a^4)*(a^2*b)^(2/3)*
log(a*b*x^2 - (a^2*b)^(2/3)*x + (a^2*b)^(1/3)*a) + 20*(b^4*x^12 + 4*a*b^3*x
^9 + 6*a^2*b^2*x^6 + 4*a^3*b*x^3 + a^4)*(a^2*b)^(2/3)*log(a*b*x + (a^2*b)^(
2/3)))/(a^4*b^7*x^12 + 4*a^5*b^6*x^9 + 6*a^6*b^5*x^6 + 4*a^7*b^4*x^3 + a^8*
b^3)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

maple [B] time = 0.02, size = 519, normalized size = 1.45

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x)
```

```
[Out] 1/2916*(-20*3^(1/2)*arctan(1/3*3^(1/2)*(-2*x+(a/b)^(1/3))/(a/b)^(1/3))*x^12
*b^4+20*ln(x+(a/b)^(1/3))*x^12*b^4-10*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))*x^1
2*b^4+30*(a/b)^(2/3)*x^10*b^4-80*3^(1/2)*arctan(1/3*3^(1/2)*(-2*x+(a/b)^(1/
3))/(a/b)^(1/3))*x^9*a*b^3+80*ln(x+(a/b)^(1/3))*x^9*a*b^3-40*ln(x^2-(a/b)^(
1/3)*x+(a/b)^(2/3))*x^9*a*b^3+108*(a/b)^(2/3)*x^7*a*b^3-120*3^(1/2)*arctan(
1/3*3^(1/2)*(-2*x+(a/b)^(1/3))/(a/b)^(1/3))*x^6*a^2*b^2+120*ln(x+(a/b)^(1/3
))*x^6*a^2*b^2-60*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))*x^6*a^2*b^2-225*(a/b)^(
2/3)*x^4*a^2*b^2-80*3^(1/2)*arctan(1/3*3^(1/2)*(-2*x+(a/b)^(1/3))/(a/b)^(1/
3))*x^3*a^3*b+80*ln(x+(a/b)^(1/3))*x^3*a^3*b-40*ln(x^2-(a/b)^(1/3)*x+(a/b)^
(2/3))*x^3*a^3*b-60*(a/b)^(2/3)*x*a^3*b-20*3^(1/2)*arctan(1/3*3^(1/2)*(-2*x
+(a/b)^(1/3))/(a/b)^(1/3))*a^4+20*ln(x+(a/b)^(1/3))*a^4-10*ln(x^2-(a/b)^(1/
3)*x+(a/b)^(2/3))*a^4)*(b*x^3+a)/(a/b)^(2/3)/b^3/a^2/((b*x^3+a)^2)^(5/2)
```

maxima [A] time = 1.35, size = 195, normalized size = 0.54

$$\frac{10\,b^3x^{10}+36\,ab^2x^7-75\,a^2bx^4-20\,a^3x}{972\left(a^2b^6x^{12}+4\,a^3b^5x^9+6\,a^4b^4x^6+4\,a^5b^3x^3+a^6b^2\right)}+\frac{5\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{729\,a^2b^3\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{5\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{1458\,a^2b^3\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{5\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{729\,a^2b^3\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/972*(10*b^3*x^10 + 36*a*b^2*x^7 - 75*a^2*b*x^4 - 20*a^3*x)/(a^2*b^6*x^12
+ 4*a^3*b^5*x^9 + 6*a^4*b^4*x^6 + 4*a^5*b^3*x^3 + a^6*b^2) + 5/729*sqrt(3)*
arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a^2*b^3*(a/b)^(2/3)) -
 5/1458*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a^2*b^3*(a/b)^(2/3)) + 5/72
9*log(x + (a/b)^(1/3))/(a^2*b^3*(a/b)^(2/3))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^6}{{\left(a^2+2\,a\,b\,x^3+b^2\,x^6\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2),x)
```

```
[Out] int(x^6/(a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{6}}{\left(\left(a + b x^{3}\right)^{2}\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(b**2*x**6+2*a*b*x**3+a**2)**(5/2),x)
```

```
[Out] Integral(x**6/((a + b*x**3)**2)**(5/2), x)
```

3.108 $\int \frac{x^5}{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}\,dx$

Optimal. Leaf size=78

$$\frac{a}{12b^2\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}}-\frac{1}{9b^2\left(a+bx^3\right)^2\sqrt{a^2+2abx^3+b^2x^6}}$$

Rubi [A] time = 0.05, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1355, 266, 43}

$$\frac{a}{12b^2\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}}-\frac{1}{9b^2\left(a+bx^3\right)^2\sqrt{a^2+2abx^3+b^2x^6}}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
```

```
[Out] a/(12*b^2*(a + b*x^3)^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) - 1/(9*b^2*(a + b*
x^3)^2*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^5}{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}\,dx &= \frac{\left(b^4\left(ab+b^2x^3\right)\right)\int \frac{x^5}{\left(ab+b^2x^3\right)^5}\,dx}{\sqrt{a^2+2abx^3+b^2x^6}}\\
&= \frac{\left(b^4\left(ab+b^2x^3\right)\right)\text{Subst}\left(\int \frac{x}{\left(ab+b^2x\right)^5}\,dx,x,x^3\right)}{3\sqrt{a^2+2abx^3+b^2x^6}}\\
&= \frac{\left(b^4\left(ab+b^2x^3\right)\right)\text{Subst}\left(\int \left(-\frac{a}{b^6(a+bx)^5}+\frac{1}{b^6(a+bx)^4}\right)dx,x,x^3\right)}{3\sqrt{a^2+2abx^3+b^2x^6}}\\
&= \frac{a}{12b^2\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}}-\frac{1}{9b^2\left(a+bx^3\right)^2\sqrt{a^2+2abx^3+b^2x^6}}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 39, normalized size = 0.50

$$\frac{-a-4bx^3}{36b^2\left(a+bx^3\right)^3\sqrt{\left(a+bx^3\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
```

```
[Out] (-a - 4*b*x^3)/(36*b^2*(a + b*x^3)^3*Sqrt[(a + b*x^3)^2])
```

IntegrateAlgebraic [B] time = 0.77, size = 229, normalized size = 2.94

$$\frac{-2\left(3a^5b-ab^5x^{12}-4b^6x^{15}\right)-2\sqrt{b^2}\sqrt{a^2+2abx^3+b^2x^6}\left(3a^4-3a^3bx^3+3a^2b^2x^6-3ab^3x^9+4b^4x^{12}\right)}{9x^{12}\sqrt{a^2+2abx^3+b^2x^6}\left(-8a^3b^7-24a^2b^8x^3-24ab^9x^6-8b^{10}x^9\right)+9\sqrt{b^2}x^{12}\left(8a^4b^6+32a^3b^7x^3+48a^2b^8x^6+32ab^9x^9+8b^{10}x^{12}\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^5/(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
```

```
[Out] (-2*Sqrt[b^2]*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]*(3*a^4 - 3*a^3*b*x^3 + 3*a^2*
b^2*x^6 - 3*a*b^3*x^9 + 4*b^4*x^12) - 2*(3*a^5*b - a*b^5*x^12 - 4*b^6*x^15)
)/(9*x^12*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]*(-8*a^3*b^7 - 24*a^2*b^8*x^3 - 24
*a*b^9*x^6 - 8*b^10*x^9) + 9*Sqrt[b^2]*x^12*(8*a^4*b^6 + 32*a^3*b^7*x^3 + 4
8*a^2*b^8*x^6 + 32*a*b^9*x^9 + 8*b^10*x^12))
```

fricas [A] time = 1.18, size = 58, normalized size = 0.74

$$-\frac{4\,bx^3+a}{36\,\left(b^6x^{12}+4\,ab^5x^9+6\,a^2b^4x^6+4\,a^3b^3x^3+a^4b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/36*(4*b*x^3 + a)/(b^6*x^12 + 4*a*b^5*x^9 + 6*a^2*b^4*x^6 + 4*a^3*b^3*x^3
 + a^4*b^2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

maple [A] time = 0.01, size = 32, normalized size = 0.41

$$-\frac{\left(b\,x^3+a\right)\left(4b\,x^3+a\right)}{36\left(\left(b\,x^3+a\right)^2\right)^{\frac{5}{2}}b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x)
```

```
[Out] -1/36*(b*x^3+a)*(4*b*x^3+a)/b^2/((b*x^3+a)^2)^(5/2)
```

maxima [A] time = 0.86, size = 43, normalized size = 0.55

$$-\frac{1}{9\left(b^2x^6+2\,abx^3+a^2\right)^{\frac{3}{2}}b^2}+\frac{a}{12\left(x^3+\frac{a}{b}\right)^4b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="maxima")
[Out] -1/9/((b^2*x^6 + 2*a*b*x^3 + a^2)^(3/2)*b^2) + 1/12*a/((x^3 + a/b)^4*b^6)
```

mupad [B] time = 1.28, size = 42, normalized size = 0.54

$$-\frac{\left(4\,b\,x^3+a\right)\,\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{36\,b^2\,\left(b\,x^3+a\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2),x)
[Out] -((a + 4*b*x^3)*(a^2 + b^2*x^6 + 2*a*b*x^3)^(1/2))/(36*b^2*(a + b*x^3)^5)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^5}{\left(\left(a+bx^3\right)^2\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(b**2*x**6+2*a*b*x**3+a**2)**(5/2),x)
[Out] Integral(x**5/((a + b*x**3)**2)**(5/2), x)
```

3.109 $\int \frac{x^4}{\left(a^2+2abx^3+b^2x^6\right)^{5/2}} \, dx$

Optimal. Leaf size=368

$$\frac{7x^2}{324a^2b\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}}+\frac{x^2}{54ab\left(a+bx^3\right)^2\sqrt{a^2+2abx^3+b^2x^6}}-\frac{x^2}{12b\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}}$$

Rubi [A] time = 0.19, antiderivative size = 368, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 9, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.346, Rules used = {1355, 288, 290, 292, 31, 634, 617, 204, 628}

$$\frac{7x^2}{243a^3b\sqrt{a^2+2abx^3+b^2x^6}}+\frac{7x^2}{324a^2b\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}}+\frac{x^2}{54ab\left(a+bx^3\right)^2\sqrt{a^2+2abx^3+b^2x^6}}-\frac{x^2}{12b\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}}-\frac{7\left(a+bx^3\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{729a^{10/3}b^{5/3}\sqrt{a^2+2abx^3+b^2x^6}}+\frac{7\left(a+bx^3\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{1458a^{10/3}b^{5/3}\sqrt{a^2+2abx^3+b^2x^6}}-\frac{7\left(a+bx^3\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{243\sqrt{3}a^{10/3}b^{5/3}\sqrt{a^2+2abx^3+b^2x^6}}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
```

```
[Out] (7*x^2)/(243*a^3*b*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) - x^2/(12*b*(a + b*x^3)
^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) + x^2/(54*a*b*(a + b*x^3)^2*Sqrt[a^2 +
2*a*b*x^3 + b^2*x^6]) + (7*x^2)/(324*a^2*b*(a + b*x^3)*Sqrt[a^2 + 2*a*b*x^3
 + b^2*x^6]) - (7*(a + b*x^3)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/
3))])/(243*Sqrt[3]*a^(10/3)*b^(5/3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) - (7*(
a + b*x^3)*Log[a^(1/3) + b^(1/3)*x])/(729*a^(10/3)*b^(5/3)*Sqrt[a^2 + 2*a*b
*x^3 + b^2*x^6]) + (7*(a + b*x^3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)
*x^2])/(1458*a^(10/3)*b^(5/3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
```

```
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\int \frac{x^4}{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}\,dx = \frac{\left(b^4\left(ab+b^2x^3\right)\right)\int \frac{x^4}{\left(ab+b^2x^3\right)^5}\,dx}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$= -\frac{x^2}{12b\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}} + \frac{\left(b^2\left(ab+b^2x^3\right)\right)\int \frac{x}{\left(ab+b^2x^3\right)^4}\,dx}{6\sqrt{a^2+2abx^3+b^2x^6}}$$

$$= -\frac{x^2}{12b\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}} + \frac{x^2}{54ab\left(a+bx^3\right)^2\sqrt{a^2+2abx^3+b^2x^6}} + \cdots$$

$$= -\frac{x^2}{12b\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}} + \frac{x^2}{54ab\left(a+bx^3\right)^2\sqrt{a^2+2abx^3+b^2x^6}} + \cdots$$

$$= \frac{7x^2}{243a^3b\sqrt{a^2+2abx^3+b^2x^6}} - \frac{x^2}{12b\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}} + \frac{\cdots}{54ab\left(a+b\cdots\right.}$$

$$= \frac{7x^2}{243a^3b\sqrt{a^2+2abx^3+b^2x^6}} - \frac{x^2}{12b\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}} + \frac{\cdots}{54ab\left(a+b\cdots\right.}$$

$$= \frac{7x^2}{243a^3b\sqrt{a^2+2abx^3+b^2x^6}} - \frac{x^2}{12b\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}} + \frac{\cdots}{54ab\left(a+b\cdots\right.}$$

$$= \frac{7x^2}{243a^3b\sqrt{a^2+2abx^3+b^2x^6}} - \frac{x^2}{12b\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}} + \frac{\cdots}{54ab\left(a+b\cdots\right.}$$

$$= \frac{7x^2}{243a^3b\sqrt{a^2+2abx^3+b^2x^6}} - \frac{x^2}{12b\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}} + \frac{\cdots}{54ab\left(a+b\cdots\right.}$$

Mathematica [A] time = 0.14, size = 229, normalized size = 0.62

$$\frac{\left(a+bx^3\right)\left(-243a^{10/3}b^{2/3}x^2+63a^{4/3}b^{2/3}x^2\left(a+bx^3\right)^2+54a^{7/3}b^{2/3}x^2\left(a+bx^3\right)+14\left(a+bx^3\right)^4\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)+84\sqrt[3]{a}b^{2/3}x^2\left(a+bx^3\right)^3-28\left(a+bx^3\right)^4\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)+28\sqrt{3}\left(a+bx^3\right)^4\tan^{-1}\left(\frac{2\sqrt[3]{b}x-\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)\right)}{2916a^{10/3}b^{5/3}\left(\left(a+bx^3\right)^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
```

```
[Out] ((a + b*x^3)*(-243*a^(10/3)*b^(2/3)*x^2 + 54*a^(7/3)*b^(2/3)*x^2*(a + b*x^3
) + 63*a^(4/3)*b^(2/3)*x^2*(a + b*x^3)^2 + 84*a^(1/3)*b^(2/3)*x^2*(a + b*x^
3)^3 + 28*Sqrt[3]*(a + b*x^3)^4*ArcTan[(-a^(1/3) + 2*b^(1/3)*x)/(Sqrt[3]*a^
(1/3))] - 28*(a + b*x^3)^4*Log[a^(1/3) + b^(1/3)*x] + 14*(a + b*x^3)^4*Log[
a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]))/(2916*a^(10/3)*b^(5/3)*((a + b
*x^3)^2)^(5/2))
```

IntegrateAlgebraic [A] time = 26.11, size = 191, normalized size = 0.52

$$\frac{\left(a+bx^3\right)\left(\frac{7\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{1458a^{10/3}b^{5/3}}-\frac{7\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{729a^{10/3}b^{5/3}}-\frac{7\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{243\sqrt{3}a^{10/3}b^{5/3}}+\frac{-14a^3x^2+144a^2bx^5+105ab^2x^8+28b^3x^{11}}{972a^3b\left(a+bx^3\right)^4}\right)}{\sqrt{\left(a+bx^3\right)^2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^4/(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
```

```
[Out] ((a + b*x^3)*((-14*a^3*x^2 + 144*a^2*b*x^5 + 105*a*b^2*x^8 + 28*b^3*x^11)/(
972*a^3*b*(a + b*x^3)^4) - (7*ArcTan[1/Sqrt[3] - (2*b^(1/3)*x)/(Sqrt[3]*a^(
1/3))])/(243*Sqrt[3]*a^(10/3)*b^(5/3)) - (7*Log[a^(1/3) + b^(1/3)*x])/(729*
a^(10/3)*b^(5/3)) + (7*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(145
8*a^(10/3)*b^(5/3))))/Sqrt[(a + b*x^3)^2]
```

fricas [A] time = 1.30, size = 734, normalized size = 1.99

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/2916*(84*a*b^5*x^11 + 315*a^2*b^4*x^8 + 432*a^3*b^3*x^5 - 42*a^4*b^2*x^2
 + 42*sqrt(1/3)*(a*b^5*x^12 + 4*a^2*b^4*x^9 + 6*a^3*b^3*x^6 + 4*a^4*b^2*x^3
 + a^5*b)*sqrt((-a*b^2)^(1/3)/a)*log((2*b^2*x^3 - a*b + 3*sqrt(1/3)*(a*b*x
+ 2*(-a*b^2)^(2/3)*x^2 + (-a*b^2)^(1/3)*a)*sqrt((-a*b^2)^(1/3)/a) - 3*(-a*b
^2)^(2/3)*x)/(b*x^3 + a)) + 14*(b^4*x^12 + 4*a*b^3*x^9 + 6*a^2*b^2*x^6 + 4*
a^3*b*x^3 + a^4)*(-a*b^2)^(2/3)*log(b^2*x^2 + (-a*b^2)^(1/3)*b*x + (-a*b^2)
^(2/3)) - 28*(b^4*x^12 + 4*a*b^3*x^9 + 6*a^2*b^2*x^6 + 4*a^3*b*x^3 + a^4)*(
-a*b^2)^(2/3)*log(b*x - (-a*b^2)^(1/3)))/(a^4*b^7*x^12 + 4*a^5*b^6*x^9 + 6*
a^6*b^5*x^6 + 4*a^7*b^4*x^3 + a^8*b^3), 1/2916*(84*a*b^5*x^11 + 315*a^2*b^4
*x^8 + 432*a^3*b^3*x^5 - 42*a^4*b^2*x^2 + 84*sqrt(1/3)*(a*b^5*x^12 + 4*a^2*
b^4*x^9 + 6*a^3*b^3*x^6 + 4*a^4*b^2*x^3 + a^5*b)*sqrt(-(-a*b^2)^(1/3)/a)*ar
ctan(sqrt(1/3)*(2*b*x + (-a*b^2)^(1/3))*sqrt(-(-a*b^2)^(1/3)/a)/b) + 14*(b^
4*x^12 + 4*a*b^3*x^9 + 6*a^2*b^2*x^6 + 4*a^3*b*x^3 + a^4)*(-a*b^2)^(2/3)*lo
g(b^2*x^2 + (-a*b^2)^(1/3)*b*x + (-a*b^2)^(2/3)) - 28*(b^4*x^12 + 4*a*b^3*x
^9 + 6*a^2*b^2*x^6 + 4*a^3*b*x^3 + a^4)*(-a*b^2)^(2/3)*log(b*x - (-a*b^2)^(
1/3)))/(a^4*b^7*x^12 + 4*a^5*b^6*x^9 + 6*a^6*b^5*x^6 + 4*a^7*b^4*x^3 + a^8*
b^3)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

maple [B] time = 0.02, size = 521, normalized size = 1.42

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x)
```

```
[Out] 1/2916*(-28*3^(1/2)*b^4*x^12*arctan(1/3*3^(1/2)*(-2*x+(a/b)^(1/3))/(a/b)^(1
/3))-28*b^4*x^12*ln(x+(a/b)^(1/3))+14*b^4*x^12*ln(x^2-(a/b)^(1/3)*x+(a/b)^(
2/3))+84*(a/b)^(1/3)*x^11*b^4-112*3^(1/2)*a*b^3*x^9*arctan(1/3*3^(1/2)*(-2*
x+(a/b)^(1/3))/(a/b)^(1/3))-112*a*b^3*x^9*ln(x+(a/b)^(1/3))+56*a*b^3*x^9*ln
(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+315*(a/b)^(1/3)*x^8*a*b^3-168*3^(1/2)*a^2*b
^2*x^6*arctan(1/3*3^(1/2)*(-2*x+(a/b)^(1/3))/(a/b)^(1/3))-168*a^2*b^2*x^6*l
n(x+(a/b)^(1/3))+84*a^2*b^2*x^6*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+432*(a/b)
^(1/3)*x^5*a^2*b^2-112*3^(1/2)*a^3*b*x^3*arctan(1/3*3^(1/2)*(-2*x+(a/b)^(1/
```

```
3))/(a/b)^(1/3))-112*a^3*b*x^3*ln(x+(a/b)^(1/3))+56*a^3*b*x^3*ln(x^2-(a/b)^
(1/3)*x+(a/b)^(2/3))-42*(a/b)^(1/3)*x^2*a^3*b-28*3^(1/2)*a^4*arctan(1/3*3^(
1/2)*(-2*x+(a/b)^(1/3))/(a/b)^(1/3))-28*a^4*ln(x+(a/b)^(1/3))+14*a^4*ln(x^2
-(a/b)^(1/3)*x+(a/b)^(2/3)))*(b*x^3+a)/(a/b)^(1/3)/b^2/a^3/((b*x^3+a)^2)^(5
/2)
```

maxima [A] time = 1.77, size = 195, normalized size = 0.53

$$\frac{28\,b^{3}x^{11}+105\,ab^{2}x^{8}+144\,a^{2}bx^{5}-14\,a^{3}x^{2}}{972\left(a^{3}b^{5}x^{12}+4\,a^{4}b^{4}x^{9}+6\,a^{5}b^{3}x^{6}+4\,a^{6}b^{2}x^{3}+a^{7}b\right)}+\frac{7\,\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{729\,a^{3}b^{2}\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{7\,\log\left(x^{2}-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{1458\,a^{3}b^{2}\left(\frac{a}{b}\right)^{\frac{1}{3}}}-\frac{7\,\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{729\,a^{3}b^{2}\left(\frac{a}{b}\right)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/972*(28*b^3*x^11 + 105*a*b^2*x^8 + 144*a^2*b*x^5 - 14*a^3*x^2)/(a^3*b^5*x
^12 + 4*a^4*b^4*x^9 + 6*a^5*b^3*x^6 + 4*a^6*b^2*x^3 + a^7*b) + 7/729*sqrt(3
)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a^3*b^2*(a/b)^(1/3))
 + 7/1458*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a^3*b^2*(a/b)^(1/3)) - 7/
729*log(x + (a/b)^(1/3))/(a^3*b^2*(a/b)^(1/3))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^4}{\left(a^2+2\,a\,b\,x^3+b^2\,x^6\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2),x)
```

```
[Out] int(x^4/(a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{4}}{\left(\left(a+bx^{3}\right)^{2}\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(b**2*x**6+2*a*b*x**3+a**2)**(5/2),x)
```

```
[Out] Integral(x**4/((a + b*x**3)**2)**(5/2), x)
```

3.110 $\int \frac{x^3}{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}\,dx$

Optimal. Leaf size=360

$$\frac{x}{81a^2b\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}}+\frac{x}{108ab\left(a+bx^3\right)^2\sqrt{a^2+2abx^3+b^2x^6}}-\frac{x}{12b\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}}$$

Rubi [A] time = 0.18, antiderivative size = 360, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 9, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.346, Rules used = {1355, 288, 199, 200, 31, 634, 617, 204, 628}

$$\frac{5x}{243a^3b\sqrt{a^2+2abx^3+b^2x^6}}+\frac{x}{81a^2b\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}}+\frac{x}{108ab\left(a+bx^3\right)^2\sqrt{a^2+2abx^3+b^2x^6}}-\frac{x}{12b\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}}+\frac{10\left(a+bx^3\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{729a^{11/3}b^{4/3}\sqrt{a^2+2abx^3+b^2x^6}}-\frac{5\left(a+bx^3\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{729a^{11/3}b^{4/3}\sqrt{a^2+2abx^3+b^2x^6}}-\frac{10\left(a+bx^3\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{243\sqrt{3}a^{11/3}b^{4/3}\sqrt{a^2+2abx^3+b^2x^6}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
```

```
[Out] (5*x)/(243*a^3*b*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) - x/(12*b*(a + b*x^3)^3*S
qrt[a^2 + 2*a*b*x^3 + b^2*x^6]) + x/(108*a*b*(a + b*x^3)^2*Sqrt[a^2 + 2*a*b
*x^3 + b^2*x^6]) + x/(81*a^2*b*(a + b*x^3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])
 - (10*(a + b*x^3)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(243*
Sqrt[3]*a^(11/3)*b^(4/3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) + (10*(a + b*x^3)
*Log[a^(1/3) + b^(1/3)*x])/(729*a^(11/3)*b^(4/3)*Sqrt[a^2 + 2*a*b*x^3 + b^2
*x^6]) - (5*(a + b*x^3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(72
9*a^(11/3)*b^(4/3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
```

```
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\int \frac{x^3}{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}\,dx = \frac{\left(b^4\left(ab+b^2x^3\right)\right)\int \frac{x^3}{\left(ab+b^2x^3\right)^5}\,dx}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$= -\frac{x}{12b\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}} + \frac{\left(b^2\left(ab+b^2x^3\right)\right)\int \frac{1}{\left(ab+b^2x^3\right)^4}\,dx}{12\sqrt{a^2+2abx^3+b^2x^6}}$$

$$= -\frac{x}{12b\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}} + \frac{x}{108ab\left(a+bx^3\right)^2\sqrt{a^2+2abx^3+b^2x^6}}$$

$$= -\frac{x}{12b\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}} + \frac{x}{108ab\left(a+bx^3\right)^2\sqrt{a^2+2abx^3+b^2x^6}}$$

$$= \frac{5x}{243a^3b\sqrt{a^2+2abx^3+b^2x^6}} - \frac{x}{12b\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}} + \frac{\quad}{108ab\left(a\right.}$$

$$= \frac{5x}{243a^3b\sqrt{a^2+2abx^3+b^2x^6}} - \frac{x}{12b\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}} + \frac{\quad}{108ab\left(a\right.}$$

$$= \frac{5x}{243a^3b\sqrt{a^2+2abx^3+b^2x^6}} - \frac{x}{12b\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}} + \frac{\quad}{108ab\left(a\right.}$$

$$= \frac{5x}{243a^3b\sqrt{a^2+2abx^3+b^2x^6}} - \frac{x}{12b\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}} + \frac{\quad}{108ab\left(a\right.}$$

$$= \frac{5x}{243a^3b\sqrt{a^2+2abx^3+b^2x^6}} - \frac{x}{12b\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}} + \frac{\quad}{108ab\left(a\right.}$$

Mathematica [A] time = 0.13, size = 221, normalized size = 0.61

$$\frac{\left(a+bx^3\right)\left(-20\left(a+bx^3\right)^4\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)+60a^{2/3}\sqrt[3]{b}x\left(a+bx^3\right)^3+36a^{5/3}\sqrt[3]{b}x\left(a+bx^3\right)^2+27a^{8/3}\sqrt[3]{b}x\left(a+bx^3\right)-243a^{11/3}\sqrt[3]{b}x+40\left(a+bx^3\right)^4\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)+40\sqrt{3}\left(a+bx^3\right)^4\tan^{-1}\left(\frac{2\sqrt[3]{b}x-\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)\right)}{2916a^{11/3}b^{4/3}\left(\left(a+bx^3\right)^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
```

```
[Out] ((a + b*x^3)*(-243*a^(11/3)*b^(1/3)*x + 27*a^(8/3)*b^(1/3)*x*(a + b*x^3) +
36*a^(5/3)*b^(1/3)*x*(a + b*x^3)^2 + 60*a^(2/3)*b^(1/3)*x*(a + b*x^3)^3 + 4
0*Sqrt[3]*(a + b*x^3)^4*ArcTan[(-a^(1/3) + 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))]
+ 40*(a + b*x^3)^4*Log[a^(1/3) + b^(1/3)*x] - 20*(a + b*x^3)^4*Log[a^(2/3)
- a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]))/(2916*a^(11/3)*b^(4/3)*((a + b*x^3)^2)
^(5/2))
```

IntegrateAlgebraic [A] time = 26.81, size = 189, normalized size = 0.52

$$\frac{\left(a+bx^3\right)\left(-\frac{5\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{729a^{11/3}b^{4/3}}+\frac{10\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{729a^{11/3}b^{4/3}}-\frac{10\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{243\sqrt{3}a^{11/3}b^{4/3}}+\frac{-40a^3x+93a^2bx^4+72ab^2x^7+20b^3x^{10}}{972a^3b\left(a+bx^3\right)^4}\right)}{\sqrt{\left(a+bx^3\right)^2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^3/(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
```

```
[Out] ((a + b*x^3)*((-40*a^3*x + 93*a^2*b*x^4 + 72*a*b^2*x^7 + 20*b^3*x^10)/(972*
a^3*b*(a + b*x^3)^4) - (10*ArcTan[1/Sqrt[3] - (2*b^(1/3)*x)/(Sqrt[3]*a^(1/3
))])/(243*Sqrt[3]*a^(11/3)*b^(4/3)) + (10*Log[a^(1/3) + b^(1/3)*x])/(729*a^
(11/3)*b^(4/3)) - (5*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(729*a
^(11/3)*b^(4/3))))/Sqrt[(a + b*x^3)^2]
```

fricas [A] time = 1.23, size = 723, normalized size = 2.01

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/2916*(60*a^2*b^4*x^10 + 216*a^3*b^3*x^7 + 279*a^4*b^2*x^4 - 120*a^5*b*x
+ 60*sqrt(1/3)*(a*b^5*x^12 + 4*a^2*b^4*x^9 + 6*a^3*b^3*x^6 + 4*a^4*b^2*x^3
+ a^5*b)*sqrt(-(a^2*b)^(1/3)/b)*log((2*a*b*x^3 - 3*(a^2*b)^(1/3)*a*x - a^2
+ 3*sqrt(1/3)*(2*a*b*x^2 + (a^2*b)^(2/3)*x - (a^2*b)^(1/3)*a)*sqrt(-(a^2*b)
^(1/3)/b))/(b*x^3 + a)) - 20*(b^4*x^12 + 4*a*b^3*x^9 + 6*a^2*b^2*x^6 + 4*a^
3*b*x^3 + a^4)*(a^2*b)^(2/3)*log(a*b*x^2 - (a^2*b)^(2/3)*x + (a^2*b)^(1/3)*
a) + 40*(b^4*x^12 + 4*a*b^3*x^9 + 6*a^2*b^2*x^6 + 4*a^3*b*x^3 + a^4)*(a^2*b
)^(2/3)*log(a*b*x + (a^2*b)^(2/3)))/(a^5*b^6*x^12 + 4*a^6*b^5*x^9 + 6*a^7*b
^4*x^6 + 4*a^8*b^3*x^3 + a^9*b^2), 1/2916*(60*a^2*b^4*x^10 + 216*a^3*b^3*x^
7 + 279*a^4*b^2*x^4 - 120*a^5*b*x + 120*sqrt(1/3)*(a*b^5*x^12 + 4*a^2*b^4*x
^9 + 6*a^3*b^3*x^6 + 4*a^4*b^2*x^3 + a^5*b)*sqrt((a^2*b)^(1/3)/b)*arctan(sq
rt(1/3)*(2*(a^2*b)^(2/3)*x - (a^2*b)^(1/3)*a)*sqrt((a^2*b)^(1/3)/b)/a^2) -
20*(b^4*x^12 + 4*a*b^3*x^9 + 6*a^2*b^2*x^6 + 4*a^3*b*x^3 + a^4)*(a^2*b)^(2/
3)*log(a*b*x^2 - (a^2*b)^(2/3)*x + (a^2*b)^(1/3)*a) + 40*(b^4*x^12 + 4*a*b^
3*x^9 + 6*a^2*b^2*x^6 + 4*a^3*b*x^3 + a^4)*(a^2*b)^(2/3)*log(a*b*x + (a^2*b
)^(2/3)))/(a^5*b^6*x^12 + 4*a^6*b^5*x^9 + 6*a^7*b^4*x^6 + 4*a^8*b^3*x^3 + a
^9*b^2)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

maple [B] time = 0.02, size = 519, normalized size = 1.44

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x)
```

```
[Out] 1/2916*(-40*3^(1/2)*b^4*x^12*arctan(1/3*3^(1/2)*(-2*x+(a/b)^(1/3))/(a/b)^(1
/3))+40*b^4*x^12*ln(x+(a/b)^(1/3))-20*b^4*x^12*ln(x^2-(a/b)^(1/3)*x+(a/b)^(
2/3))+60*(a/b)^(2/3)*b^4*x^10-160*3^(1/2)*a*b^3*x^9*arctan(1/3*3^(1/2)*(-2*
x+(a/b)^(1/3))/(a/b)^(1/3))+160*a*b^3*x^9*ln(x+(a/b)^(1/3))-80*a*b^3*x^9*ln
(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+216*(a/b)^(2/3)*a*b^3*x^7-240*3^(1/2)*a^2*b
^2*x^6*arctan(1/3*3^(1/2)*(-2*x+(a/b)^(1/3))/(a/b)^(1/3))+240*a^2*b^2*x^6*l
n(x+(a/b)^(1/3))-120*a^2*b^2*x^6*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+279*(a/b
)^(2/3)*a^2*b^2*x^4-160*3^(1/2)*a^3*b*x^3*arctan(1/3*3^(1/2)*(-2*x+(a/b)^(1
```

```
/3))/(a/b)^(1/3))+160*a^3*b*x^3*ln(x+(a/b)^(1/3))-80*a^3*b*x^3*ln(x^2-(a/b)
^(1/3)*x+(a/b)^(2/3))-120*(a/b)^(2/3)*a^3*b*x-40*3^(1/2)*a^4*arctan(1/3*3^(
1/2)*(-2*x+(a/b)^(1/3))/(a/b)^(1/3))+40*a^4*ln(x+(a/b)^(1/3))-20*a^4*ln(x^2
-(a/b)^(1/3)*x+(a/b)^(2/3)))*(b*x^3+a)/(a/b)^(2/3)/b^2/a^3/((b*x^3+a)^2)^(5
/2)
```

maxima [A] time = 2.28, size = 193, normalized size = 0.54

$$\frac{20\,b^3x^{10}+72\,ab^2x^7+93\,a^2bx^4-40\,a^3x}{972\left(a^3b^5x^{12}+4\,a^4b^4x^9+6\,a^5b^3x^6+4\,a^6b^2x^3+a^7b\right)}+\frac{10\,\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{729\,a^3b^2\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{5\,\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{729\,a^3b^2\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{10\,\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{729\,a^3b^2\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/972*(20*b^3*x^10 + 72*a*b^2*x^7 + 93*a^2*b*x^4 - 40*a^3*x)/(a^3*b^5*x^12
+ 4*a^4*b^4*x^9 + 6*a^5*b^3*x^6 + 4*a^6*b^2*x^3 + a^7*b) + 10/729*sqrt(3)*a
rctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a^3*b^2*(a/b)^(2/3)) -
5/729*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a^3*b^2*(a/b)^(2/3)) + 10/729
*log(x + (a/b)^(1/3))/(a^3*b^2*(a/b)^(2/3))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3}{{\left(a^2+2\,a\,b\,x^3+b^2\,x^6\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2),x)
```

```
[Out] int(x^3/(a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\left(\left(a+bx^3\right)^2\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(b**2*x**6+2*a*b*x**3+a**2)**(5/2),x)
```

```
[Out] Integral(x**3/((a + b*x**3)**2)**(5/2), x)
```

3.111 $\int \frac{x^2}{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}\,dx$

Optimal. Leaf size=38

$$-\frac{1}{12b\left(a+bx^3\right)\left(a^2+2abx^3+b^2x^6\right)^{3/2}}$$

Rubi [A] time = 0.03, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1352, 607}

$$-\frac{1}{12b\left(a+bx^3\right)\left(a^2+2abx^3+b^2x^6\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
```

```
[Out] -1/(12*b*(a + b*x^3)*(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2))
```

Rule 607

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(2*(a + b*x
+ c*x^2)^(p + 1))/((2*p + 1)*(b + 2*c*x)), x] /; FreeQ[{a, b, c, p}, x] &&
EqQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 1352

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a,
b, c, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2}{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}\,dx &= \frac{1}{3}\operatorname{Subst}\left(\int \frac{1}{\left(a^2+2abx+b^2x^2\right)^{5/2}}\,dx, x, x^3\right)\\ &= -\frac{1}{12b\left(a+bx^3\right)\left(a^2+2abx^3+b^2x^6\right)^{3/2}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 27, normalized size = 0.71

$$-\frac{a+bx^3}{12b\left(\left(a+bx^3\right)^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
```

```
[Out] -1/12*(a + b*x^3)/(b*((a + b*x^3)^2)^(5/2))
```

IntegrateAlgebraic [B] time = 0.83, size = 209, normalized size = 5.50

$$\frac{-2\left(-a^4b-b^5x^{12}\right)-2\sqrt{b^2}\sqrt{a^2+2abx^3+b^2x^6}\left(-a^3+a^2bx^3-ab^2x^6+b^3x^9\right)}{3bx^{12}\sqrt{a^2+2abx^3+b^2x^6}\left(-8a^3b^5-24a^2b^6x^3-24ab^7x^6-8b^8x^9\right)+3b\sqrt{b^2}\,x^{12}\left(8a^4b^4+32a^3b^5x^3+48a^2b^6x^6+32ab^7x^9+8b^8x^{12}\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^2/(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
```

```
[Out] (-2*Sqrt[b^2]*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]*(-a^3 + a^2*b*x^3 - a*b^2*x^6
 + b^3*x^9) - 2*(-(a^4*b) - b^5*x^12))/(3*b*x^12*Sqrt[a^2 + 2*a*b*x^3 + b^2
*x^6]*(-8*a^3*b^5 - 24*a^2*b^6*x^3 - 24*a*b^7*x^6 - 8*b^8*x^9) + 3*b*Sqrt[b
^2]*x^12*(8*a^4*b^4 + 32*a^3*b^5*x^3 + 48*a^2*b^6*x^6 + 32*a*b^7*x^9 + 8*b^
8*x^12))
```

fricas [A] time = 1.15, size = 48, normalized size = 1.26

$$-\frac{1}{12\left(b^{5}x^{12}+4\,ab^{4}x^{9}+6\,a^{2}b^{3}x^{6}+4\,a^{3}b^{2}x^{3}+a^{4}b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/12/(b^5*x^12 + 4*a*b^4*x^9 + 6*a^2*b^3*x^6 + 4*a^3*b^2*x^3 + a^4*b)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

maple [A] time = 0.01, size = 24, normalized size = 0.63

$$-\frac{b\,x^{3}+a}{12\left(\left(b\,x^{3}+a\right)^{2}\right)^{\frac{5}{2}}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x)
```

```
[Out] -1/12*(b*x^3+a)/b/((b*x^3+a)^2)^(5/2)
```

maxima [A] time = 0.51, size = 16, normalized size = 0.42

$$-\frac{1}{12\left(x^{3}+\frac{a}{b}\right)^{4}b^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/12/((x^3 + a/b)^4*b^5)
```

mupad [B] time = 1.26, size = 34, normalized size = 0.89

$$-\frac{\sqrt{a^{2}+2\,a\,b\,x^{3}+b^{2}\,x^{6}}}{12\,b\left(b\,x^{3}+a\right)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2),x)
```

```
[Out] -(a^2 + b^2*x^6 + 2*a*b*x^3)^(1/2)/(12*b*(a + b*x^3)^5)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(\left(a + bx^3\right)^2\right)^{\frac{5}{2}}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b**2*x**6+2*a*b*x**3+a**2)**(5/2),x)
[Out] Integral(x**2/((a + b*x**3)**2)**(5/2), x)
```

3.112 $\int \frac{x}{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}\,dx$

Optimal. Leaf size=359

$$\frac{5x^2}{54a^2\left(a+bx^3\right)^2\sqrt{a^2+2abx^3+b^2x^6}}+\frac{x^2}{12a\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}}-\frac{35\left(a+bx^3\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{729a^{13/3}b^{2/3}\sqrt{a^2+2abx^3+b^2x^6}}-$$

Rubi [A] time = 0.19, antiderivative size = 359, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1355, 290, 292, 31, 634, 617, 204, 628}

$$\frac{35x^2}{324a^3\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}}+\frac{5x^2}{54a^2\left(a+bx^3\right)^2\sqrt{a^2+2abx^3+b^2x^6}}+\frac{x^2}{12a\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}}+\frac{35x^2}{243a^4\sqrt{a^2+2abx^3+b^2x^6}}-\frac{35\left(a+bx^3\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{729a^{13/3}b^{2/3}\sqrt{a^2+2abx^3+b^2x^6}}+\frac{35\left(a+bx^3\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{1458a^{13/3}b^{2/3}\sqrt{a^2+2abx^3+b^2x^6}}-\frac{35\left(a+bx^3\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\sqrt[3]{a}}\right)}{243\sqrt{3}\,a^{13/3}b^{2/3}\sqrt{a^2+2abx^3+b^2x^6}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
```

```
[Out] (35*x^2)/(243*a^4*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) + x^2/(12*a*(a + b*x^3)^
3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) + (5*x^2)/(54*a^2*(a + b*x^3)^2*Sqrt[a^2
 + 2*a*b*x^3 + b^2*x^6]) + (35*x^2)/(324*a^3*(a + b*x^3)*Sqrt[a^2 + 2*a*b*x
^3 + b^2*x^6]) - (35*(a + b*x^3)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^
(1/3))])/(243*Sqrt[3]*a^(13/3)*b^(2/3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) - (
35*(a + b*x^3)*Log[a^(1/3) + b^(1/3)*x])/(729*a^(13/3)*b^(2/3)*Sqrt[a^2 + 2
*a*b*x^3 + b^2*x^6]) + (35*(a + b*x^3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^
(2/3)*x^2])/(1458*a^(13/3)*b^(2/3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
```

```
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\int \frac{x}{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}\,dx = \frac{\left(b^4\left(ab+b^2x^3\right)\right)\int \frac{x}{\left(ab+b^2x^3\right)^5}\,dx}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$= \frac{x^2}{12a\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}} + \frac{\left(5b^3\left(ab+b^2x^3\right)\right)\int \frac{x}{\left(ab+b^2x^3\right)^4}\,dx}{6a\sqrt{a^2+2abx^3+b^2x^6}}$$

$$= \frac{x^2}{12a\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}} + \frac{5x^2}{54a^2\left(a+bx^3\right)^2\sqrt{a^2+2abx^3+b^2x^6}} +$$

$$= \frac{x^2}{12a\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}} + \frac{5x^2}{54a^2\left(a+bx^3\right)^2\sqrt{a^2+2abx^3+b^2x^6}} +$$

$$= \frac{35x^2}{243a^4\sqrt{a^2+2abx^3+b^2x^6}} + \frac{x^2}{12a\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}} + \frac{}{54a^2\left(a+\right.}$$

$$= \frac{35x^2}{243a^4\sqrt{a^2+2abx^3+b^2x^6}} + \frac{x^2}{12a\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}} + \frac{}{54a^2\left(a+\right.}$$

$$= \frac{35x^2}{243a^4\sqrt{a^2+2abx^3+b^2x^6}} + \frac{x^2}{12a\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}} + \frac{}{54a^2\left(a+\right.}$$

$$= \frac{35x^2}{243a^4\sqrt{a^2+2abx^3+b^2x^6}} + \frac{x^2}{12a\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}} + \frac{}{54a^2\left(a+\right.}$$

$$= \frac{35x^2}{243a^4\sqrt{a^2+2abx^3+b^2x^6}} + \frac{x^2}{12a\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}} + \frac{}{54a^2\left(a+\right.}$$

Mathematica [A] time = 0.12, size = 219, normalized size = 0.61

$$\frac{\left(a+bx^3\right)\left(\frac{70\left(a+bx^3\right)^4\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{b^{2/3}}+315a^{4/3}x^2\left(a+bx^3\right)^2+270a^{7/3}x^2\left(a+bx^3\right)+243a^{10/3}x^2-\frac{140\left(a+bx^3\right)^4\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{b^{2/3}}+\frac{140\sqrt{3}\left(a+bx^3\right)^4\tan^{-1}\left(\frac{2\sqrt[3]{b}x-\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{b^{2/3}}+420\sqrt[3]{a}x^2\left(a+bx^3\right)^3\right)}{2916a^{13/3}\left(\left(a+bx^3\right)^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
```

```
[Out] ((a + b*x^3)*(243*a^(10/3)*x^2 + 270*a^(7/3)*x^2*(a + b*x^3) + 315*a^(4/3)*
x^2*(a + b*x^3)^2 + 420*a^(1/3)*x^2*(a + b*x^3)^3 + (140*Sqrt[3]*(a + b*x^3
)^4*ArcTan[(-a^(1/3) + 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/b^(2/3) - (140*(a +
 b*x^3)^4*Log[a^(1/3) + b^(1/3)*x])/b^(2/3) + (70*(a + b*x^3)^4*Log[a^(2/3)
 - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/b^(2/3)))/(2916*a^(13/3)*((a + b*x^3)^
2)^(5/2))
```

IntegrateAlgebraic [A] time = 28.09, size = 188, normalized size = 0.52

$$\frac{\left(a+bx^3\right)\left(\frac{35\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{1458a^{13/3}b^{2/3}}-\frac{35\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{729a^{13/3}b^{2/3}}-\frac{35\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{243\sqrt{3}a^{13/3}b^{2/3}}+\frac{x^2\left(416a^3+720a^2bx^3+525ab^2x^6+140b^3x^9\right)}{972a^4\left(a+bx^3\right)^4}\right)}{\sqrt{\left(a+bx^3\right)^2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x/(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
```

```
[Out] ((a + b*x^3)*((x^2*(416*a^3 + 720*a^2*b*x^3 + 525*a*b^2*x^6 + 140*b^3*x^9))
/(972*a^4*(a + b*x^3)^4) - (35*ArcTan[1/Sqrt[3] - (2*b^(1/3)*x)/(Sqrt[3]*a^
(1/3))])/(243*Sqrt[3]*a^(13/3)*b^(2/3)) - (35*Log[a^(1/3) + b^(1/3)*x])/(72
9*a^(13/3)*b^(2/3)) + (35*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(
1458*a^(13/3)*b^(2/3))))/Sqrt[(a + b*x^3)^2]
```

fricas [A] time = 1.27, size = 734, normalized size = 2.04

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/2916*(420*a*b^5*x^11 + 1575*a^2*b^4*x^8 + 2160*a^3*b^3*x^5 + 1248*a^4*b^
2*x^2 + 210*sqrt(1/3)*(a*b^5*x^12 + 4*a^2*b^4*x^9 + 6*a^3*b^3*x^6 + 4*a^4*b
^2*x^3 + a^5*b)*sqrt((-a*b^2)^(1/3)/a)*log((2*b^2*x^3 - a*b + 3*sqrt(1/3)*(
a*b*x + 2*(-a*b^2)^(2/3)*x^2 + (-a*b^2)^(1/3)*a)*sqrt((-a*b^2)^(1/3)/a) - 3
*(-a*b^2)^(2/3)*x)/(b*x^3 + a)) + 70*(b^4*x^12 + 4*a*b^3*x^9 + 6*a^2*b^2*x^
6 + 4*a^3*b*x^3 + a^4)*(-a*b^2)^(2/3)*log(b^2*x^2 + (-a*b^2)^(1/3)*b*x + (-
a*b^2)^(2/3)) - 140*(b^4*x^12 + 4*a*b^3*x^9 + 6*a^2*b^2*x^6 + 4*a^3*b*x^3 +
 a^4)*(-a*b^2)^(2/3)*log(b*x - (-a*b^2)^(1/3)))/(a^5*b^6*x^12 + 4*a^6*b^5*x
^9 + 6*a^7*b^4*x^6 + 4*a^8*b^3*x^3 + a^9*b^2), 1/2916*(420*a*b^5*x^11 + 157
5*a^2*b^4*x^8 + 2160*a^3*b^3*x^5 + 1248*a^4*b^2*x^2 + 420*sqrt(1/3)*(a*b^5*
x^12 + 4*a^2*b^4*x^9 + 6*a^3*b^3*x^6 + 4*a^4*b^2*x^3 + a^5*b)*sqrt(-(-a*b^2
)^(1/3)/a)*arctan(sqrt(1/3)*(2*b*x + (-a*b^2)^(1/3))*sqrt(-(-a*b^2)^(1/3)/a
)/b) + 70*(b^4*x^12 + 4*a*b^3*x^9 + 6*a^2*b^2*x^6 + 4*a^3*b*x^3 + a^4)*(-a*
b^2)^(2/3)*log(b^2*x^2 + (-a*b^2)^(1/3)*b*x + (-a*b^2)^(2/3)) - 140*(b^4*x^
12 + 4*a*b^3*x^9 + 6*a^2*b^2*x^6 + 4*a^3*b*x^3 + a^4)*(-a*b^2)^(2/3)*log(b*
x - (-a*b^2)^(1/3)))/(a^5*b^6*x^12 + 4*a^6*b^5*x^9 + 6*a^7*b^4*x^6 + 4*a^8*
b^3*x^3 + a^9*b^2)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

maple [B] time = 0.01, size = 521, normalized size = 1.45

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x)
```

```
[Out] 1/2916*(-140*3^(1/2)*b^4*x^12*arctan(1/3*3^(1/2)*(-2*x+(a/b)^(1/3))/(a/b)^(
1/3))-140*b^4*x^12*ln(x+(a/b)^(1/3))+70*b^4*x^12*ln(x^2-(a/b)^(1/3)*x+(a/b)
^(2/3))+420*(a/b)^(1/3)*b^4*x^11-560*3^(1/2)*a*b^3*x^9*arctan(1/3*3^(1/2)*(
-2*x+(a/b)^(1/3))/(a/b)^(1/3))-560*a*b^3*x^9*ln(x+(a/b)^(1/3))+280*a*b^3*x^
9*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1575*(a/b)^(1/3)*a*b^3*x^8-840*3^(1/2)*
a^2*b^2*x^6*arctan(1/3*3^(1/2)*(-2*x+(a/b)^(1/3))/(a/b)^(1/3))-840*a^2*b^2*
x^6*ln(x+(a/b)^(1/3))+420*a^2*b^2*x^6*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+216
0*(a/b)^(1/3)*a^2*b^2*x^5-560*3^(1/2)*a^3*b*x^3*arctan(1/3*3^(1/2)*(-2*x+(a
```

```
/b)^(1/3))/(a/b)^(1/3))-560*a^3*b*x^3*ln(x+(a/b)^(1/3))+280*a^3*b*x^3*ln(x^
2-(a/b)^(1/3)*x+(a/b)^(2/3))+1248*(a/b)^(1/3)*a^3*b*x^2-140*3^(1/2)*a^4*arc
tan(1/3*3^(1/2)*(-2*x+(a/b)^(1/3))/(a/b)^(1/3))-140*a^4*ln(x+(a/b)^(1/3))+7
0*a^4*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3)))*(b*x^3+a)/(a/b)^(1/3)/b/a^4/((b*x^
3+a)^2)^(5/2)
```

maxima [A] time = 2.18, size = 191, normalized size = 0.53

$$\frac{140\, b^{3} x^{11} + 525\, a b^{2} x^{8} + 720\, a^{2} b x^{5} + 416\, a^{3} x^{2}}{972\, \left(a^{4} b^{4} x^{12} + 4\, a^{5} b^{3} x^{9} + 6\, a^{6} b^{2} x^{6} + 4\, a^{7} b x^{3} + a^{8}\right)} + \frac{35\, \sqrt{3} \arctan\left(\frac{\sqrt{3} \left(2\, x - \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\, \left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{729\, a^{4} b \left(\frac{a}{b}\right)^{\frac{1}{3}}} + \frac{35\, \log\left(x^{2} - x \left(\frac{a}{b}\right)^{\frac{1}{3}} + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{1458\, a^{4} b \left(\frac{a}{b}\right)^{\frac{1}{3}}} - \frac{35\, \log\left(x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{729\, a^{4} b \left(\frac{a}{b}\right)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/972*(140*b^3*x^11 + 525*a*b^2*x^8 + 720*a^2*b*x^5 + 416*a^3*x^2)/(a^4*b^4
*x^12 + 4*a^5*b^3*x^9 + 6*a^6*b^2*x^6 + 4*a^7*b*x^3 + a^8) + 35/729*sqrt(3)
*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a^4*b*(a/b)^(1/3)) +
35/1458*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a^4*b*(a/b)^(1/3)) - 35/729
*log(x + (a/b)^(1/3))/(a^4*b*(a/b)^(1/3))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x}{{\left(a^2+2\,a\,b\,x^3+b^2\,x^6\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2),x)
```

```
[Out] int(x/(a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(\left(a + b x^{3}\right)^{2}\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b**2*x**6+2*a*b*x**3+a**2)**(5/2),x)
```

```
[Out] Integral(x/((a + b*x**3)**2)**(5/2), x)
```

3.113 $\int \frac{1}{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}\,dx$

Optimal. Leaf size=364

$$\frac{11x\left(a+bx^3\right)^2}{108a^2\left(a^2+2abx^3+b^2x^6\right)^{5/2}}+\frac{x\left(a+bx^3\right)}{12a\left(a^2+2abx^3+b^2x^6\right)^{5/2}}+\frac{110\left(a+bx^3\right)^5\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{729a^{14/3}\sqrt[3]{b}\left(a^2+2abx^3+b^2x^6\right)^{5/2}}-\frac{110\left(a+bx^3\right)}{243\sqrt{3}\,a^{14/3}\sqrt[3]{b}\,(}$$

Rubi [A] time = 0.20, antiderivative size = 364, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {1343, 199, 200, 31, 634, 617, 204, 628}

$$\frac{55x\left(a+bx^3\right)^4}{243a^4\left(a^2+2abx^3+b^2x^6\right)^{5/2}}+\frac{11x\left(a+bx^3\right)^3}{81a^3\left(a^2+2abx^3+b^2x^6\right)^{5/2}}+\frac{11x\left(a+bx^3\right)^2}{108a^2\left(a^2+2abx^3+b^2x^6\right)^{5/2}}+\frac{x\left(a+bx^3\right)}{12a\left(a^2+2abx^3+b^2x^6\right)^{5/2}}+\frac{110\left(a+bx^3\right)^5\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{729a^{14/3}\sqrt[3]{b}\left(a^2+2abx^3+b^2x^6\right)^{5/2}}-\frac{55\left(a+bx^3\right)^5\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{729a^{14/3}\sqrt[3]{b}\left(a^2+2abx^3+b^2x^6\right)^{5/2}}-\frac{110\left(a+bx^3\right)^5\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\sqrt[3]{a}}\right)}{243\sqrt{3}\,a^{14/3}\sqrt[3]{b}\left(a^2+2abx^3+b^2x^6\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(-5/2),x]
```

```
[Out] (x*(a + b*x^3))/(12*a*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)) + (11*x*(a + b*x^3
)^2)/(108*a^2*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)) + (11*x*(a + b*x^3)^3)/(81
*a^3*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)) + (55*x*(a + b*x^3)^4)/(243*a^4*(a^
2 + 2*a*b*x^3 + b^2*x^6)^(5/2)) - (110*(a + b*x^3)^5*ArcTan[(a^(1/3) - 2*b^
(1/3)*x)/(Sqrt[3]*a^(1/3))])/(243*Sqrt[3]*a^(14/3)*b^(1/3)*(a^2 + 2*a*b*x^3
 + b^2*x^6)^(5/2)) + (110*(a + b*x^3)^5*Log[a^(1/3) + b^(1/3)*x])/(729*a^(1
4/3)*b^(1/3)*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)) - (55*(a + b*x^3)^5*Log[a^(
2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(729*a^(14/3)*b^(1/3)*(a^2 + 2*a*b
*x^3 + b^2*x^6)^(5/2))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
```

```
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1343

```
Int[((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_), x_Symbol] :> Dist[(
a + b*x^n + c*x^(2*n))^p/(b + 2*c*x^n)^(2*p), Int[(b + 2*c*x^n)^(2*p), x],
x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{1}{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}\,dx = \frac{\left(2ab+2b^2x^3\right)^5 \int \frac{1}{\left(2ab+2b^2x^3\right)^5}\,dx}{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}$$

$$= \frac{x\left(a+bx^3\right)}{12a\left(a^2+2abx^3+b^2x^6\right)^{5/2}} + \frac{\left(11\left(2ab+2b^2x^3\right)^5\right)\int \frac{1}{\left(2ab+2b^2x^3\right)^4}\,dx}{24ab\left(a^2+2abx^3+b^2x^6\right)^{5/2}}$$

$$= \frac{x\left(a+bx^3\right)}{12a\left(a^2+2abx^3+b^2x^6\right)^{5/2}} + \frac{11x\left(a+bx^3\right)^2}{108a^2\left(a^2+2abx^3+b^2x^6\right)^{5/2}} + \frac{\left(11\left(2ab+2b^2x^3\right)^5}{54a^2b^2\left(a^2+2a}$$

$$= \frac{x\left(a+bx^3\right)}{12a\left(a^2+2abx^3+b^2x^6\right)^{5/2}} + \frac{11x\left(a+bx^3\right)^2}{108a^2\left(a^2+2abx^3+b^2x^6\right)^{5/2}} + \frac{11x\left(a+bx^3\right.}{81a^3\left(a^2+2abx^3+\right.}$$

$$= \frac{x\left(a+bx^3\right)}{12a\left(a^2+2abx^3+b^2x^6\right)^{5/2}} + \frac{11x\left(a+bx^3\right)^2}{108a^2\left(a^2+2abx^3+b^2x^6\right)^{5/2}} + \frac{11x\left(a+bx^3\right.}{81a^3\left(a^2+2abx^3+\right.}$$

$$= \frac{x\left(a+bx^3\right)}{12a\left(a^2+2abx^3+b^2x^6\right)^{5/2}} + \frac{11x\left(a+bx^3\right)^2}{108a^2\left(a^2+2abx^3+b^2x^6\right)^{5/2}} + \frac{11x\left(a+bx^3\right.}{81a^3\left(a^2+2abx^3+\right.}$$

$$= \frac{x\left(a+bx^3\right)}{12a\left(a^2+2abx^3+b^2x^6\right)^{5/2}} + \frac{11x\left(a+bx^3\right)^2}{108a^2\left(a^2+2abx^3+b^2x^6\right)^{5/2}} + \frac{11x\left(a+bx^3\right.}{81a^3\left(a^2+2abx^3+\right.}$$

$$= \frac{x\left(a+bx^3\right)}{12a\left(a^2+2abx^3+b^2x^6\right)^{5/2}} + \frac{11x\left(a+bx^3\right)^2}{108a^2\left(a^2+2abx^3+b^2x^6\right)^{5/2}} + \frac{11x\left(a+bx^3\right.}{81a^3\left(a^2+2abx^3+\right.}$$

$$= \frac{x\left(a+bx^3\right)}{12a\left(a^2+2abx^3+b^2x^6\right)^{5/2}} + \frac{11x\left(a+bx^3\right)^2}{108a^2\left(a^2+2abx^3+b^2x^6\right)^{5/2}} + \frac{11x\left(a+bx^3\right.}{81a^3\left(a^2+2abx^3+\right.}$$

Mathematica [A] time = 0.11, size = 211, normalized size = 0.58

$$\frac{\left(a+bx^3\right)\left(-\frac{220\left(a+bx^3\right)^4\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{\sqrt[3]{b}}+660a^{2/3}x\left(a+bx^3\right)^3+396a^{5/3}x\left(a+bx^3\right)^2+297a^{8/3}x\left(a+bx^3\right)+243a^{11/3}x+\frac{440\left(a+bx^3\right)^4\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}}+\frac{440\sqrt{3}\left(a+bx^3\right)^4\tan^{-1}\left(\frac{2\sqrt[3]{b}x-\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt[3]{b}}\right)}{2916a^{14/3}\left(\left(a+bx^3\right)^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(-5/2),x]
```

```
[Out] ((a + b*x^3)*(243*a^(11/3)*x + 297*a^(8/3)*x*(a + b*x^3) + 396*a^(5/3)*x*(a
 + b*x^3)^2 + 660*a^(2/3)*x*(a + b*x^3)^3 + (440*Sqrt[3]*(a + b*x^3)^4*ArcT
an[(-a^(1/3) + 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/b^(1/3) + (440*(a + b*x^3)^
4*Log[a^(1/3) + b^(1/3)*x])/b^(1/3) - (220*(a + b*x^3)^4*Log[a^(2/3) - a^(1
/3)*b^(1/3)*x + b^(2/3)*x^2])/b^(1/3)))/(2916*a^(14/3)*((a + b*x^3)^2)^(5/2
))
```

IntegrateAlgebraic [A] time = 34.13, size = 186, normalized size = 0.51

$$\frac{\left(a+bx^3\right)\left(-\frac{55\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{729a^{14/3}\sqrt[3]{b}}+\frac{110\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{729a^{14/3}\sqrt[3]{b}}-\frac{110\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{243\sqrt{3}a^{14/3}\sqrt[3]{b}}+\frac{x\left(532a^3+1023a^2bx^3+792ab^2x^6+220b^3x^9\right)}{972a^4\left(a+bx^3\right)^4}\right)}{\sqrt{\left(a+bx^3\right)^2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a^2 + 2*a*b*x^3 + b^2*x^6)^(-5/2),x]
```

```
[Out] ((a + b*x^3)*((x*(532*a^3 + 1023*a^2*b*x^3 + 792*a*b^2*x^6 + 220*b^3*x^9))/
(972*a^4*(a + b*x^3)^4) - (110*ArcTan[1/Sqrt[3] - (2*b^(1/3)*x)/(Sqrt[3]*a^
(1/3))])/(243*Sqrt[3]*a^(14/3)*b^(1/3)) + (110*Log[a^(1/3) + b^(1/3)*x])/(7
29*a^(14/3)*b^(1/3)) - (55*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/
(729*a^(14/3)*b^(1/3))))/Sqrt[(a + b*x^3)^2]
```

fricas [A] time = 1.24, size = 719, normalized size = 1.98

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/2916*(660*a^2*b^4*x^10 + 2376*a^3*b^3*x^7 + 3069*a^4*b^2*x^4 + 1596*a^5*
b*x + 660*sqrt(1/3)*(a*b^5*x^12 + 4*a^2*b^4*x^9 + 6*a^3*b^3*x^6 + 4*a^4*b^2
*x^3 + a^5*b)*sqrt(-(a^2*b)^(1/3)/b)*log((2*a*b*x^3 - 3*(a^2*b)^(1/3)*a*x -
 a^2 + 3*sqrt(1/3)*(2*a*b*x^2 + (a^2*b)^(2/3)*x - (a^2*b)^(1/3)*a)*sqrt(-(a
^2*b)^(1/3)/b))/(b*x^3 + a)) - 220*(b^4*x^12 + 4*a*b^3*x^9 + 6*a^2*b^2*x^6
+ 4*a^3*b*x^3 + a^4)*(a^2*b)^(2/3)*log(a*b*x^2 - (a^2*b)^(2/3)*x + (a^2*b)^
(1/3)*a) + 440*(b^4*x^12 + 4*a*b^3*x^9 + 6*a^2*b^2*x^6 + 4*a^3*b*x^3 + a^4)
*(a^2*b)^(2/3)*log(a*b*x + (a^2*b)^(2/3)))/(a^6*b^5*x^12 + 4*a^7*b^4*x^9 +
6*a^8*b^3*x^6 + 4*a^9*b^2*x^3 + a^10*b), 1/2916*(660*a^2*b^4*x^10 + 2376*a^
3*b^3*x^7 + 3069*a^4*b^2*x^4 + 1596*a^5*b*x + 1320*sqrt(1/3)*(a*b^5*x^12 +
4*a^2*b^4*x^9 + 6*a^3*b^3*x^6 + 4*a^4*b^2*x^3 + a^5*b)*sqrt((a^2*b)^(1/3)/b
)*arctan(sqrt(1/3)*(2*(a^2*b)^(2/3)*x - (a^2*b)^(1/3)*a)*sqrt((a^2*b)^(1/3)
/b)/a^2) - 220*(b^4*x^12 + 4*a*b^3*x^9 + 6*a^2*b^2*x^6 + 4*a^3*b*x^3 + a^4)
*(a^2*b)^(2/3)*log(a*b*x^2 - (a^2*b)^(2/3)*x + (a^2*b)^(1/3)*a) + 440*(b^4*
x^12 + 4*a*b^3*x^9 + 6*a^2*b^2*x^6 + 4*a^3*b*x^3 + a^4)*(a^2*b)^(2/3)*log(a
*b*x + (a^2*b)^(2/3)))/(a^6*b^5*x^12 + 4*a^7*b^4*x^9 + 6*a^8*b^3*x^6 + 4*a^
9*b^2*x^3 + a^10*b)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

maple [A] time = 0.01, size = 519, normalized size = 1.43

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x)
```

```
[Out] 1/2916*(-440*3^(1/2)*b^4*x^12*arctan(1/3*3^(1/2)*(-2*x+(a/b)^(1/3))/(a/b)^(
1/3))+440*b^4*x^12*ln(x+(a/b)^(1/3))-220*b^4*x^12*ln(x^2-(a/b)^(1/3)*x+(a/b
)^(2/3))+660*(a/b)^(2/3)*b^4*x^10-1760*3^(1/2)*a*b^3*x^9*arctan(1/3*3^(1/2)
*(-2*x+(a/b)^(1/3))/(a/b)^(1/3))+1760*a*b^3*x^9*ln(x+(a/b)^(1/3))-880*a*b^3
*x^9*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+2376*(a/b)^(2/3)*a*b^3*x^7-2640*3^(1
/2)*a^2*b^2*x^6*arctan(1/3*3^(1/2)*(-2*x+(a/b)^(1/3))/(a/b)^(1/3))+2640*a^2
*b^2*x^6*ln(x+(a/b)^(1/3))-1320*a^2*b^2*x^6*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3
))+3069*(a/b)^(2/3)*a^2*b^2*x^4-1760*3^(1/2)*a^3*b*x^3*arctan(1/3*3^(1/2)*(
-2*x+(a/b)^(1/3))/(a/b)^(1/3))+1760*a^3*b*x^3*ln(x+(a/b)^(1/3))-880*a^3*b*x
^3*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1596*(a/b)^(2/3)*a^3*b*x-440*3^(1/2)*a
^4*arctan(1/3*3^(1/2)*(-2*x+(a/b)^(1/3))/(a/b)^(1/3))+440*a^4*ln(x+(a/b)^(1
/3))-220*a^4*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3)))*(b*x^3+a)/(a/b)^(2/3)/b/a^4
/((b*x^3+a)^2)^(5/2)
```

maxima [A] time = 2.15, size = 189, normalized size = 0.52

$$\frac{220\,b^3x^{10}+792\,ab^2x^7+1023\,a^2bx^4+532\,a^3x}{972\left(a^4b^4x^{12}+4\,a^5b^3x^9+6\,a^6b^2x^6+4\,a^7bx^3+a^8\right)}+\frac{110\,\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{729\,a^4b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{55\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{729\,a^4b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{110\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{729\,a^4b\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/972*(220*b^3*x^10 + 792*a*b^2*x^7 + 1023*a^2*b*x^4 + 532*a^3*x)/(a^4*b^4*
x^12 + 4*a^5*b^3*x^9 + 6*a^6*b^2*x^6 + 4*a^7*b*x^3 + a^8) + 110/729*sqrt(3)
*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a^4*b*(a/b)^(2/3)) -
55/729*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a^4*b*(a/b)^(2/3)) + 110/729
*log(x + (a/b)^(1/3))/(a^4*b*(a/b)^(2/3))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{{\left(a^2+2\,a\,b\,x^3+b^2\,x^6\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2),x)
```

```
[Out] int(1/(a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a^{2}+2abx^{3}+b^{2}x^{6}\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b**2*x**6+2*a*b*x**3+a**2)**(5/2),x)
```

```
[Out] Integral((a**2 + 2*a*b*x**3 + b**2*x**6)**(-5/2), x)
```

3.114 $\int \frac{1}{x\left(a^2+2abx^3+b^2x^6\right)^{5/2}}\,dx$

Optimal. Leaf size=223

$$\frac{1}{9a^2\left(a+bx^3\right)^2\sqrt{a^2+2abx^3+b^2x^6}}+\frac{1}{12a\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}}+\frac{\log(x)\left(a+bx^3\right)}{a^5\sqrt{a^2+2abx^3+b^2x^6}}-\frac{\left(a+bx^3\right)\,\mathrm{l}}{3a^5\sqrt{a^2+}}$$

Rubi [A] time = 0.12, antiderivative size = 223, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1355, 266, 44}

$$\frac{1}{6a^3\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}}+\frac{1}{9a^2\left(a+bx^3\right)^2\sqrt{a^2+2abx^3+b^2x^6}}+\frac{1}{12a\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}}+\frac{1}{3a^4\sqrt{a^2+2abx^3+b^2x^6}}+\frac{\log(x)\left(a+bx^3\right)}{a^5\sqrt{a^2+2abx^3+b^2x^6}}-\frac{\left(a+bx^3\right)\log\left(a+bx^3\right)}{3a^5\sqrt{a^2+2abx^3+b^2x^6}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)),x]
```

```
[Out] 1/(3*a^4*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) + 1/(12*a*(a + b*x^3)^3*Sqrt[a^2
+ 2*a*b*x^3 + b^2*x^6]) + 1/(9*a^2*(a + b*x^3)^2*Sqrt[a^2 + 2*a*b*x^3 + b^2
*x^6]) + 1/(6*a^3*(a + b*x^3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) + ((a + b*x^
3)*Log[x])/(a^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) - ((a + b*x^3)*Log[a + b*x
^3])/(3*a^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\int \frac{1}{x\left(a^2+2abx^3+b^2x^6\right)^{5/2}}\,dx = \frac{\left(b^4\left(ab+b^2x^3\right)\right)\int \frac{1}{x\left(ab+b^2x^3\right)^5}\,dx}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$= \frac{\left(b^4\left(ab+b^2x^3\right)\right)\text{Subst}\left(\int \frac{1}{x\left(ab+b^2x\right)^5}\,dx, x, x^3\right)}{3\sqrt{a^2+2abx^3+b^2x^6}}$$

$$= \frac{\left(b^4\left(ab+b^2x^3\right)\right)\text{Subst}\left(\int\left(\frac{1}{a^5b^5x} - \frac{1}{ab^4(a+bx)^5} - \frac{1}{a^2b^4(a+bx)^4} - \frac{1}{a^3b^4(a+bx)^3} - \frac{1}{a^4b^4(a+bx}\right.\right.}{3\sqrt{a^2+2abx^3+b^2x^6}}$$

$$= \frac{1}{3a^4\sqrt{a^2+2abx^3+b^2x^6}} + \frac{1}{12a\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}} + \frac{}{9a^2\left(a+bx^3\right)^2}$$

Mathematica [A] time = 0.04, size = 96, normalized size = 0.43

$$\frac{a\left(25a^3+52a^2bx^3+42ab^2x^6+12b^3x^9\right)+36\log(x)\left(a+bx^3\right)^4-12\left(a+bx^3\right)^4\log\left(a+bx^3\right)}{36a^5\left(a+bx^3\right)^3\sqrt{\left(a+bx^3\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)),x]
```

```
[Out] (a*(25*a^3 + 52*a^2*b*x^3 + 42*a*b^2*x^6 + 12*b^3*x^9) + 36*(a + b*x^3)^4*L
og[x] - 12*(a + b*x^3)^4*Log[a + b*x^3])/(36*a^5*(a + b*x^3)^3*Sqrt[(a + b*
x^3)^2])
```

IntegrateAlgebraic [B] time = 127.75, size = 3896, normalized size = 17.47

Result too large to show

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)),x]
```

```
[Out] (-32*Sqrt[b^2]*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]*(3*a^87 - 3*a^86*b*x^3 + 3*a
^85*b^2*x^6 - 3*a^84*b^3*x^9 + 28*a^83*b^4*x^12 + 4074*a^82*b^5*x^15 + 3283
92*a^81*b^6*x^18 + 17416344*a^80*b^7*x^21 + 684119040*a^79*b^8*x^24 + 21226
212480*a^78*b^9*x^27 + 541798692864*a^77*b^10*x^30 + 11700119129088*a^76*b^
11*x^33 + 218177660073984*a^75*b^12*x^36 + 3568299926191104*a^74*b^13*x^39
+ 51815594461802496*a^73*b^14*x^42 + 674671446755364864*a^72*b^15*x^45 + 79
41063256075862016*a^71*b^16*x^48 + 85067029477553700864*a^70*b^17*x^51 + 83
4123879538833555456*a^69*b^18*x^54 + 7523474448566234578944*a^68*b^19*x^57
+ 62685534036478928879616*a^67*b^20*x^60 + 484263949539613361700864*a^66*b^
21*x^63 + 3479930411745645615906816*a^65*b^22*x^66 + 23327835134055094829997
3504*a^64*b^23*x^69 + 146249615315137820985655296*a^63*b^24*x^72 + 85943205
912257082461415014 4*a^62*b^25*x^75 + 4743516432354559922458853376*a^61*b^26
*x^78 + 24634603647181064055495327744*a^60*b^27*x^81 + 120573287346692375306
185998336*a^59*b^28*x^84 + 556993306425737520204547620864*a^58*b^29*x^87 +
 2431710423525963544007685439488*a^57*b^30*x^90 + 100449908116746060810190
87945728*a^56*b^31*x^93 + 39302890124942958048947321438208*a^55*b^32*x^96 +
145797964967106670900820916043776*a^54*b^33*x^99 + 51321880813862815441651
8496518144*a^53*b^34*x^102 + 1715578603609902451791538956533760*a^52*b^35*x^
105 + 5449694039783497785024773904924672*a^51*b^36*x^108 + 1646075331496819
230532900076126 2080*a^50*b^37*x^111 + 47301435355834570726904893777379328*a
^49*b^38*x^114 + 129374002716573907248459868192899072*a^48*b^39*x^117 + 336
```

```
9306366465496963815274138231111168*a^47*b^40*x^120 + 835796283354984631071335990534602752*a^46*b^41*x^123 + 1975361005020021175749282788611719168*a^45*b^42*x^126 + 4449110127870895658771635322027507712*a^44*b^43*x^129 + 9551037404260216847866175737338789888*a^43*b^44*x^132 + 19544508416407422206340829051147517952*a^42*b^45*x^135 + 38125736076483998462868401157222432768*a^41*b^46*x^138 + 70897347674846732883050381793665482752*a^40*b^47*x^141 + 125671127881529846697796348885994569728*a^39*b^48*x^144 + 212317935948353209887693339475650281472*a^38*b^49*x^147 + 341830976243852079520987249876704165888*a^37*b^50*x^150 + 524340224188047425154680645061540052992*a^36*b^51*x^153 + 766073281583947528164240486507938316288*a^35*b^52*x^156 + 1065700613371013117842709614376603615232*a^34*b^53*x^159 + 1411020168689744925287295159943475232768*a^33*b^54*x^162 + 1777302527925165461341561537146824687616*a^32*b^55*x^165 + 2128564412652177688937195991485965664256*a^31*b^56*x^168 + 2422395406390342874120214563859260768256*a^30*b^57*x^171 + 2617807829735370087962525106611234537472*a^29*b^58*x^174 + 2684295777372515995379929097395626835968*a^28*b^59*x^177 + 2609450207845959606905876323113652715520*a^27*b^60*x^180 + 2402576508782798093149416471150968438784*a^26*b^61*x^183 + 2092916507617038227414227202520498831360*a^25*b^62*x^186 + 1722894322550200785772462871077208457216*a^24*b^63*x^189 + 1338524199162955156171842117301576925184*a^23*b^64*x^192 + 979986658223088202764220685370425081856*a^22*b^65*x^195 + 675048463780981537495401786462871486464*a^21*b^66*x^198 + 436701745764302470805819212429950713856*a^20*b^67*x^201 + 264784901454740686099176120664272666624*a^19*b^68*x^204 + 150133777416926053796721610962266750976*a^18*b^69*x^207 + 79403252661685074394167343394234302464*a^17*b^70*x^210 + 39059779966378067682904413566775853056*a^16*b^71*x^213 + 17813285973748092394987873783737483264*a^15*b^72*x^216 + 7503690680834688849004036334253244416*a^14*b^73*x^219 + 2907226777830056762419313398234742784*a^13*b^74*x^222 + 1030915112359687157926188398124466176*a^12*b^75*x^225 + 332671302995605405712650389373845504*a^11*b^76*x^228 + 97031595508821255286739673941016576*a^10*b^77*x^231 + 25374257060868870577491562299654144*a^9*b^78*x^234 + 5890738842117298555432264066400256*a^8*b^79*x^237 + 1199289224076687944555612256337920*a^7*b^80*x^240 + 210809734722622322011205101682688*a^6*b^81*x^243 + 31346364513998779407019781652480*a^5*b^82*x^246 + 3833971628290179974483895386112*a^4*b^83*x^249 + 370352006987302418412887605248*a^3*b^84*x^252 + 26492400411034983734511140864*a^2*b^85*x^255 + 1247611445842297308296773632*a*b^86*x^258 + 29014219670751100192948224*b^87*x^261) - 32*(3*a^88*b - 25*a^84*b^5*x^12 - 4102*a^83*b^6*x^15 - 332466*a^82*b^7*x^18 - 17744736*a^81*b^8*x^21 - 701535384*a^80*b^9*x^24 - 21910331520*a^79*b^10*x^27 - 563024905344*a^78*b^11*x^30 - 12241917821952*a^77*b^12*x^33 - 229877779203072*a^76*b^13*x^36 - 3786477586265088*a^75*b^14*x^39 - 55383894387993600*a^74*b^15*x^42 - 726487041217167360*a^73*b^16*x^45 - 8615734702831226880*a^72*b^17*x^48 - 93008092733629562880*a^71*b^18*x^51 - 919190909016387256320*a^70*b^19*x^54 - 8357598328105068134400*a^69*b^20*x^57 - 70209008485045163458560*a^68*b^21*x^60 - 546949483576092290580480*a^67*b^22*x^63 - 3964194361285258977607680*a^66*b^23*x^66 - 26807765545800740445880320*a^65*b^24*x^69 - 169577450449192915815628800*a^64*b^25*x^72 - 1005681674437708645599805440*a^63*b^26*x^75 - 5602948491477130747073003520*a^62*b^27*x^78 - 29378120079535623977954181120*a^61*b^28*x^81 - 145207890993873439361681326080*a^60*b^29*x^84 - 677566593772429895510733619200*a^59*b^30*x^87 - 2988703729951701064212233060352*a^58*b^31*x^90 - 12476701235200569625026773385216*a^57*b^32*x^93 - 49347880936617564129966409383936*a^56*b^33*x^96 - 185100855092049628949768237481984*a^55*b^34*x^99 - 659016773105734825317339412561920*a^54*b^35*x^102 - 2228797411748530606208057453051904*a^53*b^36*x^105 - 7165272643393400236816312861458432*a^52*b^37*x^108 - 21910447354751690090353774666186752*a^51*b^38*x^111 - 63762188670802763032233894538641408*a^50*b^39*x^114 - 176675438072408477975364761970278400*a^49*b^40*x^117 - 466304639363123603629987282016010240*a^48*b^41*x^120 - 1172726920001534327452863404357713920*a^47*b^42*x^123 - 2811157288375005806820618779146321920*a^46*b^43*x^126 - 6424471132890916834520918110639226880*a^45*b^44*x^129 - 14000147532131112506637811059366297600*a^44*b^45*x
```

^132 - 290955458206676390542070047884863078400*a^43*b^46*x^135 - 57670244492
891420669209230208369950720*a^42*b^47*x^138 - 10902308375133073134591878295
0887915520*a^41*b^48*x^141 - 196568475556376579580846730679660052480*a^40*b
^49*x^144 - 337989063829883056585489688361644851200*a^39*b^50*x^147 - 55414
891219220528940868058935235447360*a^38*b^51*x^150 - 8661712004318995046756
67894938244218880*a^37*b^52*x^153 - 129041350577199495331892113156947836928
0*a^36*b^53*x^156 - 1831773894954960646006950100884541931520*a^35*b^54*x^15
9 - 2476720782060758043130004774320078848000*a^34*b^55*x^162 - 318832269661
4910386628856697090299920384*a^33*b^56*x^165 - 3905866940577343150278757528
632790351872*a^32*b^57*x^168 - 4550959819042520563057410555345226432512*a^3
1*b^58*x^171 - 5040203236125712962082739670470495305728*a^30*b^59*x^174 - 5
302103607107886083342454204006861373440*a^29*b^60*x^177 - 52937459852184756
02285805420509279551488*a^28*b^61*x^180 - 501202671662875770005529279426462
1154304*a^27*b^62*x^183 - 4495493016399836320563643673671467270144*a^26*b^6
3*x^186 - 3815810830167239013186690073597707288576*a^25*b^64*x^189 - 306141
8521713155941944304988378785382400*a^24*b^65*x^192 - 2318510857386043358936
062802672002007040*a^23*b^66*x^195 - 16550351220040697402596224718332965683
20*a^22*b^67*x^198 - 1111750209545284008301220998892822200320*a^21*b^68*x^2
01 - 701486647219043156904995333094223380480*a^20*b^69*x^204 - 414918678871
666739895897731626539417600*a^19*b^70*x^207 - 22953703007861112819088895435
6501053440*a^18*b^71*x^210 - 118463032628063142077071756961010155520*a^17*b
^72*x^213 - 56873065940126160077892287350513336320*a^16*b^73*x^216 - 253169
76654582781243991910117990727680*a^15*b^74*x^219 - 104109174586647456114233
49732487987200*a^14*b^75*x^222 - 3938141890189743920345501796359208960*a^13
*b^76*x^225 - 1363586415355292563638838787498311680*a^12*b^77*x^228 - 42970
2898504426660999390063314862080*a^11*b^78*x^231 - 1224058525696901258642312
36240670720*a^10*b^79*x^234 - 31264995902986169132923826366054400*a^9*b^80*
x^237 - 7090028066193986499987876322738176*a^8*b^81*x^240 - 141009895879931
0266566817358020608*a^7*b^82*x^243 - 242156099236621101418224883335168*a^6*
b^83*x^246 - 35180336142288959381503677038592*a^5*b^84*x^249 - 420432363527
7482392896782991360*a^4*b^85*x^252 - 396844407398337402147398746112*a^3*b^8
6*x^255 - 27740011856877281042807914496*a^2*b^87*x^258 - 127662566551304840
8489721856*a*b^88*x^261 - 29014219670751100192948224*b^89*x^264))/(9*a^4*b*
Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]*(128*a^84*b^4*x^12 + 21120*a^83*b^5*x^15 +
1721472*a^82*b^6*x^18 + 92406656*a^81*b^7*x^21 + 3674439936*a^80*b^8*x^24 +
115431837696*a^79*b^9*x^27 + 2983781007360*a^78*b^10*x^30 + 65264862633984
*a^77*b^11*x^33 + 1232951257055232*a^76*b^12*x^36 + 20432990676713472*a^75*
b^13*x^39 + 300716130445885440*a^74*b^14*x^42 + 3969238746518323200*a^73*b^
15*x^45 + 47370458487200808960*a^72*b^16*x^48 + 514638307833890734080*a^71*
b^17*x^51 + 5118983772087653498880*a^70*b^18*x^54 + 46847576245279590973440
*a^69*b^19*x^57 + 396147929845363900416000*a^68*b^20*x^60 + 310672195963271
5288412160*a^67*b^21*x^63 + 22669023436505501851975680*a^66*b^22*x^66 + 154
345386528573397590343680*a^65*b^23*x^69 + 983080641935232659210895360*a^64*
b^24*x^72 + 5870874650091141471613747200*a^63*b^25*x^75 + 32939251658045736
840163491840*a^62*b^26*x^78 + 173944179132957033591283384320*a^61*b^27*x^81
+ 865962850098397774705978245120*a^60*b^28*x^84 + 407022939653487004583170
1987328*a^59*b^29*x^87 + 18086007219228693734214252625920*a^58*b^30*x^90 +
76065544328296704454991881961472*a^57*b^31*x^93 + 3031243771788375588352918
35334656*a^56*b^32*x^96 + 1145674020859022985294059076059136*a^55*b^33*x^99
+ 4110416795701119981416309783003136*a^54*b^34*x^102 + 1400985167651859764
4792572002959360*a^53*b^35*x^105 + 45394822957658124638063332524294144*a^52
*b^36*x^108 + 139917789751193392435587350802726912*a^51*b^37*x^111 + 410459
873858094378326686815750193152*a^50*b^38*x^114 + 11465881887909049103717479
76404008960*a^49*b^39*x^117 + 3051146630923992568193710004423884800*a^48*b^
40*x^120 + 7737324336869628583877642484858224640*a^47*b^41*x^123 + 18703305
019991137633578654068606238720*a^46*b^42*x^126 + 43107103684238284774282301
309630545920*a^45*b^43*x^129 + 94746125967065558286369665929351004160*a^44*
b^44*x^132 + 198615000002424286648098587431403520000*a^43*b^45*x^135 + 3971
30692504847361152873125569091338240*a^42*b^46*x^138 + 757418302509245017132

523963169657323520*a^41*b^47*x^141 + 1377864478653181794400277391407075819
520*a^40*b^48*x^144 + 2390618413362513124890580419918786723840*a^39*b^49*x^1
47 + 3955385015068042115189991389350644940800*a^38*b^50*x^150 + 62396517372
15173635140320005662399528960*a^37*b^51*x^153 + 938251880913198683328764636
9303054254080*a^36*b^52*x^156 + 13444227011136623928559678656620034785280*a
^35*b^53*x^159 + 18350748809159245654215695088179610648576*a^34*b^54*x^162
+ 23850139665038281564949273182080325386240*a^33*b^55*x^165 + 2950101144237
0924678193433541945719783424*a^32*b^56*x^168 + 347097796391295265167998666З
9524772708352*a^31*b^57*x^171 + 38820803891602154416762573083734716710912*a
^30*b^58*x^174 + 41244886350924311041670535470940728328192*a^29*b^59*x^177
+ 41593718455288022589388471161144339333120*a^28*b^60*x^180 + 3977933777171
4841625416239365458407456768*a^27*b^61*x^183 + 3604434839423862440091822325
5712630308864*a^26*b^62*x^186 + 30909917019058539460410388414124412370944*a
^25*b^63*x^189 + 25056451871195099922047049138792730460160*a^24*b^64*x^192
+ 19174574965327876626062519403677535436800*a^23*b^65*x^195 + 1383173129744
2951886921500151546055229440*a^22*b^66*x^198 + 9389906208869529129738482371
217451909120*a^21*b^67*x^201 + 5988100438665955998606600464981368504320*a^2
0*b^68*x^204 + 3579959214285221715179046877486708162560*a^19*b^69*x^207 + 2
001897592698205886195342610222022656000*a^18*b^70*x^210 + 10444153669223527
00482947391151763619840*a^17*b^71*x^213 + 506902354822510510083032763037195
960320*a^16*b^72*x^216 + 228129429525827274249013529575497400320*a^15*b^73*
x^219 + 94849133060508440059978288214562570240*a^14*b^74*x^222 + 3627706898
6647020067202576304360652800*a^13*b^75*x^225 + 1270106131021082442296761676
9422786560*a^12*b^76*x^228 + 4047255710028362687341349707846778880*a^11*b^7
7*x^231 + 1165860312209840554524188084357038080*a^10*b^78*x^234 + 301141170
995507566848412970190372864*a^9*b^79*x^237 + 690623155850836457839339931015
57760*a^8*b^80*x^240 + 13891106442322941803683011662708736*a^7*b^81*x^243 +
 2412615694461848378203398972899328*a^6*b^82*x^246 + 3544937629702993310826
28280352768*a^5*b^83*x^249 + 42847967496007858756144393617408*a^4*b^84*x^25
2 + 4090618117313628445869793607680*a^3*b^85*x^255 + 2892137416780469667233
07896832*a^2*b^86*x^258 + 13462597927228510489527975936*a*b^87*x^261 + 3094
85009821345068724781056*b^88*x^264) + 9*a^4*b*Sqrt[b^2]*(-128*a^85*b^3*x^12
 - 21248*a^84*b^4*x^15 - 1742592*a^83*b^5*x^18 - 94128128*a^82*b^6*x^21 - 3
766846592*a^81*b^7*x^24 - 119106277632*a^80*b^8*x^27 - 3099212845056*a^79*b
^9*x^30 - 68248643641344*a^78*b^10*x^33 - 1298216119689216*a^77*b^11*x^36 -
 21665941933768704*a^76*b^12*x^39 - 321149121122598912*a^75*b^13*x^42 - 426
9954876964208640*a^74*b^14*x^45 - 51339697233719132160*a^73*b^15*x^48 - 562
008766321091543040*a^72*b^16*x^51 - 5633622079921544232960*a^71*b^17*x^54 -
 51966560017367244472320*a^70*b^18*x^57 - 442995506090643491389440*a^69*b^1
9*x^60 - 3502869889478079188828160*a^68*b^20*x^63 - 25775745396138217140387
840*a^67*b^21*x^66 - 177014409965078899442319360*a^66*b^22*x^69 - 113742602
8463806056801239040*a^65*b^23*x^72 - 6853955292026374130824642560*a^64*b^24
*x^75 - 38810126308136878311777239040*a^63*b^25*x^78 - 20688343079100277043
1446876160*a^62*b^26*x^81 - 1039907029231354808297261629440*a^61*b^27*x^84
- 4936192246633267820537680232448*a^60*b^28*x^87 - 221562366157635637800459
54613248*a^59*b^29*x^90 - 94151551547525398189206134587392*a^58*b^30*x^93 -
 379189921507134263290283717296128*a^57*b^31*x^96 - 14487983980378605441293
50911393792*a^56*b^32*x^99 - 5256090816560142966710368859062272*a^55*b^33*x
^102 - 18120268472219717626208881785962496*a^54*b^34*x^105 - 59404674634176
722282855904527253504*a^53*b^35*x^108 - 18531261270885151707365068332702105
6*a^52*b^36*x^111 - 550377663609287770762274166552920064*a^51*b^37*x^114 -
1557048062648999288698434792154202112*a^50*b^38*x^117 - 4197734819714897478
565457980827893760*a^49*b^39*x^120 - 10788470967793621152071352489282109440
*a^48*b^40*x^123 - 26440629356860766217456296553464463360*a^47*b^41*x^126 -
 61810408704229422407860955378236784640*a^46*b^42*x^129 - 13785322965130384
3060651967238981550080*a^45*b^43*x^132 - 2933611259694898449344682533607545
24160*a^44*b^44*x^135 - 595745692507271647800971713000494858240*a^43*b^45*x
^138 - 1154548995014092378285397088738748661760*a^42*b^46*x^141 - 213528278
1162426811532801354576733143040*a^41*b^47*x^144 - 3768482892015694919290857

```
811325862543360*a^40*b^48*x^147 - 634600342843055524008057180926943166464 0*
a^39*b^49*x^150 - 1019503675228321575033031139501304446976 0*a^38*b^50*x^153
 - 1562217054634716046842796637496545378304 0*a^37*b^51*x^156 - 228267458202
686107618473250259230890393 60*a^36*b^52*x^159 - 31794975820295869582775373 7
44799645433856*a^35*b^53*x^162 - 422008884741975272191649682702599360348 16*
a^34*b^54*x^165 - 533511511074092062431427067240260451696 64*a^33*b^55*x^168
 - 642107910815004511949933001814704924917 76*a^32*b^56*x^171 - 735305835307
31680933562439723259489419264*a^31*b^57*x^174 - 800656902425264654584331085
54675445039104*a^30*b^58*x^177 - 828386048062123336310590066320850676613 12*
a^29*b^59*x^180 - 813730562270028642148047105266027467898 88*a^28*b^60*x^183
 - 758236861659534660263344626211710377656 32*a^27*b^61*x^186 - 669542654132
97163861328611669837042679808*a^26*b^62*x^189 - 559663688902536393824574375
52917142831104*a^25*b^63*x^192 - 442310268365229765481095685424702658969 60*
a^24*b^64*x^195 - 330063062627708285129840195552235906662 40*a^23*b^65*x^198
 - 232216375063124810166599825227635071385 60*a^22*b^66*x^201 - 153780066475
35485128345082836198820413440*a^21*b^67*x^204 - 956805965295117771378564734
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19*b^69*x^210 - 304631295962055858667829000137378627584 0*a^18*b^70*x^213 -
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84332046292612693360640*a^16*b^72*x^219 - 322978562586335714308991817790059
970560*a^15*b^73*x^222 - 131126202047155460127180864518923223040*a^14*b^74*
x^225 - 48978130296857844490170193073783439360*a^13*b^75*x^228 - 1674831702
0239187110308966477269565440*a^12*b^76*x^231 - 5213116022238203241865537792
203816960*a^11*b^77*x^234 - 1467001483205348121372601054547410944*a^10*b^78
*x^237 - 370203486580591212632346963291930624*a^9*b^79*x^240 - 829534220274
06587587617004764266496*a^8*b^80*x^243 - 163037221367847901818864106356080 6
4*a^7*b^81*x^246 - 2767109457432147709286027253252096*a^6*b^82*x^249 - 3973
41730466307189838772673970176*a^5*b^83*x^252 - 469385856133214872020141872 2
5088*a^4*b^84*x^255 - 4379831858991675412593101504512*a^3*b^85*x^258 - 3026
763396052754772128358727 68*a^2*b^86*x^261 - 137720829370498555582527569 92*a
*b^87*x^264 - 3094850098213450687247810 56*b^88*x^267)) + (2*ArcTanh[(Sqrt[b
^2]*x^3)/a - Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]/a])/(3*a^5)
```

fricas [A] time = 1.02, size = 178, normalized size = 0.80

$$\frac{12\,ab^3x^9+42\,a^2b^2x^6+52\,a^3bx^3+25\,a^4-12\left(b^4x^{12}+4\,ab^3x^9+6\,a^2b^2x^6+4\,a^3bx^3+a^4\right)\log\left(bx^3+a\right)+36\left(b^4x^{12}+4\,ab^3x^9+6\,a^2b^2x^6+4\,a^3bx^3+a^4\right)\log(x)}{36\left(a^5b^4x^{12}+4\,a^6b^3x^9+6\,a^7b^2x^6+4\,a^8bx^3+a^9\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/36*(12*a*b^3*x^9 + 42*a^2*b^2*x^6 + 52*a^3*b*x^3 + 25*a^4 - 12*(b^4*x^12
+ 4*a*b^3*x^9 + 6*a^2*b^2*x^6 + 4*a^3*b*x^3 + a^4)*log(b*x^3 + a) + 36*(b^4
*x^12 + 4*a*b^3*x^9 + 6*a^2*b^2*x^6 + 4*a^3*b*x^3 + a^4)*log(x))/(a^5*b^4*x
^12 + 4*a^6*b^3*x^9 + 6*a^7*b^2*x^6 + 4*a^8*b*x^3 + a^9)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

maple [A] time = 0.02, size = 193, normalized size = 0.87

$$\frac{\left(36b^4x^{12}\ln(x)-12b^4x^{12}\ln\left(bx^3+a\right)+144ab^3x^9\ln(x)-48ab^3x^9\ln\left(bx^3+a\right)+12ab^3x^9+216a^2b^2x^6\ln(x)-72a^2b^2x^6\ln\left(bx^3+a\right)+42a^2b^2x^6+144a^3bx^3\ln(x)-48a^3bx^3\ln\left(bx^3+a\right)+52a^3bx^3+36a^4\ln(x)-12a^4\ln\left(bx^3+a\right)+25a^4\right)\left(bx^3+a\right)}{36\left(\left(bx^3+a\right)^2\right)^{\frac{5}{2}}a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x)
```

```
[Out] 1/36*(36*ln(x)*x^12*b^4-12*ln(b*x^3+a)*x^12*b^4+144*ln(x)*x^9*a*b^3-48*ln(b
*x^3+a)*x^9*a*b^3+12*x^9*a*b^3+216*ln(x)*x^6*a^2*b^2-72*ln(b*x^3+a)*x^6*a^2
*b^2+42*x^6*a^2*b^2+144*ln(x)*x^3*a^3*b-48*ln(b*x^3+a)*x^3*a^3*b+52*x^3*a^3
*b+36*ln(x)*a^4-12*ln(b*x^3+a)*a^4+25*a^4)*(b*x^3+a)/a^5/((b*x^3+a)^2)^(5/2
)
```

maxima [A] time = 0.88, size = 132, normalized size = 0.59

$$-\frac{(-1)^{2\,abx^3+2\,a^2}\log\left(\frac{2\,abx}{|x|}+\frac{2\,a^2}{x^2|x|}\right)}{3\,a^5}+\frac{1}{9\left(b^2x^6+2\,abx^3+a^2\right)^{\frac{3}{2}}a^2}+\frac{1}{3\,\sqrt{b^2x^6+2\,abx^3+a^2}\,a^4}+\frac{1}{6\left(x^3+\frac{a}{b}\right)^2a^3b^2}+\frac{1}{12\left(x^3+\frac{a}{b}\right)^4ab^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/3*(-1)^(2*a*b*x^3 + 2*a^2)*log(2*a*b*x/abs(x) + 2*a^2/(x^2*abs(x)))/a^5
+ 1/9/((b^2*x^6 + 2*a*b*x^3 + a^2)^(3/2)*a^2) + 1/3/(sqrt(b^2*x^6 + 2*a*b*x
^3 + a^2)*a^4) + 1/6/((x^3 + a/b)^2*a^3*b^2) + 1/12/((x^3 + a/b)^4*a*b^4)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x\left(a^2+2\,a\,b\,x^3+b^2\,x^6\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2)),x)
```

```
[Out] int(1/(x*(a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(\left(a+bx^3\right)^2\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b**2*x**6+2*a*b*x**3+a**2)**(5/2),x)
```

```
[Out] Integral(1/(x*((a + b*x**3)**2)**(5/2)), x)
```

3.115 $\int \frac{1}{x^2\left(a^2+2abx^3+b^2x^6\right)^{5/2}}\,dx$

Optimal. Leaf size=398

$$\frac{13}{108a^2x\sqrt{a^2+2abx^3+b^2x^6}\,\left(a+bx^3\right)^2}+\frac{1}{12ax\sqrt{a^2+2abx^3+b^2x^6}\,\left(a+bx^3\right)^3}+\frac{455\sqrt[3]{b}\,\left(a+bx^3\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{729a^{16/3}\sqrt{a^2+2abx^3+b^2x^6}}$$

Rubi [A] time = 0.22, antiderivative size = 398, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.346, Rules used = {1355, 290, 325, 292, 31, 634, 617, 204, 628}

$$-\frac{455\left(a+bx^3\right)}{243a^5x\sqrt{a^2+2abx^3+b^2x^6}}+\frac{455}{972a^4x\sqrt{a^2+2abx^3+b^2x^6}}+\frac{65}{324a^3x\sqrt{a^2+2abx^3+b^2x^6}\,\left(a+bx^3\right)}+\frac{13}{108a^2x\sqrt{a^2+2abx^3+b^2x^6}\,\left(a+bx^3\right)^2}+\frac{1}{12ax\sqrt{a^2+2abx^3+b^2x^6}\,\left(a+bx^3\right)^3}+\frac{455\sqrt[3]{b}\,\left(a+bx^3\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{729a^{16/3}\sqrt{a^2+2abx^3+b^2x^6}}-\frac{455\sqrt[3]{b}\,\left(a+bx^3\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{1458a^{16/3}\sqrt{a^2+2abx^3+b^2x^6}}+\frac{455\sqrt[3]{b}\,\left(a+bx^3\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\sqrt[3]{a}}\right)}{243\sqrt{3}\,a^{16/3}\sqrt{a^2+2abx^3+b^2x^6}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)),x]
```

```
[Out] 455/(972*a^4*x*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) + 1/(12*a*x*(a + b*x^3)^3*S
qrt[a^2 + 2*a*b*x^3 + b^2*x^6]) + 13/(108*a^2*x*(a + b*x^3)^2*Sqrt[a^2 + 2*
a*b*x^3 + b^2*x^6]) + 65/(324*a^3*x*(a + b*x^3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*
x^6]) - (455*(a + b*x^3))/(243*a^5*x*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) + (45
5*b^(1/3)*(a + b*x^3)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(2
43*Sqrt[3]*a^(16/3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) + (455*b^(1/3)*(a + b*
x^3)*Log[a^(1/3) + b^(1/3)*x])/(729*a^(16/3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6
]) - (455*b^(1/3)*(a + b*x^3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2
])/(1458*a^(16/3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
```

```
b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\int \frac{1}{x^2\left(a^2+2abx^3+b^2x^6\right)^{5/2}}\,dx = \frac{\left(b^4\left(ab+b^2x^3\right)\right)\int \frac{1}{x^2\left(ab+b^2x^3\right)^5}\,dx}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$= \frac{1}{12ax\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}} + \frac{\left(13b^3\left(ab+b^2x^3\right)\right)\int \frac{1}{x^2\left(ab+b^2x^3\right)^4}\,dx}{12a\sqrt{a^2+2abx^3+b^2x^6}}$$

$$= \frac{1}{12ax\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}} + \frac{13}{108a^2x\left(a+bx^3\right)^2\sqrt{a^2+2abx^3+b^2x^6}}$$

$$= \frac{1}{12ax\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}} + \frac{13}{108a^2x\left(a+bx^3\right)^2\sqrt{a^2+2abx^3+b^2x^6}}$$

$$= \frac{455}{972a^4x\sqrt{a^2+2abx^3+b^2x^6}} + \frac{1}{12ax\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}} + \frac{}{108a^2x}$$

$$= \frac{455}{972a^4x\sqrt{a^2+2abx^3+b^2x^6}} + \frac{1}{12ax\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}} + \frac{}{108a^2x}$$

$$= \frac{455}{972a^4x\sqrt{a^2+2abx^3+b^2x^6}} + \frac{1}{12ax\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}} + \frac{}{108a^2x}$$

$$= \frac{455}{972a^4x\sqrt{a^2+2abx^3+b^2x^6}} + \frac{1}{12ax\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}} + \frac{}{108a^2x}$$

$$= \frac{455}{972a^4x\sqrt{a^2+2abx^3+b^2x^6}} + \frac{1}{12ax\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}} + \frac{}{108a^2x}$$

$$= \frac{455}{972a^4x\sqrt{a^2+2abx^3+b^2x^6}} + \frac{1}{12ax\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}} + \frac{}{108a^2x}$$

Mathematica [A] time = 0.13, size = 242, normalized size = 0.61

$$\frac{\left(a+bx^3\right)\left(-910\sqrt[3]{b}\left(a+bx^3\right)^4\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)-243a^{10/3}bx^2-1179a^{4/3}bx^2\left(a+bx^3\right)^2-594a^{7/3}bx^2\left(a+bx^3\right)-\frac{2916\sqrt[3]{a}\left(a+bx^3\right)^4}{x}+1820\sqrt[3]{b}\left(a+bx^3\right)^4\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)-1820\sqrt{3}\sqrt[3]{b}\left(a+bx^3\right)^4\tan^{-1}\left(\frac{2\sqrt[3]{b}x-\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)-2544\sqrt[3]{a}bx^2\left(a+bx^3\right)^3\right)}{2916a^{16/3}\left(\left(a+bx^3\right)^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)),x]
```

```
[Out] ((a + b*x^3)*(-243*a^(10/3)*b*x^2 - 594*a^(7/3)*b*x^2*(a + b*x^3) - 1179*a^
(4/3)*b*x^2*(a + b*x^3)^2 - 2544*a^(1/3)*b*x^2*(a + b*x^3)^3 - (2916*a^(1/3
)*(a + b*x^3)^4)/x - 1820*Sqrt[3]*b^(1/3)*(a + b*x^3)^4*ArcTan[(-a^(1/3) +
2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))] + 1820*b^(1/3)*(a + b*x^3)^4*Log[a^(1/3) +
b^(1/3)*x] - 910*b^(1/3)*(a + b*x^3)^4*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^
(2/3)*x^2]))/(2916*a^(16/3)*((a + b*x^3)^2)^(5/2))
```

IntegrateAlgebraic [A] time = 35.25, size = 199, normalized size = 0.50

$$\frac{\left(a+bx^3\right)\left(-\frac{455\sqrt[3]{b}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{1458a^{16/3}}+\frac{455\sqrt[3]{b}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{729a^{16/3}}+\frac{455\sqrt[3]{b}\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{243\sqrt{3}a^{16/3}}+\frac{-972a^4-5408a^3bx^3-9360a^2b^2x^6-6825ab^3x^9-1820b^4x^{12}}{972a^5x\left(a+bx^3\right)^4}\right)}{\sqrt{\left(a+bx^3\right)^2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^2*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)),x]
```

```
[Out] ((a + b*x^3)*((-972*a^4 - 5408*a^3*b*x^3 - 9360*a^2*b^2*x^6 - 6825*a*b^3*x^
9 - 1820*b^4*x^12)/(972*a^5*x*(a + b*x^3)^4) + (455*b^(1/3)*ArcTan[1/Sqrt[3
] - (2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(243*Sqrt[3]*a^(16/3)) + (455*b^(1/3)
*Log[a^(1/3) + b^(1/3)*x])/(729*a^(16/3)) - (455*b^(1/3)*Log[a^(2/3) - a^(1
/3)*b^(1/3)*x + b^(2/3)*x^2])/(1458*a^(16/3))))/Sqrt[(a + b*x^3)^2]
```

fricas [A] time = 0.97, size = 311, normalized size = 0.78

$$-\frac{5460\,b^4x^{12}+20475\,ab^3x^9+28080\,a^2b^2x^6+16224\,a^3bx^3+2916\,a^4+1820\,\sqrt{3}\left(b^4x^{13}+4\,ab^3x^{10}+6\,a^2b^2x^7+4\,a^3bx^4+a^4x\right)\left(\frac{b}{a}\right)^{\frac{1}{3}}\arctan\left(\frac{2}{3}\,\sqrt{3}x\left(\frac{b}{a}\right)^{\frac{1}{3}}-\frac{1}{3}\,\sqrt{3}\right)+910\left(b^4x^{13}+4\,ab^3x^{10}+6\,a^2b^2x^7+4\,a^3bx^4+a^4x\right)\left(\frac{b}{a}\right)^{\frac{1}{3}}\log\left(bx^2-ax\left(\frac{b}{a}\right)^{\frac{2}{3}}+a\left(\frac{b}{a}\right)^{\frac{1}{3}}\right)-1820\left(b^4x^{13}+4\,ab^3x^{10}+6\,a^2b^2x^7+4\,a^3bx^4+a^4x\right)\left(\frac{b}{a}\right)^{\frac{1}{3}}\log\left(bx+a\left(\frac{b}{a}\right)^{\frac{2}{3}}\right)}{2916\left(a^5b^4x^{13}+4\,a^6b^3x^{10}+6\,a^7b^2x^7+4\,a^8bx^4+a^9x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/2916*(5460*b^4*x^12 + 20475*a*b^3*x^9 + 28080*a^2*b^2*x^6 + 16224*a^3*b*
x^3 + 2916*a^4 + 1820*sqrt(3)*(b^4*x^13 + 4*a*b^3*x^10 + 6*a^2*b^2*x^7 + 4*
a^3*b*x^4 + a^4*x)*(b/a)^(1/3)*arctan(2/3*sqrt(3)*x*(b/a)^(1/3) - 1/3*sqrt(
3)) + 910*(b^4*x^13 + 4*a*b^3*x^10 + 6*a^2*b^2*x^7 + 4*a^3*b*x^4 + a^4*x)*(
b/a)^(1/3)*log(b*x^2 - a*x*(b/a)^(2/3) + a*(b/a)^(1/3)) - 1820*(b^4*x^13 +
4*a*b^3*x^10 + 6*a^2*b^2*x^7 + 4*a^3*b*x^4 + a^4*x)*(b/a)^(1/3)*log(b*x + a
*(b/a)^(2/3)))/(a^5*b^4*x^13 + 4*a^6*b^3*x^10 + 6*a^7*b^2*x^7 + 4*a^8*b*x^4
 + a^9*x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

maple [B] time = 0.02, size = 536, normalized size = 1.35

$$-\frac{\left(-1820\sqrt{3}\,b^4x^{13}\arctan\left(\frac{\sqrt{3}\left(-2x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)-1820b^4x^{13}\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)+910b^4x^{13}\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)+5460\left(\frac{a}{b}\right)^{\frac{1}{3}}b^4x^{12}-7280\sqrt{3}\,ab^3x^{10}\arctan\left(\frac{\sqrt{3}\left(-2x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)-7280ab^3x^{10}\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)+3640ab^3x^{10}\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)+20475\left(\frac{a}{b}\right)^{\frac{1}{3}}ab^3x^9-10920\sqrt{3}\,a^2b^2x^7\arctan\left(\frac{\sqrt{3}\left(-2x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)-10920a^2b^2x^7\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)+5460a^2b^2x^7\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)+28080\left(\frac{a}{b}\right)^{\frac{1}{3}}a^2b^2x^6-7280\sqrt{3}\,a^3bx^4\arctan\left(\frac{\sqrt{3}\left(-2x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)-7280a^3bx^4\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)+3640a^3bx^4\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)+16224\left(\frac{a}{b}\right)^{\frac{1}{3}}a^3bx^3-1820\sqrt{3}\,a^4x\arctan\left(\frac{\sqrt{3}\left(-2x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)-1820a^4x\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)+910a^4x\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)+2916\left(\frac{a}{b}\right)^{\frac{1}{3}}a^4\right)\left(bx^3+a\right)}{2916\left(\frac{a}{b}\right)^{\frac{1}{3}}\left(\left(bx^3+a\right)^2\right)^{\frac{5}{2}}a^5x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x)
```

```
[Out] -1/2916*(-1820*3^(1/2)*arctan(1/3*3^(1/2)*(-2*x+(a/b)^(1/3))/(a/b)^(1/3))*x
^13*b^4-1820*ln(x+(a/b)^(1/3))*x^13*b^4+910*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3
))*x^13*b^4+5460*(a/b)^(1/3)*x^12*b^4-7280*3^(1/2)*arctan(1/3*3^(1/2)*(-2*x
+(a/b)^(1/3))/(a/b)^(1/3))*x^10*a*b^3-7280*ln(x+(a/b)^(1/3))*x^10*a*b^3+364
0*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))*x^10*a*b^3+20475*(a/b)^(1/3)*x^9*a*b^3-
10920*3^(1/2)*arctan(1/3*3^(1/2)*(-2*x+(a/b)^(1/3))/(a/b)^(1/3))*x^7*a^2*b^
2-10920*ln(x+(a/b)^(1/3))*x^7*a^2*b^2+5460*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3)
)*x^7*a^2*b^2+28080*(a/b)^(1/3)*x^6*a^2*b^2-7280*3^(1/2)*arctan(1/3*3^(1/2)
*(-2*x+(a/b)^(1/3))/(a/b)^(1/3))*x^4*a^3*b-7280*ln(x+(a/b)^(1/3))*x^4*a^3*b
+3640*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))*x^4*a^3*b+16224*(a/b)^(1/3)*x^3*a^3
```

```
*b-1820*3^(1/2)*arctan(1/3*3^(1/2)*(-2*x+(a/b)^(1/3))/(a/b)^(1/3))*x*a^4-18
20*ln(x+(a/b)^(1/3))*x*a^4+910*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))*x*a^4+2916
*(a/b)^(1/3)*a^4)*(b*x^3+a)/(a/b)^(1/3)/x/a^5/((b*x^3+a)^2)^(5/2)
```

maxima [A] time = 2.33, size = 192, normalized size = 0.48

$$-\frac{1820\,b^4x^{12}+6825\,ab^3x^9+9360\,a^2b^2x^6+5408\,a^3bx^3+972\,a^4}{972\left(a^5b^4x^{13}+4\,a^6b^3x^{10}+6\,a^7b^2x^7+4\,a^8bx^4+a^9x\right)}-\frac{455\,\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{729\,a^5\left(\frac{a}{b}\right)^{\frac{1}{3}}}-\frac{455\,\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{1458\,a^5\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{455\,\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{729\,a^5\left(\frac{a}{b}\right)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/972*(1820*b^4*x^12 + 6825*a*b^3*x^9 + 9360*a^2*b^2*x^6 + 5408*a^3*b*x^3
+ 972*a^4)/(a^5*b^4*x^13 + 4*a^6*b^3*x^10 + 6*a^7*b^2*x^7 + 4*a^8*b*x^4 + a
^9*x) - 455/729*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))
/(a^5*(a/b)^(1/3)) - 455/1458*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a^5*(
a/b)^(1/3)) + 455/729*log(x + (a/b)^(1/3))/(a^5*(a/b)^(1/3))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^2\left(a^2+2\,a\,b\,x^3+b^2\,x^6\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2)),x)
```

```
[Out] int(1/(x^2*(a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2\left(\left(a+bx^3\right)^2\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b**2*x**6+2*a*b*x**3+a**2)**(5/2),x)
```

```
[Out] Integral(1/(x**2*((a + b*x**3)**2)**(5/2)), x)
```

3.116 $\int \frac{1}{x^3\left(a^2+2abx^3+b^2x^6\right)^{5/2}}\,dx$

Optimal. Leaf size=398

$$\frac{7}{54a^2x^2\sqrt{a^2+2abx^3+b^2x^6}\left(a+bx^3\right)^2}+\frac{1}{12ax^2\sqrt{a^2+2abx^3+b^2x^6}\left(a+bx^3\right)^3}-\frac{770b^{2/3}\left(a+bx^3\right)\log\left(\sqrt[3]{a}+\sqrt[3]{\ }\right.}{729a^{17/3}\sqrt{a^2+2abx^3+b^2x^{\ }}}$$

Rubi [A] time = 0.21, antiderivative size = 398, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.346, Rules used = {1355, 290, 325, 200, 31, 634, 617, 204, 628}

$$-\frac{385\left(a+bx^3\right)}{243a^5x^2\sqrt{a^2+2abx^3+b^2x^6}}+\frac{154}{243a^4x^2\sqrt{a^2+2abx^3+b^2x^6}}+\frac{77}{324a^3x^2\sqrt{a^2+2abx^3+b^2x^6}\left(a+bx^3\right)}+\frac{7}{54a^2x^2\sqrt{a^2+2abx^3+b^2x^6}\left(a+bx^3\right)^2}+\frac{1}{12ax^2\sqrt{a^2+2abx^3+b^2x^6}\left(a+bx^3\right)^3}-\frac{770b^{2/3}\left(a+bx^3\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{729a^{17/3}\sqrt{a^2+2abx^3+b^2x^6}}+\frac{385b^{2/3}\left(a+bx^3\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{729a^{17/3}\sqrt{a^2+2abx^3+b^2x^6}}+\frac{770b^{2/3}\left(a+bx^3\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{243\sqrt{3}a^{17/3}\sqrt{a^2+2abx^3+b^2x^6}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)),x]
```

```
[Out] 154/(243*a^4*x^2*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) + 1/(12*a*x^2*(a + b*x^3)
^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) + 7/(54*a^2*x^2*(a + b*x^3)^2*Sqrt[a^2
+ 2*a*b*x^3 + b^2*x^6]) + 77/(324*a^3*x^2*(a + b*x^3)*Sqrt[a^2 + 2*a*b*x^3
+ b^2*x^6]) - (385*(a + b*x^3))/(243*a^5*x^2*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6
]) + (770*b^(2/3)*(a + b*x^3)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/
3))])/(243*Sqrt[3]*a^(17/3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) - (770*b^(2/3)
*(a + b*x^3)*Log[a^(1/3) + b^(1/3)*x])/(729*a^(17/3)*Sqrt[a^2 + 2*a*b*x^3 +
 b^2*x^6]) + (385*b^(2/3)*(a + b*x^3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(
2/3)*x^2])/(729*a^(17/3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
```

```
b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\int \frac{1}{x^3\left(a^2+2abx^3+b^2x^6\right)^{5/2}}\,dx = \frac{\left(b^4\left(ab+b^2x^3\right)\right)\int \frac{1}{x^3\left(ab+b^2x^3\right)^5}\,dx}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$= \frac{1}{12ax^2\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}} + \frac{\left(7b^3\left(ab+b^2x^3\right)\right)\int \frac{1}{x^3\left(ab+b^2x^3\right)^4}\,dx}{6a\sqrt{a^2+2abx^3+b^2x^6}}$$

$$= \frac{1}{12ax^2\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}} + \frac{7}{54a^2x^2\left(a+bx^3\right)^2\sqrt{a^2+2abx^3+b}}$$

$$= \frac{1}{12ax^2\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}} + \frac{7}{54a^2x^2\left(a+bx^3\right)^2\sqrt{a^2+2abx^3+b}}$$

$$= \frac{154}{243a^4x^2\sqrt{a^2+2abx^3+b^2x^6}} + \frac{1}{12ax^2\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}} + \frac{}{54}$$

$$= \frac{154}{243a^4x^2\sqrt{a^2+2abx^3+b^2x^6}} + \frac{1}{12ax^2\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}} + \frac{}{54}$$

$$= \frac{154}{243a^4x^2\sqrt{a^2+2abx^3+b^2x^6}} + \frac{1}{12ax^2\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}} + \frac{}{54}$$

$$= \frac{154}{243a^4x^2\sqrt{a^2+2abx^3+b^2x^6}} + \frac{1}{12ax^2\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}} + \frac{}{54}$$

$$= \frac{154}{243a^4x^2\sqrt{a^2+2abx^3+b^2x^6}} + \frac{1}{12ax^2\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}} + \frac{}{54}$$

$$= \frac{154}{243a^4x^2\sqrt{a^2+2abx^3+b^2x^6}} + \frac{1}{12ax^2\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}} + \frac{}{54}$$

Mathematica [A] time = 0.14, size = 234, normalized size = 0.59

$$\frac{\left(a+bx^3\right)\left(1540b^{2/3}\left(a+bx^3\right)^4\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)-3162a^{2/3}bx\left(a+bx^3\right)^3-1314a^{5/3}bx\left(a+bx^3\right)^2-621a^{8/3}bx\left(a+bx^3\right)-\frac{1458a^{2/3}\left(a+bx^3\right)^4}{x^2}-243a^{11/3}bx-3080b^{2/3}\left(a+bx^3\right)^4\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)-3080\sqrt{3}b^{2/3}\left(a+bx^3\right)^4\tan^{-1}\left(\frac{2\sqrt[3]{b}x-\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)\right)}{2916a^{17/3}\left(\left(a+bx^3\right)^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)),x]
```

```
[Out] ((a + b*x^3)*(-243*a^(11/3)*b*x - 621*a^(8/3)*b*x*(a + b*x^3) - 1314*a^(5/3
)*b*x*(a + b*x^3)^2 - 3162*a^(2/3)*b*x*(a + b*x^3)^3 - (1458*a^(2/3)*(a + b
*x^3)^4)/x^2 - 3080*Sqrt[3]*b^(2/3)*(a + b*x^3)^4*ArcTan[(-a^(1/3) + 2*b^(1
/3)*x)/(Sqrt[3]*a^(1/3))] - 3080*b^(2/3)*(a + b*x^3)^4*Log[a^(1/3) + b^(1/3
)*x] + 1540*b^(2/3)*(a + b*x^3)^4*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)
*x^2]))/(2916*a^(17/3)*((a + b*x^3)^2)^(5/2))
```

IntegrateAlgebraic [A] time = 42.12, size = 199, normalized size = 0.50

$$\frac{(a+bx^3)\left(\frac{385b^{2/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{729a^{17/3}}-\frac{770b^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{729a^{17/3}}+\frac{770b^{2/3}\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{243\sqrt{3}a^{17/3}}+\frac{-486a^4-3724a^3bx^3-7161a^2b^2x^6-5544ab^3x^9-1540b^4x^{12}}{972a^5x^2(a+bx^3)^4}\right)}{\sqrt{(a+bx^3)^2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^3*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)),x]
```

```
[Out] ((a + b*x^3)*((-486*a^4 - 3724*a^3*b*x^3 - 7161*a^2*b^2*x^6 - 5544*a*b^3*x^
9 - 1540*b^4*x^12)/(972*a^5*x^2*(a + b*x^3)^4) + (770*b^(2/3)*ArcTan[1/Sqrt
[3] - (2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(243*Sqrt[3]*a^(17/3)) - (770*b^(2/
3)*Log[a^(1/3) + b^(1/3)*x])/(729*a^(17/3)) + (385*b^(2/3)*Log[a^(2/3) - a^
(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(729*a^(17/3))))/Sqrt[(a + b*x^3)^2]
```

fricas [A] time = 1.30, size = 352, normalized size = 0.88

$$-\frac{4620\,b^4x^{12}+16632\,ab^3x^9+21483\,a^2b^2x^6+11172\,a^3bx^3+1458\,a^4-3080\,\sqrt{3}\left(b^4x^{14}+4\,ab^3x^{11}+6\,a^2b^2x^8+4\,a^3bx^5+a^4x^2\right)\left(-\frac{b^2}{a^2}\right)^{\frac{1}{3}}\arctan\left(\frac{2\,\sqrt{3}\,ax\left(-\frac{b^2}{a^2}\right)^{\frac{2}{3}}-\sqrt{3}\,b}{3\,b}\right)+1540\left(b^4x^{14}+4\,ab^3x^{11}+6\,a^2b^2x^8+4\,a^3bx^5+a^4x^2\right)\left(-\frac{b^2}{a^2}\right)^{\frac{1}{3}}\log\left(b^2x^2+abx\left(-\frac{b^2}{a^2}\right)^{\frac{1}{3}}+a^2\left(-\frac{b^2}{a^2}\right)^{\frac{2}{3}}\right)-3080\left(b^4x^{14}+4\,ab^3x^{11}+6\,a^2b^2x^8+4\,a^3bx^5+a^4x^2\right)\left(-\frac{b^2}{a^2}\right)^{\frac{1}{3}}\log\left(bx-a\left(-\frac{b^2}{a^2}\right)^{\frac{1}{3}}\right)}{2916\left(a^5b^4x^{14}+4\,a^6b^3x^{11}+6\,a^7b^2x^8+4\,a^8bx^5+a^9x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/2916*(4620*b^4*x^12 + 16632*a*b^3*x^9 + 21483*a^2*b^2*x^6 + 11172*a^3*b*
x^3 + 1458*a^4 - 3080*sqrt(3)*(b^4*x^14 + 4*a*b^3*x^11 + 6*a^2*b^2*x^8 + 4*
a^3*b*x^5 + a^4*x^2)*(-b^2/a^2)^(1/3)*arctan(1/3*(2*sqrt(3)*a*x*(-b^2/a^2)^
(2/3) - sqrt(3)*b)/b) + 1540*(b^4*x^14 + 4*a*b^3*x^11 + 6*a^2*b^2*x^8 + 4*a
^3*b*x^5 + a^4*x^2)*(-b^2/a^2)^(1/3)*log(b^2*x^2 + a*b*x*(-b^2/a^2)^(1/3) +
 a^2*(-b^2/a^2)^(2/3)) - 3080*(b^4*x^14 + 4*a*b^3*x^11 + 6*a^2*b^2*x^8 + 4*
a^3*b*x^5 + a^4*x^2)*(-b^2/a^2)^(1/3)*log(b*x - a*(-b^2/a^2)^(1/3)))/(a^5*b
^4*x^14 + 4*a^6*b^3*x^11 + 6*a^7*b^2*x^8 + 4*a^8*b*x^5 + a^9*x^2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

maple [B] time = 0.02, size = 542, normalized size = 1.36

$$-\frac{\left(-3080\sqrt{3}\,b^4x^{14}\arctan\left(\frac{\sqrt{3}\left(-2x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)+3080b^4x^{14}\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)-1540b^4x^{14}\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)-12320\sqrt{3}\,ab^3x^{11}\arctan\left(\frac{\sqrt{3}\left(-2x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)+12320ab^3x^{11}\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)-6160ab^3x^{11}\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)+4620\left(\frac{a}{b}\right)^{\frac{2}{3}}b^4x^{12}-18480\sqrt{3}\,a^2b^2x^8\arctan\left(\frac{\sqrt{3}\left(-2x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)+18480a^2b^2x^8\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)-9240a^2b^2x^8\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)+16632\left(\frac{a}{b}\right)^{\frac{2}{3}}ab^3x^9-12320\sqrt{3}\,a^3bx^5\arctan\left(\frac{\sqrt{3}\left(-2x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)+12320a^3bx^5\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)-6160a^3bx^5\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)+21483\left(\frac{a}{b}\right)^{\frac{2}{3}}a^2b^2x^6-3080\sqrt{3}\,a^4x^2\arctan\left(\frac{\sqrt{3}\left(-2x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)+3080a^4x^2\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)-1540a^4x^2\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)+11172\left(\frac{a}{b}\right)^{\frac{2}{3}}a^3bx^3+1458\left(\frac{a}{b}\right)^{\frac{2}{3}}a^4\right)\left(bx^3+a\right)}{2916\left(\frac{a}{b}\right)^{\frac{2}{3}}\left(\left(bx^3+a\right)^2\right)^{\frac{5}{2}}a^5x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x)
```

```
[Out] -1/2916*(-3080*3^(1/2)*arctan(1/3*3^(1/2)*(-2*x+(a/b)^(1/3))/(a/b)^(1/3))*x
^14*b^4+3080*ln(x+(a/b)^(1/3))*x^14*b^4-1540*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/
3))*x^14*b^4+4620*(a/b)^(2/3)*x^12*b^4-12320*3^(1/2)*arctan(1/3*3^(1/2)*(-2
*x+(a/b)^(1/3))/(a/b)^(1/3))*x^11*a*b^3+12320*ln(x+(a/b)^(1/3))*x^11*a*b^3-
6160*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))*x^11*a*b^3+16632*(a/b)^(2/3)*x^9*a*b
^3-18480*3^(1/2)*arctan(1/3*3^(1/2)*(-2*x+(a/b)^(1/3))/(a/b)^(1/3))*x^8*a^2
*b^2+18480*ln(x+(a/b)^(1/3))*x^8*a^2*b^2-9240*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2
/3))*x^8*a^2*b^2+21483*(a/b)^(2/3)*x^6*a^2*b^2-12320*3^(1/2)*arctan(1/3*3^(
1/2)*(-2*x+(a/b)^(1/3))/(a/b)^(1/3))*x^5*a^3*b+12320*ln(x+(a/b)^(1/3))*x^5*
a^3*b-6160*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))*x^5*a^3*b+11172*(a/b)^(2/3)*x^
```

```
3*a^3*b-3080*3^(1/2)*arctan(1/3*3^(1/2)*(-2*x+(a/b)^(1/3))/(a/b)^(1/3))*x^2
*a^4+3080*ln(x+(a/b)^(1/3))*x^2*a^4-1540*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))*
x^2*a^4+1458*(a/b)^(2/3)*a^4)*(b*x^3+a)/(a/b)^(2/3)/x^2/a^5/((b*x^3+a)^2)^(
5/2)
```

maxima [A] time = 2.06, size = 194, normalized size = 0.49

$$-\frac{1540\,b^{4}x^{12}+5544\,ab^{3}x^{9}+7161\,a^{2}b^{2}x^{6}+3724\,a^{3}bx^{3}+486\,a^{4}}{972\,\left(a^{5}b^{4}x^{14}+4\,a^{6}b^{3}x^{11}+6\,a^{7}b^{2}x^{8}+4\,a^{8}bx^{5}+a^{9}x^{2}\right)}-\frac{770\,\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{729\,a^{5}\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{385\,\log\left(x^{2}-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{729\,a^{5}\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{770\,\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{729\,a^{5}\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/972*(1540*b^4*x^12 + 5544*a*b^3*x^9 + 7161*a^2*b^2*x^6 + 3724*a^3*b*x^3
+ 486*a^4)/(a^5*b^4*x^14 + 4*a^6*b^3*x^11 + 6*a^7*b^2*x^8 + 4*a^8*b*x^5 + a
^9*x^2) - 770/729*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3
))/(a^5*(a/b)^(2/3)) + 385/729*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a^5*
(a/b)^(2/3)) - 770/729*log(x + (a/b)^(1/3))/(a^5*(a/b)^(2/3))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^3\,{\left(a^2+2\,a\,b\,x^3+b^2\,x^6\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2)),x)
```

```
[Out] int(1/(x^3*(a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{3} \left(\left(a + b x^{3}\right)^{2}\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(b**2*x**6+2*a*b*x**3+a**2)**(5/2),x)
```

```
[Out] Integral(1/(x**3*((a + b*x**3)**2)**(5/2)), x)
```

3.117 $\int \frac{1}{x^4\left(a^2+2abx^3+b^2x^6\right)^{5/2}}\,dx$

Optimal. Leaf size=269

$$-\frac{b}{12a^2\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}}-\frac{5b\log(x)\left(a+bx^3\right)}{a^6\sqrt{a^2+2abx^3+b^2x^6}}+\frac{5b\left(a+bx^3\right)\log\left(a+bx^3\right)}{3a^6\sqrt{a^2+2abx^3+b^2x^6}}-\frac{4b}{3a^5\sqrt{a^2+2abx^3+b^2x^6}}$$

Rubi [A] time = 0.14, antiderivative size = 269, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {1355, 266, 44}

$$-\frac{b}{2a^4\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}}-\frac{2b}{9a^3\left(a+bx^3\right)^2\sqrt{a^2+2abx^3+b^2x^6}}-\frac{b}{12a^2\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}}-\frac{4b}{3a^5\sqrt{a^2+2abx^3+b^2x^6}}-\frac{a+bx^3}{3a^5x^3\sqrt{a^2+2abx^3+b^2x^6}}-\frac{5b\log(x)\left(a+bx^3\right)}{a^6\sqrt{a^2+2abx^3+b^2x^6}}+\frac{5b\left(a+bx^3\right)\log\left(a+bx^3\right)}{3a^6\sqrt{a^2+2abx^3+b^2x^6}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)),x]
```

```
[Out] (-4*b)/(3*a^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) - b/(12*a^2*(a + b*x^3)^3*Sq
rt[a^2 + 2*a*b*x^3 + b^2*x^6]) - (2*b)/(9*a^3*(a + b*x^3)^2*Sqrt[a^2 + 2*a*
b*x^3 + b^2*x^6]) - b/(2*a^4*(a + b*x^3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) -
 (a + b*x^3)/(3*a^5*x^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) - (5*b*(a + b*x^3)
*Log[x])/(a^6*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) + (5*b*(a + b*x^3)*Log[a + b
*x^3])/(3*a^6*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\int \frac{1}{x^4\left(a^2+2abx^3+b^2x^6\right)^{5/2}}\,dx = \frac{\left(b^4\left(ab+b^2x^3\right)\right)\int \frac{1}{x^4\left(ab+b^2x^3\right)^5}\,dx}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$= \frac{\left(b^4\left(ab+b^2x^3\right)\right)\text{Subst}\left(\int \frac{1}{x^2\left(ab+b^2x\right)^5}\,dx, x, x^3\right)}{3\sqrt{a^2+2abx^3+b^2x^6}}$$

$$= \frac{\left(b^4\left(ab+b^2x^3\right)\right)\text{Subst}\left(\int \left(\frac{1}{a^5b^5x^2}-\frac{5}{a^6b^4x}+\frac{1}{a^2b^3(a+bx)^5}+\frac{2}{a^3b^3(a+bx)^4}+\frac{3}{a^4b^3(a+b}\right.\right.}{3\sqrt{a^2+2abx^3+b^2x^6}}$$

$$= -\frac{4b}{3a^5\sqrt{a^2+2abx^3+b^2x^6}}-\frac{b}{12a^2\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}}-\frac{}{9a^3\left(a\right.}$$

Mathematica [A] time = 0.05, size = 119, normalized size = 0.44

$$\frac{-a\left(12a^4+125a^3bx^3+260a^2b^2x^6+210ab^3x^9+60b^4x^{12}\right)-180bx^3\log(x)\left(a+bx^3\right)^4+60bx^3\left(a+bx^3\right)^4\log\left(a+bx^3\right)}{36a^6x^3\left(a+bx^3\right)^3\sqrt{\left(a+bx^3\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)),x]
```

```
[Out] (-(a*(12*a^4 + 125*a^3*b*x^3 + 260*a^2*b^2*x^6 + 210*a*b^3*x^9 + 60*b^4*x^1
2)) - 180*b*x^3*(a + b*x^3)^4*Log[x] + 60*b*x^3*(a + b*x^3)^4*Log[a + b*x^3
])/(36*a^6*x^3*(a + b*x^3)^3*Sqrt[(a + b*x^3)^2])
```

IntegrateAlgebraic [B] time = 81.82, size = 2850, normalized size = 10.59

Result too large to show

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^4*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)),x]
```

```
[Out] (-2*Sqrt[b^2]*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]*(3*a^63 - 3*a^62*b*x^3 + 3*a^
61*b^2*x^6 + 9*a^60*b^3*x^9 + 1484*a^59*b^4*x^12 + 89634*a^58*b^5*x^15 + 35
47176*a^57*b^6*x^18 + 103238584*a^56*b^7*x^21 + 2356838400*a^55*b^8*x^24 +
43953744576*a^54*b^9*x^27 + 688626915840*a^53*b^10*x^30 + 9250107586560*a^5
2*b^11*x^33 + 108195839032320*a^51*b^12*x^36 + 1115441253588480*a^50*b^13*x
^39 + 10234958239733760*a^49*b^14*x^42 + 84253639955097600*a^48*b^15*x^45 +
 626363783779123200*a^47*b^16*x^48 + 4228757765620531200*a^46*b^17*x^51 + 2
6049151725611581440*a^45*b^18*x^54 + 147000114743606968320*a^44*b^19*x^57 +
 762584480826139607040*a^43*b^20*x^60 + 3647550970994505154560*a^42*b^21*x^
63 + 16127976156648442429440*a^41*b^22*x^66 + 66068580639182534737920*a^40*
b^23*x^69 + 251239072713580814008320*a^39*b^24*x^72 + 888344859983663773777
920*a^38*b^25*x^75 + 2924829168272464863559680*a^37*b^26*x^78 + 89778749655
7690352566272*a^36*b^27*x^81 + 25718516470160904041791488*a^35*b^28*x^84 +
 68815279531254829887258624*a^34*b^29*x^87 + 172101662127535626374873088*a^
33*b^30*x^90 + 402506093314484489003466752*a^32*b^31*x^93 + 880658309537180
769413234688*a^31*b^32*x^96 + 1802963185421958307337207808*a^30*b^33*x^99 +
 3454182779430019135581454336*a^29*b^34*x^102 + 619235088586204268311923916
8*a^28*b^35*x^105 + 10385517381568557089430700032*a^27*b^36*x^108 + 1628959
2602873600309238693888*a^26*b^37*x^111 + 23882708470211058375623442432*a^25
*b^38*x^114 + 32708573827319329343938756608*a^24*b^39*x^117 + 4181050390018
0714625288896512*a^23*b^40*x^120 + 49833178729962384051916505088*a^22*b^41*
```

x^123 + 55314738880465273152142835712*a^21*b^42*x^126 + 5710049573385088084
0216608768*a^20*b^43*x^129 + 54726941348404022924701335552*a^19*b^44*x^132
+ 48606655891944070942417747968*a^18*b^45*x^135 + 3991760800870678803443299
1232*a^17*b^46*x^138 + 30234237658617949398937632768*a^16*b^47*x^141 + 2105
8000972731372092000305152*a^15*b^48*x^144 + 13441018440774824592175792128*a
^14*b^49*x^147 + 7830928399966132744489009152*a^13*b^50*x^150 + 41450870054
08504523991810048*a^12*b^51*x^153 + 1982450466391073623482826752*a^11*b^52*
x^156 + 851082047429972494978646016*a^10*b^53*x^159 + 325392797217001837833
486336*a^9*b^54*x^162 + 109726425313175461183881216*a^8*b^55*x^165 + 322436
24862748916500660224*a^7*b^56*x^168 + 8130508717424292308975616*a^6*b^57*x^
171 + 1723944790864168421949440*a^5*b^58*x^174 + 298927614217018297810944*a
^4*b^59*x^177 + 40705334872025491046400*a^3*b^60*x^180 + 408206270224861757
4400*a^2*b^61*x^183 + 268054249821091921920*a*b^62*x^186 + 8646911284551352
320*b^63*x^189) - 2*(3*a^64*b - 12*a^61*b^4*x^9 - 1493*a^60*b^5*x^12 - 9111
8*a^59*b^6*x^15 - 3636810*a^58*b^7*x^18 - 106785760*a^57*b^8*x^21 - 2460076
984*a^56*b^9*x^24 - 46310582976*a^55*b^10*x^27 - 732580660416*a^54*b^11*x^3
0 - 9938734502400*a^53*b^12*x^33 - 117445946618880*a^52*b^13*x^36 - 1223637
092620800*a^51*b^14*x^39 - 11350399493322240*a^50*b^15*x^42 - 9448859819483
1360*a^49*b^16*x^45 - 710617423734220800*a^48*b^17*x^48 - 48551215493996544
00*a^47*b^18*x^51 - 30277909491232112640*a^46*b^19*x^54 - 17304926646921854
9760*a^45*b^20*x^57 - 909584595569746575360*a^44*b^21*x^60 - 44101354518206
44761600*a^43*b^22*x^63 - 19775527127642947584000*a^42*b^23*x^66 - 82196556
795830977167360*a^41*b^24*x^69 - 317307653352763348746240*a^40*b^25*x^72 -
1139583932697244587786240*a^39*b^26*x^75 - 3813174028256128637337600*a^38*b
^27*x^78 - 11902704133849368389222400*a^37*b^28*x^81 - 34696391435737807567
454208*a^36*b^29*x^84 - 94533796001415733929050112*a^35*b^30*x^87 - 2409169
41658790456262131712*a^34*b^31*x^90 - 574607755442020115378339840*a^33*b^32
*x^93 - 1283164402851665258416701440*a^32*b^33*x^96 - 268362149495913907675
0442496*a^31*b^34*x^99 - 5257145964851977442918662144*a^30*b^35*x^102 - 964
6533665292061818700693504*a^29*b^36*x^105 - 16577868267430599772549939200*a
^28*b^37*x^108 - 26675109984442157398669393920*a^27*b^38*x^111 - 4017230107
3084658684862136320*a^26*b^39*x^114 - 565912822297530387719562199040*a^25*b^
40*x^117 - 74519077727500043969227653120*a^24*b^41*x^120 - 9164368263014309
8677205401600*a^23*b^42*x^123 - 105147917610427657204059340800*a^22*b^43*x^
126 - 112415234614316153992359444480*a^21*b^44*x^129 - 11182743708225490376
4917944320*a^20*b^45*x^132 - 103333597240348093867119083520*a^19*b^46*x^135
- 88524263900650858976850739200*a^18*b^47*x^138 - 7015184566732473743337O6
24000*a^17*b^48*x^141 - 51292238631349321490937937920*a^16*b^49*x^144 - 344
99019413506196684176097280*a^15*b^50*x^147 - 21271946840740957336664801280*
a^14*b^51*x^150 - 11976015405374637268480819200*a^13*b^52*x^153 - 612753747
1799578147474636800*a^12*b^53*x^156 - 2833532513821046118461472768*a^11*b^5
4*x^159 - 1176474844646974332812132352*a^10*b^55*x^162 - 435119222530177299
017367552*a^9*b^56*x^165 - 141970050175924377684541440*a^8*b^57*x^168 - 403
74133580173208809635840*a^7*b^58*x^171 - 9854453508288460730925056*a^6*b^59
*x^174 - 2022872405081186719760384*a^5*b^60*x^177 - 339632949089043788857734
4*a^4*b^61*x^180 - 44787397574274108620800*a^3*b^62*x^183 - 435011695206970
9496320*a^2*b^63*x^186 - 276701161105643274240*a*b^64*x^189 - 8646911284551
352320*b^65*x^192))/(9*a^5*x^12*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]*(-8*a^60*b^
4 - 936*a^59*b^5*x^3 - 53832*a^58*b^6*x^6 - 2028600*a^57*b^7*x^9 - 56333328
*a^56*b^8*x^12 - 1229251968*a^55*b^9*x^15 - 21948838912*a^54*b^10*x^18 - 32
9736109824*a^53*b^11*x^21 - 4253142643200*a^52*b^12*x^24 - 47832699273216*a
^51*b^13*x^27 - 474726976948224*a^50*b^14*x^30 - 4198198392760320*a^49*b^15
*x^33 - 33343446890557440*a^48*b^16*x^36 - 239403280048128000*a^47*b^17*x^3
9 - 1562459053231964160*a^46*b^18*x^42 - 9312555821128089600*a^45*b^19*x^45
- 50890640751160197120*a^44*b^20*x^48 - 255857896219504803840*a^43*b^21*x^
51 - 1186944307588300800000*a^42*b^22*x^54 - 5093751076470545448960*a^41*b^
23*x^57 - 20266419321282499706880*a^40*b^24*x^60 - 74898737599223200481280*
a^39*b^25*x^63 - 257538426184575127388160*a^38*b^26*x^66 - 8250678812240322
23232000*a^37*b^27*x^69 - 2465674112032552349859840*a^36*b^28*x^72 - 688041

```
4330081729610514432*a^35*b^29*x^75 - 17942536074222169724289024*a^34*b^30*x
^78 - 4375510777792062666047488*a^33*b^31*x^81 - 9983129760884363561533440
0*a^32*b^32*x^84 - 213181721876762624526385152*a^31*b^33*x^87 - 42615515677
5629047172431872*a^30*b^34*x^90 - 797530838364556999815856128*a^29*b^35*x^9
3 - 139718904772772213310201856*a^28*b^36*x^96 - 2290841170528074923297996
800*a^27*b^37*x^99 - 3514050815948758604845154304*a^26*b^38*x^102 - 5040462
441364212140879118336*a^25*b^39*x^105 - 6756005325714055365069373440*a^24*b
^40*x^108 - 8454788378368570414312980480*a^23*b^41*x^111 - 9868868050118054
737084416000*a^22*b^42*x^114 - 10731458193725856494093598720*a^21*b^43*x^11
7 - 10855754855129860210792857600*a^20*b^44*x^120 - 10198877550514546149257
379840*a^19*b^45*x^123 - 8881831640526360274670714880*a^18*b^46*x^126 - 715
3973538484081655808000000*a^17*b^47*x^129 - 5315869468116817954964766720*a^
16*b^48*x^132 - 3633267961926811266066677760*a^15*b^49*x^135 - 227628260141
4251560210268160*a^14*b^50*x^138 - 1302044770810745855693291520*a^13*b^51*x
^141 - 676809132086273815609344000*a^12*b^52*x^144 - 3179452545867346780933
32480*a^11*b^53*x^147 - 134101230649370624636485632*a^10*b^54*x^150 - 50381
470381939849118613504*a^9*b^55*x^153 - 16697916908414960085762048*a^8*b^56*
x^156 - 4823541649938374354534400*a^7*b^57*x^159 - 119588743031903034225459
2*a^6*b^58*x^162 - 249357249723288646582272*a^5*b^59*x^165 - 42526806734116
233412608*a^4*b^60*x^168 - 5696585154262430908416*a^3*b^61*x^171 - 56204923
3495837900800*a^2*b^62*x^174 - 36317027395115679744*a*b^63*x^177 - 11529215
04606846976*b^64*x^180) + 9*a^5*Sqrt[b^2]*x^12*(8*a^61*b^3 + 944*a^60*b^4*x
^3 + 54768*a^59*b^5*x^6 + 2082432*a^58*b^6*x^9 + 58361928*a^57*b^7*x^12 + 1
285585296*a^56*b^8*x^15 + 23178090880*a^55*b^9*x^18 + 351684948736*a^54*b^1
0*x^21 + 4582878753024*a^53*b^11*x^24 + 52085841916416*a^52*b^12*x^27 + 522
559676221440*a^51*b^13*x^30 + 4672925369708544*a^50*b^14*x^33 + 37541645283
317760*a^49*b^15*x^36 + 272746726938685440*a^48*b^16*x^39 + 180186233328009
2160*a^47*b^17*x^42 + 10875014874360053760*a^46*b^18*x^45 + 602031965722882
86720*a^45*b^19*x^48 + 306748536970665000960*a^44*b^20*x^51 + 1442802203807
805603840*a^43*b^21*x^54 + 6280695384058846248960*a^42*b^22*x^57 + 25360170
397753045155840*a^41*b^23*x^60 + 95165156920505700188160*a^40*b^24*x^63 + 3
32437163783798327869440*a^39*b^25*x^66 + 1082606307408607350620160*a^38*b^2
6*x^69 + 3290741993256584573091840*a^37*b^27*x^72 + 93460884421142819603742
72*a^36*b^28*x^75 + 24822950404303899334803456*a^35*b^29*x^78 + 61697643852
014232390336512*a^34*b^30*x^81 + 143586405386635698281381888*a^33*b^31*x^84
 + 313013019485606260141719552*a^32*b^32*x^87 + 639336878652391671698817024
*a^31*b^33*x^90 + 1223685995140186046988288000*a^30*b^34*x^93 + 21947198860
92279213126057984*a^29*b^35*x^96 + 3688030218255797136608198656*a^28*b^36*x
^99 + 5804891986476833528143151104*a^27*b^37*x^102 + 8554513257312970745724
272640*a^26*b^38*x^105 + 11796467767078267505948491776*a^25*b^39*x^108 + 15
210793704082625779382353920*a^24*b^40*x^111 + 18323656428486625151397396480
*a^23*b^41*x^114 + 20600326243843911231178014720*a^22*b^42*x^117 + 21587213
048855716704886456320*a^21*b^43*x^120 + 21054632405644406360050237440*a^20*
b^44*x^123 + 19080709191040906423928094720*a^19*b^45*x^126 + 16035805179010
441930478714880*a^18*b^46*x^129 + 12469843006600899610772766720*a^17*b^47*x
^132 + 8949137430043629221031444480*a^16*b^48*x^135 + 590955056334106282627
6945920*a^15*b^49*x^138 + 3578327372224997415903559680*a^14*b^50*x^141 + 19
78853902897019671302635520*a^13*b^51*x^144 + 994754386673008493702676480*a^
12*b^52*x^147 + 452046485236105302729818112*a^11*b^53*x^150 + 1844827010313
10473755099136*a^10*b^54*x^153 + 67079387290354809204375552*a^9*b^55*x^156
+ 21521458558353334440296448*a^8*b^56*x^159 + 6019429080257404696788992*a^7
*b^57*x^162 + 1445244680042318988836864*a^6*b^58*x^165 + 291884056457404879
994880*a^5*b^59*x^168 + 48223391888378664321024*a^4*b^60*x^171 + 6258634387
758268809216*a^3*b^61*x^174 + 598366260890953580544*a^2*b^62*x^177 + 374699
48899722526720*a*b^63*x^180 + 1152921504606846976*b^64*x^183)) - (10*b*ArcT
anh[(Sqrt[b^2]*x^3)/a - Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]/a])/(3*a^6)
```

fricas [A] time = 1.27, size = 207, normalized size = 0.77

$$-\frac{60\,ab^4x^{12}+210\,a^2b^3x^9+260\,a^3b^2x^6+125\,a^4bx^3+12\,a^5-60\left(b^5x^{15}+4\,ab^4x^{12}+6\,a^2b^3x^9+4\,a^3b^2x^6+a^4bx^3\right)\log\left(bx^3+a\right)+180\left(b^5x^{15}+4\,ab^4x^{12}+6\,a^2b^3x^9+4\,a^3b^2x^6+a^4bx^3\right)\log(x)}{36\left(a^6b^4x^{15}+4\,a^7b^3x^{12}+6\,a^8b^2x^9+4\,a^9bx^6+a^{10}x^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/36*(60*a*b^4*x^12 + 210*a^2*b^3*x^9 + 260*a^3*b^2*x^6 + 125*a^4*b*x^3 +
12*a^5 - 60*(b^5*x^15 + 4*a*b^4*x^12 + 6*a^2*b^3*x^9 + 4*a^3*b^2*x^6 + a^4*
b*x^3)*log(b*x^3 + a) + 180*(b^5*x^15 + 4*a*b^4*x^12 + 6*a^2*b^3*x^9 + 4*a^
3*b^2*x^6 + a^4*b*x^3)*log(x))/(a^6*b^4*x^15 + 4*a^7*b^3*x^12 + 6*a^8*b^2*x
^9 + 4*a^9*b*x^6 + a^10*x^3)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

maple [A] time = 0.02, size = 219, normalized size = 0.81

$$-\frac{\left(180 b^{5} x^{15} \ln(x) - 60 b^{5} x^{15} \ln\left(b x^{3}+a\right) + 720 a b^{4} x^{12} \ln(x) - 240 a b^{4} x^{12} \ln\left(b x^{3}+a\right) + 60 a b^{4} x^{12} + 1080 a^{2} b^{3} x^{9} \ln(x) - 360 a^{2} b^{3} x^{9} \ln\left(b x^{3}+a\right) + 210 a^{2} b^{3} x^{9} + 720 a^{3} b^{2} x^{6} \ln(x) - 240 a^{3} b^{2} x^{6} \ln\left(b x^{3}+a\right) + 260 a^{3} b^{2} x^{6} + 180 a^{4} b x^{3} \ln(x) - 60 a^{4} b x^{3} \ln\left(b x^{3}+a\right) + 125 a^{4} b x^{3} + 12 a^{5}\right) \left(b x^{3}+a\right)}{36 \left(\left(b x^{3}+a\right)^{2}\right)^{\frac{5}{2}} a^{6} x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x)
```

```
[Out] -1/36*(180*b^5*x^15*ln(x)-60*ln(b*x^3+a)*x^15*b^5+720*a*b^4*x^12*ln(x)-240*
ln(b*x^3+a)*x^12*a*b^4+60*a*b^4*x^12+1080*a^2*b^3*x^9*ln(x)-360*ln(b*x^3+a)
*x^9*a^2*b^3+210*a^2*b^3*x^9+720*a^3*b^2*x^6*ln(x)-240*ln(b*x^3+a)*x^6*a^3*
b^2+260*a^3*b^2*x^6+180*a^4*b*x^3*ln(x)-60*ln(b*x^3+a)*x^3*a^4*b+125*a^4*b*
x^3+12*a^5)*(b*x^3+a)/x^3/a^6/((b*x^3+a)^2)^(5/2)
```

maxima [A] time = 0.85, size = 163, normalized size = 0.61

$$\frac{5 (-1)^{2abx^3+2a^2} b \log\left(\frac{2abx}{|x|} + \frac{2a^2}{x^2|x|}\right)}{3a^6} - \frac{5b}{9\left(b^2x^6+2abx^3+a^2\right)^{\frac{3}{2}} a^3} - \frac{5b}{3\sqrt{b^2x^6+2abx^3+a^2}\, a^5} - \frac{1}{3\left(b^2x^6+2abx^3+a^2\right)^{\frac{3}{2}} a^2x^3} - \frac{5}{6\left(x^3+\frac{a}{b}\right)^2 a^4 b} - \frac{1}{12\left(x^3+\frac{a}{b}\right)^4 a^2 b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="maxima")
```

```
[Out] 5/3*(-1)^(2*a*b*x^3 + 2*a^2)*b*log(2*a*b*x/abs(x) + 2*a^2/(x^2*abs(x)))/a^6
 - 5/9*b/((b^2*x^6 + 2*a*b*x^3 + a^2)^(3/2)*a^3) - 5/3*b/(sqrt(b^2*x^6 + 2*
a*b*x^3 + a^2)*a^5) - 1/3/((b^2*x^6 + 2*a*b*x^3 + a^2)^(3/2)*a^2*x^3) - 5/6
/((x^3 + a/b)^2*a^4*b) - 1/12/((x^3 + a/b)^4*a^2*b^3)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^4 \left(a^2 + 2\,a\,b\,x^3 + b^2\,x^6\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2)),x)
```

```
[Out] int(1/(x^4*(a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^4 \left(\left(a + bx^3\right)^2\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(b**2*x**6+2*a*b*x**3+a**2)**(5/2),x)
```

```
[Out] Integral(1/(x**4*((a + b*x**3)**2)**(5/2)), x)
```

3.118 $\int (dx)^m \left(a^2+2abx^3+b^2x^6\right)^{5/2} \, dx$

Optimal. Leaf size=313

$$\frac{b^5\sqrt{a^2+2abx^3+b^2x^6}\,(dx)^{m+16}}{d^{16}(m+16)\left(a+bx^3\right)}+\frac{5ab^4\sqrt{a^2+2abx^3+b^2x^6}\,(dx)^{m+13}}{d^{13}(m+13)\left(a+bx^3\right)}+\frac{10a^2b^3\sqrt{a^2+2abx^3+b^2x^6}\,(dx)^{m+10}}{d^{10}(m+10)\left(a+bx^3\right)}+\frac{a^5\sqrt{a}\cdots}{\cdots}$$

Rubi [A] time = 0.14, antiderivative size = 313, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {1355, 270}

$$\frac{5a^4b\sqrt{a^2+2abx^3+b^2x^6}\,(dx)^{m+4}}{d^4(m+4)\left(a+bx^3\right)}+\frac{10a^3b^2\sqrt{a^2+2abx^3+b^2x^6}\,(dx)^{m+7}}{d^7(m+7)\left(a+bx^3\right)}+\frac{10a^2b^3\sqrt{a^2+2abx^3+b^2x^6}\,(dx)^{m+10}}{d^{10}(m+10)\left(a+bx^3\right)}+\frac{5ab^4\sqrt{a^2+2abx^3+b^2x^6}\,(dx)^{m+13}}{d^{13}(m+13)\left(a+bx^3\right)}+\frac{b^5\sqrt{a^2+2abx^3+b^2x^6}\,(dx)^{m+16}}{d^{16}(m+16)\left(a+bx^3\right)}+\frac{a^5\sqrt{a^2+2abx^3+b^2x^6}\,(dx)^{m+1}}{d(m+1)\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^m*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
```

```
[Out] (a^5*(d*x)^(1 + m)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(d*(1 + m)*(a + b*x^3))
 + (5*a^4*b*(d*x)^(4 + m)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(d^4*(4 + m)*(a
+ b*x^3)) + (10*a^3*b^2*(d*x)^(7 + m)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(d^7
*(7 + m)*(a + b*x^3)) + (10*a^2*b^3*(d*x)^(10 + m)*Sqrt[a^2 + 2*a*b*x^3 + b
^2*x^6])/(d^10*(10 + m)*(a + b*x^3)) + (5*a*b^4*(d*x)^(13 + m)*Sqrt[a^2 + 2
*a*b*x^3 + b^2*x^6])/(d^13*(13 + m)*(a + b*x^3)) + (b^5*(d*x)^(16 + m)*Sqrt
[a^2 + 2*a*b*x^3 + b^2*x^6])/(d^16*(16 + m)*(a + b*x^3))
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\int (dx)^m \left(a^2+2abx^3+b^2x^6\right)^{5/2} dx = \frac{\sqrt{a^2+2abx^3+b^2x^6}\int (dx)^m \left(ab+b^2x^3\right)^5 dx}{b^4\left(ab+b^2x^3\right)}$$

$$= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \left(a^5b^5(dx)^m+\frac{5a^4b^6(dx)^{3+m}}{d^3}+\frac{10a^3b^7(dx)^{6+m}}{d^6}+\frac{10a^2b^8(dx)^{9+}\cdots}{d^9}\cdots\right.}{b^4\left(ab+b^2x^3\right)}$$

$$= \frac{a^5(dx)^{1+m}\sqrt{a^2+2abx^3+b^2x^6}}{d(1+m)\left(a+bx^3\right)}+\frac{5a^4b(dx)^{4+m}\sqrt{a^2+2abx^3+b^2x^6}}{d^4(4+m)\left(a+bx^3\right)}+\frac{10a^3b\cdots}{\cdots}$$

Mathematica [A] time = 0.09, size = 111, normalized size = 0.35

$$\frac{x\left(\left(a+bx^3\right)^2\right)^{5/2}(dx)^m\left(\frac{a^5}{m+1}+\frac{5a^4bx^3}{m+4}+\frac{10a^3b^2x^6}{m+7}+\frac{10a^2b^3x^9}{m+10}+\frac{5ab^4x^{12}}{m+13}+\frac{b^5x^{15}}{m+16}\right)}{\left(a+bx^3\right)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^m*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
```

```
[Out] (x*(d*x)^m*((a + b*x^3)^2)^(5/2)*(a^5/(1 + m) + (5*a^4*b*x^3)/(4 + m) + (10
*a^3*b^2*x^6)/(7 + m) + (10*a^2*b^3*x^9)/(10 + m) + (5*a*b^4*x^12)/(13 + m)
 + (b^5*x^15)/(16 + m)))/(a + b*x^3)^5
```

IntegrateAlgebraic [F] time = 3.28, size = 0, normalized size = 0.00

$$\int (dx)^m \left(a^2 + 2abx^3 + b^2x^6\right)^{5/2} \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(d*x)^m*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
```

```
[Out] Defer[IntegrateAlgebraic][(d*x)^m*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2), x]
```

fricas [A] time = 1.91, size = 369, normalized size = 1.18

$$\frac{\left({\left(b^{5} m^{5} + 35 \, b^{5} m^{4} + 445 \, b^{5} m^{3} + 2485 \, b^{5} m^{2} + 5714 \, b^{5} m + 3640 \, b^{5}\right)} x^{16} + 5 \, {\left(a b^{4} m^{5} + 38 \, a b^{4} m^{4} + 511 \, a b^{4} m^{3} + 2962 \, a b^{4} m^{2} + 6968 \, a b^{4} m + 4480 \, a b^{4}\right)} x^{13} + 10 \, {\left(a^{2} b^{3} m^{5} + 41 \, a^{2} b^{3} m^{4} + 595 \, a^{2} b^{3} m^{3} + 3655 \, a^{2} b^{3} m^{2} + 8924 \, a^{2} b^{3} m + 5824 \, a^{2} b^{3}\right)} x^{10} + 10 \, {\left(a^{3} b^{2} m^{5} + 44 \, a^{3} b^{2} m^{4} + 697 \, a^{3} b^{2} m^{3} + 4726 \, a^{3} b^{2} m^{2} + 12392 \, a^{3} b^{2} m + 8320 \, a^{3} b^{2}\right)} x^{7} + 5 \, {\left(a^{4} b m^{5} + 47 \, a^{4} b m^{4} + 817 \, a^{4} b m^{3} + 6337 \, a^{4} b m^{2} + 20126 \, a^{4} b m + 14560 \, a^{4} b\right)} x^{4} + {\left(a^{5} m^{5} + 50 \, a^{5} m^{4} + 955 \, a^{5} m^{3} + 8650 \, a^{5} m^{2} + 36824 \, a^{5} m + 58240 \, a^{5}\right)} x\right)} \left(d x\right)^{m}}{m^{6} + 51 \, m^{5} + 1005 \, m^{4} + 9605 \, m^{3} + 45474 \, m^{2} + 95064 \, m + 58240}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="fricas")
```

```
[Out] ((b^5*m^5 + 35*b^5*m^4 + 445*b^5*m^3 + 2485*b^5*m^2 + 5714*b^5*m + 3640*b^5
)*x^16 + 5*(a*b^4*m^5 + 38*a*b^4*m^4 + 511*a*b^4*m^3 + 2962*a*b^4*m^2 + 696
8*a*b^4*m + 4480*a*b^4)*x^13 + 10*(a^2*b^3*m^5 + 41*a^2*b^3*m^4 + 595*a^2*b
^3*m^3 + 3655*a^2*b^3*m^2 + 8924*a^2*b^3*m + 5824*a^2*b^3)*x^10 + 10*(a^3*b
^2*m^5 + 44*a^3*b^2*m^4 + 697*a^3*b^2*m^3 + 4726*a^3*b^2*m^2 + 12392*a^3*b^
2*m + 8320*a^3*b^2)*x^7 + 5*(a^4*b*m^5 + 47*a^4*b*m^4 + 817*a^4*b*m^3 + 633
7*a^4*b*m^2 + 20126*a^4*b*m + 14560*a^4*b)*x^4 + (a^5*m^5 + 50*a^5*m^4 + 95
5*a^5*m^3 + 8650*a^5*m^2 + 36824*a^5*m + 58240*a^5)*x)*(d*x)^m/(m^6 + 51*m^
5 + 1005*m^4 + 9605*m^3 + 45474*m^2 + 95064*m + 58240)
```

giac [B] time = 0.67, size = 900, normalized size = 2.88

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="giac")
```

```
[Out] ((d*x)^m*b^5*m^5*x^16*sgn(b*x^3 + a) + 35*(d*x)^m*b^5*m^4*x^16*sgn(b*x^3 +
a) + 445*(d*x)^m*b^5*m^3*x^16*sgn(b*x^3 + a) + 5*(d*x)^m*a*b^4*m^5*x^13*sgn
(b*x^3 + a) + 2485*(d*x)^m*b^5*m^2*x^16*sgn(b*x^3 + a) + 190*(d*x)^m*a*b^4*
m^4*x^13*sgn(b*x^3 + a) + 5714*(d*x)^m*b^5*m*x^16*sgn(b*x^3 + a) + 2555*(d*
x)^m*a*b^4*m^3*x^13*sgn(b*x^3 + a) + 3640*(d*x)^m*b^5*x^16*sgn(b*x^3 + a) +
 10*(d*x)^m*a^2*b^3*m^5*x^10*sgn(b*x^3 + a) + 14810*(d*x)^m*a*b^4*m^2*x^13*
sgn(b*x^3 + a) + 410*(d*x)^m*a^2*b^3*m^4*x^10*sgn(b*x^3 + a) + 34840*(d*x)^
m*a*b^4*m*x^13*sgn(b*x^3 + a) + 5950*(d*x)^m*a^2*b^3*m^3*x^10*sgn(b*x^3 + a
) + 22400*(d*x)^m*a*b^4*x^13*sgn(b*x^3 + a) + 10*(d*x)^m*a^3*b^2*m^5*x^7*sg
n(b*x^3 + a) + 36550*(d*x)^m*a^2*b^3*m^2*x^10*sgn(b*x^3 + a) + 440*(d*x)^m*
a^3*b^2*m^4*x^7*sgn(b*x^3 + a) + 89240*(d*x)^m*a^2*b^3*m*x^10*sgn(b*x^3 + a
) + 6970*(d*x)^m*a^3*b^2*m^3*x^7*sgn(b*x^3 + a) + 58240*(d*x)^m*a^2*b^3*x^1
0*sgn(b*x^3 + a) + 5*(d*x)^m*a^4*b*m^5*x^4*sgn(b*x^3 + a) + 47260*(d*x)^m*a
^3*b^2*m^2*x^7*sgn(b*x^3 + a) + 235*(d*x)^m*a^4*b*m^4*x^4*sgn(b*x^3 + a) +
123920*(d*x)^m*a^3*b^2*m*x^7*sgn(b*x^3 + a) + 4085*(d*x)^m*a^4*b*m^3*x^4*sg
n(b*x^3 + a) + 83200*(d*x)^m*a^3*b^2*x^7*sgn(b*x^3 + a) + (d*x)^m*a^5*m^5*x
*sgn(b*x^3 + a) + 31685*(d*x)^m*a^4*b*m^2*x^4*sgn(b*x^3 + a) + 50*(d*x)^m*a
^5*m^4*x*sgn(b*x^3 + a) + 100630*(d*x)^m*a^4*b*m*x^4*sgn(b*x^3 + a) + 955*(
d*x)^m*a^5*m^3*x*sgn(b*x^3 + a) + 72800*(d*x)^m*a^4*b*x^4*sgn(b*x^3 + a) +
```

```
8650*(d*x)^m*a^5*m^2*x*sgn(b*x^3 + a) + 36824*(d*x)^m*a^5*m*x*sgn(b*x^3 + a
) + 58240*(d*x)^m*a^5*x*sgn(b*x^3 + a))/(m^6 + 51*m^5 + 1005*m^4 + 9605*m^3
 + 45474*m^2 + 95064*m + 58240)
```

maple [A] time = 0.01, size = 453, normalized size = 1.45

$$\frac{\left(b^5m^5x^{15}+35b^5m^4x^{15}+445b^5m^3x^{15}+5a\,b^4m^5x^{12}+2485b^5m^2x^{15}+190a\,b^4m^4x^{12}+5714b^5m\,x^{15}+2555a\,b^4m^3x^{12}+3640b^5x^{15}+10a^2b^3m^5x^9+14810a\,b^4m^2x^{12}+410a^2b^3m^4x^9+34840a\,b^4m\,x^{12}+5950a^2b^3m^3x^9+22400a\,b^4x^{12}+10a^3b^2m^5x^6+36550a^2b^3m^2x^9+440a^3b^2m^4x^6+89240a^2b^3m\,x^9+6970a^3b^2m^3x^6+58240a^2b^3x^9+5a^4b\,m^5x^3+47260a^3b^2m^2x^6+235a^4b\,m^4x^3+123920a^3b^2m\,x^6+4085a^4b\,m^3x^3+83200a^3b^2x^6+a^5m^5+31685a^4b\,m^2x^3+50a^5m^4+100630a^4bm\,x^3+955a^5m^3+72800a^4b\,x^3+8650a^5m^2+36824a^5m+58240a^5\right)\left(\left(b\,x^3+a\right)^2\right)^{\frac{5}{2}}x\,(dx)^m}{(m+1)\,(m+4)\,(m+7)\,(m+10)\,(m+13)\,(m+16)\left(b\,x^3+a\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x)
```

```
[Out] x*(b^5*m^5*x^15+35*b^5*m^4*x^15+445*b^5*m^3*x^15+5*a*b^4*m^5*x^12+2485*b^5*
m^2*x^15+190*a*b^4*m^4*x^12+5714*b^5*m*x^15+2555*a*b^4*m^3*x^12+3640*b^5*x^
15+10*a^2*b^3*m^5*x^9+14810*a*b^4*m^2*x^12+410*a^2*b^3*m^4*x^9+34840*a*b^4*
m*x^12+5950*a^2*b^3*m^3*x^9+22400*a*b^4*x^12+10*a^3*b^2*m^5*x^6+36550*a^2*b
^3*m^2*x^9+440*a^3*b^2*m^4*x^6+89240*a^2*b^3*m*x^9+6970*a^3*b^2*m^3*x^6+582
40*a^2*b^3*x^9+5*a^4*b*m^5*x^3+47260*a^3*b^2*m^2*x^6+235*a^4*b*m^4*x^3+1239
20*a^3*b^2*m*x^6+4085*a^4*b*m^3*x^3+83200*a^3*b^2*x^6+a^5*m^5+31685*a^4*b*m
^2*x^3+50*a^5*m^4+100630*a^4*b*m*x^3+955*a^5*m^3+72800*a^4*b*x^3+8650*a^5*m
^2+36824*a^5*m+58240*a^5)*(d*x)^m*((b*x^3+a)^2)^(5/2)/(m+1)/(m+4)/(m+7)/(m+
10)/(m+13)/(16+m)/(b*x^3+a)^5
```

maxima [A] time = 0.95, size = 243, normalized size = 0.78

$$\frac{\left(\left(m^5+35\,m^4+445\,m^3+2485\,m^2+5714\,m+3640\right)b^5d^mx^{16}+5\left(m^5+38\,m^4+511\,m^3+2962\,m^2+6968\,m+4480\right)ab^4d^mx^{13}+10\left(m^5+41\,m^4+595\,m^3+3655\,m^2+8924\,m+5824\right)a^2b^3d^mx^{10}+10\left(m^5+44\,m^4+697\,m^3+4726\,m^2+12392\,m+8320\right)a^3b^2d^mx^7+5\left(m^5+47\,m^4+817\,m^3+6337\,m^2+20126\,m+14560\right)a^4bd^mx^4+\left(m^5+50\,m^4+955\,m^3+8650\,m^2+36824\,m+58240\right)a^5d^mx\right)x^m}{m^6+51\,m^5+1005\,m^4+9605\,m^3+45474\,m^2+95064\,m+58240}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="maxima")
```

```
[Out] ((m^5 + 35*m^4 + 445*m^3 + 2485*m^2 + 5714*m + 3640)*b^5*d^m*x^16 + 5*(m^5
+ 38*m^4 + 511*m^3 + 2962*m^2 + 6968*m + 4480)*a*b^4*d^m*x^13 + 10*(m^5 + 4
1*m^4 + 595*m^3 + 3655*m^2 + 8924*m + 5824)*a^2*b^3*d^m*x^10 + 10*(m^5 + 44
*m^4 + 697*m^3 + 4726*m^2 + 12392*m + 8320)*a^3*b^2*d^m*x^7 + 5*(m^5 + 47*m
^4 + 817*m^3 + 6337*m^2 + 20126*m + 14560)*a^4*b*d^m*x^4 + (m^5 + 50*m^4 +
955*m^3 + 8650*m^2 + 36824*m + 58240)*a^5*d^m*x)*x^m/(m^6 + 51*m^5 + 1005*m
^4 + 9605*m^3 + 45474*m^2 + 95064*m + 58240)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int {\left(d\,x\right)}^m\,{\left(a^2+2\,a\,b\,x^3+b^2\,x^6\right)}^{5/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2),x)
```

```
[Out] int((d*x)^m*(a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(dx\right)^{m} \left(\left(a + bx^{3}\right)^{2}\right)^{\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(b**2*x**6+2*a*b*x**3+a**2)**(5/2),x)
```

```
[Out] Integral((d*x)**m*((a + b*x**3)**2)**(5/2), x)
```

3.119 $\int (dx)^m \left(a^2+2abx^3+b^2x^6\right)^{3/2} dx$

Optimal. Leaf size=205

$$\frac{3ab^2\sqrt{a^2+2abx^3+b^2x^6}\,(dx)^{m+7}}{d^7(m+7)\left(a+bx^3\right)}+\frac{3a^2b\sqrt{a^2+2abx^3+b^2x^6}\,(dx)^{m+4}}{d^4(m+4)\left(a+bx^3\right)}+\frac{b^3\sqrt{a^2+2abx^3+b^2x^6}\,(dx)^{m+10}}{d^{10}(m+10)\left(a+bx^3\right)}+\frac{a^3\sqrt{a^2\ldots}}{\ldots}$$

Rubi [A] time = 0.09, antiderivative size = 205, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {1355, 270}

$$\frac{3a^2b\sqrt{a^2+2abx^3+b^2x^6}\,(dx)^{m+4}}{d^4(m+4)\left(a+bx^3\right)}+\frac{3ab^2\sqrt{a^2+2abx^3+b^2x^6}\,(dx)^{m+7}}{d^7(m+7)\left(a+bx^3\right)}+\frac{b^3\sqrt{a^2+2abx^3+b^2x^6}\,(dx)^{m+10}}{d^{10}(m+10)\left(a+bx^3\right)}+\frac{a^3\sqrt{a^2+2abx^3+b^2x^6}\,(dx)^{m+1}}{d(m+1)\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^m*(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2),x]
```

```
[Out] (a^3*(d*x)^(1 + m)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(d*(1 + m)*(a + b*x^3))
 + (3*a^2*b*(d*x)^(4 + m)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(d^4*(4 + m)*(a
+ b*x^3)) + (3*a*b^2*(d*x)^(7 + m)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(d^7*(7
 + m)*(a + b*x^3)) + (b^3*(d*x)^(10 + m)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(
d^10*(10 + m)*(a + b*x^3))
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\begin{aligned}\int (dx)^m\left(a^2+2abx^3+b^2x^6\right)^{3/2}dx &= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int (dx)^m\left(ab+b^2x^3\right)^3dx}{b^2\left(ab+b^2x^3\right)}\\ &= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int\left(a^3b^3(dx)^m+\frac{3a^2b^4(dx)^{3+m}}{d^3}+\frac{3ab^5(dx)^{6+m}}{d^6}+\frac{b^6(dx)^{9+m}}{d^9}\right)\ldots}{b^2\left(ab+b^2x^3\right)}\\ &= \frac{a^3(dx)^{1+m}\sqrt{a^2+2abx^3+b^2x^6}}{d(1+m)\left(a+bx^3\right)}+\frac{3a^2b(dx)^{4+m}\sqrt{a^2+2abx^3+b^2x^6}}{d^4(4+m)\left(a+bx^3\right)}+\frac{3a\ldots}{\ldots}\end{aligned}$$

Mathematica [A] time = 0.06, size = 131, normalized size = 0.64

$$\frac{x\sqrt{\left(a+bx^3\right)^2}\,(dx)^m\left(a^3\left(m^3+21m^2+138m+280\right)+3a^2b\left(m^3+18m^2+87m+70\right)x^3+3ab^2\left(m^3+15m^2+54m+40\right)x^6+b^3\left(m^3+12m^2+39m+28\right)x^9\right)}{(m+1)(m+4)(m+7)(m+10)\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^m*(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2),x]
```

```
[Out] (x*(d*x)^m*Sqrt[(a + b*x^3)^2]*(a^3*(280 + 138*m + 21*m^2 + m^3) + 3*a^2*b*
(70 + 87*m + 18*m^2 + m^3)*x^3 + 3*a*b^2*(40 + 54*m + 15*m^2 + m^3)*x^6 + b
^3*(28 + 39*m + 12*m^2 + m^3)*x^9))/((1 + m)*(4 + m)*(7 + m)*(10 + m)*(a +
b*x^3))
```

IntegrateAlgebraic [F] time = 1.80, size = 0, normalized size = 0.00

$$\int (dx)^m \left(a^2 + 2abx^3 + b^2x^6\right)^{3/2} dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(d*x)^m*(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2),x]
```

```
[Out] Defer[IntegrateAlgebraic][(d*x)^m*(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2), x]
```

fricas [A] time = 1.20, size = 159, normalized size = 0.78

$$\frac{\left(\left(b^3m^3 + 12\,b^3m^2 + 39\,b^3m + 28\,b^3\right)x^{10} + 3\left(ab^2m^3 + 15\,ab^2m^2 + 54\,ab^2m + 40\,ab^2\right)x^7 + 3\left(a^2bm^3 + 18\,a^2bm^2 + 87\,a^2bm + 70\,a^2b\right)x^4 + \left(a^3m^3 + 21\,a^3m^2 + 138\,a^3m + 280\,a^3\right)x\right)(dx)^m}{m^4 + 22\,m^3 + 159\,m^2 + 418\,m + 280}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="fricas")
```

```
[Out] ((b^3*m^3 + 12*b^3*m^2 + 39*b^3*m + 28*b^3)*x^10 + 3*(a*b^2*m^3 + 15*a*b^2*
m^2 + 54*a*b^2*m + 40*a*b^2)*x^7 + 3*(a^2*b*m^3 + 18*a^2*b*m^2 + 87*a^2*b*m
 + 70*a^2*b)*x^4 + (a^3*m^3 + 21*a^3*m^2 + 138*a^3*m + 280*a^3)*x)*(d*x)^m/
(m^4 + 22*m^3 + 159*m^2 + 418*m + 280)
```

giac [B] time = 0.52, size = 384, normalized size = 1.87

$$\frac{(dx)^m b^3m^3x^{10}\mathrm{sgn}\left(bx^3+a\right) + 12\,(dx)^m b^3m^2x^{10}\mathrm{sgn}\left(bx^3+a\right) + 39\,(dx)^m b^3mx^{10}\mathrm{sgn}\left(bx^3+a\right) + 3\,(dx)^m ab^2m^3x^7\mathrm{sgn}\left(bx^3+a\right) + 28\,(dx)^m b^3x^{10}\mathrm{sgn}\left(bx^3+a\right) + 45\,(dx)^m ab^2m^2x^7\mathrm{sgn}\left(bx^3+a\right) + 162\,(dx)^m ab^2mx^7\mathrm{sgn}\left(bx^3+a\right) + 3\,(dx)^m a^2bm^3x^4\mathrm{sgn}\left(bx^3+a\right) + 120\,(dx)^m ab^2x^7\mathrm{sgn}\left(bx^3+a\right) + 54\,(dx)^m a^2bm^2x^4\mathrm{sgn}\left(bx^3+a\right) + 261\,(dx)^m a^2bmx^4\mathrm{sgn}\left(bx^3+a\right) + (dx)^m a^3m^3x\,\mathrm{sgn}\left(bx^3+a\right) + 210\,(dx)^m a^2bx^4\mathrm{sgn}\left(bx^3+a\right) + 21\,(dx)^m a^3m^2x\,\mathrm{sgn}\left(bx^3+a\right) + 138\,(dx)^m a^3mx\,\mathrm{sgn}\left(bx^3+a\right) + 280\,(dx)^m a^3x\,\mathrm{sgn}\left(bx^3+a\right)}{m^4 + 22\,m^3 + 159\,m^2 + 418\,m + 280}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="giac")
```

```
[Out] ((d*x)^m*b^3*m^3*x^10*sgn(b*x^3 + a) + 12*(d*x)^m*b^3*m^2*x^10*sgn(b*x^3 +
a) + 39*(d*x)^m*b^3*m*x^10*sgn(b*x^3 + a) + 3*(d*x)^m*a*b^2*m^3*x^7*sgn(b*x
^3 + a) + 28*(d*x)^m*b^3*x^10*sgn(b*x^3 + a) + 45*(d*x)^m*a*b^2*m^2*x^7*sgn
(b*x^3 + a) + 162*(d*x)^m*a*b^2*m*x^7*sgn(b*x^3 + a) + 3*(d*x)^m*a^2*b*m^3*
x^4*sgn(b*x^3 + a) + 120*(d*x)^m*a*b^2*x^7*sgn(b*x^3 + a) + 54*(d*x)^m*a^2*
b*m^2*x^4*sgn(b*x^3 + a) + 261*(d*x)^m*a^2*b*m*x^4*sgn(b*x^3 + a) + (d*x)^m
*a^3*m^3*x*sgn(b*x^3 + a) + 210*(d*x)^m*a^2*b*x^4*sgn(b*x^3 + a) + 21*(d*x)
^m*a^3*m^2*x*sgn(b*x^3 + a) + 138*(d*x)^m*a^3*m*x*sgn(b*x^3 + a) + 280*(d*x
)^m*a^3*x*sgn(b*x^3 + a))/(m^4 + 22*m^3 + 159*m^2 + 418*m + 280)
```

maple [A] time = 0.02, size = 199, normalized size = 0.97

$$\frac{\left(b^3m^3x^9 + 12b^3m^2x^9 + 39b^3m\,x^9 + 3a\,b^2m^3x^6 + 28b^3x^9 + 45a\,b^2m^2x^6 + 162a\,b^2m\,x^6 + 3a^2b\,m^3x^3 + 120a\,b^2x^6 + 54a^2b\,m^2x^3 + 261a^2bm\,x^3 + a^3m^3 + 210a^2b\,x^3 + 21a^3m^2 + 138a^3m + 280a^3\right)\left(\left(b\,x^3+a\right)^2\right)^{\frac{3}{2}} x\,(dx)^m}{(m+10)\,(m+7)\,(m+4)\,(m+1)\left(b\,x^3+a\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x)
```

```
[Out] x*(b^3*m^3*x^9+12*b^3*m^2*x^9+39*b^3*m*x^9+3*a*b^2*m^3*x^6+28*b^3*x^9+45*a*
b^2*m^2*x^6+162*a*b^2*m*x^6+3*a^2*b*m^3*x^3+120*a*b^2*x^6+54*a^2*b*m^2*x^3+
261*a^2*b*m*x^3+a^3*m^3+210*a^2*b*x^3+21*a^3*m^2+138*a^3*m+280*a^3)*(d*x)^m
*((b*x^3+a)^2)^(3/2)/(m+10)/(m+7)/(m+4)/(m+1)/(b*x^3+a)^3
```

maxima [A] time = 1.04, size = 119, normalized size = 0.58

$$\frac{\left(\left(m^3+12\,m^2+39\,m+28\right)b^3d^mx^{10}+3\left(m^3+15\,m^2+54\,m+40\right)ab^2d^mx^7+3\left(m^3+18\,m^2+87\,m+70\right)a^2bd^mx^4+\left(m^3+21\,m^2+138\,m+280\right)a^3d^mx\right)x^m}{m^4+22\,m^3+159\,m^2+418\,m+280}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="maxima")
```

```
[Out] ((m^3 + 12*m^2 + 39*m + 28)*b^3*d^m*x^10 + 3*(m^3 + 15*m^2 + 54*m + 40)*a*b
^2*d^m*x^7 + 3*(m^3 + 18*m^2 + 87*m + 70)*a^2*b*d^m*x^4 + (m^3 + 21*m^2 + 1
38*m + 280)*a^3*d^m*x)*x^m/(m^4 + 22*m^3 + 159*m^2 + 418*m + 280)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int (d\,x)^m\,\left(a^2+2\,a\,b\,x^3+b^2\,x^6\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2),x)
```

```
[Out] int((d*x)^m*(a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx)^m \left(\left(a+bx^3\right)^2\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(b**2*x**6+2*a*b*x**3+a**2)**(3/2),x)
```

```
[Out] Integral((d*x)**m*((a + b*x**3)**2)**(3/2), x)
```

3.120 $\int (dx)^m \sqrt{a^2+2abx^3+b^2x^6}\, dx$

Optimal. Leaf size=97

$$\frac{b\sqrt{a^2+2abx^3+b^2x^6}\,(dx)^{m+4}}{d^4(m+4)\left(a+bx^3\right)}+\frac{a\sqrt{a^2+2abx^3+b^2x^6}\,(dx)^{m+1}}{d(m+1)\left(a+bx^3\right)}$$

Rubi [A] time = 0.04, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {1355, 14}

$$\frac{b\sqrt{a^2+2abx^3+b^2x^6}\,(dx)^{m+4}}{d^4(m+4)\left(a+bx^3\right)}+\frac{a\sqrt{a^2+2abx^3+b^2x^6}\,(dx)^{m+1}}{d(m+1)\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^m*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6],x]
```

```
[Out] (a*(d*x)^(1 + m)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(d*(1 + m)*(a + b*x^3)) +
 (b*(d*x)^(4 + m)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(d^4*(4 + m)*(a + b*x^3)
)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\begin{aligned}
\int (dx)^m\sqrt{a^2+2abx^3+b^2x^6}\,dx &= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int (dx)^m\left(ab+b^2x^3\right)dx}{ab+b^2x^3}\\
&= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \left(ab(dx)^m+\frac{b^2(dx)^{3+m}}{d^3}\right)dx}{ab+b^2x^3}\\
&= \frac{a(dx)^{1+m}\sqrt{a^2+2abx^3+b^2x^6}}{d(1+m)\left(a+bx^3\right)}+\frac{b(dx)^{4+m}\sqrt{a^2+2abx^3+b^2x^6}}{d^4(4+m)\left(a+bx^3\right)}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 53, normalized size = 0.55

$$\frac{x\sqrt{\left(a+bx^3\right)^2}\,(dx)^m\left(a(m+4)+b(m+1)x^3\right)}{(m+1)(m+4)\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^m*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6],x]
```

```
[Out] (x*(d*x)^m*Sqrt[(a + b*x^3)^2]*(a*(4 + m) + b*(1 + m)*x^3))/((1 + m)*(4 + m
)*(a + b*x^3))
```

IntegrateAlgebraic [F] time = 1.21, size = 0, normalized size = 0.00

$$\int (dx)^m \sqrt{a^2 + 2abx^3 + b^2x^6}\, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(d*x)^m*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6],x]
[Out] Defer[IntegrateAlgebraic][(d*x)^m*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6], x]
```

fricas [A] time = 1.28, size = 35, normalized size = 0.36

$$\frac{\left((bm + b)x^4 + (am + 4\,a)x\right)(dx)^m}{m^2 + 5\,m + 4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(b^2*x^6+2*a*b*x^3+a^2)^(1/2),x, algorithm="fricas")
[Out] ((b*m + b)*x^4 + (a*m + 4*a)*x)*(d*x)^m/(m^2 + 5*m + 4)
```

giac [A] time = 0.45, size = 83, normalized size = 0.86

$$\frac{(dx)^m\, bmx^4 \mathrm{sgn}\left(bx^3 + a\right) + (dx)^m\, bx^4 \mathrm{sgn}\left(bx^3 + a\right) + (dx)^m\, amx \mathrm{sgn}\left(bx^3 + a\right) + 4\,(dx)^m\, ax \mathrm{sgn}\left(bx^3 + a\right)}{m^2 + 5\,m + 4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(b^2*x^6+2*a*b*x^3+a^2)^(1/2),x, algorithm="giac")
[Out] ((d*x)^m*b*m*x^4*sgn(b*x^3 + a) + (d*x)^m*b*x^4*sgn(b*x^3 + a) + (d*x)^m*a*
m*x*sgn(b*x^3 + a) + 4*(d*x)^m*a*x*sgn(b*x^3 + a))/(m^2 + 5*m + 4)
```

maple [A] time = 0.00, size = 56, normalized size = 0.58

$$\frac{\left(bm\,x^3 + b\,x^3 + am + 4a\right)\sqrt{\left(b\,x^3 + a\right)^2}\; x\,(dx)^m}{(m + 4)\,(m + 1)\left(b\,x^3 + a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(b^2*x^6+2*a*b*x^3+a^2)^(1/2),x)
[Out] x*(b*m*x^3+b*x^3+a*m+4*a)*(d*x)^m*((b*x^3+a)^2)^(1/2)/(m+4)/(m+1)/(b*x^3+a)
```

maxima [A] time = 1.03, size = 35, normalized size = 0.36

$$\frac{\left(bd^m(m + 1)x^4 + ad^m(m + 4)x\right)x^m}{m^2 + 5\,m + 4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(b^2*x^6+2*a*b*x^3+a^2)^(1/2),x, algorithm="maxima")
[Out] (b*d^m*(m + 1)*x^4 + a*d^m*(m + 4)*x)*x^m/(m^2 + 5*m + 4)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (d\,x)^m\,\sqrt{a^2 + 2\,a\,b\,x^3 + b^2\,x^6}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(a^2 + b^2*x^6 + 2*a*b*x^3)^(1/2),x)
```

```
[Out] int((d*x)^m*(a^2 + b^2*x^6 + 2*a*b*x^3)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx)^m \sqrt{\left(a+bx^3\right)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(b**2*x**6+2*a*b*x**3+a**2)**(1/2),x)
```

```
[Out] Integral((d*x)**m*sqrt((a + b*x**3)**2), x)
```

3.121 $\int x^{11} \left(a^2+2abx^3+b^2x^6\right)^p\,dx$

Optimal. Leaf size=172

$$\frac{\left(a+bx^3\right)^4\left(a^2+2abx^3+b^2x^6\right)^p}{6b^4(p+2)}-\frac{a\left(a+bx^3\right)^3\left(a^2+2abx^3+b^2x^6\right)^p}{b^4(2p+3)}+\frac{a^2\left(a+bx^3\right)^2\left(a^2+2abx^3+b^2x^6\right)^p}{2b^4(p+1)}-\frac{a^3\left(\right.}{}$$

Rubi [A] time = 0.11, antiderivative size = 172, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1356, 266, 43}

$$\frac{\left(a+bx^3\right)^4\left(a^2+2abx^3+b^2x^6\right)^p}{6b^4(p+2)}-\frac{a\left(a+bx^3\right)^3\left(a^2+2abx^3+b^2x^6\right)^p}{b^4(2p+3)}+\frac{a^2\left(a+bx^3\right)^2\left(a^2+2abx^3+b^2x^6\right)^p}{2b^4(p+1)}-\frac{a^3\left(a+bx^3\right)\left(a^2+2abx^3+b^2x^6\right)^p}{3b^4(2p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[x^11*(a^2 + 2*a*b*x^3 + b^2*x^6)^p,x]
```

```
[Out] -(a^3*(a + b*x^3)*(a^2 + 2*a*b*x^3 + b^2*x^6)^p)/(3*b^4*(1 + 2*p)) + (a^2*(
a + b*x^3)^2*(a^2 + 2*a*b*x^3 + b^2*x^6)^p)/(2*b^4*(1 + p)) - (a*(a + b*x^3
)^3*(a^2 + 2*a*b*x^3 + b^2*x^6)^p)/(b^4*(3 + 2*p)) + ((a + b*x^3)^4*(a^2 +
2*a*b*x^3 + b^2*x^6)^p)/(6*b^4*(2 + p))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1356

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n + c*x^(2*n))^FracPart[p])/(1 + (
2*c*x^n)/b)^(2*FracPart[p]), Int[(d*x)^m*(1 + (2*c*x^n)/b)^(2*p), x], x] /;
 FreeQ[{a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] &&
 !IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}\int x^{11}\left(a^2+2abx^3+b^2x^6\right)^p\,dx &= \left(\left(1+\frac{bx^3}{a}\right)^{-2p}\left(a^2+2abx^3+b^2x^6\right)^p\right)\int x^{11}\left(1+\frac{bx^3}{a}\right)^{2p}\,dx\\ &= \frac{1}{3}\left(\left(1+\frac{bx^3}{a}\right)^{-2p}\left(a^2+2abx^3+b^2x^6\right)^p\right)\text{Subst}\left(\int x^3\left(1+\frac{bx}{a}\right)^{2p}\,dx,x,x^3\right)\\ &= \frac{1}{3}\left(\left(1+\frac{bx^3}{a}\right)^{-2p}\left(a^2+2abx^3+b^2x^6\right)^p\right)\text{Subst}\left(\int\left(-\frac{a^3\left(1+\frac{bx}{a}\right)^{2p}}{b^3}+\frac{3a^3\left(1+\right.}{}\right.\right.\\ &= -\frac{a^3\left(a+bx^3\right)\left(a^2+2abx^3+b^2x^6\right)^p}{3b^4(1+2p)}+\frac{a^2\left(a+bx^3\right)^2\left(a^2+2abx^3+b^2x^6\right)^p}{2b^4(1+p)}-\end{aligned}$$

Mathematica [A] time = 0.06, size = 110, normalized size = 0.64

$$\frac{(a+bx^3)\left((a+bx^3)^2\right)^p\left(-3a^3+3a^2b(2p+1)x^3-3ab^2\left(2p^2+3p+1\right)x^6+b^3\left(4p^3+12p^2+11p+3\right)x^9\right)}{6b^4(p+1)(p+2)(2p+1)(2p+3)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^11*(a^2 + 2*a*b*x^3 + b^2*x^6)^p,x]
```

```
[Out] ((a + b*x^3)*((a + b*x^3)^2)^p*(-3*a^3 + 3*a^2*b*(1 + 2*p)*x^3 - 3*a*b^2*(1
 + 3*p + 2*p^2)*x^6 + b^3*(3 + 11*p + 12*p^2 + 4*p^3)*x^9))/(6*b^4*(1 + p)*
(2 + p)*(1 + 2*p)*(3 + 2*p))
```

IntegrateAlgebraic [F] time = 0.53, size = 0, normalized size = 0.00

$$\int x^{11}\left(a^2+2abx^3+b^2x^6\right)^p\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^11*(a^2 + 2*a*b*x^3 + b^2*x^6)^p,x]
```

```
[Out] Defer[IntegrateAlgebraic][x^11*(a^2 + 2*a*b*x^3 + b^2*x^6)^p, x]
```

fricas [A] time = 1.17, size = 163, normalized size = 0.95

$$\frac{\left((4\,b^4p^3+12\,b^4p^2+11\,b^4p+3\,b^4)x^{12}+2\left(2\,ab^3p^3+3\,ab^3p^2+ab^3p\right)x^9+6\,a^3bpx^3-3\left(2\,a^2b^2p^2+a^2b^2p\right)x^6-3\,a^4\right)\left(b^2x^6+2\,abx^3+a^2\right)^p}{6\left(4\,b^4p^4+20\,b^4p^3+35\,b^4p^2+25\,b^4p+6\,b^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(b^2*x^6+2*a*b*x^3+a^2)^p,x, algorithm="fricas")
```

```
[Out] 1/6*((4*b^4*p^3 + 12*b^4*p^2 + 11*b^4*p + 3*b^4)*x^12 + 2*(2*a*b^3*p^3 + 3*
a*b^3*p^2 + a*b^3*p)*x^9 + 6*a^3*b*p*x^3 - 3*(2*a^2*b^2*p^2 + a^2*b^2*p)*x^
6 - 3*a^4)*(b^2*x^6 + 2*a*b*x^3 + a^2)^p/(4*b^4*p^4 + 20*b^4*p^3 + 35*b^4*p
^2 + 25*b^4*p + 6*b^4)
```

giac [B] time = 0.53, size = 375, normalized size = 2.18

$$\frac{4\left(b^2x^6+2\,abx^3+a^2\right)^pb^4p^3x^{12}+12\left(b^2x^6+2\,abx^3+a^2\right)^pb^4p^2x^{12}+11\left(b^2x^6+2\,abx^3+a^2\right)^pb^4px^{12}+4\left(b^2x^6+2\,abx^3+a^2\right)^pab^3p^3x^9+3\left(b^2x^6+2\,abx^3+a^2\right)^pb^4x^{12}+6\left(b^2x^6+2\,abx^3+a^2\right)^pab^3p^2x^9+2\left(b^2x^6+2\,abx^3+a^2\right)^pab^3px^9-6\left(b^2x^6+2\,abx^3+a^2\right)^pa^2b^2p^2x^6-3\left(b^2x^6+2\,abx^3+a^2\right)^pa^2b^2px^6+6\left(b^2x^6+2\,abx^3+a^2\right)^pa^3bpx^3-3\left(b^2x^6+2\,abx^3+a^2\right)^pa^4}{6\left(4\,b^4p^4+20\,b^4p^3+35\,b^4p^2+25\,b^4p+6\,b^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(b^2*x^6+2*a*b*x^3+a^2)^p,x, algorithm="giac")
```

```
[Out] 1/6*(4*(b^2*x^6 + 2*a*b*x^3 + a^2)^p*b^4*p^3*x^12 + 12*(b^2*x^6 + 2*a*b*x^3
 + a^2)^p*b^4*p^2*x^12 + 11*(b^2*x^6 + 2*a*b*x^3 + a^2)^p*b^4*p*x^12 + 4*(b
^2*x^6 + 2*a*b*x^3 + a^2)^p*a*b^3*p^3*x^9 + 3*(b^2*x^6 + 2*a*b*x^3 + a^2)^p
*b^4*x^12 + 6*(b^2*x^6 + 2*a*b*x^3 + a^2)^p*a*b^3*p^2*x^9 + 2*(b^2*x^6 + 2*
a*b*x^3 + a^2)^p*a*b^3*p*x^9 - 6*(b^2*x^6 + 2*a*b*x^3 + a^2)^p*a^2*b^2*p^2*
x^6 - 3*(b^2*x^6 + 2*a*b*x^3 + a^2)^p*a^2*b^2*p*x^6 + 6*(b^2*x^6 + 2*a*b*x^
3 + a^2)^p*a^3*b*p*x^3 - 3*(b^2*x^6 + 2*a*b*x^3 + a^2)^p*a^4)/(4*b^4*p^4 +
20*b^4*p^3 + 35*b^4*p^2 + 25*b^4*p + 6*b^4)
```

maple [A] time = 0.01, size = 150, normalized size = 0.87

$$-\frac{\left(-4b^3p^3x^9-12b^3p^2x^9-11b^3p\,x^9-3b^3x^9+6a\,b^2p^2x^6+9a\,b^2p\,x^6+3a\,b^2x^6-6a^2bp\,x^3-3a^2b\,x^3+3a^3\right)\left(b\,x^3+a\right)\left(b^2x^6+2ab\,x^3+a^2\right)^p}{6\left(4p^4+20p^3+35p^2+25p+6\right)b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11*(b^2*x^6+2*a*b*x^3+a^2)^p,x)
```

```
[Out] -1/6*(b^2*x^6+2*a*b*x^3+a^2)^p*(-4*b^3*p^3*x^9-12*b^3*p^2*x^9-11*b^3*p*x^9-
3*b^3*x^9+6*a*b^2*p^2*x^6+9*a*b^2*p*x^6+3*a*b^2*x^6-6*a^2*b*p*x^3-3*a^2*b*x
^3+3*a^3)*(b*x^3+a)/b^4/(4*p^4+20*p^3+35*p^2+25*p+6)
```

maxima [A] time = 0.81, size = 115, normalized size = 0.67

$$\frac{\left(\left(4\,p^{3}+12\,p^{2}+11\,p+3\right)b^{4}x^{12}+2\left(2\,p^{3}+3\,p^{2}+p\right)ab^{3}x^{9}-3\left(2\,p^{2}+p\right)a^{2}b^{2}x^{6}+6\,a^{3}bpx^{3}-3\,a^{4}\right)\left(bx^{3}+a\right)^{2\,p}}{6\left(4\,p^{4}+20\,p^{3}+35\,p^{2}+25\,p+6\right)b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(b^2*x^6+2*a*b*x^3+a^2)^p,x, algorithm="maxima")
```

```
[Out] 1/6*((4*p^3 + 12*p^2 + 11*p + 3)*b^4*x^12 + 2*(2*p^3 + 3*p^2 + p)*a*b^3*x^9
 - 3*(2*p^2 + p)*a^2*b^2*x^6 + 6*a^3*b*p*x^3 - 3*a^4)*(b*x^3 + a)^(2*p)/((4
*p^4 + 20*p^3 + 35*p^2 + 25*p + 6)*b^4)
```

mupad [B] time = 1.31, size = 207, normalized size = 1.20

$$\left(a^2+2\,a\,b\,x^3+b^2\,x^6\right)^p\,\left(\frac{x^{12}\,\left(4\,p^3+12\,p^2+11\,p+3\right)}{6\,\left(4\,p^4+20\,p^3+35\,p^2+25\,p+6\right)}-\frac{a^4}{2\,b^4\,\left(4\,p^4+20\,p^3+35\,p^2+25\,p+6\right)}+\frac{a^3\,p\,x^3}{b^3\,\left(4\,p^4+20\,p^3+35\,p^2+25\,p+6\right)}+\frac{a\,p\,x^9\,\left(2\,p^2+3\,p+1\right)}{3\,b\,\left(4\,p^4+20\,p^3+35\,p^2+25\,p+6\right)}-\frac{a^2\,p\,x^6\,\left(2\,p+1\right)}{2\,b^2\,\left(4\,p^4+20\,p^3+35\,p^2+25\,p+6\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11*(a^2 + b^2*x^6 + 2*a*b*x^3)^p,x)
```

```
[Out] (a^2 + b^2*x^6 + 2*a*b*x^3)^p*((x^12*(11*p + 12*p^2 + 4*p^3 + 3))/(6*(25*p
+ 35*p^2 + 20*p^3 + 4*p^4 + 6)) - a^4/(2*b^4*(25*p + 35*p^2 + 20*p^3 + 4*p^
4 + 6)) + (a^3*p*x^3)/(b^3*(25*p + 35*p^2 + 20*p^3 + 4*p^4 + 6)) + (a*p*x^9
*(3*p + 2*p^2 + 1))/(3*b*(25*p + 35*p^2 + 20*p^3 + 4*p^4 + 6)) - (a^2*p*x^6
*(2*p + 1))/(2*b^2*(25*p + 35*p^2 + 20*p^3 + 4*p^4 + 6)))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**11*(b**2*x**6+2*a*b*x**3+a**2)**p,x)
```

```
[Out] Piecewise((x**12*(a**2)**p/12, Eq(b, 0)), (6*a**3*log(-(-1)**(1/3)*a**(1/3)
*(1/b)**(1/3) + x)/(18*a**3*b**4 + 54*a**2*b**5*x**3 + 54*a*b**6*x**6 + 18*
b**7*x**9) + 6*a**3*log(4*(-1)**(2/3)*a**(2/3)*(1/b)**(2/3) + 4*(-1)**(1/3)
*a**(1/3)*x*(1/b)**(1/3) + 4*x**2)/(18*a**3*b**4 + 54*a**2*b**5*x**3 + 54*a
*b**6*x**6 + 18*b**7*x**9) - 12*a**3*log(2)/(18*a**3*b**4 + 54*a**2*b**5*x*
*3 + 54*a*b**6*x**6 + 18*b**7*x**9) + 11*a**3/(18*a**3*b**4 + 54*a**2*b**5*
x**3 + 54*a*b**6*x**6 + 18*b**7*x**9) + 18*a**2*b*x**3*log(-(-1)**(1/3)*a**
(1/3)*(1/b)**(1/3) + x)/(18*a**3*b**4 + 54*a**2*b**5*x**3 + 54*a*b**6*x**6
+ 18*b**7*x**9) + 18*a**2*b*x**3*log(4*(-1)**(2/3)*a**(2/3)*(1/b)**(2/3) +
4*(-1)**(1/3)*a**(1/3)*x*(1/b)**(1/3) + 4*x**2)/(18*a**3*b**4 + 54*a**2*b**
5*x**3 + 54*a*b**6*x**6 + 18*b**7*x**9) - 36*a**2*b*x**3*log(2)/(18*a**3*b*
*4 + 54*a**2*b**5*x**3 + 54*a*b**6*x**6 + 18*b**7*x**9) + 27*a**2*b*x**3/(1
8*a**3*b**4 + 54*a**2*b**5*x**3 + 54*a*b**6*x**6 + 18*b**7*x**9) + 18*a*b**
2*x**6*log(-(-1)**(1/3)*a**(1/3)*(1/b)**(1/3) + x)/(18*a**3*b**4 + 54*a**2*
b**5*x**3 + 54*a*b**6*x**6 + 18*b**7*x**9) + 18*a*b**2*x**6*log(4*(-1)**(2/
3)*a**(2/3)*(1/b)**(2/3) + 4*(-1)**(1/3)*a**(1/3)*x*(1/b)**(1/3) + 4*x**2)/
(18*a**3*b**4 + 54*a**2*b**5*x**3 + 54*a*b**6*x**6 + 18*b**7*x**9) - 36*a*b
**2*x**6*log(2)/(18*a**3*b**4 + 54*a**2*b**5*x**3 + 54*a*b**6*x**6 + 18*b**
7*x**9) + 18*a*b**2*x**6/(18*a**3*b**4 + 54*a**2*b**5*x**3 + 54*a*b**6*x**6
 + 18*b**7*x**9) + 6*b**3*x**9*log(-(-1)**(1/3)*a**(1/3)*(1/b)**(1/3) + x)/
(18*a**3*b**4 + 54*a**2*b**5*x**3 + 54*a*b**6*x**6 + 18*b**7*x**9) + 6*b**3
*x**9*log(4*(-1)**(2/3)*a**(2/3)*(1/b)**(2/3) + 4*(-1)**(1/3)*a**(1/3)*x*(1
/b)**(1/3) + 4*x**2)/(18*a**3*b**4 + 54*a**2*b**5*x**3 + 54*a*b**6*x**6 + 1
8*b**7*x**9) - 12*b**3*x**9*log(2)/(18*a**3*b**4 + 54*a**2*b**5*x**3 + 54*a
```

```
*b**6*x**6 + 18*b**7*x**9), Eq(p, -2)), (Integral(x**11/((a + b*x**3)**2)**
(3/2), x), Eq(p, -3/2)), (6*a**3*log(-(-1)**(1/3)*a**(1/3)*(1/b)**(1/3) + x
)/(6*a*b**4 + 6*b**5*x**3) + 6*a**3*log(4*(-1)**(2/3)*a**(2/3)*(1/b)**(2/3)
 + 4*(-1)**(1/3)*a**(1/3)*x*(1/b)**(1/3) + 4*x**2)/(6*a*b**4 + 6*b**5*x**3)
 - 12*a**3*log(2)/(6*a*b**4 + 6*b**5*x**3) + 6*a**3/(6*a*b**4 + 6*b**5*x**3
) + 6*a**2*b*x**3*log(-(-1)**(1/3)*a**(1/3)*(1/b)**(1/3) + x)/(6*a*b**4 + 6
*b**5*x**3) + 6*a**2*b*x**3*log(4*(-1)**(2/3)*a**(2/3)*(1/b)**(2/3) + 4*(-1
)**(1/3)*a**(1/3)*x*(1/b)**(1/3) + 4*x**2)/(6*a*b**4 + 6*b**5*x**3) - 12*a*
*2*b*x**3*log(2)/(6*a*b**4 + 6*b**5*x**3) - 3*a*b**2*x**6/(6*a*b**4 + 6*b**
5*x**3) + b**3*x**9/(6*a*b**4 + 6*b**5*x**3), Eq(p, -1)), (Integral(x**11/s
qrt((a + b*x**3)**2), x), Eq(p, -1/2)), (-3*a**4*(a**2 + 2*a*b*x**3 + b**2*
x**6)**p/(24*b**4*p**4 + 120*b**4*p**3 + 210*b**4*p**2 + 150*b**4*p + 36*b*
*4) + 6*a**3*b*p*x**3*(a**2 + 2*a*b*x**3 + b**2*x**6)**p/(24*b**4*p**4 + 12
0*b**4*p**3 + 210*b**4*p**2 + 150*b**4*p + 36*b**4) - 6*a**2*b**2*p**2*x**6
*(a**2 + 2*a*b*x**3 + b**2*x**6)**p/(24*b**4*p**4 + 120*b**4*p**3 + 210*b**
4*p**2 + 150*b**4*p + 36*b**4) - 3*a**2*b**2*p*x**6*(a**2 + 2*a*b*x**3 + b*
*2*x**6)**p/(24*b**4*p**4 + 120*b**4*p**3 + 210*b**4*p**2 + 150*b**4*p + 36
*b**4) + 4*a*b**3*p**3*x**9*(a**2 + 2*a*b*x**3 + b**2*x**6)**p/(24*b**4*p**
4 + 120*b**4*p**3 + 210*b**4*p**2 + 150*b**4*p + 36*b**4) + 6*a*b**3*p**2*x
**9*(a**2 + 2*a*b*x**3 + b**2*x**6)**p/(24*b**4*p**4 + 120*b**4*p**3 + 210*
b**4*p**2 + 150*b**4*p + 36*b**4) + 2*a*b**3*p*x**9*(a**2 + 2*a*b*x**3 + b*
*2*x**6)**p/(24*b**4*p**4 + 120*b**4*p**3 + 210*b**4*p**2 + 150*b**4*p + 36
*b**4) + 4*b**4*p**3*x**12*(a**2 + 2*a*b*x**3 + b**2*x**6)**p/(24*b**4*p**4
 + 120*b**4*p**3 + 210*b**4*p**2 + 150*b**4*p + 36*b**4) + 12*b**4*p**2*x**
12*(a**2 + 2*a*b*x**3 + b**2*x**6)**p/(24*b**4*p**4 + 120*b**4*p**3 + 210*b
**4*p**2 + 150*b**4*p + 36*b**4) + 11*b**4*p*x**12*(a**2 + 2*a*b*x**3 + b**
2*x**6)**p/(24*b**4*p**4 + 120*b**4*p**3 + 210*b**4*p**2 + 150*b**4*p + 36*
b**4) + 3*b**4*x**12*(a**2 + 2*a*b*x**3 + b**2*x**6)**p/(24*b**4*p**4 + 120
*b**4*p**3 + 210*b**4*p**2 + 150*b**4*p + 36*b**4), True))
```

3.122 $\int x^8 \left(a^2+2abx^3+b^2x^6\right)^p \, dx$

Optimal. Leaf size=130

$$\frac{\left(a+bx^3\right)^3 \left(a^2+2abx^3+b^2x^6\right)^p}{3b^3(2p+3)}-\frac{a\left(a+bx^3\right)^2 \left(a^2+2abx^3+b^2x^6\right)^p}{3b^3(p+1)}+\frac{a^2\left(a+bx^3\right)\left(a^2+2abx^3+b^2x^6\right)^p}{3b^3(2p+1)}$$

Rubi [A] time = 0.08, antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1356, 266, 43}

$$\frac{\left(a+bx^3\right)^3 \left(a^2+2abx^3+b^2x^6\right)^p}{3b^3(2p+3)}-\frac{a\left(a+bx^3\right)^2 \left(a^2+2abx^3+b^2x^6\right)^p}{3b^3(p+1)}+\frac{a^2\left(a+bx^3\right)\left(a^2+2abx^3+b^2x^6\right)^p}{3b^3(2p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[x^8*(a^2 + 2*a*b*x^3 + b^2*x^6)^p,x]
```

```
[Out] (a^2*(a + b*x^3)*(a^2 + 2*a*b*x^3 + b^2*x^6)^p)/(3*b^3*(1 + 2*p)) - (a*(a +
 b*x^3)^2*(a^2 + 2*a*b*x^3 + b^2*x^6)^p)/(3*b^3*(1 + p)) + ((a + b*x^3)^3*(
a^2 + 2*a*b*x^3 + b^2*x^6)^p)/(3*b^3*(3 + 2*p))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1356

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n + c*x^(2*n))^FracPart[p])/(1 + (
2*c*x^n)/b)^(2*FracPart[p]), Int[(d*x)^m*(1 + (2*c*x^n)/b)^(2*p), x], x] /;
 FreeQ[{a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] &&
 !IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}
\int x^8 \left(a^2+2abx^3+b^2x^6\right)^p \, dx &= \left(\left(1+\frac{bx^3}{a}\right)^{-2p}\left(a^2+2abx^3+b^2x^6\right)^p\right)\int x^8\left(1+\frac{bx^3}{a}\right)^{2p} dx\\
&= \frac{1}{3}\left(\left(1+\frac{bx^3}{a}\right)^{-2p}\left(a^2+2abx^3+b^2x^6\right)^p\right)\text{Subst}\left(\int x^2\left(1+\frac{bx}{a}\right)^{2p} dx, x, x^3\right)\\
&= \frac{1}{3}\left(\left(1+\frac{bx^3}{a}\right)^{-2p}\left(a^2+2abx^3+b^2x^6\right)^p\right)\text{Subst}\left(\int\left(\frac{a^2\left(1+\frac{bx}{a}\right)^{2p}}{b^2}-\frac{2a^2\left(1+\frac{b}{a}\right.}{b^2}\right.\right.\\
&= \frac{a^2\left(a+bx^3\right)\left(a^2+2abx^3+b^2x^6\right)^p}{3b^3(1+2p)}-\frac{a\left(a+bx^3\right)^2\left(a^2+2abx^3+b^2x^6\right)^p}{3b^3(1+p)}+\frac{(a}{}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 77, normalized size = 0.59

$$\frac{\left(a+bx^3\right)\left(\left(a+bx^3\right)^2\right)^p\left(a^2-ab(2p+1)x^3+b^2\left(2p^2+3p+1\right)x^6\right)}{3b^3(p+1)(2p+1)(2p+3)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8*(a^2 + 2*a*b*x^3 + b^2*x^6)^p,x]
```

```
[Out] ((a + b*x^3)*((a + b*x^3)^2)^p*(a^2 - a*b*(1 + 2*p)*x^3 + b^2*(1 + 3*p + 2*
p^2)*x^6))/(3*b^3*(1 + p)*(1 + 2*p)*(3 + 2*p))
```

IntegrateAlgebraic [F] time = 0.49, size = 0, normalized size = 0.00

$$\int x^8\left(a^2+2abx^3+b^2x^6\right)^p\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^8*(a^2 + 2*a*b*x^3 + b^2*x^6)^p,x]
```

```
[Out] Defer[IntegrateAlgebraic][x^8*(a^2 + 2*a*b*x^3 + b^2*x^6)^p, x]
```

fricas [A] time = 1.13, size = 108, normalized size = 0.83

$$\frac{\left(\left(2\,b^3p^2+3\,b^3p+b^3\right)x^9-2\,a^2bpx^3+\left(2\,ab^2p^2+ab^2p\right)x^6+a^3\right)\left(b^2x^6+2\,abx^3+a^2\right)^p}{3\left(4\,b^3p^3+12\,b^3p^2+11\,b^3p+3\,b^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(b^2*x^6+2*a*b*x^3+a^2)^p,x, algorithm="fricas")
```

```
[Out] 1/3*((2*b^3*p^2 + 3*b^3*p + b^3)*x^9 - 2*a^2*b*p*x^3 + (2*a*b^2*p^2 + a*b^2
*p)*x^6 + a^3)*(b^2*x^6 + 2*a*b*x^3 + a^2)^p/(4*b^3*p^3 + 12*b^3*p^2 + 11*b
^3*p + 3*b^3)
```

giac [A] time = 0.44, size = 235, normalized size = 1.81

$$\frac{2\left(b^2x^6+2\,abx^3+a^2\right)^pb^3p^2x^9+3\left(b^2x^6+2\,abx^3+a^2\right)^pb^3px^9+\left(b^2x^6+2\,abx^3+a^2\right)^pb^3x^9+2\left(b^2x^6+2\,abx^3+a^2\right)^pab^2p^2x^6+\left(b^2x^6+2\,abx^3+a^2\right)^pab^2px^6-2\left(b^2x^6+2\,abx^3+a^2\right)^pa^2bpx^3+\left(b^2x^6+2\,abx^3+a^2\right)^pa^3}{3\left(4\,b^3p^3+12\,b^3p^2+11\,b^3p+3\,b^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(b^2*x^6+2*a*b*x^3+a^2)^p,x, algorithm="giac")
```

```
[Out] 1/3*(2*(b^2*x^6 + 2*a*b*x^3 + a^2)^p*b^3*p^2*x^9 + 3*(b^2*x^6 + 2*a*b*x^3 +
 a^2)^p*b^3*p*x^9 + (b^2*x^6 + 2*a*b*x^3 + a^2)^p*b^3*x^9 + 2*(b^2*x^6 + 2*
a*b*x^3 + a^2)^p*a*b^2*p^2*x^6 + (b^2*x^6 + 2*a*b*x^3 + a^2)^p*a*b^2*p*x^6
- 2*(b^2*x^6 + 2*a*b*x^3 + a^2)^p*a^2*b*p*x^3 + (b^2*x^6 + 2*a*b*x^3 + a^2)
^p*a^3)/(4*b^3*p^3 + 12*b^3*p^2 + 11*b^3*p + 3*b^3)
```

maple [A] time = 0.01, size = 96, normalized size = 0.74

$$\frac{\left(b\,x^3+a\right)\left(2b^2p^2x^6+3b^2p\,x^6+b^2x^6-2abp\,x^3-ab\,x^3+a^2\right)\left(b^2x^6+2ab\,x^3+a^2\right)^p}{3\left(4p^3+12p^2+11p+3\right)b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*(b^2*x^6+2*a*b*x^3+a^2)^p,x)
```

```
[Out] 1/3*(b*x^3+a)*(2*b^2*p^2*x^6+3*b^2*p*x^6+b^2*x^6-2*a*b*p*x^3-a*b*x^3+a^2)*(
b^2*x^6+2*a*b*x^3+a^2)^p/b^3/(4*p^3+12*p^2+11*p+3)
```

maxima [A] time = 1.22, size = 79, normalized size = 0.61

$$\frac{\left(\left(2p^2+3p+1\right)b^3x^9+\left(2p^2+p\right)ab^2x^6-2a^2bpx^3+a^3\right)\left(bx^3+a\right)^{2p}}{3\left(4p^3+12p^2+11p+3\right)b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(b^2*x^6+2*a*b*x^3+a^2)^p,x, algorithm="maxima")
```

```
[Out] 1/3*((2*p^2 + 3*p + 1)*b^3*x^9 + (2*p^2 + p)*a*b^2*x^6 - 2*a^2*b*p*x^3 + a^
3)*(b*x^3 + a)^(2*p)/((4*p^3 + 12*p^2 + 11*p + 3)*b^3)
```

mupad [B] time = 1.22, size = 137, normalized size = 1.05

$$\left(a^2+2\,a\,b\,x^3+b^2\,x^6\right)^p\left(\frac{x^9\,\left(\frac{2\,p^2}{3}+p+\frac{1}{3}\right)}{4\,p^3+12\,p^2+11\,p+3}+\frac{a^3}{3\,b^3\,\left(4\,p^3+12\,p^2+11\,p+3\right)}-\frac{2\,a^2\,p\,x^3}{3\,b^2\,\left(4\,p^3+12\,p^2+11\,p+3\right)}+\frac{a\,p\,x^6\,\left(2\,p+1\right)}{3\,b\,\left(4\,p^3+12\,p^2+11\,p+3\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*(a^2 + b^2*x^6 + 2*a*b*x^3)^p,x)
```

```
[Out] (a^2 + b^2*x^6 + 2*a*b*x^3)^p*((x^9*(p + (2*p^2)/3 + 1/3))/(11*p + 12*p^2 +
 4*p^3 + 3) + a^3/(3*b^3*(11*p + 12*p^2 + 4*p^3 + 3)) - (2*a^2*p*x^3)/(3*b^
2*(11*p + 12*p^2 + 4*p^3 + 3)) + (a*p*x^6*(2*p + 1))/(3*b*(11*p + 12*p^2 +
4*p^3 + 3)))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\begin{cases} \frac{x^9\left(a^2\right)^p}{9} & \text{for } b=0 \\ \int \frac{x^8}{\left(\left(a+bx^3\right)^2\right)^{\frac{3}{2}}}\,dx & \text{for } p=-\frac{3}{2} \\ -\frac{2a^2\log\left(-\sqrt[3]{-1}\,\sqrt[3]{a}\,\sqrt[3]{\frac{1}{b}}+x\right)}{3ab^3+3b^4x^3}-\frac{2a^2\log\left(4(-1)^{\frac{2}{3}}a^{\frac{2}{3}}\left(\frac{1}{b}\right)^{\frac{2}{3}}+4\sqrt[3]{-1}\,\sqrt[3]{a}\,x\sqrt[3]{\frac{1}{b}}+4x^2\right)}{3ab^3+3b^4x^3}-\frac{2a^2}{3ab^3+3b^4x^3}+\frac{4a^2\log(2)}{3ab^3+3b^4x^3}-\frac{2abx^3\log\left(-\sqrt[3]{-1}\,\sqrt[3]{a}\,\sqrt[3]{\frac{1}{b}}+x\right)}{3ab^3+3b^4x^3}-\frac{2abx^3\log\left(4(-1)^{\frac{2}{3}}a^{\frac{2}{3}}\left(\frac{1}{b}\right)^{\frac{2}{3}}+4\sqrt[3]{-1}\,\sqrt[3]{a}\,x\sqrt[3]{\frac{1}{b}}+4x^2\right)}{3ab^3+3b^4x^3}+\frac{4abx^3\log(2)}{3ab^3+3b^4x^3}+\frac{b^2x^6}{3ab^3+3b^4x^3} & \text{for } p=-1 \\ \int \frac{x^8}{\sqrt{\left(a+bx^3\right)^2}}\,dx & \text{for } p=-\frac{1}{2} \\ \frac{a^3\left(a^2+2abx^3+b^2x^6\right)^p}{12b^3p^3+36b^3p^2+33b^3p+9b^3}-\frac{2a^2bpx^3\left(a^2+2abx^3+b^2x^6\right)^p}{12b^3p^3+36b^3p^2+33b^3p+9b^3}+\frac{2ab^2p^2x^6\left(a^2+2abx^3+b^2x^6\right)^p}{12b^3p^3+36b^3p^2+33b^3p+9b^3}+\frac{ab^2px^6\left(a^2+2abx^3+b^2x^6\right)^p}{12b^3p^3+36b^3p^2+33b^3p+9b^3}+\frac{2b^3p^2x^9\left(a^2+2abx^3+b^2x^6\right)^p}{12b^3p^3+36b^3p^2+33b^3p+9b^3}+\frac{3b^3px^9\left(a^2+2abx^3+b^2x^6\right)^p}{12b^3p^3+36b^3p^2+33b^3p+9b^3}+\frac{b^3x^9\left(a^2+2abx^3+b^2x^6\right)^p}{12b^3p^3+36b^3p^2+33b^3p+9b^3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8*(b**2*x**6+2*a*b*x**3+a**2)**p,x)
```

```
[Out] Piecewise((x**9*(a**2)**p/9, Eq(b, 0)), (Integral(x**8/((a + b*x**3)**2)**(
3/2), x), Eq(p, -3/2)), (-2*a**2*log(-(-1)**(1/3)*a**(1/3)*(1/b)**(1/3) + x
)/(3*a*b**3 + 3*b**4*x**3) - 2*a**2*log(4*(-1)**(2/3)*a**(2/3)*(1/b)**(2/3)
 + 4*(-1)**(1/3)*a**(1/3)*x*(1/b)**(1/3) + 4*x**2)/(3*a*b**3 + 3*b**4*x**3)
 - 2*a**2/(3*a*b**3 + 3*b**4*x**3) + 4*a**2*log(2)/(3*a*b**3 + 3*b**4*x**3)
 - 2*a*b*x**3*log(-(-1)**(1/3)*a**(1/3)*(1/b)**(1/3) + x)/(3*a*b**3 + 3*b**
4*x**3) - 2*a*b*x**3*log(4*(-1)**(2/3)*a**(2/3)*(1/b)**(2/3) + 4*(-1)**(1/3
)*a**(1/3)*x*(1/b)**(1/3) + 4*x**2)/(3*a*b**3 + 3*b**4*x**3) + 4*a*b*x**3*l
og(2)/(3*a*b**3 + 3*b**4*x**3) + b**2*x**6/(3*a*b**3 + 3*b**4*x**3), Eq(p,
-1)), (Integral(x**8/sqrt((a + b*x**3)**2), x), Eq(p, -1/2)), (a**3*(a**2 +
 2*a*b*x**3 + b**2*x**6)**p/(12*b**3*p**3 + 36*b**3*p**2 + 33*b**3*p + 9*b*
*3) - 2*a**2*b*p*x**3*(a**2 + 2*a*b*x**3 + b**2*x**6)**p/(12*b**3*p**3 + 36
*b**3*p**2 + 33*b**3*p + 9*b**3) + 2*a*b**2*p**2*x**6*(a**2 + 2*a*b*x**3 +
b**2*x**6)**p/(12*b**3*p**3 + 36*b**3*p**2 + 33*b**3*p + 9*b**3) + a*b**2*p
*x**6*(a**2 + 2*a*b*x**3 + b**2*x**6)**p/(12*b**3*p**3 + 36*b**3*p**2 + 33*
b**3*p + 9*b**3) + 2*b**3*p**2*x**9*(a**2 + 2*a*b*x**3 + b**2*x**6)**p/(12*
b**3*p**3 + 36*b**3*p**2 + 33*b**3*p + 9*b**3) + 3*b**3*p*x**9*(a**2 + 2*a*
b*x**3 + b**2*x**6)**p/(12*b**3*p**3 + 36*b**3*p**2 + 33*b**3*p + 9*b**3) +
 b**3*x**9*(a**2 + 2*a*b*x**3 + b**2*x**6)**p/(12*b**3*p**3 + 36*b**3*p**2
+ 33*b**3*p + 9*b**3), True))
```

3.123 $\int x^5 \left(a^2+2abx^3+b^2x^6\right)^p \, dx$

Optimal. Leaf size=84

$$\frac{\left(a+bx^3\right)^2 \left(a^2+2abx^3+b^2x^6\right)^p}{6b^2(p+1)}-\frac{a\left(a+bx^3\right)\left(a^2+2abx^3+b^2x^6\right)^p}{3b^2(2p+1)}$$

Rubi [A] time = 0.06, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1356, 266, 43}

$$\frac{\left(a+bx^3\right)^2 \left(a^2+2abx^3+b^2x^6\right)^p}{6b^2(p+1)}-\frac{a\left(a+bx^3\right)\left(a^2+2abx^3+b^2x^6\right)^p}{3b^2(2p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*(a^2 + 2*a*b*x^3 + b^2*x^6)^p,x]
```

```
[Out] -(a*(a + b*x^3)*(a^2 + 2*a*b*x^3 + b^2*x^6)^p)/(3*b^2*(1 + 2*p)) + ((a + b*
x^3)^2*(a^2 + 2*a*b*x^3 + b^2*x^6)^p)/(6*b^2*(1 + p))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1356

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n + c*x^(2*n))^FracPart[p])/(1 + (
2*c*x^n)/b)^(2*FracPart[p]), Int[(d*x)^m*(1 + (2*c*x^n)/b)^(2*p), x], x] /;
 FreeQ[{a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] &&
 !IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}
\int x^5 \left(a^2+2abx^3+b^2x^6\right)^p \, dx &= \left(\left(1+\frac{bx^3}{a}\right)^{-2p} \left(a^2+2abx^3+b^2x^6\right)^p\right) \int x^5 \left(1+\frac{bx^3}{a}\right)^{2p} \, dx\\
&= \frac{1}{3}\left(\left(1+\frac{bx^3}{a}\right)^{-2p} \left(a^2+2abx^3+b^2x^6\right)^p\right) \text{Subst}\left(\int x \left(1+\frac{bx}{a}\right)^{2p} \, dx, x, x^3\right)\\
&= \frac{1}{3}\left(\left(1+\frac{bx^3}{a}\right)^{-2p} \left(a^2+2abx^3+b^2x^6\right)^p\right) \text{Subst}\left(\int \left(-\frac{a\left(1+\frac{bx}{a}\right)^{2p}}{b}+\frac{a\left(1+\frac{bx}{a}\right)^{1+2}}{b}\right.\right.\\
&= -\frac{a\left(a+bx^3\right)\left(a^2+2abx^3+b^2x^6\right)^p}{3b^2(1+2p)}+\frac{\left(a+bx^3\right)^2 \left(a^2+2abx^3+b^2x^6\right)^p}{6b^2(1+p)}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 51, normalized size = 0.61

$$\frac{(a+bx^3)\left((a+bx^3)^2\right)^p (b(2p+1)x^3-a)}{6b^2(p+1)(2p+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*(a^2 + 2*a*b*x^3 + b^2*x^6)^p,x]
```

```
[Out] ((a + b*x^3)*((a + b*x^3)^2)^p*(-a + b*(1 + 2*p)*x^3))/(6*b^2*(1 + p)*(1 + 
2*p))
```

IntegrateAlgebraic [F] time = 0.29, size = 0, normalized size = 0.00

$$\int x^5 \left(a^2+2abx^3+b^2x^6\right)^p \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^5*(a^2 + 2*a*b*x^3 + b^2*x^6)^p,x]
```

```
[Out] Defer[IntegrateAlgebraic][x^5*(a^2 + 2*a*b*x^3 + b^2*x^6)^p, x]
```

fricas [A] time = 1.12, size = 70, normalized size = 0.83

$$\frac{\left((2\,b^2p+b^2)x^6+2\,abpx^3-a^2\right)\left(b^2x^6+2\,abx^3+a^2\right)^p}{6\left(2\,b^2p^2+3\,b^2p+b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b^2*x^6+2*a*b*x^3+a^2)^p,x, algorithm="fricas")
```

```
[Out] 1/6*((2*b^2*p + b^2)*x^6 + 2*a*b*p*x^3 - a^2)*(b^2*x^6 + 2*a*b*x^3 + a^2)^p
/(2*b^2*p^2 + 3*b^2*p + b^2)
```

giac [A] time = 0.48, size = 132, normalized size = 1.57

$$\frac{2\left(b^2x^6+2\,abx^3+a^2\right)^p b^2px^6+\left(b^2x^6+2\,abx^3+a^2\right)^p b^2x^6+2\left(b^2x^6+2\,abx^3+a^2\right)^p abpx^3-\left(b^2x^6+2\,abx^3+a^2\right)^p a^2}{6\left(2\,b^2p^2+3\,b^2p+b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b^2*x^6+2*a*b*x^3+a^2)^p,x, algorithm="giac")
```

```
[Out] 1/6*(2*(b^2*x^6 + 2*a*b*x^3 + a^2)^p*b^2*p*x^6 + (b^2*x^6 + 2*a*b*x^3 + a^2
)^p*b^2*x^6 + 2*(b^2*x^6 + 2*a*b*x^3 + a^2)^p*a*b*p*x^3 - (b^2*x^6 + 2*a*b*
x^3 + a^2)^p*a^2)/(2*b^2*p^2 + 3*b^2*p + b^2)
```

maple [A] time = 0.01, size = 60, normalized size = 0.71

$$-\frac{\left(-2x^3pb-b\,x^3+a\right)\left(b\,x^3+a\right)\left(b^2x^6+2ab\,x^3+a^2\right)^p}{6\left(2p^2+3p+1\right)b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(b^2*x^6+2*a*b*x^3+a^2)^p,x)
```

```
[Out] -1/6*(b^2*x^6+2*a*b*x^3+a^2)^p*(-2*b*p*x^3-b*x^3+a)*(b*x^3+a)/b^2/(2*p^2+3*
p+1)
```

maxima [A] time = 0.63, size = 54, normalized size = 0.64

$$\frac{\left(b^{2}\left(2\,p+1\right)x^{6}+2\,abpx^{3}-a^{2}\right)\left(bx^{3}+a\right)^{2\,p}}{6\,\left(2\,p^{2}+3\,p+1\right)b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b^2*x^6+2*a*b*x^3+a^2)^p,x, algorithm="maxima")
```

```
[Out] 1/6*(b^2*(2*p + 1)*x^6 + 2*a*b*p*x^3 - a^2)*(b*x^3 + a)^(2*p)/((2*p^2 + 3*p
 + 1)*b^2)
```

mupad [B] time = 1.19, size = 85, normalized size = 1.01

$$\left(a^{2}+2\,a\,b\,x^{3}+b^{2}\,x^{6}\right)^{p}\,\left(\frac{x^{6}\,\left(2\,p+1\right)}{6\,\left(2\,p^{2}+3\,p+1\right)}-\frac{a^{2}}{6\,b^{2}\,\left(2\,p^{2}+3\,p+1\right)}+\frac{a\,p\,x^{3}}{3\,b\,\left(2\,p^{2}+3\,p+1\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a^2 + b^2*x^6 + 2*a*b*x^3)^p,x)
```

```
[Out] (a^2 + b^2*x^6 + 2*a*b*x^3)^p*((x^6*(2*p + 1))/(6*(3*p + 2*p^2 + 1)) - a^2/
(6*b^2*(3*p + 2*p^2 + 1)) + (a*p*x^3)/(3*b*(3*p + 2*p^2 + 1)))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\begin{cases} \frac{x^{6}\left(a^{2}\right)^{p}}{6} & \text{for}\: b = 0 \\ \frac{a \log{\left(- \sqrt[3]{-1} \sqrt[3]{a} \sqrt[3]{\frac{1}{b}} + x \right)}}{3ab^{2}+3b^{3}x^{3}} + \frac{a \log{\left(4(-1)^{\frac{2}{3}} a^{\frac{2}{3}} \left(\frac{1}{b}\right)^{\frac{2}{3}} + 4 \sqrt[3]{-1} \sqrt[3]{a} x \sqrt[3]{\frac{1}{b}} + 4x^{2} \right)}}{3ab^{2}+3b^{3}x^{3}} - \frac{2a \log{\left(2 \right)}}{3ab^{2}+3b^{3}x^{3}} + \frac{a}{3ab^{2}+3b^{3}x^{3}} + \frac{bx^{3} \log{\left(- \sqrt[3]{-1} \sqrt[3]{a} \sqrt[3]{\frac{1}{b}} + x \right)}}{3ab^{2}+3b^{3}x^{3}} + \frac{bx^{3} \log{\left(4(-1)^{\frac{2}{3}} a^{\frac{2}{3}} \left(\frac{1}{b}\right)^{\frac{2}{3}} + 4 \sqrt[3]{-1} \sqrt[3]{a} x \sqrt[3]{\frac{1}{b}} + 4x^{2} \right)}}{3ab^{2}+3b^{3}x^{3}} - \frac{2bx^{3} \log{\left(2 \right)}}{3ab^{2}+3b^{3}x^{3}} & \text{for}\: p = -1 \\ \int \frac{x^{5}}{\sqrt{\left(a+bx^{3}\right)^{2}}}\, dx & \text{for}\: p = - \frac{1}{2} \\ - \frac{a^{2}\left(a^{2}+2abx^{3}+b^{2}x^{6}\right)^{p}}{12b^{2}p^{2}+18b^{2}p+6b^{2}} + \frac{2abpx^{3}\left(a^{2}+2abx^{3}+b^{2}x^{6}\right)^{p}}{12b^{2}p^{2}+18b^{2}p+6b^{2}} + \frac{2b^{2}px^{6}\left(a^{2}+2abx^{3}+b^{2}x^{6}\right)^{p}}{12b^{2}p^{2}+18b^{2}p+6b^{2}} + \frac{b^{2}x^{6}\left(a^{2}+2abx^{3}+b^{2}x^{6}\right)^{p}}{12b^{2}p^{2}+18b^{2}p+6b^{2}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(b**2*x**6+2*a*b*x**3+a**2)**p,x)
```

```
[Out] Piecewise((x**6*(a**2)**p/6, Eq(b, 0)), (a*log(-(-1)**(1/3)*a**(1/3)*(1/b)*
*(1/3) + x)/(3*a*b**2 + 3*b**3*x**3) + a*log(4*(-1)**(2/3)*a**(2/3)*(1/b)**
(2/3) + 4*(-1)**(1/3)*a**(1/3)*x*(1/b)**(1/3) + 4*x**2)/(3*a*b**2 + 3*b**3*
x**3) - 2*a*log(2)/(3*a*b**2 + 3*b**3*x**3) + a/(3*a*b**2 + 3*b**3*x**3) +
b*x**3*log(-(-1)**(1/3)*a**(1/3)*(1/b)**(1/3) + x)/(3*a*b**2 + 3*b**3*x**3)
 + b*x**3*log(4*(-1)**(2/3)*a**(2/3)*(1/b)**(2/3) + 4*(-1)**(1/3)*a**(1/3)*
x*(1/b)**(1/3) + 4*x**2)/(3*a*b**2 + 3*b**3*x**3) - 2*b*x**3*log(2)/(3*a*b*
*2 + 3*b**3*x**3), Eq(p, -1)), (Integral(x**5/sqrt((a + b*x**3)**2), x), Eq
(p, -1/2)), (-a**2*(a**2 + 2*a*b*x**3 + b**2*x**6)**p/(12*b**2*p**2 + 18*b*
*2*p + 6*b**2) + 2*a*b*p*x**3*(a**2 + 2*a*b*x**3 + b**2*x**6)**p/(12*b**2*p
**2 + 18*b**2*p + 6*b**2) + 2*b**2*p*x**6*(a**2 + 2*a*b*x**3 + b**2*x**6)**
p/(12*b**2*p**2 + 18*b**2*p + 6*b**2) + b**2*x**6*(a**2 + 2*a*b*x**3 + b**2
*x**6)**p/(12*b**2*p**2 + 18*b**2*p + 6*b**2), True))
```

3.124 $\int x^2 \left(a^2+2abx^3+b^2x^6\right)^p \, dx$

Optimal. Leaf size=41

$$\frac{\left(a+bx^3\right)\left(a^2+2abx^3+b^2x^6\right)^p}{3b(2p+1)}$$

Rubi [A] time = 0.03, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1352, 609}

$$\frac{\left(a+bx^3\right)\left(a^2+2abx^3+b^2x^6\right)^p}{3b(2p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a^2 + 2*a*b*x^3 + b^2*x^6)^p,x]
```

```
[Out] ((a + b*x^3)*(a^2 + 2*a*b*x^3 + b^2*x^6)^p)/(3*b*(1 + 2*p))
```

Rule 609

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x)
*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] /; FreeQ[{a, b, c, p}, x] && EqQ[
b^2 - 4*a*c, 0] && NeQ[p, -2^(-1)]
```

Rule 1352

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a,
b, c, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

Rubi steps

$$\begin{aligned}\int x^2 \left(a^2+2abx^3+b^2x^6\right)^p \, dx &= \frac{1}{3}\text{Subst}\left(\int \left(a^2+2abx+b^2x^2\right)^p \, dx, x, x^3\right)\\ &= \frac{\left(a+bx^3\right)\left(a^2+2abx^3+b^2x^6\right)^p}{3b(1+2p)}\end{aligned}$$

Mathematica [A] time = 0.01, size = 32, normalized size = 0.78

$$\frac{\left(a+bx^3\right)\left(\left(a+bx^3\right)^2\right)^p}{3b(2p+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a^2 + 2*a*b*x^3 + b^2*x^6)^p,x]
```

```
[Out] ((a + b*x^3)*((a + b*x^3)^2)^p)/(3*b*(1 + 2*p))
```

IntegrateAlgebraic [F] time = 0.17, size = 0, normalized size = 0.00

$$\int x^2 \left(a^2+2abx^3+b^2x^6\right)^p \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^2*(a^2 + 2*a*b*x^3 + b^2*x^6)^p,x]
```

```
[Out] Defer[IntegrateAlgebraic][x^2*(a^2 + 2*a*b*x^3 + b^2*x^6)^p, x]
```

fricas [A] time = 1.39, size = 37, normalized size = 0.90

$$\frac{\left(bx^3+a\right)\left(b^2x^6+2\,abx^3+a^2\right)^p}{3\left(2\,bp+b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b^2*x^6+2*a*b*x^3+a^2)^p,x, algorithm="fricas")
```

```
[Out] 1/3*(b*x^3 + a)*(b^2*x^6 + 2*a*b*x^3 + a^2)^p/(2*b*p + b)
```

giac [A] time = 0.48, size = 58, normalized size = 1.41

$$\frac{\left(b^2x^6+2\,abx^3+a^2\right)^p bx^3+\left(b^2x^6+2\,abx^3+a^2\right)^p a}{3\left(2\,bp+b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b^2*x^6+2*a*b*x^3+a^2)^p,x, algorithm="giac")
```

```
[Out] 1/3*((b^2*x^6 + 2*a*b*x^3 + a^2)^p*b*x^3 + (b^2*x^6 + 2*a*b*x^3 + a^2)^p*a)
/(2*b*p + b)
```

maple [A] time = 0.01, size = 40, normalized size = 0.98

$$\frac{\left(b\,x^3+a\right)\left(b^2x^6+2ab\,x^3+a^2\right)^p}{3\left(2p+1\right)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b^2*x^6+2*a*b*x^3+a^2)^p,x)
```

```
[Out] 1/3*(b*x^3+a)*(b^2*x^6+2*a*b*x^3+a^2)^p/b/(1+2*p)
```

maxima [A] time = 0.95, size = 30, normalized size = 0.73

$$\frac{\left(bx^3+a\right)\left(bx^3+a\right)^{2\,p}}{3\,b\left(2\,p+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b^2*x^6+2*a*b*x^3+a^2)^p,x, algorithm="maxima")
```

```
[Out] 1/3*(b*x^3 + a)*(b*x^3 + a)^(2*p)/(b*(2*p + 1))
```

mupad [B] time = 1.16, size = 46, normalized size = 1.12

$$\left(\frac{x^3}{3\,\left(2\,p+1\right)}+\frac{a}{3\,b\,\left(2\,p+1\right)}\right)\,{\left(a^2+2\,a\,b\,x^3+b^2\,x^6\right)}^p$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a^2 + b^2*x^6 + 2*a*b*x^3)^p,x)
```

```
[Out] (x^3/(3*(2*p + 1)) + a/(3*b*(2*p + 1)))*(a^2 + b^2*x^6 + 2*a*b*x^3)^p
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\begin{cases} \frac{x^3}{3\sqrt{a^2}} & \text{for } b = 0 \wedge p = -\frac{1}{2} \\ \frac{x^3(a^2)^p}{3} & \text{for } b = 0 \\ \int \frac{x^2}{\sqrt{(a+bx^3)^2}}\,dx & \text{for } p = -\frac{1}{2} \\ \frac{a(a^2+2abx^3+b^2x^6)^p}{6bp+3b} + \frac{bx^3(a^2+2abx^3+b^2x^6)^p}{6bp+3b} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b**2*x**6+2*a*b*x**3+a**2)**p,x)
```

```
[Out] Piecewise((x**3/(3*sqrt(a**2)), Eq(b, 0) & Eq(p, -1/2)), (x**3*(a**2)**p/3,
 Eq(b, 0)), (Integral(x**2/sqrt((a + b*x**3)**2), x), Eq(p, -1/2)), (a*(a**
2 + 2*a*b*x**3 + b**2*x**6)**p/(6*b*p + 3*b) + b*x**3*(a**2 + 2*a*b*x**3 +
b**2*x**6)**p/(6*b*p + 3*b), True))
```

3.125 $\int \frac{x^8}{a+bx^3+cx^6} \, dx$

Optimal. Leaf size=81

$$-\frac{(b^2-2ac)\tanh^{-1}\left(\frac{b+2cx^3}{\sqrt{b^2-4ac}}\right)}{3c^2\sqrt{b^2-4ac}}-\frac{b\log\left(a+bx^3+cx^6\right)}{6c^2}+\frac{x^3}{3c}$$

Rubi [A] time = 0.08, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1357, 703, 634, 618, 206, 628}

$$-\frac{(b^2-2ac)\tanh^{-1}\left(\frac{b+2cx^3}{\sqrt{b^2-4ac}}\right)}{3c^2\sqrt{b^2-4ac}}-\frac{b\log\left(a+bx^3+cx^6\right)}{6c^2}+\frac{x^3}{3c}$$

Antiderivative was successfully verified.

```
[In] Int[x^8/(a + b*x^3 + c*x^6),x]
```

```
[Out] x^3/(3*c) - ((b^2 - 2*a*c)*ArcTanh[(b + 2*c*x^3)/Sqrt[b^2 - 4*a*c]])/(3*c^2
*Sqrt[b^2 - 4*a*c]) - (b*Log[a + b*x^3 + c*x^6])/(6*c^2)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 703

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Simp[(e*(d + e*x)^(m - 1))/(c*(m - 1)), x] + Dist[1/c, Int[((d + e*x)^
(m - 2)*Simp[c*d^2 - a*e^2 + e*(2*c*d - b*e)*x, x])/(a + b*x + c*x^2), x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && GtQ[m, 1]
```

Rule 1357

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
```

```
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^8}{a+bx^3+cx^6}\,dx &= \frac{1}{3}\operatorname{Subst}\left(\int \frac{x^2}{a+bx+cx^2}\,dx, x, x^3\right)\\
&= \frac{x^3}{3c}+\frac{\operatorname{Subst}\left(\int \frac{-a-bx}{a+bx+cx^2}\,dx, x, x^3\right)}{3c}\\
&= \frac{x^3}{3c}-\frac{b\operatorname{Subst}\left(\int \frac{b+2cx}{a+bx+cx^2}\,dx, x, x^3\right)}{6c^2}+\frac{\left(b^2-2ac\right)\operatorname{Subst}\left(\int \frac{1}{a+bx+cx^2}\,dx, x, x^3\right)}{6c^2}\\
&= \frac{x^3}{3c}-\frac{b\log\left(a+bx^3+cx^6\right)}{6c^2}-\frac{\left(b^2-2ac\right)\operatorname{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx, x, b+2cx^3\right)}{3c^2}\\
&= \frac{x^3}{3c}-\frac{\left(b^2-2ac\right)\tanh^{-1}\left(\frac{b+2cx^3}{\sqrt{b^2-4ac}}\right)}{3c^2\sqrt{b^2-4ac}}-\frac{b\log\left(a+bx^3+cx^6\right)}{6c^2}
\end{aligned}$$

Mathematica [A] time = 0.05, size = 78, normalized size = 0.96

$$\frac{\frac{2\left(b^2-2ac\right)\tan^{-1}\left(\frac{b+2cx^3}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}-b\log\left(a+bx^3+cx^6\right)+2cx^3}{6c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8/(a + b*x^3 + c*x^6),x]
```

```
[Out] (2*c*x^3 + (2*(b^2 - 2*a*c)*ArcTan[(b + 2*c*x^3)/Sqrt[-b^2 + 4*a*c]])/Sqrt[
-b^2 + 4*a*c] - b*Log[a + b*x^3 + c*x^6])/(6*c^2)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^8}{a+bx^3+cx^6}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^8/(a + b*x^3 + c*x^6),x]
```

```
[Out] IntegrateAlgebraic[x^8/(a + b*x^3 + c*x^6), x]
```

fricas [A] time = 1.29, size = 254, normalized size = 3.14

$$\left[\frac{2\left(b^2c-4ac^2\right)x^3-\left(b^2-2ac\right)\sqrt{b^2-4ac}\log\left(\frac{2c^2x^6+2bcx^3+b^2-2ac+\left(2cx^3+b\right)\sqrt{b^2-4ac}}{cx^6+bx^3+a}\right)-\left(b^3-4abc\right)\log\left(cx^6+bx^3+a\right)}{6\left(b^2c^2-4ac^3\right)}, \frac{2\left(b^2c-4ac^2\right)x^3-2\left(b^2-2ac\right)\sqrt{-b^2+4ac}\arctan\left(-\frac{\left(2cx^3+b\right)\sqrt{-b^2+4ac}}{b^2-4ac}\right)-\left(b^3-4abc\right)\log\left(cx^6+bx^3+a\right)}{6\left(b^2c^2-4ac^3\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(c*x^6+b*x^3+a),x, algorithm="fricas")
```

```
[Out] [1/6*(2*(b^2*c - 4*a*c^2)*x^3 - (b^2 - 2*a*c)*sqrt(b^2 - 4*a*c)*log((2*c^2*
x^6 + 2*b*c*x^3 + b^2 - 2*a*c + (2*c*x^3 + b)*sqrt(b^2 - 4*a*c))/(c*x^6 + b
*x^3 + a)) - (b^3 - 4*a*b*c)*log(c*x^6 + b*x^3 + a))/(b^2*c^2 - 4*a*c^3), 1
/6*(2*(b^2*c - 4*a*c^2)*x^3 - 2*(b^2 - 2*a*c)*sqrt(-b^2 + 4*a*c)*arctan(-(2
*c*x^3 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) - (b^3 - 4*a*b*c)*log(c*x^6 +
 b*x^3 + a))/(b^2*c^2 - 4*a*c^3)]
```

giac [A] time = 1.14, size = 75, normalized size = 0.93

$$\frac{x^3}{3\,c} - \frac{b\log\left(cx^6 + bx^3 + a\right)}{6\,c^2} + \frac{\left(b^2 - 2\,ac\right)\arctan\left(\frac{2\,cx^3+b}{\sqrt{-b^2+4\,ac}}\right)}{3\,\sqrt{-b^2 + 4\,ac}\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(c*x^6+b*x^3+a),x, algorithm="giac")
```

```
[Out] 1/3*x^3/c - 1/6*b*log(c*x^6 + b*x^3 + a)/c^2 + 1/3*(b^2 - 2*a*c)*arctan((2*
c*x^3 + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*c^2)
```

maple [A] time = 0.01, size = 111, normalized size = 1.37

$$\frac{x^3}{3c} - \frac{2a\arctan\left(\frac{2c\,x^3+b}{\sqrt{4ac-b^2}}\right)}{3\sqrt{4ac-b^2}\;c} + \frac{b^2\arctan\left(\frac{2c\,x^3+b}{\sqrt{4ac-b^2}}\right)}{3\sqrt{4ac-b^2}\;c^2} - \frac{b\ln\left(c\,x^6 + b\,x^3 + a\right)}{6c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(c*x^6+b*x^3+a),x)
```

```
[Out] 1/3*x^3/c-1/6*b*ln(c*x^6+b*x^3+a)/c^2-2/3/c/(4*a*c-b^2)^(1/2)*arctan((2*c*x
^3+b)/(4*a*c-b^2)^(1/2))*a+1/3/c^2/(4*a*c-b^2)^(1/2)*arctan((2*c*x^3+b)/(4*
a*c-b^2)^(1/2))*b^2
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(c*x^6+b*x^3+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

mupad [B] time = 1.98, size = 1758, normalized size = 21.70

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(a + b*x^3 + c*x^6),x)
```

```
[Out] x^3/(3*c) + (log(a + b*x^3 + c*x^6)*(3*b^3 - 12*a*b*c))/(2*(36*a*c^3 - 9*b^
2*c^2)) + (atan((4*c^3*x^3*(4*a*c - b^2)^(3/2)*((b*((b^5 + a^2*b*c^2 - 2*a*
b^3*c)/c^3 + ((3*b^3 - 12*a*b*c)*((6*a^2*c^4 + 12*b^4*c^2 - 18*a*b^2*c^3)/c
^3 + ((3*b^3 - 12*a*b*c)*((45*b^3*c^4 - 36*a*b*c^5)/c^3 + (27*b^2*c^3*(3*b^
3 - 12*a*b*c))/(36*a*c^3 - 9*b^2*c^2)))/(2*(36*a*c^3 - 9*b^2*c^2))))/(2*(36
*a*c^3 - 9*b^2*c^2)) - ((((2*a*c - b^2)*((45*b^3*c^4 - 36*a*b*c^5)/c^3 + (2
7*b^2*c^3*(3*b^3 - 12*a*b*c))/(36*a*c^3 - 9*b^2*c^2)))/(6*c^2*(4*a*c - b^2)
^(1/2)) + (9*b^2*c*(3*b^3 - 12*a*b*c)*(2*a*c - b^2))/(2*(4*a*c - b^2)^(1/2)
*(36*a*c^3 - 9*b^2*c^2)))*(2*a*c - b^2))/(6*c^2*(4*a*c - b^2)^(1/2)) - (3*b
^2*(3*b^3 - 12*a*b*c)*(2*a*c - b^2)^2)/(4*c*(4*a*c - b^2)*(36*a*c^3 - 9*b^2
*c^2))))/(4*a^2*c) + ((2*a*c - b^2)*(((3*b^3 - 12*a*b*c)*(((2*a*c - b^2)*((
45*b^3*c^4 - 36*a*b*c^5)/c^3 + (27*b^2*c^3*(3*b^3 - 12*a*b*c))/(36*a*c^3 -
9*b^2*c^2)))/(6*c^2*(4*a*c - b^2)^(1/2)) + (9*b^2*c*(3*b^3 - 12*a*b*c)*(2*a
*c - b^2))/(2*(4*a*c - b^2)^(1/2)*(36*a*c^3 - 9*b^2*c^2))))/(2*(36*a*c^3 -
9*b^2*c^2)) - (b^2*(2*a*c - b^2)^3)/(4*c^3*(4*a*c - b^2)^(3/2)) + (((6*a^2*
c^4 + 12*b^4*c^2 - 18*a*b^2*c^3)/c^3 + ((3*b^3 - 12*a*b*c)*((45*b^3*c^4 - 3
```

```
6*a*b*c^5)/c^3 + (27*b^2*c^3*(3*b^3 - 12*a*b*c))/(36*a*c^3 - 9*b^2*c^2)))/(
2*(36*a*c^3 - 9*b^2*c^2)))*(2*a*c - b^2))/(6*c^2*(4*a*c - b^2)^(1/2))))/(4*
a^2*c*(4*a*c - b^2)^(1/2))))/(b^6 - 8*a^3*c^3 + 12*a^2*b^2*c^2 - 6*a*b^4*c)
 - (c^2*(2*a*c - b^2)*(4*a*c - b^2)*(((3*b^3 - 12*a*b*c)*((((36*a^2*c^5 - 7
2*a*b^2*c^4)/c^3 - (54*a*b*c^3*(3*b^3 - 12*a*b*c))/(36*a*c^3 - 9*b^2*c^2))*
(2*a*c - b^2))/(6*c^2*(4*a*c - b^2)^(1/2)) - (9*a*b*c*(3*b^3 - 12*a*b*c)*(2
*a*c - b^2))/((4*a*c - b^2)^(1/2)*(36*a*c^3 - 9*b^2*c^2))))/(2*(36*a*c^3 -
9*b^2*c^2)) - (((15*a*b^3*c^2 - 12*a^2*b*c^3)/c^3 - ((3*b^3 - 12*a*b*c)*((3
6*a^2*c^5 - 72*a*b^2*c^4)/c^3 - (54*a*b*c^3*(3*b^3 - 12*a*b*c))/(36*a*c^3 -
 9*b^2*c^2)))/(2*(36*a*c^3 - 9*b^2*c^2)))*(2*a*c - b^2))/(6*c^2*(4*a*c - b^
2)^(1/2)) + (a*b*(2*a*c - b^2)^3)/(2*c^3*(4*a*c - b^2)^(3/2))))/(a^2*(b^6 -
 8*a^3*c^3 + 12*a^2*b^2*c^2 - 6*a*b^4*c)) + (b*c^2*(4*a*c - b^2)^(3/2)*((a*
b^4 - a^2*b^2*c)/c^3 + ((3*b^3 - 12*a*b*c)*((15*a*b^3*c^2 - 12*a^2*b*c^3)/c
^3 - ((3*b^3 - 12*a*b*c)*((36*a^2*c^5 - 72*a*b^2*c^4)/c^3 - (54*a*b*c^3*(3*
b^3 - 12*a*b*c))/(36*a*c^3 - 9*b^2*c^2)))/(2*(36*a*c^3 - 9*b^2*c^2))))/(2*(
36*a*c^3 - 9*b^2*c^2)) + (((((36*a^2*c^5 - 72*a*b^2*c^4)/c^3 - (54*a*b*c^3*
(3*b^3 - 12*a*b*c))/(36*a*c^3 - 9*b^2*c^2))*(2*a*c - b^2))/(6*c^2*(4*a*c -
b^2)^(1/2)) - (9*a*b*c*(3*b^3 - 12*a*b*c)*(2*a*c - b^2))/((4*a*c - b^2)^(1/
2)*(36*a*c^3 - 9*b^2*c^2)))*(2*a*c - b^2))/(6*c^2*(4*a*c - b^2)^(1/2)) - (3
*a*b*(3*b^3 - 12*a*b*c)*(2*a*c - b^2)^2)/(2*c*(4*a*c - b^2)*(36*a*c^3 - 9*b
^2*c^2))))/(a^2*(b^6 - 8*a^3*c^3 + 12*a^2*b^2*c^2 - 6*a*b^4*c)))*(2*a*c - b
^2))/(3*c^2*(4*a*c - b^2)^(1/2))
```

sympy [B] time = 2.70, size = 316, normalized size = 3.90

$$\left(-\frac{b}{6c^2} - \frac{\sqrt{-4ac+b^2}\,\left(2ac-b^2\right)}{6c^2\left(4ac-b^2\right)}\right)\log\left(x^3 + \frac{-ab-12ac^2\left(-\frac{b}{6c^2}-\frac{\sqrt{-4ac+b^2}(2ac-b^2)}{6c^2(4ac-b^2)}\right)+3b^2c\left(-\frac{b}{6c^2}-\frac{\sqrt{-4ac+b^2}(2ac-b^2)}{6c^2(4ac-b^2)}\right)}{2ac-b^2}\right) + \left(-\frac{b}{6c^2} + \frac{\sqrt{-4ac+b^2}\,\left(2ac-b^2\right)}{6c^2\left(4ac-b^2\right)}\right)\log\left(x^3 + \frac{-ab-12ac^2\left(-\frac{b}{6c^2}+\frac{\sqrt{-4ac+b^2}(2ac-b^2)}{6c^2(4ac-b^2)}\right)+3b^2c\left(-\frac{b}{6c^2}+\frac{\sqrt{-4ac+b^2}(2ac-b^2)}{6c^2(4ac-b^2)}\right)}{2ac-b^2}\right) + \frac{x^3}{3c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8/(c*x**6+b*x**3+a),x)
```

```
[Out] (-b/(6*c**2) - sqrt(-4*a*c + b**2)*(2*a*c - b**2)/(6*c**2*(4*a*c - b**2)))*
log(x**3 + (-a*b - 12*a*c**2*(-b/(6*c**2) - sqrt(-4*a*c + b**2)*(2*a*c - b*
*2)/(6*c**2*(4*a*c - b**2))) + 3*b**2*c*(-b/(6*c**2) - sqrt(-4*a*c + b**2)*
(2*a*c - b**2)/(6*c**2*(4*a*c - b**2))))/(2*a*c - b**2)) + (-b/(6*c**2) + s
qrt(-4*a*c + b**2)*(2*a*c - b**2)/(6*c**2*(4*a*c - b**2)))*log(x**3 + (-a*b
 - 12*a*c**2*(-b/(6*c**2) + sqrt(-4*a*c + b**2)*(2*a*c - b**2)/(6*c**2*(4*a
*c - b**2))) + 3*b**2*c*(-b/(6*c**2) + sqrt(-4*a*c + b**2)*(2*a*c - b**2)/(
6*c**2*(4*a*c - b**2))))/(2*a*c - b**2)) + x**3/(3*c)
```

3.126 $\int \frac{x^5}{a+bx^3+cx^6}\,dx$

Optimal. Leaf size=63

$$\frac{b\tanh^{-1}\left(\frac{b+2cx^3}{\sqrt{b^2-4ac}}\right)}{3c\sqrt{b^2-4ac}}+\frac{\log\left(a+bx^3+cx^6\right)}{6c}$$

Rubi [A] time = 0.06, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {1357, 634, 618, 206, 628}

$$\frac{b\tanh^{-1}\left(\frac{b+2cx^3}{\sqrt{b^2-4ac}}\right)}{3c\sqrt{b^2-4ac}}+\frac{\log\left(a+bx^3+cx^6\right)}{6c}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/(a + b*x^3 + c*x^6),x]
```

```
[Out] (b*ArcTanh[(b + 2*c*x^3)/Sqrt[b^2 - 4*a*c]])/(3*c*Sqrt[b^2 - 4*a*c]) + Log[
a + b*x^3 + c*x^6]/(6*c)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1357

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^5}{a+bx^3+cx^6}\,dx &= \frac{1}{3}\operatorname{Subst}\left(\int \frac{x}{a+bx+cx^2}\,dx,x,x^3\right)\\
&= \frac{\operatorname{Subst}\left(\int \frac{b+2cx}{a+bx+cx^2}\,dx,x,x^3\right)}{6c} - \frac{b\operatorname{Subst}\left(\int \frac{1}{a+bx+cx^2}\,dx,x,x^3\right)}{6c}\\
&= \frac{\log\left(a+bx^3+cx^6\right)}{6c} + \frac{b\operatorname{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx,x,b+2cx^3\right)}{3c}\\
&= \frac{b\tanh^{-1}\left(\frac{b+2cx^3}{\sqrt{b^2-4ac}}\right)}{3c\sqrt{b^2-4ac}} + \frac{\log\left(a+bx^3+cx^6\right)}{6c}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 62, normalized size = 0.98

$$\frac{\log\left(a+bx^3+cx^6\right) - \frac{2b\tan^{-1}\left(\frac{b+2cx^3}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}}{6c}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/(a + b*x^3 + c*x^6),x]
```

```
[Out] ((-2*b*ArcTan[(b + 2*c*x^3)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c] + Log[a
 + b*x^3 + c*x^6])/(6*c)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5}{a+bx^3+cx^6}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^5/(a + b*x^3 + c*x^6),x]
```

```
[Out] IntegrateAlgebraic[x^5/(a + b*x^3 + c*x^6), x]
```

fricas [A] time = 0.83, size = 197, normalized size = 3.13

$$\left[\frac{\sqrt{b^2-4ac}\,b\log\left(\frac{2c^2x^6+2bcx^3+b^2-2ac+\left(2cx^3+b\right)\sqrt{b^2-4ac}}{cx^6+bx^3+a}\right)+\left(b^2-4ac\right)\log\left(cx^6+bx^3+a\right)}{6\left(b^2c-4ac^2\right)}, \frac{2\sqrt{-b^2+4ac}\,b\arctan\left(-\frac{\left(2cx^3+b\right)\sqrt{-b^2+4ac}}{b^2-4ac}\right)+\left(b^2-4ac\right)\log\left(cx^6+bx^3+a\right)}{6\left(b^2c-4ac^2\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(c*x^6+b*x^3+a),x, algorithm="fricas")
```

```
[Out] [1/6*(sqrt(b^2 - 4*a*c)*b*log((2*c^2*x^6 + 2*b*c*x^3 + b^2 - 2*a*c + (2*c*x
^3 + b)*sqrt(b^2 - 4*a*c))/(c*x^6 + b*x^3 + a)) + (b^2 - 4*a*c)*log(c*x^6 +
 b*x^3 + a))/(b^2*c - 4*a*c^2), 1/6*(2*sqrt(-b^2 + 4*a*c)*b*arctan(-(2*c*x^
3 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) + (b^2 - 4*a*c)*log(c*x^6 + b*x^3
+ a))/(b^2*c - 4*a*c^2)]
```

giac [A] time = 1.19, size = 59, normalized size = 0.94

$$-\frac{b\arctan\left(\frac{2cx^3+b}{\sqrt{-b^2+4ac}}\right)}{3\sqrt{-b^2+4ac}\,c} + \frac{\log\left(cx^6+bx^3+a\right)}{6c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(c*x^6+b*x^3+a),x, algorithm="giac")
```

```
[Out] -1/3*b*arctan((2*c*x^3 + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*c) + 1/
6*log(c*x^6 + b*x^3 + a)/c
```

maple [A] time = 0.00, size = 60, normalized size = 0.95

$$-\frac{b \arctan\left(\frac{2c x^3+b}{\sqrt{4ac-b^2}}\right)}{3\sqrt{4ac-b^2}\, c}+\frac{\ln\left(c x^6+b x^3+a\right)}{6c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(c*x^6+b*x^3+a),x)
```

```
[Out] 1/6*ln(c*x^6+b*x^3+a)/c-1/3*b/c/(4*a*c-b^2)^(1/2)*arctan((2*c*x^3+b)/(4*a*c
-b^2)^(1/2))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(c*x^6+b*x^3+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

mupad [B] time = 1.80, size = 1199, normalized size = 19.03

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(a + b*x^3 + c*x^6),x)
```

```
[Out] (log(a + b*x^3 + c*x^6)*(12*a*c - 3*b^2))/(2*(36*a*c^2 - 9*b^2*c)) + (b*ata
n((4*x^3*((b*(b^2 - ((12*b^2*c - ((45*b^2*c^2 - (27*b^2*c^3*(12*a*c - 3*b^2
))/(36*a*c^2 - 9*b^2*c))*(12*a*c - 3*b^2))/(2*(36*a*c^2 - 9*b^2*c)))*(12*a*
c - 3*b^2))/(2*(36*a*c^2 - 9*b^2*c)) - (b*((b*(45*b^2*c^2 - (27*b^2*c^3*(12
*a*c - 3*b^2))/(36*a*c^2 - 9*b^2*c)))/(6*c*(4*a*c - b^2)^(1/2)) - (9*b^3*c^
2*(12*a*c - 3*b^2))/(2*(36*a*c^2 - 9*b^2*c)*(4*a*c - b^2)^(1/2))))/(6*c*(4*
a*c - b^2)^(1/2)) + (3*b^4*c*(12*a*c - 3*b^2))/(4*(36*a*c^2 - 9*b^2*c)*(4*a
*c - b^2))))/(4*a^2*c) + ((2*a*c - b^2)*(b^5/(4*(4*a*c - b^2)^(3/2)) + ((12
*a*c - 3*b^2)*((b*(45*b^2*c^2 - (27*b^2*c^3*(12*a*c - 3*b^2))/(36*a*c^2 - 9
*b^2*c)))/(6*c*(4*a*c - b^2)^(1/2)) - (9*b^3*c^2*(12*a*c - 3*b^2))/(2*(36*a
*c^2 - 9*b^2*c)*(4*a*c - b^2)^(1/2))))/(2*(36*a*c^2 - 9*b^2*c)) - (b*(12*b^
2*c - ((45*b^2*c^2 - (27*b^2*c^3*(12*a*c - 3*b^2))/(36*a*c^2 - 9*b^2*c))*(1
2*a*c - 3*b^2))/(2*(36*a*c^2 - 9*b^2*c))))/(6*c*(4*a*c - b^2)^(1/2))))/(4*a
^2*c*(4*a*c - b^2)^(1/2)))*(4*a*c - b^2)^(3/2))/b^3 + ((4*a*c - b^2)^(3/2)*
(a*b + ((((72*a*b*c^2 - (54*a*b*c^3*(12*a*c - 3*b^2))/(36*a*c^2 - 9*b^2*c))
*(12*a*c - 3*b^2))/(2*(36*a*c^2 - 9*b^2*c)) - 15*a*b*c)*(12*a*c - 3*b^2))/(
2*(36*a*c^2 - 9*b^2*c)) - (b*((b*(72*a*b*c^2 - (54*a*b*c^3*(12*a*c - 3*b^2)
)/(36*a*c^2 - 9*b^2*c)))/(6*c*(4*a*c - b^2)^(1/2)) - (9*a*b^2*c^2*(12*a*c -
 3*b^2))/((36*a*c^2 - 9*b^2*c)*(4*a*c - b^2)^(1/2))))/(6*c*(4*a*c - b^2)^(1
/2)) + (3*a*b^3*c*(12*a*c - 3*b^2))/(2*(36*a*c^2 - 9*b^2*c)*(4*a*c - b^2)))
)/(a^2*b^2*c) + ((2*a*c - b^2)*(4*a*c - b^2)*((a*b^4)/(2*(4*a*c - b^2)^(3/2
)) + (((b*(72*a*b*c^2 - (54*a*b*c^3*(12*a*c - 3*b^2))/(36*a*c^2 - 9*b^2*c))
)/(6*c*(4*a*c - b^2)^(1/2)) - (9*a*b^2*c^2*(12*a*c - 3*b^2))/((36*a*c^2 - 9
*b^2*c)*(4*a*c - b^2)^(1/2)))*(12*a*c - 3*b^2))/(2*(36*a*c^2 - 9*b^2*c)) +
(b*(((72*a*b*c^2 - (54*a*b*c^3*(12*a*c - 3*b^2))/(36*a*c^2 - 9*b^2*c))*(12*
```

```
a*c - 3*b^2))/(2*(36*a*c^2 - 9*b^2*c)) - 15*a*b*c))/(6*c*(4*a*c - b^2)^(1/2
))))/(a^2*b^3*c)))/(3*c*(4*a*c - b^2)^(1/2))
```

sympy [B] time = 1.37, size = 223, normalized size = 3.54

$$\left(-\frac{b\sqrt{-4ac+b^2}}{6c\left(4ac-b^2\right)}+\frac{1}{6c}\right)\log\left(x^3+\frac{-12ac\left(-\frac{b\sqrt{-4ac+b^2}}{6c(4ac-b^2)}+\frac{1}{6c}\right)+2a+3b^2\left(-\frac{b\sqrt{-4ac+b^2}}{6c(4ac-b^2)}+\frac{1}{6c}\right)}{b}\right)+\left(\frac{b\sqrt{-4ac+b^2}}{6c\left(4ac-b^2\right)}+\frac{1}{6c}\right)\log\left(x^3+\frac{-12ac\left(\frac{b\sqrt{-4ac+b^2}}{6c(4ac-b^2)}+\frac{1}{6c}\right)+2a+3b^2\left(\frac{b\sqrt{-4ac+b^2}}{6c(4ac-b^2)}+\frac{1}{6c}\right)}{b}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(c*x**6+b*x**3+a),x)
```

```
[Out] (-b*sqrt(-4*a*c + b**2)/(6*c*(4*a*c - b**2)) + 1/(6*c))*log(x**3 + (-12*a*c
*(-b*sqrt(-4*a*c + b**2)/(6*c*(4*a*c - b**2)) + 1/(6*c)) + 2*a + 3*b**2*(-b
*sqrt(-4*a*c + b**2)/(6*c*(4*a*c - b**2)) + 1/(6*c)))/b) + (b*sqrt(-4*a*c +
 b**2)/(6*c*(4*a*c - b**2)) + 1/(6*c))*log(x**3 + (-12*a*c*(b*sqrt(-4*a*c +
 b**2)/(6*c*(4*a*c - b**2)) + 1/(6*c)) + 2*a + 3*b**2*(b*sqrt(-4*a*c + b**2
)/(6*c*(4*a*c - b**2)) + 1/(6*c)))/b)
```

3.127 $\int \frac{x^2}{a+bx^3+cx^6}\,dx$

Optimal. Leaf size=38

$$-\frac{2\tanh^{-1}\left(\frac{b+2cx^3}{\sqrt{b^2-4ac}}\right)}{3\sqrt{b^2-4ac}}$$

Rubi [A] time = 0.04, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1352, 618, 206}

$$-\frac{2\tanh^{-1}\left(\frac{b+2cx^3}{\sqrt{b^2-4ac}}\right)}{3\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a + b*x^3 + c*x^6),x]
```

```
[Out] (-2*ArcTanh[(b + 2*c*x^3)/Sqrt[b^2 - 4*a*c]])/(3*Sqrt[b^2 - 4*a*c])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1352

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a,
b, c, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2}{a+bx^3+cx^6}\,dx &= \frac{1}{3}\operatorname{Subst}\left(\int \frac{1}{a+bx+cx^2}\,dx, x, x^3\right)\\
&= -\left(\frac{2}{3}\operatorname{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx, x, b+2cx^3\right)\right)\\
&= -\frac{2\tanh^{-1}\left(\frac{b+2cx^3}{\sqrt{b^2-4ac}}\right)}{3\sqrt{b^2-4ac}}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 42, normalized size = 1.11

$$\frac{2\tan^{-1}\left(\frac{b+2cx^3}{\sqrt{4ac-b^2}}\right)}{3\sqrt{4ac-b^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a + b*x^3 + c*x^6),x]
```

```
[Out] (2*ArcTan[(b + 2*c*x^3)/Sqrt[-b^2 + 4*a*c]])/(3*Sqrt[-b^2 + 4*a*c])
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{a + bx^3 + cx^6}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^2/(a + b*x^3 + c*x^6),x]
```

```
[Out] IntegrateAlgebraic[x^2/(a + b*x^3 + c*x^6), x]
```

fricas [A] time = 1.31, size = 129, normalized size = 3.39

$$\left[\frac{\log\left(\frac{2\,c^2x^6+2\,bcx^3+b^2-2\,ac-\left(2\,cx^3+b\right)\sqrt{b^2-4\,ac}}{cx^6+bx^3+a}\right)}{3\,\sqrt{b^2-4\,ac}}, -\frac{2\,\sqrt{-b^2+4\,ac}\,\arctan\left(-\frac{\left(2\,cx^3+b\right)\sqrt{-b^2+4\,ac}}{b^2-4\,ac}\right)}{3\,\left(b^2-4\,ac\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^6+b*x^3+a),x, algorithm="fricas")
```

```
[Out] [1/3*log((2*c^2*x^6 + 2*b*c*x^3 + b^2 - 2*a*c - (2*c*x^3 + b)*sqrt(b^2 - 4*
a*c))/(c*x^6 + b*x^3 + a))/sqrt(b^2 - 4*a*c), -2/3*sqrt(-b^2 + 4*a*c)*arcta
n(-(2*c*x^3 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c))/(b^2 - 4*a*c)]
```

giac [A] time = 0.98, size = 36, normalized size = 0.95

$$\frac{2\,\arctan\left(\frac{2\,cx^3+b}{\sqrt{-b^2+4\,ac}}\right)}{3\,\sqrt{-b^2+4\,ac}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^6+b*x^3+a),x, algorithm="giac")
```

```
[Out] 2/3*arctan((2*c*x^3 + b)/sqrt(-b^2 + 4*a*c))/sqrt(-b^2 + 4*a*c)
```

maple [A] time = 0.00, size = 37, normalized size = 0.97

$$\frac{2\arctan\left(\frac{2c\,x^3+b}{\sqrt{4ac-b^2}}\right)}{3\sqrt{4ac-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(c*x^6+b*x^3+a),x)
```

```
[Out] 2/3/(4*a*c-b^2)^(1/2)*arctan((2*c*x^3+b)/(4*a*c-b^2)^(1/2))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^6+b*x^3+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
```

```
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

mupad [B] time = 1.23, size = 174, normalized size = 4.58

$$-\frac{2\,\mathrm{atan}\left(\frac{\frac{x^3\,{\left(4\,a\,c-b^2\right)}^4}{2}+a\,b\,{\left(4\,a\,c-b^2\right)}^3+a\,b^3\,{\left(4\,a\,c-b^2\right)}^2+b^2\,x^3\,{\left(4\,a\,c-b^2\right)}^3+\frac{b^4\,x^3\,{\left(4\,a\,c-b^2\right)}^2}{2}}{b^2\,\left(32\,a^3\,c^2\,\sqrt{4\,a\,c-b^2}-4\,a^2\,b^2\,c\,\sqrt{4\,a\,c-b^2}\right)-64\,a^4\,c^3\,\sqrt{4\,a\,c-b^2}}\right)}{3\,\sqrt{4\,a\,c-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a + b*x^3 + c*x^6),x)
```

```
[Out] -(2*atan(((x^3*(4*a*c - b^2)^4)/2 + a*b*(4*a*c - b^2)^3 + a*b^3*(4*a*c - b^
2)^2 + b^2*x^3*(4*a*c - b^2)^3 + (b^4*x^3*(4*a*c - b^2)^2)/2)/(b^2*(32*a^3*
c^2*(4*a*c - b^2)^(1/2) - 4*a^2*b^2*c*(4*a*c - b^2)^(1/2)) - 64*a^4*c^3*(4*
a*c - b^2)^(1/2))))/(3*(4*a*c - b^2)^(1/2))
```

sympy [B] time = 0.64, size = 131, normalized size = 3.45

$$-\frac{\sqrt{-\frac{1}{4ac-b^2}}\log\left(x^3+\frac{-4ac\sqrt{-\frac{1}{4ac-b^2}}+b^2\sqrt{-\frac{1}{4ac-b^2}}+b}{2c}\right)}{3}+\frac{\sqrt{-\frac{1}{4ac-b^2}}\log\left(x^3+\frac{4ac\sqrt{-\frac{1}{4ac-b^2}}-b^2\sqrt{-\frac{1}{4ac-b^2}}+b}{2c}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(c*x**6+b*x**3+a),x)
```

```
[Out] -sqrt(-1/(4*a*c - b**2))*log(x**3 + (-4*a*c*sqrt(-1/(4*a*c - b**2)) + b**2*
sqrt(-1/(4*a*c - b**2)) + b)/(2*c))/3 + sqrt(-1/(4*a*c - b**2))*log(x**3 +
(4*a*c*sqrt(-1/(4*a*c - b**2)) - b**2*sqrt(-1/(4*a*c - b**2)) + b)/(2*c))/3
```

3.128 $\int \frac{1}{x\left(a+bx^3+cx^6\right)}\,dx$

Optimal. Leaf size=69

$$\frac{b\tanh^{-1}\left(\frac{b+2cx^3}{\sqrt{b^2-4ac}}\right)}{3a\sqrt{b^2-4ac}}-\frac{\log\left(a+bx^3+cx^6\right)}{6a}+\frac{\log(x)}{a}$$

Rubi [A] time = 0.07, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389, Rules used = {1357, 705, 29, 634, 618, 206, 628}

$$\frac{b\tanh^{-1}\left(\frac{b+2cx^3}{\sqrt{b^2-4ac}}\right)}{3a\sqrt{b^2-4ac}}-\frac{\log\left(a+bx^3+cx^6\right)}{6a}+\frac{\log(x)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a + b*x^3 + c*x^6)),x]
```

```
[Out] (b*ArcTanh[(b + 2*c*x^3)/Sqrt[b^2 - 4*a*c]])/(3*a*Sqrt[b^2 - 4*a*c]) + Log[
x]/a - Log[a + b*x^3 + c*x^6]/(6*a)
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 705

```
Int[1/(((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)), x_Symbol]
 :> Dist[e^2/(c*d^2 - b*d*e + a*e^2), Int[1/(d + e*x), x], x] + Dist[1/(c*d
^2 - b*d*e + a*e^2), Int[(c*d - b*e - c*e*x)/(a + b*x + c*x^2), x], x] /; F
reeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^
2, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1357

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x\left(a+bx^3+cx^6\right)}\,dx &= \frac{1}{3}\,\mathrm{Subst}\left(\int \frac{1}{x\left(a+bx+cx^2\right)}\,dx,x,x^3\right)\\
&= \frac{\mathrm{Subst}\left(\int \frac{1}{x}\,dx,x,x^3\right)}{3a}+\frac{\mathrm{Subst}\left(\int \frac{-b-cx}{a+bx+cx^2}\,dx,x,x^3\right)}{3a}\\
&= \frac{\log(x)}{a}-\frac{\mathrm{Subst}\left(\int \frac{b+2cx}{a+bx+cx^2}\,dx,x,x^3\right)}{6a}-\frac{b\,\mathrm{Subst}\left(\int \frac{1}{a+bx+cx^2}\,dx,x,x^3\right)}{6a}\\
&= \frac{\log(x)}{a}-\frac{\log\left(a+bx^3+cx^6\right)}{6a}+\frac{b\,\mathrm{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx,x,b+2cx^3\right)}{3a}\\
&= \frac{b\tanh^{-1}\left(\frac{b+2cx^3}{\sqrt{b^2-4ac}}\right)}{3a\sqrt{b^2-4ac}}+\frac{\log(x)}{a}-\frac{\log\left(a+bx^3+cx^6\right)}{6a}
\end{aligned}
$$

Mathematica [C] time = 0.02, size = 66, normalized size = 0.96

$$\frac{\log(x)}{a}-\frac{\mathrm{RootSum}\left[\#1^6c+\#1^3b+a\&,\frac{\#1^3c\log(x-\#1)+b\log(x-\#1)}{2\#1^3c+b}\&\right]}{3a}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a + b*x^3 + c*x^6)),x]
```

```
[Out] Log[x]/a - RootSum[a + b*#1^3 + c*#1^6 & , (b*Log[x - #1] + c*Log[x - #1]*#
1^3)/(b + 2*c*#1^3) & ]/(3*a)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(a+bx^3+cx^6\right)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x*(a + b*x^3 + c*x^6)),x]
```

```
[Out] IntegrateAlgebraic[1/(x*(a + b*x^3 + c*x^6)), x]
```

fricas [A] time = 1.41, size = 223, normalized size = 3.23

$$\left[\frac{\sqrt{b^2-4ac}\,b\log\left(\frac{2c^2x^6+2bcx^3+b^2-2ac+\left(2cx^3+b\right)\sqrt{b^2-4ac}}{cx^6+bx^3+a}\right)-\left(b^2-4ac\right)\log\left(cx^6+bx^3+a\right)+6\left(b^2-4ac\right)\log(x)}{6\left(ab^2-4a^2c\right)},\frac{2\sqrt{-b^2+4ac}\,b\arctan\left(-\frac{\left(2cx^3+b\right)\sqrt{-b^2+4ac}}{b^2-4ac}\right)-\left(b^2-4ac\right)\log\left(cx^6+bx^3+a\right)+6\left(b^2-4ac\right)\log(x)}{6\left(ab^2-4a^2c\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^6+b*x^3+a),x, algorithm="fricas")
```

```
[Out] [1/6*(sqrt(b^2 - 4*a*c)*b*log((2*c^2*x^6 + 2*b*c*x^3 + b^2 - 2*a*c + (2*c*x
^3 + b)*sqrt(b^2 - 4*a*c))/(c*x^6 + b*x^3 + a)) - (b^2 - 4*a*c)*log(c*x^6 +
```

```
b*x^3 + a) + 6*(b^2 - 4*a*c)*log(x))/(a*b^2 - 4*a^2*c), 1/6*(2*sqrt(-b^2 +
4*a*c)*b*arctan(-(2*c*x^3 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) - (b^2 -
4*a*c)*log(c*x^6 + b*x^3 + a) + 6*(b^2 - 4*a*c)*log(x))/(a*b^2 - 4*a^2*c)]
```

giac [A] time = 1.00, size = 66, normalized size = 0.96

$$-\frac{b \arctan\left(\frac{2 \, c x^{3} + b}{\sqrt{-b^{2} + 4 \, a c}}\right)}{3 \, \sqrt{-b^{2} + 4 \, a c} a} - \frac{\log\left(c x^{6} + b x^{3} + a\right)}{6 \, a} + \frac{\log\left({\left| x \right|}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^6+b*x^3+a),x, algorithm="giac")
```

```
[Out] -1/3*b*arctan((2*c*x^3 + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*a) - 1/
6*log(c*x^6 + b*x^3 + a)/a + log(abs(x))/a
```

maple [A] time = 0.01, size = 66, normalized size = 0.96

$$-\frac{b \arctan\left(\frac{2c\,x^{3}+b}{\sqrt{4ac-b^{2}}}\right)}{3\sqrt{4ac-b^{2}}\,a}+\frac{\ln(x)}{a}-\frac{\ln\left(c\,x^{6}+b\,x^{3}+a\right)}{6a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(c*x^6+b*x^3+a),x)
```

```
[Out] ln(x)/a-1/6*ln(c*x^6+b*x^3+a)/a-1/3/a*b/(4*a*c-b^2)^(1/2)*arctan((2*c*x^3+b
)/(4*a*c-b^2)^(1/2))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^6+b*x^3+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

mupad [B] time = 1.92, size = 1362, normalized size = 19.74

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b*x^3 + c*x^6)),x)
```

```
[Out] log(x)/a + (log(a + b*x^3 + c*x^6)*(12*a*c - 3*b^2))/(2*(9*a*b^2 - 36*a^2*c
)) - (b*atan((3*(4*a*c - b^2)^2*(4*b^4 + 7*a^2*c^2 - 15*a*b^2*c)*((b^3*(27*
b^3*c^3 - (27*a*b^3*c^3*(12*a*c - 3*b^2))/(2*(9*a*b^2 - 36*a^2*c))))/(216*a
^3*(4*a*c - b^2)^(3/2)) + (9*b^4*c^3*(12*a*c - 3*b^2)^3)/(16*(9*a*b^2 - 36*
a^2*c)^3*(4*a*c - b^2)^(1/2)) - (3*b^6*c^3*(12*a*c - 3*b^2))/(16*a^2*(9*a*b
^2 - 36*a^2*c)*(4*a*c - b^2)^(3/2)) - (b*(12*a*c - 3*b^2)^2*(27*b^3*c^3 - (
27*a*b^3*c^3*(12*a*c - 3*b^2))/(2*(9*a*b^2 - 36*a^2*c))))/(8*a*(9*a*b^2 - 3
6*a^2*c)^2*(4*a*c - b^2)^(1/2))))/(b^3*c^6*(49*a*c - 12*b^2)) - (3*(4*a*c -
 b^2)^(3/2)*(4*b^5 + 29*a^2*b*c^2 - 23*a*b^3*c)*(((12*a*c - 3*b^2)^3*(27*b^
3*c^3 - (27*a*b^3*c^3*(12*a*c - 3*b^2))/(2*(9*a*b^2 - 36*a^2*c))))/(8*(9*a*
b^2 - 36*a^2*c)^3) - (b^7*c^3)/(48*a^3*(4*a*c - b^2)^2) - (b^2*(12*a*c - 3*
b^2)*(27*b^3*c^3 - (27*a*b^3*c^3*(12*a*c - 3*b^2))/(2*(9*a*b^2 - 36*a^2*c))
))/(24*a^2*(9*a*b^2 - 36*a^2*c)*(4*a*c - b^2)) + (9*b^5*c^3*(12*a*c - 3*b^2
```

```
)^2)/(16*a*(9*a*b^2 - 36*a^2*c)^2*(4*a*c - b^2))))/(b^3*c^6*(49*a*c - 12*b^
2)) + (48*a^4*x^3*(((4*b^4 + 7*a^2*c^2 - 15*a*b^2*c)*((b^3*(63*b^2*c^4 - ((
108*b^4*c^3 - 378*a*b^2*c^4)*(12*a*c - 3*b^2))/(2*(9*a*b^2 - 36*a^2*c))))/(
216*a^3*(4*a*c - b^2)^(3/2)) + (b*(108*b^4*c^3 - 378*a*b^2*c^4)*(12*a*c - 3
*b^2)^3)/(48*a*(9*a*b^2 - 36*a^2*c)^3*(4*a*c - b^2)^(1/2)) - (b^3*(108*b^4*
c^3 - 378*a*b^2*c^4)*(12*a*c - 3*b^2))/(144*a^3*(9*a*b^2 - 36*a^2*c)*(4*a*c
 - b^2)^(3/2)) - (b*(63*b^2*c^4 - ((108*b^4*c^3 - 378*a*b^2*c^4)*(12*a*c -
3*b^2))/(2*(9*a*b^2 - 36*a^2*c)))*(12*a*c - 3*b^2)^2)/(8*a*(9*a*b^2 - 36*a^
2*c)^2*(4*a*c - b^2)^(1/2))))/(16*a^4*c^3*(49*a*c - 12*b^2)) - ((4*b^5 + 29
*a^2*b*c^2 - 23*a*b^3*c)*(((63*b^2*c^4 - ((108*b^4*c^3 - 378*a*b^2*c^4)*(12
*a*c - 3*b^2))/(2*(9*a*b^2 - 36*a^2*c)))*(12*a*c - 3*b^2)^3)/(8*(9*a*b^2 -
36*a^2*c)^3) - (b^4*(108*b^4*c^3 - 378*a*b^2*c^4))/(1296*a^4*(4*a*c - b^2)^
2) + (b^2*(108*b^4*c^3 - 378*a*b^2*c^4)*(12*a*c - 3*b^2)^2)/(48*a^2*(9*a*b^
2 - 36*a^2*c)^2*(4*a*c - b^2)) - (b^2*(63*b^2*c^4 - ((108*b^4*c^3 - 378*a*b
^2*c^4)*(12*a*c - 3*b^2))/(2*(9*a*b^2 - 36*a^2*c)))*(12*a*c - 3*b^2))/(24*a
^2*(9*a*b^2 - 36*a^2*c)*(4*a*c - b^2))))/(16*a^4*c^3*(4*a*c - b^2)^(1/2)*(4
9*a*c - 12*b^2)))*(4*a*c - b^2)^2)/(b^3*c^3)))/(3*a*(4*a*c - b^2)^(1/2))
```

sympy [B] time = 6.73, size = 253, normalized size = 3.67

$$\left(-\frac{b\sqrt{-4ac+b^2}}{6a\left(4ac-b^2\right)}-\frac{1}{6a}\right)\log\left(x^3+\frac{-12a^2c\left(-\frac{b\sqrt{-4ac+b^2}}{6a(4ac-b^2)}-\frac{1}{6a}\right)+3ab^2\left(-\frac{b\sqrt{-4ac+b^2}}{6a(4ac-b^2)}-\frac{1}{6a}\right)-2ac+b^2}{bc}\right)+\left(\frac{b\sqrt{-4ac+b^2}}{6a\left(4ac-b^2\right)}-\frac{1}{6a}\right)\log\left(x^3+\frac{-12a^2c\left(\frac{b\sqrt{-4ac+b^2}}{6a(4ac-b^2)}-\frac{1}{6a}\right)+3ab^2\left(\frac{b\sqrt{-4ac+b^2}}{6a(4ac-b^2)}-\frac{1}{6a}\right)-2ac+b^2}{bc}\right)+\frac{\log(x)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x**6+b*x**3+a),x)
```

```
[Out] (-b*sqrt(-4*a*c + b**2)/(6*a*(4*a*c - b**2)) - 1/(6*a))*log(x**3 + (-12*a**
2*c*(-b*sqrt(-4*a*c + b**2)/(6*a*(4*a*c - b**2)) - 1/(6*a)) + 3*a*b**2*(-b*
sqrt(-4*a*c + b**2)/(6*a*(4*a*c - b**2)) - 1/(6*a)) - 2*a*c + b**2)/(b*c))
+ (b*sqrt(-4*a*c + b**2)/(6*a*(4*a*c - b**2)) - 1/(6*a))*log(x**3 + (-12*a*
*2*c*(b*sqrt(-4*a*c + b**2)/(6*a*(4*a*c - b**2)) - 1/(6*a)) + 3*a*b**2*(b*s
qrt(-4*a*c + b**2)/(6*a*(4*a*c - b**2)) - 1/(6*a)) - 2*a*c + b**2)/(b*c)) +
 log(x)/a
```

3.129 $\int \frac{1}{x^4(a+bx^3+cx^6)}\,dx$

Optimal. Leaf size=89

$$-\frac{(b^2-2ac)\tanh^{-1}\left(\frac{b+2cx^3}{\sqrt{b^2-4ac}}\right)}{3a^2\sqrt{b^2-4ac}}+\frac{b\log\left(a+bx^3+cx^6\right)}{6a^2}-\frac{b\log(x)}{a^2}-\frac{1}{3ax^3}$$

Rubi [A] time = 0.13, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389, Rules used = {1357, 709, 800, 634, 618, 206, 628}

$$-\frac{(b^2-2ac)\tanh^{-1}\left(\frac{b+2cx^3}{\sqrt{b^2-4ac}}\right)}{3a^2\sqrt{b^2-4ac}}+\frac{b\log\left(a+bx^3+cx^6\right)}{6a^2}-\frac{b\log(x)}{a^2}-\frac{1}{3ax^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(a + b*x^3 + c*x^6)),x]
```

```
[Out] -1/(3*a*x^3) - ((b^2 - 2*a*c)*ArcTanh[(b + 2*c*x^3)/Sqrt[b^2 - 4*a*c]])/(3*
a^2*Sqrt[b^2 - 4*a*c]) - (b*Log[x])/a^2 + (b*Log[a + b*x^3 + c*x^6])/(6*a^2
)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 709

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Simp[(e*(d + e*x)^(m + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dis
t[1/(c*d^2 - b*d*e + a*e^2), Int[((d + e*x)^(m + 1)*Simp[c*d - b*e - c*e*x,
 x])/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 -
 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[m
, -1]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 1357

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^4\left(a+bx^3+cx^6\right)}\,dx &= \frac{1}{3}\,\mathrm{Subst}\left(\int \frac{1}{x^2\left(a+bx+cx^2\right)}\,dx, x, x^3\right)\\
&= -\frac{1}{3ax^3}+\frac{\mathrm{Subst}\left(\int \frac{-b-cx}{x\left(a+bx+cx^2\right)}\,dx, x, x^3\right)}{3a}\\
&= -\frac{1}{3ax^3}+\frac{\mathrm{Subst}\left(\int \left(-\frac{b}{ax}+\frac{b^2-ac+bcx}{a\left(a+bx+cx^2\right)}\right)dx, x, x^3\right)}{3a}\\
&= -\frac{1}{3ax^3}-\frac{b\log(x)}{a^2}+\frac{\mathrm{Subst}\left(\int \frac{b^2-ac+bcx}{a+bx+cx^2}\,dx, x, x^3\right)}{3a^2}\\
&= -\frac{1}{3ax^3}-\frac{b\log(x)}{a^2}+\frac{b\,\mathrm{Subst}\left(\int \frac{b+2cx}{a+bx+cx^2}\,dx, x, x^3\right)}{6a^2}+\frac{\left(b^2-2ac\right)\mathrm{Subst}\left(\int \frac{1}{a+bx+cx^2}\,dx, \ldots\right.}{6a^2}\\
&= -\frac{1}{3ax^3}-\frac{b\log(x)}{a^2}+\frac{b\log\left(a+bx^3+cx^6\right)}{6a^2}-\frac{\left(b^2-2ac\right)\mathrm{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx, x, b+2\ldots\right.}{3a^2}\\
&= -\frac{1}{3ax^3}-\frac{\left(b^2-2ac\right)\tanh^{-1}\left(\frac{b+2cx^3}{\sqrt{b^2-4ac}}\right)}{3a^2\sqrt{b^2-4ac}}-\frac{b\log(x)}{a^2}+\frac{b\log\left(a+bx^3+cx^6\right)}{6a^2}
\end{aligned}$$

Mathematica [C] time = 0.03, size = 92, normalized size = 1.03

$$\frac{\mathrm{RootSum}\left[\#1^6c+\#1^3b+a\&,\frac{\#1^3bc\log(x-\#1)-ac\log(x-\#1)+b^2\log(x-\#1)}{2\#1^3c+b}\&\right]}{3a^2}-\frac{b\log(x)}{a^2}-\frac{1}{3ax^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(a + b*x^3 + c*x^6)),x]
```

```
[Out] -1/3*1/(a*x^3) - (b*Log[x])/a^2 + RootSum[a + b*#1^3 + c*#1^6 & , (b^2*Log[
x - #1] - a*c*Log[x - #1] + b*c*Log[x - #1]*#1^3)/(b + 2*c*#1^3) & ]/(3*a^2
)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^4\left(a+bx^3+cx^6\right)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x^4*(a + b*x^3 + c*x^6)),x]
```

```
[Out] IntegrateAlgebraic[1/(x^4*(a + b*x^3 + c*x^6)), x]
```

fricas [A] time = 1.38, size = 293, normalized size = 3.29

$$\left[-\frac{\left(b^{2}-2\,ac\right)\sqrt{b^{2}-4\,ac}\,x^{3}\log\left(\frac{2\,c^{2}x^{6}+2\,bcx^{3}+b^{2}-2\,ac+\left(2\,cx^{3}+b\right)\sqrt{b^{2}-4\,ac}}{cx^{6}+bx^{3}+a}\right)-\left(b^{3}-4\,abc\right)x^{3}\log\left(cx^{6}+bx^{3}+a\right)+6\left(b^{3}-4\,abc\right)x^{3}\log(x)+2\,ab^{2}-8\,a^{2}c}{6\left(a^{2}b^{2}-4\,a^{3}c\right)x^{3}},-\frac{2\left(b^{2}-2\,ac\right)\sqrt{-b^{2}+4\,ac}\,x^{3}\arctan\left(-\frac{\left(2\,cx^{3}+b\right)\sqrt{-b^{2}+4\,ac}}{b^{2}-4\,ac}\right)-\left(b^{3}-4\,abc\right)x^{3}\log\left(cx^{6}+bx^{3}+a\right)+6\left(b^{3}-4\,abc\right)x^{3}\log(x)+2\,ab^{2}-8\,a^{2}c}{6\left(a^{2}b^{2}-4\,a^{3}c\right)x^{3}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(c*x^6+b*x^3+a),x, algorithm="fricas")
```

```
[Out] [-1/6*((b^2 - 2*a*c)*sqrt(b^2 - 4*a*c)*x^3*log((2*c^2*x^6 + 2*b*c*x^3 + b^2
 - 2*a*c + (2*c*x^3 + b)*sqrt(b^2 - 4*a*c))/(c*x^6 + b*x^3 + a)) - (b^3 - 4
*a*b*c)*x^3*log(c*x^6 + b*x^3 + a) + 6*(b^3 - 4*a*b*c)*x^3*log(x) + 2*a*b^2
 - 8*a^2*c)/((a^2*b^2 - 4*a^3*c)*x^3), -1/6*(2*(b^2 - 2*a*c)*sqrt(-b^2 + 4*
a*c)*x^3*arctan(-(2*c*x^3 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) - (b^3 - 4
*a*b*c)*x^3*log(c*x^6 + b*x^3 + a) + 6*(b^3 - 4*a*b*c)*x^3*log(x) + 2*a*b^2
 - 8*a^2*c)/((a^2*b^2 - 4*a^3*c)*x^3)]
```

giac [A] time = 1.14, size = 93, normalized size = 1.04

$$\frac{b\log\left(cx^{6}+bx^{3}+a\right)}{6\,a^{2}}-\frac{b\log\left(|x|\right)}{a^{2}}+\frac{\left(b^{2}-2\,ac\right)\arctan\left(\frac{2\,cx^{3}+b}{\sqrt{-b^{2}+4\,ac}}\right)}{3\,\sqrt{-b^{2}+4\,ac}\,a^{2}}+\frac{bx^{3}-a}{3\,a^{2}x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(c*x^6+b*x^3+a),x, algorithm="giac")
```

```
[Out] 1/6*b*log(c*x^6 + b*x^3 + a)/a^2 - b*log(abs(x))/a^2 + 1/3*(b^2 - 2*a*c)*ar
ctan((2*c*x^3 + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*a^2) + 1/3*(b*x^
3 - a)/(a^2*x^3)
```

maple [A] time = 0.01, size = 119, normalized size = 1.34

$$-\frac{2c\arctan\left(\frac{2c\,x^{3}+b}{\sqrt{4ac-b^{2}}}\right)}{3\sqrt{4ac-b^{2}}\;a}+\frac{b^{2}\arctan\left(\frac{2c\,x^{3}+b}{\sqrt{4ac-b^{2}}}\right)}{3\sqrt{4ac-b^{2}}\;a^{2}}-\frac{b\ln(x)}{a^{2}}+\frac{b\ln\left(c\,x^{6}+b\,x^{3}+a\right)}{6a^{2}}-\frac{1}{3a\,x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(c*x^6+b*x^3+a),x)
```

```
[Out] -1/3/a/x^3-b*ln(x)/a^2+1/6*b*ln(c*x^6+b*x^3+a)/a^2-2/3/a/(4*a*c-b^2)^(1/2)*
arctan((2*c*x^3+b)/(4*a*c-b^2)^(1/2))*c+1/3/a^2/(4*a*c-b^2)^(1/2)*arctan((2
*c*x^3+b)/(4*a*c-b^2)^(1/2))*b^2
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(c*x^6+b*x^3+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

mupad [B] time = 2.03, size = 4281, normalized size = 48.10

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(a + b*x^3 + c*x^6)),x)
```

```
[Out] (atan((48*a^8*x^3*(((4*b^5 + 9*a^2*b*c^2 - 16*a*b^3*c)*(((3*b^3 - 12*a*b*c)
*(((3*b^3 - 12*a*b*c)*(((2*a*c - b^2)*((252*a^4*b*c^5 - 18*a^3*b^3*c^4)/a^4
 - ((3*b^3 - 12*a*b*c)*(108*a^4*b^4*c^3 - 378*a^5*b^2*c^4))/(2*a^4*(36*a^3*
c - 9*a^2*b^2))))/(6*a^2*(4*a*c - b^2)^(1/2)) - ((3*b^3 - 12*a*b*c)*(2*a*c
- b^2)*(108*a^4*b^4*c^3 - 378*a^5*b^2*c^4))/(12*a^6*(4*a*c - b^2)^(1/2)*(36
*a^3*c - 9*a^2*b^2))))/(2*(36*a^3*c - 9*a^2*b^2)) + (((42*a^3*c^6 + 33*a^2*
b^2*c^5)/a^4 + ((3*b^3 - 12*a*b*c)*((252*a^4*b*c^5 - 18*a^3*b^3*c^4)/a^4 -
((3*b^3 - 12*a*b*c)*(108*a^4*b^4*c^3 - 378*a^5*b^2*c^4))/(2*a^4*(36*a^3*c -
 9*a^2*b^2))))/(2*(36*a^3*c - 9*a^2*b^2)))*(2*a*c - b^2))/(6*a^2*(4*a*c - b
^2)^(1/2))))/(2*(36*a^3*c - 9*a^2*b^2)) - ((((((2*a*c - b^2)*((252*a^4*b*c^
5 - 18*a^3*b^3*c^4)/a^4 - ((3*b^3 - 12*a*b*c)*(108*a^4*b^4*c^3 - 378*a^5*b^
2*c^4))/(2*a^4*(36*a^3*c - 9*a^2*b^2))))/(6*a^2*(4*a*c - b^2)^(1/2)) - ((3*
b^3 - 12*a*b*c)*(2*a*c - b^2)*(108*a^4*b^4*c^3 - 378*a^5*b^2*c^4))/(12*a^6*
(4*a*c - b^2)^(1/2)*(36*a^3*c - 9*a^2*b^2)))*(2*a*c - b^2))/(6*a^2*(4*a*c -
 b^2)^(1/2)) - ((3*b^3 - 12*a*b*c)*(2*a*c - b^2)^2*(108*a^4*b^4*c^3 - 378*a
^5*b^2*c^4))/(72*a^8*(4*a*c - b^2)*(36*a^3*c - 9*a^2*b^2)))*(2*a*c - b^2))/
(6*a^2*(4*a*c - b^2)^(1/2)) + ((2*a*c - b^2)*(((3*b^3 - 12*a*b*c)*((42*a^3*
c^6 + 33*a^2*b^2*c^5)/a^4 + ((3*b^3 - 12*a*b*c)*((252*a^4*b*c^5 - 18*a^3*b^
3*c^4)/a^4 - ((3*b^3 - 12*a*b*c)*(108*a^4*b^4*c^3 - 378*a^5*b^2*c^4))/(2*a^
4*(36*a^3*c - 9*a^2*b^2))))/(2*(36*a^3*c - 9*a^2*b^2))))/(2*(36*a^3*c - 9*a
^2*b^2)) + (12*b*c^6)/a^3))/(6*a^2*(4*a*c - b^2)^(1/2)) + ((3*b^3 - 12*a*b*
c)*(2*a*c - b^2)^3*(108*a^4*b^4*c^3 - 378*a^5*b^2*c^4))/(432*a^10*(4*a*c -
b^2)^(3/2)*(36*a^3*c - 9*a^2*b^2))))/(16*a^4*c^3*(a^2*c^2 - 12*b^4 + 48*a*b
^2*c)) + ((4*b^6 - 2*a^3*c^3 + 33*a^2*b^2*c^2 - 24*a*b^4*c)*(((3*b^3 - 12*a
*b*c)*(((((2*a*c - b^2)*((252*a^4*b*c^5 - 18*a^3*b^3*c^4)/a^4 - ((3*b^3 - 1
2*a*b*c)*(108*a^4*b^4*c^3 - 378*a^5*b^2*c^4))/(2*a^4*(36*a^3*c - 9*a^2*b^2)
)))/(6*a^2*(4*a*c - b^2)^(1/2)) - ((3*b^3 - 12*a*b*c)*(2*a*c - b^2)*(108*a^
4*b^4*c^3 - 378*a^5*b^2*c^4))/(12*a^6*(4*a*c - b^2)^(1/2)*(36*a^3*c - 9*a^2
*b^2)))*(2*a*c - b^2))/(6*a^2*(4*a*c - b^2)^(1/2)) - ((3*b^3 - 12*a*b*c)*(2
*a*c - b^2)^2*(108*a^4*b^4*c^3 - 378*a^5*b^2*c^4))/(72*a^8*(4*a*c - b^2)*(3
6*a^3*c - 9*a^2*b^2))))/(2*(36*a^3*c - 9*a^2*b^2)) - c^7/a^4 - ((3*b^3 - 12
*a*b*c)*(((3*b^3 - 12*a*b*c)*((42*a^3*c^6 + 33*a^2*b^2*c^5)/a^4 + ((3*b^3 -
 12*a*b*c)*((252*a^4*b*c^5 - 18*a^3*b^3*c^4)/a^4 - ((3*b^3 - 12*a*b*c)*(108
*a^4*b^4*c^3 - 378*a^5*b^2*c^4))/(2*a^4*(36*a^3*c - 9*a^2*b^2))))/(2*(36*a^
3*c - 9*a^2*b^2))))/(2*(36*a^3*c - 9*a^2*b^2)) + (12*b*c^6)/a^3))/(2*(36*a^
3*c - 9*a^2*b^2)) + ((2*a*c - b^2)*(((3*b^3 - 12*a*b*c)*(((2*a*c - b^2)*((2
52*a^4*b*c^5 - 18*a^3*b^3*c^4)/a^4 - ((3*b^3 - 12*a*b*c)*(108*a^4*b^4*c^3 -
 378*a^5*b^2*c^4))/(2*a^4*(36*a^3*c - 9*a^2*b^2))))/(6*a^2*(4*a*c - b^2)^(1
/2)) - ((3*b^3 - 12*a*b*c)*(2*a*c - b^2)*(108*a^4*b^4*c^3 - 378*a^5*b^2*c^4
))/(12*a^6*(4*a*c - b^2)^(1/2)*(36*a^3*c - 9*a^2*b^2))))/(2*(36*a^3*c - 9*a
^2*b^2)) + (((42*a^3*c^6 + 33*a^2*b^2*c^5)/a^4 + ((3*b^3 - 12*a*b*c)*((252*
a^4*b*c^5 - 18*a^3*b^3*c^4)/a^4 - ((3*b^3 - 12*a*b*c)*(108*a^4*b^4*c^3 - 37
8*a^5*b^2*c^4))/(2*a^4*(36*a^3*c - 9*a^2*b^2))))/(2*(36*a^3*c - 9*a^2*b^2))
)*(2*a*c - b^2))/(6*a^2*(4*a*c - b^2)^(1/2))))/(6*a^2*(4*a*c - b^2)^(1/2))
+ ((2*a*c - b^2)^4*(108*a^4*b^4*c^3 - 378*a^5*b^2*c^4))/(1296*a^12*(4*a*c -
 b^2)^2)))/(16*a^4*c^3*(4*a*c - b^2)^(1/2)*(a^2*c^2 - 12*b^4 + 48*a*b^2*c))
)*(4*a*c - b^2)^2)/(8*a^3*c^6 - b^6*c^3 + 6*a*b^4*c^4 - 12*a^2*b^2*c^5) + (
3*a^4*(4*a*c - b^2)^2*(4*b^5 + 9*a^2*b*c^2 - 16*a*b^3*c)*(((3*b^3 - 12*a*b*
c)*(((3*b^3 - 12*a*b*c)*((((27*a^3*b^4*c^3 - 27*a^4*b^2*c^4)/a^4 - (27*a*b^
3*c^3*(3*b^3 - 12*a*b*c))/(2*(36*a^3*c - 9*a^2*b^2)))*(2*a*c - b^2))/(6*a^2
*(4*a*c - b^2)^(1/2)) - (9*b^3*c^3*(3*b^3 - 12*a*b*c)*(2*a*c - b^2))/(4*a*(
4*a*c - b^2)^(1/2)*(36*a^3*c - 9*a^2*b^2))))/(2*(36*a^3*c - 9*a^2*b^2)) - (
(2*a*c - b^2)*((9*a^3*b*c^5 - 27*a^2*b^3*c^4)/a^4 - ((3*b^3 - 12*a*b*c)*((2
7*a^3*b^4*c^3 - 27*a^4*b^2*c^4)/a^4 - (27*a*b^3*c^3*(3*b^3 - 12*a*b*c))/(2*
(36*a^3*c - 9*a^2*b^2))))/(2*(36*a^3*c - 9*a^2*b^2))))/(6*a^2*(4*a*c - b^2)
^(1/2))))/(2*(36*a^3*c - 9*a^2*b^2)) - (((a^2*c^6 - 9*a*b^2*c^5)/a^4 + ((3*
b^3 - 12*a*b*c)*((9*a^3*b*c^5 - 27*a^2*b^3*c^4)/a^4 - ((3*b^3 - 12*a*b*c)*(
(27*a^3*b^4*c^3 - 27*a^4*b^2*c^4)/a^4 - (27*a*b^3*c^3*(3*b^3 - 12*a*b*c))/(
```

```
2*(36*a^3*c - 9*a^2*b^2))))/(2*(36*a^3*c - 9*a^2*b^2))))/(2*(36*a^3*c - 9*a
^2*b^2)))*(2*a*c - b^2))/(6*a^2*(4*a*c - b^2)^(1/2)) - (((((((27*a^3*b^4*c^
3 - 27*a^4*b^2*c^4)/a^4 - (27*a*b^3*c^3*(3*b^3 - 12*a*b*c))/(2*(36*a^3*c -
9*a^2*b^2)))*(2*a*c - b^2))/(6*a^2*(4*a*c - b^2)^(1/2)) - (9*b^3*c^3*(3*b^3
 - 12*a*b*c)*(2*a*c - b^2))/(4*a*(4*a*c - b^2)^(1/2)*(36*a^3*c - 9*a^2*b^2)
))*(2*a*c - b^2))/(6*a^2*(4*a*c - b^2)^(1/2)) - (3*b^3*c^3*(3*b^3 - 12*a*b*
c)*(2*a*c - b^2)^2)/(8*a^3*(4*a*c - b^2)*(36*a^3*c - 9*a^2*b^2)))*(2*a*c -
b^2))/(6*a^2*(4*a*c - b^2)^(1/2)) + (b^3*c^3*(3*b^3 - 12*a*b*c)*(2*a*c - b^
2)^3)/(16*a^5*(4*a*c - b^2)^(3/2)*(36*a^3*c - 9*a^2*b^2))))/(c^3*(a^2*c^2 -
 12*b^4 + 48*a*b^2*c)*(8*a^3*c^6 - b^6*c^3 + 6*a*b^4*c^4 - 12*a^2*b^2*c^5))
 + (3*a^4*(4*a*c - b^2)^(3/2)*(4*b^6 - 2*a^3*c^3 + 33*a^2*b^2*c^2 - 24*a*b^
4*c)*(((3*b^3 - 12*a*b*c)*((((((27*a^3*b^4*c^3 - 27*a^4*b^2*c^4)/a^4 - (27*
a*b^3*c^3*(3*b^3 - 12*a*b*c))/(2*(36*a^3*c - 9*a^2*b^2)))*(2*a*c - b^2))/(6
*a^2*(4*a*c - b^2)^(1/2)) - (9*b^3*c^3*(3*b^3 - 12*a*b*c)*(2*a*c - b^2))/(4
*a*(4*a*c - b^2)^(1/2)*(36*a^3*c - 9*a^2*b^2)))*(2*a*c - b^2))/(6*a^2*(4*a*
c - b^2)^(1/2)) - (3*b^3*c^3*(3*b^3 - 12*a*b*c)*(2*a*c - b^2)^2)/(8*a^3*(4*
a*c - b^2)*(36*a^3*c - 9*a^2*b^2))))/(2*(36*a^3*c - 9*a^2*b^2)) - (b*c^6)/a
^4 + ((3*b^3 - 12*a*b*c)*((a^2*c^6 - 9*a*b^2*c^5)/a^4 + ((3*b^3 - 12*a*b*c)
*((9*a^3*b*c^5 - 27*a^2*b^3*c^4)/a^4 - ((3*b^3 - 12*a*b*c)*((27*a^3*b^4*c^3
 - 27*a^4*b^2*c^4)/a^4 - (27*a*b^3*c^3*(3*b^3 - 12*a*b*c))/(2*(36*a^3*c - 9
*a^2*b^2))))/(2*(36*a^3*c - 9*a^2*b^2))))/(2*(36*a^3*c - 9*a^2*b^2))))/(2*(
36*a^3*c - 9*a^2*b^2)) + ((2*a*c - b^2)*(((3*b^3 - 12*a*b*c)*((((27*a^3*b^4
*c^3 - 27*a^4*b^2*c^4)/a^4 - (27*a*b^3*c^3*(3*b^3 - 12*a*b*c))/(2*(36*a^3*c
 - 9*a^2*b^2)))*(2*a*c - b^2))/(6*a^2*(4*a*c - b^2)^(1/2)) - (9*b^3*c^3*(3*
b^3 - 12*a*b*c)*(2*a*c - b^2))/(4*a*(4*a*c - b^2)^(1/2)*(36*a^3*c - 9*a^2*b
^2))))/(2*(36*a^3*c - 9*a^2*b^2)) - ((2*a*c - b^2)*((9*a^3*b*c^5 - 27*a^2*b
^3*c^4)/a^4 - ((3*b^3 - 12*a*b*c)*((27*a^3*b^4*c^3 - 27*a^4*b^2*c^4)/a^4 -
(27*a*b^3*c^3*(3*b^3 - 12*a*b*c))/(2*(36*a^3*c - 9*a^2*b^2))))/(2*(36*a^3*c
 - 9*a^2*b^2))))/(6*a^2*(4*a*c - b^2)^(1/2))))/(6*a^2*(4*a*c - b^2)^(1/2))
+ (b^3*c^3*(2*a*c - b^2)^4)/(48*a^7*(4*a*c - b^2)^2)))/(c^3*(a^2*c^2 - 12*b
^4 + 48*a*b^2*c)*(8*a^3*c^6 - b^6*c^3 + 6*a*b^4*c^4 - 12*a^2*b^2*c^5)))*(2*
a*c - b^2))/(3*a^2*(4*a*c - b^2)^(1/2)) - (b*log(x))/a^2 - (log(a + b*x^3 +
 c*x^6)*(3*b^3 - 12*a*b*c))/(2*(36*a^3*c - 9*a^2*b^2)) - 1/(3*a*x^3)
```

sympy [B] time = 113.79, size = 345, normalized size = 3.88

$$\left(\frac{b}{6a^2} - \frac{\sqrt{-4ac+b^2}\left(2ac-b^2\right)}{6a^2\left(4ac-b^2\right)}\right)\log\left(x^3 + \frac{-12a^3c\left(\frac{b}{6a^2} - \frac{\sqrt{-4ac+b^2}\left(2ac-b^2\right)}{6a^2\left(4ac-b^2\right)}\right) + 3a^2b^2\left(\frac{b}{6a^2} - \frac{\sqrt{-4ac+b^2}\left(2ac-b^2\right)}{6a^2\left(4ac-b^2\right)}\right) + 3abc - b^3}{2ac^2 - b^2c}\right) + \left(\frac{b}{6a^2} + \frac{\sqrt{-4ac+b^2}\left(2ac-b^2\right)}{6a^2\left(4ac-b^2\right)}\right)\log\left(x^3 + \frac{-12a^3c\left(\frac{b}{6a^2} + \frac{\sqrt{-4ac+b^2}\left(2ac-b^2\right)}{6a^2\left(4ac-b^2\right)}\right) + 3a^2b^2\left(\frac{b}{6a^2} + \frac{\sqrt{-4ac+b^2}\left(2ac-b^2\right)}{6a^2\left(4ac-b^2\right)}\right) + 3abc - b^3}{2ac^2 - b^2c}\right) - \frac{1}{3ax^3} - \frac{b\log\left(x\right)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(c*x**6+b*x**3+a),x)
```

```
[Out] (b/(6*a**2) - sqrt(-4*a*c + b**2)*(2*a*c - b**2)/(6*a**2*(4*a*c - b**2)))*l
og(x**3 + (-12*a**3*c*(b/(6*a**2) - sqrt(-4*a*c + b**2)*(2*a*c - b**2)/(6*a
**2*(4*a*c - b**2))) + 3*a**2*b**2*(b/(6*a**2) - sqrt(-4*a*c + b**2)*(2*a*c
 - b**2)/(6*a**2*(4*a*c - b**2))) + 3*a*b*c - b**3)/(2*a*c**2 - b**2*c)) +
(b/(6*a**2) + sqrt(-4*a*c + b**2)*(2*a*c - b**2)/(6*a**2*(4*a*c - b**2)))*l
og(x**3 + (-12*a**3*c*(b/(6*a**2) + sqrt(-4*a*c + b**2)*(2*a*c - b**2)/(6*a
**2*(4*a*c - b**2))) + 3*a**2*b**2*(b/(6*a**2) + sqrt(-4*a*c + b**2)*(2*a*c
 - b**2)/(6*a**2*(4*a*c - b**2))) + 3*a*b*c - b**3)/(2*a*c**2 - b**2*c)) -
1/(3*a*x**3) - b*log(x)/a**2
```

3.130 $\int \frac{x^7}{a+bx^3+cx^6}\,dx$

Optimal. Leaf size=636

$$-\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\log\left(-\sqrt[3]{2}\sqrt[3]{c}\,x\sqrt[3]{b-\sqrt{b^2-4ac}}+\left(b-\sqrt{b^2-4ac}\right)^{2/3}+2^{2/3}c^{2/3}x^2\right)}{6\,2^{2/3}c^{5/3}\sqrt[3]{b-\sqrt{b^2-4ac}}}-\frac{\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\log\left(-\sqrt[3]{2}\sqrt[3]{c}\,x \cdots\right.}{6 \cdots}$$

Rubi [A] time = 1.25, antiderivative size = 636, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 8, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {1367, 1510, 292, 31, 634, 617, 204, 628}

$$-\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\log\left(-\sqrt[3]{2}\sqrt[3]{c}\,x\sqrt[3]{b-\sqrt{b^2-4ac}}+\left(b-\sqrt{b^2-4ac}\right)^{2/3}+2^{2/3}c^{2/3}x^2\right)}{6\,2^{2/3}c^{5/3}\sqrt[3]{b-\sqrt{b^2-4ac}}}-\frac{\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\log\left(-\sqrt[3]{2}\sqrt[3]{c}\,x\sqrt[3]{\sqrt{b^2-4ac}+b}+\left(\sqrt{b^2-4ac}+b\right)^{2/3}+2^{2/3}c^{2/3}x^2\right)}{6\,2^{2/3}c^{5/3}\sqrt[3]{\sqrt{b^2-4ac}+b}}+\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}\,x\right)}{3\,2^{2/3}c^{5/3}\sqrt[3]{b-\sqrt{b^2-4ac}}}+\frac{\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\log\left(\sqrt[3]{\sqrt{b^2-4ac}+b}+\sqrt[3]{2}\sqrt[3]{c}\,x\right)}{3\,2^{2/3}c^{5/3}\sqrt[3]{\sqrt{b^2-4ac}+b}}+\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}\,x}{\sqrt[3]{b-\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}\,c^{5/3}\sqrt[3]{b-\sqrt{b^2-4ac}}}+\frac{\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}\,x}{\sqrt[3]{\sqrt{b^2-4ac}+b}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}\,c^{5/3}\sqrt[3]{\sqrt{b^2-4ac}+b}}+\frac{x^2}{2c}$$

Antiderivative was successfully verified.

```
[In] Int[x^7/(a + b*x^3 + c*x^6),x]
```

```
[Out] x^2/(2*c) + ((b - (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(1 - (2*2^(1/3)*c
^(1/3)*x)/(b - Sqrt[b^2 - 4*a*c])^(1/3))/Sqrt[3]])/(2^(2/3)*Sqrt[3]*c^(5/3)
*(b - Sqrt[b^2 - 4*a*c])^(1/3)) + ((b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*Ar
cTan[(1 - (2*2^(1/3)*c^(1/3)*x)/(b + Sqrt[b^2 - 4*a*c])^(1/3))/Sqrt[3]])/(2
^(2/3)*Sqrt[3]*c^(5/3)*(b + Sqrt[b^2 - 4*a*c])^(1/3)) + ((b - (b^2 - 2*a*c)
/Sqrt[b^2 - 4*a*c])*Log[(b - Sqrt[b^2 - 4*a*c])^(1/3) + 2^(1/3)*c^(1/3)*x])
/(3*2^(2/3)*c^(5/3)*(b - Sqrt[b^2 - 4*a*c])^(1/3)) + ((b + (b^2 - 2*a*c)/Sq
rt[b^2 - 4*a*c])*Log[(b + Sqrt[b^2 - 4*a*c])^(1/3) + 2^(1/3)*c^(1/3)*x])/(3
*2^(2/3)*c^(5/3)*(b + Sqrt[b^2 - 4*a*c])^(1/3)) - ((b - (b^2 - 2*a*c)/Sqrt[
b^2 - 4*a*c])*Log[(b - Sqrt[b^2 - 4*a*c])^(2/3) - 2^(1/3)*c^(1/3)*(b - Sqrt
[b^2 - 4*a*c])^(1/3)*x + 2^(2/3)*c^(2/3)*x^2])/(6*2^(2/3)*c^(5/3)*(b - Sqrt
[b^2 - 4*a*c])^(1/3)) - ((b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*Log[(b + Sqr
t[b^2 - 4*a*c])^(2/3) - 2^(1/3)*c^(1/3)*(b + Sqrt[b^2 - 4*a*c])^(1/3)*x + 2
^(2/3)*c^(2/3)*x^2])/(6*2^(2/3)*c^(5/3)*(b + Sqrt[b^2 - 4*a*c])^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
```

```
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1367

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Simp[(d^(2*n - 1)*(d*x)^(m - 2*n + 1)*(a + b*x^n + c*x^(2*n))^(
p + 1))/(c*(m + 2*n*p + 1)), x] - Dist[d^(2*n)/(c*(m + 2*n*p + 1)), Int[(d*
x)^(m - 2*n)*Simp[a*(m - 2*n + 1) + b*(m + n*(p - 1) + 1)*x^n, x]*(a + b*x^
n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && EqQ[n2, 2*n] && Ne
Q[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GtQ[m, 2*n - 1] && NeQ[m + 2*n*p + 1, 0]
 && IntegerQ[p]
```

Rule 1510

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_)))/((a_) + (b_.)*(x_)^(n_) +
 (c_.)*(x_)^(n2_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 +
(2*c*d - b*e)/(2*q), Int[(f*x)^m/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (
2*c*d - b*e)/(2*q), Int[(f*x)^m/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b
, c, d, e, f, m}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{x^7}{a+bx^3+cx^6}\,dx = \frac{x^2}{2c} - \frac{\int \frac{x(2a+2bx^3)}{a+bx^3+cx^6}\,dx}{2c}$$

$$= \frac{x^2}{2c} - \frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\int \frac{x}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^3}\,dx}{2c} - \frac{\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\int \frac{x}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^3}\,dx}{2c}$$

$$= \frac{x^2}{2c} + \frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\frac{\sqrt[3]{b-\sqrt{b^2-4ac}}}{\sqrt[3]{2}}+\sqrt[3]{c}\,x}\,dx}{3\,2^{2/3}c^{4/3}\sqrt[3]{b-\sqrt{b^2-4ac}}} - \frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\int \frac{\frac{\sqrt[3]{b-\sqrt{b^2-4ac}}}{\sqrt[3]{2}}+\sqrt[3]{c}\,x}{\frac{\left(b-\sqrt{b^2-4ac}\right)^{2/3}}{2^{2/3}}-\frac{\sqrt[3]{c}\,\sqrt[3]{b-\sqrt{b^2-4ac}}\,x}{\sqrt[3]{2}}+c^{2/3}x}}{3\,2^{2/3}c^{4/3}\sqrt[3]{b-\sqrt{b^2-4ac}}}$$

$$= \frac{x^2}{2c} + \frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\,\sqrt[3]{c}\,x\right)}{3\,2^{2/3}c^{5/3}\sqrt[3]{b-\sqrt{b^2-4ac}}} + \frac{\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{b+\sqrt{b^2-4}}\right.}{3\,2^{2/3}c^{5/3}\sqrt[3]{b+\sqrt{b^2-}}}$$

$$= \frac{x^2}{2c} + \frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\,\sqrt[3]{c}\,x\right)}{3\,2^{2/3}c^{5/3}\sqrt[3]{b-\sqrt{b^2-4ac}}} + \frac{\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{b+\sqrt{b^2-4}}\right.}{3\,2^{2/3}c^{5/3}\sqrt[3]{b+\sqrt{b^2-}}}$$

$$= \frac{x^2}{2c} + \frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\,\sqrt[3]{c}\,x}{\sqrt[3]{b-\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}\,c^{5/3}\sqrt[3]{b-\sqrt{b^2-4ac}}} + \frac{\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\,\sqrt[3]{c}\,x}{\sqrt[3]{b+\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}\,c^{5/3}\sqrt[3]{b+\sqrt{b^2-4ac}}} + \frac{\left(b-\frac{b^2-}{\sqrt{b^2}}\right.}{}$$

Mathematica [C] time = 0.03, size = 70, normalized size = 0.11

$$\frac{3x^2 - 2\text{RootSum}\left[\#1^6c+\#1^3b+a\&,\frac{\#1^3b\log(x-\#1)+a\log(x-\#1)}{2\#1^4c+\#1b}\&\right]}{6c}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7/(a + b*x^3 + c*x^6),x]
```

```
[Out] (3*x^2 - 2*RootSum[a + b*#1^3 + c*#1^6 & , (a*Log[x - #1] + b*Log[x - #1]*#
1^3)/(b*#1 + 2*c*#1^4) & ])/(6*c)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^7}{a+bx^3+cx^6}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^7/(a + b*x^3 + c*x^6),x]
```

```
[Out] IntegrateAlgebraic[x^7/(a + b*x^3 + c*x^6), x]
```

fricas [B] time = 4.09, size = 5601, normalized size = 8.81

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(c*x^6+b*x^3+a),x, algorithm="fricas")
```

```
[Out] -1/6*(4*sqrt(3)*(1/2)^(1/3)*c*((b^4 - 3*a*b^2*c + a^2*c^2 + (b^2*c^5 - 4*a*
c^6)*sqrt((b^10 - 10*a*b^8*c + 35*a^2*b^6*c^2 - 50*a^3*b^4*c^3 + 25*a^4*b^2
*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))/(b^2*c^5
 - 4*a*c^6))^(1/3)*arctan(-1/3*((1/2)^(5/6)*(sqrt(3)*(b^6*c^5 - 10*a*b^4*c^
6 + 32*a^2*b^2*c^7 - 32*a^3*c^8)*sqrt((b^10 - 10*a*b^8*c + 35*a^2*b^6*c^2 -
 50*a^3*b^4*c^3 + 25*a^4*b^2*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^
12 - 64*a^3*c^13)) - sqrt(3)*(b^8 - 9*a*b^6*c + 25*a^2*b^4*c^2 - 20*a^3*b^2
*c^3))*((b^4 - 3*a*b^2*c + a^2*c^2 + (b^2*c^5 - 4*a*c^6)*sqrt((b^10 - 10*a*
b^8*c + 35*a^2*b^6*c^2 - 50*a^3*b^4*c^3 + 25*a^4*b^2*c^4)/(b^6*c^10 - 12*a*
b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))/(b^2*c^5 - 4*a*c^6))^(1/3)*sqrt
((2*(a^3*b^5 - 5*a^4*b^3*c + 5*a^5*b*c^2)*x^2 + (1/2)^(2/3)*((b^8*c^5 - 13*
a*b^6*c^6 + 60*a^2*b^4*c^7 - 112*a^3*b^2*c^8 + 64*a^4*c^9)*x*sqrt((b^10 - 1
0*a*b^8*c + 35*a^2*b^6*c^2 - 50*a^3*b^4*c^3 + 25*a^4*b^2*c^4)/(b^6*c^10 - 1
2*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)) - (b^10 - 12*a*b^8*c + 52*a^
2*b^6*c^2 - 95*a^3*b^4*c^3 + 60*a^4*b^2*c^4)*x)*((b^4 - 3*a*b^2*c + a^2*c^2
 + (b^2*c^5 - 4*a*c^6)*sqrt((b^10 - 10*a*b^8*c + 35*a^2*b^6*c^2 - 50*a^3*b^
4*c^3 + 25*a^4*b^2*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^
3*c^13)))/(b^2*c^5 - 4*a*c^6))^(2/3) - (1/2)^(1/3)*(a^2*b^7 - 9*a^3*b^5*c +
 25*a^4*b^3*c^2 - 20*a^5*b*c^3 - (a^2*b^5*c^5 - 8*a^3*b^3*c^6 + 16*a^4*b*c^
7)*sqrt((b^10 - 10*a*b^8*c + 35*a^2*b^6*c^2 - 50*a^3*b^4*c^3 + 25*a^4*b^2*c
^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))*((b^4 - 3*
a*b^2*c + a^2*c^2 + (b^2*c^5 - 4*a*c^6)*sqrt((b^10 - 10*a*b^8*c + 35*a^2*b^
6*c^2 - 50*a^3*b^4*c^3 + 25*a^4*b^2*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2
*b^2*c^12 - 64*a^3*c^13)))/(b^2*c^5 - 4*a*c^6))^(1/3))/(a^3*b^5 - 5*a^4*b^3
*c + 5*a^5*b*c^2)) - (1/2)^(1/3)*(sqrt(3)*(b^6*c^5 - 10*a*b^4*c^6 + 32*a^2*
b^2*c^7 - 32*a^3*c^8)*x*sqrt((b^10 - 10*a*b^8*c + 35*a^2*b^6*c^2 - 50*a^3*b
^4*c^3 + 25*a^4*b^2*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a
^3*c^13)) - sqrt(3)*(b^8 - 9*a*b^6*c + 25*a^2*b^4*c^2 - 20*a^3*b^2*c^3)*x)*
((b^4 - 3*a*b^2*c + a^2*c^2 + (b^2*c^5 - 4*a*c^6)*sqrt((b^10 - 10*a*b^8*c +
 35*a^2*b^6*c^2 - 50*a^3*b^4*c^3 + 25*a^4*b^2*c^4)/(b^6*c^10 - 12*a*b^4*c^1
1 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))/(b^2*c^5 - 4*a*c^6))^(1/3) + sqrt(3)*(
a^2*b^5 - 5*a^3*b^3*c + 5*a^4*b*c^2))/(a^2*b^5 - 5*a^3*b^3*c + 5*a^4*b*c^2)
) - 4*sqrt(3)*(1/2)^(1/3)*c*((b^4 - 3*a*b^2*c + a^2*c^2 - (b^2*c^5 - 4*a*c^
6)*sqrt((b^10 - 10*a*b^8*c + 35*a^2*b^6*c^2 - 50*a^3*b^4*c^3 + 25*a^4*b^2*c
^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))/(b^2*c^5 -
 4*a*c^6))^(1/3)*arctan(-1/3*((1/2)^(5/6)*(sqrt(3)*(b^6*c^5 - 10*a*b^4*c^6
+ 32*a^2*b^2*c^7 - 32*a^3*c^8)*sqrt((b^10 - 10*a*b^8*c + 35*a^2*b^6*c^2 - 5
0*a^3*b^4*c^3 + 25*a^4*b^2*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12
 - 64*a^3*c^13)) + sqrt(3)*(b^8 - 9*a*b^6*c + 25*a^2*b^4*c^2 - 20*a^3*b^2*c
^3))*((b^4 - 3*a*b^2*c + a^2*c^2 - (b^2*c^5 - 4*a*c^6)*sqrt((b^10 - 10*a*b^
8*c + 35*a^2*b^6*c^2 - 50*a^3*b^4*c^3 + 25*a^4*b^2*c^4)/(b^6*c^10 - 12*a*b^
4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))/(b^2*c^5 - 4*a*c^6))^(1/3)*sqrt((
2*(a^3*b^5 - 5*a^4*b^3*c + 5*a^5*b*c^2)*x^2 - (1/2)^(2/3)*((b^8*c^5 - 13*a*
b^6*c^6 + 60*a^2*b^4*c^7 - 112*a^3*b^2*c^8 + 64*a^4*c^9)*x*sqrt((b^10 - 10*
a*b^8*c + 35*a^2*b^6*c^2 - 50*a^3*b^4*c^3 + 25*a^4*b^2*c^4)/(b^6*c^10 - 12*
a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)) + (b^10 - 12*a*b^8*c + 52*a^2*
b^6*c^2 - 95*a^3*b^4*c^3 + 60*a^4*b^2*c^4)*x)*((b^4 - 3*a*b^2*c + a^2*c^2 -
 (b^2*c^5 - 4*a*c^6)*sqrt((b^10 - 10*a*b^8*c + 35*a^2*b^6*c^2 - 50*a^3*b^4*
c^3 + 25*a^4*b^2*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*
c^13)))/(b^2*c^5 - 4*a*c^6))^(2/3) - (1/2)^(1/3)*(a^2*b^7 - 9*a^3*b^5*c + 2
5*a^4*b^3*c^2 - 20*a^5*b*c^3 + (a^2*b^5*c^5 - 8*a^3*b^3*c^6 + 16*a^4*b*c^7)
*sqrt((b^10 - 10*a*b^8*c + 35*a^2*b^6*c^2 - 50*a^3*b^4*c^3 + 25*a^4*b^2*c^4
)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))*((b^4 - 3*a*
b^2*c + a^2*c^2 - (b^2*c^5 - 4*a*c^6)*sqrt((b^10 - 10*a*b^8*c + 35*a^2*b^6*
c^2 - 50*a^3*b^4*c^3 + 25*a^4*b^2*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b
^2*c^12 - 64*a^3*c^13)))/(b^2*c^5 - 4*a*c^6))^(1/3))/(a^3*b^5 - 5*a^4*b^3*c
 + 5*a^5*b*c^2)) - (1/2)^(1/3)*(sqrt(3)*(b^6*c^5 - 10*a*b^4*c^6 + 32*a^2*b^
2*c^7 - 32*a^3*c^8)*x*sqrt((b^10 - 10*a*b^8*c + 35*a^2*b^6*c^2 - 50*a^3*b^4
```

```
*c^3 + 25*a^4*b^2*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3
*c^13)) + sqrt(3)*(b^8 - 9*a*b^6*c + 25*a^2*b^4*c^2 - 20*a^3*b^2*c^3)*x)*((
b^4 - 3*a*b^2*c + a^2*c^2 - (b^2*c^5 - 4*a*c^6)*sqrt((b^10 - 10*a*b^8*c + 3
5*a^2*b^6*c^2 - 50*a^3*b^4*c^3 + 25*a^4*b^2*c^4)/(b^6*c^10 - 12*a*b^4*c^11
+ 48*a^2*b^2*c^12 - 64*a^3*c^13)))/(b^2*c^5 - 4*a*c^6))^(1/3) - sqrt(3)*(a^
2*b^5 - 5*a^3*b^3*c + 5*a^4*b*c^2))/(a^2*b^5 - 5*a^3*b^3*c + 5*a^4*b*c^2))
+ (1/2)^(1/3)*c*((b^4 - 3*a*b^2*c + a^2*c^2 + (b^2*c^5 - 4*a*c^6)*sqrt((b^1
0 - 10*a*b^8*c + 35*a^2*b^6*c^2 - 50*a^3*b^4*c^3 + 25*a^4*b^2*c^4)/(b^6*c^1
0 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))/(b^2*c^5 - 4*a*c^6))^(
1/3)*log(2*(a^3*b^5 - 5*a^4*b^3*c + 5*a^5*b*c^2)*x^2 + (1/2)^(2/3)*((b^8*c^
5 - 13*a*b^6*c^6 + 60*a^2*b^4*c^7 - 112*a^3*b^2*c^8 + 64*a^4*c^9)*x*sqrt((b
^10 - 10*a*b^8*c + 35*a^2*b^6*c^2 - 50*a^3*b^4*c^3 + 25*a^4*b^2*c^4)/(b^6*c
^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)) - (b^10 - 12*a*b^8*c
+ 52*a^2*b^6*c^2 - 95*a^3*b^4*c^3 + 60*a^4*b^2*c^4)*x)*((b^4 - 3*a*b^2*c +
a^2*c^2 + (b^2*c^5 - 4*a*c^6)*sqrt((b^10 - 10*a*b^8*c + 35*a^2*b^6*c^2 - 50
*a^3*b^4*c^3 + 25*a^4*b^2*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12
- 64*a^3*c^13)))/(b^2*c^5 - 4*a*c^6))^(2/3) - (1/2)^(1/3)*(a^2*b^7 - 9*a^3*
b^5*c + 25*a^4*b^3*c^2 - 20*a^5*b*c^3 - (a^2*b^5*c^5 - 8*a^3*b^3*c^6 + 16*a
^4*b*c^7)*sqrt((b^10 - 10*a*b^8*c + 35*a^2*b^6*c^2 - 50*a^3*b^4*c^3 + 25*a^
4*b^2*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))*((b
^4 - 3*a*b^2*c + a^2*c^2 + (b^2*c^5 - 4*a*c^6)*sqrt((b^10 - 10*a*b^8*c + 35
*a^2*b^6*c^2 - 50*a^3*b^4*c^3 + 25*a^4*b^2*c^4)/(b^6*c^10 - 12*a*b^4*c^11 +
 48*a^2*b^2*c^12 - 64*a^3*c^13)))/(b^2*c^5 - 4*a*c^6))^(1/3)) + (1/2)^(1/3)
*c*((b^4 - 3*a*b^2*c + a^2*c^2 - (b^2*c^5 - 4*a*c^6)*sqrt((b^10 - 10*a*b^8*
c + 35*a^2*b^6*c^2 - 50*a^3*b^4*c^3 + 25*a^4*b^2*c^4)/(b^6*c^10 - 12*a*b^4*
c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))/(b^2*c^5 - 4*a*c^6))^(1/3)*log(2*(a
^3*b^5 - 5*a^4*b^3*c + 5*a^5*b*c^2)*x^2 - (1/2)^(2/3)*((b^8*c^5 - 13*a*b^6*
c^6 + 60*a^2*b^4*c^7 - 112*a^3*b^2*c^8 + 64*a^4*c^9)*x*sqrt((b^10 - 10*a*b^
8*c + 35*a^2*b^6*c^2 - 50*a^3*b^4*c^3 + 25*a^4*b^2*c^4)/(b^6*c^10 - 12*a*b^
4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)) + (b^10 - 12*a*b^8*c + 52*a^2*b^6*
c^2 - 95*a^3*b^4*c^3 + 60*a^4*b^2*c^4)*x)*((b^4 - 3*a*b^2*c + a^2*c^2 - (b^
2*c^5 - 4*a*c^6)*sqrt((b^10 - 10*a*b^8*c + 35*a^2*b^6*c^2 - 50*a^3*b^4*c^3
+ 25*a^4*b^2*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13
)))/(b^2*c^5 - 4*a*c^6))^(2/3) - (1/2)^(1/3)*(a^2*b^7 - 9*a^3*b^5*c + 25*a^
4*b^3*c^2 - 20*a^5*b*c^3 + (a^2*b^5*c^5 - 8*a^3*b^3*c^6 + 16*a^4*b*c^7)*sqr
t((b^10 - 10*a*b^8*c + 35*a^2*b^6*c^2 - 50*a^3*b^4*c^3 + 25*a^4*b^2*c^4)/(b
^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))*((b^4 - 3*a*b^2*
c + a^2*c^2 - (b^2*c^5 - 4*a*c^6)*sqrt((b^10 - 10*a*b^8*c + 35*a^2*b^6*c^2
- 50*a^3*b^4*c^3 + 25*a^4*b^2*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c
^12 - 64*a^3*c^13)))/(b^2*c^5 - 4*a*c^6))^(1/3)) - 2*(1/2)^(1/3)*c*((b^4 -
3*a*b^2*c + a^2*c^2 + (b^2*c^5 - 4*a*c^6)*sqrt((b^10 - 10*a*b^8*c + 35*a^2*
b^6*c^2 - 50*a^3*b^4*c^3 + 25*a^4*b^2*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a
^2*b^2*c^12 - 64*a^3*c^13)))/(b^2*c^5 - 4*a*c^6))^(1/3)*log((1/2)^(2/3)*(b^
10 - 12*a*b^8*c + 52*a^2*b^6*c^2 - 95*a^3*b^4*c^3 + 60*a^4*b^2*c^4 - (b^8*c
^5 - 13*a*b^6*c^6 + 60*a^2*b^4*c^7 - 112*a^3*b^2*c^8 + 64*a^4*c^9)*sqrt((b^
10 - 10*a*b^8*c + 35*a^2*b^6*c^2 - 50*a^3*b^4*c^3 + 25*a^4*b^2*c^4)/(b^6*c^
10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))*((b^4 - 3*a*b^2*c + a
^2*c^2 + (b^2*c^5 - 4*a*c^6)*sqrt((b^10 - 10*a*b^8*c + 35*a^2*b^6*c^2 - 50*
a^3*b^4*c^3 + 25*a^4*b^2*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 -
 64*a^3*c^13)))/(b^2*c^5 - 4*a*c^6))^(2/3) + 2*(a^3*b^5 - 5*a^4*b^3*c + 5*a
^5*b*c^2)*x) - 2*(1/2)^(1/3)*c*((b^4 - 3*a*b^2*c + a^2*c^2 - (b^2*c^5 - 4*a
*c^6)*sqrt((b^10 - 10*a*b^8*c + 35*a^2*b^6*c^2 - 50*a^3*b^4*c^3 + 25*a^4*b^
2*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))/(b^2*c^
5 - 4*a*c^6))^(1/3)*log((1/2)^(2/3)*(b^10 - 12*a*b^8*c + 52*a^2*b^6*c^2 - 9
5*a^3*b^4*c^3 + 60*a^4*b^2*c^4 + (b^8*c^5 - 13*a*b^6*c^6 + 60*a^2*b^4*c^7 -
 112*a^3*b^2*c^8 + 64*a^4*c^9)*sqrt((b^10 - 10*a*b^8*c + 35*a^2*b^6*c^2 - 5
0*a^3*b^4*c^3 + 25*a^4*b^2*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12
 - 64*a^3*c^13)))*((b^4 - 3*a*b^2*c + a^2*c^2 - (b^2*c^5 - 4*a*c^6)*sqrt((b
^10 - 10*a*b^8*c + 35*a^2*b^6*c^2 - 50*a^3*b^4*c^3 + 25*a^4*b^2*c^4)/(b^6*c
```

```
^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))/(b^2*c^5 - 4*a*c^6))
^(2/3) + 2*(a^3*b^5 - 5*a^4*b^3*c + 5*a^5*b*c^2)*x) - 3*x^2)/c
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^7}{cx^6 + bx^3 + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(c*x^6+b*x^3+a),x, algorithm="giac")
```

```
[Out] integrate(x^7/(c*x^6 + b*x^3 + a), x)
```

maple [C] time = 0.14, size = 61, normalized size = 0.10

$$\frac{x^2}{2c} - \frac{\left(\mathrm{RootOf}\left(c\_Z^6 + b\_Z^3 + a\right)^4 b + \mathrm{RootOf}\left(c\_Z^6 + b\_Z^3 + a\right) a\right) \ln\left(-\mathrm{RootOf}\left(c\_Z^6 + b\_Z^3 + a\right) + x\right)}{3c\left(2\,\mathrm{RootOf}\left(c\_Z^6 + b\_Z^3 + a\right)^5 c + \mathrm{RootOf}\left(c\_Z^6 + b\_Z^3 + a\right)^2 b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(c*x^6+b*x^3+a),x)
```

```
[Out] 1/2*x^2/c-1/3/c*sum((_R^4*b+_R*a)/(2*_R^5*c+_R^2*b)*ln(x-_R),_R=RootOf(_Z^6
*c+_Z^3*b+a))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{x^2}{2\,c} - \frac{\int \frac{bx^4+ax}{cx^6+bx^3+a}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(c*x^6+b*x^3+a),x, algorithm="maxima")
```

```
[Out] 1/2*x^2/c - integrate((b*x^4 + a*x)/(c*x^6 + b*x^3 + a), x)/c
```

mupad [B] time = 12.15, size = 4069, normalized size = 6.40

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(a + b*x^3 + c*x^6),x)
```

```
[Out] log((2^(1/3)*((2^(2/3)*(27*a^2*c*x*(b^4 + 8*a^2*c^2 - 6*a*b^2*c) + (27*2^(1
/3)*a*b*c^3*(4*a*c - b^2)^2*(-(b^8 + 16*a^4*c^4 + b^5*(-(4*a*c - b^2)^3)^(1
/2) + 41*a^2*b^4*c^2 - 56*a^3*b^2*c^3 - 11*a*b^6*c + 5*a^2*b*c^2*(-(4*a*c -
 b^2)^3)^(1/2) - 5*a*b^3*c*(-(4*a*c - b^2)^3)^(1/2))/(c^5*(4*a*c - b^2)^3))
^(2/3))/2)*(-(b^8 + 16*a^4*c^4 + b^5*(-(4*a*c - b^2)^3)^(1/2) + 41*a^2*b^4*
c^2 - 56*a^3*b^2*c^3 - 11*a*b^6*c + 5*a^2*b*c^2*(-(4*a*c - b^2)^3)^(1/2) -
5*a*b^3*c*(-(4*a*c - b^2)^3)^(1/2))/(c^5*(4*a*c - b^2)^3))^(1/3))/6 - (9*a*
b*(b^6 - 12*a^3*c^3 + 19*a^2*b^2*c^2 - 8*a*b^4*c))/c^2)*(-(b^8 + 16*a^4*c^4
 + b^5*(-(4*a*c - b^2)^3)^(1/2) + 41*a^2*b^4*c^2 - 56*a^3*b^2*c^3 - 11*a*b^
6*c + 5*a^2*b*c^2*(-(4*a*c - b^2)^3)^(1/2) - 5*a*b^3*c*(-(4*a*c - b^2)^3)^(
1/2))/(c^5*(4*a*c - b^2)^3))^(2/3))/18 + (a^4*x*(a*c - b^2))/c^2)*(-(b^8 +
16*a^4*c^4 + b^5*(-(4*a*c - b^2)^3)^(1/2) + 41*a^2*b^4*c^2 - 56*a^3*b^2*c^3
 - 11*a*b^6*c + 5*a^2*b*c^2*(-(4*a*c - b^2)^3)^(1/2) - 5*a*b^3*c*(-(4*a*c -
 b^2)^3)^(1/2))/(54*(64*a^3*c^8 - b^6*c^5 + 12*a*b^4*c^6 - 48*a^2*b^2*c^7))
)^(1/3) + log((2^(1/3)*((2^(2/3)*(27*a^2*c*x*(b^4 + 8*a^2*c^2 - 6*a*b^2*c)
+ (27*2^(1/3)*a*b*c^3*(4*a*c - b^2)^2*(-(b^8 + 16*a^4*c^4 - b^5*(-(4*a*c -
b^2)^3)^(1/2) + 41*a^2*b^4*c^2 - 56*a^3*b^2*c^3 - 11*a*b^6*c - 5*a^2*b*c^2*
```

```
(-(4*a*c - b^2)^3)^(1/2) + 5*a*b^3*c*(-(4*a*c - b^2)^3)^(1/2))/(c^5*(4*a*c
- b^2)^3))^(2/3))/2)*(-(b^8 + 16*a^4*c^4 - b^5*(-(4*a*c - b^2)^3)^(1/2) + 4
1*a^2*b^4*c^2 - 56*a^3*b^2*c^3 - 11*a*b^6*c - 5*a^2*b*c^2*(-(4*a*c - b^2)^3
)^(1/2) + 5*a*b^3*c*(-(4*a*c - b^2)^3)^(1/2))/(c^5*(4*a*c - b^2)^3))^(1/3))
/6 - (9*a*b*(b^6 - 12*a^3*c^3 + 19*a^2*b^2*c^2 - 8*a*b^4*c))/c^2)*(-(b^8 +
16*a^4*c^4 - b^5*(-(4*a*c - b^2)^3)^(1/2) + 41*a^2*b^4*c^2 - 56*a^3*b^2*c^3
 - 11*a*b^6*c - 5*a^2*b*c^2*(-(4*a*c - b^2)^3)^(1/2) + 5*a*b^3*c*(-(4*a*c -
 b^2)^3)^(1/2))/(c^5*(4*a*c - b^2)^3))^(2/3))/18 + (a^4*x*(a*c - b^2))/c^2)
*(-(b^8 + 16*a^4*c^4 - b^5*(-(4*a*c - b^2)^3)^(1/2) + 41*a^2*b^4*c^2 - 56*a
^3*b^2*c^3 - 11*a*b^6*c - 5*a^2*b*c^2*(-(4*a*c - b^2)^3)^(1/2) + 5*a*b^3*c*
(-(4*a*c - b^2)^3)^(1/2))/(54*(64*a^3*c^8 - b^6*c^5 + 12*a*b^4*c^6 - 48*a^2
*b^2*c^7)))^(1/3) + x^2/(2*c) - log((a^4*x*(a*c - b^2))/c^2 - (2^(1/3)*(3^(
1/2)*1i - 1)*((2^(2/3)*(3^(1/2)*1i + 1)*(27*a^2*c*x*(b^4 + 8*a^2*c^2 - 6*a*
b^2*c) + (27*2^(1/3)*a*b*c^3*(3^(1/2)*1i - 1)*(4*a*c - b^2)^2*(-(b^8 + 16*a
^4*c^4 + b^5*(-(4*a*c - b^2)^3)^(1/2) + 41*a^2*b^4*c^2 - 56*a^3*b^2*c^3 - 1
1*a*b^6*c + 5*a^2*b*c^2*(-(4*a*c - b^2)^3)^(1/2) - 5*a*b^3*c*(-(4*a*c - b^2
)^3)^(1/2))/(c^5*(4*a*c - b^2)^3))^(2/3))/4)*(-(b^8 + 16*a^4*c^4 + b^5*(-(4
*a*c - b^2)^3)^(1/2) + 41*a^2*b^4*c^2 - 56*a^3*b^2*c^3 - 11*a*b^6*c + 5*a^2
*b*c^2*(-(4*a*c - b^2)^3)^(1/2) - 5*a*b^3*c*(-(4*a*c - b^2)^3)^(1/2))/(c^5*
(4*a*c - b^2)^3))^(1/3))/12 + (9*a*b*(b^6 - 12*a^3*c^3 + 19*a^2*b^2*c^2 - 8
*a*b^4*c))/c^2)*(-(b^8 + 16*a^4*c^4 + b^5*(-(4*a*c - b^2)^3)^(1/2) + 41*a^2
*b^4*c^2 - 56*a^3*b^2*c^3 - 11*a*b^6*c + 5*a^2*b*c^2*(-(4*a*c - b^2)^3)^(1/
2) - 5*a*b^3*c*(-(4*a*c - b^2)^3)^(1/2))/(c^5*(4*a*c - b^2)^3))^(2/3))/36)*
((3^(1/2)*1i)/2 + 1/2)*(-(b^8 + 16*a^4*c^4 + b^5*(-(4*a*c - b^2)^3)^(1/2) +
 41*a^2*b^4*c^2 - 56*a^3*b^2*c^3 - 11*a*b^6*c + 5*a^2*b*c^2*(-(4*a*c - b^2)
^3)^(1/2) - 5*a*b^3*c*(-(4*a*c - b^2)^3)^(1/2))/(54*(64*a^3*c^8 - b^6*c^5 +
 12*a*b^4*c^6 - 48*a^2*b^2*c^7)))^(1/3) + log((a^4*x*(a*c - b^2))/c^2 - (2^
(1/3)*(3^(1/2)*1i + 1)*((2^(2/3)*(3^(1/2)*1i - 1)*(27*a^2*c*x*(b^4 + 8*a^2*
c^2 - 6*a*b^2*c) - (27*2^(1/3)*a*b*c^3*(3^(1/2)*1i + 1)*(4*a*c - b^2)^2*(-(
b^8 + 16*a^4*c^4 + b^5*(-(4*a*c - b^2)^3)^(1/2) + 41*a^2*b^4*c^2 - 56*a^3*b
^2*c^3 - 11*a*b^6*c + 5*a^2*b*c^2*(-(4*a*c - b^2)^3)^(1/2) - 5*a*b^3*c*(-(4
*a*c - b^2)^3)^(1/2))/(c^5*(4*a*c - b^2)^3))^(2/3))/4)*(-(b^8 + 16*a^4*c^4
+ b^5*(-(4*a*c - b^2)^3)^(1/2) + 41*a^2*b^4*c^2 - 56*a^3*b^2*c^3 - 11*a*b^6
*c + 5*a^2*b*c^2*(-(4*a*c - b^2)^3)^(1/2) - 5*a*b^3*c*(-(4*a*c - b^2)^3)^(1
/2))/(c^5*(4*a*c - b^2)^3))^(1/3))/12 - (9*a*b*(b^6 - 12*a^3*c^3 + 19*a^2*b
^2*c^2 - 8*a*b^4*c))/c^2)*(-(b^8 + 16*a^4*c^4 + b^5*(-(4*a*c - b^2)^3)^(1/2
) + 41*a^2*b^4*c^2 - 56*a^3*b^2*c^3 - 11*a*b^6*c + 5*a^2*b*c^2*(-(4*a*c - b
^2)^3)^(1/2) - 5*a*b^3*c*(-(4*a*c - b^2)^3)^(1/2))/(c^5*(4*a*c - b^2)^3))^(
2/3))/36)*((3^(1/2)*1i)/2 - 1/2)*(-(b^8 + 16*a^4*c^4 + b^5*(-(4*a*c - b^2)^
3)^(1/2) + 41*a^2*b^4*c^2 - 56*a^3*b^2*c^3 - 11*a*b^6*c + 5*a^2*b*c^2*(-(4*
a*c - b^2)^3)^(1/2) - 5*a*b^3*c*(-(4*a*c - b^2)^3)^(1/2))/(54*(64*a^3*c^8 -
 b^6*c^5 + 12*a*b^4*c^6 - 48*a^2*b^2*c^7)))^(1/3) - log((a^4*x*(a*c - b^2))
/c^2 - (2^(1/3)*(3^(1/2)*1i - 1)*((2^(2/3)*(3^(1/2)*1i + 1)*(27*a^2*c*x*(b^
4 + 8*a^2*c^2 - 6*a*b^2*c) + (27*2^(1/3)*a*b*c^3*(3^(1/2)*1i - 1)*(4*a*c -
b^2)^2*(-(b^8 + 16*a^4*c^4 - b^5*(-(4*a*c - b^2)^3)^(1/2) + 41*a^2*b^4*c^2
- 56*a^3*b^2*c^3 - 11*a*b^6*c - 5*a^2*b*c^2*(-(4*a*c - b^2)^3)^(1/2) + 5*a*
b^3*c*(-(4*a*c - b^2)^3)^(1/2))/(c^5*(4*a*c - b^2)^3))^(2/3))/4)*(-(b^8 + 1
6*a^4*c^4 - b^5*(-(4*a*c - b^2)^3)^(1/2) + 41*a^2*b^4*c^2 - 56*a^3*b^2*c^3
- 11*a*b^6*c - 5*a^2*b*c^2*(-(4*a*c - b^2)^3)^(1/2) + 5*a*b^3*c*(-(4*a*c -
b^2)^3)^(1/2))/(c^5*(4*a*c - b^2)^3))^(1/3))/12 + (9*a*b*(b^6 - 12*a^3*c^3
+ 19*a^2*b^2*c^2 - 8*a*b^4*c))/c^2)*(-(b^8 + 16*a^4*c^4 - b^5*(-(4*a*c - b^
2)^3)^(1/2) + 41*a^2*b^4*c^2 - 56*a^3*b^2*c^3 - 11*a*b^6*c - 5*a^2*b*c^2*(-
(4*a*c - b^2)^3)^(1/2) + 5*a*b^3*c*(-(4*a*c - b^2)^3)^(1/2))/(c^5*(4*a*c -
b^2)^3))^(2/3))/36)*((3^(1/2)*1i)/2 + 1/2)*(-(b^8 + 16*a^4*c^4 - b^5*(-(4*a
*c - b^2)^3)^(1/2) + 41*a^2*b^4*c^2 - 56*a^3*b^2*c^3 - 11*a*b^6*c - 5*a^2*b
*c^2*(-(4*a*c - b^2)^3)^(1/2) + 5*a*b^3*c*(-(4*a*c - b^2)^3)^(1/2))/(54*(64
*a^3*c^8 - b^6*c^5 + 12*a*b^4*c^6 - 48*a^2*b^2*c^7)))^(1/3) + log((a^4*x*(a
*c - b^2))/c^2 - (2^(1/3)*(3^(1/2)*1i + 1)*((2^(2/3)*(3^(1/2)*1i - 1)*(27*a
^2*c*x*(b^4 + 8*a^2*c^2 - 6*a*b^2*c) - (27*2^(1/3)*a*b*c^3*(3^(1/2)*1i + 1)
```

```
*(4*a*c - b^2)^2*(-(b^8 + 16*a^4*c^4 - b^5*(-(4*a*c - b^2)^3)^(1/2) + 41*a^
2*b^4*c^2 - 56*a^3*b^2*c^3 - 11*a*b^6*c - 5*a^2*b*c^2*(-(4*a*c - b^2)^3)^(1
/2) + 5*a*b^3*c*(-(4*a*c - b^2)^3)^(1/2))/(c^5*(4*a*c - b^2)^3))^(2/3))/4)*
(-(b^8 + 16*a^4*c^4 - b^5*(-(4*a*c - b^2)^3)^(1/2) + 41*a^2*b^4*c^2 - 56*a^
3*b^2*c^3 - 11*a*b^6*c - 5*a^2*b*c^2*(-(4*a*c - b^2)^3)^(1/2) + 5*a*b^3*c*(
-(4*a*c - b^2)^3)^(1/2))/(c^5*(4*a*c - b^2)^3))^(1/3))/12 - (9*a*b*(b^6 - 1
2*a^3*c^3 + 19*a^2*b^2*c^2 - 8*a*b^4*c))/c^2)*(-(b^8 + 16*a^4*c^4 - b^5*(-(
4*a*c - b^2)^3)^(1/2) + 41*a^2*b^4*c^2 - 56*a^3*b^2*c^3 - 11*a*b^6*c - 5*a^
2*b*c^2*(-(4*a*c - b^2)^3)^(1/2) + 5*a*b^3*c*(-(4*a*c - b^2)^3)^(1/2))/(c^5
*(4*a*c - b^2)^3))^(2/3))/36)*((3^(1/2)*1i)/2 - 1/2)*(-(b^8 + 16*a^4*c^4 -
b^5*(-(4*a*c - b^2)^3)^(1/2) + 41*a^2*b^4*c^2 - 56*a^3*b^2*c^3 - 11*a*b^6*c
 - 5*a^2*b*c^2*(-(4*a*c - b^2)^3)^(1/2) + 5*a*b^3*c*(-(4*a*c - b^2)^3)^(1/2
))/(54*(64*a^3*c^8 - b^6*c^5 + 12*a*b^4*c^6 - 48*a^2*b^2*c^7)))^(1/3)
```

sympy [A] time = 14.58, size = 279, normalized size = 0.44

$$\operatorname{RootSum}\left(t^{6}\left(46656a^{3}c^{8}-34992a^{2}b^{2}c^{7}+8748ab^{4}c^{6}-729b^{6}c^{5}\right)+t^{3}\left(432a^{4}c^{4}-1512a^{3}b^{2}c^{3}+1107a^{2}b^{4}c^{2}-297ab^{6}c+27b^{8}\right)+a^{5},\left(t\mapsto t\log\left(x+\frac{-15552t^{5}a^{4}c^{9}+27216t^{5}a^{3}b^{2}c^{8}-14580t^{5}a^{2}b^{4}c^{7}+3159t^{5}ab^{6}c^{6}-243t^{5}b^{8}c^{5}-72t^{2}a^{5}c^{5}+594t^{2}a^{4}b^{2}c^{4}-864t^{2}a^{3}b^{4}c^{3}+468t^{2}a^{2}b^{6}c^{2}-108t^{2}ab^{8}c+9t^{2}b^{10}}{5a^{5}bc^{2}-5a^{4}b^{3}c+a^{3}b^{5}}\right)\right)\right)+\frac{x^{2}}{2c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7/(c*x**6+b*x**3+a),x)
```

```
[Out] RootSum(_t**6*(46656*a**3*c**8 - 34992*a**2*b**2*c**7 + 8748*a*b**4*c**6 -
729*b**6*c**5) + _t**3*(432*a**4*c**4 - 1512*a**3*b**2*c**3 + 1107*a**2*b**
4*c**2 - 297*a*b**6*c + 27*b**8) + a**5, Lambda(_t, _t*log(x + (-15552*_t**
5*a**4*c**9 + 27216*_t**5*a**3*b**2*c**8 - 14580*_t**5*a**2*b**4*c**7 + 315
9*_t**5*a*b**6*c**6 - 243*_t**5*b**8*c**5 - 72*_t**2*a**5*c**5 + 594*_t**2*
a**4*b**2*c**4 - 864*_t**2*a**3*b**4*c**3 + 468*_t**2*a**2*b**6*c**2 - 108*
_t**2*a*b**8*c + 9*_t**2*b**10)/(5*a**5*b*c**2 - 5*a**4*b**3*c + a**3*b**5)
))) + x**2/(2*c)
```

3.131 $\int \frac{x^6}{a+bx^3+cx^6}\,dx$

Optimal. Leaf size=631

$$\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\log\left(-\sqrt[3]{2}\sqrt[3]{c}\,x\sqrt[3]{b-\sqrt{b^2-4ac}}+\left(b-\sqrt{b^2-4ac}\right)^{2/3}+2^{2/3}c^{2/3}x^2\right)}{6\sqrt[3]{2}\,c^{4/3}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}+\frac{\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\log\left(-\sqrt[3]{2}\sqrt[3]{c}\,x\ldots\right.}{6\sqrt[3]{\ldots}}$$

Rubi [A] time = 1.02, antiderivative size = 631, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 8, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {1367, 1422, 200, 31, 634, 617, 204, 628}

$$\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\log\left(-\sqrt[3]{2}\sqrt[3]{c}\,x\sqrt[3]{b-\sqrt{b^2-4ac}}+\left(b-\sqrt{b^2-4ac}\right)^{2/3}+2^{2/3}c^{2/3}x^2\right)}{6\sqrt[3]{2}\,c^{4/3}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}+\frac{\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\log\left(-\sqrt[3]{2}\sqrt[3]{c}\,x\sqrt[3]{\sqrt{b^2-4ac}+b}+\left(\sqrt{b^2-4ac}+b\right)^{2/3}+2^{2/3}c^{2/3}x^2\right)}{6\sqrt[3]{2}\,c^{4/3}\left(\sqrt{b^2-4ac}+b\right)^{2/3}}-\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}\,x\right)}{3\sqrt[3]{2}\,c^{4/3}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}-\frac{\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\log\left(\sqrt[3]{\sqrt{b^2-4ac}+b}+\sqrt[3]{2}\sqrt[3]{c}\,x\right)}{3\sqrt[3]{2}\,c^{4/3}\left(\sqrt{b^2-4ac}+b\right)^{2/3}}+\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}\,x}{\sqrt[3]{b-\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}\,c^{4/3}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}+\frac{\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}\,x}{\sqrt[3]{\sqrt{b^2-4ac}+b}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}\,c^{4/3}\left(\sqrt{b^2-4ac}+b\right)^{2/3}}+\frac{x}{c}$$

Antiderivative was successfully verified.

```
[In] Int[x^6/(a + b*x^3 + c*x^6),x]
```

```
[Out] x/c + ((b - (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(1 - (2*2^(1/3)*c^(1/3)
*x)/(b - Sqrt[b^2 - 4*a*c])^(1/3))/Sqrt[3]])/(2^(1/3)*Sqrt[3]*c^(4/3)*(b -
Sqrt[b^2 - 4*a*c])^(2/3)) + ((b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(
1 - (2*2^(1/3)*c^(1/3)*x)/(b + Sqrt[b^2 - 4*a*c])^(1/3))/Sqrt[3]])/(2^(1/3)
*Sqrt[3]*c^(4/3)*(b + Sqrt[b^2 - 4*a*c])^(2/3)) - ((b - (b^2 - 2*a*c)/Sqrt[
b^2 - 4*a*c])*Log[(b - Sqrt[b^2 - 4*a*c])^(1/3) + 2^(1/3)*c^(1/3)*x])/(3*2^
(1/3)*c^(4/3)*(b - Sqrt[b^2 - 4*a*c])^(2/3)) - ((b + (b^2 - 2*a*c)/Sqrt[b^2
 - 4*a*c])*Log[(b + Sqrt[b^2 - 4*a*c])^(1/3) + 2^(1/3)*c^(1/3)*x])/(3*2^(1/
3)*c^(4/3)*(b + Sqrt[b^2 - 4*a*c])^(2/3)) + ((b - (b^2 - 2*a*c)/Sqrt[b^2 -
4*a*c])*Log[(b - Sqrt[b^2 - 4*a*c])^(2/3) - 2^(1/3)*c^(1/3)*(b - Sqrt[b^2 -
 4*a*c])^(1/3)*x + 2^(2/3)*c^(2/3)*x^2])/(6*2^(1/3)*c^(4/3)*(b - Sqrt[b^2 -
 4*a*c])^(2/3)) + ((b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*Log[(b + Sqrt[b^2
- 4*a*c])^(2/3) - 2^(1/3)*c^(1/3)*(b + Sqrt[b^2 - 4*a*c])^(1/3)*x + 2^(2/3)
*c^(2/3)*x^2])/(6*2^(1/3)*c^(4/3)*(b + Sqrt[b^2 - 4*a*c])^(2/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
```

```
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1367

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Simp[(d^(2*n - 1)*(d*x)^(m - 2*n + 1)*(a + b*x^n + c*x^(2*n))^(
p + 1))/(c*(m + 2*n*p + 1)), x] - Dist[d^(2*n)/(c*(m + 2*n*p + 1)), Int[(d*
x)^(m - 2*n)*Simp[a*(m - 2*n + 1) + b*(m + n*(p - 1) + 1)*x^n, x]*(a + b*x^
n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && EqQ[n2, 2*n] && Ne
Q[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GtQ[m, 2*n - 1] && NeQ[m + 2*n*p + 1, 0]
 && IntegerQ[p]
```

Rule 1422

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q),
Int[1/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(
b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (PosQ[b^2 - 4*a
*c] ||  !IGtQ[n/2, 0])
```

Rubi steps

$$\int \frac{x^6}{a+bx^3+cx^6}\,dx = \frac{x}{c} - \frac{\int \frac{a+bx^3}{a+bx^3+cx^6}\,dx}{c}$$

$$= \frac{x}{c} - \frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^3}\,dx}{2c} - \frac{\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^3}\,dx}{2c}$$

$$= \frac{x}{c} - \frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\frac{\sqrt[3]{b-\sqrt{b^2-4ac}}}{\sqrt[3]{2}}+\sqrt[3]{c}\,x}\,dx}{3\sqrt[3]{2}\,c\left(b-\sqrt{b^2-4ac}\right)^{2/3}} - \frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\int \frac{2^{2/3}\sqrt[3]{b-\sqrt{b^2-4ac}}-\sqrt[3]{c}\,x}{\frac{\left(b-\sqrt{b^2-4ac}\right)^{2/3}}{2^{2/3}}-\frac{\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}\,x}{\sqrt[3]{2}}+c^{2/3}x^2}}{3\sqrt[3]{2}\,c\left(b-\sqrt{b^2-4ac}\right)^{2/3}}$$

$$= \frac{x}{c} - \frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}\,x\right)}{3\sqrt[3]{2}\,c^{4/3}\left(b-\sqrt{b^2-4ac}\right)^{2/3}} - \frac{\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{b+\sqrt{b^2-4a}}\right)}{3\sqrt[3]{2}\,c^{4/3}\left(b+\sqrt{b^2-4ac}\right)}$$

$$= \frac{x}{c} - \frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}\,x\right)}{3\sqrt[3]{2}\,c^{4/3}\left(b-\sqrt{b^2-4ac}\right)^{2/3}} - \frac{\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{b+\sqrt{b^2-4a}}\right)}{3\sqrt[3]{2}\,c^{4/3}\left(b+\sqrt{b^2-4ac}\right)}$$

$$= \frac{x}{c} + \frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}\,x}{\sqrt[3]{b-\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}\,c^{4/3}\left(b-\sqrt{b^2-4ac}\right)^{2/3}} + \frac{\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}\,x}{\sqrt[3]{b+\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}\,c^{4/3}\left(b+\sqrt{b^2-4ac}\right)^{2/3}} - \frac{\left(b-\frac{b^2-2}{\sqrt{b^2-}}\right)}{}$$

Mathematica [C] time = 0.03, size = 70, normalized size = 0.11

$$\frac{x}{c} - \frac{\text{RootSum}\left[\#1^6c+\#1^3b+a\&,\frac{\#1^3b\log(x-\#1)+a\log(x-\#1)}{2\#1^5c+\#1^2b}\&\right]}{3c}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6/(a + b*x^3 + c*x^6),x]
```

```
[Out] x/c - RootSum[a + b*#1^3 + c*#1^6 & , (a*Log[x - #1] + b*Log[x - #1]*#1^3)/
(b*#1^2 + 2*c*#1^5) & ]/(3*c)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6}{a+bx^3+cx^6}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^6/(a + b*x^3 + c*x^6),x]
```

```
[Out] IntegrateAlgebraic[x^6/(a + b*x^3 + c*x^6), x]
```

fricas [B] time = 3.18, size = 5260, normalized size = 8.34

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(c*x^6+b*x^3+a),x, algorithm="fricas")
```

```
[Out] 1/6*(4*sqrt(3)*(1/2)^(1/3)*c*(-(b^3 - 2*a*b*c + (b^2*c^4 - 4*a*c^5)*sqrt((b
^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(b^6*c^8 - 12
*a*b^4*c^9 + 48*a^2*b^2*c^10 - 64*a^3*c^11)))/(b^2*c^4 - 4*a*c^5))^(1/3)*ar
ctan(-1/6*(2*(1/2)^(2/3)*(sqrt(3)*(b^8*c^4 - 13*a*b^6*c^5 + 60*a^2*b^4*c^6
- 112*a^3*b^2*c^7 + 64*a^4*c^8)*x*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 -
16*a^3*b^2*c^3 + 4*a^4*c^4)/(b^6*c^8 - 12*a*b^4*c^9 + 48*a^2*b^2*c^10 - 64*
a^3*c^11)) - sqrt(3)*(b^9 - 11*a*b^7*c + 42*a^2*b^5*c^2 - 62*a^3*b^3*c^3 +
24*a^4*b*c^4)*x)*(-(b^3 - 2*a*b*c + (b^2*c^4 - 4*a*c^5)*sqrt((b^8 - 8*a*b^6
*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(b^6*c^8 - 12*a*b^4*c^9 +
 48*a^2*b^2*c^10 - 64*a^3*c^11)))/(b^2*c^4 - 4*a*c^5))^(2/3) - (1/2)^(1/6)*
(sqrt(3)*(b^8*c^4 - 13*a*b^6*c^5 + 60*a^2*b^4*c^6 - 112*a^3*b^2*c^7 + 64*a^
4*c^8)*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)
/(b^6*c^8 - 12*a*b^4*c^9 + 48*a^2*b^2*c^10 - 64*a^3*c^11)) - sqrt(3)*(b^9 -
 11*a*b^7*c + 42*a^2*b^5*c^2 - 62*a^3*b^3*c^3 + 24*a^4*b*c^4))*(-(b^3 - 2*a
*b*c + (b^2*c^4 - 4*a*c^5)*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*
b^2*c^3 + 4*a^4*c^4)/(b^6*c^8 - 12*a*b^4*c^9 + 48*a^2*b^2*c^10 - 64*a^3*c^1
1)))/(b^2*c^4 - 4*a*c^5))^(2/3)*sqrt((2*(a^2*b^4 - 4*a^3*b^2*c + 2*a^4*c^2)
*x^2 + (1/2)^(2/3)*(b^8 - 10*a*b^6*c + 34*a^2*b^4*c^2 - 44*a^3*b^2*c^3 + 16
*a^4*c^4 - (b^7*c^4 - 12*a*b^5*c^5 + 48*a^2*b^3*c^6 - 64*a^3*b*c^7)*sqrt((b
^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(b^6*c^8 - 12
*a*b^4*c^9 + 48*a^2*b^2*c^10 - 64*a^3*c^11)))*(-(b^3 - 2*a*b*c + (b^2*c^4 -
 4*a*c^5)*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c
^4)/(b^6*c^8 - 12*a*b^4*c^9 + 48*a^2*b^2*c^10 - 64*a^3*c^11)))/(b^2*c^4 - 4
*a*c^5))^(2/3) + (1/2)^(1/3)*((a*b^5*c^4 - 8*a^2*b^3*c^5 + 16*a^3*b*c^6)*x*
sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(b^6*c
^8 - 12*a*b^4*c^9 + 48*a^2*b^2*c^10 - 64*a^3*c^11)) - (a*b^6 - 8*a^2*b^4*c
+ 18*a^3*b^2*c^2 - 8*a^4*c^3)*x)*(-(b^3 - 2*a*b*c + (b^2*c^4 - 4*a*c^5)*sqr
t((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(b^6*c^8
- 12*a*b^4*c^9 + 48*a^2*b^2*c^10 - 64*a^3*c^11)))/(b^2*c^4 - 4*a*c^5))^(1/3
))/(a^2*b^4 - 4*a^3*b^2*c + 2*a^4*c^2)) + 2*sqrt(3)*(a^3*b^4 - 4*a^4*b^2*c
+ 2*a^5*c^2))/(a^3*b^4 - 4*a^4*b^2*c + 2*a^5*c^2)) - 4*sqrt(3)*(1/2)^(1/3)*
c*(-(b^3 - 2*a*b*c - (b^2*c^4 - 4*a*c^5)*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4
*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(b^6*c^8 - 12*a*b^4*c^9 + 48*a^2*b^2*c^1
0 - 64*a^3*c^11)))/(b^2*c^4 - 4*a*c^5))^(1/3)*arctan(-1/6*(2*(1/2)^(2/3)*(s
qrt(3)*(b^8*c^4 - 13*a*b^6*c^5 + 60*a^2*b^4*c^6 - 112*a^3*b^2*c^7 + 64*a^4*
c^8)*x*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)
/(b^6*c^8 - 12*a*b^4*c^9 + 48*a^2*b^2*c^10 - 64*a^3*c^11)) + sqrt(3)*(b^9 -
 11*a*b^7*c + 42*a^2*b^5*c^2 - 62*a^3*b^3*c^3 + 24*a^4*b*c^4)*x)*(-(b^3 - 2
*a*b*c - (b^2*c^4 - 4*a*c^5)*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^
3*b^2*c^3 + 4*a^4*c^4)/(b^6*c^8 - 12*a*b^4*c^9 + 48*a^2*b^2*c^10 - 64*a^3*c
^11)))/(b^2*c^4 - 4*a*c^5))^(2/3) - (1/2)^(1/6)*(sqrt(3)*(b^8*c^4 - 13*a*b^
6*c^5 + 60*a^2*b^4*c^6 - 112*a^3*b^2*c^7 + 64*a^4*c^8)*sqrt((b^8 - 8*a*b^6*
c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(b^6*c^8 - 12*a*b^4*c^9 +
48*a^2*b^2*c^10 - 64*a^3*c^11)) + sqrt(3)*(b^9 - 11*a*b^7*c + 42*a^2*b^5*c^
2 - 62*a^3*b^3*c^3 + 24*a^4*b*c^4))*(-(b^3 - 2*a*b*c - (b^2*c^4 - 4*a*c^5)*
sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(b^6*c
^8 - 12*a*b^4*c^9 + 48*a^2*b^2*c^10 - 64*a^3*c^11)))/(b^2*c^4 - 4*a*c^5))^(
2/3)*sqrt((2*(a^2*b^4 - 4*a^3*b^2*c + 2*a^4*c^2)*x^2 + (1/2)^(2/3)*(b^8 - 1
0*a*b^6*c + 34*a^2*b^4*c^2 - 44*a^3*b^2*c^3 + 16*a^4*c^4 + (b^7*c^4 - 12*a*
b^5*c^5 + 48*a^2*b^3*c^6 - 64*a^3*b*c^7)*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4
*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(b^6*c^8 - 12*a*b^4*c^9 + 48*a^2*b^2*c^1
0 - 64*a^3*c^11)))*(-(b^3 - 2*a*b*c - (b^2*c^4 - 4*a*c^5)*sqrt((b^8 - 8*a*b
^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(b^6*c^8 - 12*a*b^4*c^9
 + 48*a^2*b^2*c^10 - 64*a^3*c^11)))/(b^2*c^4 - 4*a*c^5))^(2/3) - (1/2)^(1/3
)*((a*b^5*c^4 - 8*a^2*b^3*c^5 + 16*a^3*b*c^6)*x*sqrt((b^8 - 8*a*b^6*c + 20*
a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(b^6*c^8 - 12*a*b^4*c^9 + 48*a^2*
b^2*c^10 - 64*a^3*c^11)) + (a*b^6 - 8*a^2*b^4*c + 18*a^3*b^2*c^2 - 8*a^4*c^
3)*x)*(-(b^3 - 2*a*b*c - (b^2*c^4 - 4*a*c^5)*sqrt((b^8 - 8*a*b^6*c + 20*a^2
```

```
*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(b^6*c^8 - 12*a*b^4*c^9 + 48*a^2*b^2
*c^10 - 64*a^3*c^11)))/(b^2*c^4 - 4*a*c^5))^(1/3))/(a^2*b^4 - 4*a^3*b^2*c +
 2*a^4*c^2)) - 2*sqrt(3)*(a^3*b^4 - 4*a^4*b^2*c + 2*a^5*c^2))/(a^3*b^4 - 4*
a^4*b^2*c + 2*a^5*c^2)) - (1/2)^(1/3)*c*(-(b^3 - 2*a*b*c + (b^2*c^4 - 4*a*c
^5)*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(b
^6*c^8 - 12*a*b^4*c^9 + 48*a^2*b^2*c^10 - 64*a^3*c^11)))/(b^2*c^4 - 4*a*c^5
))^(1/3)*log(2*(a^2*b^4 - 4*a^3*b^2*c + 2*a^4*c^2)*x^2 + (1/2)^(2/3)*(b^8 -
 10*a*b^6*c + 34*a^2*b^4*c^2 - 44*a^3*b^2*c^3 + 16*a^4*c^4 - (b^7*c^4 - 12*
a*b^5*c^5 + 48*a^2*b^3*c^6 - 64*a^3*b*c^7)*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b
^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(b^6*c^8 - 12*a*b^4*c^9 + 48*a^2*b^2*c
^10 - 64*a^3*c^11)))*(-(b^3 - 2*a*b*c + (b^2*c^4 - 4*a*c^5)*sqrt((b^8 - 8*a
*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(b^6*c^8 - 12*a*b^4*c
^9 + 48*a^2*b^2*c^10 - 64*a^3*c^11)))/(b^2*c^4 - 4*a*c^5))^(2/3) + (1/2)^(1
/3)*((a*b^5*c^4 - 8*a^2*b^3*c^5 + 16*a^3*b*c^6)*x*sqrt((b^8 - 8*a*b^6*c + 2
0*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(b^6*c^8 - 12*a*b^4*c^9 + 48*a^
2*b^2*c^10 - 64*a^3*c^11)) - (a*b^6 - 8*a^2*b^4*c + 18*a^3*b^2*c^2 - 8*a^4*
c^3)*x)*(-(b^3 - 2*a*b*c + (b^2*c^4 - 4*a*c^5)*sqrt((b^8 - 8*a*b^6*c + 20*a
^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(b^6*c^8 - 12*a*b^4*c^9 + 48*a^2*b
^2*c^10 - 64*a^3*c^11)))/(b^2*c^4 - 4*a*c^5))^(1/3)) - (1/2)^(1/3)*c*(-(b^3
 - 2*a*b*c - (b^2*c^4 - 4*a*c^5)*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 1
6*a^3*b^2*c^3 + 4*a^4*c^4)/(b^6*c^8 - 12*a*b^4*c^9 + 48*a^2*b^2*c^10 - 64*a
^3*c^11)))/(b^2*c^4 - 4*a*c^5))^(1/3)*log(2*(a^2*b^4 - 4*a^3*b^2*c + 2*a^4*
c^2)*x^2 + (1/2)^(2/3)*(b^8 - 10*a*b^6*c + 34*a^2*b^4*c^2 - 44*a^3*b^2*c^3
+ 16*a^4*c^4 + (b^7*c^4 - 12*a*b^5*c^5 + 48*a^2*b^3*c^6 - 64*a^3*b*c^7)*sqr
t((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(b^6*c^8
- 12*a*b^4*c^9 + 48*a^2*b^2*c^10 - 64*a^3*c^11)))*(-(b^3 - 2*a*b*c - (b^2*c
^4 - 4*a*c^5)*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a
^4*c^4)/(b^6*c^8 - 12*a*b^4*c^9 + 48*a^2*b^2*c^10 - 64*a^3*c^11)))/(b^2*c^4
 - 4*a*c^5))^(2/3) - (1/2)^(1/3)*((a*b^5*c^4 - 8*a^2*b^3*c^5 + 16*a^3*b*c^6
)*x*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(b
^6*c^8 - 12*a*b^4*c^9 + 48*a^2*b^2*c^10 - 64*a^3*c^11)) + (a*b^6 - 8*a^2*b^
4*c + 18*a^3*b^2*c^2 - 8*a^4*c^3)*x)*(-(b^3 - 2*a*b*c - (b^2*c^4 - 4*a*c^5)
*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(b^6*
c^8 - 12*a*b^4*c^9 + 48*a^2*b^2*c^10 - 64*a^3*c^11)))/(b^2*c^4 - 4*a*c^5))^
(1/3)) + 2*(1/2)^(1/3)*c*(-(b^3 - 2*a*b*c + (b^2*c^4 - 4*a*c^5)*sqrt((b^8 -
 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(b^6*c^8 - 12*a*b
^4*c^9 + 48*a^2*b^2*c^10 - 64*a^3*c^11)))/(b^2*c^4 - 4*a*c^5))^(1/3)*log(2*
(a*b^4 - 4*a^2*b^2*c + 2*a^3*c^2)*x + (1/2)^(1/3)*(b^6 - 8*a*b^4*c + 18*a^2
*b^2*c^2 - 8*a^3*c^3 - (b^5*c^4 - 8*a*b^3*c^5 + 16*a^2*b*c^6)*sqrt((b^8 - 8
*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(b^6*c^8 - 12*a*b^4
*c^9 + 48*a^2*b^2*c^10 - 64*a^3*c^11)))*(-(b^3 - 2*a*b*c + (b^2*c^4 - 4*a*c
^5)*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(b
^6*c^8 - 12*a*b^4*c^9 + 48*a^2*b^2*c^10 - 64*a^3*c^11)))/(b^2*c^4 - 4*a*c^5
))^(1/3)) + 2*(1/2)^(1/3)*c*(-(b^3 - 2*a*b*c - (b^2*c^4 - 4*a*c^5)*sqrt((b^
8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(b^6*c^8 - 12*
a*b^4*c^9 + 48*a^2*b^2*c^10 - 64*a^3*c^11)))/(b^2*c^4 - 4*a*c^5))^(1/3)*log
(2*(a*b^4 - 4*a^2*b^2*c + 2*a^3*c^2)*x + (1/2)^(1/3)*(b^6 - 8*a*b^4*c + 18*
a^2*b^2*c^2 - 8*a^3*c^3 + (b^5*c^4 - 8*a*b^3*c^5 + 16*a^2*b*c^6)*sqrt((b^8
- 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(b^6*c^8 - 12*a*
b^4*c^9 + 48*a^2*b^2*c^10 - 64*a^3*c^11)))*(-(b^3 - 2*a*b*c - (b^2*c^4 - 4*
a*c^5)*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)
/(b^6*c^8 - 12*a*b^4*c^9 + 48*a^2*b^2*c^10 - 64*a^3*c^11)))/(b^2*c^4 - 4*a*
c^5))^(1/3)) + 6*x)/c
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6}{cx^6 + bx^3 + a} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(c*x^6+b*x^3+a),x, algorithm="giac")
```

```
[Out] integrate(x^6/(c*x^6 + b*x^3 + a), x)
```

maple [C] time = 0.00, size = 59, normalized size = 0.09

$$\frac{x}{c}+\frac{\left(-\mathrm{RootOf}\left(c\_Z^{6}+b\_Z^{3}+a\right)^{3}b-a\right)\ln\left(-\mathrm{RootOf}\left(c\_Z^{6}+b\_Z^{3}+a\right)+x\right)}{3c\left(2\,\mathrm{RootOf}\left(c\_Z^{6}+b\_Z^{3}+a\right)^{5}c+\mathrm{RootOf}\left(c\_Z^{6}+b\_Z^{3}+a\right)^{2}b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(c*x^6+b*x^3+a),x)
```

```
[Out] x/c+1/3/c*sum((-_R^3*b-a)/(2*_R^5*c+_R^2*b)*ln(-_R+x),_R=RootOf(_Z^6*c+_Z^3
*b+a))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{x}{c}-\frac{\int\frac{bx^{3}+a}{cx^{6}+bx^{3}+a}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(c*x^6+b*x^3+a),x, algorithm="maxima")
```

```
[Out] x/c - integrate((b*x^3 + a)/(c*x^6 + b*x^3 + a), x)/c
```

mupad [B] time = 3.40, size = 2280, normalized size = 3.61

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(a + b*x^3 + c*x^6),x)
```

```
[Out] log((3*a^2*x*(b^4 + 2*a^2*c^2 - 4*a*b^2*c))/c - (3*2^(2/3)*a*(-(b^4*(-(4*a*
c - b^2)^3)^(1/2) - b^7 + 32*a^3*b*c^3 - 32*a^2*b^3*c^2 + 2*a^2*c^2*(-(4*a*
c - b^2)^3)^(1/2) + 10*a*b^5*c - 4*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(c^4*(
4*a*c - b^2)^3))^(1/3)*(b^4 + 2*a^2*c^2 - 4*a*b^2*c)*(b*(-(4*a*c - b^2)^3)^
(1/2) + b^4 + 16*a^2*c^2 - 8*a*b^2*c))/(4*c*(4*a*c - b^2)))*(-(b^4*(-(4*a*c
 - b^2)^3)^(1/2) - b^7 + 32*a^3*b*c^3 - 32*a^2*b^3*c^2 + 2*a^2*c^2*(-(4*a*c
 - b^2)^3)^(1/2) + 10*a*b^5*c - 4*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(54*(64
*a^3*c^7 - b^6*c^4 + 12*a*b^4*c^5 - 48*a^2*b^2*c^6)))^(1/3) + x/c + log((3*
a^2*x*(b^4 + 2*a^2*c^2 - 4*a*b^2*c))/c + (3*2^(2/3)*a*((b^7 + b^4*(-(4*a*c
- b^2)^3)^(1/2) - 32*a^3*b*c^3 + 32*a^2*b^3*c^2 + 2*a^2*c^2*(-(4*a*c - b^2)
^3)^(1/2) - 10*a*b^5*c - 4*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(c^4*(4*a*c -
b^2)^3))^(1/3)*(b^4 + 2*a^2*c^2 - 4*a*b^2*c)*(b*(-(4*a*c - b^2)^3)^(1/2) -
b^4 - 16*a^2*c^2 + 8*a*b^2*c))/(4*c*(4*a*c - b^2)))*((b^7 + b^4*(-(4*a*c -
b^2)^3)^(1/2) - 32*a^3*b*c^3 + 32*a^2*b^3*c^2 + 2*a^2*c^2*(-(4*a*c - b^2)^3
)^(1/2) - 10*a*b^5*c - 4*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(54*(64*a^3*c^7
- b^6*c^4 + 12*a*b^4*c^5 - 48*a^2*b^2*c^6)))^(1/3) + log((3*a^2*x*(b^4 + 2*
a^2*c^2 - 4*a*b^2*c))/c + (3*2^(2/3)*a*(3^(1/2)*1i - 1)*((b^7 + b^4*(-(4*a*
c - b^2)^3)^(1/2) - 32*a^3*b*c^3 + 32*a^2*b^3*c^2 + 2*a^2*c^2*(-(4*a*c - b^
2)^3)^(1/2) - 10*a*b^5*c - 4*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(c^4*(4*a*c
- b^2)^3))^(1/3)*(b^4 + 2*a^2*c^2 - 4*a*b^2*c)*(b*(-(4*a*c - b^2)^3)^(1/2)
- b^4 - 16*a^2*c^2 + 8*a*b^2*c))/(8*c*(4*a*c - b^2)))*((3^(1/2)*1i)/2 - 1/2
)*((b^7 + b^4*(-(4*a*c - b^2)^3)^(1/2) - 32*a^3*b*c^3 + 32*a^2*b^3*c^2 + 2*
a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^5*c - 4*a*b^2*c*(-(4*a*c - b^2)^3
)^(1/2))/(54*(64*a^3*c^7 - b^6*c^4 + 12*a*b^4*c^5 - 48*a^2*b^2*c^6)))^(1/3)
 - log((3*a^2*x*(b^4 + 2*a^2*c^2 - 4*a*b^2*c))/c - (3*2^(2/3)*a*(3^(1/2)*1i
 + 1)*((b^7 + b^4*(-(4*a*c - b^2)^3)^(1/2) - 32*a^3*b*c^3 + 32*a^2*b^3*c^2
+ 2*a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^5*c - 4*a*b^2*c*(-(4*a*c - b^
```

```
2)^3)^(1/2))/(c^4*(4*a*c - b^2)^3))^(1/3)*(b^4 + 2*a^2*c^2 - 4*a*b^2*c)*(b*
(-(4*a*c - b^2)^3)^(1/2) - b^4 - 16*a^2*c^2 + 8*a*b^2*c))/(8*c*(4*a*c - b^2
)))*((3^(1/2)*1i)/2 + 1/2)*((b^7 + b^4*(-(4*a*c - b^2)^3)^(1/2) - 32*a^3*b*
c^3 + 32*a^2*b^3*c^2 + 2*a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^5*c - 4*
a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(54*(64*a^3*c^7 - b^6*c^4 + 12*a*b^4*c^5
- 48*a^2*b^2*c^6)))^(1/3) + log((3*a^2*x*(b^4 + 2*a^2*c^2 - 4*a*b^2*c))/c -
 (3*2^(2/3)*a*(3^(1/2)*1i - 1)*(-(b^4*(-(4*a*c - b^2)^3)^(1/2) - b^7 + 32*a
^3*b*c^3 - 32*a^2*b^3*c^2 + 2*a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) + 10*a*b^5*c
 - 4*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(c^4*(4*a*c - b^2)^3))^(1/3)*(b^4 +
2*a^2*c^2 - 4*a*b^2*c)*(b*(-(4*a*c - b^2)^3)^(1/2) + b^4 + 16*a^2*c^2 - 8*a
*b^2*c))/(8*c*(4*a*c - b^2)))*((3^(1/2)*1i)/2 - 1/2)*(-(b^4*(-(4*a*c - b^2)
^3)^(1/2) - b^7 + 32*a^3*b*c^3 - 32*a^2*b^3*c^2 + 2*a^2*c^2*(-(4*a*c - b^2)
^3)^(1/2) + 10*a*b^5*c - 4*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(54*(64*a^3*c^
7 - b^6*c^4 + 12*a*b^4*c^5 - 48*a^2*b^2*c^6)))^(1/3) - log((3*a^2*x*(b^4 +
2*a^2*c^2 - 4*a*b^2*c))/c + (3*2^(2/3)*a*(3^(1/2)*1i + 1)*(-(b^4*(-(4*a*c -
 b^2)^3)^(1/2) - b^7 + 32*a^3*b*c^3 - 32*a^2*b^3*c^2 + 2*a^2*c^2*(-(4*a*c -
 b^2)^3)^(1/2) + 10*a*b^5*c - 4*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(c^4*(4*a
*c - b^2)^3))^(1/3)*(b^4 + 2*a^2*c^2 - 4*a*b^2*c)*(b*(-(4*a*c - b^2)^3)^(1/
2) + b^4 + 16*a^2*c^2 - 8*a*b^2*c))/(8*c*(4*a*c - b^2)))*((3^(1/2)*1i)/2 +
1/2)*(-(b^4*(-(4*a*c - b^2)^3)^(1/2) - b^7 + 32*a^3*b*c^3 - 32*a^2*b^3*c^2
+ 2*a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) + 10*a*b^5*c - 4*a*b^2*c*(-(4*a*c - b^
2)^3)^(1/2))/(54*(64*a^3*c^7 - b^6*c^4 + 12*a*b^4*c^5 - 48*a^2*b^2*c^6)))^(
1/3)
```

sympy [A] time = 6.90, size = 196, normalized size = 0.31

$$\operatorname{RootSum}\left(t^{6}\left(46656a^{3}c^{7}-34992a^{2}b^{2}c^{6}+8748ab^{4}c^{5}-729b^{6}c^{4}\right)+t^{3}\left(864a^{3}bc^{3}-864a^{2}b^{3}c^{2}+270ab^{5}c-27b^{7}\right)+a^{4},\left(t\mapsto t\log\left(x+\frac{1296t^{4}a^{2}bc^{6}-648t^{4}ab^{3}c^{5}+81t^{4}b^{5}c^{4}-12ta^{3}c^{3}+39ta^{2}b^{2}c^{2}-21tab^{4}c+3tb^{6}}{2a^{3}c^{2}-4a^{2}b^{2}c+ab^{4}}\right)\right)\right)+\frac{x}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(c*x**6+b*x**3+a),x)
```

```
[Out] RootSum(_t**6*(46656*a**3*c**7 - 34992*a**2*b**2*c**6 + 8748*a*b**4*c**5 -
729*b**6*c**4) + _t**3*(864*a**3*b*c**3 - 864*a**2*b**3*c**2 + 270*a*b**5*c
 - 27*b**7) + a**4, Lambda(_t, _t*log(x + (1296*_t**4*a**2*b*c**6 - 648*_t*
*4*a*b**3*c**5 + 81*_t**4*b**5*c**4 - 12*_t*a**3*c**3 + 39*_t*a**2*b**2*c**
2 - 21*_t*a*b**4*c + 3*_t*b**6)/(2*a**3*c**2 - 4*a**2*b**2*c + a*b**4)))) +
 x/c
```

3.132 $\int \frac{x^4}{a+bx^3+cx^6} dx$

Optimal. Leaf size=558

$$-\frac{\left(b-\sqrt{b^2-4ac}\right)^{2/3}\log\left(-\sqrt[3]{2}\sqrt[3]{c}\,x\sqrt[3]{b-\sqrt{b^2-4ac}}+\left(b-\sqrt{b^2-4ac}\right)^{2/3}+2^{2/3}c^{2/3}x^2\right)}{6\,2^{2/3}c^{2/3}\sqrt{b^2-4ac}}+\frac{\left(\sqrt{b^2-4ac}+b\right)^{2/3}\,\mathrm{l}}{}$$

Rubi [A] time = 0.52, antiderivative size = 558, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389, Rules used = {1374, 292, 31, 634, 617, 204, 628}

$$-\frac{\left(b-\sqrt{b^2-4ac}\right)^{2/3}\log\left(-\sqrt[3]{2}\sqrt[3]{c}\,x\sqrt[3]{b-\sqrt{b^2-4ac}}+\left(b-\sqrt{b^2-4ac}\right)^{2/3}+2^{2/3}c^{2/3}x^2\right)}{6\,2^{2/3}c^{2/3}\sqrt{b^2-4ac}}+\frac{\left(\sqrt{b^2-4ac}+b\right)^{2/3}\log\left(-\sqrt[3]{2}\sqrt[3]{c}\,x\sqrt[3]{\sqrt{b^2-4ac}+b}+\left(\sqrt{b^2-4ac}+b\right)^{2/3}+2^{2/3}c^{2/3}x^2\right)}{6\,2^{2/3}c^{2/3}\sqrt{b^2-4ac}}+\frac{\left(b-\sqrt{b^2-4ac}\right)^{2/3}\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}\,x\right)}{3\,2^{2/3}c^{2/3}\sqrt{b^2-4ac}}-\frac{\left(\sqrt{b^2-4ac}+b\right)^{2/3}\log\left(\sqrt[3]{\sqrt{b^2-4ac}+b}+\sqrt[3]{2}\sqrt[3]{c}\,x\right)}{3\,2^{2/3}c^{2/3}\sqrt{b^2-4ac}}+\frac{\left(b-\sqrt{b^2-4ac}\right)^{2/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}\,x}{\sqrt[3]{b-\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}\,c^{2/3}\sqrt{b^2-4ac}}-\frac{\left(\sqrt{b^2-4ac}+b\right)^{2/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}\,x}{\sqrt[3]{\sqrt{b^2-4ac}+b}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}\,c^{2/3}\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(a + b*x^3 + c*x^6),x]
```

```
[Out] ((b - Sqrt[b^2 - 4*a*c])^(2/3)*ArcTan[(1 - (2*2^(1/3)*c^(1/3)*x)/(b - Sqrt[
b^2 - 4*a*c])^(1/3))/Sqrt[3]])/(2^(2/3)*Sqrt[3]*c^(2/3)*Sqrt[b^2 - 4*a*c])
- ((b + Sqrt[b^2 - 4*a*c])^(2/3)*ArcTan[(1 - (2*2^(1/3)*c^(1/3)*x)/(b + Sqr
t[b^2 - 4*a*c])^(1/3))/Sqrt[3]])/(2^(2/3)*Sqrt[3]*c^(2/3)*Sqrt[b^2 - 4*a*c]
) + ((b - Sqrt[b^2 - 4*a*c])^(2/3)*Log[(b - Sqrt[b^2 - 4*a*c])^(1/3) + 2^(1
/3)*c^(1/3)*x])/(3*2^(2/3)*c^(2/3)*Sqrt[b^2 - 4*a*c]) - ((b + Sqrt[b^2 - 4*
a*c])^(2/3)*Log[(b + Sqrt[b^2 - 4*a*c])^(1/3) + 2^(1/3)*c^(1/3)*x])/(3*2^(2
/3)*c^(2/3)*Sqrt[b^2 - 4*a*c]) - ((b - Sqrt[b^2 - 4*a*c])^(2/3)*Log[(b - Sq
rt[b^2 - 4*a*c])^(2/3) - 2^(1/3)*c^(1/3)*(b - Sqrt[b^2 - 4*a*c])^(1/3)*x +
2^(2/3)*c^(2/3)*x^2])/(6*2^(2/3)*c^(2/3)*Sqrt[b^2 - 4*a*c]) + ((b + Sqrt[b^
2 - 4*a*c])^(2/3)*Log[(b + Sqrt[b^2 - 4*a*c])^(2/3) - 2^(1/3)*c^(1/3)*(b +
Sqrt[b^2 - 4*a*c])^(1/3)*x + 2^(2/3)*c^(2/3)*x^2])/(6*2^(2/3)*c^(2/3)*Sqrt[
b^2 - 4*a*c])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1374

```
Int[((d_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(d^n*(b/q + 1))/2, Int[(d*x)^(m -
 n)/(b/2 + q/2 + c*x^n), x], x] - Dist[(d^n*(b/q - 1))/2, Int[(d*x)^(m - n)
/(b/2 - q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[n2, 2*n] &&
NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GeQ[m, n]
```

Rubi steps

$$\int \frac{x^4}{a+bx^3+cx^6}\,dx = -\left(\frac{1}{2}\left(-1+\frac{b}{\sqrt{b^2-4ac}}\right)\int \frac{x}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^3}\,dx\right)+\frac{1}{2}\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)\int \frac{}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-}}$$

$$=\frac{\left(b-\sqrt{b^2-4ac}\right)^{2/3}\int \frac{1}{\frac{\sqrt[3]{b-\sqrt{b^2-4ac}}}{\sqrt[3]{2}}+\sqrt[3]{c}\,x}\,dx}{3\;2^{2/3}\sqrt[3]{c}\sqrt{b^2-4ac}}-\frac{\left(b-\sqrt{b^2-4ac}\right)^{2/3}\int \frac{\frac{\sqrt[3]{b-\sqrt{b^2-4ac}}}{\sqrt[3]{2}}+\sqrt[3]{c}\,x}{\frac{\left(b-\sqrt{b^2-4ac}\right)^{2/3}}{2^{2/3}}-\frac{\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}\,x}{\sqrt[3]{2}}}}{3\;2^{2/3}\sqrt[3]{c}\sqrt{b^2-4ac}}$$

$$=\frac{\left(b-\sqrt{b^2-4ac}\right)^{2/3}\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}\,x\right)}{3\;2^{2/3}c^{2/3}\sqrt{b^2-4ac}}-\frac{\left(b+\sqrt{b^2-4ac}\right)^{2/3}\log\left(\sqrt[3]{b+\sqrt{b}}\right.}{3\;2^{2/3}c^{2/3}\sqrt{b^2-}}$$

$$=\frac{\left(b-\sqrt{b^2-4ac}\right)^{2/3}\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}\,x\right)}{3\;2^{2/3}c^{2/3}\sqrt{b^2-4ac}}-\frac{\left(b+\sqrt{b^2-4ac}\right)^{2/3}\log\left(\sqrt[3]{b+\sqrt{b}}\right.}{3\;2^{2/3}c^{2/3}\sqrt{b^2-}}$$

$$=\frac{\left(b-\sqrt{b^2-4ac}\right)^{2/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}\,x}{\sqrt[3]{b-\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}\,c^{2/3}\sqrt{b^2-4ac}}-\frac{\left(b+\sqrt{b^2-4ac}\right)^{2/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}\,x}{\sqrt[3]{b+\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}\,c^{2/3}\sqrt{b^2-4ac}}+\frac{(b}{}$$

Mathematica [C] time = 0.02, size = 44, normalized size = 0.08

$$\frac{1}{3}\text{RootSum}\left[\#1^6c+\#1^3b+a\&,\frac{\#1^2\log(x-\#1)}{2\#1^3c+b}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(a + b*x^3 + c*x^6),x]
```

```
[Out] RootSum[a + b*#1^3 + c*#1^6 & , (Log[x - #1]*#1^2)/(b + 2*c*#1^3) & ]/3
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{a+bx^3+cx^6}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^4/(a + b*x^3 + c*x^6),x]
```

```
[Out] IntegrateAlgebraic[x^4/(a + b*x^3 + c*x^6), x]
```

fricas [B] time = 1.78, size = 3799, normalized size = 6.81

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(c*x^6+b*x^3+a),x, algorithm="fricas")
```

```
[Out] 2/3*sqrt(3)*(1/2)^(1/3)*(-((b^2*c^2 - 4*a*c^3)*sqrt((b^4 - 4*a*b^2*c + 4*a^
2*c^2)/(b^6*c^4 - 12*a*b^4*c^5 + 48*a^2*b^2*c^6 - 64*a^3*c^7)) + b)/(b^2*c^
2 - 4*a*c^3))^(1/3)*arctan(-1/3*((1/2)^(5/6)*(sqrt(3)*(b^5*c^2 - 8*a*b^3*c^
3 + 16*a^2*b*c^4)*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/(b^6*c^4 - 12*a*b^4*c^
5 + 48*a^2*b^2*c^6 - 64*a^3*c^7)) - sqrt(3)*(b^4 - 6*a*b^2*c + 8*a^2*c^2))*
sqrt((2*(a*b^2 - 2*a^2*c)*x^2 + (1/2)^(2/3)*((b^6*c^2 - 12*a*b^4*c^3 + 48*a
^2*b^2*c^4 - 64*a^3*c^5)*x*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/(b^6*c^4 - 12
*a*b^4*c^5 + 48*a^2*b^2*c^6 - 64*a^3*c^7)) - (b^5 - 6*a*b^3*c + 8*a^2*b*c^2
)*x)*(-((b^2*c^2 - 4*a*c^3)*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/(b^6*c^4 - 1
2*a*b^4*c^5 + 48*a^2*b^2*c^6 - 64*a^3*c^7)) + b)/(b^2*c^2 - 4*a*c^3))^(2/3)
 - 2*(1/2)^(1/3)*(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*sqrt((b^4 - 4*a*b
^2*c + 4*a^2*c^2)/(b^6*c^4 - 12*a*b^4*c^5 + 48*a^2*b^2*c^6 - 64*a^3*c^7))*(
-((b^2*c^2 - 4*a*c^3)*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/(b^6*c^4 - 12*a*b^
4*c^5 + 48*a^2*b^2*c^6 - 64*a^3*c^7)) + b)/(b^2*c^2 - 4*a*c^3))^(1/3))/(a*b
^2 - 2*a^2*c))*(-((b^2*c^2 - 4*a*c^3)*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/(b
^6*c^4 - 12*a*b^4*c^5 + 48*a^2*b^2*c^6 - 64*a^3*c^7)) + b)/(b^2*c^2 - 4*a*c
^3))^(1/3) - (1/2)^(1/3)*(sqrt(3)*(b^5*c^2 - 8*a*b^3*c^3 + 16*a^2*b*c^4)*x*
sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/(b^6*c^4 - 12*a*b^4*c^5 + 48*a^2*b^2*c^6
 - 64*a^3*c^7)) - sqrt(3)*(b^4 - 6*a*b^2*c + 8*a^2*c^2)*x)*(-((b^2*c^2 - 4*
a*c^3)*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/(b^6*c^4 - 12*a*b^4*c^5 + 48*a^2*
b^2*c^6 - 64*a^3*c^7)) + b)/(b^2*c^2 - 4*a*c^3))^(1/3) - sqrt(3)*(a*b^2 - 2
*a^2*c))/(a*b^2 - 2*a^2*c)) - 2/3*sqrt(3)*(1/2)^(1/3)*(((b^2*c^2 - 4*a*c^3)
*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/(b^6*c^4 - 12*a*b^4*c^5 + 48*a^2*b^2*c^
6 - 64*a^3*c^7)) - b)/(b^2*c^2 - 4*a*c^3))^(1/3)*arctan(-1/3*((1/2)^(5/6)*(
sqrt(3)*(b^5*c^2 - 8*a*b^3*c^3 + 16*a^2*b*c^4)*sqrt((b^4 - 4*a*b^2*c + 4*a^
2*c^2)/(b^6*c^4 - 12*a*b^4*c^5 + 48*a^2*b^2*c^6 - 64*a^3*c^7)) + sqrt(3)*(b
^4 - 6*a*b^2*c + 8*a^2*c^2))*sqrt((2*(a*b^2 - 2*a^2*c)*x^2 - (1/2)^(2/3)*((
b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*x*sqrt((b^4 - 4*a*b^2
*c + 4*a^2*c^2)/(b^6*c^4 - 12*a*b^4*c^5 + 48*a^2*b^2*c^6 - 64*a^3*c^7)) + (
b^5 - 6*a*b^3*c + 8*a^2*b*c^2)*x)*(((b^2*c^2 - 4*a*c^3)*sqrt((b^4 - 4*a*b^2
*c + 4*a^2*c^2)/(b^6*c^4 - 12*a*b^4*c^5 + 48*a^2*b^2*c^6 - 64*a^3*c^7)) - b
)/(b^2*c^2 - 4*a*c^3))^(2/3) + 2*(1/2)^(1/3)*(a*b^4*c^2 - 8*a^2*b^2*c^3 + 1
6*a^3*c^4)*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/(b^6*c^4 - 12*a*b^4*c^5 + 48*
a^2*b^2*c^6 - 64*a^3*c^7))*(((b^2*c^2 - 4*a*c^3)*sqrt((b^4 - 4*a*b^2*c + 4*
a^2*c^2)/(b^6*c^4 - 12*a*b^4*c^5 + 48*a^2*b^2*c^6 - 64*a^3*c^7)) - b)/(b^2*
c^2 - 4*a*c^3))^(1/3))/(a*b^2 - 2*a^2*c))*(((b^2*c^2 - 4*a*c^3)*sqrt((b^4 -
 4*a*b^2*c + 4*a^2*c^2)/(b^6*c^4 - 12*a*b^4*c^5 + 48*a^2*b^2*c^6 - 64*a^3*c
^7)) - b)/(b^2*c^2 - 4*a*c^3))^(1/3) - (1/2)^(1/3)*(sqrt(3)*(b^5*c^2 - 8*a*
b^3*c^3 + 16*a^2*b*c^4)*x*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/(b^6*c^4 - 12*
a*b^4*c^5 + 48*a^2*b^2*c^6 - 64*a^3*c^7)) + sqrt(3)*(b^4 - 6*a*b^2*c + 8*a^
2*c^2)*x)*(((b^2*c^2 - 4*a*c^3)*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/(b^6*c^4
 - 12*a*b^4*c^5 + 48*a^2*b^2*c^6 - 64*a^3*c^7)) - b)/(b^2*c^2 - 4*a*c^3))^(
```

```
1/3) + sqrt(3)*(a*b^2 - 2*a^2*c))/(a*b^2 - 2*a^2*c)) - 1/6*(1/2)^(1/3)*(-((
b^2*c^2 - 4*a*c^3)*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/(b^6*c^4 - 12*a*b^4*c
^5 + 48*a^2*b^2*c^6 - 64*a^3*c^7)) + b)/(b^2*c^2 - 4*a*c^3))^(1/3)*log(-2*(
a*b^2 - 2*a^2*c)*x^2 - (1/2)^(2/3)*((b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^
4 - 64*a^3*c^5)*x*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/(b^6*c^4 - 12*a*b^4*c^
5 + 48*a^2*b^2*c^6 - 64*a^3*c^7)) - (b^5 - 6*a*b^3*c + 8*a^2*b*c^2)*x)*(-((
b^2*c^2 - 4*a*c^3)*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/(b^6*c^4 - 12*a*b^4*c
^5 + 48*a^2*b^2*c^6 - 64*a^3*c^7)) + b)/(b^2*c^2 - 4*a*c^3))^(2/3) + 2*(1/2
)^(1/3)*(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*sqrt((b^4 - 4*a*b^2*c + 4*
a^2*c^2)/(b^6*c^4 - 12*a*b^4*c^5 + 48*a^2*b^2*c^6 - 64*a^3*c^7))*(-((b^2*c^
2 - 4*a*c^3)*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/(b^6*c^4 - 12*a*b^4*c^5 + 4
8*a^2*b^2*c^6 - 64*a^3*c^7)) + b)/(b^2*c^2 - 4*a*c^3))^(1/3)) - 1/6*(1/2)^(
1/3)*(((b^2*c^2 - 4*a*c^3)*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/(b^6*c^4 - 12
*a*b^4*c^5 + 48*a^2*b^2*c^6 - 64*a^3*c^7)) - b)/(b^2*c^2 - 4*a*c^3))^(1/3)*
log(-2*(a*b^2 - 2*a^2*c)*x^2 + (1/2)^(2/3)*((b^6*c^2 - 12*a*b^4*c^3 + 48*a^
2*b^2*c^4 - 64*a^3*c^5)*x*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/(b^6*c^4 - 12*
a*b^4*c^5 + 48*a^2*b^2*c^6 - 64*a^3*c^7)) + (b^5 - 6*a*b^3*c + 8*a^2*b*c^2)
*x)*(((b^2*c^2 - 4*a*c^3)*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/(b^6*c^4 - 12*
a*b^4*c^5 + 48*a^2*b^2*c^6 - 64*a^3*c^7)) - b)/(b^2*c^2 - 4*a*c^3))^(2/3) -
 2*(1/2)^(1/3)*(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*sqrt((b^4 - 4*a*b^2
*c + 4*a^2*c^2)/(b^6*c^4 - 12*a*b^4*c^5 + 48*a^2*b^2*c^6 - 64*a^3*c^7))*(((
b^2*c^2 - 4*a*c^3)*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/(b^6*c^4 - 12*a*b^4*c
^5 + 48*a^2*b^2*c^6 - 64*a^3*c^7)) - b)/(b^2*c^2 - 4*a*c^3))^(1/3)) + 1/3*(
1/2)^(1/3)*(-((b^2*c^2 - 4*a*c^3)*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/(b^6*c
^4 - 12*a*b^4*c^5 + 48*a^2*b^2*c^6 - 64*a^3*c^7)) + b)/(b^2*c^2 - 4*a*c^3))
^(1/3)*log(-(1/2)^(2/3)*(b^5 - 6*a*b^3*c + 8*a^2*b*c^2 - (b^6*c^2 - 12*a*b^
4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/(b^
6*c^4 - 12*a*b^4*c^5 + 48*a^2*b^2*c^6 - 64*a^3*c^7)))*(-((b^2*c^2 - 4*a*c^3
)*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/(b^6*c^4 - 12*a*b^4*c^5 + 48*a^2*b^2*c
^6 - 64*a^3*c^7)) + b)/(b^2*c^2 - 4*a*c^3))^(2/3) - 2*(a*b^2 - 2*a^2*c)*x)
+ 1/3*(1/2)^(1/3)*(((b^2*c^2 - 4*a*c^3)*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/
(b^6*c^4 - 12*a*b^4*c^5 + 48*a^2*b^2*c^6 - 64*a^3*c^7)) - b)/(b^2*c^2 - 4*a
*c^3))^(1/3)*log(-(1/2)^(2/3)*(b^5 - 6*a*b^3*c + 8*a^2*b*c^2 + (b^6*c^2 - 1
2*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^
2)/(b^6*c^4 - 12*a*b^4*c^5 + 48*a^2*b^2*c^6 - 64*a^3*c^7)))*(((b^2*c^2 - 4*
a*c^3)*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/(b^6*c^4 - 12*a*b^4*c^5 + 48*a^2*
b^2*c^6 - 64*a^3*c^7)) - b)/(b^2*c^2 - 4*a*c^3))^(2/3) - 2*(a*b^2 - 2*a^2*c
)*x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{cx^6 + bx^3 + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(c*x^6+b*x^3+a),x, algorithm="giac")
```

```
[Out] integrate(x^4/(c*x^6 + b*x^3 + a), x)
```

maple [C] time = 0.00, size = 43, normalized size = 0.08

$$\frac{\mathrm{RootOf}\left(c\_Z^6 + b\_Z^3 + a\right)^4 \ln\left(-\mathrm{RootOf}\left(c\_Z^6 + b\_Z^3 + a\right) + x\right)}{6\,\mathrm{RootOf}\left(c\_Z^6 + b\_Z^3 + a\right)^5 c + 3\,\mathrm{RootOf}\left(c\_Z^6 + b\_Z^3 + a\right)^2 b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(c*x^6+b*x^3+a),x)
```

```
[Out] 1/3*sum(_R^4/(2*_R^5*c+_R^2*b)*ln(-_R+x),_R=RootOf(_Z^6*c+_Z^3*b+a))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{cx^6 + bx^3 + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(c*x^6+b*x^3+a),x, algorithm="maxima")
```

```
[Out] integrate(x^4/(c*x^6 + b*x^3 + a), x)
```

mupad [B] time = 8.11, size = 2695, normalized size = 4.83

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(a + b*x^3 + c*x^6),x)
```

```
[Out] log((2^(1/3)*((b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*
c - 2*a*c*(-(4*a*c - b^2)^3)^(1/2))/(c^2*(4*a*c - b^2)^3))^(2/3)*(36*a^3*c^
3 - (2^(2/3)*(54*a^2*c^3*x*(4*a*c - b^2) - (27*2^(1/3)*a*b*c^3*(4*a*c - b^2
)^2*((b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c - 2*a*c
*(-(4*a*c - b^2)^3)^(1/2))/(c^2*(4*a*c - b^2)^3))^(2/3))/2)*((b^5 + b^2*(-(
4*a*c - b^2)^3)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c - 2*a*c*(-(4*a*c - b^2)^3)
^(1/2))/(c^2*(4*a*c - b^2)^3))^(1/3))/6 - 45*a^2*b^2*c^2 + 9*a*b^4*c))/18 +
 a^2*b*c*x)*((b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c
 - 2*a*c*(-(4*a*c - b^2)^3)^(1/2))/(54*(64*a^3*c^5 - b^6*c^2 + 12*a*b^4*c^3
 - 48*a^2*b^2*c^4)))^(1/3) + log((2^(1/3)*((b^5 - b^2*(-(4*a*c - b^2)^3)^(1
/2) + 16*a^2*b*c^2 - 8*a*b^3*c + 2*a*c*(-(4*a*c - b^2)^3)^(1/2))/(c^2*(4*a*
c - b^2)^3))^(2/3)*(36*a^3*c^3 - (2^(2/3)*(54*a^2*c^3*x*(4*a*c - b^2) - (27
*2^(1/3)*a*b*c^3*(4*a*c - b^2)^2*((b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 16*
a^2*b*c^2 - 8*a*b^3*c + 2*a*c*(-(4*a*c - b^2)^3)^(1/2))/(c^2*(4*a*c - b^2)^
3))^(2/3))/2)*((b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3
*c + 2*a*c*(-(4*a*c - b^2)^3)^(1/2))/(c^2*(4*a*c - b^2)^3))^(1/3))/6 - 45*a
^2*b^2*c^2 + 9*a*b^4*c))/18 + a^2*b*c*x)*((b^5 - b^2*(-(4*a*c - b^2)^3)^(1/
2) + 16*a^2*b*c^2 - 8*a*b^3*c + 2*a*c*(-(4*a*c - b^2)^3)^(1/2))/(54*(64*a^3
*c^5 - b^6*c^2 + 12*a*b^4*c^3 - 48*a^2*b^2*c^4)))^(1/3) - log((2^(1/3)*(3^(
1/2)*1i - 1)*((b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*
c - 2*a*c*(-(4*a*c - b^2)^3)^(1/2))/(c^2*(4*a*c - b^2)^3))^(2/3)*(36*a^3*c^
3 - 45*a^2*b^2*c^2 + 9*a*b^4*c + (2^(2/3)*(3^(1/2)*1i + 1)*(54*a^2*c^3*x*(4
*a*c - b^2) - (27*2^(1/3)*a*b*c^3*(3^(1/2)*1i - 1)*(4*a*c - b^2)^2*((b^5 +
b^2*(-(4*a*c - b^2)^3)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c - 2*a*c*(-(4*a*c -
b^2)^3)^(1/2))/(c^2*(4*a*c - b^2)^3))^(2/3))/4)*((b^5 + b^2*(-(4*a*c - b^2)
^3)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c - 2*a*c*(-(4*a*c - b^2)^3)^(1/2))/(c^2
*(4*a*c - b^2)^3))^(1/3))/12))/36 + a^2*b*c*x)*((3^(1/2)*1i)/2 + 1/2)*((b^5
 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c - 2*a*c*(-(4*a*c
 - b^2)^3)^(1/2))/(54*(64*a^3*c^5 - b^6*c^2 + 12*a*b^4*c^3 - 48*a^2*b^2*c^4
)))^(1/3) + log((2^(1/3)*(3^(1/2)*1i + 1)*((b^5 + b^2*(-(4*a*c - b^2)^3)^(1
/2) + 16*a^2*b*c^2 - 8*a*b^3*c - 2*a*c*(-(4*a*c - b^2)^3)^(1/2))/(c^2*(4*a*
c - b^2)^3))^(2/3)*(36*a^3*c^3 - 45*a^2*b^2*c^2 + 9*a*b^4*c - (2^(2/3)*(3^(
1/2)*1i - 1)*(54*a^2*c^3*x*(4*a*c - b^2) + (27*2^(1/3)*a*b*c^3*(3^(1/2)*1i
+ 1)*(4*a*c - b^2)^2*((b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 16*a^2*b*c^2 -
8*a*b^3*c - 2*a*c*(-(4*a*c - b^2)^3)^(1/2))/(c^2*(4*a*c - b^2)^3))^(2/3))/4
)*((b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c - 2*a*c*(
-(4*a*c - b^2)^3)^(1/2))/(c^2*(4*a*c - b^2)^3))^(1/3))/12))/36 - a^2*b*c*x)
*((3^(1/2)*1i)/2 - 1/2)*((b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 16*a^2*b*c^2
 - 8*a*b^3*c - 2*a*c*(-(4*a*c - b^2)^3)^(1/2))/(54*(64*a^3*c^5 - b^6*c^2 +
12*a*b^4*c^3 - 48*a^2*b^2*c^4)))^(1/3) - log((2^(1/3)*(3^(1/2)*1i - 1)*((b^
5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c + 2*a*c*(-(4*a*
c - b^2)^3)^(1/2))/(c^2*(4*a*c - b^2)^3))^(2/3)*(36*a^3*c^3 - 45*a^2*b^2*c^
2 + 9*a*b^4*c + (2^(2/3)*(3^(1/2)*1i + 1)*(54*a^2*c^3*x*(4*a*c - b^2) - (27
```

```
*2^(1/3)*a*b*c^3*(3^(1/2)*1i - 1)*(4*a*c - b^2)^2*((b^5 - b^2*(-(4*a*c - b^
2)^3)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c + 2*a*c*(-(4*a*c - b^2)^3)^(1/2))/(c
^2*(4*a*c - b^2)^3))^(2/3))/4)*((b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 16*a^
2*b*c^2 - 8*a*b^3*c + 2*a*c*(-(4*a*c - b^2)^3)^(1/2))/(c^2*(4*a*c - b^2)^3)
)^(1/3))/12))/36 + a^2*b*c*x)*((3^(1/2)*1i)/2 + 1/2)*((b^5 - b^2*(-(4*a*c -
 b^2)^3)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c + 2*a*c*(-(4*a*c - b^2)^3)^(1/2))
/(54*(64*a^3*c^5 - b^6*c^2 + 12*a*b^4*c^3 - 48*a^2*b^2*c^4)))^(1/3) + log((
2^(1/3)*(3^(1/2)*1i + 1)*((b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 16*a^2*b*c^
2 - 8*a*b^3*c + 2*a*c*(-(4*a*c - b^2)^3)^(1/2))/(c^2*(4*a*c - b^2)^3))^(2/3
)*(36*a^3*c^3 - 45*a^2*b^2*c^2 + 9*a*b^4*c - (2^(2/3)*(3^(1/2)*1i - 1)*(54*
a^2*c^3*x*(4*a*c - b^2) + (27*2^(1/3)*a*b*c^3*(3^(1/2)*1i + 1)*(4*a*c - b^2
)^2*((b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c + 2*a*c
*(-(4*a*c - b^2)^3)^(1/2))/(c^2*(4*a*c - b^2)^3))^(2/3))/4)*((b^5 - b^2*(-(
4*a*c - b^2)^3)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c + 2*a*c*(-(4*a*c - b^2)^3)
^(1/2))/(c^2*(4*a*c - b^2)^3))^(1/3))/12))/36 - a^2*b*c*x)*((3^(1/2)*1i)/2
- 1/2)*((b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c + 2*
a*c*(-(4*a*c - b^2)^3)^(1/2))/(54*(64*a^3*c^5 - b^6*c^2 + 12*a*b^4*c^3 - 48
*a^2*b^2*c^4)))^(1/3)
```

sympy [A] time = 2.18, size = 175, normalized size = 0.31

$$\operatorname{RootSum}\left(t^{6}\left(46656a^{3}c^{5} - 34992a^{2}b^{2}c^{4} + 8748ab^{4}c^{3} - 729b^{6}c^{2}\right) + t^{3}\left(-432a^{2}bc^{2} + 216ab^{3}c - 27b^{5}\right) + a^{2}, \left(t \mapsto t\log\left(x + \frac{15552t^{5}a^{3}c^{5} - 11664t^{5}a^{2}b^{2}c^{4} + 2916t^{5}ab^{4}c^{3} - 243t^{5}b^{6}c^{2} - 108t^{2}a^{2}bc^{2} + 63t^{2}ab^{3}c - 9t^{2}b^{5}}{2a^{2}c - ab^{2}}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(c*x**6+b*x**3+a),x)
```

```
[Out] RootSum(_t**6*(46656*a**3*c**5 - 34992*a**2*b**2*c**4 + 8748*a*b**4*c**3 -
729*b**6*c**2) + _t**3*(-432*a**2*b*c**2 + 216*a*b**3*c - 27*b**5) + a**2,
Lambda(_t, _t*log(x + (15552*_t**5*a**3*c**5 - 11664*_t**5*a**2*b**2*c**4 +
 2916*_t**5*a*b**4*c**3 - 243*_t**5*b**6*c**2 - 108*_t**2*a**2*b*c**2 + 63*
_t**2*a*b**3*c - 9*_t**2*b**5)/(2*a**2*c - a*b**2))))
```

3.133 $\int \frac{x^3}{a+bx^3+cx^6} \, dx$

Optimal. Leaf size=558

$$\frac{\sqrt[3]{b-\sqrt{b^2-4ac}} \log\left(-\sqrt[3]{2}\sqrt[3]{c}x\sqrt[3]{b-\sqrt{b^2-4ac}}+\left(b-\sqrt{b^2-4ac}\right)^{2/3}+2^{2/3}c^{2/3}x^2\right)}{6\sqrt[3]{2}\sqrt[3]{c}\sqrt{b^2-4ac}}-\frac{\sqrt[3]{\sqrt{b^2-4ac}+b}\log\left(-\right.}{}$$

Rubi [A] time = 0.57, antiderivative size = 558, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389, Rules used = {1374, 200, 31, 634, 617, 204, 628}

$$\frac{\sqrt[3]{b-\sqrt{b^2-4ac}}\log\left(-\sqrt[3]{2}\sqrt[3]{c}x\sqrt[3]{b-\sqrt{b^2-4ac}}+\left(b-\sqrt{b^2-4ac}\right)^{2/3}+2^{2/3}c^{2/3}x^2\right)}{6\sqrt[3]{2}\sqrt[3]{c}\sqrt{b^2-4ac}}-\frac{\sqrt[3]{\sqrt{b^2-4ac}+b}\log\left(-\sqrt[3]{2}\sqrt[3]{c}x\sqrt[3]{\sqrt{b^2-4ac}+b}+\left(\sqrt{b^2-4ac}+b\right)^{2/3}+2^{2/3}c^{2/3}x^2\right)}{6\sqrt[3]{2}\sqrt[3]{c}\sqrt{b^2-4ac}}-\frac{\sqrt[3]{b-\sqrt{b^2-4ac}}\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}x\right)}{3\sqrt[3]{2}\sqrt[3]{c}\sqrt{b^2-4ac}}+\frac{\sqrt[3]{\sqrt{b^2-4ac}+b}\log\left(\sqrt[3]{\sqrt{b^2-4ac}+b}+\sqrt[3]{2}\sqrt[3]{c}x\right)}{3\sqrt[3]{2}\sqrt[3]{c}\sqrt{b^2-4ac}}+\frac{\sqrt[3]{b-\sqrt{b^2-4ac}}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b-\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}\sqrt[3]{c}\sqrt{b^2-4ac}}-\frac{\sqrt[3]{\sqrt{b^2-4ac}+b}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{\sqrt{b^2-4ac}+b}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}\sqrt[3]{c}\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(a + b*x^3 + c*x^6),x]
```

```
[Out] ((b - Sqrt[b^2 - 4*a*c])^(1/3)*ArcTan[(1 - (2*2^(1/3)*c^(1/3)*x)/(b - Sqrt[
b^2 - 4*a*c])^(1/3))/Sqrt[3]])/(2^(1/3)*Sqrt[3]*c^(1/3)*Sqrt[b^2 - 4*a*c])
- ((b + Sqrt[b^2 - 4*a*c])^(1/3)*ArcTan[(1 - (2*2^(1/3)*c^(1/3)*x)/(b + Sqr
t[b^2 - 4*a*c])^(1/3))/Sqrt[3]])/(2^(1/3)*Sqrt[3]*c^(1/3)*Sqrt[b^2 - 4*a*c]
) - ((b - Sqrt[b^2 - 4*a*c])^(1/3)*Log[(b - Sqrt[b^2 - 4*a*c])^(1/3) + 2^(1
/3)*c^(1/3)*x])/(3*2^(1/3)*c^(1/3)*Sqrt[b^2 - 4*a*c]) + ((b + Sqrt[b^2 - 4*
a*c])^(1/3)*Log[(b + Sqrt[b^2 - 4*a*c])^(1/3) + 2^(1/3)*c^(1/3)*x])/(3*2^(1
/3)*c^(1/3)*Sqrt[b^2 - 4*a*c]) + ((b - Sqrt[b^2 - 4*a*c])^(1/3)*Log[(b - Sq
rt[b^2 - 4*a*c])^(2/3) - 2^(1/3)*c^(1/3)*(b - Sqrt[b^2 - 4*a*c])^(1/3)*x +
2^(2/3)*c^(2/3)*x^2])/(6*2^(1/3)*c^(1/3)*Sqrt[b^2 - 4*a*c]) - ((b + Sqrt[b^
2 - 4*a*c])^(1/3)*Log[(b + Sqrt[b^2 - 4*a*c])^(2/3) - 2^(1/3)*c^(1/3)*(b +
Sqrt[b^2 - 4*a*c])^(1/3)*x + 2^(2/3)*c^(2/3)*x^2])/(6*2^(1/3)*c^(1/3)*Sqrt[
b^2 - 4*a*c])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1374

```
Int[((d_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(d^n*(b/q + 1))/2, Int[(d*x)^(m -
 n)/(b/2 + q/2 + c*x^n), x], x] - Dist[(d^n*(b/q - 1))/2, Int[(d*x)^(m - n)
/(b/2 - q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[n2, 2*n] &&
NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GeQ[m, n]
```

Rubi steps

$$\int \frac{x^3}{a+bx^3+cx^6}\,dx = -\left(\frac{1}{2}\left(-1+\frac{b}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^3}\,dx\right)+\frac{1}{2}\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)\int \frac{}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-}}$$

$$= -\frac{\sqrt[3]{b-\sqrt{b^2-4ac}}\int \frac{1}{\frac{\sqrt[3]{b-\sqrt{b^2-4ac}}}{\sqrt[3]{2}}+\sqrt[3]{c}\,x}\,dx}{3\sqrt[3]{2}\sqrt{b^2-4ac}} - \frac{\sqrt[3]{b-\sqrt{b^2-4ac}}\int \frac{2^{2/3}\sqrt[3]{b-\sqrt{b^2-4ac}}-\sqrt[3]{c}\,x}{\frac{\left(b-\sqrt{b^2-4ac}\right)^{2/3}}{2^{2/3}}-\frac{\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}\,x}{\sqrt[3]{2}}+c}}{3\sqrt[3]{2}\sqrt{b^2-4ac}}$$

$$= -\frac{\sqrt[3]{b-\sqrt{b^2-4ac}}\,\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}\,x\right)}{3\sqrt[3]{2}\sqrt[3]{c}\sqrt{b^2-4ac}} + \frac{\sqrt[3]{b+\sqrt{b^2-4ac}}\,\log\left(\sqrt[3]{b+\sqrt{b^2}}\right.}{3\sqrt[3]{2}\sqrt[3]{c}\sqrt{b^2-4a}}$$

$$= -\frac{\sqrt[3]{b-\sqrt{b^2-4ac}}\,\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}\,x\right)}{3\sqrt[3]{2}\sqrt[3]{c}\sqrt{b^2-4ac}} + \frac{\sqrt[3]{b+\sqrt{b^2-4ac}}\,\log\left(\sqrt[3]{b+\sqrt{b^2}}\right.}{3\sqrt[3]{2}\sqrt[3]{c}\sqrt{b^2-4a}}$$

$$= \frac{\sqrt[3]{b-\sqrt{b^2-4ac}}\,\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}\,x}{\sqrt[3]{b-\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}\sqrt[3]{c}\sqrt{b^2-4ac}} - \frac{\sqrt[3]{b+\sqrt{b^2-4ac}}\,\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}\,x}{\sqrt[3]{b+\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}\sqrt[3]{c}\sqrt{b^2-4ac}} - \frac{\sqrt[3]{b-}}{}$$

Mathematica [C] time = 0.02, size = 42, normalized size = 0.08

$$\frac{1}{3}\text{RootSum}\left[\#1^6 c+\#1^3 b+a\&,\frac{\#1\log(x-\#1)}{2\#1^3 c+b}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(a + b*x^3 + c*x^6),x]
```

```
[Out] RootSum[a + b*#1^3 + c*#1^6 & , (Log[x - #1]*#1)/(b + 2*c*#1^3) & ]/3
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{a + bx^3 + cx^6}\, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^3/(a + b*x^3 + c*x^6),x]
```

```
[Out] IntegrateAlgebraic[x^3/(a + b*x^3 + c*x^6), x]
```

fricas [B] time = 1.49, size = 2551, normalized size = 4.57

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^6+b*x^3+a),x, algorithm="fricas")
```

```
[Out] -2/3*sqrt(3)*(1/2)^(1/3)*(((b^2*c - 4*a*c^2)*sqrt(b^2/(b^6*c^2 - 12*a*b^4*c
^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)) + 1)/(b^2*c - 4*a*c^2))^(1/3)*arctan(-1/
3*((1/2)^(2/3)*(sqrt(3)*(b^6*c - 12*a*b^4*c^2 + 48*a^2*b^2*c^3 - 64*a^3*c^4
)*sqrt(b^2/(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)) - sqrt(3
)*(b^4 - 4*a*b^2*c))*(((b^2*c - 4*a*c^2)*sqrt(b^2/(b^6*c^2 - 12*a*b^4*c^3 +
 48*a^2*b^2*c^4 - 64*a^3*c^5)) + 1)/(b^2*c - 4*a*c^2))^(2/3)*sqrt(-((1/2)^(
1/3)*(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*sqrt(b^2/(b^6*c^2 - 12*a*b^4*c^3 +
48*a^2*b^2*c^4 - 64*a^3*c^5))*x*(((b^2*c - 4*a*c^2)*sqrt(b^2/(b^6*c^2 - 12*
a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)) + 1)/(b^2*c - 4*a*c^2))^(1/3) - b
*x^2 - (1/2)^(2/3)*(b^3 - 4*a*b*c)*(((b^2*c - 4*a*c^2)*sqrt(b^2/(b^6*c^2 -
12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)) + 1)/(b^2*c - 4*a*c^2))^(2/3))
/b) - sqrt(3)*a*b - (1/2)^(2/3)*(sqrt(3)*(b^6*c - 12*a*b^4*c^2 + 48*a^2*b^2
*c^3 - 64*a^3*c^4)*sqrt(b^2/(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a
^3*c^5))*x - sqrt(3)*(b^4 - 4*a*b^2*c)*x)*(((b^2*c - 4*a*c^2)*sqrt(b^2/(b^6
*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)) + 1)/(b^2*c - 4*a*c^2))
^(2/3))/(a*b)) + 2/3*sqrt(3)*(1/2)^(1/3)*(-((b^2*c - 4*a*c^2)*sqrt(b^2/(b^6
*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)) - 1)/(b^2*c - 4*a*c^2))
^(1/3)*arctan(-1/3*((1/2)^(2/3)*(sqrt(3)*(b^6*c - 12*a*b^4*c^2 + 48*a^2*b^2
*c^3 - 64*a^3*c^4)*sqrt(b^2/(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a
^3*c^5)) + sqrt(3)*(b^4 - 4*a*b^2*c))*(-((b^2*c - 4*a*c^2)*sqrt(b^2/(b^6*c^
2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)) - 1)/(b^2*c - 4*a*c^2))^(2
/3)*sqrt(((1/2)^(1/3)*(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*sqrt(b^2/(b^6*c^2
- 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5))*x*(-((b^2*c - 4*a*c^2)*sqrt(
b^2/(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)) - 1)/(b^2*c - 4
*a*c^2))^(1/3) + b*x^2 + (1/2)^(2/3)*(b^3 - 4*a*b*c)*(-((b^2*c - 4*a*c^2)*s
qrt(b^2/(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)) - 1)/(b^2*c
 - 4*a*c^2))^(2/3))/b) + sqrt(3)*a*b - (1/2)^(2/3)*(sqrt(3)*(b^6*c - 12*a*b
^4*c^2 + 48*a^2*b^2*c^3 - 64*a^3*c^4)*sqrt(b^2/(b^6*c^2 - 12*a*b^4*c^3 + 48
*a^2*b^2*c^4 - 64*a^3*c^5))*x + sqrt(3)*(b^4 - 4*a*b^2*c)*x)*(-((b^2*c - 4*
a*c^2)*sqrt(b^2/(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)) - 1
)/(b^2*c - 4*a*c^2))^(2/3))/(a*b)) - 1/6*(1/2)^(1/3)*(((b^2*c - 4*a*c^2)*sq
rt(b^2/(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)) + 1)/(b^2*c
- 4*a*c^2))^(1/3)*log(-(1/2)^(1/3)*(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*sqrt(
b^2/(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5))*x*(((b^2*c - 4*
a*c^2)*sqrt(b^2/(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)) + 1
)/(b^2*c - 4*a*c^2))^(1/3) + b*x^2 + (1/2)^(2/3)*(b^3 - 4*a*b*c)*(((b^2*c -
 4*a*c^2)*sqrt(b^2/(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5))
+ 1)/(b^2*c - 4*a*c^2))^(2/3)) - 1/6*(1/2)^(1/3)*(-((b^2*c - 4*a*c^2)*sqrt(
b^2/(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)) - 1)/(b^2*c - 4
*a*c^2))^(1/3)*log((1/2)^(1/3)*(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*sqrt(b^2/
(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5))*x*(-((b^2*c - 4*a*c
^2)*sqrt(b^2/(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)) - 1)/(
b^2*c - 4*a*c^2))^(1/3) + b*x^2 + (1/2)^(2/3)*(b^3 - 4*a*b*c)*(-((b^2*c - 4
```

```
*a*c^2)*sqrt(b^2/(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)) -
1)/(b^2*c - 4*a*c^2))^(2/3)) + 1/3*(1/2)^(1/3)*(((b^2*c - 4*a*c^2)*sqrt(b^2
/(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)) + 1)/(b^2*c - 4*a*
c^2))^(1/3)*log((1/2)^(1/3)*(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*sqrt(b^2/(b^
6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5))*(((b^2*c - 4*a*c^2)*sq
rt(b^2/(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)) + 1)/(b^2*c
- 4*a*c^2))^(1/3) + b*x) + 1/3*(1/2)^(1/3)*(-((b^2*c - 4*a*c^2)*sqrt(b^2/(b
^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)) - 1)/(b^2*c - 4*a*c^2
))^(1/3)*log(-(1/2)^(1/3)*(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*sqrt(b^2/(b^6*
c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5))*(-((b^2*c - 4*a*c^2)*sqr
t(b^2/(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)) - 1)/(b^2*c -
 4*a*c^2))^(1/3) + b*x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{cx^6 + bx^3 + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^6+b*x^3+a),x, algorithm="giac")
```

```
[Out] integrate(x^3/(c*x^6 + b*x^3 + a), x)
```

maple [C] time = 0.00, size = 43, normalized size = 0.08

$$\frac{\mathrm{RootOf}\left(c\_Z^6 + b\_Z^3 + a\right)^3 \ln\left(-\mathrm{RootOf}\left(c\_Z^6 + b\_Z^3 + a\right) + x\right)}{6\,\mathrm{RootOf}\left(c\_Z^6 + b\_Z^3 + a\right)^5 c + 3\,\mathrm{RootOf}\left(c\_Z^6 + b\_Z^3 + a\right)^2 b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(c*x^6+b*x^3+a),x)
```

```
[Out] 1/3*sum(_R^3/(2*_R^5*c+_R^2*b)*ln(-_R+x),_R=RootOf(_Z^6*c+_Z^3*b+a))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{cx^6 + bx^3 + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^6+b*x^3+a),x, algorithm="maxima")
```

```
[Out] integrate(x^3/(c*x^6 + b*x^3 + a), x)
```

mupad [B] time = 7.71, size = 2129, normalized size = 3.82

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a + b*x^3 + c*x^6),x)
```

```
[Out] log((2^(2/3)*(-(b*(-(4*a*c - b^2)^3)^(1/2) + b^4 + 16*a^2*c^2 - 8*a*b^2*c)/
(c*(4*a*c - b^2)^3))^(1/3)*(9*a*b^3*c^2 - 36*a^2*b*c^3 + (9*2^(1/3)*a*c^3*(
4*a*c - b^2)^2*(x - (2^(2/3)*b*(-(b*(-(4*a*c - b^2)^3)^(1/2) + b^4 + 16*a^2
*c^2 - 8*a*b^2*c)/(c*(4*a*c - b^2)^3))^(1/3))/2)*(-(b*(-(4*a*c - b^2)^3)^(1
/2) + b^4 + 16*a^2*c^2 - 8*a*b^2*c)/(c*(4*a*c - b^2)^3))^(2/3))/2))/6 + 3*a
*c^2*x*(2*a*c - b^2))*((b*(-(4*a*c - b^2)^3)^(1/2) + b^4 + 16*a^2*c^2 - 8*a
*b^2*c)/(54*(b^6*c - 64*a^3*c^4 - 12*a*b^4*c^2 + 48*a^2*b^2*c^3)))^(1/3) +
log((2^(2/3)*(9*a*b^3*c^2 - 36*a^2*b*c^3 + (9*2^(1/3)*a*c^3*(x - (2^(2/3)*b
*((b*(-(4*a*c - b^2)^3)^(1/2) - b^4 - 16*a^2*c^2 + 8*a*b^2*c)/(c*(4*a*c - b
```

```
^2)^3))^(1/3))/2)*(4*a*c - b^2)^2*((b*(-(4*a*c - b^2)^3)^(1/2) - b^4 - 16*a
^2*c^2 + 8*a*b^2*c)/(c*(4*a*c - b^2)^3))^(2/3))/2)*((b*(-(4*a*c - b^2)^3)^(
1/2) - b^4 - 16*a^2*c^2 + 8*a*b^2*c)/(c*(4*a*c - b^2)^3))^(1/3))/6 + 3*a*c^
2*x*(2*a*c - b^2))*(-(b*(-(4*a*c - b^2)^3)^(1/2) - b^4 - 16*a^2*c^2 + 8*a*b
^2*c)/(54*(b^6*c - 64*a^3*c^4 - 12*a*b^4*c^2 + 48*a^2*b^2*c^3)))^(1/3) + lo
g((2^(2/3)*(3^(1/2)*1i - 1)*(36*a^2*b*c^3 - 9*a*b^3*c^2 + (2^(1/3)*(3^(1/2)
*1i + 1)*(81*a*c^3*x*(4*a*c - b^2)^2 - (81*2^(2/3)*a*b*c^3*(3^(1/2)*1i - 1)
*(4*a*c - b^2)^2*(-(b*(-(4*a*c - b^2)^3)^(1/2) + b^4 + 16*a^2*c^2 - 8*a*b^2
*c)/(c*(4*a*c - b^2)^3))^(1/3))/4)*(-(b*(-(4*a*c - b^2)^3)^(1/2) + b^4 + 16
*a^2*c^2 - 8*a*b^2*c)/(c*(4*a*c - b^2)^3))^(2/3))/36)*(-(b*(-(4*a*c - b^2)^
3)^(1/2) + b^4 + 16*a^2*c^2 - 8*a*b^2*c)/(c*(4*a*c - b^2)^3))^(1/3))/12 - 3
*a*c^2*x*(2*a*c - b^2))*((3^(1/2)*1i)/2 - 1/2)*((b*(-(4*a*c - b^2)^3)^(1/2)
 + b^4 + 16*a^2*c^2 - 8*a*b^2*c)/(54*(b^6*c - 64*a^3*c^4 - 12*a*b^4*c^2 + 4
8*a^2*b^2*c^3)))^(1/3) - log((2^(2/3)*(3^(1/2)*1i + 1)*(9*a*b^3*c^2 - 36*a^
2*b*c^3 + (2^(1/3)*(3^(1/2)*1i - 1)*(81*a*c^3*x*(4*a*c - b^2)^2 + (81*2^(2/
3)*a*b*c^3*(3^(1/2)*1i + 1)*(4*a*c - b^2)^2*(-(b*(-(4*a*c - b^2)^3)^(1/2) +
 b^4 + 16*a^2*c^2 - 8*a*b^2*c)/(c*(4*a*c - b^2)^3))^(1/3))/4)*(-(b*(-(4*a*c
 - b^2)^3)^(1/2) + b^4 + 16*a^2*c^2 - 8*a*b^2*c)/(c*(4*a*c - b^2)^3))^(2/3)
)/36)*(-(b*(-(4*a*c - b^2)^3)^(1/2) + b^4 + 16*a^2*c^2 - 8*a*b^2*c)/(c*(4*a
*c - b^2)^3))^(1/3))/12 - 3*a*c^2*x*(2*a*c - b^2))*((3^(1/2)*1i)/2 + 1/2)*(
(b*(-(4*a*c - b^2)^3)^(1/2) + b^4 + 16*a^2*c^2 - 8*a*b^2*c)/(54*(b^6*c - 64
*a^3*c^4 - 12*a*b^4*c^2 + 48*a^2*b^2*c^3)))^(1/3) + log((2^(2/3)*(3^(1/2)*1
i - 1)*(36*a^2*b*c^3 - 9*a*b^3*c^2 + (2^(1/3)*(81*a*c^3*x*(4*a*c - b^2)^2 -
 (81*2^(2/3)*a*b*c^3*(3^(1/2)*1i - 1)*(4*a*c - b^2)^2*((b*(-(4*a*c - b^2)^3
)^(1/2) - b^4 - 16*a^2*c^2 + 8*a*b^2*c)/(c*(4*a*c - b^2)^3))^(1/3))/4)*(3^(
1/2)*1i + 1)*((b*(-(4*a*c - b^2)^3)^(1/2) - b^4 - 16*a^2*c^2 + 8*a*b^2*c)/(
c*(4*a*c - b^2)^3))^(2/3))/36)*((b*(-(4*a*c - b^2)^3)^(1/2) - b^4 - 16*a^2*
c^2 + 8*a*b^2*c)/(c*(4*a*c - b^2)^3))^(1/3))/12 - 3*a*c^2*x*(2*a*c - b^2))*
((3^(1/2)*1i)/2 - 1/2)*(-(b*(-(4*a*c - b^2)^3)^(1/2) - b^4 - 16*a^2*c^2 + 8
*a*b^2*c)/(54*(b^6*c - 64*a^3*c^4 - 12*a*b^4*c^2 + 48*a^2*b^2*c^3)))^(1/3)
- log((2^(2/3)*(3^(1/2)*1i + 1)*(9*a*b^3*c^2 - 36*a^2*b*c^3 + (2^(1/3)*(81*
a*c^3*x*(4*a*c - b^2)^2 + (81*2^(2/3)*a*b*c^3*(3^(1/2)*1i + 1)*(4*a*c - b^2
)^2*((b*(-(4*a*c - b^2)^3)^(1/2) - b^4 - 16*a^2*c^2 + 8*a*b^2*c)/(c*(4*a*c
- b^2)^3))^(1/3))/4)*(3^(1/2)*1i - 1)*((b*(-(4*a*c - b^2)^3)^(1/2) - b^4 -
16*a^2*c^2 + 8*a*b^2*c)/(c*(4*a*c - b^2)^3))^(2/3))/36)*((b*(-(4*a*c - b^2)
^3)^(1/2) - b^4 - 16*a^2*c^2 + 8*a*b^2*c)/(c*(4*a*c - b^2)^3))^(1/3))/12 -
3*a*c^2*x*(2*a*c - b^2))*((3^(1/2)*1i)/2 + 1/2)*(-(b*(-(4*a*c - b^2)^3)^(1/
2) - b^4 - 16*a^2*c^2 + 8*a*b^2*c)/(54*(b^6*c - 64*a^3*c^4 - 12*a*b^4*c^2 +
 48*a^2*b^2*c^3)))^(1/3)
```

sympy [A] time = 1.80, size = 122, normalized size = 0.22

$$\mathrm{RootSum}\left(t^6\left(46656a^3c^4 - 34992a^2b^2c^3 + 8748ab^4c^2 - 729b^6c\right) + t^3\left(432a^2c^2 - 216ab^2c + 27b^4\right) + a, \left(t \mapsto t\log\left(x + \frac{2592t^4a^2c^3 - 1296t^4ab^2c^2 + 162t^4b^4c + 12tac - 3tb^2}{b}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(c*x**6+b*x**3+a),x)
```

```
[Out] RootSum(_t**6*(46656*a**3*c**4 - 34992*a**2*b**2*c**3 + 8748*a*b**4*c**2 -
729*b**6*c) + _t**3*(432*a**2*c**2 - 216*a*b**2*c + 27*b**4) + a, Lambda(_t
, _t*log(x + (2592*_t**4*a**2*c**3 - 1296*_t**4*a*b**2*c**2 + 162*_t**4*b**
4*c + 12*_t*a*c - 3*_t*b**2)/b)))
```

3.134 $\int \frac{x}{a+bx^3+cx^6}\,dx$

Optimal. Leaf size=558

$$\frac{\sqrt[3]{c}\log\left(-\sqrt[3]{2}\sqrt[3]{c}\,x\sqrt[3]{b-\sqrt{b^2-4ac}}+\left(b-\sqrt{b^2-4ac}\right)^{2/3}+2^{2/3}c^{2/3}x^2\right)}{3\,2^{2/3}\sqrt{b^2-4ac}\sqrt[3]{b-\sqrt{b^2-4ac}}}-\frac{\sqrt[3]{c}\log\left(-\sqrt[3]{2}\sqrt[3]{c}\,x\sqrt[3]{\sqrt{b^2-4ac}+b}+\left(\cdots\right.\right.}{3\,2^{2/3}\sqrt{b^2-4ac}\sqrt[3]{\sqrt{b\cdots}}}$$

Rubi [A] time = 0.47, antiderivative size = 558, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {1375, 292, 31, 634, 617, 204, 628}

$$\frac{\sqrt[3]{c}\log\left(-\sqrt[3]{2}\sqrt[3]{c}\,x\sqrt[3]{b-\sqrt{b^2-4ac}}+\left(b-\sqrt{b^2-4ac}\right)^{2/3}+2^{2/3}c^{2/3}x^2\right)}{3\,2^{2/3}\sqrt{b^2-4ac}\sqrt[3]{b-\sqrt{b^2-4ac}}}-\frac{\sqrt[3]{c}\log\left(-\sqrt[3]{2}\sqrt[3]{c}\,x\sqrt[3]{\sqrt{b^2-4ac}+b}+\left(\sqrt{b^2-4ac}+b\right)^{2/3}+2^{2/3}c^{2/3}x^2\right)}{3\,2^{2/3}\sqrt{b^2-4ac}\sqrt[3]{\sqrt{b^2-4ac}+b}}-\frac{\sqrt[3]{2}\sqrt[3]{c}\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}\,x\right)}{3\sqrt{b^2-4ac}\sqrt[3]{b-\sqrt{b^2-4ac}}}+\frac{\sqrt[3]{2}\sqrt[3]{c}\log\left(\sqrt[3]{\sqrt{b^2-4ac}+b}+\sqrt[3]{2}\sqrt[3]{c}\,x\right)}{3\sqrt{b^2-4ac}\sqrt[3]{\sqrt{b^2-4ac}+b}}-\frac{\sqrt[3]{2}\sqrt[3]{c}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}\,x}{\sqrt[3]{b-\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{\sqrt{3}\sqrt{b^2-4ac}\sqrt[3]{b-\sqrt{b^2-4ac}}}+\frac{\sqrt[3]{2}\sqrt[3]{c}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}\,x}{\sqrt[3]{\sqrt{b^2-4ac}+b}}}{\sqrt{3}}\right)}{\sqrt{3}\sqrt{b^2-4ac}\sqrt[3]{\sqrt{b^2-4ac}+b}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a + b*x^3 + c*x^6),x]
```

```
[Out] -((2^(1/3)*c^(1/3)*ArcTan[(1 - (2*2^(1/3)*c^(1/3)*x)/(b - Sqrt[b^2 - 4*a*c]
)^(1/3))/Sqrt[3]])/(Sqrt[3]*Sqrt[b^2 - 4*a*c]*(b - Sqrt[b^2 - 4*a*c])^(1/3)
)) + (2^(1/3)*c^(1/3)*ArcTan[(1 - (2*2^(1/3)*c^(1/3)*x)/(b + Sqrt[b^2 - 4*a
*c])^(1/3))/Sqrt[3]])/(Sqrt[3]*Sqrt[b^2 - 4*a*c]*(b + Sqrt[b^2 - 4*a*c])^(1
/3)) - (2^(1/3)*c^(1/3)*Log[(b - Sqrt[b^2 - 4*a*c])^(1/3) + 2^(1/3)*c^(1/3)
*x])/(3*Sqrt[b^2 - 4*a*c]*(b - Sqrt[b^2 - 4*a*c])^(1/3)) + (2^(1/3)*c^(1/3)
*Log[(b + Sqrt[b^2 - 4*a*c])^(1/3) + 2^(1/3)*c^(1/3)*x])/(3*Sqrt[b^2 - 4*a*
c]*(b + Sqrt[b^2 - 4*a*c])^(1/3)) + (c^(1/3)*Log[(b - Sqrt[b^2 - 4*a*c])^(2
/3) - 2^(1/3)*c^(1/3)*(b - Sqrt[b^2 - 4*a*c])^(1/3)*x + 2^(2/3)*c^(2/3)*x^2
])/(3*2^(2/3)*Sqrt[b^2 - 4*a*c]*(b - Sqrt[b^2 - 4*a*c])^(1/3)) - (c^(1/3)*L
og[(b + Sqrt[b^2 - 4*a*c])^(2/3) - 2^(1/3)*c^(1/3)*(b + Sqrt[b^2 - 4*a*c])^
(1/3)*x + 2^(2/3)*c^(2/3)*x^2])/(3*2^(2/3)*Sqrt[b^2 - 4*a*c]*(b + Sqrt[b^2
- 4*a*c])^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1375

```
Int[((d_.)*(x_))^(m_.)/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symb
ol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[c/q, Int[(d*x)^m/(b/2 - q/2 + c*
x^n), x], x] - Dist[c/q, Int[(d*x)^m/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[
{a, b, c, d, m}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{x}{a+bx^3+cx^6}\,dx = \frac{c\int \frac{x}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^3}\,dx}{\sqrt{b^2-4ac}} - \frac{c\int \frac{x}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^3}\,dx}{\sqrt{b^2-4ac}}$$

$$= -\frac{\left(\sqrt[3]{2}\,c^{2/3}\right)\int \frac{1}{\frac{\sqrt[3]{b-\sqrt{b^2-4ac}}}{\sqrt[3]{2}}+\sqrt[3]{c}\,x}\,dx}{3\sqrt{b^2-4ac}\,\sqrt[3]{b-\sqrt{b^2-4ac}}} + \frac{\left(\sqrt[3]{2}\,c^{2/3}\right)\int \frac{\frac{\sqrt[3]{b-\sqrt{b^2-4ac}}}{\sqrt[3]{2}}+\sqrt[3]{c}\,x}{\frac{\left(b-\sqrt{b^2-4ac}\right)^{2/3}}{2^{2/3}}-\frac{\sqrt[3]{c}\,\sqrt[3]{b-\sqrt{b^2-4ac}}\,x}{\sqrt[3]{2}}+c^{2/3}x^2}\,dx}{3\sqrt{b^2-4ac}\,\sqrt[3]{b-\sqrt{b^2-4ac}}} + \cdots$$

$$= -\frac{\sqrt[3]{2}\,\sqrt[3]{c}\,\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\,\sqrt[3]{c}\,x\right)}{3\sqrt{b^2-4ac}\,\sqrt[3]{b-\sqrt{b^2-4ac}}} + \frac{\sqrt[3]{2}\,\sqrt[3]{c}\,\log\left(\sqrt[3]{b+\sqrt{b^2-4ac}}+\sqrt[3]{2}\,\sqrt[3]{c}\,x \cdots\right.}{3\sqrt{b^2-4ac}\,\sqrt[3]{b+\sqrt{b^2-4ac}}} \cdots$$

$$= -\frac{\sqrt[3]{2}\,\sqrt[3]{c}\,\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\,\sqrt[3]{c}\,x\right)}{3\sqrt{b^2-4ac}\,\sqrt[3]{b-\sqrt{b^2-4ac}}} + \frac{\sqrt[3]{2}\,\sqrt[3]{c}\,\log\left(\sqrt[3]{b+\sqrt{b^2-4ac}}+\sqrt[3]{2}\,\sqrt[3]{c}\,x \cdots\right.}{3\sqrt{b^2-4ac}\,\sqrt[3]{b+\sqrt{b^2-4ac}}} \cdots$$

$$= -\frac{\sqrt[3]{2}\,\sqrt[3]{c}\,\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\,\sqrt[3]{c}\,x}{\sqrt[3]{b-\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{\sqrt{3}\,\sqrt{b^2-4ac}\,\sqrt[3]{b-\sqrt{b^2-4ac}}} + \frac{\sqrt[3]{2}\,\sqrt[3]{c}\,\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\,\sqrt[3]{c}\,x}{\sqrt[3]{b+\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{\sqrt{3}\,\sqrt{b^2-4ac}\,\sqrt[3]{b+\sqrt{b^2-4ac}}} - \frac{\sqrt[3]{2}\,\sqrt[3]{c}\,\log\left(\sqrt[3]{b \cdots}\right.}{3\sqrt{b^2-4a \cdots}} \cdots$$

Mathematica [C] time = 0.02, size = 43, normalized size = 0.08

$$\frac{1}{3}\text{RootSum}\left[\#1^6 c+\#1^3 b+a\&,\frac{\log(x-\#1)}{2\#1^4 c+\#1 b}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a + b*x^3 + c*x^6),x]
```

```
[Out] RootSum[a + b*#1^3 + c*#1^6 & , Log[x - #1]/(b*#1 + 2*c*#1^4) & ]/3
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{a + bx^3 + cx^6}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x/(a + b*x^3 + c*x^6),x]
```

```
[Out] IntegrateAlgebraic[x/(a + b*x^3 + c*x^6), x]
```

fricas [B] time = 1.70, size = 2875, normalized size = 5.15

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^6+b*x^3+a),x, algorithm="fricas")
```

```
[Out] -2/3*sqrt(3)*(1/2)^(1/3)*(-((a*b^2 - 4*a^2*c)*sqrt(b^2/(a^2*b^6 - 12*a^3*b^
4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3)) + 1)/(a*b^2 - 4*a^2*c))^(1/3)*arctan(-1
/3*(2*sqrt(3)*(1/2)^(1/3)*(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)*sqrt(b^2/(a^2*
b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3))*x*(-((a*b^2 - 4*a^2*c)*s
qrt(b^2/(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3)) + 1)/(a*b^2
 - 4*a^2*c))^(1/3) - 2*sqrt(3)*(1/2)^(5/6)*(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^
2)*sqrt(b^2/(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3))*(-((a*b
^2 - 4*a^2*c)*sqrt(b^2/(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^
3)) + 1)/(a*b^2 - 4*a^2*c))^(1/3)*sqrt((2*b*c*x^2 + (1/2)^(2/3)*((a*b^6 - 1
2*a^2*b^4*c + 48*a^3*b^2*c^2 - 64*a^4*c^3)*sqrt(b^2/(a^2*b^6 - 12*a^3*b^4*c
 + 48*a^4*b^2*c^2 - 64*a^5*c^3))*x - (b^4 - 4*a*b^2*c)*x)*(-((a*b^2 - 4*a^2
*c)*sqrt(b^2/(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3)) + 1)/(
a*b^2 - 4*a^2*c))^(2/3) + (1/2)^(1/3)*(b^3 - 4*a*b*c - (a*b^5 - 8*a^2*b^3*c
 + 16*a^3*b*c^2)*sqrt(b^2/(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5
*c^3)))*(-((a*b^2 - 4*a^2*c)*sqrt(b^2/(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*
c^2 - 64*a^5*c^3)) + 1)/(a*b^2 - 4*a^2*c))^(1/3))/(b*c)) + sqrt(3)*b)/b) +
2/3*sqrt(3)*(1/2)^(1/3)*(((a*b^2 - 4*a^2*c)*sqrt(b^2/(a^2*b^6 - 12*a^3*b^4*
c + 48*a^4*b^2*c^2 - 64*a^5*c^3)) - 1)/(a*b^2 - 4*a^2*c))^(1/3)*arctan(-1/3
*(2*sqrt(3)*(1/2)^(1/3)*(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)*sqrt(b^2/(a^2*b^
6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3))*x*(((a*b^2 - 4*a^2*c)*sqrt
(b^2/(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3)) - 1)/(a*b^2 -
4*a^2*c))^(1/3) - 2*sqrt(3)*(1/2)^(5/6)*(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)*
sqrt(b^2/(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3))*(((a*b^2 -
 4*a^2*c)*sqrt(b^2/(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3))
- 1)/(a*b^2 - 4*a^2*c))^(1/3)*sqrt((2*b*c*x^2 - (1/2)^(2/3)*((a*b^6 - 12*a^
2*b^4*c + 48*a^3*b^2*c^2 - 64*a^4*c^3)*sqrt(b^2/(a^2*b^6 - 12*a^3*b^4*c + 4
8*a^4*b^2*c^2 - 64*a^5*c^3))*x + (b^4 - 4*a*b^2*c)*x)*(((a*b^2 - 4*a^2*c)*s
qrt(b^2/(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3)) - 1)/(a*b^2
 - 4*a^2*c))^(2/3) + (1/2)^(1/3)*(b^3 - 4*a*b*c + (a*b^5 - 8*a^2*b^3*c + 16
*a^3*b*c^2)*sqrt(b^2/(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3)
))*(((a*b^2 - 4*a^2*c)*sqrt(b^2/(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 -
64*a^5*c^3)) - 1)/(a*b^2 - 4*a^2*c))^(1/3))/(b*c)) - sqrt(3)*b)/b) - 1/6*(1
/2)^(1/3)*(-((a*b^2 - 4*a^2*c)*sqrt(b^2/(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^
2*c^2 - 64*a^5*c^3)) + 1)/(a*b^2 - 4*a^2*c))^(1/3)*log(2*b*c*x^2 + (1/2)^(2
/3)*((a*b^6 - 12*a^2*b^4*c + 48*a^3*b^2*c^2 - 64*a^4*c^3)*sqrt(b^2/(a^2*b^6
 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3))*x - (b^4 - 4*a*b^2*c)*x)*(-
((a*b^2 - 4*a^2*c)*sqrt(b^2/(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a
^5*c^3)) + 1)/(a*b^2 - 4*a^2*c))^(2/3) + (1/2)^(1/3)*(b^3 - 4*a*b*c - (a*b^
5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*sqrt(b^2/(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b
^2*c^2 - 64*a^5*c^3)))*(-((a*b^2 - 4*a^2*c)*sqrt(b^2/(a^2*b^6 - 12*a^3*b^4*
c + 48*a^4*b^2*c^2 - 64*a^5*c^3)) + 1)/(a*b^2 - 4*a^2*c))^(1/3)) - 1/6*(1/2
)^(1/3)*(((a*b^2 - 4*a^2*c)*sqrt(b^2/(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c
```

```
^2 - 64*a^5*c^3)) - 1)/(a*b^2 - 4*a^2*c))^(1/3)*log(2*b*c*x^2 - (1/2)^(2/3)
*((a*b^6 - 12*a^2*b^4*c + 48*a^3*b^2*c^2 - 64*a^4*c^3)*sqrt(b^2/(a^2*b^6 -
12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3))*x + (b^4 - 4*a*b^2*c)*x)*(((a*
b^2 - 4*a^2*c)*sqrt(b^2/(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c
^3)) - 1)/(a*b^2 - 4*a^2*c))^(2/3) + (1/2)^(1/3)*(b^3 - 4*a*b*c + (a*b^5 -
8*a^2*b^3*c + 16*a^3*b*c^2)*sqrt(b^2/(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c
^2 - 64*a^5*c^3)))*(((a*b^2 - 4*a^2*c)*sqrt(b^2/(a^2*b^6 - 12*a^3*b^4*c + 4
8*a^4*b^2*c^2 - 64*a^5*c^3)) - 1)/(a*b^2 - 4*a^2*c))^(1/3)) + 1/3*(1/2)^(1/
3)*(-((a*b^2 - 4*a^2*c)*sqrt(b^2/(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 -
 64*a^5*c^3)) + 1)/(a*b^2 - 4*a^2*c))^(1/3)*log(2*b*c*x + (1/2)^(2/3)*(b^4
- 4*a*b^2*c - (a*b^6 - 12*a^2*b^4*c + 48*a^3*b^2*c^2 - 64*a^4*c^3)*sqrt(b^2
/(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3)))*(-((a*b^2 - 4*a^2
*c)*sqrt(b^2/(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3)) + 1)/(
a*b^2 - 4*a^2*c))^(2/3)) + 1/3*(1/2)^(1/3)*(((a*b^2 - 4*a^2*c)*sqrt(b^2/(a^
2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3)) - 1)/(a*b^2 - 4*a^2*c)
)^(1/3)*log(2*b*c*x + (1/2)^(2/3)*(b^4 - 4*a*b^2*c + (a*b^6 - 12*a^2*b^4*c
+ 48*a^3*b^2*c^2 - 64*a^4*c^3)*sqrt(b^2/(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^
2*c^2 - 64*a^5*c^3)))*(((a*b^2 - 4*a^2*c)*sqrt(b^2/(a^2*b^6 - 12*a^3*b^4*c
+ 48*a^4*b^2*c^2 - 64*a^5*c^3)) - 1)/(a*b^2 - 4*a^2*c))^(2/3))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{cx^6 + bx^3 + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^6+b*x^3+a),x, algorithm="giac")
```

```
[Out] integrate(x/(c*x^6 + b*x^3 + a), x)
```

maple [C] time = 0.00, size = 41, normalized size = 0.07

$$\frac{\mathrm{RootOf}\left(c\_Z^6 + b\_Z^3 + a\right)\ln\left(-\mathrm{RootOf}\left(c\_Z^6 + b\_Z^3 + a\right) + x\right)}{6\,\mathrm{RootOf}\left(c\_Z^6 + b\_Z^3 + a\right)^5 c + 3\,\mathrm{RootOf}\left(c\_Z^6 + b\_Z^3 + a\right)^2 b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(c*x^6+b*x^3+a),x)
```

```
[Out] 1/3*sum(_R/(2*_R^5*c+_R^2*b)*ln(-_R+x),_R=RootOf(_Z^6*c+_Z^3*b+a))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{cx^6 + bx^3 + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^6+b*x^3+a),x, algorithm="maxima")
```

```
[Out] integrate(x/(c*x^6 + b*x^3 + a), x)
```

mupad [B] time = 5.39, size = 1543, normalized size = 2.77

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a + b*x^3 + c*x^6),x)
```

```
[Out] log(c^4*x - ((27*c^3*x*(b^4 + 8*a^2*c^2 - 6*a*b^2*c) + (27*2^(1/3)*a*b*c^3*
(4*a*c - b^2)^2*((b*(-(4*a*c - b^2)^3)^(1/2) + b^4 + 16*a^2*c^2 - 8*a*b^2*c
)/(a*(4*a*c - b^2)^3))^(2/3))/2)*(b*(-(4*a*c - b^2)^3)^(1/2) + b^4 + 16*a^2
```

```
*c^2 - 8*a*b^2*c))/(54*a*(4*a*c - b^2)^3))*(-(b*(-(4*a*c - b^2)^3)^(1/2) +
b^4 + 16*a^2*c^2 - 8*a*b^2*c)/(54*(a*b^6 - 64*a^4*c^3 - 12*a^2*b^4*c + 48*a
^3*b^2*c^2)))^(1/3) + log(c^4*x + ((27*c^3*x*(b^4 + 8*a^2*c^2 - 6*a*b^2*c)
+ (27*2^(1/3)*a*b*c^3*(4*a*c - b^2)^2*(-(b*(-(4*a*c - b^2)^3)^(1/2) - b^4 -
 16*a^2*c^2 + 8*a*b^2*c)/(a*(4*a*c - b^2)^3))^(2/3))/2)*(b*(-(4*a*c - b^2)^
3)^(1/2) - b^4 - 16*a^2*c^2 + 8*a*b^2*c))/(54*a*(4*a*c - b^2)^3))*((b*(-(4*
a*c - b^2)^3)^(1/2) - b^4 - 16*a^2*c^2 + 8*a*b^2*c)/(54*(a*b^6 - 64*a^4*c^3
 - 12*a^2*b^4*c + 48*a^3*b^2*c^2)))^(1/3) - log(c^4*x - ((27*c^3*x*(b^4 + 8
*a^2*c^2 - 6*a*b^2*c) + (27*2^(1/3)*a*b*c^3*(3^(1/2)*1i - 1)*(4*a*c - b^2)^
2*((b*(-(4*a*c - b^2)^3)^(1/2) + b^4 + 16*a^2*c^2 - 8*a*b^2*c)/(a*(4*a*c -
b^2)^3))^(2/3))/4)*(b*(-(4*a*c - b^2)^3)^(1/2) + b^4 + 16*a^2*c^2 - 8*a*b^2
*c))/(54*a*(4*a*c - b^2)^3))*((3^(1/2)*1i)/2 + 1/2)*(-(b*(-(4*a*c - b^2)^3)
^(1/2) + b^4 + 16*a^2*c^2 - 8*a*b^2*c)/(54*(a*b^6 - 64*a^4*c^3 - 12*a^2*b^4
*c + 48*a^3*b^2*c^2)))^(1/3) + log(c^4*x - ((27*c^3*x*(b^4 + 8*a^2*c^2 - 6*
a*b^2*c) - (27*2^(1/3)*a*b*c^3*(3^(1/2)*1i + 1)*(4*a*c - b^2)^2*((b*(-(4*a*
c - b^2)^3)^(1/2) + b^4 + 16*a^2*c^2 - 8*a*b^2*c)/(a*(4*a*c - b^2)^3))^(2/3
))/4)*(b*(-(4*a*c - b^2)^3)^(1/2) + b^4 + 16*a^2*c^2 - 8*a*b^2*c))/(54*a*(4
*a*c - b^2)^3))*((3^(1/2)*1i)/2 - 1/2)*(-(b*(-(4*a*c - b^2)^3)^(1/2) + b^4
+ 16*a^2*c^2 - 8*a*b^2*c)/(54*(a*b^6 - 64*a^4*c^3 - 12*a^2*b^4*c + 48*a^3*b
^2*c^2)))^(1/3) - log(c^4*x + ((27*c^3*x*(b^4 + 8*a^2*c^2 - 6*a*b^2*c) + (2
7*2^(1/3)*a*b*c^3*(3^(1/2)*1i - 1)*(4*a*c - b^2)^2*(-(b*(-(4*a*c - b^2)^3)^
(1/2) - b^4 - 16*a^2*c^2 + 8*a*b^2*c)/(a*(4*a*c - b^2)^3))^(2/3))/4)*(b*(-(
4*a*c - b^2)^3)^(1/2) - b^4 - 16*a^2*c^2 + 8*a*b^2*c))/(54*a*(4*a*c - b^2)^
3))*((3^(1/2)*1i)/2 + 1/2)*((b*(-(4*a*c - b^2)^3)^(1/2) - b^4 - 16*a^2*c^2
+ 8*a*b^2*c)/(54*(a*b^6 - 64*a^4*c^3 - 12*a^2*b^4*c + 48*a^3*b^2*c^2)))^(1/
3) + log(c^4*x + ((27*c^3*x*(b^4 + 8*a^2*c^2 - 6*a*b^2*c) - (27*2^(1/3)*a*b
*c^3*(3^(1/2)*1i + 1)*(4*a*c - b^2)^2*(-(b*(-(4*a*c - b^2)^3)^(1/2) - b^4 -
 16*a^2*c^2 + 8*a*b^2*c)/(a*(4*a*c - b^2)^3))^(2/3))/4)*(b*(-(4*a*c - b^2)^
3)^(1/2) - b^4 - 16*a^2*c^2 + 8*a*b^2*c))/(54*a*(4*a*c - b^2)^3))*((3^(1/2)
*1i)/2 - 1/2)*((b*(-(4*a*c - b^2)^3)^(1/2) - b^4 - 16*a^2*c^2 + 8*a*b^2*c)/
(54*(a*b^6 - 64*a^4*c^3 - 12*a^2*b^4*c + 48*a^3*b^2*c^2)))^(1/3)
```

sympy [A] time = 1.53, size = 158, normalized size = 0.28

$$\operatorname{RootSum}\left(t^6\left(46656a^4c^3-34992a^3b^2c^2+8748a^2b^4c-729ab^6\right)+t^3\left(-432a^2c^2+216ab^2c-27b^4\right)+c,\left(t\mapsto t\log\left(x+\frac{-15552t^5a^4c^3+11664t^5a^3b^2c^2-2916t^5a^2b^4c+243t^5ab^6+72t^2a^2c^2-54t^2ab^2c+9t^2b^4}{bc}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x**6+b*x**3+a),x)
```

```
[Out] RootSum(_t**6*(46656*a**4*c**3 - 34992*a**3*b**2*c**2 + 8748*a**2*b**4*c -
729*a*b**6) + _t**3*(-432*a**2*c**2 + 216*a*b**2*c - 27*b**4) + c, Lambda(_
t, _t*log(x + (-15552*_t**5*a**4*c**3 + 11664*_t**5*a**3*b**2*c**2 - 2916*_
t**5*a**2*b**4*c + 243*_t**5*a*b**6 + 72*_t**2*a**2*c**2 - 54*_t**2*a*b**2*
c + 9*_t**2*b**4)/(b*c))))
```

3.135 $\int \frac{1}{a+bx^3+cx^6}\,dx$

Optimal. Leaf size=558

$$-\frac{c^{2/3}\log\left(-\sqrt[3]{2}\,\sqrt[3]{c}\,x\sqrt[3]{b-\sqrt{b^2-4ac}}+\left(b-\sqrt{b^2-4ac}\right)^{2/3}+2^{2/3}c^{2/3}x^2\right)}{3\sqrt[3]{2}\sqrt{b^2-4ac}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}+\frac{c^{2/3}\log\left(-\sqrt[3]{2}\,\sqrt[3]{c}\,x\sqrt[3]{\sqrt{b^2-4ac}+b}\right.}{3\sqrt[3]{2}\sqrt{b^2-4ac}\left(\right.}$$

Rubi [A] time = 0.60, antiderivative size = 558, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 7, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {1347, 200, 31, 634, 617, 204, 628}

$$-\frac{c^{2/3}\log\left(-\sqrt[3]{2}\,\sqrt[3]{c}\,x\sqrt[3]{b-\sqrt{b^2-4ac}}+\left(b-\sqrt{b^2-4ac}\right)^{2/3}+2^{2/3}c^{2/3}x^2\right)}{3\sqrt[3]{2}\sqrt{b^2-4ac}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}+\frac{c^{2/3}\log\left(-\sqrt[3]{2}\,\sqrt[3]{c}\,x\sqrt[3]{\sqrt{b^2-4ac}+b}+\left(\sqrt{b^2-4ac}+b\right)^{2/3}+2^{2/3}c^{2/3}x^2\right)}{3\sqrt[3]{2}\sqrt{b^2-4ac}\left(\sqrt{b^2-4ac}+b\right)^{2/3}}+\frac{2^{2/3}c^{2/3}\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\,\sqrt[3]{c}\,x\right)}{3\sqrt{b^2-4ac}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}-\frac{2^{2/3}c^{2/3}\log\left(\sqrt[3]{\sqrt{b^2-4ac}+b}+\sqrt[3]{2}\,\sqrt[3]{c}\,x\right)}{3\sqrt{b^2-4ac}\left(\sqrt{b^2-4ac}+b\right)^{2/3}}-\frac{2^{2/3}c^{2/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b-\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{\sqrt{3}\sqrt{b^2-4ac}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}+\frac{2^{2/3}c^{2/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{\sqrt{b^2-4ac}+b}}}{\sqrt{3}}\right)}{\sqrt{3}\sqrt{b^2-4ac}\left(\sqrt{b^2-4ac}+b\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3 + c*x^6)^(-1),x]
```

```
[Out] -((2^(2/3)*c^(2/3)*ArcTan[(1 - (2*2^(1/3)*c^(1/3)*x)/(b - Sqrt[b^2 - 4*a*c]
)^(1/3))/Sqrt[3]])/(Sqrt[3]*Sqrt[b^2 - 4*a*c]*(b - Sqrt[b^2 - 4*a*c])^(2/3)
)) + (2^(2/3)*c^(2/3)*ArcTan[(1 - (2*2^(1/3)*c^(1/3)*x)/(b + Sqrt[b^2 - 4*a
*c])^(1/3))/Sqrt[3]])/(Sqrt[3]*Sqrt[b^2 - 4*a*c]*(b + Sqrt[b^2 - 4*a*c])^(2
/3)) + (2^(2/3)*c^(2/3)*Log[(b - Sqrt[b^2 - 4*a*c])^(1/3) + 2^(1/3)*c^(1/3)
*x])/(3*Sqrt[b^2 - 4*a*c]*(b - Sqrt[b^2 - 4*a*c])^(2/3)) - (2^(2/3)*c^(2/3)
*Log[(b + Sqrt[b^2 - 4*a*c])^(1/3) + 2^(1/3)*c^(1/3)*x])/(3*Sqrt[b^2 - 4*a*
c]*(b + Sqrt[b^2 - 4*a*c])^(2/3)) - (c^(2/3)*Log[(b - Sqrt[b^2 - 4*a*c])^(2
/3) - 2^(1/3)*c^(1/3)*(b - Sqrt[b^2 - 4*a*c])^(1/3)*x + 2^(2/3)*c^(2/3)*x^2
])/(3*2^(1/3)*Sqrt[b^2 - 4*a*c]*(b - Sqrt[b^2 - 4*a*c])^(2/3)) + (c^(2/3)*L
og[(b + Sqrt[b^2 - 4*a*c])^(2/3) - 2^(1/3)*c^(1/3)*(b + Sqrt[b^2 - 4*a*c])^
(1/3)*x + 2^(2/3)*c^(2/3)*x^2])/(3*2^(1/3)*Sqrt[b^2 - 4*a*c]*(b + Sqrt[b^2
- 4*a*c])^(2/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1347

```
Int[((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^(-1), x_Symbol] :> With[{q
= Rt[b^2 - 4*a*c, 2]}, Dist[c/q, Int[1/(b/2 - q/2 + c*x^n), x], x] - Dist[c
/q, Int[1/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*
n] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{1}{a+bx^3+cx^6}\,dx = \frac{c\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^3}\,dx}{\sqrt{b^2-4ac}} - \frac{c\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^3}\,dx}{\sqrt{b^2-4ac}}$$

$$= \frac{\left(2^{2/3}c\right)\int \frac{1}{\frac{\sqrt[3]{b-\sqrt{b^2-4ac}}}{\sqrt[3]{2}}+\sqrt[3]{c}\,x}\,dx}{3\sqrt{b^2-4ac}\left(b-\sqrt{b^2-4ac}\right)^{2/3}} + \frac{\left(2^{2/3}c\right)\int \frac{2^{2/3}\sqrt[3]{b-\sqrt{b^2-4ac}}-\sqrt[3]{c}\,x}{\frac{\left(b-\sqrt{b^2-4ac}\right)^{2/3}}{2^{2/3}}-\frac{\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}\,x}{\sqrt[3]{2}}+c^{2/3}x^2}\,dx}{3\sqrt{b^2-4ac}\left(b-\sqrt{b^2-4ac}\right)^{2/3}} - \frac{\left(2^{2/3}c\right)}{3\sqrt{b^2-}}$$

$$= \frac{2^{2/3}c^{2/3}\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}\,x\right)}{3\sqrt{b^2-4ac}\left(b-\sqrt{b^2-4ac}\right)^{2/3}} - \frac{2^{2/3}c^{2/3}\log\left(\sqrt[3]{b+\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}\,x\right)}{3\sqrt{b^2-4ac}\left(b+\sqrt{b^2-4ac}\right)^{2/3}} -$$

$$= \frac{2^{2/3}c^{2/3}\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}\,x\right)}{3\sqrt{b^2-4ac}\left(b-\sqrt{b^2-4ac}\right)^{2/3}} - \frac{2^{2/3}c^{2/3}\log\left(\sqrt[3]{b+\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}\,x\right)}{3\sqrt{b^2-4ac}\left(b+\sqrt{b^2-4ac}\right)^{2/3}} -$$

$$= -\frac{2^{2/3}c^{2/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}\,x}{\sqrt[3]{b-\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{\sqrt{3}\sqrt{b^2-4ac}\left(b-\sqrt{b^2-4ac}\right)^{2/3}} + \frac{2^{2/3}c^{2/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}\,x}{\sqrt[3]{b+\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{\sqrt{3}\sqrt{b^2-4ac}\left(b+\sqrt{b^2-4ac}\right)^{2/3}} + \frac{2^{2/3}c^{2/3}\log\left(\sqrt[3]{\right.}}{3\sqrt{b^2-4a}}$$

Mathematica [C] time = 0.02, size = 45, normalized size = 0.08

$$\frac{1}{3}\text{RootSum}\left[\#1^6c+\#1^3b+a\&,\frac{\log(x-\#1)}{2\#1^5c+\#1^2b}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3 + c*x^6)^(-1),x]
```

```
[Out] RootSum[a + b*#1^3 + c*#1^6 & , Log[x - #1]/(b*#1^2 + 2*c*#1^5) & ]/3
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{a + bx^3 + cx^6} dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x^3 + c*x^6)^(-1),x]
```

```
[Out] IntegrateAlgebraic[(a + b*x^3 + c*x^6)^(-1), x]
```

fricas [B] time = 2.02, size = 3978, normalized size = 7.13

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^6+b*x^3+a),x, algorithm="fricas")
```

```
[Out] 2/3*sqrt(3)*(1/2)^(1/3)*(((a^2*b^2 - 4*a^3*c)*sqrt((b^4 - 4*a*b^2*c + 4*a^2
*c^2)/(a^4*b^6 - 12*a^5*b^4*c + 48*a^6*b^2*c^2 - 64*a^7*c^3)) + b)/(a^2*b^2
 - 4*a^3*c))^(1/3)*arctan(-1/6*(2*(1/2)^(2/3)*(sqrt(3)*(a^2*b^6 - 12*a^3*b^
4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3)*x*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/(a^
4*b^6 - 12*a^5*b^4*c + 48*a^6*b^2*c^2 - 64*a^7*c^3)) - sqrt(3)*(b^5 - 6*a*b
^3*c + 8*a^2*b*c^2)*x)*(((a^2*b^2 - 4*a^3*c)*sqrt((b^4 - 4*a*b^2*c + 4*a^2*
c^2)/(a^4*b^6 - 12*a^5*b^4*c + 48*a^6*b^2*c^2 - 64*a^7*c^3)) + b)/(a^2*b^2
- 4*a^3*c))^(2/3) - (1/2)^(1/6)*(sqrt(3)*(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b
^2*c^2 - 64*a^5*c^3)*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/(a^4*b^6 - 12*a^5*b
^4*c + 48*a^6*b^2*c^2 - 64*a^7*c^3)) - sqrt(3)*(b^5 - 6*a*b^3*c + 8*a^2*b*c
^2))*sqrt((2*(b^2*c^2 - 2*a*c^3)*x^2 + (1/2)^(2/3)*(b^6 - 8*a*b^4*c + 20*a^
2*b^2*c^2 - 16*a^3*c^3 - (a^2*b^7 - 12*a^3*b^5*c + 48*a^4*b^3*c^2 - 64*a^5*
b*c^3)*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/(a^4*b^6 - 12*a^5*b^4*c + 48*a^6*
b^2*c^2 - 64*a^7*c^3)))*(((a^2*b^2 - 4*a^3*c)*sqrt((b^4 - 4*a*b^2*c + 4*a^2
*c^2)/(a^4*b^6 - 12*a^5*b^4*c + 48*a^6*b^2*c^2 - 64*a^7*c^3)) + b)/(a^2*b^2
 - 4*a^3*c))^(2/3) - (1/2)^(1/3)*((a^2*b^5*c - 8*a^3*b^3*c^2 + 16*a^4*b*c^3
)*x*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/(a^4*b^6 - 12*a^5*b^4*c + 48*a^6*b^2
*c^2 - 64*a^7*c^3)) - (b^4*c - 6*a*b^2*c^2 + 8*a^2*c^3)*x)*(((a^2*b^2 - 4*a
^3*c)*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/(a^4*b^6 - 12*a^5*b^4*c + 48*a^6*b
^2*c^2 - 64*a^7*c^3)) + b)/(a^2*b^2 - 4*a^3*c))^(1/3))/(b^2*c^2 - 2*a*c^3))
*(((a^2*b^2 - 4*a^3*c)*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/(a^4*b^6 - 12*a^5
*b^4*c + 48*a^6*b^2*c^2 - 64*a^7*c^3)) + b)/(a^2*b^2 - 4*a^3*c))^(2/3) + 2*
sqrt(3)*(b^2*c - 2*a*c^2))/(b^2*c - 2*a*c^2)) - 2/3*sqrt(3)*(1/2)^(1/3)*(-(
(a^2*b^2 - 4*a^3*c)*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/(a^4*b^6 - 12*a^5*b^
4*c + 48*a^6*b^2*c^2 - 64*a^7*c^3)) - b)/(a^2*b^2 - 4*a^3*c))^(1/3)*arctan(
-1/6*(2*(1/2)^(2/3)*(sqrt(3)*(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*
a^5*c^3)*x*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/(a^4*b^6 - 12*a^5*b^4*c + 48*
a^6*b^2*c^2 - 64*a^7*c^3)) + sqrt(3)*(b^5 - 6*a*b^3*c + 8*a^2*b*c^2)*x)*(-(
(a^2*b^2 - 4*a^3*c)*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/(a^4*b^6 - 12*a^5*b^
4*c + 48*a^6*b^2*c^2 - 64*a^7*c^3)) - b)/(a^2*b^2 - 4*a^3*c))^(2/3) - (1/2)
^(1/6)*(sqrt(3)*(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3)*sqrt
((b^4 - 4*a*b^2*c + 4*a^2*c^2)/(a^4*b^6 - 12*a^5*b^4*c + 48*a^6*b^2*c^2 - 6
4*a^7*c^3)) + sqrt(3)*(b^5 - 6*a*b^3*c + 8*a^2*b*c^2))*sqrt((2*(b^2*c^2 - 2
*a*c^3)*x^2 + (1/2)^(2/3)*(b^6 - 8*a*b^4*c + 20*a^2*b^2*c^2 - 16*a^3*c^3 +
(a^2*b^7 - 12*a^3*b^5*c + 48*a^4*b^3*c^2 - 64*a^5*b*c^3)*sqrt((b^4 - 4*a*b^
2*c + 4*a^2*c^2)/(a^4*b^6 - 12*a^5*b^4*c + 48*a^6*b^2*c^2 - 64*a^7*c^3)))*(
-((a^2*b^2 - 4*a^3*c)*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/(a^4*b^6 - 12*a^5*
b^4*c + 48*a^6*b^2*c^2 - 64*a^7*c^3)) - b)/(a^2*b^2 - 4*a^3*c))^(2/3) + (1/
2)^(1/3)*((a^2*b^5*c - 8*a^3*b^3*c^2 + 16*a^4*b*c^3)*x*sqrt((b^4 - 4*a*b^2*
c + 4*a^2*c^2)/(a^4*b^6 - 12*a^5*b^4*c + 48*a^6*b^2*c^2 - 64*a^7*c^3)) + (b
```

```
^4*c - 6*a*b^2*c^2 + 8*a^2*c^3)*x)*(-((a^2*b^2 - 4*a^3*c)*sqrt((b^4 - 4*a*b
^2*c + 4*a^2*c^2)/(a^4*b^6 - 12*a^5*b^4*c + 48*a^6*b^2*c^2 - 64*a^7*c^3)) -
 b)/(a^2*b^2 - 4*a^3*c))^(1/3))/(b^2*c^2 - 2*a*c^3))*(-((a^2*b^2 - 4*a^3*c)
*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/(a^4*b^6 - 12*a^5*b^4*c + 48*a^6*b^2*c^
2 - 64*a^7*c^3)) - b)/(a^2*b^2 - 4*a^3*c))^(2/3) - 2*sqrt(3)*(b^2*c - 2*a*c
^2))/(b^2*c - 2*a*c^2)) - 1/6*(1/2)^(1/3)*(((a^2*b^2 - 4*a^3*c)*sqrt((b^4 -
 4*a*b^2*c + 4*a^2*c^2)/(a^4*b^6 - 12*a^5*b^4*c + 48*a^6*b^2*c^2 - 64*a^7*c
^3)) + b)/(a^2*b^2 - 4*a^3*c))^(1/3)*log(-2*(b^2*c^2 - 2*a*c^3)*x^2 - (1/2)
^(2/3)*(b^6 - 8*a*b^4*c + 20*a^2*b^2*c^2 - 16*a^3*c^3 - (a^2*b^7 - 12*a^3*b
^5*c + 48*a^4*b^3*c^2 - 64*a^5*b*c^3)*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/(a
^4*b^6 - 12*a^5*b^4*c + 48*a^6*b^2*c^2 - 64*a^7*c^3)))*(((a^2*b^2 - 4*a^3*c
)*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/(a^4*b^6 - 12*a^5*b^4*c + 48*a^6*b^2*c
^2 - 64*a^7*c^3)) + b)/(a^2*b^2 - 4*a^3*c))^(2/3) + (1/2)^(1/3)*((a^2*b^5*c
 - 8*a^3*b^3*c^2 + 16*a^4*b*c^3)*x*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/(a^4*
b^6 - 12*a^5*b^4*c + 48*a^6*b^2*c^2 - 64*a^7*c^3)) - (b^4*c - 6*a*b^2*c^2 +
 8*a^2*c^3)*x)*(((a^2*b^2 - 4*a^3*c)*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/(a^
4*b^6 - 12*a^5*b^4*c + 48*a^6*b^2*c^2 - 64*a^7*c^3)) + b)/(a^2*b^2 - 4*a^3*
c))^(1/3)) - 1/6*(1/2)^(1/3)*(-((a^2*b^2 - 4*a^3*c)*sqrt((b^4 - 4*a*b^2*c +
 4*a^2*c^2)/(a^4*b^6 - 12*a^5*b^4*c + 48*a^6*b^2*c^2 - 64*a^7*c^3)) - b)/(a
^2*b^2 - 4*a^3*c))^(1/3)*log(-2*(b^2*c^2 - 2*a*c^3)*x^2 - (1/2)^(2/3)*(b^6
- 8*a*b^4*c + 20*a^2*b^2*c^2 - 16*a^3*c^3 + (a^2*b^7 - 12*a^3*b^5*c + 48*a^
4*b^3*c^2 - 64*a^5*b*c^3)*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/(a^4*b^6 - 12*
a^5*b^4*c + 48*a^6*b^2*c^2 - 64*a^7*c^3)))*(-((a^2*b^2 - 4*a^3*c)*sqrt((b^4
 - 4*a*b^2*c + 4*a^2*c^2)/(a^4*b^6 - 12*a^5*b^4*c + 48*a^6*b^2*c^2 - 64*a^7
*c^3)) - b)/(a^2*b^2 - 4*a^3*c))^(2/3) - (1/2)^(1/3)*((a^2*b^5*c - 8*a^3*b^
3*c^2 + 16*a^4*b*c^3)*x*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/(a^4*b^6 - 12*a^
5*b^4*c + 48*a^6*b^2*c^2 - 64*a^7*c^3)) + (b^4*c - 6*a*b^2*c^2 + 8*a^2*c^3)
*x)*(-((a^2*b^2 - 4*a^3*c)*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/(a^4*b^6 - 12
*a^5*b^4*c + 48*a^6*b^2*c^2 - 64*a^7*c^3)) - b)/(a^2*b^2 - 4*a^3*c))^(1/3))
 + 1/3*(1/2)^(1/3)*(((a^2*b^2 - 4*a^3*c)*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)
/(a^4*b^6 - 12*a^5*b^4*c + 48*a^6*b^2*c^2 - 64*a^7*c^3)) + b)/(a^2*b^2 - 4*
a^3*c))^(1/3)*log(-2*(b^2*c - 2*a*c^2)*x + (1/2)^(1/3)*(b^4 - 6*a*b^2*c + 8
*a^2*c^2 - (a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*sqrt((b^4 - 4*a*b^2*c + 4
*a^2*c^2)/(a^4*b^6 - 12*a^5*b^4*c + 48*a^6*b^2*c^2 - 64*a^7*c^3)))*(((a^2*b
^2 - 4*a^3*c)*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/(a^4*b^6 - 12*a^5*b^4*c +
48*a^6*b^2*c^2 - 64*a^7*c^3)) + b)/(a^2*b^2 - 4*a^3*c))^(1/3)) + 1/3*(1/2)^
(1/3)*(-((a^2*b^2 - 4*a^3*c)*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/(a^4*b^6 -
12*a^5*b^4*c + 48*a^6*b^2*c^2 - 64*a^7*c^3)) - b)/(a^2*b^2 - 4*a^3*c))^(1/3
)*log(-2*(b^2*c - 2*a*c^2)*x + (1/2)^(1/3)*(b^4 - 6*a*b^2*c + 8*a^2*c^2 + (
a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/(a
^4*b^6 - 12*a^5*b^4*c + 48*a^6*b^2*c^2 - 64*a^7*c^3)))*(-((a^2*b^2 - 4*a^3*
c)*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/(a^4*b^6 - 12*a^5*b^4*c + 48*a^6*b^2*
c^2 - 64*a^7*c^3)) - b)/(a^2*b^2 - 4*a^3*c))^(1/3))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{cx^6 + bx^3 + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^6+b*x^3+a),x, algorithm="giac")
```

```
[Out] integrate(1/(c*x^6 + b*x^3 + a), x)
```

maple [C] time = 0.00, size = 40, normalized size = 0.07

$$\frac{\ln\left(-\mathrm{RootOf}\left(c\_Z^6 + b\_Z^3 + a\right) + x\right)}{6\,\mathrm{RootOf}\left(c\_Z^6 + b\_Z^3 + a\right)^5 c + 3\,\mathrm{RootOf}\left(c\_Z^6 + b\_Z^3 + a\right)^2 b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x^6+b*x^3+a),x)
```

```
[Out] 1/3*sum(1/(2*_R^5*c+_R^2*b)*ln(-_R+x),_R=RootOf(_Z^6*c+_Z^3*b+a))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{cx^6 + bx^3 + a} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^6+b*x^3+a),x, algorithm="maxima")
```

```
[Out] integrate(1/(c*x^6 + b*x^3 + a), x)
```

mupad [B] time = 8.49, size = 2597, normalized size = 4.65

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*x^3 + c*x^6),x)
```

```
[Out] log(6*c^5*x + (2^(2/3)*(-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 16*a^2*b*c^2
 - 8*a*b^3*c - 2*a*c*(-(4*a*c - b^2)^3)^(1/2))/(a^2*(4*a*c - b^2)^3))^(1/3)
*(36*a*c^5 - 9*b^2*c^4 + (9*2^(1/3)*b*c^3*(x + (2^(2/3)*a*(-(b^5 + b^2*(-(4
*a*c - b^2)^3)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c - 2*a*c*(-(4*a*c - b^2)^3)^
(1/2))/(a^2*(4*a*c - b^2)^3))^(1/3))/2)*(4*a*c - b^2)^2*(-(b^5 + b^2*(-(4*a
*c - b^2)^3)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c - 2*a*c*(-(4*a*c - b^2)^3)^(1
/2))/(a^2*(4*a*c - b^2)^3))^(2/3))/2))/6)*((b^5 + b^2*(-(4*a*c - b^2)^3)^(1
/2) + 16*a^2*b*c^2 - 8*a*b^3*c - 2*a*c*(-(4*a*c - b^2)^3)^(1/2))/(54*(a^2*b
^6 - 64*a^5*c^3 - 12*a^3*b^4*c + 48*a^4*b^2*c^2)))^(1/3) + log(6*c^5*x + (2
^(2/3)*(-(b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c + 2
*a*c*(-(4*a*c - b^2)^3)^(1/2))/(a^2*(4*a*c - b^2)^3))^(1/3)*(36*a*c^5 - 9*b
^2*c^4 + (9*2^(1/3)*b*c^3*(x + (2^(2/3)*a*(-(b^5 - b^2*(-(4*a*c - b^2)^3)^(
1/2) + 16*a^2*b*c^2 - 8*a*b^3*c + 2*a*c*(-(4*a*c - b^2)^3)^(1/2))/(a^2*(4*a
*c - b^2)^3))^(1/3))/2)*(4*a*c - b^2)^2*(-(b^5 - b^2*(-(4*a*c - b^2)^3)^(1/
2) + 16*a^2*b*c^2 - 8*a*b^3*c + 2*a*c*(-(4*a*c - b^2)^3)^(1/2))/(a^2*(4*a*c
 - b^2)^3))^(2/3))/2))/6)*((b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 16*a^2*b*c
^2 - 8*a*b^3*c + 2*a*c*(-(4*a*c - b^2)^3)^(1/2))/(54*(a^2*b^6 - 64*a^5*c^3
- 12*a^3*b^4*c + 48*a^4*b^2*c^2)))^(1/3) + log(6*c^5*x - (2^(2/3)*(3^(1/2)*
1i - 1)*(-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c -
2*a*c*(-(4*a*c - b^2)^3)^(1/2))/(a^2*(4*a*c - b^2)^3))^(1/3)*(9*b^2*c^4 - 3
6*a*c^5 + (2^(1/3)*(3^(1/2)*1i + 1)*(81*b*c^3*x*(4*a*c - b^2)^2 + (81*2^(2/
3)*a*b*c^3*(3^(1/2)*1i - 1)*(4*a*c - b^2)^2*(-(b^5 + b^2*(-(4*a*c - b^2)^3)
^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c - 2*a*c*(-(4*a*c - b^2)^3)^(1/2))/(a^2*(4
*a*c - b^2)^3))^(1/3))/4)*(-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 16*a^2*b*
c^2 - 8*a*b^3*c - 2*a*c*(-(4*a*c - b^2)^3)^(1/2))/(a^2*(4*a*c - b^2)^3))^(2
/3))/36))/12)*((3^(1/2)*1i)/2 - 1/2)*((b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) +
 16*a^2*b*c^2 - 8*a*b^3*c - 2*a*c*(-(4*a*c - b^2)^3)^(1/2))/(54*(a^2*b^6 -
64*a^5*c^3 - 12*a^3*b^4*c + 48*a^4*b^2*c^2)))^(1/3) - log(6*c^5*x - (2^(2/3
)*(3^(1/2)*1i + 1)*(-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 16*a^2*b*c^2 - 8
*a*b^3*c - 2*a*c*(-(4*a*c - b^2)^3)^(1/2))/(a^2*(4*a*c - b^2)^3))^(1/3)*(36
*a*c^5 - 9*b^2*c^4 + (2^(1/3)*(3^(1/2)*1i - 1)*(81*b*c^3*x*(4*a*c - b^2)^2
- (81*2^(2/3)*a*b*c^3*(3^(1/2)*1i + 1)*(4*a*c - b^2)^2*(-(b^5 + b^2*(-(4*a*
c - b^2)^3)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c - 2*a*c*(-(4*a*c - b^2)^3)^(1/
2))/(a^2*(4*a*c - b^2)^3))^(1/3))/4)*(-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2)
+ 16*a^2*b*c^2 - 8*a*b^3*c - 2*a*c*(-(4*a*c - b^2)^3)^(1/2))/(a^2*(4*a*c -
b^2)^3))^(2/3))/36))/12)*((3^(1/2)*1i)/2 + 1/2)*((b^5 + b^2*(-(4*a*c - b^2)
^3)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c - 2*a*c*(-(4*a*c - b^2)^3)^(1/2))/(54*
(a^2*b^6 - 64*a^5*c^3 - 12*a^3*b^4*c + 48*a^4*b^2*c^2)))^(1/3) + log(6*c^5*
x - (2^(2/3)*(3^(1/2)*1i - 1)*(-(b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 16*a^
2*b*c^2 - 8*a*b^3*c + 2*a*c*(-(4*a*c - b^2)^3)^(1/2))/(a^2*(4*a*c - b^2)^3)
```

```
)^(1/3)*(9*b^2*c^4 - 36*a*c^5 + (2^(1/3)*(3^(1/2)*1i + 1)*(81*b*c^3*x*(4*a*
c - b^2)^2 + (81*2^(2/3)*a*b*c^3*(3^(1/2)*1i - 1)*(4*a*c - b^2)^2*(-(b^5 -
b^2*(-(4*a*c - b^2)^3)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c + 2*a*c*(-(4*a*c -
b^2)^3)^(1/2))/(a^2*(4*a*c - b^2)^3))^(1/3))/4)*(-(b^5 - b^2*(-(4*a*c - b^2
)^3)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c + 2*a*c*(-(4*a*c - b^2)^3)^(1/2))/(a^
2*(4*a*c - b^2)^3))^(2/3))/36))/12)*((3^(1/2)*1i)/2 - 1/2)*((b^5 - b^2*(-(4
*a*c - b^2)^3)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c + 2*a*c*(-(4*a*c - b^2)^3)^
(1/2))/(54*(a^2*b^6 - 64*a^5*c^3 - 12*a^3*b^4*c + 48*a^4*b^2*c^2)))^(1/3) -
 log(6*c^5*x - (2^(2/3)*(3^(1/2)*1i + 1)*(-(b^5 - b^2*(-(4*a*c - b^2)^3)^(1
/2) + 16*a^2*b*c^2 - 8*a*b^3*c + 2*a*c*(-(4*a*c - b^2)^3)^(1/2))/(a^2*(4*a*
c - b^2)^3))^(1/3)*(36*a*c^5 - 9*b^2*c^4 + (2^(1/3)*(3^(1/2)*1i - 1)*(81*b*
c^3*x*(4*a*c - b^2)^2 - (81*2^(2/3)*a*b*c^3*(3^(1/2)*1i + 1)*(4*a*c - b^2)^
2*(-(b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c + 2*a*c*
(-(4*a*c - b^2)^3)^(1/2))/(a^2*(4*a*c - b^2)^3))^(1/3))/4)*(-(b^5 - b^2*(-(
4*a*c - b^2)^3)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c + 2*a*c*(-(4*a*c - b^2)^3)
^(1/2))/(a^2*(4*a*c - b^2)^3))^(2/3))/36))/12)*((3^(1/2)*1i)/2 + 1/2)*((b^5
 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c + 2*a*c*(-(4*a*c
 - b^2)^3)^(1/2))/(54*(a^2*b^6 - 64*a^5*c^3 - 12*a^3*b^4*c + 48*a^4*b^2*c^2
)))^(1/3)
```

sympy [A] time = 4.35, size = 155, normalized size = 0.28

$$\operatorname{RootSum}\left(t^{6}\left(46656a^{5}c^{3}-34992a^{4}b^{2}c^{2}+8748a^{3}b^{4}c-729a^{2}b^{6}\right)+t^{3}\left(432a^{2}bc^{2}-216ab^{3}c+27b^{5}\right)+c^{2},\left(t\mapsto t\log\left(x+\frac{-1296t^{4}a^{4}bc^{2}+648t^{4}a^{3}b^{3}c-81t^{4}a^{2}b^{5}+12ta^{2}c^{2}-15tab^{2}c+3tb^{4}}{2ac^{2}-b^{2}c}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x**6+b*x**3+a),x)
```

```
[Out] RootSum(_t**6*(46656*a**5*c**3 - 34992*a**4*b**2*c**2 + 8748*a**3*b**4*c -
729*a**2*b**6) + _t**3*(432*a**2*b*c**2 - 216*a*b**3*c + 27*b**5) + c**2, L
ambda(_t, _t*log(x + (-1296*_t**4*a**4*b*c**2 + 648*_t**4*a**3*b**3*c - 81*
_t**4*a**2*b**5 + 12*_t*a**2*c**2 - 15*_t*a*b**2*c + 3*_t*b**4)/(2*a*c**2 -
 b**2*c))))
```

3.136 $\int \frac{1}{x^2(a+bx^3+cx^6)}\,dx$

Optimal. Leaf size=610

$$-\frac{\sqrt[3]{c}\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\log\left(-\sqrt[3]{2}\sqrt[3]{c}\,x\sqrt[3]{b-\sqrt{b^2-4ac}}+\left(b-\sqrt{b^2-4ac}\right)^{2/3}+2^{2/3}c^{2/3}x^2\right)}{6\,2^{2/3}a\sqrt[3]{b-\sqrt{b^2-4ac}}}-\frac{\sqrt[3]{c}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\log$$

Rubi [A] time = 0.82, antiderivative size = 610, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 8, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {1368, 1510, 292, 31, 634, 617, 204, 628}

$$-\frac{\sqrt[3]{c}\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\log\left(-\sqrt[3]{2}\sqrt[3]{c}\,x\sqrt[3]{b-\sqrt{b^2-4ac}}+\left(b-\sqrt{b^2-4ac}\right)^{2/3}+2^{2/3}c^{2/3}x^2\right)}{6\,2^{2/3}a\sqrt[3]{b-\sqrt{b^2-4ac}}}-\frac{\sqrt[3]{c}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\log\left(-\sqrt[3]{2}\sqrt[3]{c}\,x\sqrt[3]{\sqrt{b^2-4ac}+b}+\left(\sqrt{b^2-4ac}+b\right)^{2/3}+2^{2/3}c^{2/3}x^2\right)}{6\,2^{2/3}a\sqrt[3]{\sqrt{b^2-4ac}+b}}+\frac{\sqrt[3]{c}\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}\,x\right)}{3\,2^{2/3}a\sqrt[3]{b-\sqrt{b^2-4ac}}}+\frac{\sqrt[3]{c}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{\sqrt{b^2-4ac}+b}+\sqrt[3]{2}\sqrt[3]{c}\,x\right)}{3\,2^{2/3}a\sqrt[3]{\sqrt{b^2-4ac}+b}}+\frac{\sqrt[3]{c}\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}\,x}{\sqrt[3]{b-\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}\,a\sqrt[3]{b-\sqrt{b^2-4ac}}}+\frac{\sqrt[3]{c}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}\,x}{\sqrt[3]{\sqrt{b^2-4ac}+b}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}\,a\sqrt[3]{\sqrt{b^2-4ac}+b}}-\frac{1}{ax}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a + b*x^3 + c*x^6)),x]
```

```
[Out] -(1/(a*x)) + (c^(1/3)*(1 + b/Sqrt[b^2 - 4*a*c])*ArcTan[(1 - (2*2^(1/3)*c^(1
/3)*x)/(b - Sqrt[b^2 - 4*a*c])^(1/3))/Sqrt[3]])/(2^(2/3)*Sqrt[3]*a*(b - Sqr
t[b^2 - 4*a*c])^(1/3)) + (c^(1/3)*(1 - b/Sqrt[b^2 - 4*a*c])*ArcTan[(1 - (2*
2^(1/3)*c^(1/3)*x)/(b + Sqrt[b^2 - 4*a*c])^(1/3))/Sqrt[3]])/(2^(2/3)*Sqrt[3
]*a*(b + Sqrt[b^2 - 4*a*c])^(1/3)) + (c^(1/3)*(1 + b/Sqrt[b^2 - 4*a*c])*Log
[(b - Sqrt[b^2 - 4*a*c])^(1/3) + 2^(1/3)*c^(1/3)*x])/(3*2^(2/3)*a*(b - Sqrt
[b^2 - 4*a*c])^(1/3)) + (c^(1/3)*(1 - b/Sqrt[b^2 - 4*a*c])*Log[(b + Sqrt[b^
2 - 4*a*c])^(1/3) + 2^(1/3)*c^(1/3)*x])/(3*2^(2/3)*a*(b + Sqrt[b^2 - 4*a*c]
)^(1/3)) - (c^(1/3)*(1 + b/Sqrt[b^2 - 4*a*c])*Log[(b - Sqrt[b^2 - 4*a*c])^(
2/3) - 2^(1/3)*c^(1/3)*(b - Sqrt[b^2 - 4*a*c])^(1/3)*x + 2^(2/3)*c^(2/3)*x^
2])/(6*2^(2/3)*a*(b - Sqrt[b^2 - 4*a*c])^(1/3)) - (c^(1/3)*(1 - b/Sqrt[b^2
- 4*a*c])*Log[(b + Sqrt[b^2 - 4*a*c])^(2/3) - 2^(1/3)*c^(1/3)*(b + Sqrt[b^2
 - 4*a*c])^(1/3)*x + 2^(2/3)*c^(2/3)*x^2])/(6*2^(2/3)*a*(b + Sqrt[b^2 - 4*a
*c])^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
```

```
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1368

```
Int[((d_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_
Symbol] :> Simp[((d*x)^(m + 1)*(a + b*x^n + c*x^(2*n))^(p + 1))/(a*d*(m + 1
)), x] - Dist[1/(a*d^n*(m + 1)), Int[(d*x)^(m + n)*(b*(m + n*(p + 1) + 1) +
 c*(m + 2*n*(p + 1) + 1)*x^n)*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a
, b, c, d, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && L
tQ[m, -1] && IntegerQ[p]
```

Rule 1510

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_)))/((a_) + (b_.)*(x_)^(n_) +
 (c_.)*(x_)^(n2_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 +
(2*c*d - b*e)/(2*q), Int[(f*x)^m/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (
2*c*d - b*e)/(2*q), Int[(f*x)^m/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b
, c, d, e, f, m}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{1}{x^2\left(a+bx^3+cx^6\right)}\,dx = -\frac{1}{ax} + \frac{\int \frac{x\left(-b-cx^3\right)}{a+bx^3+cx^6}\,dx}{a}$$

$$= -\frac{1}{ax} - \frac{\left(c\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\right)\int \frac{x}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^3}\,dx}{2a} - \frac{\left(c\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)\right)\int \frac{x}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^3}}{2a}$$

$$= -\frac{1}{ax} + \frac{\left(c^{2/3}\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)\right)\int \frac{1}{\frac{\sqrt[3]{b-\sqrt{b^2-4ac}}}{\sqrt[3]{2}}+\sqrt[3]{c}\,x}\,dx}{3\,2^{2/3}a\sqrt[3]{b-\sqrt{b^2-4ac}}} - \frac{\left(c^{2/3}\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)\right)\int \frac{}{\frac{\left(b-\sqrt{b^2-4ac}\right)}{2^{2/3}}}}{3\,2^{2/3}a\sqrt[3]{b-\sqrt{}}}$$

$$= -\frac{1}{ax} + \frac{\sqrt[3]{c}\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}\,x\right)}{3\,2^{2/3}a\sqrt[3]{b-\sqrt{b^2-4ac}}} + \frac{\sqrt[3]{c}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\mathrm{lo}}{3\,2^{2/3}}$$

$$= -\frac{1}{ax} + \frac{\sqrt[3]{c}\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}\,x\right)}{3\,2^{2/3}a\sqrt[3]{b-\sqrt{b^2-4ac}}} + \frac{\sqrt[3]{c}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\mathrm{lo}}{3\,2^{2/3}}$$

$$= -\frac{1}{ax} + \frac{\sqrt[3]{c}\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}\,x}{\sqrt[3]{b-\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}\,a\sqrt[3]{b-\sqrt{b^2-4ac}}} + \frac{\sqrt[3]{c}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}}{\sqrt[3]{b+\sqrt{b^2}}}}{\sqrt{3}}\right.}{2^{2/3}\sqrt{3}\,a\sqrt[3]{b+\sqrt{b^2-4ac}}}$$

Mathematica [C] time = 0.03, size = 71, normalized size = 0.12

$$-\frac{\text{RootSum}\left[\#1^6c+\#1^3b+a\&,\frac{\#1^3c\log(x-\#1)+b\log(x-\#1)}{2\#1^4c+\#1b}\&\right]}{3a}-\frac{1}{ax}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a + b*x^3 + c*x^6)),x]
```

```
[Out] -(1/(a*x)) - RootSum[a + b*#1^3 + c*#1^6 & , (b*Log[x - #1] + c*Log[x - #1]
*#1^3)/(b*#1 + 2*c*#1^4) & ]/(3*a)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2\left(a+bx^3+cx^6\right)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x^2*(a + b*x^3 + c*x^6)),x]
```

```
[Out] IntegrateAlgebraic[1/(x^2*(a + b*x^3 + c*x^6)), x]
```

fricas [B] time = 3.99, size = 5266, normalized size = 8.63

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(c*x^6+b*x^3+a),x, algorithm="fricas")
```

```
[Out] 1/6*(4*sqrt(3)*(1/2)^(1/3)*a*x*((b^3 - 2*a*b*c + (a^4*b^2 - 4*a^5*c)*sqrt((
b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(a^8*b^6 - 1
2*a^9*b^4*c + 48*a^10*b^2*c^2 - 64*a^11*c^3)))/(a^4*b^2 - 4*a^5*c))^(1/3)*a
rctan(1/3*((1/2)^(5/6)*(sqrt(3)*(a^4*b^5 - 8*a^5*b^3*c + 16*a^6*b*c^2)*sqrt
((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(a^8*b^6 -
 12*a^9*b^4*c + 48*a^10*b^2*c^2 - 64*a^11*c^3)) - sqrt(3)*(b^6 - 8*a*b^4*c
+ 18*a^2*b^2*c^2 - 8*a^3*c^3))*((b^3 - 2*a*b*c + (a^4*b^2 - 4*a^5*c)*sqrt((
b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(a^8*b^6 - 1
2*a^9*b^4*c + 48*a^10*b^2*c^2 - 64*a^11*c^3)))/(a^4*b^2 - 4*a^5*c))^(1/3)*s
qrt((2*(b^4*c^3 - 4*a*b^2*c^4 + 2*a^2*c^5)*x^2 + (1/2)^(2/3)*((a^4*b^8 - 13
*a^5*b^6*c + 60*a^6*b^4*c^2 - 112*a^7*b^2*c^3 + 64*a^8*c^4)*x*sqrt((b^8 - 8
*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(a^8*b^6 - 12*a^9*b
^4*c + 48*a^10*b^2*c^2 - 64*a^11*c^3)) - (b^9 - 11*a*b^7*c + 42*a^2*b^5*c^2
 - 62*a^3*b^3*c^3 + 24*a^4*b*c^4)*x)*((b^3 - 2*a*b*c + (a^4*b^2 - 4*a^5*c)*
sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(a^8*b
^6 - 12*a^9*b^4*c + 48*a^10*b^2*c^2 - 64*a^11*c^3)))/(a^4*b^2 - 4*a^5*c))^(
2/3) - (1/2)^(1/3)*(b^7*c - 8*a*b^5*c^2 + 18*a^2*b^3*c^3 - 8*a^3*b*c^4 - (a
^4*b^6*c - 10*a^5*b^4*c^2 + 32*a^6*b^2*c^3 - 32*a^7*c^4)*sqrt((b^8 - 8*a*b^
6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(a^8*b^6 - 12*a^9*b^4*c
+ 48*a^10*b^2*c^2 - 64*a^11*c^3)))*((b^3 - 2*a*b*c + (a^4*b^2 - 4*a^5*c)*sq
rt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(a^8*b^6
 - 12*a^9*b^4*c + 48*a^10*b^2*c^2 - 64*a^11*c^3)))/(a^4*b^2 - 4*a^5*c))^(1/
3))/(b^4*c^3 - 4*a*b^2*c^4 + 2*a^2*c^5)) - (1/2)^(1/3)*(sqrt(3)*(a^4*b^5 -
8*a^5*b^3*c + 16*a^6*b*c^2)*x*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a
^3*b^2*c^3 + 4*a^4*c^4)/(a^8*b^6 - 12*a^9*b^4*c + 48*a^10*b^2*c^2 - 64*a^11
*c^3)) - sqrt(3)*(b^6 - 8*a*b^4*c + 18*a^2*b^2*c^2 - 8*a^3*c^3)*x)*((b^3 -
2*a*b*c + (a^4*b^2 - 4*a^5*c)*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a
^3*b^2*c^3 + 4*a^4*c^4)/(a^8*b^6 - 12*a^9*b^4*c + 48*a^10*b^2*c^2 - 64*a^11
*c^3)))/(a^4*b^2 - 4*a^5*c))^(1/3) + sqrt(3)*(b^4*c - 4*a*b^2*c^2 + 2*a^2*c
^3))/(b^4*c - 4*a*b^2*c^2 + 2*a^2*c^3)) - 4*sqrt(3)*(1/2)^(1/3)*a*x*((b^3 -
 2*a*b*c - (a^4*b^2 - 4*a^5*c)*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*
a^3*b^2*c^3 + 4*a^4*c^4)/(a^8*b^6 - 12*a^9*b^4*c + 48*a^10*b^2*c^2 - 64*a^1
1*c^3)))/(a^4*b^2 - 4*a^5*c))^(1/3)*arctan(1/3*((1/2)^(5/6)*(sqrt(3)*(a^4*b
^5 - 8*a^5*b^3*c + 16*a^6*b*c^2)*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 1
6*a^3*b^2*c^3 + 4*a^4*c^4)/(a^8*b^6 - 12*a^9*b^4*c + 48*a^10*b^2*c^2 - 64*a
^11*c^3)) + sqrt(3)*(b^6 - 8*a*b^4*c + 18*a^2*b^2*c^2 - 8*a^3*c^3))*((b^3 -
 2*a*b*c - (a^4*b^2 - 4*a^5*c)*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*
a^3*b^2*c^3 + 4*a^4*c^4)/(a^8*b^6 - 12*a^9*b^4*c + 48*a^10*b^2*c^2 - 64*a^1
1*c^3)))/(a^4*b^2 - 4*a^5*c))^(1/3)*sqrt((2*(b^4*c^3 - 4*a*b^2*c^4 + 2*a^2*
c^5)*x^2 - (1/2)^(2/3)*((a^4*b^8 - 13*a^5*b^6*c + 60*a^6*b^4*c^2 - 112*a^7*
b^2*c^3 + 64*a^8*c^4)*x*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2
*c^3 + 4*a^4*c^4)/(a^8*b^6 - 12*a^9*b^4*c + 48*a^10*b^2*c^2 - 64*a^11*c^3))
 + (b^9 - 11*a*b^7*c + 42*a^2*b^5*c^2 - 62*a^3*b^3*c^3 + 24*a^4*b*c^4)*x)*(
(b^3 - 2*a*b*c - (a^4*b^2 - 4*a^5*c)*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2
 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(a^8*b^6 - 12*a^9*b^4*c + 48*a^10*b^2*c^2 -
64*a^11*c^3)))/(a^4*b^2 - 4*a^5*c))^(2/3) - (1/2)^(1/3)*(b^7*c - 8*a*b^5*c^
2 + 18*a^2*b^3*c^3 - 8*a^3*b*c^4 + (a^4*b^6*c - 10*a^5*b^4*c^2 + 32*a^6*b^2
*c^3 - 32*a^7*c^4)*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3
+ 4*a^4*c^4)/(a^8*b^6 - 12*a^9*b^4*c + 48*a^10*b^2*c^2 - 64*a^11*c^3)))*((b
^3 - 2*a*b*c - (a^4*b^2 - 4*a^5*c)*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 -
 16*a^3*b^2*c^3 + 4*a^4*c^4)/(a^8*b^6 - 12*a^9*b^4*c + 48*a^10*b^2*c^2 - 64
*a^11*c^3)))/(a^4*b^2 - 4*a^5*c))^(1/3))/(b^4*c^3 - 4*a*b^2*c^4 + 2*a^2*c^5
)) - (1/2)^(1/3)*(sqrt(3)*(a^4*b^5 - 8*a^5*b^3*c + 16*a^6*b*c^2)*x*sqrt((b^
8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(a^8*b^6 - 12*
a^9*b^4*c + 48*a^10*b^2*c^2 - 64*a^11*c^3)) + sqrt(3)*(b^6 - 8*a*b^4*c + 18
*a^2*b^2*c^2 - 8*a^3*c^3)*x)*((b^3 - 2*a*b*c - (a^4*b^2 - 4*a^5*c)*sqrt((b^
8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(a^8*b^6 - 12*
a^9*b^4*c + 48*a^10*b^2*c^2 - 64*a^11*c^3)))/(a^4*b^2 - 4*a^5*c))^(1/3) - s
```

```
qrt(3)*(b^4*c - 4*a*b^2*c^2 + 2*a^2*c^3))/(b^4*c - 4*a*b^2*c^2 + 2*a^2*c^3)
) - (1/2)^(1/3)*a*x*((b^3 - 2*a*b*c + (a^4*b^2 - 4*a^5*c)*sqrt((b^8 - 8*a*b
^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(a^8*b^6 - 12*a^9*b^4*c
 + 48*a^10*b^2*c^2 - 64*a^11*c^3)))/(a^4*b^2 - 4*a^5*c))^(1/3)*log(2*(b^4*c
^3 - 4*a*b^2*c^4 + 2*a^2*c^5)*x^2 + (1/2)^(2/3)*((a^4*b^8 - 13*a^5*b^6*c +
60*a^6*b^4*c^2 - 112*a^7*b^2*c^3 + 64*a^8*c^4)*x*sqrt((b^8 - 8*a*b^6*c + 20
*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(a^8*b^6 - 12*a^9*b^4*c + 48*a^1
0*b^2*c^2 - 64*a^11*c^3)) - (b^9 - 11*a*b^7*c + 42*a^2*b^5*c^2 - 62*a^3*b^3
*c^3 + 24*a^4*b*c^4)*x)*((b^3 - 2*a*b*c + (a^4*b^2 - 4*a^5*c)*sqrt((b^8 - 8
*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(a^8*b^6 - 12*a^9*b
^4*c + 48*a^10*b^2*c^2 - 64*a^11*c^3)))/(a^4*b^2 - 4*a^5*c))^(2/3) - (1/2)^
(1/3)*(b^7*c - 8*a*b^5*c^2 + 18*a^2*b^3*c^3 - 8*a^3*b*c^4 - (a^4*b^6*c - 10
*a^5*b^4*c^2 + 32*a^6*b^2*c^3 - 32*a^7*c^4)*sqrt((b^8 - 8*a*b^6*c + 20*a^2*
b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(a^8*b^6 - 12*a^9*b^4*c + 48*a^10*b^2
*c^2 - 64*a^11*c^3)))*((b^3 - 2*a*b*c + (a^4*b^2 - 4*a^5*c)*sqrt((b^8 - 8*a
*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(a^8*b^6 - 12*a^9*b^4
*c + 48*a^10*b^2*c^2 - 64*a^11*c^3)))/(a^4*b^2 - 4*a^5*c))^(1/3)) - (1/2)^(
1/3)*a*x*((b^3 - 2*a*b*c - (a^4*b^2 - 4*a^5*c)*sqrt((b^8 - 8*a*b^6*c + 20*a
^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(a^8*b^6 - 12*a^9*b^4*c + 48*a^10*
b^2*c^2 - 64*a^11*c^3)))/(a^4*b^2 - 4*a^5*c))^(1/3)*log(2*(b^4*c^3 - 4*a*b^
2*c^4 + 2*a^2*c^5)*x^2 - (1/2)^(2/3)*((a^4*b^8 - 13*a^5*b^6*c + 60*a^6*b^4*
c^2 - 112*a^7*b^2*c^3 + 64*a^8*c^4)*x*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^
2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(a^8*b^6 - 12*a^9*b^4*c + 48*a^10*b^2*c^2 -
 64*a^11*c^3)) + (b^9 - 11*a*b^7*c + 42*a^2*b^5*c^2 - 62*a^3*b^3*c^3 + 24*a
^4*b*c^4)*x)*((b^3 - 2*a*b*c - (a^4*b^2 - 4*a^5*c)*sqrt((b^8 - 8*a*b^6*c +
20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(a^8*b^6 - 12*a^9*b^4*c + 48*a
^10*b^2*c^2 - 64*a^11*c^3)))/(a^4*b^2 - 4*a^5*c))^(2/3) - (1/2)^(1/3)*(b^7*
c - 8*a*b^5*c^2 + 18*a^2*b^3*c^3 - 8*a^3*b*c^4 + (a^4*b^6*c - 10*a^5*b^4*c^
2 + 32*a^6*b^2*c^3 - 32*a^7*c^4)*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 1
6*a^3*b^2*c^3 + 4*a^4*c^4)/(a^8*b^6 - 12*a^9*b^4*c + 48*a^10*b^2*c^2 - 64*a
^11*c^3)))*((b^3 - 2*a*b*c - (a^4*b^2 - 4*a^5*c)*sqrt((b^8 - 8*a*b^6*c + 20
*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(a^8*b^6 - 12*a^9*b^4*c + 48*a^1
0*b^2*c^2 - 64*a^11*c^3)))/(a^4*b^2 - 4*a^5*c))^(1/3)) + 2*(1/2)^(1/3)*a*x*
((b^3 - 2*a*b*c + (a^4*b^2 - 4*a^5*c)*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^
2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(a^8*b^6 - 12*a^9*b^4*c + 48*a^10*b^2*c^2 -
 64*a^11*c^3)))/(a^4*b^2 - 4*a^5*c))^(1/3)*log((1/2)^(2/3)*(b^9 - 11*a*b^7*
c + 42*a^2*b^5*c^2 - 62*a^3*b^3*c^3 + 24*a^4*b*c^4 - (a^4*b^8 - 13*a^5*b^6*
c + 60*a^6*b^4*c^2 - 112*a^7*b^2*c^3 + 64*a^8*c^4)*sqrt((b^8 - 8*a*b^6*c +
20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(a^8*b^6 - 12*a^9*b^4*c + 48*a
^10*b^2*c^2 - 64*a^11*c^3)))*((b^3 - 2*a*b*c + (a^4*b^2 - 4*a^5*c)*sqrt((b^
8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(a^8*b^6 - 12*
a^9*b^4*c + 48*a^10*b^2*c^2 - 64*a^11*c^3)))/(a^4*b^2 - 4*a^5*c))^(2/3) + 2
*(b^4*c^3 - 4*a*b^2*c^4 + 2*a^2*c^5)*x) + 2*(1/2)^(1/3)*a*x*((b^3 - 2*a*b*c
 - (a^4*b^2 - 4*a^5*c)*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*
c^3 + 4*a^4*c^4)/(a^8*b^6 - 12*a^9*b^4*c + 48*a^10*b^2*c^2 - 64*a^11*c^3)))
/(a^4*b^2 - 4*a^5*c))^(1/3)*log((1/2)^(2/3)*(b^9 - 11*a*b^7*c + 42*a^2*b^5*
c^2 - 62*a^3*b^3*c^3 + 24*a^4*b*c^4 + (a^4*b^8 - 13*a^5*b^6*c + 60*a^6*b^4*
c^2 - 112*a^7*b^2*c^3 + 64*a^8*c^4)*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2
- 16*a^3*b^2*c^3 + 4*a^4*c^4)/(a^8*b^6 - 12*a^9*b^4*c + 48*a^10*b^2*c^2 - 6
4*a^11*c^3)))*((b^3 - 2*a*b*c - (a^4*b^2 - 4*a^5*c)*sqrt((b^8 - 8*a*b^6*c +
 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(a^8*b^6 - 12*a^9*b^4*c + 48*
a^10*b^2*c^2 - 64*a^11*c^3)))/(a^4*b^2 - 4*a^5*c))^(2/3) + 2*(b^4*c^3 - 4*a
*b^2*c^4 + 2*a^2*c^5)*x) - 6)/(a*x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(cx^6 + bx^3 + a\right)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(c*x^6+b*x^3+a),x, algorithm="giac")
```

```
[Out] integrate(1/((c*x^6 + b*x^3 + a)*x^2), x)
```

maple [C] time = 0.01, size = 61, normalized size = 0.10

$$-\frac{\left(\operatorname{RootOf}\left(c\_Z^6+b\_Z^3+a\right)^4 c+\operatorname{RootOf}\left(c\_Z^6+b\_Z^3+a\right) b\right) \ln \left(-\operatorname{RootOf}\left(c\_Z^6+b\_Z^3+a\right)+x\right)}{3 a\left(2 \operatorname{RootOf}\left(c\_Z^6+b\_Z^3+a\right)^5 c+\operatorname{RootOf}\left(c\_Z^6+b\_Z^3+a\right)^2 b\right)}-\frac{1}{a x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(c*x^6+b*x^3+a),x)
```

```
[Out] -1/3/a*sum((_R^4*c+_R*b)/(2*_R^5*c+_R^2*b)*ln(-_R+x),_R=RootOf(_Z^6*c+_Z^3*
b+a))-1/a/x
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{cx^4+bx}{cx^6+bx^3+a}\, dx}{a}-\frac{1}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(c*x^6+b*x^3+a),x, algorithm="maxima")
```

```
[Out] -integrate((c*x^4 + b*x)/(c*x^6 + b*x^3 + a), x)/a - 1/(a*x)
```

mupad [B] time = 6.89, size = 2978, normalized size = 4.88

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a + b*x^3 + c*x^6)),x)
```

```
[Out] log(36*a^9*c^6 + 9*a^7*b^4*c^4 - 45*a^8*b^2*c^5 - (2^(2/3)*(27*a^7*c^3*x*(b
^6 - 8*a^3*c^3 + 18*a^2*b^2*c^2 - 8*a*b^4*c) + (27*2^(1/3)*a^10*b*c^3*(4*a*
c - b^2)^2*(-(b^7 + b^4*(-(4*a*c - b^2)^3)^(1/2) - 32*a^3*b*c^3 + 32*a^2*b^
3*c^2 + 2*a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^5*c - 4*a*b^2*c*(-(4*a*
c - b^2)^3)^(1/2))/(a^4*(4*a*c - b^2)^3))^(2/3))/2)*(-(b^7 + b^4*(-(4*a*c -
 b^2)^3)^(1/2) - 32*a^3*b*c^3 + 32*a^2*b^3*c^2 + 2*a^2*c^2*(-(4*a*c - b^2)^
3)^(1/2) - 10*a*b^5*c - 4*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(a^4*(4*a*c - b
^2)^3))^(1/3))/6)*((b^7 + b^4*(-(4*a*c - b^2)^3)^(1/2) - 32*a^3*b*c^3 + 32*
a^2*b^3*c^2 + 2*a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^5*c - 4*a*b^2*c*(
-(4*a*c - b^2)^3)^(1/2))/(54*(a^4*b^6 - 64*a^7*c^3 - 12*a^5*b^4*c + 48*a^6*
b^2*c^2)))^(1/3) + log(36*a^9*c^6 + 9*a^7*b^4*c^4 - 45*a^8*b^2*c^5 - (2^(2/
3)*(27*a^7*c^3*x*(b^6 - 8*a^3*c^3 + 18*a^2*b^2*c^2 - 8*a*b^4*c) + (27*2^(1/
3)*a^10*b*c^3*(4*a*c - b^2)^2*((b^4*(-(4*a*c - b^2)^3)^(1/2) - b^7 + 32*a^3
*b*c^3 - 32*a^2*b^3*c^2 + 2*a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) + 10*a*b^5*c -
 4*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(a^4*(4*a*c - b^2)^3))^(2/3))/2)*((b^4
*(-(4*a*c - b^2)^3)^(1/2) - b^7 + 32*a^3*b*c^3 - 32*a^2*b^3*c^2 + 2*a^2*c^2
*(-(4*a*c - b^2)^3)^(1/2) + 10*a*b^5*c - 4*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2)
)/(a^4*(4*a*c - b^2)^3))^(1/3))/6)*(-(b^4*(-(4*a*c - b^2)^3)^(1/2) - b^7 +
32*a^3*b*c^3 - 32*a^2*b^3*c^2 + 2*a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) + 10*a*b
^5*c - 4*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(54*(a^4*b^6 - 64*a^7*c^3 - 12*a
^5*b^4*c + 48*a^6*b^2*c^2)))^(1/3) - 1/(a*x) + log(36*a^9*c^6 + 9*a^7*b^4*c
^4 - 45*a^8*b^2*c^5 - (2^(2/3)*(3^(1/2)*1i - 1)*(27*a^7*c^3*x*(b^6 - 8*a^3*
c^3 + 18*a^2*b^2*c^2 - 8*a*b^4*c) - (27*2^(1/3)*a^10*b*c^3*(3^(1/2)*1i + 1)
*(4*a*c - b^2)^2*(-(b^7 + b^4*(-(4*a*c - b^2)^3)^(1/2) - 32*a^3*b*c^3 + 32*
a^2*b^3*c^2 + 2*a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^5*c - 4*a*b^2*c*(
-(4*a*c - b^2)^3)^(1/2))/(a^4*(4*a*c - b^2)^3))^(2/3))/4)*(-(b^7 + b^4*(-(4
*a*c - b^2)^3)^(1/2) - 32*a^3*b*c^3 + 32*a^2*b^3*c^2 + 2*a^2*c^2*(-(4*a*c -
```

```
 b^2)^3)^(1/2) - 10*a*b^5*c - 4*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(a^4*(4*a
*c - b^2)^3))^(1/3))/12)*((3^(1/2)*1i)/2 - 1/2)*((b^7 + b^4*(-(4*a*c - b^2)
^3)^(1/2) - 32*a^3*b*c^3 + 32*a^2*b^3*c^2 + 2*a^2*c^2*(-(4*a*c - b^2)^3)^(1
/2) - 10*a*b^5*c - 4*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(54*(a^4*b^6 - 64*a^
7*c^3 - 12*a^5*b^4*c + 48*a^6*b^2*c^2)))^(1/3) - log(36*a^9*c^6 + 9*a^7*b^4
*c^4 - 45*a^8*b^2*c^5 + (2^(2/3)*(3^(1/2)*1i + 1)*(27*a^7*c^3*x*(b^6 - 8*a^
3*c^3 + 18*a^2*b^2*c^2 - 8*a*b^4*c) + (27*2^(1/3)*a^10*b*c^3*(3^(1/2)*1i -
1)*(4*a*c - b^2)^2*(-(b^7 + b^4*(-(4*a*c - b^2)^3)^(1/2) - 32*a^3*b*c^3 + 3
2*a^2*b^3*c^2 + 2*a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^5*c - 4*a*b^2*c
*(-(4*a*c - b^2)^3)^(1/2))/(a^4*(4*a*c - b^2)^3))^(2/3))/4)*(-(b^7 + b^4*(-
(4*a*c - b^2)^3)^(1/2) - 32*a^3*b*c^3 + 32*a^2*b^3*c^2 + 2*a^2*c^2*(-(4*a*c
 - b^2)^3)^(1/2) - 10*a*b^5*c - 4*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(a^4*(4
*a*c - b^2)^3))^(1/3))/12)*((3^(1/2)*1i)/2 + 1/2)*((b^7 + b^4*(-(4*a*c - b^
2)^3)^(1/2) - 32*a^3*b*c^3 + 32*a^2*b^3*c^2 + 2*a^2*c^2*(-(4*a*c - b^2)^3)^
(1/2) - 10*a*b^5*c - 4*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(54*(a^4*b^6 - 64*
a^7*c^3 - 12*a^5*b^4*c + 48*a^6*b^2*c^2)))^(1/3) - log(36*a^9*c^6 + 9*a^7*b
^4*c^4 - 45*a^8*b^2*c^5 + (2^(2/3)*(27*a^7*c^3*x*(b^6 - 8*a^3*c^3 + 18*a^2*
b^2*c^2 - 8*a*b^4*c) + (27*2^(1/3)*a^10*b*c^3*(3^(1/2)*1i - 1)*(4*a*c - b^2
)^2*((b^4*(-(4*a*c - b^2)^3)^(1/2) - b^7 + 32*a^3*b*c^3 - 32*a^2*b^3*c^2 +
2*a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) + 10*a*b^5*c - 4*a*b^2*c*(-(4*a*c - b^2)
^3)^(1/2))/(a^4*(4*a*c - b^2)^3))^(2/3))/4)*(3^(1/2)*1i + 1)*((b^4*(-(4*a*c
 - b^2)^3)^(1/2) - b^7 + 32*a^3*b*c^3 - 32*a^2*b^3*c^2 + 2*a^2*c^2*(-(4*a*c
 - b^2)^3)^(1/2) + 10*a*b^5*c - 4*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(a^4*(4
*a*c - b^2)^3))^(1/3))/12)*((3^(1/2)*1i)/2 + 1/2)*(-(b^4*(-(4*a*c - b^2)^3)
^(1/2) - b^7 + 32*a^3*b*c^3 - 32*a^2*b^3*c^2 + 2*a^2*c^2*(-(4*a*c - b^2)^3)
^(1/2) + 10*a*b^5*c - 4*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(54*(a^4*b^6 - 64
*a^7*c^3 - 12*a^5*b^4*c + 48*a^6*b^2*c^2)))^(1/3) + log(36*a^9*c^6 + 9*a^7*
b^4*c^4 - 45*a^8*b^2*c^5 - (2^(2/3)*(27*a^7*c^3*x*(b^6 - 8*a^3*c^3 + 18*a^2
*b^2*c^2 - 8*a*b^4*c) - (27*2^(1/3)*a^10*b*c^3*(3^(1/2)*1i + 1)*(4*a*c - b^
2)^2*((b^4*(-(4*a*c - b^2)^3)^(1/2) - b^7 + 32*a^3*b*c^3 - 32*a^2*b^3*c^2 +
 2*a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) + 10*a*b^5*c - 4*a*b^2*c*(-(4*a*c - b^2
)^3)^(1/2))/(a^4*(4*a*c - b^2)^3))^(2/3))/4)*(3^(1/2)*1i - 1)*((b^4*(-(4*a*
c - b^2)^3)^(1/2) - b^7 + 32*a^3*b*c^3 - 32*a^2*b^3*c^2 + 2*a^2*c^2*(-(4*a*
c - b^2)^3)^(1/2) + 10*a*b^5*c - 4*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(a^4*(
4*a*c - b^2)^3))^(1/3))/12)*((3^(1/2)*1i)/2 - 1/2)*(-(b^4*(-(4*a*c - b^2)^3
)^(1/2) - b^7 + 32*a^3*b*c^3 - 32*a^2*b^3*c^2 + 2*a^2*c^2*(-(4*a*c - b^2)^3
)^(1/2) + 10*a*b^5*c - 4*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(54*(a^4*b^6 - 6
4*a^7*c^3 - 12*a^5*b^4*c + 48*a^6*b^2*c^2)))^(1/3)
```

sympy [A] time = 3.19, size = 252, normalized size = 0.41

$$\operatorname{RootSum}\left(t^6\left(46656a^7c^3-34992a^6b^2c^2+8748a^5b^4c-729a^4b^6\right)+t^3\left(-864a^3bc^3+864a^2b^3c^2-270ab^5c+27b^7\right)+c^4,\left(t\mapsto t\log\left(x+\frac{-15552t^5a^8c^4+27216t^5a^7b^2c^3-14580t^5a^6b^4c^2+3159t^5a^5b^6c-243t^5a^4b^8+252t^2a^4bc^4-567t^2a^3b^3c^3+378t^2a^2b^5c^2-99t^2ab^7c+9t^2b^9}{2a^2c^5-4ab^2c^4+b^4c^3}\right)\right)\right)-\frac{1}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(c*x**6+b*x**3+a),x)
```

```
[Out] RootSum(_t**6*(46656*a**7*c**3 - 34992*a**6*b**2*c**2 + 8748*a**5*b**4*c -
729*a**4*b**6) + _t**3*(-864*a**3*b*c**3 + 864*a**2*b**3*c**2 - 270*a*b**5*
c + 27*b**7) + c**4, Lambda(_t, _t*log(x + (-15552*_t**5*a**8*c**4 + 27216*
_t**5*a**7*b**2*c**3 - 14580*_t**5*a**6*b**4*c**2 + 3159*_t**5*a**5*b**6*c
- 243*_t**5*a**4*b**8 + 252*_t**2*a**4*b*c**4 - 567*_t**2*a**3*b**3*c**3 +
378*_t**2*a**2*b**5*c**2 - 99*_t**2*a*b**7*c + 9*_t**2*b**9)/(2*a**2*c**5 -
 4*a*b**2*c**4 + b**4*c**3)))) - 1/(a*x)
```

3.137 $\int \frac{1}{x^3\left(a+bx^3+cx^6\right)}\,dx$

Optimal. Leaf size=612

$$\frac{c^{2/3}\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\log\left(-\sqrt[3]{2}\,\sqrt[3]{c}\,x\sqrt[3]{b-\sqrt{b^2-4ac}}+\left(b-\sqrt{b^2-4ac}\right)^{2/3}+2^{2/3}c^{2/3}x^2\right)}{6\sqrt[3]{2}\,a\left(b-\sqrt{b^2-4ac}\right)^{2/3}}+\frac{c^{2/3}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\log\left(-\sqrt[3]{\vphantom{x}}\right.}{}$$

Rubi [A] time = 0.82, antiderivative size = 612, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 8, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {1368, 1422, 200, 31, 634, 617, 204, 628}

$$\frac{c^{2/3}\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\log\left(-\sqrt[3]{2}\,\sqrt[3]{c}\,x\sqrt[3]{b-\sqrt{b^2-4ac}}+\left(b-\sqrt{b^2-4ac}\right)^{2/3}+2^{2/3}c^{2/3}x^2\right)}{6\sqrt[3]{2}\,a\left(b-\sqrt{b^2-4ac}\right)^{2/3}}+\frac{c^{2/3}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\log\left(-\sqrt[3]{2}\,\sqrt[3]{c}\,x\sqrt[3]{\sqrt{b^2-4ac}+b}+\left(\sqrt{b^2-4ac}+b\right)^{2/3}+2^{2/3}c^{2/3}x^2\right)}{6\sqrt[3]{2}\,a\left(\sqrt{b^2-4ac}+b\right)^{2/3}}-\frac{c^{2/3}\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\,\sqrt[3]{c}\,x\right)}{3\sqrt[3]{2}\,a\left(b-\sqrt{b^2-4ac}\right)^{2/3}}-\frac{c^{2/3}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{\sqrt{b^2-4ac}+b}+\sqrt[3]{2}\,\sqrt[3]{c}\,x\right)}{3\sqrt[3]{2}\,a\left(\sqrt{b^2-4ac}+b\right)^{2/3}}+\frac{c^{2/3}\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\,\sqrt[3]{c}\,x}{\sqrt[3]{b-\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\,\sqrt{3}\,a\left(b-\sqrt{b^2-4ac}\right)^{2/3}}+\frac{c^{2/3}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\,\sqrt[3]{c}\,x}{\sqrt[3]{\sqrt{b^2-4ac}+b}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\,\sqrt{3}\,a\left(\sqrt{b^2-4ac}+b\right)^{2/3}}-\frac{1}{2ax^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(a + b*x^3 + c*x^6)),x]
```

```
[Out] -1/(2*a*x^2) + (c^(2/3)*(1 + b/Sqrt[b^2 - 4*a*c])*ArcTan[(1 - (2*2^(1/3)*c^
(1/3)*x)/(b - Sqrt[b^2 - 4*a*c])^(1/3))/Sqrt[3]])/(2^(1/3)*Sqrt[3]*a*(b - S
qrt[b^2 - 4*a*c])^(2/3)) + (c^(2/3)*(1 - b/Sqrt[b^2 - 4*a*c])*ArcTan[(1 - (
2*2^(1/3)*c^(1/3)*x)/(b + Sqrt[b^2 - 4*a*c])^(1/3))/Sqrt[3]])/(2^(1/3)*Sqrt
[3]*a*(b + Sqrt[b^2 - 4*a*c])^(2/3)) - (c^(2/3)*(1 + b/Sqrt[b^2 - 4*a*c])*L
og[(b - Sqrt[b^2 - 4*a*c])^(1/3) + 2^(1/3)*c^(1/3)*x])/(3*2^(1/3)*a*(b - Sq
rt[b^2 - 4*a*c])^(2/3)) - (c^(2/3)*(1 - b/Sqrt[b^2 - 4*a*c])*Log[(b + Sqrt[
b^2 - 4*a*c])^(1/3) + 2^(1/3)*c^(1/3)*x])/(3*2^(1/3)*a*(b + Sqrt[b^2 - 4*a*
c])^(2/3)) + (c^(2/3)*(1 + b/Sqrt[b^2 - 4*a*c])*Log[(b - Sqrt[b^2 - 4*a*c])
^(2/3) - 2^(1/3)*c^(1/3)*(b - Sqrt[b^2 - 4*a*c])^(1/3)*x + 2^(2/3)*c^(2/3)*
x^2])/(6*2^(1/3)*a*(b - Sqrt[b^2 - 4*a*c])^(2/3)) + (c^(2/3)*(1 - b/Sqrt[b^
2 - 4*a*c])*Log[(b + Sqrt[b^2 - 4*a*c])^(2/3) - 2^(1/3)*c^(1/3)*(b + Sqrt[b
^2 - 4*a*c])^(1/3)*x + 2^(2/3)*c^(2/3)*x^2])/(6*2^(1/3)*a*(b + Sqrt[b^2 - 4
*a*c])^(2/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
```

```
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1368

```
Int[((d_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_
Symbol] :> Simp[((d*x)^(m + 1)*(a + b*x^n + c*x^(2*n))^(p + 1))/(a*d*(m + 1
)), x] - Dist[1/(a*d^n*(m + 1)), Int[(d*x)^(m + n)*(b*(m + n*(p + 1) + 1) +
 c*(m + 2*n*(p + 1) + 1)*x^n)*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a
, b, c, d, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && L
tQ[m, -1] && IntegerQ[p]
```

Rule 1422

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q),
Int[1/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(
b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (PosQ[b^2 - 4*a
*c] ||  !IGtQ[n/2, 0])
```

Rubi steps

$$\int \frac{1}{x^3\left(a+bx^3+cx^6\right)}\,dx = -\frac{1}{2ax^2}+\frac{\int \frac{-2b-2cx^3}{a+bx^3+cx^6}\,dx}{2a}$$

$$= -\frac{1}{2ax^2}-\frac{\left(c\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\right)\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^3}\,dx}{2a}-\frac{\left(c\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)\right)\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^3}}{2a}$$

$$= -\frac{1}{2ax^2}-\frac{\left(c\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)\right)\int \frac{1}{\frac{\sqrt[3]{b-\sqrt{b^2-4ac}}}{\sqrt[3]{2}}+\sqrt[3]{c}\,x}\,dx}{3\sqrt[3]{2}\,a\left(b-\sqrt{b^2-4ac}\right)^{2/3}}-\frac{\left(c\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)\right)\int \frac{2^{2/3}\sqrt[3]{b-\sqrt{}}}{\frac{\left(b-\sqrt{b^2-4ac}\right)^{2/3}}{2^{2/3}}-\sqrt[3]{c}}}{3\sqrt[3]{2}\,a\left(b-\sqrt{b^2-4a}\right.}$$

$$= -\frac{1}{2ax^2}-\frac{c^{2/3}\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\,\sqrt[3]{c}\,x\right)}{3\sqrt[3]{2}\,a\left(b-\sqrt{b^2-4ac}\right)^{2/3}}-\frac{c^{2/3}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\log}{3\sqrt[3]{2}\,a\left(\right.}$$

$$= -\frac{1}{2ax^2}-\frac{c^{2/3}\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\,\sqrt[3]{c}\,x\right)}{3\sqrt[3]{2}\,a\left(b-\sqrt{b^2-4ac}\right)^{2/3}}-\frac{c^{2/3}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\log}{3\sqrt[3]{2}\,a\left(\right.}$$

$$= -\frac{1}{2ax^2}+\frac{c^{2/3}\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}\,x}{\sqrt[3]{b-\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}\,a\left(b-\sqrt{b^2-4ac}\right)^{2/3}}+\frac{c^{2/3}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{}}{\sqrt[3]{b+\sqrt{b^2}}}}{\sqrt{3}}\right.}{\sqrt[3]{2}\sqrt{3}\,a\left(b+\sqrt{b^2-4ac}\right)^{2/3}}$$

Mathematica [C] time = 0.03, size = 75, normalized size = 0.12

$$-\frac{\text{RootSum}\left[\#1^6c+\#1^3b+a\&,\frac{\#1^3c\log(x-\#1)+b\log(x-\#1)}{2\#1^5c+\#1^2b}\&\right]}{3a}-\frac{1}{2ax^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(a + b*x^3 + c*x^6)),x]
```

```
[Out] -1/2*1/(a*x^2) - RootSum[a + b*#1^3 + c*#1^6 & , (b*Log[x - #1] + c*Log[x -
 #1]*#1^3)/(b*#1^2 + 2*c*#1^5) & ]/(3*a)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3\left(a+bx^3+cx^6\right)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x^3*(a + b*x^3 + c*x^6)),x]
```

```
[Out] IntegrateAlgebraic[1/(x^3*(a + b*x^3 + c*x^6)), x]
```

fricas [B] time = 3.76, size = 5771, normalized size = 9.43

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(c*x^6+b*x^3+a),x, algorithm="fricas")
```

```
[Out] 1/6*(4*sqrt(3)*(1/2)^(1/3)*a*x^2*(-(b^4 - 3*a*b^2*c + a^2*c^2 + (a^5*b^2 -
4*a^6*c)*sqrt((b^10 - 10*a*b^8*c + 35*a^2*b^6*c^2 - 50*a^3*b^4*c^3 + 25*a^4
*b^2*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))/(a^5
*b^2 - 4*a^6*c))^(1/3)*arctan(-1/6*(2*(1/2)^(2/3)*(sqrt(3)*(a^5*b^8 - 13*a^
6*b^6*c + 60*a^7*b^4*c^2 - 112*a^8*b^2*c^3 + 64*a^9*c^4)*x*sqrt((b^10 - 10*
a*b^8*c + 35*a^2*b^6*c^2 - 50*a^3*b^4*c^3 + 25*a^4*b^2*c^4)/(a^10*b^6 - 12*
a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)) - sqrt(3)*(b^10 - 12*a*b^8*c +
 52*a^2*b^6*c^2 - 95*a^3*b^4*c^3 + 60*a^4*b^2*c^4)*x)*(-(b^4 - 3*a*b^2*c +
a^2*c^2 + (a^5*b^2 - 4*a^6*c)*sqrt((b^10 - 10*a*b^8*c + 35*a^2*b^6*c^2 - 50
*a^3*b^4*c^3 + 25*a^4*b^2*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2
- 64*a^13*c^3)))/(a^5*b^2 - 4*a^6*c))^(2/3) - (1/2)^(1/6)*(sqrt(3)*(a^5*b^8
 - 13*a^6*b^6*c + 60*a^7*b^4*c^2 - 112*a^8*b^2*c^3 + 64*a^9*c^4)*sqrt((b^10
 - 10*a*b^8*c + 35*a^2*b^6*c^2 - 50*a^3*b^4*c^3 + 25*a^4*b^2*c^4)/(a^10*b^6
 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)) - sqrt(3)*(b^10 - 12*a*b
^8*c + 52*a^2*b^6*c^2 - 95*a^3*b^4*c^3 + 60*a^4*b^2*c^4))*(-(b^4 - 3*a*b^2*
c + a^2*c^2 + (a^5*b^2 - 4*a^6*c)*sqrt((b^10 - 10*a*b^8*c + 35*a^2*b^6*c^2
- 50*a^3*b^4*c^3 + 25*a^4*b^2*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*
c^2 - 64*a^13*c^3)))/(a^5*b^2 - 4*a^6*c))^(2/3)*sqrt((2*(b^5*c^4 - 5*a*b^3*
c^5 + 5*a^2*b*c^6)*x^2 + (1/2)^(2/3)*(b^11 - 13*a*b^9*c + 63*a^2*b^7*c^2 -
138*a^3*b^5*c^3 + 130*a^4*b^3*c^4 - 40*a^5*b*c^5 - (a^5*b^9 - 14*a^6*b^7*c
+ 72*a^7*b^5*c^2 - 160*a^8*b^3*c^3 + 128*a^9*b*c^4)*sqrt((b^10 - 10*a*b^8*c
 + 35*a^2*b^6*c^2 - 50*a^3*b^4*c^3 + 25*a^4*b^2*c^4)/(a^10*b^6 - 12*a^11*b^
4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))*(-(b^4 - 3*a*b^2*c + a^2*c^2 + (a^5*
b^2 - 4*a^6*c)*sqrt((b^10 - 10*a*b^8*c + 35*a^2*b^6*c^2 - 50*a^3*b^4*c^3 +
25*a^4*b^2*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3))
)/(a^5*b^2 - 4*a^6*c))^(2/3) + (1/2)^(1/3)*((a^5*b^6*c^2 - 10*a^6*b^4*c^3 +
 32*a^7*b^2*c^4 - 32*a^8*c^5)*x*sqrt((b^10 - 10*a*b^8*c + 35*a^2*b^6*c^2 -
50*a^3*b^4*c^3 + 25*a^4*b^2*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^
2 - 64*a^13*c^3)) - (b^8*c^2 - 9*a*b^6*c^3 + 25*a^2*b^4*c^4 - 20*a^3*b^2*c^
5)*x)*(-(b^4 - 3*a*b^2*c + a^2*c^2 + (a^5*b^2 - 4*a^6*c)*sqrt((b^10 - 10*a*
b^8*c + 35*a^2*b^6*c^2 - 50*a^3*b^4*c^3 + 25*a^4*b^2*c^4)/(a^10*b^6 - 12*a^
11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))/(a^5*b^2 - 4*a^6*c))^(1/3))/(b^
5*c^4 - 5*a*b^3*c^5 + 5*a^2*b*c^6)) + 2*sqrt(3)*(b^5*c^3 - 5*a*b^3*c^4 + 5*
a^2*b*c^5))/(b^5*c^3 - 5*a*b^3*c^4 + 5*a^2*b*c^5)) - 4*sqrt(3)*(1/2)^(1/3)*
a*x^2*(-(b^4 - 3*a*b^2*c + a^2*c^2 - (a^5*b^2 - 4*a^6*c)*sqrt((b^10 - 10*a*
b^8*c + 35*a^2*b^6*c^2 - 50*a^3*b^4*c^3 + 25*a^4*b^2*c^4)/(a^10*b^6 - 12*a^
11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))/(a^5*b^2 - 4*a^6*c))^(1/3)*arct
an(-1/6*(2*(1/2)^(2/3)*(sqrt(3)*(a^5*b^8 - 13*a^6*b^6*c + 60*a^7*b^4*c^2 -
112*a^8*b^2*c^3 + 64*a^9*c^4)*x*sqrt((b^10 - 10*a*b^8*c + 35*a^2*b^6*c^2 -
50*a^3*b^4*c^3 + 25*a^4*b^2*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^
2 - 64*a^13*c^3)) + sqrt(3)*(b^10 - 12*a*b^8*c + 52*a^2*b^6*c^2 - 95*a^3*b^
4*c^3 + 60*a^4*b^2*c^4)*x)*(-(b^4 - 3*a*b^2*c + a^2*c^2 - (a^5*b^2 - 4*a^6*
c)*sqrt((b^10 - 10*a*b^8*c + 35*a^2*b^6*c^2 - 50*a^3*b^4*c^3 + 25*a^4*b^2*c
^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))/(a^5*b^2 -
 4*a^6*c))^(2/3) - (1/2)^(1/6)*(sqrt(3)*(a^5*b^8 - 13*a^6*b^6*c + 60*a^7*b^
4*c^2 - 112*a^8*b^2*c^3 + 64*a^9*c^4)*sqrt((b^10 - 10*a*b^8*c + 35*a^2*b^6*
c^2 - 50*a^3*b^4*c^3 + 25*a^4*b^2*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*
b^2*c^2 - 64*a^13*c^3)) + sqrt(3)*(b^10 - 12*a*b^8*c + 52*a^2*b^6*c^2 - 95*
a^3*b^4*c^3 + 60*a^4*b^2*c^4))*(-(b^4 - 3*a*b^2*c + a^2*c^2 - (a^5*b^2 - 4*
a^6*c)*sqrt((b^10 - 10*a*b^8*c + 35*a^2*b^6*c^2 - 50*a^3*b^4*c^3 + 25*a^4*b
^2*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))/(a^5*b
^2 - 4*a^6*c))^(2/3)*sqrt((2*(b^5*c^4 - 5*a*b^3*c^5 + 5*a^2*b*c^6)*x^2 + (1
/2)^(2/3)*(b^11 - 13*a*b^9*c + 63*a^2*b^7*c^2 - 138*a^3*b^5*c^3 + 130*a^4*b
^3*c^4 - 40*a^5*b*c^5 + (a^5*b^9 - 14*a^6*b^7*c + 72*a^7*b^5*c^2 - 160*a^8*
b^3*c^3 + 128*a^9*b*c^4)*sqrt((b^10 - 10*a*b^8*c + 35*a^2*b^6*c^2 - 50*a^3*
b^4*c^3 + 25*a^4*b^2*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*
```

```
a^13*c^3)))*(-(b^4 - 3*a*b^2*c + a^2*c^2 - (a^5*b^2 - 4*a^6*c)*sqrt((b^10 -
 10*a*b^8*c + 35*a^2*b^6*c^2 - 50*a^3*b^4*c^3 + 25*a^4*b^2*c^4)/(a^10*b^6 -
 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))/(a^5*b^2 - 4*a^6*c))^(2/3
) - (1/2)^(1/3)*((a^5*b^6*c^2 - 10*a^6*b^4*c^3 + 32*a^7*b^2*c^4 - 32*a^8*c^
5)*x*sqrt((b^10 - 10*a*b^8*c + 35*a^2*b^6*c^2 - 50*a^3*b^4*c^3 + 25*a^4*b^2
*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)) + (b^8*c^
2 - 9*a*b^6*c^3 + 25*a^2*b^4*c^4 - 20*a^3*b^2*c^5)*x)*(-(b^4 - 3*a*b^2*c +
a^2*c^2 - (a^5*b^2 - 4*a^6*c)*sqrt((b^10 - 10*a*b^8*c + 35*a^2*b^6*c^2 - 50
*a^3*b^4*c^3 + 25*a^4*b^2*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2
 - 64*a^13*c^3)))/(a^5*b^2 - 4*a^6*c))^(1/3))/(b^5*c^4 - 5*a*b^3*c^5 + 5*a^2
*b*c^6)) - 2*sqrt(3)*(b^5*c^3 - 5*a*b^3*c^4 + 5*a^2*b*c^5))/(b^5*c^3 - 5*a*
b^3*c^4 + 5*a^2*b*c^5)) - (1/2)^(1/3)*a*x^2*(-(b^4 - 3*a*b^2*c + a^2*c^2 +
(a^5*b^2 - 4*a^6*c)*sqrt((b^10 - 10*a*b^8*c + 35*a^2*b^6*c^2 - 50*a^3*b^4*c
^3 + 25*a^4*b^2*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*
c^3)))/(a^5*b^2 - 4*a^6*c))^(1/3)*log(2*(b^5*c^4 - 5*a*b^3*c^5 + 5*a^2*b*c^
6)*x^2 + (1/2)^(2/3)*(b^11 - 13*a*b^9*c + 63*a^2*b^7*c^2 - 138*a^3*b^5*c^3
+ 130*a^4*b^3*c^4 - 40*a^5*b*c^5 - (a^5*b^9 - 14*a^6*b^7*c + 72*a^7*b^5*c^2
 - 160*a^8*b^3*c^3 + 128*a^9*b*c^4)*sqrt((b^10 - 10*a*b^8*c + 35*a^2*b^6*c^
2 - 50*a^3*b^4*c^3 + 25*a^4*b^2*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^
2*c^2 - 64*a^13*c^3)))*(-(b^4 - 3*a*b^2*c + a^2*c^2 + (a^5*b^2 - 4*a^6*c)*s
qrt((b^10 - 10*a*b^8*c + 35*a^2*b^6*c^2 - 50*a^3*b^4*c^3 + 25*a^4*b^2*c^4)/
(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))/(a^5*b^2 - 4*a
^6*c))^(2/3) + (1/2)^(1/3)*((a^5*b^6*c^2 - 10*a^6*b^4*c^3 + 32*a^7*b^2*c^4
- 32*a^8*c^5)*x*sqrt((b^10 - 10*a*b^8*c + 35*a^2*b^6*c^2 - 50*a^3*b^4*c^3 +
 25*a^4*b^2*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)
) - (b^8*c^2 - 9*a*b^6*c^3 + 25*a^2*b^4*c^4 - 20*a^3*b^2*c^5)*x)*(-(b^4 - 3
*a*b^2*c + a^2*c^2 + (a^5*b^2 - 4*a^6*c)*sqrt((b^10 - 10*a*b^8*c + 35*a^2*b
^6*c^2 - 50*a^3*b^4*c^3 + 25*a^4*b^2*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^
12*b^2*c^2 - 64*a^13*c^3)))/(a^5*b^2 - 4*a^6*c))^(1/3)) - (1/2)^(1/3)*a*x^2
*(-(b^4 - 3*a*b^2*c + a^2*c^2 - (a^5*b^2 - 4*a^6*c)*sqrt((b^10 - 10*a*b^8*c
 + 35*a^2*b^6*c^2 - 50*a^3*b^4*c^3 + 25*a^4*b^2*c^4)/(a^10*b^6 - 12*a^11*b^
4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))/(a^5*b^2 - 4*a^6*c))^(1/3)*log(2*(b^
5*c^4 - 5*a*b^3*c^5 + 5*a^2*b*c^6)*x^2 + (1/2)^(2/3)*(b^11 - 13*a*b^9*c + 6
3*a^2*b^7*c^2 - 138*a^3*b^5*c^3 + 130*a^4*b^3*c^4 - 40*a^5*b*c^5 + (a^5*b^9
 - 14*a^6*b^7*c + 72*a^7*b^5*c^2 - 160*a^8*b^3*c^3 + 128*a^9*b*c^4)*sqrt((b
^10 - 10*a*b^8*c + 35*a^2*b^6*c^2 - 50*a^3*b^4*c^3 + 25*a^4*b^2*c^4)/(a^10*
b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))*(-(b^4 - 3*a*b^2*c +
 a^2*c^2 - (a^5*b^2 - 4*a^6*c)*sqrt((b^10 - 10*a*b^8*c + 35*a^2*b^6*c^2 - 5
0*a^3*b^4*c^3 + 25*a^4*b^2*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2
 - 64*a^13*c^3)))/(a^5*b^2 - 4*a^6*c))^(2/3) - (1/2)^(1/3)*((a^5*b^6*c^2 -
10*a^6*b^4*c^3 + 32*a^7*b^2*c^4 - 32*a^8*c^5)*x*sqrt((b^10 - 10*a*b^8*c + 3
5*a^2*b^6*c^2 - 50*a^3*b^4*c^3 + 25*a^4*b^2*c^4)/(a^10*b^6 - 12*a^11*b^4*c
+ 48*a^12*b^2*c^2 - 64*a^13*c^3)) + (b^8*c^2 - 9*a*b^6*c^3 + 25*a^2*b^4*c^4
 - 20*a^3*b^2*c^5)*x)*(-(b^4 - 3*a*b^2*c + a^2*c^2 - (a^5*b^2 - 4*a^6*c)*sq
rt((b^10 - 10*a*b^8*c + 35*a^2*b^6*c^2 - 50*a^3*b^4*c^3 + 25*a^4*b^2*c^4)/(
a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))/(a^5*b^2 - 4*a^
6*c))^(1/3)) + 2*(1/2)^(1/3)*a*x^2*(-(b^4 - 3*a*b^2*c + a^2*c^2 + (a^5*b^2
- 4*a^6*c)*sqrt((b^10 - 10*a*b^8*c + 35*a^2*b^6*c^2 - 50*a^3*b^4*c^3 + 25*a
^4*b^2*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))/(a
^5*b^2 - 4*a^6*c))^(1/3)*log(2*(b^5*c^2 - 5*a*b^3*c^3 + 5*a^2*b*c^4)*x + (1
/2)^(1/3)*(b^8 - 9*a*b^6*c + 25*a^2*b^4*c^2 - 20*a^3*b^2*c^3 - (a^5*b^6 - 1
0*a^6*b^4*c + 32*a^7*b^2*c^2 - 32*a^8*c^3)*sqrt((b^10 - 10*a*b^8*c + 35*a^2
*b^6*c^2 - 50*a^3*b^4*c^3 + 25*a^4*b^2*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*
a^12*b^2*c^2 - 64*a^13*c^3)))*(-(b^4 - 3*a*b^2*c + a^2*c^2 + (a^5*b^2 - 4*a
^6*c)*sqrt((b^10 - 10*a*b^8*c + 35*a^2*b^6*c^2 - 50*a^3*b^4*c^3 + 25*a^4*b^
2*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))/(a^5*b^
2 - 4*a^6*c))^(1/3)) + 2*(1/2)^(1/3)*a*x^2*(-(b^4 - 3*a*b^2*c + a^2*c^2 - (
a^5*b^2 - 4*a^6*c)*sqrt((b^10 - 10*a*b^8*c + 35*a^2*b^6*c^2 - 50*a^3*b^4*c^
3 + 25*a^4*b^2*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c
```

```
^3)))/(a^5*b^2 - 4*a^6*c))^(1/3)*log(2*(b^5*c^2 - 5*a*b^3*c^3 + 5*a^2*b*c^4
)*x + (1/2)^(1/3)*(b^8 - 9*a*b^6*c + 25*a^2*b^4*c^2 - 20*a^3*b^2*c^3 + (a^5
*b^6 - 10*a^6*b^4*c + 32*a^7*b^2*c^2 - 32*a^8*c^3)*sqrt((b^10 - 10*a*b^8*c
+ 35*a^2*b^6*c^2 - 50*a^3*b^4*c^3 + 25*a^4*b^2*c^4)/(a^10*b^6 - 12*a^11*b^4
*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))*(-(b^4 - 3*a*b^2*c + a^2*c^2 - (a^5*b
^2 - 4*a^6*c)*sqrt((b^10 - 10*a*b^8*c + 35*a^2*b^6*c^2 - 50*a^3*b^4*c^3 + 2
5*a^4*b^2*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))
/(a^5*b^2 - 4*a^6*c))^(1/3)) - 3)/(a*x^2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(cx^6 + bx^3 + a\right)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(c*x^6+b*x^3+a),x, algorithm="giac")
```

```
[Out] integrate(1/((c*x^6 + b*x^3 + a)*x^3), x)
```

maple [C] time = 0.00, size = 62, normalized size = 0.10

$$\frac{\left(-\mathrm{RootOf}\left(c\_Z^6 + b\_Z^3 + a\right)^3 c - b\right)\ln\left(-\mathrm{RootOf}\left(c\_Z^6 + b\_Z^3 + a\right) + x\right)}{3a\left(2\,\mathrm{RootOf}\left(c\_Z^6 + b\_Z^3 + a\right)^5 c + \mathrm{RootOf}\left(c\_Z^6 + b\_Z^3 + a\right)^2 b\right)} - \frac{1}{2a\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(c*x^6+b*x^3+a),x)
```

```
[Out] 1/3/a*sum((-_R^3*c-b)/(2*_R^5*c+_R^2*b)*ln(-_R+x),_R=RootOf(_Z^6*c+_Z^3*b+a
))-1/2/a/x^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{cx^3+b}{cx^6+bx^3+a}\,dx}{a} - \frac{1}{2\,ax^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(c*x^6+b*x^3+a),x, algorithm="maxima")
```

```
[Out] -integrate((c*x^3 + b)/(c*x^6 + b*x^3 + a), x)/a - 1/2/(a*x^2)
```

mupad [B] time = 10.65, size = 4063, normalized size = 6.64

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a + b*x^3 + c*x^6)),x)
```

```
[Out] log((2^(2/3)*((b^8 + 16*a^4*c^4 + b^5*(-(4*a*c - b^2)^3)^(1/2) + 41*a^2*b^4
*c^2 - 56*a^3*b^2*c^3 - 11*a*b^6*c + 5*a^2*b*c^2*(-(4*a*c - b^2)^3)^(1/2) -
 5*a*b^3*c*(-(4*a*c - b^2)^3)^(1/2))/(a^5*(4*a*c - b^2)^3))^(1/3)*(72*a^8*b
*c^6 + (2^(1/3)*(81*a^8*c^3*x*(a*c - b^2)*(4*a*c - b^2)^2 + (81*2^(2/3)*a^1
0*b*c^3*(4*a*c - b^2)^2*((b^8 + 16*a^4*c^4 + b^5*(-(4*a*c - b^2)^3)^(1/2) +
 41*a^2*b^4*c^2 - 56*a^3*b^2*c^3 - 11*a*b^6*c + 5*a^2*b*c^2*(-(4*a*c - b^2)
^3)^(1/2) - 5*a*b^3*c*(-(4*a*c - b^2)^3)^(1/2))/(a^5*(4*a*c - b^2)^3))^(1/3
))/2)*((b^8 + 16*a^4*c^4 + b^5*(-(4*a*c - b^2)^3)^(1/2) + 41*a^2*b^4*c^2 -
56*a^3*b^2*c^3 - 11*a*b^6*c + 5*a^2*b*c^2*(-(4*a*c - b^2)^3)^(1/2) - 5*a*b^
3*c*(-(4*a*c - b^2)^3)^(1/2))/(a^5*(4*a*c - b^2)^3))^(2/3))/18 + 9*a^6*b^5*
c^4 - 54*a^7*b^3*c^5))/6 - 3*a^6*c^6*x*(2*a*c - b^2))*(-(b^8 + 16*a^4*c^4 +
```

```
 b^5*(-(4*a*c - b^2)^3)^(1/2) + 41*a^2*b^4*c^2 - 56*a^3*b^2*c^3 - 11*a*b^6*
c + 5*a^2*b*c^2*(-(4*a*c - b^2)^3)^(1/2) - 5*a*b^3*c*(-(4*a*c - b^2)^3)^(1/
2))/(54*(a^5*b^6 - 64*a^8*c^3 - 12*a^6*b^4*c + 48*a^7*b^2*c^2)))^(1/3) + lo
g((2^(2/3)*((b^8 + 16*a^4*c^4 - b^5*(-(4*a*c - b^2)^3)^(1/2) + 41*a^2*b^4*c
^2 - 56*a^3*b^2*c^3 - 11*a*b^6*c - 5*a^2*b*c^2*(-(4*a*c - b^2)^3)^(1/2) + 5
*a*b^3*c*(-(4*a*c - b^2)^3)^(1/2))/(a^5*(4*a*c - b^2)^3))^(1/3)*(72*a^8*b*c
^6 + (2^(1/3)*(81*a^8*c^3*x*(a*c - b^2)*(4*a*c - b^2)^2 + (81*2^(2/3)*a^10*
b*c^3*(4*a*c - b^2)^2*((b^8 + 16*a^4*c^4 - b^5*(-(4*a*c - b^2)^3)^(1/2) + 4
1*a^2*b^4*c^2 - 56*a^3*b^2*c^3 - 11*a*b^6*c - 5*a^2*b*c^2*(-(4*a*c - b^2)^3
)^(1/2) + 5*a*b^3*c*(-(4*a*c - b^2)^3)^(1/2))/(a^5*(4*a*c - b^2)^3))^(1/3))
/2)*((b^8 + 16*a^4*c^4 - b^5*(-(4*a*c - b^2)^3)^(1/2) + 41*a^2*b^4*c^2 - 56
*a^3*b^2*c^3 - 11*a*b^6*c - 5*a^2*b*c^2*(-(4*a*c - b^2)^3)^(1/2) + 5*a*b^3*
c*(-(4*a*c - b^2)^3)^(1/2))/(a^5*(4*a*c - b^2)^3))^(2/3))/18 + 9*a^6*b^5*c^
4 - 54*a^7*b^3*c^5))/6 - 3*a^6*c^6*x*(2*a*c - b^2))*(-(b^8 + 16*a^4*c^4 - b
^5*(-(4*a*c - b^2)^3)^(1/2) + 41*a^2*b^4*c^2 - 56*a^3*b^2*c^3 - 11*a*b^6*c
- 5*a^2*b*c^2*(-(4*a*c - b^2)^3)^(1/2) + 5*a*b^3*c*(-(4*a*c - b^2)^3)^(1/2)
)/(54*(a^5*b^6 - 64*a^8*c^3 - 12*a^6*b^4*c + 48*a^7*b^2*c^2)))^(1/3) - 1/(2
*a*x^2) + log((2^(2/3)*(3^(1/2)*1i - 1)*((b^8 + 16*a^4*c^4 + b^5*(-(4*a*c -
 b^2)^3)^(1/2) + 41*a^2*b^4*c^2 - 56*a^3*b^2*c^3 - 11*a*b^6*c + 5*a^2*b*c^2
*(-(4*a*c - b^2)^3)^(1/2) - 5*a*b^3*c*(-(4*a*c - b^2)^3)^(1/2))/(a^5*(4*a*c
 - b^2)^3))^(1/3)*(72*a^8*b*c^6 + 9*a^6*b^5*c^4 - 54*a^7*b^3*c^5 - (2^(1/3)
*(3^(1/2)*1i + 1)*(81*a^8*c^3*x*(a*c - b^2)*(4*a*c - b^2)^2 + (81*2^(2/3)*a
^10*b*c^3*(3^(1/2)*1i - 1)*(4*a*c - b^2)^2*((b^8 + 16*a^4*c^4 + b^5*(-(4*a*
c - b^2)^3)^(1/2) + 41*a^2*b^4*c^2 - 56*a^3*b^2*c^3 - 11*a*b^6*c + 5*a^2*b*
c^2*(-(4*a*c - b^2)^3)^(1/2) - 5*a*b^3*c*(-(4*a*c - b^2)^3)^(1/2))/(a^5*(4*
a*c - b^2)^3))^(1/3))/4)*((b^8 + 16*a^4*c^4 + b^5*(-(4*a*c - b^2)^3)^(1/2)
+ 41*a^2*b^4*c^2 - 56*a^3*b^2*c^3 - 11*a*b^6*c + 5*a^2*b*c^2*(-(4*a*c - b^2
)^3)^(1/2) - 5*a*b^3*c*(-(4*a*c - b^2)^3)^(1/2))/(a^5*(4*a*c - b^2)^3))^(2/
3))/36))/12 - 3*a^6*c^6*x*(2*a*c - b^2))*((3^(1/2)*1i)/2 - 1/2)*(-(b^8 + 16
*a^4*c^4 + b^5*(-(4*a*c - b^2)^3)^(1/2) + 41*a^2*b^4*c^2 - 56*a^3*b^2*c^3 -
 11*a*b^6*c + 5*a^2*b*c^2*(-(4*a*c - b^2)^3)^(1/2) - 5*a*b^3*c*(-(4*a*c - b
^2)^3)^(1/2))/(54*(a^5*b^6 - 64*a^8*c^3 - 12*a^6*b^4*c + 48*a^7*b^2*c^2)))^
(1/3) - log((2^(2/3)*(3^(1/2)*1i + 1)*((b^8 + 16*a^4*c^4 + b^5*(-(4*a*c - b
^2)^3)^(1/2) + 41*a^2*b^4*c^2 - 56*a^3*b^2*c^3 - 11*a*b^6*c + 5*a^2*b*c^2*(
-(4*a*c - b^2)^3)^(1/2) - 5*a*b^3*c*(-(4*a*c - b^2)^3)^(1/2))/(a^5*(4*a*c -
 b^2)^3))^(1/3)*(72*a^8*b*c^6 + 9*a^6*b^5*c^4 - 54*a^7*b^3*c^5 + (2^(1/3)*(
3^(1/2)*1i - 1)*(81*a^8*c^3*x*(a*c - b^2)*(4*a*c - b^2)^2 - (81*2^(2/3)*a^1
0*b*c^3*(3^(1/2)*1i + 1)*(4*a*c - b^2)^2*((b^8 + 16*a^4*c^4 + b^5*(-(4*a*c
- b^2)^3)^(1/2) + 41*a^2*b^4*c^2 - 56*a^3*b^2*c^3 - 11*a*b^6*c + 5*a^2*b*c^
2*(-(4*a*c - b^2)^3)^(1/2) - 5*a*b^3*c*(-(4*a*c - b^2)^3)^(1/2))/(a^5*(4*a*
c - b^2)^3))^(1/3))/4)*((b^8 + 16*a^4*c^4 + b^5*(-(4*a*c - b^2)^3)^(1/2) +
41*a^2*b^4*c^2 - 56*a^3*b^2*c^3 - 11*a*b^6*c + 5*a^2*b*c^2*(-(4*a*c - b^2)^
3)^(1/2) - 5*a*b^3*c*(-(4*a*c - b^2)^3)^(1/2))/(a^5*(4*a*c - b^2)^3))^(2/3)
)/36))/12 + 3*a^6*c^6*x*(2*a*c - b^2))*((3^(1/2)*1i)/2 + 1/2)*(-(b^8 + 16*a
^4*c^4 + b^5*(-(4*a*c - b^2)^3)^(1/2) + 41*a^2*b^4*c^2 - 56*a^3*b^2*c^3 - 1
1*a*b^6*c + 5*a^2*b*c^2*(-(4*a*c - b^2)^3)^(1/2) - 5*a*b^3*c*(-(4*a*c - b^2
)^3)^(1/2))/(54*(a^5*b^6 - 64*a^8*c^3 - 12*a^6*b^4*c + 48*a^7*b^2*c^2)))^(1
/3) + log((2^(2/3)*(3^(1/2)*1i - 1)*((b^8 + 16*a^4*c^4 - b^5*(-(4*a*c - b^2
)^3)^(1/2) + 41*a^2*b^4*c^2 - 56*a^3*b^2*c^3 - 11*a*b^6*c - 5*a^2*b*c^2*(-(
4*a*c - b^2)^3)^(1/2) + 5*a*b^3*c*(-(4*a*c - b^2)^3)^(1/2))/(a^5*(4*a*c - b
^2)^3))^(1/3)*(72*a^8*b*c^6 + 9*a^6*b^5*c^4 - 54*a^7*b^3*c^5 - (2^(1/3)*(3^
(1/2)*1i + 1)*(81*a^8*c^3*x*(a*c - b^2)*(4*a*c - b^2)^2 + (81*2^(2/3)*a^10*
b*c^3*(3^(1/2)*1i - 1)*(4*a*c - b^2)^2*((b^8 + 16*a^4*c^4 - b^5*(-(4*a*c -
b^2)^3)^(1/2) + 41*a^2*b^4*c^2 - 56*a^3*b^2*c^3 - 11*a*b^6*c - 5*a^2*b*c^2*
(-(4*a*c - b^2)^3)^(1/2) + 5*a*b^3*c*(-(4*a*c - b^2)^3)^(1/2))/(a^5*(4*a*c
- b^2)^3))^(1/3))/4)*((b^8 + 16*a^4*c^4 - b^5*(-(4*a*c - b^2)^3)^(1/2) + 41
*a^2*b^4*c^2 - 56*a^3*b^2*c^3 - 11*a*b^6*c - 5*a^2*b*c^2*(-(4*a*c - b^2)^3)
^(1/2) + 5*a*b^3*c*(-(4*a*c - b^2)^3)^(1/2))/(a^5*(4*a*c - b^2)^3))^(2/3))/
36))/12 - 3*a^6*c^6*x*(2*a*c - b^2))*((3^(1/2)*1i)/2 - 1/2)*(-(b^8 + 16*a^4
```

```
*c^4 - b^5*(-(4*a*c - b^2)^3)^(1/2) + 41*a^2*b^4*c^2 - 56*a^3*b^2*c^3 - 11*
a*b^6*c - 5*a^2*b*c^2*(-(4*a*c - b^2)^3)^(1/2) + 5*a*b^3*c*(-(4*a*c - b^2)^
3)^(1/2))/(54*(a^5*b^6 - 64*a^8*c^3 - 12*a^6*b^4*c + 48*a^7*b^2*c^2)))^(1/3
) - log((2^(2/3)*(3^(1/2)*1i + 1)*((b^8 + 16*a^4*c^4 - b^5*(-(4*a*c - b^2)^
3)^(1/2) + 41*a^2*b^4*c^2 - 56*a^3*b^2*c^3 - 11*a*b^6*c - 5*a^2*b*c^2*(-(4*
a*c - b^2)^3)^(1/2) + 5*a*b^3*c*(-(4*a*c - b^2)^3)^(1/2))/(a^5*(4*a*c - b^2
)^3))^(1/3)*(72*a^8*b*c^6 + 9*a^6*b^5*c^4 - 54*a^7*b^3*c^5 + (2^(1/3)*(3^(1
/2)*1i - 1)*(81*a^8*c^3*x*(a*c - b^2)*(4*a*c - b^2)^2 - (81*2^(2/3)*a^10*b*
c^3*(3^(1/2)*1i + 1)*(4*a*c - b^2)^2*((b^8 + 16*a^4*c^4 - b^5*(-(4*a*c - b^
2)^3)^(1/2) + 41*a^2*b^4*c^2 - 56*a^3*b^2*c^3 - 11*a*b^6*c - 5*a^2*b*c^2*(-
(4*a*c - b^2)^3)^(1/2) + 5*a*b^3*c*(-(4*a*c - b^2)^3)^(1/2))/(a^5*(4*a*c -
b^2)^3))^(1/3))/4)*((b^8 + 16*a^4*c^4 - b^5*(-(4*a*c - b^2)^3)^(1/2) + 41*a
^2*b^4*c^2 - 56*a^3*b^2*c^3 - 11*a*b^6*c - 5*a^2*b*c^2*(-(4*a*c - b^2)^3)^(
1/2) + 5*a*b^3*c*(-(4*a*c - b^2)^3)^(1/2))/(a^5*(4*a*c - b^2)^3))^(2/3))/36
))/12 + 3*a^6*c^6*x*(2*a*c - b^2))*((3^(1/2)*1i)/2 + 1/2)*(-(b^8 + 16*a^4*c
^4 - b^5*(-(4*a*c - b^2)^3)^(1/2) + 41*a^2*b^4*c^2 - 56*a^3*b^2*c^3 - 11*a*
b^6*c - 5*a^2*b*c^2*(-(4*a*c - b^2)^3)^(1/2) + 5*a*b^3*c*(-(4*a*c - b^2)^3)
^(1/2))/(54*(a^5*b^6 - 64*a^8*c^3 - 12*a^6*b^4*c + 48*a^7*b^2*c^2)))^(1/3)
```

sympy [A] time = 52.50, size = 241, normalized size = 0.39

$$\operatorname{RootSum}\left(t^{6}\left(46656a^{8}c^{3}-34992a^{7}b^{2}c^{2}+8748a^{6}b^{4}c-729a^{5}b^{6}\right)+t^{3}\left(-432a^{4}c^{4}+1512a^{3}b^{2}c^{3}-1107a^{2}b^{4}c^{2}+297ab^{6}c-27b^{8}\right)+c^{5},\left(t\mapsto t\log\left(x+\frac{-2592t^{4}a^{8}c^{3}+2592t^{4}a^{7}b^{2}c^{2}-810t^{4}a^{6}b^{4}c+81t^{4}a^{5}b^{6}+12ta^{4}c^{4}-75ta^{3}b^{2}c^{3}+78ta^{2}b^{4}c^{2}-27tab^{6}c+3tb^{8}}{5a^{2}bc^{4}-5ab^{3}c^{3}+b^{5}c^{2}}\right)\right)\right)-\frac{1}{2ax^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(c*x**6+b*x**3+a),x)
```

```
[Out] RootSum(_t**6*(46656*a**8*c**3 - 34992*a**7*b**2*c**2 + 8748*a**6*b**4*c -
729*a**5*b**6) + _t**3*(-432*a**4*c**4 + 1512*a**3*b**2*c**3 - 1107*a**2*b*
*4*c**2 + 297*a*b**6*c - 27*b**8) + c**5, Lambda(_t, _t*log(x + (-2592*_t**
4*a**8*c**3 + 2592*_t**4*a**7*b**2*c**2 - 810*_t**4*a**6*b**4*c + 81*_t**4*
a**5*b**6 + 12*_t*a**4*c**4 - 75*_t*a**3*b**2*c**3 + 78*_t*a**2*b**4*c**2 -
 27*_t*a*b**6*c + 3*_t*b**8)/(5*a**2*b*c**4 - 5*a*b**3*c**3 + b**5*c**2))))
 - 1/(2*a*x**2)
```

3.138 $\int \frac{x^{11}}{3+4x^3+x^6} \, dx$

Optimal. Leaf size=35

$$\frac{x^6}{6}-\frac{4x^3}{3}-\frac{1}{6}\log\left(x^3+1\right)+\frac{9}{2}\log\left(x^3+3\right)$$

Rubi [A] time = 0.02, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1357, 701, 632, 31}

$$\frac{x^6}{6}-\frac{4x^3}{3}-\frac{1}{6}\log\left(x^3+1\right)+\frac{9}{2}\log\left(x^3+3\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^11/(3 + 4*x^3 + x^6),x]
```

```
[Out] (-4*x^3)/3 + x^6/6 - Log[1 + x^3]/6 + (9*Log[3 + x^3])/2
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 632

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rule 701

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Int[PolynomialDivide[(d + e*x)^m, a + b*x + c*x^2, x], x] /; FreeQ[{a,
 b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
 NeQ[2*c*d - b*e, 0] && IGtQ[m, 1] && (NeQ[d, 0] || GtQ[m, 2])
```

Rule 1357

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^{11}}{3+4x^3+x^6}\,dx &= \frac{1}{3}\operatorname{Subst}\left(\int \frac{x^3}{3+4x+x^2}\,dx,x,x^3\right)\\
&= \frac{1}{3}\operatorname{Subst}\left(\int \left(-4+x+\frac{12+13x}{3+4x+x^2}\right)dx,x,x^3\right)\\
&= -\frac{4x^3}{3}+\frac{x^6}{6}+\frac{1}{3}\operatorname{Subst}\left(\int \frac{12+13x}{3+4x+x^2}\,dx,x,x^3\right)\\
&= -\frac{4x^3}{3}+\frac{x^6}{6}-\frac{1}{6}\operatorname{Subst}\left(\int \frac{1}{1+x}\,dx,x,x^3\right)+\frac{9}{2}\operatorname{Subst}\left(\int \frac{1}{3+x}\,dx,x,x^3\right)\\
&= -\frac{4x^3}{3}+\frac{x^6}{6}-\frac{1}{6}\log\left(1+x^3\right)+\frac{9}{2}\log\left(3+x^3\right)
\end{aligned}
$$

Mathematica [A] time = 0.01, size = 35, normalized size = 1.00

$$\frac{x^6}{6}-\frac{4x^3}{3}-\frac{1}{6}\log\left(x^3+1\right)+\frac{9}{2}\log\left(x^3+3\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^11/(3 + 4*x^3 + x^6),x]
[Out] (-4*x^3)/3 + x^6/6 - Log[1 + x^3]/6 + (9*Log[3 + x^3])/2
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{11}}{3+4x^3+x^6}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^11/(3 + 4*x^3 + x^6),x]
[Out] IntegrateAlgebraic[x^11/(3 + 4*x^3 + x^6), x]
```

fricas [A] time = 1.29, size = 27, normalized size = 0.77

$$\frac{1}{6}\,x^6-\frac{4}{3}\,x^3+\frac{9}{2}\,\log\left(x^3+3\right)-\frac{1}{6}\,\log\left(x^3+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(x^6+4*x^3+3),x, algorithm="fricas")
[Out] 1/6*x^6 - 4/3*x^3 + 9/2*log(x^3 + 3) - 1/6*log(x^3 + 1)
```

giac [A] time = 0.39, size = 29, normalized size = 0.83

$$\frac{1}{6}\,x^6-\frac{4}{3}\,x^3+\frac{9}{2}\,\log\left(\left|x^3+3\right|\right)-\frac{1}{6}\,\log\left(\left|x^3+1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(x^6+4*x^3+3),x, algorithm="giac")
[Out] 1/6*x^6 - 4/3*x^3 + 9/2*log(abs(x^3 + 3)) - 1/6*log(abs(x^3 + 1))
```

maple [A] time = 0.00, size = 28, normalized size = 0.80

$$\frac{x^6}{6}-\frac{4x^3}{3}-\frac{\ln\left(x^3+1\right)}{6}+\frac{9\ln\left(x^3+3\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11/(x^6+4*x^3+3),x)
```

```
[Out] -4/3*x^3+1/6*x^6-1/6*ln(x^3+1)+9/2*ln(x^3+3)
```

maxima [A] time = 0.49, size = 27, normalized size = 0.77

$$\frac{1}{6}\,x^6 - \frac{4}{3}\,x^3 + \frac{9}{2}\,\log\left(x^3+3\right) - \frac{1}{6}\,\log\left(x^3+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(x^6+4*x^3+3),x, algorithm="maxima")
```

```
[Out] 1/6*x^6 - 4/3*x^3 + 9/2*log(x^3 + 3) - 1/6*log(x^3 + 1)
```

mupad [B] time = 1.25, size = 27, normalized size = 0.77

$$\frac{9\,\ln\left(x^3+3\right)}{2} - \frac{\ln\left(x^3+1\right)}{6} - \frac{4\,x^3}{3} + \frac{x^6}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11/(4*x^3 + x^6 + 3),x)
```

```
[Out] (9*log(x^3 + 3))/2 - log(x^3 + 1)/6 - (4*x^3)/3 + x^6/6
```

sympy [A] time = 0.13, size = 29, normalized size = 0.83

$$\frac{x^6}{6} - \frac{4x^3}{3} - \frac{\log\left(x^3+1\right)}{6} + \frac{9\log\left(x^3+3\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**11/(x**6+4*x**3+3),x)
```

```
[Out] x**6/6 - 4*x**3/3 - log(x**3 + 1)/6 + 9*log(x**3 + 3)/2
```

3.139 $\int \frac{x^8}{3+4x^3+x^6} dx$

Optimal. Leaf size=28

$$\frac{x^3}{3} + \frac{1}{6}\log\left(x^3+1\right) - \frac{3}{2}\log\left(x^3+3\right)$$

Rubi [A] time = 0.02, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1357, 703, 632, 31}

$$\frac{x^3}{3} + \frac{1}{6}\log\left(x^3+1\right) - \frac{3}{2}\log\left(x^3+3\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^8/(3 + 4*x^3 + x^6),x]
```

```
[Out] x^3/3 + Log[1 + x^3]/6 - (3*Log[3 + x^3])/2
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 632

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rule 703

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Simp[(e*(d + e*x)^(m - 1))/(c*(m - 1)), x] + Dist[1/c, Int[((d + e*x)^
(m - 2)*Simp[c*d^2 - a*e^2 + e*(2*c*d - b*e)*x, x])/(a + b*x + c*x^2), x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && GtQ[m, 1]
```

Rule 1357

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^8}{3+4x^3+x^6} dx &= \frac{1}{3}\text{Subst}\left(\int \frac{x^2}{3+4x+x^2} dx, x, x^3\right)\\
&= \frac{x^3}{3} + \frac{1}{3}\text{Subst}\left(\int \frac{-3-4x}{3+4x+x^2} dx, x, x^3\right)\\
&= \frac{x^3}{3} + \frac{1}{6}\text{Subst}\left(\int \frac{1}{1+x} dx, x, x^3\right) - \frac{3}{2}\text{Subst}\left(\int \frac{1}{3+x} dx, x, x^3\right)\\
&= \frac{x^3}{3} + \frac{1}{6}\log\left(1+x^3\right) - \frac{3}{2}\log\left(3+x^3\right)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 28, normalized size = 1.00

$$\frac{x^3}{3}+\frac{1}{6}\log\left(x^3+1\right)-\frac{3}{2}\log\left(x^3+3\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8/(3 + 4*x^3 + x^6),x]
[Out] x^3/3 + Log[1 + x^3]/6 - (3*Log[3 + x^3])/2
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^8}{3+4x^3+x^6}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^8/(3 + 4*x^3 + x^6),x]
[Out] IntegrateAlgebraic[x^8/(3 + 4*x^3 + x^6), x]
```

fricas [A] time = 1.13, size = 22, normalized size = 0.79

$$\frac{1}{3}x^3-\frac{3}{2}\log\left(x^3+3\right)+\frac{1}{6}\log\left(x^3+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(x^6+4*x^3+3),x, algorithm="fricas")
[Out] 1/3*x^3 - 3/2*log(x^3 + 3) + 1/6*log(x^3 + 1)
```

giac [A] time = 0.34, size = 24, normalized size = 0.86

$$\frac{1}{3}x^3-\frac{3}{2}\log\left(\left|x^3+3\right|\right)+\frac{1}{6}\log\left(\left|x^3+1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(x^6+4*x^3+3),x, algorithm="giac")
[Out] 1/3*x^3 - 3/2*log(abs(x^3 + 3)) + 1/6*log(abs(x^3 + 1))
```

maple [A] time = 0.01, size = 23, normalized size = 0.82

$$\frac{x^3}{3}+\frac{\ln\left(x^3+1\right)}{6}-\frac{3\ln\left(x^3+3\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(x^6+4*x^3+3),x)
[Out] 1/3*x^3+1/6*ln(x^3+1)-3/2*ln(x^3+3)
```

maxima [A] time = 0.49, size = 22, normalized size = 0.79

$$\frac{1}{3}x^3-\frac{3}{2}\log\left(x^3+3\right)+\frac{1}{6}\log\left(x^3+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(x^6+4*x^3+3),x, algorithm="maxima")
[Out] 1/3*x^3 - 3/2*log(x^3 + 3) + 1/6*log(x^3 + 1)
```

mupad [**B**] time = 0.05, size = 22, normalized size = 0.79

$$\frac{\ln\left(x^3+1\right)}{6}-\frac{3\,\ln\left(x^3+3\right)}{2}+\frac{x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(4*x^3 + x^6 + 3),x)
[Out] log(x^3 + 1)/6 - (3*log(x^3 + 3))/2 + x^3/3
```

sympy [**A**] time = 0.12, size = 22, normalized size = 0.79

$$\frac{x^3}{3}+\frac{\log\left(x^3+1\right)}{6}-\frac{3\log\left(x^3+3\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8/(x**6+4*x**3+3),x)
[Out] x**3/3 + log(x**3 + 1)/6 - 3*log(x**3 + 3)/2
```

3.140 $\int \frac{x^5}{3+4x^3+x^6}\,dx$

Optimal. Leaf size=21

$$\frac{1}{2}\log\left(x^3+3\right)-\frac{1}{6}\log\left(x^3+1\right)$$

Rubi [A] time = 0.01, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {1357, 632, 31}

$$\frac{1}{2}\log\left(x^3+3\right)-\frac{1}{6}\log\left(x^3+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^5/(3 + 4*x^3 + x^6),x]
[Out] -Log[1 + x^3]/6 + Log[3 + x^3]/2
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 632

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1357

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x^5}{3+4x^3+x^6}\,dx &= \frac{1}{3}\operatorname{Subst}\left(\int \frac{x}{3+4x+x^2}\,dx,x,x^3\right)\\ &= -\left(\frac{1}{6}\operatorname{Subst}\left(\int \frac{1}{1+x}\,dx,x,x^3\right)\right)+\frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{3+x}\,dx,x,x^3\right)\\ &= -\frac{1}{6}\log\left(1+x^3\right)+\frac{1}{2}\log\left(3+x^3\right)\end{aligned}$$

Mathematica [A] time = 0.00, size = 21, normalized size = 1.00

$$\frac{1}{2}\log\left(x^3+3\right)-\frac{1}{6}\log\left(x^3+1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/(3 + 4*x^3 + x^6),x]
[Out] -1/6*Log[1 + x^3] + Log[3 + x^3]/2
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5}{3+4x^3+x^6}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^5/(3 + 4*x^3 + x^6),x]
[Out] IntegrateAlgebraic[x^5/(3 + 4*x^3 + x^6), x]
```

fricas [A] time = 1.21, size = 17, normalized size = 0.81

$$\frac{1}{2}\,\log\left(x^3+3\right)-\frac{1}{6}\,\log\left(x^3+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(x^6+4*x^3+3),x, algorithm="fricas")
[Out] 1/2*log(x^3 + 3) - 1/6*log(x^3 + 1)
```

giac [A] time = 0.36, size = 19, normalized size = 0.90

$$\frac{1}{2}\,\log\left(\left|x^3+3\right|\right)-\frac{1}{6}\,\log\left(\left|x^3+1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(x^6+4*x^3+3),x, algorithm="giac")
[Out] 1/2*log(abs(x^3 + 3)) - 1/6*log(abs(x^3 + 1))
```

maple [A] time = 0.00, size = 18, normalized size = 0.86

$$-\frac{\ln\left(x^3+1\right)}{6}+\frac{\ln\left(x^3+3\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(x^6+4*x^3+3),x)
[Out] -1/6*ln(x^3+1)+1/2*ln(x^3+3)
```

maxima [A] time = 0.52, size = 17, normalized size = 0.81

$$\frac{1}{2}\,\log\left(x^3+3\right)-\frac{1}{6}\,\log\left(x^3+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(x^6+4*x^3+3),x, algorithm="maxima")
[Out] 1/2*log(x^3 + 3) - 1/6*log(x^3 + 1)
```

mupad [B] time = 0.05, size = 17, normalized size = 0.81

$$\frac{\ln\left(x^3+3\right)}{2}-\frac{\ln\left(x^3+1\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(4*x^3 + x^6 + 3),x)
[Out] log(x^3 + 3)/2 - log(x^3 + 1)/6
```

<u>**sympy**</u> **[A]** time = 0.12, size = 15, normalized size = 0.71

$$-\frac{\log\left(x^3+1\right)}{6}+\frac{\log\left(x^3+3\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(x**6+4*x**3+3),x)
```

```
[Out] -log(x**3 + 1)/6 + log(x**3 + 3)/2
```

3.141 $\int \frac{x^2}{3+4x^3+x^6} \, dx$

Optimal. Leaf size=10

$$-\frac{1}{3}\tanh^{-1}\left(x^3+2\right)$$

Rubi [B] time = 0.01, antiderivative size = 21, normalized size of antiderivative = 2.10, number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {1352, 616, 31}

$$\frac{1}{6}\log\left(x^3+1\right)-\frac{1}{6}\log\left(x^3+3\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(3 + 4*x^3 + x^6),x]
```

```
[Out] Log[1 + x^3]/6 - Log[3 + x^3]/6
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 616

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = Rt[b^2
 - 4*a*c, 2]}, Dist[c/q, Int[1/Simp[b/2 - q/2 + c*x, x], x], x] - Dist[c/q,
 Int[1/Simp[b/2 + q/2 + c*x, x], x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[b^2 - 4*a*c] && PerfectSquareQ[b^2 - 4*a*c]
```

Rule 1352

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a,
b, c, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2}{3+4x^3+x^6}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{1}{3+4x+x^2}\,dx,x,x^3\right)\\
&= \frac{1}{6}\text{Subst}\left(\int \frac{1}{1+x}\,dx,x,x^3\right)-\frac{1}{6}\text{Subst}\left(\int \frac{1}{3+x}\,dx,x,x^3\right)\\
&= \frac{1}{6}\log\left(1+x^3\right)-\frac{1}{6}\log\left(3+x^3\right)
\end{aligned}$$

Mathematica [B] time = 0.00, size = 21, normalized size = 2.10

$$\frac{1}{6}\log\left(x^3+1\right)-\frac{1}{6}\log\left(x^3+3\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(3 + 4*x^3 + x^6),x]
```

```
[Out] Log[1 + x^3]/6 - Log[3 + x^3]/6
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{3+4x^3+x^6}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^2/(3 + 4*x^3 + x^6),x]
[Out] IntegrateAlgebraic[x^2/(3 + 4*x^3 + x^6), x]
```

fricas [B] time = 1.21, size = 17, normalized size = 1.70

$$-\frac{1}{6}\,\log\left(x^3+3\right)+\frac{1}{6}\,\log\left(x^3+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^6+4*x^3+3),x, algorithm="fricas")
[Out] -1/6*log(x^3 + 3) + 1/6*log(x^3 + 1)
```

giac [B] time = 0.33, size = 19, normalized size = 1.90

$$-\frac{1}{6}\,\log\left(\left|x^3+3\right|\right)+\frac{1}{6}\,\log\left(\left|x^3+1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^6+4*x^3+3),x, algorithm="giac")
[Out] -1/6*log(abs(x^3 + 3)) + 1/6*log(abs(x^3 + 1))
```

maple [B] time = 0.00, size = 18, normalized size = 1.80

$$\frac{\ln\left(x^3+1\right)}{6}-\frac{\ln\left(x^3+3\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(x^6+4*x^3+3),x)
[Out] 1/6*ln(x^3+1)-1/6*ln(x^3+3)
```

maxima [B] time = 0.47, size = 17, normalized size = 1.70

$$-\frac{1}{6}\,\log\left(x^3+3\right)+\frac{1}{6}\,\log\left(x^3+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^6+4*x^3+3),x, algorithm="maxima")
[Out] -1/6*log(x^3 + 3) + 1/6*log(x^3 + 1)
```

mupad [B] time = 0.38, size = 16, normalized size = 1.60

$$\frac{\mathrm{atanh}\left(\frac{9}{2\left(8x^3+6\right)}+\frac{5}{4}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(4*x^3 + x^6 + 3),x)
```

```
[Out] atanh(9/(2*(8*x^3 + 6)) + 5/4)/3
```

sympy [A] time = 0.11, size = 15, normalized size = 1.50

$$\frac{\log\left(x^3+1\right)}{6}-\frac{\log\left(x^3+3\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(x**6+4*x**3+3),x)
[Out] log(x**3 + 1)/6 - log(x**3 + 3)/6
```

3.142 $\int \frac{1}{x(3+4x^3+x^6)}\,dx$

Optimal. Leaf size=27

$$-\frac{1}{6}\log\left(x^3+1\right)+\frac{1}{18}\log\left(x^3+3\right)+\frac{\log(x)}{3}$$

Rubi [A] time = 0.02, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {1357, 705, 29, 632, 31}

$$-\frac{1}{6}\log\left(x^3+1\right)+\frac{1}{18}\log\left(x^3+3\right)+\frac{\log(x)}{3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(3 + 4*x^3 + x^6)),x]
```

```
[Out] Log[x]/3 - Log[1 + x^3]/6 + Log[3 + x^3]/18
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 632

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rule 705

```
Int[1/(((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)), x_Symbol]
 :> Dist[e^2/(c*d^2 - b*d*e + a*e^2), Int[1/(d + e*x), x], x] + Dist[1/(c*d
^2 - b*d*e + a*e^2), Int[(c*d - b*e - c*e*x)/(a + b*x + c*x^2), x], x] /; F
reeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^
2, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1357

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{1}{x\left(3+4x^3+x^6\right)}\,dx = \frac{1}{3}\operatorname{Subst}\left(\int \frac{1}{x\left(3+4x+x^2\right)}\,dx, x, x^3\right)$$
$$= \frac{1}{9}\operatorname{Subst}\left(\int \frac{1}{x}\,dx, x, x^3\right) + \frac{1}{9}\operatorname{Subst}\left(\int \frac{-4-x}{3+4x+x^2}\,dx, x, x^3\right)$$
$$= \frac{\log(x)}{3} + \frac{1}{18}\operatorname{Subst}\left(\int \frac{1}{3+x}\,dx, x, x^3\right) - \frac{1}{6}\operatorname{Subst}\left(\int \frac{1}{1+x}\,dx, x, x^3\right)$$
$$= \frac{\log(x)}{3} - \frac{1}{6}\log\left(1+x^3\right) + \frac{1}{18}\log\left(3+x^3\right)$$

Mathematica [A] time = 0.01, size = 27, normalized size = 1.00

$$-\frac{1}{6}\log\left(x^3+1\right) + \frac{1}{18}\log\left(x^3+3\right) + \frac{\log(x)}{3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(3 + 4*x^3 + x^6)),x]
[Out] Log[x]/3 - Log[1 + x^3]/6 + Log[3 + x^3]/18
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(3+4x^3+x^6\right)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x*(3 + 4*x^3 + x^6)),x]
[Out] IntegrateAlgebraic[1/(x*(3 + 4*x^3 + x^6)), x]
```

fricas [A] time = 1.25, size = 21, normalized size = 0.78

$$\frac{1}{18}\log\left(x^3+3\right) - \frac{1}{6}\log\left(x^3+1\right) + \frac{1}{3}\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x^6+4*x^3+3),x, algorithm="fricas")
[Out] 1/18*log(x^3 + 3) - 1/6*log(x^3 + 1) + 1/3*log(x)
```

giac [A] time = 0.35, size = 24, normalized size = 0.89

$$\frac{1}{18}\log\left(\left|x^3+3\right|\right) - \frac{1}{6}\log\left(\left|x^3+1\right|\right) + \frac{1}{3}\log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x^6+4*x^3+3),x, algorithm="giac")
[Out] 1/18*log(abs(x^3 + 3)) - 1/6*log(abs(x^3 + 1)) + 1/3*log(abs(x))
```

maple [A] time = 0.01, size = 31, normalized size = 1.15

$$\frac{\ln(x)}{3} - \frac{\ln(x+1)}{6} + \frac{\ln\left(x^3+3\right)}{18} - \frac{\ln\left(x^2-x+1\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(x^6+4*x^3+3),x)
```

```
[Out] 1/3*ln(x)-1/6*ln(x+1)+1/18*ln(x^3+3)-1/6*ln(x^2-x+1)
```

maxima [A] time = 0.45, size = 23, normalized size = 0.85

$$\frac{1}{18}\log\left(x^3+3\right)-\frac{1}{6}\log\left(x^3+1\right)+\frac{1}{9}\log\left(x^3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x^6+4*x^3+3),x, algorithm="maxima")
```

```
[Out] 1/18*log(x^3 + 3) - 1/6*log(x^3 + 1) + 1/9*log(x^3)
```

mupad [B] time = 1.26, size = 21, normalized size = 0.78

$$\frac{\ln\left(x^3+3\right)}{18}-\frac{\ln\left(x^3+1\right)}{6}+\frac{\ln(x)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(4*x^3 + x^6 + 3)),x)
```

```
[Out] log(x^3 + 3)/18 - log(x^3 + 1)/6 + log(x)/3
```

sympy [A] time = 0.14, size = 20, normalized size = 0.74

$$\frac{\log(x)}{3}-\frac{\log\left(x^3+1\right)}{6}+\frac{\log\left(x^3+3\right)}{18}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x**6+4*x**3+3),x)
```

```
[Out] log(x)/3 - log(x**3 + 1)/6 + log(x**3 + 3)/18
```

3.143 $\int \frac{1}{x^4\left(3+4x^3+x^6\right)}\,dx$

Optimal. Leaf size=34

$$-\frac{1}{9x^3}+\frac{1}{6}\log\left(x^3+1\right)-\frac{1}{54}\log\left(x^3+3\right)-\frac{4\log(x)}{9}$$

Rubi [A] time = 0.03, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {1357, 709, 800}

$$-\frac{1}{9x^3}+\frac{1}{6}\log\left(x^3+1\right)-\frac{1}{54}\log\left(x^3+3\right)-\frac{4\log(x)}{9}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(3 + 4*x^3 + x^6)),x]
```

```
[Out] -1/(9*x^3) - (4*Log[x])/9 + Log[1 + x^3]/6 - Log[3 + x^3]/54
```

Rule 709

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Simp[(e*(d + e*x)^(m + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dis
t[1/(c*d^2 - b*d*e + a*e^2), Int[((d + e*x)^(m + 1)*Simp[c*d - b*e - c*e*x,
 x])/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 -
 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[m
, -1]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 1357

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^4\left(3+4x^3+x^6\right)}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{1}{x^2\left(3+4x+x^2\right)}\,dx,x,x^3\right)\\
&= -\frac{1}{9x^3}+\frac{1}{9}\text{Subst}\left(\int \frac{-4-x}{x\left(3+4x+x^2\right)}\,dx,x,x^3\right)\\
&= -\frac{1}{9x^3}+\frac{1}{9}\text{Subst}\left(\int \left(-\frac{4}{3x}+\frac{3}{2(1+x)}-\frac{1}{6(3+x)}\right)dx,x,x^3\right)\\
&= -\frac{1}{9x^3}-\frac{4\log(x)}{9}+\frac{1}{6}\log\left(1+x^3\right)-\frac{1}{54}\log\left(3+x^3\right)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 34, normalized size = 1.00

$$-\frac{1}{9x^3}+\frac{1}{6}\log\left(x^3+1\right)-\frac{1}{54}\log\left(x^3+3\right)-\frac{4\log(x)}{9}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(3 + 4*x^3 + x^6)),x]
[Out] -1/9*1/x^3 - (4*Log[x])/9 + Log[1 + x^3]/6 - Log[3 + x^3]/54
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^4\left(3+4x^3+x^6\right)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x^4*(3 + 4*x^3 + x^6)),x]
[Out] IntegrateAlgebraic[1/(x^4*(3 + 4*x^3 + x^6)), x]
```

fricas [A] time = 1.27, size = 35, normalized size = 1.03

$$-\frac{x^3\log\left(x^3+3\right)-9\,x^3\log\left(x^3+1\right)+24\,x^3\log(x)+6}{54\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(x^6+4*x^3+3),x, algorithm="fricas")
[Out] -1/54*(x^3*log(x^3 + 3) - 9*x^3*log(x^3 + 1) + 24*x^3*log(x) + 6)/x^3
```

giac [A] time = 0.30, size = 36, normalized size = 1.06

$$\frac{4\,x^3-3}{27\,x^3}-\frac{1}{54}\log\left(\left|x^3+3\right|\right)+\frac{1}{6}\log\left(\left|x^3+1\right|\right)-\frac{4}{9}\log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(x^6+4*x^3+3),x, algorithm="giac")
[Out] 1/27*(4*x^3 - 3)/x^3 - 1/54*log(abs(x^3 + 3)) + 1/6*log(abs(x^3 + 1)) - 4/9
*log(abs(x))
```

maple [A] time = 0.01, size = 36, normalized size = 1.06

$$-\frac{4\ln(x)}{9}+\frac{\ln(x+1)}{6}-\frac{\ln\left(x^3+3\right)}{54}+\frac{\ln\left(x^2-x+1\right)}{6}-\frac{1}{9x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(x^6+4*x^3+3),x)
[Out] -1/9/x^3-4/9*ln(x)+1/6*ln(x+1)-1/54*ln(x^3+3)+1/6*ln(x^2-x+1)
```

maxima [A] time = 0.68, size = 28, normalized size = 0.82

$$-\frac{1}{9\,x^3}-\frac{1}{54}\log\left(x^3+3\right)+\frac{1}{6}\log\left(x^3+1\right)-\frac{4}{27}\log\left(x^3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(x^6+4*x^3+3),x, algorithm="maxima")
```

```
[Out] -1/9/x^3 - 1/54*log(x^3 + 3) + 1/6*log(x^3 + 1) - 4/27*log(x^3)
```

mupad [B] time = 1.23, size = 26, normalized size = 0.76

$$\frac{\ln\left(x^3+1\right)}{6}-\frac{\ln\left(x^3+3\right)}{54}-\frac{4\,\ln(x)}{9}-\frac{1}{9\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(4*x^3 + x^6 + 3)),x)
[Out] log(x^3 + 1)/6 - log(x^3 + 3)/54 - (4*log(x))/9 - 1/(9*x^3)
```

sympy [A] time = 0.17, size = 29, normalized size = 0.85

$$-\frac{4\log(x)}{9}+\frac{\log\left(x^3+1\right)}{6}-\frac{\log\left(x^3+3\right)}{54}-\frac{1}{9x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(x**6+4*x**3+3),x)
[Out] -4*log(x)/9 + log(x**3 + 1)/6 - log(x**3 + 3)/54 - 1/(9*x**3)
```

3.144 $\int \frac{1}{x^7\left(3+4x^3+x^6\right)}\,dx$

Optimal. Leaf size=41

$$-\frac{1}{18x^6}+\frac{4}{27x^3}-\frac{1}{6}\log\left(x^3+1\right)+\frac{1}{162}\log\left(x^3+3\right)+\frac{13\log(x)}{27}$$

Rubi [A] time = 0.04, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {1357, 709, 800}

$$\frac{4}{27x^3}-\frac{1}{18x^6}-\frac{1}{6}\log\left(x^3+1\right)+\frac{1}{162}\log\left(x^3+3\right)+\frac{13\log(x)}{27}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^7*(3 + 4*x^3 + x^6)),x]
```

```
[Out] -1/(18*x^6) + 4/(27*x^3) + (13*Log[x])/27 - Log[1 + x^3]/6 + Log[3 + x^3]/1
62
```

Rule 709

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Simp[(e*(d + e*x)^(m + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dis
t[1/(c*d^2 - b*d*e + a*e^2), Int[((d + e*x)^(m + 1)*Simp[c*d - b*e - c*e*x,
 x])/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 -
 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[m
, -1]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 1357

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^7\left(3+4x^3+x^6\right)}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{1}{x^3\left(3+4x+x^2\right)}\,dx,x,x^3\right)\\
&= -\frac{1}{18x^6}+\frac{1}{9}\text{Subst}\left(\int \frac{-4-x}{x^2\left(3+4x+x^2\right)}\,dx,x,x^3\right)\\
&= -\frac{1}{18x^6}+\frac{1}{9}\text{Subst}\left(\int \left(-\frac{4}{3x^2}+\frac{13}{9x}-\frac{3}{2(1+x)}+\frac{1}{18(3+x)}\right)dx,x,x^3\right)\\
&= -\frac{1}{18x^6}+\frac{4}{27x^3}+\frac{13\log(x)}{27}-\frac{1}{6}\log\left(1+x^3\right)+\frac{1}{162}\log\left(3+x^3\right)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 41, normalized size = 1.00

$$-\frac{1}{18x^6}+\frac{4}{27x^3}-\frac{1}{6}\log\left(x^3+1\right)+\frac{1}{162}\log\left(x^3+3\right)+\frac{13\log(x)}{27}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^7*(3 + 4*x^3 + x^6)),x]
```

```
[Out] -1/18*1/x^6 + 4/(27*x^3) + (13*Log[x])/27 - Log[1 + x^3]/6 + Log[3 + x^3]/1
62
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^7\left(3+4x^3+x^6\right)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x^7*(3 + 4*x^3 + x^6)),x]
```

```
[Out] IntegrateAlgebraic[1/(x^7*(3 + 4*x^3 + x^6)), x]
```

fricas [A] time = 1.21, size = 40, normalized size = 0.98

$$\frac{x^6\log\left(x^3+3\right)-27\,x^6\log\left(x^3+1\right)+78\,x^6\log(x)+24\,x^3-9}{162\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(x^6+4*x^3+3),x, algorithm="fricas")
```

```
[Out] 1/162*(x^6*log(x^3 + 3) - 27*x^6*log(x^3 + 1) + 78*x^6*log(x) + 24*x^3 - 9)
/x^6
```

giac [A] time = 0.37, size = 41, normalized size = 1.00

$$-\frac{13\,x^6-8\,x^3+3}{54\,x^6}+\frac{1}{162}\log\left(\left|x^3+3\right|\right)-\frac{1}{6}\log\left(\left|x^3+1\right|\right)+\frac{13}{27}\log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(x^6+4*x^3+3),x, algorithm="giac")
```

```
[Out] -1/54*(13*x^6 - 8*x^3 + 3)/x^6 + 1/162*log(abs(x^3 + 3)) - 1/6*log(abs(x^3
+ 1)) + 13/27*log(abs(x))
```

maple [A] time = 0.01, size = 41, normalized size = 1.00

$$\frac{13\ln(x)}{27}-\frac{\ln(x+1)}{6}+\frac{\ln\left(x^3+3\right)}{162}-\frac{\ln\left(x^2-x+1\right)}{6}+\frac{4}{27x^3}-\frac{1}{18x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^7/(x^6+4*x^3+3),x)
```

```
[Out] -1/18/x^6+4/27/x^3+13/27*ln(x)-1/6*ln(x+1)+1/162*ln(x^3+3)-1/6*ln(x^2-x+1)
```

maxima [A] time = 0.59, size = 35, normalized size = 0.85

$$\frac{8\,x^3-3}{54\,x^6}+\frac{1}{162}\log\left(x^3+3\right)-\frac{1}{6}\log\left(x^3+1\right)+\frac{13}{81}\log\left(x^3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(x^6+4*x^3+3),x, algorithm="maxima")
```

```
[Out] 1/54*(8*x^3 - 3)/x^6 + 1/162*log(x^3 + 3) - 1/6*log(x^3 + 1) + 13/81*log(x^
3)
```

mupad [B] time = 0.04, size = 32, normalized size = 0.78

$$\frac{\ln\left(x^3+3\right)}{162}-\frac{\ln\left(x^3+1\right)}{6}+\frac{13\,\ln(x)}{27}+\frac{\frac{4\,x^3}{27}-\frac{1}{18}}{x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^7*(4*x^3 + x^6 + 3)),x)
```

```
[Out] log(x^3 + 3)/162 - log(x^3 + 1)/6 + (13*log(x))/27 + ((4*x^3)/27 - 1/18)/x^
6
```

sympy [A] time = 0.19, size = 34, normalized size = 0.83

$$\frac{13\log(x)}{27}-\frac{\log\left(x^3+1\right)}{6}+\frac{\log\left(x^3+3\right)}{162}+\frac{8x^3-3}{54x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**7/(x**6+4*x**3+3),x)
```

```
[Out] 13*log(x)/27 - log(x**3 + 1)/6 + log(x**3 + 3)/162 + (8*x**3 - 3)/(54*x**6)
```

3.145 $\int \frac{x^{10}}{3+4x^3+x^6}\,dx$

Optimal. Leaf size=124

$$\frac{x^5}{5}-2x^2-\frac{1}{12}\log\left(x^2-x+1\right)+\frac{3}{4}3^{2/3}\log\left(x^2-\sqrt[3]{3}\,x+3^{2/3}\right)+\frac{1}{6}\log(x+1)-\frac{3}{2}3^{2/3}\log\left(x+\sqrt[3]{3}\right)+\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}}-\frac{9}{2}$$

Rubi [A] time = 0.11, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 10, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.625, Rules used = {1367, 1502, 1510, 292, 31, 634, 618, 204, 628, 617}

$$\frac{x^5}{5}-2x^2-\frac{1}{12}\log\left(x^2-x+1\right)+\frac{3}{4}3^{2/3}\log\left(x^2-\sqrt[3]{3}\,x+3^{2/3}\right)+\frac{1}{6}\log(x+1)-\frac{3}{2}3^{2/3}\log\left(x+\sqrt[3]{3}\right)+\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}}-\frac{9}{2}\sqrt[6]{3}\tan^{-1}\left(\frac{\sqrt[3]{3}-2x}{3^{5/6}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^10/(3 + 4*x^3 + x^6),x]
```

```
[Out] -2*x^2 + x^5/5 + ArcTan[(1 - 2*x)/Sqrt[3]]/(2*Sqrt[3]) - (9*3^(1/6)*ArcTan[
(3^(1/3) - 2*x)/3^(5/6)])/2 + Log[1 + x]/6 - (3*3^(2/3)*Log[3^(1/3) + x])/2
 - Log[1 - x + x^2]/12 + (3*3^(2/3)*Log[3^(2/3) - 3^(1/3)*x + x^2])/4
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1367

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Simp[(d^(2*n - 1)*(d*x)^(m - 2*n + 1)*(a + b*x^n + c*x^(2*n))^(
p + 1))/(c*(m + 2*n*p + 1)), x] - Dist[d^(2*n)/(c*(m + 2*n*p + 1)), Int[(d*
x)^(m - 2*n)*Simp[a*(m - 2*n + 1) + b*(m + n*(p - 1) + 1)*x^n, x]*(a + b*x^
n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && EqQ[n2, 2*n] && Ne
Q[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GtQ[m, 2*n - 1] && NeQ[m + 2*n*p + 1, 0]
 && IntegerQ[p]
```

Rule 1502

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (
c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Simp[(e*f^(n - 1)*(f*x)^(m - n + 1)*(a
+ b*x^n + c*x^(2*n))^(p + 1))/(c*(m + n*(2*p + 1) + 1)), x] - Dist[f^n/(c*(
m + n*(2*p + 1) + 1)), Int[(f*x)^(m - n)*(a + b*x^n + c*x^(2*n))^p*Simp[a*e
*(m - n + 1) + (b*e*(m + n*p + 1) - c*d*(m + n*(2*p + 1) + 1))*x^n, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c,
0] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*(2*p + 1) + 1, 0] && Integer
Q[p]
```

Rule 1510

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_)))/((a_) + (b_.)*(x_)^(n_) +
 (c_.)*(x_)^(n2_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 +
(2*c*d - b*e)/(2*q), Int[(f*x)^m/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (
2*c*d - b*e)/(2*q), Int[(f*x)^m/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b
, c, d, e, f, m}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^{10}}{3+4x^3+x^6}\,dx &= \frac{x^5}{5}-\frac{1}{5}\int \frac{x^4\left(15+20x^3\right)}{3+4x^3+x^6}\,dx\\
&= -2x^2+\frac{x^5}{5}+\frac{1}{10}\int \frac{x\left(120+130x^3\right)}{3+4x^3+x^6}\,dx\\
&= -2x^2+\frac{x^5}{5}-\frac{1}{2}\int \frac{x}{1+x^3}\,dx+\frac{27}{2}\int \frac{x}{3+x^3}\,dx\\
&= -2x^2+\frac{x^5}{5}+\frac{1}{6}\int \frac{1}{1+x}\,dx-\frac{1}{6}\int \frac{1+x}{1-x+x^2}\,dx-\frac{1}{2}\left(3\,3^{2/3}\right)\int \frac{1}{\sqrt[3]{3}+x}\,dx+\frac{1}{2}\left(3\,3^{2/3}\right)\int\\
&= -2x^2+\frac{x^5}{5}+\frac{1}{6}\log(1+x)-\frac{3}{2}3^{2/3}\log\left(\sqrt[3]{3}+x\right)-\frac{1}{12}\int \frac{-1+2x}{1-x+x^2}\,dx-\frac{1}{4}\int \frac{1}{1-x+x^2}\,dx\\
&= -2x^2+\frac{x^5}{5}+\frac{1}{6}\log(1+x)-\frac{3}{2}3^{2/3}\log\left(\sqrt[3]{3}+x\right)-\frac{1}{12}\log\left(1-x+x^2\right)+\frac{3}{4}3^{2/3}\log\left(3^{2/3}-\right.\\
&= -2x^2+\frac{x^5}{5}+\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}}-\frac{9}{2}\sqrt[6]{3}\tan^{-1}\left(\frac{\sqrt[3]{3}-2x}{3^{5/6}}\right)+\frac{1}{6}\log(1+x)-\frac{3}{2}3^{2/3}\log\left(\sqrt[3]{3}+x\right)-
\end{aligned}$$

Mathematica [A] time = 0.05, size = 118, normalized size = 0.95

$$\frac{1}{60}\left(12x^5-120x^2-5\log\left(x^2-x+1\right)+45\,3^{2/3}\log\left(\sqrt[3]{3}\,x^2-3^{2/3}x+3\right)+10\log(x+1)-90\,3^{2/3}\log\left(3^{2/3}x+3\right)-270\sqrt[6]{3}\,\tan^{-1}\left(\frac{\sqrt[3]{3}-2x}{3^{5/6}}\right)-10\sqrt{3}\,\tan^{-1}\left(\frac{2x-1}{\sqrt{3}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^10/(3 + 4*x^3 + x^6),x]
```

```
[Out] (-120*x^2 + 12*x^5 - 270*3^(1/6)*ArcTan[(3^(1/3) - 2*x)/3^(5/6)] - 10*Sqrt[
3]*ArcTan[(-1 + 2*x)/Sqrt[3]] + 10*Log[1 + x] - 90*3^(2/3)*Log[3 + 3^(2/3)*
x] - 5*Log[1 - x + x^2] + 45*3^(2/3)*Log[3 - 3^(2/3)*x + 3^(1/3)*x^2])/60
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{10}}{3+4x^3+x^6}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^10/(3 + 4*x^3 + x^6),x]
```

```
[Out] IntegrateAlgebraic[x^10/(3 + 4*x^3 + x^6), x]
```

fricas [A] time = 1.30, size = 102, normalized size = 0.82

$$\frac{1}{5}x^5-2x^2+\frac{3}{2}\sqrt{3}\,(-9)^{\frac{1}{3}}\arctan\left(\frac{1}{9}\sqrt{3}\left(2\,(-9)^{\frac{1}{3}}x+3\right)\right)-\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\,(2x-1)\right)-\frac{3}{4}(-9)^{\frac{1}{3}}\log\left(3x^2-(-9)^{\frac{2}{3}}x-3\,(-9)^{\frac{1}{3}}\right)+\frac{3}{2}(-9)^{\frac{1}{3}}\log\left(3x+(-9)^{\frac{2}{3}}\right)-\frac{1}{12}\log\left(x^2-x+1\right)+\frac{1}{6}\log\left(x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10/(x^6+4*x^3+3),x, algorithm="fricas")
```

```
[Out] 1/5*x^5 - 2*x^2 + 3/2*sqrt(3)*(-9)^(1/3)*arctan(1/9*sqrt(3)*(2*(-9)^(1/3)*x
 + 3)) - 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) - 3/4*(-9)^(1/3)*log(3*x
^2 - (-9)^(2/3)*x - 3*(-9)^(1/3)) + 3/2*(-9)^(1/3)*log(3*x + (-9)^(2/3)) -
1/12*log(x^2 - x + 1) + 1/6*log(x + 1)
```

giac [A] time = 0.37, size = 96, normalized size = 0.77

$$\frac{1}{5}x^5-2x^2+\frac{3}{4}\cdot 3^{\frac{2}{3}}\log\left(x^2-3^{\frac{1}{3}}x+3^{\frac{2}{3}}\right)-\frac{3}{2}\cdot 3^{\frac{2}{3}}\log\left(\left|x+3^{\frac{1}{3}}\right|\right)-\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\,(2x-1)\right)+\frac{9}{2}\cdot 3^{\frac{1}{6}}\arctan\left(\frac{1}{3}\cdot 3^{\frac{1}{6}}\left(2x-3^{\frac{1}{3}}\right)\right)-\frac{1}{12}\log\left(x^2-x+1\right)+\frac{1}{6}\log\left(|x+1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10/(x^6+4*x^3+3),x, algorithm="giac")
```

```
[Out] 1/5*x^5 - 2*x^2 + 3/4*3^(2/3)*log(x^2 - 3^(1/3)*x + 3^(2/3)) - 3/2*3^(2/3)*
log(abs(x + 3^(1/3))) - 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + 9/2*3^(
1/6)*arctan(1/3*3^(1/6)*(2*x - 3^(1/3))) - 1/12*log(x^2 - x + 1) + 1/6*log(
abs(x + 1))
```

maple [A] time = 0.01, size = 94, normalized size = 0.76

$$\frac{x^5}{5}-2x^2+\frac{9\,3^{\frac{1}{6}}\arctan\left(\frac{\sqrt{3}\left(\frac{2\,3^{\frac{2}{3}}x}{3}-1\right)}{3}\right)}{2}-\frac{\sqrt{3}\,\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{6}+\frac{\ln\left(x+1\right)}{6}-\frac{3\,3^{\frac{2}{3}}\ln\left(x+3^{\frac{1}{3}}\right)}{2}+\frac{3\,3^{\frac{2}{3}}\ln\left(x^2-3^{\frac{1}{3}}x+3^{\frac{2}{3}}\right)}{4}-\frac{\ln\left(x^2-x+1\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^10/(x^6+4*x^3+3),x)
```

```
[Out] 1/5*x^5-2*x^2+1/6*ln(x+1)-3/2*3^(2/3)*ln(3^(1/3)+x)+3/4*3^(2/3)*ln(3^(2/3)-
3^(1/3)*x+x^2)+9/2*3^(1/6)*arctan(1/3*3^(1/2)*(2/3*3^(2/3)*x-1))-1/12*ln(x^
2-x+1)-1/6*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))
```

maxima [A] time = 1.23, size = 94, normalized size = 0.76

$$\frac{1}{5}x^5 - 2x^2 + \frac{3}{4}\cdot 3^{\frac{2}{3}}\log\left(x^2 - 3^{\frac{1}{3}}x + 3^{\frac{2}{3}}\right) - \frac{3}{2}\cdot 3^{\frac{2}{3}}\log\left(x + 3^{\frac{1}{3}}\right) - \frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right) + \frac{9}{2}\cdot 3^{\frac{1}{6}}\arctan\left(\frac{1}{3}\cdot 3^{\frac{1}{6}}\left(2x - 3^{\frac{1}{3}}\right)\right) - \frac{1}{12}\log\left(x^2 - x + 1\right) + \frac{1}{6}\log\left(x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10/(x^6+4*x^3+3),x, algorithm="maxima")
```

```
[Out] 1/5*x^5 - 2*x^2 + 3/4*3^(2/3)*log(x^2 - 3^(1/3)*x + 3^(2/3)) - 3/2*3^(2/3)*
log(x + 3^(1/3)) - 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + 9/2*3^(1/6)*
arctan(1/3*3^(1/6)*(2*x - 3^(1/3))) - 1/12*log(x^2 - x + 1) + 1/6*log(x + 1
)
```

mupad [B] time = 0.24, size = 124, normalized size = 1.00

$$\frac{\ln(x+1)}{6} - \frac{3\,3^{2/3}\ln\left(x+3^{1/3}\right)}{2} + \ln\left(x - \frac{1}{2} - \frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(-\frac{1}{12} + \frac{\sqrt{3}\,1\mathrm{i}}{12}\right) - \ln\left(x - \frac{1}{2} + \frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{12} + \frac{\sqrt{3}\,1\mathrm{i}}{12}\right) - 2x^2 + \frac{x^5}{5} - \frac{3(-1)^{1/3}\ln\left(x - \frac{(-1)^{1/3}3^{1/3}}{2} - \frac{(-1)^{1/6}3^{5/6}}{2} + \frac{3^{1/3}}{2}\right)\left(3^{2/3} + 3^{1/6}\,3\mathrm{i}\right)}{4} + \frac{3(-1)^{1/3}3^{2/3}\ln\left(x + (-1)^{2/3}3^{1/3}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^10/(4*x^3 + x^6 + 3),x)
```

```
[Out] log(x + 1)/6 - (3*3^(2/3)*log(x + 3^(1/3)))/2 + log(x - (3^(1/2)*1i)/2 - 1/
2)*((3^(1/2)*1i)/12 - 1/12) - log(x + (3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/1
2 + 1/12) - 2*x^2 + x^5/5 - (3*(-1)^(1/3)*log(x - ((-1)^(1/3)*3^(1/3))/2 -
((-1)^(1/6)*3^(5/6))/2 + 3^(1/3)/2)*(3^(2/3) + 3^(1/6)*3i))/4 + (3*(-1)^(1/
3)*3^(2/3)*log(x + (-1)^(2/3)*3^(1/3)))/2
```

sympy [C] time = 0.62, size = 144, normalized size = 1.16

$$\frac{x^5}{5} - 2x^2 + \frac{\log(x+1)}{6} + \left(-\frac{1}{12} - \frac{\sqrt{3}i}{12}\right)\log\left(x + \frac{3872\left(-\frac{1}{12} - \frac{\sqrt{3}i}{12}\right)^5}{3281} + \frac{3188648\left(-\frac{1}{12} - \frac{\sqrt{3}i}{12}\right)^2}{88587}\right) + \left(-\frac{1}{12} + \frac{\sqrt{3}i}{12}\right)\log\left(x + \frac{3188648\left(-\frac{1}{12} + \frac{\sqrt{3}i}{12}\right)^2}{88587} + \frac{3872\left(-\frac{1}{12} + \frac{\sqrt{3}i}{12}\right)^5}{3281}\right) + \operatorname{RootSum}\left(8t^3 + 243, \left(t \mapsto t\log\left(\frac{3872t^5}{3281} + \frac{3188648t^2}{88587} + x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**10/(x**6+4*x**3+3),x)
```

```
[Out] x**5/5 - 2*x**2 + log(x + 1)/6 + (-1/12 - sqrt(3)*I/12)*log(x + 3872*(-1/12
 - sqrt(3)*I/12)**5/3281 + 3188648*(-1/12 - sqrt(3)*I/12)**2/88587) + (-1/1
2 + sqrt(3)*I/12)*log(x + 3188648*(-1/12 + sqrt(3)*I/12)**2/88587 + 3872*(-
1/12 + sqrt(3)*I/12)**5/3281) + RootSum(8*_t**3 + 243, Lambda(_t, _t*log(38
72*_t**5/3281 + 3188648*_t**2/88587 + x)))
```

3.146 $\int \frac{x^9}{3+4x^3+x^6}\,dx$

Optimal. Leaf size=122

$$\frac{x^4}{4}+\frac{1}{12}\log\left(x^2-x+1\right)-\frac{3}{4}\sqrt[3]{3}\log\left(x^2-\sqrt[3]{3}x+3^{2/3}\right)-4x-\frac{1}{6}\log(x+1)+\frac{3}{2}\sqrt[3]{3}\log\left(x+\sqrt[3]{3}\right)+\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}}-\frac{3}{2}3$$

Rubi [A] time = 0.09, antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 10, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.625, Rules used = {1367, 1502, 1422, 200, 31, 634, 618, 204, 628, 617}

$$\frac{x^4}{4}+\frac{1}{12}\log\left(x^2-x+1\right)-\frac{3}{4}\sqrt[3]{3}\log\left(x^2-\sqrt[3]{3}x+3^{2/3}\right)-4x-\frac{1}{6}\log(x+1)+\frac{3}{2}\sqrt[3]{3}\log\left(x+\sqrt[3]{3}\right)+\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}}-\frac{3}{2}3^{5/6}\tan^{-1}\left(\frac{\sqrt[3]{3}-2x}{3^{5/6}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^9/(3 + 4*x^3 + x^6),x]
```

```
[Out] -4*x + x^4/4 + ArcTan[(1 - 2*x)/Sqrt[3]]/(2*Sqrt[3]) - (3*3^(5/6)*ArcTan[(3
^(1/3) - 2*x)/3^(5/6)])/2 - Log[1 + x]/6 + (3*3^(1/3)*Log[3^(1/3) + x])/2 +
 Log[1 - x + x^2]/12 - (3*3^(1/3)*Log[3^(2/3) - 3^(1/3)*x + x^2])/4
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1367

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Simp[(d^(2*n - 1)*(d*x)^(m - 2*n + 1)*(a + b*x^n + c*x^(2*n))^(
p + 1))/(c*(m + 2*n*p + 1)), x] - Dist[d^(2*n)/(c*(m + 2*n*p + 1)), Int[(d*
x)^(m - 2*n)*Simp[a*(m - 2*n + 1) + b*(m + n*(p - 1) + 1)*x^n, x]*(a + b*x^
n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && EqQ[n2, 2*n] && Ne
Q[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GtQ[m, 2*n - 1] && NeQ[m + 2*n*p + 1, 0]
 && IntegerQ[p]
```

Rule 1422

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q),
Int[1/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(
b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (PosQ[b^2 - 4*a
*c] ||  !IGtQ[n/2, 0])
```

Rule 1502

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (
c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Simp[(e*f^(n - 1)*(f*x)^(m - n + 1)*(a
+ b*x^n + c*x^(2*n))^(p + 1))/(c*(m + n*(2*p + 1) + 1)), x] - Dist[f^n/(c*(
m + n*(2*p + 1) + 1)), Int[(f*x)^(m - n)*(a + b*x^n + c*x^(2*n))^p*Simp[a*e
*(m - n + 1) + (b*e*(m + n*p + 1) - c*d*(m + n*(2*p + 1) + 1))*x^n, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c,
0] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*(2*p + 1) + 1, 0] && Integer
Q[p]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^9}{3+4x^3+x^6}\,dx &= \frac{x^4}{4}-\frac{1}{4}\int \frac{x^3\left(12+16x^3\right)}{3+4x^3+x^6}\,dx\\
&= -4x+\frac{x^4}{4}+\frac{1}{4}\int \frac{48+52x^3}{3+4x^3+x^6}\,dx\\
&= -4x+\frac{x^4}{4}-\frac{1}{2}\int \frac{1}{1+x^3}\,dx+\frac{27}{2}\int \frac{1}{3+x^3}\,dx\\
&= -4x+\frac{x^4}{4}-\frac{1}{6}\int \frac{1}{1+x}\,dx-\frac{1}{6}\int \frac{2-x}{1-x+x^2}\,dx+\frac{1}{2}\left(3\sqrt[3]{3}\right)\int \frac{1}{\sqrt[3]{3}+x}\,dx+\frac{1}{2}\left(3\sqrt[3]{3}\right)\int \frac{}{3^{2/}}\\
&= -4x+\frac{x^4}{4}-\frac{1}{6}\log(1+x)+\frac{3}{2}\sqrt[3]{3}\log\left(\sqrt[3]{3}+x\right)+\frac{1}{12}\int \frac{-1+2x}{1-x+x^2}\,dx-\frac{1}{4}\int \frac{1}{1-x+x^2}\,dx\\
&= -4x+\frac{x^4}{4}-\frac{1}{6}\log(1+x)+\frac{3}{2}\sqrt[3]{3}\log\left(\sqrt[3]{3}+x\right)+\frac{1}{12}\log\left(1-x+x^2\right)-\frac{3}{4}\sqrt[3]{3}\log\left(3^{2/3}-\sqrt[3]{3}\right.\\
&= -4x+\frac{x^4}{4}+\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}}-\frac{3}{2}3^{5/6}\tan^{-1}\left(\frac{\sqrt[3]{3}-2x}{3^{5/6}}\right)-\frac{1}{6}\log(1+x)+\frac{3}{2}\sqrt[3]{3}\log\left(\sqrt[3]{3}+x\right)+
\end{aligned}$$

Mathematica [A] time = 0.03, size = 114, normalized size = 0.93

$$\frac{1}{12}\left(3x^4+\log\left(x^2-x+1\right)-9\sqrt[3]{3}\log\left(\sqrt[3]{3}x^2-3^{2/3}x+3\right)-48x-2\log(x+1)+18\sqrt[3]{3}\log\left(3^{2/3}x+3\right)-18\,3^{5/6}\tan^{-1}\left(\frac{\sqrt[3]{3}-2x}{3^{5/6}}\right)-2\sqrt{3}\tan^{-1}\left(\frac{2x-1}{\sqrt{3}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^9/(3 + 4*x^3 + x^6),x]
```

```
[Out] (-48*x + 3*x^4 - 18*3^(5/6)*ArcTan[(3^(1/3) - 2*x)/3^(5/6)] - 2*Sqrt[3]*Arc
Tan[(-1 + 2*x)/Sqrt[3]] - 2*Log[1 + x] + 18*3^(1/3)*Log[3 + 3^(2/3)*x] + Lo
g[1 - x + x^2] - 9*3^(1/3)*Log[3 - 3^(2/3)*x + 3^(1/3)*x^2])/12
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^9}{3+4x^3+x^6}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^9/(3 + 4*x^3 + x^6),x]
```

```
[Out] IntegrateAlgebraic[x^9/(3 + 4*x^3 + x^6), x]
```

fricas [A] time = 1.12, size = 90, normalized size = 0.74

$$\frac{1}{4}x^4+\frac{3}{2}\cdot 3^{\frac{5}{6}}\arctan\left(\frac{2}{3}\cdot 3^{\frac{1}{6}}x-\frac{1}{3}\sqrt{3}\right)-\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)-\frac{3}{4}\cdot 3^{\frac{1}{3}}\log\left(x^2-3^{\frac{1}{3}}x+3^{\frac{2}{3}}\right)+\frac{3}{2}\cdot 3^{\frac{1}{3}}\log\left(x+3^{\frac{1}{3}}\right)-4x+\frac{1}{12}\log\left(x^2-x+1\right)-\frac{1}{6}\log(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(x^6+4*x^3+3),x, algorithm="fricas")
```

```
[Out] 1/4*x^4 + 3/2*3^(5/6)*arctan(2/3*3^(1/6)*x - 1/3*sqrt(3)) - 1/6*sqrt(3)*arc
tan(1/3*sqrt(3)*(2*x - 1)) - 3/4*3^(1/3)*log(x^2 - 3^(1/3)*x + 3^(2/3)) + 3
/2*3^(1/3)*log(x + 3^(1/3)) - 4*x + 1/12*log(x^2 - x + 1) - 1/6*log(x + 1)
```

giac [A] time = 0.43, size = 94, normalized size = 0.77

$$\frac{1}{4}x^4+\frac{3}{2}\cdot 3^{\frac{5}{6}}\arctan\left(\frac{1}{3}\cdot 3^{\frac{1}{6}}\left(2x-3^{\frac{1}{3}}\right)\right)-\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)-\frac{3}{4}\cdot 3^{\frac{1}{3}}\log\left(x^2-3^{\frac{1}{3}}x+3^{\frac{2}{3}}\right)+\frac{3}{2}\cdot 3^{\frac{1}{3}}\log\left(\left|x+3^{\frac{1}{3}}\right|\right)-4x+\frac{1}{12}\log\left(x^2-x+1\right)-\frac{1}{6}\log(|x+1|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(x^6+4*x^3+3),x, algorithm="giac")
```

```
[Out] 1/4*x^4 + 3/2*3^(5/6)*arctan(1/3*3^(1/6)*(2*x - 3^(1/3))) - 1/6*sqrt(3)*arc
tan(1/3*sqrt(3)*(2*x - 1)) - 3/4*3^(1/3)*log(x^2 - 3^(1/3)*x + 3^(2/3)) + 3
/2*3^(1/3)*log(abs(x + 3^(1/3))) - 4*x + 1/12*log(x^2 - x + 1) - 1/6*log(ab
s(x + 1))
```

maple [A] time = 0.01, size = 92, normalized size = 0.75

$$\frac{x^4}{4}-4x+\frac{3\,3^{\frac{5}{6}}\arctan\left(\frac{\sqrt{3}\left(\frac{2\,3^{\frac{2}{3}}x}{3}-1\right)}{3}\right)}{2}-\frac{\sqrt{3}\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{6}-\frac{\ln(x+1)}{6}+\frac{3\,3^{\frac{1}{3}}\ln\left(x+3^{\frac{1}{3}}\right)}{2}-\frac{3\,3^{\frac{1}{3}}\ln\left(x^2-3^{\frac{1}{3}}x+3^{\frac{2}{3}}\right)}{4}+\frac{\ln\left(x^2-x+1\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9/(x^6+4*x^3+3),x)
```

```
[Out] 1/4*x^4-4*x-1/6*ln(x+1)+3/2*3^(1/3)*ln(x+3^(1/3))-3/4*3^(1/3)*ln(x^2-3^(1/3
)*x+3^(2/3))+3/2*3^(5/6)*arctan(1/3*3^(1/2)*(2/3*3^(2/3)*x-1))+1/12*ln(x^2-
x+1)-1/6*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))
```

maxima [A] time = 1.33, size = 92, normalized size = 0.75

$$\frac{1}{4}x^4 + \frac{3}{2}\cdot 3^{\frac{5}{6}}\arctan\left(\frac{1}{3}\cdot 3^{\frac{1}{6}}\left(2x - 3^{\frac{1}{3}}\right)\right) - \frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right) - \frac{3}{4}\cdot 3^{\frac{1}{3}}\log\left(x^2 - 3^{\frac{1}{3}}x + 3^{\frac{2}{3}}\right) + \frac{3}{2}\cdot 3^{\frac{1}{3}}\log\left(x+3^{\frac{1}{3}}\right) - 4x + \frac{1}{12}\log\left(x^2-x+1\right) - \frac{1}{6}\log(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(x^6+4*x^3+3),x, algorithm="maxima")
```

```
[Out] 1/4*x^4 + 3/2*3^(5/6)*arctan(1/3*3^(1/6)*(2*x - 3^(1/3))) - 1/6*sqrt(3)*arc
tan(1/3*sqrt(3)*(2*x - 1)) - 3/4*3^(1/3)*log(x^2 - 3^(1/3)*x + 3^(2/3)) + 3
/2*3^(1/3)*log(x + 3^(1/3)) - 4*x + 1/12*log(x^2 - x + 1) - 1/6*log(x + 1)
```

mupad [B] time = 1.42, size = 119, normalized size = 0.98

$$\frac{3\,3^{1/3}\ln\left(x+3^{1/3}\right)}{2} - \frac{\ln(x+1)}{6} - 4x + \ln\left(x - \frac{1}{2} - \frac{\sqrt{3}\,1i}{2}\right)\left(\frac{1}{12}+\frac{\sqrt{3}\,1i}{12}\right) - \ln\left(x-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\left(-\frac{1}{12}+\frac{\sqrt{3}\,1i}{12}\right) + \frac{x^4}{4} - \ln\left(x - \frac{3^{1/3}}{2} - \frac{3^{5/6}\,1i}{2}\right)\left(\frac{3\,3^{1/3}}{4}+\frac{3^{5/6}\,3i}{4}\right) + 3^{1/3}\ln\left(x-\frac{3^{1/3}}{2}+\frac{3^{5/6}\,1i}{2}\right)\left(-\frac{3}{4}+\frac{\sqrt{3}\,3i}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9/(4*x^3 + x^6 + 3),x)
```

```
[Out] (3*3^(1/3)*log(x + 3^(1/3)))/2 - log(x + 1)/6 - 4*x + log(x - (3^(1/2)*1i)/
2 - 1/2)*((3^(1/2)*1i)/12 + 1/12) - log(x + (3^(1/2)*1i)/2 - 1/2)*((3^(1/2)
*1i)/12 - 1/12) + x^4/4 - log(x - 3^(1/3)/2 - (3^(5/6)*1i)/2)*((3*3^(1/3))/
4 + (3^(5/6)*3i)/4) + 3^(1/3)*log(x - 3^(1/3)/2 + (3^(5/6)*1i)/2)*((3^(1/2)
*3i)/4 - 3/4)
```

sympy [C] time = 0.61, size = 129, normalized size = 1.06

$$\frac{x^4}{4} - 4x - \frac{\log(x+1)}{6} + \left(\frac{1}{12}+\frac{\sqrt{3}i}{12}\right)\log\left(x - \frac{9841}{19692} - \frac{9841\sqrt{3}i}{19692} + \frac{360\left(\frac{1}{12}+\frac{\sqrt{3}i}{12}\right)^4}{547}\right) + \left(\frac{1}{12}-\frac{\sqrt{3}i}{12}\right)\log\left(x - \frac{9841}{19692} + \frac{360\left(\frac{1}{12}-\frac{\sqrt{3}i}{12}\right)^4}{547} + \frac{9841\sqrt{3}i}{19692}\right) + \operatorname{RootSum}\left(8t^3-81, \left(t\mapsto t\log\left(\frac{360t^4}{547}-\frac{9841t}{1641}+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**9/(x**6+4*x**3+3),x)
```

```
[Out] x**4/4 - 4*x - log(x + 1)/6 + (1/12 + sqrt(3)*I/12)*log(x - 9841/19692 - 98
41*sqrt(3)*I/19692 + 360*(1/12 + sqrt(3)*I/12)**4/547) + (1/12 - sqrt(3)*I/
12)*log(x - 9841/19692 + 360*(1/12 - sqrt(3)*I/12)**4/547 + 9841*sqrt(3)*I/
19692) + RootSum(8*_t**3 - 81, Lambda(_t, _t*log(360*_t**4/547 - 9841*_t/16
41 + x)))
```

3.147 $\int \frac{x^7}{3+4x^3+x^6}\,dx$

Optimal. Leaf size=119

$$\frac{x^2}{2}+\frac{1}{12}\log\left(x^2-x+1\right)-\frac{1}{4}3^{2/3}\log\left(x^2-\sqrt[3]{3}x+3^{2/3}\right)-\frac{1}{6}\log(x+1)+\frac{1}{2}3^{2/3}\log\left(x+\sqrt[3]{3}\right)-\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}}+\frac{3}{2}\sqrt[6]{3}\ \text{t}$$

Rubi [A] time = 0.08, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 9, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.562, Rules used = {1367, 1510, 292, 31, 634, 618, 204, 628, 617}

$$\frac{x^2}{2}+\frac{1}{12}\log\left(x^2-x+1\right)-\frac{1}{4}3^{2/3}\log\left(x^2-\sqrt[3]{3}x+3^{2/3}\right)-\frac{1}{6}\log(x+1)+\frac{1}{2}3^{2/3}\log\left(x+\sqrt[3]{3}\right)-\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}}+\frac{3}{2}\sqrt[6]{3}\tan^{-1}\left(\frac{\sqrt[3]{3}-2x}{3^{5/6}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^7/(3 + 4*x^3 + x^6),x]
```

```
[Out] x^2/2 - ArcTan[(1 - 2*x)/Sqrt[3]]/(2*Sqrt[3]) + (3*3^(1/6)*ArcTan[(3^(1/3)
- 2*x)/3^(5/6)])/2 - Log[1 + x]/6 + (3^(2/3)*Log[3^(1/3) + x])/2 + Log[1 -
x + x^2]/12 - (3^(2/3)*Log[3^(2/3) - 3^(1/3)*x + x^2])/4
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1367

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Simp[(d^(2*n - 1)*(d*x)^(m - 2*n + 1)*(a + b*x^n + c*x^(2*n))^(
p + 1))/(c*(m + 2*n*p + 1)), x] - Dist[d^(2*n)/(c*(m + 2*n*p + 1)), Int[(d*
x)^(m - 2*n)*Simp[a*(m - 2*n + 1) + b*(m + n*(p - 1) + 1)*x^n, x]*(a + b*x^
n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && EqQ[n2, 2*n] && Ne
Q[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GtQ[m, 2*n - 1] && NeQ[m + 2*n*p + 1, 0]
 && IntegerQ[p]
```

Rule 1510

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_)))/((a_) + (b_.)*(x_)^(n_) +
 (c_.)*(x_)^(n2_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 +
(2*c*d - b*e)/(2*q), Int[(f*x)^m/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (
2*c*d - b*e)/(2*q), Int[(f*x)^m/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b
, c, d, e, f, m}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^7}{3+4x^3+x^6}\,dx &= \frac{x^2}{2}-\frac{1}{2}\int \frac{x\left(6+8x^3\right)}{3+4x^3+x^6}\,dx\\
&= \frac{x^2}{2}+\frac{1}{2}\int \frac{x}{1+x^3}\,dx-\frac{9}{2}\int \frac{x}{3+x^3}\,dx\\
&= \frac{x^2}{2}-\frac{1}{6}\int \frac{1}{1+x}\,dx+\frac{1}{6}\int \frac{1+x}{1-x+x^2}\,dx+\frac{1}{2}3^{2/3}\int \frac{1}{\sqrt[3]{3}+x}\,dx-\frac{1}{2}3^{2/3}\int \frac{\sqrt[3]{3}+x}{3^{2/3}-\sqrt[3]{3}x+x^2}\\
&= \frac{x^2}{2}-\frac{1}{6}\log(1+x)+\frac{1}{2}3^{2/3}\log\left(\sqrt[3]{3}+x\right)+\frac{1}{12}\int \frac{-1+2x}{1-x+x^2}\,dx+\frac{1}{4}\int \frac{1}{1-x+x^2}\,dx-\frac{9}{4}\int\\
&= \frac{x^2}{2}-\frac{1}{6}\log(1+x)+\frac{1}{2}3^{2/3}\log\left(\sqrt[3]{3}+x\right)+\frac{1}{12}\log\left(1-x+x^2\right)-\frac{1}{4}3^{2/3}\log\left(3^{2/3}-\sqrt[3]{3}x+x^2\right.\\
&= \frac{x^2}{2}-\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}}+\frac{3}{2}\sqrt[6]{3}\tan^{-1}\left(\frac{\sqrt[3]{3}-2x}{3^{5/6}}\right)-\frac{1}{6}\log(1+x)+\frac{1}{2}3^{2/3}\log\left(\sqrt[3]{3}+x\right)+\frac{1}{12}\log
\end{aligned}
$$

Mathematica [A] time = 0.03, size = 111, normalized size = 0.93

$$\frac{1}{12}\left(6x^2+\log\left(x^2-x+1\right)-3\,3^{2/3}\log\left(\sqrt[3]{3}x^2-3^{2/3}x+3\right)-2\log(x+1)+6\,3^{2/3}\log\left(3^{2/3}x+3\right)+18\sqrt[6]{3}\tan^{-1}\left(\frac{\sqrt[3]{3}-2x}{3^{5/6}}\right)+2\sqrt{3}\tan^{-1}\left(\frac{2x-1}{\sqrt{3}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7/(3 + 4*x^3 + x^6),x]
```

```
[Out] (6*x^2 + 18*3^(1/6)*ArcTan[(3^(1/3) - 2*x)/3^(5/6)] + 2*Sqrt[3]*ArcTan[(-1
+ 2*x)/Sqrt[3]] - 2*Log[1 + x] + 6*3^(2/3)*Log[3 + 3^(2/3)*x] + Log[1 - x +
 x^2] - 3*3^(2/3)*Log[3 - 3^(2/3)*x + 3^(1/3)*x^2])/12
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^7}{3+4x^3+x^6}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^7/(3 + 4*x^3 + x^6),x]
```

```
[Out] IntegrateAlgebraic[x^7/(3 + 4*x^3 + x^6), x]
```

fricas [A] time = 1.18, size = 99, normalized size = 0.83

$$\frac{1}{2}\,x^2-\frac{1}{2}\cdot 9^{\frac{1}{3}}\sqrt{3}\,\arctan\left(\frac{2}{9}\cdot 9^{\frac{1}{3}}\sqrt{3}\,x-\frac{1}{3}\,\sqrt{3}\right)+\frac{1}{6}\,\sqrt{3}\,\arctan\left(\frac{1}{3}\,\sqrt{3}\,(2\,x-1)\right)-\frac{1}{4}\cdot 9^{\frac{1}{3}}\log\left(3\,x^2-9^{\frac{2}{3}}x+3\cdot 9^{\frac{1}{3}}\right)+\frac{1}{2}\cdot 9^{\frac{1}{3}}\log\left(3\,x+9^{\frac{2}{3}}\right)+\frac{1}{12}\,\log\left(x^2-x+1\right)-\frac{1}{6}\,\log\left(x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(x^6+4*x^3+3),x, algorithm="fricas")
```

```
[Out] 1/2*x^2 - 1/2*9^(1/3)*sqrt(3)*arctan(2/9*9^(1/3)*sqrt(3)*x - 1/3*sqrt(3)) +
 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) - 1/4*9^(1/3)*log(3*x^2 - 9^(2/3
)*x + 3*9^(1/3)) + 1/2*9^(1/3)*log(3*x + 9^(2/3)) + 1/12*log(x^2 - x + 1) -
 1/6*log(x + 1)
```

giac [A] time = 0.34, size = 91, normalized size = 0.76

$$\frac{1}{2}\,x^2-\frac{1}{4}\cdot 3^{\frac{2}{3}}\log\left(x^2-3^{\frac{1}{3}}x+3^{\frac{2}{3}}\right)+\frac{1}{2}\cdot 3^{\frac{2}{3}}\log\left(\left|x+3^{\frac{1}{3}}\right|\right)+\frac{1}{6}\,\sqrt{3}\,\arctan\left(\frac{1}{3}\,\sqrt{3}\,(2\,x-1)\right)-\frac{3}{2}\cdot 3^{\frac{1}{6}}\arctan\left(\frac{1}{3}\cdot 3^{\frac{1}{6}}\left(2\,x-3^{\frac{1}{3}}\right)\right)+\frac{1}{12}\,\log\left(x^2-x+1\right)-\frac{1}{6}\,\log\left(|x+1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(x^6+4*x^3+3),x, algorithm="giac")
```

```
[Out] 1/2*x^2 - 1/4*3^(2/3)*log(x^2 - 3^(1/3)*x + 3^(2/3)) + 1/2*3^(2/3)*log(abs(
x + 3^(1/3))) + 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) - 3/2*3^(1/6)*arc
tan(1/3*3^(1/6)*(2*x - 3^(1/3))) + 1/12*log(x^2 - x + 1) - 1/6*log(abs(x +
1))
```

maple [A] time = 0.01, size = 89, normalized size = 0.75

$$\frac{x^2}{2}-\frac{3\,3^{\frac{1}{6}}\arctan\left(\frac{\sqrt{3}\left(\frac{2\,3^{\frac{2}{3}}x}{3}-1\right)}{3}\right)}{2}+\frac{\sqrt{3}\,\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{6}-\frac{\ln\left(x+1\right)}{6}+\frac{3^{\frac{2}{3}}\ln\left(x+3^{\frac{1}{3}}\right)}{2}-\frac{3^{\frac{2}{3}}\ln\left(x^2-3^{\frac{1}{3}}x+3^{\frac{2}{3}}\right)}{4}+\frac{\ln\left(x^2-x+1\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(x^6+4*x^3+3),x)
```

```
[Out] 1/2*x^2-1/6*ln(x+1)+1/2*3^(2/3)*ln(x+3^(1/3))-1/4*3^(2/3)*ln(x^2-3^(1/3)*x+
3^(2/3))-3/2*3^(1/6)*arctan(1/3*3^(1/2)*(2/3*3^(2/3)*x-1))+1/12*ln(x^2-x+1)
+1/6*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))
```

maxima [A] time = 1.31, size = 89, normalized size = 0.75

$$\frac{1}{2}\,x^2-\frac{1}{4}\cdot 3^{\frac{2}{3}}\log\left(x^2-3^{\frac{1}{3}}x+3^{\frac{2}{3}}\right)+\frac{1}{2}\cdot 3^{\frac{2}{3}}\log\left(x+3^{\frac{1}{3}}\right)+\frac{1}{6}\,\sqrt{3}\,\arctan\left(\frac{1}{3}\,\sqrt{3}\,(2\,x-1)\right)-\frac{3}{2}\cdot 3^{\frac{1}{6}}\arctan\left(\frac{1}{3}\cdot 3^{\frac{1}{6}}\left(2\,x-3^{\frac{1}{3}}\right)\right)+\frac{1}{12}\,\log\left(x^2-x+1\right)-\frac{1}{6}\,\log\left(x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(x^6+4*x^3+3),x, algorithm="maxima")
```

```
[Out] 1/2*x^2 - 1/4*3^(2/3)*log(x^2 - 3^(1/3)*x + 3^(2/3)) + 1/2*3^(2/3)*log(x +
3^(1/3)) + 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) - 3/2*3^(1/6)*arctan(1
/3*3^(1/6)*(2*x - 3^(1/3))) + 1/12*log(x^2 - x + 1) - 1/6*log(x + 1)
```

mupad [B] time = 0.19, size = 118, normalized size = 0.99

$$\frac{3^{2/3}\,\ln\left(x+3^{1/3}\right)}{2}-\frac{\ln\left(x+1\right)}{6}-\ln\left(x-\frac{1}{2}-\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,\left(-\frac{1}{12}+\frac{\sqrt{3}\,1{}\mathrm{i}}{12}\right)+\ln\left(x-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,\left(\frac{1}{12}+\frac{\sqrt{3}\,1{}\mathrm{i}}{12}\right)+\frac{x^2}{2}-\ln\left(x-\frac{3^{1/3}}{2}-\frac{3^{5/6}\,1{}\mathrm{i}}{2}\right)\,\left(\frac{3^{2/3}}{4}-\frac{3^{1/6}\,3{}\mathrm{i}}{4}\right)-\ln\left(x-\frac{3^{1/3}}{2}+\frac{3^{5/6}\,1{}\mathrm{i}}{2}\right)\,\left(\frac{3^{2/3}}{4}+\frac{3^{1/6}\,3{}\mathrm{i}}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(4*x^3 + x^6 + 3),x)
```

```
[Out] (3^(2/3)*log(x + 3^(1/3)))/2 - log(x + 1)/6 - log(x - (3^(1/2)*1i)/2 - 1/2)
*((3^(1/2)*1i)/12 - 1/12) + log(x + (3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/12
+ 1/12) + x^2/2 - log(x - 3^(1/3)/2 - (3^(5/6)*1i)/2)*(3^(2/3)/4 - (3^(1/6)
*3i)/4) - log(x - 3^(1/3)/2 + (3^(5/6)*1i)/2)*(3^(2/3)/4 + (3^(1/6)*3i)/4)
```

sympy [C] time = 0.61, size = 134, normalized size = 1.13

$$\frac{x^2}{2}-\frac{\log(x+1)}{6}+\left(\frac{1}{12}-\frac{\sqrt{3}\,i}{12}\right)\log\left(x+\frac{6562\left(\frac{1}{12}-\frac{\sqrt{3}\,i}{12}\right)^2}{183}-\frac{1872\left(\frac{1}{12}-\frac{\sqrt{3}\,i}{12}\right)^5}{61}\right)+\left(\frac{1}{12}+\frac{\sqrt{3}\,i}{12}\right)\log\left(x-\frac{1872\left(\frac{1}{12}+\frac{\sqrt{3}\,i}{12}\right)^5}{61}+\frac{6562\left(\frac{1}{12}+\frac{\sqrt{3}\,i}{12}\right)^2}{183}\right)+\operatorname{RootSum}\left(8t^3-9,\left(t\mapsto t\log\left(-\frac{1872t^5}{61}+\frac{6562t^2}{183}+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7/(x**6+4*x**3+3),x)
```

```
[Out] x**2/2 - log(x + 1)/6 + (1/12 - sqrt(3)*I/12)*log(x + 6562*(1/12 - sqrt(3)*
I/12)**2/183 - 1872*(1/12 - sqrt(3)*I/12)**5/61) + (1/12 + sqrt(3)*I/12)*lo
g(x - 1872*(1/12 + sqrt(3)*I/12)**5/61 + 6562*(1/12 + sqrt(3)*I/12)**2/183)
 + RootSum(8*_t**3 - 9, Lambda(_t, _t*log(-1872*_t**5/61 + 6562*_t**2/183 +
 x)))
```

3.148 $\int \frac{x^6}{3+4x^3+x^6}\,dx$

Optimal. Leaf size=113

$$-\frac{1}{12}\log\left(x^2-x+1\right)+\frac{1}{4}\sqrt[3]{3}\log\left(x^2-\sqrt[3]{3}x+3^{2/3}\right)+x+\frac{1}{6}\log(x+1)-\frac{1}{2}\sqrt[3]{3}\log\left(x+\sqrt[3]{3}\right)-\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}}+\frac{1}{2}3^{5/6}\text{ t}$$

Rubi [A] time = 0.07, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 9, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.562, Rules used = {1367, 1422, 200, 31, 634, 618, 204, 628, 617}

$$-\frac{1}{12}\log\left(x^2-x+1\right)+\frac{1}{4}\sqrt[3]{3}\log\left(x^2-\sqrt[3]{3}x+3^{2/3}\right)+x+\frac{1}{6}\log(x+1)-\frac{1}{2}\sqrt[3]{3}\log\left(x+\sqrt[3]{3}\right)-\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}}+\frac{1}{2}3^{5/6}\tan^{-1}\left(\frac{\sqrt[3]{3}-2x}{3^{5/6}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^6/(3 + 4*x^3 + x^6),x]
```

```
[Out] x - ArcTan[(1 - 2*x)/Sqrt[3]]/(2*Sqrt[3]) + (3^(5/6)*ArcTan[(3^(1/3) - 2*x)
/3^(5/6)])/2 + Log[1 + x]/6 - (3^(1/3)*Log[3^(1/3) + x])/2 - Log[1 - x + x^
2]/12 + (3^(1/3)*Log[3^(2/3) - 3^(1/3)*x + x^2])/4
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1367

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Simp[(d^(2*n - 1)*(d*x)^(m - 2*n + 1)*(a + b*x^n + c*x^(2*n))^(
p + 1))/(c*(m + 2*n*p + 1)), x] - Dist[d^(2*n)/(c*(m + 2*n*p + 1)), Int[(d*
x)^(m - 2*n)*Simp[a*(m - 2*n + 1) + b*(m + n*(p - 1) + 1)*x^n, x]*(a + b*x^
n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && EqQ[n2, 2*n] && Ne
Q[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GtQ[m, 2*n - 1] && NeQ[m + 2*n*p + 1, 0]
 && IntegerQ[p]
```

Rule 1422

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q),
Int[1/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(
b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (PosQ[b^2 - 4*a
*c] ||  !IGtQ[n/2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{x^6}{3+4x^3+x^6}\,dx &= x-\int \frac{3+4x^3}{3+4x^3+x^6}\,dx\\
&= x+\frac{1}{2}\int \frac{1}{1+x^3}\,dx-\frac{9}{2}\int \frac{1}{3+x^3}\,dx\\
&= x+\frac{1}{6}\int \frac{1}{1+x}\,dx+\frac{1}{6}\int \frac{2-x}{1-x+x^2}\,dx-\frac{1}{2}\sqrt[3]{3}\int \frac{1}{\sqrt[3]{3}+x}\,dx-\frac{1}{2}\sqrt[3]{3}\int \frac{2\sqrt[3]{3}-x}{3^{2/3}-\sqrt[3]{3}x+x^2}\,d\\
&= x+\frac{1}{6}\log(1+x)-\frac{1}{2}\sqrt[3]{3}\log\left(\sqrt[3]{3}+x\right)-\frac{1}{12}\int \frac{-1+2x}{1-x+x^2}\,dx+\frac{1}{4}\int \frac{1}{1-x+x^2}\,dx+\frac{1}{4}\sqrt[3]{3}\\
&= x+\frac{1}{6}\log(1+x)-\frac{1}{2}\sqrt[3]{3}\log\left(\sqrt[3]{3}+x\right)-\frac{1}{12}\log\left(1-x+x^2\right)+\frac{1}{4}\sqrt[3]{3}\log\left(3^{2/3}-\sqrt[3]{3}x+x^2\right)\\
&= x-\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}}+\frac{1}{2}3^{5/6}\tan^{-1}\left(\frac{\sqrt[3]{3}-2x}{3^{5/6}}\right)+\frac{1}{6}\log(1+x)-\frac{1}{2}\sqrt[3]{3}\log\left(\sqrt[3]{3}+x\right)-\frac{1}{12}\log(
\end{aligned}$$

Mathematica [A] time = 0.03, size = 111, normalized size = 0.98

$$\frac{1}{12}\left(-\log\left(x^2-x+1\right)+3\sqrt[3]{3}\log\left(\sqrt[3]{3}x^2-3^{2/3}x+3\right)+12x+2\log(x+1)-6\sqrt[3]{3}\log\left(3^{2/3}x+3\right)+6\,3^{5/6}\tan^{-1}\left(\frac{\sqrt[3]{3}-2x}{3^{5/6}}\right)+2\sqrt{3}\tan^{-1}\left(\frac{2x-1}{\sqrt{3}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6/(3 + 4*x^3 + x^6),x]
```

```
[Out] (12*x + 6*3^(5/6)*ArcTan[(3^(1/3) - 2*x)/3^(5/6)] + 2*Sqrt[3]*ArcTan[(-1 +
2*x)/Sqrt[3]] + 2*Log[1 + x] - 6*3^(1/3)*Log[3 + 3^(2/3)*x] - Log[1 - x + x
^2] + 3*3^(1/3)*Log[3 - 3^(2/3)*x + 3^(1/3)*x^2])/12
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6}{3+4x^3+x^6}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^6/(3 + 4*x^3 + x^6),x]
```

```
[Out] IntegrateAlgebraic[x^6/(3 + 4*x^3 + x^6), x]
```

fricas [A] time = 1.51, size = 88, normalized size = 0.78

$$\frac{1}{2}\,\sqrt{3}\,(-3)^{\frac{1}{3}}\arctan\left(\frac{1}{9}\,\sqrt{3}\left(2\,(-3)^{\frac{2}{3}}\,x-3\right)\right)+\frac{1}{6}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}\,(2\,x-1)\right)-\frac{1}{4}\,(-3)^{\frac{1}{3}}\log\left(x^2+(-3)^{\frac{1}{3}}\,x+(-3)^{\frac{2}{3}}\right)+\frac{1}{2}\,(-3)^{\frac{1}{3}}\log\left(x-(-3)^{\frac{1}{3}}\right)+x-\frac{1}{12}\log\left(x^2-x+1\right)+\frac{1}{6}\log\left(x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(x^6+4*x^3+3),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(3)*(-3)^(1/3)*arctan(1/9*sqrt(3)*(2*(-3)^(2/3)*x - 3)) + 1/6*sqrt(
3)*arctan(1/3*sqrt(3)*(2*x - 1)) - 1/4*(-3)^(1/3)*log(x^2 + (-3)^(1/3)*x +
(-3)^(2/3)) + 1/2*(-3)^(1/3)*log(x - (-3)^(1/3)) + x - 1/12*log(x^2 - x + 1
) + 1/6*log(x + 1)
```

giac [A] time = 0.45, size = 87, normalized size = 0.77

$$-\frac{1}{2}\cdot 3^{\frac{5}{6}}\arctan\left(\frac{1}{3}\cdot 3^{\frac{1}{6}}\left(2\,x-3^{\frac{1}{3}}\right)\right)+\frac{1}{6}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}\,(2\,x-1)\right)+\frac{1}{4}\cdot 3^{\frac{1}{3}}\log\left(x^2-3^{\frac{1}{3}}x+3^{\frac{2}{3}}\right)-\frac{1}{2}\cdot 3^{\frac{1}{3}}\log\left(\left|x+3^{\frac{1}{3}}\right|\right)+x-\frac{1}{12}\log\left(x^2-x+1\right)+\frac{1}{6}\log\left(|x+1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(x^6+4*x^3+3),x, algorithm="giac")
```

```
[Out] -1/2*3^(5/6)*arctan(1/3*3^(1/6)*(2*x - 3^(1/3))) + 1/6*sqrt(3)*arctan(1/3*s
qrt(3)*(2*x - 1)) + 1/4*3^(1/3)*log(x^2 - 3^(1/3)*x + 3^(2/3)) - 1/2*3^(1/3
)*log(abs(x + 3^(1/3))) + x - 1/12*log(x^2 - x + 1) + 1/6*log(abs(x + 1))
```

maple [A] time = 0.01, size = 85, normalized size = 0.75

$$x-\frac{3^{\frac{5}{6}}\arctan\left(\frac{\sqrt{3}\left(\frac{2\,3^{\frac{2}{3}}x}{3}-1\right)}{3}\right)}{2}+\frac{\sqrt{3}\,\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{6}+\frac{\ln\left(x+1\right)}{6}-\frac{3^{\frac{1}{3}}\ln\left(x+3^{\frac{1}{3}}\right)}{2}+\frac{3^{\frac{1}{3}}\ln\left(x^2-3^{\frac{1}{3}}x+3^{\frac{2}{3}}\right)}{4}-\frac{\ln\left(x^2-x+1\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(x^6+4*x^3+3),x)
```

```
[Out] x+1/6*ln(x+1)-1/2*3^(1/3)*ln(x+3^(1/3))+1/4*3^(1/3)*ln(x^2-3^(1/3)*x+3^(2/3
))-1/2*3^(5/6)*arctan(1/3*3^(1/2)*(2/3*3^(2/3)*x-1))-1/12*ln(x^2-x+1)+1/6*3
^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))
```

maxima [A] time = 1.23, size = 85, normalized size = 0.75

$$-\frac{1}{2}\cdot 3^{\frac{5}{6}}\arctan\left(\frac{1}{3}\cdot 3^{\frac{1}{6}}\left(2\,x-3^{\frac{1}{3}}\right)\right)+\frac{1}{6}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}\,(2\,x-1)\right)+\frac{1}{4}\cdot 3^{\frac{1}{3}}\log\left(x^2-3^{\frac{1}{3}}x+3^{\frac{2}{3}}\right)-\frac{1}{2}\cdot 3^{\frac{1}{3}}\log\left(x+3^{\frac{1}{3}}\right)+x-\frac{1}{12}\log\left(x^2-x+1\right)+\frac{1}{6}\log\left(x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(x^6+4*x^3+3),x, algorithm="maxima")
```

```
[Out] -1/2*3^(5/6)*arctan(1/3*3^(1/6)*(2*x - 3^(1/3))) + 1/6*sqrt(3)*arctan(1/3*s
qrt(3)*(2*x - 1)) + 1/4*3^(1/3)*log(x^2 - 3^(1/3)*x + 3^(2/3)) - 1/2*3^(1/3
)*log(x + 3^(1/3)) + x - 1/12*log(x^2 - x + 1) + 1/6*log(x + 1)
```

mupad [B] time = 0.16, size = 104, normalized size = 0.92

$$x+\frac{\ln(x+1)}{6}-\frac{3^{1/3}\ln\left(x+3^{1/3}\right)}{2}-\ln\left(x-\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{12}+\frac{\sqrt{3}\,1\mathrm{i}}{12}\right)+\ln\left(x-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(-\frac{1}{12}+\frac{\sqrt{3}\,1\mathrm{i}}{12}\right)+\ln\left(x-\frac{3^{1/3}}{2}+\frac{3^{5/6}\,1\mathrm{i}}{2}\right)\left(\frac{3^{1/3}}{4}-\frac{3^{5/6}\,1\mathrm{i}}{4}\right)+\frac{(-1)^{1/3}\,3^{1/3}\ln\left(x-(-1)^{1/3}\,3^{1/3}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(4*x^3 + x^6 + 3),x)
```

```
[Out] x + log(x + 1)/6 - (3^(1/3)*log(x + 3^(1/3)))/2 - log(x - (3^(1/2)*1i)/2 -
1/2)*((3^(1/2)*1i)/12 + 1/12) + log(x + (3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)
/12 - 1/12) + log(x - 3^(1/3)/2 + (3^(5/6)*1i)/2)*(3^(1/3)/4 - (3^(5/6)*1i)
/4) + ((-1)^(1/3)*3^(1/3)*log(x - (-1)^(1/3)*3^(1/3)))/2
```

sympy [C] time = 0.61, size = 126, normalized size = 1.12

$$x+\frac{\log(x+1)}{6}+\left(-\frac{1}{12}-\frac{\sqrt{3}\,i}{12}\right)\log\left(x-\frac{121}{246}-\frac{121\sqrt{3}\,i}{246}+\frac{864\left(-\frac{1}{12}-\frac{\sqrt{3}\,i}{12}\right)^4}{41}\right)+\left(-\frac{1}{12}+\frac{\sqrt{3}\,i}{12}\right)\log\left(x-\frac{121}{246}+\frac{864\left(-\frac{1}{12}+\frac{\sqrt{3}\,i}{12}\right)^4}{41}+\frac{121\sqrt{3}\,i}{246}\right)+\operatorname{RootSum}\left(8t^3+3,\left(t\mapsto t\log\left(\frac{864t^4}{41}+\frac{242t}{41}+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(x**6+4*x**3+3),x)
```

```
[Out] x + log(x + 1)/6 + (-1/12 - sqrt(3)*I/12)*log(x - 121/246 - 121*sqrt(3)*I/2
46 + 864*(-1/12 - sqrt(3)*I/12)**4/41) + (-1/12 + sqrt(3)*I/12)*log(x - 121
/246 + 864*(-1/12 + sqrt(3)*I/12)**4/41 + 121*sqrt(3)*I/246) + RootSum(8*_t
**3 + 3, Lambda(_t, _t*log(864*_t**4/41 + 242*_t/41 + x)))
```

3.149 $\int \frac{x^4}{3+4x^3+x^6}\,dx$

Optimal. Leaf size=112

$$-\frac{1}{12}\log\left(x^2-x+1\right)+\frac{\log\left(x^2-\sqrt[3]{3}x+3^{2/3}\right)}{4\sqrt[3]{3}}+\frac{1}{6}\log(x+1)-\frac{\log\left(x+\sqrt[3]{3}\right)}{2\sqrt[3]{3}}+\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}}-\frac{1}{2}\sqrt[6]{3}\tan^{-1}\left(\frac{\sqrt[3]{3}-2x}{3^{5/6}}\right)$$

Rubi [A] time = 0.07, antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 8, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {1374, 292, 31, 634, 617, 204, 628, 618}

$$-\frac{1}{12}\log\left(x^2-x+1\right)+\frac{\log\left(x^2-\sqrt[3]{3}x+3^{2/3}\right)}{4\sqrt[3]{3}}+\frac{1}{6}\log(x+1)-\frac{\log\left(x+\sqrt[3]{3}\right)}{2\sqrt[3]{3}}+\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}}-\frac{1}{2}\sqrt[6]{3}\tan^{-1}\left(\frac{\sqrt[3]{3}-2x}{3^{5/6}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(3 + 4*x^3 + x^6),x]
```

```
[Out] ArcTan[(1 - 2*x)/Sqrt[3]]/(2*Sqrt[3]) - (3^(1/6)*ArcTan[(3^(1/3) - 2*x)/3^(
5/6)])/2 + Log[1 + x]/6 - Log[3^(1/3) + x]/(2*3^(1/3)) - Log[1 - x + x^2]/1
2 + Log[3^(2/3) - 3^(1/3)*x + x^2]/(4*3^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1374

```
Int[((d_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(d^n*(b/q + 1))/2, Int[(d*x)^(m -
 n)/(b/2 + q/2 + c*x^n), x], x] - Dist[(d^n*(b/q - 1))/2, Int[(d*x)^(m - n)
/(b/2 - q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[n2, 2*n] &&
NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GeQ[m, n]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^4}{3+4x^3+x^6}\,dx &= -\left(\frac{1}{2}\int \frac{x}{1+x^3}\,dx\right)+\frac{3}{2}\int \frac{x}{3+x^3}\,dx\\
&= \frac{1}{6}\int \frac{1}{1+x}\,dx-\frac{1}{6}\int \frac{1+x}{1-x+x^2}\,dx-\frac{\int \frac{1}{\sqrt[3]{3}+x}\,dx}{2\sqrt[3]{3}}+\frac{\int \frac{\sqrt[3]{3}+x}{3^{2/3}-\sqrt[3]{3}x+x^2}\,dx}{2\sqrt[3]{3}}\\
&= \frac{1}{6}\log(1+x)-\frac{\log\left(\sqrt[3]{3}+x\right)}{2\sqrt[3]{3}}-\frac{1}{12}\int \frac{-1+2x}{1-x+x^2}\,dx-\frac{1}{4}\int \frac{1}{1-x+x^2}\,dx+\frac{3}{4}\int \frac{1}{3^{2/3}-\sqrt[3]{3}
\\
&= \frac{1}{6}\log(1+x)-\frac{\log\left(\sqrt[3]{3}+x\right)}{2\sqrt[3]{3}}-\frac{1}{12}\log\left(1-x+x^2\right)+\frac{\log\left(3^{2/3}-\sqrt[3]{3}x+x^2\right)}{4\sqrt[3]{3}}+\frac{1}{2}\text{Subst}\Bigg(\int
\\
&= \frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}}-\frac{1}{2}\sqrt[6]{3}\tan^{-1}\left(\frac{\sqrt[3]{3}-2x}{3^{5/6}}\right)+\frac{1}{6}\log(1+x)-\frac{\log\left(\sqrt[3]{3}+x\right)}{2\sqrt[3]{3}}-\frac{1}{12}\log\left(1-x+x^2
\end{aligned}
$$

Mathematica [A] time = 0.02, size = 107, normalized size = 0.96

$$\frac{1}{12}\left(-\log\left(x^2-x+1\right)+3^{2/3}\log\left(\sqrt[3]{3}x^2-3^{2/3}x+3\right)+2\log(x+1)-2\,3^{2/3}\log\left(3^{2/3}x+3\right)-6\sqrt[6]{3}\tan^{-1}\left(\frac{\sqrt[3]{3}-2x}{3^{5/6}}\right)-2\sqrt{3}\tan^{-1}\left(\frac{2x-1}{\sqrt{3}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(3 + 4*x^3 + x^6),x]
```

```
[Out] (-6*3^(1/6)*ArcTan[(3^(1/3) - 2*x)/3^(5/6)] - 2*Sqrt[3]*ArcTan[(-1 + 2*x)/S
qrt[3]] + 2*Log[1 + x] - 2*3^(2/3)*Log[3 + 3^(2/3)*x] - Log[1 - x + x^2] +
3^(2/3)*Log[3 - 3^(2/3)*x + 3^(1/3)*x^2])/12
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{3+4x^3+x^6}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^4/(3 + 4*x^3 + x^6),x]
```

```
[Out] IntegrateAlgebraic[x^4/(3 + 4*x^3 + x^6), x]
```

fricas [A] time = 1.47, size = 106, normalized size = 0.95

$$-\frac{1}{12}\cdot 3^{\frac{2}{3}}\left(-1\right)^{\frac{1}{3}}\log\left(-3^{\frac{1}{3}}\left(-1\right)^{\frac{2}{3}}x+x^2-3^{\frac{2}{3}}\left(-1\right)^{\frac{1}{3}}\right)+\frac{1}{6}\cdot 3^{\frac{2}{3}}\left(-1\right)^{\frac{1}{3}}\log\left(3^{\frac{1}{3}}\left(-1\right)^{\frac{2}{3}}+x\right)-\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x-1\right)\right)+\frac{1}{2}\cdot 3^{\frac{1}{6}}\left(-1\right)^{\frac{1}{3}}\arctan\left(\frac{1}{3}\cdot 3^{\frac{1}{6}}\left(2\left(-1\right)^{\frac{1}{3}}x+3^{\frac{1}{3}}\right)\right)-\frac{1}{12}\log\left(x^2-x+1\right)+\frac{1}{6}\log\left(x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(x^6+4*x^3+3),x, algorithm="fricas")
```

```
[Out] -1/12*3^(2/3)*(-1)^(1/3)*log(-3^(1/3)*(-1)^(2/3)*x + x^2 - 3^(2/3)*(-1)^(1/
3)) + 1/6*3^(2/3)*(-1)^(1/3)*log(3^(1/3)*(-1)^(2/3) + x) - 1/6*sqrt(3)*arct
an(1/3*sqrt(3)*(2*x - 1)) + 1/2*3^(1/6)*(-1)^(1/3)*arctan(1/3*3^(1/6)*(2*(-
1)^(1/3)*x + 3^(1/3))) - 1/12*log(x^2 - x + 1) + 1/6*log(x + 1)
```

giac [A] time = 0.30, size = 86, normalized size = 0.77

$$\frac{1}{12}\cdot 3^{\frac{2}{3}}\log\left(x^2 - 3^{\frac{1}{3}}x + 3^{\frac{2}{3}}\right) - \frac{1}{6}\cdot 3^{\frac{2}{3}}\log\left(\left|x+3^{\frac{1}{3}}\right|\right) - \frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right) + \frac{1}{2}\cdot 3^{\frac{1}{6}}\arctan\left(\frac{1}{3}\cdot 3^{\frac{1}{6}}\left(2x-3^{\frac{1}{3}}\right)\right) - \frac{1}{12}\log\left(x^2-x+1\right)+\frac{1}{6}\log\left(|x+1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(x^6+4*x^3+3),x, algorithm="giac")
```

```
[Out] 1/12*3^(2/3)*log(x^2 - 3^(1/3)*x + 3^(2/3)) - 1/6*3^(2/3)*log(abs(x + 3^(1/
3))) - 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + 1/2*3^(1/6)*arctan(1/3*3
^(1/6)*(2*x - 3^(1/3))) - 1/12*log(x^2 - x + 1) + 1/6*log(abs(x + 1))
```

maple [A] time = 0.01, size = 84, normalized size = 0.75

$$\frac{3^{\frac{1}{6}}\arctan\left(\frac{\sqrt{3}\left(\frac{2\,3^{\frac{2}{3}}x}{3}-1\right)}{3}\right)}{2} - \frac{\sqrt{3}\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{6} + \frac{\ln(x+1)}{6} - \frac{3^{\frac{2}{3}}\ln\left(x+3^{\frac{1}{3}}\right)}{6} + \frac{3^{\frac{2}{3}}\ln\left(x^2-3^{\frac{1}{3}}x+3^{\frac{2}{3}}\right)}{12} - \frac{\ln\left(x^2-x+1\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(x^6+4*x^3+3),x)
```

```
[Out] 1/6*ln(x+1)-1/6*3^(2/3)*ln(x+3^(1/3))+1/12*3^(2/3)*ln(x^2-3^(1/3)*x+3^(2/3)
)+1/2*3^(1/6)*arctan(1/3*3^(1/2)*(2/3*3^(2/3)*x-1))-1/12*ln(x^2-x+1)-1/6*3^
(1/2)*arctan(1/3*(2*x-1)*3^(1/2))
```

maxima [A] time = 1.61, size = 84, normalized size = 0.75

$$\frac{1}{12}\cdot 3^{\frac{2}{3}}\log\left(x^2 - 3^{\frac{1}{3}}x + 3^{\frac{2}{3}}\right) - \frac{1}{6}\cdot 3^{\frac{2}{3}}\log\left(x+3^{\frac{1}{3}}\right) - \frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right) + \frac{1}{2}\cdot 3^{\frac{1}{6}}\arctan\left(\frac{1}{3}\cdot 3^{\frac{1}{6}}\left(2x-3^{\frac{1}{3}}\right)\right) - \frac{1}{12}\log\left(x^2-x+1\right)+\frac{1}{6}\log\left(x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(x^6+4*x^3+3),x, algorithm="maxima")
```

```
[Out] 1/12*3^(2/3)*log(x^2 - 3^(1/3)*x + 3^(2/3)) - 1/6*3^(2/3)*log(x + 3^(1/3))
- 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + 1/2*3^(1/6)*arctan(1/3*3^(1/6
)*(2*x - 3^(1/3))) - 1/12*log(x^2 - x + 1) + 1/6*log(x + 1)
```

mupad [B] time = 1.37, size = 114, normalized size = 1.02

$$\frac{\ln(x+1)}{6} - \frac{3^{2/3}\ln\left(x+3^{1/3}\right)}{6} + \ln\left(x-\frac{1}{2}-\frac{\sqrt{3}\,1i}{2}\right)\left(-\frac{1}{12}+\frac{\sqrt{3}\,1i}{12}\right) - \ln\left(x-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\left(\frac{1}{12}+\frac{\sqrt{3}\,1i}{12}\right) - \frac{(-1)^{1/3}\ln\left(x-\frac{(-1)^{1/3}3^{1/3}}{2}-\frac{(-1)^{1/6}3^{5/6}}{2}+\frac{3^{1/3}}{2}\right)\left(3^{2/3}+3^{1/6}\,3i\right)}{12} + \frac{(-1)^{1/3}3^{2/3}\ln\left(x+(-1)^{2/3}3^{1/3}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(4*x^3 + x^6 + 3),x)
```

```
[Out] log(x + 1)/6 - (3^(2/3)*log(x + 3^(1/3)))/6 + log(x - (3^(1/2)*1i)/2 - 1/2)
*((3^(1/2)*1i)/12 - 1/12) - log(x + (3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/12
+ 1/12) - ((-1)^(1/3)*log(x - ((-1)^(1/3)*3^(1/3))/2 - ((-1)^(1/6)*3^(5/6))
/2 + 3^(1/3)/2)*(3^(2/3) + 3^(1/6)*3i))/12 + ((-1)^(1/3)*3^(2/3)*log(x + (-
1)^(2/3)*3^(1/3)))/6
```

sympy [C] time = 0.60, size = 134, normalized size = 1.20

$$\frac{\log(x+1)}{6}+\left(-\frac{1}{12}-\frac{\sqrt{3}\,i}{12}\right)\log\left(x+\frac{2592\left(-\frac{1}{12}-\frac{\sqrt{3}\,i}{12}\right)^{5}}{5}+\frac{168\left(-\frac{1}{12}-\frac{\sqrt{3}\,i}{12}\right)^{2}}{5}\right)+\left(-\frac{1}{12}+\frac{\sqrt{3}\,i}{12}\right)\log\left(x+\frac{168\left(-\frac{1}{12}+\frac{\sqrt{3}\,i}{12}\right)^{2}}{5}+\frac{2592\left(-\frac{1}{12}+\frac{\sqrt{3}\,i}{12}\right)^{5}}{5}\right)+\operatorname{RootSum}\left(24t^{3}+1,\left(t\mapsto t\log\left(\frac{2592t^{5}}{5}+\frac{168t^{2}}{5}+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(x**6+4*x**3+3),x)
```

```
[Out] log(x + 1)/6 + (-1/12 - sqrt(3)*I/12)*log(x + 2592*(-1/12 - sqrt(3)*I/12)**
5/5 + 168*(-1/12 - sqrt(3)*I/12)**2/5) + (-1/12 + sqrt(3)*I/12)*log(x + 168
*(-1/12 + sqrt(3)*I/12)**2/5 + 2592*(-1/12 + sqrt(3)*I/12)**5/5) + RootSum(
24*_t**3 + 1, Lambda(_t, _t*log(2592*_t**5/5 + 168*_t**2/5 + x)))
```

3.150 $\int \frac{x^3}{3+4x^3+x^6}\,dx$

Optimal. Leaf size=112

$$\frac{1}{12}\log\left(x^2-x+1\right)-\frac{\log\left(x^2-\sqrt[3]{3}x+3^{2/3}\right)}{4\ 3^{2/3}}-\frac{1}{6}\log(x+1)+\frac{\log\left(x+\sqrt[3]{3}\right)}{2\ 3^{2/3}}+\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}}-\frac{\tan^{-1}\left(\frac{\sqrt[3]{3}-2x}{3^{5/6}}\right)}{2\sqrt[6]{3}}$$

Rubi [A] time = 0.07, antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 8, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {1374, 200, 31, 634, 617, 204, 628, 618}

$$\frac{1}{12}\log\left(x^2-x+1\right)-\frac{\log\left(x^2-\sqrt[3]{3}x+3^{2/3}\right)}{4\ 3^{2/3}}-\frac{1}{6}\log(x+1)+\frac{\log\left(x+\sqrt[3]{3}\right)}{2\ 3^{2/3}}+\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}}-\frac{\tan^{-1}\left(\frac{\sqrt[3]{3}-2x}{3^{5/6}}\right)}{2\sqrt[6]{3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(3 + 4*x^3 + x^6),x]
```

```
[Out] ArcTan[(1 - 2*x)/Sqrt[3]]/(2*Sqrt[3]) - ArcTan[(3^(1/3) - 2*x)/3^(5/6)]/(2*
3^(1/6)) - Log[1 + x]/6 + Log[3^(1/3) + x]/(2*3^(2/3)) + Log[1 - x + x^2]/1
2 - Log[3^(2/3) - 3^(1/3)*x + x^2]/(4*3^(2/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1374

```
Int[((d_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(d^n*(b/q + 1))/2, Int[(d*x)^(m -
 n)/(b/2 + q/2 + c*x^n), x], x] - Dist[(d^n*(b/q - 1))/2, Int[(d*x)^(m - n)
/(b/2 - q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[n2, 2*n] &&
NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GeQ[m, n]
```

Rubi steps

$$\int \frac{x^3}{3+4x^3+x^6}\,dx = -\left(\frac{1}{2}\int \frac{1}{1+x^3}\,dx\right)+\frac{3}{2}\int \frac{1}{3+x^3}\,dx$$

$$= -\left(\frac{1}{6}\int \frac{1}{1+x}\,dx\right)-\frac{1}{6}\int \frac{2-x}{1-x+x^2}\,dx+\frac{\int \frac{1}{\sqrt[3]{3}+x}\,dx}{2\, 3^{2/3}}+\frac{\int \frac{2\sqrt[3]{3}-x}{3^{2/3}-\sqrt[3]{3}x+x^2}\,dx}{2\, 3^{2/3}}$$

$$= -\frac{1}{6}\log(1+x)+\frac{\log\left(\sqrt[3]{3}+x\right)}{2\, 3^{2/3}}+\frac{1}{12}\int \frac{-1+2x}{1-x+x^2}\,dx-\frac{1}{4}\int \frac{1}{1-x+x^2}\,dx-\frac{\int \frac{-\sqrt[3]{3}+2x}{3^{2/3}-\sqrt[3]{3}x+x^2}}{4\, 3^{2/3}}$$

$$= -\frac{1}{6}\log(1+x)+\frac{\log\left(\sqrt[3]{3}+x\right)}{2\, 3^{2/3}}+\frac{1}{12}\log\left(1-x+x^2\right)-\frac{\log\left(3^{2/3}-\sqrt[3]{3}x+x^2\right)}{4\, 3^{2/3}}+\frac{1}{2}\text{Subst}\Big($$

$$= \frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}}-\frac{\tan^{-1}\left(\frac{\sqrt[3]{3}-2x}{3^{5/6}}\right)}{2\sqrt[6]{3}}-\frac{1}{6}\log(1+x)+\frac{\log\left(\sqrt[3]{3}+x\right)}{2\, 3^{2/3}}+\frac{1}{12}\log\left(1-x+x^2\right)-\frac{\log}{}$$

Mathematica [A] time = 0.02, size = 106, normalized size = 0.95

$$\frac{1}{12}\left(\log\left(x^2-x+1\right)-\sqrt[3]{3}\log\left(\sqrt[3]{3}x^2-3^{2/3}x+3\right)-2\log(x+1)+2\sqrt[3]{3}\log\left(3^{2/3}x+3\right)-2\, 3^{5/6}\tan^{-1}\left(\frac{\sqrt[3]{3}-2x}{3^{5/6}}\right)-2\sqrt{3}\tan^{-1}\left(\frac{2x-1}{\sqrt{3}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(3 + 4*x^3 + x^6),x]
```

```
[Out] (-2*3^(5/6)*ArcTan[(3^(1/3) - 2*x)/3^(5/6)] - 2*Sqrt[3]*ArcTan[(-1 + 2*x)/S
qrt[3]] - 2*Log[1 + x] + 2*3^(1/3)*Log[3 + 3^(2/3)*x] + Log[1 - x + x^2] -
3^(1/3)*Log[3 - 3^(2/3)*x + 3^(1/3)*x^2])/12
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{3+4x^3+x^6}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^3/(3 + 4*x^3 + x^6),x]
```

```
[Out] IntegrateAlgebraic[x^3/(3 + 4*x^3 + x^6), x]
```

fricas [A] time = 1.31, size = 102, normalized size = 0.91

$$\frac{1}{6} \cdot 9^{\frac{1}{6}} \sqrt{3} \arctan\left(\frac{1}{27} \cdot 9^{\frac{1}{6}} \left(2 \cdot 9^{\frac{2}{3}} \sqrt{3} x - 3 \cdot 9^{\frac{1}{3}} \sqrt{3}\right)\right) - \frac{1}{36} \cdot 9^{\frac{2}{3}} \log\left(3 x^{2} - 9^{\frac{2}{3}} x + 3 \cdot 9^{\frac{1}{3}}\right) + \frac{1}{18} \cdot 9^{\frac{2}{3}} \log\left(3 x + 9^{\frac{2}{3}}\right) - \frac{1}{6} \sqrt{3} \arctan\left(\frac{1}{3} \sqrt{3} (2 x - 1)\right) + \frac{1}{12} \log\left(x^{2} - x + 1\right) - \frac{1}{6} \log(x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(x^6+4*x^3+3),x, algorithm="fricas")
```

```
[Out] 1/6*9^(1/6)*sqrt(3)*arctan(1/27*9^(1/6)*(2*9^(2/3)*sqrt(3)*x - 3*9^(1/3)*sq
rt(3))) - 1/36*9^(2/3)*log(3*x^2 - 9^(2/3)*x + 3*9^(1/3)) + 1/18*9^(2/3)*lo
g(3*x + 9^(2/3)) - 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + 1/12*log(x^2
 - x + 1) - 1/6*log(x + 1)
```

giac [A] time = 0.34, size = 86, normalized size = 0.77

$$\frac{1}{6} \cdot 3^{\frac{5}{6}} \arctan\left(\frac{1}{3} \cdot 3^{\frac{1}{6}} \left(2 x - 3^{\frac{1}{3}}\right)\right) - \frac{1}{6} \sqrt{3} \arctan\left(\frac{1}{3} \sqrt{3} (2 x - 1)\right) - \frac{1}{12} \cdot 3^{\frac{1}{3}} \log\left(x^{2} - 3^{\frac{1}{3}} x + 3^{\frac{2}{3}}\right) + \frac{1}{6} \cdot 3^{\frac{1}{3}} \log\left(\left|x + 3^{\frac{1}{3}}\right|\right) + \frac{1}{12} \log\left(x^{2} - x + 1\right) - \frac{1}{6} \log(|x + 1|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(x^6+4*x^3+3),x, algorithm="giac")
```

```
[Out] 1/6*3^(5/6)*arctan(1/3*3^(1/6)*(2*x - 3^(1/3))) - 1/6*sqrt(3)*arctan(1/3*sq
rt(3)*(2*x - 1)) - 1/12*3^(1/3)*log(x^2 - 3^(1/3)*x + 3^(2/3)) + 1/6*3^(1/3
)*log(abs(x + 3^(1/3))) + 1/12*log(x^2 - x + 1) - 1/6*log(abs(x + 1))
```

maple [A] time = 0.01, size = 84, normalized size = 0.75

$$\frac{3^{\frac{5}{6}} \arctan\left(\frac{\sqrt{3}\left(\frac{2 3^{\frac{2}{3}} x}{3}-1\right)}{3}\right)}{6} - \frac{\sqrt{3}\, \arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{6} - \frac{\ln(x+1)}{6} + \frac{3^{\frac{1}{3}} \ln\left(x + 3^{\frac{1}{3}}\right)}{6} - \frac{3^{\frac{1}{3}} \ln\left(x^{2} - 3^{\frac{1}{3}} x + 3^{\frac{2}{3}}\right)}{12} + \frac{\ln\left(x^{2} - x + 1\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(x^6+4*x^3+3),x)
```

```
[Out] -1/6*ln(x+1)+1/6*3^(1/3)*ln(x+3^(1/3))-1/12*3^(1/3)*ln(x^2-3^(1/3)*x+3^(2/3
))+1/6*3^(5/6)*arctan(1/3*3^(1/2)*(2/3*3^(2/3)*x-1))+1/12*ln(x^2-x+1)-1/6*3
^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))
```

maxima [A] time = 1.71, size = 84, normalized size = 0.75

$$\frac{1}{6} \cdot 3^{\frac{5}{6}} \arctan\left(\frac{1}{3} \cdot 3^{\frac{1}{6}} \left(2 x - 3^{\frac{1}{3}}\right)\right) - \frac{1}{6} \sqrt{3} \arctan\left(\frac{1}{3} \sqrt{3} (2 x - 1)\right) - \frac{1}{12} \cdot 3^{\frac{1}{3}} \log\left(x^{2} - 3^{\frac{1}{3}} x + 3^{\frac{2}{3}}\right) + \frac{1}{6} \cdot 3^{\frac{1}{3}} \log\left(x + 3^{\frac{1}{3}}\right) + \frac{1}{12} \log\left(x^{2} - x + 1\right) - \frac{1}{6} \log(x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(x^6+4*x^3+3),x, algorithm="maxima")
```

```
[Out] 1/6*3^(5/6)*arctan(1/3*3^(1/6)*(2*x - 3^(1/3))) - 1/6*sqrt(3)*arctan(1/3*sq
rt(3)*(2*x - 1)) - 1/12*3^(1/3)*log(x^2 - 3^(1/3)*x + 3^(2/3)) + 1/6*3^(1/3
)*log(x + 3^(1/3)) + 1/12*log(x^2 - x + 1) - 1/6*log(x + 1)
```

mupad [B] time = 1.36, size = 113, normalized size = 1.01

$$\frac{3^{1/3} \ln\left(x + 3^{1/3}\right)}{6} - \frac{\ln(x+1)}{6} + \ln\left(x - \frac{1}{2} - \frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{12} + \frac{\sqrt{3}\,1\mathrm{i}}{12}\right) - \ln\left(x - \frac{1}{2} + \frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(-\frac{1}{12} + \frac{\sqrt{3}\,1\mathrm{i}}{12}\right) - \ln\left(x - \frac{3^{1/3}}{2} - \frac{3^{5/6}\,1\mathrm{i}}{2}\right)\left(\frac{3^{1/3}}{12} + \frac{3^{5/6}\,1\mathrm{i}}{12}\right) - \ln\left(x - \frac{3^{1/3}}{2} + \frac{3^{5/6}\,1\mathrm{i}}{2}\right)\left(\frac{3^{1/3}}{12} - \frac{3^{5/6}\,1\mathrm{i}}{12}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(4*x^3 + x^6 + 3),x)
```

```
[Out] (3^(1/3)*log(x + 3^(1/3)))/6 - log(x + 1)/6 + log(x - (3^(1/2)*1i)/2 - 1/2)
*((3^(1/2)*1i)/12 + 1/12) - log(x + (3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/12
```

```
- 1/12) - log(x - 3^(1/3)/2 - (3^(5/6)*1i)/2)*(3^(1/3)/12 + (3^(5/6)*1i)/12
) - log(x - 3^(1/3)/2 + (3^(5/6)*1i)/2)*(3^(1/3)/12 - (3^(5/6)*1i)/12)
```

sympy [C] time = 0.59, size = 110, normalized size = 0.98

$$-\frac{\log(x+1)}{6}+\left(\frac{1}{12}-\frac{\sqrt{3}\,i}{12}\right)\log\left(x-\frac{1}{4}+648\left(\frac{1}{12}-\frac{\sqrt{3}\,i}{12}\right)^4+\frac{\sqrt{3}\,i}{4}\right)+\left(\frac{1}{12}+\frac{\sqrt{3}\,i}{12}\right)\log\left(x-\frac{1}{4}+648\left(\frac{1}{12}+\frac{\sqrt{3}\,i}{12}\right)^4-\frac{\sqrt{3}\,i}{4}\right)+\operatorname{RootSum}\left(72t^3-1,\left(t\mapsto t\log\left(648t^4-3t+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(x**6+4*x**3+3),x)
```

```
[Out] -log(x + 1)/6 + (1/12 - sqrt(3)*I/12)*log(x - 1/4 + 648*(1/12 - sqrt(3)*I/1
2)**4 + sqrt(3)*I/4) + (1/12 + sqrt(3)*I/12)*log(x - 1/4 + 648*(1/12 + sqrt
(3)*I/12)**4 - sqrt(3)*I/4) + RootSum(72*_t**3 - 1, Lambda(_t, _t*log(648*_
t**4 - 3*_t + x)))
```

3.151 $\int \frac{x}{3+4x^3+x^6} \, dx$

Optimal. Leaf size=112

$$\frac{1}{12}\log\left(x^2-x+1\right)-\frac{\log\left(x^2-\sqrt[3]{3}x+3^{2/3}\right)}{12\sqrt[3]{3}}-\frac{1}{6}\log(x+1)+\frac{\log\left(x+\sqrt[3]{3}\right)}{6\sqrt[3]{3}}-\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}}+\frac{\tan^{-1}\left(\frac{\sqrt[3]{3}-2x}{3^{5/6}}\right)}{2\ 3^{5/6}}$$

Rubi [A] time = 0.07, antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 8, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {1375, 292, 31, 634, 618, 204, 628, 617}

$$\frac{1}{12}\log\left(x^2-x+1\right)-\frac{\log\left(x^2-\sqrt[3]{3}x+3^{2/3}\right)}{12\sqrt[3]{3}}-\frac{1}{6}\log(x+1)+\frac{\log\left(x+\sqrt[3]{3}\right)}{6\sqrt[3]{3}}-\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}}+\frac{\tan^{-1}\left(\frac{\sqrt[3]{3}-2x}{3^{5/6}}\right)}{2\ 3^{5/6}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(3 + 4*x^3 + x^6),x]
```

```
[Out] -ArcTan[(1 - 2*x)/Sqrt[3]]/(2*Sqrt[3]) + ArcTan[(3^(1/3) - 2*x)/3^(5/6)]/(2
*3^(5/6)) - Log[1 + x]/6 + Log[3^(1/3) + x]/(6*3^(1/3)) + Log[1 - x + x^2]/
12 - Log[3^(2/3) - 3^(1/3)*x + x^2]/(12*3^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1375

```
Int[((d_.)*(x_))^(m_.)/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symb
ol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[c/q, Int[(d*x)^m/(b/2 - q/2 + c*
x^n), x], x] - Dist[c/q, Int[(d*x)^m/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[
{a, b, c, d, m}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{x}{3+4x^3+x^6}\,dx = \frac{1}{2}\int \frac{x}{1+x^3}\,dx - \frac{1}{2}\int \frac{x}{3+x^3}\,dx$$

$$= -\left(\frac{1}{6}\int \frac{1}{1+x}\,dx\right) + \frac{1}{6}\int \frac{1+x}{1-x+x^2}\,dx + \frac{\int \frac{1}{\sqrt[3]{3}+x}\,dx}{6\sqrt[3]{3}} - \frac{\int \frac{\sqrt[3]{3}+x}{3^{2/3}-\sqrt[3]{3}x+x^2}\,dx}{6\sqrt[3]{3}}$$

$$= -\frac{1}{6}\log(1+x) + \frac{\log\left(\sqrt[3]{3}+x\right)}{6\sqrt[3]{3}} + \frac{1}{12}\int \frac{-1+2x}{1-x+x^2}\,dx + \frac{1}{4}\int \frac{1}{1-x+x^2}\,dx - \frac{1}{4}\int \frac{1}{3^{2/3}-\sqrt[3]{}}$$

$$= -\frac{1}{6}\log(1+x) + \frac{\log\left(\sqrt[3]{3}+x\right)}{6\sqrt[3]{3}} + \frac{1}{12}\log\left(1-x+x^2\right) - \frac{\log\left(3^{2/3}-\sqrt[3]{3}x+x^2\right)}{12\sqrt[3]{3}} - \frac{1}{2}\text{Subst}\Big($$

$$= -\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}} + \frac{\tan^{-1}\left(\frac{\sqrt[3]{3}-2x}{3^{5/6}}\right)}{2\ 3^{5/6}} - \frac{1}{6}\log(1+x) + \frac{\log\left(\sqrt[3]{3}+x\right)}{6\sqrt[3]{3}} + \frac{1}{12}\log\left(1-x+x^2\right) - \frac{\text{lo}}{}$$

Mathematica [A] time = 0.03, size = 108, normalized size = 0.96

$$\frac{1}{36}\left(3\log\left(x^2-x+1\right)-3^{2/3}\log\left(\sqrt[3]{3}x^2-3^{2/3}x+3\right)-6\log(x+1)+2\ 3^{2/3}\log\left(3^{2/3}x+3\right)+6\sqrt[6]{3}\tan^{-1}\left(\frac{\sqrt[3]{3}-2x}{3^{5/6}}\right)+6\sqrt{3}\tan^{-1}\left(\frac{2x-1}{\sqrt{3}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(3 + 4*x^3 + x^6),x]
```

```
[Out] (6*3^(1/6)*ArcTan[(3^(1/3) - 2*x)/3^(5/6)] + 6*Sqrt[3]*ArcTan[(-1 + 2*x)/Sq
rt[3]] - 6*Log[1 + x] + 2*3^(2/3)*Log[3 + 3^(2/3)*x] + 3*Log[1 - x + x^2] -
 3^(2/3)*Log[3 - 3^(2/3)*x + 3^(1/3)*x^2])/36
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{3+4x^3+x^6}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x/(3 + 4*x^3 + x^6),x]
```

```
[Out] IntegrateAlgebraic[x/(3 + 4*x^3 + x^6), x]
```

fricas [A] time = 1.02, size = 84, normalized size = 0.75

$$-\frac{1}{36}\cdot 3^{\frac{2}{3}}\log\left(x^2-3^{\frac{1}{3}}x+3^{\frac{2}{3}}\right)+\frac{1}{18}\cdot 3^{\frac{2}{3}}\log\left(x+3^{\frac{1}{3}}\right)+\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)+\frac{1}{6}\cdot 3^{\frac{1}{6}}\arctan\left(-\frac{1}{3}\cdot 3^{\frac{1}{6}}\left(2x-3^{\frac{1}{3}}\right)\right)+\frac{1}{12}\log\left(x^2-x+1\right)-\frac{1}{6}\log(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^6+4*x^3+3),x, algorithm="fricas")
```

```
[Out] -1/36*3^(2/3)*log(x^2 - 3^(1/3)*x + 3^(2/3)) + 1/18*3^(2/3)*log(x + 3^(1/3)
) + 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + 1/6*3^(1/6)*arctan(-1/3*3^(
1/6)*(2*x - 3^(1/3))) + 1/12*log(x^2 - x + 1) - 1/6*log(x + 1)
```

giac [A] time = 0.36, size = 86, normalized size = 0.77

$$-\frac{1}{36}\cdot 3^{\frac{2}{3}}\log\left(x^2-3^{\frac{1}{3}}x+3^{\frac{2}{3}}\right)+\frac{1}{18}\cdot 3^{\frac{2}{3}}\log\left(\left|x+3^{\frac{1}{3}}\right|\right)+\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)-\frac{1}{6}\cdot 3^{\frac{1}{6}}\arctan\left(\frac{1}{3}\cdot 3^{\frac{1}{6}}\left(2x-3^{\frac{1}{3}}\right)\right)+\frac{1}{12}\log\left(x^2-x+1\right)-\frac{1}{6}\log(|x+1|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^6+4*x^3+3),x, algorithm="giac")
```

```
[Out] -1/36*3^(2/3)*log(x^2 - 3^(1/3)*x + 3^(2/3)) + 1/18*3^(2/3)*log(abs(x + 3^(
1/3))) + 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) - 1/6*3^(1/6)*arctan(1/3
*3^(1/6)*(2*x - 3^(1/3))) + 1/12*log(x^2 - x + 1) - 1/6*log(abs(x + 1))
```

maple [A] time = 0.00, size = 84, normalized size = 0.75

$$-\frac{3^{\frac{1}{6}}\arctan\left(\frac{\sqrt{3}\left(\frac{2\,3^{\frac{2}{3}}x}{3}-1\right)}{3}\right)}{6}+\frac{\sqrt{3}\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{6}-\frac{\ln(x+1)}{6}+\frac{3^{\frac{2}{3}}\ln\left(x+3^{\frac{1}{3}}\right)}{18}-\frac{3^{\frac{2}{3}}\ln\left(x^2-3^{\frac{1}{3}}x+3^{\frac{2}{3}}\right)}{36}+\frac{\ln\left(x^2-x+1\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x^6+4*x^3+3),x)
```

```
[Out] -1/6*ln(x+1)+1/18*3^(2/3)*ln(x+3^(1/3))-1/36*3^(2/3)*ln(x^2-3^(1/3)*x+3^(2/
3))-1/6*3^(1/6)*arctan(1/3*3^(1/2)*(2/3*3^(2/3)*x-1))+1/12*ln(x^2-x+1)+1/6*
3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))
```

maxima [A] time = 1.60, size = 84, normalized size = 0.75

$$-\frac{1}{36}\cdot 3^{\frac{2}{3}}\log\left(x^2-3^{\frac{1}{3}}x+3^{\frac{2}{3}}\right)+\frac{1}{18}\cdot 3^{\frac{2}{3}}\log\left(x+3^{\frac{1}{3}}\right)+\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)-\frac{1}{6}\cdot 3^{\frac{1}{6}}\arctan\left(\frac{1}{3}\cdot 3^{\frac{1}{6}}\left(2x-3^{\frac{1}{3}}\right)\right)+\frac{1}{12}\log\left(x^2-x+1\right)-\frac{1}{6}\log(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^6+4*x^3+3),x, algorithm="maxima")
```

```
[Out] -1/36*3^(2/3)*log(x^2 - 3^(1/3)*x + 3^(2/3)) + 1/18*3^(2/3)*log(x + 3^(1/3)
) + 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) - 1/6*3^(1/6)*arctan(1/3*3^(1
/6)*(2*x - 3^(1/3))) + 1/12*log(x^2 - x + 1) - 1/6*log(x + 1)
```

mupad [B] time = 1.36, size = 113, normalized size = 1.01

$$\frac{3^{2/3}\ln\left(x+3^{1/3}\right)}{18}-\frac{\ln(x+1)}{6}-\ln\left(x-\frac{1}{2}-\frac{\sqrt{3}\,1i}{2}\right)\left(-\frac{1}{12}+\frac{\sqrt{3}\,1i}{12}\right)+\ln\left(x-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\left(\frac{1}{12}+\frac{\sqrt{3}\,1i}{12}\right)-\ln\left(x-\frac{3^{1/3}}{2}-\frac{3^{5/6}\,1i}{2}\right)\left(\frac{3^{2/3}}{36}-\frac{3^{1/6}\,1i}{12}\right)-\ln\left(x-\frac{3^{1/3}}{2}+\frac{3^{5/6}\,1i}{2}\right)\left(\frac{3^{2/3}}{36}+\frac{3^{1/6}\,1i}{12}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(4*x^3 + x^6 + 3),x)
```

```
[Out] (3^(2/3)*log(x + 3^(1/3)))/18 - log(x + 1)/6 - log(x - (3^(1/2)*1i)/2 - 1/2
)*((3^(1/2)*1i)/12 - 1/12) + log(x + (3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/12
```

```
 + 1/12) - log(x - 3^(1/3)/2 - (3^(5/6)*1i)/2)*(3^(2/3)/36 - (3^(1/6)*1i)/1
2) - log(x - 3^(1/3)/2 + (3^(5/6)*1i)/2)*(3^(2/3)/36 + (3^(1/6)*1i)/12)
```

sympy [C] time = 1.86, size = 119, normalized size = 1.06

$$-\frac{\log(x+1)}{6}+\left(\frac{1}{12}-\frac{\sqrt{3}\,i}{12}\right)\log\left(x+90\left(\frac{1}{12}-\frac{\sqrt{3}\,i}{12}\right)^2+11664\left(\frac{1}{12}-\frac{\sqrt{3}\,i}{12}\right)^5\right)+\left(\frac{1}{12}+\frac{\sqrt{3}\,i}{12}\right)\log\left(x+11664\left(\frac{1}{12}+\frac{\sqrt{3}\,i}{12}\right)^5+90\left(\frac{1}{12}+\frac{\sqrt{3}\,i}{12}\right)^2\right)+\operatorname{RootSum}\left(648t^3-1,\left(t\mapsto t\log\left(11664t^5+90t^2+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x**6+4*x**3+3),x)
```

```
[Out] -log(x + 1)/6 + (1/12 - sqrt(3)*I/12)*log(x + 90*(1/12 - sqrt(3)*I/12)**2 +
 11664*(1/12 - sqrt(3)*I/12)**5) + (1/12 + sqrt(3)*I/12)*log(x + 11664*(1/1
2 + sqrt(3)*I/12)**5 + 90*(1/12 + sqrt(3)*I/12)**2) + RootSum(648*_t**3 - 1
, Lambda(_t, _t*log(11664*_t**5 + 90*_t**2 + x)))
```

3.152 $\int \frac{1}{3+4x^3+x^6} \, dx$

Optimal. Leaf size=112

$$-\frac{1}{12}\log\left(x^2-x+1\right)+\frac{\log\left(x^2-\sqrt[3]{3}x+3^{2/3}\right)}{12\ 3^{2/3}}+\frac{1}{6}\log(x+1)-\frac{\log\left(x+\sqrt[3]{3}\right)}{6\ 3^{2/3}}-\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}}+\frac{\tan^{-1}\left(\frac{\sqrt[3]{3}-2x}{3^{5/6}}\right)}{6\sqrt[6]{3}}$$

Rubi [A] time = 0.07, antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 8, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {1347, 200, 31, 634, 618, 204, 628, 617}

$$-\frac{1}{12}\log\left(x^2-x+1\right)+\frac{\log\left(x^2-\sqrt[3]{3}x+3^{2/3}\right)}{12\ 3^{2/3}}+\frac{1}{6}\log(x+1)-\frac{\log\left(x+\sqrt[3]{3}\right)}{6\ 3^{2/3}}-\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}}+\frac{\tan^{-1}\left(\frac{\sqrt[3]{3}-2x}{3^{5/6}}\right)}{6\sqrt[6]{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(3 + 4*x^3 + x^6)^(-1),x]
```

```
[Out] -ArcTan[(1 - 2*x)/Sqrt[3]]/(2*Sqrt[3]) + ArcTan[(3^(1/3) - 2*x)/3^(5/6)]/(6
*3^(1/6)) + Log[1 + x]/6 - Log[3^(1/3) + x]/(6*3^(2/3)) - Log[1 - x + x^2]/
12 + Log[3^(2/3) - 3^(1/3)*x + x^2]/(12*3^(2/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1347

```
Int[((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^(-1), x_Symbol] :> With[{q
= Rt[b^2 - 4*a*c, 2]}, Dist[c/q, Int[1/(b/2 - q/2 + c*x^n), x], x] - Dist[c
/q, Int[1/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*
n] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{1}{3+4x^3+x^6}\,dx = \frac{1}{2}\int \frac{1}{1+x^3}\,dx - \frac{1}{2}\int \frac{1}{3+x^3}\,dx$$

$$= \frac{1}{6}\int \frac{1}{1+x}\,dx + \frac{1}{6}\int \frac{2-x}{1-x+x^2}\,dx - \frac{\int \frac{1}{\sqrt[3]{3}+x}\,dx}{6\ 3^{2/3}} - \frac{\int \frac{2\sqrt[3]{3}-x}{3^{2/3}-\sqrt[3]{3}x+x^2}\,dx}{6\ 3^{2/3}}$$

$$= \frac{1}{6}\log(1+x) - \frac{\log\left(\sqrt[3]{3}+x\right)}{6\ 3^{2/3}} - \frac{1}{12}\int \frac{-1+2x}{1-x+x^2}\,dx + \frac{1}{4}\int \frac{1}{1-x+x^2}\,dx + \frac{\int \frac{-\sqrt[3]{3}+2x}{3^{2/3}-\sqrt[3]{3}x+x^2}\,d}{12\ 3^{2/3}}$$

$$= \frac{1}{6}\log(1+x) - \frac{\log\left(\sqrt[3]{3}+x\right)}{6\ 3^{2/3}} - \frac{1}{12}\log\left(1-x+x^2\right) + \frac{\log\left(3^{2/3}-\sqrt[3]{3}x+x^2\right)}{12\ 3^{2/3}} - \frac{1}{2}\text{Subst}\Big(\int$$

$$= -\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}} + \frac{\tan^{-1}\left(\frac{\sqrt[3]{3}-2x}{3^{5/6}}\right)}{6\sqrt[6]{3}} + \frac{1}{6}\log(1+x) - \frac{\log\left(\sqrt[3]{3}+x\right)}{6\ 3^{2/3}} - \frac{1}{12}\log\left(1-x+x^2\right) + \frac{\text{lo}}{}$$

Mathematica [A] time = 0.02, size = 107, normalized size = 0.96

$$\frac{1}{36}\left(-3\log\left(x^2-x+1\right)+\sqrt[3]{3}\log\left(\sqrt[3]{3}x^2-3^{2/3}x+3\right)+6\log(x+1)-2\sqrt[3]{3}\log\left(3^{2/3}x+3\right)+2\ 3^{5/6}\tan^{-1}\left(\frac{\sqrt[3]{3}-2x}{3^{5/6}}\right)+6\sqrt{3}\tan^{-1}\left(\frac{2x-1}{\sqrt{3}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 + 4*x^3 + x^6)^(-1),x]
```

```
[Out] (2*3^(5/6)*ArcTan[(3^(1/3) - 2*x)/3^(5/6)] + 6*Sqrt[3]*ArcTan[(-1 + 2*x)/Sq
rt[3]] + 6*Log[1 + x] - 2*3^(1/3)*Log[3 + 3^(2/3)*x] - 3*Log[1 - x + x^2] +
 3^(1/3)*Log[3 - 3^(2/3)*x + 3^(1/3)*x^2])/36
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{3+4x^3+x^6}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(3 + 4*x^3 + x^6)^(-1),x]
```

```
[Out] IntegrateAlgebraic[(3 + 4*x^3 + x^6)^(-1), x]
```

fricas [A] time = 1.42, size = 124, normalized size = 1.11

$$\frac{1}{18} \cdot 9^{\frac{1}{6}} \sqrt{3} \left(-1\right)^{\frac{1}{3}} \arctan\left(\frac{1}{27} \cdot 9^{\frac{1}{6}} \left(2 \cdot 9^{\frac{2}{3}} \sqrt{3} \left(-1\right)^{\frac{2}{3}} x - 3 \cdot 9^{\frac{1}{3}} \sqrt{3}\right)\right) - \frac{1}{108} \cdot 9^{\frac{2}{3}} \left(-1\right)^{\frac{1}{3}} \log\left(9^{\frac{2}{3}} \left(-1\right)^{\frac{1}{3}} x + 3 x^{2} + 3 \cdot 9^{\frac{1}{3}} \left(-1\right)^{\frac{2}{3}}\right) + \frac{1}{54} \cdot 9^{\frac{2}{3}} \left(-1\right)^{\frac{1}{3}} \log\left(-9^{\frac{2}{3}} \left(-1\right)^{\frac{1}{3}} + 3 x\right) + \frac{1}{6} \sqrt{3} \arctan\left(\frac{1}{3} \sqrt{3} \left(2 x - 1\right)\right) - \frac{1}{12} \log\left(x^{2} - x + 1\right) + \frac{1}{6} \log\left(x + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^6+4*x^3+3),x, algorithm="fricas")
```

```
[Out] 1/18*9^(1/6)*sqrt(3)*(-1)^(1/3)*arctan(1/27*9^(1/6)*(2*9^(2/3)*sqrt(3)*(-1)
^(2/3)*x - 3*9^(1/3)*sqrt(3))) - 1/108*9^(2/3)*(-1)^(1/3)*log(9^(2/3)*(-1)^
(1/3)*x + 3*x^2 + 3*9^(1/3)*(-1)^(2/3)) + 1/54*9^(2/3)*(-1)^(1/3)*log(-9^(2
/3)*(-1)^(1/3) + 3*x) + 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) - 1/12*lo
g(x^2 - x + 1) + 1/6*log(x + 1)
```

giac [A] time = 0.34, size = 86, normalized size = 0.77

$$-\frac{1}{18} \cdot 3^{\frac{5}{6}} \arctan\left(\frac{1}{3} \cdot 3^{\frac{1}{6}} \left(2 x - 3^{\frac{1}{3}}\right)\right) + \frac{1}{6} \sqrt{3} \arctan\left(\frac{1}{3} \sqrt{3} \left(2 x - 1\right)\right) + \frac{1}{36} \cdot 3^{\frac{1}{3}} \log\left(x^{2} - 3^{\frac{1}{3}} x + 3^{\frac{2}{3}}\right) - \frac{1}{18} \cdot 3^{\frac{1}{3}} \log\left(\left|x + 3^{\frac{1}{3}}\right|\right) - \frac{1}{12} \log\left(x^{2} - x + 1\right) + \frac{1}{6} \log\left(\left|x + 1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^6+4*x^3+3),x, algorithm="giac")
```

```
[Out] -1/18*3^(5/6)*arctan(1/3*3^(1/6)*(2*x - 3^(1/3))) + 1/6*sqrt(3)*arctan(1/3*
sqrt(3)*(2*x - 1)) + 1/36*3^(1/3)*log(x^2 - 3^(1/3)*x + 3^(2/3)) - 1/18*3^(
1/3)*log(abs(x + 3^(1/3))) - 1/12*log(x^2 - x + 1) + 1/6*log(abs(x + 1))
```

maple [A] time = 0.01, size = 84, normalized size = 0.75

$$-\frac{3^{\frac{5}{6}} \arctan\left(\frac{\sqrt{3}\, \left(\frac{2\, 3^{\frac{2}{3}} x}{3}-1\right)}{3}\right)}{18}+\frac{\sqrt{3}\, \arctan\left(\frac{\left(2 x -1\right) \sqrt{3}}{3}\right)}{6}+\frac{\ln \left(x +1\right)}{6}-\frac{3^{\frac{1}{3}} \ln \left(x +3^{\frac{1}{3}}\right)}{18}+\frac{3^{\frac{1}{3}} \ln \left(x^{2}-3^{\frac{1}{3}} x +3^{\frac{2}{3}}\right)}{36}-\frac{\ln \left(x^{2}-x +1\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^6+4*x^3+3),x)
```

```
[Out] 1/6*ln(x+1)-1/18*3^(1/3)*ln(x+3^(1/3))+1/36*3^(1/3)*ln(x^2-3^(1/3)*x+3^(2/3
))-1/18*3^(5/6)*arctan(1/3*3^(1/2)*(2/3*3^(2/3)*x-1))-1/12*ln(x^2-x+1)+1/6*
3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))
```

maxima [A] time = 1.28, size = 84, normalized size = 0.75

$$-\frac{1}{18} \cdot 3^{\frac{5}{6}} \arctan\left(\frac{1}{3} \cdot 3^{\frac{1}{6}} \left(2 x - 3^{\frac{1}{3}}\right)\right) + \frac{1}{6} \sqrt{3} \arctan\left(\frac{1}{3} \sqrt{3} \left(2 x - 1\right)\right) + \frac{1}{36} \cdot 3^{\frac{1}{3}} \log\left(x^{2} - 3^{\frac{1}{3}} x + 3^{\frac{2}{3}}\right) - \frac{1}{18} \cdot 3^{\frac{1}{3}} \log\left(x + 3^{\frac{1}{3}}\right) - \frac{1}{12} \log\left(x^{2} - x + 1\right) + \frac{1}{6} \log\left(x + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^6+4*x^3+3),x, algorithm="maxima")
```

```
[Out] -1/18*3^(5/6)*arctan(1/3*3^(1/6)*(2*x - 3^(1/3))) + 1/6*sqrt(3)*arctan(1/3*
sqrt(3)*(2*x - 1)) + 1/36*3^(1/3)*log(x^2 - 3^(1/3)*x + 3^(2/3)) - 1/18*3^(
1/3)*log(x + 3^(1/3)) - 1/12*log(x^2 - x + 1) + 1/6*log(x + 1)
```

mupad [B] time = 0.23, size = 110, normalized size = 0.98

$$\frac{\ln\left(x+1\right)}{6}-\frac{3^{1/3}\,\ln\left(x+3^{1/3}\right)}{18}-\ln\left(x-\frac{1}{2}-\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,\left(\frac{1}{12}+\frac{\sqrt{3}\,1{}\mathrm{i}}{12}\right)+\ln\left(x-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,\left(-\frac{1}{12}+\frac{\sqrt{3}\,1{}\mathrm{i}}{12}\right)+\frac{{\left(-1\right)}^{1/3}\,3^{1/3}\,\ln\left(x-{\left(-1\right)}^{1/3}\,3^{1/3}\right)}{18}-\frac{{\left(-1\right)}^{1/3}\,\ln\left(x+\frac{{\left(-1\right)}^{1/3}\,3^{1/3}}{2}+\frac{{\left(-1\right)}^{1/3}\,3^{5/6}\,1{}\mathrm{i}}{2}\right)\,\left(3^{1/3}+3^{5/6}\,1{}\mathrm{i}\right)}{36}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(4*x^3 + x^6 + 3),x)
```

```
[Out] log(x + 1)/6 - (3^(1/3)*log(x + 3^(1/3)))/18 - log(x - (3^(1/2)*1i)/2 - 1/2
)*((3^(1/2)*1i)/12 + 1/12) + log(x + (3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/12
```

```
 - 1/12) + ((-1)^(1/3)*3^(1/3)*log(x - (-1)^(1/3)*3^(1/3)))/18 - ((-1)^(1/3
)*log(x + ((-1)^(1/3)*3^(1/3))/2 + ((-1)^(1/3)*3^(5/6)*1i)/2)*(3^(1/3) + 3^
(5/6)*1i))/36
```

sympy [C] time = 1.82, size = 124, normalized size = 1.11

$$\frac{\log(x+1)}{6} + \left(-\frac{1}{12} + \frac{\sqrt{3}\,i}{12}\right)\log\left(x + \frac{13}{10} - \frac{13\sqrt{3}\,i}{10} + \frac{23328\left(-\frac{1}{12} + \frac{\sqrt{3}\,i}{12}\right)^4}{5}\right) + \left(-\frac{1}{12} - \frac{\sqrt{3}\,i}{12}\right)\log\left(x + \frac{13}{10} + \frac{23328\left(-\frac{1}{12} - \frac{\sqrt{3}\,i}{12}\right)^4}{5} + \frac{13\sqrt{3}\,i}{10}\right) + \operatorname{RootSum}\left(1944t^3 + 1, \left(t \mapsto t\log\left(\frac{23328t^4}{5} - \frac{78t}{5} + x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**6+4*x**3+3),x)
```

```
[Out] log(x + 1)/6 + (-1/12 + sqrt(3)*I/12)*log(x + 13/10 - 13*sqrt(3)*I/10 + 233
28*(-1/12 + sqrt(3)*I/12)**4/5) + (-1/12 - sqrt(3)*I/12)*log(x + 13/10 + 23
328*(-1/12 - sqrt(3)*I/12)**4/5 + 13*sqrt(3)*I/10) + RootSum(1944*_t**3 + 1
, Lambda(_t, _t*log(23328*_t**4/5 - 78*_t/5 + x)))
```

3.153 $\int \frac{1}{x^2\left(3+4x^3+x^6\right)}\,dx$

Optimal. Leaf size=119

$$-\frac{1}{12}\log\left(x^2-x+1\right)+\frac{\log\left(x^2-\sqrt[3]{3}\,x+3^{2/3}\right)}{36\sqrt[3]{3}}-\frac{1}{3x}+\frac{1}{6}\log(x+1)-\frac{\log\left(x+\sqrt[3]{3}\right)}{18\sqrt[3]{3}}+\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}}-\frac{\tan^{-1}\left(\frac{\sqrt[3]{3}-2x}{3^{5/6}}\right)}{6\,3^{5/6}}$$

Rubi [A] time = 0.08, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 9, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.562, Rules used = {1368, 1510, 292, 31, 634, 618, 204, 628, 617}

$$-\frac{1}{12}\log\left(x^2-x+1\right)+\frac{\log\left(x^2-\sqrt[3]{3}\,x+3^{2/3}\right)}{36\sqrt[3]{3}}-\frac{1}{3x}+\frac{1}{6}\log(x+1)-\frac{\log\left(x+\sqrt[3]{3}\right)}{18\sqrt[3]{3}}+\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}}-\frac{\tan^{-1}\left(\frac{\sqrt[3]{3}-2x}{3^{5/6}}\right)}{6\,3^{5/6}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(3 + 4*x^3 + x^6)),x]
```

```
[Out] -1/(3*x) + ArcTan[(1 - 2*x)/Sqrt[3]]/(2*Sqrt[3]) - ArcTan[(3^(1/3) - 2*x)/3
^(5/6)]/(6*3^(5/6)) + Log[1 + x]/6 - Log[3^(1/3) + x]/(18*3^(1/3)) - Log[1
- x + x^2]/12 + Log[3^(2/3) - 3^(1/3)*x + x^2]/(36*3^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1368

```
Int[((d_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_
Symbol] :> Simp[((d*x)^(m + 1)*(a + b*x^n + c*x^(2*n))^(p + 1))/(a*d*(m + 1
)), x] - Dist[1/(a*d^n*(m + 1)), Int[(d*x)^(m + n)*(b*(m + n*(p + 1) + 1) +
 c*(m + 2*n*(p + 1) + 1)*x^n)*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a
, b, c, d, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && L
tQ[m, -1] && IntegerQ[p]
```

Rule 1510

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_)))/((a_) + (b_.)*(x_)^(n_) +
 (c_.)*(x_)^(n2_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 +
(2*c*d - b*e)/(2*q), Int[(f*x)^m/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (
2*c*d - b*e)/(2*q), Int[(f*x)^m/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b
, c, d, e, f, m}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^2\left(3+4x^3+x^6\right)}\,dx &= -\frac{1}{3x}+\frac{1}{3}\int \frac{x\left(-4-x^3\right)}{3+4x^3+x^6}\,dx\\
&= -\frac{1}{3x}+\frac{1}{6}\int \frac{x}{3+x^3}\,dx-\frac{1}{2}\int \frac{x}{1+x^3}\,dx\\
&= -\frac{1}{3x}+\frac{1}{6}\int \frac{1}{1+x}\,dx-\frac{1}{6}\int \frac{1+x}{1-x+x^2}\,dx-\frac{\int \frac{1}{\sqrt[3]{3}+x}\,dx}{18\sqrt[3]{3}}+\frac{\int \frac{\sqrt[3]{3}+x}{3^{2/3}-\sqrt[3]{3}x+x^2}\,dx}{18\sqrt[3]{3}}\\
&= -\frac{1}{3x}+\frac{1}{6}\log(1+x)-\frac{\log\left(\sqrt[3]{3}+x\right)}{18\sqrt[3]{3}}-\frac{1}{12}\int \frac{-1+2x}{1-x+x^2}\,dx+\frac{1}{12}\int \frac{1}{3^{2/3}-\sqrt[3]{3}x+x^2}\,d\\
&= -\frac{1}{3x}+\frac{1}{6}\log(1+x)-\frac{\log\left(\sqrt[3]{3}+x\right)}{18\sqrt[3]{3}}-\frac{1}{12}\log\left(1-x+x^2\right)+\frac{\log\left(3^{2/3}-\sqrt[3]{3}x+x^2\right)}{36\sqrt[3]{3}}+\\
&= -\frac{1}{3x}+\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}}-\frac{\tan^{-1}\left(\frac{\sqrt[3]{3}-2x}{3^{5/6}}\right)}{6\,3^{5/6}}+\frac{1}{6}\log(1+x)-\frac{\log\left(\sqrt[3]{3}+x\right)}{18\sqrt[3]{3}}-\frac{1}{12}\log\left(1-x\right.
\end{aligned}$$

Mathematica [A] time = 0.04, size = 118, normalized size = 0.99

$$-\frac{9x\log\left(x^2-x+1\right)-3^{2/3}x\log\left(\sqrt[3]{3}x^2-3^{2/3}x+3\right)-18x\log(x+1)+2\,3^{2/3}x\log\left(3^{2/3}x+3\right)+6\sqrt[6]{3}x\tan^{-1}\left(\frac{\sqrt[3]{3}-2x}{3^{5/6}}\right)+18\sqrt{3}x\tan^{-1}\left(\frac{2x-1}{\sqrt{3}}\right)+36}{108x}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(3 + 4*x^3 + x^6)),x]
```

```
[Out] -1/108*(36 + 6*3^(1/6)*x*ArcTan[(3^(1/3) - 2*x)/3^(5/6)] + 18*Sqrt[3]*x*Arc
Tan[(-1 + 2*x)/Sqrt[3]] - 18*x*Log[1 + x] + 2*3^(2/3)*x*Log[3 + 3^(2/3)*x]
+ 9*x*Log[1 - x + x^2] - 3^(2/3)*x*Log[3 - 3^(2/3)*x + 3^(1/3)*x^2])/x
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2\left(3+4x^3+x^6\right)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x^2*(3 + 4*x^3 + x^6)),x]
```

```
[Out] IntegrateAlgebraic[1/(x^2*(3 + 4*x^3 + x^6)), x]
```

fricas [A] time = 1.18, size = 117, normalized size = 0.98

$$-\frac{3^{\frac{2}{3}} (-1)^{\frac{1}{3}} x \log\left(-3^{\frac{1}{3}} (-1)^{\frac{2}{3}} x + x^2 - 3^{\frac{2}{3}} (-1)^{\frac{1}{3}}\right) - 2 \cdot 3^{\frac{2}{3}} (-1)^{\frac{1}{3}} x \log\left(3^{\frac{1}{3}} (-1)^{\frac{2}{3}} + x\right) + 18 \sqrt{3} x \arctan\left(\frac{1}{3} \sqrt{3} (2x-1)\right) - 6 \cdot 3^{\frac{1}{6}} (-1)^{\frac{1}{3}} x \arctan\left(\frac{1}{3} \cdot 3^{\frac{1}{6}} \left(2 (-1)^{\frac{1}{3}} x + 3^{\frac{1}{3}}\right)\right) + 9 x \log\left(x^2-x+1\right) - 18 x \log\left(x+1\right) + 36}{108 x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(x^6+4*x^3+3),x, algorithm="fricas")
```

```
[Out] -1/108*(3^(2/3)*(-1)^(1/3)*x*log(-3^(1/3)*(-1)^(2/3)*x + x^2 - 3^(2/3)*(-1)
^(1/3)) - 2*3^(2/3)*(-1)^(1/3)*x*log(3^(1/3)*(-1)^(2/3) + x) + 18*sqrt(3)*x
*arctan(1/3*sqrt(3)*(2*x - 1)) - 6*3^(1/6)*(-1)^(1/3)*x*arctan(1/3*3^(1/6)*
(2*(-1)^(1/3)*x + 3^(1/3))) + 9*x*log(x^2 - x + 1) - 18*x*log(x + 1) + 36)/
x
```

giac [A] time = 0.41, size = 91, normalized size = 0.76

$$\frac{1}{108} \cdot 3^{\frac{2}{3}} \log\left(x^2 - 3^{\frac{1}{3}} x + 3^{\frac{2}{3}}\right) - \frac{1}{54} \cdot 3^{\frac{2}{3}} \log\left(\left|x + 3^{\frac{1}{3}}\right|\right) - \frac{1}{6} \sqrt{3} \arctan\left(\frac{1}{3} \sqrt{3} (2x-1)\right) + \frac{1}{18} \cdot 3^{\frac{1}{6}} \arctan\left(\frac{1}{3} \cdot 3^{\frac{1}{6}} \left(2x - 3^{\frac{1}{3}}\right)\right) - \frac{1}{3x} - \frac{1}{12} \log\left(x^2-x+1\right) + \frac{1}{6} \log\left(|x+1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(x^6+4*x^3+3),x, algorithm="giac")
```

```
[Out] 1/108*3^(2/3)*log(x^2 - 3^(1/3)*x + 3^(2/3)) - 1/54*3^(2/3)*log(abs(x + 3^(
1/3))) - 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + 1/18*3^(1/6)*arctan(1/
3*3^(1/6)*(2*x - 3^(1/3))) - 1/3/x - 1/12*log(x^2 - x + 1) + 1/6*log(abs(x
+ 1))
```

maple [A] time = 0.01, size = 89, normalized size = 0.75

$$\frac{3^{\frac{1}{6}} \arctan\left(\frac{\sqrt{3}\left(\frac{2 3^{\frac{2}{3}} x}{3}-1\right)}{3}\right)}{18} - \frac{\sqrt{3}\ \arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{6} + \frac{\ln\left(x+1\right)}{6} - \frac{3^{\frac{2}{3}} \ln\left(x+3^{\frac{1}{3}}\right)}{54} + \frac{3^{\frac{2}{3}} \ln\left(x^2 - 3^{\frac{1}{3}} x + 3^{\frac{2}{3}}\right)}{108} - \frac{\ln\left(x^2-x+1\right)}{12} - \frac{1}{3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(x^6+4*x^3+3),x)
```

```
[Out] 1/6*ln(x+1)-1/3/x-1/54*3^(2/3)*ln(x+3^(1/3))+1/108*3^(2/3)*ln(x^2-3^(1/3)*x
+3^(2/3))+1/18*3^(1/6)*arctan(1/3*3^(1/2)*(2/3*3^(2/3)*x-1))-1/12*ln(x^2-x+
1)-1/6*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))
```

maxima [A] time = 1.09, size = 89, normalized size = 0.75

$$\frac{1}{108} \cdot 3^{\frac{2}{3}} \log\left(x^2 - 3^{\frac{1}{3}} x + 3^{\frac{2}{3}}\right) - \frac{1}{54} \cdot 3^{\frac{2}{3}} \log\left(x + 3^{\frac{1}{3}}\right) - \frac{1}{6} \sqrt{3} \arctan\left(\frac{1}{3} \sqrt{3} (2x-1)\right) + \frac{1}{18} \cdot 3^{\frac{1}{6}} \arctan\left(\frac{1}{3} \cdot 3^{\frac{1}{6}} \left(2x - 3^{\frac{1}{3}}\right)\right) - \frac{1}{3x} - \frac{1}{12} \log\left(x^2-x+1\right) + \frac{1}{6} \log\left(x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(x^6+4*x^3+3),x, algorithm="maxima")
```

```
[Out] 1/108*3^(2/3)*log(x^2 - 3^(1/3)*x + 3^(2/3)) - 1/54*3^(2/3)*log(x + 3^(1/3)
) - 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + 1/18*3^(1/6)*arctan(1/3*3^(
1/6)*(2*x - 3^(1/3))) - 1/3/x - 1/12*log(x^2 - x + 1) + 1/6*log(x + 1)
```

mupad [B] time = 1.38, size = 119, normalized size = 1.00

$$\frac{\ln(x+1)}{6}-\frac{3^{2/3}\ln\left(x+3^{1/3}\right)}{54}+\ln\left(x-\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(-\frac{1}{12}+\frac{\sqrt{3}\,1\mathrm{i}}{12}\right)-\ln\left(x-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{12}+\frac{\sqrt{3}\,1\mathrm{i}}{12}\right)-\frac{1}{3\,x}-\frac{(-1)^{1/3}\ln\left(x-\frac{(-1)^{1/3}\,3^{1/3}}{2}-\frac{(-1)^{1/6}\,3^{5/6}}{2}+\frac{3^{1/3}}{2}\right)\left(3^{2/3}+3^{1/6}\,3\mathrm{i}\right)}{108}+\frac{(-1)^{1/3}\,3^{2/3}\ln\left(x+(-1)^{2/3}\,3^{1/3}\right)}{54}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(4*x^3 + x^6 + 3)),x)
```

```
[Out] log(x + 1)/6 - (3^(2/3)*log(x + 3^(1/3)))/54 + log(x - (3^(1/2)*1i)/2 - 1/2
)*((3^(1/2)*1i)/12 - 1/12) - log(x + (3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/12
 + 1/12) - 1/(3*x) - ((-1)^(1/3)*log(x - ((-1)^(1/3)*3^(1/3))/2 - ((-1)^(1/
6)*3^(5/6))/2 + 3^(1/3)/2)*(3^(2/3) + 3^(1/6)*3i))/108 + ((-1)^(1/3)*3^(2/3
)*log(x + (-1)^(2/3)*3^(1/3)))/54
```

sympy [C] time = 1.83, size = 139, normalized size = 1.17

$$\frac{\log(x+1)}{6}+\left(-\frac{1}{12}-\frac{\sqrt{3}\,i}{12}\right)\log\left(x-\frac{8188128\left(-\frac{1}{12}-\frac{\sqrt{3}\,i}{12}\right)^{5}}{41}+\frac{39384\left(-\frac{1}{12}-\frac{\sqrt{3}\,i}{12}\right)^{2}}{41}\right)+\left(-\frac{1}{12}+\frac{\sqrt{3}\,i}{12}\right)\log\left(x+\frac{39384\left(-\frac{1}{12}+\frac{\sqrt{3}\,i}{12}\right)^{2}}{41}-\frac{8188128\left(-\frac{1}{12}+\frac{\sqrt{3}\,i}{12}\right)^{5}}{41}\right)+\operatorname{RootSum}\left(17496t^{3}+1,\left(t\mapsto t\log\left(-\frac{8188128t^{5}}{41}+\frac{39384t^{2}}{41}+x\right)\right)\right)-\frac{1}{3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(x**6+4*x**3+3),x)
```

```
[Out] log(x + 1)/6 + (-1/12 - sqrt(3)*I/12)*log(x - 8188128*(-1/12 - sqrt(3)*I/12
)**5/41 + 39384*(-1/12 - sqrt(3)*I/12)**2/41) + (-1/12 + sqrt(3)*I/12)*log(
x + 39384*(-1/12 + sqrt(3)*I/12)**2/41 - 8188128*(-1/12 + sqrt(3)*I/12)**5/
41) + RootSum(17496*_t**3 + 1, Lambda(_t, _t*log(-8188128*_t**5/41 + 39384*
_t**2/41 + x))) - 1/(3*x)
```

3.154 $\int \frac{1}{x^3(3+4x^3+x^6)}\,dx$

Optimal. Leaf size=119

$$-\frac{1}{6x^2}+\frac{1}{12}\log\left(x^2-x+1\right)-\frac{\log\left(x^2-\sqrt[3]{3}x+3^{2/3}\right)}{36\ 3^{2/3}}-\frac{1}{6}\log(x+1)+\frac{\log\left(x+\sqrt[3]{3}\right)}{18\ 3^{2/3}}+\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}}-\frac{\tan^{-1}\left(\frac{\sqrt[3]{3}-2x}{3^{5/6}}\right)}{18\sqrt[6]{3}}$$

Rubi [A] time = 0.08, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 9, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.562, Rules used = {1368, 1422, 200, 31, 634, 618, 204, 628, 617}

$$-\frac{1}{6x^2}+\frac{1}{12}\log\left(x^2-x+1\right)-\frac{\log\left(x^2-\sqrt[3]{3}x+3^{2/3}\right)}{36\ 3^{2/3}}-\frac{1}{6}\log(x+1)+\frac{\log\left(x+\sqrt[3]{3}\right)}{18\ 3^{2/3}}+\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}}-\frac{\tan^{-1}\left(\frac{\sqrt[3]{3}-2x}{3^{5/6}}\right)}{18\sqrt[6]{3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(3 + 4*x^3 + x^6)),x]
```

```
[Out] -1/(6*x^2) + ArcTan[(1 - 2*x)/Sqrt[3]]/(2*Sqrt[3]) - ArcTan[(3^(1/3) - 2*x)
/3^(5/6)]/(18*3^(1/6)) - Log[1 + x]/6 + Log[3^(1/3) + x]/(18*3^(2/3)) + Log
[1 - x + x^2]/12 - Log[3^(2/3) - 3^(1/3)*x + x^2]/(36*3^(2/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1368

```
Int[((d_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_
Symbol] :> Simp[((d*x)^(m + 1)*(a + b*x^n + c*x^(2*n))^(p + 1))/(a*d*(m + 1
)), x] - Dist[1/(a*d^n*(m + 1)), Int[(d*x)^(m + n)*(b*(m + n*(p + 1) + 1) +
 c*(m + 2*n*(p + 1) + 1)*x^n)*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a
, b, c, d, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && L
tQ[m, -1] && IntegerQ[p]
```

Rule 1422

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q),
Int[1/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(
b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (PosQ[b^2 - 4*a
*c] ||  !IGtQ[n/2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^3\left(3+4x^3+x^6\right)}\,dx &= -\frac{1}{6x^2}+\frac{1}{6}\int \frac{-8-2x^3}{3+4x^3+x^6}\,dx\\
&= -\frac{1}{6x^2}+\frac{1}{6}\int \frac{1}{3+x^3}\,dx-\frac{1}{2}\int \frac{1}{1+x^3}\,dx\\
&= -\frac{1}{6x^2}-\frac{1}{6}\int \frac{1}{1+x}\,dx-\frac{1}{6}\int \frac{2-x}{1-x+x^2}\,dx+\frac{\int \frac{1}{\sqrt[3]{3}+x}\,dx}{18\ 3^{2/3}}+\frac{\int \frac{2\sqrt[3]{3}-x}{3^{2/3}-\sqrt[3]{3}x+x^2}\,dx}{18\ 3^{2/3}}\\
&= -\frac{1}{6x^2}-\frac{1}{6}\log(1+x)+\frac{\log\left(\sqrt[3]{3}+x\right)}{18\ 3^{2/3}}+\frac{1}{12}\int \frac{-1+2x}{1-x+x^2}\,dx-\frac{1}{4}\int \frac{1}{1-x+x^2}\,dx-\frac{\int}{}\\
&= -\frac{1}{6x^2}-\frac{1}{6}\log(1+x)+\frac{\log\left(\sqrt[3]{3}+x\right)}{18\ 3^{2/3}}+\frac{1}{12}\log\left(1-x+x^2\right)-\frac{\log\left(3^{2/3}-\sqrt[3]{3}x+x^2\right)}{36\ 3^{2/3}}+\\
&= -\frac{1}{6x^2}+\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}}-\frac{\tan^{-1}\left(\frac{\sqrt[3]{3}-2x}{3^{5/6}}\right)}{18\sqrt[6]{3}}-\frac{1}{6}\log(1+x)+\frac{\log\left(\sqrt[3]{3}+x\right)}{18\ 3^{2/3}}+\frac{1}{12}\log\left(1-x\right.
\end{aligned}$$

Mathematica [A] time = 0.05, size = 113, normalized size = 0.95

$$\frac{1}{108}\left(-\frac{18}{x^2}+9\log\left(x^2-x+1\right)-\sqrt[3]{3}\log\left(\sqrt[3]{3}x^2-3^{2/3}x+3\right)-18\log(x+1)+2\sqrt[3]{3}\log\left(3^{2/3}x+3\right)-2\,3^{5/6}\tan^{-1}\left(\frac{\sqrt[3]{3}-2x}{3^{5/6}}\right)-18\sqrt{3}\tan^{-1}\left(\frac{2x-1}{\sqrt{3}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(3 + 4*x^3 + x^6)),x]
```

```
[Out] (-18/x^2 - 2*3^(5/6)*ArcTan[(3^(1/3) - 2*x)/3^(5/6)] - 18*Sqrt[3]*ArcTan[(-
1 + 2*x)/Sqrt[3]] - 18*Log[1 + x] + 2*3^(1/3)*Log[3 + 3^(2/3)*x] + 9*Log[1
- x + x^2] - 3^(1/3)*Log[3 - 3^(2/3)*x + 3^(1/3)*x^2])/108
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3 \left(3 + 4x^3 + x^6\right)} \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x^3*(3 + 4*x^3 + x^6)),x]
```

```
[Out] IntegrateAlgebraic[1/(x^3*(3 + 4*x^3 + x^6)), x]
```

fricas [A] time = 1.16, size = 126, normalized size = 1.06

$$\frac{6 \cdot 9^{\frac{1}{6}} \sqrt{3} x^2 \arctan\left(\frac{1}{27} \cdot 9^{\frac{1}{6}} \left(2 \cdot 9^{\frac{2}{3}} \sqrt{3} x - 3 \cdot 9^{\frac{1}{3}} \sqrt{3}\right)\right) - 9^{\frac{2}{3}} x^2 \log\left(3 x^2 - 9^{\frac{2}{3}} x + 3 \cdot 9^{\frac{1}{3}}\right) + 2 \cdot 9^{\frac{2}{3}} x^2 \log\left(3 x + 9^{\frac{2}{3}}\right) - 54 \sqrt{3} x^2 \arctan\left(\frac{1}{3} \sqrt{3} (2 x - 1)\right) + 27 x^2 \log\left(x^2 - x + 1\right) - 54 x^2 \log(x + 1) - 54}{324 x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(x^6+4*x^3+3),x, algorithm="fricas")
```

```
[Out] 1/324*(6*9^(1/6)*sqrt(3)*x^2*arctan(1/27*9^(1/6)*(2*9^(2/3)*sqrt(3)*x - 3*9
^(1/3)*sqrt(3))) - 9^(2/3)*x^2*log(3*x^2 - 9^(2/3)*x + 3*9^(1/3)) + 2*9^(2/
3)*x^2*log(3*x + 9^(2/3)) - 54*sqrt(3)*x^2*arctan(1/3*sqrt(3)*(2*x - 1)) +
27*x^2*log(x^2 - x + 1) - 54*x^2*log(x + 1) - 54)/x^2
```

giac [A] time = 0.41, size = 91, normalized size = 0.76

$$\frac{1}{54} \cdot 3^{\frac{5}{6}} \arctan\left(\frac{1}{3} \cdot 3^{\frac{1}{6}} \left(2 x - 3^{\frac{1}{3}}\right)\right) - \frac{1}{6} \sqrt{3} \arctan\left(\frac{1}{3} \sqrt{3} (2 x - 1)\right) - \frac{1}{108} \cdot 3^{\frac{1}{3}} \log\left(x^2 - 3^{\frac{1}{3}} x + 3^{\frac{2}{3}}\right) + \frac{1}{54} \cdot 3^{\frac{1}{3}} \log\left(\left|x + 3^{\frac{1}{3}}\right|\right) - \frac{1}{6 x^2} + \frac{1}{12} \log\left(x^2 - x + 1\right) - \frac{1}{6} \log\left(|x + 1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(x^6+4*x^3+3),x, algorithm="giac")
```

```
[Out] 1/54*3^(5/6)*arctan(1/3*3^(1/6)*(2*x - 3^(1/3))) - 1/6*sqrt(3)*arctan(1/3*s
qrt(3)*(2*x - 1)) - 1/108*3^(1/3)*log(x^2 - 3^(1/3)*x + 3^(2/3)) + 1/54*3^(
1/3)*log(abs(x + 3^(1/3))) - 1/6/x^2 + 1/12*log(x^2 - x + 1) - 1/6*log(abs(
x + 1))
```

maple [A] time = 0.01, size = 89, normalized size = 0.75

$$\frac{3^{\frac{5}{6}} \arctan\left(\frac{\sqrt{3}\left(\frac{2 \, 3^{\frac{2}{3}} x}{3} - 1\right)}{3}\right)}{54} - \frac{\sqrt{3}\, \arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{6} - \frac{\ln(x+1)}{6} + \frac{3^{\frac{1}{3}} \ln\left(x + 3^{\frac{1}{3}}\right)}{54} - \frac{3^{\frac{1}{3}} \ln\left(x^2 - 3^{\frac{1}{3}} x + 3^{\frac{2}{3}}\right)}{108} + \frac{\ln\left(x^2 - x + 1\right)}{12} - \frac{1}{6x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(x^6+4*x^3+3),x)
```

```
[Out] -1/6*ln(x+1)+1/54*3^(1/3)*ln(x+3^(1/3))-1/108*3^(1/3)*ln(x^2-3^(1/3)*x+3^(2
/3))+1/54*3^(5/6)*arctan(1/3*3^(1/2)*(2/3*3^(2/3)*x-1))-1/6/x^2+1/12*ln(x^2
-x+1)-1/6*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))
```

maxima [A] time = 1.27, size = 89, normalized size = 0.75

$$\frac{1}{54} \cdot 3^{\frac{5}{6}} \arctan\left(\frac{1}{3} \cdot 3^{\frac{1}{6}} \left(2 x - 3^{\frac{1}{3}}\right)\right) - \frac{1}{6} \sqrt{3} \arctan\left(\frac{1}{3} \sqrt{3} (2 x - 1)\right) - \frac{1}{108} \cdot 3^{\frac{1}{3}} \log\left(x^2 - 3^{\frac{1}{3}} x + 3^{\frac{2}{3}}\right) + \frac{1}{54} \cdot 3^{\frac{1}{3}} \log\left(x + 3^{\frac{1}{3}}\right) - \frac{1}{6 x^2} + \frac{1}{12} \log\left(x^2 - x + 1\right) - \frac{1}{6} \log(x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(x^6+4*x^3+3),x, algorithm="maxima")
```

```
[Out] 1/54*3^(5/6)*arctan(1/3*3^(1/6)*(2*x - 3^(1/3))) - 1/6*sqrt(3)*arctan(1/3*s
qrt(3)*(2*x - 1)) - 1/108*3^(1/3)*log(x^2 - 3^(1/3)*x + 3^(2/3)) + 1/54*3^(
1/3)*log(x + 3^(1/3)) - 1/6/x^2 + 1/12*log(x^2 - x + 1) - 1/6*log(x + 1)
```

mupad [B] time = 1.36, size = 118, normalized size = 0.99

$$\frac{3^{1/3}\,\ln\left(x+3^{1/3}\right)}{54}-\frac{\ln\left(x+1\right)}{6}+\ln\left(x-\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{12}+\frac{\sqrt{3}\,1\mathrm{i}}{12}\right)-\ln\left(x-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(-\frac{1}{12}+\frac{\sqrt{3}\,1\mathrm{i}}{12}\right)-\frac{1}{6\,x^2}-\ln\left(x-\frac{3^{1/3}}{2}-\frac{3^{5/6}\,1\mathrm{i}}{2}\right)\left(\frac{3^{1/3}}{108}+\frac{3^{5/6}\,1\mathrm{i}}{108}\right)-\ln\left(x-\frac{3^{1/3}}{2}+\frac{3^{5/6}\,1\mathrm{i}}{2}\right)\left(\frac{3^{1/3}}{108}-\frac{3^{5/6}\,1\mathrm{i}}{108}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(4*x^3 + x^6 + 3)),x)
```

```
[Out] (3^(1/3)*log(x + 3^(1/3)))/54 - log(x + 1)/6 + log(x - (3^(1/2)*1i)/2 - 1/2
)*((3^(1/2)*1i)/12 + 1/12) - log(x + (3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/12
 - 1/12) - 1/(6*x^2) - log(x - 3^(1/3)/2 - (3^(5/6)*1i)/2)*(3^(1/3)/108 + (
3^(5/6)*1i)/108) - log(x - 3^(1/3)/2 + (3^(5/6)*1i)/2)*(3^(1/3)/108 - (3^(5
/6)*1i)/108)
```

sympy [C] time = 1.74, size = 128, normalized size = 1.08

$$-\frac{\log\left(x+1\right)}{6}+\left(\frac{1}{12}-\frac{\sqrt{3}\,i}{12}\right)\log\left(x+\frac{1093}{244}-\frac{1093\sqrt{3}\,i}{244}+\frac{787320\left(\frac{1}{12}-\frac{\sqrt{3}\,i}{12}\right)^4}{61}\right)+\left(\frac{1}{12}+\frac{\sqrt{3}\,i}{12}\right)\log\left(x+\frac{1093}{244}+\frac{787320\left(\frac{1}{12}+\frac{\sqrt{3}\,i}{12}\right)^4}{61}+\frac{1093\sqrt{3}\,i}{244}\right)+\operatorname{RootSum}\left(52488t^3-1,\left(t\mapsto t\log\left(\frac{787320t^4}{61}+\frac{3279t}{61}+x\right)\right)\right)-\frac{1}{6x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(x**6+4*x**3+3),x)
```

```
[Out] -log(x + 1)/6 + (1/12 - sqrt(3)*I/12)*log(x + 1093/244 - 1093*sqrt(3)*I/244
 + 787320*(1/12 - sqrt(3)*I/12)**4/61) + (1/12 + sqrt(3)*I/12)*log(x + 1093
/244 + 787320*(1/12 + sqrt(3)*I/12)**4/61 + 1093*sqrt(3)*I/244) + RootSum(5
2488*_t**3 - 1, Lambda(_t, _t*log(787320*_t**4/61 + 3279*_t/61 + x))) - 1/(
6*x**2)
```

3.155 $\int \frac{1}{x^5\left(3+4x^3+x^6\right)}\,dx$

Optimal. Leaf size=126

$$-\frac{1}{12x^4}+\frac{1}{12}\log\left(x^2-x+1\right)-\frac{\log\left(x^2-\sqrt[3]{3}x+3^{2/3}\right)}{108\sqrt[3]{3}}+\frac{4}{9x}-\frac{1}{6}\log(x+1)+\frac{\log\left(x+\sqrt[3]{3}\right)}{54\sqrt[3]{3}}-\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}}+\frac{\tan^{-1}\left(\cdots\right.}{18\,3^{\cdots}}$$

Rubi [A] time = 0.10, antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 10, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.625, Rules used = {1368, 1504, 1510, 292, 31, 634, 618, 204, 628, 617}

$$-\frac{1}{12x^4}+\frac{1}{12}\log\left(x^2-x+1\right)-\frac{\log\left(x^2-\sqrt[3]{3}x+3^{2/3}\right)}{108\sqrt[3]{3}}+\frac{4}{9x}-\frac{1}{6}\log(x+1)+\frac{\log\left(x+\sqrt[3]{3}\right)}{54\sqrt[3]{3}}-\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}}+\frac{\tan^{-1}\left(\frac{\sqrt[3]{3}-2x}{3^{5/6}}\right)}{18\,3^{5/6}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^5*(3 + 4*x^3 + x^6)),x]
```

```
[Out] -1/(12*x^4) + 4/(9*x) - ArcTan[(1 - 2*x)/Sqrt[3]]/(2*Sqrt[3]) + ArcTan[(3^(
1/3) - 2*x)/3^(5/6)]/(18*3^(5/6)) - Log[1 + x]/6 + Log[3^(1/3) + x]/(54*3^(
1/3)) + Log[1 - x + x^2]/12 - Log[3^(2/3) - 3^(1/3)*x + x^2]/(108*3^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1368

```
Int[((d_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_
Symbol] :> Simp[((d*x)^(m + 1)*(a + b*x^n + c*x^(2*n))^(p + 1))/(a*d*(m + 1
)), x] - Dist[1/(a*d^n*(m + 1)), Int[(d*x)^(m + n)*(b*(m + n*(p + 1) + 1) +
 c*(m + 2*n*(p + 1) + 1)*x^n)*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a
, b, c, d, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && L
tQ[m, -1] && IntegerQ[p]
```

Rule 1504

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (
c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Simp[(d*(f*x)^(m + 1)*(a + b*x^n + c*x^
(2*n))^(p + 1))/(a*f*(m + 1)), x] + Dist[1/(a*f^n*(m + 1)), Int[(f*x)^(m +
n)*(a + b*x^n + c*x^(2*n))^p*Simp[a*e*(m + 1) - b*d*(m + n*(p + 1) + 1) - c
*d*(m + 2*n*(p + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x]
 && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && LtQ[m, -1] && Inte
gerQ[p]
```

Rule 1510

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_)))/((a_) + (b_.)*(x_)^(n_) +
 (c_.)*(x_)^(n2_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 +
(2*c*d - b*e)/(2*q), Int[(f*x)^m/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (
2*c*d - b*e)/(2*q), Int[(f*x)^m/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b
, c, d, e, f, m}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x^5\left(3+4x^3+x^6\right)}\,dx &= -\frac{1}{12x^4}+\frac{1}{12}\int \frac{-16-4x^3}{x^2\left(3+4x^3+x^6\right)}\,dx \\
&= -\frac{1}{12x^4}+\frac{4}{9x}-\frac{1}{36}\int \frac{x\left(-52-16x^3\right)}{3+4x^3+x^6}\,dx \\
&= -\frac{1}{12x^4}+\frac{4}{9x}-\frac{1}{18}\int \frac{x}{3+x^3}\,dx+\frac{1}{2}\int \frac{x}{1+x^3}\,dx \\
&= -\frac{1}{12x^4}+\frac{4}{9x}-\frac{1}{6}\int \frac{1}{1+x}\,dx+\frac{1}{6}\int \frac{1+x}{1-x+x^2}\,dx+\frac{\int \frac{1}{\sqrt[3]{3}+x}\,dx}{54\sqrt[3]{3}}-\frac{\int \frac{\sqrt[3]{3}+x}{3^{2/3}-\sqrt[3]{3}x+x^2}\,dx}{54\sqrt[3]{3}} \\
&= -\frac{1}{12x^4}+\frac{4}{9x}-\frac{1}{6}\log(1+x)+\frac{\log\left(\sqrt[3]{3}+x\right)}{54\sqrt[3]{3}}-\frac{1}{36}\int \frac{1}{3^{2/3}-\sqrt[3]{3}\,x+x^2}\,dx+\frac{1}{12}\int \cdots \\
&= -\frac{1}{12x^4}+\frac{4}{9x}-\frac{1}{6}\log(1+x)+\frac{\log\left(\sqrt[3]{3}+x\right)}{54\sqrt[3]{3}}+\frac{1}{12}\log\left(1-x+x^2\right)-\frac{\log\left(3^{2/3}-\sqrt[3]{3}\cdots\right)}{108\sqrt[3]{3}} \\
&= -\frac{1}{12x^4}+\frac{4}{9x}-\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}}+\frac{\tan^{-1}\left(\frac{\sqrt[3]{3}-2x}{3^{5/6}}\right)}{18\;3^{5/6}}-\frac{1}{6}\log(1+x)+\frac{\log\left(\sqrt[3]{3}+x\right)}{54\sqrt[3]{3}}+\frac{1}{12}\cdots
\end{aligned}
$$

Mathematica [A] time = 0.05, size = 118, normalized size = 0.94

$$\frac{1}{324}\left(-\frac{27}{x^4}+27\log\left(x^2-x+1\right)-3^{2/3}\log\left(\sqrt[3]{3}x^2-3^{2/3}x+3\right)+\frac{144}{x}-54\log(x+1)+2\,3^{2/3}\log\left(3^{2/3}x+3\right)+6\sqrt[6]{3}\tan^{-1}\left(\frac{\sqrt[3]{3}-2x}{3^{5/6}}\right)+54\sqrt{3}\tan^{-1}\left(\frac{2x-1}{\sqrt{3}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^5*(3 + 4*x^3 + x^6)),x]
```

```
[Out] (-27/x^4 + 144/x + 6*3^(1/6)*ArcTan[(3^(1/3) - 2*x)/3^(5/6)] + 54*Sqrt[3]*A
rcTan[(-1 + 2*x)/Sqrt[3]] - 54*Log[1 + x] + 2*3^(2/3)*Log[3 + 3^(2/3)*x] +
27*Log[1 - x + x^2] - 3^(2/3)*Log[3 - 3^(2/3)*x + 3^(1/3)*x^2])/324
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^5\left(3+4x^3+x^6\right)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x^5*(3 + 4*x^3 + x^6)),x]
```

```
[Out] IntegrateAlgebraic[1/(x^5*(3 + 4*x^3 + x^6)), x]
```

fricas [A] time = 1.16, size = 112, normalized size = 0.89

$$-\frac{3^{\frac{2}{3}}x^4\log\left(x^2-3^{\frac{1}{3}}x+3^{\frac{2}{3}}\right)-2\cdot 3^{\frac{2}{3}}x^4\log\left(x+3^{\frac{1}{3}}\right)-54\sqrt{3}x^4\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)-6\cdot 3^{\frac{1}{6}}x^4\arctan\left(-\frac{1}{3}\cdot 3^{\frac{1}{6}}\left(2x-3^{\frac{1}{3}}\right)\right)-27x^4\log\left(x^2-x+1\right)+54x^4\log(x+1)-144x^3+27}{324x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(x^6+4*x^3+3),x, algorithm="fricas")
```

```
[Out] -1/324*(3^(2/3)*x^4*log(x^2 - 3^(1/3)*x + 3^(2/3)) - 2*3^(2/3)*x^4*log(x +
3^(1/3)) - 54*sqrt(3)*x^4*arctan(1/3*sqrt(3)*(2*x - 1)) - 6*3^(1/6)*x^4*arc
tan(-1/3*3^(1/6)*(2*x - 3^(1/3))) - 27*x^4*log(x^2 - x + 1) + 54*x^4*log(x
+ 1) - 144*x^3 + 27)/x^4
```

giac [A] time = 0.36, size = 98, normalized size = 0.78

$$-\frac{1}{324} \cdot 3^{\frac{2}{3}} \log\left(x^2 - 3^{\frac{1}{3}}x + 3^{\frac{2}{3}}\right) + \frac{1}{162} \cdot 3^{\frac{2}{3}} \log\left(\left|x + 3^{\frac{1}{3}}\right|\right) + \frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right) - \frac{1}{54} \cdot 3^{\frac{1}{6}}\arctan\left(\frac{1}{3} \cdot 3^{\frac{1}{6}}\left(2x - 3^{\frac{1}{3}}\right)\right) + \frac{16x^3 - 3}{36x^4} + \frac{1}{12}\log\left(x^2 - x + 1\right) - \frac{1}{6}\log\left(|x+1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(x^6+4*x^3+3),x, algorithm="giac")
```

```
[Out] -1/324*3^(2/3)*log(x^2 - 3^(1/3)*x + 3^(2/3)) + 1/162*3^(2/3)*log(abs(x + 3
^(1/3))) + 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) - 1/54*3^(1/6)*arctan(
1/3*3^(1/6)*(2*x - 3^(1/3))) + 1/36*(16*x^3 - 3)/x^4 + 1/12*log(x^2 - x + 1
) - 1/6*log(abs(x + 1))
```

maple [A] time = 0.01, size = 94, normalized size = 0.75

$$-\frac{3^{\frac{1}{6}}\arctan\left(\frac{\sqrt{3}\left(\frac{2\,3^{\frac{2}{3}}x}{3}-1\right)}{3}\right)}{54} + \frac{\sqrt{3}\,\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{6} - \frac{\ln(x+1)}{6} + \frac{3^{\frac{2}{3}}\ln\left(x + 3^{\frac{1}{3}}\right)}{162} - \frac{3^{\frac{2}{3}}\ln\left(x^2 - 3^{\frac{1}{3}}x + 3^{\frac{2}{3}}\right)}{324} + \frac{\ln\left(x^2 - x + 1\right)}{12} + \frac{4}{9x} - \frac{1}{12x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^5/(x^6+4*x^3+3),x)
```

```
[Out] -1/12/x^4+4/9/x-1/6*ln(x+1)+1/162*3^(2/3)*ln(x+3^(1/3))-1/324*3^(2/3)*ln(x^
2-3^(1/3)*x+3^(2/3))-1/54*3^(1/6)*arctan(1/3*3^(1/2)*(2/3*3^(2/3)*x-1))+1/1
2*ln(x^2-x+1)+1/6*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))
```

maxima [A] time = 1.20, size = 96, normalized size = 0.76

$$-\frac{1}{324} \cdot 3^{\frac{2}{3}} \log\left(x^2 - 3^{\frac{1}{3}}x + 3^{\frac{2}{3}}\right) + \frac{1}{162} \cdot 3^{\frac{2}{3}} \log\left(x + 3^{\frac{1}{3}}\right) + \frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right) - \frac{1}{54} \cdot 3^{\frac{1}{6}}\arctan\left(\frac{1}{3} \cdot 3^{\frac{1}{6}}\left(2x - 3^{\frac{1}{3}}\right)\right) + \frac{16x^3 - 3}{36x^4} + \frac{1}{12}\log\left(x^2 - x + 1\right) - \frac{1}{6}\log(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(x^6+4*x^3+3),x, algorithm="maxima")
```

```
[Out] -1/324*3^(2/3)*log(x^2 - 3^(1/3)*x + 3^(2/3)) + 1/162*3^(2/3)*log(x + 3^(1/
3)) + 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) - 1/54*3^(1/6)*arctan(1/3*3
^(1/6)*(2*x - 3^(1/3))) + 1/36*(16*x^3 - 3)/x^4 + 1/12*log(x^2 - x + 1) - 1
/6*log(x + 1)
```

mupad [B] time = 0.19, size = 124, normalized size = 0.98

$$\frac{3^{2/3}\ln\left(x + 3^{1/3}\right)}{162} - \frac{\ln(x+1)}{6} - \ln\left(x - \frac{1}{2} - \frac{\sqrt{3}\,1i}{2}\right)\left(-\frac{1}{12} + \frac{\sqrt{3}\,1i}{12}\right) + \ln\left(x - \frac{1}{2} + \frac{\sqrt{3}\,1i}{2}\right)\left(\frac{1}{12} + \frac{\sqrt{3}\,1i}{12}\right) + \frac{\frac{4x^3}{9} - \frac{1}{12}}{x^4} - \ln\left(x - \frac{3^{1/3}}{2} - \frac{3^{5/6}\,1i}{2}\right)\left(\frac{3^{2/3}}{324} - \frac{3^{1/6}\,1i}{108}\right) - \ln\left(x - \frac{3^{1/3}}{2} + \frac{3^{5/6}\,1i}{2}\right)\left(\frac{3^{2/3}}{324} + \frac{3^{1/6}\,1i}{108}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^5*(4*x^3 + x^6 + 3)),x)
```

```
[Out] (3^(2/3)*log(x + 3^(1/3)))/162 - log(x + 1)/6 - log(x - (3^(1/2)*1i)/2 - 1/
2)*((3^(1/2)*1i)/12 - 1/12) + log(x + (3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/1
2 + 1/12) + ((4*x^3)/9 - 1/12)/x^4 - log(x - 3^(1/3)/2 - (3^(5/6)*1i)/2)*(3
^(2/3)/324 - (3^(1/6)*1i)/108) - log(x - 3^(1/3)/2 + (3^(5/6)*1i)/2)*(3^(2/
3)/324 + (3^(1/6)*1i)/108)
```

sympy [C] time = 1.84, size = 141, normalized size = 1.12

$$-\frac{\log(x+1)}{6} + \left(\frac{1}{12} - \frac{\sqrt{3}i}{12}\right)\log\left(x + \frac{4782978\left(\frac{1}{12} - \frac{\sqrt{3}i}{12}\right)^2}{547} + \frac{1028869776\left(\frac{1}{12} - \frac{\sqrt{3}i}{12}\right)^5}{547}\right) + \left(\frac{1}{12} + \frac{\sqrt{3}i}{12}\right)\log\left(x + \frac{1028869776\left(\frac{1}{12} + \frac{\sqrt{3}i}{12}\right)^5}{547} + \frac{4782978\left(\frac{1}{12} + \frac{\sqrt{3}i}{12}\right)^2}{547}\right) + \operatorname{RootSum}\left(472392t^3 - 1, \left(t \mapsto t\log\left(\frac{1028869776t^5}{547} + \frac{4782978t^2}{547} + x\right)\right)\right) + \frac{16x^3 - 3}{36x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**5/(x**6+4*x**3+3),x)
```

```
[Out] -log(x + 1)/6 + (1/12 - sqrt(3)*I/12)*log(x + 4782978*(1/12 - sqrt(3)*I/12)
**2/547 + 1028869776*(1/12 - sqrt(3)*I/12)**5/547) + (1/12 + sqrt(3)*I/12)*
log(x + 1028869776*(1/12 + sqrt(3)*I/12)**5/547 + 4782978*(1/12 + sqrt(3)*I
/12)**2/547) + RootSum(472392*_t**3 - 1, Lambda(_t, _t*log(1028869776*_t**5
/547 + 4782978*_t**2/547 + x))) + (16*x**3 - 3)/(36*x**4)
```

3.156 $\int \frac{1}{x^6\left(3+4x^3+x^6\right)}\,dx$

Optimal. Leaf size=126

$$-\frac{1}{15x^5}+\frac{2}{9x^2}-\frac{1}{12}\log\left(x^2-x+1\right)+\frac{\log\left(x^2-\sqrt[3]{3}\,x+3^{2/3}\right)}{108\ 3^{2/3}}+\frac{1}{6}\log(x+1)-\frac{\log\left(x+\sqrt[3]{3}\right)}{54\ 3^{2/3}}-\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}}+\frac{\tan^{-1}\left(\frac{\sqrt[3]{3}\ldots}{3^{5\ldots}}\right)}{54\sqrt[6]{3}}$$

Rubi [A] time = 0.10, antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 10, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.625, Rules used = {1368, 1504, 1422, 200, 31, 634, 618, 204, 628, 617}

$$\frac{2}{9x^2}-\frac{1}{15x^5}-\frac{1}{12}\log\left(x^2-x+1\right)+\frac{\log\left(x^2-\sqrt[3]{3}\,x+3^{2/3}\right)}{108\ 3^{2/3}}+\frac{1}{6}\log(x+1)-\frac{\log\left(x+\sqrt[3]{3}\right)}{54\ 3^{2/3}}-\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}}+\frac{\tan^{-1}\left(\frac{\sqrt[3]{3}-2x}{3^{5/6}}\right)}{54\sqrt[6]{3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^6*(3 + 4*x^3 + x^6)),x]
```

```
[Out] -1/(15*x^5) + 2/(9*x^2) - ArcTan[(1 - 2*x)/Sqrt[3]]/(2*Sqrt[3]) + ArcTan[(3
^(1/3) - 2*x)/3^(5/6)]/(54*3^(1/6)) + Log[1 + x]/6 - Log[3^(1/3) + x]/(54*3
^(2/3)) - Log[1 - x + x^2]/12 + Log[3^(2/3) - 3^(1/3)*x + x^2]/(108*3^(2/3)
)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1368

```
Int[((d_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_
Symbol] :> Simp[((d*x)^(m + 1)*(a + b*x^n + c*x^(2*n))^(p + 1))/(a*d*(m + 1
)), x] - Dist[1/(a*d^n*(m + 1)), Int[(d*x)^(m + n)*(b*(m + n*(p + 1) + 1) +
 c*(m + 2*n*(p + 1) + 1)*x^n)*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a
, b, c, d, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && L
tQ[m, -1] && IntegerQ[p]
```

Rule 1422

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q),
Int[1/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(
b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (PosQ[b^2 - 4*a
*c] ||  !IGtQ[n/2, 0])
```

Rule 1504

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (
c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Simp[(d*(f*x)^(m + 1)*(a + b*x^n + c*x^
(2*n))^(p + 1))/(a*f*(m + 1)), x] + Dist[1/(a*f^n*(m + 1)), Int[(f*x)^(m +
n)*(a + b*x^n + c*x^(2*n))^p*Simp[a*e*(m + 1) - b*d*(m + n*(p + 1) + 1) - c
*d*(m + 2*n*(p + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x]
 && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && LtQ[m, -1] && Inte
gerQ[p]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x^6\left(3+4x^3+x^6\right)}\,dx &= -\frac{1}{15x^5}+\frac{1}{15}\int \frac{-20-5x^3}{x^3\left(3+4x^3+x^6\right)}\,dx \\
&= -\frac{1}{15x^5}+\frac{2}{9x^2}-\frac{1}{90}\int \frac{-130-40x^3}{3+4x^3+x^6}\,dx \\
&= -\frac{1}{15x^5}+\frac{2}{9x^2}-\frac{1}{18}\int \frac{1}{3+x^3}\,dx+\frac{1}{2}\int \frac{1}{1+x^3}\,dx \\
&= -\frac{1}{15x^5}+\frac{2}{9x^2}+\frac{1}{6}\int \frac{1}{1+x}\,dx+\frac{1}{6}\int \frac{2-x}{1-x+x^2}\,dx-\frac{\int \frac{1}{\sqrt[3]{3}+x}\,dx}{54\ 3^{2/3}}-\frac{\int \frac{2\sqrt[3]{3}-x}{3^{2/3}-\sqrt[3]{3}x+x^2}\,dx}{54\ 3^{2/3}} \\
&= -\frac{1}{15x^5}+\frac{2}{9x^2}+\frac{1}{6}\log(1+x)-\frac{\log\left(\sqrt[3]{3}+x\right)}{54\ 3^{2/3}}-\frac{1}{12}\int \frac{-1+2x}{1-x+x^2}\,dx+\frac{1}{4}\int \frac{1}{1-x+x^2} \\
&= -\frac{1}{15x^5}+\frac{2}{9x^2}+\frac{1}{6}\log(1+x)-\frac{\log\left(\sqrt[3]{3}+x\right)}{54\ 3^{2/3}}-\frac{1}{12}\log\left(1-x+x^2\right)+\frac{\log\left(3^{2/3}-\sqrt[3]{3}x\right.}{108\ 3^{2/3}} \\
&= -\frac{1}{15x^5}+\frac{2}{9x^2}-\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}}+\frac{\tan^{-1}\left(\frac{\sqrt[3]{3}-2x}{3^{5/6}}\right)}{54\sqrt[6]{3}}+\frac{1}{6}\log(1+x)-\frac{\log\left(\sqrt[3]{3}+x\right)}{54\ 3^{2/3}}-\frac{1}{12}\text{lo}
\end{aligned}
$$

Mathematica [A] time = 0.06, size = 118, normalized size = 0.94

$$
\frac{-\frac{108}{x^5}+\frac{360}{x^2}-135\log\left(x^2-x+1\right)+5\sqrt[3]{3}\log\left(\sqrt[3]{3}x^2-3^{2/3}x+3\right)+270\log(x+1)-10\sqrt[3]{3}\log\left(3^{2/3}x+3\right)+10\ 3^{5/6}\tan^{-1}\left(\frac{\sqrt[3]{3}-2x}{3^{5/6}}\right)+270\sqrt{3}\tan^{-1}\left(\frac{2x-1}{\sqrt{3}}\right)}{1620}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^6*(3 + 4*x^3 + x^6)),x]
```

```
[Out] (-108/x^5 + 360/x^2 + 10*3^(5/6)*ArcTan[(3^(1/3) - 2*x)/3^(5/6)] + 270*Sqrt
[3]*ArcTan[(-1 + 2*x)/Sqrt[3]] + 270*Log[1 + x] - 10*3^(1/3)*Log[3 + 3^(2/3
)*x] - 135*Log[1 - x + x^2] + 5*3^(1/3)*Log[3 - 3^(2/3)*x + 3^(1/3)*x^2])/1
620
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$
\int \frac{1}{x^6\left(3+4x^3+x^6\right)}\,dx
$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x^6*(3 + 4*x^3 + x^6)),x]
```

```
[Out] IntegrateAlgebraic[1/(x^6*(3 + 4*x^3 + x^6)), x]
```

fricas [A] time = 1.15, size = 153, normalized size = 1.21

$$
\frac{30\cdot 9^{\frac{1}{6}}\sqrt{3}\,(-1)^{\frac{1}{3}}x^5\arctan\left(\frac{1}{27}\cdot 9^{\frac{1}{6}}\left(2\cdot 9^{\frac{2}{3}}\sqrt{3}\,(-1)^{\frac{2}{3}}x-3\cdot 9^{\frac{1}{3}}\sqrt{3}\right)\right)-5\cdot 9^{\frac{2}{3}}(-1)^{\frac{1}{3}}x^5\log\left(9^{\frac{2}{3}}(-1)^{\frac{1}{3}}x+3x^2+3\cdot 9^{\frac{1}{3}}(-1)^{\frac{2}{3}}\right)+10\cdot 9^{\frac{2}{3}}(-1)^{\frac{1}{3}}x^5\log\left(-9^{\frac{2}{3}}(-1)^{\frac{1}{3}}+3x\right)+810\sqrt{3}x^5\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)-405x^5\log\left(x^2-x+1\right)+810x^5\log(x+1)+1080x^3-324}{4860x^5}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(x^6+4*x^3+3),x, algorithm="fricas")
```

```
[Out] 1/4860*(30*9^(1/6)*sqrt(3)*(-1)^(1/3)*x^5*arctan(1/27*9^(1/6)*(2*9^(2/3)*sq
rt(3)*(-1)^(2/3)*x - 3*9^(1/3)*sqrt(3))) - 5*9^(2/3)*(-1)^(1/3)*x^5*log(9^(
2/3)*(-1)^(1/3)*x + 3*x^2 + 3*9^(1/3)*(-1)^(2/3)) + 10*9^(2/3)*(-1)^(1/3)*x
^5*log(-9^(2/3)*(-1)^(1/3) + 3*x) + 810*sqrt(3)*x^5*arctan(1/3*sqrt(3)*(2*x
```

```
- 1)) - 405*x^5*log(x^2 - x + 1) + 810*x^5*log(x + 1) + 1080*x^3 - 324)/x^
5
```

giac [A] time = 0.36, size = 98, normalized size = 0.78

$$-\frac{1}{162} \cdot 3^{\frac{5}{6}} \arctan\left(\frac{1}{3} \cdot 3^{\frac{1}{6}} \left(2 x - 3^{\frac{1}{3}}\right)\right) + \frac{1}{6} \sqrt{3} \arctan\left(\frac{1}{3} \sqrt{3} (2 x - 1)\right) + \frac{1}{324} \cdot 3^{\frac{1}{3}} \log\left(x^2 - 3^{\frac{1}{3}} x + 3^{\frac{2}{3}}\right) - \frac{1}{162} \cdot 3^{\frac{1}{3}} \log\left(\left|x + 3^{\frac{1}{3}}\right|\right) + \frac{10 x^3 - 3}{45 x^5} - \frac{1}{12} \log\left(x^2 - x + 1\right) + \frac{1}{6} \log\left(|x + 1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(x^6+4*x^3+3),x, algorithm="giac")
```

```
[Out] -1/162*3^(5/6)*arctan(1/3*3^(1/6)*(2*x - 3^(1/3))) + 1/6*sqrt(3)*arctan(1/3
*sqrt(3)*(2*x - 1)) + 1/324*3^(1/3)*log(x^2 - 3^(1/3)*x + 3^(2/3)) - 1/162*
3^(1/3)*log(abs(x + 3^(1/3))) + 1/45*(10*x^3 - 3)/x^5 - 1/12*log(x^2 - x +
1) + 1/6*log(abs(x + 1))
```

maple [A] time = 0.01, size = 94, normalized size = 0.75

$$-\frac{3^{\frac{5}{6}} \arctan\left(\frac{\sqrt{3}\left(\frac{2\, 3^{\frac{2}{3}} x}{3} - 1\right)}{3}\right)}{162} + \frac{\sqrt{3}\, \arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{6} + \frac{\ln(x+1)}{6} - \frac{3^{\frac{1}{3}} \ln\left(x + 3^{\frac{1}{3}}\right)}{162} + \frac{3^{\frac{1}{3}} \ln\left(x^2 - 3^{\frac{1}{3}} x + 3^{\frac{2}{3}}\right)}{324} - \frac{\ln\left(x^2 - x + 1\right)}{12} + \frac{2}{9x^2} - \frac{1}{15x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^6/(x^6+4*x^3+3),x)
```

```
[Out] -1/15/x^5+2/9/x^2+1/6*ln(x+1)-1/162*3^(1/3)*ln(x+3^(1/3))+1/324*3^(1/3)*ln(
x^2-3^(1/3)*x+3^(2/3))-1/162*3^(5/6)*arctan(1/3*3^(1/2)*(2/3*3^(2/3)*x-1))-
1/12*ln(x^2-x+1)+1/6*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))
```

maxima [A] time = 1.66, size = 96, normalized size = 0.76

$$-\frac{1}{162} \cdot 3^{\frac{5}{6}} \arctan\left(\frac{1}{3} \cdot 3^{\frac{1}{6}} \left(2 x - 3^{\frac{1}{3}}\right)\right) + \frac{1}{6} \sqrt{3} \arctan\left(\frac{1}{3} \sqrt{3} (2 x - 1)\right) + \frac{1}{324} \cdot 3^{\frac{1}{3}} \log\left(x^2 - 3^{\frac{1}{3}} x + 3^{\frac{2}{3}}\right) - \frac{1}{162} \cdot 3^{\frac{1}{3}} \log\left(x + 3^{\frac{1}{3}}\right) + \frac{10 x^3 - 3}{45 x^5} - \frac{1}{12} \log\left(x^2 - x + 1\right) + \frac{1}{6} \log\left(x + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(x^6+4*x^3+3),x, algorithm="maxima")
```

```
[Out] -1/162*3^(5/6)*arctan(1/3*3^(1/6)*(2*x - 3^(1/3))) + 1/6*sqrt(3)*arctan(1/3
*sqrt(3)*(2*x - 1)) + 1/324*3^(1/3)*log(x^2 - 3^(1/3)*x + 3^(2/3)) - 1/162*
3^(1/3)*log(x + 3^(1/3)) + 1/45*(10*x^3 - 3)/x^5 - 1/12*log(x^2 - x + 1) +
1/6*log(x + 1)
```

mupad [B] time = 1.40, size = 121, normalized size = 0.96

$$\frac{\ln(x+1)}{6} - \frac{3^{1/3} \ln\left(x + 3^{1/3}\right)}{162} - \ln\left(x - \frac{1}{2} - \frac{\sqrt{3}\, 1\mathrm{i}}{2}\right)\left(\frac{1}{12} + \frac{\sqrt{3}\, 1\mathrm{i}}{12}\right) + \ln\left(x - \frac{1}{2} + \frac{\sqrt{3}\, 1\mathrm{i}}{2}\right)\left(-\frac{1}{12} + \frac{\sqrt{3}\, 1\mathrm{i}}{12}\right) + \frac{\frac{2 x^3}{9} - \frac{1}{15}}{x^5} + \frac{(-1)^{1/3}\, 3^{1/3} \ln\left(x - (-1)^{1/3}\, 3^{1/3}\right)}{162} - \frac{(-1)^{1/3} \ln\left(x + \frac{(-1)^{1/3}\, 3^{1/3}}{2} + \frac{(-1)^{1/3}\, 3^{5/6}\, 1\mathrm{i}}{2}\right)\left(3^{1/3} + 3^{5/6}\, 1\mathrm{i}\right)}{324}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^6*(4*x^3 + x^6 + 3)),x)
```

```
[Out] log(x + 1)/6 - (3^(1/3)*log(x + 3^(1/3)))/162 - log(x - (3^(1/2)*1i)/2 - 1/
2)*((3^(1/2)*1i)/12 + 1/12) + log(x + (3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/1
2 - 1/12) + ((2*x^3)/9 - 1/15)/x^5 + ((-1)^(1/3)*3^(1/3)*log(x - (-1)^(1/3)
*3^(1/3)))/162 - ((-1)^(1/3)*log(x + ((-1)^(1/3)*3^(1/3))/2 + ((-1)^(1/3)*3
^(5/6)*1i)/2)*(3^(1/3) + 3^(5/6)*1i))/324
```

sympy [C] time = 1.78, size = 136, normalized size = 1.08

$$\frac{\log(x+1)}{6} + \left(-\frac{1}{12} + \frac{\sqrt{3} i}{12}\right) \log\left(x + \frac{88573}{6562} - \frac{88573\sqrt{3} i}{6562} + \frac{119042784 \left(-\frac{1}{12} + \frac{\sqrt{3} i}{12}\right)^4}{3281}\right) + \left(-\frac{1}{12} - \frac{\sqrt{3} i}{12}\right) \log\left(x + \frac{88573}{6562} + \frac{119042784 \left(-\frac{1}{12} - \frac{\sqrt{3} i}{12}\right)^4}{3281} + \frac{88573\sqrt{3} i}{6562}\right) + \operatorname{RootSum}\left(1417176 t^3 + 1, \left(t \mapsto t \log\left(\frac{119042784 t^4}{3281} - \frac{531438 t}{3281} + x\right)\right)\right) + \frac{10 x^3 - 3}{45 x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**6/(x**6+4*x**3+3),x)
```

```
[Out] log(x + 1)/6 + (-1/12 + sqrt(3)*I/12)*log(x + 88573/6562 - 88573*sqrt(3)*I/
6562 + 119042784*(-1/12 + sqrt(3)*I/12)**4/3281) + (-1/12 - sqrt(3)*I/12)*l
og(x + 88573/6562 + 119042784*(-1/12 - sqrt(3)*I/12)**4/3281 + 88573*sqrt(3
)*I/6562) + RootSum(1417176*_t**3 + 1, Lambda(_t, _t*log(119042784*_t**4/32
81 - 531438*_t/3281 + x))) + (10*x**3 - 3)/(45*x**5)
```

3.157 $\int \frac{x^6}{1-x^3+x^6}\,dx$

Optimal. Leaf size=412

$$-\frac{\left(3-i\sqrt{3}\right)\log\left(2^{2/3}x^2+\sqrt[3]{2\left(1-i\sqrt{3}\right)}\,x+\left(1-i\sqrt{3}\right)^{2/3}\right)}{18\sqrt[3]{2}\left(1-i\sqrt{3}\right)^{2/3}}-\frac{\left(3+i\sqrt{3}\right)\log\left(2^{2/3}x^2+\sqrt[3]{2\left(1+i\sqrt{3}\right)}\,x+\left(1+i\sqrt{3}\right.\right.}{18\sqrt[3]{2}\left(1+i\sqrt{3}\right)^{2/3}}$$

Rubi [A] time = 0.43, antiderivative size = 412, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 8, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {1367, 1422, 200, 31, 634, 617, 204, 628}

$$-\frac{\left(3-i\sqrt{3}\right)\log\left(2^{2/3}x^2+\sqrt[3]{2\left(1-i\sqrt{3}\right)}\,x+\left(1-i\sqrt{3}\right)^{2/3}\right)}{18\sqrt[3]{2}\left(1-i\sqrt{3}\right)^{2/3}}-\frac{\left(3+i\sqrt{3}\right)\log\left(2^{2/3}x^2+\sqrt[3]{2\left(1+i\sqrt{3}\right)}\,x+\left(1+i\sqrt{3}\right)^{2/3}\right)}{18\sqrt[3]{2}\left(1+i\sqrt{3}\right)^{2/3}}+x+\frac{\left(3-i\sqrt{3}\right)\log\left(-\sqrt[3]{2}x+\sqrt[3]{1-i\sqrt{3}}\right)}{9\sqrt[3]{2}\left(1-i\sqrt{3}\right)^{2/3}}+\frac{\left(3+i\sqrt{3}\right)\log\left(-\sqrt[3]{2}x+\sqrt[3]{1+i\sqrt{3}}\right)}{9\sqrt[3]{2}\left(1+i\sqrt{3}\right)^{2/3}}+\frac{\left(-\sqrt{3}+i\right)\tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\sqrt[3]{2}\left(1-i\sqrt{3}\right)^{2/3}}-\frac{\left(\sqrt{3}+i\right)\tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\sqrt[3]{2}\left(1+i\sqrt{3}\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^6/(1 - x^3 + x^6),x]
```

```
[Out] x + ((I - Sqrt[3])*ArcTan[(1 + (2*x)/((1 - I*Sqrt[3])/2)^(1/3))/Sqrt[3]])/(
3*2^(1/3)*(1 - I*Sqrt[3])^(2/3)) - ((I + Sqrt[3])*ArcTan[(1 + (2*x)/((1 + I
*Sqrt[3])/2)^(1/3))/Sqrt[3]])/(3*2^(1/3)*(1 + I*Sqrt[3])^(2/3)) + ((3 - I*S
qrt[3])*Log[(1 - I*Sqrt[3])^(1/3) - 2^(1/3)*x])/(9*2^(1/3)*(1 - I*Sqrt[3])^
(2/3)) + ((3 + I*Sqrt[3])*Log[(1 + I*Sqrt[3])^(1/3) - 2^(1/3)*x])/(9*2^(1/3
)*(1 + I*Sqrt[3])^(2/3)) - ((3 - I*Sqrt[3])*Log[(1 - I*Sqrt[3])^(2/3) + (2*
(1 - I*Sqrt[3]))^(1/3)*x + 2^(2/3)*x^2])/(18*2^(1/3)*(1 - I*Sqrt[3])^(2/3))
 - ((3 + I*Sqrt[3])*Log[(1 + I*Sqrt[3])^(2/3) + (2*(1 + I*Sqrt[3]))^(1/3)*x
 + 2^(2/3)*x^2])/(18*2^(1/3)*(1 + I*Sqrt[3])^(2/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1367

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Simp[(d^(2*n - 1)*(d*x)^(m - 2*n + 1)*(a + b*x^n + c*x^(2*n))^(
p + 1))/(c*(m + 2*n*p + 1)), x] - Dist[d^(2*n)/(c*(m + 2*n*p + 1)), Int[(d*
x)^(m - 2*n)*Simp[a*(m - 2*n + 1) + b*(m + n*(p - 1) + 1)*x^n, x]*(a + b*x^
n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && EqQ[n2, 2*n] && Ne
Q[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GtQ[m, 2*n - 1] && NeQ[m + 2*n*p + 1, 0]
 && IntegerQ[p]
```

Rule 1422

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q),
Int[1/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(
b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (PosQ[b^2 - 4*a
*c] ||  !IGtQ[n/2, 0])
```

Rubi steps

$$\int \frac{x^6}{1-x^3+x^6}\,dx = x-\int \frac{1-x^3}{1-x^3+x^6}\,dx$$

$$= x-\frac{1}{6}\left(-3+i\sqrt{3}\right)\int \frac{1}{-\frac{1}{2}+\frac{i\sqrt{3}}{2}+x^3}\,dx+\frac{1}{6}\left(3+i\sqrt{3}\right)\int \frac{1}{-\frac{1}{2}-\frac{i\sqrt{3}}{2}+x^3}\,dx$$

$$= x+\frac{\left(3-i\sqrt{3}\right)\int \frac{1}{-\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}+x}\,dx}{9\sqrt[3]{2}\left(1-i\sqrt{3}\right)^{2/3}}+\frac{\left(3-i\sqrt{3}\right)\int \frac{-2^{2/3}\sqrt[3]{1-i\sqrt{3}}-x}{\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}+\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}\,x+x^2}\,dx}{9\sqrt[3]{2}\left(1-i\sqrt{3}\right)^{2/3}}+\frac{\left(3+i\sqrt{3}\right)}{9\sqrt[3]{2}}$$

$$= x+\frac{\left(3-i\sqrt{3}\right)\log\left(\sqrt[3]{1-i\sqrt{3}}-\sqrt[3]{2}\,x\right)}{9\sqrt[3]{2}\left(1-i\sqrt{3}\right)^{2/3}}+\frac{\left(3+i\sqrt{3}\right)\log\left(\sqrt[3]{1+i\sqrt{3}}-\sqrt[3]{2}\,x\right)}{9\sqrt[3]{2}\left(1+i\sqrt{3}\right)^{2/3}}-\frac{\left(3-i\sqrt{3}\right)\int}{}$$

$$= x+\frac{\left(3-i\sqrt{3}\right)\log\left(\sqrt[3]{1-i\sqrt{3}}-\sqrt[3]{2}\,x\right)}{9\sqrt[3]{2}\left(1-i\sqrt{3}\right)^{2/3}}+\frac{\left(3+i\sqrt{3}\right)\log\left(\sqrt[3]{1+i\sqrt{3}}-\sqrt[3]{2}\,x\right)}{9\sqrt[3]{2}\left(1+i\sqrt{3}\right)^{2/3}}-\frac{\left(3-i\sqrt{3}\right)\mathrm{lo}}{}$$

$$= x+\frac{\left(i-\sqrt{3}\right)\tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\sqrt[3]{2}\left(1-i\sqrt{3}\right)^{2/3}}-\frac{\left(i+\sqrt{3}\right)\tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\sqrt[3]{2}\left(1+i\sqrt{3}\right)^{2/3}}+\frac{\left(3-i\sqrt{3}\right)\log\left(\sqrt[3]{1-i}\right.}{9\sqrt[3]{2}\left(1-i\sqrt{3}\right.}$$

Mathematica [C] time = 0.01, size = 59, normalized size = 0.14

$$\frac{1}{3}\text{RootSum}\left[\#1^6-\#1^3+1\&,\frac{\#1^3\log(x-\#1)-\log(x-\#1)}{2\#1^5-\#1^2}\&\right]+x$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6/(1 - x^3 + x^6),x]
```

```
[Out] x + RootSum[1 - #1^3 + #1^6 & , (-Log[x - #1] + Log[x - #1]*#1^3)/(-#1^2 +
2*#1^5) & ]/3
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6}{1-x^3+x^6}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^6/(1 - x^3 + x^6),x]
```

```
[Out] IntegrateAlgebraic[x^6/(1 - x^3 + x^6), x]
```

fricas [B] time = 1.38, size = 1028, normalized size = 2.50

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(x^6-x^3+1),x, algorithm="fricas")
```

```
[Out] 1/54*18^(2/3)*12^(1/6)*cos(2/3*arctan(sqrt(3) - 2))*log(18^(2/3)*12^(1/6)*s
qrt(3)*x*sin(2/3*arctan(sqrt(3) - 2)) - 3*18^(2/3)*12^(1/6)*x*cos(2/3*arcta
n(sqrt(3) - 2)) + 3*18^(1/3)*12^(1/3)*cos(2/3*arctan(sqrt(3) - 2))^2 + 3*18
^(1/3)*12^(1/3)*sin(2/3*arctan(sqrt(3) - 2))^2 + 18*x^2) + 2/27*18^(2/3)*12
^(1/6)*arctan(-1/108*(6*18^(1/3)*12^(5/6)*sqrt(3)*x*cos(2/3*arctan(sqrt(3)
- 2)) - 108*sqrt(3)*cos(2/3*arctan(sqrt(3) - 2))^2 - 108*sqrt(3)*sin(2/3*ar
ctan(sqrt(3) - 2))^2 - 18*(18^(1/3)*12^(5/6)*x - 24*cos(2/3*arctan(sqrt(3)
- 2)))*sin(2/3*arctan(sqrt(3) - 2)) - sqrt(18^(2/3)*12^(1/6)*sqrt(3)*x*sin(
2/3*arctan(sqrt(3) - 2)) - 3*18^(2/3)*12^(1/6)*x*cos(2/3*arctan(sqrt(3) - 2
)) + 3*18^(1/3)*12^(1/3)*cos(2/3*arctan(sqrt(3) - 2))^2 + 3*18^(1/3)*12^(1/
3)*sin(2/3*arctan(sqrt(3) - 2))^2 + 18*x^2)*(18^(1/3)*12^(5/6)*sqrt(3)*sqrt
(2)*cos(2/3*arctan(sqrt(3) - 2)) - 3*18^(1/3)*12^(5/6)*sqrt(2)*sin(2/3*arct
an(sqrt(3) - 2))))/(cos(2/3*arctan(sqrt(3) - 2))^2 - 3*sin(2/3*arctan(sqrt(
3) - 2))^2))*sin(2/3*arctan(sqrt(3) - 2)) - 1/27*(18^(2/3)*12^(1/6)*sqrt(3)
*cos(2/3*arctan(sqrt(3) - 2)) - 18^(2/3)*12^(1/6)*sin(2/3*arctan(sqrt(3) -
2)))*arctan(1/108*(6*18^(1/3)*12^(5/6)*sqrt(3)*x*cos(2/3*arctan(sqrt(3) - 2
)) + 108*sqrt(3)*cos(2/3*arctan(sqrt(3) - 2))^2 + 108*sqrt(3)*sin(2/3*arcta
n(sqrt(3) - 2))^2 + 18*(18^(1/3)*12^(5/6)*x + 24*cos(2/3*arctan(sqrt(3) - 2
)))*sin(2/3*arctan(sqrt(3) - 2)) - sqrt(18^(2/3)*12^(1/6)*sqrt(3)*x*sin(2/3
*arctan(sqrt(3) - 2)) + 3*18^(2/3)*12^(1/6)*x*cos(2/3*arctan(sqrt(3) - 2))
+ 3*18^(1/3)*12^(1/3)*cos(2/3*arctan(sqrt(3) - 2))^2 + 3*18^(1/3)*12^(1/3)*
sin(2/3*arctan(sqrt(3) - 2))^2 + 18*x^2)*(18^(1/3)*12^(5/6)*sqrt(3)*sqrt(2)
*cos(2/3*arctan(sqrt(3) - 2)) + 3*18^(1/3)*12^(5/6)*sqrt(2)*sin(2/3*arctan(
sqrt(3) - 2))))/(cos(2/3*arctan(sqrt(3) - 2))^2 - 3*sin(2/3*arctan(sqrt(3)
- 2))^2)) + 1/27*(18^(2/3)*12^(1/6)*sqrt(3)*cos(2/3*arctan(sqrt(3) - 2)) +
18^(2/3)*12^(1/6)*sin(2/3*arctan(sqrt(3) - 2)))*arctan(1/216*(18^(1/3)*12^(
5/6)*sqrt(3)*sqrt(2)*sqrt(-2*18^(2/3)*12^(1/6)*sqrt(3)*x*sin(2/3*arctan(sqr
t(3) - 2)) + 3*18^(1/3)*12^(1/3)*cos(2/3*arctan(sqrt(3) - 2))^2 + 3*18^(1/3
)*12^(1/3)*sin(2/3*arctan(sqrt(3) - 2))^2 + 18*x^2) - 6*18^(1/3)*12^(5/6)*s
qrt(3)*x + 216*sin(2/3*arctan(sqrt(3) - 2)))/cos(2/3*arctan(sqrt(3) - 2)))
- 1/108*(18^(2/3)*12^(1/6)*sqrt(3)*sin(2/3*arctan(sqrt(3) - 2)) + 18^(2/3)*
12^(1/6)*cos(2/3*arctan(sqrt(3) - 2)))*log(18^(2/3)*12^(1/6)*sqrt(3)*x*sin(
```

```
2/3*arctan(sqrt(3) - 2)) + 3*18^(2/3)*12^(1/6)*x*cos(2/3*arctan(sqrt(3) - 2
)) + 3*18^(1/3)*12^(1/3)*cos(2/3*arctan(sqrt(3) - 2))^2 + 3*18^(1/3)*12^(1/
3)*sin(2/3*arctan(sqrt(3) - 2))^2 + 18*x^2) + 1/108*(18^(2/3)*12^(1/6)*sqrt
(3)*sin(2/3*arctan(sqrt(3) - 2)) - 18^(2/3)*12^(1/6)*cos(2/3*arctan(sqrt(3)
 - 2)))*log(-2*18^(2/3)*12^(1/6)*sqrt(3)*x*sin(2/3*arctan(sqrt(3) - 2)) + 3
*18^(1/3)*12^(1/3)*cos(2/3*arctan(sqrt(3) - 2))^2 + 3*18^(1/3)*12^(1/3)*sin
(2/3*arctan(sqrt(3) - 2))^2 + 18*x^2) + x
```

giac [B] time = 0.52, size = 638, normalized size = 1.55

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(x^6-x^3+1),x, algorithm="giac")
```

```
[Out] -1/9*(sqrt(3)*cos(4/9*pi)^4 - 6*sqrt(3)*cos(4/9*pi)^2*sin(4/9*pi)^2 + sqrt(
3)*sin(4/9*pi)^4 + 4*cos(4/9*pi)^3*sin(4/9*pi) - 4*cos(4/9*pi)*sin(4/9*pi)^
3 + 2*sqrt(3)*cos(4/9*pi) + 2*sin(4/9*pi))*arctan(-((sqrt(3)*i + 1)*cos(4/9
*pi) - 2*x)/((sqrt(3)*i + 1)*sin(4/9*pi))) - 1/9*(sqrt(3)*cos(2/9*pi)^4 - 6
*sqrt(3)*cos(2/9*pi)^2*sin(2/9*pi)^2 + sqrt(3)*sin(2/9*pi)^4 + 4*cos(2/9*pi
)^3*sin(2/9*pi) - 4*cos(2/9*pi)*sin(2/9*pi)^3 + 2*sqrt(3)*cos(2/9*pi) + 2*s
in(2/9*pi))*arctan(-((sqrt(3)*i + 1)*cos(2/9*pi) - 2*x)/((sqrt(3)*i + 1)*si
n(2/9*pi))) - 1/9*(sqrt(3)*cos(1/9*pi)^4 - 6*sqrt(3)*cos(1/9*pi)^2*sin(1/9*
pi)^2 + sqrt(3)*sin(1/9*pi)^4 - 4*cos(1/9*pi)^3*sin(1/9*pi) + 4*cos(1/9*pi)
*sin(1/9*pi)^3 - 2*sqrt(3)*cos(1/9*pi) + 2*sin(1/9*pi))*arctan(((sqrt(3)*i
+ 1)*cos(1/9*pi) + 2*x)/((sqrt(3)*i + 1)*sin(1/9*pi))) - 1/18*(4*sqrt(3)*co
s(4/9*pi)^3*sin(4/9*pi) - 4*sqrt(3)*cos(4/9*pi)*sin(4/9*pi)^3 - cos(4/9*pi)
^4 + 6*cos(4/9*pi)^2*sin(4/9*pi)^2 - sin(4/9*pi)^4 + 2*sqrt(3)*sin(4/9*pi)
- 2*cos(4/9*pi))*log(-(sqrt(3)*i*cos(4/9*pi) + cos(4/9*pi))*x + x^2 + 1) -
1/18*(4*sqrt(3)*cos(2/9*pi)^3*sin(2/9*pi) - 4*sqrt(3)*cos(2/9*pi)*sin(2/9*p
i)^3 - cos(2/9*pi)^4 + 6*cos(2/9*pi)^2*sin(2/9*pi)^2 - sin(2/9*pi)^4 + 2*sq
rt(3)*sin(2/9*pi) - 2*cos(2/9*pi))*log(-(sqrt(3)*i*cos(2/9*pi) + cos(2/9*pi
))*x + x^2 + 1) + 1/18*(4*sqrt(3)*cos(1/9*pi)^3*sin(1/9*pi) - 4*sqrt(3)*cos
(1/9*pi)*sin(1/9*pi)^3 + cos(1/9*pi)^4 - 6*cos(1/9*pi)^2*sin(1/9*pi)^2 + si
n(1/9*pi)^4 - 2*sqrt(3)*sin(1/9*pi) - 2*cos(1/9*pi))*log((sqrt(3)*i*cos(1/9
*pi) + cos(1/9*pi))*x + x^2 + 1) + x
```

maple [C] time = 0.01, size = 44, normalized size = 0.11

$$x+\frac{\left(\operatorname{RootOf}\left(\_Z^{6}-\_Z^{3}+1\right)^{3}-1\right)\ln\left(-\operatorname{RootOf}\left(\_Z^{6}-\_Z^{3}+1\right)+x\right)}{6\operatorname{RootOf}\left(\_Z^{6}-\_Z^{3}+1\right)^{5}-3\operatorname{RootOf}\left(\_Z^{6}-\_Z^{3}+1\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(x^6-x^3+1),x)
```

```
[Out] x+1/3*sum((_R^3-1)/(2*_R^5-_R^2)*ln(-_R+x),_R=RootOf(_Z^6-_Z^3+1))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$x+\int\frac{x^{3}-1}{x^{6}-x^{3}+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(x^6-x^3+1),x, algorithm="maxima")
```

```
[Out] x + integrate((x^3 - 1)/(x^6 - x^3 + 1), x)
```

mupad [B] time = 1.82, size = 320, normalized size = 0.78

$$x+\frac{\ln\left(x+\frac{\left(-\frac{27}{2}+\frac{\sqrt{3}\,9\mathrm{i}}{2}\right)\left(36+\sqrt{3}\,12\mathrm{i}\right)^{1/3}}{54}\right)\left(36+\sqrt{3}\,12\mathrm{i}\right)^{1/3}}{18}+\frac{\ln\left(x-\frac{\left(\frac{27}{2}+\frac{\sqrt{3}\,9\mathrm{i}}{2}\right)\left(36-\sqrt{3}\,12\mathrm{i}\right)^{1/3}}{54}\right)\left(36-\sqrt{3}\,12\mathrm{i}\right)^{1/3}}{18}-\frac{2^{2/3}\ln\left(x-\frac{2^{2/3}\left(3-\sqrt{3}\,1\mathrm{i}\right)^{1/3}\left(3^{1/3}-3^{5/6}\,1\mathrm{i}\right)\left(\frac{3\left(-3+\sqrt{3}\,1\mathrm{i}\right)\left(3^{1/3}-3^{5/6}\,1\mathrm{i}\right)^{3}}{16}-27\right)}{108}\right)\left(3-\sqrt{3}\,1\mathrm{i}\right)^{1/3}\left(3^{1/3}-3^{5/6}\,1\mathrm{i}\right)}{36}-\frac{2^{2/3}\ln\left(x+\frac{2^{2/3}\left(3+\sqrt{3}\,1\mathrm{i}\right)^{1/3}\left(3^{1/3}+3^{5/6}\,1\mathrm{i}\right)\left(\frac{3\left(3+\sqrt{3}\,1\mathrm{i}\right)\left(3^{1/3}+3^{5/6}\,1\mathrm{i}\right)^{3}}{16}+27\right)}{108}\right)\left(3+\sqrt{3}\,1\mathrm{i}\right)^{1/3}\left(3^{1/3}+3^{5/6}\,1\mathrm{i}\right)}{36}-\frac{2^{2/3}\ln\left(x+\frac{2^{2/3}\,3^{5/6}\left(3-\sqrt{3}\,1\mathrm{i}\right)^{1/3}\,1\mathrm{i}}{6}\right)\left(3-\sqrt{3}\,1\mathrm{i}\right)^{1/3}\left(3^{1/3}+3^{5/6}\,1\mathrm{i}\right)}{36}-\frac{2^{2/3}\ln\left(x-\frac{2^{2/3}\,3^{5/6}\left(3+\sqrt{3}\,1\mathrm{i}\right)^{1/3}\,1\mathrm{i}}{6}\right)\left(3+\sqrt{3}\,1\mathrm{i}\right)^{1/3}\left(3^{1/3}-3^{5/6}\,1\mathrm{i}\right)}{36}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(x^6 - x^3 + 1),x)
```

```
[Out] x + (log(x + (((3^(1/2)*9i)/2 - 27/2)*(3^(1/2)*12i + 36)^(1/3))/54)*(3^(1/2
)*12i + 36)^(1/3))/18 + (log(x - (((3^(1/2)*9i)/2 + 27/2)*(36 - 3^(1/2)*12i
)^(1/3))/54)*(36 - 3^(1/2)*12i)^(1/3))/18 - (2^(2/3)*log(x - (2^(2/3)*(3 -
3^(1/2)*1i)^(1/3)*(3^(1/3) - 3^(5/6)*1i)*((3*(3^(1/2)*1i - 3)*(3^(1/3) - 3^
(5/6)*1i)^3)/16 - 27))/108)*(3 - 3^(1/2)*1i)^(1/3)*(3^(1/3) - 3^(5/6)*1i))/
36 - (2^(2/3)*log(x + (2^(2/3)*(3^(1/2)*1i + 3)^(1/3)*(3^(1/3) + 3^(5/6)*1i
)*((3*(3^(1/2)*1i + 3)*(3^(1/3) + 3^(5/6)*1i)^3)/16 + 27))/108)*(3^(1/2)*1i
 + 3)^(1/3)*(3^(1/3) + 3^(5/6)*1i))/36 - (2^(2/3)*log(x + (2^(2/3)*3^(5/6)*
(3 - 3^(1/2)*1i)^(1/3)*1i)/6)*(3 - 3^(1/2)*1i)^(1/3)*(3^(1/3) + 3^(5/6)*1i)
)/36 - (2^(2/3)*log(x - (2^(2/3)*3^(5/6)*(3^(1/2)*1i + 3)^(1/3)*1i)/6)*(3^(
1/2)*1i + 3)^(1/3)*(3^(1/3) - 3^(5/6)*1i))/36
```

sympy [A] time = 0.18, size = 26, normalized size = 0.06

$$x + \operatorname{RootSum}\left(19683t^6 - 243t^3 + 1, \left(t \mapsto t\log\left(729t^4 - 9t + x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(x**6-x**3+1),x)
```

```
[Out] x + RootSum(19683*_t**6 - 243*_t**3 + 1, Lambda(_t, _t*log(729*_t**4 - 9*_t
 + x)))
```

3.158 $\int \frac{x^5}{1-x^3+x^6} \, dx$

Optimal. Leaf size=39

$$\frac{1}{6}\log\left(x^6-x^3+1\right)-\frac{\tan^{-1}\left(\frac{1-2x^3}{\sqrt{3}}\right)}{3\sqrt{3}}$$

Rubi [A] time = 0.04, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {1357, 634, 618, 204, 628}

$$\frac{1}{6}\log\left(x^6-x^3+1\right)-\frac{\tan^{-1}\left(\frac{1-2x^3}{\sqrt{3}}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/(1 - x^3 + x^6),x]
```

```
[Out] -ArcTan[(1 - 2*x^3)/Sqrt[3]]/(3*Sqrt[3]) + Log[1 - x^3 + x^6]/6
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1357

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x^5}{1-x^3+x^6}\,dx &= \frac{1}{3}\,\text{Subst}\left(\int \frac{x}{1-x+x^2}\,dx,x,x^3\right)\\ &= \frac{1}{6}\,\text{Subst}\left(\int \frac{1}{1-x+x^2}\,dx,x,x^3\right)+\frac{1}{6}\,\text{Subst}\left(\int \frac{-1+2x}{1-x+x^2}\,dx,x,x^3\right)\\ &= \frac{1}{6}\log\left(1-x^3+x^6\right)-\frac{1}{3}\,\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,-1+2x^3\right)\\ &= -\frac{\tan^{-1}\left(\frac{1-2x^3}{\sqrt{3}}\right)}{3\sqrt{3}}+\frac{1}{6}\log\left(1-x^3+x^6\right)\end{aligned}$$

Mathematica [A] time = 0.01, size = 39, normalized size = 1.00

$$\frac{\tan^{-1}\left(\frac{2x^3-1}{\sqrt{3}}\right)}{3\sqrt{3}}+\frac{1}{6}\log\left(x^6-x^3+1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/(1 - x^3 + x^6),x]
[Out] ArcTan[(-1 + 2*x^3)/Sqrt[3]]/(3*Sqrt[3]) + Log[1 - x^3 + x^6]/6
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5}{1-x^3+x^6}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^5/(1 - x^3 + x^6),x]
[Out] IntegrateAlgebraic[x^5/(1 - x^3 + x^6), x]
```

fricas [A] time = 1.18, size = 32, normalized size = 0.82

$$\frac{1}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^3-1\right)\right)+\frac{1}{6}\log\left(x^6-x^3+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(x^6-x^3+1),x, algorithm="fricas")
[Out] 1/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^3 - 1)) + 1/6*log(x^6 - x^3 + 1)
```

giac [A] time = 0.43, size = 32, normalized size = 0.82

$$\frac{1}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^3-1\right)\right)+\frac{1}{6}\log\left(x^6-x^3+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(x^6-x^3+1),x, algorithm="giac")
[Out] 1/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^3 - 1)) + 1/6*log(x^6 - x^3 + 1)
```

maple [A] time = 0.00, size = 33, normalized size = 0.85

$$\frac{\sqrt{3}\arctan\left(\frac{\left(2x^3-1\right)\sqrt{3}}{3}\right)}{9}+\frac{\ln\left(x^6-x^3+1\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(x^6-x^3+1),x)
```

```
[Out] 1/6*ln(x^6-x^3+1)+1/9*3^(1/2)*arctan(1/3*(2*x^3-1)*3^(1/2))
```

maxima [A] time = 1.04, size = 32, normalized size = 0.82

$$\frac{1}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^3-1\right)\right)+\frac{1}{6}\log\left(x^6-x^3+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(x^6-x^3+1),x, algorithm="maxima")
```

```
[Out] 1/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^3 - 1)) + 1/6*log(x^6 - x^3 + 1)
```

mupad [B] time = 1.21, size = 34, normalized size = 0.87

$$\frac{\ln\left(x^6-x^3+1\right)}{6}-\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{\sqrt{3}}{3}-\frac{2\sqrt{3}\,x^3}{3}\right)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(x^6 - x^3 + 1),x)
```

```
[Out] log(x^6 - x^3 + 1)/6 - (3^(1/2)*atan(3^(1/2)/3 - (2*3^(1/2)*x^3)/3))/9
```

sympy [A] time = 0.13, size = 37, normalized size = 0.95

$$\frac{\log\left(x^6-x^3+1\right)}{6}+\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x^3}{3}-\frac{\sqrt{3}}{3}\right)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(x**6-x**3+1),x)
```

```
[Out] log(x**6 - x**3 + 1)/6 + sqrt(3)*atan(2*sqrt(3)*x**3/3 - sqrt(3)/3)/9
```

3.159 $\int \frac{x^4}{1-x^3+x^6}\,dx$

Optimal. Leaf size=411

$$-\frac{\left(3+i\sqrt{3}\right)\log\left(2^{2/3}x^2+\sqrt[3]{2\left(1-i\sqrt{3}\right)}\,x+\left(1-i\sqrt{3}\right)^{2/3}\right)}{18\ 2^{2/3}\sqrt[3]{1-i\sqrt{3}}}-\frac{\left(3-i\sqrt{3}\right)\log\left(2^{2/3}x^2+\sqrt[3]{2\left(1+i\sqrt{3}\right)}\,x+\left(1+i\sqrt{3}\right.\right.}{18\ 2^{2/3}\sqrt[3]{1+i\sqrt{3}}}$$

Rubi [A] time = 0.28, antiderivative size = 411, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {1374, 292, 31, 634, 617, 204, 628}

$$-\frac{\left(3+i\sqrt{3}\right)\log\left(2^{2/3}x^2+\sqrt[3]{2\left(1-i\sqrt{3}\right)}\,x+\left(1-i\sqrt{3}\right)^{2/3}\right)}{18\ 2^{2/3}\sqrt[3]{1-i\sqrt{3}}}-\frac{\left(3-i\sqrt{3}\right)\log\left(2^{2/3}x^2+\sqrt[3]{2\left(1+i\sqrt{3}\right)}\,x+\left(1+i\sqrt{3}\right)^{2/3}\right)}{18\ 2^{2/3}\sqrt[3]{1+i\sqrt{3}}}+\frac{\left(3+i\sqrt{3}\right)\log\left(-\sqrt[3]{2}x+\sqrt[3]{1-i\sqrt{3}}\right)}{9\ 2^{2/3}\sqrt[3]{1-i\sqrt{3}}}+\frac{\left(3-i\sqrt{3}\right)\log\left(-\sqrt[3]{2}x+\sqrt[3]{1+i\sqrt{3}}\right)}{9\ 2^{2/3}\sqrt[3]{1+i\sqrt{3}}}+\frac{\left(\sqrt{3}+i\right)\tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\ 2^{2/3}\sqrt[3]{1-i\sqrt{3}}}-\frac{\left(-\sqrt{3}+i\right)\tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\ 2^{2/3}\sqrt[3]{1+i\sqrt{3}}}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(1 - x^3 + x^6),x]
```

```
[Out] ((I + Sqrt[3])*ArcTan[(1 + (2*x)/((1 - I*Sqrt[3])/2)^(1/3))/Sqrt[3]])/(3*2^
(2/3)*(1 - I*Sqrt[3])^(1/3)) - ((I - Sqrt[3])*ArcTan[(1 + (2*x)/((1 + I*Sqr
t[3])/2)^(1/3))/Sqrt[3]])/(3*2^(2/3)*(1 + I*Sqrt[3])^(1/3)) + ((3 + I*Sqrt[
3])*Log[(1 - I*Sqrt[3])^(1/3) - 2^(1/3)*x])/(9*2^(2/3)*(1 - I*Sqrt[3])^(1/3
)) + ((3 - I*Sqrt[3])*Log[(1 + I*Sqrt[3])^(1/3) - 2^(1/3)*x])/(9*2^(2/3)*(1
 + I*Sqrt[3])^(1/3)) - ((3 + I*Sqrt[3])*Log[(1 - I*Sqrt[3])^(2/3) + (2*(1 -
 I*Sqrt[3]))^(1/3)*x + 2^(2/3)*x^2])/(18*2^(2/3)*(1 - I*Sqrt[3])^(1/3)) - (
(3 - I*Sqrt[3])*Log[(1 + I*Sqrt[3])^(2/3) + (2*(1 + I*Sqrt[3]))^(1/3)*x + 2
^(2/3)*x^2])/(18*2^(2/3)*(1 + I*Sqrt[3])^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1374

```
Int[((d_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(d^n*(b/q + 1))/2, Int[(d*x)^(m -
 n)/(b/2 + q/2 + c*x^n), x], x] - Dist[(d^n*(b/q - 1))/2, Int[(d*x)^(m - n)
/(b/2 - q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[n2, 2*n] &&
NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GeQ[m, n]
```

Rubi steps

$$\int \frac{x^4}{1-x^3+x^6}\,dx = -\left(\frac{1}{6}\left(-3+i\sqrt{3}\right)\int \frac{x}{-\frac{1}{2}-\frac{i\sqrt{3}}{2}+x^3}\,dx\right)+\frac{1}{6}\left(3+i\sqrt{3}\right)\int \frac{x}{-\frac{1}{2}+\frac{i\sqrt{3}}{2}+x^3}\,dx$$

$$= -\left(-\frac{\left(-3-i\sqrt{3}\right)\int \frac{-\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}+x}{\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}+\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}\,x+x^2}\,dx}{9\,2^{2/3}\sqrt[3]{1-i\sqrt{3}}}\right)+\frac{\left(3-i\sqrt{3}\right)\int \frac{1}{-\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}+x}\,dx}{9\,2^{2/3}\sqrt[3]{1+i\sqrt{3}}}-\frac{\left(3-i\right.}{}$$

$$= \frac{\left(3+i\sqrt{3}\right)\log\left(\sqrt[3]{1-i\sqrt{3}}-\sqrt[3]{2}\,x\right)}{9\,2^{2/3}\sqrt[3]{1-i\sqrt{3}}}+\frac{\left(3-i\sqrt{3}\right)\log\left(\sqrt[3]{1+i\sqrt{3}}-\sqrt[3]{2}\,x\right)}{9\,2^{2/3}\sqrt[3]{1+i\sqrt{3}}}--\frac{\left(-3-i\sqrt{3}\right)\int}{1}$$

$$= \frac{\left(3+i\sqrt{3}\right)\log\left(\sqrt[3]{1-i\sqrt{3}}-\sqrt[3]{2}\,x\right)}{9\,2^{2/3}\sqrt[3]{1-i\sqrt{3}}}+\frac{\left(3-i\sqrt{3}\right)\log\left(\sqrt[3]{1+i\sqrt{3}}-\sqrt[3]{2}\,x\right)}{9\,2^{2/3}\sqrt[3]{1+i\sqrt{3}}}-\frac{\left(3+i\sqrt{3}\right)\log\left(\right.}{}$$

$$= \frac{\left(i+\sqrt{3}\right)\tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\,2^{2/3}\sqrt[3]{1-i\sqrt{3}}}-\frac{\left(i-\sqrt{3}\right)\tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\,2^{2/3}\sqrt[3]{1+i\sqrt{3}}}+\frac{\left(3+i\sqrt{3}\right)\log\left(\sqrt[3]{1-i\sqrt{3}}\right.}{9\,2^{2/3}\sqrt[3]{1-i\sqrt{3}}}$$

Mathematica [C] time = 0.01, size = 41, normalized size = 0.10

$$\frac{1}{3}\text{RootSum}\left[\#1^6-\#1^3+1\&,\frac{\#1^2\log(x-\#1)}{2\#1^3-1}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(1 - x^3 + x^6),x]
```

```
[Out] RootSum[1 - #1^3 + #1^6 & , (Log[x - #1]*#1^2)/(-1 + 2*#1^3) & ]/3
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{1-x^3+x^6}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^4/(1 - x^3 + x^6),x]
```

```
[Out] IntegrateAlgebraic[x^4/(1 - x^3 + x^6), x]
```

fricas [B] time = 1.73, size = 1583, normalized size = 3.85

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(x^6-x^3+1),x, algorithm="fricas")
```

```
[Out] 1/54*18^(2/3)*12^(1/6)*cos(2/3*arctan(sqrt(3) + 2))*log(18^(2/3)*12^(2/3)*c
os(2/3*arctan(sqrt(3) + 2))^4 + 18^(2/3)*12^(2/3)*sin(2/3*arctan(sqrt(3) +
2))^4 - 12*18^(1/3)*12^(1/3)*sqrt(3)*x*cos(2/3*arctan(sqrt(3) + 2))*sin(2/3
*arctan(sqrt(3) + 2)) + 6*18^(1/3)*12^(1/3)*x*cos(2/3*arctan(sqrt(3) + 2))^
2 + 2*(18^(2/3)*12^(2/3)*cos(2/3*arctan(sqrt(3) + 2))^2 - 3*18^(1/3)*12^(1/
3)*x)*sin(2/3*arctan(sqrt(3) + 2))^2 + 36*x^2) + 2/27*18^(2/3)*12^(1/6)*arc
tan(1/108*(6*18^(2/3)*12^(2/3)*sqrt(3)*x*cos(2/3*arctan(sqrt(3) + 2))^2 + 1
08*sqrt(3)*cos(2/3*arctan(sqrt(3) + 2))^4 + 108*sqrt(3)*sin(2/3*arctan(sqrt
(3) + 2))^4 + 864*cos(2/3*arctan(sqrt(3) + 2))*sin(2/3*arctan(sqrt(3) + 2))
^3 - 6*(18^(2/3)*12^(2/3)*sqrt(3)*x - 36*sqrt(3)*cos(2/3*arctan(sqrt(3) + 2
))^2)*sin(2/3*arctan(sqrt(3) + 2))^2 - 12*(18^(2/3)*12^(2/3)*x*cos(2/3*arct
an(sqrt(3) + 2)) + 72*cos(2/3*arctan(sqrt(3) + 2))^3)*sin(2/3*arctan(sqrt(3
) + 2)) - sqrt(18^(2/3)*12^(2/3)*cos(2/3*arctan(sqrt(3) + 2))^4 + 18^(2/3)*
12^(2/3)*sin(2/3*arctan(sqrt(3) + 2))^4 - 12*18^(1/3)*12^(1/3)*sqrt(3)*x*co
s(2/3*arctan(sqrt(3) + 2))*sin(2/3*arctan(sqrt(3) + 2)) + 6*18^(1/3)*12^(1/
3)*x*cos(2/3*arctan(sqrt(3) + 2))^2 + 2*(18^(2/3)*12^(2/3)*cos(2/3*arctan(s
qrt(3) + 2))^2 - 3*18^(1/3)*12^(1/3)*x)*sin(2/3*arctan(sqrt(3) + 2))^2 + 36
*x^2)*(18^(2/3)*12^(2/3)*sqrt(3)*cos(2/3*arctan(sqrt(3) + 2))^2 - 18^(2/3)*
12^(2/3)*sqrt(3)*sin(2/3*arctan(sqrt(3) + 2))^2 - 2*18^(2/3)*12^(2/3)*cos(2
/3*arctan(sqrt(3) + 2))*sin(2/3*arctan(sqrt(3) + 2))))/(3*cos(2/3*arctan(sq
rt(3) + 2))^4 - 10*cos(2/3*arctan(sqrt(3) + 2))^2*sin(2/3*arctan(sqrt(3) +
2))^2 + 3*sin(2/3*arctan(sqrt(3) + 2))^4))*sin(2/3*arctan(sqrt(3) + 2)) - 1
/27*(18^(2/3)*12^(1/6)*sqrt(3)*cos(2/3*arctan(sqrt(3) + 2)) - 18^(2/3)*12^(
1/6)*sin(2/3*arctan(sqrt(3) + 2)))*arctan(1/108*(6*18^(2/3)*12^(2/3)*sqrt(3
)*x*cos(2/3*arctan(sqrt(3) + 2))^2 + 108*sqrt(3)*cos(2/3*arctan(sqrt(3) + 2
))^4 + 108*sqrt(3)*sin(2/3*arctan(sqrt(3) + 2))^4 - 864*cos(2/3*arctan(sqrt
(3) + 2))*sin(2/3*arctan(sqrt(3) + 2))^3 - 6*(18^(2/3)*12^(2/3)*sqrt(3)*x -
 36*sqrt(3)*cos(2/3*arctan(sqrt(3) + 2))^2)*sin(2/3*arctan(sqrt(3) + 2))^2
+ 12*(18^(2/3)*12^(2/3)*x*cos(2/3*arctan(sqrt(3) + 2)) + 72*cos(2/3*arctan(
sqrt(3) + 2))^3)*sin(2/3*arctan(sqrt(3) + 2)) - sqrt(18^(2/3)*12^(2/3)*cos(
2/3*arctan(sqrt(3) + 2))^4 + 18^(2/3)*12^(2/3)*sin(2/3*arctan(sqrt(3) + 2))
^4 + 12*18^(1/3)*12^(1/3)*sqrt(3)*x*cos(2/3*arctan(sqrt(3) + 2))*sin(2/3*ar
ctan(sqrt(3) + 2)) + 6*18^(1/3)*12^(1/3)*x*cos(2/3*arctan(sqrt(3) + 2))^2 +
 2*(18^(2/3)*12^(2/3)*cos(2/3*arctan(sqrt(3) + 2))^2 - 3*18^(1/3)*12^(1/3)*
x)*sin(2/3*arctan(sqrt(3) + 2))^2 + 36*x^2)*(18^(2/3)*12^(2/3)*sqrt(3)*cos(
2/3*arctan(sqrt(3) + 2))^2 - 18^(2/3)*12^(2/3)*sqrt(3)*sin(2/3*arctan(sqrt(
3) + 2))^2 + 2*18^(2/3)*12^(2/3)*cos(2/3*arctan(sqrt(3) + 2))*sin(2/3*arcta
n(sqrt(3) + 2))))/(3*cos(2/3*arctan(sqrt(3) + 2))^4 - 10*cos(2/3*arctan(sqr
t(3) + 2))^2*sin(2/3*arctan(sqrt(3) + 2))^2 + 3*sin(2/3*arctan(sqrt(3) + 2)
)^4)) - 1/27*(18^(2/3)*12^(1/6)*sqrt(3)*cos(2/3*arctan(sqrt(3) + 2)) + 18^(
2/3)*12^(1/6)*sin(2/3*arctan(sqrt(3) + 2)))*arctan(-1/432*(6*18^(2/3)*12^(2
/3)*x - 216*cos(2/3*arctan(sqrt(3) + 2))^2 + 216*sin(2/3*arctan(sqrt(3) + 2
))^2 - 18^(2/3)*12^(2/3)*sqrt(18^(2/3)*12^(2/3)*cos(2/3*arctan(sqrt(3) + 2)
```

```
)^4 + 18^(2/3)*12^(2/3)*sin(2/3*arctan(sqrt(3) + 2))^4 - 12*18^(1/3)*12^(1/
3)*x*cos(2/3*arctan(sqrt(3) + 2))^2 + 2*(18^(2/3)*12^(2/3)*cos(2/3*arctan(s
qrt(3) + 2))^2 + 6*18^(1/3)*12^(1/3)*x)*sin(2/3*arctan(sqrt(3) + 2))^2 + 36
*x^2))/(cos(2/3*arctan(sqrt(3) + 2))*sin(2/3*arctan(sqrt(3) + 2)))) - 1/108
*(18^(2/3)*12^(1/6)*sqrt(3)*sin(2/3*arctan(sqrt(3) + 2)) + 18^(2/3)*12^(1/6
)*cos(2/3*arctan(sqrt(3) + 2)))*log(18^(2/3)*12^(2/3)*cos(2/3*arctan(sqrt(3
) + 2))^4 + 18^(2/3)*12^(2/3)*sin(2/3*arctan(sqrt(3) + 2))^4 + 12*18^(1/3)*
12^(1/3)*sqrt(3)*x*cos(2/3*arctan(sqrt(3) + 2))*sin(2/3*arctan(sqrt(3) + 2)
) + 6*18^(1/3)*12^(1/3)*x*cos(2/3*arctan(sqrt(3) + 2))^2 + 2*(18^(2/3)*12^(
2/3)*cos(2/3*arctan(sqrt(3) + 2))^2 - 3*18^(1/3)*12^(1/3)*x)*sin(2/3*arctan
(sqrt(3) + 2))^2 + 36*x^2) + 1/108*(18^(2/3)*12^(1/6)*sqrt(3)*sin(2/3*arcta
n(sqrt(3) + 2)) - 18^(2/3)*12^(1/6)*cos(2/3*arctan(sqrt(3) + 2)))*log(18^(2
/3)*12^(2/3)*cos(2/3*arctan(sqrt(3) + 2))^4 + 18^(2/3)*12^(2/3)*sin(2/3*arc
tan(sqrt(3) + 2))^4 - 12*18^(1/3)*12^(1/3)*x*cos(2/3*arctan(sqrt(3) + 2))^2
 + 2*(18^(2/3)*12^(2/3)*cos(2/3*arctan(sqrt(3) + 2))^2 + 6*18^(1/3)*12^(1/3
)*x)*sin(2/3*arctan(sqrt(3) + 2))^2 + 36*x^2)
```

giac [B] time = 0.69, size = 824, normalized size = 2.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(x^6-x^3+1),x, algorithm="giac")
```

```
[Out] -1/9*(2*sqrt(3)*cos(4/9*pi)^5 - 20*sqrt(3)*cos(4/9*pi)^3*sin(4/9*pi)^2 + 10
*sqrt(3)*cos(4/9*pi)*sin(4/9*pi)^4 - 10*cos(4/9*pi)^4*sin(4/9*pi) + 20*cos(
4/9*pi)^2*sin(4/9*pi)^3 - 2*sin(4/9*pi)^5 + sqrt(3)*cos(4/9*pi)^2 - sqrt(3)
*sin(4/9*pi)^2 - 2*cos(4/9*pi)*sin(4/9*pi))*arctan(-((sqrt(3)*i + 1)*cos(4/
9*pi) - 2*x)/((sqrt(3)*i + 1)*sin(4/9*pi))) - 1/9*(2*sqrt(3)*cos(2/9*pi)^5
- 20*sqrt(3)*cos(2/9*pi)^3*sin(2/9*pi)^2 + 10*sqrt(3)*cos(2/9*pi)*sin(2/9*p
i)^4 - 10*cos(2/9*pi)^4*sin(2/9*pi) + 20*cos(2/9*pi)^2*sin(2/9*pi)^3 - 2*si
n(2/9*pi)^5 + sqrt(3)*cos(2/9*pi)^2 - sqrt(3)*sin(2/9*pi)^2 - 2*cos(2/9*pi)
*sin(2/9*pi))*arctan(-((sqrt(3)*i + 1)*cos(2/9*pi) - 2*x)/((sqrt(3)*i + 1)*
sin(2/9*pi))) + 1/9*(2*sqrt(3)*cos(1/9*pi)^5 - 20*sqrt(3)*cos(1/9*pi)^3*sin
(1/9*pi)^2 + 10*sqrt(3)*cos(1/9*pi)*sin(1/9*pi)^4 + 10*cos(1/9*pi)^4*sin(1/
9*pi) - 20*cos(1/9*pi)^2*sin(1/9*pi)^3 + 2*sin(1/9*pi)^5 - sqrt(3)*cos(1/9*
pi)^2 + sqrt(3)*sin(1/9*pi)^2 - 2*cos(1/9*pi)*sin(1/9*pi))*arctan(((sqrt(3)
*i + 1)*cos(1/9*pi) + 2*x)/((sqrt(3)*i + 1)*sin(1/9*pi))) - 1/18*(10*sqrt(3
)*cos(4/9*pi)^4*sin(4/9*pi) - 20*sqrt(3)*cos(4/9*pi)^2*sin(4/9*pi)^3 + 2*sq
rt(3)*sin(4/9*pi)^5 + 2*cos(4/9*pi)^5 - 20*cos(4/9*pi)^3*sin(4/9*pi)^2 + 10
*cos(4/9*pi)*sin(4/9*pi)^4 + 2*sqrt(3)*cos(4/9*pi)*sin(4/9*pi) + cos(4/9*pi
)^2 - sin(4/9*pi)^2)*log(-(sqrt(3)*i*cos(4/9*pi) + cos(4/9*pi))*x + x^2 + 1
) - 1/18*(10*sqrt(3)*cos(2/9*pi)^4*sin(2/9*pi) - 20*sqrt(3)*cos(2/9*pi)^2*s
in(2/9*pi)^3 + 2*sqrt(3)*sin(2/9*pi)^5 + 2*cos(2/9*pi)^5 - 20*cos(2/9*pi)^3
*sin(2/9*pi)^2 + 10*cos(2/9*pi)*sin(2/9*pi)^4 + 2*sqrt(3)*cos(2/9*pi)*sin(2
/9*pi) + cos(2/9*pi)^2 - sin(2/9*pi)^2)*log(-(sqrt(3)*i*cos(2/9*pi) + cos(2
/9*pi))*x + x^2 + 1) - 1/18*(10*sqrt(3)*cos(1/9*pi)^4*sin(1/9*pi) - 20*sqrt
(3)*cos(1/9*pi)^2*sin(1/9*pi)^3 + 2*sqrt(3)*sin(1/9*pi)^5 - 2*cos(1/9*pi)^5
 + 20*cos(1/9*pi)^3*sin(1/9*pi)^2 - 10*cos(1/9*pi)*sin(1/9*pi)^4 - 2*sqrt(3
)*cos(1/9*pi)*sin(1/9*pi) + cos(1/9*pi)^2 - sin(1/9*pi)^2)*log((sqrt(3)*i*c
os(1/9*pi) + cos(1/9*pi))*x + x^2 + 1)
```

maple [C] time = 0.01, size = 40, normalized size = 0.10

$$\frac{\mathrm{RootOf}\left(\_Z^6-\_Z^3+1\right)^4\ln\left(-\mathrm{RootOf}\left(\_Z^6-\_Z^3+1\right)+x\right)}{6\,\mathrm{RootOf}\left(\_Z^6-\_Z^3+1\right)^5-3\,\mathrm{RootOf}\left(\_Z^6-\_Z^3+1\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(x^6-x^3+1),x)
```

[Out] 1/3*sum(_R^4/(2*_R^5-_R^2)*ln(-_R+x),_R=RootOf(_Z^6-_Z^3+1))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{x^6 - x^3 + 1} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4/(x^6-x^3+1),x, algorithm="maxima")

[Out] integrate(x^4/(x^6 - x^3 + 1), x)

mupad [B] time = 1.72, size = 304, normalized size = 0.74

$$\frac{\ln\left(x+\left(162\,x+\frac{27\,{\left(-36+\sqrt{3}\,12\mathrm{i}\right)}^{2/3}}{4}\right)\,\left(-\frac{1}{162}+\frac{\sqrt{3}\,1\mathrm{i}}{486}\right)\right)\,{\left(-36+\sqrt{3}\,12\mathrm{i}\right)}^{1/3}}{18}+\frac{\ln\left(x-\left(162\,x+\frac{27\,{\left(-36-\sqrt{3}\,12\mathrm{i}\right)}^{2/3}}{4}\right)\,\left(\frac{1}{162}+\frac{\sqrt{3}\,1\mathrm{i}}{486}\right)\right)\,{\left(-36-\sqrt{3}\,12\mathrm{i}\right)}^{1/3}}{18}-\frac{2^{2/3}\,\ln\left(x+\frac{2^{1/3}\,3^{2/3}\,{\left(-3-\sqrt{3}\,1\mathrm{i}\right)}^{2/3}}{12}+\frac{2^{1/3}\,3^{1/6}\,{\left(-3-\sqrt{3}\,1\mathrm{i}\right)}^{2/3}\,1\mathrm{i}}{4}\right)\,{\left(-3-\sqrt{3}\,1\mathrm{i}\right)}^{1/3}\,\left(3^{1/3}+3^{5/6}\,1\mathrm{i}\right)}{36}-\frac{2^{2/3}\,\ln\left(x+\frac{2^{1/3}\,3^{2/3}\,{\left(-3+\sqrt{3}\,1\mathrm{i}\right)}^{2/3}}{12}-\frac{2^{1/3}\,3^{1/6}\,{\left(-3+\sqrt{3}\,1\mathrm{i}\right)}^{2/3}\,1\mathrm{i}}{4}\right)\,{\left(-3+\sqrt{3}\,1\mathrm{i}\right)}^{1/3}\,\left(3^{1/3}-3^{5/6}\,1\mathrm{i}\right)}{36}-\frac{2^{2/3}\,\ln\left(x-\frac{2^{1/3}\,3^{2/3}\,{\left(-3-\sqrt{3}\,1\mathrm{i}\right)}^{2/3}}{6}\right)\,{\left(-3-\sqrt{3}\,1\mathrm{i}\right)}^{1/3}\,\left(3^{1/3}-3^{5/6}\,1\mathrm{i}\right)}{36}-\frac{2^{2/3}\,\ln\left(x-\frac{2^{1/3}\,3^{2/3}\,{\left(-3+\sqrt{3}\,1\mathrm{i}\right)}^{2/3}}{6}\right)\,{\left(-3+\sqrt{3}\,1\mathrm{i}\right)}^{1/3}\,\left(3^{1/3}+3^{5/6}\,1\mathrm{i}\right)}{36}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4/(x^6 - x^3 + 1),x)

[Out] (log(x + (162*x + (27*(3^(1/2)*12i - 36)^(2/3))/4)*((3^(1/2)*1i)/486 - 1/162))*(3^(1/2)*12i - 36)^(1/3))/18 + (log(x - (162*x + (27*(- 3^(1/2)*12i - 36)^(2/3))/4)*((3^(1/2)*1i)/486 + 1/162))*(- 3^(1/2)*12i - 36)^(1/3))/18 - (2^(2/3)*log(x + (2^(1/3)*3^(2/3)*(- 3^(1/2)*1i - 3)^(2/3))/12 + (2^(1/3)*3^(1/6)*(- 3^(1/2)*1i - 3)^(2/3)*1i)/4)*(- 3^(1/2)*1i - 3)^(1/3)*(3^(1/3) + 3^(5/6)*1i))/36 - (2^(2/3)*log(x + (2^(1/3)*3^(2/3)*(3^(1/2)*1i - 3)^(2/3))/12 - (2^(1/3)*3^(1/6)*(3^(1/2)*1i - 3)^(2/3)*1i)/4)*(3^(1/2)*1i - 3)^(1/3)*(3^(1/3) - 3^(5/6)*1i))/36 - (2^(2/3)*log(x - (2^(1/3)*3^(2/3)*(- 3^(1/2)*1i - 3)^(2/3))/6)*(- 3^(1/2)*1i - 3)^(1/3)*(3^(1/3) - 3^(5/6)*1i))/36 - (2^(2/3)*log(x - (2^(1/3)*3^(2/3)*(3^(1/2)*1i - 3)^(2/3))/6)*(3^(1/2)*1i - 3)^(1/3)*(3^(1/3) + 3^(5/6)*1i))/36

sympy [A] time = 0.18, size = 26, normalized size = 0.06

$$\operatorname{RootSum}\left(19683t^6 + 243t^3 + 1, \left(t \mapsto t \log\left(6561t^5 + 54t^2 + x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**4/(x**6-x**3+1),x)

[Out] RootSum(19683*_t**6 + 243*_t**3 + 1, Lambda(_t, _t*log(6561*_t**5 + 54*_t**2 + x)))

3.160 $\int \frac{x^3}{1-x^3+x^6}\,dx$

Optimal. Leaf size=411

$$-\frac{\left(3+i\sqrt{3}\right)\log\left(2^{2/3}x^2+\sqrt[3]{2\left(1-i\sqrt{3}\right)}\,x+\left(1-i\sqrt{3}\right)^{2/3}\right)}{18\sqrt[3]{2}\left(1-i\sqrt{3}\right)^{2/3}}-\frac{\left(3-i\sqrt{3}\right)\log\left(2^{2/3}x^2+\sqrt[3]{2\left(1+i\sqrt{3}\right)}\,x+\left(1+i\sqrt{3}\right)\right.}{18\sqrt[3]{2}\left(1+i\sqrt{3}\right)^{2/3}}$$

Rubi [A] time = 0.28, antiderivative size = 411, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {1374, 200, 31, 634, 617, 204, 628}

$$-\frac{\left(3+i\sqrt{3}\right)\log\left(2^{2/3}x^2+\sqrt[3]{2\left(1-i\sqrt{3}\right)}\,x+\left(1-i\sqrt{3}\right)^{2/3}\right)}{18\sqrt[3]{2}\left(1-i\sqrt{3}\right)^{2/3}}-\frac{\left(3-i\sqrt{3}\right)\log\left(2^{2/3}x^2+\sqrt[3]{2\left(1+i\sqrt{3}\right)}\,x+\left(1+i\sqrt{3}\right)^{2/3}\right)}{18\sqrt[3]{2}\left(1+i\sqrt{3}\right)^{2/3}}+\frac{\left(3+i\sqrt{3}\right)\log\left(-\sqrt[3]{2}x+\sqrt[3]{1-i\sqrt{3}}\right)}{9\sqrt[3]{2}\left(1-i\sqrt{3}\right)^{2/3}}+\frac{\left(3-i\sqrt{3}\right)\log\left(-\sqrt[3]{2}x+\sqrt[3]{1+i\sqrt{3}}\right)}{9\sqrt[3]{2}\left(1+i\sqrt{3}\right)^{2/3}}-\frac{\left(\sqrt{3}+i\right)\tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\sqrt[3]{2}\left(1-i\sqrt{3}\right)^{2/3}}+\frac{\left(-\sqrt{3}+i\right)\tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\sqrt[3]{2}\left(1+i\sqrt{3}\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(1 - x^3 + x^6),x]
```

```
[Out] -((I + Sqrt[3])*ArcTan[(1 + (2*x)/((1 - I*Sqrt[3])/2)^(1/3))/Sqrt[3]])/(3*2
^(1/3)*(1 - I*Sqrt[3])^(2/3)) + ((I - Sqrt[3])*ArcTan[(1 + (2*x)/((1 + I*Sq
rt[3])/2)^(1/3))/Sqrt[3]])/(3*2^(1/3)*(1 + I*Sqrt[3])^(2/3)) + ((3 + I*Sqrt
[3])*Log[(1 - I*Sqrt[3])^(1/3) - 2^(1/3)*x])/(9*2^(1/3)*(1 - I*Sqrt[3])^(2/
3)) + ((3 - I*Sqrt[3])*Log[(1 + I*Sqrt[3])^(1/3) - 2^(1/3)*x])/(9*2^(1/3)*(
1 + I*Sqrt[3])^(2/3)) - ((3 + I*Sqrt[3])*Log[(1 - I*Sqrt[3])^(2/3) + (2*(1
- I*Sqrt[3]))^(1/3)*x + 2^(2/3)*x^2])/(18*2^(1/3)*(1 - I*Sqrt[3])^(2/3)) -
((3 - I*Sqrt[3])*Log[(1 + I*Sqrt[3])^(2/3) + (2*(1 + I*Sqrt[3]))^(1/3)*x +
2^(2/3)*x^2])/(18*2^(1/3)*(1 + I*Sqrt[3])^(2/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1374

```
Int[((d_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(d^n*(b/q + 1))/2, Int[(d*x)^(m -
 n)/(b/2 + q/2 + c*x^n), x], x] - Dist[(d^n*(b/q - 1))/2, Int[(d*x)^(m - n)
/(b/2 - q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[n2, 2*n] &&
NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GeQ[m, n]
```

Rubi steps

$$\int \frac{x^3}{1-x^3+x^6}\,dx = -\left(\frac{1}{6}\left(-3+i\sqrt{3}\right)\int \frac{1}{-\frac{1}{2}-\frac{i\sqrt{3}}{2}+x^3}\,dx\right)+\frac{1}{6}\left(3+i\sqrt{3}\right)\int \frac{1}{-\frac{1}{2}+\frac{i\sqrt{3}}{2}+x^3}\,dx$$

$$= \frac{\left(3-i\sqrt{3}\right)\int \frac{1}{-\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}+x}\,dx}{9\sqrt[3]{2}\left(1+i\sqrt{3}\right)^{2/3}}+\frac{\left(3-i\sqrt{3}\right)\int \frac{-2^{2/3}\sqrt[3]{1+i\sqrt{3}}-x}{\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}+\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}x+x^2}\,dx}{9\sqrt[3]{2}\left(1+i\sqrt{3}\right)^{2/3}}+\frac{\left(3+i\sqrt{3}\right)\cdots}{9\sqrt[3]{2}\cdots}$$

$$= \frac{\left(3+i\sqrt{3}\right)\log\left(\sqrt[3]{1-i\sqrt{3}}-\sqrt[3]{2}x\right)}{9\sqrt[3]{2}\left(1-i\sqrt{3}\right)^{2/3}}+\frac{\left(3-i\sqrt{3}\right)\log\left(\sqrt[3]{1+i\sqrt{3}}-\sqrt[3]{2}x\right)}{9\sqrt[3]{2}\left(1+i\sqrt{3}\right)^{2/3}}-\frac{\left(3-i\sqrt{3}\right)\int\cdots}{1\cdots}$$

$$= \frac{\left(3+i\sqrt{3}\right)\log\left(\sqrt[3]{1-i\sqrt{3}}-\sqrt[3]{2}x\right)}{9\sqrt[3]{2}\left(1-i\sqrt{3}\right)^{2/3}}+\frac{\left(3-i\sqrt{3}\right)\log\left(\sqrt[3]{1+i\sqrt{3}}-\sqrt[3]{2}x\right)}{9\sqrt[3]{2}\left(1+i\sqrt{3}\right)^{2/3}}-\frac{\left(3+i\sqrt{3}\right)\log\cdots}{\cdots}$$

$$= -\frac{\left(i+\sqrt{3}\right)\tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\sqrt[3]{2}\left(1-i\sqrt{3}\right)^{2/3}}+\frac{\left(i-\sqrt{3}\right)\tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\sqrt[3]{2}\left(1+i\sqrt{3}\right)^{2/3}}+\frac{\left(3+i\sqrt{3}\right)\log\left(\sqrt[3]{1-\cdots}\right.}{9\sqrt[3]{2}\left(1-i\sqrt{3}\cdots\right.}$$

Mathematica [C] time = 0.01, size = 39, normalized size = 0.09

$$\frac{1}{3}\text{RootSum}\left[\#1^6-\#1^3+1\&,\frac{\#1\log(x-\#1)}{2\#1^3-1}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(1 - x^3 + x^6),x]
[Out] RootSum[1 - #1^3 + #1^6 & , (Log[x - #1]*#1)/(-1 + 2*#1^3) & ]/3
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{1-x^3+x^6}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^3/(1 - x^3 + x^6),x]
```

```
[Out] IntegrateAlgebraic[x^3/(1 - x^3 + x^6), x]
```

fricas [B] time = 1.43, size = 1031, normalized size = 2.51

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(x^6-x^3+1),x, algorithm="fricas")
```

```
[Out] 1/54*18^(2/3)*12^(1/6)*cos(2/3*arctan(sqrt(3) + 2))*log(2*18^(2/3)*12^(1/6)
*sqrt(3)*x*sin(2/3*arctan(sqrt(3) + 2)) + 3*18^(1/3)*12^(1/3)*cos(2/3*arcta
n(sqrt(3) + 2))^2 + 3*18^(1/3)*12^(1/3)*sin(2/3*arctan(sqrt(3) + 2))^2 + 18
*x^2) + 2/27*18^(2/3)*12^(1/6)*arctan(1/216*(18^(1/3)*12^(5/6)*sqrt(3)*sqrt
(2)*sqrt(2*18^(2/3)*12^(1/6)*sqrt(3)*x*sin(2/3*arctan(sqrt(3) + 2)) + 3*18^
(1/3)*12^(1/3)*cos(2/3*arctan(sqrt(3) + 2))^2 + 3*18^(1/3)*12^(1/3)*sin(2/3
*arctan(sqrt(3) + 2))^2 + 18*x^2) - 6*18^(1/3)*12^(5/6)*sqrt(3)*x - 216*sin
(2/3*arctan(sqrt(3) + 2)))/cos(2/3*arctan(sqrt(3) + 2)))*sin(2/3*arctan(sqr
t(3) + 2)) - 1/27*(18^(2/3)*12^(1/6)*sqrt(3)*cos(2/3*arctan(sqrt(3) + 2)) -
 18^(2/3)*12^(1/6)*sin(2/3*arctan(sqrt(3) + 2)))*arctan(-1/108*(6*18^(1/3)*
12^(5/6)*sqrt(3)*x*cos(2/3*arctan(sqrt(3) + 2)) + 108*sqrt(3)*cos(2/3*arcta
n(sqrt(3) + 2))^2 + 108*sqrt(3)*sin(2/3*arctan(sqrt(3) + 2))^2 - 18*(18^(1/
3)*12^(5/6)*x + 24*cos(2/3*arctan(sqrt(3) + 2)))*sin(2/3*arctan(sqrt(3) + 2
)) - sqrt(-18^(2/3)*12^(1/6)*sqrt(3)*x*sin(2/3*arctan(sqrt(3) + 2)) + 3*18^
(2/3)*12^(1/6)*x*cos(2/3*arctan(sqrt(3) + 2)) + 3*18^(1/3)*12^(1/3)*cos(2/3
*arctan(sqrt(3) + 2))^2 + 3*18^(1/3)*12^(1/3)*sin(2/3*arctan(sqrt(3) + 2))^
2 + 18*x^2)*(18^(1/3)*12^(5/6)*sqrt(3)*sqrt(2)*cos(2/3*arctan(sqrt(3) + 2))
 - 3*18^(1/3)*12^(5/6)*sqrt(2)*sin(2/3*arctan(sqrt(3) + 2))))/(cos(2/3*arct
an(sqrt(3) + 2))^2 - 3*sin(2/3*arctan(sqrt(3) + 2))^2)) - 1/27*(18^(2/3)*12
^(1/6)*sqrt(3)*cos(2/3*arctan(sqrt(3) + 2)) + 18^(2/3)*12^(1/6)*sin(2/3*arc
tan(sqrt(3) + 2)))*arctan(1/108*(6*18^(1/3)*12^(5/6)*sqrt(3)*x*cos(2/3*arct
an(sqrt(3) + 2)) - 108*sqrt(3)*cos(2/3*arctan(sqrt(3) + 2))^2 - 108*sqrt(3)
*sin(2/3*arctan(sqrt(3) + 2))^2 + 18*(18^(1/3)*12^(5/6)*x - 24*cos(2/3*arct
an(sqrt(3) + 2)))*sin(2/3*arctan(sqrt(3) + 2)) - sqrt(-18^(2/3)*12^(1/6)*sq
rt(3)*x*sin(2/3*arctan(sqrt(3) + 2)) - 3*18^(2/3)*12^(1/6)*x*cos(2/3*arctan
(sqrt(3) + 2)) + 3*18^(1/3)*12^(1/3)*cos(2/3*arctan(sqrt(3) + 2))^2 + 3*18^
(1/3)*12^(1/3)*sin(2/3*arctan(sqrt(3) + 2))^2 + 18*x^2)*(18^(1/3)*12^(5/6)*
sqrt(3)*sqrt(2)*cos(2/3*arctan(sqrt(3) + 2)) + 3*18^(1/3)*12^(5/6)*sqrt(2)*
sin(2/3*arctan(sqrt(3) + 2))))/(cos(2/3*arctan(sqrt(3) + 2))^2 - 3*sin(2/3*
arctan(sqrt(3) + 2))^2)) - 1/108*(18^(2/3)*12^(1/6)*sqrt(3)*sin(2/3*arctan(
sqrt(3) + 2)) + 18^(2/3)*12^(1/6)*cos(2/3*arctan(sqrt(3) + 2)))*log(-18^(2/
3)*12^(1/6)*sqrt(3)*x*sin(2/3*arctan(sqrt(3) + 2)) + 3*18^(2/3)*12^(1/6)*x*
cos(2/3*arctan(sqrt(3) + 2)) + 3*18^(1/3)*12^(1/3)*cos(2/3*arctan(sqrt(3) +
 2))^2 + 3*18^(1/3)*12^(1/3)*sin(2/3*arctan(sqrt(3) + 2))^2 + 18*x^2) + 1/1
08*(18^(2/3)*12^(1/6)*sqrt(3)*sin(2/3*arctan(sqrt(3) + 2)) - 18^(2/3)*12^(1
/6)*cos(2/3*arctan(sqrt(3) + 2)))*log(-18^(2/3)*12^(1/6)*sqrt(3)*x*sin(2/3*
arctan(sqrt(3) + 2)) - 3*18^(2/3)*12^(1/6)*x*cos(2/3*arctan(sqrt(3) + 2)) +
 3*18^(1/3)*12^(1/3)*cos(2/3*arctan(sqrt(3) + 2))^2 + 3*18^(1/3)*12^(1/3)*s
in(2/3*arctan(sqrt(3) + 2))^2 + 18*x^2)
```

giac [B] time = 0.57, size = 637, normalized size = 1.55

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(x^6-x^3+1),x, algorithm="giac")
```

```
[Out] -1/9*(2*sqrt(3)*cos(4/9*pi)^4 - 12*sqrt(3)*cos(4/9*pi)^2*sin(4/9*pi)^2 + 2*
sqrt(3)*sin(4/9*pi)^4 + 8*cos(4/9*pi)^3*sin(4/9*pi) - 8*cos(4/9*pi)*sin(4/9
```

```
*pi)^3 + sqrt(3)*cos(4/9*pi) + sin(4/9*pi))*arctan(-((sqrt(3)*i + 1)*cos(4/
9*pi) - 2*x)/((sqrt(3)*i + 1)*sin(4/9*pi))) - 1/9*(2*sqrt(3)*cos(2/9*pi)^4
- 12*sqrt(3)*cos(2/9*pi)^2*sin(2/9*pi)^2 + 2*sqrt(3)*sin(2/9*pi)^4 + 8*cos(
2/9*pi)^3*sin(2/9*pi) - 8*cos(2/9*pi)*sin(2/9*pi)^3 + sqrt(3)*cos(2/9*pi) +
 sin(2/9*pi))*arctan(-((sqrt(3)*i + 1)*cos(2/9*pi) - 2*x)/((sqrt(3)*i + 1)*
sin(2/9*pi))) - 1/9*(2*sqrt(3)*cos(1/9*pi)^4 - 12*sqrt(3)*cos(1/9*pi)^2*sin
(1/9*pi)^2 + 2*sqrt(3)*sin(1/9*pi)^4 - 8*cos(1/9*pi)^3*sin(1/9*pi) + 8*cos(
1/9*pi)*sin(1/9*pi)^3 - sqrt(3)*cos(1/9*pi) + sin(1/9*pi))*arctan(((sqrt(3)
*i + 1)*cos(1/9*pi) + 2*x)/((sqrt(3)*i + 1)*sin(1/9*pi))) - 1/18*(8*sqrt(3)
*cos(4/9*pi)^3*sin(4/9*pi) - 8*sqrt(3)*cos(4/9*pi)*sin(4/9*pi)^3 - 2*cos(4/
9*pi)^4 + 12*cos(4/9*pi)^2*sin(4/9*pi)^2 - 2*sin(4/9*pi)^4 + sqrt(3)*sin(4/
9*pi) - cos(4/9*pi))*log(-(sqrt(3)*i*cos(4/9*pi) + cos(4/9*pi))*x + x^2 + 1
) - 1/18*(8*sqrt(3)*cos(2/9*pi)^3*sin(2/9*pi) - 8*sqrt(3)*cos(2/9*pi)*sin(2
/9*pi)^3 - 2*cos(2/9*pi)^4 + 12*cos(2/9*pi)^2*sin(2/9*pi)^2 - 2*sin(2/9*pi)
^4 + sqrt(3)*sin(2/9*pi) - cos(2/9*pi))*log(-(sqrt(3)*i*cos(2/9*pi) + cos(2
/9*pi))*x + x^2 + 1) + 1/18*(8*sqrt(3)*cos(1/9*pi)^3*sin(1/9*pi) - 8*sqrt(3
)*cos(1/9*pi)*sin(1/9*pi)^3 + 2*cos(1/9*pi)^4 - 12*cos(1/9*pi)^2*sin(1/9*pi
)^2 + 2*sin(1/9*pi)^4 - sqrt(3)*sin(1/9*pi) - cos(1/9*pi))*log((sqrt(3)*i*c
os(1/9*pi) + cos(1/9*pi))*x + x^2 + 1)
```

maple [C] time = 0.01, size = 40, normalized size = 0.10

$$\frac{\mathrm{RootOf}\left(\_Z^6-\_Z^3+1\right)^3\ln\left(-\mathrm{RootOf}\left(\_Z^6-\_Z^3+1\right)+x\right)}{6\,\mathrm{RootOf}\left(\_Z^6-\_Z^3+1\right)^5-3\,\mathrm{RootOf}\left(\_Z^6-\_Z^3+1\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(x^6-x^3+1),x)
```

```
[Out] 1/3*sum(_R^3/(2*_R^5-_R^2)*ln(-_R+x),_R=RootOf(_Z^6-_Z^3+1))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^3}{x^6-x^3+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(x^6-x^3+1),x, algorithm="maxima")
```

```
[Out] integrate(x^3/(x^6 - x^3 + 1), x)
```

mupad [B] time = 1.84, size = 327, normalized size = 0.80

$$\frac{\ln\left(x+\frac{2^{2/3}\,3^{5/6}\,{\left(-3-\sqrt{3}\,1{}\mathrm{i}\right)}^{1/3}\,1{}\mathrm{i}}{6}\right)\,{\left(-36-\sqrt{3}\,12{}\mathrm{i}\right)}^{1/3}}{18}+\frac{\ln\left(x-\frac{2^{2/3}\,3^{5/6}\,{\left(-3+\sqrt{3}\,1{}\mathrm{i}\right)}^{1/3}\,1{}\mathrm{i}}{6}\right)\,{\left(-36+\sqrt{3}\,12{}\mathrm{i}\right)}^{1/3}}{18}-\frac{2^{2/3}\,\ln\left(x+\frac{2^{2/3}\,3^{1/3}\,{\left(-3-\sqrt{3}\,1{}\mathrm{i}\right)}^{1/3}}{2}+\frac{2^{2/3}\,3^{1/3}\,{\left(-3-\sqrt{3}\,1{}\mathrm{i}\right)}^{4/3}}{12}\right)\,{\left(-3-\sqrt{3}\,1{}\mathrm{i}\right)}^{1/3}\,\left(3^{1/3}+3^{5/6}\,1{}\mathrm{i}\right)}{36}-\frac{2^{2/3}\,\ln\left(x+\frac{2^{2/3}\,3^{1/3}\,{\left(-3+\sqrt{3}\,1{}\mathrm{i}\right)}^{1/3}}{2}+\frac{2^{2/3}\,3^{1/3}\,{\left(-3+\sqrt{3}\,1{}\mathrm{i}\right)}^{4/3}}{12}\right)\,{\left(-3+\sqrt{3}\,1{}\mathrm{i}\right)}^{1/3}\,\left(3^{1/3}-3^{5/6}\,1{}\mathrm{i}\right)}{36}-\frac{2^{2/3}\,\ln\left(x-\frac{2^{2/3}\,3^{1/3}\,{\left(-3-\sqrt{3}\,1{}\mathrm{i}\right)}^{1/3}}{4}-\frac{2^{2/3}\,3^{5/6}\,{\left(-3-\sqrt{3}\,1{}\mathrm{i}\right)}^{1/3}\,1{}\mathrm{i}}{12}\right)\,{\left(-3-\sqrt{3}\,1{}\mathrm{i}\right)}^{1/3}\,\left(3^{1/3}-3^{5/6}\,1{}\mathrm{i}\right)}{36}-\frac{2^{2/3}\,\ln\left(x-\frac{2^{2/3}\,3^{1/3}\,{\left(-3+\sqrt{3}\,1{}\mathrm{i}\right)}^{1/3}}{4}+\frac{2^{2/3}\,3^{5/6}\,{\left(-3+\sqrt{3}\,1{}\mathrm{i}\right)}^{1/3}\,1{}\mathrm{i}}{12}\right)\,{\left(-3+\sqrt{3}\,1{}\mathrm{i}\right)}^{1/3}\,\left(3^{1/3}+3^{5/6}\,1{}\mathrm{i}\right)}{36}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(x^6 - x^3 + 1),x)
```

```
[Out] (log(x + (2^(2/3)*3^(5/6)*(- 3^(1/2)*1i - 3)^(1/3)*1i)/6)*(- 3^(1/2)*12i -
36)^(1/3))/18 + (log(x - (2^(2/3)*3^(5/6)*(3^(1/2)*1i - 3)^(1/3)*1i)/6)*(3^
(1/2)*12i - 36)^(1/3))/18 - (2^(2/3)*log(x + (2^(2/3)*3^(1/3)*(- 3^(1/2)*1i
 - 3)^(1/3))/2 + (2^(2/3)*3^(1/3)*(- 3^(1/2)*1i - 3)^(4/3))/12)*(- 3^(1/2)*
1i - 3)^(1/3)*(3^(1/3) + 3^(5/6)*1i))/36 - (2^(2/3)*log(x + (2^(2/3)*3^(1/3
)*(3^(1/2)*1i - 3)^(1/3))/2 + (2^(2/3)*3^(1/3)*(3^(1/2)*1i - 3)^(4/3))/12)*
(3^(1/2)*1i - 3)^(1/3)*(3^(1/3) - 3^(5/6)*1i))/36 - (2^(2/3)*log(x - (2^(2/
3)*3^(1/3)*(- 3^(1/2)*1i - 3)^(1/3))/4 - (2^(2/3)*3^(5/6)*(- 3^(1/2)*1i - 3
)^(1/3)*1i)/12)*(- 3^(1/2)*1i - 3)^(1/3)*(3^(1/3) - 3^(5/6)*1i))/36 - (2^(2
/3)*log(x - (2^(2/3)*3^(1/3)*(3^(1/2)*1i - 3)^(1/3))/4 + (2^(2/3)*3^(5/6)*(
3^(1/2)*1i - 3)^(1/3)*1i)/12)*(3^(1/2)*1i - 3)^(1/3)*(3^(1/3) + 3^(5/6)*1i)
)/36
```

sympy [A] time = 0.18, size = 24, normalized size = 0.06

$$\text{RootSum}\left(19683t^6 + 243t^3 + 1, \left(t \mapsto t\log\left(-1458t^4 - 9t + x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(x**6-x**3+1),x)
```

```
[Out] RootSum(19683*_t**6 + 243*_t**3 + 1, Lambda(_t, _t*log(-1458*_t**4 - 9*_t +
 x)))
```

3.161 $\int \frac{x^2}{1-x^3+x^6} \, dx$

Optimal. Leaf size=23

$$-\frac{2\tan^{-1}\left(\frac{1-2x^3}{\sqrt{3}}\right)}{3\sqrt{3}}$$

Rubi [A] time = 0.02, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {1352, 618, 204}

$$-\frac{2\tan^{-1}\left(\frac{1-2x^3}{\sqrt{3}}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(1 - x^3 + x^6),x]
```

```
[Out] (-2*ArcTan[(1 - 2*x^3)/Sqrt[3]])/(3*Sqrt[3])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1352

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a,
b, c, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2}{1-x^3+x^6} \, dx &= \frac{1}{3}\text{Subst}\left(\int \frac{1}{1-x+x^2} \, dx, x, x^3\right) \\
&= -\left(\frac{2}{3}\text{Subst}\left(\int \frac{1}{-3-x^2} \, dx, x, -1+2x^3\right)\right) \\
&= -\frac{2\tan^{-1}\left(\frac{1-2x^3}{\sqrt{3}}\right)}{3\sqrt{3}}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 23, normalized size = 1.00

$$\frac{2\tan^{-1}\left(\frac{2x^3-1}{\sqrt{3}}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(1 - x^3 + x^6),x]
[Out] (2*ArcTan[(-1 + 2*x^3)/Sqrt[3]])/(3*Sqrt[3])
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{1-x^3+x^6}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^2/(1 - x^3 + x^6),x]
[Out] IntegrateAlgebraic[x^2/(1 - x^3 + x^6), x]
```

fricas [A] time = 1.13, size = 18, normalized size = 0.78

$$\frac{2}{9}\,\sqrt{3}\,\arctan\left(\frac{1}{3}\,\sqrt{3}\left(2\,x^3-1\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^6-x^3+1),x, algorithm="fricas")
[Out] 2/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^3 - 1))
```

giac [A] time = 0.49, size = 18, normalized size = 0.78

$$\frac{2}{9}\,\sqrt{3}\,\arctan\left(\frac{1}{3}\,\sqrt{3}\left(2\,x^3-1\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^6-x^3+1),x, algorithm="giac")
[Out] 2/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^3 - 1))
```

maple [A] time = 0.00, size = 19, normalized size = 0.83

$$\frac{2\sqrt{3}\;\arctan\left(\frac{\left(2x^3-1\right)\sqrt{3}}{3}\right)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(x^6-x^3+1),x)
[Out] 2/9*3^(1/2)*arctan(1/3*(2*x^3-1)*3^(1/2))
```

maxima [A] time = 1.14, size = 18, normalized size = 0.78

$$\frac{2}{9}\,\sqrt{3}\,\arctan\left(\frac{1}{3}\,\sqrt{3}\left(2\,x^3-1\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^6-x^3+1),x, algorithm="maxima")
[Out] 2/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^3 - 1))
```

mupad [B] time = 1.22, size = 20, normalized size = 0.87

$$-\frac{2\,\sqrt{3}\,\mathrm{atan}\left(\frac{\sqrt{3}}{3}-\frac{2\,\sqrt{3}\,x^3}{3}\right)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(x^6 - x^3 + 1),x)
[Out] -(2*3^(1/2)*atan(3^(1/2)/3 - (2*3^(1/2)*x^3)/3))/9
```

sympy [A] time = 0.12, size = 27, normalized size = 1.17

$$\frac{2\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}\,x^3}{3}-\frac{\sqrt{3}}{3}\right)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(x**6-x**3+1),x)
[Out] 2*sqrt(3)*atan(2*sqrt(3)*x**3/3 - sqrt(3)/3)/9
```

3.162 $\int \frac{x}{1-x^3+x^6}\,dx$

Optimal. Leaf size=375

$$-\frac{i\log\left(2^{2/3}x^2+\sqrt[3]{2\left(1-i\sqrt{3}\right)}\,x+\left(1-i\sqrt{3}\right)^{2/3}\right)}{3\,2^{2/3}\sqrt{3}\,\sqrt[3]{1-i\sqrt{3}}}+\frac{i\log\left(2^{2/3}x^2+\sqrt[3]{2\left(1+i\sqrt{3}\right)}\,x+\left(1+i\sqrt{3}\right)^{2/3}\right)}{3\,2^{2/3}\sqrt{3}\,\sqrt[3]{1+i\sqrt{3}}}+\frac{i\log\left(-\sqrt[3]{2}\,x\right.}{3\sqrt{3}\,\sqrt[3]{\frac{1}{2}\left(\right.}}$$

Rubi [A] time = 0.25, antiderivative size = 375, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 7, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {1375, 292, 31, 634, 617, 204, 628}

$$-\frac{i\log\left(2^{2/3}x^2+\sqrt[3]{2\left(1-i\sqrt{3}\right)}\,x+\left(1-i\sqrt{3}\right)^{2/3}\right)}{3\,2^{2/3}\sqrt{3}\,\sqrt[3]{1-i\sqrt{3}}}+\frac{i\log\left(2^{2/3}x^2+\sqrt[3]{2\left(1+i\sqrt{3}\right)}\,x+\left(1+i\sqrt{3}\right)^{2/3}\right)}{3\,2^{2/3}\sqrt{3}\,\sqrt[3]{1+i\sqrt{3}}}+\frac{i\log\left(-\sqrt[3]{2}\,x+\sqrt[3]{1-i\sqrt{3}}\right)}{3\sqrt{3}\,\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}-\frac{i\log\left(-\sqrt[3]{2}\,x+\sqrt[3]{1+i\sqrt{3}}\right)}{3\sqrt{3}\,\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}+\frac{i\tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}-\frac{i\tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(1 - x^3 + x^6),x]
```

```
[Out] ((I/3)*ArcTan[(1 + (2*x)/((1 - I*Sqrt[3])/2)^(1/3))/Sqrt[3]])/((1 - I*Sqrt[
3])/2)^(1/3) - ((I/3)*ArcTan[(1 + (2*x)/((1 + I*Sqrt[3])/2)^(1/3))/Sqrt[3]]
)/((1 + I*Sqrt[3])/2)^(1/3) + ((I/3)*Log[(1 - I*Sqrt[3])^(1/3) - 2^(1/3)*x]
)/(Sqrt[3]*((1 - I*Sqrt[3])/2)^(1/3)) - ((I/3)*Log[(1 + I*Sqrt[3])^(1/3) -
2^(1/3)*x])/(Sqrt[3]*((1 + I*Sqrt[3])/2)^(1/3)) - ((I/3)*Log[(1 - I*Sqrt[3]
)^(2/3) + (2*(1 - I*Sqrt[3]))^(1/3)*x + 2^(2/3)*x^2])/(2^(2/3)*Sqrt[3]*(1 -
 I*Sqrt[3])^(1/3)) + ((I/3)*Log[(1 + I*Sqrt[3])^(2/3) + (2*(1 + I*Sqrt[3]))
^(1/3)*x + 2^(2/3)*x^2])/(2^(2/3)*Sqrt[3]*(1 + I*Sqrt[3])^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1375

```
Int[((d_.)*(x_))^(m_.)/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symb
ol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[c/q, Int[(d*x)^m/(b/2 - q/2 + c*
x^n), x], x] - Dist[c/q, Int[(d*x)^m/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[
{a, b, c, d, m}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{x}{1-x^3+x^6}\,dx = -\frac{i\int \frac{x}{-\frac{1}{2}-\frac{i\sqrt{3}}{2}+x^3}\,dx}{\sqrt{3}}+\frac{i\int \frac{x}{-\frac{1}{2}+\frac{i\sqrt{3}}{2}+x^3}\,dx}{\sqrt{3}}$$

$$= \frac{i\int \frac{1}{-\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}+x}\,dx}{3\sqrt{3}\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}} - \frac{i\int \frac{-\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}+x}{\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}+\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}x+x^2}\,dx}{3\sqrt{3}\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}} - \frac{i\int \frac{1}{-\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}+x}\,dx}{3\sqrt{3}\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}} + \frac{i\int \cdots}{\cdots}$$

$$= \frac{i\log\left(\sqrt[3]{1-i\sqrt{3}}-\sqrt[3]{2}x\right)}{3\sqrt{3}\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}} - \frac{i\log\left(\sqrt[3]{1+i\sqrt{3}}-\sqrt[3]{2}x\right)}{3\sqrt{3}\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}} + \frac{i\int \frac{1}{\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}+\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}x+x^2}\,d\cdots}{2\sqrt{3}}\cdots$$

$$= \frac{i\log\left(\sqrt[3]{1-i\sqrt{3}}-\sqrt[3]{2}x\right)}{3\sqrt{3}\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}} - \frac{i\log\left(\sqrt[3]{1+i\sqrt{3}}-\sqrt[3]{2}x\right)}{3\sqrt{3}\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}} - \frac{i\log\left(\left(1-i\sqrt{3}\right)^{2/3}+\sqrt[3]{2\left(1-i\sqrt{\cdots}\right.}\cdots\right.}{3\,2^{2/3}\sqrt{3}\sqrt[3]{1-i\sqrt{3}}\cdots}\cdots$$

$$= \frac{i\tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}} - \frac{i\tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}} + \frac{i\log\left(\sqrt[3]{1-i\sqrt{3}}-\sqrt[3]{2}x\right)}{3\sqrt{3}\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}} - \frac{i\log\left(\sqrt[3]{1+\cdots}\right.}{3\sqrt{3}\sqrt[3]{\frac{1}{2}\cdots}}\cdots$$

Mathematica [C] time = 0.01, size = 40, normalized size = 0.11

$$\frac{1}{3}\text{RootSum}\left[\#1^6-\#1^3+1\&,\frac{\log(x-\#1)}{2\#1^4-\#1}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(1 - x^3 + x^6),x]
```

```
[Out] RootSum[1 - #1^3 + #1^6 & , Log[x - #1]/(-#1 + 2*#1^4) & ]/3
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{1-x^3+x^6}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x/(1 - x^3 + x^6),x]
```

```
[Out] IntegrateAlgebraic[x/(1 - x^3 + x^6), x]
```

fricas [B] time = 1.50, size = 1583, normalized size = 4.22

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^6-x^3+1),x, algorithm="fricas")
```

```
[Out] 1/54*18^(2/3)*12^(1/6)*cos(2/3*arctan(sqrt(3) - 2))*log(18^(2/3)*12^(2/3)*c
os(2/3*arctan(sqrt(3) - 2))^4 + 18^(2/3)*12^(2/3)*sin(2/3*arctan(sqrt(3) -
2))^4 + 12*18^(1/3)*12^(1/3)*sqrt(3)*x*cos(2/3*arctan(sqrt(3) - 2))*sin(2/3
*arctan(sqrt(3) - 2)) + 6*18^(1/3)*12^(1/3)*x*cos(2/3*arctan(sqrt(3) - 2))^
2 + 2*(18^(2/3)*12^(2/3)*cos(2/3*arctan(sqrt(3) - 2))^2 - 3*18^(1/3)*12^(1/
3)*x)*sin(2/3*arctan(sqrt(3) - 2))^2 + 36*x^2) - 2/27*18^(2/3)*12^(1/6)*arc
tan(1/108*(6*18^(2/3)*12^(2/3)*sqrt(3)*x*cos(2/3*arctan(sqrt(3) - 2))^2 + 1
08*sqrt(3)*cos(2/3*arctan(sqrt(3) - 2))^4 + 108*sqrt(3)*sin(2/3*arctan(sqrt
(3) - 2))^4 - 864*cos(2/3*arctan(sqrt(3) - 2))*sin(2/3*arctan(sqrt(3) - 2))
^3 - 6*(18^(2/3)*12^(2/3)*sqrt(3)*x - 36*sqrt(3)*cos(2/3*arctan(sqrt(3) - 2
))^2)*sin(2/3*arctan(sqrt(3) - 2))^2 + 12*(18^(2/3)*12^(2/3)*x*cos(2/3*arct
an(sqrt(3) - 2)) + 72*cos(2/3*arctan(sqrt(3) - 2))^3)*sin(2/3*arctan(sqrt(3
) - 2)) - sqrt(18^(2/3)*12^(2/3)*cos(2/3*arctan(sqrt(3) - 2))^4 + 18^(2/3)*
12^(2/3)*sin(2/3*arctan(sqrt(3) - 2))^4 + 12*18^(1/3)*12^(1/3)*sqrt(3)*x*co
s(2/3*arctan(sqrt(3) - 2))*sin(2/3*arctan(sqrt(3) - 2)) + 6*18^(1/3)*12^(1/
3)*x*cos(2/3*arctan(sqrt(3) - 2))^2 + 2*(18^(2/3)*12^(2/3)*cos(2/3*arctan(s
qrt(3) - 2))^2 - 3*18^(1/3)*12^(1/3)*x)*sin(2/3*arctan(sqrt(3) - 2))^2 + 36
*x^2)*(18^(2/3)*12^(2/3)*sqrt(3)*cos(2/3*arctan(sqrt(3) - 2))^2 - 18^(2/3)*
12^(2/3)*sqrt(3)*sin(2/3*arctan(sqrt(3) - 2))^2 + 2*18^(2/3)*12^(2/3)*cos(2
/3*arctan(sqrt(3) - 2))*sin(2/3*arctan(sqrt(3) - 2))))/(3*cos(2/3*arctan(sq
rt(3) - 2))^4 - 10*cos(2/3*arctan(sqrt(3) - 2))^2*sin(2/3*arctan(sqrt(3) -
2))^2 + 3*sin(2/3*arctan(sqrt(3) - 2))^4))*sin(2/3*arctan(sqrt(3) - 2)) - 1
/27*(18^(2/3)*12^(1/6)*sqrt(3)*cos(2/3*arctan(sqrt(3) - 2)) + 18^(2/3)*12^(
1/6)*sin(2/3*arctan(sqrt(3) - 2)))*arctan(1/108*(6*18^(2/3)*12^(2/3)*sqrt(3
)*x*cos(2/3*arctan(sqrt(3) - 2))^2 + 108*sqrt(3)*cos(2/3*arctan(sqrt(3) - 2
))^4 + 108*sqrt(3)*sin(2/3*arctan(sqrt(3) - 2))^4 + 864*cos(2/3*arctan(sqrt
(3) - 2))*sin(2/3*arctan(sqrt(3) - 2))^3 - 6*(18^(2/3)*12^(2/3)*sqrt(3)*x -
 36*sqrt(3)*cos(2/3*arctan(sqrt(3) - 2))^2)*sin(2/3*arctan(sqrt(3) - 2))^2
- 12*(18^(2/3)*12^(2/3)*x*cos(2/3*arctan(sqrt(3) - 2)) + 72*cos(2/3*arctan(
sqrt(3) - 2))^3)*sin(2/3*arctan(sqrt(3) - 2)) - sqrt(18^(2/3)*12^(2/3)*cos(
2/3*arctan(sqrt(3) - 2))^4 + 18^(2/3)*12^(2/3)*sin(2/3*arctan(sqrt(3) - 2))
^4 - 12*18^(1/3)*12^(1/3)*sqrt(3)*x*cos(2/3*arctan(sqrt(3) - 2))*sin(2/3*ar
ctan(sqrt(3) - 2)) + 6*18^(1/3)*12^(1/3)*x*cos(2/3*arctan(sqrt(3) - 2))^2 +
 2*(18^(2/3)*12^(2/3)*cos(2/3*arctan(sqrt(3) - 2))^2 - 3*18^(1/3)*12^(1/3)*
x)*sin(2/3*arctan(sqrt(3) - 2))^2 + 36*x^2)*(18^(2/3)*12^(2/3)*sqrt(3)*cos(
2/3*arctan(sqrt(3) - 2))^2 - 18^(2/3)*12^(2/3)*sqrt(3)*sin(2/3*arctan(sqrt(
3) - 2))^2 - 2*18^(2/3)*12^(2/3)*cos(2/3*arctan(sqrt(3) - 2))*sin(2/3*arcta
n(sqrt(3) - 2))))/(3*cos(2/3*arctan(sqrt(3) - 2))^4 - 10*cos(2/3*arctan(sqr
t(3) - 2))^2*sin(2/3*arctan(sqrt(3) - 2))^2 + 3*sin(2/3*arctan(sqrt(3) - 2)
)^4)) + 1/27*(18^(2/3)*12^(1/6)*sqrt(3)*cos(2/3*arctan(sqrt(3) - 2)) - 18^(
2/3)*12^(1/6)*sin(2/3*arctan(sqrt(3) - 2)))*arctan(-1/432*(6*18^(2/3)*12^(2
/3)*x - 216*cos(2/3*arctan(sqrt(3) - 2))^2 + 216*sin(2/3*arctan(sqrt(3) - 2
))^2 - 18^(2/3)*12^(2/3)*sqrt(18^(2/3)*12^(2/3)*cos(2/3*arctan(sqrt(3) - 2)
)^4 + 18^(2/3)*12^(2/3)*sin(2/3*arctan(sqrt(3) - 2))^4 - 12*18^(1/3)*12^(1/
3)*x*cos(2/3*arctan(sqrt(3) - 2))^2 + 2*(18^(2/3)*12^(2/3)*cos(2/3*arctan(s
qrt(3) - 2))^2 + 6*18^(1/3)*12^(1/3)*x)*sin(2/3*arctan(sqrt(3) - 2))^2 + 36
*x^2))/(cos(2/3*arctan(sqrt(3) - 2))*sin(2/3*arctan(sqrt(3) - 2)))) + 1/108
*(18^(2/3)*12^(1/6)*sqrt(3)*sin(2/3*arctan(sqrt(3) - 2)) - 18^(2/3)*12^(1/6
```

```
)*cos(2/3*arctan(sqrt(3) - 2)))*log(18^(2/3)*12^(2/3)*cos(2/3*arctan(sqrt(3
) - 2))^4 + 18^(2/3)*12^(2/3)*sin(2/3*arctan(sqrt(3) - 2))^4 - 12*18^(1/3)*
12^(1/3)*sqrt(3)*x*cos(2/3*arctan(sqrt(3) - 2))*sin(2/3*arctan(sqrt(3) - 2)
) + 6*18^(1/3)*12^(1/3)*x*cos(2/3*arctan(sqrt(3) - 2))^2 + 2*(18^(2/3)*12^(
2/3)*cos(2/3*arctan(sqrt(3) - 2))^2 - 3*18^(1/3)*12^(1/3)*x)*sin(2/3*arctan
(sqrt(3) - 2))^2 + 36*x^2) - 1/108*(18^(2/3)*12^(1/6)*sqrt(3)*sin(2/3*arcta
n(sqrt(3) - 2)) + 18^(2/3)*12^(1/6)*cos(2/3*arctan(sqrt(3) - 2)))*log(18^(2
/3)*12^(2/3)*cos(2/3*arctan(sqrt(3) - 2))^4 + 18^(2/3)*12^(2/3)*sin(2/3*arc
tan(sqrt(3) - 2))^4 - 12*18^(1/3)*12^(1/3)*x*cos(2/3*arctan(sqrt(3) - 2))^2
 + 2*(18^(2/3)*12^(2/3)*cos(2/3*arctan(sqrt(3) - 2))^2 + 6*18^(1/3)*12^(1/3
)*x)*sin(2/3*arctan(sqrt(3) - 2))^2 + 36*x^2)
```

giac [B] time = 0.57, size = 812, normalized size = 2.17

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^6-x^3+1),x, algorithm="giac")
```

```
[Out] -1/9*(sqrt(3)*cos(4/9*pi)^5 - 10*sqrt(3)*cos(4/9*pi)^3*sin(4/9*pi)^2 + 5*sq
rt(3)*cos(4/9*pi)*sin(4/9*pi)^4 - 5*cos(4/9*pi)^4*sin(4/9*pi) + 10*cos(4/9*
pi)^2*sin(4/9*pi)^3 - sin(4/9*pi)^5 - sqrt(3)*cos(4/9*pi)^2 + sqrt(3)*sin(4
/9*pi)^2 + 2*cos(4/9*pi)*sin(4/9*pi))*arctan(-((sqrt(3)*i + 1)*cos(4/9*pi)
- 2*x)/((sqrt(3)*i + 1)*sin(4/9*pi))) - 1/9*(sqrt(3)*cos(2/9*pi)^5 - 10*sqr
t(3)*cos(2/9*pi)^3*sin(2/9*pi)^2 + 5*sqrt(3)*cos(2/9*pi)*sin(2/9*pi)^4 - 5*
cos(2/9*pi)^4*sin(2/9*pi) + 10*cos(2/9*pi)^2*sin(2/9*pi)^3 - sin(2/9*pi)^5
- sqrt(3)*cos(2/9*pi)^2 + sqrt(3)*sin(2/9*pi)^2 + 2*cos(2/9*pi)*sin(2/9*pi)
)*arctan(-((sqrt(3)*i + 1)*cos(2/9*pi) - 2*x)/((sqrt(3)*i + 1)*sin(2/9*pi))
) + 1/9*(sqrt(3)*cos(1/9*pi)^5 - 10*sqrt(3)*cos(1/9*pi)^3*sin(1/9*pi)^2 + 5
*sqrt(3)*cos(1/9*pi)*sin(1/9*pi)^4 + 5*cos(1/9*pi)^4*sin(1/9*pi) - 10*cos(1
/9*pi)^2*sin(1/9*pi)^3 + sin(1/9*pi)^5 + sqrt(3)*cos(1/9*pi)^2 - sqrt(3)*si
n(1/9*pi)^2 + 2*cos(1/9*pi)*sin(1/9*pi))*arctan(((sqrt(3)*i + 1)*cos(1/9*pi
) + 2*x)/((sqrt(3)*i + 1)*sin(1/9*pi))) - 1/18*(5*sqrt(3)*cos(4/9*pi)^4*sin
(4/9*pi) - 10*sqrt(3)*cos(4/9*pi)^2*sin(4/9*pi)^3 + sqrt(3)*sin(4/9*pi)^5 +
 cos(4/9*pi)^5 - 10*cos(4/9*pi)^3*sin(4/9*pi)^2 + 5*cos(4/9*pi)*sin(4/9*pi)
^4 - 2*sqrt(3)*cos(4/9*pi)*sin(4/9*pi) - cos(4/9*pi)^2 + sin(4/9*pi)^2)*log
(-(sqrt(3)*i*cos(4/9*pi) + cos(4/9*pi))*x + x^2 + 1) - 1/18*(5*sqrt(3)*cos(
2/9*pi)^4*sin(2/9*pi) - 10*sqrt(3)*cos(2/9*pi)^2*sin(2/9*pi)^3 + sqrt(3)*si
n(2/9*pi)^5 + cos(2/9*pi)^5 - 10*cos(2/9*pi)^3*sin(2/9*pi)^2 + 5*cos(2/9*pi
)*sin(2/9*pi)^4 - 2*sqrt(3)*cos(2/9*pi)*sin(2/9*pi) - cos(2/9*pi)^2 + sin(2
/9*pi)^2)*log(-(sqrt(3)*i*cos(2/9*pi) + cos(2/9*pi))*x + x^2 + 1) - 1/18*(5
*sqrt(3)*cos(1/9*pi)^4*sin(1/9*pi) - 10*sqrt(3)*cos(1/9*pi)^2*sin(1/9*pi)^3
 + sqrt(3)*sin(1/9*pi)^5 - cos(1/9*pi)^5 + 10*cos(1/9*pi)^3*sin(1/9*pi)^2 -
 5*cos(1/9*pi)*sin(1/9*pi)^4 + 2*sqrt(3)*cos(1/9*pi)*sin(1/9*pi) - cos(1/9*
pi)^2 + sin(1/9*pi)^2)*log((sqrt(3)*i*cos(1/9*pi) + cos(1/9*pi))*x + x^2 +
1)
```

maple [C] time = 0.01, size = 38, normalized size = 0.10

$$\frac{\mathrm{RootOf}\left(\_Z^6-\_Z^3+1\right)\ln\left(-\mathrm{RootOf}\left(\_Z^6-\_Z^3+1\right)+x\right)}{6\,\mathrm{RootOf}\left(\_Z^6-\_Z^3+1\right)^5-3\,\mathrm{RootOf}\left(\_Z^6-\_Z^3+1\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x^6-x^3+1),x)
```

```
[Out] 1/3*sum(_R/(2*_R^5-_R^2)*ln(-_R+x),_R=RootOf(_Z^6-_Z^3+1))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{x^6-x^3+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^6-x^3+1),x, algorithm="maxima")
```

```
[Out] integrate(x/(x^6 - x^3 + 1), x)
```

mupad [B] time = 0.45, size = 304, normalized size = 0.81

$$\frac{\ln\left(x+\left(81\,x-\frac{27\,\left(36-\sqrt{3}\,12\mathrm{i}\right)^{2/3}}{4}\right)\,\left(-\frac{1}{162}+\frac{\sqrt{3}\,1\mathrm{i}}{486}\right)\right)\,\left(36-\sqrt{3}\,12\mathrm{i}\right)^{1/3}}{18}+\frac{\ln\left(x-\left(81\,x-\frac{27\,\left(36+\sqrt{3}\,12\mathrm{i}\right)^{2/3}}{4}\right)\,\left(\frac{1}{162}+\frac{\sqrt{3}\,1\mathrm{i}}{486}\right)\right)\,\left(36+\sqrt{3}\,12\mathrm{i}\right)^{1/3}}{18}-\frac{2^{2/3}\,\ln\left(x+\frac{2^{1/3}\,3^{2/3}\,\left(3-\sqrt{3}\,1\mathrm{i}\right)^{2/3}}{12}+\frac{2^{1/3}\,3^{1/6}\,\left(3-\sqrt{3}\,1\mathrm{i}\right)^{2/3}\,1\mathrm{i}}{4}\right)\,\left(3-\sqrt{3}\,1\mathrm{i}\right)^{1/3}\,\left(3^{1/3}+3^{5/6}\,1\mathrm{i}\right)}{36}-\frac{2^{2/3}\,\ln\left(x+\frac{2^{1/3}\,3^{2/3}\,\left(3+\sqrt{3}\,1\mathrm{i}\right)^{2/3}}{12}-\frac{2^{1/3}\,3^{1/6}\,\left(3+\sqrt{3}\,1\mathrm{i}\right)^{2/3}\,1\mathrm{i}}{4}\right)\,\left(3+\sqrt{3}\,1\mathrm{i}\right)^{1/3}\,\left(3^{1/3}-3^{5/6}\,1\mathrm{i}\right)}{36}-\frac{2^{2/3}\,\ln\left(x-\frac{2^{1/3}\,3^{2/3}\,\left(3-\sqrt{3}\,1\mathrm{i}\right)^{2/3}}{6}\right)\,\left(3-\sqrt{3}\,1\mathrm{i}\right)^{1/3}\,\left(3^{1/3}-3^{5/6}\,1\mathrm{i}\right)}{36}-\frac{2^{2/3}\,\ln\left(x-\frac{2^{1/3}\,3^{2/3}\,\left(3+\sqrt{3}\,1\mathrm{i}\right)^{2/3}}{6}\right)\,\left(3+\sqrt{3}\,1\mathrm{i}\right)^{1/3}\,\left(3^{1/3}+3^{5/6}\,1\mathrm{i}\right)}{36}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x^6 - x^3 + 1),x)
```

```
[Out] (log(x + (81*x - (27*(36 - 3^(1/2)*12i)^(2/3))/4)*((3^(1/2)*1i)/486 - 1/162
))*(36 - 3^(1/2)*12i)^(1/3))/18 + (log(x - (81*x - (27*(3^(1/2)*12i + 36)^(
2/3))/4)*((3^(1/2)*1i)/486 + 1/162))*(3^(1/2)*12i + 36)^(1/3))/18 - (2^(2/3
)*log(x + (2^(1/3)*3^(2/3)*(3 - 3^(1/2)*1i)^(2/3))/12 + (2^(1/3)*3^(1/6)*(3
 - 3^(1/2)*1i)^(2/3)*1i)/4)*(3 - 3^(1/2)*1i)^(1/3)*(3^(1/3) + 3^(5/6)*1i))/
36 - (2^(2/3)*log(x + (2^(1/3)*3^(2/3)*(3^(1/2)*1i + 3)^(2/3))/12 - (2^(1/3
)*3^(1/6)*(3^(1/2)*1i + 3)^(2/3)*1i)/4)*(3^(1/2)*1i + 3)^(1/3)*(3^(1/3) - 3
^(5/6)*1i))/36 - (2^(2/3)*log(x - (2^(1/3)*3^(2/3)*(3 - 3^(1/2)*1i)^(2/3))/
6)*(3 - 3^(1/2)*1i)^(1/3)*(3^(1/3) - 3^(5/6)*1i))/36 - (2^(2/3)*log(x - (2^
(1/3)*3^(2/3)*(3^(1/2)*1i + 3)^(2/3))/6)*(3^(1/2)*1i + 3)^(1/3)*(3^(1/3) +
3^(5/6)*1i))/36
```

sympy [A] time = 0.18, size = 26, normalized size = 0.07

$$\operatorname{RootSum}\left(19683t^6-243t^3+1,\left(t\mapsto t\log\left(6561t^5-27t^2+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x**6-x**3+1),x)
```

```
[Out] RootSum(19683*_t**6 - 243*_t**3 + 1, Lambda(_t, _t*log(6561*_t**5 - 27*_t**
2 + x)))
```

3.163 $\int \frac{1}{1-x^3+x^6}\,dx$

Optimal. Leaf size=186

$$-\frac{(-1)^{5/18}\left(3\log\left(\sqrt[9]{-1}-x\right)+\log(2)\right)}{9\sqrt{3}}+\frac{(-1)^{13/18}\log\left(-\sqrt[3]{2}\left(x+(-1)^{8/9}\right)\right)}{3\sqrt{3}}-\frac{(-1)^{13/18}\log\left(-2^{2/3}\left(\left((-1)^{8/9}-x\right)x\right.\right.}{6\sqrt{3}}$$

Rubi [C] time = 0.24, antiderivative size = 375, normalized size of antiderivative = 2.02, number of steps used = 13, number of rules used = 7, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.583, Rules used = {1347, 200, 31, 634, 617, 204, 628}

$$-\frac{i\log\left(2^{2/3}x^2+\sqrt[3]{2\left(1-i\sqrt{3}\right)}x+\left(1-i\sqrt{3}\right)^{2/3}\right)}{3\sqrt[3]{2}\sqrt{3}\left(1-i\sqrt{3}\right)^{2/3}}+\frac{i\log\left(2^{2/3}x^2+\sqrt[3]{2\left(1+i\sqrt{3}\right)}x+\left(1+i\sqrt{3}\right)^{2/3}\right)}{3\sqrt[3]{2}\sqrt{3}\left(1+i\sqrt{3}\right)^{2/3}}+\frac{i\log\left(-\sqrt[3]{2}x+\sqrt[3]{1-i\sqrt{3}}\right)}{3\sqrt{3}\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}-\frac{i\log\left(-\sqrt[3]{2}x+\sqrt[3]{1+i\sqrt{3}}\right)}{3\sqrt{3}\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}-\frac{i\tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}+\frac{i\tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x^3 + x^6)^(-1),x]
```

```
[Out] ((-I/3)*ArcTan[(1 + (2*x)/((1 - I*Sqrt[3])/2)^(1/3))/Sqrt[3]])/((1 - I*Sqrt
[3])/2)^(2/3) + ((I/3)*ArcTan[(1 + (2*x)/((1 + I*Sqrt[3])/2)^(1/3))/Sqrt[3]
])/((1 + I*Sqrt[3])/2)^(2/3) + ((I/3)*Log[(1 - I*Sqrt[3])^(1/3) - 2^(1/3)*x
])/(Sqrt[3]*((1 - I*Sqrt[3])/2)^(2/3)) - ((I/3)*Log[(1 + I*Sqrt[3])^(1/3) -
 2^(1/3)*x])/(Sqrt[3]*((1 + I*Sqrt[3])/2)^(2/3)) - ((I/3)*Log[(1 - I*Sqrt[3
])^(2/3) + (2*(1 - I*Sqrt[3]))^(1/3)*x + 2^(2/3)*x^2])/(2^(1/3)*Sqrt[3]*(1
- I*Sqrt[3])^(2/3)) + ((I/3)*Log[(1 + I*Sqrt[3])^(2/3) + (2*(1 + I*Sqrt[3])
)^(1/3)*x + 2^(2/3)*x^2])/(2^(1/3)*Sqrt[3]*(1 + I*Sqrt[3])^(2/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1347

```
Int[((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^(-1), x_Symbol] :> With[{q
= Rt[b^2 - 4*a*c, 2]}, Dist[c/q, Int[1/(b/2 - q/2 + c*x^n), x], x] - Dist[c
/q, Int[1/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*
n] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{1}{1-x^3+x^6}\,dx = -\frac{i\int \frac{1}{-\frac{1}{2}-\frac{i\sqrt{3}}{2}+x^3}\,dx}{\sqrt{3}}+\frac{i\int \frac{1}{-\frac{1}{2}+\frac{i\sqrt{3}}{2}+x^3}\,dx}{\sqrt{3}}$$

$$=\frac{i\int \frac{1}{-\sqrt[3]{\frac{1}{2}(1-i\sqrt{3})}+x}\,dx}{3\sqrt{3}\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}+\frac{i\int \frac{-2^{2/3}\sqrt[3]{1-i\sqrt{3}}-x}{\left(\frac{1}{2}(1-i\sqrt{3})\right)^{2/3}+\sqrt[3]{\frac{1}{2}(1-i\sqrt{3})}\,x+x^2}\,dx}{3\sqrt{3}\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}-\frac{i\int \frac{1}{-\sqrt[3]{\frac{1}{2}(1+i\sqrt{3})}+x}\,dx}{3\sqrt{3}\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}-\frac{i\int \frac{}{\left(\frac{1}{2}\right.}}{}$$

$$=\frac{i\log\left(\sqrt[3]{1-i\sqrt{3}}-\sqrt[3]{2}\,x\right)}{3\sqrt{3}\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}-\frac{i\log\left(\sqrt[3]{1+i\sqrt{3}}-\sqrt[3]{2}\,x\right)}{3\sqrt{3}\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}-\frac{i\int \frac{\sqrt[3]{\frac{1}{2}(1-i\sqrt{3})}+2x}{\left(\frac{1}{2}(1-i\sqrt{3})\right)^{2/3}+\sqrt[3]{\frac{1}{2}(1-i\sqrt{3})}\,x+x^2}\,dx}{3\sqrt[3]{2}\sqrt{3}\left(1-i\sqrt{3}\right)^{2/3}}-$$

$$=\frac{i\log\left(\sqrt[3]{1-i\sqrt{3}}-\sqrt[3]{2}\,x\right)}{3\sqrt{3}\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}-\frac{i\log\left(\sqrt[3]{1+i\sqrt{3}}-\sqrt[3]{2}\,x\right)}{3\sqrt{3}\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}-\frac{i\log\left(\left(1-i\sqrt{3}\right)^{2/3}+\sqrt[3]{2\left(1-i\sqrt{3}\right)}\right.}{3\sqrt[3]{2}\sqrt{3}\left(1-i\sqrt{3}\right)^{2/3}}$$

$$=-\frac{i\tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}(1-i\sqrt{3})}}}{\sqrt{3}}\right)}{3\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}+\frac{i\tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}(1+i\sqrt{3})}}}{\sqrt{3}}\right)}{3\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}+\frac{i\log\left(\sqrt[3]{1-i\sqrt{3}}-\sqrt[3]{2}\,x\right)}{3\sqrt{3}\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}-\frac{i\log\left(\sqrt[3]{1+i\right.}{3\sqrt{3}\left(\frac{1}{2}\left(1\right.\right.}$$

Mathematica [C] time = 0.01, size = 42, normalized size = 0.23

$$\frac{1}{3}\text{RootSum}\left[\#1^6-\#1^3+1\&,\frac{\log(x-\#1)}{2\#1^5-\#1^2}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x^3 + x^6)^(-1),x]
[Out] RootSum[1 - #1^3 + #1^6 & , Log[x - #1]/(-#1^2 + 2*#1^5) & ]/3
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{1-x^3+x^6}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(1 - x^3 + x^6)^(-1),x]
```

```
[Out] IntegrateAlgebraic[(1 - x^3 + x^6)^(-1), x]
```

fricas [B] time = 1.29, size = 1027, normalized size = 5.52

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^6-x^3+1),x, algorithm="fricas")
```

```
[Out] 1/54*18^(2/3)*12^(1/6)*cos(2/3*arctan(sqrt(3) - 2))*log(18^(2/3)*12^(1/6)*s
qrt(3)*x*sin(2/3*arctan(sqrt(3) - 2)) + 3*18^(2/3)*12^(1/6)*x*cos(2/3*arcta
n(sqrt(3) - 2)) + 3*18^(1/3)*12^(1/3)*cos(2/3*arctan(sqrt(3) - 2))^2 + 3*18
^(1/3)*12^(1/3)*sin(2/3*arctan(sqrt(3) - 2))^2 + 18*x^2) - 2/27*18^(2/3)*12
^(1/6)*arctan(1/108*(6*18^(1/3)*12^(5/6)*sqrt(3)*x*cos(2/3*arctan(sqrt(3) -
 2)) + 108*sqrt(3)*cos(2/3*arctan(sqrt(3) - 2))^2 + 108*sqrt(3)*sin(2/3*arc
tan(sqrt(3) - 2))^2 + 18*(18^(1/3)*12^(5/6)*x + 24*cos(2/3*arctan(sqrt(3) -
 2)))*sin(2/3*arctan(sqrt(3) - 2)) - sqrt(18^(2/3)*12^(1/6)*sqrt(3)*x*sin(2
/3*arctan(sqrt(3) - 2)) + 3*18^(2/3)*12^(1/6)*x*cos(2/3*arctan(sqrt(3) - 2)
) + 3*18^(1/3)*12^(1/3)*cos(2/3*arctan(sqrt(3) - 2))^2 + 3*18^(1/3)*12^(1/3
)*sin(2/3*arctan(sqrt(3) - 2))^2 + 18*x^2)*(18^(1/3)*12^(5/6)*sqrt(3)*sqrt(
2)*cos(2/3*arctan(sqrt(3) - 2)) + 3*18^(1/3)*12^(5/6)*sqrt(2)*sin(2/3*arcta
n(sqrt(3) - 2))))/(cos(2/3*arctan(sqrt(3) - 2))^2 - 3*sin(2/3*arctan(sqrt(3
) - 2))^2))*sin(2/3*arctan(sqrt(3) - 2)) - 1/27*(18^(2/3)*12^(1/6)*sqrt(3)*
cos(2/3*arctan(sqrt(3) - 2)) + 18^(2/3)*12^(1/6)*sin(2/3*arctan(sqrt(3) - 2
)))*arctan(-1/108*(6*18^(1/3)*12^(5/6)*sqrt(3)*x*cos(2/3*arctan(sqrt(3) - 2
)) - 108*sqrt(3)*cos(2/3*arctan(sqrt(3) - 2))^2 - 108*sqrt(3)*sin(2/3*arcta
n(sqrt(3) - 2))^2 - 18*(18^(1/3)*12^(5/6)*x - 24*cos(2/3*arctan(sqrt(3) - 2
)))*sin(2/3*arctan(sqrt(3) - 2)) - sqrt(18^(2/3)*12^(1/6)*sqrt(3)*x*sin(2/3
*arctan(sqrt(3) - 2)) - 3*18^(2/3)*12^(1/6)*x*cos(2/3*arctan(sqrt(3) - 2))
+ 3*18^(1/3)*12^(1/3)*cos(2/3*arctan(sqrt(3) - 2))^2 + 3*18^(1/3)*12^(1/3)*
sin(2/3*arctan(sqrt(3) - 2))^2 + 18*x^2)*(18^(1/3)*12^(5/6)*sqrt(3)*sqrt(2)
*cos(2/3*arctan(sqrt(3) - 2)) - 3*18^(1/3)*12^(5/6)*sqrt(2)*sin(2/3*arctan(
sqrt(3) - 2))))/(cos(2/3*arctan(sqrt(3) - 2))^2 - 3*sin(2/3*arctan(sqrt(3)
- 2))^2)) - 1/27*(18^(2/3)*12^(1/6)*sqrt(3)*cos(2/3*arctan(sqrt(3) - 2)) -
18^(2/3)*12^(1/6)*sin(2/3*arctan(sqrt(3) - 2)))*arctan(1/216*(18^(1/3)*12^(
5/6)*sqrt(3)*sqrt(2)*sqrt(-2*18^(2/3)*12^(1/6)*sqrt(3)*x*sin(2/3*arctan(sqr
t(3) - 2)) + 3*18^(1/3)*12^(1/3)*cos(2/3*arctan(sqrt(3) - 2))^2 + 3*18^(1/3
)*12^(1/3)*sin(2/3*arctan(sqrt(3) - 2))^2 + 18*x^2) - 6*18^(1/3)*12^(5/6)*s
qrt(3)*x + 216*sin(2/3*arctan(sqrt(3) - 2)))/cos(2/3*arctan(sqrt(3) - 2)))
+ 1/108*(18^(2/3)*12^(1/6)*sqrt(3)*sin(2/3*arctan(sqrt(3) - 2)) - 18^(2/3)*
12^(1/6)*cos(2/3*arctan(sqrt(3) - 2)))*log(18^(2/3)*12^(1/6)*sqrt(3)*x*sin(
2/3*arctan(sqrt(3) - 2)) - 3*18^(2/3)*12^(1/6)*x*cos(2/3*arctan(sqrt(3) - 2
)) + 3*18^(1/3)*12^(1/3)*cos(2/3*arctan(sqrt(3) - 2))^2 + 3*18^(1/3)*12^(1/
3)*sin(2/3*arctan(sqrt(3) - 2))^2 + 18*x^2) - 1/108*(18^(2/3)*12^(1/6)*sqrt
(3)*sin(2/3*arctan(sqrt(3) - 2)) + 18^(2/3)*12^(1/6)*cos(2/3*arctan(sqrt(3)
 - 2)))*log(-2*18^(2/3)*12^(1/6)*sqrt(3)*x*sin(2/3*arctan(sqrt(3) - 2)) + 3
*18^(1/3)*12^(1/3)*cos(2/3*arctan(sqrt(3) - 2))^2 + 3*18^(1/3)*12^(1/3)*sin
(2/3*arctan(sqrt(3) - 2))^2 + 18*x^2)
```

giac [B] time = 0.50, size = 629, normalized size = 3.38

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^6-x^3+1),x, algorithm="giac")
```

```
[Out] -1/9*(sqrt(3)*cos(4/9*pi)^4 - 6*sqrt(3)*cos(4/9*pi)^2*sin(4/9*pi)^2 + sqrt(
3)*sin(4/9*pi)^4 + 4*cos(4/9*pi)^3*sin(4/9*pi) - 4*cos(4/9*pi)*sin(4/9*pi)^
3 - sqrt(3)*cos(4/9*pi) - sin(4/9*pi))*arctan(-((sqrt(3)*i + 1)*cos(4/9*pi)
 - 2*x)/((sqrt(3)*i + 1)*sin(4/9*pi))) - 1/9*(sqrt(3)*cos(2/9*pi)^4 - 6*sqr
```

```
t(3)*cos(2/9*pi)^2*sin(2/9*pi)^2 + sqrt(3)*sin(2/9*pi)^4 + 4*cos(2/9*pi)^3*
sin(2/9*pi) - 4*cos(2/9*pi)*sin(2/9*pi)^3 - sqrt(3)*cos(2/9*pi) - sin(2/9*p
i))*arctan(-((sqrt(3)*i + 1)*cos(2/9*pi) - 2*x)/((sqrt(3)*i + 1)*sin(2/9*pi
))) - 1/9*(sqrt(3)*cos(1/9*pi)^4 - 6*sqrt(3)*cos(1/9*pi)^2*sin(1/9*pi)^2 +
sqrt(3)*sin(1/9*pi)^4 - 4*cos(1/9*pi)^3*sin(1/9*pi) + 4*cos(1/9*pi)*sin(1/9
*pi)^3 + sqrt(3)*cos(1/9*pi) - sin(1/9*pi))*arctan(((sqrt(3)*i + 1)*cos(1/9
*pi) + 2*x)/((sqrt(3)*i + 1)*sin(1/9*pi))) - 1/18*(4*sqrt(3)*cos(4/9*pi)^3*
sin(4/9*pi) - 4*sqrt(3)*cos(4/9*pi)*sin(4/9*pi)^3 - cos(4/9*pi)^4 + 6*cos(4
/9*pi)^2*sin(4/9*pi)^2 - sin(4/9*pi)^4 - sqrt(3)*sin(4/9*pi) + cos(4/9*pi))
*log(-(sqrt(3)*i*cos(4/9*pi) + cos(4/9*pi))*x + x^2 + 1) - 1/18*(4*sqrt(3)*
cos(2/9*pi)^3*sin(2/9*pi) - 4*sqrt(3)*cos(2/9*pi)*sin(2/9*pi)^3 - cos(2/9*p
i)^4 + 6*cos(2/9*pi)^2*sin(2/9*pi)^2 - sin(2/9*pi)^4 - sqrt(3)*sin(2/9*pi)
+ cos(2/9*pi))*log(-(sqrt(3)*i*cos(2/9*pi) + cos(2/9*pi))*x + x^2 + 1) + 1/
18*(4*sqrt(3)*cos(1/9*pi)^3*sin(1/9*pi) - 4*sqrt(3)*cos(1/9*pi)*sin(1/9*pi)
^3 + cos(1/9*pi)^4 - 6*cos(1/9*pi)^2*sin(1/9*pi)^2 + sin(1/9*pi)^4 + sqrt(3
)*sin(1/9*pi) + cos(1/9*pi))*log((sqrt(3)*i*cos(1/9*pi) + cos(1/9*pi))*x +
x^2 + 1)
```

maple [C] time = 0.01, size = 37, normalized size = 0.20

$$\frac{\ln\left(-\mathrm{RootOf}\left(\_Z^{6}-\_Z^{3}+1\right)+x\right)}{6\,\mathrm{RootOf}\left(\_Z^{6}-\_Z^{3}+1\right)^{5}-3\,\mathrm{RootOf}\left(\_Z^{6}-\_Z^{3}+1\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^6-x^3+1),x)
```

```
[Out] 1/3*sum(1/(2*_R^5-_R^2)*ln(-_R+x),_R=RootOf(_Z^6-_Z^3+1))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{6}-x^{3}+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^6-x^3+1),x, algorithm="maxima")
```

```
[Out] integrate(1/(x^6 - x^3 + 1), x)
```

mupad [B] time = 1.79, size = 327, normalized size = 1.76

$$\frac{\ln\left(x+\frac{2^{2/3}\,3^{1/3}\,{\left(3-\sqrt{3}\,1\mathrm{i}\right)}^{1/3}}{4}-\frac{2^{2/3}\,3^{5/6}\,{\left(3-\sqrt{3}\,1\mathrm{i}\right)}^{1/3}\,1\mathrm{i}}{12}\right)\,{\left(36-\sqrt{3}\,12\mathrm{i}\right)}^{1/3}}{18}+\frac{\ln\left(x+\frac{2^{2/3}\,3^{1/3}\,{\left(3+\sqrt{3}\,1\mathrm{i}\right)}^{1/3}}{4}+\frac{2^{2/3}\,3^{5/6}\,{\left(3+\sqrt{3}\,1\mathrm{i}\right)}^{1/3}\,1\mathrm{i}}{12}\right)\,{\left(36+\sqrt{3}\,12\mathrm{i}\right)}^{1/3}}{18}-\frac{2^{2/3}\,\ln\left(x-\frac{2^{2/3}\,3^{1/3}\,{\left(3-\sqrt{3}\,1\mathrm{i}\right)}^{1/3}}{2}+\frac{2^{2/3}\,3^{1/3}\,{\left(3-\sqrt{3}\,1\mathrm{i}\right)}^{4/3}}{12}\right)\,{\left(3-\sqrt{3}\,1\mathrm{i}\right)}^{1/3}\,\left(3^{1/3}+3^{5/6}\,1\mathrm{i}\right)}{36}-\frac{2^{2/3}\,\ln\left(x-\frac{2^{2/3}\,3^{1/3}\,{\left(3+\sqrt{3}\,1\mathrm{i}\right)}^{1/3}}{2}+\frac{2^{2/3}\,3^{1/3}\,{\left(3+\sqrt{3}\,1\mathrm{i}\right)}^{4/3}}{12}\right)\,{\left(3+\sqrt{3}\,1\mathrm{i}\right)}^{1/3}\,\left(3^{1/3}-3^{5/6}\,1\mathrm{i}\right)}{36}-\frac{2^{2/3}\,\ln\left(x+\frac{2^{2/3}\,3^{5/6}\,{\left(3-\sqrt{3}\,1\mathrm{i}\right)}^{1/3}\,1\mathrm{i}}{6}\right)\,{\left(3-\sqrt{3}\,1\mathrm{i}\right)}^{1/3}\,\left(3^{1/3}-3^{5/6}\,1\mathrm{i}\right)}{36}-\frac{2^{2/3}\,\ln\left(x-\frac{2^{2/3}\,3^{5/6}\,{\left(3+\sqrt{3}\,1\mathrm{i}\right)}^{1/3}\,1\mathrm{i}}{6}\right)\,{\left(3+\sqrt{3}\,1\mathrm{i}\right)}^{1/3}\,\left(3^{1/3}+3^{5/6}\,1\mathrm{i}\right)}{36}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^6 - x^3 + 1),x)
```

```
[Out] (log(x + (2^(2/3)*3^(1/3)*(3 - 3^(1/2)*1i)^(1/3))/4 - (2^(2/3)*3^(5/6)*(3 -
 3^(1/2)*1i)^(1/3)*1i)/12)*(36 - 3^(1/2)*12i)^(1/3))/18 + (log(x + (2^(2/3)
*3^(1/3)*(3^(1/2)*1i + 3)^(1/3))/4 + (2^(2/3)*3^(5/6)*(3^(1/2)*1i + 3)^(1/3
)*1i)/12)*(3^(1/2)*12i + 36)^(1/3))/18 - (2^(2/3)*log(x - (2^(2/3)*3^(1/3)*
(3 - 3^(1/2)*1i)^(1/3))/2 + (2^(2/3)*3^(1/3)*(3 - 3^(1/2)*1i)^(4/3))/12)*(3
 - 3^(1/2)*1i)^(1/3)*(3^(1/3) + 3^(5/6)*1i))/36 - (2^(2/3)*log(x - (2^(2/3)
*3^(1/3)*(3^(1/2)*1i + 3)^(1/3))/2 + (2^(2/3)*3^(1/3)*(3^(1/2)*1i + 3)^(4/3
))/12)*(3^(1/2)*1i + 3)^(1/3)*(3^(1/3) - 3^(5/6)*1i))/36 - (2^(2/3)*log(x +
 (2^(2/3)*3^(5/6)*(3 - 3^(1/2)*1i)^(1/3)*1i)/6)*(3 - 3^(1/2)*1i)^(1/3)*(3^(
1/3) - 3^(5/6)*1i))/36 - (2^(2/3)*log(x - (2^(2/3)*3^(5/6)*(3^(1/2)*1i + 3)
^(1/3)*1i)/6)*(3^(1/2)*1i + 3)^(1/3)*(3^(1/3) + 3^(5/6)*1i))/36
```

sympy [A] time = 0.18, size = 20, normalized size = 0.11

$$\mathrm{RootSum}\left(19683t^{6}-243t^{3}+1,\left(t\mapsto t\log\left(729t^{4}+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**6-x**3+1),x)
```

```
[Out] RootSum(19683*_t**6 - 243*_t**3 + 1, Lambda(_t, _t*log(729*_t**4 + x)))
```

3.164 $\int \frac{1}{x(1-x^3+x^6)}\,dx$

Optimal. Leaf size=41

$$-\frac{\tan^{-1}\left(\frac{1-2x^3}{\sqrt{3}}\right)}{3\sqrt{3}}-\frac{1}{6}\log\left(x^6-x^3+1\right)+\log(x)$$

Rubi [A] time = 0.04, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {1357, 705, 29, 634, 618, 204, 628}

$$-\frac{1}{6}\log\left(x^6-x^3+1\right)-\frac{\tan^{-1}\left(\frac{1-2x^3}{\sqrt{3}}\right)}{3\sqrt{3}}+\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(1 - x^3 + x^6)),x]
```

```
[Out] -ArcTan[(1 - 2*x^3)/Sqrt[3]]/(3*Sqrt[3]) + Log[x] - Log[1 - x^3 + x^6]/6
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 705

```
Int[1/(((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)), x_Symbol]
 :> Dist[e^2/(c*d^2 - b*d*e + a*e^2), Int[1/(d + e*x), x], x] + Dist[1/(c*d
^2 - b*d*e + a*e^2), Int[(c*d - b*e - c*e*x)/(a + b*x + c*x^2), x], x] /; F
reeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^
2, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1357

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x\left(1-x^3+x^6\right)}\,dx &= \frac{1}{3}\operatorname{Subst}\left(\int \frac{1}{x\left(1-x+x^2\right)}\,dx,x,x^3\right)\\
&= \frac{1}{3}\operatorname{Subst}\left(\int \frac{1}{x}\,dx,x,x^3\right)+\frac{1}{3}\operatorname{Subst}\left(\int \frac{1-x}{1-x+x^2}\,dx,x,x^3\right)\\
&= \log(x)+\frac{1}{6}\operatorname{Subst}\left(\int \frac{1}{1-x+x^2}\,dx,x,x^3\right)-\frac{1}{6}\operatorname{Subst}\left(\int \frac{-1+2x}{1-x+x^2}\,dx,x,x^3\right)\\
&= \log(x)-\frac{1}{6}\log\left(1-x^3+x^6\right)-\frac{1}{3}\operatorname{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,-1+2x^3\right)\\
&= -\frac{\tan^{-1}\left(\frac{1-2x^3}{\sqrt{3}}\right)}{3\sqrt{3}}+\log(x)-\frac{1}{6}\log\left(1-x^3+x^6\right)
\end{aligned}$$

Mathematica [C] time = 0.01, size = 55, normalized size = 1.34

$$\log(x)-\frac{1}{3}\operatorname{RootSum}\left[\#1^6-\#1^3+1\&,\frac{\#1^3\log(x-\#1)-\log(x-\#1)}{2\#1^3-1}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(1 - x^3 + x^6)),x]
[Out] Log[x] - RootSum[1 - #1^3 + #1^6 & , (-Log[x - #1] + Log[x - #1]*#1^3)/(-1
+ 2*#1^3) & ]/3
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(1-x^3+x^6\right)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x*(1 - x^3 + x^6)),x]
[Out] IntegrateAlgebraic[1/(x*(1 - x^3 + x^6)), x]
```

fricas [A] time = 1.23, size = 34, normalized size = 0.83

$$\frac{1}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^3-1\right)\right)-\frac{1}{6}\log\left(x^6-x^3+1\right)+\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x^6-x^3+1),x, algorithm="fricas")
[Out] 1/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^3 - 1)) - 1/6*log(x^6 - x^3 + 1) + log(
x)
```

giac [A] time = 0.35, size = 35, normalized size = 0.85

$$\frac{1}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^3-1\right)\right)-\frac{1}{6}\log\left(x^6-x^3+1\right)+\log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x^6-x^3+1),x, algorithm="giac")
```

```
[Out] 1/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^3 - 1)) - 1/6*log(x^6 - x^3 + 1) + log(
abs(x))
```

maple [A] time = 0.01, size = 35, normalized size = 0.85

$$\frac{\sqrt{3}\ \arctan\left(\frac{(2x^3-1)\sqrt{3}}{3}\right)}{9}+\ln(x)-\frac{\ln\left(x^6-x^3+1\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(x^6-x^3+1),x)
```

```
[Out] -1/6*ln(x^6-x^3+1)+1/9*3^(1/2)*arctan(1/3*(2*x^3-1)*3^(1/2))+ln(x)
```

maxima [A] time = 1.45, size = 38, normalized size = 0.93

$$\frac{1}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^3-1\right)\right)-\frac{1}{6}\log\left(x^6-x^3+1\right)+\frac{1}{3}\log\left(x^3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x^6-x^3+1),x, algorithm="maxima")
```

```
[Out] 1/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^3 - 1)) - 1/6*log(x^6 - x^3 + 1) + 1/3*
log(x^3)
```

mupad [B] time = 1.23, size = 36, normalized size = 0.88

$$\ln(x)-\frac{\ln\left(x^6-x^3+1\right)}{6}-\frac{\sqrt{3}\ \mathrm{atan}\left(\frac{\sqrt{3}}{3}-\frac{2\sqrt{3}\,x^3}{3}\right)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(x^6 - x^3 + 1)),x)
```

```
[Out] log(x) - log(x^6 - x^3 + 1)/6 - (3^(1/2)*atan(3^(1/2)/3 - (2*3^(1/2)*x^3)/3
))/9
```

sympy [A] time = 0.15, size = 41, normalized size = 1.00

$$\log(x)-\frac{\log\left(x^6-x^3+1\right)}{6}+\frac{\sqrt{3}\ \mathrm{atan}\left(\frac{2\sqrt{3}x^3}{3}-\frac{\sqrt{3}}{3}\right)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x**6-x**3+1),x)
```

```
[Out] log(x) - log(x**6 - x**3 + 1)/6 + sqrt(3)*atan(2*sqrt(3)*x**3/3 - sqrt(3)/3
)/9
```

3.165 $\int \frac{1}{x^2(1-x^3+x^6)}\,dx$

Optimal. Leaf size=416

$$\frac{\left(3-i\sqrt{3}\right)\log\left(2^{2/3}x^2+\sqrt[3]{2\left(1-i\sqrt{3}\right)}\,x+\left(1-i\sqrt{3}\right)^{2/3}\right)}{18\,2^{2/3}\sqrt[3]{1-i\sqrt{3}}}+\frac{\left(3+i\sqrt{3}\right)\log\left(2^{2/3}x^2+\sqrt[3]{2\left(1+i\sqrt{3}\right)}\,x+\left(1+i\sqrt{3}\right.\right.}{18\,2^{2/3}\sqrt[3]{1+i\sqrt{3}}}$$

Rubi [A] time = 0.30, antiderivative size = 416, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 8, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {1368, 1510, 292, 31, 634, 617, 204, 628}

$$\frac{\left(3-i\sqrt{3}\right)\log\left(2^{2/3}x^2+\sqrt[3]{2\left(1-i\sqrt{3}\right)}\,x+\left(1-i\sqrt{3}\right)^{2/3}\right)}{18\,2^{2/3}\sqrt[3]{1-i\sqrt{3}}}+\frac{\left(3+i\sqrt{3}\right)\log\left(2^{2/3}x^2+\sqrt[3]{2\left(1+i\sqrt{3}\right)}\,x+\left(1+i\sqrt{3}\right)^{2/3}\right)}{18\,2^{2/3}\sqrt[3]{1+i\sqrt{3}}}-\frac{1}{x}-\frac{\left(3-i\sqrt{3}\right)\log\left(-\sqrt[3]{2}\,x+\sqrt[3]{1-i\sqrt{3}}\right)}{9\,2^{2/3}\sqrt[3]{1-i\sqrt{3}}}-\frac{\left(3+i\sqrt{3}\right)\log\left(-\sqrt[3]{2}\,x+\sqrt[3]{1+i\sqrt{3}}\right)}{9\,2^{2/3}\sqrt[3]{1+i\sqrt{3}}}+\frac{\left(-\sqrt{3}+i\right)\tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\,2^{2/3}\sqrt[3]{1-i\sqrt{3}}}-\frac{\left(\sqrt{3}+i\right)\tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\,2^{2/3}\sqrt[3]{1+i\sqrt{3}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(1 - x^3 + x^6)),x]
```

```
[Out] -x^(-1) + ((I - Sqrt[3])*ArcTan[(1 + (2*x)/((1 - I*Sqrt[3])/2)^(1/3))/Sqrt[
3]])/(3*2^(2/3)*(1 - I*Sqrt[3])^(1/3)) - ((I + Sqrt[3])*ArcTan[(1 + (2*x)/(
(1 + I*Sqrt[3])/2)^(1/3))/Sqrt[3]])/(3*2^(2/3)*(1 + I*Sqrt[3])^(1/3)) - ((3
 - I*Sqrt[3])*Log[(1 - I*Sqrt[3])^(1/3) - 2^(1/3)*x])/(9*2^(2/3)*(1 - I*Sqr
t[3])^(1/3)) - ((3 + I*Sqrt[3])*Log[(1 + I*Sqrt[3])^(1/3) - 2^(1/3)*x])/(9*
2^(2/3)*(1 + I*Sqrt[3])^(1/3)) + ((3 - I*Sqrt[3])*Log[(1 - I*Sqrt[3])^(2/3)
 + (2*(1 - I*Sqrt[3]))^(1/3)*x + 2^(2/3)*x^2])/(18*2^(2/3)*(1 - I*Sqrt[3])^
(1/3)) + ((3 + I*Sqrt[3])*Log[(1 + I*Sqrt[3])^(2/3) + (2*(1 + I*Sqrt[3]))^(
1/3)*x + 2^(2/3)*x^2])/(18*2^(2/3)*(1 + I*Sqrt[3])^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1368

```
Int[((d_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_
Symbol] :> Simp[((d*x)^(m + 1)*(a + b*x^n + c*x^(2*n))^(p + 1))/(a*d*(m + 1
)), x] - Dist[1/(a*d^n*(m + 1)), Int[(d*x)^(m + n)*(b*(m + n*(p + 1) + 1) +
 c*(m + 2*n*(p + 1) + 1)*x^n)*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a
, b, c, d, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && L
tQ[m, -1] && IntegerQ[p]
```

Rule 1510

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_)))/((a_) + (b_.)*(x_)^(n_) +
 (c_.)*(x_)^(n2_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 +
(2*c*d - b*e)/(2*q), Int[(f*x)^m/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (
2*c*d - b*e)/(2*q), Int[(f*x)^m/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b
, c, d, e, f, m}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{1}{x^2\left(1-x^3+x^6\right)}\,dx = -\frac{1}{x}+\int \frac{x\left(1-x^3\right)}{1-x^3+x^6}\,dx$$

$$= -\frac{1}{x}+\frac{1}{6}\left(-3+i\sqrt{3}\right)\int \frac{x}{-\frac{1}{2}+\frac{i\sqrt{3}}{2}+x^3}\,dx-\frac{1}{6}\left(3+i\sqrt{3}\right)\int \frac{x}{-\frac{1}{2}-\frac{i\sqrt{3}}{2}+x^3}\,dx$$

$$= -\frac{1}{x}-\frac{\left(3-i\sqrt{3}\right)\int \frac{1}{-\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}+x}\,dx}{9\ 2^{2/3}\sqrt[3]{1-i\sqrt{3}}}+\frac{\left(3-i\sqrt{3}\right)\int \frac{-\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}+x}{\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}+\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}\,x+x^2}\,dx}{9\ 2^{2/3}\sqrt[3]{1-i\sqrt{3}}}-\frac{(3}{}$$

$$= -\frac{1}{x}-\frac{\left(3-i\sqrt{3}\right)\log\left(\sqrt[3]{1-i\sqrt{3}}-\sqrt[3]{2}\,x\right)}{9\ 2^{2/3}\sqrt[3]{1-i\sqrt{3}}}-\frac{\left(3+i\sqrt{3}\right)\log\left(\sqrt[3]{1+i\sqrt{3}}-\sqrt[3]{2}\,x\right)}{9\ 2^{2/3}\sqrt[3]{1+i\sqrt{3}}}+\frac{(3-i}{}$$

$$= -\frac{1}{x}-\frac{\left(3-i\sqrt{3}\right)\log\left(\sqrt[3]{1-i\sqrt{3}}-\sqrt[3]{2}\,x\right)}{9\ 2^{2/3}\sqrt[3]{1-i\sqrt{3}}}-\frac{\left(3+i\sqrt{3}\right)\log\left(\sqrt[3]{1+i\sqrt{3}}-\sqrt[3]{2}\,x\right)}{9\ 2^{2/3}\sqrt[3]{1+i\sqrt{3}}}+\frac{(3-i}{}$$

$$= -\frac{1}{x}+\frac{\left(i-\sqrt{3}\right)\tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\ 2^{2/3}\sqrt[3]{1-i\sqrt{3}}}-\frac{\left(i+\sqrt{3}\right)\tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\ 2^{2/3}\sqrt[3]{1+i\sqrt{3}}}-\frac{\left(3-i\sqrt{3}\right)\log(}{9\ 2^{2/3}}$$

Mathematica [C] time = 0.01, size = 61, normalized size = 0.15

$$-\frac{1}{3}\text{RootSum}\left[\#1^6-\#1^3+1\&,\frac{\#1^3\log(x-\#1)-\log(x-\#1)}{2\#1^4-\#1}\&\right]-\frac{1}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(1 - x^3 + x^6)),x]
```

```
[Out] -x^(-1) - RootSum[1 - #1^3 + #1^6 & , (-Log[x - #1] + Log[x - #1]*#1^3)/(-#
1 + 2*#1^4) & ]/3
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2(1-x^3+x^6)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x^2*(1 - x^3 + x^6)),x]
```

```
[Out] IntegrateAlgebraic[1/(x^2*(1 - x^3 + x^6)), x]
```

fricas [B] time = 1.65, size = 1598, normalized size = 3.84

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(x^6-x^3+1),x, algorithm="fricas")
```

```
[Out] 1/108*(2*18^(2/3)*12^(1/6)*x*cos(2/3*arctan(sqrt(3) + 2))*log(18^(2/3)*12^(
2/3)*cos(2/3*arctan(sqrt(3) + 2))^4 + 18^(2/3)*12^(2/3)*sin(2/3*arctan(sqrt
(3) + 2))^4 - 12*18^(1/3)*12^(1/3)*x*cos(2/3*arctan(sqrt(3) + 2))^2 + 2*(18
^(2/3)*12^(2/3)*cos(2/3*arctan(sqrt(3) + 2))^2 + 6*18^(1/3)*12^(1/3)*x)*sin
(2/3*arctan(sqrt(3) + 2))^2 + 36*x^2) + 8*18^(2/3)*12^(1/6)*x*arctan(-1/432
*(6*18^(2/3)*12^(2/3)*x - 216*cos(2/3*arctan(sqrt(3) + 2))^2 + 216*sin(2/3*
arctan(sqrt(3) + 2))^2 - 18^(2/3)*12^(2/3)*sqrt(18^(2/3)*12^(2/3)*cos(2/3*a
rctan(sqrt(3) + 2))^4 + 18^(2/3)*12^(2/3)*sin(2/3*arctan(sqrt(3) + 2))^4 -
12*18^(1/3)*12^(1/3)*x*cos(2/3*arctan(sqrt(3) + 2))^2 + 2*(18^(2/3)*12^(2/3
)*cos(2/3*arctan(sqrt(3) + 2))^2 + 6*18^(1/3)*12^(1/3)*x)*sin(2/3*arctan(sq
rt(3) + 2))^2 + 36*x^2))/(cos(2/3*arctan(sqrt(3) + 2))*sin(2/3*arctan(sqrt(
3) + 2))))*sin(2/3*arctan(sqrt(3) + 2)) + 4*(18^(2/3)*12^(1/6)*sqrt(3)*x*co
s(2/3*arctan(sqrt(3) + 2)) - 18^(2/3)*12^(1/6)*x*sin(2/3*arctan(sqrt(3) + 2
)))*arctan(1/108*(6*18^(2/3)*12^(2/3)*sqrt(3)*x*cos(2/3*arctan(sqrt(3) + 2)
)^2 + 108*sqrt(3)*cos(2/3*arctan(sqrt(3) + 2))^4 + 108*sqrt(3)*sin(2/3*arct
an(sqrt(3) + 2))^4 + 864*cos(2/3*arctan(sqrt(3) + 2))*sin(2/3*arctan(sqrt(3
) + 2))^3 - 6*(18^(2/3)*12^(2/3)*sqrt(3)*x - 36*sqrt(3)*cos(2/3*arctan(sqrt
(3) + 2))^2)*sin(2/3*arctan(sqrt(3) + 2))^2 - 12*(18^(2/3)*12^(2/3)*x*cos(2
/3*arctan(sqrt(3) + 2)) + 72*cos(2/3*arctan(sqrt(3) + 2))^3)*sin(2/3*arctan
(sqrt(3) + 2)) - sqrt(18^(2/3)*12^(2/3)*cos(2/3*arctan(sqrt(3) + 2))^4 + 18
^(2/3)*12^(2/3)*sin(2/3*arctan(sqrt(3) + 2))^4 - 12*18^(1/3)*12^(1/3)*sqrt(
3)*x*cos(2/3*arctan(sqrt(3) + 2))*sin(2/3*arctan(sqrt(3) + 2)) + 6*18^(1/3)
*12^(1/3)*x*cos(2/3*arctan(sqrt(3) + 2))^2 + 2*(18^(2/3)*12^(2/3)*cos(2/3*a
rctan(sqrt(3) + 2))^2 - 3*18^(1/3)*12^(1/3)*x)*sin(2/3*arctan(sqrt(3) + 2))
^2 + 36*x^2)*(18^(2/3)*12^(2/3)*sqrt(3)*cos(2/3*arctan(sqrt(3) + 2))^2 - 18
^(2/3)*12^(2/3)*sqrt(3)*sin(2/3*arctan(sqrt(3) + 2))^2 - 2*18^(2/3)*12^(2/3
)*cos(2/3*arctan(sqrt(3) + 2))*sin(2/3*arctan(sqrt(3) + 2))))/(3*cos(2/3*ar
ctan(sqrt(3) + 2))^4 - 10*cos(2/3*arctan(sqrt(3) + 2))^2*sin(2/3*arctan(sqr
t(3) + 2))^2 + 3*sin(2/3*arctan(sqrt(3) + 2))^4)) + 4*(18^(2/3)*12^(1/6)*sq
rt(3)*x*cos(2/3*arctan(sqrt(3) + 2)) + 18^(2/3)*12^(1/6)*x*sin(2/3*arctan(s
qrt(3) + 2)))*arctan(1/108*(6*18^(2/3)*12^(2/3)*sqrt(3)*x*cos(2/3*arctan(sq
```

```
rt(3) + 2))^2 + 108*sqrt(3)*cos(2/3*arctan(sqrt(3) + 2))^4 + 108*sqrt(3)*si
n(2/3*arctan(sqrt(3) + 2))^4 - 864*cos(2/3*arctan(sqrt(3) + 2))*sin(2/3*arc
tan(sqrt(3) + 2))^3 - 6*(18^(2/3)*12^(2/3)*sqrt(3)*x - 36*sqrt(3)*cos(2/3*a
rctan(sqrt(3) + 2))^2)*sin(2/3*arctan(sqrt(3) + 2))^2 + 12*(18^(2/3)*12^(2/
3)*x*cos(2/3*arctan(sqrt(3) + 2)) + 72*cos(2/3*arctan(sqrt(3) + 2))^3)*sin(
2/3*arctan(sqrt(3) + 2)) - sqrt(18^(2/3)*12^(2/3)*cos(2/3*arctan(sqrt(3) +
2))^4 + 18^(2/3)*12^(2/3)*sin(2/3*arctan(sqrt(3) + 2))^4 + 12*18^(1/3)*12^(
1/3)*sqrt(3)*x*cos(2/3*arctan(sqrt(3) + 2))*sin(2/3*arctan(sqrt(3) + 2)) +
6*18^(1/3)*12^(1/3)*x*cos(2/3*arctan(sqrt(3) + 2))^2 + 2*(18^(2/3)*12^(2/3)
*cos(2/3*arctan(sqrt(3) + 2))^2 - 3*18^(1/3)*12^(1/3)*x)*sin(2/3*arctan(sqr
t(3) + 2))^2 + 36*x^2)*(18^(2/3)*12^(2/3)*sqrt(3)*cos(2/3*arctan(sqrt(3) +
2))^2 - 18^(2/3)*12^(2/3)*sqrt(3)*sin(2/3*arctan(sqrt(3) + 2))^2 + 2*18^(2/
3)*12^(2/3)*cos(2/3*arctan(sqrt(3) + 2))*sin(2/3*arctan(sqrt(3) + 2))))/(3*
cos(2/3*arctan(sqrt(3) + 2))^4 - 10*cos(2/3*arctan(sqrt(3) + 2))^2*sin(2/3*
arctan(sqrt(3) + 2))^2 + 3*sin(2/3*arctan(sqrt(3) + 2))^4)) + (18^(2/3)*12^
(1/6)*sqrt(3)*x*sin(2/3*arctan(sqrt(3) + 2)) - 18^(2/3)*12^(1/6)*x*cos(2/3*
arctan(sqrt(3) + 2)))*log(18^(2/3)*12^(2/3)*cos(2/3*arctan(sqrt(3) + 2))^4
+ 18^(2/3)*12^(2/3)*sin(2/3*arctan(sqrt(3) + 2))^4 + 12*18^(1/3)*12^(1/3)*s
qrt(3)*x*cos(2/3*arctan(sqrt(3) + 2))*sin(2/3*arctan(sqrt(3) + 2)) + 6*18^(
1/3)*12^(1/3)*x*cos(2/3*arctan(sqrt(3) + 2))^2 + 2*(18^(2/3)*12^(2/3)*cos(2
/3*arctan(sqrt(3) + 2))^2 - 3*18^(1/3)*12^(1/3)*x)*sin(2/3*arctan(sqrt(3) +
 2))^2 + 36*x^2) - (18^(2/3)*12^(1/6)*sqrt(3)*x*sin(2/3*arctan(sqrt(3) + 2)
) + 18^(2/3)*12^(1/6)*x*cos(2/3*arctan(sqrt(3) + 2)))*log(18^(2/3)*12^(2/3)
*cos(2/3*arctan(sqrt(3) + 2))^4 + 18^(2/3)*12^(2/3)*sin(2/3*arctan(sqrt(3)
+ 2))^4 - 12*18^(1/3)*12^(1/3)*sqrt(3)*x*cos(2/3*arctan(sqrt(3) + 2))*sin(2
/3*arctan(sqrt(3) + 2)) + 6*18^(1/3)*12^(1/3)*x*cos(2/3*arctan(sqrt(3) + 2)
)^2 + 2*(18^(2/3)*12^(2/3)*cos(2/3*arctan(sqrt(3) + 2))^2 - 3*18^(1/3)*12^(
1/3)*x)*sin(2/3*arctan(sqrt(3) + 2))^2 + 36*x^2) - 108)/x
```

giac [B] time = 0.55, size = 826, normalized size = 1.99

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(x^6-x^3+1),x, algorithm="giac")
```

```
[Out] 1/9*(sqrt(3)*cos(4/9*pi)^5 - 10*sqrt(3)*cos(4/9*pi)^3*sin(4/9*pi)^2 + 5*sqr
t(3)*cos(4/9*pi)*sin(4/9*pi)^4 - 5*cos(4/9*pi)^4*sin(4/9*pi) + 10*cos(4/9*p
i)^2*sin(4/9*pi)^3 - sin(4/9*pi)^5 + 2*sqrt(3)*cos(4/9*pi)^2 - 2*sqrt(3)*si
n(4/9*pi)^2 - 4*cos(4/9*pi)*sin(4/9*pi))*arctan(-((sqrt(3)*i + 1)*cos(4/9*p
i) - 2*x)/((sqrt(3)*i + 1)*sin(4/9*pi))) + 1/9*(sqrt(3)*cos(2/9*pi)^5 - 10*
sqrt(3)*cos(2/9*pi)^3*sin(2/9*pi)^2 + 5*sqrt(3)*cos(2/9*pi)*sin(2/9*pi)^4 -
 5*cos(2/9*pi)^4*sin(2/9*pi) + 10*cos(2/9*pi)^2*sin(2/9*pi)^3 - sin(2/9*pi)
^5 + 2*sqrt(3)*cos(2/9*pi)^2 - 2*sqrt(3)*sin(2/9*pi)^2 - 4*cos(2/9*pi)*sin(
2/9*pi))*arctan(-((sqrt(3)*i + 1)*cos(2/9*pi) - 2*x)/((sqrt(3)*i + 1)*sin(2
/9*pi))) - 1/9*(sqrt(3)*cos(1/9*pi)^5 - 10*sqrt(3)*cos(1/9*pi)^3*sin(1/9*pi
)^2 + 5*sqrt(3)*cos(1/9*pi)*sin(1/9*pi)^4 + 5*cos(1/9*pi)^4*sin(1/9*pi) - 1
0*cos(1/9*pi)^2*sin(1/9*pi)^3 + sin(1/9*pi)^5 - 2*sqrt(3)*cos(1/9*pi)^2 + 2
*sqrt(3)*sin(1/9*pi)^2 - 4*cos(1/9*pi)*sin(1/9*pi))*arctan(((sqrt(3)*i + 1)
*cos(1/9*pi) + 2*x)/((sqrt(3)*i + 1)*sin(1/9*pi))) + 1/18*(5*sqrt(3)*cos(4/
9*pi)^4*sin(4/9*pi) - 10*sqrt(3)*cos(4/9*pi)^2*sin(4/9*pi)^3 + sqrt(3)*sin(
4/9*pi)^5 + cos(4/9*pi)^5 - 10*cos(4/9*pi)^3*sin(4/9*pi)^2 + 5*cos(4/9*pi)*
sin(4/9*pi)^4 + 4*sqrt(3)*cos(4/9*pi)*sin(4/9*pi) + 2*cos(4/9*pi)^2 - 2*sin
(4/9*pi)^2)*log(-(sqrt(3)*i*cos(4/9*pi) + cos(4/9*pi))*x + x^2 + 1) + 1/18*
(5*sqrt(3)*cos(2/9*pi)^4*sin(2/9*pi) - 10*sqrt(3)*cos(2/9*pi)^2*sin(2/9*pi)
^3 + sqrt(3)*sin(2/9*pi)^5 + cos(2/9*pi)^5 - 10*cos(2/9*pi)^3*sin(2/9*pi)^2
 + 5*cos(2/9*pi)*sin(2/9*pi)^4 + 4*sqrt(3)*cos(2/9*pi)*sin(2/9*pi) + 2*cos(
2/9*pi)^2 - 2*sin(2/9*pi)^2)*log(-(sqrt(3)*i*cos(2/9*pi) + cos(2/9*pi))*x +
 x^2 + 1) + 1/18*(5*sqrt(3)*cos(1/9*pi)^4*sin(1/9*pi) - 10*sqrt(3)*cos(1/9*
pi)^2*sin(1/9*pi)^3 + sqrt(3)*sin(1/9*pi)^5 - cos(1/9*pi)^5 + 10*cos(1/9*pi
```

```
)^3*sin(1/9*pi)^2 - 5*cos(1/9*pi)*sin(1/9*pi)^4 - 4*sqrt(3)*cos(1/9*pi)*sin
(1/9*pi) + 2*cos(1/9*pi)^2 - 2*sin(1/9*pi)^2)*log((sqrt(3)*i*cos(1/9*pi) +
cos(1/9*pi))*x + x^2 + 1) - 1/x
```

maple [C] time = 0.01, size = 50, normalized size = 0.12

$$-\frac{\left(\operatorname{RootOf}\left(\_Z^6-\_Z^3+1\right)^4-\operatorname{RootOf}\left(\_Z^6-\_Z^3+1\right)\right)\ln\left(-\operatorname{RootOf}\left(\_Z^6-\_Z^3+1\right)+x\right)}{3\left(2\operatorname{RootOf}\left(\_Z^6-\_Z^3+1\right)^5-\operatorname{RootOf}\left(\_Z^6-\_Z^3+1\right)^2\right)}-\frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(x^6-x^3+1),x)
```

```
[Out] -1/3*sum((_R^4-_R)/(2*_R^5-_R^2)*ln(-_R+x),_R=RootOf(_Z^6-_Z^3+1))-1/x
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{x}-\int\frac{x^4-x}{x^6-x^3+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(x^6-x^3+1),x, algorithm="maxima")
```

```
[Out] -1/x - integrate((x^4 - x)/(x^6 - x^3 + 1), x)
```

mupad [B] time = 1.66, size = 286, normalized size = 0.69

$$\frac{\ln\left(x-\frac{2^{1/3}\,3^{2/3}\,{\left(-3+\sqrt{3}\,1\mathrm{i}\right)}^{2/3}}{6}\right)\,{\left(-36+\sqrt{3}\,12\mathrm{i}\right)}^{1/3}}{18}-\frac{1}{x}+\frac{\ln\left(x-\frac{{\left(-36-\sqrt{3}\,12\mathrm{i}\right)}^{2/3}}{12}\right)\,{\left(-36-\sqrt{3}\,12\mathrm{i}\right)}^{1/3}}{18}-\frac{2^{2/3}\,\ln\left(x-\frac{2^{1/3}\,{\left(-3-\sqrt{3}\,1\mathrm{i}\right)}^{2/3}\,{\left(3^{1/3}-3^{5/6}\,1\mathrm{i}\right)}^2}{24}\right)\,{\left(-3-\sqrt{3}\,1\mathrm{i}\right)}^{1/3}\,\left(3^{1/3}-3^{5/6}\,1\mathrm{i}\right)}{36}-\frac{2^{2/3}\,\ln\left(x-\frac{2^{1/3}\,{\left(-3-\sqrt{3}\,1\mathrm{i}\right)}^{2/3}\,{\left(3^{1/3}+3^{5/6}\,1\mathrm{i}\right)}^2}{24}\right)\,{\left(-3-\sqrt{3}\,1\mathrm{i}\right)}^{1/3}\,\left(3^{1/3}+3^{5/6}\,1\mathrm{i}\right)}{36}-\frac{2^{2/3}\,\ln\left(x-\frac{2^{1/3}\,{\left(-3+\sqrt{3}\,1\mathrm{i}\right)}^{2/3}\,{\left(3^{1/3}-3^{5/6}\,1\mathrm{i}\right)}^2}{24}\right)\,{\left(-3+\sqrt{3}\,1\mathrm{i}\right)}^{1/3}\,\left(3^{1/3}-3^{5/6}\,1\mathrm{i}\right)}{36}-\frac{2^{2/3}\,\ln\left(x-\frac{2^{1/3}\,{\left(-3+\sqrt{3}\,1\mathrm{i}\right)}^{2/3}\,{\left(3^{1/3}+3^{5/6}\,1\mathrm{i}\right)}^2}{24}\right)\,{\left(-3+\sqrt{3}\,1\mathrm{i}\right)}^{1/3}\,\left(3^{1/3}+3^{5/6}\,1\mathrm{i}\right)}{36}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(x^6 - x^3 + 1)),x)
```

```
[Out] (log(x - (2^(1/3)*3^(2/3)*(3^(1/2)*1i - 3)^(2/3))/6)*(3^(1/2)*12i - 36)^(1/
3))/18 - 1/x + (log(x - (- 3^(1/2)*12i - 36)^(2/3)/12)*(- 3^(1/2)*12i - 36)
^(1/3))/18 - (2^(2/3)*log(x - (2^(1/3)*(- 3^(1/2)*1i - 3)^(2/3)*(3^(1/3) -
3^(5/6)*1i)^2)/24)*(- 3^(1/2)*1i - 3)^(1/3)*(3^(1/3) - 3^(5/6)*1i))/36 - (2
^(2/3)*log(x - (2^(1/3)*(- 3^(1/2)*1i - 3)^(2/3)*(3^(1/3) + 3^(5/6)*1i)^2)/
24)*(- 3^(1/2)*1i - 3)^(1/3)*(3^(1/3) + 3^(5/6)*1i))/36 - (2^(2/3)*log(x -
(2^(1/3)*(3^(1/2)*1i - 3)^(2/3)*(3^(1/3) - 3^(5/6)*1i)^2)/24)*(3^(1/2)*1i -
 3)^(1/3)*(3^(1/3) - 3^(5/6)*1i))/36 - (2^(2/3)*log(x - (2^(1/3)*(3^(1/2)*1
i - 3)^(2/3)*(3^(1/3) + 3^(5/6)*1i)^2)/24)*(3^(1/2)*1i - 3)^(1/3)*(3^(1/3)
+ 3^(5/6)*1i))/36
```

sympy [A] time = 0.20, size = 24, normalized size = 0.06

$$\operatorname{RootSum}\left(19683t^6+243t^3+1,\left(t\mapsto t\log\left(-27t^2+x\right)\right)\right)-\frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(x**6-x**3+1),x)
```

```
[Out] RootSum(19683*_t**6 + 243*_t**3 + 1, Lambda(_t, _t*log(-27*_t**2 + x))) - 1
/x
```

3.166 $\int \frac{1}{x^3(1-x^3+x^6)}\,dx$

Optimal. Leaf size=418

$$-\frac{1}{2x^2}+\frac{\left(3-i\sqrt{3}\right)\log\left(2^{2/3}x^2+\sqrt[3]{2\left(1-i\sqrt{3}\right)}\,x+\left(1-i\sqrt{3}\right)^{2/3}\right)}{18\sqrt[3]{2}\left(1-i\sqrt{3}\right)^{2/3}}+\frac{\left(3+i\sqrt{3}\right)\log\left(2^{2/3}x^2+\sqrt[3]{2\left(1+i\sqrt{3}\right)}\,x+\left(1+\right.\right.}{18\sqrt[3]{2}\left(1+i\sqrt{3}\right)^{2/3}}$$

Rubi [A] time = 0.34, antiderivative size = 418, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 8, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {1368, 1422, 200, 31, 634, 617, 204, 628}

$$-\frac{1}{2x^2}+\frac{\left(3-i\sqrt{3}\right)\log\left(2^{2/3}x^2+\sqrt[3]{2\left(1-i\sqrt{3}\right)}\,x+\left(1-i\sqrt{3}\right)^{2/3}\right)}{18\sqrt[3]{2}\left(1-i\sqrt{3}\right)^{2/3}}+\frac{\left(3+i\sqrt{3}\right)\log\left(2^{2/3}x^2+\sqrt[3]{2\left(1+i\sqrt{3}\right)}\,x+\left(1+i\sqrt{3}\right)^{2/3}\right)}{18\sqrt[3]{2}\left(1+i\sqrt{3}\right)^{2/3}}-\frac{\left(3-i\sqrt{3}\right)\log\left(-\sqrt[3]{2}x+\sqrt[3]{1-i\sqrt{3}}\right)}{9\sqrt[3]{2}\left(1-i\sqrt{3}\right)^{2/3}}-\frac{\left(3+i\sqrt{3}\right)\log\left(-\sqrt[3]{2}x+\sqrt[3]{1+i\sqrt{3}}\right)}{9\sqrt[3]{2}\left(1+i\sqrt{3}\right)^{2/3}}-\frac{\left(-\sqrt{3}+i\right)\tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\sqrt[3]{2}\left(1-i\sqrt{3}\right)^{2/3}}+\frac{\left(\sqrt{3}+i\right)\tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\sqrt[3]{2}\left(1+i\sqrt{3}\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(1 - x^3 + x^6)),x]
```

```
[Out] -1/(2*x^2) - ((I - Sqrt[3])*ArcTan[(1 + (2*x)/((1 - I*Sqrt[3])/2)^(1/3))/Sq
rt[3]])/(3*2^(1/3)*(1 - I*Sqrt[3])^(2/3)) + ((I + Sqrt[3])*ArcTan[(1 + (2*x
)/((1 + I*Sqrt[3])/2)^(1/3))/Sqrt[3]])/(3*2^(1/3)*(1 + I*Sqrt[3])^(2/3)) -
((3 - I*Sqrt[3])*Log[(1 - I*Sqrt[3])^(1/3) - 2^(1/3)*x])/(9*2^(1/3)*(1 - I*
Sqrt[3])^(2/3)) - ((3 + I*Sqrt[3])*Log[(1 + I*Sqrt[3])^(1/3) - 2^(1/3)*x])/
(9*2^(1/3)*(1 + I*Sqrt[3])^(2/3)) + ((3 - I*Sqrt[3])*Log[(1 - I*Sqrt[3])^(2
/3) + (2*(1 - I*Sqrt[3]))^(1/3)*x + 2^(2/3)*x^2])/(18*2^(1/3)*(1 - I*Sqrt[3
])^(2/3)) + ((3 + I*Sqrt[3])*Log[(1 + I*Sqrt[3])^(2/3) + (2*(1 + I*Sqrt[3])
)^(1/3)*x + 2^(2/3)*x^2])/(18*2^(1/3)*(1 + I*Sqrt[3])^(2/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1368

```
Int[((d_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_
Symbol] :> Simp[((d*x)^(m + 1)*(a + b*x^n + c*x^(2*n))^(p + 1))/(a*d*(m + 1
)), x] - Dist[1/(a*d^n*(m + 1)), Int[(d*x)^(m + n)*(b*(m + n*(p + 1) + 1) +
 c*(m + 2*n*(p + 1) + 1)*x^n)*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a
, b, c, d, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && L
tQ[m, -1] && IntegerQ[p]
```

Rule 1422

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q),
Int[1/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(
b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (PosQ[b^2 - 4*a
*c] ||  !IGtQ[n/2, 0])
```

Rubi steps

$$\int \frac{1}{x^3\left(1-x^3+x^6\right)}\,dx = -\frac{1}{2x^2}+\frac{1}{2}\int \frac{2-2x^3}{1-x^3+x^6}\,dx$$

$$= -\frac{1}{2x^2}+\frac{1}{6}\left(-3+i\sqrt{3}\right)\int \frac{1}{-\frac{1}{2}+\frac{i\sqrt{3}}{2}+x^3}\,dx-\frac{1}{6}\left(3+i\sqrt{3}\right)\int \frac{1}{-\frac{1}{2}-\frac{i\sqrt{3}}{2}+x^3}\,dx$$

$$= -\frac{1}{2x^2}-\frac{\left(3-i\sqrt{3}\right)\int \frac{1}{-\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}+x}\,dx}{9\sqrt[3]{2}\left(1-i\sqrt{3}\right)^{2/3}}-\frac{\left(3-i\sqrt{3}\right)\int \frac{-2^{2/3}\sqrt[3]{1-i\sqrt{3}}-x}{\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}+\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}\,x+x^2}\,dx}{9\sqrt[3]{2}\left(1-i\sqrt{3}\right)^{2/3}} \ldots$$

$$= -\frac{1}{2x^2}-\frac{\left(3-i\sqrt{3}\right)\log\left(\sqrt[3]{1-i\sqrt{3}}-\sqrt[3]{2}\,x\right)}{9\sqrt[3]{2}\left(1-i\sqrt{3}\right)^{2/3}}-\frac{\left(3+i\sqrt{3}\right)\log\left(\sqrt[3]{1+i\sqrt{3}}-\sqrt[3]{2}\,x\right)}{9\sqrt[3]{2}\left(1+i\sqrt{3}\right)^{2/3}}+\ldots$$

$$= -\frac{1}{2x^2}-\frac{\left(3-i\sqrt{3}\right)\log\left(\sqrt[3]{1-i\sqrt{3}}-\sqrt[3]{2}\,x\right)}{9\sqrt[3]{2}\left(1-i\sqrt{3}\right)^{2/3}}-\frac{\left(3+i\sqrt{3}\right)\log\left(\sqrt[3]{1+i\sqrt{3}}-\sqrt[3]{2}\,x\right)}{9\sqrt[3]{2}\left(1+i\sqrt{3}\right)^{2/3}}+\ldots$$

$$= -\frac{1}{2x^2}-\frac{\left(i-\sqrt{3}\right)\tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\sqrt[3]{2}\left(1-i\sqrt{3}\right)^{2/3}}+\frac{\left(i+\sqrt{3}\right)\tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\sqrt[3]{2}\left(1+i\sqrt{3}\right)^{2/3}}-\frac{\left(3-i\sqrt{3}\right)\ldots}{9\ldots}$$

Mathematica [C] time = 0.01, size = 65, normalized size = 0.16

$$-\frac{1}{3}\text{RootSum}\left[\#1^6-\#1^3+1\&,\frac{\#1^3\log(x-\#1)-\log(x-\#1)}{2\#1^5-\#1^2}\&\right]-\frac{1}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(1 - x^3 + x^6)),x]
```

```
[Out] -1/2*1/x^2 - RootSum[1 - #1^3 + #1^6 & , (-Log[x - #1] + Log[x - #1]*#1^3)/
(-#1^2 + 2*#1^5) & ]/3
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3(1-x^3+x^6)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x^3*(1 - x^3 + x^6)),x]
```

```
[Out] IntegrateAlgebraic[1/(x^3*(1 - x^3 + x^6)), x]
```

fricas [B] time = 1.45, size = 1066, normalized size = 2.55

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(x^6-x^3+1),x, algorithm="fricas")
```

```
[Out] 1/108*(2*18^(2/3)*12^(1/6)*x^2*cos(2/3*arctan(sqrt(3) + 2))*log(-18^(2/3)*1
2^(1/6)*sqrt(3)*x*sin(2/3*arctan(sqrt(3) + 2)) + 3*18^(2/3)*12^(1/6)*x*cos(
2/3*arctan(sqrt(3) + 2)) + 3*18^(1/3)*12^(1/3)*cos(2/3*arctan(sqrt(3) + 2))
^2 + 3*18^(1/3)*12^(1/3)*sin(2/3*arctan(sqrt(3) + 2))^2 + 18*x^2) - 8*18^(2
/3)*12^(1/6)*x^2*arctan(-1/108*(6*18^(1/3)*12^(5/6)*sqrt(3)*x*cos(2/3*arcta
n(sqrt(3) + 2)) + 108*sqrt(3)*cos(2/3*arctan(sqrt(3) + 2))^2 + 108*sqrt(3)*
sin(2/3*arctan(sqrt(3) + 2))^2 - 18*(18^(1/3)*12^(5/6)*x + 24*cos(2/3*arcta
n(sqrt(3) + 2)))*sin(2/3*arctan(sqrt(3) + 2)) - sqrt(-18^(2/3)*12^(1/6)*sqr
t(3)*x*sin(2/3*arctan(sqrt(3) + 2)) + 3*18^(2/3)*12^(1/6)*x*cos(2/3*arctan(
sqrt(3) + 2)) + 3*18^(1/3)*12^(1/3)*cos(2/3*arctan(sqrt(3) + 2))^2 + 3*18^(
1/3)*12^(1/3)*sin(2/3*arctan(sqrt(3) + 2))^2 + 18*x^2)*(18^(1/3)*12^(5/6)*s
qrt(3)*sqrt(2)*cos(2/3*arctan(sqrt(3) + 2)) - 3*18^(1/3)*12^(5/6)*sqrt(2)*s
in(2/3*arctan(sqrt(3) + 2))))/(cos(2/3*arctan(sqrt(3) + 2))^2 - 3*sin(2/3*a
rctan(sqrt(3) + 2))^2))*sin(2/3*arctan(sqrt(3) + 2)) + 4*(18^(2/3)*12^(1/6)
*sqrt(3)*x^2*cos(2/3*arctan(sqrt(3) + 2)) - 18^(2/3)*12^(1/6)*x^2*sin(2/3*a
rctan(sqrt(3) + 2)))*arctan(1/108*(6*18^(1/3)*12^(5/6)*sqrt(3)*x*cos(2/3*ar
ctan(sqrt(3) + 2)) - 108*sqrt(3)*cos(2/3*arctan(sqrt(3) + 2))^2 - 108*sqrt(
3)*sin(2/3*arctan(sqrt(3) + 2))^2 + 18*(18^(1/3)*12^(5/6)*x - 24*cos(2/3*ar
ctan(sqrt(3) + 2)))*sin(2/3*arctan(sqrt(3) + 2)) - sqrt(-18^(2/3)*12^(1/6)*
sqrt(3)*x*sin(2/3*arctan(sqrt(3) + 2)) - 3*18^(2/3)*12^(1/6)*x*cos(2/3*arct
an(sqrt(3) + 2)) + 3*18^(1/3)*12^(1/3)*cos(2/3*arctan(sqrt(3) + 2))^2 + 3*1
8^(1/3)*12^(1/3)*sin(2/3*arctan(sqrt(3) + 2))^2 + 18*x^2)*(18^(1/3)*12^(5/6
)*sqrt(3)*sqrt(2)*cos(2/3*arctan(sqrt(3) + 2)) + 3*18^(1/3)*12^(5/6)*sqrt(2
)*sin(2/3*arctan(sqrt(3) + 2))))/(cos(2/3*arctan(sqrt(3) + 2))^2 - 3*sin(2/
3*arctan(sqrt(3) + 2))^2)) - 4*(18^(2/3)*12^(1/6)*sqrt(3)*x^2*cos(2/3*arcta
n(sqrt(3) + 2)) + 18^(2/3)*12^(1/6)*x^2*sin(2/3*arctan(sqrt(3) + 2)))*arcta
n(1/216*(18^(1/3)*12^(5/6)*sqrt(3)*sqrt(2)*sqrt(2*18^(2/3)*12^(1/6)*sqrt(3)
*x*sin(2/3*arctan(sqrt(3) + 2)) + 3*18^(1/3)*12^(1/3)*cos(2/3*arctan(sqrt(3
) + 2))^2 + 3*18^(1/3)*12^(1/3)*sin(2/3*arctan(sqrt(3) + 2))^2 + 18*x^2) -
6*18^(1/3)*12^(5/6)*sqrt(3)*x - 216*sin(2/3*arctan(sqrt(3) + 2)))/cos(2/3*a
rctan(sqrt(3) + 2))) + (18^(2/3)*12^(1/6)*sqrt(3)*x^2*sin(2/3*arctan(sqrt(3
```

```
) + 2)) - 18^(2/3)*12^(1/6)*x^2*cos(2/3*arctan(sqrt(3) + 2)))*log(2*18^(2/3
)*12^(1/6)*sqrt(3)*x*sin(2/3*arctan(sqrt(3) + 2)) + 3*18^(1/3)*12^(1/3)*cos
(2/3*arctan(sqrt(3) + 2))^2 + 3*18^(1/3)*12^(1/3)*sin(2/3*arctan(sqrt(3) +
2))^2 + 18*x^2) - (18^(2/3)*12^(1/6)*sqrt(3)*x^2*sin(2/3*arctan(sqrt(3) + 2
)) + 18^(2/3)*12^(1/6)*x^2*cos(2/3*arctan(sqrt(3) + 2)))*log(-18^(2/3)*12^(
1/6)*sqrt(3)*x*sin(2/3*arctan(sqrt(3) + 2)) - 3*18^(2/3)*12^(1/6)*x*cos(2/3
*arctan(sqrt(3) + 2)) + 3*18^(1/3)*12^(1/3)*cos(2/3*arctan(sqrt(3) + 2))^2
+ 3*18^(1/3)*12^(1/3)*sin(2/3*arctan(sqrt(3) + 2))^2 + 18*x^2) - 54)/x^2
```

giac [B] time = 0.55, size = 642, normalized size = 1.54

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(x^6-x^3+1),x, algorithm="giac")
```

```
[Out] 1/9*(sqrt(3)*cos(4/9*pi)^4 - 6*sqrt(3)*cos(4/9*pi)^2*sin(4/9*pi)^2 + sqrt(3
)*sin(4/9*pi)^4 + 4*cos(4/9*pi)^3*sin(4/9*pi) - 4*cos(4/9*pi)*sin(4/9*pi)^3
 + 2*sqrt(3)*cos(4/9*pi) + 2*sin(4/9*pi))*arctan(-((sqrt(3)*i + 1)*cos(4/9*
pi) - 2*x)/((sqrt(3)*i + 1)*sin(4/9*pi))) + 1/9*(sqrt(3)*cos(2/9*pi)^4 - 6*
sqrt(3)*cos(2/9*pi)^2*sin(2/9*pi)^2 + sqrt(3)*sin(2/9*pi)^4 + 4*cos(2/9*pi)
^3*sin(2/9*pi) - 4*cos(2/9*pi)*sin(2/9*pi)^3 + 2*sqrt(3)*cos(2/9*pi) + 2*si
n(2/9*pi))*arctan(-((sqrt(3)*i + 1)*cos(2/9*pi) - 2*x)/((sqrt(3)*i + 1)*sin
(2/9*pi))) + 1/9*(sqrt(3)*cos(1/9*pi)^4 - 6*sqrt(3)*cos(1/9*pi)^2*sin(1/9*p
i)^2 + sqrt(3)*sin(1/9*pi)^4 - 4*cos(1/9*pi)^3*sin(1/9*pi) + 4*cos(1/9*pi)*
sin(1/9*pi)^3 - 2*sqrt(3)*cos(1/9*pi) + 2*sin(1/9*pi))*arctan(((sqrt(3)*i +
 1)*cos(1/9*pi) + 2*x)/((sqrt(3)*i + 1)*sin(1/9*pi))) + 1/18*(4*sqrt(3)*cos
(4/9*pi)^3*sin(4/9*pi) - 4*sqrt(3)*cos(4/9*pi)*sin(4/9*pi)^3 - cos(4/9*pi)^
4 + 6*cos(4/9*pi)^2*sin(4/9*pi)^2 - sin(4/9*pi)^4 + 2*sqrt(3)*sin(4/9*pi) -
 2*cos(4/9*pi))*log(-(sqrt(3)*i*cos(4/9*pi) + cos(4/9*pi))*x + x^2 + 1) + 1
/18*(4*sqrt(3)*cos(2/9*pi)^3*sin(2/9*pi) - 4*sqrt(3)*cos(2/9*pi)*sin(2/9*pi
)^3 - cos(2/9*pi)^4 + 6*cos(2/9*pi)^2*sin(2/9*pi)^2 - sin(2/9*pi)^4 + 2*sqr
t(3)*sin(2/9*pi) - 2*cos(2/9*pi))*log(-(sqrt(3)*i*cos(2/9*pi) + cos(2/9*pi)
)*x + x^2 + 1) - 1/18*(4*sqrt(3)*cos(1/9*pi)^3*sin(1/9*pi) - 4*sqrt(3)*cos(
1/9*pi)*sin(1/9*pi)^3 + cos(1/9*pi)^4 - 6*cos(1/9*pi)^2*sin(1/9*pi)^2 + sin
(1/9*pi)^4 - 2*sqrt(3)*sin(1/9*pi) - 2*cos(1/9*pi))*log((sqrt(3)*i*cos(1/9*
pi) + cos(1/9*pi))*x + x^2 + 1) - 1/2/x^2
```

maple [C] time = 0.01, size = 50, normalized size = 0.12

$$\frac{\left(-\mathrm{RootOf}\left(\_Z^6-\_Z^3+1\right)^3+1\right)\ln\left(-\mathrm{RootOf}\left(\_Z^6-\_Z^3+1\right)+x\right)}{6\,\mathrm{RootOf}\left(\_Z^6-\_Z^3+1\right)^5-3\,\mathrm{RootOf}\left(\_Z^6-\_Z^3+1\right)^2}-\frac{1}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(x^6-x^3+1),x)
```

```
[Out] 1/3*sum((-_R^3+1)/(2*_R^5-_R^2)*ln(-_R+x),_R=RootOf(_Z^6-_Z^3+1))-1/2/x^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{2\,x^2}-\int\frac{x^3-1}{x^6-x^3+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(x^6-x^3+1),x, algorithm="maxima")
```

```
[Out] -1/2/x^2 - integrate((x^3 - 1)/(x^6 - x^3 + 1), x)
```

mupad [B] time = 1.72, size = 324, normalized size = 0.78

$$\frac{\ln\left(x-\frac{\left(-\frac{27}{2}+\frac{\sqrt{3}\,9\mathrm{i}}{2}\right)\left(-36-\sqrt{3}\,12\mathrm{i}\right)^{1/3}}{54}\right)\left(-36-\sqrt{3}\,12\mathrm{i}\right)^{1/3}}{18}+\frac{\ln\left(x+\frac{\left(\frac{27}{2}+\frac{\sqrt{3}\,9\mathrm{i}}{2}\right)\left(-36+\sqrt{3}\,12\mathrm{i}\right)^{1/3}}{54}\right)\left(-36+\sqrt{3}\,12\mathrm{i}\right)^{1/3}}{18}-\frac{1}{2\,x^2}-\frac{2^{2/3}\ln\left(x-\frac{2^{2/3}\left(-3-\sqrt{3}\,1\mathrm{i}\right)^{1/3}\left(3^{1/3}+3^{5/6}\,1\mathrm{i}\right)\left(\frac{3\left(3+\sqrt{3}\,1\mathrm{i}\right)\left(3^{1/3}+3^{5/6}\,1\mathrm{i}\right)^3}{16}+27\right)}{108}\right)\left(-3-\sqrt{3}\,1\mathrm{i}\right)^{1/3}\left(3^{1/3}+3^{5/6}\,1\mathrm{i}\right)}{36}-\frac{2^{2/3}\ln\left(x+\frac{2^{2/3}\left(-3+\sqrt{3}\,1\mathrm{i}\right)^{1/3}\left(3^{1/3}-3^{5/6}\,1\mathrm{i}\right)\left(\frac{3\left(-3+\sqrt{3}\,1\mathrm{i}\right)\left(3^{1/3}-3^{5/6}\,1\mathrm{i}\right)^3}{16}-27\right)}{108}\right)\left(-3+\sqrt{3}\,1\mathrm{i}\right)^{1/3}\left(3^{1/3}-3^{5/6}\,1\mathrm{i}\right)}{36}-\frac{2^{2/3}\ln\left(x+\frac{2^{2/3}\,3^{5/6}\left(-3-\sqrt{3}\,1\mathrm{i}\right)^{1/3}\,1\mathrm{i}}{6}\right)\left(-3-\sqrt{3}\,1\mathrm{i}\right)^{1/3}\left(3^{1/3}-3^{5/6}\,1\mathrm{i}\right)}{36}-\frac{2^{2/3}\ln\left(x-\frac{2^{2/3}\,3^{5/6}\left(-3+\sqrt{3}\,1\mathrm{i}\right)^{1/3}\,1\mathrm{i}}{6}\right)\left(-3+\sqrt{3}\,1\mathrm{i}\right)^{1/3}\left(3^{1/3}+3^{5/6}\,1\mathrm{i}\right)}{36}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(x^6 - x^3 + 1)),x)
```

```
[Out] (log(x - (((3^(1/2)*9i)/2 - 27/2)*(- 3^(1/2)*12i - 36)^(1/3))/54)*(- 3^(1/2
)*12i - 36)^(1/3))/18 + (log(x + (((3^(1/2)*9i)/2 + 27/2)*(3^(1/2)*12i - 36
)^(1/3))/54)*(3^(1/2)*12i - 36)^(1/3))/18 - 1/(2*x^2) - (2^(2/3)*log(x - (2
^(2/3)*(- 3^(1/2)*1i - 3)^(1/3)*(3^(1/3) + 3^(5/6)*1i)*((3*(3^(1/2)*1i + 3)
*(3^(1/3) + 3^(5/6)*1i)^3)/16 + 27))/108)*(- 3^(1/2)*1i - 3)^(1/3)*(3^(1/3)
 + 3^(5/6)*1i))/36 - (2^(2/3)*log(x + (2^(2/3)*(3^(1/2)*1i - 3)^(1/3)*(3^(1
/3) - 3^(5/6)*1i)*((3*(3^(1/2)*1i - 3)*(3^(1/3) - 3^(5/6)*1i)^3)/16 - 27))/
108)*(3^(1/2)*1i - 3)^(1/3)*(3^(1/3) - 3^(5/6)*1i))/36 - (2^(2/3)*log(x + (
2^(2/3)*3^(5/6)*(- 3^(1/2)*1i - 3)^(1/3)*1i)/6)*(- 3^(1/2)*1i - 3)^(1/3)*(3
^(1/3) - 3^(5/6)*1i))/36 - (2^(2/3)*log(x - (2^(2/3)*3^(5/6)*(3^(1/2)*1i -
3)^(1/3)*1i)/6)*(3^(1/2)*1i - 3)^(1/3)*(3^(1/3) + 3^(5/6)*1i))/36
```

sympy [A] time = 0.21, size = 31, normalized size = 0.07

$$\operatorname{RootSum}\left(19683t^6+243t^3+1,\left(t\mapsto t\log\left(729t^4+9t+x\right)\right)\right)-\frac{1}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(x**6-x**3+1),x)
```

```
[Out] RootSum(19683*_t**6 + 243*_t**3 + 1, Lambda(_t, _t*log(729*_t**4 + 9*_t + x
))) - 1/(2*x**2)
```

3.167 $\int \frac{1}{x^4(1-x^3+x^6)} dx$

Optimal. Leaf size=48

$$-\frac{1}{3x^3}+\frac{\tan^{-1}\left(\frac{1-2x^3}{\sqrt{3}}\right)}{3\sqrt{3}}-\frac{1}{6}\log\left(x^6-x^3+1\right)+\log(x)$$

Rubi [A] time = 0.05, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {1357, 709, 800, 634, 618, 204, 628}

$$-\frac{1}{3x^3}-\frac{1}{6}\log\left(x^6-x^3+1\right)+\frac{\tan^{-1}\left(\frac{1-2x^3}{\sqrt{3}}\right)}{3\sqrt{3}}+\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(1 - x^3 + x^6)),x]
```

```
[Out] -1/(3*x^3) + ArcTan[(1 - 2*x^3)/Sqrt[3]]/(3*Sqrt[3]) + Log[x] - Log[1 - x^3
 + x^6]/6
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 709

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Simp[(e*(d + e*x)^(m + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dis
t[1/(c*d^2 - b*d*e + a*e^2), Int[((d + e*x)^(m + 1)*Simp[c*d - b*e - c*e*x,
 x])/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 -
 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[m
, -1]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 1357

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^4\left(1-x^3+x^6\right)}\,dx &= \frac{1}{3}\operatorname{Subst}\left(\int \frac{1}{x^2\left(1-x+x^2\right)}\,dx,x,x^3\right)\\
&= -\frac{1}{3x^3}+\frac{1}{3}\operatorname{Subst}\left(\int \frac{1-x}{x\left(1-x+x^2\right)}\,dx,x,x^3\right)\\
&= -\frac{1}{3x^3}+\frac{1}{3}\operatorname{Subst}\left(\int \left(\frac{1}{x}-\frac{x}{1-x+x^2}\right)dx,x,x^3\right)\\
&= -\frac{1}{3x^3}+\log(x)-\frac{1}{3}\operatorname{Subst}\left(\int \frac{x}{1-x+x^2}\,dx,x,x^3\right)\\
&= -\frac{1}{3x^3}+\log(x)-\frac{1}{6}\operatorname{Subst}\left(\int \frac{1}{1-x+x^2}\,dx,x,x^3\right)-\frac{1}{6}\operatorname{Subst}\left(\int \frac{-1+2x}{1-x+x^2}\,dx,x,x^3\right)\\
&= -\frac{1}{3x^3}+\log(x)-\frac{1}{6}\log\left(1-x^3+x^6\right)+\frac{1}{3}\operatorname{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,-1+2x^3\right)\\
&= -\frac{1}{3x^3}+\frac{\tan^{-1}\left(\frac{1-2x^3}{\sqrt{3}}\right)}{3\sqrt{3}}+\log(x)-\frac{1}{6}\log\left(1-x^3+x^6\right)
\end{aligned}$$

Mathematica [C] time = 0.01, size = 51, normalized size = 1.06

$$-\frac{1}{3}\operatorname{RootSum}\left[\#1^6-\#1^3+1\&,\frac{\#1^3\log(x-\#1)}{2\#1^3-1}\&\right]-\frac{1}{3x^3}+\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(1 - x^3 + x^6)),x]
```

```
[Out] -1/3*1/x^3 + Log[x] - RootSum[1 - #1^3 + #1^6 & , (Log[x - #1]*#1^3)/(-1 +
2*#1^3) & ]/3
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^4\left(1-x^3+x^6\right)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x^4*(1 - x^3 + x^6)),x]
```

```
[Out] IntegrateAlgebraic[1/(x^4*(1 - x^3 + x^6)), x]
```

fricas [A] time = 1.17, size = 51, normalized size = 1.06

$$-\frac{2\sqrt{3}x^3\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^3-1\right)\right)+3x^3\log\left(x^6-x^3+1\right)-18x^3\log(x)+6}{18x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(x^6-x^3+1),x, algorithm="fricas")
```

```
[Out] -1/18*(2*sqrt(3)*x^3*arctan(1/3*sqrt(3)*(2*x^3 - 1)) + 3*x^3*log(x^6 - x^3
+ 1) - 18*x^3*log(x) + 6)/x^3
```

giac [A] time = 0.42, size = 45, normalized size = 0.94

$$-\frac{1}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^3-1\right)\right)-\frac{x^3+1}{3x^3}-\frac{1}{6}\log\left(x^6-x^3+1\right)+\log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(x^6-x^3+1),x, algorithm="giac")
```

```
[Out] -1/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^3 - 1)) - 1/3*(x^3 + 1)/x^3 - 1/6*log(
x^6 - x^3 + 1) + log(abs(x))
```

maple [A] time = 0.01, size = 40, normalized size = 0.83

$$-\frac{\sqrt{3}\arctan\left(\frac{\left(2x^3-1\right)\sqrt{3}}{3}\right)}{9}+\ln(x)-\frac{\ln\left(x^6-x^3+1\right)}{6}-\frac{1}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(x^6-x^3+1),x)
```

```
[Out] -1/6*ln(x^6-x^3+1)-1/9*3^(1/2)*arctan(1/3*(2*x^3-1)*3^(1/2))-1/3/x^3+ln(x)
```

maxima [A] time = 1.13, size = 43, normalized size = 0.90

$$-\frac{1}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^3-1\right)\right)-\frac{1}{3x^3}-\frac{1}{6}\log\left(x^6-x^3+1\right)+\frac{1}{3}\log\left(x^3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(x^6-x^3+1),x, algorithm="maxima")
```

```
[Out] -1/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^3 - 1)) - 1/3/x^3 - 1/6*log(x^6 - x^3
+ 1) + 1/3*log(x^3)
```

mupad [B] time = 0.06, size = 41, normalized size = 0.85

$$\ln(x)-\frac{\ln\left(x^6-x^3+1\right)}{6}+\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{\sqrt{3}}{3}-\frac{2\sqrt{3}x^3}{3}\right)}{9}-\frac{1}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(x^6 - x^3 + 1)),x)
```

```
[Out] log(x) - log(x^6 - x^3 + 1)/6 + (3^(1/2)*atan(3^(1/2)/3 - (2*3^(1/2)*x^3)/3
))/9 - 1/(3*x^3)
```

sympy [A] time = 0.17, size = 48, normalized size = 1.00

$$\log(x) - \frac{\log(x^6 - x^3 + 1)}{6} - \frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}\,x^3}{3} - \frac{\sqrt{3}}{3}\right)}{9} - \frac{1}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(x**6-x**3+1),x)
```

```
[Out] log(x) - log(x**6 - x**3 + 1)/6 - sqrt(3)*atan(2*sqrt(3)*x**3/3 - sqrt(3)/3
)/9 - 1/(3*x**3)
```

3.168 $\int \frac{1}{x^5(1-x^3+x^6)}\,dx$

Optimal. Leaf size=423

$$-\frac{1}{4x^4}+\frac{\left(3+i\sqrt{3}\right)\log\left(2^{2/3}x^2+\sqrt[3]{2\left(1-i\sqrt{3}\right)}\,x+\left(1-i\sqrt{3}\right)^{2/3}\right)}{18\ 2^{2/3}\sqrt[3]{1-i\sqrt{3}}}+\frac{\left(3-i\sqrt{3}\right)\log\left(2^{2/3}x^2+\sqrt[3]{2\left(1+i\sqrt{3}\right)}\,x+\left(\right.\right.}{18\ 2^{2/3}\sqrt[3]{1+i\sqrt{3}}}$$

Rubi [A] time = 0.37, antiderivative size = 423, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 10, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.625, Rules used = {1368, 1504, 12, 1374, 292, 31, 634, 617, 204, 628}

$$-\frac{1}{4x^4}+\frac{\left(3+i\sqrt{3}\right)\log\left(2^{2/3}x^2+\sqrt[3]{2\left(1-i\sqrt{3}\right)}\,x+\left(1-i\sqrt{3}\right)^{2/3}\right)}{18\ 2^{2/3}\sqrt[3]{1-i\sqrt{3}}}+\frac{\left(3-i\sqrt{3}\right)\log\left(2^{2/3}x^2+\sqrt[3]{2\left(1+i\sqrt{3}\right)}\,x+\left(1+i\sqrt{3}\right)^{2/3}\right)}{18\ 2^{2/3}\sqrt[3]{1+i\sqrt{3}}}-\frac{1}{x}-\frac{\left(3+i\sqrt{3}\right)\log\left(-\sqrt[3]{2}x+\sqrt[3]{1-i\sqrt{3}}\right)}{9\ 2^{2/3}\sqrt[3]{1-i\sqrt{3}}}-\frac{\left(3-i\sqrt{3}\right)\log\left(-\sqrt[3]{2}x+\sqrt[3]{1+i\sqrt{3}}\right)}{9\ 2^{2/3}\sqrt[3]{1+i\sqrt{3}}}-\frac{\left(\sqrt{3}+i\right)\tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\ 2^{2/3}\sqrt[3]{1-i\sqrt{3}}}+\frac{\left(-\sqrt{3}+i\right)\tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\ 2^{2/3}\sqrt[3]{1+i\sqrt{3}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^5*(1 - x^3 + x^6)),x]
```

```
[Out] -1/(4*x^4) - x^(-1) - ((I + Sqrt[3])*ArcTan[(1 + (2*x)/((1 - I*Sqrt[3])/2)^
(1/3))/Sqrt[3]])/(3*2^(2/3)*(1 - I*Sqrt[3])^(1/3)) + ((I - Sqrt[3])*ArcTan[
(1 + (2*x)/((1 + I*Sqrt[3])/2)^(1/3))/Sqrt[3]])/(3*2^(2/3)*(1 + I*Sqrt[3])^
(1/3)) - ((3 + I*Sqrt[3])*Log[(1 - I*Sqrt[3])^(1/3) - 2^(1/3)*x])/(9*2^(2/3
)*(1 - I*Sqrt[3])^(1/3)) - ((3 - I*Sqrt[3])*Log[(1 + I*Sqrt[3])^(1/3) - 2^(
1/3)*x])/(9*2^(2/3)*(1 + I*Sqrt[3])^(1/3)) + ((3 + I*Sqrt[3])*Log[(1 - I*Sq
rt[3])^(2/3) + (2*(1 - I*Sqrt[3]))^(1/3)*x + 2^(2/3)*x^2])/(18*2^(2/3)*(1 -
 I*Sqrt[3])^(1/3)) + ((3 - I*Sqrt[3])*Log[(1 + I*Sqrt[3])^(2/3) + (2*(1 + I
*Sqrt[3]))^(1/3)*x + 2^(2/3)*x^2])/(18*2^(2/3)*(1 + I*Sqrt[3])^(1/3))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
```

```
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1368

```
Int[((d_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_
Symbol] :> Simp[((d*x)^(m + 1)*(a + b*x^n + c*x^(2*n))^(p + 1))/(a*d*(m + 1
)), x] - Dist[1/(a*d^n*(m + 1)), Int[(d*x)^(m + n)*(b*(m + n*(p + 1) + 1) +
 c*(m + 2*n*(p + 1) + 1)*x^n)*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a
, b, c, d, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && L
tQ[m, -1] && IntegerQ[p]
```

Rule 1374

```
Int[((d_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(d^n*(b/q + 1))/2, Int[(d*x)^(m -
 n)/(b/2 + q/2 + c*x^n), x], x] - Dist[(d^n*(b/q - 1))/2, Int[(d*x)^(m - n)
/(b/2 - q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[n2, 2*n] &&
NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GeQ[m, n]
```

Rule 1504

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (
c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Simp[(d*(f*x)^(m + 1)*(a + b*x^n + c*x^
(2*n))^(p + 1))/(a*f*(m + 1)), x] + Dist[1/(a*f^n*(m + 1)), Int[(f*x)^(m +
n)*(a + b*x^n + c*x^(2*n))^p*Simp[a*e*(m + 1) - b*d*(m + n*(p + 1) + 1) - c
*d*(m + 2*n*(p + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x]
 && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && LtQ[m, -1] && Inte
gerQ[p]
```

Rubi steps

$$\int \frac{1}{x^5\left(1-x^3+x^6\right)}\,dx = -\frac{1}{4x^4}+\frac{1}{4}\int \frac{4-4x^3}{x^2\left(1-x^3+x^6\right)}\,dx$$
$$= -\frac{1}{4x^4}-\frac{1}{x}-\frac{1}{4}\int \frac{4x^4}{1-x^3+x^6}\,dx$$
$$= -\frac{1}{4x^4}-\frac{1}{x}-\int \frac{x^4}{1-x^3+x^6}\,dx$$
$$= -\frac{1}{4x^4}-\frac{1}{x}+\frac{1}{6}\left(-3+i\sqrt{3}\right)\int \frac{x}{-\frac{1}{2}-\frac{i\sqrt{3}}{2}+x^3}\,dx-\frac{1}{6}\left(3+i\sqrt{3}\right)\int \frac{x}{-\frac{1}{2}+\frac{i\sqrt{3}}{2}+x^3}\,dx$$
$$= -\frac{1}{4x^4}-\frac{1}{x}+-\frac{\left(-3-i\sqrt{3}\right)\int \frac{-\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}+x}{\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}+\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}\,x+x^2}\,dx}{9\ 2^{2/3}\sqrt[3]{1-i\sqrt{3}}}-\frac{\left(3-i\sqrt{3}\right)\int \frac{1}{-\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right.}}}{9\ 2^{2/3}\sqrt[3]{1+i\sqrt{3}}}$$
$$= -\frac{1}{4x^4}-\frac{1}{x}-\frac{\left(3+i\sqrt{3}\right)\log\left(\sqrt[3]{1-i\sqrt{3}}-\sqrt[3]{2}\,x\right)}{9\ 2^{2/3}\sqrt[3]{1-i\sqrt{3}}}-\frac{\left(3-i\sqrt{3}\right)\log\left(\sqrt[3]{1+i\sqrt{3}}-\sqrt[3]{2}\,x\right.}{9\ 2^{2/3}\sqrt[3]{1+i\sqrt{3}}}$$
$$= -\frac{1}{4x^4}-\frac{1}{x}-\frac{\left(3+i\sqrt{3}\right)\log\left(\sqrt[3]{1-i\sqrt{3}}-\sqrt[3]{2}\,x\right)}{9\ 2^{2/3}\sqrt[3]{1-i\sqrt{3}}}-\frac{\left(3-i\sqrt{3}\right)\log\left(\sqrt[3]{1+i\sqrt{3}}-\sqrt[3]{2}\,x\right.}{9\ 2^{2/3}\sqrt[3]{1+i\sqrt{3}}}$$
$$= -\frac{1}{4x^4}-\frac{1}{x}-\frac{\left(i+\sqrt{3}\right)\tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\ 2^{2/3}\sqrt[3]{1-i\sqrt{3}}}+\frac{\left(i-\sqrt{3}\right)\tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\ 2^{2/3}\sqrt[3]{1+i\sqrt{3}}}-\frac{\left(3+i\right.}{}$$

Mathematica [C] time = 0.01, size = 54, normalized size = 0.13

$$-\frac{1}{3}\text{RootSum}\left[\#1^6-\#1^3+1\&,\frac{\#1^2\log(x-\#1)}{2\#1^3-1}\&\right]-\frac{1}{4x^4}-\frac{1}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^5*(1 - x^3 + x^6)),x]
```

```
[Out] -1/4*1/x^4 - x^(-1) - RootSum[1 - #1^3 + #1^6 & , (Log[x - #1]*#1^2)/(-1 +
2*#1^3) & ]/3
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^5\left(1-x^3+x^6\right)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x^5*(1 - x^3 + x^6)),x]
```

```
[Out] IntegrateAlgebraic[1/(x^5*(1 - x^3 + x^6)), x]
```

fricas [B] time = 1.62, size = 1623, normalized size = 3.84

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(x^6-x^3+1),x, algorithm="fricas")
```

```
[Out] 1/108*(2*18^(2/3)*12^(1/6)*x^4*cos(2/3*arctan(sqrt(3) - 2))*log(18^(2/3)*12
^(2/3)*cos(2/3*arctan(sqrt(3) - 2))^4 + 18^(2/3)*12^(2/3)*sin(2/3*arctan(sq
rt(3) - 2))^4 - 12*18^(1/3)*12^(1/3)*sqrt(3)*x*cos(2/3*arctan(sqrt(3) - 2))
*sin(2/3*arctan(sqrt(3) - 2)) + 6*18^(1/3)*12^(1/3)*x*cos(2/3*arctan(sqrt(3
) - 2))^2 + 2*(18^(2/3)*12^(2/3)*cos(2/3*arctan(sqrt(3) - 2))^2 - 3*18^(1/3
)*12^(1/3)*x)*sin(2/3*arctan(sqrt(3) - 2))^2 + 36*x^2) + 8*18^(2/3)*12^(1/6
)*x^4*arctan(1/108*(6*18^(2/3)*12^(2/3)*sqrt(3)*x*cos(2/3*arctan(sqrt(3) -
2))^2 + 108*sqrt(3)*cos(2/3*arctan(sqrt(3) - 2))^4 + 108*sqrt(3)*sin(2/3*ar
ctan(sqrt(3) - 2))^4 + 864*cos(2/3*arctan(sqrt(3) - 2))*sin(2/3*arctan(sqrt
(3) - 2))^3 - 6*(18^(2/3)*12^(2/3)*sqrt(3)*x - 36*sqrt(3)*cos(2/3*arctan(sq
rt(3) - 2))^2)*sin(2/3*arctan(sqrt(3) - 2))^2 - 12*(18^(2/3)*12^(2/3)*x*cos
(2/3*arctan(sqrt(3) - 2)) + 72*cos(2/3*arctan(sqrt(3) - 2))^3)*sin(2/3*arct
an(sqrt(3) - 2)) - sqrt(18^(2/3)*12^(2/3)*cos(2/3*arctan(sqrt(3) - 2))^4 +
18^(2/3)*12^(2/3)*sin(2/3*arctan(sqrt(3) - 2))^4 - 12*18^(1/3)*12^(1/3)*sqr
t(3)*x*cos(2/3*arctan(sqrt(3) - 2))*sin(2/3*arctan(sqrt(3) - 2)) + 6*18^(1/
3)*12^(1/3)*x*cos(2/3*arctan(sqrt(3) - 2))^2 + 2*(18^(2/3)*12^(2/3)*cos(2/3
*arctan(sqrt(3) - 2))^2 - 3*18^(1/3)*12^(1/3)*x)*sin(2/3*arctan(sqrt(3) - 2
))^2 + 36*x^2)*(18^(2/3)*12^(2/3)*sqrt(3)*cos(2/3*arctan(sqrt(3) - 2))^2 -
18^(2/3)*12^(2/3)*sqrt(3)*sin(2/3*arctan(sqrt(3) - 2))^2 - 2*18^(2/3)*12^(2
/3)*cos(2/3*arctan(sqrt(3) - 2))*sin(2/3*arctan(sqrt(3) - 2))))/(3*cos(2/3*
arctan(sqrt(3) - 2))^4 - 10*cos(2/3*arctan(sqrt(3) - 2))^2*sin(2/3*arctan(s
qrt(3) - 2))^2 + 3*sin(2/3*arctan(sqrt(3) - 2))^4))*sin(2/3*arctan(sqrt(3)
- 2)) - 108*x^3 - 4*(18^(2/3)*12^(1/6)*sqrt(3)*x^4*cos(2/3*arctan(sqrt(3) -
 2)) - 18^(2/3)*12^(1/6)*x^4*sin(2/3*arctan(sqrt(3) - 2)))*arctan(1/108*(6*
18^(2/3)*12^(2/3)*sqrt(3)*x*cos(2/3*arctan(sqrt(3) - 2))^2 + 108*sqrt(3)*co
s(2/3*arctan(sqrt(3) - 2))^4 + 108*sqrt(3)*sin(2/3*arctan(sqrt(3) - 2))^4 -
 864*cos(2/3*arctan(sqrt(3) - 2))*sin(2/3*arctan(sqrt(3) - 2))^3 - 6*(18^(2
/3)*12^(2/3)*sqrt(3)*x - 36*sqrt(3)*cos(2/3*arctan(sqrt(3) - 2))^2)*sin(2/3
*arctan(sqrt(3) - 2))^2 + 12*(18^(2/3)*12^(2/3)*x*cos(2/3*arctan(sqrt(3) -
2)) + 72*cos(2/3*arctan(sqrt(3) - 2))^3)*sin(2/3*arctan(sqrt(3) - 2)) - sqr
t(18^(2/3)*12^(2/3)*cos(2/3*arctan(sqrt(3) - 2))^4 + 18^(2/3)*12^(2/3)*sin(
2/3*arctan(sqrt(3) - 2))^4 + 12*18^(1/3)*12^(1/3)*sqrt(3)*x*cos(2/3*arctan(
sqrt(3) - 2))*sin(2/3*arctan(sqrt(3) - 2)) + 6*18^(1/3)*12^(1/3)*x*cos(2/3*
arctan(sqrt(3) - 2))^2 + 2*(18^(2/3)*12^(2/3)*cos(2/3*arctan(sqrt(3) - 2))^
2 - 3*18^(1/3)*12^(1/3)*x)*sin(2/3*arctan(sqrt(3) - 2))^2 + 36*x^2)*(18^(2/
3)*12^(2/3)*sqrt(3)*cos(2/3*arctan(sqrt(3) - 2))^2 - 18^(2/3)*12^(2/3)*sqrt
(3)*sin(2/3*arctan(sqrt(3) - 2))^2 + 2*18^(2/3)*12^(2/3)*cos(2/3*arctan(sqr
t(3) - 2))*sin(2/3*arctan(sqrt(3) - 2))))/(3*cos(2/3*arctan(sqrt(3) - 2))^4
 - 10*cos(2/3*arctan(sqrt(3) - 2))^2*sin(2/3*arctan(sqrt(3) - 2))^2 + 3*sin
(2/3*arctan(sqrt(3) - 2))^4)) - 4*(18^(2/3)*12^(1/6)*sqrt(3)*x^4*cos(2/3*ar
ctan(sqrt(3) - 2)) + 18^(2/3)*12^(1/6)*x^4*sin(2/3*arctan(sqrt(3) - 2)))*ar
ctan(-1/432*(6*18^(2/3)*12^(2/3)*x - 216*cos(2/3*arctan(sqrt(3) - 2))^2 + 2
16*sin(2/3*arctan(sqrt(3) - 2))^2 - 18^(2/3)*12^(2/3)*sqrt(18^(2/3)*12^(2/3
)*cos(2/3*arctan(sqrt(3) - 2))^4 + 18^(2/3)*12^(2/3)*sin(2/3*arctan(sqrt(3)
 - 2))^4 - 12*18^(1/3)*12^(1/3)*x*cos(2/3*arctan(sqrt(3) - 2))^2 + 2*(18^(2
/3)*12^(2/3)*cos(2/3*arctan(sqrt(3) - 2))^2 + 6*18^(1/3)*12^(1/3)*x)*sin(2/
3*arctan(sqrt(3) - 2))^2 + 36*x^2))/(cos(2/3*arctan(sqrt(3) - 2))*sin(2/3*a
rctan(sqrt(3) - 2)))) - (18^(2/3)*12^(1/6)*sqrt(3)*x^4*sin(2/3*arctan(sqrt(
3) - 2)) + 18^(2/3)*12^(1/6)*x^4*cos(2/3*arctan(sqrt(3) - 2)))*log(18^(2/3)
*12^(2/3)*cos(2/3*arctan(sqrt(3) - 2))^4 + 18^(2/3)*12^(2/3)*sin(2/3*arctan
(sqrt(3) - 2))^4 + 12*18^(1/3)*12^(1/3)*sqrt(3)*x*cos(2/3*arctan(sqrt(3) -
2))*sin(2/3*arctan(sqrt(3) - 2)) + 6*18^(1/3)*12^(1/3)*x*cos(2/3*arctan(sqr
t(3) - 2))^2 + 2*(18^(2/3)*12^(2/3)*cos(2/3*arctan(sqrt(3) - 2))^2 - 3*18^(
1/3)*12^(1/3)*x)*sin(2/3*arctan(sqrt(3) - 2))^2 + 36*x^2) + (18^(2/3)*12^(1
/6)*sqrt(3)*x^4*sin(2/3*arctan(sqrt(3) - 2)) - 18^(2/3)*12^(1/6)*x^4*cos(2/
3*arctan(sqrt(3) - 2)))*log(18^(2/3)*12^(2/3)*cos(2/3*arctan(sqrt(3) - 2))^
```

```
4 + 18^(2/3)*12^(2/3)*sin(2/3*arctan(sqrt(3) - 2))^4 - 12*18^(1/3)*12^(1/3)
*x*cos(2/3*arctan(sqrt(3) - 2))^2 + 2*(18^(2/3)*12^(2/3)*cos(2/3*arctan(sqr
t(3) - 2))^2 + 6*18^(1/3)*12^(1/3)*x)*sin(2/3*arctan(sqrt(3) - 2))^2 + 36*x
^2) - 27)/x^4
```

giac [B] time = 0.55, size = 836, normalized size = 1.98

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(x^6-x^3+1),x, algorithm="giac")
```

```
[Out] 1/9*(2*sqrt(3)*cos(4/9*pi)^5 - 20*sqrt(3)*cos(4/9*pi)^3*sin(4/9*pi)^2 + 10*
sqrt(3)*cos(4/9*pi)*sin(4/9*pi)^4 - 10*cos(4/9*pi)^4*sin(4/9*pi) + 20*cos(4
/9*pi)^2*sin(4/9*pi)^3 - 2*sin(4/9*pi)^5 + sqrt(3)*cos(4/9*pi)^2 - sqrt(3)*
sin(4/9*pi)^2 - 2*cos(4/9*pi)*sin(4/9*pi))*arctan(-((sqrt(3)*i + 1)*cos(4/9
*pi) - 2*x)/((sqrt(3)*i + 1)*sin(4/9*pi))) + 1/9*(2*sqrt(3)*cos(2/9*pi)^5 -
 20*sqrt(3)*cos(2/9*pi)^3*sin(2/9*pi)^2 + 10*sqrt(3)*cos(2/9*pi)*sin(2/9*pi
)^4 - 10*cos(2/9*pi)^4*sin(2/9*pi) + 20*cos(2/9*pi)^2*sin(2/9*pi)^3 - 2*sin
(2/9*pi)^5 + sqrt(3)*cos(2/9*pi)^2 - sqrt(3)*sin(2/9*pi)^2 - 2*cos(2/9*pi)*
sin(2/9*pi))*arctan(-((sqrt(3)*i + 1)*cos(2/9*pi) - 2*x)/((sqrt(3)*i + 1)*s
in(2/9*pi))) - 1/9*(2*sqrt(3)*cos(1/9*pi)^5 - 20*sqrt(3)*cos(1/9*pi)^3*sin(
1/9*pi)^2 + 10*sqrt(3)*cos(1/9*pi)*sin(1/9*pi)^4 + 10*cos(1/9*pi)^4*sin(1/9
*pi) - 20*cos(1/9*pi)^2*sin(1/9*pi)^3 + 2*sin(1/9*pi)^5 - sqrt(3)*cos(1/9*p
i)^2 + sqrt(3)*sin(1/9*pi)^2 - 2*cos(1/9*pi)*sin(1/9*pi))*arctan(((sqrt(3)*
i + 1)*cos(1/9*pi) + 2*x)/((sqrt(3)*i + 1)*sin(1/9*pi))) + 1/18*(10*sqrt(3)
*cos(4/9*pi)^4*sin(4/9*pi) - 20*sqrt(3)*cos(4/9*pi)^2*sin(4/9*pi)^3 + 2*sqr
t(3)*sin(4/9*pi)^5 + 2*cos(4/9*pi)^5 - 20*cos(4/9*pi)^3*sin(4/9*pi)^2 + 10*
cos(4/9*pi)*sin(4/9*pi)^4 + 2*sqrt(3)*cos(4/9*pi)*sin(4/9*pi) + cos(4/9*pi)
^2 - sin(4/9*pi)^2)*log(-(sqrt(3)*i*cos(4/9*pi) + cos(4/9*pi))*x + x^2 + 1)
 + 1/18*(10*sqrt(3)*cos(2/9*pi)^4*sin(2/9*pi) - 20*sqrt(3)*cos(2/9*pi)^2*si
n(2/9*pi)^3 + 2*sqrt(3)*sin(2/9*pi)^5 + 2*cos(2/9*pi)^5 - 20*cos(2/9*pi)^3*
sin(2/9*pi)^2 + 10*cos(2/9*pi)*sin(2/9*pi)^4 + 2*sqrt(3)*cos(2/9*pi)*sin(2/
9*pi) + cos(2/9*pi)^2 - sin(2/9*pi)^2)*log(-(sqrt(3)*i*cos(2/9*pi) + cos(2/
9*pi))*x + x^2 + 1) + 1/18*(10*sqrt(3)*cos(1/9*pi)^4*sin(1/9*pi) - 20*sqrt(
3)*cos(1/9*pi)^2*sin(1/9*pi)^3 + 2*sqrt(3)*sin(1/9*pi)^5 - 2*cos(1/9*pi)^5
+ 20*cos(1/9*pi)^3*sin(1/9*pi)^2 - 10*cos(1/9*pi)*sin(1/9*pi)^4 - 2*sqrt(3)
*cos(1/9*pi)*sin(1/9*pi) + cos(1/9*pi)^2 - sin(1/9*pi)^2)*log((sqrt(3)*i*co
s(1/9*pi) + cos(1/9*pi))*x + x^2 + 1) - 1/4*(4*x^3 + 1)/x^4
```

maple [C] time = 0.01, size = 51, normalized size = 0.12

$$-\frac{\mathrm{RootOf}\left(\_Z^{6}-\_Z^{3}+1\right)^{4} \ln\left(-\mathrm{RootOf}\left(\_Z^{6}-\_Z^{3}+1\right)+x\right)}{3\left(2\,\mathrm{RootOf}\left(\_Z^{6}-\_Z^{3}+1\right)^{5}-\mathrm{RootOf}\left(\_Z^{6}-\_Z^{3}+1\right)^{2}\right)}-\frac{1}{x}-\frac{1}{4 x^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^5/(x^6-x^3+1),x)
```

```
[Out] -1/3*sum(1/(2*_R^5-_R^2)*_R^4*ln(-_R+x),_R=RootOf(_Z^6-_Z^3+1))-1/4/x^4-1/x
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{4\,x^{3}+1}{4\,x^{4}}-\int \frac{x^{4}}{x^{6}-x^{3}+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(x^6-x^3+1),x, algorithm="maxima")
```

```
[Out] -1/4*(4*x^3 + 1)/x^4 - integrate(x^4/(x^6 - x^3 + 1), x)
```

mupad [B] time = 1.59, size = 318, normalized size = 0.75

$$\frac{\ln\left(-x+\left(162\,x+\frac{27\,{\left(36+\sqrt{3}\,12\mathrm{i}\right)}^{2/3}}{4}\right)\,\left(\frac{1}{162}+\frac{\sqrt{3}\,1\mathrm{i}}{486}\right)\right)\,{\left(36+\sqrt{3}\,12\mathrm{i}\right)}^{1/3}}{18}+\frac{\ln\left(-x-\left(162\,x+\frac{27\,{\left(36-\sqrt{3}\,12\mathrm{i}\right)}^{2/3}}{4}\right)\,\left(-\frac{1}{162}+\frac{\sqrt{3}\,1\mathrm{i}}{486}\right)\right)\,{\left(36-\sqrt{3}\,12\mathrm{i}\right)}^{1/3}}{18}-\frac{x^3+\frac{1}{4}}{x^4}-\frac{2^{2/3}\,\ln\left(x+\frac{2^{1/3}\,3^{2/3}\,{\left(3-\sqrt{3}\,1\mathrm{i}\right)}^{2/3}}{12}-\frac{2^{1/3}\,3^{1/6}\,{\left(3-\sqrt{3}\,1\mathrm{i}\right)}^{2/3}\,1\mathrm{i}}{4}\right)\,{\left(3-\sqrt{3}\,1\mathrm{i}\right)}^{1/3}\,\left(3^{1/3}-3^{5/6}\,1\mathrm{i}\right)}{36}-\frac{2^{2/3}\,\ln\left(x+\frac{2^{1/3}\,3^{2/3}\,{\left(3+\sqrt{3}\,1\mathrm{i}\right)}^{2/3}}{12}+\frac{2^{1/3}\,3^{1/6}\,{\left(3+\sqrt{3}\,1\mathrm{i}\right)}^{2/3}\,1\mathrm{i}}{4}\right)\,{\left(3+\sqrt{3}\,1\mathrm{i}\right)}^{1/3}\,\left(3^{1/3}+3^{5/6}\,1\mathrm{i}\right)}{36}-\frac{2^{2/3}\,\ln\left(x-\frac{2^{1/3}\,3^{2/3}\,{\left(3-\sqrt{3}\,1\mathrm{i}\right)}^{2/3}}{6}\right)\,{\left(3-\sqrt{3}\,1\mathrm{i}\right)}^{1/3}\,\left(3^{1/3}+3^{5/6}\,1\mathrm{i}\right)}{36}-\frac{2^{2/3}\,\ln\left(x-\frac{2^{1/3}\,3^{2/3}\,{\left(3+\sqrt{3}\,1\mathrm{i}\right)}^{2/3}}{6}\right)\,{\left(3+\sqrt{3}\,1\mathrm{i}\right)}^{1/3}\,\left(3^{1/3}-3^{5/6}\,1\mathrm{i}\right)}{36}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^5*(x^6 - x^3 + 1)),x)
```

```
[Out] (log((162*x + (27*(3^(1/2)*12i + 36)^(2/3))/4)*((3^(1/2)*1i)/486 + 1/162) -
 x)*(3^(1/2)*12i + 36)^(1/3))/18 + (log(- x - (162*x + (27*(36 - 3^(1/2)*12
i)^(2/3))/4)*((3^(1/2)*1i)/486 - 1/162))*(36 - 3^(1/2)*12i)^(1/3))/18 - (x^
3 + 1/4)/x^4 - (2^(2/3)*log(x + (2^(1/3)*3^(2/3)*(3 - 3^(1/2)*1i)^(2/3))/12
 - (2^(1/3)*3^(1/6)*(3 - 3^(1/2)*1i)^(2/3)*1i)/4)*(3 - 3^(1/2)*1i)^(1/3)*(3
^(1/3) - 3^(5/6)*1i))/36 - (2^(2/3)*log(x + (2^(1/3)*3^(2/3)*(3^(1/2)*1i +
3)^(2/3))/12 + (2^(1/3)*3^(1/6)*(3^(1/2)*1i + 3)^(2/3)*1i)/4)*(3^(1/2)*1i +
 3)^(1/3)*(3^(1/3) + 3^(5/6)*1i))/36 - (2^(2/3)*log(x - (2^(1/3)*3^(2/3)*(3
 - 3^(1/2)*1i)^(2/3))/6)*(3 - 3^(1/2)*1i)^(1/3)*(3^(1/3) + 3^(5/6)*1i))/36
- (2^(2/3)*log(x - (2^(1/3)*3^(2/3)*(3^(1/2)*1i + 3)^(2/3))/6)*(3^(1/2)*1i
+ 3)^(1/3)*(3^(1/3) - 3^(5/6)*1i))/36
```

sympy [A] time = 0.22, size = 39, normalized size = 0.09

$$\operatorname{RootSum}\left(19683t^6-243t^3+1,\left(t\mapsto t\log\left(-6561t^5+54t^2+x\right)\right)\right)+\frac{-4x^3-1}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**5/(x**6-x**3+1),x)
```

```
[Out] RootSum(19683*_t**6 - 243*_t**3 + 1, Lambda(_t, _t*log(-6561*_t**5 + 54*_t*
*2 + x))) + (-4*x**3 - 1)/(4*x**4)
```

3.169 $\int \frac{1}{2+x^3+x^6} \, dx$

Optimal. Leaf size=381

$$\frac{i \log \left(2^{2/3} x^2-\sqrt[3]{2 \left(1-i \sqrt{7}\right)} x+\left(1-i \sqrt{7}\right)^{2/3}\right)}{3 \sqrt[3]{2} \sqrt{7} \left(1-i \sqrt{7}\right)^{2/3}}-\frac{i \log \left(2^{2/3} x^2-\sqrt[3]{2 \left(1+i \sqrt{7}\right)} x+\left(1+i \sqrt{7}\right)^{2/3}\right)}{3 \sqrt[3]{2} \sqrt{7} \left(1+i \sqrt{7}\right)^{2/3}}-\frac{i \log \left(\sqrt[3]{2} x + \right.}{3 \sqrt{7} \left(\frac{1}{2} \left(1 \right.\right.}$$

Rubi [A] time = 0.40, antiderivative size = 381, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 7, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.700, Rules used = {1347, 200, 31, 634, 617, 204, 628}

$$\frac{i \log \left(2^{2/3} x^2-\sqrt[3]{2 \left(1-i \sqrt{7}\right)} x+\left(1-i \sqrt{7}\right)^{2/3}\right)}{3 \sqrt[3]{2} \sqrt{7} \left(1-i \sqrt{7}\right)^{2/3}}-\frac{i \log \left(2^{2/3} x^2-\sqrt[3]{2 \left(1+i \sqrt{7}\right)} x+\left(1+i \sqrt{7}\right)^{2/3}\right)}{3 \sqrt[3]{2} \sqrt{7} \left(1+i \sqrt{7}\right)^{2/3}}-\frac{i \log \left(\sqrt[3]{2} x+\sqrt[3]{1-i \sqrt{7}}\right)}{3 \sqrt{7} \left(\frac{1}{2} \left(1-i \sqrt{7}\right)\right)^{2/3}}+\frac{i \log \left(\sqrt[3]{2} x+\sqrt[3]{1+i \sqrt{7}}\right)}{3 \sqrt{7} \left(\frac{1}{2} \left(1+i \sqrt{7}\right)\right)^{2/3}}+\frac{i \tan ^{-1}\left(\frac{1-\frac{2 x}{\sqrt[3]{\frac{1}{2} \left(1-i \sqrt{7}\right)}}}{\sqrt{3}}\right)}{\sqrt{21} \left(\frac{1}{2} \left(1-i \sqrt{7}\right)\right)^{2/3}}-\frac{i \tan ^{-1}\left(\frac{1-\frac{2 x}{\sqrt[3]{\frac{1}{2} \left(1+i \sqrt{7}\right)}}}{\sqrt{3}}\right)}{\sqrt{21} \left(\frac{1}{2} \left(1+i \sqrt{7}\right)\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + x^3 + x^6)^(-1),x]
```

```
[Out] (I*ArcTan[(1 - (2*x)/((1 - I*Sqrt[7])/2)^(1/3))/Sqrt[3]])/(Sqrt[21]*((1 - I
*Sqrt[7])/2)^(2/3)) - (I*ArcTan[(1 - (2*x)/((1 + I*Sqrt[7])/2)^(1/3))/Sqrt[
3]])/(Sqrt[21]*((1 + I*Sqrt[7])/2)^(2/3)) - ((I/3)*Log[(1 - I*Sqrt[7])^(1/3
) + 2^(1/3)*x])/(Sqrt[7]*((1 - I*Sqrt[7])/2)^(2/3)) + ((I/3)*Log[(1 + I*Sqr
t[7])^(1/3) + 2^(1/3)*x])/(Sqrt[7]*((1 + I*Sqrt[7])/2)^(2/3)) + ((I/3)*Log[
(1 - I*Sqrt[7])^(2/3) - (2*(1 - I*Sqrt[7]))^(1/3)*x + 2^(2/3)*x^2])/(2^(1/3
)*Sqrt[7]*(1 - I*Sqrt[7])^(2/3)) - ((I/3)*Log[(1 + I*Sqrt[7])^(2/3) - (2*(1
 + I*Sqrt[7]))^(1/3)*x + 2^(2/3)*x^2])/(2^(1/3)*Sqrt[7]*(1 + I*Sqrt[7])^(2/
3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1347

```
Int[((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^(-1), x_Symbol] :> With[{q
= Rt[b^2 - 4*a*c, 2]}, Dist[c/q, Int[1/(b/2 - q/2 + c*x^n), x], x] - Dist[c
/q, Int[1/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*
n] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{1}{2+x^3+x^6}\,dx = -\frac{i\int \frac{1}{\frac{1}{2}-\frac{i\sqrt{7}}{2}+x^3}\,dx}{\sqrt{7}} + \frac{i\int \frac{1}{\frac{1}{2}+\frac{i\sqrt{7}}{2}+x^3}\,dx}{\sqrt{7}}$$

$$= -\frac{i\int \frac{1}{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{7}\right)}+x}\,dx}{3\sqrt{7}\left(\frac{1}{2}\left(1-i\sqrt{7}\right)\right)^{2/3}} - \frac{i\int \frac{2^{2/3}\sqrt[3]{1-i\sqrt{7}}-x}{\left(\frac{1}{2}\left(1-i\sqrt{7}\right)\right)^{2/3}-\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{7}\right)}x+x^2}\,dx}{3\sqrt{7}\left(\frac{1}{2}\left(1-i\sqrt{7}\right)\right)^{2/3}} + \frac{i\int \frac{1}{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{7}\right)}+x}\,dx}{3\sqrt{7}\left(\frac{1}{2}\left(1+i\sqrt{7}\right)\right)^{2/3}} + \ldots$$

$$= -\frac{i\log\left(\sqrt[3]{1-i\sqrt{7}}+\sqrt[3]{2}x\right)}{3\sqrt{7}\left(\frac{1}{2}\left(1-i\sqrt{7}\right)\right)^{2/3}} + \frac{i\log\left(\sqrt[3]{1+i\sqrt{7}}+\sqrt[3]{2}x\right)}{3\sqrt{7}\left(\frac{1}{2}\left(1+i\sqrt{7}\right)\right)^{2/3}} + \frac{i\int \frac{-\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{7}\right)}+2x}{\left(\frac{1}{2}\left(1-i\sqrt{7}\right)\right)^{2/3}-\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{7}\right)}x+x^2}\,dx}{3\sqrt[3]{2}\sqrt{7}\left(1-i\sqrt{7}\right)^{2/3}} \ldots$$

$$= -\frac{i\log\left(\sqrt[3]{1-i\sqrt{7}}+\sqrt[3]{2}x\right)}{3\sqrt{7}\left(\frac{1}{2}\left(1-i\sqrt{7}\right)\right)^{2/3}} + \frac{i\log\left(\sqrt[3]{1+i\sqrt{7}}+\sqrt[3]{2}x\right)}{3\sqrt{7}\left(\frac{1}{2}\left(1+i\sqrt{7}\right)\right)^{2/3}} + \frac{i\log\left(\left(1-i\sqrt{7}\right)^{2/3}-\sqrt[3]{2\left(1-i\sqrt{7}\right.}\ldots\right.}{3\sqrt[3]{2}\sqrt{7}\left(1-i\sqrt{7}\right)^{2/\ldots}} \ldots$$

$$= \frac{i\tan^{-1}\left(\frac{1-\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{7}\right)}}}{\sqrt{3}}\right)}{\sqrt{21}\left(\frac{1}{2}\left(1-i\sqrt{7}\right)\right)^{2/3}} - \frac{i\tan^{-1}\left(\frac{1-\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{7}\right)}}}{\sqrt{3}}\right)}{\sqrt{21}\left(\frac{1}{2}\left(1+i\sqrt{7}\right)\right)^{2/3}} - \frac{i\log\left(\sqrt[3]{1-i\sqrt{7}}+\sqrt[3]{2}x\right)}{3\sqrt{7}\left(\frac{1}{2}\left(1-i\sqrt{7}\right)\right)^{2/3}} + \frac{i\log\left(\sqrt[3]{1+i\ldots\right.}{3\sqrt{7}\left(\frac{1}{2}\left(1+\ldots\right.\right.} \ldots$$

Mathematica [C] time = 0.01, size = 38, normalized size = 0.10

$$\frac{1}{3}\text{RootSum}\left[\#1^6+\#1^3+2\&,\frac{\log(x-\#1)}{2\#1^5+\#1^2}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + x^3 + x^6)^(-1),x]
```

```
[Out] RootSum[2 + #1^3 + #1^6 & , Log[x - #1]/(#1^2 + 2*#1^5) & ]/3
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{2 + x^3 + x^6} \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(2 + x^3 + x^6)^(-1),x]
```

```
[Out] IntegrateAlgebraic[(2 + x^3 + x^6)^(-1), x]
```

fricas [B] time = 3.21, size = 1996, normalized size = 5.24

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^6+x^3+2),x, algorithm="fricas")
```

```
[Out] 1/294*112^(1/6)*49^(2/3)*cos(2/3*arctan(1/3*sqrt(7) + 4/3))*log(112^(1/6)*4
9^(2/3)*sqrt(7)*x*sin(2/3*arctan(1/3*sqrt(7) + 4/3)) + 7*112^(1/6)*49^(2/3)
*x*cos(2/3*arctan(1/3*sqrt(7) + 4/3)) + 14*49^(1/3)*14^(1/3)*cos(2/3*arctan
(1/3*sqrt(7) + 4/3))^2 + 14*49^(1/3)*14^(1/3)*sin(2/3*arctan(1/3*sqrt(7) +
4/3))^2 + 98*x^2) - 2/147*112^(1/6)*49^(2/3)*arctan(1/2744*(14*112^(5/6)*49
^(1/3)*sqrt(7)*x*cos(2/3*arctan(1/3*sqrt(7) + 4/3)) + 2744*sqrt(7)*cos(2/3*
arctan(1/3*sqrt(7) + 4/3))^2 + 2744*sqrt(7)*sin(2/3*arctan(1/3*sqrt(7) + 4/
3))^2 + 98*(112^(5/6)*49^(1/3)*x + 224*cos(2/3*arctan(1/3*sqrt(7) + 4/3)))*
sin(2/3*arctan(1/3*sqrt(7) + 4/3)) - sqrt(112^(1/6)*49^(2/3)*sqrt(7)*x*sin(
2/3*arctan(1/3*sqrt(7) + 4/3)) + 7*112^(1/6)*49^(2/3)*x*cos(2/3*arctan(1/3*
sqrt(7) + 4/3)) + 14*49^(1/3)*14^(1/3)*cos(2/3*arctan(1/3*sqrt(7) + 4/3))^2
 + 14*49^(1/3)*14^(1/3)*sin(2/3*arctan(1/3*sqrt(7) + 4/3))^2 + 98*x^2)*(112
^(5/6)*49^(1/3)*sqrt(7)*sqrt(2)*cos(2/3*arctan(1/3*sqrt(7) + 4/3)) + 7*112^
(5/6)*49^(1/3)*sqrt(2)*sin(2/3*arctan(1/3*sqrt(7) + 4/3))))/(cos(2/3*arctan
(1/3*sqrt(7) + 4/3))^2 - 7*sin(2/3*arctan(1/3*sqrt(7) + 4/3))^2))*sin(2/3*a
rctan(1/3*sqrt(7) + 4/3)) + 1/147*(112^(1/6)*49^(2/3)*sqrt(3)*cos(2/3*arcta
n(1/3*sqrt(7) + 4/3)) + 112^(1/6)*49^(2/3)*sin(2/3*arctan(1/3*sqrt(7) + 4/3
)))*arctan(1/5488*(70*112^(5/6)*49^(1/3)*(sqrt(7)*x + 7*sqrt(3)*x)*cos(2/3*
arctan(1/3*sqrt(7) + 4/3))^3 - 27440*(sqrt(7) + 2*sqrt(3))*cos(2/3*arctan(1
/3*sqrt(7) + 4/3))^4 - 5488*(sqrt(7) - 2*sqrt(3))*sin(2/3*arctan(1/3*sqrt(7
) + 4/3))^4 - 14*(112^(5/6)*49^(1/3)*(sqrt(7)*sqrt(3)*x - 7*x) - 1568*(sqrt
(7)*sqrt(3) - 5)*cos(2/3*arctan(1/3*sqrt(7) + 4/3)))*sin(2/3*arctan(1/3*sqr
t(7) + 4/3))^3 + 14*(112^(5/6)*49^(1/3)*(13*sqrt(7)*x - 21*sqrt(3)*x)*cos(2
/3*arctan(1/3*sqrt(7) + 4/3)) - 784*(3*sqrt(7) + 4*sqrt(3))*cos(2/3*arctan(
1/3*sqrt(7) + 4/3))^2)*sin(2/3*arctan(1/3*sqrt(7) + 4/3))^2 - 14*(112^(5/6)
*49^(1/3)*(9*sqrt(7)*sqrt(3)*x + 49*x)*cos(2/3*arctan(1/3*sqrt(7) + 4/3))^2
 - 1568*(sqrt(7)*sqrt(3) + 11)*cos(2/3*arctan(1/3*sqrt(7) + 4/3))^3)*sin(2/
3*arctan(1/3*sqrt(7) + 4/3)) - (5*112^(5/6)*49^(1/3)*(sqrt(7) + 7*sqrt(3))*
cos(2/3*arctan(1/3*sqrt(7) + 4/3))^3 - 112^(5/6)*49^(1/3)*(9*sqrt(7)*sqrt(3
) + 49)*cos(2/3*arctan(1/3*sqrt(7) + 4/3))^2*sin(2/3*arctan(1/3*sqrt(7) + 4
/3)) + 112^(5/6)*49^(1/3)*(13*sqrt(7) - 21*sqrt(3))*cos(2/3*arctan(1/3*sqrt
(7) + 4/3))*sin(2/3*arctan(1/3*sqrt(7) + 4/3))^2 - 112^(5/6)*49^(1/3)*(sqrt
(7)*sqrt(3) - 7)*sin(2/3*arctan(1/3*sqrt(7) + 4/3))^3)*sqrt(-112^(1/6)*49^(
2/3)*(sqrt(7)*sqrt(3)*x + 7*x)*cos(2/3*arctan(1/3*sqrt(7) + 4/3)) - 112^(1/
6)*49^(2/3)*(sqrt(7)*x - 7*sqrt(3)*x)*sin(2/3*arctan(1/3*sqrt(7) + 4/3)) +
28*49^(1/3)*14^(1/3)*cos(2/3*arctan(1/3*sqrt(7) + 4/3))^2 + 28*49^(1/3)*14^
(1/3)*sin(2/3*arctan(1/3*sqrt(7) + 4/3))^2 + 196*x^2))/(25*cos(2/3*arctan(1
/3*sqrt(7) + 4/3))^4 - 38*cos(2/3*arctan(1/3*sqrt(7) + 4/3))^2*sin(2/3*arct
an(1/3*sqrt(7) + 4/3))^2 + sin(2/3*arctan(1/3*sqrt(7) + 4/3))^4)) + 1/147*(
112^(1/6)*49^(2/3)*sqrt(3)*cos(2/3*arctan(1/3*sqrt(7) + 4/3)) - 112^(1/6)*4
9^(2/3)*sin(2/3*arctan(1/3*sqrt(7) + 4/3)))*arctan(-1/5488*(70*112^(5/6)*49
^(1/3)*(sqrt(7)*x - 7*sqrt(3)*x)*cos(2/3*arctan(1/3*sqrt(7) + 4/3))^3 - 274
40*(sqrt(7) - 2*sqrt(3))*cos(2/3*arctan(1/3*sqrt(7) + 4/3))^4 - 5488*(sqrt(
```

```
7) + 2*sqrt(3))*sin(2/3*arctan(1/3*sqrt(7) + 4/3))^4 + 14*(112^(5/6)*49^(1/
3)*(sqrt(7)*sqrt(3)*x + 7*x) - 1568*(sqrt(7)*sqrt(3) + 5)*cos(2/3*arctan(1/
3*sqrt(7) + 4/3)))*sin(2/3*arctan(1/3*sqrt(7) + 4/3))^3 + 14*(112^(5/6)*49^
(1/3)*(13*sqrt(7)*x + 21*sqrt(3)*x)*cos(2/3*arctan(1/3*sqrt(7) + 4/3)) - 78
4*(3*sqrt(7) - 4*sqrt(3))*cos(2/3*arctan(1/3*sqrt(7) + 4/3))^2)*sin(2/3*arc
tan(1/3*sqrt(7) + 4/3))^2 + 14*(112^(5/6)*49^(1/3)*(9*sqrt(7)*sqrt(3)*x - 4
9*x)*cos(2/3*arctan(1/3*sqrt(7) + 4/3))^2 - 1568*(sqrt(7)*sqrt(3) - 11)*cos
(2/3*arctan(1/3*sqrt(7) + 4/3))^3)*sin(2/3*arctan(1/3*sqrt(7) + 4/3)) - (5*
112^(5/6)*49^(1/3)*(sqrt(7) - 7*sqrt(3))*cos(2/3*arctan(1/3*sqrt(7) + 4/3))
^3 + 112^(5/6)*49^(1/3)*(9*sqrt(7)*sqrt(3) - 49)*cos(2/3*arctan(1/3*sqrt(7)
 + 4/3))^2*sin(2/3*arctan(1/3*sqrt(7) + 4/3)) + 112^(5/6)*49^(1/3)*(13*sqrt
(7) + 21*sqrt(3))*cos(2/3*arctan(1/3*sqrt(7) + 4/3))*sin(2/3*arctan(1/3*sqr
t(7) + 4/3))^2 + 112^(5/6)*49^(1/3)*(sqrt(7)*sqrt(3) + 7)*sin(2/3*arctan(1/
3*sqrt(7) + 4/3))^3)*sqrt(112^(1/6)*49^(2/3)*(sqrt(7)*sqrt(3)*x - 7*x)*cos(
2/3*arctan(1/3*sqrt(7) + 4/3)) - 112^(1/6)*49^(2/3)*(sqrt(7)*x + 7*sqrt(3)*
x)*sin(2/3*arctan(1/3*sqrt(7) + 4/3)) + 28*49^(1/3)*14^(1/3)*cos(2/3*arctan
(1/3*sqrt(7) + 4/3))^2 + 28*49^(1/3)*14^(1/3)*sin(2/3*arctan(1/3*sqrt(7) +
4/3))^2 + 196*x^2))/(25*cos(2/3*arctan(1/3*sqrt(7) + 4/3))^4 - 38*cos(2/3*a
rctan(1/3*sqrt(7) + 4/3))^2*sin(2/3*arctan(1/3*sqrt(7) + 4/3))^2 + sin(2/3*
arctan(1/3*sqrt(7) + 4/3))^4)) + 1/588*(112^(1/6)*49^(2/3)*sqrt(3)*sin(2/3*
arctan(1/3*sqrt(7) + 4/3)) - 112^(1/6)*49^(2/3)*cos(2/3*arctan(1/3*sqrt(7)
+ 4/3)))*log(-112^(1/6)*49^(2/3)*(sqrt(7)*sqrt(3)*x + 7*x)*cos(2/3*arctan(1
/3*sqrt(7) + 4/3)) - 112^(1/6)*49^(2/3)*(sqrt(7)*x - 7*sqrt(3)*x)*sin(2/3*a
rctan(1/3*sqrt(7) + 4/3)) + 28*49^(1/3)*14^(1/3)*cos(2/3*arctan(1/3*sqrt(7)
 + 4/3))^2 + 28*49^(1/3)*14^(1/3)*sin(2/3*arctan(1/3*sqrt(7) + 4/3))^2 + 19
6*x^2) - 1/588*(112^(1/6)*49^(2/3)*sqrt(3)*sin(2/3*arctan(1/3*sqrt(7) + 4/3
)) + 112^(1/6)*49^(2/3)*cos(2/3*arctan(1/3*sqrt(7) + 4/3)))*log(112^(1/6)*4
9^(2/3)*(sqrt(7)*sqrt(3)*x - 7*x)*cos(2/3*arctan(1/3*sqrt(7) + 4/3)) - 112^
(1/6)*49^(2/3)*(sqrt(7)*x + 7*sqrt(3)*x)*sin(2/3*arctan(1/3*sqrt(7) + 4/3))
 + 28*49^(1/3)*14^(1/3)*cos(2/3*arctan(1/3*sqrt(7) + 4/3))^2 + 28*49^(1/3)*
14^(1/3)*sin(2/3*arctan(1/3*sqrt(7) + 4/3))^2 + 196*x^2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^6 + x^3 + 2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^6+x^3+2),x, algorithm="giac")
```

```
[Out] integrate(1/(x^6 + x^3 + 2), x)
```

maple [C] time = 0.01, size = 33, normalized size = 0.09

$$\frac{\ln\left(-\operatorname{RootOf}\left(\_Z^6 + \_Z^3 + 2\right) + x\right)}{6\operatorname{RootOf}\left(\_Z^6 + \_Z^3 + 2\right)^5 + 3\operatorname{RootOf}\left(\_Z^6 + \_Z^3 + 2\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^6+x^3+2),x)
```

```
[Out] 1/3*sum(1/(2*_R^5+_R^2)*ln(-_R+x),_R=RootOf(_Z^6+_Z^3+2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^6 + x^3 + 2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^6+x^3+2),x, algorithm="maxima")
```

```
[Out] integrate(1/(x^6 + x^3 + 2), x)
```

mupad [B] time = 2.61, size = 513, normalized size = 1.35

$$\frac{\ln\left(x+\frac{7^{1/3}\,\left(-7-\sqrt{7}\,3\mathrm{i}\right)^{1/3}}{4}+\frac{7^{5/6}\,\left(-7-\sqrt{7}\,3\mathrm{i}\right)^{1/3}\,1\mathrm{i}}{28}\right)\,\left(-49-\sqrt{7}\,21\mathrm{i}\right)^{1/3}}{42}+\frac{\ln\left(x+\frac{7^{1/3}\,\left(-7+\sqrt{7}\,3\mathrm{i}\right)^{1/3}}{4}-\frac{7^{5/6}\,\left(-7+\sqrt{7}\,3\mathrm{i}\right)^{1/3}\,1\mathrm{i}}{28}\right)\,\left(-49+\sqrt{7}\,21\mathrm{i}\right)^{1/3}}{42}+\frac{7^{1/3}\,\ln\left(6\,x+\frac{7^{1/3}\,\left(-1+\sqrt{3}\,1\mathrm{i}\right)\,\left(-7-\sqrt{7}\,3\mathrm{i}\right)^{1/3}\,\left(\frac{7^{2/3}\,\left(-1+\sqrt{3}\,1\mathrm{i}\right)^2\,\left(-7-\sqrt{7}\,3\mathrm{i}\right)^{2/3}\,\left(3969\,x+\frac{567\,7^{1/3}\,\left(-1+\sqrt{3}\,1\mathrm{i}\right)\,\left(-7-\sqrt{7}\,3\mathrm{i}\right)^{1/3}}{2}\right)}{7056}+63\right)}{84}\right)\,\left(-1+\sqrt{3}\,1\mathrm{i}\right)\,\left(-7-\sqrt{7}\,3\mathrm{i}\right)^{1/3}}{84}+\frac{7^{1/3}\,\ln\left(6\,x+\frac{7^{1/3}\,\left(-1+\sqrt{3}\,1\mathrm{i}\right)\,\left(-7+\sqrt{7}\,3\mathrm{i}\right)^{1/3}\,\left(\frac{7^{2/3}\,\left(-1+\sqrt{3}\,1\mathrm{i}\right)^2\,\left(-7+\sqrt{7}\,3\mathrm{i}\right)^{2/3}\,\left(3969\,x+\frac{567\,7^{1/3}\,\left(-1+\sqrt{3}\,1\mathrm{i}\right)\,\left(-7+\sqrt{7}\,3\mathrm{i}\right)^{1/3}}{2}\right)}{7056}+63\right)}{84}\right)\,\left(-1+\sqrt{3}\,1\mathrm{i}\right)\,\left(-7+\sqrt{7}\,3\mathrm{i}\right)^{1/3}}{84}-\frac{7^{1/3}\,\ln\left(6\,x-\frac{7^{1/3}\,\left(1+\sqrt{3}\,1\mathrm{i}\right)\,\left(-7-\sqrt{7}\,3\mathrm{i}\right)^{1/3}\,\left(\frac{7^{2/3}\,\left(1+\sqrt{3}\,1\mathrm{i}\right)^2\,\left(-7-\sqrt{7}\,3\mathrm{i}\right)^{2/3}\,\left(3969\,x-\frac{567\,7^{1/3}\,\left(1+\sqrt{3}\,1\mathrm{i}\right)\,\left(-7-\sqrt{7}\,3\mathrm{i}\right)^{1/3}}{2}\right)}{7056}+63\right)}{84}\right)\,\left(1+\sqrt{3}\,1\mathrm{i}\right)\,\left(-7-\sqrt{7}\,3\mathrm{i}\right)^{1/3}}{84}-\frac{7^{1/3}\,\ln\left(6\,x-\frac{7^{1/3}\,\left(1+\sqrt{3}\,1\mathrm{i}\right)\,\left(-7+\sqrt{7}\,3\mathrm{i}\right)^{1/3}\,\left(\frac{7^{2/3}\,\left(1+\sqrt{3}\,1\mathrm{i}\right)^2\,\left(-7+\sqrt{7}\,3\mathrm{i}\right)^{2/3}\,\left(3969\,x-\frac{567\,7^{1/3}\,\left(1+\sqrt{3}\,1\mathrm{i}\right)\,\left(-7+\sqrt{7}\,3\mathrm{i}\right)^{1/3}}{2}\right)}{7056}+63\right)}{84}\right)\,\left(1+\sqrt{3}\,1\mathrm{i}\right)\,\left(-7+\sqrt{7}\,3\mathrm{i}\right)^{1/3}}{84}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3 + x^6 + 2),x)
```

```
[Out] (log(x + (7^(1/3)*(- 7^(1/2)*3i - 7)^(1/3))/4 + (7^(5/6)*(- 7^(1/2)*3i - 7)
^(1/3)*1i)/28)*(- 7^(1/2)*21i - 49)^(1/3))/42 + (log(x + (7^(1/3)*(7^(1/2)*
3i - 7)^(1/3))/4 - (7^(5/6)*(7^(1/2)*3i - 7)^(1/3)*1i)/28)*(7^(1/2)*21i - 4
9)^(1/3))/42 + (7^(1/3)*log(6*x + (7^(1/3)*(3^(1/2)*1i - 1)*(- 7^(1/2)*3i -
 7)^(1/3)*((7^(2/3)*(3^(1/2)*1i - 1)^2*(- 7^(1/2)*3i - 7)^(2/3)*(3969*x + (
567*7^(1/3)*(3^(1/2)*1i - 1)*(- 7^(1/2)*3i - 7)^(1/3))/2))/7056 + 63))/84)*
(3^(1/2)*1i - 1)*(- 7^(1/2)*3i - 7)^(1/3))/84 + (7^(1/3)*log(6*x + (7^(1/3)
*(3^(1/2)*1i - 1)*(7^(1/2)*3i - 7)^(1/3)*((7^(2/3)*(3^(1/2)*1i - 1)^2*(7^(1
/2)*3i - 7)^(2/3)*(3969*x + (567*7^(1/3)*(3^(1/2)*1i - 1)*(7^(1/2)*3i - 7)^
(1/3))/2))/7056 + 63))/84)*(3^(1/2)*1i - 1)*(7^(1/2)*3i - 7)^(1/3))/84 - (7
^(1/3)*log(6*x - (7^(1/3)*(3^(1/2)*1i + 1)*(- 7^(1/2)*3i - 7)^(1/3)*((7^(2/
3)*(3^(1/2)*1i + 1)^2*(- 7^(1/2)*3i - 7)^(2/3)*(3969*x - (567*7^(1/3)*(3^(1
/2)*1i + 1)*(- 7^(1/2)*3i - 7)^(1/3))/2))/7056 + 63))/84)*(3^(1/2)*1i + 1)*
(- 7^(1/2)*3i - 7)^(1/3))/84 - (7^(1/3)*log(6*x - (7^(1/3)*(3^(1/2)*1i + 1)
*(7^(1/2)*3i - 7)^(1/3)*((7^(2/3)*(3^(1/2)*1i + 1)^2*(7^(1/2)*3i - 7)^(2/3)
*(3969*x - (567*7^(1/3)*(3^(1/2)*1i + 1)*(7^(1/2)*3i - 7)^(1/3))/2))/7056 +
 63))/84)*(3^(1/2)*1i + 1)*(7^(1/2)*3i - 7)^(1/3))/84
```

sympy [A] time = 0.15, size = 24, normalized size = 0.06

$$\operatorname{RootSum}\left(1000188t^{6}+1323t^{3}+1,\left(t\mapsto t\log\left(-5292t^{4}+7t+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**6+x**3+2),x)
```

```
[Out] RootSum(1000188*_t**6 + 1323*_t**3 + 1, Lambda(_t, _t*log(-5292*_t**4 + 7*_
t + x)))
```

3.170 $\int \frac{x^2}{2+x^3+x^6}\,dx$

Optimal. Leaf size=23

$$\frac{2\tan^{-1}\left(\frac{2x^3+1}{\sqrt{7}}\right)}{3\sqrt{7}}$$

Rubi [A] time = 0.02, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {1352, 618, 204}

$$\frac{2\tan^{-1}\left(\frac{2x^3+1}{\sqrt{7}}\right)}{3\sqrt{7}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(2 + x^3 + x^6),x]
```

```
[Out] (2*ArcTan[(1 + 2*x^3)/Sqrt[7]])/(3*Sqrt[7])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1352

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a,
b, c, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2}{2+x^3+x^6}\,dx &= \frac{1}{3}\operatorname{Subst}\left(\int \frac{1}{2+x+x^2}\,dx, x, x^3\right)\\ &= -\left(\frac{2}{3}\operatorname{Subst}\left(\int \frac{1}{-7-x^2}\,dx, x, 1+2x^3\right)\right)\\ &= \frac{2\tan^{-1}\left(\frac{1+2x^3}{\sqrt{7}}\right)}{3\sqrt{7}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 23, normalized size = 1.00

$$\frac{2\tan^{-1}\left(\frac{2x^3+1}{\sqrt{7}}\right)}{3\sqrt{7}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(2 + x^3 + x^6),x]
[Out] (2*ArcTan[(1 + 2*x^3)/Sqrt[7]])/(3*Sqrt[7])
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{2+x^3+x^6}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^2/(2 + x^3 + x^6),x]
[Out] IntegrateAlgebraic[x^2/(2 + x^3 + x^6), x]
```

fricas [A] time = 1.19, size = 18, normalized size = 0.78

$$\frac{2}{21}\,\sqrt{7}\arctan\left(\frac{1}{7}\,\sqrt{7}\left(2\,x^3+1\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^6+x^3+2),x, algorithm="fricas")
[Out] 2/21*sqrt(7)*arctan(1/7*sqrt(7)*(2*x^3 + 1))
```

giac [A] time = 0.50, size = 18, normalized size = 0.78

$$\frac{2}{21}\,\sqrt{7}\arctan\left(\frac{1}{7}\,\sqrt{7}\left(2\,x^3+1\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^6+x^3+2),x, algorithm="giac")
[Out] 2/21*sqrt(7)*arctan(1/7*sqrt(7)*(2*x^3 + 1))
```

maple [A] time = 0.00, size = 19, normalized size = 0.83

$$\frac{2\sqrt{7}\,\arctan\left(\frac{\left(2x^3+1\right)\sqrt{7}}{7}\right)}{21}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(x^6+x^3+2),x)
[Out] 2/21*arctan(1/7*(2*x^3+1)*7^(1/2))*7^(1/2)
```

maxima [A] time = 1.36, size = 18, normalized size = 0.78

$$\frac{2}{21}\,\sqrt{7}\arctan\left(\frac{1}{7}\,\sqrt{7}\left(2\,x^3+1\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^6+x^3+2),x, algorithm="maxima")
[Out] 2/21*sqrt(7)*arctan(1/7*sqrt(7)*(2*x^3 + 1))
```

mupad [B] time = 0.05, size = 20, normalized size = 0.87

$$\frac{2\,\sqrt{7}\,\mathrm{atan}\left(\frac{2\,\sqrt{7}\,x^3}{7}+\frac{\sqrt{7}}{7}\right)}{21}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(x^3 + x^6 + 2),x)
```

```
[Out] (2*7^(1/2)*atan(7^(1/2)/7 + (2*7^(1/2)*x^3)/7))/21
```

sympy [A] time = 0.11, size = 27, normalized size = 1.17

$$\frac{2\sqrt{7}\operatorname{atan}\left(\frac{2\sqrt{7}x^3}{7}+\frac{\sqrt{7}}{7}\right)}{21}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(x**6+x**3+2),x)
```

```
[Out] 2*sqrt(7)*atan(2*sqrt(7)*x**3/7 + sqrt(7)/7)/21
```

3.171 $\int \frac{x^3}{2+x^3+x^6}\,dx$

Optimal. Leaf size=399

$$-\frac{\left(7+i\sqrt{7}\right)\log\left(2^{2/3}x^2-\sqrt[3]{2\left(1-i\sqrt{7}\right)}\,x+\left(1-i\sqrt{7}\right)^{2/3}\right)}{42\sqrt[3]{2}\left(1-i\sqrt{7}\right)^{2/3}}-\frac{\left(7-i\sqrt{7}\right)\log\left(2^{2/3}x^2-\sqrt[3]{2\left(1+i\sqrt{7}\right)}\,x+\left(1+i\sqrt{\ldots}\right.\right.}{42\sqrt[3]{2}\left(1+i\sqrt{7}\right)^{2/3}}$$

Rubi [A] time = 0.31, antiderivative size = 399, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 7, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {1374, 200, 31, 634, 617, 204, 628}

$$-\frac{\left(7+i\sqrt{7}\right)\log\left(2^{2/3}x^2-\sqrt[3]{2\left(1-i\sqrt{7}\right)}\,x+\left(1-i\sqrt{7}\right)^{2/3}\right)}{42\sqrt[3]{2}\left(1-i\sqrt{7}\right)^{2/3}}-\frac{\left(7-i\sqrt{7}\right)\log\left(2^{2/3}x^2-\sqrt[3]{2\left(1+i\sqrt{7}\right)}\,x+\left(1+i\sqrt{7}\right)^{2/3}\right)}{42\sqrt[3]{2}\left(1+i\sqrt{7}\right)^{2/3}}+\frac{\left(7+i\sqrt{7}\right)\log\left(\sqrt[3]{2}x+\sqrt[3]{1-i\sqrt{7}}\right)}{21\sqrt[3]{2}\left(1-i\sqrt{7}\right)^{2/3}}+\frac{\left(7-i\sqrt{7}\right)\log\left(\sqrt[3]{2}x+\sqrt[3]{1+i\sqrt{7}}\right)}{21\sqrt[3]{2}\left(1+i\sqrt{7}\right)^{2/3}}-\frac{i\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{7}\right)}\tan^{-1}\left(\frac{1-\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{7}\right)}}}{\sqrt{3}}\right)}{\sqrt{21}}+\frac{i\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{7}\right)}\tan^{-1}\left(\frac{1-\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{7}\right)}}}{\sqrt{3}}\right)}{\sqrt{21}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(2 + x^3 + x^6),x]
```

```
[Out] ((-I)*((1 - I*Sqrt[7])/2)^(1/3)*ArcTan[(1 - (2*x)/((1 - I*Sqrt[7])/2)^(1/3)
)/Sqrt[3]])/Sqrt[21] + (I*((1 + I*Sqrt[7])/2)^(1/3)*ArcTan[(1 - (2*x)/((1 +
 I*Sqrt[7])/2)^(1/3))/Sqrt[3]])/Sqrt[21] + ((7 + I*Sqrt[7])*Log[(1 - I*Sqrt
[7])^(1/3) + 2^(1/3)*x])/(21*2^(1/3)*(1 - I*Sqrt[7])^(2/3)) + ((7 - I*Sqrt[
7])*Log[(1 + I*Sqrt[7])^(1/3) + 2^(1/3)*x])/(21*2^(1/3)*(1 + I*Sqrt[7])^(2/
3)) - ((7 + I*Sqrt[7])*Log[(1 - I*Sqrt[7])^(2/3) - (2*(1 - I*Sqrt[7]))^(1/3
)*x + 2^(2/3)*x^2])/(42*2^(1/3)*(1 - I*Sqrt[7])^(2/3)) - ((7 - I*Sqrt[7])*L
og[(1 + I*Sqrt[7])^(2/3) - (2*(1 + I*Sqrt[7]))^(1/3)*x + 2^(2/3)*x^2])/(42*
2^(1/3)*(1 + I*Sqrt[7])^(2/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1374

```
Int[((d_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(d^n*(b/q + 1))/2, Int[(d*x)^(m -
 n)/(b/2 + q/2 + c*x^n), x], x] - Dist[(d^n*(b/q - 1))/2, Int[(d*x)^(m - n)
/(b/2 - q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[n2, 2*n] &&
NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GeQ[m, n]
```

Rubi steps

$$\int \frac{x^3}{2+x^3+x^6}\,dx = \frac{1}{14}\left(7-i\sqrt{7}\right)\int \frac{1}{\frac{1}{2}+\frac{i\sqrt{7}}{2}+x^3}\,dx + \frac{1}{14}\left(7+i\sqrt{7}\right)\int \frac{1}{\frac{1}{2}-\frac{i\sqrt{7}}{2}+x^3}\,dx$$

$$= \frac{\left(7-i\sqrt{7}\right)\int \frac{1}{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{7}\right)}+x}\,dx}{21\sqrt[3]{2}\left(1+i\sqrt{7}\right)^{2/3}} + \frac{\left(7-i\sqrt{7}\right)\int \frac{2^{2/3}\sqrt[3]{1+i\sqrt{7}}-x}{\left(\frac{1}{2}\left(1+i\sqrt{7}\right)\right)^{2/3}-\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{7}\right)}\,x+x^2}\,dx}{21\sqrt[3]{2}\left(1+i\sqrt{7}\right)^{2/3}} + \frac{\left(7+i\sqrt{7}\right)\int}{21\sqrt[3]{2}\left(1\right.}$$

$$= \frac{\left(7+i\sqrt{7}\right)\log\left(\sqrt[3]{1-i\sqrt{7}}+\sqrt[3]{2}\,x\right)}{21\sqrt[3]{2}\left(1-i\sqrt{7}\right)^{2/3}} + \frac{\left(7-i\sqrt{7}\right)\log\left(\sqrt[3]{1+i\sqrt{7}}+\sqrt[3]{2}\,x\right)}{21\sqrt[3]{2}\left(1+i\sqrt{7}\right)^{2/3}} - \frac{\left(7-i\sqrt{7}\right)\int}{42}$$

$$= \frac{\left(7+i\sqrt{7}\right)\log\left(\sqrt[3]{1-i\sqrt{7}}+\sqrt[3]{2}\,x\right)}{21\sqrt[3]{2}\left(1-i\sqrt{7}\right)^{2/3}} + \frac{\left(7-i\sqrt{7}\right)\log\left(\sqrt[3]{1+i\sqrt{7}}+\sqrt[3]{2}\,x\right)}{21\sqrt[3]{2}\left(1+i\sqrt{7}\right)^{2/3}} - \frac{\left(7+i\sqrt{7}\right)\log\left(\right.}{}$$

$$= -\frac{i\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{7}\right)}\,\tan^{-1}\left(\frac{1-\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{7}\right)}}}{\sqrt{3}}\right)}{\sqrt{21}} + \frac{i\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{7}\right)}\,\tan^{-1}\left(\frac{1-\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{7}\right)}}}{\sqrt{3}}\right)}{\sqrt{21}} + \frac{\left(7+i\sqrt{7}\right)\mathrm{lo}}{21\sqrt[3]{}}$$

Mathematica [C] time = 0.01, size = 37, normalized size = 0.09

$$\frac{1}{3}\text{RootSum}\left[\#1^6+\#1^3+2\&,\frac{\#1\log(x-\#1)}{2\#1^3+1}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(2 + x^3 + x^6),x]
[Out] RootSum[2 + #1^3 + #1^6 & , (Log[x - #1]*#1)/(1 + 2*#1^3) & ]/3
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{2+x^3+x^6}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^3/(2 + x^3 + x^6),x]
```

```
[Out] IntegrateAlgebraic[x^3/(2 + x^3 + x^6), x]
```

fricas [B] time = 1.51, size = 1435, normalized size = 3.60

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(x^6+x^3+2),x, algorithm="fricas")
```

```
[Out] 1/294*98^(2/3)*56^(1/6)*cos(2/3*arctan(2/7*sqrt(14)*sqrt(7) + sqrt(7)))*log
(-2*98^(2/3)*56^(1/6)*sqrt(7)*x*sin(2/3*arctan(2/7*sqrt(14)*sqrt(7) + sqrt(
7))) + 14*98^(1/3)*7^(1/3)*cos(2/3*arctan(2/7*sqrt(14)*sqrt(7) + sqrt(7)))^
2 + 14*98^(1/3)*7^(1/3)*sin(2/3*arctan(2/7*sqrt(14)*sqrt(7) + sqrt(7)))^2 +
 98*x^2) - 2/147*98^(2/3)*56^(1/6)*arctan(1/5488*(98^(1/3)*56^(5/6)*sqrt(7)
*sqrt(2)*sqrt(-2*98^(2/3)*56^(1/6)*sqrt(7)*x*sin(2/3*arctan(2/7*sqrt(14)*sq
rt(7) + sqrt(7))) + 14*98^(1/3)*7^(1/3)*cos(2/3*arctan(2/7*sqrt(14)*sqrt(7)
 + sqrt(7)))^2 + 14*98^(1/3)*7^(1/3)*sin(2/3*arctan(2/7*sqrt(14)*sqrt(7) +
sqrt(7)))^2 + 98*x^2) - 14*98^(1/3)*56^(5/6)*sqrt(7)*x + 5488*sin(2/3*arcta
n(2/7*sqrt(14)*sqrt(7) + sqrt(7))))/cos(2/3*arctan(2/7*sqrt(14)*sqrt(7) + s
qrt(7))))*sin(2/3*arctan(2/7*sqrt(14)*sqrt(7) + sqrt(7))) + 1/147*(98^(2/3)
*56^(1/6)*sqrt(3)*cos(2/3*arctan(2/7*sqrt(14)*sqrt(7) + sqrt(7))) + 98^(2/3
)*56^(1/6)*sin(2/3*arctan(2/7*sqrt(14)*sqrt(7) + sqrt(7))))*arctan(1/2744*(
14*98^(1/3)*56^(5/6)*sqrt(7)*x*cos(2/3*arctan(2/7*sqrt(14)*sqrt(7) + sqrt(7
))) + 2744*sqrt(3)*cos(2/3*arctan(2/7*sqrt(14)*sqrt(7) + sqrt(7)))^2 + 2744
*sqrt(3)*sin(2/3*arctan(2/7*sqrt(14)*sqrt(7) + sqrt(7)))^2 + 14*(98^(1/3)*5
6^(5/6)*sqrt(7)*sqrt(3)*x + 784*cos(2/3*arctan(2/7*sqrt(14)*sqrt(7) + sqrt(
7))))*sin(2/3*arctan(2/7*sqrt(14)*sqrt(7) + sqrt(7))) - sqrt(98^(2/3)*56^(1
/6)*sqrt(7)*sqrt(3)*x*cos(2/3*arctan(2/7*sqrt(14)*sqrt(7) + sqrt(7))) + 98^
(2/3)*56^(1/6)*sqrt(7)*x*sin(2/3*arctan(2/7*sqrt(14)*sqrt(7) + sqrt(7))) +
14*98^(1/3)*7^(1/3)*cos(2/3*arctan(2/7*sqrt(14)*sqrt(7) + sqrt(7)))^2 + 14*
98^(1/3)*7^(1/3)*sin(2/3*arctan(2/7*sqrt(14)*sqrt(7) + sqrt(7)))^2 + 98*x^2
)*(98^(1/3)*56^(5/6)*sqrt(7)*sqrt(3)*sqrt(2)*sin(2/3*arctan(2/7*sqrt(14)*sq
rt(7) + sqrt(7))) + 98^(1/3)*56^(5/6)*sqrt(7)*sqrt(2)*cos(2/3*arctan(2/7*sq
rt(14)*sqrt(7) + sqrt(7)))))/(cos(2/3*arctan(2/7*sqrt(14)*sqrt(7) + sqrt(7)
))^2 - 3*sin(2/3*arctan(2/7*sqrt(14)*sqrt(7) + sqrt(7)))^2)) + 1/147*(98^(2
/3)*56^(1/6)*sqrt(3)*cos(2/3*arctan(2/7*sqrt(14)*sqrt(7) + sqrt(7))) - 98^(
2/3)*56^(1/6)*sin(2/3*arctan(2/7*sqrt(14)*sqrt(7) + sqrt(7))))*arctan(-1/27
44*(14*98^(1/3)*56^(5/6)*sqrt(7)*x*cos(2/3*arctan(2/7*sqrt(14)*sqrt(7) + sq
rt(7))) - 2744*sqrt(3)*cos(2/3*arctan(2/7*sqrt(14)*sqrt(7) + sqrt(7)))^2 -
2744*sqrt(3)*sin(2/3*arctan(2/7*sqrt(14)*sqrt(7) + sqrt(7)))^2 - 14*(98^(1/
3)*56^(5/6)*sqrt(7)*sqrt(3)*x - 784*cos(2/3*arctan(2/7*sqrt(14)*sqrt(7) + s
qrt(7))))*sin(2/3*arctan(2/7*sqrt(14)*sqrt(7) + sqrt(7))) + sqrt(-98^(2/3)*
56^(1/6)*sqrt(7)*sqrt(3)*x*cos(2/3*arctan(2/7*sqrt(14)*sqrt(7) + sqrt(7)))
+ 98^(2/3)*56^(1/6)*sqrt(7)*x*sin(2/3*arctan(2/7*sqrt(14)*sqrt(7) + sqrt(7)
)) + 14*98^(1/3)*7^(1/3)*cos(2/3*arctan(2/7*sqrt(14)*sqrt(7) + sqrt(7)))^2
+ 14*98^(1/3)*7^(1/3)*sin(2/3*arctan(2/7*sqrt(14)*sqrt(7) + sqrt(7)))^2 + 9
8*x^2)*(98^(1/3)*56^(5/6)*sqrt(7)*sqrt(3)*sqrt(2)*sin(2/3*arctan(2/7*sqrt(1
4)*sqrt(7) + sqrt(7))) - 98^(1/3)*56^(5/6)*sqrt(7)*sqrt(2)*cos(2/3*arctan(2
/7*sqrt(14)*sqrt(7) + sqrt(7)))))/(cos(2/3*arctan(2/7*sqrt(14)*sqrt(7) + sq
rt(7)))^2 - 3*sin(2/3*arctan(2/7*sqrt(14)*sqrt(7) + sqrt(7)))^2)) + 1/588*(
98^(2/3)*56^(1/6)*sqrt(3)*sin(2/3*arctan(2/7*sqrt(14)*sqrt(7) + sqrt(7))) -
 98^(2/3)*56^(1/6)*cos(2/3*arctan(2/7*sqrt(14)*sqrt(7) + sqrt(7))))*log(98^
(2/3)*56^(1/6)*sqrt(7)*sqrt(3)*x*cos(2/3*arctan(2/7*sqrt(14)*sqrt(7) + sqrt
(7))) + 98^(2/3)*56^(1/6)*sqrt(7)*x*sin(2/3*arctan(2/7*sqrt(14)*sqrt(7) + s
qrt(7))) + 14*98^(1/3)*7^(1/3)*cos(2/3*arctan(2/7*sqrt(14)*sqrt(7) + sqrt(7
)))^2 + 14*98^(1/3)*7^(1/3)*sin(2/3*arctan(2/7*sqrt(14)*sqrt(7) + sqrt(7)))
^2 + 98*x^2) - 1/588*(98^(2/3)*56^(1/6)*sqrt(3)*sin(2/3*arctan(2/7*sqrt(14)
```

```
*sqrt(7) + sqrt(7))) + 98^(2/3)*56^(1/6)*cos(2/3*arctan(2/7*sqrt(14)*sqrt(7
) + sqrt(7))))*log(-98^(2/3)*56^(1/6)*sqrt(7)*sqrt(3)*x*cos(2/3*arctan(2/7*
sqrt(14)*sqrt(7) + sqrt(7))) + 98^(2/3)*56^(1/6)*sqrt(7)*x*sin(2/3*arctan(2
/7*sqrt(14)*sqrt(7) + sqrt(7))) + 14*98^(1/3)*7^(1/3)*cos(2/3*arctan(2/7*sq
rt(14)*sqrt(7) + sqrt(7)))^2 + 14*98^(1/3)*7^(1/3)*sin(2/3*arctan(2/7*sqrt(
14)*sqrt(7) + sqrt(7)))^2 + 98*x^2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{x^6 + x^3 + 2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(x^6+x^3+2),x, algorithm="giac")
```

```
[Out] integrate(x^3/(x^6 + x^3 + 2), x)
```

maple [C] time = 0.01, size = 36, normalized size = 0.09

$$\frac{\mathrm{RootOf}\left(\_Z^6 + \_Z^3 + 2\right)^3 \ln\left(-\mathrm{RootOf}\left(\_Z^6 + \_Z^3 + 2\right) + x\right)}{6\,\mathrm{RootOf}\left(\_Z^6 + \_Z^3 + 2\right)^5 + 3\,\mathrm{RootOf}\left(\_Z^6 + \_Z^3 + 2\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(x^6+x^3+2),x)
```

```
[Out] 1/3*sum(_R^3/(2*_R^5+_R^2)*ln(-_R+x),_R=RootOf(_Z^6+_Z^3+2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{x^6 + x^3 + 2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(x^6+x^3+2),x, algorithm="maxima")
```

```
[Out] integrate(x^3/(x^6 + x^3 + 2), x)
```

mupad [B] time = 2.61, size = 351, normalized size = 0.88

$$\frac{\ln\left(x - \frac{2^{2/3}\,7^{5/6}\,\left(-7-\sqrt{7}\,1i\right)^{1/3}\,1i}{14}\right)\left(-196-\sqrt{7}\,28i\right)^{1/3}}{42} + \frac{2^{2/3}\,7^{1/3}\,\ln\left(x + \frac{2^{2/3}\,7^{5/6}\,\left(-7+\sqrt{7}\,1i\right)^{1/3}\,1i}{14}\right)\left(-7+\sqrt{7}\,1i\right)^{1/3}}{42} - \frac{2^{2/3}\,7^{1/3}\,\ln\left(x + \frac{2^{2/3}\,7^{5/6}\,\left(-7-\sqrt{7}\,1i\right)^{1/3}\,1i}{28} - \frac{2^{2/3}\,\sqrt{3}\,7^{5/6}\,\left(-7-\sqrt{7}\,1i\right)^{1/3}}{28}\right)\left(1+\sqrt{3}\,1i\right)\left(-7-\sqrt{7}\,1i\right)^{1/3}}{84} + \frac{2^{2/3}\,7^{1/3}\,\ln\left(x + \frac{2^{2/3}\,7^{5/6}\,\left(-7-\sqrt{7}\,1i\right)^{1/3}\,1i}{28} + \frac{2^{2/3}\,\sqrt{3}\,7^{5/6}\,\left(-7-\sqrt{7}\,1i\right)^{1/3}}{28}\right)\left(-1+\sqrt{3}\,1i\right)\left(-7-\sqrt{7}\,1i\right)^{1/3}}{84} + \frac{2^{2/3}\,7^{1/3}\,\ln\left(x - \frac{2^{2/3}\,7^{5/6}\,\left(-7+\sqrt{7}\,1i\right)^{1/3}\,1i}{28} - \frac{2^{2/3}\,\sqrt{3}\,7^{5/6}\,\left(-7+\sqrt{7}\,1i\right)^{1/3}}{28}\right)\left(-1+\sqrt{3}\,1i\right)\left(-7+\sqrt{7}\,1i\right)^{1/3}}{84} - \frac{2^{2/3}\,7^{1/3}\,\ln\left(x - \frac{2^{2/3}\,7^{5/6}\,\left(-7+\sqrt{7}\,1i\right)^{1/3}\,1i}{28} + \frac{2^{2/3}\,\sqrt{3}\,7^{5/6}\,\left(-7+\sqrt{7}\,1i\right)^{1/3}}{28}\right)\left(1+\sqrt{3}\,1i\right)\left(-7+\sqrt{7}\,1i\right)^{1/3}}{84}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(x^3 + x^6 + 2),x)
```

```
[Out] (log(x - (2^(2/3)*7^(5/6)*(- 7^(1/2)*1i - 7)^(1/3)*1i)/14)*(- 7^(1/2)*28i -
 196)^(1/3))/42 + (2^(2/3)*7^(1/3)*log(x + (2^(2/3)*7^(5/6)*(7^(1/2)*1i - 7
)^(1/3)*1i)/14)*(7^(1/2)*1i - 7)^(1/3))/42 - (2^(2/3)*7^(1/3)*log(x + (2^(2
/3)*7^(5/6)*(- 7^(1/2)*1i - 7)^(1/3)*1i)/28 - (2^(2/3)*3^(1/2)*7^(5/6)*(- 7
^(1/2)*1i - 7)^(1/3))/28)*(3^(1/2)*1i + 1)*(- 7^(1/2)*1i - 7)^(1/3))/84 + (
2^(2/3)*7^(1/3)*log(x + (2^(2/3)*7^(5/6)*(- 7^(1/2)*1i - 7)^(1/3)*1i)/28 +
(2^(2/3)*3^(1/2)*7^(5/6)*(- 7^(1/2)*1i - 7)^(1/3))/28)*(3^(1/2)*1i - 1)*(-
7^(1/2)*1i - 7)^(1/3))/84 + (2^(2/3)*7^(1/3)*log(x - (2^(2/3)*7^(5/6)*(7^(1
/2)*1i - 7)^(1/3)*1i)/28 - (2^(2/3)*3^(1/2)*7^(5/6)*(7^(1/2)*1i - 7)^(1/3))
/28)*(3^(1/2)*1i - 1)*(7^(1/2)*1i - 7)^(1/3))/84 - (2^(2/3)*7^(1/3)*log(x -
 (2^(2/3)*7^(5/6)*(7^(1/2)*1i - 7)^(1/3)*1i)/28 + (2^(2/3)*3^(1/2)*7^(5/6)*
(7^(1/2)*1i - 7)^(1/3))/28)*(3^(1/2)*1i + 1)*(7^(1/2)*1i - 7)^(1/3))/84
```

sympy [**A**] time = 0.15, size = 24, normalized size = 0.06

$$\text{RootSum}\left(250047t^6 + 1323t^3 + 2, \left(t \mapsto t\log\left(7938t^4 + 21t + x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(x**6+x**3+2),x)
```

```
[Out] RootSum(250047*_t**6 + 1323*_t**3 + 2, Lambda(_t, _t*log(7938*_t**4 + 21*_t
 + x)))
```

3.172 $\int x^{14}\sqrt{a+bx^3+cx^6}\,dx$

Optimal. Leaf size=231

$$-\frac{(b^2-4ac)\left(16a^2c^2-56ab^2c+21b^4\right)\tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{3072c^{11/2}}+\frac{\left(16a^2c^2-56ab^2c+21b^4\right)\left(b+2cx^3\right)\sqrt{a+bx^3+}}{1536c^5}$$

Rubi [A] time = 0.30, antiderivative size = 231, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.350, Rules used = {1357, 742, 832, 779, 612, 621, 206}

$$\frac{\left(16a^2c^2-56ab^2c+21b^4\right)\left(b+2cx^3\right)\sqrt{a+bx^3+cx^6}}{1536c^5}-\frac{(b^2-4ac)\left(16a^2c^2-56ab^2c+21b^4\right)\tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{3072c^{11/2}}-\frac{\left(7b\left(15b^2-28ac\right)-6cx^3\left(21b^2-20ac\right)\right)\left(a+bx^3+cx^6\right)^{3/2}}{2880c^4}-\frac{bx^6\left(a+bx^3+cx^6\right)^{3/2}}{20c^2}+\frac{x^9\left(a+bx^3+cx^6\right)^{3/2}}{18c}$$

Antiderivative was successfully verified.

```
[In] Int[x^14*Sqrt[a + b*x^3 + c*x^6],x]
```

```
[Out] ((21*b^4 - 56*a*b^2*c + 16*a^2*c^2)*(b + 2*c*x^3)*Sqrt[a + b*x^3 + c*x^6])/
(1536*c^5) - (b*x^6*(a + b*x^3 + c*x^6)^(3/2))/(20*c^2) + (x^9*(a + b*x^3 +
 c*x^6)^(3/2))/(18*c) - ((7*b*(15*b^2 - 28*a*c) - 6*c*(21*b^2 - 20*a*c)*x^3
)*(a + b*x^3 + c*x^6)^(3/2))/(2880*c^4) - ((b^2 - 4*a*c)*(21*b^4 - 56*a*b^2
*c + 16*a^2*c^2)*ArcTanh[(b + 2*c*x^3)/(2*Sqrt[c]*Sqrt[a + b*x^3 + c*x^6])]
)/(3072*c^(11/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 742

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(m + 2*p
+ 1)), x] + Dist[1/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m +
 2*p + 1) - e*(a*e*(m - 1) + b*d*(p + 1)) + e*(2*c*d - b*e)*(m + p)*x, x]*(
a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && If[Rat
ionalQ[m], GtQ[m, 1], SumSimplerQ[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuad
raticQ[a, b, c, d, e, m, p, x]
```

Rule 779

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> -Simp[((b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x)*(a + b*x + c*x^2)^(p + 1))/(2*c^2*(p + 1)*(2*p + 3)), x
```

```
] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p +
3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] && !LeQ[p, -1]
```

Rule 832

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + b*x + c*x^2)^(p +
1))/(c*(m + 2*p + 2)), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)
*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m*
(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p]
 || IntegersQ[2*m, 2*p]) && !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 1357

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int x^{14}\sqrt{a+bx^3+cx^6}\,dx = \frac{1}{3}\text{Subst}\left(\int x^4\sqrt{a+bx+cx^2}\,dx,x,x^3\right)$$
$$= \frac{x^9\left(a+bx^3+cx^6\right)^{3/2}}{18c} + \frac{\text{Subst}\left(\int x^2\left(-3a-\frac{9bx}{2}\right)\sqrt{a+bx+cx^2}\,dx,x,x^3\right)}{18c}$$
$$= -\frac{bx^6\left(a+bx^3+cx^6\right)^{3/2}}{20c^2} + \frac{x^9\left(a+bx^3+cx^6\right)^{3/2}}{18c} + \frac{\text{Subst}\left(\int x\left(9ab+\frac{3}{4}\left(21b^2-20\right.\right.\right.}{90}$$
$$= -\frac{bx^6\left(a+bx^3+cx^6\right)^{3/2}}{20c^2} + \frac{x^9\left(a+bx^3+cx^6\right)^{3/2}}{18c} - \frac{\left(7b\left(15b^2-28ac\right)-6c\left(21b^2-\right.\right.}{2880c}$$
$$= \frac{\left(21b^4-56ab^2c+16a^2c^2\right)\left(b+2cx^3\right)\sqrt{a+bx^3+cx^6}}{1536c^5} - \frac{bx^6\left(a+bx^3+cx^6\right)^{3/2}}{20c^2} + \frac{x}{}$$
$$= \frac{\left(21b^4-56ab^2c+16a^2c^2\right)\left(b+2cx^3\right)\sqrt{a+bx^3+cx^6}}{1536c^5} - \frac{bx^6\left(a+bx^3+cx^6\right)^{3/2}}{20c^2} + \frac{x}{}$$
$$= \frac{\left(21b^4-56ab^2c+16a^2c^2\right)\left(b+2cx^3\right)\sqrt{a+bx^3+cx^6}}{1536c^5} - \frac{bx^6\left(a+bx^3+cx^6\right)^{3/2}}{20c^2} + \frac{x}{}$$

Mathematica [A] time = 0.16, size = 208, normalized size = 0.90

$$\frac{2\sqrt{c}\sqrt{a+bx^3+cx^6}\left(16bc^2\left(113a^2-34acx^6+8c^2x^{12}\right)+160c^3x^3\left(-3a^2+2acx^6+8c^2x^{12}\right)+168b^3c\left(cx^6-10a\right)+16b^2c^2x^3\left(56a-9cx^6\right)+315b^5-210b^4cx^3\right)-15\left(-64a^3c^3+240a^2b^2c^2-140ab^4c+21b^6\right)\tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{46080c^{11/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^14*Sqrt[a + b*x^3 + c*x^6],x]
```

```
[Out] (2*Sqrt[c]*Sqrt[a + b*x^3 + c*x^6]*(315*b^5 - 210*b^4*c*x^3 + 16*b^2*c^2*x^
3*(56*a - 9*c*x^6) + 168*b^3*c*(-10*a + c*x^6) + 16*b*c^2*(113*a^2 - 34*a*c
*x^6 + 8*c^2*x^12) + 160*c^3*x^3*(-3*a^2 + 2*a*c*x^6 + 8*c^2*x^12)) - 15*(2
```

```
1*b^6 - 140*a*b^4*c + 240*a^2*b^2*c^2 - 64*a^3*c^3)*ArcTanh[(b + 2*c*x^3)/(
2*Sqrt[c]*Sqrt[a + b*x^3 + c*x^6])])/(46080*c^(11/2))
```

IntegrateAlgebraic [A] time = 0.80, size = 209, normalized size = 0.90

$$\frac{\sqrt{a+bx^3+cx^6}\left(1808a^2bc^2-480a^2c^3x^3-1680ab^3c+896ab^2c^2x^3-544abc^3x^6+320ac^4x^9+315b^5-210b^4cx^3+168b^3c^2x^6-144b^2c^3x^9+128bc^4x^{12}+1280c^5x^{15}\right)}{23040c^5}+\frac{\left(-64a^3c^3+240a^2b^2c^2-140ab^4c+21b^6\right)\log\left(-2\sqrt{c}\sqrt{a+bx^3+cx^6}+b+2cx^3\right)}{3072c^{11/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^14*Sqrt[a + b*x^3 + c*x^6],x]
```

```
[Out] (Sqrt[a + b*x^3 + c*x^6]*(315*b^5 - 1680*a*b^3*c + 1808*a^2*b*c^2 - 210*b^4
*c*x^3 + 896*a*b^2*c^2*x^3 - 480*a^2*c^3*x^3 + 168*b^3*c^2*x^6 - 544*a*b*c^
3*x^6 - 144*b^2*c^3*x^9 + 320*a*c^4*x^9 + 128*b*c^4*x^12 + 1280*c^5*x^15))/
(23040*c^5) + ((21*b^6 - 140*a*b^4*c + 240*a^2*b^2*c^2 - 64*a^3*c^3)*Log[b
+ 2*c*x^3 - 2*Sqrt[c]*Sqrt[a + b*x^3 + c*x^6]])/(3072*c^(11/2))
```

fricas [A] time = 1.35, size = 451, normalized size = 1.95

$$\left[-\frac{15\left(21b^6-140ab^4c+240a^2b^2c^2-64a^3c^3\right)\sqrt{c}\log\left(-8c^2x^6-8bcx^3-b^2-4\sqrt{cx^6+bx^3+a}\left(2cx^3+b\right)\sqrt{c}-4ac\right)-4\left(1280c^6x^{15}+128bc^5x^{12}-16\left(9b^2c^4-20ac^5\right)x^9+8\left(21b^3c^3-68abc^4\right)x^6+315b^5c-1680ab^3c^2+1808a^2bc^3-2\left(105b^4c^2-448ab^2c^3+240a^2c^4\right)x^3\right)\sqrt{cx^6+bx^3+a}}{92160c^6}, \frac{15\left(21b^6-140ab^4c+240a^2b^2c^2-64a^3c^3\right)\sqrt{-c}\arctan\left(\frac{\sqrt{cx^6+bx^3+a}\left(2cx^3+b\right)\sqrt{-c}}{2\left(c^2x^6+bcx^3+ac\right)}\right)+2\left(1280c^6x^{15}+128bc^5x^{12}-16\left(9b^2c^4-20ac^5\right)x^9+8\left(21b^3c^3-68abc^4\right)x^6+315b^5c-1680ab^3c^2+1808a^2bc^3-2\left(105b^4c^2-448ab^2c^3+240a^2c^4\right)x^3\right)\sqrt{cx^6+bx^3+a}}{46080c^6}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^14*(c*x^6+b*x^3+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/92160*(15*(21*b^6 - 140*a*b^4*c + 240*a^2*b^2*c^2 - 64*a^3*c^3)*sqrt(c)
*log(-8*c^2*x^6 - 8*b*c*x^3 - b^2 - 4*sqrt(c*x^6 + b*x^3 + a)*(2*c*x^3 + b)
*sqrt(c) - 4*a*c) - 4*(1280*c^6*x^15 + 128*b*c^5*x^12 - 16*(9*b^2*c^4 - 20*
a*c^5)*x^9 + 8*(21*b^3*c^3 - 68*a*b*c^4)*x^6 + 315*b^5*c - 1680*a*b^3*c^2 +
 1808*a^2*b*c^3 - 2*(105*b^4*c^2 - 448*a*b^2*c^3 + 240*a^2*c^4)*x^3)*sqrt(c
*x^6 + b*x^3 + a))/c^6, 1/46080*(15*(21*b^6 - 140*a*b^4*c + 240*a^2*b^2*c^2
 - 64*a^3*c^3)*sqrt(-c)*arctan(1/2*sqrt(c*x^6 + b*x^3 + a)*(2*c*x^3 + b)*sq
rt(-c)/(c^2*x^6 + b*c*x^3 + a*c)) + 2*(1280*c^6*x^15 + 128*b*c^5*x^12 - 16*
(9*b^2*c^4 - 20*a*c^5)*x^9 + 8*(21*b^3*c^3 - 68*a*b*c^4)*x^6 + 315*b^5*c -
1680*a*b^3*c^2 + 1808*a^2*b*c^3 - 2*(105*b^4*c^2 - 448*a*b^2*c^3 + 240*a^2*
c^4)*x^3)*sqrt(c*x^6 + b*x^3 + a))/c^6]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{cx^6+bx^3+a}\,x^{14}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^14*(c*x^6+b*x^3+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x^6 + b*x^3 + a)*x^14, x)
```

maple [F] time = 0.05, size = 0, normalized size = 0.00

$$\int \sqrt{c\,x^6+b\,x^3+a}\;x^{14}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^14*(c*x^6+b*x^3+a)^(1/2),x)
```

```
[Out] int(x^14*(c*x^6+b*x^3+a)^(1/2),x)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^14*(c*x^6+b*x^3+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive, negative or zero?
```

mupad [B] time = 2.94, size = 543, normalized size = 2.35

$$\frac{x^9\left(cx^6+bx^3+a\right)^{3/2}}{18c}-\frac{b\left(\frac{x^6\left(cx^6+bx^3+a\right)^{3/2}}{5c}+\frac{7b\left(\frac{a\left(\left(\frac{b}{4c}+\frac{x^3}{2}\right)\sqrt{cx^6+bx^3+a}+\frac{\ln\left(\sqrt{cx^6+bx^3+a}+\frac{cx^3+\frac{b}{2}}{\sqrt{c}}\right)\left(ac-\frac{b^2}{4}\right)}{2c^{3/2}}\right)}{4c}-\frac{x^3\left(cx^6+bx^3+a\right)^{3/2}}{4c}+\frac{5b\left(\frac{\left(8c\left(cx^6+a\right)-3b^2+2bcx^3\right)\sqrt{cx^6+bx^3+a}}{24c^2}+\frac{\ln\left(2\sqrt{cx^6+bx^3+a}+\frac{2cx^3+b}{\sqrt{c}}\right)\left(b^3-4abc\right)}{16c^{5/2}}\right)}{8c}\right)}{10c}-\frac{2a\left(\frac{\left(8c\left(cx^6+a\right)-3b^2+2bcx^3\right)\sqrt{cx^6+bx^3+a}}{24c^2}+\frac{\ln\left(2\sqrt{cx^6+bx^3+a}+\frac{2cx^3+b}{\sqrt{c}}\right)\left(b^3-4abc\right)}{16c^{5/2}}\right)}{5c}\right)}{4c}+\frac{a\left(\frac{a\left(\left(\frac{b}{4c}+\frac{x^3}{2}\right)\sqrt{cx^6+bx^3+a}+\frac{\ln\left(\sqrt{cx^6+bx^3+a}+\frac{cx^3+\frac{b}{2}}{\sqrt{c}}\right)\left(ac-\frac{b^2}{4}\right)}{2c^{3/2}}\right)}{4c}-\frac{x^3\left(cx^6+bx^3+a\right)^{3/2}}{4c}+\frac{5b\left(\frac{\left(8c\left(cx^6+a\right)-3b^2+2bcx^3\right)\sqrt{cx^6+bx^3+a}}{24c^2}+\frac{\ln\left(2\sqrt{cx^6+bx^3+a}+\frac{2cx^3+b}{\sqrt{c}}\right)\left(b^3-4abc\right)}{16c^{5/2}}\right)}{8c}\right)}{6c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^14*(a + b*x^3 + c*x^6)^(1/2),x)
```

```
[Out] (x^9*(a + b*x^3 + c*x^6)^(3/2))/(18*c) - (b*((x^6*(a + b*x^3 + c*x^6)^(3/2)
)/(5*c) + (7*b*((a*((b/(4*c) + x^3/2)*(a + b*x^3 + c*x^6)^(1/2) + (log((a +
 b*x^3 + c*x^6)^(1/2) + (b/2 + c*x^3)/c^(1/2))*(a*c - b^2/4))/(2*c^(3/2))))
/(4*c) - (x^3*(a + b*x^3 + c*x^6)^(3/2))/(4*c) + (5*b*(((8*c*(a + c*x^6) -
3*b^2 + 2*b*c*x^3)*(a + b*x^3 + c*x^6)^(1/2))/(24*c^2) + (log(2*(a + b*x^3
+ c*x^6)^(1/2) + (b + 2*c*x^3)/c^(1/2))*(b^3 - 4*a*b*c))/(16*c^(5/2))))/(8*
c)))/(10*c) - (2*a*(((8*c*(a + c*x^6) - 3*b^2 + 2*b*c*x^3)*(a + b*x^3 + c*x
^6)^(1/2))/(24*c^2) + (log(2*(a + b*x^3 + c*x^6)^(1/2) + (b + 2*c*x^3)/c^(1
/2))*(b^3 - 4*a*b*c))/(16*c^(5/2))))/(5*c)))/(4*c) + (a*((a*((b/(4*c) + x^3
/2)*(a + b*x^3 + c*x^6)^(1/2) + (log((a + b*x^3 + c*x^6)^(1/2) + (b/2 + c*x
^3)/c^(1/2))*(a*c - b^2/4))/(2*c^(3/2))))/(4*c) - (x^3*(a + b*x^3 + c*x^6)^
(3/2))/(4*c) + (5*b*(((8*c*(a + c*x^6) - 3*b^2 + 2*b*c*x^3)*(a + b*x^3 + c*
x^6)^(1/2))/(24*c^2) + (log(2*(a + b*x^3 + c*x^6)^(1/2) + (b + 2*c*x^3)/c^(
1/2))*(b^3 - 4*a*b*c))/(16*c^(5/2))))/(8*c)))/(6*c)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^{14}\sqrt{a+bx^3+cx^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**14*(c*x**6+b*x**3+a)**(1/2),x)
```

```
[Out] Integral(x**14*sqrt(a + b*x**3 + c*x**6), x)
```

3.173 $\int x^{11}\sqrt{a+bx^3+cx^6}\,dx$

Optimal. Leaf size=171

$$\frac{b\left(7b^2-12ac\right)\left(b^2-4ac\right)\tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{768c^{9/2}}-\frac{b\left(7b^2-12ac\right)\left(b+2cx^3\right)\sqrt{a+bx^3+cx^6}}{384c^4}+\frac{\left(-32ac+35b^2-4\right.}{}$$

Rubi [A] time = 0.15, antiderivative size = 171, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {1357, 742, 779, 612, 621, 206}

$$\frac{\left(-32ac+35b^2-42bcx^3\right)\left(a+bx^3+cx^6\right)^{3/2}}{720c^3}-\frac{b\left(7b^2-12ac\right)\left(b+2cx^3\right)\sqrt{a+bx^3+cx^6}}{384c^4}+\frac{b\left(7b^2-12ac\right)\left(b^2-4ac\right)\tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{768c^{9/2}}+\frac{x^6\left(a+bx^3+cx^6\right)^{3/2}}{15c}$$

Antiderivative was successfully verified.

```
[In] Int[x^11*Sqrt[a + b*x^3 + c*x^6],x]
```

```
[Out] -(b*(7*b^2 - 12*a*c)*(b + 2*c*x^3)*Sqrt[a + b*x^3 + c*x^6])/(384*c^4) + (x^
6*(a + b*x^3 + c*x^6)^(3/2))/(15*c) + ((35*b^2 - 32*a*c - 42*b*c*x^3)*(a +
b*x^3 + c*x^6)^(3/2))/(720*c^3) + (b*(7*b^2 - 12*a*c)*(b^2 - 4*a*c)*ArcTanh
[(b + 2*c*x^3)/(2*Sqrt[c]*Sqrt[a + b*x^3 + c*x^6])])/(768*c^(9/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 742

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(m + 2*p
+ 1)), x] + Dist[1/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m +
 2*p + 1) - e*(a*e*(m - 1) + b*d*(p + 1)) + e*(2*c*d - b*e)*(m + p)*x, x]*(
a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && If[Rat
ionalQ[m], GtQ[m, 1], SumSimplerQ[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuad
raticQ[a, b, c, d, e, m, p, x]
```

Rule 779

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> -Simp[((b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x)*(a + b*x + c*x^2)^(p + 1))/(2*c^2*(p + 1)*(2*p + 3)), x
] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p +
3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
```

```
, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 1357

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int x^{11}\sqrt{a+bx^3+cx^6}\,dx = \frac{1}{3}\operatorname{Subst}\left(\int x^3\sqrt{a+bx+cx^2}\,dx, x, x^3\right)$$

$$= \frac{x^6\left(a+bx^3+cx^6\right)^{3/2}}{15c} + \frac{\operatorname{Subst}\left(\int x\left(-2a-\frac{7bx}{2}\right)\sqrt{a+bx+cx^2}\,dx, x, x^3\right)}{15c}$$

$$= \frac{x^6\left(a+bx^3+cx^6\right)^{3/2}}{15c} + \frac{\left(35b^2-32ac-42bcx^3\right)\left(a+bx^3+cx^6\right)^{3/2}}{720c^3} - \frac{\left(b\left(7b^2-12a\right.\right.}{}$$

$$= -\frac{b\left(7b^2-12ac\right)\left(b+2cx^3\right)\sqrt{a+bx^3+cx^6}}{384c^4} + \frac{x^6\left(a+bx^3+cx^6\right)^{3/2}}{15c} + \frac{\left(35b^2-32a\right.}{}$$

$$= -\frac{b\left(7b^2-12ac\right)\left(b+2cx^3\right)\sqrt{a+bx^3+cx^6}}{384c^4} + \frac{x^6\left(a+bx^3+cx^6\right)^{3/2}}{15c} + \frac{\left(35b^2-32a\right.}{}$$

$$= -\frac{b\left(7b^2-12ac\right)\left(b+2cx^3\right)\sqrt{a+bx^3+cx^6}}{384c^4} + \frac{x^6\left(a+bx^3+cx^6\right)^{3/2}}{15c} + \frac{\left(35b^2-32a\right.}{}$$

Mathematica [A] time = 0.13, size = 164, normalized size = 0.96

$$\frac{-\frac{\left(32ac-35b^2+42bcx^3\right)\left(a+bx^3+cx^6\right)^{3/2}}{48c^2}+\frac{5\left(12abc-7b^3\right)\left(2\sqrt{c}\left(b+2cx^3\right)\sqrt{a+bx^3+cx^6}-\left(b^2-4ac\right)\tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)\right)}{256c^{7/2}}+x^6\left(a+bx^3+cx^6\right)^{3/2}}{15c}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^11*Sqrt[a + b*x^3 + c*x^6],x]
```

```
[Out] (x^6*(a + b*x^3 + c*x^6)^(3/2) - ((-35*b^2 + 32*a*c + 42*b*c*x^3)*(a + b*x^
3 + c*x^6)^(3/2))/(48*c^2) + (5*(-7*b^3 + 12*a*b*c)*(2*Sqrt[c]*(b + 2*c*x^3
)*Sqrt[a + b*x^3 + c*x^6] - (b^2 - 4*a*c)*ArcTanh[(b + 2*c*x^3)/(2*Sqrt[c]*
Sqrt[a + b*x^3 + c*x^6])]))/(256*c^(7/2)))/(15*c)
```

IntegrateAlgebraic [A] time = 0.58, size = 170, normalized size = 0.99

$$\frac{\left(-48a^2bc^2+40ab^3c-7b^5\right)\log\left(-2c^{9/2}\sqrt{a+bx^3+cx^6}+bc^4+2c^5x^3\right)}{768c^{9/2}}+\frac{\sqrt{a+bx^3+cx^6}\left(-256a^2c^2+460ab^2c-232abc^2x^3+128ac^3x^6-105b^4+70b^3cx^3-56b^2c^2x^6+48bc^3x^9+384c^4x^{12}\right)}{5760c^4}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^11*Sqrt[a + b*x^3 + c*x^6],x]
```

```
[Out] (Sqrt[a + b*x^3 + c*x^6]*(-105*b^4 + 460*a*b^2*c - 256*a^2*c^2 + 70*b^3*c*x
^3 - 232*a*b*c^2*x^3 - 56*b^2*c^2*x^6 + 128*a*c^3*x^6 + 48*b*c^3*x^9 + 384*
c^4*x^12))/(5760*c^4) + ((-7*b^5 + 40*a*b^3*c - 48*a^2*b*c^2)*Log[b*c^4 + 2
*c^5*x^3 - 2*c^(9/2)*Sqrt[a + b*x^3 + c*x^6]])/(768*c^(9/2))
```

fricas [A] time = 1.75, size = 367, normalized size = 2.15

$$\left[\frac{15\left(7b^5-40ab^3c+48a^2bc^2\right)\sqrt{c}\log\left(-8c^2x^6-8bcx^3-b^2-4\sqrt{cx^6+bx^3+a}\left(2cx^3+b\right)\sqrt{c}-4ac\right)+4\left(384c^5x^{12}+48bc^4x^9-8\left(7b^2c^3-16ac^4\right)x^6-105b^4c+460ab^2c^2-256a^2c^3+2\left(35b^3c^2-116abc^3\right)x^3\right)\sqrt{cx^6+bx^3+a}}{23040c^5}, -\frac{15\left(7b^5-40ab^3c+48a^2bc^2\right)\sqrt{-c}\arctan\left(\frac{\sqrt{cx^6+bx^3+a}\left(2cx^3+b\right)\sqrt{-c}}{2\left(c^2x^6+bcx^3+ac\right)}\right)-2\left(384c^5x^{12}+48bc^4x^9-8\left(7b^2c^3-16ac^4\right)x^6-105b^4c+460ab^2c^2-256a^2c^3+2\left(35b^3c^2-116abc^3\right)x^3\right)\sqrt{cx^6+bx^3+a}}{11520c^5}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(c*x^6+b*x^3+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/23040*(15*(7*b^5 - 40*a*b^3*c + 48*a^2*b*c^2)*sqrt(c)*log(-8*c^2*x^6 - 8
*b*c*x^3 - b^2 - 4*sqrt(c*x^6 + b*x^3 + a)*(2*c*x^3 + b)*sqrt(c) - 4*a*c) +
 4*(384*c^5*x^12 + 48*b*c^4*x^9 - 8*(7*b^2*c^3 - 16*a*c^4)*x^6 - 105*b^4*c
+ 460*a*b^2*c^2 - 256*a^2*c^3 + 2*(35*b^3*c^2 - 116*a*b*c^3)*x^3)*sqrt(c*x^
6 + b*x^3 + a))/c^5, -1/11520*(15*(7*b^5 - 40*a*b^3*c + 48*a^2*b*c^2)*sqrt(
-c)*arctan(1/2*sqrt(c*x^6 + b*x^3 + a)*(2*c*x^3 + b)*sqrt(-c)/(c^2*x^6 + b*
c*x^3 + a*c)) - 2*(384*c^5*x^12 + 48*b*c^4*x^9 - 8*(7*b^2*c^3 - 16*a*c^4)*x
^6 - 105*b^4*c + 460*a*b^2*c^2 - 256*a^2*c^3 + 2*(35*b^3*c^2 - 116*a*b*c^3)
*x^3)*sqrt(c*x^6 + b*x^3 + a))/c^5]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{cx^6 + bx^3 + a}\, x^{11}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(c*x^6+b*x^3+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x^6 + b*x^3 + a)*x^11, x)
```

maple [F] time = 0.05, size = 0, normalized size = 0.00

$$\int \sqrt{c\,x^6 + b\,x^3 + a}\; x^{11}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11*(c*x^6+b*x^3+a)^(1/2),x)
```

```
[Out] int(x^11*(c*x^6+b*x^3+a)^(1/2),x)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(c*x^6+b*x^3+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive, negative or zero?
```

mupad [B] time = 1.87, size = 315, normalized size = 1.84

$$\frac{x^6\left(c\,x^6+b\,x^3+a\right)^{3/2}}{15\,c}+\frac{7\,b\left(\frac{a\left(\left(\frac{b}{4\,c}+\frac{x^3}{2}\right)\sqrt{c\,x^6+b\,x^3+a}+\frac{\ln\left(\sqrt{c\,x^6+b\,x^3+a}+\frac{c\,x^3+\frac{b}{2}}{\sqrt{c}}\right)\left(a\,c-\frac{b^2}{4}\right)}{2\,c^{3/2}}\right)}{4\,c}-\frac{x^3\left(c\,x^6+b\,x^3+a\right)^{3/2}}{4\,c}+\frac{5\,b\left(\frac{\left(8\,c\left(c\,x^6+a\right)-3\,b^2+2\,b\,c\,x^3\right)\sqrt{c\,x^6+b\,x^3+a}}{24\,c^2}+\frac{\ln\left(2\,\sqrt{c\,x^6+b\,x^3+a}+\frac{2\,c\,x^3+b}{\sqrt{c}}\right)\left(b^3-4\,a\,b\,c\right)}{16\,c^{5/2}}\right)}{8\,c}\right)}{30\,c}-\frac{2\,a\left(\frac{\left(8\,c\left(c\,x^6+a\right)-3\,b^2+2\,b\,c\,x^3\right)\sqrt{c\,x^6+b\,x^3+a}}{24\,c^2}+\frac{\ln\left(2\,\sqrt{c\,x^6+b\,x^3+a}+\frac{2\,c\,x^3+b}{\sqrt{c}}\right)\left(b^3-4\,a\,b\,c\right)}{16\,c^{5/2}}\right)}{15\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11*(a + b*x^3 + c*x^6)^(1/2),x)
```

```
[Out] (x^6*(a + b*x^3 + c*x^6)^(3/2))/(15*c) + (7*b*((a*((b/(4*c) + x^3/2)*(a + b
*x^3 + c*x^6)^(1/2) + (log((a + b*x^3 + c*x^6)^(1/2) + (b/2 + c*x^3)/c^(1/2
))*(a*c - b^2/4))/(2*c^(3/2))))/(4*c) - (x^3*(a + b*x^3 + c*x^6)^(3/2))/(4*
c) + (5*b*(((8*c*(a + c*x^6) - 3*b^2 + 2*b*c*x^3)*(a + b*x^3 + c*x^6)^(1/2)
```

```
)/(24*c^2) + (log(2*(a + b*x^3 + c*x^6)^(1/2) + (b + 2*c*x^3)/c^(1/2))*(b^3
 - 4*a*b*c))/(16*c^(5/2)))))/(8*c)))/(30*c) - (2*a*(((8*c*(a + c*x^6) - 3*b^
2 + 2*b*c*x^3)*(a + b*x^3 + c*x^6)^(1/2))/(24*c^2) + (log(2*(a + b*x^3 + c*
x^6)^(1/2) + (b + 2*c*x^3)/c^(1/2))*(b^3 - 4*a*b*c))/(16*c^(5/2))))/(15*c)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^{11}\sqrt{a+bx^3+cx^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**11*(c*x**6+b*x**3+a)**(1/2),x)
```

```
[Out] Integral(x**11*sqrt(a + b*x**3 + c*x**6), x)
```

3.174 $\int x^8\sqrt{a+bx^3+cx^6}\,dx$

Optimal. Leaf size=153

$$-\frac{\left(b^2-4ac\right)\left(5b^2-4ac\right)\tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{384c^{7/2}}+\frac{\left(5b^2-4ac\right)\left(b+2cx^3\right)\sqrt{a+bx^3+cx^6}}{192c^3}-\frac{5b\left(a+bx^3+cx^6\right)^{3/2}}{72c^2}+$$

Rubi [A] time = 0.14, antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {1357, 742, 640, 612, 621, 206}

$$\frac{\left(5b^2-4ac\right)\left(b+2cx^3\right)\sqrt{a+bx^3+cx^6}}{192c^3}-\frac{\left(b^2-4ac\right)\left(5b^2-4ac\right)\tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{384c^{7/2}}-\frac{5b\left(a+bx^3+cx^6\right)^{3/2}}{72c^2}+\frac{x^3\left(a+bx^3+cx^6\right)^{3/2}}{12c}$$

Antiderivative was successfully verified.

```
[In] Int[x^8*Sqrt[a + b*x^3 + c*x^6],x]
```

```
[Out] ((5*b^2 - 4*a*c)*(b + 2*c*x^3)*Sqrt[a + b*x^3 + c*x^6])/(192*c^3) - (5*b*(a
 + b*x^3 + c*x^6)^(3/2))/(72*c^2) + (x^3*(a + b*x^3 + c*x^6)^(3/2))/(12*c)
- ((b^2 - 4*a*c)*(5*b^2 - 4*a*c)*ArcTanh[(b + 2*c*x^3)/(2*Sqrt[c]*Sqrt[a +
b*x^3 + c*x^6])])/(384*c^(7/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 742

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(m + 2*p
+ 1)), x] + Dist[1/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m +
 2*p + 1) - e*(a*e*(m - 1) + b*d*(p + 1)) + e*(2*c*d - b*e)*(m + p)*x, x]*(
a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && If[Rat
ionalQ[m], GtQ[m, 1], SumSimplerQ[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuad
raticQ[a, b, c, d, e, m, p, x]
```

Rule 1357

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int x^8\sqrt{a+bx^3+cx^6}\,dx = \frac{1}{3}\,\text{Subst}\left(\int x^2\sqrt{a+bx+cx^2}\,dx,x,x^3\right)$$
$$= \frac{x^3\left(a+bx^3+cx^6\right)^{3/2}}{12c} + \frac{\text{Subst}\left(\int\left(-a-\frac{5bx}{2}\right)\sqrt{a+bx+cx^2}\,dx,x,x^3\right)}{12c}$$
$$= -\frac{5b\left(a+bx^3+cx^6\right)^{3/2}}{72c^2} + \frac{x^3\left(a+bx^3+cx^6\right)^{3/2}}{12c} + \frac{\left(5b^2-4ac\right)\text{Subst}\left(\int\sqrt{a+bx+c\right.}{48c^2}$$
$$= \frac{\left(5b^2-4ac\right)\left(b+2cx^3\right)\sqrt{a+bx^3+cx^6}}{192c^3} - \frac{5b\left(a+bx^3+cx^6\right)^{3/2}}{72c^2} + \frac{x^3\left(a+bx^3+cx^6\right.}{12c}$$
$$= \frac{\left(5b^2-4ac\right)\left(b+2cx^3\right)\sqrt{a+bx^3+cx^6}}{192c^3} - \frac{5b\left(a+bx^3+cx^6\right)^{3/2}}{72c^2} + \frac{x^3\left(a+bx^3+cx^6\right.}{12c}$$
$$= \frac{\left(5b^2-4ac\right)\left(b+2cx^3\right)\sqrt{a+bx^3+cx^6}}{192c^3} - \frac{5b\left(a+bx^3+cx^6\right)^{3/2}}{72c^2} + \frac{x^3\left(a+bx^3+cx^6\right.}{12c}$$

Mathematica [A] time = 0.07, size = 136, normalized size = 0.89

$$\frac{2\sqrt{c}\sqrt{a+bx^3+cx^6}\left(b\left(8c^2x^6-52ac\right)+24c^2x^3\left(a+2cx^6\right)+15b^3-10b^2cx^3\right)-3\left(16a^2c^2-24ab^2c+5b^4\right)\tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{1152c^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8*Sqrt[a + b*x^3 + c*x^6],x]
```

```
[Out] (2*Sqrt[c]*Sqrt[a + b*x^3 + c*x^6]*(15*b^3 - 10*b^2*c*x^3 + 24*c^2*x^3*(a +
 2*c*x^6) + b*(-52*a*c + 8*c^2*x^6)) - 3*(5*b^4 - 24*a*b^2*c + 16*a^2*c^2)*
ArcTanh[(b + 2*c*x^3)/(2*Sqrt[c]*Sqrt[a + b*x^3 + c*x^6])])/(1152*c^(7/2))
```

IntegrateAlgebraic [A] time = 0.42, size = 132, normalized size = 0.86

$$\frac{\left(16a^2c^2-24ab^2c+5b^4\right)\log\left(-2\sqrt{c}\sqrt{a+bx^3+cx^6}+b+2cx^3\right)}{384c^{7/2}} + \frac{\sqrt{a+bx^3+cx^6}\left(-52abc+24ac^2x^3+15b^3-10b^2cx^3+8bc^2x^6+48c^3x^9\right)}{576c^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^8*Sqrt[a + b*x^3 + c*x^6],x]
```

```
[Out] (Sqrt[a + b*x^3 + c*x^6]*(15*b^3 - 52*a*b*c - 10*b^2*c*x^3 + 24*a*c^2*x^3 +
 8*b*c^2*x^6 + 48*c^3*x^9))/(576*c^3) + ((5*b^4 - 24*a*b^2*c + 16*a^2*c^2)*
Log[b + 2*c*x^3 - 2*Sqrt[c]*Sqrt[a + b*x^3 + c*x^6]])/(384*c^(7/2))
```

fricas [A] time = 1.22, size = 303, normalized size = 1.98

$$\left[\frac{3\left(5b^4-24ab^2c+16a^2c^2\right)\sqrt{c}\log\left(-8c^2x^6-8bcx^3-b^2+4\sqrt{cx^6+bx^3+a}\left(2cx^3+b\right)\sqrt{c}-4ac\right)+4\left(48c^4x^9+8bc^3x^6+15b^3c-52abc^2-2\left(5b^2c^2-12ac^3\right)x^3\right)\sqrt{cx^6+bx^3+a}}{2304c^4}, \frac{3\left(5b^4-24ab^2c+16a^2c^2\right)\sqrt{-c}\arctan\left(\frac{\sqrt{cx^6+bx^3+a}\left(2cx^3+b\right)\sqrt{-c}}{2\left(c^2x^6+bcx^3+ac\right)}\right)+2\left(48c^4x^9+8bc^3x^6+15b^3c-52abc^2-2\left(5b^2c^2-12ac^3\right)x^3\right)\sqrt{cx^6+bx^3+a}}{1152c^4}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(c*x^6+b*x^3+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2304*(3*(5*b^4 - 24*a*b^2*c + 16*a^2*c^2)*sqrt(c)*log(-8*c^2*x^6 - 8*b*c
*x^3 - b^2 + 4*sqrt(c*x^6 + b*x^3 + a)*(2*c*x^3 + b)*sqrt(c) - 4*a*c) + 4*(
48*c^4*x^9 + 8*b*c^3*x^6 + 15*b^3*c - 52*a*b*c^2 - 2*(5*b^2*c^2 - 12*a*c^3)
*x^3)*sqrt(c*x^6 + b*x^3 + a))/c^4, 1/1152*(3*(5*b^4 - 24*a*b^2*c + 16*a^2*
c^2)*sqrt(-c)*arctan(1/2*sqrt(c*x^6 + b*x^3 + a)*(2*c*x^3 + b)*sqrt(-c)/(c^
2*x^6 + b*c*x^3 + a*c)) + 2*(48*c^4*x^9 + 8*b*c^3*x^6 + 15*b^3*c - 52*a*b*c
^2 - 2*(5*b^2*c^2 - 12*a*c^3)*x^3)*sqrt(c*x^6 + b*x^3 + a))/c^4]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{cx^6 + bx^3 + a}\, x^8\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(c*x^6+b*x^3+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x^6 + b*x^3 + a)*x^8, x)
```

maple [F] time = 0.03, size = 0, normalized size = 0.00

$$\int \sqrt{c\,x^6 + b\,x^3 + a}\; x^8\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*(c*x^6+b*x^3+a)^(1/2),x)
```

```
[Out] int(x^8*(c*x^6+b*x^3+a)^(1/2),x)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(c*x^6+b*x^3+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive, negative or zero?
```

mupad [B] time = 1.59, size = 193, normalized size = 1.26

$$\frac{x^3\,{\left(c\,x^6+b\,x^3+a\right)}^{3/2}}{12\,c}-\frac{a\,\left(\left(\frac{b}{4\,c}+\frac{x^3}{2}\right)\,\sqrt{c\,x^6+b\,x^3+a}+\frac{\ln\left(\sqrt{c\,x^6+b\,x^3+a}+\frac{c\,x^3+\frac{b}{2}}{\sqrt{c}}\right)\,\left(a\,c-\frac{b^2}{4}\right)}{2\,c^{3/2}}\right)}{12\,c}-\frac{5\,b\,\left(\frac{\left(8\,c\,\left(c\,x^6+a\right)-3\,b^2+2\,b\,c\,x^3\right)\,\sqrt{c\,x^6+b\,x^3+a}}{24\,c^2}+\frac{\ln\left(2\,\sqrt{c\,x^6+b\,x^3+a}+\frac{2\,c\,x^3+b}{\sqrt{c}}\right)\,\left(b^3-4\,a\,b\,c\right)}{16\,c^{5/2}}\right)}{24\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*(a + b*x^3 + c*x^6)^(1/2),x)
```

```
[Out] (x^3*(a + b*x^3 + c*x^6)^(3/2))/(12*c) - (a*((b/(4*c) + x^3/2)*(a + b*x^3 +
 c*x^6)^(1/2) + (log((a + b*x^3 + c*x^6)^(1/2) + (b/2 + c*x^3)/c^(1/2))*(a*
c - b^2/4))/(2*c^(3/2))))/(12*c) - (5*b*(((8*c*(a + c*x^6) - 3*b^2 + 2*b*c*
x^3)*(a + b*x^3 + c*x^6)^(1/2))/(24*c^2) + (log(2*(a + b*x^3 + c*x^6)^(1/2)
 + (b + 2*c*x^3)/c^(1/2))*(b^3 - 4*a*b*c))/(16*c^(5/2))))/(24*c)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^{8} \sqrt{a + bx^{3} + cx^{6}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8*(c*x**6+b*x**3+a)**(1/2),x)
```

```
[Out] Integral(x**8*sqrt(a + b*x**3 + c*x**6), x)
```

3.175 $\int x^5\sqrt{a+bx^3+cx^6}\,dx$

Optimal. Leaf size=108

$$\frac{b\left(b^2-4ac\right)\tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{48c^{5/2}}-\frac{b\left(b+2cx^3\right)\sqrt{a+bx^3+cx^6}}{24c^2}+\frac{\left(a+bx^3+cx^6\right)^{3/2}}{9c}$$

Rubi [A] time = 0.09, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1357, 640, 612, 621, 206}

$$\frac{b\left(b^2-4ac\right)\tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{48c^{5/2}}-\frac{b\left(b+2cx^3\right)\sqrt{a+bx^3+cx^6}}{24c^2}+\frac{\left(a+bx^3+cx^6\right)^{3/2}}{9c}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*Sqrt[a + b*x^3 + c*x^6],x]
```

```
[Out] -(b*(b + 2*c*x^3)*Sqrt[a + b*x^3 + c*x^6])/(24*c^2) + (a + b*x^3 + c*x^6)^(
3/2)/(9*c) + (b*(b^2 - 4*a*c)*ArcTanh[(b + 2*c*x^3)/(2*Sqrt[c]*Sqrt[a + b*x
^3 + c*x^6])])/(48*c^(5/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1357

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\int x^5\sqrt{a+bx^3+cx^6}\,dx &= \frac{1}{3}\operatorname{Subst}\left(\int x\sqrt{a+bx+cx^2}\,dx,x,x^3\right)\\
&= \frac{\left(a+bx^3+cx^6\right)^{3/2}}{9c} - \frac{b\operatorname{Subst}\left(\int\sqrt{a+bx+cx^2}\,dx,x,x^3\right)}{6c}\\
&= -\frac{b\left(b+2cx^3\right)\sqrt{a+bx^3+cx^6}}{24c^2} + \frac{\left(a+bx^3+cx^6\right)^{3/2}}{9c} + \frac{\left(b\left(b^2-4ac\right)\right)\operatorname{Subst}\left(\int\frac{\quad}{\sqrt{a+b}}\right.}{48c^2}\\
&= -\frac{b\left(b+2cx^3\right)\sqrt{a+bx^3+cx^6}}{24c^2} + \frac{\left(a+bx^3+cx^6\right)^{3/2}}{9c} + \frac{\left(b\left(b^2-4ac\right)\right)\operatorname{Subst}\left(\int\frac{1}{4c-x^2}\right.}{24c^2}\\
&= -\frac{b\left(b+2cx^3\right)\sqrt{a+bx^3+cx^6}}{24c^2} + \frac{\left(a+bx^3+cx^6\right)^{3/2}}{9c} + \frac{b\left(b^2-4ac\right)\tanh^{-1}\left(\frac{b+2c}{2\sqrt{c}\sqrt{a+b}}\right.}{48c^{5/2}}
\end{aligned}
$$

Mathematica [A] time = 0.07, size = 99, normalized size = 0.92

$$
\frac{\left(b^3-4abc\right)\tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{48c^{5/2}}+\frac{\sqrt{a+bx^3+cx^6}\left(8c\left(a+cx^6\right)-3b^2+2bcx^3\right)}{72c^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*Sqrt[a + b*x^3 + c*x^6],x]
```

```
[Out] (Sqrt[a + b*x^3 + c*x^6]*(-3*b^2 + 2*b*c*x^3 + 8*c*(a + c*x^6)))/(72*c^2) +
 ((b^3 - 4*a*b*c)*ArcTanh[(b + 2*c*x^3)/(2*Sqrt[c]*Sqrt[a + b*x^3 + c*x^6])
])/(48*c^(5/2))
```

IntegrateAlgebraic [A] time = 0.32, size = 107, normalized size = 0.99

$$
\frac{\left(4abc-b^3\right)\log\left(-2c^{5/2}\sqrt{a+bx^3+cx^6}+bc^2+2c^3x^3\right)}{48c^{5/2}}+\frac{\sqrt{a+bx^3+cx^6}\left(8ac-3b^2+2bcx^3+8c^2x^6\right)}{72c^2}
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^5*Sqrt[a + b*x^3 + c*x^6],x]
```

```
[Out] (Sqrt[a + b*x^3 + c*x^6]*(-3*b^2 + 8*a*c + 2*b*c*x^3 + 8*c^2*x^6))/(72*c^2)
 + ((-b^3 + 4*a*b*c)*Log[b*c^2 + 2*c^3*x^3 - 2*c^(5/2)*Sqrt[a + b*x^3 + c*x
^6]])/(48*c^(5/2))
```

fricas [A] time = 1.22, size = 237, normalized size = 2.19

$$
\left[-\frac{3\left(b^3-4abc\right)\sqrt{c}\log\left(-8c^2x^6-8bcx^3-b^2+4\sqrt{cx^6+bx^3+a}\left(2cx^3+b\right)\sqrt{c}-4ac\right)-4\left(8c^3x^6+2bc^2x^3-3b^2c+8ac^2\right)\sqrt{cx^6+bx^3+a}}{288c^3}, -\frac{3\left(b^3-4abc\right)\sqrt{-c}\arctan\left(\frac{\sqrt{cx^6+bx^3+a}\left(2cx^3+b\right)\sqrt{-c}}{2\left(c^2x^6+bcx^3+ac\right)}\right)-2\left(8c^3x^6+2bc^2x^3-3b^2c+8ac^2\right)\sqrt{cx^6+bx^3+a}}{144c^3}\right]
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(c*x^6+b*x^3+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/288*(3*(b^3 - 4*a*b*c)*sqrt(c)*log(-8*c^2*x^6 - 8*b*c*x^3 - b^2 + 4*sqr
t(c*x^6 + b*x^3 + a)*(2*c*x^3 + b)*sqrt(c) - 4*a*c) - 4*(8*c^3*x^6 + 2*b*c^
2*x^3 - 3*b^2*c + 8*a*c^2)*sqrt(c*x^6 + b*x^3 + a))/c^3, -1/144*(3*(b^3 - 4
*a*b*c)*sqrt(-c)*arctan(1/2*sqrt(c*x^6 + b*x^3 + a)*(2*c*x^3 + b)*sqrt(-c)/
(c^2*x^6 + b*c*x^3 + a*c)) - 2*(8*c^3*x^6 + 2*b*c^2*x^3 - 3*b^2*c + 8*a*c^2
)*sqrt(c*x^6 + b*x^3 + a))/c^3]
```

giac [**A**] time = 0.49, size = 98, normalized size = 0.91

$$\frac{1}{72}\sqrt{cx^6+bx^3+a}\left(2\left(4x^3+\frac{b}{c}\right)x^3-\frac{3b^2-8ac}{c^2}\right)-\frac{\left(b^3-4abc\right)\log\left(\left|-2\left(\sqrt{c}x^3-\sqrt{cx^6+bx^3+a}\right)\sqrt{c}-b\right|\right)}{48c^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(c*x^6+b*x^3+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/72*sqrt(c*x^6 + b*x^3 + a)*(2*(4*x^3 + b/c)*x^3 - (3*b^2 - 8*a*c)/c^2) -
1/48*(b^3 - 4*a*b*c)*log(abs(-2*(sqrt(c)*x^3 - sqrt(c*x^6 + b*x^3 + a))*sqr
t(c) - b))/c^(5/2)
```

maple [**F**] time = 0.03, size = 0, normalized size = 0.00

$$\int \sqrt{c\,x^6+b\,x^3+a}\;x^5\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(c*x^6+b*x^3+a)^(1/2),x)
```

```
[Out] int(x^5*(c*x^6+b*x^3+a)^(1/2),x)
```

maxima [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(c*x^6+b*x^3+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive, negative or zero?
```

mupad [**B**] time = 1.39, size = 87, normalized size = 0.81

$$\frac{\left(8\,c\,\left(c\,x^6+a\right)-3\,b^2+2\,b\,c\,x^3\right)\,\sqrt{c\,x^6+b\,x^3+a}}{72\,c^2}+\frac{\ln\left(2\,\sqrt{c\,x^6+b\,x^3+a}+\frac{2\,c\,x^3+b}{\sqrt{c}}\right)\,\left(b^3-4\,a\,b\,c\right)}{48\,c^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a + b*x^3 + c*x^6)^(1/2),x)
```

```
[Out] ((8*c*(a + c*x^6) - 3*b^2 + 2*b*c*x^3)*(a + b*x^3 + c*x^6)^(1/2))/(72*c^2)
+ (log(2*(a + b*x^3 + c*x^6)^(1/2) + (b + 2*c*x^3)/c^(1/2))*(b^3 - 4*a*b*c)
)/(48*c^(5/2))
```

sympy [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int x^5\sqrt{a+bx^3+cx^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(c*x**6+b*x**3+a)**(1/2),x)
```

```
[Out] Integral(x**5*sqrt(a + b*x**3 + c*x**6), x)
```

3.176 $\int x^2\sqrt{a+bx^3+cx^6}\,dx$

Optimal. Leaf size=83

$$\frac{(b+2cx^3)\sqrt{a+bx^3+cx^6}}{12c}-\frac{(b^2-4ac)\tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{24c^{3/2}}$$

Rubi [A] time = 0.06, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1352, 612, 621, 206}

$$\frac{(b+2cx^3)\sqrt{a+bx^3+cx^6}}{12c}-\frac{(b^2-4ac)\tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{24c^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Sqrt[a + b*x^3 + c*x^6],x]
```

```
[Out] ((b + 2*c*x^3)*Sqrt[a + b*x^3 + c*x^6])/(12*c) - ((b^2 - 4*a*c)*ArcTanh[(b
+ 2*c*x^3)/(2*Sqrt[c]*Sqrt[a + b*x^3 + c*x^6])])/(24*c^(3/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1352

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a,
b, c, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

Rubi steps

$$\int x^2\sqrt{a+bx^3+cx^6}\,dx = \frac{1}{3}\text{Subst}\left(\int \sqrt{a+bx+cx^2}\,dx, x, x^3\right)$$
$$= \frac{\left(b+2cx^3\right)\sqrt{a+bx^3+cx^6}}{12c} - \frac{\left(b^2-4ac\right)\text{Subst}\left(\int \frac{1}{\sqrt{a+bx+cx^2}}\,dx, x, x^3\right)}{24c}$$
$$= \frac{\left(b+2cx^3\right)\sqrt{a+bx^3+cx^6}}{12c} - \frac{\left(b^2-4ac\right)\text{Subst}\left(\int \frac{1}{4c-x^2}\,dx, x, \frac{b+2cx^3}{\sqrt{a+bx^3+cx^6}}\right)}{12c}$$
$$= \frac{\left(b+2cx^3\right)\sqrt{a+bx^3+cx^6}}{12c} - \frac{\left(b^2-4ac\right)\tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{24c^{3/2}}$$

Mathematica [A] time = 0.01, size = 87, normalized size = 1.05

$$\frac{1}{3}\left(\frac{\left(b+2cx^3\right)\sqrt{a+bx^3+cx^6}}{4c} - \frac{\left(b^2-4ac\right)\tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{8c^{3/2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Sqrt[a + b*x^3 + c*x^6],x]
```

```
[Out] (((b + 2*c*x^3)*Sqrt[a + b*x^3 + c*x^6])/(4*c) - ((b^2 - 4*a*c)*ArcTanh[(b
+ 2*c*x^3)/(2*Sqrt[c]*Sqrt[a + b*x^3 + c*x^6])])/(8*c^(3/2)))/3
```

IntegrateAlgebraic [A] time = 0.25, size = 85, normalized size = 1.02

$$\frac{\left(b^2-4ac\right)\log\left(-2c^{3/2}\sqrt{a+bx^3+cx^6}+bc+2c^2x^3\right)}{24c^{3/2}} + \frac{\left(b+2cx^3\right)\sqrt{a+bx^3+cx^6}}{12c}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^2*Sqrt[a + b*x^3 + c*x^6],x]
```

```
[Out] ((b + 2*c*x^3)*Sqrt[a + b*x^3 + c*x^6])/(12*c) + ((b^2 - 4*a*c)*Log[b*c + 2
*c^2*x^3 - 2*c^(3/2)*Sqrt[a + b*x^3 + c*x^6]])/(24*c^(3/2))
```

fricas [A] time = 1.34, size = 197, normalized size = 2.37

$$\left[-\frac{\left(b^2-4\,ac\right)\sqrt{c}\log\left(-8\,c^2x^6-8\,bcx^3-b^2-4\,\sqrt{cx^6+bx^3+a}\left(2\,cx^3+b\right)\sqrt{c}-4\,ac\right)-4\,\sqrt{cx^6+bx^3+a}\left(2\,c^2x^3+bc\right)}{48\,c^2}, \frac{\left(b^2-4\,ac\right)\sqrt{-c}\arctan\left(\frac{\sqrt{cx^6+bx^3+a}\left(2\,cx^3+b\right)\sqrt{-c}}{2\left(c^2x^6+bcx^3+ac\right)}\right)+2\,\sqrt{cx^6+bx^3+a}\left(2\,c^2x^3+bc\right)}{24\,c^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^6+b*x^3+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/48*((b^2 - 4*a*c)*sqrt(c)*log(-8*c^2*x^6 - 8*b*c*x^3 - b^2 - 4*sqrt(c*x
^6 + b*x^3 + a)*(2*c*x^3 + b)*sqrt(c) - 4*a*c) - 4*sqrt(c*x^6 + b*x^3 + a)*
(2*c^2*x^3 + b*c))/c^2, 1/24*((b^2 - 4*a*c)*sqrt(-c)*arctan(1/2*sqrt(c*x^6
+ b*x^3 + a)*(2*c*x^3 + b)*sqrt(-c)/(c^2*x^6 + b*c*x^3 + a*c)) + 2*sqrt(c*x
^6 + b*x^3 + a)*(2*c^2*x^3 + b*c))/c^2]
```

giac [A] time = 0.56, size = 76, normalized size = 0.92

$$\frac{1}{12}\sqrt{cx^6+bx^3+a}\left(2\,x^3+\frac{b}{c}\right) + \frac{\left(b^2-4\,ac\right)\log\left(\left|-2\left(\sqrt{c}\,x^3-\sqrt{cx^6+bx^3+a}\right)\sqrt{c}-b\right|\right)}{24\,c^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^6+b*x^3+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/12*sqrt(c*x^6 + b*x^3 + a)*(2*x^3 + b/c) + 1/24*(b^2 - 4*a*c)*log(abs(-2*
(sqrt(c)*x^3 - sqrt(c*x^6 + b*x^3 + a))*sqrt(c) - b))/c^(3/2)
```

maple [F] time = 0.03, size = 0, normalized size = 0.00

$$\int \sqrt{c\,x^{6}+b\,x^{3}+a}\;x^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(c*x^6+b*x^3+a)^(1/2),x)
```

```
[Out] int(x^2*(c*x^6+b*x^3+a)^(1/2),x)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^6+b*x^3+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive, negative or zero?
```

mupad [B] time = 1.39, size = 72, normalized size = 0.87

$$\frac{\left(\frac{b}{4\,c}+\frac{x^{3}}{2}\right)\,\sqrt{c\,x^{6}+b\,x^{3}+a}}{3}+\frac{\ln\left(\sqrt{c\,x^{6}+b\,x^{3}+a}+\frac{c\,x^{3}+\frac{b}{2}}{\sqrt{c}}\right)\,\left(a\,c-\frac{b^{2}}{4}\right)}{6\,c^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*x^3 + c*x^6)^(1/2),x)
```

```
[Out] ((b/(4*c) + x^3/2)*(a + b*x^3 + c*x^6)^(1/2))/3 + (log((a + b*x^3 + c*x^6)^
(1/2) + (b/2 + c*x^3)/c^(1/2))*(a*c - b^2/4))/(6*c^(3/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^{2}\sqrt{a+bx^{3}+cx^{6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(c*x**6+b*x**3+a)**(1/2),x)
```

```
[Out] Integral(x**2*sqrt(a + b*x**3 + c*x**6), x)
```

3.177 $\int \frac{\sqrt{a+bx^3+cx^6}}{x}\,dx$

Optimal. Leaf size=109

$$\frac{1}{3}\sqrt{a+bx^3+cx^6}-\frac{1}{3}\sqrt{a}\tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)+\frac{b\tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{6\sqrt{c}}$$

Rubi [A] time = 0.11, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {1357, 734, 843, 621, 206, 724}

$$\frac{1}{3}\sqrt{a+bx^3+cx^6}-\frac{1}{3}\sqrt{a}\tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)+\frac{b\tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{6\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x^3 + c*x^6]/x,x]
```

```
[Out] Sqrt[a + b*x^3 + c*x^6]/3 - (Sqrt[a]*ArcTanh[(2*a + b*x^3)/(2*Sqrt[a]*Sqrt[
a + b*x^3 + c*x^6])])/3 + (b*ArcTanh[(b + 2*c*x^3)/(2*Sqrt[c]*Sqrt[a + b*x^
3 + c*x^6])])/(6*Sqrt[c])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 734

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 2*p + 1)), x
] - Dist[p/(e*(m + 2*p + 1)), Int[(d + e*x)^m*Simp[b*d - 2*a*e + (2*c*d - b
*e)*x, x]*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e
, 0] && GtQ[p, 0] && NeQ[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &
&  !ILtQ[m + 2*p, 0] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
```

```
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1357

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{a+bx^3+cx^6}}{x}\,dx &= \frac{1}{3}\,\text{Subst}\left(\int \frac{\sqrt{a+bx+cx^2}}{x}\,dx, x, x^3\right)\\ &= \frac{1}{3}\sqrt{a+bx^3+cx^6} - \frac{1}{6}\,\text{Subst}\left(\int \frac{-2a-bx}{x\sqrt{a+bx+cx^2}}\,dx, x, x^3\right)\\ &= \frac{1}{3}\sqrt{a+bx^3+cx^6} + \frac{1}{3}a\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx+cx^2}}\,dx, x, x^3\right) + \frac{1}{6}b\,\text{Subst}\left(\int \frac{1}{\sqrt{a+bx}}\right.\\ &= \frac{1}{3}\sqrt{a+bx^3+cx^6} - \frac{1}{3}(2a)\,\text{Subst}\left(\int \frac{1}{4a-x^2}\,dx, x, \frac{2a+bx^3}{\sqrt{a+bx^3+cx^6}}\right) + \frac{1}{3}b\,\text{Subst}\left(\int\right.\\ &= \frac{1}{3}\sqrt{a+bx^3+cx^6} - \frac{1}{3}\sqrt{a}\,\tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right) + \frac{b\tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{6\sqrt{c}}\end{aligned}$$

Mathematica [A] time = 0.04, size = 106, normalized size = 0.97

$$\frac{1}{6}\left(2\sqrt{a+bx^3+cx^6} - 2\sqrt{a}\,\tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right) + \frac{b\tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{\sqrt{c}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x^3 + c*x^6]/x,x]
```

```
[Out] (2*Sqrt[a + b*x^3 + c*x^6] - 2*Sqrt[a]*ArcTanh[(2*a + b*x^3)/(2*Sqrt[a]*Sqr
t[a + b*x^3 + c*x^6])] + (b*ArcTanh[(b + 2*c*x^3)/(2*Sqrt[c]*Sqrt[a + b*x^3
 + c*x^6])])/Sqrt[c])/6
```

IntegrateAlgebraic [A] time = 0.29, size = 111, normalized size = 1.02

$$\frac{1}{3}\sqrt{a+bx^3+cx^6} - \frac{b\log\left(-2\sqrt{c}\sqrt{a+bx^3+cx^6} + b + 2cx^3\right)}{6\sqrt{c}} + \frac{2}{3}\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{c}\,x^3}{\sqrt{a}} - \frac{\sqrt{a+bx^3+cx^6}}{\sqrt{a}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[a + b*x^3 + c*x^6]/x,x]
```

```
[Out] Sqrt[a + b*x^3 + c*x^6]/3 + (2*Sqrt[a]*ArcTanh[(Sqrt[c]*x^3)/Sqrt[a] - Sqrt
[a + b*x^3 + c*x^6]/Sqrt[a]])/3 - (b*Log[b + 2*c*x^3 - 2*Sqrt[c]*Sqrt[a + b
*x^3 + c*x^6]])/(6*Sqrt[c])
```

fricas [A] time = 1.37, size = 566, normalized size = 5.19

$$\left[\frac{b\sqrt{c}\log\left(-8c^2x^6 - 8bcx^3 - b^2 - 4\sqrt{cx^6+bx^3+a}\left(2cx^3+b\right)\sqrt{c} - 4ac\right) + 2\sqrt{a}c\log\left(-\frac{(b^2+4ac)x^6+8abx^3-4\sqrt{cx^6+bx^3+a}\left(bx^3+2a\right)\sqrt{a}+8a^2}{x^6}\right) + 4\sqrt{cx^6+bx^3+a}\,c}{12c}, -\frac{b\sqrt{-c}\arctan\left(\frac{\sqrt{cx^6+bx^3+a}\left(2cx^3+b\right)\sqrt{-c}}{2\left(c^2x^6+bcx^3+ac\right)}\right) - \sqrt{a}c\log\left(-\frac{(b^2+4ac)x^6+8abx^3-4\sqrt{cx^6+bx^3+a}\left(bx^3+2a\right)\sqrt{a}+8a^2}{x^6}\right) - 2\sqrt{cx^6+bx^3+a}\,c}{6c}, \frac{4\sqrt{-a}c\arctan\left(\frac{\sqrt{cx^6+bx^3+a}\left(bx^3+2a\right)\sqrt{-a}}{2\left(acx^6+abx^3+a^2\right)}\right) + b\sqrt{c}\log\left(-8c^2x^6 - 8bcx^3 - b^2 - 4\sqrt{cx^6+bx^3+a}\left(2cx^3+b\right)\sqrt{c} - 4ac\right) + 4\sqrt{cx^6+bx^3+a}\,c}{12c}, \frac{2\sqrt{-a}c\arctan\left(\frac{\sqrt{cx^6+bx^3+a}\left(bx^3+2a\right)\sqrt{-a}}{2\left(acx^6+abx^3+a^2\right)}\right) - b\sqrt{-c}\arctan\left(\frac{\sqrt{cx^6+bx^3+a}\left(2cx^3+b\right)\sqrt{-c}}{2\left(c^2x^6+bcx^3+ac\right)}\right) + 2\sqrt{cx^6+bx^3+a}\,c}{6c}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)^(1/2)/x,x, algorithm="fricas")
```

```
[Out] [1/12*(b*sqrt(c)*log(-8*c^2*x^6 - 8*b*c*x^3 - b^2 - 4*sqrt(c*x^6 + b*x^3 +
a)*(2*c*x^3 + b)*sqrt(c) - 4*a*c) + 2*sqrt(a)*c*log(-((b^2 + 4*a*c)*x^6 + 8
*a*b*x^3 - 4*sqrt(c*x^6 + b*x^3 + a)*(b*x^3 + 2*a)*sqrt(a) + 8*a^2)/x^6) +
4*sqrt(c*x^6 + b*x^3 + a)*c)/c, -1/6*(b*sqrt(-c)*arctan(1/2*sqrt(c*x^6 + b*
x^3 + a)*(2*c*x^3 + b)*sqrt(-c)/(c^2*x^6 + b*c*x^3 + a*c)) - sqrt(a)*c*log(
-((b^2 + 4*a*c)*x^6 + 8*a*b*x^3 - 4*sqrt(c*x^6 + b*x^3 + a)*(b*x^3 + 2*a)*s
qrt(a) + 8*a^2)/x^6) - 2*sqrt(c*x^6 + b*x^3 + a)*c)/c, 1/12*(4*sqrt(-a)*c*a
rctan(1/2*sqrt(c*x^6 + b*x^3 + a)*(b*x^3 + 2*a)*sqrt(-a)/(a*c*x^6 + a*b*x^3
 + a^2)) + b*sqrt(c)*log(-8*c^2*x^6 - 8*b*c*x^3 - b^2 - 4*sqrt(c*x^6 + b*x^
3 + a)*(2*c*x^3 + b)*sqrt(c) - 4*a*c) + 4*sqrt(c*x^6 + b*x^3 + a)*c)/c, 1/6
*(2*sqrt(-a)*c*arctan(1/2*sqrt(c*x^6 + b*x^3 + a)*(b*x^3 + 2*a)*sqrt(-a)/(a
*c*x^6 + a*b*x^3 + a^2)) - b*sqrt(-c)*arctan(1/2*sqrt(c*x^6 + b*x^3 + a)*(2
*c*x^3 + b)*sqrt(-c)/(c^2*x^6 + b*c*x^3 + a*c)) + 2*sqrt(c*x^6 + b*x^3 + a)
*c)/c]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cx^6 + bx^3 + a}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)^(1/2)/x,x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x^6 + b*x^3 + a)/x, x)
```

maple [F] time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c\,x^6 + b\,x^3 + a}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^6+b*x^3+a)^(1/2)/x,x)
```

```
[Out] int((c*x^6+b*x^3+a)^(1/2)/x,x)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)^(1/2)/x,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive, negative or zero?
```

mupad [B] time = 1.36, size = 88, normalized size = 0.81

$$\frac{\sqrt{c\,x^6 + b\,x^3 + a}}{3} - \frac{\sqrt{a}\,\ln\left(\frac{b}{2} + \frac{a}{x^3} + \frac{\sqrt{a}\,\sqrt{c\,x^6+b\,x^3+a}}{x^3}\right)}{3} + \frac{b\,\ln\left(\sqrt{c\,x^6 + b\,x^3 + a} + \frac{c\,x^3+\frac{b}{2}}{\sqrt{c}}\right)}{6\,\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3 + c*x^6)^(1/2)/x,x)
```

```
[Out] (a + b*x^3 + c*x^6)^(1/2)/3 - (a^(1/2)*log(b/2 + a/x^3 + (a^(1/2)*(a + b*x^
3 + c*x^6)^(1/2))/x^3))/3 + (b*log((a + b*x^3 + c*x^6)^(1/2) + (b/2 + c*x^3
)/c^(1/2)))/(6*c^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a + bx^3 + cx^6}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**6+b*x**3+a)**(1/2)/x,x)
```

```
[Out] Integral(sqrt(a + b*x**3 + c*x**6)/x, x)
```

3.178 $\int \frac{\sqrt{a+bx^3+cx^6}}{x^4} dx$

Optimal. Leaf size=112

$$-\frac{\sqrt{a+bx^3+cx^6}}{3x^3}-\frac{b\tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{6\sqrt{a}}+\frac{1}{3}\sqrt{c}\tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)$$

Rubi [A] time = 0.11, antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {1357, 732, 843, 621, 206, 724}

$$-\frac{\sqrt{a+bx^3+cx^6}}{3x^3}-\frac{b\tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{6\sqrt{a}}+\frac{1}{3}\sqrt{c}\tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x^3 + c*x^6]/x^4,x]
```

```
[Out] -Sqrt[a + b*x^3 + c*x^6]/(3*x^3) - (b*ArcTanh[(2*a + b*x^3)/(2*Sqrt[a]*Sqrt
[a + b*x^3 + c*x^6])])/(6*Sqrt[a]) + (Sqrt[c]*ArcTanh[(b + 2*c*x^3)/(2*Sqrt
[c]*Sqrt[a + b*x^3 + c*x^6])])/3
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 732

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 1)), x] - Di
st[p/(e*(m + 1)), Int[(d + e*x)^(m + 1)*(b + 2*c*x)*(a + b*x + c*x^2)^(p -
1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*
d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && GtQ[p, 0] && (IntegerQ[p]
 || LtQ[m, -1]) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && IntQuadraticQ[a,
 b, c, d, e, m, p, x]
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
```

```
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1357

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt{a+bx^3+cx^6}}{x^4}\,dx &= \frac{1}{3}\operatorname{Subst}\left(\int \frac{\sqrt{a+bx+cx^2}}{x^2}\,dx, x, x^3\right)\\
&= -\frac{\sqrt{a+bx^3+cx^6}}{3x^3} + \frac{1}{6}\operatorname{Subst}\left(\int \frac{b+2cx}{x\sqrt{a+bx+cx^2}}\,dx, x, x^3\right)\\
&= -\frac{\sqrt{a+bx^3+cx^6}}{3x^3} + \frac{1}{6}b\operatorname{Subst}\left(\int \frac{1}{x\sqrt{a+bx+cx^2}}\,dx, x, x^3\right) + \frac{1}{3}c\operatorname{Subst}\left(\int \frac{1}{\sqrt{a+bx}\cdots}\right.\\
&= -\frac{\sqrt{a+bx^3+cx^6}}{3x^3} - \frac{1}{3}b\operatorname{Subst}\left(\int \frac{1}{4a-x^2}\,dx, x, \frac{2a+bx^3}{\sqrt{a+bx^3+cx^6}}\right) + \frac{1}{3}(2c)\operatorname{Subst}\left(\int \cdots\right.\\
&= -\frac{\sqrt{a+bx^3+cx^6}}{3x^3} - \frac{b\tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{6\sqrt{a}} + \frac{1}{3}\sqrt{c}\tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)
\end{aligned}
$$

Mathematica [A] time = 0.05, size = 112, normalized size = 1.00

$$
-\frac{\sqrt{a+bx^3+cx^6}}{3x^3} - \frac{b\tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{6\sqrt{a}} + \frac{1}{3}\sqrt{c}\tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x^3 + c*x^6]/x^4,x]
```

```
[Out] -1/3*Sqrt[a + b*x^3 + c*x^6]/x^3 - (b*ArcTanh[(2*a + b*x^3)/(2*Sqrt[a]*Sqrt
[a + b*x^3 + c*x^6])])/(6*Sqrt[a]) + (Sqrt[c]*ArcTanh[(b + 2*c*x^3)/(2*Sqrt
[c]*Sqrt[a + b*x^3 + c*x^6])])/3
```

IntegrateAlgebraic [A] time = 0.30, size = 114, normalized size = 1.02

$$
-\frac{\sqrt{a+bx^3+cx^6}}{3x^3} - \frac{1}{3}\sqrt{c}\log\left(-2\sqrt{c}\sqrt{a+bx^3+cx^6}+b+2cx^3\right) + \frac{b\tanh^{-1}\left(\frac{\sqrt{c}x^3}{\sqrt{a}} - \frac{\sqrt{a+bx^3+cx^6}}{\sqrt{a}}\right)}{3\sqrt{a}}
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[a + b*x^3 + c*x^6]/x^4,x]
```

```
[Out] -1/3*Sqrt[a + b*x^3 + c*x^6]/x^3 + (b*ArcTanh[(Sqrt[c]*x^3)/Sqrt[a] - Sqrt[
a + b*x^3 + c*x^6]/Sqrt[a]])/(3*Sqrt[a]) - (Sqrt[c]*Log[b + 2*c*x^3 - 2*Sqr
t[c]*Sqrt[a + b*x^3 + c*x^6]])/3
```

fricas [A] time = 1.57, size = 601, normalized size = 5.37

$$
\left[\frac{2a\sqrt{c}x^3\log\left(-8c^2x^6-8bcx^3-b^2-4\sqrt{cx^6+bx^3+a}\left(2cx^3+b\right)\sqrt{c}-4ac\right)+\sqrt{a}bx^3\log\left(-\frac{\left(b^2+4ac\right)x^6+8abx^3-4\sqrt{cx^6+bx^3+a}\left(bx^3+2a\right)\sqrt{a}+8a^2}{x^6}\right)-4\sqrt{cx^6+bx^3+a}\,a}{12ax^3}, -\frac{4a\sqrt{-c}x^3\arctan\left(\frac{\sqrt{cx^6+bx^3+a}\left(2cx^3+b\right)\sqrt{-c}}{2\left(c^2x^6+bcx^3+ac\right)}\right)-\sqrt{a}bx^3\log\left(-\frac{\left(b^2+4ac\right)x^6+8abx^3-4\sqrt{cx^6+bx^3+a}\left(bx^3+2a\right)\sqrt{a}+8a^2}{x^6}\right)+4\sqrt{cx^6+bx^3+a}\,a}{12ax^3}, \frac{\sqrt{-a}bx^3\arctan\left(\frac{\sqrt{cx^6+bx^3+a}\left(bx^3+2a\right)\sqrt{-a}}{2\left(acx^6+abx^3+a^2\right)}\right)+a\sqrt{c}x^3\log\left(-8c^2x^6-8bcx^3-b^2-4\sqrt{cx^6+bx^3+a}\left(2cx^3+b\right)\sqrt{c}-4ac\right)-2\sqrt{cx^6+bx^3+a}\,a}{6ax^3}, \frac{\sqrt{-a}bx^3\arctan\left(\frac{\sqrt{cx^6+bx^3+a}\left(bx^3+2a\right)\sqrt{-a}}{2\left(acx^6+abx^3+a^2\right)}\right)-2a\sqrt{-c}x^3\arctan\left(\frac{\sqrt{cx^6+bx^3+a}\left(2cx^3+b\right)\sqrt{-c}}{2\left(c^2x^6+bcx^3+ac\right)}\right)-2\sqrt{cx^6+bx^3+a}\,a}{6ax^3}\right]
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)^(1/2)/x^4,x, algorithm="fricas")
```

```
[Out] [1/12*(2*a*sqrt(c)*x^3*log(-8*c^2*x^6 - 8*b*c*x^3 - b^2 - 4*sqrt(c*x^6 + b*
x^3 + a)*(2*c*x^3 + b)*sqrt(c) - 4*a*c) + sqrt(a)*b*x^3*log(-((b^2 + 4*a*c)
*x^6 + 8*a*b*x^3 - 4*sqrt(c*x^6 + b*x^3 + a)*(b*x^3 + 2*a)*sqrt(a) + 8*a^2)
/x^6) - 4*sqrt(c*x^6 + b*x^3 + a)*a)/(a*x^3), -1/12*(4*a*sqrt(-c)*x^3*arcta
n(1/2*sqrt(c*x^6 + b*x^3 + a)*(2*c*x^3 + b)*sqrt(-c)/(c^2*x^6 + b*c*x^3 + a
*c)) - sqrt(a)*b*x^3*log(-((b^2 + 4*a*c)*x^6 + 8*a*b*x^3 - 4*sqrt(c*x^6 + b
*x^3 + a)*(b*x^3 + 2*a)*sqrt(a) + 8*a^2)/x^6) + 4*sqrt(c*x^6 + b*x^3 + a)*a
)/(a*x^3), 1/6*(sqrt(-a)*b*x^3*arctan(1/2*sqrt(c*x^6 + b*x^3 + a)*(b*x^3 +
2*a)*sqrt(-a)/(a*c*x^6 + a*b*x^3 + a^2)) + a*sqrt(c)*x^3*log(-8*c^2*x^6 - 8
*b*c*x^3 - b^2 - 4*sqrt(c*x^6 + b*x^3 + a)*(2*c*x^3 + b)*sqrt(c) - 4*a*c) -
 2*sqrt(c*x^6 + b*x^3 + a)*a)/(a*x^3), 1/6*(sqrt(-a)*b*x^3*arctan(1/2*sqrt(
c*x^6 + b*x^3 + a)*(b*x^3 + 2*a)*sqrt(-a)/(a*c*x^6 + a*b*x^3 + a^2)) - 2*a*
sqrt(-c)*x^3*arctan(1/2*sqrt(c*x^6 + b*x^3 + a)*(2*c*x^3 + b)*sqrt(-c)/(c^2
*x^6 + b*c*x^3 + a*c)) - 2*sqrt(c*x^6 + b*x^3 + a)*a)/(a*x^3)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cx^6 + bx^3 + a}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)^(1/2)/x^4,x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x^6 + b*x^3 + a)/x^4, x)
```

maple [F] time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c\,x^6 + b\,x^3 + a}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^6+b*x^3+a)^(1/2)/x^4,x)
```

```
[Out] int((c*x^6+b*x^3+a)^(1/2)/x^4,x)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)^(1/2)/x^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive, negative or zero?
```

mupad [B] time = 1.55, size = 91, normalized size = 0.81

$$\frac{\sqrt{c}\,\ln\left(\sqrt{c\,x^6+b\,x^3+a}+\frac{c\,x^3+\frac{b}{2}}{\sqrt{c}}\right)}{3}-\frac{\sqrt{c\,x^6+b\,x^3+a}}{3\,x^3}-\frac{b\,\ln\left(\frac{b}{2}+\frac{a}{x^3}+\frac{\sqrt{a}\,\sqrt{c\,x^6+b\,x^3+a}}{x^3}\right)}{6\,\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3 + c*x^6)^(1/2)/x^4,x)
```

```
[Out] (c^(1/2)*log((a + b*x^3 + c*x^6)^(1/2) + (b/2 + c*x^3)/c^(1/2)))/3 - (a + b
*x^3 + c*x^6)^(1/2)/(3*x^3) - (b*log(b/2 + a/x^3 + (a^(1/2)*(a + b*x^3 + c*
x^6)^(1/2))/x^3))/(6*a^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a + bx^3 + cx^6}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**6+b*x**3+a)**(1/2)/x**4,x)
```

```
[Out] Integral(sqrt(a + b*x**3 + c*x**6)/x**4, x)
```

3.179 $\int \frac{\sqrt{a+bx^3+cx^6}}{x^7} \, dx$

Optimal. Leaf size=88

$$\frac{\left(b^2-4ac\right)\tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{24a^{3/2}}-\frac{\left(2a+bx^3\right)\sqrt{a+bx^3+cx^6}}{12ax^6}$$

Rubi [A] time = 0.07, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1357, 720, 724, 206}

$$\frac{\left(b^2-4ac\right)\tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{24a^{3/2}}-\frac{\left(2a+bx^3\right)\sqrt{a+bx^3+cx^6}}{12ax^6}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x^3 + c*x^6]/x^7,x]
```

```
[Out] -((2*a + b*x^3)*Sqrt[a + b*x^3 + c*x^6])/(12*a*x^6) + ((b^2 - 4*a*c)*ArcTan
h[(2*a + b*x^3)/(2*Sqrt[a]*Sqrt[a + b*x^3 + c*x^6])])/(24*a^(3/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 720

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> -Simp[((d + e*x)^(m + 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*(a + b*x
 + c*x^2)^p)/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[(p*(b^2 - 4*a*c
))/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 2)*(a + b*x + c*
x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] &&
 NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && EqQ[m + 2*p + 2, 0
] && GtQ[p, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1357

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{\sqrt{a+bx^3+cx^6}}{x^7}\,dx = \frac{1}{3}\,\text{Subst}\left(\int \frac{\sqrt{a+bx+cx^2}}{x^3}\,dx, x, x^3\right)$$
$$= -\frac{\left(2a+bx^3\right)\sqrt{a+bx^3+cx^6}}{12ax^6} - \frac{\left(b^2-4ac\right)\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx+cx^2}}\,dx, x, x^3\right)}{24a}$$
$$= -\frac{\left(2a+bx^3\right)\sqrt{a+bx^3+cx^6}}{12ax^6} + \frac{\left(b^2-4ac\right)\text{Subst}\left(\int \frac{1}{4a-x^2}\,dx, x, \frac{2a+bx^3}{\sqrt{a+bx^3+cx^6}}\right)}{12a}$$
$$= -\frac{\left(2a+bx^3\right)\sqrt{a+bx^3+cx^6}}{12ax^6} + \frac{\left(b^2-4ac\right)\tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{24a^{3/2}}$$

Mathematica [**A**] time = 0.04, size = 89, normalized size = 1.01

$$\frac{\left(b^2-4ac\right)\tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right) - \frac{2\sqrt{a}\left(2a+bx^3\right)\sqrt{a+bx^3+cx^6}}{x^6}}{24a^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x^3 + c*x^6]/x^7,x]
```

```
[Out] ((-2*Sqrt[a]*(2*a + b*x^3)*Sqrt[a + b*x^3 + c*x^6])/x^6 + (b^2 - 4*a*c)*Arc
Tanh[(2*a + b*x^3)/(2*Sqrt[a]*Sqrt[a + b*x^3 + c*x^6])])/(24*a^(3/2))
```

IntegrateAlgebraic [**A**] time = 0.38, size = 91, normalized size = 1.03

$$\frac{\left(4ac-b^2\right)\tanh^{-1}\left(\frac{\sqrt{c}\,x^3-\sqrt{a+bx^3+cx^6}}{\sqrt{a}}\right)}{12a^{3/2}} + \frac{\left(-2a-bx^3\right)\sqrt{a+bx^3+cx^6}}{12ax^6}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[a + b*x^3 + c*x^6]/x^7,x]
```

```
[Out] ((-2*a - b*x^3)*Sqrt[a + b*x^3 + c*x^6])/(12*a*x^6) + ((-b^2 + 4*a*c)*ArcTa
nh[(Sqrt[c]*x^3 - Sqrt[a + b*x^3 + c*x^6])/Sqrt[a]])/(12*a^(3/2))
```

fricas [**A**] time = 1.37, size = 215, normalized size = 2.44

$$\left[-\frac{\left(b^2-4\,ac\right)\sqrt{a}\,x^6\log\left(-\frac{\left(b^2+4\,ac\right)x^6+8\,abx^3-4\,\sqrt{cx^6+bx^3+a}\left(bx^3+2\,a\right)\sqrt{a}+8\,a^2}{x^6}\right)+4\,\sqrt{cx^6+bx^3+a}\left(abx^3+2\,a^2\right)}{48\,a^2x^6}, -\frac{\left(b^2-4\,ac\right)\sqrt{-a}\,x^6\arctan\left(\frac{\sqrt{cx^6+bx^3+a}\left(bx^3+2\,a\right)\sqrt{-a}}{2\left(acx^6+abx^3+a^2\right)}\right)+2\,\sqrt{cx^6+bx^3+a}\left(abx^3+2\,a^2\right)}{24\,a^2x^6}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)^(1/2)/x^7,x, algorithm="fricas")
```

```
[Out] [-1/48*((b^2 - 4*a*c)*sqrt(a)*x^6*log(-((b^2 + 4*a*c)*x^6 + 8*a*b*x^3 - 4*s
qrt(c*x^6 + b*x^3 + a)*(b*x^3 + 2*a)*sqrt(a) + 8*a^2)/x^6) + 4*sqrt(c*x^6 +
 b*x^3 + a)*(a*b*x^3 + 2*a^2))/(a^2*x^6), -1/24*((b^2 - 4*a*c)*sqrt(-a)*x^6
*arctan(1/2*sqrt(c*x^6 + b*x^3 + a)*(b*x^3 + 2*a)*sqrt(-a)/(a*c*x^6 + a*b*x
^3 + a^2)) + 2*sqrt(c*x^6 + b*x^3 + a)*(a*b*x^3 + 2*a^2))/(a^2*x^6)]
```

giac [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cx^6+bx^3+a}}{x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)^(1/2)/x^7,x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x^6 + b*x^3 + a)/x^7, x)
```

maple [F] time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c\,x^{6}+b\,x^{3}+a}}{x^{7}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^6+b*x^3+a)^(1/2)/x^7,x)
```

```
[Out] int((c*x^6+b*x^3+a)^(1/2)/x^7,x)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)^(1/2)/x^7,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive, negative or zero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c\,x^{6}+b\,x^{3}+a}}{x^{7}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3 + c*x^6)^(1/2)/x^7,x)
```

```
[Out] int((a + b*x^3 + c*x^6)^(1/2)/x^7, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a + bx^{3} + cx^{6}}}{x^{7}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**6+b*x**3+a)**(1/2)/x**7,x)
```

```
[Out] Integral(sqrt(a + b*x**3 + c*x**6)/x**7, x)
```

3.180 $\int \frac{\sqrt{a+bx^3+cx^6}}{x^{10}} \, dx$

Optimal. Leaf size=116

$$-\frac{b\left(b^2-4ac\right)\tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{48a^{5/2}}+\frac{b\left(2a+bx^3\right)\sqrt{a+bx^3+cx^6}}{24a^2x^6}-\frac{\left(a+bx^3+cx^6\right)^{3/2}}{9ax^9}$$

Rubi [A] time = 0.10, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1357, 730, 720, 724, 206}

$$-\frac{b\left(b^2-4ac\right)\tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{48a^{5/2}}+\frac{b\left(2a+bx^3\right)\sqrt{a+bx^3+cx^6}}{24a^2x^6}-\frac{\left(a+bx^3+cx^6\right)^{3/2}}{9ax^9}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x^3 + c*x^6]/x^10,x]
```

```
[Out] (b*(2*a + b*x^3)*Sqrt[a + b*x^3 + c*x^6])/(24*a^2*x^6) - (a + b*x^3 + c*x^6
)^(3/2)/(9*a*x^9) - (b*(b^2 - 4*a*c)*ArcTanh[(2*a + b*x^3)/(2*Sqrt[a]*Sqrt[
a + b*x^3 + c*x^6])])/(48*a^(5/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 720

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> -Simp[((d + e*x)^(m + 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*(a + b*x
 + c*x^2)^p)/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[(p*(b^2 - 4*a*c
))/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 2)*(a + b*x + c*
x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] &&
 NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && EqQ[m + 2*p + 2, 0
] && GtQ[p, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 730

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 - b*d*e + a*e^2)), x] + Dist[(2*c*d - b*e)/(2*(c*d^2 - b*d*e + a*e^2)),
 Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e,
 m, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2
*c*d - b*e, 0] && EqQ[m + 2*p + 3, 0]
```

Rule 1357

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
```

```
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{\sqrt{a+bx^3+cx^6}}{x^{10}}\,dx = \frac{1}{3}\,\text{Subst}\left(\int \frac{\sqrt{a+bx+cx^2}}{x^4}\,dx, x, x^3\right)$$

$$= -\frac{\left(a+bx^3+cx^6\right)^{3/2}}{9ax^9} - \frac{b\,\text{Subst}\left(\int \frac{\sqrt{a+bx+cx^2}}{x^3}\,dx, x, x^3\right)}{6a}$$

$$= \frac{b\left(2a+bx^3\right)\sqrt{a+bx^3+cx^6}}{24a^2x^6} - \frac{\left(a+bx^3+cx^6\right)^{3/2}}{9ax^9} + \frac{\left(b\left(b^2-4ac\right)\right)\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx+cx^2}}\right.}{48a^2}$$

$$= \frac{b\left(2a+bx^3\right)\sqrt{a+bx^3+cx^6}}{24a^2x^6} - \frac{\left(a+bx^3+cx^6\right)^{3/2}}{9ax^9} - \frac{\left(b\left(b^2-4ac\right)\right)\text{Subst}\left(\int \frac{1}{4a-x^2}\,dx, x,\right.}{24a^2}$$

$$= \frac{b\left(2a+bx^3\right)\sqrt{a+bx^3+cx^6}}{24a^2x^6} - \frac{\left(a+bx^3+cx^6\right)^{3/2}}{9ax^9} - \frac{b\left(b^2-4ac\right)\tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right.}{48a^{5/2}}$$

Mathematica [A] time = 0.07, size = 108, normalized size = 0.93

$$-\frac{b\left(b^2-4ac\right)\tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{48a^{5/2}} - \frac{\sqrt{a+bx^3+cx^6}\left(8a^2+2ax^3\left(b+4cx^3\right)-3b^2x^6\right)}{72a^2x^9}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x^3 + c*x^6]/x^10,x]
```

```
[Out] -1/72*(Sqrt[a + b*x^3 + c*x^6]*(8*a^2 - 3*b^2*x^6 + 2*a*x^3*(b + 4*c*x^3)))
/(a^2*x^9) - (b*(b^2 - 4*a*c)*ArcTanh[(2*a + b*x^3)/(2*Sqrt[a]*Sqrt[a + b*x
^3 + c*x^6])])/(48*a^(5/2))
```

IntegrateAlgebraic [A] time = 0.65, size = 108, normalized size = 0.93

$$\frac{\left(b^3-4abc\right)\tanh^{-1}\left(\frac{\sqrt{c}\,x^3-\sqrt{a+bx^3+cx^6}}{\sqrt{a}}\right)}{24a^{5/2}} + \frac{\sqrt{a+bx^3+cx^6}\left(-8a^2-2abx^3-8acx^6+3b^2x^6\right)}{72a^2x^9}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[a + b*x^3 + c*x^6]/x^10,x]
```

```
[Out] (Sqrt[a + b*x^3 + c*x^6]*(-8*a^2 - 2*a*b*x^3 + 3*b^2*x^6 - 8*a*c*x^6))/(72*
a^2*x^9) + ((b^3 - 4*a*b*c)*ArcTanh[(Sqrt[c]*x^3 - Sqrt[a + b*x^3 + c*x^6])
/Sqrt[a]])/(24*a^(5/2))
```

fricas [A] time = 1.36, size = 259, normalized size = 2.23

$$\left[-\frac{3\left(b^3-4\,abc\right)\sqrt{a}\,x^9\log\left(-\frac{\left(b^2+4\,ac\right)x^6+8\,abx^3+4\,\sqrt{cx^6+bx^3+a}\left(bx^3+2\,a\right)\sqrt{a}+8\,a^2}{x^6}\right)-4\left(\left(3\,ab^2-8\,a^2c\right)x^6-2\,a^2bx^3-8\,a^3\right)\sqrt{cx^6+bx^3+a}}{288\,a^3x^9}, \frac{3\left(b^3-4\,abc\right)\sqrt{-a}\,x^9\arctan\left(\frac{\sqrt{cx^6+bx^3+a}\left(bx^3+2\,a\right)\sqrt{-a}}{2\left(acx^6+abx^3+a^2\right)}\right)+2\left(\left(3\,ab^2-8\,a^2c\right)x^6-2\,a^2bx^3-8\,a^3\right)\sqrt{cx^6+bx^3+a}}{144\,a^3x^9}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)^(1/2)/x^10,x, algorithm="fricas")
```

```
[Out] [-1/288*(3*(b^3 - 4*a*b*c)*sqrt(a)*x^9*log(-((b^2 + 4*a*c)*x^6 + 8*a*b*x^3
+ 4*sqrt(c*x^6 + b*x^3 + a)*(b*x^3 + 2*a)*sqrt(a) + 8*a^2)/x^6) - 4*((3*a*b
^2 - 8*a^2*c)*x^6 - 2*a^2*b*x^3 - 8*a^3)*sqrt(c*x^6 + b*x^3 + a))/(a^3*x^9)
, 1/144*(3*(b^3 - 4*a*b*c)*sqrt(-a)*x^9*arctan(1/2*sqrt(c*x^6 + b*x^3 + a)*
(b*x^3 + 2*a)*sqrt(-a)/(a*c*x^6 + a*b*x^3 + a^2)) + 2*((3*a*b^2 - 8*a^2*c)*
x^6 - 2*a^2*b*x^3 - 8*a^3)*sqrt(c*x^6 + b*x^3 + a))/(a^3*x^9)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cx^6 + bx^3 + a}}{x^{10}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)^(1/2)/x^10,x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x^6 + b*x^3 + a)/x^10, x)
```

maple [F] time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c\,x^6 + b\,x^3 + a}}{x^{10}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^6+b*x^3+a)^(1/2)/x^10,x)
```

```
[Out] int((c*x^6+b*x^3+a)^(1/2)/x^10,x)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)^(1/2)/x^10,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive, negative or zero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c\,x^6 + b\,x^3 + a}}{x^{10}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3 + c*x^6)^(1/2)/x^10,x)
```

```
[Out] int((a + b*x^3 + c*x^6)^(1/2)/x^10, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a + bx^3 + cx^6}}{x^{10}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**6+b*x**3+a)**(1/2)/x**10,x)
```

```
[Out] Integral(sqrt(a + b*x**3 + c*x**6)/x**10, x)
```

3.181 $\int \frac{\sqrt{a+bx^3+cx^6}}{x^{13}}\,dx$

Optimal. Leaf size=161

$$\frac{(b^2-4ac)(5b^2-4ac)\tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{384a^{7/2}}-\frac{(5b^2-4ac)(2a+bx^3)\sqrt{a+bx^3+cx^6}}{192a^3x^6}+\frac{5b(a+bx^3+cx^6)^{3/2}}{72a^2x^9}-\cdots$$

Rubi [A] time = 0.15, antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {1357, 744, 806, 720, 724, 206}

$$-\frac{(5b^2-4ac)(2a+bx^3)\sqrt{a+bx^3+cx^6}}{192a^3x^6}+\frac{(b^2-4ac)(5b^2-4ac)\tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{384a^{7/2}}+\frac{5b(a+bx^3+cx^6)^{3/2}}{72a^2x^9}-\frac{(a+bx^3+cx^6)^{3/2}}{12ax^{12}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x^3 + c*x^6]/x^13,x]
```

```
[Out] -((5*b^2 - 4*a*c)*(2*a + b*x^3)*Sqrt[a + b*x^3 + c*x^6])/(192*a^3*x^6) - (a
 + b*x^3 + c*x^6)^(3/2)/(12*a*x^12) + (5*b*(a + b*x^3 + c*x^6)^(3/2))/(72*a
^2*x^9) + ((b^2 - 4*a*c)*(5*b^2 - 4*a*c)*ArcTanh[(2*a + b*x^3)/(2*Sqrt[a]*S
qrt[a + b*x^3 + c*x^6])])/(384*a^(7/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 720

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> -Simp[((d + e*x)^(m + 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*(a + b*x
 + c*x^2)^p)/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[(p*(b^2 - 4*a*c
))/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 2)*(a + b*x + c*
x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] &&
 NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && EqQ[m + 2*p + 2, 0
] && GtQ[p, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 744

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b*d*e + a*e^2)), Int[(
d + e*x)^(m + 1)*Simp[c*d*(m + 1) - b*e*(m + p + 2) - c*e*(m + 2*p + 3)*x,
x]*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^
2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && Ne
Q[m, -1] && ((LtQ[m, -1] && IntQuadraticQ[a, b, c, d, e, m, p, x]) || (SumS
implerQ[m, 1] && IntegerQ[p]) || ILtQ[Simplify[m + 2*p + 3], 0])
```

Rule 806

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b
*x + c*x^2)^(p + 1))/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[(b*(e*f
 + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m
+ 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &
& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m +
2*p + 3], 0]
```

Rule 1357

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{a+bx^3+cx^6}}{x^{13}}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{\sqrt{a+bx+cx^2}}{x^5}\,dx, x, x^3\right) \\
&= -\frac{\left(a+bx^3+cx^6\right)^{3/2}}{12ax^{12}} - \frac{\text{Subst}\left(\int \frac{\left(\frac{5b}{2}+cx\right)\sqrt{a+bx+cx^2}}{x^4}\,dx, x, x^3\right)}{12a} \\
&= -\frac{\left(a+bx^3+cx^6\right)^{3/2}}{12ax^{12}} + \frac{5b\left(a+bx^3+cx^6\right)^{3/2}}{72a^2x^9} + \frac{\left(5b^2-4ac\right)\text{Subst}\left(\int \frac{\sqrt{a+bx+cx^2}}{x^3}\,dx, x, \right.}{48a^2} \\
&= -\frac{\left(5b^2-4ac\right)\left(2a+bx^3\right)\sqrt{a+bx^3+cx^6}}{192a^3x^6} - \frac{\left(a+bx^3+cx^6\right)^{3/2}}{12ax^{12}} + \frac{5b\left(a+bx^3+cx^6\right)^{3/2}}{72a^2x^9} \\
&= -\frac{\left(5b^2-4ac\right)\left(2a+bx^3\right)\sqrt{a+bx^3+cx^6}}{192a^3x^6} - \frac{\left(a+bx^3+cx^6\right)^{3/2}}{12ax^{12}} + \frac{5b\left(a+bx^3+cx^6\right)^{3/2}}{72a^2x^9} \\
&= -\frac{\left(5b^2-4ac\right)\left(2a+bx^3\right)\sqrt{a+bx^3+cx^6}}{192a^3x^6} - \frac{\left(a+bx^3+cx^6\right)^{3/2}}{12ax^{12}} + \frac{5b\left(a+bx^3+cx^6\right)^{3/2}}{72a^2x^9}
\end{aligned}$$

Mathematica [A] time = 0.09, size = 139, normalized size = 0.86

$$\frac{\left(16a^2c^2-24ab^2c+5b^4\right)\tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{384a^{7/2}} - \frac{\sqrt{a+bx^3+cx^6}\left(48a^3+8a^2x^3\left(b+3cx^3\right)-2abx^6\left(5b+26cx^3\right)+15b^3x^9\right)}{576a^3x^{12}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x^3 + c*x^6]/x^13,x]
```

```
[Out] -1/576*(Sqrt[a + b*x^3 + c*x^6]*(48*a^3 + 15*b^3*x^9 + 8*a^2*x^3*(b + 3*c*x
^3) - 2*a*b*x^6*(5*b + 26*c*x^3)))/(a^3*x^12) + ((5*b^4 - 24*a*b^2*c + 16*a
^2*c^2)*ArcTanh[(2*a + b*x^3)/(2*Sqrt[a]*Sqrt[a + b*x^3 + c*x^6])])/(384*a^
(7/2))
```

IntegrateAlgebraic [A] time = 0.84, size = 141, normalized size = 0.88

$$\frac{\left(-16a^2c^2+24ab^2c-5b^4\right)\tanh^{-1}\left(\frac{\sqrt{c}x^3-\sqrt{a+bx^3+cx^6}}{\sqrt{a}}\right)}{192a^{7/2}} + \frac{\sqrt{a+bx^3+cx^6}\left(-48a^3-8a^2bx^3-24a^2cx^6+10ab^2x^6+52abcx^9-15b^3x^9\right)}{576a^3x^{12}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[a + b*x^3 + c*x^6]/x^13,x]
```

```
[Out] (Sqrt[a + b*x^3 + c*x^6]*(-48*a^3 - 8*a^2*b*x^3 + 10*a*b^2*x^6 - 24*a^2*c*x
^6 - 15*b^3*x^9 + 52*a*b*c*x^9))/(576*a^3*x^12) + ((-5*b^4 + 24*a*b^2*c - 1
6*a^2*c^2)*ArcTanh[(Sqrt[c]*x^3 - Sqrt[a + b*x^3 + c*x^6])/Sqrt[a]])/(192*a
^(7/2))
```

fricas [A] time = 1.40, size = 325, normalized size = 2.02

$$\left[\frac{3\left(5b^4-24ab^2c+16a^2c^2\right)\sqrt{a}x^{12}\log\left(-\frac{\left(b^2+4ac\right)x^6+8abx^3+4\sqrt{cx^6+bx^3+a}\left(bx^3+2a\right)\sqrt{a}+8a^2}{x^6}\right)-4\left(\left(15ab^3-52a^2bc\right)x^9+8a^3bx^3-2\left(5a^2b^2-12a^3c\right)x^6+48a^4\right)\sqrt{cx^6+bx^3+a}}{2304a^4x^{12}},-\frac{3\left(5b^4-24ab^2c+16a^2c^2\right)\sqrt{-a}x^{12}\arctan\left(\frac{\sqrt{cx^6+bx^3+a}\left(bx^3+2a\right)\sqrt{-a}}{2\left(acx^6+abx^3+a^2\right)}\right)+2\left(\left(15ab^3-52a^2bc\right)x^9+8a^3bx^3-2\left(5a^2b^2-12a^3c\right)x^6+48a^4\right)\sqrt{cx^6+bx^3+a}}{1152a^4x^{12}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)^(1/2)/x^13,x, algorithm="fricas")
```

```
[Out] [1/2304*(3*(5*b^4 - 24*a*b^2*c + 16*a^2*c^2)*sqrt(a)*x^12*log(-((b^2 + 4*a*
c)*x^6 + 8*a*b*x^3 + 4*sqrt(c*x^6 + b*x^3 + a)*(b*x^3 + 2*a)*sqrt(a) + 8*a^
2)/x^6) - 4*((15*a*b^3 - 52*a^2*b*c)*x^9 + 8*a^3*b*x^3 - 2*(5*a^2*b^2 - 12*
a^3*c)*x^6 + 48*a^4)*sqrt(c*x^6 + b*x^3 + a))/(a^4*x^12), -1/1152*(3*(5*b^4
 - 24*a*b^2*c + 16*a^2*c^2)*sqrt(-a)*x^12*arctan(1/2*sqrt(c*x^6 + b*x^3 + a
)*(b*x^3 + 2*a)*sqrt(-a)/(a*c*x^6 + a*b*x^3 + a^2)) + 2*((15*a*b^3 - 52*a^2
*b*c)*x^9 + 8*a^3*b*x^3 - 2*(5*a^2*b^2 - 12*a^3*c)*x^6 + 48*a^4)*sqrt(c*x^6
 + b*x^3 + a))/(a^4*x^12)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cx^6+bx^3+a}}{x^{13}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)^(1/2)/x^13,x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x^6 + b*x^3 + a)/x^13, x)
```

maple [F] time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c\,x^6+b\,x^3+a}}{x^{13}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^6+b*x^3+a)^(1/2)/x^13,x)
```

```
[Out] int((c*x^6+b*x^3+a)^(1/2)/x^13,x)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)^(1/2)/x^13,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive, negative or zero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c\,x^6+b\,x^3+a}}{x^{13}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3 + c*x^6)^(1/2)/x^13,x)
```

```
[Out] int((a + b*x^3 + c*x^6)^(1/2)/x^13, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a + bx^3 + cx^6}}{x^{13}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**6+b*x**3+a)**(1/2)/x**13,x)
```

```
[Out] Integral(sqrt(a + b*x**3 + c*x**6)/x**13, x)
```

3.182 $\int \frac{\sqrt{a+bx^3+cx^6}}{x^{16}}\,dx$

Optimal. Leaf size=199

$$-\frac{b\left(7b^2-12ac\right)\left(b^2-4ac\right)\tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{768a^{9/2}}+\frac{b\left(7b^2-12ac\right)\left(2a+bx^3\right)\sqrt{a+bx^3+cx^6}}{384a^4x^6}-\frac{\left(35b^2-32ac\right)\left(a\right.}{720a}$$

Rubi [A] time = 0.23, antiderivative size = 199, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.350, Rules used = {1357, 744, 834, 806, 720, 724, 206}

$$-\frac{\left(35b^2-32ac\right)\left(a+bx^3+cx^6\right)^{3/2}}{720a^3x^9}+\frac{b\left(7b^2-12ac\right)\left(2a+bx^3\right)\sqrt{a+bx^3+cx^6}}{384a^4x^6}-\frac{b\left(7b^2-12ac\right)\left(b^2-4ac\right)\tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{768a^{9/2}}+\frac{7b\left(a+bx^3+cx^6\right)^{3/2}}{120a^2x^{12}}-\frac{\left(a+bx^3+cx^6\right)^{3/2}}{15ax^{15}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x^3 + c*x^6]/x^16,x]
```

```
[Out] (b*(7*b^2 - 12*a*c)*(2*a + b*x^3)*Sqrt[a + b*x^3 + c*x^6])/(384*a^4*x^6) -
(a + b*x^3 + c*x^6)^(3/2)/(15*a*x^15) + (7*b*(a + b*x^3 + c*x^6)^(3/2))/(12
0*a^2*x^12) - ((35*b^2 - 32*a*c)*(a + b*x^3 + c*x^6)^(3/2))/(720*a^3*x^9) -
 (b*(7*b^2 - 12*a*c)*(b^2 - 4*a*c)*ArcTanh[(2*a + b*x^3)/(2*Sqrt[a]*Sqrt[a
+ b*x^3 + c*x^6])])/(768*a^(9/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 720

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> -Simp[((d + e*x)^(m + 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*(a + b*x
 + c*x^2)^p)/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[(p*(b^2 - 4*a*c
))/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 2)*(a + b*x + c*
x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] &&
 NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && EqQ[m + 2*p + 2, 0
] && GtQ[p, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 744

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b*d*e + a*e^2)), Int[(
d + e*x)^(m + 1)*Simp[c*d*(m + 1) - b*e*(m + p + 2) - c*e*(m + 2*p + 3)*x,
x]*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^
2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && Ne
Q[m, -1] && ((LtQ[m, -1] && IntQuadraticQ[a, b, c, d, e, m, p, x]) || (SumS
implerQ[m, 1] && IntegerQ[p]) || ILtQ[Simplify[m + 2*p + 3], 0])
```

Rule 806

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b
*x + c*x^2)^(p + 1))/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[(b*(e*f
 + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m
+ 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &
& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m +
2*p + 3], 0]
```

Rule 834

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b*
x + c*x^2)^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[(
c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m +
 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] ||
 IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 1357

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{\sqrt{a+bx^3+cx^6}}{x^{16}}\,dx = \frac{1}{3}\text{Subst}\left(\int \frac{\sqrt{a+bx+cx^2}}{x^6}\,dx, x, x^3\right)$$

$$= -\frac{\left(a+bx^3+cx^6\right)^{3/2}}{15ax^{15}} - \frac{\text{Subst}\left(\int \frac{\left(\frac{7b}{2}+2cx\right)\sqrt{a+bx+cx^2}}{x^5}\,dx, x, x^3\right)}{15a}$$

$$= -\frac{\left(a+bx^3+cx^6\right)^{3/2}}{15ax^{15}} + \frac{7b\left(a+bx^3+cx^6\right)^{3/2}}{120a^2x^{12}} + \frac{\text{Subst}\left(\int \frac{\left(\frac{1}{4}\left(35b^2-32ac\right)+\frac{7bcx}{2}\right)\sqrt{a+bx+cx^2}}{x^4}\,d\right.}{60a^2}$$

$$= -\frac{\left(a+bx^3+cx^6\right)^{3/2}}{15ax^{15}} + \frac{7b\left(a+bx^3+cx^6\right)^{3/2}}{120a^2x^{12}} - \frac{\left(35b^2-32ac\right)\left(a+bx^3+cx^6\right)^{3/2}}{720a^3x^9} - \frac{(b}{}$$

$$= \frac{b\left(7b^2-12ac\right)\left(2a+bx^3\right)\sqrt{a+bx^3+cx^6}}{384a^4x^6} - \frac{\left(a+bx^3+cx^6\right)^{3/2}}{15ax^{15}} + \frac{7b\left(a+bx^3+cx^6\right)^{3}}{120a^2x^{12}}$$

$$= \frac{b\left(7b^2-12ac\right)\left(2a+bx^3\right)\sqrt{a+bx^3+cx^6}}{384a^4x^6} - \frac{\left(a+bx^3+cx^6\right)^{3/2}}{15ax^{15}} + \frac{7b\left(a+bx^3+cx^6\right)^{3}}{120a^2x^{12}}$$

$$= \frac{b\left(7b^2-12ac\right)\left(2a+bx^3\right)\sqrt{a+bx^3+cx^6}}{384a^4x^6} - \frac{\left(a+bx^3+cx^6\right)^{3/2}}{15ax^{15}} + \frac{7b\left(a+bx^3+cx^6\right)^{3}}{120a^2x^{12}}$$

Mathematica [A] time = 0.12, size = 173, normalized size = 0.87

$$-\frac{b\left(48a^2c^2-40ab^2c+7b^4\right)\tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{768a^{9/2}} - \frac{\sqrt{a+bx^3+cx^6}\left(384a^4+16a^3\left(3bx^3+8cx^6\right)-8a^2x^6\left(7b^2+29bcx^3+32c^2x^6\right)+10ab^2x^9\left(7b+46cx^3\right)-105b^4x^{12}\right)}{5760a^4x^{15}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x^3 + c*x^6]/x^16,x]
```

```
[Out] -1/5760*(Sqrt[a + b*x^3 + c*x^6]*(384*a^4 - 105*b^4*x^12 + 10*a*b^2*x^9*(7*
b + 46*c*x^3) + 16*a^3*(3*b*x^3 + 8*c*x^6) - 8*a^2*x^6*(7*b^2 + 29*b*c*x^3
+ 32*c^2*x^6)))/(a^4*x^15) - (b*(7*b^4 - 40*a*b^2*c + 48*a^2*c^2)*ArcTanh[(
2*a + b*x^3)/(2*Sqrt[a]*Sqrt[a + b*x^3 + c*x^6])])/(768*a^(9/2))
```

IntegrateAlgebraic [A] time = 1.20, size = 176, normalized size = 0.88

$$\frac{\left(48a^2bc^2-40ab^3c+7b^5\right)\tanh^{-1}\left(\frac{\sqrt{c}x^3-\sqrt{a+bx^3+cx^6}}{\sqrt{a}}\right)}{384a^{9/2}}+\frac{\sqrt{a+bx^3+cx^6}\left(-384a^4-48a^3bx^3-128a^3cx^6+56a^2b^2x^6+232a^2bcx^9+256a^2c^2x^{12}-70ab^3x^9-460ab^2cx^{12}+105b^4x^{12}\right)}{5760a^4x^{15}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[a + b*x^3 + c*x^6]/x^16,x]
```

```
[Out] (Sqrt[a + b*x^3 + c*x^6]*(-384*a^4 - 48*a^3*b*x^3 + 56*a^2*b^2*x^6 - 128*a^
3*c*x^6 - 70*a*b^3*x^9 + 232*a^2*b*c*x^9 + 105*b^4*x^12 - 460*a*b^2*c*x^12
+ 256*a^2*c^2*x^12))/(5760*a^4*x^15) + ((7*b^5 - 40*a*b^3*c + 48*a^2*b*c^2)
*ArcTanh[(Sqrt[c]*x^3 - Sqrt[a + b*x^3 + c*x^6])/Sqrt[a]])/(384*a^(9/2))
```

fricas [A] time = 1.52, size = 389, normalized size = 1.95

$$\left[\frac{15\left(7b^5-40ab^3c+48a^2bc^2\right)\sqrt{a}x^{15}\log\left(-\frac{\left(b^2+4ac\right)x^6+8abx^3-4\sqrt{cx^6+bx^3+a}\left(bx^3+2a\right)\sqrt{a}+8a^2}{x^6}\right)+4\left(\left(105ab^4-460a^2b^2c+256a^3c^2\right)x^{12}-2\left(35a^2b^3-116a^3bc\right)x^9-48a^4bx^3+8\left(7a^3b^2-16a^4c\right)x^6-384a^5\right)\sqrt{cx^6+bx^3+a}}{23040a^5x^{15}},\ \frac{15\left(7b^5-40ab^3c+48a^2bc^2\right)\sqrt{-a}x^{15}\arctan\left(\frac{\sqrt{cx^6+bx^3+a}\left(bx^3+2a\right)\sqrt{-a}}{2\left(acx^6+abx^3+a^2\right)}\right)+2\left(\left(105ab^4-460a^2b^2c+256a^3c^2\right)x^{12}-2\left(35a^2b^3-116a^3bc\right)x^9-48a^4bx^3+8\left(7a^3b^2-16a^4c\right)x^6-384a^5\right)\sqrt{cx^6+bx^3+a}}{11520a^5x^{15}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)^(1/2)/x^16,x, algorithm="fricas")
```

```
[Out] [1/23040*(15*(7*b^5 - 40*a*b^3*c + 48*a^2*b*c^2)*sqrt(a)*x^15*log(-((b^2 +
4*a*c)*x^6 + 8*a*b*x^3 - 4*sqrt(c*x^6 + b*x^3 + a)*(b*x^3 + 2*a)*sqrt(a) +
8*a^2)/x^6) + 4*((105*a*b^4 - 460*a^2*b^2*c + 256*a^3*c^2)*x^12 - 2*(35*a^2
*b^3 - 116*a^3*b*c)*x^9 - 48*a^4*b*x^3 + 8*(7*a^3*b^2 - 16*a^4*c)*x^6 - 384
*a^5)*sqrt(c*x^6 + b*x^3 + a))/(a^5*x^15), 1/11520*(15*(7*b^5 - 40*a*b^3*c
+ 48*a^2*b*c^2)*sqrt(-a)*x^15*arctan(1/2*sqrt(c*x^6 + b*x^3 + a)*(b*x^3 + 2
*a)*sqrt(-a)/(a*c*x^6 + a*b*x^3 + a^2)) + 2*((105*a*b^4 - 460*a^2*b^2*c + 2
56*a^3*c^2)*x^12 - 2*(35*a^2*b^3 - 116*a^3*b*c)*x^9 - 48*a^4*b*x^3 + 8*(7*a
^3*b^2 - 16*a^4*c)*x^6 - 384*a^5)*sqrt(c*x^6 + b*x^3 + a))/(a^5*x^15)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cx^6+bx^3+a}}{x^{16}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)^(1/2)/x^16,x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x^6 + b*x^3 + a)/x^16, x)
```

maple [F] time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c\,x^6+b\,x^3+a}}{x^{16}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^6+b*x^3+a)^(1/2)/x^16,x)
```

```
[Out] int((c*x^6+b*x^3+a)^(1/2)/x^16,x)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)^(1/2)/x^16,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive, negative or zero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c\,x^6+b\,x^3+a}}{x^{16}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3 + c*x^6)^(1/2)/x^16,x)
```

```
[Out] int((a + b*x^3 + c*x^6)^(1/2)/x^16, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a+bx^3+cx^6}}{x^{16}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**6+b*x**3+a)**(1/2)/x**16,x)
```

```
[Out] Integral(sqrt(a + b*x**3 + c*x**6)/x**16, x)
```

3.183 $\int x^{14} \left(a+bx^3+cx^6\right)^{3/2} \, dx$

Optimal. Leaf size=293

$$\frac{\left(b^2-4ac\right)^2 \left(16a^2c^2-72ab^2c+33b^4\right) \tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{32768c^{13/2}}-\frac{\left(b^2-4ac\right)\left(16a^2c^2-72ab^2c+33b^4\right)\left(b+2cx^3\right)\sqrt{}}{16384c^6}$$

Rubi [A] time = 0.40, antiderivative size = 293, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.350$, Rules used = {1357, 742, 832, 779, 612, 621, 206}

$$\frac{\left(16a^2c^2-72ab^2c+33b^4\right)\left(b+2cx^3\right)\left(a+bx^3+cx^6\right)^{3/2}}{6144c^5}-\frac{\left(b^2-4ac\right)\left(16a^2c^2-72ab^2c+33b^4\right)\left(b+2cx^3\right)\sqrt{a+bx^3+cx^6}}{16384c^6}+\frac{\left(b^2-4ac\right)^2\left(16a^2c^2-72ab^2c+33b^4\right)\tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{32768c^{13/2}}-\frac{\left(3b\left(77b^2-124ac\right)-10cx^3\left(33b^2-28ac\right)\right)\left(a+bx^3+cx^6\right)^{5/2}}{13440c^4}-\frac{11bx^6\left(a+bx^3+cx^6\right)^{5/2}}{336c^2}+\frac{x^9\left(a+bx^3+cx^6\right)^{5/2}}{24c}$$

Antiderivative was successfully verified.

```
[In] Int[x^14*(a + b*x^3 + c*x^6)^(3/2),x]
```

```
[Out] -((b^2 - 4*a*c)*(33*b^4 - 72*a*b^2*c + 16*a^2*c^2)*(b + 2*c*x^3)*Sqrt[a + b
*x^3 + c*x^6])/(16384*c^6) + ((33*b^4 - 72*a*b^2*c + 16*a^2*c^2)*(b + 2*c*x
^3)*(a + b*x^3 + c*x^6)^(3/2))/(6144*c^5) - (11*b*x^6*(a + b*x^3 + c*x^6)^(
5/2))/(336*c^2) + (x^9*(a + b*x^3 + c*x^6)^(5/2))/(24*c) - ((3*b*(77*b^2 -
124*a*c) - 10*c*(33*b^2 - 28*a*c)*x^3)*(a + b*x^3 + c*x^6)^(5/2))/(13440*c^
4) + ((b^2 - 4*a*c)^2*(33*b^4 - 72*a*b^2*c + 16*a^2*c^2)*ArcTanh[(b + 2*c*x
^3)/(2*Sqrt[c]*Sqrt[a + b*x^3 + c*x^6])])/(32768*c^(13/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 742

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(m + 2*p
+ 1)), x] + Dist[1/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m +
 2*p + 1) - e*(a*e*(m - 1) + b*d*(p + 1)) + e*(2*c*d - b*e)*(m + p)*x, x]*(
a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && If[Rat
ionalQ[m], GtQ[m, 1], SumSimplerQ[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuad
raticQ[a, b, c, d, e, m, p, x]
```

Rule 779

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> -Simp[((b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
```

```
2*c*e*g*(p + 1)*x)*(a + b*x + c*x^2)^(p + 1))/(2*c^2*(p + 1)*(2*p + 3)), x
] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p +
3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 832

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + b*x + c*x^2)^(p +
1))/(c*(m + 2*p + 2)), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)
*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m*
(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p]
 || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 1357

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int x^{14}\left(a+bx^3+cx^6\right)^{3/2}\,dx=\frac{1}{3}\,\text{Subst}\left(\int x^4\left(a+bx+cx^2\right)^{3/2}\,dx,x,x^3\right)$$

$$=\frac{x^9\left(a+bx^3+cx^6\right)^{5/2}}{24c}+\frac{\text{Subst}\left(\int x^2\left(-3a-\frac{11bx}{2}\right)\left(a+bx+cx^2\right)^{3/2}\,dx,x,x^3\right)}{24c}$$

$$=-\frac{11bx^6\left(a+bx^3+cx^6\right)^{5/2}}{336c^2}+\frac{x^9\left(a+bx^3+cx^6\right)^{5/2}}{24c}+\frac{\text{Subst}\left(\int x\left(11ab+\frac{3}{4}\left(33b\right.\right.\right.}{}$$

$$=-\frac{11bx^6\left(a+bx^3+cx^6\right)^{5/2}}{336c^2}+\frac{x^9\left(a+bx^3+cx^6\right)^{5/2}}{24c}-\frac{\left(3b\left(77b^2-124ac\right)-10c\left(\right.\right.}{}$$

$$=\frac{\left(33b^4-72ab^2c+16a^2c^2\right)\left(b+2cx^3\right)\left(a+bx^3+cx^6\right)^{3/2}}{6144c^5}-\frac{11bx^6\left(a+bx^3+cx^6\right)}{336c^2}$$

$$=-\frac{\left(b^2-4ac\right)\left(33b^4-72ab^2c+16a^2c^2\right)\left(b+2cx^3\right)\sqrt{a+bx^3+cx^6}}{16384c^6}+\frac{\left(33b^4-72\right.}{}$$

$$=-\frac{\left(b^2-4ac\right)\left(33b^4-72ab^2c+16a^2c^2\right)\left(b+2cx^3\right)\sqrt{a+bx^3+cx^6}}{16384c^6}+\frac{\left(33b^4-72\right.}{}$$

$$=-\frac{\left(b^2-4ac\right)\left(33b^4-72ab^2c+16a^2c^2\right)\left(b+2cx^3\right)\sqrt{a+bx^3+cx^6}}{16384c^6}+\frac{\left(33b^4-72\right.}{}$$

Mathematica [A] time = 0.35, size = 241, normalized size = 0.82

$$\frac{\frac{\left(16a^2c^2-72ab^2c+33b^4\right)\left(2\sqrt{c}\left(b+2cx^3\right)\sqrt{a+bx^3+cx^6}\left(4c\left(5a+2cx^6\right)-3b^2+8bcx^3\right)+3\left(b^2-4ac\right)^2\tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)\right)}{4096c^{11/2}}+\frac{\left(372abc-280ac^2x^3-231b^3+330b^2cx^3\right)\left(a+bx^3+cx^6\right)^{5/2}}{560c^3}-\frac{11bx^6\left(a+bx^3+cx^6\right)^{5/2}}{14c}+x^9\left(a+bx^3+cx^6\right)^{5/2}}{24c}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^14*(a + b*x^3 + c*x^6)^(3/2),x]
```

[Out] ((-11*b*x^6*(a + b*x^3 + c*x^6)^(5/2))/(14*c) + x^9*(a + b*x^3 + c*x^6)^(5/
2) + ((-231*b^3 + 372*a*b*c + 330*b^2*c*x^3 - 280*a*c^2*x^3)*(a + b*x^3 + c
*x^6)^(5/2))/(560*c^3) + ((33*b^4 - 72*a*b^2*c + 16*a^2*c^2)*(2*Sqrt[c]*(b
+ 2*c*x^3)*Sqrt[a + b*x^3 + c*x^6]*(-3*b^2 + 8*b*c*x^3 + 4*c*(5*a + 2*c*x^6
)) + 3*(b^2 - 4*a*c)^2*ArcTanh[(b + 2*c*x^3)/(2*Sqrt[c]*Sqrt[a + b*x^3 + c*
x^6])]))/(4096*c^(11/2)))/(24*c)

IntegrateAlgebraic [A] time = 1.40, size = 314, normalized size = 1.07

$$\frac{\sqrt{a+bx^3+cx^6}\left(58816a^3bc^3-13440a^3c^4x^3-81648a^2b^3c^2+37792a^2b^2c^3x^3-19328a^2bc^4x^6+8960a^2c^5x^9+30660ab^5c-17976ab^4c^2x^3+12480ab^3c^3x^6-9088ab^2c^4x^9+6656abc^5x^{12}+107520ac^6x^{15}-3465b^7+2310b^6cx^3-1848b^5c^2x^6+1584b^4c^3x^9-1408b^3c^4x^{12}+1280b^2c^5x^{15}+87040bc^6x^{18}+71680c^7x^{21}\right)}{1720320c^6}+\frac{\left(-256a^4c^4+1280a^3b^2c^3-1120a^2b^4c^2+336ab^6c-33b^8\right)\log\left(-2\sqrt{c}\sqrt{a+bx^3+cx^6}+b+2cx^3\right)}{32768c^{13/2}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^14*(a + b*x^3 + c*x^6)^(3/2),x]

[Out] (Sqrt[a + b*x^3 + c*x^6]*(-3465*b^7 + 30660*a*b^5*c - 81648*a^2*b^3*c^2 + 5
8816*a^3*b*c^3 + 2310*b^6*c*x^3 - 17976*a*b^4*c^2*x^3 + 37792*a^2*b^2*c^3*x
^3 - 13440*a^3*c^4*x^3 - 1848*b^5*c^2*x^6 + 12480*a*b^3*c^3*x^6 - 19328*a^2
*b*c^4*x^6 + 1584*b^4*c^3*x^9 - 9088*a*b^2*c^4*x^9 + 8960*a^2*c^5*x^9 - 140
8*b^3*c^4*x^12 + 6656*a*b*c^5*x^12 + 1280*b^2*c^5*x^15 + 107520*a*c^6*x^15
+ 87040*b*c^6*x^18 + 71680*c^7*x^21))/(1720320*c^6) + ((-33*b^8 + 336*a*b^6
*c - 1120*a^2*b^4*c^2 + 1280*a^3*b^2*c^3 - 256*a^4*c^4)*Log[b + 2*c*x^3 - 2
*Sqrt[c]*Sqrt[a + b*x^3 + c*x^6]])/(32768*c^(13/2))

fricas [A] time = 1.56, size = 641, normalized size = 2.19

$$\left[\frac{105\left(33b^8-336ab^6c+1120a^2b^4c^2-1280a^3b^2c^3+256a^4c^4\right)\sqrt{c}\log\left(-8c^2x^6-8bcx^3-b^2-4\sqrt{cx^6+bx^3+a}\left(2cx^3+b\right)\sqrt{c}-4ac\right)+4\left(71680c^8x^{21}+87040bc^7x^{18}+1280\left(b^2c^6+84ac^7\right)x^{15}-128\left(11b^3c^5-52abc^6\right)x^{12}+16\left(99b^4c^4-568ab^2c^5+560a^2c^6\right)x^9-3465b^7c+30660ab^5c^2-81648a^2b^3c^3+58816a^3bc^4-8\left(231b^5c^3-1560ab^3c^4+2416a^2bc^5\right)x^6+2\left(1155b^6c^2-8988ab^4c^3+18896a^2b^2c^4-6720a^3c^5\right)x^3\right)\sqrt{cx^6+bx^3+a}}{6881280c^7}, -\frac{105\left(33b^8-336ab^6c+1120a^2b^4c^2-1280a^3b^2c^3+256a^4c^4\right)\sqrt{-c}\arctan\left(\frac{\sqrt{cx^6+bx^3+a}\left(2cx^3+b\right)\sqrt{-c}}{2\left(c^2x^6+bcx^3+ac\right)}\right)-2\left(71680c^8x^{21}+87040bc^7x^{18}+1280\left(b^2c^6+84ac^7\right)x^{15}-128\left(11b^3c^5-52abc^6\right)x^{12}+16\left(99b^4c^4-568ab^2c^5+560a^2c^6\right)x^9-3465b^7c+30660ab^5c^2-81648a^2b^3c^3+58816a^3bc^4-8\left(231b^5c^3-1560ab^3c^4+2416a^2bc^5\right)x^6+2\left(1155b^6c^2-8988ab^4c^3+18896a^2b^2c^4-6720a^3c^5\right)x^3\right)\sqrt{cx^6+bx^3+a}}{3440640c^7}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^14*(c*x^6+b*x^3+a)^(3/2),x, algorithm="fricas")

[Out] [1/6881280*(105*(33*b^8 - 336*a*b^6*c + 1120*a^2*b^4*c^2 - 1280*a^3*b^2*c^3
 + 256*a^4*c^4)*sqrt(c)*log(-8*c^2*x^6 - 8*b*c*x^3 - b^2 - 4*sqrt(c*x^6 + b
x^3 + a)(2*c*x^3 + b)*sqrt(c) - 4*a*c) + 4*(71680*c^8*x^21 + 87040*b*c^7*
x^18 + 1280*(b^2*c^6 + 84*a*c^7)*x^15 - 128*(11*b^3*c^5 - 52*a*b*c^6)*x^12
+ 16*(99*b^4*c^4 - 568*a*b^2*c^5 + 560*a^2*c^6)*x^9 - 3465*b^7*c + 30660*a*
b^5*c^2 - 81648*a^2*b^3*c^3 + 58816*a^3*b*c^4 - 8*(231*b^5*c^3 - 1560*a*b^3
*c^4 + 2416*a^2*b*c^5)*x^6 + 2*(1155*b^6*c^2 - 8988*a*b^4*c^3 + 18896*a^2*b
^2*c^4 - 6720*a^3*c^5)*x^3)*sqrt(c*x^6 + b*x^3 + a))/c^7, -1/3440640*(105*(
33*b^8 - 336*a*b^6*c + 1120*a^2*b^4*c^2 - 1280*a^3*b^2*c^3 + 256*a^4*c^4)*s
qrt(-c)*arctan(1/2*sqrt(c*x^6 + b*x^3 + a)*(2*c*x^3 + b)*sqrt(-c)/(c^2*x^6
+ b*c*x^3 + a*c)) - 2*(71680*c^8*x^21 + 87040*b*c^7*x^18 + 1280*(b^2*c^6 +
84*a*c^7)*x^15 - 128*(11*b^3*c^5 - 52*a*b*c^6)*x^12 + 16*(99*b^4*c^4 - 568*
a*b^2*c^5 + 560*a^2*c^6)*x^9 - 3465*b^7*c + 30660*a*b^5*c^2 - 81648*a^2*b^3
*c^3 + 58816*a^3*b*c^4 - 8*(231*b^5*c^3 - 1560*a*b^3*c^4 + 2416*a^2*b*c^5)*
x^6 + 2*(1155*b^6*c^2 - 8988*a*b^4*c^3 + 18896*a^2*b^2*c^4 - 6720*a^3*c^5)*
x^3)*sqrt(c*x^6 + b*x^3 + a))/c^7]

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(cx^6+bx^3+a\right)^{\frac{3}{2}}x^{14}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^14*(c*x^6+b*x^3+a)^(3/2),x, algorithm="giac")

[Out] integrate((c*x^6 + b*x^3 + a)^(3/2)*x^14, x)

maple [F] time = 0.05, size = 0, normalized size = 0.00

$$\int \left(c\,x^6+b\,x^3+a\right)^{\frac{3}{2}}x^{14}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^14*(c*x^6+b*x^3+a)^(3/2),x)
```

```
[Out] int(x^14*(c*x^6+b*x^3+a)^(3/2),x)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^14*(c*x^6+b*x^3+a)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive, negative or zero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^{14} \left(c x^6 + b x^3 + a\right)^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^14*(a + b*x^3 + c*x^6)^(3/2),x)
```

```
[Out] int(x^14*(a + b*x^3 + c*x^6)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^{14} \left(a + bx^3 + cx^6\right)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**14*(c*x**6+b*x**3+a)**(3/2),x)
```

```
[Out] Integral(x**14*(a + b*x**3 + c*x**6)**(3/2), x)
```

3.184 $\int x^{11} \left(a + bx^3 + cx^6\right)^{3/2} \, dx$

Optimal. Leaf size=223

$$-\frac{b\left(b^2-4ac\right)^2\left(3b^2-4ac\right)\tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{2048c^{11/2}}+\frac{b\left(b^2-4ac\right)\left(3b^2-4ac\right)\left(b+2cx^3\right)\sqrt{a+bx^3+cx^6}}{1024c^5}-\frac{b\left(3b^2-\right.}{}$$

Rubi [A] time = 0.21, antiderivative size = 223, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {1357, 742, 779, 612, 621, 206}

$$\frac{\left(-16ac+21b^2-30bcx^3\right)\left(a+bx^3+cx^6\right)^{5/2}}{840c^3}-\frac{b\left(3b^2-4ac\right)\left(b+2cx^3\right)\left(a+bx^3+cx^6\right)^{3/2}}{384c^4}+\frac{b\left(b^2-4ac\right)\left(3b^2-4ac\right)\left(b+2cx^3\right)\sqrt{a+bx^3+cx^6}}{1024c^5}-\frac{b\left(b^2-4ac\right)^2\left(3b^2-4ac\right)\tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{2048c^{11/2}}+\frac{x^6\left(a+bx^3+cx^6\right)^{5/2}}{21c}$$

Antiderivative was successfully verified.

```
[In] Int[x^11*(a + b*x^3 + c*x^6)^(3/2),x]
```

```
[Out] (b*(b^2 - 4*a*c)*(3*b^2 - 4*a*c)*(b + 2*c*x^3)*Sqrt[a + b*x^3 + c*x^6])/(10
24*c^5) - (b*(3*b^2 - 4*a*c)*(b + 2*c*x^3)*(a + b*x^3 + c*x^6)^(3/2))/(384*
c^4) + (x^6*(a + b*x^3 + c*x^6)^(5/2))/(21*c) + ((21*b^2 - 16*a*c - 30*b*c*
x^3)*(a + b*x^3 + c*x^6)^(5/2))/(840*c^3) - (b*(b^2 - 4*a*c)^2*(3*b^2 - 4*a
*c)*ArcTanh[(b + 2*c*x^3)/(2*Sqrt[c]*Sqrt[a + b*x^3 + c*x^6])])/(2048*c^(11
/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 742

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(m + 2*p
+ 1)), x] + Dist[1/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m +
 2*p + 1) - e*(a*e*(m - 1) + b*d*(p + 1)) + e*(2*c*d - b*e)*(m + p)*x, x]*(
a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && If[Rat
ionalQ[m], GtQ[m, 1], SumSimplerQ[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuad
raticQ[a, b, c, d, e, m, p, x]
```

Rule 779

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> -Simp[((b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x)*(a + b*x + c*x^2)^(p + 1))/(2*c^2*(p + 1)*(2*p + 3)), x
```

```
] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p +
3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 1357

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int x^{11}\left(a+bx^3+cx^6\right)^{3/2}dx &= \frac{1}{3}\,\mathrm{Subst}\left(\int x^3\left(a+bx+cx^2\right)^{3/2}dx,x,x^3\right)\\
&= \frac{x^6\left(a+bx^3+cx^6\right)^{5/2}}{21c}+\frac{\mathrm{Subst}\left(\int x\left(-2a-\frac{9bx}{2}\right)\left(a+bx+cx^2\right)^{3/2}dx,x,x^3\right)}{21c}\\
&= \frac{x^6\left(a+bx^3+cx^6\right)^{5/2}}{21c}+\frac{\left(21b^2-16ac-30bcx^3\right)\left(a+bx^3+cx^6\right)^{5/2}}{840c^3}-\frac{\left(b\left(3b^2-4\right.\right.}{}\\
&= -\frac{b\left(3b^2-4ac\right)\left(b+2cx^3\right)\left(a+bx^3+cx^6\right)^{3/2}}{384c^4}+\frac{x^6\left(a+bx^3+cx^6\right)^{5/2}}{21c}+\frac{\left(21b^2-\right.}{}\\
&= \frac{b\left(b^2-4ac\right)\left(3b^2-4ac\right)\left(b+2cx^3\right)\sqrt{a+bx^3+cx^6}}{1024c^5}-\frac{b\left(3b^2-4ac\right)\left(b+2cx^3\right)\left(\right.}{384c^4}\\
&= \frac{b\left(b^2-4ac\right)\left(3b^2-4ac\right)\left(b+2cx^3\right)\sqrt{a+bx^3+cx^6}}{1024c^5}-\frac{b\left(3b^2-4ac\right)\left(b+2cx^3\right)\left(\right.}{384c^4}\\
&= \frac{b\left(b^2-4ac\right)\left(3b^2-4ac\right)\left(b+2cx^3\right)\sqrt{a+bx^3+cx^6}}{1024c^5}-\frac{b\left(3b^2-4ac\right)\left(b+2cx^3\right)\left(\right.}{384c^4}
\end{aligned}$$

Mathematica [A] time = 0.19, size = 192, normalized size = 0.86

$$\frac{-\frac{\left(16ac-21b^2+30bcx^3\right)\left(a+bx^3+cx^6\right)^{5/2}}{40c^2}+\frac{7\left(4abc-3b^3\right)\left(2\sqrt{c}\left(b+2cx^3\right)\sqrt{a+bx^3+cx^6}\left(4c\left(5a+2cx^6\right)-3b^2+8bcx^3\right)+3\left(b^2-4ac\right)^2\tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)\right)}{2048c^{9/2}}+x^6\left(a+bx^3+cx^6\right)^{5/2}}{21c}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^11*(a + b*x^3 + c*x^6)^(3/2),x]
```

```
[Out] (x^6*(a + b*x^3 + c*x^6)^(5/2) - ((-21*b^2 + 16*a*c + 30*b*c*x^3)*(a + b*x^
3 + c*x^6)^(5/2))/(40*c^2) + (7*(-3*b^3 + 4*a*b*c)*(2*Sqrt[c]*(b + 2*c*x^3)
*Sqrt[a + b*x^3 + c*x^6]*(-3*b^2 + 8*b*c*x^3 + 4*c*(5*a + 2*c*x^6)) + 3*(b^
2 - 4*a*c)^2*ArcTanh[(b + 2*c*x^3)/(2*Sqrt[c]*Sqrt[a + b*x^3 + c*x^6])]))/(
2048*c^(9/2)))/(21*c)
```

IntegrateAlgebraic [A] time = 1.13, size = 255, normalized size = 1.14

$$\frac{\left(-64a^3bc^3+80a^2b^3c^2-28ab^5c+3b^7\right)\log\left(-2\sqrt{c}\sqrt{a+bx^3+cx^6}+b+2cx^3\right)}{2048c^{11/2}}+\frac{\sqrt{a+bx^3+cx^6}\left(-2048a^3c^3+5488a^2b^2c^2-2336a^2bc^3x^3+1024a^2c^4x^6-2520ab^4c+1456ab^3c^2x^3-992ab^2c^3x^6+704abc^4x^9+8192ac^5x^{12}+315b^6-210b^5cx^3+168b^4c^2x^6-144b^3c^3x^9+128b^2c^4x^{12}+6400bc^5x^{15}+5120c^6x^{18}\right)}{107520c^5}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^11*(a + b*x^3 + c*x^6)^(3/2),x]
```

```
[Out] (Sqrt[a + b*x^3 + c*x^6]*(315*b^6 - 2520*a*b^4*c + 5488*a^2*b^2*c^2 - 2048*
a^3*c^3 - 210*b^5*c*x^3 + 1456*a*b^3*c^2*x^3 - 2336*a^2*b*c^3*x^3 + 168*b^4
*c^2*x^6 - 992*a*b^2*c^3*x^6 + 1024*a^2*c^4*x^6 - 144*b^3*c^3*x^9 + 704*a*b
*c^4*x^9 + 128*b^2*c^4*x^12 + 8192*a*c^5*x^12 + 6400*b*c^5*x^15 + 5120*c^6*
x^18))/(107520*c^5) + ((3*b^7 - 28*a*b^5*c + 80*a^2*b^3*c^2 - 64*a^3*b*c^3)
*Log[b + 2*c*x^3 - 2*Sqrt[c]*Sqrt[a + b*x^3 + c*x^6]])/(2048*c^(11/2))
```

fricas [A] time = 1.24, size = 535, normalized size = 2.40

$$\left[-\frac{105\left(3 b^{7}-28 a b^{5} c+80 a^{2} b^{3} c^{2}-64 a^{3} b c^{3}\right) \sqrt{c} \log\left(-8 c^{2} x^{6}-8 b c x^{3}-b^{2}-4 \sqrt{c x^{6}+b x^{3}+a}\left(2 c x^{3}+b\right) \sqrt{c}-4 a c\right)-4\left(5120 c^{7} x^{18}+6400 b c^{6} x^{15}+128\left(b^{2} c^{5}+64 a c^{6}\right) x^{12}-16\left(9 b^{3} c^{4}-44 a b c^{5}\right) x^{9}+315 b^{6} c-2520 a b^{4} c^{2}+5488 a^{2} b^{2} c^{3}-2048 a^{3} c^{4}+8\left(21 b^{4} c^{3}-124 a b^{2} c^{4}+128 a^{2} c^{5}\right) x^{6}-2\left(105 b^{5} c^{2}-728 a b^{3} c^{3}+1168 a^{2} b c^{4}\right) x^{3}\right) \sqrt{c x^{6}+b x^{3}+a}}{430080 c^{6}}, \frac{105\left(3 b^{7}-28 a b^{5} c+80 a^{2} b^{3} c^{2}-64 a^{3} b c^{3}\right) \sqrt{-c} \arctan\left(\frac{\sqrt{c x^{6}+b x^{3}+a}\left(2 c x^{3}+b\right) \sqrt{-c}}{2\left(c^{2} x^{6}+b c x^{3}+a c\right)}\right)+2\left(5120 c^{7} x^{18}+6400 b c^{6} x^{15}+128\left(b^{2} c^{5}+64 a c^{6}\right) x^{12}-16\left(9 b^{3} c^{4}-44 a b c^{5}\right) x^{9}+315 b^{6} c-2520 a b^{4} c^{2}+5488 a^{2} b^{2} c^{3}-2048 a^{3} c^{4}+8\left(21 b^{4} c^{3}-124 a b^{2} c^{4}+128 a^{2} c^{5}\right) x^{6}-2\left(105 b^{5} c^{2}-728 a b^{3} c^{3}+1168 a^{2} b c^{4}\right) x^{3}\right) \sqrt{c x^{6}+b x^{3}+a}}{215040 c^{6}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(c*x^6+b*x^3+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/430080*(105*(3*b^7 - 28*a*b^5*c + 80*a^2*b^3*c^2 - 64*a^3*b*c^3)*sqrt(c
)*log(-8*c^2*x^6 - 8*b*c*x^3 - b^2 - 4*sqrt(c*x^6 + b*x^3 + a)*(2*c*x^3 + b
)*sqrt(c) - 4*a*c) - 4*(5120*c^7*x^18 + 6400*b*c^6*x^15 + 128*(b^2*c^5 + 64
*a*c^6)*x^12 - 16*(9*b^3*c^4 - 44*a*b*c^5)*x^9 + 315*b^6*c - 2520*a*b^4*c^2
 + 5488*a^2*b^2*c^3 - 2048*a^3*c^4 + 8*(21*b^4*c^3 - 124*a*b^2*c^4 + 128*a^
2*c^5)*x^6 - 2*(105*b^5*c^2 - 728*a*b^3*c^3 + 1168*a^2*b*c^4)*x^3)*sqrt(c*x
^6 + b*x^3 + a))/c^6, 1/215040*(105*(3*b^7 - 28*a*b^5*c + 80*a^2*b^3*c^2 -
64*a^3*b*c^3)*sqrt(-c)*arctan(1/2*sqrt(c*x^6 + b*x^3 + a)*(2*c*x^3 + b)*sqr
t(-c)/(c^2*x^6 + b*c*x^3 + a*c)) + 2*(5120*c^7*x^18 + 6400*b*c^6*x^15 + 128
*(b^2*c^5 + 64*a*c^6)*x^12 - 16*(9*b^3*c^4 - 44*a*b*c^5)*x^9 + 315*b^6*c -
2520*a*b^4*c^2 + 5488*a^2*b^2*c^3 - 2048*a^3*c^4 + 8*(21*b^4*c^3 - 124*a*b^
2*c^4 + 128*a^2*c^5)*x^6 - 2*(105*b^5*c^2 - 728*a*b^3*c^3 + 1168*a^2*b*c^4)
*x^3)*sqrt(c*x^6 + b*x^3 + a))/c^6]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(c x^{6} + b x^{3} + a\right)^{\frac{3}{2}} x^{11}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(c*x^6+b*x^3+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((c*x^6 + b*x^3 + a)^(3/2)*x^11, x)
```

maple [F] time = 0.03, size = 0, normalized size = 0.00

$$\int \left(c\,x^{6}+b\,x^{3}+a\right)^{\frac{3}{2}} x^{11}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11*(c*x^6+b*x^3+a)^(3/2),x)
```

```
[Out] int(x^11*(c*x^6+b*x^3+a)^(3/2),x)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(c*x^6+b*x^3+a)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive, negative or zero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^{11} \left(c x^6 + b x^3 + a\right)^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11*(a + b*x^3 + c*x^6)^(3/2),x)
```

```
[Out] int(x^11*(a + b*x^3 + c*x^6)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^{11} \left(a + bx^3 + cx^6\right)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**11*(c*x**6+b*x**3+a)**(3/2),x)
```

```
[Out] Integral(x**11*(a + b*x**3 + c*x**6)**(3/2), x)
```

3.185 $\int x^8 \left(a + bx^3 + cx^6\right)^{3/2} dx$

Optimal. Leaf size=204

$$\frac{\left(b^2-4ac\right)^2\left(7b^2-4ac\right)\tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{3072c^{9/2}}-\frac{\left(b^2-4ac\right)\left(7b^2-4ac\right)\left(b+2cx^3\right)\sqrt{a+bx^3+cx^6}}{1536c^4}+\frac{\left(7b^2-4ac\right)\cdots}{}$$

Rubi [A] time = 0.19, antiderivative size = 204, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = $\{1357, 742, 640, 612, 621, 206\}$

$$\frac{\left(7b^2-4ac\right)\left(b+2cx^3\right)\left(a+bx^3+cx^6\right)^{3/2}}{576c^3}-\frac{\left(b^2-4ac\right)\left(7b^2-4ac\right)\left(b+2cx^3\right)\sqrt{a+bx^3+cx^6}}{1536c^4}+\frac{\left(b^2-4ac\right)^2\left(7b^2-4ac\right)\tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{3072c^{9/2}}-\frac{7b\left(a+bx^3+cx^6\right)^{5/2}}{180c^2}+\frac{x^3\left(a+bx^3+cx^6\right)^{5/2}}{18c}$$

Antiderivative was successfully verified.

```
[In] Int[x^8*(a + b*x^3 + c*x^6)^(3/2),x]
```

```
[Out] -((b^2 - 4*a*c)*(7*b^2 - 4*a*c)*(b + 2*c*x^3)*Sqrt[a + b*x^3 + c*x^6])/(153
6*c^4) + ((7*b^2 - 4*a*c)*(b + 2*c*x^3)*(a + b*x^3 + c*x^6)^(3/2))/(576*c^3
) - (7*b*(a + b*x^3 + c*x^6)^(5/2))/(180*c^2) + (x^3*(a + b*x^3 + c*x^6)^(5
/2))/(18*c) + ((b^2 - 4*a*c)^2*(7*b^2 - 4*a*c)*ArcTanh[(b + 2*c*x^3)/(2*Sqr
t[c]*Sqrt[a + b*x^3 + c*x^6])])/(3072*c^(9/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 742

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(m + 2*p
+ 1)), x] + Dist[1/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m +
 2*p + 1) - e*(a*e*(m - 1) + b*d*(p + 1)) + e*(2*c*d - b*e)*(m + p)*x, x]*(
a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && If[Rat
ionalQ[m], GtQ[m, 1], SumSimplerQ[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuad
```

```
raticQ[a, b, c, d, e, m, p, x]
```

Rule 1357

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int x^8\left(a+bx^3+cx^6\right)^{3/2}\,dx=\frac{1}{3}\text{Subst}\left(\int x^2\left(a+bx+cx^2\right)^{3/2}\,dx,x,x^3\right)$$
$$=\frac{x^3\left(a+bx^3+cx^6\right)^{5/2}}{18c}+\frac{\text{Subst}\left(\int\left(-a-\frac{7bx}{2}\right)\left(a+bx+cx^2\right)^{3/2}\,dx,x,x^3\right)}{18c}$$
$$=-\frac{7b\left(a+bx^3+cx^6\right)^{5/2}}{180c^2}+\frac{x^3\left(a+bx^3+cx^6\right)^{5/2}}{18c}+\frac{\left(7b^2-4ac\right)\text{Subst}\left(\int\left(a+bx+\right.\right.}{72c^2}$$
$$=\frac{\left(7b^2-4ac\right)\left(b+2cx^3\right)\left(a+bx^3+cx^6\right)^{3/2}}{576c^3}-\frac{7b\left(a+bx^3+cx^6\right)^{5/2}}{180c^2}+\frac{x^3\left(a+bx^3\right.}{18c}$$
$$=-\frac{\left(b^2-4ac\right)\left(7b^2-4ac\right)\left(b+2cx^3\right)\sqrt{a+bx^3+cx^6}}{1536c^4}+\frac{\left(7b^2-4ac\right)\left(b+2cx^3\right)\left(a\right.}{576c^3}$$
$$=-\frac{\left(b^2-4ac\right)\left(7b^2-4ac\right)\left(b+2cx^3\right)\sqrt{a+bx^3+cx^6}}{1536c^4}+\frac{\left(7b^2-4ac\right)\left(b+2cx^3\right)\left(a\right.}{576c^3}$$
$$=-\frac{\left(b^2-4ac\right)\left(7b^2-4ac\right)\left(b+2cx^3\right)\sqrt{a+bx^3+cx^6}}{1536c^4}+\frac{\left(7b^2-4ac\right)\left(b+2cx^3\right)\left(a\right.}{576c^3}$$

Mathematica [A] time = 0.16, size = 175, normalized size = 0.86

$$\frac{\frac{\left(7b^2-4ac\right)\left(2\sqrt{c}\left(b+2cx^3\right)\sqrt{a+bx^3+cx^6}\left(4c\left(5a+2cx^6\right)-3b^2+8bcx^3\right)+3\left(b^2-4ac\right)^2\tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)\right)}{512c^{7/2}}+x^3\left(a+bx^3+cx^6\right)^{5/2}-\frac{7b\left(a+bx^3+cx^6\right)^{5/2}}{10c}}{18c}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8*(a + b*x^3 + c*x^6)^(3/2),x]
```

```
[Out] ((-7*b*(a + b*x^3 + c*x^6)^(5/2))/(10*c) + x^3*(a + b*x^3 + c*x^6)^(5/2) +
((7*b^2 - 4*a*c)*(2*Sqrt[c]*(b + 2*c*x^3)*Sqrt[a + b*x^3 + c*x^6]*(-3*b^2 +
 8*b*c*x^3 + 4*c*(5*a + 2*c*x^6)) + 3*(b^2 - 4*a*c)^2*ArcTanh[(b + 2*c*x^3)
/(2*Sqrt[c]*Sqrt[a + b*x^3 + c*x^6])]))/(512*c^(7/2)))/(18*c)
```

IntegrateAlgebraic [A] time = 0.88, size = 209, normalized size = 1.02

$$\frac{\sqrt{a+bx^3+cx^6}\left(-1296a^2bc^2+480a^2c^3x^3+760ab^3c-432ab^2c^2x^3+288abc^3x^6+2240ac^4x^9-105b^5+70b^4cx^3-56b^3c^2x^6+48b^2c^3x^9+1664bc^4x^{12}+1280c^5x^{15}\right)}{23040c^4}+\frac{\left(64a^3c^3-144a^2b^2c^2+60ab^4c-7b^6\right)\log\left(-2\sqrt{c}\sqrt{a+bx^3+cx^6}+b+2cx^3\right)}{3072c^{9/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^8*(a + b*x^3 + c*x^6)^(3/2),x]
```

```
[Out] (Sqrt[a + b*x^3 + c*x^6]*(-105*b^5 + 760*a*b^3*c - 1296*a^2*b*c^2 + 70*b^4*
c*x^3 - 432*a*b^2*c^2*x^3 + 480*a^2*c^3*x^3 - 56*b^3*c^2*x^6 + 288*a*b*c^3*
x^6 + 48*b^2*c^3*x^9 + 2240*a*c^4*x^9 + 1664*b*c^4*x^12 + 1280*c^5*x^15))/(
```

```
23040*c^4) + ((-7*b^6 + 60*a*b^4*c - 144*a^2*b^2*c^2 + 64*a^3*c^3)*Log[b +
2*c*x^3 - 2*Sqrt[c]*Sqrt[a + b*x^3 + c*x^6]])/(3072*c^(9/2))
```

fricas [A] time = 1.30, size = 451, normalized size = 2.21

$$\left[-\frac{15\left(7b^{6}-60ab^{4}c+144a^{2}b^{2}c^{2}-64a^{3}c^{3}\right)\sqrt{c}\log\left(-8c^{2}x^{6}-8bcx^{3}-b^{2}+4\sqrt{cx^{6}+bx^{3}+a}\left(2cx^{3}+b\right)\sqrt{c}-4ac\right)-4\left(1280c^{6}x^{15}+1664bc^{5}x^{12}+16\left(3b^{2}c^{4}+140ac^{5}\right)x^{9}-8\left(7b^{3}c^{3}-36abc^{4}\right)x^{6}-105b^{5}c+760ab^{3}c^{2}-1296a^{2}bc^{3}+2\left(35b^{4}c^{2}-216ab^{2}c^{3}+240a^{2}c^{4}\right)x^{3}\right)\sqrt{cx^{6}+bx^{3}+a}}{92160c^{5}}, -\frac{15\left(7b^{6}-60ab^{4}c+144a^{2}b^{2}c^{2}-64a^{3}c^{3}\right)\sqrt{-c}\arctan\left(\frac{\sqrt{cx^{6}+bx^{3}+a}\left(2cx^{3}+b\right)\sqrt{-c}}{2\left(c^{2}x^{6}+bcx^{3}+ac\right)}\right)-2\left(1280c^{6}x^{15}+1664bc^{5}x^{12}+16\left(3b^{2}c^{4}+140ac^{5}\right)x^{9}-8\left(7b^{3}c^{3}-36abc^{4}\right)x^{6}-105b^{5}c+760ab^{3}c^{2}-1296a^{2}bc^{3}+2\left(35b^{4}c^{2}-216ab^{2}c^{3}+240a^{2}c^{4}\right)x^{3}\right)\sqrt{cx^{6}+bx^{3}+a}}{46080c^{5}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(c*x^6+b*x^3+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/92160*(15*(7*b^6 - 60*a*b^4*c + 144*a^2*b^2*c^2 - 64*a^3*c^3)*sqrt(c)*l
og(-8*c^2*x^6 - 8*b*c*x^3 - b^2 + 4*sqrt(c*x^6 + b*x^3 + a)*(2*c*x^3 + b)*s
qrt(c) - 4*a*c) - 4*(1280*c^6*x^15 + 1664*b*c^5*x^12 + 16*(3*b^2*c^4 + 140*
a*c^5)*x^9 - 8*(7*b^3*c^3 - 36*a*b*c^4)*x^6 - 105*b^5*c + 760*a*b^3*c^2 - 1
296*a^2*b*c^3 + 2*(35*b^4*c^2 - 216*a*b^2*c^3 + 240*a^2*c^4)*x^3)*sqrt(c*x^
6 + b*x^3 + a))/c^5, -1/46080*(15*(7*b^6 - 60*a*b^4*c + 144*a^2*b^2*c^2 - 6
4*a^3*c^3)*sqrt(-c)*arctan(1/2*sqrt(c*x^6 + b*x^3 + a)*(2*c*x^3 + b)*sqrt(-
c)/(c^2*x^6 + b*c*x^3 + a*c)) - 2*(1280*c^6*x^15 + 1664*b*c^5*x^12 + 16*(3*
b^2*c^4 + 140*a*c^5)*x^9 - 8*(7*b^3*c^3 - 36*a*b*c^4)*x^6 - 105*b^5*c + 760
*a*b^3*c^2 - 1296*a^2*b*c^3 + 2*(35*b^4*c^2 - 216*a*b^2*c^3 + 240*a^2*c^4)*
x^3)*sqrt(c*x^6 + b*x^3 + a))/c^5]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(cx^{6}+bx^{3}+a\right)^{\frac{3}{2}}x^{8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(c*x^6+b*x^3+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((c*x^6 + b*x^3 + a)^(3/2)*x^8, x)
```

maple [F] time = 0.03, size = 0, normalized size = 0.00

$$\int\left(c\,x^{6}+b\,x^{3}+a\right)^{\frac{3}{2}}x^{8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*(c*x^6+b*x^3+a)^(3/2),x)
```

```
[Out] int(x^8*(c*x^6+b*x^3+a)^(3/2),x)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(c*x^6+b*x^3+a)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive, negative or zero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^{8}\,\left(c\,x^{6}+b\,x^{3}+a\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*(a + b*x^3 + c*x^6)^(3/2),x)
```

```
[Out] int(x^8*(a + b*x^3 + c*x^6)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^8 \left(a + bx^3 + cx^6\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8*(c*x**6+b*x**3+a)**(3/2),x)
```

```
[Out] Integral(x**8*(a + b*x**3 + c*x**6)**(3/2), x)
```

3.186 $\int x^5 \left(a+bx^3+cx^6\right)^{3/2} \, dx$

Optimal. Leaf size=150

$$-\frac{b\left(b^2-4ac\right)^2 \tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{256c^{7/2}}+\frac{b\left(b^2-4ac\right)\left(b+2cx^3\right)\sqrt{a+bx^3+cx^6}}{128c^3}-\frac{b\left(b+2cx^3\right)\left(a+bx^3+cx^6\right)^{3/2}}{48c^2}$$

Rubi [A] time = 0.12, antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1357, 640, 612, 621, 206}

$$\frac{b\left(b^2-4ac\right)\left(b+2cx^3\right)\sqrt{a+bx^3+cx^6}}{128c^3}-\frac{b\left(b^2-4ac\right)^2 \tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{256c^{7/2}}-\frac{b\left(b+2cx^3\right)\left(a+bx^3+cx^6\right)^{3/2}}{48c^2}+\frac{\left(a+bx^3+cx^6\right)^{5/2}}{15c}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*(a + b*x^3 + c*x^6)^(3/2),x]
```

```
[Out] (b*(b^2 - 4*a*c)*(b + 2*c*x^3)*Sqrt[a + b*x^3 + c*x^6])/(128*c^3) - (b*(b +
 2*c*x^3)*(a + b*x^3 + c*x^6)^(3/2))/(48*c^2) + (a + b*x^3 + c*x^6)^(5/2)/(
15*c) - (b*(b^2 - 4*a*c)^2*ArcTanh[(b + 2*c*x^3)/(2*Sqrt[c]*Sqrt[a + b*x^3
+ c*x^6])])/(256*c^(7/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1357

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\int x^5\left(a+bx^3+cx^6\right)^{3/2}\,dx &= \frac{1}{3}\,\text{Subst}\left(\int x\left(a+bx+cx^2\right)^{3/2}\,dx,x,x^3\right)\\
&= \frac{\left(a+bx^3+cx^6\right)^{5/2}}{15c}-\frac{b\,\text{Subst}\left(\int\left(a+bx+cx^2\right)^{3/2}\,dx,x,x^3\right)}{6c}\\
&= -\frac{b\left(b+2cx^3\right)\left(a+bx^3+cx^6\right)^{3/2}}{48c^2}+\frac{\left(a+bx^3+cx^6\right)^{5/2}}{15c}+\frac{\left(b\left(b^2-4ac\right)\right)\text{Subst}\left(\int\right.}{32\ldots}\\
&= \frac{b\left(b^2-4ac\right)\left(b+2cx^3\right)\sqrt{a+bx^3+cx^6}}{128c^3}-\frac{b\left(b+2cx^3\right)\left(a+bx^3+cx^6\right)^{3/2}}{48c^2}+\frac{(a+\ldots}{\ldots}\\
&= \frac{b\left(b^2-4ac\right)\left(b+2cx^3\right)\sqrt{a+bx^3+cx^6}}{128c^3}-\frac{b\left(b+2cx^3\right)\left(a+bx^3+cx^6\right)^{3/2}}{48c^2}+\frac{(a+\ldots}{\ldots}\\
&= \frac{b\left(b^2-4ac\right)\left(b+2cx^3\right)\sqrt{a+bx^3+cx^6}}{128c^3}-\frac{b\left(b+2cx^3\right)\left(a+bx^3+cx^6\right)^{3/2}}{48c^2}+\frac{(a+\ldots}{\ldots}
\end{aligned}
$$

Mathematica [**A**] time = 0.15, size = 149, normalized size = 0.99

$$
-\frac{b\left(b^2-4ac\right)\left(\left(b^2-4ac\right)\tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)-2\sqrt{c}\left(b+2cx^3\right)\sqrt{a+bx^3+cx^6}\right)}{256c^{7/2}}-\frac{b\left(b+2cx^3\right)\left(a+bx^3+cx^6\right)^{3/2}}{48c^2}+\frac{\left(a+bx^3+cx^6\right)^{5/2}}{15c}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*(a + b*x^3 + c*x^6)^(3/2),x]
```

```
[Out] -1/48*(b*(b + 2*c*x^3)*(a + b*x^3 + c*x^6)^(3/2))/c^2 + (a + b*x^3 + c*x^6)
^(5/2)/(15*c) - (b*(b^2 - 4*a*c)*(-2*Sqrt[c]*(b + 2*c*x^3)*Sqrt[a + b*x^3 +
 c*x^6] + (b^2 - 4*a*c)*ArcTanh[(b + 2*c*x^3)/(2*Sqrt[c]*Sqrt[a + b*x^3 + c
*x^6])]))/(256*c^(7/2))
```

IntegrateAlgebraic [**A**] time = 0.66, size = 162, normalized size = 1.08

$$
\frac{\left(16a^2bc^2-8ab^3c+b^5\right)\log\left(-2\sqrt{c}\sqrt{a+bx^3+cx^6}+b+2cx^3\right)}{256c^{7/2}}+\frac{\sqrt{a+bx^3+cx^6}\left(128a^2c^2-100ab^2c+56abc^2x^3+256ac^3x^6+15b^4-10b^3cx^3+8b^2c^2x^6+176bc^3x^9+128c^4x^{12}\right)}{1920c^3}
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^5*(a + b*x^3 + c*x^6)^(3/2),x]
```

```
[Out] (Sqrt[a + b*x^3 + c*x^6]*(15*b^4 - 100*a*b^2*c + 128*a^2*c^2 - 10*b^3*c*x^3
 + 56*a*b*c^2*x^3 + 8*b^2*c^2*x^6 + 256*a*c^3*x^6 + 176*b*c^3*x^9 + 128*c^4
*x^12))/(1920*c^3) + ((b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*Log[b + 2*c*x^3 - 2*
Sqrt[c]*Sqrt[a + b*x^3 + c*x^6]])/(256*c^(7/2))
```

fricas [**A**] time = 1.21, size = 361, normalized size = 2.41

$$
\left[\frac{15\left(b^5-8ab^3c+16a^2bc^2\right)\sqrt{c}\log\left(-8c^2x^6-8bcx^3-b^2+4\sqrt{cx^6+bx^3+a}\left(2cx^3+b\right)\sqrt{c}-4ac\right)+4\left(128c^5x^{12}+176bc^4x^9+8\left(b^2c^3+32ac^4\right)x^6+15b^4c-100ab^2c^2+128a^2c^3-2\left(5b^3c^2-28abc^3\right)x^3\right)\sqrt{cx^6+bx^3+a}}{7680c^4},\ \frac{15\left(b^5-8ab^3c+16a^2bc^2\right)\sqrt{-c}\arctan\left(\frac{\sqrt{cx^6+bx^3+a}\left(2cx^3+b\right)\sqrt{-c}}{2\left(c^2x^6+bcx^3+ac\right)}\right)+2\left(128c^5x^{12}+176bc^4x^9+8\left(b^2c^3+32ac^4\right)x^6+15b^4c-100ab^2c^2+128a^2c^3-2\left(5b^3c^2-28abc^3\right)x^3\right)\sqrt{cx^6+bx^3+a}}{3840c^4}\right]
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(c*x^6+b*x^3+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/7680*(15*(b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*sqrt(c)*log(-8*c^2*x^6 - 8*b*c
*x^3 - b^2 + 4*sqrt(c*x^6 + b*x^3 + a)*(2*c*x^3 + b)*sqrt(c) - 4*a*c) + 4*(
128*c^5*x^12 + 176*b*c^4*x^9 + 8*(b^2*c^3 + 32*a*c^4)*x^6 + 15*b^4*c - 100*
a*b^2*c^2 + 128*a^2*c^3 - 2*(5*b^3*c^2 - 28*a*b*c^3)*x^3)*sqrt(c*x^6 + b*x^
3 + a))/c^4, 1/3840*(15*(b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*sqrt(-c)*arctan(1/
```

```
2*sqrt(c*x^6 + b*x^3 + a)*(2*c*x^3 + b)*sqrt(-c)/(c^2*x^6 + b*c*x^3 + a*c))
 + 2*(128*c^5*x^12 + 176*b*c^4*x^9 + 8*(b^2*c^3 + 32*a*c^4)*x^6 + 15*b^4*c
- 100*a*b^2*c^2 + 128*a^2*c^3 - 2*(5*b^3*c^2 - 28*a*b*c^3)*x^3)*sqrt(c*x^6
+ b*x^3 + a))/c^4]
```

giac [A] time = 0.56, size = 172, normalized size = 1.15

$$\frac{1}{1920}\sqrt{cx^6+bx^3+a}\left(2\left(4\left(2\left(8\,cx^3+11\,b\right)x^3+\frac{b^2c^3+32\,ac^4}{c^4}\right)x^3-\frac{5\,b^3c^2-28\,abc^3}{c^4}\right)x^3+\frac{15\,b^4c-100\,ab^2c^2+128\,a^2c^3}{c^4}\right)+\frac{\left(b^5-8\,ab^3c+16\,a^2bc^2\right)\log\left(\left|-2\left(\sqrt{c}x^3-\sqrt{cx^6+bx^3+a}\right)\sqrt{c}-b\right|\right)}{256\,c^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(c*x^6+b*x^3+a)^(3/2),x, algorithm="giac")
```

```
[Out] 1/1920*sqrt(c*x^6 + b*x^3 + a)*(2*(4*(2*(8*c*x^3 + 11*b)*x^3 + (b^2*c^3 + 3
2*a*c^4)/c^4)*x^3 - (5*b^3*c^2 - 28*a*b*c^3)/c^4)*x^3 + (15*b^4*c - 100*a*b
^2*c^2 + 128*a^2*c^3)/c^4) + 1/256*(b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*log(abs
(-2*(sqrt(c)*x^3 - sqrt(c*x^6 + b*x^3 + a))*sqrt(c) - b))/c^(7/2)
```

maple [F] time = 0.04, size = 0, normalized size = 0.00

$$\int\left(c\,x^6+b\,x^3+a\right)^{\frac{3}{2}}x^5\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(c*x^6+b*x^3+a)^(3/2),x)
```

```
[Out] int(x^5*(c*x^6+b*x^3+a)^(3/2),x)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(c*x^6+b*x^3+a)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive, negative or zero?
```

mupad [B] time = 1.58, size = 223, normalized size = 1.49

$$\frac{\left(c\,x^6+b\,x^3+a\right)^{5/2}}{15\,c}-\frac{b\left(\frac{3\,a\left(\ln\left(\sqrt{c\,x^6+b\,x^3+a}+\frac{c\,x^3+\frac{b}{2}}{\sqrt{c}}\right)\left(\frac{a}{2\,\sqrt{c}}-\frac{b^2}{8\,c^{3/2}}\right)+\frac{\left(2\,c\,x^3+b\right)\sqrt{c\,x^6+b\,x^3+a}}{4\,c}\right)}{4}+\frac{x^3\left(c\,x^6+b\,x^3+a\right)^{3/2}}{4}-\frac{3\,b^2\left(\ln\left(\sqrt{c\,x^6+b\,x^3+a}+\frac{c\,x^3+\frac{b}{2}}{\sqrt{c}}\right)\left(\frac{a}{2\,\sqrt{c}}-\frac{b^2}{8\,c^{3/2}}\right)+\frac{\left(2\,c\,x^3+b\right)\sqrt{c\,x^6+b\,x^3+a}}{4\,c}\right)}{16\,c}+\frac{b\left(c\,x^6+b\,x^3+a\right)^{3/2}}{8\,c}\right)}{6\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a + b*x^3 + c*x^6)^(3/2),x)
```

```
[Out] (a + b*x^3 + c*x^6)^(5/2)/(15*c) - (b*((3*a*(log((a + b*x^3 + c*x^6)^(1/2)
+ (b/2 + c*x^3)/c^(1/2))*(a/(2*c^(1/2)) - b^2/(8*c^(3/2))) + ((b + 2*c*x^3)
*(a + b*x^3 + c*x^6)^(1/2))/(4*c)))/4 + (x^3*(a + b*x^3 + c*x^6)^(3/2))/4 -
 (3*b^2*(log((a + b*x^3 + c*x^6)^(1/2) + (b/2 + c*x^3)/c^(1/2))*(a/(2*c^(1/
2)) - b^2/(8*c^(3/2))) + ((b + 2*c*x^3)*(a + b*x^3 + c*x^6)^(1/2))/(4*c)))/
(16*c) + (b*(a + b*x^3 + c*x^6)^(3/2))/(8*c)))/(6*c)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^5\left(a+bx^3+cx^6\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(c*x**6+b*x**3+a)**(3/2),x)
```

```
[Out] Integral(x**5*(a + b*x**3 + c*x**6)**(3/2), x)
```

3.187 $\int x^2 \left(a+bx^3+cx^6\right)^{3/2} dx$

Optimal. Leaf size=124

$$\frac{\left(b^2-4ac\right)^2 \tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{128c^{5/2}}-\frac{\left(b^2-4ac\right)\left(b+2cx^3\right)\sqrt{a+bx^3+cx^6}}{64c^2}+\frac{\left(b+2cx^3\right)\left(a+bx^3+cx^6\right)^{3/2}}{24c}$$

Rubi [A] time = 0.09, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1352, 612, 621, 206}

$$-\frac{\left(b^2-4ac\right)\left(b+2cx^3\right)\sqrt{a+bx^3+cx^6}}{64c^2}+\frac{\left(b^2-4ac\right)^2 \tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{128c^{5/2}}+\frac{\left(b+2cx^3\right)\left(a+bx^3+cx^6\right)^{3/2}}{24c}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*x^3 + c*x^6)^(3/2),x]
```

```
[Out] -((b^2 - 4*a*c)*(b + 2*c*x^3)*Sqrt[a + b*x^3 + c*x^6])/(64*c^2) + ((b + 2*c
*x^3)*(a + b*x^3 + c*x^6)^(3/2))/(24*c) + ((b^2 - 4*a*c)^2*ArcTanh[(b + 2*c
*x^3)/(2*Sqrt[c]*Sqrt[a + b*x^3 + c*x^6])])/(128*c^(5/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1352

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a,
b, c, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

Rubi steps

$$\int x^2\left(a+bx^3+cx^6\right)^{3/2}dx=\frac{1}{3}\text{Subst}\left(\int\left(a+bx+cx^2\right)^{3/2}dx,x,x^3\right)$$
$$=\frac{\left(b+2cx^3\right)\left(a+bx^3+cx^6\right)^{3/2}}{24c}-\frac{\left(b^2-4ac\right)\text{Subst}\left(\int\sqrt{a+bx+cx^2}\,dx,x,x^3\right)}{16c}$$
$$=-\frac{\left(b^2-4ac\right)\left(b+2cx^3\right)\sqrt{a+bx^3+cx^6}}{64c^2}+\frac{\left(b+2cx^3\right)\left(a+bx^3+cx^6\right)^{3/2}}{24c}+\frac{\left(b^2-\right.}{}$$
$$=-\frac{\left(b^2-4ac\right)\left(b+2cx^3\right)\sqrt{a+bx^3+cx^6}}{64c^2}+\frac{\left(b+2cx^3\right)\left(a+bx^3+cx^6\right)^{3/2}}{24c}+\frac{\left(b^2-\right.}{}$$
$$=-\frac{\left(b^2-4ac\right)\left(b+2cx^3\right)\sqrt{a+bx^3+cx^6}}{64c^2}+\frac{\left(b+2cx^3\right)\left(a+bx^3+cx^6\right)^{3/2}}{24c}+\frac{\left(b^2-\right.}{}$$

Mathematica [A] time = 0.09, size = 126, normalized size = 1.02

$$\frac{\frac{3\left(b^2-4ac\right)\left(\left(b^2-4ac\right)\tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)-2\sqrt{c}\left(b+2cx^3\right)\sqrt{a+bx^3+cx^6}\right)}{8c^{3/2}}+2\left(b+2cx^3\right)\left(a+bx^3+cx^6\right)^{3/2}}{48c}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*x^3 + c*x^6)^(3/2),x]
```

```
[Out] (2*(b + 2*c*x^3)*(a + b*x^3 + c*x^6)^(3/2) + (3*(b^2 - 4*a*c)*(-2*Sqrt[c]*(
b + 2*c*x^3)*Sqrt[a + b*x^3 + c*x^6] + (b^2 - 4*a*c)*ArcTanh[(b + 2*c*x^3)/
(2*Sqrt[c]*Sqrt[a + b*x^3 + c*x^6])]))/(8*c^(3/2)))/(48*c)
```

IntegrateAlgebraic [A] time = 0.50, size = 132, normalized size = 1.06

$$\frac{\left(-16a^2c^2+8ab^2c-b^4\right)\log\left(-2\sqrt{c}\sqrt{a+bx^3+cx^6}+b+2cx^3\right)}{128c^{5/2}}+\frac{\sqrt{a+bx^3+cx^6}\left(20abc+40ac^2x^3-3b^3+2b^2cx^3+24bc^2x^6+16c^3x^9\right)}{192c^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^2*(a + b*x^3 + c*x^6)^(3/2),x]
```

```
[Out] (Sqrt[a + b*x^3 + c*x^6]*(-3*b^3 + 20*a*b*c + 2*b^2*c*x^3 + 40*a*c^2*x^3 +
24*b*c^2*x^6 + 16*c^3*x^9))/(192*c^2) + ((-b^4 + 8*a*b^2*c - 16*a^2*c^2)*Lo
g[b + 2*c*x^3 - 2*Sqrt[c]*Sqrt[a + b*x^3 + c*x^6]])/(128*c^(5/2))
```

fricas [A] time = 1.38, size = 297, normalized size = 2.40

$$\left[\frac{3\left(b^4-8ab^2c+16a^2c^2\right)\sqrt{c}\log\left(-8c^2x^6-8bcx^3-b^2-4\sqrt{cx^6+bx^3+a}\left(2cx^3+b\right)\sqrt{c}-4ac\right)+4\left(16c^4x^9+24bc^3x^6-3b^3c+20abc^2+2\left(b^2c^2+20ac^3\right)x^3\right)\sqrt{cx^6+bx^3+a}}{768c^3},-\frac{3\left(b^4-8ab^2c+16a^2c^2\right)\sqrt{-c}\arctan\left(\frac{\sqrt{cx^6+bx^3+a}\left(2cx^3+b\right)\sqrt{-c}}{2\left(c^2x^6+bcx^3+ac\right)}\right)-2\left(16c^4x^9+24bc^3x^6-3b^3c+20abc^2+2\left(b^2c^2+20ac^3\right)x^3\right)\sqrt{cx^6+bx^3+a}}{384c^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^6+b*x^3+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/768*(3*(b^4 - 8*a*b^2*c + 16*a^2*c^2)*sqrt(c)*log(-8*c^2*x^6 - 8*b*c*x^3
 - b^2 - 4*sqrt(c*x^6 + b*x^3 + a)*(2*c*x^3 + b)*sqrt(c) - 4*a*c) + 4*(16*c
^4*x^9 + 24*b*c^3*x^6 - 3*b^3*c + 20*a*b*c^2 + 2*(b^2*c^2 + 20*a*c^3)*x^3)*
sqrt(c*x^6 + b*x^3 + a))/c^3, -1/384*(3*(b^4 - 8*a*b^2*c + 16*a^2*c^2)*sqrt
(-c)*arctan(1/2*sqrt(c*x^6 + b*x^3 + a)*(2*c*x^3 + b)*sqrt(-c)/(c^2*x^6 + b
*c*x^3 + a*c)) - 2*(16*c^4*x^9 + 24*b*c^3*x^6 - 3*b^3*c + 20*a*b*c^2 + 2*(b
^2*c^2 + 20*a*c^3)*x^3)*sqrt(c*x^6 + b*x^3 + a))/c^3]
```

giac [A] time = 0.61, size = 135, normalized size = 1.09

$$\frac{1}{192}\sqrt{cx^6+bx^3+a}\left(2\left(4\left(2\,cx^3+3\,b\right)x^3+\frac{b^2c^2+20\,ac^3}{c^3}\right)x^3-\frac{3\,b^3c-20\,abc^2}{c^3}\right)-\frac{\left(b^4-8\,ab^2c+16\,a^2c^2\right)\log\left(\left|-2\left(\sqrt{c}x^3-\sqrt{cx^6+bx^3+a}\right)\sqrt{c}-b\right|\right)}{128\,c^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^6+b*x^3+a)^(3/2),x, algorithm="giac")
```

```
[Out] 1/192*sqrt(c*x^6 + b*x^3 + a)*(2*(4*(2*c*x^3 + 3*b)*x^3 + (b^2*c^2 + 20*a*c
^3)/c^3)*x^3 - (3*b^3*c - 20*a*b*c^2)/c^3) - 1/128*(b^4 - 8*a*b^2*c + 16*a^
2*c^2)*log(abs(-2*(sqrt(c)*x^3 - sqrt(c*x^6 + b*x^3 + a))*sqrt(c) - b))/c^(
5/2)
```

maple [F] time = 0.03, size = 0, normalized size = 0.00

$$\int \left(c\,x^6+b\,x^3+a\right)^{\frac{3}{2}} x^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(c*x^6+b*x^3+a)^(3/2),x)
```

```
[Out] int(x^2*(c*x^6+b*x^3+a)^(3/2),x)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^6+b*x^3+a)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive, negative or zero?
```

mupad [B] time = 1.44, size = 115, normalized size = 0.93

$$\frac{\left(c\,x^3+\frac{b}{2}\right)\left(c\,x^6+b\,x^3+a\right)^{3/2}}{12\,c}+\frac{\left(3\,a\,c-\frac{3\,b^2}{4}\right)\left(\left(\frac{b}{4\,c}+\frac{x^3}{2}\right)\sqrt{c\,x^6+b\,x^3+a}+\frac{\ln\left(\sqrt{c\,x^6+b\,x^3+a}+\frac{c\,x^3+\frac{b}{2}}{\sqrt{c}}\right)\left(a\,c-\frac{b^2}{4}\right)}{2\,c^{3/2}}\right)}{12\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*x^3 + c*x^6)^(3/2),x)
```

```
[Out] ((b/2 + c*x^3)*(a + b*x^3 + c*x^6)^(3/2))/(12*c) + ((3*a*c - (3*b^2)/4)*((b
/(4*c) + x^3/2)*(a + b*x^3 + c*x^6)^(1/2) + (log((a + b*x^3 + c*x^6)^(1/2)
+ (b/2 + c*x^3)/c^(1/2))*(a*c - b^2/4))/(2*c^(3/2))))/(12*c)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2\left(a+bx^3+cx^6\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(c*x**6+b*x**3+a)**(3/2),x)
```

```
[Out] Integral(x**2*(a + b*x**3 + c*x**6)**(3/2), x)
```

3.188 $\int \frac{(a+bx^3+cx^6)^{3/2}}{x}\,dx$

Optimal. Leaf size=155

$$-\frac{1}{3}a^{3/2}\tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)-\frac{b\left(b^2-12ac\right)\tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{48c^{3/2}}+\frac{\left(8ac+b^2+2bcx^3\right)\sqrt{a+bx^3+}}{24c}$$

Rubi [A] time = 0.18, antiderivative size = 155, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.350, Rules used = {1357, 734, 814, 843, 621, 206, 724}

$$-\frac{1}{3}a^{3/2}\tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)-\frac{b\left(b^2-12ac\right)\tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{48c^{3/2}}+\frac{\left(8ac+b^2+2bcx^3\right)\sqrt{a+bx^3+cx^6}}{24c}+\frac{1}{9}\left(a+bx^3+cx^6\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3 + c*x^6)^(3/2)/x,x]
```

```
[Out] ((b^2 + 8*a*c + 2*b*c*x^3)*Sqrt[a + b*x^3 + c*x^6])/(24*c) + (a + b*x^3 + c
*x^6)^(3/2)/9 - (a^(3/2)*ArcTanh[(2*a + b*x^3)/(2*Sqrt[a]*Sqrt[a + b*x^3 +
c*x^6])])/3 - (b*(b^2 - 12*a*c)*ArcTanh[(b + 2*c*x^3)/(2*Sqrt[c]*Sqrt[a + b
*x^3 + c*x^6])])/(48*c^(3/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 734

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 2*p + 1)), x
] - Dist[p/(e*(m + 2*p + 1)), Int[(d + e*x)^m*Simp[b*d - 2*a*e + (2*c*d - b
*e)*x, x]*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e
, 0] && GtQ[p, 0] && NeQ[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &
&  !ILtQ[m + 2*p, 0] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 814

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2
) - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*(a + b*x + c*x^2)^p)
/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
```

```
 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
|| IntegersQ[2*m, 2*p])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1357

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{\left(a+bx^3+cx^6\right)^{3/2}}{x}\,dx = \frac{1}{3}\operatorname{Subst}\left(\int \frac{\left(a+bx+cx^2\right)^{3/2}}{x}\,dx, x, x^3\right)$$

$$= \frac{1}{9}\left(a+bx^3+cx^6\right)^{3/2} - \frac{1}{6}\operatorname{Subst}\left(\int \frac{(-2a-bx)\sqrt{a+bx+cx^2}}{x}\,dx, x, x^3\right)$$

$$= \frac{\left(b^2+8ac+2bcx^3\right)\sqrt{a+bx^3+cx^6}}{24c} + \frac{1}{9}\left(a+bx^3+cx^6\right)^{3/2} + \frac{\operatorname{Subst}\left(\int \frac{8a^2c-\frac{1}{2}b\left(b^2-12ac\right)\ldots}{x\sqrt{a+bx+cx^2}}\ldots\right)}{24c}$$

$$= \frac{\left(b^2+8ac+2bcx^3\right)\sqrt{a+bx^3+cx^6}}{24c} + \frac{1}{9}\left(a+bx^3+cx^6\right)^{3/2} + \frac{1}{3}a^2\operatorname{Subst}\left(\int \frac{1}{x\sqrt{a+bx\ldots}}\ldots\right)$$

$$= \frac{\left(b^2+8ac+2bcx^3\right)\sqrt{a+bx^3+cx^6}}{24c} + \frac{1}{9}\left(a+bx^3+cx^6\right)^{3/2} - \frac{1}{3}\left(2a^2\right)\operatorname{Subst}\left(\int \frac{1}{4a-x\ldots}\ldots\right)$$

$$= \frac{\left(b^2+8ac+2bcx^3\right)\sqrt{a+bx^3+cx^6}}{24c} + \frac{1}{9}\left(a+bx^3+cx^6\right)^{3/2} - \frac{1}{3}a^{3/2}\tanh^{-1}\left(\frac{2a+\ldots}{2\sqrt{a}\sqrt{a+\ldots}}\ldots\right)$$

Mathematica [A] time = 0.14, size = 143, normalized size = 0.92

$$\frac{1}{144}\left(-48a^{3/2}\tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)-\frac{3b\left(b^2-12ac\right)\tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{c^{3/2}}+\frac{2\sqrt{a+bx^3+cx^6}\left(8c\left(4a+cx^6\right)+3b^2+14bcx^3\right)}{c}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3 + c*x^6)^(3/2)/x,x]
```

```
[Out] ((2*Sqrt[a + b*x^3 + c*x^6]*(3*b^2 + 14*b*c*x^3 + 8*c*(4*a + c*x^6)))/c - 4
8*a^(3/2)*ArcTanh[(2*a + b*x^3)/(2*Sqrt[a]*Sqrt[a + b*x^3 + c*x^6])] - (3*b
*(b^2 - 12*a*c)*ArcTanh[(b + 2*c*x^3)/(2*Sqrt[c]*Sqrt[a + b*x^3 + c*x^6])])
/c^(3/2))/144
```

IntegrateAlgebraic [A] time = 0.70, size = 151, normalized size = 0.97

$$\frac{2}{3}a^{3/2}\tanh^{-1}\left(\frac{\sqrt{c}x^3}{\sqrt{a}}-\frac{\sqrt{a+bx^3+cx^6}}{\sqrt{a}}\right)+\frac{\left(b^3-12abc\right)\log\left(-2c^{3/2}\sqrt{a+bx^3+cx^6}+bc+2c^2x^3\right)}{48c^{3/2}}+\frac{\sqrt{a+bx^3+cx^6}\left(32ac+3b^2+14bcx^3+8c^2x^6\right)}{72c}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x^3 + c*x^6)^(3/2)/x,x]
```

```
[Out] (Sqrt[a + b*x^3 + c*x^6]*(3*b^2 + 32*a*c + 14*b*c*x^3 + 8*c^2*x^6))/(72*c)
+ (2*a^(3/2)*ArcTanh[(Sqrt[c]*x^3)/Sqrt[a] - Sqrt[a + b*x^3 + c*x^6]/Sqrt[a
]])/3 + ((b^3 - 12*a*b*c)*Log[b*c + 2*c^2*x^3 - 2*c^(3/2)*Sqrt[a + b*x^3 +
c*x^6]])/(48*c^(3/2))
```

fricas [A] time = 1.82, size = 727, normalized size = 4.69

$$\left[\frac{48\,a^{\frac{3}{2}}c^2\log\left(-\frac{\left(b^2+4ac\right)x^6+8abx^3-4\sqrt{cx^6+bx^3+a}\left(bx^3+2a\right)\sqrt{a}+8a^2}{x^6}\right)-3\left(b^3-12abc\right)\sqrt{c}\log\left(-8c^2x^6-8bcx^3-b^2-4\sqrt{cx^6+bx^3+a}\left(2cx^3+b\right)\sqrt{c}-4ac\right)+4\left(8c^3x^6+14bc^2x^3+3b^2c+32ac^2\right)\sqrt{cx^6+bx^3+a}}{288c^2},\ \frac{24\,a^{\frac{3}{2}}c^2\log\left(-\frac{\left(b^2+4ac\right)x^6+8abx^3-4\sqrt{cx^6+bx^3+a}\left(bx^3+2a\right)\sqrt{a}+8a^2}{x^6}\right)+3\left(b^3-12abc\right)\sqrt{-c}\arctan\left(\frac{\sqrt{cx^6+bx^3+a}\left(2cx^3+b\right)\sqrt{-c}}{2\left(c^2x^6+bcx^3+ac\right)}\right)+2\left(8c^3x^6+14bc^2x^3+3b^2c+32ac^2\right)\sqrt{cx^6+bx^3+a}}{144c^2},\ \frac{96\sqrt{-a}\,ac^2\arctan\left(\frac{\sqrt{cx^6+bx^3+a}\left(bx^3+2a\right)\sqrt{-a}}{2\left(acx^6+abx^3+a^2\right)}\right)-3\left(b^3-12abc\right)\sqrt{c}\log\left(-8c^2x^6-8bcx^3-b^2-4\sqrt{cx^6+bx^3+a}\left(2cx^3+b\right)\sqrt{c}-4ac\right)+4\left(8c^3x^6+14bc^2x^3+3b^2c+32ac^2\right)\sqrt{cx^6+bx^3+a}}{288c^2},\ \frac{48\sqrt{-a}\,ac^2\arctan\left(\frac{\sqrt{cx^6+bx^3+a}\left(bx^3+2a\right)\sqrt{-a}}{2\left(acx^6+abx^3+a^2\right)}\right)+3\left(b^3-12abc\right)\sqrt{-c}\arctan\left(\frac{\sqrt{cx^6+bx^3+a}\left(2cx^3+b\right)\sqrt{-c}}{2\left(c^2x^6+bcx^3+ac\right)}\right)+2\left(8c^3x^6+14bc^2x^3+3b^2c+32ac^2\right)\sqrt{cx^6+bx^3+a}}{144c^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)^(3/2)/x,x, algorithm="fricas")
```

```
[Out] [1/288*(48*a^(3/2)*c^2*log(-((b^2 + 4*a*c)*x^6 + 8*a*b*x^3 - 4*sqrt(c*x^6 +
 b*x^3 + a)*(b*x^3 + 2*a)*sqrt(a) + 8*a^2)/x^6) - 3*(b^3 - 12*a*b*c)*sqrt(c
)*log(-8*c^2*x^6 - 8*b*c*x^3 - b^2 - 4*sqrt(c*x^6 + b*x^3 + a)*(2*c*x^3 + b
)*sqrt(c) - 4*a*c) + 4*(8*c^3*x^6 + 14*b*c^2*x^3 + 3*b^2*c + 32*a*c^2)*sqrt
(c*x^6 + b*x^3 + a))/c^2, 1/144*(24*a^(3/2)*c^2*log(-((b^2 + 4*a*c)*x^6 + 8
*a*b*x^3 - 4*sqrt(c*x^6 + b*x^3 + a)*(b*x^3 + 2*a)*sqrt(a) + 8*a^2)/x^6) +
3*(b^3 - 12*a*b*c)*sqrt(-c)*arctan(1/2*sqrt(c*x^6 + b*x^3 + a)*(2*c*x^3 + b
)*sqrt(-c)/(c^2*x^6 + b*c*x^3 + a*c)) + 2*(8*c^3*x^6 + 14*b*c^2*x^3 + 3*b^2
*c + 32*a*c^2)*sqrt(c*x^6 + b*x^3 + a))/c^2, 1/288*(96*sqrt(-a)*a*c^2*arcta
n(1/2*sqrt(c*x^6 + b*x^3 + a)*(b*x^3 + 2*a)*sqrt(-a)/(a*c*x^6 + a*b*x^3 + a
^2)) - 3*(b^3 - 12*a*b*c)*sqrt(c)*log(-8*c^2*x^6 - 8*b*c*x^3 - b^2 - 4*sqrt
(c*x^6 + b*x^3 + a)*(2*c*x^3 + b)*sqrt(c) - 4*a*c) + 4*(8*c^3*x^6 + 14*b*c^
2*x^3 + 3*b^2*c + 32*a*c^2)*sqrt(c*x^6 + b*x^3 + a))/c^2, 1/144*(48*sqrt(-a
)*a*c^2*arctan(1/2*sqrt(c*x^6 + b*x^3 + a)*(b*x^3 + 2*a)*sqrt(-a)/(a*c*x^6
+ a*b*x^3 + a^2)) + 3*(b^3 - 12*a*b*c)*sqrt(-c)*arctan(1/2*sqrt(c*x^6 + b*x
^3 + a)*(2*c*x^3 + b)*sqrt(-c)/(c^2*x^6 + b*c*x^3 + a*c)) + 2*(8*c^3*x^6 +
14*b*c^2*x^3 + 3*b^2*c + 32*a*c^2)*sqrt(c*x^6 + b*x^3 + a))/c^2]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cx^6+bx^3+a\right)^{\frac{3}{2}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)^(3/2)/x,x, algorithm="giac")
```

```
[Out] integrate((c*x^6 + b*x^3 + a)^(3/2)/x, x)
```

maple [F] time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{\left(c\,x^6+b\,x^3+a\right)^{\frac{3}{2}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^6+b*x^3+a)^(3/2)/x,x)
```

```
[Out] int((c*x^6+b*x^3+a)^(3/2)/x,x)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)^(3/2)/x,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive, negative or zero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(c\,x^6+b\,x^3+a\right)^{3/2}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3 + c*x^6)^(3/2)/x,x)
```

```
[Out] int((a + b*x^3 + c*x^6)^(3/2)/x, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+bx^3+cx^6\right)^{\frac{3}{2}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**6+b*x**3+a)**(3/2)/x,x)
```

```
[Out] Integral((a + b*x**3 + c*x**6)**(3/2)/x, x)
```

3.189 $\int \frac{(a+bx^3+cx^6)^{3/2}}{x^4}\,dx$

Optimal. Leaf size=150

$$\frac{(4ac+b^2)\tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{8\sqrt{c}}-\frac{(a+bx^3+cx^6)^{3/2}}{3x^3}+\frac{1}{4}(3b+2cx^3)\sqrt{a+bx^3+cx^6}-\frac{1}{2}\sqrt{a}\,b\tanh^{-1}\left(\frac{\quad}{2\sqrt{a}\ \cdots}\right.$$

Rubi [A] time = 0.17, antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.350, Rules used = {1357, 732, 814, 843, 621, 206, 724}

$$\frac{(4ac+b^2)\tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{8\sqrt{c}}-\frac{(a+bx^3+cx^6)^{3/2}}{3x^3}+\frac{1}{4}(3b+2cx^3)\sqrt{a+bx^3+cx^6}-\frac{1}{2}\sqrt{a}\,b\tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3 + c*x^6)^(3/2)/x^4,x]
```

```
[Out] ((3*b + 2*c*x^3)*Sqrt[a + b*x^3 + c*x^6])/4 - (a + b*x^3 + c*x^6)^(3/2)/(3*
x^3) - (Sqrt[a]*b*ArcTanh[(2*a + b*x^3)/(2*Sqrt[a]*Sqrt[a + b*x^3 + c*x^6])
])/2 + ((b^2 + 4*a*c)*ArcTanh[(b + 2*c*x^3)/(2*Sqrt[c]*Sqrt[a + b*x^3 + c*x
^6])])/(8*Sqrt[c])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 732

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 1)), x] - Di
st[p/(e*(m + 1)), Int[(d + e*x)^(m + 1)*(b + 2*c*x)*(a + b*x + c*x^2)^(p -
1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*
d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && GtQ[p, 0] && (IntegerQ[p]
 || LtQ[m, -1]) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && IntQuadraticQ[a,
 b, c, d, e, m, p, x]
```

Rule 814

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2
) - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*(a + b*x + c*x^2)^p)
/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
```

```
2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
|| IntegersQ[2*m, 2*p])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1357

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(a+bx^3+cx^6\right)^{3/2}}{x^4}\,dx &= \frac{1}{3}\,\mathrm{Subst}\left(\int \frac{\left(a+bx+cx^2\right)^{3/2}}{x^2}\,dx, x, x^3\right)\\
&= -\frac{\left(a+bx^3+cx^6\right)^{3/2}}{3x^3}+\frac{1}{2}\,\mathrm{Subst}\left(\int \frac{(b+2cx)\sqrt{a+bx+cx^2}}{x}\,dx, x, x^3\right)\\
&= \frac{1}{4}\left(3b+2cx^3\right)\sqrt{a+bx^3+cx^6}-\frac{\left(a+bx^3+cx^6\right)^{3/2}}{3x^3}-\frac{\mathrm{Subst}\left(\int \frac{-4abc-c\left(b^2+4ac\right)x}{x\sqrt{a+bx+cx^2}}\,dx, x, x
\end{aligned}$$

$$\begin{aligned}
&= \frac{1}{4}\left(3b+2cx^3\right)\sqrt{a+bx^3+cx^6}-\frac{\left(a+bx^3+cx^6\right)^{3/2}}{3x^3}+\frac{1}{2}(ab)\,\mathrm{Subst}\left(\int \frac{1}{x\sqrt{a+bx+cx^2}}\right.\\
&= \frac{1}{4}\left(3b+2cx^3\right)\sqrt{a+bx^3+cx^6}-\frac{\left(a+bx^3+cx^6\right)^{3/2}}{3x^3}-(ab)\,\mathrm{Subst}\left(\int \frac{1}{4a-x^2}\,dx, x, \right.\\
&= \frac{1}{4}\left(3b+2cx^3\right)\sqrt{a+bx^3+cx^6}-\frac{\left(a+bx^3+cx^6\right)^{3/2}}{3x^3}-\frac{1}{2}\sqrt{a}\,b\tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+}}\right.
\end{aligned}$$

Mathematica [A] time = 0.11, size = 134, normalized size = 0.89

$$\frac{1}{24}\left(\frac{3\left(4ac+b^2\right)\tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{\sqrt{c}}+\frac{2\sqrt{a+bx^3+cx^6}\left(-4a+5bx^3+2cx^6\right)}{x^3}-12\sqrt{a}\,b\tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3 + c*x^6)^(3/2)/x^4,x]
```

```
[Out] ((2*Sqrt[a + b*x^3 + c*x^6]*(-4*a + 5*b*x^3 + 2*c*x^6))/x^3 - 12*Sqrt[a]*b*
ArcTanh[(2*a + b*x^3)/(2*Sqrt[a]*Sqrt[a + b*x^3 + c*x^6])] + (3*(b^2 + 4*a*
c)*ArcTanh[(b + 2*c*x^3)/(2*Sqrt[c]*Sqrt[a + b*x^3 + c*x^6])])/Sqrt[c])/24
```

IntegrateAlgebraic [A] time = 0.73, size = 137, normalized size = 0.91

$$\frac{\left(-4ac-b^2\right)\log\left(-2\sqrt{c}\sqrt{a+bx^3+cx^6}+b+2cx^3\right)}{8\sqrt{c}}+\frac{\sqrt{a+bx^3+cx^6}\left(-4a+5bx^3+2cx^6\right)}{12x^3}+\sqrt{a}\,b\tanh^{-1}\left(\frac{\sqrt{c}\,x^3}{\sqrt{a}}-\frac{\sqrt{a+bx^3+cx^6}}{\sqrt{a}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x^3 + c*x^6)^(3/2)/x^4,x]
```

```
[Out] (Sqrt[a + b*x^3 + c*x^6]*(-4*a + 5*b*x^3 + 2*c*x^6))/(12*x^3) + Sqrt[a]*b*A
rcTanh[(Sqrt[c]*x^3)/Sqrt[a] - Sqrt[a + b*x^3 + c*x^6]/Sqrt[a]] + ((-b^2 -
4*a*c)*Log[b + 2*c*x^3 - 2*Sqrt[c]*Sqrt[a + b*x^3 + c*x^6]])/(8*Sqrt[c])
```

fricas [A] time = 1.55, size = 713, normalized size = 4.75

$$\left[\frac{12\sqrt{a}bcx^3\log\left(-\frac{(b^2+4ac)x^6+8abx^3-4\sqrt{cx^6+bx^3+a}(bx^3+2a)\sqrt{a}+8a^2}{x^6}\right)+3\left(b^2+4ac\right)\sqrt{c}x^3\log\left(-8c^2x^6-8bcx^3-b^2-4\sqrt{cx^6+bx^3+a}\left(2cx^3+b\right)\sqrt{c}-4ac\right)+4\left(2c^2x^6+5bcx^3-4ac\right)\sqrt{cx^6+bx^3+a}}{48cx^3}, \frac{6\sqrt{a}bcx^3\log\left(-\frac{(b^2+4ac)x^6+8abx^3-4\sqrt{cx^6+bx^3+a}(bx^3+2a)\sqrt{a}+8a^2}{x^6}\right)-3\left(b^2+4ac\right)\sqrt{-c}x^3\arctan\left(\frac{\sqrt{cx^6+bx^3+a}(2cx^3+b)\sqrt{-c}}{2(c^2x^6+bcx^3+ac)}\right)+2\left(2c^2x^6+5bcx^3-4ac\right)\sqrt{cx^6+bx^3+a}}{24cx^3}, \frac{24\sqrt{-a}bcx^3\arctan\left(\frac{\sqrt{cx^6+bx^3+a}(bx^3+2a)\sqrt{-a}}{2(acx^6+abx^3+a^2)}\right)+3\left(b^2+4ac\right)\sqrt{c}x^3\log\left(-8c^2x^6-8bcx^3-b^2-4\sqrt{cx^6+bx^3+a}\left(2cx^3+b\right)\sqrt{c}-4ac\right)+4\left(2c^2x^6+5bcx^3-4ac\right)\sqrt{cx^6+bx^3+a}}{48cx^3}, \frac{12\sqrt{-a}bcx^3\arctan\left(\frac{\sqrt{cx^6+bx^3+a}(bx^3+2a)\sqrt{-a}}{2(acx^6+abx^3+a^2)}\right)-3\left(b^2+4ac\right)\sqrt{-c}x^3\arctan\left(\frac{\sqrt{cx^6+bx^3+a}(2cx^3+b)\sqrt{-c}}{2(c^2x^6+bcx^3+ac)}\right)+2\left(2c^2x^6+5bcx^3-4ac\right)\sqrt{cx^6+bx^3+a}}{24cx^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)^(3/2)/x^4,x, algorithm="fricas")
```

```
[Out] [1/48*(12*sqrt(a)*b*c*x^3*log(-((b^2 + 4*a*c)*x^6 + 8*a*b*x^3 - 4*sqrt(c*x^
6 + b*x^3 + a)*(b*x^3 + 2*a)*sqrt(a) + 8*a^2)/x^6) + 3*(b^2 + 4*a*c)*sqrt(c
)*x^3*log(-8*c^2*x^6 - 8*b*c*x^3 - b^2 - 4*sqrt(c*x^6 + b*x^3 + a)*(2*c*x^3
 + b)*sqrt(c) - 4*a*c) + 4*(2*c^2*x^6 + 5*b*c*x^3 - 4*a*c)*sqrt(c*x^6 + b*x
^3 + a))/(c*x^3), 1/24*(6*sqrt(a)*b*c*x^3*log(-((b^2 + 4*a*c)*x^6 + 8*a*b*x
^3 - 4*sqrt(c*x^6 + b*x^3 + a)*(b*x^3 + 2*a)*sqrt(a) + 8*a^2)/x^6) - 3*(b^2
 + 4*a*c)*sqrt(-c)*x^3*arctan(1/2*sqrt(c*x^6 + b*x^3 + a)*(2*c*x^3 + b)*sqr
t(-c)/(c^2*x^6 + b*c*x^3 + a*c)) + 2*(2*c^2*x^6 + 5*b*c*x^3 - 4*a*c)*sqrt(c
*x^6 + b*x^3 + a))/(c*x^3), 1/48*(24*sqrt(-a)*b*c*x^3*arctan(1/2*sqrt(c*x^6
 + b*x^3 + a)*(b*x^3 + 2*a)*sqrt(-a)/(a*c*x^6 + a*b*x^3 + a^2)) + 3*(b^2 +
4*a*c)*sqrt(c)*x^3*log(-8*c^2*x^6 - 8*b*c*x^3 - b^2 - 4*sqrt(c*x^6 + b*x^3
+ a)*(2*c*x^3 + b)*sqrt(c) - 4*a*c) + 4*(2*c^2*x^6 + 5*b*c*x^3 - 4*a*c)*sqr
t(c*x^6 + b*x^3 + a))/(c*x^3), 1/24*(12*sqrt(-a)*b*c*x^3*arctan(1/2*sqrt(c*
x^6 + b*x^3 + a)*(b*x^3 + 2*a)*sqrt(-a)/(a*c*x^6 + a*b*x^3 + a^2)) - 3*(b^2
 + 4*a*c)*sqrt(-c)*x^3*arctan(1/2*sqrt(c*x^6 + b*x^3 + a)*(2*c*x^3 + b)*sqr
t(-c)/(c^2*x^6 + b*c*x^3 + a*c)) + 2*(2*c^2*x^6 + 5*b*c*x^3 - 4*a*c)*sqrt(c
*x^6 + b*x^3 + a))/(c*x^3)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cx^6+bx^3+a\right)^{\frac{3}{2}}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)^(3/2)/x^4,x, algorithm="giac")
```

```
[Out] integrate((c*x^6 + b*x^3 + a)^(3/2)/x^4, x)
```

maple [F] time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{\left(c\,x^6+b\,x^3+a\right)^{\frac{3}{2}}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^6+b*x^3+a)^(3/2)/x^4,x)
```

```
[Out] int((c*x^6+b*x^3+a)^(3/2)/x^4,x)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)^(3/2)/x^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive, negative or zero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(c\,x^6+b\,x^3+a\right)^{3/2}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3 + c*x^6)^(3/2)/x^4,x)
```

```
[Out] int((a + b*x^3 + c*x^6)^(3/2)/x^4, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+bx^3+cx^6\right)^{\frac{3}{2}}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**6+b*x**3+a)**(3/2)/x**4,x)
```

```
[Out] Integral((a + b*x**3 + c*x**6)**(3/2)/x**4, x)
```

3.190 $\int \frac{(a+bx^3+cx^6)^{3/2}}{x^7}\,dx$

Optimal. Leaf size=151

$$-\frac{(4ac+b^2)\tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{8\sqrt{a}}-\frac{(a+bx^3+cx^6)^{3/2}}{6x^6}-\frac{(b-2cx^3)\sqrt{a+bx^3+cx^6}}{4x^3}+\frac{1}{2}b\sqrt{c}\tanh^{-1}\left(\frac{b}{2\sqrt{c}\sqrt{a}}\right.$$

Rubi [A] time = 0.16, antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.350, Rules used = {1357, 732, 812, 843, 621, 206, 724}

$$-\frac{(4ac+b^2)\tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{8\sqrt{a}}-\frac{(a+bx^3+cx^6)^{3/2}}{6x^6}-\frac{(b-2cx^3)\sqrt{a+bx^3+cx^6}}{4x^3}+\frac{1}{2}b\sqrt{c}\tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3 + c*x^6)^(3/2)/x^7,x]
```

```
[Out] -((b - 2*c*x^3)*Sqrt[a + b*x^3 + c*x^6])/(4*x^3) - (a + b*x^3 + c*x^6)^(3/2
)/(6*x^6) - ((b^2 + 4*a*c)*ArcTanh[(2*a + b*x^3)/(2*Sqrt[a]*Sqrt[a + b*x^3
+ c*x^6])])/(8*Sqrt[a]) + (b*Sqrt[c]*ArcTanh[(b + 2*c*x^3)/(2*Sqrt[c]*Sqrt[
a + b*x^3 + c*x^6])])/2
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 732

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 1)), x] - Di
st[p/(e*(m + 1)), Int[(d + e*x)^(m + 1)*(b + 2*c*x)*(a + b*x + c*x^2)^(p -
1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*
d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && GtQ[p, 0] && (IntegerQ[p]
 || LtQ[m, -1]) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && IntQuadraticQ[a,
 b, c, d, e, m, p, x]
```

Rule 812

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(e*f*(m + 2*p + 2)
- d*g*(2*p + 1) + e*g*(m + 1)*x)*(a + b*x + c*x^2)^p)/(e^2*(m + 1)*(m + 2*p
```

```
 + 2)), x] + Dist[p/(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a +
 b*x + c*x^2)^(p - 1)*Simp[g*(b*d + 2*a*e + 2*a*e*m + 2*b*d*p) - f*b*e*(m +
 2*p + 2) + (g*(2*c*d + b*e + b*e*m + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] && RationalQ[p] && p > 0 && (LtQ[m, -1] || Eq
Q[p, 1] || (IntegerQ[p] &&  !RationalQ[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p
 + 1, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1357

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(a+bx^3+cx^6\right)^{3/2}}{x^7}\,dx &= \frac{1}{3}\operatorname{Subst}\left(\int \frac{\left(a+bx+cx^2\right)^{3/2}}{x^3}\,dx,x,x^3\right)\\
&= -\frac{\left(a+bx^3+cx^6\right)^{3/2}}{6x^6}+\frac{1}{4}\operatorname{Subst}\left(\int \frac{(b+2cx)\sqrt{a+bx+cx^2}}{x^2}\,dx,x,x^3\right)\\
&= -\frac{\left(b-2cx^3\right)\sqrt{a+bx^3+cx^6}}{4x^3}-\frac{\left(a+bx^3+cx^6\right)^{3/2}}{6x^6}-\frac{1}{8}\operatorname{Subst}\left(\int \frac{-b^2-4ac-4bcx}{x\sqrt{a+bx+cx^2}}\,dx,\right.\\
&= -\frac{\left(b-2cx^3\right)\sqrt{a+bx^3+cx^6}}{4x^3}-\frac{\left(a+bx^3+cx^6\right)^{3/2}}{6x^6}+\frac{1}{2}(bc)\operatorname{Subst}\left(\int \frac{1}{\sqrt{a+bx+cx^2}}\,d\right.\\
&= -\frac{\left(b-2cx^3\right)\sqrt{a+bx^3+cx^6}}{4x^3}-\frac{\left(a+bx^3+cx^6\right)^{3/2}}{6x^6}+(bc)\operatorname{Subst}\left(\int \frac{1}{4c-x^2}\,dx,x,\frac{b}{\sqrt{a+}}\right.\\
&= -\frac{\left(b-2cx^3\right)\sqrt{a+bx^3+cx^6}}{4x^3}-\frac{\left(a+bx^3+cx^6\right)^{3/2}}{6x^6}-\frac{\left(b^2+4ac\right)\tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right.}{8\sqrt{a}}
\end{aligned}$$

Mathematica [A] time = 0.17, size = 134, normalized size = 0.89

$$\frac{1}{24}\left(-\frac{3\left(4ac+b^2\right)\tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{\sqrt{a}}-\frac{2\sqrt{a+bx^3+cx^6}\left(2a+5bx^3-4cx^6\right)}{x^6}+12b\sqrt{c}\tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3 + c*x^6)^(3/2)/x^7,x]
```

```
[Out] ((-2*(2*a + 5*b*x^3 - 4*c*x^6)*Sqrt[a + b*x^3 + c*x^6])/x^6 - (3*(b^2 + 4*a
*c)*ArcTanh[(2*a + b*x^3)/(2*Sqrt[a]*Sqrt[a + b*x^3 + c*x^6])])/Sqrt[a] + 1
2*b*Sqrt[c]*ArcTanh[(b + 2*c*x^3)/(2*Sqrt[c]*Sqrt[a + b*x^3 + c*x^6])])/24
```

IntegrateAlgebraic [A] time = 0.80, size = 134, normalized size = 0.89

$$\frac{(4ac+b^2)\tanh^{-1}\left(\frac{\sqrt{c}x^3-\sqrt{a+bx^3+cx^6}}{\sqrt{a}}\right)}{4\sqrt{a}}+\frac{\sqrt{a+bx^3+cx^6}\,(-2a-5bx^3+4cx^6)}{12x^6}-\frac{1}{2}b\sqrt{c}\log\left(-2\sqrt{c}\sqrt{a+bx^3+cx^6}+b+2cx^3\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x^3 + c*x^6)^(3/2)/x^7,x]
```

```
[Out] (Sqrt[a + b*x^3 + c*x^6]*(-2*a - 5*b*x^3 + 4*c*x^6))/(12*x^6) + ((b^2 + 4*a
*c)*ArcTanh[(Sqrt[c]*x^3 - Sqrt[a + b*x^3 + c*x^6])/Sqrt[a]])/(4*Sqrt[a]) -
 (b*Sqrt[c]*Log[b + 2*c*x^3 - 2*Sqrt[c]*Sqrt[a + b*x^3 + c*x^6]])/2
```

fricas [A] time = 1.46, size = 713, normalized size = 4.72

$$\left[\frac{12ab\sqrt{c}x^6\log\left(-8c^2x^6-8bcx^3-b^2-4\sqrt{cx^6+bx^3+a}(2cx^3+b)\sqrt{c}-4ac\right)+3(b^2+4ac)\sqrt{a}x^6\log\left(-\frac{(b^2+4ac)x^6+8abx^3-4\sqrt{cx^6+bx^3+a}(bx^3+2a)\sqrt{a}+8a^2}{x^6}\right)+4(4acx^6-5abx^3-2a^2)\sqrt{cx^6+bx^3+a}}{48ax^6},\ -\frac{24ab\sqrt{-c}x^6\arctan\left(\frac{\sqrt{cx^6+bx^3+a}(2cx^3+b)\sqrt{-c}}{2(c^2x^6+bcx^3+ac)}\right)-3(b^2+4ac)\sqrt{a}x^6\log\left(-\frac{(b^2+4ac)x^6+8abx^3-4\sqrt{cx^6+bx^3+a}(bx^3+2a)\sqrt{a}+8a^2}{x^6}\right)-4(4acx^6-5abx^3-2a^2)\sqrt{cx^6+bx^3+a}}{48ax^6},\ \frac{6ab\sqrt{c}x^6\log\left(-8c^2x^6-8bcx^3-b^2-4\sqrt{cx^6+bx^3+a}(2cx^3+b)\sqrt{c}-4ac\right)+3(b^2+4ac)\sqrt{-a}x^6\arctan\left(\frac{\sqrt{cx^6+bx^3+a}(bx^3+2a)\sqrt{-a}}{2(acx^6+abx^3+a^2)}\right)+2(4acx^6-5abx^3-2a^2)\sqrt{cx^6+bx^3+a}}{24ax^6},\ -\frac{12ab\sqrt{-c}x^6\arctan\left(\frac{\sqrt{cx^6+bx^3+a}(2cx^3+b)\sqrt{-c}}{2(c^2x^6+bcx^3+ac)}\right)-3(b^2+4ac)\sqrt{-a}x^6\arctan\left(\frac{\sqrt{cx^6+bx^3+a}(bx^3+2a)\sqrt{-a}}{2(acx^6+abx^3+a^2)}\right)-2(4acx^6-5abx^3-2a^2)\sqrt{cx^6+bx^3+a}}{24ax^6}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)^(3/2)/x^7,x, algorithm="fricas")
```

```
[Out] [1/48*(12*a*b*sqrt(c)*x^6*log(-8*c^2*x^6 - 8*b*c*x^3 - b^2 - 4*sqrt(c*x^6 +
 b*x^3 + a)*(2*c*x^3 + b)*sqrt(c) - 4*a*c) + 3*(b^2 + 4*a*c)*sqrt(a)*x^6*lo
g(-((b^2 + 4*a*c)*x^6 + 8*a*b*x^3 - 4*sqrt(c*x^6 + b*x^3 + a)*(b*x^3 + 2*a)
*sqrt(a) + 8*a^2)/x^6) + 4*(4*a*c*x^6 - 5*a*b*x^3 - 2*a^2)*sqrt(c*x^6 + b*x
^3 + a))/(a*x^6), -1/48*(24*a*b*sqrt(-c)*x^6*arctan(1/2*sqrt(c*x^6 + b*x^3
+ a)*(2*c*x^3 + b)*sqrt(-c)/(c^2*x^6 + b*c*x^3 + a*c)) - 3*(b^2 + 4*a*c)*sq
rt(a)*x^6*log(-((b^2 + 4*a*c)*x^6 + 8*a*b*x^3 - 4*sqrt(c*x^6 + b*x^3 + a)*(
b*x^3 + 2*a)*sqrt(a) + 8*a^2)/x^6) - 4*(4*a*c*x^6 - 5*a*b*x^3 - 2*a^2)*sqrt
(c*x^6 + b*x^3 + a))/(a*x^6), 1/24*(6*a*b*sqrt(c)*x^6*log(-8*c^2*x^6 - 8*b*
c*x^3 - b^2 - 4*sqrt(c*x^6 + b*x^3 + a)*(2*c*x^3 + b)*sqrt(c) - 4*a*c) + 3*
(b^2 + 4*a*c)*sqrt(-a)*x^6*arctan(1/2*sqrt(c*x^6 + b*x^3 + a)*(b*x^3 + 2*a)
*sqrt(-a)/(a*c*x^6 + a*b*x^3 + a^2)) + 2*(4*a*c*x^6 - 5*a*b*x^3 - 2*a^2)*sq
rt(c*x^6 + b*x^3 + a))/(a*x^6), -1/24*(12*a*b*sqrt(-c)*x^6*arctan(1/2*sqrt(
c*x^6 + b*x^3 + a)*(2*c*x^3 + b)*sqrt(-c)/(c^2*x^6 + b*c*x^3 + a*c)) - 3*(b
^2 + 4*a*c)*sqrt(-a)*x^6*arctan(1/2*sqrt(c*x^6 + b*x^3 + a)*(b*x^3 + 2*a)*s
qrt(-a)/(a*c*x^6 + a*b*x^3 + a^2)) - 2*(4*a*c*x^6 - 5*a*b*x^3 - 2*a^2)*sqrt
(c*x^6 + b*x^3 + a))/(a*x^6)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(cx^6+bx^3+a)^{\frac{3}{2}}}{x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)^(3/2)/x^7,x, algorithm="giac")
```

```
[Out] integrate((c*x^6 + b*x^3 + a)^(3/2)/x^7, x)
```

maple [F] time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{(c\,x^6+b\,x^3+a)^{\frac{3}{2}}}{x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^6+b*x^3+a)^(3/2)/x^7,x)
```

```
[Out] int((c*x^6+b*x^3+a)^(3/2)/x^7,x)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)^(3/2)/x^7,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive, negative or zero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(c\,x^6+b\,x^3+a\right)^{3/2}}{x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3 + c*x^6)^(3/2)/x^7,x)
```

```
[Out] int((a + b*x^3 + c*x^6)^(3/2)/x^7, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+bx^3+cx^6\right)^{\frac{3}{2}}}{x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**6+b*x**3+a)**(3/2)/x**7,x)
```

```
[Out] Integral((a + b*x**3 + c*x**6)**(3/2)/x**7, x)
```

3.191 $\int \frac{(a+bx^3+cx^6)^{3/2}}{x^{10}}\,dx$

Optimal. Leaf size=163

$$\frac{b\left(b^2-12ac\right)\tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{48a^{3/2}}-\frac{\left(x^3\left(8ac+b^2\right)+2ab\right)\sqrt{a+bx^3+cx^6}}{24ax^6}+\frac{1}{3}c^{3/2}\tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+}}\right.$$

Rubi [A] time = 0.18, antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.350$, Rules used = {1357, 732, 810, 843, 621, 206, 724}

$$\frac{b\left(b^2-12ac\right)\tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{48a^{3/2}}-\frac{\left(x^3\left(8ac+b^2\right)+2ab\right)\sqrt{a+bx^3+cx^6}}{24ax^6}+\frac{1}{3}c^{3/2}\tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)-\frac{\left(a+bx^3+cx^6\right)^{3/2}}{9x^9}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3 + c*x^6)^(3/2)/x^10,x]
```

```
[Out] -((2*a*b + (b^2 + 8*a*c)*x^3)*Sqrt[a + b*x^3 + c*x^6])/(24*a*x^6) - (a + b*
x^3 + c*x^6)^(3/2)/(9*x^9) + (b*(b^2 - 12*a*c)*ArcTanh[(2*a + b*x^3)/(2*Sqr
t[a]*Sqrt[a + b*x^3 + c*x^6])])/(48*a^(3/2)) + (c^(3/2)*ArcTanh[(b + 2*c*x^
3)/(2*Sqrt[c]*Sqrt[a + b*x^3 + c*x^6])])/3
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 732

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 1)), x] - Di
st[p/(e*(m + 1)), Int[(d + e*x)^(m + 1)*(b + 2*c*x)*(a + b*x + c*x^2)^(p -
1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*
d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && GtQ[p, 0] && (IntegerQ[p]
 || LtQ[m, -1]) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && IntQuadraticQ[a,
 b, c, d, e, m, p, x]
```

Rule 810

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^
p*((d*g - e*f*(m + 2))*(c*d^2 - b*d*e + a*e^2) - d*p*(2*c*d - b*e)*(e*f - d
*g) - e*(g*(m + 1)*(c*d^2 - b*d*e + a*e^2) + p*(2*c*d - b*e)*(e*f - d*g))*x
```

```
))/(e^2*(m + 1)*(m + 2)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[p/(e^2*(m + 1)*
(m + 2)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)^(
p - 1)*Simp[2*a*c*e*(e*f - d*g)*(m + 2) + b^2*e*(d*g*(p + 1) - e*f*(m + p +
 2)) + b*(a*e^2*g*(m + 1) - c*d*(d*g*(2*p + 1) - e*f*(m + 2*p + 2))) - c*(2
*c*d*(d*g*(2*p + 1) - e*f*(m + 2*p + 2)) - e*(2*a*e*g*(m + 1) - b*(d*g*(m -
 2*p) + e*f*(m + 2*p + 2))))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g},
x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[p, 0] &&
LtQ[m, -2] && LtQ[m + 2*p, 0] &&  !ILtQ[m + 2*p + 3, 0]
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1357

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{\left(a+bx^3+cx^6\right)^{3/2}}{x^{10}}\,dx = \frac{1}{3}\text{Subst}\left(\int \frac{\left(a+bx+cx^2\right)^{3/2}}{x^4}\,dx,x,x^3\right)$$
$$= -\frac{\left(a+bx^3+cx^6\right)^{3/2}}{9x^9}+\frac{1}{6}\text{Subst}\left(\int \frac{(b+2cx)\sqrt{a+bx+cx^2}}{x^3}\,dx,x,x^3\right)$$
$$= -\frac{\left(2ab+\left(b^2+8ac\right)x^3\right)\sqrt{a+bx^3+cx^6}}{24ax^6}-\frac{\left(a+bx^3+cx^6\right)^{3/2}}{9x^9}-\frac{\text{Subst}\left(\int \frac{\frac{1}{2}b\left(b^2-12ac\right)-8a}{x\sqrt{a+bx+cx^2}}\right.}{24a}$$
$$= -\frac{\left(2ab+\left(b^2+8ac\right)x^3\right)\sqrt{a+bx^3+cx^6}}{24ax^6}-\frac{\left(a+bx^3+cx^6\right)^{3/2}}{9x^9}+\frac{1}{3}c^2\,\text{Subst}\left(\int \frac{1}{\sqrt{a+b}}\right.$$
$$= -\frac{\left(2ab+\left(b^2+8ac\right)x^3\right)\sqrt{a+bx^3+cx^6}}{24ax^6}-\frac{\left(a+bx^3+cx^6\right)^{3/2}}{9x^9}+\frac{1}{3}\left(2c^2\right)\text{Subst}\left(\int \frac{1}{4c-}\right.$$
$$= -\frac{\left(2ab+\left(b^2+8ac\right)x^3\right)\sqrt{a+bx^3+cx^6}}{24ax^6}-\frac{\left(a+bx^3+cx^6\right)^{3/2}}{9x^9}+\frac{b\left(b^2-12ac\right)\tanh^{-1}\left(\right.}{48a^{3/2}}$$

Mathematica [A] time = 0.21, size = 149, normalized size = 0.91

$$\frac{1}{144}\left(\frac{3b\left(b^2-12ac\right)\tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{a^{3/2}}-\frac{2\sqrt{a+bx^3+cx^6}\left(8a^2+14abx^3+32acx^6+3b^2x^6\right)}{ax^9}+48c^{3/2}\tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3 + c*x^6)^(3/2)/x^10,x]
```

```
[Out] ((-2*Sqrt[a + b*x^3 + c*x^6]*(8*a^2 + 14*a*b*x^3 + 3*b^2*x^6 + 32*a*c*x^6))
/(a*x^9) + (3*b*(b^2 - 12*a*c)*ArcTanh[(2*a + b*x^3)/(2*Sqrt[a]*Sqrt[a + b*
x^3 + c*x^6])])/a^(3/2) + 48*c^(3/2)*ArcTanh[(b + 2*c*x^3)/(2*Sqrt[c]*Sqrt[
a + b*x^3 + c*x^6])])/144
```

IntegrateAlgebraic [A] time = 0.99, size = 148, normalized size = 0.91

$$\frac{\left(b^3-12abc\right)\tanh^{-1}\left(\frac{\sqrt{a+bx^3+cx^6}-\sqrt{c}x^3}{\sqrt{a}}\right)}{24a^{3/2}}+\frac{\sqrt{a+bx^3+cx^6}\left(-8a^2-14abx^3-32acx^6-3b^2x^6\right)}{72ax^9}-\frac{1}{3}c^{3/2}\log\left(-2\sqrt{c}\sqrt{a+bx^3+cx^6}+b+2cx^3\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x^3 + c*x^6)^(3/2)/x^10,x]
```

```
[Out] (Sqrt[a + b*x^3 + c*x^6]*(-8*a^2 - 14*a*b*x^3 - 3*b^2*x^6 - 32*a*c*x^6))/(7
2*a*x^9) + ((b^3 - 12*a*b*c)*ArcTanh[(-(Sqrt[c]*x^3) + Sqrt[a + b*x^3 + c*x
^6])/Sqrt[a]])/(24*a^(3/2)) - (c^(3/2)*Log[b + 2*c*x^3 - 2*Sqrt[c]*Sqrt[a +
 b*x^3 + c*x^6]])/3
```

fricas [A] time = 1.60, size = 771, normalized size = 4.73

$$\left[\frac{48a^2c^{\frac{3}{2}}x^9\log\left(-8c^2x^6-8bcx^3-b^2-4\sqrt{cx^6+bx^3+a}\left(2cx^3+b\right)\sqrt{c}-4ac\right)-3\left(b^3-12abc\right)\sqrt{a}x^9\log\left(-\frac{\left(b^2+4ac\right)x^6+8abx^3-4\sqrt{cx^6+bx^3+a}\left(bx^3+2a\right)\sqrt{a}+8a^2}{x^6}\right)-4\left(\left(3ab^2+32a^2c\right)x^6+14a^2bx^3+8a^3\right)\sqrt{cx^6+bx^3+a}}{288a^2x^9}, -\frac{96a^2\sqrt{-c}cx^9\arctan\left(\frac{\sqrt{cx^6+bx^3+a}\left(2cx^3+b\right)\sqrt{-c}}{2\left(c^2x^6+bcx^3+ac\right)}\right)+3\left(b^3-12abc\right)\sqrt{a}x^9\log\left(-\frac{\left(b^2+4ac\right)x^6+8abx^3-4\sqrt{cx^6+bx^3+a}\left(bx^3+2a\right)\sqrt{a}+8a^2}{x^6}\right)+4\left(\left(3ab^2+32a^2c\right)x^6+14a^2bx^3+8a^3\right)\sqrt{cx^6+bx^3+a}}{288a^2x^9}, \frac{24a^2c^{\frac{3}{2}}x^9\log\left(-8c^2x^6-8bcx^3-b^2-4\sqrt{cx^6+bx^3+a}\left(2cx^3+b\right)\sqrt{c}-4ac\right)-3\left(b^3-12abc\right)\sqrt{-a}x^9\arctan\left(\frac{\sqrt{cx^6+bx^3+a}\left(bx^3+2a\right)\sqrt{-a}}{2\left(acx^6+abx^3+a^2\right)}\right)-2\left(\left(3ab^2+32a^2c\right)x^6+14a^2bx^3+8a^3\right)\sqrt{cx^6+bx^3+a}}{144a^2x^9}, -\frac{48a^2\sqrt{-c}cx^9\arctan\left(\frac{\sqrt{cx^6+bx^3+a}\left(2cx^3+b\right)\sqrt{-c}}{2\left(c^2x^6+bcx^3+ac\right)}\right)+3\left(b^3-12abc\right)\sqrt{-a}x^9\arctan\left(\frac{\sqrt{cx^6+bx^3+a}\left(bx^3+2a\right)\sqrt{-a}}{2\left(acx^6+abx^3+a^2\right)}\right)+2\left(\left(3ab^2+32a^2c\right)x^6+14a^2bx^3+8a^3\right)\sqrt{cx^6+bx^3+a}}{144a^2x^9}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)^(3/2)/x^10,x, algorithm="fricas")
```

```
[Out] [1/288*(48*a^2*c^(3/2)*x^9*log(-8*c^2*x^6 - 8*b*c*x^3 - b^2 - 4*sqrt(c*x^6
+ b*x^3 + a)*(2*c*x^3 + b)*sqrt(c) - 4*a*c) - 3*(b^3 - 12*a*b*c)*sqrt(a)*x^
9*log(-((b^2 + 4*a*c)*x^6 + 8*a*b*x^3 - 4*sqrt(c*x^6 + b*x^3 + a)*(b*x^3 +
2*a)*sqrt(a) + 8*a^2)/x^6) - 4*((3*a*b^2 + 32*a^2*c)*x^6 + 14*a^2*b*x^3 + 8
*a^3)*sqrt(c*x^6 + b*x^3 + a))/(a^2*x^9), -1/288*(96*a^2*sqrt(-c)*c*x^9*arc
tan(1/2*sqrt(c*x^6 + b*x^3 + a)*(2*c*x^3 + b)*sqrt(-c)/(c^2*x^6 + b*c*x^3 +
 a*c)) + 3*(b^3 - 12*a*b*c)*sqrt(a)*x^9*log(-((b^2 + 4*a*c)*x^6 + 8*a*b*x^3
 - 4*sqrt(c*x^6 + b*x^3 + a)*(b*x^3 + 2*a)*sqrt(a) + 8*a^2)/x^6) + 4*((3*a*
b^2 + 32*a^2*c)*x^6 + 14*a^2*b*x^3 + 8*a^3)*sqrt(c*x^6 + b*x^3 + a))/(a^2*x
^9), 1/144*(24*a^2*c^(3/2)*x^9*log(-8*c^2*x^6 - 8*b*c*x^3 - b^2 - 4*sqrt(c*
x^6 + b*x^3 + a)*(2*c*x^3 + b)*sqrt(c) - 4*a*c) - 3*(b^3 - 12*a*b*c)*sqrt(-
a)*x^9*arctan(1/2*sqrt(c*x^6 + b*x^3 + a)*(b*x^3 + 2*a)*sqrt(-a)/(a*c*x^6 +
 a*b*x^3 + a^2)) - 2*((3*a*b^2 + 32*a^2*c)*x^6 + 14*a^2*b*x^3 + 8*a^3)*sqrt
(c*x^6 + b*x^3 + a))/(a^2*x^9), -1/144*(48*a^2*sqrt(-c)*c*x^9*arctan(1/2*sq
rt(c*x^6 + b*x^3 + a)*(2*c*x^3 + b)*sqrt(-c)/(c^2*x^6 + b*c*x^3 + a*c)) + 3
*(b^3 - 12*a*b*c)*sqrt(-a)*x^9*arctan(1/2*sqrt(c*x^6 + b*x^3 + a)*(b*x^3 +
2*a)*sqrt(-a)/(a*c*x^6 + a*b*x^3 + a^2)) + 2*((3*a*b^2 + 32*a^2*c)*x^6 + 14
*a^2*b*x^3 + 8*a^3)*sqrt(c*x^6 + b*x^3 + a))/(a^2*x^9)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(cx^6+bx^3+a\right)^{\frac{3}{2}}}{x^{10}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)^(3/2)/x^10,x, algorithm="giac")
```

```
[Out] integrate((c*x^6 + b*x^3 + a)^(3/2)/x^10, x)
```

maple [F] time = 0.06, size = 0, normalized size = 0.00

$$\int\frac{\left(c\,x^6+b\,x^3+a\right)^{\frac{3}{2}}}{x^{10}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c*x^6+b*x^3+a)^(3/2)/x^10,x)

[Out] int((c*x^6+b*x^3+a)^(3/2)/x^10,x)

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^6+b*x^3+a)^(3/2)/x^10,x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive, negative or zero?

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(c\,x^6+b\,x^3+a\right)^{3/2}}{x^{10}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b*x^3 + c*x^6)^(3/2)/x^10,x)

[Out] int((a + b*x^3 + c*x^6)^(3/2)/x^10, x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a + bx^{3} + cx^{6}\right)^{\frac{3}{2}}}{x^{10}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x**6+b*x**3+a)**(3/2)/x**10,x)

[Out] Integral((a + b*x**3 + c*x**6)**(3/2)/x**10, x)

3.192 $\int \frac{\left(a+bx^3+cx^6\right)^{3/2}}{x^{13}} \, dx$

Optimal. Leaf size=133

$$-\frac{\left(b^2-4ac\right)^2 \tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{128a^{5/2}}+\frac{\left(b^2-4ac\right)\left(2a+bx^3\right)\sqrt{a+bx^3+cx^6}}{64a^2x^6}-\frac{\left(2a+bx^3\right)\left(a+bx^3+cx^6\right)^{3/2}}{24ax^{12}}$$

Rubi [A] time = 0.11, antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1357, 720, 724, 206}

$$\frac{\left(b^2-4ac\right)\left(2a+bx^3\right)\sqrt{a+bx^3+cx^6}}{64a^2x^6}-\frac{\left(b^2-4ac\right)^2 \tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{128a^{5/2}}-\frac{\left(2a+bx^3\right)\left(a+bx^3+cx^6\right)^{3/2}}{24ax^{12}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3 + c*x^6)^(3/2)/x^13,x]
```

```
[Out] ((b^2 - 4*a*c)*(2*a + b*x^3)*Sqrt[a + b*x^3 + c*x^6])/(64*a^2*x^6) - ((2*a
+ b*x^3)*(a + b*x^3 + c*x^6)^(3/2))/(24*a*x^12) - ((b^2 - 4*a*c)^2*ArcTanh[
(2*a + b*x^3)/(2*Sqrt[a]*Sqrt[a + b*x^3 + c*x^6])])/(128*a^(5/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 720

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> -Simp[((d + e*x)^(m + 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*(a + b*x
 + c*x^2)^p)/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[(p*(b^2 - 4*a*c
))/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 2)*(a + b*x + c*
x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] &&
 NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && EqQ[m + 2*p + 2, 0
] && GtQ[p, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1357

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{\left(a+bx^3+cx^6\right)^{3/2}}{x^{13}}\,dx=\frac{1}{3}\text{Subst}\left(\int \frac{\left(a+bx+cx^2\right)^{3/2}}{x^5}\,dx,x,x^3\right)$$
$$=-\frac{\left(2a+bx^3\right)\left(a+bx^3+cx^6\right)^{3/2}}{24ax^{12}}-\frac{\left(b^2-4ac\right)\text{Subst}\left(\int \frac{\sqrt{a+bx+cx^2}}{x^3}\,dx,x,x^3\right)}{16a}$$
$$=\frac{\left(b^2-4ac\right)\left(2a+bx^3\right)\sqrt{a+bx^3+cx^6}}{64a^2x^6}-\frac{\left(2a+bx^3\right)\left(a+bx^3+cx^6\right)^{3/2}}{24ax^{12}}+\frac{\left(b^2-4ac\right)^2 \text{S}}{}$$
$$=\frac{\left(b^2-4ac\right)\left(2a+bx^3\right)\sqrt{a+bx^3+cx^6}}{64a^2x^6}-\frac{\left(2a+bx^3\right)\left(a+bx^3+cx^6\right)^{3/2}}{24ax^{12}}-\frac{\left(b^2-4ac\right)^2 \text{S}}{}$$
$$=\frac{\left(b^2-4ac\right)\left(2a+bx^3\right)\sqrt{a+bx^3+cx^6}}{64a^2x^6}-\frac{\left(2a+bx^3\right)\left(a+bx^3+cx^6\right)^{3/2}}{24ax^{12}}-\frac{\left(b^2-4ac\right)^2 \text{ta}}{}$$

Mathematica [A] time = 0.16, size = 138, normalized size = 1.04

$$-\frac{\frac{3\left(b^2-4ac\right)\left(x^6\left(b^2-4ac\right)\tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)-2\sqrt{a}\left(2a+bx^3\right)\sqrt{a+bx^3+cx^6}\right)}{8a^{3/2}x^6}+\frac{2\left(2a+bx^3\right)\left(a+bx^3+cx^6\right)^{3/2}}{x^{12}}}{48a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3 + c*x^6)^(3/2)/x^13,x]
```

```
[Out] -1/48*((2*(2*a + b*x^3)*(a + b*x^3 + c*x^6)^(3/2))/x^12 + (3*(b^2 - 4*a*c)*
(-2*Sqrt[a]*(2*a + b*x^3)*Sqrt[a + b*x^3 + c*x^6] + (b^2 - 4*a*c)*x^6*ArcTa
nh[(2*a + b*x^3)/(2*Sqrt[a]*Sqrt[a + b*x^3 + c*x^6])]))/(8*a^(3/2)*x^6))/a
```

IntegrateAlgebraic [A] time = 1.11, size = 139, normalized size = 1.05

$$\frac{\left(16a^2c^2-8ab^2c+b^4\right)\tanh^{-1}\left(\frac{\sqrt{c}x^3-\sqrt{a+bx^3+cx^6}}{\sqrt{a}}\right)}{64a^{5/2}}+\frac{\sqrt{a+bx^3+cx^6}\left(-16a^3-24a^2bx^3-40a^2cx^6-2ab^2x^6-20abcx^9+3b^3x^9\right)}{192a^2x^{12}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x^3 + c*x^6)^(3/2)/x^13,x]
```

```
[Out] (Sqrt[a + b*x^3 + c*x^6]*(-16*a^3 - 24*a^2*b*x^3 - 2*a*b^2*x^6 - 40*a^2*c*x
^6 + 3*b^3*x^9 - 20*a*b*c*x^9))/(192*a^2*x^12) + ((b^4 - 8*a*b^2*c + 16*a^2
*c^2)*ArcTanh[(Sqrt[c]*x^3 - Sqrt[a + b*x^3 + c*x^6])/Sqrt[a]])/(64*a^(5/2)
)
```

fricas [A] time = 1.40, size = 319, normalized size = 2.40

$$\left[\frac{3\left(b^4-8ab^2c+16a^2c^2\right)\sqrt{a}x^{12}\log\left(-\frac{\left(b^2+4ac\right)x^6+8abx^3-4\sqrt{cx^6+bx^3+a}\left(bx^3+2a\right)\sqrt{a}+8a^2}{x^6}\right)+4\left(\left(3ab^3-20a^2bc\right)x^9-24a^3bx^3-2\left(a^2b^2+20a^3c\right)x^6-16a^4\right)\sqrt{cx^6+bx^3+a}}{768a^3x^{12}},\frac{3\left(b^4-8ab^2c+16a^2c^2\right)\sqrt{-a}x^{12}\arctan\left(\frac{\sqrt{cx^6+bx^3+a}\left(bx^3+2a\right)\sqrt{-a}}{2\left(acx^6+abx^3+a^2\right)}\right)+2\left(\left(3ab^3-20a^2bc\right)x^9-24a^3bx^3-2\left(a^2b^2+20a^3c\right)x^6-16a^4\right)\sqrt{cx^6+bx^3+a}}{384a^3x^{12}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)^(3/2)/x^13,x, algorithm="fricas")
```

```
[Out] [1/768*(3*(b^4 - 8*a*b^2*c + 16*a^2*c^2)*sqrt(a)*x^12*log(-((b^2 + 4*a*c)*x
^6 + 8*a*b*x^3 - 4*sqrt(c*x^6 + b*x^3 + a)*(b*x^3 + 2*a)*sqrt(a) + 8*a^2)/x
^6) + 4*((3*a*b^3 - 20*a^2*b*c)*x^9 - 24*a^3*b*x^3 - 2*(a^2*b^2 + 20*a^3*c)
*x^6 - 16*a^4)*sqrt(c*x^6 + b*x^3 + a))/(a^3*x^12), 1/384*(3*(b^4 - 8*a*b^2
*c + 16*a^2*c^2)*sqrt(-a)*x^12*arctan(1/2*sqrt(c*x^6 + b*x^3 + a)*(b*x^3 +
```

```
2*a)*sqrt(-a)/(a*c*x^6 + a*b*x^3 + a^2)) + 2*((3*a*b^3 - 20*a^2*b*c)*x^9 -
24*a^3*b*x^3 - 2*(a^2*b^2 + 20*a^3*c)*x^6 - 16*a^4)*sqrt(c*x^6 + b*x^3 + a)
)/(a^3*x^12)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cx^6 + bx^3 + a\right)^{\frac{3}{2}}}{x^{13}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)^(3/2)/x^13,x, algorithm="giac")
[Out] integrate((c*x^6 + b*x^3 + a)^(3/2)/x^13, x)
```

maple [F] time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{\left(c\,x^6 + b\,x^3 + a\right)^{\frac{3}{2}}}{x^{13}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^6+b*x^3+a)^(3/2)/x^13,x)
[Out] int((c*x^6+b*x^3+a)^(3/2)/x^13,x)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)^(3/2)/x^13,x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive, negative or zero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(c\,x^6 + b\,x^3 + a\right)}^{3/2}}{x^{13}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3 + c*x^6)^(3/2)/x^13,x)
[Out] int((a + b*x^3 + c*x^6)^(3/2)/x^13, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a + bx^3 + cx^6\right)^{\frac{3}{2}}}{x^{13}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**6+b*x**3+a)**(3/2)/x**13,x)
[Out] Integral((a + b*x**3 + c*x**6)**(3/2)/x**13, x)
```

3.193 $\int \frac{(a+bx^3+cx^6)^{3/2}}{x^{16}}\,dx$

Optimal. Leaf size=162

$$\frac{b(b^2-4ac)^2\tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{256a^{7/2}}-\frac{b(b^2-4ac)(2a+bx^3)\sqrt{a+bx^3+cx^6}}{128a^3x^6}+\frac{b(2a+bx^3)(a+bx^3+cx^6)^{3/2}}{48a^2x^{12}}-$$

Rubi [A] time = 0.15, antiderivative size = 162, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1357, 730, 720, 724, 206}

$$-\frac{b(b^2-4ac)(2a+bx^3)\sqrt{a+bx^3+cx^6}}{128a^3x^6}+\frac{b(b^2-4ac)^2\tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{256a^{7/2}}+\frac{b(2a+bx^3)(a+bx^3+cx^6)^{3/2}}{48a^2x^{12}}-\frac{(a+bx^3+cx^6)^{5/2}}{15ax^{15}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3 + c*x^6)^(3/2)/x^16,x]
```

```
[Out] -(b*(b^2 - 4*a*c)*(2*a + b*x^3)*Sqrt[a + b*x^3 + c*x^6])/(128*a^3*x^6) + (b
*(2*a + b*x^3)*(a + b*x^3 + c*x^6)^(3/2))/(48*a^2*x^12) - (a + b*x^3 + c*x^
6)^(5/2)/(15*a*x^15) + (b*(b^2 - 4*a*c)^2*ArcTanh[(2*a + b*x^3)/(2*Sqrt[a]*
Sqrt[a + b*x^3 + c*x^6])])/(256*a^(7/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 720

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> -Simp[((d + e*x)^(m + 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*(a + b*x
 + c*x^2)^p)/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[(p*(b^2 - 4*a*c
))/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 2)*(a + b*x + c*
x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] &&
 NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && EqQ[m + 2*p + 2, 0
] && GtQ[p, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 730

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 - b*d*e + a*e^2)), x] + Dist[(2*c*d - b*e)/(2*(c*d^2 - b*d*e + a*e^2)),
 Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e,
 m, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2
*c*d - b*e, 0] && EqQ[m + 2*p + 3, 0]
```

Rule 1357

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
```

```
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{\left(a+bx^3+cx^6\right)^{3/2}}{x^{16}}\,dx=\frac{1}{3}\operatorname{Subst}\left(\int \frac{\left(a+bx+cx^2\right)^{3/2}}{x^6}\,dx,x,x^3\right)$$

$$=-\frac{\left(a+bx^3+cx^6\right)^{5/2}}{15ax^{15}}-\frac{b\operatorname{Subst}\left(\int \frac{\left(a+bx+cx^2\right)^{3/2}}{x^5}\,dx,x,x^3\right)}{6a}$$

$$=\frac{b\left(2a+bx^3\right)\left(a+bx^3+cx^6\right)^{3/2}}{48a^2x^{12}}-\frac{\left(a+bx^3+cx^6\right)^{5/2}}{15ax^{15}}+\frac{\left(b\left(b^2-4ac\right)\right)\operatorname{Subst}\left(\int \frac{\sqrt{a+\ldots}}{\ldots}\right.}{32a^2}$$

$$=-\frac{b\left(b^2-4ac\right)\left(2a+bx^3\right)\sqrt{a+bx^3+cx^6}}{128a^3x^6}+\frac{b\left(2a+bx^3\right)\left(a+bx^3+cx^6\right)^{3/2}}{48a^2x^{12}}-\frac{(a+\ldots}{\ldots}$$

$$=-\frac{b\left(b^2-4ac\right)\left(2a+bx^3\right)\sqrt{a+bx^3+cx^6}}{128a^3x^6}+\frac{b\left(2a+bx^3\right)\left(a+bx^3+cx^6\right)^{3/2}}{48a^2x^{12}}-\frac{(a+\ldots}{\ldots}$$

$$=-\frac{b\left(b^2-4ac\right)\left(2a+bx^3\right)\sqrt{a+bx^3+cx^6}}{128a^3x^6}+\frac{b\left(2a+bx^3\right)\left(a+bx^3+cx^6\right)^{3/2}}{48a^2x^{12}}-\frac{(a+\ldots}{\ldots}$$

Mathematica [A] time = 0.14, size = 167, normalized size = 1.03

$$\frac{b\left(16a^{3/2}\left(2a+bx^3\right)\left(a+bx^3+cx^6\right)^{3/2}-3x^6\left(b^2-4ac\right)\left(2\sqrt{a}\left(2a+bx^3\right)\sqrt{a+bx^3+cx^6}-x^6\left(b^2-4ac\right)\tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)\right)\right)}{768a^{7/2}x^{12}}-\frac{\left(a+bx^3+cx^6\right)^{5/2}}{15ax^{15}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3 + c*x^6)^(3/2)/x^16,x]
```

```
[Out] -1/15*(a + b*x^3 + c*x^6)^(5/2)/(a*x^15) + (b*(16*a^(3/2)*(2*a + b*x^3)*(a
+ b*x^3 + c*x^6)^(3/2) - 3*(b^2 - 4*a*c)*x^6*(2*Sqrt[a]*(2*a + b*x^3)*Sqrt[
a + b*x^3 + c*x^6] - (b^2 - 4*a*c)*x^6*ArcTanh[(2*a + b*x^3)/(2*Sqrt[a]*Sqr
t[a + b*x^3 + c*x^6])])))/(768*a^(7/2)*x^12)
```

IntegrateAlgebraic [A] time = 1.55, size = 176, normalized size = 1.09

$$\frac{\left(-16a^2bc^2+8ab^3c-b^5\right)\tanh^{-1}\left(\frac{\sqrt{c}x^3-\sqrt{a+bx^3+cx^6}}{\sqrt{a}}\right)}{128a^{7/2}}+\frac{\sqrt{a+bx^3+cx^6}\left(-128a^4-176a^3bx^3-256a^3cx^6-8a^2b^2x^6-56a^2bcx^9-128a^2c^2x^{12}+10ab^3x^9+100ab^2cx^{12}-15b^4x^{12}\right)}{1920a^3x^{15}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x^3 + c*x^6)^(3/2)/x^16,x]
```

```
[Out] (Sqrt[a + b*x^3 + c*x^6]*(-128*a^4 - 176*a^3*b*x^3 - 8*a^2*b^2*x^6 - 256*a^
3*c*x^6 + 10*a*b^3*x^9 - 56*a^2*b*c*x^9 - 15*b^4*x^12 + 100*a*b^2*c*x^12 -
128*a^2*c^2*x^12))/(1920*a^3*x^15) + ((-b^5 + 8*a*b^3*c - 16*a^2*b*c^2)*Arc
Tanh[(Sqrt[c]*x^3 - Sqrt[a + b*x^3 + c*x^6])/Sqrt[a]])/(128*a^(7/2))
```

fricas [A] time = 1.67, size = 383, normalized size = 2.36

$$\left[\frac{15\left(b^5-8ab^3c+16a^2bc^2\right)\sqrt{a}x^{15}\log\left(-\frac{\left(b^2+4ac\right)x^6+8abx^3+4\sqrt{cx^6+bx^3+a}\left(bx^3+2a\right)\sqrt{a}+8a^2}{x^6}\right)-4\left(\left(15ab^4-100a^2b^2c+128a^3c^2\right)x^{12}-2\left(5a^2b^3-28a^3bc\right)x^9+176a^4bx^3+8\left(a^3b^2+32a^4c\right)x^6+128a^5\right)\sqrt{cx^6+bx^3+a}}{7680a^4x^{15}},-\frac{15\left(b^5-8ab^3c+16a^2bc^2\right)\sqrt{-a}x^{15}\arctan\left(\frac{\sqrt{cx^6+bx^3+a}\left(bx^3+2a\right)\sqrt{-a}}{2\left(acx^6+abx^3+a^2\right)}\right)+2\left(\left(15ab^4-100a^2b^2c+128a^3c^2\right)x^{12}-2\left(5a^2b^3-28a^3bc\right)x^9+176a^4bx^3+8\left(a^3b^2+32a^4c\right)x^6+128a^5\right)\sqrt{cx^6+bx^3+a}}{3840a^4x^{15}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)^(3/2)/x^16,x, algorithm="fricas")
```

```
[Out] [1/7680*(15*(b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*sqrt(a)*x^15*log(-((b^2 + 4*a*
c)*x^6 + 8*a*b*x^3 + 4*sqrt(c*x^6 + b*x^3 + a)*(b*x^3 + 2*a)*sqrt(a) + 8*a^
2)/x^6) - 4*((15*a*b^4 - 100*a^2*b^2*c + 128*a^3*c^2)*x^12 - 2*(5*a^2*b^3 -
 28*a^3*b*c)*x^9 + 176*a^4*b*x^3 + 8*(a^3*b^2 + 32*a^4*c)*x^6 + 128*a^5)*sq
rt(c*x^6 + b*x^3 + a))/(a^4*x^15), -1/3840*(15*(b^5 - 8*a*b^3*c + 16*a^2*b*
c^2)*sqrt(-a)*x^15*arctan(1/2*sqrt(c*x^6 + b*x^3 + a)*(b*x^3 + 2*a)*sqrt(-a
)/(a*c*x^6 + a*b*x^3 + a^2)) + 2*((15*a*b^4 - 100*a^2*b^2*c + 128*a^3*c^2)*
x^12 - 2*(5*a^2*b^3 - 28*a^3*b*c)*x^9 + 176*a^4*b*x^3 + 8*(a^3*b^2 + 32*a^4
*c)*x^6 + 128*a^5)*sqrt(c*x^6 + b*x^3 + a))/(a^4*x^15)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cx^6+bx^3+a\right)^{\frac{3}{2}}}{x^{16}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)^(3/2)/x^16,x, algorithm="giac")
```

```
[Out] integrate((c*x^6 + b*x^3 + a)^(3/2)/x^16, x)
```

maple [F] time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{\left(c\,x^6+b\,x^3+a\right)^{\frac{3}{2}}}{x^{16}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^6+b*x^3+a)^(3/2)/x^16,x)
```

```
[Out] int((c*x^6+b*x^3+a)^(3/2)/x^16,x)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)^(3/2)/x^16,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive, negative or zero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(c\,x^6+b\,x^3+a\right)}^{3/2}}{x^{16}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3 + c*x^6)^(3/2)/x^16,x)
```

```
[Out] int((a + b*x^3 + c*x^6)^(3/2)/x^16, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+bx^3+cx^6\right)^{\frac{3}{2}}}{x^{16}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**6+b*x**3+a)**(3/2)/x**16,x)
```

```
[Out] Integral((a + b*x**3 + c*x**6)**(3/2)/x**16, x)
```

3.194 $\int \frac{(a+bx^3+cx^6)^{3/2}}{x^{19}}\,dx$

Optimal. Leaf size=216

$$-\frac{(b^2-4ac)^2(7b^2-4ac)\tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{3072a^{9/2}}+\frac{(b^2-4ac)(7b^2-4ac)(2a+bx^3)\sqrt{a+bx^3+cx^6}}{1536a^4x^6}-\frac{(7b^2-4ac}{}$$

Rubi [A] time = 0.21, antiderivative size = 216, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {1357, 744, 806, 720, 724, 206}

$$-\frac{(7b^2-4ac)(2a+bx^3)(a+bx^3+cx^6)^{3/2}}{576a^3x^{12}}+\frac{(b^2-4ac)(7b^2-4ac)(2a+bx^3)\sqrt{a+bx^3+cx^6}}{1536a^4x^6}-\frac{(b^2-4ac)^2(7b^2-4ac)\tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{3072a^{9/2}}+\frac{7b(a+bx^3+cx^6)^{5/2}}{180a^2x^{15}}-\frac{(a+bx^3+cx^6)^{5/2}}{18ax^{18}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3 + c*x^6)^(3/2)/x^19,x]
```

```
[Out] ((b^2 - 4*a*c)*(7*b^2 - 4*a*c)*(2*a + b*x^3)*Sqrt[a + b*x^3 + c*x^6])/(1536
*a^4*x^6) - ((7*b^2 - 4*a*c)*(2*a + b*x^3)*(a + b*x^3 + c*x^6)^(3/2))/(576*
a^3*x^12) - (a + b*x^3 + c*x^6)^(5/2)/(18*a*x^18) + (7*b*(a + b*x^3 + c*x^6
)^(5/2))/(180*a^2*x^15) - ((b^2 - 4*a*c)^2*(7*b^2 - 4*a*c)*ArcTanh[(2*a + b
*x^3)/(2*Sqrt[a]*Sqrt[a + b*x^3 + c*x^6])])/(3072*a^(9/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 720

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> -Simp[((d + e*x)^(m + 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*(a + b*x
 + c*x^2)^p)/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[(p*(b^2 - 4*a*c
))/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 2)*(a + b*x + c*
x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] &&
 NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && EqQ[m + 2*p + 2, 0
] && GtQ[p, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 744

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b*d*e + a*e^2)), Int[(
d + e*x)^(m + 1)*Simp[c*d*(m + 1) - b*e*(m + p + 2) - c*e*(m + 2*p + 3)*x,
x]*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^
2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && Ne
Q[m, -1] && ((LtQ[m, -1] && IntQuadraticQ[a, b, c, d, e, m, p, x]) || (SumS
implerQ[m, 1] && IntegerQ[p]) || ILtQ[Simplify[m + 2*p + 3], 0])
```

Rule 806

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b
*x + c*x^2)^(p + 1))/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[(b*(e*f
 + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m
+ 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &
& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m +
2*p + 3], 0]
```

Rule 1357

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{\left(a+bx^3+cx^6\right)^{3/2}}{x^{19}}\,dx = \frac{1}{3}\,\text{Subst}\left(\int \frac{\left(a+bx+cx^2\right)^{3/2}}{x^7}\,dx, x, x^3\right)$$

$$= -\frac{\left(a+bx^3+cx^6\right)^{5/2}}{18ax^{18}} - \frac{\text{Subst}\left(\int \frac{\left(\frac{7b}{2}+cx\right)\left(a+bx+cx^2\right)^{3/2}}{x^6}\,dx, x, x^3\right)}{18a}$$

$$= -\frac{\left(a+bx^3+cx^6\right)^{5/2}}{18ax^{18}} + \frac{7b\left(a+bx^3+cx^6\right)^{5/2}}{180a^2x^{15}} + \frac{\left(7b^2-4ac\right)\text{Subst}\left(\int \frac{\left(a+bx+cx^2\right)^{3/2}}{x^5}\,dx \right.}{72a^2}$$

$$= -\frac{\left(7b^2-4ac\right)\left(2a+bx^3\right)\left(a+bx^3+cx^6\right)^{3/2}}{576a^3x^{12}} - \frac{\left(a+bx^3+cx^6\right)^{5/2}}{18ax^{18}} + \frac{7b\left(a+bx^3+cx\right.}{180a^2x^{15}}$$

$$= \frac{\left(b^2-4ac\right)\left(7b^2-4ac\right)\left(2a+bx^3\right)\sqrt{a+bx^3+cx^6}}{1536a^4x^6} - \frac{\left(7b^2-4ac\right)\left(2a+bx^3\right)\left(a+bx\right.}{576a^3x^{12}}$$

$$= \frac{\left(b^2-4ac\right)\left(7b^2-4ac\right)\left(2a+bx^3\right)\sqrt{a+bx^3+cx^6}}{1536a^4x^6} - \frac{\left(7b^2-4ac\right)\left(2a+bx^3\right)\left(a+bx\right.}{576a^3x^{12}}$$

$$= \frac{\left(b^2-4ac\right)\left(7b^2-4ac\right)\left(2a+bx^3\right)\sqrt{a+bx^3+cx^6}}{1536a^4x^6} - \frac{\left(7b^2-4ac\right)\left(2a+bx^3\right)\left(a+bx\right.}{576a^3x^{12}}$$

Mathematica [A] time = 0.21, size = 206, normalized size = 0.95

$$-\frac{\frac{\left(\frac{7b^2}{2}-2ac\right)\left(16a^{3/2}\left(2a+bx^3\right)\left(a+bx^3+cx^6\right)^{3/2}-3x^6\left(b^2-4ac\right)\left(2\sqrt{a}\left(2a+bx^3\right)\sqrt{a+bx^3+cx^6}-x^6\left(b^2-4ac\right)\tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)\right)\right)}{256a^{7/2}x^{12}}+\frac{\left(a+bx^3+cx^6\right)^{5/2}}{x^{18}}-\frac{7b\left(a+bx^3+cx^6\right)^{5/2}}{10ax^{15}}}{18a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3 + c*x^6)^(3/2)/x^19,x]
```

```
[Out] -1/18*((a + b*x^3 + c*x^6)^(5/2)/x^18 - (7*b*(a + b*x^3 + c*x^6)^(5/2))/(10
*a*x^15) + (((7*b^2)/2 - 2*a*c)*(16*a^(3/2)*(2*a + b*x^3)*(a + b*x^3 + c*x^
6)^(3/2) - 3*(b^2 - 4*a*c)*x^6*(2*Sqrt[a]*(2*a + b*x^3)*Sqrt[a + b*x^3 + c*
x^6] - (b^2 - 4*a*c)*x^6*ArcTanh[(2*a + b*x^3)/(2*Sqrt[a]*Sqrt[a + b*x^3 +
c*x^6])])))/(256*a^(7/2)*x^12))/a
```

IntegrateAlgebraic [A] time = 2.27, size = 221, normalized size = 1.02

$$\frac{\left(-64a^3c^3+144a^2b^2c^2-60ab^4c+7b^6\right)\tanh^{-1}\left(\frac{\sqrt{c}x^3-\sqrt{a+bx^3+cx^6}}{\sqrt{a}}\right)}{1536a^{9/2}}+\frac{\sqrt{a+bx^3+cx^6}\left(-1280a^5-1664a^4bx^3-2240a^4cx^6-48a^3b^2x^6-288a^3bcx^9-480a^3c^2x^{12}+56a^2b^3x^9+432a^2b^2cx^{12}+1296a^2bc^2x^{15}-70ab^4x^{12}-760ab^3cx^{15}+105b^5x^{15}\right)}{23040a^4x^{18}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x^3 + c*x^6)^(3/2)/x^19,x]
```

```
[Out] (Sqrt[a + b*x^3 + c*x^6]*(-1280*a^5 - 1664*a^4*b*x^3 - 48*a^3*b^2*x^6 - 224
0*a^4*c*x^6 + 56*a^2*b^3*x^9 - 288*a^3*b*c*x^9 - 70*a*b^4*x^12 + 432*a^2*b^
2*c*x^12 - 480*a^3*c^2*x^12 + 105*b^5*x^15 - 760*a*b^3*c*x^15 + 1296*a^2*b*
c^2*x^15))/(23040*a^4*x^18) + ((7*b^6 - 60*a*b^4*c + 144*a^2*b^2*c^2 - 64*a
^3*c^3)*ArcTanh[(Sqrt[c]*x^3 - Sqrt[a + b*x^3 + c*x^6])/Sqrt[a]])/(1536*a^(
9/2))
```

fricas [A] time = 2.09, size = 473, normalized size = 2.19

$$\left[-\frac{15\left(7b^6-60ab^4c+144a^2b^2c^2-64a^3c^3\right)\sqrt{a}x^{18}\log\left(-\frac{(b^2+4ac)x^6+8abx^3+4\sqrt{cx^6+bx^3+a}(bx^3+2a)\sqrt{a}+8a^2}{x^6}\right)-4\left((105ab^5-760a^2b^3c+1296a^3bc^2)x^{15}-2\left(35a^2b^4-216a^3b^2c+240a^4c^2\right)x^{12}+8\left(7a^3b^3-36a^4bc\right)x^9-1664a^5bx^3-16\left(3a^4b^2+140a^5c\right)x^6-1280a^6\right)\sqrt{cx^6+bx^3+a}}{92160a^5x^{18}}, \frac{15\left(7b^6-60ab^4c+144a^2b^2c^2-64a^3c^3\right)\sqrt{-a}x^{18}\arctan\left(\frac{\sqrt{cx^6+bx^3+a}(bx^3+2a)\sqrt{-a}}{2(acx^6+abx^3+a^2)}\right)+2\left((105ab^5-760a^2b^3c+1296a^3bc^2)x^{15}-2\left(35a^2b^4-216a^3b^2c+240a^4c^2\right)x^{12}+8\left(7a^3b^3-36a^4bc\right)x^9-1664a^5bx^3-16\left(3a^4b^2+140a^5c\right)x^6-1280a^6\right)\sqrt{cx^6+bx^3+a}}{46080a^5x^{18}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)^(3/2)/x^19,x, algorithm="fricas")
```

```
[Out] [-1/92160*(15*(7*b^6 - 60*a*b^4*c + 144*a^2*b^2*c^2 - 64*a^3*c^3)*sqrt(a)*x
^18*log(-((b^2 + 4*a*c)*x^6 + 8*a*b*x^3 + 4*sqrt(c*x^6 + b*x^3 + a)*(b*x^3
+ 2*a)*sqrt(a) + 8*a^2)/x^6) - 4*((105*a*b^5 - 760*a^2*b^3*c + 1296*a^3*b*c
^2)*x^15 - 2*(35*a^2*b^4 - 216*a^3*b^2*c + 240*a^4*c^2)*x^12 + 8*(7*a^3*b^3
 - 36*a^4*b*c)*x^9 - 1664*a^5*b*x^3 - 16*(3*a^4*b^2 + 140*a^5*c)*x^6 - 1280
*a^6)*sqrt(c*x^6 + b*x^3 + a))/(a^5*x^18), 1/46080*(15*(7*b^6 - 60*a*b^4*c
+ 144*a^2*b^2*c^2 - 64*a^3*c^3)*sqrt(-a)*x^18*arctan(1/2*sqrt(c*x^6 + b*x^3
 + a)*(b*x^3 + 2*a)*sqrt(-a)/(a*c*x^6 + a*b*x^3 + a^2)) + 2*((105*a*b^5 - 7
60*a^2*b^3*c + 1296*a^3*b*c^2)*x^15 - 2*(35*a^2*b^4 - 216*a^3*b^2*c + 240*a
^4*c^2)*x^12 + 8*(7*a^3*b^3 - 36*a^4*b*c)*x^9 - 1664*a^5*b*x^3 - 16*(3*a^4*
b^2 + 140*a^5*c)*x^6 - 1280*a^6)*sqrt(c*x^6 + b*x^3 + a))/(a^5*x^18)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cx^6+bx^3+a\right)^{\frac{3}{2}}}{x^{19}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)^(3/2)/x^19,x, algorithm="giac")
```

```
[Out] integrate((c*x^6 + b*x^3 + a)^(3/2)/x^19, x)
```

maple [F] time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{\left(c\,x^6+b\,x^3+a\right)^{\frac{3}{2}}}{x^{19}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^6+b*x^3+a)^(3/2)/x^19,x)
```

```
[Out] int((c*x^6+b*x^3+a)^(3/2)/x^19,x)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)^(3/2)/x^19,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive, negative or zero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(c\,x^6+b\,x^3+a\right)^{3/2}}{x^{19}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3 + c*x^6)^(3/2)/x^19,x)
```

```
[Out] int((a + b*x^3 + c*x^6)^(3/2)/x^19, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+bx^3+cx^6\right)^{\frac{3}{2}}}{x^{19}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**6+b*x**3+a)**(3/2)/x**19,x)
```

```
[Out] Integral((a + b*x**3 + c*x**6)**(3/2)/x**19, x)
```

3.195 $\int \frac{\left(a+bx^3+cx^6\right)^{3/2}}{x^{22}}\,dx$

Optimal. Leaf size=255

$$\frac{b\left(b^2-4ac\right)^2\left(3b^2-4ac\right)\tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{2048a^{11/2}}-\frac{b\left(b^2-4ac\right)\left(3b^2-4ac\right)\left(2a+bx^3\right)\sqrt{a+bx^3+cx^6}}{1024a^5x^6}+b\left(3b^2-\right.$$

Rubi [A] time = 0.31, antiderivative size = 255, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.350$, Rules used = {1357, 744, 834, 806, 720, 724, 206}

$$-\frac{\left(21b^2-16ac\right)\left(a+bx^3+cx^6\right)^{5/2}}{840a^3x^{15}}+\frac{b\left(3b^2-4ac\right)\left(2a+bx^3\right)\left(a+bx^3+cx^6\right)^{3/2}}{384a^4x^{12}}-\frac{b\left(b^2-4ac\right)\left(3b^2-4ac\right)\left(2a+bx^3\right)\sqrt{a+bx^3+cx^6}}{1024a^5x^6}+\frac{b\left(b^2-4ac\right)^2\left(3b^2-4ac\right)\tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{2048a^{11/2}}+\frac{b\left(a+bx^3+cx^6\right)^{5/2}}{28a^2x^{18}}-\frac{\left(a+bx^3+cx^6\right)^{5/2}}{21ax^{21}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3 + c*x^6)^(3/2)/x^22,x]
```

```
[Out] -(b*(b^2 - 4*a*c)*(3*b^2 - 4*a*c)*(2*a + b*x^3)*Sqrt[a + b*x^3 + c*x^6])/(1
024*a^5*x^6) + (b*(3*b^2 - 4*a*c)*(2*a + b*x^3)*(a + b*x^3 + c*x^6)^(3/2))/
(384*a^4*x^12) - (a + b*x^3 + c*x^6)^(5/2)/(21*a*x^21) + (b*(a + b*x^3 + c*
x^6)^(5/2))/(28*a^2*x^18) - ((21*b^2 - 16*a*c)*(a + b*x^3 + c*x^6)^(5/2))/(
840*a^3*x^15) + (b*(b^2 - 4*a*c)^2*(3*b^2 - 4*a*c)*ArcTanh[(2*a + b*x^3)/(2
*Sqrt[a]*Sqrt[a + b*x^3 + c*x^6])])/(2048*a^(11/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 720

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> -Simp[((d + e*x)^(m + 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*(a + b*x
 + c*x^2)^p)/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[(p*(b^2 - 4*a*c
))/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 2)*(a + b*x + c*
x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] &&
 NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && EqQ[m + 2*p + 2, 0
] && GtQ[p, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 744

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b*d*e + a*e^2)), Int[(
d + e*x)^(m + 1)*Simp[c*d*(m + 1) - b*e*(m + p + 2) - c*e*(m + 2*p + 3)*x,
x]*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^
2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && Ne
Q[m, -1] && ((LtQ[m, -1] && IntQuadraticQ[a, b, c, d, e, m, p, x]) || (SumS
implerQ[m, 1] && IntegerQ[p]) || ILtQ[Simplify[m + 2*p + 3], 0])
```

Rule 806

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b
*x + c*x^2)^(p + 1))/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[(b*(e*f
 + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m
+ 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &
& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m +
2*p + 3], 0]
```

Rule 834

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b*
x + c*x^2)^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[(
c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m +
 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] ||
 IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 1357

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{\left(a+bx^3+cx^6\right)^{3/2}}{x^{22}}\,dx = \frac{1}{3}\text{Subst}\left(\int \frac{\left(a+bx+cx^2\right)^{3/2}}{x^8}\,dx, x, x^3\right)$$

$$= -\frac{\left(a+bx^3+cx^6\right)^{5/2}}{21ax^{21}} - \frac{\text{Subst}\left(\int \frac{\left(\frac{9b}{2}+2cx\right)\left(a+bx+cx^2\right)^{3/2}}{x^7}\,dx, x, x^3\right)}{21a}$$

$$= -\frac{\left(a+bx^3+cx^6\right)^{5/2}}{21ax^{21}} + \frac{b\left(a+bx^3+cx^6\right)^{5/2}}{28a^2x^{18}} + \frac{\text{Subst}\left(\int \frac{\left(\frac{3}{4}\left(21b^2-16ac\right)+\frac{9bcx}{2}\right)\left(a+bx+cx^2\right)^{3/\cdots}}{x^6}\cdots\right.}{126a^2}$$

$$= -\frac{\left(a+bx^3+cx^6\right)^{5/2}}{21ax^{21}} + \frac{b\left(a+bx^3+cx^6\right)^{5/2}}{28a^2x^{18}} - \frac{\left(21b^2-16ac\right)\left(a+bx^3+cx^6\right)^{5/2}}{840a^3x^{15}} - \frac{(b\cdots}{\cdots}$$

$$= \frac{b\left(3b^2-4ac\right)\left(2a+bx^3\right)\left(a+bx^3+cx^6\right)^{3/2}}{384a^4x^{12}} - \frac{\left(a+bx^3+cx^6\right)^{5/2}}{21ax^{21}} + \frac{b\left(a+bx^3+cx^{6}\cdots\right.}{28a^2x^{18}}$$

$$= -\frac{b\left(b^2-4ac\right)\left(3b^2-4ac\right)\left(2a+bx^3\right)\sqrt{a+bx^3+cx^6}}{1024a^5x^6} + \frac{b\left(3b^2-4ac\right)\left(2a+bx^3\right)\left(a\cdots\right.}{384a^4x^{12}}$$

$$= -\frac{b\left(b^2-4ac\right)\left(3b^2-4ac\right)\left(2a+bx^3\right)\sqrt{a+bx^3+cx^6}}{1024a^5x^6} + \frac{b\left(3b^2-4ac\right)\left(2a+bx^3\right)\left(a\cdots\right.}{384a^4x^{12}}$$

$$= -\frac{b\left(b^2-4ac\right)\left(3b^2-4ac\right)\left(2a+bx^3\right)\sqrt{a+bx^3+cx^6}}{1024a^5x^6} + \frac{b\left(3b^2-4ac\right)\left(2a+bx^3\right)\left(a\cdots\right.}{384a^4x^{12}}$$

Mathematica [A] time = 0.49, size = 243, normalized size = 0.95

$$-\frac{\frac{\left(21abc-\frac{63b^3}{4}\right)\left(16a^{3/2}\left(2a+bx^3\right)\left(a+bx^3+cx^6\right)^{3/2}-3x^6\left(b^2-4ac\right)\left(2\sqrt{a}\left(2a+bx^3\right)\sqrt{a+bx^3+cx^6}-x^6\left(b^2-4ac\right)\tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)\right)\right)}{1536a^{9/2}x^{12}}+\frac{\left(21b^2-16ac\right)\left(a+bx^3+cx^6\right)^{5/2}}{40a^2x^{15}}+\frac{\left(a+bx^3+cx^6\right)^{5/2}}{x^{21}}-\frac{3b\left(a+bx^3+cx^6\right)^{5/2}}{4ax^{18}}}{21a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3 + c*x^6)^(3/2)/x^22,x]
```

```
[Out] -1/21*((a + b*x^3 + c*x^6)^(5/2)/x^21 - (3*b*(a + b*x^3 + c*x^6)^(5/2))/(4*
a*x^18) + ((21*b^2 - 16*a*c)*(a + b*x^3 + c*x^6)^(5/2))/(40*a^2*x^15) + (((
-63*b^3)/4 + 21*a*b*c)*(16*a^(3/2)*(2*a + b*x^3)*(a + b*x^3 + c*x^6)^(3/2)
- 3*(b^2 - 4*a*c)*x^6*(2*Sqrt[a]*(2*a + b*x^3)*Sqrt[a + b*x^3 + c*x^6] - (b
^2 - 4*a*c)*x^6*ArcTanh[(2*a + b*x^3)/(2*Sqrt[a]*Sqrt[a + b*x^3 + c*x^6])])
))/(1536*a^(9/2)*x^12))/a
```

IntegrateAlgebraic [A] time = 2.48, size = 270, normalized size = 1.06

$$\frac{\left(64a^3bc^3-80a^2b^3c^2+28ab^5c-3b^7\right)\tanh^{-1}\left(\frac{\sqrt{c}x^3-\sqrt{a+bx^3+cx^6}}{\sqrt{a}}\right)}{1024a^{11/2}}+\frac{\sqrt{a+bx^3+cx^6}\left(-5120a^6-6400a^5bx^3-8192a^5cx^6-128a^4b^2x^6-704a^4bcx^9-1024a^4c^2x^{12}+144a^3b^3x^9+992a^3b^2cx^{12}+2336a^3bc^2x^{15}+2048a^3c^3x^{18}-168a^2b^4x^{12}-1456a^2b^3cx^{15}-5488a^2b^2c^2x^{18}+210ab^5x^{15}+2520ab^4cx^{18}-315b^6x^{18}\right)}{107520a^5x^{21}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x^3 + c*x^6)^(3/2)/x^22,x]
```

```
[Out] (Sqrt[a + b*x^3 + c*x^6]*(-5120*a^6 - 6400*a^5*b*x^3 - 128*a^4*b^2*x^6 - 81
92*a^5*c*x^6 + 144*a^3*b^3*x^9 - 704*a^4*b*c*x^9 - 168*a^2*b^4*x^12 + 992*a
^3*b^2*c*x^12 - 1024*a^4*c^2*x^12 + 210*a*b^5*x^15 - 1456*a^2*b^3*c*x^15 +
2336*a^3*b*c^2*x^15 - 315*b^6*x^18 + 2520*a*b^4*c*x^18 - 5488*a^2*b^2*c^2*x
^18 + 2048*a^3*c^3*x^18))/(107520*a^5*x^21) + ((-3*b^7 + 28*a*b^5*c - 80*a^
2*b^3*c^2 + 64*a^3*b*c^3)*ArcTanh[(Sqrt[c]*x^3 - Sqrt[a + b*x^3 + c*x^6])/S
qrt[a]])/(1024*a^(11/2))
```

fricas [A] time = 2.63, size = 557, normalized size = 2.18

$$\left[-\frac{105\left(3b^7-28ab^5c+80a^2b^3c^2-64a^3bc^3\right)\sqrt{a}x^{21}\log\left(-\frac{\left(b^2+4ac\right)x^6+8abx^3-4\sqrt{cx^6+bx^3+a}\left(bx^3+2a\right)\sqrt{a}+8a^2}{x^6}\right)+4\left(\left(315ab^6-2520a^2b^4c+5488a^3b^2c^2-2048a^4c^3\right)x^{18}-2\left(105a^2b^5-728a^3b^3c+1168a^4bc^2\right)x^{15}+8\left(21a^3b^4-124a^4b^2c+128a^5c^2\right)x^{12}+6400a^6bx^3-16\left(9a^4b^3-44a^5bc\right)x^9+5120a^7+128\left(a^5b^2+64a^6c\right)x^6\right)\sqrt{cx^6+bx^3+a}}{430080a^6x^{21}},\ -\frac{105\left(3b^7-28ab^5c+80a^2b^3c^2-64a^3bc^3\right)\sqrt{-a}x^{21}\arctan\left(\frac{\sqrt{cx^6+bx^3+a}\left(bx^3+2a\right)\sqrt{-a}}{2\left(acx^6+abx^3+a^2\right)}\right)+2\left(\left(315ab^6-2520a^2b^4c+5488a^3b^2c^2-2048a^4c^3\right)x^{18}-2\left(105a^2b^5-728a^3b^3c+1168a^4bc^2\right)x^{15}+8\left(21a^3b^4-124a^4b^2c+128a^5c^2\right)x^{12}+6400a^6bx^3-16\left(9a^4b^3-44a^5bc\right)x^9+5120a^7+128\left(a^5b^2+64a^6c\right)x^6\right)\sqrt{cx^6+bx^3+a}}{215040a^6x^{21}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)^(3/2)/x^22,x, algorithm="fricas")
```

```
[Out] [-1/430080*(105*(3*b^7 - 28*a*b^5*c + 80*a^2*b^3*c^2 - 64*a^3*b*c^3)*sqrt(a
)*x^21*log(-((b^2 + 4*a*c)*x^6 + 8*a*b*x^3 - 4*sqrt(c*x^6 + b*x^3 + a)*(b*x
^3 + 2*a)*sqrt(a) + 8*a^2)/x^6) + 4*((315*a*b^6 - 2520*a^2*b^4*c + 5488*a^3
*b^2*c^2 - 2048*a^4*c^3)*x^18 - 2*(105*a^2*b^5 - 728*a^3*b^3*c + 1168*a^4*b
*c^2)*x^15 + 8*(21*a^3*b^4 - 124*a^4*b^2*c + 128*a^5*c^2)*x^12 + 6400*a^6*b
*x^3 - 16*(9*a^4*b^3 - 44*a^5*b*c)*x^9 + 5120*a^7 + 128*(a^5*b^2 + 64*a^6*c
)*x^6)*sqrt(c*x^6 + b*x^3 + a))/(a^6*x^21), -1/215040*(105*(3*b^7 - 28*a*b^
5*c + 80*a^2*b^3*c^2 - 64*a^3*b*c^3)*sqrt(-a)*x^21*arctan(1/2*sqrt(c*x^6 +
b*x^3 + a)*(b*x^3 + 2*a)*sqrt(-a)/(a*c*x^6 + a*b*x^3 + a^2)) + 2*((315*a*b^
6 - 2520*a^2*b^4*c + 5488*a^3*b^2*c^2 - 2048*a^4*c^3)*x^18 - 2*(105*a^2*b^5
 - 728*a^3*b^3*c + 1168*a^4*b*c^2)*x^15 + 8*(21*a^3*b^4 - 124*a^4*b^2*c + 1
28*a^5*c^2)*x^12 + 6400*a^6*b*x^3 - 16*(9*a^4*b^3 - 44*a^5*b*c)*x^9 + 5120*
a^7 + 128*(a^5*b^2 + 64*a^6*c)*x^6)*sqrt(c*x^6 + b*x^3 + a))/(a^6*x^21)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cx^6+bx^3+a\right)^{\frac{3}{2}}}{x^{22}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)^(3/2)/x^22,x, algorithm="giac")
```

```
[Out] integrate((c*x^6 + b*x^3 + a)^(3/2)/x^22, x)
```

maple [F] time = 0.09, size = 0, normalized size = 0.00

$$\int \frac{\left(c\,x^{6}+b\,x^{3}+a\right)^{\frac{3}{2}}}{x^{22}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^6+b*x^3+a)^(3/2)/x^22,x)
```

```
[Out] int((c*x^6+b*x^3+a)^(3/2)/x^22,x)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)^(3/2)/x^22,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive, negative or zero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(c\,x^6+b\,x^3+a\right)}^{3/2}}{x^{22}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3 + c*x^6)^(3/2)/x^22,x)
```

```
[Out] int((a + b*x^3 + c*x^6)^(3/2)/x^22, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a + bx^{3} + cx^{6}\right)^{\frac{3}{2}}}{x^{22}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**6+b*x**3+a)**(3/2)/x**22,x)
```

```
[Out] Integral((a + b*x**3 + c*x**6)**(3/2)/x**22, x)
```

3.196 $\int \frac{x^{14}}{\sqrt{a+bx^3+cx^6}}\,dx$

Optimal. Leaf size=171

$$\frac{\left(48a^2c^2-120ab^2c+35b^4\right)\tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{384c^{9/2}}-\frac{\left(5b\left(21b^2-44ac\right)-2cx^3\left(35b^2-36ac\right)\right)\sqrt{a+bx^3+cx^6}}{576c^4}-7$$

Rubi [A] time = 0.22, antiderivative size = 171, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {1357, 742, 832, 779, 621, 206}

$$\frac{\left(48a^2c^2-120ab^2c+35b^4\right)\tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{384c^{9/2}}-\frac{\left(5b\left(21b^2-44ac\right)-2cx^3\left(35b^2-36ac\right)\right)\sqrt{a+bx^3+cx^6}}{576c^4}-\frac{7bx^6\sqrt{a+bx^3+cx^6}}{72c^2}+\frac{x^9\sqrt{a+bx^3+cx^6}}{12c}$$

Antiderivative was successfully verified.

```
[In] Int[x^14/Sqrt[a + b*x^3 + c*x^6],x]
```

```
[Out] (-7*b*x^6*Sqrt[a + b*x^3 + c*x^6])/(72*c^2) + (x^9*Sqrt[a + b*x^3 + c*x^6])
/(12*c) - ((5*b*(21*b^2 - 44*a*c) - 2*c*(35*b^2 - 36*a*c)*x^3)*Sqrt[a + b*x
^3 + c*x^6])/(576*c^4) + ((35*b^4 - 120*a*b^2*c + 48*a^2*c^2)*ArcTanh[(b +
2*c*x^3)/(2*Sqrt[c]*Sqrt[a + b*x^3 + c*x^6])])/(384*c^(9/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 742

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(m + 2*p
+ 1)), x] + Dist[1/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m +
 2*p + 1) - e*(a*e*(m - 1) + b*d*(p + 1)) + e*(2*c*d - b*e)*(m + p)*x, x]*(
a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && If[Rat
ionalQ[m], GtQ[m, 1], SumSimplerQ[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuad
raticQ[a, b, c, d, e, m, p, x]
```

Rule 779

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> -Simp[((b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x)*(a + b*x + c*x^2)^(p + 1))/(2*c^2*(p + 1)*(2*p + 3)), x
] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p +
3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 832

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + b*x + c*x^2)^(p +
```

```
1))/(c*(m + 2*p + 2)), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)
*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m*
(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p]
 || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 1357

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^{14}}{\sqrt{a+bx^3+cx^6}}\,dx = \frac{1}{3}\text{Subst}\left(\int \frac{x^4}{\sqrt{a+bx+cx^2}}\,dx, x, x^3\right)$$

$$= \frac{x^9\sqrt{a+bx^3+cx^6}}{12c} + \frac{\text{Subst}\left(\int \frac{x^2\left(-3a-\frac{7bx}{2}\right)}{\sqrt{a+bx+cx^2}}\,dx, x, x^3\right)}{12c}$$

$$= -\frac{7bx^6\sqrt{a+bx^3+cx^6}}{72c^2} + \frac{x^9\sqrt{a+bx^3+cx^6}}{12c} + \frac{\text{Subst}\left(\int \frac{x\left(7ab+\frac{1}{4}\left(35b^2-36ac\right)x\right)}{\sqrt{a+bx+cx^2}}\,dx, x, x^3\right)}{36c^2}$$

$$= -\frac{7bx^6\sqrt{a+bx^3+cx^6}}{72c^2} + \frac{x^9\sqrt{a+bx^3+cx^6}}{12c} - \frac{\left(5b\left(21b^2-44ac\right)-2c\left(35b^2-36ac\right)x \ldots}{576c^4}$$

$$= -\frac{7bx^6\sqrt{a+bx^3+cx^6}}{72c^2} + \frac{x^9\sqrt{a+bx^3+cx^6}}{12c} - \frac{\left(5b\left(21b^2-44ac\right)-2c\left(35b^2-36ac\right)x \ldots}{576c^4}$$

$$= -\frac{7bx^6\sqrt{a+bx^3+cx^6}}{72c^2} + \frac{x^9\sqrt{a+bx^3+cx^6}}{12c} - \frac{\left(5b\left(21b^2-44ac\right)-2c\left(35b^2-36ac\right)x \ldots}{576c^4}$$

Mathematica [A] time = 0.11, size = 137, normalized size = 0.80

$$\frac{3\left(48a^2c^2-120ab^2c+35b^4\right)\tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)+2\sqrt{c}\sqrt{a+bx^3+cx^6}\left(4bc\left(55a-14cx^6\right)+24c^2x^3\left(2cx^6-3a\right)-105b^3+70b^2cx^3\right)}{1152c^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^14/Sqrt[a + b*x^3 + c*x^6],x]
```

```
[Out] (2*Sqrt[c]*Sqrt[a + b*x^3 + c*x^6]*(-105*b^3 + 70*b^2*c*x^3 + 4*b*c*(55*a -
 14*c*x^6) + 24*c^2*x^3*(-3*a + 2*c*x^6)) + 3*(35*b^4 - 120*a*b^2*c + 48*a^
2*c^2)*ArcTanh[(b + 2*c*x^3)/(2*Sqrt[c]*Sqrt[a + b*x^3 + c*x^6])])/(1152*c^
(9/2))
```

IntegrateAlgebraic [A] time = 0.42, size = 138, normalized size = 0.81

$$\frac{\left(-48a^2c^2+120ab^2c-35b^4\right)\log\left(-2c^{9/2}\sqrt{a+bx^3+cx^6}+bc^4+2c^5x^3\right)}{384c^{9/2}}+\frac{\sqrt{a+bx^3+cx^6}\left(220abc-72ac^2x^3-105b^3+70b^2cx^3-56bc^2x^6+48c^3x^9\right)}{576c^4}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^14/Sqrt[a + b*x^3 + c*x^6],x]
```

```
[Out] (Sqrt[a + b*x^3 + c*x^6]*(-105*b^3 + 220*a*b*c + 70*b^2*c*x^3 - 72*a*c^2*x^
3 - 56*b*c^2*x^6 + 48*c^3*x^9))/(576*c^4) + ((-35*b^4 + 120*a*b^2*c - 48*a^
2*c^2)*Log[b*c^4 + 2*c^5*x^3 - 2*c^(9/2)*Sqrt[a + b*x^3 + c*x^6]])/(384*c^(
9/2))
```

fricas [A] time = 0.64, size = 303, normalized size = 1.77

$$\left[\frac{3\left(35\,b^{4}-120\,ab^{2}c+48\,a^{2}c^{2}\right)\sqrt{c}\,\log\left(-8\,c^{2}x^{6}-8\,bcx^{3}-b^{2}-4\,\sqrt{cx^{6}+bx^{3}+a}\left(2\,cx^{3}+b\right)\sqrt{c}-4\,ac\right)+4\left(48\,c^{4}x^{9}-56\,bc^{3}x^{6}-105\,b^{3}c+220\,abc^{2}+2\left(35\,b^{2}c^{2}-36\,ac^{3}\right)x^{3}\right)\sqrt{cx^{6}+bx^{3}+a}}{2304\,c^{5}},-\frac{3\left(35\,b^{4}-120\,ab^{2}c+48\,a^{2}c^{2}\right)\sqrt{-c}\,\arctan\left(\frac{\sqrt{cx^{6}+bx^{3}+a}\left(2\,cx^{3}+b\right)\sqrt{-c}}{2\left(c^{2}x^{6}+bcx^{3}+ac\right)}\right)-2\left(48\,c^{4}x^{9}-56\,bc^{3}x^{6}-105\,b^{3}c+220\,abc^{2}+2\left(35\,b^{2}c^{2}-36\,ac^{3}\right)x^{3}\right)\sqrt{cx^{6}+bx^{3}+a}}{1152\,c^{5}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^14/(c*x^6+b*x^3+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2304*(3*(35*b^4 - 120*a*b^2*c + 48*a^2*c^2)*sqrt(c)*log(-8*c^2*x^6 - 8*b
*c*x^3 - b^2 - 4*sqrt(c*x^6 + b*x^3 + a)*(2*c*x^3 + b)*sqrt(c) - 4*a*c) + 4
*(48*c^4*x^9 - 56*b*c^3*x^6 - 105*b^3*c + 220*a*b*c^2 + 2*(35*b^2*c^2 - 36*
a*c^3)*x^3)*sqrt(c*x^6 + b*x^3 + a))/c^5, -1/1152*(3*(35*b^4 - 120*a*b^2*c
+ 48*a^2*c^2)*sqrt(-c)*arctan(1/2*sqrt(c*x^6 + b*x^3 + a)*(2*c*x^3 + b)*sqr
t(-c)/(c^2*x^6 + b*c*x^3 + a*c)) - 2*(48*c^4*x^9 - 56*b*c^3*x^6 - 105*b^3*c
 + 220*a*b*c^2 + 2*(35*b^2*c^2 - 36*a*c^3)*x^3)*sqrt(c*x^6 + b*x^3 + a))/c^
5]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{14}}{\sqrt{cx^{6}+bx^{3}+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^14/(c*x^6+b*x^3+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^14/sqrt(c*x^6 + b*x^3 + a), x)
```

maple [F] time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{x^{14}}{\sqrt{c\,x^{6}+b\,x^{3}+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^14/(c*x^6+b*x^3+a)^(1/2),x)
```

```
[Out] int(x^14/(c*x^6+b*x^3+a)^(1/2),x)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^14/(c*x^6+b*x^3+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 zero or nonzero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{14}}{\sqrt{c\,x^{6}+b\,x^{3}+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^14/(a + b*x^3 + c*x^6)^(1/2),x)
```

```
[Out] int(x^14/(a + b*x^3 + c*x^6)^(1/2), x)
```

sympy [F]   time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{14}}{\sqrt{a+bx^3+cx^6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**14/(c*x**6+b*x**3+a)**(1/2),x)
```

```
[Out] Integral(x**14/sqrt(a + b*x**3 + c*x**6), x)
```

3.197 $\int \frac{x^{11}}{\sqrt{a+bx^3+cx^6}}\,dx$

Optimal. Leaf size=121

$$-\frac{b\left(5b^2-12ac\right)\tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{48c^{7/2}}+\frac{\left(-16ac+15b^2-10bcx^3\right)\sqrt{a+bx^3+cx^6}}{72c^3}+\frac{x^6\sqrt{a+bx^3+cx^6}}{9c}$$

Rubi [A] time = 0.11, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1357, 742, 779, 621, 206}

$$\frac{\left(-16ac+15b^2-10bcx^3\right)\sqrt{a+bx^3+cx^6}}{72c^3}-\frac{b\left(5b^2-12ac\right)\tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{48c^{7/2}}+\frac{x^6\sqrt{a+bx^3+cx^6}}{9c}$$

Antiderivative was successfully verified.

```
[In] Int[x^11/Sqrt[a + b*x^3 + c*x^6],x]
```

```
[Out] (x^6*Sqrt[a + b*x^3 + c*x^6])/(9*c) + ((15*b^2 - 16*a*c - 10*b*c*x^3)*Sqrt[
a + b*x^3 + c*x^6])/(72*c^3) - (b*(5*b^2 - 12*a*c)*ArcTanh[(b + 2*c*x^3)/(2
*Sqrt[c]*Sqrt[a + b*x^3 + c*x^6])])/(48*c^(7/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 742

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(m + 2*p
+ 1)), x] + Dist[1/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m +
 2*p + 1) - e*(a*e*(m - 1) + b*d*(p + 1)) + e*(2*c*d - b*e)*(m + p)*x, x]*(
a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && If[Rat
ionalQ[m], GtQ[m, 1], SumSimplerQ[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuad
raticQ[a, b, c, d, e, m, p, x]
```

Rule 779

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> -Simp[((b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x)*(a + b*x + c*x^2)^(p + 1))/(2*c^2*(p + 1)*(2*p + 3)), x
] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p +
3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 1357

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
```

```
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^{11}}{\sqrt{a+bx^3+cx^6}}\,dx = \frac{1}{3}\,\text{Subst}\left(\int \frac{x^3}{\sqrt{a+bx+cx^2}}\,dx, x, x^3\right)$$
$$= \frac{x^6\sqrt{a+bx^3+cx^6}}{9c} + \frac{\text{Subst}\left(\int \frac{x\left(-2a-\frac{5bx}{2}\right)}{\sqrt{a+bx+cx^2}}\,dx, x, x^3\right)}{9c}$$
$$= \frac{x^6\sqrt{a+bx^3+cx^6}}{9c} + \frac{\left(15b^2-16ac-10bcx^3\right)\sqrt{a+bx^3+cx^6}}{72c^3} - \frac{\left(b\left(5b^2-12ac\right)\right)\text{Sub}}{}$$
$$= \frac{x^6\sqrt{a+bx^3+cx^6}}{9c} + \frac{\left(15b^2-16ac-10bcx^3\right)\sqrt{a+bx^3+cx^6}}{72c^3} - \frac{\left(b\left(5b^2-12ac\right)\right)\text{Sub}}{}$$
$$= \frac{x^6\sqrt{a+bx^3+cx^6}}{9c} + \frac{\left(15b^2-16ac-10bcx^3\right)\sqrt{a+bx^3+cx^6}}{72c^3} - \frac{b\left(5b^2-12ac\right)\tanh^{-}}{48c^{7/}}$$

Mathematica [A] time = 0.05, size = 104, normalized size = 0.86

$$\frac{\left(36abc-15b^3\right)\tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)+2\sqrt{c}\sqrt{a+bx^3+cx^6}\left(8c\left(cx^6-2a\right)+15b^2-10bcx^3\right)}{144c^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^11/Sqrt[a + b*x^3 + c*x^6],x]
```

```
[Out] (2*Sqrt[c]*Sqrt[a + b*x^3 + c*x^6]*(15*b^2 - 10*b*c*x^3 + 8*c*(-2*a + c*x^6
)) + (-15*b^3 + 36*a*b*c)*ArcTanh[(b + 2*c*x^3)/(2*Sqrt[c]*Sqrt[a + b*x^3 +
 c*x^6])])/(144*c^(7/2))
```

IntegrateAlgebraic [A] time = 0.32, size = 101, normalized size = 0.83

$$\frac{\left(5b^3-12abc\right)\log\left(-2\sqrt{c}\sqrt{a+bx^3+cx^6}+b+2cx^3\right)}{48c^{7/2}}+\frac{\sqrt{a+bx^3+cx^6}\left(-16ac+15b^2-10bcx^3+8c^2x^6\right)}{72c^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^11/Sqrt[a + b*x^3 + c*x^6],x]
```

```
[Out] (Sqrt[a + b*x^3 + c*x^6]*(15*b^2 - 16*a*c - 10*b*c*x^3 + 8*c^2*x^6))/(72*c^
3) + ((5*b^3 - 12*a*b*c)*Log[b + 2*c*x^3 - 2*Sqrt[c]*Sqrt[a + b*x^3 + c*x^6
]])/(48*c^(7/2))
```

fricas [A] time = 1.36, size = 241, normalized size = 1.99

$$\left[-\frac{3\left(5b^3-12abc\right)\sqrt{c}\log\left(-8c^2x^6-8bcx^3-b^2-4\sqrt{cx^6+bx^3+a}\left(2cx^3+b\right)\sqrt{c}-4ac\right)-4\left(8c^3x^6-10bc^2x^3+15b^2c-16ac^2\right)\sqrt{cx^6+bx^3+a}}{288c^4}, \frac{3\left(5b^3-12abc\right)\sqrt{-c}\arctan\left(\frac{\sqrt{cx^6+bx^3+a}\left(2cx^3+b\right)\sqrt{-c}}{2\left(c^2x^6+bcx^3+ac\right)}\right)+2\left(8c^3x^6-10bc^2x^3+15b^2c-16ac^2\right)\sqrt{cx^6+bx^3+a}}{144c^4}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(c*x^6+b*x^3+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/288*(3*(5*b^3 - 12*a*b*c)*sqrt(c)*log(-8*c^2*x^6 - 8*b*c*x^3 - b^2 - 4*
sqrt(c*x^6 + b*x^3 + a)*(2*c*x^3 + b)*sqrt(c) - 4*a*c) - 4*(8*c^3*x^6 - 10*
```

```
b*c^2*x^3 + 15*b^2*c - 16*a*c^2)*sqrt(c*x^6 + b*x^3 + a))/c^4, 1/144*(3*(5*
b^3 - 12*a*b*c)*sqrt(-c)*arctan(1/2*sqrt(c*x^6 + b*x^3 + a)*(2*c*x^3 + b)*s
qrt(-c)/(c^2*x^6 + b*c*x^3 + a*c)) + 2*(8*c^3*x^6 - 10*b*c^2*x^3 + 15*b^2*c
 - 16*a*c^2)*sqrt(c*x^6 + b*x^3 + a))/c^4]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{11}}{\sqrt{cx^6 + bx^3 + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(c*x^6+b*x^3+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^11/sqrt(c*x^6 + b*x^3 + a), x)
```

maple [F] time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{x^{11}}{\sqrt{c\,x^6 + b\,x^3 + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11/(c*x^6+b*x^3+a)^(1/2),x)
```

```
[Out] int(x^11/(c*x^6+b*x^3+a)^(1/2),x)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(c*x^6+b*x^3+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 zero or nonzero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{11}}{\sqrt{c\,x^6 + b\,x^3 + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11/(a + b*x^3 + c*x^6)^(1/2),x)
```

```
[Out] int(x^11/(a + b*x^3 + c*x^6)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{11}}{\sqrt{a + bx^3 + cx^6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**11/(c*x**6+b*x**3+a)**(1/2),x)
```

```
[Out] Integral(x**11/sqrt(a + b*x**3 + c*x**6), x)
```

3.198 $\int \frac{x^8}{\sqrt{a+bx^3+cx^6}}\,dx$

Optimal. Leaf size=104

$$\frac{(3b^2-4ac)\tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{24c^{5/2}}-\frac{b\sqrt{a+bx^3+cx^6}}{4c^2}+\frac{x^3\sqrt{a+bx^3+cx^6}}{6c}$$

Rubi [A] time = 0.09, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1357, 742, 640, 621, 206}

$$\frac{(3b^2-4ac)\tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{24c^{5/2}}-\frac{b\sqrt{a+bx^3+cx^6}}{4c^2}+\frac{x^3\sqrt{a+bx^3+cx^6}}{6c}$$

Antiderivative was successfully verified.

```
[In] Int[x^8/Sqrt[a + b*x^3 + c*x^6],x]
```

```
[Out] -(b*Sqrt[a + b*x^3 + c*x^6])/(4*c^2) + (x^3*Sqrt[a + b*x^3 + c*x^6])/(6*c)
+ ((3*b^2 - 4*a*c)*ArcTanh[(b + 2*c*x^3)/(2*Sqrt[c]*Sqrt[a + b*x^3 + c*x^6]
)])/(24*c^(5/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 742

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(m + 2*p
+ 1)), x] + Dist[1/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m +
 2*p + 1) - e*(a*e*(m - 1) + b*d*(p + 1)) + e*(2*c*d - b*e)*(m + p)*x, x]*(
a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && If[Rat
ionalQ[m], GtQ[m, 1], SumSimplerQ[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuad
raticQ[a, b, c, d, e, m, p, x]
```

Rule 1357

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
```

```
4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^8}{\sqrt{a+bx^3+cx^6}}\,dx &= \frac{1}{3}\,\text{Subst}\left(\int \frac{x^2}{\sqrt{a+bx+cx^2}}\,dx, x, x^3\right)\\
&= \frac{x^3\sqrt{a+bx^3+cx^6}}{6c} + \frac{\text{Subst}\left(\int \frac{-a-\frac{3bx}{2}}{\sqrt{a+bx+cx^2}}\,dx, x, x^3\right)}{6c}\\
&= -\frac{b\sqrt{a+bx^3+cx^6}}{4c^2} + \frac{x^3\sqrt{a+bx^3+cx^6}}{6c} + \frac{\left(3b^2-4ac\right)\text{Subst}\left(\int \frac{1}{\sqrt{a+bx+cx^2}}\,dx, x, x^3\right)}{24c^2}\\
&= -\frac{b\sqrt{a+bx^3+cx^6}}{4c^2} + \frac{x^3\sqrt{a+bx^3+cx^6}}{6c} + \frac{\left(3b^2-4ac\right)\text{Subst}\left(\int \frac{1}{4c-x^2}\,dx, x, \frac{b+2cx^3}{\sqrt{a+bx^3+cx^6}}\right)}{12c^2}\\
&= -\frac{b\sqrt{a+bx^3+cx^6}}{4c^2} + \frac{x^3\sqrt{a+bx^3+cx^6}}{6c} + \frac{\left(3b^2-4ac\right)\tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{24c^{5/2}}
\end{aligned}
$$

Mathematica [A] time = 0.03, size = 88, normalized size = 0.85

$$
\frac{\left(3b^2-4ac\right)\tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)+2\sqrt{c}\left(2cx^3-3b\right)\sqrt{a+bx^3+cx^6}}{24c^{5/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8/Sqrt[a + b*x^3 + c*x^6],x]
```

```
[Out] (2*Sqrt[c]*(-3*b + 2*c*x^3)*Sqrt[a + b*x^3 + c*x^6] + (3*b^2 - 4*a*c)*ArcTa
nh[(b + 2*c*x^3)/(2*Sqrt[c]*Sqrt[a + b*x^3 + c*x^6])])/(24*c^(5/2))
```

IntegrateAlgebraic [A] time = 0.29, size = 91, normalized size = 0.88

$$
\frac{\left(4ac-3b^2\right)\log\left(-2c^{5/2}\sqrt{a+bx^3+cx^6}+bc^2+2c^3x^3\right)}{24c^{5/2}}+\frac{\left(2cx^3-3b\right)\sqrt{a+bx^3+cx^6}}{12c^2}
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^8/Sqrt[a + b*x^3 + c*x^6],x]
```

```
[Out] ((-3*b + 2*c*x^3)*Sqrt[a + b*x^3 + c*x^6])/(12*c^2) + ((-3*b^2 + 4*a*c)*Log
[b*c^2 + 2*c^3*x^3 - 2*c^(5/2)*Sqrt[a + b*x^3 + c*x^6]])/(24*c^(5/2))
```

fricas [A] time = 1.46, size = 203, normalized size = 1.95

$$
\left[-\frac{\left(3b^2-4ac\right)\sqrt{c}\log\left(-8c^2x^6-8bcx^3-b^2+4\sqrt{cx^6+bx^3+a}\left(2cx^3+b\right)\sqrt{c}-4ac\right)-4\sqrt{cx^6+bx^3+a}\left(2c^2x^3-3bc\right)}{48c^3}, -\frac{\left(3b^2-4ac\right)\sqrt{-c}\arctan\left(\frac{\sqrt{cx^6+bx^3+a}\left(2cx^3+b\right)\sqrt{-c}}{2\left(c^2x^6+bcx^3+ac\right)}\right)-2\sqrt{cx^6+bx^3+a}\left(2c^2x^3-3bc\right)}{24c^3}\right]
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(c*x^6+b*x^3+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/48*((3*b^2 - 4*a*c)*sqrt(c)*log(-8*c^2*x^6 - 8*b*c*x^3 - b^2 + 4*sqrt(c
*x^6 + b*x^3 + a)*(2*c*x^3 + b)*sqrt(c) - 4*a*c) - 4*sqrt(c*x^6 + b*x^3 + a
)*(2*c^2*x^3 - 3*b*c))/c^3, -1/24*((3*b^2 - 4*a*c)*sqrt(-c)*arctan(1/2*sqrt
(c*x^6 + b*x^3 + a)*(2*c*x^3 + b)*sqrt(-c)/(c^2*x^6 + b*c*x^3 + a*c)) - 2*s
qrt(c*x^6 + b*x^3 + a)*(2*c^2*x^3 - 3*b*c))/c^3]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^8}{\sqrt{cx^6 + bx^3 + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(c*x^6+b*x^3+a)^(1/2),x, algorithm="giac")
[Out] integrate(x^8/sqrt(c*x^6 + b*x^3 + a), x)
```

maple [F] time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{x^8}{\sqrt{c\,x^6 + b\,x^3 + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(c*x^6+b*x^3+a)^(1/2),x)
[Out] int(x^8/(c*x^6+b*x^3+a)^(1/2),x)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(c*x^6+b*x^3+a)^(1/2),x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 zero or nonzero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^8}{\sqrt{c\,x^6 + b\,x^3 + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(a + b*x^3 + c*x^6)^(1/2),x)
[Out] int(x^8/(a + b*x^3 + c*x^6)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^8}{\sqrt{a + bx^3 + cx^6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8/(c*x**6+b*x**3+a)**(1/2),x)
[Out] Integral(x**8/sqrt(a + b*x**3 + c*x**6), x)
```

3.199 $\int \frac{x^5}{\sqrt{a+bx^3+cx^6}}\,dx$

Optimal. Leaf size=68

$$\frac{\sqrt{a+bx^3+cx^6}}{3c}-\frac{b\tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{6c^{3/2}}$$

Rubi [A] time = 0.06, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1357, 640, 621, 206}

$$\frac{\sqrt{a+bx^3+cx^6}}{3c}-\frac{b\tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{6c^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/Sqrt[a + b*x^3 + c*x^6],x]
```

```
[Out] Sqrt[a + b*x^3 + c*x^6]/(3*c) - (b*ArcTanh[(b + 2*c*x^3)/(2*Sqrt[c]*Sqrt[a
+ b*x^3 + c*x^6])])/(6*c^(3/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1357

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^5}{\sqrt{a+bx^3+cx^6}}\,dx = \frac{1}{3}\operatorname{Subst}\left(\int \frac{x}{\sqrt{a+bx+cx^2}}\,dx, x, x^3\right)$$
$$= \frac{\sqrt{a+bx^3+cx^6}}{3c} - \frac{b\operatorname{Subst}\left(\int \frac{1}{\sqrt{a+bx+cx^2}}\,dx, x, x^3\right)}{6c}$$
$$= \frac{\sqrt{a+bx^3+cx^6}}{3c} - \frac{b\operatorname{Subst}\left(\int \frac{1}{4c-x^2}\,dx, x, \frac{b+2cx^3}{\sqrt{a+bx^3+cx^6}}\right)}{3c}$$
$$= \frac{\sqrt{a+bx^3+cx^6}}{3c} - \frac{b\tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{6c^{3/2}}$$

Mathematica [A] time = 0.02, size = 68, normalized size = 1.00

$$\frac{\sqrt{a+bx^3+cx^6}}{3c} - \frac{b\tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{6c^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/Sqrt[a + b*x^3 + c*x^6],x]
```

```
[Out] Sqrt[a + b*x^3 + c*x^6]/(3*c) - (b*ArcTanh[(b + 2*c*x^3)/(2*Sqrt[c]*Sqrt[a
+ b*x^3 + c*x^6])])/(6*c^(3/2))
```

IntegrateAlgebraic [A] time = 0.19, size = 70, normalized size = 1.03

$$\frac{b\log\left(-2c^{3/2}\sqrt{a+bx^3+cx^6}+bc+2c^2x^3\right)}{6c^{3/2}} + \frac{\sqrt{a+bx^3+cx^6}}{3c}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^5/Sqrt[a + b*x^3 + c*x^6],x]
```

```
[Out] Sqrt[a + b*x^3 + c*x^6]/(3*c) + (b*Log[b*c + 2*c^2*x^3 - 2*c^(3/2)*Sqrt[a +
 b*x^3 + c*x^6]])/(6*c^(3/2))
```

fricas [A] time = 1.35, size = 161, normalized size = 2.37

$$\left[\frac{b\sqrt{c}\log\left(-8\,c^2x^6 - 8\,bcx^3 - b^2 + 4\,\sqrt{cx^6+bx^3+a}\left(2\,cx^3+b\right)\sqrt{c} - 4\,ac\right) + 4\,\sqrt{cx^6+bx^3+a}\,c}{12\,c^2}, \frac{b\sqrt{-c}\arctan\left(\frac{\sqrt{cx^6+bx^3+a}\left(2\,cx^3+b\right)\sqrt{-c}}{2\left(c^2x^6+bcx^3+ac\right)}\right) + 2\,\sqrt{cx^6+bx^3+a}\,c}{6\,c^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(c*x^6+b*x^3+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/12*(b*sqrt(c)*log(-8*c^2*x^6 - 8*b*c*x^3 - b^2 + 4*sqrt(c*x^6 + b*x^3 +
a)*(2*c*x^3 + b)*sqrt(c) - 4*a*c) + 4*sqrt(c*x^6 + b*x^3 + a)*c)/c^2, 1/6*(
b*sqrt(-c)*arctan(1/2*sqrt(c*x^6 + b*x^3 + a)*(2*c*x^3 + b)*sqrt(-c)/(c^2*x
^6 + b*c*x^3 + a*c)) + 2*sqrt(c*x^6 + b*x^3 + a)*c)/c^2]
```

giac [A] time = 0.67, size = 61, normalized size = 0.90

$$\frac{b\log\left(\left|-2\left(\sqrt{c}\,x^3 - \sqrt{cx^6+bx^3+a}\right)\sqrt{c} - b\right|\right)}{6\,c^{\frac{3}{2}}} + \frac{\sqrt{cx^6+bx^3+a}}{3\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(c*x^6+b*x^3+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/6*b*log(abs(-2*(sqrt(c)*x^3 - sqrt(c*x^6 + b*x^3 + a))*sqrt(c) - b))/c^(3
/2) + 1/3*sqrt(c*x^6 + b*x^3 + a)/c
```

maple [F] time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{x^5}{\sqrt{c\,x^6+b\,x^3+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(c*x^6+b*x^3+a)^(1/2),x)
```

```
[Out] int(x^5/(c*x^6+b*x^3+a)^(1/2),x)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(c*x^6+b*x^3+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 zero or nonzero?
```

mupad [B] time = 1.49, size = 55, normalized size = 0.81

$$\frac{\sqrt{c\,x^6+b\,x^3+a}}{3\,c}-\frac{b\,\ln\left(\sqrt{c\,x^6+b\,x^3+a}+\frac{c\,x^3+\frac{b}{2}}{\sqrt{c}}\right)}{6\,c^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(a + b*x^3 + c*x^6)^(1/2),x)
```

```
[Out] (a + b*x^3 + c*x^6)^(1/2)/(3*c) - (b*log((a + b*x^3 + c*x^6)^(1/2) + (b/2 +
 c*x^3)/c^(1/2)))/(6*c^(3/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5}{\sqrt{a+bx^3+cx^6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(c*x**6+b*x**3+a)**(1/2),x)
```

```
[Out] Integral(x**5/sqrt(a + b*x**3 + c*x**6), x)
```

3.200 $\int \frac{x^2}{\sqrt{a+bx^3+cx^6}}\,dx$

Optimal. Leaf size=43

$$\frac{\tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{3\sqrt{c}}$$

Rubi [A] time = 0.03, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {1352, 621, 206}

$$\frac{\tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{3\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/Sqrt[a + b*x^3 + c*x^6],x]
```

```
[Out] ArcTanh[(b + 2*c*x^3)/(2*Sqrt[c]*Sqrt[a + b*x^3 + c*x^6])]/(3*Sqrt[c])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1352

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a,
b, c, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2}{\sqrt{a+bx^3+cx^6}}\,dx &= \frac{1}{3}\operatorname{Subst}\left(\int \frac{1}{\sqrt{a+bx+cx^2}}\,dx, x, x^3\right)\\
&= \frac{2}{3}\operatorname{Subst}\left(\int \frac{1}{4c-x^2}\,dx, x, \frac{b+2cx^3}{\sqrt{a+bx^3+cx^6}}\right)\\
&= \frac{\tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{3\sqrt{c}}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 43, normalized size = 1.00

$$\frac{\tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{3\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/Sqrt[a + b*x^3 + c*x^6],x]
```

```
[Out] ArcTanh[(b + 2*c*x^3)/(2*Sqrt[c]*Sqrt[a + b*x^3 + c*x^6])]/(3*Sqrt[c])
```

IntegrateAlgebraic [A] time = 0.14, size = 41, normalized size = 0.95

$$-\frac{\log\left(-2\sqrt{c}\sqrt{a+bx^3+cx^6}+b+2cx^3\right)}{3\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^2/Sqrt[a + b*x^3 + c*x^6],x]
```

```
[Out] -1/3*Log[b + 2*c*x^3 - 2*Sqrt[c]*Sqrt[a + b*x^3 + c*x^6]]/Sqrt[c]
```

fricas [A] time = 1.21, size = 118, normalized size = 2.74

$$\left[\frac{\log\left(-8\,c^2x^6-8\,bcx^3-b^2-4\,\sqrt{cx^6+bx^3+a}\left(2\,cx^3+b\right)\sqrt{c}-4\,ac\right)}{6\,\sqrt{c}},-\frac{\sqrt{-c}\arctan\left(\frac{\sqrt{cx^6+bx^3+a}\left(2\,cx^3+b\right)\sqrt{-c}}{2\left(c^2x^6+bcx^3+ac\right)}\right)}{3\,c}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^6+b*x^3+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/6*log(-8*c^2*x^6 - 8*b*c*x^3 - b^2 - 4*sqrt(c*x^6 + b*x^3 + a)*(2*c*x^3
+ b)*sqrt(c) - 4*a*c)/sqrt(c), -1/3*sqrt(-c)*arctan(1/2*sqrt(c*x^6 + b*x^3
+ a)*(2*c*x^3 + b)*sqrt(-c)/(c^2*x^6 + b*c*x^3 + a*c))/c]
```

giac [A] time = 0.68, size = 40, normalized size = 0.93

$$-\frac{\log\left(\left|-2\left(\sqrt{c}x^3-\sqrt{cx^6+bx^3+a}\right)\sqrt{c}-b\right|\right)}{3\,\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^6+b*x^3+a)^(1/2),x, algorithm="giac")
```

```
[Out] -1/3*log(abs(-2*(sqrt(c)*x^3 - sqrt(c*x^6 + b*x^3 + a))*sqrt(c) - b))/sqrt(
c)
```

maple [F] time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\sqrt{c\,x^6+b\,x^3+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(c*x^6+b*x^3+a)^(1/2),x)
```

```
[Out] int(x^2/(c*x^6+b*x^3+a)^(1/2),x)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^6+b*x^3+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 zero or nonzero?
```

mupad [B] time = 1.58, size = 34, normalized size = 0.79

$$\frac{\ln\left(\sqrt{c\,x^6+b\,x^3+a}+\frac{c\,x^3+\frac{b}{2}}{\sqrt{c}}\right)}{3\,\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a + b*x^3 + c*x^6)^(1/2),x)
```

```
[Out] log((a + b*x^3 + c*x^6)^(1/2) + (b/2 + c*x^3)/c^(1/2))/(3*c^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\sqrt{a+bx^3+cx^6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(c*x**6+b*x**3+a)**(1/2),x)
```

```
[Out] Integral(x**2/sqrt(a + b*x**3 + c*x**6), x)
```

3.201 $\int \frac{1}{x\sqrt{a+bx^3+cx^6}}\,dx$

Optimal. Leaf size=44

$$-\frac{\tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{3\sqrt{a}}$$

Rubi [A] time = 0.04, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {1357, 724, 206}

$$-\frac{\tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{3\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Sqrt[a + b*x^3 + c*x^6]),x]
```

```
[Out] -ArcTanh[(2*a + b*x^3)/(2*Sqrt[a]*Sqrt[a + b*x^3 + c*x^6])]/(3*Sqrt[a])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/((((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1357

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x\sqrt{a+bx^3+cx^6}}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx+cx^2}}\,dx, x, x^3\right) \\
&= -\left(\frac{2}{3}\text{Subst}\left(\int \frac{1}{4a-x^2}\,dx, x, \frac{2a+bx^3}{\sqrt{a+bx^3+cx^6}}\right)\right) \\
&= -\frac{\tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{3\sqrt{a}}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 44, normalized size = 1.00

$$-\frac{\tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{3\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Sqrt[a + b*x^3 + c*x^6]),x]
[Out] -1/3*ArcTanh[(2*a + b*x^3)/(2*Sqrt[a]*Sqrt[a + b*x^3 + c*x^6])]/Sqrt[a]
```

IntegrateAlgebraic [A] time = 0.13, size = 48, normalized size = 1.09

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{c}x^3}{\sqrt{a}}-\frac{\sqrt{a+bx^3+cx^6}}{\sqrt{a}}\right)}{3\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x*Sqrt[a + b*x^3 + c*x^6]),x]
[Out] (2*ArcTanh[(Sqrt[c]*x^3)/Sqrt[a] - Sqrt[a + b*x^3 + c*x^6]/Sqrt[a]])/(3*Sqr
t[a])
```

fricas [A] time = 1.10, size = 124, normalized size = 2.82

$$\left[\frac{\log\left(-\frac{(b^2+4\,ac)x^6+8\,abx^3-4\,\sqrt{cx^6+bx^3+a}\,(bx^3+2\,a)\sqrt{a}+8\,a^2}{x^6}\right)}{6\,\sqrt{a}},\ \frac{\sqrt{-a}\,\arctan\left(\frac{\sqrt{cx^6+bx^3+a}\,(bx^3+2\,a)\sqrt{-a}}{2\,(acx^6+abx^3+a^2)}\right)}{3\,a}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^6+b*x^3+a)^(1/2),x, algorithm="fricas")
[Out] [1/6*log(-((b^2 + 4*a*c)*x^6 + 8*a*b*x^3 - 4*sqrt(c*x^6 + b*x^3 + a)*(b*x^3
 + 2*a)*sqrt(a) + 8*a^2)/x^6)/sqrt(a), 1/3*sqrt(-a)*arctan(1/2*sqrt(c*x^6 +
 b*x^3 + a)*(b*x^3 + 2*a)*sqrt(-a)/(a*c*x^6 + a*b*x^3 + a^2))/a]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{cx^6+bx^3+a}\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^6+b*x^3+a)^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(c*x^6 + b*x^3 + a)*x), x)
```

maple [F] time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{c\,x^6+b\,x^3+a}\;x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(c*x^6+b*x^3+a)^(1/2),x)
[Out] int(1/x/(c*x^6+b*x^3+a)^(1/2),x)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^6+b*x^3+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive, negative or zero?
```

mupad [B] time = 1.57, size = 36, normalized size = 0.82

$$-\frac{\ln\left(\frac{b}{2}+\frac{a}{x^3}+\frac{\sqrt{a}\,\sqrt{c\,x^6+b\,x^3+a}}{x^3}\right)}{3\,\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b*x^3 + c*x^6)^(1/2)),x)
[Out] -log(b/2 + a/x^3 + (a^(1/2)*(a + b*x^3 + c*x^6)^(1/2))/x^3)/(3*a^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\sqrt{a+bx^3+cx^6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x**6+b*x**3+a)**(1/2),x)
[Out] Integral(1/(x*sqrt(a + b*x**3 + c*x**6)), x)
```

3.202 $\int \frac{1}{x^4\sqrt{a+bx^3+cx^6}}\,dx$

Optimal. Leaf size=72

$$\frac{b\tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{6a^{3/2}}-\frac{\sqrt{a+bx^3+cx^6}}{3ax^3}$$

Rubi [A] time = 0.06, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1357, 730, 724, 206}

$$\frac{b\tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{6a^{3/2}}-\frac{\sqrt{a+bx^3+cx^6}}{3ax^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*Sqrt[a + b*x^3 + c*x^6]),x]
```

```
[Out] -Sqrt[a + b*x^3 + c*x^6]/(3*a*x^3) + (b*ArcTanh[(2*a + b*x^3)/(2*Sqrt[a]*Sq
rt[a + b*x^3 + c*x^6])])/(6*a^(3/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 730

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 - b*d*e + a*e^2)), x] + Dist[(2*c*d - b*e)/(2*(c*d^2 - b*d*e + a*e^2)),
 Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e,
 m, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2
*c*d - b*e, 0] && EqQ[m + 2*p + 3, 0]
```

Rule 1357

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{1}{x^4\sqrt{a+bx^3+cx^6}}\,dx = \frac{1}{3}\text{Subst}\left(\int \frac{1}{x^2\sqrt{a+bx+cx^2}}\,dx, x, x^3\right)$$
$$= -\frac{\sqrt{a+bx^3+cx^6}}{3ax^3} - \frac{b\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx+cx^2}}\,dx, x, x^3\right)}{6a}$$
$$= -\frac{\sqrt{a+bx^3+cx^6}}{3ax^3} + \frac{b\,\text{Subst}\left(\int \frac{1}{4a-x^2}\,dx, x, \frac{2a+bx^3}{\sqrt{a+bx^3+cx^6}}\right)}{3a}$$
$$= -\frac{\sqrt{a+bx^3+cx^6}}{3ax^3} + \frac{b\tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{6a^{3/2}}$$

Mathematica [A] time = 0.02, size = 72, normalized size = 1.00

$$\frac{b\tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{6a^{3/2}} - \frac{\sqrt{a+bx^3+cx^6}}{3ax^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*Sqrt[a + b*x^3 + c*x^6]),x]
```

```
[Out] -1/3*Sqrt[a + b*x^3 + c*x^6]/(a*x^3) + (b*ArcTanh[(2*a + b*x^3)/(2*Sqrt[a]*
Sqrt[a + b*x^3 + c*x^6])])/(6*a^(3/2))
```

IntegrateAlgebraic [A] time = 0.21, size = 76, normalized size = 1.06

$$-\frac{b\tanh^{-1}\left(\frac{\sqrt{c}x^3}{\sqrt{a}} - \frac{\sqrt{a+bx^3+cx^6}}{\sqrt{a}}\right)}{3a^{3/2}} - \frac{\sqrt{a+bx^3+cx^6}}{3ax^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^4*Sqrt[a + b*x^3 + c*x^6]),x]
```

```
[Out] -1/3*Sqrt[a + b*x^3 + c*x^6]/(a*x^3) - (b*ArcTanh[(Sqrt[c]*x^3)/Sqrt[a] - S
qrt[a + b*x^3 + c*x^6]/Sqrt[a]])/(3*a^(3/2))
```

fricas [A] time = 1.36, size = 179, normalized size = 2.49

$$\left[\frac{\sqrt{a}bx^3\log\left(-\frac{(b^2+4\,ac)x^6+8\,abx^3+4\,\sqrt{cx^6+bx^3+a}\,(bx^3+2\,a)\sqrt{a}+8\,a^2}{x^6}\right) - 4\,\sqrt{cx^6+bx^3+a}\,a}{12\,a^2x^3}, -\frac{\sqrt{-a}\,bx^3\arctan\left(\frac{\sqrt{cx^6+bx^3+a}\,(bx^3+2\,a)\sqrt{-a}}{2\,(acx^6+abx^3+a^2)}\right) + 2\,\sqrt{cx^6+bx^3+a}\,a}{6\,a^2x^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(c*x^6+b*x^3+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/12*(sqrt(a)*b*x^3*log(-((b^2 + 4*a*c)*x^6 + 8*a*b*x^3 + 4*sqrt(c*x^6 + b
*x^3 + a)*(b*x^3 + 2*a)*sqrt(a) + 8*a^2)/x^6) - 4*sqrt(c*x^6 + b*x^3 + a)*a
)/(a^2*x^3), -1/6*(sqrt(-a)*b*x^3*arctan(1/2*sqrt(c*x^6 + b*x^3 + a)*(b*x^3
 + 2*a)*sqrt(-a)/(a*c*x^6 + a*b*x^3 + a^2)) + 2*sqrt(c*x^6 + b*x^3 + a)*a)/
(a^2*x^3)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{cx^6+bx^3+a}\,x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(c*x^6+b*x^3+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(c*x^6 + b*x^3 + a)*x^4), x)
```

maple [F] time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{c\,x^6+b\,x^3+a}\;x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(c*x^6+b*x^3+a)^(1/2),x)
```

```
[Out] int(1/x^4/(c*x^6+b*x^3+a)^(1/2),x)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(c*x^6+b*x^3+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive, negative or zero?
```

mupad [B] time = 1.56, size = 56, normalized size = 0.78

$$\frac{b\,\mathrm{atanh}\left(\frac{\frac{b\,x^3}{2}+a}{\sqrt{a}\,\sqrt{c\,x^6+b\,x^3+a}}\right)}{6\,a^{3/2}}-\frac{\sqrt{c\,x^6+b\,x^3+a}}{3\,a\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(a + b*x^3 + c*x^6)^(1/2)),x)
```

```
[Out] (b*atanh((a + (b*x^3)/2)/(a^(1/2)*(a + b*x^3 + c*x^6)^(1/2))))/(6*a^(3/2))
- (a + b*x^3 + c*x^6)^(1/2)/(3*a*x^3)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^4\sqrt{a+bx^3+cx^6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(c*x**6+b*x**3+a)**(1/2),x)
```

```
[Out] Integral(1/(x**4*sqrt(a + b*x**3 + c*x**6)), x)
```

3.203 $\int \frac{1}{x^7\sqrt{a+bx^3+cx^6}}\,dx$

Optimal. Leaf size=108

$$-\frac{(3b^2-4ac)\tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{24a^{5/2}}+\frac{b\sqrt{a+bx^3+cx^6}}{4a^2x^3}-\frac{\sqrt{a+bx^3+cx^6}}{6ax^6}$$

Rubi [A] time = 0.10, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1357, 744, 806, 724, 206}

$$-\frac{(3b^2-4ac)\tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{24a^{5/2}}+\frac{b\sqrt{a+bx^3+cx^6}}{4a^2x^3}-\frac{\sqrt{a+bx^3+cx^6}}{6ax^6}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^7*Sqrt[a + b*x^3 + c*x^6]),x]
```

```
[Out] -Sqrt[a + b*x^3 + c*x^6]/(6*a*x^6) + (b*Sqrt[a + b*x^3 + c*x^6])/(4*a^2*x^3
) - ((3*b^2 - 4*a*c)*ArcTanh[(2*a + b*x^3)/(2*Sqrt[a]*Sqrt[a + b*x^3 + c*x^
6])])/(24*a^(5/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 744

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b*d*e + a*e^2)), Int[(
d + e*x)^(m + 1)*Simp[c*d*(m + 1) - b*e*(m + p + 2) - c*e*(m + 2*p + 3)*x,
x]*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^
2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && Ne
Q[m, -1] && ((LtQ[m, -1] && IntQuadraticQ[a, b, c, d, e, m, p, x]) || (SumS
implerQ[m, 1] && IntegerQ[p]) || ILtQ[Simplify[m + 2*p + 3], 0])
```

Rule 806

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b
*x + c*x^2)^(p + 1))/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[(b*(e*f
 + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m
+ 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &
& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m +
2*p + 3], 0]
```

Rule 1357

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^7\sqrt{a+bx^3+cx^6}}\,dx &= \frac{1}{3}\operatorname{Subst}\left(\int \frac{1}{x^3\sqrt{a+bx+cx^2}}\,dx, x, x^3\right)\\
&= -\frac{\sqrt{a+bx^3+cx^6}}{6ax^6} - \frac{\operatorname{Subst}\left(\int \frac{\frac{3b}{2}+cx}{x^2\sqrt{a+bx+cx^2}}\,dx, x, x^3\right)}{6a}\\
&= -\frac{\sqrt{a+bx^3+cx^6}}{6ax^6} + \frac{b\sqrt{a+bx^3+cx^6}}{4a^2x^3} + \frac{\left(3b^2-4ac\right)\operatorname{Subst}\left(\int \frac{1}{x\sqrt{a+bx+cx^2}}\,dx, x, x^3\right)}{24a^2}\\
&= -\frac{\sqrt{a+bx^3+cx^6}}{6ax^6} + \frac{b\sqrt{a+bx^3+cx^6}}{4a^2x^3} - \frac{\left(3b^2-4ac\right)\operatorname{Subst}\left(\int \frac{1}{4a-x^2}\,dx, x, \frac{2a+bx^3}{\sqrt{a+bx^3+cx^6}}\right)}{12a^2}\\
&= -\frac{\sqrt{a+bx^3+cx^6}}{6ax^6} + \frac{b\sqrt{a+bx^3+cx^6}}{4a^2x^3} - \frac{\left(3b^2-4ac\right)\tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{24a^{5/2}}\end{aligned}$$

Mathematica [A] time = 0.06, size = 92, normalized size = 0.85

$$\frac{\left(4ac-3b^2\right)\tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)+\frac{2\sqrt{a}\left(3bx^3-2a\right)\sqrt{a+bx^3+cx^6}}{x^6}}{24a^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^7*Sqrt[a + b*x^3 + c*x^6]),x]
```

```
[Out] ((2*Sqrt[a]*(-2*a + 3*b*x^3)*Sqrt[a + b*x^3 + c*x^6])/x^6 + (-3*b^2 + 4*a*c
)*ArcTanh[(2*a + b*x^3)/(2*Sqrt[a]*Sqrt[a + b*x^3 + c*x^6])])/(24*a^(5/2))
```

IntegrateAlgebraic [A] time = 0.34, size = 91, normalized size = 0.84

$$\frac{\left(3b^2-4ac\right)\tanh^{-1}\left(\frac{\sqrt{c}\,x^3-\sqrt{a+bx^3+cx^6}}{\sqrt{a}}\right)}{12a^{5/2}}+\frac{\left(3bx^3-2a\right)\sqrt{a+bx^3+cx^6}}{12a^2x^6}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^7*Sqrt[a + b*x^3 + c*x^6]),x]
```

```
[Out] ((-2*a + 3*b*x^3)*Sqrt[a + b*x^3 + c*x^6])/(12*a^2*x^6) + ((3*b^2 - 4*a*c)*
ArcTanh[(Sqrt[c]*x^3 - Sqrt[a + b*x^3 + c*x^6])/Sqrt[a]])/(12*a^(5/2))
```

fricas [A] time = 1.41, size = 221, normalized size = 2.05

$$\left[-\frac{\left(3\,b^2-4\,ac\right)\sqrt{a}\,x^6\log\left(-\frac{\left(b^2+4\,ac\right)x^6+8\,abx^3+4\,\sqrt{cx^6+bx^3+a}\left(bx^3+2\,a\right)\sqrt{a}+8\,a^2}{x^6}\right)-4\,\sqrt{cx^6+bx^3+a}\left(3\,abx^3-2\,a^2\right)}{48\,a^3x^6},\ \frac{\left(3\,b^2-4\,ac\right)\sqrt{-a}\,x^6\arctan\left(\frac{\sqrt{cx^6+bx^3+a}\left(bx^3+2\,a\right)\sqrt{-a}}{2\left(acx^6+abx^3+a^2\right)}\right)+2\,\sqrt{cx^6+bx^3+a}\left(3\,abx^3-2\,a^2\right)}{24\,a^3x^6}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(c*x^6+b*x^3+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/48*((3*b^2 - 4*a*c)*sqrt(a)*x^6*log(-((b^2 + 4*a*c)*x^6 + 8*a*b*x^3 + 4
*sqrt(c*x^6 + b*x^3 + a)*(b*x^3 + 2*a)*sqrt(a) + 8*a^2)/x^6) - 4*sqrt(c*x^6
 + b*x^3 + a)*(3*a*b*x^3 - 2*a^2))/(a^3*x^6), 1/24*((3*b^2 - 4*a*c)*sqrt(-a
)*x^6*arctan(1/2*sqrt(c*x^6 + b*x^3 + a)*(b*x^3 + 2*a)*sqrt(-a)/(a*c*x^6 +
a*b*x^3 + a^2)) + 2*sqrt(c*x^6 + b*x^3 + a)*(3*a*b*x^3 - 2*a^2))/(a^3*x^6)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{cx^6 + bx^3 + a}\,x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(c*x^6+b*x^3+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(c*x^6 + b*x^3 + a)*x^7), x)
```

maple [F] time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{c\,x^6 + b\,x^3 + a}\;x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^7/(c*x^6+b*x^3+a)^(1/2),x)
```

```
[Out] int(1/x^7/(c*x^6+b*x^3+a)^(1/2),x)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(c*x^6+b*x^3+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive, negative or zero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^7\,\sqrt{c\,x^6 + b\,x^3 + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^7*(a + b*x^3 + c*x^6)^(1/2)),x)
```

```
[Out] int(1/(x^7*(a + b*x^3 + c*x^6)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^7\sqrt{a + bx^3 + cx^6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**7/(c*x**6+b*x**3+a)**(1/2),x)
```

```
[Out] Integral(1/(x**7*sqrt(a + b*x**3 + c*x**6)), x)
```

3.204 $\int \frac{1}{x^{10}\sqrt{a+bx^3+cx^6}}\,dx$

Optimal. Leaf size=145

$$\frac{b\left(5b^2-12ac\right)\tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{48a^{7/2}}-\frac{\left(15b^2-16ac\right)\sqrt{a+bx^3+cx^6}}{72a^3x^3}+\frac{5b\sqrt{a+bx^3+cx^6}}{36a^2x^6}-\frac{\sqrt{a+bx^3+cx^6}}{9ax^9}$$

Rubi [A] time = 0.16, antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {1357, 744, 834, 806, 724, 206}

$$-\frac{\left(15b^2-16ac\right)\sqrt{a+bx^3+cx^6}}{72a^3x^3}+\frac{b\left(5b^2-12ac\right)\tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{48a^{7/2}}+\frac{5b\sqrt{a+bx^3+cx^6}}{36a^2x^6}-\frac{\sqrt{a+bx^3+cx^6}}{9ax^9}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^10*Sqrt[a + b*x^3 + c*x^6]),x]
```

```
[Out] -Sqrt[a + b*x^3 + c*x^6]/(9*a*x^9) + (5*b*Sqrt[a + b*x^3 + c*x^6])/(36*a^2*
x^6) - ((15*b^2 - 16*a*c)*Sqrt[a + b*x^3 + c*x^6])/(72*a^3*x^3) + (b*(5*b^2
 - 12*a*c)*ArcTanh[(2*a + b*x^3)/(2*Sqrt[a]*Sqrt[a + b*x^3 + c*x^6])])/(48*
a^(7/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 744

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b*d*e + a*e^2)), Int[(
d + e*x)^(m + 1)*Simp[c*d*(m + 1) - b*e*(m + p + 2) - c*e*(m + 2*p + 3)*x,
x]*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^
2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && Ne
Q[m, -1] && ((LtQ[m, -1] && IntQuadraticQ[a, b, c, d, e, m, p, x]) || (SumS
implerQ[m, 1] && IntegerQ[p]) || ILtQ[Simplify[m + 2*p + 3], 0])
```

Rule 806

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b
*x + c*x^2)^(p + 1))/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[(b*(e*f
 + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m
+ 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &
& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m +
2*p + 3], 0]
```

Rule 834

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b*
x + c*x^2)^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[(
c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m +
 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] ||
 IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 1357

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x^{10}\sqrt{a+bx^3+cx^6}}\,dx &= \frac{1}{3}\operatorname{Subst}\left(\int \frac{1}{x^4\sqrt{a+bx+cx^2}}\,dx,x,x^3\right)\\
&= -\frac{\sqrt{a+bx^3+cx^6}}{9ax^9}-\frac{\operatorname{Subst}\left(\int \frac{\frac{5b}{2}+2cx}{x^3\sqrt{a+bx+cx^2}}\,dx,x,x^3\right)}{9a}\\
&= -\frac{\sqrt{a+bx^3+cx^6}}{9ax^9}+\frac{5b\sqrt{a+bx^3+cx^6}}{36a^2x^6}+\frac{\operatorname{Subst}\left(\int \frac{\frac{1}{4}\left(15b^2-16ac\right)+\frac{5bcx}{2}}{x^2\sqrt{a+bx+cx^2}}\,dx,x,x^3\right)}{18a^2}\\
&= -\frac{\sqrt{a+bx^3+cx^6}}{9ax^9}+\frac{5b\sqrt{a+bx^3+cx^6}}{36a^2x^6}-\frac{\left(15b^2-16ac\right)\sqrt{a+bx^3+cx^6}}{72a^3x^3}-\frac{\left(b\left(5b^2-\right.\right.}{}\\
&= -\frac{\sqrt{a+bx^3+cx^6}}{9ax^9}+\frac{5b\sqrt{a+bx^3+cx^6}}{36a^2x^6}-\frac{\left(15b^2-16ac\right)\sqrt{a+bx^3+cx^6}}{72a^3x^3}+\frac{\left(b\left(5b^2-\right.\right.}{}\\
&= -\frac{\sqrt{a+bx^3+cx^6}}{9ax^9}+\frac{5b\sqrt{a+bx^3+cx^6}}{36a^2x^6}-\frac{\left(15b^2-16ac\right)\sqrt{a+bx^3+cx^6}}{72a^3x^3}+\frac{b\left(5b^2-1\right.}{}
\end{aligned}
$$

Mathematica [A] time = 0.08, size = 112, normalized size = 0.77

$$
\frac{b\left(5b^2-12ac\right)\tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{48a^{7/2}}+\frac{\sqrt{a+bx^3+cx^6}\left(-8a^2+2a\left(5bx^3+8cx^6\right)-15b^2x^6\right)}{72a^3x^9}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^10*Sqrt[a + b*x^3 + c*x^6]),x]
```

```
[Out] (Sqrt[a + b*x^3 + c*x^6]*(-8*a^2 - 15*b^2*x^6 + 2*a*(5*b*x^3 + 8*c*x^6)))/(
72*a^3*x^9) + (b*(5*b^2 - 12*a*c)*ArcTanh[(2*a + b*x^3)/(2*Sqrt[a]*Sqrt[a +
 b*x^3 + c*x^6])])/(48*a^(7/2))
```

IntegrateAlgebraic [A] time = 0.53, size = 110, normalized size = 0.76

$$
\frac{\left(12abc-5b^3\right)\tanh^{-1}\left(\frac{\sqrt{c}\,x^3-\sqrt{a+bx^3+cx^6}}{\sqrt{a}}\right)}{24a^{7/2}}+\frac{\sqrt{a+bx^3+cx^6}\left(-8a^2+10abx^3+16acx^6-15b^2x^6\right)}{72a^3x^9}
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^10*Sqrt[a + b*x^3 + c*x^6]),x]
```

```
[Out] (Sqrt[a + b*x^3 + c*x^6]*(-8*a^2 + 10*a*b*x^3 - 15*b^2*x^6 + 16*a*c*x^6))/(
72*a^3*x^9) + ((-5*b^3 + 12*a*b*c)*ArcTanh[(Sqrt[c]*x^3 - Sqrt[a + b*x^3 +
c*x^6])/Sqrt[a]])/(24*a^(7/2))
```

fricas [A] time = 1.28, size = 263, normalized size = 1.81

$$\left[-\frac{3 \left(5 b^{3}-12 a b c\right) \sqrt{a} x^{9} \log \left(-\frac{\left(b^{2}+4 a c\right) x^{6}+8 a b x^{3}-4 \sqrt{c x^{6}+b x^{3}+a} \left(b x^{3}+2 a\right) \sqrt{a}+8 a^{2}}{x^{6}}\right)+4 \left(\left(15 a b^{2}-16 a^{2} c\right) x^{6}-10 a^{2} b x^{3}+8 a^{3}\right) \sqrt{c x^{6}+b x^{3}+a}}{288 a^{4} x^{9}}, -\frac{3 \left(5 b^{3}-12 a b c\right) \sqrt{-a} x^{9} \arctan \left(\frac{\sqrt{c x^{6}+b x^{3}+a} \left(b x^{3}+2 a\right) \sqrt{-a}}{2 \left(a c x^{6}+a b x^{3}+a^{2}\right)}\right)+2 \left(\left(15 a b^{2}-16 a^{2} c\right) x^{6}-10 a^{2} b x^{3}+8 a^{3}\right) \sqrt{c x^{6}+b x^{3}+a}}{144 a^{4} x^{9}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^10/(c*x^6+b*x^3+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/288*(3*(5*b^3 - 12*a*b*c)*sqrt(a)*x^9*log(-((b^2 + 4*a*c)*x^6 + 8*a*b*x
^3 - 4*sqrt(c*x^6 + b*x^3 + a)*(b*x^3 + 2*a)*sqrt(a) + 8*a^2)/x^6) + 4*((15
*a*b^2 - 16*a^2*c)*x^6 - 10*a^2*b*x^3 + 8*a^3)*sqrt(c*x^6 + b*x^3 + a))/(a^
4*x^9), -1/144*(3*(5*b^3 - 12*a*b*c)*sqrt(-a)*x^9*arctan(1/2*sqrt(c*x^6 + b
*x^3 + a)*(b*x^3 + 2*a)*sqrt(-a)/(a*c*x^6 + a*b*x^3 + a^2)) + 2*((15*a*b^2
- 16*a^2*c)*x^6 - 10*a^2*b*x^3 + 8*a^3)*sqrt(c*x^6 + b*x^3 + a))/(a^4*x^9)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{c x^{6}+b x^{3}+a}\, x^{10}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^10/(c*x^6+b*x^3+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(c*x^6 + b*x^3 + a)*x^10), x)
```

maple [F] time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{c\,x^{6}+b\,x^{3}+a}\;x^{10}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^10/(c*x^6+b*x^3+a)^(1/2),x)
```

```
[Out] int(1/x^10/(c*x^6+b*x^3+a)^(1/2),x)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^10/(c*x^6+b*x^3+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive, negative or zero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^{10}\,\sqrt{c\,x^{6}+b\,x^{3}+a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^10*(a + b*x^3 + c*x^6)^(1/2)),x)
```

```
[Out] int(1/(x^10*(a + b*x^3 + c*x^6)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{10}\sqrt{a + bx^3 + cx^6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**10/(c*x**6+b*x**3+a)**(1/2),x)
```

```
[Out] Integral(1/(x**10*sqrt(a + b*x**3 + c*x**6)), x)
```

3.205 $\int \frac{1}{x^{13}\sqrt{a+bx^3+cx^6}}\,dx$

Optimal. Leaf size=192

$$\frac{5b\left(21b^2-44ac\right)\sqrt{a+bx^3+cx^6}}{576a^4x^3}-\frac{\left(35b^2-36ac\right)\sqrt{a+bx^3+cx^6}}{288a^3x^6}+\frac{7b\sqrt{a+bx^3+cx^6}}{72a^2x^9}-\frac{\left(48a^2c^2-120ab^2c+3\right.}{3}$$

Rubi [A] time = 0.23, antiderivative size = 192, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {1357, 744, 834, 806, 724, 206}

$$-\frac{\left(48a^2c^2-120ab^2c+35b^4\right)\tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{384a^{9/2}}+\frac{5b\left(21b^2-44ac\right)\sqrt{a+bx^3+cx^6}}{576a^4x^3}-\frac{\left(35b^2-36ac\right)\sqrt{a+bx^3+cx^6}}{288a^3x^6}+\frac{7b\sqrt{a+bx^3+cx^6}}{72a^2x^9}-\frac{\sqrt{a+bx^3+cx^6}}{12ax^{12}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^13*Sqrt[a + b*x^3 + c*x^6]),x]
```

```
[Out] -Sqrt[a + b*x^3 + c*x^6]/(12*a*x^12) + (7*b*Sqrt[a + b*x^3 + c*x^6])/(72*a^
2*x^9) - ((35*b^2 - 36*a*c)*Sqrt[a + b*x^3 + c*x^6])/(288*a^3*x^6) + (5*b*(
21*b^2 - 44*a*c)*Sqrt[a + b*x^3 + c*x^6])/(576*a^4*x^3) - ((35*b^4 - 120*a*
b^2*c + 48*a^2*c^2)*ArcTanh[(2*a + b*x^3)/(2*Sqrt[a]*Sqrt[a + b*x^3 + c*x^6
])])/(384*a^(9/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 744

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b*d*e + a*e^2)), Int[(
d + e*x)^(m + 1)*Simp[c*d*(m + 1) - b*e*(m + p + 2) - c*e*(m + 2*p + 3)*x,
x]*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^
2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && Ne
Q[m, -1] && ((LtQ[m, -1] && IntQuadraticQ[a, b, c, d, e, m, p, x]) || (SumS
implerQ[m, 1] && IntegerQ[p]) || ILtQ[Simplify[m + 2*p + 3], 0])
```

Rule 806

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b
*x + c*x^2)^(p + 1))/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[(b*(e*f
 + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m
+ 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &
& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m +
2*p + 3], 0]
```

Rule 834

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b*
x + c*x^2)^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[(
c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m +
 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] ||
 IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 1357

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{1}{x^{13}\sqrt{a+bx^3+cx^6}}\,dx = \frac{1}{3}\text{Subst}\left(\int \frac{1}{x^5\sqrt{a+bx+cx^2}}\,dx, x, x^3\right)$$

$$= -\frac{\sqrt{a+bx^3+cx^6}}{12ax^{12}} - \frac{\text{Subst}\left(\int \frac{\frac{7b}{2}+3cx}{x^4\sqrt{a+bx+cx^2}}\,dx, x, x^3\right)}{12a}$$

$$= -\frac{\sqrt{a+bx^3+cx^6}}{12ax^{12}} + \frac{7b\sqrt{a+bx^3+cx^6}}{72a^2x^9} + \frac{\text{Subst}\left(\int \frac{\frac{1}{4}\left(35b^2-36ac\right)+7bcx}{x^3\sqrt{a+bx+cx^2}}\,dx, x, x^3\right)}{36a^2}$$

$$= -\frac{\sqrt{a+bx^3+cx^6}}{12ax^{12}} + \frac{7b\sqrt{a+bx^3+cx^6}}{72a^2x^9} - \frac{\left(35b^2-36ac\right)\sqrt{a+bx^3+cx^6}}{288a^3x^6} - \frac{\text{Subst}\left(\int\right.}{}$$

$$= -\frac{\sqrt{a+bx^3+cx^6}}{12ax^{12}} + \frac{7b\sqrt{a+bx^3+cx^6}}{72a^2x^9} - \frac{\left(35b^2-36ac\right)\sqrt{a+bx^3+cx^6}}{288a^3x^6} + \frac{5b\left(21b^2\right.}{}$$

$$= -\frac{\sqrt{a+bx^3+cx^6}}{12ax^{12}} + \frac{7b\sqrt{a+bx^3+cx^6}}{72a^2x^9} - \frac{\left(35b^2-36ac\right)\sqrt{a+bx^3+cx^6}}{288a^3x^6} + \frac{5b\left(21b^2\right.}{}$$

$$= -\frac{\sqrt{a+bx^3+cx^6}}{12ax^{12}} + \frac{7b\sqrt{a+bx^3+cx^6}}{72a^2x^9} - \frac{\left(35b^2-36ac\right)\sqrt{a+bx^3+cx^6}}{288a^3x^6} + \frac{5b\left(21b^2\right.}{}$$

Mathematica [A] time = 0.10, size = 141, normalized size = 0.73

$$\frac{\sqrt{a+bx^3+cx^6}\left(-48a^3+8a^2\left(7bx^3+9cx^6\right)-10abx^6\left(7b+22cx^3\right)+105b^3x^9\right)}{576a^4x^{12}} - \frac{\left(48a^2c^2-120ab^2c+35b^4\right)\tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{384a^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^13*Sqrt[a + b*x^3 + c*x^6]),x]
```

```
[Out] (Sqrt[a + b*x^3 + c*x^6]*(-48*a^3 + 105*b^3*x^9 - 10*a*b*x^6*(7*b + 22*c*x^
3) + 8*a^2*(7*b*x^3 + 9*c*x^6)))/(576*a^4*x^12) - ((35*b^4 - 120*a*b^2*c +
48*a^2*c^2)*ArcTanh[(2*a + b*x^3)/(2*Sqrt[a]*Sqrt[a + b*x^3 + c*x^6])])/(38
4*a^(9/2))
```

IntegrateAlgebraic [A] time = 0.78, size = 141, normalized size = 0.73

$$\frac{\left(48a^2c^2-120ab^2c+35b^4\right)\tanh^{-1}\left(\frac{\sqrt{c}x^3-\sqrt{a+bx^3+cx^6}}{\sqrt{a}}\right)}{192a^{9/2}} + \frac{\sqrt{a+bx^3+cx^6}\left(-48a^3+56a^2bx^3+72a^2cx^6-70ab^2x^6-220abcx^9+105b^3x^9\right)}{576a^4x^{12}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^13*Sqrt[a + b*x^3 + c*x^6]),x]
```

```
[Out] (Sqrt[a + b*x^3 + c*x^6]*(-48*a^3 + 56*a^2*b*x^3 - 70*a*b^2*x^6 + 72*a^2*c*
x^6 + 105*b^3*x^9 - 220*a*b*c*x^9))/(576*a^4*x^12) + ((35*b^4 - 120*a*b^2*c
 + 48*a^2*c^2)*ArcTanh[(Sqrt[c]*x^3 - Sqrt[a + b*x^3 + c*x^6])/Sqrt[a]])/(1
92*a^(9/2))
```

fricas [A] time = 1.78, size = 327, normalized size = 1.70

$$\left[\frac{3\left(35\,b^4 - 120\,ab^2c + 48\,a^2c^2\right)\sqrt{a}\,x^{12}\log\left(-\frac{\left(b^2+4\,ac\right)x^6+8\,abx^3-4\,\sqrt{cx^6+bx^3+a}\left(bx^3+2\,a\right)\sqrt{a}+8\,a^2}{x^6}\right) + 4\left(5\left(21\,ab^3 - 44\,a^2bc\right)x^9 + 56\,a^3bx^3 - 2\left(35\,a^2b^2 - 36\,a^3c\right)x^6 - 48\,a^4\right)\sqrt{cx^6 + bx^3 + a}}{2304\,a^5x^{12}}, \frac{3\left(35\,b^4 - 120\,ab^2c + 48\,a^2c^2\right)\sqrt{-a}\,x^{12}\arctan\left(\frac{\sqrt{cx^6+bx^3+a}\left(bx^3+2\,a\right)\sqrt{-a}}{2\left(acx^6+abx^3+a^2\right)}\right) + 2\left(5\left(21\,ab^3 - 44\,a^2bc\right)x^9 + 56\,a^3bx^3 - 2\left(35\,a^2b^2 - 36\,a^3c\right)x^6 - 48\,a^4\right)\sqrt{cx^6 + bx^3 + a}}{1152\,a^5x^{12}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^13/(c*x^6+b*x^3+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2304*(3*(35*b^4 - 120*a*b^2*c + 48*a^2*c^2)*sqrt(a)*x^12*log(-((b^2 + 4*
a*c)*x^6 + 8*a*b*x^3 - 4*sqrt(c*x^6 + b*x^3 + a)*(b*x^3 + 2*a)*sqrt(a) + 8*
a^2)/x^6) + 4*(5*(21*a*b^3 - 44*a^2*b*c)*x^9 + 56*a^3*b*x^3 - 2*(35*a^2*b^2
 - 36*a^3*c)*x^6 - 48*a^4)*sqrt(c*x^6 + b*x^3 + a))/(a^5*x^12), 1/1152*(3*(
35*b^4 - 120*a*b^2*c + 48*a^2*c^2)*sqrt(-a)*x^12*arctan(1/2*sqrt(c*x^6 + b*
x^3 + a)*(b*x^3 + 2*a)*sqrt(-a)/(a*c*x^6 + a*b*x^3 + a^2)) + 2*(5*(21*a*b^3
 - 44*a^2*b*c)*x^9 + 56*a^3*b*x^3 - 2*(35*a^2*b^2 - 36*a^3*c)*x^6 - 48*a^4)
*sqrt(c*x^6 + b*x^3 + a))/(a^5*x^12)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{cx^6 + bx^3 + a}\,x^{13}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^13/(c*x^6+b*x^3+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(c*x^6 + b*x^3 + a)*x^13), x)
```

maple [F] time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{c\,x^6 + b\,x^3 + a}\;x^{13}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^13/(c*x^6+b*x^3+a)^(1/2),x)
```

```
[Out] int(1/x^13/(c*x^6+b*x^3+a)^(1/2),x)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^13/(c*x^6+b*x^3+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive, negative or zero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^{13}\,\sqrt{c\,x^6 + b\,x^3 + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^13*(a + b*x^3 + c*x^6)^(1/2)),x)
[Out] int(1/(x^13*(a + b*x^3 + c*x^6)^(1/2)), x)
```

sympy [F]   time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{13}\sqrt{a + bx^3 + cx^6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**13/(c*x**6+b*x**3+a)**(1/2),x)
[Out] Integral(1/(x**13*sqrt(a + b*x**3 + c*x**6)), x)
```

3.206 $\int \frac{x^{14}}{\left(a+bx^3+cx^6\right)^{3/2}}\,dx$

Optimal. Leaf size=195

$$\frac{\left(5b^2-4ac\right)\tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{8c^{7/2}}-\frac{\left(b\left(15b^2-52ac\right)-2cx^3\left(5b^2-12ac\right)\right)\sqrt{a+bx^3+cx^6}}{12c^3\left(b^2-4ac\right)}-\frac{2bx^6\sqrt{a+bx^3+}}{3c\left(b^2-4ac\right.}$$

Rubi [A] time = 0.23, antiderivative size = 195, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {1357, 738, 832, 779, 621, 206}

$$-\frac{\left(b\left(15b^2-52ac\right)-2cx^3\left(5b^2-12ac\right)\right)\sqrt{a+bx^3+cx^6}}{12c^3\left(b^2-4ac\right)}+\frac{\left(5b^2-4ac\right)\tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{8c^{7/2}}+\frac{2x^9\left(2a+bx^3\right)}{3\left(b^2-4ac\right)\sqrt{a+bx^3+cx^6}}-\frac{2bx^6\sqrt{a+bx^3+cx^6}}{3c\left(b^2-4ac\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x^14/(a + b*x^3 + c*x^6)^(3/2),x]
```

```
[Out] (2*x^9*(2*a + b*x^3))/(3*(b^2 - 4*a*c)*Sqrt[a + b*x^3 + c*x^6]) - (2*b*x^6*
Sqrt[a + b*x^3 + c*x^6])/(3*c*(b^2 - 4*a*c)) - ((b*(15*b^2 - 52*a*c) - 2*c*
(5*b^2 - 12*a*c)*x^3)*Sqrt[a + b*x^3 + c*x^6])/(12*c^3*(b^2 - 4*a*c)) + ((5
*b^2 - 4*a*c)*ArcTanh[(b + 2*c*x^3)/(2*Sqrt[c]*Sqrt[a + b*x^3 + c*x^6])])/(
8*c^(7/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 738

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m - 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*(a + b*x
+ c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*
c)), Int[(d + e*x)^(m - 2)*Simp[e*(2*a*e*(m - 1) + b*d*(2*p - m + 4)) - 2*c
*d^2*(2*p + 3) + e*(b*e - 2*d*c)*(m + 2*p + 2)*x, x]*(a + b*x + c*x^2)^(p +
 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^
2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p, -1] && GtQ[m, 1] &&
IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 779

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> -Simp[((b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x)*(a + b*x + c*x^2)^(p + 1))/(2*c^2*(p + 1)*(2*p + 3)), x
] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p +
3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 832

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + b*x + c*x^2)^(p +
1))/(c*(m + 2*p + 2)), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)
*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m*
(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p]
 || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 1357

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^{14}}{\left(a+bx^3+cx^6\right)^{3/2}}\,dx = \frac{1}{3}\,\text{Subst}\left(\int \frac{x^4}{\left(a+bx+cx^2\right)^{3/2}}\,dx,x,x^3\right)$$

$$= \frac{2x^9\left(2a+bx^3\right)}{3\left(b^2-4ac\right)\sqrt{a+bx^3+cx^6}} - \frac{2\,\text{Subst}\left(\int \frac{x^2(6a+3bx)}{\sqrt{a+bx+cx^2}}\,dx,x,x^3\right)}{3\left(b^2-4ac\right)}$$

$$= \frac{2x^9\left(2a+bx^3\right)}{3\left(b^2-4ac\right)\sqrt{a+bx^3+cx^6}} - \frac{2bx^6\sqrt{a+bx^3+cx^6}}{3c\left(b^2-4ac\right)} - \frac{2\,\text{Subst}\left(\int \frac{x\left(-6ab-\frac{3}{2}\left(5b^2-12ac\right)x\right)}{\sqrt{a+bx+cx^2}}\,d\right.}{9c\left(b^2-4ac\right)}$$

$$= \frac{2x^9\left(2a+bx^3\right)}{3\left(b^2-4ac\right)\sqrt{a+bx^3+cx^6}} - \frac{2bx^6\sqrt{a+bx^3+cx^6}}{3c\left(b^2-4ac\right)} - \frac{\left(b\left(15b^2-52ac\right)-2c\left(5b^2-12\right.\right.}{12c^3\left(b^2-4\right.}$$

$$= \frac{2x^9\left(2a+bx^3\right)}{3\left(b^2-4ac\right)\sqrt{a+bx^3+cx^6}} - \frac{2bx^6\sqrt{a+bx^3+cx^6}}{3c\left(b^2-4ac\right)} - \frac{\left(b\left(15b^2-52ac\right)-2c\left(5b^2-12\right.\right.}{12c^3\left(b^2-4\right.}$$

$$= \frac{2x^9\left(2a+bx^3\right)}{3\left(b^2-4ac\right)\sqrt{a+bx^3+cx^6}} - \frac{2bx^6\sqrt{a+bx^3+cx^6}}{3c\left(b^2-4ac\right)} - \frac{\left(b\left(15b^2-52ac\right)-2c\left(5b^2-12\right.\right.}{12c^3\left(b^2-4\right.}$$

Mathematica [A] time = 0.18, size = 181, normalized size = 0.93

$$\frac{\frac{2\sqrt{c}\left(4a^2c\left(6cx^3-13b\right)+a\left(15b^3-62b^2cx^3-20bc^2x^6+8c^3x^9\right)+b^2x^3\left(15b^2+5bcx^3-2c^2x^6\right)\right)}{\sqrt{a+bx^3+cx^6}}-3\left(16a^2c^2-24ab^2c+5b^4\right)\tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{24c^{7/2}\left(4ac-b^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^14/(a + b*x^3 + c*x^6)^(3/2),x]
```

```
[Out] ((2*Sqrt[c]*(4*a^2*c*(-13*b + 6*c*x^3) + b^2*x^3*(15*b^2 + 5*b*c*x^3 - 2*c^
2*x^6) + a*(15*b^3 - 62*b^2*c*x^3 - 20*b*c^2*x^6 + 8*c^3*x^9)))/Sqrt[a + b*
x^3 + c*x^6] - 3*(5*b^4 - 24*a*b^2*c + 16*a^2*c^2)*ArcTanh[(b + 2*c*x^3)/(2
*Sqrt[c]*Sqrt[a + b*x^3 + c*x^6]])])/(24*c^(7/2)*(-b^2 + 4*a*c))
```

IntegrateAlgebraic [A] time = 0.96, size = 175, normalized size = 0.90

$$\frac{-52a^2bc+24a^2c^2x^3+15ab^3-62ab^2cx^3-20abc^2x^6+8ac^3x^9+15b^4x^3+5b^3cx^6-2b^2c^2x^9}{12c^3\left(4ac-b^2\right)\sqrt{a+bx^3+cx^6}}+\frac{\left(4ac-5b^2\right)\log\left(-2c^{7/2}\sqrt{a+bx^3+cx^6}+bc^3+2c^4x^3\right)}{8c^{7/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^14/(a + b*x^3 + c*x^6)^(3/2),x]
```

```
[Out] (15*a*b^3 - 52*a^2*b*c + 15*b^4*x^3 - 62*a*b^2*c*x^3 + 24*a^2*c^2*x^3 + 5*b
^3*c*x^6 - 20*a*b*c^2*x^6 - 2*b^2*c^2*x^9 + 8*a*c^3*x^9)/(12*c^3*(-b^2 + 4*
a*c)*Sqrt[a + b*x^3 + c*x^6]) + ((-5*b^2 + 4*a*c)*Log[b*c^3 + 2*c^4*x^3 - 2
*c^(7/2)*Sqrt[a + b*x^3 + c*x^6]])/(8*c^(7/2))
```

fricas [A] time = 1.91, size = 591, normalized size = 3.03

$$\left[-\frac{3\left(\left(5b^4c-24ab^2c^2+16a^2c^3\right)x^6+5ab^4-24a^2b^2c+16a^3c^2+\left(5b^5-24ab^3c+16a^2bc^2\right)x^3\right)\sqrt{c}\log\left(-8c^2x^6-8bcx^3-b^2+4\sqrt{cx^6+bx^3+a}\left(2cx^3+b\right)\sqrt{c}-4ac\right)-4\left(2\left(b^2c^3-4ac^4\right)x^9-5\left(b^3c^2-4abc^3\right)x^6-15ab^3c+52a^2bc^2-\left(15b^4c-62ab^2c^2+24a^2c^3\right)x^3\right)\sqrt{cx^6+bx^3+a}}{48\left(ab^2c^4-4a^2c^5+\left(b^2c^5-4ac^6\right)x^6+\left(b^3c^4-4abc^5\right)x^3\right)}, -\frac{3\left(\left(5b^4c-24ab^2c^2+16a^2c^3\right)x^6+5ab^4-24a^2b^2c+16a^3c^2+\left(5b^5-24ab^3c+16a^2bc^2\right)x^3\right)\sqrt{-c}\arctan\left(\frac{\sqrt{cx^6+bx^3+a}\left(2cx^3+b\right)\sqrt{-c}}{2\left(c^2x^6+bcx^3+ac\right)}\right)-2\left(2\left(b^2c^3-4ac^4\right)x^9-5\left(b^3c^2-4abc^3\right)x^6-15ab^3c+52a^2bc^2-\left(15b^4c-62ab^2c^2+24a^2c^3\right)x^3\right)\sqrt{cx^6+bx^3+a}}{24\left(ab^2c^4-4a^2c^5+\left(b^2c^5-4ac^6\right)x^6+\left(b^3c^4-4abc^5\right)x^3\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^14/(c*x^6+b*x^3+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/48*(3*((5*b^4*c - 24*a*b^2*c^2 + 16*a^2*c^3)*x^6 + 5*a*b^4 - 24*a^2*b^2
*c + 16*a^3*c^2 + (5*b^5 - 24*a*b^3*c + 16*a^2*b*c^2)*x^3)*sqrt(c)*log(-8*c
^2*x^6 - 8*b*c*x^3 - b^2 + 4*sqrt(c*x^6 + b*x^3 + a)*(2*c*x^3 + b)*sqrt(c)
- 4*a*c) - 4*(2*(b^2*c^3 - 4*a*c^4)*x^9 - 5*(b^3*c^2 - 4*a*b*c^3)*x^6 - 15*
a*b^3*c + 52*a^2*b*c^2 - (15*b^4*c - 62*a*b^2*c^2 + 24*a^2*c^3)*x^3)*sqrt(c
*x^6 + b*x^3 + a))/(a*b^2*c^4 - 4*a^2*c^5 + (b^2*c^5 - 4*a*c^6)*x^6 + (b^3*
c^4 - 4*a*b*c^5)*x^3), -1/24*(3*((5*b^4*c - 24*a*b^2*c^2 + 16*a^2*c^3)*x^6
+ 5*a*b^4 - 24*a^2*b^2*c + 16*a^3*c^2 + (5*b^5 - 24*a*b^3*c + 16*a^2*b*c^2)
*x^3)*sqrt(-c)*arctan(1/2*sqrt(c*x^6 + b*x^3 + a)*(2*c*x^3 + b)*sqrt(-c)/(c
^2*x^6 + b*c*x^3 + a*c)) - 2*(2*(b^2*c^3 - 4*a*c^4)*x^9 - 5*(b^3*c^2 - 4*a*
b*c^3)*x^6 - 15*a*b^3*c + 52*a^2*b*c^2 - (15*b^4*c - 62*a*b^2*c^2 + 24*a^2*
c^3)*x^3)*sqrt(c*x^6 + b*x^3 + a))/(a*b^2*c^4 - 4*a^2*c^5 + (b^2*c^5 - 4*a*
c^6)*x^6 + (b^3*c^4 - 4*a*b*c^5)*x^3)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{14}}{\left(cx^6+bx^3+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^14/(c*x^6+b*x^3+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x^14/(c*x^6 + b*x^3 + a)^(3/2), x)
```

maple [F] time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{x^{14}}{\left(c\,x^6+b\,x^3+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^14/(c*x^6+b*x^3+a)^(3/2),x)
```

```
[Out] int(x^14/(c*x^6+b*x^3+a)^(3/2),x)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^14/(c*x^6+b*x^3+a)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 zero or nonzero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{14}}{\left(c\,x^6+b\,x^3+a\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^14/(a + b*x^3 + c*x^6)^(3/2),x)
```

```
[Out] int(x^14/(a + b*x^3 + c*x^6)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{14}}{\left(a + bx^{3} + cx^{6}\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**14/(c*x**6+b*x**3+a)**(3/2),x)
```

```
[Out] Integral(x**14/(a + b*x**3 + c*x**6)**(3/2), x)
```

3.207 $\int \frac{x^{11}}{\left(a+bx^3+cx^6\right)^{3/2}}\,dx$

Optimal. Leaf size=137

$$\frac{\left(-8ac+3b^2-2bcx^3\right)\sqrt{a+bx^3+cx^6}}{3c^2\left(b^2-4ac\right)}+\frac{2x^6\left(2a+bx^3\right)}{3\left(b^2-4ac\right)\sqrt{a+bx^3+cx^6}}-\frac{b\tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{2c^{5/2}}$$

Rubi [A] time = 0.11, antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1357, 738, 779, 621, 206}

$$\frac{\left(-8ac+3b^2-2bcx^3\right)\sqrt{a+bx^3+cx^6}}{3c^2\left(b^2-4ac\right)}+\frac{2x^6\left(2a+bx^3\right)}{3\left(b^2-4ac\right)\sqrt{a+bx^3+cx^6}}-\frac{b\tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{2c^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^11/(a + b*x^3 + c*x^6)^(3/2),x]
```

```
[Out] (2*x^6*(2*a + b*x^3))/(3*(b^2 - 4*a*c)*Sqrt[a + b*x^3 + c*x^6]) + ((3*b^2 -
 8*a*c - 2*b*c*x^3)*Sqrt[a + b*x^3 + c*x^6])/(3*c^2*(b^2 - 4*a*c)) - (b*Arc
Tanh[(b + 2*c*x^3)/(2*Sqrt[c]*Sqrt[a + b*x^3 + c*x^6])])/(2*c^(5/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 738

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m - 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*(a + b*x
+ c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*
c)), Int[(d + e*x)^(m - 2)*Simp[e*(2*a*e*(m - 1) + b*d*(2*p - m + 4)) - 2*c
*d^2*(2*p + 3) + e*(b*e - 2*d*c)*(m + 2*p + 2)*x, x]*(a + b*x + c*x^2)^(p +
 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^
2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p, -1] && GtQ[m, 1] &&
IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 779

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> -Simp[((b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x)*(a + b*x + c*x^2)^(p + 1))/(2*c^2*(p + 1)*(2*p + 3)), x
] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p +
3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 1357

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^{11}}{\left(a+bx^3+cx^6\right)^{3/2}}\,dx = \frac{1}{3}\,\mathrm{Subst}\left(\int \frac{x^3}{\left(a+bx+cx^2\right)^{3/2}}\,dx, x, x^3\right)$$

$$= \frac{2x^6\left(2a+bx^3\right)}{3\left(b^2-4ac\right)\sqrt{a+bx^3+cx^6}} - \frac{2\,\mathrm{Subst}\left(\int \frac{x(4a+2bx)}{\sqrt{a+bx+cx^2}}\,dx, x, x^3\right)}{3\left(b^2-4ac\right)}$$

$$= \frac{2x^6\left(2a+bx^3\right)}{3\left(b^2-4ac\right)\sqrt{a+bx^3+cx^6}} + \frac{\left(3b^2-8ac-2bcx^3\right)\sqrt{a+bx^3+cx^6}}{3c^2\left(b^2-4ac\right)} - \frac{b\,\mathrm{Subst}\left(\int \frac{\cdots}{\sqrt{a+\cdots}}\cdots\right.}{2\cdots}$$

$$= \frac{2x^6\left(2a+bx^3\right)}{3\left(b^2-4ac\right)\sqrt{a+bx^3+cx^6}} + \frac{\left(3b^2-8ac-2bcx^3\right)\sqrt{a+bx^3+cx^6}}{3c^2\left(b^2-4ac\right)} - \frac{b\,\mathrm{Subst}\left(\int \frac{1}{4c-x\cdots}\cdots\right.}{\cdots}$$

$$= \frac{2x^6\left(2a+bx^3\right)}{3\left(b^2-4ac\right)\sqrt{a+bx^3+cx^6}} + \frac{\left(3b^2-8ac-2bcx^3\right)\sqrt{a+bx^3+cx^6}}{3c^2\left(b^2-4ac\right)} - \frac{b\tanh^{-1}\left(\frac{b\cdots}{2\sqrt{c}\sqrt{\cdots}}\right.}{2c^{5/2}}$$

Mathematica [A] time = 0.12, size = 137, normalized size = 1.00

$$\frac{\frac{2\sqrt{c}\left(8a^2c+a\left(-3b^2+10bcx^3+4c^2x^6\right)-b^2x^3\left(3b+cx^3\right)\right)}{\sqrt{a+bx^3+cx^6}}+3b\left(b^2-4ac\right)\tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{6c^{5/2}\left(4ac-b^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^11/(a + b*x^3 + c*x^6)^(3/2),x]
```

```
[Out] ((2*Sqrt[c]*(8*a^2*c - b^2*x^3*(3*b + c*x^3) + a*(-3*b^2 + 10*b*c*x^3 + 4*c
^2*x^6)))/Sqrt[a + b*x^3 + c*x^6] + 3*b*(b^2 - 4*a*c)*ArcTanh[(b + 2*c*x^3)
/(2*Sqrt[c]*Sqrt[a + b*x^3 + c*x^6])])/(6*c^(5/2)*(-b^2 + 4*a*c))
```

IntegrateAlgebraic [A] time = 0.67, size = 131, normalized size = 0.96

$$\frac{8a^2c-3ab^2+10abcx^3+4ac^2x^6-3b^3x^3-b^2cx^6}{3c^2\left(4ac-b^2\right)\sqrt{a+bx^3+cx^6}}+\frac{b\log\left(-2c^{5/2}\sqrt{a+bx^3+cx^6}+bc^2+2c^3x^3\right)}{2c^{5/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^11/(a + b*x^3 + c*x^6)^(3/2),x]
```

```
[Out] (-3*a*b^2 + 8*a^2*c - 3*b^3*x^3 + 10*a*b*c*x^3 - b^2*c*x^6 + 4*a*c^2*x^6)/(
3*c^2*(-b^2 + 4*a*c)*Sqrt[a + b*x^3 + c*x^6]) + (b*Log[b*c^2 + 2*c^3*x^3 -
2*c^(5/2)*Sqrt[a + b*x^3 + c*x^6]])/(2*c^(5/2))
```

fricas [A] time = 1.37, size = 459, normalized size = 3.35

$$\left[\frac{3\left(\left(b^3c-4abc^2\right)x^6+ab^3-4a^2bc+\left(b^4-4ab^2c\right)x^3\right)\sqrt{c}\log\left(-8c^2x^6-8bcx^3-b^2+4\sqrt{cx^6+bx^3+a}\left(2cx^3+b\right)\sqrt{c}-4ac\right)+4\left(\left(b^2c^2-4ac^3\right)x^6+3ab^2c-8a^2c^2+\left(3b^3c-10abc^2\right)x^3\right)\sqrt{cx^6+bx^3+a}}{12\left(\left(b^2c^4-4ac^5\right)x^6+ab^2c^3-4a^2c^4+\left(b^3c^3-4abc^4\right)x^3\right)}, \frac{3\left(\left(b^3c-4abc^2\right)x^6+ab^3-4a^2bc+\left(b^4-4ab^2c\right)x^3\right)\sqrt{-c}\arctan\left(\frac{\sqrt{cx^6+bx^3+a}\left(2cx^3+b\right)\sqrt{-c}}{2\left(c^2x^6+bcx^3+ac\right)}\right)+2\left(\left(b^2c^2-4ac^3\right)x^6+3ab^2c-8a^2c^2+\left(3b^3c-10abc^2\right)x^3\right)\sqrt{cx^6+bx^3+a}}{6\left(\left(b^2c^4-4ac^5\right)x^6+ab^2c^3-4a^2c^4+\left(b^3c^3-4abc^4\right)x^3\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(c*x^6+b*x^3+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/12*(3*((b^3*c - 4*a*b*c^2)*x^6 + a*b^3 - 4*a^2*b*c + (b^4 - 4*a*b^2*c)*x
^3)*sqrt(c)*log(-8*c^2*x^6 - 8*b*c*x^3 - b^2 + 4*sqrt(c*x^6 + b*x^3 + a)*(2
*c*x^3 + b)*sqrt(c) - 4*a*c) + 4*((b^2*c^2 - 4*a*c^3)*x^6 + 3*a*b^2*c - 8*a
^2*c^2 + (3*b^3*c - 10*a*b*c^2)*x^3)*sqrt(c*x^6 + b*x^3 + a))/((b^2*c^4 - 4
*a*c^5)*x^6 + a*b^2*c^3 - 4*a^2*c^4 + (b^3*c^3 - 4*a*b*c^4)*x^3), 1/6*(3*((
b^3*c - 4*a*b*c^2)*x^6 + a*b^3 - 4*a^2*b*c + (b^4 - 4*a*b^2*c)*x^3)*sqrt(-c
)*arctan(1/2*sqrt(c*x^6 + b*x^3 + a)*(2*c*x^3 + b)*sqrt(-c)/(c^2*x^6 + b*c*
x^3 + a*c)) + 2*((b^2*c^2 - 4*a*c^3)*x^6 + 3*a*b^2*c - 8*a^2*c^2 + (3*b^3*c
 - 10*a*b*c^2)*x^3)*sqrt(c*x^6 + b*x^3 + a))/((b^2*c^4 - 4*a*c^5)*x^6 + a*b
^2*c^3 - 4*a^2*c^4 + (b^3*c^3 - 4*a*b*c^4)*x^3)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{11}}{\left(cx^6 + bx^3 + a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(c*x^6+b*x^3+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x^11/(c*x^6 + b*x^3 + a)^(3/2), x)
```

maple [F] time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{x^{11}}{\left(c\,x^6 + b\,x^3 + a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11/(c*x^6+b*x^3+a)^(3/2),x)
```

```
[Out] int(x^11/(c*x^6+b*x^3+a)^(3/2),x)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(c*x^6+b*x^3+a)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 zero or nonzero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{11}}{{\left(c\,x^6+b\,x^3+a\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11/(a + b*x^3 + c*x^6)^(3/2),x)
```

```
[Out] int(x^11/(a + b*x^3 + c*x^6)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{11}}{\left(a+bx^3+cx^6\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**11/(c*x**6+b*x**3+a)**(3/2),x)
```

```
[Out] Integral(x**11/(a + b*x**3 + c*x**6)**(3/2), x)
```

3.208 $\int \frac{x^8}{\left(a+bx^3+cx^6\right)^{3/2}}\,dx$

Optimal. Leaf size=120

$$\frac{2x^3\left(2a+bx^3\right)}{3\left(b^2-4ac\right)\sqrt{a+bx^3+cx^6}}-\frac{2b\sqrt{a+bx^3+cx^6}}{3c\left(b^2-4ac\right)}+\frac{\tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{3c^{3/2}}$$

Rubi [A] time = 0.09, antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1357, 738, 640, 621, 206}

$$\frac{2x^3\left(2a+bx^3\right)}{3\left(b^2-4ac\right)\sqrt{a+bx^3+cx^6}}-\frac{2b\sqrt{a+bx^3+cx^6}}{3c\left(b^2-4ac\right)}+\frac{\tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{3c^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^8/(a + b*x^3 + c*x^6)^(3/2),x]
```

```
[Out] (2*x^3*(2*a + b*x^3))/(3*(b^2 - 4*a*c)*Sqrt[a + b*x^3 + c*x^6]) - (2*b*Sqrt
[a + b*x^3 + c*x^6])/(3*c*(b^2 - 4*a*c)) + ArcTanh[(b + 2*c*x^3)/(2*Sqrt[c]
*Sqrt[a + b*x^3 + c*x^6])]/(3*c^(3/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 738

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m - 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*(a + b*x
+ c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*
c)), Int[(d + e*x)^(m - 2)*Simp[e*(2*a*e*(m - 1) + b*d*(2*p - m + 4)) - 2*c
*d^2*(2*p + 3) + e*(b*e - 2*d*c)*(m + 2*p + 2)*x, x]*(a + b*x + c*x^2)^(p +
 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^
2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p, -1] && GtQ[m, 1] &&
IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 1357

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
```

```
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^8}{\left(a+bx^3+cx^6\right)^{3/2}}\,dx &= \frac{1}{3}\,\text{Subst}\left(\int \frac{x^2}{\left(a+bx+cx^2\right)^{3/2}}\,dx,x,x^3\right)\\
&= \frac{2x^3\left(2a+bx^3\right)}{3\left(b^2-4ac\right)\sqrt{a+bx^3+cx^6}}-\frac{2\,\text{Subst}\left(\int \frac{2a+bx}{\sqrt{a+bx+cx^2}}\,dx,x,x^3\right)}{3\left(b^2-4ac\right)}\\
&= \frac{2x^3\left(2a+bx^3\right)}{3\left(b^2-4ac\right)\sqrt{a+bx^3+cx^6}}-\frac{2b\sqrt{a+bx^3+cx^6}}{3c\left(b^2-4ac\right)}+\frac{\text{Subst}\left(\int \frac{1}{\sqrt{a+bx+cx^2}}\,dx,x,x^3\right)}{3c}\\
&= \frac{2x^3\left(2a+bx^3\right)}{3\left(b^2-4ac\right)\sqrt{a+bx^3+cx^6}}-\frac{2b\sqrt{a+bx^3+cx^6}}{3c\left(b^2-4ac\right)}+\frac{2\,\text{Subst}\left(\int \frac{1}{4c-x^2}\,dx,x,\frac{b+2cx^3}{\sqrt{a+bx^3+cx^6}}\right)}{3c}\\
&= \frac{2x^3\left(2a+bx^3\right)}{3\left(b^2-4ac\right)\sqrt{a+bx^3+cx^6}}-\frac{2b\sqrt{a+bx^3+cx^6}}{3c\left(b^2-4ac\right)}+\frac{\tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{3c^{3/2}}
\end{aligned}
$$

Mathematica [A] time = 0.10, size = 107, normalized size = 0.89

$$
\frac{\frac{2\sqrt{c}\left(a\left(b-2cx^3\right)+b^2x^3\right)}{\sqrt{a+bx^3+cx^6}}-\left(b^2-4ac\right)\tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{3c^{3/2}\left(4ac-b^2\right)}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8/(a + b*x^3 + c*x^6)^(3/2),x]
```

```
[Out] ((2*Sqrt[c]*(b^2*x^3 + a*(b - 2*c*x^3)))/Sqrt[a + b*x^3 + c*x^6] - (b^2 - 4
*a*c)*ArcTanh[(b + 2*c*x^3)/(2*Sqrt[c]*Sqrt[a + b*x^3 + c*x^6])])/(3*c^(3/2
)*(-b^2 + 4*a*c))
```

IntegrateAlgebraic [A] time = 0.52, size = 99, normalized size = 0.82

$$
\frac{2\left(ab-2acx^3+b^2x^3\right)}{3c\left(4ac-b^2\right)\sqrt{a+bx^3+cx^6}}-\frac{\log\left(-2c^{3/2}\sqrt{a+bx^3+cx^6}+bc+2c^2x^3\right)}{3c^{3/2}}
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^8/(a + b*x^3 + c*x^6)^(3/2),x]
```

```
[Out] (2*(a*b + b^2*x^3 - 2*a*c*x^3))/(3*c*(-b^2 + 4*a*c)*Sqrt[a + b*x^3 + c*x^6]
) - Log[b*c + 2*c^2*x^3 - 2*c^(3/2)*Sqrt[a + b*x^3 + c*x^6]]/(3*c^(3/2))
```

fricas [A] time = 1.34, size = 387, normalized size = 3.22

$$
\left[\frac{\left(\left(b^2c-4ac^2\right)x^6+\left(b^3-4abc\right)x^3+ab^2-4a^2c\right)\sqrt{c}\log\left(-8c^2x^6-8bcx^3-b^2-4\sqrt{cx^6+bx^3+a}\left(2cx^3+b\right)\sqrt{c}-4ac\right)-4\sqrt{cx^6+bx^3+a}\left(\left(b^2c-2ac^2\right)x^3+abc\right)}{6\left(\left(b^2c^3-4ac^4\right)x^6+ab^2c^2-4a^2c^3+\left(b^3c^2-4abc^3\right)x^3\right)},-\frac{\left(\left(b^2c-4ac^2\right)x^6+\left(b^3-4abc\right)x^3+ab^2-4a^2c\right)\sqrt{-c}\arctan\left(\frac{\sqrt{cx^6+bx^3+a}\left(2cx^3+b\right)\sqrt{-c}}{2\left(c^2x^6+bcx^3+ac\right)}\right)+2\sqrt{cx^6+bx^3+a}\left(\left(b^2c-2ac^2\right)x^3+abc\right)}{3\left(\left(b^2c^3-4ac^4\right)x^6+ab^2c^2-4a^2c^3+\left(b^3c^2-4abc^3\right)x^3\right)}\right]
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(c*x^6+b*x^3+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/6*(((b^2*c - 4*a*c^2)*x^6 + (b^3 - 4*a*b*c)*x^3 + a*b^2 - 4*a^2*c)*sqrt(
c)*log(-8*c^2*x^6 - 8*b*c*x^3 - b^2 - 4*sqrt(c*x^6 + b*x^3 + a)*(2*c*x^3 +
b)*sqrt(c) - 4*a*c) - 4*sqrt(c*x^6 + b*x^3 + a)*((b^2*c - 2*a*c^2)*x^3 + a*
b*c))/((b^2*c^3 - 4*a*c^4)*x^6 + a*b^2*c^2 - 4*a^2*c^3 + (b^3*c^2 - 4*a*b*c
^3)*x^3), -1/3*(((b^2*c - 4*a*c^2)*x^6 + (b^3 - 4*a*b*c)*x^3 + a*b^2 - 4*a^
2*c)*sqrt(-c)*arctan(1/2*sqrt(c*x^6 + b*x^3 + a)*(2*c*x^3 + b)*sqrt(-c)/(c^
2*x^6 + b*c*x^3 + a*c)) + 2*sqrt(c*x^6 + b*x^3 + a)*((b^2*c - 2*a*c^2)*x^3
+ a*b*c))/((b^2*c^3 - 4*a*c^4)*x^6 + a*b^2*c^2 - 4*a^2*c^3 + (b^3*c^2 - 4*a
*b*c^3)*x^3)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^8}{\left(cx^6+bx^3+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(c*x^6+b*x^3+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x^8/(c*x^6 + b*x^3 + a)^(3/2), x)
```

maple [F] time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{x^8}{\left(c\,x^6+b\,x^3+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(c*x^6+b*x^3+a)^(3/2),x)
```

```
[Out] int(x^8/(c*x^6+b*x^3+a)^(3/2),x)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(c*x^6+b*x^3+a)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 zero or nonzero?
```

mupad [B] time = 1.66, size = 84, normalized size = 0.70

$$\frac{\ln\left(\sqrt{c\,x^6+b\,x^3+a}+\frac{c\,x^3+\frac{b}{2}}{\sqrt{c}}\right)}{3\,c^{3/2}}+\frac{\frac{a\,b}{2}-x^3\,\left(a\,c-\frac{b^2}{2}\right)}{3\,c\,\left(a\,c-\frac{b^2}{4}\right)\,\sqrt{c\,x^6+b\,x^3+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(a + b*x^3 + c*x^6)^(3/2),x)
```

```
[Out] log((a + b*x^3 + c*x^6)^(1/2) + (b/2 + c*x^3)/c^(1/2))/(3*c^(3/2)) + ((a*b)
/2 - x^3*(a*c - b^2/2))/(3*c*(a*c - b^2/4)*(a + b*x^3 + c*x^6)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^8}{\left(a+bx^3+cx^6\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8/(c*x**6+b*x**3+a)**(3/2),x)
```

```
[Out] Integral(x**8/(a + b*x**3 + c*x**6)**(3/2), x)
```

3.209 $\int \frac{x^5}{(a+bx^3+cx^6)^{3/2}}\,dx$

Optimal. Leaf size=39

$$\frac{2(2a+bx^3)}{3(b^2-4ac)\sqrt{a+bx^3+cx^6}}$$

Rubi [A] time = 0.03, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {1357, 636}

$$\frac{2(2a+bx^3)}{3(b^2-4ac)\sqrt{a+bx^3+cx^6}}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/(a + b*x^3 + c*x^6)^(3/2),x]
```

```
[Out] (2*(2*a + b*x^3))/(3*(b^2 - 4*a*c)*Sqrt[a + b*x^3 + c*x^6])
```

Rule 636

```
Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(3/2), x_Symbo
l] :> Simp[(-2*(b*d - 2*a*e + (2*c*d - b*e)*x))/((b^2 - 4*a*c)*Sqrt[a + b*x
 + c*x^2]), x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b
^2 - 4*a*c, 0]
```

Rule 1357

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x^5}{(a+bx^3+cx^6)^{3/2}}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{x}{(a+bx+cx^2)^{3/2}}\,dx, x, x^3\right)\\ &= \frac{2(2a+bx^3)}{3(b^2-4ac)\sqrt{a+bx^3+cx^6}}\end{aligned}$$

Mathematica [A] time = 0.10, size = 41, normalized size = 1.05

$$-\frac{2(2a+bx^3)}{3(4ac-b^2)\sqrt{a+bx^3+cx^6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/(a + b*x^3 + c*x^6)^(3/2),x]
```

```
[Out] (-2*(2*a + b*x^3))/(3*(-b^2 + 4*a*c)*Sqrt[a + b*x^3 + c*x^6])
```

IntegrateAlgebraic [A] time = 0.39, size = 39, normalized size = 1.00

$$\frac{2\left(2a+bx^3\right)}{3\left(b^2-4ac\right)\sqrt{a+bx^3+cx^6}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^5/(a + b*x^3 + c*x^6)^(3/2),x]
[Out] (2*(2*a + b*x^3))/(3*(b^2 - 4*a*c)*Sqrt[a + b*x^3 + c*x^6])
```

fricas [A] time = 0.95, size = 68, normalized size = 1.74

$$\frac{2\sqrt{cx^6+bx^3+a}\left(bx^3+2a\right)}{3\left(\left(b^2c-4ac^2\right)x^6+\left(b^3-4abc\right)x^3+ab^2-4a^2c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(c*x^6+b*x^3+a)^(3/2),x, algorithm="fricas")
[Out] 2/3*sqrt(c*x^6 + b*x^3 + a)*(b*x^3 + 2*a)/((b^2*c - 4*a*c^2)*x^6 + (b^3 - 4
*a*b*c)*x^3 + a*b^2 - 4*a^2*c)
```

giac [A] time = 1.32, size = 45, normalized size = 1.15

$$\frac{2\left(\frac{bx^3}{b^2-4ac}+\frac{2a}{b^2-4ac}\right)}{3\sqrt{cx^6+bx^3+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(c*x^6+b*x^3+a)^(3/2),x, algorithm="giac")
[Out] 2/3*(b*x^3/(b^2 - 4*a*c) + 2*a/(b^2 - 4*a*c))/sqrt(c*x^6 + b*x^3 + a)
```

maple [A] time = 0.01, size = 38, normalized size = 0.97

$$-\frac{2\left(bx^3+2a\right)}{3\sqrt{cx^6+bx^3+a}\,\left(4ac-b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(c*x^6+b*x^3+a)^(3/2),x)
[Out] -2/3/(c*x^6+b*x^3+a)^(1/2)*(b*x^3+2*a)/(4*a*c-b^2)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(c*x^6+b*x^3+a)^(3/2),x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 zero or nonzero?
```

mupad [B] time = 1.43, size = 38, normalized size = 0.97

$$-\frac{2bx^3+4a}{\left(12ac-3b^2\right)\sqrt{cx^6+bx^3+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(a + b*x^3 + c*x^6)^(3/2),x)
```

```
[Out] -(4*a + 2*b*x^3)/((12*a*c - 3*b^2)*(a + b*x^3 + c*x^6)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5}{\left(a + bx^3 + cx^6\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(c*x**6+b*x**3+a)**(3/2),x)
```

```
[Out] Integral(x**5/(a + b*x**3 + c*x**6)**(3/2), x)
```

3.210 $\int \frac{x^2}{(a+bx^3+cx^6)^{3/2}}\,dx$

Optimal. Leaf size=38

$$-\frac{2(b+2cx^3)}{3(b^2-4ac)\sqrt{a+bx^3+cx^6}}$$

Rubi [A] time = 0.02, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {1352, 613}

$$-\frac{2(b+2cx^3)}{3(b^2-4ac)\sqrt{a+bx^3+cx^6}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a + b*x^3 + c*x^6)^(3/2),x]
```

```
[Out] (-2*(b + 2*c*x^3))/(3*(b^2 - 4*a*c)*Sqrt[a + b*x^3 + c*x^6])
```

Rule 613

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[(-2*(b +
2*c*x))/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2]), x] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0]
```

Rule 1352

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a,
b, c, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2}{(a+bx^3+cx^6)^{3/2}}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{1}{(a+bx+cx^2)^{3/2}}\,dx, x, x^3\right)\\ &= -\frac{2(b+2cx^3)}{3(b^2-4ac)\sqrt{a+bx^3+cx^6}}\end{aligned}$$

Mathematica [A] time = 0.02, size = 38, normalized size = 1.00

$$-\frac{2(b+2cx^3)}{3(b^2-4ac)\sqrt{a+bx^3+cx^6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a + b*x^3 + c*x^6)^(3/2),x]
```

```
[Out] (-2*(b + 2*c*x^3))/(3*(b^2 - 4*a*c)*Sqrt[a + b*x^3 + c*x^6])
```

IntegrateAlgebraic [A] time = 0.32, size = 38, normalized size = 1.00

$$-\frac{2(b+2cx^3)}{3(b^2-4ac)\sqrt{a+bx^3+cx^6}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^2/(a + b*x^3 + c*x^6)^(3/2),x]
```

```
[Out] (-2*(b + 2*c*x^3))/(3*(b^2 - 4*a*c)*Sqrt[a + b*x^3 + c*x^6])
```

fricas [A] time = 1.20, size = 67, normalized size = 1.76

$$-\frac{2 \sqrt{cx^6 + bx^3 + a}\left(2 cx^3 + b\right)}{3 \left(\left(b^2c - 4 ac^2\right)x^6 + \left(b^3 - 4 abc\right)x^3 + ab^2 - 4 a^2c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^6+b*x^3+a)^(3/2),x, algorithm="fricas")
```

```
[Out] -2/3*sqrt(c*x^6 + b*x^3 + a)*(2*c*x^3 + b)/((b^2*c - 4*a*c^2)*x^6 + (b^3 -
4*a*b*c)*x^3 + a*b^2 - 4*a^2*c)
```

giac [A] time = 1.41, size = 45, normalized size = 1.18

$$-\frac{2 \left(\frac{2 cx^3}{b^2 - 4 ac} + \frac{b}{b^2 - 4 ac}\right)}{3 \sqrt{cx^6 + bx^3 + a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^6+b*x^3+a)^(3/2),x, algorithm="giac")
```

```
[Out] -2/3*(2*c*x^3/(b^2 - 4*a*c) + b/(b^2 - 4*a*c))/sqrt(c*x^6 + b*x^3 + a)
```

maple [A] time = 0.01, size = 37, normalized size = 0.97

$$\frac{\frac{4c x^3}{3} + \frac{2b}{3}}{\sqrt{c x^6 + b x^3 + a}\, \left(4ac - b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(c*x^6+b*x^3+a)^(3/2),x)
```

```
[Out] 2/3/(c*x^6+b*x^3+a)^(1/2)*(2*c*x^3+b)/(4*a*c-b^2)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^6+b*x^3+a)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 zero or nonzero?
```

mupad [B] time = 1.37, size = 37, normalized size = 0.97

$$\frac{4\,c\,x^3 + 2\,b}{\left(12\,a\,c - 3\,b^2\right)\,\sqrt{c\,x^6 + b\,x^3 + a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a + b*x^3 + c*x^6)^(3/2),x)
```

```
[Out] (2*b + 4*c*x^3)/((12*a*c - 3*b^2)*(a + b*x^3 + c*x^6)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(a + bx^3 + cx^6\right)^{\frac{3}{2}}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(c*x**6+b*x**3+a)**(3/2),x)
[Out] Integral(x**2/(a + b*x**3 + c*x**6)**(3/2), x)
```

3.211 $\int \frac{1}{x\left(a+bx^3+cx^6\right)^{3/2}}\,dx$

Optimal. Leaf size=92

$$\frac{2\left(-2ac+b^2+bcx^3\right)}{3a\left(b^2-4ac\right)\sqrt{a+bx^3+cx^6}}-\frac{\tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{3a^{3/2}}$$

Rubi [A] time = 0.08, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1357, 740, 12, 724, 206}

$$\frac{2\left(-2ac+b^2+bcx^3\right)}{3a\left(b^2-4ac\right)\sqrt{a+bx^3+cx^6}}-\frac{\tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{3a^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a + b*x^3 + c*x^6)^(3/2)),x]
```

```
[Out] (2*(b^2 - 2*a*c + b*c*x^3))/(3*a*(b^2 - 4*a*c)*Sqrt[a + b*x^3 + c*x^6]) - A
rcTanh[(2*a + b*x^3)/(2*Sqrt[a]*Sqrt[a + b*x^3 + c*x^6])]/(3*a^(3/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 740

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e
)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e
^2)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d +
 e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p +
 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p,
-1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 1357

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
```

```
4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x\left(a+bx^3+cx^6\right)^{3/2}}\,dx &= \frac{1}{3}\operatorname{Subst}\left(\int \frac{1}{x\left(a+bx+cx^2\right)^{3/2}}\,dx,x,x^3\right)\\
&= \frac{2\left(b^2-2ac+bcx^3\right)}{3a\left(b^2-4ac\right)\sqrt{a+bx^3+cx^6}}-\frac{2\operatorname{Subst}\left(\int \frac{-\frac{b^2}{2}+2ac}{x\sqrt{a+bx+cx^2}}\,dx,x,x^3\right)}{3a\left(b^2-4ac\right)}\\
&= \frac{2\left(b^2-2ac+bcx^3\right)}{3a\left(b^2-4ac\right)\sqrt{a+bx^3+cx^6}}+\frac{\operatorname{Subst}\left(\int \frac{1}{x\sqrt{a+bx+cx^2}}\,dx,x,x^3\right)}{3a}\\
&= \frac{2\left(b^2-2ac+bcx^3\right)}{3a\left(b^2-4ac\right)\sqrt{a+bx^3+cx^6}}-\frac{2\operatorname{Subst}\left(\int \frac{1}{4a-x^2}\,dx,x,\frac{2a+bx^3}{\sqrt{a+bx^3+cx^6}}\right)}{3a}\\
&= \frac{2\left(b^2-2ac+bcx^3\right)}{3a\left(b^2-4ac\right)\sqrt{a+bx^3+cx^6}}-\frac{\tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{3a^{3/2}}
\end{aligned}$$

Mathematica [A] time = 0.12, size = 92, normalized size = 1.00

$$\frac{1}{3}\left(\frac{2\left(-2ac+b^2+bcx^3\right)}{a\left(b^2-4ac\right)\sqrt{a+bx^3+cx^6}}-\frac{\tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{a^{3/2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a + b*x^3 + c*x^6)^(3/2)),x]
```

```
[Out] ((2*(b^2 - 2*a*c + b*c*x^3))/(a*(b^2 - 4*a*c)*Sqrt[a + b*x^3 + c*x^6]) - Ar
cTanh[(2*a + b*x^3)/(2*Sqrt[a]*Sqrt[a + b*x^3 + c*x^6])]/a^(3/2))/3
```

IntegrateAlgebraic [A] time = 0.54, size = 101, normalized size = 1.10

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{c}x^3}{\sqrt{a}}-\frac{\sqrt{a+bx^3+cx^6}}{\sqrt{a}}\right)}{3a^{3/2}}+\frac{2\left(2ac-b^2-bcx^3\right)}{3a\left(4ac-b^2\right)\sqrt{a+bx^3+cx^6}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x*(a + b*x^3 + c*x^6)^(3/2)),x]
```

```
[Out] (2*(-b^2 + 2*a*c - b*c*x^3))/(3*a*(-b^2 + 4*a*c)*Sqrt[a + b*x^3 + c*x^6]) +
 (2*ArcTanh[(Sqrt[c]*x^3)/Sqrt[a] - Sqrt[a + b*x^3 + c*x^6]/Sqrt[a]])/(3*a^
(3/2))
```

fricas [B] time = 1.67, size = 389, normalized size = 4.23

$$\left[\frac{\left(\left(b^2c-4ac^2\right)x^6+\left(b^3-4abc\right)x^3+ab^2-4a^2c\right)\sqrt{a}\log\left(-\frac{\left(b^2+4ac\right)x^6+8abx^3-4\sqrt{cx^6+bx^3+a}\left(bx^3+2a\right)\sqrt{a}+8a^2}{x^6}\right)+4\sqrt{cx^6+bx^3+a}\left(abcx^3+ab^2-2a^2c\right)}{6\left(\left(a^2b^2c-4a^3c^2\right)x^6+a^3b^2-4a^4c+\left(a^2b^3-4a^3bc\right)x^3\right)},\ \frac{\left(\left(b^2c-4ac^2\right)x^6+\left(b^3-4abc\right)x^3+ab^2-4a^2c\right)\sqrt{-a}\arctan\left(\frac{\sqrt{cx^6+bx^3+a}\left(bx^3+2a\right)\sqrt{-a}}{2\left(acx^6+abx^3+a^2\right)}\right)+2\sqrt{cx^6+bx^3+a}\left(abcx^3+ab^2-2a^2c\right)}{3\left(\left(a^2b^2c-4a^3c^2\right)x^6+a^3b^2-4a^4c+\left(a^2b^3-4a^3bc\right)x^3\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^6+b*x^3+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/6*(((b^2*c - 4*a*c^2)*x^6 + (b^3 - 4*a*b*c)*x^3 + a*b^2 - 4*a^2*c)*sqrt(
a)*log(-((b^2 + 4*a*c)*x^6 + 8*a*b*x^3 - 4*sqrt(c*x^6 + b*x^3 + a)*(b*x^3 +
 2*a)*sqrt(a) + 8*a^2)/x^6) + 4*sqrt(c*x^6 + b*x^3 + a)*(a*b*c*x^3 + a*b^2
- 2*a^2*c))/((a^2*b^2*c - 4*a^3*c^2)*x^6 + a^3*b^2 - 4*a^4*c + (a^2*b^3 - 4
*a^3*b*c)*x^3), 1/3*(((b^2*c - 4*a*c^2)*x^6 + (b^3 - 4*a*b*c)*x^3 + a*b^2 -
 4*a^2*c)*sqrt(-a)*arctan(1/2*sqrt(c*x^6 + b*x^3 + a)*(b*x^3 + 2*a)*sqrt(-a
)/(a*c*x^6 + a*b*x^3 + a^2)) + 2*sqrt(c*x^6 + b*x^3 + a)*(a*b*c*x^3 + a*b^2
 - 2*a^2*c))/((a^2*b^2*c - 4*a^3*c^2)*x^6 + a^3*b^2 - 4*a^4*c + (a^2*b^3 -
4*a^3*b*c)*x^3)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(cx^6 + bx^3 + a\right)^{\frac{3}{2}} x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^6+b*x^3+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((c*x^6 + b*x^3 + a)^(3/2)*x), x)
```

maple [F] time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(c\,x^6 + b\,x^3 + a\right)^{\frac{3}{2}} x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(c*x^6+b*x^3+a)^(3/2),x)
```

```
[Out] int(1/x/(c*x^6+b*x^3+a)^(3/2),x)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^6+b*x^3+a)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive, negative or zero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x\,{\left(c\,x^6+b\,x^3+a\right)}^{3/2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b*x^3 + c*x^6)^(3/2)),x)
```

```
[Out] int(1/(x*(a + b*x^3 + c*x^6)^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x \left(a + bx^3 + cx^6\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x**6+b*x**3+a)**(3/2),x)
```

```
[Out] Integral(1/(x*(a + b*x**3 + c*x**6)**(3/2)), x)
```

3.212 $\int \frac{1}{x^4\left(a+bx^3+cx^6\right)^{3/2}}\,dx$

Optimal. Leaf size=142

$$\frac{b\tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{2a^{5/2}}-\frac{\left(3b^2-8ac\right)\sqrt{a+bx^3+cx^6}}{3a^2x^3\left(b^2-4ac\right)}+\frac{2\left(-2ac+b^2+bcx^3\right)}{3ax^3\left(b^2-4ac\right)\sqrt{a+bx^3+cx^6}}$$

Rubi [A] time = 0.12, antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1357, 740, 806, 724, 206}

$$-\frac{\left(3b^2-8ac\right)\sqrt{a+bx^3+cx^6}}{3a^2x^3\left(b^2-4ac\right)}+\frac{b\tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{2a^{5/2}}+\frac{2\left(-2ac+b^2+bcx^3\right)}{3ax^3\left(b^2-4ac\right)\sqrt{a+bx^3+cx^6}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(a + b*x^3 + c*x^6)^(3/2)),x]
```

```
[Out] (2*(b^2 - 2*a*c + b*c*x^3))/(3*a*(b^2 - 4*a*c)*x^3*Sqrt[a + b*x^3 + c*x^6])
 - ((3*b^2 - 8*a*c)*Sqrt[a + b*x^3 + c*x^6])/(3*a^2*(b^2 - 4*a*c)*x^3) + (b
*ArcTanh[(2*a + b*x^3)/(2*Sqrt[a]*Sqrt[a + b*x^3 + c*x^6])])/(2*a^(5/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 740

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e
)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e
^2)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d +
 e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p +
 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p,
-1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 806

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b
*x + c*x^2)^(p + 1))/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[(b*(e*f
 + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m
+ 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &
& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m +
2*p + 3], 0]
```

Rule 1357

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{1}{x^4\left(a+bx^3+cx^6\right)^{3/2}}\,dx = \frac{1}{3}\,\text{Subst}\left(\int \frac{1}{x^2\left(a+bx+cx^2\right)^{3/2}}\,dx, x, x^3\right)$$

$$= \frac{2\left(b^2-2ac+bcx^3\right)}{3a\left(b^2-4ac\right)x^3\sqrt{a+bx^3+cx^6}} - \frac{2\,\text{Subst}\left(\int \frac{\frac{1}{2}\left(-3b^2+8ac\right)-bcx}{x^2\sqrt{a+bx+cx^2}}\,dx, x, x^3\right)}{3a\left(b^2-4ac\right)}$$

$$= \frac{2\left(b^2-2ac+bcx^3\right)}{3a\left(b^2-4ac\right)x^3\sqrt{a+bx^3+cx^6}} - \frac{\left(3b^2-8ac\right)\sqrt{a+bx^3+cx^6}}{3a^2\left(b^2-4ac\right)x^3} - \frac{b\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}} \cdots\right.}{2a^2}$$

$$= \frac{2\left(b^2-2ac+bcx^3\right)}{3a\left(b^2-4ac\right)x^3\sqrt{a+bx^3+cx^6}} - \frac{\left(3b^2-8ac\right)\sqrt{a+bx^3+cx^6}}{3a^2\left(b^2-4ac\right)x^3} + \frac{b\,\text{Subst}\left(\int \frac{1}{4a-x^2} \cdots\right.}{a \cdots}$$

$$= \frac{2\left(b^2-2ac+bcx^3\right)}{3a\left(b^2-4ac\right)x^3\sqrt{a+bx^3+cx^6}} - \frac{\left(3b^2-8ac\right)\sqrt{a+bx^3+cx^6}}{3a^2\left(b^2-4ac\right)x^3} + \frac{b\tanh^{-1}\left(\frac{2a+\cdots}{2\sqrt{a}\sqrt{a+\cdots}}\right.}{2a^{5/2}}$$

Mathematica [A] time = 0.08, size = 137, normalized size = 0.96

$$\frac{\frac{2\sqrt{a}\left(-4a^2c+a\left(b^2-10bcx^3-8c^2x^6\right)+3b^2x^3\left(b+cx^3\right)\right)}{x^3\sqrt{a+bx^3+cx^6}}-3b\left(b^2-4ac\right)\tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{6a^{5/2}\left(4ac-b^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(a + b*x^3 + c*x^6)^(3/2)),x]
```

```
[Out] ((2*Sqrt[a]*(-4*a^2*c + 3*b^2*x^3*(b + c*x^3) + a*(b^2 - 10*b*c*x^3 - 8*c^2
*x^6)))/(x^3*Sqrt[a + b*x^3 + c*x^6]) - 3*b*(b^2 - 4*a*c)*ArcTanh[(2*a + b*
x^3)/(2*Sqrt[a]*Sqrt[a + b*x^3 + c*x^6])])/(6*a^(5/2)*(-b^2 + 4*a*c))
```

IntegrateAlgebraic [A] time = 0.68, size = 132, normalized size = 0.93

$$\frac{-4a^2c+ab^2-10abcx^3-8ac^2x^6+3b^3x^3+3b^2cx^6}{3a^2x^3\left(4ac-b^2\right)\sqrt{a+bx^3+cx^6}}-\frac{b\tanh^{-1}\left(\frac{\sqrt{c}\,x^3}{\sqrt{a}}-\frac{\sqrt{a+bx^3+cx^6}}{\sqrt{a}}\right)}{a^{5/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^4*(a + b*x^3 + c*x^6)^(3/2)),x]
```

```
[Out] (a*b^2 - 4*a^2*c + 3*b^3*x^3 - 10*a*b*c*x^3 + 3*b^2*c*x^6 - 8*a*c^2*x^6)/(3
*a^2*(-b^2 + 4*a*c)*x^3*Sqrt[a + b*x^3 + c*x^6]) - (b*ArcTanh[(Sqrt[c]*x^3)
/Sqrt[a] - Sqrt[a + b*x^3 + c*x^6]/Sqrt[a]])/a^(5/2)
```

fricas [A] time = 1.53, size = 485, normalized size = 3.42

$$\left[\frac{3\left(\left(b^3c-4abc^2\right)x^9+\left(b^4-4ab^2c\right)x^6+\left(ab^3-4a^2bc\right)x^3\right)\sqrt{a}\log\left(-\frac{\left(b^2+4ac\right)x^6+8abx^3+4\sqrt{cx^6+bx^3+a}\left(bx^3+2a\right)\sqrt{a}+8a^2}{x^6}\right)-4\left(\left(3ab^2c-8a^2c^2\right)x^6+a^2b^2-4a^3c+\left(3ab^3-10a^2bc\right)x^3\right)\sqrt{cx^6+bx^3+a}}{12\left(\left(a^3b^2c-4a^4c^2\right)x^9+\left(a^3b^3-4a^4bc\right)x^6+\left(a^4b^2-4a^5c\right)x^3\right)}, -\frac{3\left(\left(b^3c-4abc^2\right)x^9+\left(b^4-4ab^2c\right)x^6+\left(ab^3-4a^2bc\right)x^3\right)\sqrt{-a}\arctan\left(\frac{\sqrt{cx^6+bx^3+a}\left(bx^3+2a\right)\sqrt{-a}}{2\left(acx^6+abx^3+a^2\right)}\right)+2\left(\left(3ab^2c-8a^2c^2\right)x^6+a^2b^2-4a^3c+\left(3ab^3-10a^2bc\right)x^3\right)\sqrt{cx^6+bx^3+a}}{6\left(\left(a^3b^2c-4a^4c^2\right)x^9+\left(a^3b^3-4a^4bc\right)x^6+\left(a^4b^2-4a^5c\right)x^3\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(c*x^6+b*x^3+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/12*(3*((b^3*c - 4*a*b*c^2)*x^9 + (b^4 - 4*a*b^2*c)*x^6 + (a*b^3 - 4*a^2*
b*c)*x^3)*sqrt(a)*log(-((b^2 + 4*a*c)*x^6 + 8*a*b*x^3 + 4*sqrt(c*x^6 + b*x^
3 + a)*(b*x^3 + 2*a)*sqrt(a) + 8*a^2)/x^6) - 4*((3*a*b^2*c - 8*a^2*c^2)*x^6
 + a^2*b^2 - 4*a^3*c + (3*a*b^3 - 10*a^2*b*c)*x^3)*sqrt(c*x^6 + b*x^3 + a))
/((a^3*b^2*c - 4*a^4*c^2)*x^9 + (a^3*b^3 - 4*a^4*b*c)*x^6 + (a^4*b^2 - 4*a^
5*c)*x^3), -1/6*(3*((b^3*c - 4*a*b*c^2)*x^9 + (b^4 - 4*a*b^2*c)*x^6 + (a*b^
3 - 4*a^2*b*c)*x^3)*sqrt(-a)*arctan(1/2*sqrt(c*x^6 + b*x^3 + a)*(b*x^3 + 2*
a)*sqrt(-a)/(a*c*x^6 + a*b*x^3 + a^2)) + 2*((3*a*b^2*c - 8*a^2*c^2)*x^6 + a
^2*b^2 - 4*a^3*c + (3*a*b^3 - 10*a^2*b*c)*x^3)*sqrt(c*x^6 + b*x^3 + a))/((a
^3*b^2*c - 4*a^4*c^2)*x^9 + (a^3*b^3 - 4*a^4*b*c)*x^6 + (a^4*b^2 - 4*a^5*c)
*x^3)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(cx^6 + bx^3 + a\right)^{\frac{3}{2}} x^4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(c*x^6+b*x^3+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((c*x^6 + b*x^3 + a)^(3/2)*x^4), x)
```

maple [F] time = 0.09, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(c\,x^6 + b\,x^3 + a\right)^{\frac{3}{2}} x^4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(c*x^6+b*x^3+a)^(3/2),x)
```

```
[Out] int(1/x^4/(c*x^6+b*x^3+a)^(3/2),x)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(c*x^6+b*x^3+a)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive, negative or zero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^4 \left(c\,x^6 + b\,x^3 + a\right)^{3/2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(a + b*x^3 + c*x^6)^(3/2)),x)
```

```
[Out] int(1/(x^4*(a + b*x^3 + c*x^6)^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^4 \left(a + bx^3 + cx^6\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(c*x**6+b*x**3+a)**(3/2),x)
```

```
[Out] Integral(1/(x**4*(a + b*x**3 + c*x**6)**(3/2)), x)
```

3.213 $\int \frac{1}{x^7\left(a+bx^3+cx^6\right)^{3/2}}\,dx$

Optimal. Leaf size=198

$$-\frac{\left(5b^2-4ac\right)\tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{8a^{7/2}}+\frac{b\left(15b^2-52ac\right)\sqrt{a+bx^3+cx^6}}{12a^3x^3\left(b^2-4ac\right)}-\frac{\left(5b^2-12ac\right)\sqrt{a+bx^3+cx^6}}{6a^2x^6\left(b^2-4ac\right)}+\frac{2}{3ax^6\left(\right.}$$

Rubi [A] time = 0.20, antiderivative size = 198, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {1357, 740, 834, 806, 724, 206}

$$\frac{b\left(15b^2-52ac\right)\sqrt{a+bx^3+cx^6}}{12a^3x^3\left(b^2-4ac\right)}-\frac{\left(5b^2-12ac\right)\sqrt{a+bx^3+cx^6}}{6a^2x^6\left(b^2-4ac\right)}-\frac{\left(5b^2-4ac\right)\tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{8a^{7/2}}+\frac{2\left(-2ac+b^2+bcx^3\right)}{3ax^6\left(b^2-4ac\right)\sqrt{a+bx^3+cx^6}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^7*(a + b*x^3 + c*x^6)^(3/2)),x]
```

```
[Out] (2*(b^2 - 2*a*c + b*c*x^3))/(3*a*(b^2 - 4*a*c)*x^6*Sqrt[a + b*x^3 + c*x^6])
 - ((5*b^2 - 12*a*c)*Sqrt[a + b*x^3 + c*x^6])/(6*a^2*(b^2 - 4*a*c)*x^6) + (
b*(15*b^2 - 52*a*c)*Sqrt[a + b*x^3 + c*x^6])/(12*a^3*(b^2 - 4*a*c)*x^3) - (
(5*b^2 - 4*a*c)*ArcTanh[(2*a + b*x^3)/(2*Sqrt[a]*Sqrt[a + b*x^3 + c*x^6])])
/(8*a^(7/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 740

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e
)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e
^2)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d +
 e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p +
 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p,
-1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 806

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b
*x + c*x^2)^(p + 1))/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[(b*(e*f
 + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m
+ 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &
& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m +
```

```
2*p + 3], 0]
```

Rule 834

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b*
x + c*x^2)^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[(
c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m +
 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] ||
 IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 1357

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{1}{x^7\left(a+bx^3+cx^6\right)^{3/2}}\,dx = \frac{1}{3}\text{Subst}\left(\int \frac{1}{x^3\left(a+bx+cx^2\right)^{3/2}}\,dx, x, x^3\right)$$

$$= \frac{2\left(b^2-2ac+bcx^3\right)}{3a\left(b^2-4ac\right)x^6\sqrt{a+bx^3+cx^6}} - \frac{2\,\text{Subst}\left(\int \frac{\frac{1}{2}\left(-5b^2+12ac\right)-2bcx}{x^3\sqrt{a+bx+cx^2}}\,dx, x, x^3\right)}{3a\left(b^2-4ac\right)}$$

$$= \frac{2\left(b^2-2ac+bcx^3\right)}{3a\left(b^2-4ac\right)x^6\sqrt{a+bx^3+cx^6}} - \frac{\left(5b^2-12ac\right)\sqrt{a+bx^3+cx^6}}{6a^2\left(b^2-4ac\right)x^6} + \frac{\text{Subst}\left(\int \frac{-\frac{1}{4}b(15b}{}\right.}{3}$$

$$= \frac{2\left(b^2-2ac+bcx^3\right)}{3a\left(b^2-4ac\right)x^6\sqrt{a+bx^3+cx^6}} - \frac{\left(5b^2-12ac\right)\sqrt{a+bx^3+cx^6}}{6a^2\left(b^2-4ac\right)x^6} + \frac{b\left(15b^2-52ac\right)}{12a^3\left(b^2-\right.}$$

$$= \frac{2\left(b^2-2ac+bcx^3\right)}{3a\left(b^2-4ac\right)x^6\sqrt{a+bx^3+cx^6}} - \frac{\left(5b^2-12ac\right)\sqrt{a+bx^3+cx^6}}{6a^2\left(b^2-4ac\right)x^6} + \frac{b\left(15b^2-52ac\right)}{12a^3\left(b^2-\right.}$$

$$= \frac{2\left(b^2-2ac+bcx^3\right)}{3a\left(b^2-4ac\right)x^6\sqrt{a+bx^3+cx^6}} - \frac{\left(5b^2-12ac\right)\sqrt{a+bx^3+cx^6}}{6a^2\left(b^2-4ac\right)x^6} + \frac{b\left(15b^2-52ac\right)}{12a^3\left(b^2-\right.}$$

Mathematica [A] time = 0.13, size = 179, normalized size = 0.90

$$\frac{3\left(16a^2c^2-24ab^2c+5b^4\right)\tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)+\frac{2\sqrt{a}\left(-8a^3c+2a^2\left(b^2+10bcx^3-12c^2x^6\right)+abx^3\left(-5b^2+62bcx^3+52c^2x^6\right)-15b^3x^6\left(b+cx^3\right)\right)}{x^6\sqrt{a+bx^3+cx^6}}}{24a^{7/2}\left(4ac-b^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^7*(a + b*x^3 + c*x^6)^(3/2)),x]
```

```
[Out] ((2*Sqrt[a]*(-8*a^3*c - 15*b^3*x^6*(b + c*x^3) + 2*a^2*(b^2 + 10*b*c*x^3 -
12*c^2*x^6) + a*b*x^3*(-5*b^2 + 62*b*c*x^3 + 52*c^2*x^6)))/(x^6*Sqrt[a + b*
```

```
x^3 + c*x^6]) + 3*(5*b^4 - 24*a*b^2*c + 16*a^2*c^2)*ArcTanh[(2*a + b*x^3)/(
2*Sqrt[a]*Sqrt[a + b*x^3 + c*x^6])])/(24*a^(7/2)*(-b^2 + 4*a*c))
```

IntegrateAlgebraic [A] time = 1.01, size = 175, normalized size = 0.88

$$\frac{\left(5b^2-4ac\right)\tanh^{-1}\left(\frac{\sqrt{c}x^3-\sqrt{a+bx^3+cx^6}}{\sqrt{a}}\right)}{4a^{7/2}}+\frac{-8a^3c+2a^2b^2+20a^2bcx^3-24a^2c^2x^6-5ab^3x^3+62ab^2cx^6+52abc^2x^9-15b^4x^6-15b^3cx^9}{12a^3x^6\left(4ac-b^2\right)\sqrt{a+bx^3+cx^6}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^7*(a + b*x^3 + c*x^6)^(3/2)),x]
```

```
[Out] (2*a^2*b^2 - 8*a^3*c - 5*a*b^3*x^3 + 20*a^2*b*c*x^3 - 15*b^4*x^6 + 62*a*b^2
*c*x^6 - 24*a^2*c^2*x^6 - 15*b^3*c*x^9 + 52*a*b*c^2*x^9)/(12*a^3*(-b^2 + 4*
a*c)*x^6*Sqrt[a + b*x^3 + c*x^6]) + ((5*b^2 - 4*a*c)*ArcTanh[(Sqrt[c]*x^3 -
 Sqrt[a + b*x^3 + c*x^6])/Sqrt[a]])/(4*a^(7/2))
```

fricas [A] time = 1.85, size = 615, normalized size = 3.11

$$\left[-\frac{3\left(\left(5b^4c-24ab^2c^2+16a^2c^3\right)x^{12}+\left(5b^5-24ab^3c+16a^2bc^2\right)x^9+\left(5ab^4-24a^2b^2c+16a^3c^2\right)x^6\right)\sqrt{a}\log\left(-\frac{\left(b^2+4ac\right)x^6+8abx^3+4\sqrt{cx^6+bx^3+a}\left(bx^3+2a\right)\sqrt{a}+8a^2}{x^6}\right)-4\left(\left(15ab^3c-52a^2bc^2\right)x^9+\left(15ab^4-62a^2b^2c+24a^3c^2\right)x^6-2a^3b^2+8a^4c+5\left(a^2b^3-4a^3bc\right)x^3\right)\sqrt{cx^6+bx^3+a}}{48\left(\left(a^4b^2c-4a^5c^2\right)x^{12}+\left(a^4b^3-4a^5bc\right)x^9+\left(a^5b^2-4a^6c\right)x^6\right)}, \frac{3\left(\left(5b^4c-24ab^2c^2+16a^2c^3\right)x^{12}+\left(5b^5-24ab^3c+16a^2bc^2\right)x^9+\left(5ab^4-24a^2b^2c+16a^3c^2\right)x^6\right)\sqrt{-a}\arctan\left(\frac{\sqrt{cx^6+bx^3+a}\left(bx^3+2a\right)\sqrt{-a}}{2\left(acx^6+abx^3+a^2\right)}\right)+2\left(\left(15ab^3c-52a^2bc^2\right)x^9+\left(15ab^4-62a^2b^2c+24a^3c^2\right)x^6-2a^3b^2+8a^4c+5\left(a^2b^3-4a^3bc\right)x^3\right)\sqrt{cx^6+bx^3+a}}{24\left(\left(a^4b^2c-4a^5c^2\right)x^{12}+\left(a^4b^3-4a^5bc\right)x^9+\left(a^5b^2-4a^6c\right)x^6\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(c*x^6+b*x^3+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/48*(3*((5*b^4*c - 24*a*b^2*c^2 + 16*a^2*c^3)*x^12 + (5*b^5 - 24*a*b^3*c
 + 16*a^2*b*c^2)*x^9 + (5*a*b^4 - 24*a^2*b^2*c + 16*a^3*c^2)*x^6)*sqrt(a)*l
og(-((b^2 + 4*a*c)*x^6 + 8*a*b*x^3 + 4*sqrt(c*x^6 + b*x^3 + a)*(b*x^3 + 2*a
)*sqrt(a) + 8*a^2)/x^6) - 4*((15*a*b^3*c - 52*a^2*b*c^2)*x^9 + (15*a*b^4 -
62*a^2*b^2*c + 24*a^3*c^2)*x^6 - 2*a^3*b^2 + 8*a^4*c + 5*(a^2*b^3 - 4*a^3*b
*c)*x^3)*sqrt(c*x^6 + b*x^3 + a))/((a^4*b^2*c - 4*a^5*c^2)*x^12 + (a^4*b^3
- 4*a^5*b*c)*x^9 + (a^5*b^2 - 4*a^6*c)*x^6), 1/24*(3*((5*b^4*c - 24*a*b^2*c
^2 + 16*a^2*c^3)*x^12 + (5*b^5 - 24*a*b^3*c + 16*a^2*b*c^2)*x^9 + (5*a*b^4
- 24*a^2*b^2*c + 16*a^3*c^2)*x^6)*sqrt(-a)*arctan(1/2*sqrt(c*x^6 + b*x^3 +
a)*(b*x^3 + 2*a)*sqrt(-a)/(a*c*x^6 + a*b*x^3 + a^2)) + 2*((15*a*b^3*c - 52*
a^2*b*c^2)*x^9 + (15*a*b^4 - 62*a^2*b^2*c + 24*a^3*c^2)*x^6 - 2*a^3*b^2 + 8
*a^4*c + 5*(a^2*b^3 - 4*a^3*b*c)*x^3)*sqrt(c*x^6 + b*x^3 + a))/((a^4*b^2*c
- 4*a^5*c^2)*x^12 + (a^4*b^3 - 4*a^5*b*c)*x^9 + (a^5*b^2 - 4*a^6*c)*x^6)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(cx^6+bx^3+a\right)^{\frac{3}{2}}x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(c*x^6+b*x^3+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((c*x^6 + b*x^3 + a)^(3/2)*x^7), x)
```

maple [F] time = 0.10, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(c\,x^6+b\,x^3+a\right)^{\frac{3}{2}}\,x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^7/(c*x^6+b*x^3+a)^(3/2),x)
```

```
[Out] int(1/x^7/(c*x^6+b*x^3+a)^(3/2),x)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(c*x^6+b*x^3+a)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive, negative or zero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^7 \left(c\,x^6+b\,x^3+a\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^7*(a + b*x^3 + c*x^6)^(3/2)),x)
```

```
[Out] int(1/(x^7*(a + b*x^3 + c*x^6)^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{7} \left(a + bx^{3} + cx^{6}\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**7/(c*x**6+b*x**3+a)**(3/2),x)
```

```
[Out] Integral(1/(x**7*(a + b*x**3 + c*x**6)**(3/2)), x)
```

3.214 $\int \frac{1}{x^{10}\left(a+bx^3+cx^6\right)^{3/2}} \, dx$

Optimal. Leaf size=256

$$\frac{5b\left(7b^2-12ac\right)\tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{48a^{9/2}}+\frac{b\left(35b^2-116ac\right)\sqrt{a+bx^3+cx^6}}{36a^3x^6\left(b^2-4ac\right)}-\frac{\left(7b^2-16ac\right)\sqrt{a+bx^3+cx^6}}{9a^2x^9\left(b^2-4ac\right)}-\frac{(2}{}$$

Rubi [A] time = 0.29, antiderivative size = 256, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {1357, 740, 834, 806, 724, 206}

$$-\frac{\left(256a^2c^2-460ab^2c+105b^4\right)\sqrt{a+bx^3+cx^6}}{72a^4x^3\left(b^2-4ac\right)}+\frac{b\left(35b^2-116ac\right)\sqrt{a+bx^3+cx^6}}{36a^3x^6\left(b^2-4ac\right)}-\frac{\left(7b^2-16ac\right)\sqrt{a+bx^3+cx^6}}{9a^2x^9\left(b^2-4ac\right)}+\frac{5b\left(7b^2-12ac\right)\tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{48a^{9/2}}+\frac{2\left(-2ac+b^2+bcx^3\right)}{3ax^9\left(b^2-4ac\right)\sqrt{a+bx^3+cx^6}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^10*(a + b*x^3 + c*x^6)^(3/2)),x]
```

```
[Out] (2*(b^2 - 2*a*c + b*c*x^3))/(3*a*(b^2 - 4*a*c)*x^9*Sqrt[a + b*x^3 + c*x^6])
 - ((7*b^2 - 16*a*c)*Sqrt[a + b*x^3 + c*x^6])/(9*a^2*(b^2 - 4*a*c)*x^9) + (
b*(35*b^2 - 116*a*c)*Sqrt[a + b*x^3 + c*x^6])/(36*a^3*(b^2 - 4*a*c)*x^6) -
((105*b^4 - 460*a*b^2*c + 256*a^2*c^2)*Sqrt[a + b*x^3 + c*x^6])/(72*a^4*(b^
2 - 4*a*c)*x^3) + (5*b*(7*b^2 - 12*a*c)*ArcTanh[(2*a + b*x^3)/(2*Sqrt[a]*Sq
rt[a + b*x^3 + c*x^6])])/(48*a^(9/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 740

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e
)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e
^2)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d +
 e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p +
 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p,
-1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 806

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b
*x + c*x^2)^(p + 1))/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[(b*(e*f
 + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m
+ 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &
& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m +
```

```
2*p + 3], 0]
```

Rule 834

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b*
x + c*x^2)^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[(
c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m +
 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] ||
 IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 1357

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{1}{x^{10}\left(a+bx^3+cx^6\right)^{3/2}}\,dx = \frac{1}{3}\,\text{Subst}\left(\int \frac{1}{x^4\left(a+bx+cx^2\right)^{3/2}}\,dx, x, x^3\right)$$

$$= \frac{2\left(b^2-2ac+bcx^3\right)}{3a\left(b^2-4ac\right)x^9\sqrt{a+bx^3+cx^6}} - \frac{2\,\text{Subst}\left(\int \frac{\frac{1}{2}\left(-7b^2+16ac\right)-3bcx}{x^4\sqrt{a+bx+cx^2}}\,dx, x, x^3\right)}{3a\left(b^2-4ac\right)}$$

$$= \frac{2\left(b^2-2ac+bcx^3\right)}{3a\left(b^2-4ac\right)x^9\sqrt{a+bx^3+cx^6}} - \frac{\left(7b^2-16ac\right)\sqrt{a+bx^3+cx^6}}{9a^2\left(b^2-4ac\right)x^9} + \frac{2\,\text{Subst}\left(\int \frac{-\frac{1}{4}b(\ldots}{\ldots}\right.}{\ldots}$$

$$= \frac{2\left(b^2-2ac+bcx^3\right)}{3a\left(b^2-4ac\right)x^9\sqrt{a+bx^3+cx^6}} - \frac{\left(7b^2-16ac\right)\sqrt{a+bx^3+cx^6}}{9a^2\left(b^2-4ac\right)x^9} + \frac{b\left(35b^2-116ac\ldots\right.}{36a^3\left(b^2\ldots\right.}$$

$$= \frac{2\left(b^2-2ac+bcx^3\right)}{3a\left(b^2-4ac\right)x^9\sqrt{a+bx^3+cx^6}} - \frac{\left(7b^2-16ac\right)\sqrt{a+bx^3+cx^6}}{9a^2\left(b^2-4ac\right)x^9} + \frac{b\left(35b^2-116ac\ldots\right.}{36a^3\left(b^2\ldots\right.}$$

$$= \frac{2\left(b^2-2ac+bcx^3\right)}{3a\left(b^2-4ac\right)x^9\sqrt{a+bx^3+cx^6}} - \frac{\left(7b^2-16ac\right)\sqrt{a+bx^3+cx^6}}{9a^2\left(b^2-4ac\right)x^9} + \frac{b\left(35b^2-116ac\ldots\right.}{36a^3\left(b^2\ldots\right.}$$

$$= \frac{2\left(b^2-2ac+bcx^3\right)}{3a\left(b^2-4ac\right)x^9\sqrt{a+bx^3+cx^6}} - \frac{\left(7b^2-16ac\right)\sqrt{a+bx^3+cx^6}}{9a^2\left(b^2-4ac\right)x^9} + \frac{b\left(35b^2-116ac\ldots\right.}{36a^3\left(b^2\ldots\right.}$$

Mathematica [A] time = 0.18, size = 223, normalized size = 0.87

$$\frac{\frac{2\sqrt{a}\left(-32a^4c+8a^3\left(b^2+7bcx^3+16c^2x^6\right)+2a^2x^3\left(-7b^3-86b^2cx^3+244bc^2x^6+128c^3x^9\right)+5ab^2x^6\left(7b^2-106bcx^3-92c^2x^6\right)+105b^4x^9\left(b+cx^3\right)\right)}{x^9\sqrt{a+bx^3+cx^6}}-15b\left(48a^2c^2-40ab^2c+7b^4\right)\tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{144a^{9/2}\left(4ac-b^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^10*(a + b*x^3 + c*x^6)^(3/2)),x]
```

[Out] ((2*Sqrt[a]*(-32*a^4*c + 105*b^4*x^9*(b + c*x^3) + 5*a*b^2*x^6*(7*b^2 - 106*b*c*x^3 - 92*c^2*x^6) + 8*a^3*(b^2 + 7*b*c*x^3 + 16*c^2*x^6) + 2*a^2*x^3*(-7*b^3 - 86*b^2*c*x^3 + 244*b*c^2*x^6 + 128*c^3*x^9)))/(x^9*Sqrt[a + b*x^3 + c*x^6]) - 15*b*(7*b^4 - 40*a*b^2*c + 48*a^2*c^2)*ArcTanh[(2*a + b*x^3)/(2*Sqrt[a]*Sqrt[a + b*x^3 + c*x^6])])/(144*a^(9/2)*(-b^2 + 4*a*c))

IntegrateAlgebraic [A] time = 1.35, size = 224, normalized size = 0.88

$$\frac{5\left(12abc-7b^3\right)\tanh^{-1}\left(\frac{\sqrt{c}\,x^3-\sqrt{a+bx^3+cx^6}}{\sqrt{a}}\right)}{24a^{9/2}}+\frac{-32a^4c+8a^3b^2+56a^3bcx^3+128a^3c^2x^6-14a^2b^3x^3-172a^2b^2cx^6+488a^2bc^2x^9+256a^2c^3x^{12}+35ab^4x^6-530ab^3cx^9-460ab^2c^2x^{12}+105b^5x^9+105b^4cx^{12}}{72a^4x^9\left(4ac-b^2\right)\sqrt{a+bx^3+cx^6}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[1/(x^10*(a + b*x^3 + c*x^6)^(3/2)),x]

[Out] (8*a^3*b^2 - 32*a^4*c - 14*a^2*b^3*x^3 + 56*a^3*b*c*x^3 + 35*a*b^4*x^6 - 172*a^2*b^2*c*x^6 + 128*a^3*c^2*x^6 + 105*b^5*x^9 - 530*a*b^3*c*x^9 + 488*a^2*b*c^2*x^9 + 105*b^4*c*x^12 - 460*a*b^2*c^2*x^12 + 256*a^2*c^3*x^12)/(72*a^4*(-b^2 + 4*a*c)*x^9*Sqrt[a + b*x^3 + c*x^6]) + (5*(-7*b^3 + 12*a*b*c)*ArcTanh[(Sqrt[c]*x^3 - Sqrt[a + b*x^3 + c*x^6])/Sqrt[a]])/(24*a^(9/2))

fricas [A] time = 2.10, size = 705, normalized size = 2.75

$$\left[-\frac{15\left(\left(7b^5c-40ab^3c^2+48a^2bc^3\right)x^{15}+\left(7b^6-40ab^4c+48a^2b^2c^2\right)x^{12}+\left(7ab^5-40a^2b^3c+48a^3bc^2\right)x^9\right)\sqrt{a}\log\left(-\frac{\left(b^2+4ac\right)x^6+8abx^3-4\sqrt{cx^6+bx^3+a}\left(bx^3+2a\right)\sqrt{a}+8a^2}{x^6}\right)+4\left(\left(105ab^4c-460a^2b^2c^2+256a^3c^3\right)x^{12}+\left(105ab^5-530a^2b^3c+488a^3bc^2\right)x^9+\left(35a^2b^4-172a^3b^2c+128a^4c^2\right)x^6+8a^4b^2-32a^5c-14\left(a^3b^3-4a^4bc\right)x^3\right)\sqrt{cx^6+bx^3+a}}{288\left(\left(a^5b^2c-4a^6c^2\right)x^{15}+\left(a^5b^3-4a^6bc\right)x^{12}+\left(a^6b^2-4a^7c\right)x^9\right)},-\frac{15\left(\left(7b^5c-40ab^3c^2+48a^2bc^3\right)x^{15}+\left(7b^6-40ab^4c+48a^2b^2c^2\right)x^{12}+\left(7ab^5-40a^2b^3c+48a^3bc^2\right)x^9\right)\sqrt{-a}\arctan\left(\frac{\sqrt{cx^6+bx^3+a}\left(bx^3+2a\right)\sqrt{-a}}{2\left(acx^6+abx^3+a^2\right)}\right)+2\left(\left(105ab^4c-460a^2b^2c^2+256a^3c^3\right)x^{12}+\left(105ab^5-530a^2b^3c+488a^3bc^2\right)x^9+\left(35a^2b^4-172a^3b^2c+128a^4c^2\right)x^6+8a^4b^2-32a^5c-14\left(a^3b^3-4a^4bc\right)x^3\right)\sqrt{cx^6+bx^3+a}}{144\left(\left(a^5b^2c-4a^6c^2\right)x^{15}+\left(a^5b^3-4a^6bc\right)x^{12}+\left(a^6b^2-4a^7c\right)x^9\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^10/(c*x^6+b*x^3+a)^(3/2),x, algorithm="fricas")

[Out] [-1/288*(15*((7*b^5*c - 40*a*b^3*c^2 + 48*a^2*b*c^3)*x^15 + (7*b^6 - 40*a*b^4*c + 48*a^2*b^2*c^2)*x^12 + (7*a*b^5 - 40*a^2*b^3*c + 48*a^3*b*c^2)*x^9)*sqrt(a)*log(-((b^2 + 4*a*c)*x^6 + 8*a*b*x^3 - 4*sqrt(c*x^6 + b*x^3 + a)*(b*x^3 + 2*a)*sqrt(a) + 8*a^2)/x^6) + 4*((105*a*b^4*c - 460*a^2*b^2*c^2 + 256*a^3*c^3)*x^12 + (105*a*b^5 - 530*a^2*b^3*c + 488*a^3*b*c^2)*x^9 + (35*a^2*b^4 - 172*a^3*b^2*c + 128*a^4*c^2)*x^6 + 8*a^4*b^2 - 32*a^5*c - 14*(a^3*b^3 - 4*a^4*b*c)*x^3)*sqrt(c*x^6 + b*x^3 + a))/((a^5*b^2*c - 4*a^6*c^2)*x^15 + (a^5*b^3 - 4*a^6*b*c)*x^12 + (a^6*b^2 - 4*a^7*c)*x^9), -1/144*(15*((7*b^5*c - 40*a*b^3*c^2 + 48*a^2*b*c^3)*x^15 + (7*b^6 - 40*a*b^4*c + 48*a^2*b^2*c^2)*x^12 + (7*a*b^5 - 40*a^2*b^3*c + 48*a^3*b*c^2)*x^9)*sqrt(-a)*arctan(1/2*sqrt(c*x^6 + b*x^3 + a)*(b*x^3 + 2*a)*sqrt(-a)/(a*c*x^6 + a*b*x^3 + a^2)) + 2*((105*a*b^4*c - 460*a^2*b^2*c^2 + 256*a^3*c^3)*x^12 + (105*a*b^5 - 530*a^2*b^3*c + 488*a^3*b*c^2)*x^9 + (35*a^2*b^4 - 172*a^3*b^2*c + 128*a^4*c^2)*x^6 + 8*a^4*b^2 - 32*a^5*c - 14*(a^3*b^3 - 4*a^4*b*c)*x^3)*sqrt(c*x^6 + b*x^3 + a))/((a^5*b^2*c - 4*a^6*c^2)*x^15 + (a^5*b^3 - 4*a^6*b*c)*x^12 + (a^6*b^2 - 4*a^7*c)*x^9)]

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(cx^6+bx^3+a\right)^{\frac{3}{2}} x^{10}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^10/(c*x^6+b*x^3+a)^(3/2),x, algorithm="giac")

[Out] integrate(1/((c*x^6 + b*x^3 + a)^(3/2)*x^10), x)

maple [F] time = 0.13, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(c\,x^6+b\,x^3+a\right)^{\frac{3}{2}} x^{10}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^10/(c*x^6+b*x^3+a)^(3/2),x)
```

```
[Out] int(1/x^10/(c*x^6+b*x^3+a)^(3/2),x)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^10/(c*x^6+b*x^3+a)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive, negative or zero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^{10}\left(c\,x^6+b\,x^3+a\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^10*(a + b*x^3 + c*x^6)^(3/2)),x)
```

```
[Out] int(1/(x^10*(a + b*x^3 + c*x^6)^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{10}\left(a+bx^3+cx^6\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**10/(c*x**6+b*x**3+a)**(3/2),x)
```

```
[Out] Integral(1/(x**10*(a + b*x**3 + c*x**6)**(3/2)), x)
```

3.215 $\int (dx)^m \left(a+bx^3+cx^6\right)^2 \, dx$

Optimal. Leaf size=101

$$\frac{a^2(dx)^{m+1}}{d(m+1)}+\frac{\left(2ac+b^2\right)(dx)^{m+7}}{d^7(m+7)}+\frac{2ab(dx)^{m+4}}{d^4(m+4)}+\frac{2bc(dx)^{m+10}}{d^{10}(m+10)}+\frac{c^2(dx)^{m+13}}{d^{13}(m+13)}$$

Rubi [A] time = 0.06, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {1353}

$$\frac{a^2(dx)^{m+1}}{d(m+1)}+\frac{\left(2ac+b^2\right)(dx)^{m+7}}{d^7(m+7)}+\frac{2ab(dx)^{m+4}}{d^4(m+4)}+\frac{2bc(dx)^{m+10}}{d^{10}(m+10)}+\frac{c^2(dx)^{m+13}}{d^{13}(m+13)}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^m*(a + b*x^3 + c*x^6)^2,x]
```

```
[Out] (a^2*(d*x)^(1 + m))/(d*(1 + m)) + (2*a*b*(d*x)^(4 + m))/(d^4*(4 + m)) + ((b
^2 + 2*a*c)*(d*x)^(7 + m))/(d^7*(7 + m)) + (2*b*c*(d*x)^(10 + m))/(d^10*(10
 + m)) + (c^2*(d*x)^(13 + m))/(d^13*(13 + m))
```

Rule 1353

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.),
x_Symbol] :> Int[ExpandIntegrand[(d*x)^m*(a + b*x^n + c*x^(2*n))^p, x], x]
/; FreeQ[{a, b, c, d, m, n}, x] && EqQ[n2, 2*n] && IGtQ[p, 0] &&  !IntegerQ
[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int (dx)^m \left(a+bx^3+cx^6\right)^2 \, dx &= \int \left(a^2(dx)^m+\frac{2ab(dx)^{3+m}}{d^3}+\frac{\left(b^2+2ac\right)(dx)^{6+m}}{d^6}+\frac{2bc(dx)^{9+m}}{d^9}+\frac{c^2(dx)^{12+m}}{d^{12}}\right) \\ &= \frac{a^2(dx)^{1+m}}{d(1+m)}+\frac{2ab(dx)^{4+m}}{d^4(4+m)}+\frac{\left(b^2+2ac\right)(dx)^{7+m}}{d^7(7+m)}+\frac{2bc(dx)^{10+m}}{d^{10}(10+m)}+\frac{c^2(dx)^{13+m}}{d^{13}(13+m)}\end{aligned}$$

Mathematica [A] time = 0.07, size = 70, normalized size = 0.69

$$x(dx)^m\left(\frac{a^2}{m+1}+\frac{x^6\left(2ac+b^2\right)}{m+7}+\frac{2abx^3}{m+4}+\frac{2bcx^9}{m+10}+\frac{c^2x^{12}}{m+13}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^m*(a + b*x^3 + c*x^6)^2,x]
```

```
[Out] x*(d*x)^m*(a^2/(1 + m) + (2*a*b*x^3)/(4 + m) + ((b^2 + 2*a*c)*x^6)/(7 + m)
+ (2*b*c*x^9)/(10 + m) + (c^2*x^12)/(13 + m))
```

IntegrateAlgebraic [F] time = 1.17, size = 0, normalized size = 0.00

$$\int (dx)^m \left(a+bx^3+cx^6\right)^2 \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(d*x)^m*(a + b*x^3 + c*x^6)^2,x]
```

```
[Out] Defer[IntegrateAlgebraic][(d*x)^m*(a + b*x^3 + c*x^6)^2, x]
```

fricas [B] time = 1.08, size = 241, normalized size = 2.39

$$\frac{\left(\left(c^2m^4 + 22\,c^2m^3 + 159\,c^2m^2 + 418\,c^2m + 280\,c^2\right)x^{13} + 2\left(bcm^4 + 25\,bcm^3 + 195\,bcm^2 + 535\,bcm + 364\,bc\right)x^{10} + \left(\left(b^2 + 2\,ac\right)m^4 + 28\left(b^2 + 2\,ac\right)m^3 + 249\left(b^2 + 2\,ac\right)m^2 + 520\,b^2 + 1040\,ac + 742\left(b^2 + 2\,ac\right)m\right)x^7 + 2\left(abm^4 + 31\,abm^3 + 321\,abm^2 + 1201\,abm + 910\,ab\right)x^4 + \left(a^2m^4 + 34\,a^2m^3 + 411\,a^2m^2 + 2074\,a^2m + 3640\,a^2\right)x\right)(dx)^m}{m^5 + 35\,m^4 + 445\,m^3 + 2485\,m^2 + 5714\,m + 3640}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(c*x^6+b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] ((c^2*m^4 + 22*c^2*m^3 + 159*c^2*m^2 + 418*c^2*m + 280*c^2)*x^13 + 2*(b*c*m
^4 + 25*b*c*m^3 + 195*b*c*m^2 + 535*b*c*m + 364*b*c)*x^10 + ((b^2 + 2*a*c)*
m^4 + 28*(b^2 + 2*a*c)*m^3 + 249*(b^2 + 2*a*c)*m^2 + 520*b^2 + 1040*a*c + 7
42*(b^2 + 2*a*c)*m)*x^7 + 2*(a*b*m^4 + 31*a*b*m^3 + 321*a*b*m^2 + 1201*a*b*
m + 910*a*b)*x^4 + (a^2*m^4 + 34*a^2*m^3 + 411*a^2*m^2 + 2074*a^2*m + 3640*
a^2)*x)*(d*x)^m/(m^5 + 35*m^4 + 445*m^3 + 2485*m^2 + 5714*m + 3640)
```

giac [B] time = 0.47, size = 449, normalized size = 4.45

$$\frac{(dx)^m c^2m^4x^{13} + 22\,(dx)^m c^2m^3x^{13} + 159\,(dx)^m c^2m^2x^{13} + 2\,(dx)^m bcm^4x^{10} + 418\,(dx)^m c^2mx^{13} + 50\,(dx)^m bcm^3x^{10} + 280\,(dx)^m c^2x^{13} + 390\,(dx)^m bcm^2x^{10} + (dx)^m b^2m^4x^7 + 2\,(dx)^m acm^4x^7 + 1070\,(dx)^m bcmx^{10} + 28\,(dx)^m b^2m^3x^7 + 56\,(dx)^m acm^3x^7 + 728\,(dx)^m bcx^{10} + 249\,(dx)^m b^2m^2x^7 + 498\,(dx)^m acm^2x^7 + 2\,(dx)^m abm^4x^4 + 742\,(dx)^m b^2mx^7 + 1484\,(dx)^m acmx^7 + 62\,(dx)^m abm^3x^4 + 520\,(dx)^m b^2x^7 + 1040\,(dx)^m acx^7 + 642\,(dx)^m abm^2x^4 + (dx)^m a^2m^4x + 2402\,(dx)^m abmx^4 + 34\,(dx)^m a^2m^3x + 1820\,(dx)^m abx^4 + 411\,(dx)^m a^2m^2x + 2074\,(dx)^m a^2mx + 3640\,(dx)^m a^2x}{m^5 + 35\,m^4 + 445\,m^3 + 2485\,m^2 + 5714\,m + 3640}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(c*x^6+b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] ((d*x)^m*c^2*m^4*x^13 + 22*(d*x)^m*c^2*m^3*x^13 + 159*(d*x)^m*c^2*m^2*x^13
+ 2*(d*x)^m*b*c*m^4*x^10 + 418*(d*x)^m*c^2*m*x^13 + 50*(d*x)^m*b*c*m^3*x^10
 + 280*(d*x)^m*c^2*x^13 + 390*(d*x)^m*b*c*m^2*x^10 + (d*x)^m*b^2*m^4*x^7 +
2*(d*x)^m*a*c*m^4*x^7 + 1070*(d*x)^m*b*c*m*x^10 + 28*(d*x)^m*b^2*m^3*x^7 +
56*(d*x)^m*a*c*m^3*x^7 + 728*(d*x)^m*b*c*x^10 + 249*(d*x)^m*b^2*m^2*x^7 + 4
98*(d*x)^m*a*c*m^2*x^7 + 2*(d*x)^m*a*b*m^4*x^4 + 742*(d*x)^m*b^2*m*x^7 + 14
84*(d*x)^m*a*c*m*x^7 + 62*(d*x)^m*a*b*m^3*x^4 + 520*(d*x)^m*b^2*x^7 + 1040*
(d*x)^m*a*c*x^7 + 642*(d*x)^m*a*b*m^2*x^4 + (d*x)^m*a^2*m^4*x + 2402*(d*x)^
m*a*b*m*x^4 + 34*(d*x)^m*a^2*m^3*x + 1820*(d*x)^m*a*b*x^4 + 411*(d*x)^m*a^2
*m^2*x + 2074*(d*x)^m*a^2*m*x + 3640*(d*x)^m*a^2*x)/(m^5 + 35*m^4 + 445*m^3
 + 2485*m^2 + 5714*m + 3640)
```

maple [B] time = 0.01, size = 301, normalized size = 2.98

$$\frac{\left(c^2m^4x^{12} + 22c^2m^3x^{12} + 159c^2m^2x^{12} + 2bc\,m^4x^9 + 418c^2m\,x^{12} + 50bc\,m^3x^9 + 280c^2x^{12} + 390bc\,m^2x^9 + 2ac\,m^4x^6 + b^2m^4x^6 + 1070bcm\,x^9 + 56ac\,m^3x^6 + 28b^2m^3x^6 + 728bc\,x^9 + 498ac\,m^2x^6 + 249b^2m^2x^6 + 2ab\,m^4x^3 + 1484acm\,x^6 + 742b^2m\,x^6 + 62ab\,m^3x^3 + 1040ac\,x^6 + 520b^2x^6 + 642ab\,m^2x^3 + a^2m^4 + 2402abm\,x^3 + 34a^2m^3 + 1820ab\,x^3 + 411a^2m^2 + 2074a^2m + 3640a^2\right)x\,(dx)^m}{(m+13)\,(m+10)\,(m+7)\,(m+4)\,(m+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(c*x^6+b*x^3+a)^2,x)
```

```
[Out] x*(c^2*m^4*x^12+22*c^2*m^3*x^12+159*c^2*m^2*x^12+2*b*c*m^4*x^9+418*c^2*m*x^
12+50*b*c*m^3*x^9+280*c^2*x^12+390*b*c*m^2*x^9+2*a*c*m^4*x^6+b^2*m^4*x^6+10
70*b*c*m*x^9+56*a*c*m^3*x^6+28*b^2*m^3*x^6+728*b*c*x^9+498*a*c*m^2*x^6+249*
b^2*m^2*x^6+2*a*b*m^4*x^3+1484*a*c*m*x^6+742*b^2*m*x^6+62*a*b*m^3*x^3+1040*
a*c*x^6+520*b^2*x^6+642*a*b*m^2*x^3+a^2*m^4+2402*a*b*m*x^3+34*a^2*m^3+1820*
a*b*x^3+411*a^2*m^2+2074*a^2*m+3640*a^2)*(d*x)^m/(m+13)/(m+10)/(m+7)/(m+4)/
(m+1)
```

maxima [A] time = 1.10, size = 110, normalized size = 1.09

$$\frac{c^2d^mx^{13}x^m}{m+13} + \frac{2\,bcd^mx^{10}x^m}{m+10} + \frac{b^2d^mx^7x^m}{m+7} + \frac{2\,acd^mx^7x^m}{m+7} + \frac{2\,abd^mx^4x^m}{m+4} + \frac{(dx)^{m+1}\,a^2}{d(m+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(c*x^6+b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] c^2*d^m*x^13*x^m/(m + 13) + 2*b*c*d^m*x^10*x^m/(m + 10) + b^2*d^m*x^7*x^m/(
m + 7) + 2*a*c*d^m*x^7*x^m/(m + 7) + 2*a*b*d^m*x^4*x^m/(m + 4) + (d*x)^(m +
 1)*a^2/(d*(m + 1))
```

mupad [B] time = 1.52, size = 260, normalized size = 2.57

$$(d\,x)^m\,\left(\frac{c^2\,x^{13}\,\left(m^4+22\,m^3+159\,m^2+418\,m+280\right)}{m^5+35\,m^4+445\,m^3+2485\,m^2+5714\,m+3640}+\frac{x^7\,\left(b^2+2\,a\,c\right)\,\left(m^4+28\,m^3+249\,m^2+742\,m+520\right)}{m^5+35\,m^4+445\,m^3+2485\,m^2+5714\,m+3640}+\frac{a^2\,x\,\left(m^4+34\,m^3+411\,m^2+2074\,m+3640\right)}{m^5+35\,m^4+445\,m^3+2485\,m^2+5714\,m+3640}+\frac{2\,a\,b\,x^4\,\left(m^4+31\,m^3+321\,m^2+1201\,m+910\right)}{m^5+35\,m^4+445\,m^3+2485\,m^2+5714\,m+3640}+\frac{2\,b\,c\,x^{10}\,\left(m^4+25\,m^3+195\,m^2+535\,m+364\right)}{m^5+35\,m^4+445\,m^3+2485\,m^2+5714\,m+3640}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(a + b*x^3 + c*x^6)^2,x)
```

```
[Out] (d*x)^m*((c^2*x^13*(418*m + 159*m^2 + 22*m^3 + m^4 + 280))/(5714*m + 2485*m
^2 + 445*m^3 + 35*m^4 + m^5 + 3640) + (x^7*(2*a*c + b^2)*(742*m + 249*m^2 +
 28*m^3 + m^4 + 520))/(5714*m + 2485*m^2 + 445*m^3 + 35*m^4 + m^5 + 3640) +
 (a^2*x*(2074*m + 411*m^2 + 34*m^3 + m^4 + 3640))/(5714*m + 2485*m^2 + 445*
m^3 + 35*m^4 + m^5 + 3640) + (2*a*b*x^4*(1201*m + 321*m^2 + 31*m^3 + m^4 +
910))/(5714*m + 2485*m^2 + 445*m^3 + 35*m^4 + m^5 + 3640) + (2*b*c*x^10*(53
5*m + 195*m^2 + 25*m^3 + m^4 + 364))/(5714*m + 2485*m^2 + 445*m^3 + 35*m^4
+ m^5 + 3640))
```

sympy [A] time = 5.82, size = 1510, normalized size = 14.95

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(c*x**6+b*x**3+a)**2,x)
```

```
[Out] Piecewise(((-a**2/(12*x**12) - 2*a*b/(9*x**9) - a*c/(3*x**6) - b**2/(6*x**6
) - 2*b*c/(3*x**3) + c**2*log(x))/d**13, Eq(m, -13)), ((-a**2/(9*x**9) - a*
b/(3*x**6) - 2*a*c/(3*x**3) - b**2/(3*x**3) + 2*b*c*log(x) + c**2*x**3/3)/d
**10, Eq(m, -10)), ((-a**2/(6*x**6) - 2*a*b/(3*x**3) + 2*a*c*log(x) + b**2*
log(x) + 2*b*c*x**3/3 + c**2*x**6/6)/d**7, Eq(m, -7)), ((-a**2/(3*x**3) + 2
*a*b*log(x) + 2*a*c*x**3/3 + b**2*x**3/3 + b*c*x**6/3 + c**2*x**9/9)/d**4,
Eq(m, -4)), ((a**2*log(x) + 2*a*b*x**3/3 + a*c*x**6/3 + b**2*x**6/6 + 2*b*c
*x**9/9 + c**2*x**12/12)/d, Eq(m, -1)), (a**2*d**m*m**4*x*x**m/(m**5 + 35*m
**4 + 445*m**3 + 2485*m**2 + 5714*m + 3640) + 34*a**2*d**m*m**3*x*x**m/(m**
5 + 35*m**4 + 445*m**3 + 2485*m**2 + 5714*m + 3640) + 411*a**2*d**m*m**2*x*
x**m/(m**5 + 35*m**4 + 445*m**3 + 2485*m**2 + 5714*m + 3640) + 2074*a**2*d*
*m*m*x*x**m/(m**5 + 35*m**4 + 445*m**3 + 2485*m**2 + 5714*m + 3640) + 3640*
a**2*d**m*x*x**m/(m**5 + 35*m**4 + 445*m**3 + 2485*m**2 + 5714*m + 3640) +
2*a*b*d**m*m**4*x**4*x**m/(m**5 + 35*m**4 + 445*m**3 + 2485*m**2 + 5714*m +
 3640) + 62*a*b*d**m*m**3*x**4*x**m/(m**5 + 35*m**4 + 445*m**3 + 2485*m**2
+ 5714*m + 3640) + 642*a*b*d**m*m**2*x**4*x**m/(m**5 + 35*m**4 + 445*m**3 +
 2485*m**2 + 5714*m + 3640) + 2402*a*b*d**m*m*x**4*x**m/(m**5 + 35*m**4 + 4
45*m**3 + 2485*m**2 + 5714*m + 3640) + 1820*a*b*d**m*x**4*x**m/(m**5 + 35*m
**4 + 445*m**3 + 2485*m**2 + 5714*m + 3640) + 2*a*c*d**m*m**4*x**7*x**m/(m*
*5 + 35*m**4 + 445*m**3 + 2485*m**2 + 5714*m + 3640) + 56*a*c*d**m*m**3*x**
7*x**m/(m**5 + 35*m**4 + 445*m**3 + 2485*m**2 + 5714*m + 3640) + 498*a*c*d*
*m*m**2*x**7*x**m/(m**5 + 35*m**4 + 445*m**3 + 2485*m**2 + 5714*m + 3640) +
 1484*a*c*d**m*m*x**7*x**m/(m**5 + 35*m**4 + 445*m**3 + 2485*m**2 + 5714*m
+ 3640) + 1040*a*c*d**m*x**7*x**m/(m**5 + 35*m**4 + 445*m**3 + 2485*m**2 +
5714*m + 3640) + b**2*d**m*m**4*x**7*x**m/(m**5 + 35*m**4 + 445*m**3 + 2485
*m**2 + 5714*m + 3640) + 28*b**2*d**m*m**3*x**7*x**m/(m**5 + 35*m**4 + 445*
m**3 + 2485*m**2 + 5714*m + 3640) + 249*b**2*d**m*m**2*x**7*x**m/(m**5 + 35
*m**4 + 445*m**3 + 2485*m**2 + 5714*m + 3640) + 742*b**2*d**m*m*x**7*x**m/(
m**5 + 35*m**4 + 445*m**3 + 2485*m**2 + 5714*m + 3640) + 520*b**2*d**m*x**7
*x**m/(m**5 + 35*m**4 + 445*m**3 + 2485*m**2 + 5714*m + 3640) + 2*b*c*d**m*
m**4*x**10*x**m/(m**5 + 35*m**4 + 445*m**3 + 2485*m**2 + 5714*m + 3640) + 5
0*b*c*d**m*m**3*x**10*x**m/(m**5 + 35*m**4 + 445*m**3 + 2485*m**2 + 5714*m
+ 3640) + 390*b*c*d**m*m**2*x**10*x**m/(m**5 + 35*m**4 + 445*m**3 + 2485*m*
```

```
*2 + 5714*m + 3640) + 1070*b*c*d**m*m*x**10*x**m/(m**5 + 35*m**4 + 445*m**3
 + 2485*m**2 + 5714*m + 3640) + 728*b*c*d**m*x**10*x**m/(m**5 + 35*m**4 + 4
45*m**3 + 2485*m**2 + 5714*m + 3640) + c**2*d**m*m**4*x**13*x**m/(m**5 + 35
*m**4 + 445*m**3 + 2485*m**2 + 5714*m + 3640) + 22*c**2*d**m*m**3*x**13*x**
m/(m**5 + 35*m**4 + 445*m**3 + 2485*m**2 + 5714*m + 3640) + 159*c**2*d**m*m
**2*x**13*x**m/(m**5 + 35*m**4 + 445*m**3 + 2485*m**2 + 5714*m + 3640) + 41
8*c**2*d**m*m*x**13*x**m/(m**5 + 35*m**4 + 445*m**3 + 2485*m**2 + 5714*m +
3640) + 280*c**2*d**m*x**13*x**m/(m**5 + 35*m**4 + 445*m**3 + 2485*m**2 + 5
714*m + 3640), True))
```

3.216 $\int (dx)^m \left(a+bx^3+cx^6\right) dx$

Optimal. Leaf size=52

$$\frac{a(dx)^{m+1}}{d(m+1)}+\frac{b(dx)^{m+4}}{d^4(m+4)}+\frac{c(dx)^{m+7}}{d^7(m+7)}$$

Rubi [A] time = 0.02, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {14}

$$\frac{a(dx)^{m+1}}{d(m+1)}+\frac{b(dx)^{m+4}}{d^4(m+4)}+\frac{c(dx)^{m+7}}{d^7(m+7)}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^m*(a + b*x^3 + c*x^6),x]
```

```
[Out] (a*(d*x)^(1 + m))/(d*(1 + m)) + (b*(d*x)^(4 + m))/(d^4*(4 + m)) + (c*(d*x)^
(7 + m))/(d^7*(7 + m))
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int (dx)^m \left(a+bx^3+cx^6\right) dx &= \int \left(a(dx)^m+\frac{b(dx)^{3+m}}{d^3}+\frac{c(dx)^{6+m}}{d^6}\right) dx\\ &= \frac{a(dx)^{1+m}}{d(1+m)}+\frac{b(dx)^{4+m}}{d^4(4+m)}+\frac{c(dx)^{7+m}}{d^7(7+m)}\end{aligned}$$

Mathematica [A] time = 0.03, size = 35, normalized size = 0.67

$$x(dx)^m\left(\frac{a}{m+1}+\frac{bx^3}{m+4}+\frac{cx^6}{m+7}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^m*(a + b*x^3 + c*x^6),x]
```

```
[Out] x*(d*x)^m*(a/(1 + m) + (b*x^3)/(4 + m) + (c*x^6)/(7 + m))
```

IntegrateAlgebraic [F] time = 0.21, size = 0, normalized size = 0.00

$$\int (dx)^m \left(a+bx^3+cx^6\right) dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(d*x)^m*(a + b*x^3 + c*x^6),x]
```

```
[Out] Defer[IntegrateAlgebraic][(d*x)^m*(a + b*x^3 + c*x^6), x]
```

fricas [A] time = 1.31, size = 71, normalized size = 1.37

$$\frac{\left(\left(cm^2+5\,cm+4\,c\right)x^7+\left(bm^2+8\,bm+7\,b\right)x^4+\left(am^2+11\,am+28\,a\right)x\right)(dx)^m}{m^3+12\,m^2+39\,m+28}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(c*x^6+b*x^3+a),x, algorithm="fricas")
```

```
[Out] ((c*m^2 + 5*c*m + 4*c)*x^7 + (b*m^2 + 8*b*m + 7*b)*x^4 + (a*m^2 + 11*a*m +
28*a)*x)*(d*x)^m/(m^3 + 12*m^2 + 39*m + 28)
```

giac [B] time = 0.35, size = 119, normalized size = 2.29

$$\frac{(dx)^m cm^2x^7 + 5\,(dx)^m cmx^7 + 4\,(dx)^m cx^7 + (dx)^m bm^2x^4 + 8\,(dx)^m bmx^4 + 7\,(dx)^m bx^4 + (dx)^m am^2x + 11\,(dx)^m amx + 28\,(dx)^m ax}{m^3 + 12\,m^2 + 39\,m + 28}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(c*x^6+b*x^3+a),x, algorithm="giac")
```

```
[Out] ((d*x)^m*c*m^2*x^7 + 5*(d*x)^m*c*m*x^7 + 4*(d*x)^m*c*x^7 + (d*x)^m*b*m^2*x^
4 + 8*(d*x)^m*b*m*x^4 + 7*(d*x)^m*b*x^4 + (d*x)^m*a*m^2*x + 11*(d*x)^m*a*m*
x + 28*(d*x)^m*a*x)/(m^3 + 12*m^2 + 39*m + 28)
```

maple [A] time = 0.00, size = 78, normalized size = 1.50

$$\frac{\left(c\,m^2x^6 + 5cm\,x^6 + 4c\,x^6 + b\,m^2x^3 + 8bm\,x^3 + 7b\,x^3 + a\,m^2 + 11am + 28a\right)x\,(dx)^m}{(m+7)\,(m+4)\,(m+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(c*x^6+b*x^3+a),x)
```

```
[Out] x*(c*m^2*x^6+5*c*m*x^6+4*c*x^6+b*m^2*x^3+8*b*m*x^3+7*b*x^3+a*m^2+11*a*m+28*
a)*(d*x)^m/(m+7)/(m+4)/(m+1)
```

maxima [A] time = 1.16, size = 50, normalized size = 0.96

$$\frac{cd^mx^7x^m}{m+7} + \frac{bd^mx^4x^m}{m+4} + \frac{(dx)^{m+1}\,a}{d(m+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(c*x^6+b*x^3+a),x, algorithm="maxima")
```

```
[Out] c*d^m*x^7*x^m/(m + 7) + b*d^m*x^4*x^m/(m + 4) + (d*x)^(m + 1)*a/(d*(m + 1))
```

mupad [B] time = 1.36, size = 89, normalized size = 1.71

$$(d\,x)^m\,\left(\frac{b\,x^4\,\left(m^2+8\,m+7\right)}{m^3+12\,m^2+39\,m+28}+\frac{c\,x^7\,\left(m^2+5\,m+4\right)}{m^3+12\,m^2+39\,m+28}+\frac{a\,x\,\left(m^2+11\,m+28\right)}{m^3+12\,m^2+39\,m+28}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(a + b*x^3 + c*x^6),x)
```

```
[Out] (d*x)^m*((b*x^4*(8*m + m^2 + 7))/(39*m + 12*m^2 + m^3 + 28) + (c*x^7*(5*m +
 m^2 + 4))/(39*m + 12*m^2 + m^3 + 28) + (a*x*(11*m + m^2 + 28))/(39*m + 12*
m^2 + m^3 + 28))
```

sympy [A] time = 1.47, size = 314, normalized size = 6.04

$$\begin{cases} \frac{- \frac{a}{6x^6} - \frac{b}{3x^3} + c\log(x)}{d^7} & \text{for}\: m = -7 \\ \frac{- \frac{a}{3x^3} + b\log(x) + \frac{cx^3}{3}}{d^4} & \text{for}\: m = -4 \\ \frac{a\log(x) + \frac{bx^3}{3} + \frac{cx^6}{6}}{d} & \text{for}\: m = -1 \\ \frac{ad^mm^2xx^m}{m^3+12m^2+39m+28} + \frac{11ad^mmxx^m}{m^3+12m^2+39m+28} + \frac{28ad^mxx^m}{m^3+12m^2+39m+28} + \frac{bd^mm^2x^4x^m}{m^3+12m^2+39m+28} + \frac{8bd^mmx^4x^m}{m^3+12m^2+39m+28} + \frac{7bd^mx^4x^m}{m^3+12m^2+39m+28} + \frac{cd^mm^2x^7x^m}{m^3+12m^2+39m+28} + \frac{5cd^mmx^7x^m}{m^3+12m^2+39m+28} + \frac{4cd^mx^7x^m}{m^3+12m^2+39m+28} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(c*x**6+b*x**3+a),x)
```

```
[Out] Piecewise(((-a/(6*x**6) - b/(3*x**3) + c*log(x))/d**7, Eq(m, -7)), ((-a/(3*
x**3) + b*log(x) + c*x**3/3)/d**4, Eq(m, -4)), ((a*log(x) + b*x**3/3 + c*x*
*6/6)/d, Eq(m, -1)), (a*d**m*m**2*x*x**m/(m**3 + 12*m**2 + 39*m + 28) + 11*
a*d**m*m*x*x**m/(m**3 + 12*m**2 + 39*m + 28) + 28*a*d**m*x*x**m/(m**3 + 12*
m**2 + 39*m + 28) + b*d**m*m**2*x**4*x**m/(m**3 + 12*m**2 + 39*m + 28) + 8*
b*d**m*m*x**4*x**m/(m**3 + 12*m**2 + 39*m + 28) + 7*b*d**m*x**4*x**m/(m**3
+ 12*m**2 + 39*m + 28) + c*d**m*m**2*x**7*x**m/(m**3 + 12*m**2 + 39*m + 28)
 + 5*c*d**m*m*x**7*x**m/(m**3 + 12*m**2 + 39*m + 28) + 4*c*d**m*x**7*x**m/(
m**3 + 12*m**2 + 39*m + 28), True))
```

3.217 $\int \frac{x^9}{1+2x^4+x^8}\,dx$

Optimal. Leaf size=30

$$\frac{3x^2}{4}-\frac{3}{4}\tan^{-1}\left(x^2\right)-\frac{x^6}{4\left(x^4+1\right)}$$

Rubi [A] time = 0.01, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {28, 275, 288, 321, 203}

$$-\frac{x^6}{4\left(x^4+1\right)}+\frac{3x^2}{4}-\frac{3}{4}\tan^{-1}\left(x^2\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^9/(1 + 2*x^4 + x^8),x]
```

```
[Out] (3*x^2)/4 - x^6/(4*(1 + x^4)) - (3*ArcTan[x^2])/4
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}\int \frac{x^9}{1+2x^4+x^8}\,dx &= \int \frac{x^9}{\left(1+x^4\right)^2}\,dx\\ &= \frac{1}{2}\text{Subst}\left(\int \frac{x^4}{\left(1+x^2\right)^2}\,dx,x,x^2\right)\\ &= -\frac{x^6}{4\left(1+x^4\right)}+\frac{3}{4}\text{Subst}\left(\int \frac{x^2}{1+x^2}\,dx,x,x^2\right)\\ &= \frac{3x^2}{4}-\frac{x^6}{4\left(1+x^4\right)}-\frac{3}{4}\text{Subst}\left(\int \frac{1}{1+x^2}\,dx,x,x^2\right)\\ &= \frac{3x^2}{4}-\frac{x^6}{4\left(1+x^4\right)}-\frac{3}{4}\tan^{-1}\left(x^2\right)\end{aligned}$$

Mathematica [A] time = 0.01, size = 24, normalized size = 0.80

$$\frac{1}{4}\left(x^2\left(\frac{1}{x^4+1}+2\right)-3\tan^{-1}\left(x^2\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^9/(1 + 2*x^4 + x^8),x]
[Out] (x^2*(2 + (1 + x^4)^(-1)) - 3*ArcTan[x^2])/4
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^9}{1+2x^4+x^8}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^9/(1 + 2*x^4 + x^8),x]
[Out] IntegrateAlgebraic[x^9/(1 + 2*x^4 + x^8), x]
```

fricas [A] time = 0.93, size = 31, normalized size = 1.03

$$\frac{2\,x^6+3\,x^2-3\left(x^4+1\right)\arctan\left(x^2\right)}{4\left(x^4+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(x^8+2*x^4+1),x, algorithm="fricas")
[Out] 1/4*(2*x^6 + 3*x^2 - 3*(x^4 + 1)*arctan(x^2))/(x^4 + 1)
```

giac [A] time = 0.34, size = 24, normalized size = 0.80

$$\frac{1}{2}\,x^2+\frac{x^2}{4\left(x^4+1\right)}-\frac{3}{4}\,\arctan\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(x^8+2*x^4+1),x, algorithm="giac")
[Out] 1/2*x^2 + 1/4*x^2/(x^4 + 1) - 3/4*arctan(x^2)
```

maple [A] time = 0.01, size = 25, normalized size = 0.83

$$\frac{x^2}{2} + \frac{x^2}{4x^4 + 4} - \frac{3\arctan\left(x^2\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9/(x^8+2*x^4+1),x)
[Out] 1/2*x^2+1/4*x^2/(x^4+1)-3/4*arctan(x^2)
```

maxima [A] time = 2.16, size = 24, normalized size = 0.80

$$\frac{1}{2}x^2 + \frac{x^2}{4\left(x^4 + 1\right)} - \frac{3}{4}\arctan\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(x^8+2*x^4+1),x, algorithm="maxima")
[Out] 1/2*x^2 + 1/4*x^2/(x^4 + 1) - 3/4*arctan(x^2)
```

mupad [B] time = 0.05, size = 25, normalized size = 0.83

$$\frac{x^2}{4\left(x^4 + 1\right)} - \frac{3\operatorname{atan}\left(x^2\right)}{4} + \frac{x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9/(2*x^4 + x^8 + 1),x)
[Out] x^2/(4*(x^4 + 1)) - (3*atan(x^2))/4 + x^2/2
```

sympy [A] time = 0.13, size = 22, normalized size = 0.73

$$\frac{x^2}{2} + \frac{x^2}{4x^4 + 4} - \frac{3\operatorname{atan}\left(x^2\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**9/(x**8+2*x**4+1),x)
[Out] x**2/2 + x**2/(4*x**4 + 4) - 3*atan(x**2)/4
```

3.218 $\int \frac{x^7}{1+2x^4+x^8} \, dx$

Optimal. Leaf size=22

$$\frac{1}{4\left(x^4+1\right)}+\frac{1}{4}\log\left(x^4+1\right)$$

Rubi [A] time = 0.01, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {28, 266, 43}

$$\frac{1}{4\left(x^4+1\right)}+\frac{1}{4}\log\left(x^4+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^7/(1 + 2*x^4 + x^8),x]
```

```
[Out] 1/(4*(1 + x^4)) + Log[1 + x^4]/4
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^7}{1+2x^4+x^8}\,dx &= \int \frac{x^7}{\left(1+x^4\right)^2}\,dx\\
&= \frac{1}{4}\text{Subst}\left(\int \frac{x}{(1+x)^2}\,dx, x, x^4\right)\\
&= \frac{1}{4}\text{Subst}\left(\int \left(-\frac{1}{(1+x)^2}+\frac{1}{1+x}\right)dx, x, x^4\right)\\
&= \frac{1}{4\left(1+x^4\right)}+\frac{1}{4}\log\left(1+x^4\right)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 18, normalized size = 0.82

$$\frac{1}{4}\left(\frac{1}{x^4+1}+\log\left(x^4+1\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7/(1 + 2*x^4 + x^8),x]
[Out] ((1 + x^4)^(-1) + Log[1 + x^4])/4
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^7}{1+2x^4+x^8}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^7/(1 + 2*x^4 + x^8),x]
[Out] IntegrateAlgebraic[x^7/(1 + 2*x^4 + x^8), x]
```

fricas [A] time = 0.94, size = 23, normalized size = 1.05

$$\frac{(x^4+1)\log(x^4+1)+1}{4(x^4+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(x^8+2*x^4+1),x, algorithm="fricas")
[Out] 1/4*((x^4 + 1)*log(x^4 + 1) + 1)/(x^4 + 1)
```

giac [A] time = 0.28, size = 18, normalized size = 0.82

$$\frac{1}{4(x^4+1)}+\frac{1}{4}\log(x^4+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(x^8+2*x^4+1),x, algorithm="giac")
[Out] 1/4/(x^4 + 1) + 1/4*log(x^4 + 1)
```

maple [A] time = 0.01, size = 19, normalized size = 0.86

$$\frac{\ln(x^4+1)}{4}+\frac{1}{4x^4+4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(x^8+2*x^4+1),x)
[Out] 1/4/(x^4+1)+1/4*ln(x^4+1)
```

maxima [A] time = 0.98, size = 18, normalized size = 0.82

$$\frac{1}{4(x^4+1)}+\frac{1}{4}\log(x^4+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(x^8+2*x^4+1),x, algorithm="maxima")
[Out] 1/4/(x^4 + 1) + 1/4*log(x^4 + 1)
```

mupad [B] time = 1.32, size = 18, normalized size = 0.82

$$\frac{\ln(x^4+1)}{4}+\frac{1}{4(x^4+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(2*x^4 + x^8 + 1),x)
```

```
[Out] log(x^4 + 1)/4 + 1/(4*(x^4 + 1))
```

sympy [A] time = 0.11, size = 15, normalized size = 0.68

$$\frac{\log\left(x^4+1\right)}{4}+\frac{1}{4x^4+4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7/(x**8+2*x**4+1),x)
```

```
[Out] log(x**4 + 1)/4 + 1/(4*x**4 + 4)
```

3.219 $\int \frac{x^5}{1+2x^4+x^8}\,dx$

Optimal. Leaf size=23

$$\frac{1}{4}\tan^{-1}\left(x^2\right)-\frac{x^2}{4\left(x^4+1\right)}$$

Rubi [A] time = 0.01, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {28, 275, 288, 203}

$$\frac{1}{4}\tan^{-1}\left(x^2\right)-\frac{x^2}{4\left(x^4+1\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/(1 + 2*x^4 + x^8),x]
```

```
[Out] -x^2/(4*(1 + x^4)) + ArcTan[x^2]/4
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int \frac{x^5}{1+2x^4+x^8}\,dx = \int \frac{x^5}{\left(1+x^4\right)^2}\,dx$$
$$= \frac{1}{2}\,\text{Subst}\left(\int \frac{x^2}{\left(1+x^2\right)^2}\,dx, x, x^2\right)$$
$$= -\frac{x^2}{4\left(1+x^4\right)} + \frac{1}{4}\,\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, x^2\right)$$
$$= -\frac{x^2}{4\left(1+x^4\right)} + \frac{1}{4}\tan^{-1}\left(x^2\right)$$

Mathematica [A] time = 0.01, size = 23, normalized size = 1.00

$$\frac{1}{4}\tan^{-1}\left(x^2\right) - \frac{x^2}{4\left(x^4+1\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/(1 + 2*x^4 + x^8),x]
[Out] -1/4*x^2/(1 + x^4) + ArcTan[x^2]/4
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5}{1+2x^4+x^8}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^5/(1 + 2*x^4 + x^8),x]
[Out] IntegrateAlgebraic[x^5/(1 + 2*x^4 + x^8), x]
```

fricas [A] time = 0.94, size = 24, normalized size = 1.04

$$-\frac{x^2 - \left(x^4+1\right)\arctan\left(x^2\right)}{4\left(x^4+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(x^8+2*x^4+1),x, algorithm="fricas")
[Out] -1/4*(x^2 - (x^4 + 1)*arctan(x^2))/(x^4 + 1)
```

giac [A] time = 0.37, size = 19, normalized size = 0.83

$$-\frac{x^2}{4\left(x^4+1\right)} + \frac{1}{4}\arctan\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(x^8+2*x^4+1),x, algorithm="giac")
[Out] -1/4*x^2/(x^4 + 1) + 1/4*arctan(x^2)
```

maple [A] time = 0.01, size = 20, normalized size = 0.87

$$-\frac{x^{2}}{4\left(x^{4}+1\right)}+\frac{\arctan\left(x^{2}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(x^8+2*x^4+1),x)
[Out] -1/4/(x^4+1)*x^2+1/4*arctan(x^2)
```

maxima [A] time = 2.02, size = 19, normalized size = 0.83

$$-\frac{x^{2}}{4\left(x^{4}+1\right)}+\frac{1}{4}\arctan\left(x^{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(x^8+2*x^4+1),x, algorithm="maxima")
[Out] -1/4*x^2/(x^4 + 1) + 1/4*arctan(x^2)
```

mupad [B] time = 1.37, size = 21, normalized size = 0.91

$$\frac{\operatorname{atan}\left(x^{2}\right)}{4}-\frac{x^{2}}{4\left(x^{4}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(2*x^4 + x^8 + 1),x)
[Out] atan(x^2)/4 - x^2/(4*(x^4 + 1))
```

sympy [A] time = 0.12, size = 15, normalized size = 0.65

$$-\frac{x^{2}}{4x^{4}+4}+\frac{\operatorname{atan}\left(x^{2}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(x**8+2*x**4+1),x)
[Out] -x**2/(4*x**4 + 4) + atan(x**2)/4
```

3.220 $\int \frac{x^3}{1+2x^4+x^8} \, dx$

Optimal. Leaf size=11

$$-\frac{1}{4(x^4+1)}$$

Rubi [A] time = 0.00, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {28, 261}

$$-\frac{1}{4(x^4+1)}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(1 + 2*x^4 + x^8),x]
```

```
[Out] -1/(4*(1 + x^4))
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{x^3}{1+2x^4+x^8} \, dx &= \int \frac{x^3}{\left(1+x^4\right)^2} \, dx\\ &= -\frac{1}{4\left(1+x^4\right)}\end{aligned}$$

Mathematica [A] time = 0.00, size = 11, normalized size = 1.00

$$-\frac{1}{4(x^4+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(1 + 2*x^4 + x^8),x]
```

```
[Out] -1/4*1/(1 + x^4)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{1+2x^4+x^8} \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^3/(1 + 2*x^4 + x^8),x]
[Out] IntegrateAlgebraic[x^3/(1 + 2*x^4 + x^8), x]
```

fricas [A] time = 0.86, size = 9, normalized size = 0.82

$$-\frac{1}{4\left(x^4+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(x^8+2*x^4+1),x, algorithm="fricas")
[Out] -1/4/(x^4 + 1)
```

giac [A] time = 0.36, size = 9, normalized size = 0.82

$$-\frac{1}{4\left(x^4+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(x^8+2*x^4+1),x, algorithm="giac")
[Out] -1/4/(x^4 + 1)
```

maple [A] time = 0.00, size = 10, normalized size = 0.91

$$-\frac{1}{4\left(x^4+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(x^8+2*x^4+1),x)
[Out] -1/4/(x^4+1)
```

maxima [A] time = 0.89, size = 9, normalized size = 0.82

$$-\frac{1}{4\left(x^4+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(x^8+2*x^4+1),x, algorithm="maxima")
[Out] -1/4/(x^4 + 1)
```

mupad [B] time = 0.02, size = 11, normalized size = 1.00

$$-\frac{1}{4\,\left(x^4+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(2*x^4 + x^8 + 1),x)
[Out] -1/(4*(x^4 + 1))
```

sympy [A] time = 0.09, size = 8, normalized size = 0.73

$$-\frac{1}{4x^4+4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(x**8+2*x**4+1),x)
[Out] -1/(4*x**4 + 4)
```

3.221 $\int \frac{x}{1+2x^4+x^8}\,dx$

Optimal. Leaf size=23

$$\frac{1}{4}\tan^{-1}\left(x^2\right)+\frac{x^2}{4\left(x^4+1\right)}$$

Rubi [A] time = 0.01, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {28, 275, 199, 203}

$$\frac{x^2}{4\left(x^4+1\right)}+\frac{1}{4}\tan^{-1}\left(x^2\right)$$

Antiderivative was successfully verified.

```
[In] Int[x/(1 + 2*x^4 + x^8),x]
```

```
[Out] x^2/(4*(1 + x^4)) + ArcTan[x^2]/4
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\int \frac{x}{1+2x^4+x^8}\,dx &= \int \frac{x}{\left(1+x^4\right)^2}\,dx\\
&= \frac{1}{2}\text{Subst}\left(\int \frac{1}{\left(1+x^2\right)^2}\,dx,x,x^2\right)\\
&= \frac{x^2}{4\left(1+x^4\right)}+\frac{1}{4}\text{Subst}\left(\int \frac{1}{1+x^2}\,dx,x,x^2\right)\\
&= \frac{x^2}{4\left(1+x^4\right)}+\frac{1}{4}\tan^{-1}\left(x^2\right)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 20, normalized size = 0.87

$$\frac{1}{4}\left(\tan^{-1}\left(x^2\right)+\frac{x^2}{x^4+1}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(1 + 2*x^4 + x^8),x]
[Out] (x^2/(1 + x^4) + ArcTan[x^2])/4
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{1+2x^4+x^8}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x/(1 + 2*x^4 + x^8),x]
[Out] IntegrateAlgebraic[x/(1 + 2*x^4 + x^8), x]
```

fricas [A] time = 1.12, size = 23, normalized size = 1.00

$$\frac{x^2+\left(x^4+1\right)\arctan\left(x^2\right)}{4\left(x^4+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^8+2*x^4+1),x, algorithm="fricas")
[Out] 1/4*(x^2 + (x^4 + 1)*arctan(x^2))/(x^4 + 1)
```

giac [A] time = 0.39, size = 19, normalized size = 0.83

$$\frac{x^2}{4\left(x^4+1\right)}+\frac{1}{4}\arctan\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^8+2*x^4+1),x, algorithm="giac")
[Out] 1/4*x^2/(x^4 + 1) + 1/4*arctan(x^2)
```

maple [A] time = 0.00, size = 20, normalized size = 0.87

$$\frac{x^2}{4x^4+4}+\frac{\arctan\left(x^2\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x^8+2*x^4+1),x)
[Out] 1/4/(x^4+1)*x^2+1/4*arctan(x^2)
```

maxima [A] time = 2.17, size = 19, normalized size = 0.83

$$\frac{x^2}{4\left(x^4+1\right)}+\frac{1}{4}\arctan\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^8+2*x^4+1),x, algorithm="maxima")
```

```
[Out] 1/4*x^2/(x^4 + 1) + 1/4*arctan(x^2)
```

mupad [B] time = 0.03, size = 20, normalized size = 0.87

$$\frac{\operatorname{atan}\left(x^2\right)}{4}+\frac{x^2}{4\,\left(x^4+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(2*x^4 + x^8 + 1),x)
```

```
[Out] atan(x^2)/4 + x^2/(4*(x^4 + 1))
```

sympy [A] time = 0.12, size = 15, normalized size = 0.65

$$\frac{x^2}{4x^4+4}+\frac{\operatorname{atan}\left(x^2\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x**8+2*x**4+1),x)
```

```
[Out] x**2/(4*x**4 + 4) + atan(x**2)/4
```

3.222 $\int \frac{1}{x(1+2x^4+x^8)}\,dx$

Optimal. Leaf size=24

$$\frac{1}{4\left(x^4+1\right)}-\frac{1}{4}\log\left(x^4+1\right)+\log(x)$$

Rubi [A] time = 0.01, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {28, 266, 44}

$$\frac{1}{4\left(x^4+1\right)}-\frac{1}{4}\log\left(x^4+1\right)+\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(1 + 2*x^4 + x^8)),x]
```

```
[Out] 1/(4*(1 + x^4)) + Log[x] - Log[1 + x^4]/4
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x\left(1+2x^4+x^8\right)}\,dx &= \int \frac{1}{x\left(1+x^4\right)^2}\,dx\\ &= \frac{1}{4}\text{Subst}\left(\int \frac{1}{x(1+x)^2}\,dx,x,x^4\right)\\ &= \frac{1}{4}\text{Subst}\left(\int \left(\frac{1}{-1-x}+\frac{1}{x}-\frac{1}{(1+x)^2}\right)dx,x,x^4\right)\\ &= \frac{1}{4\left(1+x^4\right)}+\log(x)-\frac{1}{4}\log\left(1+x^4\right)\end{aligned}$$

Mathematica [A] time = 0.01, size = 24, normalized size = 1.00

$$\frac{1}{4\left(x^4+1\right)}-\frac{1}{4}\log\left(x^4+1\right)+\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(1 + 2*x^4 + x^8)),x]
[Out] 1/(4*(1 + x^4)) + Log[x] - Log[1 + x^4]/4
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(1+2x^4+x^8\right)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x*(1 + 2*x^4 + x^8)),x]
[Out] IntegrateAlgebraic[1/(x*(1 + 2*x^4 + x^8)), x]
```

fricas [A] time = 0.78, size = 32, normalized size = 1.33

$$-\frac{\left(x^4+1\right)\log\left(x^4+1\right)-4\left(x^4+1\right)\log(x)-1}{4\left(x^4+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x^8+2*x^4+1),x, algorithm="fricas")
[Out] -1/4*((x^4 + 1)*log(x^4 + 1) - 4*(x^4 + 1)*log(x) - 1)/(x^4 + 1)
```

giac [A] time = 0.36, size = 29, normalized size = 1.21

$$\frac{x^4+2}{4\left(x^4+1\right)}-\frac{1}{4}\log\left(x^4+1\right)+\frac{1}{4}\log\left(x^4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x^8+2*x^4+1),x, algorithm="giac")
[Out] 1/4*(x^4 + 2)/(x^4 + 1) - 1/4*log(x^4 + 1) + 1/4*log(x^4)
```

maple [A] time = 0.01, size = 21, normalized size = 0.88

$$\ln(x)-\frac{\ln\left(x^4+1\right)}{4}+\frac{1}{4x^4+4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(x^8+2*x^4+1),x)
[Out] 1/4/(x^4+1)+ln(x)-1/4*ln(x^4+1)
```

maxima [A] time = 0.97, size = 24, normalized size = 1.00

$$\frac{1}{4\left(x^4+1\right)}-\frac{1}{4}\log\left(x^4+1\right)+\frac{1}{4}\log\left(x^4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x^8+2*x^4+1),x, algorithm="maxima")
[Out] 1/4/(x^4 + 1) - 1/4*log(x^4 + 1) + 1/4*log(x^4)
```

mupad [B] time = 0.04, size = 20, normalized size = 0.83

$$\ln(x) - \frac{\ln\left(x^4 + 1\right)}{4} + \frac{1}{4\left(x^4 + 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(2*x^4 + x^8 + 1)),x)
```

```
[Out] log(x) - log(x^4 + 1)/4 + 1/(4*(x^4 + 1))
```

sympy [A] time = 0.13, size = 19, normalized size = 0.79

$$\log(x) - \frac{\log\left(x^4 + 1\right)}{4} + \frac{1}{4x^4 + 4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x**8+2*x**4+1),x)
```

```
[Out] log(x) - log(x**4 + 1)/4 + 1/(4*x**4 + 4)
```

3.223 $\int \frac{1}{x^3(1+2x^4+x^8)}\,dx$

Optimal. Leaf size=30

$$-\frac{3}{4x^2}-\frac{3}{4}\tan^{-1}\left(x^2\right)+\frac{1}{4x^2\left(x^4+1\right)}$$

Rubi [A] time = 0.01, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {28, 275, 290, 325, 203}

$$\frac{1}{4x^2\left(x^4+1\right)}-\frac{3}{4x^2}-\frac{3}{4}\tan^{-1}\left(x^2\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(1 + 2*x^4 + x^8)),x]
```

```
[Out] -3/(4*x^2) + 1/(4*x^2*(1 + x^4)) - (3*ArcTan[x^2])/4
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^3\left(1+2x^4+x^8\right)}\,dx &= \int \frac{1}{x^3\left(1+x^4\right)^2}\,dx\\ &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{x^2\left(1+x^2\right)^2}\,dx,x,x^2\right)\\ &= \frac{1}{4x^2\left(1+x^4\right)}+\frac{3}{4}\operatorname{Subst}\left(\int \frac{1}{x^2\left(1+x^2\right)}\,dx,x,x^2\right)\\ &= -\frac{3}{4x^2}+\frac{1}{4x^2\left(1+x^4\right)}-\frac{3}{4}\operatorname{Subst}\left(\int \frac{1}{1+x^2}\,dx,x,x^2\right)\\ &= -\frac{3}{4x^2}+\frac{1}{4x^2\left(1+x^4\right)}-\frac{3}{4}\tan^{-1}\left(x^2\right)\end{aligned}$$

Mathematica [A] time = 0.01, size = 30, normalized size = 1.00

$$-\frac{1}{2x^2}+\frac{3}{4}\tan^{-1}\left(\frac{1}{x^2}\right)-\frac{x^2}{4\left(x^4+1\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(1 + 2*x^4 + x^8)),x]
[Out] -1/2*1/x^2 - x^2/(4*(1 + x^4)) + (3*ArcTan[x^(-2)])/4
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3\left(1+2x^4+x^8\right)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x^3*(1 + 2*x^4 + x^8)),x]
[Out] IntegrateAlgebraic[1/(x^3*(1 + 2*x^4 + x^8)), x]
```

fricas [A] time = 1.02, size = 31, normalized size = 1.03

$$-\frac{3\,x^4+3\left(x^6+x^2\right)\arctan\left(x^2\right)+2}{4\left(x^6+x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(x^8+2*x^4+1),x, algorithm="fricas")
[Out] -1/4*(3*x^4 + 3*(x^6 + x^2)*arctan(x^2) + 2)/(x^6 + x^2)
```

giac [A] time = 0.34, size = 25, normalized size = 0.83

$$-\frac{3\,x^4+2}{4\left(x^6+x^2\right)}-\frac{3}{4}\arctan\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(x^8+2*x^4+1),x, algorithm="giac")
[Out] -1/4*(3*x^4 + 2)/(x^6 + x^2) - 3/4*arctan(x^2)
```

maple [A] time = 0.01, size = 25, normalized size = 0.83

$$-\frac{x^2}{4\left(x^4+1\right)}-\frac{3\arctan\left(x^2\right)}{4}-\frac{1}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(x^8+2*x^4+1),x)
```

```
[Out] -1/2/x^2-1/4/(x^4+1)*x^2-3/4*arctan(x^2)
```

maxima [A] time = 1.98, size = 25, normalized size = 0.83

$$-\frac{3\,x^4+2}{4\left(x^6+x^2\right)}-\frac{3}{4}\,\arctan\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(x^8+2*x^4+1),x, algorithm="maxima")
```

```
[Out] -1/4*(3*x^4 + 2)/(x^6 + x^2) - 3/4*arctan(x^2)
```

mupad [B] time = 0.04, size = 25, normalized size = 0.83

$$-\frac{3\,\mathrm{atan}\left(x^2\right)}{4}-\frac{\frac{3\,x^4}{4}+\frac{1}{2}}{x^6+x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(2*x^4 + x^8 + 1)),x)
```

```
[Out] - (3*atan(x^2))/4 - ((3*x^4)/4 + 1/2)/(x^2 + x^6)
```

sympy [A] time = 0.15, size = 26, normalized size = 0.87

$$\frac{-3x^4-2}{4x^6+4x^2}-\frac{3\,\mathrm{atan}\left(x^2\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(x**8+2*x**4+1),x)
```

```
[Out] (-3*x**4 - 2)/(4*x**6 + 4*x**2) - 3*atan(x**2)/4
```

3.224 $\int \frac{1}{x^5(1+2x^4+x^8)}\,dx$

Optimal. Leaf size=33

$$-\frac{1}{4\left(x^4+1\right)}-\frac{1}{4x^4}+\frac{1}{2}\log\left(x^4+1\right)-2\log(x)$$

Rubi [A] time = 0.02, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {28, 266, 44}

$$-\frac{1}{4\left(x^4+1\right)}-\frac{1}{4x^4}+\frac{1}{2}\log\left(x^4+1\right)-2\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^5*(1 + 2*x^4 + x^8)),x]
```

```
[Out] -1/(4*x^4) - 1/(4*(1 + x^4)) - 2*Log[x] + Log[1 + x^4]/2
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^5\left(1+2x^4+x^8\right)}\,dx &= \int \frac{1}{x^5\left(1+x^4\right)^2}\,dx\\
&= \frac{1}{4}\operatorname{Subst}\left(\int \frac{1}{x^2(1+x)^2}\,dx, x, x^4\right)\\
&= \frac{1}{4}\operatorname{Subst}\left(\int \left(\frac{1}{x^2}-\frac{2}{x}+\frac{1}{(1+x)^2}+\frac{2}{1+x}\right)dx, x, x^4\right)\\
&= -\frac{1}{4x^4}-\frac{1}{4\left(1+x^4\right)}-2\log(x)+\frac{1}{2}\log\left(1+x^4\right)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 33, normalized size = 1.00

$$-\frac{1}{4\left(x^4+1\right)}-\frac{1}{4x^4}+\frac{1}{2}\log\left(x^4+1\right)-2\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^5*(1 + 2*x^4 + x^8)),x]
[Out] -1/4*1/x^4 - 1/(4*(1 + x^4)) - 2*Log[x] + Log[1 + x^4]/2
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^5\left(1+2x^4+x^8\right)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x^5*(1 + 2*x^4 + x^8)),x]
[Out] IntegrateAlgebraic[1/(x^5*(1 + 2*x^4 + x^8)), x]
```

fricas [A] time = 1.13, size = 44, normalized size = 1.33

$$-\frac{2\,x^4-2\left(x^8+x^4\right)\log\left(x^4+1\right)+8\left(x^8+x^4\right)\log(x)+1}{4\left(x^8+x^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(x^8+2*x^4+1),x, algorithm="fricas")
[Out] -1/4*(2*x^4 - 2*(x^8 + x^4)*log(x^4 + 1) + 8*(x^8 + x^4)*log(x) + 1)/(x^8 +
 x^4)
```

giac [A] time = 0.35, size = 33, normalized size = 1.00

$$-\frac{2\,x^4+1}{4\left(x^8+x^4\right)}+\frac{1}{2}\,\log\left(x^4+1\right)-\frac{1}{2}\,\log\left(x^4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(x^8+2*x^4+1),x, algorithm="giac")
[Out] -1/4*(2*x^4 + 1)/(x^8 + x^4) + 1/2*log(x^4 + 1) - 1/2*log(x^4)
```

maple [A] time = 0.02, size = 28, normalized size = 0.85

$$-2\ln(x)+\frac{\ln\left(x^4+1\right)}{2}-\frac{1}{4x^4}-\frac{1}{4\left(x^4+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^5/(x^8+2*x^4+1),x)
[Out] -1/4/x^4-1/4/(x^4+1)-2*ln(x)+1/2*ln(x^4+1)
```

maxima [A] time = 0.88, size = 33, normalized size = 1.00

$$-\frac{2\,x^4+1}{4\left(x^8+x^4\right)}+\frac{1}{2}\,\log\left(x^4+1\right)-\frac{1}{2}\,\log\left(x^4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(x^8+2*x^4+1),x, algorithm="maxima")
[Out] -1/4*(2*x^4 + 1)/(x^8 + x^4) + 1/2*log(x^4 + 1) - 1/2*log(x^4)
```

mupad [**B**] time = 0.05, size = 31, normalized size = 0.94

$$\frac{\ln\left(x^4+1\right)}{2}-2\,\ln(x)-\frac{\frac{x^4}{2}+\frac{1}{4}}{x^8+x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^5*(2*x^4 + x^8 + 1)),x)
```

```
[Out] log(x^4 + 1)/2 - 2*log(x) - (x^4/2 + 1/4)/(x^4 + x^8)
```

sympy [**A**] time = 0.15, size = 31, normalized size = 0.94

$$\frac{-2x^4-1}{4x^8+4x^4}-2\log\left(x\right)+\frac{\log\left(x^4+1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**5/(x**8+2*x**4+1),x)
```

```
[Out] (-2*x**4 - 1)/(4*x**8 + 4*x**4) - 2*log(x) + log(x**4 + 1)/2
```

3.225 $\int \frac{1}{x^7(1+2x^4+x^8)}\,dx$

Optimal. Leaf size=37

$$-\frac{5}{12x^6}+\frac{5}{4x^2}+\frac{5}{4}\tan^{-1}\left(x^2\right)+\frac{1}{4x^6\left(x^4+1\right)}$$

Rubi [A] time = 0.02, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {28, 275, 290, 325, 203}

$$\frac{1}{4x^6\left(x^4+1\right)}+\frac{5}{4x^2}-\frac{5}{12x^6}+\frac{5}{4}\tan^{-1}\left(x^2\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^7*(1 + 2*x^4 + x^8)),x]
```

```
[Out] -5/(12*x^6) + 5/(4*x^2) + 1/(4*x^6*(1 + x^4)) + (5*ArcTan[x^2])/4
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x^7\left(1+2x^4+x^8\right)}\,dx &= \int \frac{1}{x^7\left(1+x^4\right)^2}\,dx \\
&= \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{x^4\left(1+x^2\right)^2}\,dx, x, x^2\right) \\
&= \frac{1}{4x^6\left(1+x^4\right)} + \frac{5}{4}\,\text{Subst}\left(\int \frac{1}{x^4\left(1+x^2\right)}\,dx, x, x^2\right) \\
&= -\frac{5}{12x^6} + \frac{1}{4x^6\left(1+x^4\right)} - \frac{5}{4}\,\text{Subst}\left(\int \frac{1}{x^2\left(1+x^2\right)}\,dx, x, x^2\right) \\
&= -\frac{5}{12x^6} + \frac{5}{4x^2} + \frac{1}{4x^6\left(1+x^4\right)} + \frac{5}{4}\,\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, x^2\right) \\
&= -\frac{5}{12x^6} + \frac{5}{4x^2} + \frac{1}{4x^6\left(1+x^4\right)} + \frac{5}{4}\tan^{-1}\left(x^2\right)
\end{aligned}
$$

Mathematica [**A**] time = 0.01, size = 33, normalized size = 0.89

$$
-\frac{1}{6x^6} + \frac{1}{x^2} - \frac{5}{4}\tan^{-1}\left(\frac{1}{x^2}\right) + \frac{x^2}{4\left(x^4+1\right)}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^7*(1 + 2*x^4 + x^8)),x]
[Out] -1/6*1/x^6 + x^(-2) + x^2/(4*(1 + x^4)) - (5*ArcTan[x^(-2)])/4
```

IntegrateAlgebraic [**F**] time = 0.00, size = 0, normalized size = 0.00

$$
\int \frac{1}{x^7\left(1+2x^4+x^8\right)}\,dx
$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x^7*(1 + 2*x^4 + x^8)),x]
[Out] IntegrateAlgebraic[1/(x^7*(1 + 2*x^4 + x^8)), x]
```

fricas [**A**] time = 1.10, size = 36, normalized size = 0.97

$$
\frac{15\,x^8 + 10\,x^4 + 15\left(x^{10}+x^6\right)\arctan\left(x^2\right) - 2}{12\left(x^{10}+x^6\right)}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(x^8+2*x^4+1),x, algorithm="fricas")
[Out] 1/12*(15*x^8 + 10*x^4 + 15*(x^10 + x^6)*arctan(x^2) - 2)/(x^10 + x^6)
```

giac [**A**] time = 0.30, size = 31, normalized size = 0.84

$$
\frac{x^2}{4\left(x^4+1\right)} + \frac{6\,x^4-1}{6\,x^6} + \frac{5}{4}\arctan\left(x^2\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(x^8+2*x^4+1),x, algorithm="giac")
[Out] 1/4*x^2/(x^4 + 1) + 1/6*(6*x^4 - 1)/x^6 + 5/4*arctan(x^2)
```

maple [A] time = 0.01, size = 28, normalized size = 0.76

$$\frac{x^2}{4x^4+4} + \frac{5\arctan\left(x^2\right)}{4} + \frac{1}{x^2} - \frac{1}{6x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^7/(x^8+2*x^4+1),x)
[Out] -1/6/x^6+1/x^2+1/4/(x^4+1)*x^2+5/4*arctan(x^2)
```

maxima [A] time = 2.49, size = 30, normalized size = 0.81

$$\frac{15\,x^8 + 10\,x^4 - 2}{12\left(x^{10} + x^6\right)} + \frac{5}{4}\arctan\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(x^8+2*x^4+1),x, algorithm="maxima")
[Out] 1/12*(15*x^8 + 10*x^4 - 2)/(x^10 + x^6) + 5/4*arctan(x^2)
```

mupad [B] time = 0.05, size = 30, normalized size = 0.81

$$\frac{5\,\mathrm{atan}\left(x^2\right)}{4} + \frac{\frac{5\,x^8}{4} + \frac{5\,x^4}{6} - \frac{1}{6}}{x^6\,\left(x^4+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^7*(2*x^4 + x^8 + 1)),x)
[Out] (5*atan(x^2))/4 + ((5*x^4)/6 + (5*x^8)/4 - 1/6)/(x^6*(x^4 + 1))
```

sympy [A] time = 0.17, size = 29, normalized size = 0.78

$$\frac{5\,\mathrm{atan}\left(x^2\right)}{4} + \frac{15x^8 + 10x^4 - 2}{12x^{10} + 12x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**7/(x**8+2*x**4+1),x)
[Out] 5*atan(x**2)/4 + (15*x**8 + 10*x**4 - 2)/(12*x**10 + 12*x**6)
```

3.226 $\int \frac{x^8}{1+2x^4+x^8}\,dx$

Optimal. Leaf size=104

$$\frac{5\log\left(x^2-\sqrt{2}\,x+1\right)}{16\sqrt{2}}-\frac{5\log\left(x^2+\sqrt{2}\,x+1\right)}{16\sqrt{2}}-\frac{x^5}{4\left(x^4+1\right)}+\frac{5x}{4}+\frac{5\tan^{-1}\left(1-\sqrt{2}\,x\right)}{8\sqrt{2}}-\frac{5\tan^{-1}\left(\sqrt{2}\,x+1\right)}{8\sqrt{2}}$$

Rubi [A] time = 0.05, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.562, Rules used = {28, 288, 321, 211, 1165, 628, 1162, 617, 204}

$$-\frac{x^5}{4\left(x^4+1\right)}+\frac{5\log\left(x^2-\sqrt{2}\,x+1\right)}{16\sqrt{2}}-\frac{5\log\left(x^2+\sqrt{2}\,x+1\right)}{16\sqrt{2}}+\frac{5x}{4}+\frac{5\tan^{-1}\left(1-\sqrt{2}\,x\right)}{8\sqrt{2}}-\frac{5\tan^{-1}\left(\sqrt{2}\,x+1\right)}{8\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^8/(1 + 2*x^4 + x^8),x]
```

```
[Out] (5*x)/4 - x^5/(4*(1 + x^4)) + (5*ArcTan[1 - Sqrt[2]*x])/(8*Sqrt[2]) - (5*Ar
cTan[1 + Sqrt[2]*x])/(8*Sqrt[2]) + (5*Log[1 - Sqrt[2]*x + x^2])/(16*Sqrt[2]
) - (5*Log[1 + Sqrt[2]*x + x^2])/(16*Sqrt[2])
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^8}{1+2x^4+x^8}\,dx &= \int \frac{x^8}{\left(1+x^4\right)^2}\,dx \\
&= -\frac{x^5}{4\left(1+x^4\right)}+\frac{5}{4}\int \frac{x^4}{1+x^4}\,dx \\
&= \frac{5x}{4}-\frac{x^5}{4\left(1+x^4\right)}-\frac{5}{4}\int \frac{1}{1+x^4}\,dx \\
&= \frac{5x}{4}-\frac{x^5}{4\left(1+x^4\right)}-\frac{5}{8}\int \frac{1-x^2}{1+x^4}\,dx-\frac{5}{8}\int \frac{1+x^2}{1+x^4}\,dx \\
&= \frac{5x}{4}-\frac{x^5}{4\left(1+x^4\right)}-\frac{5}{16}\int \frac{1}{1-\sqrt{2}\,x+x^2}\,dx-\frac{5}{16}\int \frac{1}{1+\sqrt{2}\,x+x^2}\,dx+\frac{5\int \frac{\sqrt{2}+2x}{-1-\sqrt{2}\,x-x^2}\,dx}{16\sqrt{2}} \\
&= \frac{5x}{4}-\frac{x^5}{4\left(1+x^4\right)}+\frac{5\log\left(1-\sqrt{2}\,x+x^2\right)}{16\sqrt{2}}-\frac{5\log\left(1+\sqrt{2}\,x+x^2\right)}{16\sqrt{2}}-\frac{5\,\mathrm{Subst}\left(\int \frac{1}{-1-x^2}\,dx,\right.}{8\sqrt{2}} \\
&= \frac{5x}{4}-\frac{x^5}{4\left(1+x^4\right)}+\frac{5\tan^{-1}\left(1-\sqrt{2}\,x\right)}{8\sqrt{2}}-\frac{5\tan^{-1}\left(1+\sqrt{2}\,x\right)}{8\sqrt{2}}+\frac{5\log\left(1-\sqrt{2}\,x+x^2\right)}{16\sqrt{2}}-
\end{aligned}$$

Mathematica [A] time = 0.07, size = 94, normalized size = 0.90

$$\frac{1}{32}\left(\frac{8x}{x^4+1}+5\sqrt{2}\log\left(x^2-\sqrt{2}\,x+1\right)-5\sqrt{2}\log\left(x^2+\sqrt{2}\,x+1\right)+32x+10\sqrt{2}\tan^{-1}\left(1-\sqrt{2}\,x\right)-10\sqrt{2}\tan^{-1}\left(\sqrt{2}\,x+1\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8/(1 + 2*x^4 + x^8),x]
```

```
[Out] (32*x + (8*x)/(1 + x^4) + 10*Sqrt[2]*ArcTan[1 - Sqrt[2]*x] - 10*Sqrt[2]*Arc
Tan[1 + Sqrt[2]*x] + 5*Sqrt[2]*Log[1 - Sqrt[2]*x + x^2] - 5*Sqrt[2]*Log[1 +
 Sqrt[2]*x + x^2])/32
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^8}{1+2x^4+x^8}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^8/(1 + 2*x^4 + x^8),x]
```

```
[Out] IntegrateAlgebraic[x^8/(1 + 2*x^4 + x^8), x]
```

fricas [A] time = 1.02, size = 132, normalized size = 1.27

$$\frac{32\,x^5+20\,\sqrt{2}\left(x^4+1\right)\arctan\left(-\sqrt{2}\,x+\sqrt{2}\,\sqrt{x^2+\sqrt{2}\,x+1}-1\right)+20\,\sqrt{2}\left(x^4+1\right)\arctan\left(-\sqrt{2}\,x+\sqrt{2}\,\sqrt{x^2-\sqrt{2}\,x+1}+1\right)-5\,\sqrt{2}\left(x^4+1\right)\log\left(x^2+\sqrt{2}\,x+1\right)+5\,\sqrt{2}\left(x^4+1\right)\log\left(x^2-\sqrt{2}\,x+1\right)+40\,x}{32\left(x^4+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(x^8+2*x^4+1),x, algorithm="fricas")
```

```
[Out] 1/32*(32*x^5 + 20*sqrt(2)*(x^4 + 1)*arctan(-sqrt(2)*x + sqrt(2)*sqrt(x^2 +
sqrt(2)*x + 1) - 1) + 20*sqrt(2)*(x^4 + 1)*arctan(-sqrt(2)*x + sqrt(2)*sqrt
(x^2 - sqrt(2)*x + 1) + 1) - 5*sqrt(2)*(x^4 + 1)*log(x^2 + sqrt(2)*x + 1) +
 5*sqrt(2)*(x^4 + 1)*log(x^2 - sqrt(2)*x + 1) + 40*x)/(x^4 + 1)
```

giac [A] time = 0.34, size = 83, normalized size = 0.80

$$-\frac{5}{16}\,\sqrt{2}\arctan\left(\frac{1}{2}\,\sqrt{2}\left(2\,x+\sqrt{2}\right)\right)-\frac{5}{16}\,\sqrt{2}\arctan\left(\frac{1}{2}\,\sqrt{2}\left(2\,x-\sqrt{2}\right)\right)-\frac{5}{32}\,\sqrt{2}\log\left(x^2+\sqrt{2}\,x+1\right)+\frac{5}{32}\,\sqrt{2}\log\left(x^2-\sqrt{2}\,x+1\right)+x+\frac{x}{4\left(x^4+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(x^8+2*x^4+1),x, algorithm="giac")
```

```
[Out] -5/16*sqrt(2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2))) - 5/16*sqrt(2)*arctan(1/2
*sqrt(2)*(2*x - sqrt(2))) - 5/32*sqrt(2)*log(x^2 + sqrt(2)*x + 1) + 5/32*sq
rt(2)*log(x^2 - sqrt(2)*x + 1) + x + 1/4*x/(x^4 + 1)
```

maple [A] time = 0.01, size = 69, normalized size = 0.66

$$x+\frac{x}{4x^4+4}-\frac{5\sqrt{2}\,\arctan\left(\sqrt{2}\,x-1\right)}{16}-\frac{5\sqrt{2}\,\arctan\left(\sqrt{2}\,x+1\right)}{16}-\frac{5\sqrt{2}\,\ln\left(\frac{x^2+\sqrt{2}\,x+1}{x^2-\sqrt{2}\,x+1}\right)}{32}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(x^8+2*x^4+1),x)
```

```
[Out] x+1/4*x/(x^4+1)-5/32*2^(1/2)*ln((1+x^2+2^(1/2)*x)/(1+x^2-2^(1/2)*x))-5/16*a
rctan(-1+2^(1/2)*x)*2^(1/2)-5/16*arctan(1+2^(1/2)*x)*2^(1/2)
```

maxima [A] time = 2.02, size = 83, normalized size = 0.80

$$-\frac{5}{16}\,\sqrt{2}\arctan\left(\frac{1}{2}\,\sqrt{2}\left(2\,x+\sqrt{2}\right)\right)-\frac{5}{16}\,\sqrt{2}\arctan\left(\frac{1}{2}\,\sqrt{2}\left(2\,x-\sqrt{2}\right)\right)-\frac{5}{32}\,\sqrt{2}\log\left(x^2+\sqrt{2}\,x+1\right)+\frac{5}{32}\,\sqrt{2}\log\left(x^2-\sqrt{2}\,x+1\right)+x+\frac{x}{4\left(x^4+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(x^8+2*x^4+1),x, algorithm="maxima")
```

```
[Out] -5/16*sqrt(2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2))) - 5/16*sqrt(2)*arctan(1/2
*sqrt(2)*(2*x - sqrt(2))) - 5/32*sqrt(2)*log(x^2 + sqrt(2)*x + 1) + 5/32*sq
rt(2)*log(x^2 - sqrt(2)*x + 1) + x + 1/4*x/(x^4 + 1)
```

mupad [B] time = 1.37, size = 45, normalized size = 0.43

$$x+\frac{x}{4\left(x^4+1\right)}+\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,x\,\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)\right)\,\left(-\frac{5}{16}-\frac{5}{16}\mathrm{i}\right)+\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,x\,\left(\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)\right)\,\left(-\frac{5}{16}+\frac{5}{16}\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(2*x^4 + x^8 + 1),x)
```

```
[Out] x - 2^(1/2)*atan(2^(1/2)*x*(1/2 - 1i/2))*(5/16 + 5i/16) - 2^(1/2)*atan(2^(1
/2)*x*(1/2 + 1i/2))*(5/16 - 5i/16) + x/(4*(x^4 + 1))
```

sympy [A] time = 0.18, size = 90, normalized size = 0.87

$$x+\frac{x}{4x^4+4}+\frac{5\sqrt{2}\log\left(x^2-\sqrt{2}\,x+1\right)}{32}-\frac{5\sqrt{2}\log\left(x^2+\sqrt{2}\,x+1\right)}{32}-\frac{5\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,x-1\right)}{16}-\frac{5\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,x+1\right)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8/(x**8+2*x**4+1),x)
```

```
[Out] x + x/(4*x**4 + 4) + 5*sqrt(2)*log(x**2 - sqrt(2)*x + 1)/32 - 5*sqrt(2)*log
(x**2 + sqrt(2)*x + 1)/32 - 5*sqrt(2)*atan(sqrt(2)*x - 1)/16 - 5*sqrt(2)*at
an(sqrt(2)*x + 1)/16
```

3.227 $\int \frac{x^6}{1+2x^4+x^8}\,dx$

Optimal. Leaf size=99

$$\frac{3\log\left(x^2-\sqrt{2}x+1\right)}{16\sqrt{2}}-\frac{3\log\left(x^2+\sqrt{2}x+1\right)}{16\sqrt{2}}-\frac{x^3}{4\left(x^4+1\right)}-\frac{3\tan^{-1}\left(1-\sqrt{2}x\right)}{8\sqrt{2}}+\frac{3\tan^{-1}\left(\sqrt{2}x+1\right)}{8\sqrt{2}}$$

Rubi [A] time = 0.05, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {28, 288, 297, 1162, 617, 204, 1165, 628}

$$-\frac{x^3}{4\left(x^4+1\right)}+\frac{3\log\left(x^2-\sqrt{2}x+1\right)}{16\sqrt{2}}-\frac{3\log\left(x^2+\sqrt{2}x+1\right)}{16\sqrt{2}}-\frac{3\tan^{-1}\left(1-\sqrt{2}x\right)}{8\sqrt{2}}+\frac{3\tan^{-1}\left(\sqrt{2}x+1\right)}{8\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^6/(1 + 2*x^4 + x^8),x]
```

```
[Out] -x^3/(4*(1 + x^4)) - (3*ArcTan[1 - Sqrt[2]*x])/(8*Sqrt[2]) + (3*ArcTan[1 +
Sqrt[2]*x])/(8*Sqrt[2]) + (3*Log[1 - Sqrt[2]*x + x^2])/(16*Sqrt[2]) - (3*Lo
g[1 + Sqrt[2]*x + x^2])/(16*Sqrt[2])
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{x^6}{1+2x^4+x^8}\,dx = \int \frac{x^6}{\left(1+x^4\right)^2}\,dx$$

$$= -\frac{x^3}{4\left(1+x^4\right)} + \frac{3}{4}\int \frac{x^2}{1+x^4}\,dx$$

$$= -\frac{x^3}{4\left(1+x^4\right)} - \frac{3}{8}\int \frac{1-x^2}{1+x^4}\,dx + \frac{3}{8}\int \frac{1+x^2}{1+x^4}\,dx$$

$$= -\frac{x^3}{4\left(1+x^4\right)} + \frac{3}{16}\int \frac{1}{1-\sqrt{2}x+x^2}\,dx + \frac{3}{16}\int \frac{1}{1+\sqrt{2}x+x^2}\,dx + \frac{3\int \frac{\sqrt{2}+2x}{-1-\sqrt{2}x-x^2}\,dx}{16\sqrt{2}} + \frac{3}{}$$

$$= -\frac{x^3}{4\left(1+x^4\right)} + \frac{3\log\left(1-\sqrt{2}x+x^2\right)}{16\sqrt{2}} - \frac{3\log\left(1+\sqrt{2}x+x^2\right)}{16\sqrt{2}} + \frac{3\,\mathrm{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1\right.}{8\sqrt{2}}$$

$$= -\frac{x^3}{4\left(1+x^4\right)} - \frac{3\tan^{-1}\left(1-\sqrt{2}x\right)}{8\sqrt{2}} + \frac{3\tan^{-1}\left(1+\sqrt{2}x\right)}{8\sqrt{2}} + \frac{3\log\left(1-\sqrt{2}x+x^2\right)}{16\sqrt{2}} - \frac{3\log(}{}$$

Mathematica [A] time = 0.06, size = 93, normalized size = 0.94

$$\frac{1}{32}\left(3\sqrt{2}\log\left(x^2-\sqrt{2}x+1\right)-3\sqrt{2}\log\left(x^2+\sqrt{2}x+1\right)-\frac{8x^3}{x^4+1}-6\sqrt{2}\tan^{-1}\left(1-\sqrt{2}x\right)+6\sqrt{2}\tan^{-1}\left(\sqrt{2}x+1\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6/(1 + 2*x^4 + x^8),x]
```

```
[Out] ((-8*x^3)/(1 + x^4) - 6*Sqrt[2]*ArcTan[1 - Sqrt[2]*x] + 6*Sqrt[2]*ArcTan[1
+ Sqrt[2]*x] + 3*Sqrt[2]*Log[1 - Sqrt[2]*x + x^2] - 3*Sqrt[2]*Log[1 + Sqrt[
2]*x + x^2])/32
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6}{1+2x^4+x^8}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^6/(1 + 2*x^4 + x^8),x]
```

```
[Out] IntegrateAlgebraic[x^6/(1 + 2*x^4 + x^8), x]
```

fricas [A] time = 1.37, size = 129, normalized size = 1.30

$$-\frac{8 x^{3} + 12 \sqrt{2} \left(x^{4} + 1\right) \arctan\left(-\sqrt{2} x + \sqrt{2} \sqrt{x^{2} + \sqrt{2} x + 1} - 1\right) + 12 \sqrt{2} \left(x^{4} + 1\right) \arctan\left(-\sqrt{2} x + \sqrt{2} \sqrt{x^{2} - \sqrt{2} x + 1} + 1\right) + 3 \sqrt{2} \left(x^{4} + 1\right) \log\left(x^{2} + \sqrt{2} x + 1\right) - 3 \sqrt{2} \left(x^{4} + 1\right) \log\left(x^{2} - \sqrt{2} x + 1\right)}{32 \left(x^{4} + 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(x^8+2*x^4+1),x, algorithm="fricas")
```

```
[Out] -1/32*(8*x^3 + 12*sqrt(2)*(x^4 + 1)*arctan(-sqrt(2)*x + sqrt(2)*sqrt(x^2 +
sqrt(2)*x + 1) - 1) + 12*sqrt(2)*(x^4 + 1)*arctan(-sqrt(2)*x + sqrt(2)*sqrt
(x^2 - sqrt(2)*x + 1) + 1) + 3*sqrt(2)*(x^4 + 1)*log(x^2 + sqrt(2)*x + 1) -
 3*sqrt(2)*(x^4 + 1)*log(x^2 - sqrt(2)*x + 1))/(x^4 + 1)
```

giac [A] time = 0.29, size = 84, normalized size = 0.85

$$-\frac{x^{3}}{4 \left(x^{4} + 1\right)} + \frac{3}{16} \sqrt{2} \arctan\left(\frac{1}{2} \sqrt{2} \left(2 x + \sqrt{2}\right)\right) + \frac{3}{16} \sqrt{2} \arctan\left(\frac{1}{2} \sqrt{2} \left(2 x - \sqrt{2}\right)\right) - \frac{3}{32} \sqrt{2} \log\left(x^{2} + \sqrt{2} x + 1\right) + \frac{3}{32} \sqrt{2} \log\left(x^{2} - \sqrt{2} x + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(x^8+2*x^4+1),x, algorithm="giac")
```

```
[Out] -1/4*x^3/(x^4 + 1) + 3/16*sqrt(2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2))) + 3/1
6*sqrt(2)*arctan(1/2*sqrt(2)*(2*x - sqrt(2))) - 3/32*sqrt(2)*log(x^2 + sqrt
(2)*x + 1) + 3/32*sqrt(2)*log(x^2 - sqrt(2)*x + 1)
```

maple [A] time = 0.01, size = 70, normalized size = 0.71

$$-\frac{x^{3}}{4 \left(x^{4} + 1\right)} + \frac{3 \sqrt{2}\, \arctan\left(\sqrt{2}\, x - 1\right)}{16} + \frac{3 \sqrt{2}\, \arctan\left(\sqrt{2}\, x + 1\right)}{16} + \frac{3 \sqrt{2}\, \ln\left(\frac{x^{2} - \sqrt{2}\, x + 1}{x^{2} + \sqrt{2}\, x + 1}\right)}{32}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(x^8+2*x^4+1),x)
```

```
[Out] -1/4*x^3/(x^4+1)+3/16*2^(1/2)*arctan(2^(1/2)*x-1)+3/32*2^(1/2)*ln((x^2-2^(1
/2)*x+1)/(x^2+2^(1/2)*x+1))+3/16*2^(1/2)*arctan(2^(1/2)*x+1)
```

maxima [A] time = 2.07, size = 84, normalized size = 0.85

$$-\frac{x^{3}}{4 \left(x^{4} + 1\right)} + \frac{3}{16} \sqrt{2} \arctan\left(\frac{1}{2} \sqrt{2} \left(2 x + \sqrt{2}\right)\right) + \frac{3}{16} \sqrt{2} \arctan\left(\frac{1}{2} \sqrt{2} \left(2 x - \sqrt{2}\right)\right) - \frac{3}{32} \sqrt{2} \log\left(x^{2} + \sqrt{2} x + 1\right) + \frac{3}{32} \sqrt{2} \log\left(x^{2} - \sqrt{2} x + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(x^8+2*x^4+1),x, algorithm="maxima")
```

```
[Out] -1/4*x^3/(x^4 + 1) + 3/16*sqrt(2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2))) + 3/1
6*sqrt(2)*arctan(1/2*sqrt(2)*(2*x - sqrt(2))) - 3/32*sqrt(2)*log(x^2 + sqrt
(2)*x + 1) + 3/32*sqrt(2)*log(x^2 - sqrt(2)*x + 1)
```

mupad [B] time = 1.33, size = 47, normalized size = 0.47

$$-\frac{x^{3}}{4 \left(x^{4} + 1\right)} + \sqrt{2}\, \mathrm{atan}\left(\sqrt{2}\, x \left(\frac{1}{2} - \frac{1}{2}\mathrm{i}\right)\right) \left(\frac{3}{16} - \frac{3}{16}\mathrm{i}\right) + \sqrt{2}\, \mathrm{atan}\left(\sqrt{2}\, x \left(\frac{1}{2} + \frac{1}{2}\mathrm{i}\right)\right) \left(\frac{3}{16} + \frac{3}{16}\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(2*x^4 + x^8 + 1),x)
```

```
[Out] 2^(1/2)*atan(2^(1/2)*x*(1/2 - 1i/2))*(3/16 - 3i/16) + 2^(1/2)*atan(2^(1/2)*
x*(1/2 + 1i/2))*(3/16 + 3i/16) - x^3/(4*(x^4 + 1))
```

sympy [A] time = 0.18, size = 90, normalized size = 0.91

$$-\frac{x^3}{4x^4+4}+\frac{3\sqrt{2}\log\left(x^2-\sqrt{2}\,x+1\right)}{32}-\frac{3\sqrt{2}\log\left(x^2+\sqrt{2}\,x+1\right)}{32}+\frac{3\sqrt{2}\operatorname{atan}\left(\sqrt{2}\,x-1\right)}{16}+\frac{3\sqrt{2}\operatorname{atan}\left(\sqrt{2}\,x+1\right)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(x**8+2*x**4+1),x)
```

```
[Out] -x**3/(4*x**4 + 4) + 3*sqrt(2)*log(x**2 - sqrt(2)*x + 1)/32 - 3*sqrt(2)*log
(x**2 + sqrt(2)*x + 1)/32 + 3*sqrt(2)*atan(sqrt(2)*x - 1)/16 + 3*sqrt(2)*at
an(sqrt(2)*x + 1)/16
```

3.228 $\int \frac{x^4}{1+2x^4+x^8}\,dx$

Optimal. Leaf size=97

$$-\frac{x}{4\left(x^4+1\right)}-\frac{\log\left(x^2-\sqrt{2}\,x+1\right)}{16\sqrt{2}}+\frac{\log\left(x^2+\sqrt{2}\,x+1\right)}{16\sqrt{2}}-\frac{\tan^{-1}\left(1-\sqrt{2}\,x\right)}{8\sqrt{2}}+\frac{\tan^{-1}\left(\sqrt{2}\,x+1\right)}{8\sqrt{2}}$$

Rubi [A] time = 0.05, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {28, 288, 211, 1165, 628, 1162, 617, 204}

$$-\frac{x}{4\left(x^4+1\right)}-\frac{\log\left(x^2-\sqrt{2}\,x+1\right)}{16\sqrt{2}}+\frac{\log\left(x^2+\sqrt{2}\,x+1\right)}{16\sqrt{2}}-\frac{\tan^{-1}\left(1-\sqrt{2}\,x\right)}{8\sqrt{2}}+\frac{\tan^{-1}\left(\sqrt{2}\,x+1\right)}{8\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(1 + 2*x^4 + x^8),x]
```

```
[Out] -x/(4*(1 + x^4)) - ArcTan[1 - Sqrt[2]*x]/(8*Sqrt[2]) + ArcTan[1 + Sqrt[2]*x
]/(8*Sqrt[2]) - Log[1 - Sqrt[2]*x + x^2]/(16*Sqrt[2]) + Log[1 + Sqrt[2]*x +
 x^2]/(16*Sqrt[2])
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{x^4}{1+2x^4+x^8}\,dx = \int \frac{x^4}{\left(1+x^4\right)^2}\,dx$$

$$= -\frac{x}{4\left(1+x^4\right)} + \frac{1}{4}\int \frac{1}{1+x^4}\,dx$$

$$= -\frac{x}{4\left(1+x^4\right)} + \frac{1}{8}\int \frac{1-x^2}{1+x^4}\,dx + \frac{1}{8}\int \frac{1+x^2}{1+x^4}\,dx$$

$$= -\frac{x}{4\left(1+x^4\right)} + \frac{1}{16}\int \frac{1}{1-\sqrt{2}x+x^2}\,dx + \frac{1}{16}\int \frac{1}{1+\sqrt{2}x+x^2}\,dx - \frac{\int \frac{\sqrt{2}+2x}{-1-\sqrt{2}x-x^2}\,dx}{16\sqrt{2}} - \frac{\int \ldots$$

$$= -\frac{x}{4\left(1+x^4\right)} - \frac{\log\left(1-\sqrt{2}x+x^2\right)}{16\sqrt{2}} + \frac{\log\left(1+\sqrt{2}x+x^2\right)}{16\sqrt{2}} + \frac{\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\sqrt{2}\ldots\right)}{8\sqrt{2}}$$

$$= -\frac{x}{4\left(1+x^4\right)} - \frac{\tan^{-1}\left(1-\sqrt{2}x\right)}{8\sqrt{2}} + \frac{\tan^{-1}\left(1+\sqrt{2}x\right)}{8\sqrt{2}} - \frac{\log\left(1-\sqrt{2}x+x^2\right)}{16\sqrt{2}} + \frac{\log\left(1+\sqrt{\ldots}\right.}{16\sqrt{\ldots}}$$

Mathematica [A] time = 0.06, size = 90, normalized size = 0.93

$$\frac{1}{32}\left(-\frac{8x}{x^4+1} - \sqrt{2}\log\left(x^2-\sqrt{2}x+1\right) + \sqrt{2}\log\left(x^2+\sqrt{2}x+1\right) - 2\sqrt{2}\tan^{-1}\left(1-\sqrt{2}x\right) + 2\sqrt{2}\tan^{-1}\left(\sqrt{2}x+1\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(1 + 2*x^4 + x^8),x]
```

```
[Out] ((-8*x)/(1 + x^4) - 2*Sqrt[2]*ArcTan[1 - Sqrt[2]*x] + 2*Sqrt[2]*ArcTan[1 +
Sqrt[2]*x] - Sqrt[2]*Log[1 - Sqrt[2]*x + x^2] + Sqrt[2]*Log[1 + Sqrt[2]*x +
 x^2])/32
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{1+2x^4+x^8}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^4/(1 + 2*x^4 + x^8),x]
```

```
[Out] IntegrateAlgebraic[x^4/(1 + 2*x^4 + x^8), x]
```

fricas [A] time = 1.30, size = 126, normalized size = 1.30

$$-\frac{4\sqrt{2}\left(x^4+1\right)\arctan\left(-\sqrt{2}x+\sqrt{2}\sqrt{x^2+\sqrt{2}x+1}-1\right)+4\sqrt{2}\left(x^4+1\right)\arctan\left(-\sqrt{2}x+\sqrt{2}\sqrt{x^2-\sqrt{2}x+1}+1\right)-\sqrt{2}\left(x^4+1\right)\log\left(x^2+\sqrt{2}x+1\right)+\sqrt{2}\left(x^4+1\right)\log\left(x^2-\sqrt{2}x+1\right)+8x}{32\left(x^4+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(x^8+2*x^4+1),x, algorithm="fricas")
```

```
[Out] -1/32*(4*sqrt(2)*(x^4 + 1)*arctan(-sqrt(2)*x + sqrt(2)*sqrt(x^2 + sqrt(2)*x
 + 1) - 1) + 4*sqrt(2)*(x^4 + 1)*arctan(-sqrt(2)*x + sqrt(2)*sqrt(x^2 - sqr
t(2)*x + 1) + 1) - sqrt(2)*(x^4 + 1)*log(x^2 + sqrt(2)*x + 1) + sqrt(2)*(x^
4 + 1)*log(x^2 - sqrt(2)*x + 1) + 8*x)/(x^4 + 1)
```

giac [A] time = 0.36, size = 82, normalized size = 0.85

$$\frac{1}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x+\sqrt{2}\right)\right)+\frac{1}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x-\sqrt{2}\right)\right)+\frac{1}{32}\sqrt{2}\log\left(x^2+\sqrt{2}x+1\right)-\frac{1}{32}\sqrt{2}\log\left(x^2-\sqrt{2}x+1\right)-\frac{x}{4\left(x^4+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(x^8+2*x^4+1),x, algorithm="giac")
```

```
[Out] 1/16*sqrt(2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2))) + 1/16*sqrt(2)*arctan(1/2*
sqrt(2)*(2*x - sqrt(2))) + 1/32*sqrt(2)*log(x^2 + sqrt(2)*x + 1) - 1/32*sqr
t(2)*log(x^2 - sqrt(2)*x + 1) - 1/4*x/(x^4 + 1)
```

maple [A] time = 0.01, size = 68, normalized size = 0.70

$$-\frac{x}{4\left(x^4+1\right)}+\frac{\sqrt{2}\,\arctan\left(\sqrt{2}\,x-1\right)}{16}+\frac{\sqrt{2}\,\arctan\left(\sqrt{2}\,x+1\right)}{16}+\frac{\sqrt{2}\,\ln\left(\frac{x^2+\sqrt{2}\,x+1}{x^2-\sqrt{2}\,x+1}\right)}{32}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(x^8+2*x^4+1),x)
```

```
[Out] -1/4/(x^4+1)*x+1/32*2^(1/2)*ln((x^2+2^(1/2)*x+1)/(x^2-2^(1/2)*x+1))+1/16*2^
(1/2)*arctan(2^(1/2)*x-1)+1/16*2^(1/2)*arctan(2^(1/2)*x+1)
```

maxima [A] time = 1.99, size = 82, normalized size = 0.85

$$\frac{1}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x+\sqrt{2}\right)\right)+\frac{1}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x-\sqrt{2}\right)\right)+\frac{1}{32}\sqrt{2}\log\left(x^2+\sqrt{2}x+1\right)-\frac{1}{32}\sqrt{2}\log\left(x^2-\sqrt{2}x+1\right)-\frac{x}{4\left(x^4+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(x^8+2*x^4+1),x, algorithm="maxima")
```

```
[Out] 1/16*sqrt(2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2))) + 1/16*sqrt(2)*arctan(1/2*
sqrt(2)*(2*x - sqrt(2))) + 1/32*sqrt(2)*log(x^2 + sqrt(2)*x + 1) - 1/32*sqr
t(2)*log(x^2 - sqrt(2)*x + 1) - 1/4*x/(x^4 + 1)
```

mupad [B] time = 0.08, size = 45, normalized size = 0.46

$$-\frac{x}{4\left(x^4+1\right)}+\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,x\,\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)\right)\,\left(\frac{1}{16}+\frac{1}{16}\mathrm{i}\right)+\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,x\,\left(\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)\right)\,\left(\frac{1}{16}-\frac{1}{16}\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(2*x^4 + x^8 + 1),x)
```

```
[Out] 2^(1/2)*atan(2^(1/2)*x*(1/2 - 1i/2))*(1/16 + 1i/16) + 2^(1/2)*atan(2^(1/2)*
x*(1/2 + 1i/2))*(1/16 - 1i/16) - x/(4*(x^4 + 1))
```

sympy [A] time = 0.17, size = 82, normalized size = 0.85

$$-\frac{x}{4x^4+4}-\frac{\sqrt{2}\log\left(x^2-\sqrt{2}\,x+1\right)}{32}+\frac{\sqrt{2}\log\left(x^2+\sqrt{2}\,x+1\right)}{32}+\frac{\sqrt{2}\operatorname{atan}\left(\sqrt{2}\,x-1\right)}{16}+\frac{\sqrt{2}\operatorname{atan}\left(\sqrt{2}\,x+1\right)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(x**8+2*x**4+1),x)
```

```
[Out] -x/(4*x**4 + 4) - sqrt(2)*log(x**2 - sqrt(2)*x + 1)/32 + sqrt(2)*log(x**2 +
 sqrt(2)*x + 1)/32 + sqrt(2)*atan(sqrt(2)*x - 1)/16 + sqrt(2)*atan(sqrt(2)*
x + 1)/16
```

3.229 $\int \frac{x^2}{1+2x^4+x^8}\,dx$

Optimal. Leaf size=99

$$\frac{\log\left(x^2-\sqrt{2}\,x+1\right)}{16\sqrt{2}}-\frac{\log\left(x^2+\sqrt{2}\,x+1\right)}{16\sqrt{2}}+\frac{x^3}{4\left(x^4+1\right)}-\frac{\tan^{-1}\left(1-\sqrt{2}\,x\right)}{8\sqrt{2}}+\frac{\tan^{-1}\left(\sqrt{2}\,x+1\right)}{8\sqrt{2}}$$

Rubi [A] time = 0.05, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {28, 290, 297, 1162, 617, 204, 1165, 628}

$$\frac{x^3}{4\left(x^4+1\right)}+\frac{\log\left(x^2-\sqrt{2}\,x+1\right)}{16\sqrt{2}}-\frac{\log\left(x^2+\sqrt{2}\,x+1\right)}{16\sqrt{2}}-\frac{\tan^{-1}\left(1-\sqrt{2}\,x\right)}{8\sqrt{2}}+\frac{\tan^{-1}\left(\sqrt{2}\,x+1\right)}{8\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(1 + 2*x^4 + x^8),x]
```

```
[Out] x^3/(4*(1 + x^4)) - ArcTan[1 - Sqrt[2]*x]/(8*Sqrt[2]) + ArcTan[1 + Sqrt[2]*
x]/(8*Sqrt[2]) + Log[1 - Sqrt[2]*x + x^2]/(16*Sqrt[2]) - Log[1 + Sqrt[2]*x
+ x^2]/(16*Sqrt[2])
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2}{1+2x^4+x^8}\,dx &= \int \frac{x^2}{\left(1+x^4\right)^2}\,dx \\
&= \frac{x^3}{4\left(1+x^4\right)} + \frac{1}{4}\int \frac{x^2}{1+x^4}\,dx \\
&= \frac{x^3}{4\left(1+x^4\right)} - \frac{1}{8}\int \frac{1-x^2}{1+x^4}\,dx + \frac{1}{8}\int \frac{1+x^2}{1+x^4}\,dx \\
&= \frac{x^3}{4\left(1+x^4\right)} + \frac{1}{16}\int \frac{1}{1-\sqrt{2}x+x^2}\,dx + \frac{1}{16}\int \frac{1}{1+\sqrt{2}x+x^2}\,dx + \frac{\int \frac{\sqrt{2}+2x}{-1-\sqrt{2}x-x^2}\,dx}{16\sqrt{2}} + \frac{\int \frac{}{-1}}{} \\
&= \frac{x^3}{4\left(1+x^4\right)} + \frac{\log\left(1-\sqrt{2}x+x^2\right)}{16\sqrt{2}} - \frac{\log\left(1+\sqrt{2}x+x^2\right)}{16\sqrt{2}} + \frac{\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\sqrt{2}x\right.}{8\sqrt{2}} \\
&= \frac{x^3}{4\left(1+x^4\right)} - \frac{\tan^{-1}\left(1-\sqrt{2}x\right)}{8\sqrt{2}} + \frac{\tan^{-1}\left(1+\sqrt{2}x\right)}{8\sqrt{2}} + \frac{\log\left(1-\sqrt{2}x+x^2\right)}{16\sqrt{2}} - \frac{\log\left(1+\sqrt{2}\right.}{16\sqrt{2}}
\end{aligned}$$

Mathematica [A] time = 0.05, size = 92, normalized size = 0.93

$$\frac{1}{32}\left(\sqrt{2}\log\left(x^2-\sqrt{2}x+1\right)-\sqrt{2}\log\left(x^2+\sqrt{2}x+1\right)+\frac{8x^3}{x^4+1}-2\sqrt{2}\tan^{-1}\left(1-\sqrt{2}x\right)+2\sqrt{2}\tan^{-1}\left(\sqrt{2}x+1\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(1 + 2*x^4 + x^8),x]
```

```
[Out] ((8*x^3)/(1 + x^4) - 2*Sqrt[2]*ArcTan[1 - Sqrt[2]*x] + 2*Sqrt[2]*ArcTan[1 +
 Sqrt[2]*x] + Sqrt[2]*Log[1 - Sqrt[2]*x + x^2] - Sqrt[2]*Log[1 + Sqrt[2]*x
+ x^2])/32
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{1+2x^4+x^8}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^2/(1 + 2*x^4 + x^8),x]
```

```
[Out] IntegrateAlgebraic[x^2/(1 + 2*x^4 + x^8), x]
```

fricas [A] time = 1.11, size = 128, normalized size = 1.29

$$\frac{8\,x^{3} - 4\,\sqrt{2}\left(x^{4} + 1\right)\arctan\left(-\sqrt{2}\,x + \sqrt{2}\,\sqrt{x^{2} + \sqrt{2}\,x + 1} - 1\right) - 4\,\sqrt{2}\left(x^{4} + 1\right)\arctan\left(-\sqrt{2}\,x + \sqrt{2}\,\sqrt{x^{2} - \sqrt{2}\,x + 1} + 1\right) - \sqrt{2}\left(x^{4} + 1\right)\log\left(x^{2} + \sqrt{2}\,x + 1\right) + \sqrt{2}\left(x^{4} + 1\right)\log\left(x^{2} - \sqrt{2}\,x + 1\right)}{32\left(x^{4} + 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^8+2*x^4+1),x, algorithm="fricas")
```

```
[Out] 1/32*(8*x^3 - 4*sqrt(2)*(x^4 + 1)*arctan(-sqrt(2)*x + sqrt(2)*sqrt(x^2 + sq
rt(2)*x + 1) - 1) - 4*sqrt(2)*(x^4 + 1)*arctan(-sqrt(2)*x + sqrt(2)*sqrt(x^
2 - sqrt(2)*x + 1) + 1) - sqrt(2)*(x^4 + 1)*log(x^2 + sqrt(2)*x + 1) + sqrt
(2)*(x^4 + 1)*log(x^2 - sqrt(2)*x + 1))/(x^4 + 1)
```

giac [A] time = 0.35, size = 84, normalized size = 0.85

$$\frac{x^{3}}{4\left(x^{4} + 1\right)} + \frac{1}{16}\,\sqrt{2}\arctan\left(\frac{1}{2}\,\sqrt{2}\left(2\,x + \sqrt{2}\right)\right) + \frac{1}{16}\,\sqrt{2}\arctan\left(\frac{1}{2}\,\sqrt{2}\left(2\,x - \sqrt{2}\right)\right) - \frac{1}{32}\,\sqrt{2}\log\left(x^{2} + \sqrt{2}\,x + 1\right) + \frac{1}{32}\,\sqrt{2}\log\left(x^{2} - \sqrt{2}\,x + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^8+2*x^4+1),x, algorithm="giac")
```

```
[Out] 1/4*x^3/(x^4 + 1) + 1/16*sqrt(2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2))) + 1/16
*sqrt(2)*arctan(1/2*sqrt(2)*(2*x - sqrt(2))) - 1/32*sqrt(2)*log(x^2 + sqrt(
2)*x + 1) + 1/32*sqrt(2)*log(x^2 - sqrt(2)*x + 1)
```

maple [A] time = 0.01, size = 70, normalized size = 0.71

$$\frac{x^{3}}{4x^{4}+4}+\frac{\sqrt{2}\,\arctan\left(\sqrt{2}\,x -1\right)}{16}+\frac{\sqrt{2}\,\arctan\left(\sqrt{2}\,x +1\right)}{16}+\frac{\sqrt{2}\,\ln\left(\frac{x^{2}-\sqrt{2}\,x +1}{x^{2}+\sqrt{2}\,x +1}\right)}{32}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(x^8+2*x^4+1),x)
```

```
[Out] 1/4/(x^4+1)*x^3+1/16*2^(1/2)*arctan(2^(1/2)*x-1)+1/32*2^(1/2)*ln((x^2-2^(1/
2)*x+1)/(x^2+2^(1/2)*x+1))+1/16*2^(1/2)*arctan(2^(1/2)*x+1)
```

maxima [A] time = 2.06, size = 84, normalized size = 0.85

$$\frac{x^{3}}{4\left(x^{4} + 1\right)} + \frac{1}{16}\,\sqrt{2}\arctan\left(\frac{1}{2}\,\sqrt{2}\left(2\,x + \sqrt{2}\right)\right) + \frac{1}{16}\,\sqrt{2}\arctan\left(\frac{1}{2}\,\sqrt{2}\left(2\,x - \sqrt{2}\right)\right) - \frac{1}{32}\,\sqrt{2}\log\left(x^{2} + \sqrt{2}\,x + 1\right) + \frac{1}{32}\,\sqrt{2}\log\left(x^{2} - \sqrt{2}\,x + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^8+2*x^4+1),x, algorithm="maxima")
```

```
[Out] 1/4*x^3/(x^4 + 1) + 1/16*sqrt(2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2))) + 1/16
*sqrt(2)*arctan(1/2*sqrt(2)*(2*x - sqrt(2))) - 1/32*sqrt(2)*log(x^2 + sqrt(
2)*x + 1) + 1/32*sqrt(2)*log(x^2 - sqrt(2)*x + 1)
```

mupad [B] time = 0.05, size = 46, normalized size = 0.46

$$\frac{x^3}{4\,\left(x^4+1\right)}+\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,x\,\left(\frac{1}{2}-\frac{1}{2}{}\mathrm{i}\right)\right)\,\left(\frac{1}{16}-\frac{1}{16}{}\mathrm{i}\right)+\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,x\,\left(\frac{1}{2}+\frac{1}{2}{}\mathrm{i}\right)\right)\,\left(\frac{1}{16}+\frac{1}{16}{}\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(2*x^4 + x^8 + 1),x)
```

```
[Out] 2^(1/2)*atan(2^(1/2)*x*(1/2 - 1i/2))*(1/16 - 1i/16) + 2^(1/2)*atan(2^(1/2)*
x*(1/2 + 1i/2))*(1/16 + 1i/16) + x^3/(4*(x^4 + 1))
```

sympy [A] time = 0.17, size = 83, normalized size = 0.84

$$\frac{x^3}{4x^4+4}+\frac{\sqrt{2}\log\left(x^2-\sqrt{2}\,x+1\right)}{32}-\frac{\sqrt{2}\log\left(x^2+\sqrt{2}\,x+1\right)}{32}+\frac{\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,x-1\right)}{16}+\frac{\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,x+1\right)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(x**8+2*x**4+1),x)
```

```
[Out] x**3/(4*x**4 + 4) + sqrt(2)*log(x**2 - sqrt(2)*x + 1)/32 - sqrt(2)*log(x**2
 + sqrt(2)*x + 1)/32 + sqrt(2)*atan(sqrt(2)*x - 1)/16 + sqrt(2)*atan(sqrt(2
)*x + 1)/16
```

3.230 $\int \frac{1}{1+2x^4+x^8} dx$

Optimal. Leaf size=97

$$\frac{x}{4(x^4+1)}-\frac{3\log\left(x^2-\sqrt{2}x+1\right)}{16\sqrt{2}}+\frac{3\log\left(x^2+\sqrt{2}x+1\right)}{16\sqrt{2}}-\frac{3\tan^{-1}\left(1-\sqrt{2}x\right)}{8\sqrt{2}}+\frac{3\tan^{-1}\left(\sqrt{2}x+1\right)}{8\sqrt{2}}$$

Rubi [A] time = 0.05, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {28, 199, 211, 1165, 628, 1162, 617, 204}

$$\frac{x}{4(x^4+1)}-\frac{3\log\left(x^2-\sqrt{2}x+1\right)}{16\sqrt{2}}+\frac{3\log\left(x^2+\sqrt{2}x+1\right)}{16\sqrt{2}}-\frac{3\tan^{-1}\left(1-\sqrt{2}x\right)}{8\sqrt{2}}+\frac{3\tan^{-1}\left(\sqrt{2}x+1\right)}{8\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 2*x^4 + x^8)^(-1),x]
```

```
[Out] x/(4*(1 + x^4)) - (3*ArcTan[1 - Sqrt[2]*x])/(8*Sqrt[2]) + (3*ArcTan[1 + Sqr
t[2]*x])/(8*Sqrt[2]) - (3*Log[1 - Sqrt[2]*x + x^2])/(16*Sqrt[2]) + (3*Log[1
 + Sqrt[2]*x + x^2])/(16*Sqrt[2])
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{1}{1+2x^4+x^8}\,dx = \int \frac{1}{\left(1+x^4\right)^2}\,dx$$

$$= \frac{x}{4\left(1+x^4\right)} + \frac{3}{4}\int \frac{1}{1+x^4}\,dx$$

$$= \frac{x}{4\left(1+x^4\right)} + \frac{3}{8}\int \frac{1-x^2}{1+x^4}\,dx + \frac{3}{8}\int \frac{1+x^2}{1+x^4}\,dx$$

$$= \frac{x}{4\left(1+x^4\right)} + \frac{3}{16}\int \frac{1}{1-\sqrt{2}\,x+x^2}\,dx + \frac{3}{16}\int \frac{1}{1+\sqrt{2}\,x+x^2}\,dx - \frac{3\int \frac{\sqrt{2}+2x}{-1-\sqrt{2}\,x-x^2}\,dx}{16\sqrt{2}} - \frac{3\int}{}$$

$$= \frac{x}{4\left(1+x^4\right)} - \frac{3\log\left(1-\sqrt{2}\,x+x^2\right)}{16\sqrt{2}} + \frac{3\log\left(1+\sqrt{2}\,x+x^2\right)}{16\sqrt{2}} + \frac{3\,\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\right.}{8\sqrt{2}}$$

$$= \frac{x}{4\left(1+x^4\right)} - \frac{3\tan^{-1}\left(1-\sqrt{2}\,x\right)}{8\sqrt{2}} + \frac{3\tan^{-1}\left(1+\sqrt{2}\,x\right)}{8\sqrt{2}} - \frac{3\log\left(1-\sqrt{2}\,x+x^2\right)}{16\sqrt{2}} + \frac{3\log\left(1\right.}{}$$

Mathematica [A] time = 0.05, size = 91, normalized size = 0.94

$$\frac{1}{32}\left(\frac{8x}{x^4+1} - 3\sqrt{2}\log\left(x^2-\sqrt{2}\,x+1\right) + 3\sqrt{2}\log\left(x^2+\sqrt{2}\,x+1\right) - 6\sqrt{2}\tan^{-1}\left(1-\sqrt{2}\,x\right) + 6\sqrt{2}\tan^{-1}\left(\sqrt{2}\,x+1\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 2*x^4 + x^8)^(-1),x]
```

```
[Out] ((8*x)/(1 + x^4) - 6*Sqrt[2]*ArcTan[1 - Sqrt[2]*x] + 6*Sqrt[2]*ArcTan[1 + S
qrt[2]*x] - 3*Sqrt[2]*Log[1 - Sqrt[2]*x + x^2] + 3*Sqrt[2]*Log[1 + Sqrt[2]*
x + x^2])/32
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{1+2x^4+x^8}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(1 + 2*x^4 + x^8)^(-1),x]
```

```
[Out] IntegrateAlgebraic[(1 + 2*x^4 + x^8)^(-1), x]
```

fricas [A] time = 1.13, size = 127, normalized size = 1.31

$$-\frac{12\sqrt{2}\left(x^4+1\right)\arctan\left(-\sqrt{2}x+\sqrt{2}\sqrt{x^2+\sqrt{2}x+1}-1\right)+12\sqrt{2}\left(x^4+1\right)\arctan\left(-\sqrt{2}x+\sqrt{2}\sqrt{x^2-\sqrt{2}x+1}+1\right)-3\sqrt{2}\left(x^4+1\right)\log\left(x^2+\sqrt{2}x+1\right)+3\sqrt{2}\left(x^4+1\right)\log\left(x^2-\sqrt{2}x+1\right)-8x}{32\left(x^4+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^8+2*x^4+1),x, algorithm="fricas")
```

```
[Out] -1/32*(12*sqrt(2)*(x^4 + 1)*arctan(-sqrt(2)*x + sqrt(2)*sqrt(x^2 + sqrt(2)*
x + 1) - 1) + 12*sqrt(2)*(x^4 + 1)*arctan(-sqrt(2)*x + sqrt(2)*sqrt(x^2 - s
qrt(2)*x + 1) + 1) - 3*sqrt(2)*(x^4 + 1)*log(x^2 + sqrt(2)*x + 1) + 3*sqrt(
2)*(x^4 + 1)*log(x^2 - sqrt(2)*x + 1) - 8*x)/(x^4 + 1)
```

giac [A] time = 0.34, size = 82, normalized size = 0.85

$$\frac{3}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x+\sqrt{2}\right)\right)+\frac{3}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x-\sqrt{2}\right)\right)+\frac{3}{32}\sqrt{2}\log\left(x^2+\sqrt{2}x+1\right)-\frac{3}{32}\sqrt{2}\log\left(x^2-\sqrt{2}x+1\right)+\frac{x}{4\left(x^4+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^8+2*x^4+1),x, algorithm="giac")
```

```
[Out] 3/16*sqrt(2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2))) + 3/16*sqrt(2)*arctan(1/2*
sqrt(2)*(2*x - sqrt(2))) + 3/32*sqrt(2)*log(x^2 + sqrt(2)*x + 1) - 3/32*sqr
t(2)*log(x^2 - sqrt(2)*x + 1) + 1/4*x/(x^4 + 1)
```

maple [A] time = 0.00, size = 68, normalized size = 0.70

$$\frac{x}{4x^4+4}+\frac{3\sqrt{2}\ \arctan\left(\sqrt{2}\ x-1\right)}{16}+\frac{3\sqrt{2}\ \arctan\left(\sqrt{2}\ x+1\right)}{16}+\frac{3\sqrt{2}\ \ln\left(\frac{x^2+\sqrt{2}\ x+1}{x^2-\sqrt{2}\ x+1}\right)}{32}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^8+2*x^4+1),x)
```

```
[Out] 1/4/(x^4+1)*x+3/16*2^(1/2)*arctan(2^(1/2)*x-1)+3/32*2^(1/2)*ln((x^2+2^(1/2)
*x+1)/(x^2-2^(1/2)*x+1))+3/16*2^(1/2)*arctan(2^(1/2)*x+1)
```

maxima [A] time = 2.00, size = 82, normalized size = 0.85

$$\frac{3}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x+\sqrt{2}\right)\right)+\frac{3}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x-\sqrt{2}\right)\right)+\frac{3}{32}\sqrt{2}\log\left(x^2+\sqrt{2}x+1\right)-\frac{3}{32}\sqrt{2}\log\left(x^2-\sqrt{2}x+1\right)+\frac{x}{4\left(x^4+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^8+2*x^4+1),x, algorithm="maxima")
```

```
[Out] 3/16*sqrt(2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2))) + 3/16*sqrt(2)*arctan(1/2*
sqrt(2)*(2*x - sqrt(2))) + 3/32*sqrt(2)*log(x^2 + sqrt(2)*x + 1) - 3/32*sqr
t(2)*log(x^2 - sqrt(2)*x + 1) + 1/4*x/(x^4 + 1)
```

mupad [B] time = 1.31, size = 44, normalized size = 0.45

$$\frac{x}{4\left(x^4+1\right)}+\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,x\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)\right)\left(\frac{3}{16}+\frac{3}{16}\mathrm{i}\right)+\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,x\left(\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)\right)\left(\frac{3}{16}-\frac{3}{16}\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*x^4 + x^8 + 1),x)
```

```
[Out] 2^(1/2)*atan(2^(1/2)*x*(1/2 - 1i/2))*(3/16 + 3i/16) + 2^(1/2)*atan(2^(1/2)*
x*(1/2 + 1i/2))*(3/16 - 3i/16) + x/(4*(x^4 + 1))
```

sympy [A] time = 0.20, size = 88, normalized size = 0.91

$$\frac{x}{4x^4+4}-\frac{3\sqrt{2}\log\left(x^2-\sqrt{2}\,x+1\right)}{32}+\frac{3\sqrt{2}\log\left(x^2+\sqrt{2}\,x+1\right)}{32}+\frac{3\sqrt{2}\operatorname{atan}\left(\sqrt{2}\,x-1\right)}{16}+\frac{3\sqrt{2}\operatorname{atan}\left(\sqrt{2}\,x+1\right)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**8+2*x**4+1),x)
```

```
[Out] x/(4*x**4 + 4) - 3*sqrt(2)*log(x**2 - sqrt(2)*x + 1)/32 + 3*sqrt(2)*log(x**
2 + sqrt(2)*x + 1)/32 + 3*sqrt(2)*atan(sqrt(2)*x - 1)/16 + 3*sqrt(2)*atan(s
qrt(2)*x + 1)/16
```

3.231 $\int \frac{1}{x^2(1+2x^4+x^8)}\,dx$

Optimal. Leaf size=106

$$\frac{1}{4x\left(x^4+1\right)}-\frac{5\log\left(x^2-\sqrt{2}x+1\right)}{16\sqrt{2}}+\frac{5\log\left(x^2+\sqrt{2}x+1\right)}{16\sqrt{2}}-\frac{5}{4x}+\frac{5\tan^{-1}\left(1-\sqrt{2}x\right)}{8\sqrt{2}}-\frac{5\tan^{-1}\left(\sqrt{2}x+1\right)}{8\sqrt{2}}$$

Rubi [A] time = 0.05, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.562, Rules used = {28, 290, 325, 297, 1162, 617, 204, 1165, 628}

$$\frac{1}{4x\left(x^4+1\right)}-\frac{5\log\left(x^2-\sqrt{2}x+1\right)}{16\sqrt{2}}+\frac{5\log\left(x^2+\sqrt{2}x+1\right)}{16\sqrt{2}}-\frac{5}{4x}+\frac{5\tan^{-1}\left(1-\sqrt{2}x\right)}{8\sqrt{2}}-\frac{5\tan^{-1}\left(\sqrt{2}x+1\right)}{8\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(1 + 2*x^4 + x^8)),x]
```

```
[Out] -5/(4*x) + 1/(4*x*(1 + x^4)) + (5*ArcTan[1 - Sqrt[2]*x])/(8*Sqrt[2]) - (5*A
rcTan[1 + Sqrt[2]*x])/(8*Sqrt[2]) - (5*Log[1 - Sqrt[2]*x + x^2])/(16*Sqrt[2
]) + (5*Log[1 + Sqrt[2]*x + x^2])/(16*Sqrt[2])
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^2\left(1+2x^4+x^8\right)}\,dx &= \int \frac{1}{x^2\left(1+x^4\right)^2}\,dx\\
&= \frac{1}{4x\left(1+x^4\right)} + \frac{5}{4}\int \frac{1}{x^2\left(1+x^4\right)}\,dx\\
&= -\frac{5}{4x} + \frac{1}{4x\left(1+x^4\right)} - \frac{5}{4}\int \frac{x^2}{1+x^4}\,dx\\
&= -\frac{5}{4x} + \frac{1}{4x\left(1+x^4\right)} + \frac{5}{8}\int \frac{1-x^2}{1+x^4}\,dx - \frac{5}{8}\int \frac{1+x^2}{1+x^4}\,dx\\
&= -\frac{5}{4x} + \frac{1}{4x\left(1+x^4\right)} - \frac{5}{16}\int \frac{1}{1-\sqrt{2}\,x+x^2}\,dx - \frac{5}{16}\int \frac{1}{1+\sqrt{2}\,x+x^2}\,dx - \frac{5\int \frac{\sqrt{2}+}{-1-\sqrt{2}}}{16\sqrt{}}\\
&= -\frac{5}{4x} + \frac{1}{4x\left(1+x^4\right)} - \frac{5\log\left(1-\sqrt{2}\,x+x^2\right)}{16\sqrt{2}} + \frac{5\log\left(1+\sqrt{2}\,x+x^2\right)}{16\sqrt{2}} - \frac{5\,\mathrm{Subst}\left(\int \right.}{}\\
&= -\frac{5}{4x} + \frac{1}{4x\left(1+x^4\right)} + \frac{5\tan^{-1}\left(1-\sqrt{2}\,x\right)}{8\sqrt{2}} - \frac{5\tan^{-1}\left(1+\sqrt{2}\,x\right)}{8\sqrt{2}} - \frac{5\log\left(1-\sqrt{2}\,x+x\right.}{16\sqrt{2}}
\end{aligned}$$

Mathematica [A] time = 0.07, size = 98, normalized size = 0.92

$$\frac{1}{32}\left(-5\sqrt{2}\log\left(x^2-\sqrt{2}\,x+1\right)+5\sqrt{2}\log\left(x^2+\sqrt{2}\,x+1\right)-\frac{8x^3}{x^4+1}-\frac{32}{x}+10\sqrt{2}\tan^{-1}\left(1-\sqrt{2}\,x\right)-10\sqrt{2}\tan^{-1}\left(\sqrt{2}\,x+1\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(1 + 2*x^4 + x^8)),x]
```

```
[Out] (-32/x - (8*x^3)/(1 + x^4) + 10*Sqrt[2]*ArcTan[1 - Sqrt[2]*x] - 10*Sqrt[2]*
ArcTan[1 + Sqrt[2]*x] - 5*Sqrt[2]*Log[1 - Sqrt[2]*x + x^2] + 5*Sqrt[2]*Log[
1 + Sqrt[2]*x + x^2])/32
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2 \left(1 + 2x^4 + x^8\right)} \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x^2*(1 + 2*x^4 + x^8)),x]
```

```
[Out] IntegrateAlgebraic[1/(x^2*(1 + 2*x^4 + x^8)), x]
```

fricas [A] time = 1.28, size = 130, normalized size = 1.23

$$-\frac{40 x^4 - 20 \sqrt{2} \left(x^5 + x\right) \arctan\left(-\sqrt{2} x + \sqrt{2} \sqrt{x^2 + \sqrt{2} x + 1} - 1\right) - 20 \sqrt{2} \left(x^5 + x\right) \arctan\left(-\sqrt{2} x + \sqrt{2} \sqrt{x^2 - \sqrt{2} x + 1} + 1\right) - 5 \sqrt{2} \left(x^5 + x\right) \log\left(x^2 + \sqrt{2} x + 1\right) + 5 \sqrt{2} \left(x^5 + x\right) \log\left(x^2 - \sqrt{2} x + 1\right) + 32}{32 \left(x^5 + x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(x^8+2*x^4+1),x, algorithm="fricas")
```

```
[Out] -1/32*(40*x^4 - 20*sqrt(2)*(x^5 + x)*arctan(-sqrt(2)*x + sqrt(2)*sqrt(x^2 +
 sqrt(2)*x + 1) - 1) - 20*sqrt(2)*(x^5 + x)*arctan(-sqrt(2)*x + sqrt(2)*sqr
t(x^2 - sqrt(2)*x + 1) + 1) - 5*sqrt(2)*(x^5 + x)*log(x^2 + sqrt(2)*x + 1)
+ 5*sqrt(2)*(x^5 + x)*log(x^2 - sqrt(2)*x + 1) + 32)/(x^5 + x)
```

giac [A] time = 0.36, size = 88, normalized size = 0.83

$$-\frac{5}{16} \sqrt{2} \arctan\left(\frac{1}{2} \sqrt{2} \left(2 x + \sqrt{2}\right)\right) - \frac{5}{16} \sqrt{2} \arctan\left(\frac{1}{2} \sqrt{2} \left(2 x - \sqrt{2}\right)\right) + \frac{5}{32} \sqrt{2} \log\left(x^2 + \sqrt{2} x + 1\right) - \frac{5}{32} \sqrt{2} \log\left(x^2 - \sqrt{2} x + 1\right) - \frac{5 x^4 + 4}{4 \left(x^5 + x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(x^8+2*x^4+1),x, algorithm="giac")
```

```
[Out] -5/16*sqrt(2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2))) - 5/16*sqrt(2)*arctan(1/2
*sqrt(2)*(2*x - sqrt(2))) + 5/32*sqrt(2)*log(x^2 + sqrt(2)*x + 1) - 5/32*sq
rt(2)*log(x^2 - sqrt(2)*x + 1) - 1/4*(5*x^4 + 4)/(x^5 + x)
```

maple [A] time = 0.01, size = 75, normalized size = 0.71

$$-\frac{x^3}{4 \left(x^4 + 1\right)} - \frac{5\sqrt{2}\ \arctan\left(\sqrt{2}\ x - 1\right)}{16} - \frac{5\sqrt{2}\ \arctan\left(\sqrt{2}\ x + 1\right)}{16} - \frac{5\sqrt{2}\ \ln\left(\frac{x^2 - \sqrt{2}\ x + 1}{x^2 + \sqrt{2}\ x + 1}\right)}{32} - \frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(x^8+2*x^4+1),x)
```

```
[Out] -1/x-1/4/(x^4+1)*x^3-5/16*2^(1/2)*arctan(2^(1/2)*x-1)-5/32*2^(1/2)*ln((x^2-
2^(1/2)*x+1)/(x^2+2^(1/2)*x+1))-5/16*2^(1/2)*arctan(2^(1/2)*x+1)
```

maxima [A] time = 2.03, size = 88, normalized size = 0.83

$$-\frac{5}{16} \sqrt{2} \arctan\left(\frac{1}{2} \sqrt{2} \left(2 x + \sqrt{2}\right)\right) - \frac{5}{16} \sqrt{2} \arctan\left(\frac{1}{2} \sqrt{2} \left(2 x - \sqrt{2}\right)\right) + \frac{5}{32} \sqrt{2} \log\left(x^2 + \sqrt{2} x + 1\right) - \frac{5}{32} \sqrt{2} \log\left(x^2 - \sqrt{2} x + 1\right) - \frac{5 x^4 + 4}{4 \left(x^5 + x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(x^8+2*x^4+1),x, algorithm="maxima")
```

```
[Out] -5/16*sqrt(2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2))) - 5/16*sqrt(2)*arctan(1/2
*sqrt(2)*(2*x - sqrt(2))) + 5/32*sqrt(2)*log(x^2 + sqrt(2)*x + 1) - 5/32*sq
rt(2)*log(x^2 - sqrt(2)*x + 1) - 1/4*(5*x^4 + 4)/(x^5 + x)
```

mupad [B] time = 1.31, size = 49, normalized size = 0.46

$$-\frac{\frac{5x^4}{4}+1}{x^5+x}+\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,x\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)\right)\left(-\frac{5}{16}+\frac{5}{16}\mathrm{i}\right)+\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,x\left(\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)\right)\left(-\frac{5}{16}-\frac{5}{16}\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(2*x^4 + x^8 + 1)),x)
```

```
[Out] - ((5*x^4)/4 + 1)/(x + x^5) - 2^(1/2)*atan(2^(1/2)*x*(1/2 - 1i/2))*(5/16 -
5i/16) - 2^(1/2)*atan(2^(1/2)*x*(1/2 + 1i/2))*(5/16 + 5i/16)
```

sympy [A] time = 0.21, size = 97, normalized size = 0.92

$$\frac{-5x^4-4}{4x^5+4x}-\frac{5\sqrt{2}\log\left(x^2-\sqrt{2}x+1\right)}{32}+\frac{5\sqrt{2}\log\left(x^2+\sqrt{2}x+1\right)}{32}-\frac{5\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}x-1\right)}{16}-\frac{5\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}x+1\right)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(x**8+2*x**4+1),x)
```

```
[Out] (-5*x**4 - 4)/(4*x**5 + 4*x) - 5*sqrt(2)*log(x**2 - sqrt(2)*x + 1)/32 + 5*s
qrt(2)*log(x**2 + sqrt(2)*x + 1)/32 - 5*sqrt(2)*atan(sqrt(2)*x - 1)/16 - 5*
sqrt(2)*atan(sqrt(2)*x + 1)/16
```

3.232 $\int \frac{1}{x^4(1+2x^4+x^8)}\,dx$

Optimal. Leaf size=106

$$-\frac{7}{12x^3}+\frac{7\log\left(x^2-\sqrt{2}\,x+1\right)}{16\sqrt{2}}-\frac{7\log\left(x^2+\sqrt{2}\,x+1\right)}{16\sqrt{2}}+\frac{1}{4x^3\left(x^4+1\right)}+\frac{7\tan^{-1}\left(1-\sqrt{2}\,x\right)}{8\sqrt{2}}-\frac{7\tan^{-1}\left(\sqrt{2}\,x+1\right)}{8\sqrt{2}}$$

Rubi [A] time = 0.05, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.562, Rules used = {28, 290, 325, 211, 1165, 628, 1162, 617, 204}

$$\frac{1}{4x^3\left(x^4+1\right)}-\frac{7}{12x^3}+\frac{7\log\left(x^2-\sqrt{2}\,x+1\right)}{16\sqrt{2}}-\frac{7\log\left(x^2+\sqrt{2}\,x+1\right)}{16\sqrt{2}}+\frac{7\tan^{-1}\left(1-\sqrt{2}\,x\right)}{8\sqrt{2}}-\frac{7\tan^{-1}\left(\sqrt{2}\,x+1\right)}{8\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(1 + 2*x^4 + x^8)),x]
```

```
[Out] -7/(12*x^3) + 1/(4*x^3*(1 + x^4)) + (7*ArcTan[1 - Sqrt[2]*x])/(8*Sqrt[2]) -
 (7*ArcTan[1 + Sqrt[2]*x])/(8*Sqrt[2]) + (7*Log[1 - Sqrt[2]*x + x^2])/(16*S
qrt[2]) - (7*Log[1 + Sqrt[2]*x + x^2])/(16*Sqrt[2])
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x^4\left(1+2x^4+x^8\right)}\,dx &= \int \frac{1}{x^4\left(1+x^4\right)^2}\,dx\\
&= \frac{1}{4x^3\left(1+x^4\right)}+\frac{7}{4}\int \frac{1}{x^4\left(1+x^4\right)}\,dx\\
&= -\frac{7}{12x^3}+\frac{1}{4x^3\left(1+x^4\right)}-\frac{7}{4}\int \frac{1}{1+x^4}\,dx\\
&= -\frac{7}{12x^3}+\frac{1}{4x^3\left(1+x^4\right)}-\frac{7}{8}\int \frac{1-x^2}{1+x^4}\,dx-\frac{7}{8}\int \frac{1+x^2}{1+x^4}\,dx\\
&= -\frac{7}{12x^3}+\frac{1}{4x^3\left(1+x^4\right)}-\frac{7}{16}\int \frac{1}{1-\sqrt{2}\,x+x^2}\,dx-\frac{7}{16}\int \frac{1}{1+\sqrt{2}\,x+x^2}\,dx+\frac{7\int \frac{\cdots}{-1-\cdots}}{1\cdots}\\
&= -\frac{7}{12x^3}+\frac{1}{4x^3\left(1+x^4\right)}+\frac{7\log\left(1-\sqrt{2}\,x+x^2\right)}{16\sqrt{2}}-\frac{7\log\left(1+\sqrt{2}\,x+x^2\right)}{16\sqrt{2}}-\frac{7\,\mathrm{Subst}\left(\cdots\right.}{\cdots}\\
&= -\frac{7}{12x^3}+\frac{1}{4x^3\left(1+x^4\right)}+\frac{7\tan^{-1}\left(1-\sqrt{2}\,x\right)}{8\sqrt{2}}-\frac{7\tan^{-1}\left(1+\sqrt{2}\,x\right)}{8\sqrt{2}}+\frac{7\log\left(1-\sqrt{2}\,x\cdots\right.}{16\sqrt{2}}
\end{aligned}
$$

Mathematica [A] time = 0.07, size = 96, normalized size = 0.91

$$\frac{1}{96}\left(-\frac{24x}{x^4+1}-\frac{32}{x^3}+21\sqrt{2}\log\left(x^2-\sqrt{2}\,x+1\right)-21\sqrt{2}\log\left(x^2+\sqrt{2}\,x+1\right)+42\sqrt{2}\tan^{-1}\left(1-\sqrt{2}\,x\right)-42\sqrt{2}\tan^{-1}\left(\sqrt{2}\,x+1\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(1 + 2*x^4 + x^8)),x]
```

```
[Out] (-32/x^3 - (24*x)/(1 + x^4) + 42*Sqrt[2]*ArcTan[1 - Sqrt[2]*x] - 42*Sqrt[2]
*ArcTan[1 + Sqrt[2]*x] + 21*Sqrt[2]*Log[1 - Sqrt[2]*x + x^2] - 21*Sqrt[2]*L
og[1 + Sqrt[2]*x + x^2])/96
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^4 \left(1 + 2x^4 + x^8\right)}\, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x^4*(1 + 2*x^4 + x^8)),x]
```

```
[Out] IntegrateAlgebraic[1/(x^4*(1 + 2*x^4 + x^8)), x]
```

fricas [A] time = 1.31, size = 140, normalized size = 1.32

$$-\frac{56\,x^4 - 84\,\sqrt{2}\left(x^7 + x^3\right)\arctan\left(-\sqrt{2}\,x + \sqrt{2}\,\sqrt{x^2 + \sqrt{2}\,x + 1} - 1\right) - 84\,\sqrt{2}\left(x^7 + x^3\right)\arctan\left(-\sqrt{2}\,x + \sqrt{2}\,\sqrt{x^2 - \sqrt{2}\,x + 1} + 1\right) + 21\,\sqrt{2}\left(x^7 + x^3\right)\log\left(x^2 + \sqrt{2}\,x + 1\right) - 21\,\sqrt{2}\left(x^7 + x^3\right)\log\left(x^2 - \sqrt{2}\,x + 1\right) + 32}{96\left(x^7 + x^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(x^8+2*x^4+1),x, algorithm="fricas")
```

```
[Out] -1/96*(56*x^4 - 84*sqrt(2)*(x^7 + x^3)*arctan(-sqrt(2)*x + sqrt(2)*sqrt(x^2
 + sqrt(2)*x + 1) - 1) - 84*sqrt(2)*(x^7 + x^3)*arctan(-sqrt(2)*x + sqrt(2)
*sqrt(x^2 - sqrt(2)*x + 1) + 1) + 21*sqrt(2)*(x^7 + x^3)*log(x^2 + sqrt(2)*
x + 1) - 21*sqrt(2)*(x^7 + x^3)*log(x^2 - sqrt(2)*x + 1) + 32)/(x^7 + x^3)
```

giac [A] time = 0.44, size = 87, normalized size = 0.82

$$-\frac{7}{16}\,\sqrt{2}\arctan\left(\frac{1}{2}\,\sqrt{2}\left(2\,x + \sqrt{2}\right)\right) - \frac{7}{16}\,\sqrt{2}\arctan\left(\frac{1}{2}\,\sqrt{2}\left(2\,x - \sqrt{2}\right)\right) - \frac{7}{32}\,\sqrt{2}\log\left(x^2 + \sqrt{2}\,x + 1\right) + \frac{7}{32}\,\sqrt{2}\log\left(x^2 - \sqrt{2}\,x + 1\right) - \frac{x}{4\left(x^4 + 1\right)} - \frac{1}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(x^8+2*x^4+1),x, algorithm="giac")
```

```
[Out] -7/16*sqrt(2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2))) - 7/16*sqrt(2)*arctan(1/2
*sqrt(2)*(2*x - sqrt(2))) - 7/32*sqrt(2)*log(x^2 + sqrt(2)*x + 1) + 7/32*sq
rt(2)*log(x^2 - sqrt(2)*x + 1) - 1/4*x/(x^4 + 1) - 1/3/x^3
```

maple [A] time = 0.01, size = 73, normalized size = 0.69

$$-\frac{x}{4\left(x^4+1\right)} - \frac{7\sqrt{2}\ \arctan\left(\sqrt{2}\ x-1\right)}{16} - \frac{7\sqrt{2}\ \arctan\left(\sqrt{2}\ x+1\right)}{16} - \frac{7\sqrt{2}\ \ln\left(\frac{x^2+\sqrt{2}\ x+1}{x^2-\sqrt{2}\ x+1}\right)}{32} - \frac{1}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(x^8+2*x^4+1),x)
```

```
[Out] -1/3/x^3-1/4/(x^4+1)*x-7/32*2^(1/2)*ln((x^2+2^(1/2)*x+1)/(x^2-2^(1/2)*x+1))
-7/16*2^(1/2)*arctan(2^(1/2)*x+1)-7/16*2^(1/2)*arctan(2^(1/2)*x-1)
```

maxima [A] time = 2.20, size = 90, normalized size = 0.85

$$-\frac{7}{16}\,\sqrt{2}\arctan\left(\frac{1}{2}\,\sqrt{2}\left(2\,x + \sqrt{2}\right)\right) - \frac{7}{16}\,\sqrt{2}\arctan\left(\frac{1}{2}\,\sqrt{2}\left(2\,x - \sqrt{2}\right)\right) - \frac{7}{32}\,\sqrt{2}\log\left(x^2 + \sqrt{2}\,x + 1\right) + \frac{7}{32}\,\sqrt{2}\log\left(x^2 - \sqrt{2}\,x + 1\right) - \frac{7\,x^4 + 4}{12\left(x^7 + x^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(x^8+2*x^4+1),x, algorithm="maxima")
```

```
[Out] -7/16*sqrt(2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2))) - 7/16*sqrt(2)*arctan(1/2
*sqrt(2)*(2*x - sqrt(2))) - 7/32*sqrt(2)*log(x^2 + sqrt(2)*x + 1) + 7/32*sq
rt(2)*log(x^2 - sqrt(2)*x + 1) - 1/12*(7*x^4 + 4)/(x^7 + x^3)
```

mupad [B] time = 1.36, size = 51, normalized size = 0.48

$$-\frac{\frac{7x^4}{12}+\frac{1}{3}}{x^7+x^3}+\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,x\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)\right)\left(-\frac{7}{16}-\frac{7}{16}\mathrm{i}\right)+\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,x\left(\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)\right)\left(-\frac{7}{16}+\frac{7}{16}\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(2*x^4 + x^8 + 1)),x)
```

```
[Out] - 2^(1/2)*atan(2^(1/2)*x*(1/2 - 1i/2))*(7/16 + 7i/16) - 2^(1/2)*atan(2^(1/2
)*x*(1/2 + 1i/2))*(7/16 - 7i/16) - ((7*x^4)/12 + 1/3)/(x^3 + x^7)
```

sympy [A] time = 0.22, size = 99, normalized size = 0.93

$$\frac{-7x^4-4}{12x^7+12x^3}+\frac{7\sqrt{2}\log\left(x^2-\sqrt{2}x+1\right)}{32}-\frac{7\sqrt{2}\log\left(x^2+\sqrt{2}x+1\right)}{32}-\frac{7\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}x-1\right)}{16}-\frac{7\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}x+1\right)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(x**8+2*x**4+1),x)
```

```
[Out] (-7*x**4 - 4)/(12*x**7 + 12*x**3) + 7*sqrt(2)*log(x**2 - sqrt(2)*x + 1)/32
- 7*sqrt(2)*log(x**2 + sqrt(2)*x + 1)/32 - 7*sqrt(2)*atan(sqrt(2)*x - 1)/16
 - 7*sqrt(2)*atan(sqrt(2)*x + 1)/16
```

3.233 $\int \frac{1}{x^6(1+2x^4+x^8)}\,dx$

Optimal. Leaf size=113

$$-\frac{9}{20x^5}+\frac{9\log\left(x^2-\sqrt{2}\,x+1\right)}{16\sqrt{2}}-\frac{9\log\left(x^2+\sqrt{2}\,x+1\right)}{16\sqrt{2}}+\frac{1}{4x^5\left(x^4+1\right)}+\frac{9}{4x}-\frac{9\tan^{-1}\left(1-\sqrt{2}\,x\right)}{8\sqrt{2}}+\frac{9\tan^{-1}\left(\sqrt{2}\,x+1\right)}{8\sqrt{2}}$$

Rubi [A] time = 0.05, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.562, Rules used = {28, 290, 325, 297, 1162, 617, 204, 1165, 628}

$$\frac{1}{4x^5\left(x^4+1\right)}-\frac{9}{20x^5}+\frac{9\log\left(x^2-\sqrt{2}\,x+1\right)}{16\sqrt{2}}-\frac{9\log\left(x^2+\sqrt{2}\,x+1\right)}{16\sqrt{2}}+\frac{9}{4x}-\frac{9\tan^{-1}\left(1-\sqrt{2}\,x\right)}{8\sqrt{2}}+\frac{9\tan^{-1}\left(\sqrt{2}\,x+1\right)}{8\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^6*(1 + 2*x^4 + x^8)),x]
```

```
[Out] -9/(20*x^5) + 9/(4*x) + 1/(4*x^5*(1 + x^4)) - (9*ArcTan[1 - Sqrt[2]*x])/(8*
Sqrt[2]) + (9*ArcTan[1 + Sqrt[2]*x])/(8*Sqrt[2]) + (9*Log[1 - Sqrt[2]*x + x
^2])/(16*Sqrt[2]) - (9*Log[1 + Sqrt[2]*x + x^2])/(16*Sqrt[2])
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{1}{x^6\left(1+2x^4+x^8\right)}\,dx = \int \frac{1}{x^6\left(1+x^4\right)^2}\,dx$$

$$= \frac{1}{4x^5\left(1+x^4\right)} + \frac{9}{4}\int \frac{1}{x^6\left(1+x^4\right)}\,dx$$

$$= -\frac{9}{20x^5} + \frac{1}{4x^5\left(1+x^4\right)} - \frac{9}{4}\int \frac{1}{x^2\left(1+x^4\right)}\,dx$$

$$= -\frac{9}{20x^5} + \frac{9}{4x} + \frac{1}{4x^5\left(1+x^4\right)} + \frac{9}{4}\int \frac{x^2}{1+x^4}\,dx$$

$$= -\frac{9}{20x^5} + \frac{9}{4x} + \frac{1}{4x^5\left(1+x^4\right)} - \frac{9}{8}\int \frac{1-x^2}{1+x^4}\,dx + \frac{9}{8}\int \frac{1+x^2}{1+x^4}\,dx$$

$$= -\frac{9}{20x^5} + \frac{9}{4x} + \frac{1}{4x^5\left(1+x^4\right)} + \frac{9}{16}\int \frac{1}{1-\sqrt{2}x+x^2}\,dx + \frac{9}{16}\int \frac{1}{1+\sqrt{2}x+x^2}\,dx + \cdots$$

$$= -\frac{9}{20x^5} + \frac{9}{4x} + \frac{1}{4x^5\left(1+x^4\right)} + \frac{9\log\left(1-\sqrt{2}x+x^2\right)}{16\sqrt{2}} - \frac{9\log\left(1+\sqrt{2}x+x^2\right)}{16\sqrt{2}} + \frac{9\,\mathrm{Su}\cdots}{\cdots}$$

$$= -\frac{9}{20x^5} + \frac{9}{4x} + \frac{1}{4x^5\left(1+x^4\right)} - \frac{9\tan^{-1}\left(1-\sqrt{2}x\right)}{8\sqrt{2}} + \frac{9\tan^{-1}\left(1+\sqrt{2}x\right)}{8\sqrt{2}} + \frac{9\log\left(1-\cdots\right.}{16\cdots}$$

Mathematica [A] time = 0.08, size = 103, normalized size = 0.91

$$\frac{1}{160}\left(-\frac{32}{x^5}+45\sqrt{2}\log\left(x^2-\sqrt{2}x+1\right)-45\sqrt{2}\log\left(x^2+\sqrt{2}x+1\right)+\frac{40x^3}{x^4+1}+\frac{320}{x}-90\sqrt{2}\tan^{-1}\left(1-\sqrt{2}x\right)+90\sqrt{2}\tan^{-1}\left(\sqrt{2}x+1\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^6*(1 + 2*x^4 + x^8)),x]
```

```
[Out] (-32/x^5 + 320/x + (40*x^3)/(1 + x^4) - 90*Sqrt[2]*ArcTan[1 - Sqrt[2]*x] +
90*Sqrt[2]*ArcTan[1 + Sqrt[2]*x] + 45*Sqrt[2]*Log[1 - Sqrt[2]*x + x^2] - 45
*Sqrt[2]*Log[1 + Sqrt[2]*x + x^2])/160
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^6 \left(1 + 2x^4 + x^8\right)} \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x^6*(1 + 2*x^4 + x^8)),x]
```

```
[Out] IntegrateAlgebraic[1/(x^6*(1 + 2*x^4 + x^8)), x]
```

fricas [A] time = 1.28, size = 145, normalized size = 1.28

$$\frac{360\,x^8 + 288\,x^4 - 180\,\sqrt{2}\left(x^9 + x^5\right)\arctan\left(-\sqrt{2}\,x + \sqrt{2}\,\sqrt{x^2 + \sqrt{2}\,x + 1} - 1\right) - 180\,\sqrt{2}\left(x^9 + x^5\right)\arctan\left(-\sqrt{2}\,x + \sqrt{2}\,\sqrt{x^2 - \sqrt{2}\,x + 1} + 1\right) - 45\,\sqrt{2}\left(x^9 + x^5\right)\log\left(x^2 + \sqrt{2}\,x + 1\right) + 45\,\sqrt{2}\left(x^9 + x^5\right)\log\left(x^2 - \sqrt{2}\,x + 1\right) - 32}{160\left(x^9 + x^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(x^8+2*x^4+1),x, algorithm="fricas")
```

```
[Out] 1/160*(360*x^8 + 288*x^4 - 180*sqrt(2)*(x^9 + x^5)*arctan(-sqrt(2)*x + sqrt
(2)*sqrt(x^2 + sqrt(2)*x + 1) - 1) - 180*sqrt(2)*(x^9 + x^5)*arctan(-sqrt(2
)*x + sqrt(2)*sqrt(x^2 - sqrt(2)*x + 1) + 1) - 45*sqrt(2)*(x^9 + x^5)*log(x
^2 + sqrt(2)*x + 1) + 45*sqrt(2)*(x^9 + x^5)*log(x^2 - sqrt(2)*x + 1) - 32)
/(x^9 + x^5)
```

giac [A] time = 0.30, size = 96, normalized size = 0.85

$$\frac{x^3}{4\left(x^4 + 1\right)} + \frac{9}{16}\,\sqrt{2}\,\arctan\left(\frac{1}{2}\,\sqrt{2}\left(2\,x + \sqrt{2}\right)\right) + \frac{9}{16}\,\sqrt{2}\,\arctan\left(\frac{1}{2}\,\sqrt{2}\left(2\,x - \sqrt{2}\right)\right) - \frac{9}{32}\,\sqrt{2}\,\log\left(x^2 + \sqrt{2}\,x + 1\right) + \frac{9}{32}\,\sqrt{2}\,\log\left(x^2 - \sqrt{2}\,x + 1\right) + \frac{10\,x^4 - 1}{5\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(x^8+2*x^4+1),x, algorithm="giac")
```

```
[Out] 1/4*x^3/(x^4 + 1) + 9/16*sqrt(2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2))) + 9/16
*sqrt(2)*arctan(1/2*sqrt(2)*(2*x - sqrt(2))) - 9/32*sqrt(2)*log(x^2 + sqrt(
2)*x + 1) + 9/32*sqrt(2)*log(x^2 - sqrt(2)*x + 1) + 1/5*(10*x^4 - 1)/x^5
```

maple [A] time = 0.01, size = 80, normalized size = 0.71

$$\frac{x^3}{4x^4 + 4} + \frac{9\sqrt{2}\ \arctan\left(\sqrt{2}\ x - 1\right)}{16} + \frac{9\sqrt{2}\ \arctan\left(\sqrt{2}\ x + 1\right)}{16} + \frac{9\sqrt{2}\ \ln\left(\frac{x^2 - \sqrt{2}\ x + 1}{x^2 + \sqrt{2}\ x + 1}\right)}{32} + \frac{2}{x} - \frac{1}{5x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^6/(x^8+2*x^4+1),x)
```

```
[Out] -1/5/x^5+2/x+1/4/(x^4+1)*x^3+9/32*2^(1/2)*ln((x^2-2^(1/2)*x+1)/(x^2+2^(1/2)
*x+1))+9/16*2^(1/2)*arctan(2^(1/2)*x+1)+9/16*2^(1/2)*arctan(2^(1/2)*x-1)
```

maxima [A] time = 2.01, size = 95, normalized size = 0.84

$$\frac{9}{16}\,\sqrt{2}\,\arctan\left(\frac{1}{2}\,\sqrt{2}\left(2\,x + \sqrt{2}\right)\right) + \frac{9}{16}\,\sqrt{2}\,\arctan\left(\frac{1}{2}\,\sqrt{2}\left(2\,x - \sqrt{2}\right)\right) - \frac{9}{32}\,\sqrt{2}\,\log\left(x^2 + \sqrt{2}\,x + 1\right) + \frac{9}{32}\,\sqrt{2}\,\log\left(x^2 - \sqrt{2}\,x + 1\right) + \frac{45\,x^8 + 36\,x^4 - 4}{20\left(x^9 + x^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(x^8+2*x^4+1),x, algorithm="maxima")
```

```
[Out] 9/16*sqrt(2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2))) + 9/16*sqrt(2)*arctan(1/2*
sqrt(2)*(2*x - sqrt(2))) - 9/32*sqrt(2)*log(x^2 + sqrt(2)*x + 1) + 9/32*sqr
t(2)*log(x^2 - sqrt(2)*x + 1) + 1/20*(45*x^8 + 36*x^4 - 4)/(x^9 + x^5)
```

mupad [B] time = 0.09, size = 55, normalized size = 0.49

$$\frac{\frac{9x^8}{4}+\frac{9x^4}{5}-\frac{1}{5}}{x^9+x^5}+\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,x\,\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)\right)\left(\frac{9}{16}-\frac{9}{16}\mathrm{i}\right)+\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,x\,\left(\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)\right)\left(\frac{9}{16}+\frac{9}{16}\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^6*(2*x^4 + x^8 + 1)),x)
```

```
[Out] 2^(1/2)*atan(2^(1/2)*x*(1/2 - 1i/2))*(9/16 - 9i/16) + 2^(1/2)*atan(2^(1/2)*
x*(1/2 + 1i/2))*(9/16 + 9i/16) + ((9*x^4)/5 + (9*x^8)/4 - 1/5)/(x^5 + x^9)
```

sympy [A] time = 0.23, size = 102, normalized size = 0.90

$$\frac{9\sqrt{2}\log\left(x^2-\sqrt{2}x+1\right)}{32}-\frac{9\sqrt{2}\log\left(x^2+\sqrt{2}x+1\right)}{32}+\frac{9\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}x-1\right)}{16}+\frac{9\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}x+1\right)}{16}+\frac{45x^8+36x^4-4}{20x^9+20x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**6/(x**8+2*x**4+1),x)
```

```
[Out] 9*sqrt(2)*log(x**2 - sqrt(2)*x + 1)/32 - 9*sqrt(2)*log(x**2 + sqrt(2)*x + 1
)/32 + 9*sqrt(2)*atan(sqrt(2)*x - 1)/16 + 9*sqrt(2)*atan(sqrt(2)*x + 1)/16
+ (45*x**8 + 36*x**4 - 4)/(20*x**9 + 20*x**5)
```

3.234 $\int \frac{1}{x^8(1+2x^4+x^8)}\,dx$

Optimal. Leaf size=113

$$-\frac{11}{28x^7}+\frac{11}{12x^3}-\frac{11\log\left(x^2-\sqrt{2}x+1\right)}{16\sqrt{2}}+\frac{11\log\left(x^2+\sqrt{2}x+1\right)}{16\sqrt{2}}+\frac{1}{4x^7\left(x^4+1\right)}-\frac{11\tan^{-1}\left(1-\sqrt{2}x\right)}{8\sqrt{2}}+\frac{11\tan^{-1}\left(\sqrt{2}\right.}{8\sqrt{2}}$$

Rubi [A] time = 0.05, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.562, Rules used = {28, 290, 325, 211, 1165, 628, 1162, 617, 204}

$$\frac{1}{4x^7\left(x^4+1\right)}+\frac{11}{12x^3}-\frac{11}{28x^7}-\frac{11\log\left(x^2-\sqrt{2}x+1\right)}{16\sqrt{2}}+\frac{11\log\left(x^2+\sqrt{2}x+1\right)}{16\sqrt{2}}-\frac{11\tan^{-1}\left(1-\sqrt{2}x\right)}{8\sqrt{2}}+\frac{11\tan^{-1}\left(\sqrt{2}x+1\right)}{8\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^8*(1 + 2*x^4 + x^8)),x]
```

```
[Out] -11/(28*x^7) + 11/(12*x^3) + 1/(4*x^7*(1 + x^4)) - (11*ArcTan[1 - Sqrt[2]*x
])/(8*Sqrt[2]) + (11*ArcTan[1 + Sqrt[2]*x])/(8*Sqrt[2]) - (11*Log[1 - Sqrt[
2]*x + x^2])/(16*Sqrt[2]) + (11*Log[1 + Sqrt[2]*x + x^2])/(16*Sqrt[2])
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x^8\left(1+2x^4+x^8\right)}\,dx &= \int \frac{1}{x^8\left(1+x^4\right)^2}\,dx\\
&= \frac{1}{4x^7\left(1+x^4\right)}+\frac{11}{4}\int \frac{1}{x^8\left(1+x^4\right)}\,dx\\
&= -\frac{11}{28x^7}+\frac{1}{4x^7\left(1+x^4\right)}-\frac{11}{4}\int \frac{1}{x^4\left(1+x^4\right)}\,dx\\
&= -\frac{11}{28x^7}+\frac{11}{12x^3}+\frac{1}{4x^7\left(1+x^4\right)}+\frac{11}{4}\int \frac{1}{1+x^4}\,dx\\
&= -\frac{11}{28x^7}+\frac{11}{12x^3}+\frac{1}{4x^7\left(1+x^4\right)}+\frac{11}{8}\int \frac{1-x^2}{1+x^4}\,dx+\frac{11}{8}\int \frac{1+x^2}{1+x^4}\,dx\\
&= -\frac{11}{28x^7}+\frac{11}{12x^3}+\frac{1}{4x^7\left(1+x^4\right)}+\frac{11}{16}\int \frac{1}{1-\sqrt{2}\,x+x^2}\,dx+\frac{11}{16}\int \frac{1}{1+\sqrt{2}\,x+x^2}\,dx\ -\\
&= -\frac{11}{28x^7}+\frac{11}{12x^3}+\frac{1}{4x^7\left(1+x^4\right)}-\frac{11\log\left(1-\sqrt{2}\,x+x^2\right)}{16\sqrt{2}}+\frac{11\log\left(1+\sqrt{2}\,x+x^2\right)}{16\sqrt{2}}+\\
&= -\frac{11}{28x^7}+\frac{11}{12x^3}+\frac{1}{4x^7\left(1+x^4\right)}-\frac{11\tan^{-1}\left(1-\sqrt{2}\,x\right)}{8\sqrt{2}}+\frac{11\tan^{-1}\left(1+\sqrt{2}\,x\right)}{8\sqrt{2}}-\ 11\log
\end{aligned}
$$

Mathematica [A] time = 0.08, size = 101, normalized size = 0.89

$$\frac{1}{672}\left(-\frac{96}{x^7}+\frac{168x}{x^4+1}+\frac{448}{x^3}-231\sqrt{2}\log\left(x^2-\sqrt{2}\,x+1\right)+231\sqrt{2}\log\left(x^2+\sqrt{2}\,x+1\right)-462\sqrt{2}\tan^{-1}\left(1-\sqrt{2}\,x\right)+462\sqrt{2}\tan^{-1}\left(\sqrt{2}\,x+1\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^8*(1 + 2*x^4 + x^8)),x]
```

```
[Out] (-96/x^7 + 448/x^3 + (168*x)/(1 + x^4) - 462*Sqrt[2]*ArcTan[1 - Sqrt[2]*x]
+ 462*Sqrt[2]*ArcTan[1 + Sqrt[2]*x] - 231*Sqrt[2]*Log[1 - Sqrt[2]*x + x^2]
+ 231*Sqrt[2]*Log[1 + Sqrt[2]*x + x^2])/672
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^8 \left(1+2x^4+x^8\right)} \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x^8*(1 + 2*x^4 + x^8)),x]
```

```
[Out] IntegrateAlgebraic[1/(x^8*(1 + 2*x^4 + x^8)), x]
```

fricas [A] time = 1.23, size = 145, normalized size = 1.28

$$\frac{616 x^8 + 352 x^4 - 924 \sqrt{2} \left(x^{11} + x^7\right) \arctan\left(-\sqrt{2} x + \sqrt{2} \sqrt{x^2 + \sqrt{2} x + 1} - 1\right) - 924 \sqrt{2} \left(x^{11} + x^7\right) \arctan\left(-\sqrt{2} x + \sqrt{2} \sqrt{x^2 - \sqrt{2} x + 1} + 1\right) + 231 \sqrt{2} \left(x^{11} + x^7\right) \log\left(x^2 + \sqrt{2} x + 1\right) - 231 \sqrt{2} \left(x^{11} + x^7\right) \log\left(x^2 - \sqrt{2} x + 1\right) - 96}{672 \left(x^{11} + x^7\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^8/(x^8+2*x^4+1),x, algorithm="fricas")
```

```
[Out] 1/672*(616*x^8 + 352*x^4 - 924*sqrt(2)*(x^11 + x^7)*arctan(-sqrt(2)*x + sqr
t(2)*sqrt(x^2 + sqrt(2)*x + 1) - 1) - 924*sqrt(2)*(x^11 + x^7)*arctan(-sqrt
(2)*x + sqrt(2)*sqrt(x^2 - sqrt(2)*x + 1) + 1) + 231*sqrt(2)*(x^11 + x^7)*l
og(x^2 + sqrt(2)*x + 1) - 231*sqrt(2)*(x^11 + x^7)*log(x^2 - sqrt(2)*x + 1)
 - 96)/(x^11 + x^7)
```

giac [A] time = 0.36, size = 94, normalized size = 0.83

$$\frac{11}{16} \sqrt{2} \arctan\left(\frac{1}{2} \sqrt{2} \left(2 x + \sqrt{2}\right)\right) + \frac{11}{16} \sqrt{2} \arctan\left(\frac{1}{2} \sqrt{2} \left(2 x - \sqrt{2}\right)\right) + \frac{11}{32} \sqrt{2} \log\left(x^2 + \sqrt{2} x + 1\right) - \frac{11}{32} \sqrt{2} \log\left(x^2 - \sqrt{2} x + 1\right) + \frac{x}{4 \left(x^4 + 1\right)} + \frac{14 x^4 - 3}{21 x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^8/(x^8+2*x^4+1),x, algorithm="giac")
```

```
[Out] 11/16*sqrt(2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2))) + 11/16*sqrt(2)*arctan(1/
2*sqrt(2)*(2*x - sqrt(2))) + 11/32*sqrt(2)*log(x^2 + sqrt(2)*x + 1) - 11/32
*sqrt(2)*log(x^2 - sqrt(2)*x + 1) + 1/4*x/(x^4 + 1) + 1/21*(14*x^4 - 3)/x^7
```

maple [A] time = 0.01, size = 78, normalized size = 0.69

$$\frac{x}{4x^4+4} + \frac{11\sqrt{2}\ \arctan\left(\sqrt{2}\ x-1\right)}{16} + \frac{11\sqrt{2}\ \arctan\left(\sqrt{2}\ x+1\right)}{16} + \frac{11\sqrt{2}\ \ln\left(\frac{x^2+\sqrt{2}\ x+1}{x^2-\sqrt{2}\ x+1}\right)}{32} + \frac{2}{3x^3} - \frac{1}{7x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^8/(x^8+2*x^4+1),x)
```

```
[Out] -1/7/x^7+2/3/x^3+1/4/(x^4+1)*x+11/16*2^(1/2)*arctan(2^(1/2)*x-1)+11/32*2^(1
/2)*ln((x^2+2^(1/2)*x+1)/(x^2-2^(1/2)*x+1))+11/16*2^(1/2)*arctan(2^(1/2)*x+
1)
```

maxima [A] time = 1.93, size = 95, normalized size = 0.84

$$\frac{11}{16} \sqrt{2} \arctan\left(\frac{1}{2} \sqrt{2} \left(2 x + \sqrt{2}\right)\right) + \frac{11}{16} \sqrt{2} \arctan\left(\frac{1}{2} \sqrt{2} \left(2 x - \sqrt{2}\right)\right) + \frac{11}{32} \sqrt{2} \log\left(x^2 + \sqrt{2} x + 1\right) - \frac{11}{32} \sqrt{2} \log\left(x^2 - \sqrt{2} x + 1\right) + \frac{77 x^8 + 44 x^4 - 12}{84 \left(x^{11} + x^7\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^8/(x^8+2*x^4+1),x, algorithm="maxima")
```

```
[Out] 11/16*sqrt(2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2))) + 11/16*sqrt(2)*arctan(1/
2*sqrt(2)*(2*x - sqrt(2))) + 11/32*sqrt(2)*log(x^2 + sqrt(2)*x + 1) - 11/32
*sqrt(2)*log(x^2 - sqrt(2)*x + 1) + 1/84*(77*x^8 + 44*x^4 - 12)/(x^11 + x^7
)
```

mupad [B] time = 0.10, size = 55, normalized size = 0.49

$$\frac{\frac{11\,x^8}{12}+\frac{11\,x^4}{21}-\frac{1}{7}}{x^{11}+x^7}+\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,x\,\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)\right)\,\left(\frac{11}{16}+\frac{11}{16}\mathrm{i}\right)+\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,x\,\left(\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)\right)\,\left(\frac{11}{16}-\frac{11}{16}\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^8*(2*x^4 + x^8 + 1)),x)
```

```
[Out] 2^(1/2)*atan(2^(1/2)*x*(1/2 - 1i/2))*(11/16 + 11i/16) + 2^(1/2)*atan(2^(1/2
)*x*(1/2 + 1i/2))*(11/16 - 11i/16) + ((11*x^4)/21 + (11*x^8)/12 - 1/7)/(x^7
 + x^11)
```

sympy [A] time = 0.23, size = 102, normalized size = 0.90

$$-\frac{11\sqrt{2}\log\left(x^2-\sqrt{2}x+1\right)}{32}+\frac{11\sqrt{2}\log\left(x^2+\sqrt{2}x+1\right)}{32}+\frac{11\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}x-1\right)}{16}+\frac{11\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}x+1\right)}{16}+\frac{77x^8+44x^4-12}{84x^{11}+84x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**8/(x**8+2*x**4+1),x)
```

```
[Out] -11*sqrt(2)*log(x**2 - sqrt(2)*x + 1)/32 + 11*sqrt(2)*log(x**2 + sqrt(2)*x
+ 1)/32 + 11*sqrt(2)*atan(sqrt(2)*x - 1)/16 + 11*sqrt(2)*atan(sqrt(2)*x + 1
)/16 + (77*x**8 + 44*x**4 - 12)/(84*x**11 + 84*x**7)
```

3.235 $\int \frac{x^9}{1-2x^4+x^8} dx$

Optimal. Leaf size=32

$$\frac{3x^2}{4} - \frac{3}{4}\tanh^{-1}\left(x^2\right) + \frac{x^6}{4\left(1-x^4\right)}$$

Rubi [A] time = 0.01, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {28, 275, 288, 321, 207}

$$\frac{x^6}{4\left(1-x^4\right)} + \frac{3x^2}{4} - \frac{3}{4}\tanh^{-1}\left(x^2\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^9/(1 - 2*x^4 + x^8),x]
```

```
[Out] (3*x^2)/4 + x^6/(4*(1 - x^4)) - (3*ArcTanh[x^2])/4
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}\int \frac{x^9}{1-2x^4+x^8}\,dx &= \int \frac{x^9}{\left(-1+x^4\right)^2}\,dx\\ &= \frac{1}{2}\,\text{Subst}\left(\int \frac{x^4}{\left(-1+x^2\right)^2}\,dx, x, x^2\right)\\ &= \frac{x^6}{4\left(1-x^4\right)}+\frac{3}{4}\,\text{Subst}\left(\int \frac{x^2}{-1+x^2}\,dx, x, x^2\right)\\ &= \frac{3x^2}{4}+\frac{x^6}{4\left(1-x^4\right)}+\frac{3}{4}\,\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, x^2\right)\\ &= \frac{3x^2}{4}+\frac{x^6}{4\left(1-x^4\right)}-\frac{3}{4}\tanh^{-1}\left(x^2\right)\end{aligned}$$

Mathematica [A] time = 0.02, size = 39, normalized size = 1.22

$$\frac{1}{8}\left(3\log\left(1-x^2\right)-3\log\left(x^2+1\right)+2\left(\frac{1}{1-x^4}+2\right)x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^9/(1 - 2*x^4 + x^8),x]
[Out] (2*x^2*(2 + (1 - x^4)^(-1)) + 3*Log[1 - x^2] - 3*Log[1 + x^2])/8
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^9}{1-2x^4+x^8}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^9/(1 - 2*x^4 + x^8),x]
[Out] IntegrateAlgebraic[x^9/(1 - 2*x^4 + x^8), x]
```

fricas [A] time = 1.14, size = 46, normalized size = 1.44

$$\frac{4\,x^6-6\,x^2-3\left(x^4-1\right)\log\left(x^2+1\right)+3\left(x^4-1\right)\log\left(x^2-1\right)}{8\left(x^4-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(x^8-2*x^4+1),x, algorithm="fricas")
[Out] 1/8*(4*x^6 - 6*x^2 - 3*(x^4 - 1)*log(x^2 + 1) + 3*(x^4 - 1)*log(x^2 - 1))/(
x^4 - 1)
```

giac [A] time = 0.33, size = 35, normalized size = 1.09

$$\frac{1}{2}\,x^2-\frac{x^2}{4\left(x^4-1\right)}-\frac{3}{8}\,\log\left(x^2+1\right)+\frac{3}{8}\,\log\left(\left|x^2-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(x^8-2*x^4+1),x, algorithm="giac")
```

```
[Out] 1/2*x^2 - 1/4*x^2/(x^4 - 1) - 3/8*log(x^2 + 1) + 3/8*log(abs(x^2 - 1))
```

maple [A] time = 0.01, size = 41, normalized size = 1.28

$$\frac{x^2}{2} + \frac{3\ln\left(x^2-1\right)}{8} - \frac{3\ln\left(x^2+1\right)}{8} - \frac{1}{8\left(x^2+1\right)} - \frac{1}{8\left(x^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9/(x^8-2*x^4+1),x)
[Out] 1/2*x^2-1/8/(x^2+1)-3/8*ln(x^2+1)-1/8/(x^2-1)+3/8*ln(x^2-1)
```

maxima [A] time = 1.03, size = 34, normalized size = 1.06

$$\frac{1}{2}x^2 - \frac{x^2}{4\left(x^4-1\right)} - \frac{3}{8}\log\left(x^2+1\right) + \frac{3}{8}\log\left(x^2-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(x^8-2*x^4+1),x, algorithm="maxima")
[Out] 1/2*x^2 - 1/4*x^2/(x^4 - 1) - 3/8*log(x^2 + 1) + 3/8*log(x^2 - 1)
```

mupad [B] time = 0.05, size = 26, normalized size = 0.81

$$\frac{x^2}{2} - \frac{x^2}{4\left(x^4-1\right)} - \frac{3\,\mathrm{atanh}\left(x^2\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9/(x^8 - 2*x^4 + 1),x)
[Out] x^2/2 - x^2/(4*(x^4 - 1)) - (3*atanh(x^2))/4
```

sympy [A] time = 0.12, size = 34, normalized size = 1.06

$$\frac{x^2}{2} - \frac{x^2}{4x^4-4} + \frac{3\log\left(x^2-1\right)}{8} - \frac{3\log\left(x^2+1\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**9/(x**8-2*x**4+1),x)
[Out] x**2/2 - x**2/(4*x**4 - 4) + 3*log(x**2 - 1)/8 - 3*log(x**2 + 1)/8
```

3.236 $\int \frac{x^7}{1-2x^4+x^8} \, dx$

Optimal. Leaf size=26

$$\frac{1}{4\left(1-x^4\right)}+\frac{1}{4}\log\left(1-x^4\right)$$

Rubi [A] time = 0.01, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {28, 266, 43}

$$\frac{1}{4\left(1-x^4\right)}+\frac{1}{4}\log\left(1-x^4\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^7/(1 - 2*x^4 + x^8),x]
```

```
[Out] 1/(4*(1 - x^4)) + Log[1 - x^4]/4
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^7}{1-2x^4+x^8}\,dx &= \int \frac{x^7}{\left(-1+x^4\right)^2}\,dx\\
&= \frac{1}{4}\text{Subst}\left(\int \frac{x}{(-1+x)^2}\,dx,x,x^4\right)\\
&= \frac{1}{4}\text{Subst}\left(\int \left(\frac{1}{(-1+x)^2}+\frac{1}{-1+x}\right)dx,x,x^4\right)\\
&= \frac{1}{4\left(1-x^4\right)}+\frac{1}{4}\log\left(1-x^4\right)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 22, normalized size = 0.85

$$\frac{1}{4}\log\left(x^4-1\right)-\frac{1}{4\left(x^4-1\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7/(1 - 2*x^4 + x^8),x]
[Out] -1/4*1/(-1 + x^4) + Log[-1 + x^4]/4
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^7}{1-2x^4+x^8}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^7/(1 - 2*x^4 + x^8),x]
[Out] IntegrateAlgebraic[x^7/(1 - 2*x^4 + x^8), x]
```

fricas [A] time = 1.27, size = 23, normalized size = 0.88

$$\frac{(x^4-1)\log(x^4-1)-1}{4(x^4-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(x^8-2*x^4+1),x, algorithm="fricas")
[Out] 1/4*((x^4 - 1)*log(x^4 - 1) - 1)/(x^4 - 1)
```

giac [A] time = 0.41, size = 19, normalized size = 0.73

$$-\frac{1}{4(x^4-1)}+\frac{1}{4}\log(|x^4-1|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(x^8-2*x^4+1),x, algorithm="giac")
[Out] -1/4/(x^4 - 1) + 1/4*log(abs(x^4 - 1))
```

maple [A] time = 0.01, size = 19, normalized size = 0.73

$$\frac{\ln(x^4-1)}{4}-\frac{1}{4(x^4-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(x^8-2*x^4+1),x)
[Out] -1/4/(x^4-1)+1/4*ln(x^4-1)
```

maxima [A] time = 1.13, size = 18, normalized size = 0.69

$$-\frac{1}{4(x^4-1)}+\frac{1}{4}\log(x^4-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(x^8-2*x^4+1),x, algorithm="maxima")
[Out] -1/4/(x^4 - 1) + 1/4*log(x^4 - 1)
```

mupad [B] time = 0.05, size = 20, normalized size = 0.77

$$\frac{\ln(x^4-1)}{4}-\frac{1}{4(x^4-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(x^8 - 2*x^4 + 1),x)
[Out] log(x^4 - 1)/4 - 1/(4*(x^4 - 1))
```

sympy [A] time = 0.10, size = 15, normalized size = 0.58

$$\frac{\log\left(x^4-1\right)}{4}-\frac{1}{4x^4-4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7/(x**8-2*x**4+1),x)
[Out] log(x**4 - 1)/4 - 1/(4*x**4 - 4)
```

3.237 $\int \frac{x^5}{1-2x^4+x^8}\,dx$

Optimal. Leaf size=25

$$\frac{x^2}{4\left(1-x^4\right)}-\frac{1}{4}\tanh^{-1}\left(x^2\right)$$

Rubi [A] time = 0.01, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {28, 275, 288, 207}

$$\frac{x^2}{4\left(1-x^4\right)}-\frac{1}{4}\tanh^{-1}\left(x^2\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^5/(1 - 2*x^4 + x^8),x]
```

```
[Out] x^2/(4*(1 - x^4)) - ArcTanh[x^2]/4
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}\int \frac{x^5}{1-2x^4+x^8}\,dx &= \int \frac{x^5}{\left(-1+x^4\right)^2}\,dx\\ &= \frac{1}{2}\,\text{Subst}\left(\int \frac{x^2}{\left(-1+x^2\right)^2}\,dx,x,x^2\right)\\ &= \frac{x^2}{4\left(1-x^4\right)}+\frac{1}{4}\,\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx,x,x^2\right)\\ &= \frac{x^2}{4\left(1-x^4\right)}-\frac{1}{4}\tanh^{-1}\left(x^2\right)\end{aligned}$$

Mathematica [A] time = 0.01, size = 33, normalized size = 1.32

$$\frac{1}{8}\left(\log\left(1-x^2\right)-\log\left(x^2+1\right)-\frac{2x^2}{x^4-1}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/(1 - 2*x^4 + x^8),x]
[Out] ((-2*x^2)/(-1 + x^4) + Log[1 - x^2] - Log[1 + x^2])/8
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5}{1-2x^4+x^8}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^5/(1 - 2*x^4 + x^8),x]
[Out] IntegrateAlgebraic[x^5/(1 - 2*x^4 + x^8), x]
```

fricas [B] time = 1.17, size = 40, normalized size = 1.60

$$-\frac{2\,x^2+\left(x^4-1\right)\log\left(x^2+1\right)-\left(x^4-1\right)\log\left(x^2-1\right)}{8\left(x^4-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(x^8-2*x^4+1),x, algorithm="fricas")
[Out] -1/8*(2*x^2 + (x^4 - 1)*log(x^2 + 1) - (x^4 - 1)*log(x^2 - 1))/(x^4 - 1)
```

giac [A] time = 0.50, size = 30, normalized size = 1.20

$$-\frac{x^2}{4\left(x^4-1\right)}-\frac{1}{8}\,\log\left(x^2+1\right)+\frac{1}{8}\,\log\left(\left|x^2-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(x^8-2*x^4+1),x, algorithm="giac")
[Out] -1/4*x^2/(x^4 - 1) - 1/8*log(x^2 + 1) + 1/8*log(abs(x^2 - 1))
```

maple [A] time = 0.01, size = 36, normalized size = 1.44

$$\frac{\ln\left(x^2-1\right)}{8}-\frac{\ln\left(x^2+1\right)}{8}-\frac{1}{8\left(x^2+1\right)}-\frac{1}{8\left(x^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(x^8-2*x^4+1),x)
```

```
[Out] -1/8/(x^2+1)-1/8*ln(x^2+1)-1/8/(x^2-1)+1/8*ln(x^2-1)
```

maxima [A] time = 0.91, size = 29, normalized size = 1.16

$$-\frac{x^2}{4\left(x^4-1\right)}-\frac{1}{8}\log\left(x^2+1\right)+\frac{1}{8}\log\left(x^2-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(x^8-2*x^4+1),x, algorithm="maxima")
```

```
[Out] -1/4*x^2/(x^4 - 1) - 1/8*log(x^2 + 1) + 1/8*log(x^2 - 1)
```

mupad [B] time = 1.27, size = 21, normalized size = 0.84

$$-\frac{\operatorname{atanh}\left(x^2\right)}{4}-\frac{x^2}{4\left(x^4-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(x^8 - 2*x^4 + 1),x)
```

```
[Out] - atanh(x^2)/4 - x^2/(4*(x^4 - 1))
```

sympy [A] time = 0.12, size = 26, normalized size = 1.04

$$-\frac{x^2}{4x^4-4}+\frac{\log\left(x^2-1\right)}{8}-\frac{\log\left(x^2+1\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(x**8-2*x**4+1),x)
```

```
[Out] -x**2/(4*x**4 - 4) + log(x**2 - 1)/8 - log(x**2 + 1)/8
```

3.238 $\int \frac{x^3}{1-2x^4+x^8} dx$

Optimal. Leaf size=13

$$\frac{1}{4(1-x^4)}$$

Rubi [A] time = 0.00, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {28, 261}

$$\frac{1}{4(1-x^4)}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(1 - 2*x^4 + x^8),x]
```

```
[Out] 1/(4*(1 - x^4))
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{x^3}{1-2x^4+x^8} dx &= \int \frac{x^3}{\left(-1+x^4\right)^2} dx\\ &= \frac{1}{4(1-x^4)}\end{aligned}$$

Mathematica [A] time = 0.00, size = 11, normalized size = 0.85

$$-\frac{1}{4(x^4-1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(1 - 2*x^4 + x^8),x]
```

```
[Out] -1/4*1/(-1 + x^4)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{1-2x^4+x^8} dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^3/(1 - 2*x^4 + x^8),x]
```

```
[Out] IntegrateAlgebraic[x^3/(1 - 2*x^4 + x^8), x]
```

fricas [A] time = 1.15, size = 9, normalized size = 0.69

$$-\frac{1}{4\left(x^4-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(x^8-2*x^4+1),x, algorithm="fricas")
```

```
[Out] -1/4/(x^4 - 1)
```

giac [A] time = 0.50, size = 9, normalized size = 0.69

$$-\frac{1}{4\left(x^4-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(x^8-2*x^4+1),x, algorithm="giac")
```

```
[Out] -1/4/(x^4 - 1)
```

maple [A] time = 0.00, size = 10, normalized size = 0.77

$$-\frac{1}{4\left(x^4-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(x^8-2*x^4+1),x)
```

```
[Out] -1/4/(x^4-1)
```

maxima [A] time = 0.88, size = 9, normalized size = 0.69

$$-\frac{1}{4\left(x^4-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(x^8-2*x^4+1),x, algorithm="maxima")
```

```
[Out] -1/4/(x^4 - 1)
```

mupad [B] time = 0.02, size = 11, normalized size = 0.85

$$-\frac{1}{4\,\left(x^4-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(x^8 - 2*x^4 + 1),x)
```

```
[Out] -1/(4*(x^4 - 1))
```

sympy [A] time = 0.10, size = 8, normalized size = 0.62

$$-\frac{1}{4x^4-4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(x**8-2*x**4+1),x)
```

```
[Out] -1/(4*x**4 - 4)
```

3.239 $\int \frac{x}{1-2x^4+x^8} \, dx$

Optimal. Leaf size=25

$$\frac{1}{4}\tanh^{-1}\left(x^2\right)+\frac{x^2}{4\left(1-x^4\right)}$$

Rubi [A] time = 0.01, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {28, 275, 199, 207}

$$\frac{x^2}{4\left(1-x^4\right)}+\frac{1}{4}\tanh^{-1}\left(x^2\right)$$

Antiderivative was successfully verified.

```
[In] Int[x/(1 - 2*x^4 + x^8),x]
```

```
[Out] x^2/(4*(1 - x^4)) + ArcTanh[x^2]/4
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\int \frac{x}{1-2x^4+x^8} \, dx &= \int \frac{x}{\left(-1+x^4\right)^2} \, dx \\
&= \frac{1}{2}\text{Subst}\left(\int \frac{1}{\left(-1+x^2\right)^2} \, dx, x, x^2\right) \\
&= \frac{x^2}{4\left(1-x^4\right)}-\frac{1}{4}\text{Subst}\left(\int \frac{1}{-1+x^2} \, dx, x, x^2\right) \\
&= \frac{x^2}{4\left(1-x^4\right)}+\frac{1}{4}\tanh^{-1}\left(x^2\right)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 33, normalized size = 1.32

$$\frac{1}{8}\left(-\log\left(1-x^2\right)+\log\left(x^2+1\right)-\frac{2x^2}{x^4-1}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(1 - 2*x^4 + x^8),x]
[Out] ((-2*x^2)/(-1 + x^4) - Log[1 - x^2] + Log[1 + x^2])/8
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{1-2x^4+x^8}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x/(1 - 2*x^4 + x^8),x]
[Out] IntegrateAlgebraic[x/(1 - 2*x^4 + x^8), x]
```

fricas [B] time = 1.26, size = 40, normalized size = 1.60

$$-\frac{2\,x^2-\left(x^4-1\right)\log\left(x^2+1\right)+\left(x^4-1\right)\log\left(x^2-1\right)}{8\left(x^4-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^8-2*x^4+1),x, algorithm="fricas")
[Out] -1/8*(2*x^2 - (x^4 - 1)*log(x^2 + 1) + (x^4 - 1)*log(x^2 - 1))/(x^4 - 1)
```

giac [A] time = 0.47, size = 30, normalized size = 1.20

$$-\frac{x^2}{4\left(x^4-1\right)}+\frac{1}{8}\log\left(x^2+1\right)-\frac{1}{8}\log\left(\left|x^2-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^8-2*x^4+1),x, algorithm="giac")
[Out] -1/4*x^2/(x^4 - 1) + 1/8*log(x^2 + 1) - 1/8*log(abs(x^2 - 1))
```

maple [A] time = 0.01, size = 36, normalized size = 1.44

$$-\frac{\ln\left(x^2-1\right)}{8}+\frac{\ln\left(x^2+1\right)}{8}-\frac{1}{8\left(x^2+1\right)}-\frac{1}{8\left(x^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x^8-2*x^4+1),x)
[Out] -1/8/(x^2+1)+1/8*ln(x^2+1)-1/8/(x^2-1)-1/8*ln(x^2-1)
```

maxima [A] time = 0.92, size = 29, normalized size = 1.16

$$-\frac{x^2}{4\left(x^4-1\right)}+\frac{1}{8}\log\left(x^2+1\right)-\frac{1}{8}\log\left(x^2-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^8-2*x^4+1),x, algorithm="maxima")
```

```
[Out] -1/4*x^2/(x^4 - 1) + 1/8*log(x^2 + 1) - 1/8*log(x^2 - 1)
```

mupad [B] time = 0.03, size = 21, normalized size = 0.84

$$\frac{\operatorname{atanh}\left(x^2\right)}{4}-\frac{x^2}{4\left(x^4-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x^8 - 2*x^4 + 1),x)
```

```
[Out] atanh(x^2)/4 - x^2/(4*(x^4 - 1))
```

sympy [A] time = 0.12, size = 26, normalized size = 1.04

$$-\frac{x^2}{4x^4-4}-\frac{\log\left(x^2-1\right)}{8}+\frac{\log\left(x^2+1\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x**8-2*x**4+1),x)
```

```
[Out] -x**2/(4*x**4 - 4) - log(x**2 - 1)/8 + log(x**2 + 1)/8
```

3.240 $\int \frac{1}{x(1-2x^4+x^8)} dx$

Optimal. Leaf size=28

$$\frac{1}{4(1-x^4)} - \frac{1}{4}\log(1-x^4) + \log(x)$$

Rubi [A] time = 0.01, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {28, 266, 44}

$$\frac{1}{4(1-x^4)} - \frac{1}{4}\log(1-x^4) + \log(x)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(1 - 2*x^4 + x^8)),x]
```

```
[Out] 1/(4*(1 - x^4)) + Log[x] - Log[1 - x^4]/4
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x(1-2x^4+x^8)} dx &= \int \frac{1}{x(-1+x^4)^2} dx \\
&= \frac{1}{4}\text{Subst}\left(\int \frac{1}{(-1+x)^2 x} dx, x, x^4\right) \\
&= \frac{1}{4}\text{Subst}\left(\int \left(\frac{1}{1-x} + \frac{1}{(-1+x)^2} + \frac{1}{x}\right) dx, x, x^4\right) \\
&= \frac{1}{4(1-x^4)} + \log(x) - \frac{1}{4}\log(1-x^4)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 26, normalized size = 0.93

$$-\frac{1}{4(x^4-1)} - \frac{1}{4}\log(1-x^4) + \log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(1 - 2*x^4 + x^8)),x]
[Out] -1/4*1/(-1 + x^4) + Log[x] - Log[1 - x^4]/4
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(1-2x^4+x^8\right)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x*(1 - 2*x^4 + x^8)),x]
[Out] IntegrateAlgebraic[1/(x*(1 - 2*x^4 + x^8)), x]
```

fricas [A] time = 0.98, size = 32, normalized size = 1.14

$$-\frac{\left(x^4-1\right)\log\left(x^4-1\right)-4\left(x^4-1\right)\log(x)+1}{4\left(x^4-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x^8-2*x^4+1),x, algorithm="fricas")
[Out] -1/4*((x^4 - 1)*log(x^4 - 1) - 4*(x^4 - 1)*log(x) + 1)/(x^4 - 1)
```

giac [A] time = 0.38, size = 30, normalized size = 1.07

$$\frac{x^4-2}{4\left(x^4-1\right)}+\frac{1}{4}\log\left(x^4\right)-\frac{1}{4}\log\left(\left|x^4-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x^8-2*x^4+1),x, algorithm="giac")
[Out] 1/4*(x^4 - 2)/(x^4 - 1) + 1/4*log(x^4) - 1/4*log(abs(x^4 - 1))
```

maple [A] time = 0.02, size = 47, normalized size = 1.68

$$\ln(x)-\frac{\ln(x-1)}{4}-\frac{\ln(x+1)}{4}-\frac{\ln\left(x^2+1\right)}{4}+\frac{1}{16x+16}+\frac{1}{8x^2+8}-\frac{1}{16(x-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(x^8-2*x^4+1),x)
[Out] ln(x)+1/16/(x+1)-1/4*ln(x+1)-1/4*ln(x^2+1)+1/8/(x^2+1)-1/16/(x-1)-1/4*ln(x-
1)
```

maxima [A] time = 0.94, size = 24, normalized size = 0.86

$$-\frac{1}{4\left(x^4-1\right)}-\frac{1}{4}\log\left(x^4-1\right)+\frac{1}{4}\log\left(x^4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x^8-2*x^4+1),x, algorithm="maxima")
[Out] -1/4/(x^4 - 1) - 1/4*log(x^4 - 1) + 1/4*log(x^4)
```

mupad [B] time = 0.06, size = 22, normalized size = 0.79

$$\ln(x) - \frac{\ln\left(x^4 - 1\right)}{4} - \frac{1}{4\left(x^4 - 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(x^8 - 2*x^4 + 1)),x)
[Out] log(x) - log(x^4 - 1)/4 - 1/(4*(x^4 - 1))
```

sympy [A] time = 0.13, size = 19, normalized size = 0.68

$$\log(x) - \frac{\log\left(x^4 - 1\right)}{4} - \frac{1}{4x^4 - 4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x**8-2*x**4+1),x)
[Out] log(x) - log(x**4 - 1)/4 - 1/(4*x**4 - 4)
```

3.241 $\int \frac{1}{x^3(1-2x^4+x^8)} dx$

Optimal. Leaf size=32

$$-\frac{3}{4x^2}+\frac{3}{4}\tanh^{-1}\left(x^2\right)+\frac{1}{4x^2\left(1-x^4\right)}$$

Rubi [A] time = 0.01, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {28, 275, 290, 325, 207}

$$\frac{1}{4x^2\left(1-x^4\right)}-\frac{3}{4x^2}+\frac{3}{4}\tanh^{-1}\left(x^2\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(1 - 2*x^4 + x^8)),x]
```

```
[Out] -3/(4*x^2) + 1/(4*x^2*(1 - x^4)) + (3*ArcTanh[x^2])/4
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rubi steps

$$\int \frac{1}{x^3 (1-2x^4+x^8)} dx = \int \frac{1}{x^3 (-1+x^4)^2} dx$$
$$= \frac{1}{2} \text{Subst}\left(\int \frac{1}{x^2 (-1+x^2)^2} dx, x, x^2\right)$$
$$= \frac{1}{4x^2 (1-x^4)} - \frac{3}{4} \text{Subst}\left(\int \frac{1}{x^2 (-1+x^2)} dx, x, x^2\right)$$
$$= -\frac{3}{4x^2} + \frac{1}{4x^2 (1-x^4)} - \frac{3}{4} \text{Subst}\left(\int \frac{1}{-1+x^2} dx, x, x^2\right)$$
$$= -\frac{3}{4x^2} + \frac{1}{4x^2 (1-x^4)} + \frac{3}{4} \tanh^{-1}(x^2)$$

Mathematica [A] time = 0.02, size = 41, normalized size = 1.28

$$\frac{1}{8}\left(-3\log(1-x^2) + 3\log(x^2+1) + \frac{4-6x^4}{x^2 (x^4-1)}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(1 - 2*x^4 + x^8)),x]
[Out] ((4 - 6*x^4)/(x^2*(-1 + x^4)) - 3*Log[1 - x^2] + 3*Log[1 + x^2])/8
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3 (1-2x^4+x^8)} dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x^3*(1 - 2*x^4 + x^8)),x]
[Out] IntegrateAlgebraic[1/(x^3*(1 - 2*x^4 + x^8)), x]
```

fricas [B] time = 1.16, size = 54, normalized size = 1.69

$$-\frac{6x^4 - 3(x^6-x^2)\log(x^2+1) + 3(x^6-x^2)\log(x^2-1) - 4}{8(x^6-x^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(x^8-2*x^4+1),x, algorithm="fricas")
[Out] -1/8*(6*x^4 - 3*(x^6 - x^2)*log(x^2 + 1) + 3*(x^6 - x^2)*log(x^2 - 1) - 4)/
(x^6 - x^2)
```

giac [A] time = 0.41, size = 38, normalized size = 1.19

$$-\frac{3x^4-2}{4(x^6-x^2)} + \frac{3}{8}\log(x^2+1) - \frac{3}{8}\log(|x^2-1|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(x^8-2*x^4+1),x, algorithm="giac")
```

[Out] -1/4*(3*x^4 - 2)/(x^6 - x^2) + 3/8*log(x^2 + 1) - 3/8*log(abs(x^2 - 1))

maple [A] time = 0.02, size = 50, normalized size = 1.56

$$-\frac{3\ln(x-1)}{8}-\frac{3\ln(x+1)}{8}+\frac{3\ln\left(x^2+1\right)}{8}-\frac{1}{2x^2}+\frac{1}{16x+16}-\frac{1}{8\left(x^2+1\right)}-\frac{1}{16(x-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^3/(x^8-2*x^4+1),x)

[Out] -1/2/x^2+1/16/(x+1)-3/8*ln(x+1)+3/8*ln(x^2+1)-1/8/(x^2+1)-1/16/(x-1)-3/8*ln(x-1)

maxima [A] time = 0.95, size = 37, normalized size = 1.16

$$-\frac{3x^4-2}{4\left(x^6-x^2\right)}+\frac{3}{8}\log\left(x^2+1\right)-\frac{3}{8}\log\left(x^2-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^3/(x^8-2*x^4+1),x, algorithm="maxima")

[Out] -1/4*(3*x^4 - 2)/(x^6 - x^2) + 3/8*log(x^2 + 1) - 3/8*log(x^2 - 1)

mupad [B] time = 0.04, size = 26, normalized size = 0.81

$$\frac{3\,\mathrm{atanh}\left(x^2\right)}{4}+\frac{\frac{3x^4}{4}-\frac{1}{2}}{x^2-x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^3*(x^8 - 2*x^4 + 1)),x)

[Out] (3*atanh(x^2))/4 + ((3*x^4)/4 - 1/2)/(x^2 - x^6)

sympy [A] time = 0.15, size = 36, normalized size = 1.12

$$\frac{2-3x^4}{4x^6-4x^2}-\frac{3\log\left(x^2-1\right)}{8}+\frac{3\log\left(x^2+1\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x**3/(x**8-2*x**4+1),x)

[Out] (2 - 3*x**4)/(4*x**6 - 4*x**2) - 3*log(x**2 - 1)/8 + 3*log(x**2 + 1)/8

3.242 $\int \frac{1}{x^5(1-2x^4+x^8)}\,dx$

Optimal. Leaf size=37

$$\frac{1}{4(1-x^4)}-\frac{1}{4x^4}-\frac{1}{2}\log\left(1-x^4\right)+2\log(x)$$

Rubi [A] time = 0.02, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {28, 266, 44}

$$\frac{1}{4(1-x^4)}-\frac{1}{4x^4}-\frac{1}{2}\log\left(1-x^4\right)+2\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^5*(1 - 2*x^4 + x^8)),x]
```

```
[Out] -1/(4*x^4) + 1/(4*(1 - x^4)) + 2*Log[x] - Log[1 - x^4]/2
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^5\left(1-2x^4+x^8\right)}\,dx &= \int \frac{1}{x^5\left(-1+x^4\right)^2}\,dx\\ &= \frac{1}{4}\operatorname{Subst}\left(\int \frac{1}{(-1+x)^2x^2}\,dx,x,x^4\right)\\ &= \frac{1}{4}\operatorname{Subst}\left(\int \left(\frac{1}{(-1+x)^2}-\frac{2}{-1+x}+\frac{1}{x^2}+\frac{2}{x}\right)dx,x,x^4\right)\\ &= -\frac{1}{4x^4}+\frac{1}{4\left(1-x^4\right)}+2\log(x)-\frac{1}{2}\log\left(1-x^4\right)\end{aligned}$$

Mathematica [A] time = 0.01, size = 35, normalized size = 0.95

$$-\frac{1}{4\left(x^4-1\right)}-\frac{1}{4x^4}-\frac{1}{2}\log\left(1-x^4\right)+2\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^5*(1 - 2*x^4 + x^8)),x]
```

```
[Out] -1/4*1/x^4 - 1/(4*(-1 + x^4)) + 2*Log[x] - Log[1 - x^4]/2
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^5 (1 - 2x^4 + x^8)} \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x^5*(1 - 2*x^4 + x^8)),x]
```

```
[Out] IntegrateAlgebraic[1/(x^5*(1 - 2*x^4 + x^8)), x]
```

fricas [A] time = 0.86, size = 50, normalized size = 1.35

$$-\frac{2\,x^4 + 2\left(x^8 - x^4\right)\log\left(x^4 - 1\right) - 8\left(x^8 - x^4\right)\log(x) - 1}{4\left(x^8 - x^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(x^8-2*x^4+1),x, algorithm="fricas")
```

```
[Out] -1/4*(2*x^4 + 2*(x^8 - x^4)*log(x^4 - 1) - 8*(x^8 - x^4)*log(x) - 1)/(x^8 -
 x^4)
```

giac [A] time = 0.45, size = 36, normalized size = 0.97

$$-\frac{2\,x^4 - 1}{4\left(x^8 - x^4\right)} + \frac{1}{2}\,\log\left(x^4\right) - \frac{1}{2}\,\log\left(\left|x^4 - 1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(x^8-2*x^4+1),x, algorithm="giac")
```

```
[Out] -1/4*(2*x^4 - 1)/(x^8 - x^4) + 1/2*log(x^4) - 1/2*log(abs(x^4 - 1))
```

maple [A] time = 0.02, size = 54, normalized size = 1.46

$$2\ln(x) - \frac{\ln(x-1)}{2} - \frac{\ln(x+1)}{2} - \frac{\ln\left(x^2+1\right)}{2} - \frac{1}{4x^4} + \frac{1}{16x+16} + \frac{1}{8x^2+8} - \frac{1}{16(x-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^5/(x^8-2*x^4+1),x)
```

```
[Out] -1/4/x^4+2*ln(x)+1/16/(x+1)-1/2*ln(x+1)-1/2*ln(x^2+1)+1/8/(x^2+1)-1/16/(x-1
)-1/2*ln(x-1)
```

maxima [A] time = 0.83, size = 35, normalized size = 0.95

$$-\frac{2\,x^4 - 1}{4\left(x^8 - x^4\right)} - \frac{1}{2}\,\log\left(x^4 - 1\right) + \frac{1}{2}\,\log\left(x^4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(x^8-2*x^4+1),x, algorithm="maxima")
```

```
[Out] -1/4*(2*x^4 - 1)/(x^8 - x^4) - 1/2*log(x^4 - 1) + 1/2*log(x^4)
```

mupad [B] time = 0.05, size = 32, normalized size = 0.86

$$2\ \ln(x) - \frac{\ln\left(x^4 - 1\right)}{2} + \frac{\frac{x^4}{2} - \frac{1}{4}}{x^4 - x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^5*(x^8 - 2*x^4 + 1)),x)
[Out] 2*log(x) - log(x^4 - 1)/2 + (x^4/2 - 1/4)/(x^4 - x^8)
```

sympy [A] time = 0.15, size = 29, normalized size = 0.78

$$\frac{1 - 2x^4}{4x^8 - 4x^4} + 2\log\left(x\right) - \frac{\log\left(x^4 - 1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**5/(x**8-2*x**4+1),x)
[Out] (1 - 2*x**4)/(4*x**8 - 4*x**4) + 2*log(x) - log(x**4 - 1)/2
```

3.243 $\int \frac{1}{x^7(1-2x^4+x^8)}\,dx$

Optimal. Leaf size=39

$$-\frac{5}{12x^6}-\frac{5}{4x^2}+\frac{5}{4}\tanh^{-1}\left(x^2\right)+\frac{1}{4x^6\left(1-x^4\right)}$$

Rubi [A] time = 0.02, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {28, 275, 290, 325, 207}

$$\frac{1}{4x^6\left(1-x^4\right)}-\frac{5}{4x^2}-\frac{5}{12x^6}+\frac{5}{4}\tanh^{-1}\left(x^2\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^7*(1 - 2*x^4 + x^8)),x]
```

```
[Out] -5/(12*x^6) - 5/(4*x^2) + 1/(4*x^6*(1 - x^4)) + (5*ArcTanh[x^2])/4
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rubi steps

$$\int \frac{1}{x^7 \left(1-2x^4+x^8\right)} \, dx = \int \frac{1}{x^7 \left(-1+x^4\right)^2} \, dx$$
$$= \frac{1}{2} \text{Subst}\left(\int \frac{1}{x^4 \left(-1+x^2\right)^2} \, dx, x, x^2\right)$$
$$= \frac{1}{4x^6 \left(1-x^4\right)} - \frac{5}{4} \text{Subst}\left(\int \frac{1}{x^4 \left(-1+x^2\right)} \, dx, x, x^2\right)$$
$$= -\frac{5}{12x^6} + \frac{1}{4x^6 \left(1-x^4\right)} - \frac{5}{4} \text{Subst}\left(\int \frac{1}{x^2 \left(-1+x^2\right)} \, dx, x, x^2\right)$$
$$= -\frac{5}{12x^6} - \frac{5}{4x^2} + \frac{1}{4x^6 \left(1-x^4\right)} - \frac{5}{4} \text{Subst}\left(\int \frac{1}{-1+x^2} \, dx, x, x^2\right)$$
$$= -\frac{5}{12x^6} - \frac{5}{4x^2} + \frac{1}{4x^6 \left(1-x^4\right)} + \frac{5}{4} \tanh^{-1}\left(x^2\right)$$

Mathematica [A] time = 0.01, size = 49, normalized size = 1.26

$$-\frac{1}{6x^6} - \frac{1}{x^2} - \frac{5}{8} \log\left(1-x^2\right) + \frac{5}{8} \log\left(x^2+1\right) - \frac{x^2}{4\left(x^4-1\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^7*(1 - 2*x^4 + x^8)),x]
```

```
[Out] -1/6*1/x^6 - x^(-2) - x^2/(4*(-1 + x^4)) - (5*Log[1 - x^2])/8 + (5*Log[1 + 
x^2])/8
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^7 \left(1-2x^4+x^8\right)} \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x^7*(1 - 2*x^4 + x^8)),x]
```

```
[Out] IntegrateAlgebraic[1/(x^7*(1 - 2*x^4 + x^8)), x]
```

fricas [B] time = 1.23, size = 59, normalized size = 1.51

$$-\frac{30\,x^8 - 20\,x^4 - 15\left(x^{10}-x^6\right)\log\left(x^2+1\right) + 15\left(x^{10}-x^6\right)\log\left(x^2-1\right) - 4}{24\left(x^{10}-x^6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(x^8-2*x^4+1),x, algorithm="fricas")
```

```
[Out] -1/24*(30*x^8 - 20*x^4 - 15*(x^10 - x^6)*log(x^2 + 1) + 15*(x^10 - x^6)*log
(x^2 - 1) - 4)/(x^10 - x^6)
```

giac [A] time = 0.33, size = 42, normalized size = 1.08

$$-\frac{x^2}{4\left(x^4-1\right)} - \frac{6\,x^4+1}{6\,x^6} + \frac{5}{8}\,\log\left(x^2+1\right) - \frac{5}{8}\,\log\left(\left|x^2-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(x^8-2*x^4+1),x, algorithm="giac")
```

```
[Out] -1/4*x^2/(x^4 - 1) - 1/6*(6*x^4 + 1)/x^6 + 5/8*log(x^2 + 1) - 5/8*log(abs(x
^2 - 1))
```

maple [A] time = 0.02, size = 55, normalized size = 1.41

$$-\frac{5\ln(x-1)}{8}-\frac{5\ln(x+1)}{8}+\frac{5\ln\left(x^2+1\right)}{8}-\frac{1}{x^2}-\frac{1}{6x^6}+\frac{1}{16x+16}-\frac{1}{8\left(x^2+1\right)}-\frac{1}{16(x-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^7/(x^8-2*x^4+1),x)
```

```
[Out] -1/6/x^6-1/x^2+1/16/(x+1)-5/8*ln(x+1)+5/8*ln(x^2+1)-1/8/(x^2+1)-1/16/(x-1)-
5/8*ln(x-1)
```

maxima [A] time = 0.94, size = 42, normalized size = 1.08

$$-\frac{15\,x^8-10\,x^4-2}{12\left(x^{10}-x^6\right)}+\frac{5}{8}\log\left(x^2+1\right)-\frac{5}{8}\log\left(x^2-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(x^8-2*x^4+1),x, algorithm="maxima")
```

```
[Out] -1/12*(15*x^8 - 10*x^4 - 2)/(x^10 - x^6) + 5/8*log(x^2 + 1) - 5/8*log(x^2 -
 1)
```

mupad [B] time = 0.05, size = 32, normalized size = 0.82

$$\frac{5\,\mathrm{atanh}\left(x^2\right)}{4}-\frac{-\frac{5\,x^8}{4}+\frac{5\,x^4}{6}+\frac{1}{6}}{x^6-x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^7*(x^8 - 2*x^4 + 1)),x)
```

```
[Out] (5*atanh(x^2))/4 - ((5*x^4)/6 - (5*x^8)/4 + 1/6)/(x^6 - x^10)
```

sympy [A] time = 0.18, size = 41, normalized size = 1.05

$$-\frac{5\log\left(x^2-1\right)}{8}+\frac{5\log\left(x^2+1\right)}{8}+\frac{-15x^8+10x^4+2}{12x^{10}-12x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**7/(x**8-2*x**4+1),x)
```

```
[Out] -5*log(x**2 - 1)/8 + 5*log(x**2 + 1)/8 + (-15*x**8 + 10*x**4 + 2)/(12*x**10
 - 12*x**6)
```

3.244 $\int \frac{x^8}{1-2x^4+x^8}\,dx$

Optimal. Leaf size=34

$$\frac{x^5}{4\left(1-x^4\right)}+\frac{5x}{4}-\frac{5}{8}\tan^{-1}(x)-\frac{5}{8}\tanh^{-1}(x)$$

Rubi [A] time = 0.01, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {28, 288, 321, 212, 206, 203}

$$\frac{x^5}{4\left(1-x^4\right)}+\frac{5x}{4}-\frac{5}{8}\tan^{-1}(x)-\frac{5}{8}\tanh^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[x^8/(1 - 2*x^4 + x^8),x]
```

```
[Out] (5*x)/4 + x^5/(4*(1 - x^4)) - (5*ArcTan[x])/8 - (5*ArcTanh[x])/8
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
```

```
+ 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}\int \frac{x^8}{1-2x^4+x^8}\,dx &= \int \frac{x^8}{\left(-1+x^4\right)^2}\,dx\\ &= \frac{x^5}{4\left(1-x^4\right)}+\frac{5}{4}\int \frac{x^4}{-1+x^4}\,dx\\ &= \frac{5x}{4}+\frac{x^5}{4\left(1-x^4\right)}+\frac{5}{4}\int \frac{1}{-1+x^4}\,dx\\ &= \frac{5x}{4}+\frac{x^5}{4\left(1-x^4\right)}-\frac{5}{8}\int \frac{1}{1-x^2}\,dx-\frac{5}{8}\int \frac{1}{1+x^2}\,dx\\ &= \frac{5x}{4}+\frac{x^5}{4\left(1-x^4\right)}-\frac{5}{8}\tan^{-1}(x)-\frac{5}{8}\tanh^{-1}(x)\end{aligned}$$

Mathematica [A] time = 0.02, size = 38, normalized size = 1.12

$$-\frac{x}{4\left(x^4-1\right)}+x+\frac{5}{16}\log(1-x)-\frac{5}{16}\log(x+1)-\frac{5}{8}\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8/(1 - 2*x^4 + x^8),x]
[Out] x - x/(4*(-1 + x^4)) - (5*ArcTan[x])/8 + (5*Log[1 - x])/16 - (5*Log[1 + x])
/16
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^8}{1-2x^4+x^8}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^8/(1 - 2*x^4 + x^8),x]
[Out] IntegrateAlgebraic[x^8/(1 - 2*x^4 + x^8), x]
```

fricas [B] time = 1.19, size = 49, normalized size = 1.44

$$\frac{16\,x^5-10\left(x^4-1\right)\arctan(x)-5\left(x^4-1\right)\log\left(x+1\right)+5\left(x^4-1\right)\log\left(x-1\right)-20\,x}{16\left(x^4-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(x^8-2*x^4+1),x, algorithm="fricas")
[Out] 1/16*(16*x^5 - 10*(x^4 - 1)*arctan(x) - 5*(x^4 - 1)*log(x + 1) + 5*(x^4 - 1
)*log(x - 1) - 20*x)/(x^4 - 1)
```

giac [A] time = 0.28, size = 30, normalized size = 0.88

$$x-\frac{x}{4\left(x^4-1\right)}-\frac{5}{8}\arctan(x)-\frac{5}{16}\log\left(|x+1|\right)+\frac{5}{16}\log\left(|x-1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(x^8-2*x^4+1),x, algorithm="giac")
```

```
[Out] x - 1/4*x/(x^4 - 1) - 5/8*arctan(x) - 5/16*log(abs(x + 1)) + 5/16*log(abs(x
 - 1))
```

maple [A] time = 0.01, size = 43, normalized size = 1.26

$$x+\frac{x}{8x^2+8}-\frac{5\arctan(x)}{8}+\frac{5\ln(x-1)}{16}-\frac{5\ln(x+1)}{16}-\frac{1}{16(x+1)}-\frac{1}{16(x-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(x^8-2*x^4+1),x)
```

```
[Out] x-1/16/(x+1)-5/16*ln(x+1)+1/8/(x^2+1)*x-5/8*arctan(x)-1/16/(x-1)+5/16*ln(x-
1)
```

maxima [A] time = 2.13, size = 28, normalized size = 0.82

$$x-\frac{x}{4\left(x^4-1\right)}-\frac{5}{8}\arctan(x)-\frac{5}{16}\log(x+1)+\frac{5}{16}\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(x^8-2*x^4+1),x, algorithm="maxima")
```

```
[Out] x - 1/4*x/(x^4 - 1) - 5/8*arctan(x) - 5/16*log(x + 1) + 5/16*log(x - 1)
```

mupad [B] time = 1.29, size = 26, normalized size = 0.76

$$x-\frac{5\,\mathrm{atan}(x)}{8}-\frac{x}{4\left(x^4-1\right)}+\frac{\mathrm{atan}\left(x\,1\mathrm{i}\right)\,5\mathrm{i}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(x^8 - 2*x^4 + 1),x)
```

```
[Out] x + (atan(x*1i)*5i)/8 - (5*atan(x))/8 - x/(4*(x^4 - 1))
```

sympy [A] time = 0.15, size = 32, normalized size = 0.94

$$x-\frac{x}{4x^4-4}+\frac{5\log(x-1)}{16}-\frac{5\log(x+1)}{16}-\frac{5\,\mathrm{atan}(x)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8/(x**8-2*x**4+1),x)
```

```
[Out] x - x/(4*x**4 - 4) + 5*log(x - 1)/16 - 5*log(x + 1)/16 - 5*atan(x)/8
```

3.245 $\int \frac{x^6}{1-2x^4+x^8}\,dx$

Optimal. Leaf size=29

$$\frac{x^3}{4(1-x^4)}+\frac{3}{8}\tan^{-1}(x)-\frac{3}{8}\tanh^{-1}(x)$$

Rubi [A] time = 0.01, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {28, 288, 298, 203, 206}

$$\frac{x^3}{4(1-x^4)}+\frac{3}{8}\tan^{-1}(x)-\frac{3}{8}\tanh^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[x^6/(1 - 2*x^4 + x^8),x]
```

```
[Out] x^3/(4*(1 - x^4)) + (3*ArcTan[x])/8 - (3*ArcTanh[x])/8
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rubi steps

$$\int \frac{x^6}{1-2x^4+x^8}\,dx = \int \frac{x^6}{\left(-1+x^4\right)^2}\,dx$$
$$= \frac{x^3}{4\left(1-x^4\right)} + \frac{3}{4}\int \frac{x^2}{-1+x^4}\,dx$$
$$= \frac{x^3}{4\left(1-x^4\right)} - \frac{3}{8}\int \frac{1}{1-x^2}\,dx + \frac{3}{8}\int \frac{1}{1+x^2}\,dx$$
$$= \frac{x^3}{4\left(1-x^4\right)} + \frac{3}{8}\tan^{-1}(x) - \frac{3}{8}\tanh^{-1}(x)$$

Mathematica [A] time = 0.01, size = 35, normalized size = 1.21

$$\frac{1}{16}\left(-\frac{4x^3}{x^4-1} + 3\log(1-x) - 3\log(x+1) + 6\tan^{-1}(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6/(1 - 2*x^4 + x^8),x]
```

```
[Out] ((-4*x^3)/(-1 + x^4) + 6*ArcTan[x] + 3*Log[1 - x] - 3*Log[1 + x])/16
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6}{1-2x^4+x^8}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^6/(1 - 2*x^4 + x^8),x]
```

```
[Out] IntegrateAlgebraic[x^6/(1 - 2*x^4 + x^8), x]
```

fricas [B] time = 1.23, size = 46, normalized size = 1.59

$$-\frac{4\,x^3 - 6\left(x^4-1\right)\arctan(x) + 3\left(x^4-1\right)\log\left(x+1\right) - 3\left(x^4-1\right)\log\left(x-1\right)}{16\left(x^4-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(x^8-2*x^4+1),x, algorithm="fricas")
```

```
[Out] -1/16*(4*x^3 - 6*(x^4 - 1)*arctan(x) + 3*(x^4 - 1)*log(x + 1) - 3*(x^4 - 1)
*log(x - 1))/(x^4 - 1)
```

giac [A] time = 0.40, size = 31, normalized size = 1.07

$$-\frac{x^3}{4\left(x^4-1\right)} + \frac{3}{8}\arctan(x) - \frac{3}{16}\log\left(|x+1|\right) + \frac{3}{16}\log\left(|x-1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(x^8-2*x^4+1),x, algorithm="giac")
```

```
[Out] -1/4*x^3/(x^4 - 1) + 3/8*arctan(x) - 3/16*log(abs(x + 1)) + 3/16*log(abs(x
- 1))
```

maple [A] time = 0.02, size = 42, normalized size = 1.45

$$-\frac{x}{8\left(x^2+1\right)}+\frac{3\arctan(x)}{8}+\frac{3\ln\left(x-1\right)}{16}-\frac{3\ln\left(x+1\right)}{16}-\frac{1}{16\left(x+1\right)}-\frac{1}{16\left(x-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(x^8-2*x^4+1),x)
```

```
[Out] -1/16/(x+1)-3/16*ln(x+1)-1/8/(x^2+1)*x+3/8*arctan(x)-1/16/(x-1)+3/16*ln(x-1
)
```

maxima [A] time = 2.03, size = 29, normalized size = 1.00

$$-\frac{x^3}{4\left(x^4-1\right)}+\frac{3}{8}\arctan(x)-\frac{3}{16}\log\left(x+1\right)+\frac{3}{16}\log\left(x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(x^8-2*x^4+1),x, algorithm="maxima")
```

```
[Out] -1/4*x^3/(x^4 - 1) + 3/8*arctan(x) - 3/16*log(x + 1) + 3/16*log(x - 1)
```

mupad [B] time = 0.03, size = 23, normalized size = 0.79

$$\frac{3\,\mathrm{atan}(x)}{8}-\frac{3\,\mathrm{atanh}(x)}{8}-\frac{x^3}{4\,\left(x^4-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(x^8 - 2*x^4 + 1),x)
```

```
[Out] (3*atan(x))/8 - (3*atanh(x))/8 - x^3/(4*(x^4 - 1))
```

sympy [A] time = 0.15, size = 32, normalized size = 1.10

$$-\frac{x^3}{4x^4-4}+\frac{3\log\left(x-1\right)}{16}-\frac{3\log\left(x+1\right)}{16}+\frac{3\,\mathrm{atan}\left(x\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(x**8-2*x**4+1),x)
```

```
[Out] -x**3/(4*x**4 - 4) + 3*log(x - 1)/16 - 3*log(x + 1)/16 + 3*atan(x)/8
```

3.246 $\int \frac{x^4}{1-2x^4+x^8}\,dx$

Optimal. Leaf size=27

$$\frac{x}{4\left(1-x^4\right)}-\frac{1}{8}\tan^{-1}(x)-\frac{1}{8}\tanh^{-1}(x)$$

Rubi [A] time = 0.01, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {28, 288, 212, 206, 203}

$$\frac{x}{4\left(1-x^4\right)}-\frac{1}{8}\tan^{-1}(x)-\frac{1}{8}\tanh^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(1 - 2*x^4 + x^8),x]
```

```
[Out] x/(4*(1 - x^4)) - ArcTan[x]/8 - ArcTanh[x]/8
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int \frac{x^4}{1-2x^4+x^8}\,dx = \int \frac{x^4}{\left(-1+x^4\right)^2}\,dx$$
$$= \frac{x}{4\left(1-x^4\right)} + \frac{1}{4}\int \frac{1}{-1+x^4}\,dx$$
$$= \frac{x}{4\left(1-x^4\right)} - \frac{1}{8}\int \frac{1}{1-x^2}\,dx - \frac{1}{8}\int \frac{1}{1+x^2}\,dx$$
$$= \frac{x}{4\left(1-x^4\right)} - \frac{1}{8}\tan^{-1}(x) - \frac{1}{8}\tanh^{-1}(x)$$

Mathematica [A] time = 0.01, size = 31, normalized size = 1.15

$$\frac{1}{16}\left(-\frac{4x}{x^4-1} + \log(1-x) - \log(x+1) - 2\tan^{-1}(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(1 - 2*x^4 + x^8),x]
[Out] ((-4*x)/(-1 + x^4) - 2*ArcTan[x] + Log[1 - x] - Log[1 + x])/16
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{1-2x^4+x^8}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^4/(1 - 2*x^4 + x^8),x]
[Out] IntegrateAlgebraic[x^4/(1 - 2*x^4 + x^8), x]
```

fricas [B] time = 1.27, size = 43, normalized size = 1.59

$$-\frac{2\left(x^4-1\right)\arctan(x) + \left(x^4-1\right)\log(x+1) - \left(x^4-1\right)\log(x-1) + 4x}{16\left(x^4-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(x^8-2*x^4+1),x, algorithm="fricas")
[Out] -1/16*(2*(x^4 - 1)*arctan(x) + (x^4 - 1)*log(x + 1) - (x^4 - 1)*log(x - 1)
+ 4*x)/(x^4 - 1)
```

giac [A] time = 0.51, size = 29, normalized size = 1.07

$$-\frac{x}{4\left(x^4-1\right)} - \frac{1}{8}\arctan(x) - \frac{1}{16}\log\left(|x+1|\right) + \frac{1}{16}\log\left(|x-1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(x^8-2*x^4+1),x, algorithm="giac")
[Out] -1/4*x/(x^4 - 1) - 1/8*arctan(x) - 1/16*log(abs(x + 1)) + 1/16*log(abs(x -
1))
```

maple [A] time = 0.01, size = 42, normalized size = 1.56

$$\frac{x}{8x^2+8}-\frac{\arctan(x)}{8}+\frac{\ln(x-1)}{16}-\frac{\ln(x+1)}{16}-\frac{1}{16(x+1)}-\frac{1}{16(x-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(x^8-2*x^4+1),x)
```

```
[Out] -1/16/(x+1)-1/16*ln(x+1)+1/8/(x^2+1)*x-1/8*arctan(x)-1/16/(x-1)+1/16*ln(x-1
)
```

maxima [A] time = 1.92, size = 27, normalized size = 1.00

$$-\frac{x}{4\left(x^4-1\right)}-\frac{1}{8}\arctan(x)-\frac{1}{16}\log(x+1)+\frac{1}{16}\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(x^8-2*x^4+1),x, algorithm="maxima")
```

```
[Out] -1/4*x/(x^4 - 1) - 1/8*arctan(x) - 1/16*log(x + 1) + 1/16*log(x - 1)
```

mupad [B] time = 0.03, size = 21, normalized size = 0.78

$$-\frac{\operatorname{atan}(x)}{8}-\frac{\operatorname{atanh}(x)}{8}-\frac{x}{4\left(x^4-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(x^8 - 2*x^4 + 1),x)
```

```
[Out] - atan(x)/8 - atanh(x)/8 - x/(4*(x^4 - 1))
```

sympy [A] time = 0.15, size = 26, normalized size = 0.96

$$-\frac{x}{4x^4-4}+\frac{\log(x-1)}{16}-\frac{\log(x+1)}{16}-\frac{\operatorname{atan}(x)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(x**8-2*x**4+1),x)
```

```
[Out] -x/(4*x**4 - 4) + log(x - 1)/16 - log(x + 1)/16 - atan(x)/8
```

3.247 $\int \frac{x^2}{1-2x^4+x^8} dx$

Optimal. Leaf size=29

$$\frac{x^3}{4\left(1-x^4\right)} - \frac{1}{8}\tan^{-1}(x) + \frac{1}{8}\tanh^{-1}(x)$$

Rubi [A] time = 0.01, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {28, 290, 298, 203, 206}

$$\frac{x^3}{4\left(1-x^4\right)} - \frac{1}{8}\tan^{-1}(x) + \frac{1}{8}\tanh^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(1 - 2*x^4 + x^8),x]
```

```
[Out] x^3/(4*(1 - x^4)) - ArcTan[x]/8 + ArcTanh[x]/8
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2}{1-2x^4+x^8}\,dx &= \int \frac{x^2}{\left(-1+x^4\right)^2}\,dx\\ &= \frac{x^3}{4\left(1-x^4\right)} - \frac{1}{4}\int \frac{x^2}{-1+x^4}\,dx\\ &= \frac{x^3}{4\left(1-x^4\right)} + \frac{1}{8}\int \frac{1}{1-x^2}\,dx - \frac{1}{8}\int \frac{1}{1+x^2}\,dx\\ &= \frac{x^3}{4\left(1-x^4\right)} - \frac{1}{8}\tan^{-1}(x) + \frac{1}{8}\tanh^{-1}(x)\end{aligned}$$

Mathematica [A] time = 0.01, size = 33, normalized size = 1.14

$$\frac{1}{16}\left(-\frac{4x^3}{x^4-1} - \log(1-x) + \log(x+1) - 2\tan^{-1}(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(1 - 2*x^4 + x^8),x]
[Out] ((-4*x^3)/(-1 + x^4) - 2*ArcTan[x] - Log[1 - x] + Log[1 + x])/16
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{1-2x^4+x^8}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^2/(1 - 2*x^4 + x^8),x]
[Out] IntegrateAlgebraic[x^2/(1 - 2*x^4 + x^8), x]
```

fricas [B] time = 1.22, size = 45, normalized size = 1.55

$$-\frac{4\,x^3 + 2\left(x^4-1\right)\arctan(x) - \left(x^4-1\right)\log\left(x+1\right) + \left(x^4-1\right)\log\left(x-1\right)}{16\left(x^4-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^8-2*x^4+1),x, algorithm="fricas")
[Out] -1/16*(4*x^3 + 2*(x^4 - 1)*arctan(x) - (x^4 - 1)*log(x + 1) + (x^4 - 1)*log
(x - 1))/(x^4 - 1)
```

giac [A] time = 0.34, size = 31, normalized size = 1.07

$$-\frac{x^3}{4\left(x^4-1\right)} - \frac{1}{8}\arctan(x) + \frac{1}{16}\log\left(|x+1|\right) - \frac{1}{16}\log\left(|x-1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^8-2*x^4+1),x, algorithm="giac")
[Out] -1/4*x^3/(x^4 - 1) - 1/8*arctan(x) + 1/16*log(abs(x + 1)) - 1/16*log(abs(x
- 1))
```

maple [A] time = 0.01, size = 42, normalized size = 1.45

$$-\frac{x}{8\left(x^{2}+1\right)}-\frac{\arctan(x)}{8}-\frac{\ln\left(x-1\right)}{16}+\frac{\ln\left(x+1\right)}{16}-\frac{1}{16\left(x+1\right)}-\frac{1}{16\left(x-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(x^8-2*x^4+1),x)
```

```
[Out] -1/16/(x+1)+1/16*ln(x+1)-1/8/(x^2+1)*x-1/8*arctan(x)-1/16/(x-1)-1/16*ln(x-1
)
```

maxima [A] time = 1.71, size = 29, normalized size = 1.00

$$-\frac{x^{3}}{4\left(x^{4}-1\right)}-\frac{1}{8}\arctan(x)+\frac{1}{16}\log\left(x+1\right)-\frac{1}{16}\log\left(x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^8-2*x^4+1),x, algorithm="maxima")
```

```
[Out] -1/4*x^3/(x^4 - 1) - 1/8*arctan(x) + 1/16*log(x + 1) - 1/16*log(x - 1)
```

mupad [B] time = 0.03, size = 23, normalized size = 0.79

$$\frac{\mathrm{atanh}(x)}{8}-\frac{\mathrm{atan}(x)}{8}-\frac{x^{3}}{4\left(x^{4}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(x^8 - 2*x^4 + 1),x)
```

```
[Out] atanh(x)/8 - atan(x)/8 - x^3/(4*(x^4 - 1))
```

sympy [A] time = 0.15, size = 27, normalized size = 0.93

$$-\frac{x^{3}}{4x^{4}-4}-\frac{\log\left(x-1\right)}{16}+\frac{\log\left(x+1\right)}{16}-\frac{\mathrm{atan}\left(x\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(x**8-2*x**4+1),x)
```

```
[Out] -x**3/(4*x**4 - 4) - log(x - 1)/16 + log(x + 1)/16 - atan(x)/8
```

3.248 $\int \frac{1}{1-2x^4+x^8} dx$

Optimal. Leaf size=27

$$\frac{x}{4(1-x^4)} + \frac{3}{8}\tan^{-1}(x) + \frac{3}{8}\tanh^{-1}(x)$$

Rubi [A] time = 0.01, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {28, 199, 212, 206, 203}

$$\frac{x}{4(1-x^4)} + \frac{3}{8}\tan^{-1}(x) + \frac{3}{8}\tanh^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - 2*x^4 + x^8)^(-1),x]
```

```
[Out] x/(4*(1 - x^4)) + (3*ArcTan[x])/8 + (3*ArcTanh[x])/8
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{1-2x^4+x^8}\,dx &= \int \frac{1}{\left(-1+x^4\right)^2}\,dx\\ &= \frac{x}{4\left(1-x^4\right)}-\frac{3}{4}\int \frac{1}{-1+x^4}\,dx\\ &= \frac{x}{4\left(1-x^4\right)}+\frac{3}{8}\int \frac{1}{1-x^2}\,dx+\frac{3}{8}\int \frac{1}{1+x^2}\,dx\\ &= \frac{x}{4\left(1-x^4\right)}+\frac{3}{8}\tan^{-1}(x)+\frac{3}{8}\tanh^{-1}(x)\end{aligned}$$

Mathematica [A] time = 0.01, size = 33, normalized size = 1.22

$$\frac{1}{16}\left(-\frac{4x}{x^4-1}-3\log(1-x)+3\log(x+1)+6\tan^{-1}(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - 2*x^4 + x^8)^(-1),x]
[Out] ((-4*x)/(-1 + x^4) + 6*ArcTan[x] - 3*Log[1 - x] + 3*Log[1 + x])/16
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{1-2x^4+x^8}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(1 - 2*x^4 + x^8)^(-1),x]
[Out] IntegrateAlgebraic[(1 - 2*x^4 + x^8)^(-1), x]
```

fricas [B] time = 1.30, size = 44, normalized size = 1.63

$$\frac{6\left(x^4-1\right)\arctan(x)+3\left(x^4-1\right)\log(x+1)-3\left(x^4-1\right)\log(x-1)-4x}{16\left(x^4-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^8-2*x^4+1),x, algorithm="fricas")
[Out] 1/16*(6*(x^4 - 1)*arctan(x) + 3*(x^4 - 1)*log(x + 1) - 3*(x^4 - 1)*log(x -
1) - 4*x)/(x^4 - 1)
```

giac [A] time = 0.37, size = 29, normalized size = 1.07

$$-\frac{x}{4\left(x^4-1\right)}+\frac{3}{8}\arctan(x)+\frac{3}{16}\log\left(|x+1|\right)-\frac{3}{16}\log\left(|x-1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^8-2*x^4+1),x, algorithm="giac")
[Out] -1/4*x/(x^4 - 1) + 3/8*arctan(x) + 3/16*log(abs(x + 1)) - 3/16*log(abs(x -
1))
```

maple [A] time = 0.01, size = 42, normalized size = 1.56

$$\frac{x}{8x^2+8}+\frac{3\arctan(x)}{8}-\frac{3\ln(x-1)}{16}+\frac{3\ln(x+1)}{16}-\frac{1}{16(x+1)}-\frac{1}{16(x-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^8-2*x^4+1),x)
```

```
[Out] -1/16/(x+1)+3/16*ln(x+1)+1/8/(x^2+1)*x+3/8*arctan(x)-1/16/(x-1)-3/16*ln(x-1
)
```

maxima [A] time = 2.13, size = 27, normalized size = 1.00

$$-\frac{x}{4\left(x^4-1\right)}+\frac{3}{8}\arctan(x)+\frac{3}{16}\log(x+1)-\frac{3}{16}\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^8-2*x^4+1),x, algorithm="maxima")
```

```
[Out] -1/4*x/(x^4 - 1) + 3/8*arctan(x) + 3/16*log(x + 1) - 3/16*log(x - 1)
```

mupad [B] time = 0.03, size = 21, normalized size = 0.78

$$\frac{3\,\mathrm{atan}(x)}{8}+\frac{3\,\mathrm{atanh}(x)}{8}-\frac{x}{4\left(x^4-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^8 - 2*x^4 + 1),x)
```

```
[Out] (3*atan(x))/8 + (3*atanh(x))/8 - x/(4*(x^4 - 1))
```

sympy [A] time = 0.16, size = 31, normalized size = 1.15

$$-\frac{x}{4x^4-4}-\frac{3\log(x-1)}{16}+\frac{3\log(x+1)}{16}+\frac{3\,\mathrm{atan}(x)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**8-2*x**4+1),x)
```

```
[Out] -x/(4*x**4 - 4) - 3*log(x - 1)/16 + 3*log(x + 1)/16 + 3*atan(x)/8
```

3.249 $\int \frac{1}{x^2\left(1-2x^4+x^8\right)}\,dx$

Optimal. Leaf size=36

$$\frac{1}{4x\left(1-x^4\right)}-\frac{5}{4x}-\frac{5}{8}\tan^{-1}(x)+\frac{5}{8}\tanh^{-1}(x)$$

Rubi [A] time = 0.01, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {28, 290, 325, 298, 203, 206}

$$\frac{1}{4x\left(1-x^4\right)}-\frac{5}{4x}-\frac{5}{8}\tan^{-1}(x)+\frac{5}{8}\tanh^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(1 - 2*x^4 + x^8)),x]
```

```
[Out] -5/(4*x) + 1/(4*x*(1 - x^4)) - (5*ArcTan[x])/8 + (5*ArcTanh[x])/8
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
```

```
x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x^2\left(1-2x^4+x^8\right)}\,dx &= \int \frac{1}{x^2\left(-1+x^4\right)^2}\,dx\\
&= \frac{1}{4x\left(1-x^4\right)}-\frac{5}{4}\int \frac{1}{x^2\left(-1+x^4\right)}\,dx\\
&= -\frac{5}{4x}+\frac{1}{4x\left(1-x^4\right)}-\frac{5}{4}\int \frac{x^2}{-1+x^4}\,dx\\
&= -\frac{5}{4x}+\frac{1}{4x\left(1-x^4\right)}+\frac{5}{8}\int \frac{1}{1-x^2}\,dx-\frac{5}{8}\int \frac{1}{1+x^2}\,dx\\
&= -\frac{5}{4x}+\frac{1}{4x\left(1-x^4\right)}-\frac{5}{8}\tan^{-1}(x)+\frac{5}{8}\tanh^{-1}(x)
\end{aligned}
$$

Mathematica [A] time = 0.02, size = 40, normalized size = 1.11

$$\frac{1}{16}\left(-\frac{4x^3}{x^4-1}-\frac{16}{x}-5\log(1-x)+5\log(x+1)-10\tan^{-1}(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(1 - 2*x^4 + x^8)),x]
[Out] (-16/x - (4*x^3)/(-1 + x^4) - 10*ArcTan[x] - 5*Log[1 - x] + 5*Log[1 + x])/1
6
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2\left(1-2x^4+x^8\right)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x^2*(1 - 2*x^4 + x^8)),x]
[Out] IntegrateAlgebraic[1/(x^2*(1 - 2*x^4 + x^8)), x]
```

fricas [B] time = 1.26, size = 55, normalized size = 1.53

$$-\frac{20\,x^4+10\left(x^5-x\right)\arctan(x)-5\left(x^5-x\right)\log\left(x+1\right)+5\left(x^5-x\right)\log\left(x-1\right)-16}{16\left(x^5-x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(x^8-2*x^4+1),x, algorithm="fricas")
[Out] -1/16*(20*x^4 + 10*(x^5 - x)*arctan(x) - 5*(x^5 - x)*log(x + 1) + 5*(x^5 -
x)*log(x - 1) - 16)/(x^5 - x)
```

giac [A] time = 0.37, size = 37, normalized size = 1.03

$$-\frac{5\,x^4-4}{4\left(x^5-x\right)}-\frac{5}{8}\arctan(x)+\frac{5}{16}\log\left(|x+1|\right)-\frac{5}{16}\log\left(|x-1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(x^8-2*x^4+1),x, algorithm="giac")
```

```
[Out] -1/4*(5*x^4 - 4)/(x^5 - x) - 5/8*arctan(x) + 5/16*log(abs(x + 1)) - 5/16*lo
g(abs(x - 1))
```

maple [A] time = 0.01, size = 47, normalized size = 1.31

$$-\frac{x}{8\left(x^{2}+1\right)}-\frac{5\arctan(x)}{8}-\frac{5\ln(x-1)}{16}+\frac{5\ln(x+1)}{16}-\frac{1}{x}-\frac{1}{16\,(x+1)}-\frac{1}{16\,(x-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(x^8-2*x^4+1),x)
```

```
[Out] -1/x-1/16/(x+1)+5/16*ln(x+1)-1/8/(x^2+1)*x-5/8*arctan(x)-1/16/(x-1)-5/16*ln
(x-1)
```

maxima [A] time = 2.09, size = 35, normalized size = 0.97

$$-\frac{5\,x^{4}-4}{4\left(x^{5}-x\right)}-\frac{5}{8}\arctan(x)+\frac{5}{16}\log(x+1)-\frac{5}{16}\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(x^8-2*x^4+1),x, algorithm="maxima")
```

```
[Out] -1/4*(5*x^4 - 4)/(x^5 - x) - 5/8*arctan(x) + 5/16*log(x + 1) - 5/16*log(x -
 1)
```

mupad [B] time = 0.04, size = 26, normalized size = 0.72

$$\frac{5\,\mathrm{atanh}(x)}{8}-\frac{5\,\mathrm{atan}(x)}{8}+\frac{\frac{5\,x^{4}}{4}-1}{x-x^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(x^8 - 2*x^4 + 1)),x)
```

```
[Out] (5*atanh(x))/8 - (5*atan(x))/8 + ((5*x^4)/4 - 1)/(x - x^5)
```

sympy [A] time = 0.18, size = 37, normalized size = 1.03

$$\frac{4-5x^{4}}{4x^{5}-4x}-\frac{5\log(x-1)}{16}+\frac{5\log(x+1)}{16}-\frac{5\,\mathrm{atan}(x)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(x**8-2*x**4+1),x)
```

```
[Out] (4 - 5*x**4)/(4*x**5 - 4*x) - 5*log(x - 1)/16 + 5*log(x + 1)/16 - 5*atan(x)
/8
```

3.250 $\int \frac{1}{x^4(1-2x^4+x^8)}\,dx$

Optimal. Leaf size=36

$$-\frac{7}{12x^3}+\frac{1}{4x^3\left(1-x^4\right)}+\frac{7}{8}\tan^{-1}(x)+\frac{7}{8}\tanh^{-1}(x)$$

Rubi [A] time = 0.01, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {28, 290, 325, 212, 206, 203}

$$\frac{1}{4x^3\left(1-x^4\right)}-\frac{7}{12x^3}+\frac{7}{8}\tan^{-1}(x)+\frac{7}{8}\tanh^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(1 - 2*x^4 + x^8)),x]
```

```
[Out] -7/(12*x^3) + 1/(4*x^3*(1 - x^4)) + (7*ArcTan[x])/8 + (7*ArcTanh[x])/8
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
```

```
x]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^4\left(1-2x^4+x^8\right)}\,dx &= \int \frac{1}{x^4\left(-1+x^4\right)^2}\,dx\\
&= \frac{1}{4x^3\left(1-x^4\right)} - \frac{7}{4}\int \frac{1}{x^4\left(-1+x^4\right)}\,dx\\
&= -\frac{7}{12x^3} + \frac{1}{4x^3\left(1-x^4\right)} - \frac{7}{4}\int \frac{1}{-1+x^4}\,dx\\
&= -\frac{7}{12x^3} + \frac{1}{4x^3\left(1-x^4\right)} + \frac{7}{8}\int \frac{1}{1-x^2}\,dx + \frac{7}{8}\int \frac{1}{1+x^2}\,dx\\
&= -\frac{7}{12x^3} + \frac{1}{4x^3\left(1-x^4\right)} + \frac{7}{8}\tan^{-1}(x) + \frac{7}{8}\tanh^{-1}(x)
\end{aligned}$$

Mathematica [A] time = 0.02, size = 38, normalized size = 1.06

$$\frac{1}{48}\left(-\frac{12x}{x^4-1}-\frac{16}{x^3}-21\log(1-x)+21\log(x+1)+42\tan^{-1}(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(1 - 2*x^4 + x^8)),x]
```

```
[Out] (-16/x^3 - (12*x)/(-1 + x^4) + 42*ArcTan[x] - 21*Log[1 - x] + 21*Log[1 + x]
)/48
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^4\left(1-2x^4+x^8\right)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x^4*(1 - 2*x^4 + x^8)),x]
```

```
[Out] IntegrateAlgebraic[1/(x^4*(1 - 2*x^4 + x^8)), x]
```

fricas [B] time = 1.22, size = 63, normalized size = 1.75

$$-\frac{28\,x^4-42\left(x^7-x^3\right)\arctan(x)-21\left(x^7-x^3\right)\log\left(x+1\right)+21\left(x^7-x^3\right)\log\left(x-1\right)-16}{48\left(x^7-x^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(x^8-2*x^4+1),x, algorithm="fricas")
```

```
[Out] -1/48*(28*x^4 - 42*(x^7 - x^3)*arctan(x) - 21*(x^7 - x^3)*log(x + 1) + 21*(
x^7 - x^3)*log(x - 1) - 16)/(x^7 - x^3)
```

giac [A] time = 0.46, size = 34, normalized size = 0.94

$$-\frac{x}{4\left(x^4-1\right)}-\frac{1}{3\,x^3}+\frac{7}{8}\arctan(x)+\frac{7}{16}\log\left(|x+1|\right)-\frac{7}{16}\log\left(|x-1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(x^8-2*x^4+1),x, algorithm="giac")
```

```
[Out] -1/4*x/(x^4 - 1) - 1/3/x^3 + 7/8*arctan(x) + 7/16*log(abs(x + 1)) - 7/16*lo
g(abs(x - 1))
```

maple [A] time = 0.01, size = 47, normalized size = 1.31

$$\frac{x}{8x^2+8}+\frac{7\arctan(x)}{8}-\frac{7\ln(x-1)}{16}+\frac{7\ln(x+1)}{16}-\frac{1}{3x^3}-\frac{1}{16(x+1)}-\frac{1}{16(x-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(x^8-2*x^4+1),x)
```

```
[Out] -1/3/x^3-1/16/(x+1)+7/16*ln(x+1)+1/8/(x^2+1)*x+7/8*arctan(x)-1/16/(x-1)-7/1
6*ln(x-1)
```

maxima [A] time = 1.98, size = 37, normalized size = 1.03

$$-\frac{7x^4-4}{12\left(x^7-x^3\right)}+\frac{7}{8}\arctan(x)+\frac{7}{16}\log(x+1)-\frac{7}{16}\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(x^8-2*x^4+1),x, algorithm="maxima")
```

```
[Out] -1/12*(7*x^4 - 4)/(x^7 - x^3) + 7/8*arctan(x) + 7/16*log(x + 1) - 7/16*log(
x - 1)
```

mupad [B] time = 1.29, size = 28, normalized size = 0.78

$$\frac{7\,\mathrm{atan}(x)}{8}+\frac{7\,\mathrm{atanh}(x)}{8}+\frac{\frac{7x^4}{12}-\frac{1}{3}}{x^3-x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(x^8 - 2*x^4 + 1)),x)
```

```
[Out] (7*atan(x))/8 + (7*atanh(x))/8 + ((7*x^4)/12 - 1/3)/(x^3 - x^7)
```

sympy [A] time = 0.18, size = 39, normalized size = 1.08

$$\frac{4-7x^4}{12x^7-12x^3}-\frac{7\log(x-1)}{16}+\frac{7\log(x+1)}{16}+\frac{7\,\mathrm{atan}(x)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(x**8-2*x**4+1),x)
```

```
[Out] (4 - 7*x**4)/(12*x**7 - 12*x**3) - 7*log(x - 1)/16 + 7*log(x + 1)/16 + 7*at
an(x)/8
```

3.251 $\int \frac{1}{x^6(1-2x^4+x^8)}\,dx$

Optimal. Leaf size=43

$$-\frac{9}{20x^5}+\frac{1}{4x^5(1-x^4)}-\frac{9}{4x}-\frac{9}{8}\tan^{-1}(x)+\frac{9}{8}\tanh^{-1}(x)$$

Rubi [A] time = 0.01, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {28, 290, 325, 298, 203, 206}

$$\frac{1}{4x^5(1-x^4)}-\frac{9}{20x^5}-\frac{9}{4x}-\frac{9}{8}\tan^{-1}(x)+\frac{9}{8}\tanh^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^6*(1 - 2*x^4 + x^8)),x]
```

```
[Out] -9/(20*x^5) - 9/(4*x) + 1/(4*x^5*(1 - x^4)) - (9*ArcTan[x])/8 + (9*ArcTanh[
x])/8
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
```

```
b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
x]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^6\left(1-2x^4+x^8\right)}\,dx &= \int \frac{1}{x^6\left(-1+x^4\right)^2}\,dx\\
&= \frac{1}{4x^5\left(1-x^4\right)}-\frac{9}{4}\int \frac{1}{x^6\left(-1+x^4\right)}\,dx\\
&= -\frac{9}{20x^5}+\frac{1}{4x^5\left(1-x^4\right)}-\frac{9}{4}\int \frac{1}{x^2\left(-1+x^4\right)}\,dx\\
&= -\frac{9}{20x^5}-\frac{9}{4x}+\frac{1}{4x^5\left(1-x^4\right)}-\frac{9}{4}\int \frac{x^2}{-1+x^4}\,dx\\
&= -\frac{9}{20x^5}-\frac{9}{4x}+\frac{1}{4x^5\left(1-x^4\right)}+\frac{9}{8}\int \frac{1}{1-x^2}\,dx-\frac{9}{8}\int \frac{1}{1+x^2}\,dx\\
&= -\frac{9}{20x^5}-\frac{9}{4x}+\frac{1}{4x^5\left(1-x^4\right)}-\frac{9}{8}\tan^{-1}(x)+\frac{9}{8}\tanh^{-1}(x)
\end{aligned}$$

Mathematica [A] time = 0.02, size = 51, normalized size = 1.19

$$-\frac{1}{5x^5}-\frac{x^3}{4\left(x^4-1\right)}-\frac{2}{x}-\frac{9}{16}\log(1-x)+\frac{9}{16}\log(x+1)-\frac{9}{8}\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^6*(1 - 2*x^4 + x^8)),x]
```

```
[Out] -1/5*1/x^5 - 2/x - x^3/(4*(-1 + x^4)) - (9*ArcTan[x])/8 - (9*Log[1 - x])/16
 + (9*Log[1 + x])/16
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^6\left(1-2x^4+x^8\right)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x^6*(1 - 2*x^4 + x^8)),x]
```

```
[Out] IntegrateAlgebraic[1/(x^6*(1 - 2*x^4 + x^8)), x]
```

fricas [B] time = 1.36, size = 68, normalized size = 1.58

$$-\frac{180\,x^8-144\,x^4+90\left(x^9-x^5\right)\arctan(x)-45\left(x^9-x^5\right)\log\left(x+1\right)+45\left(x^9-x^5\right)\log\left(x-1\right)-16}{80\left(x^9-x^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(x^8-2*x^4+1),x, algorithm="fricas")
```

```
[Out] -1/80*(180*x^8 - 144*x^4 + 90*(x^9 - x^5)*arctan(x) - 45*(x^9 - x^5)*log(x
+ 1) + 45*(x^9 - x^5)*log(x - 1) - 16)/(x^9 - x^5)
```

giac [A] time = 0.45, size = 43, normalized size = 1.00

$$-\frac{x^3}{4\left(x^4-1\right)}-\frac{10\,x^4+1}{5\,x^5}-\frac{9}{8}\arctan(x)+\frac{9}{16}\log\left(|x+1|\right)-\frac{9}{16}\log\left(|x-1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(x^8-2*x^4+1),x, algorithm="giac")
```

```
[Out] -1/4*x^3/(x^4 - 1) - 1/5*(10*x^4 + 1)/x^5 - 9/8*arctan(x) + 9/16*log(abs(x
+ 1)) - 9/16*log(abs(x - 1))
```

maple [A] time = 0.02, size = 52, normalized size = 1.21

$$-\frac{x}{8\left(x^2+1\right)}-\frac{9\arctan(x)}{8}-\frac{9\ln\left(x-1\right)}{16}+\frac{9\ln\left(x+1\right)}{16}-\frac{2}{x}-\frac{1}{5x^5}-\frac{1}{16\left(x+1\right)}-\frac{1}{16\left(x-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^6/(x^8-2*x^4+1),x)
```

```
[Out] -1/5/x^5-2/x-1/16/(x+1)+9/16*ln(x+1)-1/8/(x^2+1)*x-9/8*arctan(x)-1/16/(x-1)
-9/16*ln(x-1)
```

maxima [A] time = 2.43, size = 42, normalized size = 0.98

$$-\frac{45\,x^8-36\,x^4-4}{20\left(x^9-x^5\right)}-\frac{9}{8}\arctan(x)+\frac{9}{16}\log\left(x+1\right)-\frac{9}{16}\log\left(x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(x^8-2*x^4+1),x, algorithm="maxima")
```

```
[Out] -1/20*(45*x^8 - 36*x^4 - 4)/(x^9 - x^5) - 9/8*arctan(x) + 9/16*log(x + 1) -
 9/16*log(x - 1)
```

mupad [B] time = 0.04, size = 34, normalized size = 0.79

$$\frac{9\,\mathrm{atanh}(x)}{8}-\frac{9\,\mathrm{atan}(x)}{8}-\frac{-\frac{9\,x^8}{4}+\frac{9\,x^4}{5}+\frac{1}{5}}{x^5-x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^6*(x^8 - 2*x^4 + 1)),x)
```

```
[Out] (9*atanh(x))/8 - (9*atan(x))/8 - ((9*x^4)/5 - (9*x^8)/4 + 1/5)/(x^5 - x^9)
```

sympy [A] time = 0.20, size = 44, normalized size = 1.02

$$-\frac{9\log\left(x-1\right)}{16}+\frac{9\log\left(x+1\right)}{16}-\frac{9\,\mathrm{atan}\left(x\right)}{8}+\frac{-45x^8+36x^4+4}{20x^9-20x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**6/(x**8-2*x**4+1),x)
```

```
[Out] -9*log(x - 1)/16 + 9*log(x + 1)/16 - 9*atan(x)/8 + (-45*x**8 + 36*x**4 + 4)
/(20*x**9 - 20*x**5)
```

3.252 $\int \frac{1}{x^8(1-2x^4+x^8)}\,dx$

Optimal. Leaf size=43

$$-\frac{11}{28x^7}-\frac{11}{12x^3}+\frac{1}{4x^7\left(1-x^4\right)}+\frac{11}{8}\tan^{-1}(x)+\frac{11}{8}\tanh^{-1}(x)$$

Rubi [A] time = 0.01, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {28, 290, 325, 212, 206, 203}

$$\frac{1}{4x^7\left(1-x^4\right)}-\frac{11}{12x^3}-\frac{11}{28x^7}+\frac{11}{8}\tan^{-1}(x)+\frac{11}{8}\tanh^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^8*(1 - 2*x^4 + x^8)),x]
```

```
[Out] -11/(28*x^7) - 11/(12*x^3) + 1/(4*x^7*(1 - x^4)) + (11*ArcTan[x])/8 + (11*A
rcTanh[x])/8
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
```

```
b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^8\left(1-2x^4+x^8\right)}\,dx &= \int \frac{1}{x^8\left(-1+x^4\right)^2}\,dx\\ &= \frac{1}{4x^7\left(1-x^4\right)}-\frac{11}{4}\int \frac{1}{x^8\left(-1+x^4\right)}\,dx\\ &= -\frac{11}{28x^7}+\frac{1}{4x^7\left(1-x^4\right)}-\frac{11}{4}\int \frac{1}{x^4\left(-1+x^4\right)}\,dx\\ &= -\frac{11}{28x^7}-\frac{11}{12x^3}+\frac{1}{4x^7\left(1-x^4\right)}-\frac{11}{4}\int \frac{1}{-1+x^4}\,dx\\ &= -\frac{11}{28x^7}-\frac{11}{12x^3}+\frac{1}{4x^7\left(1-x^4\right)}+\frac{11}{8}\int \frac{1}{1-x^2}\,dx+\frac{11}{8}\int \frac{1}{1+x^2}\,dx\\ &= -\frac{11}{28x^7}-\frac{11}{12x^3}+\frac{1}{4x^7\left(1-x^4\right)}+\frac{11}{8}\tan^{-1}(x)+\frac{11}{8}\tanh^{-1}(x)\end{aligned}$$

Mathematica [A] time = 0.02, size = 43, normalized size = 1.00

$$\frac{1}{336}\left(-\frac{48}{x^7}-\frac{84x}{x^4-1}-\frac{224}{x^3}-231\log(1-x)+231\log(x+1)+462\tan^{-1}(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^8*(1 - 2*x^4 + x^8)),x]
```

```
[Out] (-48/x^7 - 224/x^3 - (84*x)/(-1 + x^4) + 462*ArcTan[x] - 231*Log[1 - x] + 2
31*Log[1 + x])/336
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^8\left(1-2x^4+x^8\right)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x^8*(1 - 2*x^4 + x^8)),x]
```

```
[Out] IntegrateAlgebraic[1/(x^8*(1 - 2*x^4 + x^8)), x]
```

fricas [B] time = 1.18, size = 68, normalized size = 1.58

$$-\frac{308\,x^8-176\,x^4-462\left(x^{11}-x^7\right)\arctan(x)-231\left(x^{11}-x^7\right)\log\left(x+1\right)+231\left(x^{11}-x^7\right)\log\left(x-1\right)-48}{336\left(x^{11}-x^7\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^8/(x^8-2*x^4+1),x, algorithm="fricas")
```

```
[Out] -1/336*(308*x^8 - 176*x^4 - 462*(x^11 - x^7)*arctan(x) - 231*(x^11 - x^7)*l
og(x + 1) + 231*(x^11 - x^7)*log(x - 1) - 48)/(x^11 - x^7)
```

giac [A] time = 0.45, size = 41, normalized size = 0.95

$$-\frac{x}{4\left(x^4-1\right)}-\frac{14\,x^4+3}{21\,x^7}+\frac{11}{8}\,\arctan(x)+\frac{11}{16}\,\log\left(|x+1|\right)-\frac{11}{16}\,\log\left(|x-1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^8/(x^8-2*x^4+1),x, algorithm="giac")
```

```
[Out] -1/4*x/(x^4 - 1) - 1/21*(14*x^4 + 3)/x^7 + 11/8*arctan(x) + 11/16*log(abs(x
 + 1)) - 11/16*log(abs(x - 1))
```

maple [A] time = 0.02, size = 52, normalized size = 1.21

$$\frac{x}{8x^2+8}+\frac{11\arctan(x)}{8}-\frac{11\ln(x-1)}{16}+\frac{11\ln(x+1)}{16}-\frac{2}{3x^3}-\frac{1}{7x^7}-\frac{1}{16\,(x+1)}-\frac{1}{16\,(x-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^8/(x^8-2*x^4+1),x)
```

```
[Out] -1/7/x^7-2/3/x^3-1/16/(x+1)+11/16*ln(x+1)+1/8/(x^2+1)*x+11/8*arctan(x)-1/16
/(x-1)-11/16*ln(x-1)
```

maxima [A] time = 1.94, size = 42, normalized size = 0.98

$$-\frac{77\,x^8-44\,x^4-12}{84\left(x^{11}-x^7\right)}+\frac{11}{8}\,\arctan(x)+\frac{11}{16}\,\log\left(x+1\right)-\frac{11}{16}\,\log\left(x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^8/(x^8-2*x^4+1),x, algorithm="maxima")
```

```
[Out] -1/84*(77*x^8 - 44*x^4 - 12)/(x^11 - x^7) + 11/8*arctan(x) + 11/16*log(x +
1) - 11/16*log(x - 1)
```

mupad [B] time = 0.05, size = 34, normalized size = 0.79

$$\frac{11\,\mathrm{atan}(x)}{8}+\frac{11\,\mathrm{atanh}(x)}{8}-\frac{-\frac{11\,x^8}{12}+\frac{11\,x^4}{21}+\frac{1}{7}}{x^7-x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^8*(x^8 - 2*x^4 + 1)),x)
```

```
[Out] (11*atan(x))/8 + (11*atanh(x))/8 - ((11*x^4)/21 - (11*x^8)/12 + 1/7)/(x^7 -
 x^11)
```

sympy [A] time = 0.21, size = 44, normalized size = 1.02

$$-\frac{11\log\left(x-1\right)}{16}+\frac{11\log\left(x+1\right)}{16}+\frac{11\,\mathrm{atan}\left(x\right)}{8}+\frac{-77x^8+44x^4+12}{84x^{11}-84x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**8/(x**8-2*x**4+1),x)
```

```
[Out] -11*log(x - 1)/16 + 11*log(x + 1)/16 + 11*atan(x)/8 + (-77*x**8 + 44*x**4 +
 12)/(84*x**11 - 84*x**7)
```

3.253 $\int \frac{x^{11}}{a+bx^4+cx^8}\,dx$

Optimal. Leaf size=81

$$-\frac{\left(b^2-2ac\right)\tanh^{-1}\left(\frac{b+2cx^4}{\sqrt{b^2-4ac}}\right)}{4c^2\sqrt{b^2-4ac}}-\frac{b\log\left(a+bx^4+cx^8\right)}{8c^2}+\frac{x^4}{4c}$$

Rubi [A] time = 0.08, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1357, 703, 634, 618, 206, 628}

$$-\frac{\left(b^2-2ac\right)\tanh^{-1}\left(\frac{b+2cx^4}{\sqrt{b^2-4ac}}\right)}{4c^2\sqrt{b^2-4ac}}-\frac{b\log\left(a+bx^4+cx^8\right)}{8c^2}+\frac{x^4}{4c}$$

Antiderivative was successfully verified.

```
[In] Int[x^11/(a + b*x^4 + c*x^8),x]
```

```
[Out] x^4/(4*c) - ((b^2 - 2*a*c)*ArcTanh[(b + 2*c*x^4)/Sqrt[b^2 - 4*a*c]])/(4*c^2
*Sqrt[b^2 - 4*a*c]) - (b*Log[a + b*x^4 + c*x^8])/(8*c^2)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 703

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Simp[(e*(d + e*x)^(m - 1))/(c*(m - 1)), x] + Dist[1/c, Int[((d + e*x)^
(m - 2)*Simp[c*d^2 - a*e^2 + e*(2*c*d - b*e)*x, x])/(a + b*x + c*x^2), x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && GtQ[m, 1]
```

Rule 1357

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
```

```
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^{11}}{a+bx^4+cx^8}\,dx &= \frac{1}{4}\text{Subst}\left(\int \frac{x^2}{a+bx+cx^2}\,dx, x, x^4\right)\\
&= \frac{x^4}{4c} + \frac{\text{Subst}\left(\int \frac{-a-bx}{a+bx+cx^2}\,dx, x, x^4\right)}{4c}\\
&= \frac{x^4}{4c} - \frac{b\,\text{Subst}\left(\int \frac{b+2cx}{a+bx+cx^2}\,dx, x, x^4\right)}{8c^2} + \frac{\left(b^2-2ac\right)\text{Subst}\left(\int \frac{1}{a+bx+cx^2}\,dx, x, x^4\right)}{8c^2}\\
&= \frac{x^4}{4c} - \frac{b\log\left(a+bx^4+cx^8\right)}{8c^2} - \frac{\left(b^2-2ac\right)\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx, x, b+2cx^4\right)}{4c^2}\\
&= \frac{x^4}{4c} - \frac{\left(b^2-2ac\right)\tanh^{-1}\left(\frac{b+2cx^4}{\sqrt{b^2-4ac}}\right)}{4c^2\sqrt{b^2-4ac}} - \frac{b\log\left(a+bx^4+cx^8\right)}{8c^2}
\end{aligned}
$$

Mathematica [A] time = 0.05, size = 78, normalized size = 0.96

$$\frac{\frac{2\left(b^2-2ac\right)\tan^{-1}\left(\frac{b+2cx^4}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}} - b\log\left(a+bx^4+cx^8\right) + 2cx^4}{8c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^11/(a + b*x^4 + c*x^8),x]
```

```
[Out] (2*c*x^4 + (2*(b^2 - 2*a*c)*ArcTan[(b + 2*c*x^4)/Sqrt[-b^2 + 4*a*c]])/Sqrt[
-b^2 + 4*a*c] - b*Log[a + b*x^4 + c*x^8])/(8*c^2)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{11}}{a+bx^4+cx^8}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^11/(a + b*x^4 + c*x^8),x]
```

```
[Out] IntegrateAlgebraic[x^11/(a + b*x^4 + c*x^8), x]
```

fricas [A] time = 1.37, size = 254, normalized size = 3.14

$$\left[\frac{2\left(b^2c-4ac^2\right)x^4-\left(b^2-2ac\right)\sqrt{b^2-4ac}\log\left(\frac{2c^2x^8+2bcx^4+b^2-2ac+\left(2cx^4+b\right)\sqrt{b^2-4ac}}{cx^8+bx^4+a}\right)-\left(b^3-4abc\right)\log\left(cx^8+bx^4+a\right)}{8\left(b^2c^2-4ac^3\right)}, \frac{2\left(b^2c-4ac^2\right)x^4-2\left(b^2-2ac\right)\sqrt{-b^2+4ac}\arctan\left(-\frac{\left(2cx^4+b\right)\sqrt{-b^2+4ac}}{b^2-4ac}\right)-\left(b^3-4abc\right)\log\left(cx^8+bx^4+a\right)}{8\left(b^2c^2-4ac^3\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(c*x^8+b*x^4+a),x, algorithm="fricas")
```

```
[Out] [1/8*(2*(b^2*c - 4*a*c^2)*x^4 - (b^2 - 2*a*c)*sqrt(b^2 - 4*a*c)*log((2*c^2*
x^8 + 2*b*c*x^4 + b^2 - 2*a*c + (2*c*x^4 + b)*sqrt(b^2 - 4*a*c))/(c*x^8 + b
*x^4 + a)) - (b^3 - 4*a*b*c)*log(c*x^8 + b*x^4 + a))/(b^2*c^2 - 4*a*c^3), 1
/8*(2*(b^2*c - 4*a*c^2)*x^4 - 2*(b^2 - 2*a*c)*sqrt(-b^2 + 4*a*c)*arctan(-(2
*c*x^4 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) - (b^3 - 4*a*b*c)*log(c*x^8 +
 b*x^4 + a))/(b^2*c^2 - 4*a*c^3)]
```

giac [A] time = 17.07, size = 75, normalized size = 0.93

$$\frac{x^4}{4c} - \frac{b\log\left(cx^8 + bx^4 + a\right)}{8c^2} + \frac{\left(b^2 - 2ac\right)\arctan\left(\frac{2cx^4+b}{\sqrt{-b^2+4ac}}\right)}{4\sqrt{-b^2+4ac}\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(c*x^8+b*x^4+a),x, algorithm="giac")
```

```
[Out] 1/4*x^4/c - 1/8*b*log(c*x^8 + b*x^4 + a)/c^2 + 1/4*(b^2 - 2*a*c)*arctan((2*
c*x^4 + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*c^2)
```

maple [A] time = 0.01, size = 111, normalized size = 1.37

$$\frac{x^4}{4c} - \frac{a\arctan\left(\frac{2cx^4+b}{\sqrt{4ac-b^2}}\right)}{2\sqrt{4ac-b^2}\;c} + \frac{b^2\arctan\left(\frac{2cx^4+b}{\sqrt{4ac-b^2}}\right)}{4\sqrt{4ac-b^2}\;c^2} - \frac{b\ln\left(cx^8+bx^4+a\right)}{8c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11/(c*x^8+b*x^4+a),x)
```

```
[Out] 1/4*x^4/c-1/8*b*ln(c*x^8+b*x^4+a)/c^2-1/2/c/(4*a*c-b^2)^(1/2)*arctan((2*c*x
^4+b)/(4*a*c-b^2)^(1/2))*a+1/4/c^2/(4*a*c-b^2)^(1/2)*arctan((2*c*x^4+b)/(4*
a*c-b^2)^(1/2))*b^2
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(c*x^8+b*x^4+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

mupad [B] time = 2.69, size = 3916, normalized size = 48.35

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11/(a + b*x^4 + c*x^8),x)
```

```
[Out] x^4/(4*c) + (log(a + b*x^4 + c*x^8)*(4*b^3 - 16*a*b*c))/(2*(64*a*c^3 - 16*b
^2*c^2)) - (atan((8*c^4*x^4*(((a*c - b^2)*(((((2*a*c - b^2)*((((448*b^4*c^6
 - 384*a*b^2*c^7)/c^4 + (256*b^3*c^4*(4*b^3 - 16*a*b*c))/(64*a*c^3 - 16*b^2
*c^2))*(2*a*c - b^2))/(8*c^2*(4*a*c - b^2)^(1/2)) + (32*b^3*c^2*(4*b^3 - 16
*a*b*c)*(2*a*c - b^2))/((4*a*c - b^2)^(1/2)*(64*a*c^3 - 16*b^2*c^2))))/(8*c
^2*(4*a*c - b^2)^(1/2)) + (4*b^3*(4*b^3 - 16*a*b*c)*(2*a*c - b^2)^2)/((4*a*
c - b^2)*(64*a*c^3 - 16*b^2*c^2)))*(2*a*c - b^2))/(8*c^2*(4*a*c - b^2)^(1/2
)) - ((4*b^3 - 16*a*b*c)*(((4*b^3 - 16*a*b*c)*((((448*b^4*c^6 - 384*a*b^2*c
^7)/c^4 + (256*b^3*c^4*(4*b^3 - 16*a*b*c))/(64*a*c^3 - 16*b^2*c^2))*(2*a*c
- b^2))/(8*c^2*(4*a*c - b^2)^(1/2)) + (32*b^3*c^2*(4*b^3 - 16*a*b*c)*(2*a*c
 - b^2))/((4*a*c - b^2)^(1/2)*(64*a*c^3 - 16*b^2*c^2))))/(2*(64*a*c^3 - 16*
b^2*c^2)) + ((2*a*c - b^2)*((144*b^5*c^4 - 240*a*b^3*c^5 + 96*a^2*b*c^6)/c^
4 + ((4*b^3 - 16*a*b*c)*((448*b^4*c^6 - 384*a*b^2*c^7)/c^4 + (256*b^3*c^4*(
4*b^3 - 16*a*b*c))/(64*a*c^3 - 16*b^2*c^2)))/(2*(64*a*c^3 - 16*b^2*c^2))))/
(8*c^2*(4*a*c - b^2)^(1/2))))/(2*(64*a*c^3 - 16*b^2*c^2)) + (((8*a^3*c^5 -
20*b^6*c^2 + 48*a*b^4*c^3 - 36*a^2*b^2*c^4)/c^4 - ((4*b^3 - 16*a*b*c)*((144
```

*b^5*c^4 - 240*a*b^3*c^5 + 96*a^2*b*c^6)/c^4 + ((4*b^3 - 16*a*b*c)*((448*b^
4*c^6 - 384*a*b^2*c^7)/c^4 + (256*b^3*c^4*(4*b^3 - 16*a*b*c))/(64*a*c^3 - 1
6*b^2*c^2)))/(2*(64*a*c^3 - 16*b^2*c^2))))/(2*(64*a*c^3 - 16*b^2*c^2)))*(2*
a*c - b^2))/(8*c^2*(4*a*c - b^2)^(1/2)) + (b^3*(4*b^3 - 16*a*b*c)*(2*a*c -
b^2)^3)/(2*c^2*(4*a*c - b^2)^(3/2)*(64*a*c^3 - 16*b^2*c^2))))/(8*a^3*c^2) -
((b^3 - 3*a*b*c)*(((4*b^3 - 16*a*b*c)*((8*a^3*c^5 - 20*b^6*c^2 + 48*a*b^4*
c^3 - 36*a^2*b^2*c^4)/c^4 - ((4*b^3 - 16*a*b*c)*((144*b^5*c^4 - 240*a*b^3*c
^5 + 96*a^2*b*c^6)/c^4 + ((4*b^3 - 16*a*b*c)*((448*b^4*c^6 - 384*a*b^2*c^7)
/c^4 + (256*b^3*c^4*(4*b^3 - 16*a*b*c))/(64*a*c^3 - 16*b^2*c^2)))/(2*(64*a*
c^3 - 16*b^2*c^2))))/(2*(64*a*c^3 - 16*b^2*c^2))))/(2*(64*a*c^3 - 16*b^2*c^
2)) - (b^7 - a^3*b*c^3 + 3*a^2*b^3*c^2 - 3*a*b^5*c)/c^4 + ((4*b^3 - 16*a*b*
c)*(((2*a*c - b^2)*((((448*b^4*c^6 - 384*a*b^2*c^7)/c^4 + (256*b^3*c^4*(4*b
^3 - 16*a*b*c))/(64*a*c^3 - 16*b^2*c^2))*(2*a*c - b^2))/(8*c^2*(4*a*c - b^2
)^(1/2)) + (32*b^3*c^2*(4*b^3 - 16*a*b*c)*(2*a*c - b^2))/((4*a*c - b^2)^(1/
2)*(64*a*c^3 - 16*b^2*c^2))))/(8*c^2*(4*a*c - b^2)^(1/2)) + (4*b^3*(4*b^3 -
16*a*b*c)*(2*a*c - b^2)^2)/((4*a*c - b^2)*(64*a*c^3 - 16*b^2*c^2))))/(2*(6
4*a*c^3 - 16*b^2*c^2)) + ((((4*b^3 - 16*a*b*c)*((((448*b^4*c^6 - 384*a*b^2*
c^7)/c^4 + (256*b^3*c^4*(4*b^3 - 16*a*b*c))/(64*a*c^3 - 16*b^2*c^2))*(2*a*c
- b^2))/(8*c^2*(4*a*c - b^2)^(1/2)) + (32*b^3*c^2*(4*b^3 - 16*a*b*c)*(2*a*
c - b^2))/((4*a*c - b^2)^(1/2)*(64*a*c^3 - 16*b^2*c^2))))/(2*(64*a*c^3 - 16
*b^2*c^2)) + ((2*a*c - b^2)*((144*b^5*c^4 - 240*a*b^3*c^5 + 96*a^2*b*c^6)/c
^4 + ((4*b^3 - 16*a*b*c)*((448*b^4*c^6 - 384*a*b^2*c^7)/c^4 + (256*b^3*c^4*
(4*b^3 - 16*a*b*c))/(64*a*c^3 - 16*b^2*c^2)))/(2*(64*a*c^3 - 16*b^2*c^2))))
/(8*c^2*(4*a*c - b^2)^(1/2)))*(2*a*c - b^2))/(8*c^2*(4*a*c - b^2)^(1/2)) -
(b^3*(2*a*c - b^2)^4)/(8*c^4*(4*a*c - b^2)^2)))/(8*a^3*c^2*(4*a*c - b^2)^(1
/2)))*(4*a*c - b^2)^2)/(b^8 + 16*a^4*c^4 + 24*a^2*b^4*c^2 - 32*a^3*b^2*c^3
- 8*a*b^6*c) - (c^2*(a*c - b^2)*(4*a*c - b^2)^2*(((4*b^3 - 16*a*b*c)*(((4*b
^3 - 16*a*b*c)*(((2*a*c - b^2)*((768*a*b^3*c^6 - 512*a^2*b*c^7)/c^4 + (512*
a*b^2*c^4*(4*b^3 - 16*a*b*c))/(64*a*c^3 - 16*b^2*c^2)))/(8*c^2*(4*a*c - b^2
)^(1/2)) + (64*a*b^2*c^2*(4*b^3 - 16*a*b*c)*(2*a*c - b^2))/((4*a*c - b^2)^(
1/2)*(64*a*c^3 - 16*b^2*c^2))))/(2*(64*a*c^3 - 16*b^2*c^2)) + (((64*a^3*c^6
+ 208*a*b^4*c^4 - 256*a^2*b^2*c^5)/c^4 + ((4*b^3 - 16*a*b*c)*((768*a*b^3*c
^6 - 512*a^2*b*c^7)/c^4 + (512*a*b^2*c^4*(4*b^3 - 16*a*b*c))/(64*a*c^3 - 16
*b^2*c^2)))/(2*(64*a*c^3 - 16*b^2*c^2)))*(2*a*c - b^2))/(8*c^2*(4*a*c - b^2
)^(1/2))))/(2*(64*a*c^3 - 16*b^2*c^2)) + ((2*a*c - b^2)*((24*a*b^5*c^2 + 16
*a^3*b*c^4 - 40*a^2*b^3*c^3)/c^4 + ((4*b^3 - 16*a*b*c)*((64*a^3*c^6 + 208*a
*b^4*c^4 - 256*a^2*b^2*c^5)/c^4 + ((4*b^3 - 16*a*b*c)*((768*a*b^3*c^6 - 512
*a^2*b*c^7)/c^4 + (512*a*b^2*c^4*(4*b^3 - 16*a*b*c))/(64*a*c^3 - 16*b^2*c^2
)))/(2*(64*a*c^3 - 16*b^2*c^2))))/(2*(64*a*c^3 - 16*b^2*c^2))))/(8*c^2*(4*a
*c - b^2)^(1/2)) - ((2*a*c - b^2)*(((((2*a*c - b^2)*((768*a*b^3*c^6 - 512*a
^2*b*c^7)/c^4 + (512*a*b^2*c^4*(4*b^3 - 16*a*b*c))/(64*a*c^3 - 16*b^2*c^2))
)/(8*c^2*(4*a*c - b^2)^(1/2)) + (64*a*b^2*c^2*(4*b^3 - 16*a*b*c)*(2*a*c - b
^2))/((4*a*c - b^2)^(1/2)*(64*a*c^3 - 16*b^2*c^2)))*(2*a*c - b^2))/(8*c^2*(
4*a*c - b^2)^(1/2)) + (8*a*b^2*(4*b^3 - 16*a*b*c)*(2*a*c - b^2)^2)/((4*a*c
- b^2)*(64*a*c^3 - 16*b^2*c^2))))/(8*c^2*(4*a*c - b^2)^(1/2)) - (a*b^2*(4*b
^3 - 16*a*b*c)*(2*a*c - b^2)^3)/(c^2*(4*a*c - b^2)^(3/2)*(64*a*c^3 - 16*b^2
c^2))))/(a^3(b^8 + 16*a^4*c^4 + 24*a^2*b^4*c^2 - 32*a^3*b^2*c^3 - 8*a*b^6
c)) + (c^2(4*a*c - b^2)^(3/2)*(b^3 - 3*a*b*c)*((a*b^6 - 2*a^2*b^4*c + a^3
*b^2*c^2)/c^4 + ((4*b^3 - 16*a*b*c)*((24*a*b^5*c^2 + 16*a^3*b*c^4 - 40*a^2*
b^3*c^3)/c^4 + ((4*b^3 - 16*a*b*c)*((64*a^3*c^6 + 208*a*b^4*c^4 - 256*a^2*b
^2*c^5)/c^4 + ((4*b^3 - 16*a*b*c)*((768*a*b^3*c^6 - 512*a^2*b*c^7)/c^4 + (5
12*a*b^2*c^4*(4*b^3 - 16*a*b*c))/(64*a*c^3 - 16*b^2*c^2)))/(2*(64*a*c^3 - 1
6*b^2*c^2))))/(2*(64*a*c^3 - 16*b^2*c^2))))/(2*(64*a*c^3 - 16*b^2*c^2)) - (
(4*b^3 - 16*a*b*c)*(((((2*a*c - b^2)*((768*a*b^3*c^6 - 512*a^2*b*c^7)/c^4 +
(512*a*b^2*c^4*(4*b^3 - 16*a*b*c))/(64*a*c^3 - 16*b^2*c^2)))/(8*c^2*(4*a*c
- b^2)^(1/2)) + (64*a*b^2*c^2*(4*b^3 - 16*a*b*c)*(2*a*c - b^2))/((4*a*c -
b^2)^(1/2)*(64*a*c^3 - 16*b^2*c^2)))*(2*a*c - b^2))/(8*c^2*(4*a*c - b^2)^(1
/2)) + (8*a*b^2*(4*b^3 - 16*a*b*c)*(2*a*c - b^2)^2)/((4*a*c - b^2)*(64*a*c^
3 - 16*b^2*c^2))))/(2*(64*a*c^3 - 16*b^2*c^2)) - ((2*a*c - b^2)*(((4*b^3 -

```
16*a*b*c)*(((2*a*c - b^2)*((768*a*b^3*c^6 - 512*a^2*b*c^7)/c^4 + (512*a*b^2
*c^4*(4*b^3 - 16*a*b*c))/(64*a*c^3 - 16*b^2*c^2)))/(8*c^2*(4*a*c - b^2)^(1/
2)) + (64*a*b^2*c^2*(4*b^3 - 16*a*b*c)*(2*a*c - b^2))/((4*a*c - b^2)^(1/2)*
(64*a*c^3 - 16*b^2*c^2))))/(2*(64*a*c^3 - 16*b^2*c^2)) + (((64*a^3*c^6 + 20
8*a*b^4*c^4 - 256*a^2*b^2*c^5)/c^4 + ((4*b^3 - 16*a*b*c)*((768*a*b^3*c^6 -
512*a^2*b*c^7)/c^4 + (512*a*b^2*c^4*(4*b^3 - 16*a*b*c))/(64*a*c^3 - 16*b^2*
c^2)))/(2*(64*a*c^3 - 16*b^2*c^2)))*(2*a*c - b^2))/(8*c^2*(4*a*c - b^2)^(1/
2))))/(8*c^2*(4*a*c - b^2)^(1/2)) + (a*b^2*(2*a*c - b^2)^4)/(4*c^4*(4*a*c -
 b^2)^2)))/(a^3*(b^8 + 16*a^4*c^4 + 24*a^2*b^4*c^2 - 32*a^3*b^2*c^3 - 8*a*b
^6*c)))*(2*a*c - b^2))/(4*c^2*(4*a*c - b^2)^(1/2))
```

sympy [B] time = 4.11, size = 316, normalized size = 3.90

$$\left(-\frac{b}{8c^2}-\frac{\sqrt{-4ac+b^2}\,(2ac-b^2)}{8c^2(4ac-b^2)}\right)\log\left(x^4+\frac{-ab-16ac^2\left(-\frac{b}{8c^2}-\frac{\sqrt{-4ac+b^2}\,(2ac-b^2)}{8c^2(4ac-b^2)}\right)+4b^2c\left(-\frac{b}{8c^2}-\frac{\sqrt{-4ac+b^2}\,(2ac-b^2)}{8c^2(4ac-b^2)}\right)}{2ac-b^2}\right)+\left(-\frac{b}{8c^2}+\frac{\sqrt{-4ac+b^2}\,(2ac-b^2)}{8c^2(4ac-b^2)}\right)\log\left(x^4+\frac{-ab-16ac^2\left(-\frac{b}{8c^2}+\frac{\sqrt{-4ac+b^2}\,(2ac-b^2)}{8c^2(4ac-b^2)}\right)+4b^2c\left(-\frac{b}{8c^2}+\frac{\sqrt{-4ac+b^2}\,(2ac-b^2)}{8c^2(4ac-b^2)}\right)}{2ac-b^2}\right)+\frac{x^4}{4c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**11/(c*x**8+b*x**4+a),x)
```

```
[Out] (-b/(8*c**2) - sqrt(-4*a*c + b**2)*(2*a*c - b**2)/(8*c**2*(4*a*c - b**2)))*
log(x**4 + (-a*b - 16*a*c**2*(-b/(8*c**2) - sqrt(-4*a*c + b**2)*(2*a*c - b*
*2)/(8*c**2*(4*a*c - b**2))) + 4*b**2*c*(-b/(8*c**2) - sqrt(-4*a*c + b**2)*
(2*a*c - b**2)/(8*c**2*(4*a*c - b**2))))/(2*a*c - b**2)) + (-b/(8*c**2) + s
qrt(-4*a*c + b**2)*(2*a*c - b**2)/(8*c**2*(4*a*c - b**2)))*log(x**4 + (-a*b
 - 16*a*c**2*(-b/(8*c**2) + sqrt(-4*a*c + b**2)*(2*a*c - b**2)/(8*c**2*(4*a
*c - b**2))) + 4*b**2*c*(-b/(8*c**2) + sqrt(-4*a*c + b**2)*(2*a*c - b**2)/(
8*c**2*(4*a*c - b**2))))/(2*a*c - b**2)) + x**4/(4*c)
```

3.254 $\int \frac{x^9}{a+bx^4+cx^8} \, dx$

Optimal. Leaf size=192

$$-\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x^2}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\,c^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x^2}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{2\sqrt{2}\,c^{3/2}\sqrt{\sqrt{b^2-4ac}+b}}+\frac{x^2}{2c}$$

Rubi [A] time = 0.34, antiderivative size = 192, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {1359, 1122, 1166, 205}

$$-\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x^2}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\,c^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x^2}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{2\sqrt{2}\,c^{3/2}\sqrt{\sqrt{b^2-4ac}+b}}+\frac{x^2}{2c}$$

Antiderivative was successfully verified.

```
[In] Int[x^9/(a + b*x^4 + c*x^8),x]
```

```
[Out] x^2/(2*c) - ((b - (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*
x^2)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*c^(3/2)*Sqrt[b - Sqrt[b^2 - 4
*a*c]]) - ((b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x^
2)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*c^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a
*c]])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1122

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Simp[(d^3*(d*x)^(m - 3)*(a + b*x^2 + c*x^4)^(p + 1))/(c*(m + 4*p + 1)),
 x] - Dist[d^4/(c*(m + 4*p + 1)), Int[(d*x)^(m - 4)*Simp[a*(m - 3) + b*(m +
 2*p - 1)*x^2, x]*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x
] && NeQ[b^2 - 4*a*c, 0] && GtQ[m, 3] && NeQ[m + 4*p + 1, 0] && IntegerQ[2*
p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1359

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*
x^(n/k) + c*x^((2*n)/k))^p, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, c, p
}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\int \frac{x^9}{a+bx^4+cx^8}\,dx = \frac{1}{2}\operatorname{Subst}\left(\int \frac{x^4}{a+bx^2+cx^4}\,dx, x, x^2\right)$$
$$= \frac{x^2}{2c} - \frac{\operatorname{Subst}\left(\int \frac{a+bx^2}{a+bx^2+cx^4}\,dx, x, x^2\right)}{2c}$$
$$= \frac{x^2}{2c} - \frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\operatorname{Subst}\left(\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx, x, x^2\right)}{4c} - \frac{\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\operatorname{Subst}\left(\int \frac{}{\frac{b}{2}+\frac{1}{2}\sqrt{b}}\right.}{4c}$$
$$= \frac{x^2}{2c} - \frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x^2}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}c^{3/2}\sqrt{b-\sqrt{b^2-4ac}}} - \frac{\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x^2}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}c^{3/2}\sqrt{b+\sqrt{b^2-4ac}}}$$

Mathematica [**A**] time = 0.12, size = 210, normalized size = 1.09

$$\frac{-\frac{\sqrt{2}\left(b\sqrt{b^2-4ac}+2ac-b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x^2}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}} - \frac{\sqrt{2}\left(b\sqrt{b^2-4ac}-2ac+b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x^2}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4ac}+b}} + 2\sqrt{c}x^2}{4c^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^9/(a + b*x^4 + c*x^8),x]
```

```
[Out] (2*Sqrt[c]*x^2 - (Sqrt[2]*(-b^2 + 2*a*c + b*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt
[2]*Sqrt[c]*x^2)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2 - 4*a*c]*Sqrt[b -
Sqrt[b^2 - 4*a*c]]) - (Sqrt[2]*(b^2 - 2*a*c + b*Sqrt[b^2 - 4*a*c])*ArcTan[(
Sqrt[2]*Sqrt[c]*x^2)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2 - 4*a*c]*Sqrt[
b + Sqrt[b^2 - 4*a*c]]))/(4*c^(3/2))
```

IntegrateAlgebraic [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^9}{a+bx^4+cx^8}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^9/(a + b*x^4 + c*x^8),x]
```

```
[Out] IntegrateAlgebraic[x^9/(a + b*x^4 + c*x^8), x]
```

fricas [**B**] time = 1.44, size = 1071, normalized size = 5.58

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(c*x^8+b*x^4+a),x, algorithm="fricas")
```

```
[Out] -1/4*(sqrt(1/2)*c*sqrt(-(b^3 - 3*a*b*c + (b^2*c^3 - 4*a*c^4)*sqrt((b^4 - 2*
a*b^2*c + a^2*c^2)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4))*log(-(a*b^2 -
 a^2*c)*x^2 + 1/2*sqrt(1/2)*(b^4 - 5*a*b^2*c + 4*a^2*c^2 - (b^3*c^3 - 4*a*b
*c^4)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^2*c^6 - 4*a*c^7)))*sqrt(-(b^3 - 3
*a*b*c + (b^2*c^3 - 4*a*c^4)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^2*c^6 - 4*
a*c^7)))/(b^2*c^3 - 4*a*c^4))) - sqrt(1/2)*c*sqrt(-(b^3 - 3*a*b*c + (b^2*c^
3 - 4*a*c^4)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^
3 - 4*a*c^4))*log(-(a*b^2 - a^2*c)*x^2 - 1/2*sqrt(1/2)*(b^4 - 5*a*b^2*c + 4
```

```
*a^2*c^2 - (b^3*c^3 - 4*a*b*c^4)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^2*c^6
- 4*a*c^7)))*sqrt(-(b^3 - 3*a*b*c + (b^2*c^3 - 4*a*c^4)*sqrt((b^4 - 2*a*b^2
*c + a^2*c^2)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4))) + sqrt(1/2)*c*sqr
t(-(b^3 - 3*a*b*c - (b^2*c^3 - 4*a*c^4)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b
^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4))*log(-(a*b^2 - a^2*c)*x^2 + 1/2*sqr
t(1/2)*(b^4 - 5*a*b^2*c + 4*a^2*c^2 + (b^3*c^3 - 4*a*b*c^4)*sqrt((b^4 - 2*a
*b^2*c + a^2*c^2)/(b^2*c^6 - 4*a*c^7)))*sqrt(-(b^3 - 3*a*b*c - (b^2*c^3 - 4
*a*c^4)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4
*a*c^4))) - sqrt(1/2)*c*sqrt(-(b^3 - 3*a*b*c - (b^2*c^3 - 4*a*c^4)*sqrt((b^
4 - 2*a*b^2*c + a^2*c^2)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4))*log(-(a
*b^2 - a^2*c)*x^2 - 1/2*sqrt(1/2)*(b^4 - 5*a*b^2*c + 4*a^2*c^2 + (b^3*c^3 -
 4*a*b*c^4)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^2*c^6 - 4*a*c^7)))*sqrt(-(b
^3 - 3*a*b*c - (b^2*c^3 - 4*a*c^4)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^2*c^
6 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4))) - 2*x^2)/c
```

giac [B] time = 18.17, size = 2043, normalized size = 10.64

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(c*x^8+b*x^4+a),x, algorithm="giac")
```

```
[Out] 1/2*x^2/c + 1/8*(2*a*b^3*c^3 - 8*a^2*b*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt
(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
- sqrt(b^2 - 4*a*c)*c)*a^2*b*c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - s
qrt(b^2 - 4*a*c)*c)*a*b^2*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b
^2 - 4*a*c)*c)*a*b*c^3 - 2*(b^2 - 4*a*c)*a*b*c^3 - (sqrt(2)*sqrt(b*c - sqrt
(b^2 - 4*a*c)*c)*b^5 - 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c -
2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4*c + 2*b^5*c + 16*sqrt(2)*sqrt
(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^2 + 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a
*c)*c)*a*b^2*c^2 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c^2 - 16*a*b
^3*c^2 - 4*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^3 + 32*a^2*b*c^3 -
 2*(b^2 - 4*a*c)*b^3*c + 8*(b^2 - 4*a*c)*a*b*c^2)*x^4*abs(c) + (2*b^4*c^3 -
 8*a*b^2*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^
4*c + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2
 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c^2 - sq
rt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^2*c^3 - 2*(b^2 -
4*a*c)*b^2*c^3)*x^4 - (sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4 - 8*sq
rt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c - 2*sqrt(2)*sqrt(b*c - sqrt
(b^2 - 4*a*c)*c)*a*b^3*c + 2*a*b^4*c + 16*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a
*c)*c)*a^3*c^2 + 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^2 + sqrt
(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 - 16*a^2*b^2*c^2 - 4*sqrt(2)*
sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*c^3 + 32*a^3*c^3 - 2*(b^2 - 4*a*c)*a*b^
2*c + 8*(b^2 - 4*a*c)*a^2*c^2)*abs(c))*arctan(2*sqrt(1/2)*x^2/sqrt((b*c + s
qrt(b^2*c^2 - 4*a*c^3))/c^2))/((a*b^4*c - 8*a^2*b^2*c^2 - 2*a*b^3*c^2 + 16*
a^3*c^3 + 8*a^2*b*c^3 + a*b^2*c^3 - 4*a^2*c^4)*c^2) + 1/8*(2*a*b^3*c^3 - 8*
a^2*b*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3
*c + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^2
+ 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 - s
qrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^3 - 2*(b^2 -
 4*a*c)*a*b*c^3 - (sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5 - 8*sqrt(2)*
sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4
*a*c)*c)*b^4*c - 2*b^5*c + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b
*c^2 + 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 + sqrt(2)*sqrt(b
*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^2 + 16*a*b^3*c^2 - 4*sqrt(2)*sqrt(b*c + sqr
t(b^2 - 4*a*c)*c)*a*b*c^3 - 32*a^2*b*c^3 + 2*(b^2 - 4*a*c)*b^3*c - 8*(b^2 -
 4*a*c)*a*b*c^2)*x^4*abs(c) + (2*b^4*c^3 - 8*a*b^2*c^4 - sqrt(2)*sqrt(b^2 -
 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c + 4*sqrt(2)*sqrt(b^2 - 4*a*c)
*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sq
rt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
```

```
+ sqrt(b^2 - 4*a*c)*c)*b^2*c^3 - 2*(b^2 - 4*a*c)*b^2*c^3)*x^4 - (sqrt(2)*sq
rt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4 - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c
)*c)*a^2*b^2*c - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c - 2*a*b^
4*c + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*c^2 + 8*sqrt(2)*sqrt(b
*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^2 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*
c)*a*b^2*c^2 + 16*a^2*b^2*c^2 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a
^2*c^3 - 32*a^3*c^3 + 2*(b^2 - 4*a*c)*a*b^2*c - 8*(b^2 - 4*a*c)*a^2*c^2)*ab
s(c))*arctan(2*sqrt(1/2)*x^2/sqrt((b*c - sqrt(b^2*c^2 - 4*a*c^3))/c^2))/((a
*b^4*c - 8*a^2*b^2*c^2 - 2*a*b^3*c^2 + 16*a^3*c^3 + 8*a^2*b*c^3 + a*b^2*c^3
 - 4*a^2*c^4)*c^2)
```

maple [B] time = 0.03, size = 360, normalized size = 1.88

$$\frac{\sqrt{2}\, a \operatorname{arctanh}\left(\frac{\sqrt{2}\, c x^2}{\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}\right)}{2\sqrt{-4ac+b^2}\,\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}+\frac{\sqrt{2}\, a \arctan\left(\frac{\sqrt{2}\, c x^2}{\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}\right)}{2\sqrt{-4ac+b^2}\,\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}-\frac{\sqrt{2}\, b^2 \operatorname{arctanh}\left(\frac{\sqrt{2}\, c x^2}{\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}\right)}{4\sqrt{-4ac+b^2}\,\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}\, c}-\frac{\sqrt{2}\, b^2 \arctan\left(\frac{\sqrt{2}\, c x^2}{\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}\right)}{4\sqrt{-4ac+b^2}\,\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}\, c}+\frac{\sqrt{2}\, b \operatorname{arctanh}\left(\frac{\sqrt{2}\, c x^2}{\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}\right)}{4\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}\, c}-\frac{\sqrt{2}\, b \arctan\left(\frac{\sqrt{2}\, c x^2}{\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}\right)}{4\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}\, c}+\frac{x^2}{2c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9/(c*x^8+b*x^4+a),x)
```

```
[Out] 1/2/c*x^2+1/4/c*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x^2*2^(
1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b+1/2/(-4*a*c+b^2)^(1/2)*2^(1/2)/((
-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x^2*2^(1/2)/((-b+(-4*a*c+b^2)^(1/
2))*c)^(1/2))*a-1/4/(-4*a*c+b^2)^(1/2)/c*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c
)^(1/2)*arctanh(c*x^2*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b^2-1/4/c*
2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x^2*2^(1/2)/((b+(-4*a*c+b
^2)^(1/2))*c)^(1/2))*b+1/2/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2
))*c)^(1/2)*arctan(c*x^2*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*a-1/4/(-
4*a*c+b^2)^(1/2)/c*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x^2*2^
(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(c*x^8+b*x^4+a),x, algorithm="maxima")
```

```
[Out] 1/2*x^2/c - integrate((b*x^4 + a)*x/(c*x^8 + b*x^4 + a), x)/c
```

mupad [B] time = 3.07, size = 5659, normalized size = 29.47

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9/(a + b*x^4 + c*x^8),x)
```

```
[Out] atan(((((16*(a*b^8 + 4*a^5*c^4 - 8*a^2*b^6*c + 20*a^3*b^4*c^2 - 16*a^4*b^2*
c^3))/c^2 + (((16*(32*a*b^7*c^3 - 256*a^4*b*c^6 - 256*a^2*b^5*c^4 + 576*a^3
*b^3*c^5))/c^2 - ((256*a*b^6*c^6 - 2048*a^2*b^4*c^7 + 4096*a^3*b^2*c^8)*(b^
5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c
- b^2)^3)^(1/2)))/(2*c^2*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))*(-(b^5 + b^
2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)
^3)^(1/2))/(32*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2) - (4*x^2*(64*a^
4*b*c^5 + 16*a^2*b^5*c^3 - 80*a^3*b^3*c^4))/c^2)*(-(b^5 + b^2*(-(4*a*c - b^
2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(32*
(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2))*(-(b^5 + b^2*(-(4*a*c - b^2)^
3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(32*(16
*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2) - (4*x^2*(a^2*b^6 - 2*a^5*c^3 - 5
*a^3*b^4*c + 6*a^4*b^2*c^2))/c^2)*(-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 1
2*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(32*(16*a^2*c^5 + b
```

^4*c^3 - 8*a*b^2*c^4)))^(1/2)*1i - (((16*(a*b^8 + 4*a^5*c^4 - 8*a^2*b^6*c + 20*a^3*b^4*c^2 - 16*a^4*b^2*c^3))/c^2 + (((16*(32*a*b^7*c^3 - 256*a^4*b*c^6 - 256*a^2*b^5*c^4 + 576*a^3*b^3*c^5))/c^2 - ((256*a*b^6*c^6 - 2048*a^2*b^4*c^7 + 4096*a^3*b^2*c^8)*(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2)))/(2*c^2*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))*(-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(32*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2) + (4*x^2*(64*a^4*b*c^5 + 16*a^2*b^5*c^3 - 80*a^3*b^3*c^4))/c^2)*(-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(32*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2))*(-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(32*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2) + (4*x^2*(a^2*b^6 - 2*a^5*c^3 - 5*a^3*b^4*c + 6*a^4*b^2*c^2))/c^2)*(-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(32*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2)*1i)/((((16*(a*b^8 + 4*a^5*c^4 - 8*a^2*b^6*c + 20*a^3*b^4*c^2 - 16*a^4*b^2*c^3))/c^2 + (((16*(32*a*b^7*c^3 - 256*a^4*b*c^6 - 256*a^2*b^5*c^4 + 576*a^3*b^3*c^5))/c^2 - ((256*a*b^6*c^6 - 2048*a^2*b^4*c^7 + 4096*a^3*b^2*c^8)*(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2)))/(2*c^2*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))*(-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(32*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2) - (4*x^2*(64*a^4*b*c^5 + 16*a^2*b^5*c^3 - 80*a^3*b^3*c^4))/c^2)*(-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(32*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2))*(-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(32*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2) - (4*x^2*(a^2*b^6 - 2*a^5*c^3 - 5*a^3*b^4*c + 6*a^4*b^2*c^2))/c^2)*(-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(32*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2) + (((16*(a*b^8 + 4*a^5*c^4 - 8*a^2*b^6*c + 20*a^3*b^4*c^2 - 16*a^4*b^2*c^3))/c^2 + (((16*(32*a*b^7*c^3 - 256*a^4*b*c^6 - 256*a^2*b^5*c^4 + 576*a^3*b^3*c^5))/c^2 - ((256*a*b^6*c^6 - 2048*a^2*b^4*c^7 + 4096*a^3*b^2*c^8)*(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2)))/(2*c^2*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))*(-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(32*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2) + (4*x^2*(64*a^4*b*c^5 + 16*a^2*b^5*c^3 - 80*a^3*b^3*c^4))/c^2)*(-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(32*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2))*(-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(32*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2) + (4*x^2*(a^2*b^6 - 2*a^5*c^3 - 5*a^3*b^4*c + 6*a^4*b^2*c^2))/c^2)*(-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(32*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2)))*(-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(32*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2)*2i + atan(((((16*(a*b^8 + 4*a^5*c^4 - 8*a^2*b^6*c + 20*a^3*b^4*c^2 - 16*a^4*b^2*c^3))/c^2 + (((16*(32*a*b^7*c^3 - 256*a^4*b*c^6 - 256*a^2*b^5*c^4 + 576*a^3*b^3*c^5))/c^2 - ((256*a*b^6*c^6 - 2048*a^2*b^4*c^7 + 4096*a^3*b^2*c^8)*(b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2)))/(2*c^2*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))*(-(b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2))/(32*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2) - (4*x^2*(64*a^4*b*c^5 + 16*a^2*b^5*c^3 - 80*a^3*b^3*c^4))/c^2)*(-(b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2))/(32*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2))*(-(b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2))/(32*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2) - (4*x^2*(a^2*b^6 - 2*a^5*c^3 - 5*a^3*b^4*c + 6*a^4*b^2*c^2))/c^2)*(-(b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(

```
4*a*c - b^2)^3)^(1/2))/(32*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2)*1i
- (((16*(a*b^8 + 4*a^5*c^4 - 8*a^2*b^6*c + 20*a^3*b^4*c^2 - 16*a^4*b^2*c^3)
)/c^2 + (((16*(32*a*b^7*c^3 - 256*a^4*b*c^6 - 256*a^2*b^5*c^4 + 576*a^3*b^3
*c^5))/c^2 - ((256*a*b^6*c^6 - 2048*a^2*b^4*c^7 + 4096*a^3*b^2*c^8)*(b^5 -
b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^
2)^3)^(1/2)))/(2*c^2*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))*(-(b^5 - b^2*(-
(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^
(1/2))/(32*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2) + (4*x^2*(64*a^4*b*
c^5 + 16*a^2*b^5*c^3 - 80*a^3*b^3*c^4))/c^2)*(-(b^5 - b^2*(-(4*a*c - b^2)^3
)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2))/(32*(16*
a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2))*(-(b^5 - b^2*(-(4*a*c - b^2)^3)^(
1/2) + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2))/(32*(16*a^2
*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2) + (4*x^2*(a^2*b^6 - 2*a^5*c^3 - 5*a^3
*b^4*c + 6*a^4*b^2*c^2))/c^2)*(-(b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^
2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2))/(32*(16*a^2*c^5 + b^4*c
^3 - 8*a*b^2*c^4)))^(1/2)*1i)/((((16*(a*b^8 + 4*a^5*c^4 - 8*a^2*b^6*c + 20*
a^3*b^4*c^2 - 16*a^4*b^2*c^3))/c^2 + (((16*(32*a*b^7*c^3 - 256*a^4*b*c^6 -
256*a^2*b^5*c^4 + 576*a^3*b^3*c^5))/c^2 - ((256*a*b^6*c^6 - 2048*a^2*b^4*c^
7 + 4096*a^3*b^2*c^8)*(b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 -
7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2)))/(2*c^2*(16*a^2*c^5 + b^4*c^3 - 8
*a*b^2*c^4)))*(-(b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^
3*c + a*c*(-(4*a*c - b^2)^3)^(1/2))/(32*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4
)))^(1/2) - (4*x^2*(64*a^4*b*c^5 + 16*a^2*b^5*c^3 - 80*a^3*b^3*c^4))/c^2)*(
-(b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*
a*c - b^2)^3)^(1/2))/(32*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2))*(-(b
^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c
 - b^2)^3)^(1/2))/(32*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2) - (4*x^2
*(a^2*b^6 - 2*a^5*c^3 - 5*a^3*b^4*c + 6*a^4*b^2*c^2))/c^2)*(-(b^5 - b^2*(-(
4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(
1/2))/(32*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2) + (((16*(a*b^8 + 4*a
^5*c^4 - 8*a^2*b^6*c + 20*a^3*b^4*c^2 - 16*a^4*b^2*c^3))/c^2 + (((16*(32*a*
b^7*c^3 - 256*a^4*b*c^6 - 256*a^2*b^5*c^4 + 576*a^3*b^3*c^5))/c^2 - ((256*a
*b^6*c^6 - 2048*a^2*b^4*c^7 + 4096*a^3*b^2*c^8)*(b^5 - b^2*(-(4*a*c - b^2)^
3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2)))/(2*c^2
*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))*(-(b^5 - b^2*(-(4*a*c - b^2)^3)^(1/
2) + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2))/(32*(16*a^2*c
^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2) + (4*x^2*(64*a^4*b*c^5 + 16*a^2*b^5*c^3
 - 80*a^3*b^3*c^4))/c^2)*(-(b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c
^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2))/(32*(16*a^2*c^5 + b^4*c^3 -
8*a*b^2*c^4)))^(1/2))*(-(b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2
- 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2))/(32*(16*a^2*c^5 + b^4*c^3 - 8*a
*b^2*c^4)))^(1/2) + (4*x^2*(a^2*b^6 - 2*a^5*c^3 - 5*a^3*b^4*c + 6*a^4*b^2*c
^2))/c^2)*(-(b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c
+ a*c*(-(4*a*c - b^2)^3)^(1/2))/(32*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^
(1/2)))*(-(b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c +
a*c*(-(4*a*c - b^2)^3)^(1/2))/(32*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1
/2)*2i + x^2/(2*c)
```

sympy [A] time = 4.42, size = 134, normalized size = 0.70

$$\text{RootSum}\left(t^4\left(4096a^2c^5-2048ab^2c^4+256b^4c^3\right)+t^2\left(192a^2bc^2-112ab^3c+16b^5\right)+a^3,\left(t\mapsto t\log\left(x^2+\frac{256t^3abc^4-64t^3b^3c^3-8ta^2c^2+16tab^2c-4tb^4}{a^2c-ab^2}\right)\right)\right)+\frac{x^2}{2c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**9/(c*x**8+b*x**4+a),x)
```

```
[Out] RootSum(_t**4*(4096*a**2*c**5 - 2048*a*b**2*c**4 + 256*b**4*c**3) + _t**2*(
192*a**2*b*c**2 - 112*a*b**3*c + 16*b**5) + a**3, Lambda(_t, _t*log(x**2 +
(256*_t**3*a*b*c**4 - 64*_t**3*b**3*c**3 - 8*_t*a**2*c**2 + 16*_t*a*b**2*c
- 4*_t*b**4)/(a**2*c - a*b**2)))) + x**2/(2*c)
```

3.255 $\int \frac{x^7}{a+bx^4+cx^8}\,dx$

Optimal. Leaf size=63

$$\frac{b\tanh^{-1}\left(\frac{b+2cx^4}{\sqrt{b^2-4ac}}\right)}{4c\sqrt{b^2-4ac}}+\frac{\log\left(a+bx^4+cx^8\right)}{8c}$$

Rubi [A] time = 0.06, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {1357, 634, 618, 206, 628}

$$\frac{b\tanh^{-1}\left(\frac{b+2cx^4}{\sqrt{b^2-4ac}}\right)}{4c\sqrt{b^2-4ac}}+\frac{\log\left(a+bx^4+cx^8\right)}{8c}$$

Antiderivative was successfully verified.

```
[In] Int[x^7/(a + b*x^4 + c*x^8),x]
```

```
[Out] (b*ArcTanh[(b + 2*c*x^4)/Sqrt[b^2 - 4*a*c]])/(4*c*Sqrt[b^2 - 4*a*c]) + Log[
a + b*x^4 + c*x^8]/(8*c)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1357

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^7}{a+bx^4+cx^8}\,dx &= \frac{1}{4}\operatorname{Subst}\left(\int \frac{x}{a+bx+cx^2}\,dx,x,x^4\right)\\
&= \frac{\operatorname{Subst}\left(\int \frac{b+2cx}{a+bx+cx^2}\,dx,x,x^4\right)}{8c} - \frac{b\operatorname{Subst}\left(\int \frac{1}{a+bx+cx^2}\,dx,x,x^4\right)}{8c}\\
&= \frac{\log\left(a+bx^4+cx^8\right)}{8c} + \frac{b\operatorname{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx,x,b+2cx^4\right)}{4c}\\
&= \frac{b\tanh^{-1}\left(\frac{b+2cx^4}{\sqrt{b^2-4ac}}\right)}{4c\sqrt{b^2-4ac}} + \frac{\log\left(a+bx^4+cx^8\right)}{8c}
\end{aligned}
$$

Mathematica [A] time = 0.02, size = 62, normalized size = 0.98

$$
\frac{\log\left(a+bx^4+cx^8\right) - \frac{2b\tan^{-1}\left(\frac{b+2cx^4}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}}{8c}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7/(a + b*x^4 + c*x^8),x]
```

```
[Out] ((-2*b*ArcTan[(b + 2*c*x^4)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c] + Log[a
 + b*x^4 + c*x^8])/(8*c)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$
\int \frac{x^7}{a+bx^4+cx^8}\,dx
$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^7/(a + b*x^4 + c*x^8),x]
```

```
[Out] IntegrateAlgebraic[x^7/(a + b*x^4 + c*x^8), x]
```

fricas [A] time = 1.42, size = 197, normalized size = 3.13

$$
\left[\frac{\sqrt{b^2-4ac}\,b\log\left(\frac{2c^2x^8+2bcx^4+b^2-2ac+\left(2cx^4+b\right)\sqrt{b^2-4ac}}{cx^8+bx^4+a}\right)+\left(b^2-4ac\right)\log\left(cx^8+bx^4+a\right)}{8\left(b^2c-4ac^2\right)}, \frac{2\sqrt{-b^2+4ac}\,b\arctan\left(-\frac{\left(2cx^4+b\right)\sqrt{-b^2+4ac}}{b^2-4ac}\right)+\left(b^2-4ac\right)\log\left(cx^8+bx^4+a\right)}{8\left(b^2c-4ac^2\right)}\right]
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(c*x^8+b*x^4+a),x, algorithm="fricas")
```

```
[Out] [1/8*(sqrt(b^2 - 4*a*c)*b*log((2*c^2*x^8 + 2*b*c*x^4 + b^2 - 2*a*c + (2*c*x
^4 + b)*sqrt(b^2 - 4*a*c))/(c*x^8 + b*x^4 + a)) + (b^2 - 4*a*c)*log(c*x^8 +
 b*x^4 + a))/(b^2*c - 4*a*c^2), 1/8*(2*sqrt(-b^2 + 4*a*c)*b*arctan(-(2*c*x^
4 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) + (b^2 - 4*a*c)*log(c*x^8 + b*x^4
+ a))/(b^2*c - 4*a*c^2)]
```

giac [A] time = 17.12, size = 59, normalized size = 0.94

$$
-\frac{b\arctan\left(\frac{2cx^4+b}{\sqrt{-b^2+4ac}}\right)}{4\sqrt{-b^2+4ac}\,c} + \frac{\log\left(cx^8+bx^4+a\right)}{8c}
$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^7/(c*x^8+b*x^4+a),x, algorithm="giac")

[Out] -1/4*b*arctan((2*c*x^4 + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*c) + 1/8*log(c*x^8 + b*x^4 + a)/c

maple [A] time = 0.00, size = 60, normalized size = 0.95

$$-\frac{b \arctan\left(\frac{2c\,x^4+b}{\sqrt{4ac-b^2}}\right)}{4\sqrt{4ac-b^2}\;c}+\frac{\ln\left(c\,x^8+b\,x^4+a\right)}{8c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^7/(c*x^8+b*x^4+a),x)

[Out] 1/8*ln(c*x^8+b*x^4+a)/c-1/4*b/c/(4*a*c-b^2)^(1/2)*arctan((2*c*x^4+b)/(4*a*c-b^2)^(1/2))

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^7/(c*x^8+b*x^4+a),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for more details)Is 4*a*c-b^2 positive or negative?

mupad [B] time = 2.61, size = 2654, normalized size = 42.13

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^7/(a + b*x^4 + c*x^8),x)

[Out] (log(a + b*x^4 + c*x^8)*(16*a*c - 4*b^2))/(2*(64*a*c^2 - 16*b^2*c)) - (b*atan((8*x^4*(((a*c - b^2)*(((((16*a*c - 4*b^2)*((b*(448*b^3*c^3 - (256*b^3*c^4*(16*a*c - 4*b^2))/(64*a*c^2 - 16*b^2*c)))/(8*c*(4*a*c - b^2)^(1/2)) - (32*b^4*c^3*(16*a*c - 4*b^2))/((64*a*c^2 - 16*b^2*c)*(4*a*c - b^2)^(1/2))))/(2*(64*a*c^2 - 16*b^2*c)) - (b*(144*b^3*c^2 - ((448*b^3*c^3 - (256*b^3*c^4*(16*a*c - 4*b^2))/(64*a*c^2 - 16*b^2*c))*(16*a*c - 4*b^2))/(2*(64*a*c^2 - 16*b^2*c))))/(8*c*(4*a*c - b^2)^(1/2)))*(16*a*c - 4*b^2))/(2*(64*a*c^2 - 16*b^2*c)) - (b*((b*((b*(448*b^3*c^3 - (256*b^3*c^4*(16*a*c - 4*b^2))/(64*a*c^2 - 16*b^2*c)))/(8*c*(4*a*c - b^2)^(1/2)) - (32*b^4*c^3*(16*a*c - 4*b^2))/((64*a*c^2 - 16*b^2*c)*(4*a*c - b^2)^(1/2))))/(8*c*(4*a*c - b^2)^(1/2)) - (4*b^5*c^2*(16*a*c - 4*b^2))/((64*a*c^2 - 16*b^2*c)*(4*a*c - b^2))))/(8*c*(4*a*c - b^2)^(1/2)) + (b*(20*b^3*c - ((144*b^3*c^2 - ((448*b^3*c^3 - (256*b^3*c^4*(16*a*c - 4*b^2))/(64*a*c^2 - 16*b^2*c))*(16*a*c - 4*b^2))/(2*(64*a*c^2 - 16*b^2*c)))*(16*a*c - 4*b^2))/(2*(64*a*c^2 - 16*b^2*c))))/(8*c*(4*a*c - b^2)^(1/2)) + (b^6*c*(16*a*c - 4*b^2))/(2*(64*a*c^2 - 16*b^2*c)*(4*a*c - b^2)^(3/2))))/(8*a^3*c^2) + ((b^3 - 3*a*b*c)*(b^7/(8*(4*a*c - b^2)^2) + b^3 - ((20*b^3*c - ((144*b^3*c^2 - ((448*b^3*c^3 - (256*b^3*c^4*(16*a*c - 4*b^2))/(64*a*c^2 - 16*b^2*c))*(16*a*c - 4*b^2))/(2*(64*a*c^2 - 16*b^2*c)))*(16*a*c - 4*b^2))/(2*(64*a*c^2 - 16*b^2*c)))*(16*a*c - 4*b^2))/(2*(64*a*c^2 - 16*b^2*c)) + ((16*a*c - 4*b^2)*((b*((b*(448*b^3*c^3 - (256*b^3*c^4*(16*a*c - 4*b^2))/(64*a*c^2 - 16*b^2*c)))/(8*c*(4*a*c - b^2)^(1/2)) - (32*b^4*c^3*(16*a*c - 4*b^2))/((64*a*c^2 - 16*b^2*c)*(4*a*c - b^2)^(1/2))))/(8*c*(4*a*c - b^2)^(1/2)) - (4*b^5*c^2*(16*a*c - 4*b^2))/((64*a*c^2 - 16*b^2*c)*(4*a*c - b^2))))/(2*(64*a*c^2 - 16*b^2*c)) + (b*(((16*a*c - 4*b^2)*((b*(448*b^3*c^3 - (256*b^3*c^4*(16*a*c - 4*b^2))/(64*a*c^2 - 16*b^2*c)))/(8*c*(4*a*c - b^2)^

```
(1/2)) - (32*b^4*c^3*(16*a*c - 4*b^2))/((64*a*c^2 - 16*b^2*c)*(4*a*c - b^2)
^(1/2))))/(2*(64*a*c^2 - 16*b^2*c)) - (b*(144*b^3*c^2 - ((448*b^3*c^3 - (25
6*b^3*c^4*(16*a*c - 4*b^2))/(64*a*c^2 - 16*b^2*c))*(16*a*c - 4*b^2))/(2*(64
*a*c^2 - 16*b^2*c))))/(8*c*(4*a*c - b^2)^(1/2))))/(8*c*(4*a*c - b^2)^(1/2))
))/(8*a^3*c^2*(4*a*c - b^2)^(1/2)))*(4*a*c - b^2)^2)/b^4 + ((4*a*c - b^2)^(
3/2)*(b^3 - 3*a*b*c)*(a*b^2 + (((b*((b*(768*a*b^2*c^3 - (512*a*b^2*c^4*(16*
a*c - 4*b^2))/(64*a*c^2 - 16*b^2*c)))/(8*c*(4*a*c - b^2)^(1/2)) - (64*a*b^3
*c^3*(16*a*c - 4*b^2))/((64*a*c^2 - 16*b^2*c)*(4*a*c - b^2)^(1/2))))/(8*c*(
4*a*c - b^2)^(1/2)) - (8*a*b^4*c^2*(16*a*c - 4*b^2))/((64*a*c^2 - 16*b^2*c)
*(4*a*c - b^2)))*(16*a*c - 4*b^2))/(2*(64*a*c^2 - 16*b^2*c)) + (a*b^6)/(4*(
4*a*c - b^2)^2) - ((16*a*c - 4*b^2)*(((16*a*c - 4*b^2)*(((768*a*b^2*c^3 - (
512*a*b^2*c^4*(16*a*c - 4*b^2))/(64*a*c^2 - 16*b^2*c))*(16*a*c - 4*b^2))/(2
*(64*a*c^2 - 16*b^2*c)) - 208*a*b^2*c^2))/(2*(64*a*c^2 - 16*b^2*c)) + 24*a*
b^2*c))/(2*(64*a*c^2 - 16*b^2*c)) + (b*(((16*a*c - 4*b^2)*((b*(768*a*b^2*c^
3 - (512*a*b^2*c^4*(16*a*c - 4*b^2))/(64*a*c^2 - 16*b^2*c)))/(8*c*(4*a*c -
b^2)^(1/2)) - (64*a*b^3*c^3*(16*a*c - 4*b^2))/((64*a*c^2 - 16*b^2*c)*(4*a*c
 - b^2)^(1/2))))/(2*(64*a*c^2 - 16*b^2*c)) + (b*(((768*a*b^2*c^3 - (512*a*b
^2*c^4*(16*a*c - 4*b^2))/(64*a*c^2 - 16*b^2*c))*(16*a*c - 4*b^2))/(2*(64*a*
c^2 - 16*b^2*c)) - 208*a*b^2*c^2))/(8*c*(4*a*c - b^2)^(1/2))))/(8*c*(4*a*c
- b^2)^(1/2))))/(a^3*b^4*c^2) + ((a*c - b^2)*(4*a*c - b^2)^2*(((((16*a*c -
4*b^2)*((b*(768*a*b^2*c^3 - (512*a*b^2*c^4*(16*a*c - 4*b^2))/(64*a*c^2 - 16
*b^2*c)))/(8*c*(4*a*c - b^2)^(1/2)) - (64*a*b^3*c^3*(16*a*c - 4*b^2))/((64*
a*c^2 - 16*b^2*c)*(4*a*c - b^2)^(1/2))))/(2*(64*a*c^2 - 16*b^2*c)) + (b*(((
768*a*b^2*c^3 - (512*a*b^2*c^4*(16*a*c - 4*b^2))/(64*a*c^2 - 16*b^2*c))*(16
*a*c - 4*b^2))/(2*(64*a*c^2 - 16*b^2*c)) - 208*a*b^2*c^2))/(8*c*(4*a*c - b^
2)^(1/2)))*(16*a*c - 4*b^2))/(2*(64*a*c^2 - 16*b^2*c)) - (b*((b*((b*(768*a*
b^2*c^3 - (512*a*b^2*c^4*(16*a*c - 4*b^2))/(64*a*c^2 - 16*b^2*c)))/(8*c*(4*
a*c - b^2)^(1/2)) - (64*a*b^3*c^3*(16*a*c - 4*b^2))/((64*a*c^2 - 16*b^2*c)*
(4*a*c - b^2)^(1/2))))/(8*c*(4*a*c - b^2)^(1/2)) - (8*a*b^4*c^2*(16*a*c - 4
*b^2))/((64*a*c^2 - 16*b^2*c)*(4*a*c - b^2))))/(8*c*(4*a*c - b^2)^(1/2)) +
(b*(((16*a*c - 4*b^2)*(((768*a*b^2*c^3 - (512*a*b^2*c^4*(16*a*c - 4*b^2))/(
64*a*c^2 - 16*b^2*c))*(16*a*c - 4*b^2))/(2*(64*a*c^2 - 16*b^2*c)) - 208*a*b
^2*c^2))/(2*(64*a*c^2 - 16*b^2*c)) + 24*a*b^2*c))/(8*c*(4*a*c - b^2)^(1/2))
 + (a*b^5*c*(16*a*c - 4*b^2))/((64*a*c^2 - 16*b^2*c)*(4*a*c - b^2)^(3/2))))
/(a^3*b^4*c^2)))/(4*c*(4*a*c - b^2)^(1/2))
```

sympy [**B**] time = 2.18, size = 223, normalized size = 3.54

$$\left(-\frac{b\sqrt{-4ac+b^2}}{8c\left(4ac-b^2\right)}+\frac{1}{8c}\right)\log\left(x^4+\frac{-16ac\left(-\frac{b\sqrt{-4ac+b^2}}{8c(4ac-b^2)}+\frac{1}{8c}\right)+2a+4b^2\left(-\frac{b\sqrt{-4ac+b^2}}{8c(4ac-b^2)}+\frac{1}{8c}\right)}{b}\right)+\left(\frac{b\sqrt{-4ac+b^2}}{8c\left(4ac-b^2\right)}+\frac{1}{8c}\right)\log\left(x^4+\frac{-16ac\left(\frac{b\sqrt{-4ac+b^2}}{8c(4ac-b^2)}+\frac{1}{8c}\right)+2a+4b^2\left(\frac{b\sqrt{-4ac+b^2}}{8c(4ac-b^2)}+\frac{1}{8c}\right)}{b}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7/(c*x**8+b*x**4+a),x)
```

```
[Out] (-b*sqrt(-4*a*c + b**2)/(8*c*(4*a*c - b**2)) + 1/(8*c))*log(x**4 + (-16*a*c
*(-b*sqrt(-4*a*c + b**2)/(8*c*(4*a*c - b**2)) + 1/(8*c)) + 2*a + 4*b**2*(-b
*sqrt(-4*a*c + b**2)/(8*c*(4*a*c - b**2)) + 1/(8*c)))/b) + (b*sqrt(-4*a*c +
 b**2)/(8*c*(4*a*c - b**2)) + 1/(8*c))*log(x**4 + (-16*a*c*(b*sqrt(-4*a*c +
 b**2)/(8*c*(4*a*c - b**2)) + 1/(8*c)) + 2*a + 4*b**2*(b*sqrt(-4*a*c + b**2
)/(8*c*(4*a*c - b**2)) + 1/(8*c)))/b)
```

3.256 $\int \frac{x^5}{a+bx^4+cx^8}\,dx$

Optimal. Leaf size=159

$$\frac{\sqrt{\sqrt{b^2-4ac}+b}\,\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x^2}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{2\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}}-\frac{\sqrt{b-\sqrt{b^2-4ac}}\,\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x^2}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}}$$

Rubi [A] time = 0.13, antiderivative size = 159, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1359, 1130, 205}

$$\frac{\sqrt{\sqrt{b^2-4ac}+b}\,\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x^2}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{2\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}}-\frac{\sqrt{b-\sqrt{b^2-4ac}}\,\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x^2}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/(a + b*x^4 + c*x^8),x]
```

```
[Out] -(Sqrt[b - Sqrt[b^2 - 4*a*c]]*ArcTan[(Sqrt[2]*Sqrt[c]*x^2)/Sqrt[b - Sqrt[b^
2 - 4*a*c]]])/(2*Sqrt[2]*Sqrt[c]*Sqrt[b^2 - 4*a*c]) + (Sqrt[b + Sqrt[b^2 -
4*a*c]]*ArcTan[(Sqrt[2]*Sqrt[c]*x^2)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[
2]*Sqrt[c]*Sqrt[b^2 - 4*a*c])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1130

```
Int[((d_.)*(x_))^(m_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Wi
th[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(d^2*(b/q + 1))/2, Int[(d*x)^(m - 2)/(b/2
 + q/2 + c*x^2), x], x] - Dist[(d^2*(b/q - 1))/2, Int[(d*x)^(m - 2)/(b/2 -
q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && G
eQ[m, 2]
```

Rule 1359

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*
x^(n/k) + c*x^((2*n)/k))^p, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, c, p
}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\int \frac{x^5}{a+bx^4+cx^8}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{x^2}{a+bx^2+cx^4}\,dx,x,x^2\right)\\ &= \frac{1}{4}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\text{Subst}\left(\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx,x,x^2\right)+\frac{1}{4}\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)\text{Subs}\\ &= -\frac{\sqrt{b-\sqrt{b^2-4ac}}\,\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x^2}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}}+\frac{\sqrt{b+\sqrt{b^2-4ac}}\,\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x^2}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}}\end{aligned}$$

Mathematica [A] time = 0.09, size = 171, normalized size = 1.08

$$\frac{\left(\sqrt{b^2-4ac}-b\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x^2}{\sqrt{b-\sqrt{b^2-4ac}}}\right)+\sqrt{b-\sqrt{b^2-4ac}}\sqrt{\sqrt{b^2-4ac}+b}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x^2}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{2\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/(a + b*x^4 + c*x^8),x]
```

```
[Out] ((-b + Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x^2)/Sqrt[b - Sqrt[b^2 -
4*a*c]]] + Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[b + Sqrt[b^2 - 4*a*c]]*ArcTan[(
Sqrt[2]*Sqrt[c]*x^2)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*Sqrt[c]*Sqrt[
b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]])
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5}{a+bx^4+cx^8}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^5/(a + b*x^4 + c*x^8),x]
```

```
[Out] IntegrateAlgebraic[x^5/(a + b*x^4 + c*x^8), x]
```

fricas [B] time = 1.25, size = 567, normalized size = 3.57

$$\frac{1}{4}\sqrt{\frac{1}{2}}\sqrt{-\frac{b+\frac{b^2c-4ac^2}{\sqrt{b^2c^2-4ac^3}}}{b^2c-4ac^2}}\log\left(x^2+\frac{\sqrt{\frac{1}{2}}\left(b^2c-4ac^2\right)\sqrt{-\frac{b+\frac{b^2c-4ac^2}{\sqrt{b^2c^2-4ac^3}}}{b^2c-4ac^2}}}{\sqrt{b^2c^2-4ac^3}}\right)-\frac{1}{4}\sqrt{\frac{1}{2}}\sqrt{-\frac{b+\frac{b^2c-4ac^2}{\sqrt{b^2c^2-4ac^3}}}{b^2c-4ac^2}}\log\left(x^2-\frac{\sqrt{\frac{1}{2}}\left(b^2c-4ac^2\right)\sqrt{-\frac{b+\frac{b^2c-4ac^2}{\sqrt{b^2c^2-4ac^3}}}{b^2c-4ac^2}}}{\sqrt{b^2c^2-4ac^3}}\right)-\frac{1}{4}\sqrt{\frac{1}{2}}\sqrt{-\frac{b-\frac{b^2c-4ac^2}{\sqrt{b^2c^2-4ac^3}}}{b^2c-4ac^2}}\log\left(x^2+\frac{\sqrt{\frac{1}{2}}\left(b^2c-4ac^2\right)\sqrt{-\frac{b-\frac{b^2c-4ac^2}{\sqrt{b^2c^2-4ac^3}}}{b^2c-4ac^2}}}{\sqrt{b^2c^2-4ac^3}}\right)+\frac{1}{4}\sqrt{\frac{1}{2}}\sqrt{-\frac{b-\frac{b^2c-4ac^2}{\sqrt{b^2c^2-4ac^3}}}{b^2c-4ac^2}}\log\left(x^2-\frac{\sqrt{\frac{1}{2}}\left(b^2c-4ac^2\right)\sqrt{-\frac{b-\frac{b^2c-4ac^2}{\sqrt{b^2c^2-4ac^3}}}{b^2c-4ac^2}}}{\sqrt{b^2c^2-4ac^3}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(c*x^8+b*x^4+a),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(1/2)*sqrt(-(b + (b^2*c - 4*a*c^2)/sqrt(b^2*c^2 - 4*a*c^3))/(b^2*c
- 4*a*c^2))*log(x^2 + sqrt(1/2)*(b^2*c - 4*a*c^2)*sqrt(-(b + (b^2*c - 4*a*c
^2)/sqrt(b^2*c^2 - 4*a*c^3))/(b^2*c - 4*a*c^2))/sqrt(b^2*c^2 - 4*a*c^3)) -
1/4*sqrt(1/2)*sqrt(-(b + (b^2*c - 4*a*c^2)/sqrt(b^2*c^2 - 4*a*c^3))/(b^2*c
- 4*a*c^2))*log(x^2 - sqrt(1/2)*(b^2*c - 4*a*c^2)*sqrt(-(b + (b^2*c - 4*a*c
^2)/sqrt(b^2*c^2 - 4*a*c^3))/(b^2*c - 4*a*c^2))/sqrt(b^2*c^2 - 4*a*c^3)) -
1/4*sqrt(1/2)*sqrt(-(b - (b^2*c - 4*a*c^2)/sqrt(b^2*c^2 - 4*a*c^3))/(b^2*c
- 4*a*c^2))*log(x^2 + sqrt(1/2)*(b^2*c - 4*a*c^2)*sqrt(-(b - (b^2*c - 4*a*c
^2)/sqrt(b^2*c^2 - 4*a*c^3))/(b^2*c - 4*a*c^2))/sqrt(b^2*c^2 - 4*a*c^3)) +
1/4*sqrt(1/2)*sqrt(-(b - (b^2*c - 4*a*c^2)/sqrt(b^2*c^2 - 4*a*c^3))/(b^2*c
- 4*a*c^2))*log(x^2 - sqrt(1/2)*(b^2*c - 4*a*c^2)*sqrt(-(b - (b^2*c - 4*a*c
^2)/sqrt(b^2*c^2 - 4*a*c^3))/(b^2*c - 4*a*c^2))/sqrt(b^2*c^2 - 4*a*c^3))
```

giac [B] time = 18.74, size = 1036, normalized size = 6.52

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(c*x^8+b*x^4+a),x, algorithm="giac")
```

```
[Out] 1/8*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4 - 8*sqrt(2)*sqrt(b*c + sqr
t(b^2 - 4*a*c)*c)*a*b^2*c - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c
 - 2*b^4*c + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^2 + 8*sqrt(2)
*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^2 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*
a*c)*c)*b^2*c^2 + 16*a*b^2*c^2 + 2*b^3*c^2 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2
```

```
- 4*a*c)*c)*a*c^3 - 32*a^2*c^3 - 8*a*b*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt
(b*c + sqrt(b^2 - 4*a*c)*c)*b^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sq
rt(b^2 - 4*a*c)*c)*a*b*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
- 4*a*c)*c)*b^2*c - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*
c)*b*c^2 + 2*(b^2 - 4*a*c)*b^2*c - 8*(b^2 - 4*a*c)*a*c^2 - 2*(b^2 - 4*a*c)*
b*c^2)*x^4*arctan(2*sqrt(1/2)*x^2/sqrt((b + sqrt(b^2 - 4*a*c))/c))/((a*b^4
- 8*a^2*b^2*c - 2*a*b^3*c + 16*a^3*c^2 + 8*a^2*b*c^2 + a*b^2*c^2 - 4*a^2*c^
3)*abs(c)) + 1/8*(sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4 - 8*sqrt(2)*s
qrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c - 2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*
a*c)*c)*b^3*c + 2*b^4*c + 16*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*c^
2 + 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^2 + sqrt(2)*sqrt(b*c -
sqrt(b^2 - 4*a*c)*c)*b^2*c^2 - 16*a*b^2*c^2 - 2*b^3*c^2 - 4*sqrt(2)*sqrt(b*
c - sqrt(b^2 - 4*a*c)*c)*a*c^3 + 32*a^2*c^3 + 8*a*b*c^3 + sqrt(2)*sqrt(b^2
- 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3 - 4*sqrt(2)*sqrt(b^2 - 4*a*c)*
sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c - 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*
c - sqrt(b^2 - 4*a*c)*c)*b^2*c + sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(
b^2 - 4*a*c)*c)*b*c^2 - 2*(b^2 - 4*a*c)*b^2*c + 8*(b^2 - 4*a*c)*a*c^2 + 2*(
b^2 - 4*a*c)*b*c^2)*x^4*arctan(2*sqrt(1/2)*x^2/sqrt((b - sqrt(b^2 - 4*a*c))
/c))/((a*b^4 - 8*a^2*b^2*c - 2*a*b^3*c + 16*a^3*c^2 + 8*a^2*b*c^2 + a*b^2*c
^2 - 4*a^2*c^3)*abs(c))
```

maple [A] time = 0.02, size = 216, normalized size = 1.36

$$\frac{\sqrt{2}\, b \operatorname{arctanh}\left(\frac{\sqrt{2}\, c x^2}{\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}\right)}{4\sqrt{-4ac+b^2}\,\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}+\frac{\sqrt{2}\, b \arctan\left(\frac{\sqrt{2}\, c x^2}{\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}\right)}{4\sqrt{-4ac+b^2}\,\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}-\frac{\sqrt{2}\ \operatorname{arctanh}\left(\frac{\sqrt{2}\, c x^2}{\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}\right)}{4\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}+\frac{\sqrt{2}\ \arctan\left(\frac{\sqrt{2}\, c x^2}{\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}\right)}{4\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(c*x^8+b*x^4+a),x)
```

```
[Out] -1/4*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c
+b^2)^(1/2))*c)^(1/2)*c*x^2)+1/4/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^
2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x^2)
*b+1/4*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+
b^2)^(1/2))*c)^(1/2)*c*x^2)+1/4/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)
^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x^2)*b
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5}{cx^8+bx^4+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(c*x^8+b*x^4+a),x, algorithm="maxima")
```

```
[Out] integrate(x^5/(c*x^8 + b*x^4 + a), x)
```

mupad [B] time = 2.81, size = 1220, normalized size = 7.67

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(a + b*x^4 + c*x^8),x)
```

```
[Out] atan((x^2*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2)*1i + b^3*x
^2*1i - a*b*c*x^2*4i)/(8*b^4*(((b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^
4*c)^(1/2) - b^3 + 4*a*b*c)/(32*b^4*c + 512*a^2*c^3 - 256*a*b^2*c^2))^(1/2)
 + 128*b^5*c*(((b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2) - b^3
```

```
 + 4*a*b*c)/(32*b^4*c + 512*a^2*c^3 - 256*a*b^2*c^2))^(3/2) + 64*a^2*c^2*((
(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2) - b^3 + 4*a*b*c)/(32
*b^4*c + 512*a^2*c^3 - 256*a*b^2*c^2))^(1/2) - 1024*a*b^3*c^2*(((b^6 - 64*a
^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2) - b^3 + 4*a*b*c)/(32*b^4*c + 51
2*a^2*c^3 - 256*a*b^2*c^2))^(3/2) + 2048*a^2*b*c^3*(((b^6 - 64*a^3*c^3 + 48
*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2) - b^3 + 4*a*b*c)/(32*b^4*c + 512*a^2*c^3 -
 256*a*b^2*c^2))^(3/2) - 48*a*b^2*c*(((b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 -
12*a*b^4*c)^(1/2) - b^3 + 4*a*b*c)/(32*b^4*c + 512*a^2*c^3 - 256*a*b^2*c^2)
)^(1/2)))*(((b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2) - b^3 +
4*a*b*c)/(32*b^4*c + 512*a^2*c^3 - 256*a*b^2*c^2))^(1/2)*2i - atan((x^2*(b^
6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2)*1i - b^3*x^2*1i + a*b*c
*x^2*4i)/(8*b^4*(-(b^3 + (b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(
1/2) - 4*a*b*c)/(32*b^4*c + 512*a^2*c^3 - 256*a*b^2*c^2))^(1/2) + 128*b^5*c
*(-(b^3 + (b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2) - 4*a*b*c)
/(32*b^4*c + 512*a^2*c^3 - 256*a*b^2*c^2))^(3/2) + 64*a^2*c^2*(-(b^3 + (b^6
 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2) - 4*a*b*c)/(32*b^4*c + 5
12*a^2*c^3 - 256*a*b^2*c^2))^(1/2) - 48*a*b^2*c*(-(b^3 + (b^6 - 64*a^3*c^3
+ 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2) - 4*a*b*c)/(32*b^4*c + 512*a^2*c^3 - 2
56*a*b^2*c^2))^(1/2) - 1024*a*b^3*c^2*(-(b^3 + (b^6 - 64*a^3*c^3 + 48*a^2*b
^2*c^2 - 12*a*b^4*c)^(1/2) - 4*a*b*c)/(32*b^4*c + 512*a^2*c^3 - 256*a*b^2*c
^2))^(3/2) + 2048*a^2*b*c^3*(-(b^3 + (b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 1
2*a*b^4*c)^(1/2) - 4*a*b*c)/(32*b^4*c + 512*a^2*c^3 - 256*a*b^2*c^2))^(3/2)
))*(-(b^3 + (b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2) - 4*a*b*
c)/(32*b^4*c + 512*a^2*c^3 - 256*a*b^2*c^2))^(1/2)*2i
```

sympy [A] time = 2.51, size = 76, normalized size = 0.48

$$\text{RootSum}\left(t^4\left(4096a^2c^3 - 2048ab^2c^2 + 256b^4c\right) + t^2\left(-64abc + 16b^3\right) + a, \left(t \mapsto t\log\left(512t^3ac^2 - 128t^3b^2c - 4tb + x^2\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(c*x**8+b*x**4+a),x)
```

```
[Out] RootSum(_t**4*(4096*a**2*c**3 - 2048*a*b**2*c**2 + 256*b**4*c) + _t**2*(-64
*a*b*c + 16*b**3) + a, Lambda(_t, _t*log(512*_t**3*a*c**2 - 128*_t**3*b**2*
c - 4*_t*b + x**2)))
```

3.257 $\int \frac{x^3}{a+bx^4+cx^8} \, dx$

Optimal. Leaf size=38

$$-\frac{\tanh^{-1}\left(\frac{b+2cx^4}{\sqrt{b^2-4ac}}\right)}{2\sqrt{b^2-4ac}}$$

Rubi [A] time = 0.03, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1352, 618, 206}

$$-\frac{\tanh^{-1}\left(\frac{b+2cx^4}{\sqrt{b^2-4ac}}\right)}{2\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(a + b*x^4 + c*x^8),x]
```

```
[Out] -ArcTanh[(b + 2*c*x^4)/Sqrt[b^2 - 4*a*c]]/(2*Sqrt[b^2 - 4*a*c])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1352

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a,
b, c, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^3}{a+bx^4+cx^8} \, dx &= \frac{1}{4}\text{Subst}\left(\int \frac{1}{a+bx+cx^2} \, dx, x, x^4\right) \\
&= -\left(\frac{1}{2}\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2} \, dx, x, b+2cx^4\right)\right) \\
&= -\frac{\tanh^{-1}\left(\frac{b+2cx^4}{\sqrt{b^2-4ac}}\right)}{2\sqrt{b^2-4ac}}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 42, normalized size = 1.11

$$\frac{\tan^{-1}\left(\frac{b+2cx^4}{\sqrt{4ac-b^2}}\right)}{2\sqrt{4ac-b^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(a + b*x^4 + c*x^8),x]
```

```
[Out] ArcTan[(b + 2*c*x^4)/Sqrt[-b^2 + 4*a*c]]/(2*Sqrt[-b^2 + 4*a*c])
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{a+bx^4+cx^8}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^3/(a + b*x^4 + c*x^8),x]
```

```
[Out] IntegrateAlgebraic[x^3/(a + b*x^4 + c*x^8), x]
```

fricas [A] time = 1.29, size = 129, normalized size = 3.39

$$\left[\frac{\log\left(\frac{2\,c^2x^8+2\,bcx^4+b^2-2\,ac-\left(2\,cx^4+b\right)\sqrt{b^2-4\,ac}}{cx^8+bx^4+a}\right)}{4\,\sqrt{b^2-4\,ac}},\,-\frac{\sqrt{-b^2+4\,ac}\,\arctan\left(-\frac{\left(2\,cx^4+b\right)\sqrt{-b^2+4\,ac}}{b^2-4\,ac}\right)}{2\,\left(b^2-4\,ac\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^8+b*x^4+a),x, algorithm="fricas")
```

```
[Out] [1/4*log((2*c^2*x^8 + 2*b*c*x^4 + b^2 - 2*a*c - (2*c*x^4 + b)*sqrt(b^2 - 4*
a*c))/(c*x^8 + b*x^4 + a))/sqrt(b^2 - 4*a*c), -1/2*sqrt(-b^2 + 4*a*c)*arcta
n(-(2*c*x^4 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c))/(b^2 - 4*a*c)]
```

giac [A] time = 17.34, size = 36, normalized size = 0.95

$$\frac{\arctan\left(\frac{2\,cx^4+b}{\sqrt{-b^2+4\,ac}}\right)}{2\,\sqrt{-b^2+4\,ac}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^8+b*x^4+a),x, algorithm="giac")
```

```
[Out] 1/2*arctan((2*c*x^4 + b)/sqrt(-b^2 + 4*a*c))/sqrt(-b^2 + 4*a*c)
```

maple [A] time = 0.00, size = 37, normalized size = 0.97

$$\frac{\arctan\left(\frac{2c\,x^4+b}{\sqrt{4ac-b^2}}\right)}{2\sqrt{4ac-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(c*x^8+b*x^4+a),x)
```

```
[Out] 1/2/(4*a*c-b^2)^(1/2)*arctan((2*c*x^4+b)/(4*a*c-b^2)^(1/2))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^8+b*x^4+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
```

```
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

mupad [B] time = 1.37, size = 260, normalized size = 6.84

$$-\frac{\operatorname{atan}\left(\frac{(4ac-b^2)^2\left(\frac{\left(\frac{4ac^4}{4ac-b^2}-\frac{4ab^2c^4}{(4ac-b^2)^2}\right)(b^3-3abc)}{8a^3c^2\sqrt{4ac-b^2}}-x^4\left(\frac{\left(\frac{2c^4}{\sqrt{4ac-b^2}}-\frac{6b^2c^4}{(4ac-b^2)^{3/2}}\right)(ac-b^2)}{8a^3c^2}-\frac{(b^3-3abc)\left(\frac{6bc^4}{4ac-b^2}-\frac{2b^3c^4}{(4ac-b^2)^2}\right)}{8a^3c^2\sqrt{4ac-b^2}}+\frac{bc^2(ac-b^2)}{a^2(4ac-b^2)^{3/2}}\right)\right)}{2c^4}\right)}{2\sqrt{4ac-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a + b*x^4 + c*x^8),x)
```

```
[Out] -atan(((4*a*c - b^2)^2*((((4*a*c^4)/(4*a*c - b^2) - (4*a*b^2*c^4)/(4*a*c -
b^2)^2)*(b^3 - 3*a*b*c))/(8*a^3*c^2*(4*a*c - b^2)^(1/2)) - x^4*((((2*c^4)/(
4*a*c - b^2)^(1/2) - (6*b^2*c^4)/(4*a*c - b^2)^(3/2))*(a*c - b^2))/(8*a^3*c
^2) - ((b^3 - 3*a*b*c)*((6*b*c^4)/(4*a*c - b^2) - (2*b^3*c^4)/(4*a*c - b^2)
^2))/(8*a^3*c^2*(4*a*c - b^2)^(1/2))) + (b*c^2*(a*c - b^2))/(a^2*(4*a*c - b
^2)^(3/2)))))/(2*c^4))/(2*(4*a*c - b^2)^(1/2))
```

sympy [B] time = 0.77, size = 131, normalized size = 3.45

$$-\frac{\sqrt{-\frac{1}{4ac-b^2}}\log\left(x^4+\frac{-4ac\sqrt{-\frac{1}{4ac-b^2}}+b^2\sqrt{-\frac{1}{4ac-b^2}}+b}{2c}\right)}{4}+\frac{\sqrt{-\frac{1}{4ac-b^2}}\log\left(x^4+\frac{4ac\sqrt{-\frac{1}{4ac-b^2}}-b^2\sqrt{-\frac{1}{4ac-b^2}}+b}{2c}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(c*x**8+b*x**4+a),x)
```

```
[Out] -sqrt(-1/(4*a*c - b**2))*log(x**4 + (-4*a*c*sqrt(-1/(4*a*c - b**2)) + b**2*
sqrt(-1/(4*a*c - b**2)) + b)/(2*c))/4 + sqrt(-1/(4*a*c - b**2))*log(x**4 +
(4*a*c*sqrt(-1/(4*a*c - b**2)) - b**2*sqrt(-1/(4*a*c - b**2)) + b)/(2*c))/4
```

3.258 $\int \frac{x}{a+bx^4+cx^8}\,dx$

Optimal. Leaf size=154

$$\frac{\sqrt{c}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x^2}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{c}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x^2}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4ac}+b}}$$

Rubi [A] time = 0.09, antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {1359, 1093, 205}

$$\frac{\sqrt{c}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x^2}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{c}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x^2}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4ac}+b}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a + b*x^4 + c*x^8),x]
```

```
[Out] (Sqrt[c]*ArcTan[(Sqrt[2]*Sqrt[c]*x^2)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2
]*Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - (Sqrt[c]*ArcTan[(Sqrt[2]
*Sqrt[c]*x^2)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[b^2 - 4*a*c]*Sqrt
[b + Sqrt[b^2 - 4*a*c]])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1093

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[b^
2 - 4*a*c, 2]}, Dist[c/q, Int[1/(b/2 - q/2 + c*x^2), x], x] - Dist[c/q, Int
[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
0] && PosQ[b^2 - 4*a*c]
```

Rule 1359

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*
x^(n/k) + c*x^((2*n)/k))^p, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, c, p
}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\int \frac{x}{a+bx^4+cx^8}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{a+bx^2+cx^4}\,dx,x,x^2\right)\\ &= \frac{c\,\text{Subst}\left(\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx,x,x^2\right)}{2\sqrt{b^2-4ac}}-\frac{c\,\text{Subst}\left(\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx,x,x^2\right)}{2\sqrt{b^2-4ac}}\\ &= \frac{\sqrt{c}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x^2}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{c}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x^2}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{b^2-4ac}\sqrt{b+\sqrt{b^2-4ac}}}\end{aligned}$$

Mathematica [A] time = 0.08, size = 133, normalized size = 0.86

$$\frac{\sqrt{c}\left(\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x^2}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x^2}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a + b*x^4 + c*x^8),x]
```

```
[Out] (Sqrt[c]*(ArcTan[(Sqrt[2]*Sqrt[c]*x^2)/Sqrt[b - Sqrt[b^2 - 4*a*c]]]/Sqrt[b
 - Sqrt[b^2 - 4*a*c]] - ArcTan[(Sqrt[2]*Sqrt[c]*x^2)/Sqrt[b + Sqrt[b^2 - 4*a
*c]]]/Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(Sqrt[2]*Sqrt[b^2 - 4*a*c])
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{a+bx^4+cx^8}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x/(a + b*x^4 + c*x^8),x]
```

```
[Out] IntegrateAlgebraic[x/(a + b*x^4 + c*x^8), x]
```

fricas [B] time = 1.25, size = 619, normalized size = 4.02

$$-\frac{1}{4}\sqrt{\frac{1}{2}}\sqrt{-\frac{b+\frac{ab^2-4a^2c}{\sqrt{a^2b^2-4a^3c}}}{ab^2-4a^2c}}\log\left(cx^2+\frac{1}{2}\sqrt{\frac{1}{2}}\left(b^2-4ac-\frac{ab^3-4a^2bc}{\sqrt{a^2b^2-4a^3c}}\right)\sqrt{-\frac{b+\frac{ab^2-4a^2c}{\sqrt{a^2b^2-4a^3c}}}{ab^2-4a^2c}}\right)+\frac{1}{4}\sqrt{\frac{1}{2}}\sqrt{-\frac{b+\frac{ab^2-4a^2c}{\sqrt{a^2b^2-4a^3c}}}{ab^2-4a^2c}}\log\left(cx^2-\frac{1}{2}\sqrt{\frac{1}{2}}\left(b^2-4ac-\frac{ab^3-4a^2bc}{\sqrt{a^2b^2-4a^3c}}\right)\sqrt{-\frac{b+\frac{ab^2-4a^2c}{\sqrt{a^2b^2-4a^3c}}}{ab^2-4a^2c}}\right)-\frac{1}{4}\sqrt{\frac{1}{2}}\sqrt{-\frac{b-\frac{ab^2-4a^2c}{\sqrt{a^2b^2-4a^3c}}}{ab^2-4a^2c}}\log\left(cx^2+\frac{1}{2}\sqrt{\frac{1}{2}}\left(b^2-4ac+\frac{ab^3-4a^2bc}{\sqrt{a^2b^2-4a^3c}}\right)\sqrt{-\frac{b-\frac{ab^2-4a^2c}{\sqrt{a^2b^2-4a^3c}}}{ab^2-4a^2c}}\right)+\frac{1}{4}\sqrt{\frac{1}{2}}\sqrt{-\frac{b-\frac{ab^2-4a^2c}{\sqrt{a^2b^2-4a^3c}}}{ab^2-4a^2c}}\log\left(cx^2-\frac{1}{2}\sqrt{\frac{1}{2}}\left(b^2-4ac+\frac{ab^3-4a^2bc}{\sqrt{a^2b^2-4a^3c}}\right)\sqrt{-\frac{b-\frac{ab^2-4a^2c}{\sqrt{a^2b^2-4a^3c}}}{ab^2-4a^2c}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^8+b*x^4+a),x, algorithm="fricas")
```

```
[Out] -1/4*sqrt(1/2)*sqrt(-(b + (a*b^2 - 4*a^2*c)/sqrt(a^2*b^2 - 4*a^3*c))/(a*b^2
 - 4*a^2*c))*log(c*x^2 + 1/2*sqrt(1/2)*(b^2 - 4*a*c - (a*b^3 - 4*a^2*b*c)/s
qrt(a^2*b^2 - 4*a^3*c))*sqrt(-(b + (a*b^2 - 4*a^2*c)/sqrt(a^2*b^2 - 4*a^3*c
))/(a*b^2 - 4*a^2*c))) + 1/4*sqrt(1/2)*sqrt(-(b + (a*b^2 - 4*a^2*c)/sqrt(a^
2*b^2 - 4*a^3*c))/(a*b^2 - 4*a^2*c))*log(c*x^2 - 1/2*sqrt(1/2)*(b^2 - 4*a*c
 - (a*b^3 - 4*a^2*b*c)/sqrt(a^2*b^2 - 4*a^3*c))*sqrt(-(b + (a*b^2 - 4*a^2*c
)/sqrt(a^2*b^2 - 4*a^3*c))/(a*b^2 - 4*a^2*c))) - 1/4*sqrt(1/2)*sqrt(-(b - (
a*b^2 - 4*a^2*c)/sqrt(a^2*b^2 - 4*a^3*c))/(a*b^2 - 4*a^2*c))*log(c*x^2 + 1/
2*sqrt(1/2)*(b^2 - 4*a*c + (a*b^3 - 4*a^2*b*c)/sqrt(a^2*b^2 - 4*a^3*c))*sqr
t(-(b - (a*b^2 - 4*a^2*c)/sqrt(a^2*b^2 - 4*a^3*c))/(a*b^2 - 4*a^2*c))) + 1/
4*sqrt(1/2)*sqrt(-(b - (a*b^2 - 4*a^2*c)/sqrt(a^2*b^2 - 4*a^3*c))/(a*b^2 -
4*a^2*c))*log(c*x^2 - 1/2*sqrt(1/2)*(b^2 - 4*a*c + (a*b^3 - 4*a^2*b*c)/sqrt
(a^2*b^2 - 4*a^3*c))*sqrt(-(b - (a*b^2 - 4*a^2*c)/sqrt(a^2*b^2 - 4*a^3*c))/
(a*b^2 - 4*a^2*c)))
```

giac [B] time = 19.50, size = 1030, normalized size = 6.69

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^8+b*x^4+a),x, algorithm="giac")
```

```
[Out] 1/8*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4 - 8*sqrt(2)*sqrt(b*c + sqr
t(b^2 - 4*a*c)*c)*a*b^2*c - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c
 - 2*b^4*c + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^2 + 8*sqrt(2)
*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^2 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*
```

```
a*c)*c)*b^2*c^2 + 16*a*b^2*c^2 + 2*b^3*c^2 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2
- 4*a*c)*c)*a*c^3 - 32*a^2*c^3 - 8*a*b*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt
(b*c + sqrt(b^2 - 4*a*c)*c)*b^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sq
rt(b^2 - 4*a*c)*c)*a*b*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
- 4*a*c)*c)*b^2*c - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*
c)*b*c^2 + 2*(b^2 - 4*a*c)*b^2*c - 8*(b^2 - 4*a*c)*a*c^2 - 2*(b^2 - 4*a*c)*
b*c^2)*arctan(2*sqrt(1/2)*x^2/sqrt((b + sqrt(b^2 - 4*a*c))/c))/((a*b^4 - 8*
a^2*b^2*c - 2*a*b^3*c + 16*a^3*c^2 + 8*a^2*b*c^2 + a*b^2*c^2 - 4*a^2*c^3)*a
bs(c)) + 1/8*(sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4 - 8*sqrt(2)*sqrt(
b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c - 2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)
*c)*b^3*c + 2*b^4*c + 16*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*c^2 +
8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^2 + sqrt(2)*sqrt(b*c - sqrt
(b^2 - 4*a*c)*c)*b^2*c^2 - 16*a*b^2*c^2 - 2*b^3*c^2 - 4*sqrt(2)*sqrt(b*c -
sqrt(b^2 - 4*a*c)*c)*a*c^3 + 32*a^2*c^3 + 8*a*b*c^3 + sqrt(2)*sqrt(b^2 - 4*
a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3 - 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt
(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c - 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c -
sqrt(b^2 - 4*a*c)*c)*b^2*c + sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2
- 4*a*c)*c)*b*c^2 - 2*(b^2 - 4*a*c)*b^2*c + 8*(b^2 - 4*a*c)*a*c^2 + 2*(b^2
- 4*a*c)*b*c^2)*arctan(2*sqrt(1/2)*x^2/sqrt((b - sqrt(b^2 - 4*a*c))/c))/((a
*b^4 - 8*a^2*b^2*c - 2*a*b^3*c + 16*a^3*c^2 + 8*a^2*b*c^2 + a*b^2*c^2 - 4*a
^2*c^3)*abs(c))
```

maple [A] time = 0.01, size = 120, normalized size = 0.78

$$-\frac{\sqrt{2}\,c\,\mathrm{arctanh}\left(\frac{\sqrt{2}\,c\,x^{2}}{\sqrt{\left(-b+\sqrt{-4ac+b^{2}}\right)c}}\right)}{2\sqrt{-4ac+b^{2}}\,\sqrt{\left(-b+\sqrt{-4ac+b^{2}}\right)c}}-\frac{\sqrt{2}\,c\arctan\left(\frac{\sqrt{2}\,c\,x^{2}}{\sqrt{\left(b+\sqrt{-4ac+b^{2}}\right)c}}\right)}{2\sqrt{-4ac+b^{2}}\,\sqrt{\left(b+\sqrt{-4ac+b^{2}}\right)c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(c*x^8+b*x^4+a),x)
```

```
[Out] -1/2*c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh
(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x^2)-1/2*c/(-4*a*c+b^2)^(1/2)*
2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1
/2))*c)^(1/2)*c*x^2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{cx^{8}+bx^{4}+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^8+b*x^4+a),x, algorithm="maxima")
```

```
[Out] integrate(x/(c*x^8 + b*x^4 + a), x)
```

mupad [B] time = 2.27, size = 1105, normalized size = 7.18

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a + b*x^4 + c*x^8),x)
```

```
[Out] atan((b^4*x^2*1i + b*x^2*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(
1/2)*1i + a^2*c^2*x^2*8i - a*b^2*c*x^2*6i)/(128*a^2*b^5*(-(b^3 + (b^6 - 64*
a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2) - 4*a*b*c)/(32*a*b^4 + 512*a^3
*c^2 - 256*a^2*b^2*c))^(3/2) - 64*a^3*c^2*(-(b^3 + (b^6 - 64*a^3*c^3 + 48*a
```

```
^2*b^2*c^2 - 12*a*b^4*c)^(1/2) - 4*a*b*c)/(32*a*b^4 + 512*a^3*c^2 - 256*a^2
*b^2*c))^(1/2) + 16*a^2*b^2*c*(-(b^3 + (b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 -
 12*a*b^4*c)^(1/2) - 4*a*b*c)/(32*a*b^4 + 512*a^3*c^2 - 256*a^2*b^2*c))^(1/
2) - 1024*a^3*b^3*c*(-(b^3 + (b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*
c)^(1/2) - 4*a*b*c)/(32*a*b^4 + 512*a^3*c^2 - 256*a^2*b^2*c))^(3/2) + 2048*
a^4*b*c^2*(-(b^3 + (b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2) -
 4*a*b*c)/(32*a*b^4 + 512*a^3*c^2 - 256*a^2*b^2*c))^(3/2)))*(-(b^3 + (b^6 -
 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2) - 4*a*b*c)/(32*a*b^4 + 512
*a^3*c^2 - 256*a^2*b^2*c))^(1/2)*2i + atan((b^4*x^2*1i - b*x^2*(b^6 - 64*a^
3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2)*1i + a^2*c^2*x^2*8i - a*b^2*c*x^
2*6i)/(128*a^2*b^5*(((b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2)
 - b^3 + 4*a*b*c)/(32*a*b^4 + 512*a^3*c^2 - 256*a^2*b^2*c))^(3/2) - 64*a^3*
c^2*(((b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2) - b^3 + 4*a*b*
c)/(32*a*b^4 + 512*a^3*c^2 - 256*a^2*b^2*c))^(1/2) + 16*a^2*b^2*c*(((b^6 -
64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2) - b^3 + 4*a*b*c)/(32*a*b^4
+ 512*a^3*c^2 - 256*a^2*b^2*c))^(1/2) - 1024*a^3*b^3*c*(((b^6 - 64*a^3*c^3
+ 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2) - b^3 + 4*a*b*c)/(32*a*b^4 + 512*a^3*c
^2 - 256*a^2*b^2*c))^(3/2) + 2048*a^4*b*c^2*(((b^6 - 64*a^3*c^3 + 48*a^2*b^
2*c^2 - 12*a*b^4*c)^(1/2) - b^3 + 4*a*b*c)/(32*a*b^4 + 512*a^3*c^2 - 256*a^
2*b^2*c))^(3/2)))*(((b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2)
- b^3 + 4*a*b*c)/(32*a*b^4 + 512*a^3*c^2 - 256*a^2*b^2*c))^(1/2)*2i
```

sympy [A] time = 3.36, size = 88, normalized size = 0.57

$$\operatorname{RootSum}\left(t^4\left(4096a^3c^2-2048a^2b^2c+256ab^4\right)+t^2\left(-64abc+16b^3\right)+c,\left(t\mapsto t\log\left(x^2+\frac{256t^3a^2bc-64t^3ab^3+8tac-4tb^2}{c}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x**8+b*x**4+a),x)
```

```
[Out] RootSum(_t**4*(4096*a**3*c**2 - 2048*a**2*b**2*c + 256*a*b**4) + _t**2*(-64
*a*b*c + 16*b**3) + c, Lambda(_t, _t*log(x**2 + (256*_t**3*a**2*b*c - 64*_t
**3*a*b**3 + 8*_t*a*c - 4*_t*b**2)/c)))
```

3.259 $\int \frac{1}{x(a+bx^4+cx^8)} dx$

Optimal. Leaf size=69

$$\frac{b\tanh^{-1}\left(\frac{b+2cx^4}{\sqrt{b^2-4ac}}\right)}{4a\sqrt{b^2-4ac}}-\frac{\log\left(a+bx^4+cx^8\right)}{8a}+\frac{\log(x)}{a}$$

Rubi [A] time = 0.07, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389, Rules used = {1357, 705, 29, 634, 618, 206, 628}

$$\frac{b\tanh^{-1}\left(\frac{b+2cx^4}{\sqrt{b^2-4ac}}\right)}{4a\sqrt{b^2-4ac}}-\frac{\log\left(a+bx^4+cx^8\right)}{8a}+\frac{\log(x)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a + b*x^4 + c*x^8)),x]
```

```
[Out] (b*ArcTanh[(b + 2*c*x^4)/Sqrt[b^2 - 4*a*c]])/(4*a*Sqrt[b^2 - 4*a*c]) + Log[
x]/a - Log[a + b*x^4 + c*x^8]/(8*a)
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 705

```
Int[1/(((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)), x_Symbol]
 :> Dist[e^2/(c*d^2 - b*d*e + a*e^2), Int[1/(d + e*x), x], x] + Dist[1/(c*d
^2 - b*d*e + a*e^2), Int[(c*d - b*e - c*e*x)/(a + b*x + c*x^2), x], x] /; F
reeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^
2, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1357

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x\left(a+bx^4+cx^8\right)}\,dx &= \frac{1}{4}\operatorname{Subst}\left(\int \frac{1}{x\left(a+bx+cx^2\right)}\,dx, x, x^4\right) \\
&= \frac{\operatorname{Subst}\left(\int \frac{1}{x}\,dx, x, x^4\right)}{4a} + \frac{\operatorname{Subst}\left(\int \frac{-b-cx}{a+bx+cx^2}\,dx, x, x^4\right)}{4a} \\
&= \frac{\log(x)}{a} - \frac{\operatorname{Subst}\left(\int \frac{b+2cx}{a+bx+cx^2}\,dx, x, x^4\right)}{8a} - \frac{b\operatorname{Subst}\left(\int \frac{1}{a+bx+cx^2}\,dx, x, x^4\right)}{8a} \\
&= \frac{\log(x)}{a} - \frac{\log\left(a+bx^4+cx^8\right)}{8a} + \frac{b\operatorname{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx, x, b+2cx^4\right)}{4a} \\
&= \frac{b\tanh^{-1}\left(\frac{b+2cx^4}{\sqrt{b^2-4ac}}\right)}{4a\sqrt{b^2-4ac}} + \frac{\log(x)}{a} - \frac{\log\left(a+bx^4+cx^8\right)}{8a}
\end{aligned}$$

Mathematica [C] time = 0.02, size = 66, normalized size = 0.96

$$\frac{\log(x)}{a} - \frac{\operatorname{RootSum}\left[\#1^8c+\#1^4b+a\&, \frac{\#1^4c\log(x-\#1)+b\log(x-\#1)}{2\#1^4c+b}\&\right]}{4a}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a + b*x^4 + c*x^8)),x]
```

```
[Out] Log[x]/a - RootSum[a + b*#1^4 + c*#1^8 & , (b*Log[x - #1] + c*Log[x - #1]*#
1^4)/(b + 2*c*#1^4) & ]/(4*a)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(a+bx^4+cx^8\right)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x*(a + b*x^4 + c*x^8)),x]
```

```
[Out] IntegrateAlgebraic[1/(x*(a + b*x^4 + c*x^8)), x]
```

fricas [A] time = 1.24, size = 223, normalized size = 3.23

$$\left[\frac{\sqrt{b^2-4ac}\,b\log\left(\frac{2c^2x^8+2bcx^4+b^2-2ac+\left(2cx^4+b\right)\sqrt{b^2-4ac}}{cx^8+bx^4+a}\right)-\left(b^2-4ac\right)\log\left(cx^8+bx^4+a\right)+8\left(b^2-4ac\right)\log(x)}{8\left(ab^2-4a^2c\right)}, \frac{2\sqrt{-b^2+4ac}\,b\arctan\left(-\frac{\left(2cx^4+b\right)\sqrt{-b^2+4ac}}{b^2-4ac}\right)-\left(b^2-4ac\right)\log\left(cx^8+bx^4+a\right)+8\left(b^2-4ac\right)\log(x)}{8\left(ab^2-4a^2c\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^8+b*x^4+a),x, algorithm="fricas")
```

```
[Out] [1/8*(sqrt(b^2 - 4*a*c)*b*log((2*c^2*x^8 + 2*b*c*x^4 + b^2 - 2*a*c + (2*c*x
^4 + b)*sqrt(b^2 - 4*a*c))/(c*x^8 + b*x^4 + a)) - (b^2 - 4*a*c)*log(c*x^8 +
```

```
b*x^4 + a) + 8*(b^2 - 4*a*c)*log(x))/(a*b^2 - 4*a^2*c), 1/8*(2*sqrt(-b^2 +
4*a*c)*b*arctan(-(2*c*x^4 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) - (b^2 -
4*a*c)*log(c*x^8 + b*x^4 + a) + 8*(b^2 - 4*a*c)*log(x))/(a*b^2 - 4*a^2*c)]
```

giac [**A**] time = 16.28, size = 68, normalized size = 0.99

$$-\frac{b \arctan\left(\frac{2 \, c x^{4}+b}{\sqrt{-b^{2}+4 \, a c}}\right)}{4 \, \sqrt{-b^{2}+4 \, a c} a}-\frac{\log\left(c x^{8}+b x^{4}+a\right)}{8 \, a}+\frac{\log\left(x^{4}\right)}{4 \, a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^8+b*x^4+a),x, algorithm="giac")
```

```
[Out] -1/4*b*arctan((2*c*x^4 + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*a) - 1/
8*log(c*x^8 + b*x^4 + a)/a + 1/4*log(x^4)/a
```

maple [**A**] time = 0.01, size = 66, normalized size = 0.96

$$-\frac{b \arctan\left(\frac{2 c \,x^{4}+b}{\sqrt{4 a c -b^{2}}}\right)}{4 \sqrt{4 a c -b^{2}}\, a}+\frac{\ln\left(x\right)}{a}-\frac{\ln\left(c \,x^{8}+b \,x^{4}+a\right)}{8 a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(c*x^8+b*x^4+a),x)
```

```
[Out] 1/a*ln(x)-1/8*ln(c*x^8+b*x^4+a)/a-1/4/a*b/(4*a*c-b^2)^(1/2)*arctan((2*c*x^4
+b)/(4*a*c-b^2)^(1/2))
```

maxima [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^8+b*x^4+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

mupad [**B**] time = 2.18, size = 1690, normalized size = 24.49

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b*x^4 + c*x^8)),x)
```

```
[Out] log(x)/a + (log(a + b*x^4 + c*x^8)*(16*a*c - 4*b^2))/(2*(16*a*b^2 - 64*a^2*
c)) - (b*atan((4*(4*a*c - b^2)^2*(5*b^6 - 18*a^3*c^3 + 61*a^2*b^2*c^2 - 34*
a*b^4*c)*((b^9*c^4)/(128*a^4*(4*a*c - b^2)^(5/2)) + (2*b^5*c^4*(16*a*c - 4*
b^2)^4)/((16*a*b^2 - 64*a^2*c)^4*(4*a*c - b^2)^(1/2)) - (b*(16*a*c - 4*b^2)
^3*(256*b^4*c^4 - (128*a*b^4*c^4*(16*a*c - 4*b^2))/(16*a*b^2 - 64*a^2*c)))/
(16*a*(16*a*b^2 - 64*a^2*c)^3*(4*a*c - b^2)^(1/2)) + (b^3*(16*a*c - 4*b^2)*
(256*b^4*c^4 - (128*a*b^4*c^4*(16*a*c - 4*b^2))/(16*a*b^2 - 64*a^2*c)))/(25
6*a^3*(16*a*b^2 - 64*a^2*c)*(4*a*c - b^2)^(3/2)) - (3*b^7*c^4*(16*a*c - 4*b
^2)^2)/(4*a^2*(16*a*b^2 - 64*a^2*c)^2*(4*a*c - b^2)^(3/2))))/(b^4*c^8*(81*a
*c - 20*b^2)) + (128*a^5*x^4*(((5*b^5 + 23*a^2*b*c^2 - 24*a*b^3*c)*(((576*b
^3*c^5 - ((1280*b^5*c^4 - 4608*a*b^3*c^5)*(16*a*c - 4*b^2))/(2*(16*a*b^2 -
64*a^2*c)))*(16*a*c - 4*b^2)^4)/(16*(16*a*b^2 - 64*a^2*c)^4) + (b^4*(576*b^
3*c^5 - ((1280*b^5*c^4 - 4608*a*b^3*c^5)*(16*a*c - 4*b^2))/(2*(16*a*b^2 - 6
```

```
4*a^2*c))))/(4096*a^4*(4*a*c - b^2)^2) + (b^2*(1280*b^5*c^4 - 4608*a*b^3*c^
5)*(16*a*c - 4*b^2)^3)/(128*a^2*(16*a*b^2 - 64*a^2*c)^3*(4*a*c - b^2)) - (3
*b^2*(576*b^3*c^5 - ((1280*b^5*c^4 - 4608*a*b^3*c^5)*(16*a*c - 4*b^2))/(2*(
16*a*b^2 - 64*a^2*c)))*(16*a*c - 4*b^2)^2)/(128*a^2*(16*a*b^2 - 64*a^2*c)^2
*(4*a*c - b^2)) - (b^4*(1280*b^5*c^4 - 4608*a*b^3*c^5)*(16*a*c - 4*b^2))/(2
048*a^4*(16*a*b^2 - 64*a^2*c)*(4*a*c - b^2)^2)))/(32*a^5*c^4*(81*a*c - 20*b
^2)) + ((5*b^6 - 18*a^3*c^3 + 61*a^2*b^2*c^2 - 34*a*b^4*c)*((b^5*(1280*b^5*
c^4 - 4608*a*b^3*c^5))/(32768*a^5*(4*a*c - b^2)^(5/2)) - (3*b^3*(1280*b^5*c
^4 - 4608*a*b^3*c^5)*(16*a*c - 4*b^2)^2)/(1024*a^3*(16*a*b^2 - 64*a^2*c)^2*
(4*a*c - b^2)^(3/2)) + (b*(1280*b^5*c^4 - 4608*a*b^3*c^5)*(16*a*c - 4*b^2)^
4)/(128*a*(16*a*b^2 - 64*a^2*c)^4*(4*a*c - b^2)^(1/2)) - (b*(576*b^3*c^5 -
((1280*b^5*c^4 - 4608*a*b^3*c^5)*(16*a*c - 4*b^2))/(2*(16*a*b^2 - 64*a^2*c)
))*(16*a*c - 4*b^2)^3)/(16*a*(16*a*b^2 - 64*a^2*c)^3*(4*a*c - b^2)^(1/2)) +
 (b^3*(576*b^3*c^5 - ((1280*b^5*c^4 - 4608*a*b^3*c^5)*(16*a*c - 4*b^2))/(2*
(16*a*b^2 - 64*a^2*c)))*(16*a*c - 4*b^2))/(256*a^3*(16*a*b^2 - 64*a^2*c)*(4
*a*c - b^2)^(3/2))))/(32*a^5*c^4*(4*a*c - b^2)^(1/2)*(81*a*c - 20*b^2)))*(4
*a*c - b^2)^(5/2))/(b^4*c^4) + (4*(4*a*c - b^2)^(5/2)*(5*b^5 + 23*a^2*b*c^2
 - 24*a*b^3*c)*(((16*a*c - 4*b^2)^4*(256*b^4*c^4 - (128*a*b^4*c^4*(16*a*c -
 4*b^2))/(16*a*b^2 - 64*a^2*c)))/(16*(16*a*b^2 - 64*a^2*c)^4) + (b^4*(256*b
^4*c^4 - (128*a*b^4*c^4*(16*a*c - 4*b^2))/(16*a*b^2 - 64*a^2*c)))/(4096*a^4
*(4*a*c - b^2)^2) - (b^8*c^4*(16*a*c - 4*b^2))/(8*a^3*(16*a*b^2 - 64*a^2*c)
*(4*a*c - b^2)^2) + (2*b^6*c^4*(16*a*c - 4*b^2)^3)/(a*(16*a*b^2 - 64*a^2*c)
^3*(4*a*c - b^2)) - (3*b^2*(16*a*c - 4*b^2)^2*(256*b^4*c^4 - (128*a*b^4*c^4
*(16*a*c - 4*b^2))/(16*a*b^2 - 64*a^2*c)))/(128*a^2*(16*a*b^2 - 64*a^2*c)^2
*(4*a*c - b^2))))/(b^4*c^8*(81*a*c - 20*b^2))))/(4*a*(4*a*c - b^2)^(1/2))
```

sympy [B] time = 14.47, size = 253, normalized size = 3.67

$$\left(-\frac{b\sqrt{-4ac+b^2}}{8a\left(4ac-b^2\right)}-\frac{1}{8a}\right)\log\left(x^4+\frac{-16a^2c\left(-\frac{b\sqrt{-4ac+b^2}}{8a\left(4ac-b^2\right)}-\frac{1}{8a}\right)+4ab^2\left(-\frac{b\sqrt{-4ac+b^2}}{8a\left(4ac-b^2\right)}-\frac{1}{8a}\right)-2ac+b^2}{bc}\right)+\left(\frac{b\sqrt{-4ac+b^2}}{8a\left(4ac-b^2\right)}-\frac{1}{8a}\right)\log\left(x^4+\frac{-16a^2c\left(\frac{b\sqrt{-4ac+b^2}}{8a\left(4ac-b^2\right)}-\frac{1}{8a}\right)+4ab^2\left(\frac{b\sqrt{-4ac+b^2}}{8a\left(4ac-b^2\right)}-\frac{1}{8a}\right)-2ac+b^2}{bc}\right)+\frac{\log\left(x\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x**8+b*x**4+a),x)
```

```
[Out] (-b*sqrt(-4*a*c + b**2)/(8*a*(4*a*c - b**2)) - 1/(8*a))*log(x**4 + (-16*a**
2*c*(-b*sqrt(-4*a*c + b**2)/(8*a*(4*a*c - b**2)) - 1/(8*a)) + 4*a*b**2*(-b*
sqrt(-4*a*c + b**2)/(8*a*(4*a*c - b**2)) - 1/(8*a)) - 2*a*c + b**2)/(b*c))
+ (b*sqrt(-4*a*c + b**2)/(8*a*(4*a*c - b**2)) - 1/(8*a))*log(x**4 + (-16*a*
*2*c*(b*sqrt(-4*a*c + b**2)/(8*a*(4*a*c - b**2)) - 1/(8*a)) + 4*a*b**2*(b*s
qrt(-4*a*c + b**2)/(8*a*(4*a*c - b**2)) - 1/(8*a)) - 2*a*c + b**2)/(b*c)) +
 log(x)/a
```

3.260 $\int \frac{1}{x^3\left(a+bx^4+cx^8\right)}\,dx$

Optimal. Leaf size=184

$$-\frac{\sqrt{c}\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x^2}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\,a\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{c}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x^2}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{2\sqrt{2}\,a\sqrt{\sqrt{b^2-4ac}+b}}-\frac{1}{2ax^2}$$

Rubi [A] time = 0.22, antiderivative size = 184, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {1359, 1123, 1166, 205}

$$-\frac{\sqrt{c}\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x^2}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\,a\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{c}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x^2}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{2\sqrt{2}\,a\sqrt{\sqrt{b^2-4ac}+b}}-\frac{1}{2ax^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(a + b*x^4 + c*x^8)),x]
```

```
[Out] -1/(2*a*x^2) - (Sqrt[c]*(1 + b/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x
^2)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a*Sqrt[b - Sqrt[b^2 - 4*a*c]])
 - (Sqrt[c]*(1 - b/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x^2)/Sqrt[b +
 Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1123

```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*x^2 + c*x^4)^(p + 1))/(a*d*(m + 1)), x] - Dis
t[1/(a*d^2*(m + 1)), Int[(d*x)^(m + 2)*(b*(m + 2*p + 3) + c*(m + 4*p + 5)*x
^2)*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b^2 -
 4*a*c, 0] && LtQ[m, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1359

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*
x^(n/k) + c*x^((2*n)/k))^p, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, c, p
}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\int \frac{1}{x^3\left(a+bx^4+cx^8\right)}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{x^2\left(a+bx^2+cx^4\right)}\,dx, x, x^2\right)$$

$$= -\frac{1}{2ax^2} + \frac{\text{Subst}\left(\int \frac{-b-cx^2}{a+bx^2+cx^4}\,dx, x, x^2\right)}{2a}$$

$$= -\frac{1}{2ax^2} - \frac{\left(c\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\right)\text{Subst}\left(\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx, x, x^2\right)}{4a} - \frac{\left(c\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)\right)\text{Sub}}{}$$

$$= -\frac{1}{2ax^2} - \frac{\sqrt{c}\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x^2}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\,a\sqrt{b-\sqrt{b^2-4ac}}} - \frac{\sqrt{c}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x^2}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\,a\sqrt{b+\sqrt{b^2-4ac}}}$$

Mathematica [C] time = 0.03, size = 75, normalized size = 0.41

$$-\frac{\text{RootSum}\left[\#1^8c+\#1^4b+a\&, \frac{\#1^4c\log(x-\#1)+b\log(x-\#1)}{2\#1^6c+\#1^2b}\&\right]}{4a} - \frac{1}{2ax^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(a + b*x^4 + c*x^8)),x]
[Out] -1/2*1/(a*x^2) - RootSum[a + b*#1^4 + c*#1^8 & , (b*Log[x - #1] + c*Log[x -
 #1]*#1^4)/(b*#1^2 + 2*c*#1^6) & ]/(4*a)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3\left(a+bx^4+cx^8\right)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x^3*(a + b*x^4 + c*x^8)),x]
[Out] IntegrateAlgebraic[1/(x^3*(a + b*x^4 + c*x^8)), x]
```

fricas [B] time = 1.40, size = 1134, normalized size = 6.16

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(c*x^8+b*x^4+a),x, algorithm="fricas")
[Out] -1/4*(sqrt(1/2)*a*x^2*sqrt(-(b^3 - 3*a*b*c + (a^3*b^2 - 4*a^4*c)*sqrt((b^4
- 2*a*b^2*c + a^2*c^2)/(a^6*b^2 - 4*a^7*c)))/(a^3*b^2 - 4*a^4*c))*log(-(b^2
*c^2 - a*c^3)*x^2 + 1/2*sqrt(1/2)*(b^5 - 5*a*b^3*c + 4*a^2*b*c^2 - (a^3*b^4
 - 6*a^4*b^2*c + 8*a^5*c^2)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^2 - 4*a
^7*c)))*sqrt(-(b^3 - 3*a*b*c + (a^3*b^2 - 4*a^4*c)*sqrt((b^4 - 2*a*b^2*c +
a^2*c^2)/(a^6*b^2 - 4*a^7*c)))/(a^3*b^2 - 4*a^4*c))) - sqrt(1/2)*a*x^2*sqrt
(-(b^3 - 3*a*b*c + (a^3*b^2 - 4*a^4*c)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(a^
6*b^2 - 4*a^7*c)))/(a^3*b^2 - 4*a^4*c))*log(-(b^2*c^2 - a*c^3)*x^2 - 1/2*sq
rt(1/2)*(b^5 - 5*a*b^3*c + 4*a^2*b*c^2 - (a^3*b^4 - 6*a^4*b^2*c + 8*a^5*c^2
)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^2 - 4*a^7*c)))*sqrt(-(b^3 - 3*a*b
*c + (a^3*b^2 - 4*a^4*c)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^2 - 4*a^7*
c)))/(a^3*b^2 - 4*a^4*c))) + sqrt(1/2)*a*x^2*sqrt(-(b^3 - 3*a*b*c - (a^3*b^
```

```
2 - 4*a^4*c)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^2 - 4*a^7*c)))/(a^3*b^
2 - 4*a^4*c))*log(-(b^2*c^2 - a*c^3)*x^2 + 1/2*sqrt(1/2)*(b^5 - 5*a*b^3*c +
 4*a^2*b*c^2 + (a^3*b^4 - 6*a^4*b^2*c + 8*a^5*c^2)*sqrt((b^4 - 2*a*b^2*c +
a^2*c^2)/(a^6*b^2 - 4*a^7*c)))*sqrt(-(b^3 - 3*a*b*c - (a^3*b^2 - 4*a^4*c)*s
qrt((b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^2 - 4*a^7*c)))/(a^3*b^2 - 4*a^4*c)))
 - sqrt(1/2)*a*x^2*sqrt(-(b^3 - 3*a*b*c - (a^3*b^2 - 4*a^4*c)*sqrt((b^4 - 2
*a*b^2*c + a^2*c^2)/(a^6*b^2 - 4*a^7*c)))/(a^3*b^2 - 4*a^4*c))*log(-(b^2*c^
2 - a*c^3)*x^2 - 1/2*sqrt(1/2)*(b^5 - 5*a*b^3*c + 4*a^2*b*c^2 + (a^3*b^4 -
6*a^4*b^2*c + 8*a^5*c^2)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^2 - 4*a^7*
c)))*sqrt(-(b^3 - 3*a*b*c - (a^3*b^2 - 4*a^4*c)*sqrt((b^4 - 2*a*b^2*c + a^2
*c^2)/(a^6*b^2 - 4*a^7*c)))/(a^3*b^2 - 4*a^4*c))) + 2)/(a*x^2)
```

giac [B] time = 15.89, size = 2055, normalized size = 11.17

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(c*x^8+b*x^4+a),x, algorithm="giac")
```

```
[Out] -1/8*(2*a*b^4*c^2 - 8*a^2*b^2*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sq
rt(b^2 - 4*a*c)*c)*a*b^4 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
- 4*a*c)*c)*a^2*b^2*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4
*a*c)*c)*a*b^3*c - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c
)*a*b^2*c^2 - 2*(b^2 - 4*a*c)*a*b^2*c^2 + (sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*
a*c)*c)*b^4*c - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 - 2*sqr
t(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^2 - 2*b^4*c^2 + 16*sqrt(2)*sqrt(
b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^3 + 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)
*c)*a*b*c^3 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c^3 + 16*a*b^2*c^
3 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*c^4 - 32*a^2*c^4 + 2*(b^2 -
 4*a*c)*b^2*c^2 - 8*(b^2 - 4*a*c)*a*c^3)*x^4*abs(a) + (2*a*b^3*c^3 - 8*a^2*
b*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c +
 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^2 + 2*
sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 - sqrt(
2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^3 - 2*(b^2 - 4*a
*c)*a*b*c^3)*x^4 + (sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5 - 8*sqrt(2)
*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 -
4*a*c)*c)*b^4*c - 2*b^5*c + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*
b*c^2 + 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 + sqrt(2)*sqrt(
b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^2 + 16*a*b^3*c^2 - 4*sqrt(2)*sqrt(b*c + sq
rt(b^2 - 4*a*c)*c)*a*b*c^3 - 32*a^2*b*c^3 + 2*(b^2 - 4*a*c)*b^3*c - 8*(b^2
- 4*a*c)*a*b*c^2)*abs(a))*arctan(2*sqrt(1/2)*x^2/sqrt((a*b + sqrt(a^2*b^2 -
 4*a^3*c))/(a*c)))/((a^2*b^4 - 8*a^3*b^2*c - 2*a^2*b^3*c + 16*a^4*c^2 + 8*a
^3*b*c^2 + a^2*b^2*c^2 - 4*a^3*c^3)*abs(a)*abs(c)) + 1/8*(2*a*b^4*c^2 - 8*a
^2*b^2*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^
4 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c +
 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c - sqrt
(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 - 2*(b^2 -
4*a*c)*a*b^2*c^2 - (sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4*c - 8*sqrt(
2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 - 2*sqrt(2)*sqrt(b*c - sqrt(b^
2 - 4*a*c)*c)*b^3*c^2 + 2*b^4*c^2 + 16*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)
*c)*a^2*c^3 + 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^3 + sqrt(2)*s
qrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^2*c^3 - 16*a*b^2*c^3 - 4*sqrt(2)*sqrt(b*c
- sqrt(b^2 - 4*a*c)*c)*a*c^4 + 32*a^2*c^4 - 2*(b^2 - 4*a*c)*b^2*c^2 + 8*(b^
2 - 4*a*c)*a*c^3)*x^4*abs(a) + (2*a*b^3*c^3 - 8*a^2*b*c^4 - sqrt(2)*sqrt(b^
2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c + 4*sqrt(2)*sqrt(b^2 - 4
*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*
c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sq
rt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^3 - 2*(b^2 - 4*a*c)*a*b*c^3)*x^4 - (sqr
t(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^5 - 8*sqrt(2)*sqrt(b*c - sqrt(b^2 -
4*a*c)*c)*a*b^3*c - 2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4*c + 2*b^5
```

```
*c + 16*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^2 + 8*sqrt(2)*sqrt(
b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)
*c)*b^3*c^2 - 16*a*b^3*c^2 - 4*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*
c^3 + 32*a^2*b*c^3 - 2*(b^2 - 4*a*c)*b^3*c + 8*(b^2 - 4*a*c)*a*b*c^2)*abs(a
))*arctan(2*sqrt(1/2)*x^2/sqrt((a*b - sqrt(a^2*b^2 - 4*a^3*c))/(a*c)))/((a^
2*b^4 - 8*a^3*b^2*c - 2*a^2*b^3*c + 16*a^4*c^2 + 8*a^3*b*c^2 + a^2*b^2*c^2
- 4*a^3*c^3)*abs(a)*abs(c)) - 1/2/(a*x^2)
```

maple [A] time = 0.02, size = 240, normalized size = 1.30

$$\frac{\sqrt{2}\, bc \operatorname{arctanh}\left(\frac{\sqrt{2}\, c x^2}{\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}\right)}{4\sqrt{-4ac+b^2}\,\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}\; a}+\frac{\sqrt{2}\, bc \arctan\left(\frac{\sqrt{2}\, c x^2}{\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}\right)}{4\sqrt{-4ac+b^2}\,\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}\; a}+\frac{\sqrt{2}\, c \operatorname{arctanh}\left(\frac{\sqrt{2}\, c x^2}{\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}\right)}{4\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}\; a}-\frac{\sqrt{2}\, c \arctan\left(\frac{\sqrt{2}\, c x^2}{\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}\right)}{4\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}\; a}-\frac{1}{2a x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(c*x^8+b*x^4+a),x)
```

```
[Out] 1/4/a*c*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*
a*c+b^2)^(1/2))*c)^(1/2)*c*x^2)+1/4/a*c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4
*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)
*c*x^2)*b-1/4/a*c*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/(
(b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x^2)+1/4/a*c/(-4*a*c+b^2)^(1/2)*2^(1/2)/(
(b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(
1/2)*c*x^2)*b-1/2/a/x^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(c*x^8+b*x^4+a),x, algorithm="maxima")
```

```
[Out] -integrate((c*x^4 + b)*x/(c*x^8 + b*x^4 + a), x)/a - 1/2/(a*x^2)
```

mupad [B] time = 2.42, size = 5451, normalized size = 29.62

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a + b*x^4 + c*x^8)),x)
```

```
[Out] - atan((((64*a^10*c^8 + ((-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c
^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(32*(a^3*b^4 + 16*a^5*c^2 -
8*a^4*b^2*c)))^(1/2)*(((-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2
 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(32*(a^3*b^4 + 16*a^5*c^2 - 8*
a^4*b^2*c)))^(1/2)*(4096*a^12*b^6*c^4 - 32768*a^13*b^4*c^5 + 65536*a^14*b^2
*c^6) + x^2*(16384*a^13*b*c^7 - 1024*a^10*b^7*c^4 + 9216*a^11*b^5*c^5 - 245
76*a^12*b^3*c^6))*(-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*
a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(32*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b
^2*c)))^(1/2) + 4096*a^12*b*c^7 + 512*a^10*b^5*c^5 - 3072*a^11*b^3*c^6) + x
^2*(512*a^11*c^8 - 64*a^8*b^6*c^5 + 448*a^9*b^4*c^6 - 896*a^10*b^2*c^7))*(-
(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a
*c - b^2)^3)^(1/2))/(32*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2) + 16*a
^8*b^4*c^6 - 64*a^9*b^2*c^7)*(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b
*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))*1i)/(32*(a^3*b^4 + 16*a^5*
c^2 - 8*a^4*b^2*c)) - ((64*a^10*c^8 + ((-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2
) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(32*(a^3*b^4 +
 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2)*(((-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2)
+ 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(32*(a^3*b^4 + 1
6*a^5*c^2 - 8*a^4*b^2*c)))^(1/2)*(4096*a^12*b^6*c^4 - 32768*a^13*b^4*c^5 +
65536*a^14*b^2*c^6) - x^2*(16384*a^13*b*c^7 - 1024*a^10*b^7*c^4 + 9216*a^11
```

```
*b^5*c^5 - 24576*a^12*b^3*c^6))*(-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*
a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(32*(a^3*b^4 + 16*a^5
*c^2 - 8*a^4*b^2*c)))^(1/2) + 4096*a^12*b*c^7 + 512*a^10*b^5*c^5 - 3072*a^1
1*b^3*c^6) - x^2*(512*a^11*c^8 - 64*a^8*b^6*c^5 + 448*a^9*b^4*c^6 - 896*a^1
0*b^2*c^7))*(-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*
c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(32*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c))
)^(1/2) + 16*a^8*b^4*c^6 - 64*a^9*b^2*c^7)*(b^5 + b^2*(-(4*a*c - b^2)^3)^(1
/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))*1i)/(32*(a^3
*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))/(((64*a^10*c^8 + ((-(b^5 + b^2*(-(4*a*c
- b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/
(32*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2)*(((-(b^5 + b^2*(-(4*a*c -
b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(3
2*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2)*(4096*a^12*b^6*c^4 - 32768*a
^13*b^4*c^5 + 65536*a^14*b^2*c^6) + x^2*(16384*a^13*b*c^7 - 1024*a^10*b^7*c
^4 + 9216*a^11*b^5*c^5 - 24576*a^12*b^3*c^6))*(-(b^5 + b^2*(-(4*a*c - b^2)^
3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(32*(a^
3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2) + 4096*a^12*b*c^7 + 512*a^10*b^5*
c^5 - 3072*a^11*b^3*c^6) + x^2*(512*a^11*c^8 - 64*a^8*b^6*c^5 + 448*a^9*b^4
*c^6 - 896*a^10*b^2*c^7))*(-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*
c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(32*(a^3*b^4 + 16*a^5*c^2 -
 8*a^4*b^2*c)))^(1/2) + 16*a^8*b^4*c^6 - 64*a^9*b^2*c^7)*(b^5 + b^2*(-(4*a*
c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2)
))/(32*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)) + ((64*a^10*c^8 + ((-(b^5 + b^
2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)
^3)^(1/2))/(32*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2)*(((-(b^5 + b^2*
(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3
)^(1/2))/(32*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2)*(4096*a^12*b^6*c^
4 - 32768*a^13*b^4*c^5 + 65536*a^14*b^2*c^6) - x^2*(16384*a^13*b*c^7 - 1024
*a^10*b^7*c^4 + 9216*a^11*b^5*c^5 - 24576*a^12*b^3*c^6))*(-(b^5 + b^2*(-(4*
a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/
2))/(32*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2) + 4096*a^12*b*c^7 + 51
2*a^10*b^5*c^5 - 3072*a^11*b^3*c^6) - x^2*(512*a^11*c^8 - 64*a^8*b^6*c^5 +
448*a^9*b^4*c^6 - 896*a^10*b^2*c^7))*(-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2)
+ 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(32*(a^3*b^4 + 1
6*a^5*c^2 - 8*a^4*b^2*c)))^(1/2) + 16*a^8*b^4*c^6 - 64*a^9*b^2*c^7)*(b^5 +
b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^
2)^3)^(1/2)))/(32*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c))))*(-(b^5 + b^2*(-(4
*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1
/2))/(32*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2)*2i - atan((((64*a^10*
c^8 + ((-(b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c + a
*c*(-(4*a*c - b^2)^3)^(1/2))/(32*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/
2)*(((-(b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c + a*c
*(-(4*a*c - b^2)^3)^(1/2))/(32*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2)
*(4096*a^12*b^6*c^4 - 32768*a^13*b^4*c^5 + 65536*a^14*b^2*c^6) + x^2*(16384
*a^13*b*c^7 - 1024*a^10*b^7*c^4 + 9216*a^11*b^5*c^5 - 24576*a^12*b^3*c^6))*
(-(b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4
*a*c - b^2)^3)^(1/2))/(32*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2) + 40
96*a^12*b*c^7 + 512*a^10*b^5*c^5 - 3072*a^11*b^3*c^6) + x^2*(512*a^11*c^8 -
 64*a^8*b^6*c^5 + 448*a^9*b^4*c^6 - 896*a^10*b^2*c^7))*(-(b^5 - b^2*(-(4*a*
c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2)
)/(32*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2) + 16*a^8*b^4*c^6 - 64*a^
9*b^2*c^7)*(b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c +
 a*c*(-(4*a*c - b^2)^3)^(1/2))*1i)/(32*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)
) - ((64*a^10*c^8 + ((-(b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 -
 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2))/(32*(a^3*b^4 + 16*a^5*c^2 - 8*a^
4*b^2*c)))^(1/2)*(((-(b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7
*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2))/(32*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*
b^2*c)))^(1/2)*(4096*a^12*b^6*c^4 - 32768*a^13*b^4*c^5 + 65536*a^14*b^2*c^6
) - x^2*(16384*a^13*b*c^7 - 1024*a^10*b^7*c^4 + 9216*a^11*b^5*c^5 - 24576*a
```

```
^12*b^3*c^6))*(-(b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^
3*c + a*c*(-(4*a*c - b^2)^3)^(1/2))/(32*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c
)))^(1/2) + 4096*a^12*b*c^7 + 512*a^10*b^5*c^5 - 3072*a^11*b^3*c^6) - x^2*(
512*a^11*c^8 - 64*a^8*b^6*c^5 + 448*a^9*b^4*c^6 - 896*a^10*b^2*c^7))*(-(b^5
 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c -
 b^2)^3)^(1/2))/(32*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2) + 16*a^8*b
^4*c^6 - 64*a^9*b^2*c^7)*(b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2
 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2))*1i)/(32*(a^3*b^4 + 16*a^5*c^2
- 8*a^4*b^2*c)))/(((64*a^10*c^8 + ((-(b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) +
12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2))/(32*(a^3*b^4 + 16*
a^5*c^2 - 8*a^4*b^2*c)))^(1/2)*(((-(b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 12
*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2))/(32*(a^3*b^4 + 16*a^
5*c^2 - 8*a^4*b^2*c)))^(1/2)*(4096*a^12*b^6*c^4 - 32768*a^13*b^4*c^5 + 6553
6*a^14*b^2*c^6) + x^2*(16384*a^13*b*c^7 - 1024*a^10*b^7*c^4 + 9216*a^11*b^5
*c^5 - 24576*a^12*b^3*c^6))*(-(b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*
b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2))/(32*(a^3*b^4 + 16*a^5*c^2
 - 8*a^4*b^2*c)))^(1/2) + 4096*a^12*b*c^7 + 512*a^10*b^5*c^5 - 3072*a^11*b^
3*c^6) + x^2*(512*a^11*c^8 - 64*a^8*b^6*c^5 + 448*a^9*b^4*c^6 - 896*a^10*b^
2*c^7))*(-(b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c +
a*c*(-(4*a*c - b^2)^3)^(1/2))/(32*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1
/2) + 16*a^8*b^4*c^6 - 64*a^9*b^2*c^7)*(b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2)
+ 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2)))/(32*(a^3*b^4 +
16*a^5*c^2 - 8*a^4*b^2*c)) + ((64*a^10*c^8 + ((-(b^5 - b^2*(-(4*a*c - b^2)^
3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2))/(32*(a^
3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2)*(((-(b^5 - b^2*(-(4*a*c - b^2)^3)
^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2))/(32*(a^3*
b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2)*(4096*a^12*b^6*c^4 - 32768*a^13*b^4
*c^5 + 65536*a^14*b^2*c^6) - x^2*(16384*a^13*b*c^7 - 1024*a^10*b^7*c^4 + 92
16*a^11*b^5*c^5 - 24576*a^12*b^3*c^6))*(-(b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2
) + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2))/(32*(a^3*b^4 +
 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2) + 4096*a^12*b*c^7 + 512*a^10*b^5*c^5 - 3
072*a^11*b^3*c^6) - x^2*(512*a^11*c^8 - 64*a^8*b^6*c^5 + 448*a^9*b^4*c^6 -
896*a^10*b^2*c^7))*(-(b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7
*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2))/(32*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*
b^2*c)))^(1/2) + 16*a^8*b^4*c^6 - 64*a^9*b^2*c^7)*(b^5 - b^2*(-(4*a*c - b^2
)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2)))/(32*
(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c))))*(-(b^5 - b^2*(-(4*a*c - b^2)^3)^(1/
2) + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2))/(32*(a^3*b^4
+ 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2)*2i - 1/(2*a*x^2)
```

sympy [A] time = 15.34, size = 153, normalized size = 0.83

$$\operatorname{RootSum}\left(t^4\left(4096a^5c^2-2048a^4b^2c+256a^3b^4\right)+t^2\left(192a^2bc^2-112ab^3c+16b^5\right)+c^3,\left(t\mapsto t\log\left(x^2+\frac{-512t^3a^5c^2+384t^3a^4b^2c-64t^3a^3b^4-20ta^2bc^2+20tab^3c-4tb^5}{ac^3-b^2c^2}\right)\right)\right)-\frac{1}{2ax^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(c*x**8+b*x**4+a),x)
```

```
[Out] RootSum(_t**4*(4096*a**5*c**2 - 2048*a**4*b**2*c + 256*a**3*b**4) + _t**2*(
192*a**2*b*c**2 - 112*a*b**3*c + 16*b**5) + c**3, Lambda(_t, _t*log(x**2 +
(-512*_t**3*a**5*c**2 + 384*_t**3*a**4*b**2*c - 64*_t**3*a**3*b**4 - 20*_t*
a**2*b*c**2 + 20*_t*a*b**3*c - 4*_t*b**5)/(a*c**3 - b**2*c**2)))) - 1/(2*a*
x**2)
```

3.261 $\int \frac{1}{x^5\left(a+bx^4+cx^8\right)}\,dx$

Optimal. Leaf size=89

$$-\frac{\left(b^2-2ac\right)\tanh^{-1}\left(\frac{b+2cx^4}{\sqrt{b^2-4ac}}\right)}{4a^2\sqrt{b^2-4ac}}+\frac{b\log\left(a+bx^4+cx^8\right)}{8a^2}-\frac{b\log(x)}{a^2}-\frac{1}{4ax^4}$$

Rubi [A] time = 0.12, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389, Rules used = {1357, 709, 800, 634, 618, 206, 628}

$$-\frac{\left(b^2-2ac\right)\tanh^{-1}\left(\frac{b+2cx^4}{\sqrt{b^2-4ac}}\right)}{4a^2\sqrt{b^2-4ac}}+\frac{b\log\left(a+bx^4+cx^8\right)}{8a^2}-\frac{b\log(x)}{a^2}-\frac{1}{4ax^4}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^5*(a + b*x^4 + c*x^8)),x]
```

```
[Out] -1/(4*a*x^4) - ((b^2 - 2*a*c)*ArcTanh[(b + 2*c*x^4)/Sqrt[b^2 - 4*a*c]])/(4*
a^2*Sqrt[b^2 - 4*a*c]) - (b*Log[x])/a^2 + (b*Log[a + b*x^4 + c*x^8])/(8*a^2
)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 709

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Simp[(e*(d + e*x)^(m + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dis
t[1/(c*d^2 - b*d*e + a*e^2), Int[((d + e*x)^(m + 1)*Simp[c*d - b*e - c*e*x,
 x])/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 -
 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[m
, -1]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 1357

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x^5\left(a+bx^4+cx^8\right)}\,dx &= \frac{1}{4}\,\text{Subst}\left(\int \frac{1}{x^2\left(a+bx+cx^2\right)}\,dx, x, x^4\right)\\
&= -\frac{1}{4ax^4}+\frac{\text{Subst}\left(\int \frac{-b-cx}{x\left(a+bx+cx^2\right)}\,dx, x, x^4\right)}{4a}\\
&= -\frac{1}{4ax^4}+\frac{\text{Subst}\left(\int \left(-\frac{b}{ax}+\frac{b^2-ac+bcx}{a\left(a+bx+cx^2\right)}\right)dx, x, x^4\right)}{4a}\\
&= -\frac{1}{4ax^4}-\frac{b\log(x)}{a^2}+\frac{\text{Subst}\left(\int \frac{b^2-ac+bcx}{a+bx+cx^2}\,dx, x, x^4\right)}{4a^2}\\
&= -\frac{1}{4ax^4}-\frac{b\log(x)}{a^2}+\frac{b\,\text{Subst}\left(\int \frac{b+2cx}{a+bx+cx^2}\,dx, x, x^4\right)}{8a^2}+\frac{\left(b^2-2ac\right)\text{Subst}\left(\int \frac{1}{a+bx+cx^2}\,dx, \ldots\right.}{8a^2}\\
&= -\frac{1}{4ax^4}-\frac{b\log(x)}{a^2}+\frac{b\log\left(a+bx^4+cx^8\right)}{8a^2}-\frac{\left(b^2-2ac\right)\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx, x, b+2\ldots\right.}{4a^2}\\
&= -\frac{1}{4ax^4}-\frac{\left(b^2-2ac\right)\tanh^{-1}\left(\frac{b+2cx^4}{\sqrt{b^2-4ac}}\right)}{4a^2\sqrt{b^2-4ac}}-\frac{b\log(x)}{a^2}+\frac{b\log\left(a+bx^4+cx^8\right)}{8a^2}
\end{aligned}
$$

Mathematica [C] time = 0.03, size = 92, normalized size = 1.03

$$
\frac{\text{RootSum}\left[\#1^8c+\#1^4b+a\&,\frac{\#1^4bc\log(x-\#1)-ac\log(x-\#1)+b^2\log(x-\#1)}{2\#1^4c+b}\&\right]}{4a^2}-\frac{b\log(x)}{a^2}-\frac{1}{4ax^4}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^5*(a + b*x^4 + c*x^8)),x]
```

```
[Out] -1/4*1/(a*x^4) - (b*Log[x])/a^2 + RootSum[a + b*#1^4 + c*#1^8 & , (b^2*Log[
x - #1] - a*c*Log[x - #1] + b*c*Log[x - #1]*#1^4)/(b + 2*c*#1^4) & ]/(4*a^2
)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$
\int \frac{1}{x^5\left(a+bx^4+cx^8\right)}\,dx
$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x^5*(a + b*x^4 + c*x^8)),x]
```

```
[Out] IntegrateAlgebraic[1/(x^5*(a + b*x^4 + c*x^8)), x]
```

fricas [A] time = 1.88, size = 293, normalized size = 3.29

$$\left[-\frac{\left(b^{2}-2\,ac\right)\sqrt{b^{2}-4\,ac}\,x^{4}\log\left(\frac{2\,c^{2}x^{8}+2\,bcx^{4}+b^{2}-2\,ac+\left(2\,cx^{4}+b\right)\sqrt{b^{2}-4\,ac}}{cx^{8}+bx^{4}+a}\right)-\left(b^{3}-4\,abc\right)x^{4}\log\left(cx^{8}+bx^{4}+a\right)+8\left(b^{3}-4\,abc\right)x^{4}\log(x)+2\,ab^{2}-8\,a^{2}c}{8\left(a^{2}b^{2}-4\,a^{3}c\right)x^{4}},\ -\frac{2\left(b^{2}-2\,ac\right)\sqrt{-b^{2}+4\,ac}\,x^{4}\arctan\left(-\frac{\left(2\,cx^{4}+b\right)\sqrt{-b^{2}+4\,ac}}{b^{2}-4\,ac}\right)-\left(b^{3}-4\,abc\right)x^{4}\log\left(cx^{8}+bx^{4}+a\right)+8\left(b^{3}-4\,abc\right)x^{4}\log(x)+2\,ab^{2}-8\,a^{2}c}{8\left(a^{2}b^{2}-4\,a^{3}c\right)x^{4}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(c*x^8+b*x^4+a),x, algorithm="fricas")
```

```
[Out] [-1/8*((b^2 - 2*a*c)*sqrt(b^2 - 4*a*c)*x^4*log((2*c^2*x^8 + 2*b*c*x^4 + b^2
 - 2*a*c + (2*c*x^4 + b)*sqrt(b^2 - 4*a*c))/(c*x^8 + b*x^4 + a)) - (b^3 - 4
*a*b*c)*x^4*log(c*x^8 + b*x^4 + a) + 8*(b^3 - 4*a*b*c)*x^4*log(x) + 2*a*b^2
 - 8*a^2*c)/((a^2*b^2 - 4*a^3*c)*x^4), -1/8*(2*(b^2 - 2*a*c)*sqrt(-b^2 + 4*
a*c)*x^4*arctan(-(2*c*x^4 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) - (b^3 - 4
*a*b*c)*x^4*log(c*x^8 + b*x^4 + a) + 8*(b^3 - 4*a*b*c)*x^4*log(x) + 2*a*b^2
 - 8*a^2*c)/((a^2*b^2 - 4*a^3*c)*x^4)]
```

giac [A] time = 14.50, size = 94, normalized size = 1.06

$$\frac{b\log\left(cx^{8}+bx^{4}+a\right)}{8\,a^{2}}-\frac{b\log\left(x^{4}\right)}{4\,a^{2}}+\frac{\left(b^{2}-2\,ac\right)\arctan\left(\frac{2\,cx^{4}+b}{\sqrt{-b^{2}+4\,ac}}\right)}{4\,\sqrt{-b^{2}+4\,ac}\,a^{2}}+\frac{bx^{4}-a}{4\,a^{2}x^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(c*x^8+b*x^4+a),x, algorithm="giac")
```

```
[Out] 1/8*b*log(c*x^8 + b*x^4 + a)/a^2 - 1/4*b*log(x^4)/a^2 + 1/4*(b^2 - 2*a*c)*a
rctan((2*c*x^4 + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*a^2) + 1/4*(b*x
^4 - a)/(a^2*x^4)
```

maple [A] time = 0.01, size = 119, normalized size = 1.34

$$-\frac{c\arctan\left(\frac{2c\,x^{4}+b}{\sqrt{4ac-b^{2}}}\right)}{2\sqrt{4ac-b^{2}}\;a}+\frac{b^{2}\arctan\left(\frac{2c\,x^{4}+b}{\sqrt{4ac-b^{2}}}\right)}{4\sqrt{4ac-b^{2}}\;a^{2}}-\frac{b\ln(x)}{a^{2}}+\frac{b\ln\left(c\,x^{8}+b\,x^{4}+a\right)}{8a^{2}}-\frac{1}{4a\,x^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^5/(c*x^8+b*x^4+a),x)
```

```
[Out] -1/4/x^4/a-1/a^2*b*ln(x)+1/8*b*ln(c*x^8+b*x^4+a)/a^2-1/2/a/(4*a*c-b^2)^(1/2
)*arctan((2*c*x^4+b)/(4*a*c-b^2)^(1/2))*c+1/4/a^2/(4*a*c-b^2)^(1/2)*arctan(
(2*c*x^4+b)/(4*a*c-b^2)^(1/2))*b^2
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(c*x^8+b*x^4+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

mupad [B] time = 2.79, size = 8817, normalized size = 99.07

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^5*(a + b*x^4 + c*x^8)),x)
```

```
[Out] (atan((4*a^5*(4*a*c - b^2)^2*(5*b^7 - 23*a^3*b*c^3 + 66*a^2*b^3*c^2 - 35*a*
b^5*c)*(((4*b^3 - 16*a*b*c)*((((((((256*a^4*b^5*c^4 - 256*a^5*b^3*c^5)/a^5
- (128*a*b^4*c^4*(4*b^3 - 16*a*b*c))/(64*a^3*c - 16*a^2*b^2))*(2*a*c - b^2)
)/(8*a^2*(4*a*c - b^2)^(1/2)) - (16*b^4*c^4*(4*b^3 - 16*a*b*c)*(2*a*c - b^2
))/(a*(4*a*c - b^2)^(1/2)*(64*a^3*c - 16*a^2*b^2)))*(2*a*c - b^2))/(8*a^2*(
4*a*c - b^2)^(1/2)) - (2*b^4*c^4*(4*b^3 - 16*a*b*c)*(2*a*c - b^2)^2)/(a^3*(
4*a*c - b^2)*(64*a^3*c - 16*a^2*b^2)))*(2*a*c - b^2))/(8*a^2*(4*a*c - b^2)^
(1/2)) - (b^4*c^4*(4*b^3 - 16*a*b*c)*(2*a*c - b^2)^3)/(4*a^5*(4*a*c - b^2)^
(3/2)*(64*a^3*c - 16*a^2*b^2))))/(2*(64*a^3*c - 16*a^2*b^2)) - ((4*b^3 - 16
*a*b*c)*(((4*b^3 - 16*a*b*c)*(((4*b^3 - 16*a*b*c)*((((256*a^4*b^5*c^4 - 256
*a^5*b^3*c^5)/a^5 - (128*a*b^4*c^4*(4*b^3 - 16*a*b*c))/(64*a^3*c - 16*a^2*b
^2))*(2*a*c - b^2))/(8*a^2*(4*a*c - b^2)^(1/2)) - (16*b^4*c^4*(4*b^3 - 16*a
*b*c)*(2*a*c - b^2))/(a*(4*a*c - b^2)^(1/2)*(64*a^3*c - 16*a^2*b^2))))/(2*(
64*a^3*c - 16*a^2*b^2)) + (((256*a^3*b^4*c^5 - 96*a^4*b^2*c^6)/a^5 + ((4*b^
3 - 16*a*b*c)*((256*a^4*b^5*c^4 - 256*a^5*b^3*c^5)/a^5 - (128*a*b^4*c^4*(4*
b^3 - 16*a*b*c))/(64*a^3*c - 16*a^2*b^2)))/(2*(64*a^3*c - 16*a^2*b^2)))*(2*
a*c - b^2))/(8*a^2*(4*a*c - b^2)^(1/2))))/(2*(64*a^3*c - 16*a^2*b^2)) - (((
16*a^3*b*c^7 - 96*a^2*b^3*c^6)/a^5 - ((4*b^3 - 16*a*b*c)*((256*a^3*b^4*c^5
- 96*a^4*b^2*c^6)/a^5 + ((4*b^3 - 16*a*b*c)*((256*a^4*b^5*c^4 - 256*a^5*b^3
*c^5)/a^5 - (128*a*b^4*c^4*(4*b^3 - 16*a*b*c))/(64*a^3*c - 16*a^2*b^2)))/(2
*(64*a^3*c - 16*a^2*b^2))))/(2*(64*a^3*c - 16*a^2*b^2)))*(2*a*c - b^2))/(8*
a^2*(4*a*c - b^2)^(1/2))))/(2*(64*a^3*c - 16*a^2*b^2)) + ((2*a*c - b^2)*(((
4*b^3 - 16*a*b*c)*((((((256*a^4*b^5*c^4 - 256*a^5*b^3*c^5)/a^5 - (128*a*b^4
*c^4*(4*b^3 - 16*a*b*c))/(64*a^3*c - 16*a^2*b^2))*(2*a*c - b^2))/(8*a^2*(4*
a*c - b^2)^(1/2)) - (16*b^4*c^4*(4*b^3 - 16*a*b*c)*(2*a*c - b^2))/(a*(4*a*c
 - b^2)^(1/2)*(64*a^3*c - 16*a^2*b^2)))*(2*a*c - b^2))/(8*a^2*(4*a*c - b^2)
^(1/2)) - (2*b^4*c^4*(4*b^3 - 16*a*b*c)*(2*a*c - b^2)^2)/(a^3*(4*a*c - b^2)
*(64*a^3*c - 16*a^2*b^2))))/(2*(64*a^3*c - 16*a^2*b^2)) + ((((4*b^3 - 16*a*
b*c)*((((256*a^4*b^5*c^4 - 256*a^5*b^3*c^5)/a^5 - (128*a*b^4*c^4*(4*b^3 - 1
6*a*b*c))/(64*a^3*c - 16*a^2*b^2))*(2*a*c - b^2))/(8*a^2*(4*a*c - b^2)^(1/2
)) - (16*b^4*c^4*(4*b^3 - 16*a*b*c)*(2*a*c - b^2))/(a*(4*a*c - b^2)^(1/2)*(
64*a^3*c - 16*a^2*b^2))))/(2*(64*a^3*c - 16*a^2*b^2)) + (((256*a^3*b^4*c^5
- 96*a^4*b^2*c^6)/a^5 + ((4*b^3 - 16*a*b*c)*((256*a^4*b^5*c^4 - 256*a^5*b^3
*c^5)/a^5 - (128*a*b^4*c^4*(4*b^3 - 16*a*b*c))/(64*a^3*c - 16*a^2*b^2)))/(2
*(64*a^3*c - 16*a^2*b^2)))*(2*a*c - b^2))/(8*a^2*(4*a*c - b^2)^(1/2)))*(2*a
*c - b^2))/(8*a^2*(4*a*c - b^2)^(1/2))))/(8*a^2*(4*a*c - b^2)^(1/2)) + (((a
^2*c^8 - 16*a*b^2*c^7)/a^5 + ((4*b^3 - 16*a*b*c)*((16*a^3*b*c^7 - 96*a^2*b^
3*c^6)/a^5 - ((4*b^3 - 16*a*b*c)*((256*a^3*b^4*c^5 - 96*a^4*b^2*c^6)/a^5 +
((4*b^3 - 16*a*b*c)*((256*a^4*b^5*c^4 - 256*a^5*b^3*c^5)/a^5 - (128*a*b^4*c
^4*(4*b^3 - 16*a*b*c))/(64*a^3*c - 16*a^2*b^2)))/(2*(64*a^3*c - 16*a^2*b^2)
)))/(2*(64*a^3*c - 16*a^2*b^2))))/(2*(64*a^3*c - 16*a^2*b^2)))*(2*a*c - b^2
))/(8*a^2*(4*a*c - b^2)^(1/2)) + (b^4*c^4*(2*a*c - b^2)^5)/(128*a^9*(4*a*c
- b^2)^(5/2))))/(c^4*(a^2*c^2 - 20*b^4 + 80*a*b^2*c)*(16*a^4*c^8 + b^8*c^4
- 8*a*b^6*c^5 + 24*a^2*b^4*c^6 - 32*a^3*b^2*c^7)) - (128*a^10*x^4*(((5*b^6
- a^3*c^3 + 26*a^2*b^2*c^2 - 25*a*b^4*c)*(c^9/a^5 + ((4*b^3 - 16*a*b*c)*((2
0*b*c^8)/a^4 + ((4*b^3 - 16*a*b*c)*((72*a^3*c^8 + 124*a^2*b^2*c^7)/a^5 + ((
4*b^3 - 16*a*b*c)*((864*a^4*b*c^7 + 208*a^3*b^3*c^6)/a^5 - ((4*b^3 - 16*a*b
*c)*((448*a^4*b^4*c^5 - 3456*a^5*b^2*c^6)/a^5 + ((4*b^3 - 16*a*b*c)*(1280*a
^5*b^5*c^4 - 4608*a^6*b^3*c^5))/(2*a^5*(64*a^3*c - 16*a^2*b^2))))/(2*(64*a^
3*c - 16*a^2*b^2))))/(2*(64*a^3*c - 16*a^2*b^2))))/(2*(64*a^3*c - 16*a^2*b^
2))))/(2*(64*a^3*c - 16*a^2*b^2)) + ((4*b^3 - 16*a*b*c)*(((4*b^3 - 16*a*b*c
)*(((2*a*c - b^2)*((((448*a^4*b^4*c^5 - 3456*a^5*b^2*c^6)/a^5 + ((4*b^3 - 1
6*a*b*c)*(1280*a^5*b^5*c^4 - 4608*a^6*b^3*c^5))/(2*a^5*(64*a^3*c - 16*a^2*b
^2)))*(2*a*c - b^2))/(8*a^2*(4*a*c - b^2)^(1/2)) + ((4*b^3 - 16*a*b*c)*(2*a
*c - b^2)*(1280*a^5*b^5*c^4 - 4608*a^6*b^3*c^5))/(16*a^7*(4*a*c - b^2)^(1/2
)*(64*a^3*c - 16*a^2*b^2))))/(8*a^2*(4*a*c - b^2)^(1/2)) + ((4*b^3 - 16*a*b
*c)*(2*a*c - b^2)^2*(1280*a^5*b^5*c^4 - 4608*a^6*b^3*c^5))/(128*a^9*(4*a*c
- b^2)*(64*a^3*c - 16*a^2*b^2))))/(2*(64*a^3*c - 16*a^2*b^2)) + ((2*a*c - b
```

```
^2)*(((4*b^3 - 16*a*b*c)*((((448*a^4*b^4*c^5 - 3456*a^5*b^2*c^6)/a^5 + ((4*
b^3 - 16*a*b*c)*(1280*a^5*b^5*c^4 - 4608*a^6*b^3*c^5))/(2*a^5*(64*a^3*c - 1
6*a^2*b^2)))*(2*a*c - b^2))/(8*a^2*(4*a*c - b^2)^(1/2)) + ((4*b^3 - 16*a*b*
c)*(2*a*c - b^2)*(1280*a^5*b^5*c^4 - 4608*a^6*b^3*c^5))/(16*a^7*(4*a*c - b^
2)^(1/2)*(64*a^3*c - 16*a^2*b^2))))/(2*(64*a^3*c - 16*a^2*b^2)) - (((864*a^
4*b*c^7 + 208*a^3*b^3*c^6)/a^5 - ((4*b^3 - 16*a*b*c)*((448*a^4*b^4*c^5 - 34
56*a^5*b^2*c^6)/a^5 + ((4*b^3 - 16*a*b*c)*(1280*a^5*b^5*c^4 - 4608*a^6*b^3*
c^5))/(2*a^5*(64*a^3*c - 16*a^2*b^2))))/(2*(64*a^3*c - 16*a^2*b^2)))*(2*a*c
 - b^2))/(8*a^2*(4*a*c - b^2)^(1/2))))/(8*a^2*(4*a*c - b^2)^(1/2))))/(2*(64
*a^3*c - 16*a^2*b^2)) + ((2*a*c - b^2)*(((4*b^3 - 16*a*b*c)*(((4*b^3 - 16*a
*b*c)*((((448*a^4*b^4*c^5 - 3456*a^5*b^2*c^6)/a^5 + ((4*b^3 - 16*a*b*c)*(12
80*a^5*b^5*c^4 - 4608*a^6*b^3*c^5))/(2*a^5*(64*a^3*c - 16*a^2*b^2)))*(2*a*c
 - b^2))/(8*a^2*(4*a*c - b^2)^(1/2)) + ((4*b^3 - 16*a*b*c)*(2*a*c - b^2)*(1
280*a^5*b^5*c^4 - 4608*a^6*b^3*c^5))/(16*a^7*(4*a*c - b^2)^(1/2)*(64*a^3*c
- 16*a^2*b^2))))/(2*(64*a^3*c - 16*a^2*b^2)) - (((864*a^4*b*c^7 + 208*a^3*b
^3*c^6)/a^5 - ((4*b^3 - 16*a*b*c)*((448*a^4*b^4*c^5 - 3456*a^5*b^2*c^6)/a^5
 + ((4*b^3 - 16*a*b*c)*(1280*a^5*b^5*c^4 - 4608*a^6*b^3*c^5))/(2*a^5*(64*a^
3*c - 16*a^2*b^2))))/(2*(64*a^3*c - 16*a^2*b^2)))*(2*a*c - b^2))/(8*a^2*(4*
a*c - b^2)^(1/2))))/(2*(64*a^3*c - 16*a^2*b^2)) - (((72*a^3*c^8 + 124*a^2*b
^2*c^7)/a^5 + ((4*b^3 - 16*a*b*c)*((864*a^4*b*c^7 + 208*a^3*b^3*c^6)/a^5 -
((4*b^3 - 16*a*b*c)*((448*a^4*b^4*c^5 - 3456*a^5*b^2*c^6)/a^5 + ((4*b^3 - 1
6*a*b*c)*(1280*a^5*b^5*c^4 - 4608*a^6*b^3*c^5))/(2*a^5*(64*a^3*c - 16*a^2*b
^2))))/(2*(64*a^3*c - 16*a^2*b^2))))/(2*(64*a^3*c - 16*a^2*b^2)))*(2*a*c -
b^2))/(8*a^2*(4*a*c - b^2)^(1/2))))/(8*a^2*(4*a*c - b^2)^(1/2)) - ((((2*a*c
 - b^2)*(((2*a*c - b^2)*((((448*a^4*b^4*c^5 - 3456*a^5*b^2*c^6)/a^5 + ((4*b
^3 - 16*a*b*c)*(1280*a^5*b^5*c^4 - 4608*a^6*b^3*c^5))/(2*a^5*(64*a^3*c - 16
*a^2*b^2)))*(2*a*c - b^2))/(8*a^2*(4*a*c - b^2)^(1/2)) + ((4*b^3 - 16*a*b*c
)*(2*a*c - b^2)*(1280*a^5*b^5*c^4 - 4608*a^6*b^3*c^5))/(16*a^7*(4*a*c - b^2
)^(1/2)*(64*a^3*c - 16*a^2*b^2))))/(8*a^2*(4*a*c - b^2)^(1/2)) + ((4*b^3 -
16*a*b*c)*(2*a*c - b^2)^2*(1280*a^5*b^5*c^4 - 4608*a^6*b^3*c^5))/(128*a^9*(
4*a*c - b^2)*(64*a^3*c - 16*a^2*b^2))))/(8*a^2*(4*a*c - b^2)^(1/2)) + ((4*b
^3 - 16*a*b*c)*(2*a*c - b^2)^3*(1280*a^5*b^5*c^4 - 4608*a^6*b^3*c^5))/(1024
*a^11*(4*a*c - b^2)^(3/2)*(64*a^3*c - 16*a^2*b^2)))*(2*a*c - b^2))/(8*a^2*(
4*a*c - b^2)^(1/2)) - ((4*b^3 - 16*a*b*c)*(2*a*c - b^2)^4*(1280*a^5*b^5*c^4
 - 4608*a^6*b^3*c^5))/(8192*a^13*(4*a*c - b^2)^2*(64*a^3*c - 16*a^2*b^2))))
/(32*a^5*c^4*(a^2*c^2 - 20*b^4 + 80*a*b^2*c)) + ((5*b^7 - 23*a^3*b*c^3 + 66
*a^2*b^3*c^2 - 35*a*b^5*c)*(((4*b^3 - 16*a*b*c)*(((2*a*c - b^2)*(((2*a*c -
b^2)*((((448*a^4*b^4*c^5 - 3456*a^5*b^2*c^6)/a^5 + ((4*b^3 - 16*a*b*c)*(128
0*a^5*b^5*c^4 - 4608*a^6*b^3*c^5))/(2*a^5*(64*a^3*c - 16*a^2*b^2)))*(2*a*c
- b^2))/(8*a^2*(4*a*c - b^2)^(1/2)) + ((4*b^3 - 16*a*b*c)*(2*a*c - b^2)*(12
80*a^5*b^5*c^4 - 4608*a^6*b^3*c^5))/(16*a^7*(4*a*c - b^2)^(1/2)*(64*a^3*c -
 16*a^2*b^2))))/(8*a^2*(4*a*c - b^2)^(1/2)) + ((4*b^3 - 16*a*b*c)*(2*a*c -
b^2)^2*(1280*a^5*b^5*c^4 - 4608*a^6*b^3*c^5))/(128*a^9*(4*a*c - b^2)*(64*a^
3*c - 16*a^2*b^2))))/(8*a^2*(4*a*c - b^2)^(1/2)) + ((4*b^3 - 16*a*b*c)*(2*a
*c - b^2)^3*(1280*a^5*b^5*c^4 - 4608*a^6*b^3*c^5))/(1024*a^11*(4*a*c - b^2)
^(3/2)*(64*a^3*c - 16*a^2*b^2))))/(2*(64*a^3*c - 16*a^2*b^2)) - ((4*b^3 - 1
6*a*b*c)*(((4*b^3 - 16*a*b*c)*(((4*b^3 - 16*a*b*c)*((((448*a^4*b^4*c^5 - 34
56*a^5*b^2*c^6)/a^5 + ((4*b^3 - 16*a*b*c)*(1280*a^5*b^5*c^4 - 4608*a^6*b^3*
c^5))/(2*a^5*(64*a^3*c - 16*a^2*b^2)))*(2*a*c - b^2))/(8*a^2*(4*a*c - b^2)^
(1/2)) + ((4*b^3 - 16*a*b*c)*(2*a*c - b^2)*(1280*a^5*b^5*c^4 - 4608*a^6*b^3
*c^5))/(16*a^7*(4*a*c - b^2)^(1/2)*(64*a^3*c - 16*a^2*b^2))))/(2*(64*a^3*c
- 16*a^2*b^2)) - (((864*a^4*b*c^7 + 208*a^3*b^3*c^6)/a^5 - ((4*b^3 - 16*a*b
*c)*((448*a^4*b^4*c^5 - 3456*a^5*b^2*c^6)/a^5 + ((4*b^3 - 16*a*b*c)*(1280*a
^5*b^5*c^4 - 4608*a^6*b^3*c^5))/(2*a^5*(64*a^3*c - 16*a^2*b^2))))/(2*(64*a^
3*c - 16*a^2*b^2)))*(2*a*c - b^2))/(8*a^2*(4*a*c - b^2)^(1/2))))/(2*(64*a^3
*c - 16*a^2*b^2)) - (((72*a^3*c^8 + 124*a^2*b^2*c^7)/a^5 + ((4*b^3 - 16*a*b
*c)*((864*a^4*b*c^7 + 208*a^3*b^3*c^6)/a^5 - ((4*b^3 - 16*a*b*c)*((448*a^4*
b^4*c^5 - 3456*a^5*b^2*c^6)/a^5 + ((4*b^3 - 16*a*b*c)*(1280*a^5*b^5*c^4 - 4
608*a^6*b^3*c^5))/(2*a^5*(64*a^3*c - 16*a^2*b^2))))/(2*(64*a^3*c - 16*a^2*b
```

^2))))/(2*(64*a^3*c - 16*a^2*b^2)))*(2*a*c - b^2))/(8*a^2*(4*a*c - b^2)^(1/
2))))/(2*(64*a^3*c - 16*a^2*b^2)) + (((20*b*c^8)/a^4 + ((4*b^3 - 16*a*b*c)*
((72*a^3*c^8 + 124*a^2*b^2*c^7)/a^5 + ((4*b^3 - 16*a*b*c)*((864*a^4*b*c^7 +
208*a^3*b^3*c^6)/a^5 - ((4*b^3 - 16*a*b*c)*((448*a^4*b^4*c^5 - 3456*a^5*b^
2*c^6)/a^5 + ((4*b^3 - 16*a*b*c)*(1280*a^5*b^5*c^4 - 4608*a^6*b^3*c^5))/(2*
a^5*(64*a^3*c - 16*a^2*b^2))))/(2*(64*a^3*c - 16*a^2*b^2))))/(2*(64*a^3*c -
16*a^2*b^2))))/(2*(64*a^3*c - 16*a^2*b^2)))*(2*a*c - b^2))/(8*a^2*(4*a*c -
b^2)^(1/2)) + ((((4*b^3 - 16*a*b*c)*(((2*a*c - b^2)*((((448*a^4*b^4*c^5 -
3456*a^5*b^2*c^6)/a^5 + ((4*b^3 - 16*a*b*c)*(1280*a^5*b^5*c^4 - 4608*a^6*b^
3*c^5))/(2*a^5*(64*a^3*c - 16*a^2*b^2)))*(2*a*c - b^2))/(8*a^2*(4*a*c - b^2
)^(1/2)) + ((4*b^3 - 16*a*b*c)*(2*a*c - b^2)*(1280*a^5*b^5*c^4 - 4608*a^6*b
^3*c^5))/(16*a^7*(4*a*c - b^2)^(1/2)*(64*a^3*c - 16*a^2*b^2))))/(8*a^2*(4*a
*c - b^2)^(1/2)) + ((4*b^3 - 16*a*b*c)*(2*a*c - b^2)^2*(1280*a^5*b^5*c^4 -
4608*a^6*b^3*c^5))/(128*a^9*(4*a*c - b^2)*(64*a^3*c - 16*a^2*b^2))))/(2*(64
*a^3*c - 16*a^2*b^2)) + ((2*a*c - b^2)*(((4*b^3 - 16*a*b*c)*((((448*a^4*b^4
*c^5 - 3456*a^5*b^2*c^6)/a^5 + ((4*b^3 - 16*a*b*c)*(1280*a^5*b^5*c^4 - 4608
*a^6*b^3*c^5))/(2*a^5*(64*a^3*c - 16*a^2*b^2)))*(2*a*c - b^2))/(8*a^2*(4*a*
c - b^2)^(1/2)) + ((4*b^3 - 16*a*b*c)*(2*a*c - b^2)*(1280*a^5*b^5*c^4 - 460
8*a^6*b^3*c^5))/(16*a^7*(4*a*c - b^2)^(1/2)*(64*a^3*c - 16*a^2*b^2))))/(2*(
64*a^3*c - 16*a^2*b^2)) - (((864*a^4*b*c^7 + 208*a^3*b^3*c^6)/a^5 - ((4*b^3
- 16*a*b*c)*((448*a^4*b^4*c^5 - 3456*a^5*b^2*c^6)/a^5 + ((4*b^3 - 16*a*b*c
)*(1280*a^5*b^5*c^4 - 4608*a^6*b^3*c^5))/(2*a^5*(64*a^3*c - 16*a^2*b^2))))/
(2*(64*a^3*c - 16*a^2*b^2)))*(2*a*c - b^2))/(8*a^2*(4*a*c - b^2)^(1/2))))/(
8*a^2*(4*a*c - b^2)^(1/2)))*(2*a*c - b^2))/(8*a^2*(4*a*c - b^2)^(1/2)) - ((
2*a*c - b^2)^5*(1280*a^5*b^5*c^4 - 4608*a^6*b^3*c^5))/(32768*a^15*(4*a*c -
b^2)^(5/2))))/(32*a^5*c^4*(4*a*c - b^2)^(1/2)*(a^2*c^2 - 20*b^4 + 80*a*b^2*
c)))*(4*a*c - b^2)^(5/2))/(16*a^4*c^8 + b^8*c^4 - 8*a*b^6*c^5 + 24*a^2*b^4*
c^6 - 32*a^3*b^2*c^7) + (4*a^5*(4*a*c - b^2)^(5/2)*(5*b^6 - a^3*c^3 + 26*a^
2*b^2*c^2 - 25*a*b^4*c)*(((4*b^3 - 16*a*b*c)*(((4*b^3 - 16*a*b*c)*((((((256
*a^4*b^5*c^4 - 256*a^5*b^3*c^5)/a^5 - (128*a*b^4*c^4*(4*b^3 - 16*a*b*c))/(6
4*a^3*c - 16*a^2*b^2))*(2*a*c - b^2))/(8*a^2*(4*a*c - b^2)^(1/2)) - (16*b^4
c^4(4*b^3 - 16*a*b*c)*(2*a*c - b^2))/(a*(4*a*c - b^2)^(1/2)*(64*a^3*c - 1
6*a^2*b^2)))*(2*a*c - b^2))/(8*a^2*(4*a*c - b^2)^(1/2)) - (2*b^4*c^4*(4*b^3
- 16*a*b*c)*(2*a*c - b^2)^2)/(a^3*(4*a*c - b^2)*(64*a^3*c - 16*a^2*b^2))))
/(2*(64*a^3*c - 16*a^2*b^2)) + ((((4*b^3 - 16*a*b*c)*((((256*a^4*b^5*c^4 -
256*a^5*b^3*c^5)/a^5 - (128*a*b^4*c^4*(4*b^3 - 16*a*b*c))/(64*a^3*c - 16*a^
2*b^2))*(2*a*c - b^2))/(8*a^2*(4*a*c - b^2)^(1/2)) - (16*b^4*c^4*(4*b^3 - 1
6*a*b*c)*(2*a*c - b^2))/(a*(4*a*c - b^2)^(1/2)*(64*a^3*c - 16*a^2*b^2))))/(
2*(64*a^3*c - 16*a^2*b^2)) + (((256*a^3*b^4*c^5 - 96*a^4*b^2*c^6)/a^5 + ((4
*b^3 - 16*a*b*c)*((256*a^4*b^5*c^4 - 256*a^5*b^3*c^5)/a^5 - (128*a*b^4*c^4*
(4*b^3 - 16*a*b*c))/(64*a^3*c - 16*a^2*b^2)))/(2*(64*a^3*c - 16*a^2*b^2)))*
(2*a*c - b^2))/(8*a^2*(4*a*c - b^2)^(1/2)))*(2*a*c - b^2))/(8*a^2*(4*a*c -
b^2)^(1/2))))/(2*(64*a^3*c - 16*a^2*b^2)) - (b*c^8)/a^5 + ((4*b^3 - 16*a*b*
c)*((a^2*c^8 - 16*a*b^2*c^7)/a^5 + ((4*b^3 - 16*a*b*c)*((16*a^3*b*c^7 - 96*
a^2*b^3*c^6)/a^5 - ((4*b^3 - 16*a*b*c)*((256*a^3*b^4*c^5 - 96*a^4*b^2*c^6)/
a^5 + ((4*b^3 - 16*a*b*c)*((256*a^4*b^5*c^4 - 256*a^5*b^3*c^5)/a^5 - (128*a
*b^4*c^4*(4*b^3 - 16*a*b*c))/(64*a^3*c - 16*a^2*b^2)))/(2*(64*a^3*c - 16*a^
2*b^2))))/(2*(64*a^3*c - 16*a^2*b^2))))/(2*(64*a^3*c - 16*a^2*b^2))))/(2*(6
4*a^3*c - 16*a^2*b^2)) + ((((4*b^3 - 16*a*b*c)*(((4*b^3 - 16*a*b*c)*((((256
*a^4*b^5*c^4 - 256*a^5*b^3*c^5)/a^5 - (128*a*b^4*c^4*(4*b^3 - 16*a*b*c))/(6
4*a^3*c - 16*a^2*b^2))*(2*a*c - b^2))/(8*a^2*(4*a*c - b^2)^(1/2)) - (16*b^4
c^4(4*b^3 - 16*a*b*c)*(2*a*c - b^2))/(a*(4*a*c - b^2)^(1/2)*(64*a^3*c - 1
6*a^2*b^2))))/(2*(64*a^3*c - 16*a^2*b^2)) + (((256*a^3*b^4*c^5 - 96*a^4*b^2
*c^6)/a^5 + ((4*b^3 - 16*a*b*c)*((256*a^4*b^5*c^4 - 256*a^5*b^3*c^5)/a^5 -
(128*a*b^4*c^4*(4*b^3 - 16*a*b*c))/(64*a^3*c - 16*a^2*b^2)))/(2*(64*a^3*c -
16*a^2*b^2)))*(2*a*c - b^2))/(8*a^2*(4*a*c - b^2)^(1/2))))/(2*(64*a^3*c -
16*a^2*b^2)) - (((16*a^3*b*c^7 - 96*a^2*b^3*c^6)/a^5 - ((4*b^3 - 16*a*b*c)*
((256*a^3*b^4*c^5 - 96*a^4*b^2*c^6)/a^5 + ((4*b^3 - 16*a*b*c)*((256*a^4*b^5
*c^4 - 256*a^5*b^3*c^5)/a^5 - (128*a*b^4*c^4*(4*b^3 - 16*a*b*c))/(64*a^3*c

```
- 16*a^2*b^2)))/(2*(64*a^3*c - 16*a^2*b^2))))/(2*(64*a^3*c - 16*a^2*b^2)))*
(2*a*c - b^2))/(8*a^2*(4*a*c - b^2)^(1/2)))*(2*a*c - b^2))/(8*a^2*(4*a*c -
b^2)^(1/2)) - ((((((((((256*a^4*b^5*c^4 - 256*a^5*b^3*c^5)/a^5 - (128*a*b^4*
c^4*(4*b^3 - 16*a*b*c))/(64*a^3*c - 16*a^2*b^2))*(2*a*c - b^2))/(8*a^2*(4*a
*c - b^2)^(1/2)) - (16*b^4*c^4*(4*b^3 - 16*a*b*c)*(2*a*c - b^2))/(a*(4*a*c
- b^2)^(1/2)*(64*a^3*c - 16*a^2*b^2)))*(2*a*c - b^2))/(8*a^2*(4*a*c - b^2)^
(1/2)) - (2*b^4*c^4*(4*b^3 - 16*a*b*c)*(2*a*c - b^2)^2)/(a^3*(4*a*c - b^2)*
(64*a^3*c - 16*a^2*b^2)))*(2*a*c - b^2))/(8*a^2*(4*a*c - b^2)^(1/2)) - (b^4
*c^4*(4*b^3 - 16*a*b*c)*(2*a*c - b^2)^3)/(4*a^5*(4*a*c - b^2)^(3/2)*(64*a^3
*c - 16*a^2*b^2)))*(2*a*c - b^2))/(8*a^2*(4*a*c - b^2)^(1/2)) + (b^4*c^4*(4
*b^3 - 16*a*b*c)*(2*a*c - b^2)^4)/(32*a^7*(4*a*c - b^2)^2*(64*a^3*c - 16*a^
2*b^2))))/(c^4*(a^2*c^2 - 20*b^4 + 80*a*b^2*c)*(16*a^4*c^8 + b^8*c^4 - 8*a*
b^6*c^5 + 24*a^2*b^4*c^6 - 32*a^3*b^2*c^7)))*(2*a*c - b^2))/(4*a^2*(4*a*c -
 b^2)^(1/2)) - (b*log(x))/a^2 - (log(a + b*x^4 + c*x^8)*(4*b^3 - 16*a*b*c))
/(2*(64*a^3*c - 16*a^2*b^2)) - 1/(4*a*x^4)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**5/(c*x**8+b*x**4+a),x)
[Out] Timed out
```

3.262 $\int \frac{x^{10}}{a+bx^4+cx^8}\,dx$

Optimal. Leaf size=381

$$-\frac{\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\,x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2\,2^{3/4}c^{7/4}\sqrt[4]{-\sqrt{b^2-4ac}-b}}-\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\,x}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{2\,2^{3/4}c^{7/4}\sqrt[4]{\sqrt{b^2-4ac}-b}}+\frac{\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\,x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2\,2^{3/4}c^{7/4}\sqrt[4]{-\sqrt{b^2-4ac}-b}}+$$

Rubi [A] time = 0.65, antiderivative size = 381, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {1367, 1510, 298, 205, 208}

$$-\frac{\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\,x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2\,2^{3/4}c^{7/4}\sqrt[4]{-\sqrt{b^2-4ac}-b}}-\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\,x}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{2\,2^{3/4}c^{7/4}\sqrt[4]{\sqrt{b^2-4ac}-b}}+\frac{\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\,x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2\,2^{3/4}c^{7/4}\sqrt[4]{-\sqrt{b^2-4ac}-b}}+\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\,x}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{2\,2^{3/4}c^{7/4}\sqrt[4]{\sqrt{b^2-4ac}-b}}+\frac{x^3}{3c}$$

Antiderivative was successfully verified.

```
[In] Int[x^10/(a + b*x^4 + c*x^8),x]
```

```
[Out] x^3/(3*c) - ((b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(2^(1/4)*c^(1/4)*
x)/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(2*2^(3/4)*c^(7/4)*(-b - Sqrt[b^2 - 4*a
*c])^(1/4)) - ((b - (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(2^(1/4)*c^(1/4
)*x)/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(2*2^(3/4)*c^(7/4)*(-b + Sqrt[b^2 - 4
*a*c])^(1/4)) + ((b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTanh[(2^(1/4)*c^(
1/4)*x)/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(2*2^(3/4)*c^(7/4)*(-b - Sqrt[b^2
- 4*a*c])^(1/4)) + ((b - (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTanh[(2^(1/4)*
c^(1/4)*x)/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(2*2^(3/4)*c^(7/4)*(-b + Sqrt[b
^2 - 4*a*c])^(1/4))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 1367

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Simp[(d^(2*n - 1)*(d*x)^(m - 2*n + 1)*(a + b*x^n + c*x^(2*n))^(
p + 1))/(c*(m + 2*n*p + 1)), x] - Dist[d^(2*n)/(c*(m + 2*n*p + 1)), Int[(d*
x)^(m - 2*n)*Simp[a*(m - 2*n + 1) + b*(m + n*(p - 1) + 1)*x^n, x]*(a + b*x^
n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && EqQ[n2, 2*n] && Ne
Q[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GtQ[m, 2*n - 1] && NeQ[m + 2*n*p + 1, 0]
 && IntegerQ[p]
```

Rule 1510

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_)))/((a_) + (b_.)*(x_)^(n_) +
 (c_.)*(x_)^(n2_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 +
(2*c*d - b*e)/(2*q), Int[(f*x)^m/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (
2*c*d - b*e)/(2*q), Int[(f*x)^m/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b
, c, d, e, f, m}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{x^{10}}{a+bx^4+cx^8}\,dx = \frac{x^3}{3c}-\frac{\int \frac{x^2(3a+3bx^4)}{a+bx^4+cx^8}\,dx}{3c}$$
$$= \frac{x^3}{3c}-\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\int \frac{x^2}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^4}\,dx}{2c}-\frac{\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\int \frac{x^2}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^4}\,dx}{2c}$$
$$= \frac{x^3}{3c}+\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\sqrt{-b+\sqrt{b^2-4ac}}-\sqrt{2}\sqrt{c}x^2}\,dx}{2\sqrt{2}c^{3/2}}-\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\sqrt{-b+\sqrt{b^2-4ac}}+\sqrt{2}\sqrt{c}x^2}\,d}{2\sqrt{2}c^{3/2}}$$
$$= \frac{x^3}{3c}-\frac{\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{2\,2^{3/4}c^{7/4}\sqrt[4]{-b-\sqrt{b^2-4ac}}}-\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b+\sqrt{b^2-4ac}}}\right)}{2\,2^{3/4}c^{7/4}\sqrt[4]{-b+\sqrt{b^2-4ac}}}+\frac{(b+}{2}$$

Mathematica [C] time = 0.04, size = 70, normalized size = 0.18

$$\frac{4x^3-3\text{RootSum}\left[\#1^8c+\#1^4b+a\&,\frac{\#1^4b\log(x-\#1)+a\log(x-\#1)}{2\#1^5c+\#1b}\&\right]}{12c}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^10/(a + b*x^4 + c*x^8),x]
```

```
[Out] (4*x^3 - 3*RootSum[a + b*#1^4 + c*#1^8 & , (a*Log[x - #1] + b*Log[x - #1]*#
1^4)/(b*#1 + 2*c*#1^5) & ])/(12*c)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{10}}{a+bx^4+cx^8}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^10/(a + b*x^4 + c*x^8),x]
```

```
[Out] IntegrateAlgebraic[x^10/(a + b*x^4 + c*x^8), x]
```

fricas [B] time = 5.66, size = 6296, normalized size = 16.52

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10/(c*x^8+b*x^4+a),x, algorithm="fricas")
```

```
[Out] 1/12*(4*x^3 + 12*c*sqrt(sqrt(1/2)*sqrt(-(b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 -
 7*a^3*b*c^3 + (b^4*c^7 - 8*a*b^2*c^8 + 16*a^2*c^9)*sqrt((b^12 - 10*a*b^10*
c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6
*c^6)/(b^6*c^14 - 12*a*b^4*c^15 + 48*a^2*b^2*c^16 - 64*a^3*c^17)))/(b^4*c^7
 - 8*a*b^2*c^8 + 16*a^2*c^9)))*arctan(-1/2*((b^6*c^7 - 10*a*b^4*c^8 + 32*a^
```

2*b^2*c^9 - 32*a^3*c^10)*x*sqrt((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(b^6*c^14 - 12*a*b^4*c^15 + 48*a^2*b^2*c^16 - 64*a^3*c^17)) - (b^9 - 9*a*b^7*c + 26*a^2*b^5*c^2 - 25*a^3*b^3*c^3 + 4*a^4*b*c^4)*x + sqrt(1/2)*(b^9 - 9*a*b^7*c + 26*a^2*b^5*c^2 - 25*a^3*b^3*c^3 + 4*a^4*b*c^4 - (b^6*c^7 - 10*a*b^4*c^8 + 32*a^2*b^2*c^9 - 32*a^3*c^10)*sqrt((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(b^6*c^14 - 12*a*b^4*c^15 + 48*a^2*b^2*c^16 - 64*a^3*c^17)))*sqrt((2*(a^3*b^6 - 5*a^4*b^4*c + 6*a^5*b^2*c^2 - a^6*c^3)*x^2 - sqrt(1/2)*(b^11 - 12*a*b^9*c + 53*a^2*b^7*c^2 - 103*a^3*b^5*c^3 + 79*a^4*b^3*c^4 - 12*a^5*b*c^5 - (b^8*c^7 - 13*a*b^6*c^8 + 60*a^2*b^4*c^9 - 112*a^3*b^2*c^10 + 64*a^4*c^11)*sqrt((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(b^6*c^14 - 12*a*b^4*c^15 + 48*a^2*b^2*c^16 - 64*a^3*c^17)))*sqrt(-(b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c^3 + (b^4*c^7 - 8*a*b^2*c^8 + 16*a^2*c^9)*sqrt((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(b^6*c^14 - 12*a*b^4*c^15 + 48*a^2*b^2*c^16 - 64*a^3*c^17)))/(b^4*c^7 - 8*a*b^2*c^8 + 16*a^2*c^9)))/(a^3*b^6 - 5*a^4*b^4*c + 6*a^5*b^2*c^2 - a^6*c^3)))*sqrt(sqrt(1/2)*sqrt(-(b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c^3 + (b^4*c^7 - 8*a*b^2*c^8 + 16*a^2*c^9)*sqrt((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(b^6*c^14 - 12*a*b^4*c^15 + 48*a^2*b^2*c^16 - 64*a^3*c^17)))/(b^4*c^7 - 8*a*b^2*c^8 + 16*a^2*c^9)))/(a^2*b^6 - 5*a^3*b^4*c + 6*a^4*b^2*c^2 - a^5*c^3)) - 12*c*sqrt(sqrt(1/2)*sqrt(-(b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c^3 - (b^4*c^7 - 8*a*b^2*c^8 + 16*a^2*c^9)*sqrt((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(b^6*c^14 - 12*a*b^4*c^15 + 48*a^2*b^2*c^16 - 64*a^3*c^17)))/(b^4*c^7 - 8*a*b^2*c^8 + 16*a^2*c^9)))*arctan(1/2*(sqrt(1/2)*(b^9 - 9*a*b^7*c + 26*a^2*b^5*c^2 - 25*a^3*b^3*c^3 + 4*a^4*b*c^4 + (b^6*c^7 - 10*a*b^4*c^8 + 32*a^2*b^2*c^9 - 32*a^3*c^10)*sqrt((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(b^6*c^14 - 12*a*b^4*c^15 + 48*a^2*b^2*c^16 - 64*a^3*c^17)))*sqrt(sqrt(1/2)*sqrt(-(b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c^3 - (b^4*c^7 - 8*a*b^2*c^8 + 16*a^2*c^9)*sqrt((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(b^6*c^14 - 12*a*b^4*c^15 + 48*a^2*b^2*c^16 - 64*a^3*c^17)))/(b^4*c^7 - 8*a*b^2*c^8 + 16*a^2*c^9)))*sqrt((2*(a^3*b^6 - 5*a^4*b^4*c + 6*a^5*b^2*c^2 - a^6*c^3)*x^2 - sqrt(1/2)*(b^11 - 12*a*b^9*c + 53*a^2*b^7*c^2 - 103*a^3*b^5*c^3 + 79*a^4*b^3*c^4 - 12*a^5*b*c^5 + (b^8*c^7 - 13*a*b^6*c^8 + 60*a^2*b^4*c^9 - 112*a^3*b^2*c^10 + 64*a^4*c^11)*sqrt((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(b^6*c^14 - 12*a*b^4*c^15 + 48*a^2*b^2*c^16 - 64*a^3*c^17)))*sqrt(-(b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c^3 - (b^4*c^7 - 8*a*b^2*c^8 + 16*a^2*c^9)*sqrt((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(b^6*c^14 - 12*a*b^4*c^15 + 48*a^2*b^2*c^16 - 64*a^3*c^17)))/(b^4*c^7 - 8*a*b^2*c^8 + 16*a^2*c^9)))/(a^3*b^6 - 5*a^4*b^4*c + 6*a^5*b^2*c^2 - a^6*c^3)) - ((b^6*c^7 - 10*a*b^4*c^8 + 32*a^2*b^2*c^9 - 32*a^3*c^10)*x*sqrt((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(b^6*c^14 - 12*a*b^4*c^15 + 48*a^2*b^2*c^16 - 64*a^3*c^17)) + (b^9 - 9*a*b^7*c + 26*a^2*b^5*c^2 - 25*a^3*b^3*c^3 + 4*a^4*b*c^4)*x)*sqrt(sqrt(1/2)*sqrt(-(b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c^3 - (b^4*c^7 - 8*a*b^2*c^8 + 16*a^2*c^9)*sqrt((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(b^6*c^14 - 12*a*b^4*c^15 + 48*a^2*b^2*c^16 - 64*a^3*c^17)))/(b^4*c^7 - 8*a*b^2*c^8 + 16*a^2*c^9))))/(a^2*b^6 - 5*a^3*b^4*c + 6*a^4*b^2*c^2 - a^5*c^3)) - 3*c*sqrt(sqrt(1/2)*sqrt(-(b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c^3 + (b^4*c^7 - 8*a*b^2*c^8 + 16*a^2*c^9)*sqrt((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(b^6*c^14 - 12*a*b^4*c^15 + 48*a^2*b^2*c^16 - 64*a^3*c^17)))/(b^4*c^7 - 8*a*b^2*c^8 + 16*a^2*c^9)))*log(1/2*sqrt(1/2)*(b^14 - 16*a*b^12*c + 102*a^2*b^10*c^2 - 328*a^3*b^8*c^3 + 553*

```
a^4*b^6*c^4 - 457*a^5*b^4*c^5 + 152*a^6*b^2*c^6 - 16*a^7*c^7 - (b^11*c^7 -
17*a*b^9*c^8 + 113*a^2*b^7*c^9 - 364*a^3*b^5*c^10 + 560*a^4*b^3*c^11 - 320*
a^5*b*c^12)*sqrt((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46
*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(b^6*c^14 - 12*a*b^4*c^15 + 48*a^2
*b^2*c^16 - 64*a^3*c^17)))*sqrt(sqrt(1/2)*sqrt(-(b^7 - 7*a*b^5*c + 14*a^2*b
^3*c^2 - 7*a^3*b*c^3 + (b^4*c^7 - 8*a*b^2*c^8 + 16*a^2*c^9)*sqrt((b^12 - 10
*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c
^5 + a^6*c^6)/(b^6*c^14 - 12*a*b^4*c^15 + 48*a^2*b^2*c^16 - 64*a^3*c^17)))/
(b^4*c^7 - 8*a*b^2*c^8 + 16*a^2*c^9)))*sqrt(-(b^7 - 7*a*b^5*c + 14*a^2*b^3*
c^2 - 7*a^3*b*c^3 + (b^4*c^7 - 8*a*b^2*c^8 + 16*a^2*c^9)*sqrt((b^12 - 10*a*
b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5
+ a^6*c^6)/(b^6*c^14 - 12*a*b^4*c^15 + 48*a^2*b^2*c^16 - 64*a^3*c^17)))/(b^
4*c^7 - 8*a*b^2*c^8 + 16*a^2*c^9)) - (a^5*b^6 - 5*a^6*b^4*c + 6*a^7*b^2*c^2
 - a^8*c^3)*x) + 3*c*sqrt(sqrt(1/2)*sqrt(-(b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2
 - 7*a^3*b*c^3 + (b^4*c^7 - 8*a*b^2*c^8 + 16*a^2*c^9)*sqrt((b^12 - 10*a*b^1
0*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a
^6*c^6)/(b^6*c^14 - 12*a*b^4*c^15 + 48*a^2*b^2*c^16 - 64*a^3*c^17)))/(b^4*c
^7 - 8*a*b^2*c^8 + 16*a^2*c^9)))*log(-1/2*sqrt(1/2)*(b^14 - 16*a*b^12*c + 1
02*a^2*b^10*c^2 - 328*a^3*b^8*c^3 + 553*a^4*b^6*c^4 - 457*a^5*b^4*c^5 + 152
*a^6*b^2*c^6 - 16*a^7*c^7 - (b^11*c^7 - 17*a*b^9*c^8 + 113*a^2*b^7*c^9 - 36
4*a^3*b^5*c^10 + 560*a^4*b^3*c^11 - 320*a^5*b*c^12)*sqrt((b^12 - 10*a*b^10*
c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6
*c^6)/(b^6*c^14 - 12*a*b^4*c^15 + 48*a^2*b^2*c^16 - 64*a^3*c^17)))*sqrt(sqr
t(1/2)*sqrt(-(b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c^3 + (b^4*c^7 - 8
*a*b^2*c^8 + 16*a^2*c^9)*sqrt((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3
*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(b^6*c^14 - 12*a*b^4*
c^15 + 48*a^2*b^2*c^16 - 64*a^3*c^17)))/(b^4*c^7 - 8*a*b^2*c^8 + 16*a^2*c^9
)))*sqrt(-(b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c^3 + (b^4*c^7 - 8*a*
b^2*c^8 + 16*a^2*c^9)*sqrt((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^
6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(b^6*c^14 - 12*a*b^4*c^1
5 + 48*a^2*b^2*c^16 - 64*a^3*c^17)))/(b^4*c^7 - 8*a*b^2*c^8 + 16*a^2*c^9))
- (a^5*b^6 - 5*a^6*b^4*c + 6*a^7*b^2*c^2 - a^8*c^3)*x) - 3*c*sqrt(sqrt(1/2)
*sqrt(-(b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c^3 - (b^4*c^7 - 8*a*b^2
*c^8 + 16*a^2*c^9)*sqrt((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c
^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(b^6*c^14 - 12*a*b^4*c^15 +
 48*a^2*b^2*c^16 - 64*a^3*c^17)))/(b^4*c^7 - 8*a*b^2*c^8 + 16*a^2*c^9)))*lo
g(1/2*sqrt(1/2)*(b^14 - 16*a*b^12*c + 102*a^2*b^10*c^2 - 328*a^3*b^8*c^3 +
553*a^4*b^6*c^4 - 457*a^5*b^4*c^5 + 152*a^6*b^2*c^6 - 16*a^7*c^7 + (b^11*c^
7 - 17*a*b^9*c^8 + 113*a^2*b^7*c^9 - 364*a^3*b^5*c^10 + 560*a^4*b^3*c^11 -
320*a^5*b*c^12)*sqrt((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3
+ 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(b^6*c^14 - 12*a*b^4*c^15 + 48
*a^2*b^2*c^16 - 64*a^3*c^17)))*sqrt(sqrt(1/2)*sqrt(-(b^7 - 7*a*b^5*c + 14*a
^2*b^3*c^2 - 7*a^3*b*c^3 - (b^4*c^7 - 8*a*b^2*c^8 + 16*a^2*c^9)*sqrt((b^12
- 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b
^2*c^5 + a^6*c^6)/(b^6*c^14 - 12*a*b^4*c^15 + 48*a^2*b^2*c^16 - 64*a^3*c^17
)))/(b^4*c^7 - 8*a*b^2*c^8 + 16*a^2*c^9)))*sqrt(-(b^7 - 7*a*b^5*c + 14*a^2*
b^3*c^2 - 7*a^3*b*c^3 - (b^4*c^7 - 8*a*b^2*c^8 + 16*a^2*c^9)*sqrt((b^12 - 1
0*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*
c^5 + a^6*c^6)/(b^6*c^14 - 12*a*b^4*c^15 + 48*a^2*b^2*c^16 - 64*a^3*c^17)))
/(b^4*c^7 - 8*a*b^2*c^8 + 16*a^2*c^9)) - (a^5*b^6 - 5*a^6*b^4*c + 6*a^7*b^2
*c^2 - a^8*c^3)*x) + 3*c*sqrt(sqrt(1/2)*sqrt(-(b^7 - 7*a*b^5*c + 14*a^2*b^3
*c^2 - 7*a^3*b*c^3 - (b^4*c^7 - 8*a*b^2*c^8 + 16*a^2*c^9)*sqrt((b^12 - 10*a
*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5
 + a^6*c^6)/(b^6*c^14 - 12*a*b^4*c^15 + 48*a^2*b^2*c^16 - 64*a^3*c^17)))/(b
^4*c^7 - 8*a*b^2*c^8 + 16*a^2*c^9)))*log(-1/2*sqrt(1/2)*(b^14 - 16*a*b^12*c
 + 102*a^2*b^10*c^2 - 328*a^3*b^8*c^3 + 553*a^4*b^6*c^4 - 457*a^5*b^4*c^5 +
 152*a^6*b^2*c^6 - 16*a^7*c^7 + (b^11*c^7 - 17*a*b^9*c^8 + 113*a^2*b^7*c^9
- 364*a^3*b^5*c^10 + 560*a^4*b^3*c^11 - 320*a^5*b*c^12)*sqrt((b^12 - 10*a*b
^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 +
```

```
 a^6*c^6)/(b^6*c^14 - 12*a*b^4*c^15 + 48*a^2*b^2*c^16 - 64*a^3*c^17)))*sqrt
(sqrt(1/2)*sqrt(-(b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c^3 - (b^4*c^7
 - 8*a*b^2*c^8 + 16*a^2*c^9)*sqrt((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62
*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(b^6*c^14 - 12*a*
b^4*c^15 + 48*a^2*b^2*c^16 - 64*a^3*c^17)))/(b^4*c^7 - 8*a*b^2*c^8 + 16*a^2
*c^9)))*sqrt(-(b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c^3 - (b^4*c^7 -
8*a*b^2*c^8 + 16*a^2*c^9)*sqrt((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^
3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(b^6*c^14 - 12*a*b^4
*c^15 + 48*a^2*b^2*c^16 - 64*a^3*c^17)))/(b^4*c^7 - 8*a*b^2*c^8 + 16*a^2*c^
9)) - (a^5*b^6 - 5*a^6*b^4*c + 6*a^7*b^2*c^2 - a^8*c^3)*x))/c
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10/(c*x^8+b*x^4+a),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Evaluation time: 19.53Unable to convert to r
eal 1/4 Error: Bad Argument Value
```

maple [C] time = 0.04, size = 63, normalized size = 0.17

$$\frac{x^3}{3c} - \frac{\left(\mathrm{RootOf}\left(\_Z^8 c + b\_Z^4 + a\right)^6 b + \mathrm{RootOf}\left(\_Z^8 c + b\_Z^4 + a\right)^2 a\right) \ln\left(-\mathrm{RootOf}\left(\_Z^8 c + b\_Z^4 + a\right) + x\right)}{4c\left(2\,\mathrm{RootOf}\left(\_Z^8 c + b\_Z^4 + a\right)^7 c + \mathrm{RootOf}\left(\_Z^8 c + b\_Z^4 + a\right)^3 b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^10/(c*x^8+b*x^4+a),x)
```

```
[Out] 1/3/c*x^3-1/4/c*sum((_R^6*b+_R^2*a)/(2*_R^7*c+_R^3*b)*ln(-_R+x),_R=RootOf(_
Z^8*c+_Z^4*b+a))
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10/(c*x^8+b*x^4+a),x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [B] time = 3.49, size = 12709, normalized size = 33.36

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^10/(a + b*x^4 + c*x^8),x)
```

```
[Out] atan(((((8192*a^6*b*c^6 - 256*a^3*b^7*c^3 + 2560*a^4*b^5*c^4 - 8192*a^5*b^3
*c^5)/c^3 - (4*x*(-(b^11 + b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 8
6*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 - a^3*c^3*(-(4*a*c - b^2)
^5)^(1/2) - 15*a*b^9*c + 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 5*a*b^4*c
*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^11 + b^8*c^7 - 16*a*b^6*c^8 + 96
*a^2*b^4*c^9 - 256*a^3*b^2*c^10)))^(1/4)*(8192*a^6*c^8 - 256*a^3*b^6*c^5 +
2560*a^4*b^4*c^6 - 8192*a^5*b^2*c^7))/c^3)*(-(b^11 + b^6*(-(4*a*c - b^2)^5)
^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4
 - a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c + 6*a^2*b^2*c^2*(-(4*a*c -
 b^2)^5)^(1/2) - 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^11 + b
```

^8*c^7 - 16*a*b^6*c^8 + 96*a^2*b^4*c^9 - 256*a^3*b^2*c^10)))^(3/4) + (4*x*(
a^5*b^5 - 5*a^6*b^3*c + 5*a^7*b*c^2))/c^3)*(-(b^11 + b^6*(-(4*a*c - b^2)^5)
^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4
 - a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c + 6*a^2*b^2*c^2*(-(4*a*c -
 b^2)^5)^(1/2) - 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^11 + b
^8*c^7 - 16*a*b^6*c^8 + 96*a^2*b^4*c^9 - 256*a^3*b^2*c^10)))^(1/4)*1i - (((
8192*a^6*b*c^6 - 256*a^3*b^7*c^3 + 2560*a^4*b^5*c^4 - 8192*a^5*b^3*c^5)/c^3
 + (4*x*(-(b^11 + b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7
*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 - a^3*c^3*(-(4*a*c - b^2)^5)^(1/2)
 - 15*a*b^9*c + 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 5*a*b^4*c*(-(4*a*c
 - b^2)^5)^(1/2))/(512*(256*a^4*c^11 + b^8*c^7 - 16*a*b^6*c^8 + 96*a^2*b^4*
c^9 - 256*a^3*b^2*c^10)))^(1/4)*(8192*a^6*c^8 - 256*a^3*b^6*c^5 + 2560*a^4*
b^4*c^6 - 8192*a^5*b^2*c^7))/c^3)*(-(b^11 + b^6*(-(4*a*c - b^2)^5)^(1/2) -
112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 - a^3*c^
3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c + 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^
(1/2) - 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^11 + b^8*c^7 -
16*a*b^6*c^8 + 96*a^2*b^4*c^9 - 256*a^3*b^2*c^10)))^(3/4) - (4*x*(a^5*b^5 -
 5*a^6*b^3*c + 5*a^7*b*c^2))/c^3)*(-(b^11 + b^6*(-(4*a*c - b^2)^5)^(1/2) -
112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 - a^3*c^
3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c + 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^
(1/2) - 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^11 + b^8*c^7 -
16*a*b^6*c^8 + 96*a^2*b^4*c^9 - 256*a^3*b^2*c^10)))^(1/4)*1i)/((((8192*a^6*
b*c^6 - 256*a^3*b^7*c^3 + 2560*a^4*b^5*c^4 - 8192*a^5*b^3*c^5)/c^3 - (4*x*(
-(b^11 + b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 23
1*a^3*b^5*c^3 + 280*a^4*b^3*c^4 - a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b
^9*c + 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 5*a*b^4*c*(-(4*a*c - b^2)^5
)^(1/2))/(512*(256*a^4*c^11 + b^8*c^7 - 16*a*b^6*c^8 + 96*a^2*b^4*c^9 - 256
*a^3*b^2*c^10)))^(1/4)*(8192*a^6*c^8 - 256*a^3*b^6*c^5 + 2560*a^4*b^4*c^6 -
 8192*a^5*b^2*c^7))/c^3)*(-(b^11 + b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b
*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 - a^3*c^3*(-(4*a*
c - b^2)^5)^(1/2) - 15*a*b^9*c + 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 5
*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^11 + b^8*c^7 - 16*a*b^6*
c^8 + 96*a^2*b^4*c^9 - 256*a^3*b^2*c^10)))^(3/4) + (4*x*(a^5*b^5 - 5*a^6*b^
3*c + 5*a^7*b*c^2))/c^3)*(-(b^11 + b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b
*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 - a^3*c^3*(-(4*a*
c - b^2)^5)^(1/2) - 15*a*b^9*c + 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 5
*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^11 + b^8*c^7 - 16*a*b^6*
c^8 + 96*a^2*b^4*c^9 - 256*a^3*b^2*c^10)))^(1/4) + (((8192*a^6*b*c^6 - 256*
a^3*b^7*c^3 + 2560*a^4*b^5*c^4 - 8192*a^5*b^3*c^5)/c^3 + (4*x*(-(b^11 + b^6
*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^
3 + 280*a^4*b^3*c^4 - a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c + 6*a^2
*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(51
2*(256*a^4*c^11 + b^8*c^7 - 16*a*b^6*c^8 + 96*a^2*b^4*c^9 - 256*a^3*b^2*c^1
0)))^(1/4)*(8192*a^6*c^8 - 256*a^3*b^6*c^5 + 2560*a^4*b^4*c^6 - 8192*a^5*b^
2*c^7))/c^3)*(-(b^11 + b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^
2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 - a^3*c^3*(-(4*a*c - b^2)^5)^
(1/2) - 15*a*b^9*c + 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 5*a*b^4*c*(-(
4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^11 + b^8*c^7 - 16*a*b^6*c^8 + 96*a^2
*b^4*c^9 - 256*a^3*b^2*c^10)))^(3/4) - (4*x*(a^5*b^5 - 5*a^6*b^3*c + 5*a^7*
b*c^2))/c^3)*(-(b^11 + b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^
2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 - a^3*c^3*(-(4*a*c - b^2)^5)^
(1/2) - 15*a*b^9*c + 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 5*a*b^4*c*(-(
4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^11 + b^8*c^7 - 16*a*b^6*c^8 + 96*a^2
*b^4*c^9 - 256*a^3*b^2*c^10)))^(1/4) - (2*(a^8*c - a^7*b^2))/c^3))*(-(b^11
+ b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b
^5*c^3 + 280*a^4*b^3*c^4 - a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c +
6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2)
)/(512*(256*a^4*c^11 + b^8*c^7 - 16*a*b^6*c^8 + 96*a^2*b^4*c^9 - 256*a^3*b^
2*c^10)))^(1/4)*2i + atan(((((8192*a^6*b*c^6 - 256*a^3*b^7*c^3 + 2560*a^4*b

```
^5*c^4 - 8192*a^5*b^3*c^5)/c^3 - (4*x*(-(b^11 - b^6*(-(4*a*c - b^2)^5)^(1/2
) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 + a^
3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c - 6*a^2*b^2*c^2*(-(4*a*c - b^2)
^5)^(1/2) + 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^11 + b^8*c^
7 - 16*a*b^6*c^8 + 96*a^2*b^4*c^9 - 256*a^3*b^2*c^10)))^(1/4)*(8192*a^6*c^8
 - 256*a^3*b^6*c^5 + 2560*a^4*b^4*c^6 - 8192*a^5*b^2*c^7))/c^3)*(-(b^11 - b
^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*
c^3 + 280*a^4*b^3*c^4 + a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c - 6*a
^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) + 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(
512*(256*a^4*c^11 + b^8*c^7 - 16*a*b^6*c^8 + 96*a^2*b^4*c^9 - 256*a^3*b^2*c
^10)))^(3/4) + (4*x*(a^5*b^5 - 5*a^6*b^3*c + 5*a^7*b*c^2))/c^3)*(-(b^11 - b
^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*
c^3 + 280*a^4*b^3*c^4 + a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c - 6*a
^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) + 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(
512*(256*a^4*c^11 + b^8*c^7 - 16*a*b^6*c^8 + 96*a^2*b^4*c^9 - 256*a^3*b^2*c
^10)))^(1/4)*1i - (((8192*a^6*b*c^6 - 256*a^3*b^7*c^3 + 2560*a^4*b^5*c^4 -
8192*a^5*b^3*c^5)/c^3 + (4*x*(-(b^11 - b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a
^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 + a^3*c^3*(-(
4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c - 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2)
 + 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^11 + b^8*c^7 - 16*a*
b^6*c^8 + 96*a^2*b^4*c^9 - 256*a^3*b^2*c^10)))^(1/4)*(8192*a^6*c^8 - 256*a^
3*b^6*c^5 + 2560*a^4*b^4*c^6 - 8192*a^5*b^2*c^7))/c^3)*(-(b^11 - b^6*(-(4*a
*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280
*a^4*b^3*c^4 + a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c - 6*a^2*b^2*c^
2*(-(4*a*c - b^2)^5)^(1/2) + 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*
a^4*c^11 + b^8*c^7 - 16*a*b^6*c^8 + 96*a^2*b^4*c^9 - 256*a^3*b^2*c^10)))^(3
/4) - (4*x*(a^5*b^5 - 5*a^6*b^3*c + 5*a^7*b*c^2))/c^3)*(-(b^11 - b^6*(-(4*a
*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280
*a^4*b^3*c^4 + a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c - 6*a^2*b^2*c^
2*(-(4*a*c - b^2)^5)^(1/2) + 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*
a^4*c^11 + b^8*c^7 - 16*a*b^6*c^8 + 96*a^2*b^4*c^9 - 256*a^3*b^2*c^10)))^(1
/4)*1i)/((((8192*a^6*b*c^6 - 256*a^3*b^7*c^3 + 2560*a^4*b^5*c^4 - 8192*a^5*
b^3*c^5)/c^3 - (4*x*(-(b^11 - b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5
+ 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 + a^3*c^3*(-(4*a*c - b
^2)^5)^(1/2) - 15*a*b^9*c - 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) + 5*a*b^
4*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^11 + b^8*c^7 - 16*a*b^6*c^8 +
 96*a^2*b^4*c^9 - 256*a^3*b^2*c^10)))^(1/4)*(8192*a^6*c^8 - 256*a^3*b^6*c^5
 + 2560*a^4*b^4*c^6 - 8192*a^5*b^2*c^7))/c^3)*(-(b^11 - b^6*(-(4*a*c - b^2)
^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*
c^4 + a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c - 6*a^2*b^2*c^2*(-(4*a*
c - b^2)^5)^(1/2) + 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^11
+ b^8*c^7 - 16*a*b^6*c^8 + 96*a^2*b^4*c^9 - 256*a^3*b^2*c^10)))^(3/4) + (4*
x*(a^5*b^5 - 5*a^6*b^3*c + 5*a^7*b*c^2))/c^3)*(-(b^11 - b^6*(-(4*a*c - b^2)
^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*
c^4 + a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c - 6*a^2*b^2*c^2*(-(4*a*
c - b^2)^5)^(1/2) + 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^11
+ b^8*c^7 - 16*a*b^6*c^8 + 96*a^2*b^4*c^9 - 256*a^3*b^2*c^10)))^(1/4) + (((
8192*a^6*b*c^6 - 256*a^3*b^7*c^3 + 2560*a^4*b^5*c^4 - 8192*a^5*b^3*c^5)/c^3
 + (4*x*(-(b^11 - b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7
*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 + a^3*c^3*(-(4*a*c - b^2)^5)^(1/2)
 - 15*a*b^9*c - 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) + 5*a*b^4*c*(-(4*a*c
 - b^2)^5)^(1/2))/(512*(256*a^4*c^11 + b^8*c^7 - 16*a*b^6*c^8 + 96*a^2*b^4*
c^9 - 256*a^3*b^2*c^10)))^(1/4)*(8192*a^6*c^8 - 256*a^3*b^6*c^5 + 2560*a^4*
b^4*c^6 - 8192*a^5*b^2*c^7))/c^3)*(-(b^11 - b^6*(-(4*a*c - b^2)^5)^(1/2) -
112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 + a^3*c^
3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c - 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^
(1/2) + 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^11 + b^8*c^7 -
16*a*b^6*c^8 + 96*a^2*b^4*c^9 - 256*a^3*b^2*c^10)))^(3/4) - (4*x*(a^5*b^5 -
 5*a^6*b^3*c + 5*a^7*b*c^2))/c^3)*(-(b^11 - b^6*(-(4*a*c - b^2)^5)^(1/2) -
```

112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 + a^3*c^
3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c - 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^
(1/2) + 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^11 + b^8*c^7 -
16*a*b^6*c^8 + 96*a^2*b^4*c^9 - 256*a^3*b^2*c^10)))^(1/4) - (2*(a^8*c - a^7
b^2))/c^3))(-(b^11 - b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^
2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 + a^3*c^3*(-(4*a*c - b^2)^5)^
(1/2) - 15*a*b^9*c - 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) + 5*a*b^4*c*(-(
4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^11 + b^8*c^7 - 16*a*b^6*c^8 + 96*a^2
*b^4*c^9 - 256*a^3*b^2*c^10)))^(1/4)*2i + 2*atan((((8192*a^6*b*c^6 - 256*a
^3*b^7*c^3 + 2560*a^4*b^5*c^4 - 8192*a^5*b^3*c^5)/c^3 - (x*(-(b^11 + b^6*(-
(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 +
280*a^4*b^3*c^4 - a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c + 6*a^2*b^
2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(
256*a^4*c^11 + b^8*c^7 - 16*a*b^6*c^8 + 96*a^2*b^4*c^9 - 256*a^3*b^2*c^10))
)^(1/4)*(8192*a^6*c^8 - 256*a^3*b^6*c^5 + 2560*a^4*b^4*c^6 - 8192*a^5*b^2*c
^7)*4i)/c^3)*(-(b^11 + b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^
2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 - a^3*c^3*(-(4*a*c - b^2)^5)^
(1/2) - 15*a*b^9*c + 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 5*a*b^4*c*(-(
4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^11 + b^8*c^7 - 16*a*b^6*c^8 + 96*a^2
*b^4*c^9 - 256*a^3*b^2*c^10)))^(3/4)*1i - (4*x*(a^5*b^5 - 5*a^6*b^3*c + 5*a
^7*b*c^2))/c^3)*(-(b^11 + b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86
*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 - a^3*c^3*(-(4*a*c - b^2)^
5)^(1/2) - 15*a*b^9*c + 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 5*a*b^4*c*
(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^11 + b^8*c^7 - 16*a*b^6*c^8 + 96*
a^2*b^4*c^9 - 256*a^3*b^2*c^10)))^(1/4) - (((8192*a^6*b*c^6 - 256*a^3*b^7*c
^3 + 2560*a^4*b^5*c^4 - 8192*a^5*b^3*c^5)/c^3 + (x*(-(b^11 + b^6*(-(4*a*c -
b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4
*b^3*c^4 - a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c + 6*a^2*b^2*c^2*(-
(4*a*c - b^2)^5)^(1/2) - 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*
c^11 + b^8*c^7 - 16*a*b^6*c^8 + 96*a^2*b^4*c^9 - 256*a^3*b^2*c^10)))^(1/4)*
(8192*a^6*c^8 - 256*a^3*b^6*c^5 + 2560*a^4*b^4*c^6 - 8192*a^5*b^2*c^7)*4i)/
c^3)*(-(b^11 + b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^
2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 - a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) -
15*a*b^9*c + 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 5*a*b^4*c*(-(4*a*c -
b^2)^5)^(1/2))/(512*(256*a^4*c^11 + b^8*c^7 - 16*a*b^6*c^8 + 96*a^2*b^4*c^9
- 256*a^3*b^2*c^10)))^(3/4)*1i + (4*x*(a^5*b^5 - 5*a^6*b^3*c + 5*a^7*b*c^2
))/c^3)*(-(b^11 + b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7
*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 - a^3*c^3*(-(4*a*c - b^2)^5)^(1/2)
- 15*a*b^9*c + 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 5*a*b^4*c*(-(4*a*c
- b^2)^5)^(1/2))/(512*(256*a^4*c^11 + b^8*c^7 - 16*a*b^6*c^8 + 96*a^2*b^4*
c^9 - 256*a^3*b^2*c^10)))^(1/4))/((((8192*a^6*b*c^6 - 256*a^3*b^7*c^3 + 256
0*a^4*b^5*c^4 - 8192*a^5*b^3*c^5)/c^3 - (x*(-(b^11 + b^6*(-(4*a*c - b^2)^5)
^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4
- a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c + 6*a^2*b^2*c^2*(-(4*a*c -
b^2)^5)^(1/2) - 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^11 + b
^8*c^7 - 16*a*b^6*c^8 + 96*a^2*b^4*c^9 - 256*a^3*b^2*c^10)))^(1/4)*(8192*a^
6*c^8 - 256*a^3*b^6*c^5 + 2560*a^4*b^4*c^6 - 8192*a^5*b^2*c^7)*4i)/c^3)*(-(
b^11 + b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*
a^3*b^5*c^3 + 280*a^4*b^3*c^4 - a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9
*c + 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 5*a*b^4*c*(-(4*a*c - b^2)^5)^
(1/2))/(512*(256*a^4*c^11 + b^8*c^7 - 16*a*b^6*c^8 + 96*a^2*b^4*c^9 - 256*a
^3*b^2*c^10)))^(3/4)*1i - (4*x*(a^5*b^5 - 5*a^6*b^3*c + 5*a^7*b*c^2))/c^3)*
(-(b^11 + b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 2
31*a^3*b^5*c^3 + 280*a^4*b^3*c^4 - a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*
b^9*c + 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 5*a*b^4*c*(-(4*a*c - b^2)^
5)^(1/2))/(512*(256*a^4*c^11 + b^8*c^7 - 16*a*b^6*c^8 + 96*a^2*b^4*c^9 - 25
6*a^3*b^2*c^10)))^(1/4)*1i + (((8192*a^6*b*c^6 - 256*a^3*b^7*c^3 + 2560*a^4
*b^5*c^4 - 8192*a^5*b^3*c^5)/c^3 + (x*(-(b^11 + b^6*(-(4*a*c - b^2)^5)^(1/2
) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 - a^

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3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c + 6*a^2*b^2*c^2*(-(4*a*c - b^2)
^5)^(1/2) - 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^11 + b^8*c^
7 - 16*a*b^6*c^8 + 96*a^2*b^4*c^9 - 256*a^3*b^2*c^10)))^(1/4)*(8192*a^6*c^8
 - 256*a^3*b^6*c^5 + 2560*a^4*b^4*c^6 - 8192*a^5*b^2*c^7)*4i)/c^3)*(-(b^11
+ b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b
^5*c^3 + 280*a^4*b^3*c^4 - a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c +
6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2)
)/(512*(256*a^4*c^11 + b^8*c^7 - 16*a*b^6*c^8 + 96*a^2*b^4*c^9 - 256*a^3*b^
2*c^10)))^(3/4)*1i + (4*x*(a^5*b^5 - 5*a^6*b^3*c + 5*a^7*b*c^2))/c^3)*(-(b^
11 + b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^
3*b^5*c^3 + 280*a^4*b^3*c^4 - a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c
 + 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1
/2))/(512*(256*a^4*c^11 + b^8*c^7 - 16*a*b^6*c^8 + 96*a^2*b^4*c^9 - 256*a^3
*b^2*c^10)))^(1/4)*1i + (2*(a^8*c - a^7*b^2))/c^3))*(-(b^11 + b^6*(-(4*a*c
- b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^
4*b^3*c^4 - a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c + 6*a^2*b^2*c^2*(
-(4*a*c - b^2)^5)^(1/2) - 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4
*c^11 + b^8*c^7 - 16*a*b^6*c^8 + 96*a^2*b^4*c^9 - 256*a^3*b^2*c^10)))^(1/4)
 + 2*atan(((((8192*a^6*b*c^6 - 256*a^3*b^7*c^3 + 2560*a^4*b^5*c^4 - 8192*a^
5*b^3*c^5)/c^3 - (x*(-(b^11 - b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5
+ 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 + a^3*c^3*(-(4*a*c - b
^2)^5)^(1/2) - 15*a*b^9*c - 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) + 5*a*b^
4*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^11 + b^8*c^7 - 16*a*b^6*c^8 +
 96*a^2*b^4*c^9 - 256*a^3*b^2*c^10)))^(1/4)*(8192*a^6*c^8 - 256*a^3*b^6*c^5
 + 2560*a^4*b^4*c^6 - 8192*a^5*b^2*c^7)*4i)/c^3)*(-(b^11 - b^6*(-(4*a*c - b
^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b
^3*c^4 + a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c - 6*a^2*b^2*c^2*(-(4
*a*c - b^2)^5)^(1/2) + 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^
11 + b^8*c^7 - 16*a*b^6*c^8 + 96*a^2*b^4*c^9 - 256*a^3*b^2*c^10)))^(3/4)*1i
 - (4*x*(a^5*b^5 - 5*a^6*b^3*c + 5*a^7*b*c^2))/c^3)*(-(b^11 - b^6*(-(4*a*c
- b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^
4*b^3*c^4 + a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c - 6*a^2*b^2*c^2*(
-(4*a*c - b^2)^5)^(1/2) + 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4
*c^11 + b^8*c^7 - 16*a*b^6*c^8 + 96*a^2*b^4*c^9 - 256*a^3*b^2*c^10)))^(1/4)
 - (((8192*a^6*b*c^6 - 256*a^3*b^7*c^3 + 2560*a^4*b^5*c^4 - 8192*a^5*b^3*c^
5)/c^3 + (x*(-(b^11 - b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2
*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 + a^3*c^3*(-(4*a*c - b^2)^5)^(
1/2) - 15*a*b^9*c - 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) + 5*a*b^4*c*(-(4
*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^11 + b^8*c^7 - 16*a*b^6*c^8 + 96*a^2*
b^4*c^9 - 256*a^3*b^2*c^10)))^(1/4)*(8192*a^6*c^8 - 256*a^3*b^6*c^5 + 2560*
a^4*b^4*c^6 - 8192*a^5*b^2*c^7)*4i)/c^3)*(-(b^11 - b^6*(-(4*a*c - b^2)^5)^(
1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 +
 a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c - 6*a^2*b^2*c^2*(-(4*a*c - b
^2)^5)^(1/2) + 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^11 + b^8
*c^7 - 16*a*b^6*c^8 + 96*a^2*b^4*c^9 - 256*a^3*b^2*c^10)))^(3/4)*1i + (4*x*
(a^5*b^5 - 5*a^6*b^3*c + 5*a^7*b*c^2))/c^3)*(-(b^11 - b^6*(-(4*a*c - b^2)^5
)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^
4 + a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c - 6*a^2*b^2*c^2*(-(4*a*c
- b^2)^5)^(1/2) + 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^11 +
b^8*c^7 - 16*a*b^6*c^8 + 96*a^2*b^4*c^9 - 256*a^3*b^2*c^10)))^(1/4))/((((81
92*a^6*b*c^6 - 256*a^3*b^7*c^3 + 2560*a^4*b^5*c^4 - 8192*a^5*b^3*c^5)/c^3 -
 (x*(-(b^11 - b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2
 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 + a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 1
5*a*b^9*c - 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) + 5*a*b^4*c*(-(4*a*c - b
^2)^5)^(1/2))/(512*(256*a^4*c^11 + b^8*c^7 - 16*a*b^6*c^8 + 96*a^2*b^4*c^9
- 256*a^3*b^2*c^10)))^(1/4)*(8192*a^6*c^8 - 256*a^3*b^6*c^5 + 2560*a^4*b^4*
c^6 - 8192*a^5*b^2*c^7)*4i)/c^3)*(-(b^11 - b^6*(-(4*a*c - b^2)^5)^(1/2) - 1
12*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 + a^3*c^3
*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c - 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(
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1/2) + 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^11 + b^8*c^7 - 1
6*a*b^6*c^8 + 96*a^2*b^4*c^9 - 256*a^3*b^2*c^10)))^(3/4)*1i - (4*x*(a^5*b^5
 - 5*a^6*b^3*c + 5*a^7*b*c^2))/c^3)*(-(b^11 - b^6*(-(4*a*c - b^2)^5)^(1/2)
- 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 + a^3*
c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c - 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5
)^(1/2) + 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^11 + b^8*c^7
- 16*a*b^6*c^8 + 96*a^2*b^4*c^9 - 256*a^3*b^2*c^10)))^(1/4)*1i + (((8192*a^
6*b*c^6 - 256*a^3*b^7*c^3 + 2560*a^4*b^5*c^4 - 8192*a^5*b^3*c^5)/c^3 + (x*(
-(b^11 - b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 23
1*a^3*b^5*c^3 + 280*a^4*b^3*c^4 + a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b
^9*c - 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) + 5*a*b^4*c*(-(4*a*c - b^2)^5
)^(1/2))/(512*(256*a^4*c^11 + b^8*c^7 - 16*a*b^6*c^8 + 96*a^2*b^4*c^9 - 256
*a^3*b^2*c^10)))^(1/4)*(8192*a^6*c^8 - 256*a^3*b^6*c^5 + 2560*a^4*b^4*c^6 -
 8192*a^5*b^2*c^7)*4i)/c^3)*(-(b^11 - b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^
5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 + a^3*c^3*(-(4
*a*c - b^2)^5)^(1/2) - 15*a*b^9*c - 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2)
+ 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^11 + b^8*c^7 - 16*a*b
^6*c^8 + 96*a^2*b^4*c^9 - 256*a^3*b^2*c^10)))^(3/4)*1i + (4*x*(a^5*b^5 - 5*
a^6*b^3*c + 5*a^7*b*c^2))/c^3)*(-(b^11 - b^6*(-(4*a*c - b^2)^5)^(1/2) - 112
*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 + a^3*c^3*(
-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c - 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/
2) + 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^11 + b^8*c^7 - 16*
a*b^6*c^8 + 96*a^2*b^4*c^9 - 256*a^3*b^2*c^10)))^(1/4)*1i + (2*(a^8*c - a^7
*b^2))/c^3))*(-(b^11 - b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^
2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 + a^3*c^3*(-(4*a*c - b^2)^5)^
(1/2) - 15*a*b^9*c - 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) + 5*a*b^4*c*(-(
4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^11 + b^8*c^7 - 16*a*b^6*c^8 + 96*a^2
*b^4*c^9 - 256*a^3*b^2*c^10)))^(1/4) + x^3/(3*c)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**10/(c*x**8+b*x**4+a),x)
```

```
[Out] Timed out
```

3.263 $\int \frac{x^8}{a+bx^4+cx^8}\,dx$

Optimal. Leaf size=376

$$\frac{\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\,x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2\sqrt[4]{2}\,c^{5/4}\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}+\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\,x}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{2\sqrt[4]{2}\,c^{5/4}\left(\sqrt{b^2-4ac}-b\right)^{3/4}}+\frac{\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\,x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2\sqrt[4]{2}\,c^{5/4}\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}+$$

Rubi [A] time = 0.57, antiderivative size = 376, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {1367, 1422, 212, 208, 205}

$$\frac{\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\,x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2\sqrt[4]{2}\,c^{5/4}\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}+\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\,x}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{2\sqrt[4]{2}\,c^{5/4}\left(\sqrt{b^2-4ac}-b\right)^{3/4}}+\frac{\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\,x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2\sqrt[4]{2}\,c^{5/4}\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}+\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\,x}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{2\sqrt[4]{2}\,c^{5/4}\left(\sqrt{b^2-4ac}-b\right)^{3/4}}+\frac{x}{c}$$

Antiderivative was successfully verified.

```
[In] Int[x^8/(a + b*x^4 + c*x^8),x]
```

```
[Out] x/c + ((b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(2^(1/4)*c^(1/4)*x)/(-b
 - Sqrt[b^2 - 4*a*c])^(1/4)])/(2*2^(1/4)*c^(5/4)*(-b - Sqrt[b^2 - 4*a*c])^(
3/4)) + ((b - (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(2^(1/4)*c^(1/4)*x)/(
-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(2*2^(1/4)*c^(5/4)*(-b + Sqrt[b^2 - 4*a*c])
^(3/4)) + ((b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTanh[(2^(1/4)*c^(1/4)*x
)/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(2*2^(1/4)*c^(5/4)*(-b - Sqrt[b^2 - 4*a*
c])^(3/4)) + ((b - (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTanh[(2^(1/4)*c^(1/4
)*x)/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(2*2^(1/4)*c^(5/4)*(-b + Sqrt[b^2 - 4
*a*c])^(3/4))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 1367

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Simp[(d^(2*n - 1)*(d*x)^(m - 2*n + 1)*(a + b*x^n + c*x^(2*n))^(
p + 1))/(c*(m + 2*n*p + 1)), x] - Dist[d^(2*n)/(c*(m + 2*n*p + 1)), Int[(d*
x)^(m - 2*n)*Simp[a*(m - 2*n + 1) + b*(m + n*(p - 1) + 1)*x^n, x]*(a + b*x^
n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && EqQ[n2, 2*n] && Ne
Q[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GtQ[m, 2*n - 1] && NeQ[m + 2*n*p + 1, 0]
 && IntegerQ[p]
```

Rule 1422

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q),
Int[1/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(
b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (PosQ[b^2 - 4*a
*c] ||  !IGtQ[n/2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^8}{a+bx^4+cx^8}\,dx &= \frac{x}{c}-\frac{\int \frac{a+bx^4}{a+bx^4+cx^8}\,dx}{c}\\
&= \frac{x}{c}-\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^4}\,dx}{2c}-\frac{\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^4}\,dx}{2c}\\
&= \frac{x}{c}+\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\sqrt{-b+\sqrt{b^2-4ac}}-\sqrt{2}\sqrt{c}x^2}\,dx}{2c\sqrt{-b+\sqrt{b^2-4ac}}}+\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\sqrt{-b+\sqrt{b^2-4ac}}+\sqrt{2}\sqrt{c}x^2}\,d
\\
&= \frac{x}{c}+\frac{\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{2\sqrt[4]{2}c^{5/4}\left(-b-\sqrt{b^2-4ac}\right)^{3/4}}+\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b+\sqrt{b^2-4ac}}}\right)}{2\sqrt[4]{2}c^{5/4}\left(-b+\sqrt{b^2-4ac}\right)^{3/4}}+\frac{(b+}{2}
\end{aligned}$$

Mathematica [C] time = 0.04, size = 70, normalized size = 0.19

$$\frac{x}{c}-\frac{\text{RootSum}\left[\#1^8c+\#1^4b+a\&,\frac{\#1^4b\log(x-\#1)+a\log(x-\#1)}{2\#1^7c+\#1^3b}\&\right]}{4c}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8/(a + b*x^4 + c*x^8),x]
```

```
[Out] x/c - RootSum[a + b*#1^4 + c*#1^8 & , (a*Log[x - #1] + b*Log[x - #1]*#1^4)/
(b*#1^3 + 2*c*#1^7) & ]/(4*c)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^8}{a+bx^4+cx^8}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^8/(a + b*x^4 + c*x^8),x]
```

```
[Out] IntegrateAlgebraic[x^8/(a + b*x^4 + c*x^8), x]
```

fricas [B] time = 2.61, size = 5082, normalized size = 13.52

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(c*x^8+b*x^4+a),x, algorithm="fricas")
```

```
[Out] -1/4*(4*c*sqrt(sqrt(1/2)*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 - (b^4*c^5 -
8*a*b^2*c^6 + 16*a^2*c^7)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^
2*c^3 + a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)
```

```
))/(b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)))*arctan(1/4*(2*sqrt(1/2)*((b^10*c^
5 - 16*a*b^8*c^6 + 98*a^2*b^6*c^7 - 280*a^3*b^4*c^8 + 352*a^4*b^2*c^9 - 128
*a^5*c^10)*x*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c
^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)) + (b^11 - 1
3*a*b^9*c + 63*a^2*b^7*c^2 - 138*a^3*b^5*c^3 + 128*a^4*b^3*c^4 - 32*a^5*b*c
^5)*x)*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 - (b^4*c^5 - 8*a*b^2*c^6 + 16*a
^2*c^7)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(
b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))/(b^4*c^5 - 8*a*
b^2*c^6 + 16*a^2*c^7)) - (b^11 - 13*a*b^9*c + 63*a^2*b^7*c^2 - 138*a^3*b^5*
c^3 + 128*a^4*b^3*c^4 - 32*a^5*b*c^5 + (b^10*c^5 - 16*a*b^8*c^6 + 98*a^2*b^
6*c^7 - 280*a^3*b^4*c^8 + 352*a^4*b^2*c^9 - 128*a^5*c^10)*sqrt((b^8 - 6*a*b
^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^11
 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 - (
b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2
 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64
*a^3*c^13)))/(b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7))*sqrt((2*(a^2*b^4 - 3*a^3
*b^2*c + a^4*c^2)*x^2 + sqrt(1/2)*(b^8 - 9*a*b^6*c + 27*a^2*b^4*c^2 - 30*a^
3*b^2*c^3 + 8*a^4*c^4 + (b^7*c^5 - 12*a*b^5*c^6 + 48*a^2*b^3*c^7 - 64*a^3*b
*c^8)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^
6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))*sqrt(-(b^5 - 5*a*
b^3*c + 5*a^2*b*c^2 - (b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)*sqrt((b^8 - 6*a*
b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^11
 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))/(b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)))/
(a^2*b^4 - 3*a^3*b^2*c + a^4*c^2)))*sqrt(sqrt(1/2)*sqrt(-(b^5 - 5*a*b^3*c +
 5*a^2*b*c^2 - (b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)*sqrt((b^8 - 6*a*b^6*c +
 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a
^2*b^2*c^12 - 64*a^3*c^13)))/(b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)))/(a^4*b^
4 - 3*a^5*b^2*c + a^6*c^2)) - 4*c*sqrt(sqrt(1/2)*sqrt(-(b^5 - 5*a*b^3*c + 5
*a^2*b*c^2 + (b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)*sqrt((b^8 - 6*a*b^6*c + 1
1*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2
*b^2*c^12 - 64*a^3*c^13)))/(b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)))*arctan(1/
4*(2*sqrt(1/2)*((b^10*c^5 - 16*a*b^8*c^6 + 98*a^2*b^6*c^7 - 280*a^3*b^4*c^8
 + 352*a^4*b^2*c^9 - 128*a^5*c^10)*x*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2
 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 6
4*a^3*c^13)) - (b^11 - 13*a*b^9*c + 63*a^2*b^7*c^2 - 138*a^3*b^5*c^3 + 128*
a^4*b^3*c^4 - 32*a^5*b*c^5)*x)*sqrt(sqrt(1/2)*sqrt(-(b^5 - 5*a*b^3*c + 5*a^
2*b*c^2 + (b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)*sqrt((b^8 - 6*a*b^6*c + 11*a
^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^
2*c^12 - 64*a^3*c^13)))/(b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)))*sqrt(-(b^5 -
 5*a*b^3*c + 5*a^2*b*c^2 + (b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)*sqrt((b^8 -
 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^6*c^10 - 12*a*b^4
*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))/(b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^
7)) + (b^11 - 13*a*b^9*c + 63*a^2*b^7*c^2 - 138*a^3*b^5*c^3 + 128*a^4*b^3*c
^4 - 32*a^5*b*c^5 - (b^10*c^5 - 16*a*b^8*c^6 + 98*a^2*b^6*c^7 - 280*a^3*b^4
*c^8 + 352*a^4*b^2*c^9 - 128*a^5*c^10)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c
^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 -
 64*a^3*c^13)))*sqrt(sqrt(1/2)*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 + (b^4*
c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*
a^3*b^2*c^3 + a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3
*c^13)))/(b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)))*sqrt(-(b^5 - 5*a*b^3*c + 5*
a^2*b*c^2 + (b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)*sqrt((b^8 - 6*a*b^6*c + 11
*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*
b^2*c^12 - 64*a^3*c^13)))/(b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7))*sqrt((2*(a^
2*b^4 - 3*a^3*b^2*c + a^4*c^2)*x^2 + sqrt(1/2)*(b^8 - 9*a*b^6*c + 27*a^2*b^
4*c^2 - 30*a^3*b^2*c^3 + 8*a^4*c^4 - (b^7*c^5 - 12*a*b^5*c^6 + 48*a^2*b^3*c
^7 - 64*a^3*b*c^8)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 +
 a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))*sqrt
(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 + (b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)*sqr
t((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^6*c^10 -
```

```
12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))/(b^4*c^5 - 8*a*b^2*c^6 + 1
6*a^2*c^7)))/(a^2*b^4 - 3*a^3*b^2*c + a^4*c^2)))/(a^4*b^4 - 3*a^5*b^2*c + a
^6*c^2)) - c*sqrt(sqrt(1/2)*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 + (b^4*c^5
 - 8*a*b^2*c^6 + 16*a^2*c^7)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3
*b^2*c^3 + a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^
13)))/(b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)))*log((a*b^4 - 3*a^2*b^2*c + a^3
*c^2)*x + 1/2*(b^6 - 7*a*b^4*c + 13*a^2*b^2*c^2 - 4*a^3*c^3 - (b^5*c^5 - 8*
a*b^3*c^6 + 16*a^2*b*c^7)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^
2*c^3 + a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)
))*sqrt(sqrt(1/2)*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 + (b^4*c^5 - 8*a*b^2
*c^6 + 16*a^2*c^7)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 +
 a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))/(b^4
*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)))) + c*sqrt(sqrt(1/2)*sqrt(-(b^5 - 5*a*b^3
*c + 5*a^2*b*c^2 + (b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)*sqrt((b^8 - 6*a*b^6
*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^11 +
48*a^2*b^2*c^12 - 64*a^3*c^13)))/(b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)))*log
((a*b^4 - 3*a^2*b^2*c + a^3*c^2)*x - 1/2*(b^6 - 7*a*b^4*c + 13*a^2*b^2*c^2
- 4*a^3*c^3 - (b^5*c^5 - 8*a*b^3*c^6 + 16*a^2*b*c^7)*sqrt((b^8 - 6*a*b^6*c
+ 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*
a^2*b^2*c^12 - 64*a^3*c^13)))*sqrt(sqrt(1/2)*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2
*b*c^2 + (b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)*sqrt((b^8 - 6*a*b^6*c + 11*a^
2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2
*c^12 - 64*a^3*c^13)))/(b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)))) - c*sqrt(sqr
t(1/2)*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 - (b^4*c^5 - 8*a*b^2*c^6 + 16*a
^2*c^7)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(
b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))/(b^4*c^5 - 8*a*
b^2*c^6 + 16*a^2*c^7)))*log((a*b^4 - 3*a^2*b^2*c + a^3*c^2)*x + 1/2*(b^6 -
7*a*b^4*c + 13*a^2*b^2*c^2 - 4*a^3*c^3 + (b^5*c^5 - 8*a*b^3*c^6 + 16*a^2*b*
c^7)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^6
*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))*sqrt(sqrt(1/2)*sqr
t(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 - (b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)*sq
rt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^6*c^10 -
 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))/(b^4*c^5 - 8*a*b^2*c^6 +
16*a^2*c^7)))) + c*sqrt(sqrt(1/2)*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 - (b
^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 -
 6*a^3*b^2*c^3 + a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*
a^3*c^13)))/(b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)))*log((a*b^4 - 3*a^2*b^2*c
 + a^3*c^2)*x - 1/2*(b^6 - 7*a*b^4*c + 13*a^2*b^2*c^2 - 4*a^3*c^3 + (b^5*c^
5 - 8*a*b^3*c^6 + 16*a^2*b*c^7)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*
a^3*b^2*c^3 + a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3
*c^13)))*sqrt(sqrt(1/2)*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 - (b^4*c^5 - 8
*a*b^2*c^6 + 16*a^2*c^7)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2
*c^3 + a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13))
)/(b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)))) - 4*x)/c
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(c*x^8+b*x^4+a),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Evaluation time: 14.73Unable to convert to r
eal 1/4 Error: Bad Argument Value
```

maple [C] time = 0.01, size = 59, normalized size = 0.16

$$\frac{x}{c}+\frac{\left(-\mathrm{RootOf}\left(\_Z^{8}c+b\_Z^{4}+a\right)^{4}b-a\right)\ln\left(-\mathrm{RootOf}\left(\_Z^{8}c+b\_Z^{4}+a\right)+x\right)}{4c\left(2\,\mathrm{RootOf}\left(\_Z^{8}c+b\_Z^{4}+a\right)^{7}c+\mathrm{RootOf}\left(\_Z^{8}c+b\_Z^{4}+a\right)^{3}b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(c*x^8+b*x^4+a),x)
```

```
[Out] 1/c*x+1/4/c*sum((-_R^4*b-a)/(2*_R^7*c+_R^3*b)*ln(-_R+x),_R=RootOf(_Z^8*c+_Z
^4*b+a))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{x}{c}-\frac{\int\frac{bx^4+a}{cx^8+bx^4+a}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(c*x^8+b*x^4+a),x, algorithm="maxima")
```

```
[Out] x/c - integrate((b*x^4 + a)/(c*x^8 + b*x^4 + a), x)/c
```

mupad [B] time = 3.97, size = 10382, normalized size = 27.61

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(a + b*x^4 + c*x^8),x)
```

```
[Out] atan(((((16*(a^3*b^6 - 4*a^6*c^3 - 7*a^4*b^4*c + 13*a^5*b^2*c^2))/c - (4*x*
(-(b^9 + b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120
*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c - 3*a*b^2*c*(-
(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2
*b^4*c^7 - 256*a^3*b^2*c^8)))^(3/4)*(4096*a^5*b*c^6 + 256*a^3*b^5*c^4 - 204
8*a^4*b^3*c^5))/c)*(-(b^9 + b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 6
1*a^2*b^5*c^2 - 120*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b
^7*c - 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16
*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4) - (4*x*(a^4*b^4 + 2*
a^6*c^2 - 4*a^5*b^2*c))/c)*(-(b^9 + b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b
*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^(1/2)
- 13*a*b^7*c - 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*
c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4)*1i - (((16*(
a^3*b^6 - 4*a^6*c^3 - 7*a^4*b^4*c + 13*a^5*b^2*c^2))/c + (4*x*(-(b^9 + b^4*
(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3
+ a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c - 3*a*b^2*c*(-(4*a*c - b^2)
^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 25
6*a^3*b^2*c^8)))^(3/4)*(4096*a^5*b*c^6 + 256*a^3*b^5*c^4 - 2048*a^4*b^3*c^5
))/c)*(-(b^9 + b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2
 - 120*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c - 3*a*b^
2*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 +
96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4) + (4*x*(a^4*b^4 + 2*a^6*c^2 - 4*a
^5*b^2*c))/c)*(-(b^9 + b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2
*b^5*c^2 - 120*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c
- 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^
6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4)*1i)/((((16*(a^3*b^6 - 4*a
^6*c^3 - 7*a^4*b^4*c + 13*a^5*b^2*c^2))/c - (4*x*(-(b^9 + b^4*(-(4*a*c - b^
2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 + a^2*c^2*(-(
4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c - 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(5
12*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8
```

```
)))^(3/4)*(4096*a^5*b*c^6 + 256*a^3*b^5*c^4 - 2048*a^4*b^3*c^5))/c)*(-(b^9
+ b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^
3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c - 3*a*b^2*c*(-(4*a*c
- b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^
7 - 256*a^3*b^2*c^8)))^(1/4) - (4*x*(a^4*b^4 + 2*a^6*c^2 - 4*a^5*b^2*c))/c)
*(-(b^9 + b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 12
0*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c - 3*a*b^2*c*(
-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^
2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4) + (((16*(a^3*b^6 - 4*a^6*c^3 - 7*a^4*b
^4*c + 13*a^5*b^2*c^2))/c + (4*x*(-(b^9 + b^4*(-(4*a*c - b^2)^5)^(1/2) + 80
*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^
(1/2) - 13*a*b^7*c - 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9
+ b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(3/4)*(4096*
a^5*b*c^6 + 256*a^3*b^5*c^4 - 2048*a^4*b^3*c^5))/c)*(-(b^9 + b^4*(-(4*a*c -
 b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 + a^2*c^2*
(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c - 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))
/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*
c^8)))^(1/4) + (4*x*(a^4*b^4 + 2*a^6*c^2 - 4*a^5*b^2*c))/c)*(-(b^9 + b^4*(-
(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 +
a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c - 3*a*b^2*c*(-(4*a*c - b^2)^5
)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*
a^3*b^2*c^8)))^(1/4)))*(-(b^9 + b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4
 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13
*a*b^7*c - 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5
- 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4)*2i + atan((((16
*(a^3*b^6 - 4*a^6*c^3 - 7*a^4*b^4*c + 13*a^5*b^2*c^2))/c - (4*x*(-(b^9 - b^
4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^
3 - a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c + 3*a*b^2*c*(-(4*a*c - b^
2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 -
256*a^3*b^2*c^8)))^(3/4)*(4096*a^5*b*c^6 + 256*a^3*b^5*c^4 - 2048*a^4*b^3*c
^5))/c)*(-(b^9 - b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c
^2 - 120*a^3*b^3*c^3 - a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c + 3*a*
b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6
+ 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4) - (4*x*(a^4*b^4 + 2*a^6*c^2 - 4
*a^5*b^2*c))/c)*(-(b^9 - b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a
^2*b^5*c^2 - 120*a^3*b^3*c^3 - a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*
c + 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*
b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4)*1i - (((16*(a^3*b^6 - 4
*a^6*c^3 - 7*a^4*b^4*c + 13*a^5*b^2*c^2))/c + (4*x*(-(b^9 - b^4*(-(4*a*c -
b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 - a^2*c^2*(
-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c + 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/
(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c
^8)))^(3/4)*(4096*a^5*b*c^6 + 256*a^3*b^5*c^4 - 2048*a^4*b^3*c^5))/c)*(-(b^
9 - b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*
b^3*c^3 - a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c + 3*a*b^2*c*(-(4*a*
c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*
c^7 - 256*a^3*b^2*c^8)))^(1/4) + (4*x*(a^4*b^4 + 2*a^6*c^2 - 4*a^5*b^2*c))/
c)*(-(b^9 - b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 -
120*a^3*b^3*c^3 - a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c + 3*a*b^2*c
*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*
a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4)*1i)/((((16*(a^3*b^6 - 4*a^6*c^3 - 7*
a^4*b^4*c + 13*a^5*b^2*c^2))/c - (4*x*(-(b^9 - b^4*(-(4*a*c - b^2)^5)^(1/2)
 + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 - a^2*c^2*(-(4*a*c - b^2
)^5)^(1/2) - 13*a*b^7*c + 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4
*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(3/4)*(
4096*a^5*b*c^6 + 256*a^3*b^5*c^4 - 2048*a^4*b^3*c^5))/c)*(-(b^9 - b^4*(-(4*
a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 - a^2
*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c + 3*a*b^2*c*(-(4*a*c - b^2)^5)^(
1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3
```

```
*b^2*c^8)))^(1/4) - (4*x*(a^4*b^4 + 2*a^6*c^2 - 4*a^5*b^2*c))/c)*(-(b^9 - b
^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c
^3 - a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c + 3*a*b^2*c*(-(4*a*c - b
^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 -
 256*a^3*b^2*c^8)))^(1/4) + (((16*(a^3*b^6 - 4*a^6*c^3 - 7*a^4*b^4*c + 13*a
^5*b^2*c^2))/c + (4*x*(-(b^9 - b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4
+ 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 - a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*
a*b^7*c + 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 -
 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(3/4)*(4096*a^5*b*c^6 +
 256*a^3*b^5*c^4 - 2048*a^4*b^3*c^5))/c)*(-(b^9 - b^4*(-(4*a*c - b^2)^5)^(1
/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 - a^2*c^2*(-(4*a*c -
b^2)^5)^(1/2) - 13*a*b^7*c + 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*
a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4
) + (4*x*(a^4*b^4 + 2*a^6*c^2 - 4*a^5*b^2*c))/c)*(-(b^9 - b^4*(-(4*a*c - b^
2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 - a^2*c^2*(-(
4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c + 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(5
12*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8
)))^(1/4)))*(-(b^9 - b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b
^5*c^2 - 120*a^3*b^3*c^3 - a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c +
3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*
c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4)*2i - 2*atan(((((16*(a^3*b^6
 - 4*a^6*c^3 - 7*a^4*b^4*c + 13*a^5*b^2*c^2))/c - (x*(-(b^9 + b^4*(-(4*a*c
- b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 + a^2*c^2
*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c - 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2)
)/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2
*c^8)))^(3/4)*(4096*a^5*b*c^6 + 256*a^3*b^5*c^4 - 2048*a^4*b^3*c^5)*4i)/c)*
(-(b^9 + b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120
*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c - 3*a*b^2*c*(-
(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2
*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4)*1i + (4*x*(a^4*b^4 + 2*a^6*c^2 - 4*a^5*
b^2*c))/c)*(-(b^9 + b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^
5*c^2 - 120*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c - 3
*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c
^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4) - (((16*(a^3*b^6 - 4*a^6*c^3
 - 7*a^4*b^4*c + 13*a^5*b^2*c^2))/c + (x*(-(b^9 + b^4*(-(4*a*c - b^2)^5)^(1
/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c -
b^2)^5)^(1/2) - 13*a*b^7*c - 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*
a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(3/4
)*(4096*a^5*b*c^6 + 256*a^3*b^5*c^4 - 2048*a^4*b^3*c^5)*4i)/c)*(-(b^9 + b^4
*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3
 + a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c - 3*a*b^2*c*(-(4*a*c - b^2
)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 2
56*a^3*b^2*c^8)))^(1/4)*1i - (4*x*(a^4*b^4 + 2*a^6*c^2 - 4*a^5*b^2*c))/c)*(
-(b^9 + b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*
a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c - 3*a*b^2*c*(-(
4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*
b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4))/((((16*(a^3*b^6 - 4*a^6*c^3 - 7*a^4*b^4
*c + 13*a^5*b^2*c^2))/c - (x*(-(b^9 + b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4
*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^(1/2
) - 13*a*b^7*c - 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^
8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(3/4)*(4096*a^5*
b*c^6 + 256*a^3*b^5*c^4 - 2048*a^4*b^3*c^5)*4i)/c)*(-(b^9 + b^4*(-(4*a*c -
b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 + a^2*c^2*(
-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c - 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/
(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c
^8)))^(1/4)*1i + (4*x*(a^4*b^4 + 2*a^6*c^2 - 4*a^5*b^2*c))/c)*(-(b^9 + b^4*
(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3
+ a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c - 3*a*b^2*c*(-(4*a*c - b^2)
^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 25
```

```
6*a^3*b^2*c^8)))^(1/4)*1i + (((16*(a^3*b^6 - 4*a^6*c^3 - 7*a^4*b^4*c + 13*a
^5*b^2*c^2))/c + (x*(-(b^9 + b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 +
61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*
b^7*c - 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 1
6*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(3/4)*(4096*a^5*b*c^6 + 2
56*a^3*b^5*c^4 - 2048*a^4*b^3*c^5)*4i)/c)*(-(b^9 + b^4*(-(4*a*c - b^2)^5)^(
1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c -
 b^2)^5)^(1/2) - 13*a*b^7*c - 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256
*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/
4)*1i - (4*x*(a^4*b^4 + 2*a^6*c^2 - 4*a^5*b^2*c))/c)*(-(b^9 + b^4*(-(4*a*c
- b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 + a^2*c^2
*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c - 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2)
)/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2
*c^8)))^(1/4)*1i))*(-(b^9 + b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 6
1*a^2*b^5*c^2 - 120*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b
^7*c - 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16
*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4) - 2*atan(((((16*(a^3
*b^6 - 4*a^6*c^3 - 7*a^4*b^4*c + 13*a^5*b^2*c^2))/c - (x*(-(b^9 - b^4*(-(4*
a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 - a^2
*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c + 3*a*b^2*c*(-(4*a*c - b^2)^5)^(
1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3
*b^2*c^8)))^(3/4)*(4096*a^5*b*c^6 + 256*a^3*b^5*c^4 - 2048*a^4*b^3*c^5)*4i)
/c)*(-(b^9 - b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 -
 120*a^3*b^3*c^3 - a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c + 3*a*b^2*
c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96
*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4)*1i + (4*x*(a^4*b^4 + 2*a^6*c^2 - 4*
a^5*b^2*c))/c)*(-(b^9 - b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^
2*b^5*c^2 - 120*a^3*b^3*c^3 - a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c
 + 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b
^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4) - (((16*(a^3*b^6 - 4*a^6
*c^3 - 7*a^4*b^4*c + 13*a^5*b^2*c^2))/c + (x*(-(b^9 - b^4*(-(4*a*c - b^2)^5
)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 - a^2*c^2*(-(4*a*
c - b^2)^5)^(1/2) - 13*a*b^7*c + 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(
256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^
(3/4)*(4096*a^5*b*c^6 + 256*a^3*b^5*c^4 - 2048*a^4*b^3*c^5)*4i)/c)*(-(b^9 -
 b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3
*c^3 - a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c + 3*a*b^2*c*(-(4*a*c -
 b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7
 - 256*a^3*b^2*c^8)))^(1/4)*1i - (4*x*(a^4*b^4 + 2*a^6*c^2 - 4*a^5*b^2*c))/
c)*(-(b^9 - b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 -
120*a^3*b^3*c^3 - a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c + 3*a*b^2*c
*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*
a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4))/((((16*(a^3*b^6 - 4*a^6*c^3 - 7*a^4
*b^4*c + 13*a^5*b^2*c^2))/c - (x*(-(b^9 - b^4*(-(4*a*c - b^2)^5)^(1/2) + 80
*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 - a^2*c^2*(-(4*a*c - b^2)^5)^
(1/2) - 13*a*b^7*c + 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9
+ b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(3/4)*(4096*
a^5*b*c^6 + 256*a^3*b^5*c^4 - 2048*a^4*b^3*c^5)*4i)/c)*(-(b^9 - b^4*(-(4*a*
c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 - a^2*c
^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c + 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/
2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b
^2*c^8)))^(1/4)*1i + (4*x*(a^4*b^4 + 2*a^6*c^2 - 4*a^5*b^2*c))/c)*(-(b^9 -
b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*
c^3 - a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c + 3*a*b^2*c*(-(4*a*c -
b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7
- 256*a^3*b^2*c^8)))^(1/4)*1i + (((16*(a^3*b^6 - 4*a^6*c^3 - 7*a^4*b^4*c +
13*a^5*b^2*c^2))/c + (x*(-(b^9 - b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^
4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 - a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 1
3*a*b^7*c + 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5
```

```
 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(3/4)*(4096*a^5*b*c^6
 + 256*a^3*b^5*c^4 - 2048*a^4*b^3*c^5)*4i)/c)*(-(b^9 - b^4*(-(4*a*c - b^2)^
5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 - a^2*c^2*(-(4*a
*c - b^2)^5)^(1/2) - 13*a*b^7*c + 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(512*
(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))
^(1/4)*1i - (4*x*(a^4*b^4 + 2*a^6*c^2 - 4*a^5*b^2*c))/c)*(-(b^9 - b^4*(-(4*
a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 - a^2
*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c + 3*a*b^2*c*(-(4*a*c - b^2)^5)^(
1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3
*b^2*c^8)))^(1/4)*1i))*(-(b^9 - b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4
 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 - a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13
*a*b^7*c + 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5
- 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4) + x/c
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8/(c*x**8+b*x**4+a),x)
```

```
[Out] Timed out
```

3.264 $\int \frac{x^6}{a+bx^4+cx^8}\,dx$

Optimal. Leaf size=325

$$-\frac{\left(-\sqrt{b^2-4ac}-b\right)^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\,x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2\,2^{3/4}c^{3/4}\sqrt{b^2-4ac}}+\frac{\left(\sqrt{b^2-4ac}-b\right)^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\,x}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{2\,2^{3/4}c^{3/4}\sqrt{b^2-4ac}}+\frac{\left(-\sqrt{b^2-4ac}-b\right)^{3/4}\,\mathrm{ta}}{2\,2^{3/4}c^{3/4}\sqrt{}}$$

Rubi [A] time = 0.31, antiderivative size = 325, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {1374, 298, 205, 208}

$$-\frac{\left(-\sqrt{b^2-4ac}-b\right)^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\,x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2\,2^{3/4}c^{3/4}\sqrt{b^2-4ac}}+\frac{\left(\sqrt{b^2-4ac}-b\right)^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\,x}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{2\,2^{3/4}c^{3/4}\sqrt{b^2-4ac}}+\frac{\left(-\sqrt{b^2-4ac}-b\right)^{3/4}\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\,x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2\,2^{3/4}c^{3/4}\sqrt{b^2-4ac}}-\frac{\left(\sqrt{b^2-4ac}-b\right)^{3/4}\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\,x}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{2\,2^{3/4}c^{3/4}\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In] Int[x^6/(a + b*x^4 + c*x^8),x]
```

```
[Out] -((-b - Sqrt[b^2 - 4*a*c])^(3/4)*ArcTan[(2^(1/4)*c^(1/4)*x)/(-b - Sqrt[b^2
- 4*a*c])^(1/4)])/(2*2^(3/4)*c^(3/4)*Sqrt[b^2 - 4*a*c]) + ((-b + Sqrt[b^2 -
 4*a*c])^(3/4)*ArcTan[(2^(1/4)*c^(1/4)*x)/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/
(2*2^(3/4)*c^(3/4)*Sqrt[b^2 - 4*a*c]) + ((-b - Sqrt[b^2 - 4*a*c])^(3/4)*Arc
Tanh[(2^(1/4)*c^(1/4)*x)/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(2*2^(3/4)*c^(3/4
)*Sqrt[b^2 - 4*a*c]) - ((-b + Sqrt[b^2 - 4*a*c])^(3/4)*ArcTanh[(2^(1/4)*c^(
1/4)*x)/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(2*2^(3/4)*c^(3/4)*Sqrt[b^2 - 4*a*
c])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 1374

```
Int[((d_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(d^n*(b/q + 1))/2, Int[(d*x)^(m -
 n)/(b/2 + q/2 + c*x^n), x], x] - Dist[(d^n*(b/q - 1))/2, Int[(d*x)^(m - n)
/(b/2 - q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[n2, 2*n] &&
NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GeQ[m, n]
```

Rubi steps

$$\int \frac{x^6}{a+bx^4+cx^8}\,dx = -\left(\frac{1}{2}\left(-1+\frac{b}{\sqrt{b^2-4ac}}\right)\int \frac{x^2}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^4}\,dx\right)+\frac{1}{2}\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)\int \frac{x}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-}}$$

$$= -\frac{\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\sqrt{-b+\sqrt{b^2-4ac}}-\sqrt{2}\sqrt{c}x^2}\,dx}{2\sqrt{2}\sqrt{c}}+\frac{\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\sqrt{-b+\sqrt{b^2-4ac}}+\sqrt{2}\sqrt{c}x^2}\,dx}{2\sqrt{2}\sqrt{c}}-\frac{(1}{}$$

$$= -\frac{\left(-b-\sqrt{b^2-4ac}\right)^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{2\,2^{3/4}c^{3/4}\sqrt{b^2-4ac}}+\frac{\left(-b+\sqrt{b^2-4ac}\right)^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b+\sqrt{b^2-4ac}}}\right)}{2\,2^{3/4}c^{3/4}\sqrt{b^2-4ac}}$$

Mathematica [C] time = 0.02, size = 44, normalized size = 0.14

$$\frac{1}{4}\text{RootSum}\left[\#1^8c+\#1^4b+a\&,\frac{\#1^3\log(x-\#1)}{2\#1^4c+b}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6/(a + b*x^4 + c*x^8),x]
[Out] RootSum[a + b*#1^4 + c*#1^8 & , (Log[x - #1]*#1^3)/(b + 2*c*#1^4) & ]/4
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6}{a+bx^4+cx^8}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^6/(a + b*x^4 + c*x^8),x]
[Out] IntegrateAlgebraic[x^6/(a + b*x^4 + c*x^8), x]
```

fricas [B] time = 1.93, size = 3912, normalized size = 12.04

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(c*x^8+b*x^4+a),x, algorithm="fricas")
[Out] -sqrt(sqrt(1/2)*sqrt(-(b^3 - 3*a*b*c - (b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)
*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^6*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8
- 64*a^3*c^9)))/(b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)))*arctan(1/2*((b^5*c^3
 - 8*a*b^3*c^4 + 16*a^2*b*c^5)*x*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^6*c^6
- 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64*a^3*c^9)) + (b^4 - 5*a*b^2*c + 4*a^2*c
^2)*x - sqrt(1/2)*(b^4 - 5*a*b^2*c + 4*a^2*c^2 + (b^5*c^3 - 8*a*b^3*c^4 + 1
6*a^2*b*c^5)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^6*c^6 - 12*a*b^4*c^7 + 48*
a^2*b^2*c^8 - 64*a^3*c^9)))*sqrt((2*(a*b^2 - a^2*c)*x^2 - sqrt(1/2)*(b^5 -
5*a*b^3*c + 4*a^2*b*c^2 + (b^6*c^3 - 12*a*b^4*c^4 + 48*a^2*b^2*c^5 - 64*a^3
*c^6)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^6*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2
*c^8 - 64*a^3*c^9)))*sqrt(-(b^3 - 3*a*b*c - (b^4*c^3 - 8*a*b^2*c^4 + 16*a^2
*c^5)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^6*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2
*c^8 - 64*a^3*c^9)))/(b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)))/(a*b^2 - a^2*c)
))*sqrt(sqrt(1/2)*sqrt(-(b^3 - 3*a*b*c - (b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^
5)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^6*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^
8 - 64*a^3*c^9)))/(b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)))/(a*b^2 - a^2*c)) +
 sqrt(sqrt(1/2)*sqrt(-(b^3 - 3*a*b*c + (b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)
*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^6*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8
```

```
- 64*a^3*c^9)))/(b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)))*arctan(1/2*(sqrt(1/2
)*(b^4 - 5*a*b^2*c + 4*a^2*c^2 - (b^5*c^3 - 8*a*b^3*c^4 + 16*a^2*b*c^5)*sqr
t((b^4 - 2*a*b^2*c + a^2*c^2)/(b^6*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64
*a^3*c^9)))*sqrt(sqrt(1/2)*sqrt(-(b^3 - 3*a*b*c + (b^4*c^3 - 8*a*b^2*c^4 +
16*a^2*c^5)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^6*c^6 - 12*a*b^4*c^7 + 48*a
^2*b^2*c^8 - 64*a^3*c^9)))/(b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)))*sqrt((2*(
a*b^2 - a^2*c)*x^2 - sqrt(1/2)*(b^5 - 5*a*b^3*c + 4*a^2*b*c^2 - (b^6*c^3 -
12*a*b^4*c^4 + 48*a^2*b^2*c^5 - 64*a^3*c^6)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2
)/(b^6*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64*a^3*c^9)))*sqrt(-(b^3 - 3*a
*b*c + (b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2
)/(b^6*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64*a^3*c^9)))/(b^4*c^3 - 8*a*b
^2*c^4 + 16*a^2*c^5)))/(a*b^2 - a^2*c)) + ((b^5*c^3 - 8*a*b^3*c^4 + 16*a^2*
b*c^5)*x*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^6*c^6 - 12*a*b^4*c^7 + 48*a^2*
b^2*c^8 - 64*a^3*c^9)) - (b^4 - 5*a*b^2*c + 4*a^2*c^2)*x)*sqrt(sqrt(1/2)*sq
rt(-(b^3 - 3*a*b*c + (b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)*sqrt((b^4 - 2*a*b
^2*c + a^2*c^2)/(b^6*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64*a^3*c^9)))/(b
^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5))))/(a*b^2 - a^2*c)) + 1/4*sqrt(sqrt(1/2)
*sqrt(-(b^3 - 3*a*b*c + (b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)*sqrt((b^4 - 2*
a*b^2*c + a^2*c^2)/(b^6*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64*a^3*c^9)))
/(b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)))*log(1/2*sqrt(1/2)*(b^7 - 9*a*b^5*c
+ 24*a^2*b^3*c^2 - 16*a^3*b*c^3 - (b^8*c^3 - 14*a*b^6*c^4 + 72*a^2*b^4*c^5
- 160*a^3*b^2*c^6 + 128*a^4*c^7)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^6*c^6
- 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64*a^3*c^9)))*sqrt(sqrt(1/2)*sqrt(-(b^3 -
 3*a*b*c + (b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)*sqrt((b^4 - 2*a*b^2*c + a^2
*c^2)/(b^6*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64*a^3*c^9)))/(b^4*c^3 - 8
*a*b^2*c^4 + 16*a^2*c^5)))*sqrt(-(b^3 - 3*a*b*c + (b^4*c^3 - 8*a*b^2*c^4 +
16*a^2*c^5)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^6*c^6 - 12*a*b^4*c^7 + 48*a
^2*b^2*c^8 - 64*a^3*c^9)))/(b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)) - (a^2*b^2
 - a^3*c)*x) - 1/4*sqrt(sqrt(1/2)*sqrt(-(b^3 - 3*a*b*c + (b^4*c^3 - 8*a*b^2
*c^4 + 16*a^2*c^5)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^6*c^6 - 12*a*b^4*c^7
 + 48*a^2*b^2*c^8 - 64*a^3*c^9)))/(b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)))*lo
g(-1/2*sqrt(1/2)*(b^7 - 9*a*b^5*c + 24*a^2*b^3*c^2 - 16*a^3*b*c^3 - (b^8*c^
3 - 14*a*b^6*c^4 + 72*a^2*b^4*c^5 - 160*a^3*b^2*c^6 + 128*a^4*c^7)*sqrt((b^
4 - 2*a*b^2*c + a^2*c^2)/(b^6*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64*a^3*
c^9)))*sqrt(sqrt(1/2)*sqrt(-(b^3 - 3*a*b*c + (b^4*c^3 - 8*a*b^2*c^4 + 16*a^
2*c^5)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^6*c^6 - 12*a*b^4*c^7 + 48*a^2*b^
2*c^8 - 64*a^3*c^9)))/(b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)))*sqrt(-(b^3 - 3
*a*b*c + (b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)*sqrt((b^4 - 2*a*b^2*c + a^2*c
^2)/(b^6*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64*a^3*c^9)))/(b^4*c^3 - 8*a
*b^2*c^4 + 16*a^2*c^5)) - (a^2*b^2 - a^3*c)*x) + 1/4*sqrt(sqrt(1/2)*sqrt(-(
b^3 - 3*a*b*c - (b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)*sqrt((b^4 - 2*a*b^2*c
+ a^2*c^2)/(b^6*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64*a^3*c^9)))/(b^4*c^
3 - 8*a*b^2*c^4 + 16*a^2*c^5)))*log(1/2*sqrt(1/2)*(b^7 - 9*a*b^5*c + 24*a^2
*b^3*c^2 - 16*a^3*b*c^3 + (b^8*c^3 - 14*a*b^6*c^4 + 72*a^2*b^4*c^5 - 160*a^
3*b^2*c^6 + 128*a^4*c^7)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^6*c^6 - 12*a*b
^4*c^7 + 48*a^2*b^2*c^8 - 64*a^3*c^9)))*sqrt(sqrt(1/2)*sqrt(-(b^3 - 3*a*b*c
 - (b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b
^6*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64*a^3*c^9)))/(b^4*c^3 - 8*a*b^2*c
^4 + 16*a^2*c^5)))*sqrt(-(b^3 - 3*a*b*c - (b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c
^5)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^6*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c
^8 - 64*a^3*c^9)))/(b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)) - (a^2*b^2 - a^3*c
)*x) - 1/4*sqrt(sqrt(1/2)*sqrt(-(b^3 - 3*a*b*c - (b^4*c^3 - 8*a*b^2*c^4 + 1
6*a^2*c^5)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^6*c^6 - 12*a*b^4*c^7 + 48*a^
2*b^2*c^8 - 64*a^3*c^9)))/(b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)))*log(-1/2*s
qrt(1/2)*(b^7 - 9*a*b^5*c + 24*a^2*b^3*c^2 - 16*a^3*b*c^3 + (b^8*c^3 - 14*a
*b^6*c^4 + 72*a^2*b^4*c^5 - 160*a^3*b^2*c^6 + 128*a^4*c^7)*sqrt((b^4 - 2*a*
b^2*c + a^2*c^2)/(b^6*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64*a^3*c^9)))*s
qrt(sqrt(1/2)*sqrt(-(b^3 - 3*a*b*c - (b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)*s
qrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^6*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 -
```

```
64*a^3*c^9)))/(b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)))*sqrt(-(b^3 - 3*a*b*c -
 (b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^6
*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64*a^3*c^9)))/(b^4*c^3 - 8*a*b^2*c^4
 + 16*a^2*c^5)) - (a^2*b^2 - a^3*c)*x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6}{cx^8 + bx^4 + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(c*x^8+b*x^4+a),x, algorithm="giac")
[Out] integrate(x^6/(c*x^8 + b*x^4 + a), x)
```

maple [C] time = 0.00, size = 43, normalized size = 0.13

$$\frac{\mathrm{RootOf}\left(\_Z^8 c + b\_Z^4 + a\right)^6 \ln\left(-\mathrm{RootOf}\left(\_Z^8 c + b\_Z^4 + a\right) + x\right)}{8\,\mathrm{RootOf}\left(\_Z^8 c + b\_Z^4 + a\right)^7 c + 4\,\mathrm{RootOf}\left(\_Z^8 c + b\_Z^4 + a\right)^3 b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(c*x^8+b*x^4+a),x)
[Out] 1/4*sum(_R^6/(2*_R^7*c+_R^3*b)*ln(-_R+x),_R=RootOf(_Z^8*c+_Z^4*b+a))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6}{cx^8 + bx^4 + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(c*x^8+b*x^4+a),x, algorithm="maxima")
[Out] integrate(x^6/(c*x^8 + b*x^4 + a), x)
```

mupad [B] time = 3.51, size = 8033, normalized size = 24.72

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(a + b*x^4 + c*x^8),x)
[Out] atan((((-(b^7 + b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^
2 - 11*a*b^5*c - a*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^7 + b^8*c^3
- 16*a*b^6*c^4 + 96*a^2*b^4*c^5 - 256*a^3*b^2*c^6)))^(3/4)*(4096*a^5*c^5 +
256*a^3*b^4*c^3 - 2048*a^4*b^2*c^4 + x*(-(b^7 + b^2*(-(4*a*c - b^2)^5)^(1/2
) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c - a*c*(-(4*a*c - b^2)^5)^(1/
2))/(512*(256*a^4*c^7 + b^8*c^3 - 16*a*b^6*c^4 + 96*a^2*b^4*c^5 - 256*a^3*b
^2*c^6)))^(1/4)*(32768*a^5*c^6 + 2048*a^3*b^4*c^4 - 16384*a^4*b^2*c^5)) + x
*(4*a^3*b^3*c - 12*a^4*b*c^2))*(-(b^7 + b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a
^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c - a*c*(-(4*a*c - b^2)^5)^(1/2))/(512
*(256*a^4*c^7 + b^8*c^3 - 16*a*b^6*c^4 + 96*a^2*b^4*c^5 - 256*a^3*b^2*c^6))
)^(1/4)*1i - ((-(b^7 + b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2
*b^3*c^2 - 11*a*b^5*c - a*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^7 + b
^8*c^3 - 16*a*b^6*c^4 + 96*a^2*b^4*c^5 - 256*a^3*b^2*c^6)))^(3/4)*(4096*a^5
*c^5 + 256*a^3*b^4*c^3 - 2048*a^4*b^2*c^4 - x*(-(b^7 + b^2*(-(4*a*c - b^2)^
5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c - a*c*(-(4*a*c - b^2)
^5)^(1/2))/(512*(256*a^4*c^7 + b^8*c^3 - 16*a*b^6*c^4 + 96*a^2*b^4*c^5 - 25
6*a^3*b^2*c^6)))^(1/4)*(32768*a^5*c^6 + 2048*a^3*b^4*c^4 - 16384*a^4*b^2*c^
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5)) - x*(4*a^3*b^3*c - 12*a^4*b*c^2))*(-(b^7 + b^2*(-(4*a*c - b^2)^5)^(1/2)
 - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c - a*c*(-(4*a*c - b^2)^5)^(1/2
))/(512*(256*a^4*c^7 + b^8*c^3 - 16*a*b^6*c^4 + 96*a^2*b^4*c^5 - 256*a^3*b^
2*c^6)))^(1/4)*1i)/(((-(b^7 + b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 +
 40*a^2*b^3*c^2 - 11*a*b^5*c - a*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*
c^7 + b^8*c^3 - 16*a*b^6*c^4 + 96*a^2*b^4*c^5 - 256*a^3*b^2*c^6)))^(3/4)*(4
096*a^5*c^5 + 256*a^3*b^4*c^3 - 2048*a^4*b^2*c^4 + x*(-(b^7 + b^2*(-(4*a*c
- b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c - a*c*(-(4*a*c
 - b^2)^5)^(1/2))/(512*(256*a^4*c^7 + b^8*c^3 - 16*a*b^6*c^4 + 96*a^2*b^4*c
^5 - 256*a^3*b^2*c^6)))^(1/4)*(32768*a^5*c^6 + 2048*a^3*b^4*c^4 - 16384*a^4
*b^2*c^5)) + x*(4*a^3*b^3*c - 12*a^4*b*c^2))*(-(b^7 + b^2*(-(4*a*c - b^2)^5
)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c - a*c*(-(4*a*c - b^2)^
5)^(1/2))/(512*(256*a^4*c^7 + b^8*c^3 - 16*a*b^6*c^4 + 96*a^2*b^4*c^5 - 256
*a^3*b^2*c^6)))^(1/4) + ((-(b^7 + b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c
^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c - a*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*
a^4*c^7 + b^8*c^3 - 16*a*b^6*c^4 + 96*a^2*b^4*c^5 - 256*a^3*b^2*c^6)))^(3/4
)*(4096*a^5*c^5 + 256*a^3*b^4*c^3 - 2048*a^4*b^2*c^4 - x*(-(b^7 + b^2*(-(4*
a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c - a*c*(-(4
*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^7 + b^8*c^3 - 16*a*b^6*c^4 + 96*a^2*b
^4*c^5 - 256*a^3*b^2*c^6)))^(1/4)*(32768*a^5*c^6 + 2048*a^3*b^4*c^4 - 16384
*a^4*b^2*c^5)) - x*(4*a^3*b^3*c - 12*a^4*b*c^2))*(-(b^7 + b^2*(-(4*a*c - b^
2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c - a*c*(-(4*a*c - b
^2)^5)^(1/2))/(512*(256*a^4*c^7 + b^8*c^3 - 16*a*b^6*c^4 + 96*a^2*b^4*c^5 -
 256*a^3*b^2*c^6)))^(1/4) - 2*a^4*b*c))*(-(b^7 + b^2*(-(4*a*c - b^2)^5)^(1/
2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c - a*c*(-(4*a*c - b^2)^5)^(1
/2))/(512*(256*a^4*c^7 + b^8*c^3 - 16*a*b^6*c^4 + 96*a^2*b^4*c^5 - 256*a^3*
b^2*c^6)))^(1/4)*2i + atan((((-(b^7 - b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3
*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c + a*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(
256*a^4*c^7 + b^8*c^3 - 16*a*b^6*c^4 + 96*a^2*b^4*c^5 - 256*a^3*b^2*c^6)))^
(3/4)*(4096*a^5*c^5 + 256*a^3*b^4*c^3 - 2048*a^4*b^2*c^4 + x*(-(b^7 - b^2*(
-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c + a*c*
(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^7 + b^8*c^3 - 16*a*b^6*c^4 + 96*a
^2*b^4*c^5 - 256*a^3*b^2*c^6)))^(1/4)*(32768*a^5*c^6 + 2048*a^3*b^4*c^4 - 1
6384*a^4*b^2*c^5)) + x*(4*a^3*b^3*c - 12*a^4*b*c^2))*(-(b^7 - b^2*(-(4*a*c
- b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c + a*c*(-(4*a*c
 - b^2)^5)^(1/2))/(512*(256*a^4*c^7 + b^8*c^3 - 16*a*b^6*c^4 + 96*a^2*b^4*c
^5 - 256*a^3*b^2*c^6)))^(1/4)*1i - ((-(b^7 - b^2*(-(4*a*c - b^2)^5)^(1/2) -
 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c + a*c*(-(4*a*c - b^2)^5)^(1/2))
/(512*(256*a^4*c^7 + b^8*c^3 - 16*a*b^6*c^4 + 96*a^2*b^4*c^5 - 256*a^3*b^2*
c^6)))^(3/4)*(4096*a^5*c^5 + 256*a^3*b^4*c^3 - 2048*a^4*b^2*c^4 - x*(-(b^7
- b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c
 + a*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^7 + b^8*c^3 - 16*a*b^6*c^4
 + 96*a^2*b^4*c^5 - 256*a^3*b^2*c^6)))^(1/4)*(32768*a^5*c^6 + 2048*a^3*b^4*
c^4 - 16384*a^4*b^2*c^5)) - x*(4*a^3*b^3*c - 12*a^4*b*c^2))*(-(b^7 - b^2*(-
(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c + a*c*(
-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^7 + b^8*c^3 - 16*a*b^6*c^4 + 96*a^
2*b^4*c^5 - 256*a^3*b^2*c^6)))^(1/4)*1i)/(((-(b^7 - b^2*(-(4*a*c - b^2)^5)^
(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c + a*c*(-(4*a*c - b^2)^5)
^(1/2))/(512*(256*a^4*c^7 + b^8*c^3 - 16*a*b^6*c^4 + 96*a^2*b^4*c^5 - 256*a
^3*b^2*c^6)))^(3/4)*(4096*a^5*c^5 + 256*a^3*b^4*c^3 - 2048*a^4*b^2*c^4 + x*
(-(b^7 - b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*
a*b^5*c + a*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^7 + b^8*c^3 - 16*a*
b^6*c^4 + 96*a^2*b^4*c^5 - 256*a^3*b^2*c^6)))^(1/4)*(32768*a^5*c^6 + 2048*a
^3*b^4*c^4 - 16384*a^4*b^2*c^5)) + x*(4*a^3*b^3*c - 12*a^4*b*c^2))*(-(b^7 -
 b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c
+ a*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^7 + b^8*c^3 - 16*a*b^6*c^4
+ 96*a^2*b^4*c^5 - 256*a^3*b^2*c^6)))^(1/4) + ((-(b^7 - b^2*(-(4*a*c - b^2)
^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c + a*c*(-(4*a*c - b^2
)^5)^(1/2))/(512*(256*a^4*c^7 + b^8*c^3 - 16*a*b^6*c^4 + 96*a^2*b^4*c^5 - 2
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56*a^3*b^2*c^6)))^(3/4)*(4096*a^5*c^5 + 256*a^3*b^4*c^3 - 2048*a^4*b^2*c^4 - x*(-(b^7 - b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c + a*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^7 + b^8*c^3 - 16*a*b^6*c^4 + 96*a^2*b^4*c^5 - 256*a^3*b^2*c^6)))^(1/4)*(32768*a^5*c^6 + 2048*a^3*b^4*c^4 - 16384*a^4*b^2*c^5)) - x*(4*a^3*b^3*c - 12*a^4*b*c^2))*(-(b^7 - b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c + a*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^7 + b^8*c^3 - 16*a*b^6*c^4 + 96*a^2*b^4*c^5 - 256*a^3*b^2*c^6)))^(1/4) - 2*a^4*b*c))*(-(b^7 - b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c + a*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^7 + b^8*c^3 - 16*a*b^6*c^4 + 96*a^2*b^4*c^5 - 256*a^3*b^2*c^6)))^(1/4)*2i - 2*atan((((-(b^7 + b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c - a*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^7 + b^8*c^3 - 16*a*b^6*c^4 + 96*a^2*b^4*c^5 - 256*a^3*b^2*c^6)))^(3/4)*(4096*a^5*c^5 + 256*a^3*b^4*c^3 - 2048*a^4*b^2*c^4 - x*(-(b^7 + b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c - a*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^7 + b^8*c^3 - 16*a*b^6*c^4 + 96*a^2*b^4*c^5 - 256*a^3*b^2*c^6)))^(1/4)*(32768*a^5*c^6 + 2048*a^3*b^4*c^4 - 16384*a^4*b^2*c^5)*1i)*1i + x*(4*a^3*b^3*c - 12*a^4*b*c^2))*(-(b^7 + b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c - a*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^7 + b^8*c^3 - 16*a*b^6*c^4 + 96*a^2*b^4*c^5 - 256*a^3*b^2*c^6)))^(1/4) - ((-(b^7 + b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c - a*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^7 + b^8*c^3 - 16*a*b^6*c^4 + 96*a^2*b^4*c^5 - 256*a^3*b^2*c^6)))^(3/4)*(4096*a^5*c^5 + 256*a^3*b^4*c^3 - 2048*a^4*b^2*c^4 + x*(-(b^7 + b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c - a*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^7 + b^8*c^3 - 16*a*b^6*c^4 + 96*a^2*b^4*c^5 - 256*a^3*b^2*c^6)))^(1/4)*(32768*a^5*c^6 + 2048*a^3*b^4*c^4 - 16384*a^4*b^2*c^5)*1i)*1i - x*(4*a^3*b^3*c - 12*a^4*b*c^2))*(-(b^7 + b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c - a*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^7 + b^8*c^3 - 16*a*b^6*c^4 + 96*a^2*b^4*c^5 - 256*a^3*b^2*c^6)))^(1/4))/(((-(b^7 + b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c - a*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^7 + b^8*c^3 - 16*a*b^6*c^4 + 96*a^2*b^4*c^5 - 256*a^3*b^2*c^6)))^(3/4)*(4096*a^5*c^5 + 256*a^3*b^4*c^3 - 2048*a^4*b^2*c^4 - x*(-(b^7 + b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c - a*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^7 + b^8*c^3 - 16*a*b^6*c^4 + 96*a^2*b^4*c^5 - 256*a^3*b^2*c^6)))^(1/4)*(32768*a^5*c^6 + 2048*a^3*b^4*c^4 - 16384*a^4*b^2*c^5)*1i)*1i + x*(4*a^3*b^3*c - 12*a^4*b*c^2))*(-(b^7 + b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c - a*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^7 + b^8*c^3 - 16*a*b^6*c^4 + 96*a^2*b^4*c^5 - 256*a^3*b^2*c^6)))^(1/4)*1i + ((-(b^7 + b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c - a*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^7 + b^8*c^3 - 16*a*b^6*c^4 + 96*a^2*b^4*c^5 - 256*a^3*b^2*c^6)))^(3/4)*(4096*a^5*c^5 + 256*a^3*b^4*c^3 - 2048*a^4*b^2*c^4 + x*(-(b^7 + b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c - a*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^7 + b^8*c^3 - 16*a*b^6*c^4 + 96*a^2*b^4*c^5 - 256*a^3*b^2*c^6)))^(1/4)*(32768*a^5*c^6 + 2048*a^3*b^4*c^4 - 16384*a^4*b^2*c^5)*1i)*1i - x*(4*a^3*b^3*c - 12*a^4*b*c^2))*(-(b^7 + b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c - a*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^7 + b^8*c^3 - 16*a*b^6*c^4 + 96*a^2*b^4*c^5 - 256*a^3*b^2*c^6)))^(1/4)*1i + 2*a^4*b*c))*(-(b^7 + b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c - a*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^7 + b^8*c^3 - 16*a*b^6*c^4 + 96*a^2*b^4*c^5 - 256*a^3*b^2*c^6)))^(1/4) - 2*atan((((-(b^7 - b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c + a*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^7 + b^8*c^3 - 16*a*b^6*c^4 + 96*a^2*b^4*c^5 - 256*a^3*b^2*c^6)))^(3/4)*(4096*a^5*c^5 + 256*a^3*b^4*c^3 - 2048*a^4*b^2*c^4 - x*(-(b^7 - b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c + a*c*(-(4*a

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*c - b^2)^5)^(1/2))/(512*(256*a^4*c^7 + b^8*c^3 - 16*a*b^6*c^4 + 96*a^2*b^4
*c^5 - 256*a^3*b^2*c^6)))^(1/4)*(32768*a^5*c^6 + 2048*a^3*b^4*c^4 - 16384*a
^4*b^2*c^5)*1i)*1i + x*(4*a^3*b^3*c - 12*a^4*b*c^2))*(-(b^7 - b^2*(-(4*a*c
- b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c + a*c*(-(4*a*c
 - b^2)^5)^(1/2))/(512*(256*a^4*c^7 + b^8*c^3 - 16*a*b^6*c^4 + 96*a^2*b^4*c
^5 - 256*a^3*b^2*c^6)))^(1/4) - ((-(b^7 - b^2*(-(4*a*c - b^2)^5)^(1/2) - 48
*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c + a*c*(-(4*a*c - b^2)^5)^(1/2))/(5
12*(256*a^4*c^7 + b^8*c^3 - 16*a*b^6*c^4 + 96*a^2*b^4*c^5 - 256*a^3*b^2*c^6
)))^(3/4)*(4096*a^5*c^5 + 256*a^3*b^4*c^3 - 2048*a^4*b^2*c^4 + x*(-(b^7 - b
^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c +
a*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^7 + b^8*c^3 - 16*a*b^6*c^4 +
96*a^2*b^4*c^5 - 256*a^3*b^2*c^6)))^(1/4)*(32768*a^5*c^6 + 2048*a^3*b^4*c^4
 - 16384*a^4*b^2*c^5)*1i)*1i - x*(4*a^3*b^3*c - 12*a^4*b*c^2))*(-(b^7 - b^2
*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c + a*
c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^7 + b^8*c^3 - 16*a*b^6*c^4 + 96
*a^2*b^4*c^5 - 256*a^3*b^2*c^6)))^(1/4))/(((-(b^7 - b^2*(-(4*a*c - b^2)^5)^
(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c + a*c*(-(4*a*c - b^2)^5)
^(1/2))/(512*(256*a^4*c^7 + b^8*c^3 - 16*a*b^6*c^4 + 96*a^2*b^4*c^5 - 256*a
^3*b^2*c^6)))^(3/4)*(4096*a^5*c^5 + 256*a^3*b^4*c^3 - 2048*a^4*b^2*c^4 - x*
(-(b^7 - b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*
a*b^5*c + a*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^7 + b^8*c^3 - 16*a*
b^6*c^4 + 96*a^2*b^4*c^5 - 256*a^3*b^2*c^6)))^(1/4)*(32768*a^5*c^6 + 2048*a
^3*b^4*c^4 - 16384*a^4*b^2*c^5)*1i)*1i + x*(4*a^3*b^3*c - 12*a^4*b*c^2))*(-
(b^7 - b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*
b^5*c + a*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^7 + b^8*c^3 - 16*a*b^
6*c^4 + 96*a^2*b^4*c^5 - 256*a^3*b^2*c^6)))^(1/4)*1i + ((-(b^7 - b^2*(-(4*a
*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c + a*c*(-(4*
a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^7 + b^8*c^3 - 16*a*b^6*c^4 + 96*a^2*b^
4*c^5 - 256*a^3*b^2*c^6)))^(3/4)*(4096*a^5*c^5 + 256*a^3*b^4*c^3 - 2048*a^4
*b^2*c^4 + x*(-(b^7 - b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*
b^3*c^2 - 11*a*b^5*c + a*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^7 + b^
8*c^3 - 16*a*b^6*c^4 + 96*a^2*b^4*c^5 - 256*a^3*b^2*c^6)))^(1/4)*(32768*a^5
*c^6 + 2048*a^3*b^4*c^4 - 16384*a^4*b^2*c^5)*1i)*1i - x*(4*a^3*b^3*c - 12*a
^4*b*c^2))*(-(b^7 - b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^
3*c^2 - 11*a*b^5*c + a*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^7 + b^8*
c^3 - 16*a*b^6*c^4 + 96*a^2*b^4*c^5 - 256*a^3*b^2*c^6)))^(1/4)*1i + 2*a^4*b
*c))*(-(b^7 - b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2
- 11*a*b^5*c + a*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^7 + b^8*c^3 -
16*a*b^6*c^4 + 96*a^2*b^4*c^5 - 256*a^3*b^2*c^6)))^(1/4)
```

sympy [A] time = 61.58, size = 230, normalized size = 0.71

$$\operatorname{RootSum}\left(t^8\left(16777216a^4c^7-16777216a^3b^2c^6+6291456a^2b^4c^5-1048576ab^6c^4+65536b^8c^3\right)+t^4\left(-12288a^3bc^3+10240a^2b^3c^2-2816ab^5c+256b^7\right)+a^3,\left(t\mapsto t\log\left(x+\frac{2097152t^7a^4c^7-2621440t^7a^3b^2c^6+1179648t^7a^2b^4c^5-229376t^7ab^6c^4+16384t^7b^8c^3-1280t^3a^3bc^3+1600t^3a^2b^3c^2-576t^3ab^5c+64t^3b^7}{a^3c-a^2b^2}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(c*x**8+b*x**4+a),x)
```

```
[Out] RootSum(_t**8*(16777216*a**4*c**7 - 16777216*a**3*b**2*c**6 + 6291456*a**2*
b**4*c**5 - 1048576*a*b**6*c**4 + 65536*b**8*c**3) + _t**4*(-12288*a**3*b*c
**3 + 10240*a**2*b**3*c**2 - 2816*a*b**5*c + 256*b**7) + a**3, Lambda(_t, _
t*log(x + (2097152*_t**7*a**4*c**7 - 2621440*_t**7*a**3*b**2*c**6 + 1179648
*_t**7*a**2*b**4*c**5 - 229376*_t**7*a*b**6*c**4 + 16384*_t**7*b**8*c**3 -
1280*_t**3*a**3*b*c**3 + 1600*_t**3*a**2*b**3*c**2 - 576*_t**3*a*b**5*c + 6
4*_t**3*b**7)/(a**3*c - a**2*b**2))))
```

3.265 $\int \frac{x^4}{a+bx^4+cx^8}\,dx$

Optimal. Leaf size=325

$$\frac{\sqrt[4]{-\sqrt{b^2-4ac}-b}\,\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\,x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2\sqrt[4]{2}\sqrt[4]{c}\sqrt{b^2-4ac}}-\frac{\sqrt[4]{\sqrt{b^2-4ac}-b}\,\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\,x}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{2\sqrt[4]{2}\sqrt[4]{c}\sqrt{b^2-4ac}}+\frac{\sqrt[4]{-\sqrt{b^2-4ac}-b}\,\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\,x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2\sqrt[4]{2}\sqrt[4]{c}\sqrt{b^2-4ac}}$$

Rubi [A] time = 0.30, antiderivative size = 325, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {1374, 212, 208, 205}

$$\frac{\sqrt[4]{-\sqrt{b^2-4ac}-b}\,\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\,x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2\sqrt[4]{2}\sqrt[4]{c}\sqrt{b^2-4ac}}-\frac{\sqrt[4]{\sqrt{b^2-4ac}-b}\,\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\,x}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{2\sqrt[4]{2}\sqrt[4]{c}\sqrt{b^2-4ac}}+\frac{\sqrt[4]{-\sqrt{b^2-4ac}-b}\,\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\,x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2\sqrt[4]{2}\sqrt[4]{c}\sqrt{b^2-4ac}}-\frac{\sqrt[4]{\sqrt{b^2-4ac}-b}\,\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\,x}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{2\sqrt[4]{2}\sqrt[4]{c}\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(a + b*x^4 + c*x^8),x]
```

```
[Out] ((-b - Sqrt[b^2 - 4*a*c])^(1/4)*ArcTan[(2^(1/4)*c^(1/4)*x)/(-b - Sqrt[b^2 -
 4*a*c])^(1/4)])/(2*2^(1/4)*c^(1/4)*Sqrt[b^2 - 4*a*c]) - ((-b + Sqrt[b^2 -
4*a*c])^(1/4)*ArcTan[(2^(1/4)*c^(1/4)*x)/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(
2*2^(1/4)*c^(1/4)*Sqrt[b^2 - 4*a*c]) + ((-b - Sqrt[b^2 - 4*a*c])^(1/4)*ArcT
anh[(2^(1/4)*c^(1/4)*x)/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(2*2^(1/4)*c^(1/4)
*Sqrt[b^2 - 4*a*c]) - ((-b + Sqrt[b^2 - 4*a*c])^(1/4)*ArcTanh[(2^(1/4)*c^(1
/4)*x)/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(2*2^(1/4)*c^(1/4)*Sqrt[b^2 - 4*a*c
])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 1374

```
Int[((d_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(d^n*(b/q + 1))/2, Int[(d*x)^(m -
 n)/(b/2 + q/2 + c*x^n), x], x] - Dist[(d^n*(b/q - 1))/2, Int[(d*x)^(m - n)
/(b/2 - q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[n2, 2*n] &&
NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GeQ[m, n]
```

Rubi steps

$$\int \frac{x^4}{a+bx^4+cx^8}\,dx = -\left(\frac{1}{2}\left(-1+\frac{b}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^4}\,dx\right)+\frac{1}{2}\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)\int \frac{}{\frac{b}{2}+\frac{1}{2}\sqrt{}}$$

$$= \frac{\sqrt{-b-\sqrt{b^2-4ac}}\int \frac{1}{\sqrt{-b-\sqrt{b^2-4ac}}-\sqrt{2}\sqrt{c}x^2}\,dx}{2\sqrt{b^2-4ac}}+\frac{\sqrt{-b-\sqrt{b^2-4ac}}\int \frac{1}{\sqrt{-b-\sqrt{b^2-4ac}}+\sqrt{2}\sqrt{}}}{2\sqrt{b^2-4ac}}$$

$$= \frac{\sqrt[4]{-b-\sqrt{b^2-4ac}}\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{2\sqrt[4]{2}\sqrt[4]{c}\sqrt{b^2-4ac}}-\frac{\sqrt[4]{-b+\sqrt{b^2-4ac}}\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b+\sqrt{b^2-4ac}}}\right)}{2\sqrt[4]{2}\sqrt[4]{c}\sqrt{b^2-4ac}}+$$

Mathematica [C] time = 0.02, size = 42, normalized size = 0.13

$$\frac{1}{4}\text{RootSum}\left[\#1^8c+\#1^4b+a\&,\frac{\#1\log(x-\#1)}{2\#1^4c+b}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(a + b*x^4 + c*x^8),x]
```

```
[Out] RootSum[a + b*#1^4 + c*#1^8 & , (Log[x - #1]*#1)/(b + 2*c*#1^4) & ]/4
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{a+bx^4+cx^8}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^4/(a + b*x^4 + c*x^8),x]
```

```
[Out] IntegrateAlgebraic[x^4/(a + b*x^4 + c*x^8), x]
```

fricas [B] time = 1.53, size = 2479, normalized size = 7.63

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(c*x^8+b*x^4+a),x, algorithm="fricas")
```

```
[Out] -sqrt(sqrt(1/2)*sqrt(-(b + (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)/sqrt(b^6*c^2
- 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5))/(b^4*c - 8*a*b^2*c^2 + 16*a^
2*c^3)))*arctan(1/2*(sqrt(1/2)*(b^4 - 8*a*b^2*c + 16*a^2*c^2 - (b^7*c - 12*
a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)/sqrt(b^6*c^2 - 12*a*b^4*c^3 + 48
*a^2*b^2*c^4 - 64*a^3*c^5))*sqrt(x^2 + sqrt(1/2)*(b^2 - 4*a*c)*sqrt(-(b + (
b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)/sqrt(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*
c^4 - 64*a^3*c^5))/(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)))*sqrt(-(b + (b^4*c -
 8*a*b^2*c^2 + 16*a^2*c^3)/sqrt(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 6
4*a^3*c^5))/(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)) - sqrt(1/2)*((b^4 - 8*a*b^2
*c + 16*a^2*c^2)*x - (b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)
*x/sqrt(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5))*sqrt(-(b + (
b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)/sqrt(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*
c^4 - 64*a^3*c^5))/(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)))*sqrt(sqrt(1/2)*sqrt
(-(b + (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)/sqrt(b^6*c^2 - 12*a*b^4*c^3 + 48*
a^2*b^2*c^4 - 64*a^3*c^5))/(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)))/a) + sqrt(s
qrt(1/2)*sqrt(-(b - (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)/sqrt(b^6*c^2 - 12*a*
b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5))/(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3))
)*arctan(-1/2*(sqrt(1/2)*(b^4 - 8*a*b^2*c + 16*a^2*c^2 + (b^7*c - 12*a*b^5*
```

```
c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)/sqrt(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b
^2*c^4 - 64*a^3*c^5))*sqrt(x^2 + sqrt(1/2)*(b^2 - 4*a*c)*sqrt(-(b - (b^4*c
- 8*a*b^2*c^2 + 16*a^2*c^3)/sqrt(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 -
64*a^3*c^5))/(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)))*sqrt(-(b - (b^4*c - 8*a*b
^2*c^2 + 16*a^2*c^3)/sqrt(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*
c^5))/(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)) - sqrt(1/2)*((b^4 - 8*a*b^2*c + 1
6*a^2*c^2)*x + (b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*x/sqr
t(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5))*sqrt(-(b - (b^4*c
- 8*a*b^2*c^2 + 16*a^2*c^3)/sqrt(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 -
64*a^3*c^5))/(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)))*sqrt(sqrt(1/2)*sqrt(-(b -
 (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)/sqrt(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^
2*c^4 - 64*a^3*c^5))/(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)))/a) + 1/4*sqrt(sqr
t(1/2)*sqrt(-(b + (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)/sqrt(b^6*c^2 - 12*a*b^
4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5))/(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)))*
log(x + (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*sqrt(sqrt(1/2)*sqrt(-(b + (b^4*c
 - 8*a*b^2*c^2 + 16*a^2*c^3)/sqrt(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 -
 64*a^3*c^5))/(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)))/sqrt(b^6*c^2 - 12*a*b^4*
c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)) - 1/4*sqrt(sqrt(1/2)*sqrt(-(b + (b^4*c
- 8*a*b^2*c^2 + 16*a^2*c^3)/sqrt(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 -
64*a^3*c^5))/(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)))*log(x - (b^4*c - 8*a*b^2*
c^2 + 16*a^2*c^3)*sqrt(sqrt(1/2)*sqrt(-(b + (b^4*c - 8*a*b^2*c^2 + 16*a^2*c
^3)/sqrt(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5))/(b^4*c - 8*
a*b^2*c^2 + 16*a^2*c^3)))/sqrt(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64
*a^3*c^5)) - 1/4*sqrt(sqrt(1/2)*sqrt(-(b - (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^
3)/sqrt(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5))/(b^4*c - 8*a
*b^2*c^2 + 16*a^2*c^3)))*log(x + (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*sqrt(sq
rt(1/2)*sqrt(-(b - (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)/sqrt(b^6*c^2 - 12*a*b
^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5))/(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)))
/sqrt(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)) + 1/4*sqrt(sqr
t(1/2)*sqrt(-(b - (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)/sqrt(b^6*c^2 - 12*a*b^
4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5))/(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)))*
log(x - (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*sqrt(sqrt(1/2)*sqrt(-(b - (b^4*c
 - 8*a*b^2*c^2 + 16*a^2*c^3)/sqrt(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 -
 64*a^3*c^5))/(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)))/sqrt(b^6*c^2 - 12*a*b^4*
c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{cx^8 + bx^4 + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(c*x^8+b*x^4+a),x, algorithm="giac")
```

```
[Out] integrate(x^4/(c*x^8 + b*x^4 + a), x)
```

maple [C] time = 0.00, size = 43, normalized size = 0.13

$$\frac{\mathrm{RootOf}\left(\_Z^8 c + b\,\_Z^4 + a\right)^4 \ln\left(-\mathrm{RootOf}\left(\_Z^8 c + b\,\_Z^4 + a\right) + x\right)}{8\,\mathrm{RootOf}\left(\_Z^8 c + b\,\_Z^4 + a\right)^7 c + 4\,\mathrm{RootOf}\left(\_Z^8 c + b\,\_Z^4 + a\right)^3 b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(c*x^8+b*x^4+a),x)
```

```
[Out] 1/4*sum(_R^4/(2*_R^7*c+_R^3*b)*ln(-_R+x),_R=RootOf(_Z^8*c+_Z^4*b+a))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{cx^8 + bx^4 + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(c*x^8+b*x^4+a),x, algorithm="maxima")
```

```
[Out] integrate(x^4/(c*x^8 + b*x^4 + a), x)
```

mupad [B] time = 3.63, size = 8169, normalized size = 25.14

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(a + b*x^4 + c*x^8),x)
```

```
[Out] - atan((((-(b^5 + (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(512
*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)))^
(1/4)*(((-(b^5 + (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(512*
(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)))^(
1/4)*(262144*a^5*c^7 - 4096*a^2*b^6*c^4 + 49152*a^3*b^4*c^5 - 196608*a^4*b^
2*c^6) + x*(16384*a^4*b*c^6 + 1024*a^2*b^5*c^4 - 8192*a^3*b^3*c^5))*(-(b^5
+ (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(512*(b^8*c + 256*a^
4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)))^(3/4) + 64*a^3*b
*c^4 - 16*a^2*b^3*c^3) - x*(8*a^3*c^4 - 4*a^2*b^2*c^3))*(-(b^5 + (-(4*a*c -
 b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(512*(b^8*c + 256*a^4*c^5 - 16*a
*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)))^(1/4)*1i - ((-(b^5 + (-(4*a*
c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(512*(b^8*c + 256*a^4*c^5 - 1
6*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)))^(1/4)*(((-(b^5 + (-(4*a*c
 - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(512*(b^8*c + 256*a^4*c^5 - 16
*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)))^(1/4)*(262144*a^5*c^7 - 40
96*a^2*b^6*c^4 + 49152*a^3*b^4*c^5 - 196608*a^4*b^2*c^6) - x*(16384*a^4*b*c
^6 + 1024*a^2*b^5*c^4 - 8192*a^3*b^3*c^5))*(-(b^5 + (-(4*a*c - b^2)^5)^(1/2
) + 16*a^2*b*c^2 - 8*a*b^3*c)/(512*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96
*a^2*b^4*c^3 - 256*a^3*b^2*c^4)))^(3/4) + 64*a^3*b*c^4 - 16*a^2*b^3*c^3) +
x*(8*a^3*c^4 - 4*a^2*b^2*c^3))*(-(b^5 + (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b
*c^2 - 8*a*b^3*c)/(512*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3
 - 256*a^3*b^2*c^4)))^(1/4)*1i)/(((-(b^5 + (-(4*a*c - b^2)^5)^(1/2) + 16*a^
2*b*c^2 - 8*a*b^3*c)/(512*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*
c^3 - 256*a^3*b^2*c^4)))^(1/4)*(((-(b^5 + (-(4*a*c - b^2)^5)^(1/2) + 16*a^2
*b*c^2 - 8*a*b^3*c)/(512*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c
^3 - 256*a^3*b^2*c^4)))^(1/4)*(262144*a^5*c^7 - 4096*a^2*b^6*c^4 + 49152*a^
3*b^4*c^5 - 196608*a^4*b^2*c^6) + x*(16384*a^4*b*c^6 + 1024*a^2*b^5*c^4 - 8
192*a^3*b^3*c^5))*(-(b^5 + (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^
3*c)/(512*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^
2*c^4)))^(3/4) + 64*a^3*b*c^4 - 16*a^2*b^3*c^3) - x*(8*a^3*c^4 - 4*a^2*b^2*
c^3))*(-(b^5 + (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(512*(b
^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)))^(1/
4) + ((-(b^5 + (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(512*(b
^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)))^(1/
4)*(((-(b^5 + (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(512*(b^
8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)))^(1/4
)*(262144*a^5*c^7 - 4096*a^2*b^6*c^4 + 49152*a^3*b^4*c^5 - 196608*a^4*b^2*c
^6) - x*(16384*a^4*b*c^6 + 1024*a^2*b^5*c^4 - 8192*a^3*b^3*c^5))*(-(b^5 + (
-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(512*(b^8*c + 256*a^4*c
^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)))^(3/4) + 64*a^3*b*c^
4 - 16*a^2*b^3*c^3) + x*(8*a^3*c^4 - 4*a^2*b^2*c^3))*(-(b^5 + (-(4*a*c - b^
2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(512*(b^8*c + 256*a^4*c^5 - 16*a*b^
6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)))^(1/4)))*(-(b^5 + (-(4*a*c - b^2
)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(512*(b^8*c + 256*a^4*c^5 - 16*a*b^6
*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)))^(1/4)*2i - 2*atan((((-(b^5 + (-(
4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(512*(b^8*c + 256*a^4*c^5
 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)))^(1/4)*(((-(b^5 + (-(4
*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(512*(b^8*c + 256*a^4*c^5
```

- 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)))^(1/4)*(262144*a^5*c^7
- 4096*a^2*b^6*c^4 + 49152*a^3*b^4*c^5 - 196608*a^4*b^2*c^6)*1i + x*(16384*
a^4*b*c^6 + 1024*a^2*b^5*c^4 - 8192*a^3*b^3*c^5))*(-(b^5 + (-(4*a*c - b^2)^
5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(512*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c
^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)))^(3/4)*1i - 64*a^3*b*c^4 + 16*a^2*b
^3*c^3)*1i + x*(8*a^3*c^4 - 4*a^2*b^2*c^3))*(-(b^5 + (-(4*a*c - b^2)^5)^(1/
2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(512*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 9
6*a^2*b^4*c^3 - 256*a^3*b^2*c^4)))^(1/4) - ((-(b^5 + (-(4*a*c - b^2)^5)^(1/
2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(512*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 9
6*a^2*b^4*c^3 - 256*a^3*b^2*c^4)))^(1/4)*(((-(b^5 + (-(4*a*c - b^2)^5)^(1/2
) + 16*a^2*b*c^2 - 8*a*b^3*c)/(512*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96
*a^2*b^4*c^3 - 256*a^3*b^2*c^4)))^(1/4)*(262144*a^5*c^7 - 4096*a^2*b^6*c^4
+ 49152*a^3*b^4*c^5 - 196608*a^4*b^2*c^6)*1i - x*(16384*a^4*b*c^6 + 1024*a^
2*b^5*c^4 - 8192*a^3*b^3*c^5))*(-(b^5 + (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b
*c^2 - 8*a*b^3*c)/(512*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3
 - 256*a^3*b^2*c^4)))^(3/4)*1i - 64*a^3*b*c^4 + 16*a^2*b^3*c^3)*1i - x*(8*a
^3*c^4 - 4*a^2*b^2*c^3))*(-(b^5 + (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 -
 8*a*b^3*c)/(512*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256
*a^3*b^2*c^4)))^(1/4))/(((-(b^5 + (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 -
 8*a*b^3*c)/(512*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256
*a^3*b^2*c^4)))^(1/4)*(((-(b^5 + (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 -
8*a*b^3*c)/(512*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*
a^3*b^2*c^4)))^(1/4)*(262144*a^5*c^7 - 4096*a^2*b^6*c^4 + 49152*a^3*b^4*c^5
 - 196608*a^4*b^2*c^6)*1i + x*(16384*a^4*b*c^6 + 1024*a^2*b^5*c^4 - 8192*a^
3*b^3*c^5))*(-(b^5 + (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(
512*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)
))^(3/4)*1i - 64*a^3*b*c^4 + 16*a^2*b^3*c^3)*1i + x*(8*a^3*c^4 - 4*a^2*b^2*
c^3))*(-(b^5 + (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(512*(b
^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)))^(1/
4)*1i + ((-(b^5 + (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(512
*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)))^
(1/4)*(((-(b^5 + (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(512*
(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)))^(
1/4)*(262144*a^5*c^7 - 4096*a^2*b^6*c^4 + 49152*a^3*b^4*c^5 - 196608*a^4*b^
2*c^6)*1i - x*(16384*a^4*b*c^6 + 1024*a^2*b^5*c^4 - 8192*a^3*b^3*c^5))*(-(b
^5 + (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(512*(b^8*c + 256
*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)))^(3/4)*1i - 64
*a^3*b*c^4 + 16*a^2*b^3*c^3)*1i - x*(8*a^3*c^4 - 4*a^2*b^2*c^3))*(-(b^5 + (
-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(512*(b^8*c + 256*a^4*c
^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)))^(1/4)*1i))*(-(b^5 +
 (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(512*(b^8*c + 256*a^4
*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)))^(1/4) - atan(((((
(-(b^5 - (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(512*(b^8*c +
 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)))^(1/4)*(26
2144*a^5*c^7 - 4096*a^2*b^6*c^4 + 49152*a^3*b^4*c^5 - 196608*a^4*b^2*c^6) +
 x*(16384*a^4*b*c^6 + 1024*a^2*b^5*c^4 - 8192*a^3*b^3*c^5))*(-(b^5 - (-(4*a
*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(512*(b^8*c + 256*a^4*c^5 -
16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)))^(3/4) + 64*a^3*b*c^4 - 1
6*a^2*b^3*c^3)*(-(b^5 - (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c
)/(512*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c
^4)))^(1/4) - x*(8*a^3*c^4 - 4*a^2*b^2*c^3))*(-(b^5 - (-(4*a*c - b^2)^5)^(1
/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(512*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 +
96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)))^(1/4)*1i - ((((-(b^5 - (-(4*a*c - b^2)^
5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(512*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c
^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)))^(1/4)*(262144*a^5*c^7 - 4096*a^2*b
^6*c^4 + 49152*a^3*b^4*c^5 - 196608*a^4*b^2*c^6) - x*(16384*a^4*b*c^6 + 102
4*a^2*b^5*c^4 - 8192*a^3*b^3*c^5))*(-(b^5 - (-(4*a*c - b^2)^5)^(1/2) + 16*a
^2*b*c^2 - 8*a*b^3*c)/(512*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4
*c^3 - 256*a^3*b^2*c^4)))^(3/4) + 64*a^3*b*c^4 - 16*a^2*b^3*c^3)*(-(b^5 - (

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-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(512*(b^8*c + 256*a^4*c
^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)))^(1/4) + x*(8*a^3*c^
4 - 4*a^2*b^2*c^3))*(-(b^5 - (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*
b^3*c)/(512*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*
b^2*c^4)))^(1/4)*1i)/(((((-(b^5 - (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 -
 8*a*b^3*c)/(512*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256
*a^3*b^2*c^4)))^(1/4)*(262144*a^5*c^7 - 4096*a^2*b^6*c^4 + 49152*a^3*b^4*c^
5 - 196608*a^4*b^2*c^6) + x*(16384*a^4*b*c^6 + 1024*a^2*b^5*c^4 - 8192*a^3*
b^3*c^5))*(-(b^5 - (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(51
2*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)))
^(3/4) + 64*a^3*b*c^4 - 16*a^2*b^3*c^3)*(-(b^5 - (-(4*a*c - b^2)^5)^(1/2) +
 16*a^2*b*c^2 - 8*a*b^3*c)/(512*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^
2*b^4*c^3 - 256*a^3*b^2*c^4)))^(1/4) - x*(8*a^3*c^4 - 4*a^2*b^2*c^3))*(-(b^
5 - (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(512*(b^8*c + 256*
a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)))^(1/4) + ((((-(
b^5 - (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(512*(b^8*c + 25
6*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)))^(1/4)*(26214
4*a^5*c^7 - 4096*a^2*b^6*c^4 + 49152*a^3*b^4*c^5 - 196608*a^4*b^2*c^6) - x*
(16384*a^4*b*c^6 + 1024*a^2*b^5*c^4 - 8192*a^3*b^3*c^5))*(-(b^5 - (-(4*a*c
- b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(512*(b^8*c + 256*a^4*c^5 - 16*
a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)))^(3/4) + 64*a^3*b*c^4 - 16*a
^2*b^3*c^3)*(-(b^5 - (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(
512*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)
))^(1/4) + x*(8*a^3*c^4 - 4*a^2*b^2*c^3))*(-(b^5 - (-(4*a*c - b^2)^5)^(1/2)
 + 16*a^2*b*c^2 - 8*a*b^3*c)/(512*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*
a^2*b^4*c^3 - 256*a^3*b^2*c^4)))^(1/4)))*(-(b^5 - (-(4*a*c - b^2)^5)^(1/2)
+ 16*a^2*b*c^2 - 8*a*b^3*c)/(512*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a
^2*b^4*c^3 - 256*a^3*b^2*c^4)))^(1/4)*2i - 2*atan((((((-(b^5 - (-(4*a*c - b
^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(512*(b^8*c + 256*a^4*c^5 - 16*a*b
^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)))^(1/4)*(262144*a^5*c^7 - 4096*a
^2*b^6*c^4 + 49152*a^3*b^4*c^5 - 196608*a^4*b^2*c^6)*1i + x*(16384*a^4*b*c^
6 + 1024*a^2*b^5*c^4 - 8192*a^3*b^3*c^5))*(-(b^5 - (-(4*a*c - b^2)^5)^(1/2)
 + 16*a^2*b*c^2 - 8*a*b^3*c)/(512*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*
a^2*b^4*c^3 - 256*a^3*b^2*c^4)))^(3/4)*1i - 64*a^3*b*c^4 + 16*a^2*b^3*c^3)*
(-(b^5 - (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(512*(b^8*c +
 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)))^(1/4)*1i
+ x*(8*a^3*c^4 - 4*a^2*b^2*c^3))*(-(b^5 - (-(4*a*c - b^2)^5)^(1/2) + 16*a^2
*b*c^2 - 8*a*b^3*c)/(512*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c
^3 - 256*a^3*b^2*c^4)))^(1/4) - ((((-(b^5 - (-(4*a*c - b^2)^5)^(1/2) + 16*a
^2*b*c^2 - 8*a*b^3*c)/(512*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4
*c^3 - 256*a^3*b^2*c^4)))^(1/4)*(262144*a^5*c^7 - 4096*a^2*b^6*c^4 + 49152*
a^3*b^4*c^5 - 196608*a^4*b^2*c^6)*1i - x*(16384*a^4*b*c^6 + 1024*a^2*b^5*c^
4 - 8192*a^3*b^3*c^5))*(-(b^5 - (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8
*a*b^3*c)/(512*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a
^3*b^2*c^4)))^(3/4)*1i - 64*a^3*b*c^4 + 16*a^2*b^3*c^3)*(-(b^5 - (-(4*a*c -
 b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(512*(b^8*c + 256*a^4*c^5 - 16*a
*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)))^(1/4)*1i - x*(8*a^3*c^4 - 4*
a^2*b^2*c^3))*(-(b^5 - (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)
/(512*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^
4)))^(1/4))/(((((-(b^5 - (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*
c)/(512*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*
c^4)))^(1/4)*(262144*a^5*c^7 - 4096*a^2*b^6*c^4 + 49152*a^3*b^4*c^5 - 19660
8*a^4*b^2*c^6)*1i + x*(16384*a^4*b*c^6 + 1024*a^2*b^5*c^4 - 8192*a^3*b^3*c^
5))*(-(b^5 - (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(512*(b^8
*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)))^(3/4)
*1i - 64*a^3*b*c^4 + 16*a^2*b^3*c^3)*(-(b^5 - (-(4*a*c - b^2)^5)^(1/2) + 16
*a^2*b*c^2 - 8*a*b^3*c)/(512*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b
^4*c^3 - 256*a^3*b^2*c^4)))^(1/4)*1i + x*(8*a^3*c^4 - 4*a^2*b^2*c^3))*(-(b^
5 - (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(512*(b^8*c + 256*
```

```
a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)))^(1/4)*1i + (((
(-(b^5 - (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(512*(b^8*c +
 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)))^(1/4)*(26
2144*a^5*c^7 - 4096*a^2*b^6*c^4 + 49152*a^3*b^4*c^5 - 196608*a^4*b^2*c^6)*1
i - x*(16384*a^4*b*c^6 + 1024*a^2*b^5*c^4 - 8192*a^3*b^3*c^5))*(-(b^5 - (-(
4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(512*(b^8*c + 256*a^4*c^5
 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)))^(3/4)*1i - 64*a^3*b*c
^4 + 16*a^2*b^3*c^3)*(-(b^5 - (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a
*b^3*c)/(512*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3
*b^2*c^4)))^(1/4)*1i - x*(8*a^3*c^4 - 4*a^2*b^2*c^3))*(-(b^5 - (-(4*a*c - b
^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(512*(b^8*c + 256*a^4*c^5 - 16*a*b
^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)))^(1/4)*1i))*(-(b^5 - (-(4*a*c -
 b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(512*(b^8*c + 256*a^4*c^5 - 16*a
*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)))^(1/4)
```

sympy [A] time = 2.92, size = 126, normalized size = 0.39

$$\operatorname{RootSum}\left(t^{8}\left(16777216a^{4}c^{5}-16777216a^{3}b^{2}c^{4}+6291456a^{2}b^{4}c^{3}-1048576ab^{6}c^{2}+65536b^{8}c\right)+t^{4}\left(4096a^{2}bc^{2}-2048ab^{3}c+256b^{5}\right)+a,\left(t\mapsto t\log\left(-32768t^{5}a^{2}c^{3}+16384t^{5}ab^{2}c^{2}-2048t^{5}b^{4}c-4tb+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(c*x**8+b*x**4+a),x)
```

```
[Out] RootSum(_t**8*(16777216*a**4*c**5 - 16777216*a**3*b**2*c**4 + 6291456*a**2*
b**4*c**3 - 1048576*a*b**6*c**2 + 65536*b**8*c) + _t**4*(4096*a**2*b*c**2 -
 2048*a*b**3*c + 256*b**5) + a, Lambda(_t, _t*log(-32768*_t**5*a**2*c**3 +
16384*_t**5*a*b**2*c**2 - 2048*_t**5*b**4*c - 4*_t*b + x)))
```

3.266 $\int \frac{x^2}{a+bx^4+cx^8} \, dx$

Optimal. Leaf size=315

$$-\frac{\sqrt[4]{c}\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2^{3/4}\sqrt{b^2-4ac}\sqrt[4]{-\sqrt{b^2-4ac}-b}}+\frac{\sqrt[4]{c}\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{2^{3/4}\sqrt{b^2-4ac}\sqrt[4]{\sqrt{b^2-4ac}-b}}+\frac{\sqrt[4]{c}\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2^{3/4}\sqrt{b^2-4ac}\sqrt[4]{-\sqrt{b^2-4ac}-b}}-\frac{\sqrt[4]{}}{2^{3/4}\sqrt{}}$$

Rubi [A] time = 0.29, antiderivative size = 315, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {1375, 298, 205, 208}

$$-\frac{\sqrt[4]{c}\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2^{3/4}\sqrt{b^2-4ac}\sqrt[4]{-\sqrt{b^2-4ac}-b}}+\frac{\sqrt[4]{c}\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{2^{3/4}\sqrt{b^2-4ac}\sqrt[4]{\sqrt{b^2-4ac}-b}}+\frac{\sqrt[4]{c}\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2^{3/4}\sqrt{b^2-4ac}\sqrt[4]{-\sqrt{b^2-4ac}-b}}-\frac{\sqrt[4]{c}\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{2^{3/4}\sqrt{b^2-4ac}\sqrt[4]{\sqrt{b^2-4ac}-b}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a + b*x^4 + c*x^8),x]
```

```
[Out] -((c^(1/4)*ArcTan[(2^(1/4)*c^(1/4)*x)/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(2^(
3/4)*Sqrt[b^2 - 4*a*c]*(-b - Sqrt[b^2 - 4*a*c])^(1/4))) + (c^(1/4)*ArcTan[(
2^(1/4)*c^(1/4)*x)/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(2^(3/4)*Sqrt[b^2 - 4*a
*c]*(-b + Sqrt[b^2 - 4*a*c])^(1/4)) + (c^(1/4)*ArcTanh[(2^(1/4)*c^(1/4)*x)/
(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(2^(3/4)*Sqrt[b^2 - 4*a*c]*(-b - Sqrt[b^2
- 4*a*c])^(1/4)) - (c^(1/4)*ArcTanh[(2^(1/4)*c^(1/4)*x)/(-b + Sqrt[b^2 - 4*
a*c])^(1/4)])/(2^(3/4)*Sqrt[b^2 - 4*a*c]*(-b + Sqrt[b^2 - 4*a*c])^(1/4))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 1375

```
Int[((d_.)*(x_))^(m_.)/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symb
ol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[c/q, Int[(d*x)^m/(b/2 - q/2 + c*
x^n), x], x] - Dist[c/q, Int[(d*x)^m/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[
{a, b, c, d, m}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{x^2}{a+bx^4+cx^8}\,dx = \frac{c\int \frac{x^2}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^4}\,dx}{\sqrt{b^2-4ac}} - \frac{c\int \frac{x^2}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^4}\,dx}{\sqrt{b^2-4ac}}$$

$$= \frac{\sqrt{c}\int \frac{1}{\sqrt{-b-\sqrt{b^2-4ac}}-\sqrt{2}\sqrt{c}x^2}\,dx}{\sqrt{2}\sqrt{b^2-4ac}} - \frac{\sqrt{c}\int \frac{1}{\sqrt{-b+\sqrt{b^2-4ac}}-\sqrt{2}\sqrt{c}x^2}\,dx}{\sqrt{2}\sqrt{b^2-4ac}} - \frac{\sqrt{c}\int \frac{1}{\sqrt{-b-\sqrt{b^2-4ac}}+\sqrt{2}\sqrt{c}\ldots}}{\sqrt{2}\sqrt{b^2-4ac}}$$

$$= -\frac{\sqrt[4]{c}\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{2^{3/4}\sqrt{b^2-4ac}\sqrt[4]{-b-\sqrt{b^2-4ac}}} + \frac{\sqrt[4]{c}\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b+\sqrt{b^2-4ac}}}\right)}{2^{3/4}\sqrt{b^2-4ac}\sqrt[4]{-b+\sqrt{b^2-4ac}}} + \frac{\sqrt[4]{c}\tanh^{-1}\left(\ldots\right.}{2^{3/4}\sqrt{b^2-4ac}\sqrt[4]{\ldots}}$$

Mathematica [C] time = 0.02, size = 43, normalized size = 0.14

$$\frac{1}{4}\text{RootSum}\left[\#1^8c+\#1^4b+a\&,\frac{\log(x-\#1)}{2\#1^5c+\#1b}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a + b*x^4 + c*x^8),x]
```

```
[Out] RootSum[a + b*#1^4 + c*#1^8 & , Log[x - #1]/(b*#1 + 2*c*#1^5) & ]/4
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{a+bx^4+cx^8}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^2/(a + b*x^4 + c*x^8),x]
```

```
[Out] IntegrateAlgebraic[x^2/(a + b*x^4 + c*x^8), x]
```

fricas [B] time = 1.66, size = 2746, normalized size = 8.72

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^8+b*x^4+a),x, algorithm="fricas")
```

```
[Out] sqrt(sqrt(1/2)*sqrt(-(b - (a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)/sqrt(a^2*b^6 -
 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3))/(a*b^4 - 8*a^2*b^2*c + 16*a^3
*c^2)))*arctan(-(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)*x*sqrt(sqrt(1/2)*sqrt(-(
b - (a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)/sqrt(a^2*b^6 - 12*a^3*b^4*c + 48*a^4
*b^2*c^2 - 64*a^5*c^3))/(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)))/sqrt(a^2*b^6 -
 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3) + sqrt(1/2)*(a*b^4 - 8*a^2*b^2
*c + 16*a^3*c^2)*sqrt(sqrt(1/2)*sqrt(-(b - (a*b^4 - 8*a^2*b^2*c + 16*a^3*c^
2)/sqrt(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3))/(a*b^4 - 8*a
^2*b^2*c + 16*a^3*c^2)))*sqrt((2*c*x^2 - sqrt(1/2)*(b^3 - 4*a*b*c + (a*b^6
- 12*a^2*b^4*c + 48*a^3*b^2*c^2 - 64*a^4*c^3)/sqrt(a^2*b^6 - 12*a^3*b^4*c +
 48*a^4*b^2*c^2 - 64*a^5*c^3))*sqrt(-(b - (a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2
)/sqrt(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3))/(a*b^4 - 8*a^
2*b^2*c + 16*a^3*c^2)))/c)/sqrt(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 6
4*a^5*c^3)) - sqrt(sqrt(1/2)*sqrt(-(b + (a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)/
sqrt(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3))/(a*b^4 - 8*a^2*
b^2*c + 16*a^3*c^2)))*arctan(-((a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)*x/sqrt(a^
2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3) - sqrt(1/2)*(a*b^4 - 8*
```

```
a^2*b^2*c + 16*a^3*c^2)*sqrt((2*c*x^2 - sqrt(1/2)*(b^3 - 4*a*b*c - (a*b^6 -
 12*a^2*b^4*c + 48*a^3*b^2*c^2 - 64*a^4*c^3)/sqrt(a^2*b^6 - 12*a^3*b^4*c +
48*a^4*b^2*c^2 - 64*a^5*c^3))*sqrt(-(b + (a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)
/sqrt(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3))/(a*b^4 - 8*a^2
*b^2*c + 16*a^3*c^2)))/c)/sqrt(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64
*a^5*c^3))*sqrt(sqrt(1/2)*sqrt(-(b + (a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)/sqr
t(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3))/(a*b^4 - 8*a^2*b^2
*c + 16*a^3*c^2)))) - 1/4*sqrt(sqrt(1/2)*sqrt(-(b + (a*b^4 - 8*a^2*b^2*c +
16*a^3*c^2)/sqrt(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3))/(a*
b^4 - 8*a^2*b^2*c + 16*a^3*c^2)))*log(1/2*sqrt(1/2)*(b^4 - 8*a*b^2*c + 16*a
^2*c^2 - (a*b^7 - 12*a^2*b^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3)/sqrt(a^2*b^
6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3))*sqrt(sqrt(1/2)*sqrt(-(b +
(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)/sqrt(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2
*c^2 - 64*a^5*c^3))/(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)))*sqrt(-(b + (a*b^4
- 8*a^2*b^2*c + 16*a^3*c^2)/sqrt(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 -
64*a^5*c^3))/(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)) + c*x) + 1/4*sqrt(sqrt(1/2
)*sqrt(-(b + (a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)/sqrt(a^2*b^6 - 12*a^3*b^4*c
 + 48*a^4*b^2*c^2 - 64*a^5*c^3))/(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)))*log(-
1/2*sqrt(1/2)*(b^4 - 8*a*b^2*c + 16*a^2*c^2 - (a*b^7 - 12*a^2*b^5*c + 48*a^
3*b^3*c^2 - 64*a^4*b*c^3)/sqrt(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64
*a^5*c^3))*sqrt(sqrt(1/2)*sqrt(-(b + (a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)/sqr
t(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3))/(a*b^4 - 8*a^2*b^2
*c + 16*a^3*c^2)))*sqrt(-(b + (a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)/sqrt(a^2*b
^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3))/(a*b^4 - 8*a^2*b^2*c + 16
*a^3*c^2)) + c*x) - 1/4*sqrt(sqrt(1/2)*sqrt(-(b - (a*b^4 - 8*a^2*b^2*c + 16
*a^3*c^2)/sqrt(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3))/(a*b^
4 - 8*a^2*b^2*c + 16*a^3*c^2)))*log(1/2*sqrt(1/2)*(b^4 - 8*a*b^2*c + 16*a^2
*c^2 + (a*b^7 - 12*a^2*b^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3)/sqrt(a^2*b^6
- 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3))*sqrt(sqrt(1/2)*sqrt(-(b - (a
*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)/sqrt(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c
^2 - 64*a^5*c^3))/(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)))*sqrt(-(b - (a*b^4 -
8*a^2*b^2*c + 16*a^3*c^2)/sqrt(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64
*a^5*c^3))/(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)) + c*x) + 1/4*sqrt(sqrt(1/2)*
sqrt(-(b - (a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)/sqrt(a^2*b^6 - 12*a^3*b^4*c +
 48*a^4*b^2*c^2 - 64*a^5*c^3))/(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)))*log(-1/
2*sqrt(1/2)*(b^4 - 8*a*b^2*c + 16*a^2*c^2 + (a*b^7 - 12*a^2*b^5*c + 48*a^3*
b^3*c^2 - 64*a^4*b*c^3)/sqrt(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a
^5*c^3))*sqrt(sqrt(1/2)*sqrt(-(b - (a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)/sqrt(
a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3))/(a*b^4 - 8*a^2*b^2*c
 + 16*a^3*c^2)))*sqrt(-(b - (a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)/sqrt(a^2*b^6
 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3))/(a*b^4 - 8*a^2*b^2*c + 16*a
^3*c^2)) + c*x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{cx^8 + bx^4 + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^8+b*x^4+a),x, algorithm="giac")
```

```
[Out] integrate(x^2/(c*x^8 + b*x^4 + a), x)
```

maple [C] time = 0.00, size = 43, normalized size = 0.14

$$\frac{\mathrm{RootOf}\left(\_Z^8 c + b\_Z^4 + a\right)^2 \ln\left(-\mathrm{RootOf}\left(\_Z^8 c + b\_Z^4 + a\right) + x\right)}{8\,\mathrm{RootOf}\left(\_Z^8 c + b\_Z^4 + a\right)^7 c + 4\,\mathrm{RootOf}\left(\_Z^8 c + b\_Z^4 + a\right)^3 b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(c*x^8+b*x^4+a),x)
```

```
[Out] 1/4*sum(_R^2/(2*_R^7*c+_R^3*b)*ln(-_R+x),_R=RootOf(_Z^8*c+_Z^4*b+a))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{cx^8 + bx^4 + a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^8+b*x^4+a),x, algorithm="maxima")
```

```
[Out] integrate(x^2/(c*x^8 + b*x^4 + a), x)
```

mupad [B] time = 2.34, size = 6067, normalized size = 19.26

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a + b*x^4 + c*x^8),x)
```

```
[Out] 2*atan((((-(b^5 - (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(512
*(a*b^8 + 256*a^5*c^4 - 16*a^2*b^6*c + 96*a^3*b^4*c^2 - 256*a^4*b^2*c^3)))^
(3/4)*(256*a*b^5*c^4 + 4096*a^3*b*c^6 - x*(-(b^5 - (-(4*a*c - b^2)^5)^(1/2)
 + 16*a^2*b*c^2 - 8*a*b^3*c)/(512*(a*b^8 + 256*a^5*c^4 - 16*a^2*b^6*c + 96*
a^3*b^4*c^2 - 256*a^4*b^2*c^3)))^(1/4)*(32768*a^4*c^7 - 1024*a*b^6*c^4 + 10
240*a^2*b^4*c^5 - 32768*a^3*b^2*c^6)*1i - 2048*a^2*b^3*c^5)*1i - 4*a*b*c^5*
x)*(-(b^5 - (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(512*(a*b^
8 + 256*a^5*c^4 - 16*a^2*b^6*c + 96*a^3*b^4*c^2 - 256*a^4*b^2*c^3)))^(1/4)
- ((-(b^5 - (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(512*(a*b^
8 + 256*a^5*c^4 - 16*a^2*b^6*c + 96*a^3*b^4*c^2 - 256*a^4*b^2*c^3)))^(3/4)*
(256*a*b^5*c^4 + 4096*a^3*b*c^6 + x*(-(b^5 - (-(4*a*c - b^2)^5)^(1/2) + 16*
a^2*b*c^2 - 8*a*b^3*c)/(512*(a*b^8 + 256*a^5*c^4 - 16*a^2*b^6*c + 96*a^3*b^
4*c^2 - 256*a^4*b^2*c^3)))^(1/4)*(32768*a^4*c^7 - 1024*a*b^6*c^4 + 10240*a^
2*b^4*c^5 - 32768*a^3*b^2*c^6)*1i - 2048*a^2*b^3*c^5)*1i + 4*a*b*c^5*x)*(-(
b^5 - (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(512*(a*b^8 + 25
6*a^5*c^4 - 16*a^2*b^6*c + 96*a^3*b^4*c^2 - 256*a^4*b^2*c^3)))^(1/4))/(((-(
b^5 - (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(512*(a*b^8 + 25
6*a^5*c^4 - 16*a^2*b^6*c + 96*a^3*b^4*c^2 - 256*a^4*b^2*c^3)))^(3/4)*(256*a
*b^5*c^4 + 4096*a^3*b*c^6 - x*(-(b^5 - (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*
c^2 - 8*a*b^3*c)/(512*(a*b^8 + 256*a^5*c^4 - 16*a^2*b^6*c + 96*a^3*b^4*c^2
- 256*a^4*b^2*c^3)))^(1/4)*(32768*a^4*c^7 - 1024*a*b^6*c^4 + 10240*a^2*b^4*
c^5 - 32768*a^3*b^2*c^6)*1i - 2048*a^2*b^3*c^5)*1i - 4*a*b*c^5*x)*(-(b^5 -
(-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(512*(a*b^8 + 256*a^5*
c^4 - 16*a^2*b^6*c + 96*a^3*b^4*c^2 - 256*a^4*b^2*c^3)))^(1/4)*1i + ((-(b^5
 - (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(512*(a*b^8 + 256*a
^5*c^4 - 16*a^2*b^6*c + 96*a^3*b^4*c^2 - 256*a^4*b^2*c^3)))^(3/4)*(256*a*b^
5*c^4 + 4096*a^3*b*c^6 + x*(-(b^5 - (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2
 - 8*a*b^3*c)/(512*(a*b^8 + 256*a^5*c^4 - 16*a^2*b^6*c + 96*a^3*b^4*c^2 - 2
56*a^4*b^2*c^3)))^(1/4)*(32768*a^4*c^7 - 1024*a*b^6*c^4 + 10240*a^2*b^4*c^5
 - 32768*a^3*b^2*c^6)*1i - 2048*a^2*b^3*c^5)*1i + 4*a*b*c^5*x)*(-(b^5 - (-(
4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(512*(a*b^8 + 256*a^5*c^4
 - 16*a^2*b^6*c + 96*a^3*b^4*c^2 - 256*a^4*b^2*c^3)))^(1/4)*1i - 2*a*c^5))*
(-(b^5 - (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(512*(a*b^8 +
 256*a^5*c^4 - 16*a^2*b^6*c + 96*a^3*b^4*c^2 - 256*a^4*b^2*c^3)))^(1/4) - a
tan((((-(b^5 - (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(512*(a
*b^8 + 256*a^5*c^4 - 16*a^2*b^6*c + 96*a^3*b^4*c^2 - 256*a^4*b^2*c^3)))^(3/
4)*(256*a*b^5*c^4 + 4096*a^3*b*c^6 + x*(-(b^5 - (-(4*a*c - b^2)^5)^(1/2) +
16*a^2*b*c^2 - 8*a*b^3*c)/(512*(a*b^8 + 256*a^5*c^4 - 16*a^2*b^6*c + 96*a^3
*b^4*c^2 - 256*a^4*b^2*c^3)))^(1/4)*(32768*a^4*c^7 - 1024*a*b^6*c^4 + 10240
*a^2*b^4*c^5 - 32768*a^3*b^2*c^6) - 2048*a^2*b^3*c^5) - 4*a*b*c^5*x)*(-(b^5
```

```
 - (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(512*(a*b^8 + 256*a
^5*c^4 - 16*a^2*b^6*c + 96*a^3*b^4*c^2 - 256*a^4*b^2*c^3)))^(1/4)*1i - ((-(
b^5 - (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(512*(a*b^8 + 25
6*a^5*c^4 - 16*a^2*b^6*c + 96*a^3*b^4*c^2 - 256*a^4*b^2*c^3)))^(3/4)*(256*a
*b^5*c^4 + 4096*a^3*b*c^6 - x*(-(b^5 - (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*
c^2 - 8*a*b^3*c)/(512*(a*b^8 + 256*a^5*c^4 - 16*a^2*b^6*c + 96*a^3*b^4*c^2
- 256*a^4*b^2*c^3)))^(1/4)*(32768*a^4*c^7 - 1024*a*b^6*c^4 + 10240*a^2*b^4*
c^5 - 32768*a^3*b^2*c^6) - 2048*a^2*b^3*c^5) + 4*a*b*c^5*x)*(-(b^5 - (-(4*a
*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(512*(a*b^8 + 256*a^5*c^4 -
16*a^2*b^6*c + 96*a^3*b^4*c^2 - 256*a^4*b^2*c^3)))^(1/4)*1i)/(((-(b^5 - (-(
4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(512*(a*b^8 + 256*a^5*c^4
 - 16*a^2*b^6*c + 96*a^3*b^4*c^2 - 256*a^4*b^2*c^3)))^(3/4)*(256*a*b^5*c^4
+ 4096*a^3*b*c^6 + x*(-(b^5 - (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a
*b^3*c)/(512*(a*b^8 + 256*a^5*c^4 - 16*a^2*b^6*c + 96*a^3*b^4*c^2 - 256*a^4
*b^2*c^3)))^(1/4)*(32768*a^4*c^7 - 1024*a*b^6*c^4 + 10240*a^2*b^4*c^5 - 327
68*a^3*b^2*c^6) - 2048*a^2*b^3*c^5) - 4*a*b*c^5*x)*(-(b^5 - (-(4*a*c - b^2)
^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(512*(a*b^8 + 256*a^5*c^4 - 16*a^2*b^
6*c + 96*a^3*b^4*c^2 - 256*a^4*b^2*c^3)))^(1/4) + ((-(b^5 - (-(4*a*c - b^2)
^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(512*(a*b^8 + 256*a^5*c^4 - 16*a^2*b^
6*c + 96*a^3*b^4*c^2 - 256*a^4*b^2*c^3)))^(3/4)*(256*a*b^5*c^4 + 4096*a^3*b
*c^6 - x*(-(b^5 - (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(512
*(a*b^8 + 256*a^5*c^4 - 16*a^2*b^6*c + 96*a^3*b^4*c^2 - 256*a^4*b^2*c^3)))^
(1/4)*(32768*a^4*c^7 - 1024*a*b^6*c^4 + 10240*a^2*b^4*c^5 - 32768*a^3*b^2*c
^6) - 2048*a^2*b^3*c^5) + 4*a*b*c^5*x)*(-(b^5 - (-(4*a*c - b^2)^5)^(1/2) +
16*a^2*b*c^2 - 8*a*b^3*c)/(512*(a*b^8 + 256*a^5*c^4 - 16*a^2*b^6*c + 96*a^3
*b^4*c^2 - 256*a^4*b^2*c^3)))^(1/4) + 2*a*c^5))*(-(b^5 - (-(4*a*c - b^2)^5)
^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(512*(a*b^8 + 256*a^5*c^4 - 16*a^2*b^6*c
 + 96*a^3*b^4*c^2 - 256*a^4*b^2*c^3)))^(1/4)*2i - atan((((-(b^5 + (-(4*a*c
- b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(512*(a*b^8 + 256*a^5*c^4 - 16*
a^2*b^6*c + 96*a^3*b^4*c^2 - 256*a^4*b^2*c^3)))^(3/4)*(x*(-(b^5 + (-(4*a*c
- b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(512*(a*b^8 + 256*a^5*c^4 - 16*
a^2*b^6*c + 96*a^3*b^4*c^2 - 256*a^4*b^2*c^3)))^(1/4)*(32768*a^4*c^7 - 1024
*a*b^6*c^4 + 10240*a^2*b^4*c^5 - 32768*a^3*b^2*c^6) - 256*a*b^5*c^4 - 4096*
a^3*b*c^6 + 2048*a^2*b^3*c^5) - 4*a*b*c^5*x)*(-(b^5 + (-(4*a*c - b^2)^5)^(1
/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(512*(a*b^8 + 256*a^5*c^4 - 16*a^2*b^6*c +
96*a^3*b^4*c^2 - 256*a^4*b^2*c^3)))^(1/4)*1i + ((-(b^5 + (-(4*a*c - b^2)^5)
^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(512*(a*b^8 + 256*a^5*c^4 - 16*a^2*b^6*c
 + 96*a^3*b^4*c^2 - 256*a^4*b^2*c^3)))^(3/4)*(x*(-(b^5 + (-(4*a*c - b^2)^5)
^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(512*(a*b^8 + 256*a^5*c^4 - 16*a^2*b^6*c
 + 96*a^3*b^4*c^2 - 256*a^4*b^2*c^3)))^(1/4)*(32768*a^4*c^7 - 1024*a*b^6*c^
4 + 10240*a^2*b^4*c^5 - 32768*a^3*b^2*c^6) + 256*a*b^5*c^4 + 4096*a^3*b*c^6
 - 2048*a^2*b^3*c^5) - 4*a*b*c^5*x)*(-(b^5 + (-(4*a*c - b^2)^5)^(1/2) + 16*
a^2*b*c^2 - 8*a*b^3*c)/(512*(a*b^8 + 256*a^5*c^4 - 16*a^2*b^6*c + 96*a^3*b^
4*c^2 - 256*a^4*b^2*c^3)))^(1/4)*1i)/(((-(b^5 + (-(4*a*c - b^2)^5)^(1/2) +
16*a^2*b*c^2 - 8*a*b^3*c)/(512*(a*b^8 + 256*a^5*c^4 - 16*a^2*b^6*c + 96*a^3
*b^4*c^2 - 256*a^4*b^2*c^3)))^(3/4)*(x*(-(b^5 + (-(4*a*c - b^2)^5)^(1/2) +
16*a^2*b*c^2 - 8*a*b^3*c)/(512*(a*b^8 + 256*a^5*c^4 - 16*a^2*b^6*c + 96*a^3
*b^4*c^2 - 256*a^4*b^2*c^3)))^(1/4)*(32768*a^4*c^7 - 1024*a*b^6*c^4 + 10240
*a^2*b^4*c^5 - 32768*a^3*b^2*c^6) + 256*a*b^5*c^4 + 4096*a^3*b*c^6 - 2048*a
^2*b^3*c^5) - 4*a*b*c^5*x)*(-(b^5 + (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2
 - 8*a*b^3*c)/(512*(a*b^8 + 256*a^5*c^4 - 16*a^2*b^6*c + 96*a^3*b^4*c^2 - 2
56*a^4*b^2*c^3)))^(1/4) - ((-(b^5 + (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2
 - 8*a*b^3*c)/(512*(a*b^8 + 256*a^5*c^4 - 16*a^2*b^6*c + 96*a^3*b^4*c^2 - 2
56*a^4*b^2*c^3)))^(3/4)*(x*(-(b^5 + (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2
 - 8*a*b^3*c)/(512*(a*b^8 + 256*a^5*c^4 - 16*a^2*b^6*c + 96*a^3*b^4*c^2 - 2
56*a^4*b^2*c^3)))^(1/4)*(32768*a^4*c^7 - 1024*a*b^6*c^4 + 10240*a^2*b^4*c^5
 - 32768*a^3*b^2*c^6) - 256*a*b^5*c^4 - 4096*a^3*b*c^6 + 2048*a^2*b^3*c^5)
- 4*a*b*c^5*x)*(-(b^5 + (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c
)/(512*(a*b^8 + 256*a^5*c^4 - 16*a^2*b^6*c + 96*a^3*b^4*c^2 - 256*a^4*b^2*c
```

```
^3)))^(1/4) + 2*a*c^5))*(-(b^5 + (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 -
8*a*b^3*c)/(512*(a*b^8 + 256*a^5*c^4 - 16*a^2*b^6*c + 96*a^3*b^4*c^2 - 256*
a^4*b^2*c^3)))^(1/4)*2i + 2*atan((((-(b^5 + (-(4*a*c - b^2)^5)^(1/2) + 16*a
^2*b*c^2 - 8*a*b^3*c)/(512*(a*b^8 + 256*a^5*c^4 - 16*a^2*b^6*c + 96*a^3*b^4
*c^2 - 256*a^4*b^2*c^3)))^(3/4)*(x*(-(b^5 + (-(4*a*c - b^2)^5)^(1/2) + 16*a
^2*b*c^2 - 8*a*b^3*c)/(512*(a*b^8 + 256*a^5*c^4 - 16*a^2*b^6*c + 96*a^3*b^4
*c^2 - 256*a^4*b^2*c^3)))^(1/4)*(32768*a^4*c^7 - 1024*a*b^6*c^4 + 10240*a^2
*b^4*c^5 - 32768*a^3*b^2*c^6)*1i - 256*a*b^5*c^4 - 4096*a^3*b*c^6 + 2048*a^
2*b^3*c^5)*1i + 4*a*b*c^5*x)*(-(b^5 + (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c
^2 - 8*a*b^3*c)/(512*(a*b^8 + 256*a^5*c^4 - 16*a^2*b^6*c + 96*a^3*b^4*c^2 -
 256*a^4*b^2*c^3)))^(1/4) + ((-(b^5 + (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c
^2 - 8*a*b^3*c)/(512*(a*b^8 + 256*a^5*c^4 - 16*a^2*b^6*c + 96*a^3*b^4*c^2 -
 256*a^4*b^2*c^3)))^(3/4)*(x*(-(b^5 + (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c
^2 - 8*a*b^3*c)/(512*(a*b^8 + 256*a^5*c^4 - 16*a^2*b^6*c + 96*a^3*b^4*c^2 -
 256*a^4*b^2*c^3)))^(1/4)*(32768*a^4*c^7 - 1024*a*b^6*c^4 + 10240*a^2*b^4*c
^5 - 32768*a^3*b^2*c^6)*1i + 256*a*b^5*c^4 + 4096*a^3*b*c^6 - 2048*a^2*b^3*
c^5)*1i + 4*a*b*c^5*x)*(-(b^5 + (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8
*a*b^3*c)/(512*(a*b^8 + 256*a^5*c^4 - 16*a^2*b^6*c + 96*a^3*b^4*c^2 - 256*a
^4*b^2*c^3)))^(1/4))/(((-(b^5 + (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8
*a*b^3*c)/(512*(a*b^8 + 256*a^5*c^4 - 16*a^2*b^6*c + 96*a^3*b^4*c^2 - 256*a
^4*b^2*c^3)))^(3/4)*(x*(-(b^5 + (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8
*a*b^3*c)/(512*(a*b^8 + 256*a^5*c^4 - 16*a^2*b^6*c + 96*a^3*b^4*c^2 - 256*a
^4*b^2*c^3)))^(1/4)*(32768*a^4*c^7 - 1024*a*b^6*c^4 + 10240*a^2*b^4*c^5 - 3
2768*a^3*b^2*c^6)*1i - 256*a*b^5*c^4 - 4096*a^3*b*c^6 + 2048*a^2*b^3*c^5)*1
i + 4*a*b*c^5*x)*(-(b^5 + (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3
*c)/(512*(a*b^8 + 256*a^5*c^4 - 16*a^2*b^6*c + 96*a^3*b^4*c^2 - 256*a^4*b^2
*c^3)))^(1/4)*1i - ((-(b^5 + (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*
b^3*c)/(512*(a*b^8 + 256*a^5*c^4 - 16*a^2*b^6*c + 96*a^3*b^4*c^2 - 256*a^4*
b^2*c^3)))^(3/4)*(x*(-(b^5 + (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*
b^3*c)/(512*(a*b^8 + 256*a^5*c^4 - 16*a^2*b^6*c + 96*a^3*b^4*c^2 - 256*a^4*
b^2*c^3)))^(1/4)*(32768*a^4*c^7 - 1024*a*b^6*c^4 + 10240*a^2*b^4*c^5 - 3276
8*a^3*b^2*c^6)*1i + 256*a*b^5*c^4 + 4096*a^3*b*c^6 - 2048*a^2*b^3*c^5)*1i +
 4*a*b*c^5*x)*(-(b^5 + (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)
/(512*(a*b^8 + 256*a^5*c^4 - 16*a^2*b^6*c + 96*a^3*b^4*c^2 - 256*a^4*b^2*c^
3)))^(1/4)*1i + 2*a*c^5))*(-(b^5 + (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2
- 8*a*b^3*c)/(512*(a*b^8 + 256*a^5*c^4 - 16*a^2*b^6*c + 96*a^3*b^4*c^2 - 25
6*a^4*b^2*c^3)))^(1/4)
```

sympy [A] time = 4.10, size = 172, normalized size = 0.55

$$\operatorname{RootSum}\left(t^{8}\left(16777216a^{5}c^{4} - 16777216a^{4}b^{2}c^{3} + 6291456a^{3}b^{4}c^{2} - 1048576a^{2}b^{6}c + 65536ab^{8}\right) + t^{4}\left(4096a^{2}bc^{2} - 2048ab^{3}c + 256b^{5}\right) + c, \left(t \mapsto t\log\left(x + \frac{1048576t^{7}a^{4}bc^{3} - 786432t^{7}a^{3}b^{3}c^{2} + 196608t^{7}a^{2}b^{5}c - 16384t^{7}ab^{7} - 512t^{3}a^{2}c^{2} + 384t^{3}ab^{2}c - 64t^{3}b^{4}}{c}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(c*x**8+b*x**4+a),x)
```

```
[Out] RootSum(_t**8*(16777216*a**5*c**4 - 16777216*a**4*b**2*c**3 + 6291456*a**3*
b**4*c**2 - 1048576*a**2*b**6*c + 65536*a*b**8) + _t**4*(4096*a**2*b*c**2 -
 2048*a*b**3*c + 256*b**5) + c, Lambda(_t, _t*log(x + (1048576*_t**7*a**4*b
*c**3 - 786432*_t**7*a**3*b**3*c**2 + 196608*_t**7*a**2*b**5*c - 16384*_t**
7*a*b**7 - 512*_t**3*a**2*c**2 + 384*_t**3*a*b**2*c - 64*_t**3*b**4)/c)))
```

3.267 $\int \frac{1}{a+bx^4+cx^8}\,dx$

Optimal. Leaf size=315

$$\frac{c^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{\sqrt[4]{2}\sqrt{b^2-4ac}\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}-\frac{c^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{\sqrt[4]{2}\sqrt{b^2-4ac}\left(\sqrt{b^2-4ac}-b\right)^{3/4}}+\frac{c^{3/4}\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{\sqrt[4]{2}\sqrt{b^2-4ac}\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}-$$

Rubi [A] time = 0.30, antiderivative size = 315, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {1347, 212, 208, 205}

$$\frac{c^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{\sqrt[4]{2}\sqrt{b^2-4ac}\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}-\frac{c^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{\sqrt[4]{2}\sqrt{b^2-4ac}\left(\sqrt{b^2-4ac}-b\right)^{3/4}}+\frac{c^{3/4}\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{\sqrt[4]{2}\sqrt{b^2-4ac}\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}-\frac{c^{3/4}\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{\sqrt[4]{2}\sqrt{b^2-4ac}\left(\sqrt{b^2-4ac}-b\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^4 + c*x^8)^(-1),x]
```

```
[Out] (c^(3/4)*ArcTan[(2^(1/4)*c^(1/4)*x)/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(2^(1/
4)*Sqrt[b^2 - 4*a*c]*(-b - Sqrt[b^2 - 4*a*c])^(3/4)) - (c^(3/4)*ArcTan[(2^(
1/4)*c^(1/4)*x)/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(2^(1/4)*Sqrt[b^2 - 4*a*c]
*(-b + Sqrt[b^2 - 4*a*c])^(3/4)) + (c^(3/4)*ArcTanh[(2^(1/4)*c^(1/4)*x)/(-b
 - Sqrt[b^2 - 4*a*c])^(1/4)])/(2^(1/4)*Sqrt[b^2 - 4*a*c]*(-b - Sqrt[b^2 - 4
*a*c])^(3/4)) - (c^(3/4)*ArcTanh[(2^(1/4)*c^(1/4)*x)/(-b + Sqrt[b^2 - 4*a*c
])^(1/4)])/(2^(1/4)*Sqrt[b^2 - 4*a*c]*(-b + Sqrt[b^2 - 4*a*c])^(3/4))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 1347

```
Int[((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^(-1), x_Symbol] :> With[{q
= Rt[b^2 - 4*a*c, 2]}, Dist[c/q, Int[1/(b/2 - q/2 + c*x^n), x], x] - Dist[c
/q, Int[1/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*
n] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{1}{a+bx^4+cx^8}\,dx = \frac{c\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^4}\,dx}{\sqrt{b^2-4ac}} - \frac{c\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^4}\,dx}{\sqrt{b^2-4ac}}$$

$$= \frac{c\int \frac{1}{\sqrt{-b-\sqrt{b^2-4ac}}-\sqrt{2}\sqrt{c}x^2}\,dx}{\sqrt{b^2-4ac}\sqrt{-b-\sqrt{b^2-4ac}}} + \frac{c\int \frac{1}{\sqrt{-b-\sqrt{b^2-4ac}}+\sqrt{2}\sqrt{c}x^2}\,dx}{\sqrt{b^2-4ac}\sqrt{-b-\sqrt{b^2-4ac}}} - \frac{c\int \frac{1}{\sqrt{-b+\sqrt{b^2-4ac}}-\sqrt{2}\sqrt{c}x^2}}{\sqrt{b^2-4ac}\sqrt{-b+\sqrt{b^2-}}}$$

$$= \frac{c^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{\sqrt[4]{2}\sqrt{b^2-4ac}\left(-b-\sqrt{b^2-4ac}\right)^{3/4}} - \frac{c^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b+\sqrt{b^2-4ac}}}\right)}{\sqrt[4]{2}\sqrt{b^2-4ac}\left(-b+\sqrt{b^2-4ac}\right)^{3/4}} + \frac{c^{3/4}\tanh}{\sqrt[4]{2}\sqrt{b^2-4ac}}$$

Mathematica [C] time = 0.02, size = 45, normalized size = 0.14

$$\frac{1}{4}\text{RootSum}\left[\#1^8c+\#1^4b+a\&,\frac{\log(x-\#1)}{2\#1^7c+\#1^3b}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^4 + c*x^8)^(-1),x]
[Out] RootSum[a + b*#1^4 + c*#1^8 & , Log[x - #1]/(b*#1^3 + 2*c*#1^7) & ]/4
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{a+bx^4+cx^8}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x^4 + c*x^8)^(-1),x]
[Out] IntegrateAlgebraic[(a + b*x^4 + c*x^8)^(-1), x]
```

fricas [B] time = 1.87, size = 3929, normalized size = 12.47

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^8+b*x^4+a),x, algorithm="fricas")
[Out] -sqrt(sqrt(1/2)*sqrt(-(b^3 - 3*a*b*c + (a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)
*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2
- 64*a^9*c^3)))/(a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)))*arctan(1/4*(2*sqrt(1
/2)*((a^3*b^8 - 14*a^4*b^6*c + 72*a^5*b^4*c^2 - 160*a^6*b^2*c^3 + 128*a^7*c
^4)*x*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2
*c^2 - 64*a^9*c^3)) - (b^7 - 9*a*b^5*c + 24*a^2*b^3*c^2 - 16*a^3*b*c^3)*x)*
sqrt(-(b^3 - 3*a*b*c + (a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*sqrt((b^4 - 2*a
*b^2*c + a^2*c^2)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))/
(a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)) + (b^7 - 9*a*b^5*c + 24*a^2*b^3*c^2 -
 16*a^3*b*c^3 - (a^3*b^8 - 14*a^4*b^6*c + 72*a^5*b^4*c^2 - 160*a^6*b^2*c^3
+ 128*a^7*c^4)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^6 - 12*a^7*b^4*c + 4
8*a^8*b^2*c^2 - 64*a^9*c^3)))*sqrt(-(b^3 - 3*a*b*c + (a^3*b^4 - 8*a^4*b^2*c
 + 16*a^5*c^2)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^6 - 12*a^7*b^4*c + 4
8*a^8*b^2*c^2 - 64*a^9*c^3)))/(a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2))*sqrt((2
*(b^2*c^2 - a*c^3)*x^2 + sqrt(1/2)*(b^6 - 7*a*b^4*c + 14*a^2*b^2*c^2 - 8*a^
3*c^3 - (a^3*b^7 - 12*a^4*b^5*c + 48*a^5*b^3*c^2 - 64*a^6*b*c^3)*sqrt((b^4
- 2*a*b^2*c + a^2*c^2)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^
```

```
3)))*sqrt(-(b^3 - 3*a*b*c + (a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*sqrt((b^4
- 2*a*b^2*c + a^2*c^2)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^
3)))/(a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)))/(b^2*c^2 - a*c^3)))*sqrt(sqrt(1
/2)*sqrt(-(b^3 - 3*a*b*c + (a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*sqrt((b^4 -
 2*a*b^2*c + a^2*c^2)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3
)))/(a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)))/(b^2*c^2 - a*c^3)) + sqrt(sqrt(1
/2)*sqrt(-(b^3 - 3*a*b*c - (a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*sqrt((b^4 -
 2*a*b^2*c + a^2*c^2)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3
)))/(a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)))*arctan(1/4*(2*sqrt(1/2)*((a^3*b^
8 - 14*a^4*b^6*c + 72*a^5*b^4*c^2 - 160*a^6*b^2*c^3 + 128*a^7*c^4)*x*sqrt((
b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^
9*c^3)) + (b^7 - 9*a*b^5*c + 24*a^2*b^3*c^2 - 16*a^3*b*c^3)*x)*sqrt(sqrt(1/
2)*sqrt(-(b^3 - 3*a*b*c - (a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*sqrt((b^4 -
2*a*b^2*c + a^2*c^2)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)
))/(a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)))*sqrt(-(b^3 - 3*a*b*c - (a^3*b^4 -
 8*a^4*b^2*c + 16*a^5*c^2)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^6 - 12*a
^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))/(a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c
^2)) - (b^7 - 9*a*b^5*c + 24*a^2*b^3*c^2 - 16*a^3*b*c^3 + (a^3*b^8 - 14*a^4
*b^6*c + 72*a^5*b^4*c^2 - 160*a^6*b^2*c^3 + 128*a^7*c^4)*sqrt((b^4 - 2*a*b^
2*c + a^2*c^2)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))*sqr
t(sqrt(1/2)*sqrt(-(b^3 - 3*a*b*c - (a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*sqr
t((b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64
*a^9*c^3)))/(a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)))*sqrt(-(b^3 - 3*a*b*c - (
a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b
^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))/(a^3*b^4 - 8*a^4*b^2*c +
 16*a^5*c^2))*sqrt((2*(b^2*c^2 - a*c^3)*x^2 + sqrt(1/2)*(b^6 - 7*a*b^4*c +
14*a^2*b^2*c^2 - 8*a^3*c^3 + (a^3*b^7 - 12*a^4*b^5*c + 48*a^5*b^3*c^2 - 64*
a^6*b*c^3)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^
8*b^2*c^2 - 64*a^9*c^3)))*sqrt(-(b^3 - 3*a*b*c - (a^3*b^4 - 8*a^4*b^2*c + 1
6*a^5*c^2)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^
8*b^2*c^2 - 64*a^9*c^3)))/(a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)))/(b^2*c^2 -
 a*c^3)))/(b^2*c^2 - a*c^3)) + 1/4*sqrt(sqrt(1/2)*sqrt(-(b^3 - 3*a*b*c + (a
^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^
6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))/(a^3*b^4 - 8*a^4*b^2*c +
16*a^5*c^2)))*log(-(b^2*c - a*c^2)*x + 1/2*(b^4 - 5*a*b^2*c + 4*a^2*c^2 - (
a^3*b^5 - 8*a^4*b^3*c + 16*a^5*b*c^2)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(a^6
*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))*sqrt(sqrt(1/2)*sqrt(-(
b^3 - 3*a*b*c + (a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*sqrt((b^4 - 2*a*b^2*c
+ a^2*c^2)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))/(a^3*b^
4 - 8*a^4*b^2*c + 16*a^5*c^2)))) - 1/4*sqrt(sqrt(1/2)*sqrt(-(b^3 - 3*a*b*c
+ (a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(a^
6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))/(a^3*b^4 - 8*a^4*b^2*
c + 16*a^5*c^2)))*log(-(b^2*c - a*c^2)*x - 1/2*(b^4 - 5*a*b^2*c + 4*a^2*c^2
 - (a^3*b^5 - 8*a^4*b^3*c + 16*a^5*b*c^2)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/
(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))*sqrt(sqrt(1/2)*sqr
t(-(b^3 - 3*a*b*c + (a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*sqrt((b^4 - 2*a*b^
2*c + a^2*c^2)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))/(a^
3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)))) + 1/4*sqrt(sqrt(1/2)*sqrt(-(b^3 - 3*a*
b*c - (a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)
/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))/(a^3*b^4 - 8*a^4*
b^2*c + 16*a^5*c^2)))*log(-(b^2*c - a*c^2)*x + 1/2*(b^4 - 5*a*b^2*c + 4*a^2
*c^2 + (a^3*b^5 - 8*a^4*b^3*c + 16*a^5*b*c^2)*sqrt((b^4 - 2*a*b^2*c + a^2*c
^2)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))*sqrt(sqrt(1/2)
*sqrt(-(b^3 - 3*a*b*c - (a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*sqrt((b^4 - 2*
a*b^2*c + a^2*c^2)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))
/(a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)))) - 1/4*sqrt(sqrt(1/2)*sqrt(-(b^3 -
3*a*b*c - (a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*sqrt((b^4 - 2*a*b^2*c + a^2*
c^2)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))/(a^3*b^4 - 8*
a^4*b^2*c + 16*a^5*c^2)))*log(-(b^2*c - a*c^2)*x - 1/2*(b^4 - 5*a*b^2*c + 4
```

```
*a^2*c^2 + (a^3*b^5 - 8*a^4*b^3*c + 16*a^5*b*c^2)*sqrt((b^4 - 2*a*b^2*c + a
^2*c^2)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))*sqrt(sqrt(
1/2)*sqrt(-(b^3 - 3*a*b*c - (a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*sqrt((b^4
- 2*a*b^2*c + a^2*c^2)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^
3)))/(a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2))))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{cx^8 + bx^4 + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^8+b*x^4+a),x, algorithm="giac")
```

```
[Out] integrate(1/(c*x^8 + b*x^4 + a), x)
```

maple [C] time = 0.00, size = 40, normalized size = 0.13

$$\frac{\ln\left(-\mathrm{RootOf}\left(\_Z^8 c + b\_Z^4 + a\right) + x\right)}{8\,\mathrm{RootOf}\left(\_Z^8 c + b\_Z^4 + a\right)^7 c + 4\,\mathrm{RootOf}\left(\_Z^8 c + b\_Z^4 + a\right)^3 b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x^8+b*x^4+a),x)
```

```
[Out] 1/4*sum(1/(2*_R^7*c+_R^3*b)*ln(-_R+x),_R=RootOf(_Z^8*c+_Z^4*b+a))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{cx^8 + bx^4 + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^8+b*x^4+a),x, algorithm="maxima")
```

```
[Out] integrate(1/(c*x^8 + b*x^4 + a), x)
```

mupad [B] time = 3.42, size = 10337, normalized size = 32.82

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*x^4 + c*x^8),x)
```

```
[Out] - atan((((-(b^7 + b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*
c^2 - 11*a*b^5*c - a*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^3*b^8 + 256*a^7*c^
4 - 16*a^4*b^6*c + 96*a^5*b^4*c^2 - 256*a^6*b^2*c^3)))^(1/4)*(64*a*c^7 + ((
-(b^7 + b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a
*b^5*c - a*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^3*b^8 + 256*a^7*c^4 - 16*a^4
*b^6*c + 96*a^5*b^4*c^2 - 256*a^6*b^2*c^3)))^(1/4)*(4096*a*b^7*c^4 - 262144
*a^4*b*c^7 - 49152*a^2*b^5*c^5 + 196608*a^3*b^3*c^6) + x*(1024*b^7*c^4 - 11
264*a*b^5*c^5 - 49152*a^3*b*c^7 + 40960*a^2*b^3*c^6))*(-(b^7 + b^2*(-(4*a*c
 - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c - a*c*(-(4*a*
c - b^2)^5)^(1/2))/(512*(a^3*b^8 + 256*a^7*c^4 - 16*a^4*b^6*c + 96*a^5*b^4*
c^2 - 256*a^6*b^2*c^3)))^(3/4) - 16*b^2*c^6) + 8*c^7*x)*(-(b^7 + b^2*(-(4*a
*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c - a*c*(-(4*
a*c - b^2)^5)^(1/2))/(512*(a^3*b^8 + 256*a^7*c^4 - 16*a^4*b^6*c + 96*a^5*b^
4*c^2 - 256*a^6*b^2*c^3)))^(1/4)*1i - ((-(b^7 + b^2*(-(4*a*c - b^2)^5)^(1/2
) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c - a*c*(-(4*a*c - b^2)^5)^(1/
2))/(512*(a^3*b^8 + 256*a^7*c^4 - 16*a^4*b^6*c + 96*a^5*b^4*c^2 - 256*a^6*b
```

```
^2*c^3)))^(1/4)*(64*a*c^7 + ((-(b^7 + b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3
*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c - a*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(
a^3*b^8 + 256*a^7*c^4 - 16*a^4*b^6*c + 96*a^5*b^4*c^2 - 256*a^6*b^2*c^3)))^
(1/4)*(4096*a*b^7*c^4 - 262144*a^4*b*c^7 - 49152*a^2*b^5*c^5 + 196608*a^3*b
^3*c^6) - x*(1024*b^7*c^4 - 11264*a*b^5*c^5 - 49152*a^3*b*c^7 + 40960*a^2*b
^3*c^6))*(-(b^7 + b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*
c^2 - 11*a*b^5*c - a*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^3*b^8 + 256*a^7*c^
4 - 16*a^4*b^6*c + 96*a^5*b^4*c^2 - 256*a^6*b^2*c^3)))^(3/4) - 16*b^2*c^6)
 - 8*c^7*x)*(-(b^7 + b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^
3*c^2 - 11*a*b^5*c - a*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^3*b^8 + 256*a^7*
c^4 - 16*a^4*b^6*c + 96*a^5*b^4*c^2 - 256*a^6*b^2*c^3)))^(1/4)*1i)/(((-(b^7
 + b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*
c - a*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^3*b^8 + 256*a^7*c^4 - 16*a^4*b^6*
c + 96*a^5*b^4*c^2 - 256*a^6*b^2*c^3)))^(1/4)*(64*a*c^7 + ((-(b^7 + b^2*(-(
4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c - a*c*(-
(4*a*c - b^2)^5)^(1/2))/(512*(a^3*b^8 + 256*a^7*c^4 - 16*a^4*b^6*c + 96*a^5
*b^4*c^2 - 256*a^6*b^2*c^3)))^(1/4)*(4096*a*b^7*c^4 - 262144*a^4*b*c^7 - 49
152*a^2*b^5*c^5 + 196608*a^3*b^3*c^6) + x*(1024*b^7*c^4 - 11264*a*b^5*c^5 -
 49152*a^3*b*c^7 + 40960*a^2*b^3*c^6))*(-(b^7 + b^2*(-(4*a*c - b^2)^5)^(1/2
) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c - a*c*(-(4*a*c - b^2)^5)^(1/
2))/(512*(a^3*b^8 + 256*a^7*c^4 - 16*a^4*b^6*c + 96*a^5*b^4*c^2 - 256*a^6*b
^2*c^3)))^(3/4) - 16*b^2*c^6) + 8*c^7*x)*(-(b^7 + b^2*(-(4*a*c - b^2)^5)^(1
/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c - a*c*(-(4*a*c - b^2)^5)^(
1/2))/(512*(a^3*b^8 + 256*a^7*c^4 - 16*a^4*b^6*c + 96*a^5*b^4*c^2 - 256*a^6
*b^2*c^3)))^(1/4) + ((-(b^7 + b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 +
 40*a^2*b^3*c^2 - 11*a*b^5*c - a*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^3*b^8
+ 256*a^7*c^4 - 16*a^4*b^6*c + 96*a^5*b^4*c^2 - 256*a^6*b^2*c^3)))^(1/4)*(6
4*a*c^7 + ((-(b^7 + b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^
3*c^2 - 11*a*b^5*c - a*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^3*b^8 + 256*a^7*
c^4 - 16*a^4*b^6*c + 96*a^5*b^4*c^2 - 256*a^6*b^2*c^3)))^(1/4)*(4096*a*b^7*
c^4 - 262144*a^4*b*c^7 - 49152*a^2*b^5*c^5 + 196608*a^3*b^3*c^6) - x*(1024*
b^7*c^4 - 11264*a*b^5*c^5 - 49152*a^3*b*c^7 + 40960*a^2*b^3*c^6))*(-(b^7 +
b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c -
 a*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^3*b^8 + 256*a^7*c^4 - 16*a^4*b^6*c +
 96*a^5*b^4*c^2 - 256*a^6*b^2*c^3)))^(3/4) - 16*b^2*c^6) - 8*c^7*x)*(-(b^7
+ b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c
 - a*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^3*b^8 + 256*a^7*c^4 - 16*a^4*b^6*c
 + 96*a^5*b^4*c^2 - 256*a^6*b^2*c^3)))^(1/4)))*(-(b^7 + b^2*(-(4*a*c - b^2)
^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c - a*c*(-(4*a*c - b^2
)^5)^(1/2))/(512*(a^3*b^8 + 256*a^7*c^4 - 16*a^4*b^6*c + 96*a^5*b^4*c^2 - 2
56*a^6*b^2*c^3)))^(1/4)*2i - atan((((-(b^7 - b^2*(-(4*a*c - b^2)^5)^(1/2) -
 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c + a*c*(-(4*a*c - b^2)^5)^(1/2))
/(512*(a^3*b^8 + 256*a^7*c^4 - 16*a^4*b^6*c + 96*a^5*b^4*c^2 - 256*a^6*b^2*
c^3)))^(1/4)*(64*a*c^7 + ((-(b^7 - b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*
c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c + a*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^3
*b^8 + 256*a^7*c^4 - 16*a^4*b^6*c + 96*a^5*b^4*c^2 - 256*a^6*b^2*c^3)))^(1/
4)*(4096*a*b^7*c^4 - 262144*a^4*b*c^7 - 49152*a^2*b^5*c^5 + 196608*a^3*b^3*
c^6) + x*(1024*b^7*c^4 - 11264*a*b^5*c^5 - 49152*a^3*b*c^7 + 40960*a^2*b^3*
c^6))*(-(b^7 - b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2
 - 11*a*b^5*c + a*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^3*b^8 + 256*a^7*c^4 -
 16*a^4*b^6*c + 96*a^5*b^4*c^2 - 256*a^6*b^2*c^3)))^(3/4) - 16*b^2*c^6) + 8
*c^7*x)*(-(b^7 - b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c
^2 - 11*a*b^5*c + a*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^3*b^8 + 256*a^7*c^4
 - 16*a^4*b^6*c + 96*a^5*b^4*c^2 - 256*a^6*b^2*c^3)))^(1/4)*1i - ((-(b^7 -
b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c +
 a*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^3*b^8 + 256*a^7*c^4 - 16*a^4*b^6*c +
 96*a^5*b^4*c^2 - 256*a^6*b^2*c^3)))^(1/4)*(64*a*c^7 + ((-(b^7 - b^2*(-(4*a
*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c + a*c*(-(4*
a*c - b^2)^5)^(1/2))/(512*(a^3*b^8 + 256*a^7*c^4 - 16*a^4*b^6*c + 96*a^5*b^
```

```
4*c^2 - 256*a^6*b^2*c^3)))^(1/4)*(4096*a*b^7*c^4 - 262144*a^4*b*c^7 - 49152
*a^2*b^5*c^5 + 196608*a^3*b^3*c^6) - x*(1024*b^7*c^4 - 11264*a*b^5*c^5 - 49
152*a^3*b*c^7 + 40960*a^2*b^3*c^6))*(-(b^7 - b^2*(-(4*a*c - b^2)^5)^(1/2) -
 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c + a*c*(-(4*a*c - b^2)^5)^(1/2))
/(512*(a^3*b^8 + 256*a^7*c^4 - 16*a^4*b^6*c + 96*a^5*b^4*c^2 - 256*a^6*b^2*
c^3)))^(3/4) - 16*b^2*c^6) - 8*c^7*x)*(-(b^7 - b^2*(-(4*a*c - b^2)^5)^(1/2)
 - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c + a*c*(-(4*a*c - b^2)^5)^(1/2
))/(512*(a^3*b^8 + 256*a^7*c^4 - 16*a^4*b^6*c + 96*a^5*b^4*c^2 - 256*a^6*b^
2*c^3)))^(1/4)*1i)/(((-(b^7 - b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 +
 40*a^2*b^3*c^2 - 11*a*b^5*c + a*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^3*b^8
+ 256*a^7*c^4 - 16*a^4*b^6*c + 96*a^5*b^4*c^2 - 256*a^6*b^2*c^3)))^(1/4)*(6
4*a*c^7 + ((-(b^7 - b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^
3*c^2 - 11*a*b^5*c + a*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^3*b^8 + 256*a^7*
c^4 - 16*a^4*b^6*c + 96*a^5*b^4*c^2 - 256*a^6*b^2*c^3)))^(1/4)*(4096*a*b^7*
c^4 - 262144*a^4*b*c^7 - 49152*a^2*b^5*c^5 + 196608*a^3*b^3*c^6) + x*(1024*
b^7*c^4 - 11264*a*b^5*c^5 - 49152*a^3*b*c^7 + 40960*a^2*b^3*c^6))*(-(b^7 -
b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c +
 a*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^3*b^8 + 256*a^7*c^4 - 16*a^4*b^6*c +
 96*a^5*b^4*c^2 - 256*a^6*b^2*c^3)))^(3/4) - 16*b^2*c^6) + 8*c^7*x)*(-(b^7
- b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c
 + a*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^3*b^8 + 256*a^7*c^4 - 16*a^4*b^6*c
 + 96*a^5*b^4*c^2 - 256*a^6*b^2*c^3)))^(1/4) + ((-(b^7 - b^2*(-(4*a*c - b^2
)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c + a*c*(-(4*a*c - b^
2)^5)^(1/2))/(512*(a^3*b^8 + 256*a^7*c^4 - 16*a^4*b^6*c + 96*a^5*b^4*c^2 -
256*a^6*b^2*c^3)))^(1/4)*(64*a*c^7 + ((-(b^7 - b^2*(-(4*a*c - b^2)^5)^(1/2)
 - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c + a*c*(-(4*a*c - b^2)^5)^(1/2
))/(512*(a^3*b^8 + 256*a^7*c^4 - 16*a^4*b^6*c + 96*a^5*b^4*c^2 - 256*a^6*b^
2*c^3)))^(1/4)*(4096*a*b^7*c^4 - 262144*a^4*b*c^7 - 49152*a^2*b^5*c^5 + 196
608*a^3*b^3*c^6) - x*(1024*b^7*c^4 - 11264*a*b^5*c^5 - 49152*a^3*b*c^7 + 40
960*a^2*b^3*c^6))*(-(b^7 - b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40
*a^2*b^3*c^2 - 11*a*b^5*c + a*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^3*b^8 + 2
56*a^7*c^4 - 16*a^4*b^6*c + 96*a^5*b^4*c^2 - 256*a^6*b^2*c^3)))^(3/4) - 16*
b^2*c^6) - 8*c^7*x)*(-(b^7 - b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 +
40*a^2*b^3*c^2 - 11*a*b^5*c + a*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^3*b^8 +
 256*a^7*c^4 - 16*a^4*b^6*c + 96*a^5*b^4*c^2 - 256*a^6*b^2*c^3)))^(1/4)))*(
-(b^7 - b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a
*b^5*c + a*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^3*b^8 + 256*a^7*c^4 - 16*a^4
*b^6*c + 96*a^5*b^4*c^2 - 256*a^6*b^2*c^3)))^(1/4)*2i - 2*atan((((-(b^7 + b
^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c -
a*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^3*b^8 + 256*a^7*c^4 - 16*a^4*b^6*c +
96*a^5*b^4*c^2 - 256*a^6*b^2*c^3)))^(1/4)*(((-(b^7 + b^2*(-(4*a*c - b^2)^5)
^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c - a*c*(-(4*a*c - b^2)^5
)^(1/2))/(512*(a^3*b^8 + 256*a^7*c^4 - 16*a^4*b^6*c + 96*a^5*b^4*c^2 - 256*
a^6*b^2*c^3)))^(1/4)*(4096*a*b^7*c^4 - 262144*a^4*b*c^7 - 49152*a^2*b^5*c^5
 + 196608*a^3*b^3*c^6)*1i + x*(1024*b^7*c^4 - 11264*a*b^5*c^5 - 49152*a^3*b
*c^7 + 40960*a^2*b^3*c^6))*(-(b^7 + b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b
*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c - a*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^
3*b^8 + 256*a^7*c^4 - 16*a^4*b^6*c + 96*a^5*b^4*c^2 - 256*a^6*b^2*c^3)))^(3
/4)*1i - 64*a*c^7 + 16*b^2*c^6)*1i - 8*c^7*x)*(-(b^7 + b^2*(-(4*a*c - b^2)^
5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c - a*c*(-(4*a*c - b^2)
^5)^(1/2))/(512*(a^3*b^8 + 256*a^7*c^4 - 16*a^4*b^6*c + 96*a^5*b^4*c^2 - 25
6*a^6*b^2*c^3)))^(1/4) - ((-(b^7 + b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*
c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c - a*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^3
*b^8 + 256*a^7*c^4 - 16*a^4*b^6*c + 96*a^5*b^4*c^2 - 256*a^6*b^2*c^3)))^(1/
4)*(((-(b^7 + b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2
- 11*a*b^5*c - a*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^3*b^8 + 256*a^7*c^4 -
16*a^4*b^6*c + 96*a^5*b^4*c^2 - 256*a^6*b^2*c^3)))^(1/4)*(4096*a*b^7*c^4 -
262144*a^4*b*c^7 - 49152*a^2*b^5*c^5 + 196608*a^3*b^3*c^6)*1i - x*(1024*b^7
*c^4 - 11264*a*b^5*c^5 - 49152*a^3*b*c^7 + 40960*a^2*b^3*c^6))*(-(b^7 + b^2
```

```
*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c - a*
c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^3*b^8 + 256*a^7*c^4 - 16*a^4*b^6*c + 96
*a^5*b^4*c^2 - 256*a^6*b^2*c^3)))^(3/4)*1i - 64*a*c^7 + 16*b^2*c^6)*1i + 8*
c^7*x)*(-(b^7 + b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^
2 - 11*a*b^5*c - a*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^3*b^8 + 256*a^7*c^4
- 16*a^4*b^6*c + 96*a^5*b^4*c^2 - 256*a^6*b^2*c^3)))^(1/4))/(((-(b^7 + b^2*
(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c - a*c
*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^3*b^8 + 256*a^7*c^4 - 16*a^4*b^6*c + 96*
a^5*b^4*c^2 - 256*a^6*b^2*c^3)))^(1/4)*(((-(b^7 + b^2*(-(4*a*c - b^2)^5)^(1
/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c - a*c*(-(4*a*c - b^2)^5)^(
1/2))/(512*(a^3*b^8 + 256*a^7*c^4 - 16*a^4*b^6*c + 96*a^5*b^4*c^2 - 256*a^6
*b^2*c^3)))^(1/4)*(4096*a*b^7*c^4 - 262144*a^4*b*c^7 - 49152*a^2*b^5*c^5 +
196608*a^3*b^3*c^6)*1i + x*(1024*b^7*c^4 - 11264*a*b^5*c^5 - 49152*a^3*b*c^
7 + 40960*a^2*b^3*c^6))*(-(b^7 + b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^
3 + 40*a^2*b^3*c^2 - 11*a*b^5*c - a*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^3*b
^8 + 256*a^7*c^4 - 16*a^4*b^6*c + 96*a^5*b^4*c^2 - 256*a^6*b^2*c^3)))^(3/4)
*1i - 64*a*c^7 + 16*b^2*c^6)*1i - 8*c^7*x)*(-(b^7 + b^2*(-(4*a*c - b^2)^5)^
(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c - a*c*(-(4*a*c - b^2)^5)
^(1/2))/(512*(a^3*b^8 + 256*a^7*c^4 - 16*a^4*b^6*c + 96*a^5*b^4*c^2 - 256*a
^6*b^2*c^3)))^(1/4)*1i + ((-(b^7 + b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*
c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c - a*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^3
*b^8 + 256*a^7*c^4 - 16*a^4*b^6*c + 96*a^5*b^4*c^2 - 256*a^6*b^2*c^3)))^(1/
4)*(((-(b^7 + b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2
- 11*a*b^5*c - a*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^3*b^8 + 256*a^7*c^4 -
16*a^4*b^6*c + 96*a^5*b^4*c^2 - 256*a^6*b^2*c^3)))^(1/4)*(4096*a*b^7*c^4 -
262144*a^4*b*c^7 - 49152*a^2*b^5*c^5 + 196608*a^3*b^3*c^6)*1i - x*(1024*b^7
*c^4 - 11264*a*b^5*c^5 - 49152*a^3*b*c^7 + 40960*a^2*b^3*c^6))*(-(b^7 + b^2
*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c - a*
c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^3*b^8 + 256*a^7*c^4 - 16*a^4*b^6*c + 96
*a^5*b^4*c^2 - 256*a^6*b^2*c^3)))^(3/4)*1i - 64*a*c^7 + 16*b^2*c^6)*1i + 8*
c^7*x)*(-(b^7 + b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^
2 - 11*a*b^5*c - a*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^3*b^8 + 256*a^7*c^4
- 16*a^4*b^6*c + 96*a^5*b^4*c^2 - 256*a^6*b^2*c^3)))^(1/4)*1i))*(-(b^7 + b^
2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c - a
*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^3*b^8 + 256*a^7*c^4 - 16*a^4*b^6*c + 9
6*a^5*b^4*c^2 - 256*a^6*b^2*c^3)))^(1/4) - 2*atan((((-(b^7 - b^2*(-(4*a*c -
 b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c + a*c*(-(4*a*c
- b^2)^5)^(1/2))/(512*(a^3*b^8 + 256*a^7*c^4 - 16*a^4*b^6*c + 96*a^5*b^4*c^
2 - 256*a^6*b^2*c^3)))^(1/4)*(((-(b^7 - b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a
^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c + a*c*(-(4*a*c - b^2)^5)^(1/2))/(512
*(a^3*b^8 + 256*a^7*c^4 - 16*a^4*b^6*c + 96*a^5*b^4*c^2 - 256*a^6*b^2*c^3))
)^(1/4)*(4096*a*b^7*c^4 - 262144*a^4*b*c^7 - 49152*a^2*b^5*c^5 + 196608*a^3
*b^3*c^6)*1i + x*(1024*b^7*c^4 - 11264*a*b^5*c^5 - 49152*a^3*b*c^7 + 40960*
a^2*b^3*c^6))*(-(b^7 - b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2
*b^3*c^2 - 11*a*b^5*c + a*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^3*b^8 + 256*a
^7*c^4 - 16*a^4*b^6*c + 96*a^5*b^4*c^2 - 256*a^6*b^2*c^3)))^(3/4)*1i - 64*a
*c^7 + 16*b^2*c^6)*1i - 8*c^7*x)*(-(b^7 - b^2*(-(4*a*c - b^2)^5)^(1/2) - 48
*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c + a*c*(-(4*a*c - b^2)^5)^(1/2))/(5
12*(a^3*b^8 + 256*a^7*c^4 - 16*a^4*b^6*c + 96*a^5*b^4*c^2 - 256*a^6*b^2*c^3
)))^(1/4) - ((-(b^7 - b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*
b^3*c^2 - 11*a*b^5*c + a*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^3*b^8 + 256*a^
7*c^4 - 16*a^4*b^6*c + 96*a^5*b^4*c^2 - 256*a^6*b^2*c^3)))^(1/4)*(((-(b^7 -
 b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c
+ a*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^3*b^8 + 256*a^7*c^4 - 16*a^4*b^6*c
+ 96*a^5*b^4*c^2 - 256*a^6*b^2*c^3)))^(1/4)*(4096*a*b^7*c^4 - 262144*a^4*b*
c^7 - 49152*a^2*b^5*c^5 + 196608*a^3*b^3*c^6)*1i - x*(1024*b^7*c^4 - 11264*
a*b^5*c^5 - 49152*a^3*b*c^7 + 40960*a^2*b^3*c^6))*(-(b^7 - b^2*(-(4*a*c - b
^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c + a*c*(-(4*a*c -
b^2)^5)^(1/2))/(512*(a^3*b^8 + 256*a^7*c^4 - 16*a^4*b^6*c + 96*a^5*b^4*c^2
```

```
- 256*a^6*b^2*c^3)))^(3/4)*1i - 64*a*c^7 + 16*b^2*c^6)*1i + 8*c^7*x)*(-(b^7
 - b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*
c + a*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^3*b^8 + 256*a^7*c^4 - 16*a^4*b^6*
c + 96*a^5*b^4*c^2 - 256*a^6*b^2*c^3)))^(1/4))/(((-(b^7 - b^2*(-(4*a*c - b^
2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c + a*c*(-(4*a*c - b
^2)^5)^(1/2))/(512*(a^3*b^8 + 256*a^7*c^4 - 16*a^4*b^6*c + 96*a^5*b^4*c^2 -
 256*a^6*b^2*c^3)))^(1/4)*(((-(b^7 - b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*
b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c + a*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a
^3*b^8 + 256*a^7*c^4 - 16*a^4*b^6*c + 96*a^5*b^4*c^2 - 256*a^6*b^2*c^3)))^(
1/4)*(4096*a*b^7*c^4 - 262144*a^4*b*c^7 - 49152*a^2*b^5*c^5 + 196608*a^3*b^
3*c^6)*1i + x*(1024*b^7*c^4 - 11264*a*b^5*c^5 - 49152*a^3*b*c^7 + 40960*a^2
*b^3*c^6))*(-(b^7 - b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^
3*c^2 - 11*a*b^5*c + a*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^3*b^8 + 256*a^7*
c^4 - 16*a^4*b^6*c + 96*a^5*b^4*c^2 - 256*a^6*b^2*c^3)))^(3/4)*1i - 64*a*c^
7 + 16*b^2*c^6)*1i - 8*c^7*x)*(-(b^7 - b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^
3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c + a*c*(-(4*a*c - b^2)^5)^(1/2))/(512*
(a^3*b^8 + 256*a^7*c^4 - 16*a^4*b^6*c + 96*a^5*b^4*c^2 - 256*a^6*b^2*c^3)))
^(1/4)*1i + ((-(b^7 - b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*
b^3*c^2 - 11*a*b^5*c + a*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^3*b^8 + 256*a^
7*c^4 - 16*a^4*b^6*c + 96*a^5*b^4*c^2 - 256*a^6*b^2*c^3)))^(1/4)*(((-(b^7 -
 b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c
+ a*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^3*b^8 + 256*a^7*c^4 - 16*a^4*b^6*c
+ 96*a^5*b^4*c^2 - 256*a^6*b^2*c^3)))^(1/4)*(4096*a*b^7*c^4 - 262144*a^4*b*
c^7 - 49152*a^2*b^5*c^5 + 196608*a^3*b^3*c^6)*1i - x*(1024*b^7*c^4 - 11264*
a*b^5*c^5 - 49152*a^3*b*c^7 + 40960*a^2*b^3*c^6))*(-(b^7 - b^2*(-(4*a*c - b
^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c + a*c*(-(4*a*c -
b^2)^5)^(1/2))/(512*(a^3*b^8 + 256*a^7*c^4 - 16*a^4*b^6*c + 96*a^5*b^4*c^2
- 256*a^6*b^2*c^3)))^(3/4)*1i - 64*a*c^7 + 16*b^2*c^6)*1i + 8*c^7*x)*(-(b^7
 - b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*
c + a*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^3*b^8 + 256*a^7*c^4 - 16*a^4*b^6*
c + 96*a^5*b^4*c^2 - 256*a^6*b^2*c^3)))^(1/4)*1i))*(-(b^7 - b^2*(-(4*a*c -
b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c + a*c*(-(4*a*c -
 b^2)^5)^(1/2))/(512*(a^3*b^8 + 256*a^7*c^4 - 16*a^4*b^6*c + 96*a^5*b^4*c^2
 - 256*a^6*b^2*c^3)))^(1/4)
```

sympy [A] time = 19.79, size = 177, normalized size = 0.56

$$\operatorname{RootSum}\left(t^{8}\left(16777216a^{7}c^{4}-16777216a^{6}b^{2}c^{3}+6291456a^{5}b^{4}c^{2}-1048576a^{4}b^{6}c+65536a^{3}b^{8}\right)+t^{4}\left(-12288a^{3}bc^{3}+10240a^{2}b^{3}c^{2}-2816ab^{5}c+256b^{7}\right)+c^{3},\left(t\mapsto t\log\left(x+\frac{16384t^{5}a^{5}bc^{2}-8192t^{5}a^{4}b^{3}c+1024t^{5}a^{3}b^{5}+8ta^{2}c^{2}-16tab^{2}c+4tb^{4}}{ac^{2}-b^{2}c}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x**8+b*x**4+a),x)
```

```
[Out] RootSum(_t**8*(16777216*a**7*c**4 - 16777216*a**6*b**2*c**3 + 6291456*a**5*
b**4*c**2 - 1048576*a**4*b**6*c + 65536*a**3*b**8) + _t**4*(-12288*a**3*b*c
**3 + 10240*a**2*b**3*c**2 - 2816*a*b**5*c + 256*b**7) + c**3, Lambda(_t, _
t*log(x + (16384*_t**5*a**5*b*c**2 - 8192*_t**5*a**4*b**3*c + 1024*_t**5*a*
*3*b**5 + 8*_t*a**2*c**2 - 16*_t*a*b**2*c + 4*_t*b**4)/(a*c**2 - b**2*c))))
```

3.268 $\int \frac{1}{x^2\left(a+bx^4+cx^8\right)}\,dx$

Optimal. Leaf size=363

$$-\frac{\sqrt[4]{c}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2\,2^{3/4}a\sqrt[4]{-\sqrt{b^2-4ac}-b}}-\frac{\sqrt[4]{c}\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{2\,2^{3/4}a\sqrt[4]{\sqrt{b^2-4ac}-b}}+\frac{\sqrt[4]{c}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\tanh^{-1}\left(\ldots\right.}{2\,2^{3/4}a\sqrt[4]{-\sqrt{b^2-4ac}\ldots}}$$

Rubi [A] time = 0.41, antiderivative size = 363, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {1368, 1510, 298, 205, 208}

$$-\frac{\sqrt[4]{c}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2\,2^{3/4}a\sqrt[4]{-\sqrt{b^2-4ac}-b}}-\frac{\sqrt[4]{c}\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{2\,2^{3/4}a\sqrt[4]{\sqrt{b^2-4ac}-b}}+\frac{\sqrt[4]{c}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2\,2^{3/4}a\sqrt[4]{-\sqrt{b^2-4ac}-b}}+\frac{\sqrt[4]{c}\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{2\,2^{3/4}a\sqrt[4]{\sqrt{b^2-4ac}-b}}-\frac{1}{ax}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a + b*x^4 + c*x^8)),x]
```

```
[Out] -(1/(a*x)) - (c^(1/4)*(1 - b/Sqrt[b^2 - 4*a*c])*ArcTan[(2^(1/4)*c^(1/4)*x)/
(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(2*2^(3/4)*a*(-b - Sqrt[b^2 - 4*a*c])^(1/4
)) - (c^(1/4)*(1 + b/Sqrt[b^2 - 4*a*c])*ArcTan[(2^(1/4)*c^(1/4)*x)/(-b + Sq
rt[b^2 - 4*a*c])^(1/4)])/(2*2^(3/4)*a*(-b + Sqrt[b^2 - 4*a*c])^(1/4)) + (c^
(1/4)*(1 - b/Sqrt[b^2 - 4*a*c])*ArcTanh[(2^(1/4)*c^(1/4)*x)/(-b - Sqrt[b^2
- 4*a*c])^(1/4)])/(2*2^(3/4)*a*(-b - Sqrt[b^2 - 4*a*c])^(1/4)) + (c^(1/4)*(
1 + b/Sqrt[b^2 - 4*a*c])*ArcTanh[(2^(1/4)*c^(1/4)*x)/(-b + Sqrt[b^2 - 4*a*c
])^(1/4)])/(2*2^(3/4)*a*(-b + Sqrt[b^2 - 4*a*c])^(1/4))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 1368

```
Int[((d_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_
Symbol] :> Simp[((d*x)^(m + 1)*(a + b*x^n + c*x^(2*n))^(p + 1))/(a*d*(m + 1
)), x] - Dist[1/(a*d^n*(m + 1)), Int[(d*x)^(m + n)*(b*(m + n*(p + 1) + 1) +
 c*(m + 2*n*(p + 1) + 1)*x^n)*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a
, b, c, d, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && L
tQ[m, -1] && IntegerQ[p]
```

Rule 1510

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_)))/((a_) + (b_.)*(x_)^(n_) +
 (c_.)*(x_)^(n2_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 +
```

```
(2*c*d - b*e)/(2*q), Int[(f*x)^m/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (
2*c*d - b*e)/(2*q), Int[(f*x)^m/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b
, c, d, e, f, m}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{1}{x^2\left(a+bx^4+cx^8\right)}\,dx = -\frac{1}{ax}+\frac{\int \frac{x^2\left(-b-cx^4\right)}{a+bx^4+cx^8}\,dx}{a}$$

$$= -\frac{1}{ax}-\frac{\left(c\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\right)\int \frac{x^2}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^4}\,dx}{2a}-\frac{\left(c\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)\right)\int \frac{x^2}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^4}\,dx}{2a}$$

$$= -\frac{1}{ax}+\frac{\left(\sqrt{c}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\right)\int \frac{1}{\sqrt{-b-\sqrt{b^2-4ac}}-\sqrt{2}\sqrt{c}x^2}\,dx}{2\sqrt{2}a}-\frac{\left(\sqrt{c}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\right)\int \frac{}{\sqrt{-b-\sqrt{b}}}}{2\sqrt{2}a}$$

$$= -\frac{1}{ax}-\frac{\sqrt[4]{c}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{2\,2^{3/4}a\sqrt[4]{-b-\sqrt{b^2-4ac}}}-\frac{\sqrt[4]{c}\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b+\sqrt{b^2-4a}}}\right)}{2\,2^{3/4}a\sqrt[4]{-b+\sqrt{b^2-4ac}}}$$

Mathematica [C] time = 0.04, size = 71, normalized size = 0.20

$$-\frac{\text{RootSum}\left[\#1^8c+\#1^4b+a\&,\frac{\#1^4c\log(x-\#1)+b\log(x-\#1)}{2\#1^5c+\#1b}\&\right]}{4a}-\frac{1}{ax}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a + b*x^4 + c*x^8)),x]
```

```
[Out] -(1/(a*x)) - RootSum[a + b*#1^4 + c*#1^8 & , (b*Log[x - #1] + c*Log[x - #1]
*#1^4)/(b*#1 + 2*c*#1^5) & ]/(4*a)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2\left(a+bx^4+cx^8\right)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x^2*(a + b*x^4 + c*x^8)),x]
```

```
[Out] IntegrateAlgebraic[1/(x^2*(a + b*x^4 + c*x^8)), x]
```

fricas [B] time = 3.47, size = 5125, normalized size = 14.12

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(c*x^8+b*x^4+a),x, algorithm="fricas")
```

```
[Out] -1/4*(4*a*x*sqrt(sqrt(1/2)*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 - (a^5*b^4
- 8*a^6*b^2*c + 16*a^7*c^2)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*
b^2*c^3 + a^4*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^
3)))/(a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)))*arctan(-1/2*((a^5*b^5 - 8*a^6*b
^3*c + 16*a^7*b*c^2)*x*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c
^3 + a^4*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)) +
```

(b^6 - 7*a*b^4*c + 13*a^2*b^2*c^2 - 4*a^3*c^3)*x - sqrt(1/2)*(b^6 - 7*a*b^4*c + 13*a^2*b^2*c^2 - 4*a^3*c^3 + (a^5*b^5 - 8*a^6*b^3*c + 16*a^7*b*c^2)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))*sqrt((2*(b^4*c^3 - 3*a*b^2*c^4 + a^2*c^5)*x^2 - sqrt(1/2)*(b^9 - 10*a*b^7*c + 34*a^2*b^5*c^2 - 43*a^3*b^3*c^3 + 12*a^4*b*c^4 + (a^5*b^8 - 13*a^6*b^6*c + 60*a^7*b^4*c^2 - 112*a^8*b^2*c^3 + 64*a^9*c^4)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 - (a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))/(a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)))/(b^4*c^3 - 3*a*b^2*c^4 + a^2*c^5)))*sqrt(sqrt(1/2)*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 - (a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))/(a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)))/(b^4*c - 3*a*b^2*c^2 + a^2*c^3)) - 4*a*x*sqrt(sqrt(1/2)*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 + (a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))/(a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)))*arctan(-1/2*(sqrt(1/2)*(b^6 - 7*a*b^4*c + 13*a^2*b^2*c^2 - 4*a^3*c^3 - (a^5*b^5 - 8*a^6*b^3*c + 16*a^7*b*c^2)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))*sqrt(sqrt(1/2)*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 + (a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))/(a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)))*sqrt((2*(b^4*c^3 - 3*a*b^2*c^4 + a^2*c^5)*x^2 - sqrt(1/2)*(b^9 - 10*a*b^7*c + 34*a^2*b^5*c^2 - 43*a^3*b^3*c^3 + 12*a^4*b*c^4 - (a^5*b^8 - 13*a^6*b^6*c + 60*a^7*b^4*c^2 - 112*a^8*b^2*c^3 + 64*a^9*c^4)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 + (a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))/(a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)))/(b^4*c^3 - 3*a*b^2*c^4 + a^2*c^5)) + ((a^5*b^5 - 8*a^6*b^3*c + 16*a^7*b*c^2)*x*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)) - (b^6 - 7*a*b^4*c + 13*a^2*b^2*c^2 - 4*a^3*c^3)*x)*sqrt(sqrt(1/2)*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 + (a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))/(a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2))))/(b^4*c - 3*a*b^2*c^2 + a^2*c^3)) - a*x*sqrt(sqrt(1/2)*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 + (a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))/(a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)))*log(1/2*sqrt(1/2)*(b^11 - 13*a*b^9*c + 63*a^2*b^7*c^2 - 138*a^3*b^5*c^3 + 128*a^4*b^3*c^4 - 32*a^5*b*c^5 - (a^5*b^10 - 16*a^6*b^8*c + 98*a^7*b^6*c^2 - 280*a^8*b^4*c^3 + 352*a^9*b^2*c^4 - 128*a^10*c^5)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))*sqrt(sqrt(1/2)*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 + (a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))/(a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)))*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 + (a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))/(a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)) + (b^4*c^4 - 3*a*b^2*c^5 + a^2*c^6)*x) + a*x*sqrt(sqrt(1/2)*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 + (a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))/(a^5*b^4 - 8*a^6*b^2*c + 1

```
6*a^7*c^2)))*log(-1/2*sqrt(1/2)*(b^11 - 13*a*b^9*c + 63*a^2*b^7*c^2 - 138*a
^3*b^5*c^3 + 128*a^4*b^3*c^4 - 32*a^5*b*c^5 - (a^5*b^10 - 16*a^6*b^8*c + 98
*a^7*b^6*c^2 - 280*a^8*b^4*c^3 + 352*a^9*b^2*c^4 - 128*a^10*c^5)*sqrt((b^8
- 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(a^10*b^6 - 12*a^11
*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))*sqrt(sqrt(1/2)*sqrt(-(b^5 - 5*a*b
^3*c + 5*a^2*b*c^2 + (a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*sqrt((b^8 - 6*a*b
^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(a^10*b^6 - 12*a^11*b^4*c
+ 48*a^12*b^2*c^2 - 64*a^13*c^3)))/(a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)))*s
qrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 + (a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*
sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(a^10*b^6
 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))/(a^5*b^4 - 8*a^6*b^2*c
+ 16*a^7*c^2)) + (b^4*c^4 - 3*a*b^2*c^5 + a^2*c^6)*x) - a*x*sqrt(sqrt(1/2)*
sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 - (a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)
*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(a^10*b^
6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))/(a^5*b^4 - 8*a^6*b^2*c
 + 16*a^7*c^2)))*log(1/2*sqrt(1/2)*(b^11 - 13*a*b^9*c + 63*a^2*b^7*c^2 - 13
8*a^3*b^5*c^3 + 128*a^4*b^3*c^4 - 32*a^5*b*c^5 + (a^5*b^10 - 16*a^6*b^8*c +
 98*a^7*b^6*c^2 - 280*a^8*b^4*c^3 + 352*a^9*b^2*c^4 - 128*a^10*c^5)*sqrt((b
^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(a^10*b^6 - 12*a
^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))*sqrt(sqrt(1/2)*sqrt(-(b^5 - 5*
a*b^3*c + 5*a^2*b*c^2 - (a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*sqrt((b^8 - 6*
a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(a^10*b^6 - 12*a^11*b^4
*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))/(a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2))
)*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 - (a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^
2)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(a^10*
b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))/(a^5*b^4 - 8*a^6*b^2
*c + 16*a^7*c^2)) + (b^4*c^4 - 3*a*b^2*c^5 + a^2*c^6)*x) + a*x*sqrt(sqrt(1/
2)*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 - (a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c
^2)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(a^10
*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))/(a^5*b^4 - 8*a^6*b^
2*c + 16*a^7*c^2)))*log(-1/2*sqrt(1/2)*(b^11 - 13*a*b^9*c + 63*a^2*b^7*c^2
- 138*a^3*b^5*c^3 + 128*a^4*b^3*c^4 - 32*a^5*b*c^5 + (a^5*b^10 - 16*a^6*b^8
*c + 98*a^7*b^6*c^2 - 280*a^8*b^4*c^3 + 352*a^9*b^2*c^4 - 128*a^10*c^5)*sqr
t((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(a^10*b^6 -
12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))*sqrt(sqrt(1/2)*sqrt(-(b^5
- 5*a*b^3*c + 5*a^2*b*c^2 - (a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*sqrt((b^8
- 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(a^10*b^6 - 12*a^11
*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))/(a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c
^2)))*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 - (a^5*b^4 - 8*a^6*b^2*c + 16*a^
7*c^2)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(a
^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))/(a^5*b^4 - 8*a^6
*b^2*c + 16*a^7*c^2)) + (b^4*c^4 - 3*a*b^2*c^5 + a^2*c^6)*x) + 4)/(a*x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(c*x^8+b*x^4+a),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Evaluation time: 14.78Unable to convert to r
eal 1/4 Error: Bad Argument Value
```

maple [C] time = 0.01, size = 63, normalized size = 0.17

$$-\frac{\left(\mathrm{RootOf}\left(\_Z^8c+b\_Z^4+a\right)^6c+\mathrm{RootOf}\left(\_Z^8c+b\_Z^4+a\right)^2b\right)\ln\left(-\mathrm{RootOf}\left(\_Z^8c+b\_Z^4+a\right)+x\right)}{4a\left(2\,\mathrm{RootOf}\left(\_Z^8c+b\_Z^4+a\right)^7c+\mathrm{RootOf}\left(\_Z^8c+b\_Z^4+a\right)^3b\right)}-\frac{1}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(c*x^8+b*x^4+a),x)
```

```
[Out] -1/a/x-1/4/a*sum((_R^6*c+_R^2*b)/(2*_R^7*c+_R^3*b)*ln(-_R+x),_R=RootOf(_Z^8
*c+_Z^4*b+a))
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(c*x^8+b*x^4+a),x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [B] time = 2.81, size = 10509, normalized size = 28.95

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a + b*x^4 + c*x^8)),x)
```

```
[Out] 2*atan((((-(b^9 + b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*
c^2 - 120*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c - 3*a
*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c
 + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3)))^(3/4)*(4096*a^15*c^8 - x*(-(b^9 + b^
4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^
3 + a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c - 3*a*b^2*c*(-(4*a*c - b^
2)^5)^(1/2))/(512*(a^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 -
256*a^8*b^2*c^3)))^(1/4)*(32768*a^16*c^8 + 1024*a^12*b^8*c^4 - 12288*a^13*b
^6*c^5 + 51200*a^14*b^4*c^6 - 81920*a^15*b^2*c^7)*1i + 256*a^11*b^8*c^4 - 2
816*a^12*b^6*c^5 + 10496*a^13*b^4*c^6 - 14336*a^14*b^2*c^7)*1i + 4*a^11*b*c
^8*x)*(-(b^9 + b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2
 - 120*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c - 3*a*b^
2*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c +
96*a^7*b^4*c^2 - 256*a^8*b^2*c^3)))^(1/4) - ((-(b^9 + b^4*(-(4*a*c - b^2)^5
)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 + a^2*c^2*(-(4*a*
c - b^2)^5)^(1/2) - 13*a*b^7*c - 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(
a^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3)))^
(3/4)*(4096*a^15*c^8 + x*(-(b^9 + b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c
^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) -
13*a*b^7*c - 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^5*b^8 + 256*a^9*c^
4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3)))^(1/4)*(32768*a^16*c^
8 + 1024*a^12*b^8*c^4 - 12288*a^13*b^6*c^5 + 51200*a^14*b^4*c^6 - 81920*a^1
5*b^2*c^7)*1i + 256*a^11*b^8*c^4 - 2816*a^12*b^6*c^5 + 10496*a^13*b^4*c^6 -
 14336*a^14*b^2*c^7)*1i - 4*a^11*b*c^8*x)*(-(b^9 + b^4*(-(4*a*c - b^2)^5)^(
1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c -
 b^2)^5)^(1/2) - 13*a*b^7*c - 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^5
*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3)))^(1/
4))/(((-(b^9 + b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2
 - 120*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c - 3*a*b^
2*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c +
96*a^7*b^4*c^2 - 256*a^8*b^2*c^3)))^(3/4)*(4096*a^15*c^8 - x*(-(b^9 + b^4*(
-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 +
 a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c - 3*a*b^2*c*(-(4*a*c - b^2)^
5)^(1/2))/(512*(a^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256
*a^8*b^2*c^3)))^(1/4)*(32768*a^16*c^8 + 1024*a^12*b^8*c^4 - 12288*a^13*b^6*
c^5 + 51200*a^14*b^4*c^6 - 81920*a^15*b^2*c^7)*1i + 256*a^11*b^8*c^4 - 2816
*a^12*b^6*c^5 + 10496*a^13*b^4*c^6 - 14336*a^14*b^2*c^7)*1i + 4*a^11*b*c^8*
x)*(-(b^9 + b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 -
```

```
120*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c - 3*a*b^2*c
*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c + 96*
a^7*b^4*c^2 - 256*a^8*b^2*c^3)))^(1/4)*1i + ((-(b^9 + b^4*(-(4*a*c - b^2)^5
)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 + a^2*c^2*(-(4*a*
c - b^2)^5)^(1/2) - 13*a*b^7*c - 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(
a^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3)))^
(3/4)*(4096*a^15*c^8 + x*(-(b^9 + b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c
^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) -
13*a*b^7*c - 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^5*b^8 + 256*a^9*c^
4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3)))^(1/4)*(32768*a^16*c^
8 + 1024*a^12*b^8*c^4 - 12288*a^13*b^6*c^5 + 51200*a^14*b^4*c^6 - 81920*a^1
5*b^2*c^7)*1i + 256*a^11*b^8*c^4 - 2816*a^12*b^6*c^5 + 10496*a^13*b^4*c^6 -
 14336*a^14*b^2*c^7)*1i - 4*a^11*b*c^8*x)*(-(b^9 + b^4*(-(4*a*c - b^2)^5)^(
1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c -
 b^2)^5)^(1/2) - 13*a*b^7*c - 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^5
*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3)))^(1/
4)*1i))*(-(b^9 + b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c
^2 - 120*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c - 3*a*
b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c
+ 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3)))^(1/4) - atan((((-(b^9 - b^4*(-(4*a*c
- b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 - a^2*c^2
*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c + 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2)
)/(512*(a^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2
*c^3)))^(3/4)*(4096*a^15*c^8 + x*(-(b^9 - b^4*(-(4*a*c - b^2)^5)^(1/2) + 80
*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 - a^2*c^2*(-(4*a*c - b^2)^5)^
(1/2) - 13*a*b^7*c + 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^5*b^8 + 25
6*a^9*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3)))^(1/4)*(32768
*a^16*c^8 + 1024*a^12*b^8*c^4 - 12288*a^13*b^6*c^5 + 51200*a^14*b^4*c^6 - 8
1920*a^15*b^2*c^7) + 256*a^11*b^8*c^4 - 2816*a^12*b^6*c^5 + 10496*a^13*b^4*
c^6 - 14336*a^14*b^2*c^7) + 4*a^11*b*c^8*x)*(-(b^9 - b^4*(-(4*a*c - b^2)^5)
^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 - a^2*c^2*(-(4*a*c
 - b^2)^5)^(1/2) - 13*a*b^7*c + 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a
^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3)))^(
1/4)*1i - ((-(b^9 - b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^
5*c^2 - 120*a^3*b^3*c^3 - a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c + 3
*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6
*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3)))^(3/4)*(4096*a^15*c^8 - x*(-(b^9 -
b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*
c^3 - a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c + 3*a*b^2*c*(-(4*a*c -
b^2)^5)^(1/2))/(512*(a^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2
- 256*a^8*b^2*c^3)))^(1/4)*(32768*a^16*c^8 + 1024*a^12*b^8*c^4 - 12288*a^13
*b^6*c^5 + 51200*a^14*b^4*c^6 - 81920*a^15*b^2*c^7) + 256*a^11*b^8*c^4 - 28
16*a^12*b^6*c^5 + 10496*a^13*b^4*c^6 - 14336*a^14*b^2*c^7) - 4*a^11*b*c^8*x
)*(-(b^9 - b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 1
20*a^3*b^3*c^3 - a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c + 3*a*b^2*c*
(-(4*a*c - b^2)^5)^(1/2))/(512*(a^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c + 96*a
^7*b^4*c^2 - 256*a^8*b^2*c^3)))^(1/4)*1i)/(((-(b^9 - b^4*(-(4*a*c - b^2)^5)
^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 - a^2*c^2*(-(4*a*c
 - b^2)^5)^(1/2) - 13*a*b^7*c + 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a
^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3)))^(
3/4)*(4096*a^15*c^8 + x*(-(b^9 - b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^
4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 - a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 1
3*a*b^7*c + 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^5*b^8 + 256*a^9*c^4
 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3)))^(1/4)*(32768*a^16*c^8
 + 1024*a^12*b^8*c^4 - 12288*a^13*b^6*c^5 + 51200*a^14*b^4*c^6 - 81920*a^15
*b^2*c^7) + 256*a^11*b^8*c^4 - 2816*a^12*b^6*c^5 + 10496*a^13*b^4*c^6 - 143
36*a^14*b^2*c^7) + 4*a^11*b*c^8*x)*(-(b^9 - b^4*(-(4*a*c - b^2)^5)^(1/2) +
80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 - a^2*c^2*(-(4*a*c - b^2)^5
)^(1/2) - 13*a*b^7*c + 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^5*b^8 +
```

256*a^9*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3)))^(1/4) + ((
-(b^9 - b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*
a^3*b^3*c^3 - a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c + 3*a*b^2*c*(-(
4*a*c - b^2)^5)^(1/2))/(512*(a^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c + 96*a^7*
b^4*c^2 - 256*a^8*b^2*c^3)))^(3/4)*(4096*a^15*c^8 - x*(-(b^9 - b^4*(-(4*a*c
 - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 - a^2*c^
2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c + 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2
))/(512*(a^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^
2*c^3)))^(1/4)*(32768*a^16*c^8 + 1024*a^12*b^8*c^4 - 12288*a^13*b^6*c^5 + 5
1200*a^14*b^4*c^6 - 81920*a^15*b^2*c^7) + 256*a^11*b^8*c^4 - 2816*a^12*b^6*
c^5 + 10496*a^13*b^4*c^6 - 14336*a^14*b^2*c^7) - 4*a^11*b*c^8*x)*(-(b^9 - b
^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c
^3 - a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c + 3*a*b^2*c*(-(4*a*c - b
^2)^5)^(1/2))/(512*(a^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 -
 256*a^8*b^2*c^3)))^(1/4)))*(-(b^9 - b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*
b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 - a^2*c^2*(-(4*a*c - b^2)^5)^(1/2)
 - 13*a*b^7*c + 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^5*b^8 + 256*a^9
*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3)))^(1/4)*2i - atan((
((-(b^9 + b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 12
0*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c - 3*a*b^2*c*(
-(4*a*c - b^2)^5)^(1/2))/(512*(a^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c + 96*a^
7*b^4*c^2 - 256*a^8*b^2*c^3)))^(3/4)*(4096*a^15*c^8 + x*(-(b^9 + b^4*(-(4*a
*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 + a^2*
c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c - 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1
/2))/(512*(a^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*
b^2*c^3)))^(1/4)*(32768*a^16*c^8 + 1024*a^12*b^8*c^4 - 12288*a^13*b^6*c^5 +
 51200*a^14*b^4*c^6 - 81920*a^15*b^2*c^7) + 256*a^11*b^8*c^4 - 2816*a^12*b^
6*c^5 + 10496*a^13*b^4*c^6 - 14336*a^14*b^2*c^7) + 4*a^11*b*c^8*x)*(-(b^9 +
 b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3
*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c - 3*a*b^2*c*(-(4*a*c -
 b^2)^5)^(1/2))/(512*(a^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2
 - 256*a^8*b^2*c^3)))^(1/4)*1i - ((-(b^9 + b^4*(-(4*a*c - b^2)^5)^(1/2) + 8
0*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)
^(1/2) - 13*a*b^7*c - 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^5*b^8 + 2
56*a^9*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3)))^(3/4)*(4096
*a^15*c^8 - x*(-(b^9 + b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2
*b^5*c^2 - 120*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c
- 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^5*b^8 + 256*a^9*c^4 - 16*a^6*
b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3)))^(1/4)*(32768*a^16*c^8 + 1024*a^
12*b^8*c^4 - 12288*a^13*b^6*c^5 + 51200*a^14*b^4*c^6 - 81920*a^15*b^2*c^7)
+ 256*a^11*b^8*c^4 - 2816*a^12*b^6*c^5 + 10496*a^13*b^4*c^6 - 14336*a^14*b^
2*c^7) - 4*a^11*b*c^8*x)*(-(b^9 + b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c
^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) -
13*a*b^7*c - 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^5*b^8 + 256*a^9*c^
4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3)))^(1/4)*1i)/(((-(b^9 +
 b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3
*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c - 3*a*b^2*c*(-(4*a*c -
 b^2)^5)^(1/2))/(512*(a^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2
 - 256*a^8*b^2*c^3)))^(3/4)*(4096*a^15*c^8 + x*(-(b^9 + b^4*(-(4*a*c - b^2)
^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 + a^2*c^2*(-(4*
a*c - b^2)^5)^(1/2) - 13*a*b^7*c - 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(512
*(a^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3))
)^(1/4)*(32768*a^16*c^8 + 1024*a^12*b^8*c^4 - 12288*a^13*b^6*c^5 + 51200*a^
14*b^4*c^6 - 81920*a^15*b^2*c^7) + 256*a^11*b^8*c^4 - 2816*a^12*b^6*c^5 + 1
0496*a^13*b^4*c^6 - 14336*a^14*b^2*c^7) + 4*a^11*b*c^8*x)*(-(b^9 + b^4*(-(4
*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 + a^
2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c - 3*a*b^2*c*(-(4*a*c - b^2)^5)^
(1/2))/(512*(a^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^
8*b^2*c^3)))^(1/4) + ((-(b^9 + b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4

+ 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*
a*b^7*c - 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^5*b^8 + 256*a^9*c^4 -
16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3)))^(3/4)*(4096*a^15*c^8 -
x*(-(b^9 + b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 1
20*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c - 3*a*b^2*c*
(-(4*a*c - b^2)^5)^(1/2))/(512*(a^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c + 96*a
^7*b^4*c^2 - 256*a^8*b^2*c^3)))^(1/4)*(32768*a^16*c^8 + 1024*a^12*b^8*c^4 -
12288*a^13*b^6*c^5 + 51200*a^14*b^4*c^6 - 81920*a^15*b^2*c^7) + 256*a^11*b
^8*c^4 - 2816*a^12*b^6*c^5 + 10496*a^13*b^4*c^6 - 14336*a^14*b^2*c^7) - 4*a
^11*b*c^8*x)*(-(b^9 + b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*
b^5*c^2 - 120*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c -
3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^5*b^8 + 256*a^9*c^4 - 16*a^6*b
^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3)))^(1/4))*(-(b^9 + b^4*(-(4*a*c -
b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 + a^2*c^2*(
-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c - 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/
(512*(a^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c
^3)))^(1/4)*2i + 2*atan((((-(b^9 - b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*
c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 - a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) -
13*a*b^7*c + 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^5*b^8 + 256*a^9*c
^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3)))^(3/4)*(4096*a^15*c^
8 - x*(-(b^9 - b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2
- 120*a^3*b^3*c^3 - a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c + 3*a*b^
2*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c +
96*a^7*b^4*c^2 - 256*a^8*b^2*c^3)))^(1/4)*(32768*a^16*c^8 + 1024*a^12*b^8*c
^4 - 12288*a^13*b^6*c^5 + 51200*a^14*b^4*c^6 - 81920*a^15*b^2*c^7)*1i + 256
*a^11*b^8*c^4 - 2816*a^12*b^6*c^5 + 10496*a^13*b^4*c^6 - 14336*a^14*b^2*c^7
)*1i + 4*a^11*b*c^8*x)*(-(b^9 - b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4
+ 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 - a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13
*a*b^7*c + 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^5*b^8 + 256*a^9*c^4
- 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3)))^(1/4) - ((-(b^9 - b^4*
(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3
- a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c + 3*a*b^2*c*(-(4*a*c - b^2)
^5)^(1/2))/(512*(a^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 25
6*a^8*b^2*c^3)))^(3/4)*(4096*a^15*c^8 + x*(-(b^9 - b^4*(-(4*a*c - b^2)^5)^(
1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 - a^2*c^2*(-(4*a*c -
b^2)^5)^(1/2) - 13*a*b^7*c + 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^5
*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3)))^(1/
4)*(32768*a^16*c^8 + 1024*a^12*b^8*c^4 - 12288*a^13*b^6*c^5 + 51200*a^14*b^
4*c^6 - 81920*a^15*b^2*c^7)*1i + 256*a^11*b^8*c^4 - 2816*a^12*b^6*c^5 + 104
96*a^13*b^4*c^6 - 14336*a^14*b^2*c^7)*1i - 4*a^11*b*c^8*x)*(-(b^9 - b^4*(-(
4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 - a
^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c + 3*a*b^2*c*(-(4*a*c - b^2)^5)
^(1/2))/(512*(a^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a
^8*b^2*c^3)))^(1/4))/(((-(b^9 - b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4
+ 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 - a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13
*a*b^7*c + 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^5*b^8 + 256*a^9*c^4
- 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3)))^(3/4)*(4096*a^15*c^8 -
x*(-(b^9 - b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 -
120*a^3*b^3*c^3 - a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c + 3*a*b^2*c
*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c + 96*
a^7*b^4*c^2 - 256*a^8*b^2*c^3)))^(1/4)*(32768*a^16*c^8 + 1024*a^12*b^8*c^4
- 12288*a^13*b^6*c^5 + 51200*a^14*b^4*c^6 - 81920*a^15*b^2*c^7)*1i + 256*a^
11*b^8*c^4 - 2816*a^12*b^6*c^5 + 10496*a^13*b^4*c^6 - 14336*a^14*b^2*c^7)*1
i + 4*a^11*b*c^8*x)*(-(b^9 - b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 +
61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 - a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*
b^7*c + 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^5*b^8 + 256*a^9*c^4 - 1
6*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3)))^(1/4)*1i + ((-(b^9 - b^4*
(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3
- a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c + 3*a*b^2*c*(-(4*a*c - b^2)

```
^5)^(1/2))/(512*(a^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 25
6*a^8*b^2*c^3)))^(3/4)*(4096*a^15*c^8 + x*(-(b^9 - b^4*(-(4*a*c - b^2)^5)^(
1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 - a^2*c^2*(-(4*a*c -
 b^2)^5)^(1/2) - 13*a*b^7*c + 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^5
*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3)))^(1/
4)*(32768*a^16*c^8 + 1024*a^12*b^8*c^4 - 12288*a^13*b^6*c^5 + 51200*a^14*b^
4*c^6 - 81920*a^15*b^2*c^7)*1i + 256*a^11*b^8*c^4 - 2816*a^12*b^6*c^5 + 104
96*a^13*b^4*c^6 - 14336*a^14*b^2*c^7)*1i - 4*a^11*b*c^8*x)*(-(b^9 - b^4*(-(
4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 - a
^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c + 3*a*b^2*c*(-(4*a*c - b^2)^5)
^(1/2))/(512*(a^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a
^8*b^2*c^3)))^(1/4)*1i))*(-(b^9 - b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c
^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 - a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) -
13*a*b^7*c + 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^5*b^8 + 256*a^9*c^
4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3)))^(1/4) - 1/(a*x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(c*x**8+b*x**4+a),x)
```

```
[Out] Timed out
```

3.269 $\int \frac{1}{x^4(a+bx^4+cx^8)}\,dx$

Optimal. Leaf size=365

$$\frac{c^{3/4}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\,x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2\sqrt[4]{2}\,a\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}+\frac{c^{3/4}\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\,x}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{2\sqrt[4]{2}\,a\left(\sqrt{b^2-4ac}-b\right)^{3/4}}+\frac{c^{3/4}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\cdots}{\sqrt[4]{-\sqrt{b}\cdots}}\right.}{2\sqrt[4]{2}\,a\left(-\sqrt{b^2-4ac}-b\right)^{3\cdots}}$$

Rubi [A] time = 0.40, antiderivative size = 365, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {1368, 1422, 212, 208, 205}

$$\frac{c^{3/4}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\,x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2\sqrt[4]{2}\,a\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}+\frac{c^{3/4}\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\,x}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{2\sqrt[4]{2}\,a\left(\sqrt{b^2-4ac}-b\right)^{3/4}}+\frac{c^{3/4}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\,x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2\sqrt[4]{2}\,a\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}+\frac{c^{3/4}\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\,x}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{2\sqrt[4]{2}\,a\left(\sqrt{b^2-4ac}-b\right)^{3/4}}-\frac{1}{3ax^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(a + b*x^4 + c*x^8)),x]
```

```
[Out] -1/(3*a*x^3) + (c^(3/4)*(1 - b/Sqrt[b^2 - 4*a*c])*ArcTan[(2^(1/4)*c^(1/4)*x
)/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(2*2^(1/4)*a*(-b - Sqrt[b^2 - 4*a*c])^(3
/4)) + (c^(3/4)*(1 + b/Sqrt[b^2 - 4*a*c])*ArcTan[(2^(1/4)*c^(1/4)*x)/(-b +
Sqrt[b^2 - 4*a*c])^(1/4)])/(2*2^(1/4)*a*(-b + Sqrt[b^2 - 4*a*c])^(3/4)) + (
c^(3/4)*(1 - b/Sqrt[b^2 - 4*a*c])*ArcTanh[(2^(1/4)*c^(1/4)*x)/(-b - Sqrt[b^
2 - 4*a*c])^(1/4)])/(2*2^(1/4)*a*(-b - Sqrt[b^2 - 4*a*c])^(3/4)) + (c^(3/4)
*(1 + b/Sqrt[b^2 - 4*a*c])*ArcTanh[(2^(1/4)*c^(1/4)*x)/(-b + Sqrt[b^2 - 4*a
*c])^(1/4)])/(2*2^(1/4)*a*(-b + Sqrt[b^2 - 4*a*c])^(3/4))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 1368

```
Int[((d_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_
Symbol] :> Simp[((d*x)^(m + 1)*(a + b*x^n + c*x^(2*n))^(p + 1))/(a*d*(m + 1
)), x] - Dist[1/(a*d^n*(m + 1)), Int[(d*x)^(m + n)*(b*(m + n*(p + 1) + 1) +
 c*(m + 2*n*(p + 1) + 1)*x^n)*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a
, b, c, d, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && L
tQ[m, -1] && IntegerQ[p]
```

Rule 1422

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q),
```

```
Int[1/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(
b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (PosQ[b^2 - 4*a
*c] ||  !IGtQ[n/2, 0])
```

Rubi steps

$$\int \frac{1}{x^4\left(a+bx^4+cx^8\right)}\,dx = -\frac{1}{3ax^3}+\frac{\int \frac{-3b-3cx^4}{a+bx^4+cx^8}\,dx}{3a}$$

$$= -\frac{1}{3ax^3}-\frac{\left(c\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\right)\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^4}\,dx}{2a}-\frac{\left(c\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)\right)\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+\ldots}}{2a}$$

$$= -\frac{1}{3ax^3}+\frac{\left(c\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\right)\int \frac{1}{\sqrt{-b-\sqrt{b^2-4ac}}-\sqrt{2}\sqrt{c}x^2}\,dx}{2a\sqrt{-b-\sqrt{b^2-4ac}}}+\frac{\left(c\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\right)\int \frac{\ldots}{\sqrt{-b-\ldots}}}{2a\sqrt{-b-\sqrt{b^2\ldots}}}$$

$$= -\frac{1}{3ax^3}+\frac{c^{3/4}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{2\sqrt[4]{2}a\left(-b-\sqrt{b^2-4ac}\right)^{3/4}}+\frac{c^{3/4}\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\ldots}{\sqrt[4]{-b+\ldots}}\right)}{2\sqrt[4]{2}a\left(-b+\sqrt{b^2-4ac}\right)\ldots}$$

Mathematica [C] time = 0.04, size = 75, normalized size = 0.21

$$-\frac{\text{RootSum}\left[\#1^8c+\#1^4b+a\&,\frac{\#1^4c\log(x-\#1)+b\log(x-\#1)}{2\#1^7c+\#1^3b}\&\right]}{4a}-\frac{1}{3ax^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(a + b*x^4 + c*x^8)),x]
```

```
[Out] -1/3*1/(a*x^3) - RootSum[a + b*#1^4 + c*#1^8 & , (b*Log[x - #1] + c*Log[x -
 #1]*#1^4)/(b*#1^3 + 2*c*#1^7) & ]/(4*a)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^4\left(a+bx^4+cx^8\right)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x^4*(a + b*x^4 + c*x^8)),x]
```

```
[Out] IntegrateAlgebraic[1/(x^4*(a + b*x^4 + c*x^8)), x]
```

fricas [B] time = 4.08, size = 6324, normalized size = 17.33

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(c*x^8+b*x^4+a),x, algorithm="fricas")
```

```
[Out] 1/12*(12*a*x^3*sqrt(sqrt(1/2)*sqrt(-(b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a
^3*b*c^3 - (a^7*b^4 - 8*a^8*b^2*c + 16*a^9*c^2)*sqrt((b^12 - 10*a*b^10*c +
37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6
)/(a^14*b^6 - 12*a^15*b^4*c + 48*a^16*b^2*c^2 - 64*a^17*c^3)))/(a^7*b^4 - 8
```

*a^8*b^2*c + 16*a^9*c^2)))*arctan(-1/4*(2*sqrt(1/2)*((a^7*b^11 - 17*a^8*b^9*c + 113*a^9*b^7*c^2 - 364*a^10*b^5*c^3 + 560*a^11*b^3*c^4 - 320*a^12*b*c^5)*x*sqrt((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(a^14*b^6 - 12*a^15*b^4*c + 48*a^16*b^2*c^2 - 64*a^17*c^3)) + (b^14 - 16*a*b^12*c + 102*a^2*b^10*c^2 - 328*a^3*b^8*c^3 + 553*a^4*b^6*c^4 - 457*a^5*b^4*c^5 + 152*a^6*b^2*c^6 - 16*a^7*c^7)*x)*sqrt(-(b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c^3 - (a^7*b^4 - 8*a^8*b^2*c + 16*a^9*c^2)*sqrt((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(a^14*b^6 - 12*a^15*b^4*c + 48*a^16*b^2*c^2 - 64*a^17*c^3)))/(a^7*b^4 - 8*a^8*b^2*c + 16*a^9*c^2)) - (b^14 - 16*a*b^12*c + 102*a^2*b^10*c^2 - 328*a^3*b^8*c^3 + 553*a^4*b^6*c^4 - 457*a^5*b^4*c^5 + 152*a^6*b^2*c^6 - 16*a^7*c^7 + (a^7*b^11 - 17*a^8*b^9*c + 113*a^9*b^7*c^2 - 364*a^10*b^5*c^3 + 560*a^11*b^3*c^4 - 320*a^12*b*c^5)*sqrt((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(a^14*b^6 - 12*a^15*b^4*c + 48*a^16*b^2*c^2 - 64*a^17*c^3)))*sqrt(-(b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c^3 - (a^7*b^4 - 8*a^8*b^2*c + 16*a^9*c^2)*sqrt((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(a^14*b^6 - 12*a^15*b^4*c + 48*a^16*b^2*c^2 - 64*a^17*c^3)))/(a^7*b^4 - 8*a^8*b^2*c + 16*a^9*c^2))*sqrt((2*(b^6*c^4 - 5*a*b^4*c^5 + 6*a^2*b^2*c^6 - a^3*c^7)*x^2 + sqrt(1/2)*(b^12 - 13*a*b^10*c + 64*a^2*b^8*c^2 - 147*a^3*b^6*c^3 + 156*a^4*b^4*c^4 - 66*a^5*b^2*c^5 + 8*a^6*c^6 + (a^7*b^9 - 14*a^8*b^7*c + 72*a^9*b^5*c^2 - 160*a^10*b^3*c^3 + 128*a^11*b*c^4)*sqrt((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(a^14*b^6 - 12*a^15*b^4*c + 48*a^16*b^2*c^2 - 64*a^17*c^3)))*sqrt(-(b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c^3 - (a^7*b^4 - 8*a^8*b^2*c + 16*a^9*c^2)*sqrt((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(a^14*b^6 - 12*a^15*b^4*c + 48*a^16*b^2*c^2 - 64*a^17*c^3)))/(a^7*b^4 - 8*a^8*b^2*c + 16*a^9*c^2)))/(b^6*c^4 - 5*a*b^4*c^5 + 6*a^2*b^2*c^6 - a^3*c^7)))*sqrt(sqrt(1/2)*sqrt(-(b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c^3 - (a^7*b^4 - 8*a^8*b^2*c + 16*a^9*c^2)*sqrt((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(a^14*b^6 - 12*a^15*b^4*c + 48*a^16*b^2*c^2 - 64*a^17*c^3)))/(a^7*b^4 - 8*a^8*b^2*c + 16*a^9*c^2)))/(b^6*c^5 - 5*a*b^4*c^6 + 6*a^2*b^2*c^7 - a^3*c^8)) - 12*a*x^3*sqrt(sqrt(1/2)*sqrt(-(b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c^3 + (a^7*b^4 - 8*a^8*b^2*c + 16*a^9*c^2)*sqrt((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(a^14*b^6 - 12*a^15*b^4*c + 48*a^16*b^2*c^2 - 64*a^17*c^3)))/(a^7*b^4 - 8*a^8*b^2*c + 16*a^9*c^2)))*arctan(-1/4*(2*sqrt(1/2)*((a^7*b^11 - 17*a^8*b^9*c + 113*a^9*b^7*c^2 - 364*a^10*b^5*c^3 + 560*a^11*b^3*c^4 - 320*a^12*b*c^5)*x*sqrt((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(a^14*b^6 - 12*a^15*b^4*c + 48*a^16*b^2*c^2 - 64*a^17*c^3)) - (b^14 - 16*a*b^12*c + 102*a^2*b^10*c^2 - 328*a^3*b^8*c^3 + 553*a^4*b^6*c^4 - 457*a^5*b^4*c^5 + 152*a^6*b^2*c^6 - 16*a^7*c^7)*x)*sqrt(sqrt(1/2)*sqrt(-(b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c^3 + (a^7*b^4 - 8*a^8*b^2*c + 16*a^9*c^2)*sqrt((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(a^14*b^6 - 12*a^15*b^4*c + 48*a^16*b^2*c^2 - 64*a^17*c^3)))/(a^7*b^4 - 8*a^8*b^2*c + 16*a^9*c^2)))*sqrt(-(b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c^3 + (a^7*b^4 - 8*a^8*b^2*c + 16*a^9*c^2)*sqrt((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(a^14*b^6 - 12*a^15*b^4*c + 48*a^16*b^2*c^2 - 64*a^17*c^3)))/(a^7*b^4 - 8*a^8*b^2*c + 16*a^9*c^2)) + (b^14 - 16*a*b^12*c + 102*a^2*b^10*c^2 - 328*a^3*b^8*c^3 + 553*a^4*b^6*c^4 - 457*a^5*b^4*c^5 + 152*a^6*b^2*c^6 - 16*a^7*c^7 - (a^7*b^11 - 17*a^8*b^9*c + 113*a^9*b^7*c^2 - 364*a^10*b^5*c^3 + 560*a^11*b^3*c^4 - 320*a^12*b*c^5)*sqrt((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(a^14*b^6 - 12*a^15*b^4*c + 48*a^16*b^2*c^2 - 64*a^17*c^3)))*sqrt(sqrt(1/2)*sqrt(-(b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c^3 + (a^7*b^4 - 8*a^8*b^2*c + 16*a^9*c^2)*sqrt((b^

12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^
5*b^2*c^5 + a^6*c^6)/(a^14*b^6 - 12*a^15*b^4*c + 48*a^16*b^2*c^2 - 64*a^17*
c^3)))/(a^7*b^4 - 8*a^8*b^2*c + 16*a^9*c^2)))*sqrt(-(b^7 - 7*a*b^5*c + 14*a
^2*b^3*c^2 - 7*a^3*b*c^3 + (a^7*b^4 - 8*a^8*b^2*c + 16*a^9*c^2)*sqrt((b^12
- 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b
^2*c^5 + a^6*c^6)/(a^14*b^6 - 12*a^15*b^4*c + 48*a^16*b^2*c^2 - 64*a^17*c^3
)))/(a^7*b^4 - 8*a^8*b^2*c + 16*a^9*c^2))*sqrt((2*(b^6*c^4 - 5*a*b^4*c^5 +
6*a^2*b^2*c^6 - a^3*c^7)*x^2 + sqrt(1/2)*(b^12 - 13*a*b^10*c + 64*a^2*b^8*c
^2 - 147*a^3*b^6*c^3 + 156*a^4*b^4*c^4 - 66*a^5*b^2*c^5 + 8*a^6*c^6 - (a^7*
b^9 - 14*a^8*b^7*c + 72*a^9*b^5*c^2 - 160*a^10*b^3*c^3 + 128*a^11*b*c^4)*sq
rt((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 -
12*a^5*b^2*c^5 + a^6*c^6)/(a^14*b^6 - 12*a^15*b^4*c + 48*a^16*b^2*c^2 - 64
*a^17*c^3)))*sqrt(-(b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c^3 + (a^7*b
^4 - 8*a^8*b^2*c + 16*a^9*c^2)*sqrt((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 -
62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(a^14*b^6 - 12*
a^15*b^4*c + 48*a^16*b^2*c^2 - 64*a^17*c^3)))/(a^7*b^4 - 8*a^8*b^2*c + 16*a
^9*c^2)))/(b^6*c^4 - 5*a*b^4*c^5 + 6*a^2*b^2*c^6 - a^3*c^7)))/(b^6*c^5 - 5*
a*b^4*c^6 + 6*a^2*b^2*c^7 - a^3*c^8)) - 3*a*x^3*sqrt(sqrt(1/2)*sqrt(-(b^7 -
7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c^3 + (a^7*b^4 - 8*a^8*b^2*c + 16*a^9
*c^2)*sqrt((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b
^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(a^14*b^6 - 12*a^15*b^4*c + 48*a^16*b^2*
c^2 - 64*a^17*c^3)))/(a^7*b^4 - 8*a^8*b^2*c + 16*a^9*c^2)))*log(-(b^6*c^2 -
5*a*b^4*c^3 + 6*a^2*b^2*c^4 - a^3*c^5)*x + 1/2*(b^9 - 9*a*b^7*c + 26*a^2*b
^5*c^2 - 25*a^3*b^3*c^3 + 4*a^4*b*c^4 - (a^7*b^6 - 10*a^8*b^4*c + 32*a^9*b^
2*c^2 - 32*a^10*c^3)*sqrt((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6
*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(a^14*b^6 - 12*a^15*b^4*c
+ 48*a^16*b^2*c^2 - 64*a^17*c^3)))*sqrt(sqrt(1/2)*sqrt(-(b^7 - 7*a*b^5*c +
14*a^2*b^3*c^2 - 7*a^3*b*c^3 + (a^7*b^4 - 8*a^8*b^2*c + 16*a^9*c^2)*sqrt((
b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*
a^5*b^2*c^5 + a^6*c^6)/(a^14*b^6 - 12*a^15*b^4*c + 48*a^16*b^2*c^2 - 64*a^1
7*c^3)))/(a^7*b^4 - 8*a^8*b^2*c + 16*a^9*c^2)))) + 3*a*x^3*sqrt(sqrt(1/2)*s
qrt(-(b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c^3 + (a^7*b^4 - 8*a^8*b^2
*c + 16*a^9*c^2)*sqrt((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3
+ 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(a^14*b^6 - 12*a^15*b^4*c + 4
8*a^16*b^2*c^2 - 64*a^17*c^3)))/(a^7*b^4 - 8*a^8*b^2*c + 16*a^9*c^2)))*log(
-(b^6*c^2 - 5*a*b^4*c^3 + 6*a^2*b^2*c^4 - a^3*c^5)*x - 1/2*(b^9 - 9*a*b^7*c
+ 26*a^2*b^5*c^2 - 25*a^3*b^3*c^3 + 4*a^4*b*c^4 - (a^7*b^6 - 10*a^8*b^4*c
+ 32*a^9*b^2*c^2 - 32*a^10*c^3)*sqrt((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 -
62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(a^14*b^6 - 12
*a^15*b^4*c + 48*a^16*b^2*c^2 - 64*a^17*c^3)))*sqrt(sqrt(1/2)*sqrt(-(b^7 -
7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c^3 + (a^7*b^4 - 8*a^8*b^2*c + 16*a^9*
c^2)*sqrt((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^
4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(a^14*b^6 - 12*a^15*b^4*c + 48*a^16*b^2*c
^2 - 64*a^17*c^3)))/(a^7*b^4 - 8*a^8*b^2*c + 16*a^9*c^2)))) - 3*a*x^3*sqrt(
sqrt(1/2)*sqrt(-(b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c^3 - (a^7*b^4
- 8*a^8*b^2*c + 16*a^9*c^2)*sqrt((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*
a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(a^14*b^6 - 12*a^1
5*b^4*c + 48*a^16*b^2*c^2 - 64*a^17*c^3)))/(a^7*b^4 - 8*a^8*b^2*c + 16*a^9*
c^2)))*log(-(b^6*c^2 - 5*a*b^4*c^3 + 6*a^2*b^2*c^4 - a^3*c^5)*x + 1/2*(b^9
- 9*a*b^7*c + 26*a^2*b^5*c^2 - 25*a^3*b^3*c^3 + 4*a^4*b*c^4 + (a^7*b^6 - 10
*a^8*b^4*c + 32*a^9*b^2*c^2 - 32*a^10*c^3)*sqrt((b^12 - 10*a*b^10*c + 37*a^
2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(a^
14*b^6 - 12*a^15*b^4*c + 48*a^16*b^2*c^2 - 64*a^17*c^3)))*sqrt(sqrt(1/2)*sq
rt(-(b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c^3 - (a^7*b^4 - 8*a^8*b^2*
c + 16*a^9*c^2)*sqrt((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3
+ 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(a^14*b^6 - 12*a^15*b^4*c + 48
*a^16*b^2*c^2 - 64*a^17*c^3)))/(a^7*b^4 - 8*a^8*b^2*c + 16*a^9*c^2)))) + 3*
a*x^3*sqrt(sqrt(1/2)*sqrt(-(b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c^3
- (a^7*b^4 - 8*a^8*b^2*c + 16*a^9*c^2)*sqrt((b^12 - 10*a*b^10*c + 37*a^2*b^

```
8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(a^14*b
^6 - 12*a^15*b^4*c + 48*a^16*b^2*c^2 - 64*a^17*c^3)))/(a^7*b^4 - 8*a^8*b^2*
c + 16*a^9*c^2)))*log(-(b^6*c^2 - 5*a*b^4*c^3 + 6*a^2*b^2*c^4 - a^3*c^5)*x
- 1/2*(b^9 - 9*a*b^7*c + 26*a^2*b^5*c^2 - 25*a^3*b^3*c^3 + 4*a^4*b*c^4 + (a
^7*b^6 - 10*a^8*b^4*c + 32*a^9*b^2*c^2 - 32*a^10*c^3)*sqrt((b^12 - 10*a*b^1
0*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a
^6*c^6)/(a^14*b^6 - 12*a^15*b^4*c + 48*a^16*b^2*c^2 - 64*a^17*c^3)))*sqrt(s
qrt(1/2)*sqrt(-(b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c^3 - (a^7*b^4 -
 8*a^8*b^2*c + 16*a^9*c^2)*sqrt((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a
^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(a^14*b^6 - 12*a^15
*b^4*c + 48*a^16*b^2*c^2 - 64*a^17*c^3)))/(a^7*b^4 - 8*a^8*b^2*c + 16*a^9*c
^2)))) - 4)/(a*x^3)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(c*x^8+b*x^4+a),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Evaluation time: 21.84Unable to convert to r
eal 1/4 Error: Bad Argument Value
```

maple [C] time = 0.01, size = 62, normalized size = 0.17

$$\frac{\left(-\mathrm{RootOf}\left(\_Z^8 c+b\_Z^4+a\right)^4 c-b\right)\ln\left(-\mathrm{RootOf}\left(\_Z^8 c+b\_Z^4+a\right)+x\right)}{4a\left(2\,\mathrm{RootOf}\left(\_Z^8 c+b\_Z^4+a\right)^7 c+\mathrm{RootOf}\left(\_Z^8 c+b\_Z^4+a\right)^3 b\right)}-\frac{1}{3a\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(c*x^8+b*x^4+a),x)
```

```
[Out] -1/3/a/x^3+1/4/a*sum((-_R^4*c-b)/(2*_R^7*c+_R^3*b)*ln(-_R+x),_R=RootOf(_Z^8
*c+_Z^4*b+a))
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(c*x^8+b*x^4+a),x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [B] time = 5.57, size = 16497, normalized size = 45.20

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(a + b*x^4 + c*x^8)),x)
```

```
[Out] 2*atan(-(((-(b^11 + b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b
^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 - a^3*c^3*(-(4*a*c - b^2)^5)^(1/
2) - 15*a*b^9*c + 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 5*a*b^4*c*(-(4*a
*c - b^2)^5)^(1/2))/(512*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^
4*c^2 - 256*a^10*b^2*c^3)))^(1/4)*((x*(81920*a^15*b*c^8 + 1024*a^11*b^9*c^4
 - 13312*a^12*b^7*c^5 + 62464*a^13*b^5*c^6 - 122880*a^14*b^3*c^7) - (-(b^11
 + b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*
```

```
b^5*c^3 + 280*a^4*b^3*c^4 - a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c +
 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2
))/(512*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*
b^2*c^3)))^(1/4)*(262144*a^17*c^8 + 4096*a^13*b^8*c^4 - 53248*a^14*b^6*c^5
+ 245760*a^15*b^4*c^6 - 458752*a^16*b^2*c^7)*1i)*(-(b^11 + b^6*(-(4*a*c - b
^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b
^3*c^4 - a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c + 6*a^2*b^2*c^2*(-(4
*a*c - b^2)^5)^(1/2) - 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^7*b^8 +
256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(3/4)*1i
 - 128*a^11*b*c^9 - 16*a^9*b^5*c^7 + 96*a^10*b^3*c^8)*1i + x*(8*a^10*c^10 -
 4*a^9*b^2*c^9))*(-(b^11 + b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 8
6*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 - a^3*c^3*(-(4*a*c - b^2)
^5)^(1/2) - 15*a*b^9*c + 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 5*a*b^4*c
*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96
*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(1/4) + ((-(b^11 + b^6*(-(4*a*c - b^2)^5
)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^
4 - a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c + 6*a^2*b^2*c^2*(-(4*a*c
- b^2)^5)^(1/2) - 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^7*b^8 + 256*a
^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(1/4)*((x*(81
920*a^15*b*c^8 + 1024*a^11*b^9*c^4 - 13312*a^12*b^7*c^5 + 62464*a^13*b^5*c^
6 - 122880*a^14*b^3*c^7) + (-(b^11 + b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5
*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 - a^3*c^3*(-(4*
a*c - b^2)^5)^(1/2) - 15*a*b^9*c + 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) -
 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*
b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(1/4)*(262144*a^17*c^8 + 4096*
a^13*b^8*c^4 - 53248*a^14*b^6*c^5 + 245760*a^15*b^4*c^6 - 458752*a^16*b^2*c
^7)*1i)*(-(b^11 + b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7
*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 - a^3*c^3*(-(4*a*c - b^2)^5)^(1/2)
 - 15*a*b^9*c + 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 5*a*b^4*c*(-(4*a*c
 - b^2)^5)^(1/2))/(512*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*
c^2 - 256*a^10*b^2*c^3)))^(3/4)*1i + 128*a^11*b*c^9 + 16*a^9*b^5*c^7 - 96*a
^10*b^3*c^8)*1i + x*(8*a^10*c^10 - 4*a^9*b^2*c^9))*(-(b^11 + b^6*(-(4*a*c -
 b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4
*b^3*c^4 - a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c + 6*a^2*b^2*c^2*(-
(4*a*c - b^2)^5)^(1/2) - 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^7*b^8
+ 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(1/4))
/(((-(b^11 + b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2
- 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 - a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15
*a*b^9*c + 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 5*a*b^4*c*(-(4*a*c - b^
2)^5)^(1/2))/(512*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 -
 256*a^10*b^2*c^3)))^(1/4)*((x*(81920*a^15*b*c^8 + 1024*a^11*b^9*c^4 - 1331
2*a^12*b^7*c^5 + 62464*a^13*b^5*c^6 - 122880*a^14*b^3*c^7) - (-(b^11 + b^6*
(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3
 + 280*a^4*b^3*c^4 - a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c + 6*a^2*
b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(512
*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3
)))^(1/4)*(262144*a^17*c^8 + 4096*a^13*b^8*c^4 - 53248*a^14*b^6*c^5 + 24576
0*a^15*b^4*c^6 - 458752*a^16*b^2*c^7)*1i)*(-(b^11 + b^6*(-(4*a*c - b^2)^5)^
(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4
- a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c + 6*a^2*b^2*c^2*(-(4*a*c -
b^2)^5)^(1/2) - 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^7*b^8 + 256*a^1
1*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(3/4)*1i - 128*
a^11*b*c^9 - 16*a^9*b^5*c^7 + 96*a^10*b^3*c^8)*1i + x*(8*a^10*c^10 - 4*a^9*
b^2*c^9))*(-(b^11 + b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b
^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 - a^3*c^3*(-(4*a*c - b^2)^5)^(1/
2) - 15*a*b^9*c + 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 5*a*b^4*c*(-(4*a
*c - b^2)^5)^(1/2))/(512*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^
4*c^2 - 256*a^10*b^2*c^3)))^(1/4)*1i - ((-(b^11 + b^6*(-(4*a*c - b^2)^5)^(1
/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 -
```

a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c + 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(1/4)*((x*(81920*a^15*b*c^8 + 1024*a^11*b^9*c^4 - 13312*a^12*b^7*c^5 + 62464*a^13*b^5*c^6 - 122880*a^14*b^3*c^7) + (-(b^11 + b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 - a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c + 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(1/4)*(262144*a^17*c^8 + 4096*a^13*b^8*c^4 - 53248*a^14*b^6*c^5 + 245760*a^15*b^4*c^6 - 458752*a^16*b^2*c^7)*1i)*(-(b^11 + b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 - a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c + 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(3/4)*1i + 128*a^11*b*c^9 + 16*a^9*b^5*c^7 - 96*a^10*b^3*c^8)*1i + x*(8*a^10*c^10 - 4*a^9*b^2*c^9))*(-(b^11 + b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 - a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c + 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(1/4)*1i))*(-(b^11 + b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 - a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c + 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(1/4) - atan(-(((-(b^11 - b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 + a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c - 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) + 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(1/4)*((x*(81920*a^15*b*c^8 + 1024*a^11*b^9*c^4 - 13312*a^12*b^7*c^5 + 62464*a^13*b^5*c^6 - 122880*a^14*b^3*c^7) + (-(b^11 - b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 + a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c - 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) + 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(1/4)*(262144*a^17*c^8 + 4096*a^13*b^8*c^4 - 53248*a^14*b^6*c^5 + 245760*a^15*b^4*c^6 - 458752*a^16*b^2*c^7))*(-(b^11 - b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 + a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c - 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) + 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(3/4) - 128*a^11*b*c^9 - 16*a^9*b^5*c^7 + 96*a^10*b^3*c^8) - x*(8*a^10*c^10 - 4*a^9*b^2*c^9))*(-(b^11 - b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 + a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c - 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) + 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(1/4)*1i + ((-(b^11 - b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 + a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c - 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) + 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(1/4)*((x*(81920*a^15*b*c^8 + 1024*a^11*b^9*c^4 - 13312*a^12*b^7*c^5 + 62464*a^13*b^5*c^6 - 122880*a^14*b^3*c^7) - (-(b^11 - b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 + a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c - 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) + 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(1/4)*(262144*a^17*c^8 + 4096*a^13*b^8*c^4 - 53248*a^14*b^6*c^5 + 245760*a^15*b^4*c^6 - 458752*a^16*b^2*c^7))*(-(b^11 - b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 + a^3*c^3*(-(4*a*c

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 - b^2)^5)^(1/2) - 15*a*b^9*c - 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) + 5*
a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6
*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(3/4) + 128*a^11*b*c^9 + 16*a^9*b
^5*c^7 - 96*a^10*b^3*c^8) - x*(8*a^10*c^10 - 4*a^9*b^2*c^9))*(-(b^11 - b^6*
(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3
 + 280*a^4*b^3*c^4 + a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c - 6*a^2*
b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) + 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(512
*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3
)))^(1/4)*1i)/(((-(b^11 - b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86
*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 + a^3*c^3*(-(4*a*c - b^2)^
5)^(1/2) - 15*a*b^9*c - 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) + 5*a*b^4*c*
(-(4*a*c - b^2)^5)^(1/2))/(512*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*
a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(1/4)*((x*(81920*a^15*b*c^8 + 1024*a^11*b
^9*c^4 - 13312*a^12*b^7*c^5 + 62464*a^13*b^5*c^6 - 122880*a^14*b^3*c^7) + (
-(b^11 - b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 23
1*a^3*b^5*c^3 + 280*a^4*b^3*c^4 + a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b
^9*c - 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) + 5*a*b^4*c*(-(4*a*c - b^2)^5
)^(1/2))/(512*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256
*a^10*b^2*c^3)))^(1/4)*(262144*a^17*c^8 + 4096*a^13*b^8*c^4 - 53248*a^14*b^
6*c^5 + 245760*a^15*b^4*c^6 - 458752*a^16*b^2*c^7))*(-(b^11 - b^6*(-(4*a*c
 - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^
4*b^3*c^4 + a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c - 6*a^2*b^2*c^2*(
-(4*a*c - b^2)^5)^(1/2) + 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^7*b^8
 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(3/4)
 - 128*a^11*b*c^9 - 16*a^9*b^5*c^7 + 96*a^10*b^3*c^8) - x*(8*a^10*c^10 - 4*
a^9*b^2*c^9))*(-(b^11 - b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a
^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 + a^3*c^3*(-(4*a*c - b^2)^5)
^(1/2) - 15*a*b^9*c - 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) + 5*a*b^4*c*(-
(4*a*c - b^2)^5)^(1/2))/(512*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^
9*b^4*c^2 - 256*a^10*b^2*c^3)))^(1/4) - ((-(b^11 - b^6*(-(4*a*c - b^2)^5)^(
1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 +
 a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c - 6*a^2*b^2*c^2*(-(4*a*c - b
^2)^5)^(1/2) + 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^7*b^8 + 256*a^11
*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(1/4)*((x*(81920
*a^15*b*c^8 + 1024*a^11*b^9*c^4 - 13312*a^12*b^7*c^5 + 62464*a^13*b^5*c^6 -
 122880*a^14*b^3*c^7) - (-(b^11 - b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*
c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 + a^3*c^3*(-(4*a*c
 - b^2)^5)^(1/2) - 15*a*b^9*c - 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) + 5*
a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6
*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(1/4)*(262144*a^17*c^8 + 4096*a^1
3*b^8*c^4 - 53248*a^14*b^6*c^5 + 245760*a^15*b^4*c^6 - 458752*a^16*b^2*c^7)
)*(-(b^11 - b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 -
 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 + a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*
a*b^9*c - 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) + 5*a*b^4*c*(-(4*a*c - b^2
)^5)^(1/2))/(512*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 -
256*a^10*b^2*c^3)))^(3/4) + 128*a^11*b*c^9 + 16*a^9*b^5*c^7 - 96*a^10*b^3*c
^8) - x*(8*a^10*c^10 - 4*a^9*b^2*c^9))*(-(b^11 - b^6*(-(4*a*c - b^2)^5)^(1/
2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 + a
^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c - 6*a^2*b^2*c^2*(-(4*a*c - b^2
)^5)^(1/2) + 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^7*b^8 + 256*a^11*c
^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(1/4)))*(-(b^11 -
b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5
*c^3 + 280*a^4*b^3*c^4 + a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c - 6*
a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) + 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/
(512*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2
*c^3)))^(1/4)*2i - atan(-(((-(b^11 + b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5
*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 - a^3*c^3*(-(4*
a*c - b^2)^5)^(1/2) - 15*a*b^9*c + 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) -
 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*
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b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(1/4)*((x*(81920*a^15*b*c^8 + 1024*a^11*b^9*c^4 - 13312*a^12*b^7*c^5 + 62464*a^13*b^5*c^6 - 122880*a^14*b^3*c^7) + (-(b^11 + b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 - a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c + 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(1/4)*(262144*a^17*c^8 + 4096*a^13*b^8*c^4 - 53248*a^14*b^6*c^5 + 245760*a^15*b^4*c^6 - 458752*a^16*b^2*c^7))*(-(b^11 + b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 - a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c + 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(3/4) - 128*a^11*b*c^9 - 16*a^9*b^5*c^7 + 96*a^10*b^3*c^8) - x*(8*a^10*c^10 - 4*a^9*b^2*c^9))*(-(b^11 + b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 - a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c + 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(1/4)*1i + ((-(b^11 + b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 - a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c + 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(1/4)*((x*(81920*a^15*b*c^8 + 1024*a^11*b^9*c^4 - 13312*a^12*b^7*c^5 + 62464*a^13*b^5*c^6 - 122880*a^14*b^3*c^7) - (-(b^11 + b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 - a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c + 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(1/4)*(262144*a^17*c^8 + 4096*a^13*b^8*c^4 - 53248*a^14*b^6*c^5 + 245760*a^15*b^4*c^6 - 458752*a^16*b^2*c^7))*(-(b^11 + b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 - a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c + 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(3/4) + 128*a^11*b*c^9 + 16*a^9*b^5*c^7 - 96*a^10*b^3*c^8) - x*(8*a^10*c^10 - 4*a^9*b^2*c^9))*(-(b^11 + b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 - a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c + 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(1/4)*1i)/(((-(b^11 + b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 - a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c + 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(1/4)*((x*(81920*a^15*b*c^8 + 1024*a^11*b^9*c^4 - 13312*a^12*b^7*c^5 + 62464*a^13*b^5*c^6 - 122880*a^14*b^3*c^7) + (-(b^11 + b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 - a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c + 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(1/4)*(262144*a^17*c^8 + 4096*a^13*b^8*c^4 - 53248*a^14*b^6*c^5 + 245760*a^15*b^4*c^6 - 458752*a^16*b^2*c^7))*(-(b^11 + b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 - a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c + 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(3/4) - 128*a^11*b*c^9 - 16*a^9*b^5*c^7 + 96*a^10*b^3*c^8) - x*(8*a^10*c^10 - 4*a^9*b^2*c^9))*(-(b^11 + b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 - a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c + 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 5*a*b^4*c*(-(4*a*c -

b^2)^5)^(1/2))/(512*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(1/4) - ((-(b^11 + b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 - a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c + 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(1/4)*((x*(81920*a^15*b*c^8 + 1024*a^11*b^9*c^4 - 13312*a^12*b^7*c^5 + 62464*a^13*b^5*c^6 - 122880*a^14*b^3*c^7) - (-(b^11 + b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 - a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c + 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(1/4)*(262144*a^17*c^8 + 4096*a^13*b^8*c^4 - 53248*a^14*b^6*c^5 + 245760*a^15*b^4*c^6 - 458752*a^16*b^2*c^7))*(-(b^11 + b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 - a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c + 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(3/4) + 128*a^11*b*c^9 + 16*a^9*b^5*c^7 - 96*a^10*b^3*c^8) - x*(8*a^10*c^10 - 4*a^9*b^2*c^9))*(-(b^11 + b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 - a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c + 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(1/4)))*(-(b^11 + b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 - a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c + 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(1/4)*2i + 2*atan(-(((-(b^11 - b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 + a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c - 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) + 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(1/4)*((x*(81920*a^15*b*c^8 + 1024*a^11*b^9*c^4 - 13312*a^12*b^7*c^5 + 62464*a^13*b^5*c^6 - 122880*a^14*b^3*c^7) - (-(b^11 - b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 + a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c - 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) + 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(1/4)*(262144*a^17*c^8 + 4096*a^13*b^8*c^4 - 53248*a^14*b^6*c^5 + 245760*a^15*b^4*c^6 - 458752*a^16*b^2*c^7)*1i)*(-(b^11 - b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 + a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c - 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) + 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(3/4)*1i - 128*a^11*b*c^9 - 16*a^9*b^5*c^7 + 96*a^10*b^3*c^8)*1i + x*(8*a^10*c^10 - 4*a^9*b^2*c^9))*(-(b^11 - b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 + a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c - 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) + 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(1/4) + ((-(b^11 - b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 + a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c - 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) + 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(1/4)*((x*(81920*a^15*b*c^8 + 1024*a^11*b^9*c^4 - 13312*a^12*b^7*c^5 + 62464*a^13*b^5*c^6 - 122880*a^14*b^3*c^7) + (-(b^11 - b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 + a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c - 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) + 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(1/4)*(262144*a^17*

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c^8 + 4096*a^13*b^8*c^4 - 53248*a^14*b^6*c^5 + 245760*a^15*b^4*c^6 - 458752
*a^16*b^2*c^7)*1i)*(-(b^11 - b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 +
 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 + a^3*c^3*(-(4*a*c - b^
2)^5)^(1/2) - 15*a*b^9*c - 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) + 5*a*b^4
*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c +
96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(3/4)*1i + 128*a^11*b*c^9 + 16*a^9*b^5
*c^7 - 96*a^10*b^3*c^8)*1i + x*(8*a^10*c^10 - 4*a^9*b^2*c^9))*(-(b^11 - b^6
*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^
3 + 280*a^4*b^3*c^4 + a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c - 6*a^2
*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) + 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(51
2*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^
3)))^(1/4))/(((-(b^11 - b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a
^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 + a^3*c^3*(-(4*a*c - b^2)^5)
^(1/2) - 15*a*b^9*c - 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) + 5*a*b^4*c*(-
(4*a*c - b^2)^5)^(1/2))/(512*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^
9*b^4*c^2 - 256*a^10*b^2*c^3)))^(1/4)*((x*(81920*a^15*b*c^8 + 1024*a^11*b^9
*c^4 - 13312*a^12*b^7*c^5 + 62464*a^13*b^5*c^6 - 122880*a^14*b^3*c^7) - (-(
b^11 - b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*
a^3*b^5*c^3 + 280*a^4*b^3*c^4 + a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9
*c - 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) + 5*a*b^4*c*(-(4*a*c - b^2)^5)^
(1/2))/(512*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a
^10*b^2*c^3)))^(1/4)*(262144*a^17*c^8 + 4096*a^13*b^8*c^4 - 53248*a^14*b^6*
c^5 + 245760*a^15*b^4*c^6 - 458752*a^16*b^2*c^7)*1i)*(-(b^11 - b^6*(-(4*a*c
 - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a
^4*b^3*c^4 + a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c - 6*a^2*b^2*c^2*
(-(4*a*c - b^2)^5)^(1/2) + 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^7*b^
8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(3/4
)*1i - 128*a^11*b*c^9 - 16*a^9*b^5*c^7 + 96*a^10*b^3*c^8)*1i + x*(8*a^10*c^
10 - 4*a^9*b^2*c^9))*(-(b^11 - b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5
 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 + a^3*c^3*(-(4*a*c -
b^2)^5)^(1/2) - 15*a*b^9*c - 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) + 5*a*b
^4*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c
+ 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(1/4)*1i - ((-(b^11 - b^6*(-(4*a*c -
 b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4
*b^3*c^4 + a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c - 6*a^2*b^2*c^2*(-
(4*a*c - b^2)^5)^(1/2) + 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^7*b^8
+ 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(1/4)*
((x*(81920*a^15*b*c^8 + 1024*a^11*b^9*c^4 - 13312*a^12*b^7*c^5 + 62464*a^13
*b^5*c^6 - 122880*a^14*b^3*c^7) + (-(b^11 - b^6*(-(4*a*c - b^2)^5)^(1/2) -
112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 + a^3*c^
3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c - 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^
(1/2) + 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^7*b^8 + 256*a^11*c^4 -
16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(1/4)*(262144*a^17*c^8
+ 4096*a^13*b^8*c^4 - 53248*a^14*b^6*c^5 + 245760*a^15*b^4*c^6 - 458752*a^1
6*b^2*c^7)*1i)*(-(b^11 - b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*
a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 + a^3*c^3*(-(4*a*c - b^2)^5
)^(1/2) - 15*a*b^9*c - 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) + 5*a*b^4*c*(
-(4*a*c - b^2)^5)^(1/2))/(512*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a
^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(3/4)*1i + 128*a^11*b*c^9 + 16*a^9*b^5*c^7
 - 96*a^10*b^3*c^8)*1i + x*(8*a^10*c^10 - 4*a^9*b^2*c^9))*(-(b^11 - b^6*(-(
4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 +
280*a^4*b^3*c^4 + a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c - 6*a^2*b^2
*c^2*(-(4*a*c - b^2)^5)^(1/2) + 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a
^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))
^(1/4)*1i))*(-(b^11 - b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2
*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 + a^3*c^3*(-(4*a*c - b^2)^5)^(
1/2) - 15*a*b^9*c - 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) + 5*a*b^4*c*(-(4
*a*c - b^2)^5)^(1/2))/(512*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*
b^4*c^2 - 256*a^10*b^2*c^3)))^(1/4) - 1/(3*a*x^3)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(c*x**8+b*x**4+a),x)
[Out] Timed out
```

3.270 $\int \frac{x^{11}}{1+x^4+x^8} \, dx$

Optimal. Leaf size=44

$$\frac{x^4}{4}-\frac{\tan^{-1}\left(\frac{2x^4+1}{\sqrt{3}}\right)}{4\sqrt{3}}-\frac{1}{8}\log\left(x^8+x^4+1\right)$$

Rubi [A] time = 0.04, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {1357, 703, 634, 618, 204, 628}

$$\frac{x^4}{4}-\frac{1}{8}\log\left(x^8+x^4+1\right)-\frac{\tan^{-1}\left(\frac{2x^4+1}{\sqrt{3}}\right)}{4\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^11/(1 + x^4 + x^8),x]
```

```
[Out] x^4/4 - ArcTan[(1 + 2*x^4)/Sqrt[3]]/(4*Sqrt[3]) - Log[1 + x^4 + x^8]/8
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 703

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Simp[(e*(d + e*x)^(m - 1))/(c*(m - 1)), x] + Dist[1/c, Int[((d + e*x)^
(m - 2)*Simp[c*d^2 - a*e^2 + e*(2*c*d - b*e)*x, x])/(a + b*x + c*x^2), x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && GtQ[m, 1]
```

Rule 1357

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
```

```
4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x^{11}}{1+x^4+x^8}\,dx &= \frac{1}{4}\,\text{Subst}\left(\int \frac{x^2}{1+x+x^2}\,dx,x,x^4\right)\\ &= \frac{x^4}{4}+\frac{1}{4}\,\text{Subst}\left(\int \frac{-1-x}{1+x+x^2}\,dx,x,x^4\right)\\ &= \frac{x^4}{4}-\frac{1}{8}\,\text{Subst}\left(\int \frac{1}{1+x+x^2}\,dx,x,x^4\right)-\frac{1}{8}\,\text{Subst}\left(\int \frac{1+2x}{1+x+x^2}\,dx,x,x^4\right)\\ &= \frac{x^4}{4}-\frac{1}{8}\log\left(1+x^4+x^8\right)+\frac{1}{4}\,\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,1+2x^4\right)\\ &= \frac{x^4}{4}-\frac{\tan^{-1}\left(\frac{1+2x^4}{\sqrt{3}}\right)}{4\sqrt{3}}-\frac{1}{8}\log\left(1+x^4+x^8\right)\end{aligned}$$

Mathematica [A] time = 0.01, size = 44, normalized size = 1.00

$$\frac{x^4}{4}-\frac{\tan^{-1}\left(\frac{2x^4+1}{\sqrt{3}}\right)}{4\sqrt{3}}-\frac{1}{8}\log\left(x^8+x^4+1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^11/(1 + x^4 + x^8),x]
[Out] x^4/4 - ArcTan[(1 + 2*x^4)/Sqrt[3]]/(4*Sqrt[3]) - Log[1 + x^4 + x^8]/8
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{11}}{1+x^4+x^8}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^11/(1 + x^4 + x^8),x]
[Out] IntegrateAlgebraic[x^11/(1 + x^4 + x^8), x]
```

fricas [A] time = 1.42, size = 35, normalized size = 0.80

$$\frac{1}{4}\,x^4-\frac{1}{12}\,\sqrt{3}\,\arctan\left(\frac{1}{3}\,\sqrt{3}\left(2\,x^4+1\right)\right)-\frac{1}{8}\,\log\left(x^8+x^4+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(x^8+x^4+1),x, algorithm="fricas")
[Out] 1/4*x^4 - 1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^4 + 1)) - 1/8*log(x^8 + x^4
+ 1)
```

giac [A] time = 0.34, size = 35, normalized size = 0.80

$$\frac{1}{4}\,x^4-\frac{1}{12}\,\sqrt{3}\,\arctan\left(\frac{1}{3}\,\sqrt{3}\left(2\,x^4+1\right)\right)-\frac{1}{8}\,\log\left(x^8+x^4+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(x^8+x^4+1),x, algorithm="giac")
```

```
[Out] 1/4*x^4 - 1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^4 + 1)) - 1/8*log(x^8 + x^4
+ 1)
```

maple [A] time = 0.00, size = 36, normalized size = 0.82

$$\frac{x^4}{4} - \frac{\sqrt{3}\ \arctan\left(\frac{\left(2x^4+1\right)\sqrt{3}}{3}\right)}{12} - \frac{\ln\left(x^8+x^4+1\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11/(x^8+x^4+1),x)
```

```
[Out] 1/4*x^4-1/8*ln(x^8+x^4+1)-1/12*arctan(1/3*(2*x^4+1)*3^(1/2))*3^(1/2)
```

maxima [A] time = 2.42, size = 35, normalized size = 0.80

$$\frac{1}{4}\,x^4 - \frac{1}{12}\,\sqrt{3}\,\arctan\left(\frac{1}{3}\,\sqrt{3}\left(2\,x^4+1\right)\right) - \frac{1}{8}\,\log\left(x^8+x^4+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(x^8+x^4+1),x, algorithm="maxima")
```

```
[Out] 1/4*x^4 - 1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^4 + 1)) - 1/8*log(x^8 + x^4
+ 1)
```

mupad [B] time = 0.05, size = 37, normalized size = 0.84

$$\frac{x^4}{4} - \frac{\sqrt{3}\ \mathrm{atan}\left(\frac{2\,\sqrt{3}\,x^4}{3}+\frac{\sqrt{3}}{3}\right)}{12} - \frac{\ln\left(x^8+x^4+1\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11/(x^4 + x^8 + 1),x)
```

```
[Out] x^4/4 - (3^(1/2)*atan(3^(1/2)/3 + (2*3^(1/2)*x^4)/3))/12 - log(x^4 + x^8 +
1)/8
```

sympy [A] time = 0.14, size = 42, normalized size = 0.95

$$\frac{x^4}{4} - \frac{\log\left(x^8+x^4+1\right)}{8} - \frac{\sqrt{3}\ \mathrm{atan}\left(\frac{2\sqrt{3}\,x^4}{3}+\frac{\sqrt{3}}{3}\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**11/(x**8+x**4+1),x)
```

```
[Out] x**4/4 - log(x**8 + x**4 + 1)/8 - sqrt(3)*atan(2*sqrt(3)*x**4/3 + sqrt(3)/3
)/12
```

3.271 $\int \frac{x^9}{1+x^4+x^8} \, dx$

Optimal. Leaf size=54

$$\frac{x^2}{2}+\frac{\tan^{-1}\left(\frac{1-2x^2}{\sqrt{3}}\right)}{2\sqrt{3}}-\frac{\tan^{-1}\left(\frac{2x^2+1}{\sqrt{3}}\right)}{2\sqrt{3}}$$

Rubi [A] time = 0.06, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {1359, 1122, 1161, 618, 204}

$$\frac{x^2}{2}+\frac{\tan^{-1}\left(\frac{1-2x^2}{\sqrt{3}}\right)}{2\sqrt{3}}-\frac{\tan^{-1}\left(\frac{2x^2+1}{\sqrt{3}}\right)}{2\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^9/(1 + x^4 + x^8),x]
```

```
[Out] x^2/2 + ArcTan[(1 - 2*x^2)/Sqrt[3]]/(2*Sqrt[3]) - ArcTan[(1 + 2*x^2)/Sqrt[3
]]/(2*Sqrt[3])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1122

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Simp[(d^3*(d*x)^(m - 3)*(a + b*x^2 + c*x^4)^(p + 1))/(c*(m + 4*p + 1)),
 x] - Dist[d^4/(c*(m + 4*p + 1)), Int[(d*x)^(m - 4)*Simp[a*(m - 3) + b*(m +
 2*p - 1)*x^2, x]*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x
] && NeQ[b^2 - 4*a*c, 0] && GtQ[m, 3] && NeQ[m + 4*p + 1, 0] && IntegerQ[2*
p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1161

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[(2*d)/e - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2
, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (
GtQ[(2*d)/e - b/c, 0] || ( !LtQ[(2*d)/e - b/c, 0] && EqQ[d - e*Rt[a/c, 2],
0]))
```

Rule 1359

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*
x^(n/k) + c*x^((2*n)/k))^p, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, c, p
}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^9}{1+x^4+x^8}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{x^4}{1+x^2+x^4}\,dx,x,x^2\right)\\
&= \frac{x^2}{2}-\frac{1}{2}\,\text{Subst}\left(\int \frac{1+x^2}{1+x^2+x^4}\,dx,x,x^2\right)\\
&= \frac{x^2}{2}-\frac{1}{4}\,\text{Subst}\left(\int \frac{1}{1-x+x^2}\,dx,x,x^2\right)-\frac{1}{4}\,\text{Subst}\left(\int \frac{1}{1+x+x^2}\,dx,x,x^2\right)\\
&= \frac{x^2}{2}+\frac{1}{2}\,\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,-1+2x^2\right)+\frac{1}{2}\,\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,1+2x^2\right)\\
&= \frac{x^2}{2}+\frac{\tan^{-1}\left(\frac{1-2x^2}{\sqrt{3}}\right)}{2\sqrt{3}}-\frac{\tan^{-1}\left(\frac{1+2x^2}{\sqrt{3}}\right)}{2\sqrt{3}}
\end{aligned}$$

Mathematica [C] time = 0.17, size = 98, normalized size = 1.81

$$\frac{x^2}{2}-\frac{\left(\sqrt{3}+i\right)\tan^{-1}\left(\frac{1}{2}\left(\sqrt{3}-i\right)x^2\right)}{2\sqrt{6+6i\sqrt{3}}}-\frac{\left(\sqrt{3}-i\right)\tan^{-1}\left(\frac{1}{2}\left(\sqrt{3}+i\right)x^2\right)}{2\sqrt{6-6i\sqrt{3}}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^9/(1 + x^4 + x^8),x]
```

```
[Out] x^2/2 - ((I + Sqrt[3])*ArcTan[((-I + Sqrt[3])*x^2)/2])/(2*Sqrt[6 + (6*I)*Sq
rt[3]]) - ((-I + Sqrt[3])*ArcTan[((I + Sqrt[3])*x^2)/2])/(2*Sqrt[6 - (6*I)*
Sqrt[3]])
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^9}{1+x^4+x^8}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^9/(1 + x^4 + x^8),x]
```

```
[Out] IntegrateAlgebraic[x^9/(1 + x^4 + x^8), x]
```

fricas [A] time = 1.23, size = 40, normalized size = 0.74

$$\frac{1}{2}\,x^2-\frac{1}{6}\,\sqrt{3}\,\arctan\left(\frac{1}{3}\,\sqrt{3}x^2\right)-\frac{1}{6}\,\sqrt{3}\,\arctan\left(\frac{1}{3}\,\sqrt{3}\left(x^6+2\,x^2\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(x^8+x^4+1),x, algorithm="fricas")
```

```
[Out] 1/2*x^2 - 1/6*sqrt(3)*arctan(1/3*sqrt(3)*x^2) - 1/6*sqrt(3)*arctan(1/3*sqrt
(3)*(x^6 + 2*x^2))
```

giac [A] time = 0.32, size = 42, normalized size = 0.78

$$\frac{1}{2}\,x^2-\frac{1}{6}\,\sqrt{3}\,\arctan\left(\frac{1}{3}\,\sqrt{3}\left(2\,x^2+1\right)\right)-\frac{1}{6}\,\sqrt{3}\,\arctan\left(\frac{1}{3}\,\sqrt{3}\left(2\,x^2-1\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(x^8+x^4+1),x, algorithm="giac")
```

```
[Out] 1/2*x^2 - 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^2 + 1)) - 1/6*sqrt(3)*arctan(
1/3*sqrt(3)*(2*x^2 - 1))
```

maple [A] time = 0.01, size = 43, normalized size = 0.80

$$\frac{x^2}{2} - \frac{\sqrt{3}\ \arctan\left(\frac{(2x^2-1)\sqrt{3}}{3}\right)}{6} - \frac{\sqrt{3}\ \arctan\left(\frac{(2x^2+1)\sqrt{3}}{3}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9/(x^8+x^4+1),x)
```

```
[Out] 1/2*x^2-1/6*3^(1/2)*arctan(1/3*(2*x^2+1)*3^(1/2))-1/6*3^(1/2)*arctan(1/3*(2
*x^2-1)*3^(1/2))
```

maxima [A] time = 2.39, size = 42, normalized size = 0.78

$$\frac{1}{2}\,x^2 - \frac{1}{6}\,\sqrt{3}\,\arctan\left(\frac{1}{3}\,\sqrt{3}\left(2\,x^2+1\right)\right) - \frac{1}{6}\,\sqrt{3}\,\arctan\left(\frac{1}{3}\,\sqrt{3}\left(2\,x^2-1\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(x^8+x^4+1),x, algorithm="maxima")
```

```
[Out] 1/2*x^2 - 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^2 + 1)) - 1/6*sqrt(3)*arctan(
1/3*sqrt(3)*(2*x^2 - 1))
```

mupad [B] time = 0.04, size = 43, normalized size = 0.80

$$\frac{x^2}{2} - \frac{\sqrt{3}\,\left(2\,\mathrm{atan}\left(\frac{\sqrt{3}\,x^6}{3} + \frac{2\,\sqrt{3}\,x^2}{3}\right) + 2\,\mathrm{atan}\left(\frac{\sqrt{3}\,x^2}{3}\right)\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9/(x^4 + x^8 + 1),x)
```

```
[Out] x^2/2 - (3^(1/2)*(2*atan((2*3^(1/2)*x^2)/3 + (3^(1/2)*x^6)/3) + 2*atan((3^(
1/2)*x^2)/3)))/12
```

sympy [A] time = 0.14, size = 51, normalized size = 0.94

$$\frac{x^2}{2} + \frac{\sqrt{3}\,\left(-2\,\mathrm{atan}\left(\frac{\sqrt{3}\,x^2}{3}\right) - 2\,\mathrm{atan}\left(\frac{\sqrt{3}\,x^6}{3} + \frac{2\,\sqrt{3}\,x^2}{3}\right)\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**9/(x**8+x**4+1),x)
```

```
[Out] x**2/2 + sqrt(3)*(-2*atan(sqrt(3)*x**2/3) - 2*atan(sqrt(3)*x**6/3 + 2*sqrt(
3)*x**2/3))/12
```

3.272 $\int \frac{x^7}{1+x^4+x^8} \, dx$

Optimal. Leaf size=37

$$\frac{1}{8} \log \left(x^8+x^4+1\right)-\frac{\tan^{-1}\left(\frac{2x^4+1}{\sqrt{3}}\right)}{4\sqrt{3}}$$

Rubi [A] time = 0.03, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {1357, 634, 618, 204, 628}

$$\frac{1}{8} \log \left(x^8+x^4+1\right)-\frac{\tan^{-1}\left(\frac{2x^4+1}{\sqrt{3}}\right)}{4\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^7/(1 + x^4 + x^8),x]
```

```
[Out] -ArcTan[(1 + 2*x^4)/Sqrt[3]]/(4*Sqrt[3]) + Log[1 + x^4 + x^8]/8
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1357

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^7}{1+x^4+x^8}\,dx = \frac{1}{4}\,\text{Subst}\left(\int \frac{x}{1+x+x^2}\,dx,x,x^4\right)$$
$$= -\left(\frac{1}{8}\,\text{Subst}\left(\int \frac{1}{1+x+x^2}\,dx,x,x^4\right)\right)+\frac{1}{8}\,\text{Subst}\left(\int \frac{1+2x}{1+x+x^2}\,dx,x,x^4\right)$$
$$= \frac{1}{8}\log\left(1+x^4+x^8\right)+\frac{1}{4}\,\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,1+2x^4\right)$$
$$= -\frac{\tan^{-1}\left(\frac{1+2x^4}{\sqrt{3}}\right)}{4\sqrt{3}}+\frac{1}{8}\log\left(1+x^4+x^8\right)$$

Mathematica [A] time = 0.01, size = 37, normalized size = 1.00

$$\frac{1}{8}\log\left(x^8+x^4+1\right)-\frac{\tan^{-1}\left(\frac{2x^4+1}{\sqrt{3}}\right)}{4\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7/(1 + x^4 + x^8),x]
[Out] -1/4*ArcTan[(1 + 2*x^4)/Sqrt[3]]/Sqrt[3] + Log[1 + x^4 + x^8]/8
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^7}{1+x^4+x^8}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^7/(1 + x^4 + x^8),x]
[Out] IntegrateAlgebraic[x^7/(1 + x^4 + x^8), x]
```

fricas [A] time = 1.21, size = 30, normalized size = 0.81

$$-\frac{1}{12}\sqrt{3}\,\arctan\left(\frac{1}{3}\sqrt{3}\left(2\,x^4+1\right)\right)+\frac{1}{8}\,\log\left(x^8+x^4+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(x^8+x^4+1),x, algorithm="fricas")
[Out] -1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^4 + 1)) + 1/8*log(x^8 + x^4 + 1)
```

giac [A] time = 0.39, size = 30, normalized size = 0.81

$$-\frac{1}{12}\sqrt{3}\,\arctan\left(\frac{1}{3}\sqrt{3}\left(2\,x^4+1\right)\right)+\frac{1}{8}\,\log\left(x^8+x^4+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(x^8+x^4+1),x, algorithm="giac")
[Out] -1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^4 + 1)) + 1/8*log(x^8 + x^4 + 1)
```

maple [A] time = 0.00, size = 31, normalized size = 0.84

$$-\frac{\sqrt{3}\,\arctan\left(\frac{\left(2x^4+1\right)\sqrt{3}}{3}\right)}{12}+\frac{\ln\left(x^8+x^4+1\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(x^8+x^4+1),x)
```

```
[Out] 1/8*ln(x^8+x^4+1)-1/12*3^(1/2)*arctan(1/3*(2*x^4+1)*3^(1/2))
```

maxima [A] time = 2.43, size = 30, normalized size = 0.81

$$-\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^4+1\right)\right)+\frac{1}{8}\log\left(x^8+x^4+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(x^8+x^4+1),x, algorithm="maxima")
```

```
[Out] -1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^4 + 1)) + 1/8*log(x^8 + x^4 + 1)
```

mupad [B] time = 0.04, size = 32, normalized size = 0.86

$$\frac{\ln\left(x^8+x^4+1\right)}{8}-\frac{\sqrt{3}\operatorname{atan}\left(\frac{2\sqrt{3}x^4}{3}+\frac{\sqrt{3}}{3}\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(x^4 + x^8 + 1),x)
```

```
[Out] log(x^4 + x^8 + 1)/8 - (3^(1/2)*atan(3^(1/2)/3 + (2*3^(1/2)*x^4)/3))/12
```

sympy [A] time = 0.13, size = 37, normalized size = 1.00

$$\frac{\log\left(x^8+x^4+1\right)}{8}-\frac{\sqrt{3}\operatorname{atan}\left(\frac{2\sqrt{3}x^4}{3}+\frac{\sqrt{3}}{3}\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7/(x**8+x**4+1),x)
```

```
[Out] log(x**8 + x**4 + 1)/8 - sqrt(3)*atan(2*sqrt(3)*x**4/3 + sqrt(3)/3)/12
```

3.273 $\int \frac{x^5}{1+x^4+x^8}\,dx$

Optimal. Leaf size=75

$$-\frac{\tan^{-1}\left(\frac{1-2x^2}{\sqrt{3}}\right)}{4\sqrt{3}}+\frac{\tan^{-1}\left(\frac{2x^2+1}{\sqrt{3}}\right)}{4\sqrt{3}}+\frac{1}{8}\log\left(x^4-x^2+1\right)-\frac{1}{8}\log\left(x^4+x^2+1\right)$$

Rubi [A] time = 0.08, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {1359, 1127, 1161, 618, 204, 1164, 628}

$$\frac{1}{8}\log\left(x^4-x^2+1\right)-\frac{1}{8}\log\left(x^4+x^2+1\right)-\frac{\tan^{-1}\left(\frac{1-2x^2}{\sqrt{3}}\right)}{4\sqrt{3}}+\frac{\tan^{-1}\left(\frac{2x^2+1}{\sqrt{3}}\right)}{4\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/(1 + x^4 + x^8),x]
```

```
[Out] -ArcTan[(1 - 2*x^2)/Sqrt[3]]/(4*Sqrt[3]) + ArcTan[(1 + 2*x^2)/Sqrt[3]]/(4*S
qrt[3]) + Log[1 - x^2 + x^4]/8 - Log[1 + x^2 + x^4]/8
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1127

```
Int[(x_)^2/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a/c, 2]}, Dist[1/2, Int[(q + x^2)/(a + b*x^2 + c*x^4), x], x] - Dist[1/2, I
nt[(q - x^2)/(a + b*x^2 + c*x^4), x], x]] /; FreeQ[{a, b, c}, x] && LtQ[b^2
 - 4*a*c, 0] && PosQ[a*c]
```

Rule 1161

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[(2*d)/e - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2
, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (
GtQ[(2*d)/e - b/c, 0] || ( !LtQ[(2*d)/e - b/c, 0] && EqQ[d - e*Rt[a/c, 2],
0]))
```

Rule 1164

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[(-2*d)/e - b/c, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e
```

```
+ q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^
2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c
*d^2 - a*e^2, 0] &&  !GtQ[b^2 - 4*a*c, 0]
```

Rule 1359

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*
x^(n/k) + c*x^((2*n)/k))^p, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, c, p
}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^5}{1+x^4+x^8}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{x^2}{1+x^2+x^4}\,dx,x,x^2\right)\\
&= -\left(\frac{1}{4}\operatorname{Subst}\left(\int \frac{1-x^2}{1+x^2+x^4}\,dx,x,x^2\right)\right)+\frac{1}{4}\operatorname{Subst}\left(\int \frac{1+x^2}{1+x^2+x^4}\,dx,x,x^2\right)\\
&= \frac{1}{8}\operatorname{Subst}\left(\int \frac{1+2x}{-1-x-x^2}\,dx,x,x^2\right)+\frac{1}{8}\operatorname{Subst}\left(\int \frac{1-2x}{-1+x-x^2}\,dx,x,x^2\right)+\frac{1}{8}\operatorname{Subst}\left(\int \frac{1}{1-x
\end{aligned}$$

$$\begin{aligned}
&= \frac{1}{8}\log\left(1-x^2+x^4\right)-\frac{1}{8}\log\left(1+x^2+x^4\right)-\frac{1}{4}\operatorname{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,-1+2x^2\right)-\frac{1}{4}\operatorname{Subst}\left(
\end{aligned}$$

$$= -\frac{\tan^{-1}\left(\frac{1-2x^2}{\sqrt{3}}\right)}{4\sqrt{3}}+\frac{\tan^{-1}\left(\frac{1+2x^2}{\sqrt{3}}\right)}{4\sqrt{3}}+\frac{1}{8}\log\left(1-x^2+x^4\right)-\frac{1}{8}\log\left(1+x^2+x^4\right)$$

Mathematica [C] time = 0.12, size = 94, normalized size = 1.25

$$\frac{\sqrt{1-i\sqrt{3}}\left(\sqrt{3}-i\right)\tan^{-1}\left(\frac{1}{2}\left(\sqrt{3}-i\right)x^2\right)+\sqrt{1+i\sqrt{3}}\left(\sqrt{3}+i\right)\tan^{-1}\left(\frac{1}{2}\left(\sqrt{3}+i\right)x^2\right)}{4\sqrt{6}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^5/(1 + x^4 + x^8),x]
[Out] (Sqrt[1 - I*Sqrt[3]]*(-I + Sqrt[3])*ArcTan[((-I + Sqrt[3])*x^2)/2] + Sqrt[1
 + I*Sqrt[3]]*(I + Sqrt[3])*ArcTan[((I + Sqrt[3])*x^2)/2])/(4*Sqrt[6])
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5}{1+x^4+x^8}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^5/(1 + x^4 + x^8),x]
[Out] IntegrateAlgebraic[x^5/(1 + x^4 + x^8), x]
```

fricas [A] time = 0.97, size = 61, normalized size = 0.81

$$\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^2+1\right)\right)+\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^2-1\right)\right)-\frac{1}{8}\log\left(x^4+x^2+1\right)+\frac{1}{8}\log\left(x^4-x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(x^8+x^4+1),x, algorithm="fricas")
```

```
[Out] 1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^2 + 1)) + 1/12*sqrt(3)*arctan(1/3*sqrt
(3)*(2*x^2 - 1)) - 1/8*log(x^4 + x^2 + 1) + 1/8*log(x^4 - x^2 + 1)
```

giac [A] time = 0.39, size = 61, normalized size = 0.81

$$\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^2+1\right)\right)+\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^2-1\right)\right)-\frac{1}{8}\log\left(x^4+x^2+1\right)+\frac{1}{8}\log\left(x^4-x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(x^8+x^4+1),x, algorithm="giac")
```

```
[Out] 1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^2 + 1)) + 1/12*sqrt(3)*arctan(1/3*sqrt
(3)*(2*x^2 - 1)) - 1/8*log(x^4 + x^2 + 1) + 1/8*log(x^4 - x^2 + 1)
```

maple [A] time = 0.00, size = 62, normalized size = 0.83

$$\frac{\sqrt{3}\ \arctan\left(\frac{\left(2x^2-1\right)\sqrt{3}}{3}\right)}{12}+\frac{\sqrt{3}\ \arctan\left(\frac{\left(2x^2+1\right)\sqrt{3}}{3}\right)}{12}+\frac{\ln\left(x^4-x^2+1\right)}{8}-\frac{\ln\left(x^4+x^2+1\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(x^8+x^4+1),x)
```

```
[Out] -1/8*ln(x^4+x^2+1)+1/12*3^(1/2)*arctan(1/3*(2*x^2+1)*3^(1/2))+1/8*ln(x^4-x^
2+1)+1/12*3^(1/2)*arctan(1/3*(2*x^2-1)*3^(1/2))
```

maxima [A] time = 2.49, size = 61, normalized size = 0.81

$$\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^2+1\right)\right)+\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^2-1\right)\right)-\frac{1}{8}\log\left(x^4+x^2+1\right)+\frac{1}{8}\log\left(x^4-x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(x^8+x^4+1),x, algorithm="maxima")
```

```
[Out] 1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^2 + 1)) + 1/12*sqrt(3)*arctan(1/3*sqrt
(3)*(2*x^2 - 1)) - 1/8*log(x^4 + x^2 + 1) + 1/8*log(x^4 - x^2 + 1)
```

mupad [B] time = 0.09, size = 51, normalized size = 0.68

$$\operatorname{atanh}\left(\frac{2x^2}{-1+\sqrt{3}\,1\mathrm{i}}\right)\left(\frac{1}{4}+\frac{\sqrt{3}\,1\mathrm{i}}{12}\right)+\operatorname{atanh}\left(\frac{2x^2}{1+\sqrt{3}\,1\mathrm{i}}\right)\left(-\frac{1}{4}+\frac{\sqrt{3}\,1\mathrm{i}}{12}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(x^4 + x^8 + 1),x)
```

```
[Out] atanh((2*x^2)/(3^(1/2)*1i - 1))*((3^(1/2)*1i)/12 + 1/4) + atanh((2*x^2)/(3^
(1/2)*1i + 1))*((3^(1/2)*1i)/12 - 1/4)
```

sympy [A] time = 0.21, size = 76, normalized size = 1.01

$$\frac{\log\left(x^4-x^2+1\right)}{8}-\frac{\log\left(x^4+x^2+1\right)}{8}+\frac{\sqrt{3}\ \operatorname{atan}\left(\frac{2\sqrt{3}x^2}{3}-\frac{\sqrt{3}}{3}\right)}{12}+\frac{\sqrt{3}\ \operatorname{atan}\left(\frac{2\sqrt{3}x^2}{3}+\frac{\sqrt{3}}{3}\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(x**8+x**4+1),x)
```

```
[Out] log(x**4 - x**2 + 1)/8 - log(x**4 + x**2 + 1)/8 + sqrt(3)*atan(2*sqrt(3)*x*
*2/3 - sqrt(3)/3)/12 + sqrt(3)*atan(2*sqrt(3)*x**2/3 + sqrt(3)/3)/12
```

3.274 $\int \frac{x^3}{1+x^4+x^8} dx$

Optimal. Leaf size=23

$$\frac{\tan^{-1}\left(\frac{2x^4+1}{\sqrt{3}}\right)}{2\sqrt{3}}$$

Rubi [A] time = 0.02, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {1352, 618, 204}

$$\frac{\tan^{-1}\left(\frac{2x^4+1}{\sqrt{3}}\right)}{2\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(1 + x^4 + x^8),x]
```

```
[Out] ArcTan[(1 + 2*x^4)/Sqrt[3]]/(2*Sqrt[3])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1352

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a,
b, c, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^3}{1+x^4+x^8} dx &= \frac{1}{4}\text{Subst}\left(\int \frac{1}{1+x+x^2} dx, x, x^4\right)\\
&= -\left(\frac{1}{2}\text{Subst}\left(\int \frac{1}{-3-x^2} dx, x, 1+2x^4\right)\right)\\
&= \frac{\tan^{-1}\left(\frac{1+2x^4}{\sqrt{3}}\right)}{2\sqrt{3}}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 23, normalized size = 1.00

$$\frac{\tan^{-1}\left(\frac{2x^4+1}{\sqrt{3}}\right)}{2\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(1 + x^4 + x^8),x]
[Out] ArcTan[(1 + 2*x^4)/Sqrt[3]]/(2*Sqrt[3])
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{1+x^4+x^8}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^3/(1 + x^4 + x^8),x]
[Out] IntegrateAlgebraic[x^3/(1 + x^4 + x^8), x]
```

fricas [A] time = 1.14, size = 18, normalized size = 0.78

$$\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^4+1\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(x^8+x^4+1),x, algorithm="fricas")
[Out] 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^4 + 1))
```

giac [A] time = 0.36, size = 18, normalized size = 0.78

$$\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^4+1\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(x^8+x^4+1),x, algorithm="giac")
[Out] 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^4 + 1))
```

maple [A] time = 0.00, size = 19, normalized size = 0.83

$$\frac{\sqrt{3}\,\arctan\left(\frac{\left(2x^4+1\right)\sqrt{3}}{3}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(x^8+x^4+1),x)
[Out] 1/6*3^(1/2)*arctan(1/3*(2*x^4+1)*3^(1/2))
```

maxima [A] time = 2.47, size = 18, normalized size = 0.78

$$\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^4+1\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(x^8+x^4+1),x, algorithm="maxima")
[Out] 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^4 + 1))
```

mupad [B] time = 1.30, size = 17, normalized size = 0.74

$$\frac{\sqrt{3}\,\mathrm{atan}\left(\sqrt{3}\,\left(\frac{2x^4}{3}+\frac{1}{3}\right)\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(x^4 + x^8 + 1),x)
```

```
[Out] (3^(1/2)*atan(3^(1/2)*((2*x^4)/3 + 1/3)))/6
```

sympy [**A**] time = 0.12, size = 26, normalized size = 1.13

$$\frac{\sqrt{3}\operatorname{atan}\left(\frac{2\sqrt{3}x^4}{3}+\frac{\sqrt{3}}{3}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(x**8+x**4+1),x)
```

```
[Out] sqrt(3)*atan(2*sqrt(3)*x**4/3 + sqrt(3)/3)/6
```

3.275 $\int \frac{x}{1+x^4+x^8} \, dx$

Optimal. Leaf size=75

$$-\frac{\tan^{-1}\left(\frac{1-2x^2}{\sqrt{3}}\right)}{4\sqrt{3}}+\frac{\tan^{-1}\left(\frac{2x^2+1}{\sqrt{3}}\right)}{4\sqrt{3}}-\frac{1}{8}\log\left(x^4-x^2+1\right)+\frac{1}{8}\log\left(x^4+x^2+1\right)$$

Rubi [A] time = 0.06, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {1359, 1094, 634, 618, 204, 628}

$$-\frac{1}{8}\log\left(x^4-x^2+1\right)+\frac{1}{8}\log\left(x^4+x^2+1\right)-\frac{\tan^{-1}\left(\frac{1-2x^2}{\sqrt{3}}\right)}{4\sqrt{3}}+\frac{\tan^{-1}\left(\frac{2x^2+1}{\sqrt{3}}\right)}{4\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(1 + x^4 + x^8),x]
```

```
[Out] -ArcTan[(1 - 2*x^2)/Sqrt[3]]/(4*Sqrt[3]) + ArcTan[(1 + 2*x^2)/Sqrt[3]]/(4*S
qrt[3]) - Log[1 - x^2 + x^4]/8 + Log[1 + x^2 + x^4]/8
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1094

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[a/
c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int[(r - x)/(q - r*x
 + x^2), x], x] + Dist[1/(2*c*q*r), Int[(r + x)/(q + r*x + x^2), x], x]]] /
; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[b^2 - 4*a*c]
```

Rule 1359

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*
x^(n/k) + c*x^((2*n)/k))^p, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, c, p
```

```
}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\int \frac{x}{1+x^4+x^8}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{1+x^2+x^4}\,dx,x,x^2\right)\\ &= \frac{1}{4}\operatorname{Subst}\left(\int \frac{1-x}{1-x+x^2}\,dx,x,x^2\right)+\frac{1}{4}\operatorname{Subst}\left(\int \frac{1+x}{1+x+x^2}\,dx,x,x^2\right)\\ &= \frac{1}{8}\operatorname{Subst}\left(\int \frac{1}{1-x+x^2}\,dx,x,x^2\right)-\frac{1}{8}\operatorname{Subst}\left(\int \frac{-1+2x}{1-x+x^2}\,dx,x,x^2\right)+\frac{1}{8}\operatorname{Subst}\left(\int \frac{1}{1+x+x\right.\\ &= -\frac{1}{8}\log\left(1-x^2+x^4\right)+\frac{1}{8}\log\left(1+x^2+x^4\right)-\frac{1}{4}\operatorname{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,-1+2x^2\right)-\frac{1}{4}\operatorname{Subs}\\ &= -\frac{\tan^{-1}\left(\frac{1-2x^2}{\sqrt{3}}\right)}{4\sqrt{3}}+\frac{\tan^{-1}\left(\frac{1+2x^2}{\sqrt{3}}\right)}{4\sqrt{3}}-\frac{1}{8}\log\left(1-x^2+x^4\right)+\frac{1}{8}\log\left(1+x^2+x^4\right)\end{aligned}$$

Mathematica [C] time = 0.05, size = 79, normalized size = 1.05

$$\frac{i\left(\sqrt{1-i\sqrt{3}}\,\tan^{-1}\left(\frac{1}{2}\left(\sqrt{3}-i\right)x^2\right)-\sqrt{1+i\sqrt{3}}\,\tan^{-1}\left(\frac{1}{2}\left(\sqrt{3}+i\right)x^2\right)\right)}{2\sqrt{6}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x/(1 + x^4 + x^8),x]
[Out] ((I/2)*(Sqrt[1 - I*Sqrt[3]]*ArcTan[((-I + Sqrt[3])*x^2)/2] - Sqrt[1 + I*Sqr
t[3]]*ArcTan[((I + Sqrt[3])*x^2)/2]))/Sqrt[6]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{1+x^4+x^8}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x/(1 + x^4 + x^8),x]
[Out] IntegrateAlgebraic[x/(1 + x^4 + x^8), x]
```

fricas [A] time = 1.50, size = 61, normalized size = 0.81

$$\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^2+1\right)\right)+\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^2-1\right)\right)+\frac{1}{8}\log\left(x^4+x^2+1\right)-\frac{1}{8}\log\left(x^4-x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^8+x^4+1),x, algorithm="fricas")
[Out] 1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^2 + 1)) + 1/12*sqrt(3)*arctan(1/3*sqrt
(3)*(2*x^2 - 1)) + 1/8*log(x^4 + x^2 + 1) - 1/8*log(x^4 - x^2 + 1)
```

giac [A] time = 0.31, size = 61, normalized size = 0.81

$$\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^2+1\right)\right)+\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^2-1\right)\right)+\frac{1}{8}\log\left(x^4+x^2+1\right)-\frac{1}{8}\log\left(x^4-x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^8+x^4+1),x, algorithm="giac")
```

```
[Out] 1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^2 + 1)) + 1/12*sqrt(3)*arctan(1/3*sqrt
(3)*(2*x^2 - 1)) + 1/8*log(x^4 + x^2 + 1) - 1/8*log(x^4 - x^2 + 1)
```

maple [A] time = 0.00, size = 62, normalized size = 0.83

$$\frac{\sqrt{3}\ \arctan\left(\frac{\left(2x^2-1\right)\sqrt{3}}{3}\right)}{12}+\frac{\sqrt{3}\ \arctan\left(\frac{\left(2x^2+1\right)\sqrt{3}}{3}\right)}{12}-\frac{\ln\left(x^4-x^2+1\right)}{8}+\frac{\ln\left(x^4+x^2+1\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x^8+x^4+1),x)
```

```
[Out] 1/8*ln(x^4+x^2+1)+1/12*3^(1/2)*arctan(1/3*(2*x^2+1)*3^(1/2))-1/8*ln(x^4-x^2
+1)+1/12*3^(1/2)*arctan(1/3*(2*x^2-1)*3^(1/2))
```

maxima [A] time = 2.59, size = 61, normalized size = 0.81

$$\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^2+1\right)\right)+\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^2-1\right)\right)+\frac{1}{8}\log\left(x^4+x^2+1\right)-\frac{1}{8}\log\left(x^4-x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^8+x^4+1),x, algorithm="maxima")
```

```
[Out] 1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^2 + 1)) + 1/12*sqrt(3)*arctan(1/3*sqrt
(3)*(2*x^2 - 1)) + 1/8*log(x^4 + x^2 + 1) - 1/8*log(x^4 - x^2 + 1)
```

mupad [B] time = 1.28, size = 51, normalized size = 0.68

$$\mathrm{atan}\left(\frac{\sqrt{3}\ x^2}{2}-\frac{x^2\,1\mathrm{i}}{2}\right)\left(\frac{\sqrt{3}}{12}+\frac{1}{4}\mathrm{i}\right)+\mathrm{atan}\left(\frac{\sqrt{3}\ x^2}{2}+\frac{x^2\,1\mathrm{i}}{2}\right)\left(\frac{\sqrt{3}}{12}-\frac{1}{4}\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x^4 + x^8 + 1),x)
```

```
[Out] atan((3^(1/2)*x^2)/2 - (x^2*1i)/2)*(3^(1/2)/12 + 1i/4) + atan((3^(1/2)*x^2)
/2 + (x^2*1i)/2)*(3^(1/2)/12 - 1i/4)
```

sympy [A] time = 0.20, size = 76, normalized size = 1.01

$$-\frac{\log\left(x^4-x^2+1\right)}{8}+\frac{\log\left(x^4+x^2+1\right)}{8}+\frac{\sqrt{3}\ \mathrm{atan}\left(\frac{2\sqrt{3}\,x^2}{3}-\frac{\sqrt{3}}{3}\right)}{12}+\frac{\sqrt{3}\ \mathrm{atan}\left(\frac{2\sqrt{3}\,x^2}{3}+\frac{\sqrt{3}}{3}\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x**8+x**4+1),x)
```

```
[Out] -log(x**4 - x**2 + 1)/8 + log(x**4 + x**2 + 1)/8 + sqrt(3)*atan(2*sqrt(3)*x
**2/3 - sqrt(3)/3)/12 + sqrt(3)*atan(2*sqrt(3)*x**2/3 + sqrt(3)/3)/12
```

3.276 $\int \frac{1}{x(1+x^4+x^8)}\,dx$

Optimal. Leaf size=39

$$-\frac{\tan^{-1}\left(\frac{2x^4+1}{\sqrt{3}}\right)}{4\sqrt{3}}-\frac{1}{8}\log\left(x^8+x^4+1\right)+\log(x)$$

Rubi [A] time = 0.03, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {1357, 705, 29, 634, 618, 204, 628}

$$-\frac{1}{8}\log\left(x^8+x^4+1\right)-\frac{\tan^{-1}\left(\frac{2x^4+1}{\sqrt{3}}\right)}{4\sqrt{3}}+\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(1 + x^4 + x^8)),x]
```

```
[Out] -ArcTan[(1 + 2*x^4)/Sqrt[3]]/(4*Sqrt[3]) + Log[x] - Log[1 + x^4 + x^8]/8
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 705

```
Int[1/(((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)), x_Symbol]
 :> Dist[e^2/(c*d^2 - b*d*e + a*e^2), Int[1/(d + e*x), x], x] + Dist[1/(c*d
^2 - b*d*e + a*e^2), Int[(c*d - b*e - c*e*x)/(a + b*x + c*x^2), x], x] /; F
reeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^
2, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1357

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x\left(1+x^4+x^8\right)}\,dx &= \frac{1}{4}\text{Subst}\left(\int \frac{1}{x\left(1+x+x^2\right)}\,dx,x,x^4\right)\\
&= \frac{1}{4}\text{Subst}\left(\int \frac{1}{x}\,dx,x,x^4\right)+\frac{1}{4}\text{Subst}\left(\int \frac{-1-x}{1+x+x^2}\,dx,x,x^4\right)\\
&= \log(x)-\frac{1}{8}\text{Subst}\left(\int \frac{1}{1+x+x^2}\,dx,x,x^4\right)-\frac{1}{8}\text{Subst}\left(\int \frac{1+2x}{1+x+x^2}\,dx,x,x^4\right)\\
&= \log(x)-\frac{1}{8}\log\left(1+x^4+x^8\right)+\frac{1}{4}\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,1+2x^4\right)\\
&= -\frac{\tan^{-1}\left(\frac{1+2x^4}{\sqrt{3}}\right)}{4\sqrt{3}}+\log(x)-\frac{1}{8}\log\left(1+x^4+x^8\right)
\end{aligned}
$$

Mathematica [C] time = 0.08, size = 138, normalized size = 3.54

$$\frac{1}{24}\left(-\sqrt{3}\left(\sqrt{3}-i\right)\log\left(x^2-\frac{i\sqrt{3}}{2}-\frac{1}{2}\right)-\sqrt{3}\left(\sqrt{3}+i\right)\log\left(x^2+\frac{1}{2}i\left(\sqrt{3}+i\right)\right)-3\log\left(x^2-x+1\right)-3\log\left(x^2+x+1\right)+24\log(x)+2\sqrt{3}\tan^{-1}\left(\frac{2x-1}{\sqrt{3}}\right)-2\sqrt{3}\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(1 + x^4 + x^8)),x]
```

```
[Out] (2*Sqrt[3]*ArcTan[(-1 + 2*x)/Sqrt[3]] - 2*Sqrt[3]*ArcTan[(1 + 2*x)/Sqrt[3]]
 + 24*Log[x] - Sqrt[3]*(-I + Sqrt[3])*Log[-1/2 - (I/2)*Sqrt[3] + x^2] - Sqr
t[3]*(I + Sqrt[3])*Log[(I/2)*(I + Sqrt[3]) + x^2] - 3*Log[1 - x + x^2] - 3*
Log[1 + x + x^2])/24
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(1+x^4+x^8\right)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x*(1 + x^4 + x^8)),x]
```

```
[Out] IntegrateAlgebraic[1/(x*(1 + x^4 + x^8)), x]
```

fricas [A] time = 1.26, size = 32, normalized size = 0.82

$$-\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^4+1\right)\right)-\frac{1}{8}\log\left(x^8+x^4+1\right)+\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x^8+x^4+1),x, algorithm="fricas")
```

```
[Out] -1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^4 + 1)) - 1/8*log(x^8 + x^4 + 1) + lo
g(x)
```

giac [A] time = 0.38, size = 36, normalized size = 0.92

$$-\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^4+1\right)\right)-\frac{1}{8}\log\left(x^8+x^4+1\right)+\frac{1}{4}\log\left(x^4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x^8+x^4+1),x, algorithm="giac")
```

```
[Out] -1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^4 + 1)) - 1/8*log(x^8 + x^4 + 1) + 1/
4*log(x^4)
```

maple [B] time = 0.01, size = 87, normalized size = 2.23

$$-\frac{\sqrt{3}\arctan\left(\frac{(2x+1)\sqrt{3}}{3}\right)}{12}+\frac{\sqrt{3}\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{12}-\frac{\sqrt{3}\arctan\left(\frac{(2x^2-1)\sqrt{3}}{3}\right)}{12}+\ln(x)-\frac{\ln\left(x^2-x+1\right)}{8}-\frac{\ln\left(x^2+x+1\right)}{8}-\frac{\ln\left(x^4-x^2+1\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(x^8+x^4+1),x)
```

```
[Out] ln(x)-1/8*ln(x^2+x+1)-1/12*3^(1/2)*arctan(1/3*(2*x+1)*3^(1/2))-1/8*ln(x^4-x
^2+1)-1/12*3^(1/2)*arctan(1/3*(2*x^2-1)*3^(1/2))-1/8*ln(x^2-x+1)+1/12*3^(1/
2)*arctan(1/3*(2*x-1)*3^(1/2))
```

maxima [A] time = 3.05, size = 36, normalized size = 0.92

$$-\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^4+1\right)\right)-\frac{1}{8}\log\left(x^8+x^4+1\right)+\frac{1}{4}\log\left(x^4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x^8+x^4+1),x, algorithm="maxima")
```

```
[Out] -1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^4 + 1)) - 1/8*log(x^8 + x^4 + 1) + 1/
4*log(x^4)
```

mupad [B] time = 1.30, size = 34, normalized size = 0.87

$$\ln(x)-\frac{\ln\left(x^8+x^4+1\right)}{8}-\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}\,x^4}{3}+\frac{\sqrt{3}}{3}\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(x^4 + x^8 + 1)),x)
```

```
[Out] log(x) - log(x^4 + x^8 + 1)/8 - (3^(1/2)*atan(3^(1/2)/3 + (2*3^(1/2)*x^4)/3
))/12
```

sympy [A] time = 0.15, size = 41, normalized size = 1.05

$$\log(x)-\frac{\log\left(x^8+x^4+1\right)}{8}-\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x^4}{3}+\frac{\sqrt{3}}{3}\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x**8+x**4+1),x)
```

```
[Out] log(x) - log(x**8 + x**4 + 1)/8 - sqrt(3)*atan(2*sqrt(3)*x**4/3 + sqrt(3)/3
)/12
```

3.277 $\int \frac{1}{x^3(1+x^4+x^8)}\,dx$

Optimal. Leaf size=54

$$-\frac{1}{2x^2}+\frac{\tan^{-1}\left(\frac{1-2x^2}{\sqrt{3}}\right)}{2\sqrt{3}}-\frac{\tan^{-1}\left(\frac{2x^2+1}{\sqrt{3}}\right)}{2\sqrt{3}}$$

Rubi [A] time = 0.05, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {1359, 1123, 1161, 618, 204}

$$-\frac{1}{2x^2}+\frac{\tan^{-1}\left(\frac{1-2x^2}{\sqrt{3}}\right)}{2\sqrt{3}}-\frac{\tan^{-1}\left(\frac{2x^2+1}{\sqrt{3}}\right)}{2\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(1 + x^4 + x^8)),x]
```

```
[Out] -1/(2*x^2) + ArcTan[(1 - 2*x^2)/Sqrt[3]]/(2*Sqrt[3]) - ArcTan[(1 + 2*x^2)/S
qrt[3]]/(2*Sqrt[3])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1123

```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*x^2 + c*x^4)^(p + 1))/(a*d*(m + 1)), x] - Dis
t[1/(a*d^2*(m + 1)), Int[(d*x)^(m + 2)*(b*(m + 2*p + 3) + c*(m + 4*p + 5)*x
^2)*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b^2 -
 4*a*c, 0] && LtQ[m, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1161

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[(2*d)/e - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2
, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (
GtQ[(2*d)/e - b/c, 0] || ( !LtQ[(2*d)/e - b/c, 0] && EqQ[d - e*Rt[a/c, 2],
0]))
```

Rule 1359

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*
x^(n/k) + c*x^((2*n)/k))^p, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, c, p
}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\int \frac{1}{x^3\left(1+x^4+x^8\right)}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{1}{x^2\left(1+x^2+x^4\right)}\,dx,x,x^2\right)$$
$$= -\frac{1}{2x^2} + \frac{1}{2}\text{Subst}\left(\int \frac{-1-x^2}{1+x^2+x^4}\,dx,x,x^2\right)$$
$$= -\frac{1}{2x^2} - \frac{1}{4}\text{Subst}\left(\int \frac{1}{1-x+x^2}\,dx,x,x^2\right) - \frac{1}{4}\text{Subst}\left(\int \frac{1}{1+x+x^2}\,dx,x,x^2\right)$$
$$= -\frac{1}{2x^2} + \frac{1}{2}\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,-1+2x^2\right) + \frac{1}{2}\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,1+2x^2\right)$$
$$= -\frac{1}{2x^2} + \frac{\tan^{-1}\left(\frac{1-2x^2}{\sqrt{3}}\right)}{2\sqrt{3}} - \frac{\tan^{-1}\left(\frac{1+2x^2}{\sqrt{3}}\right)}{2\sqrt{3}}$$

Mathematica [C] time = 0.05, size = 100, normalized size = 1.85

$$\frac{1}{12}\left(-\frac{6}{x^2} + i\sqrt{3}\log\left(2x^2 - i\sqrt{3} - 1\right) - i\sqrt{3}\log\left(2x^2 + i\sqrt{3} - 1\right) - 2\sqrt{3}\tan^{-1}\left(\frac{2x-1}{\sqrt{3}}\right) + 2\sqrt{3}\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(1 + x^4 + x^8)),x]
```

```
[Out] (-6/x^2 - 2*Sqrt[3]*ArcTan[(-1 + 2*x)/Sqrt[3]] + 2*Sqrt[3]*ArcTan[(1 + 2*x)
/Sqrt[3]] + I*Sqrt[3]*Log[-1 - I*Sqrt[3] + 2*x^2] - I*Sqrt[3]*Log[-1 + I*Sq
rt[3] + 2*x^2])/12
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3\left(1+x^4+x^8\right)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x^3*(1 + x^4 + x^8)),x]
```

```
[Out] IntegrateAlgebraic[1/(x^3*(1 + x^4 + x^8)), x]
```

fricas [A] time = 1.08, size = 45, normalized size = 0.83

$$-\frac{\sqrt{3}\,x^2\arctan\left(\frac{1}{3}\sqrt{3}\,x^2\right) + \sqrt{3}\,x^2\arctan\left(\frac{1}{3}\sqrt{3}\left(x^6+2\,x^2\right)\right) + 3}{6\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(x^8+x^4+1),x, algorithm="fricas")
```

```
[Out] -1/6*(sqrt(3)*x^2*arctan(1/3*sqrt(3)*x^2) + sqrt(3)*x^2*arctan(1/3*sqrt(3)*
(x^6 + 2*x^2)) + 3)/x^2
```

giac [A] time = 0.38, size = 42, normalized size = 0.78

$$-\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\,x^2+1\right)\right) - \frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\,x^2-1\right)\right) - \frac{1}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(x^8+x^4+1),x, algorithm="giac")
```

```
[Out] -1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^2 + 1)) - 1/6*sqrt(3)*arctan(1/3*sqrt(
3)*(2*x^2 - 1)) - 1/2/x^2
```

maple [A] time = 0.01, size = 57, normalized size = 1.06

$$\frac{\sqrt{3}\,\arctan\left(\frac{(2x+1)\sqrt{3}}{3}\right)}{6}-\frac{\sqrt{3}\,\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{6}-\frac{\sqrt{3}\,\arctan\left(\frac{(2x^2-1)\sqrt{3}}{3}\right)}{6}-\frac{1}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(x^8+x^4+1),x)
```

```
[Out] 1/6*3^(1/2)*arctan(1/3*(2*x+1)*3^(1/2))-1/6*3^(1/2)*arctan(1/3*(2*x^2-1)*3^
(1/2))-1/2/x^2-1/6*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))
```

maxima [A] time = 2.39, size = 42, normalized size = 0.78

$$-\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^2+1\right)\right)-\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^2-1\right)\right)-\frac{1}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(x^8+x^4+1),x, algorithm="maxima")
```

```
[Out] -1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^2 + 1)) - 1/6*sqrt(3)*arctan(1/3*sqrt(
3)*(2*x^2 - 1)) - 1/2/x^2
```

mupad [B] time = 0.04, size = 43, normalized size = 0.80

$$-\frac{\sqrt{3}\,\left(2\,\mathrm{atan}\left(\frac{\sqrt{3}\,x^6}{3}+\frac{2\,\sqrt{3}\,x^2}{3}\right)+2\,\mathrm{atan}\left(\frac{\sqrt{3}\,x^2}{3}\right)\right)}{12}-\frac{1}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(x^4 + x^8 + 1)),x)
```

```
[Out] - (3^(1/2)*(2*atan((2*3^(1/2)*x^2)/3 + (3^(1/2)*x^6)/3) + 2*atan((3^(1/2)*x
^2)/3)))/12 - 1/(2*x^2)
```

sympy [A] time = 0.16, size = 53, normalized size = 0.98

$$\frac{\sqrt{3}\left(-2\,\mathrm{atan}\left(\frac{\sqrt{3}x^2}{3}\right)-2\,\mathrm{atan}\left(\frac{\sqrt{3}x^6}{3}+\frac{2\sqrt{3}x^2}{3}\right)\right)}{12}-\frac{1}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(x**8+x**4+1),x)
```

```
[Out] sqrt(3)*(-2*atan(sqrt(3)*x**2/3) - 2*atan(sqrt(3)*x**6/3 + 2*sqrt(3)*x**2/3
))/12 - 1/(2*x**2)
```

3.278 $\int \frac{1}{x^5(1+x^4+x^8)}\,dx$

Optimal. Leaf size=48

$$-\frac{1}{4x^4}-\frac{\tan^{-1}\left(\frac{2x^4+1}{\sqrt{3}}\right)}{4\sqrt{3}}+\frac{1}{8}\log\left(x^8+x^4+1\right)-\log(x)$$

Rubi [A] time = 0.05, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {1357, 709, 800, 634, 618, 204, 628}

$$-\frac{1}{4x^4}+\frac{1}{8}\log\left(x^8+x^4+1\right)-\frac{\tan^{-1}\left(\frac{2x^4+1}{\sqrt{3}}\right)}{4\sqrt{3}}-\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^5*(1 + x^4 + x^8)),x]
```

```
[Out] -1/(4*x^4) - ArcTan[(1 + 2*x^4)/Sqrt[3]]/(4*Sqrt[3]) - Log[x] + Log[1 + x^4
 + x^8]/8
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 709

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Simp[(e*(d + e*x)^(m + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dis
t[1/(c*d^2 - b*d*e + a*e^2), Int[((d + e*x)^(m + 1)*Simp[c*d - b*e - c*e*x,
 x])/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 -
 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[m
, -1]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 1357

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^5\left(1+x^4+x^8\right)}\,dx &= \frac{1}{4}\text{Subst}\left(\int \frac{1}{x^2\left(1+x+x^2\right)}\,dx,x,x^4\right)\\
&= -\frac{1}{4x^4}+\frac{1}{4}\text{Subst}\left(\int \frac{-1-x}{x\left(1+x+x^2\right)}\,dx,x,x^4\right)\\
&= -\frac{1}{4x^4}+\frac{1}{4}\text{Subst}\left(\int \left(-\frac{1}{x}+\frac{x}{1+x+x^2}\right)dx,x,x^4\right)\\
&= -\frac{1}{4x^4}-\log(x)+\frac{1}{4}\text{Subst}\left(\int \frac{x}{1+x+x^2}\,dx,x,x^4\right)\\
&= -\frac{1}{4x^4}-\log(x)-\frac{1}{8}\text{Subst}\left(\int \frac{1}{1+x+x^2}\,dx,x,x^4\right)+\frac{1}{8}\text{Subst}\left(\int \frac{1+2x}{1+x+x^2}\,dx,x,x^4\right)\\
&= -\frac{1}{4x^4}-\log(x)+\frac{1}{8}\log\left(1+x^4+x^8\right)+\frac{1}{4}\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,1+2x^4\right)\\
&= -\frac{1}{4x^4}-\frac{\tan^{-1}\left(\frac{1+2x^4}{\sqrt{3}}\right)}{4\sqrt{3}}-\log(x)+\frac{1}{8}\log\left(1+x^4+x^8\right)
\end{aligned}$$

Mathematica [C] time = 0.10, size = 141, normalized size = 2.94

$$\frac{1}{24}\left(-\frac{6}{x^4}+\sqrt{3}\left(\sqrt{3}+i\right)\log\left(x^2-\frac{i\sqrt{3}}{2}-\frac{1}{2}\right)+\sqrt{3}\left(\sqrt{3}-i\right)\log\left(x^2+\frac{1}{2}i\left(\sqrt{3}+i\right)\right)+3\log\left(x^2-x+1\right)+3\log\left(x^2+x+1\right)-24\log(x)+2\sqrt{3}\tan^{-1}\left(\frac{2x-1}{\sqrt{3}}\right)-2\sqrt{3}\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^5*(1 + x^4 + x^8)),x]
```

```
[Out] (-6/x^4 + 2*Sqrt[3]*ArcTan[(-1 + 2*x)/Sqrt[3]] - 2*Sqrt[3]*ArcTan[(1 + 2*x)
/Sqrt[3]] - 24*Log[x] + Sqrt[3]*(I + Sqrt[3])*Log[-1/2 - (I/2)*Sqrt[3] + x^
2] + Sqrt[3]*(-I + Sqrt[3])*Log[(I/2)*(I + Sqrt[3]) + x^2] + 3*Log[1 - x +
x^2] + 3*Log[1 + x + x^2])/24
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^5\left(1+x^4+x^8\right)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x^5*(1 + x^4 + x^8)),x]
```

```
[Out] IntegrateAlgebraic[1/(x^5*(1 + x^4 + x^8)), x]
```

fricas [A] time = 1.08, size = 49, normalized size = 1.02

$$-\frac{2\sqrt{3}x^4\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^4+1\right)\right)-3x^4\log\left(x^8+x^4+1\right)+24x^4\log(x)+6}{24x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(x^8+x^4+1),x, algorithm="fricas")
```

```
[Out] -1/24*(2*sqrt(3)*x^4*arctan(1/3*sqrt(3)*(2*x^4 + 1)) - 3*x^4*log(x^8 + x^4
+ 1) + 24*x^4*log(x) + 6)/x^4
```

giac [A] time = 0.31, size = 46, normalized size = 0.96

$$-\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^4+1\right)\right)+\frac{x^4-1}{4x^4}+\frac{1}{8}\log\left(x^8+x^4+1\right)-\frac{1}{4}\log\left(x^4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(x^8+x^4+1),x, algorithm="giac")
```

```
[Out] -1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^4 + 1)) + 1/4*(x^4 - 1)/x^4 + 1/8*log
(x^8 + x^4 + 1) - 1/4*log(x^4)
```

maple [B] time = 0.01, size = 94, normalized size = 1.96

$$-\frac{\sqrt{3}\arctan\left(\frac{(2x+1)\sqrt{3}}{3}\right)}{12}+\frac{\sqrt{3}\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{12}-\frac{\sqrt{3}\arctan\left(\frac{(2x^2-1)\sqrt{3}}{3}\right)}{12}-\ln(x)+\frac{\ln\left(x^2-x+1\right)}{8}+\frac{\ln\left(x^2+x+1\right)}{8}+\frac{\ln\left(x^4-x^2+1\right)}{8}-\frac{1}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^5/(x^8+x^4+1),x)
```

```
[Out] -1/4/x^4-ln(x)+1/8*ln(x^2+x+1)-1/12*3^(1/2)*arctan(1/3*(2*x+1)*3^(1/2))+1/8
*ln(x^4-x^2+1)-1/12*3^(1/2)*arctan(1/3*(2*x^2-1)*3^(1/2))+1/8*ln(x^2-x+1)+1
/12*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))
```

maxima [A] time = 2.79, size = 41, normalized size = 0.85

$$-\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^4+1\right)\right)-\frac{1}{4x^4}+\frac{1}{8}\log\left(x^8+x^4+1\right)-\frac{1}{4}\log\left(x^4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(x^8+x^4+1),x, algorithm="maxima")
```

```
[Out] -1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^4 + 1)) - 1/4/x^4 + 1/8*log(x^8 + x^4
 + 1) - 1/4*log(x^4)
```

mupad [B] time = 0.06, size = 41, normalized size = 0.85

$$\frac{\ln\left(x^8+x^4+1\right)}{8}-\ln(x)-\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x^4}{3}+\frac{\sqrt{3}}{3}\right)}{12}-\frac{1}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^5*(x^4 + x^8 + 1)),x)
```

```
[Out] log(x^4 + x^8 + 1)/8 - log(x) - (3^(1/2)*atan(3^(1/2)/3 + (2*3^(1/2)*x^4)/3
))/12 - 1/(4*x^4)
```

sympy [A] time = 0.18, size = 48, normalized size = 1.00

$$-\log(x)+\frac{\log(x^8+x^4+1)}{8}-\frac{\sqrt{3}\,\operatorname{atan}\left(\frac{2\sqrt{3}\,x^4}{3}+\frac{\sqrt{3}}{3}\right)}{12}-\frac{1}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**5/(x**8+x**4+1),x)
```

```
[Out] -log(x) + log(x**8 + x**4 + 1)/8 - sqrt(3)*atan(2*sqrt(3)*x**4/3 + sqrt(3)/
3)/12 - 1/(4*x**4)
```

3.279 $\int \frac{1}{x^7(1+x^4+x^8)}\,dx$

Optimal. Leaf size=89

$$-\frac{1}{6x^6}+\frac{1}{2x^2}-\frac{\tan^{-1}\left(\frac{1-2x^2}{\sqrt{3}}\right)}{4\sqrt{3}}+\frac{\tan^{-1}\left(\frac{2x^2+1}{\sqrt{3}}\right)}{4\sqrt{3}}+\frac{1}{8}\log\left(x^4-x^2+1\right)-\frac{1}{8}\log\left(x^4+x^2+1\right)$$

Rubi [A] time = 0.10, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.714, Rules used = {1359, 1123, 1281, 12, 1127, 1161, 618, 204, 1164, 628}

$$\frac{1}{2x^2}-\frac{1}{6x^6}+\frac{1}{8}\log\left(x^4-x^2+1\right)-\frac{1}{8}\log\left(x^4+x^2+1\right)-\frac{\tan^{-1}\left(\frac{1-2x^2}{\sqrt{3}}\right)}{4\sqrt{3}}+\frac{\tan^{-1}\left(\frac{2x^2+1}{\sqrt{3}}\right)}{4\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^7*(1 + x^4 + x^8)),x]
```

```
[Out] -1/(6*x^6) + 1/(2*x^2) - ArcTan[(1 - 2*x^2)/Sqrt[3]]/(4*Sqrt[3]) + ArcTan[(
1 + 2*x^2)/Sqrt[3]]/(4*Sqrt[3]) + Log[1 - x^2 + x^4]/8 - Log[1 + x^2 + x^4]
/8
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1123

```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*x^2 + c*x^4)^(p + 1))/(a*d*(m + 1)), x] - Dis
t[1/(a*d^2*(m + 1)), Int[(d*x)^(m + 2)*(b*(m + 2*p + 3) + c*(m + 4*p + 5)*x
^2)*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b^2 -
 4*a*c, 0] && LtQ[m, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1127

```
Int[(x_)^2/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a/c, 2]}, Dist[1/2, Int[(q + x^2)/(a + b*x^2 + c*x^4), x], x] - Dist[1/2, I
nt[(q - x^2)/(a + b*x^2 + c*x^4), x], x]] /; FreeQ[{a, b, c}, x] && LtQ[b^2
```

```
 - 4*a*c, 0] && PosQ[a*c]
```

Rule 1161

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[(2*d)/e - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2
, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (
GtQ[(2*d)/e - b/c, 0] || ( !LtQ[(2*d)/e - b/c, 0] && EqQ[d - e*Rt[a/c, 2],
0]))
```

Rule 1164

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[(-2*d)/e - b/c, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e
+ q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^
2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c
*d^2 - a*e^2, 0] &&  !GtQ[b^2 - 4*a*c, 0]
```

Rule 1281

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[(d*(f*x)^(m + 1)*(a + b*x^2 + c*x^4)^(p + 1)
)/(a*f*(m + 1)), x] + Dist[1/(a*f^2*(m + 1)), Int[(f*x)^(m + 2)*(a + b*x^2
+ c*x^4)^p*Simp[a*e*(m + 1) - b*d*(m + 2*p + 3) - c*d*(m + 4*p + 5)*x^2, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[m
, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1359

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*
x^(n/k) + c*x^((2*n)/k))^p, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, c, p
}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x^7\left(1+x^4+x^8\right)}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{x^4\left(1+x^2+x^4\right)}\,dx,x,x^2\right)\\
&= -\frac{1}{6x^6}+\frac{1}{6}\,\text{Subst}\left(\int \frac{-3-3x^2}{x^2\left(1+x^2+x^4\right)}\,dx,x,x^2\right)\\
&= -\frac{1}{6x^6}+\frac{1}{2x^2}-\frac{1}{6}\,\text{Subst}\left(\int -\frac{3x^2}{1+x^2+x^4}\,dx,x,x^2\right)\\
&= -\frac{1}{6x^6}+\frac{1}{2x^2}+\frac{1}{2}\,\text{Subst}\left(\int \frac{x^2}{1+x^2+x^4}\,dx,x,x^2\right)\\
&= -\frac{1}{6x^6}+\frac{1}{2x^2}-\frac{1}{4}\,\text{Subst}\left(\int \frac{1-x^2}{1+x^2+x^4}\,dx,x,x^2\right)+\frac{1}{4}\,\text{Subst}\left(\int \frac{1+x^2}{1+x^2+x^4}\,dx,x,x^2\right)\\
&= -\frac{1}{6x^6}+\frac{1}{2x^2}+\frac{1}{8}\,\text{Subst}\left(\int \frac{1+2x}{-1-x-x^2}\,dx,x,x^2\right)+\frac{1}{8}\,\text{Subst}\left(\int \frac{1-2x}{-1+x-x^2}\,dx,x,x^2\right)+\\
&= -\frac{1}{6x^6}+\frac{1}{2x^2}+\frac{1}{8}\log\left(1-x^2+x^4\right)-\frac{1}{8}\log\left(1+x^2+x^4\right)-\frac{1}{4}\,\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,-1\right.\\
&= -\frac{1}{6x^6}+\frac{1}{2x^2}-\frac{\tan^{-1}\left(\frac{1-2x^2}{\sqrt{3}}\right)}{4\sqrt{3}}+\frac{\tan^{-1}\left(\frac{1+2x^2}{\sqrt{3}}\right)}{4\sqrt{3}}+\frac{1}{8}\log\left(1-x^2+x^4\right)-\frac{1}{8}\log\left(1+x^2+x^4\right.
\end{aligned}
$$

Mathematica [C] time = 0.11, size = 142, normalized size = 1.60

$$\frac{1}{24}\left(-\frac{4}{x^6}+\frac{12}{x^2}+\sqrt{3}\left(\sqrt{3}-i\right)\log\left(x^2-\frac{i\sqrt{3}}{2}-\frac{1}{2}\right)+\sqrt{3}\left(\sqrt{3}+i\right)\log\left(x^2+\frac{1}{2}i\left(\sqrt{3}+i\right)\right)-3\log\left(x^2-x+1\right)-3\log\left(x^2+x+1\right)+2\sqrt{3}\tan^{-1}\left(\frac{2x-1}{\sqrt{3}}\right)-2\sqrt{3}\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^7*(1 + x^4 + x^8)),x]
```

```
[Out] (-4/x^6 + 12/x^2 + 2*Sqrt[3]*ArcTan[(-1 + 2*x)/Sqrt[3]] - 2*Sqrt[3]*ArcTan[
(1 + 2*x)/Sqrt[3]] + Sqrt[3]*(-I + Sqrt[3])*Log[-1/2 - (I/2)*Sqrt[3] + x^2]
 + Sqrt[3]*(I + Sqrt[3])*Log[(I/2)*(I + Sqrt[3]) + x^2] - 3*Log[1 - x + x^2
] - 3*Log[1 + x + x^2])/24
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^7\left(1+x^4+x^8\right)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x^7*(1 + x^4 + x^8)),x]
```

```
[Out] IntegrateAlgebraic[1/(x^7*(1 + x^4 + x^8)), x]
```

fricas [A] time = 1.46, size = 84, normalized size = 0.94

$$\frac{2\sqrt{3}x^6\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^2+1\right)\right)+2\sqrt{3}x^6\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^2-1\right)\right)-3x^6\log\left(x^4+x^2+1\right)+3x^6\log\left(x^4-x^2+1\right)+12x^4-4}{24x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(x^8+x^4+1),x, algorithm="fricas")
```

```
[Out] 1/24*(2*sqrt(3)*x^6*arctan(1/3*sqrt(3)*(2*x^2 + 1)) + 2*sqrt(3)*x^6*arctan(
1/3*sqrt(3)*(2*x^2 - 1)) - 3*x^6*log(x^4 + x^2 + 1) + 3*x^6*log(x^4 - x^2 +
 1) + 12*x^4 - 4)/x^6
```

giac [A] time = 0.27, size = 73, normalized size = 0.82

$$\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^2+1\right)\right)+\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^2-1\right)\right)+\frac{3x^4-1}{6x^6}-\frac{1}{8}\log\left(x^4+x^2+1\right)+\frac{1}{8}\log\left(x^4-x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(x^8+x^4+1),x, algorithm="giac")
```

```
[Out] 1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^2 + 1)) + 1/12*sqrt(3)*arctan(1/3*sqrt
(3)*(2*x^2 - 1)) + 1/6*(3*x^4 - 1)/x^6 - 1/8*log(x^4 + x^2 + 1) + 1/8*log(x
^4 - x^2 + 1)
```

maple [A] time = 0.01, size = 95, normalized size = 1.07

$$-\frac{\sqrt{3}\arctan\left(\frac{(2x+1)\sqrt{3}}{3}\right)}{12}+\frac{\sqrt{3}\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{12}+\frac{\sqrt{3}\arctan\left(\frac{(2x^2-1)\sqrt{3}}{3}\right)}{12}-\frac{\ln\left(x^2-x+1\right)}{8}-\frac{\ln\left(x^2+x+1\right)}{8}+\frac{\ln\left(x^4-x^2+1\right)}{8}+\frac{1}{2x^2}-\frac{1}{6x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^7/(x^8+x^4+1),x)
```

```
[Out] -1/6/x^6+1/2/x^2-1/8*ln(x^2+x+1)-1/12*3^(1/2)*arctan(1/3*(2*x+1)*3^(1/2))+1
/8*ln(x^4-x^2+1)+1/12*3^(1/2)*arctan(1/3*(2*x^2-1)*3^(1/2))-1/8*ln(x^2-x+1)
+1/12*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))
```

maxima [A] time = 2.37, size = 73, normalized size = 0.82

$$\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^2+1\right)\right)+\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^2-1\right)\right)+\frac{3x^4-1}{6x^6}-\frac{1}{8}\log\left(x^4+x^2+1\right)+\frac{1}{8}\log\left(x^4-x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(x^8+x^4+1),x, algorithm="maxima")
```

```
[Out] 1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^2 + 1)) + 1/12*sqrt(3)*arctan(1/3*sqrt
(3)*(2*x^2 - 1)) + 1/6*(3*x^4 - 1)/x^6 - 1/8*log(x^4 + x^2 + 1) + 1/8*log(x
^4 - x^2 + 1)
```

mupad [B] time = 0.04, size = 62, normalized size = 0.70

$$\mathrm{atanh}\left(\frac{2x^2}{-1+\sqrt{3}\,1\mathrm{i}}\right)\left(\frac{1}{4}+\frac{\sqrt{3}\,1\mathrm{i}}{12}\right)+\mathrm{atanh}\left(\frac{2x^2}{1+\sqrt{3}\,1\mathrm{i}}\right)\left(-\frac{1}{4}+\frac{\sqrt{3}\,1\mathrm{i}}{12}\right)+\frac{\frac{x^4}{2}-\frac{1}{6}}{x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^7*(x^4 + x^8 + 1)),x)
```

```
[Out] atanh((2*x^2)/(3^(1/2)*1i - 1))*((3^(1/2)*1i)/12 + 1/4) + atanh((2*x^2)/(3^
(1/2)*1i + 1))*((3^(1/2)*1i)/12 - 1/4) + (x^4/2 - 1/6)/x^6
```

sympy [A] time = 0.25, size = 88, normalized size = 0.99

$$\frac{\log\left(x^4-x^2+1\right)}{8}-\frac{\log\left(x^4+x^2+1\right)}{8}+\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x^2}{3}-\frac{\sqrt{3}}{3}\right)}{12}+\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x^2}{3}+\frac{\sqrt{3}}{3}\right)}{12}+\frac{3x^4-1}{6x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**7/(x**8+x**4+1),x)
```

```
[Out] log(x**4 - x**2 + 1)/8 - log(x**4 + x**2 + 1)/8 + sqrt(3)*atan(2*sqrt(3)*x*
*2/3 - sqrt(3)/3)/12 + sqrt(3)*atan(2*sqrt(3)*x**2/3 + sqrt(3)/3)/12 + (3*x
**4 - 1)/(6*x**6)
```

3.280 $\int \frac{x^8}{1+x^4+x^8} dx$

Optimal. Leaf size=141

$$\frac{1}{8}\log\left(x^2-x+1\right)-\frac{1}{8}\log\left(x^2+x+1\right)+\frac{\log\left(x^2-\sqrt{3}x+1\right)}{8\sqrt{3}}-\frac{\log\left(x^2+\sqrt{3}x+1\right)}{8\sqrt{3}}+x+\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{4\sqrt{3}}+\frac{1}{4}\tan^{-1}\left(\sqrt{3}\right.$$

Rubi [A] time = 0.10, antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 7, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {1367, 1419, 1094, 634, 618, 204, 628}

$$\frac{1}{8}\log\left(x^2-x+1\right)-\frac{1}{8}\log\left(x^2+x+1\right)+\frac{\log\left(x^2-\sqrt{3}x+1\right)}{8\sqrt{3}}-\frac{\log\left(x^2+\sqrt{3}x+1\right)}{8\sqrt{3}}+x+\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{4\sqrt{3}}+\frac{1}{4}\tan^{-1}\left(\sqrt{3}-2x\right)-\frac{\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{4\sqrt{3}}-\frac{1}{4}\tan^{-1}\left(2x+\sqrt{3}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^8/(1 + x^4 + x^8),x]
```

```
[Out] x + ArcTan[(1 - 2*x)/Sqrt[3]]/(4*Sqrt[3]) + ArcTan[Sqrt[3] - 2*x]/4 - ArcTa
n[(1 + 2*x)/Sqrt[3]]/(4*Sqrt[3]) - ArcTan[Sqrt[3] + 2*x]/4 + Log[1 - x + x^
2]/8 - Log[1 + x + x^2]/8 + Log[1 - Sqrt[3]*x + x^2]/(8*Sqrt[3]) - Log[1 +
Sqrt[3]*x + x^2]/(8*Sqrt[3])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1094

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[a/
c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int[(r - x)/(q - r*x
 + x^2), x], x] + Dist[1/(2*c*q*r), Int[(r + x)/(q + r*x + x^2), x], x]]] /
; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[b^2 - 4*a*c]
```

Rule 1367

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Simp[(d^(2*n - 1)*(d*x)^(m - 2*n + 1)*(a + b*x^n + c*x^(2*n))^(
```

```
p + 1))/(c*(m + 2*n*p + 1)), x] - Dist[d^(2*n)/(c*(m + 2*n*p + 1)), Int[(d*
x)^(m - 2*n)*Simp[a*(m - 2*n + 1) + b*(m + n*(p - 1) + 1)*x^n, x]*(a + b*x^
n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && EqQ[n2, 2*n] && Ne
Q[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GtQ[m, 2*n - 1] && NeQ[m + 2*n*p + 1, 0]
 && IntegerQ[p]
```

Rule 1419

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[(2*d)/e - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e +
 q*x^(n/2) + x^n, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x^(n/2) + x
^n, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4
*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && IGtQ[n/2, 0] && (GtQ[(2*d)/e - b/c, 0]
 || ( !LtQ[(2*d)/e - b/c, 0] && EqQ[d, e*Rt[a/c, 2]]))
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^8}{1+x^4+x^8}\,dx &= x-\int \frac{1+x^4}{1+x^4+x^8}\,dx\\
&= x-\frac{1}{2}\int \frac{1}{1-x^2+x^4}\,dx-\frac{1}{2}\int \frac{1}{1+x^2+x^4}\,dx\\
&= x-\frac{1}{4}\int \frac{1-x}{1-x+x^2}\,dx-\frac{1}{4}\int \frac{1+x}{1+x+x^2}\,dx-\frac{\int \frac{\sqrt{3}-x}{1-\sqrt{3}x+x^2}\,dx}{4\sqrt{3}}-\frac{\int \frac{\sqrt{3}+x}{1+\sqrt{3}x+x^2}\,dx}{4\sqrt{3}}\\
&= x-\frac{1}{8}\int \frac{1}{1-x+x^2}\,dx+\frac{1}{8}\int \frac{-1+2x}{1-x+x^2}\,dx-\frac{1}{8}\int \frac{1}{1+x+x^2}\,dx-\frac{1}{8}\int \frac{1+2x}{1+x+x^2}\,dx-\\
&= x+\frac{1}{8}\log\left(1-x+x^2\right)-\frac{1}{8}\log\left(1+x+x^2\right)+\frac{\log\left(1-\sqrt{3}x+x^2\right)}{8\sqrt{3}}-\frac{\log\left(1+\sqrt{3}x+x^2\right)}{8\sqrt{3}}\\
&= x+\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{4\sqrt{3}}+\frac{1}{4}\tan^{-1}\left(\sqrt{3}-2x\right)-\frac{\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)}{4\sqrt{3}}-\frac{1}{4}\tan^{-1}\left(\sqrt{3}+2x\right)+\frac{1}{8}\log\left(1-\right.
\end{aligned}
$$

Mathematica [C] time = 0.28, size = 139, normalized size = 0.99

$$\frac{1}{24}\left(3\log\left(x^2-x+1\right)-3\log\left(x^2+x+1\right)+24x-2\sqrt{3}\tan^{-1}\left(\frac{2x-1}{\sqrt{3}}\right)-2\sqrt{3}\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)\right)-\frac{i\tan^{-1}\left(\frac{1}{2}\left(1-i\sqrt{3}\right)x\right)}{\sqrt{-6+6i\sqrt{3}}}+\frac{i\tan^{-1}\left(\frac{1}{2}\left(1+i\sqrt{3}\right)x\right)}{\sqrt{-6-6i\sqrt{3}}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^8/(1 + x^4 + x^8),x]
```

```
[Out] ((-I)*ArcTan[((1 - I*Sqrt[3])*x)/2])/Sqrt[-6 + (6*I)*Sqrt[3]] + (I*ArcTan[(
(1 + I*Sqrt[3])*x)/2])/Sqrt[-6 - (6*I)*Sqrt[3]] + (24*x - 2*Sqrt[3]*ArcTan[
(-1 + 2*x)/Sqrt[3]] - 2*Sqrt[3]*ArcTan[(1 + 2*x)/Sqrt[3]] + 3*Log[1 - x + x
^2] - 3*Log[1 + x + x^2])/24
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^8}{1+x^4+x^8}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^8/(1 + x^4 + x^8),x]
```

```
[Out] IntegrateAlgebraic[x^8/(1 + x^4 + x^8), x]
```

fricas [A] time = 0.95, size = 212, normalized size = 1.50

$$\frac{1}{12}\sqrt{6}\sqrt{3}\sqrt{2}\arctan\left(-\frac{1}{3}\sqrt{6}\sqrt{3}\sqrt{2}x+\frac{1}{3}\sqrt{6}\sqrt{3}\sqrt{\sqrt{6}\sqrt{2}x+2x^2+2}-\sqrt{3}\right)+\frac{1}{12}\sqrt{6}\sqrt{3}\sqrt{2}\arctan\left(-\frac{1}{3}\sqrt{6}\sqrt{3}\sqrt{2}x+\frac{1}{3}\sqrt{6}\sqrt{3}\sqrt{-\sqrt{6}\sqrt{2}x+2x^2+2}+\sqrt{3}\right)-\frac{1}{48}\sqrt{6}\sqrt{2}\log\left(\sqrt{6}\sqrt{2}x+2x^2+2\right)+\frac{1}{48}\sqrt{6}\sqrt{2}\log\left(-\sqrt{6}\sqrt{2}x+2x^2+2\right)-\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)-\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)+x-\frac{1}{8}\log\left(x^2+x+1\right)+\frac{1}{8}\log\left(x^2-x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(x^8+x^4+1),x, algorithm="fricas")
```

```
[Out] 1/12*sqrt(6)*sqrt(3)*sqrt(2)*arctan(-1/3*sqrt(6)*sqrt(3)*sqrt(2)*x + 1/3*sq
rt(6)*sqrt(3)*sqrt(sqrt(6)*sqrt(2)*x + 2*x^2 + 2) - sqrt(3)) + 1/12*sqrt(6)
*sqrt(3)*sqrt(2)*arctan(-1/3*sqrt(6)*sqrt(3)*sqrt(2)*x + 1/3*sqrt(6)*sqrt(3
)*sqrt(-sqrt(6)*sqrt(2)*x + 2*x^2 + 2) + sqrt(3)) - 1/48*sqrt(6)*sqrt(2)*lo
g(sqrt(6)*sqrt(2)*x + 2*x^2 + 2) + 1/48*sqrt(6)*sqrt(2)*log(-sqrt(6)*sqrt(2
)*x + 2*x^2 + 2) - 1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 1/12*sqrt(3
)*arctan(1/3*sqrt(3)*(2*x - 1)) + x - 1/8*log(x^2 + x + 1) + 1/8*log(x^2 -
x + 1)
```

giac [A] time = 0.39, size = 109, normalized size = 0.77

$$-\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)-\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)-\frac{1}{24}\sqrt{3}\log\left(x^2+\sqrt{3}x+1\right)+\frac{1}{24}\sqrt{3}\log\left(x^2-\sqrt{3}x+1\right)+x-\frac{1}{4}\arctan\left(2x+\sqrt{3}\right)-\frac{1}{4}\arctan\left(2x-\sqrt{3}\right)-\frac{1}{8}\log\left(x^2+x+1\right)+\frac{1}{8}\log\left(x^2-x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(x^8+x^4+1),x, algorithm="giac")
```

```
[Out] -1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 1/12*sqrt(3)*arctan(1/3*sqrt(
3)*(2*x - 1)) - 1/24*sqrt(3)*log(x^2 + sqrt(3)*x + 1) + 1/24*sqrt(3)*log(x^
2 - sqrt(3)*x + 1) + x - 1/4*arctan(2*x + sqrt(3)) - 1/4*arctan(2*x - sqrt(
3)) - 1/8*log(x^2 + x + 1) + 1/8*log(x^2 - x + 1)
```

maple [A] time = 0.04, size = 110, normalized size = 0.78

$$x-\frac{\sqrt{3}\,\arctan\left(\frac{(2x+1)\sqrt{3}}{3}\right)}{12}-\frac{\sqrt{3}\,\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{12}-\frac{\arctan\left(2x-\sqrt{3}\right)}{4}-\frac{\arctan\left(2x+\sqrt{3}\right)}{4}+\frac{\sqrt{3}\,\ln\left(x^2-\sqrt{3}\,x+1\right)}{24}-\frac{\sqrt{3}\,\ln\left(x^2+\sqrt{3}\,x+1\right)}{24}+\frac{\ln\left(x^2-x+1\right)}{8}-\frac{\ln\left(x^2+x+1\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(x^8+x^4+1),x)
```

```
[Out] x-1/8*ln(x^2+x+1)-1/12*3^(1/2)*arctan(1/3*(2*x+1)*3^(1/2))+1/24*ln(1+x^2-x*
3^(1/2))*3^(1/2)-1/4*arctan(2*x-3^(1/2))-1/24*ln(1+x^2+x*3^(1/2))*3^(1/2)-1
/4*arctan(2*x+3^(1/2))+1/8*ln(x^2-x+1)-1/12*3^(1/2)*arctan(1/3*(2*x-1)*3^(1
/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)-\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)+x-\frac{1}{2}\int\frac{1}{x^4-x^2+1}\,dx-\frac{1}{8}\log\left(x^2+x+1\right)+\frac{1}{8}\log\left(x^2-x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(x^8+x^4+1),x, algorithm="maxima")
```

```
[Out] -1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 1/12*sqrt(3)*arctan(1/3*sqrt(
3)*(2*x - 1)) + x - 1/2*integrate(1/(x^4 - x^2 + 1), x) - 1/8*log(x^2 + x +
 1) + 1/8*log(x^2 - x + 1)
```

mupad [B] time = 0.10, size = 100, normalized size = 0.71

$$x-\mathrm{atan}\left(\frac{2x}{-1+\sqrt{3}\,1\mathrm{i}}\right)\left(-\frac{1}{4}+\frac{\sqrt{3}\,1\mathrm{i}}{12}\right)-\mathrm{atan}\left(\frac{2x}{1+\sqrt{3}\,1\mathrm{i}}\right)\left(\frac{1}{4}+\frac{\sqrt{3}\,1\mathrm{i}}{12}\right)-\mathrm{atan}\left(\frac{x\,2\mathrm{i}}{-1+\sqrt{3}\,1\mathrm{i}}\right)\left(\frac{\sqrt{3}}{12}+\frac{1}{4}\mathrm{i}\right)-\mathrm{atan}\left(\frac{x\,2\mathrm{i}}{1+\sqrt{3}\,1\mathrm{i}}\right)\left(\frac{\sqrt{3}}{12}-\frac{1}{4}\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^8/(x^4 + x^8 + 1),x)

[Out] x - atan((2*x)/(3^(1/2)*1i - 1))*((3^(1/2)*1i)/12 - 1/4) - atan((2*x)/(3^(1/2)*1i + 1))*((3^(1/2)*1i)/12 + 1/4) - atan((x*2i)/(3^(1/2)*1i - 1))*(3^(1/2)/12 + 1i/4) - atan((x*2i)/(3^(1/2)*1i + 1))*(3^(1/2)/12 - 1i/4)

sympy [C] time = 0.71, size = 192, normalized size = 1.36

$$x+\left(\frac{1}{8}+\frac{\sqrt{3}\,i}{24}\right)\log\left(x-1-\frac{\sqrt{3}\,i}{3}-9216\left(\frac{1}{8}+\frac{\sqrt{3}\,i}{24}\right)^{5}\right)+\left(\frac{1}{8}-\frac{\sqrt{3}\,i}{24}\right)\log\left(x-1-9216\left(\frac{1}{8}-\frac{\sqrt{3}\,i}{24}\right)^{5}+\frac{\sqrt{3}\,i}{3}\right)+\left(-\frac{1}{8}+\frac{\sqrt{3}\,i}{24}\right)\log\left(x+1-\frac{\sqrt{3}\,i}{3}-9216\left(-\frac{1}{8}+\frac{\sqrt{3}\,i}{24}\right)^{5}\right)+\left(-\frac{1}{8}-\frac{\sqrt{3}\,i}{24}\right)\log\left(x+1-9216\left(-\frac{1}{8}-\frac{\sqrt{3}\,i}{24}\right)^{5}+\frac{\sqrt{3}\,i}{3}\right)+\operatorname{RootSum}\left(2304t^{4}+48t^{2}+1,\left(t\mapsto t\log\left(-9216t^{5}-8t+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**8/(x**8+x**4+1),x)

[Out] x + (1/8 + sqrt(3)*I/24)*log(x - 1 - sqrt(3)*I/3 - 9216*(1/8 + sqrt(3)*I/24)**5) + (1/8 - sqrt(3)*I/24)*log(x - 1 - 9216*(1/8 - sqrt(3)*I/24)**5 + sqrt(3)*I/3) + (-1/8 + sqrt(3)*I/24)*log(x + 1 - sqrt(3)*I/3 - 9216*(-1/8 + sqrt(3)*I/24)**5) + (-1/8 - sqrt(3)*I/24)*log(x + 1 - 9216*(-1/8 - sqrt(3)*I/24)**5 + sqrt(3)*I/3) + RootSum(2304*_t**4 + 48*_t**2 + 1, Lambda(_t, _t*log(-9216*_t**5 - 8*_t + x)))

3.281 $\int \frac{x^6}{1+x^4+x^8} \, dx$

Optimal. Leaf size=88

$$\frac{\log\left(x^2-\sqrt{3}\,x+1\right)}{4\sqrt{3}}-\frac{\log\left(x^2+\sqrt{3}\,x+1\right)}{4\sqrt{3}}-\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}}+\frac{\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{2\sqrt{3}}$$

Rubi [A] time = 0.06, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {1372, 1164, 628, 1161, 618, 204}

$$\frac{\log\left(x^2-\sqrt{3}\,x+1\right)}{4\sqrt{3}}-\frac{\log\left(x^2+\sqrt{3}\,x+1\right)}{4\sqrt{3}}-\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}}+\frac{\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{2\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^6/(1 + x^4 + x^8),x]
```

```
[Out] -ArcTan[(1 - 2*x)/Sqrt[3]]/(2*Sqrt[3]) + ArcTan[(1 + 2*x)/Sqrt[3]]/(2*Sqrt[
3]) + Log[1 - Sqrt[3]*x + x^2]/(4*Sqrt[3]) - Log[1 + Sqrt[3]*x + x^2]/(4*Sq
rt[3])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1161

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[(2*d)/e - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2
, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (
GtQ[(2*d)/e - b/c, 0] || ( !LtQ[(2*d)/e - b/c, 0] && EqQ[d - e*Rt[a/c, 2],
0]))
```

Rule 1164

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[(-2*d)/e - b/c, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e
+ q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^
2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c
*d^2 - a*e^2, 0] &&  !GtQ[b^2 - 4*a*c, 0]
```

Rule 1372

```
Int[(x_)^(m_.)/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symbol] :> W
ith[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, -Dist[1/(2*c*r), Int[(x^
(m - 3*(n/2))*(q - r*x^(n/2)))/(q - r*x^(n/2) + x^n), x], x] + Dist[1/(2*c*
r), Int[(x^(m - 3*(n/2))*(q + r*x^(n/2)))/(q + r*x^(n/2) + x^n), x], x]]] /
; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n/2, 0
] && IGtQ[m, 0] && GeQ[m, (3*n)/2] && LtQ[m, 2*n] && NegQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}\int \frac{x^6}{1+x^4+x^8}\,dx &= -\left(\frac{1}{2}\int \frac{1-x^2}{1-x^2+x^4}\,dx\right)+\frac{1}{2}\int \frac{1+x^2}{1+x^2+x^4}\,dx\\ &= \frac{1}{4}\int \frac{1}{1-x+x^2}\,dx+\frac{1}{4}\int \frac{1}{1+x+x^2}\,dx+\frac{\int \frac{\sqrt{3}+2x}{-1-\sqrt{3}x-x^2}\,dx}{4\sqrt{3}}+\frac{\int \frac{\sqrt{3}-2x}{-1+\sqrt{3}x-x^2}\,dx}{4\sqrt{3}}\\ &= \frac{\log\left(1-\sqrt{3}x+x^2\right)}{4\sqrt{3}}-\frac{\log\left(1+\sqrt{3}x+x^2\right)}{4\sqrt{3}}-\frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,-1+2x\right)-\frac{1}{2}\operatorname{Sul}\\ &= -\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}}+\frac{\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)}{2\sqrt{3}}+\frac{\log\left(1-\sqrt{3}x+x^2\right)}{4\sqrt{3}}-\frac{\log\left(1+\sqrt{3}x+x^2\right)}{4\sqrt{3}}\end{aligned}$$

Mathematica [A] time = 0.02, size = 68, normalized size = 0.77

$$\frac{\log\left(-x^2+\sqrt{3}x-1\right)-\log\left(x^2+\sqrt{3}x+1\right)+2\tan^{-1}\left(\frac{2x-1}{\sqrt{3}}\right)+2\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{4\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6/(1 + x^4 + x^8),x]
[Out] (2*ArcTan[(-1 + 2*x)/Sqrt[3]] + 2*ArcTan[(1 + 2*x)/Sqrt[3]] + Log[-1 + Sqrt
[3]*x - x^2] - Log[1 + Sqrt[3]*x + x^2])/(4*Sqrt[3])
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6}{1+x^4+x^8}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^6/(1 + x^4 + x^8),x]
[Out] IntegrateAlgebraic[x^6/(1 + x^4 + x^8), x]
```

fricas [A] time = 1.21, size = 70, normalized size = 0.80

$$\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(x^3+2x\right)\right)+\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}x\right)+\frac{1}{12}\sqrt{3}\log\left(\frac{x^4+5x^2-2\sqrt{3}\left(x^3+x\right)+1}{x^4-x^2+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(x^8+x^4+1),x, algorithm="fricas")
[Out] 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(x^3 + 2*x)) + 1/6*sqrt(3)*arctan(1/3*sqrt(3
)*x) + 1/12*sqrt(3)*log((x^4 + 5*x^2 - 2*sqrt(3)*(x^3 + x) + 1)/(x^4 - x^2
+ 1))
```

giac [A] time = 0.33, size = 66, normalized size = 0.75

$$\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)+\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)-\frac{1}{12}\sqrt{3}\log\left(x^2+\sqrt{3}x+1\right)+\frac{1}{12}\sqrt{3}\log\left(x^2-\sqrt{3}x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(x^8+x^4+1),x, algorithm="giac")
```

```
[Out] 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/6*sqrt(3)*arctan(1/3*sqrt(3)*
(2*x - 1)) - 1/12*sqrt(3)*log(x^2 + sqrt(3)*x + 1) + 1/12*sqrt(3)*log(x^2 -
 sqrt(3)*x + 1)
```

maple [A] time = 0.01, size = 67, normalized size = 0.76

$$\frac{\sqrt{3}\arctan\left(\frac{(2x+1)\sqrt{3}}{3}\right)}{6}+\frac{\sqrt{3}\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{6}+\frac{\sqrt{3}\ln\left(x^2-\sqrt{3}x+1\right)}{12}-\frac{\sqrt{3}\ln\left(x^2+\sqrt{3}x+1\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(x^8+x^4+1),x)
```

```
[Out] 1/6*3^(1/2)*arctan(1/3*(2*x+1)*3^(1/2))+1/12*3^(1/2)*ln(x^2-3^(1/2)*x+1)-1/
12*3^(1/2)*ln(x^2+3^(1/2)*x+1)+1/6*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)+\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)+\frac{1}{2}\int\frac{x^2-1}{x^4-x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(x^8+x^4+1),x, algorithm="maxima")
```

```
[Out] 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/6*sqrt(3)*arctan(1/3*sqrt(3)*
(2*x - 1)) + 1/2*integrate((x^2 - 1)/(x^4 - x^2 + 1), x)
```

mupad [B] time = 1.31, size = 38, normalized size = 0.43

$$-\frac{\sqrt{3}\left(\mathrm{atan}\left(\frac{2\sqrt{3}x}{3\left(\frac{2x^2}{3}-\frac{2}{3}\right)}\right)+\mathrm{atanh}\left(\frac{2\sqrt{3}x}{3\left(\frac{2x^2}{3}+\frac{2}{3}\right)}\right)\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(x^4 + x^8 + 1),x)
```

```
[Out] -(3^(1/2)*(atan((2*3^(1/2)*x)/(3*((2*x^2)/3 - 2/3))) + atanh((2*3^(1/2)*x)/
(3*((2*x^2)/3 + 2/3)))))/6
```

sympy [A] time = 0.18, size = 82, normalized size = 0.93

$$\frac{\sqrt{3}\left(2\,\mathrm{atan}\left(\frac{\sqrt{3}x}{3}\right)+2\,\mathrm{atan}\left(\frac{\sqrt{3}x^3}{3}+\frac{2\sqrt{3}x}{3}\right)\right)}{12}+\frac{\sqrt{3}\log\left(x^2-\sqrt{3}x+1\right)}{12}-\frac{\sqrt{3}\log\left(x^2+\sqrt{3}x+1\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(x**8+x**4+1),x)
```

```
[Out] sqrt(3)*(2*atan(sqrt(3)*x/3) + 2*atan(sqrt(3)*x**3/3 + 2*sqrt(3)*x/3))/12 +
 sqrt(3)*log(x**2 - sqrt(3)*x + 1)/12 - sqrt(3)*log(x**2 + sqrt(3)*x + 1)/1
2
```

3.282 $\int \frac{x^4}{1+x^4+x^8} \, dx$

Optimal. Leaf size=140

$$-\frac{1}{8}\log\left(x^2-x+1\right)+\frac{1}{8}\log\left(x^2+x+1\right)+\frac{\log\left(x^2-\sqrt{3}x+1\right)}{8\sqrt{3}}-\frac{\log\left(x^2+\sqrt{3}x+1\right)}{8\sqrt{3}}+\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{4\sqrt{3}}-\frac{1}{4}\tan^{-1}\left(\sqrt{\right.$$

Rubi [A] time = 0.11, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 7, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {1373, 1127, 1161, 618, 204, 1164, 628}

$$-\frac{1}{8}\log\left(x^2-x+1\right)+\frac{1}{8}\log\left(x^2+x+1\right)+\frac{\log\left(x^2-\sqrt{3}x+1\right)}{8\sqrt{3}}-\frac{\log\left(x^2+\sqrt{3}x+1\right)}{8\sqrt{3}}+\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{4\sqrt{3}}-\frac{1}{4}\tan^{-1}\left(\sqrt{3}-2x\right)-\frac{\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{4\sqrt{3}}+\frac{1}{4}\tan^{-1}\left(2x+\sqrt{3}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(1 + x^4 + x^8),x]
```

```
[Out] ArcTan[(1 - 2*x)/Sqrt[3]]/(4*Sqrt[3]) - ArcTan[Sqrt[3] - 2*x]/4 - ArcTan[(1
 + 2*x)/Sqrt[3]]/(4*Sqrt[3]) + ArcTan[Sqrt[3] + 2*x]/4 - Log[1 - x + x^2]/8
 + Log[1 + x + x^2]/8 + Log[1 - Sqrt[3]*x + x^2]/(8*Sqrt[3]) - Log[1 + Sqrt
[3]*x + x^2]/(8*Sqrt[3])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1127

```
Int[(x_)^2/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a/c, 2]}, Dist[1/2, Int[(q + x^2)/(a + b*x^2 + c*x^4), x], x] - Dist[1/2, I
nt[(q - x^2)/(a + b*x^2 + c*x^4), x], x]] /; FreeQ[{a, b, c}, x] && LtQ[b^2
 - 4*a*c, 0] && PosQ[a*c]
```

Rule 1161

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[(2*d)/e - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2
, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (
GtQ[(2*d)/e - b/c, 0] || ( !LtQ[(2*d)/e - b/c, 0] && EqQ[d - e*Rt[a/c, 2],
0]))
```

Rule 1164

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[(-2*d)/e - b/c, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e
+ q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^
2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c
*d^2 - a*e^2, 0] &&  !GtQ[b^2 - 4*a*c, 0]
```

Rule 1373

```
Int[(x_)^(m_.)/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symbol] :> W
ith[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*r), Int[x^(m
 - n/2)/(q - r*x^(n/2) + x^n), x], x] - Dist[1/(2*c*r), Int[x^(m - n/2)/(q
+ r*x^(n/2) + x^n), x], x]]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[n/2, 0] && IGtQ[m, 0] && GeQ[m, n/2] && LtQ[m, (3*n
)/2] && NegQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^4}{1+x^4+x^8}\,dx &= \frac{1}{2}\int \frac{x^2}{1-x^2+x^4}\,dx - \frac{1}{2}\int \frac{x^2}{1+x^2+x^4}\,dx \\
&= -\left(\frac{1}{4}\int \frac{1-x^2}{1-x^2+x^4}\,dx\right) + \frac{1}{4}\int \frac{1+x^2}{1-x^2+x^4}\,dx + \frac{1}{4}\int \frac{1-x^2}{1+x^2+x^4}\,dx - \frac{1}{4}\int \frac{1+x^2}{1+x^2+x^4}\,dx \\
&= -\left(\frac{1}{8}\int \frac{1+2x}{-1-x-x^2}\,dx\right) - \frac{1}{8}\int \frac{1-2x}{-1+x-x^2}\,dx - \frac{1}{8}\int \frac{1}{1-x+x^2}\,dx - \frac{1}{8}\int \frac{1}{1+x+x^2}\,dx + \\
&= -\frac{1}{8}\log\left(1-x+x^2\right) + \frac{1}{8}\log\left(1+x+x^2\right) + \frac{\log\left(1-\sqrt{3}x+x^2\right)}{8\sqrt{3}} - \frac{\log\left(1+\sqrt{3}x+x^2\right)}{8\sqrt{3}} + \frac{1}{4}\mathrm{S} \\
&= \frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{4\sqrt{3}} - \frac{1}{4}\tan^{-1}\left(\sqrt{3}-2x\right) - \frac{\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)}{4\sqrt{3}} + \frac{1}{4}\tan^{-1}\left(\sqrt{3}+2x\right) - \frac{1}{8}\log\left(1-x+x^2\right)
\end{aligned}$$

Mathematica [C] time = 0.16, size = 135, normalized size = 0.96

$$\frac{1}{24}\left(-3\log\left(x^2-x+1\right)+3\log\left(x^2+x+1\right)-2i\sqrt{-6+6i\sqrt{3}}\,\tan^{-1}\left(\frac{1}{2}\left(1-i\sqrt{3}\right)x\right)+2i\sqrt{-6-6i\sqrt{3}}\,\tan^{-1}\left(\frac{1}{2}\left(1+i\sqrt{3}\right)x\right)-2\sqrt{3}\tan^{-1}\left(\frac{2x-1}{\sqrt{3}}\right)-2\sqrt{3}\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^4/(1 + x^4 + x^8),x]
```

```
[Out] ((-2*I)*Sqrt[-6 + (6*I)*Sqrt[3]]*ArcTan[((1 - I*Sqrt[3])*x)/2] + (2*I)*Sqrt
[-6 - (6*I)*Sqrt[3]]*ArcTan[((1 + I*Sqrt[3])*x)/2] - 2*Sqrt[3]*ArcTan[(-1 +
 2*x)/Sqrt[3]] - 2*Sqrt[3]*ArcTan[(1 + 2*x)/Sqrt[3]] - 3*Log[1 - x + x^2] +
 3*Log[1 + x + x^2])/24
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{1+x^4+x^8}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^4/(1 + x^4 + x^8),x]
```

```
[Out] IntegrateAlgebraic[x^4/(1 + x^4 + x^8), x]
```

fricas [A] time = 1.53, size = 211, normalized size = 1.51

$$-\frac{1}{12}\sqrt{6}\sqrt{3}\sqrt{2}\arctan\left(-\frac{1}{3}\sqrt{6}\sqrt{3}\sqrt{2}x+\frac{1}{3}\sqrt{6}\sqrt{3}\sqrt{\sqrt{6}\sqrt{2}x+2x^2+2}-\sqrt{3}\right)-\frac{1}{12}\sqrt{6}\sqrt{3}\sqrt{2}\arctan\left(-\frac{1}{3}\sqrt{6}\sqrt{3}\sqrt{2}x+\frac{1}{3}\sqrt{6}\sqrt{3}\sqrt{-\sqrt{6}\sqrt{2}x+2x^2+2}+\sqrt{3}\right)-\frac{1}{48}\sqrt{6}\sqrt{2}\log\left(\sqrt{6}\sqrt{2}x+2x^2+2\right)+\frac{1}{48}\sqrt{6}\sqrt{2}\log\left(-\sqrt{6}\sqrt{2}x+2x^2+2\right)-\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)-\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)+\frac{1}{8}\log\left(x^2+x+1\right)-\frac{1}{8}\log\left(x^2-x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(x^8+x^4+1),x, algorithm="fricas")
```

```
[Out] -1/12*sqrt(6)*sqrt(3)*sqrt(2)*arctan(-1/3*sqrt(6)*sqrt(3)*sqrt(2)*x + 1/3*s
qrt(6)*sqrt(3)*sqrt(sqrt(6)*sqrt(2)*x + 2*x^2 + 2) - sqrt(3)) - 1/12*sqrt(6
)*sqrt(3)*sqrt(2)*arctan(-1/3*sqrt(6)*sqrt(3)*sqrt(2)*x + 1/3*sqrt(6)*sqrt(
3)*sqrt(-sqrt(6)*sqrt(2)*x + 2*x^2 + 2) + sqrt(3)) - 1/48*sqrt(6)*sqrt(2)*l
og(sqrt(6)*sqrt(2)*x + 2*x^2 + 2) + 1/48*sqrt(6)*sqrt(2)*log(-sqrt(6)*sqrt(
2)*x + 2*x^2 + 2) - 1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 1/12*sqrt(
3)*arctan(1/3*sqrt(3)*(2*x - 1)) + 1/8*log(x^2 + x + 1) - 1/8*log(x^2 - x +
 1)
```

giac [A] time = 0.40, size = 108, normalized size = 0.77

$$-\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)-\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)-\frac{1}{24}\sqrt{3}\log\left(x^2+\sqrt{3}x+1\right)+\frac{1}{24}\sqrt{3}\log\left(x^2-\sqrt{3}x+1\right)+\frac{1}{4}\arctan\left(2x+\sqrt{3}\right)+\frac{1}{4}\arctan\left(2x-\sqrt{3}\right)+\frac{1}{8}\log\left(x^2+x+1\right)-\frac{1}{8}\log\left(x^2-x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(x^8+x^4+1),x, algorithm="giac")
```

```
[Out] -1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 1/12*sqrt(3)*arctan(1/3*sqrt(
3)*(2*x - 1)) - 1/24*sqrt(3)*log(x^2 + sqrt(3)*x + 1) + 1/24*sqrt(3)*log(x^
2 - sqrt(3)*x + 1) + 1/4*arctan(2*x + sqrt(3)) + 1/4*arctan(2*x - sqrt(3))
+ 1/8*log(x^2 + x + 1) - 1/8*log(x^2 - x + 1)
```

maple [A] time = 0.02, size = 109, normalized size = 0.78

$$-\frac{\sqrt{3}\,\arctan\left(\frac{(2x+1)\sqrt{3}}{3}\right)}{12}-\frac{\sqrt{3}\,\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{12}+\frac{\arctan\left(2x-\sqrt{3}\right)}{4}+\frac{\arctan\left(2x+\sqrt{3}\right)}{4}+\frac{\sqrt{3}\,\ln\left(x^2-\sqrt{3}\,x+1\right)}{24}-\frac{\sqrt{3}\,\ln\left(x^2+\sqrt{3}\,x+1\right)}{24}-\frac{\ln\left(x^2-x+1\right)}{8}+\frac{\ln\left(x^2+x+1\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(x^8+x^4+1),x)
```

```
[Out] 1/8*ln(x^2+x+1)-1/12*3^(1/2)*arctan(1/3*(2*x+1)*3^(1/2))+1/24*3^(1/2)*ln(x^
2-3^(1/2)*x+1)+1/4*arctan(2*x-3^(1/2))-1/24*3^(1/2)*ln(x^2+3^(1/2)*x+1)+1/4
*arctan(2*x+3^(1/2))-1/8*ln(x^2-x+1)-1/12*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2
))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)-\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)+\frac{1}{2}\int\frac{x^2}{x^4-x^2+1}\,dx+\frac{1}{8}\log\left(x^2+x+1\right)-\frac{1}{8}\log\left(x^2-x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(x^8+x^4+1),x, algorithm="maxima")
```

```
[Out] -1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 1/12*sqrt(3)*arctan(1/3*sqrt(
3)*(2*x - 1)) + 1/2*integrate(x^2/(x^4 - x^2 + 1), x) + 1/8*log(x^2 + x + 1
) - 1/8*log(x^2 - x + 1)
```

mupad [B] time = 0.07, size = 99, normalized size = 0.71

$$-\mathrm{atan}\left(\frac{2x}{-1+\sqrt{3}\,1\mathrm{i}}\right)\left(\frac{1}{4}+\frac{\sqrt{3}\,1\mathrm{i}}{12}\right)-\mathrm{atan}\left(\frac{2x}{1+\sqrt{3}\,1\mathrm{i}}\right)\left(-\frac{1}{4}+\frac{\sqrt{3}\,1\mathrm{i}}{12}\right)-\mathrm{atan}\left(\frac{x\,2\mathrm{i}}{-1+\sqrt{3}\,1\mathrm{i}}\right)\left(\frac{\sqrt{3}}{12}-\frac{1}{4}\mathrm{i}\right)-\mathrm{atan}\left(\frac{x\,2\mathrm{i}}{1+\sqrt{3}\,1\mathrm{i}}\right)\left(\frac{\sqrt{3}}{12}+\frac{1}{4}\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(x^4 + x^8 + 1),x)
```

```
[Out] - atan((2*x)/(3^(1/2)*1i - 1))*((3^(1/2)*1i)/12 + 1/4) - atan((2*x)/(3^(1/2
)*1i + 1))*((3^(1/2)*1i)/12 - 1/4) - atan((x*2i)/(3^(1/2)*1i - 1))*(3^(1/2)
/12 - 1i/4) - atan((x*2i)/(3^(1/2)*1i + 1))*(3^(1/2)/12 + 1i/4)
```

sympy [C] time = 0.72, size = 197, normalized size = 1.41

$$\left(\frac{1}{8}-\frac{\sqrt{3}i}{24}\right)\log\left(x-\frac{1}{2}+\frac{\sqrt{3}i}{6}-18432\left(\frac{1}{8}-\frac{\sqrt{3}i}{24}\right)^5\right)+\left(\frac{1}{8}+\frac{\sqrt{3}i}{24}\right)\log\left(x-\frac{1}{2}-18432\left(\frac{1}{8}+\frac{\sqrt{3}i}{24}\right)^5-\frac{\sqrt{3}i}{6}\right)+\left(-\frac{1}{8}-\frac{\sqrt{3}i}{24}\right)\log\left(x+\frac{1}{2}+\frac{\sqrt{3}i}{6}-18432\left(-\frac{1}{8}-\frac{\sqrt{3}i}{24}\right)^5\right)+\left(-\frac{1}{8}+\frac{\sqrt{3}i}{24}\right)\log\left(x+\frac{1}{2}-18432\left(-\frac{1}{8}+\frac{\sqrt{3}i}{24}\right)^5-\frac{\sqrt{3}i}{6}\right)+\operatorname{RootSum}\left(2304t^4+48t^2+1,\left(t\mapsto t\log\left(-18432t^5-4t+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(x**8+x**4+1),x)
```

```
[Out] (1/8 - sqrt(3)*I/24)*log(x - 1/2 + sqrt(3)*I/6 - 18432*(1/8 - sqrt(3)*I/24)
**5) + (1/8 + sqrt(3)*I/24)*log(x - 1/2 - 18432*(1/8 + sqrt(3)*I/24)**5 - s
qrt(3)*I/6) + (-1/8 - sqrt(3)*I/24)*log(x + 1/2 + sqrt(3)*I/6 - 18432*(-1/8
 - sqrt(3)*I/24)**5) + (-1/8 + sqrt(3)*I/24)*log(x + 1/2 - 18432*(-1/8 + sq
rt(3)*I/24)**5 - sqrt(3)*I/6) + RootSum(2304*_t**4 + 48*_t**2 + 1, Lambda(_
t, _t*log(-18432*_t**5 - 4*_t + x)))
```

3.283 $\int \frac{x^2}{1+x^4+x^8}\,dx$

Optimal. Leaf size=140

$$\frac{1}{8}\log\left(x^2-x+1\right)-\frac{1}{8}\log\left(x^2+x+1\right)-\frac{\log\left(x^2-\sqrt{3}x+1\right)}{8\sqrt{3}}+\frac{\log\left(x^2+\sqrt{3}x+1\right)}{8\sqrt{3}}+\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{4\sqrt{3}}-\frac{1}{4}\tan^{-1}\left(\sqrt{3}\right.$$

Rubi [A] time = 0.08, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {1373, 1094, 634, 618, 204, 628}

$$\frac{1}{8}\log\left(x^2-x+1\right)-\frac{1}{8}\log\left(x^2+x+1\right)-\frac{\log\left(x^2-\sqrt{3}x+1\right)}{8\sqrt{3}}+\frac{\log\left(x^2+\sqrt{3}x+1\right)}{8\sqrt{3}}+\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{4\sqrt{3}}-\frac{1}{4}\tan^{-1}\left(\sqrt{3}-2x\right)-\frac{\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{4\sqrt{3}}+\frac{1}{4}\tan^{-1}\left(2x+\sqrt{3}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(1 + x^4 + x^8),x]
```

```
[Out] ArcTan[(1 - 2*x)/Sqrt[3]]/(4*Sqrt[3]) - ArcTan[Sqrt[3] - 2*x]/4 - ArcTan[(1
 + 2*x)/Sqrt[3]]/(4*Sqrt[3]) + ArcTan[Sqrt[3] + 2*x]/4 + Log[1 - x + x^2]/8
 - Log[1 + x + x^2]/8 - Log[1 - Sqrt[3]*x + x^2]/(8*Sqrt[3]) + Log[1 + Sqrt
[3]*x + x^2]/(8*Sqrt[3])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1094

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[a/
c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int[(r - x)/(q - r*x
 + x^2), x], x] + Dist[1/(2*c*q*r), Int[(r + x)/(q + r*x + x^2), x], x]]] /
; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[b^2 - 4*a*c]
```

Rule 1373

```
Int[(x_)^(m_.)/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symbol] :> W
ith[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*r), Int[x^(m
```

```
 - n/2)/(q - r*x^(n/2) + x^n), x], x] - Dist[1/(2*c*r), Int[x^(m - n/2)/(q
+ r*x^(n/2) + x^n), x], x]]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[n/2, 0] && IGtQ[m, 0] && GeQ[m, n/2] && LtQ[m, (3*n
)/2] && NegQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2}{1+x^4+x^8}\,dx &= \frac{1}{2}\int \frac{1}{1-x^2+x^4}\,dx - \frac{1}{2}\int \frac{1}{1+x^2+x^4}\,dx \\
&= -\left(\frac{1}{4}\int \frac{1-x}{1-x+x^2}\,dx\right) - \frac{1}{4}\int \frac{1+x}{1+x+x^2}\,dx + \frac{\int \frac{\sqrt{3}-x}{1-\sqrt{3}x+x^2}\,dx}{4\sqrt{3}} + \frac{\int \frac{\sqrt{3}+x}{1+\sqrt{3}x+x^2}\,dx}{4\sqrt{3}} \\
&= -\left(\frac{1}{8}\int \frac{1}{1-x+x^2}\,dx\right) + \frac{1}{8}\int \frac{-1+2x}{1-x+x^2}\,dx - \frac{1}{8}\int \frac{1}{1+x+x^2}\,dx - \frac{1}{8}\int \frac{1+2x}{1+x+x^2}\,dx + \frac{1}{8}\int \\
&= \frac{1}{8}\log\left(1-x+x^2\right) - \frac{1}{8}\log\left(1+x+x^2\right) - \frac{\log\left(1-\sqrt{3}x+x^2\right)}{8\sqrt{3}} + \frac{\log\left(1+\sqrt{3}x+x^2\right)}{8\sqrt{3}} + \frac{1}{4}\mathrm{Su} \\
&= \frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{4\sqrt{3}} - \frac{1}{4}\tan^{-1}\left(\sqrt{3}-2x\right) - \frac{\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)}{4\sqrt{3}} + \frac{1}{4}\tan^{-1}\left(\sqrt{3}+2x\right) + \frac{1}{8}\log\left(1-x+x^2\right)
\end{aligned}$$

Mathematica [C] time = 0.15, size = 135, normalized size = 0.96

$$\frac{1}{48}\left(6\log\left(x^2-x+1\right)-6\log\left(x^2+x+1\right)+4i\sqrt{-6-6i\sqrt{3}}\tan^{-1}\left(\frac{1}{2}\left(1-i\sqrt{3}\right)x\right)-4i\sqrt{-6+6i\sqrt{3}}\tan^{-1}\left(\frac{1}{2}\left(1+i\sqrt{3}\right)x\right)-4\sqrt{3}\tan^{-1}\left(\frac{2x-1}{\sqrt{3}}\right)-4\sqrt{3}\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2/(1 + x^4 + x^8),x]
```

```
[Out] ((4*I)*Sqrt[-6 - (6*I)*Sqrt[3]]*ArcTan[((1 - I*Sqrt[3])*x)/2] - (4*I)*Sqrt[
-6 + (6*I)*Sqrt[3]]*ArcTan[((1 + I*Sqrt[3])*x)/2] - 4*Sqrt[3]*ArcTan[(-1 +
2*x)/Sqrt[3]] - 4*Sqrt[3]*ArcTan[(1 + 2*x)/Sqrt[3]] + 6*Log[1 - x + x^2] -
6*Log[1 + x + x^2])/48
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{1+x^4+x^8}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^2/(1 + x^4 + x^8),x]
```

```
[Out] IntegrateAlgebraic[x^2/(1 + x^4 + x^8), x]
```

fricas [A] time = 1.45, size = 211, normalized size = 1.51

$$-\frac{1}{12}\sqrt{6}\sqrt{3}\sqrt{2}\arctan\left(-\frac{1}{3}\sqrt{6}\sqrt{3}\sqrt{2}x+\frac{1}{3}\sqrt{6}\sqrt{3}\sqrt{\sqrt{6}\sqrt{2}x+2x^2+2}-\sqrt{3}\right)-\frac{1}{12}\sqrt{6}\sqrt{3}\sqrt{2}\arctan\left(-\frac{1}{3}\sqrt{6}\sqrt{3}\sqrt{2}x+\frac{1}{3}\sqrt{6}\sqrt{3}\sqrt{-\sqrt{6}\sqrt{2}x+2x^2+2}+\sqrt{3}\right)+\frac{1}{48}\sqrt{6}\sqrt{2}\log\left(\sqrt{6}\sqrt{2}x+2x^2+2\right)-\frac{1}{48}\sqrt{6}\sqrt{2}\log\left(-\sqrt{6}\sqrt{2}x+2x^2+2\right)-\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)-\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)-\frac{1}{8}\log\left(x^2+x+1\right)+\frac{1}{8}\log\left(x^2-x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^8+x^4+1),x, algorithm="fricas")
```

```
[Out] -1/12*sqrt(6)*sqrt(3)*sqrt(2)*arctan(-1/3*sqrt(6)*sqrt(3)*sqrt(2)*x + 1/3*s
qrt(6)*sqrt(3)*sqrt(sqrt(6)*sqrt(2)*x + 2*x^2 + 2) - sqrt(3)) - 1/12*sqrt(6
)*sqrt(3)*sqrt(2)*arctan(-1/3*sqrt(6)*sqrt(3)*sqrt(2)*x + 1/3*sqrt(6)*sqrt(
3)*sqrt(-sqrt(6)*sqrt(2)*x + 2*x^2 + 2) + sqrt(3)) + 1/48*sqrt(6)*sqrt(2)*l
```

```
og(sqrt(6)*sqrt(2)*x + 2*x^2 + 2) - 1/48*sqrt(6)*sqrt(2)*log(-sqrt(6)*sqrt(
2)*x + 2*x^2 + 2) - 1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 1/12*sqrt(
3)*arctan(1/3*sqrt(3)*(2*x - 1)) - 1/8*log(x^2 + x + 1) + 1/8*log(x^2 - x +
 1)
```

giac [A] time = 0.34, size = 108, normalized size = 0.77

$$-\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)-\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)+\frac{1}{24}\sqrt{3}\log\left(x^2+\sqrt{3}x+1\right)-\frac{1}{24}\sqrt{3}\log\left(x^2-\sqrt{3}x+1\right)+\frac{1}{4}\arctan\left(2x+\sqrt{3}\right)+\frac{1}{4}\arctan\left(2x-\sqrt{3}\right)-\frac{1}{8}\log\left(x^2+x+1\right)+\frac{1}{8}\log\left(x^2-x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^8+x^4+1),x, algorithm="giac")
```

```
[Out] -1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 1/12*sqrt(3)*arctan(1/3*sqrt(
3)*(2*x - 1)) + 1/24*sqrt(3)*log(x^2 + sqrt(3)*x + 1) - 1/24*sqrt(3)*log(x^
2 - sqrt(3)*x + 1) + 1/4*arctan(2*x + sqrt(3)) + 1/4*arctan(2*x - sqrt(3))
- 1/8*log(x^2 + x + 1) + 1/8*log(x^2 - x + 1)
```

maple [A] time = 0.01, size = 109, normalized size = 0.78

$$-\frac{\sqrt{3}\arctan\left(\frac{(2x+1)\sqrt{3}}{3}\right)}{12}-\frac{\sqrt{3}\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{12}+\frac{\arctan\left(2x-\sqrt{3}\right)}{4}+\frac{\arctan\left(2x+\sqrt{3}\right)}{4}-\frac{\sqrt{3}\ln\left(x^2-\sqrt{3}x+1\right)}{24}+\frac{\sqrt{3}\ln\left(x^2+\sqrt{3}x+1\right)}{24}+\frac{\ln\left(x^2-x+1\right)}{8}-\frac{\ln\left(x^2+x+1\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(x^8+x^4+1),x)
```

```
[Out] -1/8*ln(x^2+x+1)-1/12*3^(1/2)*arctan(1/3*(2*x+1)*3^(1/2))-1/24*3^(1/2)*ln(x
^2-3^(1/2)*x+1)+1/4*arctan(2*x-3^(1/2))+1/24*3^(1/2)*ln(x^2+3^(1/2)*x+1)+1/
4*arctan(2*x+3^(1/2))+1/8*ln(x^2-x+1)-1/12*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/
2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)-\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)+\frac{1}{2}\int\frac{1}{x^4-x^2+1}\,dx-\frac{1}{8}\log\left(x^2+x+1\right)+\frac{1}{8}\log\left(x^2-x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^8+x^4+1),x, algorithm="maxima")
```

```
[Out] -1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 1/12*sqrt(3)*arctan(1/3*sqrt(
3)*(2*x - 1)) + 1/2*integrate(1/(x^4 - x^2 + 1), x) - 1/8*log(x^2 + x + 1)
+ 1/8*log(x^2 - x + 1)
```

mupad [B] time = 1.31, size = 97, normalized size = 0.69

$$\mathrm{atan}\left(\frac{2x}{-1+\sqrt{3}\,1\mathrm{i}}\right)\left(-\frac{1}{4}+\frac{\sqrt{3}\,1\mathrm{i}}{12}\right)+\mathrm{atan}\left(\frac{2x}{1+\sqrt{3}\,1\mathrm{i}}\right)\left(\frac{1}{4}+\frac{\sqrt{3}\,1\mathrm{i}}{12}\right)-\mathrm{atan}\left(\frac{x\,2\mathrm{i}}{-1+\sqrt{3}\,1\mathrm{i}}\right)\left(\frac{\sqrt{3}}{12}+\frac{1}{4}\mathrm{i}\right)-\mathrm{atan}\left(\frac{x\,2\mathrm{i}}{1+\sqrt{3}\,1\mathrm{i}}\right)\left(\frac{\sqrt{3}}{12}-\frac{1}{4}\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(x^4 + x^8 + 1),x)
```

```
[Out] atan((2*x)/(3^(1/2)*1i - 1))*((3^(1/2)*1i)/12 - 1/4) + atan((2*x)/(3^(1/2)*
1i + 1))*((3^(1/2)*1i)/12 + 1/4) - atan((x*2i)/(3^(1/2)*1i - 1))*(3^(1/2)/1
2 + 1i/4) - atan((x*2i)/(3^(1/2)*1i + 1))*(3^(1/2)/12 - 1i/4)
```

sympy [C] time = 0.71, size = 214, normalized size = 1.53

$$\left(-\frac{1}{8}-\frac{\sqrt{3}i}{24}\right)\log\left(x+442368\left(-\frac{1}{8}-\frac{\sqrt{3}i}{24}\right)^7-192\left(-\frac{1}{8}-\frac{\sqrt{3}i}{24}\right)^3\right)+\left(-\frac{1}{8}+\frac{\sqrt{3}i}{24}\right)\log\left(x-192\left(-\frac{1}{8}+\frac{\sqrt{3}i}{24}\right)^3+442368\left(-\frac{1}{8}+\frac{\sqrt{3}i}{24}\right)^7\right)+\left(\frac{1}{8}-\frac{\sqrt{3}i}{24}\right)\log\left(x+442368\left(\frac{1}{8}-\frac{\sqrt{3}i}{24}\right)^7-192\left(\frac{1}{8}-\frac{\sqrt{3}i}{24}\right)^3\right)+\left(\frac{1}{8}+\frac{\sqrt{3}i}{24}\right)\log\left(x-192\left(\frac{1}{8}+\frac{\sqrt{3}i}{24}\right)^3+442368\left(\frac{1}{8}+\frac{\sqrt{3}i}{24}\right)^7\right)+\mathrm{RootSum}\left(2304t^4+48t^2+1,\left(t\mapsto t\log\left(442368t^7-192t^3+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(x**8+x**4+1),x)
```

```
[Out] (-1/8 - sqrt(3)*I/24)*log(x + 442368*(-1/8 - sqrt(3)*I/24)**7 - 192*(-1/8 -
 sqrt(3)*I/24)**3) + (-1/8 + sqrt(3)*I/24)*log(x - 192*(-1/8 + sqrt(3)*I/24
)**3 + 442368*(-1/8 + sqrt(3)*I/24)**7) + (1/8 - sqrt(3)*I/24)*log(x + 4423
68*(1/8 - sqrt(3)*I/24)**7 - 192*(1/8 - sqrt(3)*I/24)**3) + (1/8 + sqrt(3)*
I/24)*log(x - 192*(1/8 + sqrt(3)*I/24)**3 + 442368*(1/8 + sqrt(3)*I/24)**7)
 + RootSum(2304*_t**4 + 48*_t**2 + 1, Lambda(_t, _t*log(442368*_t**7 - 192*
_t**3 + x)))
```

3.284 $\int \frac{1}{1+x^4+x^8} dx$

Optimal. Leaf size=88

$$-\frac{\log \left(x^2-\sqrt{3} x+1\right)}{4 \sqrt{3}}+\frac{\log \left(x^2+\sqrt{3} x+1\right)}{4 \sqrt{3}}-\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}}+\frac{\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{2\sqrt{3}}$$

Rubi [A] time = 0.05, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {1346, 1164, 628, 1161, 618, 204}

$$-\frac{\log \left(x^2-\sqrt{3} x+1\right)}{4 \sqrt{3}}+\frac{\log \left(x^2+\sqrt{3} x+1\right)}{4 \sqrt{3}}-\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}}+\frac{\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{2\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^4 + x^8)^(-1),x]
```

```
[Out] -ArcTan[(1 - 2*x)/Sqrt[3]]/(2*Sqrt[3]) + ArcTan[(1 + 2*x)/Sqrt[3]]/(2*Sqrt[
3]) - Log[1 - Sqrt[3]*x + x^2]/(4*Sqrt[3]) + Log[1 + Sqrt[3]*x + x^2]/(4*Sq
rt[3])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1161

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[(2*d)/e - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2
, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (
GtQ[(2*d)/e - b/c, 0] || ( !LtQ[(2*d)/e - b/c, 0] && EqQ[d - e*Rt[a/c, 2],
0]))
```

Rule 1164

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[(-2*d)/e - b/c, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e
+ q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^
2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c
*d^2 - a*e^2, 0] &&  !GtQ[b^2 - 4*a*c, 0]
```

Rule 1346

```
Int[((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^(-1), x_Symbol] :> With[{q
= Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int[(r - x^(n
/2))/(q - r*x^(n/2) + x^n), x], x] + Dist[1/(2*c*q*r), Int[(r + x^(n/2))/(q
 + r*x^(n/2) + x^n), x], x]]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && NeQ
[b^2 - 4*a*c, 0] && IGtQ[n/2, 0] && NegQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{1}{1+x^4+x^8}\,dx = \frac{1}{2}\int \frac{1-x^2}{1-x^2+x^4}\,dx + \frac{1}{2}\int \frac{1+x^2}{1+x^2+x^4}\,dx$$

$$= \frac{1}{4}\int \frac{1}{1-x+x^2}\,dx + \frac{1}{4}\int \frac{1}{1+x+x^2}\,dx - \frac{\int \frac{\sqrt{3}+2x}{-1-\sqrt{3}x-x^2}\,dx}{4\sqrt{3}} - \frac{\int \frac{\sqrt{3}-2x}{-1+\sqrt{3}x-x^2}\,dx}{4\sqrt{3}}$$

$$= -\frac{\log\left(1-\sqrt{3}x+x^2\right)}{4\sqrt{3}} + \frac{\log\left(1+\sqrt{3}x+x^2\right)}{4\sqrt{3}} - \frac{1}{2}\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, -1+2x\right) - \frac{1}{2}\text{Subs}$$

$$= -\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}} + \frac{\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)}{2\sqrt{3}} - \frac{\log\left(1-\sqrt{3}x+x^2\right)}{4\sqrt{3}} + \frac{\log\left(1+\sqrt{3}x+x^2\right)}{4\sqrt{3}}$$

Mathematica [A] time = 0.02, size = 68, normalized size = 0.77

$$\frac{-\log\left(-x^2+\sqrt{3}x-1\right)+\log\left(x^2+\sqrt{3}x+1\right)+2\tan^{-1}\left(\frac{2x-1}{\sqrt{3}}\right)+2\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{4\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^4 + x^8)^(-1),x]
```

```
[Out] (2*ArcTan[(-1 + 2*x)/Sqrt[3]] + 2*ArcTan[(1 + 2*x)/Sqrt[3]] - Log[-1 + Sqrt
[3]*x - x^2] + Log[1 + Sqrt[3]*x + x^2])/(4*Sqrt[3])
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{1+x^4+x^8}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(1 + x^4 + x^8)^(-1),x]
```

```
[Out] IntegrateAlgebraic[(1 + x^4 + x^8)^(-1), x]
```

fricas [A] time = 1.12, size = 70, normalized size = 0.80

$$\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(x^3+2x\right)\right)+\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}x\right)+\frac{1}{12}\sqrt{3}\log\left(\frac{x^4+5x^2+2\sqrt{3}\left(x^3+x\right)+1}{x^4-x^2+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^8+x^4+1),x, algorithm="fricas")
```

```
[Out] 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(x^3 + 2*x)) + 1/6*sqrt(3)*arctan(1/3*sqrt(3
)*x) + 1/12*sqrt(3)*log((x^4 + 5*x^2 + 2*sqrt(3)*(x^3 + x) + 1)/(x^4 - x^2
+ 1))
```

giac [A] time = 0.39, size = 66, normalized size = 0.75

$$\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)+\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)+\frac{1}{12}\sqrt{3}\log\left(x^2+\sqrt{3}x+1\right)-\frac{1}{12}\sqrt{3}\log\left(x^2-\sqrt{3}x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^8+x^4+1),x, algorithm="giac")
```

```
[Out] 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/6*sqrt(3)*arctan(1/3*sqrt(3)*
(2*x - 1)) + 1/12*sqrt(3)*log(x^2 + sqrt(3)*x + 1) - 1/12*sqrt(3)*log(x^2 -
 sqrt(3)*x + 1)
```

maple [A] time = 0.01, size = 67, normalized size = 0.76

$$\frac{\sqrt{3}\,\arctan\left(\frac{(2x+1)\sqrt{3}}{3}\right)}{6}+\frac{\sqrt{3}\,\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{6}-\frac{\sqrt{3}\,\ln\left(x^2-\sqrt{3}\,x+1\right)}{12}+\frac{\sqrt{3}\,\ln\left(x^2+\sqrt{3}\,x+1\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^8+x^4+1),x)
```

```
[Out] 1/6*3^(1/2)*arctan(1/3*(2*x+1)*3^(1/2))-1/12*3^(1/2)*ln(x^2-3^(1/2)*x+1)+1/
12*3^(1/2)*ln(x^2+3^(1/2)*x+1)+1/6*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{6}\sqrt{3}\,\arctan\left(\frac{1}{3}\sqrt{3}\,(2\,x+1)\right)+\frac{1}{6}\sqrt{3}\,\arctan\left(\frac{1}{3}\sqrt{3}\,(2\,x-1)\right)-\frac{1}{2}\int\frac{x^2-1}{x^4-x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^8+x^4+1),x, algorithm="maxima")
```

```
[Out] 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/6*sqrt(3)*arctan(1/3*sqrt(3)*
(2*x - 1)) - 1/2*integrate((x^2 - 1)/(x^4 - x^2 + 1), x)
```

mupad [B] time = 0.04, size = 40, normalized size = 0.45

$$-\frac{\sqrt{3}\left(\operatorname{atan}\left(\frac{2\,\sqrt{3}\,x}{3\left(\frac{2\,x^2}{3}-\frac{2}{3}\right)}\right)-\operatorname{atanh}\left(\frac{2\,\sqrt{3}\,x}{3\left(\frac{2\,x^2}{3}+\frac{2}{3}\right)}\right)\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4 + x^8 + 1),x)
```

```
[Out] -(3^(1/2)*(atan((2*3^(1/2)*x)/(3*((2*x^2)/3 - 2/3))) - atanh((2*3^(1/2)*x)/
(3*((2*x^2)/3 + 2/3)))))/6
```

sympy [A] time = 0.18, size = 82, normalized size = 0.93

$$\frac{\sqrt{3}\left(2\operatorname{atan}\left(\frac{\sqrt{3}x}{3}\right)+2\operatorname{atan}\left(\frac{\sqrt{3}x^3}{3}+\frac{2\sqrt{3}x}{3}\right)\right)}{12}-\frac{\sqrt{3}\log\left(x^2-\sqrt{3}x+1\right)}{12}+\frac{\sqrt{3}\log\left(x^2+\sqrt{3}x+1\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**8+x**4+1),x)
```

```
[Out] sqrt(3)*(2*atan(sqrt(3)*x/3) + 2*atan(sqrt(3)*x**3/3 + 2*sqrt(3)*x/3))/12 -
 sqrt(3)*log(x**2 - sqrt(3)*x + 1)/12 + sqrt(3)*log(x**2 + sqrt(3)*x + 1)/1
2
```

3.285 $\int \frac{1}{x^2(1+x^4+x^8)}\,dx$

Optimal. Leaf size=145

$$-\frac{1}{8}\log\left(x^2-x+1\right)+\frac{1}{8}\log\left(x^2+x+1\right)-\frac{\log\left(x^2-\sqrt{3}x+1\right)}{8\sqrt{3}}+\frac{\log\left(x^2+\sqrt{3}x+1\right)}{8\sqrt{3}}-\frac{1}{x}+\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{4\sqrt{3}}+\frac{1}{4}\tan^{-1}\left(\sqrt{\right.$$

Rubi [A] time = 0.11, antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 8, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {1368, 1506, 1127, 1161, 618, 204, 1164, 628}

$$-\frac{1}{8}\log\left(x^2-x+1\right)+\frac{1}{8}\log\left(x^2+x+1\right)-\frac{\log\left(x^2-\sqrt{3}x+1\right)}{8\sqrt{3}}+\frac{\log\left(x^2+\sqrt{3}x+1\right)}{8\sqrt{3}}-\frac{1}{x}+\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{4\sqrt{3}}+\frac{1}{4}\tan^{-1}\left(\sqrt{3}-2x\right)-\frac{\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{4\sqrt{3}}-\frac{1}{4}\tan^{-1}\left(2x+\sqrt{3}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(1 + x^4 + x^8)),x]
```

```
[Out] -x^(-1) + ArcTan[(1 - 2*x)/Sqrt[3]]/(4*Sqrt[3]) + ArcTan[Sqrt[3] - 2*x]/4 -
 ArcTan[(1 + 2*x)/Sqrt[3]]/(4*Sqrt[3]) - ArcTan[Sqrt[3] + 2*x]/4 - Log[1 -
x + x^2]/8 + Log[1 + x + x^2]/8 - Log[1 - Sqrt[3]*x + x^2]/(8*Sqrt[3]) + Lo
g[1 + Sqrt[3]*x + x^2]/(8*Sqrt[3])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1127

```
Int[(x_)^2/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a/c, 2]}, Dist[1/2, Int[(q + x^2)/(a + b*x^2 + c*x^4), x], x] - Dist[1/2, I
nt[(q - x^2)/(a + b*x^2 + c*x^4), x], x]] /; FreeQ[{a, b, c}, x] && LtQ[b^2
 - 4*a*c, 0] && PosQ[a*c]
```

Rule 1161

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[(2*d)/e - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2
, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (
GtQ[(2*d)/e - b/c, 0] || ( !LtQ[(2*d)/e - b/c, 0] && EqQ[d - e*Rt[a/c, 2],
0]))
```

Rule 1164

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[(-2*d)/e - b/c, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e
+ q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^
2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c
*d^2 - a*e^2, 0] &&  !GtQ[b^2 - 4*a*c, 0]
```

Rule 1368

```
Int[((d_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_
Symbol] :> Simp[((d*x)^(m + 1)*(a + b*x^n + c*x^(2*n))^(p + 1))/(a*d*(m + 1
)), x] - Dist[1/(a*d^n*(m + 1)), Int[(d*x)^(m + n)*(b*(m + n*(p + 1) + 1) +
 c*(m + 2*n*(p + 1) + 1)*x^n)*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a
, b, c, d, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && L
tQ[m, -1] && IntegerQ[p]
```

Rule 1506

```
Int[(((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_)^(n_)))/((a_) + (b_.)*(x_)^(n_) +
(c_.)*(x_)^(n2_)), x_Symbol] :> With[{q = Rt[a*c, 2]}, With[{r = Rt[2*c*q -
 b*c, 2]}, Dist[c/(2*q*r), Int[((f*x)^m*Simp[d*r - (c*d - e*q)*x^(n/2), x])
/(q - r*x^(n/2) + c*x^n), x], x] + Dist[c/(2*q*r), Int[((f*x)^m*Simp[d*r +
(c*d - e*q)*x^(n/2), x])/(q + r*x^(n/2) + c*x^n), x], x]] /;  !LtQ[2*c*q -
b*c, 0]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[n2, 2*n] && LtQ[b^2 - 4*a*c
, 0] && IntegersQ[m, n/2] && LtQ[0, m, n] && PosQ[a*c]
```

Rubi steps

$$\int \frac{1}{x^2\left(1+x^4+x^8\right)}\,dx = -\frac{1}{x}+\int \frac{x^2\left(-1-x^4\right)}{1+x^4+x^8}\,dx$$
$$= -\frac{1}{x}-\frac{1}{2}\int \frac{x^2}{1-x^2+x^4}\,dx-\frac{1}{2}\int \frac{x^2}{1+x^2+x^4}\,dx$$
$$= -\frac{1}{x}+\frac{1}{4}\int \frac{1-x^2}{1-x^2+x^4}\,dx-\frac{1}{4}\int \frac{1+x^2}{1-x^2+x^4}\,dx+\frac{1}{4}\int \frac{1-x^2}{1+x^2+x^4}\,dx-\frac{1}{4}\int \frac{1+}{1+x^2}$$
$$= -\frac{1}{x}-\frac{1}{8}\int \frac{1+2x}{-1-x-x^2}\,dx-\frac{1}{8}\int \frac{1-2x}{-1+x-x^2}\,dx-\frac{1}{8}\int \frac{1}{1-x+x^2}\,dx-\frac{1}{8}\int \frac{1}{1+x+}$$
$$= -\frac{1}{x}-\frac{1}{8}\log\left(1-x+x^2\right)+\frac{1}{8}\log\left(1+x+x^2\right)-\frac{\log\left(1-\sqrt{3}\,x+x^2\right)}{8\sqrt{3}}+\frac{\log\left(1+\sqrt{3}\,x\right.}{8\sqrt{3}}$$
$$= -\frac{1}{x}+\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{4\sqrt{3}}+\frac{1}{4}\tan^{-1}\left(\sqrt{3}-2x\right)-\frac{\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)}{4\sqrt{3}}-\frac{1}{4}\tan^{-1}\left(\sqrt{3}+2x\right)-\frac{1}{8}\mathrm{lo}$$

Mathematica [C] time = 0.21, size = 140, normalized size = 0.97

$$\frac{1}{24}\left(-3\log\left(x^2-x+1\right)+3\log\left(x^2+x+1\right)-\frac{24}{x}+2i\sqrt{-6+6i\sqrt{3}}\,\tan^{-1}\left(\frac{1}{2}\left(1-i\sqrt{3}\right)x\right)-2i\sqrt{-6-6i\sqrt{3}}\,\tan^{-1}\left(\frac{1}{2}\left(1+i\sqrt{3}\right)x\right)-2\sqrt{3}\tan^{-1}\left(\frac{2x-1}{\sqrt{3}}\right)-2\sqrt{3}\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(x^2*(1 + x^4 + x^8)),x]
```

```
[Out] (-24/x + (2*I)*Sqrt[-6 + (6*I)*Sqrt[3]]*ArcTan[((1 - I*Sqrt[3])*x)/2] - (2*
I)*Sqrt[-6 - (6*I)*Sqrt[3]]*ArcTan[((1 + I*Sqrt[3])*x)/2] - 2*Sqrt[3]*ArcTa
n[(-1 + 2*x)/Sqrt[3]] - 2*Sqrt[3]*ArcTan[(1 + 2*x)/Sqrt[3]] - 3*Log[1 - x +
 x^2] + 3*Log[1 + x + x^2])/24
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2\left(1+x^4+x^8\right)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x^2*(1 + x^4 + x^8)),x]
```

```
[Out] IntegrateAlgebraic[1/(x^2*(1 + x^4 + x^8)), x]
```

fricas [A] time = 1.32, size = 224, normalized size = 1.54

$$\frac{4\sqrt{6}\sqrt{3}\sqrt{2}x\arctan\left(-\frac{1}{3}\sqrt{6}\sqrt{3}\sqrt{2}x+\frac{1}{3}\sqrt{6}\sqrt{3}\sqrt{\sqrt{6}\sqrt{2}x+2x^2+2}-\sqrt{3}\right)+4\sqrt{6}\sqrt{3}\sqrt{2}x\arctan\left(-\frac{1}{3}\sqrt{6}\sqrt{3}\sqrt{2}x+\frac{1}{3}\sqrt{6}\sqrt{3}\sqrt{-\sqrt{6}\sqrt{2}x+2x^2+2}+\sqrt{3}\right)+\sqrt{6}\sqrt{2}x\log\left(\sqrt{6}\sqrt{2}x+2x^2+2\right)-\sqrt{6}\sqrt{2}x\log\left(-\sqrt{6}\sqrt{2}x+2x^2+2\right)-4\sqrt{3}x\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)-4\sqrt{3}x\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)+6x\log\left(x^2+x+1\right)-6x\log\left(x^2-x+1\right)-48}{48x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(x^8+x^4+1),x, algorithm="fricas")
```

```
[Out] 1/48*(4*sqrt(6)*sqrt(3)*sqrt(2)*x*arctan(-1/3*sqrt(6)*sqrt(3)*sqrt(2)*x + 1
/3*sqrt(6)*sqrt(3)*sqrt(sqrt(6)*sqrt(2)*x + 2*x^2 + 2) - sqrt(3)) + 4*sqrt(
6)*sqrt(3)*sqrt(2)*x*arctan(-1/3*sqrt(6)*sqrt(3)*sqrt(2)*x + 1/3*sqrt(6)*sq
rt(3)*sqrt(-sqrt(6)*sqrt(2)*x + 2*x^2 + 2) + sqrt(3)) + sqrt(6)*sqrt(2)*x*l
og(sqrt(6)*sqrt(2)*x + 2*x^2 + 2) - sqrt(6)*sqrt(2)*x*log(-sqrt(6)*sqrt(2)*
x + 2*x^2 + 2) - 4*sqrt(3)*x*arctan(1/3*sqrt(3)*(2*x + 1)) - 4*sqrt(3)*x*ar
ctan(1/3*sqrt(3)*(2*x - 1)) + 6*x*log(x^2 + x + 1) - 6*x*log(x^2 - x + 1) -
 48)/x
```

giac [A] time = 0.33, size = 113, normalized size = 0.78

$$-\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)-\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)+\frac{1}{24}\sqrt{3}\log\left(x^2+\sqrt{3}x+1\right)-\frac{1}{24}\sqrt{3}\log\left(x^2-\sqrt{3}x+1\right)-\frac{1}{x}-\frac{1}{4}\arctan\left(2x+\sqrt{3}\right)-\frac{1}{4}\arctan\left(2x-\sqrt{3}\right)+\frac{1}{8}\log\left(x^2+x+1\right)-\frac{1}{8}\log\left(x^2-x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(x^8+x^4+1),x, algorithm="giac")
```

```
[Out] -1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 1/12*sqrt(3)*arctan(1/3*sqrt(
3)*(2*x - 1)) + 1/24*sqrt(3)*log(x^2 + sqrt(3)*x + 1) - 1/24*sqrt(3)*log(x^
2 - sqrt(3)*x + 1) - 1/x - 1/4*arctan(2*x + sqrt(3)) - 1/4*arctan(2*x - sqr
t(3)) + 1/8*log(x^2 + x + 1) - 1/8*log(x^2 - x + 1)
```

maple [A] time = 0.01, size = 114, normalized size = 0.79

$$-\frac{\sqrt{3}\arctan\left(\frac{(2x+1)\sqrt{3}}{3}\right)}{12}-\frac{\sqrt{3}\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{12}-\frac{\arctan\left(2x-\sqrt{3}\right)}{4}-\frac{\arctan\left(2x+\sqrt{3}\right)}{4}-\frac{\sqrt{3}\ln\left(x^2-\sqrt{3}x+1\right)}{24}+\frac{\sqrt{3}\ln\left(x^2+\sqrt{3}x+1\right)}{24}-\frac{\ln\left(x^2-x+1\right)}{8}+\frac{\ln\left(x^2+x+1\right)}{8}-\frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(x^8+x^4+1),x)
```

```
[Out] 1/8*ln(x^2+x+1)-1/12*3^(1/2)*arctan(1/3*(2*x+1)*3^(1/2))-1/x-1/24*3^(1/2)*l
n(x^2-3^(1/2)*x+1)-1/4*arctan(2*x-3^(1/2))+1/24*3^(1/2)*ln(x^2+3^(1/2)*x+1)
-1/4*arctan(2*x+3^(1/2))-1/8*ln(x^2-x+1)-1/12*3^(1/2)*arctan(1/3*(2*x-1)*3^
(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)-\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)-\frac{1}{x}-\frac{1}{2}\int\frac{x^2}{x^4-x^2+1}\,dx+\frac{1}{8}\log\left(x^2+x+1\right)-\frac{1}{8}\log\left(x^2-x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(x^8+x^4+1),x, algorithm="maxima")
```

```
[Out] -1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 1/12*sqrt(3)*arctan(1/3*sqrt(
3)*(2*x - 1)) - 1/x - 1/2*integrate(x^2/(x^4 - x^2 + 1), x) + 1/8*log(x^2 +
 x + 1) - 1/8*log(x^2 - x + 1)
```

mupad [B] time = 0.05, size = 102, normalized size = 0.70

$$\operatorname{atan}\left(\frac{2x}{-1+\sqrt{3}\,1\mathrm{i}}\right)\left(\frac{1}{4}+\frac{\sqrt{3}\,1\mathrm{i}}{12}\right)+\operatorname{atan}\left(\frac{2x}{1+\sqrt{3}\,1\mathrm{i}}\right)\left(-\frac{1}{4}+\frac{\sqrt{3}\,1\mathrm{i}}{12}\right)-\operatorname{atan}\left(\frac{x\,2\mathrm{i}}{-1+\sqrt{3}\,1\mathrm{i}}\right)\left(\frac{\sqrt{3}}{12}-\frac{1}{4}\mathrm{i}\right)-\operatorname{atan}\left(\frac{x\,2\mathrm{i}}{1+\sqrt{3}\,1\mathrm{i}}\right)\left(\frac{\sqrt{3}}{12}+\frac{1}{4}\mathrm{i}\right)-\frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(x^4 + x^8 + 1)),x)
```

```
[Out] atan((2*x)/(3^(1/2)*1i - 1))*((3^(1/2)*1i)/12 + 1/4) + atan((2*x)/(3^(1/2)*
1i + 1))*((3^(1/2)*1i)/12 - 1/4) - atan((x*2i)/(3^(1/2)*1i - 1))*(3^(1/2)/1
2 - 1i/4) - atan((x*2i)/(3^(1/2)*1i + 1))*(3^(1/2)/12 + 1i/4) - 1/x
```

sympy [C] time = 0.74, size = 218, normalized size = 1.50

$$\left(-\frac{1}{8}-\frac{\sqrt{3}i}{24}\right)\log\left(x-442368\left(-\frac{1}{8}-\frac{\sqrt{3}i}{24}\right)^{7}-384\left(-\frac{1}{8}-\frac{\sqrt{3}i}{24}\right)^{3}\right)+\left(-\frac{1}{8}+\frac{\sqrt{3}i}{24}\right)\log\left(x-384\left(-\frac{1}{8}+\frac{\sqrt{3}i}{24}\right)^{3}-442368\left(-\frac{1}{8}+\frac{\sqrt{3}i}{24}\right)^{7}\right)+\left(\frac{1}{8}-\frac{\sqrt{3}i}{24}\right)\log\left(x-442368\left(\frac{1}{8}-\frac{\sqrt{3}i}{24}\right)^{7}-384\left(\frac{1}{8}-\frac{\sqrt{3}i}{24}\right)^{3}\right)+\left(\frac{1}{8}+\frac{\sqrt{3}i}{24}\right)\log\left(x-384\left(\frac{1}{8}+\frac{\sqrt{3}i}{24}\right)^{3}-442368\left(\frac{1}{8}+\frac{\sqrt{3}i}{24}\right)^{7}\right)+\operatorname{RootSum}\left(2304t^{4}+48t^{2}+1,\left(t\mapsto t\log\left(-442368t^{7}-384t^{3}+x\right)\right)\right)-\frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(x**8+x**4+1),x)
```

```
[Out] (-1/8 - sqrt(3)*I/24)*log(x - 442368*(-1/8 - sqrt(3)*I/24)**7 - 384*(-1/8 -
 sqrt(3)*I/24)**3) + (-1/8 + sqrt(3)*I/24)*log(x - 384*(-1/8 + sqrt(3)*I/24
)**3 - 442368*(-1/8 + sqrt(3)*I/24)**7) + (1/8 - sqrt(3)*I/24)*log(x - 4423
68*(1/8 - sqrt(3)*I/24)**7 - 384*(1/8 - sqrt(3)*I/24)**3) + (1/8 + sqrt(3)*
I/24)*log(x - 384*(1/8 + sqrt(3)*I/24)**3 - 442368*(1/8 + sqrt(3)*I/24)**7)
 + RootSum(2304*_t**4 + 48*_t**2 + 1, Lambda(_t, _t*log(-442368*_t**7 - 384
*_t**3 + x))) - 1/x
```

3.286 $\int \frac{1}{x^4(1+x^4+x^8)} \, dx$

Optimal. Leaf size=147

$$-\frac{1}{3x^3}+\frac{1}{8}\log\left(x^2-x+1\right)-\frac{1}{8}\log\left(x^2+x+1\right)+\frac{\log\left(x^2-\sqrt{3}x+1\right)}{8\sqrt{3}}-\frac{\log\left(x^2+\sqrt{3}x+1\right)}{8\sqrt{3}}+\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{4\sqrt{3}}+\frac{1}{4}\tan^{-1}$$

Rubi [A] time = 0.10, antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 7, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {1368, 1419, 1094, 634, 618, 204, 628}

$$-\frac{1}{3x^3}+\frac{1}{8}\log\left(x^2-x+1\right)-\frac{1}{8}\log\left(x^2+x+1\right)+\frac{\log\left(x^2-\sqrt{3}x+1\right)}{8\sqrt{3}}-\frac{\log\left(x^2+\sqrt{3}x+1\right)}{8\sqrt{3}}+\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{4\sqrt{3}}+\frac{1}{4}\tan^{-1}\left(\sqrt{3}-2x\right)-\frac{\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{4\sqrt{3}}-\frac{1}{4}\tan^{-1}\left(2x+\sqrt{3}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(1 + x^4 + x^8)),x]
```

```
[Out] -1/(3*x^3) + ArcTan[(1 - 2*x)/Sqrt[3]]/(4*Sqrt[3]) + ArcTan[Sqrt[3] - 2*x]/
4 - ArcTan[(1 + 2*x)/Sqrt[3]]/(4*Sqrt[3]) - ArcTan[Sqrt[3] + 2*x]/4 + Log[1
 - x + x^2]/8 - Log[1 + x + x^2]/8 + Log[1 - Sqrt[3]*x + x^2]/(8*Sqrt[3]) -
 Log[1 + Sqrt[3]*x + x^2]/(8*Sqrt[3])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1094

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[a/
c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int[(r - x)/(q - r*x
 + x^2), x], x] + Dist[1/(2*c*q*r), Int[(r + x)/(q + r*x + x^2), x], x]]] /
; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[b^2 - 4*a*c]
```

Rule 1368

```
Int[((d_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_
Symbol] :> Simp[((d*x)^(m + 1)*(a + b*x^n + c*x^(2*n))^(p + 1))/(a*d*(m + 1
```

```
)), x] - Dist[1/(a*d^n*(m + 1)), Int[(d*x)^(m + n)*(b*(m + n*(p + 1) + 1) +
 c*(m + 2*n*(p + 1) + 1)*x^n)*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a
, b, c, d, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && L
tQ[m, -1] && IntegerQ[p]
```

Rule 1419

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[(2*d)/e - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e +
 q*x^(n/2) + x^n, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x^(n/2) + x
^n, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4
*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && IGtQ[n/2, 0] && (GtQ[(2*d)/e - b/c, 0]
 || ( !LtQ[(2*d)/e - b/c, 0] && EqQ[d, e*Rt[a/c, 2]]))
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^4\left(1+x^4+x^8\right)}\,dx &= -\frac{1}{3x^3}+\frac{1}{3}\int \frac{-3-3x^4}{1+x^4+x^8}\,dx\\
&= -\frac{1}{3x^3}-\frac{1}{2}\int \frac{1}{1-x^2+x^4}\,dx-\frac{1}{2}\int \frac{1}{1+x^2+x^4}\,dx\\
&= -\frac{1}{3x^3}-\frac{1}{4}\int \frac{1-x}{1-x+x^2}\,dx-\frac{1}{4}\int \frac{1+x}{1+x+x^2}\,dx-\frac{\int \frac{\sqrt{3}-x}{1-\sqrt{3}x+x^2}\,dx}{4\sqrt{3}}-\frac{\int \frac{\sqrt{3}+x}{1+\sqrt{3}x+x^2}\,dx}{4\sqrt{3}}\\
&= -\frac{1}{3x^3}-\frac{1}{8}\int \frac{1}{1-x+x^2}\,dx+\frac{1}{8}\int \frac{-1+2x}{1-x+x^2}\,dx-\frac{1}{8}\int \frac{1}{1+x+x^2}\,dx-\frac{1}{8}\int \frac{1+2x}{1+x+
\\
&= -\frac{1}{3x^3}+\frac{1}{8}\log\left(1-x+x^2\right)-\frac{1}{8}\log\left(1+x+x^2\right)+\frac{\log\left(1-\sqrt{3}x+x^2\right)}{8\sqrt{3}}-\frac{\log\left(1+\sqrt{3}\right.}{8\sqrt{3}}\\
&= -\frac{1}{3x^3}+\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{4\sqrt{3}}+\frac{1}{4}\tan^{-1}\left(\sqrt{3}-2x\right)-\frac{\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)}{4\sqrt{3}}-\frac{1}{4}\tan^{-1}\left(\sqrt{3}+2x\right)+\frac{1}{8}
\end{aligned}$$

Mathematica [C] time = 0.29, size = 148, normalized size = 1.01

$$\frac{1}{24}\left(-\frac{8}{x^3}+3\log\left(x^2-x+1\right)-3\log\left(x^2+x+1\right)-\frac{4i\tan^{-1}\left(\frac{1}{2}\left(1-i\sqrt{3}\right)x\right)}{\sqrt{\frac{1}{6}i\left(\sqrt{3}+i\right)}}+\frac{4i\tan^{-1}\left(\frac{1}{2}\left(1+i\sqrt{3}\right)x\right)}{\sqrt{-\frac{1}{6}i\left(\sqrt{3}-i\right)}}-2\sqrt{3}\tan^{-1}\left(\frac{2x-1}{\sqrt{3}}\right)-2\sqrt{3}\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(x^4*(1 + x^4 + x^8)),x]
```

```
[Out] (-8/x^3 - ((4*I)*ArcTan[((1 - I*Sqrt[3])*x)/2])/Sqrt[(I/6)*(I + Sqrt[3])] +
 ((4*I)*ArcTan[((1 + I*Sqrt[3])*x)/2])/Sqrt[(-1/6*I)*(-I + Sqrt[3])] - 2*Sq
rt[3]*ArcTan[(-1 + 2*x)/Sqrt[3]] - 2*Sqrt[3]*ArcTan[(1 + 2*x)/Sqrt[3]] + 3*
Log[1 - x + x^2] - 3*Log[1 + x + x^2])/24
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^4\left(1+x^4+x^8\right)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x^4*(1 + x^4 + x^8)),x]
```

```
[Out] IntegrateAlgebraic[1/(x^4*(1 + x^4 + x^8)), x]
```

fricas [B] time = 1.23, size = 240, normalized size = 1.63

$$\frac{4\sqrt{6}\sqrt{3}\sqrt{2}x^{3}\arctan\left(-\frac{1}{3}\sqrt{6}\sqrt{3}\sqrt{2}x+\frac{1}{3}\sqrt{6}\sqrt{3}\sqrt{\sqrt{6}\sqrt{2}x+2x^{2}+2}-\sqrt{3}\right)+4\sqrt{6}\sqrt{3}\sqrt{2}x^{3}\arctan\left(-\frac{1}{3}\sqrt{6}\sqrt{3}\sqrt{2}x+\frac{1}{3}\sqrt{6}\sqrt{3}\sqrt{-\sqrt{6}\sqrt{2}x+2x^{2}+2}+\sqrt{3}\right)-\sqrt{6}\sqrt{2}x^{3}\log\left(\sqrt{6}\sqrt{2}x+2x^{2}+2\right)+\sqrt{6}\sqrt{2}x^{3}\log\left(-\sqrt{6}\sqrt{2}x+2x^{2}+2\right)-4\sqrt{3}x^{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)-4\sqrt{3}x^{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)-6x^{3}\log\left(x^{2}+x+1\right)+6x^{3}\log\left(x^{2}-x+1\right)-16}{48x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(x^8+x^4+1),x, algorithm="fricas")
```

```
[Out] 1/48*(4*sqrt(6)*sqrt(3)*sqrt(2)*x^3*arctan(-1/3*sqrt(6)*sqrt(3)*sqrt(2)*x +
 1/3*sqrt(6)*sqrt(3)*sqrt(sqrt(6)*sqrt(2)*x + 2*x^2 + 2) - sqrt(3)) + 4*sqr
t(6)*sqrt(3)*sqrt(2)*x^3*arctan(-1/3*sqrt(6)*sqrt(3)*sqrt(2)*x + 1/3*sqrt(6
)*sqrt(3)*sqrt(-sqrt(6)*sqrt(2)*x + 2*x^2 + 2) + sqrt(3)) - sqrt(6)*sqrt(2)
*x^3*log(sqrt(6)*sqrt(2)*x + 2*x^2 + 2) + sqrt(6)*sqrt(2)*x^3*log(-sqrt(6)*
sqrt(2)*x + 2*x^2 + 2) - 4*sqrt(3)*x^3*arctan(1/3*sqrt(3)*(2*x + 1)) - 4*sq
rt(3)*x^3*arctan(1/3*sqrt(3)*(2*x - 1)) - 6*x^3*log(x^2 + x + 1) + 6*x^3*lo
g(x^2 - x + 1) - 16)/x^3
```

giac [A] time = 0.38, size = 113, normalized size = 0.77

$$-\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)-\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)-\frac{1}{24}\sqrt{3}\log\left(x^{2}+\sqrt{3}x+1\right)+\frac{1}{24}\sqrt{3}\log\left(x^{2}-\sqrt{3}x+1\right)-\frac{1}{3x^{3}}-\frac{1}{4}\arctan\left(2x+\sqrt{3}\right)-\frac{1}{4}\arctan\left(2x-\sqrt{3}\right)-\frac{1}{8}\log\left(x^{2}+x+1\right)+\frac{1}{8}\log\left(x^{2}-x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(x^8+x^4+1),x, algorithm="giac")
```

```
[Out] -1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 1/12*sqrt(3)*arctan(1/3*sqrt(
3)*(2*x - 1)) - 1/24*sqrt(3)*log(x^2 + sqrt(3)*x + 1) + 1/24*sqrt(3)*log(x^
2 - sqrt(3)*x + 1) - 1/3/x^3 - 1/4*arctan(2*x + sqrt(3)) - 1/4*arctan(2*x -
 sqrt(3)) - 1/8*log(x^2 + x + 1) + 1/8*log(x^2 - x + 1)
```

maple [A] time = 0.01, size = 114, normalized size = 0.78

$$-\frac{\sqrt{3}\,\arctan\left(\frac{(2x+1)\sqrt{3}}{3}\right)}{12}-\frac{\sqrt{3}\,\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{12}-\frac{\arctan\left(2x-\sqrt{3}\right)}{4}-\frac{\arctan\left(2x+\sqrt{3}\right)}{4}+\frac{\sqrt{3}\,\ln\left(x^{2}-\sqrt{3}\,x+1\right)}{24}-\frac{\sqrt{3}\,\ln\left(x^{2}+\sqrt{3}\,x+1\right)}{24}+\frac{\ln\left(x^{2}-x+1\right)}{8}-\frac{\ln\left(x^{2}+x+1\right)}{8}-\frac{1}{3x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(x^8+x^4+1),x)
```

```
[Out] -1/8*ln(x^2+x+1)-1/12*3^(1/2)*arctan(1/3*(2*x+1)*3^(1/2))-1/3/x^3+1/24*3^(1
/2)*ln(x^2-3^(1/2)*x+1)-1/4*arctan(2*x-3^(1/2))-1/24*3^(1/2)*ln(x^2+3^(1/2)
*x+1)-1/4*arctan(2*x+3^(1/2))+1/8*ln(x^2-x+1)-1/12*3^(1/2)*arctan(1/3*(2*x-
1)*3^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)-\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)-\frac{1}{3x^{3}}-\frac{1}{2}\int\frac{1}{x^{4}-x^{2}+1}\,dx-\frac{1}{8}\log\left(x^{2}+x+1\right)+\frac{1}{8}\log\left(x^{2}-x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(x^8+x^4+1),x, algorithm="maxima")
```

```
[Out] -1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 1/12*sqrt(3)*arctan(1/3*sqrt(
3)*(2*x - 1)) - 1/3/x^3 - 1/2*integrate(1/(x^4 - x^2 + 1), x) - 1/8*log(x^2
 + x + 1) + 1/8*log(x^2 - x + 1)
```

mupad [B] time = 0.03, size = 104, normalized size = 0.71

$$-\mathrm{atan}\left(\frac{2x}{-1+\sqrt{3}\,1\mathrm{i}}\right)\left(-\frac{1}{4}+\frac{\sqrt{3}\,1\mathrm{i}}{12}\right)-\mathrm{atan}\left(\frac{2x}{1+\sqrt{3}\,1\mathrm{i}}\right)\left(\frac{1}{4}+\frac{\sqrt{3}\,1\mathrm{i}}{12}\right)-\mathrm{atan}\left(\frac{x\,2\mathrm{i}}{-1+\sqrt{3}\,1\mathrm{i}}\right)\left(\frac{\sqrt{3}}{12}+\frac{1}{4}\mathrm{i}\right)-\mathrm{atan}\left(\frac{x\,2\mathrm{i}}{1+\sqrt{3}\,1\mathrm{i}}\right)\left(\frac{\sqrt{3}}{12}-\frac{1}{4}\mathrm{i}\right)-\frac{1}{3x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(x^4 + x^8 + 1)),x)
```

```
[Out] - atan((2*x)/(3^(1/2)*1i - 1))*((3^(1/2)*1i)/12 - 1/4) - atan((2*x)/(3^(1/2
)*1i + 1))*((3^(1/2)*1i)/12 + 1/4) - atan((x*2i)/(3^(1/2)*1i - 1))*(3^(1/2)
/12 + 1i/4) - atan((x*2i)/(3^(1/2)*1i + 1))*(3^(1/2)/12 - 1i/4) - 1/(3*x^3)
```

sympy [C] time = 0.75, size = 197, normalized size = 1.34

$$\left(\frac{1}{8}+\frac{\sqrt{3}\,i}{24}\right)\log\left(x-1-\frac{\sqrt{3}\,i}{3}-9216\left(\frac{1}{8}+\frac{\sqrt{3}\,i}{24}\right)^{5}\right)+\left(\frac{1}{8}-\frac{\sqrt{3}\,i}{24}\right)\log\left(x-1-9216\left(\frac{1}{8}-\frac{\sqrt{3}\,i}{24}\right)^{5}+\frac{\sqrt{3}\,i}{3}\right)+\left(-\frac{1}{8}+\frac{\sqrt{3}\,i}{24}\right)\log\left(x+1-\frac{\sqrt{3}\,i}{3}-9216\left(-\frac{1}{8}+\frac{\sqrt{3}\,i}{24}\right)^{5}\right)+\left(-\frac{1}{8}-\frac{\sqrt{3}\,i}{24}\right)\log\left(x+1-9216\left(-\frac{1}{8}-\frac{\sqrt{3}\,i}{24}\right)^{5}+\frac{\sqrt{3}\,i}{3}\right)+\operatorname{RootSum}\left(2304t^{4}+48t^{2}+1,\left(t\mapsto t\log\left(-9216t^{5}-8t+x\right)\right)\right)-\frac{1}{3x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(x**8+x**4+1),x)
```

```
[Out] (1/8 + sqrt(3)*I/24)*log(x - 1 - sqrt(3)*I/3 - 9216*(1/8 + sqrt(3)*I/24)**5
) + (1/8 - sqrt(3)*I/24)*log(x - 1 - 9216*(1/8 - sqrt(3)*I/24)**5 + sqrt(3)
*I/3) + (-1/8 + sqrt(3)*I/24)*log(x + 1 - sqrt(3)*I/3 - 9216*(-1/8 + sqrt(3
)*I/24)**5) + (-1/8 - sqrt(3)*I/24)*log(x + 1 - 9216*(-1/8 - sqrt(3)*I/24)*
*5 + sqrt(3)*I/3) + RootSum(2304*_t**4 + 48*_t**2 + 1, Lambda(_t, _t*log(-9
216*_t**5 - 8*_t + x))) - 1/(3*x**3)
```

3.287 $\int \frac{1}{x^6(1+x^4+x^8)}\,dx$

Optimal. Leaf size=98

$$-\frac{1}{5x^5}+\frac{\log\left(x^2-\sqrt{3}\,x+1\right)}{4\sqrt{3}}-\frac{\log\left(x^2+\sqrt{3}\,x+1\right)}{4\sqrt{3}}+\frac{1}{x}-\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}}+\frac{\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{2\sqrt{3}}$$

Rubi [A] time = 0.08, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.643, Rules used = {1368, 1504, 12, 1372, 1164, 628, 1161, 618, 204}

$$-\frac{1}{5x^5}+\frac{\log\left(x^2-\sqrt{3}\,x+1\right)}{4\sqrt{3}}-\frac{\log\left(x^2+\sqrt{3}\,x+1\right)}{4\sqrt{3}}+\frac{1}{x}-\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}}+\frac{\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{2\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^6*(1 + x^4 + x^8)),x]
```

```
[Out] -1/(5*x^5) + x^(-1) - ArcTan[(1 - 2*x)/Sqrt[3]]/(2*Sqrt[3]) + ArcTan[(1 + 2
*x)/Sqrt[3]]/(2*Sqrt[3]) + Log[1 - Sqrt[3]*x + x^2]/(4*Sqrt[3]) - Log[1 + S
qrt[3]*x + x^2]/(4*Sqrt[3])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1161

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[(2*d)/e - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2
, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (
GtQ[(2*d)/e - b/c, 0] || ( !LtQ[(2*d)/e - b/c, 0] && EqQ[d - e*Rt[a/c, 2],
0]))
```

Rule 1164

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[(-2*d)/e - b/c, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e
```

```
+ q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^
2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c
*d^2 - a*e^2, 0] &&  !GtQ[b^2 - 4*a*c, 0]
```

Rule 1368

```
Int[((d_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_
Symbol] :> Simp[((d*x)^(m + 1)*(a + b*x^n + c*x^(2*n))^(p + 1))/(a*d*(m + 1
)), x] - Dist[1/(a*d^n*(m + 1)), Int[(d*x)^(m + n)*(b*(m + n*(p + 1) + 1) +
 c*(m + 2*n*(p + 1) + 1)*x^n)*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a
, b, c, d, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && L
tQ[m, -1] && IntegerQ[p]
```

Rule 1372

```
Int[(x_)^(m_.)/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symbol] :> W
ith[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, -Dist[1/(2*c*r), Int[(x^
(m - 3*(n/2))*(q - r*x^(n/2)))/(q - r*x^(n/2) + x^n), x], x] + Dist[1/(2*c*
r), Int[(x^(m - 3*(n/2))*(q + r*x^(n/2)))/(q + r*x^(n/2) + x^n), x], x]]] /
; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n/2, 0
] && IGtQ[m, 0] && GeQ[m, (3*n)/2] && LtQ[m, 2*n] && NegQ[b^2 - 4*a*c]
```

Rule 1504

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (
c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Simp[(d*(f*x)^(m + 1)*(a + b*x^n + c*x^
(2*n))^(p + 1))/(a*f*(m + 1)), x] + Dist[1/(a*f^n*(m + 1)), Int[(f*x)^(m +
n)*(a + b*x^n + c*x^(2*n))^p*Simp[a*e*(m + 1) - b*d*(m + n*(p + 1) + 1) - c
*d*(m + 2*n*(p + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x]
 && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && LtQ[m, -1] && Inte
gerQ[p]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^6\left(1+x^4+x^8\right)}\,dx &= -\frac{1}{5x^5}+\frac{1}{5}\int \frac{-5-5x^4}{x^2\left(1+x^4+x^8\right)}\,dx\\
&= -\frac{1}{5x^5}+\frac{1}{x}-\frac{1}{5}\int -\frac{5x^6}{1+x^4+x^8}\,dx\\
&= -\frac{1}{5x^5}+\frac{1}{x}+\int \frac{x^6}{1+x^4+x^8}\,dx\\
&= -\frac{1}{5x^5}+\frac{1}{x}-\frac{1}{2}\int \frac{1-x^2}{1-x^2+x^4}\,dx+\frac{1}{2}\int \frac{1+x^2}{1+x^2+x^4}\,dx\\
&= -\frac{1}{5x^5}+\frac{1}{x}+\frac{1}{4}\int \frac{1}{1-x+x^2}\,dx+\frac{1}{4}\int \frac{1}{1+x+x^2}\,dx+\frac{\int \frac{\sqrt{3}+2x}{-1-\sqrt{3}x-x^2}\,dx}{4\sqrt{3}}+\frac{\int \frac{\sqrt{3}-2x}{-1+\sqrt{3}x-}}{4\sqrt{3}}\\
&= -\frac{1}{5x^5}+\frac{1}{x}+\frac{\log\left(1-\sqrt{3}\,x+x^2\right)}{4\sqrt{3}}-\frac{\log\left(1+\sqrt{3}\,x+x^2\right)}{4\sqrt{3}}-\frac{1}{2}\,\mathrm{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,\right.\\
&= -\frac{1}{5x^5}+\frac{1}{x}-\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}}+\frac{\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)}{2\sqrt{3}}+\frac{\log\left(1-\sqrt{3}\,x+x^2\right)}{4\sqrt{3}}-\frac{\log\left(1+\sqrt{3}\,x+x\right.}{4\sqrt{3}}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 95, normalized size = 0.97

$$\frac{1}{60}\left(-\frac{12}{x^5}+5\sqrt{3}\log\left(-x^2+\sqrt{3}x-1\right)-5\sqrt{3}\log\left(x^2+\sqrt{3}x+1\right)+\frac{60}{x}+10\sqrt{3}\tan^{-1}\left(\frac{2x-1}{\sqrt{3}}\right)+10\sqrt{3}\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^6*(1 + x^4 + x^8)),x]
```

```
[Out] (-12/x^5 + 60/x + 10*Sqrt[3]*ArcTan[(-1 + 2*x)/Sqrt[3]] + 10*Sqrt[3]*ArcTan
[(1 + 2*x)/Sqrt[3]] + 5*Sqrt[3]*Log[-1 + Sqrt[3]*x - x^2] - 5*Sqrt[3]*Log[1
 + Sqrt[3]*x + x^2])/60
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^6\left(1+x^4+x^8\right)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x^6*(1 + x^4 + x^8)),x]
```

```
[Out] IntegrateAlgebraic[1/(x^6*(1 + x^4 + x^8)), x]
```

fricas [A] time = 1.20, size = 90, normalized size = 0.92

$$\frac{10\sqrt{3}\,x^5\arctan\left(\frac{1}{3}\sqrt{3}\left(x^3+2x\right)\right)+10\sqrt{3}\,x^5\arctan\left(\frac{1}{3}\sqrt{3}\,x\right)+5\sqrt{3}\,x^5\log\left(\frac{x^4+5x^2-2\sqrt{3}\left(x^3+x\right)+1}{x^4-x^2+1}\right)+60x^4-12}{60x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(x^8+x^4+1),x, algorithm="fricas")
```

```
[Out] 1/60*(10*sqrt(3)*x^5*arctan(1/3*sqrt(3)*(x^3 + 2*x)) + 10*sqrt(3)*x^5*arcta
n(1/3*sqrt(3)*x) + 5*sqrt(3)*x^5*log((x^4 + 5*x^2 - 2*sqrt(3)*(x^3 + x) + 1
)/(x^4 - x^2 + 1)) + 60*x^4 - 12)/x^5
```

giac [A] time = 0.40, size = 100, normalized size = 1.02

$$\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)+\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)-\frac{1}{24}\sqrt{3}\log\left(x^2+\sqrt{3}x+1\right)+\frac{1}{24}\sqrt{3}\log\left(x^2-\sqrt{3}x+1\right)+\frac{5x^4-1}{5x^5}+\frac{1}{4}\arctan\left(2x+\sqrt{3}\right)+\frac{1}{4}\arctan\left(2x-\sqrt{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(x^8+x^4+1),x, algorithm="giac")
```

```
[Out] 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/6*sqrt(3)*arctan(1/3*sqrt(3)*
(2*x - 1)) - 1/24*sqrt(3)*log(x^2 + sqrt(3)*x + 1) + 1/24*sqrt(3)*log(x^2 -
 sqrt(3)*x + 1) + 1/5*(5*x^4 - 1)/x^5 + 1/4*arctan(2*x + sqrt(3)) + 1/4*arc
tan(2*x - sqrt(3))
```

maple [A] time = 0.01, size = 75, normalized size = 0.77

$$\frac{\sqrt{3}\arctan\left(\frac{(2x+1)\sqrt{3}}{3}\right)}{6}+\frac{\sqrt{3}\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{6}+\frac{\sqrt{3}\ln\left(x^2-\sqrt{3}\,x+1\right)}{12}-\frac{\sqrt{3}\ln\left(x^2+\sqrt{3}\,x+1\right)}{12}+\frac{1}{x}-\frac{1}{5x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^6/(x^8+x^4+1),x)
```

```
[Out] -1/5/x^5+1/x+1/6*3^(1/2)*arctan(1/3*(2*x+1)*3^(1/2))+1/12*3^(1/2)*ln(x^2-3^
(1/2)*x+1)-1/12*3^(1/2)*ln(x^2+3^(1/2)*x+1)+1/6*3^(1/2)*arctan(1/3*(2*x-1)*
3^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\,(2x+1)\right)+\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\,(2x-1)\right)+\frac{5x^4-1}{5x^5}+\frac{1}{2}\int\frac{x^2-1}{x^4-x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(x^8+x^4+1),x, algorithm="maxima")
```

```
[Out] 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/6*sqrt(3)*arctan(1/3*sqrt(3)*
(2*x - 1)) + 1/5*(5*x^4 - 1)/x^5 + 1/2*integrate((x^2 - 1)/(x^4 - x^2 + 1),
 x)
```

mupad [B] time = 0.04, size = 52, normalized size = 0.53

$$\frac{x^4-\frac{1}{5}}{x^5}-\frac{\sqrt{3}\,\operatorname{atanh}\left(\frac{2\sqrt{3}\,x}{3\left(\frac{2x^2}{3}+\frac{2}{3}\right)}\right)}{6}-\frac{\sqrt{3}\,\operatorname{atan}\left(\frac{2\sqrt{3}\,x}{3\left(\frac{2x^2}{3}-\frac{2}{3}\right)}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^6*(x^4 + x^8 + 1)),x)
```

```
[Out] (x^4 - 1/5)/x^5 - (3^(1/2)*atanh((2*3^(1/2)*x)/(3*((2*x^2)/3 + 2/3))))/6 -
(3^(1/2)*atan((2*3^(1/2)*x)/(3*((2*x^2)/3 - 2/3))))/6
```

sympy [A] time = 0.22, size = 94, normalized size = 0.96

$$\frac{\sqrt{3}\left(2\operatorname{atan}\left(\frac{\sqrt{3}\,x}{3}\right)+2\operatorname{atan}\left(\frac{\sqrt{3}\,x^3}{3}+\frac{2\sqrt{3}\,x}{3}\right)\right)}{12}+\frac{\sqrt{3}\log\left(x^2-\sqrt{3}\,x+1\right)}{12}-\frac{\sqrt{3}\log\left(x^2+\sqrt{3}\,x+1\right)}{12}+\frac{5x^4-1}{5x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**6/(x**8+x**4+1),x)
```

```
[Out] sqrt(3)*(2*atan(sqrt(3)*x/3) + 2*atan(sqrt(3)*x**3/3 + 2*sqrt(3)*x/3))/12 +
 sqrt(3)*log(x**2 - sqrt(3)*x + 1)/12 - sqrt(3)*log(x**2 + sqrt(3)*x + 1)/1
2 + (5*x**4 - 1)/(5*x**5)
```

3.288 $\int \frac{1}{x^8(1+x^4+x^8)}\,dx$

Optimal. Leaf size=154

$$-\frac{1}{7x^7}+\frac{1}{3x^3}-\frac{1}{8}\log\left(x^2-x+1\right)+\frac{1}{8}\log\left(x^2+x+1\right)+\frac{\log\left(x^2-\sqrt{3}\,x+1\right)}{8\sqrt{3}}-\frac{\log\left(x^2+\sqrt{3}\,x+1\right)}{8\sqrt{3}}+\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{4\sqrt{3}}-\frac{1}{4}\,\text{t}$$

Rubi [A] time = 0.14, antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 22, number of rules used = 10, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.714, Rules used = {1368, 1504, 12, 1373, 1127, 1161, 618, 204, 1164, 628}

$$\frac{1}{3x^3}-\frac{1}{7x^7}-\frac{1}{8}\log\left(x^2-x+1\right)+\frac{1}{8}\log\left(x^2+x+1\right)+\frac{\log\left(x^2-\sqrt{3}x+1\right)}{8\sqrt{3}}-\frac{\log\left(x^2+\sqrt{3}x+1\right)}{8\sqrt{3}}+\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{4\sqrt{3}}-\frac{1}{4}\tan^{-1}\left(\sqrt{3}-2x\right)-\frac{\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{4\sqrt{3}}+\frac{1}{4}\tan^{-1}\left(2x+\sqrt{3}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^8*(1 + x^4 + x^8)),x]
```

```
[Out] -1/(7*x^7) + 1/(3*x^3) + ArcTan[(1 - 2*x)/Sqrt[3]]/(4*Sqrt[3]) - ArcTan[Sqr
t[3] - 2*x]/4 - ArcTan[(1 + 2*x)/Sqrt[3]]/(4*Sqrt[3]) + ArcTan[Sqrt[3] + 2*
x]/4 - Log[1 - x + x^2]/8 + Log[1 + x + x^2]/8 + Log[1 - Sqrt[3]*x + x^2]/(
8*Sqrt[3]) - Log[1 + Sqrt[3]*x + x^2]/(8*Sqrt[3])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1127

```
Int[(x_)^2/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a/c, 2]}, Dist[1/2, Int[(q + x^2)/(a + b*x^2 + c*x^4), x], x] - Dist[1/2, I
nt[(q - x^2)/(a + b*x^2 + c*x^4), x], x]] /; FreeQ[{a, b, c}, x] && LtQ[b^2
 - 4*a*c, 0] && PosQ[a*c]
```

Rule 1161

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[(2*d)/e - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2
, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (
```

```
GtQ[(2*d)/e - b/c, 0] || ( !LtQ[(2*d)/e - b/c, 0] && EqQ[d - e*Rt[a/c, 2],
0]))
```

Rule 1164

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[(-2*d)/e - b/c, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e
+ q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^
2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c
*d^2 - a*e^2, 0] &&  !GtQ[b^2 - 4*a*c, 0]
```

Rule 1368

```
Int[((d_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_
Symbol] :> Simp[((d*x)^(m + 1)*(a + b*x^n + c*x^(2*n))^(p + 1))/(a*d*(m + 1
)), x] - Dist[1/(a*d^n*(m + 1)), Int[(d*x)^(m + n)*(b*(m + n*(p + 1) + 1) +
 c*(m + 2*n*(p + 1) + 1)*x^n)*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a
, b, c, d, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && L
tQ[m, -1] && IntegerQ[p]
```

Rule 1373

```
Int[(x_)^(m_.)/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symbol] :> W
ith[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*r), Int[x^(m
 - n/2)/(q - r*x^(n/2) + x^n), x], x] - Dist[1/(2*c*r), Int[x^(m - n/2)/(q
+ r*x^(n/2) + x^n), x], x]]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[n/2, 0] && IGtQ[m, 0] && GeQ[m, n/2] && LtQ[m, (3*n
)/2] && NegQ[b^2 - 4*a*c]
```

Rule 1504

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (
c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Simp[(d*(f*x)^(m + 1)*(a + b*x^n + c*x^
(2*n))^(p + 1))/(a*f*(m + 1)), x] + Dist[1/(a*f^n*(m + 1)), Int[(f*x)^(m +
n)*(a + b*x^n + c*x^(2*n))^p*Simp[a*e*(m + 1) - b*d*(m + n*(p + 1) + 1) - c
*d*(m + 2*n*(p + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x]
 && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && LtQ[m, -1] && Inte
gerQ[p]
```

Rubi steps

$$\int \frac{1}{x^8\left(1+x^4+x^8\right)}\,dx = -\frac{1}{7x^7}+\frac{1}{7}\int \frac{-7-7x^4}{x^4\left(1+x^4+x^8\right)}\,dx$$
$$= -\frac{1}{7x^7}+\frac{1}{3x^3}-\frac{1}{21}\int -\frac{21x^4}{1+x^4+x^8}\,dx$$
$$= -\frac{1}{7x^7}+\frac{1}{3x^3}+\int \frac{x^4}{1+x^4+x^8}\,dx$$
$$= -\frac{1}{7x^7}+\frac{1}{3x^3}+\frac{1}{2}\int \frac{x^2}{1-x^2+x^4}\,dx-\frac{1}{2}\int \frac{x^2}{1+x^2+x^4}\,dx$$
$$= -\frac{1}{7x^7}+\frac{1}{3x^3}-\frac{1}{4}\int \frac{1-x^2}{1-x^2+x^4}\,dx+\frac{1}{4}\int \frac{1+x^2}{1-x^2+x^4}\,dx+\frac{1}{4}\int \frac{1-x^2}{1+x^2+x^4}\,dx-\frac{1}{4}\int \frac{}{1}$$
$$= -\frac{1}{7x^7}+\frac{1}{3x^3}-\frac{1}{8}\int \frac{1+2x}{-1-x-x^2}\,dx-\frac{1}{8}\int \frac{1-2x}{-1+x-x^2}\,dx-\frac{1}{8}\int \frac{1}{1-x+x^2}\,dx-\frac{1}{8}\int \frac{}{1}$$
$$= -\frac{1}{7x^7}+\frac{1}{3x^3}-\frac{1}{8}\log\left(1-x+x^2\right)+\frac{1}{8}\log\left(1+x+x^2\right)+\frac{\log\left(1-\sqrt{3}x+x^2\right)}{8\sqrt{3}}-\frac{\log\left(1+\right.}{8}$$
$$= -\frac{1}{7x^7}+\frac{1}{3x^3}+\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{4\sqrt{3}}-\frac{1}{4}\tan^{-1}\left(\sqrt{3}-2x\right)-\frac{\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)}{4\sqrt{3}}+\frac{1}{4}\tan^{-1}\left(\sqrt{3}+2x\right)$$

Mathematica [C] time = 0.34, size = 171, normalized size = 1.11

$$-\frac{1}{7x^7}+\frac{1}{3x^3}-\frac{1}{8}\log\left(x^2-x+1\right)+\frac{1}{8}\log\left(x^2+x+1\right)+\frac{\left(\sqrt{3}+i\right)\tan^{-1}\left(\frac{1}{2}\left(1-i\sqrt{3}\right)x\right)}{2\sqrt{-6+6i\sqrt{3}}}+\frac{\left(\sqrt{3}-i\right)\tan^{-1}\left(\frac{1}{2}\left(1+i\sqrt{3}\right)x\right)}{2\sqrt{-6-6i\sqrt{3}}}-\frac{\tan^{-1}\left(\frac{2x-1}{\sqrt{3}}\right)}{4\sqrt{3}}-\frac{\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{4\sqrt{3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(x^8*(1 + x^4 + x^8)),x]
```

```
[Out] -1/7*1/x^7 + 1/(3*x^3) + ((I + Sqrt[3])*ArcTan[((1 - I*Sqrt[3])*x)/2])/(2*S
qrt[-6 + (6*I)*Sqrt[3]]) + ((-I + Sqrt[3])*ArcTan[((1 + I*Sqrt[3])*x)/2])/(
2*Sqrt[-6 - (6*I)*Sqrt[3]]) - ArcTan[(-1 + 2*x)/Sqrt[3]]/(4*Sqrt[3]) - ArcT
an[(1 + 2*x)/Sqrt[3]]/(4*Sqrt[3]) - Log[1 - x + x^2]/8 + Log[1 + x + x^2]/8
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^8\left(1+x^4+x^8\right)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x^8*(1 + x^4 + x^8)),x]
```

```
[Out] IntegrateAlgebraic[1/(x^8*(1 + x^4 + x^8)), x]
```

fricas [B] time = 1.20, size = 246, normalized size = 1.60

$$-\frac{28\sqrt{6}\sqrt{3}\sqrt{2}x^7\arctan\left(-\frac{1}{3}\sqrt{6}\sqrt{3}\sqrt{2}x+\frac{1}{3}\sqrt{6}\sqrt{3}\sqrt{\sqrt{6}\sqrt{2}x+2x^2+2}-\sqrt{3}\right)+28\sqrt{6}\sqrt{3}\sqrt{2}x^7\arctan\left(-\frac{1}{3}\sqrt{6}\sqrt{3}\sqrt{2}x+\frac{1}{3}\sqrt{6}\sqrt{3}\sqrt{-\sqrt{6}\sqrt{2}x+2x^2+2}+\sqrt{3}\right)+7\sqrt{6}\sqrt{2}x^7\log\left(\sqrt{6}\sqrt{2}x+2x^2+2\right)-7\sqrt{6}\sqrt{2}x^7\log\left(-\sqrt{6}\sqrt{2}x+2x^2+2\right)+28\sqrt{3}x^7\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)+28\sqrt{3}x^7\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)-42x^7\log\left(x^2+x+1\right)+42x^7\log\left(x^2-x+1\right)-112x^4+48}{336x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^8/(x^8+x^4+1),x, algorithm="fricas")
```

```
[Out] -1/336*(28*sqrt(6)*sqrt(3)*sqrt(2)*x^7*arctan(-1/3*sqrt(6)*sqrt(3)*sqrt(2)*
x + 1/3*sqrt(6)*sqrt(3)*sqrt(sqrt(6)*sqrt(2)*x + 2*x^2 + 2) - sqrt(3)) + 28
*sqrt(6)*sqrt(3)*sqrt(2)*x^7*arctan(-1/3*sqrt(6)*sqrt(3)*sqrt(2)*x + 1/3*sq
rt(6)*sqrt(3)*sqrt(-sqrt(6)*sqrt(2)*x + 2*x^2 + 2) + sqrt(3)) + 7*sqrt(6)*s
```

```
qrt(2)*x^7*log(sqrt(6)*sqrt(2)*x + 2*x^2 + 2) - 7*sqrt(6)*sqrt(2)*x^7*log(-
sqrt(6)*sqrt(2)*x + 2*x^2 + 2) + 28*sqrt(3)*x^7*arctan(1/3*sqrt(3)*(2*x + 1
)) + 28*sqrt(3)*x^7*arctan(1/3*sqrt(3)*(2*x - 1)) - 42*x^7*log(x^2 + x + 1)
 + 42*x^7*log(x^2 - x + 1) - 112*x^4 + 48)/x^7
```

giac [A] time = 0.43, size = 120, normalized size = 0.78

$$-\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)-\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)-\frac{1}{24}\sqrt{3}\log\left(x^2+\sqrt{3}x+1\right)+\frac{1}{24}\sqrt{3}\log\left(x^2-\sqrt{3}x+1\right)+\frac{7x^4-3}{21x^7}+\frac{1}{4}\arctan\left(2x+\sqrt{3}\right)+\frac{1}{4}\arctan\left(2x-\sqrt{3}\right)+\frac{1}{8}\log\left(x^2+x+1\right)-\frac{1}{8}\log\left(x^2-x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^8/(x^8+x^4+1),x, algorithm="giac")
```

```
[Out] -1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 1/12*sqrt(3)*arctan(1/3*sqrt(
3)*(2*x - 1)) - 1/24*sqrt(3)*log(x^2 + sqrt(3)*x + 1) + 1/24*sqrt(3)*log(x^
2 - sqrt(3)*x + 1) + 1/21*(7*x^4 - 3)/x^7 + 1/4*arctan(2*x + sqrt(3)) + 1/4
*arctan(2*x - sqrt(3)) + 1/8*log(x^2 + x + 1) - 1/8*log(x^2 - x + 1)
```

maple [A] time = 0.01, size = 119, normalized size = 0.77

$$-\frac{\sqrt{3}\arctan\left(\frac{(2x+1)\sqrt{3}}{3}\right)}{12}-\frac{\sqrt{3}\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{12}+\frac{\arctan\left(2x-\sqrt{3}\right)}{4}+\frac{\arctan\left(2x+\sqrt{3}\right)}{4}+\frac{\sqrt{3}\ln\left(x^2-\sqrt{3}x+1\right)}{24}-\frac{\sqrt{3}\ln\left(x^2+\sqrt{3}x+1\right)}{24}-\frac{\ln\left(x^2-x+1\right)}{8}+\frac{\ln\left(x^2+x+1\right)}{8}+\frac{1}{3x^3}-\frac{1}{7x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^8/(x^8+x^4+1),x)
```

```
[Out] -1/7/x^7+1/3/x^3+1/8*ln(x^2+x+1)-1/12*3^(1/2)*arctan(1/3*(2*x+1)*3^(1/2))+1
/24*3^(1/2)*ln(x^2-3^(1/2)*x+1)+1/4*arctan(2*x-3^(1/2))-1/24*3^(1/2)*ln(x^2
+3^(1/2)*x+1)+1/4*arctan(2*x+3^(1/2))-1/8*ln(x^2-x+1)-1/12*3^(1/2)*arctan(1
/3*(2*x-1)*3^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)-\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)+\frac{7x^4-3}{21x^7}+\frac{1}{2}\int\frac{x^2}{x^4-x^2+1}\,dx+\frac{1}{8}\log\left(x^2+x+1\right)-\frac{1}{8}\log\left(x^2-x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^8/(x^8+x^4+1),x, algorithm="maxima")
```

```
[Out] -1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 1/12*sqrt(3)*arctan(1/3*sqrt(
3)*(2*x - 1)) + 1/21*(7*x^4 - 3)/x^7 + 1/2*integrate(x^2/(x^4 - x^2 + 1), x
) + 1/8*log(x^2 + x + 1) - 1/8*log(x^2 - x + 1)
```

mupad [B] time = 0.03, size = 110, normalized size = 0.71

$$\frac{\frac{x^4}{3}-\frac{1}{7}}{x^7}-\mathrm{atan}\left(\frac{2x}{1+\sqrt{3}\,1\mathrm{i}}\right)\left(-\frac{1}{4}+\frac{\sqrt{3}\,1\mathrm{i}}{12}\right)-\mathrm{atan}\left(\frac{x\,2\mathrm{i}}{-1+\sqrt{3}\,1\mathrm{i}}\right)\left(\frac{\sqrt{3}}{12}-\frac{1}{4}\mathrm{i}\right)-\mathrm{atan}\left(\frac{x\,2\mathrm{i}}{1+\sqrt{3}\,1\mathrm{i}}\right)\left(\frac{\sqrt{3}}{12}+\frac{1}{4}\mathrm{i}\right)-\mathrm{atan}\left(\frac{2x}{-1+\sqrt{3}\,1\mathrm{i}}\right)\left(\frac{1}{4}+\frac{\sqrt{3}\,1\mathrm{i}}{12}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^8*(x^4 + x^8 + 1)),x)
```

```
[Out] (x^4/3 - 1/7)/x^7 - atan((2*x)/(3^(1/2)*1i + 1))*((3^(1/2)*1i)/12 - 1/4) -
atan((x*2i)/(3^(1/2)*1i - 1))*(3^(1/2)/12 - 1i/4) - atan((x*2i)/(3^(1/2)*1i
 + 1))*(3^(1/2)/12 + 1i/4) - atan((2*x)/(3^(1/2)*1i - 1))*((3^(1/2)*1i)/12
+ 1/4)
```

sympy [C] time = 0.76, size = 209, normalized size = 1.36

$$\left(\frac{1}{8}-\frac{\sqrt{3}i}{24}\right)\log\left(x-\frac{1}{2}+\frac{\sqrt{3}i}{6}-18432\left(\frac{1}{8}-\frac{\sqrt{3}i}{24}\right)^5\right)+\left(\frac{1}{8}+\frac{\sqrt{3}i}{24}\right)\log\left(x-\frac{1}{2}-18432\left(\frac{1}{8}+\frac{\sqrt{3}i}{24}\right)^5-\frac{\sqrt{3}i}{6}\right)+\left(-\frac{1}{8}-\frac{\sqrt{3}i}{24}\right)\log\left(x+\frac{1}{2}+\frac{\sqrt{3}i}{6}-18432\left(-\frac{1}{8}-\frac{\sqrt{3}i}{24}\right)^5\right)+\left(-\frac{1}{8}+\frac{\sqrt{3}i}{24}\right)\log\left(x+\frac{1}{2}-18432\left(-\frac{1}{8}+\frac{\sqrt{3}i}{24}\right)^5-\frac{\sqrt{3}i}{6}\right)+\operatorname{RootSum}\left(2304t^4+48t^2+1,\left(t\mapsto t\log\left(-18432t^5-4t+x\right)\right)\right)+\frac{7x^4-3}{21x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**8/(x**8+x**4+1),x)
```

```
[Out] (1/8 - sqrt(3)*I/24)*log(x - 1/2 + sqrt(3)*I/6 - 18432*(1/8 - sqrt(3)*I/24)
**5) + (1/8 + sqrt(3)*I/24)*log(x - 1/2 - 18432*(1/8 + sqrt(3)*I/24)**5 - s
qrt(3)*I/6) + (-1/8 - sqrt(3)*I/24)*log(x + 1/2 + sqrt(3)*I/6 - 18432*(-1/8
 - sqrt(3)*I/24)**5) + (-1/8 + sqrt(3)*I/24)*log(x + 1/2 - 18432*(-1/8 + sq
rt(3)*I/24)**5 - sqrt(3)*I/6) + RootSum(2304*_t**4 + 48*_t**2 + 1, Lambda(_
t, _t*log(-18432*_t**5 - 4*_t + x))) + (7*x**4 - 3)/(21*x**7)
```

3.289 $\int \frac{x^{11}}{1-x^4+x^8} \, dx$

Optimal. Leaf size=46

$$\frac{x^4}{4}+\frac{\tan^{-1}\left(\frac{1-2x^4}{\sqrt{3}}\right)}{4\sqrt{3}}+\frac{1}{8}\log\left(x^8-x^4+1\right)$$

Rubi [A] time = 0.04, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {1357, 703, 634, 618, 204, 628}

$$\frac{x^4}{4}+\frac{1}{8}\log\left(x^8-x^4+1\right)+\frac{\tan^{-1}\left(\frac{1-2x^4}{\sqrt{3}}\right)}{4\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^11/(1 - x^4 + x^8),x]
```

```
[Out] x^4/4 + ArcTan[(1 - 2*x^4)/Sqrt[3]]/(4*Sqrt[3]) + Log[1 - x^4 + x^8]/8
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 703

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Simp[(e*(d + e*x)^(m - 1))/(c*(m - 1)), x] + Dist[1/c, Int[((d + e*x)^
(m - 2)*Simp[c*d^2 - a*e^2 + e*(2*c*d - b*e)*x, x])/(a + b*x + c*x^2), x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && GtQ[m, 1]
```

Rule 1357

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
```

```
4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^{11}}{1-x^4+x^8}\,dx &= \frac{1}{4}\text{Subst}\left(\int \frac{x^2}{1-x+x^2}\,dx,x,x^4\right)\\
&= \frac{x^4}{4}+\frac{1}{4}\text{Subst}\left(\int \frac{-1+x}{1-x+x^2}\,dx,x,x^4\right)\\
&= \frac{x^4}{4}-\frac{1}{8}\text{Subst}\left(\int \frac{1}{1-x+x^2}\,dx,x,x^4\right)+\frac{1}{8}\text{Subst}\left(\int \frac{-1+2x}{1-x+x^2}\,dx,x,x^4\right)\\
&= \frac{x^4}{4}+\frac{1}{8}\log\left(1-x^4+x^8\right)+\frac{1}{4}\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,-1+2x^4\right)\\
&= \frac{x^4}{4}+\frac{\tan^{-1}\left(\frac{1-2x^4}{\sqrt{3}}\right)}{4\sqrt{3}}+\frac{1}{8}\log\left(1-x^4+x^8\right)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 46, normalized size = 1.00

$$\frac{x^4}{4}-\frac{\tan^{-1}\left(\frac{2x^4-1}{\sqrt{3}}\right)}{4\sqrt{3}}+\frac{1}{8}\log\left(x^8-x^4+1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^11/(1 - x^4 + x^8),x]
[Out] x^4/4 - ArcTan[(-1 + 2*x^4)/Sqrt[3]]/(4*Sqrt[3]) + Log[1 - x^4 + x^8]/8
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{11}}{1-x^4+x^8}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^11/(1 - x^4 + x^8),x]
[Out] IntegrateAlgebraic[x^11/(1 - x^4 + x^8), x]
```

fricas [A] time = 1.14, size = 37, normalized size = 0.80

$$\frac{1}{4}x^4-\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^4-1\right)\right)+\frac{1}{8}\log\left(x^8-x^4+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(x^8-x^4+1),x, algorithm="fricas")
[Out] 1/4*x^4 - 1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^4 - 1)) + 1/8*log(x^8 - x^4
+ 1)
```

giac [A] time = 0.34, size = 37, normalized size = 0.80

$$\frac{1}{4}x^4-\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^4-1\right)\right)+\frac{1}{8}\log\left(x^8-x^4+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(x^8-x^4+1),x, algorithm="giac")
```

```
[Out] 1/4*x^4 - 1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^4 - 1)) + 1/8*log(x^8 - x^4
+ 1)
```

maple [A] time = 0.01, size = 38, normalized size = 0.83

$$\frac{x^4}{4}-\frac{\sqrt{3}\ \arctan\left(\frac{\left(2x^4-1\right)\sqrt{3}}{3}\right)}{12}+\frac{\ln\left(x^8-x^4+1\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11/(x^8-x^4+1),x)
```

```
[Out] 1/4*x^4+1/8*ln(x^8-x^4+1)-1/12*3^(1/2)*arctan(1/3*(2*x^4-1)*3^(1/2))
```

maxima [A] time = 2.11, size = 37, normalized size = 0.80

$$\frac{1}{4}\,x^4-\frac{1}{12}\,\sqrt{3}\ \arctan\left(\frac{1}{3}\,\sqrt{3}\left(2\,x^4-1\right)\right)+\frac{1}{8}\,\log\left(x^8-x^4+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(x^8-x^4+1),x, algorithm="maxima")
```

```
[Out] 1/4*x^4 - 1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^4 - 1)) + 1/8*log(x^8 - x^4
+ 1)
```

mupad [B] time = 0.05, size = 39, normalized size = 0.85

$$\frac{\ln\left(x^8-x^4+1\right)}{8}+\frac{\sqrt{3}\ \mathrm{atan}\left(\frac{\sqrt{3}}{3}-\frac{2\,\sqrt{3}\,x^4}{3}\right)}{12}+\frac{x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11/(x^8 - x^4 + 1),x)
```

```
[Out] log(x^8 - x^4 + 1)/8 + (3^(1/2)*atan(3^(1/2)/3 - (2*3^(1/2)*x^4)/3))/12 + x
^4/4
```

sympy [A] time = 0.14, size = 42, normalized size = 0.91

$$\frac{x^4}{4}+\frac{\log\left(x^8-x^4+1\right)}{8}-\frac{\sqrt{3}\ \mathrm{atan}\left(\frac{2\,\sqrt{3}\,x^4}{3}-\frac{\sqrt{3}}{3}\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**11/(x**8-x**4+1),x)
```

```
[Out] x**4/4 + log(x**8 - x**4 + 1)/8 - sqrt(3)*atan(2*sqrt(3)*x**4/3 - sqrt(3)/3
)/12
```

3.290 $\int \frac{x^9}{1-x^4+x^8} \, dx$

Optimal. Leaf size=57

$$\frac{x^2}{2}+\frac{\log\left(x^4-\sqrt{3}x^2+1\right)}{4\sqrt{3}}-\frac{\log\left(x^4+\sqrt{3}x^2+1\right)}{4\sqrt{3}}$$

Rubi [A] time = 0.04, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1359, 1122, 1164, 628}

$$\frac{x^2}{2}+\frac{\log\left(x^4-\sqrt{3}x^2+1\right)}{4\sqrt{3}}-\frac{\log\left(x^4+\sqrt{3}x^2+1\right)}{4\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^9/(1 - x^4 + x^8),x]
```

```
[Out] x^2/2 + Log[1 - Sqrt[3]*x^2 + x^4]/(4*Sqrt[3]) - Log[1 + Sqrt[3]*x^2 + x^4]
/(4*Sqrt[3])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1122

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Simp[(d^3*(d*x)^(m - 3)*(a + b*x^2 + c*x^4)^(p + 1))/(c*(m + 4*p + 1)),
 x] - Dist[d^4/(c*(m + 4*p + 1)), Int[(d*x)^(m - 4)*Simp[a*(m - 3) + b*(m +
 2*p - 1)*x^2, x]*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x
] && NeQ[b^2 - 4*a*c, 0] && GtQ[m, 3] && NeQ[m + 4*p + 1, 0] && IntegerQ[2*
p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1164

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[(-2*d)/e - b/c, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e
+ q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^
2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c
*d^2 - a*e^2, 0] &&  !GtQ[b^2 - 4*a*c, 0]
```

Rule 1359

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*
x^(n/k) + c*x^((2*n)/k))^p, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, c, p
}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\int \frac{x^9}{1-x^4+x^8}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{x^4}{1-x^2+x^4}\,dx, x, x^2\right)$$
$$= \frac{x^2}{2} - \frac{1}{2}\text{Subst}\left(\int \frac{1-x^2}{1-x^2+x^4}\,dx, x, x^2\right)$$
$$= \frac{x^2}{2} + \frac{\text{Subst}\left(\int \frac{\sqrt{3}+2x}{-1-\sqrt{3}x-x^2}\,dx, x, x^2\right)}{4\sqrt{3}} + \frac{\text{Subst}\left(\int \frac{\sqrt{3}-2x}{-1+\sqrt{3}x-x^2}\,dx, x, x^2\right)}{4\sqrt{3}}$$
$$= \frac{x^2}{2} + \frac{\log\left(1-\sqrt{3}x^2+x^4\right)}{4\sqrt{3}} - \frac{\log\left(1+\sqrt{3}x^2+x^4\right)}{4\sqrt{3}}$$

Mathematica [A] time = 0.01, size = 55, normalized size = 0.96

$$\frac{1}{12}\left(6x^2+\sqrt{3}\log\left(-x^4+\sqrt{3}x^2-1\right)-\sqrt{3}\log\left(x^4+\sqrt{3}x^2+1\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^9/(1 - x^4 + x^8),x]
[Out] (6*x^2 + Sqrt[3]*Log[-1 + Sqrt[3]*x^2 - x^4] - Sqrt[3]*Log[1 + Sqrt[3]*x^2
+ x^4])/12
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^9}{1-x^4+x^8}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^9/(1 - x^4 + x^8),x]
[Out] IntegrateAlgebraic[x^9/(1 - x^4 + x^8), x]
```

fricas [A] time = 1.17, size = 47, normalized size = 0.82

$$\frac{1}{2}x^2+\frac{1}{12}\sqrt{3}\log\left(\frac{x^8+5x^4-2\sqrt{3}\left(x^6+x^2\right)+1}{x^8-x^4+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(x^8-x^4+1),x, algorithm="fricas")
[Out] 1/2*x^2 + 1/12*sqrt(3)*log((x^8 + 5*x^4 - 2*sqrt(3)*(x^6 + x^2) + 1)/(x^8 -
 x^4 + 1))
```

giac [B] time = 0.32, size = 99, normalized size = 1.74

$$\frac{1}{2}x^2+\frac{1}{4}\left(x^4-1\right)\arctan\left(2x^2+\sqrt{3}\right)+\frac{1}{4}\left(x^4-1\right)\arctan\left(2x^2-\sqrt{3}\right)+\frac{1}{24}\left(\sqrt{3}x^4-\sqrt{3}\right)\log\left(x^4+\sqrt{3}x^2+1\right)-\frac{1}{24}\left(\sqrt{3}x^4-\sqrt{3}\right)\log\left(x^4-\sqrt{3}x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(x^8-x^4+1),x, algorithm="giac")
[Out] 1/2*x^2 + 1/4*(x^4 - 1)*arctan(2*x^2 + sqrt(3)) + 1/4*(x^4 - 1)*arctan(2*x^
2 - sqrt(3)) + 1/24*(sqrt(3)*x^4 - sqrt(3))*log(x^4 + sqrt(3)*x^2 + 1) - 1/
24*(sqrt(3)*x^4 - sqrt(3))*log(x^4 - sqrt(3)*x^2 + 1)
```

maple [A] time = 0.01, size = 44, normalized size = 0.77

$$\frac{x^2}{2}+\frac{\sqrt{3}\ \ln\left(x^4-\sqrt{3}\ x^2+1\right)}{12}-\frac{\sqrt{3}\ \ln\left(x^4+\sqrt{3}\ x^2+1\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9/(x^8-x^4+1),x)
```

```
[Out] 1/2*x^2+1/12*ln(1+x^4-x^2*3^(1/2))*3^(1/2)-1/12*ln(1+x^4+x^2*3^(1/2))*3^(1/
2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{2}x^2+\int\frac{\left(x^4-1\right)x}{x^8-x^4+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(x^8-x^4+1),x, algorithm="maxima")
```

```
[Out] 1/2*x^2 + integrate((x^4 - 1)*x/(x^8 - x^4 + 1), x)
```

mupad [B] time = 1.31, size = 29, normalized size = 0.51

$$\frac{x^2}{2}-\frac{\sqrt{3}\ \mathrm{atanh}\left(\frac{2\sqrt{3}\ x^2}{9\left(\frac{2x^4}{9}+\frac{2}{9}\right)}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9/(x^8 - x^4 + 1),x)
```

```
[Out] x^2/2 - (3^(1/2)*atanh((2*3^(1/2)*x^2)/(9*((2*x^4)/9 + 2/9))))/6
```

sympy [A] time = 0.13, size = 48, normalized size = 0.84

$$\frac{x^2}{2}+\frac{\sqrt{3}\log\left(x^4-\sqrt{3}x^2+1\right)}{12}-\frac{\sqrt{3}\log\left(x^4+\sqrt{3}x^2+1\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**9/(x**8-x**4+1),x)
```

```
[Out] x**2/2 + sqrt(3)*log(x**4 - sqrt(3)*x**2 + 1)/12 - sqrt(3)*log(x**4 + sqrt(
3)*x**2 + 1)/12
```

3.291 $\int \frac{x^7}{1-x^4+x^8}\,dx$

Optimal. Leaf size=39

$$\frac{1}{8}\log\left(x^8-x^4+1\right)-\frac{\tan^{-1}\left(\frac{1-2x^4}{\sqrt{3}}\right)}{4\sqrt{3}}$$

Rubi [A] time = 0.03, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {1357, 634, 618, 204, 628}

$$\frac{1}{8}\log\left(x^8-x^4+1\right)-\frac{\tan^{-1}\left(\frac{1-2x^4}{\sqrt{3}}\right)}{4\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^7/(1 - x^4 + x^8),x]
```

```
[Out] -ArcTan[(1 - 2*x^4)/Sqrt[3]]/(4*Sqrt[3]) + Log[1 - x^4 + x^8]/8
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1357

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x^7}{1-x^4+x^8}\,dx &= \frac{1}{4}\,\text{Subst}\left(\int \frac{x}{1-x+x^2}\,dx,x,x^4\right)\\ &= \frac{1}{8}\,\text{Subst}\left(\int \frac{1}{1-x+x^2}\,dx,x,x^4\right)+\frac{1}{8}\,\text{Subst}\left(\int \frac{-1+2x}{1-x+x^2}\,dx,x,x^4\right)\\ &= \frac{1}{8}\log\left(1-x^4+x^8\right)-\frac{1}{4}\,\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,-1+2x^4\right)\\ &= -\frac{\tan^{-1}\left(\frac{1-2x^4}{\sqrt{3}}\right)}{4\sqrt{3}}+\frac{1}{8}\log\left(1-x^4+x^8\right)\end{aligned}$$

Mathematica [A] time = 0.01, size = 39, normalized size = 1.00

$$\frac{\tan^{-1}\left(\frac{2x^4-1}{\sqrt{3}}\right)}{4\sqrt{3}}+\frac{1}{8}\log\left(x^8-x^4+1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7/(1 - x^4 + x^8),x]
[Out] ArcTan[(-1 + 2*x^4)/Sqrt[3]]/(4*Sqrt[3]) + Log[1 - x^4 + x^8]/8
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^7}{1-x^4+x^8}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^7/(1 - x^4 + x^8),x]
[Out] IntegrateAlgebraic[x^7/(1 - x^4 + x^8), x]
```

fricas [A] time = 1.16, size = 32, normalized size = 0.82

$$\frac{1}{12}\,\sqrt{3}\,\arctan\left(\frac{1}{3}\,\sqrt{3}\left(2\,x^4-1\right)\right)+\frac{1}{8}\,\log\left(x^8-x^4+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(x^8-x^4+1),x, algorithm="fricas")
[Out] 1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^4 - 1)) + 1/8*log(x^8 - x^4 + 1)
```

giac [A] time = 0.42, size = 32, normalized size = 0.82

$$\frac{1}{12}\,\sqrt{3}\,\arctan\left(\frac{1}{3}\,\sqrt{3}\left(2\,x^4-1\right)\right)+\frac{1}{8}\,\log\left(x^8-x^4+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(x^8-x^4+1),x, algorithm="giac")
[Out] 1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^4 - 1)) + 1/8*log(x^8 - x^4 + 1)
```

maple [A] time = 0.00, size = 33, normalized size = 0.85

$$\frac{\sqrt{3}\,\arctan\left(\frac{\left(2x^4-1\right)\sqrt{3}}{3}\right)}{12}+\frac{\ln\left(x^8-x^4+1\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(x^8-x^4+1),x)
[Out] 1/8*ln(x^8-x^4+1)+1/12*3^(1/2)*arctan(1/3*(2*x^4-1)*3^(1/2))
```

maxima [A] time = 1.97, size = 32, normalized size = 0.82

$$\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^4-1\right)\right)+\frac{1}{8}\log\left(x^8-x^4+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(x^8-x^4+1),x, algorithm="maxima")
[Out] 1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^4 - 1)) + 1/8*log(x^8 - x^4 + 1)
```

mupad [B] time = 1.28, size = 34, normalized size = 0.87

$$\frac{\ln\left(x^8-x^4+1\right)}{8}-\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{\sqrt{3}}{3}-\frac{2\sqrt{3}\,x^4}{3}\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(x^8 - x^4 + 1),x)
[Out] log(x^8 - x^4 + 1)/8 - (3^(1/2)*atan(3^(1/2)/3 - (2*3^(1/2)*x^4)/3))/12
```

sympy [A] time = 0.14, size = 37, normalized size = 0.95

$$\frac{\log\left(x^8-x^4+1\right)}{8}+\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x^4}{3}-\frac{\sqrt{3}}{3}\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7/(x**8-x**4+1),x)
[Out] log(x**8 - x**4 + 1)/8 + sqrt(3)*atan(2*sqrt(3)*x**4/3 - sqrt(3)/3)/12
```

3.292 $\int \frac{x^5}{1-x^4+x^8} \, dx$

Optimal. Leaf size=82

$$-\frac{1}{4}\tan^{-1}\left(\sqrt{3}-2x^2\right)+\frac{1}{4}\tan^{-1}\left(2x^2+\sqrt{3}\right)+\frac{\log\left(x^4-\sqrt{3}x^2+1\right)}{8\sqrt{3}}-\frac{\log\left(x^4+\sqrt{3}x^2+1\right)}{8\sqrt{3}}$$

Rubi [A] time = 0.07, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {1359, 1127, 1161, 618, 204, 1164, 628}

$$\frac{\log\left(x^4-\sqrt{3}x^2+1\right)}{8\sqrt{3}}-\frac{\log\left(x^4+\sqrt{3}x^2+1\right)}{8\sqrt{3}}-\frac{1}{4}\tan^{-1}\left(\sqrt{3}-2x^2\right)+\frac{1}{4}\tan^{-1}\left(2x^2+\sqrt{3}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^5/(1 - x^4 + x^8),x]
```

```
[Out] -ArcTan[Sqrt[3] - 2*x^2]/4 + ArcTan[Sqrt[3] + 2*x^2]/4 + Log[1 - Sqrt[3]*x^
2 + x^4]/(8*Sqrt[3]) - Log[1 + Sqrt[3]*x^2 + x^4]/(8*Sqrt[3])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1127

```
Int[(x_)^2/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a/c, 2]}, Dist[1/2, Int[(q + x^2)/(a + b*x^2 + c*x^4), x], x] - Dist[1/2, I
nt[(q - x^2)/(a + b*x^2 + c*x^4), x], x]] /; FreeQ[{a, b, c}, x] && LtQ[b^2
 - 4*a*c, 0] && PosQ[a*c]
```

Rule 1161

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[(2*d)/e - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2
, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (
GtQ[(2*d)/e - b/c, 0] || ( !LtQ[(2*d)/e - b/c, 0] && EqQ[d - e*Rt[a/c, 2],
0]))
```

Rule 1164

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[(-2*d)/e - b/c, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e
```

```
+ q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^
2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c
*d^2 - a*e^2, 0] &&  !GtQ[b^2 - 4*a*c, 0]
```

Rule 1359

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*
x^(n/k) + c*x^((2*n)/k))^p, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, c, p
}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^5}{1-x^4+x^8}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{x^2}{1-x^2+x^4}\,dx,x,x^2\right)\\
&= -\left(\frac{1}{4}\text{Subst}\left(\int \frac{1-x^2}{1-x^2+x^4}\,dx,x,x^2\right)\right)+\frac{1}{4}\text{Subst}\left(\int \frac{1+x^2}{1-x^2+x^4}\,dx,x,x^2\right)\\
&= \frac{1}{8}\text{Subst}\left(\int \frac{1}{1-\sqrt{3}\,x+x^2}\,dx,x,x^2\right)+\frac{1}{8}\text{Subst}\left(\int \frac{1}{1+\sqrt{3}\,x+x^2}\,dx,x,x^2\right)+\frac{\text{Subst}\left(\int -\right.}{}\\
&= \frac{\log\left(1-\sqrt{3}\,x^2+x^4\right)}{8\sqrt{3}}-\frac{\log\left(1+\sqrt{3}\,x^2+x^4\right)}{8\sqrt{3}}-\frac{1}{4}\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx,x,-\sqrt{3}+2x^2\right)-\\
&= -\frac{1}{4}\tan^{-1}\left(\sqrt{3}-2x^2\right)+\frac{1}{4}\tan^{-1}\left(\sqrt{3}+2x^2\right)+\frac{\log\left(1-\sqrt{3}\,x^2+x^4\right)}{8\sqrt{3}}-\frac{\log\left(1+\sqrt{3}\,x^2+x\right.}{8\sqrt{3}}
\end{aligned}$$

Mathematica [C] time = 0.13, size = 98, normalized size = 1.20

$$\frac{\sqrt{-1-i\sqrt{3}}\,\left(\sqrt{3}+i\right)\tan^{-1}\left(\frac{1}{2}\left(1-i\sqrt{3}\right)x^2\right)+\sqrt{-1+i\sqrt{3}}\,\left(\sqrt{3}-i\right)\tan^{-1}\left(\frac{1}{2}\left(1+i\sqrt{3}\right)x^2\right)}{4\sqrt{6}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^5/(1 - x^4 + x^8),x]
```

```
[Out] (Sqrt[-1 - I*Sqrt[3]]*(I + Sqrt[3])*ArcTan[((1 - I*Sqrt[3])*x^2)/2] + Sqrt[
-1 + I*Sqrt[3]]*(-I + Sqrt[3])*ArcTan[((1 + I*Sqrt[3])*x^2)/2])/(4*Sqrt[6])
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5}{1-x^4+x^8}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^5/(1 - x^4 + x^8),x]
```

```
[Out] IntegrateAlgebraic[x^5/(1 - x^4 + x^8), x]
```

fricas [B] time = 1.17, size = 171, normalized size = 2.09

$$-\frac{1}{12}\sqrt{6}\sqrt{3}\sqrt{2}\arctan\left(-\frac{1}{3}\sqrt{6}\sqrt{3}\sqrt{2}x^2+\frac{1}{3}\sqrt{6}\sqrt{3}\sqrt{2x^4+\sqrt{6}\sqrt{2}x^2+2}-\sqrt{3}\right)-\frac{1}{12}\sqrt{6}\sqrt{3}\sqrt{2}\arctan\left(-\frac{1}{3}\sqrt{6}\sqrt{3}\sqrt{2}x^2+\frac{1}{3}\sqrt{6}\sqrt{3}\sqrt{2x^4-\sqrt{6}\sqrt{2}x^2+2}+\sqrt{3}\right)-\frac{1}{48}\sqrt{6}\sqrt{2}\log\left(2x^4+\sqrt{6}\sqrt{2}x^2+2\right)+\frac{1}{48}\sqrt{6}\sqrt{2}\log\left(2x^4-\sqrt{6}\sqrt{2}x^2+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(x^8-x^4+1),x, algorithm="fricas")
```

```
[Out] -1/12*sqrt(6)*sqrt(3)*sqrt(2)*arctan(-1/3*sqrt(6)*sqrt(3)*sqrt(2)*x^2 + 1/3
*sqrt(6)*sqrt(3)*sqrt(2*x^4 + sqrt(6)*sqrt(2)*x^2 + 2) - sqrt(3)) - 1/12*sq
rt(6)*sqrt(3)*sqrt(2)*arctan(-1/3*sqrt(6)*sqrt(3)*sqrt(2)*x^2 + 1/3*sqrt(6)
*sqrt(3)*sqrt(2*x^4 - sqrt(6)*sqrt(2)*x^2 + 2) + sqrt(3)) - 1/48*sqrt(6)*sq
rt(2)*log(2*x^4 + sqrt(6)*sqrt(2)*x^2 + 2) + 1/48*sqrt(6)*sqrt(2)*log(2*x^4
 - sqrt(6)*sqrt(2)*x^2 + 2)
```

giac [A] time = 0.35, size = 76, normalized size = 0.93

$$\frac{1}{24}\sqrt{3}x^4\log\left(x^4+\sqrt{3}x^2+1\right)-\frac{1}{24}\sqrt{3}x^4\log\left(x^4-\sqrt{3}x^2+1\right)+\frac{1}{4}x^4\arctan\left(2x^2+\sqrt{3}\right)+\frac{1}{4}x^4\arctan\left(2x^2-\sqrt{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(x^8-x^4+1),x, algorithm="giac")
```

```
[Out] 1/24*sqrt(3)*x^4*log(x^4 + sqrt(3)*x^2 + 1) - 1/24*sqrt(3)*x^4*log(x^4 - sq
rt(3)*x^2 + 1) + 1/4*x^4*arctan(2*x^2 + sqrt(3)) + 1/4*x^4*arctan(2*x^2 - s
qrt(3))
```

maple [A] time = 0.01, size = 65, normalized size = 0.79

$$\frac{\arctan\left(2x^2-\sqrt{3}\right)}{4}+\frac{\arctan\left(2x^2+\sqrt{3}\right)}{4}+\frac{\sqrt{3}\,\ln\left(x^4-\sqrt{3}\,x^2+1\right)}{24}-\frac{\sqrt{3}\,\ln\left(x^4+\sqrt{3}\,x^2+1\right)}{24}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(x^8-x^4+1),x)
```

```
[Out] 1/4*arctan(2*x^2-3^(1/2))+1/4*arctan(2*x^2+3^(1/2))+1/24*3^(1/2)*ln(x^4-3^(
1/2)*x^2+1)-1/24*3^(1/2)*ln(x^4+3^(1/2)*x^2+1)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^5}{x^8-x^4+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(x^8-x^4+1),x, algorithm="maxima")
```

```
[Out] integrate(x^5/(x^8 - x^4 + 1), x)
```

mupad [B] time = 0.05, size = 53, normalized size = 0.65

$$-\mathrm{atan}\left(\frac{2\,x^2}{-1+\sqrt{3}\,1\mathrm{i}}\right)\left(\frac{1}{4}+\frac{\sqrt{3}\,1\mathrm{i}}{12}\right)-\mathrm{atan}\left(\frac{2\,x^2}{1+\sqrt{3}\,1\mathrm{i}}\right)\left(-\frac{1}{4}+\frac{\sqrt{3}\,1\mathrm{i}}{12}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(x^8 - x^4 + 1),x)
```

```
[Out] - atan((2*x^2)/(3^(1/2)*1i - 1))*((3^(1/2)*1i)/12 + 1/4) - atan((2*x^2)/(3^
(1/2)*1i + 1))*((3^(1/2)*1i)/12 - 1/4)
```

sympy [A] time = 0.21, size = 70, normalized size = 0.85

$$\frac{\sqrt{3}\log\left(x^4-\sqrt{3}x^2+1\right)}{24}-\frac{\sqrt{3}\log\left(x^4+\sqrt{3}x^2+1\right)}{24}+\frac{\operatorname{atan}\left(2x^2-\sqrt{3}\right)}{4}+\frac{\operatorname{atan}\left(2x^2+\sqrt{3}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(x**8-x**4+1),x)
```

```
[Out] sqrt(3)*log(x**4 - sqrt(3)*x**2 + 1)/24 - sqrt(3)*log(x**4 + sqrt(3)*x**2 +
 1)/24 + atan(2*x**2 - sqrt(3))/4 + atan(2*x**2 + sqrt(3))/4
```

3.293 $\int \frac{x^3}{1-x^4+x^8}\,dx$

Optimal. Leaf size=23

$$-\frac{\tan^{-1}\left(\frac{1-2x^4}{\sqrt{3}}\right)}{2\sqrt{3}}$$

Rubi [A] time = 0.02, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {1352, 618, 204}

$$-\frac{\tan^{-1}\left(\frac{1-2x^4}{\sqrt{3}}\right)}{2\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(1 - x^4 + x^8),x]
```

```
[Out] -ArcTan[(1 - 2*x^4)/Sqrt[3]]/(2*Sqrt[3])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1352

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a,
b, c, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x^3}{1-x^4+x^8}\,dx &= \frac{1}{4}\operatorname{Subst}\left(\int \frac{1}{1-x+x^2}\,dx, x, x^4\right)\\ &= -\left(\frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, -1+2x^4\right)\right)\\ &= -\frac{\tan^{-1}\left(\frac{1-2x^4}{\sqrt{3}}\right)}{2\sqrt{3}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 23, normalized size = 1.00

$$\frac{\tan^{-1}\left(\frac{2x^4-1}{\sqrt{3}}\right)}{2\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(1 - x^4 + x^8),x]
[Out] ArcTan[(-1 + 2*x^4)/Sqrt[3]]/(2*Sqrt[3])
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{1-x^4+x^8}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^3/(1 - x^4 + x^8),x]
[Out] IntegrateAlgebraic[x^3/(1 - x^4 + x^8), x]
```

fricas [A] time = 1.70, size = 18, normalized size = 0.78

$$\frac{1}{6}\,\sqrt{3}\,\arctan\left(\frac{1}{3}\,\sqrt{3}\left(2\,x^4-1\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(x^8-x^4+1),x, algorithm="fricas")
[Out] 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^4 - 1))
```

giac [A] time = 0.41, size = 18, normalized size = 0.78

$$\frac{1}{6}\,\sqrt{3}\,\arctan\left(\frac{1}{3}\,\sqrt{3}\left(2\,x^4-1\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(x^8-x^4+1),x, algorithm="giac")
[Out] 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^4 - 1))
```

maple [A] time = 0.00, size = 19, normalized size = 0.83

$$\frac{\sqrt{3}\,\arctan\left(\frac{\left(2x^4-1\right)\sqrt{3}}{3}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(x^8-x^4+1),x)
[Out] 1/6*3^(1/2)*arctan(1/3*(2*x^4-1)*3^(1/2))
```

maxima [A] time = 1.94, size = 18, normalized size = 0.78

$$\frac{1}{6}\,\sqrt{3}\,\arctan\left(\frac{1}{3}\,\sqrt{3}\left(2\,x^4-1\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(x^8-x^4+1),x, algorithm="maxima")
[Out] 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^4 - 1))
```

mupad [B] time = 1.29, size = 17, normalized size = 0.74

$$\frac{\sqrt{3}\,\mathrm{atan}\left(\sqrt{3}\,\left(\frac{2\,x^4}{3}-\frac{1}{3}\right)\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(x^8 - x^4 + 1),x)
```

```
[Out] (3^(1/2)*atan(3^(1/2)*((2*x^4)/3 - 1/3)))/6
```

sympy [A] time = 0.12, size = 26, normalized size = 1.13

$$\frac{\sqrt{3}\operatorname{atan}\left(\frac{2\sqrt{3}x^4}{3} - \frac{\sqrt{3}}{3}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(x**8-x**4+1),x)
```

```
[Out] sqrt(3)*atan(2*sqrt(3)*x**4/3 - sqrt(3)/3)/6
```

3.294 $\int \frac{x}{1-x^4+x^8} \, dx$

Optimal. Leaf size=82

$$-\frac{1}{4}\tan^{-1}\left(\sqrt{3}-2x^2\right)+\frac{1}{4}\tan^{-1}\left(2x^2+\sqrt{3}\right)-\frac{\log\left(x^4-\sqrt{3}x^2+1\right)}{8\sqrt{3}}+\frac{\log\left(x^4+\sqrt{3}x^2+1\right)}{8\sqrt{3}}$$

Rubi [A] time = 0.06, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {1359, 1094, 634, 618, 204, 628}

$$-\frac{\log\left(x^4-\sqrt{3}x^2+1\right)}{8\sqrt{3}}+\frac{\log\left(x^4+\sqrt{3}x^2+1\right)}{8\sqrt{3}}-\frac{1}{4}\tan^{-1}\left(\sqrt{3}-2x^2\right)+\frac{1}{4}\tan^{-1}\left(2x^2+\sqrt{3}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x/(1 - x^4 + x^8),x]
```

```
[Out] -ArcTan[Sqrt[3] - 2*x^2]/4 + ArcTan[Sqrt[3] + 2*x^2]/4 - Log[1 - Sqrt[3]*x^
2 + x^4]/(8*Sqrt[3]) + Log[1 + Sqrt[3]*x^2 + x^4]/(8*Sqrt[3])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1094

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[a/
c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int[(r - x)/(q - r*x
 + x^2), x], x] + Dist[1/(2*c*q*r), Int[(r + x)/(q + r*x + x^2), x], x]]] /
; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[b^2 - 4*a*c]
```

Rule 1359

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*
x^(n/k) + c*x^((2*n)/k))^p, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, c, p
}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\int \frac{x}{1-x^4+x^8}\,dx = \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{1-x^2+x^4}\,dx, x, x^2\right)$$

$$= \frac{\operatorname{Subst}\left(\int \frac{\sqrt{3}-x}{1-\sqrt{3}x+x^2}\,dx, x, x^2\right)}{4\sqrt{3}} + \frac{\operatorname{Subst}\left(\int \frac{\sqrt{3}+x}{1+\sqrt{3}x+x^2}\,dx, x, x^2\right)}{4\sqrt{3}}$$

$$= \frac{1}{8}\operatorname{Subst}\left(\int \frac{1}{1-\sqrt{3}x+x^2}\,dx, x, x^2\right) + \frac{1}{8}\operatorname{Subst}\left(\int \frac{1}{1+\sqrt{3}x+x^2}\,dx, x, x^2\right) - \frac{\operatorname{Subst}\left(\int \cdots\right.}{\cdots}$$

$$= -\frac{\log\left(1-\sqrt{3}x^2+x^4\right)}{8\sqrt{3}} + \frac{\log\left(1+\sqrt{3}x^2+x^4\right)}{8\sqrt{3}} - \frac{1}{4}\operatorname{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, -\sqrt{3}+2x^2\right) \cdots$$

$$= -\frac{1}{4}\tan^{-1}\left(\sqrt{3}-2x^2\right) + \frac{1}{4}\tan^{-1}\left(\sqrt{3}+2x^2\right) - \frac{\log\left(1-\sqrt{3}x^2+x^4\right)}{8\sqrt{3}} + \frac{\log\left(1+\sqrt{3}x^2+x \cdots\right.}{8\sqrt{3}}$$

Mathematica [C] time = 0.05, size = 83, normalized size = 1.01

$$\frac{i\left(\sqrt{-1-i\sqrt{3}}\,\tan^{-1}\left(\frac{1}{2}\left(1-i\sqrt{3}\right)x^2\right)-\sqrt{-1+i\sqrt{3}}\,\tan^{-1}\left(\frac{1}{2}\left(1+i\sqrt{3}\right)x^2\right)\right)}{2\sqrt{6}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x/(1 - x^4 + x^8),x]
[Out] ((I/2)*(Sqrt[-1 - I*Sqrt[3]]*ArcTan[((1 - I*Sqrt[3])*x^2)/2] - Sqrt[-1 + I*
Sqrt[3]]*ArcTan[((1 + I*Sqrt[3])*x^2)/2]))/Sqrt[6]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{1-x^4+x^8}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x/(1 - x^4 + x^8),x]
[Out] IntegrateAlgebraic[x/(1 - x^4 + x^8), x]
```

fricas [B] time = 1.17, size = 171, normalized size = 2.09

$$-\frac{1}{12}\sqrt{6}\sqrt{3}\sqrt{2}\arctan\left(-\frac{1}{3}\sqrt{6}\sqrt{3}\sqrt{2}x^2+\frac{1}{3}\sqrt{6}\sqrt{3}\sqrt{2x^4+\sqrt{6}\sqrt{2}x^2+2}-\sqrt{3}\right)-\frac{1}{12}\sqrt{6}\sqrt{3}\sqrt{2}\arctan\left(-\frac{1}{3}\sqrt{6}\sqrt{3}\sqrt{2}x^2+\frac{1}{3}\sqrt{6}\sqrt{3}\sqrt{2x^4-\sqrt{6}\sqrt{2}x^2+2}+\sqrt{3}\right)+\frac{1}{48}\sqrt{6}\sqrt{2}\log\left(2x^4+\sqrt{6}\sqrt{2}x^2+2\right)-\frac{1}{48}\sqrt{6}\sqrt{2}\log\left(2x^4-\sqrt{6}\sqrt{2}x^2+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^8-x^4+1),x, algorithm="fricas")
[Out] -1/12*sqrt(6)*sqrt(3)*sqrt(2)*arctan(-1/3*sqrt(6)*sqrt(3)*sqrt(2)*x^2 + 1/3
*sqrt(6)*sqrt(3)*sqrt(2*x^4 + sqrt(6)*sqrt(2)*x^2 + 2) - sqrt(3)) - 1/12*sq
rt(6)*sqrt(3)*sqrt(2)*arctan(-1/3*sqrt(6)*sqrt(3)*sqrt(2)*x^2 + 1/3*sqrt(6)
*sqrt(3)*sqrt(2*x^4 - sqrt(6)*sqrt(2)*x^2 + 2) + sqrt(3)) + 1/48*sqrt(6)*sq
rt(2)*log(2*x^4 + sqrt(6)*sqrt(2)*x^2 + 2) - 1/48*sqrt(6)*sqrt(2)*log(2*x^4
 - sqrt(6)*sqrt(2)*x^2 + 2)
```

giac [A] time = 0.41, size = 64, normalized size = 0.78

$$\frac{1}{24}\sqrt{3}\log\left(x^4+\sqrt{3}x^2+1\right)-\frac{1}{24}\sqrt{3}\log\left(x^4-\sqrt{3}x^2+1\right)+\frac{1}{4}\arctan\left(2x^2+\sqrt{3}\right)+\frac{1}{4}\arctan\left(2x^2-\sqrt{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^8-x^4+1),x, algorithm="giac")
```

```
[Out] 1/24*sqrt(3)*log(x^4 + sqrt(3)*x^2 + 1) - 1/24*sqrt(3)*log(x^4 - sqrt(3)*x^
2 + 1) + 1/4*arctan(2*x^2 + sqrt(3)) + 1/4*arctan(2*x^2 - sqrt(3))
```

maple [A] time = 0.01, size = 65, normalized size = 0.79

$$\frac{\arctan\left(2x^2-\sqrt{3}\right)}{4}+\frac{\arctan\left(2x^2+\sqrt{3}\right)}{4}-\frac{\sqrt{3}\ \ln\left(x^4-\sqrt{3}\ x^2+1\right)}{24}+\frac{\sqrt{3}\ \ln\left(x^4+\sqrt{3}\ x^2+1\right)}{24}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x^8-x^4+1),x)
```

```
[Out] 1/4*arctan(2*x^2-3^(1/2))+1/4*arctan(2*x^2+3^(1/2))-1/24*3^(1/2)*ln(x^4-3^(
1/2)*x^2+1)+1/24*3^(1/2)*ln(x^4+3^(1/2)*x^2+1)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x}{x^8-x^4+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^8-x^4+1),x, algorithm="maxima")
```

```
[Out] integrate(x/(x^8 - x^4 + 1), x)
```

mupad [B] time = 0.04, size = 53, normalized size = 0.65

$$-\mathrm{atan}\left(-\frac{x^2}{2}+\frac{\sqrt{3}\,x^2\,1\mathrm{i}}{2}\right)\left(\frac{1}{4}+\frac{\sqrt{3}\,1\mathrm{i}}{12}\right)-\mathrm{atan}\left(\frac{x^2}{2}+\frac{\sqrt{3}\,x^2\,1\mathrm{i}}{2}\right)\left(-\frac{1}{4}+\frac{\sqrt{3}\,1\mathrm{i}}{12}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x^8 - x^4 + 1),x)
```

```
[Out] - atan((3^(1/2)*x^2*1i)/2 - x^2/2)*((3^(1/2)*1i)/12 + 1/4) - atan((3^(1/2)*
x^2*1i)/2 + x^2/2)*((3^(1/2)*1i)/12 - 1/4)
```

sympy [A] time = 0.21, size = 70, normalized size = 0.85

$$-\frac{\sqrt{3}\log\left(x^4-\sqrt{3}x^2+1\right)}{24}+\frac{\sqrt{3}\log\left(x^4+\sqrt{3}x^2+1\right)}{24}+\frac{\mathrm{atan}\left(2x^2-\sqrt{3}\right)}{4}+\frac{\mathrm{atan}\left(2x^2+\sqrt{3}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x**8-x**4+1),x)
```

```
[Out] -sqrt(3)*log(x**4 - sqrt(3)*x**2 + 1)/24 + sqrt(3)*log(x**4 + sqrt(3)*x**2
+ 1)/24 + atan(2*x**2 - sqrt(3))/4 + atan(2*x**2 + sqrt(3))/4
```

3.295 $\int \frac{1}{x(1-x^4+x^8)} \, dx$

Optimal. Leaf size=41

$$-\frac{\tan^{-1}\left(\frac{1-2x^4}{\sqrt{3}}\right)}{4\sqrt{3}}-\frac{1}{8}\log\left(x^8-x^4+1\right)+\log(x)$$

Rubi [A] time = 0.04, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {1357, 705, 29, 634, 618, 204, 628}

$$-\frac{1}{8}\log\left(x^8-x^4+1\right)-\frac{\tan^{-1}\left(\frac{1-2x^4}{\sqrt{3}}\right)}{4\sqrt{3}}+\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(1 - x^4 + x^8)),x]
```

```
[Out] -ArcTan[(1 - 2*x^4)/Sqrt[3]]/(4*Sqrt[3]) + Log[x] - Log[1 - x^4 + x^8]/8
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 705

```
Int[1/(((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)), x_Symbol]
 :> Dist[e^2/(c*d^2 - b*d*e + a*e^2), Int[1/(d + e*x), x], x] + Dist[1/(c*d
^2 - b*d*e + a*e^2), Int[(c*d - b*e - c*e*x)/(a + b*x + c*x^2), x], x] /; F
reeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^
2, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1357

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x\left(1-x^4+x^8\right)}\,dx &= \frac{1}{4}\,\text{Subst}\left(\int \frac{1}{x\left(1-x+x^2\right)}\,dx,x,x^4\right)\\
&= \frac{1}{4}\,\text{Subst}\left(\int \frac{1}{x}\,dx,x,x^4\right)+\frac{1}{4}\,\text{Subst}\left(\int \frac{1-x}{1-x+x^2}\,dx,x,x^4\right)\\
&= \log(x)+\frac{1}{8}\,\text{Subst}\left(\int \frac{1}{1-x+x^2}\,dx,x,x^4\right)-\frac{1}{8}\,\text{Subst}\left(\int \frac{-1+2x}{1-x+x^2}\,dx,x,x^4\right)\\
&= \log(x)-\frac{1}{8}\log\left(1-x^4+x^8\right)-\frac{1}{4}\,\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,-1+2x^4\right)\\
&= -\frac{\tan^{-1}\left(\frac{1-2x^4}{\sqrt{3}}\right)}{4\sqrt{3}}+\log(x)-\frac{1}{8}\log\left(1-x^4+x^8\right)
\end{aligned}$$

Mathematica [C] time = 0.01, size = 55, normalized size = 1.34

$$\log(x)-\frac{1}{4}\text{RootSum}\left[\#1^8-\#1^4+1\&,\frac{\#1^4\log(x-\#1)-\log(x-\#1)}{2\#1^4-1}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(1 - x^4 + x^8)),x]
```

```
[Out] Log[x] - RootSum[1 - #1^4 + #1^8 & , (-Log[x - #1] + Log[x - #1]*#1^4)/(-1
+ 2*#1^4) & ]/4
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(1-x^4+x^8\right)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x*(1 - x^4 + x^8)),x]
```

```
[Out] IntegrateAlgebraic[1/(x*(1 - x^4 + x^8)), x]
```

fricas [A] time = 1.12, size = 34, normalized size = 0.83

$$\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^4-1\right)\right)-\frac{1}{8}\log\left(x^8-x^4+1\right)+\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x^8-x^4+1),x, algorithm="fricas")
```

```
[Out] 1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^4 - 1)) - 1/8*log(x^8 - x^4 + 1) + log
(x)
```

giac [A] time = 0.35, size = 38, normalized size = 0.93

$$\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^4-1\right)\right)-\frac{1}{8}\log\left(x^8-x^4+1\right)+\frac{1}{4}\log\left(x^4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x^8-x^4+1),x, algorithm="giac")
```

```
[Out] 1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^4 - 1)) - 1/8*log(x^8 - x^4 + 1) + 1/4
*log(x^4)
```

maple [A] time = 0.01, size = 35, normalized size = 0.85

$$\frac{\sqrt{3}\ \arctan\left(\frac{\left(2x^4-1\right)\sqrt{3}}{3}\right)}{12}+\ln(x)-\frac{\ln\left(x^8-x^4+1\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(x^8-x^4+1),x)
```

```
[Out] ln(x)-1/8*ln(x^8-x^4+1)+1/12*3^(1/2)*arctan(1/3*(2*x^4-1)*3^(1/2))
```

maxima [A] time = 1.95, size = 38, normalized size = 0.93

$$\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^4-1\right)\right)-\frac{1}{8}\log\left(x^8-x^4+1\right)+\frac{1}{4}\log\left(x^4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x^8-x^4+1),x, algorithm="maxima")
```

```
[Out] 1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^4 - 1)) - 1/8*log(x^8 - x^4 + 1) + 1/4
*log(x^4)
```

mupad [B] time = 1.29, size = 36, normalized size = 0.88

$$\ln(x)-\frac{\ln\left(x^8-x^4+1\right)}{8}-\frac{\sqrt{3}\ \mathrm{atan}\left(\frac{\sqrt{3}}{3}-\frac{2\sqrt{3}\,x^4}{3}\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(x^8 - x^4 + 1)),x)
```

```
[Out] log(x) - log(x^8 - x^4 + 1)/8 - (3^(1/2)*atan(3^(1/2)/3 - (2*3^(1/2)*x^4)/3
))/12
```

sympy [A] time = 0.16, size = 41, normalized size = 1.00

$$\log\left(x\right)-\frac{\log\left(x^8-x^4+1\right)}{8}+\frac{\sqrt{3}\ \mathrm{atan}\left(\frac{2\sqrt{3}x^4}{3}-\frac{\sqrt{3}}{3}\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x**8-x**4+1),x)
```

```
[Out] log(x) - log(x**8 - x**4 + 1)/8 + sqrt(3)*atan(2*sqrt(3)*x**4/3 - sqrt(3)/3
)/12
```

3.296 $\int \frac{1}{x^3(1-x^4+x^8)}\,dx$

Optimal. Leaf size=57

$$-\frac{1}{2x^2}-\frac{\log\left(x^4-\sqrt{3}\,x^2+1\right)}{4\sqrt{3}}+\frac{\log\left(x^4+\sqrt{3}\,x^2+1\right)}{4\sqrt{3}}$$

Rubi [A] time = 0.04, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1359, 1123, 1164, 628}

$$-\frac{1}{2x^2}-\frac{\log\left(x^4-\sqrt{3}\,x^2+1\right)}{4\sqrt{3}}+\frac{\log\left(x^4+\sqrt{3}\,x^2+1\right)}{4\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(1 - x^4 + x^8)),x]
```

```
[Out] -1/(2*x^2) - Log[1 - Sqrt[3]*x^2 + x^4]/(4*Sqrt[3]) + Log[1 + Sqrt[3]*x^2 +
 x^4]/(4*Sqrt[3])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1123

```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*x^2 + c*x^4)^(p + 1))/(a*d*(m + 1)), x] - Dis
t[1/(a*d^2*(m + 1)), Int[(d*x)^(m + 2)*(b*(m + 2*p + 3) + c*(m + 4*p + 5)*x
^2)*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b^2 -
 4*a*c, 0] && LtQ[m, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1164

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[(-2*d)/e - b/c, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e
+ q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^
2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c
*d^2 - a*e^2, 0] &&  !GtQ[b^2 - 4*a*c, 0]
```

Rule 1359

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*
x^(n/k) + c*x^((2*n)/k))^p, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, c, p
}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x^3\left(1-x^4+x^8\right)}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{x^2\left(1-x^2+x^4\right)}\,dx,x,x^2\right)\\
&= -\frac{1}{2x^2}+\frac{1}{2}\text{Subst}\left(\int \frac{1-x^2}{1-x^2+x^4}\,dx,x,x^2\right)\\
&= -\frac{1}{2x^2}-\frac{\text{Subst}\left(\int \frac{\sqrt{3}+2x}{-1-\sqrt{3}x-x^2}\,dx,x,x^2\right)}{4\sqrt{3}}-\frac{\text{Subst}\left(\int \frac{\sqrt{3}-2x}{-1+\sqrt{3}x-x^2}\,dx,x,x^2\right)}{4\sqrt{3}}\\
&= -\frac{1}{2x^2}-\frac{\log\left(1-\sqrt{3}x^2+x^4\right)}{4\sqrt{3}}+\frac{\log\left(1+\sqrt{3}x^2+x^4\right)}{4\sqrt{3}}
\end{aligned}
$$

Mathematica [A] time = 0.01, size = 55, normalized size = 0.96

$$\frac{1}{12}\left(-\frac{6}{x^2}-\sqrt{3}\log\left(-x^4+\sqrt{3}x^2-1\right)+\sqrt{3}\log\left(x^4+\sqrt{3}x^2+1\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(1 - x^4 + x^8)),x]
```

```
[Out] (-6/x^2 - Sqrt[3]*Log[-1 + Sqrt[3]*x^2 - x^4] + Sqrt[3]*Log[1 + Sqrt[3]*x^2
 + x^4])/12
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3\left(1-x^4+x^8\right)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x^3*(1 - x^4 + x^8)),x]
```

```
[Out] IntegrateAlgebraic[1/(x^3*(1 - x^4 + x^8)), x]
```

fricas [A] time = 1.24, size = 50, normalized size = 0.88

$$\frac{\sqrt{3}x^2\log\left(\frac{x^8+5x^4+2\sqrt{3}\left(x^6+x^2\right)+1}{x^8-x^4+1}\right)-6}{12x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(x^8-x^4+1),x, algorithm="fricas")
```

```
[Out] 1/12*(sqrt(3)*x^2*log((x^8 + 5*x^4 + 2*sqrt(3)*(x^6 + x^2) + 1)/(x^8 - x^4
+ 1)) - 6)/x^2
```

giac [B] time = 0.31, size = 99, normalized size = 1.74

$$-\frac{1}{4}\left(x^4-1\right)\arctan\left(2x^2+\sqrt{3}\right)-\frac{1}{4}\left(x^4-1\right)\arctan\left(2x^2-\sqrt{3}\right)-\frac{1}{24}\left(\sqrt{3}x^4-\sqrt{3}\right)\log\left(x^4+\sqrt{3}x^2+1\right)+\frac{1}{24}\left(\sqrt{3}x^4-\sqrt{3}\right)\log\left(x^4-\sqrt{3}x^2+1\right)-\frac{1}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(x^8-x^4+1),x, algorithm="giac")
```

```
[Out] -1/4*(x^4 - 1)*arctan(2*x^2 + sqrt(3)) - 1/4*(x^4 - 1)*arctan(2*x^2 - sqrt(
3)) - 1/24*(sqrt(3)*x^4 - sqrt(3))*log(x^4 + sqrt(3)*x^2 + 1) + 1/24*(sqrt(
3)*x^4 - sqrt(3))*log(x^4 - sqrt(3)*x^2 + 1) - 1/2/x^2
```

maple [A] time = 0.01, size = 44, normalized size = 0.77

$$-\frac{\sqrt{3}\,\ln\left(x^4-\sqrt{3}\,x^2+1\right)}{12}+\frac{\sqrt{3}\,\ln\left(x^4+\sqrt{3}\,x^2+1\right)}{12}-\frac{1}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(x^8-x^4+1),x)
```

```
[Out] -1/2/x^2-1/12*3^(1/2)*ln(x^4-3^(1/2)*x^2+1)+1/12*3^(1/2)*ln(x^4+3^(1/2)*x^2
+1)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{2\,x^2}-\int\frac{\left(x^4-1\right)x}{x^8-x^4+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(x^8-x^4+1),x, algorithm="maxima")
```

```
[Out] -1/2/x^2 - integrate((x^4 - 1)*x/(x^8 - x^4 + 1), x)
```

mupad [B] time = 1.27, size = 29, normalized size = 0.51

$$\frac{\sqrt{3}\,\mathrm{atanh}\left(\frac{2\,\sqrt{3}\,x^2}{9\left(\frac{2\,x^4}{9}+\frac{2}{9}\right)}\right)}{6}-\frac{1}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(x^8 - x^4 + 1)),x)
```

```
[Out] (3^(1/2)*atanh((2*3^(1/2)*x^2)/(9*((2*x^4)/9 + 2/9))))/6 - 1/(2*x^2)
```

sympy [A] time = 0.15, size = 49, normalized size = 0.86

$$-\frac{\sqrt{3}\log\left(x^4-\sqrt{3}x^2+1\right)}{12}+\frac{\sqrt{3}\log\left(x^4+\sqrt{3}x^2+1\right)}{12}-\frac{1}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(x**8-x**4+1),x)
```

```
[Out] -sqrt(3)*log(x**4 - sqrt(3)*x**2 + 1)/12 + sqrt(3)*log(x**4 + sqrt(3)*x**2
+ 1)/12 - 1/(2*x**2)
```

3.297 $\int \frac{1}{x^5(1-x^4+x^8)}\,dx$

Optimal. Leaf size=48

$$-\frac{1}{4x^4}+\frac{\tan^{-1}\left(\frac{1-2x^4}{\sqrt{3}}\right)}{4\sqrt{3}}-\frac{1}{8}\log\left(x^8-x^4+1\right)+\log(x)$$

Rubi [A] time = 0.05, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {1357, 709, 800, 634, 618, 204, 628}

$$-\frac{1}{4x^4}-\frac{1}{8}\log\left(x^8-x^4+1\right)+\frac{\tan^{-1}\left(\frac{1-2x^4}{\sqrt{3}}\right)}{4\sqrt{3}}+\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^5*(1 - x^4 + x^8)),x]
```

```
[Out] -1/(4*x^4) + ArcTan[(1 - 2*x^4)/Sqrt[3]]/(4*Sqrt[3]) + Log[x] - Log[1 - x^4
 + x^8]/8
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 709

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Simp[(e*(d + e*x)^(m + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dis
t[1/(c*d^2 - b*d*e + a*e^2), Int[((d + e*x)^(m + 1)*Simp[c*d - b*e - c*e*x,
 x])/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 -
 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[m
, -1]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 1357

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x^5\left(1-x^4+x^8\right)}\,dx &= \frac{1}{4}\text{Subst}\left(\int \frac{1}{x^2\left(1-x+x^2\right)}\,dx,x,x^4\right)\\
&= -\frac{1}{4x^4}+\frac{1}{4}\text{Subst}\left(\int \frac{1-x}{x\left(1-x+x^2\right)}\,dx,x,x^4\right)\\
&= -\frac{1}{4x^4}+\frac{1}{4}\text{Subst}\left(\int \left(\frac{1}{x}-\frac{x}{1-x+x^2}\right)dx,x,x^4\right)\\
&= -\frac{1}{4x^4}+\log(x)-\frac{1}{4}\text{Subst}\left(\int \frac{x}{1-x+x^2}\,dx,x,x^4\right)\\
&= -\frac{1}{4x^4}+\log(x)-\frac{1}{8}\text{Subst}\left(\int \frac{1}{1-x+x^2}\,dx,x,x^4\right)-\frac{1}{8}\text{Subst}\left(\int \frac{-1+2x}{1-x+x^2}\,dx,x,x^4\right)\\
&= -\frac{1}{4x^4}+\log(x)-\frac{1}{8}\log\left(1-x^4+x^8\right)+\frac{1}{4}\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,-1+2x^4\right)\\
&= -\frac{1}{4x^4}+\frac{\tan^{-1}\left(\frac{1-2x^4}{\sqrt{3}}\right)}{4\sqrt{3}}+\log(x)-\frac{1}{8}\log\left(1-x^4+x^8\right)
\end{aligned}
$$

Mathematica [C] time = 0.01, size = 51, normalized size = 1.06

$$-\frac{1}{4}\text{RootSum}\left[\#1^8-\#1^4+1\&,\frac{\#1^4\log(x-\#1)}{2\#1^4-1}\&\right]-\frac{1}{4x^4}+\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^5*(1 - x^4 + x^8)),x]
```

```
[Out] -1/4*1/x^4 + Log[x] - RootSum[1 - #1^4 + #1^8 & , (Log[x - #1]*#1^4)/(-1 +
2*#1^4) & ]/4
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^5\left(1-x^4+x^8\right)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x^5*(1 - x^4 + x^8)),x]
```

```
[Out] IntegrateAlgebraic[1/(x^5*(1 - x^4 + x^8)), x]
```

fricas [A] time = 1.22, size = 51, normalized size = 1.06

$$-\frac{2\sqrt{3}\,x^4\arctan\left(\frac{1}{3}\sqrt{3}\left(2\,x^4-1\right)\right)+3\,x^4\log\left(x^8-x^4+1\right)-24\,x^4\log(x)+6}{24\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(x^8-x^4+1),x, algorithm="fricas")
```

```
[Out] -1/24*(2*sqrt(3)*x^4*arctan(1/3*sqrt(3)*(2*x^4 - 1)) + 3*x^4*log(x^8 - x^4
+ 1) - 24*x^4*log(x) + 6)/x^4
```

giac [A] time = 0.40, size = 48, normalized size = 1.00

$$-\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\,x^4-1\right)\right)-\frac{x^4+1}{4\,x^4}-\frac{1}{8}\log\left(x^8-x^4+1\right)+\frac{1}{4}\log\left(x^4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(x^8-x^4+1),x, algorithm="giac")
```

```
[Out] -1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^4 - 1)) - 1/4*(x^4 + 1)/x^4 - 1/8*log
(x^8 - x^4 + 1) + 1/4*log(x^4)
```

maple [A] time = 0.01, size = 40, normalized size = 0.83

$$-\frac{\sqrt{3}\arctan\left(\frac{\left(2x^4-1\right)\sqrt{3}}{3}\right)}{12}+\ln(x)-\frac{\ln\left(x^8-x^4+1\right)}{8}-\frac{1}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^5/(x^8-x^4+1),x)
```

```
[Out] -1/4/x^4+ln(x)-1/8*ln(x^8-x^4+1)-1/12*3^(1/2)*arctan(1/3*(2*x^4-1)*3^(1/2))
```

maxima [A] time = 1.97, size = 43, normalized size = 0.90

$$-\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\,x^4-1\right)\right)-\frac{1}{4\,x^4}-\frac{1}{8}\log\left(x^8-x^4+1\right)+\frac{1}{4}\log\left(x^4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(x^8-x^4+1),x, algorithm="maxima")
```

```
[Out] -1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^4 - 1)) - 1/4/x^4 - 1/8*log(x^8 - x^4
 + 1) + 1/4*log(x^4)
```

mupad [B] time = 0.07, size = 41, normalized size = 0.85

$$\ln(x)-\frac{\ln\left(x^8-x^4+1\right)}{8}+\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{\sqrt{3}}{3}-\frac{2\,\sqrt{3}\,x^4}{3}\right)}{12}-\frac{1}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^5*(x^8 - x^4 + 1)),x)
```

```
[Out] log(x) - log(x^8 - x^4 + 1)/8 + (3^(1/2)*atan(3^(1/2)/3 - (2*3^(1/2)*x^4)/3
))/12 - 1/(4*x^4)
```

sympy [A] time = 0.19, size = 48, normalized size = 1.00

$$\log(x) - \frac{\log\left(x^8 - x^4 + 1\right)}{8} - \frac{\sqrt{3}\,\operatorname{atan}\left(\frac{2\sqrt{3}\,x^4}{3} - \frac{\sqrt{3}}{3}\right)}{12} - \frac{1}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**5/(x**8-x**4+1),x)
```

```
[Out] log(x) - log(x**8 - x**4 + 1)/8 - sqrt(3)*atan(2*sqrt(3)*x**4/3 - sqrt(3)/3
)/12 - 1/(4*x**4)
```

3.298 $\int \frac{1}{x^7(1-x^4+x^8)}\,dx$

Optimal. Leaf size=96

$$-\frac{1}{6x^6}-\frac{1}{2x^2}+\frac{1}{4}\tan^{-1}\left(\sqrt{3}-2x^2\right)-\frac{1}{4}\tan^{-1}\left(2x^2+\sqrt{3}\right)-\frac{\log\left(x^4-\sqrt{3}x^2+1\right)}{8\sqrt{3}}+\frac{\log\left(x^4+\sqrt{3}x^2+1\right)}{8\sqrt{3}}$$

Rubi [A] time = 0.09, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.625, Rules used = {1359, 1123, 1281, 12, 1127, 1161, 618, 204, 1164, 628}

$$-\frac{1}{2x^2}-\frac{1}{6x^6}-\frac{\log\left(x^4-\sqrt{3}x^2+1\right)}{8\sqrt{3}}+\frac{\log\left(x^4+\sqrt{3}x^2+1\right)}{8\sqrt{3}}+\frac{1}{4}\tan^{-1}\left(\sqrt{3}-2x^2\right)-\frac{1}{4}\tan^{-1}\left(2x^2+\sqrt{3}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^7*(1 - x^4 + x^8)),x]
```

```
[Out] -1/(6*x^6) - 1/(2*x^2) + ArcTan[Sqrt[3] - 2*x^2]/4 - ArcTan[Sqrt[3] + 2*x^2
]/4 - Log[1 - Sqrt[3]*x^2 + x^4]/(8*Sqrt[3]) + Log[1 + Sqrt[3]*x^2 + x^4]/(
8*Sqrt[3])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1123

```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*x^2 + c*x^4)^(p + 1))/(a*d*(m + 1)), x] - Dis
t[1/(a*d^2*(m + 1)), Int[(d*x)^(m + 2)*(b*(m + 2*p + 3) + c*(m + 4*p + 5)*x
^2)*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b^2 -
 4*a*c, 0] && LtQ[m, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1127

```
Int[(x_)^2/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a/c, 2]}, Dist[1/2, Int[(q + x^2)/(a + b*x^2 + c*x^4), x], x] - Dist[1/2, I
nt[(q - x^2)/(a + b*x^2 + c*x^4), x], x]] /; FreeQ[{a, b, c}, x] && LtQ[b^2
```

```
 - 4*a*c, 0] && PosQ[a*c]
```

Rule 1161

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[(2*d)/e - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2
, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (
GtQ[(2*d)/e - b/c, 0] || ( !LtQ[(2*d)/e - b/c, 0] && EqQ[d - e*Rt[a/c, 2],
0]))
```

Rule 1164

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[(-2*d)/e - b/c, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e
+ q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^
2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c
*d^2 - a*e^2, 0] &&  !GtQ[b^2 - 4*a*c, 0]
```

Rule 1281

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[(d*(f*x)^(m + 1)*(a + b*x^2 + c*x^4)^(p + 1)
)/(a*f*(m + 1)), x] + Dist[1/(a*f^2*(m + 1)), Int[(f*x)^(m + 2)*(a + b*x^2
+ c*x^4)^p*Simp[a*e*(m + 1) - b*d*(m + 2*p + 3) - c*d*(m + 4*p + 5)*x^2, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[m
, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1359

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*
x^(n/k) + c*x^((2*n)/k))^p, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, c, p
}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x^7\left(1-x^4+x^8\right)}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{x^4\left(1-x^2+x^4\right)}\,dx, x, x^2\right) \\
&= -\frac{1}{6x^6}+\frac{1}{6}\,\text{Subst}\left(\int \frac{3-3x^2}{x^2\left(1-x^2+x^4\right)}\,dx, x, x^2\right) \\
&= -\frac{1}{6x^6}-\frac{1}{2x^2}-\frac{1}{6}\,\text{Subst}\left(\int \frac{3x^2}{1-x^2+x^4}\,dx, x, x^2\right) \\
&= -\frac{1}{6x^6}-\frac{1}{2x^2}-\frac{1}{2}\,\text{Subst}\left(\int \frac{x^2}{1-x^2+x^4}\,dx, x, x^2\right) \\
&= -\frac{1}{6x^6}-\frac{1}{2x^2}+\frac{1}{4}\,\text{Subst}\left(\int \frac{1-x^2}{1-x^2+x^4}\,dx, x, x^2\right)-\frac{1}{4}\,\text{Subst}\left(\int \frac{1+x^2}{1-x^2+x^4}\,dx, x, x^2\right) \\
&= -\frac{1}{6x^6}-\frac{1}{2x^2}-\frac{1}{8}\,\text{Subst}\left(\int \frac{1}{1-\sqrt{3}\,x+x^2}\,dx, x, x^2\right)-\frac{1}{8}\,\text{Subst}\left(\int \frac{1}{1+\sqrt{3}\,x+x^2}\,dx,
\right. \\
&= -\frac{1}{6x^6}-\frac{1}{2x^2}-\frac{\log\left(1-\sqrt{3}\,x^2+x^4\right)}{8\sqrt{3}}+\frac{\log\left(1+\sqrt{3}\,x^2+x^4\right)}{8\sqrt{3}}+\frac{1}{4}\,\text{Subst}\left(\int \frac{1}{-1-x^2}\,d
\right. \\
&= -\frac{1}{6x^6}-\frac{1}{2x^2}+\frac{1}{4}\tan^{-1}\left(\sqrt{3}-2x^2\right)-\frac{1}{4}\tan^{-1}\left(\sqrt{3}+2x^2\right)-\frac{\log\left(1-\sqrt{3}\,x^2+x^4\right)}{8\sqrt{3}}+
\end{aligned}
$$

Mathematica [C] time = 0.02, size = 56, normalized size = 0.58

$$-\frac{1}{4}\text{RootSum}\left[\#1^8-\#1^4+1\&,\frac{\#1^2\log(x-\#1)}{2\#1^4-1}\&\right]-\frac{1}{6x^6}-\frac{1}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^7*(1 - x^4 + x^8)),x]
```

```
[Out] -1/6*1/x^6 - 1/(2*x^2) - RootSum[1 - #1^4 + #1^8 & , (Log[x - #1]*#1^2)/(-1
 + 2*#1^4) & ]/4
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^7\left(1-x^4+x^8\right)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x^7*(1 - x^4 + x^8)),x]
```

```
[Out] IntegrateAlgebraic[1/(x^7*(1 - x^4 + x^8)), x]
```

fricas [B] time = 1.33, size = 193, normalized size = 2.01

$$\frac{4\sqrt{6}\sqrt{3}\sqrt{2}x^6\arctan\left(-\frac{1}{3}\sqrt{6}\sqrt{3}\sqrt{2}x^2+\frac{1}{3}\sqrt{6}\sqrt{3}\sqrt{2x^4+\sqrt{6}\sqrt{2}x^2+2}-\sqrt{3}\right)+4\sqrt{6}\sqrt{3}\sqrt{2}x^6\arctan\left(-\frac{1}{3}\sqrt{6}\sqrt{3}\sqrt{2}x^2+\frac{1}{3}\sqrt{6}\sqrt{3}\sqrt{2x^4-\sqrt{6}\sqrt{2}x^2+2}+\sqrt{3}\right)+\sqrt{6}\sqrt{2}x^6\log\left(2x^4+\sqrt{6}\sqrt{2}x^2+2\right)-\sqrt{6}\sqrt{2}x^6\log\left(2x^4-\sqrt{6}\sqrt{2}x^2+2\right)-24x^4-8}{48x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(x^8-x^4+1),x, algorithm="fricas")
```

```
[Out] 1/48*(4*sqrt(6)*sqrt(3)*sqrt(2)*x^6*arctan(-1/3*sqrt(6)*sqrt(3)*sqrt(2)*x^2
 + 1/3*sqrt(6)*sqrt(3)*sqrt(2*x^4 + sqrt(6)*sqrt(2)*x^2 + 2) - sqrt(3)) + 4
*sqrt(6)*sqrt(3)*sqrt(2)*x^6*arctan(-1/3*sqrt(6)*sqrt(3)*sqrt(2)*x^2 + 1/3*
sqrt(6)*sqrt(3)*sqrt(2*x^4 - sqrt(6)*sqrt(2)*x^2 + 2) + sqrt(3)) + sqrt(6)*
```

```
sqrt(2)*x^6*log(2*x^4 + sqrt(6)*sqrt(2)*x^2 + 2) - sqrt(6)*sqrt(2)*x^6*log(
2*x^4 - sqrt(6)*sqrt(2)*x^2 + 2) - 24*x^4 - 8)/x^6
```

giac [A] time = 0.42, size = 56, normalized size = 0.58

$$-\frac{1}{12}\sqrt{3}x^4\log\left(x^4+\sqrt{3}x^2+1\right)+\frac{1}{12}\sqrt{3}x^4\log\left(x^4-\sqrt{3}x^2+1\right)-\frac{3x^4+1}{6x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(x^8-x^4+1),x, algorithm="giac")
```

```
[Out] -1/12*sqrt(3)*x^4*log(x^4 + sqrt(3)*x^2 + 1) + 1/12*sqrt(3)*x^4*log(x^4 - s
qrt(3)*x^2 + 1) - 1/6*(3*x^4 + 1)/x^6
```

maple [A] time = 0.01, size = 75, normalized size = 0.78

$$-\frac{\arctan\left(2x^2-\sqrt{3}\right)}{4}-\frac{\arctan\left(2x^2+\sqrt{3}\right)}{4}-\frac{\sqrt{3}\,\ln\left(x^4-\sqrt{3}\,x^2+1\right)}{24}+\frac{\sqrt{3}\,\ln\left(x^4+\sqrt{3}\,x^2+1\right)}{24}-\frac{1}{2x^2}-\frac{1}{6x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^7/(x^8-x^4+1),x)
```

```
[Out] -1/6/x^6-1/2/x^2-1/4*arctan(2*x^2-3^(1/2))-1/4*arctan(2*x^2+3^(1/2))-1/24*3
^(1/2)*ln(x^4-3^(1/2)*x^2+1)+1/24*3^(1/2)*ln(x^4+3^(1/2)*x^2+1)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{3x^4+1}{6x^6}-\int\frac{x^5}{x^8-x^4+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(x^8-x^4+1),x, algorithm="maxima")
```

```
[Out] -1/6*(3*x^4 + 1)/x^6 - integrate(x^5/(x^8 - x^4 + 1), x)
```

mupad [B] time = 0.06, size = 63, normalized size = 0.66

$$\mathrm{atan}\left(\frac{2x^2}{-1+\sqrt{3}\,1\mathrm{i}}\right)\left(\frac{1}{4}+\frac{\sqrt{3}\,1\mathrm{i}}{12}\right)+\mathrm{atan}\left(\frac{2x^2}{1+\sqrt{3}\,1\mathrm{i}}\right)\left(-\frac{1}{4}+\frac{\sqrt{3}\,1\mathrm{i}}{12}\right)-\frac{\frac{x^4}{2}+\frac{1}{6}}{x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^7*(x^8 - x^4 + 1)),x)
```

```
[Out] atan((2*x^2)/(3^(1/2)*1i - 1))*((3^(1/2)*1i)/12 + 1/4) + atan((2*x^2)/(3^(1
/2)*1i + 1))*((3^(1/2)*1i)/12 - 1/4) - (x^4/2 + 1/6)/x^6
```

sympy [A] time = 0.26, size = 83, normalized size = 0.86

$$-\frac{\sqrt{3}\log\left(x^4-\sqrt{3}x^2+1\right)}{24}+\frac{\sqrt{3}\log\left(x^4+\sqrt{3}x^2+1\right)}{24}-\frac{\mathrm{atan}\left(2x^2-\sqrt{3}\right)}{4}-\frac{\mathrm{atan}\left(2x^2+\sqrt{3}\right)}{4}+\frac{-3x^4-1}{6x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**7/(x**8-x**4+1),x)
```

```
[Out] -sqrt(3)*log(x**4 - sqrt(3)*x**2 + 1)/24 + sqrt(3)*log(x**4 + sqrt(3)*x**2
+ 1)/24 - atan(2*x**2 - sqrt(3))/4 - atan(2*x**2 + sqrt(3))/4 + (-3*x**4 -
1)/(6*x**6)
```

3.299 $\int \frac{x^8}{1-x^4+x^8} \, dx$

Optimal. Leaf size=356

$$-\frac{1}{8}\sqrt{\frac{1}{3}\left(2-\sqrt{3}\right)}\log\left(x^2-\sqrt{2-\sqrt{3}}\,x+1\right)+\frac{1}{8}\sqrt{\frac{1}{3}\left(2-\sqrt{3}\right)}\log\left(x^2+\sqrt{2-\sqrt{3}}\,x+1\right)+\frac{1}{8}\sqrt{\frac{1}{3}\left(2+\sqrt{3}\right)}\log$$

Rubi [A] time = 0.33, antiderivative size = 356, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {1367, 1421, 1169, 634, 618, 204, 628}

$$-\frac{1}{8}\sqrt{\frac{1}{3}\left(2-\sqrt{3}\right)}\log\left(x^2-\sqrt{2-\sqrt{3}}\,x+1\right)+\frac{1}{8}\sqrt{\frac{1}{3}\left(2-\sqrt{3}\right)}\log\left(x^2+\sqrt{2-\sqrt{3}}\,x+1\right)+\frac{1}{8}\sqrt{\frac{1}{3}\left(2+\sqrt{3}\right)}\log\left(x^2-\sqrt{2+\sqrt{3}}\,x+1\right)-\frac{1}{8}\sqrt{\frac{1}{3}\left(2+\sqrt{3}\right)}\log\left(x^2+\sqrt{2+\sqrt{3}}\,x+1\right)+x+\frac{\tan^{-1}\left(\frac{\sqrt{2-\sqrt{3}}-2x}{\sqrt{2+\sqrt{3}}}\right)}{4\sqrt{3\left(2-\sqrt{3}\right)}}-\frac{\tan^{-1}\left(\frac{\sqrt{2+\sqrt{3}}-2x}{\sqrt{2-\sqrt{3}}}\right)}{4\sqrt{3\left(2+\sqrt{3}\right)}}-\frac{\tan^{-1}\left(\frac{2x+\sqrt{2-\sqrt{3}}}{\sqrt{2+\sqrt{3}}}\right)}{4\sqrt{3\left(2-\sqrt{3}\right)}}+\frac{\tan^{-1}\left(\frac{2x+\sqrt{2+\sqrt{3}}}{\sqrt{2-\sqrt{3}}}\right)}{4\sqrt{3\left(2+\sqrt{3}\right)}}$$

Antiderivative was successfully verified.

```
[In] Int[x^8/(1 - x^4 + x^8),x]
```

```
[Out] x + ArcTan[(Sqrt[2 - Sqrt[3]] - 2*x)/Sqrt[2 + Sqrt[3]]]/(4*Sqrt[3*(2 - Sqrt
[3])]) - ArcTan[(Sqrt[2 + Sqrt[3]] - 2*x)/Sqrt[2 - Sqrt[3]]]/(4*Sqrt[3*(2 +
 Sqrt[3])]) - ArcTan[(Sqrt[2 - Sqrt[3]] + 2*x)/Sqrt[2 + Sqrt[3]]]/(4*Sqrt[3
*(2 - Sqrt[3])]) + ArcTan[(Sqrt[2 + Sqrt[3]] + 2*x)/Sqrt[2 - Sqrt[3]]]/(4*S
qrt[3*(2 + Sqrt[3])]) - (Sqrt[(2 - Sqrt[3])/3]*Log[1 - Sqrt[2 - Sqrt[3]]*x
+ x^2])/8 + (Sqrt[(2 - Sqrt[3])/3]*Log[1 + Sqrt[2 - Sqrt[3]]*x + x^2])/8 +
(Sqrt[(2 + Sqrt[3])/3]*Log[1 - Sqrt[2 + Sqrt[3]]*x + x^2])/8 - (Sqrt[(2 + S
qrt[3])/3]*Log[1 + Sqrt[2 + Sqrt[3]]*x + x^2])/8
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1169

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
```

```
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 1367

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Simp[(d^(2*n - 1)*(d*x)^(m - 2*n + 1)*(a + b*x^n + c*x^(2*n))^(
p + 1))/(c*(m + 2*n*p + 1)), x] - Dist[d^(2*n)/(c*(m + 2*n*p + 1)), Int[(d*
x)^(m - 2*n)*Simp[a*(m - 2*n + 1) + b*(m + n*(p - 1) + 1)*x^n, x]*(a + b*x^
n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && EqQ[n2, 2*n] && Ne
Q[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GtQ[m, 2*n - 1] && NeQ[m + 2*n*p + 1, 0]
 && IntegerQ[p]
```

Rule 1421

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[(-2*d)/e - b/c, 2]}, Dist[e/(2*c*q), Int[(q - 2*x^
(n/2))/Simp[d/e + q*x^(n/2) - x^n, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*
x^(n/2))/Simp[d/e - q*x^(n/2) - x^n, x], x], x]] /; FreeQ[{a, b, c, d, e},
x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && IGtQ[
n/2, 0] &&  !GtQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{x^8}{1-x^4+x^8}\,dx = x-\int \frac{1-x^4}{1-x^4+x^8}\,dx$$

$$= x+\frac{\int \frac{\sqrt{3}+2x^2}{-1-\sqrt{3}x^2-x^4}\,dx}{2\sqrt{3}}+\frac{\int \frac{\sqrt{3}-2x^2}{-1+\sqrt{3}x^2-x^4}\,dx}{2\sqrt{3}}$$

$$= x-\frac{\int \frac{\sqrt{3\left(2-\sqrt{3}\right)}-\left(-2+\sqrt{3}\right)x}{1-\sqrt{2-\sqrt{3}}\,x+x^2}\,dx}{4\sqrt{3\left(2-\sqrt{3}\right)}}-\frac{\int \frac{\sqrt{3\left(2-\sqrt{3}\right)}+\left(-2+\sqrt{3}\right)x}{1+\sqrt{2-\sqrt{3}}\,x+x^2}\,dx}{4\sqrt{3\left(2-\sqrt{3}\right)}}-\frac{\int \frac{\sqrt{3\left(2+\sqrt{3}\right)}-\left(2+\sqrt{3}\right)x}{1-\sqrt{2+\sqrt{3}}\,x+x^2}\,dx}{4\sqrt{3\left(2+\sqrt{3}\right)}}-\frac{\int \frac{\sqrt{3(2+}}{1+}}{4\sqrt{}}$$

$$= x+\frac{1}{8}\sqrt{\frac{1}{3}\left(7-4\sqrt{3}\right)}\int \frac{1}{1-\sqrt{2+\sqrt{3}}\,x+x^2}\,dx+\frac{1}{8}\sqrt{\frac{1}{3}\left(7-4\sqrt{3}\right)}\int \frac{1}{1+\sqrt{2+\sqrt{3}}\,x+x^2}$$

$$= x-\frac{1}{8}\sqrt{\frac{1}{3}\left(2-\sqrt{3}\right)}\log\left(1-\sqrt{2-\sqrt{3}}\,x+x^2\right)+\frac{1}{8}\sqrt{\frac{2}{3}-\frac{1}{\sqrt{3}}}\log\left(1+\sqrt{2-\sqrt{3}}\,x+x^2\right)+$$

$$= x+\frac{1}{4}\sqrt{\frac{1}{3}\left(2+\sqrt{3}\right)}\tan^{-1}\left(\frac{\sqrt{2-\sqrt{3}}-2x}{\sqrt{2+\sqrt{3}}}\right)-\frac{1}{4}\sqrt{\frac{1}{3}\left(2-\sqrt{3}\right)}\tan^{-1}\left(\frac{\sqrt{2+\sqrt{3}}-2x}{\sqrt{2-\sqrt{3}}}\right)-\frac{1}{4}$$

Mathematica [C] time = 0.01, size = 59, normalized size = 0.17

$$\frac{1}{4}\text{RootSum}\left[\#1^8-\#1^4+1\&,\frac{\#1^4\log(x-\#1)-\log(x-\#1)}{2\#1^7-\#1^3}\&\right]+x$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8/(1 - x^4 + x^8),x]
```

```
[Out] x + RootSum[1 - #1^4 + #1^8 & , (-Log[x - #1] + Log[x - #1]*#1^4)/(-#1^3 +
2*#1^7) & ]/4
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^8}{1-x^4+x^8}\,dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x^8/(1 - x^4 + x^8),x]

[Out] IntegrateAlgebraic[x^8/(1 - x^4 + x^8), x]

fricas [B] time = 1.43, size = 716, normalized size = 2.01

$$-\frac{1}{48}\sqrt{6}\left(\sqrt{3}\sqrt{2}-2\sqrt{2}\right)\sqrt{\sqrt{3}+2}\log\left(12x^2+2\sqrt{6}\left(2\sqrt{3}\sqrt{2}x-3\sqrt{2}x\right)\sqrt{\sqrt{3}+2}+12\right)+\frac{1}{48}\sqrt{6}\left(\sqrt{3}\sqrt{2}-2\sqrt{2}\right)\sqrt{\sqrt{3}+2}\log\left(12x^2-2\sqrt{6}\left(2\sqrt{3}\sqrt{2}x-3\sqrt{2}x\right)\sqrt{\sqrt{3}+2}+12\right)-\frac{1}{96}\sqrt{6}\left(\sqrt{3}\sqrt{2}+2\sqrt{2}\right)\sqrt{-4\sqrt{3}+8}\log\left(12x^2+\sqrt{6}\left(2\sqrt{3}\sqrt{2}x+3\sqrt{2}x\right)\sqrt{-4\sqrt{3}+8}+12\right)+\frac{1}{96}\sqrt{6}\left(\sqrt{3}\sqrt{2}+2\sqrt{2}\right)\sqrt{-4\sqrt{3}+8}\log\left(12x^2-\sqrt{6}\left(2\sqrt{3}\sqrt{2}x+3\sqrt{2}x\right)\sqrt{-4\sqrt{3}+8}+12\right)-\frac{1}{12}\sqrt{6}\sqrt{2}\sqrt{\sqrt{3}+2}\arctan\left(\frac{1}{6}\sqrt{6}\sqrt{12x^2+2\sqrt{6}\left(2\sqrt{3}\sqrt{2}x-3\sqrt{2}x\right)\sqrt{\sqrt{3}+2}+12}\left(\sqrt{3}\sqrt{2}-2\sqrt{2}\right)\sqrt{\sqrt{3}+2}+\frac{1}{3}\sqrt{6}\left(2\sqrt{3}\sqrt{2}x-3\sqrt{2}x\right)\sqrt{\sqrt{3}+2}-\sqrt{3}+2\right)-\frac{1}{12}\sqrt{6}\sqrt{2}\sqrt{\sqrt{3}+2}\arctan\left(\frac{1}{6}\sqrt{6}\sqrt{12x^2-2\sqrt{6}\left(2\sqrt{3}\sqrt{2}x-3\sqrt{2}x\right)\sqrt{\sqrt{3}+2}+12}\left(\sqrt{3}\sqrt{2}-2\sqrt{2}\right)\sqrt{\sqrt{3}+2}+\frac{1}{3}\sqrt{6}\left(2\sqrt{3}\sqrt{2}x-3\sqrt{2}x\right)\sqrt{\sqrt{3}+2}+\sqrt{3}-2\right)-\frac{1}{24}\sqrt{6}\sqrt{2}\sqrt{-4\sqrt{3}+8}\arctan\left(\frac{1}{12}\sqrt{6}\sqrt{12x^2+\sqrt{6}\left(2\sqrt{3}\sqrt{2}x+3\sqrt{2}x\right)\sqrt{-4\sqrt{3}+8}+12}\left(\sqrt{3}\sqrt{2}+2\sqrt{2}\right)\sqrt{-4\sqrt{3}+8}-\frac{1}{6}\sqrt{6}\left(2\sqrt{3}\sqrt{2}x+3\sqrt{2}x\right)\sqrt{-4\sqrt{3}+8}-\sqrt{3}-2\right)-\frac{1}{24}\sqrt{6}\sqrt{2}\sqrt{-4\sqrt{3}+8}\arctan\left(\frac{1}{12}\sqrt{6}\sqrt{12x^2-\sqrt{6}\left(2\sqrt{3}\sqrt{2}x+3\sqrt{2}x\right)\sqrt{-4\sqrt{3}+8}+12}\left(\sqrt{3}\sqrt{2}+2\sqrt{2}\right)\sqrt{-4\sqrt{3}+8}-\frac{1}{6}\sqrt{6}\left(2\sqrt{3}\sqrt{2}x+3\sqrt{2}x\right)\sqrt{-4\sqrt{3}+8}+\sqrt{3}+2\right)+x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^8/(x^8-x^4+1),x, algorithm="fricas")

[Out] -1/48*sqrt(6)*(sqrt(3)*sqrt(2) - 2*sqrt(2))*sqrt(sqrt(3) + 2)*log(12*x^2 + 2*sqrt(6)*(2*sqrt(3)*sqrt(2)*x - 3*sqrt(2)*x)*sqrt(sqrt(3) + 2) + 12) + 1/48*sqrt(6)*(sqrt(3)*sqrt(2) - 2*sqrt(2))*sqrt(sqrt(3) + 2)*log(12*x^2 - 2*sqrt(6)*(2*sqrt(3)*sqrt(2)*x - 3*sqrt(2)*x)*sqrt(sqrt(3) + 2) + 12) - 1/96*sqrt(6)*(sqrt(3)*sqrt(2) + 2*sqrt(2))*sqrt(-4*sqrt(3) + 8)*log(12*x^2 + sqrt(6)*(2*sqrt(3)*sqrt(2)*x + 3*sqrt(2)*x)*sqrt(-4*sqrt(3) + 8) + 12) + 1/96*sqrt(6)*(sqrt(3)*sqrt(2) + 2*sqrt(2))*sqrt(-4*sqrt(3) + 8)*log(12*x^2 - sqrt(6)*(2*sqrt(3)*sqrt(2)*x + 3*sqrt(2)*x)*sqrt(-4*sqrt(3) + 8) + 12) - 1/12*sqrt(6)*sqrt(2)*sqrt(sqrt(3) + 2)*arctan(1/6*sqrt(6)*sqrt(12*x^2 + 2*sqrt(6)*(2*sqrt(3)*sqrt(2)*x - 3*sqrt(2)*x)*sqrt(sqrt(3) + 2) + 12)*(sqrt(3)*sqrt(2) - 2*sqrt(2))*sqrt(sqrt(3) + 2) + 1/3*sqrt(6)*(2*sqrt(3)*sqrt(2)*x - 3*sqrt(2)*x)*sqrt(sqrt(3) + 2) - sqrt(3) + 2) - 1/12*sqrt(6)*sqrt(2)*sqrt(sqrt(3) + 2)*arctan(1/6*sqrt(6)*sqrt(12*x^2 - 2*sqrt(6)*(2*sqrt(3)*sqrt(2)*x - 3*sqrt(2)*x)*sqrt(sqrt(3) + 2) + 12)*(sqrt(3)*sqrt(2) - 2*sqrt(2))*sqrt(sqrt(3) + 2) + 1/3*sqrt(6)*(2*sqrt(3)*sqrt(2)*x - 3*sqrt(2)*x)*sqrt(sqrt(3) + 2) + sqrt(3) - 2) - 1/24*sqrt(6)*sqrt(2)*sqrt(-4*sqrt(3) + 8)*arctan(1/12*sqrt(6)*sqrt(12*x^2 + sqrt(6)*(2*sqrt(3)*sqrt(2)*x + 3*sqrt(2)*x)*sqrt(-4*sqrt(3) + 8) + 12)*(sqrt(3)*sqrt(2) + 2*sqrt(2))*sqrt(-4*sqrt(3) + 8) - 1/6*sqrt(6)*(2*sqrt(3)*sqrt(2)*x + 3*sqrt(2)*x)*sqrt(-4*sqrt(3) + 8) - sqrt(3) - 2) - 1/24*sqrt(6)*sqrt(2)*sqrt(-4*sqrt(3) + 8)*arctan(1/12*sqrt(6)*sqrt(12*x^2 - sqrt(6)*(2*sqrt(3)*sqrt(2)*x + 3*sqrt(2)*x)*sqrt(-4*sqrt(3) + 8) + 12)*(sqrt(3)*sqrt(2) + 2*sqrt(2))*sqrt(-4*sqrt(3) + 8) - 1/6*sqrt(6)*(2*sqrt(3)*sqrt(2)*x + 3*sqrt(2)*x)*sqrt(-4*sqrt(3) + 8) + sqrt(3) + 2) + x

giac [A] time = 0.35, size = 254, normalized size = 0.71

$$-\frac{1}{24}\left(\sqrt{6}+3\sqrt{2}\right)\arctan\left(\frac{4x+\sqrt{6}-\sqrt{2}}{\sqrt{6}+\sqrt{2}}\right)-\frac{1}{24}\left(\sqrt{6}+3\sqrt{2}\right)\arctan\left(\frac{4x-\sqrt{6}+\sqrt{2}}{\sqrt{6}+\sqrt{2}}\right)-\frac{1}{24}\left(\sqrt{6}-3\sqrt{2}\right)\arctan\left(\frac{4x+\sqrt{6}+\sqrt{2}}{\sqrt{6}-\sqrt{2}}\right)-\frac{1}{24}\left(\sqrt{6}-3\sqrt{2}\right)\arctan\left(\frac{4x-\sqrt{6}-\sqrt{2}}{\sqrt{6}-\sqrt{2}}\right)-\frac{1}{48}\left(\sqrt{6}+3\sqrt{2}\right)\log\left(x^2+\frac{1}{2}x\left(\sqrt{6}+\sqrt{2}\right)+1\right)+\frac{1}{48}\left(\sqrt{6}+3\sqrt{2}\right)\log\left(x^2-\frac{1}{2}x\left(\sqrt{6}+\sqrt{2}\right)+1\right)-\frac{1}{48}\left(\sqrt{6}-3\sqrt{2}\right)\log\left(x^2+\frac{1}{2}x\left(\sqrt{6}-\sqrt{2}\right)+1\right)+\frac{1}{48}\left(\sqrt{6}-3\sqrt{2}\right)\log\left(x^2-\frac{1}{2}x\left(\sqrt{6}-\sqrt{2}\right)+1\right)+x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^8/(x^8-x^4+1),x, algorithm="giac")

[Out] -1/24*(sqrt(6) + 3*sqrt(2))*arctan((4*x + sqrt(6) - sqrt(2))/(sqrt(6) + sqrt(2))) - 1/24*(sqrt(6) + 3*sqrt(2))*arctan((4*x - sqrt(6) + sqrt(2))/(sqrt(6) + sqrt(2))) - 1/24*(sqrt(6) - 3*sqrt(2))*arctan((4*x + sqrt(6) + sqrt(2))/(sqrt(6) - sqrt(2))) - 1/24*(sqrt(6) - 3*sqrt(2))*arctan((4*x - sqrt(6) - sqrt(2))/(sqrt(6) - sqrt(2))) - 1/48*(sqrt(6) + 3*sqrt(2))*log(x^2 + 1/2*x*(sqrt(6) + sqrt(2)) + 1) + 1/48*(sqrt(6) + 3*sqrt(2))*log(x^2 - 1/2*x*(sqrt(6) + sqrt(2)) + 1) - 1/48*(sqrt(6) - 3*sqrt(2))*log(x^2 + 1/2*x*(sqrt(6) - sqrt(2)) + 1) + 1/48*(sqrt(6) - 3*sqrt(2))*log(x^2 - 1/2*x*(sqrt(6) - sqrt(2)) + 1) + x

maple [C] time = 0.02, size = 44, normalized size = 0.12

$$x+\frac{\left(\mathrm{RootOf}\left(\_Z^{8}-\_Z^{4}+1\right)^{4}-1\right)\ln\left(-\mathrm{RootOf}\left(\_Z^{8}-\_Z^{4}+1\right)+x\right)}{8\,\mathrm{RootOf}\left(\_Z^{8}-\_Z^{4}+1\right)^{7}-4\,\mathrm{RootOf}\left(\_Z^{8}-\_Z^{4}+1\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(x^8-x^4+1),x)
```

```
[Out] x+1/4*sum((_R^4-1)/(2*_R^7-_R^3)*ln(-_R+x),_R=RootOf(_Z^8-_Z^4+1))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$x+\int\frac{x^{4}-1}{x^{8}-x^{4}+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(x^8-x^4+1),x, algorithm="maxima")
```

```
[Out] x + integrate((x^4 - 1)/(x^8 - x^4 + 1), x)
```

mupad [B] time = 0.16, size = 209, normalized size = 0.59

$$x+\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{x}{\left(8-\sqrt{3}\,8\mathrm{i}\right)^{1/4}}+\frac{\sqrt{3}\,x\,1\mathrm{i}}{\left(8-\sqrt{3}\,8\mathrm{i}\right)^{1/4}}\right)\left(8-\sqrt{3}\,8\mathrm{i}\right)^{1/4}1\mathrm{i}}{12}+\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{x\,1\mathrm{i}}{\left(8-\sqrt{3}\,8\mathrm{i}\right)^{1/4}}-\frac{\sqrt{3}\,x}{\left(8-\sqrt{3}\,8\mathrm{i}\right)^{1/4}}\right)\left(8-\sqrt{3}\,8\mathrm{i}\right)^{1/4}}{12}-\frac{2^{3/4}\sqrt{3}\,\mathrm{atan}\left(\frac{2^{1/4}x}{2\left(1+\sqrt{3}\,1\mathrm{i}\right)^{1/4}}-\frac{2^{1/4}\sqrt{3}\,x\,1\mathrm{i}}{2\left(1+\sqrt{3}\,1\mathrm{i}\right)^{1/4}}\right)\left(1+\sqrt{3}\,1\mathrm{i}\right)^{1/4}1\mathrm{i}}{12}-\frac{2^{3/4}\sqrt{3}\,\mathrm{atan}\left(\frac{2^{1/4}x\,1\mathrm{i}}{2\left(1+\sqrt{3}\,1\mathrm{i}\right)^{1/4}}+\frac{2^{1/4}\sqrt{3}\,x}{2\left(1+\sqrt{3}\,1\mathrm{i}\right)^{1/4}}\right)\left(1+\sqrt{3}\,1\mathrm{i}\right)^{1/4}}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(x^8 - x^4 + 1),x)
```

```
[Out] x + (3^(1/2)*atan(x/(8 - 3^(1/2)*8i)^(1/4) + (3^(1/2)*x*1i)/(8 - 3^(1/2)*8i
)^(1/4))*(8 - 3^(1/2)*8i)^(1/4)*1i)/12 + (3^(1/2)*atan((x*1i)/(8 - 3^(1/2)*
8i)^(1/4) - (3^(1/2)*x)/(8 - 3^(1/2)*8i)^(1/4))*(8 - 3^(1/2)*8i)^(1/4))/12
- (2^(3/4)*3^(1/2)*atan((2^(1/4)*x)/(2*(3^(1/2)*1i + 1)^(1/4)) - (2^(1/4)*3
^(1/2)*x*1i)/(2*(3^(1/2)*1i + 1)^(1/4)))*(3^(1/2)*1i + 1)^(1/4)*1i)/12 - (2
^(3/4)*3^(1/2)*atan((2^(1/4)*x*1i)/(2*(3^(1/2)*1i + 1)^(1/4)) + (2^(1/4)*3^
(1/2)*x)/(2*(3^(1/2)*1i + 1)^(1/4)))*(3^(1/2)*1i + 1)^(1/4))/12
```

sympy [A] time = 3.20, size = 26, normalized size = 0.07

$$x+\mathrm{RootSum}\left(5308416t^{8}-2304t^{4}+1,\left(t\mapsto t\log\left(9216t^{5}-8t+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8/(x**8-x**4+1),x)
```

```
[Out] x + RootSum(5308416*_t**8 - 2304*_t**4 + 1, Lambda(_t, _t*log(9216*_t**5 -
8*_t + x)))
```

3.300 $\int \frac{x^6}{1-x^4+x^8}\,dx$

Optimal. Leaf size=275

$$\frac{\log\left(x^2-\sqrt{2-\sqrt{3}}\,x+1\right)}{4\sqrt{6}}-\frac{\log\left(x^2+\sqrt{2-\sqrt{3}}\,x+1\right)}{4\sqrt{6}}+\frac{\log\left(x^2-\sqrt{2+\sqrt{3}}\,x+1\right)}{4\sqrt{6}}-\frac{\log\left(x^2+\sqrt{2+\sqrt{3}}\,x+1\right)}{4\sqrt{6}}$$

Rubi [A] time = 0.24, antiderivative size = 275, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {1372, 1169, 634, 618, 204, 628}

$$\frac{\log\left(x^2-\sqrt{2-\sqrt{3}}\,x+1\right)}{4\sqrt{6}}-\frac{\log\left(x^2+\sqrt{2-\sqrt{3}}\,x+1\right)}{4\sqrt{6}}+\frac{\log\left(x^2-\sqrt{2+\sqrt{3}}\,x+1\right)}{4\sqrt{6}}-\frac{\log\left(x^2+\sqrt{2+\sqrt{3}}\,x+1\right)}{4\sqrt{6}}-\frac{\tan^{-1}\left(\frac{\sqrt{2-\sqrt{3}}-2x}{\sqrt{2+\sqrt{3}}}\right)}{2\sqrt{6}}-\frac{\tan^{-1}\left(\frac{\sqrt{2+\sqrt{3}}-2x}{\sqrt{2-\sqrt{3}}}\right)}{2\sqrt{6}}+\frac{\tan^{-1}\left(\frac{2x+\sqrt{2-\sqrt{3}}}{\sqrt{2+\sqrt{3}}}\right)}{2\sqrt{6}}+\frac{\tan^{-1}\left(\frac{2x+\sqrt{2+\sqrt{3}}}{\sqrt{2-\sqrt{3}}}\right)}{2\sqrt{6}}$$

Antiderivative was successfully verified.

```
[In] Int[x^6/(1 - x^4 + x^8),x]
```

```
[Out] -ArcTan[(Sqrt[2 - Sqrt[3]] - 2*x)/Sqrt[2 + Sqrt[3]]]/(2*Sqrt[6]) - ArcTan[(
Sqrt[2 + Sqrt[3]] - 2*x)/Sqrt[2 - Sqrt[3]]]/(2*Sqrt[6]) + ArcTan[(Sqrt[2 -
Sqrt[3]] + 2*x)/Sqrt[2 + Sqrt[3]]]/(2*Sqrt[6]) + ArcTan[(Sqrt[2 + Sqrt[3]]
+ 2*x)/Sqrt[2 - Sqrt[3]]]/(2*Sqrt[6]) + Log[1 - Sqrt[2 - Sqrt[3]]*x + x^2]/
(4*Sqrt[6]) - Log[1 + Sqrt[2 - Sqrt[3]]*x + x^2]/(4*Sqrt[6]) + Log[1 - Sqrt
[2 + Sqrt[3]]*x + x^2]/(4*Sqrt[6]) - Log[1 + Sqrt[2 + Sqrt[3]]*x + x^2]/(4*
Sqrt[6])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1169

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 1372

```
Int[(x_)^(m_.)/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symbol] :> W
ith[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, -Dist[1/(2*c*r), Int[(x^
(m - 3*(n/2))*(q - r*x^(n/2)))/(q - r*x^(n/2) + x^n), x], x] + Dist[1/(2*c*
r), Int[(x^(m - 3*(n/2))*(q + r*x^(n/2)))/(q + r*x^(n/2) + x^n), x], x]]] /
; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n/2, 0
] && IGtQ[m, 0] && GeQ[m, (3*n)/2] && LtQ[m, 2*n] && NegQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{x^6}{1-x^4+x^8}\,dx = -\frac{\int \frac{1-\sqrt{3}x^2}{1-\sqrt{3}x^2+x^4}\,dx}{2\sqrt{3}} + \frac{\int \frac{1+\sqrt{3}x^2}{1+\sqrt{3}x^2+x^4}\,dx}{2\sqrt{3}}$$

$$= \frac{\int \frac{\sqrt{2-\sqrt{3}}-\left(1-\sqrt{3}\right)x}{1-\sqrt{2-\sqrt{3}}\,x+x^2}\,dx}{4\sqrt{3\left(2-\sqrt{3}\right)}} + \frac{\int \frac{\sqrt{2-\sqrt{3}}+\left(1-\sqrt{3}\right)x}{1+\sqrt{2-\sqrt{3}}\,x+x^2}\,dx}{4\sqrt{3\left(2-\sqrt{3}\right)}} - \frac{\int \frac{\sqrt{2+\sqrt{3}}-\left(1+\sqrt{3}\right)x}{1-\sqrt{2+\sqrt{3}}\,x+x^2}\,dx}{4\sqrt{3\left(2+\sqrt{3}\right)}} - \frac{\int \frac{\sqrt{2+\sqrt{3}}+\left(1+\sqrt{3}\right)x}{1+\sqrt{2+\sqrt{3}}\,x+x^2}\,dx}{4\sqrt{3\left(2+\sqrt{3}\right)}}$$

$$= \frac{\int \frac{-\sqrt{2-\sqrt{3}}+2x}{1-\sqrt{2-\sqrt{3}}\,x+x^2}\,dx}{4\sqrt{6}} - \frac{\int \frac{\sqrt{2-\sqrt{3}}+2x}{1+\sqrt{2-\sqrt{3}}\,x+x^2}\,dx}{4\sqrt{6}} + \frac{\int \frac{-\sqrt{2+\sqrt{3}}+2x}{1-\sqrt{2+\sqrt{3}}\,x+x^2}\,dx}{4\sqrt{6}} - \frac{\int \frac{\sqrt{2+\sqrt{3}}+2x}{1+\sqrt{2+\sqrt{3}}\,x+x^2}\,dx}{4\sqrt{6}} + \cdots$$

$$= \frac{\log\left(1-\sqrt{2-\sqrt{3}}\,x+x^2\right)}{4\sqrt{6}} - \frac{\log\left(1+\sqrt{2-\sqrt{3}}\,x+x^2\right)}{4\sqrt{6}} + \frac{\log\left(1-\sqrt{2+\sqrt{3}}\,x+x^2\right)}{4\sqrt{6}} - \frac{\log \cdots}{\cdots}$$

$$= -\frac{\tan^{-1}\left(\frac{\sqrt{2-\sqrt{3}}-2x}{\sqrt{2+\sqrt{3}}}\right)}{2\sqrt{6}} - \frac{\tan^{-1}\left(\frac{\sqrt{2+\sqrt{3}}-2x}{\sqrt{2-\sqrt{3}}}\right)}{2\sqrt{6}} + \frac{\tan^{-1}\left(\frac{\sqrt{2-\sqrt{3}}+2x}{\sqrt{2+\sqrt{3}}}\right)}{2\sqrt{6}} + \frac{\tan^{-1}\left(\frac{\sqrt{2+\sqrt{3}}+2x}{\sqrt{2-\sqrt{3}}}\right)}{2\sqrt{6}} + \frac{\log \cdots}{\cdots}$$

Mathematica [C]   time = 0.01, size = 41, normalized size = 0.15

$$\frac{1}{4}\text{RootSum}\left[\#1^8-\#1^4+1\&,\frac{\#1^3\log(x-\#1)}{2\#1^4-1}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6/(1 - x^4 + x^8),x]
```

```
[Out] RootSum[1 - #1^4 + #1^8 & , (Log[x - #1]*#1^3)/(-1 + 2*#1^4) & ]/4
```

IntegrateAlgebraic [F]   time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6}{1-x^4+x^8}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^6/(1 - x^4 + x^8),x]
```

```
[Out] IntegrateAlgebraic[x^6/(1 - x^4 + x^8), x]
```

fricas [A]   time = 1.27, size = 215, normalized size = 0.78

$$-\frac{1}{6}\sqrt{3}\sqrt{2}\arctan\left(-\frac{\sqrt{3}\sqrt{2}\left(x^3-x\right)+x^2-\sqrt{x^4+\sqrt{3}\sqrt{2}\left(x^3+x\right)+3x^2+1}\left(\sqrt{3}\sqrt{2}x-2\right)}{3x^2-2}\right)-\frac{1}{6}\sqrt{3}\sqrt{2}\arctan\left(-\frac{\sqrt{3}\sqrt{2}\left(x^3-x\right)-x^2-\sqrt{x^4-\sqrt{3}\sqrt{2}\left(x^3+x\right)+3x^2+1}\left(\sqrt{3}\sqrt{2}x+2\right)}{3x^2-2}\right)-\frac{1}{24}\sqrt{3}\sqrt{2}\log\left(x^4+\sqrt{3}\sqrt{2}\left(x^3+x\right)+3x^2+1\right)+\frac{1}{24}\sqrt{3}\sqrt{2}\log\left(x^4-\sqrt{3}\sqrt{2}\left(x^3+x\right)+3x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(x^8-x^4+1),x, algorithm="fricas")
```

```
[Out] -1/6*sqrt(3)*sqrt(2)*arctan(-(sqrt(3)*sqrt(2)*(x^3 - x) + x^2 - sqrt(x^4 +
sqrt(3)*sqrt(2)*(x^3 + x) + 3*x^2 + 1)*(sqrt(3)*sqrt(2)*x - 2))/(3*x^2 - 2)
) - 1/6*sqrt(3)*sqrt(2)*arctan(-(sqrt(3)*sqrt(2)*(x^3 - x) - x^2 - sqrt(x^4
 - sqrt(3)*sqrt(2)*(x^3 + x) + 3*x^2 + 1)*(sqrt(3)*sqrt(2)*x + 2))/(3*x^2 -
 2)) - 1/24*sqrt(3)*sqrt(2)*log(x^4 + sqrt(3)*sqrt(2)*(x^3 + x) + 3*x^2 + 1
) + 1/24*sqrt(3)*sqrt(2)*log(x^4 - sqrt(3)*sqrt(2)*(x^3 + x) + 3*x^2 + 1)
```

giac [A] time = 0.40, size = 205, normalized size = 0.75

$$\frac{1}{12}\sqrt{6}\arctan\left(\frac{4x+\sqrt{6}-\sqrt{2}}{\sqrt{6}+\sqrt{2}}\right)+\frac{1}{12}\sqrt{6}\arctan\left(\frac{4x-\sqrt{6}+\sqrt{2}}{\sqrt{6}+\sqrt{2}}\right)+\frac{1}{12}\sqrt{6}\arctan\left(\frac{4x+\sqrt{6}+\sqrt{2}}{\sqrt{6}-\sqrt{2}}\right)+\frac{1}{12}\sqrt{6}\arctan\left(\frac{4x-\sqrt{6}-\sqrt{2}}{\sqrt{6}-\sqrt{2}}\right)-\frac{1}{24}\sqrt{6}\log\left(x^2+\frac{1}{2}x\left(\sqrt{6}+\sqrt{2}\right)+1\right)+\frac{1}{24}\sqrt{6}\log\left(x^2-\frac{1}{2}x\left(\sqrt{6}+\sqrt{2}\right)+1\right)-\frac{1}{24}\sqrt{6}\log\left(x^2+\frac{1}{2}x\left(\sqrt{6}-\sqrt{2}\right)+1\right)+\frac{1}{24}\sqrt{6}\log\left(x^2-\frac{1}{2}x\left(\sqrt{6}-\sqrt{2}\right)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(x^8-x^4+1),x, algorithm="giac")
```

```
[Out] 1/12*sqrt(6)*arctan((4*x + sqrt(6) - sqrt(2))/(sqrt(6) + sqrt(2))) + 1/12*s
qrt(6)*arctan((4*x - sqrt(6) + sqrt(2))/(sqrt(6) + sqrt(2))) + 1/12*sqrt(6)
*arctan((4*x + sqrt(6) + sqrt(2))/(sqrt(6) - sqrt(2))) + 1/12*sqrt(6)*arcta
n((4*x - sqrt(6) - sqrt(2))/(sqrt(6) - sqrt(2))) - 1/24*sqrt(6)*log(x^2 + 1
/2*x*(sqrt(6) + sqrt(2)) + 1) + 1/24*sqrt(6)*log(x^2 - 1/2*x*(sqrt(6) + sqr
t(2)) + 1) - 1/24*sqrt(6)*log(x^2 + 1/2*x*(sqrt(6) - sqrt(2)) + 1) + 1/24*s
qrt(6)*log(x^2 - 1/2*x*(sqrt(6) - sqrt(2)) + 1)
```

maple [C] time = 0.01, size = 32, normalized size = 0.12

$$\frac{\mathrm{RootOf}\left(9\_Z^4+1\right)\ln\left(9\,\mathrm{RootOf}\left(9\_Z^4+1\right)^3 x-3\,\mathrm{RootOf}\left(9\_Z^4+1\right)^2+x^2\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(x^8-x^4+1),x)
```

```
[Out] 1/4*sum(_R*ln(9*_R^3*x-3*_R^2+x^2),_R=RootOf(9*_Z^4+1))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6}{x^8-x^4+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(x^8-x^4+1),x, algorithm="maxima")
```

```
[Out] integrate(x^6/(x^8 - x^4 + 1), x)
```

mupad [B] time = 0.10, size = 53, normalized size = 0.19

$$\sqrt{6}\,\mathrm{atan}\left(\frac{\sqrt{6}\,x\,\left(\frac{1}{3}+\frac{1}{3}\mathrm{i}\right)}{\frac{2\,x^2}{3}-\frac{2}{3}\mathrm{i}}\right)\left(-\frac{1}{12}+\frac{1}{12}\mathrm{i}\right)+\sqrt{6}\,\mathrm{atan}\left(\frac{\sqrt{6}\,x\,\left(\frac{1}{3}-\frac{1}{3}\mathrm{i}\right)}{\frac{2\,x^2}{3}+\frac{2}{3}\mathrm{i}}\right)\left(-\frac{1}{12}-\frac{1}{12}\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(x^8 - x^4 + 1),x)
```

```
[Out] - 6^(1/2)*atan((6^(1/2)*x*(1/3 + 1i/3))/((2*x^2)/3 - 2i/3))*(1/12 - 1i/12)
- 6^(1/2)*atan((6^(1/2)*x*(1/3 - 1i/3))/((2*x^2)/3 + 2i/3))*(1/12 + 1i/12)
```

sympy [A] time = 0.22, size = 165, normalized size = 0.60

$$\frac{\sqrt{6}\left(2\operatorname{atan}\left(\frac{\sqrt{6}x}{3}-\frac{1}{3}\right)+2\operatorname{atan}\left(\sqrt{6}x^3-4x^2+2\sqrt{6}x-3\right)\right)}{24}+\frac{\sqrt{6}\left(2\operatorname{atan}\left(\frac{\sqrt{6}x}{3}+\frac{1}{3}\right)+2\operatorname{atan}\left(\sqrt{6}x^3+4x^2+2\sqrt{6}x+3\right)\right)}{24}+\frac{\sqrt{6}\log\left(x^4-\sqrt{6}x^3+3x^2-\sqrt{6}x+1\right)}{24}-\frac{\sqrt{6}\log\left(x^4+\sqrt{6}x^3+3x^2+\sqrt{6}x+1\right)}{24}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(x**8-x**4+1),x)
```

```
[Out] sqrt(6)*(2*atan(sqrt(6)*x/3 - 1/3) + 2*atan(sqrt(6)*x**3 - 4*x**2 + 2*sqrt(
6)*x - 3))/24 + sqrt(6)*(2*atan(sqrt(6)*x/3 + 1/3) + 2*atan(sqrt(6)*x**3 +
4*x**2 + 2*sqrt(6)*x + 3))/24 + sqrt(6)*log(x**4 - sqrt(6)*x**3 + 3*x**2 -
sqrt(6)*x + 1)/24 - sqrt(6)*log(x**4 + sqrt(6)*x**3 + 3*x**2 + sqrt(6)*x +
1)/24
```

3.301 $\int \frac{x^4}{1-x^4+x^8}\,dx$

Optimal. Leaf size=347

$$-\frac{\log\left(x^2-\sqrt{2-\sqrt{3}}\,x+1\right)}{8\sqrt{3\left(2-\sqrt{3}\right)}}+\frac{\log\left(x^2+\sqrt{2-\sqrt{3}}\,x+1\right)}{8\sqrt{3\left(2-\sqrt{3}\right)}}+\frac{\log\left(x^2-\sqrt{2+\sqrt{3}}\,x+1\right)}{8\sqrt{3\left(2+\sqrt{3}\right)}}-\frac{\log\left(x^2+\sqrt{2+\sqrt{3}}\,x+\right.}{8\sqrt{3\left(2+\sqrt{3}\right)}}$$

Rubi [A] time = 0.21, antiderivative size = 347, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {1373, 1127, 1161, 618, 204, 1164, 628}

$$-\frac{\log\left(x^2-\sqrt{2-\sqrt{3}}\,x+1\right)}{8\sqrt{3\left(2-\sqrt{3}\right)}}+\frac{\log\left(x^2+\sqrt{2-\sqrt{3}}\,x+1\right)}{8\sqrt{3\left(2-\sqrt{3}\right)}}+\frac{\log\left(x^2-\sqrt{2+\sqrt{3}}\,x+1\right)}{8\sqrt{3\left(2+\sqrt{3}\right)}}-\frac{\log\left(x^2+\sqrt{2+\sqrt{3}}\,x+1\right)}{8\sqrt{3\left(2+\sqrt{3}\right)}}+\frac{\tan^{-1}\left(\frac{\sqrt{2-\sqrt{3}}-2x}{\sqrt{2+\sqrt{3}}}\right)}{4\sqrt{3\left(2+\sqrt{3}\right)}}-\frac{\tan^{-1}\left(\frac{\sqrt{2+\sqrt{3}}-2x}{\sqrt{2-\sqrt{3}}}\right)}{4\sqrt{3\left(2-\sqrt{3}\right)}}-\frac{\tan^{-1}\left(\frac{2x+\sqrt{2-\sqrt{3}}}{\sqrt{2+\sqrt{3}}}\right)}{4\sqrt{3\left(2+\sqrt{3}\right)}}+\frac{\tan^{-1}\left(\frac{2x+\sqrt{2+\sqrt{3}}}{\sqrt{2-\sqrt{3}}}\right)}{4\sqrt{3\left(2-\sqrt{3}\right)}}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(1 - x^4 + x^8),x]
```

```
[Out] ArcTan[(Sqrt[2 - Sqrt[3]] - 2*x)/Sqrt[2 + Sqrt[3]]]/(4*Sqrt[3*(2 + Sqrt[3])
]) - ArcTan[(Sqrt[2 + Sqrt[3]] - 2*x)/Sqrt[2 - Sqrt[3]]]/(4*Sqrt[3*(2 - Sqr
t[3])]) - ArcTan[(Sqrt[2 - Sqrt[3]] + 2*x)/Sqrt[2 + Sqrt[3]]]/(4*Sqrt[3*(2
+ Sqrt[3])]) + ArcTan[(Sqrt[2 + Sqrt[3]] + 2*x)/Sqrt[2 - Sqrt[3]]]/(4*Sqrt[
3*(2 - Sqrt[3])]) - Log[1 - Sqrt[2 - Sqrt[3]]*x + x^2]/(8*Sqrt[3*(2 - Sqrt[
3])]) + Log[1 + Sqrt[2 - Sqrt[3]]*x + x^2]/(8*Sqrt[3*(2 - Sqrt[3])]) + Log[
1 - Sqrt[2 + Sqrt[3]]*x + x^2]/(8*Sqrt[3*(2 + Sqrt[3])]) - Log[1 + Sqrt[2 +
 Sqrt[3]]*x + x^2]/(8*Sqrt[3*(2 + Sqrt[3])])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1127

```
Int[(x_)^2/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a/c, 2]}, Dist[1/2, Int[(q + x^2)/(a + b*x^2 + c*x^4), x], x] - Dist[1/2, I
nt[(q - x^2)/(a + b*x^2 + c*x^4), x], x]] /; FreeQ[{a, b, c}, x] && LtQ[b^2
 - 4*a*c, 0] && PosQ[a*c]
```

Rule 1161

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[(2*d)/e - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2
, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; Fre
```

```
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (
GtQ[(2*d)/e - b/c, 0] || ( !LtQ[(2*d)/e - b/c, 0] && EqQ[d - e*Rt[a/c, 2],
0]))
```

Rule 1164

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[(-2*d)/e - b/c, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e
+ q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^
2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c
*d^2 - a*e^2, 0] &&  !GtQ[b^2 - 4*a*c, 0]
```

Rule 1373

```
Int[(x_)^(m_.)/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symbol] :> W
ith[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*r), Int[x^(m
 - n/2)/(q - r*x^(n/2) + x^n), x], x] - Dist[1/(2*c*r), Int[x^(m - n/2)/(q
+ r*x^(n/2) + x^n), x], x]]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[n/2, 0] && IGtQ[m, 0] && GeQ[m, n/2] && LtQ[m, (3*n
)/2] && NegQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{x^4}{1-x^4+x^8}\,dx = \frac{\int \frac{x^2}{1-\sqrt{3}x^2+x^4}\,dx}{2\sqrt{3}} - \frac{\int \frac{x^2}{1+\sqrt{3}x^2+x^4}\,dx}{2\sqrt{3}}$$

$$= -\frac{\int \frac{1-x^2}{1-\sqrt{3}x^2+x^4}\,dx}{4\sqrt{3}} + \frac{\int \frac{1+x^2}{1-\sqrt{3}x^2+x^4}\,dx}{4\sqrt{3}} + \frac{\int \frac{1-x^2}{1+\sqrt{3}x^2+x^4}\,dx}{4\sqrt{3}} - \frac{\int \frac{1+x^2}{1+\sqrt{3}x^2+x^4}\,dx}{4\sqrt{3}}$$

$$= -\frac{\int \frac{1}{1-\sqrt{2-\sqrt{3}}\,x+x^2}\,dx}{8\sqrt{3}} - \frac{\int \frac{1}{1+\sqrt{2-\sqrt{3}}\,x+x^2}\,dx}{8\sqrt{3}} + \frac{\int \frac{1}{1-\sqrt{2+\sqrt{3}}\,x+x^2}\,dx}{8\sqrt{3}} + \frac{\int \frac{1}{1+\sqrt{2+\sqrt{3}}\,x+x^2}\,dx}{8\sqrt{3}} - \frac{\int \frac{1}{-1\cdots}}{8\sqrt{\cdots}}$$

$$= -\frac{\log\left(1-\sqrt{2-\sqrt{3}}\,x+x^2\right)}{8\sqrt{3\left(2-\sqrt{3}\right)}} + \frac{\log\left(1+\sqrt{2-\sqrt{3}}\,x+x^2\right)}{8\sqrt{3\left(2-\sqrt{3}\right)}} + \frac{\log\left(1-\sqrt{2+\sqrt{3}}\,x+x^2\right)}{8\sqrt{3\left(2+\sqrt{3}\right)}} - \frac{\log\cdots}{\cdots}$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt{2-\sqrt{3}}-2x}{\sqrt{2+\sqrt{3}}}\right)}{4\sqrt{3\left(2+\sqrt{3}\right)}} - \frac{\tan^{-1}\left(\frac{\sqrt{2+\sqrt{3}}-2x}{\sqrt{2-\sqrt{3}}}\right)}{4\sqrt{3\left(2-\sqrt{3}\right)}} - \frac{\tan^{-1}\left(\frac{\sqrt{2-\sqrt{3}}+2x}{\sqrt{2+\sqrt{3}}}\right)}{4\sqrt{3\left(2+\sqrt{3}\right)}} + \frac{\tan^{-1}\left(\frac{\sqrt{2+\sqrt{3}}+2x}{\sqrt{2-\sqrt{3}}}\right)}{4\sqrt{3\left(2-\sqrt{3}\right)}} - \frac{\log\left(1\cdots\right.}{8\cdots}$$

Mathematica [C] time = 0.01, size = 39, normalized size = 0.11

$$\frac{1}{4}\text{RootSum}\left[\#1^8-\#1^4+1\&,\frac{\#1\log(x-\#1)}{2\#1^4-1}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(1 - x^4 + x^8),x]
```

```
[Out] RootSum[1 - #1^4 + #1^8 & , (Log[x - #1]*#1)/(-1 + 2*#1^4) & ]/4
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{1-x^4+x^8}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^4/(1 - x^4 + x^8),x]
```

```
[Out] IntegrateAlgebraic[x^4/(1 - x^4 + x^8), x]
```

fricas [B] time = 1.32, size = 567, normalized size = 1.63

$$\frac{1}{48}\sqrt{6}\left(\sqrt{3}\sqrt{2}-2\sqrt{2}\right)\sqrt{\sqrt{3}+2}\log\left(2\sqrt{6}\sqrt{3}\sqrt{2}x\sqrt{\sqrt{3}+2}+12x^{2}+12\right)-\frac{1}{48}\sqrt{6}\left(\sqrt{3}\sqrt{2}-2\sqrt{2}\right)\sqrt{\sqrt{3}+2}\log\left(-2\sqrt{6}\sqrt{3}\sqrt{2}x\sqrt{\sqrt{3}+2}+12x^{2}+12\right)+\frac{1}{96}\sqrt{6}\left(\sqrt{3}\sqrt{2}+2\sqrt{2}\right)\sqrt{-4\sqrt{3}+8}\log\left(\sqrt{6}\sqrt{3}\sqrt{2}x\sqrt{-4\sqrt{3}+8}+12x^{2}+12\right)-\frac{1}{96}\sqrt{6}\left(\sqrt{3}\sqrt{2}+2\sqrt{2}\right)\sqrt{-4\sqrt{3}+8}\log\left(-\sqrt{6}\sqrt{3}\sqrt{2}x\sqrt{-4\sqrt{3}+8}+12x^{2}+12\right)-\frac{1}{12}\sqrt{6}\sqrt{2}\sqrt{\sqrt{3}+2}\arctan\left(-\frac{1}{3}\sqrt{6}\sqrt{3}\sqrt{2}x\sqrt{\sqrt{3}+2}+\frac{1}{6}\sqrt{6}\sqrt{2}\sqrt{2\sqrt{6}\sqrt{3}\sqrt{2}x\sqrt{\sqrt{3}+2}+12x^{2}+12}\sqrt{\sqrt{3}+2}-\sqrt{3}-2\right)-\frac{1}{12}\sqrt{6}\sqrt{2}\sqrt{\sqrt{3}+2}\arctan\left(-\frac{1}{3}\sqrt{6}\sqrt{3}\sqrt{2}x\sqrt{\sqrt{3}+2}+\frac{1}{6}\sqrt{6}\sqrt{2}\sqrt{-2\sqrt{6}\sqrt{3}\sqrt{2}x\sqrt{\sqrt{3}+2}+12x^{2}+12}\sqrt{\sqrt{3}+2}+\sqrt{3}+2\right)+\frac{1}{24}\sqrt{6}\sqrt{2}\sqrt{-4\sqrt{3}+8}\arctan\left(-\frac{1}{6}\sqrt{6}\sqrt{3}\sqrt{2}x\sqrt{-4\sqrt{3}+8}+\frac{1}{12}\sqrt{6}\sqrt{2}\sqrt{\sqrt{6}\sqrt{3}\sqrt{2}x\sqrt{-4\sqrt{3}+8}+12x^{2}+12}\sqrt{-4\sqrt{3}+8}+\sqrt{3}-2\right)+\frac{1}{24}\sqrt{6}\sqrt{2}\sqrt{-4\sqrt{3}+8}\arctan\left(-\frac{1}{6}\sqrt{6}\sqrt{3}\sqrt{2}x\sqrt{-4\sqrt{3}+8}+\frac{1}{12}\sqrt{6}\sqrt{2}\sqrt{-\sqrt{6}\sqrt{3}\sqrt{2}x\sqrt{-4\sqrt{3}+8}+12x^{2}+12}\sqrt{-4\sqrt{3}+8}-\sqrt{3}+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(x^8-x^4+1),x, algorithm="fricas")
```

```
[Out] 1/48*sqrt(6)*(sqrt(3)*sqrt(2) - 2*sqrt(2))*sqrt(sqrt(3) + 2)*log(2*sqrt(6)*
sqrt(3)*sqrt(2)*x*sqrt(sqrt(3) + 2) + 12*x^2 + 12) - 1/48*sqrt(6)*(sqrt(3)*
sqrt(2) - 2*sqrt(2))*sqrt(sqrt(3) + 2)*log(-2*sqrt(6)*sqrt(3)*sqrt(2)*x*sqr
t(sqrt(3) + 2) + 12*x^2 + 12) + 1/96*sqrt(6)*(sqrt(3)*sqrt(2) + 2*sqrt(2))*
sqrt(-4*sqrt(3) + 8)*log(sqrt(6)*sqrt(3)*sqrt(2)*x*sqrt(-4*sqrt(3) + 8) + 1
2*x^2 + 12) - 1/96*sqrt(6)*(sqrt(3)*sqrt(2) + 2*sqrt(2))*sqrt(-4*sqrt(3) +
8)*log(-sqrt(6)*sqrt(3)*sqrt(2)*x*sqrt(-4*sqrt(3) + 8) + 12*x^2 + 12) - 1/1
2*sqrt(6)*sqrt(2)*sqrt(sqrt(3) + 2)*arctan(-1/3*sqrt(6)*sqrt(3)*sqrt(2)*x*s
qrt(sqrt(3) + 2) + 1/6*sqrt(6)*sqrt(2)*sqrt(2*sqrt(6)*sqrt(3)*sqrt(2)*x*sqr
t(sqrt(3) + 2) + 12*x^2 + 12)*sqrt(sqrt(3) + 2) - sqrt(3) - 2) - 1/12*sqrt(
6)*sqrt(2)*sqrt(sqrt(3) + 2)*arctan(-1/3*sqrt(6)*sqrt(3)*sqrt(2)*x*sqrt(sqr
t(3) + 2) + 1/6*sqrt(6)*sqrt(2)*sqrt(-2*sqrt(6)*sqrt(3)*sqrt(2)*x*sqrt(sqrt
(3) + 2) + 12*x^2 + 12)*sqrt(sqrt(3) + 2) + sqrt(3) + 2) + 1/24*sqrt(6)*sqr
t(2)*sqrt(-4*sqrt(3) + 8)*arctan(-1/6*sqrt(6)*sqrt(3)*sqrt(2)*x*sqrt(-4*sqr
t(3) + 8) + 1/12*sqrt(6)*sqrt(2)*sqrt(sqrt(6)*sqrt(3)*sqrt(2)*x*sqrt(-4*sqr
t(3) + 8) + 12*x^2 + 12)*sqrt(-4*sqrt(3) + 8) + sqrt(3) - 2) + 1/24*sqrt(6)
*sqrt(2)*sqrt(-4*sqrt(3) + 8)*arctan(-1/6*sqrt(6)*sqrt(3)*sqrt(2)*x*sqrt(-4
*sqrt(3) + 8) + 1/12*sqrt(6)*sqrt(2)*sqrt(-sqrt(6)*sqrt(3)*sqrt(2)*x*sqrt(-
4*sqrt(3) + 8) + 12*x^2 + 12)*sqrt(-4*sqrt(3) + 8) - sqrt(3) + 2)
```

giac [A] time = 0.48, size = 253, normalized size = 0.73

$$\frac{1}{24}\left(\sqrt{6}-3\sqrt{2}\right)\arctan\left(\frac{4x+\sqrt{6}-\sqrt{2}}{\sqrt{6}+\sqrt{2}}\right)+\frac{1}{24}\left(\sqrt{6}-3\sqrt{2}\right)\arctan\left(\frac{4x-\sqrt{6}+\sqrt{2}}{\sqrt{6}+\sqrt{2}}\right)+\frac{1}{24}\left(\sqrt{6}+3\sqrt{2}\right)\arctan\left(\frac{4x+\sqrt{6}+\sqrt{2}}{\sqrt{6}-\sqrt{2}}\right)+\frac{1}{24}\left(\sqrt{6}+3\sqrt{2}\right)\arctan\left(\frac{4x-\sqrt{6}-\sqrt{2}}{\sqrt{6}-\sqrt{2}}\right)+\frac{1}{48}\left(\sqrt{6}-3\sqrt{2}\right)\log\left(x^{2}+\frac{1}{2}x\left(\sqrt{6}+\sqrt{2}\right)+1\right)-\frac{1}{48}\left(\sqrt{6}-3\sqrt{2}\right)\log\left(x^{2}-\frac{1}{2}x\left(\sqrt{6}+\sqrt{2}\right)+1\right)+\frac{1}{48}\left(\sqrt{6}+3\sqrt{2}\right)\log\left(x^{2}+\frac{1}{2}x\left(\sqrt{6}-\sqrt{2}\right)+1\right)-\frac{1}{48}\left(\sqrt{6}+3\sqrt{2}\right)\log\left(x^{2}-\frac{1}{2}x\left(\sqrt{6}-\sqrt{2}\right)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(x^8-x^4+1),x, algorithm="giac")
```

```
[Out] 1/24*(sqrt(6) - 3*sqrt(2))*arctan((4*x + sqrt(6) - sqrt(2))/(sqrt(6) + sqrt
(2))) + 1/24*(sqrt(6) - 3*sqrt(2))*arctan((4*x - sqrt(6) + sqrt(2))/(sqrt(6
) + sqrt(2))) + 1/24*(sqrt(6) + 3*sqrt(2))*arctan((4*x + sqrt(6) + sqrt(2))
/(sqrt(6) - sqrt(2))) + 1/24*(sqrt(6) + 3*sqrt(2))*arctan((4*x - sqrt(6) -
sqrt(2))/(sqrt(6) - sqrt(2))) + 1/48*(sqrt(6) - 3*sqrt(2))*log(x^2 + 1/2*x*
(sqrt(6) + sqrt(2)) + 1) - 1/48*(sqrt(6) - 3*sqrt(2))*log(x^2 - 1/2*x*(sqrt
(6) + sqrt(2)) + 1) + 1/48*(sqrt(6) + 3*sqrt(2))*log(x^2 + 1/2*x*(sqrt(6) -
 sqrt(2)) + 1) - 1/48*(sqrt(6) + 3*sqrt(2))*log(x^2 - 1/2*x*(sqrt(6) - sqrt
(2)) + 1)
```

maple [C] time = 0.01, size = 40, normalized size = 0.12

$$\frac{\mathrm{RootOf}\left(\_Z^{8}-\_Z^{4}+1\right)^{4}\ln\left(-\mathrm{RootOf}\left(\_Z^{8}-\_Z^{4}+1\right)+x\right)}{8\,\mathrm{RootOf}\left(\_Z^{8}-\_Z^{4}+1\right)^{7}-4\,\mathrm{RootOf}\left(\_Z^{8}-\_Z^{4}+1\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(x^8-x^4+1),x)
```

```
[Out] 1/4*sum(_R^4/(2*_R^7-_R^3)*ln(-_R+x),_R=RootOf(_Z^8-_Z^4+1))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{x^8 - x^4 + 1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(x^8-x^4+1),x, algorithm="maxima")
```

```
[Out] integrate(x^4/(x^8 - x^4 + 1), x)
```

mupad [B] time = 1.33, size = 474, normalized size = 1.37

$$-\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{x\left(8-\sqrt{3}\,8i\right)^{1/4}}{2\left(\frac{\sqrt{3}\,\sqrt{8-\sqrt{3}\,8i}\,1i}{4}+\frac{\sqrt{8-\sqrt{3}\,8i}}{4}\right)}+\frac{\sqrt{3}\,x\left(8-\sqrt{3}\,8i\right)^{1/4}\,1i}{2\left(\frac{\sqrt{3}\,\sqrt{8-\sqrt{3}\,8i}\,1i}{4}+\frac{\sqrt{8-\sqrt{3}\,8i}}{4}\right)}\right)\left(8-\sqrt{3}\,8i\right)^{1/4}\,1i}{12}-\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{x\left(8-\sqrt{3}\,8i\right)^{1/4}\,1i}{2\left(\frac{\sqrt{3}\,\sqrt{8-\sqrt{3}\,8i}\,1i}{4}+\frac{\sqrt{8-\sqrt{3}\,8i}}{4}\right)}-\frac{\sqrt{3}\,x\left(8-\sqrt{3}\,8i\right)^{1/4}}{2\left(\frac{\sqrt{3}\,\sqrt{8-\sqrt{3}\,8i}\,1i}{4}+\frac{\sqrt{8-\sqrt{3}\,8i}}{4}\right)}\right)\left(8-\sqrt{3}\,8i\right)^{1/4}}{12}+\frac{2^{3/4}\,\sqrt{3}\,\mathrm{atan}\left(\frac{2^{3/4}\,x\left(1+\sqrt{3}\,1i\right)^{1/4}}{2\left(\frac{\sqrt{2}\,\sqrt{1+\sqrt{3}\,1i}}{2}-\frac{\sqrt{2}\,\sqrt{3}\,\sqrt{1+\sqrt{3}\,1i}\,1i}{2}\right)}-\frac{2^{3/4}\,\sqrt{3}\,x\left(1+\sqrt{3}\,1i\right)^{1/4}\,1i}{2\left(\frac{\sqrt{2}\,\sqrt{1+\sqrt{3}\,1i}}{2}-\frac{\sqrt{2}\,\sqrt{3}\,\sqrt{1+\sqrt{3}\,1i}\,1i}{2}\right)}\right)\left(1+\sqrt{3}\,1i\right)^{1/4}\,1i}{12}+\frac{2^{3/4}\,\sqrt{3}\,\mathrm{atan}\left(\frac{2^{3/4}\,x\left(1+\sqrt{3}\,1i\right)^{1/4}\,1i}{2\left(\frac{\sqrt{2}\,\sqrt{1+\sqrt{3}\,1i}}{2}-\frac{\sqrt{2}\,\sqrt{3}\,\sqrt{1+\sqrt{3}\,1i}\,1i}{2}\right)}+\frac{2^{3/4}\,\sqrt{3}\,x\left(1+\sqrt{3}\,1i\right)^{1/4}}{2\left(\frac{\sqrt{2}\,\sqrt{1+\sqrt{3}\,1i}}{2}-\frac{\sqrt{2}\,\sqrt{3}\,\sqrt{1+\sqrt{3}\,1i}\,1i}{2}\right)}\right)\left(1+\sqrt{3}\,1i\right)^{1/4}}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(x^8 - x^4 + 1),x)
```

```
[Out] (2^(3/4)*3^(1/2)*atan((2^(3/4)*x*(3^(1/2)*1i + 1)^(1/4))/(2*((2^(1/2)*(3^(1
/2)*1i + 1)^(1/2))/2 - (2^(1/2)*3^(1/2)*(3^(1/2)*1i + 1)^(1/2)*1i)/2)) - (2
^(3/4)*3^(1/2)*x*(3^(1/2)*1i + 1)^(1/4)*1i)/(2*((2^(1/2)*(3^(1/2)*1i + 1)^(
1/2))/2 - (2^(1/2)*3^(1/2)*(3^(1/2)*1i + 1)^(1/2)*1i)/2)))*(3^(1/2)*1i + 1)
^(1/4)*1i)/12 - (3^(1/2)*atan((x*(8 - 3^(1/2)*8i)^(1/4)*1i)/(2*((3^(1/2)*(8
 - 3^(1/2)*8i)^(1/2)*1i)/4 + (8 - 3^(1/2)*8i)^(1/2)/4)) - (3^(1/2)*x*(8 - 3
^(1/2)*8i)^(1/4))/(2*((3^(1/2)*(8 - 3^(1/2)*8i)^(1/2)*1i)/4 + (8 - 3^(1/2)*
8i)^(1/2)/4)))*(8 - 3^(1/2)*8i)^(1/4))/12 - (3^(1/2)*atan((x*(8 - 3^(1/2)*8
i)^(1/4))/(2*((3^(1/2)*(8 - 3^(1/2)*8i)^(1/2)*1i)/4 + (8 - 3^(1/2)*8i)^(1/2
)/4)) + (3^(1/2)*x*(8 - 3^(1/2)*8i)^(1/4)*1i)/(2*((3^(1/2)*(8 - 3^(1/2)*8i)
^(1/2)*1i)/4 + (8 - 3^(1/2)*8i)^(1/2)/4)))*(8 - 3^(1/2)*8i)^(1/4)*1i)/12 +
(2^(3/4)*3^(1/2)*atan((2^(3/4)*x*(3^(1/2)*1i + 1)^(1/4)*1i)/(2*((2^(1/2)*(3
^(1/2)*1i + 1)^(1/2))/2 - (2^(1/2)*3^(1/2)*(3^(1/2)*1i + 1)^(1/2)*1i)/2)) +
 (2^(3/4)*3^(1/2)*x*(3^(1/2)*1i + 1)^(1/4))/(2*((2^(1/2)*(3^(1/2)*1i + 1)^(
1/2))/2 - (2^(1/2)*3^(1/2)*(3^(1/2)*1i + 1)^(1/2)*1i)/2)))*(3^(1/2)*1i + 1)
^(1/4))/12
```

sympy [A] time = 3.19, size = 24, normalized size = 0.07

$$\mathrm{RootSum}\left(5308416t^8 - 2304t^4 + 1, \left(t \mapsto t \log\left(-18432t^5 + 4t + x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(x**8-x**4+1),x)
```

```
[Out] RootSum(5308416*_t**8 - 2304*_t**4 + 1, Lambda(_t, _t*log(-18432*_t**5 + 4*
_t + x)))
```

3.302 $\int \frac{x^2}{1-x^4+x^8}\,dx$

Optimal. Leaf size=355

$$\frac{\log\left(x^2-\sqrt{2-\sqrt{3}}\,x+1\right)}{8\sqrt{3\left(2-\sqrt{3}\right)}}-\frac{\log\left(x^2+\sqrt{2-\sqrt{3}}\,x+1\right)}{8\sqrt{3\left(2-\sqrt{3}\right)}}-\frac{\log\left(x^2-\sqrt{2+\sqrt{3}}\,x+1\right)}{8\sqrt{3\left(2+\sqrt{3}\right)}}+\frac{\log\left(x^2+\sqrt{2+\sqrt{3}}\,x+1\right)}{8\sqrt{3\left(2+\sqrt{3}\right)}}$$

Rubi [A] time = 0.20, antiderivative size = 355, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {1373, 1094, 634, 618, 204, 628}

$$\frac{\log\left(x^2-\sqrt{2-\sqrt{3}}\,x+1\right)}{8\sqrt{3\left(2-\sqrt{3}\right)}}-\frac{\log\left(x^2+\sqrt{2-\sqrt{3}}\,x+1\right)}{8\sqrt{3\left(2-\sqrt{3}\right)}}-\frac{\log\left(x^2-\sqrt{2+\sqrt{3}}\,x+1\right)}{8\sqrt{3\left(2+\sqrt{3}\right)}}+\frac{\log\left(x^2+\sqrt{2+\sqrt{3}}\,x+1\right)}{8\sqrt{3\left(2+\sqrt{3}\right)}}+\frac{1}{4}\sqrt{\frac{1}{3}\left(2-\sqrt{3}\right)}\tan^{-1}\left(\frac{\sqrt{2-\sqrt{3}}-2x}{\sqrt{2+\sqrt{3}}}\right)-\frac{1}{4}\sqrt{\frac{1}{3}\left(2+\sqrt{3}\right)}\tan^{-1}\left(\frac{\sqrt{2+\sqrt{3}}-2x}{\sqrt{2-\sqrt{3}}}\right)-\frac{1}{4}\sqrt{\frac{1}{3}\left(2-\sqrt{3}\right)}\tan^{-1}\left(\frac{2x+\sqrt{2-\sqrt{3}}}{\sqrt{2+\sqrt{3}}}\right)+\frac{1}{4}\sqrt{\frac{1}{3}\left(2+\sqrt{3}\right)}\tan^{-1}\left(\frac{2x+\sqrt{2+\sqrt{3}}}{\sqrt{2-\sqrt{3}}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(1 - x^4 + x^8),x]
```

```
[Out] (Sqrt[(2 - Sqrt[3])/3]*ArcTan[(Sqrt[2 - Sqrt[3]] - 2*x)/Sqrt[2 + Sqrt[3]]])
/4 - (Sqrt[(2 + Sqrt[3])/3]*ArcTan[(Sqrt[2 + Sqrt[3]] - 2*x)/Sqrt[2 - Sqrt[
3]]])/4 - (Sqrt[(2 - Sqrt[3])/3]*ArcTan[(Sqrt[2 - Sqrt[3]] + 2*x)/Sqrt[2 +
Sqrt[3]]])/4 + (Sqrt[(2 + Sqrt[3])/3]*ArcTan[(Sqrt[2 + Sqrt[3]] + 2*x)/Sqrt
[2 - Sqrt[3]]])/4 + Log[1 - Sqrt[2 - Sqrt[3]]*x + x^2]/(8*Sqrt[3*(2 - Sqrt[
3])]) - Log[1 + Sqrt[2 - Sqrt[3]]*x + x^2]/(8*Sqrt[3*(2 - Sqrt[3])]) - Log[
1 - Sqrt[2 + Sqrt[3]]*x + x^2]/(8*Sqrt[3*(2 + Sqrt[3])]) + Log[1 + Sqrt[2 +
 Sqrt[3]]*x + x^2]/(8*Sqrt[3*(2 + Sqrt[3])])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1094

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[a/
c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int[(r - x)/(q - r*x
 + x^2), x], x] + Dist[1/(2*c*q*r), Int[(r + x)/(q + r*x + x^2), x], x]]] /
; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[b^2 - 4*a*c]
```

Rule 1373

```
Int[(x_)^(m_.)/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symbol] :> W
ith[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*r), Int[x^(m
 - n/2)/(q - r*x^(n/2) + x^n), x], x] - Dist[1/(2*c*r), Int[x^(m - n/2)/(q
+ r*x^(n/2) + x^n), x], x]]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[n/2, 0] && IGtQ[m, 0] && GeQ[m, n/2] && LtQ[m, (3*n
)/2] && NegQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{x^2}{1-x^4+x^8}\,dx = \frac{\int \frac{1}{1-\sqrt{3}x^2+x^4}\,dx}{2\sqrt{3}} - \frac{\int \frac{1}{1+\sqrt{3}x^2+x^4}\,dx}{2\sqrt{3}}$$

$$= -\frac{\int \frac{\sqrt{2-\sqrt{3}}-x}{1-\sqrt{2-\sqrt{3}}\,x+x^2}\,dx}{4\sqrt{3\left(2-\sqrt{3}\right)}} - \frac{\int \frac{\sqrt{2-\sqrt{3}}+x}{1+\sqrt{2-\sqrt{3}}\,x+x^2}\,dx}{4\sqrt{3\left(2-\sqrt{3}\right)}} + \frac{\int \frac{\sqrt{2+\sqrt{3}}-x}{1-\sqrt{2+\sqrt{3}}\,x+x^2}\,dx}{4\sqrt{3\left(2+\sqrt{3}\right)}} + \frac{\int \frac{\sqrt{2+\sqrt{3}}+x}{1+\sqrt{2+\sqrt{3}}\,x+x^2}\,dx}{4\sqrt{3\left(2+\sqrt{3}\right)}}$$

$$= -\frac{\int \frac{1}{1-\sqrt{2-\sqrt{3}}\,x+x^2}\,dx}{8\sqrt{3}} - \frac{\int \frac{1}{1+\sqrt{2-\sqrt{3}}\,x+x^2}\,dx}{8\sqrt{3}} + \frac{\int \frac{1}{1-\sqrt{2+\sqrt{3}}\,x+x^2}\,dx}{8\sqrt{3}} + \frac{\int \frac{1}{1+\sqrt{2+\sqrt{3}}\,x+x^2}\,dx}{8\sqrt{3}} + \cdots$$

$$= \frac{\log\left(1-\sqrt{2-\sqrt{3}}\,x+x^2\right)}{8\sqrt{3\left(2-\sqrt{3}\right)}} - \frac{\log\left(1+\sqrt{2-\sqrt{3}}\,x+x^2\right)}{8\sqrt{3\left(2-\sqrt{3}\right)}} - \frac{\log\left(1-\sqrt{2+\sqrt{3}}\,x+x^2\right)}{8\sqrt{3\left(2+\sqrt{3}\right)}} + \cdots$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt{2-\sqrt{3}}-2x}{\sqrt{2+\sqrt{3}}}\right)}{4\sqrt{3\left(2+\sqrt{3}\right)}} - \frac{\tan^{-1}\left(\frac{\sqrt{2+\sqrt{3}}-2x}{\sqrt{2-\sqrt{3}}}\right)}{4\sqrt{3\left(2-\sqrt{3}\right)}} - \frac{\tan^{-1}\left(\frac{\sqrt{2-\sqrt{3}}+2x}{\sqrt{2+\sqrt{3}}}\right)}{4\sqrt{3\left(2+\sqrt{3}\right)}} + \frac{\tan^{-1}\left(\frac{\sqrt{2+\sqrt{3}}+2x}{\sqrt{2-\sqrt{3}}}\right)}{4\sqrt{3\left(2-\sqrt{3}\right)}} + \cdots$$

Mathematica [C] time = 0.01, size = 40, normalized size = 0.11

$$\frac{1}{4}\text{RootSum}\left[\#1^8-\#1^4+1\&,\frac{\log(x-\#1)}{2\#1^5-\#1}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(1 - x^4 + x^8),x]
```

```
[Out] RootSum[1 - #1^4 + #1^8 & , Log[x - #1]/(-#1 + 2*#1^5) & ]/4
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{1-x^4+x^8}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^2/(1 - x^4 + x^8),x]
```

```
[Out] IntegrateAlgebraic[x^2/(1 - x^4 + x^8), x]
```

fricas [B] time = 1.30, size = 567, normalized size = 1.60

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^8-x^4+1),x, algorithm="fricas")
```

```
[Out] -1/48*sqrt(6)*(sqrt(3)*sqrt(2) - 2*sqrt(2))*sqrt(sqrt(3) + 2)*log(2*sqrt(6)
*sqrt(3)*sqrt(2)*x*sqrt(sqrt(3) + 2) + 12*x^2 + 12) + 1/48*sqrt(6)*(sqrt(3)
*sqrt(2) - 2*sqrt(2))*sqrt(sqrt(3) + 2)*log(-2*sqrt(6)*sqrt(3)*sqrt(2)*x*sq
rt(sqrt(3) + 2) + 12*x^2 + 12) - 1/96*sqrt(6)*(sqrt(3)*sqrt(2) + 2*sqrt(2))
*sqrt(-4*sqrt(3) + 8)*log(sqrt(6)*sqrt(3)*sqrt(2)*x*sqrt(-4*sqrt(3) + 8) +
12*x^2 + 12) + 1/96*sqrt(6)*(sqrt(3)*sqrt(2) + 2*sqrt(2))*sqrt(-4*sqrt(3) +
 8)*log(-sqrt(6)*sqrt(3)*sqrt(2)*x*sqrt(-4*sqrt(3) + 8) + 12*x^2 + 12) - 1/
12*sqrt(6)*sqrt(2)*sqrt(sqrt(3) + 2)*arctan(-1/3*sqrt(6)*sqrt(3)*sqrt(2)*x*
sqrt(sqrt(3) + 2) + 1/6*sqrt(6)*sqrt(2)*sqrt(2*sqrt(6)*sqrt(3)*sqrt(2)*x*sq
rt(sqrt(3) + 2) + 12*x^2 + 12)*sqrt(sqrt(3) + 2) - sqrt(3) - 2) - 1/12*sqrt
(6)*sqrt(2)*sqrt(sqrt(3) + 2)*arctan(-1/3*sqrt(6)*sqrt(3)*sqrt(2)*x*sqrt(sq
rt(3) + 2) + 1/6*sqrt(6)*sqrt(2)*sqrt(-2*sqrt(6)*sqrt(3)*sqrt(2)*x*sqrt(sqr
t(3) + 2) + 12*x^2 + 12)*sqrt(sqrt(3) + 2) + sqrt(3) + 2) + 1/24*sqrt(6)*sq
rt(2)*sqrt(-4*sqrt(3) + 8)*arctan(-1/6*sqrt(6)*sqrt(3)*sqrt(2)*x*sqrt(-4*sq
rt(3) + 8) + 1/12*sqrt(6)*sqrt(2)*sqrt(sqrt(6)*sqrt(3)*sqrt(2)*x*sqrt(-4*sq
rt(3) + 8) + 12*x^2 + 12)*sqrt(-4*sqrt(3) + 8) + sqrt(3) - 2) + 1/24*sqrt(6
)*sqrt(2)*sqrt(-4*sqrt(3) + 8)*arctan(-1/6*sqrt(6)*sqrt(3)*sqrt(2)*x*sqrt(-
4*sqrt(3) + 8) + 1/12*sqrt(6)*sqrt(2)*sqrt(-sqrt(6)*sqrt(3)*sqrt(2)*x*sqrt(
-4*sqrt(3) + 8) + 12*x^2 + 12)*sqrt(-4*sqrt(3) + 8) - sqrt(3) + 2)
```

giac [A] time = 0.46, size = 253, normalized size = 0.71

$$\frac{1}{24}\left(\sqrt{6}-3\sqrt{2}\right)\arctan\left(\frac{4x+\sqrt{6}-\sqrt{2}}{\sqrt{6}+\sqrt{2}}\right)+\frac{1}{24}\left(\sqrt{6}-3\sqrt{2}\right)\arctan\left(\frac{4x-\sqrt{6}+\sqrt{2}}{\sqrt{6}+\sqrt{2}}\right)+\frac{1}{24}\left(\sqrt{6}+3\sqrt{2}\right)\arctan\left(\frac{4x+\sqrt{6}+\sqrt{2}}{\sqrt{6}-\sqrt{2}}\right)+\frac{1}{24}\left(\sqrt{6}+3\sqrt{2}\right)\arctan\left(\frac{4x-\sqrt{6}-\sqrt{2}}{\sqrt{6}-\sqrt{2}}\right)-\frac{1}{48}\left(\sqrt{6}-3\sqrt{2}\right)\log\left(x^2+\frac{1}{2}x\left(\sqrt{6}+\sqrt{2}\right)+1\right)+\frac{1}{48}\left(\sqrt{6}-3\sqrt{2}\right)\log\left(x^2-\frac{1}{2}x\left(\sqrt{6}+\sqrt{2}\right)+1\right)-\frac{1}{48}\left(\sqrt{6}+3\sqrt{2}\right)\log\left(x^2+\frac{1}{2}x\left(\sqrt{6}-\sqrt{2}\right)+1\right)+\frac{1}{48}\left(\sqrt{6}+3\sqrt{2}\right)\log\left(x^2-\frac{1}{2}x\left(\sqrt{6}-\sqrt{2}\right)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^8-x^4+1),x, algorithm="giac")
```

```
[Out] 1/24*(sqrt(6) - 3*sqrt(2))*arctan((4*x + sqrt(6) - sqrt(2))/(sqrt(6) + sqrt
(2))) + 1/24*(sqrt(6) - 3*sqrt(2))*arctan((4*x - sqrt(6) + sqrt(2))/(sqrt(6
) + sqrt(2))) + 1/24*(sqrt(6) + 3*sqrt(2))*arctan((4*x + sqrt(6) + sqrt(2))
/(sqrt(6) - sqrt(2))) + 1/24*(sqrt(6) + 3*sqrt(2))*arctan((4*x - sqrt(6) -
sqrt(2))/(sqrt(6) - sqrt(2))) - 1/48*(sqrt(6) - 3*sqrt(2))*log(x^2 + 1/2*x*
(sqrt(6) + sqrt(2)) + 1) + 1/48*(sqrt(6) - 3*sqrt(2))*log(x^2 - 1/2*x*(sqrt
(6) + sqrt(2)) + 1) - 1/48*(sqrt(6) + 3*sqrt(2))*log(x^2 + 1/2*x*(sqrt(6) -
 sqrt(2)) + 1) + 1/48*(sqrt(6) + 3*sqrt(2))*log(x^2 - 1/2*x*(sqrt(6) - sqrt
(2)) + 1)
```

maple [C] time = 0.01, size = 40, normalized size = 0.11

$$\frac{\mathrm{RootOf}\left(\_Z^8-\_Z^4+1\right)^2\ln\left(-\mathrm{RootOf}\left(\_Z^8-\_Z^4+1\right)+x\right)}{8\,\mathrm{RootOf}\left(\_Z^8-\_Z^4+1\right)^7-4\,\mathrm{RootOf}\left(\_Z^8-\_Z^4+1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(x^8-x^4+1),x)
```

```
[Out] 1/4*sum(_R^2/(2*_R^7-_R^3)*ln(-_R+x),_R=RootOf(_Z^8-_Z^4+1))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{x^8-x^4+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^8-x^4+1),x, algorithm="maxima")
```

```
[Out] integrate(x^2/(x^8 - x^4 + 1), x)
```

mupad [B] time = 0.08, size = 286, normalized size = 0.81

$$-\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{x\,{\left(8-\sqrt{3}\,8\mathrm{i}\right)}^{1/4}}{2\,\left(1+\sqrt{3}\,1\mathrm{i}\right)}+\frac{\sqrt{3}\,x\,{\left(8-\sqrt{3}\,8\mathrm{i}\right)}^{1/4}\,1\mathrm{i}}{2\,\left(1+\sqrt{3}\,1\mathrm{i}\right)}\right)\,{\left(8-\sqrt{3}\,8\mathrm{i}\right)}^{1/4}\,1\mathrm{i}}{12}+\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{x\,{\left(8-\sqrt{3}\,8\mathrm{i}\right)}^{1/4}\,1\mathrm{i}}{2\,\left(1+\sqrt{3}\,1\mathrm{i}\right)}-\frac{\sqrt{3}\,x\,{\left(8-\sqrt{3}\,8\mathrm{i}\right)}^{1/4}}{2\,\left(1+\sqrt{3}\,1\mathrm{i}\right)}\right)\,{\left(8-\sqrt{3}\,8\mathrm{i}\right)}^{1/4}}{12}-\frac{2^{3/4}\,\sqrt{3}\,\mathrm{atan}\left(\frac{2^{3/4}\,x\,{\left(1+\sqrt{3}\,1\mathrm{i}\right)}^{1/4}}{2\,\left(-1+\sqrt{3}\,1\mathrm{i}\right)}-\frac{2^{3/4}\,\sqrt{3}\,x\,{\left(1+\sqrt{3}\,1\mathrm{i}\right)}^{1/4}\,1\mathrm{i}}{2\,\left(-1+\sqrt{3}\,1\mathrm{i}\right)}\right)\,{\left(1+\sqrt{3}\,1\mathrm{i}\right)}^{1/4}\,1\mathrm{i}}{12}+\frac{2^{3/4}\,\sqrt{3}\,\mathrm{atan}\left(\frac{2^{3/4}\,x\,{\left(1+\sqrt{3}\,1\mathrm{i}\right)}^{1/4}\,1\mathrm{i}}{2\,\left(-1+\sqrt{3}\,1\mathrm{i}\right)}+\frac{2^{3/4}\,\sqrt{3}\,x\,{\left(1+\sqrt{3}\,1\mathrm{i}\right)}^{1/4}}{2\,\left(-1+\sqrt{3}\,1\mathrm{i}\right)}\right)\,{\left(1+\sqrt{3}\,1\mathrm{i}\right)}^{1/4}}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(x^8 - x^4 + 1),x)
```

```
[Out] (3^(1/2)*atan((x*(8 - 3^(1/2)*8i)^(1/4)*1i)/(2*(3^(1/2)*1i + 1)) - (3^(1/2)
*x*(8 - 3^(1/2)*8i)^(1/4))/(2*(3^(1/2)*1i + 1)))*(8 - 3^(1/2)*8i)^(1/4))/12
 - (3^(1/2)*atan((x*(8 - 3^(1/2)*8i)^(1/4))/(2*(3^(1/2)*1i + 1)) + (3^(1/2)
*x*(8 - 3^(1/2)*8i)^(1/4)*1i)/(2*(3^(1/2)*1i + 1)))*(8 - 3^(1/2)*8i)^(1/4)*
1i)/12 - (2^(3/4)*3^(1/2)*atan((2^(3/4)*x*(3^(1/2)*1i + 1)^(1/4))/(2*(3^(1/
2)*1i - 1)) - (2^(3/4)*3^(1/2)*x*(3^(1/2)*1i + 1)^(1/4)*1i)/(2*(3^(1/2)*1i
- 1)))*(3^(1/2)*1i + 1)^(1/4)*1i)/12 + (2^(3/4)*3^(1/2)*atan((2^(3/4)*x*(3^
(1/2)*1i + 1)^(1/4)*1i)/(2*(3^(1/2)*1i - 1)) + (2^(3/4)*3^(1/2)*x*(3^(1/2)*
1i + 1)^(1/4))/(2*(3^(1/2)*1i - 1)))*(3^(1/2)*1i + 1)^(1/4))/12
```

sympy [A] time = 3.28, size = 26, normalized size = 0.07

$$\mathrm{RootSum}\left(5308416t^8-2304t^4+1,\left(t\mapsto t\log\left(-442368t^7-192t^3+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(x**8-x**4+1),x)
```

```
[Out] RootSum(5308416*_t**8 - 2304*_t**4 + 1, Lambda(_t, _t*log(-442368*_t**7 - 1
92*_t**3 + x)))
```

3.303 $\int \frac{1}{1-x^4+x^8}\,dx$

Optimal. Leaf size=275

$$-\frac{\log\left(x^2-\sqrt{2-\sqrt{3}}\,x+1\right)}{4\sqrt{6}}+\frac{\log\left(x^2+\sqrt{2-\sqrt{3}}\,x+1\right)}{4\sqrt{6}}-\frac{\log\left(x^2-\sqrt{2+\sqrt{3}}\,x+1\right)}{4\sqrt{6}}+\frac{\log\left(x^2+\sqrt{2+\sqrt{3}}\,x+\right.}{4\sqrt{6}}$$

Rubi [A] time = 0.21, antiderivative size = 275, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 6, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {1346, 1169, 634, 618, 204, 628}

$$-\frac{\log\left(x^2-\sqrt{2-\sqrt{3}}\,x+1\right)}{4\sqrt{6}}+\frac{\log\left(x^2+\sqrt{2-\sqrt{3}}\,x+1\right)}{4\sqrt{6}}-\frac{\log\left(x^2-\sqrt{2+\sqrt{3}}\,x+1\right)}{4\sqrt{6}}+\frac{\log\left(x^2+\sqrt{2+\sqrt{3}}\,x+1\right)}{4\sqrt{6}}-\frac{\tan^{-1}\left(\frac{\sqrt{2-\sqrt{3}}-2x}{\sqrt{2+\sqrt{3}}}\right)}{2\sqrt{6}}-\frac{\tan^{-1}\left(\frac{\sqrt{2+\sqrt{3}}-2x}{\sqrt{2-\sqrt{3}}}\right)}{2\sqrt{6}}+\frac{\tan^{-1}\left(\frac{2x+\sqrt{2-\sqrt{3}}}{\sqrt{2+\sqrt{3}}}\right)}{2\sqrt{6}}+\frac{\tan^{-1}\left(\frac{2x+\sqrt{2+\sqrt{3}}}{\sqrt{2-\sqrt{3}}}\right)}{2\sqrt{6}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x^4 + x^8)^(-1),x]
```

```
[Out] -ArcTan[(Sqrt[2 - Sqrt[3]] - 2*x)/Sqrt[2 + Sqrt[3]]]/(2*Sqrt[6]) - ArcTan[(
Sqrt[2 + Sqrt[3]] - 2*x)/Sqrt[2 - Sqrt[3]]]/(2*Sqrt[6]) + ArcTan[(Sqrt[2 -
Sqrt[3]] + 2*x)/Sqrt[2 + Sqrt[3]]]/(2*Sqrt[6]) + ArcTan[(Sqrt[2 + Sqrt[3]]
+ 2*x)/Sqrt[2 - Sqrt[3]]]/(2*Sqrt[6]) - Log[1 - Sqrt[2 - Sqrt[3]]*x + x^2]/
(4*Sqrt[6]) + Log[1 + Sqrt[2 - Sqrt[3]]*x + x^2]/(4*Sqrt[6]) - Log[1 - Sqrt
[2 + Sqrt[3]]*x + x^2]/(4*Sqrt[6]) + Log[1 + Sqrt[2 + Sqrt[3]]*x + x^2]/(4*
Sqrt[6])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1169

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 1346

```
Int[((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^(-1), x_Symbol] :> With[{q
= Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int[(r - x^(n
/2))/(q - r*x^(n/2) + x^n), x], x] + Dist[1/(2*c*q*r), Int[(r + x^(n/2))/(q
 + r*x^(n/2) + x^n), x], x]]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && NeQ
[b^2 - 4*a*c, 0] && IGtQ[n/2, 0] && NegQ[b^2 - 4*a*c]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{1-x^4+x^8}\,dx &= \frac{\int \frac{\sqrt{3}-x^2}{1-\sqrt{3}x^2+x^4}\,dx}{2\sqrt{3}}+\frac{\int \frac{\sqrt{3}+x^2}{1+\sqrt{3}x^2+x^4}\,dx}{2\sqrt{3}}\\
&= \frac{\int \frac{\sqrt{3\left(2-\sqrt{3}\right)}-\left(-1+\sqrt{3}\right)x}{1-\sqrt{2-\sqrt{3}}\,x+x^2}\,dx}{4\sqrt{3\left(2-\sqrt{3}\right)}}+\frac{\int \frac{\sqrt{3\left(2-\sqrt{3}\right)}+\left(-1+\sqrt{3}\right)x}{1+\sqrt{2-\sqrt{3}}\,x+x^2}\,dx}{4\sqrt{3\left(2-\sqrt{3}\right)}}+\frac{\int \frac{\sqrt{3\left(2+\sqrt{3}\right)}-\left(1+\sqrt{3}\right)x}{1-\sqrt{2+\sqrt{3}}\,x+x^2}\,dx}{4\sqrt{3\left(2+\sqrt{3}\right)}}+\cdots\\
&= -\frac{\int \frac{-\sqrt{2-\sqrt{3}}+2x}{1-\sqrt{2-\sqrt{3}}\,x+x^2}\,dx}{4\sqrt{6}}+\frac{\int \frac{\sqrt{2-\sqrt{3}}+2x}{1+\sqrt{2-\sqrt{3}}\,x+x^2}\,dx}{4\sqrt{6}}-\frac{\int \frac{-\sqrt{2+\sqrt{3}}+2x}{1-\sqrt{2+\sqrt{3}}\,x+x^2}\,dx}{4\sqrt{6}}+\frac{\int \frac{\sqrt{2+\sqrt{3}}+2x}{1+\sqrt{2+\sqrt{3}}\,x+x^2}\,dx}{4\sqrt{6}}+\cdots\\
&= -\frac{\log\left(1-\sqrt{2-\sqrt{3}}\,x+x^2\right)}{4\sqrt{6}}+\frac{\log\left(1+\sqrt{2-\sqrt{3}}\,x+x^2\right)}{4\sqrt{6}}-\frac{\log\left(1-\sqrt{2+\sqrt{3}}\,x+x^2\right)}{4\sqrt{6}}+\cdots\\
&= -\frac{\tan^{-1}\left(\frac{\sqrt{2-\sqrt{3}}-2x}{\sqrt{2+\sqrt{3}}}\right)}{2\sqrt{6}}-\frac{\tan^{-1}\left(\frac{\sqrt{2+\sqrt{3}}-2x}{\sqrt{2-\sqrt{3}}}\right)}{2\sqrt{6}}+\frac{\tan^{-1}\left(\frac{\sqrt{2-\sqrt{3}}+2x}{\sqrt{2+\sqrt{3}}}\right)}{2\sqrt{6}}+\frac{\tan^{-1}\left(\frac{\sqrt{2+\sqrt{3}}+2x}{\sqrt{2-\sqrt{3}}}\right)}{2\sqrt{6}}-\cdots
\end{aligned}
$$

Mathematica [C] time = 0.01, size = 42, normalized size = 0.15

$$\frac{1}{4}\text{RootSum}\left[\#1^8-\#1^4+1\&,\frac{\log(x-\#1)}{2\#1^7-\#1^3}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x^4 + x^8)^(-1),x]
```

```
[Out] RootSum[1 - #1^4 + #1^8 & , Log[x - #1]/(-#1^3 + 2*#1^7) & ]/4
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{1-x^4+x^8}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(1 - x^4 + x^8)^(-1),x]
```

```
[Out] IntegrateAlgebraic[(1 - x^4 + x^8)^(-1), x]
```

fricas [A] time = 1.46, size = 215, normalized size = 0.78

$$-\frac{1}{6}\sqrt{3}\sqrt{2}\arctan\left(-\frac{\sqrt{3}\sqrt{2}\left(x^3-x\right)+x^2-\sqrt{x^4+\sqrt{3}\sqrt{2}\left(x^3+x\right)+3x^2+1}\left(\sqrt{3}\sqrt{2}x-2\right)}{3x^2-2}\right)-\frac{1}{6}\sqrt{3}\sqrt{2}\arctan\left(-\frac{\sqrt{3}\sqrt{2}\left(x^3-x\right)-x^2-\sqrt{x^4-\sqrt{3}\sqrt{2}\left(x^3+x\right)+3x^2+1}\left(\sqrt{3}\sqrt{2}x+2\right)}{3x^2-2}\right)+\frac{1}{24}\sqrt{3}\sqrt{2}\log\left(x^4+\sqrt{3}\sqrt{2}\left(x^3+x\right)+3x^2+1\right)-\frac{1}{24}\sqrt{3}\sqrt{2}\log\left(x^4-\sqrt{3}\sqrt{2}\left(x^3+x\right)+3x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^8-x^4+1),x, algorithm="fricas")
```

```
[Out] -1/6*sqrt(3)*sqrt(2)*arctan(-(sqrt(3)*sqrt(2)*(x^3 - x) + x^2 - sqrt(x^4 +
sqrt(3)*sqrt(2)*(x^3 + x) + 3*x^2 + 1)*(sqrt(3)*sqrt(2)*x - 2))/(3*x^2 - 2)
) - 1/6*sqrt(3)*sqrt(2)*arctan(-(sqrt(3)*sqrt(2)*(x^3 - x) - x^2 - sqrt(x^4
 - sqrt(3)*sqrt(2)*(x^3 + x) + 3*x^2 + 1)*(sqrt(3)*sqrt(2)*x + 2))/(3*x^2 -
 2)) + 1/24*sqrt(3)*sqrt(2)*log(x^4 + sqrt(3)*sqrt(2)*(x^3 + x) + 3*x^2 + 1
) - 1/24*sqrt(3)*sqrt(2)*log(x^4 - sqrt(3)*sqrt(2)*(x^3 + x) + 3*x^2 + 1)
```

giac [A] time = 0.40, size = 205, normalized size = 0.75

$$\frac{1}{12}\sqrt{6}\arctan\left(\frac{4x+\sqrt{6}-\sqrt{2}}{\sqrt{6}+\sqrt{2}}\right)+\frac{1}{12}\sqrt{6}\arctan\left(\frac{4x-\sqrt{6}+\sqrt{2}}{\sqrt{6}+\sqrt{2}}\right)+\frac{1}{12}\sqrt{6}\arctan\left(\frac{4x+\sqrt{6}+\sqrt{2}}{\sqrt{6}-\sqrt{2}}\right)+\frac{1}{12}\sqrt{6}\arctan\left(\frac{4x-\sqrt{6}-\sqrt{2}}{\sqrt{6}-\sqrt{2}}\right)+\frac{1}{24}\sqrt{6}\log\left(x^2+\frac{1}{2}x\left(\sqrt{6}+\sqrt{2}\right)+1\right)-\frac{1}{24}\sqrt{6}\log\left(x^2-\frac{1}{2}x\left(\sqrt{6}+\sqrt{2}\right)+1\right)+\frac{1}{24}\sqrt{6}\log\left(x^2+\frac{1}{2}x\left(\sqrt{6}-\sqrt{2}\right)+1\right)-\frac{1}{24}\sqrt{6}\log\left(x^2-\frac{1}{2}x\left(\sqrt{6}-\sqrt{2}\right)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^8-x^4+1),x, algorithm="giac")
```

```
[Out] 1/12*sqrt(6)*arctan((4*x + sqrt(6) - sqrt(2))/(sqrt(6) + sqrt(2))) + 1/12*s
qrt(6)*arctan((4*x - sqrt(6) + sqrt(2))/(sqrt(6) + sqrt(2))) + 1/12*sqrt(6)
*arctan((4*x + sqrt(6) + sqrt(2))/(sqrt(6) - sqrt(2))) + 1/12*sqrt(6)*arcta
n((4*x - sqrt(6) - sqrt(2))/(sqrt(6) - sqrt(2))) + 1/24*sqrt(6)*log(x^2 + 1
/2*x*(sqrt(6) + sqrt(2)) + 1) - 1/24*sqrt(6)*log(x^2 - 1/2*x*(sqrt(6) + sqr
t(2)) + 1) + 1/24*sqrt(6)*log(x^2 + 1/2*x*(sqrt(6) - sqrt(2)) + 1) - 1/24*s
qrt(6)*log(x^2 - 1/2*x*(sqrt(6) - sqrt(2)) + 1)
```

maple [C] time = 0.01, size = 30, normalized size = 0.11

$$\frac{\mathrm{RootOf}\left(9\_Z^4+1\right)\ln\left(3\,\mathrm{RootOf}\left(9\_Z^4+1\right)^2+3\,\mathrm{RootOf}\left(9\_Z^4+1\right)x+x^2\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^8-x^4+1),x)
```

```
[Out] 1/4*sum(_R*ln(3*_R^2+3*_R*x+x^2),_R=RootOf(9*_Z^4+1))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^8-x^4+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^8-x^4+1),x, algorithm="maxima")
```

```
[Out] integrate(1/(x^8 - x^4 + 1), x)
```

mupad [B] time = 0.04, size = 53, normalized size = 0.19

$$\sqrt{6}\ \mathrm{atan}\left(\frac{\sqrt{6}\ x\left(\frac{1}{3}+\frac{1}{3}\mathrm{i}\right)}{\frac{2\,x^2}{3}-\frac{2}{3}\mathrm{i}}\right)\left(-\frac{1}{12}-\frac{1}{12}\mathrm{i}\right)+\sqrt{6}\ \mathrm{atan}\left(\frac{\sqrt{6}\ x\left(\frac{1}{3}-\frac{1}{3}\mathrm{i}\right)}{\frac{2\,x^2}{3}+\frac{2}{3}\mathrm{i}}\right)\left(-\frac{1}{12}+\frac{1}{12}\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^8 - x^4 + 1),x)
```

```
[Out] - 6^(1/2)*atan((6^(1/2)*x*(1/3 + 1i/3))/((2*x^2)/3 - 2i/3))*(1/12 + 1i/12)
- 6^(1/2)*atan((6^(1/2)*x*(1/3 - 1i/3))/((2*x^2)/3 + 2i/3))*(1/12 - 1i/12)
```

sympy [A] time = 0.22, size = 165, normalized size = 0.60

$$\frac{\sqrt{6}\left(2\,\mathrm{atan}\left(\frac{\sqrt{6}\,x}{3}-\frac{1}{3}\right)+2\,\mathrm{atan}\left(\sqrt{6}\,x^3-4x^2+2\sqrt{6}\,x-3\right)\right)}{24}+\frac{\sqrt{6}\left(2\,\mathrm{atan}\left(\frac{\sqrt{6}\,x}{3}+\frac{1}{3}\right)+2\,\mathrm{atan}\left(\sqrt{6}\,x^3+4x^2+2\sqrt{6}\,x+3\right)\right)}{24}-\frac{\sqrt{6}\log\left(x^4-\sqrt{6}\,x^3+3x^2-\sqrt{6}\,x+1\right)}{24}+\frac{\sqrt{6}\log\left(x^4+\sqrt{6}\,x^3+3x^2+\sqrt{6}\,x+1\right)}{24}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**8-x**4+1),x)
```

```
[Out] sqrt(6)*(2*atan(sqrt(6)*x/3 - 1/3) + 2*atan(sqrt(6)*x**3 - 4*x**2 + 2*sqrt(
6)*x - 3))/24 + sqrt(6)*(2*atan(sqrt(6)*x/3 + 1/3) + 2*atan(sqrt(6)*x**3 +
4*x**2 + 2*sqrt(6)*x + 3))/24 - sqrt(6)*log(x**4 - sqrt(6)*x**3 + 3*x**2 -
sqrt(6)*x + 1)/24 + sqrt(6)*log(x**4 + sqrt(6)*x**3 + 3*x**2 + sqrt(6)*x +
1)/24
```

3.304 $\int \frac{1}{x^2(1-x^4+x^8)}\,dx$

Optimal. Leaf size=360

$$\frac{1}{8}\sqrt{\frac{1}{3}\left(2-\sqrt{3}\right)}\,\log\left(x^2-\sqrt{2-\sqrt{3}}\,x+1\right)-\frac{1}{8}\sqrt{\frac{1}{3}\left(2-\sqrt{3}\right)}\,\log\left(x^2+\sqrt{2-\sqrt{3}}\,x+1\right)-\frac{1}{8}\sqrt{\frac{1}{3}\left(2+\sqrt{3}\right)}\,\log\left(\right.$$

Rubi [A] time = 0.24, antiderivative size = 360, normalized size of antiderivative = 1.00, number of steps used = 22, number of rules used = 8, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {1368, 1506, 1279, 1169, 634, 618, 204, 628}

$$\frac{1}{8}\sqrt{\frac{1}{3}\left(2-\sqrt{3}\right)}\log\left(x^2-\sqrt{2-\sqrt{3}}\,x+1\right)-\frac{1}{8}\sqrt{\frac{1}{3}\left(2-\sqrt{3}\right)}\log\left(x^2+\sqrt{2-\sqrt{3}}\,x+1\right)-\frac{1}{8}\sqrt{\frac{1}{3}\left(2+\sqrt{3}\right)}\log\left(x^2-\sqrt{2+\sqrt{3}}\,x+1\right)+\frac{1}{8}\sqrt{\frac{1}{3}\left(2+\sqrt{3}\right)}\log\left(x^2+\sqrt{2+\sqrt{3}}\,x+1\right)-\frac{1}{x}+\frac{\tan^{-1}\left(\frac{\sqrt{2-\sqrt{3}}-2x}{\sqrt{2+\sqrt{3}}}\right)}{4\sqrt{3\left(2-\sqrt{3}\right)}}-\frac{\tan^{-1}\left(\frac{\sqrt{2+\sqrt{3}}-2x}{\sqrt{2-\sqrt{3}}}\right)}{4\sqrt{3\left(2+\sqrt{3}\right)}}-\frac{\tan^{-1}\left(\frac{2x+\sqrt{2-\sqrt{3}}}{\sqrt{2+\sqrt{3}}}\right)}{4\sqrt{3\left(2-\sqrt{3}\right)}}+\frac{\tan^{-1}\left(\frac{2x+\sqrt{2+\sqrt{3}}}{\sqrt{2-\sqrt{3}}}\right)}{4\sqrt{3\left(2+\sqrt{3}\right)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(1 - x^4 + x^8)),x]
```

```
[Out] -x^(-1) + ArcTan[(Sqrt[2 - Sqrt[3]] - 2*x)/Sqrt[2 + Sqrt[3]]]/(4*Sqrt[3*(2
 - Sqrt[3])]) - ArcTan[(Sqrt[2 + Sqrt[3]] - 2*x)/Sqrt[2 - Sqrt[3]]]/(4*Sqrt[
3*(2 + Sqrt[3])]) - ArcTan[(Sqrt[2 - Sqrt[3]] + 2*x)/Sqrt[2 + Sqrt[3]]]/(4*
Sqrt[3*(2 - Sqrt[3])]) + ArcTan[(Sqrt[2 + Sqrt[3]] + 2*x)/Sqrt[2 - Sqrt[3]]
]/(4*Sqrt[3*(2 + Sqrt[3])]) + (Sqrt[(2 - Sqrt[3])/3]*Log[1 - Sqrt[2 - Sqrt[
3]]*x + x^2])/8 - (Sqrt[(2 - Sqrt[3])/3]*Log[1 + Sqrt[2 - Sqrt[3]]*x + x^2]
)/8 - (Sqrt[(2 + Sqrt[3])/3]*Log[1 - Sqrt[2 + Sqrt[3]]*x + x^2])/8 + (Sqrt[
(2 + Sqrt[3])/3]*Log[1 + Sqrt[2 + Sqrt[3]]*x + x^2])/8
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1169

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
```

```
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 1279

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[(e*f*(f*x)^(m - 1)*(a + b*x^2 + c*x^4)^(p +
1))/(c*(m + 4*p + 3)), x] - Dist[f^2/(c*(m + 4*p + 3)), Int[(f*x)^(m - 2)*(
a + b*x^2 + c*x^4)^p*Simp[a*e*(m - 1) + (b*e*(m + 2*p + 1) - c*d*(m + 4*p +
 3))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c,
 0] && GtQ[m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] ||
 IntegerQ[m])
```

Rule 1368

```
Int[((d_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_
Symbol] :> Simp[((d*x)^(m + 1)*(a + b*x^n + c*x^(2*n))^(p + 1))/(a*d*(m + 1
)), x] - Dist[1/(a*d^n*(m + 1)), Int[(d*x)^(m + n)*(b*(m + n*(p + 1) + 1) +
 c*(m + 2*n*(p + 1) + 1)*x^n)*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a
, b, c, d, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && L
tQ[m, -1] && IntegerQ[p]
```

Rule 1506

```
Int[(((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_)^(n_)))/((a_) + (b_.)*(x_)^(n_) +
(c_.)*(x_)^(n2_)), x_Symbol] :> With[{q = Rt[a*c, 2]}, With[{r = Rt[2*c*q -
 b*c, 2]}, Dist[c/(2*q*r), Int[((f*x)^m*Simp[d*r - (c*d - e*q)*x^(n/2), x])
/(q - r*x^(n/2) + c*x^n), x], x] + Dist[c/(2*q*r), Int[((f*x)^m*Simp[d*r +
(c*d - e*q)*x^(n/2), x])/(q + r*x^(n/2) + c*x^n), x], x]] /;  !LtQ[2*c*q -
b*c, 0]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[n2, 2*n] && LtQ[b^2 - 4*a*c
, 0] && IntegersQ[m, n/2] && LtQ[0, m, n] && PosQ[a*c]
```

Rubi steps

$$\int \frac{1}{x^2\left(1-x^4+x^8\right)}\,dx = -\frac{1}{x}+\int \frac{x^2\left(1-x^4\right)}{1-x^4+x^8}\,dx$$

$$= -\frac{1}{x}+\frac{\int \frac{x^2\left(\sqrt{3}-2x^2\right)}{1-\sqrt{3}x^2+x^4}\,dx}{2\sqrt{3}}+\frac{\int \frac{x^2\left(\sqrt{3}+2x^2\right)}{1+\sqrt{3}x^2+x^4}\,dx}{2\sqrt{3}}$$

$$= -\frac{1}{x}-\frac{\int \frac{-2+\sqrt{3}x^2}{1-\sqrt{3}x^2+x^4}\,dx}{2\sqrt{3}}-\frac{\int \frac{2+\sqrt{3}x^2}{1+\sqrt{3}x^2+x^4}\,dx}{2\sqrt{3}}$$

$$= -\frac{1}{x}-\frac{\int \frac{2\sqrt{2-\sqrt{3}}-\left(2-\sqrt{3}\right)x}{1-\sqrt{2-\sqrt{3}}\,x+x^2}\,dx}{4\sqrt{3\left(2-\sqrt{3}\right)}}-\frac{\int \frac{2\sqrt{2-\sqrt{3}}+\left(2-\sqrt{3}\right)x}{1+\sqrt{2-\sqrt{3}}\,x+x^2}\,dx}{4\sqrt{3\left(2-\sqrt{3}\right)}}-\frac{\int \frac{-2\sqrt{2+\sqrt{3}}-\left(-2-\sqrt{3}\right)x}{1-\sqrt{2+\sqrt{3}}\,x+x^2}\,dx}{4\sqrt{3\left(2+\sqrt{3}\right)}}-\ldots$$

$$= -\frac{1}{x}+\frac{1}{8}\sqrt{\frac{1}{3}\left(7-4\sqrt{3}\right)}\int \frac{1}{1-\sqrt{2+\sqrt{3}}\,x+x^2}\,dx+\frac{1}{8}\sqrt{\frac{1}{3}\left(7-4\sqrt{3}\right)}\int \frac{1}{1+\sqrt{2+\ldots}}$$

$$= -\frac{1}{x}+\frac{1}{8}\sqrt{\frac{2}{3}-\frac{1}{\sqrt{3}}}\log\left(1-\sqrt{2-\sqrt{3}}\,x+x^2\right)-\frac{1}{8}\sqrt{\frac{1}{3}\left(2-\sqrt{3}\right)}\log\left(1+\sqrt{2-\sqrt{3}}\,x\ldots\right.$$

$$= -\frac{1}{x}+\frac{1}{4}\sqrt{\frac{1}{3}\left(2+\sqrt{3}\right)}\tan^{-1}\left(\frac{\sqrt{2-\sqrt{3}}-2x}{\sqrt{2+\sqrt{3}}}\right)-\frac{1}{4}\sqrt{\frac{1}{3}\left(2-\sqrt{3}\right)}\tan^{-1}\left(\frac{\sqrt{2+\sqrt{3}}-\ldots}{\sqrt{2-\sqrt{3}}\ldots}\right.$$

Mathematica [C] time = 0.02, size = 61, normalized size = 0.17

$$-\frac{1}{4}\text{RootSum}\left[\#1^8-\#1^4+1\&,\frac{\#1^4\log(x-\#1)-\log(x-\#1)}{2\#1^5-\#1}\&\right]-\frac{1}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(1 - x^4 + x^8)),x]
```

```
[Out] -x^(-1) - RootSum[1 - #1^4 + #1^8 & , (-Log[x - #1] + Log[x - #1]*#1^4)/(-#
1 + 2*#1^5) & ]/4
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2\left(1-x^4+x^8\right)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x^2*(1 - x^4 + x^8)),x]
```

```
[Out] IntegrateAlgebraic[1/(x^2*(1 - x^4 + x^8)), x]
```

fricas [B] time = 1.56, size = 732, normalized size = 2.03

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(x^8-x^4+1),x, algorithm="fricas")
```

```
[Out] -1/96*(8*sqrt(6)*sqrt(2)*x*sqrt(sqrt(3) + 2)*arctan(1/6*sqrt(6)*sqrt(12*x^2
 + 2*sqrt(6)*(2*sqrt(3)*sqrt(2)*x - 3*sqrt(2)*x)*sqrt(sqrt(3) + 2) + 12)*(s
qrt(3)*sqrt(2) - 2*sqrt(2))*sqrt(sqrt(3) + 2) + 1/3*sqrt(6)*(2*sqrt(3)*sqrt
(2)*x - 3*sqrt(2)*x)*sqrt(sqrt(3) + 2) - sqrt(3) + 2) + 8*sqrt(6)*sqrt(2)*x
*sqrt(sqrt(3) + 2)*arctan(1/6*sqrt(6)*sqrt(12*x^2 - 2*sqrt(6)*(2*sqrt(3)*sq
rt(2)*x - 3*sqrt(2)*x)*sqrt(sqrt(3) + 2) + 12)*(sqrt(3)*sqrt(2) - 2*sqrt(2)
)*sqrt(sqrt(3) + 2) + 1/3*sqrt(6)*(2*sqrt(3)*sqrt(2)*x - 3*sqrt(2)*x)*sqrt(
sqrt(3) + 2) + sqrt(3) - 2) + 4*sqrt(6)*sqrt(2)*x*sqrt(-4*sqrt(3) + 8)*arct
an(1/12*sqrt(6)*sqrt(12*x^2 + sqrt(6)*(2*sqrt(3)*sqrt(2)*x + 3*sqrt(2)*x)*s
qrt(-4*sqrt(3) + 8) + 12)*(sqrt(3)*sqrt(2) + 2*sqrt(2))*sqrt(-4*sqrt(3) + 8
) - 1/6*sqrt(6)*(2*sqrt(3)*sqrt(2)*x + 3*sqrt(2)*x)*sqrt(-4*sqrt(3) + 8) -
sqrt(3) - 2) + 4*sqrt(6)*sqrt(2)*x*sqrt(-4*sqrt(3) + 8)*arctan(1/12*sqrt(6)
*sqrt(12*x^2 - sqrt(6)*(2*sqrt(3)*sqrt(2)*x + 3*sqrt(2)*x)*sqrt(-4*sqrt(3)
+ 8) + 12)*(sqrt(3)*sqrt(2) + 2*sqrt(2))*sqrt(-4*sqrt(3) + 8) - 1/6*sqrt(6)
*(2*sqrt(3)*sqrt(2)*x + 3*sqrt(2)*x)*sqrt(-4*sqrt(3) + 8) + sqrt(3) + 2) -
2*sqrt(6)*(sqrt(3)*sqrt(2)*x - 2*sqrt(2)*x)*sqrt(sqrt(3) + 2)*log(12*x^2 +
2*sqrt(6)*(2*sqrt(3)*sqrt(2)*x - 3*sqrt(2)*x)*sqrt(sqrt(3) + 2) + 12) + 2*s
qrt(6)*(sqrt(3)*sqrt(2)*x - 2*sqrt(2)*x)*sqrt(sqrt(3) + 2)*log(12*x^2 - 2*s
qrt(6)*(2*sqrt(3)*sqrt(2)*x - 3*sqrt(2)*x)*sqrt(sqrt(3) + 2) + 12) - sqrt(6
)*(sqrt(3)*sqrt(2)*x + 2*sqrt(2)*x)*sqrt(-4*sqrt(3) + 8)*log(12*x^2 + sqrt(
6)*(2*sqrt(3)*sqrt(2)*x + 3*sqrt(2)*x)*sqrt(-4*sqrt(3) + 8) + 12) + sqrt(6)
*(sqrt(3)*sqrt(2)*x + 2*sqrt(2)*x)*sqrt(-4*sqrt(3) + 8)*log(12*x^2 - sqrt(6
)*(2*sqrt(3)*sqrt(2)*x + 3*sqrt(2)*x)*sqrt(-4*sqrt(3) + 8) + 12) + 96)/x
```

giac [A] time = 0.46, size = 258, normalized size = 0.72

$$-\frac{1}{24}\left(\sqrt{6}+3\sqrt{2}\right)\arctan\left(\frac{4x+\sqrt{6}-\sqrt{2}}{\sqrt{6}+\sqrt{2}}\right)-\frac{1}{24}\left(\sqrt{6}+3\sqrt{2}\right)\arctan\left(\frac{4x-\sqrt{6}+\sqrt{2}}{\sqrt{6}+\sqrt{2}}\right)-\frac{1}{24}\left(\sqrt{6}-3\sqrt{2}\right)\arctan\left(\frac{4x+\sqrt{6}+\sqrt{2}}{\sqrt{6}-\sqrt{2}}\right)-\frac{1}{24}\left(\sqrt{6}-3\sqrt{2}\right)\arctan\left(\frac{4x-\sqrt{6}-\sqrt{2}}{\sqrt{6}-\sqrt{2}}\right)+\frac{1}{48}\left(\sqrt{6}+3\sqrt{2}\right)\log\left(x^2+\frac{1}{2}x\left(\sqrt{6}+\sqrt{2}\right)+1\right)-\frac{1}{48}\left(\sqrt{6}+3\sqrt{2}\right)\log\left(x^2-\frac{1}{2}x\left(\sqrt{6}+\sqrt{2}\right)+1\right)+\frac{1}{48}\left(\sqrt{6}-3\sqrt{2}\right)\log\left(x^2+\frac{1}{2}x\left(\sqrt{6}-\sqrt{2}\right)+1\right)-\frac{1}{48}\left(\sqrt{6}-3\sqrt{2}\right)\log\left(x^2-\frac{1}{2}x\left(\sqrt{6}-\sqrt{2}\right)+1\right)-\frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(x^8-x^4+1),x, algorithm="giac")
```

```
[Out] -1/24*(sqrt(6) + 3*sqrt(2))*arctan((4*x + sqrt(6) - sqrt(2))/(sqrt(6) + sqr
t(2))) - 1/24*(sqrt(6) + 3*sqrt(2))*arctan((4*x - sqrt(6) + sqrt(2))/(sqrt(
6) + sqrt(2))) - 1/24*(sqrt(6) - 3*sqrt(2))*arctan((4*x + sqrt(6) + sqrt(2)
)/(sqrt(6) - sqrt(2))) - 1/24*(sqrt(6) - 3*sqrt(2))*arctan((4*x - sqrt(6) -
 sqrt(2))/(sqrt(6) - sqrt(2))) + 1/48*(sqrt(6) + 3*sqrt(2))*log(x^2 + 1/2*x
*(sqrt(6) + sqrt(2)) + 1) - 1/48*(sqrt(6) + 3*sqrt(2))*log(x^2 - 1/2*x*(sqr
t(6) + sqrt(2)) + 1) + 1/48*(sqrt(6) - 3*sqrt(2))*log(x^2 + 1/2*x*(sqrt(6)
- sqrt(2)) + 1) - 1/48*(sqrt(6) - 3*sqrt(2))*log(x^2 - 1/2*x*(sqrt(6) - sqr
t(2)) + 1) - 1/x
```

maple [C] time = 0.02, size = 52, normalized size = 0.14

$$-\frac{\left(\mathrm{RootOf}\left(\_Z^8-\_Z^4+1\right)^6-\mathrm{RootOf}\left(\_Z^8-\_Z^4+1\right)^2\right)\ln\left(-\mathrm{RootOf}\left(\_Z^8-\_Z^4+1\right)+x\right)}{4\left(2\,\mathrm{RootOf}\left(\_Z^8-\_Z^4+1\right)^7-\mathrm{RootOf}\left(\_Z^8-\_Z^4+1\right)^3\right)}-\frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(x^8-x^4+1),x)
```

```
[Out] -1/x-1/4*sum((_R^6-_R^2)/(2*_R^7-_R^3)*ln(-_R+x),_R=RootOf(_Z^8-_Z^4+1))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{x}-\int\frac{x^6-x^2}{x^8-x^4+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(x^8-x^4+1),x, algorithm="maxima")
```

```
[Out] -1/x - integrate((x^6 - x^2)/(x^8 - x^4 + 1), x)
```

mupad [B] time = 1.29, size = 253, normalized size = 0.70

$$-\frac{1}{x}+\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{x\,{\left(8-\sqrt{3}\,8{}\mathrm{i}\right)}^{1/4}}{2\,\left(-1+\sqrt{3}\,1{}\mathrm{i}\right)}+\frac{\sqrt{3}\,x\,{\left(8-\sqrt{3}\,8{}\mathrm{i}\right)}^{1/4}\,1{}\mathrm{i}}{2\,\left(-1+\sqrt{3}\,1{}\mathrm{i}\right)}\right)\,{\left(8-\sqrt{3}\,8{}\mathrm{i}\right)}^{1/4}\,1{}\mathrm{i}}{12}-\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{x\,{\left(8-\sqrt{3}\,8{}\mathrm{i}\right)}^{1/4}\,1{}\mathrm{i}}{2\,\left(-1+\sqrt{3}\,1{}\mathrm{i}\right)}-\frac{\sqrt{3}\,x\,{\left(8-\sqrt{3}\,8{}\mathrm{i}\right)}^{1/4}}{2\,\left(-1+\sqrt{3}\,1{}\mathrm{i}\right)}\right)\,{\left(8-\sqrt{3}\,8{}\mathrm{i}\right)}^{1/4}}{12}+\frac{2^{3/4}\,\sqrt{3}\,\mathrm{atan}\left(\frac{2^{3/4}\,x}{2\,{\left(1+\sqrt{3}\,1{}\mathrm{i}\right)}^{3/4}}-\frac{2^{3/4}\,\sqrt{3}\,x\,1{}\mathrm{i}}{2\,{\left(1+\sqrt{3}\,1{}\mathrm{i}\right)}^{3/4}}\right)\,{\left(1+\sqrt{3}\,1{}\mathrm{i}\right)}^{1/4}\,1{}\mathrm{i}}{12}-\frac{2^{3/4}\,\sqrt{3}\,\mathrm{atan}\left(\frac{2^{3/4}\,x\,1{}\mathrm{i}}{2\,{\left(1+\sqrt{3}\,1{}\mathrm{i}\right)}^{3/4}}+\frac{2^{3/4}\,\sqrt{3}\,x}{2\,{\left(1+\sqrt{3}\,1{}\mathrm{i}\right)}^{3/4}}\right)\,{\left(1+\sqrt{3}\,1{}\mathrm{i}\right)}^{1/4}}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(x^8 - x^4 + 1)),x)
```

```
[Out] (3^(1/2)*atan((x*(8 - 3^(1/2)*8i)^(1/4))/(2*(3^(1/2)*1i - 1)) + (3^(1/2)*x*
(8 - 3^(1/2)*8i)^(1/4)*1i)/(2*(3^(1/2)*1i - 1)))*(8 - 3^(1/2)*8i)^(1/4)*1i)
/12 - 1/x - (3^(1/2)*atan((x*(8 - 3^(1/2)*8i)^(1/4)*1i)/(2*(3^(1/2)*1i - 1)
) - (3^(1/2)*x*(8 - 3^(1/2)*8i)^(1/4))/(2*(3^(1/2)*1i - 1)))*(8 - 3^(1/2)*8
i)^(1/4))/12 + (2^(3/4)*3^(1/2)*atan((2^(3/4)*x)/(2*(3^(1/2)*1i + 1)^(3/4))
 - (2^(3/4)*3^(1/2)*x*1i)/(2*(3^(1/2)*1i + 1)^(3/4)))*(3^(1/2)*1i + 1)^(1/4
)*1i)/12 - (2^(3/4)*3^(1/2)*atan((2^(3/4)*x*1i)/(2*(3^(1/2)*1i + 1)^(3/4))
+ (2^(3/4)*3^(1/2)*x)/(2*(3^(1/2)*1i + 1)^(3/4)))*(3^(1/2)*1i + 1)^(1/4))/1
2
```

sympy [A] time = 3.29, size = 29, normalized size = 0.08

$$\mathrm{RootSum}\left(5308416t^{8}-2304t^{4}+1,\left(t\mapsto t\log\left(-442368t^{7}+384t^{3}+x\right)\right)\right)-\frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(x**8-x**4+1),x)
```

```
[Out] RootSum(5308416*_t**8 - 2304*_t**4 + 1, Lambda(_t, _t*log(-442368*_t**7 + 3
84*_t**3 + x))) - 1/x
```

3.305 $\int \frac{1}{x^4(1-x^4+x^8)} dx$

Optimal. Leaf size=370

$$-\frac{1}{3x^3}+\frac{1}{8}\sqrt{\frac{1}{3}\left(2-\sqrt{3}\right)}\log\left(x^2-\sqrt{2-\sqrt{3}}\,x+1\right)-\frac{1}{8}\sqrt{\frac{1}{3}\left(2-\sqrt{3}\right)}\log\left(x^2+\sqrt{2-\sqrt{3}}\,x+1\right)-\frac{1}{8}\sqrt{\frac{1}{3}\left(2+\sqrt{3}\right)}$$

Rubi [A] time = 0.24, antiderivative size = 370, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {1368, 1421, 1169, 634, 618, 204, 628}

$$-\frac{1}{3x^3}+\frac{1}{8}\sqrt{\frac{1}{3}\left(2-\sqrt{3}\right)}\log\left(x^2-\sqrt{2-\sqrt{3}}\,x+1\right)-\frac{1}{8}\sqrt{\frac{1}{3}\left(2-\sqrt{3}\right)}\log\left(x^2+\sqrt{2-\sqrt{3}}\,x+1\right)-\frac{1}{8}\sqrt{\frac{1}{3}\left(2+\sqrt{3}\right)}\log\left(x^2-\sqrt{2+\sqrt{3}}\,x+1\right)+\frac{1}{8}\sqrt{\frac{1}{3}\left(2+\sqrt{3}\right)}\log\left(x^2+\sqrt{2+\sqrt{3}}\,x+1\right)-\frac{1}{4}\sqrt{\frac{1}{3}\left(2+\sqrt{3}\right)}\tan^{-1}\left(\frac{\sqrt{2-\sqrt{3}}-2x}{\sqrt{2+\sqrt{3}}}\right)+\frac{1}{4}\sqrt{\frac{1}{3}\left(2-\sqrt{3}\right)}\tan^{-1}\left(\frac{\sqrt{2+\sqrt{3}}-2x}{\sqrt{2-\sqrt{3}}}\right)+\frac{1}{4}\sqrt{\frac{1}{3}\left(2+\sqrt{3}\right)}\tan^{-1}\left(\frac{2x+\sqrt{2-\sqrt{3}}}{\sqrt{2+\sqrt{3}}}\right)-\frac{1}{4}\sqrt{\frac{1}{3}\left(2-\sqrt{3}\right)}\tan^{-1}\left(\frac{2x+\sqrt{2+\sqrt{3}}}{\sqrt{2-\sqrt{3}}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(1 - x^4 + x^8)),x]
```

```
[Out] -1/(3*x^3) - (Sqrt[(2 + Sqrt[3])/3]*ArcTan[(Sqrt[2 - Sqrt[3]] - 2*x)/Sqrt[2
 + Sqrt[3]]])/4 + (Sqrt[(2 - Sqrt[3])/3]*ArcTan[(Sqrt[2 + Sqrt[3]] - 2*x)/S
qrt[2 - Sqrt[3]]])/4 + (Sqrt[(2 + Sqrt[3])/3]*ArcTan[(Sqrt[2 - Sqrt[3]] + 2
*x)/Sqrt[2 + Sqrt[3]]])/4 - (Sqrt[(2 - Sqrt[3])/3]*ArcTan[(Sqrt[2 + Sqrt[3]
] + 2*x)/Sqrt[2 - Sqrt[3]]])/4 + (Sqrt[(2 - Sqrt[3])/3]*Log[1 - Sqrt[2 - Sq
rt[3]]*x + x^2])/8 - (Sqrt[(2 - Sqrt[3])/3]*Log[1 + Sqrt[2 - Sqrt[3]]*x + x
^2])/8 - (Sqrt[(2 + Sqrt[3])/3]*Log[1 - Sqrt[2 + Sqrt[3]]*x + x^2])/8 + (Sq
rt[(2 + Sqrt[3])/3]*Log[1 + Sqrt[2 + Sqrt[3]]*x + x^2])/8
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1169

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 1368

```
Int[((d_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_
Symbol] :> Simp[((d*x)^(m + 1)*(a + b*x^n + c*x^(2*n))^(p + 1))/(a*d*(m + 1
)), x] - Dist[1/(a*d^n*(m + 1)), Int[(d*x)^(m + n)*(b*(m + n*(p + 1) + 1) +
 c*(m + 2*n*(p + 1) + 1)*x^n)*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a
, b, c, d, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && L
tQ[m, -1] && IntegerQ[p]
```

Rule 1421

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[(-2*d)/e - b/c, 2]}, Dist[e/(2*c*q), Int[(q - 2*x^
(n/2))/Simp[d/e + q*x^(n/2) - x^n, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*
x^(n/2))/Simp[d/e - q*x^(n/2) - x^n, x], x], x]] /; FreeQ[{a, b, c, d, e},
x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && IGtQ[
n/2, 0] &&  !GtQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{1}{x^4\left(1-x^4+x^8\right)}\,dx = -\frac{1}{3x^3}+\frac{1}{3}\int \frac{3-3x^4}{1-x^4+x^8}\,dx$$

$$= -\frac{1}{3x^3}-\frac{\int \frac{\sqrt{3}+2x^2}{-1-\sqrt{3}x^2-x^4}\,dx}{2\sqrt{3}}-\frac{\int \frac{\sqrt{3}-2x^2}{-1+\sqrt{3}x^2-x^4}\,dx}{2\sqrt{3}}$$

$$= -\frac{1}{3x^3}+\frac{\int \frac{\sqrt{3\left(2-\sqrt{3}\right)}-\left(-2+\sqrt{3}\right)x}{1-\sqrt{2-\sqrt{3}}\,x+x^2}\,dx}{4\sqrt{3\left(2-\sqrt{3}\right)}}+\frac{\int \frac{\sqrt{3\left(2-\sqrt{3}\right)}+\left(-2+\sqrt{3}\right)x}{1+\sqrt{2-\sqrt{3}}\,x+x^2}\,dx}{4\sqrt{3\left(2-\sqrt{3}\right)}}+\frac{\int \frac{\sqrt{3\left(2+\sqrt{3}\right)}-\left(2+\sqrt{3}\right)x}{1-\sqrt{2+\sqrt{3}}\,x+x^2}\,d}{4\sqrt{3\left(2+\sqrt{3}\right)}}$$

$$= -\frac{1}{3x^3}-\frac{1}{8}\sqrt{\frac{1}{3}\left(7-4\sqrt{3}\right)}\int \frac{1}{1-\sqrt{2+\sqrt{3}}\,x+x^2}\,dx-\frac{1}{8}\sqrt{\frac{1}{3}\left(7-4\sqrt{3}\right)}\int \frac{}{1+\sqrt{2-}}$$

$$= -\frac{1}{3x^3}+\frac{1}{8}\sqrt{\frac{1}{3}\left(2-\sqrt{3}\right)}\log\left(1-\sqrt{2-\sqrt{3}}\,x+x^2\right)-\frac{1}{8}\sqrt{\frac{2}{3}-\frac{1}{\sqrt{3}}}\log\left(1+\sqrt{2-\sqrt{3}}\right.$$

$$= -\frac{1}{3x^3}-\frac{1}{4}\sqrt{\frac{1}{3}\left(2+\sqrt{3}\right)}\tan^{-1}\left(\frac{\sqrt{2-\sqrt{3}}-2x}{\sqrt{2+\sqrt{3}}}\right)+\frac{1}{4}\sqrt{\frac{1}{3}\left(2-\sqrt{3}\right)}\tan^{-1}\left(\frac{\sqrt{2+\sqrt{3}}}{\sqrt{2-}}\right.$$

Mathematica [C] time = 0.01, size = 65, normalized size = 0.18

$$-\frac{1}{4}\text{RootSum}\left[\#1^8-\#1^4+1\&,\frac{\#1^4\log(x-\#1)-\log(x-\#1)}{2\#1^7-\#1^3}\&\right]-\frac{1}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(1 - x^4 + x^8)),x]
```

```
[Out] -1/3*1/x^3 - RootSum[1 - #1^4 + #1^8 & , (-Log[x - #1] + Log[x - #1]*#1^4)/
(-#1^3 + 2*#1^7) & ]/4
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^4\left(1-x^4+x^8\right)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x^4*(1 - x^4 + x^8)),x]
```

```
[Out] IntegrateAlgebraic[1/(x^4*(1 - x^4 + x^8)), x]
```

fricas [B]   time = 0.96, size = 756, normalized size = 2.04

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(x^8-x^4+1),x, algorithm="fricas")
```

```
[Out] 1/96*(8*sqrt(6)*sqrt(2)*x^3*sqrt(sqrt(3) + 2)*arctan(1/6*sqrt(6)*sqrt(12*x^
2 + 2*sqrt(6)*(2*sqrt(3)*sqrt(2)*x - 3*sqrt(2)*x)*sqrt(sqrt(3) + 2) + 12)*(
sqrt(3)*sqrt(2) - 2*sqrt(2))*sqrt(sqrt(3) + 2) + 1/3*sqrt(6)*(2*sqrt(3)*sqr
t(2)*x - 3*sqrt(2)*x)*sqrt(sqrt(3) + 2) - sqrt(3) + 2) + 8*sqrt(6)*sqrt(2)*
x^3*sqrt(sqrt(3) + 2)*arctan(1/6*sqrt(6)*sqrt(12*x^2 - 2*sqrt(6)*(2*sqrt(3)
*sqrt(2)*x - 3*sqrt(2)*x)*sqrt(sqrt(3) + 2) + 12)*(sqrt(3)*sqrt(2) - 2*sqrt
(2))*sqrt(sqrt(3) + 2) + 1/3*sqrt(6)*(2*sqrt(3)*sqrt(2)*x - 3*sqrt(2)*x)*sq
rt(sqrt(3) + 2) + sqrt(3) - 2) + 4*sqrt(6)*sqrt(2)*x^3*sqrt(-4*sqrt(3) + 8)
*arctan(1/12*sqrt(6)*sqrt(12*x^2 + sqrt(6)*(2*sqrt(3)*sqrt(2)*x + 3*sqrt(2)
*x)*sqrt(-4*sqrt(3) + 8) + 12)*(sqrt(3)*sqrt(2) + 2*sqrt(2))*sqrt(-4*sqrt(3
) + 8) - 1/6*sqrt(6)*(2*sqrt(3)*sqrt(2)*x + 3*sqrt(2)*x)*sqrt(-4*sqrt(3) +
8) - sqrt(3) - 2) + 4*sqrt(6)*sqrt(2)*x^3*sqrt(-4*sqrt(3) + 8)*arctan(1/12*
sqrt(6)*sqrt(12*x^2 - sqrt(6)*(2*sqrt(3)*sqrt(2)*x + 3*sqrt(2)*x)*sqrt(-4*s
qrt(3) + 8) + 12)*(sqrt(3)*sqrt(2) + 2*sqrt(2))*sqrt(-4*sqrt(3) + 8) - 1/6*
sqrt(6)*(2*sqrt(3)*sqrt(2)*x + 3*sqrt(2)*x)*sqrt(-4*sqrt(3) + 8) + sqrt(3)
+ 2) + 2*sqrt(6)*(sqrt(3)*sqrt(2)*x^3 - 2*sqrt(2)*x^3)*sqrt(sqrt(3) + 2)*lo
g(12*x^2 + 2*sqrt(6)*(2*sqrt(3)*sqrt(2)*x - 3*sqrt(2)*x)*sqrt(sqrt(3) + 2)
+ 12) - 2*sqrt(6)*(sqrt(3)*sqrt(2)*x^3 - 2*sqrt(2)*x^3)*sqrt(sqrt(3) + 2)*l
og(12*x^2 - 2*sqrt(6)*(2*sqrt(3)*sqrt(2)*x - 3*sqrt(2)*x)*sqrt(sqrt(3) + 2)
 + 12) + sqrt(6)*(sqrt(3)*sqrt(2)*x^3 + 2*sqrt(2)*x^3)*sqrt(-4*sqrt(3) + 8)
*log(12*x^2 + sqrt(6)*(2*sqrt(3)*sqrt(2)*x + 3*sqrt(2)*x)*sqrt(-4*sqrt(3) +
 8) + 12) - sqrt(6)*(sqrt(3)*sqrt(2)*x^3 + 2*sqrt(2)*x^3)*sqrt(-4*sqrt(3) +
 8)*log(12*x^2 - sqrt(6)*(2*sqrt(3)*sqrt(2)*x + 3*sqrt(2)*x)*sqrt(-4*sqrt(3
) + 8) + 12) - 32)/x^3
```

giac [A]   time = 0.35, size = 258, normalized size = 0.70

$$\frac{1}{24}\left(\sqrt{6}+3\sqrt{2}\right)\arctan\left(\frac{4x+\sqrt{6}-\sqrt{2}}{\sqrt{6}+\sqrt{2}}\right)+\frac{1}{24}\left(\sqrt{6}+3\sqrt{2}\right)\arctan\left(\frac{4x-\sqrt{6}+\sqrt{2}}{\sqrt{6}+\sqrt{2}}\right)+\frac{1}{24}\left(\sqrt{6}-3\sqrt{2}\right)\arctan\left(\frac{4x+\sqrt{6}+\sqrt{2}}{\sqrt{6}-\sqrt{2}}\right)+\frac{1}{24}\left(\sqrt{6}-3\sqrt{2}\right)\arctan\left(\frac{4x-\sqrt{6}-\sqrt{2}}{\sqrt{6}-\sqrt{2}}\right)+\frac{1}{48}\left(\sqrt{6}+3\sqrt{2}\right)\log\left(x^2+\frac{1}{2}x\left(\sqrt{6}+\sqrt{2}\right)+1\right)-\frac{1}{48}\left(\sqrt{6}+3\sqrt{2}\right)\log\left(x^2-\frac{1}{2}x\left(\sqrt{6}+\sqrt{2}\right)+1\right)+\frac{1}{48}\left(\sqrt{6}-3\sqrt{2}\right)\log\left(x^2+\frac{1}{2}x\left(\sqrt{6}-\sqrt{2}\right)+1\right)-\frac{1}{48}\left(\sqrt{6}-3\sqrt{2}\right)\log\left(x^2-\frac{1}{2}x\left(\sqrt{6}-\sqrt{2}\right)+1\right)-\frac{1}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(x^8-x^4+1),x, algorithm="giac")
```

```
[Out] 1/24*(sqrt(6) + 3*sqrt(2))*arctan((4*x + sqrt(6) - sqrt(2))/(sqrt(6) + sqrt
(2))) + 1/24*(sqrt(6) + 3*sqrt(2))*arctan((4*x - sqrt(6) + sqrt(2))/(sqrt(6
) + sqrt(2))) + 1/24*(sqrt(6) - 3*sqrt(2))*arctan((4*x + sqrt(6) + sqrt(2))
/(sqrt(6) - sqrt(2))) + 1/24*(sqrt(6) - 3*sqrt(2))*arctan((4*x - sqrt(6) -
sqrt(2))/(sqrt(6) - sqrt(2))) + 1/48*(sqrt(6) + 3*sqrt(2))*log(x^2 + 1/2*x*
(sqrt(6) + sqrt(2)) + 1) - 1/48*(sqrt(6) + 3*sqrt(2))*log(x^2 - 1/2*x*(sqrt
(6) + sqrt(2)) + 1) + 1/48*(sqrt(6) - 3*sqrt(2))*log(x^2 + 1/2*x*(sqrt(6) -
 sqrt(2)) + 1) - 1/48*(sqrt(6) - 3*sqrt(2))*log(x^2 - 1/2*x*(sqrt(6) - sqrt
(2)) + 1) - 1/3/x^3
```

maple [C]   time = 0.01, size = 50, normalized size = 0.14

$$\frac{\left(-\mathrm{RootOf}\left(\_Z^8-\_Z^4+1\right)^4+1\right)\ln\left(-\mathrm{RootOf}\left(\_Z^8-\_Z^4+1\right)+x\right)}{8\,\mathrm{RootOf}\left(\_Z^8-\_Z^4+1\right)^7-4\,\mathrm{RootOf}\left(\_Z^8-\_Z^4+1\right)^3}-\frac{1}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(x^8-x^4+1),x)
```

```
[Out] -1/3/x^3+1/4*sum((-_R^4+1)/(2*_R^7-_R^3)*ln(-_R+x),_R=RootOf(_Z^8-_Z^4+1))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{3\,x^{3}}-\int \frac{x^{4}-1}{x^{8}-x^{4}+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(x^8-x^4+1),x, algorithm="maxima")
```

```
[Out] -1/3/x^3 - integrate((x^4 - 1)/(x^8 - x^4 + 1), x)
```

mupad [B] time = 1.29, size = 213, normalized size = 0.58

$$-\frac{1}{3\,x^{3}}-\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{x}{\left(8-\sqrt{3}\,8\mathrm{i}\right)^{1/4}}+\frac{\sqrt{3}\,x\,1\mathrm{i}}{\left(8-\sqrt{3}\,8\mathrm{i}\right)^{1/4}}\right)\,\left(8-\sqrt{3}\,8\mathrm{i}\right)^{1/4}\,1\mathrm{i}}{12}-\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{x\,1\mathrm{i}}{\left(8-\sqrt{3}\,8\mathrm{i}\right)^{1/4}}-\frac{\sqrt{3}\,x}{\left(8-\sqrt{3}\,8\mathrm{i}\right)^{1/4}}\right)\,\left(8-\sqrt{3}\,8\mathrm{i}\right)^{1/4}}{12}+\frac{2^{3/4}\,\sqrt{3}\,\mathrm{atan}\left(\frac{2^{1/4}\,x}{2\,\left(1+\sqrt{3}\,1\mathrm{i}\right)^{1/4}}-\frac{2^{1/4}\,\sqrt{3}\,x\,1\mathrm{i}}{2\,\left(1+\sqrt{3}\,1\mathrm{i}\right)^{1/4}}\right)\,\left(1+\sqrt{3}\,1\mathrm{i}\right)^{1/4}\,1\mathrm{i}}{12}+\frac{2^{3/4}\,\sqrt{3}\,\mathrm{atan}\left(\frac{2^{1/4}\,x\,1\mathrm{i}}{2\,\left(1+\sqrt{3}\,1\mathrm{i}\right)^{1/4}}+\frac{2^{1/4}\,\sqrt{3}\,x}{2\,\left(1+\sqrt{3}\,1\mathrm{i}\right)^{1/4}}\right)\,\left(1+\sqrt{3}\,1\mathrm{i}\right)^{1/4}}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(x^8 - x^4 + 1)),x)
```

```
[Out] (2^(3/4)*3^(1/2)*atan((2^(1/4)*x)/(2*(3^(1/2)*1i + 1)^(1/4)) - (2^(1/4)*3^(
1/2)*x*1i)/(2*(3^(1/2)*1i + 1)^(1/4)))*(3^(1/2)*1i + 1)^(1/4)*1i)/12 - (3^(
1/2)*atan(x/(8 - 3^(1/2)*8i)^(1/4) + (3^(1/2)*x*1i)/(8 - 3^(1/2)*8i)^(1/4))
*(8 - 3^(1/2)*8i)^(1/4)*1i)/12 - (3^(1/2)*atan((x*1i)/(8 - 3^(1/2)*8i)^(1/4
) - (3^(1/2)*x)/(8 - 3^(1/2)*8i)^(1/4))*(8 - 3^(1/2)*8i)^(1/4))/12 - 1/(3*x
^3) + (2^(3/4)*3^(1/2)*atan((2^(1/4)*x*1i)/(2*(3^(1/2)*1i + 1)^(1/4)) + (2^
(1/4)*3^(1/2)*x)/(2*(3^(1/2)*1i + 1)^(1/4)))*(3^(1/2)*1i + 1)^(1/4))/12
```

sympy [A] time = 3.25, size = 31, normalized size = 0.08

$$\operatorname{RootSum}{\left(5308416 t^{8} - 2304 t^{4} + 1, \left(t \mapsto t \log{\left(- 9216 t^{5} + 8 t + x \right)} \right)\right)} - \frac{1}{3 x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(x**8-x**4+1),x)
```

```
[Out] RootSum(5308416*_t**8 - 2304*_t**4 + 1, Lambda(_t, _t*log(-9216*_t**5 + 8*_
t + x))) - 1/(3*x**3)
```

3.306 $\int \frac{1}{x^6\left(1-x^4+x^8\right)}\,dx$

Optimal. Leaf size=287

$$-\frac{1}{5x^5}-\frac{\log\left(x^2-\sqrt{2-\sqrt{3}}\,x+1\right)}{4\sqrt{6}}+\frac{\log\left(x^2+\sqrt{2-\sqrt{3}}\,x+1\right)}{4\sqrt{6}}-\frac{\log\left(x^2-\sqrt{2+\sqrt{3}}\,x+1\right)}{4\sqrt{6}}+\frac{\log\left(x^2+\sqrt{2+\sqrt{3}}\right.}{4\sqrt{6}}$$

Rubi [A] time = 0.24, antiderivative size = 287, normalized size of antiderivative = 1.00, number of steps used = 22, number of rules used = 9, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.562, Rules used = {1368, 1504, 12, 1372, 1169, 634, 618, 204, 628}

$$-\frac{1}{5x^5}-\frac{\log\left(x^2-\sqrt{2-\sqrt{3}}\,x+1\right)}{4\sqrt{6}}+\frac{\log\left(x^2+\sqrt{2-\sqrt{3}}\,x+1\right)}{4\sqrt{6}}-\frac{\log\left(x^2-\sqrt{2+\sqrt{3}}\,x+1\right)}{4\sqrt{6}}+\frac{\log\left(x^2+\sqrt{2+\sqrt{3}}\,x+1\right)}{4\sqrt{6}}-\frac{1}{x}+\frac{\tan^{-1}\left(\frac{\sqrt{2-\sqrt{3}}-2x}{\sqrt{2+\sqrt{3}}}\right)}{2\sqrt{6}}+\frac{\tan^{-1}\left(\frac{\sqrt{2+\sqrt{3}}-2x}{\sqrt{2-\sqrt{3}}}\right)}{2\sqrt{6}}-\frac{\tan^{-1}\left(\frac{2x+\sqrt{2-\sqrt{3}}}{\sqrt{2+\sqrt{3}}}\right)}{2\sqrt{6}}-\frac{\tan^{-1}\left(\frac{2x+\sqrt{2+\sqrt{3}}}{\sqrt{2-\sqrt{3}}}\right)}{2\sqrt{6}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^6*(1 - x^4 + x^8)),x]
```

```
[Out] -1/(5*x^5) - x^(-1) + ArcTan[(Sqrt[2 - Sqrt[3]] - 2*x)/Sqrt[2 + Sqrt[3]]]/(
2*Sqrt[6]) + ArcTan[(Sqrt[2 + Sqrt[3]] - 2*x)/Sqrt[2 - Sqrt[3]]]/(2*Sqrt[6]
) - ArcTan[(Sqrt[2 - Sqrt[3]] + 2*x)/Sqrt[2 + Sqrt[3]]]/(2*Sqrt[6]) - ArcTa
n[(Sqrt[2 + Sqrt[3]] + 2*x)/Sqrt[2 - Sqrt[3]]]/(2*Sqrt[6]) - Log[1 - Sqrt[2
 - Sqrt[3]]*x + x^2]/(4*Sqrt[6]) + Log[1 + Sqrt[2 - Sqrt[3]]*x + x^2]/(4*Sq
rt[6]) - Log[1 - Sqrt[2 + Sqrt[3]]*x + x^2]/(4*Sqrt[6]) + Log[1 + Sqrt[2 +
Sqrt[3]]*x + x^2]/(4*Sqrt[6])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1169

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 1368

```
Int[((d_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_
Symbol] :> Simp[((d*x)^(m + 1)*(a + b*x^n + c*x^(2*n))^(p + 1))/(a*d*(m + 1
)), x] - Dist[1/(a*d^n*(m + 1)), Int[(d*x)^(m + n)*(b*(m + n*(p + 1) + 1) +
 c*(m + 2*n*(p + 1) + 1)*x^n)*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a
, b, c, d, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && L
tQ[m, -1] && IntegerQ[p]
```

Rule 1372

```
Int[(x_)^(m_.)/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symbol] :> W
ith[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, -Dist[1/(2*c*r), Int[(x^
(m - 3*(n/2))*(q - r*x^(n/2)))/(q - r*x^(n/2) + x^n), x], x] + Dist[1/(2*c*
r), Int[(x^(m - 3*(n/2))*(q + r*x^(n/2)))/(q + r*x^(n/2) + x^n), x], x]]] /
; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n/2, 0
] && IGtQ[m, 0] && GeQ[m, (3*n)/2] && LtQ[m, 2*n] && NegQ[b^2 - 4*a*c]
```

Rule 1504

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (
c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Simp[(d*(f*x)^(m + 1)*(a + b*x^n + c*x^
(2*n))^(p + 1))/(a*f*(m + 1)), x] + Dist[1/(a*f^n*(m + 1)), Int[(f*x)^(m +
n)*(a + b*x^n + c*x^(2*n))^p*Simp[a*e*(m + 1) - b*d*(m + n*(p + 1) + 1) - c
*d*(m + 2*n*(p + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x]
 && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && LtQ[m, -1] && Inte
gerQ[p]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x^6\left(1-x^4+x^8\right)}\,dx &= -\frac{1}{5x^5}+\frac{1}{5}\int \frac{5-5x^4}{x^2\left(1-x^4+x^8\right)}\,dx\\
&= -\frac{1}{5x^5}-\frac{1}{x}-\frac{1}{5}\int \frac{5x^6}{1-x^4+x^8}\,dx\\
&= -\frac{1}{5x^5}-\frac{1}{x}-\int \frac{x^6}{1-x^4+x^8}\,dx\\
&= -\frac{1}{5x^5}-\frac{1}{x}+\frac{\int \frac{1-\sqrt{3}x^2}{1-\sqrt{3}x^2+x^4}\,dx}{2\sqrt{3}}-\frac{\int \frac{1+\sqrt{3}x^2}{1+\sqrt{3}x^2+x^4}\,dx}{2\sqrt{3}}\\
&= -\frac{1}{5x^5}-\frac{1}{x}-\frac{\int \frac{\sqrt{2-\sqrt{3}}-\left(1-\sqrt{3}\right)x}{1-\sqrt{2-\sqrt{3}}\,x+x^2}\,dx}{4\sqrt{3\left(2-\sqrt{3}\right)}}-\frac{\int \frac{\sqrt{2-\sqrt{3}}+\left(1-\sqrt{3}\right)x}{1+\sqrt{2-\sqrt{3}}\,x+x^2}\,dx}{4\sqrt{3\left(2-\sqrt{3}\right)}}+\frac{\int \frac{\sqrt{2+\sqrt{3}}-\left(1+\sqrt{3}\right)x}{1-\sqrt{2+\sqrt{3}}\,x+x^2}\,dx}{4\sqrt{3\left(2+\sqrt{3}\right)}}+\cdots\\
&= -\frac{1}{5x^5}-\frac{1}{x}-\frac{\int \frac{-\sqrt{2-\sqrt{3}}+2x}{1-\sqrt{2-\sqrt{3}}\,x+x^2}\,dx}{4\sqrt{6}}+\frac{\int \frac{\sqrt{2-\sqrt{3}}+2x}{1+\sqrt{2-\sqrt{3}}\,x+x^2}\,dx}{4\sqrt{6}}-\frac{\int \frac{-\sqrt{2+\sqrt{3}}+2x}{1-\sqrt{2+\sqrt{3}}\,x+x^2}\,dx}{4\sqrt{6}}+\frac{\int \frac{\sqrt{2+\sqrt{3}}\cdots}{1+\sqrt{2+\sqrt{3}}\cdots}}{4\sqrt{6}\cdots}\\
&= -\frac{1}{5x^5}-\frac{1}{x}-\frac{\log\left(1-\sqrt{2-\sqrt{3}}\,x+x^2\right)}{4\sqrt{6}}+\frac{\log\left(1+\sqrt{2-\sqrt{3}}\,x+x^2\right)}{4\sqrt{6}}-\frac{\log\left(1-\sqrt{2+\cdots}\right.}{4\sqrt{6}}\\
&= -\frac{1}{5x^5}-\frac{1}{x}+\frac{\tan^{-1}\left(\frac{\sqrt{2-\sqrt{3}}-2x}{\sqrt{2+\sqrt{3}}}\right)}{2\sqrt{6}}+\frac{\tan^{-1}\left(\frac{\sqrt{2+\sqrt{3}}-2x}{\sqrt{2-\sqrt{3}}}\right)}{2\sqrt{6}}-\frac{\tan^{-1}\left(\frac{\sqrt{2-\sqrt{3}}+2x}{\sqrt{2+\sqrt{3}}}\right)}{2\sqrt{6}}-\frac{\tan^{-1}\left(\cdots\right)}{2\sqrt{6}\cdots}
\end{aligned}
$$

Mathematica [C] time = 0.02, size = 54, normalized size = 0.19

$$
-\frac{1}{4}\text{RootSum}\left[\#1^8-\#1^4+1\&,\frac{\#1^3\log(x-\#1)}{2\#1^4-1}\&\right]-\frac{1}{5x^5}-\frac{1}{x}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^6*(1 - x^4 + x^8)),x]
```

```
[Out] -1/5*1/x^5 - x^(-1) - RootSum[1 - #1^4 + #1^8 & , (Log[x - #1]*#1^3)/(-1 + 
2*#1^4) & ]/4
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$
\int \frac{1}{x^6\left(1-x^4+x^8\right)}\,dx
$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x^6*(1 - x^4 + x^8)),x]
```

```
[Out] IntegrateAlgebraic[1/(x^6*(1 - x^4 + x^8)), x]
```

fricas [A] time = 2.13, size = 238, normalized size = 0.83

$$
\frac{20\sqrt{3}\sqrt{2}x^5\arctan\left(-\frac{\sqrt{3}\sqrt{2}\left(x^3-x\right)+x^2-\sqrt{x^4+\sqrt{3}\sqrt{2}\left(x^3+x\right)+3x^2+1}\left(\sqrt{3}\sqrt{2}x-2\right)}{3x^2-2}\right)+20\sqrt{3}\sqrt{2}x^5\arctan\left(-\frac{\sqrt{3}\sqrt{2}\left(x^3-x\right)-x^2-\sqrt{x^4-\sqrt{3}\sqrt{2}\left(x^3+x\right)+3x^2+1}\left(\sqrt{3}\sqrt{2}x+2\right)}{3x^2-2}\right)+5\sqrt{3}\sqrt{2}x^5\log\left(x^4+\sqrt{3}\sqrt{2}\left(x^3+x\right)+3x^2+1\right)-5\sqrt{3}\sqrt{2}x^5\log\left(x^4-\sqrt{3}\sqrt{2}\left(x^3+x\right)+3x^2+1\right)-120x^4-24}{120x^5}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(x^8-x^4+1),x, algorithm="fricas")
```

```
[Out] 1/120*(20*sqrt(3)*sqrt(2)*x^5*arctan(-(sqrt(3)*sqrt(2)*(x^3 - x) + x^2 - sq
rt(x^4 + sqrt(3)*sqrt(2)*(x^3 + x) + 3*x^2 + 1)*(sqrt(3)*sqrt(2)*x - 2))/(3
*x^2 - 2)) + 20*sqrt(3)*sqrt(2)*x^5*arctan(-(sqrt(3)*sqrt(2)*(x^3 - x) - x^
2 - sqrt(x^4 - sqrt(3)*sqrt(2)*(x^3 + x) + 3*x^2 + 1)*(sqrt(3)*sqrt(2)*x +
2))/(3*x^2 - 2)) + 5*sqrt(3)*sqrt(2)*x^5*log(x^4 + sqrt(3)*sqrt(2)*(x^3 + x
) + 3*x^2 + 1) - 5*sqrt(3)*sqrt(2)*x^5*log(x^4 - sqrt(3)*sqrt(2)*(x^3 + x)
+ 3*x^2 + 1) - 120*x^4 - 24)/x^5
```

giac [A]   time = 0.38, size = 217, normalized size = 0.76

$$-\frac{1}{12}\sqrt{6}\arctan\left(\frac{4x+\sqrt{6}-\sqrt{2}}{\sqrt{6}+\sqrt{2}}\right)-\frac{1}{12}\sqrt{6}\arctan\left(\frac{4x-\sqrt{6}+\sqrt{2}}{\sqrt{6}+\sqrt{2}}\right)-\frac{1}{12}\sqrt{6}\arctan\left(\frac{4x+\sqrt{6}+\sqrt{2}}{\sqrt{6}-\sqrt{2}}\right)-\frac{1}{12}\sqrt{6}\arctan\left(\frac{4x-\sqrt{6}-\sqrt{2}}{\sqrt{6}-\sqrt{2}}\right)+\frac{1}{24}\sqrt{6}\log\left(x^2+\frac{1}{2}x\left(\sqrt{6}+\sqrt{2}\right)+1\right)-\frac{1}{24}\sqrt{6}\log\left(x^2-\frac{1}{2}x\left(\sqrt{6}+\sqrt{2}\right)+1\right)+\frac{1}{24}\sqrt{6}\log\left(x^2+\frac{1}{2}x\left(\sqrt{6}-\sqrt{2}\right)+1\right)-\frac{1}{24}\sqrt{6}\log\left(x^2-\frac{1}{2}x\left(\sqrt{6}-\sqrt{2}\right)+1\right)-\frac{5x^4+1}{5x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(x^8-x^4+1),x, algorithm="giac")
```

```
[Out] -1/12*sqrt(6)*arctan((4*x + sqrt(6) - sqrt(2))/(sqrt(6) + sqrt(2))) - 1/12*
sqrt(6)*arctan((4*x - sqrt(6) + sqrt(2))/(sqrt(6) + sqrt(2))) - 1/12*sqrt(6
)*arctan((4*x + sqrt(6) + sqrt(2))/(sqrt(6) - sqrt(2))) - 1/12*sqrt(6)*arct
an((4*x - sqrt(6) - sqrt(2))/(sqrt(6) - sqrt(2))) + 1/24*sqrt(6)*log(x^2 +
1/2*x*(sqrt(6) + sqrt(2)) + 1) - 1/24*sqrt(6)*log(x^2 - 1/2*x*(sqrt(6) + sq
rt(2)) + 1) + 1/24*sqrt(6)*log(x^2 + 1/2*x*(sqrt(6) - sqrt(2)) + 1) - 1/24*
sqrt(6)*log(x^2 - 1/2*x*(sqrt(6) - sqrt(2)) + 1) - 1/5*(5*x^4 + 1)/x^5
```

maple [C]   time = 0.01, size = 43, normalized size = 0.15

$$-\frac{\mathrm{RootOf}\left(9\_Z^4+1\right)\ln\left(9\,\mathrm{RootOf}\left(9\_Z^4+1\right)^3 x-3\,\mathrm{RootOf}\left(9\_Z^4+1\right)^2+x^2\right)}{4}-\frac{1}{x}-\frac{1}{5x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^6/(x^8-x^4+1),x)
```

```
[Out] -1/5/x^5-1/x-1/4*sum(_R*ln(9*_R^3*x-3*_R^2+x^2),_R=RootOf(9*_Z^4+1))
```

maxima [F]   time = 0.00, size = 0, normalized size = 0.00

$$-\frac{5\,x^4+1}{5\,x^5}-\int\frac{x^6}{x^8-x^4+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(x^8-x^4+1),x, algorithm="maxima")
```

```
[Out] -1/5*(5*x^4 + 1)/x^5 - integrate(x^6/(x^8 - x^4 + 1), x)
```

mupad [B]   time = 1.30, size = 63, normalized size = 0.22

$$-\frac{x^4+\frac{1}{5}}{x^5}+\sqrt{6}\,\mathrm{atan}\left(\frac{\sqrt{6}\,x\,\left(\frac{1}{3}+\frac{1}{3}\mathrm{i}\right)}{\frac{2\,x^2}{3}-\frac{2}{3}\mathrm{i}}\right)\,\left(\frac{1}{12}-\frac{1}{12}\mathrm{i}\right)+\sqrt{6}\,\mathrm{atan}\left(\frac{\sqrt{6}\,x\,\left(\frac{1}{3}-\frac{1}{3}\mathrm{i}\right)}{\frac{2\,x^2}{3}+\frac{2}{3}\mathrm{i}}\right)\,\left(\frac{1}{12}+\frac{1}{12}\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^6*(x^8 - x^4 + 1)),x)
```

```
[Out] 6^(1/2)*atan((6^(1/2)*x*(1/3 + 1i/3))/((2*x^2)/3 - 2i/3))*(1/12 - 1i/12) +
6^(1/2)*atan((6^(1/2)*x*(1/3 - 1i/3))/((2*x^2)/3 + 2i/3))*(1/12 + 1i/12) -
(x^4 + 1/5)/x^5
```

sympy [**A**] time = 0.27, size = 182, normalized size = 0.63

$$\frac{\sqrt{6}\left(-2\,\mathrm{atan}\left(\frac{\sqrt{6}x}{3}-\frac{1}{3}\right)-2\,\mathrm{atan}\left(\sqrt{6}x^3-4x^2+2\sqrt{6}x-3\right)\right)}{24}+\frac{\sqrt{6}\left(-2\,\mathrm{atan}\left(\frac{\sqrt{6}x}{3}+\frac{1}{3}\right)-2\,\mathrm{atan}\left(\sqrt{6}x^3+4x^2+2\sqrt{6}x+3\right)\right)}{24}-\frac{\sqrt{6}\log\left(x^4-\sqrt{6}x^3+3x^2-\sqrt{6}x+1\right)}{24}+\frac{\sqrt{6}\log\left(x^4+\sqrt{6}x^3+3x^2+\sqrt{6}x+1\right)}{24}+\frac{-5x^4-1}{5x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**6/(x**8-x**4+1),x)
```

```
[Out] sqrt(6)*(-2*atan(sqrt(6)*x/3 - 1/3) - 2*atan(sqrt(6)*x**3 - 4*x**2 + 2*sqrt
(6)*x - 3))/24 + sqrt(6)*(-2*atan(sqrt(6)*x/3 + 1/3) - 2*atan(sqrt(6)*x**3
+ 4*x**2 + 2*sqrt(6)*x + 3))/24 - sqrt(6)*log(x**4 - sqrt(6)*x**3 + 3*x**2
- sqrt(6)*x + 1)/24 + sqrt(6)*log(x**4 + sqrt(6)*x**3 + 3*x**2 + sqrt(6)*x
+ 1)/24 + (-5*x**4 - 1)/(5*x**5)
```

3.307 $\int \frac{1}{x^8(1-x^4+x^8)}\,dx$

Optimal. Leaf size=377

$$-\frac{1}{7x^7}-\frac{1}{3x^3}+\frac{1}{8}\sqrt{\frac{1}{3}\left(2+\sqrt{3}\right)}\log\left(x^2-\sqrt{2-\sqrt{3}}\,x+1\right)-\frac{1}{8}\sqrt{\frac{1}{3}\left(2+\sqrt{3}\right)}\log\left(x^2+\sqrt{2-\sqrt{3}}\,x+1\right)-\frac{1}{8}\sqrt{\frac{1}{3}\left(2\right.}$$

Rubi [A] time = 0.29, antiderivative size = 377, normalized size of antiderivative = 1.00, number of steps used = 22, number of rules used = 10, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.625, Rules used = {1368, 1504, 12, 1373, 1127, 1161, 618, 204, 1164, 628}

$$-\frac{1}{3x^3}-\frac{1}{7x^7}+\frac{1}{8}\sqrt{\frac{1}{3}\left(2+\sqrt{3}\right)}\log\left(x^2-\sqrt{2-\sqrt{3}}\,x+1\right)-\frac{1}{8}\sqrt{\frac{1}{3}\left(2+\sqrt{3}\right)}\log\left(x^2+\sqrt{2-\sqrt{3}}\,x+1\right)-\frac{1}{8}\sqrt{\frac{1}{3}\left(2-\sqrt{3}\right)}\log\left(x^2-\sqrt{2+\sqrt{3}}\,x+1\right)+\frac{1}{8}\sqrt{\frac{1}{3}\left(2-\sqrt{3}\right)}\log\left(x^2+\sqrt{2+\sqrt{3}}\,x+1\right)-\frac{1}{4}\sqrt{\frac{1}{3}\left(2-\sqrt{3}\right)}\tan^{-1}\left(\frac{\sqrt{2-\sqrt{3}}-2x}{\sqrt{2+\sqrt{3}}}\right)+\frac{1}{4}\sqrt{\frac{1}{3}\left(2+\sqrt{3}\right)}\tan^{-1}\left(\frac{\sqrt{2+\sqrt{3}}-2x}{\sqrt{2-\sqrt{3}}}\right)+\frac{1}{4}\sqrt{\frac{1}{3}\left(2-\sqrt{3}\right)}\tan^{-1}\left(\frac{2x+\sqrt{2-\sqrt{3}}}{\sqrt{2+\sqrt{3}}}\right)-\frac{1}{4}\sqrt{\frac{1}{3}\left(2+\sqrt{3}\right)}\tan^{-1}\left(\frac{2x+\sqrt{2+\sqrt{3}}}{\sqrt{2-\sqrt{3}}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^8*(1 - x^4 + x^8)),x]
```

```
[Out] -1/(7*x^7) - 1/(3*x^3) - (Sqrt[(2 - Sqrt[3])/3]*ArcTan[(Sqrt[2 - Sqrt[3]] - 2*x)/Sqrt[2 + Sqrt[3]]])/4 + (Sqrt[(2 + Sqrt[3])/3]*ArcTan[(Sqrt[2 + Sqrt[3]] - 2*x)/Sqrt[2 - Sqrt[3]]])/4 + (Sqrt[(2 - Sqrt[3])/3]*ArcTan[(Sqrt[2 - Sqrt[3]] + 2*x)/Sqrt[2 + Sqrt[3]]])/4 - (Sqrt[(2 + Sqrt[3])/3]*ArcTan[(Sqrt[2 + Sqrt[3]] + 2*x)/Sqrt[2 - Sqrt[3]]])/4 + (Sqrt[(2 + Sqrt[3])/3]*Log[1 - Sqrt[2 - Sqrt[3]]*x + x^2])/8 - (Sqrt[(2 + Sqrt[3])/3]*Log[1 + Sqrt[2 - Sqrt[3]]*x + x^2])/8 - (Sqrt[(2 - Sqrt[3])/3]*Log[1 - Sqrt[2 + Sqrt[3]]*x + x^2])/8 + (Sqrt[(2 - Sqrt[3])/3]*Log[1 + Sqrt[2 + Sqrt[3]]*x + x^2])/8
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !MatchQ[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1127

```
Int[(x_)^2/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[a/c, 2]}, Dist[1/2, Int[(q + x^2)/(a + b*x^2 + c*x^4), x], x] - Dist[1/2, Int[(q - x^2)/(a + b*x^2 + c*x^4), x], x]] /; FreeQ[{a, b, c}, x] && LtQ[b^2 - 4*a*c, 0] && PosQ[a*c]
```

Rule 1161

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[(2*d)/e - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2
, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (
GtQ[(2*d)/e - b/c, 0] || ( !LtQ[(2*d)/e - b/c, 0] && EqQ[d - e*Rt[a/c, 2],
0]))
```

Rule 1164

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[(-2*d)/e - b/c, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e
+ q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^
2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c
*d^2 - a*e^2, 0] &&  !GtQ[b^2 - 4*a*c, 0]
```

Rule 1368

```
Int[((d_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_
Symbol] :> Simp[((d*x)^(m + 1)*(a + b*x^n + c*x^(2*n))^(p + 1))/(a*d*(m + 1
)), x] - Dist[1/(a*d^n*(m + 1)), Int[(d*x)^(m + n)*(b*(m + n*(p + 1) + 1) +
 c*(m + 2*n*(p + 1) + 1)*x^n)*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a
, b, c, d, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && L
tQ[m, -1] && IntegerQ[p]
```

Rule 1373

```
Int[(x_)^(m_.)/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symbol] :> W
ith[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*r), Int[x^(m
 - n/2)/(q - r*x^(n/2) + x^n), x], x] - Dist[1/(2*c*r), Int[x^(m - n/2)/(q
+ r*x^(n/2) + x^n), x], x]]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[n/2, 0] && IGtQ[m, 0] && GeQ[m, n/2] && LtQ[m, (3*n
)/2] && NegQ[b^2 - 4*a*c]
```

Rule 1504

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (
c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Simp[(d*(f*x)^(m + 1)*(a + b*x^n + c*x^
(2*n))^(p + 1))/(a*f*(m + 1)), x] + Dist[1/(a*f^n*(m + 1)), Int[(f*x)^(m +
n)*(a + b*x^n + c*x^(2*n))^p*Simp[a*e*(m + 1) - b*d*(m + n*(p + 1) + 1) - c
*d*(m + 2*n*(p + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x]
 && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && LtQ[m, -1] && Inte
gerQ[p]
```

Rubi steps

$$\int \frac{1}{x^8\left(1-x^4+x^8\right)}\,dx = -\frac{1}{7x^7}+\frac{1}{7}\int \frac{7-7x^4}{x^4\left(1-x^4+x^8\right)}\,dx$$
$$= -\frac{1}{7x^7}-\frac{1}{3x^3}-\frac{1}{21}\int \frac{21x^4}{1-x^4+x^8}\,dx$$
$$= -\frac{1}{7x^7}-\frac{1}{3x^3}-\int \frac{x^4}{1-x^4+x^8}\,dx$$
$$= -\frac{1}{7x^7}-\frac{1}{3x^3}-\frac{\int \frac{x^2}{1-\sqrt{3}x^2+x^4}\,dx}{2\sqrt{3}}+\frac{\int \frac{x^2}{1+\sqrt{3}x^2+x^4}\,dx}{2\sqrt{3}}$$
$$= -\frac{1}{7x^7}-\frac{1}{3x^3}+\frac{\int \frac{1-x^2}{1-\sqrt{3}x^2+x^4}\,dx}{4\sqrt{3}}-\frac{\int \frac{1+x^2}{1-\sqrt{3}x^2+x^4}\,dx}{4\sqrt{3}}-\frac{\int \frac{1-x^2}{1+\sqrt{3}x^2+x^4}\,dx}{4\sqrt{3}}+\frac{\int \frac{1+x^2}{1+\sqrt{3}x^2+x^4}\,dx}{4\sqrt{3}}$$
$$= -\frac{1}{7x^7}-\frac{1}{3x^3}+\frac{\int \frac{1}{1-\sqrt{2-\sqrt{3}}\,x+x^2}\,dx}{8\sqrt{3}}+\frac{\int \frac{1}{1+\sqrt{2-\sqrt{3}}\,x+x^2}\,dx}{8\sqrt{3}}-\frac{\int \frac{1}{1-\sqrt{2+\sqrt{3}}\,x+x^2}\,dx}{8\sqrt{3}}-\frac{\int \frac{}{1+}}{}$$
$$= -\frac{1}{7x^7}-\frac{1}{3x^3}+\frac{\log\left(1-\sqrt{2-\sqrt{3}}\,x+x^2\right)}{8\sqrt{3\left(2-\sqrt{3}\right)}}-\frac{\log\left(1+\sqrt{2-\sqrt{3}}\,x+x^2\right)}{8\sqrt{3\left(2-\sqrt{3}\right)}}-\frac{\log\left(1-\sqrt{}\right.}{8\sqrt{3}}$$
$$= -\frac{1}{7x^7}-\frac{1}{3x^3}-\frac{\tan^{-1}\left(\frac{\sqrt{2-\sqrt{3}}-2x}{\sqrt{2+\sqrt{3}}}\right)}{4\sqrt{3\left(2+\sqrt{3}\right)}}+\frac{\tan^{-1}\left(\frac{\sqrt{2+\sqrt{3}}-2x}{\sqrt{2-\sqrt{3}}}\right)}{4\sqrt{3\left(2-\sqrt{3}\right)}}+\frac{\tan^{-1}\left(\frac{\sqrt{2-\sqrt{3}}+2x}{\sqrt{2+\sqrt{3}}}\right)}{4\sqrt{3\left(2+\sqrt{3}\right)}}-\frac{\tan^{-}}{4\sqrt{}}$$

Mathematica [C] time = 0.01, size = 54, normalized size = 0.14

$$-\frac{1}{4}\text{RootSum}\left[\#1^8-\#1^4+1\&,\frac{\#1\log(x-\#1)}{2\#1^4-1}\&\right]-\frac{1}{7x^7}-\frac{1}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^8*(1 - x^4 + x^8)),x]
[Out] -1/7*1/x^7 - 1/(3*x^3) - RootSum[1 - #1^4 + #1^8 & , (Log[x - #1]*#1)/(-1 +
 2*#1^4) & ]/4
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^8\left(1-x^4+x^8\right)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x^8*(1 - x^4 + x^8)),x]
[Out] IntegrateAlgebraic[1/(x^8*(1 - x^4 + x^8)), x]
```

fricas [B] time = 1.73, size = 614, normalized size = 1.63

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^8/(x^8-x^4+1),x, algorithm="fricas")
```

```
[Out] 1/672*(56*sqrt(6)*sqrt(2)*x^7*sqrt(sqrt(3) + 2)*arctan(-1/3*sqrt(6)*sqrt(3)
*sqrt(2)*x*sqrt(sqrt(3) + 2) + 1/6*sqrt(6)*sqrt(2)*sqrt(2*sqrt(6)*sqrt(3)*s
qrt(2)*x*sqrt(sqrt(3) + 2) + 12*x^2 + 12)*sqrt(sqrt(3) + 2) - sqrt(3) - 2)
+ 56*sqrt(6)*sqrt(2)*x^7*sqrt(sqrt(3) + 2)*arctan(-1/3*sqrt(6)*sqrt(3)*sqrt
(2)*x*sqrt(sqrt(3) + 2) + 1/6*sqrt(6)*sqrt(2)*sqrt(-2*sqrt(6)*sqrt(3)*sqrt(
2)*x*sqrt(sqrt(3) + 2) + 12*x^2 + 12)*sqrt(sqrt(3) + 2) + sqrt(3) + 2) - 28
*sqrt(6)*sqrt(2)*x^7*sqrt(-4*sqrt(3) + 8)*arctan(-1/6*sqrt(6)*sqrt(3)*sqrt(
2)*x*sqrt(-4*sqrt(3) + 8) + 1/12*sqrt(6)*sqrt(2)*sqrt(sqrt(6)*sqrt(3)*sqrt(
2)*x*sqrt(-4*sqrt(3) + 8) + 12*x^2 + 12)*sqrt(-4*sqrt(3) + 8) + sqrt(3) - 2
) - 28*sqrt(6)*sqrt(2)*x^7*sqrt(-4*sqrt(3) + 8)*arctan(-1/6*sqrt(6)*sqrt(3)
*sqrt(2)*x*sqrt(-4*sqrt(3) + 8) + 1/12*sqrt(6)*sqrt(2)*sqrt(-sqrt(6)*sqrt(3
)*sqrt(2)*x*sqrt(-4*sqrt(3) + 8) + 12*x^2 + 12)*sqrt(-4*sqrt(3) + 8) - sqrt
(3) + 2) - 224*x^4 - 14*sqrt(6)*(sqrt(3)*sqrt(2)*x^7 - 2*sqrt(2)*x^7)*sqrt(
sqrt(3) + 2)*log(2*sqrt(6)*sqrt(3)*sqrt(2)*x*sqrt(sqrt(3) + 2) + 12*x^2 + 1
2) + 14*sqrt(6)*(sqrt(3)*sqrt(2)*x^7 - 2*sqrt(2)*x^7)*sqrt(sqrt(3) + 2)*log
(-2*sqrt(6)*sqrt(3)*sqrt(2)*x*sqrt(sqrt(3) + 2) + 12*x^2 + 12) - 7*sqrt(6)*
(sqrt(3)*sqrt(2)*x^7 + 2*sqrt(2)*x^7)*sqrt(-4*sqrt(3) + 8)*log(sqrt(6)*sqrt
(3)*sqrt(2)*x*sqrt(-4*sqrt(3) + 8) + 12*x^2 + 12) + 7*sqrt(6)*(sqrt(3)*sqrt
(2)*x^7 + 2*sqrt(2)*x^7)*sqrt(-4*sqrt(3) + 8)*log(-sqrt(6)*sqrt(3)*sqrt(2)*
x*sqrt(-4*sqrt(3) + 8) + 12*x^2 + 12) - 96)/x^7
```

giac [A] time = 0.38, size = 265, normalized size = 0.70

$$-\frac{1}{24}\left(\sqrt{6}-3\sqrt{2}\right)\arctan\left(\frac{4x+\sqrt{6}-\sqrt{2}}{\sqrt{6}+\sqrt{2}}\right)-\frac{1}{24}\left(\sqrt{6}-3\sqrt{2}\right)\arctan\left(\frac{4x-\sqrt{6}+\sqrt{2}}{\sqrt{6}+\sqrt{2}}\right)-\frac{1}{24}\left(\sqrt{6}+3\sqrt{2}\right)\arctan\left(\frac{4x+\sqrt{6}+\sqrt{2}}{\sqrt{6}-\sqrt{2}}\right)-\frac{1}{24}\left(\sqrt{6}+3\sqrt{2}\right)\arctan\left(\frac{4x-\sqrt{6}-\sqrt{2}}{\sqrt{6}-\sqrt{2}}\right)-\frac{1}{48}\left(\sqrt{6}-3\sqrt{2}\right)\log\left(x^2+\frac{1}{2}x\left(\sqrt{6}+\sqrt{2}\right)+1\right)+\frac{1}{48}\left(\sqrt{6}-3\sqrt{2}\right)\log\left(x^2-\frac{1}{2}x\left(\sqrt{6}+\sqrt{2}\right)+1\right)-\frac{1}{48}\left(\sqrt{6}+3\sqrt{2}\right)\log\left(x^2+\frac{1}{2}x\left(\sqrt{6}-\sqrt{2}\right)+1\right)+\frac{1}{48}\left(\sqrt{6}+3\sqrt{2}\right)\log\left(x^2-\frac{1}{2}x\left(\sqrt{6}-\sqrt{2}\right)+1\right)-\frac{7x^4+3}{21x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^8/(x^8-x^4+1),x, algorithm="giac")
```

```
[Out] -1/24*(sqrt(6) - 3*sqrt(2))*arctan((4*x + sqrt(6) - sqrt(2))/(sqrt(6) + sqr
t(2))) - 1/24*(sqrt(6) - 3*sqrt(2))*arctan((4*x - sqrt(6) + sqrt(2))/(sqrt(
6) + sqrt(2))) - 1/24*(sqrt(6) + 3*sqrt(2))*arctan((4*x + sqrt(6) + sqrt(2)
)/(sqrt(6) - sqrt(2))) - 1/24*(sqrt(6) + 3*sqrt(2))*arctan((4*x - sqrt(6) -
 sqrt(2))/(sqrt(6) - sqrt(2))) - 1/48*(sqrt(6) - 3*sqrt(2))*log(x^2 + 1/2*x
*(sqrt(6) + sqrt(2)) + 1) + 1/48*(sqrt(6) - 3*sqrt(2))*log(x^2 - 1/2*x*(sqr
t(6) + sqrt(2)) + 1) - 1/48*(sqrt(6) + 3*sqrt(2))*log(x^2 + 1/2*x*(sqrt(6)
- sqrt(2)) + 1) + 1/48*(sqrt(6) + 3*sqrt(2))*log(x^2 - 1/2*x*(sqrt(6) - sqr
t(2)) + 1) - 1/21*(7*x^4 + 3)/x^7
```

maple [C] time = 0.02, size = 51, normalized size = 0.14

$$-\frac{\mathrm{RootOf}\left(\_Z^8-\_Z^4+1\right)^4\ln\left(-\mathrm{RootOf}\left(\_Z^8-\_Z^4+1\right)+x\right)}{4\left(2\,\mathrm{RootOf}\left(\_Z^8-\_Z^4+1\right)^7-\mathrm{RootOf}\left(\_Z^8-\_Z^4+1\right)^3\right)}-\frac{1}{3x^3}-\frac{1}{7x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^8/(x^8-x^4+1),x)
```

```
[Out] -1/7/x^7-1/3/x^3-1/4*sum(1/(2*_R^7-_R^3)*_R^4*ln(-_R+x),_R=RootOf(_Z^8-_Z^4
+1))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{7\,x^4+3}{21\,x^7}-\int\frac{x^4}{x^8-x^4+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^8/(x^8-x^4+1),x, algorithm="maxima")
```

```
[Out] -1/21*(7*x^4 + 3)/x^7 - integrate(x^4/(x^8 - x^4 + 1), x)
```

mupad [B] time = 0.06, size = 486, normalized size = 1.29

$$-\frac{\frac{x^4}{3}+\frac{1}{7}}{x^7}+\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{x\left(8-\sqrt{3}\,8i\right)^{1/4}}{2\left(\frac{\sqrt{3}\,\sqrt{8-\sqrt{3}\,8i}\,1i}{4}+\frac{\sqrt{8-\sqrt{3}\,8i}}{4}\right)}+\frac{\sqrt{3}\,x\left(8-\sqrt{3}\,8i\right)^{1/4}\,1i}{2\left(\frac{\sqrt{3}\,\sqrt{8-\sqrt{3}\,8i}\,1i}{4}+\frac{\sqrt{8-\sqrt{3}\,8i}}{4}\right)}\right)\left(8-\sqrt{3}\,8i\right)^{1/4}\,1i}{12}+\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{x\left(8-\sqrt{3}\,8i\right)^{1/4}\,1i}{2\left(\frac{\sqrt{3}\,\sqrt{8-\sqrt{3}\,8i}\,1i}{4}+\frac{\sqrt{8-\sqrt{3}\,8i}}{4}\right)}-\frac{\sqrt{3}\,x\left(8-\sqrt{3}\,8i\right)^{1/4}}{2\left(\frac{\sqrt{3}\,\sqrt{8-\sqrt{3}\,8i}\,1i}{4}+\frac{\sqrt{8-\sqrt{3}\,8i}}{4}\right)}\right)\left(8-\sqrt{3}\,8i\right)^{1/4}}{12}-\frac{2^{3/4}\,\sqrt{3}\,\mathrm{atan}\left(\frac{2^{3/4}\,x\left(1+\sqrt{3}\,1i\right)^{1/4}}{2\left(\frac{\sqrt{2}\,\sqrt{1+\sqrt{3}\,1i}}{2}-\frac{\sqrt{2}\,\sqrt{3}\,\sqrt{1+\sqrt{3}\,1i}\,1i}{2}\right)}-\frac{2^{3/4}\,\sqrt{3}\,x\left(1+\sqrt{3}\,1i\right)^{1/4}\,1i}{2\left(\frac{\sqrt{2}\,\sqrt{1+\sqrt{3}\,1i}}{2}-\frac{\sqrt{2}\,\sqrt{3}\,\sqrt{1+\sqrt{3}\,1i}\,1i}{2}\right)}\right)\left(1+\sqrt{3}\,1i\right)^{1/4}\,1i}{12}-\frac{2^{3/4}\,\sqrt{3}\,\mathrm{atan}\left(\frac{2^{3/4}\,x\left(1+\sqrt{3}\,1i\right)^{1/4}\,1i}{2\left(\frac{\sqrt{2}\,\sqrt{1+\sqrt{3}\,1i}}{2}-\frac{\sqrt{2}\,\sqrt{3}\,\sqrt{1+\sqrt{3}\,1i}\,1i}{2}\right)}+\frac{2^{3/4}\,\sqrt{3}\,x\left(1+\sqrt{3}\,1i\right)^{1/4}}{2\left(\frac{\sqrt{2}\,\sqrt{1+\sqrt{3}\,1i}}{2}-\frac{\sqrt{2}\,\sqrt{3}\,\sqrt{1+\sqrt{3}\,1i}\,1i}{2}\right)}\right)\left(1+\sqrt{3}\,1i\right)^{1/4}}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^8*(x^8 - x^4 + 1)),x)
```

```
[Out] (3^(1/2)*atan((x*(8 - 3^(1/2)*8i)^(1/4))/(2*((3^(1/2)*(8 - 3^(1/2)*8i)^(1/2
)*1i)/4 + (8 - 3^(1/2)*8i)^(1/2)/4)) + (3^(1/2)*x*(8 - 3^(1/2)*8i)^(1/4)*1i
)/(2*((3^(1/2)*(8 - 3^(1/2)*8i)^(1/2)*1i)/4 + (8 - 3^(1/2)*8i)^(1/2)/4)))*(
8 - 3^(1/2)*8i)^(1/4)*1i)/12 - (x^4/3 + 1/7)/x^7 + (3^(1/2)*atan((x*(8 - 3^
(1/2)*8i)^(1/4)*1i)/(2*((3^(1/2)*(8 - 3^(1/2)*8i)^(1/2)*1i)/4 + (8 - 3^(1/2
)*8i)^(1/2)/4)) - (3^(1/2)*x*(8 - 3^(1/2)*8i)^(1/4))/(2*((3^(1/2)*(8 - 3^(1
/2)*8i)^(1/2)*1i)/4 + (8 - 3^(1/2)*8i)^(1/2)/4)))*(8 - 3^(1/2)*8i)^(1/4))/1
2 - (2^(3/4)*3^(1/2)*atan((2^(3/4)*x*(3^(1/2)*1i + 1)^(1/4))/(2*((2^(1/2)*(
3^(1/2)*1i + 1)^(1/2))/2 - (2^(1/2)*3^(1/2)*(3^(1/2)*1i + 1)^(1/2)*1i)/2))
- (2^(3/4)*3^(1/2)*x*(3^(1/2)*1i + 1)^(1/4)*1i)/(2*((2^(1/2)*(3^(1/2)*1i +
1)^(1/2))/2 - (2^(1/2)*3^(1/2)*(3^(1/2)*1i + 1)^(1/2)*1i)/2)))*(3^(1/2)*1i
+ 1)^(1/4)*1i)/12 - (2^(3/4)*3^(1/2)*atan((2^(3/4)*x*(3^(1/2)*1i + 1)^(1/4)
*1i)/(2*((2^(1/2)*(3^(1/2)*1i + 1)^(1/2))/2 - (2^(1/2)*3^(1/2)*(3^(1/2)*1i
+ 1)^(1/2)*1i)/2)) + (2^(3/4)*3^(1/2)*x*(3^(1/2)*1i + 1)^(1/4))/(2*((2^(1/2
)*(3^(1/2)*1i + 1)^(1/2))/2 - (2^(1/2)*3^(1/2)*(3^(1/2)*1i + 1)^(1/2)*1i)/2
)))*(3^(1/2)*1i + 1)^(1/4))/12
```

sympy [A] time = 3.37, size = 37, normalized size = 0.10

$$\mathrm{RootSum}\left(5308416t^8-2304t^4+1,\left(t\mapsto t\log\left(18432t^5-4t+x\right)\right)\right)+\frac{-7x^4-3}{21x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**8/(x**8-x**4+1),x)
```

```
[Out] RootSum(5308416*_t**8 - 2304*_t**4 + 1, Lambda(_t, _t*log(18432*_t**5 - 4*_
t + x))) + (-7*x**4 - 3)/(21*x**7)
```

3.308 $\int \frac{x^{11}}{1+3x^4+x^8} dx$

Optimal. Leaf size=62

$$\frac{x^4}{4} - \frac{1}{40}\left(15-7\sqrt{5}\right)\log\left(2x^4-\sqrt{5}+3\right) - \frac{1}{40}\left(15+7\sqrt{5}\right)\log\left(2x^4+\sqrt{5}+3\right)$$

Rubi [A] time = 0.06, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1357, 703, 632, 31}

$$\frac{x^4}{4} - \frac{1}{40}\left(15-7\sqrt{5}\right)\log\left(2x^4-\sqrt{5}+3\right) - \frac{1}{40}\left(15+7\sqrt{5}\right)\log\left(2x^4+\sqrt{5}+3\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^11/(1 + 3*x^4 + x^8),x]
```

```
[Out] x^4/4 - ((15 - 7*Sqrt[5])*Log[3 - Sqrt[5] + 2*x^4])/40 - ((15 + 7*Sqrt[5])*
Log[3 + Sqrt[5] + 2*x^4])/40
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 632

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rule 703

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Simp[(e*(d + e*x)^(m - 1))/(c*(m - 1)), x] + Dist[1/c, Int[((d + e*x)^
(m - 2)*Simp[c*d^2 - a*e^2 + e*(2*c*d - b*e)*x, x])/(a + b*x + c*x^2), x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && GtQ[m, 1]
```

Rule 1357

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x^{11}}{1+3x^4+x^8}\,dx &= \frac{1}{4}\operatorname{Subst}\left(\int \frac{x^2}{1+3x+x^2}\,dx, x, x^4\right)\\ &= \frac{x^4}{4}+\frac{1}{4}\operatorname{Subst}\left(\int \frac{-1-3x}{1+3x+x^2}\,dx, x, x^4\right)\\ &= \frac{x^4}{4}+\frac{1}{40}\left(-15+7\sqrt{5}\right)\operatorname{Subst}\left(\int \frac{1}{\frac{3}{2}-\frac{\sqrt{5}}{2}+x}\,dx, x, x^4\right)-\frac{1}{40}\left(15+7\sqrt{5}\right)\operatorname{Subst}\left(\int \frac{}{\frac{3}{2}+}\right.\\ &= \frac{x^4}{4}-\frac{1}{40}\left(15-7\sqrt{5}\right)\log\left(3-\sqrt{5}+2x^4\right)-\frac{1}{40}\left(15+7\sqrt{5}\right)\log\left(3+\sqrt{5}+2x^4\right)\end{aligned}$$

Mathematica [A] time = 0.03, size = 57, normalized size = 0.92

$$\frac{1}{40}\left(10x^4+\left(7\sqrt{5}-15\right)\log\left(-2x^4+\sqrt{5}-3\right)-\left(15+7\sqrt{5}\right)\log\left(2x^4+\sqrt{5}+3\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^11/(1 + 3*x^4 + x^8),x]
```

```
[Out] (10*x^4 + (-15 + 7*Sqrt[5])*Log[-3 + Sqrt[5] - 2*x^4] - (15 + 7*Sqrt[5])*Lo
g[3 + Sqrt[5] + 2*x^4])/40
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{11}}{1+3x^4+x^8}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^11/(1 + 3*x^4 + x^8),x]
```

```
[Out] IntegrateAlgebraic[x^11/(1 + 3*x^4 + x^8), x]
```

fricas [A] time = 1.48, size = 62, normalized size = 1.00

$$\frac{1}{4}x^4+\frac{7}{40}\sqrt{5}\log\left(\frac{2x^8+6x^4-\sqrt{5}\left(2x^4+3\right)+7}{x^8+3x^4+1}\right)-\frac{3}{8}\log\left(x^8+3x^4+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(x^8+3*x^4+1),x, algorithm="fricas")
```

```
[Out] 1/4*x^4 + 7/40*sqrt(5)*log((2*x^8 + 6*x^4 - sqrt(5)*(2*x^4 + 3) + 7)/(x^8 +
 3*x^4 + 1)) - 3/8*log(x^8 + 3*x^4 + 1)
```

giac [A] time = 0.50, size = 50, normalized size = 0.81

$$\frac{1}{4}x^4+\frac{7}{40}\sqrt{5}\log\left(\frac{2x^4-\sqrt{5}+3}{2x^4+\sqrt{5}+3}\right)-\frac{3}{8}\log\left(x^8+3x^4+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(x^8+3*x^4+1),x, algorithm="giac")
```

```
[Out] 1/4*x^4 + 7/40*sqrt(5)*log((2*x^4 - sqrt(5) + 3)/(2*x^4 + sqrt(5) + 3)) - 3
/8*log(x^8 + 3*x^4 + 1)
```

maple [A] time = 0.00, size = 38, normalized size = 0.61

$$\frac{x^4}{4}-\frac{7\sqrt{5}\ \operatorname{arctanh}\left(\frac{(2x^4+3)\sqrt{5}}{5}\right)}{20}-\frac{3\ln\left(x^8+3x^4+1\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11/(x^8+3*x^4+1),x)
```

```
[Out] 1/4*x^4-3/8*ln(x^8+3*x^4+1)-7/20*arctanh(1/5*(2*x^4+3)*5^(1/2))*5^(1/2)
```

maxima [A] time = 1.28, size = 50, normalized size = 0.81

$$\frac{1}{4}x^4+\frac{7}{40}\sqrt{5}\log\left(\frac{2x^4-\sqrt{5}+3}{2x^4+\sqrt{5}+3}\right)-\frac{3}{8}\log\left(x^8+3x^4+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(x^8+3*x^4+1),x, algorithm="maxima")
```

```
[Out] 1/4*x^4 + 7/40*sqrt(5)*log((2*x^4 - sqrt(5) + 3)/(2*x^4 + sqrt(5) + 3)) - 3
/8*log(x^8 + 3*x^4 + 1)
```

mupad [B] time = 0.13, size = 64, normalized size = 1.03

$$\frac{7\sqrt{5}\ \ln\left(x^4-\frac{\sqrt{5}}{2}+\frac{3}{2}\right)}{40}-\frac{3\ln\left(x^4+\frac{\sqrt{5}}{2}+\frac{3}{2}\right)}{8}-\frac{3\ln\left(x^4-\frac{\sqrt{5}}{2}+\frac{3}{2}\right)}{8}-\frac{7\sqrt{5}\ \ln\left(x^4+\frac{\sqrt{5}}{2}+\frac{3}{2}\right)}{40}+\frac{x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11/(3*x^4 + x^8 + 1),x)
```

```
[Out] (7*5^(1/2)*log(x^4 - 5^(1/2)/2 + 3/2))/40 - (3*log(5^(1/2)/2 + x^4 + 3/2))/
8 - (3*log(x^4 - 5^(1/2)/2 + 3/2))/8 - (7*5^(1/2)*log(5^(1/2)/2 + x^4 + 3/2
))/40 + x^4/4
```

sympy [A] time = 0.14, size = 60, normalized size = 0.97

$$\frac{x^4}{4}+\left(-\frac{3}{8}+\frac{7\sqrt{5}}{40}\right)\log\left(x^4-\frac{\sqrt{5}}{2}+\frac{3}{2}\right)+\left(-\frac{7\sqrt{5}}{40}-\frac{3}{8}\right)\log\left(x^4+\frac{\sqrt{5}}{2}+\frac{3}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**11/(x**8+3*x**4+1),x)
```

```
[Out] x**4/4 + (-3/8 + 7*sqrt(5)/40)*log(x**4 - sqrt(5)/2 + 3/2) + (-7*sqrt(5)/40
 - 3/8)*log(x**4 + sqrt(5)/2 + 3/2)
```

3.309 $\int \frac{x^9}{1+3x^4+x^8}\,dx$

Optimal. Leaf size=90

$$\frac{x^2}{2}-\frac{1}{2}\sqrt{\frac{1}{5}\left(9+4\sqrt{5}\right)}\tan^{-1}\left(\sqrt{\frac{2}{3+\sqrt{5}}}\,x^2\right)+\frac{1}{2}\sqrt{\frac{1}{5}\left(9-4\sqrt{5}\right)}\tan^{-1}\left(\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}\,x^2\right)$$

Rubi [A] time = 0.14, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1359, 1122, 1166, 203}

$$\frac{x^2}{2}-\frac{1}{2}\sqrt{\frac{1}{5}\left(9+4\sqrt{5}\right)}\tan^{-1}\left(\sqrt{\frac{2}{3+\sqrt{5}}}\,x^2\right)+\frac{1}{2}\sqrt{\frac{1}{5}\left(9-4\sqrt{5}\right)}\tan^{-1}\left(\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}\,x^2\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^9/(1 + 3*x^4 + x^8),x]
```

```
[Out] x^2/2 - (Sqrt[(9 + 4*Sqrt[5])/5]*ArcTan[Sqrt[2/(3 + Sqrt[5])]*x^2])/2 + (Sq
rt[(9 - 4*Sqrt[5])/5]*ArcTan[Sqrt[(3 + Sqrt[5])/2]*x^2])/2
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 1122

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Simp[(d^3*(d*x)^(m - 3)*(a + b*x^2 + c*x^4)^(p + 1))/(c*(m + 4*p + 1)),
 x] - Dist[d^4/(c*(m + 4*p + 1)), Int[(d*x)^(m - 4)*Simp[a*(m - 3) + b*(m +
 2*p - 1)*x^2, x]*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x
] && NeQ[b^2 - 4*a*c, 0] && GtQ[m, 3] && NeQ[m + 4*p + 1, 0] && IntegerQ[2*
p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1359

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*
x^(n/k) + c*x^((2*n)/k))^p, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, c, p
}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\int \frac{x^9}{1+3x^4+x^8}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{x^4}{1+3x^2+x^4}\,dx, x, x^2\right)\\ &= \frac{x^2}{2}-\frac{1}{2}\operatorname{Subst}\left(\int \frac{1+3x^2}{1+3x^2+x^4}\,dx, x, x^2\right)\\ &= \frac{x^2}{2}-\frac{1}{20}\left(15-7\sqrt{5}\right)\operatorname{Subst}\left(\int \frac{1}{\frac{3}{2}-\frac{\sqrt{5}}{2}+x^2}\,dx, x, x^2\right)-\frac{1}{20}\left(15+7\sqrt{5}\right)\operatorname{Subst}\left(\int \frac{1}{\frac{3}{2}+\frac{\sqrt{5}}{2}\cdots}\right.\\ &= \frac{x^2}{2}-\frac{1}{2}\sqrt{\frac{1}{5}\left(9+4\sqrt{5}\right)}\tan^{-1}\left(\sqrt{\frac{2}{3+\sqrt{5}}}\,x^2\right)+\frac{1}{20}\sqrt{180-80\sqrt{5}}\tan^{-1}\left(\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}\,x^2\right)\end{aligned}$$

Mathematica [A] time = 0.15, size = 97, normalized size = 1.08

$$\frac{1}{40}\left(20x^2-\sqrt{6-2\sqrt{5}}\left(15+7\sqrt{5}\right)\tan^{-1}\left(\sqrt{\frac{2}{3+\sqrt{5}}}\,x^2\right)+\sqrt{2\left(3+\sqrt{5}\right)}\left(7\sqrt{5}-15\right)\tan^{-1}\left(\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}\,x^2\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^9/(1 + 3*x^4 + x^8),x]
```

```
[Out] (20*x^2 - Sqrt[6 - 2*Sqrt[5]]*(15 + 7*Sqrt[5])*ArcTan[Sqrt[2/(3 + Sqrt[5])]
*x^2] + Sqrt[2*(3 + Sqrt[5])]*(-15 + 7*Sqrt[5])*ArcTan[Sqrt[(3 + Sqrt[5])/2
]*x^2])/40
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^9}{1+3x^4+x^8}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^9/(1 + 3*x^4 + x^8),x]
```

```
[Out] IntegrateAlgebraic[x^9/(1 + 3*x^4 + x^8), x]
```

fricas [B] time = 1.29, size = 154, normalized size = 1.71

$$\frac{1}{2}x^2-\frac{1}{5}\sqrt{5}\sqrt{-4\sqrt{5}+9}\arctan\left(\frac{1}{4}\sqrt{2x^4-\sqrt{5}+3}\left(3\sqrt{5}\sqrt{2}+7\sqrt{2}\right)\sqrt{-4\sqrt{5}+9}-\frac{1}{2}\left(3\sqrt{5}x^2+7x^2\right)\sqrt{-4\sqrt{5}+9}\right)-\frac{1}{5}\sqrt{5}\sqrt{4\sqrt{5}+9}\arctan\left(-\frac{1}{4}\left(6\sqrt{5}x^2-14x^2-\sqrt{2x^4+\sqrt{5}+3}\left(3\sqrt{5}\sqrt{2}-7\sqrt{2}\right)\right)\sqrt{4\sqrt{5}+9}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(x^8+3*x^4+1),x, algorithm="fricas")
```

```
[Out] 1/2*x^2 - 1/5*sqrt(5)*sqrt(-4*sqrt(5) + 9)*arctan(1/4*sqrt(2*x^4 - sqrt(5)
+ 3)*(3*sqrt(5)*sqrt(2) + 7*sqrt(2))*sqrt(-4*sqrt(5) + 9) - 1/2*(3*sqrt(5)*
x^2 + 7*x^2)*sqrt(-4*sqrt(5) + 9)) - 1/5*sqrt(5)*sqrt(4*sqrt(5) + 9)*arctan
(-1/4*(6*sqrt(5)*x^2 - 14*x^2 - sqrt(2*x^4 + sqrt(5) + 3)*(3*sqrt(5)*sqrt(2
) - 7*sqrt(2)))*sqrt(4*sqrt(5) + 9))
```

giac [A] time = 0.56, size = 66, normalized size = 0.73

$$\frac{1}{2}x^2-\frac{1}{20}\left(3x^4\left(\sqrt{5}-5\right)+\sqrt{5}-5\right)\arctan\left(\frac{2x^2}{\sqrt{5}+1}\right)-\frac{1}{20}\left(3x^4\left(\sqrt{5}+5\right)+\sqrt{5}+5\right)\arctan\left(\frac{2x^2}{\sqrt{5}-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(x^8+3*x^4+1),x, algorithm="giac")
```

```
[Out] 1/2*x^2 - 1/20*(3*x^4*(sqrt(5) - 5) + sqrt(5) - 5)*arctan(2*x^2/(sqrt(5) +
1)) - 1/20*(3*x^4*(sqrt(5) + 5) + sqrt(5) + 5)*arctan(2*x^2/(sqrt(5) - 1))
```

maple [B] time = 0.05, size = 117, normalized size = 1.30

$$\frac{x^2}{2}+\frac{7\sqrt{5}\,\arctan\left(\frac{4x^2}{-2+2\sqrt{5}}\right)}{5\left(-2+2\sqrt{5}\right)}-\frac{3\arctan\left(\frac{4x^2}{-2+2\sqrt{5}}\right)}{-2+2\sqrt{5}}-\frac{7\sqrt{5}\,\arctan\left(\frac{4x^2}{2+2\sqrt{5}}\right)}{5\left(2+2\sqrt{5}\right)}-\frac{3\arctan\left(\frac{4x^2}{2+2\sqrt{5}}\right)}{2+2\sqrt{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9/(x^8+3*x^4+1),x)
```

```
[Out] 1/2*x^2-7/5*5^(1/2)/(2+2*5^(1/2))*arctan(4*x^2/(2+2*5^(1/2)))-3/(2+2*5^(1/2
))*arctan(4*x^2/(2+2*5^(1/2)))+7/5*5^(1/2)/(-2+2*5^(1/2))*arctan(4*x^2/(-2+
2*5^(1/2)))-3/(-2+2*5^(1/2))*arctan(4*x^2/(-2+2*5^(1/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{2}\,x^2-\int\frac{\left(3\,x^4+1\right)x}{x^8+3\,x^4+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(x^8+3*x^4+1),x, algorithm="maxima")
```

```
[Out] 1/2*x^2 - integrate((3*x^4 + 1)*x/(x^8 + 3*x^4 + 1), x)
```

mupad [B] time = 1.34, size = 130, normalized size = 1.44

$$2\,\mathrm{atanh}\left(\frac{1280\,x^2\,\sqrt{\frac{\sqrt{5}}{20}-\frac{9}{80}}}{64\,\sqrt{5}-192}+\frac{768\,\sqrt{5}\,x^2\,\sqrt{\frac{\sqrt{5}}{20}-\frac{9}{80}}}{64\,\sqrt{5}-192}\right)\sqrt{\frac{\sqrt{5}}{20}-\frac{9}{80}}-2\,\mathrm{atanh}\left(\frac{1280\,x^2\,\sqrt{-\frac{\sqrt{5}}{20}-\frac{9}{80}}}{64\,\sqrt{5}+192}-\frac{768\,\sqrt{5}\,x^2\,\sqrt{-\frac{\sqrt{5}}{20}-\frac{9}{80}}}{64\,\sqrt{5}+192}\right)\sqrt{-\frac{\sqrt{5}}{20}-\frac{9}{80}}+\frac{x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9/(3*x^4 + x^8 + 1),x)
```

```
[Out] 2*atanh((1280*x^2*(5^(1/2)/20 - 9/80)^(1/2))/(64*5^(1/2) - 192) + (768*5^(1
/2)*x^2*(5^(1/2)/20 - 9/80)^(1/2))/(64*5^(1/2) - 192))*(5^(1/2)/20 - 9/80)^
(1/2) - 2*atanh((1280*x^2*(- 5^(1/2)/20 - 9/80)^(1/2))/(64*5^(1/2) + 192) -
 (768*5^(1/2)*x^2*(- 5^(1/2)/20 - 9/80)^(1/2))/(64*5^(1/2) + 192))*(- 5^(1/
2)/20 - 9/80)^(1/2) + x^2/2
```

sympy [A] time = 0.21, size = 54, normalized size = 0.60

$$\frac{x^2}{2}+2\left(\frac{1}{4}-\frac{\sqrt{5}}{10}\right)\operatorname{atan}\left(\frac{2x^2}{-1+\sqrt{5}}\right)-2\left(\frac{\sqrt{5}}{10}+\frac{1}{4}\right)\operatorname{atan}\left(\frac{2x^2}{1+\sqrt{5}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**9/(x**8+3*x**4+1),x)
```

```
[Out] x**2/2 + 2*(1/4 - sqrt(5)/10)*atan(2*x**2/(-1 + sqrt(5))) - 2*(sqrt(5)/10 +
 1/4)*atan(2*x**2/(1 + sqrt(5)))
```

3.310 $\int \frac{x^7}{1+3x^4+x^8}\,dx$

Optimal. Leaf size=55

$$\frac{1}{40}\left(5-3\sqrt{5}\right)\log\left(2x^4-\sqrt{5}+3\right)+\frac{1}{40}\left(5+3\sqrt{5}\right)\log\left(2x^4+\sqrt{5}+3\right)$$

Rubi [A] time = 0.03, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {1357, 632, 31}

$$\frac{1}{40}\left(5-3\sqrt{5}\right)\log\left(2x^4-\sqrt{5}+3\right)+\frac{1}{40}\left(5+3\sqrt{5}\right)\log\left(2x^4+\sqrt{5}+3\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^7/(1 + 3*x^4 + x^8),x]
```

```
[Out] ((5 - 3*Sqrt[5])*Log[3 - Sqrt[5] + 2*x^4])/40 + ((5 + 3*Sqrt[5])*Log[3 + Sq
rt[5] + 2*x^4])/40
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 632

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1357

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x^7}{1+3x^4+x^8}\,dx &= \frac{1}{4}\text{Subst}\left(\int \frac{x}{1+3x+x^2}\,dx,x,x^4\right)\\ &= \frac{1}{40}\left(5-3\sqrt{5}\right)\text{Subst}\left(\int \frac{1}{\frac{3}{2}-\frac{\sqrt{5}}{2}+x}\,dx,x,x^4\right)+\frac{1}{40}\left(5+3\sqrt{5}\right)\text{Subst}\left(\int \frac{1}{\frac{3}{2}+\frac{\sqrt{5}}{2}+x}\,dx,x\right.\\ &= \frac{1}{40}\left(5-3\sqrt{5}\right)\log\left(3-\sqrt{5}+2x^4\right)+\frac{1}{40}\left(5+3\sqrt{5}\right)\log\left(3+\sqrt{5}+2x^4\right)\end{aligned}$$

Mathematica [A] time = 0.02, size = 53, normalized size = 0.96

$$\frac{1}{40}\left(5-3\sqrt{5}\right)\log\left(-2x^4+\sqrt{5}-3\right)+\frac{1}{40}\left(5+3\sqrt{5}\right)\log\left(2x^4+\sqrt{5}+3\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7/(1 + 3*x^4 + x^8),x]
```

```
[Out] ((5 - 3*Sqrt[5])*Log[-3 + Sqrt[5] - 2*x^4])/40 + ((5 + 3*Sqrt[5])*Log[3 + S
qrt[5] + 2*x^4])/40
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^7}{1+3x^4+x^8}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^7/(1 + 3*x^4 + x^8),x]
```

```
[Out] IntegrateAlgebraic[x^7/(1 + 3*x^4 + x^8), x]
```

fricas [A] time = 1.40, size = 56, normalized size = 1.02

$$\frac{3}{40}\,\sqrt{5}\,\log\left(\frac{2\,x^8+6\,x^4+\sqrt{5}\left(2\,x^4+3\right)+7}{x^8+3\,x^4+1}\right)+\frac{1}{8}\,\log\left(x^8+3\,x^4+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(x^8+3*x^4+1),x, algorithm="fricas")
```

```
[Out] 3/40*sqrt(5)*log((2*x^8 + 6*x^4 + sqrt(5)*(2*x^4 + 3) + 7)/(x^8 + 3*x^4 + 1
)) + 1/8*log(x^8 + 3*x^4 + 1)
```

giac [A] time = 0.50, size = 45, normalized size = 0.82

$$-\frac{3}{40}\,\sqrt{5}\,\log\left(\frac{2\,x^4-\sqrt{5}+3}{2\,x^4+\sqrt{5}+3}\right)+\frac{1}{8}\,\log\left(x^8+3\,x^4+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(x^8+3*x^4+1),x, algorithm="giac")
```

```
[Out] -3/40*sqrt(5)*log((2*x^4 - sqrt(5) + 3)/(2*x^4 + sqrt(5) + 3)) + 1/8*log(x^
8 + 3*x^4 + 1)
```

maple [A] time = 0.00, size = 33, normalized size = 0.60

$$\frac{3\sqrt{5}\ \operatorname{arctanh}\left(\frac{\left(2x^4+3\right)\sqrt{5}}{5}\right)}{20}+\frac{\ln\left(x^8+3x^4+1\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(x^8+3*x^4+1),x)
```

```
[Out] 1/8*ln(x^8+3*x^4+1)+3/20*arctanh(1/5*(2*x^4+3)*5^(1/2))*5^(1/2)
```

maxima [A] time = 1.17, size = 45, normalized size = 0.82

$$-\frac{3}{40}\,\sqrt{5}\,\log\left(\frac{2\,x^4-\sqrt{5}+3}{2\,x^4+\sqrt{5}+3}\right)+\frac{1}{8}\,\log\left(x^8+3\,x^4+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(x^8+3*x^4+1),x, algorithm="maxima")
```

```
[Out] -3/40*sqrt(5)*log((2*x^4 - sqrt(5) + 3)/(2*x^4 + sqrt(5) + 3)) + 1/8*log(x^
8 + 3*x^4 + 1)
```

mupad [B] time = 1.36, size = 59, normalized size = 1.07

$$\frac{\ln\left(x^4-\frac{\sqrt{5}}{2}+\frac{3}{2}\right)}{8}+\frac{\ln\left(x^4+\frac{\sqrt{5}}{2}+\frac{3}{2}\right)}{8}-\frac{3\sqrt{5}\,\ln\left(x^4-\frac{\sqrt{5}}{2}+\frac{3}{2}\right)}{40}+\frac{3\sqrt{5}\,\ln\left(x^4+\frac{\sqrt{5}}{2}+\frac{3}{2}\right)}{40}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(3*x^4 + x^8 + 1),x)
```

```
[Out] log(x^4 - 5^(1/2)/2 + 3/2)/8 + log(5^(1/2)/2 + x^4 + 3/2)/8 - (3*5^(1/2)*lo
g(x^4 - 5^(1/2)/2 + 3/2))/40 + (3*5^(1/2)*log(5^(1/2)/2 + x^4 + 3/2))/40
```

sympy [A] time = 0.14, size = 53, normalized size = 0.96

$$\left(\frac{1}{8}-\frac{3\sqrt{5}}{40}\right)\log\left(x^4-\frac{\sqrt{5}}{2}+\frac{3}{2}\right)+\left(\frac{1}{8}+\frac{3\sqrt{5}}{40}\right)\log\left(x^4+\frac{\sqrt{5}}{2}+\frac{3}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7/(x**8+3*x**4+1),x)
```

```
[Out] (1/8 - 3*sqrt(5)/40)*log(x**4 - sqrt(5)/2 + 3/2) + (1/8 + 3*sqrt(5)/40)*log
(x**4 + sqrt(5)/2 + 3/2)
```

3.311 $\int \frac{x^5}{1+3x^4+x^8}\,dx$

Optimal. Leaf size=81

$$\frac{1}{2}\sqrt{\frac{1}{10}\left(3+\sqrt{5}\right)}\tan^{-1}\left(\sqrt{\frac{2}{3+\sqrt{5}}}\,x^2\right)-\frac{1}{2}\sqrt{\frac{1}{10}\left(3-\sqrt{5}\right)}\tan^{-1}\left(\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}\,x^2\right)$$

Rubi [A] time = 0.08, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {1359, 1130, 203}

$$\frac{1}{2}\sqrt{\frac{1}{10}\left(3+\sqrt{5}\right)}\tan^{-1}\left(\sqrt{\frac{2}{3+\sqrt{5}}}\,x^2\right)-\frac{1}{2}\sqrt{\frac{1}{10}\left(3-\sqrt{5}\right)}\tan^{-1}\left(\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}\,x^2\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^5/(1 + 3*x^4 + x^8),x]
```

```
[Out] (Sqrt[(3 + Sqrt[5])/10]*ArcTan[Sqrt[2/(3 + Sqrt[5])]*x^2])/2 - (Sqrt[(3 - S
qrt[5])/10]*ArcTan[Sqrt[(3 + Sqrt[5])/2]*x^2])/2
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 1130

```
Int[((d_.)*(x_))^(m_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Wi
th[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(d^2*(b/q + 1))/2, Int[(d*x)^(m - 2)/(b/2
 + q/2 + c*x^2), x], x] - Dist[(d^2*(b/q - 1))/2, Int[(d*x)^(m - 2)/(b/2 -
q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && G
eQ[m, 2]
```

Rule 1359

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*
x^(n/k) + c*x^((2*n)/k))^p, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, c, p
}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\int \frac{x^5}{1+3x^4+x^8}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{x^2}{1+3x^2+x^4}\,dx,x,x^2\right)\\ &= \frac{1}{20}\left(5-3\sqrt{5}\right)\text{Subst}\left(\int \frac{1}{\frac{3}{2}-\frac{\sqrt{5}}{2}+x^2}\,dx,x,x^2\right)+\frac{1}{20}\left(5+3\sqrt{5}\right)\text{Subst}\left(\int \frac{1}{\frac{3}{2}+\frac{\sqrt{5}}{2}+x^2}\right.\\ &= \frac{1}{2}\sqrt{\frac{1}{10}\left(3+\sqrt{5}\right)}\tan^{-1}\left(\sqrt{\frac{2}{3+\sqrt{5}}}\,x^2\right)-\frac{1}{2}\sqrt{\frac{1}{10}\left(3-\sqrt{5}\right)}\tan^{-1}\left(\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}\,x^2\right)\end{aligned}$$

Mathematica [A] time = 0.05, size = 75, normalized size = 0.93

$$\frac{2\sqrt{5}\tan^{-1}\left(\sqrt{\frac{2}{3+\sqrt{5}}}\,x^2\right)+\left(5-3\sqrt{5}\right)\tan^{-1}\left(\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}\,x^2\right)}{10\sqrt{6-2\sqrt{5}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/(1 + 3*x^4 + x^8),x]
```

```
[Out] (2*Sqrt[5]*ArcTan[Sqrt[2/(3 + Sqrt[5])]*x^2] + (5 - 3*Sqrt[5])*ArcTan[Sqrt[
(3 + Sqrt[5])/2]*x^2])/(10*Sqrt[6 - 2*Sqrt[5]])
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5}{1+3x^4+x^8}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^5/(1 + 3*x^4 + x^8),x]
```

```
[Out] IntegrateAlgebraic[x^5/(1 + 3*x^4 + x^8), x]
```

fricas [B] time = 1.45, size = 165, normalized size = 2.04

$$-\frac{1}{10}\sqrt{10}\sqrt{\sqrt{5}+3}\arctan\left(\frac{1}{40}\sqrt{10}\sqrt{2x^4+\sqrt{5}+3}\left(3\sqrt{5}\sqrt{2}-5\sqrt{2}\right)\sqrt{\sqrt{5}+3}-\frac{1}{20}\sqrt{10}\left(3\sqrt{5}x^2-5x^2\right)\sqrt{\sqrt{5}+3}\right)+\frac{1}{10}\sqrt{10}\sqrt{-\sqrt{5}+3}\arctan\left(\frac{1}{40}\sqrt{10}\sqrt{2x^4-\sqrt{5}+3}\left(3\sqrt{5}\sqrt{2}+5\sqrt{2}\right)\sqrt{-\sqrt{5}+3}-\frac{1}{20}\sqrt{10}\left(3\sqrt{5}x^2+5x^2\right)\sqrt{-\sqrt{5}+3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(x^8+3*x^4+1),x, algorithm="fricas")
```

```
[Out] -1/10*sqrt(10)*sqrt(sqrt(5) + 3)*arctan(1/40*sqrt(10)*sqrt(2*x^4 + sqrt(5)
+ 3)*(3*sqrt(5)*sqrt(2) - 5*sqrt(2))*sqrt(sqrt(5) + 3) - 1/20*sqrt(10)*(3*s
qrt(5)*x^2 - 5*x^2)*sqrt(sqrt(5) + 3)) + 1/10*sqrt(10)*sqrt(-sqrt(5) + 3)*a
rctan(1/40*sqrt(10)*sqrt(2*x^4 - sqrt(5) + 3)*(3*sqrt(5)*sqrt(2) + 5*sqrt(2
))*sqrt(-sqrt(5) + 3) - 1/20*sqrt(10)*(3*sqrt(5)*x^2 + 5*x^2)*sqrt(-sqrt(5)
 + 3))
```

giac [A] time = 0.55, size = 47, normalized size = 0.58

$$\frac{1}{20}\,x^4\left(\sqrt{5}-5\right)\arctan\left(\frac{2\,x^2}{\sqrt{5}+1}\right)+\frac{1}{20}\,x^4\left(\sqrt{5}+5\right)\arctan\left(\frac{2\,x^2}{\sqrt{5}-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(x^8+3*x^4+1),x, algorithm="giac")
```

```
[Out] 1/20*x^4*(sqrt(5) - 5)*arctan(2*x^2/(sqrt(5) + 1)) + 1/20*x^4*(sqrt(5) + 5)
*arctan(2*x^2/(sqrt(5) - 1))
```

maple [B] time = 0.02, size = 110, normalized size = 1.36

$$\frac{\arctan\left(\frac{4x^2}{-2+2\sqrt{5}}\right)}{-2+2\sqrt{5}}-\frac{3\sqrt{5}\,\arctan\left(\frac{4x^2}{-2+2\sqrt{5}}\right)}{5\left(-2+2\sqrt{5}\right)}+\frac{\arctan\left(\frac{4x^2}{2+2\sqrt{5}}\right)}{2+2\sqrt{5}}+\frac{3\sqrt{5}\,\arctan\left(\frac{4x^2}{2+2\sqrt{5}}\right)}{5\left(2+2\sqrt{5}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(x^8+3*x^4+1),x)
```

[Out] 1/(2+2*5^(1/2))*arctan(4/(2+2*5^(1/2))*x^2)+3/5*5^(1/2)/(2+2*5^(1/2))*arctan(4/(2+2*5^(1/2))*x^2)+1/(-2+2*5^(1/2))*arctan(4/(-2+2*5^(1/2))*x^2)-3/5*5^(1/2)/(-2+2*5^(1/2))*arctan(4/(-2+2*5^(1/2))*x^2)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5}{x^8 + 3x^4 + 1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5/(x^8+3*x^4+1),x, algorithm="maxima")

[Out] integrate(x^5/(x^8 + 3*x^4 + 1), x)

mupad [B] time = 0.12, size = 117, normalized size = 1.44

$$2\,\mathrm{atanh}\left(\frac{60\,x^2\,\sqrt{\frac{\sqrt{5}}{160}-\frac{3}{160}}}{\sqrt{5}+3}+\frac{28\,\sqrt{5}\,x^2\,\sqrt{\frac{\sqrt{5}}{160}-\frac{3}{160}}}{\sqrt{5}+3}\right)\sqrt{\frac{\sqrt{5}}{160}-\frac{3}{160}}-2\,\mathrm{atanh}\left(\frac{60\,x^2\,\sqrt{-\frac{\sqrt{5}}{160}-\frac{3}{160}}}{\sqrt{5}-3}-\frac{28\,\sqrt{5}\,x^2\,\sqrt{-\frac{\sqrt{5}}{160}-\frac{3}{160}}}{\sqrt{5}-3}\right)\sqrt{-\frac{\sqrt{5}}{160}-\frac{3}{160}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^5/(3*x^4 + x^8 + 1),x)

[Out] 2*atanh((60*x^2*(5^(1/2)/160 - 3/160)^(1/2))/(5^(1/2) + 3) + (28*5^(1/2)*x^2*(5^(1/2)/160 - 3/160)^(1/2))/(5^(1/2) + 3))*(5^(1/2)/160 - 3/160)^(1/2) - 2*atanh((60*x^2*(- 5^(1/2)/160 - 3/160)^(1/2))/(5^(1/2) - 3) - (28*5^(1/2)*x^2*(- 5^(1/2)/160 - 3/160)^(1/2))/(5^(1/2) - 3))*(- 5^(1/2)/160 - 3/160)^(1/2)

sympy [A] time = 0.20, size = 49, normalized size = 0.60

$$-2\left(\frac{1}{8}-\frac{\sqrt{5}}{40}\right)\mathrm{atan}\left(\frac{2x^2}{-1+\sqrt{5}}\right)+2\left(\frac{\sqrt{5}}{40}+\frac{1}{8}\right)\mathrm{atan}\left(\frac{2x^2}{1+\sqrt{5}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**5/(x**8+3*x**4+1),x)

[Out] -2*(1/8 - sqrt(5)/40)*atan(2*x**2/(-1 + sqrt(5))) + 2*(sqrt(5)/40 + 1/8)*atan(2*x**2/(1 + sqrt(5)))

3.312 $\int \frac{x^3}{1+3x^4+x^8} \, dx$

Optimal. Leaf size=23

$$-\frac{\tanh^{-1}\left(\frac{2x^4+3}{\sqrt{5}}\right)}{2\sqrt{5}}$$

Rubi [A] time = 0.03, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {1352, 618, 206}

$$-\frac{\tanh^{-1}\left(\frac{2x^4+3}{\sqrt{5}}\right)}{2\sqrt{5}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(1 + 3*x^4 + x^8),x]
```

```
[Out] -ArcTanh[(3 + 2*x^4)/Sqrt[5]]/(2*Sqrt[5])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1352

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a,
b, c, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^3}{1+3x^4+x^8} \, dx &= \frac{1}{4}\text{Subst}\left(\int \frac{1}{1+3x+x^2} \, dx, x, x^4\right) \\
&= -\left(\frac{1}{2}\text{Subst}\left(\int \frac{1}{5-x^2} \, dx, x, 3+2x^4\right)\right) \\
&= -\frac{\tanh^{-1}\left(\frac{3+2x^4}{\sqrt{5}}\right)}{2\sqrt{5}}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 38, normalized size = 1.65

$$\frac{\log\left(-2x^4+\sqrt{5}-3\right)-\log\left(2x^4+\sqrt{5}+3\right)}{4\sqrt{5}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(1 + 3*x^4 + x^8),x]
```

```
[Out] (Log[-3 + Sqrt[5] - 2*x^4] - Log[3 + Sqrt[5] + 2*x^4])/(4*Sqrt[5])
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{1+3x^4+x^8}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^3/(1 + 3*x^4 + x^8),x]
```

```
[Out] IntegrateAlgebraic[x^3/(1 + 3*x^4 + x^8), x]
```

fricas [B] time = 1.59, size = 43, normalized size = 1.87

$$\frac{1}{20}\,\sqrt{5}\,\log\left(\frac{2\,x^{8}+6\,x^{4}-\sqrt{5}\left(2\,x^{4}+3\right)+7}{x^{8}+3\,x^{4}+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(x^8+3*x^4+1),x, algorithm="fricas")
```

```
[Out] 1/20*sqrt(5)*log((2*x^8 + 6*x^4 - sqrt(5)*(2*x^4 + 3) + 7)/(x^8 + 3*x^4 + 1
))
```

giac [A] time = 0.60, size = 31, normalized size = 1.35

$$\frac{1}{20}\,\sqrt{5}\,\log\left(\frac{2\,x^{4}-\sqrt{5}+3}{2\,x^{4}+\sqrt{5}+3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(x^8+3*x^4+1),x, algorithm="giac")
```

```
[Out] 1/20*sqrt(5)*log((2*x^4 - sqrt(5) + 3)/(2*x^4 + sqrt(5) + 3))
```

maple [A] time = 0.00, size = 19, normalized size = 0.83

$$-\frac{\sqrt{5}\,\operatorname{arctanh}\left(\frac{\left(2x^{4}+3\right)\sqrt{5}}{5}\right)}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(x^8+3*x^4+1),x)
```

```
[Out] -1/10*arctanh(1/5*(2*x^4+3)*5^(1/2))*5^(1/2)
```

maxima [A] time = 1.48, size = 31, normalized size = 1.35

$$\frac{1}{20}\,\sqrt{5}\,\log\left(\frac{2\,x^{4}-\sqrt{5}+3}{2\,x^{4}+\sqrt{5}+3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(x^8+3*x^4+1),x, algorithm="maxima")
```

```
[Out] 1/20*sqrt(5)*log((2*x^4 - sqrt(5) + 3)/(2*x^4 + sqrt(5) + 3))
```

mupad [B] time = 1.33, size = 30, normalized size = 1.30

$$\frac{\sqrt{5}\,\mathrm{atanh}\left(\frac{8\,\sqrt{5}\,x^4+3\,\sqrt{5}}{18\,x^4+7}\right)}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(3*x^4 + x^8 + 1),x)
```

```
[Out] (5^(1/2)*atanh((3*5^(1/2) + 8*5^(1/2)*x^4)/(18*x^4 + 7)))/10
```

sympy [A] time = 0.12, size = 42, normalized size = 1.83

$$\frac{\sqrt{5}\log\left(x^4-\frac{\sqrt{5}}{2}+\frac{3}{2}\right)}{20}-\frac{\sqrt{5}\log\left(x^4+\frac{\sqrt{5}}{2}+\frac{3}{2}\right)}{20}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(x**8+3*x**4+1),x)
```

```
[Out] sqrt(5)*log(x**4 - sqrt(5)/2 + 3/2)/20 - sqrt(5)*log(x**4 + sqrt(5)/2 + 3/2
)/20
```

3.313 $\int \frac{x}{1+3x^4+x^8}\,dx$

Optimal. Leaf size=75

$$\frac{1}{2}\sqrt{\frac{1}{10}\left(3+\sqrt{5}\right)}\tan^{-1}\left(\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}x^2\right)-\frac{\tan^{-1}\left(\sqrt{\frac{2}{3+\sqrt{5}}}x^2\right)}{\sqrt{10\left(3+\sqrt{5}\right)}}$$

Rubi [A] time = 0.06, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {1359, 1093, 203}

$$\frac{1}{2}\sqrt{\frac{1}{10}\left(3+\sqrt{5}\right)}\tan^{-1}\left(\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}x^2\right)-\frac{\tan^{-1}\left(\sqrt{\frac{2}{3+\sqrt{5}}}x^2\right)}{\sqrt{10\left(3+\sqrt{5}\right)}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(1 + 3*x^4 + x^8),x]
```

```
[Out] -(ArcTan[Sqrt[2/(3 + Sqrt[5])]*x^2]/Sqrt[10*(3 + Sqrt[5])]) + (Sqrt[(3 + Sq
rt[5])/10]*ArcTan[Sqrt[(3 + Sqrt[5])/2]*x^2])/2
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 1093

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[b^
2 - 4*a*c, 2]}, Dist[c/q, Int[1/(b/2 - q/2 + c*x^2), x], x] - Dist[c/q, Int
[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
0] && PosQ[b^2 - 4*a*c]
```

Rule 1359

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*
x^(n/k) + c*x^((2*n)/k))^p, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, c, p
}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\int \frac{x}{1+3x^4+x^8}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{1+3x^2+x^4}\,dx,x,x^2\right)\\ &= \frac{\text{Subst}\left(\int \frac{1}{\frac{3}{2}-\frac{\sqrt{5}}{2}+x^2}\,dx,x,x^2\right)}{2\sqrt{5}}-\frac{\text{Subst}\left(\int \frac{1}{\frac{3}{2}+\frac{\sqrt{5}}{2}+x^2}\,dx,x,x^2\right)}{2\sqrt{5}}\\ &= -\frac{\tan^{-1}\left(\sqrt{\frac{2}{3+\sqrt{5}}}x^2\right)}{\sqrt{10\left(3+\sqrt{5}\right)}}+\frac{1}{2}\sqrt{\frac{1}{10}\left(3+\sqrt{5}\right)}\tan^{-1}\left(\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}x^2\right)\end{aligned}$$

Mathematica [A] time = 0.04, size = 74, normalized size = 0.99

$$\frac{\tan^{-1}\left(\sqrt{\frac{2}{3-\sqrt{5}}}x^2\right)}{\sqrt{10\left(3-\sqrt{5}\right)}}-\frac{\tan^{-1}\left(\sqrt{\frac{2}{3+\sqrt{5}}}x^2\right)}{\sqrt{10\left(3+\sqrt{5}\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(1 + 3*x^4 + x^8),x]
```

```
[Out] ArcTan[Sqrt[2/(3 - Sqrt[5])]*x^2]/Sqrt[10*(3 - Sqrt[5])] - ArcTan[Sqrt[2/(3
 + Sqrt[5])]*x^2]/Sqrt[10*(3 + Sqrt[5])]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{1+3x^4+x^8}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x/(1 + 3*x^4 + x^8),x]
```

```
[Out] IntegrateAlgebraic[x/(1 + 3*x^4 + x^8), x]
```

fricas [B] time = 1.36, size = 128, normalized size = 1.71

$$\frac{1}{10}\sqrt{10}\sqrt{-\sqrt{5}+3}\arctan\left(-\frac{1}{10}\sqrt{10}\sqrt{5}x^2\sqrt{-\sqrt{5}+3}+\frac{1}{20}\sqrt{10}\sqrt{5}\sqrt{2}\sqrt{2x^4+\sqrt{5}+3}\sqrt{-\sqrt{5}+3}\right)-\frac{1}{10}\sqrt{10}\sqrt{\sqrt{5}+3}\arctan\left(-\frac{1}{20}\left(2\sqrt{10}\sqrt{5}x^2-\sqrt{10}\sqrt{5}\sqrt{2}\sqrt{2x^4-\sqrt{5}+3}\right)\sqrt{\sqrt{5}+3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^8+3*x^4+1),x, algorithm="fricas")
```

```
[Out] 1/10*sqrt(10)*sqrt(-sqrt(5) + 3)*arctan(-1/10*sqrt(10)*sqrt(5)*x^2*sqrt(-sq
rt(5) + 3) + 1/20*sqrt(10)*sqrt(5)*sqrt(2)*sqrt(2*x^4 + sqrt(5) + 3)*sqrt(-
sqrt(5) + 3)) - 1/10*sqrt(10)*sqrt(sqrt(5) + 3)*arctan(-1/20*(2*sqrt(10)*sq
rt(5)*x^2 - sqrt(10)*sqrt(5)*sqrt(2)*sqrt(2*x^4 - sqrt(5) + 3))*sqrt(sqrt(5
) + 3))
```

giac [A] time = 0.42, size = 41, normalized size = 0.55

$$\frac{1}{20}\left(\sqrt{5}-5\right)\arctan\left(\frac{2x^2}{\sqrt{5}+1}\right)+\frac{1}{20}\left(\sqrt{5}+5\right)\arctan\left(\frac{2x^2}{\sqrt{5}-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^8+3*x^4+1),x, algorithm="giac")
```

```
[Out] 1/20*(sqrt(5) - 5)*arctan(2*x^2/(sqrt(5) + 1)) + 1/20*(sqrt(5) + 5)*arctan(
2*x^2/(sqrt(5) - 1))
```

maple [A] time = 0.02, size = 60, normalized size = 0.80

$$\frac{2\sqrt{5}\arctan\left(\frac{4x^2}{-2+2\sqrt{5}}\right)}{5\left(-2+2\sqrt{5}\right)}-\frac{2\sqrt{5}\arctan\left(\frac{4x^2}{2+2\sqrt{5}}\right)}{5\left(2+2\sqrt{5}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x^8+3*x^4+1),x)
```

```
[Out] -2/5*5^(1/2)/(2+2*5^(1/2))*arctan(4/(2+2*5^(1/2))*x^2)+2/5*5^(1/2)/(-2+2*5^
(1/2))*arctan(4/(-2+2*5^(1/2))*x^2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{x^8 + 3\,x^4 + 1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^8+3*x^4+1),x, algorithm="maxima")
```

```
[Out] integrate(x/(x^8 + 3*x^4 + 1), x)
```

mupad [B] time = 0.05, size = 125, normalized size = 1.67

$$2\,\mathrm{atanh}\left(\frac{160\,x^2\,\sqrt{\frac{\sqrt{5}}{160}-\frac{3}{160}}}{8\,\sqrt{5}-18}-\frac{72\,\sqrt{5}\,x^2\,\sqrt{\frac{\sqrt{5}}{160}-\frac{3}{160}}}{8\,\sqrt{5}-18}\right)\sqrt{\frac{\sqrt{5}}{160}-\frac{3}{160}}-2\,\mathrm{atanh}\left(\frac{160\,x^2\,\sqrt{-\frac{\sqrt{5}}{160}-\frac{3}{160}}}{8\,\sqrt{5}+18}+\frac{72\,\sqrt{5}\,x^2\,\sqrt{-\frac{\sqrt{5}}{160}-\frac{3}{160}}}{8\,\sqrt{5}+18}\right)\sqrt{-\frac{\sqrt{5}}{160}-\frac{3}{160}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(3*x^4 + x^8 + 1),x)
```

```
[Out] 2*atanh((160*x^2*(5^(1/2)/160 - 3/160)^(1/2))/(8*5^(1/2) - 18) - (72*5^(1/2
)*x^2*(5^(1/2)/160 - 3/160)^(1/2))/(8*5^(1/2) - 18))*(5^(1/2)/160 - 3/160)^
(1/2) - 2*atanh((160*x^2*(- 5^(1/2)/160 - 3/160)^(1/2))/(8*5^(1/2) + 18) +
(72*5^(1/2)*x^2*(- 5^(1/2)/160 - 3/160)^(1/2))/(8*5^(1/2) + 18))*(- 5^(1/2)
/160 - 3/160)^(1/2)
```

sympy [A] time = 0.20, size = 49, normalized size = 0.65

$$2\left(\frac{\sqrt{5}}{40}+\frac{1}{8}\right)\mathrm{atan}\left(\frac{2x^2}{-1+\sqrt{5}}\right)-2\left(\frac{1}{8}-\frac{\sqrt{5}}{40}\right)\mathrm{atan}\left(\frac{2x^2}{1+\sqrt{5}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x**8+3*x**4+1),x)
```

```
[Out] 2*(sqrt(5)/40 + 1/8)*atan(2*x**2/(-1 + sqrt(5))) - 2*(1/8 - sqrt(5)/40)*ata
n(2*x**2/(1 + sqrt(5)))
```

3.314 $\int \frac{1}{x(1+3x^4+x^8)}\,dx$

Optimal. Leaf size=57

$$-\frac{1}{40}\left(5+3\sqrt{5}\right)\log\left(2x^4-\sqrt{5}+3\right)-\frac{1}{40}\left(5-3\sqrt{5}\right)\log\left(2x^4+\sqrt{5}+3\right)+\log(x)$$

Rubi [A] time = 0.03, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {1357, 705, 29, 632, 31}

$$-\frac{1}{40}\left(5+3\sqrt{5}\right)\log\left(2x^4-\sqrt{5}+3\right)-\frac{1}{40}\left(5-3\sqrt{5}\right)\log\left(2x^4+\sqrt{5}+3\right)+\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(1 + 3*x^4 + x^8)),x]
```

```
[Out] Log[x] - ((5 + 3*Sqrt[5])*Log[3 - Sqrt[5] + 2*x^4])/40 - ((5 - 3*Sqrt[5])*L
og[3 + Sqrt[5] + 2*x^4])/40
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 632

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rule 705

```
Int[1/(((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)), x_Symbol]
 :> Dist[e^2/(c*d^2 - b*d*e + a*e^2), Int[1/(d + e*x), x], x] + Dist[1/(c*d
^2 - b*d*e + a*e^2), Int[(c*d - b*e - c*e*x)/(a + b*x + c*x^2), x], x] /; F
reeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^
2, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1357

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{1}{x\left(1+3x^4+x^8\right)}\,dx = \frac{1}{4}\operatorname{Subst}\left(\int \frac{1}{x\left(1+3x+x^2\right)}\,dx, x, x^4\right)$$
$$= \frac{1}{4}\operatorname{Subst}\left(\int \frac{1}{x}\,dx, x, x^4\right) + \frac{1}{4}\operatorname{Subst}\left(\int \frac{-3-x}{1+3x+x^2}\,dx, x, x^4\right)$$
$$= \log(x) + \frac{1}{40}\left(-5+3\sqrt{5}\right)\operatorname{Subst}\left(\int \frac{1}{\frac{3}{2}+\frac{\sqrt{5}}{2}+x}\,dx, x, x^4\right) - \frac{1}{40}\left(5+3\sqrt{5}\right)\operatorname{Subst}\left(\int\right.$$
$$= \log(x) - \frac{1}{40}\left(5+3\sqrt{5}\right)\log\left(3-\sqrt{5}+2x^4\right) - \frac{1}{40}\left(5-3\sqrt{5}\right)\log\left(3+\sqrt{5}+2x^4\right)$$

Mathematica [**A**] time = 0.03, size = 55, normalized size = 0.96

$$\frac{1}{40}\left(-5-3\sqrt{5}\right)\log\left(-2x^4+\sqrt{5}-3\right)+\frac{1}{40}\left(3\sqrt{5}-5\right)\log\left(2x^4+\sqrt{5}+3\right)+\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(1 + 3*x^4 + x^8)),x]
```

```
[Out] Log[x] + ((-5 - 3*Sqrt[5])*Log[-3 + Sqrt[5] - 2*x^4])/40 + ((-5 + 3*Sqrt[5]
)*Log[3 + Sqrt[5] + 2*x^4])/40
```

IntegrateAlgebraic [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(1+3x^4+x^8\right)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x*(1 + 3*x^4 + x^8)),x]
```

```
[Out] IntegrateAlgebraic[1/(x*(1 + 3*x^4 + x^8)), x]
```

fricas [**A**] time = 1.45, size = 58, normalized size = 1.02

$$\frac{3}{40}\sqrt{5}\log\left(\frac{2x^8+6x^4+\sqrt{5}\left(2x^4+3\right)+7}{x^8+3x^4+1}\right)-\frac{1}{8}\log\left(x^8+3x^4+1\right)+\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x^8+3*x^4+1),x, algorithm="fricas")
```

```
[Out] 3/40*sqrt(5)*log((2*x^8 + 6*x^4 + sqrt(5)*(2*x^4 + 3) + 7)/(x^8 + 3*x^4 + 1
)) - 1/8*log(x^8 + 3*x^4 + 1) + log(x)
```

giac [**A**] time = 0.48, size = 51, normalized size = 0.89

$$-\frac{3}{40}\sqrt{5}\log\left(\frac{2x^4-\sqrt{5}+3}{2x^4+\sqrt{5}+3}\right)-\frac{1}{8}\log\left(x^8+3x^4+1\right)+\frac{1}{4}\log\left(x^4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x^8+3*x^4+1),x, algorithm="giac")
```

```
[Out] -3/40*sqrt(5)*log((2*x^4 - sqrt(5) + 3)/(2*x^4 + sqrt(5) + 3)) - 1/8*log(x^
8 + 3*x^4 + 1) + 1/4*log(x^4)
```

maple [A] time = 0.01, size = 35, normalized size = 0.61

$$\frac{3\sqrt{5}\ \operatorname{arctanh}\left(\frac{(2x^4+3)\sqrt{5}}{5}\right)}{20}+\ln(x)-\frac{\ln\left(x^8+3x^4+1\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(x^8+3*x^4+1),x)
```

```
[Out] ln(x)-1/8*ln(x^8+3*x^4+1)+3/20*arctanh(1/5*(2*x^4+3)*5^(1/2))*5^(1/2)
```

maxima [A] time = 1.50, size = 51, normalized size = 0.89

$$-\frac{3}{40}\sqrt{5}\log\left(\frac{2x^4-\sqrt{5}+3}{2x^4+\sqrt{5}+3}\right)-\frac{1}{8}\log\left(x^8+3x^4+1\right)+\frac{1}{4}\log\left(x^4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x^8+3*x^4+1),x, algorithm="maxima")
```

```
[Out] -3/40*sqrt(5)*log((2*x^4 - sqrt(5) + 3)/(2*x^4 + sqrt(5) + 3)) - 1/8*log(x^
8 + 3*x^4 + 1) + 1/4*log(x^4)
```

mupad [B] time = 1.41, size = 42, normalized size = 0.74

$$\ln(x)-\ln\left(x^4-\frac{\sqrt{5}}{2}+\frac{3}{2}\right)\left(\frac{3\sqrt{5}}{40}+\frac{1}{8}\right)+\ln\left(x^4+\frac{\sqrt{5}}{2}+\frac{3}{2}\right)\left(\frac{3\sqrt{5}}{40}-\frac{1}{8}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(3*x^4 + x^8 + 1)),x)
```

```
[Out] log(x) - log(x^4 - 5^(1/2)/2 + 3/2)*((3*5^(1/2))/40 + 1/8) + log(5^(1/2)/2
+ x^4 + 3/2)*((3*5^(1/2))/40 - 1/8)
```

sympy [A] time = 0.16, size = 58, normalized size = 1.02

$$\log(x)+\left(-\frac{3\sqrt{5}}{40}-\frac{1}{8}\right)\log\left(x^4-\frac{\sqrt{5}}{2}+\frac{3}{2}\right)+\left(-\frac{1}{8}+\frac{3\sqrt{5}}{40}\right)\log\left(x^4+\frac{\sqrt{5}}{2}+\frac{3}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x**8+3*x**4+1),x)
```

```
[Out] log(x) + (-3*sqrt(5)/40 - 1/8)*log(x**4 - sqrt(5)/2 + 3/2) + (-1/8 + 3*sqrt
(5)/40)*log(x**4 + sqrt(5)/2 + 3/2)
```

3.315 $\int \frac{1}{x^3(1+3x^4+x^8)} dx$

Optimal. Leaf size=89

$$-\frac{1}{2x^2}+\frac{1}{2}\sqrt{\frac{1}{5}\left(9-4\sqrt{5}\right)}\tan^{-1}\left(\sqrt{\frac{2}{3+\sqrt{5}}}\,x^2\right)-\frac{\left(3+\sqrt{5}\right)^{3/2}\tan^{-1}\left(\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}\,x^2\right)}{4\sqrt{10}}$$

Rubi [A] time = 0.07, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1359, 1123, 1166, 203}

$$-\frac{1}{2x^2}+\frac{1}{2}\sqrt{\frac{1}{5}\left(9-4\sqrt{5}\right)}\tan^{-1}\left(\sqrt{\frac{2}{3+\sqrt{5}}}\,x^2\right)-\frac{\left(3+\sqrt{5}\right)^{3/2}\tan^{-1}\left(\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}\,x^2\right)}{4\sqrt{10}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(1 + 3*x^4 + x^8)),x]
```

```
[Out] -1/(2*x^2) + (Sqrt[(9 - 4*Sqrt[5])/5]*ArcTan[Sqrt[2/(3 + Sqrt[5])]*x^2])/2
- ((3 + Sqrt[5])^(3/2)*ArcTan[Sqrt[(3 + Sqrt[5])/2]*x^2])/(4*Sqrt[10])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 1123

```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*x^2 + c*x^4)^(p + 1))/(a*d*(m + 1)), x] - Dis
t[1/(a*d^2*(m + 1)), Int[(d*x)^(m + 2)*(b*(m + 2*p + 3) + c*(m + 4*p + 5)*x
^2)*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b^2 -
 4*a*c, 0] && LtQ[m, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1359

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*
x^(n/k) + c*x^((2*n)/k))^p, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, c, p
}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\int \frac{1}{x^3\left(1+3x^4+x^8\right)}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{x^2\left(1+3x^2+x^4\right)}\,dx,x,x^2\right)$$
$$= -\frac{1}{2x^2}+\frac{1}{2}\,\text{Subst}\left(\int \frac{-3-x^2}{1+3x^2+x^4}\,dx,x,x^2\right)$$
$$= -\frac{1}{2x^2}+\frac{1}{20}\left(-5+3\sqrt{5}\right)\text{Subst}\left(\int \frac{1}{\frac{3}{2}+\frac{\sqrt{5}}{2}+x^2}\,dx,x,x^2\right)-\frac{1}{20}\left(5+3\sqrt{5}\right)\text{Subst}\left(\int \frac{}{\frac{3}{2}}\right.$$
$$= -\frac{1}{2x^2}+\frac{1}{10}\sqrt{45-20\sqrt{5}}\,\tan^{-1}\left(\sqrt{\frac{2}{3+\sqrt{5}}}\,x^2\right)-\frac{\left(3+\sqrt{5}\right)^{3/2}\tan^{-1}\left(\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}\,x^2\right)}{4\sqrt{10}}$$

Mathematica [C] time = 0.02, size = 65, normalized size = 0.73

$$-\frac{1}{4}\text{RootSum}\left[\#1^8+3\#1^4+1\&,\frac{\#1^4\log(x-\#1)+3\log(x-\#1)}{2\#1^6+3\#1^2}\&\right]-\frac{1}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(1 + 3*x^4 + x^8)),x]
```

```
[Out] -1/2*1/x^2 - RootSum[1 + 3*#1^4 + #1^8 & , (3*Log[x - #1] + Log[x - #1]*#1^
4)/(3*#1^2 + 2*#1^6) & ]/4
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3\left(1+3x^4+x^8\right)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x^3*(1 + 3*x^4 + x^8)),x]
```

```
[Out] IntegrateAlgebraic[1/(x^3*(1 + 3*x^4 + x^8)), x]
```

fricas [B] time = 0.96, size = 158, normalized size = 1.78

$$-\frac{2\sqrt{5}x^2\sqrt{-4\sqrt{5}+9}\arctan\left(\frac{1}{4}\sqrt{2x^4+\sqrt{5}+3}\left(\sqrt{5}\sqrt{2}+3\sqrt{2}\right)\sqrt{-4\sqrt{5}+9}-\frac{1}{2}\left(\sqrt{5}x^2+3x^2\right)\sqrt{-4\sqrt{5}+9}\right)+2\sqrt{5}x^2\sqrt{4\sqrt{5}+9}\arctan\left(-\frac{1}{4}\left(2\sqrt{5}x^2-6x^2-\sqrt{2x^4-\sqrt{5}+3}\left(\sqrt{5}\sqrt{2}-3\sqrt{2}\right)\right)\sqrt{4\sqrt{5}+9}\right)+5}{10x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(x^8+3*x^4+1),x, algorithm="fricas")
```

```
[Out] -1/10*(2*sqrt(5)*x^2*sqrt(-4*sqrt(5) + 9)*arctan(1/4*sqrt(2*x^4 + sqrt(5) +
 3)*(sqrt(5)*sqrt(2) + 3*sqrt(2))*sqrt(-4*sqrt(5) + 9) - 1/2*(sqrt(5)*x^2 +
 3*x^2)*sqrt(-4*sqrt(5) + 9)) + 2*sqrt(5)*x^2*sqrt(4*sqrt(5) + 9)*arctan(-1
/4*(2*sqrt(5)*x^2 - 6*x^2 - sqrt(2*x^4 - sqrt(5) + 3)*(sqrt(5)*sqrt(2) - 3*
sqrt(2)))*sqrt(4*sqrt(5) + 9)) + 5)/x^2
```

giac [A] time = 0.46, size = 68, normalized size = 0.76

$$-\frac{1}{20}\left(x^4\left(\sqrt{5}-5\right)+3\sqrt{5}-15\right)\arctan\left(\frac{2x^2}{\sqrt{5}+1}\right)-\frac{1}{20}\left(x^4\left(\sqrt{5}+5\right)+3\sqrt{5}+15\right)\arctan\left(\frac{2x^2}{\sqrt{5}-1}\right)-\frac{1}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(x^8+3*x^4+1),x, algorithm="giac")
```

```
[Out] -1/20*(x^4*(sqrt(5) - 5) + 3*sqrt(5) - 15)*arctan(2*x^2/(sqrt(5) + 1)) - 1/
20*(x^4*(sqrt(5) + 5) + 3*sqrt(5) + 15)*arctan(2*x^2/(sqrt(5) - 1)) - 1/2/x
^2
```

maple [B] time = 0.03, size = 117, normalized size = 1.31

$$-\frac{\arctan\left(\frac{4x^2}{-2+2\sqrt{5}}\right)}{-2+2\sqrt{5}}-\frac{3\sqrt{5}\,\arctan\left(\frac{4x^2}{-2+2\sqrt{5}}\right)}{5\left(-2+2\sqrt{5}\right)}-\frac{\arctan\left(\frac{4x^2}{2+2\sqrt{5}}\right)}{2+2\sqrt{5}}+\frac{3\sqrt{5}\,\arctan\left(\frac{4x^2}{2+2\sqrt{5}}\right)}{5\left(2+2\sqrt{5}\right)}-\frac{1}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(x^8+3*x^4+1),x)
```

```
[Out] -1/(2+2*5^(1/2))*arctan(4/(2+2*5^(1/2))*x^2)+3/5*5^(1/2)/(2+2*5^(1/2))*arct
an(4/(2+2*5^(1/2))*x^2)-1/(-2+2*5^(1/2))*arctan(4/(-2+2*5^(1/2))*x^2)-3/5*5
^(1/2)/(-2+2*5^(1/2))*arctan(4/(-2+2*5^(1/2))*x^2)-1/2/x^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{2\,x^2}-\int\frac{\left(x^4+3\right)x}{x^8+3\,x^4+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(x^8+3*x^4+1),x, algorithm="maxima")
```

```
[Out] -1/2/x^2 - integrate((x^4 + 3)*x/(x^8 + 3*x^4 + 1), x)
```

mupad [B] time = 1.30, size = 130, normalized size = 1.46

$$2\,\mathrm{atanh}\left(\frac{26880\,x^2\,\sqrt{-\frac{\sqrt{5}}{20}-\frac{9}{80}}}{3520\,\sqrt{5}+7872}+\frac{12032\,\sqrt{5}\,x^2\,\sqrt{-\frac{\sqrt{5}}{20}-\frac{9}{80}}}{3520\,\sqrt{5}+7872}\right)\sqrt{-\frac{\sqrt{5}}{20}-\frac{9}{80}}-2\,\mathrm{atanh}\left(\frac{26880\,x^2\,\sqrt{\frac{\sqrt{5}}{20}-\frac{9}{80}}}{3520\,\sqrt{5}-7872}-\frac{12032\,\sqrt{5}\,x^2\,\sqrt{\frac{\sqrt{5}}{20}-\frac{9}{80}}}{3520\,\sqrt{5}-7872}\right)\sqrt{\frac{\sqrt{5}}{20}-\frac{9}{80}}-\frac{1}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(3*x^4 + x^8 + 1)),x)
```

```
[Out] 2*atanh((26880*x^2*(- 5^(1/2)/20 - 9/80)^(1/2))/(3520*5^(1/2) + 7872) + (12
032*5^(1/2)*x^2*(- 5^(1/2)/20 - 9/80)^(1/2))/(3520*5^(1/2) + 7872))*(- 5^(1
/2)/20 - 9/80)^(1/2) - 2*atanh((26880*x^2*(5^(1/2)/20 - 9/80)^(1/2))/(3520*
5^(1/2) - 7872) - (12032*5^(1/2)*x^2*(5^(1/2)/20 - 9/80)^(1/2))/(3520*5^(1/
2) - 7872))*(5^(1/2)/20 - 9/80)^(1/2) - 1/(2*x^2)
```

sympy [A] time = 0.24, size = 56, normalized size = 0.63

$$-2\left(\frac{\sqrt{5}}{10}+\frac{1}{4}\right)\mathrm{atan}\left(\frac{2x^2}{-1+\sqrt{5}}\right)+2\left(\frac{1}{4}-\frac{\sqrt{5}}{10}\right)\mathrm{atan}\left(\frac{2x^2}{1+\sqrt{5}}\right)-\frac{1}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(x**8+3*x**4+1),x)
```

```
[Out] -2*(sqrt(5)/10 + 1/4)*atan(2*x**2/(-1 + sqrt(5))) + 2*(1/4 - sqrt(5)/10)*at
an(2*x**2/(1 + sqrt(5))) - 1/(2*x**2)
```

3.316 $\int \frac{1}{x^5(1+3x^4+x^8)}\,dx$

Optimal. Leaf size=66

$$-\frac{1}{4x^4}+\frac{1}{40}\left(15+7\sqrt{5}\right)\log\left(2x^4-\sqrt{5}+3\right)+\frac{1}{40}\left(15-7\sqrt{5}\right)\log\left(2x^4+\sqrt{5}+3\right)-3\log(x)$$

Rubi [A] time = 0.07, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {1357, 709, 800, 632, 31}

$$-\frac{1}{4x^4}+\frac{1}{40}\left(15+7\sqrt{5}\right)\log\left(2x^4-\sqrt{5}+3\right)+\frac{1}{40}\left(15-7\sqrt{5}\right)\log\left(2x^4+\sqrt{5}+3\right)-3\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^5*(1 + 3*x^4 + x^8)),x]
```

```
[Out] -1/(4*x^4) - 3*Log[x] + ((15 + 7*Sqrt[5])*Log[3 - Sqrt[5] + 2*x^4])/40 + ((
15 - 7*Sqrt[5])*Log[3 + Sqrt[5] + 2*x^4])/40
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 632

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rule 709

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Simp[(e*(d + e*x)^(m + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dis
t[1/(c*d^2 - b*d*e + a*e^2), Int[((d + e*x)^(m + 1)*Simp[c*d - b*e - c*e*x,
 x])/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 -
 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[m
, -1]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 1357

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{1}{x^5\left(1+3x^4+x^8\right)}\,dx=\frac{1}{4}\text{Subst}\left(\int \frac{1}{x^2\left(1+3x+x^2\right)}\,dx,x,x^4\right)$$
$$=-\frac{1}{4x^4}+\frac{1}{4}\text{Subst}\left(\int \frac{-3-x}{x\left(1+3x+x^2\right)}\,dx,x,x^4\right)$$
$$=-\frac{1}{4x^4}+\frac{1}{4}\text{Subst}\left(\int \left(-\frac{3}{x}+\frac{8+3x}{1+3x+x^2}\right)dx,x,x^4\right)$$
$$=-\frac{1}{4x^4}-3\log(x)+\frac{1}{4}\text{Subst}\left(\int \frac{8+3x}{1+3x+x^2}\,dx,x,x^4\right)$$
$$=-\frac{1}{4x^4}-3\log(x)+\frac{1}{40}\left(15-7\sqrt{5}\right)\text{Subst}\left(\int \frac{1}{\frac{3}{2}+\frac{\sqrt{5}}{2}+x}\,dx,x,x^4\right)+\frac{1}{40}\left(15+7\sqrt{5}\right.$$
$$=-\frac{1}{4x^4}-3\log(x)+\frac{1}{40}\left(15+7\sqrt{5}\right)\log\left(3-\sqrt{5}+2x^4\right)+\frac{1}{40}\left(15-7\sqrt{5}\right)\log\left(3+\sqrt{}\right.$$

Mathematica [A] time = 0.04, size = 60, normalized size = 0.91

$$\frac{1}{40}\left(-\frac{10}{x^4}+\left(15+7\sqrt{5}\right)\log\left(-2x^4+\sqrt{5}-3\right)+\left(15-7\sqrt{5}\right)\log\left(2x^4+\sqrt{5}+3\right)-120\log(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^5*(1 + 3*x^4 + x^8)),x]
```

```
[Out] (-10/x^4 - 120*Log[x] + (15 + 7*Sqrt[5])*Log[-3 + Sqrt[5] - 2*x^4] + (15 -
7*Sqrt[5])*Log[3 + Sqrt[5] + 2*x^4])/40
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^5\left(1+3x^4+x^8\right)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x^5*(1 + 3*x^4 + x^8)),x]
```

```
[Out] IntegrateAlgebraic[1/(x^5*(1 + 3*x^4 + x^8)), x]
```

fricas [A] time = 1.18, size = 76, normalized size = 1.15

$$\frac{7\sqrt{5}\,x^4\log\left(\frac{2x^8+6x^4-\sqrt{5}\left(2x^4+3\right)+7}{x^8+3x^4+1}\right)+15\,x^4\log\left(x^8+3\,x^4+1\right)-120\,x^4\log(x)-10}{40\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(x^8+3*x^4+1),x, algorithm="fricas")
```

```
[Out] 1/40*(7*sqrt(5)*x^4*log((2*x^8 + 6*x^4 - sqrt(5)*(2*x^4 + 3) + 7)/(x^8 + 3*
x^4 + 1)) + 15*x^4*log(x^8 + 3*x^4 + 1) - 120*x^4*log(x) - 10)/x^4
```

giac [A] time = 0.55, size = 63, normalized size = 0.95

$$\frac{7}{40}\sqrt{5}\log\left(\frac{2\,x^4-\sqrt{5}+3}{2\,x^4+\sqrt{5}+3}\right)+\frac{3\,x^4-1}{4\,x^4}+\frac{3}{8}\log\left(x^8+3\,x^4+1\right)-\frac{3}{4}\log\left(x^4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(x^8+3*x^4+1),x, algorithm="giac")
```

```
[Out] 7/40*sqrt(5)*log((2*x^4 - sqrt(5) + 3)/(2*x^4 + sqrt(5) + 3)) + 1/4*(3*x^4
- 1)/x^4 + 3/8*log(x^8 + 3*x^4 + 1) - 3/4*log(x^4)
```

maple [A] time = 0.01, size = 42, normalized size = 0.64

$$-\frac{7\sqrt{5}\ \operatorname{arctanh}\left(\frac{(2x^4+3)\sqrt{5}}{5}\right)}{20}-3\ln(x)+\frac{3\ln\left(x^8+3x^4+1\right)}{8}-\frac{1}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^5/(x^8+3*x^4+1),x)
```

```
[Out] -1/4/x^4-3*ln(x)+3/8*ln(x^8+3*x^4+1)-7/20*arctanh(1/5*(2*x^4+3)*5^(1/2))*5^
(1/2)
```

maxima [A] time = 1.29, size = 56, normalized size = 0.85

$$\frac{7}{40}\sqrt{5}\log\left(\frac{2x^4-\sqrt{5}+3}{2x^4+\sqrt{5}+3}\right)-\frac{1}{4x^4}+\frac{3}{8}\log\left(x^8+3x^4+1\right)-\frac{3}{4}\log\left(x^4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(x^8+3*x^4+1),x, algorithm="maxima")
```

```
[Out] 7/40*sqrt(5)*log((2*x^4 - sqrt(5) + 3)/(2*x^4 + sqrt(5) + 3)) - 1/4/x^4 + 3
/8*log(x^8 + 3*x^4 + 1) - 3/4*log(x^4)
```

mupad [B] time = 1.36, size = 49, normalized size = 0.74

$$\ln\left(x^4-\frac{\sqrt{5}}{2}+\frac{3}{2}\right)\left(\frac{7\sqrt{5}}{40}+\frac{3}{8}\right)-\frac{1}{4x^4}-3\ln(x)-\ln\left(x^4+\frac{\sqrt{5}}{2}+\frac{3}{2}\right)\left(\frac{7\sqrt{5}}{40}-\frac{3}{8}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^5*(3*x^4 + x^8 + 1)),x)
```

```
[Out] log(x^4 - 5^(1/2)/2 + 3/2)*((7*5^(1/2))/40 + 3/8) - 1/(4*x^4) - 3*log(x) -
log(5^(1/2)/2 + x^4 + 3/2)*((7*5^(1/2))/40 - 3/8)
```

sympy [A] time = 0.19, size = 65, normalized size = 0.98

$$-3\log(x)+\left(\frac{3}{8}+\frac{7\sqrt{5}}{40}\right)\log\left(x^4-\frac{\sqrt{5}}{2}+\frac{3}{2}\right)+\left(\frac{3}{8}-\frac{7\sqrt{5}}{40}\right)\log\left(x^4+\frac{\sqrt{5}}{2}+\frac{3}{2}\right)-\frac{1}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**5/(x**8+3*x**4+1),x)
```

```
[Out] -3*log(x) + (3/8 + 7*sqrt(5)/40)*log(x**4 - sqrt(5)/2 + 3/2) + (3/8 - 7*sqr
t(5)/40)*log(x**4 + sqrt(5)/2 + 3/2) - 1/(4*x**4)
```

3.317 $\int \frac{1}{x^7(1+3x^4+x^8)}\,dx$

Optimal. Leaf size=97

$$-\frac{1}{6x^6}+\frac{3}{2x^2}-\frac{1}{2}\sqrt{\frac{1}{10}\left(123-55\sqrt{5}\right)}\tan^{-1}\left(\sqrt{\frac{2}{3+\sqrt{5}}}\,x^2\right)+\frac{1}{2}\sqrt{\frac{1}{10}\left(123+55\sqrt{5}\right)}\tan^{-1}\left(\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}\,x^2\right)$$

Rubi [A] time = 0.13, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {1359, 1123, 1281, 1166, 203}

$$\frac{3}{2x^2}-\frac{1}{6x^6}-\frac{1}{2}\sqrt{\frac{1}{10}\left(123-55\sqrt{5}\right)}\tan^{-1}\left(\sqrt{\frac{2}{3+\sqrt{5}}}\,x^2\right)+\frac{1}{2}\sqrt{\frac{1}{10}\left(123+55\sqrt{5}\right)}\tan^{-1}\left(\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}\,x^2\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^7*(1 + 3*x^4 + x^8)),x]
```

```
[Out] -1/(6*x^6) + 3/(2*x^2) - (Sqrt[(123 - 55*Sqrt[5])/10]*ArcTan[Sqrt[2/(3 + Sq
rt[5])]*x^2])/2 + (Sqrt[(123 + 55*Sqrt[5])/10]*ArcTan[Sqrt[(3 + Sqrt[5])/2]
*x^2])/2
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 1123

```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*x^2 + c*x^4)^(p + 1))/(a*d*(m + 1)), x] - Dis
t[1/(a*d^2*(m + 1)), Int[(d*x)^(m + 2)*(b*(m + 2*p + 3) + c*(m + 4*p + 5)*x
^2)*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b^2 -
 4*a*c, 0] && LtQ[m, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1281

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[(d*(f*x)^(m + 1)*(a + b*x^2 + c*x^4)^(p + 1)
)/(a*f*(m + 1)), x] + Dist[1/(a*f^2*(m + 1)), Int[(f*x)^(m + 2)*(a + b*x^2
+ c*x^4)^p*Simp[a*e*(m + 1) - b*d*(m + 2*p + 3) - c*d*(m + 4*p + 5)*x^2, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[m
, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1359

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*
x^(n/k) + c*x^((2*n)/k))^p, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, c, p
```

```
}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^7\left(1+3x^4+x^8\right)}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{x^4\left(1+3x^2+x^4\right)}\,dx,x,x^2\right)\\ &= -\frac{1}{6x^6}+\frac{1}{6}\,\text{Subst}\left(\int \frac{-9-3x^2}{x^2\left(1+3x^2+x^4\right)}\,dx,x,x^2\right)\\ &= -\frac{1}{6x^6}+\frac{3}{2x^2}-\frac{1}{6}\,\text{Subst}\left(\int \frac{-24-9x^2}{1+3x^2+x^4}\,dx,x,x^2\right)\\ &= -\frac{1}{6x^6}+\frac{3}{2x^2}-\frac{1}{20}\left(-15+7\sqrt{5}\right)\text{Subst}\left(\int \frac{1}{\frac{3}{2}+\frac{\sqrt{5}}{2}+x^2}\,dx,x,x^2\right)+\frac{1}{20}\left(15+7\sqrt{5}\right)\text{Su}\\ &= -\frac{1}{6x^6}+\frac{3}{2x^2}-\frac{1}{2}\sqrt{\frac{1}{10}\left(123-55\sqrt{5}\right)}\,\tan^{-1}\left(\sqrt{\frac{2}{3+\sqrt{5}}}\,x^2\right)+\frac{1}{20}\sqrt{1230+550\sqrt{5}}\,\tan\end{aligned}$$

Mathematica [C] time = 0.02, size = 73, normalized size = 0.75

$$\frac{1}{4}\text{RootSum}\left[\#1^8+3\#1^4+1\&,\frac{3\#1^4\log(x-\#1)+8\log(x-\#1)}{2\#1^6+3\#1^2}\&\right]-\frac{1}{6x^6}+\frac{3}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^7*(1 + 3*x^4 + x^8)),x]
```

```
[Out] -1/6*1/x^6 + 3/(2*x^2) + RootSum[1 + 3*#1^4 + #1^8 & , (8*Log[x - #1] + 3*L
og[x - #1]*#1^4)/(3*#1^2 + 2*#1^6) & ]/4
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^7\left(1+3x^4+x^8\right)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x^7*(1 + 3*x^4 + x^8)),x]
```

```
[Out] IntegrateAlgebraic[1/(x^7*(1 + 3*x^4 + x^8)), x]
```

fricas [B] time = 0.79, size = 180, normalized size = 1.86

$$\frac{3\sqrt{10}x^6\sqrt{-55\sqrt{5}+123}\arctan\left(\frac{1}{40}\sqrt{10}\sqrt{2x^4+\sqrt{5}+3}\left(7\sqrt{5}\sqrt{2}+15\sqrt{2}\right)\sqrt{-55\sqrt{5}+123}-\frac{1}{20}\sqrt{10}\left(7\sqrt{5}x^2+15x^2\right)\sqrt{-55\sqrt{5}+123}\right)-3\sqrt{10}x^6\sqrt{55\sqrt{5}+123}\arctan\left(\frac{1}{40}\left(\sqrt{10}\sqrt{2x^4-\sqrt{5}+3}\left(7\sqrt{5}\sqrt{2}-15\sqrt{2}\right)-2\sqrt{10}\left(7\sqrt{5}x^2-15x^2\right)\right)\sqrt{55\sqrt{5}+123}\right)+45x^4-5}{30x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(x^8+3*x^4+1),x, algorithm="fricas")
```

```
[Out] 1/30*(3*sqrt(10)*x^6*sqrt(-55*sqrt(5) + 123)*arctan(1/40*sqrt(10)*sqrt(2*x^
4 + sqrt(5) + 3)*(7*sqrt(5)*sqrt(2) + 15*sqrt(2))*sqrt(-55*sqrt(5) + 123) -
 1/20*sqrt(10)*(7*sqrt(5)*x^2 + 15*x^2)*sqrt(-55*sqrt(5) + 123)) - 3*sqrt(1
0)*x^6*sqrt(55*sqrt(5) + 123)*arctan(1/40*(sqrt(10)*sqrt(2*x^4 - sqrt(5) +
3)*(7*sqrt(5)*sqrt(2) - 15*sqrt(2)) - 2*sqrt(10)*(7*sqrt(5)*x^2 - 15*x^2))*
sqrt(55*sqrt(5) + 123)) + 45*x^4 - 5)/x^6
```

giac [A] time = 0.51, size = 77, normalized size = 0.79

$$\frac{1}{20}\left(3\,x^4\left(\sqrt{5}-5\right)+8\,\sqrt{5}-40\right)\arctan\left(\frac{2\,x^2}{\sqrt{5}+1}\right)+\frac{1}{20}\left(3\,x^4\left(\sqrt{5}+5\right)+8\,\sqrt{5}+40\right)\arctan\left(\frac{2\,x^2}{\sqrt{5}-1}\right)+\frac{9\,x^4-1}{6\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(x^8+3*x^4+1),x, algorithm="giac")
```

```
[Out] 1/20*(3*x^4*(sqrt(5) - 5) + 8*sqrt(5) - 40)*arctan(2*x^2/(sqrt(5) + 1)) + 1
/20*(3*x^4*(sqrt(5) + 5) + 8*sqrt(5) + 40)*arctan(2*x^2/(sqrt(5) - 1)) + 1/
6*(9*x^4 - 1)/x^6
```

maple [B] time = 0.03, size = 122, normalized size = 1.26

$$\frac{7\sqrt{5}\ \arctan\left(\frac{4x^2}{-2+2\sqrt{5}}\right)}{5\left(-2+2\sqrt{5}\right)}+\frac{3\arctan\left(\frac{4x^2}{-2+2\sqrt{5}}\right)}{-2+2\sqrt{5}}-\frac{7\sqrt{5}\ \arctan\left(\frac{4x^2}{2+2\sqrt{5}}\right)}{5\left(2+2\sqrt{5}\right)}+\frac{3\arctan\left(\frac{4x^2}{2+2\sqrt{5}}\right)}{2+2\sqrt{5}}+\frac{3}{2x^2}-\frac{1}{6x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^7/(x^8+3*x^4+1),x)
```

```
[Out] -1/6/x^6+3/2/x^2-7/5*5^(1/2)/(2+2*5^(1/2))*arctan(4/(2+2*5^(1/2))*x^2)+3/(2
+2*5^(1/2))*arctan(4/(2+2*5^(1/2))*x^2)+7/5*5^(1/2)/(-2+2*5^(1/2))*arctan(4
/(-2+2*5^(1/2))*x^2)+3/(-2+2*5^(1/2))*arctan(4/(-2+2*5^(1/2))*x^2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{9\,x^4-1}{6\,x^6}+\int\frac{\left(3\,x^4+8\right)x}{x^8+3\,x^4+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(x^8+3*x^4+1),x, algorithm="maxima")
```

```
[Out] 1/6*(9*x^4 - 1)/x^6 + integrate((3*x^4 + 8)*x/(x^8 + 3*x^4 + 1), x)
```

mupad [B] time = 0.12, size = 136, normalized size = 1.40

$$2\,\mathrm{atanh}\left(\frac{3327500\,x^2\,\sqrt{\frac{11\,\sqrt{5}}{32}-\frac{123}{160}}}{1140425\,\sqrt{5}-2550075}-\frac{1488300\,\sqrt{5}\,x^2\,\sqrt{\frac{11\,\sqrt{5}}{32}-\frac{123}{160}}}{1140425\,\sqrt{5}-2550075}\right)\sqrt{\frac{11\,\sqrt{5}}{32}-\frac{123}{160}}-2\,\mathrm{atanh}\left(\frac{3327500\,x^2\,\sqrt{-\frac{11\,\sqrt{5}}{32}-\frac{123}{160}}}{1140425\,\sqrt{5}+2550075}+\frac{1488300\,\sqrt{5}\,x^2\,\sqrt{-\frac{11\,\sqrt{5}}{32}-\frac{123}{160}}}{1140425\,\sqrt{5}+2550075}\right)\sqrt{-\frac{11\,\sqrt{5}}{32}-\frac{123}{160}}+\frac{\frac{3\,x^4}{2}-\frac{1}{6}}{x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^7*(3*x^4 + x^8 + 1)),x)
```

```
[Out] 2*atanh((3327500*x^2*((11*5^(1/2))/32 - 123/160)^(1/2))/(1140425*5^(1/2) -
2550075) - (1488300*5^(1/2)*x^2*((11*5^(1/2))/32 - 123/160)^(1/2))/(1140425
*5^(1/2) - 2550075))*((11*5^(1/2))/32 - 123/160)^(1/2) - 2*atanh((3327500*x
^2*(- (11*5^(1/2))/32 - 123/160)^(1/2))/(1140425*5^(1/2) + 2550075) + (1488
300*5^(1/2)*x^2*(- (11*5^(1/2))/32 - 123/160)^(1/2))/(1140425*5^(1/2) + 255
0075))*(- (11*5^(1/2))/32 - 123/160)^(1/2) + ((3*x^4)/2 - 1/6)/x^6
```

sympy [A] time = 0.27, size = 65, normalized size = 0.67

$$2\left(\frac{11\sqrt{5}}{40}+\frac{5}{8}\right)\mathrm{atan}\left(\frac{2x^2}{-1+\sqrt{5}}\right)-2\left(\frac{5}{8}-\frac{11\sqrt{5}}{40}\right)\mathrm{atan}\left(\frac{2x^2}{1+\sqrt{5}}\right)+\frac{9x^4-1}{6x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**7/(x**8+3*x**4+1),x)
```

```
[Out] 2*(11*sqrt(5)/40 + 5/8)*atan(2*x**2/(-1 + sqrt(5))) - 2*(5/8 - 11*sqrt(5)/4
0)*atan(2*x**2/(1 + sqrt(5))) + (9*x**4 - 1)/(6*x**6)
```

3.318 $\int \frac{x^8}{1+3x^4+x^8}\,dx$

Optimal. Leaf size=460

$$-\frac{\sqrt[4]{123-55\sqrt{5}}\,\log\left(2x^2-2\sqrt[4]{2\left(3-\sqrt{5}\right)}\,x+\sqrt{2\left(3-\sqrt{5}\right)}\right)}{4\,2^{3/4}\sqrt{5}}+\frac{\sqrt[4]{123-55\sqrt{5}}\,\log\left(2x^2+2\sqrt[4]{2\left(3-\sqrt{5}\right)}\,x+\sqrt{2\right.}}{4\,2^{3/4}\sqrt{5}}$$

Rubi [A] time = 0.42, antiderivative size = 440, normalized size of antiderivative = 0.96, number of steps used = 20, number of rules used = 8, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {1367, 1422, 211, 1165, 628, 1162, 617, 204}

$$-\frac{\sqrt[4]{984-440\sqrt{5}}\,\log\left(2x^2-2\sqrt[4]{2\left(3-\sqrt{5}\right)}\,x+\sqrt{2\left(3-\sqrt{5}\right)}\right)}{8\sqrt{10}}+\frac{\sqrt[4]{984-440\sqrt{5}}\,\log\left(2x^2+2\sqrt[4]{2\left(3-\sqrt{5}\right)}\,x+\sqrt{2\left(3-\sqrt{5}\right)}\right)}{8\sqrt{10}}+\frac{\sqrt[4]{123+55\sqrt{5}}\,\log\left(2x^2-2\sqrt[4]{2\left(3+\sqrt{5}\right)}\,x+\sqrt{2\left(3+\sqrt{5}\right)}\right)}{4\,2^{3/4}\sqrt{5}}-\frac{\sqrt[4]{123+55\sqrt{5}}\,\log\left(2x^2+2\sqrt[4]{2\left(3+\sqrt{5}\right)}\,x+\sqrt{2\left(3+\sqrt{5}\right)}\right)}{4\,2^{3/4}\sqrt{5}}+x-\frac{\sqrt[4]{984-440\sqrt{5}}\,\tan^{-1}\left(1-\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}\right)}{4\sqrt{10}}+\frac{\sqrt[4]{984-440\sqrt{5}}\,\tan^{-1}\left(\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}+1\right)}{4\sqrt{10}}+\frac{\sqrt[4]{123+55\sqrt{5}}\,\tan^{-1}\left(1-\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}\right)}{2\,2^{3/4}\sqrt{5}}-\frac{\sqrt[4]{123+55\sqrt{5}}\,\tan^{-1}\left(\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}+1\right)}{2\,2^{3/4}\sqrt{5}}$$

Antiderivative was successfully verified.

```
[In] Int[x^8/(1 + 3*x^4 + x^8),x]
```

```
[Out] x - ((984 - 440*Sqrt[5])^(1/4)*ArcTan[1 - (2^(3/4)*x)/(3 - Sqrt[5])^(1/4)])
/(4*Sqrt[10]) + ((984 - 440*Sqrt[5])^(1/4)*ArcTan[1 + (2^(3/4)*x)/(3 - Sqrt
[5])^(1/4)])/(4*Sqrt[10]) + ((123 + 55*Sqrt[5])^(1/4)*ArcTan[1 - (2^(3/4)*x
)/(3 + Sqrt[5])^(1/4)])/(2*2^(3/4)*Sqrt[5]) - ((123 + 55*Sqrt[5])^(1/4)*Arc
Tan[1 + (2^(3/4)*x)/(3 + Sqrt[5])^(1/4)])/(2*2^(3/4)*Sqrt[5]) - ((984 - 440
*Sqrt[5])^(1/4)*Log[Sqrt[2*(3 - Sqrt[5])] - 2*(2*(3 - Sqrt[5]))^(1/4)*x + 2
*x^2])/(8*Sqrt[10]) + ((984 - 440*Sqrt[5])^(1/4)*Log[Sqrt[2*(3 - Sqrt[5])]
+ 2*(2*(3 - Sqrt[5]))^(1/4)*x + 2*x^2])/(8*Sqrt[10]) + ((123 + 55*Sqrt[5])^
(1/4)*Log[Sqrt[2*(3 + Sqrt[5])] - 2*(2*(3 + Sqrt[5]))^(1/4)*x + 2*x^2])/(4*
2^(3/4)*Sqrt[5]) - ((123 + 55*Sqrt[5])^(1/4)*Log[Sqrt[2*(3 + Sqrt[5])] + 2*
(2*(3 + Sqrt[5]))^(1/4)*x + 2*x^2])/(4*2^(3/4)*Sqrt[5])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1367

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Simp[(d^(2*n - 1)*(d*x)^(m - 2*n + 1)*(a + b*x^n + c*x^(2*n))^(
p + 1))/(c*(m + 2*n*p + 1)), x] - Dist[d^(2*n)/(c*(m + 2*n*p + 1)), Int[(d*
x)^(m - 2*n)*Simp[a*(m - 2*n + 1) + b*(m + n*(p - 1) + 1)*x^n, x]*(a + b*x^
n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && EqQ[n2, 2*n] && Ne
Q[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GtQ[m, 2*n - 1] && NeQ[m + 2*n*p + 1, 0]
 && IntegerQ[p]
```

Rule 1422

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q),
Int[1/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(
b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (PosQ[b^2 - 4*a
*c] ||  !IGtQ[n/2, 0])
```

Rubi steps

$$\int \frac{x^8}{1+3x^4+x^8}\,dx = x-\int \frac{1+3x^4}{1+3x^4+x^8}\,dx$$

$$= x-\frac{1}{10}\left(15-7\sqrt{5}\right)\int \frac{1}{\frac{3}{2}-\frac{\sqrt{5}}{2}+x^4}\,dx-\frac{1}{10}\left(15+7\sqrt{5}\right)\int \frac{1}{\frac{3}{2}+\frac{\sqrt{5}}{2}+x^4}\,dx$$

$$= x+\frac{1}{2}\sqrt{\frac{1}{10}\left(9-4\sqrt{5}\right)}\int \frac{\sqrt{3-\sqrt{5}}-\sqrt{2}\,x^2}{\frac{3}{2}-\frac{\sqrt{5}}{2}+x^4}\,dx+\frac{1}{2}\sqrt{\frac{1}{10}\left(9-4\sqrt{5}\right)}\int \frac{\sqrt{3-\sqrt{5}}+\sqrt{2}}{\frac{3}{2}-\frac{\sqrt{5}}{2}+x^4}$$

$$= x+\frac{1}{4}\sqrt{\frac{1}{5}\left(9-4\sqrt{5}\right)}\int \frac{1}{\sqrt{\frac{1}{2}\left(3-\sqrt{5}\right)}-\sqrt[4]{2\left(3-\sqrt{5}\right)}\,x+x^2}\,dx+\frac{1}{4}\sqrt{\frac{1}{5}\left(9-4\sqrt{5}\right)}\int$$

$$= x-\frac{1}{8}\sqrt[4]{\frac{246}{25}-\frac{22}{\sqrt{5}}}\log\left(\sqrt{2\left(3-\sqrt{5}\right)}-2\sqrt[4]{2\left(3-\sqrt{5}\right)}\,x+2x^2\right)+\frac{1}{8}\sqrt[4]{\frac{246}{25}-\frac{22}{\sqrt{5}}}\log\Big($$

$$= x-\frac{\sqrt[4]{123-55\sqrt{5}}\tan^{-1}\left(1-\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}\right)}{2\,2^{3/4}\sqrt{5}}+\frac{\sqrt[4]{123-55\sqrt{5}}\tan^{-1}\left(1+\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}\right)}{2\,2^{3/4}\sqrt{5}}+\frac{\sqrt[4]{123+}}{}$$

Mathematica [C] time = 0.01, size = 58, normalized size = 0.13

$$x-\frac{1}{4}\text{RootSum}\left[\#1^8+3\#1^4+1\&,\frac{3\#1^4\log(x-\#1)+\log(x-\#1)}{2\#1^7+3\#1^3}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8/(1 + 3*x^4 + x^8),x]
```

```
[Out] x - RootSum[1 + 3*#1^4 + #1^8 & , (Log[x - #1] + 3*Log[x - #1]*#1^4)/(3*#1^
3 + 2*#1^7) & ]/4
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^8}{1+3x^4+x^8}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^8/(1 + 3*x^4 + x^8),x]
```

```
[Out] IntegrateAlgebraic[x^8/(1 + 3*x^4 + x^8), x]
```

fricas [B] time = 1.68, size = 1012, normalized size = 2.20

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(x^8+3*x^4+1),x, algorithm="fricas")
```

```
[Out] 1/80*sqrt(10)*(110*sqrt(5) + 246)^(3/4)*sqrt(55*sqrt(5) + 123)*(55*sqrt(5)
- 123)*arctan(1/80*sqrt(10)*sqrt(20*x^2 + sqrt(10)*(3*sqrt(5)*sqrt(2)*x - 5
*sqrt(2)*x)*(110*sqrt(5) + 246)^(1/4) - 5*sqrt(110*sqrt(5) + 246)*(3*sqrt(5
) - 7))*(1292*sqrt(5) - 2889)*(110*sqrt(5) + 246)^(5/4)*sqrt(55*sqrt(5) + 1
23) + 1/40*sqrt(10)*(2889*sqrt(5)*x - 6460*x)*(110*sqrt(5) + 246)^(5/4)*sqr
t(55*sqrt(5) + 123) - 1/8*(55*sqrt(5)*sqrt(2) - 123*sqrt(2))*sqrt(110*sqrt(
5) + 246)*sqrt(55*sqrt(5) + 123)) + 1/80*sqrt(10)*(110*sqrt(5) + 246)^(3/4)
*sqrt(55*sqrt(5) + 123)*(55*sqrt(5) - 123)*arctan(1/80*sqrt(10)*sqrt(20*x^2
 - sqrt(10)*(3*sqrt(5)*sqrt(2)*x - 5*sqrt(2)*x)*(110*sqrt(5) + 246)^(1/4) -
 5*sqrt(110*sqrt(5) + 246)*(3*sqrt(5) - 7))*(1292*sqrt(5) - 2889)*(110*sqrt
(5) + 246)^(5/4)*sqrt(55*sqrt(5) + 123) + 1/40*sqrt(10)*(2889*sqrt(5)*x - 6
460*x)*(110*sqrt(5) + 246)^(5/4)*sqrt(55*sqrt(5) + 123) + 1/8*(55*sqrt(5)*s
qrt(2) - 123*sqrt(2))*sqrt(110*sqrt(5) + 246)*sqrt(55*sqrt(5) + 123)) - 1/8
0*sqrt(10)*(55*sqrt(5) + 123)*sqrt(-55*sqrt(5) + 123)*(-110*sqrt(5) + 246)^
(3/4)*arctan(1/80*sqrt(10)*sqrt(20*x^2 + sqrt(10)*(3*sqrt(5)*sqrt(2)*x + 5*
sqrt(2)*x)*(-110*sqrt(5) + 246)^(1/4) + 5*(3*sqrt(5) + 7)*sqrt(-110*sqrt(5)
 + 246))*(1292*sqrt(5) + 2889)*sqrt(-55*sqrt(5) + 123)*(-110*sqrt(5) + 246)
^(5/4) - 1/40*(sqrt(10)*(2889*sqrt(5)*x + 6460*x)*(-110*sqrt(5) + 246)^(5/4
) + 5*(55*sqrt(5)*sqrt(2) + 123*sqrt(2))*sqrt(-110*sqrt(5) + 246))*sqrt(-55
*sqrt(5) + 123)) - 1/80*sqrt(10)*(55*sqrt(5) + 123)*sqrt(-55*sqrt(5) + 123)
*(-110*sqrt(5) + 246)^(3/4)*arctan(1/80*sqrt(10)*sqrt(20*x^2 - sqrt(10)*(3*
sqrt(5)*sqrt(2)*x + 5*sqrt(2)*x)*(-110*sqrt(5) + 246)^(1/4) + 5*(3*sqrt(5)
+ 7)*sqrt(-110*sqrt(5) + 246))*(1292*sqrt(5) + 2889)*sqrt(-55*sqrt(5) + 123
)*(-110*sqrt(5) + 246)^(5/4) - 1/40*(sqrt(10)*(2889*sqrt(5)*x + 6460*x)*(-1
10*sqrt(5) + 246)^(5/4) - 5*(55*sqrt(5)*sqrt(2) + 123*sqrt(2))*sqrt(-110*sq
rt(5) + 246))*sqrt(-55*sqrt(5) + 123)) - 1/80*sqrt(10)*sqrt(2)*(110*sqrt(5)
 + 246)^(1/4)*log(20*x^2 + sqrt(10)*(3*sqrt(5)*sqrt(2)*x - 5*sqrt(2)*x)*(11
0*sqrt(5) + 246)^(1/4) - 5*sqrt(110*sqrt(5) + 246)*(3*sqrt(5) - 7)) + 1/80*
sqrt(10)*sqrt(2)*(110*sqrt(5) + 246)^(1/4)*log(20*x^2 - sqrt(10)*(3*sqrt(5)
*sqrt(2)*x - 5*sqrt(2)*x)*(110*sqrt(5) + 246)^(1/4) - 5*sqrt(110*sqrt(5) +
246)*(3*sqrt(5) - 7)) + 1/80*sqrt(10)*sqrt(2)*(-110*sqrt(5) + 246)^(1/4)*lo
g(20*x^2 + sqrt(10)*(3*sqrt(5)*sqrt(2)*x + 5*sqrt(2)*x)*(-110*sqrt(5) + 246
```

```
)^(1/4) + 5*(3*sqrt(5) + 7)*sqrt(-110*sqrt(5) + 246)) - 1/80*sqrt(10)*sqrt(
2)*(-110*sqrt(5) + 246)^(1/4)*log(20*x^2 - sqrt(10)*(3*sqrt(5)*sqrt(2)*x +
5*sqrt(2)*x)*(-110*sqrt(5) + 246)^(1/4) + 5*(3*sqrt(5) + 7)*sqrt(-110*sqrt(
5) + 246)) + x
```

giac [A] time = 0.73, size = 240, normalized size = 0.52

$$-\frac{1}{80}\left(\pi + 4\arctan\left(x\sqrt{\sqrt{5}-1}+1\right)\right)\sqrt{25\sqrt{5}+55} + \frac{1}{80}\left(\pi + 4\arctan\left(-x\sqrt{\sqrt{5}-1}+1\right)\right)\sqrt{25\sqrt{5}+55} + \frac{1}{80}\left(\pi + 4\arctan\left(x\sqrt{\sqrt{5}+1}-1\right)\right)\sqrt{25\sqrt{5}-55} - \frac{1}{80}\left(\pi + 4\arctan\left(-x\sqrt{\sqrt{5}+1}-1\right)\right)\sqrt{25\sqrt{5}-55} - \frac{1}{40}\sqrt{25\sqrt{5}+55}\log\left(722500\left(x+\sqrt{\sqrt{5}+1}\right)^2+722500x^2\right) + \frac{1}{40}\sqrt{25\sqrt{5}+55}\log\left(722500\left(x-\sqrt{\sqrt{5}+1}\right)^2+722500x^2\right) + \frac{1}{40}\sqrt{25\sqrt{5}-55}\log\left(2992900\left(x+\sqrt{\sqrt{5}-1}\right)^2+2992900x^2\right) - \frac{1}{40}\sqrt{25\sqrt{5}-55}\log\left(2992900\left(x-\sqrt{\sqrt{5}-1}\right)^2+2992900x^2\right) + x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(x^8+3*x^4+1),x, algorithm="giac")
```

```
[Out] -1/80*(pi + 4*arctan(x*sqrt(sqrt(5) - 1) + 1))*sqrt(25*sqrt(5) + 55) + 1/80
*(pi + 4*arctan(-x*sqrt(sqrt(5) - 1) + 1))*sqrt(25*sqrt(5) + 55) + 1/80*(pi
 + 4*arctan(x*sqrt(sqrt(5) + 1) - 1))*sqrt(25*sqrt(5) - 55) - 1/80*(pi + 4*
arctan(-x*sqrt(sqrt(5) + 1) - 1))*sqrt(25*sqrt(5) - 55) - 1/40*sqrt(25*sqrt
(5) + 55)*log(722500*(x + sqrt(sqrt(5) + 1))^2 + 722500*x^2) + 1/40*sqrt(25
*sqrt(5) + 55)*log(722500*(x - sqrt(sqrt(5) + 1))^2 + 722500*x^2) + 1/40*sq
rt(25*sqrt(5) - 55)*log(2992900*(x + sqrt(sqrt(5) - 1))^2 + 2992900*x^2) -
1/40*sqrt(25*sqrt(5) - 55)*log(2992900*(x - sqrt(sqrt(5) - 1))^2 + 2992900*
x^2) + x
```

maple [C] time = 0.01, size = 46, normalized size = 0.10

$$x + \frac{\left(-3\,\mathrm{RootOf}\left(\_Z^8+3\_Z^4+1\right)^4-1\right)\ln\left(-\mathrm{RootOf}\left(\_Z^8+3\_Z^4+1\right)+x\right)}{8\,\mathrm{RootOf}\left(\_Z^8+3\_Z^4+1\right)^7+12\,\mathrm{RootOf}\left(\_Z^8+3\_Z^4+1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(x^8+3*x^4+1),x)
```

```
[Out] x+1/4*sum((-3*_R^4-1)/(2*_R^7+3*_R^3)*ln(-_R+x),_R=RootOf(_Z^8+3*_Z^4+1))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$x - \int \frac{3\,x^4+1}{x^8+3\,x^4+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(x^8+3*x^4+1),x, algorithm="maxima")
```

```
[Out] x - integrate((3*x^4 + 1)/(x^8 + 3*x^4 + 1), x)
```

mupad [B] time = 1.44, size = 216, normalized size = 0.47

$$x - \frac{2^{3/4}\sqrt{5}\,\mathrm{atan}\left(\frac{3\,2^{1/4}\,x}{2\,{\left(-55\,\sqrt{5}-123\right)}^{1/4}}+\frac{2^{1/4}\,\sqrt{5}\,x}{2\,{\left(-55\,\sqrt{5}-123\right)}^{1/4}}\right)\,{\left(-55\,\sqrt{5}-123\right)}^{1/4}}{20} + \frac{2^{3/4}\sqrt{5}\,\mathrm{atan}\left(\frac{3\,2^{1/4}\,x}{2\,{\left(55\,\sqrt{5}-123\right)}^{1/4}}-\frac{2^{1/4}\,\sqrt{5}\,x}{2\,{\left(55\,\sqrt{5}-123\right)}^{1/4}}\right)\,{\left(55\,\sqrt{5}-123\right)}^{1/4}}{20} + \frac{2^{3/4}\sqrt{5}\,\mathrm{atan}\left(\frac{2^{1/4}\,x\,3\mathrm{i}}{2\,{\left(-55\,\sqrt{5}-123\right)}^{1/4}}+\frac{2^{1/4}\,\sqrt{5}\,x\,1\mathrm{i}}{2\,{\left(-55\,\sqrt{5}-123\right)}^{1/4}}\right)\,{\left(-55\,\sqrt{5}-123\right)}^{1/4}\,1\mathrm{i}}{20} - \frac{2^{3/4}\sqrt{5}\,\mathrm{atan}\left(\frac{2^{1/4}\,x\,3\mathrm{i}}{2\,{\left(55\,\sqrt{5}-123\right)}^{1/4}}-\frac{2^{1/4}\,\sqrt{5}\,x\,1\mathrm{i}}{2\,{\left(55\,\sqrt{5}-123\right)}^{1/4}}\right)\,{\left(55\,\sqrt{5}-123\right)}^{1/4}\,1\mathrm{i}}{20}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(3*x^4 + x^8 + 1),x)
```

```
[Out] x - (2^(3/4)*5^(1/2)*atan((3*2^(1/4)*x)/(2*(- 55*5^(1/2) - 123)^(1/4)) + (2
^(1/4)*5^(1/2)*x)/(2*(- 55*5^(1/2) - 123)^(1/4)))*(- 55*5^(1/2) - 123)^(1/4
))/20 + (2^(3/4)*5^(1/2)*atan((3*2^(1/4)*x)/(2*(55*5^(1/2) - 123)^(1/4)) -
(2^(1/4)*5^(1/2)*x)/(2*(55*5^(1/2) - 123)^(1/4)))*(55*5^(1/2) - 123)^(1/4))
/20 + (2^(3/4)*5^(1/2)*atan((2^(1/4)*x*3i)/(2*(- 55*5^(1/2) - 123)^(1/4)) +
 (2^(1/4)*5^(1/2)*x*1i)/(2*(- 55*5^(1/2) - 123)^(1/4)))*(- 55*5^(1/2) - 123
)^(1/4)*1i)/20 - (2^(3/4)*5^(1/2)*atan((2^(1/4)*x*3i)/(2*(55*5^(1/2) - 123)
^(1/4)) - (2^(1/4)*5^(1/2)*x*1i)/(2*(55*5^(1/2) - 123)^(1/4)))*(55*5^(1/2)
- 123)^(1/4)*1i)/20
```

sympy [A] time = 1.59, size = 29, normalized size = 0.06

$$x + \text{RootSum}\left(40960000t^8 + 787200t^4 + 1, \left(t \mapsto t\log\left(\frac{15360t^5}{11} + \frac{1288t}{55} + x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8/(x**8+3*x**4+1),x)
```

```
[Out] x + RootSum(40960000*_t**8 + 787200*_t**4 + 1, Lambda(_t, _t*log(15360*_t**
5/11 + 1288*_t/55 + x)))
```

3.319 $\int \frac{x^6}{1+3x^4+x^8}\,dx$

Optimal. Leaf size=431

$$-\frac{\sqrt[4]{9-4\sqrt{5}}\,\log\left(2x^2-2\sqrt[4]{2\left(3-\sqrt{5}\right)}\,x+\sqrt{2\left(3-\sqrt{5}\right)}\right)}{4\sqrt{10}}+\frac{\sqrt[4]{9-4\sqrt{5}}\,\log\left(2x^2+2\sqrt[4]{2\left(3-\sqrt{5}\right)}\,x+\sqrt{2\left(3-\right.}\right.}{4\sqrt{10}}$$

Rubi [A] time = 0.29, antiderivative size = 431, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {1374, 297, 1162, 617, 204, 1165, 628}

$$-\frac{\sqrt[4]{9-4\sqrt{5}}\,\log\left(2x^2-2\sqrt[4]{2\left(3-\sqrt{5}\right)}\,x+\sqrt{2\left(3-\sqrt{5}\right)}\right)}{4\sqrt{10}}+\frac{\sqrt[4]{9-4\sqrt{5}}\,\log\left(2x^2+2\sqrt[4]{2\left(3-\sqrt{5}\right)}\,x+\sqrt{2\left(3-\sqrt{5}\right)}\right)}{4\sqrt{10}}+\frac{\left(3+\sqrt{5}\right)^{3/4}\log\left(2x^2-2\sqrt[4]{2\left(3+\sqrt{5}\right)}\,x+\sqrt{2\left(3+\sqrt{5}\right)}\right)}{8\sqrt[4]{2}\sqrt{5}}-\frac{\left(3+\sqrt{5}\right)^{3/4}\log\left(2x^2+2\sqrt[4]{2\left(3+\sqrt{5}\right)}\,x+\sqrt{2\left(3+\sqrt{5}\right)}\right)}{8\sqrt[4]{2}\sqrt{5}}+\frac{\sqrt[4]{9-4\sqrt{5}}\,\tan^{-1}\left(1-\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}\right)}{2\sqrt{10}}-\frac{\sqrt[4]{9-4\sqrt{5}}\,\tan^{-1}\left(\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}+1\right)}{2\sqrt{10}}-\frac{\left(3+\sqrt{5}\right)^{3/4}\tan^{-1}\left(1-\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}\right)}{4\sqrt[4]{2}\sqrt{5}}+\frac{\left(3+\sqrt{5}\right)^{3/4}\tan^{-1}\left(\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}+1\right)}{4\sqrt[4]{2}\sqrt{5}}$$

Antiderivative was successfully verified.

```
[In] Int[x^6/(1 + 3*x^4 + x^8),x]
```

```
[Out] ((9 - 4*Sqrt[5])^(1/4)*ArcTan[1 - (2^(3/4)*x)/(3 - Sqrt[5])^(1/4)])/(2*Sqrt
[10]) - ((9 - 4*Sqrt[5])^(1/4)*ArcTan[1 + (2^(3/4)*x)/(3 - Sqrt[5])^(1/4)])
/(2*Sqrt[10]) - ((3 + Sqrt[5])^(3/4)*ArcTan[1 - (2^(3/4)*x)/(3 + Sqrt[5])^(
1/4)])/(4*2^(1/4)*Sqrt[5]) + ((3 + Sqrt[5])^(3/4)*ArcTan[1 + (2^(3/4)*x)/(3
 + Sqrt[5])^(1/4)])/(4*2^(1/4)*Sqrt[5]) - ((9 - 4*Sqrt[5])^(1/4)*Log[Sqrt[2
*(3 - Sqrt[5])] - 2*(2*(3 - Sqrt[5]))^(1/4)*x + 2*x^2])/(4*Sqrt[10]) + ((9
- 4*Sqrt[5])^(1/4)*Log[Sqrt[2*(3 - Sqrt[5])] + 2*(2*(3 - Sqrt[5]))^(1/4)*x
+ 2*x^2])/(4*Sqrt[10]) + ((3 + Sqrt[5])^(3/4)*Log[Sqrt[2*(3 + Sqrt[5])] - 2
*(2*(3 + Sqrt[5]))^(1/4)*x + 2*x^2])/(8*2^(1/4)*Sqrt[5]) - ((3 + Sqrt[5])^(
3/4)*Log[Sqrt[2*(3 + Sqrt[5])] + 2*(2*(3 + Sqrt[5]))^(1/4)*x + 2*x^2])/(8*2
^(1/4)*Sqrt[5])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1374

```
Int[((d_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(d^n*(b/q + 1))/2, Int[(d*x)^(m -
 n)/(b/2 + q/2 + c*x^n), x], x] - Dist[(d^n*(b/q - 1))/2, Int[(d*x)^(m - n)
/(b/2 - q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[n2, 2*n] &&
NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GeQ[m, n]
```

Rubi steps

$$\int \frac{x^6}{1+3x^4+x^8}\,dx = -\left(\frac{1}{10}\left(-5+3\sqrt{5}\right)\int \frac{x^2}{\frac{3}{2}-\frac{\sqrt{5}}{2}+x^4}\,dx\right)+\frac{1}{10}\left(5+3\sqrt{5}\right)\int \frac{x^2}{\frac{3}{2}+\frac{\sqrt{5}}{2}+x^4}\,dx$$

$$= \frac{\left(3-\sqrt{5}\right)\int \frac{\sqrt{3-\sqrt{5}}-\sqrt{2}x^2}{\frac{3}{2}-\frac{\sqrt{5}}{2}+x^4}\,dx}{4\sqrt{10}} - \frac{\left(3-\sqrt{5}\right)\int \frac{\sqrt{3-\sqrt{5}}+\sqrt{2}x^2}{\frac{3}{2}-\frac{\sqrt{5}}{2}+x^4}\,dx}{4\sqrt{10}} - \frac{\left(3+\sqrt{5}\right)\int \frac{\sqrt{3+\sqrt{5}}-\sqrt{2}x^2}{\frac{3}{2}+\frac{\sqrt{5}}{2}+x^4}\,dx}{4\sqrt{10}}$$

$$= -\frac{\sqrt[4]{9-4\sqrt{5}}\int \frac{\sqrt[4]{2\left(3-\sqrt{5}\right)}+2x}{-\sqrt{\frac{1}{2}\left(3-\sqrt{5}\right)}-\sqrt[4]{2\left(3-\sqrt{5}\right)}x-x^2}\,dx}{4\sqrt{10}} - \frac{\sqrt[4]{9-4\sqrt{5}}\int \frac{\sqrt[4]{2\left(3-\sqrt{5}\right)}-2x}{-\sqrt{\frac{1}{2}\left(3-\sqrt{5}\right)}+\sqrt[4]{2\left(3-\sqrt{5}\right)}x-x^2}\,dx}{4\sqrt{10}} +$$

$$= -\frac{\sqrt[4]{9-4\sqrt{5}}\log\left(\sqrt{2\left(3-\sqrt{5}\right)}-2\sqrt[4]{2\left(3-\sqrt{5}\right)}x+2x^2\right)}{4\sqrt{10}} + \frac{\sqrt[4]{9-4\sqrt{5}}\log\left(\sqrt{2\left(3-\sqrt{5}\right)}\right.}{4\sqrt{1}}$$

$$= \frac{\left(3-\sqrt{5}\right)^{3/4}\tan^{-1}\left(1-\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}\right)}{4\sqrt[4]{2}\sqrt{5}} - \frac{\sqrt[4]{36-16\sqrt{5}}\tan^{-1}\left(1+\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}\right)}{4\sqrt{5}} - \frac{\left(3+\sqrt{5}\right)^{3/4}\tan^{-1}\left(\right.}{4\sqrt[4]{2}\sqrt{5}}$$

Mathematica [C] time = 0.01, size = 41, normalized size = 0.10

$$\frac{1}{4}\text{RootSum}\left[\#1^8+3\#1^4+1\&,\frac{\#1^3\log(x-\#1)}{2\#1^4+3}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6/(1 + 3*x^4 + x^8),x]
```

```
[Out] RootSum[1 + 3*#1^4 + #1^8 & , (Log[x - #1]*#1^3)/(3 + 2*#1^4) & ]/4
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6}{1+3x^4+x^8}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^6/(1 + 3*x^4 + x^8),x]
```

```
[Out] IntegrateAlgebraic[x^6/(1 + 3*x^4 + x^8), x]
```

fricas [B] time = 1.58, size = 725, normalized size = 1.68

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(x^8+3*x^4+1),x, algorithm="fricas")
```

```
[Out] 1/10*sqrt(5)*sqrt(2)*(4*sqrt(5) + 9)^(1/4)*arctan(1/2*sqrt(2*x^2 + (3*sqrt(
5)*sqrt(2)*x - 7*sqrt(2)*x)*(4*sqrt(5) + 9)^(3/4) - sqrt(4*sqrt(5) + 9)*(sq
rt(5) - 3))*(21*sqrt(5) - 47)*(4*sqrt(5) + 9)^(5/4) - 1/2*(21*sqrt(5)*sqrt(
2)*x - 47*sqrt(2)*x)*(4*sqrt(5) + 9)^(5/4) - 1) + 1/10*sqrt(5)*sqrt(2)*(4*s
qrt(5) + 9)^(1/4)*arctan(1/2*sqrt(2*x^2 - (3*sqrt(5)*sqrt(2)*x - 7*sqrt(2)*
x)*(4*sqrt(5) + 9)^(3/4) - sqrt(4*sqrt(5) + 9)*(sqrt(5) - 3))*(21*sqrt(5) -
 47)*(4*sqrt(5) + 9)^(5/4) - 1/2*(21*sqrt(5)*sqrt(2)*x - 47*sqrt(2)*x)*(4*s
qrt(5) + 9)^(5/4) + 1) + 1/10*sqrt(5)*sqrt(2)*(-4*sqrt(5) + 9)^(1/4)*arctan
(1/2*sqrt(2*x^2 + (3*sqrt(5)*sqrt(2)*x + 7*sqrt(2)*x)*(-4*sqrt(5) + 9)^(3/4
) + (sqrt(5) + 3)*sqrt(-4*sqrt(5) + 9))*(21*sqrt(5) + 47)*(-4*sqrt(5) + 9)^
(5/4) - 1/2*(21*sqrt(5)*sqrt(2)*x + 47*sqrt(2)*x)*(-4*sqrt(5) + 9)^(5/4) -
1) + 1/10*sqrt(5)*sqrt(2)*(-4*sqrt(5) + 9)^(1/4)*arctan(1/2*sqrt(2*x^2 - (3
*sqrt(5)*sqrt(2)*x + 7*sqrt(2)*x)*(-4*sqrt(5) + 9)^(3/4) + (sqrt(5) + 3)*sq
rt(-4*sqrt(5) + 9))*(21*sqrt(5) + 47)*(-4*sqrt(5) + 9)^(5/4) - 1/2*(21*sqrt
(5)*sqrt(2)*x + 47*sqrt(2)*x)*(-4*sqrt(5) + 9)^(5/4) + 1) + 1/40*sqrt(5)*sq
rt(2)*(4*sqrt(5) + 9)^(1/4)*log(2*x^2 + (3*sqrt(5)*sqrt(2)*x - 7*sqrt(2)*x)
*(4*sqrt(5) + 9)^(3/4) - sqrt(4*sqrt(5) + 9)*(sqrt(5) - 3)) - 1/40*sqrt(5)*
sqrt(2)*(4*sqrt(5) + 9)^(1/4)*log(2*x^2 - (3*sqrt(5)*sqrt(2)*x - 7*sqrt(2)*
x)*(4*sqrt(5) + 9)^(3/4) - sqrt(4*sqrt(5) + 9)*(sqrt(5) - 3)) + 1/40*sqrt(5
)*sqrt(2)*(-4*sqrt(5) + 9)^(1/4)*log(2*x^2 + (3*sqrt(5)*sqrt(2)*x + 7*sqrt(
2)*x)*(-4*sqrt(5) + 9)^(3/4) + (sqrt(5) + 3)*sqrt(-4*sqrt(5) + 9)) - 1/40*s
qrt(5)*sqrt(2)*(-4*sqrt(5) + 9)^(1/4)*log(2*x^2 - (3*sqrt(5)*sqrt(2)*x + 7*
sqrt(2)*x)*(-4*sqrt(5) + 9)^(3/4) + (sqrt(5) + 3)*sqrt(-4*sqrt(5) + 9))
```

giac [A] time = 0.66, size = 239, normalized size = 0.55

$$\frac{1}{80}\left(\pi + 4\arctan\left(x\sqrt{\sqrt{5}-1}-1\right)\right)\sqrt{10\sqrt{5}+20} - \frac{1}{80}\left(\pi + 4\arctan\left(-x\sqrt{\sqrt{5}-1}-1\right)\right)\sqrt{10\sqrt{5}+20} - \frac{1}{80}\left(\pi + 4\arctan\left(x\sqrt{\sqrt{5}+1}+1\right)\right)\sqrt{10\sqrt{5}-20} + \frac{1}{80}\left(\pi + 4\arctan\left(-x\sqrt{\sqrt{5}+1}+1\right)\right)\sqrt{10\sqrt{5}-20} - \frac{1}{40}\sqrt{10\sqrt{5}+20}\log\left(400\left(x+\sqrt{\sqrt{5}+1}\right)^2+400x^2\right) + \frac{1}{40}\sqrt{10\sqrt{5}+20}\log\left(400\left(x-\sqrt{\sqrt{5}+1}\right)^2+400x^2\right) + \frac{1}{40}\sqrt{10\sqrt{5}-20}\log\left(10000\left(x+\sqrt{\sqrt{5}-1}\right)^2+10000x^2\right) - \frac{1}{40}\sqrt{10\sqrt{5}-20}\log\left(10000\left(x-\sqrt{\sqrt{5}-1}\right)^2+10000x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(x^8+3*x^4+1),x, algorithm="giac")
```

```
[Out] 1/80*(pi + 4*arctan(x*sqrt(sqrt(5) - 1) - 1))*sqrt(10*sqrt(5) + 20) - 1/80*
(pi + 4*arctan(-x*sqrt(sqrt(5) - 1) - 1))*sqrt(10*sqrt(5) + 20) - 1/80*(pi
+ 4*arctan(x*sqrt(sqrt(5) + 1) + 1))*sqrt(10*sqrt(5) - 20) + 1/80*(pi + 4*a
rctan(-x*sqrt(sqrt(5) + 1) + 1))*sqrt(10*sqrt(5) - 20) - 1/40*sqrt(10*sqrt(
5) + 20)*log(400*(x + sqrt(sqrt(5) + 1))^2 + 400*x^2) + 1/40*sqrt(10*sqrt(5
) + 20)*log(400*(x - sqrt(sqrt(5) + 1))^2 + 400*x^2) + 1/40*sqrt(10*sqrt(5)
 - 20)*log(10000*(x + sqrt(sqrt(5) - 1))^2 + 10000*x^2) - 1/40*sqrt(10*sqrt
(5) - 20)*log(10000*(x - sqrt(sqrt(5) - 1))^2 + 10000*x^2)
```

maple [C] time = 0.01, size = 40, normalized size = 0.09

$$\frac{\mathrm{RootOf}\left(\_Z^8+3\_Z^4+1\right)^6 \ln\left(-\mathrm{RootOf}\left(\_Z^8+3\_Z^4+1\right)+x\right)}{8\,\mathrm{RootOf}\left(\_Z^8+3\_Z^4+1\right)^7+12\,\mathrm{RootOf}\left(\_Z^8+3\_Z^4+1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(x^8+3*x^4+1),x)
```

```
[Out] 1/4*sum(_R^6/(2*_R^7+3*_R^3)*ln(-_R+x),_R=RootOf(_Z^8+3*_Z^4+1))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6}{x^8+3\,x^4+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(x^8+3*x^4+1),x, algorithm="maxima")
```

```
[Out] integrate(x^6/(x^8 + 3*x^4 + 1), x)
```

mupad [B] time = 1.46, size = 149, normalized size = 0.35

$$\frac{\sqrt{5}\,\mathrm{atan}\left(\frac{16\,x\,{\left(-4\,\sqrt{5}-9\right)}^{1/4}}{8\,\sqrt{5}+24}\right)\,{\left(-4\,\sqrt{5}-9\right)}^{1/4}}{10}+\frac{\sqrt{5}\,\mathrm{atan}\left(\frac{16\,x\,{\left(4\,\sqrt{5}-9\right)}^{1/4}}{8\,\sqrt{5}-24}\right)\,{\left(4\,\sqrt{5}-9\right)}^{1/4}}{10}+\frac{\sqrt{5}\,\mathrm{atan}\left(\frac{x\,{\left(-4\,\sqrt{5}-9\right)}^{1/4}\,16{}\mathrm{i}}{8\,\sqrt{5}+24}\right)\,{\left(-4\,\sqrt{5}-9\right)}^{1/4}\,1{}\mathrm{i}}{10}+\frac{\sqrt{5}\,\mathrm{atan}\left(\frac{x\,{\left(4\,\sqrt{5}-9\right)}^{1/4}\,16{}\mathrm{i}}{8\,\sqrt{5}-24}\right)\,{\left(4\,\sqrt{5}-9\right)}^{1/4}\,1{}\mathrm{i}}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(3*x^4 + x^8 + 1),x)
```

```
[Out] (5^(1/2)*atan((16*x*(- 4*5^(1/2) - 9)^(1/4))/(8*5^(1/2) + 24))*(- 4*5^(1/2)
 - 9)^(1/4))/10 + (5^(1/2)*atan((16*x*(4*5^(1/2) - 9)^(1/4))/(8*5^(1/2) - 2
4))*(4*5^(1/2) - 9)^(1/4))/10 + (5^(1/2)*atan((x*(- 4*5^(1/2) - 9)^(1/4)*16
i)/(8*5^(1/2) + 24))*(- 4*5^(1/2) - 9)^(1/4)*1i)/10 + (5^(1/2)*atan((x*(4*5
^(1/2) - 9)^(1/4)*16i)/(8*5^(1/2) - 24))*(4*5^(1/2) - 9)^(1/4)*1i)/10
```

sympy [A] time = 1.53, size = 26, normalized size = 0.06

$$\operatorname{RootSum}{\left(40960000 t^{8} + 115200 t^{4} + 1, \left(t \mapsto t \log{\left(-1792000 t^{7} - 4920 t^{3} + x \right)} \right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(x**8+3*x**4+1),x)
```

```
[Out] RootSum(40960000*_t**8 + 115200*_t**4 + 1, Lambda(_t, _t*log(-1792000*_t**7
 - 4920*_t**3 + x)))
```

3.320 $\int \frac{x^4}{1+3x^4+x^8}\,dx$

Optimal. Leaf size=451

$$\frac{\sqrt[4]{3-\sqrt{5}}\,\log\left(2x^2-2\sqrt[4]{2\left(3-\sqrt{5}\right)}\,x+\sqrt{2\left(3-\sqrt{5}\right)}\right)}{4\,2^{3/4}\sqrt{5}}-\frac{\sqrt[4]{3-\sqrt{5}}\,\log\left(2x^2+2\sqrt[4]{2\left(3-\sqrt{5}\right)}\,x+\sqrt{2\left(3-\sqrt{5}\right.}\right.}{4\,2^{3/4}\sqrt{5}}$$

Rubi [A] time = 0.28, antiderivative size = 451, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {1374, 211, 1165, 628, 1162, 617, 204}

$$\frac{\sqrt[4]{3-\sqrt{5}}\log\left(2x^2-2\sqrt[4]{2\left(3-\sqrt{5}\right)}x+\sqrt{2\left(3-\sqrt{5}\right)}\right)}{4\,2^{3/4}\sqrt{5}}-\frac{\sqrt[4]{3-\sqrt{5}}\log\left(2x^2+2\sqrt[4]{2\left(3-\sqrt{5}\right)}x+\sqrt{2\left(3-\sqrt{5}\right)}\right)}{4\,2^{3/4}\sqrt{5}}-\frac{\sqrt[4]{3+\sqrt{5}}\log\left(2x^2-2\sqrt[4]{2\left(3+\sqrt{5}\right)}x+\sqrt{2\left(3+\sqrt{5}\right)}\right)}{4\,2^{3/4}\sqrt{5}}+\frac{\sqrt[4]{3+\sqrt{5}}\log\left(2x^2+2\sqrt[4]{2\left(3+\sqrt{5}\right)}x+\sqrt{2\left(3+\sqrt{5}\right)}\right)}{4\,2^{3/4}\sqrt{5}}+\frac{\sqrt[4]{3-\sqrt{5}}\tan^{-1}\left(1-\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}\right)}{2\,2^{3/4}\sqrt{5}}-\frac{\sqrt[4]{3-\sqrt{5}}\tan^{-1}\left(\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}+1\right)}{2\,2^{3/4}\sqrt{5}}-\frac{\sqrt[4]{3+\sqrt{5}}\tan^{-1}\left(1-\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}\right)}{2\,2^{3/4}\sqrt{5}}+\frac{\sqrt[4]{3+\sqrt{5}}\tan^{-1}\left(\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}+1\right)}{2\,2^{3/4}\sqrt{5}}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(1 + 3*x^4 + x^8),x]
```

```
[Out] ((3 - Sqrt[5])^(1/4)*ArcTan[1 - (2^(3/4)*x)/(3 - Sqrt[5])^(1/4)])/(2*2^(3/4
)*Sqrt[5]) - ((3 - Sqrt[5])^(1/4)*ArcTan[1 + (2^(3/4)*x)/(3 - Sqrt[5])^(1/4
)])/(2*2^(3/4)*Sqrt[5]) - ((3 + Sqrt[5])^(1/4)*ArcTan[1 - (2^(3/4)*x)/(3 +
Sqrt[5])^(1/4)])/(2*2^(3/4)*Sqrt[5]) + ((3 + Sqrt[5])^(1/4)*ArcTan[1 + (2^(
3/4)*x)/(3 + Sqrt[5])^(1/4)])/(2*2^(3/4)*Sqrt[5]) + ((3 - Sqrt[5])^(1/4)*Lo
g[Sqrt[2*(3 - Sqrt[5])] - 2*(2*(3 - Sqrt[5]))^(1/4)*x + 2*x^2])/(4*2^(3/4)*
Sqrt[5]) - ((3 - Sqrt[5])^(1/4)*Log[Sqrt[2*(3 - Sqrt[5])] + 2*(2*(3 - Sqrt[
5]))^(1/4)*x + 2*x^2])/(4*2^(3/4)*Sqrt[5]) - ((3 + Sqrt[5])^(1/4)*Log[Sqrt[
2*(3 + Sqrt[5])] - 2*(2*(3 + Sqrt[5]))^(1/4)*x + 2*x^2])/(4*2^(3/4)*Sqrt[5]
) + ((3 + Sqrt[5])^(1/4)*Log[Sqrt[2*(3 + Sqrt[5])] + 2*(2*(3 + Sqrt[5]))^(1
/4)*x + 2*x^2])/(4*2^(3/4)*Sqrt[5])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1374

```
Int[((d_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(d^n*(b/q + 1))/2, Int[(d*x)^(m -
 n)/(b/2 + q/2 + c*x^n), x], x] - Dist[(d^n*(b/q - 1))/2, Int[(d*x)^(m - n)
/(b/2 - q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[n2, 2*n] &&
NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GeQ[m, n]
```

Rubi steps

$$\int \frac{x^4}{1+3x^4+x^8}\,dx = -\left(\frac{1}{10}\left(-5+3\sqrt{5}\right)\int \frac{1}{\frac{3}{2}-\frac{\sqrt{5}}{2}+x^4}\,dx\right)+\frac{1}{10}\left(5+3\sqrt{5}\right)\int \frac{1}{\frac{3}{2}+\frac{\sqrt{5}}{2}+x^4}\,dx$$

$$= -\left(\frac{1}{4}\sqrt{\frac{1}{5}\left(3-\sqrt{5}\right)}\int \frac{\sqrt{3-\sqrt{5}}-\sqrt{2}\,x^2}{\frac{3}{2}-\frac{\sqrt{5}}{2}+x^4}\,dx\right)-\frac{1}{4}\sqrt{\frac{1}{5}\left(3-\sqrt{5}\right)}\int \frac{\sqrt{3-\sqrt{5}}+\sqrt{2}\,x^2}{\frac{3}{2}-\frac{\sqrt{5}}{2}+x^4}\,dx + \cdots$$

$$= -\left(\frac{1}{4}\sqrt{\frac{1}{10}\left(3-\sqrt{5}\right)}\int \frac{1}{\sqrt{\frac{1}{2}\left(3-\sqrt{5}\right)}-\sqrt[4]{2\left(3-\sqrt{5}\right)}\,x+x^2}\,dx\right)-\frac{1}{4}\sqrt{\frac{1}{10}\left(3-\sqrt{5}\right)}\int \cdots$$

$$= \frac{\sqrt[4]{3-\sqrt{5}}\,\log\left(\sqrt{2\left(3-\sqrt{5}\right)}-2\sqrt[4]{2\left(3-\sqrt{5}\right)}\,x+2x^2\right)}{4\,2^{3/4}\sqrt{5}}-\frac{\sqrt[4]{3-\sqrt{5}}\,\log\left(\sqrt{2\left(3-\sqrt{5}\right)}+\cdots\right)}{4\,2^{3/4}\sqrt{5}}\cdots$$

$$= \frac{\sqrt[4]{3-\sqrt{5}}\,\tan^{-1}\left(1-\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}\right)}{2\,2^{3/4}\sqrt{5}}-\frac{\sqrt[4]{3-\sqrt{5}}\,\tan^{-1}\left(1+\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}\right)}{2\,2^{3/4}\sqrt{5}}-\frac{\sqrt[4]{3+\sqrt{5}}\,\tan^{-1}\left(1-\cdots\right)}{2\,2^{3/4}\sqrt{5}}\cdots$$

Mathematica [C] time = 0.01, size = 39, normalized size = 0.09

$$\frac{1}{4}\text{RootSum}\left[\#1^8+3\#1^4+1\&,\frac{\#1\log(x-\#1)}{2\#1^4+3}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(1 + 3*x^4 + x^8),x]
[Out] RootSum[1 + 3*#1^4 + #1^8 & , (Log[x - #1]*#1)/(3 + 2*#1^4) & ]/4
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{1+3x^4+x^8}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^4/(1 + 3*x^4 + x^8),x]
```

```
[Out] IntegrateAlgebraic[x^4/(1 + 3*x^4 + x^8), x]
```

fricas [B] time = 1.10, size = 843, normalized size = 1.87

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(x^8+3*x^4+1),x, algorithm="fricas")
```

```
[Out] 1/80*sqrt(10)*(2*sqrt(5) + 6)^(3/4)*sqrt(sqrt(5) + 3)*(sqrt(5) - 3)*arctan(
-1/80*sqrt(10)*(7*sqrt(5)*x - 15*x)*(2*sqrt(5) + 6)^(5/4)*sqrt(sqrt(5) + 3)
 + 1/80*sqrt(sqrt(10)*sqrt(5)*sqrt(2)*x*(2*sqrt(5) + 6)^(1/4) + 10*x^2 + 5*
sqrt(2*sqrt(5) + 6))*(7*sqrt(5) - 15)*(2*sqrt(5) + 6)^(5/4)*sqrt(sqrt(5) +
3) + 1/8*(sqrt(5)*sqrt(2) - 3*sqrt(2))*sqrt(2*sqrt(5) + 6)*sqrt(sqrt(5) + 3
)) + 1/80*sqrt(10)*(2*sqrt(5) + 6)^(3/4)*sqrt(sqrt(5) + 3)*(sqrt(5) - 3)*ar
ctan(-1/80*sqrt(10)*(7*sqrt(5)*x - 15*x)*(2*sqrt(5) + 6)^(5/4)*sqrt(sqrt(5)
 + 3) + 1/80*sqrt(-sqrt(10)*sqrt(5)*sqrt(2)*x*(2*sqrt(5) + 6)^(1/4) + 10*x^
2 + 5*sqrt(2*sqrt(5) + 6))*(7*sqrt(5) - 15)*(2*sqrt(5) + 6)^(5/4)*sqrt(sqrt
(5) + 3) - 1/8*(sqrt(5)*sqrt(2) - 3*sqrt(2))*sqrt(2*sqrt(5) + 6)*sqrt(sqrt(
5) + 3)) + 1/80*sqrt(10)*(sqrt(5) + 3)*sqrt(-sqrt(5) + 3)*(-2*sqrt(5) + 6)^
(3/4)*arctan(1/80*sqrt(sqrt(10)*sqrt(5)*sqrt(2)*x*(-2*sqrt(5) + 6)^(1/4) +
10*x^2 + 5*sqrt(-2*sqrt(5) + 6))*(7*sqrt(5) + 15)*sqrt(-sqrt(5) + 3)*(-2*sq
rt(5) + 6)^(5/4) - 1/80*(sqrt(10)*(7*sqrt(5)*x + 15*x)*(-2*sqrt(5) + 6)^(5/
4) + 10*(sqrt(5)*sqrt(2) + 3*sqrt(2))*sqrt(-2*sqrt(5) + 6))*sqrt(-sqrt(5) +
 3)) + 1/80*sqrt(10)*(sqrt(5) + 3)*sqrt(-sqrt(5) + 3)*(-2*sqrt(5) + 6)^(3/4
)*arctan(1/80*sqrt(-sqrt(10)*sqrt(5)*sqrt(2)*x*(-2*sqrt(5) + 6)^(1/4) + 10*
x^2 + 5*sqrt(-2*sqrt(5) + 6))*(7*sqrt(5) + 15)*sqrt(-sqrt(5) + 3)*(-2*sqrt(
5) + 6)^(5/4) - 1/80*(sqrt(10)*(7*sqrt(5)*x + 15*x)*(-2*sqrt(5) + 6)^(5/4)
- 10*(sqrt(5)*sqrt(2) + 3*sqrt(2))*sqrt(-2*sqrt(5) + 6))*sqrt(-sqrt(5) + 3)
) + 1/80*sqrt(10)*sqrt(2)*(2*sqrt(5) + 6)^(1/4)*log(sqrt(10)*sqrt(5)*sqrt(2
)*x*(2*sqrt(5) + 6)^(1/4) + 10*x^2 + 5*sqrt(2*sqrt(5) + 6)) - 1/80*sqrt(10)
*sqrt(2)*(2*sqrt(5) + 6)^(1/4)*log(-sqrt(10)*sqrt(5)*sqrt(2)*x*(2*sqrt(5) +
 6)^(1/4) + 10*x^2 + 5*sqrt(2*sqrt(5) + 6)) - 1/80*sqrt(10)*sqrt(2)*(-2*sqr
t(5) + 6)^(1/4)*log(sqrt(10)*sqrt(5)*sqrt(2)*x*(-2*sqrt(5) + 6)^(1/4) + 10*
x^2 + 5*sqrt(-2*sqrt(5) + 6)) + 1/80*sqrt(10)*sqrt(2)*(-2*sqrt(5) + 6)^(1/4
)*log(-sqrt(10)*sqrt(5)*sqrt(2)*x*(-2*sqrt(5) + 6)^(1/4) + 10*x^2 + 5*sqrt(
-2*sqrt(5) + 6))
```

giac [A] time = 0.77, size = 239, normalized size = 0.53

$$\frac{1}{80}\left(\pi + 4\arctan\left(x\sqrt{\sqrt{5}-1}+1\right)\right)\sqrt{5\sqrt{5}+5} - \frac{1}{80}\left(\pi + 4\arctan\left(-x\sqrt{\sqrt{5}-1}+1\right)\right)\sqrt{5\sqrt{5}+5} - \frac{1}{80}\left(\pi + 4\arctan\left(x\sqrt{\sqrt{5}+1}-1\right)\right)\sqrt{5\sqrt{5}-5} + \frac{1}{80}\left(\pi + 4\arctan\left(-x\sqrt{\sqrt{5}+1}-1\right)\right)\sqrt{5\sqrt{5}-5} + \frac{1}{40}\sqrt{5\sqrt{5}+5}\log\left(625\left(x+\sqrt{\sqrt{5}+1}\right)^2+625x^2\right) - \frac{1}{40}\sqrt{5\sqrt{5}+5}\log\left(625\left(x-\sqrt{\sqrt{5}+1}\right)^2+625x^2\right) - \frac{1}{40}\sqrt{5\sqrt{5}-5}\log\left(4225\left(x+\sqrt{\sqrt{5}-1}\right)^2+4225x^2\right) + \frac{1}{40}\sqrt{5\sqrt{5}-5}\log\left(4225\left(x-\sqrt{\sqrt{5}-1}\right)^2+4225x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(x^8+3*x^4+1),x, algorithm="giac")
```

```
[Out] 1/80*(pi + 4*arctan(x*sqrt(sqrt(5) - 1) + 1))*sqrt(5*sqrt(5) + 5) - 1/80*(p
i + 4*arctan(-x*sqrt(sqrt(5) - 1) + 1))*sqrt(5*sqrt(5) + 5) - 1/80*(pi + 4*
arctan(x*sqrt(sqrt(5) + 1) - 1))*sqrt(5*sqrt(5) - 5) + 1/80*(pi + 4*arctan(
-x*sqrt(sqrt(5) + 1) - 1))*sqrt(5*sqrt(5) - 5) + 1/40*sqrt(5*sqrt(5) + 5)*l
og(625*(x + sqrt(sqrt(5) + 1))^2 + 625*x^2) - 1/40*sqrt(5*sqrt(5) + 5)*log(
625*(x - sqrt(sqrt(5) + 1))^2 + 625*x^2) - 1/40*sqrt(5*sqrt(5) - 5)*log(422
5*(x + sqrt(sqrt(5) - 1))^2 + 4225*x^2) + 1/40*sqrt(5*sqrt(5) - 5)*log(4225
*(x - sqrt(sqrt(5) - 1))^2 + 4225*x^2)
```

maple [C] time = 0.02, size = 40, normalized size = 0.09

$$\frac{\mathrm{RootOf}\left(\_Z^8+3\_Z^4+1\right)^4 \ln\left(-\mathrm{RootOf}\left(\_Z^8+3\_Z^4+1\right)+x\right)}{8\,\mathrm{RootOf}\left(\_Z^8+3\_Z^4+1\right)^7+12\,\mathrm{RootOf}\left(\_Z^8+3\_Z^4+1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(x^8+3*x^4+1),x)
```

```
[Out] 1/4*sum(_R^4/(2*_R^7+3*_R^3)*ln(-_R+x),_R=RootOf(_Z^8+3*_Z^4+1))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{x^8+3\,x^4+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(x^8+3*x^4+1),x, algorithm="maxima")
```

```
[Out] integrate(x^4/(x^8 + 3*x^4 + 1), x)
```

mupad [B] time = 0.20, size = 454, normalized size = 1.01

$$\frac{2^{3/4}\sqrt{5}\,\mathrm{atan}\left(\frac{3\,2^{3/4}\,x\,\left(-\sqrt{5}-3\right)^{1/4}}{2\left(\frac{3\sqrt{2}\sqrt{-\sqrt{5}-3}}{2}-\frac{\sqrt{2}\sqrt{5}\sqrt{-\sqrt{5}-3}}{2}\right)}-\frac{2^{3/4}\sqrt{5}\,x\,\left(-\sqrt{5}-3\right)^{1/4}}{2\left(\frac{3\sqrt{2}\sqrt{-\sqrt{5}-3}}{2}-\frac{\sqrt{2}\sqrt{5}\sqrt{-\sqrt{5}-3}}{2}\right)}\right)\left(-\sqrt{5}-3\right)^{1/4}}{20}-\frac{2^{3/4}\sqrt{5}\,\mathrm{atan}\left(\frac{2^{3/4}\,x\,\left(-\sqrt{5}-3\right)^{1/4}3\mathrm{i}}{2\left(\frac{3\sqrt{2}\sqrt{-\sqrt{5}-3}}{2}-\frac{\sqrt{2}\sqrt{5}\sqrt{-\sqrt{5}-3}}{2}\right)}-\frac{2^{3/4}\sqrt{5}\,x\,\left(-\sqrt{5}-3\right)^{1/4}1\mathrm{i}}{2\left(\frac{3\sqrt{2}\sqrt{-\sqrt{5}-3}}{2}-\frac{\sqrt{2}\sqrt{5}\sqrt{-\sqrt{5}-3}}{2}\right)}\right)\left(-\sqrt{5}-3\right)^{1/4}1\mathrm{i}}{20}-\frac{2^{3/4}\sqrt{5}\,\mathrm{atan}\left(\frac{3\,2^{3/4}\,x\,\left(\sqrt{5}-3\right)^{1/4}}{2\left(\frac{3\sqrt{2}\sqrt{\sqrt{5}-3}}{2}+\frac{\sqrt{2}\sqrt{5}\sqrt{\sqrt{5}-3}}{2}\right)}+\frac{2^{3/4}\sqrt{5}\,x\,\left(\sqrt{5}-3\right)^{1/4}}{2\left(\frac{3\sqrt{2}\sqrt{\sqrt{5}-3}}{2}+\frac{\sqrt{2}\sqrt{5}\sqrt{\sqrt{5}-3}}{2}\right)}\right)\left(\sqrt{5}-3\right)^{1/4}}{20}+\frac{2^{3/4}\sqrt{5}\,\mathrm{atan}\left(\frac{2^{3/4}\,x\,\left(\sqrt{5}-3\right)^{1/4}3\mathrm{i}}{2\left(\frac{3\sqrt{2}\sqrt{\sqrt{5}-3}}{2}+\frac{\sqrt{2}\sqrt{5}\sqrt{\sqrt{5}-3}}{2}\right)}+\frac{2^{3/4}\sqrt{5}\,x\,\left(\sqrt{5}-3\right)^{1/4}1\mathrm{i}}{2\left(\frac{3\sqrt{2}\sqrt{\sqrt{5}-3}}{2}+\frac{\sqrt{2}\sqrt{5}\sqrt{\sqrt{5}-3}}{2}\right)}\right)\left(\sqrt{5}-3\right)^{1/4}1\mathrm{i}}{20}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(3*x^4 + x^8 + 1),x)
```

```
[Out] (2^(3/4)*5^(1/2)*atan((3*2^(3/4)*x*(- 5^(1/2) - 3)^(1/4))/(2*((3*2^(1/2)*(-
 5^(1/2) - 3)^(1/2))/2 - (2^(1/2)*5^(1/2)*(- 5^(1/2) - 3)^(1/2))/2)) - (2^(
3/4)*5^(1/2)*x*(- 5^(1/2) - 3)^(1/4))/(2*((3*2^(1/2)*(- 5^(1/2) - 3)^(1/2))
/2 - (2^(1/2)*5^(1/2)*(- 5^(1/2) - 3)^(1/2))/2)))*(- 5^(1/2) - 3)^(1/4))/20
 - (2^(3/4)*5^(1/2)*atan((2^(3/4)*x*(- 5^(1/2) - 3)^(1/4)*3i)/(2*((3*2^(1/2
)*(- 5^(1/2) - 3)^(1/2))/2 - (2^(1/2)*5^(1/2)*(- 5^(1/2) - 3)^(1/2))/2)) -
(2^(3/4)*5^(1/2)*x*(- 5^(1/2) - 3)^(1/4)*1i)/(2*((3*2^(1/2)*(- 5^(1/2) - 3)
^(1/2))/2 - (2^(1/2)*5^(1/2)*(- 5^(1/2) - 3)^(1/2))/2)))*(- 5^(1/2) - 3)^(1
/4)*1i)/20 - (2^(3/4)*5^(1/2)*atan((3*2^(3/4)*x*(5^(1/2) - 3)^(1/4))/(2*((3
*2^(1/2)*(5^(1/2) - 3)^(1/2))/2 + (2^(1/2)*5^(1/2)*(5^(1/2) - 3)^(1/2))/2))
 + (2^(3/4)*5^(1/2)*x*(5^(1/2) - 3)^(1/4))/(2*((3*2^(1/2)*(5^(1/2) - 3)^(1/
2))/2 + (2^(1/2)*5^(1/2)*(5^(1/2) - 3)^(1/2))/2)))*(5^(1/2) - 3)^(1/4))/20
+ (2^(3/4)*5^(1/2)*atan((2^(3/4)*x*(5^(1/2) - 3)^(1/4)*3i)/(2*((3*2^(1/2)*(
5^(1/2) - 3)^(1/2))/2 + (2^(1/2)*5^(1/2)*(5^(1/2) - 3)^(1/2))/2)) + (2^(3/4
)*5^(1/2)*x*(5^(1/2) - 3)^(1/4)*1i)/(2*((3*2^(1/2)*(5^(1/2) - 3)^(1/2))/2 +
 (2^(1/2)*5^(1/2)*(5^(1/2) - 3)^(1/2))/2)))*(5^(1/2) - 3)^(1/4)*1i)/20
```

sympy [A] time = 1.48, size = 24, normalized size = 0.05

$$\mathrm{RootSum}\left(40960000t^8+19200t^4+1,\left(t\mapsto t\log\left(-51200t^5-12t+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(x**8+3*x**4+1),x)
```

```
[Out] RootSum(40960000*_t**8 + 19200*_t**4 + 1, Lambda(_t, _t*log(-51200*_t**5 -
12*_t + x)))
```

3.321 $\int \frac{x^2}{1+3x^4+x^8}\,dx$

Optimal. Leaf size=427

$$\frac{\log\left(2x^2-2\sqrt[4]{2\left(3-\sqrt{5}\right)}\,x+\sqrt{2\left(3-\sqrt{5}\right)}\right)}{4\sqrt{5}\sqrt[4]{2\left(3-\sqrt{5}\right)}}-\frac{\log\left(2x^2+2\sqrt[4]{2\left(3-\sqrt{5}\right)}\,x+\sqrt{2\left(3-\sqrt{5}\right)}\right)}{4\sqrt{5}\sqrt[4]{2\left(3-\sqrt{5}\right)}}-\frac{\log\left(2x^2-2\sqrt[4]{2\cdots\right.}{4\sqrt{\cdots}}$$

Rubi [A] time = 0.26, antiderivative size = 431, normalized size of antiderivative = 1.01, number of steps used = 19, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {1375, 297, 1162, 617, 204, 1165, 628}

$$\frac{\sqrt[4]{3+\sqrt{5}}\,\log\left(2x^2-2\sqrt[4]{2\left(3-\sqrt{5}\right)}\,x+\sqrt{2\left(3-\sqrt{5}\right)}\right)}{4\,2^{3/4}\sqrt{5}}-\frac{\sqrt[4]{3+\sqrt{5}}\,\log\left(2x^2+2\sqrt[4]{2\left(3-\sqrt{5}\right)}\,x+\sqrt{2\left(3-\sqrt{5}\right)}\right)}{4\,2^{3/4}\sqrt{5}}-\frac{\log\left(2x^2-2\sqrt[4]{2\left(3+\sqrt{5}\right)}\,x+\sqrt{2\left(3+\sqrt{5}\right)}\right)}{4\sqrt{5}\sqrt[4]{2\left(3+\sqrt{5}\right)}}+\frac{\log\left(2x^2+2\sqrt[4]{2\left(3+\sqrt{5}\right)}\,x+\sqrt{2\left(3+\sqrt{5}\right)}\right)}{4\sqrt{5}\sqrt[4]{2\left(3+\sqrt{5}\right)}}-\frac{\sqrt[4]{3+\sqrt{5}}\,\tan^{-1}\left(1-\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}\right)}{2\,2^{3/4}\sqrt{5}}+\frac{\sqrt[4]{3+\sqrt{5}}\,\tan^{-1}\left(\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}+1\right)}{2\,2^{3/4}\sqrt{5}}+\frac{\tan^{-1}\left(1-\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}\right)}{2\sqrt{5}\sqrt[4]{2\left(3+\sqrt{5}\right)}}-\frac{\tan^{-1}\left(\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}+1\right)}{2\sqrt{5}\sqrt[4]{2\left(3+\sqrt{5}\right)}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(1 + 3*x^4 + x^8),x]
```

```
[Out] -((3 + Sqrt[5])^(1/4)*ArcTan[1 - (2^(3/4)*x)/(3 - Sqrt[5])^(1/4)])/(2*2^(3/
4)*Sqrt[5]) + ((3 + Sqrt[5])^(1/4)*ArcTan[1 + (2^(3/4)*x)/(3 - Sqrt[5])^(1/
4)])/(2*2^(3/4)*Sqrt[5]) + ArcTan[1 - (2^(3/4)*x)/(3 + Sqrt[5])^(1/4)]/(2*S
qrt[5]*(2*(3 + Sqrt[5]))^(1/4)) - ArcTan[1 + (2^(3/4)*x)/(3 + Sqrt[5])^(1/4
)]/(2*Sqrt[5]*(2*(3 + Sqrt[5]))^(1/4)) + ((3 + Sqrt[5])^(1/4)*Log[Sqrt[2*(3
 - Sqrt[5])] - 2*(2*(3 - Sqrt[5]))^(1/4)*x + 2*x^2])/(4*2^(3/4)*Sqrt[5]) -
((3 + Sqrt[5])^(1/4)*Log[Sqrt[2*(3 - Sqrt[5])] + 2*(2*(3 - Sqrt[5]))^(1/4)*
x + 2*x^2])/(4*2^(3/4)*Sqrt[5]) - Log[Sqrt[2*(3 + Sqrt[5])] - 2*(2*(3 + Sqr
t[5]))^(1/4)*x + 2*x^2]/(4*Sqrt[5]*(2*(3 + Sqrt[5]))^(1/4)) + Log[Sqrt[2*(3
 + Sqrt[5])] + 2*(2*(3 + Sqrt[5]))^(1/4)*x + 2*x^2]/(4*Sqrt[5]*(2*(3 + Sqrt
[5]))^(1/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1375

```
Int[((d_.)*(x_))^(m_.)/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symb
ol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[c/q, Int[(d*x)^m/(b/2 - q/2 + c*
x^n), x], x] - Dist[c/q, Int[(d*x)^m/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[
{a, b, c, d, m}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{x^2}{1+3x^4+x^8}\,dx = \frac{\int \frac{x^2}{\frac{3}{2}-\frac{\sqrt{5}}{2}+x^4}\,dx}{\sqrt{5}} - \frac{\int \frac{x^2}{\frac{3}{2}+\frac{\sqrt{5}}{2}+x^4}\,dx}{\sqrt{5}}$$

$$= -\frac{\int \frac{\sqrt{3-\sqrt{5}}-\sqrt{2}x^2}{\frac{3}{2}-\frac{\sqrt{5}}{2}+x^4}\,dx}{2\sqrt{10}} + \frac{\int \frac{\sqrt{3-\sqrt{5}}+\sqrt{2}x^2}{\frac{3}{2}-\frac{\sqrt{5}}{2}+x^4}\,dx}{2\sqrt{10}} + \frac{\int \frac{\sqrt{3+\sqrt{5}}-\sqrt{2}x^2}{\frac{3}{2}+\frac{\sqrt{5}}{2}+x^4}\,dx}{2\sqrt{10}} - \frac{\int \frac{\sqrt{3+\sqrt{5}}+\sqrt{2}x^2}{\frac{3}{2}+\frac{\sqrt{5}}{2}+x^4}\,dx}{2\sqrt{10}}$$

$$= \frac{\int \frac{1}{\sqrt{\frac{1}{2}\left(3-\sqrt{5}\right)}-\sqrt[4]{2\left(3-\sqrt{5}\right)}x+x^2}\,dx}{4\sqrt{5}} + \frac{\int \frac{1}{\sqrt{\frac{1}{2}\left(3-\sqrt{5}\right)}+\sqrt[4]{2\left(3-\sqrt{5}\right)}x+x^2}\,dx}{4\sqrt{5}} - \frac{\int \frac{1}{\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}-\sqrt[4]{2\left(3+\sqrt{5}\right)}x+x^2}}{4\sqrt{5}}$$

$$= \frac{\sqrt[4]{3+\sqrt{5}}\,\log\left(\sqrt{2\left(3-\sqrt{5}\right)}-2\sqrt[4]{2\left(3-\sqrt{5}\right)}x+2x^2\right)}{4\,2^{3/4}\sqrt{5}} - \frac{\sqrt[4]{3+\sqrt{5}}\,\log\left(\sqrt{2\left(3-\sqrt{5}\right)}+\right.}{4\,2^{3/4}\sqrt{5}}$$

$$= -\frac{\tan^{-1}\left(1-\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}\right)}{2\sqrt{5}\sqrt[4]{2\left(3-\sqrt{5}\right)}} + \frac{\tan^{-1}\left(1+\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}\right)}{2\sqrt{5}\sqrt[4]{2\left(3-\sqrt{5}\right)}} + \frac{\tan^{-1}\left(1-\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}\right)}{2\sqrt{5}\sqrt[4]{2\left(3+\sqrt{5}\right)}} - \frac{\tan^{-1}\left(1+\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}\right)}{2\sqrt{5}\sqrt[4]{2\left(3+\sqrt{5}\right)}} +$$

Mathematica [C] time = 0.01, size = 40, normalized size = 0.09

$$\frac{1}{4}\text{RootSum}\left[\#1^8+3\#1^4+1\&,\frac{\log(x-\#1)}{2\#1^5+3\#1}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(1 + 3*x^4 + x^8),x]
```

```
[Out] RootSum[1 + 3*#1^4 + #1^8 & , Log[x - #1]/(3*#1 + 2*#1^5) & ]/4
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{1+3x^4+x^8}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^2/(1 + 3*x^4 + x^8),x]
```

```
[Out] IntegrateAlgebraic[x^2/(1 + 3*x^4 + x^8), x]
```

fricas [B] time = 1.86, size = 955, normalized size = 2.24

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^8+3*x^4+1),x, algorithm="fricas")
```

```
[Out] 1/80*sqrt(10)*(2*sqrt(5) + 6)^(3/4)*sqrt(sqrt(5) + 3)*(sqrt(5) - 3)*arctan(
-1/40*sqrt(10)*(3*sqrt(5)*x - 5*x)*(2*sqrt(5) + 6)^(3/4)*sqrt(sqrt(5) + 3)
+ 1/80*sqrt(sqrt(10)*(3*sqrt(5)*sqrt(2)*x - 5*sqrt(2)*x)*(2*sqrt(5) + 6)^(3
/4) + 40*x^2 - 10*sqrt(2*sqrt(5) + 6)*(sqrt(5) - 3))*(3*sqrt(5) - 5)*(2*sqr
t(5) + 6)^(3/4)*sqrt(sqrt(5) + 3) + 1/8*(sqrt(5)*sqrt(2) - 3*sqrt(2))*sqrt(
2*sqrt(5) + 6)*sqrt(sqrt(5) + 3)) + 1/80*sqrt(10)*(2*sqrt(5) + 6)^(3/4)*sqr
t(sqrt(5) + 3)*(sqrt(5) - 3)*arctan(-1/40*sqrt(10)*(3*sqrt(5)*x - 5*x)*(2*s
qrt(5) + 6)^(3/4)*sqrt(sqrt(5) + 3) + 1/80*sqrt(-sqrt(10)*(3*sqrt(5)*sqrt(2
)*x - 5*sqrt(2)*x)*(2*sqrt(5) + 6)^(3/4) + 40*x^2 - 10*sqrt(2*sqrt(5) + 6)*
(sqrt(5) - 3))*(3*sqrt(5) - 5)*(2*sqrt(5) + 6)^(3/4)*sqrt(sqrt(5) + 3) - 1/
8*(sqrt(5)*sqrt(2) - 3*sqrt(2))*sqrt(2*sqrt(5) + 6)*sqrt(sqrt(5) + 3)) + 1/
80*sqrt(10)*(sqrt(5) + 3)*sqrt(-sqrt(5) + 3)*(-2*sqrt(5) + 6)^(3/4)*arctan(
1/80*sqrt(sqrt(10)*(3*sqrt(5)*sqrt(2)*x + 5*sqrt(2)*x)*(-2*sqrt(5) + 6)^(3/
4) + 40*x^2 + 10*(sqrt(5) + 3)*sqrt(-2*sqrt(5) + 6))*(3*sqrt(5) + 5)*sqrt(-
sqrt(5) + 3)*(-2*sqrt(5) + 6)^(3/4) - 1/40*(sqrt(10)*(3*sqrt(5)*x + 5*x)*(-
2*sqrt(5) + 6)^(3/4) + 5*(sqrt(5)*sqrt(2) + 3*sqrt(2))*sqrt(-2*sqrt(5) + 6)
)*sqrt(-sqrt(5) + 3)) + 1/80*sqrt(10)*(sqrt(5) + 3)*sqrt(-sqrt(5) + 3)*(-2*
sqrt(5) + 6)^(3/4)*arctan(1/80*sqrt(-sqrt(10)*(3*sqrt(5)*sqrt(2)*x + 5*sqrt
(2)*x)*(-2*sqrt(5) + 6)^(3/4) + 40*x^2 + 10*(sqrt(5) + 3)*sqrt(-2*sqrt(5) +
 6))*(3*sqrt(5) + 5)*sqrt(-sqrt(5) + 3)*(-2*sqrt(5) + 6)^(3/4) - 1/40*(sqrt
(10)*(3*sqrt(5)*x + 5*x)*(-2*sqrt(5) + 6)^(3/4) - 5*(sqrt(5)*sqrt(2) + 3*sq
rt(2))*sqrt(-2*sqrt(5) + 6))*sqrt(-sqrt(5) + 3)) - 1/80*sqrt(10)*sqrt(2)*(2
*sqrt(5) + 6)^(1/4)*log(sqrt(10)*(3*sqrt(5)*sqrt(2)*x - 5*sqrt(2)*x)*(2*sqr
t(5) + 6)^(3/4) + 40*x^2 - 10*sqrt(2*sqrt(5) + 6)*(sqrt(5) - 3)) + 1/80*sqr
t(10)*sqrt(2)*(2*sqrt(5) + 6)^(1/4)*log(-sqrt(10)*(3*sqrt(5)*sqrt(2)*x - 5*
sqrt(2)*x)*(2*sqrt(5) + 6)^(3/4) + 40*x^2 - 10*sqrt(2*sqrt(5) + 6)*(sqrt(5)
 - 3)) + 1/80*sqrt(10)*sqrt(2)*(-2*sqrt(5) + 6)^(1/4)*log(sqrt(10)*(3*sqrt(
5)*sqrt(2)*x + 5*sqrt(2)*x)*(-2*sqrt(5) + 6)^(3/4) + 40*x^2 + 10*(sqrt(5) +
 3)*sqrt(-2*sqrt(5) + 6)) - 1/80*sqrt(10)*sqrt(2)*(-2*sqrt(5) + 6)^(1/4)*lo
g(-sqrt(10)*(3*sqrt(5)*sqrt(2)*x + 5*sqrt(2)*x)*(-2*sqrt(5) + 6)^(3/4) + 40
*x^2 + 10*(sqrt(5) + 3)*sqrt(-2*sqrt(5) + 6))
```

giac [A] time = 0.59, size = 239, normalized size = 0.56

$$\frac{1}{80}\left(\pi + 4\arctan\left(x\sqrt{\sqrt{5}+1}-1\right)\right)\sqrt{5\sqrt{5}+5} - \frac{1}{80}\left(\pi + 4\arctan\left(-x\sqrt{\sqrt{5}+1}-1\right)\right)\sqrt{5\sqrt{5}+5} - \frac{1}{80}\left(\pi + 4\arctan\left(x\sqrt{\sqrt{5}-1}+1\right)\right)\sqrt{5\sqrt{5}-5} + \frac{1}{80}\left(\pi + 4\arctan\left(-x\sqrt{\sqrt{5}-1}+1\right)\right)\sqrt{5\sqrt{5}-5} + \frac{1}{40}\sqrt{5\sqrt{5}-5}\log\left(16900\left(x+\sqrt{\sqrt{5}+1}\right)^2+16900x^2\right) - \frac{1}{40}\sqrt{5\sqrt{5}-5}\log\left(16900\left(x-\sqrt{\sqrt{5}+1}\right)^2+16900x^2\right) - \frac{1}{40}\sqrt{5\sqrt{5}+5}\log\left(2500\left(x+\sqrt{\sqrt{5}-1}\right)^2+2500x^2\right) + \frac{1}{40}\sqrt{5\sqrt{5}+5}\log\left(2500\left(x-\sqrt{\sqrt{5}-1}\right)^2+2500x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^8+3*x^4+1),x, algorithm="giac")
```

```
[Out] 1/80*(pi + 4*arctan(x*sqrt(sqrt(5) + 1) - 1))*sqrt(5*sqrt(5) + 5) - 1/80*(p
i + 4*arctan(-x*sqrt(sqrt(5) + 1) - 1))*sqrt(5*sqrt(5) + 5) - 1/80*(pi + 4*
arctan(x*sqrt(sqrt(5) - 1) + 1))*sqrt(5*sqrt(5) - 5) + 1/80*(pi + 4*arctan(
-x*sqrt(sqrt(5) - 1) + 1))*sqrt(5*sqrt(5) - 5) + 1/40*sqrt(5*sqrt(5) - 5)*l
og(16900*(x + sqrt(sqrt(5) + 1))^2 + 16900*x^2) - 1/40*sqrt(5*sqrt(5) - 5)*
log(16900*(x - sqrt(sqrt(5) + 1))^2 + 16900*x^2) - 1/40*sqrt(5*sqrt(5) + 5)
*log(2500*(x + sqrt(sqrt(5) - 1))^2 + 2500*x^2) + 1/40*sqrt(5*sqrt(5) + 5)*
log(2500*(x - sqrt(sqrt(5) - 1))^2 + 2500*x^2)
```

maple [C] time = 0.01, size = 40, normalized size = 0.09

$$\frac{\mathrm{RootOf}\left(\_Z^8+3\_Z^4+1\right)^2\ln\left(-\mathrm{RootOf}\left(\_Z^8+3\_Z^4+1\right)+x\right)}{8\,\mathrm{RootOf}\left(\_Z^8+3\_Z^4+1\right)^7+12\,\mathrm{RootOf}\left(\_Z^8+3\_Z^4+1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(x^8+3*x^4+1),x)
```

```
[Out] 1/4*sum(_R^2/(2*_R^7+3*_R^3)*ln(-_R+x),_R=RootOf(_Z^8+3*_Z^4+1))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{x^8+3\,x^4+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^8+3*x^4+1),x, algorithm="maxima")
```

```
[Out] integrate(x^2/(x^8 + 3*x^4 + 1), x)
```

mupad [B] time = 0.09, size = 275, normalized size = 0.64

$$\frac{2^{3/4}\sqrt{5}\,\mathrm{atan}\left(\frac{7\,2^{3/4}\,x\,{\left(\sqrt{5}-3\right)}^{1/4}}{2\,\left(3\,\sqrt{5}-7\right)}-\frac{3\,2^{3/4}\,\sqrt{5}\,x\,{\left(\sqrt{5}-3\right)}^{1/4}}{2\,\left(3\,\sqrt{5}-7\right)}\right)\,{\left(\sqrt{5}-3\right)}^{1/4}}{20}+\frac{2^{3/4}\sqrt{5}\,\mathrm{atan}\left(\frac{2^{3/4}\,x\,{\left(\sqrt{5}-3\right)}^{1/4}\,7\mathrm{i}}{2\,\left(3\,\sqrt{5}-7\right)}-\frac{2^{3/4}\,\sqrt{5}\,x\,{\left(\sqrt{5}-3\right)}^{1/4}\,3\mathrm{i}}{2\,\left(3\,\sqrt{5}-7\right)}\right)\,{\left(\sqrt{5}-3\right)}^{1/4}\,1\mathrm{i}}{20}+\frac{2^{3/4}\sqrt{5}\,\mathrm{atan}\left(\frac{7\,2^{3/4}\,x\,{\left(-\sqrt{5}-3\right)}^{1/4}}{2\,\left(3\,\sqrt{5}+7\right)}+\frac{3\,2^{3/4}\,\sqrt{5}\,x\,{\left(-\sqrt{5}-3\right)}^{1/4}}{2\,\left(3\,\sqrt{5}+7\right)}\right)\,{\left(-\sqrt{5}-3\right)}^{1/4}}{20}+\frac{2^{3/4}\sqrt{5}\,\mathrm{atan}\left(\frac{2^{3/4}\,x\,{\left(-\sqrt{5}-3\right)}^{1/4}\,7\mathrm{i}}{2\,\left(3\,\sqrt{5}+7\right)}+\frac{2^{3/4}\,\sqrt{5}\,x\,{\left(-\sqrt{5}-3\right)}^{1/4}\,3\mathrm{i}}{2\,\left(3\,\sqrt{5}+7\right)}\right)\,{\left(-\sqrt{5}-3\right)}^{1/4}\,1\mathrm{i}}{20}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(3*x^4 + x^8 + 1),x)
```

```
[Out] (2^(3/4)*5^(1/2)*atan((7*2^(3/4)*x*(5^(1/2) - 3)^(1/4))/(2*(3*5^(1/2) - 7))
 - (3*2^(3/4)*5^(1/2)*x*(5^(1/2) - 3)^(1/4))/(2*(3*5^(1/2) - 7)))*(5^(1/2)
- 3)^(1/4))/20 + (2^(3/4)*5^(1/2)*atan((2^(3/4)*x*(5^(1/2) - 3)^(1/4)*7i)/(
2*(3*5^(1/2) - 7)) - (2^(3/4)*5^(1/2)*x*(5^(1/2) - 3)^(1/4)*3i)/(2*(3*5^(1/
2) - 7)))*(5^(1/2) - 3)^(1/4)*1i)/20 + (2^(3/4)*5^(1/2)*atan((7*2^(3/4)*x*(
- 5^(1/2) - 3)^(1/4))/(2*(3*5^(1/2) + 7)) + (3*2^(3/4)*5^(1/2)*x*(- 5^(1/2)
 - 3)^(1/4))/(2*(3*5^(1/2) + 7)))*(- 5^(1/2) - 3)^(1/4))/20 + (2^(3/4)*5^(1
/2)*atan((2^(3/4)*x*(- 5^(1/2) - 3)^(1/4)*7i)/(2*(3*5^(1/2) + 7)) + (2^(3/4
)*5^(1/2)*x*(- 5^(1/2) - 3)^(1/4)*3i)/(2*(3*5^(1/2) + 7)))*(- 5^(1/2) - 3)^
(1/4)*1i)/20
```

sympy [A] time = 1.51, size = 26, normalized size = 0.06

$$\mathrm{RootSum}\left(40960000t^8+19200t^4+1,\left(t\mapsto t\log\left(-6144000t^7-2240t^3+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(x**8+3*x**4+1),x)
```

```
[Out] RootSum(40960000*_t**8 + 19200*_t**4 + 1, Lambda(_t, _t*log(-6144000*_t**7
- 2240*_t**3 + x)))
```

3.322 $\int \frac{1}{1+3x^4+x^8} \, dx$

Optimal. Leaf size=414

$$-\frac{\sqrt[4]{9+4\sqrt{5}}\,\log\left(2x^2-2\sqrt[4]{2\left(3-\sqrt{5}\right)}\,x+\sqrt{2\left(3-\sqrt{5}\right)}\right)}{4\sqrt{10}}+\frac{\sqrt[4]{9+4\sqrt{5}}\,\log\left(2x^2+2\sqrt[4]{2\left(3-\sqrt{5}\right)}\,x+\sqrt{2\left(3-\right.}\right.}{4\sqrt{10}}$$

Rubi [A] time = 0.26, antiderivative size = 414, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 7, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.583, Rules used = {1347, 211, 1165, 628, 1162, 617, 204}

$$-\frac{\sqrt[4]{9+4\sqrt{5}}\,\log\left(2x^2-2\sqrt[4]{2\left(3-\sqrt{5}\right)}\,x+\sqrt{2\left(3-\sqrt{5}\right)}\right)}{4\sqrt{10}}+\frac{\sqrt[4]{9+4\sqrt{5}}\,\log\left(2x^2+2\sqrt[4]{2\left(3-\sqrt{5}\right)}\,x+\sqrt{2\left(3-\sqrt{5}\right)}\right)}{4\sqrt{10}}+\frac{\log\left(2x^2-2\sqrt[4]{2\left(3+\sqrt{5}\right)}\,x+\sqrt{2\left(3+\sqrt{5}\right)}\right)}{2\sqrt{5}\left(2\left(3+\sqrt{5}\right)\right)^{3/4}}-\frac{\log\left(2x^2+2\sqrt[4]{2\left(3+\sqrt{5}\right)}\,x+\sqrt{2\left(3+\sqrt{5}\right)}\right)}{2\sqrt{5}\left(2\left(3+\sqrt{5}\right)\right)^{3/4}}-\frac{\sqrt[4]{9+4\sqrt{5}}\,\tan^{-1}\left(1-\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}\right)}{2\sqrt{10}}+\frac{\sqrt[4]{9+4\sqrt{5}}\,\tan^{-1}\left(\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}+1\right)}{2\sqrt{10}}+\frac{\tan^{-1}\left(1-\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}\right)}{\sqrt{5}\left(2\left(3+\sqrt{5}\right)\right)^{3/4}}-\frac{\tan^{-1}\left(\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}+1\right)}{\sqrt{5}\left(2\left(3+\sqrt{5}\right)\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 3*x^4 + x^8)^(-1),x]
```

```
[Out] -((9 + 4*Sqrt[5])^(1/4)*ArcTan[1 - (2^(3/4)*x)/(3 - Sqrt[5])^(1/4)])/(2*Sqr
t[10]) + ((9 + 4*Sqrt[5])^(1/4)*ArcTan[1 + (2^(3/4)*x)/(3 - Sqrt[5])^(1/4)]
)/(2*Sqrt[10]) + ArcTan[1 - (2^(3/4)*x)/(3 + Sqrt[5])^(1/4)]/(Sqrt[5]*(2*(3
 + Sqrt[5]))^(3/4)) - ArcTan[1 + (2^(3/4)*x)/(3 + Sqrt[5])^(1/4)]/(Sqrt[5]*
(2*(3 + Sqrt[5]))^(3/4)) - ((9 + 4*Sqrt[5])^(1/4)*Log[Sqrt[2*(3 - Sqrt[5])]
 - 2*(2*(3 - Sqrt[5]))^(1/4)*x + 2*x^2])/(4*Sqrt[10]) + ((9 + 4*Sqrt[5])^(1
/4)*Log[Sqrt[2*(3 - Sqrt[5])] + 2*(2*(3 - Sqrt[5]))^(1/4)*x + 2*x^2])/(4*Sq
rt[10]) + Log[Sqrt[2*(3 + Sqrt[5])] - 2*(2*(3 + Sqrt[5]))^(1/4)*x + 2*x^2]/
(2*Sqrt[5]*(2*(3 + Sqrt[5]))^(3/4)) - Log[Sqrt[2*(3 + Sqrt[5])] + 2*(2*(3 +
 Sqrt[5]))^(1/4)*x + 2*x^2]/(2*Sqrt[5]*(2*(3 + Sqrt[5]))^(3/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1347

```
Int[((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^(-1), x_Symbol] :> With[{q
= Rt[b^2 - 4*a*c, 2]}, Dist[c/q, Int[1/(b/2 - q/2 + c*x^n), x], x] - Dist[c
/q, Int[1/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*
n] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{1}{1+3x^4+x^8}\,dx = \frac{\int \frac{1}{\frac{3}{2}-\frac{\sqrt{5}}{2}+x^4}\,dx}{\sqrt{5}} - \frac{\int \frac{1}{\frac{3}{2}+\frac{\sqrt{5}}{2}+x^4}\,dx}{\sqrt{5}}$$

$$= \frac{\int \frac{\sqrt{3-\sqrt{5}}-\sqrt{2}x^2}{\frac{3}{2}-\frac{\sqrt{5}}{2}+x^4}\,dx}{2\sqrt{5\left(3-\sqrt{5}\right)}} + \frac{\int \frac{\sqrt{3-\sqrt{5}}+\sqrt{2}x^2}{\frac{3}{2}-\frac{\sqrt{5}}{2}+x^4}\,dx}{2\sqrt{5\left(3-\sqrt{5}\right)}} - \frac{\int \frac{\sqrt{3+\sqrt{5}}-\sqrt{2}x^2}{\frac{3}{2}+\frac{\sqrt{5}}{2}+x^4}\,dx}{2\sqrt{5\left(3+\sqrt{5}\right)}} - \frac{\int \frac{\sqrt{3+\sqrt{5}}+\sqrt{2}x^2}{\frac{3}{2}+\frac{\sqrt{5}}{2}+x^4}\,dx}{2\sqrt{5\left(3+\sqrt{5}\right)}}$$

$$= \frac{\int \frac{1}{\sqrt{\frac{1}{2}\left(3-\sqrt{5}\right)}-\sqrt[4]{2\left(3-\sqrt{5}\right)}x+x^2}\,dx}{2\sqrt{10\left(3-\sqrt{5}\right)}} + \frac{\int \frac{1}{\sqrt{\frac{1}{2}\left(3-\sqrt{5}\right)}+\sqrt[4]{2\left(3-\sqrt{5}\right)}x+x^2}\,dx}{2\sqrt{10\left(3-\sqrt{5}\right)}} + \frac{\int \frac{\sqrt[4]{2\left(3+\sqrt{5}\right)}+2x}{-\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}-\sqrt[4]{2\left(3+\sqrt{5}\right)}x-\ldots}}{2\sqrt{5}\left(2\left(3+\sqrt{5}\right)\right)^{3/4}}$$

$$= -\frac{\sqrt[4]{9+4\sqrt{5}}\,\log\left(\sqrt{2\left(3-\sqrt{5}\right)}-2\sqrt[4]{2\left(3-\sqrt{5}\right)}x+2x^2\right)}{4\sqrt{10}} + \frac{\sqrt[4]{9+4\sqrt{5}}\,\log\left(\sqrt{2\left(3-\sqrt{5}\ldots\right.}\right.}{4\sqrt{1\ldots}}$$

$$= -\frac{\tan^{-1}\left(1-\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}\right)}{\sqrt{5}\left(2\left(3-\sqrt{5}\right)\right)^{3/4}} + \frac{\tan^{-1}\left(1+\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}\right)}{\sqrt{5}\left(2\left(3-\sqrt{5}\right)\right)^{3/4}} + \frac{\tan^{-1}\left(1-\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}\right)}{\sqrt{5}\left(2\left(3+\sqrt{5}\right)\right)^{3/4}} - \frac{\tan^{-1}\left(1+\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}\right)}{\sqrt{5}\left(2\left(3+\sqrt{5}\right)\right)^{3/4}} \ldots$$

Mathematica [C] time = 0.01, size = 42, normalized size = 0.10

$$\frac{1}{4}\text{RootSum}\left[\#1^8+3\#1^4+1\&,\frac{\log(x-\#1)}{2\#1^7+3\#1^3}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 3*x^4 + x^8)^(-1),x]
```

```
[Out] RootSum[1 + 3*#1^4 + #1^8 & , Log[x - #1]/(3*#1^3 + 2*#1^7) & ]/4
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{1+3x^4+x^8}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(1 + 3*x^4 + x^8)^(-1),x]
```

```
[Out] IntegrateAlgebraic[(1 + 3*x^4 + x^8)^(-1), x]
```

fricas [B] time = 1.76, size = 733, normalized size = 1.77

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^8+3*x^4+1),x, algorithm="fricas")
```

```
[Out] 1/10*sqrt(5)*sqrt(2)*(4*sqrt(5) + 9)^(1/4)*arctan(1/2*sqrt(2*x^2 - sqrt(4*s
qrt(5) + 9)*(3*sqrt(5) - 7) + (sqrt(5)*sqrt(2)*x - 3*sqrt(2)*x)*(4*sqrt(5)
+ 9)^(1/4))*(4*sqrt(5) + 9)^(3/4)*(3*sqrt(5) - 7) - 1/2*(3*sqrt(5)*sqrt(2)*
x - 7*sqrt(2)*x)*(4*sqrt(5) + 9)^(3/4) - 1) + 1/10*sqrt(5)*sqrt(2)*(4*sqrt(
5) + 9)^(1/4)*arctan(1/2*sqrt(2*x^2 - sqrt(4*sqrt(5) + 9)*(3*sqrt(5) - 7) -
 (sqrt(5)*sqrt(2)*x - 3*sqrt(2)*x)*(4*sqrt(5) + 9)^(1/4))*(4*sqrt(5) + 9)^(
3/4)*(3*sqrt(5) - 7) - 1/2*(3*sqrt(5)*sqrt(2)*x - 7*sqrt(2)*x)*(4*sqrt(5) +
 9)^(3/4) + 1) + 1/10*sqrt(5)*sqrt(2)*(-4*sqrt(5) + 9)^(1/4)*arctan(1/2*sqr
t(2*x^2 + (3*sqrt(5) + 7)*sqrt(-4*sqrt(5) + 9) + (sqrt(5)*sqrt(2)*x + 3*sqr
t(2)*x)*(-4*sqrt(5) + 9)^(1/4))*(3*sqrt(5) + 7)*(-4*sqrt(5) + 9)^(3/4) - 1/
2*(3*sqrt(5)*sqrt(2)*x + 7*sqrt(2)*x)*(-4*sqrt(5) + 9)^(3/4) - 1) + 1/10*sq
rt(5)*sqrt(2)*(-4*sqrt(5) + 9)^(1/4)*arctan(1/2*sqrt(2*x^2 + (3*sqrt(5) + 7
)*sqrt(-4*sqrt(5) + 9) - (sqrt(5)*sqrt(2)*x + 3*sqrt(2)*x)*(-4*sqrt(5) + 9)
^(1/4))*(3*sqrt(5) + 7)*(-4*sqrt(5) + 9)^(3/4) - 1/2*(3*sqrt(5)*sqrt(2)*x +
 7*sqrt(2)*x)*(-4*sqrt(5) + 9)^(3/4) + 1) - 1/40*sqrt(5)*sqrt(2)*(4*sqrt(5)
 + 9)^(1/4)*log(2*x^2 - sqrt(4*sqrt(5) + 9)*(3*sqrt(5) - 7) + (sqrt(5)*sqrt
(2)*x - 3*sqrt(2)*x)*(4*sqrt(5) + 9)^(1/4)) + 1/40*sqrt(5)*sqrt(2)*(4*sqrt(
5) + 9)^(1/4)*log(2*x^2 - sqrt(4*sqrt(5) + 9)*(3*sqrt(5) - 7) - (sqrt(5)*sq
rt(2)*x - 3*sqrt(2)*x)*(4*sqrt(5) + 9)^(1/4)) - 1/40*sqrt(5)*sqrt(2)*(-4*sq
rt(5) + 9)^(1/4)*log(2*x^2 + (3*sqrt(5) + 7)*sqrt(-4*sqrt(5) + 9) + (sqrt(5
)*sqrt(2)*x + 3*sqrt(2)*x)*(-4*sqrt(5) + 9)^(1/4)) + 1/40*sqrt(5)*sqrt(2)*(
-4*sqrt(5) + 9)^(1/4)*log(2*x^2 + (3*sqrt(5) + 7)*sqrt(-4*sqrt(5) + 9) - (s
qrt(5)*sqrt(2)*x + 3*sqrt(2)*x)*(-4*sqrt(5) + 9)^(1/4))
```

giac [A] time = 0.53, size = 239, normalized size = 0.58

$$\frac{1}{80}\left(\pi + 4\arctan\left(x\sqrt{\sqrt{5}+1}+1\right)\right)\sqrt{10\sqrt{5}+20} - \frac{1}{80}\left(\pi + 4\arctan\left(-x\sqrt{\sqrt{5}+1}+1\right)\right)\sqrt{10\sqrt{5}+20} - \frac{1}{80}\left(\pi + 4\arctan\left(x\sqrt{\sqrt{5}-1}-1\right)\right)\sqrt{10\sqrt{5}-20} + \frac{1}{80}\left(\pi + 4\arctan\left(-x\sqrt{\sqrt{5}-1}-1\right)\right)\sqrt{10\sqrt{5}-20} - \frac{1}{40}\sqrt{10\sqrt{5}-20}\log\left(10000\left(x+\sqrt{\sqrt{5}+1}\right)^2 + 10000x^2\right) + \frac{1}{40}\sqrt{10\sqrt{5}-20}\log\left(10000\left(x-\sqrt{\sqrt{5}+1}\right)^2 + 10000x^2\right) + \frac{1}{40}\sqrt{10\sqrt{5}+20}\log\left(400\left(x+\sqrt{\sqrt{5}-1}\right)^2 + 400x^2\right) - \frac{1}{40}\sqrt{10\sqrt{5}+20}\log\left(400\left(x-\sqrt{\sqrt{5}-1}\right)^2 + 400x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^8+3*x^4+1),x, algorithm="giac")
```

```
[Out] 1/80*(pi + 4*arctan(x*sqrt(sqrt(5) + 1) + 1))*sqrt(10*sqrt(5) + 20) - 1/80*
(pi + 4*arctan(-x*sqrt(sqrt(5) + 1) + 1))*sqrt(10*sqrt(5) + 20) - 1/80*(pi
+ 4*arctan(x*sqrt(sqrt(5) - 1) - 1))*sqrt(10*sqrt(5) - 20) + 1/80*(pi + 4*a
rctan(-x*sqrt(sqrt(5) - 1) - 1))*sqrt(10*sqrt(5) - 20) - 1/40*sqrt(10*sqrt(
5) - 20)*log(10000*(x + sqrt(sqrt(5) + 1))^2 + 10000*x^2) + 1/40*sqrt(10*sq
rt(5) - 20)*log(10000*(x - sqrt(sqrt(5) + 1))^2 + 10000*x^2) + 1/40*sqrt(10
*sqrt(5) + 20)*log(400*(x + sqrt(sqrt(5) - 1))^2 + 400*x^2) - 1/40*sqrt(10*
sqrt(5) + 20)*log(400*(x - sqrt(sqrt(5) - 1))^2 + 400*x^2)
```

maple [C] time = 0.01, size = 37, normalized size = 0.09

$$\frac{\ln\left(-\operatorname{RootOf}\left(\_Z^8+3\_Z^4+1\right)+x\right)}{8\operatorname{RootOf}\left(\_Z^8+3\_Z^4+1\right)^7+12\operatorname{RootOf}\left(\_Z^8+3\_Z^4+1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^8+3*x^4+1),x)
```

```
[Out] 1/4*sum(1/(2*_R^7+3*_R^3)*ln(-_R+x),_R=RootOf(_Z^8+3*_Z^4+1))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^8 + 3\,x^4 + 1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^8+3*x^4+1),x, algorithm="maxima")
```

```
[Out] integrate(1/(x^8 + 3*x^4 + 1), x)
```

mupad [B] time = 0.08, size = 403, normalized size = 0.97

$$\frac{\sqrt{5}\,\mathrm{atan}\left(\frac{144\,x\,{\left(-4\,\sqrt{5}-9\right)}^{1/4}}{24\,\sqrt{5}\,\sqrt{-4\,\sqrt{5}-9}+56\,\sqrt{-4\,\sqrt{5}-9}}+\frac{64\,\sqrt{5}\,x\,{\left(-4\,\sqrt{5}-9\right)}^{1/4}}{24\,\sqrt{5}\,\sqrt{-4\,\sqrt{5}-9}+56\,\sqrt{-4\,\sqrt{5}-9}}\right)\,{\left(-4\,\sqrt{5}-9\right)}^{1/4}}{10}+\frac{\sqrt{5}\,\mathrm{atan}\left(\frac{144\,x\,{\left(4\,\sqrt{5}-9\right)}^{1/4}}{24\,\sqrt{5}\,\sqrt{4\,\sqrt{5}-9}-56\,\sqrt{4\,\sqrt{5}-9}}-\frac{64\,\sqrt{5}\,x\,{\left(4\,\sqrt{5}-9\right)}^{1/4}}{24\,\sqrt{5}\,\sqrt{4\,\sqrt{5}-9}-56\,\sqrt{4\,\sqrt{5}-9}}\right)\,{\left(4\,\sqrt{5}-9\right)}^{1/4}}{10}-\frac{\sqrt{5}\,\mathrm{atan}\left(\frac{x\,{\left(-4\,\sqrt{5}-9\right)}^{1/4}\,144{}\mathrm{i}}{24\,\sqrt{5}\,\sqrt{-4\,\sqrt{5}-9}+56\,\sqrt{-4\,\sqrt{5}-9}}+\frac{\sqrt{5}\,x\,{\left(-4\,\sqrt{5}-9\right)}^{1/4}\,64{}\mathrm{i}}{24\,\sqrt{5}\,\sqrt{-4\,\sqrt{5}-9}+56\,\sqrt{-4\,\sqrt{5}-9}}\right)\,{\left(-4\,\sqrt{5}-9\right)}^{1/4}\,1{}\mathrm{i}}{10}-\frac{\sqrt{5}\,\mathrm{atan}\left(\frac{x\,{\left(4\,\sqrt{5}-9\right)}^{1/4}\,144{}\mathrm{i}}{24\,\sqrt{5}\,\sqrt{4\,\sqrt{5}-9}-56\,\sqrt{4\,\sqrt{5}-9}}-\frac{\sqrt{5}\,x\,{\left(4\,\sqrt{5}-9\right)}^{1/4}\,64{}\mathrm{i}}{24\,\sqrt{5}\,\sqrt{4\,\sqrt{5}-9}-56\,\sqrt{4\,\sqrt{5}-9}}\right)\,{\left(4\,\sqrt{5}-9\right)}^{1/4}\,1{}\mathrm{i}}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3*x^4 + x^8 + 1),x)
```

```
[Out] (5^(1/2)*atan((144*x*(- 4*5^(1/2) - 9)^(1/4))/(24*5^(1/2)*(- 4*5^(1/2) - 9)
^(1/2) + 56*(- 4*5^(1/2) - 9)^(1/2)) + (64*5^(1/2)*x*(- 4*5^(1/2) - 9)^(1/4
))/(24*5^(1/2)*(- 4*5^(1/2) - 9)^(1/2) + 56*(- 4*5^(1/2) - 9)^(1/2)))*(- 4*
5^(1/2) - 9)^(1/4))/10 + (5^(1/2)*atan((144*x*(4*5^(1/2) - 9)^(1/4))/(24*5^
(1/2)*(4*5^(1/2) - 9)^(1/2) - 56*(4*5^(1/2) - 9)^(1/2)) - (64*5^(1/2)*x*(4*
5^(1/2) - 9)^(1/4))/(24*5^(1/2)*(4*5^(1/2) - 9)^(1/2) - 56*(4*5^(1/2) - 9)^
(1/2)))*(4*5^(1/2) - 9)^(1/4))/10 - (5^(1/2)*atan((x*(- 4*5^(1/2) - 9)^(1/4
)*144i)/(24*5^(1/2)*(- 4*5^(1/2) - 9)^(1/2) + 56*(- 4*5^(1/2) - 9)^(1/2)) +
 (5^(1/2)*x*(- 4*5^(1/2) - 9)^(1/4)*64i)/(24*5^(1/2)*(- 4*5^(1/2) - 9)^(1/2
) + 56*(- 4*5^(1/2) - 9)^(1/2)))*(- 4*5^(1/2) - 9)^(1/4)*1i)/10 - (5^(1/2)*
atan((x*(4*5^(1/2) - 9)^(1/4)*144i)/(24*5^(1/2)*(4*5^(1/2) - 9)^(1/2) - 56*
(4*5^(1/2) - 9)^(1/2)) - (5^(1/2)*x*(4*5^(1/2) - 9)^(1/4)*64i)/(24*5^(1/2)*
(4*5^(1/2) - 9)^(1/2) - 56*(4*5^(1/2) - 9)^(1/2)))*(4*5^(1/2) - 9)^(1/4)*1i
)/10
```

sympy [A] time = 1.52, size = 26, normalized size = 0.06

$$\mathrm{RootSum}\left(40960000t^8 + 115200t^4 + 1, \left(t \mapsto t \log\left(-9600t^5 - \frac{47t}{2} + x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**8+3*x**4+1),x)
```

```
[Out] RootSum(40960000*_t**8 + 115200*_t**4 + 1, Lambda(_t, _t*log(-9600*_t**5 -
47*_t/2 + x)))
```

3.323 $\int \frac{1}{x^2(1+3x^4+x^8)}\,dx$

Optimal. Leaf size=416

$$-\frac{\left(3+\sqrt{5}\right)^{5/4}\log\left(2x^2-2\sqrt[4]{2\left(3-\sqrt{5}\right)}\,x+\sqrt{2\left(3-\sqrt{5}\right)}\right)}{8\,2^{3/4}\sqrt{5}}+\frac{\left(3+\sqrt{5}\right)^{5/4}\log\left(2x^2+2\sqrt[4]{2\left(3-\sqrt{5}\right)}\,x+\sqrt{2\left(3\right.}\right.}{8\,2^{3/4}\sqrt{5}}$$

Rubi [A] time = 0.29, antiderivative size = 416, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 8, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {1368, 1510, 297, 1162, 617, 204, 1165, 628}

$$-\frac{\left(3+\sqrt{5}\right)^{5/4}\log\left(2x^2-2\sqrt[4]{2\left(3-\sqrt{5}\right)}\,x+\sqrt{2\left(3-\sqrt{5}\right)}\right)}{8\,2^{3/4}\sqrt{5}}+\frac{\left(3+\sqrt{5}\right)^{5/4}\log\left(2x^2+2\sqrt[4]{2\left(3-\sqrt{5}\right)}\,x+\sqrt{2\left(3-\sqrt{5}\right)}\right)}{8\,2^{3/4}\sqrt{5}}+\frac{1}{40}\sqrt[4]{6150-2750\sqrt{5}}\log\left(2x^2-2\sqrt[4]{2\left(3+\sqrt{5}\right)}\,x+\sqrt{2\left(3+\sqrt{5}\right)}\right)-\frac{1}{40}\sqrt[4]{6150-2750\sqrt{5}}\log\left(2x^2+2\sqrt[4]{2\left(3+\sqrt{5}\right)}\,x+\sqrt{2\left(3+\sqrt{5}\right)}\right)-\frac{1}{x}+\frac{\left(3+\sqrt{5}\right)^{5/4}\tan^{-1}\left(1-\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}\right)}{4\,2^{3/4}\sqrt{5}}-\frac{\left(3+\sqrt{5}\right)^{5/4}\tan^{-1}\left(\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}+1\right)}{4\,2^{3/4}\sqrt{5}}-\frac{1}{20}\sqrt[4]{6150-2750\sqrt{5}}\tan^{-1}\left(1-\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}\right)+\frac{1}{20}\sqrt[4]{6150-2750\sqrt{5}}\tan^{-1}\left(\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(1 + 3*x^4 + x^8)),x]
```

```
[Out] -x^(-1) + ((3 + Sqrt[5])^(5/4)*ArcTan[1 - (2^(3/4)*x)/(3 - Sqrt[5])^(1/4)])
/(4*2^(3/4)*Sqrt[5]) - ((3 + Sqrt[5])^(5/4)*ArcTan[1 + (2^(3/4)*x)/(3 - Sqr
t[5])^(1/4)])/(4*2^(3/4)*Sqrt[5]) - ((6150 - 2750*Sqrt[5])^(1/4)*ArcTan[1 -
 (2^(3/4)*x)/(3 + Sqrt[5])^(1/4)])/20 + ((6150 - 2750*Sqrt[5])^(1/4)*ArcTan
[1 + (2^(3/4)*x)/(3 + Sqrt[5])^(1/4)])/20 - ((3 + Sqrt[5])^(5/4)*Log[Sqrt[2
*(3 - Sqrt[5])] - 2*(2*(3 - Sqrt[5]))^(1/4)*x + 2*x^2])/(8*2^(3/4)*Sqrt[5])
 + ((3 + Sqrt[5])^(5/4)*Log[Sqrt[2*(3 - Sqrt[5])] + 2*(2*(3 - Sqrt[5]))^(1/
4)*x + 2*x^2])/(8*2^(3/4)*Sqrt[5]) + ((6150 - 2750*Sqrt[5])^(1/4)*Log[Sqrt[
2*(3 + Sqrt[5])] - 2*(2*(3 + Sqrt[5]))^(1/4)*x + 2*x^2])/40 - ((6150 - 2750
*Sqrt[5])^(1/4)*Log[Sqrt[2*(3 + Sqrt[5])] + 2*(2*(3 + Sqrt[5]))^(1/4)*x + 2
*x^2])/40
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1368

```
Int[((d_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_
Symbol] :> Simp[((d*x)^(m + 1)*(a + b*x^n + c*x^(2*n))^(p + 1))/(a*d*(m + 1
)), x] - Dist[1/(a*d^n*(m + 1)), Int[(d*x)^(m + n)*(b*(m + n*(p + 1) + 1) +
 c*(m + 2*n*(p + 1) + 1)*x^n)*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a
, b, c, d, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && L
tQ[m, -1] && IntegerQ[p]
```

Rule 1510

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_)))/((a_) + (b_.)*(x_)^(n_) +
 (c_.)*(x_)^(n2_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 +
(2*c*d - b*e)/(2*q), Int[(f*x)^m/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (
2*c*d - b*e)/(2*q), Int[(f*x)^m/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b
, c, d, e, f, m}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{1}{x^2\left(1+3x^4+x^8\right)}\,dx = -\frac{1}{x}+\int \frac{x^2\left(-3-x^4\right)}{1+3x^4+x^8}\,dx$$

$$= -\frac{1}{x}+\frac{1}{10}\left(-5+3\sqrt{5}\right)\int \frac{x^2}{\frac{3}{2}+\frac{\sqrt{5}}{2}+x^4}\,dx-\frac{1}{10}\left(5+3\sqrt{5}\right)\int \frac{x^2}{\frac{3}{2}-\frac{\sqrt{5}}{2}+x^4}\,dx$$

$$= -\frac{1}{x}-\frac{\left(3-\sqrt{5}\right)\int \frac{\sqrt{3+\sqrt{5}}-\sqrt{2}x^2}{\frac{3}{2}+\frac{\sqrt{5}}{2}+x^4}\,dx}{4\sqrt{10}}+\frac{\left(3-\sqrt{5}\right)\int \frac{\sqrt{3+\sqrt{5}}+\sqrt{2}x^2}{\frac{3}{2}+\frac{\sqrt{5}}{2}+x^4}\,dx}{4\sqrt{10}}+\frac{\left(3+\sqrt{5}\right)\int \frac{\sqrt{3-\sqrt{\cdots}}}{\frac{3}{2}-\cdots}}{4\sqrt{10}}\cdots$$

$$= -\frac{1}{x}-\frac{\left(3+\sqrt{5}\right)^{5/4}\int \frac{\sqrt[4]{2\left(3-\sqrt{5}\right)}+2x}{-\sqrt{\frac{1}{2}\left(3-\sqrt{5}\right)}-\sqrt[4]{2\left(3-\sqrt{5}\right)}x-x^2}\,dx}{8\,2^{3/4}\sqrt{5}}-\frac{\left(3+\sqrt{5}\right)^{5/4}\int \frac{\sqrt[4]{2\left(3-\sqrt{5}\right)}-2x}{-\sqrt{\frac{1}{2}\left(3-\sqrt{5}\right)}+\sqrt[4]{2\left(3-\sqrt{\cdots}\right)}\cdots}}{8\,2^{3/4}\sqrt{5}}\cdots$$

$$= -\frac{1}{x}-\frac{\left(3+\sqrt{5}\right)^{5/4}\log\left(\sqrt{2\left(3-\sqrt{5}\right)}-2\sqrt[4]{2\left(3-\sqrt{5}\right)}x+2x^2\right)}{8\,2^{3/4}\sqrt{5}}+\frac{\left(3+\sqrt{5}\right)^{5/4}\log\left(\sqrt{\cdots}\right.}{\cdots}$$

$$= -\frac{1}{x}+\frac{\sqrt[4]{246+110\sqrt{5}}\,\tan^{-1}\left(1-\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}\right)}{4\sqrt{5}}-\frac{\sqrt[4]{246+110\sqrt{5}}\,\tan^{-1}\left(1+\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}\right)}{4\sqrt{5}}-\frac{1}{20\cdots}\cdots$$

Mathematica [C] time = 0.02, size = 61, normalized size = 0.15

$$-\frac{1}{4}\text{RootSum}\left[\#1^8+3\#1^4+1\&,\frac{\#1^4\log(x-\#1)+3\log(x-\#1)}{2\#1^5+3\#1}\&\right]-\frac{1}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(1 + 3*x^4 + x^8)),x]
```

```
[Out] -x^(-1) - RootSum[1 + 3*#1^4 + #1^8 & , (3*Log[x - #1] + Log[x - #1]*#1^4)/
(3*#1 + 2*#1^5) & ]/4
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2\left(1+3x^4+x^8\right)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x^2*(1 + 3*x^4 + x^8)),x]
```

```
[Out] IntegrateAlgebraic[1/(x^2*(1 + 3*x^4 + x^8)), x]
```

fricas [B] time = 1.50, size = 1017, normalized size = 2.44

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(x^8+3*x^4+1),x, algorithm="fricas")
```

```
[Out] -1/80*(sqrt(10)*(55*sqrt(5)*x - 123*x)*(110*sqrt(5) + 246)^(3/4)*sqrt(55*sq
rt(5) + 123)*arctan(-1/20*sqrt(10)*(161*sqrt(5)*x - 360*x)*(110*sqrt(5) + 2
46)^(3/4)*sqrt(55*sqrt(5) + 123) + 1/40*sqrt(sqrt(10)*(47*sqrt(5)*sqrt(2)*x
 - 105*sqrt(2)*x)*(110*sqrt(5) + 246)^(3/4) + 40*x^2 - 20*sqrt(110*sqrt(5)
+ 246)*(4*sqrt(5) - 9))*(161*sqrt(5) - 360)*(110*sqrt(5) + 246)^(3/4)*sqrt(
55*sqrt(5) + 123) + 1/8*(55*sqrt(5)*sqrt(2) - 123*sqrt(2))*sqrt(110*sqrt(5)
 + 246)*sqrt(55*sqrt(5) + 123)) + sqrt(10)*(55*sqrt(5)*x - 123*x)*(110*sqrt
(5) + 246)^(3/4)*sqrt(55*sqrt(5) + 123)*arctan(-1/20*sqrt(10)*(161*sqrt(5)*
x - 360*x)*(110*sqrt(5) + 246)^(3/4)*sqrt(55*sqrt(5) + 123) + 1/40*sqrt(-sq
rt(10)*(47*sqrt(5)*sqrt(2)*x - 105*sqrt(2)*x)*(110*sqrt(5) + 246)^(3/4) + 4
0*x^2 - 20*sqrt(110*sqrt(5) + 246)*(4*sqrt(5) - 9))*(161*sqrt(5) - 360)*(11
0*sqrt(5) + 246)^(3/4)*sqrt(55*sqrt(5) + 123) - 1/8*(55*sqrt(5)*sqrt(2) - 1
23*sqrt(2))*sqrt(110*sqrt(5) + 246)*sqrt(55*sqrt(5) + 123)) + sqrt(10)*(55*
sqrt(5)*x + 123*x)*sqrt(-55*sqrt(5) + 123)*(-110*sqrt(5) + 246)^(3/4)*arcta
n(1/40*sqrt(sqrt(10)*(47*sqrt(5)*sqrt(2)*x + 105*sqrt(2)*x)*(-110*sqrt(5) +
 246)^(3/4) + 40*x^2 + 20*(4*sqrt(5) + 9)*sqrt(-110*sqrt(5) + 246))*(161*sq
rt(5) + 360)*sqrt(-55*sqrt(5) + 123)*(-110*sqrt(5) + 246)^(3/4) - 1/40*(2*s
qrt(10)*(161*sqrt(5)*x + 360*x)*(-110*sqrt(5) + 246)^(3/4) + 5*(55*sqrt(5)*
sqrt(2) + 123*sqrt(2))*sqrt(-110*sqrt(5) + 246))*sqrt(-55*sqrt(5) + 123)) +
 sqrt(10)*(55*sqrt(5)*x + 123*x)*sqrt(-55*sqrt(5) + 123)*(-110*sqrt(5) + 24
6)^(3/4)*arctan(1/40*sqrt(-sqrt(10)*(47*sqrt(5)*sqrt(2)*x + 105*sqrt(2)*x)*
(-110*sqrt(5) + 246)^(3/4) + 40*x^2 + 20*(4*sqrt(5) + 9)*sqrt(-110*sqrt(5)
+ 246))*(161*sqrt(5) + 360)*sqrt(-55*sqrt(5) + 123)*(-110*sqrt(5) + 246)^(3
/4) - 1/40*(2*sqrt(10)*(161*sqrt(5)*x + 360*x)*(-110*sqrt(5) + 246)^(3/4) -
 5*(55*sqrt(5)*sqrt(2) + 123*sqrt(2))*sqrt(-110*sqrt(5) + 246))*sqrt(-55*sq
rt(5) + 123)) - sqrt(10)*sqrt(2)*x*(110*sqrt(5) + 246)^(1/4)*log(sqrt(10)*(
47*sqrt(5)*sqrt(2)*x - 105*sqrt(2)*x)*(110*sqrt(5) + 246)^(3/4) + 40*x^2 -
20*sqrt(110*sqrt(5) + 246)*(4*sqrt(5) - 9)) + sqrt(10)*sqrt(2)*x*(110*sqrt(
5) + 246)^(1/4)*log(-sqrt(10)*(47*sqrt(5)*sqrt(2)*x - 105*sqrt(2)*x)*(110*s
qrt(5) + 246)^(3/4) + 40*x^2 - 20*sqrt(110*sqrt(5) + 246)*(4*sqrt(5) - 9))
+ sqrt(10)*sqrt(2)*x*(-110*sqrt(5) + 246)^(1/4)*log(sqrt(10)*(47*sqrt(5)*sq
```

```
rt(2)*x + 105*sqrt(2)*x)*(-110*sqrt(5) + 246)^(3/4) + 40*x^2 + 20*(4*sqrt(5
) + 9)*sqrt(-110*sqrt(5) + 246)) - sqrt(10)*sqrt(2)*x*(-110*sqrt(5) + 246)^
(1/4)*log(-sqrt(10)*(47*sqrt(5)*sqrt(2)*x + 105*sqrt(2)*x)*(-110*sqrt(5) +
246)^(3/4) + 40*x^2 + 20*(4*sqrt(5) + 9)*sqrt(-110*sqrt(5) + 246)) + 80)/x
```

giac [A] time = 0.60, size = 244, normalized size = 0.59

$$-\frac{1}{80}\left(\pi+4\arctan\left(x\sqrt{\sqrt{5}+1}-1\right)\right)\sqrt{25\sqrt{5}+55}+\frac{1}{80}\left(\pi+4\arctan\left(-x\sqrt{\sqrt{5}+1}-1\right)\right)\sqrt{25\sqrt{5}+55}+\frac{1}{80}\left(\pi+4\arctan\left(x\sqrt{\sqrt{5}-1}+1\right)\right)\sqrt{25\sqrt{5}-55}-\frac{1}{80}\left(\pi+4\arctan\left(-x\sqrt{\sqrt{5}-1}+1\right)\right)\sqrt{25\sqrt{5}-55}-\frac{1}{40}\sqrt{25\sqrt{5}-55}\log\left(748225\left(x+\sqrt{\sqrt{5}+1}\right)^2+748225x^2\right)+\frac{1}{40}\sqrt{25\sqrt{5}-55}\log\left(748225\left(x-\sqrt{\sqrt{5}+1}\right)^2+748225x^2\right)+\frac{1}{40}\sqrt{25\sqrt{5}+55}\log\left(180625\left(x+\sqrt{\sqrt{5}-1}\right)^2+180625x^2\right)-\frac{1}{40}\sqrt{25\sqrt{5}+55}\log\left(180625\left(x-\sqrt{\sqrt{5}-1}\right)^2+180625x^2\right)-\frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(x^8+3*x^4+1),x, algorithm="giac")
```

```
[Out] -1/80*(pi + 4*arctan(x*sqrt(sqrt(5) + 1) - 1))*sqrt(25*sqrt(5) + 55) + 1/80
*(pi + 4*arctan(-x*sqrt(sqrt(5) + 1) - 1))*sqrt(25*sqrt(5) + 55) + 1/80*(pi
 + 4*arctan(x*sqrt(sqrt(5) - 1) + 1))*sqrt(25*sqrt(5) - 55) - 1/80*(pi + 4*
arctan(-x*sqrt(sqrt(5) - 1) + 1))*sqrt(25*sqrt(5) - 55) - 1/40*sqrt(25*sqrt
(5) - 55)*log(748225*(x + sqrt(sqrt(5) + 1))^2 + 748225*x^2) + 1/40*sqrt(25
*sqrt(5) - 55)*log(748225*(x - sqrt(sqrt(5) + 1))^2 + 748225*x^2) + 1/40*sq
rt(25*sqrt(5) + 55)*log(180625*(x + sqrt(sqrt(5) - 1))^2 + 180625*x^2) - 1/
40*sqrt(25*sqrt(5) + 55)*log(180625*(x - sqrt(sqrt(5) - 1))^2 + 180625*x^2)
 - 1/x
```

maple [C] time = 0.01, size = 52, normalized size = 0.12

$$-\frac{\left(\mathrm{RootOf}\left(\_Z^8+3\_Z^4+1\right)^6+3\,\mathrm{RootOf}\left(\_Z^8+3\_Z^4+1\right)^2\right)\ln\left(-\mathrm{RootOf}\left(\_Z^8+3\_Z^4+1\right)+x\right)}{4\left(2\,\mathrm{RootOf}\left(\_Z^8+3\_Z^4+1\right)^7+3\,\mathrm{RootOf}\left(\_Z^8+3\_Z^4+1\right)^3\right)}-\frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(x^8+3*x^4+1),x)
```

```
[Out] -1/4*sum((_R^6+3*_R^2)/(2*_R^7+3*_R^3)*ln(-_R+x),_R=RootOf(_Z^8+3*_Z^4+1))-
1/x
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{x}-\int\frac{x^6+3\,x^2}{x^8+3\,x^4+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(x^8+3*x^4+1),x, algorithm="maxima")
```

```
[Out] -1/x - integrate((x^6 + 3*x^2)/(x^8 + 3*x^4 + 1), x)
```

mupad [B] time = 1.29, size = 292, normalized size = 0.70

$$-\frac{1}{x}-\frac{2^{3/4}\sqrt{5}\,\mathrm{atan}\left(\frac{2585\,2^{3/4}x\left(-55\sqrt{5}-123\right)^{1/4}}{2\left(3025\sqrt{5}+6765\right)}+\frac{1155\,2^{3/4}\sqrt{5}\,x\left(-55\sqrt{5}-123\right)^{1/4}}{2\left(3025\sqrt{5}+6765\right)}\right)\left(-55\sqrt{5}-123\right)^{1/4}}{20}-\frac{2^{3/4}\sqrt{5}\,\mathrm{atan}\left(\frac{2585\,2^{3/4}x\left(55\sqrt{5}-123\right)^{1/4}}{2\left(3025\sqrt{5}-6765\right)}-\frac{1155\,2^{3/4}\sqrt{5}\,x\left(55\sqrt{5}-123\right)^{1/4}}{2\left(3025\sqrt{5}-6765\right)}\right)\left(55\sqrt{5}-123\right)^{1/4}}{20}-\frac{2^{3/4}\sqrt{5}\,\mathrm{atan}\left(\frac{2^{3/4}x\left(-55\sqrt{5}-123\right)^{1/4}2585i}{2\left(3025\sqrt{5}+6765\right)}+\frac{2^{3/4}\sqrt{5}\,x\left(-55\sqrt{5}-123\right)^{1/4}1155i}{2\left(3025\sqrt{5}+6765\right)}\right)\left(-55\sqrt{5}-123\right)^{1/4}1i}{20}-\frac{2^{3/4}\sqrt{5}\,\mathrm{atan}\left(\frac{2^{3/4}x\left(55\sqrt{5}-123\right)^{1/4}2585i}{2\left(3025\sqrt{5}-6765\right)}-\frac{2^{3/4}\sqrt{5}\,x\left(55\sqrt{5}-123\right)^{1/4}1155i}{2\left(3025\sqrt{5}-6765\right)}\right)\left(55\sqrt{5}-123\right)^{1/4}1i}{20}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(3*x^4 + x^8 + 1)),x)
```

```
[Out] - 1/x - (2^(3/4)*5^(1/2)*atan((2585*2^(3/4)*x*(- 55*5^(1/2) - 123)^(1/4))/(
2*(3025*5^(1/2) + 6765)) + (1155*2^(3/4)*5^(1/2)*x*(- 55*5^(1/2) - 123)^(1/
4))/(2*(3025*5^(1/2) + 6765)))*(- 55*5^(1/2) - 123)^(1/4))/20 - (2^(3/4)*5^
(1/2)*atan((2585*2^(3/4)*x*(55*5^(1/2) - 123)^(1/4))/(2*(3025*5^(1/2) - 676
5)) - (1155*2^(3/4)*5^(1/2)*x*(55*5^(1/2) - 123)^(1/4))/(2*(3025*5^(1/2) -
6765)))*(55*5^(1/2) - 123)^(1/4))/20 - (2^(3/4)*5^(1/2)*atan((2^(3/4)*x*(-
55*5^(1/2) - 123)^(1/4)*2585i)/(2*(3025*5^(1/2) + 6765)) + (2^(3/4)*5^(1/2)
*x*(- 55*5^(1/2) - 123)^(1/4)*1155i)/(2*(3025*5^(1/2) + 6765)))*(- 55*5^(1/
```

```
2) - 123)^(1/4)*1i)/20 - (2^(3/4)*5^(1/2)*atan((2^(3/4)*x*(55*5^(1/2) - 123
)^(1/4)*2585i)/(2*(3025*5^(1/2) - 6765)) - (2^(3/4)*5^(1/2)*x*(55*5^(1/2) -
 123)^(1/4)*1155i)/(2*(3025*5^(1/2) - 6765)))*(55*5^(1/2) - 123)^(1/4)*1i)/
20
```

sympy [A] time = 1.61, size = 32, normalized size = 0.08

$$\mathrm{RootSum}\left(40960000t^8 + 787200t^4 + 1, \left(t \mapsto t\log\left(\frac{19251200t^7}{11} + \frac{369792t^3}{11} + x\right)\right)\right) - \frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(x**8+3*x**4+1),x)
```

```
[Out] RootSum(40960000*_t**8 + 787200*_t**4 + 1, Lambda(_t, _t*log(19251200*_t**7
/11 + 369792*_t**3/11 + x))) - 1/x
```

3.324 $\int \frac{1}{x^4(1+3x^4+x^8)}\,dx$

Optimal. Leaf size=466

$$-\frac{1}{3x^3}+\frac{\sqrt[4]{843+377\sqrt{5}}\,\log\left(2x^2-2\sqrt[4]{2\left(3-\sqrt{5}\right)}\,x+\sqrt{2\left(3-\sqrt{5}\right)}\right)}{4\,2^{3/4}\sqrt{5}}-\frac{\sqrt[4]{843+377\sqrt{5}}\,\log\left(2x^2+2\sqrt[4]{2\left(3-\sqrt{5}\right.}\right.}{4\,2^{3/4}\sqrt{5}}$$

Rubi [A] time = 0.37, antiderivative size = 466, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 8, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {1368, 1422, 211, 1165, 628, 1162, 617, 204}

$$-\frac{1}{3x^3}+\frac{\sqrt[4]{843+377\sqrt{5}}\,\log\left(2x^2-2\sqrt[4]{2\left(3-\sqrt{5}\right)}\,x+\sqrt{2\left(3-\sqrt{5}\right)}\right)}{4\,2^{3/4}\sqrt{5}}-\frac{\sqrt[4]{843+377\sqrt{5}}\,\log\left(2x^2+2\sqrt[4]{2\left(3-\sqrt{5}\right)}\,x+\sqrt{2\left(3-\sqrt{5}\right)}\right)}{4\,2^{3/4}\sqrt{5}}-\frac{\sqrt[4]{843-377\sqrt{5}}\,\log\left(2x^2-2\sqrt[4]{2\left(3+\sqrt{5}\right)}\,x+\sqrt{2\left(3+\sqrt{5}\right)}\right)}{4\,2^{3/4}\sqrt{5}}+\frac{\sqrt[4]{843-377\sqrt{5}}\,\log\left(2x^2+2\sqrt[4]{2\left(3+\sqrt{5}\right)}\,x+\sqrt{2\left(3+\sqrt{5}\right)}\right)}{4\,2^{3/4}\sqrt{5}}+\frac{\sqrt[4]{843+377\sqrt{5}}\,\tan^{-1}\left(1-\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}\right)}{2\,2^{3/4}\sqrt{5}}-\frac{\sqrt[4]{843+377\sqrt{5}}\,\tan^{-1}\left(\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}+1\right)}{2\,2^{3/4}\sqrt{5}}-\frac{\sqrt[4]{843-377\sqrt{5}}\,\tan^{-1}\left(1-\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}\right)}{2\,2^{3/4}\sqrt{5}}+\frac{\sqrt[4]{843-377\sqrt{5}}\,\tan^{-1}\left(\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}+1\right)}{2\,2^{3/4}\sqrt{5}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(1 + 3*x^4 + x^8)),x]
```

```
[Out] -1/(3*x^3) + ((843 + 377*Sqrt[5])^(1/4)*ArcTan[1 - (2^(3/4)*x)/(3 - Sqrt[5]
)^(1/4)])/(2*2^(3/4)*Sqrt[5]) - ((843 + 377*Sqrt[5])^(1/4)*ArcTan[1 + (2^(3
/4)*x)/(3 - Sqrt[5])^(1/4)])/(2*2^(3/4)*Sqrt[5]) - ((843 - 377*Sqrt[5])^(1/
4)*ArcTan[1 - (2^(3/4)*x)/(3 + Sqrt[5])^(1/4)])/(2*2^(3/4)*Sqrt[5]) + ((843
 - 377*Sqrt[5])^(1/4)*ArcTan[1 + (2^(3/4)*x)/(3 + Sqrt[5])^(1/4)])/(2*2^(3/
4)*Sqrt[5]) + ((843 + 377*Sqrt[5])^(1/4)*Log[Sqrt[2*(3 - Sqrt[5])] - 2*(2*(
3 - Sqrt[5]))^(1/4)*x + 2*x^2])/(4*2^(3/4)*Sqrt[5]) - ((843 + 377*Sqrt[5])^
(1/4)*Log[Sqrt[2*(3 - Sqrt[5])] + 2*(2*(3 - Sqrt[5]))^(1/4)*x + 2*x^2])/(4*
2^(3/4)*Sqrt[5]) - ((843 - 377*Sqrt[5])^(1/4)*Log[Sqrt[2*(3 + Sqrt[5])] - 2
*(2*(3 + Sqrt[5]))^(1/4)*x + 2*x^2])/(4*2^(3/4)*Sqrt[5]) + ((843 - 377*Sqrt
[5])^(1/4)*Log[Sqrt[2*(3 + Sqrt[5])] + 2*(2*(3 + Sqrt[5]))^(1/4)*x + 2*x^2]
)/(4*2^(3/4)*Sqrt[5])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1368

```
Int[((d_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_
Symbol] :> Simp[((d*x)^(m + 1)*(a + b*x^n + c*x^(2*n))^(p + 1))/(a*d*(m + 1
)), x] - Dist[1/(a*d^n*(m + 1)), Int[(d*x)^(m + n)*(b*(m + n*(p + 1) + 1) +
 c*(m + 2*n*(p + 1) + 1)*x^n)*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a
, b, c, d, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && L
tQ[m, -1] && IntegerQ[p]
```

Rule 1422

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q),
Int[1/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(
b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (PosQ[b^2 - 4*a
*c] ||  !IGtQ[n/2, 0])
```

Rubi steps

$$\int \frac{1}{x^4\left(1+3x^4+x^8\right)}\,dx = -\frac{1}{3x^3}+\frac{1}{3}\int \frac{-9-3x^4}{1+3x^4+x^8}\,dx$$

$$= -\frac{1}{3x^3}+\frac{1}{10}\left(-5+3\sqrt{5}\right)\int \frac{1}{\frac{3}{2}+\frac{\sqrt{5}}{2}+x^4}\,dx-\frac{1}{10}\left(5+3\sqrt{5}\right)\int \frac{1}{\frac{3}{2}-\frac{\sqrt{5}}{2}+x^4}\,dx$$

$$= -\frac{1}{3x^3}-\frac{\left(3+\sqrt{5}\right)^{3/2}\int \frac{\sqrt{3-\sqrt{5}}-\sqrt{2}x^2}{\frac{3}{2}-\frac{\sqrt{5}}{2}+x^4}\,dx}{8\sqrt{5}}-\frac{\left(3+\sqrt{5}\right)^{3/2}\int \frac{\sqrt{3-\sqrt{5}}+\sqrt{2}x^2}{\frac{3}{2}-\frac{\sqrt{5}}{2}+x^4}\,dx}{8\sqrt{5}}+\frac{\left(-5+3\right.}{}$$

$$= -\frac{1}{3x^3}-\frac{\sqrt[4]{843-377\sqrt{5}}\int \frac{\sqrt[4]{2\left(3+\sqrt{5}\right)}+2x}{-\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}-\sqrt[4]{2\left(3+\sqrt{5}\right)}x-x^2}\,dx}{4\,2^{3/4}\sqrt{5}}-\frac{\sqrt[4]{843-377\sqrt{5}}\int \frac{}{-\sqrt{\frac{1}{2}(3}}}{4\,2^{3/4}}$$

$$= -\frac{1}{3x^3}+\frac{\left(3+\sqrt{5}\right)^{7/4}\log\left(\sqrt{2\left(3-\sqrt{5}\right)}-2\sqrt[4]{2\left(3-\sqrt{5}\right)}x+2x^2\right)}{16\sqrt[4]{2}\sqrt{5}}-\frac{\left(3+\sqrt{5}\right)^{7/4}\mathrm{lo}}{}$$

$$= -\frac{1}{3x^3}+\frac{\left(3+\sqrt{5}\right)^{7/4}\tan^{-1}\left(1-\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}\right)}{8\sqrt[4]{2}\sqrt{5}}-\frac{\left(3+\sqrt{5}\right)^{7/4}\tan^{-1}\left(1+\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}\right)}{8\sqrt[4]{2}\sqrt{5}}-\frac{\sqrt[4]{84}}{}$$

Mathematica [C] time = 0.02, size = 65, normalized size = 0.14

$$-\frac{1}{4}\text{RootSum}\left[\#1^8+3\#1^4+1\&,\frac{\#1^4\log(x-\#1)+3\log(x-\#1)}{2\#1^7+3\#1^3}\&\right]-\frac{1}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(1 + 3*x^4 + x^8)),x]
```

```
[Out] -1/3*1/x^3 - RootSum[1 + 3*#1^4 + #1^8 & , (3*Log[x - #1] + Log[x - #1]*#1^
4)/(3*#1^3 + 2*#1^7) & ]/4
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^4(1+3x^4+x^8)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x^4*(1 + 3*x^4 + x^8)),x]
```

```
[Out] IntegrateAlgebraic[1/(x^4*(1 + 3*x^4 + x^8)), x]
```

fricas [B] time = 1.79, size = 1057, normalized size = 2.27

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(x^8+3*x^4+1),x, algorithm="fricas")
```

```
[Out] -1/240*(3*sqrt(10)*sqrt(2)*x^3*(754*sqrt(5) + 1686)^(1/4)*log(20*x^2 + sqrt
(10)*(7*sqrt(5)*sqrt(2)*x - 15*sqrt(2)*x)*(754*sqrt(5) + 1686)^(1/4) - 5*sq
rt(754*sqrt(5) + 1686)*(21*sqrt(5) - 47)) - 3*sqrt(10)*sqrt(2)*x^3*(754*sqr
t(5) + 1686)^(1/4)*log(20*x^2 - sqrt(10)*(7*sqrt(5)*sqrt(2)*x - 15*sqrt(2)*
x)*(754*sqrt(5) + 1686)^(1/4) - 5*sqrt(754*sqrt(5) + 1686)*(21*sqrt(5) - 47
)) - 3*sqrt(10)*sqrt(2)*x^3*(-754*sqrt(5) + 1686)^(1/4)*log(20*x^2 + sqrt(1
0)*(7*sqrt(5)*sqrt(2)*x + 15*sqrt(2)*x)*(-754*sqrt(5) + 1686)^(1/4) + 5*(21
*sqrt(5) + 47)*sqrt(-754*sqrt(5) + 1686)) + 3*sqrt(10)*sqrt(2)*x^3*(-754*sq
rt(5) + 1686)^(1/4)*log(20*x^2 - sqrt(10)*(7*sqrt(5)*sqrt(2)*x + 15*sqrt(2)
*x)*(-754*sqrt(5) + 1686)^(1/4) + 5*(21*sqrt(5) + 47)*sqrt(-754*sqrt(5) + 1
686)) - 3*sqrt(10)*(377*sqrt(5)*x^3 - 843*x^3)*(754*sqrt(5) + 1686)^(3/4)*s
qrt(377*sqrt(5) + 843)*arctan(1/80*sqrt(10)*sqrt(20*x^2 + sqrt(10)*(7*sqrt(
5)*sqrt(2)*x - 15*sqrt(2)*x)*(754*sqrt(5) + 1686)^(1/4) - 5*sqrt(754*sqrt(5
) + 1686)*(21*sqrt(5) - 47))*(23184*sqrt(5) - 51841)*(754*sqrt(5) + 1686)^(
5/4)*sqrt(377*sqrt(5) + 843) + 1/40*sqrt(10)*(51841*sqrt(5)*x - 115920*x)*(
754*sqrt(5) + 1686)^(5/4)*sqrt(377*sqrt(5) + 843) - 1/8*(377*sqrt(5)*sqrt(2
) - 843*sqrt(2))*sqrt(754*sqrt(5) + 1686)*sqrt(377*sqrt(5) + 843)) - 3*sqrt
(10)*(377*sqrt(5)*x^3 - 843*x^3)*(754*sqrt(5) + 1686)^(3/4)*sqrt(377*sqrt(5
) + 843)*arctan(1/80*sqrt(10)*sqrt(20*x^2 - sqrt(10)*(7*sqrt(5)*sqrt(2)*x -
 15*sqrt(2)*x)*(754*sqrt(5) + 1686)^(1/4) - 5*sqrt(754*sqrt(5) + 1686)*(21*
sqrt(5) - 47))*(23184*sqrt(5) - 51841)*(754*sqrt(5) + 1686)^(5/4)*sqrt(377*
sqrt(5) + 843) + 1/40*sqrt(10)*(51841*sqrt(5)*x - 115920*x)*(754*sqrt(5) +
1686)^(5/4)*sqrt(377*sqrt(5) + 843) + 1/8*(377*sqrt(5)*sqrt(2) - 843*sqrt(2
))*sqrt(754*sqrt(5) + 1686)*sqrt(377*sqrt(5) + 843)) + 3*sqrt(10)*(377*sqrt
(5)*x^3 + 843*x^3)*sqrt(-377*sqrt(5) + 843)*(-754*sqrt(5) + 1686)^(3/4)*arc
tan(1/80*sqrt(10)*sqrt(20*x^2 + sqrt(10)*(7*sqrt(5)*sqrt(2)*x + 15*sqrt(2)*
x)*(-754*sqrt(5) + 1686)^(1/4) + 5*(21*sqrt(5) + 47)*sqrt(-754*sqrt(5) + 16
86))*(23184*sqrt(5) + 51841)*sqrt(-377*sqrt(5) + 843)*(-754*sqrt(5) + 1686)
^(5/4) - 1/40*(sqrt(10)*(51841*sqrt(5)*x + 115920*x)*(-754*sqrt(5) + 1686)^
(5/4) + 5*(377*sqrt(5)*sqrt(2) + 843*sqrt(2))*sqrt(-754*sqrt(5) + 1686))*sq
rt(-377*sqrt(5) + 843)) + 3*sqrt(10)*(377*sqrt(5)*x^3 + 843*x^3)*sqrt(-377*
```

```
sqrt(5) + 843)*(-754*sqrt(5) + 1686)^(3/4)*arctan(1/80*sqrt(10)*sqrt(20*x^2
 - sqrt(10)*(7*sqrt(5)*sqrt(2)*x + 15*sqrt(2)*x)*(-754*sqrt(5) + 1686)^(1/4
) + 5*(21*sqrt(5) + 47)*sqrt(-754*sqrt(5) + 1686))*(23184*sqrt(5) + 51841)*
sqrt(-377*sqrt(5) + 843)*(-754*sqrt(5) + 1686)^(5/4) - 1/40*(sqrt(10)*(5184
1*sqrt(5)*x + 115920*x)*(-754*sqrt(5) + 1686)^(5/4) - 5*(377*sqrt(5)*sqrt(2
) + 843*sqrt(2))*sqrt(-754*sqrt(5) + 1686))*sqrt(-377*sqrt(5) + 843)) + 80)
/x^3
```

giac [A] time = 0.70, size = 244, normalized size = 0.52

$$-\frac{1}{80}\left(\pi+4\arctan\left(x\sqrt{\sqrt{5}+1}+1\right)\right)\sqrt{65\sqrt{5}+145}+\frac{1}{80}\left(\pi+4\arctan\left(-x\sqrt{\sqrt{5}+1}+1\right)\right)\sqrt{65\sqrt{5}+145}+\frac{1}{80}\left(\pi+4\arctan\left(x\sqrt{\sqrt{5}-1}-1\right)\right)\sqrt{65\sqrt{5}-145}-\frac{1}{80}\left(\pi+4\arctan\left(-x\sqrt{\sqrt{5}-1}-1\right)\right)\sqrt{65\sqrt{5}-145}+\frac{1}{40}\sqrt{65\sqrt{5}-145}\log\left(93122500\left(x+\sqrt{\sqrt{5}+1}\right)^{2}+93122500x^{2}\right)-\frac{1}{40}\sqrt{65\sqrt{5}-145}\log\left(93122500\left(x-\sqrt{\sqrt{5}+1}\right)^{2}+93122500x^{2}\right)-\frac{1}{40}\sqrt{65\sqrt{5}+145}\log\left(53728900\left(x+\sqrt{\sqrt{5}-1}\right)^{2}+53728900x^{2}\right)+\frac{1}{40}\sqrt{65\sqrt{5}+145}\log\left(53728900\left(x-\sqrt{\sqrt{5}-1}\right)^{2}+53728900x^{2}\right)-\frac{1}{3x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(x^8+3*x^4+1),x, algorithm="giac")
```

```
[Out] -1/80*(pi + 4*arctan(x*sqrt(sqrt(5) + 1) + 1))*sqrt(65*sqrt(5) + 145) + 1/8
0*(pi + 4*arctan(-x*sqrt(sqrt(5) + 1) + 1))*sqrt(65*sqrt(5) + 145) + 1/80*(
pi + 4*arctan(x*sqrt(sqrt(5) - 1) - 1))*sqrt(65*sqrt(5) - 145) - 1/80*(pi +
 4*arctan(-x*sqrt(sqrt(5) - 1) - 1))*sqrt(65*sqrt(5) - 145) + 1/40*sqrt(65*
sqrt(5) - 145)*log(93122500*(x + sqrt(sqrt(5) + 1))^2 + 93122500*x^2) - 1/4
0*sqrt(65*sqrt(5) - 145)*log(93122500*(x - sqrt(sqrt(5) + 1))^2 + 93122500*
x^2) - 1/40*sqrt(65*sqrt(5) + 145)*log(53728900*(x + sqrt(sqrt(5) - 1))^2 +
 53728900*x^2) + 1/40*sqrt(65*sqrt(5) + 145)*log(53728900*(x - sqrt(sqrt(5)
 - 1))^2 + 53728900*x^2) - 1/3/x^3
```

maple [C] time = 0.01, size = 50, normalized size = 0.11

$$\frac{\left(-\mathrm{RootOf}\left(\_Z^{8}+3\_Z^{4}+1\right)^{4}-3\right)\ln\left(-\mathrm{RootOf}\left(\_Z^{8}+3\_Z^{4}+1\right)+x\right)}{8\,\mathrm{RootOf}\left(\_Z^{8}+3\_Z^{4}+1\right)^{7}+12\,\mathrm{RootOf}\left(\_Z^{8}+3\_Z^{4}+1\right)^{3}}-\frac{1}{3x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(x^8+3*x^4+1),x)
```

```
[Out] 1/4*sum((-_R^4-3)/(2*_R^7+3*_R^3)*ln(-_R+x),_R=RootOf(_Z^8+3*_Z^4+1))-1/3/x
^3
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{3\,x^{3}}-\int \frac{x^{4}+3}{x^{8}+3\,x^{4}+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(x^8+3*x^4+1),x, algorithm="maxima")
```

```
[Out] -1/3/x^3 - integrate((x^4 + 3)/(x^8 + 3*x^4 + 1), x)
```

mupad [B] time = 0.18, size = 492, normalized size = 1.06

$$\frac{2^{3/4}\sqrt{5}\,\mathrm{atan}\left(\frac{46371\,2^{3/4}x\left(377\sqrt{5}-843\right)^{1/4}}{2\left(3393\sqrt{2}\sqrt{377\sqrt{5}-843}-1508\sqrt{2}\sqrt{5}\sqrt{377\sqrt{5}-843}\right)}-\frac{20735\,2^{3/4}\sqrt{5}\,x\left(377\sqrt{5}-843\right)^{1/4}}{2\left(3393\sqrt{2}\sqrt{377\sqrt{5}-843}-1508\sqrt{2}\sqrt{5}\sqrt{377\sqrt{5}-843}\right)}\right)\left(377\sqrt{5}-843\right)^{1/4}}{20}-\frac{2^{3/4}\sqrt{5}\,\mathrm{atan}\left(\frac{46371\,2^{3/4}x\left(-377\sqrt{5}-843\right)^{1/4}}{2\left(3393\sqrt{2}\sqrt{-377\sqrt{5}-843}+1508\sqrt{2}\sqrt{5}\sqrt{-377\sqrt{5}-843}\right)}+\frac{20735\,2^{3/4}\sqrt{5}\,x\left(-377\sqrt{5}-843\right)^{1/4}}{2\left(3393\sqrt{2}\sqrt{-377\sqrt{5}-843}+1508\sqrt{2}\sqrt{5}\sqrt{-377\sqrt{5}-843}\right)}\right)\left(-377\sqrt{5}-843\right)^{1/4}}{20}-\frac{1}{3x^{3}}+\frac{2^{3/4}\sqrt{5}\,\mathrm{atan}\left(\frac{2^{3/4}x\left(-377\sqrt{5}-843\right)^{1/4}46371\mathrm{i}}{2\left(3393\sqrt{2}\sqrt{-377\sqrt{5}-843}+1508\sqrt{2}\sqrt{5}\sqrt{-377\sqrt{5}-843}\right)}+\frac{2^{3/4}\sqrt{5}\,x\left(-377\sqrt{5}-843\right)^{1/4}20735\mathrm{i}}{2\left(3393\sqrt{2}\sqrt{-377\sqrt{5}-843}+1508\sqrt{2}\sqrt{5}\sqrt{-377\sqrt{5}-843}\right)}\right)\left(-377\sqrt{5}-843\right)^{1/4}1\mathrm{i}}{20}-\frac{2^{3/4}\sqrt{5}\,\mathrm{atan}\left(\frac{2^{3/4}x\left(377\sqrt{5}-843\right)^{1/4}46371\mathrm{i}}{2\left(3393\sqrt{2}\sqrt{377\sqrt{5}-843}-1508\sqrt{2}\sqrt{5}\sqrt{377\sqrt{5}-843}\right)}-\frac{2^{3/4}\sqrt{5}\,x\left(377\sqrt{5}-843\right)^{1/4}20735\mathrm{i}}{2\left(3393\sqrt{2}\sqrt{377\sqrt{5}-843}-1508\sqrt{2}\sqrt{5}\sqrt{377\sqrt{5}-843}\right)}\right)\left(377\sqrt{5}-843\right)^{1/4}1\mathrm{i}}{20}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(3*x^4 + x^8 + 1)),x)
```

```
[Out] (2^(3/4)*5^(1/2)*atan((46371*2^(3/4)*x*(377*5^(1/2) - 843)^(1/4))/(2*(3393*
2^(1/2)*(377*5^(1/2) - 843)^(1/2) - 1508*2^(1/2)*5^(1/2)*(377*5^(1/2) - 843
)^(1/2))) - (20735*2^(3/4)*5^(1/2)*x*(377*5^(1/2) - 843)^(1/4))/(2*(3393*2^
(1/2)*(377*5^(1/2) - 843)^(1/2) - 1508*2^(1/2)*5^(1/2)*(377*5^(1/2) - 843)^
(1/2))))*(377*5^(1/2) - 843)^(1/4))/20 - (2^(3/4)*5^(1/2)*atan((46371*2^(3/
```

```
4)*x*(- 377*5^(1/2) - 843)^(1/4))/(2*(3393*2^(1/2)*(- 377*5^(1/2) - 843)^(1
/2) + 1508*2^(1/2)*5^(1/2)*(- 377*5^(1/2) - 843)^(1/2))) + (20735*2^(3/4)*5
^(1/2)*x*(- 377*5^(1/2) - 843)^(1/4))/(2*(3393*2^(1/2)*(- 377*5^(1/2) - 843
)^(1/2) + 1508*2^(1/2)*5^(1/2)*(- 377*5^(1/2) - 843)^(1/2))))*(- 377*5^(1/2
) - 843)^(1/4))/20 - 1/(3*x^3) + (2^(3/4)*5^(1/2)*atan((2^(3/4)*x*(- 377*5^
(1/2) - 843)^(1/4)*46371i)/(2*(3393*2^(1/2)*(- 377*5^(1/2) - 843)^(1/2) + 1
508*2^(1/2)*5^(1/2)*(- 377*5^(1/2) - 843)^(1/2))) + (2^(3/4)*5^(1/2)*x*(- 3
77*5^(1/2) - 843)^(1/4)*20735i)/(2*(3393*2^(1/2)*(- 377*5^(1/2) - 843)^(1/2
) + 1508*2^(1/2)*5^(1/2)*(- 377*5^(1/2) - 843)^(1/2))))*(- 377*5^(1/2) - 84
3)^(1/4)*1i)/20 - (2^(3/4)*5^(1/2)*atan((2^(3/4)*x*(377*5^(1/2) - 843)^(1/4
)*46371i)/(2*(3393*2^(1/2)*(377*5^(1/2) - 843)^(1/2) - 1508*2^(1/2)*5^(1/2)
*(377*5^(1/2) - 843)^(1/2))) - (2^(3/4)*5^(1/2)*x*(377*5^(1/2) - 843)^(1/4)
*20735i)/(2*(3393*2^(1/2)*(377*5^(1/2) - 843)^(1/2) - 1508*2^(1/2)*5^(1/2)*
(377*5^(1/2) - 843)^(1/2))))*(377*5^(1/2) - 843)^(1/4)*1i)/20
```

sympy [A] time = 1.62, size = 34, normalized size = 0.07

$$\operatorname{RootSum}\left(40960000t^{8}+5395200t^{4}+1,\left(t\mapsto t\log\left(\frac{179200t^{5}}{377}+\frac{23112t}{377}+x\right)\right)\right)-\frac{1}{3x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(x**8+3*x**4+1),x)
```

```
[Out] RootSum(40960000*_t**8 + 5395200*_t**4 + 1, Lambda(_t, _t*log(179200*_t**5/
377 + 23112*_t/377 + x))) - 1/(3*x**3)
```

3.325 $\int \frac{x^{11}}{1-3x^4+x^8} dx$

Optimal. Leaf size=62

$$\frac{x^4}{4}+\frac{1}{40}\left(15-7\sqrt{5}\right)\log\left(-2x^4-\sqrt{5}+3\right)+\frac{1}{40}\left(15+7\sqrt{5}\right)\log\left(-2x^4+\sqrt{5}+3\right)$$

Rubi [A] time = 0.05, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1357, 703, 632, 31}

$$\frac{x^4}{4}+\frac{1}{40}\left(15-7\sqrt{5}\right)\log\left(-2x^4-\sqrt{5}+3\right)+\frac{1}{40}\left(15+7\sqrt{5}\right)\log\left(-2x^4+\sqrt{5}+3\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^11/(1 - 3*x^4 + x^8),x]
```

```
[Out] x^4/4 + ((15 - 7*Sqrt[5])*Log[3 - Sqrt[5] - 2*x^4])/40 + ((15 + 7*Sqrt[5])*
Log[3 + Sqrt[5] - 2*x^4])/40
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 632

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rule 703

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Simp[(e*(d + e*x)^(m - 1))/(c*(m - 1)), x] + Dist[1/c, Int[((d + e*x)^
(m - 2)*Simp[c*d^2 - a*e^2 + e*(2*c*d - b*e)*x, x])/(a + b*x + c*x^2), x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && GtQ[m, 1]
```

Rule 1357

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^{11}}{1-3x^4+x^8}\,dx = \frac{1}{4}\operatorname{Subst}\left(\int \frac{x^2}{1-3x+x^2}\,dx, x, x^4\right)$$
$$= \frac{x^4}{4} + \frac{1}{4}\operatorname{Subst}\left(\int \frac{-1+3x}{1-3x+x^2}\,dx, x, x^4\right)$$
$$= \frac{x^4}{4} + \frac{1}{40}\left(15-7\sqrt{5}\right)\operatorname{Subst}\left(\int \frac{1}{-\frac{3}{2}+\frac{\sqrt{5}}{2}+x}\,dx, x, x^4\right) + \frac{1}{40}\left(15+7\sqrt{5}\right)\operatorname{Subst}\left(\int \frac{1}{-\frac{3}{2}-\frac{\sqrt{5}}{2}+x}\,dx, x, x^4\right)$$
$$= \frac{x^4}{4} + \frac{1}{40}\left(15-7\sqrt{5}\right)\log\left(3-\sqrt{5}-2x^4\right) + \frac{1}{40}\left(15+7\sqrt{5}\right)\log\left(3+\sqrt{5}-2x^4\right)$$

Mathematica [A] time = 0.03, size = 56, normalized size = 0.90

$$\frac{1}{40}\left(10x^4+\left(15+7\sqrt{5}\right)\log\left(-2x^4+\sqrt{5}+3\right)+\left(15-7\sqrt{5}\right)\log\left(2x^4+\sqrt{5}-3\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^11/(1 - 3*x^4 + x^8),x]
```

```
[Out] (10*x^4 + (15 + 7*Sqrt[5])*Log[3 + Sqrt[5] - 2*x^4] + (15 - 7*Sqrt[5])*Log[
-3 + Sqrt[5] + 2*x^4])/40
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{11}}{1-3x^4+x^8}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^11/(1 - 3*x^4 + x^8),x]
```

```
[Out] IntegrateAlgebraic[x^11/(1 - 3*x^4 + x^8), x]
```

fricas [A] time = 0.68, size = 62, normalized size = 1.00

$$\frac{1}{4}x^4 + \frac{7}{40}\sqrt{5}\log\left(\frac{2x^8-6x^4-\sqrt{5}\left(2x^4-3\right)+7}{x^8-3x^4+1}\right) + \frac{3}{8}\log\left(x^8-3x^4+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(x^8-3*x^4+1),x, algorithm="fricas")
```

```
[Out] 1/4*x^4 + 7/40*sqrt(5)*log((2*x^8 - 6*x^4 - sqrt(5)*(2*x^4 - 3) + 7)/(x^8 -
 3*x^4 + 1)) + 3/8*log(x^8 - 3*x^4 + 1)
```

giac [A] time = 0.42, size = 53, normalized size = 0.85

$$\frac{1}{4}x^4 + \frac{7}{40}\sqrt{5}\log\left(\frac{\left|2x^4-\sqrt{5}-3\right|}{\left|2x^4+\sqrt{5}-3\right|}\right) + \frac{3}{8}\log\left(\left|x^8-3x^4+1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(x^8-3*x^4+1),x, algorithm="giac")
```

```
[Out] 1/4*x^4 + 7/40*sqrt(5)*log(abs(2*x^4 - sqrt(5) - 3)/abs(2*x^4 + sqrt(5) - 3
)) + 3/8*log(abs(x^8 - 3*x^4 + 1))
```

maple [A] time = 0.00, size = 38, normalized size = 0.61

$$\frac{x^4}{4} - \frac{7\sqrt{5}\ \operatorname{arctanh}\left(\frac{(2x^4-3)\sqrt{5}}{5}\right)}{20} + \frac{3\ln\left(x^8-3x^4+1\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11/(x^8-3*x^4+1),x)
```

```
[Out] 1/4*x^4+3/8*ln(x^8-3*x^4+1)-7/20*5^(1/2)*arctanh(1/5*(2*x^4-3)*5^(1/2))
```

maxima [A] time = 1.39, size = 50, normalized size = 0.81

$$\frac{1}{4}x^4 + \frac{7}{40}\sqrt{5}\log\left(\frac{2x^4-\sqrt{5}-3}{2x^4+\sqrt{5}-3}\right) + \frac{3}{8}\log\left(x^8-3x^4+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(x^8-3*x^4+1),x, algorithm="maxima")
```

```
[Out] 1/4*x^4 + 7/40*sqrt(5)*log((2*x^4 - sqrt(5) - 3)/(2*x^4 + sqrt(5) - 3)) + 3
/8*log(x^8 - 3*x^4 + 1)
```

mupad [B] time = 0.12, size = 64, normalized size = 1.03

$$\frac{3\ln\left(x^4-\frac{\sqrt{5}}{2}-\frac{3}{2}\right)}{8} + \frac{3\ln\left(x^4+\frac{\sqrt{5}}{2}-\frac{3}{2}\right)}{8} + \frac{7\sqrt{5}\ln\left(x^4-\frac{\sqrt{5}}{2}-\frac{3}{2}\right)}{40} - \frac{7\sqrt{5}\ln\left(x^4+\frac{\sqrt{5}}{2}-\frac{3}{2}\right)}{40} + \frac{x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11/(x^8 - 3*x^4 + 1),x)
```

```
[Out] (3*log(x^4 - 5^(1/2)/2 - 3/2))/8 + (3*log(5^(1/2)/2 + x^4 - 3/2))/8 + (7*5^
(1/2)*log(x^4 - 5^(1/2)/2 - 3/2))/40 - (7*5^(1/2)*log(5^(1/2)/2 + x^4 - 3/2
))/40 + x^4/4
```

sympy [A] time = 0.14, size = 58, normalized size = 0.94

$$\frac{x^4}{4} + \left(\frac{3}{8}+\frac{7\sqrt{5}}{40}\right)\log\left(x^4-\frac{3}{2}-\frac{\sqrt{5}}{2}\right) + \left(\frac{3}{8}-\frac{7\sqrt{5}}{40}\right)\log\left(x^4-\frac{3}{2}+\frac{\sqrt{5}}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**11/(x**8-3*x**4+1),x)
```

```
[Out] x**4/4 + (3/8 + 7*sqrt(5)/40)*log(x**4 - 3/2 - sqrt(5)/2) + (3/8 - 7*sqrt(5
)/40)*log(x**4 - 3/2 + sqrt(5)/2)
```

3.326 $\int \frac{x^9}{1-3x^4+x^8}\,dx$

Optimal. Leaf size=90

$$\frac{x^2}{2}-\frac{1}{2}\sqrt{\frac{1}{5}\left(9+4\sqrt{5}\right)}\tanh^{-1}\left(\sqrt{\frac{2}{3+\sqrt{5}}}\,x^2\right)+\frac{1}{2}\sqrt{\frac{1}{5}\left(9-4\sqrt{5}\right)}\tanh^{-1}\left(\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}\,x^2\right)$$

Rubi [A] time = 0.07, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1359, 1122, 1166, 207}

$$\frac{x^2}{2}-\frac{1}{2}\sqrt{\frac{1}{5}\left(9+4\sqrt{5}\right)}\tanh^{-1}\left(\sqrt{\frac{2}{3+\sqrt{5}}}\,x^2\right)+\frac{1}{2}\sqrt{\frac{1}{5}\left(9-4\sqrt{5}\right)}\tanh^{-1}\left(\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}\,x^2\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^9/(1 - 3*x^4 + x^8),x]
```

```
[Out] x^2/2 - (Sqrt[(9 + 4*Sqrt[5])/5]*ArcTanh[Sqrt[2/(3 + Sqrt[5])]*x^2])/2 + (S
qrt[(9 - 4*Sqrt[5])/5]*ArcTanh[Sqrt[(3 + Sqrt[5])/2]*x^2])/2
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 1122

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Simp[(d^3*(d*x)^(m - 3)*(a + b*x^2 + c*x^4)^(p + 1))/(c*(m + 4*p + 1)),
 x] - Dist[d^4/(c*(m + 4*p + 1)), Int[(d*x)^(m - 4)*Simp[a*(m - 3) + b*(m +
 2*p - 1)*x^2, x]*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x
] && NeQ[b^2 - 4*a*c, 0] && GtQ[m, 3] && NeQ[m + 4*p + 1, 0] && IntegerQ[2*
p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1359

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*
x^(n/k) + c*x^((2*n)/k))^p, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, c, p
}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\int \frac{x^9}{1-3x^4+x^8}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{x^4}{1-3x^2+x^4}\,dx,x,x^2\right)$$
$$= \frac{x^2}{2}-\frac{1}{2}\text{Subst}\left(\int \frac{1-3x^2}{1-3x^2+x^4}\,dx,x,x^2\right)$$
$$= \frac{x^2}{2}-\frac{1}{20}\left(-15+7\sqrt{5}\right)\text{Subst}\left(\int \frac{1}{-\frac{3}{2}+\frac{\sqrt{5}}{2}+x^2}\,dx,x,x^2\right)+\frac{1}{20}\left(15+7\sqrt{5}\right)\text{Subst}\left(\int \frac{}{-}\right.$$
$$= \frac{x^2}{2}-\frac{1}{2}\sqrt{\frac{1}{5}\left(9+4\sqrt{5}\right)}\tanh^{-1}\left(\sqrt{\frac{2}{3+\sqrt{5}}}\,x^2\right)+\frac{1}{20}\sqrt{180-80\sqrt{5}}\tanh^{-1}\left(\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right.}\right.$$

Mathematica [A] time = 0.05, size = 103, normalized size = 1.14

$$\frac{1}{20}\left(10x^2+\left(2\sqrt{5}-5\right)\log\left(-2x^2+\sqrt{5}-1\right)+\left(5+2\sqrt{5}\right)\log\left(-2x^2+\sqrt{5}+1\right)+\left(5-2\sqrt{5}\right)\log\left(2x^2+\sqrt{5}-1\right)-\left(5+2\sqrt{5}\right)\log\left(2x^2+\sqrt{5}+1\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^9/(1 - 3*x^4 + x^8),x]
```

```
[Out] (10*x^2 + (-5 + 2*Sqrt[5])*Log[-1 + Sqrt[5] - 2*x^2] + (5 + 2*Sqrt[5])*Log[
1 + Sqrt[5] - 2*x^2] + (5 - 2*Sqrt[5])*Log[-1 + Sqrt[5] + 2*x^2] - (5 + 2*S
qrt[5])*Log[1 + Sqrt[5] + 2*x^2])/20
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^9}{1-3x^4+x^8}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^9/(1 - 3*x^4 + x^8),x]
```

```
[Out] IntegrateAlgebraic[x^9/(1 - 3*x^4 + x^8), x]
```

fricas [B] time = 1.16, size = 114, normalized size = 1.27

$$\frac{1}{2}x^2+\frac{1}{10}\sqrt{5}\log\left(\frac{2x^4+2x^2-\sqrt{5}\left(2x^2+1\right)+3}{x^4+x^2-1}\right)+\frac{1}{10}\sqrt{5}\log\left(\frac{2x^4-2x^2-\sqrt{5}\left(2x^2-1\right)+3}{x^4-x^2-1}\right)-\frac{1}{4}\log\left(x^4+x^2-1\right)+\frac{1}{4}\log\left(x^4-x^2-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(x^8-3*x^4+1),x, algorithm="fricas")
```

```
[Out] 1/2*x^2 + 1/10*sqrt(5)*log((2*x^4 + 2*x^2 - sqrt(5)*(2*x^2 + 1) + 3)/(x^4 +
 x^2 - 1)) + 1/10*sqrt(5)*log((2*x^4 - 2*x^2 - sqrt(5)*(2*x^2 - 1) + 3)/(x^
4 - x^2 - 1)) - 1/4*log(x^4 + x^2 - 1) + 1/4*log(x^4 - x^2 - 1)
```

giac [A] time = 0.46, size = 97, normalized size = 1.08

$$\frac{1}{2}x^2+\frac{1}{10}\sqrt{5}\log\left(\frac{\left|2x^2-\sqrt{5}+1\right|}{2x^2+\sqrt{5}+1}\right)+\frac{1}{10}\sqrt{5}\log\left(\frac{\left|2x^2-\sqrt{5}-1\right|}{\left|2x^2+\sqrt{5}-1\right|}\right)-\frac{1}{4}\log\left(\left|x^4+x^2-1\right|\right)+\frac{1}{4}\log\left(\left|x^4-x^2-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(x^8-3*x^4+1),x, algorithm="giac")
```

```
[Out] 1/2*x^2 + 1/10*sqrt(5)*log(abs(2*x^2 - sqrt(5) + 1)/(2*x^2 + sqrt(5) + 1))
+ 1/10*sqrt(5)*log(abs(2*x^2 - sqrt(5) - 1)/abs(2*x^2 + sqrt(5) - 1)) - 1/4
*log(abs(x^4 + x^2 - 1)) + 1/4*log(abs(x^4 - x^2 - 1))
```

maple [A] time = 0.01, size = 67, normalized size = 0.74

$$\frac{x^2}{2}-\frac{\sqrt{5}\ \operatorname{arctanh}\left(\frac{(2x^2-1)\sqrt{5}}{5}\right)}{5}-\frac{\sqrt{5}\ \operatorname{arctanh}\left(\frac{(2x^2+1)\sqrt{5}}{5}\right)}{5}+\frac{\ln\left(x^4-x^2-1\right)}{4}-\frac{\ln\left(x^4+x^2-1\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9/(x^8-3*x^4+1),x)
```

```
[Out] 1/2*x^2-1/4*ln(x^4+x^2-1)-1/5*5^(1/2)*arctanh(1/5*(2*x^2+1)*5^(1/2))+1/4*ln
(x^4-x^2-1)-1/5*5^(1/2)*arctanh(1/5*(2*x^2-1)*5^(1/2))
```

maxima [A] time = 1.36, size = 92, normalized size = 1.02

$$\frac{1}{2}\,x^2+\frac{1}{10}\,\sqrt{5}\,\log\left(\frac{2\,x^2-\sqrt{5}+1}{2\,x^2+\sqrt{5}+1}\right)+\frac{1}{10}\,\sqrt{5}\,\log\left(\frac{2\,x^2-\sqrt{5}-1}{2\,x^2+\sqrt{5}-1}\right)-\frac{1}{4}\,\log\left(x^4+x^2-1\right)+\frac{1}{4}\,\log\left(x^4-x^2-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(x^8-3*x^4+1),x, algorithm="maxima")
```

```
[Out] 1/2*x^2 + 1/10*sqrt(5)*log((2*x^2 - sqrt(5) + 1)/(2*x^2 + sqrt(5) + 1)) + 1
/10*sqrt(5)*log((2*x^2 - sqrt(5) - 1)/(2*x^2 + sqrt(5) - 1)) - 1/4*log(x^4
+ x^2 - 1) + 1/4*log(x^4 - x^2 - 1)
```

mupad [B] time = 1.33, size = 90, normalized size = 1.00

$$\frac{x^2}{2}-\operatorname{atanh}\left(\frac{64\,x^2}{64\,\sqrt{5}+192}+\frac{64\,\sqrt{5}\,x^2}{64\,\sqrt{5}+192}\right)\left(\frac{\sqrt{5}}{5}+\frac{1}{2}\right)-\operatorname{atanh}\left(\frac{64\,x^2}{64\,\sqrt{5}-192}-\frac{64\,\sqrt{5}\,x^2}{64\,\sqrt{5}-192}\right)\left(\frac{\sqrt{5}}{5}-\frac{1}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9/(x^8 - 3*x^4 + 1),x)
```

```
[Out] x^2/2 - atanh((64*x^2)/(64*5^(1/2) + 192) + (64*5^(1/2)*x^2)/(64*5^(1/2) +
192))*(5^(1/2)/5 + 1/2) - atanh((64*x^2)/(64*5^(1/2) - 192) - (64*5^(1/2)*x
^2)/(64*5^(1/2) - 192))*(5^(1/2)/5 - 1/2)
```

sympy [B] time = 0.38, size = 170, normalized size = 1.89

$$\frac{x^2}{2}+\left(-\frac{1}{4}-\frac{\sqrt{5}}{10}\right)\log\left(x^2-\frac{47}{8}-\frac{47\sqrt{5}}{20}-120\left(-\frac{1}{4}-\frac{\sqrt{5}}{10}\right)^3\right)+\left(-\frac{1}{4}+\frac{\sqrt{5}}{10}\right)\log\left(x^2-\frac{47}{8}-120\left(-\frac{1}{4}+\frac{\sqrt{5}}{10}\right)^3+\frac{47\sqrt{5}}{20}\right)+\left(\frac{1}{4}-\frac{\sqrt{5}}{10}\right)\log\left(x^2-\frac{47\sqrt{5}}{20}-120\left(\frac{1}{4}-\frac{\sqrt{5}}{10}\right)^3+\frac{47}{8}\right)+\left(\frac{\sqrt{5}}{10}+\frac{1}{4}\right)\log\left(x^2-120\left(\frac{\sqrt{5}}{10}+\frac{1}{4}\right)^3+\frac{47\sqrt{5}}{20}+\frac{47}{8}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**9/(x**8-3*x**4+1),x)
```

```
[Out] x**2/2 + (-1/4 - sqrt(5)/10)*log(x**2 - 47/8 - 47*sqrt(5)/20 - 120*(-1/4 -
sqrt(5)/10)**3) + (-1/4 + sqrt(5)/10)*log(x**2 - 47/8 - 120*(-1/4 + sqrt(5)
/10)**3 + 47*sqrt(5)/20) + (1/4 - sqrt(5)/10)*log(x**2 - 47*sqrt(5)/20 - 12
0*(1/4 - sqrt(5)/10)**3 + 47/8) + (sqrt(5)/10 + 1/4)*log(x**2 - 120*(sqrt(5
)/10 + 1/4)**3 + 47*sqrt(5)/20 + 47/8)
```

3.327 $\int \frac{x^7}{1-3x^4+x^8} \, dx$

Optimal. Leaf size=55

$$\frac{1}{40}\left(5-3\sqrt{5}\right)\log\left(-2x^4-\sqrt{5}+3\right)+\frac{1}{40}\left(5+3\sqrt{5}\right)\log\left(-2x^4+\sqrt{5}+3\right)$$

Rubi [A] time = 0.03, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {1357, 632, 31}

$$\frac{1}{40}\left(5-3\sqrt{5}\right)\log\left(-2x^4-\sqrt{5}+3\right)+\frac{1}{40}\left(5+3\sqrt{5}\right)\log\left(-2x^4+\sqrt{5}+3\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^7/(1 - 3*x^4 + x^8),x]
```

```
[Out] ((5 - 3*Sqrt[5])*Log[3 - Sqrt[5] - 2*x^4])/40 + ((5 + 3*Sqrt[5])*Log[3 + Sq
rt[5] - 2*x^4])/40
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 632

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1357

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x^7}{1-3x^4+x^8}\,dx &= \frac{1}{4}\text{Subst}\left(\int \frac{x}{1-3x+x^2}\,dx,x,x^4\right)\\ &= \frac{1}{40}\left(5-3\sqrt{5}\right)\text{Subst}\left(\int \frac{1}{-\frac{3}{2}+\frac{\sqrt{5}}{2}+x}\,dx,x,x^4\right)+\frac{1}{40}\left(5+3\sqrt{5}\right)\text{Subst}\left(\int \frac{1}{-\frac{3}{2}-\frac{\sqrt{5}}{2}+x}\right.\\ &= \frac{1}{40}\left(5-3\sqrt{5}\right)\log\left(3-\sqrt{5}-2x^4\right)+\frac{1}{40}\left(5+3\sqrt{5}\right)\log\left(3+\sqrt{5}-2x^4\right)\end{aligned}$$

Mathematica [A] time = 0.02, size = 53, normalized size = 0.96

$$\frac{1}{40}\left(5+3\sqrt{5}\right)\log\left(-2x^4+\sqrt{5}+3\right)+\frac{1}{40}\left(5-3\sqrt{5}\right)\log\left(2x^4+\sqrt{5}-3\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7/(1 - 3*x^4 + x^8),x]
```

```
[Out] ((5 + 3*Sqrt[5])*Log[3 + Sqrt[5] - 2*x^4])/40 + ((5 - 3*Sqrt[5])*Log[-3 + S
qrt[5] + 2*x^4])/40
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^7}{1-3x^4+x^8}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^7/(1 - 3*x^4 + x^8),x]
```

```
[Out] IntegrateAlgebraic[x^7/(1 - 3*x^4 + x^8), x]
```

fricas [A] time = 1.04, size = 57, normalized size = 1.04

$$\frac{3}{40}\sqrt{5}\log\left(\frac{2x^8-6x^4-\sqrt{5}\left(2x^4-3\right)+7}{x^8-3x^4+1}\right)+\frac{1}{8}\log\left(x^8-3x^4+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(x^8-3*x^4+1),x, algorithm="fricas")
```

```
[Out] 3/40*sqrt(5)*log((2*x^8 - 6*x^4 - sqrt(5)*(2*x^4 - 3) + 7)/(x^8 - 3*x^4 + 1
)) + 1/8*log(x^8 - 3*x^4 + 1)
```

giac [A] time = 0.42, size = 48, normalized size = 0.87

$$\frac{3}{40}\sqrt{5}\log\left(\frac{\left|2x^4-\sqrt{5}-3\right|}{\left|2x^4+\sqrt{5}-3\right|}\right)+\frac{1}{8}\log\left(\left|x^8-3x^4+1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(x^8-3*x^4+1),x, algorithm="giac")
```

```
[Out] 3/40*sqrt(5)*log(abs(2*x^4 - sqrt(5) - 3)/abs(2*x^4 + sqrt(5) - 3)) + 1/8*l
og(abs(x^8 - 3*x^4 + 1))
```

maple [A] time = 0.00, size = 33, normalized size = 0.60

$$-\frac{3\sqrt{5}\,\operatorname{arctanh}\left(\frac{\left(2x^4-3\right)\sqrt{5}}{5}\right)}{20}+\frac{\ln\left(x^8-3x^4+1\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(x^8-3*x^4+1),x)
```

```
[Out] 1/8*ln(x^8-3*x^4+1)-3/20*5^(1/2)*arctanh(1/5*(2*x^4-3)*5^(1/2))
```

maxima [A] time = 1.48, size = 45, normalized size = 0.82

$$\frac{3}{40}\sqrt{5}\log\left(\frac{2x^4-\sqrt{5}-3}{2x^4+\sqrt{5}-3}\right)+\frac{1}{8}\log\left(x^8-3x^4+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(x^8-3*x^4+1),x, algorithm="maxima")
```

```
[Out] 3/40*sqrt(5)*log((2*x^4 - sqrt(5) - 3)/(2*x^4 + sqrt(5) - 3)) + 1/8*log(x^8
 - 3*x^4 + 1)
```

mupad [B] time = 0.10, size = 59, normalized size = 1.07

$$\frac{\ln\left(x^4-\frac{\sqrt{5}}{2}-\frac{3}{2}\right)}{8}+\frac{\ln\left(x^4+\frac{\sqrt{5}}{2}-\frac{3}{2}\right)}{8}+\frac{3\sqrt{5}\,\ln\left(x^4-\frac{\sqrt{5}}{2}-\frac{3}{2}\right)}{40}-\frac{3\sqrt{5}\,\ln\left(x^4+\frac{\sqrt{5}}{2}-\frac{3}{2}\right)}{40}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(x^8 - 3*x^4 + 1),x)
```

```
[Out] log(x^4 - 5^(1/2)/2 - 3/2)/8 + log(5^(1/2)/2 + x^4 - 3/2)/8 + (3*5^(1/2)*lo
g(x^4 - 5^(1/2)/2 - 3/2))/40 - (3*5^(1/2)*log(5^(1/2)/2 + x^4 - 3/2))/40
```

sympy [A] time = 0.14, size = 53, normalized size = 0.96

$$\left(\frac{1}{8}+\frac{3\sqrt{5}}{40}\right)\log\left(x^4-\frac{3}{2}-\frac{\sqrt{5}}{2}\right)+\left(\frac{1}{8}-\frac{3\sqrt{5}}{40}\right)\log\left(x^4-\frac{3}{2}+\frac{\sqrt{5}}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7/(x**8-3*x**4+1),x)
```

```
[Out] (1/8 + 3*sqrt(5)/40)*log(x**4 - 3/2 - sqrt(5)/2) + (1/8 - 3*sqrt(5)/40)*log
(x**4 - 3/2 + sqrt(5)/2)
```

3.328 $\int \frac{x^5}{1-3x^4+x^8}\,dx$

Optimal. Leaf size=81

$$\frac{1}{2}\sqrt{\frac{1}{10}\left(3-\sqrt{5}\right)}\tanh^{-1}\left(\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}x^2\right)-\frac{1}{2}\sqrt{\frac{1}{10}\left(3+\sqrt{5}\right)}\tanh^{-1}\left(\sqrt{\frac{2}{3+\sqrt{5}}}x^2\right)$$

Rubi [A] time = 0.05, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {1359, 1130, 207}

$$\frac{1}{2}\sqrt{\frac{1}{10}\left(3-\sqrt{5}\right)}\tanh^{-1}\left(\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}x^2\right)-\frac{1}{2}\sqrt{\frac{1}{10}\left(3+\sqrt{5}\right)}\tanh^{-1}\left(\sqrt{\frac{2}{3+\sqrt{5}}}x^2\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^5/(1 - 3*x^4 + x^8),x]
```

```
[Out] -(Sqrt[(3 + Sqrt[5])/10]*ArcTanh[Sqrt[2/(3 + Sqrt[5])]*x^2])/2 + (Sqrt[(3 -
 Sqrt[5])/10]*ArcTanh[Sqrt[(3 + Sqrt[5])/2]*x^2])/2
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 1130

```
Int[((d_.)*(x_))^(m_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Wi
th[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(d^2*(b/q + 1))/2, Int[(d*x)^(m - 2)/(b/2
 + q/2 + c*x^2), x], x] - Dist[(d^2*(b/q - 1))/2, Int[(d*x)^(m - 2)/(b/2 -
q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && G
eQ[m, 2]
```

Rule 1359

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*
x^(n/k) + c*x^((2*n)/k))^p, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, c, p
}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\int \frac{x^5}{1-3x^4+x^8}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{x^2}{1-3x^2+x^4}\,dx,x,x^2\right)\\ &= \frac{1}{20}\left(5-3\sqrt{5}\right)\text{Subst}\left(\int \frac{1}{-\frac{3}{2}+\frac{\sqrt{5}}{2}+x^2}\,dx,x,x^2\right)+\frac{1}{20}\left(5+3\sqrt{5}\right)\text{Subst}\left(\int \frac{1}{-\frac{3}{2}-\frac{\sqrt{5}}{2}+x^2}\right.\\ &= -\frac{1}{2}\sqrt{\frac{1}{10}\left(3+\sqrt{5}\right)}\tanh^{-1}\left(\sqrt{\frac{2}{3+\sqrt{5}}}x^2\right)+\frac{1}{2}\sqrt{\frac{1}{10}\left(3-\sqrt{5}\right)}\tanh^{-1}\left(\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}x^2\right)\end{aligned}$$

Mathematica [A] time = 0.03, size = 91, normalized size = 1.12

$$\frac{1}{40}\left(\left(\sqrt{5}-5\right)\log\left(-2x^2+\sqrt{5}-1\right)+\left(5+\sqrt{5}\right)\log\left(-2x^2+\sqrt{5}+1\right)-\left(\sqrt{5}-5\right)\log\left(2x^2+\sqrt{5}-1\right)-\left(5+\sqrt{5}\right)\log\left(2x^2+\sqrt{5}+1\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/(1 - 3*x^4 + x^8),x]
```

```
[Out] ((-5 + Sqrt[5])*Log[-1 + Sqrt[5] - 2*x^2] + (5 + Sqrt[5])*Log[1 + Sqrt[5] -
 2*x^2] - (-5 + Sqrt[5])*Log[-1 + Sqrt[5] + 2*x^2] - (5 + Sqrt[5])*Log[1 +
Sqrt[5] + 2*x^2])/40
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5}{1-3x^4+x^8}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^5/(1 - 3*x^4 + x^8),x]
```

```
[Out] IntegrateAlgebraic[x^5/(1 - 3*x^4 + x^8), x]
```

fricas [B] time = 1.17, size = 109, normalized size = 1.35

$$\frac{1}{40}\,\sqrt{5}\log\left(\frac{2\,x^4+2\,x^2-\sqrt{5}\left(2\,x^2+1\right)+3}{x^4+x^2-1}\right)+\frac{1}{40}\,\sqrt{5}\log\left(\frac{2\,x^4-2\,x^2-\sqrt{5}\left(2\,x^2-1\right)+3}{x^4-x^2-1}\right)-\frac{1}{8}\,\log\left(x^4+x^2-1\right)+\frac{1}{8}\,\log\left(x^4-x^2-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(x^8-3*x^4+1),x, algorithm="fricas")
```

```
[Out] 1/40*sqrt(5)*log((2*x^4 + 2*x^2 - sqrt(5)*(2*x^2 + 1) + 3)/(x^4 + x^2 - 1))
 + 1/40*sqrt(5)*log((2*x^4 - 2*x^2 - sqrt(5)*(2*x^2 - 1) + 3)/(x^4 - x^2 -
1)) - 1/8*log(x^4 + x^2 - 1) + 1/8*log(x^4 - x^2 - 1)
```

giac [B] time = 0.44, size = 92, normalized size = 1.14

$$\frac{1}{40}\,\sqrt{5}\log\left(\frac{\left|2\,x^2-\sqrt{5}+1\right|}{2\,x^2+\sqrt{5}+1}\right)+\frac{1}{40}\,\sqrt{5}\log\left(\frac{\left|2\,x^2-\sqrt{5}-1\right|}{\left|2\,x^2+\sqrt{5}-1\right|}\right)-\frac{1}{8}\,\log\left(\left|x^4+x^2-1\right|\right)+\frac{1}{8}\,\log\left(\left|x^4-x^2-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(x^8-3*x^4+1),x, algorithm="giac")
```

```
[Out] 1/40*sqrt(5)*log(abs(2*x^2 - sqrt(5) + 1)/(2*x^2 + sqrt(5) + 1)) + 1/40*sqr
t(5)*log(abs(2*x^2 - sqrt(5) - 1)/abs(2*x^2 + sqrt(5) - 1)) - 1/8*log(abs(x
^4 + x^2 - 1)) + 1/8*log(abs(x^4 - x^2 - 1))
```

maple [A] time = 0.00, size = 62, normalized size = 0.77

$$-\frac{\sqrt{5}\,\operatorname{arctanh}\left(\frac{\left(2x^2-1\right)\sqrt{5}}{5}\right)}{20}-\frac{\sqrt{5}\,\operatorname{arctanh}\left(\frac{\left(2x^2+1\right)\sqrt{5}}{5}\right)}{20}+\frac{\ln\left(x^4-x^2-1\right)}{8}-\frac{\ln\left(x^4+x^2-1\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(x^8-3*x^4+1),x)
```

```
[Out] -1/8*ln(x^4+x^2-1)-1/20*5^(1/2)*arctanh(1/5*(2*x^2+1)*5^(1/2))+1/8*ln(x^4-x
^2-1)-1/20*5^(1/2)*arctanh(1/5*(2*x^2-1)*5^(1/2))
```

maxima [B] time = 1.92, size = 87, normalized size = 1.07

$$\frac{1}{40}\,\sqrt{5}\log\left(\frac{2\,x^2-\sqrt{5}+1}{2\,x^2+\sqrt{5}+1}\right)+\frac{1}{40}\,\sqrt{5}\log\left(\frac{2\,x^2-\sqrt{5}-1}{2\,x^2+\sqrt{5}-1}\right)-\frac{1}{8}\,\log\left(x^4+x^2-1\right)+\frac{1}{8}\,\log\left(x^4-x^2-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(x^8-3*x^4+1),x, algorithm="maxima")
```

```
[Out] 1/40*sqrt(5)*log((2*x^2 - sqrt(5) + 1)/(2*x^2 + sqrt(5) + 1)) + 1/40*sqrt(5
)*log((2*x^2 - sqrt(5) - 1)/(2*x^2 + sqrt(5) - 1)) - 1/8*log(x^4 + x^2 - 1)
 + 1/8*log(x^4 - x^2 - 1)
```

mupad [B] time = 1.38, size = 77, normalized size = 0.95

$$-\operatorname{atanh}\left(\frac{4\,x^2}{\sqrt{5}-3}-\frac{2\,\sqrt{5}\,x^2}{\sqrt{5}-3}\right)\left(\frac{\sqrt{5}}{20}+\frac{1}{4}\right)-\operatorname{atanh}\left(\frac{4\,x^2}{\sqrt{5}+3}+\frac{2\,\sqrt{5}\,x^2}{\sqrt{5}+3}\right)\left(\frac{\sqrt{5}}{20}-\frac{1}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(x^8 - 3*x^4 + 1),x)
```

```
[Out] - atanh((4*x^2)/(5^(1/2) - 3) - (2*5^(1/2)*x^2)/(5^(1/2) - 3))*(5^(1/2)/20
+ 1/4) - atanh((4*x^2)/(5^(1/2) + 3) + (2*5^(1/2)*x^2)/(5^(1/2) + 3))*(5^(1
/2)/20 - 1/4)
```

sympy [B] time = 0.37, size = 165, normalized size = 2.04

$$\left(-\frac{1}{8}-\frac{\sqrt{5}}{40}\right)\log\left(x^2-\frac{3}{2}-\frac{3\sqrt{5}}{10}-640\left(-\frac{1}{8}-\frac{\sqrt{5}}{40}\right)^3\right)+\left(-\frac{1}{8}+\frac{\sqrt{5}}{40}\right)\log\left(x^2-\frac{3}{2}-640\left(-\frac{1}{8}+\frac{\sqrt{5}}{40}\right)^3+\frac{3\sqrt{5}}{10}\right)+\left(\frac{1}{8}-\frac{\sqrt{5}}{40}\right)\log\left(x^2-\frac{3\sqrt{5}}{10}-640\left(\frac{1}{8}-\frac{\sqrt{5}}{40}\right)^3+\frac{3}{2}\right)+\left(\frac{\sqrt{5}}{40}+\frac{1}{8}\right)\log\left(x^2-640\left(\frac{\sqrt{5}}{40}+\frac{1}{8}\right)^3+\frac{3\sqrt{5}}{10}+\frac{3}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(x**8-3*x**4+1),x)
```

```
[Out] (-1/8 - sqrt(5)/40)*log(x**2 - 3/2 - 3*sqrt(5)/10 - 640*(-1/8 - sqrt(5)/40)
**3) + (-1/8 + sqrt(5)/40)*log(x**2 - 3/2 - 640*(-1/8 + sqrt(5)/40)**3 + 3*
sqrt(5)/10) + (1/8 - sqrt(5)/40)*log(x**2 - 3*sqrt(5)/10 - 640*(1/8 - sqrt(
5)/40)**3 + 3/2) + (sqrt(5)/40 + 1/8)*log(x**2 - 640*(sqrt(5)/40 + 1/8)**3
+ 3*sqrt(5)/10 + 3/2)
```

3.329 $\int \frac{x^3}{1-3x^4+x^8}\,dx$

Optimal. Leaf size=23

$$\frac{\tanh^{-1}\left(\frac{3-2x^4}{\sqrt{5}}\right)}{2\sqrt{5}}$$

Rubi [A] time = 0.03, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {1352, 618, 206}

$$\frac{\tanh^{-1}\left(\frac{3-2x^4}{\sqrt{5}}\right)}{2\sqrt{5}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(1 - 3*x^4 + x^8),x]
```

```
[Out] ArcTanh[(3 - 2*x^4)/Sqrt[5]]/(2*Sqrt[5])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1352

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a,
b, c, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x^3}{1-3x^4+x^8}\,dx &= \frac{1}{4}\operatorname{Subst}\left(\int \frac{1}{1-3x+x^2}\,dx, x, x^4\right)\\ &= -\left(\frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{5-x^2}\,dx, x, -3+2x^4\right)\right)\\ &= \frac{\tanh^{-1}\left(\frac{3-2x^4}{\sqrt{5}}\right)}{2\sqrt{5}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 38, normalized size = 1.65

$$\frac{\log\left(-2x^4+\sqrt{5}+3\right)-\log\left(2x^4+\sqrt{5}-3\right)}{4\sqrt{5}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(1 - 3*x^4 + x^8),x]
```

```
[Out] (Log[3 + Sqrt[5] - 2*x^4] - Log[-3 + Sqrt[5] + 2*x^4])/(4*Sqrt[5])
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{1-3x^4+x^8}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^3/(1 - 3*x^4 + x^8),x]
```

```
[Out] IntegrateAlgebraic[x^3/(1 - 3*x^4 + x^8), x]
```

fricas [B] time = 1.28, size = 43, normalized size = 1.87

$$\frac{1}{20}\,\sqrt{5}\,\log\left(\frac{2\,x^8-6\,x^4-\sqrt{5}\left(2\,x^4-3\right)+7}{x^8-3\,x^4+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(x^8-3*x^4+1),x, algorithm="fricas")
```

```
[Out] 1/20*sqrt(5)*log((2*x^8 - 6*x^4 - sqrt(5)*(2*x^4 - 3) + 7)/(x^8 - 3*x^4 + 1
))
```

giac [A] time = 0.54, size = 33, normalized size = 1.43

$$\frac{1}{20}\,\sqrt{5}\,\log\left(\frac{\left|2\,x^4-\sqrt{5}-3\right|}{\left|2\,x^4+\sqrt{5}-3\right|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(x^8-3*x^4+1),x, algorithm="giac")
```

```
[Out] 1/20*sqrt(5)*log(abs(2*x^4 - sqrt(5) - 3)/abs(2*x^4 + sqrt(5) - 3))
```

maple [A] time = 0.00, size = 19, normalized size = 0.83

$$-\frac{\sqrt{5}\,\operatorname{arctanh}\left(\frac{\left(2x^4-3\right)\sqrt{5}}{5}\right)}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(x^8-3*x^4+1),x)
```

```
[Out] -1/10*5^(1/2)*arctanh(1/5*(2*x^4-3)*5^(1/2))
```

maxima [A] time = 1.41, size = 31, normalized size = 1.35

$$\frac{1}{20}\,\sqrt{5}\,\log\left(\frac{2\,x^4-\sqrt{5}-3}{2\,x^4+\sqrt{5}-3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(x^8-3*x^4+1),x, algorithm="maxima")
```

```
[Out] 1/20*sqrt(5)*log((2*x^4 - sqrt(5) - 3)/(2*x^4 + sqrt(5) - 3))
```

mupad [**B**] time = 1.57, size = 30, normalized size = 1.30

$$\frac{\sqrt{5}\,\operatorname{atanh}\left(\frac{3\,\sqrt{5}-8\,\sqrt{5}\,x^4}{18\,x^4-7}\right)}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(x^8 - 3*x^4 + 1),x)
```

```
[Out] (5^(1/2)*atanh((3*5^(1/2) - 8*5^(1/2)*x^4)/(18*x^4 - 7)))/10
```

sympy [**A**] time = 0.12, size = 42, normalized size = 1.83

$$\frac{\sqrt{5}\log\left(x^4-\frac{3}{2}-\frac{\sqrt{5}}{2}\right)}{20}-\frac{\sqrt{5}\log\left(x^4-\frac{3}{2}+\frac{\sqrt{5}}{2}\right)}{20}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(x**8-3*x**4+1),x)
```

```
[Out] sqrt(5)*log(x**4 - 3/2 - sqrt(5)/2)/20 - sqrt(5)*log(x**4 - 3/2 + sqrt(5)/2
)/20
```

3.330 $\int \frac{x}{1-3x^4+x^8}\,dx$

Optimal. Leaf size=75

$$\frac{1}{2}\sqrt{\frac{1}{10}\left(3+\sqrt{5}\right)}\tanh^{-1}\left(\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}x^2\right)-\frac{\tanh^{-1}\left(\sqrt{\frac{2}{3+\sqrt{5}}}x^2\right)}{\sqrt{10\left(3+\sqrt{5}\right)}}$$

Rubi [A] time = 0.04, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {1359, 1093, 207}

$$\frac{1}{2}\sqrt{\frac{1}{10}\left(3+\sqrt{5}\right)}\tanh^{-1}\left(\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}x^2\right)-\frac{\tanh^{-1}\left(\sqrt{\frac{2}{3+\sqrt{5}}}x^2\right)}{\sqrt{10\left(3+\sqrt{5}\right)}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(1 - 3*x^4 + x^8),x]
```

```
[Out] -(ArcTanh[Sqrt[2/(3 + Sqrt[5])]*x^2]/Sqrt[10*(3 + Sqrt[5])]) + (Sqrt[(3 + S
qrt[5])/10]*ArcTanh[Sqrt[(3 + Sqrt[5])/2]*x^2])/2
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 1093

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[b^
2 - 4*a*c, 2]}, Dist[c/q, Int[1/(b/2 - q/2 + c*x^2), x], x] - Dist[c/q, Int
[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
0] && PosQ[b^2 - 4*a*c]
```

Rule 1359

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*
x^(n/k) + c*x^((2*n)/k))^p, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, c, p
}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\int \frac{x}{1-3x^4+x^8}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{1-3x^2+x^4}\,dx,x,x^2\right)\\ &= \frac{\text{Subst}\left(\int \frac{1}{-\frac{3}{2}-\frac{\sqrt{5}}{2}+x^2}\,dx,x,x^2\right)}{2\sqrt{5}}-\frac{\text{Subst}\left(\int \frac{1}{-\frac{3}{2}+\frac{\sqrt{5}}{2}+x^2}\,dx,x,x^2\right)}{2\sqrt{5}}\\ &= -\frac{\tanh^{-1}\left(\sqrt{\frac{2}{3+\sqrt{5}}}x^2\right)}{\sqrt{10\left(3+\sqrt{5}\right)}}+\frac{1}{2}\sqrt{\frac{1}{10}\left(3+\sqrt{5}\right)}\tanh^{-1}\left(\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}x^2\right)\end{aligned}$$

Mathematica [A] time = 0.03, size = 91, normalized size = 1.21

$$\frac{1}{40}\left(-\left(\left(5+\sqrt{5}\right)\log\left(-2x^2+\sqrt{5}-1\right)\right)-\left(\sqrt{5}-5\right)\log\left(-2x^2+\sqrt{5}+1\right)+\left(5+\sqrt{5}\right)\log\left(2x^2+\sqrt{5}-1\right)+\left(\sqrt{5}-5\right)\log\left(2x^2+\sqrt{5}+1\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(1 - 3*x^4 + x^8),x]
```

```
[Out] (-((5 + Sqrt[5])*Log[-1 + Sqrt[5] - 2*x^2]) - (-5 + Sqrt[5])*Log[1 + Sqrt[5
] - 2*x^2] + (5 + Sqrt[5])*Log[-1 + Sqrt[5] + 2*x^2] + (-5 + Sqrt[5])*Log[1
 + Sqrt[5] + 2*x^2])/40
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{1-3x^4+x^8}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x/(1 - 3*x^4 + x^8),x]
```

```
[Out] IntegrateAlgebraic[x/(1 - 3*x^4 + x^8), x]
```

fricas [B] time = 1.23, size = 107, normalized size = 1.43

$$\frac{1}{40}\sqrt{5}\log\left(\frac{2x^4+2x^2+\sqrt{5}\left(2x^2+1\right)+3}{x^4+x^2-1}\right)+\frac{1}{40}\sqrt{5}\log\left(\frac{2x^4-2x^2+\sqrt{5}\left(2x^2-1\right)+3}{x^4-x^2-1}\right)-\frac{1}{8}\log\left(x^4+x^2-1\right)+\frac{1}{8}\log\left(x^4-x^2-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^8-3*x^4+1),x, algorithm="fricas")
```

```
[Out] 1/40*sqrt(5)*log((2*x^4 + 2*x^2 + sqrt(5)*(2*x^2 + 1) + 3)/(x^4 + x^2 - 1))
 + 1/40*sqrt(5)*log((2*x^4 - 2*x^2 + sqrt(5)*(2*x^2 - 1) + 3)/(x^4 - x^2 -
1)) - 1/8*log(x^4 + x^2 - 1) + 1/8*log(x^4 - x^2 - 1)
```

giac [B] time = 0.43, size = 92, normalized size = 1.23

$$-\frac{1}{40}\sqrt{5}\log\left(\frac{\left|2x^2-\sqrt{5}+1\right|}{2x^2+\sqrt{5}+1}\right)-\frac{1}{40}\sqrt{5}\log\left(\frac{\left|2x^2-\sqrt{5}-1\right|}{\left|2x^2+\sqrt{5}-1\right|}\right)-\frac{1}{8}\log\left(\left|x^4+x^2-1\right|\right)+\frac{1}{8}\log\left(\left|x^4-x^2-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^8-3*x^4+1),x, algorithm="giac")
```

```
[Out] -1/40*sqrt(5)*log(abs(2*x^2 - sqrt(5) + 1)/(2*x^2 + sqrt(5) + 1)) - 1/40*sq
rt(5)*log(abs(2*x^2 - sqrt(5) - 1)/abs(2*x^2 + sqrt(5) - 1)) - 1/8*log(abs(
x^4 + x^2 - 1)) + 1/8*log(abs(x^4 - x^2 - 1))
```

maple [A] time = 0.00, size = 62, normalized size = 0.83

$$\frac{\sqrt{5}\ \mathrm{arctanh}\left(\frac{\left(2x^2-1\right)\sqrt{5}}{5}\right)}{20}+\frac{\sqrt{5}\ \mathrm{arctanh}\left(\frac{\left(2x^2+1\right)\sqrt{5}}{5}\right)}{20}+\frac{\ln\left(x^4-x^2-1\right)}{8}-\frac{\ln\left(x^4+x^2-1\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x^8-3*x^4+1),x)
```

```
[Out] -1/8*ln(x^4+x^2-1)+1/20*5^(1/2)*arctanh(1/5*(2*x^2+1)*5^(1/2))+1/8*ln(x^4-x
^2-1)+1/20*5^(1/2)*arctanh(1/5*(2*x^2-1)*5^(1/2))
```

maxima [**B**] time = 1.45, size = 87, normalized size = 1.16

$$-\frac{1}{40}\sqrt{5}\log\left(\frac{2x^2-\sqrt{5}+1}{2x^2+\sqrt{5}+1}\right)-\frac{1}{40}\sqrt{5}\log\left(\frac{2x^2-\sqrt{5}-1}{2x^2+\sqrt{5}-1}\right)-\frac{1}{8}\log\left(x^4+x^2-1\right)+\frac{1}{8}\log\left(x^4-x^2-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^8-3*x^4+1),x, algorithm="maxima")
```

```
[Out] -1/40*sqrt(5)*log((2*x^2 - sqrt(5) + 1)/(2*x^2 + sqrt(5) + 1)) - 1/40*sqrt(
5)*log((2*x^2 - sqrt(5) - 1)/(2*x^2 + sqrt(5) - 1)) - 1/8*log(x^4 + x^2 - 1
) + 1/8*log(x^4 - x^2 - 1)
```

mupad [**B**] time = 1.30, size = 83, normalized size = 1.11

$$\operatorname{atanh}\left(\frac{29x^2}{8\sqrt{5}-18}-\frac{13\sqrt{5}\,x^2}{8\sqrt{5}-18}\right)\left(\frac{\sqrt{5}}{20}-\frac{1}{4}\right)+\operatorname{atanh}\left(\frac{29x^2}{8\sqrt{5}+18}+\frac{13\sqrt{5}\,x^2}{8\sqrt{5}+18}\right)\left(\frac{\sqrt{5}}{20}+\frac{1}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x^8 - 3*x^4 + 1),x)
```

```
[Out] atanh((29*x^2)/(8*5^(1/2) - 18) - (13*5^(1/2)*x^2)/(8*5^(1/2) - 18))*(5^(1/
2)/20 - 1/4) + atanh((29*x^2)/(8*5^(1/2) + 18) + (13*5^(1/2)*x^2)/(8*5^(1/2
) + 18))*(5^(1/2)/20 + 1/4)
```

sympy [**B**] time = 0.37, size = 165, normalized size = 2.20

$$\left(\frac{\sqrt{5}}{40}+\frac{1}{8}\right)\log\left(x^2-\frac{7}{2}-\frac{7\sqrt{5}}{10}+960\left(\frac{\sqrt{5}}{40}+\frac{1}{8}\right)^3\right)+\left(\frac{1}{8}-\frac{\sqrt{5}}{40}\right)\log\left(x^2-\frac{7}{2}+960\left(\frac{1}{8}-\frac{\sqrt{5}}{40}\right)^3+\frac{7\sqrt{5}}{10}\right)+\left(-\frac{1}{8}+\frac{\sqrt{5}}{40}\right)\log\left(x^2-\frac{7\sqrt{5}}{10}+960\left(-\frac{1}{8}+\frac{\sqrt{5}}{40}\right)^3+\frac{7}{2}\right)+\left(-\frac{1}{8}-\frac{\sqrt{5}}{40}\right)\log\left(x^2+960\left(-\frac{1}{8}-\frac{\sqrt{5}}{40}\right)^3+\frac{7\sqrt{5}}{10}+\frac{7}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x**8-3*x**4+1),x)
```

```
[Out] (sqrt(5)/40 + 1/8)*log(x**2 - 7/2 - 7*sqrt(5)/10 + 960*(sqrt(5)/40 + 1/8)**
3) + (1/8 - sqrt(5)/40)*log(x**2 - 7/2 + 960*(1/8 - sqrt(5)/40)**3 + 7*sqrt
(5)/10) + (-1/8 + sqrt(5)/40)*log(x**2 - 7*sqrt(5)/10 + 960*(-1/8 + sqrt(5)
/40)**3 + 7/2) + (-1/8 - sqrt(5)/40)*log(x**2 + 960*(-1/8 - sqrt(5)/40)**3
+ 7*sqrt(5)/10 + 7/2)
```

3.331 $\int \frac{1}{x(1-3x^4+x^8)} \, dx$

Optimal. Leaf size=57

$$-\frac{1}{40}\left(5+3\sqrt{5}\right)\log\left(-2x^4-\sqrt{5}+3\right)-\frac{1}{40}\left(5-3\sqrt{5}\right)\log\left(-2x^4+\sqrt{5}+3\right)+\log(x)$$

Rubi [A] time = 0.03, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {1357, 705, 29, 632, 31}

$$-\frac{1}{40}\left(5+3\sqrt{5}\right)\log\left(-2x^4-\sqrt{5}+3\right)-\frac{1}{40}\left(5-3\sqrt{5}\right)\log\left(-2x^4+\sqrt{5}+3\right)+\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(1 - 3*x^4 + x^8)),x]
```

```
[Out] Log[x] - ((5 + 3*Sqrt[5])*Log[3 - Sqrt[5] - 2*x^4])/40 - ((5 - 3*Sqrt[5])*L
og[3 + Sqrt[5] - 2*x^4])/40
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 632

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rule 705

```
Int[1/(((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)), x_Symbol]
 :> Dist[e^2/(c*d^2 - b*d*e + a*e^2), Int[1/(d + e*x), x], x] + Dist[1/(c*d
^2 - b*d*e + a*e^2), Int[(c*d - b*e - c*e*x)/(a + b*x + c*x^2), x], x] /; F
reeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^
2, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1357

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x\left(1-3x^4+x^8\right)}\,dx &= \frac{1}{4}\text{Subst}\left(\int \frac{1}{x\left(1-3x+x^2\right)}\,dx,x,x^4\right)\\ &= \frac{1}{4}\text{Subst}\left(\int \frac{1}{x}\,dx,x,x^4\right)+\frac{1}{4}\text{Subst}\left(\int \frac{3-x}{1-3x+x^2}\,dx,x,x^4\right)\\ &= \log(x)+\frac{1}{40}\left(-5+3\sqrt{5}\right)\text{Subst}\left(\int \frac{1}{-\frac{3}{2}-\frac{\sqrt{5}}{2}+x}\,dx,x,x^4\right)-\frac{1}{40}\left(5+3\sqrt{5}\right)\text{Subst}\left(\int \frac{1}{-\right.\right.\\ &= \log(x)-\frac{1}{40}\left(5+3\sqrt{5}\right)\log\left(3-\sqrt{5}-2x^4\right)-\frac{1}{40}\left(5-3\sqrt{5}\right)\log\left(3+\sqrt{5}-2x^4\right)\end{aligned}$$

Mathematica [A] time = 0.03, size = 55, normalized size = 0.96

$$\frac{1}{40}\left(3\sqrt{5}-5\right)\log\left(-2x^4+\sqrt{5}+3\right)+\frac{1}{40}\left(-5-3\sqrt{5}\right)\log\left(2x^4+\sqrt{5}-3\right)+\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(1 - 3*x^4 + x^8)),x]
```

```
[Out] Log[x] + ((-5 + 3*Sqrt[5])*Log[3 + Sqrt[5] - 2*x^4])/40 + ((-5 - 3*Sqrt[5])
*Log[-3 + Sqrt[5] + 2*x^4])/40
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(1-3x^4+x^8\right)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x*(1 - 3*x^4 + x^8)),x]
```

```
[Out] IntegrateAlgebraic[1/(x*(1 - 3*x^4 + x^8)), x]
```

fricas [A] time = 1.28, size = 59, normalized size = 1.04

$$\frac{3}{40}\sqrt{5}\log\left(\frac{2x^8-6x^4-\sqrt{5}\left(2x^4-3\right)+7}{x^8-3x^4+1}\right)-\frac{1}{8}\log\left(x^8-3x^4+1\right)+\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x^8-3*x^4+1),x, algorithm="fricas")
```

```
[Out] 3/40*sqrt(5)*log((2*x^8 - 6*x^4 - sqrt(5)*(2*x^4 - 3) + 7)/(x^8 - 3*x^4 + 1
)) - 1/8*log(x^8 - 3*x^4 + 1) + log(x)
```

giac [A] time = 0.48, size = 54, normalized size = 0.95

$$\frac{3}{40}\sqrt{5}\log\left(\frac{\left|2x^4-\sqrt{5}-3\right|}{\left|2x^4+\sqrt{5}-3\right|}\right)+\frac{1}{4}\log\left(x^4\right)-\frac{1}{8}\log\left(\left|x^8-3x^4+1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x^8-3*x^4+1),x, algorithm="giac")
```

```
[Out] 3/40*sqrt(5)*log(abs(2*x^4 - sqrt(5) - 3)/abs(2*x^4 + sqrt(5) - 3)) + 1/4*l
og(x^4) - 1/8*log(abs(x^8 - 3*x^4 + 1))
```

maple [A] time = 0.01, size = 64, normalized size = 1.12

$$-\frac{3\sqrt{5}\ \operatorname{arctanh}\left(\frac{\left(2x^{2}-1\right)\sqrt{5}}{5}\right)}{20}+\frac{3\sqrt{5}\ \operatorname{arctanh}\left(\frac{\left(2x^{2}+1\right)\sqrt{5}}{5}\right)}{20}+\ln(x)-\frac{\ln\left(x^{4}-x^{2}-1\right)}{8}-\frac{\ln\left(x^{4}+x^{2}-1\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(x^8-3*x^4+1),x)
```

```
[Out] ln(x)-1/8*ln(x^4-x^2-1)-3/20*5^(1/2)*arctanh(1/5*(2*x^2-1)*5^(1/2))-1/8*ln(
x^4+x^2-1)+3/20*5^(1/2)*arctanh(1/5*(2*x^2+1)*5^(1/2))
```

maxima [A] time = 1.45, size = 51, normalized size = 0.89

$$\frac{3}{40}\sqrt{5}\log\left(\frac{2x^{4}-\sqrt{5}-3}{2x^{4}+\sqrt{5}-3}\right)-\frac{1}{8}\log\left(x^{8}-3x^{4}+1\right)+\frac{1}{4}\log\left(x^{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x^8-3*x^4+1),x, algorithm="maxima")
```

```
[Out] 3/40*sqrt(5)*log((2*x^4 - sqrt(5) - 3)/(2*x^4 + sqrt(5) - 3)) - 1/8*log(x^8
 - 3*x^4 + 1) + 1/4*log(x^4)
```

mupad [B] time = 0.43, size = 42, normalized size = 0.74

$$\ln(x)+\ln\left(x^{4}-\frac{\sqrt{5}}{2}-\frac{3}{2}\right)\left(\frac{3\sqrt{5}}{40}-\frac{1}{8}\right)-\ln\left(x^{4}+\frac{\sqrt{5}}{2}-\frac{3}{2}\right)\left(\frac{3\sqrt{5}}{40}+\frac{1}{8}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(x^8 - 3*x^4 + 1)),x)
```

```
[Out] log(x) + log(x^4 - 5^(1/2)/2 - 3/2)*((3*5^(1/2))/40 - 1/8) - log(5^(1/2)/2
+ x^4 - 3/2)*((3*5^(1/2))/40 + 1/8)
```

sympy [A] time = 0.16, size = 58, normalized size = 1.02

$$\log(x)+\left(-\frac{1}{8}+\frac{3\sqrt{5}}{40}\right)\log\left(x^{4}-\frac{3}{2}-\frac{\sqrt{5}}{2}\right)+\left(-\frac{3\sqrt{5}}{40}-\frac{1}{8}\right)\log\left(x^{4}-\frac{3}{2}+\frac{\sqrt{5}}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x**8-3*x**4+1),x)
```

```
[Out] log(x) + (-1/8 + 3*sqrt(5)/40)*log(x**4 - 3/2 - sqrt(5)/2) + (-3*sqrt(5)/40
 - 1/8)*log(x**4 - 3/2 + sqrt(5)/2)
```

3.332 $\int \frac{1}{x^3(1-3x^4+x^8)}\, dx$

Optimal. Leaf size=89

$$-\frac{1}{2x^2}-\frac{1}{2}\sqrt{\frac{1}{5}\left(9-4\sqrt{5}\right)}\tanh^{-1}\left(\sqrt{\frac{2}{3+\sqrt{5}}}\,x^2\right)+\frac{\left(3+\sqrt{5}\right)^{3/2}\tanh^{-1}\left(\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}\,x^2\right)}{4\sqrt{10}}$$

Rubi [A] time = 0.06, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1359, 1123, 1166, 207}

$$-\frac{1}{2x^2}-\frac{1}{2}\sqrt{\frac{1}{5}\left(9-4\sqrt{5}\right)}\tanh^{-1}\left(\sqrt{\frac{2}{3+\sqrt{5}}}\,x^2\right)+\frac{\left(3+\sqrt{5}\right)^{3/2}\tanh^{-1}\left(\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}\,x^2\right)}{4\sqrt{10}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(1 - 3*x^4 + x^8)),x]
```

```
[Out] -1/(2*x^2) - (Sqrt[(9 - 4*Sqrt[5])/5]*ArcTanh[Sqrt[2/(3 + Sqrt[5])]*x^2])/2
 + ((3 + Sqrt[5])^(3/2)*ArcTanh[Sqrt[(3 + Sqrt[5])/2]*x^2])/(4*Sqrt[10])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 1123

```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*x^2 + c*x^4)^(p + 1))/(a*d*(m + 1)), x] - Dis
t[1/(a*d^2*(m + 1)), Int[(d*x)^(m + 2)*(b*(m + 2*p + 3) + c*(m + 4*p + 5)*x
^2)*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b^2 -
 4*a*c, 0] && LtQ[m, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1359

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*
x^(n/k) + c*x^((2*n)/k))^p, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, c, p
}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\int \frac{1}{x^3(1-3x^4+x^8)}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{1}{x^2(1-3x^2+x^4)}\,dx,x,x^2\right)$$
$$= -\frac{1}{2x^2}+\frac{1}{2}\text{Subst}\left(\int \frac{3-x^2}{1-3x^2+x^4}\,dx,x,x^2\right)$$
$$= -\frac{1}{2x^2}+\frac{1}{20}\left(-5+3\sqrt{5}\right)\text{Subst}\left(\int \frac{1}{-\frac{3}{2}-\frac{\sqrt{5}}{2}+x^2}\,dx,x,x^2\right)-\frac{1}{20}\left(5+3\sqrt{5}\right)\text{Subst}\Bigg($$
$$= -\frac{1}{2x^2}-\frac{1}{10}\sqrt{45-20\sqrt{5}}\tanh^{-1}\left(\sqrt{\frac{2}{3+\sqrt{5}}}\,x^2\right)+\frac{\left(3+\sqrt{5}\right)^{3/2}\tanh^{-1}\left(\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right.}\right.}{4\sqrt{10}}$$

Mathematica [A] time = 0.06, size = 103, normalized size = 1.16

$$\frac{1}{20}\left(-\frac{10}{x^2}-\left(\left(5+2\sqrt{5}\right)\log\left(-2x^2+\sqrt{5}-1\right)\right)+\left(5-2\sqrt{5}\right)\log\left(-2x^2+\sqrt{5}+1\right)+\left(5+2\sqrt{5}\right)\log\left(2x^2+\sqrt{5}-1\right)+\left(2\sqrt{5}-5\right)\log\left(2x^2+\sqrt{5}+1\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(1 - 3*x^4 + x^8)),x]
[Out] (-10/x^2 - (5 + 2*Sqrt[5])*Log[-1 + Sqrt[5] - 2*x^2] + (5 - 2*Sqrt[5])*Log[
1 + Sqrt[5] - 2*x^2] + (5 + 2*Sqrt[5])*Log[-1 + Sqrt[5] + 2*x^2] + (-5 + 2*
Sqrt[5])*Log[1 + Sqrt[5] + 2*x^2])/20
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3(1-3x^4+x^8)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x^3*(1 - 3*x^4 + x^8)),x]
[Out] IntegrateAlgebraic[1/(x^3*(1 - 3*x^4 + x^8)), x]
```

fricas [B] time = 1.11, size = 125, normalized size = 1.40

$$\frac{2\sqrt{5}x^2\log\left(\frac{2x^4+2x^2+\sqrt{5}(2x^2+1)+3}{x^4+x^2-1}\right)+2\sqrt{5}x^2\log\left(\frac{2x^4-2x^2+\sqrt{5}(2x^2-1)+3}{x^4-x^2-1}\right)-5x^2\log\left(x^4+x^2-1\right)+5x^2\log\left(x^4-x^2-1\right)-10}{20x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(x^8-3*x^4+1),x, algorithm="fricas")
[Out] 1/20*(2*sqrt(5)*x^2*log((2*x^4 + 2*x^2 + sqrt(5)*(2*x^2 + 1) + 3)/(x^4 + x^
2 - 1)) + 2*sqrt(5)*x^2*log((2*x^4 - 2*x^2 + sqrt(5)*(2*x^2 - 1) + 3)/(x^4
- x^2 - 1)) - 5*x^2*log(x^4 + x^2 - 1) + 5*x^2*log(x^4 - x^2 - 1) - 10)/x^2
```

giac [A] time = 0.38, size = 97, normalized size = 1.09

$$-\frac{1}{10}\sqrt{5}\log\left(\frac{\left|2x^2-\sqrt{5}+1\right|}{2x^2+\sqrt{5}+1}\right)-\frac{1}{10}\sqrt{5}\log\left(\frac{\left|2x^2-\sqrt{5}-1\right|}{\left|2x^2+\sqrt{5}-1\right|}\right)-\frac{1}{2x^2}-\frac{1}{4}\log\left(\left|x^4+x^2-1\right|\right)+\frac{1}{4}\log\left(\left|x^4-x^2-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(x^8-3*x^4+1),x, algorithm="giac")
```

```
[Out] -1/10*sqrt(5)*log(abs(2*x^2 - sqrt(5) + 1)/(2*x^2 + sqrt(5) + 1)) - 1/10*sq
rt(5)*log(abs(2*x^2 - sqrt(5) - 1)/abs(2*x^2 + sqrt(5) - 1)) - 1/2/x^2 - 1/
4*log(abs(x^4 + x^2 - 1)) + 1/4*log(abs(x^4 - x^2 - 1))
```

maple [A] time = 0.01, size = 67, normalized size = 0.75

$$\frac{\sqrt{5}\ \operatorname{arctanh}\left(\frac{(2x^2-1)\sqrt{5}}{5}\right)}{5}+\frac{\sqrt{5}\ \operatorname{arctanh}\left(\frac{(2x^2+1)\sqrt{5}}{5}\right)}{5}+\frac{\ln\left(x^4-x^2-1\right)}{4}-\frac{\ln\left(x^4+x^2-1\right)}{4}-\frac{1}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(x^8-3*x^4+1),x)
```

```
[Out] 1/4*ln(x^4-x^2-1)+1/5*5^(1/2)*arctanh(1/5*(2*x^2-1)*5^(1/2))-1/4*ln(x^4+x^2
-1)+1/5*5^(1/2)*arctanh(1/5*(2*x^2+1)*5^(1/2))-1/2/x^2
```

maxima [A] time = 1.46, size = 92, normalized size = 1.03

$$-\frac{1}{10}\sqrt{5}\log\left(\frac{2x^2-\sqrt{5}+1}{2x^2+\sqrt{5}+1}\right)-\frac{1}{10}\sqrt{5}\log\left(\frac{2x^2-\sqrt{5}-1}{2x^2+\sqrt{5}-1}\right)-\frac{1}{2x^2}-\frac{1}{4}\log\left(x^4+x^2-1\right)+\frac{1}{4}\log\left(x^4-x^2-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(x^8-3*x^4+1),x, algorithm="maxima")
```

```
[Out] -1/10*sqrt(5)*log((2*x^2 - sqrt(5) + 1)/(2*x^2 + sqrt(5) + 1)) - 1/10*sqrt(
5)*log((2*x^2 - sqrt(5) - 1)/(2*x^2 + sqrt(5) - 1)) - 1/2/x^2 - 1/4*log(x^4
 + x^2 - 1) + 1/4*log(x^4 - x^2 - 1)
```

mupad [B] time = 0.06, size = 88, normalized size = 0.99

$$\operatorname{atanh}\left(\frac{12736\,x^2}{3520\sqrt{5}-7872}-\frac{5696\sqrt{5}\,x^2}{3520\sqrt{5}-7872}\right)\left(\frac{\sqrt{5}}{5}-\frac{1}{2}\right)+\operatorname{atanh}\left(\frac{12736\,x^2}{3520\sqrt{5}+7872}+\frac{5696\sqrt{5}\,x^2}{3520\sqrt{5}+7872}\right)\left(\frac{\sqrt{5}}{5}+\frac{1}{2}\right)-\frac{1}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(x^8 - 3*x^4 + 1)),x)
```

```
[Out] atanh((12736*x^2)/(3520*5^(1/2) - 7872) - (5696*5^(1/2)*x^2)/(3520*5^(1/2)
- 7872))*(5^(1/2)/5 - 1/2) + atanh((12736*x^2)/(3520*5^(1/2) + 7872) + (569
6*5^(1/2)*x^2)/(3520*5^(1/2) + 7872))*(5^(1/2)/5 + 1/2) - 1/(2*x^2)
```

sympy [B] time = 0.41, size = 172, normalized size = 1.93

$$\left(\frac{\sqrt{5}}{10}+\frac{1}{4}\right)\log\left(x^2-\frac{123}{8}-\frac{123\sqrt{5}}{20}+280\left(\frac{\sqrt{5}}{10}+\frac{1}{4}\right)^3\right)+\left(\frac{1}{4}-\frac{\sqrt{5}}{10}\right)\log\left(x^2-\frac{123}{8}+280\left(\frac{1}{4}-\frac{\sqrt{5}}{10}\right)^3+\frac{123\sqrt{5}}{20}\right)+\left(-\frac{1}{4}+\frac{\sqrt{5}}{10}\right)\log\left(x^2-\frac{123\sqrt{5}}{20}+280\left(-\frac{1}{4}+\frac{\sqrt{5}}{10}\right)^3+\frac{123}{8}\right)+\left(-\frac{1}{4}-\frac{\sqrt{5}}{10}\right)\log\left(x^2+280\left(-\frac{1}{4}-\frac{\sqrt{5}}{10}\right)^3+\frac{123\sqrt{5}}{20}+\frac{123}{8}\right)-\frac{1}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(x**8-3*x**4+1),x)
```

```
[Out] (sqrt(5)/10 + 1/4)*log(x**2 - 123/8 - 123*sqrt(5)/20 + 280*(sqrt(5)/10 + 1/
4)**3) + (1/4 - sqrt(5)/10)*log(x**2 - 123/8 + 280*(1/4 - sqrt(5)/10)**3 +
123*sqrt(5)/20) + (-1/4 + sqrt(5)/10)*log(x**2 - 123*sqrt(5)/20 + 280*(-1/4
 + sqrt(5)/10)**3 + 123/8) + (-1/4 - sqrt(5)/10)*log(x**2 + 280*(-1/4 - sqr
t(5)/10)**3 + 123*sqrt(5)/20 + 123/8) - 1/(2*x**2)
```

3.333 $\int \frac{1}{x^5(1-3x^4+x^8)}\,dx$

Optimal. Leaf size=66

$$-\frac{1}{4x^4}-\frac{1}{40}\left(15+7\sqrt{5}\right)\log\left(-2x^4-\sqrt{5}+3\right)-\frac{1}{40}\left(15-7\sqrt{5}\right)\log\left(-2x^4+\sqrt{5}+3\right)+3\log(x)$$

Rubi [A] time = 0.06, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {1357, 709, 800, 632, 31}

$$-\frac{1}{4x^4}-\frac{1}{40}\left(15+7\sqrt{5}\right)\log\left(-2x^4-\sqrt{5}+3\right)-\frac{1}{40}\left(15-7\sqrt{5}\right)\log\left(-2x^4+\sqrt{5}+3\right)+3\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^5*(1 - 3*x^4 + x^8)),x]
```

```
[Out] -1/(4*x^4) + 3*Log[x] - ((15 + 7*Sqrt[5])*Log[3 - Sqrt[5] - 2*x^4])/40 - ((
15 - 7*Sqrt[5])*Log[3 + Sqrt[5] - 2*x^4])/40
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 632

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rule 709

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Simp[(e*(d + e*x)^(m + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dis
t[1/(c*d^2 - b*d*e + a*e^2), Int[((d + e*x)^(m + 1)*Simp[c*d - b*e - c*e*x,
 x])/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 -
 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[m
, -1]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 1357

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{1}{x^5\left(1-3x^4+x^8\right)}\,dx = \frac{1}{4}\,\text{Subst}\left(\int \frac{1}{x^2\left(1-3x+x^2\right)}\,dx, x, x^4\right)$$
$$= -\frac{1}{4x^4}+\frac{1}{4}\,\text{Subst}\left(\int \frac{3-x}{x\left(1-3x+x^2\right)}\,dx, x, x^4\right)$$
$$= -\frac{1}{4x^4}+\frac{1}{4}\,\text{Subst}\left(\int \left(\frac{3}{x}+\frac{8-3x}{1-3x+x^2}\right)dx, x, x^4\right)$$
$$= -\frac{1}{4x^4}+3\log(x)+\frac{1}{4}\,\text{Subst}\left(\int \frac{8-3x}{1-3x+x^2}\,dx, x, x^4\right)$$
$$= -\frac{1}{4x^4}+3\log(x)+\frac{1}{40}\left(-15+7\sqrt{5}\right)\text{Subst}\left(\int \frac{1}{-\frac{3}{2}-\frac{\sqrt{5}}{2}+x}\,dx, x, x^4\right)-\frac{1}{40}\left(15+7\sqrt{5}\right.$$
$$= -\frac{1}{4x^4}+3\log(x)-\frac{1}{40}\left(15+7\sqrt{5}\right)\log\left(3-\sqrt{5}-2x^4\right)-\frac{1}{40}\left(15-7\sqrt{5}\right)\log\left(3+\sqrt{5}-\right.$$

Mathematica [A] time = 0.04, size = 61, normalized size = 0.92

$$\frac{1}{40}\left(-\frac{10}{x^4}+\left(7\sqrt{5}-15\right)\log\left(-2x^4+\sqrt{5}+3\right)-\left(15+7\sqrt{5}\right)\log\left(2x^4+\sqrt{5}-3\right)+120\log(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^5*(1 - 3*x^4 + x^8)),x]
[Out] (-10/x^4 + 120*Log[x] + (-15 + 7*Sqrt[5])*Log[3 + Sqrt[5] - 2*x^4] - (15 +
7*Sqrt[5])*Log[-3 + Sqrt[5] + 2*x^4])/40
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^5\left(1-3x^4+x^8\right)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x^5*(1 - 3*x^4 + x^8)),x]
[Out] IntegrateAlgebraic[1/(x^5*(1 - 3*x^4 + x^8)), x]
```

fricas [A] time = 1.28, size = 76, normalized size = 1.15

$$\frac{7\sqrt{5}\,x^4\log\left(\frac{2x^8-6x^4-\sqrt{5}\left(2x^4-3\right)+7}{x^8-3x^4+1}\right)-15x^4\log\left(x^8-3x^4+1\right)+120x^4\log(x)-10}{40x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(x^8-3*x^4+1),x, algorithm="fricas")
[Out] 1/40*(7*sqrt(5)*x^4*log((2*x^8 - 6*x^4 - sqrt(5)*(2*x^4 - 3) + 7)/(x^8 - 3*
x^4 + 1)) - 15*x^4*log(x^8 - 3*x^4 + 1) + 120*x^4*log(x) - 10)/x^4
```

giac [A] time = 0.51, size = 66, normalized size = 1.00

$$\frac{7}{40}\sqrt{5}\log\left(\frac{\left|2x^4-\sqrt{5}-3\right|}{\left|2x^4+\sqrt{5}-3\right|}\right)-\frac{3x^4+1}{4x^4}+\frac{3}{4}\log\left(x^4\right)-\frac{3}{8}\log\left(\left|x^8-3x^4+1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(x^8-3*x^4+1),x, algorithm="giac")
```

```
[Out] 7/40*sqrt(5)*log(abs(2*x^4 - sqrt(5) - 3)/abs(2*x^4 + sqrt(5) - 3)) - 1/4*(
3*x^4 + 1)/x^4 + 3/4*log(x^4) - 3/8*log(abs(x^8 - 3*x^4 + 1))
```

maple [A] time = 0.01, size = 71, normalized size = 1.08

$$-\frac{7\sqrt{5}\,\operatorname{arctanh}\left(\frac{(2x^2-1)\sqrt{5}}{5}\right)}{20}+\frac{7\sqrt{5}\,\operatorname{arctanh}\left(\frac{(2x^2+1)\sqrt{5}}{5}\right)}{20}+3\ln(x)-\frac{3\ln\left(x^4-x^2-1\right)}{8}-\frac{3\ln\left(x^4+x^2-1\right)}{8}-\frac{1}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^5/(x^8-3*x^4+1),x)
```

```
[Out] -1/4/x^4+3*ln(x)-3/8*ln(x^4-x^2-1)-7/20*5^(1/2)*arctanh(1/5*(2*x^2-1)*5^(1/
2))-3/8*ln(x^4+x^2-1)+7/20*5^(1/2)*arctanh(1/5*(2*x^2+1)*5^(1/2))
```

maxima [A] time = 1.40, size = 56, normalized size = 0.85

$$\frac{7}{40}\sqrt{5}\log\left(\frac{2x^4-\sqrt{5}-3}{2x^4+\sqrt{5}-3}\right)-\frac{1}{4x^4}-\frac{3}{8}\log\left(x^8-3x^4+1\right)+\frac{3}{4}\log\left(x^4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(x^8-3*x^4+1),x, algorithm="maxima")
```

```
[Out] 7/40*sqrt(5)*log((2*x^4 - sqrt(5) - 3)/(2*x^4 + sqrt(5) - 3)) - 1/4/x^4 - 3
/8*log(x^8 - 3*x^4 + 1) + 3/4*log(x^4)
```

mupad [B] time = 1.35, size = 49, normalized size = 0.74

$$3\ln(x)-\frac{1}{4x^4}+\ln\left(x^4-\frac{\sqrt{5}}{2}-\frac{3}{2}\right)\left(\frac{7\sqrt{5}}{40}-\frac{3}{8}\right)-\ln\left(x^4+\frac{\sqrt{5}}{2}-\frac{3}{2}\right)\left(\frac{7\sqrt{5}}{40}+\frac{3}{8}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^5*(x^8 - 3*x^4 + 1)),x)
```

```
[Out] 3*log(x) - 1/(4*x^4) + log(x^4 - 5^(1/2)/2 - 3/2)*((7*5^(1/2))/40 - 3/8) -
log(5^(1/2)/2 + x^4 - 3/2)*((7*5^(1/2))/40 + 3/8)
```

sympy [A] time = 0.19, size = 66, normalized size = 1.00

$$3\log(x)+\left(-\frac{3}{8}+\frac{7\sqrt{5}}{40}\right)\log\left(x^4-\frac{3}{2}-\frac{\sqrt{5}}{2}\right)+\left(-\frac{7\sqrt{5}}{40}-\frac{3}{8}\right)\log\left(x^4-\frac{3}{2}+\frac{\sqrt{5}}{2}\right)-\frac{1}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**5/(x**8-3*x**4+1),x)
```

```
[Out] 3*log(x) + (-3/8 + 7*sqrt(5)/40)*log(x**4 - 3/2 - sqrt(5)/2) + (-7*sqrt(5)/
40 - 3/8)*log(x**4 - 3/2 + sqrt(5)/2) - 1/(4*x**4)
```

3.334 $\int \frac{1}{x^7(1-3x^4+x^8)}\,dx$

Optimal. Leaf size=97

$$-\frac{1}{6x^6}-\frac{3}{2x^2}-\frac{1}{2}\sqrt{\frac{1}{10}\left(123-55\sqrt{5}\right)}\tanh^{-1}\left(\sqrt{\frac{2}{3+\sqrt{5}}}\,x^2\right)+\frac{1}{2}\sqrt{\frac{1}{10}\left(123+55\sqrt{5}\right)}\tanh^{-1}\left(\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}\,x^2\right)$$

Rubi [A] time = 0.09, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {1359, 1123, 1281, 1166, 207}

$$-\frac{3}{2x^2}-\frac{1}{6x^6}-\frac{1}{2}\sqrt{\frac{1}{10}\left(123-55\sqrt{5}\right)}\tanh^{-1}\left(\sqrt{\frac{2}{3+\sqrt{5}}}\,x^2\right)+\frac{1}{2}\sqrt{\frac{1}{10}\left(123+55\sqrt{5}\right)}\tanh^{-1}\left(\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}\,x^2\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^7*(1 - 3*x^4 + x^8)),x]
```

```
[Out] -1/(6*x^6) - 3/(2*x^2) - (Sqrt[(123 - 55*Sqrt[5])/10]*ArcTanh[Sqrt[2/(3 + S
qrt[5])]*x^2])/2 + (Sqrt[(123 + 55*Sqrt[5])/10]*ArcTanh[Sqrt[(3 + Sqrt[5])/
2]*x^2])/2
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 1123

```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*x^2 + c*x^4)^(p + 1))/(a*d*(m + 1)), x] - Dis
t[1/(a*d^2*(m + 1)), Int[(d*x)^(m + 2)*(b*(m + 2*p + 3) + c*(m + 4*p + 5)*x
^2)*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b^2 -
 4*a*c, 0] && LtQ[m, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1281

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[(d*(f*x)^(m + 1)*(a + b*x^2 + c*x^4)^(p + 1)
)/(a*f*(m + 1)), x] + Dist[1/(a*f^2*(m + 1)), Int[(f*x)^(m + 2)*(a + b*x^2
+ c*x^4)^p*Simp[a*e*(m + 1) - b*d*(m + 2*p + 3) - c*d*(m + 4*p + 5)*x^2, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[m
, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1359

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*
x^(n/k) + c*x^((2*n)/k))^p, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, c, p
```

```
}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^7\left(1-3x^4+x^8\right)}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{x^4\left(1-3x^2+x^4\right)}\,dx,x,x^2\right)\\
&= -\frac{1}{6x^6}+\frac{1}{6}\text{Subst}\left(\int \frac{9-3x^2}{x^2\left(1-3x^2+x^4\right)}\,dx,x,x^2\right)\\
&= -\frac{1}{6x^6}-\frac{3}{2x^2}-\frac{1}{6}\text{Subst}\left(\int \frac{-24+9x^2}{1-3x^2+x^4}\,dx,x,x^2\right)\\
&= -\frac{1}{6x^6}-\frac{3}{2x^2}-\frac{1}{20}\left(15-7\sqrt{5}\right)\text{Subst}\left(\int \frac{1}{-\frac{3}{2}-\frac{\sqrt{5}}{2}+x^2}\,dx,x,x^2\right)-\frac{1}{20}\left(15+7\sqrt{5}\right)\text{S}\\
&= -\frac{1}{6x^6}-\frac{3}{2x^2}-\frac{1}{2}\sqrt{\frac{1}{10}\left(123-55\sqrt{5}\right)}\tanh^{-1}\left(\sqrt{\frac{2}{3+\sqrt{5}}}\,x^2\right)+\frac{1}{20}\sqrt{1230+550\sqrt{5}}
\end{aligned}$$

Mathematica [A] time = 0.07, size = 111, normalized size = 1.14

$$\frac{1}{120}\left(-\frac{20}{x^6}-\frac{180}{x^2}-3\left(25+11\sqrt{5}\right)\log\left(-2x^2+\sqrt{5}-1\right)+3\left(25-11\sqrt{5}\right)\log\left(-2x^2+\sqrt{5}+1\right)+3\left(25+11\sqrt{5}\right)\log\left(2x^2+\sqrt{5}-1\right)+3\left(11\sqrt{5}-25\right)\log\left(2x^2+\sqrt{5}+1\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^7*(1 - 3*x^4 + x^8)),x]
```

```
[Out] (-20/x^6 - 180/x^2 - 3*(25 + 11*Sqrt[5])*Log[-1 + Sqrt[5] - 2*x^2] + 3*(25
- 11*Sqrt[5])*Log[1 + Sqrt[5] - 2*x^2] + 3*(25 + 11*Sqrt[5])*Log[-1 + Sqrt[
5] + 2*x^2] + 3*(-25 + 11*Sqrt[5])*Log[1 + Sqrt[5] + 2*x^2])/120
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^7\left(1-3x^4+x^8\right)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x^7*(1 - 3*x^4 + x^8)),x]
```

```
[Out] IntegrateAlgebraic[1/(x^7*(1 - 3*x^4 + x^8)), x]
```

fricas [B] time = 1.20, size = 130, normalized size = 1.34

$$\frac{33\sqrt{5}x^6\log\left(\frac{2x^4+2x^2+\sqrt{5}\left(2x^2+1\right)+3}{x^4+x^2-1}\right)+33\sqrt{5}x^6\log\left(\frac{2x^4-2x^2+\sqrt{5}\left(2x^2-1\right)+3}{x^4-x^2-1}\right)-75x^6\log\left(x^4+x^2-1\right)+75x^6\log\left(x^4-x^2-1\right)-180x^4-20}{120x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(x^8-3*x^4+1),x, algorithm="fricas")
```

```
[Out] 1/120*(33*sqrt(5)*x^6*log((2*x^4 + 2*x^2 + sqrt(5)*(2*x^2 + 1) + 3)/(x^4 +
x^2 - 1)) + 33*sqrt(5)*x^6*log((2*x^4 - 2*x^2 + sqrt(5)*(2*x^2 - 1) + 3)/(x
^4 - x^2 - 1)) - 75*x^6*log(x^4 + x^2 - 1) + 75*x^6*log(x^4 - x^2 - 1) - 18
0*x^4 - 20)/x^6
```

giac [A] time = 0.41, size = 104, normalized size = 1.07

$$-\frac{11}{40}\sqrt{5}\log\left(\frac{\left|2x^2-\sqrt{5}+1\right|}{2x^2+\sqrt{5}+1}\right)-\frac{11}{40}\sqrt{5}\log\left(\frac{\left|2x^2-\sqrt{5}-1\right|}{\left|2x^2+\sqrt{5}-1\right|}\right)-\frac{9x^4+1}{6x^6}-\frac{5}{8}\log\left(\left|x^4+x^2-1\right|\right)+\frac{5}{8}\log\left(\left|x^4-x^2-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(x^8-3*x^4+1),x, algorithm="giac")
```

```
[Out] -11/40*sqrt(5)*log(abs(2*x^2 - sqrt(5) + 1)/(2*x^2 + sqrt(5) + 1)) - 11/40*
sqrt(5)*log(abs(2*x^2 - sqrt(5) - 1)/abs(2*x^2 + sqrt(5) - 1)) - 1/6*(9*x^4
 + 1)/x^6 - 5/8*log(abs(x^4 + x^2 - 1)) + 5/8*log(abs(x^4 - x^2 - 1))
```

maple [A] time = 0.01, size = 72, normalized size = 0.74

$$\frac{11\sqrt{5}\ \operatorname{arctanh}\left(\frac{\left(2x^2-1\right)\sqrt{5}}{5}\right)}{20}+\frac{11\sqrt{5}\ \operatorname{arctanh}\left(\frac{\left(2x^2+1\right)\sqrt{5}}{5}\right)}{20}+\frac{5\ln\left(x^4-x^2-1\right)}{8}-\frac{5\ln\left(x^4+x^2-1\right)}{8}-\frac{3}{2x^2}-\frac{1}{6x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^7/(x^8-3*x^4+1),x)
```

```
[Out] -1/6/x^6-3/2/x^2+5/8*ln(x^4-x^2-1)+11/20*5^(1/2)*arctanh(1/5*(2*x^2-1)*5^(1
/2))-5/8*ln(x^4+x^2-1)+11/20*5^(1/2)*arctanh(1/5*(2*x^2+1)*5^(1/2))
```

maxima [A] time = 1.32, size = 99, normalized size = 1.02

$$-\frac{11}{40}\sqrt{5}\log\left(\frac{2x^2-\sqrt{5}+1}{2x^2+\sqrt{5}+1}\right)-\frac{11}{40}\sqrt{5}\log\left(\frac{2x^2-\sqrt{5}-1}{2x^2+\sqrt{5}-1}\right)-\frac{9x^4+1}{6x^6}-\frac{5}{8}\log\left(x^4+x^2-1\right)+\frac{5}{8}\log\left(x^4-x^2-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(x^8-3*x^4+1),x, algorithm="maxima")
```

```
[Out] -11/40*sqrt(5)*log((2*x^2 - sqrt(5) + 1)/(2*x^2 + sqrt(5) + 1)) - 11/40*sqr
t(5)*log((2*x^2 - sqrt(5) - 1)/(2*x^2 + sqrt(5) - 1)) - 1/6*(9*x^4 + 1)/x^6
 - 5/8*log(x^4 + x^2 - 1) + 5/8*log(x^4 - x^2 - 1)
```

mupad [B] time = 1.38, size = 95, normalized size = 0.98

$$\operatorname{atanh}\left(\frac{4126100\,x^2}{1140425\sqrt{5}-2550075}-\frac{1845250\sqrt{5}\,x^2}{1140425\sqrt{5}-2550075}\right)\left(\frac{11\sqrt{5}}{20}-\frac{5}{4}\right)+\operatorname{atanh}\left(\frac{4126100\,x^2}{1140425\sqrt{5}+2550075}+\frac{1845250\sqrt{5}\,x^2}{1140425\sqrt{5}+2550075}\right)\left(\frac{11\sqrt{5}}{20}+\frac{5}{4}\right)-\frac{\frac{3x^4}{2}+\frac{1}{6}}{x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^7*(x^8 - 3*x^4 + 1)),x)
```

```
[Out] atanh((4126100*x^2)/(1140425*5^(1/2) - 2550075) - (1845250*5^(1/2)*x^2)/(11
40425*5^(1/2) - 2550075))*((11*5^(1/2))/20 - 5/4) + atanh((4126100*x^2)/(11
40425*5^(1/2) + 2550075) + (1845250*5^(1/2)*x^2)/(1140425*5^(1/2) + 2550075
))*((11*5^(1/2))/20 + 5/4) - ((3*x^4)/2 + 1/6)/x^6
```

sympy [B] time = 0.44, size = 199, normalized size = 2.05

$$\left(\frac{11\sqrt{5}}{40}+\frac{5}{8}\right)\log\left(x^2-\frac{2207}{22}-\frac{2207\sqrt{5}}{50}+\frac{1152\left(\frac{11\sqrt{5}}{40}+\frac{5}{8}\right)^3}{11}\right)+\left(\frac{5}{8}-\frac{11\sqrt{5}}{40}\right)\log\left(x^2-\frac{2207}{22}+\frac{1152\left(\frac{5}{8}-\frac{11\sqrt{5}}{40}\right)^3}{11}+\frac{2207\sqrt{5}}{50}\right)+\left(-\frac{5}{8}+\frac{11\sqrt{5}}{40}\right)\log\left(x^2-\frac{2207\sqrt{5}}{50}+\frac{1152\left(-\frac{5}{8}+\frac{11\sqrt{5}}{40}\right)^3}{11}+\frac{2207}{22}\right)+\left(-\frac{5}{8}-\frac{11\sqrt{5}}{40}\right)\log\left(x^2+\frac{1152\left(-\frac{5}{8}-\frac{11\sqrt{5}}{40}\right)^3}{11}+\frac{2207\sqrt{5}}{50}+\frac{2207}{22}\right)+\frac{-9x^4-1}{6x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**7/(x**8-3*x**4+1),x)
```

```
[Out] (11*sqrt(5)/40 + 5/8)*log(x**2 - 2207/22 - 2207*sqrt(5)/50 + 1152*(11*sqrt(
5)/40 + 5/8)**3/11) + (5/8 - 11*sqrt(5)/40)*log(x**2 - 2207/22 + 1152*(5/8
- 11*sqrt(5)/40)**3/11 + 2207*sqrt(5)/50) + (-5/8 + 11*sqrt(5)/40)*log(x**2
 - 2207*sqrt(5)/50 + 1152*(-5/8 + 11*sqrt(5)/40)**3/11 + 2207/22) + (-5/8 -
 11*sqrt(5)/40)*log(x**2 + 1152*(-5/8 - 11*sqrt(5)/40)**3/11 + 2207*sqrt(5)
/50 + 2207/22) + (-9*x**4 - 1)/(6*x**6)
```

3.335 $\int \frac{x^8}{1-3x^4+x^8}\,dx$

Optimal. Leaf size=170

$$x-\frac{\sqrt[4]{\frac{1}{2}\left(123+55\sqrt{5}\right)}\,\tan^{-1}\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}}\,x\right)}{2\sqrt{5}}+\frac{\sqrt[4]{984-440\sqrt{5}}\,\tan^{-1}\left(\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}\,x\right)}{4\sqrt{5}}-\frac{\sqrt[4]{\frac{1}{2}\left(123+55\sqrt{5}\right)}\,\tanh^{-}}{2\sqrt{5}}$$

Rubi [A] time = 0.11, antiderivative size = 170, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {1367, 1422, 212, 206, 203}

$$x-\frac{\sqrt[4]{\frac{1}{2}\left(123+55\sqrt{5}\right)}\,\tan^{-1}\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}}\,x\right)}{2\sqrt{5}}+\frac{\sqrt[4]{984-440\sqrt{5}}\,\tan^{-1}\left(\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}\,x\right)}{4\sqrt{5}}-\frac{\sqrt[4]{\frac{1}{2}\left(123+55\sqrt{5}\right)}\,\tanh^{-1}\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}}\,x\right)}{2\sqrt{5}}+\frac{\sqrt[4]{984-440\sqrt{5}}\,\tanh^{-1}\left(\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}\,x\right)}{4\sqrt{5}}$$

Antiderivative was successfully verified.

```
[In] Int[x^8/(1 - 3*x^4 + x^8),x]
```

```
[Out] x - (((123 + 55*Sqrt[5])/2)^(1/4)*ArcTan[(2/(3 + Sqrt[5]))^(1/4)*x])/(2*Sqr
t[5]) + ((984 - 440*Sqrt[5])^(1/4)*ArcTan[((3 + Sqrt[5])/2)^(1/4)*x])/(4*Sq
rt[5]) - (((123 + 55*Sqrt[5])/2)^(1/4)*ArcTanh[(2/(3 + Sqrt[5]))^(1/4)*x])/
(2*Sqrt[5]) + ((984 - 440*Sqrt[5])^(1/4)*ArcTanh[((3 + Sqrt[5])/2)^(1/4)*x]
)/(4*Sqrt[5])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 1367

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Simp[(d^(2*n - 1)*(d*x)^(m - 2*n + 1)*(a + b*x^n + c*x^(2*n))^(
p + 1))/(c*(m + 2*n*p + 1)), x] - Dist[d^(2*n)/(c*(m + 2*n*p + 1)), Int[(d*
x)^(m - 2*n)*Simp[a*(m - 2*n + 1) + b*(m + n*(p - 1) + 1)*x^n, x]*(a + b*x^
n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && EqQ[n2, 2*n] && Ne
Q[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GtQ[m, 2*n - 1] && NeQ[m + 2*n*p + 1, 0]
 && IntegerQ[p]
```

Rule 1422

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q),
Int[1/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(
b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n]
```

```
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (PosQ[b^2 - 4*a
*c] ||  !IGtQ[n/2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^8}{1-3x^4+x^8}\,dx &= x-\int \frac{1-3x^4}{1-3x^4+x^8}\,dx\\ &= x-\frac{1}{10}\left(-15+7\sqrt{5}\right)\int \frac{1}{-\frac{3}{2}+\frac{\sqrt{5}}{2}+x^4}\,dx+\frac{1}{10}\left(15+7\sqrt{5}\right)\int \frac{1}{-\frac{3}{2}-\frac{\sqrt{5}}{2}+x^4}\,dx\\ &= x+\sqrt{\frac{1}{10}\left(9-4\sqrt{5}\right)}\int \frac{1}{\sqrt{3-\sqrt{5}}-\sqrt{2}\,x^2}\,dx+\sqrt{\frac{1}{10}\left(9-4\sqrt{5}\right)}\int \frac{1}{\sqrt{3-\sqrt{5}}+\sqrt{2}\,x^2}\,dx\\ &= x-\frac{\sqrt[4]{\frac{1}{2}\left(123+55\sqrt{5}\right)}\tan^{-1}\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}}\,x\right)}{2\sqrt{5}}+\frac{\sqrt[4]{\frac{1}{2}\left(123-55\sqrt{5}\right)}\tan^{-1}\left(\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}\,x\right)}{2\sqrt{5}}-\cdots\end{aligned}$$

Mathematica [A] time = 0.27, size = 160, normalized size = 0.94

$$x+\frac{\left(\sqrt{5}-2\right)\tan^{-1}\left(\sqrt{\frac{2}{\sqrt{5}-1}}\,x\right)}{\sqrt{10\left(\sqrt{5}-1\right)}}-\frac{\left(2+\sqrt{5}\right)\tan^{-1}\left(\sqrt{\frac{2}{1+\sqrt{5}}}\,x\right)}{\sqrt{10\left(1+\sqrt{5}\right)}}+\frac{\left(\sqrt{5}-2\right)\tanh^{-1}\left(\sqrt{\frac{2}{\sqrt{5}-1}}\,x\right)}{\sqrt{10\left(\sqrt{5}-1\right)}}-\frac{\left(2+\sqrt{5}\right)\tanh^{-1}\left(\sqrt{\frac{2}{1+\sqrt{5}}}\,x\right)}{\sqrt{10\left(1+\sqrt{5}\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8/(1 - 3*x^4 + x^8),x]
```

```
[Out] x + ((-2 + Sqrt[5])*ArcTan[Sqrt[2/(-1 + Sqrt[5])]*x])/Sqrt[10*(-1 + Sqrt[5]
)] - ((2 + Sqrt[5])*ArcTan[Sqrt[2/(1 + Sqrt[5])]*x])/Sqrt[10*(1 + Sqrt[5])]
 + ((-2 + Sqrt[5])*ArcTanh[Sqrt[2/(-1 + Sqrt[5])]*x])/Sqrt[10*(-1 + Sqrt[5]
)] - ((2 + Sqrt[5])*ArcTanh[Sqrt[2/(1 + Sqrt[5])]*x])/Sqrt[10*(1 + Sqrt[5])
]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^8}{1-3x^4+x^8}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^8/(1 - 3*x^4 + x^8),x]
```

```
[Out] IntegrateAlgebraic[x^8/(1 - 3*x^4 + x^8), x]
```

fricas [B] time = 1.31, size = 304, normalized size = 1.79

$$-\frac{1}{10}\sqrt{10}\sqrt{5\sqrt{5}+11}\arctan\left(\frac{1}{20}\left(\sqrt{10}\sqrt{2x^2+\sqrt{5}+1}\left(2\sqrt{5}\sqrt{2}-5\sqrt{2}\right)-2\sqrt{10}\left(2\sqrt{5}x-5x\right)\right)\sqrt{5\sqrt{5}+11}\right)-\frac{1}{10}\sqrt{10}\sqrt{5\sqrt{5}-11}\arctan\left(\frac{1}{20}\left(\sqrt{10}\sqrt{2x^2+\sqrt{5}-1}\left(2\sqrt{5}\sqrt{2}+5\sqrt{2}\right)-2\sqrt{10}\left(2\sqrt{5}x+5x\right)\right)\sqrt{5\sqrt{5}-11}\right)+\frac{1}{40}\sqrt{10}\sqrt{5\sqrt{5}-11}\log\left(\sqrt{10}\sqrt{5\sqrt{5}-11}\left(3\sqrt{5}+5\right)+20x\right)-\frac{1}{40}\sqrt{10}\sqrt{5\sqrt{5}-11}\log\left(-\sqrt{10}\sqrt{5\sqrt{5}-11}\left(3\sqrt{5}+5\right)+20x\right)-\frac{1}{40}\sqrt{10}\sqrt{5\sqrt{5}+11}\log\left(\sqrt{10}\sqrt{5\sqrt{5}+11}\left(3\sqrt{5}-5\right)+20x\right)+\frac{1}{40}\sqrt{10}\sqrt{5\sqrt{5}+11}\log\left(-\sqrt{10}\sqrt{5\sqrt{5}+11}\left(3\sqrt{5}-5\right)+20x\right)+x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(x^8-3*x^4+1),x, algorithm="fricas")
```

```
[Out] -1/10*sqrt(10)*sqrt(5*sqrt(5) + 11)*arctan(1/20*(sqrt(10)*sqrt(2*x^2 + sqrt
(5) + 1)*(2*sqrt(5)*sqrt(2) - 5*sqrt(2)) - 2*sqrt(10)*(2*sqrt(5)*x - 5*x))*
sqrt(5*sqrt(5) + 11)) - 1/10*sqrt(10)*sqrt(5*sqrt(5) - 11)*arctan(1/20*(sqr
t(10)*sqrt(2*x^2 + sqrt(5) - 1)*(2*sqrt(5)*sqrt(2) + 5*sqrt(2)) - 2*sqrt(10
)*(2*sqrt(5)*x + 5*x))*sqrt(5*sqrt(5) - 11)) + 1/40*sqrt(10)*sqrt(5*sqrt(5)
```

```
 - 11)*log(sqrt(10)*sqrt(5*sqrt(5) - 11)*(3*sqrt(5) + 5) + 20*x) - 1/40*sqr
t(10)*sqrt(5*sqrt(5) - 11)*log(-sqrt(10)*sqrt(5*sqrt(5) - 11)*(3*sqrt(5) +
5) + 20*x) - 1/40*sqrt(10)*sqrt(5*sqrt(5) + 11)*log(sqrt(10)*sqrt(5*sqrt(5)
 + 11)*(3*sqrt(5) - 5) + 20*x) + 1/40*sqrt(10)*sqrt(5*sqrt(5) + 11)*log(-sq
rt(10)*sqrt(5*sqrt(5) + 11)*(3*sqrt(5) - 5) + 20*x) + x
```

giac [A] time = 0.68, size = 148, normalized size = 0.87

$$-\frac{1}{20}\sqrt{50\sqrt{5}+110}\arctan\left(\frac{x}{\sqrt{\frac{1}{2}\sqrt{5}+\frac{1}{2}}}\right)+\frac{1}{20}\sqrt{50\sqrt{5}-110}\arctan\left(\frac{x}{\sqrt{\frac{1}{2}\sqrt{5}-\frac{1}{2}}}\right)-\frac{1}{40}\sqrt{50\sqrt{5}+110}\log\left(\left|x+\sqrt{\frac{1}{2}\sqrt{5}+\frac{1}{2}}\right|\right)+\frac{1}{40}\sqrt{50\sqrt{5}+110}\log\left(\left|x-\sqrt{\frac{1}{2}\sqrt{5}+\frac{1}{2}}\right|\right)+\frac{1}{40}\sqrt{50\sqrt{5}-110}\log\left(\left|x+\sqrt{\frac{1}{2}\sqrt{5}-\frac{1}{2}}\right|\right)-\frac{1}{40}\sqrt{50\sqrt{5}-110}\log\left(\left|x-\sqrt{\frac{1}{2}\sqrt{5}-\frac{1}{2}}\right|\right)+x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(x^8-3*x^4+1),x, algorithm="giac")
```

```
[Out] -1/20*sqrt(50*sqrt(5) + 110)*arctan(x/sqrt(1/2*sqrt(5) + 1/2)) + 1/20*sqrt(
50*sqrt(5) - 110)*arctan(x/sqrt(1/2*sqrt(5) - 1/2)) - 1/40*sqrt(50*sqrt(5)
+ 110)*log(abs(x + sqrt(1/2*sqrt(5) + 1/2))) + 1/40*sqrt(50*sqrt(5) + 110)*
log(abs(x - sqrt(1/2*sqrt(5) + 1/2))) + 1/40*sqrt(50*sqrt(5) - 110)*log(abs
(x + sqrt(1/2*sqrt(5) - 1/2))) - 1/40*sqrt(50*sqrt(5) - 110)*log(abs(x - sq
rt(1/2*sqrt(5) - 1/2))) + x
```

maple [A] time = 0.06, size = 205, normalized size = 1.21

$$x+\frac{\operatorname{arctanh}\left(\frac{2x}{\sqrt{-2+2\sqrt{5}}}\right)}{\sqrt{-2+2\sqrt{5}}}-\frac{2\sqrt{5}\operatorname{arctanh}\left(\frac{2x}{\sqrt{-2+2\sqrt{5}}}\right)}{5\sqrt{-2+2\sqrt{5}}}-\frac{2\sqrt{5}\operatorname{arctanh}\left(\frac{2x}{\sqrt{2+2\sqrt{5}}}\right)}{5\sqrt{2+2\sqrt{5}}}-\frac{\operatorname{arctanh}\left(\frac{2x}{\sqrt{2+2\sqrt{5}}}\right)}{\sqrt{2+2\sqrt{5}}}+\frac{\arctan\left(\frac{2x}{\sqrt{-2+2\sqrt{5}}}\right)}{\sqrt{-2+2\sqrt{5}}}-\frac{2\sqrt{5}\arctan\left(\frac{2x}{\sqrt{-2+2\sqrt{5}}}\right)}{5\sqrt{-2+2\sqrt{5}}}-\frac{2\sqrt{5}\arctan\left(\frac{2x}{\sqrt{2+2\sqrt{5}}}\right)}{5\sqrt{2+2\sqrt{5}}}-\frac{\arctan\left(\frac{2x}{\sqrt{2+2\sqrt{5}}}\right)}{\sqrt{2+2\sqrt{5}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(x^8-3*x^4+1),x)
```

```
[Out] x-2/5*5^(1/2)/(2+2*5^(1/2))^(1/2)*arctanh(2*x/(2+2*5^(1/2))^(1/2))-1/(2+2*5
^(1/2))^(1/2)*arctanh(2*x/(2+2*5^(1/2))^(1/2))+1/(-2+2*5^(1/2))^(1/2)*arcta
n(2*x/(-2+2*5^(1/2))^(1/2))-2/5*5^(1/2)/(-2+2*5^(1/2))^(1/2)*arctan(2*x/(-2
+2*5^(1/2))^(1/2))+1/(-2+2*5^(1/2))^(1/2)*arctanh(2*x/(-2+2*5^(1/2))^(1/2))
-2/5*5^(1/2)/(-2+2*5^(1/2))^(1/2)*arctanh(2*x/(-2+2*5^(1/2))^(1/2))-2/5*5^(
1/2)/(2+2*5^(1/2))^(1/2)*arctan(2*x/(2+2*5^(1/2))^(1/2))-1/(2+2*5^(1/2))^(1
/2)*arctan(2*x/(2+2*5^(1/2))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$x+\frac{1}{2}\int\frac{2x^2+1}{x^4-x^2-1}\,dx-\frac{1}{2}\int\frac{2x^2-1}{x^4+x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(x^8-3*x^4+1),x, algorithm="maxima")
```

```
[Out] x + 1/2*integrate((2*x^2 + 1)/(x^4 - x^2 - 1), x) - 1/2*integrate((2*x^2 -
1)/(x^4 + x^2 - 1), x)
```

mupad [B] time = 1.44, size = 246, normalized size = 1.45

$$x-\frac{\operatorname{atan}\left(\frac{x\sqrt{-50\sqrt{5}-110}\,55\mathrm{i}}{2\left(275\sqrt{5}+605\right)}+\frac{\sqrt{5}\,x\sqrt{-50\sqrt{5}-110}\,33\mathrm{i}}{2\left(275\sqrt{5}+605\right)}\right)\sqrt{-50\sqrt{5}-110}\,1\mathrm{i}}{20}-\frac{\operatorname{atan}\left(\frac{x\sqrt{110-50\sqrt{5}}\,55\mathrm{i}}{2\left(275\sqrt{5}-605\right)}-\frac{\sqrt{5}\,x\sqrt{110-50\sqrt{5}}\,33\mathrm{i}}{2\left(275\sqrt{5}-605\right)}\right)\sqrt{110-50\sqrt{5}}\,1\mathrm{i}}{20}+\frac{\operatorname{atan}\left(\frac{x\sqrt{50\sqrt{5}-110}\,55\mathrm{i}}{2\left(275\sqrt{5}-605\right)}-\frac{\sqrt{5}\,x\sqrt{50\sqrt{5}-110}\,33\mathrm{i}}{2\left(275\sqrt{5}-605\right)}\right)\sqrt{50\sqrt{5}-110}\,1\mathrm{i}}{20}+\frac{\operatorname{atan}\left(\frac{x\sqrt{50\sqrt{5}+110}\,55\mathrm{i}}{2\left(275\sqrt{5}+605\right)}+\frac{\sqrt{5}\,x\sqrt{50\sqrt{5}+110}\,33\mathrm{i}}{2\left(275\sqrt{5}+605\right)}\right)\sqrt{50\sqrt{5}+110}\,1\mathrm{i}}{20}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(x^8 - 3*x^4 + 1),x)
```

```
[Out] x - (atan((x*(- 50*5^(1/2) - 110)^(1/2)*55i)/(2*(275*5^(1/2) + 605)) + (5^(
1/2)*x*(- 50*5^(1/2) - 110)^(1/2)*33i)/(2*(275*5^(1/2) + 605)))*(- 50*5^(1/
2) - 110)^(1/2)*1i)/20 - (atan((x*(110 - 50*5^(1/2))^(1/2)*55i)/(2*(275*5^(
1/2) - 605)) - (5^(1/2)*x*(110 - 50*5^(1/2))^(1/2)*33i)/(2*(275*5^(1/2) - 6
```

```
05)))*(110 - 50*5^(1/2))^(1/2)*1i)/20 + (atan((x*(50*5^(1/2) - 110)^(1/2)*5
5i)/(2*(275*5^(1/2) - 605)) - (5^(1/2)*x*(50*5^(1/2) - 110)^(1/2)*33i)/(2*(
275*5^(1/2) - 605)))*(50*5^(1/2) - 110)^(1/2)*1i)/20 + (atan((x*(50*5^(1/2)
 + 110)^(1/2)*55i)/(2*(275*5^(1/2) + 605)) + (5^(1/2)*x*(50*5^(1/2) + 110)^
(1/2)*33i)/(2*(275*5^(1/2) + 605)))*(50*5^(1/2) + 110)^(1/2)*1i)/20
```

sympy [A] time = 1.24, size = 58, normalized size = 0.34

$$x+\operatorname{RootSum}\left(6400t^4-880t^2-1,\left(t\mapsto t\log\left(-\frac{15360t^5}{11}+\frac{1288t}{55}+x\right)\right)\right)+\operatorname{RootSum}\left(6400t^4+880t^2-1,\left(t\mapsto t\log\left(-\frac{15360t^5}{11}+\frac{1288t}{55}+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8/(x**8-3*x**4+1),x)
```

```
[Out] x + RootSum(6400*_t**4 - 880*_t**2 - 1, Lambda(_t, _t*log(-15360*_t**5/11 +
 1288*_t/55 + x))) + RootSum(6400*_t**4 + 880*_t**2 - 1, Lambda(_t, _t*log(
-15360*_t**5/11 + 1288*_t/55 + x)))
```

3.336 $\int \frac{x^6}{1-3x^4+x^8}\,dx$

Optimal. Leaf size=167

$$\frac{\left(3+\sqrt{5}\right)^{3/4}\tan^{-1}\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}}\,x\right)}{2\,2^{3/4}\sqrt{5}}-\frac{\sqrt[4]{144-64\sqrt{5}}\,\tan^{-1}\left(\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}\,x\right)}{4\sqrt{5}}-\frac{\left(3+\sqrt{5}\right)^{3/4}\tanh^{-1}\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}}\,x\right)}{2\,2^{3/4}\sqrt{5}}+$$

Rubi [A] time = 0.08, antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1374, 298, 203, 206}

$$\frac{\left(3+\sqrt{5}\right)^{3/4}\tan^{-1}\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}}\,x\right)}{2\,2^{3/4}\sqrt{5}}-\frac{\sqrt[4]{144-64\sqrt{5}}\,\tan^{-1}\left(\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}\,x\right)}{4\sqrt{5}}-\frac{\left(3+\sqrt{5}\right)^{3/4}\tanh^{-1}\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}}\,x\right)}{2\,2^{3/4}\sqrt{5}}+\frac{\sqrt[4]{144-64\sqrt{5}}\,\tanh^{-1}\left(\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}\,x\right)}{4\sqrt{5}}$$

Antiderivative was successfully verified.

```
[In] Int[x^6/(1 - 3*x^4 + x^8),x]
```

```
[Out] ((3 + Sqrt[5])^(3/4)*ArcTan[(2/(3 + Sqrt[5]))^(1/4)*x])/(2*2^(3/4)*Sqrt[5])
 - ((144 - 64*Sqrt[5])^(1/4)*ArcTan[((3 + Sqrt[5])/2)^(1/4)*x])/(4*Sqrt[5])
 - ((3 + Sqrt[5])^(3/4)*ArcTanh[(2/(3 + Sqrt[5]))^(1/4)*x])/(2*2^(3/4)*Sqrt
[5]) + ((144 - 64*Sqrt[5])^(1/4)*ArcTanh[((3 + Sqrt[5])/2)^(1/4)*x])/(4*Sqr
t[5])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 1374

```
Int[((d_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(d^n*(b/q + 1))/2, Int[(d*x)^(m -
 n)/(b/2 + q/2 + c*x^n), x], x] - Dist[(d^n*(b/q - 1))/2, Int[(d*x)^(m - n)
/(b/2 - q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[n2, 2*n] &&
NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GeQ[m, n]
```

Rubi steps

$$\int \frac{x^6}{1-3x^4+x^8}\,dx = \frac{1}{10}\left(5-3\sqrt{5}\right)\int \frac{x^2}{-\frac{3}{2}+\frac{\sqrt{5}}{2}+x^4}\,dx + \frac{1}{10}\left(5+3\sqrt{5}\right)\int \frac{x^2}{-\frac{3}{2}-\frac{\sqrt{5}}{2}+x^4}\,dx$$

$$= \frac{\left(3-\sqrt{5}\right)\int \frac{1}{\sqrt{3-\sqrt{5}}-\sqrt{2}x^2}\,dx}{2\sqrt{10}} - \frac{\left(3-\sqrt{5}\right)\int \frac{1}{\sqrt{3-\sqrt{5}}+\sqrt{2}x^2}\,dx}{2\sqrt{10}} - \frac{\left(3+\sqrt{5}\right)\int \frac{1}{\sqrt{3+\sqrt{5}}-\sqrt{2}x^2}\,dx}{2\sqrt{10}}$$

$$= \frac{\left(3+\sqrt{5}\right)^{3/4}\tan^{-1}\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}}\,x\right)}{2\,2^{3/4}\sqrt{5}} - \frac{\left(3-\sqrt{5}\right)^{3/4}\tan^{-1}\left(\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}\,x\right)}{2\,2^{3/4}\sqrt{5}} - \frac{\left(3+\sqrt{5}\right)^{3/4}\tanh^{-1}}{2\,2^{3/4}\sqrt{}}$$

Mathematica [A] time = 0.15, size = 160, normalized size = 0.96

$$\frac{\frac{\left(\sqrt{5}-3\right)\tan^{-1}\left(\sqrt{\frac{2}{\sqrt{5}-1}}\,x\right)}{\sqrt{\sqrt{5}-1}} + \frac{\left(3+\sqrt{5}\right)\tan^{-1}\left(\sqrt{\frac{2}{1+\sqrt{5}}}\,x\right)}{\sqrt{1+\sqrt{5}}} - \frac{\left(\sqrt{5}-3\right)\tanh^{-1}\left(\sqrt{\frac{2}{\sqrt{5}-1}}\,x\right)}{\sqrt{\sqrt{5}-1}} - \frac{\left(3+\sqrt{5}\right)\tanh^{-1}\left(\sqrt{\frac{2}{1+\sqrt{5}}}\,x\right)}{\sqrt{1+\sqrt{5}}}}{2\sqrt{10}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6/(1 - 3*x^4 + x^8),x]
```

```
[Out] (((-3 + Sqrt[5])*ArcTan[Sqrt[2/(-1 + Sqrt[5])]*x])/Sqrt[-1 + Sqrt[5]] + ((3
 + Sqrt[5])*ArcTan[Sqrt[2/(1 + Sqrt[5])]*x])/Sqrt[1 + Sqrt[5]] - ((-3 + Sqr
t[5])*ArcTanh[Sqrt[2/(-1 + Sqrt[5])]*x])/Sqrt[-1 + Sqrt[5]] - ((3 + Sqrt[5]
)*ArcTanh[Sqrt[2/(1 + Sqrt[5])]*x])/Sqrt[1 + Sqrt[5]])/(2*Sqrt[10])
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6}{1-3x^4+x^8}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^6/(1 - 3*x^4 + x^8),x]
```

```
[Out] IntegrateAlgebraic[x^6/(1 - 3*x^4 + x^8), x]
```

fricas [B] time = 1.27, size = 255, normalized size = 1.53

$$\frac{1}{5}\sqrt{5}\sqrt{\sqrt{5}+2}\arctan\left(\frac{1}{4}\sqrt{2x^2+\sqrt{5}+1}\left(\sqrt{5}\sqrt{2}-3\sqrt{2}\right)\sqrt{\sqrt{5}+2}-\frac{1}{2}\left(\sqrt{5}x-3x\right)\sqrt{\sqrt{5}+2}\right)+\frac{1}{5}\sqrt{5}\sqrt{\sqrt{5}-2}\arctan\left(\frac{1}{4}\sqrt{2x^2+\sqrt{5}-1}\left(\sqrt{5}\sqrt{2}+3\sqrt{2}\right)\sqrt{\sqrt{5}-2}-\frac{1}{2}\left(\sqrt{5}x+3x\right)\sqrt{\sqrt{5}-2}\right)-\frac{1}{20}\sqrt{5}\sqrt{\sqrt{5}+2}\log\left(\sqrt{\sqrt{5}+2}\left(\sqrt{5}-1\right)+2x\right)+\frac{1}{20}\sqrt{5}\sqrt{\sqrt{5}+2}\log\left(-\sqrt{\sqrt{5}+2}\left(\sqrt{5}-1\right)+2x\right)+\frac{1}{20}\sqrt{5}\sqrt{\sqrt{5}-2}\log\left(\left(\sqrt{5}+1\right)\sqrt{\sqrt{5}-2}+2x\right)-\frac{1}{20}\sqrt{5}\sqrt{\sqrt{5}-2}\log\left(-\left(\sqrt{5}+1\right)\sqrt{\sqrt{5}-2}+2x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(x^8-3*x^4+1),x, algorithm="fricas")
```

```
[Out] 1/5*sqrt(5)*sqrt(sqrt(5) + 2)*arctan(1/4*sqrt(2*x^2 + sqrt(5) + 1)*(sqrt(5)
*sqrt(2) - 3*sqrt(2))*sqrt(sqrt(5) + 2) - 1/2*(sqrt(5)*x - 3*x)*sqrt(sqrt(5
) + 2)) + 1/5*sqrt(5)*sqrt(sqrt(5) - 2)*arctan(1/4*sqrt(2*x^2 + sqrt(5) - 1
)*(sqrt(5)*sqrt(2) + 3*sqrt(2))*sqrt(sqrt(5) - 2) - 1/2*(sqrt(5)*x + 3*x)*s
qrt(sqrt(5) - 2)) - 1/20*sqrt(5)*sqrt(sqrt(5) + 2)*log(sqrt(sqrt(5) + 2)*(s
qrt(5) - 1) + 2*x) + 1/20*sqrt(5)*sqrt(sqrt(5) + 2)*log(-sqrt(sqrt(5) + 2)*
(sqrt(5) - 1) + 2*x) + 1/20*sqrt(5)*sqrt(sqrt(5) - 2)*log((sqrt(5) + 1)*sqr
t(sqrt(5) - 2) + 2*x) - 1/20*sqrt(5)*sqrt(sqrt(5) - 2)*log(-(sqrt(5) + 1)*s
qrt(sqrt(5) - 2) + 2*x)
```

giac [A] time = 0.75, size = 147, normalized size = 0.88

$$\frac{1}{10}\sqrt{5\sqrt{5}+10}\arctan\left(\frac{x}{\sqrt{\frac{1}{2}\sqrt{5}+\frac{1}{2}}}\right)-\frac{1}{10}\sqrt{5\sqrt{5}-10}\arctan\left(\frac{x}{\sqrt{\frac{1}{2}\sqrt{5}-\frac{1}{2}}}\right)-\frac{1}{20}\sqrt{5\sqrt{5}+10}\log\left(\left|x+\sqrt{\frac{1}{2}\sqrt{5}+\frac{1}{2}}\right|\right)+\frac{1}{20}\sqrt{5\sqrt{5}+10}\log\left(\left|x-\sqrt{\frac{1}{2}\sqrt{5}+\frac{1}{2}}\right|\right)+\frac{1}{20}\sqrt{5\sqrt{5}-10}\log\left(\left|x+\sqrt{\frac{1}{2}\sqrt{5}-\frac{1}{2}}\right|\right)-\frac{1}{20}\sqrt{5\sqrt{5}-10}\log\left(\left|x-\sqrt{\frac{1}{2}\sqrt{5}-\frac{1}{2}}\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(x^8-3*x^4+1),x, algorithm="giac")
```

```
[Out] 1/10*sqrt(5*sqrt(5) + 10)*arctan(x/sqrt(1/2*sqrt(5) + 1/2)) - 1/10*sqrt(5*s
qrt(5) - 10)*arctan(x/sqrt(1/2*sqrt(5) - 1/2)) - 1/20*sqrt(5*sqrt(5) + 10)*
log(abs(x + sqrt(1/2*sqrt(5) + 1/2))) + 1/20*sqrt(5*sqrt(5) + 10)*log(abs(x
 - sqrt(1/2*sqrt(5) + 1/2))) + 1/20*sqrt(5*sqrt(5) - 10)*log(abs(x + sqrt(1
/2*sqrt(5) - 1/2))) - 1/20*sqrt(5*sqrt(5) - 10)*log(abs(x - sqrt(1/2*sqrt(5
) - 1/2)))
```

maple [A] time = 0.03, size = 206, normalized size = 1.23

$$-\frac{\operatorname{arctanh}\left(\frac{2x}{\sqrt{-2+2\sqrt{5}}}\right)}{2\sqrt{-2+2\sqrt{5}}}+\frac{3\sqrt{5}\,\operatorname{arctanh}\left(\frac{2x}{\sqrt{-2+2\sqrt{5}}}\right)}{10\sqrt{-2+2\sqrt{5}}}-\frac{\operatorname{arctanh}\left(\frac{2x}{\sqrt{2+2\sqrt{5}}}\right)}{2\sqrt{2+2\sqrt{5}}}-\frac{3\sqrt{5}\,\operatorname{arctanh}\left(\frac{2x}{\sqrt{2+2\sqrt{5}}}\right)}{10\sqrt{2+2\sqrt{5}}}+\frac{\arctan\left(\frac{2x}{\sqrt{-2+2\sqrt{5}}}\right)}{2\sqrt{-2+2\sqrt{5}}}-\frac{3\sqrt{5}\,\arctan\left(\frac{2x}{\sqrt{-2+2\sqrt{5}}}\right)}{10\sqrt{-2+2\sqrt{5}}}+\frac{\arctan\left(\frac{2x}{\sqrt{2+2\sqrt{5}}}\right)}{2\sqrt{2+2\sqrt{5}}}+\frac{3\sqrt{5}\,\arctan\left(\frac{2x}{\sqrt{2+2\sqrt{5}}}\right)}{10\sqrt{2+2\sqrt{5}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(x^8-3*x^4+1),x)
```

```
[Out] -1/2/(2+2*5^(1/2))^(1/2)*arctanh(2/(2+2*5^(1/2))^(1/2)*x)-3/10*5^(1/2)/(2+2
*5^(1/2))^(1/2)*arctanh(2/(2+2*5^(1/2))^(1/2)*x)+1/2/(-2+2*5^(1/2))^(1/2)*a
rctan(2/(-2+2*5^(1/2))^(1/2)*x)-3/10*5^(1/2)/(-2+2*5^(1/2))^(1/2)*arctan(2/
(-2+2*5^(1/2))^(1/2)*x)-1/2/(-2+2*5^(1/2))^(1/2)*arctanh(2/(-2+2*5^(1/2))^(
1/2)*x)+3/10*5^(1/2)/(-2+2*5^(1/2))^(1/2)*arctanh(2/(-2+2*5^(1/2))^(1/2)*x)
+1/2/(2+2*5^(1/2))^(1/2)*arctan(2/(2+2*5^(1/2))^(1/2)*x)+3/10*5^(1/2)/(2+2*
5^(1/2))^(1/2)*arctan(2/(2+2*5^(1/2))^(1/2)*x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6}{x^8-3\,x^4+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(x^8-3*x^4+1),x, algorithm="maxima")
```

```
[Out] integrate(x^6/(x^8 - 3*x^4 + 1), x)
```

mupad [B] time = 0.19, size = 147, normalized size = 0.88

$$\frac{\sqrt{5}\,\mathrm{atan}\left(\frac{16\,x\,\sqrt{2-\sqrt{5}}}{8\,\sqrt{5}-24}\right)\,\sqrt{\sqrt{5}-2}\,1\mathrm{i}}{10}+\frac{\sqrt{5}\,\mathrm{atan}\left(\frac{16\,x\,\sqrt{-\sqrt{5}-2}}{8\,\sqrt{5}+24}\right)\,\sqrt{\sqrt{5}+2}\,1\mathrm{i}}{10}+\frac{\sqrt{5}\,\mathrm{atan}\left(\frac{x\,\sqrt{2-\sqrt{5}}\,16\mathrm{i}}{8\,\sqrt{5}-24}\right)\,\sqrt{2-\sqrt{5}}\,1\mathrm{i}}{10}+\frac{\sqrt{5}\,\mathrm{atan}\left(\frac{x\,\sqrt{-\sqrt{5}-2}\,16\mathrm{i}}{8\,\sqrt{5}+24}\right)\,\sqrt{-\sqrt{5}-2}\,1\mathrm{i}}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(x^8 - 3*x^4 + 1),x)
```

```
[Out] (5^(1/2)*atan((16*x*(2 - 5^(1/2))^(1/2))/(8*5^(1/2) - 24))*(5^(1/2) - 2)^(1
/2)*1i)/10 + (5^(1/2)*atan((16*x*(- 5^(1/2) - 2)^(1/2))/(8*5^(1/2) + 24))*(
5^(1/2) + 2)^(1/2)*1i)/10 + (5^(1/2)*atan((x*(2 - 5^(1/2))^(1/2)*16i)/(8*5^
(1/2) - 24))*(2 - 5^(1/2))^(1/2)*1i)/10 + (5^(1/2)*atan((x*(- 5^(1/2) - 2)^
(1/2)*16i)/(8*5^(1/2) + 24))*(- 5^(1/2) - 2)^(1/2)*1i)/10
```

sympy [A] time = 1.22, size = 53, normalized size = 0.32

$$\operatorname{RootSum}\left(6400t^4-320t^2-1,\left(t\mapsto t\log\left(-1792000t^7+4920t^3+x\right)\right)\right)+\operatorname{RootSum}\left(6400t^4+320t^2-1,\left(t\mapsto t\log\left(-1792000t^7+4920t^3+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(x**8-3*x**4+1),x)
```

```
[Out] RootSum(6400*_t**4 - 320*_t**2 - 1, Lambda(_t, _t*log(-1792000*_t**7 + 4920
*_t**3 + x))) + RootSum(6400*_t**4 + 320*_t**2 - 1, Lambda(_t, _t*log(-1792
000*_t**7 + 4920*_t**3 + x)))
```

3.337 $\int \frac{x^4}{1-3x^4+x^8}\,dx$

Optimal. Leaf size=173

$$-\frac{\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}\tan^{-1}\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}}\,x\right)}{2\sqrt{5}}+\frac{\sqrt[4]{\frac{1}{2}\left(3-\sqrt{5}\right)}\tan^{-1}\left(\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}\,x\right)}{2\sqrt{5}}-\frac{\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}\tanh^{-1}\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}}\,x\right)}{2\sqrt{5}}+\ldots$$

Rubi [A] time = 0.08, antiderivative size = 173, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1374, 212, 206, 203}

$$-\frac{\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}\tan^{-1}\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}}\,x\right)}{2\sqrt{5}}+\frac{\sqrt[4]{\frac{1}{2}\left(3-\sqrt{5}\right)}\tan^{-1}\left(\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}\,x\right)}{2\sqrt{5}}-\frac{\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}\tanh^{-1}\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}}\,x\right)}{2\sqrt{5}}+\frac{\sqrt[4]{\frac{1}{2}\left(3-\sqrt{5}\right)}\tanh^{-1}\left(\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}\,x\right)}{2\sqrt{5}}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(1 - 3*x^4 + x^8),x]
```

```
[Out] -(((3 + Sqrt[5])/2)^(1/4)*ArcTan[(2/(3 + Sqrt[5]))^(1/4)*x])/(2*Sqrt[5]) +
(((3 - Sqrt[5])/2)^(1/4)*ArcTan[((3 + Sqrt[5])/2)^(1/4)*x])/(2*Sqrt[5]) - (
((3 + Sqrt[5])/2)^(1/4)*ArcTanh[(2/(3 + Sqrt[5]))^(1/4)*x])/(2*Sqrt[5]) + (
((3 - Sqrt[5])/2)^(1/4)*ArcTanh[((3 + Sqrt[5])/2)^(1/4)*x])/(2*Sqrt[5])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 1374

```
Int[((d_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(d^n*(b/q + 1))/2, Int[(d*x)^(m -
 n)/(b/2 + q/2 + c*x^n), x], x] - Dist[(d^n*(b/q - 1))/2, Int[(d*x)^(m - n)
/(b/2 - q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[n2, 2*n] &&
NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GeQ[m, n]
```

Rubi steps

$$\int \frac{x^4}{1-3x^4+x^8}\,dx = \frac{1}{10}\left(5-3\sqrt{5}\right)\int \frac{1}{-\frac{3}{2}+\frac{\sqrt{5}}{2}+x^4}\,dx+\frac{1}{10}\left(5+3\sqrt{5}\right)\int \frac{1}{-\frac{3}{2}-\frac{\sqrt{5}}{2}+x^4}\,dx$$

$$= \frac{1}{2}\sqrt{\frac{1}{5}\left(3-\sqrt{5}\right)}\int \frac{1}{\sqrt{3-\sqrt{5}}-\sqrt{2}\,x^2}\,dx+\frac{1}{2}\sqrt{\frac{1}{5}\left(3-\sqrt{5}\right)}\int \frac{1}{\sqrt{3-\sqrt{5}}+\sqrt{2}\,x^2}\,dx-$$

$$= -\frac{\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}\tan^{-1}\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}}\,x\right)}{2\sqrt{5}}+\frac{\sqrt[4]{\frac{1}{2}\left(3-\sqrt{5}\right)}\tan^{-1}\left(\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}\,x\right)}{2\sqrt{5}}-\frac{\sqrt[4]{\frac{1}{2}\left(3+\right.}}{}$$

Mathematica [A] time = 0.19, size = 132, normalized size = 0.76

$$\frac{\sqrt{\sqrt{5}-1}\tan^{-1}\left(\sqrt{\frac{2}{\sqrt{5}-1}}\,x\right)-\sqrt{1+\sqrt{5}}\tan^{-1}\left(\sqrt{\frac{2}{1+\sqrt{5}}}\,x\right)+\sqrt{\sqrt{5}-1}\tanh^{-1}\left(\sqrt{\frac{2}{\sqrt{5}-1}}\,x\right)-\sqrt{1+\sqrt{5}}\tanh^{-1}\left(\sqrt{\frac{2}{1+\sqrt{5}}}\,x\right)}{2\sqrt{10}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(1 - 3*x^4 + x^8),x]
```

```
[Out] (Sqrt[-1 + Sqrt[5]]*ArcTan[Sqrt[2/(-1 + Sqrt[5])]*x] - Sqrt[1 + Sqrt[5]]*Ar
cTan[Sqrt[2/(1 + Sqrt[5])]*x] + Sqrt[-1 + Sqrt[5]]*ArcTanh[Sqrt[2/(-1 + Sqr
t[5])]*x] - Sqrt[1 + Sqrt[5]]*ArcTanh[Sqrt[2/(1 + Sqrt[5])]*x])/(2*Sqrt[10]
)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{1-3x^4+x^8}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^4/(1 - 3*x^4 + x^8),x]
```

```
[Out] IntegrateAlgebraic[x^4/(1 - 3*x^4 + x^8), x]
```

fricas [B] time = 1.42, size = 271, normalized size = 1.57

$$-\frac{1}{10}\sqrt{10}\sqrt{\sqrt{5}+1}\arctan\left(\frac{1}{40}\sqrt{10}\sqrt{2x^2+\sqrt{5}+1}\left(\sqrt{5}\sqrt{2}-5\sqrt{2}\right)\sqrt{\sqrt{5}+1}-\frac{1}{20}\sqrt{10}\left(\sqrt{5}x-5x\right)\sqrt{\sqrt{5}+1}\right)-\frac{1}{10}\sqrt{10}\sqrt{\sqrt{5}-1}\arctan\left(\frac{1}{40}\sqrt{10}\sqrt{2x^2+\sqrt{5}-1}\left(\sqrt{5}\sqrt{2}+5\sqrt{2}\right)\sqrt{\sqrt{5}-1}-\frac{1}{20}\sqrt{10}\left(\sqrt{5}x+5x\right)\sqrt{\sqrt{5}-1}\right)-\frac{1}{40}\sqrt{10}\sqrt{\sqrt{5}+1}\log\left(\sqrt{10}\sqrt{5}\sqrt{\sqrt{5}+1}+10x\right)+\frac{1}{40}\sqrt{10}\sqrt{\sqrt{5}+1}\log\left(-\sqrt{10}\sqrt{5}\sqrt{\sqrt{5}+1}+10x\right)+\frac{1}{40}\sqrt{10}\sqrt{\sqrt{5}-1}\log\left(\sqrt{10}\sqrt{5}\sqrt{\sqrt{5}-1}+10x\right)-\frac{1}{40}\sqrt{10}\sqrt{\sqrt{5}-1}\log\left(-\sqrt{10}\sqrt{5}\sqrt{\sqrt{5}-1}+10x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(x^8-3*x^4+1),x, algorithm="fricas")
```

```
[Out] -1/10*sqrt(10)*sqrt(sqrt(5) + 1)*arctan(1/40*sqrt(10)*sqrt(2*x^2 + sqrt(5)
+ 1)*(sqrt(5)*sqrt(2) - 5*sqrt(2))*sqrt(sqrt(5) + 1) - 1/20*sqrt(10)*(sqrt(
5)*x - 5*x)*sqrt(sqrt(5) + 1)) - 1/10*sqrt(10)*sqrt(sqrt(5) - 1)*arctan(1/4
0*sqrt(10)*sqrt(2*x^2 + sqrt(5) - 1)*(sqrt(5)*sqrt(2) + 5*sqrt(2))*sqrt(sqr
t(5) - 1) - 1/20*sqrt(10)*(sqrt(5)*x + 5*x)*sqrt(sqrt(5) - 1)) - 1/40*sqrt(
10)*sqrt(sqrt(5) + 1)*log(sqrt(10)*sqrt(5)*sqrt(sqrt(5) + 1) + 10*x) + 1/40
*sqrt(10)*sqrt(sqrt(5) + 1)*log(-sqrt(10)*sqrt(5)*sqrt(sqrt(5) + 1) + 10*x)
 + 1/40*sqrt(10)*sqrt(sqrt(5) - 1)*log(sqrt(10)*sqrt(5)*sqrt(sqrt(5) - 1) +
 10*x) - 1/40*sqrt(10)*sqrt(sqrt(5) - 1)*log(-sqrt(10)*sqrt(5)*sqrt(sqrt(5)
 - 1) + 10*x)
```

giac [A] time = 0.61, size = 147, normalized size = 0.85

$$-\frac{1}{20}\sqrt{10\sqrt{5}+10}\arctan\left(\frac{x}{\sqrt{\frac{1}{2}\sqrt{5}+\frac{1}{2}}}\right)+\frac{1}{20}\sqrt{10\sqrt{5}-10}\arctan\left(\frac{x}{\sqrt{\frac{1}{2}\sqrt{5}-\frac{1}{2}}}\right)-\frac{1}{40}\sqrt{10\sqrt{5}+10}\log\left(\left|x+\sqrt{\frac{1}{2}\sqrt{5}+\frac{1}{2}}\right|\right)+\frac{1}{40}\sqrt{10\sqrt{5}+10}\log\left(\left|x-\sqrt{\frac{1}{2}\sqrt{5}+\frac{1}{2}}\right|\right)+\frac{1}{40}\sqrt{10\sqrt{5}-10}\log\left(\left|x+\sqrt{\frac{1}{2}\sqrt{5}-\frac{1}{2}}\right|\right)-\frac{1}{40}\sqrt{10\sqrt{5}-10}\log\left(\left|x-\sqrt{\frac{1}{2}\sqrt{5}-\frac{1}{2}}\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(x^8-3*x^4+1),x, algorithm="giac")
```

```
[Out] -1/20*sqrt(10*sqrt(5) + 10)*arctan(x/sqrt(1/2*sqrt(5) + 1/2)) + 1/20*sqrt(1
0*sqrt(5) - 10)*arctan(x/sqrt(1/2*sqrt(5) - 1/2)) - 1/40*sqrt(10*sqrt(5) +
10)*log(abs(x + sqrt(1/2*sqrt(5) + 1/2))) + 1/40*sqrt(10*sqrt(5) + 10)*log(
abs(x - sqrt(1/2*sqrt(5) + 1/2))) + 1/40*sqrt(10*sqrt(5) - 10)*log(abs(x +
sqrt(1/2*sqrt(5) - 1/2))) - 1/40*sqrt(10*sqrt(5) - 10)*log(abs(x - sqrt(1/2
*sqrt(5) - 1/2)))
```

maple [A] time = 0.04, size = 206, normalized size = 1.19

$$\frac{\operatorname{arctanh}\left(\frac{2x}{\sqrt{-2+2\sqrt{5}}}\right)}{2\sqrt{-2+2\sqrt{5}}}-\frac{\sqrt{5}\,\operatorname{arctanh}\left(\frac{2x}{\sqrt{-2+2\sqrt{5}}}\right)}{10\sqrt{-2+2\sqrt{5}}}-\frac{\sqrt{5}\,\operatorname{arctanh}\left(\frac{2x}{\sqrt{2+2\sqrt{5}}}\right)}{10\sqrt{2+2\sqrt{5}}}-\frac{\operatorname{arctanh}\left(\frac{2x}{\sqrt{2+2\sqrt{5}}}\right)}{2\sqrt{2+2\sqrt{5}}}+\frac{\arctan\left(\frac{2x}{\sqrt{-2+2\sqrt{5}}}\right)}{2\sqrt{-2+2\sqrt{5}}}-\frac{\sqrt{5}\,\arctan\left(\frac{2x}{\sqrt{-2+2\sqrt{5}}}\right)}{10\sqrt{-2+2\sqrt{5}}}-\frac{\sqrt{5}\,\arctan\left(\frac{2x}{\sqrt{2+2\sqrt{5}}}\right)}{10\sqrt{2+2\sqrt{5}}}-\frac{\arctan\left(\frac{2x}{\sqrt{2+2\sqrt{5}}}\right)}{2\sqrt{2+2\sqrt{5}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(x^8-3*x^4+1),x)
```

```
[Out] -1/10*5^(1/2)/(2+2*5^(1/2))^(1/2)*arctanh(2/(2+2*5^(1/2))^(1/2)*x)-1/2/(2+2
*5^(1/2))^(1/2)*arctanh(2/(2+2*5^(1/2))^(1/2)*x)+1/2/(-2+2*5^(1/2))^(1/2)*a
rctan(2/(-2+2*5^(1/2))^(1/2)*x)-1/10*5^(1/2)/(-2+2*5^(1/2))^(1/2)*arctan(2/
(-2+2*5^(1/2))^(1/2)*x)+1/2/(-2+2*5^(1/2))^(1/2)*arctanh(2/(-2+2*5^(1/2))^(
1/2)*x)-1/10*5^(1/2)/(-2+2*5^(1/2))^(1/2)*arctanh(2/(-2+2*5^(1/2))^(1/2)*x)
-1/10*5^(1/2)/(2+2*5^(1/2))^(1/2)*arctan(2/(2+2*5^(1/2))^(1/2)*x)-1/2/(2+2*
5^(1/2))^(1/2)*arctan(2/(2+2*5^(1/2))^(1/2)*x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{x^8-3\,x^4+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(x^8-3*x^4+1),x, algorithm="maxima")
```

```
[Out] integrate(x^4/(x^8 - 3*x^4 + 1), x)
```

mupad [B] time = 1.47, size = 269, normalized size = 1.55

$$\frac{\sqrt{10}\,\mathrm{atan}\left(\frac{\sqrt{10}\,x\,\sqrt{-\sqrt{5}-1}\,1{}\mathrm{i}}{2\,\left(\sqrt{5}-1\right)}-\frac{\sqrt{5}\,\sqrt{10}\,x\,\sqrt{-\sqrt{5}-1}\,3{}\mathrm{i}}{10\,\left(\sqrt{5}-1\right)}\right)\,\sqrt{-\sqrt{5}-1}\,1{}\mathrm{i}}{20}+\frac{\sqrt{10}\,\mathrm{atan}\left(\frac{\sqrt{10}\,x\,\sqrt{1-\sqrt{5}}\,1{}\mathrm{i}}{2\,\left(\sqrt{5}+1\right)}+\frac{\sqrt{5}\,\sqrt{10}\,x\,\sqrt{1-\sqrt{5}}\,3{}\mathrm{i}}{10\,\left(\sqrt{5}+1\right)}\right)\,\sqrt{1-\sqrt{5}}\,1{}\mathrm{i}}{20}-\frac{\sqrt{10}\,\mathrm{atan}\left(\frac{\sqrt{10}\,x\,\sqrt{\sqrt{5}+1}\,1{}\mathrm{i}}{2\,\left(\sqrt{5}-1\right)}-\frac{\sqrt{5}\,\sqrt{10}\,x\,\sqrt{\sqrt{5}+1}\,3{}\mathrm{i}}{10\,\left(\sqrt{5}-1\right)}\right)\,\sqrt{\sqrt{5}+1}\,1{}\mathrm{i}}{20}-\frac{\sqrt{10}\,\mathrm{atan}\left(\frac{\sqrt{10}\,x\,\sqrt{\sqrt{5}-1}\,1{}\mathrm{i}}{2\,\left(\sqrt{5}+1\right)}+\frac{\sqrt{5}\,\sqrt{10}\,x\,\sqrt{\sqrt{5}-1}\,3{}\mathrm{i}}{10\,\left(\sqrt{5}+1\right)}\right)\,\sqrt{\sqrt{5}-1}\,1{}\mathrm{i}}{20}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(x^8 - 3*x^4 + 1),x)
```

```
[Out] (10^(1/2)*atan((10^(1/2)*x*(- 5^(1/2) - 1)^(1/2)*1i)/(2*(5^(1/2) - 1)) - (5
^(1/2)*10^(1/2)*x*(- 5^(1/2) - 1)^(1/2)*3i)/(10*(5^(1/2) - 1)))*(- 5^(1/2)
- 1)^(1/2)*1i)/20 + (10^(1/2)*atan((10^(1/2)*x*(1 - 5^(1/2))^(1/2)*1i)/(2*(
5^(1/2) + 1)) + (5^(1/2)*10^(1/2)*x*(1 - 5^(1/2))^(1/2)*3i)/(10*(5^(1/2) +
1)))*(1 - 5^(1/2))^(1/2)*1i)/20 - (10^(1/2)*atan((10^(1/2)*x*(5^(1/2) + 1)^
(1/2)*1i)/(2*(5^(1/2) - 1)) - (5^(1/2)*10^(1/2)*x*(5^(1/2) + 1)^(1/2)*3i)/(
10*(5^(1/2) - 1)))*(5^(1/2) + 1)^(1/2)*1i)/20 - (10^(1/2)*atan((10^(1/2)*x*
(5^(1/2) - 1)^(1/2)*1i)/(2*(5^(1/2) + 1)) + (5^(1/2)*10^(1/2)*x*(5^(1/2) -
1)^(1/2)*3i)/(10*(5^(1/2) + 1)))*(5^(1/2) - 1)^(1/2)*1i)/20
```

sympy [A] time = 1.20, size = 49, normalized size = 0.28

$$\operatorname{RootSum}{\left(6400 t^{4} - 80 t^{2} - 1, \left(t \mapsto t \log{\left(-51200 t^{5} + 12 t + x \right)} \right)\right)} + \operatorname{RootSum}{\left(6400 t^{4} + 80 t^{2} - 1, \left(t \mapsto t \log{\left(-51200 t^{5} + 12 t + x \right)} \right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(x**8-3*x**4+1),x)
```

```
[Out] RootSum(6400*_t**4 - 80*_t**2 - 1, Lambda(_t, _t*log(-51200*_t**5 + 12*_t +
 x))) + RootSum(6400*_t**4 + 80*_t**2 - 1, Lambda(_t, _t*log(-51200*_t**5 +
 12*_t + x)))
```

3.338 $\int \frac{x^2}{1-3x^4+x^8}\,dx$

Optimal. Leaf size=145

$$\frac{1}{20}\sqrt{10\sqrt{5}-10}\,\tan^{-1}\left(\frac{1}{2}\sqrt{2\sqrt{5}-2}\,x\right)-\frac{1}{20}\sqrt{10+10\sqrt{5}}\,\tan^{-1}\left(\frac{1}{2}\sqrt{2+2\sqrt{5}}\,x\right)-\frac{1}{20}\sqrt{10\sqrt{5}-10}\,\tanh^{-1}\left(\frac{1}{2}\sqrt{2\right.$$

Rubi [A] time = 0.06, antiderivative size = 166, normalized size of antiderivative = 1.14, number of steps used = 7, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1375, 298, 203, 206}

$$\frac{\tan^{-1}\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}}\,x\right)}{2^{3/4}\sqrt{5}\sqrt[4]{3+\sqrt{5}}}-\frac{\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}\tan^{-1}\left(\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}\,x\right)}{2\sqrt{5}}-\frac{\tanh^{-1}\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}}\,x\right)}{2^{3/4}\sqrt{5}\sqrt[4]{3+\sqrt{5}}}+\frac{\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}\tanh^{-1}\left(\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}\,x\right)}{2\sqrt{5}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(1 - 3*x^4 + x^8),x]
```

```
[Out] ArcTan[(2/(3 + Sqrt[5]))^(1/4)*x]/(2^(3/4)*Sqrt[5]*(3 + Sqrt[5])^(1/4)) - (
((3 + Sqrt[5])/2)^(1/4)*ArcTan[((3 + Sqrt[5])/2)^(1/4)*x])/(2*Sqrt[5]) - Ar
cTanh[(2/(3 + Sqrt[5]))^(1/4)*x]/(2^(3/4)*Sqrt[5]*(3 + Sqrt[5])^(1/4)) + ((
(3 + Sqrt[5])/2)^(1/4)*ArcTanh[((3 + Sqrt[5])/2)^(1/4)*x])/(2*Sqrt[5])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 1375

```
Int[((d_.)*(x_))^(m_.)/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symb
ol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[c/q, Int[(d*x)^m/(b/2 - q/2 + c*
x^n), x], x] - Dist[c/q, Int[(d*x)^m/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[
{a, b, c, d, m}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{x^2}{1-3x^4+x^8}\,dx = \frac{\int \frac{x^2}{-\frac{3}{2}-\frac{\sqrt{5}}{2}+x^4}\,dx}{\sqrt{5}} - \frac{\int \frac{x^2}{-\frac{3}{2}+\frac{\sqrt{5}}{2}+x^4}\,dx}{\sqrt{5}}$$

$$= \frac{\int \frac{1}{\sqrt{3-\sqrt{5}}-\sqrt{2}x^2}\,dx}{\sqrt{10}} - \frac{\int \frac{1}{\sqrt{3+\sqrt{5}}-\sqrt{2}x^2}\,dx}{\sqrt{10}} - \frac{\int \frac{1}{\sqrt{3-\sqrt{5}}+\sqrt{2}x^2}\,dx}{\sqrt{10}} + \frac{\int \frac{1}{\sqrt{3+\sqrt{5}}+\sqrt{2}x^2}\,dx}{\sqrt{10}}$$

$$= \frac{\tan^{-1}\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}}\,x\right)}{2^{3/4}\sqrt{5}\sqrt[4]{3+\sqrt{5}}} - \frac{\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}\tan^{-1}\left(\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}\,x\right)}{2\sqrt{5}} - \frac{\tanh^{-1}\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}}\,x\right)}{2^{3/4}\sqrt{5}\sqrt[4]{3+\sqrt{5}}} + \frac{\sqrt[4]{\frac{1}{2}}}{}$$

Mathematica [A] time = 0.04, size = 131, normalized size = 0.90

$$-\frac{\tan^{-1}\left(\sqrt{\frac{2}{\sqrt{5}-1}}\,x\right)}{\sqrt{10\left(\sqrt{5}-1\right)}} + \frac{\tan^{-1}\left(\sqrt{\frac{2}{1+\sqrt{5}}}\,x\right)}{\sqrt{10\left(1+\sqrt{5}\right)}} + \frac{\tanh^{-1}\left(\sqrt{\frac{2}{\sqrt{5}-1}}\,x\right)}{\sqrt{10\left(\sqrt{5}-1\right)}} - \frac{\tanh^{-1}\left(\sqrt{\frac{2}{1+\sqrt{5}}}\,x\right)}{\sqrt{10\left(1+\sqrt{5}\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(1 - 3*x^4 + x^8),x]
```

```
[Out] -(ArcTan[Sqrt[2/(-1 + Sqrt[5])]*x]/Sqrt[10*(-1 + Sqrt[5])]) + ArcTan[Sqrt[2
/(1 + Sqrt[5])]*x]/Sqrt[10*(1 + Sqrt[5])] + ArcTanh[Sqrt[2/(-1 + Sqrt[5])]*
x]/Sqrt[10*(-1 + Sqrt[5])] - ArcTanh[Sqrt[2/(1 + Sqrt[5])]*x]/Sqrt[10*(1 +
Sqrt[5])]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{1-3x^4+x^8}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^2/(1 - 3*x^4 + x^8),x]
```

```
[Out] IntegrateAlgebraic[x^2/(1 - 3*x^4 + x^8), x]
```

fricas [B] time = 1.08, size = 255, normalized size = 1.76

$$\frac{1}{10}\sqrt{10}\sqrt{\sqrt{5}+1}\arctan\left(\frac{1}{20}\sqrt{10}\sqrt{5}\sqrt{2}\sqrt{2x^2+\sqrt{5}-1}\sqrt{\sqrt{5}+1}-\frac{1}{10}\sqrt{10}\sqrt{5}x\sqrt{\sqrt{5}+1}\right)-\frac{1}{10}\sqrt{10}\sqrt{\sqrt{5}-1}\arctan\left(\frac{1}{20}\sqrt{10}\sqrt{5}\sqrt{2}\sqrt{2x^2+\sqrt{5}+1}\sqrt{\sqrt{5}-1}-\frac{1}{10}\sqrt{10}\sqrt{5}x\sqrt{\sqrt{5}-1}\right)-\frac{1}{40}\sqrt{10}\sqrt{\sqrt{5}-1}\log\left(\sqrt{10}\left(\sqrt{5}+5\right)\sqrt{\sqrt{5}-1}+20x\right)+\frac{1}{40}\sqrt{10}\sqrt{\sqrt{5}-1}\log\left(-\sqrt{10}\left(\sqrt{5}+5\right)\sqrt{\sqrt{5}-1}+20x\right)-\frac{1}{40}\sqrt{10}\sqrt{\sqrt{5}+1}\log\left(\sqrt{10}\sqrt{\sqrt{5}+1}\left(\sqrt{5}-5\right)+20x\right)+\frac{1}{40}\sqrt{10}\sqrt{\sqrt{5}+1}\log\left(-\sqrt{10}\sqrt{\sqrt{5}+1}\left(\sqrt{5}-5\right)+20x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^8-3*x^4+1),x, algorithm="fricas")
```

```
[Out] 1/10*sqrt(10)*sqrt(sqrt(5) + 1)*arctan(1/20*sqrt(10)*sqrt(5)*sqrt(2)*sqrt(2
*x^2 + sqrt(5) - 1)*sqrt(sqrt(5) + 1) - 1/10*sqrt(10)*sqrt(5)*x*sqrt(sqrt(5
) + 1)) - 1/10*sqrt(10)*sqrt(sqrt(5) - 1)*arctan(1/20*sqrt(10)*sqrt(5)*sqrt
(2)*sqrt(2*x^2 + sqrt(5) + 1)*sqrt(sqrt(5) - 1) - 1/10*sqrt(10)*sqrt(5)*x*s
qrt(sqrt(5) - 1)) - 1/40*sqrt(10)*sqrt(sqrt(5) - 1)*log(sqrt(10)*(sqrt(5) +
 5)*sqrt(sqrt(5) - 1) + 20*x) + 1/40*sqrt(10)*sqrt(sqrt(5) - 1)*log(-sqrt(1
0)*(sqrt(5) + 5)*sqrt(sqrt(5) - 1) + 20*x) - 1/40*sqrt(10)*sqrt(sqrt(5) + 1
)*log(sqrt(10)*sqrt(sqrt(5) + 1)*(sqrt(5) - 5) + 20*x) + 1/40*sqrt(10)*sqrt
(sqrt(5) + 1)*log(-sqrt(10)*sqrt(sqrt(5) + 1)*(sqrt(5) - 5) + 20*x)
```

giac [A] time = 0.62, size = 147, normalized size = 1.01

$$\frac{1}{20}\sqrt{10\sqrt{5}-10}\arctan\left(\frac{x}{\sqrt{\frac{1}{2}\sqrt{5}+\frac{1}{2}}}\right)-\frac{1}{20}\sqrt{10\sqrt{5}+10}\arctan\left(\frac{x}{\sqrt{\frac{1}{2}\sqrt{5}-\frac{1}{2}}}\right)-\frac{1}{40}\sqrt{10\sqrt{5}-10}\log\left(\left|x+\sqrt{\frac{1}{2}\sqrt{5}+\frac{1}{2}}\right|\right)+\frac{1}{40}\sqrt{10\sqrt{5}-10}\log\left(\left|x-\sqrt{\frac{1}{2}\sqrt{5}+\frac{1}{2}}\right|\right)+\frac{1}{40}\sqrt{10\sqrt{5}+10}\log\left(\left|x+\sqrt{\frac{1}{2}\sqrt{5}-\frac{1}{2}}\right|\right)-\frac{1}{40}\sqrt{10\sqrt{5}+10}\log\left(\left|x-\sqrt{\frac{1}{2}\sqrt{5}-\frac{1}{2}}\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^8-3*x^4+1),x, algorithm="giac")
```

```
[Out] 1/20*sqrt(10*sqrt(5) - 10)*arctan(x/sqrt(1/2*sqrt(5) + 1/2)) - 1/20*sqrt(10
*sqrt(5) + 10)*arctan(x/sqrt(1/2*sqrt(5) - 1/2)) - 1/40*sqrt(10*sqrt(5) - 1
0)*log(abs(x + sqrt(1/2*sqrt(5) + 1/2))) + 1/40*sqrt(10*sqrt(5) - 10)*log(a
bs(x - sqrt(1/2*sqrt(5) + 1/2))) + 1/40*sqrt(10*sqrt(5) + 10)*log(abs(x + s
qrt(1/2*sqrt(5) - 1/2))) - 1/40*sqrt(10*sqrt(5) + 10)*log(abs(x - sqrt(1/2*
sqrt(5) - 1/2)))
```

maple [A] time = 0.03, size = 110, normalized size = 0.76

$$\frac{\sqrt{5}\,\operatorname{arctanh}\left(\frac{2x}{\sqrt{-2+2\sqrt{5}}}\right)}{5\sqrt{-2+2\sqrt{5}}}-\frac{\sqrt{5}\,\operatorname{arctanh}\left(\frac{2x}{\sqrt{2+2\sqrt{5}}}\right)}{5\sqrt{2+2\sqrt{5}}}-\frac{\sqrt{5}\,\arctan\left(\frac{2x}{\sqrt{-2+2\sqrt{5}}}\right)}{5\sqrt{-2+2\sqrt{5}}}+\frac{\sqrt{5}\,\arctan\left(\frac{2x}{\sqrt{2+2\sqrt{5}}}\right)}{5\sqrt{2+2\sqrt{5}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(x^8-3*x^4+1),x)
```

```
[Out] -1/5*5^(1/2)/(2+2*5^(1/2))^(1/2)*arctanh(2/(2+2*5^(1/2))^(1/2)*x)-1/5*5^(1/
2)/(-2+2*5^(1/2))^(1/2)*arctan(2/(-2+2*5^(1/2))^(1/2)*x)+1/5*5^(1/2)/(-2+2*
5^(1/2))^(1/2)*arctanh(2/(-2+2*5^(1/2))^(1/2)*x)+1/5*5^(1/2)/(2+2*5^(1/2))^
(1/2)*arctan(2/(2+2*5^(1/2))^(1/2)*x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{x^8-3\,x^4+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^8-3*x^4+1),x, algorithm="maxima")
```

```
[Out] integrate(x^2/(x^8 - 3*x^4 + 1), x)
```

mupad [B] time = 0.08, size = 269, normalized size = 1.86

$$\frac{\sqrt{10}\,\operatorname{atan}\left(\frac{\sqrt{10}\,x\,\sqrt{\sqrt{5}-1}\,3\mathrm{i}}{2\,\left(3\,\sqrt{5}-7\right)}-\frac{\sqrt{5}\,\sqrt{10}\,x\,\sqrt{\sqrt{5}-1}\,7\mathrm{i}}{10\,\left(3\,\sqrt{5}-7\right)}\right)\,\sqrt{\sqrt{5}-1}\,1\mathrm{i}}{20}-\frac{\sqrt{10}\,\operatorname{atan}\left(\frac{\sqrt{10}\,x\,\sqrt{\sqrt{5}+1}\,3\mathrm{i}}{2\,\left(3\,\sqrt{5}+7\right)}+\frac{\sqrt{5}\,\sqrt{10}\,x\,\sqrt{\sqrt{5}+1}\,7\mathrm{i}}{10\,\left(3\,\sqrt{5}+7\right)}\right)\,\sqrt{\sqrt{5}+1}\,1\mathrm{i}}{20}+\frac{\sqrt{10}\,\operatorname{atan}\left(\frac{\sqrt{10}\,x\,\sqrt{1-\sqrt{5}}\,3\mathrm{i}}{2\,\left(3\,\sqrt{5}-7\right)}-\frac{\sqrt{5}\,\sqrt{10}\,x\,\sqrt{1-\sqrt{5}}\,7\mathrm{i}}{10\,\left(3\,\sqrt{5}-7\right)}\right)\,\sqrt{1-\sqrt{5}}\,1\mathrm{i}}{20}-\frac{\sqrt{10}\,\operatorname{atan}\left(\frac{\sqrt{10}\,x\,\sqrt{-\sqrt{5}-1}\,3\mathrm{i}}{2\,\left(3\,\sqrt{5}+7\right)}+\frac{\sqrt{5}\,\sqrt{10}\,x\,\sqrt{-\sqrt{5}-1}\,7\mathrm{i}}{10\,\left(3\,\sqrt{5}+7\right)}\right)\,\sqrt{-\sqrt{5}-1}\,1\mathrm{i}}{20}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(x^8 - 3*x^4 + 1),x)
```

```
[Out] (10^(1/2)*atan((10^(1/2)*x*(5^(1/2) - 1)^(1/2)*3i)/(2*(3*5^(1/2) - 7)) - (5
^(1/2)*10^(1/2)*x*(5^(1/2) - 1)^(1/2)*7i)/(10*(3*5^(1/2) - 7)))*(5^(1/2) -
1)^(1/2)*1i)/20 - (10^(1/2)*atan((10^(1/2)*x*(5^(1/2) + 1)^(1/2)*3i)/(2*(3*
5^(1/2) + 7)) + (5^(1/2)*10^(1/2)*x*(5^(1/2) + 1)^(1/2)*7i)/(10*(3*5^(1/2)
+ 7)))*(5^(1/2) + 1)^(1/2)*1i)/20 + (10^(1/2)*atan((10^(1/2)*x*(1 - 5^(1/2)
)^(1/2)*3i)/(2*(3*5^(1/2) - 7)) - (5^(1/2)*10^(1/2)*x*(1 - 5^(1/2))^(1/2)*7
i)/(10*(3*5^(1/2) - 7)))*(1 - 5^(1/2))^(1/2)*1i)/20 - (10^(1/2)*atan((10^(1
/2)*x*(- 5^(1/2) - 1)^(1/2)*3i)/(2*(3*5^(1/2) + 7)) + (5^(1/2)*10^(1/2)*x*(
- 5^(1/2) - 1)^(1/2)*7i)/(10*(3*5^(1/2) + 7)))*(- 5^(1/2) - 1)^(1/2)*1i)/20
```

sympy [A] time = 1.19, size = 53, normalized size = 0.37

$$\operatorname{RootSum}\left(6400t^4-80t^2-1,\left(t\mapsto t\log\left(6144000t^7-2240t^3+x\right)\right)\right)+\operatorname{RootSum}\left(6400t^4+80t^2-1,\left(t\mapsto t\log\left(6144000t^7-2240t^3+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(x**8-3*x**4+1),x)
```

```
[Out] RootSum(6400*_t**4 - 80*_t**2 - 1, Lambda(_t, _t*log(6144000*_t**7 - 2240*_
t**3 + x))) + RootSum(6400*_t**4 + 80*_t**2 - 1, Lambda(_t, _t*log(6144000*
_t**7 - 2240*_t**3 + x)))
```

3.339 $\int \frac{1}{1-3x^4+x^8}\,dx$

Optimal. Leaf size=169

$$-\frac{\tan^{-1}\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}}\,x\right)}{\sqrt[4]{2}\sqrt{5}\left(3+\sqrt{5}\right)^{3/4}}+\frac{\left(3+\sqrt{5}\right)^{3/4}\tan^{-1}\left(\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}\,x\right)}{2\,2^{3/4}\sqrt{5}}-\frac{\tanh^{-1}\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}}\,x\right)}{\sqrt[4]{2}\sqrt{5}\left(3+\sqrt{5}\right)^{3/4}}+\frac{\left(3+\sqrt{5}\right)^{3/4}\tanh^{-1}\left(\sqrt[4]{\frac{1}{2}\left(3+\right.}\right.}{2\,2^{3/4}\sqrt{5}}$$

Rubi [A] time = 0.06, antiderivative size = 169, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1347, 212, 206, 203}

$$-\frac{\tan^{-1}\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}}\,x\right)}{\sqrt[4]{2}\sqrt{5}\left(3+\sqrt{5}\right)^{3/4}}+\frac{\left(3+\sqrt{5}\right)^{3/4}\tan^{-1}\left(\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}\,x\right)}{2\,2^{3/4}\sqrt{5}}-\frac{\tanh^{-1}\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}}\,x\right)}{\sqrt[4]{2}\sqrt{5}\left(3+\sqrt{5}\right)^{3/4}}+\frac{\left(3+\sqrt{5}\right)^{3/4}\tanh^{-1}\left(\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}\,x\right)}{2\,2^{3/4}\sqrt{5}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - 3*x^4 + x^8)^(-1),x]
```

```
[Out] -(ArcTan[(2/(3 + Sqrt[5]))^(1/4)*x]/(2^(1/4)*Sqrt[5]*(3 + Sqrt[5])^(3/4)))
+ ((3 + Sqrt[5])^(3/4)*ArcTan[((3 + Sqrt[5])/2)^(1/4)*x])/(2*2^(3/4)*Sqrt[5
]) - ArcTanh[(2/(3 + Sqrt[5]))^(1/4)*x]/(2^(1/4)*Sqrt[5]*(3 + Sqrt[5])^(3/4
)) + ((3 + Sqrt[5])^(3/4)*ArcTanh[((3 + Sqrt[5])/2)^(1/4)*x])/(2*2^(3/4)*Sq
rt[5])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 1347

```
Int[((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^(-1), x_Symbol] :> With[{q
= Rt[b^2 - 4*a*c, 2]}, Dist[c/q, Int[1/(b/2 - q/2 + c*x^n), x], x] - Dist[c
/q, Int[1/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*
n] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{1}{1-3x^4+x^8}\,dx = \frac{\int \frac{1}{-\frac{3}{2}-\frac{\sqrt{5}}{2}+x^4}\,dx}{\sqrt{5}} - \frac{\int \frac{1}{-\frac{3}{2}+\frac{\sqrt{5}}{2}+x^4}\,dx}{\sqrt{5}}$$

$$= \frac{\int \frac{1}{\sqrt{3-\sqrt{5}}-\sqrt{2}x^2}\,dx}{\sqrt{5\left(3-\sqrt{5}\right)}} + \frac{\int \frac{1}{\sqrt{3-\sqrt{5}}+\sqrt{2}x^2}\,dx}{\sqrt{5\left(3-\sqrt{5}\right)}} - \frac{\int \frac{1}{\sqrt{3+\sqrt{5}}-\sqrt{2}x^2}\,dx}{\sqrt{5\left(3+\sqrt{5}\right)}} - \frac{\int \frac{1}{\sqrt{3+\sqrt{5}}+\sqrt{2}x^2}\,dx}{\sqrt{5\left(3+\sqrt{5}\right)}}$$

$$= -\frac{\tan^{-1}\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}}\,x\right)}{\sqrt[4]{2}\sqrt{5}\left(3+\sqrt{5}\right)^{3/4}} + \frac{\left(3+\sqrt{5}\right)^{3/4}\tan^{-1}\left(\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}\,x\right)}{2\,2^{3/4}\sqrt{5}} - \frac{\tanh^{-1}\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}}\,x\right)}{\sqrt[4]{2}\sqrt{5}\left(3+\sqrt{5}\right)^{3/4}} + \ldots$$

Mathematica [A] time = 0.16, size = 160, normalized size = 0.95

$$\frac{\frac{\left(1+\sqrt{5}\right)\tan^{-1}\left(\sqrt{\frac{2}{\sqrt{5}-1}}\,x\right)}{\sqrt{\sqrt{5}-1}} - \frac{\left(\sqrt{5}-1\right)\tan^{-1}\left(\sqrt{\frac{2}{1+\sqrt{5}}}\,x\right)}{\sqrt{1+\sqrt{5}}} + \frac{\left(1+\sqrt{5}\right)\tanh^{-1}\left(\sqrt{\frac{2}{\sqrt{5}-1}}\,x\right)}{\sqrt{\sqrt{5}-1}} - \frac{\left(\sqrt{5}-1\right)\tanh^{-1}\left(\sqrt{\frac{2}{1+\sqrt{5}}}\,x\right)}{\sqrt{1+\sqrt{5}}}}{2\sqrt{10}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - 3*x^4 + x^8)^(-1),x]
```

```
[Out] (((1 + Sqrt[5])*ArcTan[Sqrt[2/(-1 + Sqrt[5])]*x])/Sqrt[-1 + Sqrt[5]] - ((-1
 + Sqrt[5])*ArcTan[Sqrt[2/(1 + Sqrt[5])]*x])/Sqrt[1 + Sqrt[5]] + ((1 + Sqrt
[5])*ArcTanh[Sqrt[2/(-1 + Sqrt[5])]*x])/Sqrt[-1 + Sqrt[5]] - ((-1 + Sqrt[5]
)*ArcTanh[Sqrt[2/(1 + Sqrt[5])]*x])/Sqrt[1 + Sqrt[5]])/(2*Sqrt[10])
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{1-3x^4+x^8}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(1 - 3*x^4 + x^8)^(-1),x]
```

```
[Out] IntegrateAlgebraic[(1 - 3*x^4 + x^8)^(-1), x]
```

fricas [B] time = 1.49, size = 251, normalized size = 1.49

$$-\frac{1}{5}\sqrt{5}\sqrt{\sqrt{5}+2}\arctan\left(\frac{1}{4}\sqrt{2x^2+\sqrt{5}-1}\left(\sqrt{5}\sqrt{2}-\sqrt{2}\right)\sqrt{\sqrt{5}+2}-\frac{1}{2}\left(\sqrt{5}x-x\right)\sqrt{\sqrt{5}+2}\right)+\frac{1}{5}\sqrt{5}\sqrt{\sqrt{5}-2}\arctan\left(\frac{1}{4}\sqrt{2x^2+\sqrt{5}+1}\left(\sqrt{5}\sqrt{2}+\sqrt{2}\right)\sqrt{\sqrt{5}-2}-\frac{1}{2}\left(\sqrt{5}x+x\right)\sqrt{\sqrt{5}-2}\right)-\frac{1}{20}\sqrt{5}\sqrt{\sqrt{5}-2}\log\left(\left(\sqrt{5}+3\right)\sqrt{\sqrt{5}-2}+2x\right)+\frac{1}{20}\sqrt{5}\sqrt{\sqrt{5}-2}\log\left(-\left(\sqrt{5}+3\right)\sqrt{\sqrt{5}-2}+2x\right)-\frac{1}{20}\sqrt{5}\sqrt{\sqrt{5}+2}\log\left(\sqrt{\sqrt{5}+2}\left(\sqrt{5}-3\right)+2x\right)+\frac{1}{20}\sqrt{5}\sqrt{\sqrt{5}+2}\log\left(-\sqrt{\sqrt{5}+2}\left(\sqrt{5}-3\right)+2x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^8-3*x^4+1),x, algorithm="fricas")
```

```
[Out] -1/5*sqrt(5)*sqrt(sqrt(5) + 2)*arctan(1/4*sqrt(2*x^2 + sqrt(5) - 1)*(sqrt(5
)*sqrt(2) - sqrt(2))*sqrt(sqrt(5) + 2) - 1/2*(sqrt(5)*x - x)*sqrt(sqrt(5) +
 2)) + 1/5*sqrt(5)*sqrt(sqrt(5) - 2)*arctan(1/4*sqrt(2*x^2 + sqrt(5) + 1)*(
sqrt(5)*sqrt(2) + sqrt(2))*sqrt(sqrt(5) - 2) - 1/2*(sqrt(5)*x + x)*sqrt(sqr
t(5) - 2)) - 1/20*sqrt(5)*sqrt(sqrt(5) - 2)*log((sqrt(5) + 3)*sqrt(sqrt(5)
- 2) + 2*x) + 1/20*sqrt(5)*sqrt(sqrt(5) - 2)*log(-(sqrt(5) + 3)*sqrt(sqrt(5
) - 2) + 2*x) - 1/20*sqrt(5)*sqrt(sqrt(5) + 2)*log(sqrt(sqrt(5) + 2)*(sqrt(
5) - 3) + 2*x) + 1/20*sqrt(5)*sqrt(sqrt(5) + 2)*log(-sqrt(sqrt(5) + 2)*(sqr
t(5) - 3) + 2*x)
```

giac [A] time = 0.48, size = 147, normalized size = 0.87

$$-\frac{1}{10}\sqrt{5\sqrt{5}-10}\arctan\left(\frac{x}{\sqrt{\frac{1}{2}\sqrt{5}+\frac{1}{2}}}\right)+\frac{1}{10}\sqrt{5\sqrt{5}+10}\arctan\left(\frac{x}{\sqrt{\frac{1}{2}\sqrt{5}-\frac{1}{2}}}\right)-\frac{1}{20}\sqrt{5\sqrt{5}-10}\log\left(\left|x+\sqrt{\frac{1}{2}\sqrt{5}+\frac{1}{2}}\right|\right)+\frac{1}{20}\sqrt{5\sqrt{5}-10}\log\left(\left|x-\sqrt{\frac{1}{2}\sqrt{5}+\frac{1}{2}}\right|\right)+\frac{1}{20}\sqrt{5\sqrt{5}+10}\log\left(\left|x+\sqrt{\frac{1}{2}\sqrt{5}-\frac{1}{2}}\right|\right)-\frac{1}{20}\sqrt{5\sqrt{5}+10}\log\left(\left|x-\sqrt{\frac{1}{2}\sqrt{5}-\frac{1}{2}}\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^8-3*x^4+1),x, algorithm="giac")
```

```
[Out] -1/10*sqrt(5*sqrt(5) - 10)*arctan(x/sqrt(1/2*sqrt(5) + 1/2)) + 1/10*sqrt(5*
sqrt(5) + 10)*arctan(x/sqrt(1/2*sqrt(5) - 1/2)) - 1/20*sqrt(5*sqrt(5) - 10)
*log(abs(x + sqrt(1/2*sqrt(5) + 1/2))) + 1/20*sqrt(5*sqrt(5) - 10)*log(abs(
x - sqrt(1/2*sqrt(5) + 1/2))) + 1/20*sqrt(5*sqrt(5) + 10)*log(abs(x + sqrt(
1/2*sqrt(5) - 1/2))) - 1/20*sqrt(5*sqrt(5) + 10)*log(abs(x - sqrt(1/2*sqrt(
5) - 1/2)))
```

maple [A] time = 0.03, size = 206, normalized size = 1.22

$$\frac{\sqrt{5}\,\mathrm{arctanh}\left(\frac{2x}{\sqrt{-2+2\sqrt{5}}}\right)}{10\sqrt{-2+2\sqrt{5}}}+\frac{\mathrm{arctanh}\left(\frac{2x}{\sqrt{-2+2\sqrt{5}}}\right)}{2\sqrt{-2+2\sqrt{5}}}-\frac{\mathrm{arctanh}\left(\frac{2x}{\sqrt{2+2\sqrt{5}}}\right)}{2\sqrt{2+2\sqrt{5}}}+\frac{\sqrt{5}\,\mathrm{arctanh}\left(\frac{2x}{\sqrt{2+2\sqrt{5}}}\right)}{10\sqrt{2+2\sqrt{5}}}+\frac{\sqrt{5}\arctan\left(\frac{2x}{\sqrt{-2+2\sqrt{5}}}\right)}{10\sqrt{-2+2\sqrt{5}}}+\frac{\arctan\left(\frac{2x}{\sqrt{-2+2\sqrt{5}}}\right)}{2\sqrt{-2+2\sqrt{5}}}-\frac{\arctan\left(\frac{2x}{\sqrt{2+2\sqrt{5}}}\right)}{2\sqrt{2+2\sqrt{5}}}+\frac{\sqrt{5}\arctan\left(\frac{2x}{\sqrt{2+2\sqrt{5}}}\right)}{10\sqrt{2+2\sqrt{5}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^8-3*x^4+1),x)
```

```
[Out] -1/2/(2+2*5^(1/2))^(1/2)*arctanh(2/(2+2*5^(1/2))^(1/2)*x)+1/10*5^(1/2)/(2+2
*5^(1/2))^(1/2)*arctanh(2/(2+2*5^(1/2))^(1/2)*x)+1/10*5^(1/2)/(-2+2*5^(1/2)
)^(1/2)*arctan(2/(-2+2*5^(1/2))^(1/2)*x)+1/2/(-2+2*5^(1/2))^(1/2)*arctan(2/
(-2+2*5^(1/2))^(1/2)*x)+1/10*5^(1/2)/(-2+2*5^(1/2))^(1/2)*arctanh(2/(-2+2*5
^(1/2))^(1/2)*x)+1/2/(-2+2*5^(1/2))^(1/2)*arctanh(2/(-2+2*5^(1/2))^(1/2)*x)
-1/2/(2+2*5^(1/2))^(1/2)*arctan(2/(2+2*5^(1/2))^(1/2)*x)+1/10*5^(1/2)/(2+2*
5^(1/2))^(1/2)*arctan(2/(2+2*5^(1/2))^(1/2)*x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^8 - 3\,x^4 + 1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^8-3*x^4+1),x, algorithm="maxima")
```

```
[Out] integrate(1/(x^8 - 3*x^4 + 1), x)
```

mupad [B] time = 0.08, size = 245, normalized size = 1.45

$$-\frac{\sqrt{5}\,\mathrm{atan}\left(\frac{x\,\sqrt{2-\sqrt{5}}\,144\mathrm{i}}{104\,\sqrt{5}-232}-\frac{\sqrt{5}\,x\,\sqrt{2-\sqrt{5}}\,64\mathrm{i}}{104\,\sqrt{5}-232}\right)\sqrt{2-\sqrt{5}}\,1\mathrm{i}}{10}+\frac{\sqrt{5}\,\mathrm{atan}\left(\frac{x\,\sqrt{-\sqrt{5}-2}\,144\mathrm{i}}{104\,\sqrt{5}+232}+\frac{\sqrt{5}\,x\,\sqrt{-\sqrt{5}-2}\,64\mathrm{i}}{104\,\sqrt{5}+232}\right)\sqrt{-\sqrt{5}-2}\,1\mathrm{i}}{10}+\frac{\sqrt{5}\,\mathrm{atan}\left(\frac{x\,\sqrt{\sqrt{5}-2}\,144\mathrm{i}}{104\,\sqrt{5}-232}-\frac{\sqrt{5}\,x\,\sqrt{\sqrt{5}-2}\,64\mathrm{i}}{104\,\sqrt{5}-232}\right)\sqrt{\sqrt{5}-2}\,1\mathrm{i}}{10}-\frac{\sqrt{5}\,\mathrm{atan}\left(\frac{x\,\sqrt{\sqrt{5}+2}\,144\mathrm{i}}{104\,\sqrt{5}+232}+\frac{\sqrt{5}\,x\,\sqrt{\sqrt{5}+2}\,64\mathrm{i}}{104\,\sqrt{5}+232}\right)\sqrt{\sqrt{5}+2}\,1\mathrm{i}}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^8 - 3*x^4 + 1),x)
```

```
[Out] (5^(1/2)*atan((x*(- 5^(1/2) - 2)^(1/2)*144i)/(104*5^(1/2) + 232) + (5^(1/2)
*x*(- 5^(1/2) - 2)^(1/2)*64i)/(104*5^(1/2) + 232))*(- 5^(1/2) - 2)^(1/2)*1i
)/10 - (5^(1/2)*atan((x*(2 - 5^(1/2))^(1/2)*144i)/(104*5^(1/2) - 232) - (5^
(1/2)*x*(2 - 5^(1/2))^(1/2)*64i)/(104*5^(1/2) - 232))*(2 - 5^(1/2))^(1/2)*1
i)/10 + (5^(1/2)*atan((x*(5^(1/2) - 2)^(1/2)*144i)/(104*5^(1/2) - 232) - (5
^(1/2)*x*(5^(1/2) - 2)^(1/2)*64i)/(104*5^(1/2) - 232))*(5^(1/2) - 2)^(1/2)*
1i)/10 - (5^(1/2)*atan((x*(5^(1/2) + 2)^(1/2)*144i)/(104*5^(1/2) + 232) + (
5^(1/2)*x*(5^(1/2) + 2)^(1/2)*64i)/(104*5^(1/2) + 232))*(5^(1/2) + 2)^(1/2)
*1i)/10
```

sympy [A] time = 1.21, size = 53, normalized size = 0.31

$$\mathrm{RootSum}\left(6400t^4 - 320t^2 - 1, \left(t \mapsto t\log\left(9600t^5 - \frac{47t}{2} + x\right)\right)\right) + \mathrm{RootSum}\left(6400t^4 + 320t^2 - 1, \left(t \mapsto t\log\left(9600t^5 - \frac{47t}{2} + x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**8-3*x**4+1),x)
```

```
[Out] RootSum(6400*_t**4 - 320*_t**2 - 1, Lambda(_t, _t*log(9600*_t**5 - 47*_t/2
+ x))) + RootSum(6400*_t**4 + 320*_t**2 - 1, Lambda(_t, _t*log(9600*_t**5 -
 47*_t/2 + x)))
```

3.340 $\int \frac{1}{x^2\left(1-3x^4+x^8\right)}\,dx$

Optimal. Leaf size=172

$$-\frac{1}{x}+\frac{\sqrt[4]{984-440\sqrt{5}}\,\tan^{-1}\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}}\,x\right)}{4\sqrt{5}}-\frac{\left(3+\sqrt{5}\right)^{5/4}\tan^{-1}\left(\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}\,x\right)}{4\sqrt[4]{2}\sqrt{5}}-\frac{\sqrt[4]{984-440\sqrt{5}}\,\tanh^{-1}\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}}\right.}{4\sqrt{5}}$$

Rubi [A] time = 0.09, antiderivative size = 172, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {1368, 1510, 298, 203, 206}

$$-\frac{1}{x}+\frac{\sqrt[4]{984-440\sqrt{5}}\,\tan^{-1}\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}}\,x\right)}{4\sqrt{5}}-\frac{\left(3+\sqrt{5}\right)^{5/4}\tan^{-1}\left(\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}\,x\right)}{4\sqrt[4]{2}\sqrt{5}}-\frac{\sqrt[4]{984-440\sqrt{5}}\,\tanh^{-1}\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}}\,x\right)}{4\sqrt{5}}+\frac{\left(3+\sqrt{5}\right)^{5/4}\tanh^{-1}\left(\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}\,x\right)}{4\sqrt[4]{2}\sqrt{5}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(1 - 3*x^4 + x^8)),x]
```

```
[Out] -x^(-1) + ((984 - 440*Sqrt[5])^(1/4)*ArcTan[(2/(3 + Sqrt[5]))^(1/4)*x])/(4*
Sqrt[5]) - ((3 + Sqrt[5])^(5/4)*ArcTan[((3 + Sqrt[5])/2)^(1/4)*x])/(4*2^(1/
4)*Sqrt[5]) - ((984 - 440*Sqrt[5])^(1/4)*ArcTanh[(2/(3 + Sqrt[5]))^(1/4)*x]
)/(4*Sqrt[5]) + ((3 + Sqrt[5])^(5/4)*ArcTanh[((3 + Sqrt[5])/2)^(1/4)*x])/(4
*2^(1/4)*Sqrt[5])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 1368

```
Int[((d_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_
Symbol] :> Simp[((d*x)^(m + 1)*(a + b*x^n + c*x^(2*n))^(p + 1))/(a*d*(m + 1
)), x] - Dist[1/(a*d^n*(m + 1)), Int[(d*x)^(m + n)*(b*(m + n*(p + 1) + 1) +
 c*(m + 2*n*(p + 1) + 1)*x^n)*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a
, b, c, d, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && L
tQ[m, -1] && IntegerQ[p]
```

Rule 1510

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_)))/((a_) + (b_.)*(x_)^(n_) +
 (c_.)*(x_)^(n2_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 +
(2*c*d - b*e)/(2*q), Int[(f*x)^m/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (
2*c*d - b*e)/(2*q), Int[(f*x)^m/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b
```

```
, c, d, e, f, m}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^2\left(1-3x^4+x^8\right)}\,dx &= -\frac{1}{x}+\int \frac{x^2\left(3-x^4\right)}{1-3x^4+x^8}\,dx\\ &= -\frac{1}{x}+\frac{1}{10}\left(-5+3\sqrt{5}\right)\int \frac{x^2}{-\frac{3}{2}-\frac{\sqrt{5}}{2}+x^4}\,dx-\frac{1}{10}\left(5+3\sqrt{5}\right)\int \frac{x^2}{-\frac{3}{2}+\frac{\sqrt{5}}{2}+x^4}\,dx\\ &= -\frac{1}{x}-\frac{\left(3-\sqrt{5}\right)\int \frac{1}{\sqrt{3+\sqrt{5}}-\sqrt{2}x^2}\,dx}{2\sqrt{10}}+\frac{\left(3-\sqrt{5}\right)\int \frac{1}{\sqrt{3+\sqrt{5}}+\sqrt{2}x^2}\,dx}{2\sqrt{10}}+\frac{\left(3+\sqrt{5}\right)\int \frac{}{\sqrt{3}}}{2\sqrt{1}}\\ &= -\frac{1}{x}+\frac{\sqrt[4]{984-440\sqrt{5}}\,\tan^{-1}\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}}\,x\right)}{4\sqrt{5}}-\frac{\left(3+\sqrt{5}\right)^{5/4}\tan^{-1}\left(\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}\,x\right)}{4\sqrt[4]{2}\sqrt{5}}-\end{aligned}$$

Mathematica [A] time = 0.27, size = 174, normalized size = 1.01

$$-\frac{1}{x}-\frac{\left(3+\sqrt{5}\right)\tan^{-1}\left(\sqrt{\frac{2}{\sqrt{5}-1}}\,x\right)}{2\sqrt{10\left(\sqrt{5}-1\right)}}-\frac{\left(\sqrt{5}-3\right)\tan^{-1}\left(\sqrt{\frac{2}{1+\sqrt{5}}}\,x\right)}{2\sqrt{10\left(1+\sqrt{5}\right)}}+\frac{\left(3+\sqrt{5}\right)\tanh^{-1}\left(\sqrt{\frac{2}{\sqrt{5}-1}}\,x\right)}{2\sqrt{10\left(\sqrt{5}-1\right)}}+\frac{\left(\sqrt{5}-3\right)\tanh^{-1}\left(\sqrt{\frac{2}{1+\sqrt{5}}}\,x\right)}{2\sqrt{10\left(1+\sqrt{5}\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(1 - 3*x^4 + x^8)),x]
```

```
[Out] -x^(-1) - ((3 + Sqrt[5])*ArcTan[Sqrt[2/(-1 + Sqrt[5])]*x])/(2*Sqrt[10*(-1 +
 Sqrt[5])]) - ((-3 + Sqrt[5])*ArcTan[Sqrt[2/(1 + Sqrt[5])]*x])/(2*Sqrt[10*(
1 + Sqrt[5])]) + ((3 + Sqrt[5])*ArcTanh[Sqrt[2/(-1 + Sqrt[5])]*x])/(2*Sqrt[
10*(-1 + Sqrt[5])]) + ((-3 + Sqrt[5])*ArcTanh[Sqrt[2/(1 + Sqrt[5])]*x])/(2*
Sqrt[10*(1 + Sqrt[5])])
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2\left(1-3x^4+x^8\right)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x^2*(1 - 3*x^4 + x^8)),x]
```

```
[Out] IntegrateAlgebraic[1/(x^2*(1 - 3*x^4 + x^8)), x]
```

fricas [B] time = 1.31, size = 313, normalized size = 1.82

$$\frac{4\sqrt{10}x\sqrt{5\sqrt{5}+11}\arctan\left(\frac{1}{40}\left(\sqrt{10}\sqrt{2x^2+\sqrt{5}-1}\left(3\sqrt{5}\sqrt{2}-5\sqrt{2}\right)-2\sqrt{10}\left(3\sqrt{5}x-5x\right)\right)\sqrt{5\sqrt{5}+11}\right)-4\sqrt{10}x\sqrt{5\sqrt{5}-11}\arctan\left(\frac{1}{40}\left(\sqrt{10}\sqrt{2x^2+\sqrt{5}+1}\left(3\sqrt{5}\sqrt{2}+5\sqrt{2}\right)-2\sqrt{10}\left(3\sqrt{5}x+5x\right)\right)\sqrt{5\sqrt{5}-11}\right)-\sqrt{10}x\sqrt{5\sqrt{5}-11}\log\left(\sqrt{10}\sqrt{5\sqrt{5}-11}\left(2\sqrt{5}+5\right)+10x\right)+\sqrt{10}x\sqrt{5\sqrt{5}-11}\log\left(-\sqrt{10}\sqrt{5\sqrt{5}-11}\left(2\sqrt{5}+5\right)+10x\right)-\sqrt{10}x\sqrt{5\sqrt{5}+11}\log\left(\sqrt{10}\sqrt{5\sqrt{5}+11}\left(2\sqrt{5}-5\right)+10x\right)+\sqrt{10}x\sqrt{5\sqrt{5}+11}\log\left(-\sqrt{10}\sqrt{5\sqrt{5}+11}\left(2\sqrt{5}-5\right)+10x\right)-40}{40x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(x^8-3*x^4+1),x, algorithm="fricas")
```

```
[Out] 1/40*(4*sqrt(10)*x*sqrt(5*sqrt(5) + 11)*arctan(1/40*(sqrt(10)*sqrt(2*x^2 +
sqrt(5) - 1)*(3*sqrt(5)*sqrt(2) - 5*sqrt(2)) - 2*sqrt(10)*(3*sqrt(5)*x - 5*
x))*sqrt(5*sqrt(5) + 11)) - 4*sqrt(10)*x*sqrt(5*sqrt(5) - 11)*arctan(1/40*(
sqrt(10)*sqrt(2*x^2 + sqrt(5) + 1)*(3*sqrt(5)*sqrt(2) + 5*sqrt(2)) - 2*sqrt
(10)*(3*sqrt(5)*x + 5*x))*sqrt(5*sqrt(5) - 11)) - sqrt(10)*x*sqrt(5*sqrt(5)
 - 11)*log(sqrt(10)*sqrt(5*sqrt(5) - 11)*(2*sqrt(5) + 5) + 10*x) + sqrt(10)
```

```
*x*sqrt(5*sqrt(5) - 11)*log(-sqrt(10)*sqrt(5*sqrt(5) - 11)*(2*sqrt(5) + 5)
+ 10*x) - sqrt(10)*x*sqrt(5*sqrt(5) + 11)*log(sqrt(10)*sqrt(5*sqrt(5) + 11)
*(2*sqrt(5) - 5) + 10*x) + sqrt(10)*x*sqrt(5*sqrt(5) + 11)*log(-sqrt(10)*sq
rt(5*sqrt(5) + 11)*(2*sqrt(5) - 5) + 10*x) - 40)/x
```

giac [A] time = 0.54, size = 152, normalized size = 0.88

$$\frac{1}{20}\sqrt{50\sqrt{5}-110}\arctan\left(\frac{x}{\sqrt{\frac{1}{2}\sqrt{5}+\frac{1}{2}}}\right)-\frac{1}{20}\sqrt{50\sqrt{5}+110}\arctan\left(\frac{x}{\sqrt{\frac{1}{2}\sqrt{5}-\frac{1}{2}}}\right)-\frac{1}{40}\sqrt{50\sqrt{5}-110}\log\left(\left|x+\sqrt{\frac{1}{2}\sqrt{5}+\frac{1}{2}}\right|\right)+\frac{1}{40}\sqrt{50\sqrt{5}-110}\log\left(\left|x-\sqrt{\frac{1}{2}\sqrt{5}+\frac{1}{2}}\right|\right)+\frac{1}{40}\sqrt{50\sqrt{5}+110}\log\left(\left|x+\sqrt{\frac{1}{2}\sqrt{5}-\frac{1}{2}}\right|\right)-\frac{1}{40}\sqrt{50\sqrt{5}+110}\log\left(\left|x-\sqrt{\frac{1}{2}\sqrt{5}-\frac{1}{2}}\right|\right)-\frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(x^8-3*x^4+1),x, algorithm="giac")
```

```
[Out] 1/20*sqrt(50*sqrt(5) - 110)*arctan(x/sqrt(1/2*sqrt(5) + 1/2)) - 1/20*sqrt(5
0*sqrt(5) + 110)*arctan(x/sqrt(1/2*sqrt(5) - 1/2)) - 1/40*sqrt(50*sqrt(5) -
 110)*log(abs(x + sqrt(1/2*sqrt(5) + 1/2))) + 1/40*sqrt(50*sqrt(5) - 110)*l
og(abs(x - sqrt(1/2*sqrt(5) + 1/2))) + 1/40*sqrt(50*sqrt(5) + 110)*log(abs(
x + sqrt(1/2*sqrt(5) - 1/2))) - 1/40*sqrt(50*sqrt(5) + 110)*log(abs(x - sqr
t(1/2*sqrt(5) - 1/2))) - 1/x
```

maple [A] time = 0.03, size = 211, normalized size = 1.23

$$\frac{\operatorname{arctanh}\left(\frac{2x}{\sqrt{-2+2\sqrt{5}}}\right)}{2\sqrt{-2+2\sqrt{5}}}+\frac{3\sqrt{5}\,\operatorname{arctanh}\left(\frac{2x}{\sqrt{-2+2\sqrt{5}}}\right)}{10\sqrt{-2+2\sqrt{5}}}+\frac{\operatorname{arctanh}\left(\frac{2x}{\sqrt{2+2\sqrt{5}}}\right)}{2\sqrt{2+2\sqrt{5}}}-\frac{3\sqrt{5}\,\operatorname{arctanh}\left(\frac{2x}{\sqrt{2+2\sqrt{5}}}\right)}{10\sqrt{2+2\sqrt{5}}}-\frac{\arctan\left(\frac{2x}{\sqrt{-2+2\sqrt{5}}}\right)}{2\sqrt{-2+2\sqrt{5}}}-\frac{3\sqrt{5}\,\arctan\left(\frac{2x}{\sqrt{-2+2\sqrt{5}}}\right)}{10\sqrt{-2+2\sqrt{5}}}-\frac{\arctan\left(\frac{2x}{\sqrt{2+2\sqrt{5}}}\right)}{2\sqrt{2+2\sqrt{5}}}+\frac{3\sqrt{5}\,\arctan\left(\frac{2x}{\sqrt{2+2\sqrt{5}}}\right)}{10\sqrt{2+2\sqrt{5}}}-\frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(x^8-3*x^4+1),x)
```

```
[Out] 1/2/(2+2*5^(1/2))^(1/2)*arctanh(2/(2+2*5^(1/2))^(1/2)*x)-3/10*5^(1/2)/(2+2*
5^(1/2))^(1/2)*arctanh(2/(2+2*5^(1/2))^(1/2)*x)-1/2/(-2+2*5^(1/2))^(1/2)*ar
ctan(2/(-2+2*5^(1/2))^(1/2)*x)-3/10*5^(1/2)/(-2+2*5^(1/2))^(1/2)*arctan(2/(
-2+2*5^(1/2))^(1/2)*x)-1/x+1/2/(-2+2*5^(1/2))^(1/2)*arctanh(2/(-2+2*5^(1/2)
)^(1/2)*x)+3/10*5^(1/2)/(-2+2*5^(1/2))^(1/2)*arctanh(2/(-2+2*5^(1/2))^(1/2)
*x)-1/2/(2+2*5^(1/2))^(1/2)*arctan(2/(2+2*5^(1/2))^(1/2)*x)+3/10*5^(1/2)/(2
+2*5^(1/2))^(1/2)*arctan(2/(2+2*5^(1/2))^(1/2)*x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{x}-\frac{1}{2}\int\frac{x^2+2}{x^4+x^2-1}\,dx-\frac{1}{2}\int\frac{x^2-2}{x^4-x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(x^8-3*x^4+1),x, algorithm="maxima")
```

```
[Out] -1/x - 1/2*integrate((x^2 + 2)/(x^4 + x^2 - 1), x) - 1/2*integrate((x^2 - 2
)/(x^4 - x^2 - 1), x)
```

mupad [B] time = 1.34, size = 250, normalized size = 1.45

$$-\frac{1}{x}-\frac{\operatorname{atan}\left(\frac{x\sqrt{-50\sqrt{5}-110}\,1155\mathrm{i}}{2\left(3025\sqrt{5}+6765\right)}+\frac{\sqrt{5}\,x\sqrt{-50\sqrt{5}-110}\,517\mathrm{i}}{2\left(3025\sqrt{5}+6765\right)}\right)\sqrt{-50\sqrt{5}-110}\,1\mathrm{i}}{20}+\frac{\operatorname{atan}\left(\frac{x\sqrt{110-50\sqrt{5}}\,1155\mathrm{i}}{2\left(3025\sqrt{5}-6765\right)}-\frac{\sqrt{5}\,x\sqrt{110-50\sqrt{5}}\,517\mathrm{i}}{2\left(3025\sqrt{5}-6765\right)}\right)\sqrt{110-50\sqrt{5}}\,1\mathrm{i}}{20}+\frac{\operatorname{atan}\left(\frac{x\sqrt{50\sqrt{5}-110}\,1155\mathrm{i}}{2\left(3025\sqrt{5}-6765\right)}-\frac{\sqrt{5}\,x\sqrt{50\sqrt{5}-110}\,517\mathrm{i}}{2\left(3025\sqrt{5}-6765\right)}\right)\sqrt{50\sqrt{5}-110}\,1\mathrm{i}}{20}-\frac{\operatorname{atan}\left(\frac{x\sqrt{50\sqrt{5}+110}\,1155\mathrm{i}}{2\left(3025\sqrt{5}+6765\right)}+\frac{\sqrt{5}\,x\sqrt{50\sqrt{5}+110}\,517\mathrm{i}}{2\left(3025\sqrt{5}+6765\right)}\right)\sqrt{50\sqrt{5}+110}\,1\mathrm{i}}{20}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(x^8 - 3*x^4 + 1)),x)
```

```
[Out] (atan((x*(110 - 50*5^(1/2))^(1/2)*1155i)/(2*(3025*5^(1/2) - 6765)) - (5^(1/
2)*x*(110 - 50*5^(1/2))^(1/2)*517i)/(2*(3025*5^(1/2) - 6765)))*(110 - 50*5^
(1/2))^(1/2)*1i)/20 - (atan((x*(- 50*5^(1/2) - 110)^(1/2)*1155i)/(2*(3025*5
^(1/2) + 6765)) + (5^(1/2)*x*(- 50*5^(1/2) - 110)^(1/2)*517i)/(2*(3025*5^(1
/2) + 6765)))*(- 50*5^(1/2) - 110)^(1/2)*1i)/20 + (atan((x*(50*5^(1/2) - 11
```

```
0)^(1/2)*1155i)/(2*(3025*5^(1/2) - 6765)) - (5^(1/2)*x*(50*5^(1/2) - 110)^(
1/2)*517i)/(2*(3025*5^(1/2) - 6765)))*(50*5^(1/2) - 110)^(1/2)*1i)/20 - (at
an((x*(50*5^(1/2) + 110)^(1/2)*1155i)/(2*(3025*5^(1/2) + 6765)) + (5^(1/2)*
x*(50*5^(1/2) + 110)^(1/2)*517i)/(2*(3025*5^(1/2) + 6765)))*(50*5^(1/2) + 1
10)^(1/2)*1i)/20 - 1/x
```

sympy [**A**] time = 1.25, size = 63, normalized size = 0.37

$$\operatorname{RootSum}\left(6400t^4-880t^2-1,\left(t\mapsto t\log\left(\frac{19251200t^7}{11}-\frac{369792t^3}{11}+x\right)\right)\right)+\operatorname{RootSum}\left(6400t^4+880t^2-1,\left(t\mapsto t\log\left(\frac{19251200t^7}{11}-\frac{369792t^3}{11}+x\right)\right)\right)-\frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(x**8-3*x**4+1),x)
```

```
[Out] RootSum(6400*_t**4 - 880*_t**2 - 1, Lambda(_t, _t*log(19251200*_t**7/11 - 3
69792*_t**3/11 + x))) + RootSum(6400*_t**4 + 880*_t**2 - 1, Lambda(_t, _t*l
og(19251200*_t**7/11 - 369792*_t**3/11 + x))) - 1/x
```

3.341 $\int \frac{1}{x^4\left(1-3x^4+x^8\right)}\,dx$

Optimal. Leaf size=182

$$-\frac{1}{3x^3}-\frac{\sqrt[4]{\frac{1}{2}\left(843-377\sqrt{5}\right)}\tan^{-1}\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}}\,x\right)}{2\sqrt{5}}+\frac{\left(3+\sqrt{5}\right)^{7/4}\tan^{-1}\left(\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}\,x\right)}{4\,2^{3/4}\sqrt{5}}-\frac{\sqrt[4]{\frac{1}{2}\left(843-377\sqrt{5}\right)}\tanh^{-1}}{2\sqrt{5}}$$

Rubi [A] time = 0.12, antiderivative size = 182, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {1368, 1422, 212, 206, 203}

$$-\frac{1}{3x^3}-\frac{\sqrt[4]{\frac{1}{2}\left(843-377\sqrt{5}\right)}\tan^{-1}\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}}\,x\right)}{2\sqrt{5}}+\frac{\left(3+\sqrt{5}\right)^{7/4}\tan^{-1}\left(\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}\,x\right)}{4\,2^{3/4}\sqrt{5}}-\frac{\sqrt[4]{\frac{1}{2}\left(843-377\sqrt{5}\right)}\tanh^{-1}\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}}\,x\right)}{2\sqrt{5}}+\frac{\left(3+\sqrt{5}\right)^{7/4}\tanh^{-1}\left(\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}\,x\right)}{4\,2^{3/4}\sqrt{5}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(1 - 3*x^4 + x^8)),x]
```

```
[Out] -1/(3*x^3) - (((843 - 377*Sqrt[5])/2)^(1/4)*ArcTan[(2/(3 + Sqrt[5]))^(1/4)*
x])/(2*Sqrt[5]) + ((3 + Sqrt[5])^(7/4)*ArcTan[((3 + Sqrt[5])/2)^(1/4)*x])/(
4*2^(3/4)*Sqrt[5]) - (((843 - 377*Sqrt[5])/2)^(1/4)*ArcTanh[(2/(3 + Sqrt[5]
))^(1/4)*x])/(2*Sqrt[5]) + ((3 + Sqrt[5])^(7/4)*ArcTanh[((3 + Sqrt[5])/2)^(
1/4)*x])/(4*2^(3/4)*Sqrt[5])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 1368

```
Int[((d_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_
Symbol] :> Simp[((d*x)^(m + 1)*(a + b*x^n + c*x^(2*n))^(p + 1))/(a*d*(m + 1
)), x] - Dist[1/(a*d^n*(m + 1)), Int[(d*x)^(m + n)*(b*(m + n*(p + 1) + 1) +
 c*(m + 2*n*(p + 1) + 1)*x^n)*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a
, b, c, d, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && L
tQ[m, -1] && IntegerQ[p]
```

Rule 1422

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q),
Int[1/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(
b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (PosQ[b^2 - 4*a
```

```
*c] ||  !IGtQ[n/2, 0])
```

Rubi steps

$$\int \frac{1}{x^4\left(1-3x^4+x^8\right)}\,dx = -\frac{1}{3x^3}+\frac{1}{3}\int \frac{9-3x^4}{1-3x^4+x^8}\,dx$$
$$= -\frac{1}{3x^3}+\frac{1}{10}\left(-5+3\sqrt{5}\right)\int \frac{1}{-\frac{3}{2}-\frac{\sqrt{5}}{2}+x^4}\,dx-\frac{1}{10}\left(5+3\sqrt{5}\right)\int \frac{1}{-\frac{3}{2}+\frac{\sqrt{5}}{2}+x^4}\,dx$$
$$= -\frac{1}{3x^3}+\frac{\left(5-3\sqrt{5}\right)\int \frac{1}{\sqrt{3+\sqrt{5}}-\sqrt{2}x^2}\,dx}{10\sqrt{3+\sqrt{5}}}+\frac{\left(5-3\sqrt{5}\right)\int \frac{1}{\sqrt{3+\sqrt{5}}+\sqrt{2}x^2}\,dx}{10\sqrt{3+\sqrt{5}}}+\frac{\left(3+\sqrt{5}\right)^{3}\ldots}{\ldots}$$
$$= -\frac{1}{3x^3}-\frac{\sqrt[4]{\frac{1}{2}\left(843-377\sqrt{5}\right)}\tan^{-1}\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}}x\right)}{2\sqrt{5}}+\frac{\sqrt[4]{\frac{1}{2}\left(843+377\sqrt{5}\right)}\tan^{-1}\left(\sqrt[4]{\frac{1}{2}\left(3\ldots\right.}\right.}{2\sqrt{5}}$$

Mathematica [A] time = 0.26, size = 166, normalized size = 0.91

$$-\frac{1}{3x^3}+\frac{\left(2+\sqrt{5}\right)\tan^{-1}\left(\sqrt{\frac{2}{\sqrt{5}-1}}x\right)}{\sqrt{10\left(\sqrt{5}-1\right)}}-\frac{\left(\sqrt{5}-2\right)\tan^{-1}\left(\sqrt{\frac{2}{1+\sqrt{5}}}x\right)}{\sqrt{10\left(1+\sqrt{5}\right)}}+\frac{\left(2+\sqrt{5}\right)\tanh^{-1}\left(\sqrt{\frac{2}{\sqrt{5}-1}}x\right)}{\sqrt{10\left(\sqrt{5}-1\right)}}-\frac{\left(\sqrt{5}-2\right)\tanh^{-1}\left(\sqrt{\frac{2}{1+\sqrt{5}}}x\right)}{\sqrt{10\left(1+\sqrt{5}\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(1 - 3*x^4 + x^8)),x]
```

```
[Out] -1/3*1/x^3 + ((2 + Sqrt[5])*ArcTan[Sqrt[2/(-1 + Sqrt[5])]*x])/Sqrt[10*(-1 +
 Sqrt[5])] - ((-2 + Sqrt[5])*ArcTan[Sqrt[2/(1 + Sqrt[5])]*x])/Sqrt[10*(1 +
Sqrt[5])] + ((2 + Sqrt[5])*ArcTanh[Sqrt[2/(-1 + Sqrt[5])]*x])/Sqrt[10*(-1 +
 Sqrt[5])] - ((-2 + Sqrt[5])*ArcTanh[Sqrt[2/(1 + Sqrt[5])]*x])/Sqrt[10*(1 +
 Sqrt[5])]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^4\left(1-3x^4+x^8\right)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x^4*(1 - 3*x^4 + x^8)),x]
```

```
[Out] IntegrateAlgebraic[1/(x^4*(1 - 3*x^4 + x^8)), x]
```

fricas [B] time = 1.40, size = 327, normalized size = 1.80

$$\frac{12\sqrt{10}x^3\sqrt{13\sqrt{5}+29}\arctan\left(\frac{1}{20}\left(\sqrt{10}\sqrt{2x^2+\sqrt{5}-1}\left(2\sqrt{5}\sqrt{2}-5\sqrt{2}\right)-2\sqrt{10}\left(2\sqrt{5}x-5x\right)\right)\sqrt{13\sqrt{5}+29}\right)+12\sqrt{10}x^3\sqrt{13\sqrt{5}-29}\arctan\left(\frac{1}{20}\left(\sqrt{10}\sqrt{2x^2+\sqrt{5}+1}\left(2\sqrt{5}\sqrt{2}+5\sqrt{2}\right)-2\sqrt{10}\left(2\sqrt{5}x+5x\right)\right)\sqrt{13\sqrt{5}-29}\right)-3\sqrt{10}x^3\sqrt{13\sqrt{5}-29}\log\left(\sqrt{10}\sqrt{13\sqrt{5}-29}\left(7\sqrt{5}+15\right)+20x\right)+3\sqrt{10}x^3\sqrt{13\sqrt{5}-29}\log\left(-\sqrt{10}\sqrt{13\sqrt{5}-29}\left(7\sqrt{5}+15\right)+20x\right)+3\sqrt{10}x^3\sqrt{13\sqrt{5}+29}\log\left(\sqrt{10}\sqrt{13\sqrt{5}+29}\left(7\sqrt{5}-15\right)+20x\right)-3\sqrt{10}x^3\sqrt{13\sqrt{5}+29}\log\left(-\sqrt{10}\sqrt{13\sqrt{5}+29}\left(7\sqrt{5}-15\right)+20x\right)-40}{120x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(x^8-3*x^4+1),x, algorithm="fricas")
```

```
[Out] 1/120*(12*sqrt(10)*x^3*sqrt(13*sqrt(5) + 29)*arctan(1/20*(sqrt(10)*sqrt(2*x
^2 + sqrt(5) - 1)*(2*sqrt(5)*sqrt(2) - 5*sqrt(2)) - 2*sqrt(10)*(2*sqrt(5)*x
 - 5*x))*sqrt(13*sqrt(5) + 29)) + 12*sqrt(10)*x^3*sqrt(13*sqrt(5) - 29)*arc
tan(1/20*(sqrt(10)*sqrt(2*x^2 + sqrt(5) + 1)*(2*sqrt(5)*sqrt(2) + 5*sqrt(2)
) - 2*sqrt(10)*(2*sqrt(5)*x + 5*x))*sqrt(13*sqrt(5) - 29)) - 3*sqrt(10)*x^3
```

```
*sqrt(13*sqrt(5) - 29)*log(sqrt(10)*sqrt(13*sqrt(5) - 29)*(7*sqrt(5) + 15)
+ 20*x) + 3*sqrt(10)*x^3*sqrt(13*sqrt(5) - 29)*log(-sqrt(10)*sqrt(13*sqrt(5
) - 29)*(7*sqrt(5) + 15) + 20*x) + 3*sqrt(10)*x^3*sqrt(13*sqrt(5) + 29)*log
(sqrt(10)*sqrt(13*sqrt(5) + 29)*(7*sqrt(5) - 15) + 20*x) - 3*sqrt(10)*x^3*s
qrt(13*sqrt(5) + 29)*log(-sqrt(10)*sqrt(13*sqrt(5) + 29)*(7*sqrt(5) - 15) +
 20*x) - 40)/x^3
```

giac [A] time = 0.67, size = 152, normalized size = 0.84

$$-\frac{1}{20}\sqrt{130\sqrt{5}-290}\arctan\left(\frac{x}{\sqrt{\frac{1}{2}\sqrt{5}+\frac{1}{2}}}\right)+\frac{1}{20}\sqrt{130\sqrt{5}+290}\arctan\left(\frac{x}{\sqrt{\frac{1}{2}\sqrt{5}-\frac{1}{2}}}\right)-\frac{1}{40}\sqrt{130\sqrt{5}-290}\log\left(\left|x+\sqrt{\frac{1}{2}\sqrt{5}+\frac{1}{2}}\right|\right)+\frac{1}{40}\sqrt{130\sqrt{5}-290}\log\left(\left|x-\sqrt{\frac{1}{2}\sqrt{5}+\frac{1}{2}}\right|\right)+\frac{1}{40}\sqrt{130\sqrt{5}+290}\log\left(\left|x+\sqrt{\frac{1}{2}\sqrt{5}-\frac{1}{2}}\right|\right)-\frac{1}{40}\sqrt{130\sqrt{5}+290}\log\left(\left|x-\sqrt{\frac{1}{2}\sqrt{5}-\frac{1}{2}}\right|\right)-\frac{1}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(x^8-3*x^4+1),x, algorithm="giac")
```

```
[Out] -1/20*sqrt(130*sqrt(5) - 290)*arctan(x/sqrt(1/2*sqrt(5) + 1/2)) + 1/20*sqrt
(130*sqrt(5) + 290)*arctan(x/sqrt(1/2*sqrt(5) - 1/2)) - 1/40*sqrt(130*sqrt(
5) - 290)*log(abs(x + sqrt(1/2*sqrt(5) + 1/2))) + 1/40*sqrt(130*sqrt(5) - 2
90)*log(abs(x - sqrt(1/2*sqrt(5) + 1/2))) + 1/40*sqrt(130*sqrt(5) + 290)*lo
g(abs(x + sqrt(1/2*sqrt(5) - 1/2))) - 1/40*sqrt(130*sqrt(5) + 290)*log(abs(
x - sqrt(1/2*sqrt(5) - 1/2))) - 1/3/x^3
```

maple [A] time = 0.03, size = 209, normalized size = 1.15

$$\frac{2\sqrt{5}\,\mathrm{arctanh}\left(\frac{2x}{\sqrt{-2+2\sqrt{5}}}\right)}{5\sqrt{-2+2\sqrt{5}}}+\frac{\mathrm{arctanh}\left(\frac{2x}{\sqrt{-2+2\sqrt{5}}}\right)}{\sqrt{-2+2\sqrt{5}}}+\frac{2\sqrt{5}\,\mathrm{arctanh}\left(\frac{2x}{\sqrt{2+2\sqrt{5}}}\right)}{5\sqrt{2+2\sqrt{5}}}-\frac{\mathrm{arctanh}\left(\frac{2x}{\sqrt{2+2\sqrt{5}}}\right)}{\sqrt{2+2\sqrt{5}}}+\frac{2\sqrt{5}\,\arctan\left(\frac{2x}{\sqrt{-2+2\sqrt{5}}}\right)}{5\sqrt{-2+2\sqrt{5}}}+\frac{\arctan\left(\frac{2x}{\sqrt{-2+2\sqrt{5}}}\right)}{\sqrt{-2+2\sqrt{5}}}+\frac{2\sqrt{5}\,\arctan\left(\frac{2x}{\sqrt{2+2\sqrt{5}}}\right)}{5\sqrt{2+2\sqrt{5}}}-\frac{\arctan\left(\frac{2x}{\sqrt{2+2\sqrt{5}}}\right)}{\sqrt{2+2\sqrt{5}}}-\frac{1}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(x^8-3*x^4+1),x)
```

```
[Out] 2/5*5^(1/2)/(2+2*5^(1/2))^(1/2)*arctanh(2/(2+2*5^(1/2))^(1/2)*x)-1/(2+2*5^(
1/2))^(1/2)*arctanh(2/(2+2*5^(1/2))^(1/2)*x)+2/5*5^(1/2)/(-2+2*5^(1/2))^(1/
2)*arctan(2/(-2+2*5^(1/2))^(1/2)*x)+1/(-2+2*5^(1/2))^(1/2)*arctan(2/(-2+2*5
^(1/2))^(1/2)*x)+2/5*5^(1/2)/(-2+2*5^(1/2))^(1/2)*arctanh(2/(-2+2*5^(1/2))^
(1/2)*x)+1/(-2+2*5^(1/2))^(1/2)*arctanh(2/(-2+2*5^(1/2))^(1/2)*x)+2/5*5^(1/
2)/(2+2*5^(1/2))^(1/2)*arctan(2/(2+2*5^(1/2))^(1/2)*x)-1/(2+2*5^(1/2))^(1/2
)*arctan(2/(2+2*5^(1/2))^(1/2)*x)-1/3/x^3
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{3\,x^3}-\frac{1}{2}\int\frac{2\,x^2+3}{x^4+x^2-1}\,dx+\frac{1}{2}\int\frac{2\,x^2-3}{x^4-x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(x^8-3*x^4+1),x, algorithm="maxima")
```

```
[Out] -1/3/x^3 - 1/2*integrate((2*x^2 + 3)/(x^4 + x^2 - 1), x) + 1/2*integrate((2
*x^2 - 3)/(x^4 - x^2 - 1), x)
```

mupad [B] time = 0.20, size = 268, normalized size = 1.47

$$\frac{\mathrm{atan}\left(\frac{x\sqrt{-130\sqrt{5}-290}\,20735\mathrm{i}}{2\left(87841\sqrt{5}+196417\right)}+\frac{\sqrt{5}\,x\sqrt{-130\sqrt{5}-290}\,46371\mathrm{i}}{10\left(87841\sqrt{5}+196417\right)}\right)\sqrt{-130\sqrt{5}-290}\,1\mathrm{i}}{20}+\frac{\mathrm{atan}\left(\frac{x\sqrt{290-130\sqrt{5}}\,20735\mathrm{i}}{2\left(87841\sqrt{5}-196417\right)}-\frac{\sqrt{5}\,x\sqrt{290-130\sqrt{5}}\,46371\mathrm{i}}{10\left(87841\sqrt{5}-196417\right)}\right)\sqrt{290-130\sqrt{5}}\,1\mathrm{i}}{20}-\frac{1}{3\,x^3}-\frac{\sqrt{10}\,\mathrm{atan}\left(\frac{\sqrt{10}\,x\sqrt{13\sqrt{5}-29}\,20735\mathrm{i}}{2\left(87841\sqrt{5}-196417\right)}-\frac{\sqrt{5}\,\sqrt{10}\,x\sqrt{13\sqrt{5}-29}\,46371\mathrm{i}}{10\left(87841\sqrt{5}-196417\right)}\right)\sqrt{13\sqrt{5}-29}\,1\mathrm{i}}{20}-\frac{\sqrt{10}\,\mathrm{atan}\left(\frac{\sqrt{10}\,x\sqrt{13\sqrt{5}+29}\,20735\mathrm{i}}{2\left(87841\sqrt{5}+196417\right)}+\frac{\sqrt{5}\,\sqrt{10}\,x\sqrt{13\sqrt{5}+29}\,46371\mathrm{i}}{10\left(87841\sqrt{5}+196417\right)}\right)\sqrt{13\sqrt{5}+29}\,1\mathrm{i}}{20}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(x^8 - 3*x^4 + 1)),x)
```

```
[Out] (atan((x*(- 130*5^(1/2) - 290)^(1/2)*20735i)/(2*(87841*5^(1/2) + 196417)) +
 (5^(1/2)*x*(- 130*5^(1/2) - 290)^(1/2)*46371i)/(10*(87841*5^(1/2) + 196417
)))*(- 130*5^(1/2) - 290)^(1/2)*1i)/20 + (atan((x*(290 - 130*5^(1/2))^(1/2)
*20735i)/(2*(87841*5^(1/2) - 196417)) - (5^(1/2)*x*(290 - 130*5^(1/2))^(1/2
```

```
)*46371i)/(10*(87841*5^(1/2) - 196417)))*(290 - 130*5^(1/2))^(1/2)*1i)/20 -
 1/(3*x^3) - (10^(1/2)*atan((10^(1/2)*x*(13*5^(1/2) - 29)^(1/2)*20735i)/(2*
(87841*5^(1/2) - 196417)) - (5^(1/2)*10^(1/2)*x*(13*5^(1/2) - 29)^(1/2)*463
71i)/(10*(87841*5^(1/2) - 196417)))*(13*5^(1/2) - 29)^(1/2)*1i)/20 - (10^(1
/2)*atan((10^(1/2)*x*(13*5^(1/2) + 29)^(1/2)*20735i)/(2*(87841*5^(1/2) + 19
6417)) + (5^(1/2)*10^(1/2)*x*(13*5^(1/2) + 29)^(1/2)*46371i)/(10*(87841*5^(
1/2) + 196417)))*(13*5^(1/2) + 29)^(1/2)*1i)/20
```

sympy [A] time = 1.28, size = 63, normalized size = 0.35

$$\operatorname{RootSum}\left(6400t^4-2320t^2-1,\left(t\mapsto t\log\left(\frac{179200t^5}{377}-\frac{23112t}{377}+x\right)\right)\right)+\operatorname{RootSum}\left(6400t^4+2320t^2-1,\left(t\mapsto t\log\left(\frac{179200t^5}{377}-\frac{23112t}{377}+x\right)\right)\right)-\frac{1}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(x**8-3*x**4+1),x)
```

```
[Out] RootSum(6400*_t**4 - 2320*_t**2 - 1, Lambda(_t, _t*log(179200*_t**5/377 - 2
3112*_t/377 + x))) + RootSum(6400*_t**4 + 2320*_t**2 - 1, Lambda(_t, _t*log
(179200*_t**5/377 - 23112*_t/377 + x))) - 1/(3*x**3)
```

3.342 $\int \frac{1}{x^6\left(1-3x^4+x^8\right)}\,dx$

Optimal. Leaf size=173

$$-\frac{1}{5x^5}-\frac{3}{x}+\frac{\sqrt[4]{2889-1292\sqrt{5}}\,\tan^{-1}\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}}\,x\right)}{2\sqrt{5}}-\frac{\sqrt[4]{2889+1292\sqrt{5}}\,\tan^{-1}\left(\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}\,x\right)}{2\sqrt{5}}-\frac{\sqrt[4]{2889-1292\sqrt{5}}}{2}$$

Rubi [A] time = 0.14, antiderivative size = 173, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {1368, 1504, 1510, 298, 203, 206}

$$-\frac{1}{5x^5}-\frac{3}{x}+\frac{\sqrt[4]{2889-1292\sqrt{5}}\,\tan^{-1}\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}}\,x\right)}{2\sqrt{5}}-\frac{\sqrt[4]{2889+1292\sqrt{5}}\,\tan^{-1}\left(\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}\,x\right)}{2\sqrt{5}}-\frac{\sqrt[4]{2889-1292\sqrt{5}}\,\tanh^{-1}\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}}\,x\right)}{2\sqrt{5}}+\frac{\sqrt[4]{2889+1292\sqrt{5}}\,\tanh^{-1}\left(\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}\,x\right)}{2\sqrt{5}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^6*(1 - 3*x^4 + x^8)),x]
```

```
[Out] -1/(5*x^5) - 3/x + ((2889 - 1292*Sqrt[5])^(1/4)*ArcTan[(2/(3 + Sqrt[5]))^(1
/4)*x])/(2*Sqrt[5]) - ((2889 + 1292*Sqrt[5])^(1/4)*ArcTan[((3 + Sqrt[5])/2)
^(1/4)*x])/(2*Sqrt[5]) - ((2889 - 1292*Sqrt[5])^(1/4)*ArcTanh[(2/(3 + Sqrt[
5]))^(1/4)*x])/(2*Sqrt[5]) + ((2889 + 1292*Sqrt[5])^(1/4)*ArcTanh[((3 + Sqr
t[5])/2)^(1/4)*x])/(2*Sqrt[5])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 1368

```
Int[((d_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_
Symbol] :> Simp[((d*x)^(m + 1)*(a + b*x^n + c*x^(2*n))^(p + 1))/(a*d*(m + 1
)), x] - Dist[1/(a*d^n*(m + 1)), Int[(d*x)^(m + n)*(b*(m + n*(p + 1) + 1) +
 c*(m + 2*n*(p + 1) + 1)*x^n)*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a
, b, c, d, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && L
tQ[m, -1] && IntegerQ[p]
```

Rule 1504

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (
c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Simp[(d*(f*x)^(m + 1)*(a + b*x^n + c*x^
(2*n))^(p + 1))/(a*f*(m + 1)), x] + Dist[1/(a*f^n*(m + 1)), Int[(f*x)^(m +
n)*(a + b*x^n + c*x^(2*n))^p*Simp[a*e*(m + 1) - b*d*(m + n*(p + 1) + 1) - c
*d*(m + 2*n*(p + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x]
```

```
 && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && LtQ[m, -1] && Inte
gerQ[p]
```

Rule 1510

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_)))/((a_) + (b_.)*(x_)^(n_) +
 (c_.)*(x_)^(n2_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 +
(2*c*d - b*e)/(2*q), Int[(f*x)^m/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (
2*c*d - b*e)/(2*q), Int[(f*x)^m/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b
, c, d, e, f, m}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{1}{x^6\left(1-3x^4+x^8\right)}\,dx = -\frac{1}{5x^5}+\frac{1}{5}\int \frac{15-5x^4}{x^2\left(1-3x^4+x^8\right)}\,dx$$

$$= -\frac{1}{5x^5}-\frac{3}{x}-\frac{1}{5}\int \frac{x^2\left(-40+15x^4\right)}{1-3x^4+x^8}\,dx$$

$$= -\frac{1}{5x^5}-\frac{3}{x}-\frac{1}{10}\left(15-7\sqrt{5}\right)\int \frac{x^2}{-\frac{3}{2}-\frac{\sqrt{5}}{2}+x^4}\,dx-\frac{1}{10}\left(15+7\sqrt{5}\right)\int \frac{x^2}{-\frac{3}{2}+\frac{\sqrt{5}}{2}+x^4}$$

$$= -\frac{1}{5x^5}-\frac{3}{x}-\frac{\left(7-3\sqrt{5}\right)\int \frac{1}{\sqrt{3+\sqrt{5}}-\sqrt{2}x^2}\,dx}{2\sqrt{10}}+\frac{\left(7-3\sqrt{5}\right)\int \frac{1}{\sqrt{3+\sqrt{5}}+\sqrt{2}x^2}\,dx}{2\sqrt{10}}+\frac{\left(7+3\right.}{}$$

$$= -\frac{1}{5x^5}-\frac{3}{x}+\frac{\sqrt[4]{46224-20672\sqrt{5}}\,\tan^{-1}\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}}\,x\right)}{4\sqrt{5}}-\frac{\sqrt[4]{46224+20672\sqrt{5}}\,\tan^{-1}}{4\sqrt{5}}$$

Mathematica [A] time = 0.27, size = 189, normalized size = 1.09

$$-\frac{1}{5x^5}-\frac{3}{x}+\frac{\left(-7-3\sqrt{5}\right)\tan^{-1}\left(\sqrt{\frac{2}{\sqrt{5}-1}}\,x\right)}{2\sqrt{10\left(\sqrt{5}-1\right)}}+\frac{\left(7-3\sqrt{5}\right)\tan^{-1}\left(\sqrt{\frac{2}{1+\sqrt{5}}}\,x\right)}{2\sqrt{10\left(1+\sqrt{5}\right)}}-\frac{\left(-7-3\sqrt{5}\right)\tanh^{-1}\left(\sqrt{\frac{2}{\sqrt{5}-1}}\,x\right)}{2\sqrt{10\left(\sqrt{5}-1\right)}}-\frac{\left(7-3\sqrt{5}\right)\tanh^{-1}\left(\sqrt{\frac{2}{1+\sqrt{5}}}\,x\right)}{2\sqrt{10\left(1+\sqrt{5}\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^6*(1 - 3*x^4 + x^8)),x]
```

```
[Out] -1/5*1/x^5 - 3/x + ((-7 - 3*Sqrt[5])*ArcTan[Sqrt[2/(-1 + Sqrt[5])]*x])/(2*S
qrt[10*(-1 + Sqrt[5])]) + ((7 - 3*Sqrt[5])*ArcTan[Sqrt[2/(1 + Sqrt[5])]*x])
/(2*Sqrt[10*(1 + Sqrt[5])]) - ((-7 - 3*Sqrt[5])*ArcTanh[Sqrt[2/(-1 + Sqrt[5
])]*x])/(2*Sqrt[10*(-1 + Sqrt[5])]) - ((7 - 3*Sqrt[5])*ArcTanh[Sqrt[2/(1 +
Sqrt[5])]*x])/(2*Sqrt[10*(1 + Sqrt[5])])
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^6\left(1-3x^4+x^8\right)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x^6*(1 - 3*x^4 + x^8)),x]
```

```
[Out] IntegrateAlgebraic[1/(x^6*(1 - 3*x^4 + x^8)), x]
```

fricas [B] time = 1.19, size = 300, normalized size = 1.73

$$-\frac{4\sqrt{5}x^5\sqrt{17\sqrt{5}+38}\arctan\left(\frac{1}{4}\left(\sqrt{2x^2+\sqrt{5}-1}\left(3\sqrt{5}\sqrt{2}-7\sqrt{2}\right)-6\sqrt{5}x+14x\right)\sqrt{17\sqrt{5}+38}\right)+4\sqrt{5}x^5\sqrt{17\sqrt{5}-38}\arctan\left(\frac{1}{4}\left(\sqrt{2x^2+\sqrt{5}+1}\left(3\sqrt{5}\sqrt{2}+7\sqrt{2}\right)-6\sqrt{5}x-14x\right)\sqrt{17\sqrt{5}-38}\right)+\sqrt{5}x^5\sqrt{17\sqrt{5}-38}\log\left(\sqrt{17\sqrt{5}-38}\left(5\sqrt{5}+11\right)+2x\right)-\sqrt{5}x^5\sqrt{17\sqrt{5}-38}\log\left(-\sqrt{17\sqrt{5}-38}\left(5\sqrt{5}+11\right)+2x\right)-\sqrt{5}x^5\sqrt{17\sqrt{5}+38}\log\left(\sqrt{17\sqrt{5}+38}\left(5\sqrt{5}-11\right)+2x\right)+\sqrt{5}x^5\sqrt{17\sqrt{5}+38}\log\left(-\sqrt{17\sqrt{5}+38}\left(5\sqrt{5}-11\right)+2x\right)+60x^4+4}{20x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(x^8-3*x^4+1),x, algorithm="fricas")
```

```
[Out] -1/20*(4*sqrt(5)*x^5*sqrt(17*sqrt(5) + 38)*arctan(1/4*(sqrt(2*x^2 + sqrt(5)
 - 1)*(3*sqrt(5)*sqrt(2) - 7*sqrt(2)) - 6*sqrt(5)*x + 14*x)*sqrt(17*sqrt(5)
 + 38)) + 4*sqrt(5)*x^5*sqrt(17*sqrt(5) - 38)*arctan(1/4*(sqrt(2*x^2 + sqrt
(5) + 1)*(3*sqrt(5)*sqrt(2) + 7*sqrt(2)) - 6*sqrt(5)*x - 14*x)*sqrt(17*sqrt
(5) - 38)) + sqrt(5)*x^5*sqrt(17*sqrt(5) - 38)*log(sqrt(17*sqrt(5) - 38)*(5
*sqrt(5) + 11) + 2*x) - sqrt(5)*x^5*sqrt(17*sqrt(5) - 38)*log(-sqrt(17*sqrt
(5) - 38)*(5*sqrt(5) + 11) + 2*x) - sqrt(5)*x^5*sqrt(17*sqrt(5) + 38)*log(s
qrt(17*sqrt(5) + 38)*(5*sqrt(5) - 11) + 2*x) + sqrt(5)*x^5*sqrt(17*sqrt(5)
+ 38)*log(-sqrt(17*sqrt(5) + 38)*(5*sqrt(5) - 11) + 2*x) + 60*x^4 + 4)/x^5
```

giac [A] time = 0.54, size = 159, normalized size = 0.92

$$\frac{1}{10}\sqrt{85\sqrt{5}-190}\arctan\left(\frac{x}{\sqrt{\frac{1}{2}\sqrt{5}+\frac{1}{2}}}\right)-\frac{1}{10}\sqrt{85\sqrt{5}+190}\arctan\left(\frac{x}{\sqrt{\frac{1}{2}\sqrt{5}-\frac{1}{2}}}\right)-\frac{1}{20}\sqrt{85\sqrt{5}-190}\log\left(\left|x+\sqrt{\frac{1}{2}\sqrt{5}+\frac{1}{2}}\right|\right)+\frac{1}{20}\sqrt{85\sqrt{5}-190}\log\left(\left|x-\sqrt{\frac{1}{2}\sqrt{5}+\frac{1}{2}}\right|\right)+\frac{1}{20}\sqrt{85\sqrt{5}+190}\log\left(\left|x+\sqrt{\frac{1}{2}\sqrt{5}-\frac{1}{2}}\right|\right)-\frac{1}{20}\sqrt{85\sqrt{5}+190}\log\left(\left|x-\sqrt{\frac{1}{2}\sqrt{5}-\frac{1}{2}}\right|\right)-\frac{15x^4+1}{5x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(x^8-3*x^4+1),x, algorithm="giac")
```

```
[Out] 1/10*sqrt(85*sqrt(5) - 190)*arctan(x/sqrt(1/2*sqrt(5) + 1/2)) - 1/10*sqrt(8
5*sqrt(5) + 190)*arctan(x/sqrt(1/2*sqrt(5) - 1/2)) - 1/20*sqrt(85*sqrt(5) -
 190)*log(abs(x + sqrt(1/2*sqrt(5) + 1/2))) + 1/20*sqrt(85*sqrt(5) - 190)*l
og(abs(x - sqrt(1/2*sqrt(5) + 1/2))) + 1/20*sqrt(85*sqrt(5) + 190)*log(abs(
x + sqrt(1/2*sqrt(5) - 1/2))) - 1/20*sqrt(85*sqrt(5) + 190)*log(abs(x - sqr
t(1/2*sqrt(5) - 1/2))) - 1/5*(15*x^4 + 1)/x^5
```

maple [A] time = 0.04, size = 216, normalized size = 1.25

$$\frac{7\sqrt{5}\,\mathrm{arctanh}\left(\frac{2x}{\sqrt{-2+2\sqrt{5}}}\right)}{10\sqrt{-2+2\sqrt{5}}}+\frac{3\,\mathrm{arctanh}\left(\frac{2x}{\sqrt{-2+2\sqrt{5}}}\right)}{2\sqrt{-2+2\sqrt{5}}}-\frac{7\sqrt{5}\,\mathrm{arctanh}\left(\frac{2x}{\sqrt{2+2\sqrt{5}}}\right)}{10\sqrt{2+2\sqrt{5}}}+\frac{3\,\mathrm{arctanh}\left(\frac{2x}{\sqrt{2+2\sqrt{5}}}\right)}{2\sqrt{2+2\sqrt{5}}}-\frac{7\sqrt{5}\,\arctan\left(\frac{2x}{\sqrt{-2+2\sqrt{5}}}\right)}{10\sqrt{-2+2\sqrt{5}}}-\frac{3\arctan\left(\frac{2x}{\sqrt{-2+2\sqrt{5}}}\right)}{2\sqrt{-2+2\sqrt{5}}}+\frac{7\sqrt{5}\,\arctan\left(\frac{2x}{\sqrt{2+2\sqrt{5}}}\right)}{10\sqrt{2+2\sqrt{5}}}-\frac{3\arctan\left(\frac{2x}{\sqrt{2+2\sqrt{5}}}\right)}{2\sqrt{2+2\sqrt{5}}}-\frac{3}{x}-\frac{1}{5x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^6/(x^8-3*x^4+1),x)
```

```
[Out] -1/5/x^5-3/x-7/10*5^(1/2)/(2+2*5^(1/2))^(1/2)*arctanh(2/(2+2*5^(1/2))^(1/2)
*x)+3/2/(2+2*5^(1/2))^(1/2)*arctanh(2/(2+2*5^(1/2))^(1/2)*x)-7/10*5^(1/2)/(
-2+2*5^(1/2))^(1/2)*arctan(2/(-2+2*5^(1/2))^(1/2)*x)-3/2/(-2+2*5^(1/2))^(1/
2)*arctan(2/(-2+2*5^(1/2))^(1/2)*x)+7/10*5^(1/2)/(-2+2*5^(1/2))^(1/2)*arcta
nh(2/(-2+2*5^(1/2))^(1/2)*x)+3/2/(-2+2*5^(1/2))^(1/2)*arctanh(2/(-2+2*5^(1/
2))^(1/2)*x)+7/10*5^(1/2)/(2+2*5^(1/2))^(1/2)*arctan(2/(2+2*5^(1/2))^(1/2)*
x)-3/2/(2+2*5^(1/2))^(1/2)*arctan(2/(2+2*5^(1/2))^(1/2)*x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{15\,x^4+1}{5\,x^5}-\frac{1}{2}\int\frac{3\,x^2+5}{x^4+x^2-1}\,dx-\frac{1}{2}\int\frac{3\,x^2-5}{x^4-x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(x^8-3*x^4+1),x, algorithm="maxima")
```

```
[Out] -1/5*(15*x^4 + 1)/x^5 - 1/2*integrate((3*x^2 + 5)/(x^4 + x^2 - 1), x) - 1/2
*integrate((3*x^2 - 5)/(x^4 - x^2 - 1), x)
```

mupad [B] time = 1.49, size = 257, normalized size = 1.49

$$-\frac{3\,x^4+\frac{1}{5}}{x^5}-\frac{\mathrm{atan}\left(\frac{x\,\sqrt{-85\,\sqrt{5}-190}\,372096{}\mathrm{i}}{2550408\,\sqrt{5}+5702888}+\frac{\sqrt{5}\,x\,\sqrt{-85\,\sqrt{5}-190}\,832048{}\mathrm{i}}{5\,\left(2550408\,\sqrt{5}+5702888\right)}\right)\,\sqrt{-85\,\sqrt{5}-190}\,1{}\mathrm{i}}{10}+\frac{\mathrm{atan}\left(\frac{x\,\sqrt{190-85\,\sqrt{5}}\,372096{}\mathrm{i}}{2550408\,\sqrt{5}-5702888}-\frac{\sqrt{5}\,x\,\sqrt{190-85\,\sqrt{5}}\,832048{}\mathrm{i}}{5\,\left(2550408\,\sqrt{5}-5702888\right)}\right)\,\sqrt{190-85\,\sqrt{5}}\,1{}\mathrm{i}}{10}+\frac{\mathrm{atan}\left(\frac{x\,\sqrt{85\,\sqrt{5}-190}\,372096{}\mathrm{i}}{2550408\,\sqrt{5}-5702888}-\frac{\sqrt{5}\,x\,\sqrt{85\,\sqrt{5}-190}\,832048{}\mathrm{i}}{5\,\left(2550408\,\sqrt{5}-5702888\right)}\right)\,\sqrt{85\,\sqrt{5}-190}\,1{}\mathrm{i}}{10}-\frac{\mathrm{atan}\left(\frac{x\,\sqrt{85\,\sqrt{5}+190}\,372096{}\mathrm{i}}{2550408\,\sqrt{5}+5702888}+\frac{\sqrt{5}\,x\,\sqrt{85\,\sqrt{5}+190}\,832048{}\mathrm{i}}{5\,\left(2550408\,\sqrt{5}+5702888\right)}\right)\,\sqrt{85\,\sqrt{5}+190}\,1{}\mathrm{i}}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^6*(x^8 - 3*x^4 + 1)),x)
```

```
[Out] (atan((x*(190 - 85*5^(1/2))^(1/2)*372096i)/(2550408*5^(1/2) - 5702888) - (5
^(1/2)*x*(190 - 85*5^(1/2))^(1/2)*832048i)/(5*(2550408*5^(1/2) - 5702888)))
*(190 - 85*5^(1/2))^(1/2)*1i)/10 - (atan((x*(- 85*5^(1/2) - 190)^(1/2)*3720
96i)/(2550408*5^(1/2) + 5702888) + (5^(1/2)*x*(- 85*5^(1/2) - 190)^(1/2)*83
2048i)/(5*(2550408*5^(1/2) + 5702888)))*(- 85*5^(1/2) - 190)^(1/2)*1i)/10 +
 (atan((x*(85*5^(1/2) - 190)^(1/2)*372096i)/(2550408*5^(1/2) - 5702888) - (
5^(1/2)*x*(85*5^(1/2) - 190)^(1/2)*832048i)/(5*(2550408*5^(1/2) - 5702888))
)*(85*5^(1/2) - 190)^(1/2)*1i)/10 - (atan((x*(85*5^(1/2) + 190)^(1/2)*37209
6i)/(2550408*5^(1/2) + 5702888) + (5^(1/2)*x*(85*5^(1/2) + 190)^(1/2)*83204
8i)/(5*(2550408*5^(1/2) + 5702888)))*(85*5^(1/2) + 190)^(1/2)*1i)/10 - (3*x
^4 + 1/5)/x^5
```

sympy [A] time = 1.29, size = 73, normalized size = 0.42

$$\operatorname{RootSum}\left(6400t^4 - 6080t^2 - 1, \left(t \mapsto t\log\left(\frac{215808000t^7}{323} - \frac{194833880t^3}{323} + x\right)\right)\right) + \operatorname{RootSum}\left(6400t^4 + 6080t^2 - 1, \left(t \mapsto t\log\left(\frac{215808000t^7}{323} - \frac{194833880t^3}{323} + x\right)\right)\right) + \frac{-15x^4 - 1}{5x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**6/(x**8-3*x**4+1),x)
```

```
[Out] RootSum(6400*_t**4 - 6080*_t**2 - 1, Lambda(_t, _t*log(215808000*_t**7/323
- 194833880*_t**3/323 + x))) + RootSum(6400*_t**4 + 6080*_t**2 - 1, Lambda(
_t, _t*log(215808000*_t**7/323 - 194833880*_t**3/323 + x))) + (-15*x**4 - 1
)/(5*x**5)
```

3.343 $\int \frac{1}{x^8(1-3x^4+x^8)}\,dx$

Optimal. Leaf size=189

$$-\frac{1}{7x^7}-\frac{1}{x^3}-\frac{\sqrt[4]{\frac{1}{2}\left(39603-17711\sqrt{5}\right)}\tan^{-1}\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}}\,x\right)}{2\sqrt{5}}+\frac{\sqrt[4]{\frac{1}{2}\left(39603+17711\sqrt{5}\right)}\tan^{-1}\left(\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}\,x\right)}{2\sqrt{5}}-\frac{\sqrt[4]{\frac{1}{2}\left(\right.}}{}$$

Rubi [A] time = 0.16, antiderivative size = 189, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {1368, 1504, 1422, 212, 206, 203}

$$-\frac{1}{x^3}-\frac{1}{7x^7}-\frac{\sqrt[4]{\frac{1}{2}\left(39603-17711\sqrt{5}\right)}\tan^{-1}\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}}\,x\right)}{2\sqrt{5}}+\frac{\sqrt[4]{\frac{1}{2}\left(39603+17711\sqrt{5}\right)}\tan^{-1}\left(\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}\,x\right)}{2\sqrt{5}}-\frac{\sqrt[4]{\frac{1}{2}\left(39603-17711\sqrt{5}\right)}\tanh^{-1}\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}}\,x\right)}{2\sqrt{5}}+\frac{\sqrt[4]{\frac{1}{2}\left(39603+17711\sqrt{5}\right)}\tanh^{-1}\left(\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}\,x\right)}{2\sqrt{5}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^8*(1 - 3*x^4 + x^8)),x]
```

```
[Out] -1/(7*x^7) - x^(-3) - (((39603 - 17711*Sqrt[5])/2)^(1/4)*ArcTan[(2/(3 + Sqr
t[5]))^(1/4)*x])/(2*Sqrt[5]) + (((39603 + 17711*Sqrt[5])/2)^(1/4)*ArcTan[((
3 + Sqrt[5])/2)^(1/4)*x])/(2*Sqrt[5]) - (((39603 - 17711*Sqrt[5])/2)^(1/4)*
ArcTanh[(2/(3 + Sqrt[5]))^(1/4)*x])/(2*Sqrt[5]) + (((39603 + 17711*Sqrt[5])
/2)^(1/4)*ArcTanh[((3 + Sqrt[5])/2)^(1/4)*x])/(2*Sqrt[5])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 1368

```
Int[((d_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_
Symbol] :> Simp[((d*x)^(m + 1)*(a + b*x^n + c*x^(2*n))^(p + 1))/(a*d*(m + 1
)), x] - Dist[1/(a*d^n*(m + 1)), Int[(d*x)^(m + n)*(b*(m + n*(p + 1) + 1) +
 c*(m + 2*n*(p + 1) + 1)*x^n)*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a
, b, c, d, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && L
tQ[m, -1] && IntegerQ[p]
```

Rule 1422

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q),
Int[1/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(
b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (PosQ[b^2 - 4*a
```

```
*c] ||  !IGtQ[n/2, 0])
```

Rule 1504

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (
c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Simp[(d*(f*x)^(m + 1)*(a + b*x^n + c*x^
(2*n))^(p + 1))/(a*f*(m + 1)), x] + Dist[1/(a*f^n*(m + 1)), Int[(f*x)^(m +
n)*(a + b*x^n + c*x^(2*n))^p*Simp[a*e*(m + 1) - b*d*(m + n*(p + 1) + 1) - c
*d*(m + 2*n*(p + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x]
 && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && LtQ[m, -1] && Inte
gerQ[p]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^8\left(1-3x^4+x^8\right)}\,dx &= -\frac{1}{7x^7}+\frac{1}{7}\int \frac{21-7x^4}{x^4\left(1-3x^4+x^8\right)}\,dx\\ &= -\frac{1}{7x^7}-\frac{1}{x^3}-\frac{1}{21}\int \frac{-168+63x^4}{1-3x^4+x^8}\,dx\\ &= -\frac{1}{7x^7}-\frac{1}{x^3}-\frac{1}{10}\left(15-7\sqrt{5}\right)\int \frac{1}{-\frac{3}{2}-\frac{\sqrt{5}}{2}+x^4}\,dx-\frac{1}{10}\left(15+7\sqrt{5}\right)\int \frac{1}{-\frac{3}{2}+\frac{\sqrt{5}}{2}+x\cdots}\\ &= -\frac{1}{7x^7}-\frac{1}{x^3}-\frac{\left(-15+7\sqrt{5}\right)\int \frac{1}{\sqrt{3+\sqrt{5}}-\sqrt{2}x^2}\,dx}{10\sqrt{3+\sqrt{5}}}-\frac{\left(-15+7\sqrt{5}\right)\int \frac{1}{\sqrt{3+\sqrt{5}}+\sqrt{2}x^2}\,dx}{10\sqrt{3+\sqrt{5}}}+\cdots\\ &= -\frac{1}{7x^7}-\frac{1}{x^3}-\frac{\sqrt[4]{\frac{1}{2}\left(39603-17711\sqrt{5}\right)}\tan^{-1}\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}}\,x\right)}{2\sqrt{5}}+\frac{\sqrt[4]{\frac{1}{2}\left(39603+17711\sqrt{5}\cdots\right.}}{2\cdots}\end{aligned}$$

Mathematica [A] time = 0.27, size = 189, normalized size = 1.00

$$-\frac{1}{7x^7}-\frac{1}{x^3}+\frac{\left(11+5\sqrt{5}\right)\tan^{-1}\left(\sqrt{\frac{2}{\sqrt{5}-1}}\,x\right)}{2\sqrt{10\left(\sqrt{5}-1\right)}}+\frac{\left(11-5\sqrt{5}\right)\tan^{-1}\left(\sqrt{\frac{2}{1+\sqrt{5}}}\,x\right)}{2\sqrt{10\left(1+\sqrt{5}\right)}}-\frac{\left(-11-5\sqrt{5}\right)\tanh^{-1}\left(\sqrt{\frac{2}{\sqrt{5}-1}}\,x\right)}{2\sqrt{10\left(\sqrt{5}-1\right)}}-\frac{\left(5\sqrt{5}-11\right)\tanh^{-1}\left(\sqrt{\frac{2}{1+\sqrt{5}}}\,x\right)}{2\sqrt{10\left(1+\sqrt{5}\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^8*(1 - 3*x^4 + x^8)),x]
```

```
[Out] -1/7*1/x^7 - x^(-3) + ((11 + 5*Sqrt[5])*ArcTan[Sqrt[2/(-1 + Sqrt[5])]*x])/(
2*Sqrt[10*(-1 + Sqrt[5])]) + ((11 - 5*Sqrt[5])*ArcTan[Sqrt[2/(1 + Sqrt[5])]
*x])/(2*Sqrt[10*(1 + Sqrt[5])]) - ((-11 - 5*Sqrt[5])*ArcTanh[Sqrt[2/(-1 + S
qrt[5])]*x])/(2*Sqrt[10*(-1 + Sqrt[5])]) - ((-11 + 5*Sqrt[5])*ArcTanh[Sqrt[
2/(1 + Sqrt[5])]*x])/(2*Sqrt[10*(1 + Sqrt[5])])
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^8\left(1-3x^4+x^8\right)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x^8*(1 - 3*x^4 + x^8)),x]
```

```
[Out] IntegrateAlgebraic[1/(x^8*(1 - 3*x^4 + x^8)), x]
```

fricas [B] time = 1.21, size = 332, normalized size = 1.76

$$\frac{28\sqrt{10}x^7\sqrt{89\sqrt{5}+199}\arctan\left(\frac{1}{40}\left(\sqrt{10}\sqrt{2x^2+\sqrt{5}-1}\left(11\sqrt{5}\sqrt{2}-25\sqrt{2}\right)-2\sqrt{10}\left(11\sqrt{5}x-25x\right)\right)\sqrt{89\sqrt{5}+199}\right)+28\sqrt{10}x^7\sqrt{89\sqrt{5}-199}\arctan\left(\frac{1}{40}\left(\sqrt{10}\sqrt{2x^2+\sqrt{5}+1}\left(11\sqrt{5}\sqrt{2}+25\sqrt{2}\right)-2\sqrt{10}\left(11\sqrt{5}x+25x\right)\right)\sqrt{89\sqrt{5}-199}\right)-7\sqrt{10}x^7\sqrt{89\sqrt{5}-199}\log\left(\sqrt{10}\sqrt{89\sqrt{5}-199}\left(9\sqrt{5}+20\right)+10x\right)+7\sqrt{10}x^7\sqrt{89\sqrt{5}-199}\log\left(-\sqrt{10}\sqrt{89\sqrt{5}-199}\left(9\sqrt{5}+20\right)+10x\right)+7\sqrt{10}x^7\sqrt{89\sqrt{5}+199}\log\left(\sqrt{10}\sqrt{89\sqrt{5}+199}\left(9\sqrt{5}-20\right)+10x\right)-7\sqrt{10}x^7\sqrt{89\sqrt{5}+199}\log\left(-\sqrt{10}\sqrt{89\sqrt{5}+199}\left(9\sqrt{5}-20\right)+10x\right)-280x^4-40}{280x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^8/(x^8-3*x^4+1),x, algorithm="fricas")
```

```
[Out] 1/280*(28*sqrt(10)*x^7*sqrt(89*sqrt(5) + 199)*arctan(1/40*(sqrt(10)*sqrt(2*
x^2 + sqrt(5) - 1)*(11*sqrt(5)*sqrt(2) - 25*sqrt(2)) - 2*sqrt(10)*(11*sqrt(
5)*x - 25*x))*sqrt(89*sqrt(5) + 199)) + 28*sqrt(10)*x^7*sqrt(89*sqrt(5) - 1
99)*arctan(1/40*(sqrt(10)*sqrt(2*x^2 + sqrt(5) + 1)*(11*sqrt(5)*sqrt(2) + 2
5*sqrt(2)) - 2*sqrt(10)*(11*sqrt(5)*x + 25*x))*sqrt(89*sqrt(5) - 199)) - 7*
sqrt(10)*x^7*sqrt(89*sqrt(5) - 199)*log(sqrt(10)*sqrt(89*sqrt(5) - 199)*(9*
sqrt(5) + 20) + 10*x) + 7*sqrt(10)*x^7*sqrt(89*sqrt(5) - 199)*log(-sqrt(10)
*sqrt(89*sqrt(5) - 199)*(9*sqrt(5) + 20) + 10*x) + 7*sqrt(10)*x^7*sqrt(89*s
qrt(5) + 199)*log(sqrt(10)*sqrt(89*sqrt(5) + 199)*(9*sqrt(5) - 20) + 10*x)
- 7*sqrt(10)*x^7*sqrt(89*sqrt(5) + 199)*log(-sqrt(10)*sqrt(89*sqrt(5) + 199
)*(9*sqrt(5) - 20) + 10*x) - 280*x^4 - 40)/x^7
```

giac [A] time = 0.56, size = 159, normalized size = 0.84

$$-\frac{1}{20}\sqrt{890\sqrt{5}-1990}\arctan\left(\frac{x}{\sqrt{\frac{1}{2}\sqrt{5}+\frac{1}{2}}}\right)+\frac{1}{20}\sqrt{890\sqrt{5}+1990}\arctan\left(\frac{x}{\sqrt{\frac{1}{2}\sqrt{5}-\frac{1}{2}}}\right)-\frac{1}{40}\sqrt{890\sqrt{5}-1990}\log\left(\left|x+\sqrt{\frac{1}{2}\sqrt{5}+\frac{1}{2}}\right|\right)+\frac{1}{40}\sqrt{890\sqrt{5}-1990}\log\left(\left|x-\sqrt{\frac{1}{2}\sqrt{5}+\frac{1}{2}}\right|\right)+\frac{1}{40}\sqrt{890\sqrt{5}+1990}\log\left(\left|x+\sqrt{\frac{1}{2}\sqrt{5}-\frac{1}{2}}\right|\right)-\frac{1}{40}\sqrt{890\sqrt{5}+1990}\log\left(\left|x-\sqrt{\frac{1}{2}\sqrt{5}-\frac{1}{2}}\right|\right)-\frac{7x^4+1}{7x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^8/(x^8-3*x^4+1),x, algorithm="giac")
```

```
[Out] -1/20*sqrt(890*sqrt(5) - 1990)*arctan(x/sqrt(1/2*sqrt(5) + 1/2)) + 1/20*sqr
t(890*sqrt(5) + 1990)*arctan(x/sqrt(1/2*sqrt(5) - 1/2)) - 1/40*sqrt(890*sqr
t(5) - 1990)*log(abs(x + sqrt(1/2*sqrt(5) + 1/2))) + 1/40*sqrt(890*sqrt(5)
- 1990)*log(abs(x - sqrt(1/2*sqrt(5) + 1/2))) + 1/40*sqrt(890*sqrt(5) + 199
0)*log(abs(x + sqrt(1/2*sqrt(5) - 1/2))) - 1/40*sqrt(890*sqrt(5) + 1990)*lo
g(abs(x - sqrt(1/2*sqrt(5) - 1/2))) - 1/7*(7*x^4 + 1)/x^7
```

maple [A] time = 0.04, size = 216, normalized size = 1.14

$$\frac{11\sqrt{5}\,\mathrm{arctanh}\left(\frac{2x}{\sqrt{-2+2\sqrt{5}}}\right)}{10\sqrt{-2+2\sqrt{5}}}+\frac{5\,\mathrm{arctanh}\left(\frac{2x}{\sqrt{-2+2\sqrt{5}}}\right)}{2\sqrt{-2+2\sqrt{5}}}+\frac{11\sqrt{5}\,\mathrm{arctanh}\left(\frac{2x}{\sqrt{2+2\sqrt{5}}}\right)}{10\sqrt{2+2\sqrt{5}}}-\frac{5\,\mathrm{arctanh}\left(\frac{2x}{\sqrt{2+2\sqrt{5}}}\right)}{2\sqrt{2+2\sqrt{5}}}+\frac{11\sqrt{5}\arctan\left(\frac{2x}{\sqrt{-2+2\sqrt{5}}}\right)}{10\sqrt{-2+2\sqrt{5}}}+\frac{5\arctan\left(\frac{2x}{\sqrt{-2+2\sqrt{5}}}\right)}{2\sqrt{-2+2\sqrt{5}}}+\frac{11\sqrt{5}\arctan\left(\frac{2x}{\sqrt{2+2\sqrt{5}}}\right)}{10\sqrt{2+2\sqrt{5}}}-\frac{5\arctan\left(\frac{2x}{\sqrt{2+2\sqrt{5}}}\right)}{2\sqrt{2+2\sqrt{5}}}-\frac{1}{x^3}-\frac{1}{7x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^8/(x^8-3*x^4+1),x)
```

```
[Out] -1/7/x^7-1/x^3+11/10*5^(1/2)/(2+2*5^(1/2))^(1/2)*arctanh(2/(2+2*5^(1/2))^(1
/2)*x)-5/2/(2+2*5^(1/2))^(1/2)*arctanh(2/(2+2*5^(1/2))^(1/2)*x)+11/10*5^(1/
2)/(-2+2*5^(1/2))^(1/2)*arctan(2/(-2+2*5^(1/2))^(1/2)*x)+5/2/(-2+2*5^(1/2))
^(1/2)*arctan(2/(-2+2*5^(1/2))^(1/2)*x)+11/10*5^(1/2)/(-2+2*5^(1/2))^(1/2)*
arctanh(2/(-2+2*5^(1/2))^(1/2)*x)+5/2/(-2+2*5^(1/2))^(1/2)*arctanh(2/(-2+2*
5^(1/2))^(1/2)*x)+11/10*5^(1/2)/(2+2*5^(1/2))^(1/2)*arctan(2/(2+2*5^(1/2))^
(1/2)*x)-5/2/(2+2*5^(1/2))^(1/2)*arctan(2/(2+2*5^(1/2))^(1/2)*x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{7\,x^4+1}{7\,x^7}-\frac{1}{2}\int\frac{5\,x^2+8}{x^4+x^2-1}\,dx+\frac{1}{2}\int\frac{5\,x^2-8}{x^4-x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^8/(x^8-3*x^4+1),x, algorithm="maxima")
```

```
[Out] -1/7*(7*x^4 + 1)/x^7 - 1/2*integrate((5*x^2 + 8)/(x^4 + x^2 - 1), x) + 1/2*
integrate((5*x^2 - 8)/(x^4 - x^2 - 1), x)
```

mupad [B] time = 0.21, size = 291, normalized size = 1.54

$$-\frac{x^4+\frac{1}{7}}{x^7}+\frac{\sqrt{10}\,\mathrm{atan}\left(\frac{\sqrt{10}\,x\,\sqrt{-89\,\sqrt{5}-199}\,6677047\mathrm{i}}{2\,\left(74049691\,\sqrt{5}+165580139\right)}+\frac{\sqrt{5}\,\sqrt{10}\,x\,\sqrt{-89\,\sqrt{5}-199}\,14930373\mathrm{i}}{10\,\left(74049691\,\sqrt{5}+165580139\right)}\right)\,\sqrt{-89\,\sqrt{5}-199}\,1\mathrm{i}}{20}+\frac{\sqrt{10}\,\mathrm{atan}\left(\frac{\sqrt{10}\,x\,\sqrt{199-89\,\sqrt{5}}\,6677047\mathrm{i}}{2\,\left(74049691\,\sqrt{5}-165580139\right)}-\frac{\sqrt{5}\,\sqrt{10}\,x\,\sqrt{199-89\,\sqrt{5}}\,14930373\mathrm{i}}{10\,\left(74049691\,\sqrt{5}-165580139\right)}\right)\,\sqrt{199-89\,\sqrt{5}}\,1\mathrm{i}}{20}-\frac{\sqrt{10}\,\mathrm{atan}\left(\frac{\sqrt{10}\,x\,\sqrt{89\,\sqrt{5}-199}\,6677047\mathrm{i}}{2\,\left(74049691\,\sqrt{5}-165580139\right)}-\frac{\sqrt{5}\,\sqrt{10}\,x\,\sqrt{89\,\sqrt{5}-199}\,14930373\mathrm{i}}{10\,\left(74049691\,\sqrt{5}-165580139\right)}\right)\,\sqrt{89\,\sqrt{5}-199}\,1\mathrm{i}}{20}-\frac{\sqrt{10}\,\mathrm{atan}\left(\frac{\sqrt{10}\,x\,\sqrt{89\,\sqrt{5}+199}\,6677047\mathrm{i}}{2\,\left(74049691\,\sqrt{5}+165580139\right)}+\frac{\sqrt{5}\,\sqrt{10}\,x\,\sqrt{89\,\sqrt{5}+199}\,14930373\mathrm{i}}{10\,\left(74049691\,\sqrt{5}+165580139\right)}\right)\,\sqrt{89\,\sqrt{5}+199}\,1\mathrm{i}}{20}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^8*(x^8 - 3*x^4 + 1)),x)
```

```
[Out] (10^(1/2)*atan((10^(1/2)*x*(- 89*5^(1/2) - 199)^(1/2)*6677047i)/(2*(7404969
1*5^(1/2) + 165580139)) + (5^(1/2)*10^(1/2)*x*(- 89*5^(1/2) - 199)^(1/2)*14
930373i)/(10*(74049691*5^(1/2) + 165580139)))*(- 89*5^(1/2) - 199)^(1/2)*1i
)/20 - (x^4 + 1/7)/x^7 + (10^(1/2)*atan((10^(1/2)*x*(199 - 89*5^(1/2))^(1/2
)*6677047i)/(2*(74049691*5^(1/2) - 165580139)) - (5^(1/2)*10^(1/2)*x*(199 -
 89*5^(1/2))^(1/2)*14930373i)/(10*(74049691*5^(1/2) - 165580139)))*(199 - 8
9*5^(1/2))^(1/2)*1i)/20 - (10^(1/2)*atan((10^(1/2)*x*(89*5^(1/2) - 199)^(1/
2)*6677047i)/(2*(74049691*5^(1/2) - 165580139)) - (5^(1/2)*10^(1/2)*x*(89*5
^(1/2) - 199)^(1/2)*14930373i)/(10*(74049691*5^(1/2) - 165580139)))*(89*5^(
1/2) - 199)^(1/2)*1i)/20 - (10^(1/2)*atan((10^(1/2)*x*(89*5^(1/2) + 199)^(1
/2)*6677047i)/(2*(74049691*5^(1/2) + 165580139)) + (5^(1/2)*10^(1/2)*x*(89*
5^(1/2) + 199)^(1/2)*14930373i)/(10*(74049691*5^(1/2) + 165580139)))*(89*5^
(1/2) + 199)^(1/2)*1i)/20
```

sympy [A] time = 1.30, size = 70, normalized size = 0.37

$$\mathrm{RootSum}\left(6400t^4-15920t^2-1,\left(t\mapsto t\log\left(\frac{460800t^5}{17711}-\frac{2842588t}{17711}+x\right)\right)\right)+\mathrm{RootSum}\left(6400t^4+15920t^2-1,\left(t\mapsto t\log\left(\frac{460800t^5}{17711}-\frac{2842588t}{17711}+x\right)\right)\right)+\frac{-7x^4-1}{7x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**8/(x**8-3*x**4+1),x)
```

```
[Out] RootSum(6400*_t**4 - 15920*_t**2 - 1, Lambda(_t, _t*log(460800*_t**5/17711
- 2842588*_t/17711 + x))) + RootSum(6400*_t**4 + 15920*_t**2 - 1, Lambda(_t
, _t*log(460800*_t**5/17711 - 2842588*_t/17711 + x))) + (-7*x**4 - 1)/(7*x*
*7)
```

3.344 $\int \frac{x^3}{2+3x^4+x^8}\,dx$

Optimal. Leaf size=21

$$\frac{1}{4}\log\left(x^4+1\right)-\frac{1}{4}\log\left(x^4+2\right)$$

Rubi [A] time = 0.01, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {1352, 616, 31}

$$\frac{1}{4}\log\left(x^4+1\right)-\frac{1}{4}\log\left(x^4+2\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(2 + 3*x^4 + x^8),x]
```

```
[Out] Log[1 + x^4]/4 - Log[2 + x^4]/4
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 616

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[c/q, Int[1/Simp[b/2 - q/2 + c*x, x], x], x] - Dist[c/q, Int[1/Simp[b/2 + q/2 + c*x, x], x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[b^2 - 4*a*c] && PerfectSquareQ[b^2 - 4*a*c]
```

Rule 1352

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x^3}{2+3x^4+x^8}\,dx &= \frac{1}{4}\operatorname{Subst}\left(\int \frac{1}{2+3x+x^2}\,dx,x,x^4\right)\\ &= \frac{1}{4}\operatorname{Subst}\left(\int \frac{1}{1+x}\,dx,x,x^4\right)-\frac{1}{4}\operatorname{Subst}\left(\int \frac{1}{2+x}\,dx,x,x^4\right)\\ &= \frac{1}{4}\log\left(1+x^4\right)-\frac{1}{4}\log\left(2+x^4\right)\end{aligned}$$

Mathematica [A] time = 0.00, size = 21, normalized size = 1.00

$$\frac{1}{4}\log\left(x^4+1\right)-\frac{1}{4}\log\left(x^4+2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(2 + 3*x^4 + x^8),x]
```

```
[Out] Log[1 + x^4]/4 - Log[2 + x^4]/4
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{2+3x^4+x^8}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^3/(2 + 3*x^4 + x^8),x]
[Out] IntegrateAlgebraic[x^3/(2 + 3*x^4 + x^8), x]
```

fricas [A] time = 1.28, size = 17, normalized size = 0.81

$$-\frac{1}{4}\,\log\left(x^4+2\right)+\frac{1}{4}\,\log\left(x^4+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(x^8+3*x^4+2),x, algorithm="fricas")
[Out] -1/4*log(x^4 + 2) + 1/4*log(x^4 + 1)
```

giac [A] time = 0.28, size = 17, normalized size = 0.81

$$-\frac{1}{4}\,\log\left(x^4+2\right)+\frac{1}{4}\,\log\left(x^4+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(x^8+3*x^4+2),x, algorithm="giac")
[Out] -1/4*log(x^4 + 2) + 1/4*log(x^4 + 1)
```

maple [A] time = 0.00, size = 18, normalized size = 0.86

$$\frac{\ln\left(x^4+1\right)}{4}-\frac{\ln\left(x^4+2\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(x^8+3*x^4+2),x)
[Out] 1/4*ln(x^4+1)-1/4*ln(x^4+2)
```

maxima [A] time = 0.59, size = 17, normalized size = 0.81

$$-\frac{1}{4}\,\log\left(x^4+2\right)+\frac{1}{4}\,\log\left(x^4+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(x^8+3*x^4+2),x, algorithm="maxima")
[Out] -1/4*log(x^4 + 2) + 1/4*log(x^4 + 1)
```

mupad [B] time = 0.06, size = 16, normalized size = 0.76

$$-\frac{\operatorname{atanh}\left(\frac{256}{9\,\left(144\,x^4+160\right)}-\frac{7}{9}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(3*x^4 + x^8 + 2),x)
```

```
[Out] -atanh(256/(9*(144*x^4 + 160)) - 7/9)/2
```

sympy [A] time = 0.12, size = 15, normalized size = 0.71

$$\frac{\log\left(x^4+1\right)}{4}-\frac{\log\left(x^4+2\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(x**8+3*x**4+2),x)
[Out] log(x**4 + 1)/4 - log(x**4 + 2)/4
```

3.345 $\int \frac{x^{11}}{2+3x^4+x^8} dx$

Optimal. Leaf size=26

$$\frac{x^4}{4} + \frac{1}{4}\log\left(x^4+1\right) - \log\left(x^4+2\right)$$

Rubi [A] time = 0.02, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1357, 703, 632, 31}

$$\frac{x^4}{4} + \frac{1}{4}\log\left(x^4+1\right) - \log\left(x^4+2\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^11/(2 + 3*x^4 + x^8),x]
```

```
[Out] x^4/4 + Log[1 + x^4]/4 - Log[2 + x^4]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 632

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rule 703

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Simp[(e*(d + e*x)^(m - 1))/(c*(m - 1)), x] + Dist[1/c, Int[((d + e*x)^
(m - 2)*Simp[c*d^2 - a*e^2 + e*(2*c*d - b*e)*x, x])/(a + b*x + c*x^2), x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && GtQ[m, 1]
```

Rule 1357

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^{11}}{2+3x^4+x^8} dx &= \frac{1}{4}\text{Subst}\left(\int \frac{x^2}{2+3x+x^2} dx, x, x^4\right)\\
&= \frac{x^4}{4} + \frac{1}{4}\text{Subst}\left(\int \frac{-2-3x}{2+3x+x^2} dx, x, x^4\right)\\
&= \frac{x^4}{4} + \frac{1}{4}\text{Subst}\left(\int \frac{1}{1+x} dx, x, x^4\right) - \text{Subst}\left(\int \frac{1}{2+x} dx, x, x^4\right)\\
&= \frac{x^4}{4} + \frac{1}{4}\log\left(1+x^4\right) - \log\left(2+x^4\right)
\end{aligned}$$

Mathematica [A] time = 0.00, size = 26, normalized size = 1.00

$$\frac{x^4}{4}+\frac{1}{4}\log\left(x^4+1\right)-\log\left(x^4+2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^11/(2 + 3*x^4 + x^8),x]
[Out] x^4/4 + Log[1 + x^4]/4 - Log[2 + x^4]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{11}}{2+3x^4+x^8}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^11/(2 + 3*x^4 + x^8),x]
[Out] IntegrateAlgebraic[x^11/(2 + 3*x^4 + x^8), x]
```

fricas [A] time = 1.20, size = 22, normalized size = 0.85

$$\frac{1}{4}\,x^4-\log\left(x^4+2\right)+\frac{1}{4}\,\log\left(x^4+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(x^8+3*x^4+2),x, algorithm="fricas")
[Out] 1/4*x^4 - log(x^4 + 2) + 1/4*log(x^4 + 1)
```

giac [A] time = 0.36, size = 22, normalized size = 0.85

$$\frac{1}{4}\,x^4-\log\left(x^4+2\right)+\frac{1}{4}\,\log\left(x^4+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(x^8+3*x^4+2),x, algorithm="giac")
[Out] 1/4*x^4 - log(x^4 + 2) + 1/4*log(x^4 + 1)
```

maple [A] time = 0.01, size = 23, normalized size = 0.88

$$\frac{x^4}{4}+\frac{\ln\left(x^4+1\right)}{4}-\ln\left(x^4+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11/(x^8+3*x^4+2),x)
[Out] 1/4*x^4+1/4*ln(x^4+1)-ln(x^4+2)
```

maxima [A] time = 0.86, size = 22, normalized size = 0.85

$$\frac{1}{4}\,x^4-\log\left(x^4+2\right)+\frac{1}{4}\,\log\left(x^4+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(x^8+3*x^4+2),x, algorithm="maxima")
[Out] 1/4*x^4 - log(x^4 + 2) + 1/4*log(x^4 + 1)
```

mupad [B] time = 1.32, size = 22, normalized size = 0.85

$$\frac{\ln\left(x^4+1\right)}{4}-\ln\left(x^4+2\right)+\frac{x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11/(3*x^4 + x^8 + 2),x)
[Out] log(x^4 + 1)/4 - log(x^4 + 2) + x^4/4
```

sympy [A] time = 0.13, size = 19, normalized size = 0.73

$$\frac{x^4}{4}+\frac{\log\left(x^4+1\right)}{4}-\log\left(x^4+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**11/(x**8+3*x**4+2),x)
[Out] x**4/4 + log(x**4 + 1)/4 - log(x**4 + 2)
```

3.346 $\int \frac{x^9}{2+x^5+x^{10}} dx$

Optimal. Leaf size=37

$$\frac{1}{10}\log\left(x^{10}+x^5+2\right)-\frac{\tan^{-1}\left(\frac{2x^5+1}{\sqrt{7}}\right)}{5\sqrt{7}}$$

Rubi [A] time = 0.03, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {1357, 634, 618, 204, 628}

$$\frac{1}{10}\log\left(x^{10}+x^5+2\right)-\frac{\tan^{-1}\left(\frac{2x^5+1}{\sqrt{7}}\right)}{5\sqrt{7}}$$

Antiderivative was successfully verified.

```
[In] Int[x^9/(2 + x^5 + x^10),x]
```

```
[Out] -ArcTan[(1 + 2*x^5)/Sqrt[7]]/(5*Sqrt[7]) + Log[2 + x^5 + x^10]/10
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1357

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^9}{2+x^5+x^{10}}\,dx = \frac{1}{5}\operatorname{Subst}\left(\int \frac{x}{2+x+x^2}\,dx, x, x^5\right)$$
$$= -\left(\frac{1}{10}\operatorname{Subst}\left(\int \frac{1}{2+x+x^2}\,dx, x, x^5\right)\right) + \frac{1}{10}\operatorname{Subst}\left(\int \frac{1+2x}{2+x+x^2}\,dx, x, x^5\right)$$
$$= \frac{1}{10}\log\left(2+x^5+x^{10}\right) + \frac{1}{5}\operatorname{Subst}\left(\int \frac{1}{-7-x^2}\,dx, x, 1+2x^5\right)$$
$$= -\frac{\tan^{-1}\left(\frac{1+2x^5}{\sqrt{7}}\right)}{5\sqrt{7}} + \frac{1}{10}\log\left(2+x^5+x^{10}\right)$$

Mathematica [A] time = 0.01, size = 37, normalized size = 1.00

$$\frac{1}{10}\log\left(x^{10}+x^5+2\right) - \frac{\tan^{-1}\left(\frac{2x^5+1}{\sqrt{7}}\right)}{5\sqrt{7}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^9/(2 + x^5 + x^10),x]
[Out] -1/5*ArcTan[(1 + 2*x^5)/Sqrt[7]]/Sqrt[7] + Log[2 + x^5 + x^10]/10
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^9}{2+x^5+x^{10}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^9/(2 + x^5 + x^10),x]
[Out] IntegrateAlgebraic[x^9/(2 + x^5 + x^10), x]
```

fricas [A] time = 1.20, size = 30, normalized size = 0.81

$$-\frac{1}{35}\sqrt{7}\arctan\left(\frac{1}{7}\sqrt{7}\left(2x^5+1\right)\right) + \frac{1}{10}\log\left(x^{10}+x^5+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(x^10+x^5+2),x, algorithm="fricas")
[Out] -1/35*sqrt(7)*arctan(1/7*sqrt(7)*(2*x^5 + 1)) + 1/10*log(x^10 + x^5 + 2)
```

giac [A] time = 2.72, size = 30, normalized size = 0.81

$$-\frac{1}{35}\sqrt{7}\arctan\left(\frac{1}{7}\sqrt{7}\left(2x^5+1\right)\right) + \frac{1}{10}\log\left(x^{10}+x^5+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(x^10+x^5+2),x, algorithm="giac")
[Out] -1/35*sqrt(7)*arctan(1/7*sqrt(7)*(2*x^5 + 1)) + 1/10*log(x^10 + x^5 + 2)
```

maple [A] time = 0.00, size = 31, normalized size = 0.84

$$-\frac{\sqrt{7}\arctan\left(\frac{\left(2x^5+1\right)\sqrt{7}}{7}\right)}{35} + \frac{\ln\left(x^{10}+x^5+2\right)}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9/(x^10+x^5+2),x)
```

```
[Out] 1/10*ln(x^10+x^5+2)-1/35*arctan(1/7*(2*x^5+1)*7^(1/2))*7^(1/2)
```

maxima [A] time = 2.05, size = 30, normalized size = 0.81

$$-\frac{1}{35}\sqrt{7}\arctan\left(\frac{1}{7}\sqrt{7}\left(2x^5+1\right)\right)+\frac{1}{10}\log\left(x^{10}+x^5+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(x^10+x^5+2),x, algorithm="maxima")
```

```
[Out] -1/35*sqrt(7)*arctan(1/7*sqrt(7)*(2*x^5 + 1)) + 1/10*log(x^10 + x^5 + 2)
```

mupad [B] time = 1.35, size = 32, normalized size = 0.86

$$\frac{\ln\left(x^{10}+x^5+2\right)}{10}-\frac{\sqrt{7}\,\mathrm{atan}\left(\frac{2\sqrt{7}\,x^5}{7}+\frac{\sqrt{7}}{7}\right)}{35}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9/(x^5 + x^10 + 2),x)
```

```
[Out] log(x^5 + x^10 + 2)/10 - (7^(1/2)*atan(7^(1/2)/7 + (2*7^(1/2)*x^5)/7))/35
```

sympy [A] time = 0.14, size = 37, normalized size = 1.00

$$\frac{\log\left(x^{10}+x^5+2\right)}{10}-\frac{\sqrt{7}\,\mathrm{atan}\left(\frac{2\sqrt{7}x^5}{7}+\frac{\sqrt{7}}{7}\right)}{35}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**9/(x**10+x**5+2),x)
```

```
[Out] log(x**10 + x**5 + 2)/10 - sqrt(7)*atan(2*sqrt(7)*x**5/7 + sqrt(7)/7)/35
```

3.347 $\int \frac{x^4}{2+x^5+x^{10}} \, dx$

Optimal. Leaf size=23

$$\frac{2\tan^{-1}\left(\frac{2x^5+1}{\sqrt{7}}\right)}{5\sqrt{7}}$$

Rubi [A] time = 0.02, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {1352, 618, 204}

$$\frac{2\tan^{-1}\left(\frac{2x^5+1}{\sqrt{7}}\right)}{5\sqrt{7}}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(2 + x^5 + x^10),x]

[Out] (2*ArcTan[(1 + 2*x^5)/Sqrt[7]])/(5*Sqrt[7])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1352

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a,
b, c, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^4}{2+x^5+x^{10}} \, dx &= \frac{1}{5}\text{Subst}\left(\int \frac{1}{2+x+x^2} \, dx, x, x^5\right) \\
&= -\left(\frac{2}{5}\text{Subst}\left(\int \frac{1}{-7-x^2} \, dx, x, 1+2x^5\right)\right) \\
&= \frac{2\tan^{-1}\left(\frac{1+2x^5}{\sqrt{7}}\right)}{5\sqrt{7}}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 23, normalized size = 1.00

$$\frac{2\tan^{-1}\left(\frac{2x^5+1}{\sqrt{7}}\right)}{5\sqrt{7}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(2 + x^5 + x^10),x]
[Out] (2*ArcTan[(1 + 2*x^5)/Sqrt[7]])/(5*Sqrt[7])
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{2 + x^5 + x^{10}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^4/(2 + x^5 + x^10),x]
[Out] IntegrateAlgebraic[x^4/(2 + x^5 + x^10), x]
```

fricas [A] time = 1.22, size = 18, normalized size = 0.78

$$\frac{2}{35}\,\sqrt{7}\,\arctan\left(\frac{1}{7}\,\sqrt{7}\,\left(2\,x^5+1\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(x^10+x^5+2),x, algorithm="fricas")
[Out] 2/35*sqrt(7)*arctan(1/7*sqrt(7)*(2*x^5 + 1))
```

giac [A] time = 2.96, size = 18, normalized size = 0.78

$$\frac{2}{35}\,\sqrt{7}\,\arctan\left(\frac{1}{7}\,\sqrt{7}\,\left(2\,x^5+1\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(x^10+x^5+2),x, algorithm="giac")
[Out] 2/35*sqrt(7)*arctan(1/7*sqrt(7)*(2*x^5 + 1))
```

maple [A] time = 0.00, size = 19, normalized size = 0.83

$$\frac{2\sqrt{7}\,\arctan\left(\frac{\left(2x^5+1\right)\sqrt{7}}{7}\right)}{35}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(x^10+x^5+2),x)
[Out] 2/35*7^(1/2)*arctan(1/7*(2*x^5+1)*7^(1/2))
```

maxima [A] time = 2.09, size = 18, normalized size = 0.78

$$\frac{2}{35}\,\sqrt{7}\,\arctan\left(\frac{1}{7}\,\sqrt{7}\,\left(2\,x^5+1\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(x^10+x^5+2),x, algorithm="maxima")
[Out] 2/35*sqrt(7)*arctan(1/7*sqrt(7)*(2*x^5 + 1))
```

mupad [B] time = 1.33, size = 20, normalized size = 0.87

$$\frac{2\,\sqrt{7}\,\mathrm{atan}\left(\frac{2\,\sqrt{7}\,x^5}{7}+\frac{\sqrt{7}}{7}\right)}{35}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(x^5 + x^10 + 2),x)
```

```
[Out] (2*7^(1/2)*atan(7^(1/2)/7 + (2*7^(1/2)*x^5)/7))/35
```

sympy [A] time = 0.13, size = 27, normalized size = 1.17

$$\frac{2\sqrt{7}\operatorname{atan}\left(\frac{2\sqrt{7}x^5}{7}+\frac{\sqrt{7}}{7}\right)}{35}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(x**10+x**5+2),x)
```

```
[Out] 2*sqrt(7)*atan(2*sqrt(7)*x**5/7 + sqrt(7)/7)/35
```

3.348 $\int \frac{1}{x(1+x^5+x^{10})} \, dx$

Optimal. Leaf size=39

$$-\frac{\tan^{-1}\left(\frac{2x^5+1}{\sqrt{3}}\right)}{5\sqrt{3}}-\frac{1}{10}\log\left(x^{10}+x^5+1\right)+\log(x)$$

Rubi [A] time = 0.04, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {1357, 705, 29, 634, 618, 204, 628}

$$-\frac{1}{10}\log\left(x^{10}+x^5+1\right)-\frac{\tan^{-1}\left(\frac{2x^5+1}{\sqrt{3}}\right)}{5\sqrt{3}}+\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(1 + x^5 + x^10)),x]
```

```
[Out] -ArcTan[(1 + 2*x^5)/Sqrt[3]]/(5*Sqrt[3]) + Log[x] - Log[1 + x^5 + x^10]/10
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 705

```
Int[1/(((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)), x_Symbol]
 :> Dist[e^2/(c*d^2 - b*d*e + a*e^2), Int[1/(d + e*x), x], x] + Dist[1/(c*d
^2 - b*d*e + a*e^2), Int[(c*d - b*e - c*e*x)/(a + b*x + c*x^2), x], x] /; F
reeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^
2, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1357

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x\left(1+x^5+x^{10}\right)}\,dx &= \frac{1}{5}\operatorname{Subst}\left(\int \frac{1}{x\left(1+x+x^2\right)}\,dx,x,x^5\right)\\
&= \frac{1}{5}\operatorname{Subst}\left(\int \frac{1}{x}\,dx,x,x^5\right)+\frac{1}{5}\operatorname{Subst}\left(\int \frac{-1-x}{1+x+x^2}\,dx,x,x^5\right)\\
&= \log(x)-\frac{1}{10}\operatorname{Subst}\left(\int \frac{1}{1+x+x^2}\,dx,x,x^5\right)-\frac{1}{10}\operatorname{Subst}\left(\int \frac{1+2x}{1+x+x^2}\,dx,x,x^5\right)\\
&= \log(x)-\frac{1}{10}\log\left(1+x^5+x^{10}\right)+\frac{1}{5}\operatorname{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,1+2x^5\right)\\
&= -\frac{\tan^{-1}\left(\frac{1+2x^5}{\sqrt{3}}\right)}{5\sqrt{3}}+\log(x)-\frac{1}{10}\log\left(1+x^5+x^{10}\right)
\end{aligned}
$$

Mathematica [C] time = 0.04, size = 197, normalized size = 5.05

$$
-\frac{1}{5}\text{RootSum}\left[\#1^8-\#1^7+\#1^5-\#1^4+\#1^3-\#1+1\&,\frac{4\#1^7\log(x-\#1)-3\#1^6\log(x-\#1)-\#1^5\log(x-\#1)+3\#1^4\log(x-\#1)-\#1^3\log(x-\#1)+2\#1^2\log(x-\#1)-\#1\log(x-\#1)}{8\#1^7-7\#1^6+5\#1^4-4\#1^3+3\#1^2-1}\&\right]-\frac{1}{10}\log\left(x^2+x+1\right)+\log(x)+\frac{\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{5\sqrt{3}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(1 + x^5 + x^10)),x]
```

```
[Out] ArcTan[(1 + 2*x)/Sqrt[3]]/(5*Sqrt[3]) + Log[x] - Log[1 + x + x^2]/10 - Root
Sum[1 - #1 + #1^3 - #1^4 + #1^5 - #1^7 + #1^8 & , (-(Log[x - #1]*#1) + 2*Lo
g[x - #1]*#1^2 - Log[x - #1]*#1^3 + 3*Log[x - #1]*#1^4 - Log[x - #1]*#1^5 -
 3*Log[x - #1]*#1^6 + 4*Log[x - #1]*#1^7)/(-1 + 3*#1^2 - 4*#1^3 + 5*#1^4 -
7*#1^6 + 8*#1^7) & ]/5
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$
\int \frac{1}{x\left(1+x^5+x^{10}\right)}\,dx
$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x*(1 + x^5 + x^10)),x]
```

```
[Out] IntegrateAlgebraic[1/(x*(1 + x^5 + x^10)), x]
```

fricas [A] time = 1.16, size = 32, normalized size = 0.82

$$
-\frac{1}{15}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^5+1\right)\right)-\frac{1}{10}\log\left(x^{10}+x^5+1\right)+\log(x)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x^10+x^5+1),x, algorithm="fricas")
```

```
[Out] -1/15*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^5 + 1)) - 1/10*log(x^10 + x^5 + 1) +
log(x)
```

giac [A] time = 0.41, size = 33, normalized size = 0.85

$$-\frac{1}{15}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}\left(2\,x^{5}+1\right)\right)-\frac{1}{10}\,\log\left(x^{10}+x^{5}+1\right)+\log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x^10+x^5+1),x, algorithm="giac")
```

```
[Out] -1/15*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^5 + 1)) - 1/10*log(x^10 + x^5 + 1) +
log(abs(x))
```

maple [B] time = 0.04, size = 66, normalized size = 1.69

$$-\frac{\sqrt{3}\arctan\left(\frac{2\sqrt{3}\,x^{5}}{3}+\frac{\sqrt{3}}{3}\right)}{15}+\ln(x)-\frac{\ln\left(x^{2}+x+1\right)}{10}-\frac{\ln\left(4x^{8}-4x^{7}+4x^{5}-4x^{4}+4x^{3}-4x+4\right)}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(x^10+x^5+1),x)
```

```
[Out] -1/10*ln(4*x^8-4*x^7+4*x^5-4*x^4+4*x^3-4*x+4)-1/15*3^(1/2)*arctan(2/3*3^(1/
2)*x^5+1/3*3^(1/2))+ln(x)-1/10*ln(x^2+x+1)
```

maxima [A] time = 2.03, size = 36, normalized size = 0.92

$$-\frac{1}{15}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}\left(2\,x^{5}+1\right)\right)-\frac{1}{10}\,\log\left(x^{10}+x^{5}+1\right)+\frac{1}{5}\,\log\left(x^{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x^10+x^5+1),x, algorithm="maxima")
```

```
[Out] -1/15*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^5 + 1)) - 1/10*log(x^10 + x^5 + 1) +
1/5*log(x^5)
```

mupad [B] time = 0.06, size = 34, normalized size = 0.87

$$\ln(x)-\frac{\ln\left(x^{10}+x^{5}+1\right)}{10}-\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\,\sqrt{3}\,x^{5}}{3}+\frac{\sqrt{3}}{3}\right)}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(x^5 + x^10 + 1)),x)
```

```
[Out] log(x) - log(x^5 + x^10 + 1)/10 - (3^(1/2)*atan(3^(1/2)/3 + (2*3^(1/2)*x^5)
/3))/15
```

sympy [A] time = 0.17, size = 41, normalized size = 1.05

$$\log\left(x\right)-\frac{\log\left(x^{10}+x^{5}+1\right)}{10}-\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}\,x^{5}}{3}+\frac{\sqrt{3}}{3}\right)}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x**10+x**5+1),x)
```

```
[Out] log(x) - log(x**10 + x**5 + 1)/10 - sqrt(3)*atan(2*sqrt(3)*x**5/3 + sqrt(3)
/3)/15
```

3.349 $\int \frac{1}{x^6(1+x^5+x^{10})} \, dx$

Optimal. Leaf size=48

$$-\frac{1}{5x^5} - \frac{\tan^{-1}\left(\frac{2x^5+1}{\sqrt{3}}\right)}{5\sqrt{3}} + \frac{1}{10}\log\left(x^{10}+x^5+1\right) - \log(x)$$

Rubi [A] time = 0.05, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {1357, 709, 800, 634, 618, 204, 628}

$$-\frac{1}{5x^5} + \frac{1}{10}\log\left(x^{10}+x^5+1\right) - \frac{\tan^{-1}\left(\frac{2x^5+1}{\sqrt{3}}\right)}{5\sqrt{3}} - \log(x)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^6*(1 + x^5 + x^10)),x]
```

```
[Out] -1/(5*x^5) - ArcTan[(1 + 2*x^5)/Sqrt[3]]/(5*Sqrt[3]) - Log[x] + Log[1 + x^5
 + x^10]/10
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 709

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Simp[(e*(d + e*x)^(m + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dis
t[1/(c*d^2 - b*d*e + a*e^2), Int[((d + e*x)^(m + 1)*Simp[c*d - b*e - c*e*x,
 x])/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 -
 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[m
, -1]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 1357

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x^6\left(1+x^5+x^{10}\right)}\,dx &= \frac{1}{5}\operatorname{Subst}\left(\int \frac{1}{x^2\left(1+x+x^2\right)}\,dx,x,x^5\right)\\
&= -\frac{1}{5x^5}+\frac{1}{5}\operatorname{Subst}\left(\int \frac{-1-x}{x\left(1+x+x^2\right)}\,dx,x,x^5\right)\\
&= -\frac{1}{5x^5}+\frac{1}{5}\operatorname{Subst}\left(\int \left(-\frac{1}{x}+\frac{x}{1+x+x^2}\right)dx,x,x^5\right)\\
&= -\frac{1}{5x^5}-\log(x)+\frac{1}{5}\operatorname{Subst}\left(\int \frac{x}{1+x+x^2}\,dx,x,x^5\right)\\
&= -\frac{1}{5x^5}-\log(x)-\frac{1}{10}\operatorname{Subst}\left(\int \frac{1}{1+x+x^2}\,dx,x,x^5\right)+\frac{1}{10}\operatorname{Subst}\left(\int \frac{1+2x}{1+x+x^2}\,dx,x,x^5\right)\\
&= -\frac{1}{5x^5}-\log(x)+\frac{1}{10}\log\left(1+x^5+x^{10}\right)+\frac{1}{5}\operatorname{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,1+2x^5\right)\\
&= -\frac{1}{5x^5}-\frac{\tan^{-1}\left(\frac{1+2x^5}{\sqrt{3}}\right)}{5\sqrt{3}}-\log(x)+\frac{1}{10}\log\left(1+x^5+x^{10}\right)
\end{aligned}
$$

Mathematica [C] time = 0.04, size = 208, normalized size = 4.33

$$
\frac{1}{30}\left(6\operatorname{RootSum}\left[\#1^8-\#1^7+\#1^5-\#1^4+\#1^3-\#1+1\&,\frac{4\#1^7\log(x-\#1)-4\#1^6\log(x-\#1)+\#1^5\log(x-\#1)+2\#1^4\log(x-\#1)-3\#1^3\log(x-\#1)+\#1^2\log(x-\#1)+\#1\log(x-\#1)-\log(x-\#1)}{8\#1^7-7\#1^6+5\#1^4-4\#1^3+3\#1^2-1}\&\right]-\frac{6}{x^5}+3\log\left(x^2+x+1\right)-30\log(x)+2\sqrt{3}\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^6*(1 + x^5 + x^10)),x]
```

```
[Out] (-6/x^5 + 2*Sqrt[3]*ArcTan[(1 + 2*x)/Sqrt[3]] - 30*Log[x] + 3*Log[1 + x + x
^2] + 6*RootSum[1 - #1 + #1^3 - #1^4 + #1^5 - #1^7 + #1^8 & , (-Log[x - #1]
 + Log[x - #1]*#1 + Log[x - #1]*#1^2 - 3*Log[x - #1]*#1^3 + 2*Log[x - #1]*#
1^4 + Log[x - #1]*#1^5 - 4*Log[x - #1]*#1^6 + 4*Log[x - #1]*#1^7)/(-1 + 3*#
1^2 - 4*#1^3 + 5*#1^4 - 7*#1^6 + 8*#1^7) & ])/30
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$
\int \frac{1}{x^6\left(1+x^5+x^{10}\right)}\,dx
$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x^6*(1 + x^5 + x^10)),x]
```

```
[Out] IntegrateAlgebraic[1/(x^6*(1 + x^5 + x^10)), x]
```

fricas [A] time = 1.27, size = 49, normalized size = 1.02

$$-\frac{2\sqrt{3}x^5\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^5+1\right)\right)-3x^5\log\left(x^{10}+x^5+1\right)+30x^5\log(x)+6}{30x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(x^10+x^5+1),x, algorithm="fricas")
```

```
[Out] -1/30*(2*sqrt(3)*x^5*arctan(1/3*sqrt(3)*(2*x^5 + 1)) - 3*x^5*log(x^10 + x^5
 + 1) + 30*x^5*log(x) + 6)/x^5
```

giac [A] time = 0.25, size = 45, normalized size = 0.94

$$-\frac{1}{15}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^5+1\right)\right)+\frac{x^5-1}{5x^5}+\frac{1}{10}\log\left(x^{10}+x^5+1\right)-\log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(x^10+x^5+1),x, algorithm="giac")
```

```
[Out] -1/15*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^5 + 1)) + 1/5*(x^5 - 1)/x^5 + 1/10*lo
g(x^10 + x^5 + 1) - log(abs(x))
```

maple [A] time = 0.02, size = 73, normalized size = 1.52

$$-\frac{\sqrt{3}\arctan\left(\frac{2\sqrt{3}x^5}{3}+\frac{\sqrt{3}}{3}\right)}{15}-\ln(x)+\frac{\ln\left(x^2+x+1\right)}{10}+\frac{\ln\left(4x^8-4x^7+4x^5-4x^4+4x^3-4x+4\right)}{10}-\frac{1}{5x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^6/(x^10+x^5+1),x)
```

```
[Out] 1/10*ln(4*x^8-4*x^7+4*x^5-4*x^4+4*x^3-4*x+4)-1/15*3^(1/2)*arctan(2/3*3^(1/2
)*x^5+1/3*3^(1/2))-1/5/x^5-ln(x)+1/10*ln(x^2+x+1)
```

maxima [A] time = 2.07, size = 41, normalized size = 0.85

$$-\frac{1}{15}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^5+1\right)\right)-\frac{1}{5x^5}+\frac{1}{10}\log\left(x^{10}+x^5+1\right)-\frac{1}{5}\log\left(x^5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(x^10+x^5+1),x, algorithm="maxima")
```

```
[Out] -1/15*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^5 + 1)) - 1/5/x^5 + 1/10*log(x^10 + x
^5 + 1) - 1/5*log(x^5)
```

mupad [B] time = 1.37, size = 41, normalized size = 0.85

$$\frac{\ln\left(x^{10}+x^5+1\right)}{10}-\ln(x)-\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x^5}{3}+\frac{\sqrt{3}}{3}\right)}{15}-\frac{1}{5x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^6*(x^5 + x^10 + 1)),x)
```

```
[Out] log(x^5 + x^10 + 1)/10 - log(x) - (3^(1/2)*atan(3^(1/2)/3 + (2*3^(1/2)*x^5)
/3))/15 - 1/(5*x^5)
```

sympy [A] time = 0.20, size = 48, normalized size = 1.00

$$-\log(x) + \frac{\log\left(x^{10} + x^5 + 1\right)}{10} - \frac{\sqrt{3}\,\operatorname{atan}\left(\frac{2\sqrt{3}\,x^5}{3} + \frac{\sqrt{3}}{3}\right)}{15} - \frac{1}{5x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**6/(x**10+x**5+1),x)
```

```
[Out] -log(x) + log(x**10 + x**5 + 1)/10 - sqrt(3)*atan(2*sqrt(3)*x**5/3 + sqrt(3
)/3)/15 - 1/(5*x**5)
```

3.350 $\int \frac{1}{x+x^6+x^{11}} \, dx$

Optimal. Leaf size=39

$$-\frac{\tan^{-1}\left(\frac{2x^5+1}{\sqrt{3}}\right)}{5\sqrt{3}}-\frac{1}{10}\log\left(x^{10}+x^5+1\right)+\log(x)$$

Rubi [A] time = 0.04, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.800, Rules used = {1594, 1357, 705, 29, 634, 618, 204, 628}

$$-\frac{1}{10}\log\left(x^{10}+x^5+1\right)-\frac{\tan^{-1}\left(\frac{2x^5+1}{\sqrt{3}}\right)}{5\sqrt{3}}+\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(x + x^6 + x^11)^(-1),x]
```

```
[Out] -ArcTan[(1 + 2*x^5)/Sqrt[3]]/(5*Sqrt[3]) + Log[x] - Log[1 + x^5 + x^10]/10
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 705

```
Int[1/(((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)), x_Symbol]
 :> Dist[e^2/(c*d^2 - b*d*e + a*e^2), Int[1/(d + e*x), x], x] + Dist[1/(c*d
^2 - b*d*e + a*e^2), Int[(c*d - b*e - c*e*x)/(a + b*x + c*x^2), x], x] /; F
reeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^
2, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1357

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1594

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(n_.), x
_Symbol] :> Int[u*x^(n*p)*(a + b*x^(q - p) + c*x^(r - p))^n, x] /; FreeQ[{a
, b, c, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && PosQ[r - p]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x+x^6+x^{11}}\,dx &= \int \frac{1}{x\left(1+x^5+x^{10}\right)}\,dx\\
&= \frac{1}{5}\operatorname{Subst}\left(\int \frac{1}{x\left(1+x+x^2\right)}\,dx,x,x^5\right)\\
&= \frac{1}{5}\operatorname{Subst}\left(\int \frac{1}{x}\,dx,x,x^5\right)+\frac{1}{5}\operatorname{Subst}\left(\int \frac{-1-x}{1+x+x^2}\,dx,x,x^5\right)\\
&= \log(x)-\frac{1}{10}\operatorname{Subst}\left(\int \frac{1}{1+x+x^2}\,dx,x,x^5\right)-\frac{1}{10}\operatorname{Subst}\left(\int \frac{1+2x}{1+x+x^2}\,dx,x,x^5\right)\\
&= \log(x)-\frac{1}{10}\log\left(1+x^5+x^{10}\right)+\frac{1}{5}\operatorname{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,1+2x^5\right)\\
&= -\frac{\tan^{-1}\left(\frac{1+2x^5}{\sqrt{3}}\right)}{5\sqrt{3}}+\log(x)-\frac{1}{10}\log\left(1+x^5+x^{10}\right)
\end{aligned}
$$

Mathematica [C] time = 0.02, size = 197, normalized size = 5.05

$$-\frac{1}{5}\text{RootSum}\left[\#1^8-\#1^7+\#1^5-\#1^4+\#1^3-\#1+1\&,\frac{4\#1^7\log(x-\#1)-3\#1^6\log(x-\#1)-\#1^5\log(x-\#1)+3\#1^4\log(x-\#1)-\#1^3\log(x-\#1)+2\#1^2\log(x-\#1)-\#1\log(x-\#1)}{8\#1^7-7\#1^6+5\#1^4-4\#1^3+3\#1^2-1}\&\right]-\frac{1}{10}\log\left(x^2+x+1\right)+\log(x)+\frac{\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{5\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x + x^6 + x^11)^(-1),x]
```

```
[Out] ArcTan[(1 + 2*x)/Sqrt[3]]/(5*Sqrt[3]) + Log[x] - Log[1 + x + x^2]/10 - Root
Sum[1 - #1 + #1^3 - #1^4 + #1^5 - #1^7 + #1^8 & , (-(Log[x - #1]*#1) + 2*Lo
g[x - #1]*#1^2 - Log[x - #1]*#1^3 + 3*Log[x - #1]*#1^4 - Log[x - #1]*#1^5 -
 3*Log[x - #1]*#1^6 + 4*Log[x - #1]*#1^7)/(-1 + 3*#1^2 - 4*#1^3 + 5*#1^4 -
7*#1^6 + 8*#1^7) & ]/5
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x+x^6+x^{11}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(x + x^6 + x^11)^(-1),x]
```

```
[Out] IntegrateAlgebraic[(x + x^6 + x^11)^(-1), x]
```

fricas [A] time = 1.06, size = 32, normalized size = 0.82

$$-\frac{1}{15}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^5+1\right)\right)-\frac{1}{10}\log\left(x^{10}+x^5+1\right)+\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^11+x^6+x),x, algorithm="fricas")
```

```
[Out] -1/15*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^5 + 1)) - 1/10*log(x^10 + x^5 + 1) +
log(x)
```

giac [A] time = 0.35, size = 33, normalized size = 0.85

$$-\frac{1}{15}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^5+1\right)\right)-\frac{1}{10}\log\left(x^{10}+x^5+1\right)+\log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^11+x^6+x),x, algorithm="giac")
```

```
[Out] -1/15*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^5 + 1)) - 1/10*log(x^10 + x^5 + 1) +
log(abs(x))
```

maple [B] time = 0.02, size = 66, normalized size = 1.69

$$-\frac{\sqrt{3}\arctan\left(\frac{2\sqrt{3}\,x^5}{3}+\frac{\sqrt{3}}{3}\right)}{15}+\ln(x)-\frac{\ln\left(x^2+x+1\right)}{10}-\frac{\ln\left(4x^8-4x^7+4x^5-4x^4+4x^3-4x+4\right)}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^11+x^6+x),x)
```

```
[Out] -1/15*3^(1/2)*arctan(2/3*3^(1/2)*x^5+1/3*3^(1/2))+ln(x)-1/10*ln(x^2+x+1)-1/
10*ln(4*x^8-4*x^7+4*x^5-4*x^4+4*x^3-4*x+4)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{15}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\,(2x+1)\right)-\frac{1}{5}\int\frac{4x^7-3x^6-x^5+3x^4-x^3+2x^2-x}{x^8-x^7+x^5-x^4+x^3-x+1}\,dx-\frac{1}{10}\log\left(x^2+x+1\right)+\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^11+x^6+x),x, algorithm="maxima")
```

```
[Out] 1/15*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 1/5*integrate((4*x^7 - 3*x^6 -
 x^5 + 3*x^4 - x^3 + 2*x^2 - x)/(x^8 - x^7 + x^5 - x^4 + x^3 - x + 1), x) -
 1/10*log(x^2 + x + 1) + log(x)
```

mupad [B] time = 0.03, size = 34, normalized size = 0.87

$$\ln(x)-\frac{\ln\left(x^{10}+x^5+1\right)}{10}-\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}\,x^5}{3}+\frac{\sqrt{3}}{3}\right)}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x + x^6 + x^11),x)
```

```
[Out] log(x) - log(x^5 + x^10 + 1)/10 - (3^(1/2)*atan(3^(1/2)/3 + (2*3^(1/2)*x^5)
/3))/15
```

sympy [A] time = 0.17, size = 41, normalized size = 1.05

$$\log\left(x\right)-\frac{\log\left(x^{10}+x^5+1\right)}{10}-\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}\,x^5}{3}+\frac{\sqrt{3}}{3}\right)}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**11+x**6+x),x)
```

```
[Out] log(x) - log(x**10 + x**5 + 1)/10 - sqrt(3)*atan(2*sqrt(3)*x**5/3 + sqrt(3)
/3)/15
```

3.351 $\int \frac{x^3}{c+\frac{a}{x^2}+\frac{b}{x}} \, dx$

Optimal. Leaf size=147

$$\frac{(a^2c^2-3ab^2c+b^4)\log(a+bx+cx^2)}{2c^5}+\frac{b(5a^2c^2-5ab^2c+b^4)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^5\sqrt{b^2-4ac}}-\frac{bx(b^2-2ac)}{c^4}+\frac{x^2(b^2-ac)}{2c^3}$$

Rubi [A] time = 0.14, antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1354, 701, 634, 618, 206, 628}

$$\frac{(a^2c^2-3ab^2c+b^4)\log(a+bx+cx^2)}{2c^5}+\frac{b(5a^2c^2-5ab^2c+b^4)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^5\sqrt{b^2-4ac}}+\frac{x^2(b^2-ac)}{2c^3}-\frac{bx(b^2-2ac)}{c^4}-\frac{bx^3}{3c^2}+\frac{x^4}{4c}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(c + a/x^2 + b/x),x]
```

```
[Out] -((b*(b^2 - 2*a*c)*x)/c^4) + ((b^2 - a*c)*x^2)/(2*c^3) - (b*x^3)/(3*c^2) +
x^4/(4*c) + (b*(b^4 - 5*a*b^2*c + 5*a^2*c^2)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 -
 4*a*c]])/(c^5*Sqrt[b^2 - 4*a*c]) + ((b^4 - 3*a*b^2*c + a^2*c^2)*Log[a + b*
x + c*x^2])/(2*c^5)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 701

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Int[PolynomialDivide[(d + e*x)^m, a + b*x + c*x^2, x], x] /; FreeQ[{a,
 b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
 NeQ[2*c*d - b*e, 0] && IGtQ[m, 1] && (NeQ[d, 0] || GtQ[m, 2])
```

Rule 1354

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> Int[x^(m + 2*n*p)*(c + b/x^n + a/x^(2*n))^p, x] /; FreeQ[{a, b, c, m, n
```

```
}, x] && EqQ[n2, 2*n] && ILtQ[p, 0] && NegQ[n]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^3}{c+\frac{a}{x^2}+\frac{b}{x}}\,dx &= \int \frac{x^5}{a+bx+cx^2}\,dx\\
&= \int \left(-\frac{b\left(b^2-2ac\right)}{c^4}+\frac{\left(b^2-ac\right)x}{c^3}-\frac{bx^2}{c^2}+\frac{x^3}{c}+\frac{ab\left(b^2-2ac\right)+\left(b^4-3ab^2c+a^2c^2\right)x}{c^4\left(a+bx+cx^2\right)}\right)dx\\
&= -\frac{b\left(b^2-2ac\right)x}{c^4}+\frac{\left(b^2-ac\right)x^2}{2c^3}-\frac{bx^3}{3c^2}+\frac{x^4}{4c}+\frac{\int \frac{ab\left(b^2-2ac\right)+\left(b^4-3ab^2c+a^2c^2\right)x}{a+bx+cx^2}\,dx}{c^4}\\
&= -\frac{b\left(b^2-2ac\right)x}{c^4}+\frac{\left(b^2-ac\right)x^2}{2c^3}-\frac{bx^3}{3c^2}+\frac{x^4}{4c}+\frac{\left(b^4-3ab^2c+a^2c^2\right)\int \frac{b+2cx}{a+bx+cx^2}\,dx}{2c^5}-\frac{\left(b\left(b^4-5ab\right.\right.}{}\\
&= -\frac{b\left(b^2-2ac\right)x}{c^4}+\frac{\left(b^2-ac\right)x^2}{2c^3}-\frac{bx^3}{3c^2}+\frac{x^4}{4c}+\frac{\left(b^4-3ab^2c+a^2c^2\right)\log\left(a+bx+cx^2\right)}{2c^5}+\frac{\left(b\left(b^4\right.\right.}{}\\
&= -\frac{b\left(b^2-2ac\right)x}{c^4}+\frac{\left(b^2-ac\right)x^2}{2c^3}-\frac{bx^3}{3c^2}+\frac{x^4}{4c}+\frac{b\left(b^4-5ab^2c+5a^2c^2\right)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^5\sqrt{b^2-4ac}}+\frac{\left(b^4-\right.}{}
\end{aligned}$$

Mathematica [A] time = 0.12, size = 140, normalized size = 0.95

$$\frac{6\left(a^2c^2-3ab^2c+b^4\right)\log(a+x(b+cx))-\frac{12b\left(5a^2c^2-5ab^2c+b^4\right)\tan^{-1}\left(\frac{b+2cx}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}+cx\left(-4bc\left(cx^2-6a\right)+3c^2x\left(cx^2-2a\right)-12b^3+6b^2cx\right)}{12c^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(c + a/x^2 + b/x),x]
```

```
[Out] (c*x*(-12*b^3 + 6*b^2*c*x - 4*b*c*(-6*a + c*x^2) + 3*c^2*x*(-2*a + c*x^2))
- (12*b*(b^4 - 5*a*b^2*c + 5*a^2*c^2)*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]
])/Sqrt[-b^2 + 4*a*c] + 6*(b^4 - 3*a*b^2*c + a^2*c^2)*Log[a + x*(b + c*x)])
/(12*c^5)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{c+\frac{a}{x^2}+\frac{b}{x}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^3/(c + a/x^2 + b/x),x]
```

```
[Out] IntegrateAlgebraic[x^3/(c + a/x^2 + b/x), x]
```

fricas [A] time = 1.30, size = 466, normalized size = 3.17

$$\left[\frac{3\left(b^2c^4-4ac^5\right)x^4-4\left(b^3c^3-4abc^4\right)x^3+6\left(b^4c^2-5ab^2c^3+4a^2c^4\right)x^2+6\left(b^5-5ab^3c+5a^2bc^2\right)\sqrt{b^2-4ac}\log\left(\frac{2c^2x^2+2bcx+b^2-2ac+\sqrt{b^2-4ac}(2cx+b)}{cx^2+bx+a}\right)-12\left(b^5c-6ab^3c^2+8a^2bc^3\right)x+6\left(b^6-7ab^4c+13a^2b^2c^2-4a^3c^3\right)\log\left(cx^2+bx+a\right)}{12\left(b^2c^5-4ac^6\right)}, \frac{3\left(b^2c^4-4ac^5\right)x^4-4\left(b^3c^3-4abc^4\right)x^3+6\left(b^4c^2-5ab^2c^3+4a^2c^4\right)x^2+12\left(b^5-5ab^3c+5a^2bc^2\right)\sqrt{-b^2+4ac}\arctan\left(-\frac{\sqrt{-b^2+4ac}(2cx+b)}{b^2-4ac}\right)-12\left(b^5c-6ab^3c^2+8a^2bc^3\right)x+6\left(b^6-7ab^4c+13a^2b^2c^2-4a^3c^3\right)\log\left(cx^2+bx+a\right)}{12\left(b^2c^5-4ac^6\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c+a/x^2+b/x),x, algorithm="fricas")
```

```
[Out] [1/12*(3*(b^2*c^4 - 4*a*c^5)*x^4 - 4*(b^3*c^3 - 4*a*b*c^4)*x^3 + 6*(b^4*c^2
 - 5*a*b^2*c^3 + 4*a^2*c^4)*x^2 + 6*(b^5 - 5*a*b^3*c + 5*a^2*b*c^2)*sqrt(b^
2 - 4*a*c)*log((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c + sqrt(b^2 - 4*a*c)*(2*c*
```

```
x + b))/(c*x^2 + b*x + a)) - 12*(b^5*c - 6*a*b^3*c^2 + 8*a^2*b*c^3)*x + 6*(
b^6 - 7*a*b^4*c + 13*a^2*b^2*c^2 - 4*a^3*c^3)*log(c*x^2 + b*x + a))/(b^2*c^
5 - 4*a*c^6), 1/12*(3*(b^2*c^4 - 4*a*c^5)*x^4 - 4*(b^3*c^3 - 4*a*b*c^4)*x^3
 + 6*(b^4*c^2 - 5*a*b^2*c^3 + 4*a^2*c^4)*x^2 + 12*(b^5 - 5*a*b^3*c + 5*a^2*
b*c^2)*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a
*c)) - 12*(b^5*c - 6*a*b^3*c^2 + 8*a^2*b*c^3)*x + 6*(b^6 - 7*a*b^4*c + 13*a
^2*b^2*c^2 - 4*a^3*c^3)*log(c*x^2 + b*x + a))/(b^2*c^5 - 4*a*c^6)]
```

giac [A] time = 0.29, size = 145, normalized size = 0.99

$$\frac{3\,c^3x^4 - 4\,bc^2x^3 + 6\,b^2cx^2 - 6\,ac^2x^2 - 12\,b^3x + 24\,abcx}{12\,c^4} + \frac{\left(b^4 - 3\,ab^2c + a^2c^2\right)\log\left(cx^2 + bx + a\right)}{2\,c^5} - \frac{\left(b^5 - 5\,ab^3c + 5\,a^2bc^2\right)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{\sqrt{-b^2 + 4\,ac}\,c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c+a/x^2+b/x),x, algorithm="giac")
```

```
[Out] 1/12*(3*c^3*x^4 - 4*b*c^2*x^3 + 6*b^2*c*x^2 - 6*a*c^2*x^2 - 12*b^3*x + 24*a
*b*c*x)/c^4 + 1/2*(b^4 - 3*a*b^2*c + a^2*c^2)*log(c*x^2 + b*x + a)/c^5 - (b
^5 - 5*a*b^3*c + 5*a^2*b*c^2)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/(sqrt(
-b^2 + 4*a*c)*c^5)
```

maple [A] time = 0.01, size = 236, normalized size = 1.61

$$\frac{x^4}{4c} - \frac{b\,x^3}{3c^2} - \frac{5a^2b\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}\,c^3} + \frac{5a\,b^3\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}\,c^4} - \frac{a\,x^2}{2c^2} - \frac{b^5\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}\,c^5} + \frac{b^2x^2}{2c^3} + \frac{a^2\ln\left(c\,x^2+bx+a\right)}{2c^3} - \frac{3a\,b^2\ln\left(c\,x^2+bx+a\right)}{2c^4} + \frac{2abx}{c^3} + \frac{b^4\ln\left(c\,x^2+bx+a\right)}{2c^5} - \frac{b^3x}{c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(c+a/x^2+b/x),x)
```

```
[Out] 1/4/c*x^4-1/3*b*x^3/c^2-1/2/c^2*x^2*a+1/2/c^3*x^2*b^2+2/c^3*a*b*x-1/c^4*b^3
*x+1/2/c^3*ln(c*x^2+b*x+a)*a^2-3/2/c^4*ln(c*x^2+b*x+a)*a*b^2+1/2/c^5*ln(c*x
^2+b*x+a)*b^4-5/c^3/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*a
^2*b+5/c^4/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*a*b^3-1/c^
5/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*b^5
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c+a/x^2+b/x),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

mupad [B] time = 0.16, size = 183, normalized size = 1.24

$$x\,\left(\frac{b\,\left(\frac{a}{c^2}-\frac{b^2}{c^3}\right)}{c}+\frac{a\,b}{c^3}\right)+\frac{x^4}{4\,c}-x^2\,\left(\frac{a}{2\,c^2}-\frac{b^2}{2\,c^3}\right)-\frac{\ln\left(c\,x^2+b\,x+a\right)\,\left(-4\,a^3\,c^3+13\,a^2\,b^2\,c^2-7\,a\,b^4\,c+b^6\right)}{2\,\left(4\,a\,c^6-b^2\,c^5\right)}-\frac{b\,x^3}{3\,c^2}-\frac{b\,\mathrm{atan}\left(\frac{b+2\,c\,x}{\sqrt{4\,a\,c-b^2}}\right)\,\left(5\,a^2\,c^2-5\,a\,b^2\,c+b^4\right)}{c^5\,\sqrt{4\,a\,c-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(c + a/x^2 + b/x),x)
```

```
[Out] x*((b*(a/c^2 - b^2/c^3))/c + (a*b)/c^3) + x^4/(4*c) - x^2*(a/(2*c^2) - b^2/
(2*c^3)) - (log(a + b*x + c*x^2)*(b^6 - 4*a^3*c^3 + 13*a^2*b^2*c^2 - 7*a*b^
4*c))/(2*(4*a*c^6 - b^2*c^5)) - (b*x^3)/(3*c^2) - (b*atan((b + 2*c*x)/(4*a*
c - b^2)^(1/2))*(b^4 + 5*a^2*c^2 - 5*a*b^2*c))/(c^5*(4*a*c - b^2)^(1/2))
```

sympy [B] time = 1.25, size = 605, normalized size = 4.12

$$-\frac{bx^3}{3c^2} + x^2\left(-\frac{a}{2c^2} + \frac{b^2}{2c^3}\right) + x\left(\frac{2ab}{c^3} - \frac{b^3}{c^4}\right) + \left(-\frac{b\sqrt{-4ac+b^2}\left(5a^2c^2 - 5ab^2c + b^4\right)}{2c^5\left(4ac-b^2\right)} + \frac{a^2c^2 - 3ab^2c + b^4}{2c^5}\right)\log\left(x + \frac{2a^3c^2 - 4a^2b^2c + ab^4 - 4ac^5\left(-\frac{b\sqrt{-4ac+b^2}\left(5a^2c^2-5ab^2c+b^4\right)}{2c^5\left(4ac-b^2\right)} + \frac{a^2c^2-3ab^2c+b^4}{2c^5}\right) + b^2c^4\left(-\frac{b\sqrt{-4ac+b^2}\left(5a^2c^2-5ab^2c+b^4\right)}{2c^5\left(4ac-b^2\right)} + \frac{a^2c^2-3ab^2c+b^4}{2c^5}\right)}{5a^2bc^2 - 5ab^3c + b^5}\right) + \left(\frac{b\sqrt{-4ac+b^2}\left(5a^2c^2 - 5ab^2c + b^4\right)}{2c^5\left(4ac-b^2\right)} + \frac{a^2c^2 - 3ab^2c + b^4}{2c^5}\right)\log\left(x + \frac{2a^3c^2 - 4a^2b^2c + ab^4 - 4ac^5\left(\frac{b\sqrt{-4ac+b^2}\left(5a^2c^2-5ab^2c+b^4\right)}{2c^5\left(4ac-b^2\right)} + \frac{a^2c^2-3ab^2c+b^4}{2c^5}\right) + b^2c^4\left(\frac{b\sqrt{-4ac+b^2}\left(5a^2c^2-5ab^2c+b^4\right)}{2c^5\left(4ac-b^2\right)} + \frac{a^2c^2-3ab^2c+b^4}{2c^5}\right)}{5a^2bc^2 - 5ab^3c + b^5}\right) + \frac{x^4}{4c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(c+a/x**2+b/x),x)
```

```
[Out] -b*x**3/(3*c**2) + x**2*(-a/(2*c**2) + b**2/(2*c**3)) + x*(2*a*b/c**3 - b**
3/c**4) + (-b*sqrt(-4*a*c + b**2)*(5*a**2*c**2 - 5*a*b**2*c + b**4)/(2*c**5
*(4*a*c - b**2)) + (a**2*c**2 - 3*a*b**2*c + b**4)/(2*c**5))*log(x + (2*a**
3*c**2 - 4*a**2*b**2*c + a*b**4 - 4*a*c**5*(-b*sqrt(-4*a*c + b**2)*(5*a**2*
c**2 - 5*a*b**2*c + b**4)/(2*c**5*(4*a*c - b**2)) + (a**2*c**2 - 3*a*b**2*c
 + b**4)/(2*c**5)) + b**2*c**4*(-b*sqrt(-4*a*c + b**2)*(5*a**2*c**2 - 5*a*b
**2*c + b**4)/(2*c**5*(4*a*c - b**2)) + (a**2*c**2 - 3*a*b**2*c + b**4)/(2*
c**5)))/(5*a**2*b*c**2 - 5*a*b**3*c + b**5)) + (b*sqrt(-4*a*c + b**2)*(5*a*
*2*c**2 - 5*a*b**2*c + b**4)/(2*c**5*(4*a*c - b**2)) + (a**2*c**2 - 3*a*b**
2*c + b**4)/(2*c**5))*log(x + (2*a**3*c**2 - 4*a**2*b**2*c + a*b**4 - 4*a*c
**5*(b*sqrt(-4*a*c + b**2)*(5*a**2*c**2 - 5*a*b**2*c + b**4)/(2*c**5*(4*a*c
 - b**2)) + (a**2*c**2 - 3*a*b**2*c + b**4)/(2*c**5)) + b**2*c**4*(b*sqrt(-
4*a*c + b**2)*(5*a**2*c**2 - 5*a*b**2*c + b**4)/(2*c**5*(4*a*c - b**2)) + (
a**2*c**2 - 3*a*b**2*c + b**4)/(2*c**5)))/(5*a**2*b*c**2 - 5*a*b**3*c + b**
5)) + x**4/(4*c)
```

3.352 $\int \frac{x^2}{c+\frac{a}{x^2}+\frac{b}{x}} dx$

Optimal. Leaf size=118

$$-\frac{(2a^2c^2-4ab^2c+b^4)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^4\sqrt{b^2-4ac}}-\frac{b(b^2-2ac)\log(a+bx+cx^2)}{2c^4}+\frac{x(b^2-ac)}{c^3}-\frac{bx^2}{2c^2}+\frac{x^3}{3c}$$

Rubi [A] time = 0.10, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1354, 701, 634, 618, 206, 628}

$$-\frac{(2a^2c^2-4ab^2c+b^4)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^4\sqrt{b^2-4ac}}-\frac{b(b^2-2ac)\log(a+bx+cx^2)}{2c^4}+\frac{x(b^2-ac)}{c^3}-\frac{bx^2}{2c^2}+\frac{x^3}{3c}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(c + a/x^2 + b/x),x]
```

```
[Out] ((b^2 - a*c)*x)/c^3 - (b*x^2)/(2*c^2) + x^3/(3*c) - ((b^4 - 4*a*b^2*c + 2*a
^2*c^2)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(c^4*Sqrt[b^2 - 4*a*c]) - (
b*(b^2 - 2*a*c)*Log[a + b*x + c*x^2])/(2*c^4)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 701

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Int[PolynomialDivide[(d + e*x)^m, a + b*x + c*x^2, x], x] /; FreeQ[{a,
 b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
 NeQ[2*c*d - b*e, 0] && IGtQ[m, 1] && (NeQ[d, 0] || GtQ[m, 2])
```

Rule 1354

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> Int[x^(m + 2*n*p)*(c + b/x^n + a/x^(2*n))^p, x] /; FreeQ[{a, b, c, m, n
```

```
}, x] && EqQ[n2, 2*n] && ILtQ[p, 0] && NegQ[n]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2}{c+\frac{a}{x^2}+\frac{b}{x}}\,dx &= \int \frac{x^4}{a+bx+cx^2}\,dx \\
&= \int \left(\frac{b^2-ac}{c^3}-\frac{bx}{c^2}+\frac{x^2}{c}-\frac{a\left(b^2-ac\right)+b\left(b^2-2ac\right)x}{c^3\left(a+bx+cx^2\right)}\right)dx \\
&= \frac{\left(b^2-ac\right)x}{c^3}-\frac{bx^2}{2c^2}+\frac{x^3}{3c}-\frac{\int \frac{a\left(b^2-ac\right)+b\left(b^2-2ac\right)x}{a+bx+cx^2}\,dx}{c^3} \\
&= \frac{\left(b^2-ac\right)x}{c^3}-\frac{bx^2}{2c^2}+\frac{x^3}{3c}-\frac{\left(b\left(b^2-2ac\right)\right)\int \frac{b+2cx}{a+bx+cx^2}\,dx}{2c^4}+\frac{\left(b^4-4ab^2c+2a^2c^2\right)\int \frac{1}{a+bx+cx^2}\,dx}{2c^4} \\
&= \frac{\left(b^2-ac\right)x}{c^3}-\frac{bx^2}{2c^2}+\frac{x^3}{3c}-\frac{b\left(b^2-2ac\right)\log\left(a+bx+cx^2\right)}{2c^4}-\frac{\left(b^4-4ab^2c+2a^2c^2\right)\mathrm{Subst}\left(\int \frac{}{b^2-}\right.}{c^4} \\
&= \frac{\left(b^2-ac\right)x}{c^3}-\frac{bx^2}{2c^2}+\frac{x^3}{3c}-\frac{\left(b^4-4ab^2c+2a^2c^2\right)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^4\sqrt{b^2-4ac}}-\frac{b\left(b^2-2ac\right)\log\left(a+bx+\right.}{2c^4}
\end{aligned}$$

Mathematica [A] time = 0.08, size = 112, normalized size = 0.95

$$\frac{\frac{6\left(2a^2c^2-4ab^2c+b^4\right)\tan^{-1}\left(\frac{b+2cx}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}-3\left(b^3-2abc\right)\log(a+x(b+cx))+cx\left(-6ac+6b^2-3bcx+2c^2x^2\right)}{6c^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(c + a/x^2 + b/x),x]
```

```
[Out] (c*x*(6*b^2 - 6*a*c - 3*b*c*x + 2*c^2*x^2) + (6*(b^4 - 4*a*b^2*c + 2*a^2*c^
2)*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c] - 3*(b^3 - 2*
a*b*c)*Log[a + x*(b + c*x)])/(6*c^4)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{c+\frac{a}{x^2}+\frac{b}{x}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^2/(c + a/x^2 + b/x),x]
```

```
[Out] IntegrateAlgebraic[x^2/(c + a/x^2 + b/x), x]
```

fricas [A] time = 1.34, size = 383, normalized size = 3.25

$$\left[\frac{2\left(b^2c^3-4ac^4\right)x^3-3\left(b^3c^2-4abc^3\right)x^2+3\left(b^4-4ab^2c+2a^2c^2\right)\sqrt{b^2-4ac}\log\left(\frac{2c^2x^2+2bcx+b^2-2ac-\sqrt{b^2-4ac}(2cx+b)}{cx^2+bx+a}\right)+6\left(b^4c-5ab^2c^2+4a^2c^3\right)x-3\left(b^5-6ab^3c+8a^2bc^2\right)\log\left(cx^2+bx+a\right)}{6\left(b^2c^4-4ac^5\right)}, \frac{2\left(b^2c^3-4ac^4\right)x^3-3\left(b^3c^2-4abc^3\right)x^2-6\left(b^4-4ab^2c+2a^2c^2\right)\sqrt{-b^2+4ac}\arctan\left(-\frac{\sqrt{-b^2+4ac}(2cx+b)}{b^2-4ac}\right)+6\left(b^4c-5ab^2c^2+4a^2c^3\right)x-3\left(b^5-6ab^3c+8a^2bc^2\right)\log\left(cx^2+bx+a\right)}{6\left(b^2c^4-4ac^5\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c+a/x^2+b/x),x, algorithm="fricas")
```

```
[Out] [1/6*(2*(b^2*c^3 - 4*a*c^4)*x^3 - 3*(b^3*c^2 - 4*a*b*c^3)*x^2 + 3*(b^4 - 4*
a*b^2*c + 2*a^2*c^2)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a
*c - sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) + 6*(b^4*c - 5*a*b^2
```

```
*c^2 + 4*a^2*c^3)*x - 3*(b^5 - 6*a*b^3*c + 8*a^2*b*c^2)*log(c*x^2 + b*x + a
))/(b^2*c^4 - 4*a*c^5), 1/6*(2*(b^2*c^3 - 4*a*c^4)*x^3 - 3*(b^3*c^2 - 4*a*b
*c^3)*x^2 - 6*(b^4 - 4*a*b^2*c + 2*a^2*c^2)*sqrt(-b^2 + 4*a*c)*arctan(-sqrt
(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) + 6*(b^4*c - 5*a*b^2*c^2 + 4*a^2*
c^3)*x - 3*(b^5 - 6*a*b^3*c + 8*a^2*b*c^2)*log(c*x^2 + b*x + a))/(b^2*c^4 -
 4*a*c^5)]
```

giac [A] time = 0.39, size = 113, normalized size = 0.96

$$\frac{2\,c^2x^3 - 3\,bcx^2 + 6\,b^2x - 6\,acx}{6\,c^3} - \frac{\left(b^3 - 2\,abc\right)\log\left(cx^2 + bx + a\right)}{2\,c^4} + \frac{\left(b^4 - 4\,ab^2c + 2\,a^2c^2\right)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{\sqrt{-b^2 + 4\,ac}\,c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c+a/x^2+b/x),x, algorithm="giac")
```

```
[Out] 1/6*(2*c^2*x^3 - 3*b*c*x^2 + 6*b^2*x - 6*a*c*x)/c^3 - 1/2*(b^3 - 2*a*b*c)*l
og(c*x^2 + b*x + a)/c^4 + (b^4 - 4*a*b^2*c + 2*a^2*c^2)*arctan((2*c*x + b)/
sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*c^4)
```

maple [A] time = 0.00, size = 190, normalized size = 1.61

$$\frac{x^3}{3c} + \frac{2a^2\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}\,c^2} - \frac{4a\,b^2\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}\,c^3} + \frac{b^4\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}\,c^4} - \frac{b\,x^2}{2c^2} + \frac{ab\ln\left(c\,x^2+bx+a\right)}{c^3} - \frac{ax}{c^2} - \frac{b^3\ln\left(c\,x^2+bx+a\right)}{2c^4} + \frac{b^2x}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(c+a/x^2+b/x),x)
```

```
[Out] 1/3/c*x^3-1/2*b*x^2/c^2-1/c^2*a*x+1/c^3*b^2*x+1/c^3*ln(c*x^2+b*x+a)*a*b-1/2
/c^4*ln(c*x^2+b*x+a)*b^3+2/c^2/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^
2)^(1/2))*a^2-4/c^3/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*a
*b^2+1/c^4/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*b^4
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c+a/x^2+b/x),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

mupad [B] time = 1.40, size = 151, normalized size = 1.28

$$\frac{x^3}{3\,c} - x\left(\frac{a}{c^2} - \frac{b^2}{c^3}\right) - \frac{b\,x^2}{2\,c^2} + \frac{\ln\left(c\,x^2 + b\,x + a\right)\left(8\,a^2\,b\,c^2 - 6\,a\,b^3\,c + b^5\right)}{2\left(4\,a\,c^5 - b^2\,c^4\right)} + \frac{\mathrm{atan}\left(\frac{b}{\sqrt{4\,a\,c-b^2}} + \frac{2\,c\,x}{\sqrt{4\,a\,c-b^2}}\right)\left(2\,a^2\,c^2 - 4\,a\,b^2\,c + b^4\right)}{c^4\,\sqrt{4\,a\,c - b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(c + a/x^2 + b/x),x)
```

```
[Out] x^3/(3*c) - x*(a/c^2 - b^2/c^3) - (b*x^2)/(2*c^2) + (log(a + b*x + c*x^2)*(
b^5 + 8*a^2*b*c^2 - 6*a*b^3*c))/(2*(4*a*c^5 - b^2*c^4)) + (atan(b/(4*a*c -
b^2)^(1/2) + (2*c*x)/(4*a*c - b^2)^(1/2))*(b^4 + 2*a^2*c^2 - 4*a*b^2*c))/(c
^4*(4*a*c - b^2)^(1/2))
```

sympy [B] time = 1.01, size = 498, normalized size = 4.22

$$- \frac{b x^{2}}{2 c^{2}} + x \left(- \frac{a}{c^{2}} + \frac{b^{2}}{c^{3}}\right) + \left(\frac{b \left(2 a c - b^{2}\right)}{2 c^{4}} - \frac{\sqrt{- 4 a c + b^{2}} \left(2 a^{2} c^{2} - 4 a b^{2} c + b^{4}\right)}{2 c^{4} \left(4 a c - b^{2}\right)}\right) \log{\left(x + \frac{- 3 a^{2} b c + a b^{3} + 4 a c^{4} \left(\frac{b \left(2 a c - b^{2}\right)}{2 c^{4}} - \frac{\sqrt{- 4 a c + b^{2}} \left(2 a^{2} c^{2} - 4 a b^{2} c + b^{4}\right)}{2 c^{4} \left(4 a c - b^{2}\right)}\right) - b^{2} c^{3} \left(\frac{b \left(2 a c - b^{2}\right)}{2 c^{4}} - \frac{\sqrt{- 4 a c + b^{2}} \left(2 a^{2} c^{2} - 4 a b^{2} c + b^{4}\right)}{2 c^{4} \left(4 a c - b^{2}\right)}\right)}{2 a^{2} c^{2} - 4 a b^{2} c + b^{4}} \right)} + \left(\frac{b \left(2 a c - b^{2}\right)}{2 c^{4}} + \frac{\sqrt{- 4 a c + b^{2}} \left(2 a^{2} c^{2} - 4 a b^{2} c + b^{4}\right)}{2 c^{4} \left(4 a c - b^{2}\right)}\right) \log{\left(x + \frac{- 3 a^{2} b c + a b^{3} + 4 a c^{4} \left(\frac{b \left(2 a c - b^{2}\right)}{2 c^{4}} + \frac{\sqrt{- 4 a c + b^{2}} \left(2 a^{2} c^{2} - 4 a b^{2} c + b^{4}\right)}{2 c^{4} \left(4 a c - b^{2}\right)}\right) - b^{2} c^{3} \left(\frac{b \left(2 a c - b^{2}\right)}{2 c^{4}} + \frac{\sqrt{- 4 a c + b^{2}} \left(2 a^{2} c^{2} - 4 a b^{2} c + b^{4}\right)}{2 c^{4} \left(4 a c - b^{2}\right)}\right)}{2 a^{2} c^{2} - 4 a b^{2} c + b^{4}} \right)} + \frac{x^{3}}{3 c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(c+a/x**2+b/x),x)
```

```
[Out] -b*x**2/(2*c**2) + x*(-a/c**2 + b**2/c**3) + (b*(2*a*c - b**2)/(2*c**4) - s
qrt(-4*a*c + b**2)*(2*a**2*c**2 - 4*a*b**2*c + b**4)/(2*c**4*(4*a*c - b**2)
))*log(x + (-3*a**2*b*c + a*b**3 + 4*a*c**4*(b*(2*a*c - b**2)/(2*c**4) - sq
rt(-4*a*c + b**2)*(2*a**2*c**2 - 4*a*b**2*c + b**4)/(2*c**4*(4*a*c - b**2))
) - b**2*c**3*(b*(2*a*c - b**2)/(2*c**4) - sqrt(-4*a*c + b**2)*(2*a**2*c**2
 - 4*a*b**2*c + b**4)/(2*c**4*(4*a*c - b**2))))/(2*a**2*c**2 - 4*a*b**2*c +
 b**4)) + (b*(2*a*c - b**2)/(2*c**4) + sqrt(-4*a*c + b**2)*(2*a**2*c**2 - 4
*a*b**2*c + b**4)/(2*c**4*(4*a*c - b**2)))*log(x + (-3*a**2*b*c + a*b**3 +
4*a*c**4*(b*(2*a*c - b**2)/(2*c**4) + sqrt(-4*a*c + b**2)*(2*a**2*c**2 - 4*
a*b**2*c + b**4)/(2*c**4*(4*a*c - b**2))) - b**2*c**3*(b*(2*a*c - b**2)/(2*
c**4) + sqrt(-4*a*c + b**2)*(2*a**2*c**2 - 4*a*b**2*c + b**4)/(2*c**4*(4*a*
c - b**2))))/(2*a**2*c**2 - 4*a*b**2*c + b**4)) + x**3/(3*c)
```

3.353 $\int \frac{x}{c+\frac{a}{x^2}+\frac{b}{x}}\,dx$

Optimal. Leaf size=89

$$\frac{(b^2-ac)\log(a+bx+cx^2)}{2c^3}+\frac{b(b^2-3ac)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^3\sqrt{b^2-4ac}}-\frac{bx}{c^2}+\frac{x^2}{2c}$$

Rubi [A] time = 0.09, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {1354, 701, 634, 618, 206, 628}

$$\frac{(b^2-ac)\log(a+bx+cx^2)}{2c^3}+\frac{b(b^2-3ac)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^3\sqrt{b^2-4ac}}-\frac{bx}{c^2}+\frac{x^2}{2c}$$

Antiderivative was successfully verified.

```
[In] Int[x/(c + a/x^2 + b/x),x]
```

```
[Out] -((b*x)/c^2) + x^2/(2*c) + (b*(b^2 - 3*a*c)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 -
4*a*c]])/(c^3*Sqrt[b^2 - 4*a*c]) + ((b^2 - a*c)*Log[a + b*x + c*x^2])/(2*c^
3)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 701

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Int[PolynomialDivide[(d + e*x)^m, a + b*x + c*x^2, x], x] /; FreeQ[{a,
 b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
 NeQ[2*c*d - b*e, 0] && IGtQ[m, 1] && (NeQ[d, 0] || GtQ[m, 2])
```

Rule 1354

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> Int[x^(m + 2*n*p)*(c + b/x^n + a/x^(2*n))^p, x] /; FreeQ[{a, b, c, m, n
```

```
}, x] && EqQ[n2, 2*n] && ILtQ[p, 0] && NegQ[n]
```

Rubi steps

$$\begin{aligned}
\int \frac{x}{c+\frac{a}{x^2}+\frac{b}{x}}\,dx &= \int \frac{x^3}{a+bx+cx^2}\,dx \\
&= \int \left(-\frac{b}{c^2}+\frac{x}{c}+\frac{ab+\left(b^2-ac\right)x}{c^2\left(a+bx+cx^2\right)}\right)dx \\
&= -\frac{bx}{c^2}+\frac{x^2}{2c}+\frac{\int \frac{ab+\left(b^2-ac\right)x}{a+bx+cx^2}\,dx}{c^2} \\
&= -\frac{bx}{c^2}+\frac{x^2}{2c}-\frac{\left(b\left(b^2-3ac\right)\right)\int \frac{1}{a+bx+cx^2}\,dx}{2c^3}+\frac{\left(b^2-ac\right)\int \frac{b+2cx}{a+bx+cx^2}\,dx}{2c^3} \\
&= -\frac{bx}{c^2}+\frac{x^2}{2c}+\frac{\left(b^2-ac\right)\log\left(a+bx+cx^2\right)}{2c^3}+\frac{\left(b\left(b^2-3ac\right)\right)\mathrm{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx,x,b+2cx\right)}{c^3} \\
&= -\frac{bx}{c^2}+\frac{x^2}{2c}+\frac{b\left(b^2-3ac\right)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^3\sqrt{b^2-4ac}}+\frac{\left(b^2-ac\right)\log\left(a+bx+cx^2\right)}{2c^3}
\end{aligned}$$

Mathematica [A] time = 0.10, size = 84, normalized size = 0.94

$$\frac{\left(b^2-ac\right)\log(a+x(b+cx))-\frac{2b\left(b^2-3ac\right)\tan^{-1}\left(\frac{b+2cx}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}+cx(cx-2b)}{2c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(c + a/x^2 + b/x),x]
```

```
[Out] (c*x*(-2*b + c*x) - (2*b*(b^2 - 3*a*c)*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c
]])/Sqrt[-b^2 + 4*a*c] + (b^2 - a*c)*Log[a + x*(b + c*x)])/(2*c^3)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{c+\frac{a}{x^2}+\frac{b}{x}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x/(c + a/x^2 + b/x),x]
```

```
[Out] IntegrateAlgebraic[x/(c + a/x^2 + b/x), x]
```

fricas [A] time = 1.41, size = 297, normalized size = 3.34

$$\left[\frac{\left(b^2c^2-4ac^3\right)x^2-\left(b^3-3abc\right)\sqrt{b^2-4ac}\log\left(\frac{2c^2x^2+2bcx+b^2-2ac-\sqrt{b^2-4ac}(2cx+b)}{cx^2+bx+a}\right)-2\left(b^3c-4abc^2\right)x+\left(b^4-5ab^2c+4a^2c^2\right)\log\left(cx^2+bx+a\right)}{2\left(b^2c^3-4ac^4\right)}, \frac{\left(b^2c^2-4ac^3\right)x^2+2\left(b^3-3abc\right)\sqrt{-b^2+4ac}\arctan\left(-\frac{\sqrt{-b^2+4ac}(2cx+b)}{b^2-4ac}\right)-2\left(b^3c-4abc^2\right)x+\left(b^4-5ab^2c+4a^2c^2\right)\log\left(cx^2+bx+a\right)}{2\left(b^2c^3-4ac^4\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c+a/x^2+b/x),x, algorithm="fricas")
```

```
[Out] [1/2*((b^2*c^2 - 4*a*c^3)*x^2 - (b^3 - 3*a*b*c)*sqrt(b^2 - 4*a*c)*log((2*c^
2*x^2 + 2*b*c*x + b^2 - 2*a*c - sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x
 + a)) - 2*(b^3*c - 4*a*b*c^2)*x + (b^4 - 5*a*b^2*c + 4*a^2*c^2)*log(c*x^2
+ b*x + a))/(b^2*c^3 - 4*a*c^4), 1/2*((b^2*c^2 - 4*a*c^3)*x^2 + 2*(b^3 - 3*
```

```
a*b*c)*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a
*c)) - 2*(b^3*c - 4*a*b*c^2)*x + (b^4 - 5*a*b^2*c + 4*a^2*c^2)*log(c*x^2 +
b*x + a))/(b^2*c^3 - 4*a*c^4)]
```

giac [A] time = 0.26, size = 86, normalized size = 0.97

$$\frac{cx^2 - 2\,bx}{2\,c^2} + \frac{\left(b^2 - ac\right)\log\left(cx^2 + bx + a\right)}{2\,c^3} - \frac{\left(b^3 - 3\,abc\right)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{\sqrt{-b^2 + 4\,ac}\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c+a/x^2+b/x),x, algorithm="giac")
```

```
[Out] 1/2*(c*x^2 - 2*b*x)/c^2 + 1/2*(b^2 - a*c)*log(c*x^2 + b*x + a)/c^3 - (b^3 -
 3*a*b*c)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*c^3)
```

maple [A] time = 0.00, size = 132, normalized size = 1.48

$$\frac{3ab\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}\;c^2} - \frac{b^3\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}\;c^3} + \frac{x^2}{2c} - \frac{a\ln\left(c\,x^2 + bx + a\right)}{2c^2} + \frac{b^2\ln\left(c\,x^2 + bx + a\right)}{2c^3} - \frac{bx}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(c+a/x^2+b/x),x)
```

```
[Out] 1/2/c*x^2-b*x/c^2-1/2/c^2*ln(c*x^2+b*x+a)*a+1/2/c^3*ln(c*x^2+b*x+a)*b^2+3/c
^2/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*a*b-1/c^3/(4*a*c-b
^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*b^3
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c+a/x^2+b/x),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

mupad [B] time = 0.13, size = 112, normalized size = 1.26

$$\frac{x^2}{2\,c} - \frac{\ln\left(c\,x^2 + b\,x + a\right)\,\left(4\,a^2\,c^2 - 5\,a\,b^2\,c + b^4\right)}{2\,\left(4\,a\,c^4 - b^2\,c^3\right)} - \frac{b\,x}{c^2} + \frac{b\,\mathrm{atan}\left(\frac{b+2\,c\,x}{\sqrt{4\,a\,c-b^2}}\right)\,\left(3\,a\,c - b^2\right)}{c^3\,\sqrt{4\,a\,c - b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(c + a/x^2 + b/x),x)
```

```
[Out] x^2/(2*c) - (log(a + b*x + c*x^2)*(b^4 + 4*a^2*c^2 - 5*a*b^2*c))/(2*(4*a*c^
4 - b^2*c^3)) - (b*x)/c^2 + (b*atan((b + 2*c*x)/(4*a*c - b^2)^(1/2))*(3*a*c
 - b^2))/(c^3*(4*a*c - b^2)^(1/2))
```

sympy [B] time = 0.84, size = 381, normalized size = 4.28

$$-\frac{bx}{c^2} + \left(-\frac{b\sqrt{-4ac+b^2}\,\left(3ac-b^2\right)}{2c^3\left(4ac-b^2\right)} - \frac{ac-b^2}{2c^3}\right)\log\left(x + \frac{2a^2c - ab^2 + 4ac^3\left(-\frac{b\sqrt{-4ac+b^2}\,(3ac-b^2)}{2c^3(4ac-b^2)} - \frac{ac-b^2}{2c^3}\right) - b^2c^2\left(-\frac{b\sqrt{-4ac+b^2}\,(3ac-b^2)}{2c^3(4ac-b^2)} - \frac{ac-b^2}{2c^3}\right)}{3abc-b^3}\right) + \left(\frac{b\sqrt{-4ac+b^2}\,\left(3ac-b^2\right)}{2c^3\left(4ac-b^2\right)} - \frac{ac-b^2}{2c^3}\right)\log\left(x + \frac{2a^2c - ab^2 + 4ac^3\left(\frac{b\sqrt{-4ac+b^2}\,(3ac-b^2)}{2c^3(4ac-b^2)} - \frac{ac-b^2}{2c^3}\right) - b^2c^2\left(\frac{b\sqrt{-4ac+b^2}\,(3ac-b^2)}{2c^3(4ac-b^2)} - \frac{ac-b^2}{2c^3}\right)}{3abc-b^3}\right) + \frac{x^2}{2c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c+a/x**2+b/x),x)
```

```
[Out] -b*x/c**2 + (-b*sqrt(-4*a*c + b**2)*(3*a*c - b**2)/(2*c**3*(4*a*c - b**2))
- (a*c - b**2)/(2*c**3))*log(x + (2*a**2*c - a*b**2 + 4*a*c**3*(-b*sqrt(-4*
a*c + b**2)*(3*a*c - b**2)/(2*c**3*(4*a*c - b**2)) - (a*c - b**2)/(2*c**3))
 - b**2*c**2*(-b*sqrt(-4*a*c + b**2)*(3*a*c - b**2)/(2*c**3*(4*a*c - b**2))
 - (a*c - b**2)/(2*c**3)))/(3*a*b*c - b**3)) + (b*sqrt(-4*a*c + b**2)*(3*a*
c - b**2)/(2*c**3*(4*a*c - b**2)) - (a*c - b**2)/(2*c**3))*log(x + (2*a**2*
c - a*b**2 + 4*a*c**3*(b*sqrt(-4*a*c + b**2)*(3*a*c - b**2)/(2*c**3*(4*a*c
- b**2)) - (a*c - b**2)/(2*c**3)) - b**2*c**2*(b*sqrt(-4*a*c + b**2)*(3*a*c
 - b**2)/(2*c**3*(4*a*c - b**2)) - (a*c - b**2)/(2*c**3)))/(3*a*b*c - b**3)
) + x**2/(2*c)
```

3.354 $\int \frac{1}{c+\frac{a}{x^2}+\frac{b}{x}}\,dx$

Optimal. Leaf size=70

$$-\frac{(b^2-2ac)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^2\sqrt{b^2-4ac}}-\frac{b\log\left(a+bx+cx^2\right)}{2c^2}+\frac{x}{c}$$

Rubi [A] time = 0.05, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {1340, 703, 634, 618, 206, 628}

$$-\frac{(b^2-2ac)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^2\sqrt{b^2-4ac}}-\frac{b\log\left(a+bx+cx^2\right)}{2c^2}+\frac{x}{c}$$

Antiderivative was successfully verified.

```
[In] Int[(c + a/x^2 + b/x)^(-1),x]
```

```
[Out] x/c - ((b^2 - 2*a*c)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(c^2*Sqrt[b^2
- 4*a*c]) - (b*Log[a + b*x + c*x^2])/(2*c^2)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 703

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Simp[(e*(d + e*x)^(m - 1))/(c*(m - 1)), x] + Dist[1/c, Int[((d + e*x)^
(m - 2)*Simp[c*d^2 - a*e^2 + e*(2*c*d - b*e)*x, x])/(a + b*x + c*x^2), x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && GtQ[m, 1]
```

Rule 1340

```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[x^
(2*n*p)*(c + b/x^n + a/x^(2*n))^p, x] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n
```

```
] && LtQ[n, 0] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{c+\frac{a}{x^2}+\frac{b}{x}}\,dx &= \int \frac{x^2}{a+bx+cx^2}\,dx\\ &= \frac{x}{c}+\frac{\int \frac{-a-bx}{a+bx+cx^2}\,dx}{c}\\ &= \frac{x}{c}-\frac{b\int \frac{b+2cx}{a+bx+cx^2}\,dx}{2c^2}+\frac{\left(b^2-2ac\right)\int \frac{1}{a+bx+cx^2}\,dx}{2c^2}\\ &= \frac{x}{c}-\frac{b\log\left(a+bx+cx^2\right)}{2c^2}-\frac{\left(b^2-2ac\right)\mathrm{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx,x,b+2cx\right)}{c^2}\\ &= \frac{x}{c}-\frac{\left(b^2-2ac\right)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^2\sqrt{b^2-4ac}}-\frac{b\log\left(a+bx+cx^2\right)}{2c^2}\end{aligned}$$

Mathematica [A] time = 0.06, size = 73, normalized size = 1.04

$$\frac{\left(b^2-2ac\right)\tan^{-1}\left(\frac{b+2cx}{\sqrt{4ac-b^2}}\right)}{c^2\sqrt{4ac-b^2}}-\frac{b\log\left(a+bx+cx^2\right)}{2c^2}+\frac{x}{c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + a/x^2 + b/x)^(-1),x]
```

```
[Out] x/c + ((b^2 - 2*a*c)*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/(c^2*Sqrt[-b^2
 + 4*a*c]) - (b*Log[a + b*x + c*x^2])/(2*c^2)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{c+\frac{a}{x^2}+\frac{b}{x}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + a/x^2 + b/x)^(-1),x]
```

```
[Out] IntegrateAlgebraic[(c + a/x^2 + b/x)^(-1), x]
```

fricas [A] time = 1.43, size = 235, normalized size = 3.36

$$\left[-\frac{\left(b^2-2\,ac\right)\sqrt{b^2-4\,ac}\,\log\left(\frac{2\,c^2x^2+2\,bcx+b^2-2\,ac+\sqrt{b^2-4\,ac}\,(2\,cx+b)}{cx^2+bx+a}\right)-2\left(b^2c-4\,ac^2\right)x+\left(b^3-4\,abc\right)\log\left(cx^2+bx+a\right)}{2\left(b^2c^2-4\,ac^3\right)},-\frac{2\left(b^2-2\,ac\right)\sqrt{-b^2+4\,ac}\,\arctan\left(-\frac{\sqrt{-b^2+4\,ac}\,(2\,cx+b)}{b^2-4\,ac}\right)-2\left(b^2c-4\,ac^2\right)x+\left(b^3-4\,abc\right)\log\left(cx^2+bx+a\right)}{2\left(b^2c^2-4\,ac^3\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x^2+b/x),x, algorithm="fricas")
```

```
[Out] [-1/2*((b^2 - 2*a*c)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a
*c + sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) - 2*(b^2*c - 4*a*c^2
)*x + (b^3 - 4*a*b*c)*log(c*x^2 + b*x + a))/(b^2*c^2 - 4*a*c^3), -1/2*(2*(b
^2 - 2*a*c)*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2
- 4*a*c)) - 2*(b^2*c - 4*a*c^2)*x + (b^3 - 4*a*b*c)*log(c*x^2 + b*x + a))/(
b^2*c^2 - 4*a*c^3)]
```

giac [A] time = 0.27, size = 67, normalized size = 0.96

$$\frac{x}{c} - \frac{b\log\left(cx^2 + bx + a\right)}{2\,c^2} + \frac{\left(b^2 - 2\,ac\right)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{\sqrt{-b^2 + 4\,ac}\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x^2+b/x),x, algorithm="giac")
```

```
[Out] x/c - 1/2*b*log(c*x^2 + b*x + a)/c^2 + (b^2 - 2*a*c)*arctan((2*c*x + b)/sqr
t(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*c^2)
```

maple [A] time = 0.00, size = 101, normalized size = 1.44

$$-\frac{2a\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}\;c} + \frac{b^2\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}\;c^2} - \frac{b\ln\left(c\,x^2 + bx + a\right)}{2c^2} + \frac{x}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c+a/x^2+b/x),x)
```

```
[Out] 1/c*x-1/2*b*ln(c*x^2+b*x+a)/c^2-2/c/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a
*c-b^2)^(1/2))*a+1/c^2/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2)
)*b^2
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x^2+b/x),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

mupad [B] time = 1.42, size = 172, normalized size = 2.46

$$\frac{x}{c} + \frac{b^3\ln\left(c\,x^2 + b\,x + a\right)}{2\,\left(4\,a\,c^3 - b^2\,c^2\right)} - \frac{2\,a\,\mathrm{atan}\left(\frac{b}{\sqrt{4\,a\,c-b^2}} + \frac{2\,c\,x}{\sqrt{4\,a\,c-b^2}}\right)}{c\,\sqrt{4\,a\,c - b^2}} + \frac{b^2\,\mathrm{atan}\left(\frac{b}{\sqrt{4\,a\,c-b^2}} + \frac{2\,c\,x}{\sqrt{4\,a\,c-b^2}}\right)}{c^2\,\sqrt{4\,a\,c - b^2}} - \frac{2\,a\,b\,c\,\ln\left(c\,x^2 + b\,x + a\right)}{4\,a\,c^3 - b^2\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c + a/x^2 + b/x),x)
```

```
[Out] x/c + (b^3*log(a + b*x + c*x^2))/(2*(4*a*c^3 - b^2*c^2)) - (2*a*atan(b/(4*a
*c - b^2)^(1/2) + (2*c*x)/(4*a*c - b^2)^(1/2)))/(c*(4*a*c - b^2)^(1/2)) + (
b^2*atan(b/(4*a*c - b^2)^(1/2) + (2*c*x)/(4*a*c - b^2)^(1/2)))/(c^2*(4*a*c
- b^2)^(1/2)) - (2*a*b*c*log(a + b*x + c*x^2))/(4*a*c^3 - b^2*c^2)
```

sympy [B] time = 0.60, size = 306, normalized size = 4.37

$$\left(-\frac{b}{2c^2} - \frac{\sqrt{-4ac + b^2}\,\left(2ac - b^2\right)}{2c^2\left(4ac - b^2\right)}\right)\log\left(x + \frac{-ab - 4ac^2\left(-\frac{b}{2c^2} - \frac{\sqrt{-4ac+b^2}\left(2ac-b^2\right)}{2c^2\left(4ac-b^2\right)}\right) + b^2c\left(-\frac{b}{2c^2} - \frac{\sqrt{-4ac+b^2}\left(2ac-b^2\right)}{2c^2\left(4ac-b^2\right)}\right)}{2ac - b^2}\right) + \left(-\frac{b}{2c^2} + \frac{\sqrt{-4ac + b^2}\,\left(2ac - b^2\right)}{2c^2\left(4ac - b^2\right)}\right)\log\left(x + \frac{-ab - 4ac^2\left(-\frac{b}{2c^2} + \frac{\sqrt{-4ac+b^2}\left(2ac-b^2\right)}{2c^2\left(4ac-b^2\right)}\right) + b^2c\left(-\frac{b}{2c^2} + \frac{\sqrt{-4ac+b^2}\left(2ac-b^2\right)}{2c^2\left(4ac-b^2\right)}\right)}{2ac - b^2}\right) + \frac{x}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x**2+b/x),x)
```

```
[Out] (-b/(2*c**2) - sqrt(-4*a*c + b**2)*(2*a*c - b**2)/(2*c**2*(4*a*c - b**2)))*
log(x + (-a*b - 4*a*c**2*(-b/(2*c**2) - sqrt(-4*a*c + b**2)*(2*a*c - b**2)/
(2*c**2*(4*a*c - b**2))) + b**2*c*(-b/(2*c**2) - sqrt(-4*a*c + b**2)*(2*a*c
 - b**2)/(2*c**2*(4*a*c - b**2))))/(2*a*c - b**2)) + (-b/(2*c**2) + sqrt(-4
*a*c + b**2)*(2*a*c - b**2)/(2*c**2*(4*a*c - b**2)))*log(x + (-a*b - 4*a*c*
*2*(-b/(2*c**2) + sqrt(-4*a*c + b**2)*(2*a*c - b**2)/(2*c**2*(4*a*c - b**2)
)) + b**2*c*(-b/(2*c**2) + sqrt(-4*a*c + b**2)*(2*a*c - b**2)/(2*c**2*(4*a*
c - b**2))))/(2*a*c - b**2)) + x/c
```

3.355 $\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)x}\,dx$

Optimal. Leaf size=56

$$\frac{b\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c\sqrt{b^2-4ac}}+\frac{\log\left(a+bx+cx^2\right)}{2c}$$

Rubi [A] time = 0.04, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {1354, 634, 618, 206, 628}

$$\frac{b\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c\sqrt{b^2-4ac}}+\frac{\log\left(a+bx+cx^2\right)}{2c}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c + a/x^2 + b/x)*x),x]
```

```
[Out] (b*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(c*Sqrt[b^2 - 4*a*c]) + Log[a +
b*x + c*x^2]/(2*c)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1354

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> Int[x^(m + 2*n*p)*(c + b/x^n + a/x^(2*n))^p, x] /; FreeQ[{a, b, c, m, n
}, x] && EqQ[n2, 2*n] && ILtQ[p, 0] && NegQ[n]
```

Rubi steps

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)x}\,dx = \int \frac{x}{a+bx+cx^2}\,dx$$

$$= \frac{\int \frac{b+2cx}{a+bx+cx^2}\,dx}{2c} - \frac{b\int \frac{1}{a+bx+cx^2}\,dx}{2c}$$

$$= \frac{\log\left(a+bx+cx^2\right)}{2c} + \frac{b\,\mathrm{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx, x, b+2cx\right)}{c}$$

$$= \frac{b\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c\sqrt{b^2-4ac}} + \frac{\log\left(a+bx+cx^2\right)}{2c}$$

Mathematica [A] time = 0.03, size = 57, normalized size = 1.02

$$\frac{\log(a+x(b+cx)) - \frac{2b\tan^{-1}\left(\frac{b+2cx}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}}{2c}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c + a/x^2 + b/x)*x),x]
```

```
[Out] ((-2*b*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c] + Log[a +
 x*(b + c*x)])/(2*c)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)x}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/((c + a/x^2 + b/x)*x),x]
```

```
[Out] IntegrateAlgebraic[1/((c + a/x^2 + b/x)*x), x]
```

fricas [A] time = 1.35, size = 185, normalized size = 3.30

$$\left[\frac{\sqrt{b^2-4\,ac}\,b\log\left(\frac{2\,c^2x^2+2\,bcx+b^2-2\,ac+\sqrt{b^2-4\,ac}\,(2\,cx+b)}{cx^2+bx+a}\right)+\left(b^2-4\,ac\right)\log\left(cx^2+bx+a\right)}{2\left(b^2c-4\,ac^2\right)}, \frac{2\sqrt{-b^2+4\,ac}\,b\arctan\left(-\frac{\sqrt{-b^2+4\,ac}\,(2\,cx+b)}{b^2-4\,ac}\right)+\left(b^2-4\,ac\right)\log\left(cx^2+bx+a\right)}{2\left(b^2c-4\,ac^2\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x^2+b/x)/x,x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(b^2 - 4*a*c)*b*log((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c + sqrt(b^2
 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) + (b^2 - 4*a*c)*log(c*x^2 + b*x +
 a))/(b^2*c - 4*a*c^2), 1/2*(2*sqrt(-b^2 + 4*a*c)*b*arctan(-sqrt(-b^2 + 4*a
*c)*(2*c*x + b)/(b^2 - 4*a*c)) + (b^2 - 4*a*c)*log(c*x^2 + b*x + a))/(b^2*c
 - 4*a*c^2)]
```

giac [A] time = 0.29, size = 55, normalized size = 0.98

$$-\frac{b\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{\sqrt{-b^2+4\,ac}\,c} + \frac{\log\left(cx^2+bx+a\right)}{2\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x^2+b/x)/x,x, algorithm="giac")
```

```
[Out] -b*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*c) + 1/2*log(
c*x^2 + b*x + a)/c
```

maple [A] time = 0.00, size = 56, normalized size = 1.00

$$-\frac{b \arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}\, c}+\frac{\ln\left(c\,x^2+bx+a\right)}{2c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c+a/x^2+b/x)/x,x)
```

```
[Out] 1/2*ln(c*x^2+b*x+a)/c-b/c/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1
/2))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x^2+b/x)/x,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

mupad [B] time = 0.17, size = 112, normalized size = 2.00

$$\frac{2\,a\,c\,\ln\left(c\,x^2+b\,x+a\right)}{4\,a\,c^2-b^2\,c}-\frac{b\,\mathrm{atan}\left(\frac{b}{\sqrt{4\,a\,c-b^2}}+\frac{2\,c\,x}{\sqrt{4\,a\,c-b^2}}\right)}{c\,\sqrt{4\,a\,c-b^2}}-\frac{b^2\,\ln\left(c\,x^2+b\,x+a\right)}{2\,\left(4\,a\,c^2-b^2\,c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(c + a/x^2 + b/x)),x)
```

```
[Out] (2*a*c*log(a + b*x + c*x^2))/(4*a*c^2 - b^2*c) - (b*atan(b/(4*a*c - b^2)^(1
/2) + (2*c*x)/(4*a*c - b^2)^(1/2)))/(c*(4*a*c - b^2)^(1/2)) - (b^2*log(a +
b*x + c*x^2))/(2*(4*a*c^2 - b^2*c))
```

sympy [B] time = 0.32, size = 216, normalized size = 3.86

$$\left(-\frac{b\sqrt{-4ac+b^2}}{2c\left(4ac-b^2\right)}+\frac{1}{2c}\right)\log\left(x+\frac{-4ac\left(-\frac{b\sqrt{-4ac+b^2}}{2c\left(4ac-b^2\right)}+\frac{1}{2c}\right)+2a+b^2\left(-\frac{b\sqrt{-4ac+b^2}}{2c\left(4ac-b^2\right)}+\frac{1}{2c}\right)}{b}\right)+\left(\frac{b\sqrt{-4ac+b^2}}{2c\left(4ac-b^2\right)}+\frac{1}{2c}\right)\log\left(x+\frac{-4ac\left(\frac{b\sqrt{-4ac+b^2}}{2c\left(4ac-b^2\right)}+\frac{1}{2c}\right)+2a+b^2\left(\frac{b\sqrt{-4ac+b^2}}{2c\left(4ac-b^2\right)}+\frac{1}{2c}\right)}{b}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x**2+b/x)/x,x)
```

```
[Out] (-b*sqrt(-4*a*c + b**2)/(2*c*(4*a*c - b**2)) + 1/(2*c))*log(x + (-4*a*c*(-b
*sqrt(-4*a*c + b**2)/(2*c*(4*a*c - b**2)) + 1/(2*c)) + 2*a + b**2*(-b*sqrt(
-4*a*c + b**2)/(2*c*(4*a*c - b**2)) + 1/(2*c)))/b) + (b*sqrt(-4*a*c + b**2)
/(2*c*(4*a*c - b**2)) + 1/(2*c))*log(x + (-4*a*c*(b*sqrt(-4*a*c + b**2)/(2*
c*(4*a*c - b**2)) + 1/(2*c)) + 2*a + b**2*(b*sqrt(-4*a*c + b**2)/(2*c*(4*a*
c - b**2)) + 1/(2*c)))/b)
```

3.356 $\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)x^2}\,dx$

Optimal. Leaf size=36

$$\frac{2\tanh^{-1}\left(\frac{\frac{2a}{x}+b}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}}$$

Rubi [A] time = 0.03, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1352, 618, 206}

$$\frac{2\tanh^{-1}\left(\frac{\frac{2a}{x}+b}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c + a/x^2 + b/x)*x^2),x]
```

```
[Out] (2*ArcTanh[(b + (2*a)/x)/Sqrt[b^2 - 4*a*c]])/Sqrt[b^2 - 4*a*c]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1352

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a,
b, c, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)x^2}\,dx &= -\text{Subst}\left(\int \frac{1}{c+bx+ax^2}\,dx, x, \frac{1}{x}\right)\\ &= 2\,\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx, x, b+\frac{2a}{x}\right)\\ &= \frac{2\tanh^{-1}\left(\frac{b+\frac{2a}{x}}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 38, normalized size = 1.06

$$\frac{2\tan^{-1}\left(\frac{b+2cx}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c + a/x^2 + b/x)*x^2),x]
[Out] (2*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)x^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/((c + a/x^2 + b/x)*x^2),x]
[Out] IntegrateAlgebraic[1/((c + a/x^2 + b/x)*x^2), x]
```

fricas [A] time = 1.24, size = 120, normalized size = 3.33

$$\left[\frac{\log\left(\frac{2\,c^2x^2+2\,bcx+b^2-2\,ac-\sqrt{b^2-4\,ac}\,(2\,cx+b)}{cx^2+bx+a}\right)}{\sqrt{b^2-4\,ac}}, -\frac{2\,\sqrt{-b^2+4\,ac}\,\arctan\left(-\frac{\sqrt{-b^2+4\,ac}\,(2\,cx+b)}{b^2-4\,ac}\right)}{b^2-4\,ac}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x^2+b/x)/x^2,x, algorithm="fricas")
[Out] [log((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c - sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c
*x^2 + b*x + a))/sqrt(b^2 - 4*a*c), -2*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b^2
 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c))/(b^2 - 4*a*c)]
```

giac [A] time = 0.35, size = 34, normalized size = 0.94

$$\frac{2\,\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{\sqrt{-b^2+4\,ac}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x^2+b/x)/x^2,x, algorithm="giac")
[Out] 2*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/sqrt(-b^2 + 4*a*c)
```

maple [A] time = 0.00, size = 35, normalized size = 0.97

$$\frac{2\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c+a/x^2+b/x)/x^2,x)
[Out] 2/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x^2+b/x)/x^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

mupad [B] time = 0.05, size = 46, normalized size = 1.28

$$\frac{2\,\mathrm{atan}\left(\frac{b}{\sqrt{4\,a\,c-b^2}}+\frac{2\,c\,x}{\sqrt{4\,a\,c-b^2}}\right)}{\sqrt{4\,a\,c-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(c + a/x^2 + b/x)),x)
```

```
[Out] (2*atan(b/(4*a*c - b^2)^(1/2) + (2*c*x)/(4*a*c - b^2)^(1/2)))/(4*a*c - b^2)
^(1/2)
```

sympy [B] time = 0.22, size = 124, normalized size = 3.44

$$-\sqrt{-\frac{1}{4ac-b^2}}\log\left(x+\frac{-4ac\sqrt{-\frac{1}{4ac-b^2}}+b^2\sqrt{-\frac{1}{4ac-b^2}}+b}{2c}\right)+\sqrt{-\frac{1}{4ac-b^2}}\log\left(x+\frac{4ac\sqrt{-\frac{1}{4ac-b^2}}-b^2\sqrt{-\frac{1}{4ac-b^2}}+b}{2c}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x**2+b/x)/x**2,x)
```

```
[Out] -sqrt(-1/(4*a*c - b**2))*log(x + (-4*a*c*sqrt(-1/(4*a*c - b**2)) + b**2*sqr
t(-1/(4*a*c - b**2)) + b)/(2*c)) + sqrt(-1/(4*a*c - b**2))*log(x + (4*a*c*s
qrt(-1/(4*a*c - b**2)) - b**2*sqrt(-1/(4*a*c - b**2)) + b)/(2*c))
```

3.357 $\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)x^3}\,dx$

Optimal. Leaf size=62

$$\frac{b\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a\sqrt{b^2-4ac}}-\frac{\log\left(a+bx+cx^2\right)}{2a}+\frac{\log(x)}{a}$$

Rubi [A] time = 0.05, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389, Rules used = {1354, 705, 29, 634, 618, 206, 628}

$$\frac{b\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a\sqrt{b^2-4ac}}-\frac{\log\left(a+bx+cx^2\right)}{2a}+\frac{\log(x)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c + a/x^2 + b/x)*x^3),x]
```

```
[Out] (b*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(a*Sqrt[b^2 - 4*a*c]) + Log[x]/a
 - Log[a + b*x + c*x^2]/(2*a)
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 705

```
Int[1/(((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)), x_Symbol]
 :> Dist[e^2/(c*d^2 - b*d*e + a*e^2), Int[1/(d + e*x), x], x] + Dist[1/(c*d
^2 - b*d*e + a*e^2), Int[(c*d - b*e - c*e*x)/(a + b*x + c*x^2), x], x] /; F
reeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^
2, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1354

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> Int[x^(m + 2*n*p)*(c + b/x^n + a/x^(2*n))^p, x] /; FreeQ[{a, b, c, m, n
}, x] && EqQ[n2, 2*n] && ILtQ[p, 0] && NegQ[n]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)x^3}\,dx &= \int \frac{1}{x\left(a+bx+cx^2\right)}\,dx\\ &= \frac{\int \frac{1}{x}\,dx}{a}+\frac{\int \frac{-b-cx}{a+bx+cx^2}\,dx}{a}\\ &= \frac{\log(x)}{a}-\frac{\int \frac{b+2cx}{a+bx+cx^2}\,dx}{2a}-\frac{b\int \frac{1}{a+bx+cx^2}\,dx}{2a}\\ &= \frac{\log(x)}{a}-\frac{\log\left(a+bx+cx^2\right)}{2a}+\frac{b\,\mathrm{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx,x,b+2cx\right)}{a}\\ &= \frac{b\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a\sqrt{b^2-4ac}}+\frac{\log(x)}{a}-\frac{\log\left(a+bx+cx^2\right)}{2a}\end{aligned}$$

Mathematica [A] time = 0.07, size = 61, normalized size = 0.98

$$-\frac{\frac{2b\tan^{-1}\left(\frac{b+2cx}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}+\log(a+x(b+cx))-2\log(x)}{2a}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c + a/x^2 + b/x)*x^3),x]
```

```
[Out] -1/2*((2*b*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c] - 2*L
og[x] + Log[a + x*(b + c*x)])/a
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)x^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/((c + a/x^2 + b/x)*x^3),x]
```

```
[Out] IntegrateAlgebraic[1/((c + a/x^2 + b/x)*x^3), x]
```

fricas [A] time = 1.41, size = 211, normalized size = 3.40

$$\left[\frac{\sqrt{b^2-4\,ac}\,b\log\left(\frac{2\,c^2x^2+2\,bcx+b^2-2\,ac+\sqrt{b^2-4\,ac}\,(2\,cx+b)}{cx^2+bx+a}\right)-\left(b^2-4\,ac\right)\log\left(cx^2+bx+a\right)+2\left(b^2-4\,ac\right)\log(x)}{2\left(ab^2-4\,a^2c\right)},\frac{2\sqrt{-b^2+4\,ac}\,b\arctan\left(-\frac{\sqrt{-b^2+4\,ac}\,(2\,cx+b)}{b^2-4\,ac}\right)-\left(b^2-4\,ac\right)\log\left(cx^2+bx+a\right)+2\left(b^2-4\,ac\right)\log(x)}{2\left(ab^2-4\,a^2c\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x^2+b/x)/x^3,x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(b^2 - 4*a*c)*b*log((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c + sqrt(b^2
 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) - (b^2 - 4*a*c)*log(c*x^2 + b*x +
```

```
a) + 2*(b^2 - 4*a*c)*log(x))/(a*b^2 - 4*a^2*c), 1/2*(2*sqrt(-b^2 + 4*a*c)*
b*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) - (b^2 - 4*a*c)*log
(c*x^2 + b*x + a) + 2*(b^2 - 4*a*c)*log(x))/(a*b^2 - 4*a^2*c)]
```

giac [A] time = 0.30, size = 62, normalized size = 1.00

$$-\frac{b\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{\sqrt{-b^2+4\,ac}\,a}-\frac{\log\left(cx^2+bx+a\right)}{2\,a}+\frac{\log\left(|x|\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x^2+b/x)/x^3,x, algorithm="giac")
```

```
[Out] -b*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*a) - 1/2*log(
c*x^2 + b*x + a)/a + log(abs(x))/a
```

maple [A] time = 0.01, size = 62, normalized size = 1.00

$$-\frac{b\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}\;a}+\frac{\ln(x)}{a}-\frac{\ln\left(c\,x^2+bx+a\right)}{2a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c+a/x^2+b/x)/x^3,x)
```

```
[Out] 1/a*ln(x)-1/2*ln(c*x^2+b*x+a)/a-1/a*b/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4
*a*c-b^2)^(1/2))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x^2+b/x)/x^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

mupad [B] time = 1.72, size = 213, normalized size = 3.44

$$\frac{\ln(x)}{a}-\ln\left(b\,c-\left(x\,\left(6\,a\,c^2-2\,b^2\,c\right)-a\,b\,c\right)\,\left(\frac{1}{2\,a}-\frac{b\,\sqrt{b^2-4\,a\,c}}{2\,\left(a\,b^2-4\,a^2\,c\right)}\right)+3\,c^2\,x\right)\,\left(\frac{1}{2\,a}-\frac{b\,\sqrt{b^2-4\,a\,c}}{2\,\left(a\,b^2-4\,a^2\,c\right)}\right)-\ln\left(\left(x\,\left(6\,a\,c^2-2\,b^2\,c\right)-a\,b\,c\right)\,\left(\frac{1}{2\,a}+\frac{b\,\sqrt{b^2-4\,a\,c}}{2\,\left(a\,b^2-4\,a^2\,c\right)}\right)-b\,c-3\,c^2\,x\right)\,\left(\frac{1}{2\,a}+\frac{b\,\sqrt{b^2-4\,a\,c}}{2\,\left(a\,b^2-4\,a^2\,c\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(c + a/x^2 + b/x)),x)
```

```
[Out] log(x)/a - log(b*c - (x*(6*a*c^2 - 2*b^2*c) - a*b*c)*(1/(2*a) - (b*(b^2 - 4
*a*c)^(1/2))/(2*(a*b^2 - 4*a^2*c))) + 3*c^2*x)*(1/(2*a) - (b*(b^2 - 4*a*c)^
(1/2))/(2*(a*b^2 - 4*a^2*c))) - log((x*(6*a*c^2 - 2*b^2*c) - a*b*c)*(1/(2*a
) + (b*(b^2 - 4*a*c)^(1/2))/(2*(a*b^2 - 4*a^2*c))) - b*c - 3*c^2*x)*(1/(2*a
) + (b*(b^2 - 4*a*c)^(1/2))/(2*(a*b^2 - 4*a^2*c)))
```

sympy [B] time = 4.08, size = 564, normalized size = 9.10

$$\left(-\frac{b\sqrt{-4ac+b^2}}{2a\left(4ac-b^2\right)}-\frac{1}{2a}\right)\log\left(x+\frac{24a^4c^2\left(-\frac{b\sqrt{-4ac+b^2}}{2a(4ac-b^2)}-\frac{1}{2a}\right)^2-14a^3b^2c\left(-\frac{b\sqrt{-4ac+b^2}}{2a(4ac-b^2)}-\frac{1}{2a}\right)^2-12a^3c^2\left(-\frac{b\sqrt{-4ac+b^2}}{2a(4ac-b^2)}-\frac{1}{2a}\right)+2a^2b^4\left(-\frac{b\sqrt{-4ac+b^2}}{2a(4ac-b^2)}-\frac{1}{2a}\right)^2+3a^2b^2c\left(-\frac{b\sqrt{-4ac+b^2}}{2a(4ac-b^2)}-\frac{1}{2a}\right)-12a^2c^2+11ab^2c-2b^4}{9abc^2-2b^3c}\right)+\left(\frac{b\sqrt{-4ac+b^2}}{2a\left(4ac-b^2\right)}-\frac{1}{2a}\right)\log\left(x+\frac{24a^4c^2\left(\frac{b\sqrt{-4ac+b^2}}{2a(4ac-b^2)}-\frac{1}{2a}\right)^2-14a^3b^2c\left(\frac{b\sqrt{-4ac+b^2}}{2a(4ac-b^2)}-\frac{1}{2a}\right)^2-12a^3c^2\left(\frac{b\sqrt{-4ac+b^2}}{2a(4ac-b^2)}-\frac{1}{2a}\right)+2a^2b^4\left(\frac{b\sqrt{-4ac+b^2}}{2a(4ac-b^2)}-\frac{1}{2a}\right)^2+3a^2b^2c\left(\frac{b\sqrt{-4ac+b^2}}{2a(4ac-b^2)}-\frac{1}{2a}\right)-12a^2c^2+11ab^2c-2b^4}{9abc^2-2b^3c}\right)+\frac{\log(x)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x**2+b/x)/x**3,x)
```

```
[Out] (-b*sqrt(-4*a*c + b**2)/(2*a*(4*a*c - b**2)) - 1/(2*a))*log(x + (24*a**4*c*
*2*(-b*sqrt(-4*a*c + b**2)/(2*a*(4*a*c - b**2)) - 1/(2*a))**2 - 14*a**3*b**
2*c*(-b*sqrt(-4*a*c + b**2)/(2*a*(4*a*c - b**2)) - 1/(2*a))**2 - 12*a**3*c*
*2*(-b*sqrt(-4*a*c + b**2)/(2*a*(4*a*c - b**2)) - 1/(2*a)) + 2*a**2*b**4*(-
b*sqrt(-4*a*c + b**2)/(2*a*(4*a*c - b**2)) - 1/(2*a))**2 + 3*a**2*b**2*c*(-
b*sqrt(-4*a*c + b**2)/(2*a*(4*a*c - b**2)) - 1/(2*a)) - 12*a**2*c**2 + 11*a
*b**2*c - 2*b**4)/(9*a*b*c**2 - 2*b**3*c)) + (b*sqrt(-4*a*c + b**2)/(2*a*(4
*a*c - b**2)) - 1/(2*a))*log(x + (24*a**4*c**2*(b*sqrt(-4*a*c + b**2)/(2*a*
(4*a*c - b**2)) - 1/(2*a))**2 - 14*a**3*b**2*c*(b*sqrt(-4*a*c + b**2)/(2*a*
(4*a*c - b**2)) - 1/(2*a))**2 - 12*a**3*c**2*(b*sqrt(-4*a*c + b**2)/(2*a*(4
*a*c - b**2)) - 1/(2*a)) + 2*a**2*b**4*(b*sqrt(-4*a*c + b**2)/(2*a*(4*a*c -
 b**2)) - 1/(2*a))**2 + 3*a**2*b**2*c*(b*sqrt(-4*a*c + b**2)/(2*a*(4*a*c -
b**2)) - 1/(2*a)) - 12*a**2*c**2 + 11*a*b**2*c - 2*b**4)/(9*a*b*c**2 - 2*b*
*3*c)) + log(x)/a
```

3.358 $\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)x^4}\,dx$

Optimal. Leaf size=81

$$-\frac{(b^2-2ac)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a^2\sqrt{b^2-4ac}}+\frac{b\log\left(a+bx+cx^2\right)}{2a^2}-\frac{b\log(x)}{a^2}-\frac{1}{ax}$$

Rubi [A] time = 0.10, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389, Rules used = {1354, 709, 800, 634, 618, 206, 628}

$$-\frac{(b^2-2ac)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a^2\sqrt{b^2-4ac}}+\frac{b\log\left(a+bx+cx^2\right)}{2a^2}-\frac{b\log(x)}{a^2}-\frac{1}{ax}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c + a/x^2 + b/x)*x^4),x]
```

```
[Out] -(1/(a*x)) - ((b^2 - 2*a*c)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(a^2*Sq
rt[b^2 - 4*a*c]) - (b*Log[x])/a^2 + (b*Log[a + b*x + c*x^2])/(2*a^2)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 709

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Simp[(e*(d + e*x)^(m + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dis
t[1/(c*d^2 - b*d*e + a*e^2), Int[((d + e*x)^(m + 1)*Simp[c*d - b*e - c*e*x,
 x])/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 -
 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[m
, -1]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 1354

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> Int[x^(m + 2*n*p)*(c + b/x^n + a/x^(2*n))^p, x] /; FreeQ[{a, b, c, m, n
}, x] && EqQ[n2, 2*n] && ILtQ[p, 0] && NegQ[n]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)x^4}\,dx &= \int \frac{1}{x^2\left(a+bx+cx^2\right)}\,dx \\
&= -\frac{1}{ax}+\frac{\int \frac{-b-cx}{x\left(a+bx+cx^2\right)}\,dx}{a} \\
&= -\frac{1}{ax}+\frac{\int \left(-\frac{b}{ax}+\frac{b^2-ac+bcx}{a\left(a+bx+cx^2\right)}\right)dx}{a} \\
&= -\frac{1}{ax}-\frac{b\log(x)}{a^2}+\frac{\int \frac{b^2-ac+bcx}{a+bx+cx^2}\,dx}{a^2} \\
&= -\frac{1}{ax}-\frac{b\log(x)}{a^2}+\frac{b\int \frac{b+2cx}{a+bx+cx^2}\,dx}{2a^2}+\frac{\left(b^2-2ac\right)\int \frac{1}{a+bx+cx^2}\,dx}{2a^2} \\
&= -\frac{1}{ax}-\frac{b\log(x)}{a^2}+\frac{b\log\left(a+bx+cx^2\right)}{2a^2}-\frac{\left(b^2-2ac\right)\mathrm{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx,x,b+2cx\right)}{a^2} \\
&= -\frac{1}{ax}-\frac{\left(b^2-2ac\right)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a^2\sqrt{b^2-4ac}}-\frac{b\log(x)}{a^2}+\frac{b\log\left(a+bx+cx^2\right)}{2a^2}
\end{aligned}$$

Mathematica [A] time = 0.08, size = 77, normalized size = 0.95

$$\frac{\frac{2\left(b^2-2ac\right)\tan^{-1}\left(\frac{b+2cx}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}+b\log(a+x(b+cx))-\frac{2a}{x}-2b\log(x)}{2a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c + a/x^2 + b/x)*x^4),x]
```

```
[Out] ((-2*a)/x + (2*(b^2 - 2*a*c)*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-
b^2 + 4*a*c] - 2*b*Log[x] + b*Log[a + x*(b + c*x)])/(2*a^2)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)x^4}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/((c + a/x^2 + b/x)*x^4),x]
```

```
[Out] IntegrateAlgebraic[1/((c + a/x^2 + b/x)*x^4), x]
```

fricas [A] time = 0.89, size = 269, normalized size = 3.32

$$\left[-\frac{\left(b^2-2\,ac\right)\sqrt{b^2-4\,ac}\,x\log\left(\frac{2\,c^2x^2+2\,bcx+b^2-2\,ac+\sqrt{b^2-4\,ac}\,(2\,cx+b)}{cx^2+bx+a}\right)+2\,ab^2-8\,a^2c-\left(b^3-4\,abc\right)x\log\left(cx^2+bx+a\right)+2\left(b^3-4\,abc\right)x\log(x)}{2\left(a^2b^2-4\,a^3c\right)x},-\frac{2\left(b^2-2\,ac\right)\sqrt{-b^2+4\,ac}\,x\arctan\left(-\frac{\sqrt{-b^2+4\,ac}\,(2\,cx+b)}{b^2-4\,ac}\right)+2\,ab^2-8\,a^2c-\left(b^3-4\,abc\right)x\log\left(cx^2+bx+a\right)+2\left(b^3-4\,abc\right)x\log(x)}{2\left(a^2b^2-4\,a^3c\right)x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x^2+b/x)/x^4,x, algorithm="fricas")
```

```
[Out] [-1/2*((b^2 - 2*a*c)*sqrt(b^2 - 4*a*c)*x*log((2*c^2*x^2 + 2*b*c*x + b^2 - 2
*a*c + sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) + 2*a*b^2 - 8*a^2*
c - (b^3 - 4*a*b*c)*x*log(c*x^2 + b*x + a) + 2*(b^3 - 4*a*b*c)*x*log(x))/((
a^2*b^2 - 4*a^3*c)*x), -1/2*(2*(b^2 - 2*a*c)*sqrt(-b^2 + 4*a*c)*x*arctan(-s
qrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) + 2*a*b^2 - 8*a^2*c - (b^3 - 4
*a*b*c)*x*log(c*x^2 + b*x + a) + 2*(b^3 - 4*a*b*c)*x*log(x))/((a^2*b^2 - 4*
a^3*c)*x)]
```

giac [A] time = 0.39, size = 79, normalized size = 0.98

$$\frac{b\log\left(cx^2+bx+a\right)}{2\,a^2}-\frac{b\log\left(|x|\right)}{a^2}+\frac{\left(b^2-2\,ac\right)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{\sqrt{-b^2+4\,ac}\,a^2}-\frac{1}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x^2+b/x)/x^4,x, algorithm="giac")
```

```
[Out] 1/2*b*log(c*x^2 + b*x + a)/a^2 - b*log(abs(x))/a^2 + (b^2 - 2*a*c)*arctan((
2*c*x + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*a^2) - 1/(a*x)
```

maple [A] time = 0.01, size = 112, normalized size = 1.38

$$-\frac{2c\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}\;a}+\frac{b^2\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}\;a^2}-\frac{b\ln(x)}{a^2}+\frac{b\ln\left(c\,x^2+bx+a\right)}{2a^2}-\frac{1}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c+a/x^2+b/x)/x^4,x)
```

```
[Out] -1/a/x-1/a^2*b*ln(x)+1/2*b*ln(c*x^2+b*x+a)/a^2-2/a/(4*a*c-b^2)^(1/2)*arctan
((2*c*x+b)/(4*a*c-b^2)^(1/2))*c+1/a^2/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4
*a*c-b^2)^(1/2))*b^2
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x^2+b/x)/x^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

mupad [B] time = 1.81, size = 339, normalized size = 4.19

$$\frac{\ln\left(2\,a\,b^3+2\,b^4\,x-2\,a\,b^2\,\sqrt{b^2-4\,a\,c}+a^2\,c\,\sqrt{b^2-4\,a\,c}-2\,b^3\,x\,\sqrt{b^2-4\,a\,c}+2\,a^2\,c^2\,x-7\,a^2\,b\,c-8\,a\,b^2\,c\,x+4\,a\,b\,c\,x\,\sqrt{b^2-4\,a\,c}\right)\left(a\,\left(2\,b\,c-c\,\sqrt{b^2-4\,a\,c}\right)-\frac{b^3}{2}+\frac{b^2\,\sqrt{b^2-4\,a\,c}}{2}\right)}{4\,a^3\,c-a^2\,b^2}-\frac{1}{a\,x}-\frac{\ln\left(2\,a\,b^3+2\,b^4\,x+2\,a\,b^2\,\sqrt{b^2-4\,a\,c}-a^2\,c\,\sqrt{b^2-4\,a\,c}+2\,b^3\,x\,\sqrt{b^2-4\,a\,c}+2\,a^2\,c^2\,x-7\,a^2\,b\,c-8\,a\,b^2\,c\,x-4\,a\,b\,c\,x\,\sqrt{b^2-4\,a\,c}\right)\left(\frac{b^3}{2}-a\,\left(2\,b\,c+c\,\sqrt{b^2-4\,a\,c}\right)+\frac{b^2\,\sqrt{b^2-4\,a\,c}}{2}\right)}{4\,a^3\,c-a^2\,b^2}-\frac{b\,\ln(x)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(c + a/x^2 + b/x)),x)
```

```
[Out] (log(2*a*b^3 + 2*b^4*x - 2*a*b^2*(b^2 - 4*a*c)^(1/2) + a^2*c*(b^2 - 4*a*c)^
(1/2) - 2*b^3*x*(b^2 - 4*a*c)^(1/2) + 2*a^2*c^2*x - 7*a^2*b*c - 8*a*b^2*c*x
 + 4*a*b*c*x*(b^2 - 4*a*c)^(1/2))*(a*(2*b*c - c*(b^2 - 4*a*c)^(1/2)) - b^3/
2 + (b^2*(b^2 - 4*a*c)^(1/2))/2))/(4*a^3*c - a^2*b^2) - 1/(a*x) - (log(2*a*
b^3 + 2*b^4*x + 2*a*b^2*(b^2 - 4*a*c)^(1/2) - a^2*c*(b^2 - 4*a*c)^(1/2) + 2
*b^3*x*(b^2 - 4*a*c)^(1/2) + 2*a^2*c^2*x - 7*a^2*b*c - 8*a*b^2*c*x - 4*a*b*
c*x*(b^2 - 4*a*c)^(1/2))*(b^3/2 - a*(2*b*c + c*(b^2 - 4*a*c)^(1/2)) + (b^2*
(b^2 - 4*a*c)^(1/2))/2))/(4*a^3*c - a^2*b^2) - (b*log(x))/a^2
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x**2+b/x)/x**4,x)
```

```
[Out] Timed out
```

3.359 $\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)x^5}\,dx$

Optimal. Leaf size=104

$$-\frac{(b^2-ac)\log(a+bx+cx^2)}{2a^3}+\frac{\log(x)(b^2-ac)}{a^3}+\frac{b(b^2-3ac)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a^3\sqrt{b^2-4ac}}+\frac{b}{a^2x}-\frac{1}{2ax^2}$$

Rubi [A] time = 0.15, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389, Rules used = {1354, 709, 800, 634, 618, 206, 628}

$$-\frac{(b^2-ac)\log(a+bx+cx^2)}{2a^3}+\frac{\log(x)(b^2-ac)}{a^3}+\frac{b(b^2-3ac)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a^3\sqrt{b^2-4ac}}+\frac{b}{a^2x}-\frac{1}{2ax^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c + a/x^2 + b/x)*x^5),x]
```

```
[Out] -1/(2*a*x^2) + b/(a^2*x) + (b*(b^2 - 3*a*c)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 -
4*a*c]])/(a^3*Sqrt[b^2 - 4*a*c]) + ((b^2 - a*c)*Log[x])/a^3 - ((b^2 - a*c)*
Log[a + b*x + c*x^2])/(2*a^3)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 709

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Simp[(e*(d + e*x)^(m + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dis
t[1/(c*d^2 - b*d*e + a*e^2), Int[((d + e*x)^(m + 1)*Simp[c*d - b*e - c*e*x,
 x])/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 -
 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[m
, -1]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 1354

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> Int[x^(m + 2*n*p)*(c + b/x^n + a/x^(2*n))^p, x] /; FreeQ[{a, b, c, m, n
}, x] && EqQ[n2, 2*n] && ILtQ[p, 0] && NegQ[n]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)x^5}\,dx &= \int \frac{1}{x^3\left(a+bx+cx^2\right)}\,dx\\
&= -\frac{1}{2ax^2}+\frac{\int \frac{-b-cx}{x^2\left(a+bx+cx^2\right)}\,dx}{a}\\
&= -\frac{1}{2ax^2}+\frac{\int \left(-\frac{b}{ax^2}+\frac{b^2-ac}{a^2x}+\frac{-b\left(b^2-2ac\right)-c\left(b^2-ac\right)x}{a^2\left(a+bx+cx^2\right)}\right)dx}{a}\\
&= -\frac{1}{2ax^2}+\frac{b}{a^2x}+\frac{\left(b^2-ac\right)\log(x)}{a^3}+\frac{\int \frac{-b\left(b^2-2ac\right)-c\left(b^2-ac\right)x}{a+bx+cx^2}\,dx}{a^3}\\
&= -\frac{1}{2ax^2}+\frac{b}{a^2x}+\frac{\left(b^2-ac\right)\log(x)}{a^3}-\frac{\left(b\left(b^2-3ac\right)\right)\int \frac{1}{a+bx+cx^2}\,dx}{2a^3}-\frac{\left(b^2-ac\right)\int \frac{b+2cx}{a+bx+cx^2}\,dx}{2a^3}\\
&= -\frac{1}{2ax^2}+\frac{b}{a^2x}+\frac{\left(b^2-ac\right)\log(x)}{a^3}-\frac{\left(b^2-ac\right)\log\left(a+bx+cx^2\right)}{2a^3}+\frac{\left(b\left(b^2-3ac\right)\right)\operatorname{Subst}\left(\right.}{}\\
&= -\frac{1}{2ax^2}+\frac{b}{a^2x}+\frac{b\left(b^2-3ac\right)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a^3\sqrt{b^2-4ac}}+\frac{\left(b^2-ac\right)\log(x)}{a^3}-\frac{\left(b^2-ac\right)\log\left(a+bx\right.}{2a^3}
\end{aligned}$$

Mathematica [A] time = 0.14, size = 102, normalized size = 0.98

$$\frac{-\frac{a^2}{x^2}+2\log(x)\left(b^2-ac\right)+\left(ac-b^2\right)\log(a+x(b+cx))-\frac{2b\left(b^2-3ac\right)\tan^{-1}\left(\frac{b+2cx}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}+\frac{2ab}{x}}{2a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c + a/x^2 + b/x)*x^5),x]
```

```
[Out] (-(a^2/x^2) + (2*a*b)/x - (2*b*(b^2 - 3*a*c)*ArcTan[(b + 2*c*x)/Sqrt[-b^2 +
 4*a*c]])/Sqrt[-b^2 + 4*a*c] + 2*(b^2 - a*c)*Log[x] + (-b^2 + a*c)*Log[a +
x*(b + c*x)])/(2*a^3)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)x^5}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/((c + a/x^2 + b/x)*x^5),x]
```

```
[Out] IntegrateAlgebraic[1/((c + a/x^2 + b/x)*x^5), x]
```

fricas [**A**] time = 1.36, size = 358, normalized size = 3.44

$$\left[-\frac{\left(b^{3}-3\,abc\right)\sqrt{b^{2}-4\,ac}\,x^{2}\log\left(\frac{2\,c^{2}x^{2}+2\,bcx+b^{2}-2\,ac-\sqrt{b^{2}-4\,ac}\,(2\,cx+b)}{cx^{2}+bx+a}\right)+a^{2}b^{2}-4\,a^{3}c+\left(b^{4}-5\,ab^{2}c+4\,a^{2}c^{2}\right)x^{2}\log\left(cx^{2}+bx+a\right)-2\left(b^{4}-5\,ab^{2}c+4\,a^{2}c^{2}\right)x^{2}\log(x)-2\left(ab^{3}-4\,a^{2}bc\right)x}{2\left(a^{3}b^{2}-4\,a^{4}c\right)x^{2}}, \frac{2\left(b^{3}-3\,abc\right)\sqrt{-b^{2}+4\,ac}\,x^{2}\arctan\left(-\frac{\sqrt{-b^{2}+4\,ac}\,(2\,cx+b)}{b^{2}-4\,ac}\right)-a^{2}b^{2}+4\,a^{3}c-\left(b^{4}-5\,ab^{2}c+4\,a^{2}c^{2}\right)x^{2}\log\left(cx^{2}+bx+a\right)+2\left(b^{4}-5\,ab^{2}c+4\,a^{2}c^{2}\right)x^{2}\log(x)+2\left(ab^{3}-4\,a^{2}bc\right)x}{2\left(a^{3}b^{2}-4\,a^{4}c\right)x^{2}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x^2+b/x)/x^5,x, algorithm="fricas")
```

```
[Out] [-1/2*((b^3 - 3*a*b*c)*sqrt(b^2 - 4*a*c)*x^2*log((2*c^2*x^2 + 2*b*c*x + b^2
 - 2*a*c - sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) + a^2*b^2 - 4*
a^3*c + (b^4 - 5*a*b^2*c + 4*a^2*c^2)*x^2*log(c*x^2 + b*x + a) - 2*(b^4 - 5
*a*b^2*c + 4*a^2*c^2)*x^2*log(x) - 2*(a*b^3 - 4*a^2*b*c)*x)/((a^3*b^2 - 4*a
^4*c)*x^2), 1/2*(2*(b^3 - 3*a*b*c)*sqrt(-b^2 + 4*a*c)*x^2*arctan(-sqrt(-b^2
 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) - a^2*b^2 + 4*a^3*c - (b^4 - 5*a*b^2*c
 + 4*a^2*c^2)*x^2*log(c*x^2 + b*x + a) + 2*(b^4 - 5*a*b^2*c + 4*a^2*c^2)*x^
2*log(x) + 2*(a*b^3 - 4*a^2*b*c)*x)/((a^3*b^2 - 4*a^4*c)*x^2)]
```

giac [**A**] time = 0.31, size = 105, normalized size = 1.01

$$-\frac{\left(b^{2}-ac\right)\log\left(cx^{2}+bx+a\right)}{2\,a^{3}}+\frac{\left(b^{2}-ac\right)\log\left(|x|\right)}{a^{3}}-\frac{\left(b^{3}-3\,abc\right)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^{2}+4\,ac}}\right)}{\sqrt{-b^{2}+4\,ac}\,a^{3}}+\frac{2\,abx-a^{2}}{2\,a^{3}x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x^2+b/x)/x^5,x, algorithm="giac")
```

```
[Out] -1/2*(b^2 - a*c)*log(c*x^2 + b*x + a)/a^3 + (b^2 - a*c)*log(abs(x))/a^3 - (
b^3 - 3*a*b*c)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*a
^3) + 1/2*(2*a*b*x - a^2)/(a^3*x^2)
```

maple [**A**] time = 0.01, size = 150, normalized size = 1.44

$$\frac{3bc\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^{2}}}\right)}{\sqrt{4ac-b^{2}}\,a^{2}}-\frac{b^{3}\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^{2}}}\right)}{\sqrt{4ac-b^{2}}\,a^{3}}-\frac{c\ln(x)}{a^{2}}+\frac{c\ln\left(c\,x^{2}+bx+a\right)}{2a^{2}}+\frac{b^{2}\ln(x)}{a^{3}}-\frac{b^{2}\ln\left(c\,x^{2}+bx+a\right)}{2a^{3}}+\frac{b}{a^{2}x}-\frac{1}{2a\,x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c+a/x^2+b/x)/x^5,x)
```

```
[Out] -1/2/a/x^2-1/a^2*ln(x)*c+1/a^3*ln(x)*b^2+b/a^2/x+1/2/a^2*c*ln(c*x^2+b*x+a)-
1/2/a^3*ln(c*x^2+b*x+a)*b^2+3/a^2/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c
-b^2)^(1/2))*b*c-1/a^3/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2)
)*b^3
```

maxima [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x^2+b/x)/x^5,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

mupad [**B**] time = 1.87, size = 447, normalized size = 4.30

$$\frac{\ln\left(2\,a\,b^{4}+2\,b^{5}\,x+6\,a^{3}\,c^{2}+2\,a\,b^{3}\,\sqrt{b^{2}-4\,a\,c}+2\,b^{4}\,x\,\sqrt{b^{2}-4\,a\,c}-9\,a^{2}\,b^{2}\,c-10\,a\,b^{3}\,c\,x-3\,a^{2}\,b\,c\,\sqrt{b^{2}-4\,a\,c}+9\,a^{2}\,b\,c^{2}\,x+3\,a^{2}\,c^{2}\,x\,\sqrt{b^{2}-4\,a\,c}-6\,a\,b^{2}\,c\,x\,\sqrt{b^{2}-4\,a\,c}\right)\left(\frac{b^{4}}{2}-a\left(\frac{5\,b^{2}\,c}{2}+\frac{3\,b\,c\,\sqrt{b^{2}-4\,a\,c}}{2}\right)+\frac{b^{3}\,\sqrt{b^{2}-4\,a\,c}}{2}+2\,a^{2}\,c^{2}\right)}{4\,a^{4}\,c-a^{3}\,b^{2}}-\frac{\ln\left(2\,a\,b^{4}+2\,b^{5}\,x+6\,a^{3}\,c^{2}-2\,a\,b^{3}\,\sqrt{b^{2}-4\,a\,c}-2\,b^{4}\,x\,\sqrt{b^{2}-4\,a\,c}-9\,a^{2}\,b^{2}\,c-10\,a\,b^{3}\,c\,x+3\,a^{2}\,b\,c\,\sqrt{b^{2}-4\,a\,c}+9\,a^{2}\,b\,c^{2}\,x-3\,a^{2}\,c^{2}\,x\,\sqrt{b^{2}-4\,a\,c}+6\,a\,b^{2}\,c\,x\,\sqrt{b^{2}-4\,a\,c}\right)\left(a\left(\frac{5\,b^{2}\,c}{2}-\frac{3\,b\,c\,\sqrt{b^{2}-4\,a\,c}}{2}\right)-\frac{b^{4}}{2}+\frac{b^{3}\,\sqrt{b^{2}-4\,a\,c}}{2}-2\,a^{2}\,c^{2}\right)}{4\,a^{4}\,c-a^{3}\,b^{2}}-\frac{\frac{1}{2\,a}-\frac{b\,x}{a^{2}}}{x^{2}}-\frac{\ln(x)\,\left(a\,c-b^{2}\right)}{a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^5*(c + a/x^2 + b/x)),x)
```

```
[Out] (log(2*a*b^4 + 2*b^5*x + 6*a^3*c^2 + 2*a*b^3*(b^2 - 4*a*c)^(1/2) + 2*b^4*x*
(b^2 - 4*a*c)^(1/2) - 9*a^2*b^2*c - 10*a*b^3*c*x - 3*a^2*b*c*(b^2 - 4*a*c)^
(1/2) + 9*a^2*b*c^2*x + 3*a^2*c^2*x*(b^2 - 4*a*c)^(1/2) - 6*a*b^2*c*x*(b^2
- 4*a*c)^(1/2))*(b^4/2 - a*((5*b^2*c)/2 + (3*b*c*(b^2 - 4*a*c)^(1/2))/2) +
(b^3*(b^2 - 4*a*c)^(1/2))/2 + 2*a^2*c^2))/(4*a^4*c - a^3*b^2) - (log(2*a*b^
4 + 2*b^5*x + 6*a^3*c^2 - 2*a*b^3*(b^2 - 4*a*c)^(1/2) - 2*b^4*x*(b^2 - 4*a*
c)^(1/2) - 9*a^2*b^2*c - 10*a*b^3*c*x + 3*a^2*b*c*(b^2 - 4*a*c)^(1/2) + 9*a
^2*b*c^2*x - 3*a^2*c^2*x*(b^2 - 4*a*c)^(1/2) + 6*a*b^2*c*x*(b^2 - 4*a*c)^(1
/2))*(a*((5*b^2*c)/2 - (3*b*c*(b^2 - 4*a*c)^(1/2))/2) - b^4/2 + (b^3*(b^2 -
 4*a*c)^(1/2))/2 - 2*a^2*c^2))/(4*a^4*c - a^3*b^2) - (1/(2*a) - (b*x)/a^2)/
x^2 - (log(x)*(a*c - b^2))/a^3
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x**2+b/x)/x**5,x)
```

```
[Out] Timed out
```

3.360 $\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)x^6}\,dx$

Optimal. Leaf size=137

$$\frac{b\left(b^2-2ac\right)\log\left(a+bx+cx^2\right)}{2a^4}-\frac{b\log(x)\left(b^2-2ac\right)}{a^4}-\frac{b^2-ac}{a^3x}+\frac{b}{2a^2x^2}-\frac{\left(2a^2c^2-4ab^2c+b^4\right)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a^4\sqrt{b^2-4ac}}-$$

Rubi [A] time = 0.19, antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389, Rules used = {1354, 709, 800, 634, 618, 206, 628}

$$-\frac{\left(2a^2c^2-4ab^2c+b^4\right)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a^4\sqrt{b^2-4ac}}+\frac{b\left(b^2-2ac\right)\log\left(a+bx+cx^2\right)}{2a^4}-\frac{b^2-ac}{a^3x}-\frac{b\log(x)\left(b^2-2ac\right)}{a^4}+\frac{b}{2a^2x^2}-\frac{1}{3ax^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c + a/x^2 + b/x)*x^6),x]
```

```
[Out] -1/(3*a*x^3) + b/(2*a^2*x^2) - (b^2 - a*c)/(a^3*x) - ((b^4 - 4*a*b^2*c + 2*
a^2*c^2)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(a^4*Sqrt[b^2 - 4*a*c]) -
(b*(b^2 - 2*a*c)*Log[x])/a^4 + (b*(b^2 - 2*a*c)*Log[a + b*x + c*x^2])/(2*a^
4)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 709

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Simp[(e*(d + e*x)^(m + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dis
t[1/(c*d^2 - b*d*e + a*e^2), Int[((d + e*x)^(m + 1)*Simp[c*d - b*e - c*e*x,
 x])/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 -
 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[m
, -1]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 1354

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> Int[x^(m + 2*n*p)*(c + b/x^n + a/x^(2*n))^p, x] /; FreeQ[{a, b, c, m, n
}, x] && EqQ[n2, 2*n] && ILtQ[p, 0] && NegQ[n]
```

Rubi steps

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)x^6}\,dx=\int \frac{1}{x^4\left(a+bx+cx^2\right)}\,dx$$

$$=-\frac{1}{3ax^3}+\frac{\int \frac{-b-cx}{x^3\left(a+bx+cx^2\right)}\,dx}{a}$$

$$=-\frac{1}{3ax^3}+\frac{\int \left(-\frac{b}{ax^3}+\frac{b^2-ac}{a^2x^2}+\frac{-b^3+2abc}{a^3x}+\frac{b^4-3ab^2c+a^2c^2+bc\left(b^2-2ac\right)x}{a^3\left(a+bx+cx^2\right)}\right)dx}{a}$$

$$=-\frac{1}{3ax^3}+\frac{b}{2a^2x^2}-\frac{b^2-ac}{a^3x}-\frac{b\left(b^2-2ac\right)\log(x)}{a^4}+\frac{\int \frac{b^4-3ab^2c+a^2c^2+bc\left(b^2-2ac\right)x}{a+bx+cx^2}\,dx}{a^4}$$

$$=-\frac{1}{3ax^3}+\frac{b}{2a^2x^2}-\frac{b^2-ac}{a^3x}-\frac{b\left(b^2-2ac\right)\log(x)}{a^4}+\frac{\left(b\left(b^2-2ac\right)\right)\int \frac{b+2cx}{a+bx+cx^2}\,dx}{2a^4}+\frac{\left(b^4-4\ldots\right.}{}$$

$$=-\frac{1}{3ax^3}+\frac{b}{2a^2x^2}-\frac{b^2-ac}{a^3x}-\frac{b\left(b^2-2ac\right)\log(x)}{a^4}+\frac{b\left(b^2-2ac\right)\log\left(a+bx+cx^2\right)}{2a^4}-\frac{\left(b^4\ldots\right.}{}$$

$$=-\frac{1}{3ax^3}+\frac{b}{2a^2x^2}-\frac{b^2-ac}{a^3x}-\frac{\left(b^4-4ab^2c+2a^2c^2\right)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a^4\sqrt{b^2-4ac}}-\frac{b\left(b^2-2ac\right)\log(x)}{a^4}$$

Mathematica [A] time = 0.10, size = 131, normalized size = 0.96

$$\frac{-\frac{2a^3}{x^3}+\frac{6\left(2a^2c^2-4ab^2c+b^4\right)\tan^{-1}\left(\frac{b+2cx}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}+\frac{3a^2b}{x^2}-6\log(x)\left(b^3-2abc\right)+3\left(b^3-2abc\right)\log(a+x(b+cx))+\frac{6a\left(ac-b^2\right)}{x}}{6a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c + a/x^2 + b/x)*x^6),x]
```

```
[Out] ((-2*a^3)/x^3 + (3*a^2*b)/x^2 + (6*a*(-b^2 + a*c))/x + (6*(b^4 - 4*a*b^2*c
+ 2*a^2*c^2)*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c] - 6
*(b^3 - 2*a*b*c)*Log[x] + 3*(b^3 - 2*a*b*c)*Log[a + x*(b + c*x)])/(6*a^4)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)x^6}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/((c + a/x^2 + b/x)*x^6),x]
```

```
[Out] IntegrateAlgebraic[1/((c + a/x^2 + b/x)*x^6), x]
```

fricas [A] time = 1.47, size = 445, normalized size = 3.25

$$\left[\frac{3\left(b^4-4ab^2c+2a^2c^2\right)\sqrt{b^2-4ac}\,x^3\log\left(\frac{2c^2x^2+2bcx+b^2-2ac-\sqrt{b^2-4ac}\,(2cx+b)}{cx^2+bx+a}\right)-2a^3b^2+8a^4c+3\left(b^5-6ab^3c+8a^2bc^2\right)x^3\log\left(cx^2+bx+a\right)-6\left(b^5-6ab^3c+8a^2bc^2\right)x^3\log(x)-6\left(ab^4-5a^2b^2c+4a^3c^2\right)x^2+3\left(a^2b^3-4a^3bc\right)x}{6\left(a^4b^2-4a^5c\right)x^3}, -\frac{6\left(b^4-4ab^2c+2a^2c^2\right)\sqrt{-b^2+4ac}\,x^3\arctan\left(-\frac{\sqrt{-b^2+4ac}\,(2cx+b)}{b^2-4ac}\right)+2a^3b^2-8a^4c-3\left(b^5-6ab^3c+8a^2bc^2\right)x^3\log\left(cx^2+bx+a\right)+6\left(b^5-6ab^3c+8a^2bc^2\right)x^3\log(x)+6\left(ab^4-5a^2b^2c+4a^3c^2\right)x^2-3\left(a^2b^3-4a^3bc\right)x}{6\left(a^4b^2-4a^5c\right)x^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x^2+b/x)/x^6,x, algorithm="fricas")
```

```
[Out] [1/6*(3*(b^4 - 4*a*b^2*c + 2*a^2*c^2)*sqrt(b^2 - 4*a*c)*x^3*log((2*c^2*x^2
+ 2*b*c*x + b^2 - 2*a*c - sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a))
 - 2*a^3*b^2 + 8*a^4*c + 3*(b^5 - 6*a*b^3*c + 8*a^2*b*c^2)*x^3*log(c*x^2 +
b*x + a) - 6*(b^5 - 6*a*b^3*c + 8*a^2*b*c^2)*x^3*log(x) - 6*(a*b^4 - 5*a^2*
b^2*c + 4*a^3*c^2)*x^2 + 3*(a^2*b^3 - 4*a^3*b*c)*x)/((a^4*b^2 - 4*a^5*c)*x^
3), -1/6*(6*(b^4 - 4*a*b^2*c + 2*a^2*c^2)*sqrt(-b^2 + 4*a*c)*x^3*arctan(-sq
rt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) + 2*a^3*b^2 - 8*a^4*c - 3*(b^5
- 6*a*b^3*c + 8*a^2*b*c^2)*x^3*log(c*x^2 + b*x + a) + 6*(b^5 - 6*a*b^3*c +
8*a^2*b*c^2)*x^3*log(x) + 6*(a*b^4 - 5*a^2*b^2*c + 4*a^3*c^2)*x^2 - 3*(a^2*
b^3 - 4*a^3*b*c)*x)/((a^4*b^2 - 4*a^5*c)*x^3)]
```

giac [A] time = 0.38, size = 136, normalized size = 0.99

$$\frac{\left(b^3-2abc\right)\log\left(cx^2+bx+a\right)}{2a^4}-\frac{\left(b^3-2abc\right)\log\left(|x|\right)}{a^4}+\frac{\left(b^4-4ab^2c+2a^2c^2\right)\arctan\left(\frac{2cx+b}{\sqrt{-b^2+4ac}}\right)}{\sqrt{-b^2+4ac}\,a^4}+\frac{3a^2bx-2a^3-6\left(ab^2-a^2c\right)x^2}{6a^4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x^2+b/x)/x^6,x, algorithm="giac")
```

```
[Out] 1/2*(b^3 - 2*a*b*c)*log(c*x^2 + b*x + a)/a^4 - (b^3 - 2*a*b*c)*log(abs(x))/
a^4 + (b^4 - 4*a*b^2*c + 2*a^2*c^2)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/
(sqrt(-b^2 + 4*a*c)*a^4) + 1/6*(3*a^2*b*x - 2*a^3 - 6*(a*b^2 - a^2*c)*x^2)/
(a^4*x^3)
```

maple [A] time = 0.01, size = 214, normalized size = 1.56

$$\frac{2c^2\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}\,a^2}-\frac{4b^2c\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}\,a^3}+\frac{b^4\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}\,a^4}+\frac{2bc\ln(x)}{a^3}-\frac{bc\ln\left(cx^2+bx+a\right)}{a^3}-\frac{b^3\ln(x)}{a^4}+\frac{b^3\ln\left(cx^2+bx+a\right)}{2a^4}+\frac{c}{a^2x}-\frac{b^2}{a^3x}+\frac{b}{2a^2x^2}-\frac{1}{3ax^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c+a/x^2+b/x)/x^6,x)
```

```
[Out] -1/3/a/x^3+1/a^2/x*c-1/a^3/x*b^2+2*b/a^3*ln(x)*c-b^3/a^4*ln(x)+1/2*b/a^2/x^
2-1/a^3*c*ln(c*x^2+b*x+a)*b+1/2/a^4*ln(c*x^2+b*x+a)*b^3+2/a^2/(4*a*c-b^2)^(
1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*c^2-4/a^3/(4*a*c-b^2)^(1/2)*arctan
((2*c*x+b)/(4*a*c-b^2)^(1/2))*b^2*c+1/a^4/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b
)/(4*a*c-b^2)^(1/2))*b^4
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x^2+b/x)/x^6,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

mupad [B] time = 1.92, size = 524, normalized size = 3.82

$$\ln\left(2ab^4\sqrt{b^2-4ac}-2b^6x-2ab^5+2b^5x\sqrt{b^2-4ac}+11a^2b^3c-13a^3bc^2+2a^3c^3x+a^3c^2\sqrt{b^2-4ac}-17a^2b^2c^2x+12ab^4cx-5a^2b^2c\sqrt{b^2-4ac}-8ab^3cx\sqrt{b^2-4ac}+7a^2bc^2x\sqrt{b^2-4ac}\right)\left(\frac{b^3}{2a^4}-\frac{b^2\sqrt{b^2-4ac}}{2a^4}-\frac{bc}{a^3}+\frac{a^2c^2\sqrt{b^2-4ac}}{4a^5c-a^4b^2}\right)+\ln\left(2ab^5+2b^6x+2ab^4\sqrt{b^2-4ac}+2b^5x\sqrt{b^2-4ac}-11a^2b^3c+13a^3bc^2-2a^3c^3x+a^3c^2\sqrt{b^2-4ac}+17a^2b^2c^2x-12ab^4cx-5a^2b^2c\sqrt{b^2-4ac}-8ab^3cx\sqrt{b^2-4ac}+7a^2bc^2x\sqrt{b^2-4ac}\right)\left(\frac{b^3}{2a^4}+\frac{b^2\sqrt{b^2-4ac}}{2a^4}-\frac{bc}{a^3}-\frac{a^2c^2\sqrt{b^2-4ac}}{4a^5c-a^4b^2}\right)+\frac{\frac{x^2\left(ac-b^2\right)}{a^3}-\frac{1}{3a}+\frac{bx}{2a^2}}{x^3}+\frac{b\ln(x)\left(2ac-b^2\right)}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^6*(c + a/x^2 + b/x)),x)
```

```
[Out] log(2*a*b^4*(b^2 - 4*a*c)^(1/2) - 2*b^6*x - 2*a*b^5 + 2*b^5*x*(b^2 - 4*a*c)
^(1/2) + 11*a^2*b^3*c - 13*a^3*b*c^2 + 2*a^3*c^3*x + a^3*c^2*(b^2 - 4*a*c)^
(1/2) - 17*a^2*b^2*c^2*x + 12*a*b^4*c*x - 5*a^2*b^2*c*(b^2 - 4*a*c)^(1/2) -
 8*a*b^3*c*x*(b^2 - 4*a*c)^(1/2) + 7*a^2*b*c^2*x*(b^2 - 4*a*c)^(1/2))*(b^3/
(2*a^4) - (b^2*(b^2 - 4*a*c)^(1/2))/(2*a^4) - (b*c)/a^3 + (a^2*c^2*(b^2 - 4
*a*c)^(1/2))/(4*a^5*c - a^4*b^2)) + log(2*a*b^5 + 2*b^6*x + 2*a*b^4*(b^2 -
4*a*c)^(1/2) + 2*b^5*x*(b^2 - 4*a*c)^(1/2) - 11*a^2*b^3*c + 13*a^3*b*c^2 -
2*a^3*c^3*x + a^3*c^2*(b^2 - 4*a*c)^(1/2) + 17*a^2*b^2*c^2*x - 12*a*b^4*c*x
 - 5*a^2*b^2*c*(b^2 - 4*a*c)^(1/2) - 8*a*b^3*c*x*(b^2 - 4*a*c)^(1/2) + 7*a^
2*b*c^2*x*(b^2 - 4*a*c)^(1/2))*(b^3/(2*a^4) + (b^2*(b^2 - 4*a*c)^(1/2))/(2*
a^4) - (b*c)/a^3 - (a^2*c^2*(b^2 - 4*a*c)^(1/2))/(4*a^5*c - a^4*b^2)) + ((x
^2*(a*c - b^2))/a^3 - 1/(3*a) + (b*x)/(2*a^2))/x^3 + (b*log(x)*(2*a*c - b^2
))/a^4
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x**2+b/x)/x**6,x)
```

```
[Out] Timed out
```

3.361 $\int \frac{x}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2}\,dx$

Optimal. Leaf size=196

$$\frac{b\left(30a^2c^2-20ab^2c+3b^4\right)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^4\left(b^2-4ac\right)^{3/2}}+\frac{\left(3b^2-2ac\right)\log\left(a+bx+cx^2\right)}{2c^4}-\frac{bx\left(3b^2-11ac\right)}{c^3\left(b^2-4ac\right)}+\frac{x^2\left(3b^2-8ac\right)}{2c^2\left(b^2-4ac\right)}$$

Rubi [A] time = 0.20, antiderivative size = 196, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {1354, 738, 800, 634, 618, 206, 628}

$$\frac{b\left(30a^2c^2-20ab^2c+3b^4\right)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^4\left(b^2-4ac\right)^{3/2}}+\frac{x^2\left(3b^2-8ac\right)}{2c^2\left(b^2-4ac\right)}+\frac{\left(3b^2-2ac\right)\log\left(a+bx+cx^2\right)}{2c^4}-\frac{bx\left(3b^2-11ac\right)}{c^3\left(b^2-4ac\right)}+\frac{x^4(2a+bx)}{\left(b^2-4ac\right)\left(a+bx+cx^2\right)}-\frac{bx^3}{c\left(b^2-4ac\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x/(c + a/x^2 + b/x)^2,x]
```

```
[Out] -((b*(3*b^2 - 11*a*c)*x)/(c^3*(b^2 - 4*a*c))) + ((3*b^2 - 8*a*c)*x^2)/(2*c^
2*(b^2 - 4*a*c)) - (b*x^3)/(c*(b^2 - 4*a*c)) + (x^4*(2*a + b*x))/((b^2 - 4*
a*c)*(a + b*x + c*x^2)) + (b*(3*b^4 - 20*a*b^2*c + 30*a^2*c^2)*ArcTanh[(b +
 2*c*x)/Sqrt[b^2 - 4*a*c]])/(c^4*(b^2 - 4*a*c)^(3/2)) + ((3*b^2 - 2*a*c)*Lo
g[a + b*x + c*x^2])/(2*c^4)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 738

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m - 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*(a + b*x
+ c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*
c)), Int[(d + e*x)^(m - 2)*Simp[e*(2*a*e*(m - 1) + b*d*(2*p - m + 4)) - 2*c
*d^2*(2*p + 3) + e*(b*e - 2*d*c)*(m + 2*p + 2)*x, x]*(a + b*x + c*x^2)^(p +
 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^
2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p, -1] && GtQ[m, 1] &&
```

```
IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 1354

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> Int[x^(m + 2*n*p)*(c + b/x^n + a/x^(2*n))^p, x] /; FreeQ[{a, b, c, m, n
}, x] && EqQ[n2, 2*n] && ILtQ[p, 0] && NegQ[n]
```

Rubi steps

$$\int \frac{x}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2}\,dx = \int \frac{x^5}{\left(a+bx+cx^2\right)^2}\,dx$$

$$= \frac{x^4(2a+bx)}{\left(b^2-4ac\right)\left(a+bx+cx^2\right)} + \frac{\int \frac{x^3(8a+3bx)}{a+bx+cx^2}\,dx}{-b^2+4ac}$$

$$= \frac{x^4(2a+bx)}{\left(b^2-4ac\right)\left(a+bx+cx^2\right)} + \frac{\int \left(\frac{b\left(3b^2-11ac\right)}{c^3} - \frac{\left(3b^2-8ac\right)x}{c^2} + \frac{3bx^2}{c} - \frac{ab\left(3b^2-11ac\right)+\left(b^2-4ac\right)\left(3b^2-2ac\right)x}{c^3\left(a+bx+cx^2\right)} \cdots}{-b^2+4ac}$$

$$= -\frac{b\left(3b^2-11ac\right)x}{c^3\left(b^2-4ac\right)} + \frac{\left(3b^2-8ac\right)x^2}{2c^2\left(b^2-4ac\right)} - \frac{bx^3}{c\left(b^2-4ac\right)} + \frac{x^4(2a+bx)}{\left(b^2-4ac\right)\left(a+bx+cx^2\right)} + \int \frac{ab\left(3b^2-\cdots\right.}{\cdots}$$

$$= -\frac{b\left(3b^2-11ac\right)x}{c^3\left(b^2-4ac\right)} + \frac{\left(3b^2-8ac\right)x^2}{2c^2\left(b^2-4ac\right)} - \frac{bx^3}{c\left(b^2-4ac\right)} + \frac{x^4(2a+bx)}{\left(b^2-4ac\right)\left(a+bx+cx^2\right)} + \frac{\left(3b^2-2a\cdots\right.}{\cdots}$$

$$= -\frac{b\left(3b^2-11ac\right)x}{c^3\left(b^2-4ac\right)} + \frac{\left(3b^2-8ac\right)x^2}{2c^2\left(b^2-4ac\right)} - \frac{bx^3}{c\left(b^2-4ac\right)} + \frac{x^4(2a+bx)}{\left(b^2-4ac\right)\left(a+bx+cx^2\right)} + \frac{\left(3b^2-2a\cdots\right.}{\cdots}$$

$$= -\frac{b\left(3b^2-11ac\right)x}{c^3\left(b^2-4ac\right)} + \frac{\left(3b^2-8ac\right)x^2}{2c^2\left(b^2-4ac\right)} - \frac{bx^3}{c\left(b^2-4ac\right)} + \frac{x^4(2a+bx)}{\left(b^2-4ac\right)\left(a+bx+cx^2\right)} + \frac{b\left(3b^4-\cdots\right.}{\cdots}$$

Mathematica [A] time = 0.22, size = 163, normalized size = 0.83

$$\frac{\frac{2b\left(30a^2c^2-20ab^2c+3b^4\right)\tan^{-1}\left(\frac{b+2cx}{\sqrt{4ac-b^2}}\right)}{\left(4ac-b^2\right)^{3/2}} + \frac{2\left(2a^3c^2+a^2bc(5cx-4b)+ab^3(b-5cx)+b^5x\right)}{\left(b^2-4ac\right)(a+x(b+cx))} + \left(3b^2-2ac\right)\log(a+x(b+cx)) - 4bcx + c^2x^2}{2c^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(c + a/x^2 + b/x)^2,x]
```

```
[Out] (-4*b*c*x + c^2*x^2 + (2*(2*a^3*c^2 + b^5*x + a*b^3*(b - 5*c*x) + a^2*b*c*(
-4*b + 5*c*x)))/((b^2 - 4*a*c)*(a + x*(b + c*x))) + (2*b*(3*b^4 - 20*a*b^2*
c + 30*a^2*c^2)*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/(-b^2 + 4*a*c)^(3/2
) + (3*b^2 - 2*a*c)*Log[a + x*(b + c*x)])/(2*c^4)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x/(c + a/x^2 + b/x)^2,x]
[Out] IntegrateAlgebraic[x/(c + a/x^2 + b/x)^2, x]
```

fricas [B] time = 1.33, size = 1029, normalized size = 5.25

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c+a/x^2+b/x)^2,x, algorithm="fricas")
[Out] [1/2*(2*a*b^6 - 16*a^2*b^4*c + 36*a^3*b^2*c^2 - 16*a^4*c^3 + (b^4*c^3 - 8*a
*b^2*c^4 + 16*a^2*c^5)*x^4 - 3*(b^5*c^2 - 8*a*b^3*c^3 + 16*a^2*b*c^4)*x^3 -
 (4*b^6*c - 33*a*b^4*c^2 + 72*a^2*b^2*c^3 - 16*a^3*c^4)*x^2 - (3*a*b^5 - 20
*a^2*b^3*c + 30*a^3*b*c^2 + (3*b^5*c - 20*a*b^3*c^2 + 30*a^2*b*c^3)*x^2 + (
3*b^6 - 20*a*b^4*c + 30*a^2*b^2*c^2)*x)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^2 +
2*b*c*x + b^2 - 2*a*c - sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) +
 2*(b^7 - 11*a*b^5*c + 41*a^2*b^3*c^2 - 52*a^3*b*c^3)*x + (3*a*b^6 - 26*a^2
*b^4*c + 64*a^3*b^2*c^2 - 32*a^4*c^3 + (3*b^6*c - 26*a*b^4*c^2 + 64*a^2*b^2
*c^3 - 32*a^3*c^4)*x^2 + (3*b^7 - 26*a*b^5*c + 64*a^2*b^3*c^2 - 32*a^3*b*c^
3)*x)*log(c*x^2 + b*x + a))/(a*b^4*c^4 - 8*a^2*b^2*c^5 + 16*a^3*c^6 + (b^4*
c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)*x^2 + (b^5*c^4 - 8*a*b^3*c^5 + 16*a^2*b*c^6
)*x), 1/2*(2*a*b^6 - 16*a^2*b^4*c + 36*a^3*b^2*c^2 - 16*a^4*c^3 + (b^4*c^3
- 8*a*b^2*c^4 + 16*a^2*c^5)*x^4 - 3*(b^5*c^2 - 8*a*b^3*c^3 + 16*a^2*b*c^4)*
x^3 - (4*b^6*c - 33*a*b^4*c^2 + 72*a^2*b^2*c^3 - 16*a^3*c^4)*x^2 + 2*(3*a*b
^5 - 20*a^2*b^3*c + 30*a^3*b*c^2 + (3*b^5*c - 20*a*b^3*c^2 + 30*a^2*b*c^3)*
x^2 + (3*b^6 - 20*a*b^4*c + 30*a^2*b^2*c^2)*x)*sqrt(-b^2 + 4*a*c)*arctan(-s
qrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) + 2*(b^7 - 11*a*b^5*c + 41*a^2
*b^3*c^2 - 52*a^3*b*c^3)*x + (3*a*b^6 - 26*a^2*b^4*c + 64*a^3*b^2*c^2 - 32*
a^4*c^3 + (3*b^6*c - 26*a*b^4*c^2 + 64*a^2*b^2*c^3 - 32*a^3*c^4)*x^2 + (3*b
^7 - 26*a*b^5*c + 64*a^2*b^3*c^2 - 32*a^3*b*c^3)*x)*log(c*x^2 + b*x + a))/(
a*b^4*c^4 - 8*a^2*b^2*c^5 + 16*a^3*c^6 + (b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^
7)*x^2 + (b^5*c^4 - 8*a*b^3*c^5 + 16*a^2*b*c^6)*x)]
```

giac [A] time = 0.33, size = 188, normalized size = 0.96

$$-\frac{\left(3\,b^{5}-20\,ab^{3}c+30\,a^{2}bc^{2}\right)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^{2}+4\,ac}}\right)}{\left(b^{2}c^{4}-4\,ac^{5}\right)\sqrt{-b^{2}+4\,ac}}+\frac{\left(3\,b^{2}-2\,ac\right)\log\left(cx^{2}+bx+a\right)}{2\,c^{4}}+\frac{c^{2}x^{2}-4\,bcx}{2\,c^{4}}+\frac{ab^{4}-4\,a^{2}b^{2}c+2\,a^{3}c^{2}+\left(b^{5}-5\,ab^{3}c+5\,a^{2}bc^{2}\right)x}{\left(cx^{2}+bx+a\right)\left(b^{2}-4\,ac\right)c^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c+a/x^2+b/x)^2,x, algorithm="giac")
[Out] -(3*b^5 - 20*a*b^3*c + 30*a^2*b*c^2)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))
/((b^2*c^4 - 4*a*c^5)*sqrt(-b^2 + 4*a*c)) + 1/2*(3*b^2 - 2*a*c)*log(c*x^2 +
 b*x + a)/c^4 + 1/2*(c^2*x^2 - 4*b*c*x)/c^4 + (a*b^4 - 4*a^2*b^2*c + 2*a^3*
c^2 + (b^5 - 5*a*b^3*c + 5*a^2*b*c^2)*x)/((c*x^2 + b*x + a)*(b^2 - 4*a*c)*c
^4)
```

maple [B] time = 0.01, size = 434, normalized size = 2.21

$$-\frac{5a^{2}bx}{\left(cx^{2}+bx+a\right)\left(4ac-b^{2}\right)c^{2}}+\frac{30a^{2}b\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^{2}}}\right)}{\left(4ac-b^{2}\right)^{\frac{3}{2}}c^{2}}+\frac{5ab^{3}x}{\left(cx^{2}+bx+a\right)\left(4ac-b^{2}\right)c^{3}}-\frac{20ab^{3}\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^{2}}}\right)}{\left(4ac-b^{2}\right)^{\frac{3}{2}}c^{3}}-\frac{b^{5}x}{\left(cx^{2}+bx+a\right)\left(4ac-b^{2}\right)c^{4}}+\frac{3b^{5}\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^{2}}}\right)}{\left(4ac-b^{2}\right)^{\frac{3}{2}}c^{4}}-\frac{2a^{3}}{\left(cx^{2}+bx+a\right)\left(4ac-b^{2}\right)c^{2}}+\frac{4a^{2}b^{2}}{\left(cx^{2}+bx+a\right)\left(4ac-b^{2}\right)c^{3}}-\frac{4a^{2}\ln\left(cx^{2}+bx+a\right)}{\left(4ac-b^{2}\right)c^{2}}-\frac{ab^{4}}{\left(cx^{2}+bx+a\right)\left(4ac-b^{2}\right)c^{4}}+\frac{7ab^{2}\ln\left(cx^{2}+bx+a\right)}{\left(4ac-b^{2}\right)c^{3}}-\frac{3b^{4}\ln\left(cx^{2}+bx+a\right)}{2\left(4ac-b^{2}\right)c^{4}}+\frac{x^{2}}{2c^{2}}-\frac{2bx}{c^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(c+a/x^2+b/x)^2,x)
```

```
[Out] 1/2/c^2*x^2-2/c^3*b*x-5/c^2/(c*x^2+b*x+a)/(4*a*c-b^2)*b*x*a^2+5/c^3/(c*x^2+
b*x+a)/(4*a*c-b^2)*b^3*x*a-1/c^4/(c*x^2+b*x+a)/(4*a*c-b^2)*b^5*x-2/c^2/(c*x
^2+b*x+a)*a^3/(4*a*c-b^2)+4/c^3/(c*x^2+b*x+a)*a^2/(4*a*c-b^2)*b^2-1/c^4/(c*
x^2+b*x+a)*a/(4*a*c-b^2)*b^4-4/c^2/(4*a*c-b^2)*ln(c*x^2+b*x+a)*a^2+7/c^3/(4
*a*c-b^2)*ln(c*x^2+b*x+a)*a*b^2-3/2/c^4/(4*a*c-b^2)*ln(c*x^2+b*x+a)*b^4+30/
c^2/(4*a*c-b^2)^(3/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*a^2*b-20/c^3/(4*a
*c-b^2)^(3/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*a*b^3+3/c^4/(4*a*c-b^2)^(
3/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*b^5
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c+a/x^2+b/x)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

mupad [B] time = 1.82, size = 382, normalized size = 1.95

$$\frac{x^2}{2c^2}-\frac{\frac{a\left(2a^2c^2-4ab^2c+b^4\right)}{c\left(4ac-b^2\right)}+\frac{bx\left(5a^2c^2-5ab^2c+b^4\right)}{c\left(4ac-b^2\right)}}{c^4x^2+bc^3x+ac^3}-\frac{\ln\left(cx^2+bx+a\right)\left(128a^4c^4-288a^3b^2c^3+168a^2b^4c^2-38ab^6c+3b^8\right)}{2\left(64a^3c^7-48a^2b^2c^6+12ab^4c^5-b^6c^4\right)}-\frac{2bx}{c^3}+\frac{b\,\mathrm{atan}\left(\frac{c^4\left(\frac{2bx\left(30a^2c^2-20ab^2c+3b^4\right)}{c^3\left(4ac-b^2\right)^3}-\frac{b\left(b^3c^3-4abc^4\right)\left(30a^2c^2-20ab^2c+3b^4\right)}{c^7\left(4ac-b^2\right)^4}\right)\left(4ac-b^2\right)^{5/2}}{30a^2bc^2-20ab^3c+3b^5}\right)\left(30a^2c^2-20ab^2c+3b^4\right)}{c^4\left(4ac-b^2\right)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(c + a/x^2 + b/x)^2,x)
```

```
[Out] x^2/(2*c^2) - ((a*(b^4 + 2*a^2*c^2 - 4*a*b^2*c))/(c*(4*a*c - b^2)) + (b*x*(
b^4 + 5*a^2*c^2 - 5*a*b^2*c))/(c*(4*a*c - b^2)))/(a*c^3 + c^4*x^2 + b*c^3*x
) - (log(a + b*x + c*x^2)*(3*b^8 + 128*a^4*c^4 + 168*a^2*b^4*c^2 - 288*a^3*
b^2*c^3 - 38*a*b^6*c))/(2*(64*a^3*c^7 - b^6*c^4 + 12*a*b^4*c^5 - 48*a^2*b^2
*c^6)) - (2*b*x)/c^3 + (b*atan((c^4*((2*b*x*(3*b^4 + 30*a^2*c^2 - 20*a*b^2*
c))/(c^3*(4*a*c - b^2)^3) - (b*(b^3*c^3 - 4*a*b*c^4)*(3*b^4 + 30*a^2*c^2 -
20*a*b^2*c))/(c^7*(4*a*c - b^2)^4))*(4*a*c - b^2)^(5/2))/(3*b^5 + 30*a^2*b*
c^2 - 20*a*b^3*c))*(3*b^4 + 30*a^2*c^2 - 20*a*b^2*c))/(c^4*(4*a*c - b^2)^(3
/2))
```

sympy [B] time = 2.37, size = 1012, normalized size = 5.16

$$-\frac{2bx}{c^3}+\left(-\frac{b\sqrt{-\left(4ac-b^2\right)^3}\left(30a^2c^2-20ab^2c+3b^4\right)}{2c^4\left(64a^3c^3-48a^2b^2c^2+12ab^4c-b^6\right)}-\frac{2ac-3b^2}{2c^4}\right)\log\left(x+\frac{16a^3c^2-17a^2b^2c+16a^2c^5\left(-\frac{b\sqrt{-(4ac-b^2)^3}(30a^2c^2-20ab^2c+3b^4)}{2c^4(64a^3c^3-48a^2b^2c^2+12ab^4c-b^6)}-\frac{2ac-3b^2}{2c^4}\right)+3ab^4-8ab^2c^4\left(-\frac{b\sqrt{-(4ac-b^2)^3}(30a^2c^2-20ab^2c+3b^4)}{2c^4(64a^3c^3-48a^2b^2c^2+12ab^4c-b^6)}-\frac{2ac-3b^2}{2c^4}\right)+b^4c^3\left(-\frac{b\sqrt{-(4ac-b^2)^3}(30a^2c^2-20ab^2c+3b^4)}{2c^4(64a^3c^3-48a^2b^2c^2+12ab^4c-b^6)}-\frac{2ac-3b^2}{2c^4}\right)}{30a^2bc^2-20ab^3c+3b^5}\right)+\left(\frac{b\sqrt{-\left(4ac-b^2\right)^3}\left(30a^2c^2-20ab^2c+3b^4\right)}{2c^4\left(64a^3c^3-48a^2b^2c^2+12ab^4c-b^6\right)}-\frac{2ac-3b^2}{2c^4}\right)\log\left(x+\frac{16a^3c^2-17a^2b^2c+16a^2c^5\left(\frac{b\sqrt{-(4ac-b^2)^3}(30a^2c^2-20ab^2c+3b^4)}{2c^4(64a^3c^3-48a^2b^2c^2+12ab^4c-b^6)}-\frac{2ac-3b^2}{2c^4}\right)+3ab^4-8ab^2c^4\left(\frac{b\sqrt{-(4ac-b^2)^3}(30a^2c^2-20ab^2c+3b^4)}{2c^4(64a^3c^3-48a^2b^2c^2+12ab^4c-b^6)}-\frac{2ac-3b^2}{2c^4}\right)+b^4c^3\left(\frac{b\sqrt{-(4ac-b^2)^3}(30a^2c^2-20ab^2c+3b^4)}{2c^4(64a^3c^3-48a^2b^2c^2+12ab^4c-b^6)}-\frac{2ac-3b^2}{2c^4}\right)}{30a^2bc^2-20ab^3c+3b^5}\right)+\frac{-2a^3c^2+4a^2b^2c-ab^4+x\left(-5a^2bc^2+5ab^3c-b^5\right)}{4a^2c^5-ab^2c^4+x^2\left(4ac^6-b^2c^5\right)+x\left(4abc^5-b^3c^4\right)}+\frac{x^2}{2c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c+a/x**2+b/x)**2,x)
```

```
[Out] -2*b*x/c**3 + (-b*sqrt(-(4*a*c - b**2)**3)*(30*a**2*c**2 - 20*a*b**2*c + 3*
b**4)/(2*c**4*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)) - (2
*a*c - 3*b**2)/(2*c**4))*log(x + (16*a**3*c**2 - 17*a**2*b**2*c + 16*a**2*c
**5*(-b*sqrt(-(4*a*c - b**2)**3)*(30*a**2*c**2 - 20*a*b**2*c + 3*b**4)/(2*c
**4*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)) - (2*a*c - 3*b
**2)/(2*c**4)) + 3*a*b**4 - 8*a*b**2*c**4*(-b*sqrt(-(4*a*c - b**2)**3)*(30*
a**2*c**2 - 20*a*b**2*c + 3*b**4)/(2*c**4*(64*a**3*c**3 - 48*a**2*b**2*c**2
 + 12*a*b**4*c - b**6)) - (2*a*c - 3*b**2)/(2*c**4)) + b**4*c**3*(-b*sqrt(-
(4*a*c - b**2)**3)*(30*a**2*c**2 - 20*a*b**2*c + 3*b**4)/(2*c**4*(64*a**3*c
```

```
**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)) - (2*a*c - 3*b**2)/(2*c**4))
)/(30*a**2*b*c**2 - 20*a*b**3*c + 3*b**5)) + (b*sqrt(-(4*a*c - b**2)**3)*(3
0*a**2*c**2 - 20*a*b**2*c + 3*b**4)/(2*c**4*(64*a**3*c**3 - 48*a**2*b**2*c*
*2 + 12*a*b**4*c - b**6)) - (2*a*c - 3*b**2)/(2*c**4))*log(x + (16*a**3*c**
2 - 17*a**2*b**2*c + 16*a**2*c**5*(b*sqrt(-(4*a*c - b**2)**3)*(30*a**2*c**2
 - 20*a*b**2*c + 3*b**4)/(2*c**4*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b
**4*c - b**6)) - (2*a*c - 3*b**2)/(2*c**4)) + 3*a*b**4 - 8*a*b**2*c**4*(b*s
qrt(-(4*a*c - b**2)**3)*(30*a**2*c**2 - 20*a*b**2*c + 3*b**4)/(2*c**4*(64*a
**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)) - (2*a*c - 3*b**2)/(2*c
**4)) + b**4*c**3*(b*sqrt(-(4*a*c - b**2)**3)*(30*a**2*c**2 - 20*a*b**2*c +
 3*b**4)/(2*c**4*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)) -
 (2*a*c - 3*b**2)/(2*c**4)))/(30*a**2*b*c**2 - 20*a*b**3*c + 3*b**5)) + (-2
*a**3*c**2 + 4*a**2*b**2*c - a*b**4 + x*(-5*a**2*b*c**2 + 5*a*b**3*c - b**5
))/(4*a**2*c**5 - a*b**2*c**4 + x**2*(4*a*c**6 - b**2*c**5) + x*(4*a*b*c**5
 - b**3*c**4)) + x**2/(2*c**2)
```

3.362 $\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2}\,dx$

Optimal. Leaf size=150

$$-\frac{2\left(6a^2c^2-6ab^2c+b^4\right)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^3\left(b^2-4ac\right)^{3/2}}+\frac{2x\left(b^2-3ac\right)}{c^2\left(b^2-4ac\right)}-\frac{bx^2}{c\left(b^2-4ac\right)}+\frac{x^3(2a+bx)}{\left(b^2-4ac\right)\left(a+bx+cx^2\right)}-\frac{b\log\left(a+bx\right.}{c^3}$$

Rubi [A] time = 0.15, antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {1340, 738, 800, 634, 618, 206, 628}

$$-\frac{2\left(6a^2c^2-6ab^2c+b^4\right)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^3\left(b^2-4ac\right)^{3/2}}+\frac{2x\left(b^2-3ac\right)}{c^2\left(b^2-4ac\right)}+\frac{x^3(2a+bx)}{\left(b^2-4ac\right)\left(a+bx+cx^2\right)}-\frac{bx^2}{c\left(b^2-4ac\right)}-\frac{b\log\left(a+bx+cx^2\right)}{c^3}$$

Antiderivative was successfully verified.

```
[In] Int[(c + a/x^2 + b/x)^(-2),x]
```

```
[Out] (2*(b^2 - 3*a*c)*x)/(c^2*(b^2 - 4*a*c)) - (b*x^2)/(c*(b^2 - 4*a*c)) + (x^3*
(2*a + b*x))/((b^2 - 4*a*c)*(a + b*x + c*x^2)) - (2*(b^4 - 6*a*b^2*c + 6*a^
2*c^2)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(c^3*(b^2 - 4*a*c)^(3/2)) -
(b*Log[a + b*x + c*x^2])/c^3
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 738

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m - 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*(a + b*x
+ c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*
c)), Int[(d + e*x)^(m - 2)*Simp[e*(2*a*e*(m - 1) + b*d*(2*p - m + 4)) - 2*c
*d^2*(2*p + 3) + e*(b*e - 2*d*c)*(m + 2*p + 2)*x, x]*(a + b*x + c*x^2)^(p +
 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^
2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p, -1] && GtQ[m, 1] &&
```

```
IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 1340

```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[x^
(2*n*p)*(c + b/x^n + a/x^(2*n))^p, x] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n
] && LtQ[n, 0] && IntegerQ[p]
```

Rubi steps

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2}\,dx = \int \frac{x^4}{\left(a+bx+cx^2\right)^2}\,dx$$

$$= \frac{x^3(2a+bx)}{\left(b^2-4ac\right)\left(a+bx+cx^2\right)} + \frac{\int \frac{x^2(6a+2bx)}{a+bx+cx^2}\,dx}{-b^2+4ac}$$

$$= \frac{x^3(2a+bx)}{\left(b^2-4ac\right)\left(a+bx+cx^2\right)} + \frac{\int \left(-\frac{2\left(b^2-3ac\right)}{c^2}+\frac{2bx}{c}+\frac{2\left(a\left(b^2-3ac\right)+b\left(b^2-4ac\right)x\right)}{c^2\left(a+bx+cx^2\right)}\right)dx}{-b^2+4ac}$$

$$= \frac{2\left(b^2-3ac\right)x}{c^2\left(b^2-4ac\right)} - \frac{bx^2}{c\left(b^2-4ac\right)} + \frac{x^3(2a+bx)}{\left(b^2-4ac\right)\left(a+bx+cx^2\right)} - \frac{2\int \frac{a\left(b^2-3ac\right)+b\left(b^2-4ac\right)x}{a+bx+cx^2}\,dx}{c^2\left(b^2-4ac\right)}$$

$$= \frac{2\left(b^2-3ac\right)x}{c^2\left(b^2-4ac\right)} - \frac{bx^2}{c\left(b^2-4ac\right)} + \frac{x^3(2a+bx)}{\left(b^2-4ac\right)\left(a+bx+cx^2\right)} - \frac{b\int \frac{b+2cx}{a+bx+cx^2}\,dx}{c^3} + \left(b^4-6ab\ldots\right.$$

$$= \frac{2\left(b^2-3ac\right)x}{c^2\left(b^2-4ac\right)} - \frac{bx^2}{c\left(b^2-4ac\right)} + \frac{x^3(2a+bx)}{\left(b^2-4ac\right)\left(a+bx+cx^2\right)} - \frac{b\log\left(a+bx+cx^2\right)}{c^3} - (2(\ldots$$

$$= \frac{2\left(b^2-3ac\right)x}{c^2\left(b^2-4ac\right)} - \frac{bx^2}{c\left(b^2-4ac\right)} + \frac{x^3(2a+bx)}{\left(b^2-4ac\right)\left(a+bx+cx^2\right)} - \frac{2\left(b^4-6ab^2c+6a^2c^2\right)\tanh\ldots}{c^3\left(b^2-4ac\right)^{3/2}}$$

Mathematica [A] time = 0.18, size = 132, normalized size = 0.88

$$\frac{-\frac{2\left(6a^2c^2-6ab^2c+b^4\right)\tan^{-1}\left(\frac{b+2cx}{\sqrt{4ac-b^2}}\right)}{\left(4ac-b^2\right)^{3/2}}+\frac{a^2c(3b-2cx)-ab^2(b-4cx)+b^4(-x)}{\left(b^2-4ac\right)(a+x(b+cx))}-b\log(a+x(b+cx))+cx}{c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + a/x^2 + b/x)^(-2),x]
```

```
[Out] (c*x + (-(b^4*x) - a*b^2*(b - 4*c*x) + a^2*c*(3*b - 2*c*x))/((b^2 - 4*a*c)*
(a + x*(b + c*x))) - (2*(b^4 - 6*a*b^2*c + 6*a^2*c^2)*ArcTan[(b + 2*c*x)/Sq
rt[-b^2 + 4*a*c]])/(-b^2 + 4*a*c)^(3/2) - b*Log[a + x*(b + c*x)])/c^3
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + a/x^2 + b/x)^(-2),x]
```

```
[Out] IntegrateAlgebraic[(c + a/x^2 + b/x)^(-2), x]
```

fricas [B] time = 1.62, size = 837, normalized size = 5.58

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x^2+b/x)^2,x, algorithm="fricas")
```

```
[Out] [-(a*b^5 - 7*a^2*b^3*c + 12*a^3*b*c^2 - (b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4
)*x^3 - (b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*x^2 + (a*b^4 - 6*a^2*b^2*c + 6
*a^3*c^2 + (b^4*c - 6*a*b^2*c^2 + 6*a^2*c^3)*x^2 + (b^5 - 6*a*b^3*c + 6*a^2
*b*c^2)*x)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c + sqrt(
b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) + (b^6 - 9*a*b^4*c + 26*a^2*b^
2*c^2 - 24*a^3*c^3)*x + (a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2 + (b^5*c - 8*a*
b^3*c^2 + 16*a^2*b*c^3)*x^2 + (b^6 - 8*a*b^4*c + 16*a^2*b^2*c^2)*x)*log(c*x
^2 + b*x + a))/(a*b^4*c^3 - 8*a^2*b^2*c^4 + 16*a^3*c^5 + (b^4*c^4 - 8*a*b^2
*c^5 + 16*a^2*c^6)*x^2 + (b^5*c^3 - 8*a*b^3*c^4 + 16*a^2*b*c^5)*x), -(a*b^5
 - 7*a^2*b^3*c + 12*a^3*b*c^2 - (b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*x^3 -
(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*x^2 + 2*(a*b^4 - 6*a^2*b^2*c + 6*a^3*c
^2 + (b^4*c - 6*a*b^2*c^2 + 6*a^2*c^3)*x^2 + (b^5 - 6*a*b^3*c + 6*a^2*b*c^2
)*x)*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c
)) + (b^6 - 9*a*b^4*c + 26*a^2*b^2*c^2 - 24*a^3*c^3)*x + (a*b^5 - 8*a^2*b^3
*c + 16*a^3*b*c^2 + (b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*x^2 + (b^6 - 8*a*b
^4*c + 16*a^2*b^2*c^2)*x)*log(c*x^2 + b*x + a))/(a*b^4*c^3 - 8*a^2*b^2*c^4
+ 16*a^3*c^5 + (b^4*c^4 - 8*a*b^2*c^5 + 16*a^2*c^6)*x^2 + (b^5*c^3 - 8*a*b^
3*c^4 + 16*a^2*b*c^5)*x)]
```

giac [A] time = 0.37, size = 161, normalized size = 1.07

$$\frac{2\left(b^4-6\,ab^2c+6\,a^2c^2\right)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{\left(b^2c^3-4\,ac^4\right)\sqrt{-b^2+4\,ac}}+\frac{x}{c^2}-\frac{b\log\left(cx^2+bx+a\right)}{c^3}-\frac{\frac{\left(b^4-4\,ab^2c+2\,a^2c^2\right)x}{c}+\frac{ab^3-3\,a^2bc}{c}}{\left(cx^2+bx+a\right)\left(b^2-4\,ac\right)c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x^2+b/x)^2,x, algorithm="giac")
```

```
[Out] 2*(b^4 - 6*a*b^2*c + 6*a^2*c^2)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/((b^
2*c^3 - 4*a*c^4)*sqrt(-b^2 + 4*a*c)) + x/c^2 - b*log(c*x^2 + b*x + a)/c^3 -
 ((b^4 - 4*a*b^2*c + 2*a^2*c^2)*x/c + (a*b^3 - 3*a^2*b*c)/c)/((c*x^2 + b*x
+ a)*(b^2 - 4*a*c)*c^2)
```

maple [B] time = 0.01, size = 352, normalized size = 2.35

$$\frac{2a^2x}{\left(cx^2+bx+a\right)\left(4ac-b^2\right)c}-\frac{12a^2\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{\left(4ac-b^2\right)^{\frac{3}{2}}c}-\frac{4ab^2x}{\left(cx^2+bx+a\right)\left(4ac-b^2\right)c^2}+\frac{12ab^2\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{\left(4ac-b^2\right)^{\frac{3}{2}}c^2}+\frac{b^4x}{\left(cx^2+bx+a\right)\left(4ac-b^2\right)c^3}-\frac{2b^4\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{\left(4ac-b^2\right)^{\frac{3}{2}}c^3}-\frac{3a^2b}{\left(cx^2+bx+a\right)\left(4ac-b^2\right)c^2}+\frac{ab^3}{\left(cx^2+bx+a\right)\left(4ac-b^2\right)c^3}-\frac{4ab\ln\left(cx^2+bx+a\right)}{\left(4ac-b^2\right)c^2}+\frac{b^3\ln\left(cx^2+bx+a\right)}{\left(4ac-b^2\right)c^3}+\frac{x}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c+a/x^2+b/x)^2,x)
```

```
[Out] x/c^2+2/c/(c*x^2+b*x+a)/(4*a*c-b^2)*x*a^2-4/c^2/(c*x^2+b*x+a)/(4*a*c-b^2)*x
*a*b^2+1/c^3/(c*x^2+b*x+a)/(4*a*c-b^2)*x*b^4-3/c^2/(c*x^2+b*x+a)*a^2*b/(4*a
*c-b^2)+1/c^3/(c*x^2+b*x+a)*a*b^3/(4*a*c-b^2)-4/c^2/(4*a*c-b^2)*ln(c*x^2+b*
x+a)*a*b+1/c^3/(4*a*c-b^2)*ln(c*x^2+b*x+a)*b^3-12/c/(4*a*c-b^2)^(3/2)*arcta
n((2*c*x+b)/(4*a*c-b^2)^(1/2))*a^2+12/c^2/(4*a*c-b^2)^(3/2)*arctan((2*c*x+b
)/(4*a*c-b^2)^(1/2))*a*b^2-2/c^3/(4*a*c-b^2)^(3/2)*arctan((2*c*x+b)/(4*a*c-
b^2)^(1/2))*b^4
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x^2+b/x)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

mupad [B] time = 1.80, size = 261, normalized size = 1.74

$$\frac{x}{c^2}+\frac{\frac{a\left(b^3-3abc\right)}{c\left(4ac-b^2\right)}+\frac{x\left(2a^2c^2-4ab^2c+b^4\right)}{c\left(4ac-b^2\right)}}{c^3x^2+bc^2x+ac^2}+\frac{\ln\left(cx^2+bx+a\right)\left(-128a^3bc^3+96a^2b^3c^2-24ab^5c+2b^7\right)}{2\left(64a^3c^6-48a^2b^2c^5+12ab^4c^4-b^6c^3\right)}-\frac{2\,\mathrm{atan}\left(\frac{2cx}{\sqrt{4ac-b^2}}-\frac{b^3c^2-4abc^3}{c^2\left(4ac-b^2\right)^{3/2}}\right)\left(6a^2c^2-6ab^2c+b^4\right)}{c^3\left(4ac-b^2\right)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c + a/x^2 + b/x)^2,x)
```

```
[Out] x/c^2 + ((a*(b^3 - 3*a*b*c))/(c*(4*a*c - b^2)) + (x*(b^4 + 2*a^2*c^2 - 4*a*
b^2*c))/(c*(4*a*c - b^2)))/(a*c^2 + c^3*x^2 + b*c^2*x) + (log(a + b*x + c*x
^2)*(2*b^7 - 128*a^3*b*c^3 + 96*a^2*b^3*c^2 - 24*a*b^5*c))/(2*(64*a^3*c^6 -
 b^6*c^3 + 12*a*b^4*c^4 - 48*a^2*b^2*c^5)) - (2*atan((2*c*x)/(4*a*c - b^2)^
(1/2) - (b^3*c^2 - 4*a*b*c^3)/(c^2*(4*a*c - b^2)^(3/2)))*(b^4 + 6*a^2*c^2 -
 6*a*b^2*c))/(c^3*(4*a*c - b^2)^(3/2))
```

sympy [B] time = 1.73, size = 842, normalized size = 5.61

$$\left(-\frac{b}{c^3}-\frac{\sqrt{-\left(4ac-b^2\right)^3}\left(6a^2c^2-6ab^2c+b^4\right)}{c^3\left(64a^3c^3-48a^2b^2c^2+12ab^4c-b^6\right)}\right)\log\left(x+\frac{-10a^2bc-16a^2c^4\left(-\frac{b}{c^3}-\frac{\sqrt{-\left(4ac-b^2\right)^3}\left(6a^2c^2-6ab^2c+b^4\right)}{c^3\left(64a^3c^3-48a^2b^2c^2+12ab^4c-b^6\right)}\right)+2ab^3+8ab^2c^3\left(-\frac{b}{c^3}-\frac{\sqrt{-\left(4ac-b^2\right)^3}\left(6a^2c^2-6ab^2c+b^4\right)}{c^3\left(64a^3c^3-48a^2b^2c^2+12ab^4c-b^6\right)}\right)-b^4c^2\left(-\frac{b}{c^3}-\frac{\sqrt{-\left(4ac-b^2\right)^3}\left(6a^2c^2-6ab^2c+b^4\right)}{c^3\left(64a^3c^3-48a^2b^2c^2+12ab^4c-b^6\right)}\right)}{12a^2c^2-12ab^2c+2b^4}\right)+\left(-\frac{b}{c^3}+\frac{\sqrt{-\left(4ac-b^2\right)^3}\left(6a^2c^2-6ab^2c+b^4\right)}{c^3\left(64a^3c^3-48a^2b^2c^2+12ab^4c-b^6\right)}\right)\log\left(x+\frac{-10a^2bc-16a^2c^4\left(-\frac{b}{c^3}+\frac{\sqrt{-\left(4ac-b^2\right)^3}\left(6a^2c^2-6ab^2c+b^4\right)}{c^3\left(64a^3c^3-48a^2b^2c^2+12ab^4c-b^6\right)}\right)+2ab^3+8ab^2c^3\left(-\frac{b}{c^3}+\frac{\sqrt{-\left(4ac-b^2\right)^3}\left(6a^2c^2-6ab^2c+b^4\right)}{c^3\left(64a^3c^3-48a^2b^2c^2+12ab^4c-b^6\right)}\right)-b^4c^2\left(-\frac{b}{c^3}+\frac{\sqrt{-\left(4ac-b^2\right)^3}\left(6a^2c^2-6ab^2c+b^4\right)}{c^3\left(64a^3c^3-48a^2b^2c^2+12ab^4c-b^6\right)}\right)}{12a^2c^2-12ab^2c+2b^4}\right)+\frac{-3a^2bc+ab^3+x\left(2a^2c^2-4ab^2c+b^4\right)}{4a^2c^4-ab^2c^3+x^2\left(4ac^5-b^2c^4\right)+x\left(4abc^4-b^3c^3\right)}+\frac{x}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x**2+b/x)**2,x)
```

```
[Out] (-b/c**3 - sqrt(-(4*a*c - b**2)**3)*(6*a**2*c**2 - 6*a*b**2*c + b**4)/(c**3
*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)))*log(x + (-10*a**
2*b*c - 16*a**2*c**4*(-b/c**3 - sqrt(-(4*a*c - b**2)**3)*(6*a**2*c**2 - 6*a
*b**2*c + b**4)/(c**3*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**
6))) + 2*a*b**3 + 8*a*b**2*c**3*(-b/c**3 - sqrt(-(4*a*c - b**2)**3)*(6*a**2
*c**2 - 6*a*b**2*c + b**4)/(c**3*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b
**4*c - b**6))) - b**4*c**2*(-b/c**3 - sqrt(-(4*a*c - b**2)**3)*(6*a**2*c**
2 - 6*a*b**2*c + b**4)/(c**3*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*
c - b**6))))/(12*a**2*c**2 - 12*a*b**2*c + 2*b**4)) + (-b/c**3 + sqrt(-(4*a
*c - b**2)**3)*(6*a**2*c**2 - 6*a*b**2*c + b**4)/(c**3*(64*a**3*c**3 - 48*a
**2*b**2*c**2 + 12*a*b**4*c - b**6)))*log(x + (-10*a**2*b*c - 16*a**2*c**4*
(-b/c**3 + sqrt(-(4*a*c - b**2)**3)*(6*a**2*c**2 - 6*a*b**2*c + b**4)/(c**3
*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6))) + 2*a*b**3 + 8*a
*b**2*c**3*(-b/c**3 + sqrt(-(4*a*c - b**2)**3)*(6*a**2*c**2 - 6*a*b**2*c +
b**4)/(c**3*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6))) - b**
4*c**2*(-b/c**3 + sqrt(-(4*a*c - b**2)**3)*(6*a**2*c**2 - 6*a*b**2*c + b**4
```

```
)/(c**3*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6))))/(12*a**2
*c**2 - 12*a*b**2*c + 2*b**4)) + (-3*a**2*b*c + a*b**3 + x*(2*a**2*c**2 - 4
*a*b**2*c + b**4))/(4*a**2*c**4 - a*b**2*c**3 + x**2*(4*a*c**5 - b**2*c**4)
 + x*(4*a*b*c**4 - b**3*c**3)) + x/c**2
```

3.363 $\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2 x} \, dx$

Optimal. Leaf size=114

$$\frac{b\left(b^2-6ac\right)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^2\left(b^2-4ac\right)^{3/2}}+\frac{x^2(2a+bx)}{\left(b^2-4ac\right)\left(a+bx+cx^2\right)}-\frac{bx}{c\left(b^2-4ac\right)}+\frac{\log\left(a+bx+cx^2\right)}{2c^2}$$

Rubi [A] time = 0.10, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389, Rules used = {1354, 738, 773, 634, 618, 206, 628}

$$\frac{b\left(b^2-6ac\right)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^2\left(b^2-4ac\right)^{3/2}}+\frac{x^2(2a+bx)}{\left(b^2-4ac\right)\left(a+bx+cx^2\right)}-\frac{bx}{c\left(b^2-4ac\right)}+\frac{\log\left(a+bx+cx^2\right)}{2c^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c + a/x^2 + b/x)^2*x),x]
```

```
[Out] -((b*x)/(c*(b^2 - 4*a*c))) + (x^2*(2*a + b*x))/((b^2 - 4*a*c)*(a + b*x + c*
x^2)) + (b*(b^2 - 6*a*c)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(c^2*(b^2
- 4*a*c)^(3/2)) + Log[a + b*x + c*x^2]/(2*c^2)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 738

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m - 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*(a + b*x
+ c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*
c)), Int[(d + e*x)^(m - 2)*Simp[e*(2*a*e*(m - 1) + b*d*(2*p - m + 4)) - 2*c
*d^2*(2*p + 3) + e*(b*e - 2*d*c)*(m + 2*p + 2)*x, x]*(a + b*x + c*x^2)^(p +
 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^
2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p, -1] && GtQ[m, 1] &&
```

```
IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 773

```
Int[(((d_.) + (e_.)*(x_))*((f_) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) + (c_.)*
(x_)^2), x_Symbol] :> Simp[(e*g*x)/c, x] + Dist[1/c, Int[(c*d*f - a*e*g + (
c*e*f + c*d*g - b*e*g)*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f, g}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1354

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> Int[x^(m + 2*n*p)*(c + b/x^n + a/x^(2*n))^p, x] /; FreeQ[{a, b, c, m, n
}, x] && EqQ[n2, 2*n] && ILtQ[p, 0] && NegQ[n]
```

Rubi steps

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2 x}\,dx = \int \frac{x^3}{\left(a+bx+cx^2\right)^2}\,dx$$

$$= \frac{x^2(2a+bx)}{\left(b^2-4ac\right)\left(a+bx+cx^2\right)} + \frac{\int \frac{x(4a+bx)}{a+bx+cx^2}\,dx}{-b^2+4ac}$$

$$= -\frac{bx}{c\left(b^2-4ac\right)} + \frac{x^2(2a+bx)}{\left(b^2-4ac\right)\left(a+bx+cx^2\right)} - \frac{\int \frac{-ab+\left(-b^2+4ac\right)x}{a+bx+cx^2}\,dx}{c\left(b^2-4ac\right)}$$

$$= -\frac{bx}{c\left(b^2-4ac\right)} + \frac{x^2(2a+bx)}{\left(b^2-4ac\right)\left(a+bx+cx^2\right)} + \frac{\int \frac{b+2cx}{a+bx+cx^2}\,dx}{2c^2} - \frac{\left(b\left(b^2-6ac\right)\right)\int \frac{1}{a+bx+cx^2}\,dx}{2c^2\left(b^2-4ac\right)}$$

$$= -\frac{bx}{c\left(b^2-4ac\right)} + \frac{x^2(2a+bx)}{\left(b^2-4ac\right)\left(a+bx+cx^2\right)} + \frac{\log\left(a+bx+cx^2\right)}{2c^2} + \frac{\left(b\left(b^2-6ac\right)\right)\text{Subst}\left(\right.}{c^2\left(b\right.}$$

$$= -\frac{bx}{c\left(b^2-4ac\right)} + \frac{x^2(2a+bx)}{\left(b^2-4ac\right)\left(a+bx+cx^2\right)} + \frac{b\left(b^2-6ac\right)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^2\left(b^2-4ac\right)^{3/2}} + \frac{\log\left(a+bx\right.}{2c^2}$$

Mathematica [A] time = 0.14, size = 109, normalized size = 0.96

$$\frac{\frac{2\left(-2a^2c+ab(b-3cx)+b^3x\right)}{\left(b^2-4ac\right)(a+x(b+cx))} + \frac{2b\left(b^2-6ac\right)\tan^{-1}\left(\frac{b+2cx}{\sqrt{4ac-b^2}}\right)}{\left(4ac-b^2\right)^{3/2}} + \log(a+x(b+cx))}{2c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c + a/x^2 + b/x)^2*x),x]
```

```
[Out] ((2*(-2*a^2*c + b^3*x + a*b*(b - 3*c*x)))/((b^2 - 4*a*c)*(a + x*(b + c*x)))
 + (2*b*(b^2 - 6*a*c)*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/(-b^2 + 4*a*c
)^(3/2) + Log[a + x*(b + c*x)])/(2*c^2)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2 x}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/((c + a/x^2 + b/x)^2*x),x]
```

```
[Out] IntegrateAlgebraic[1/((c + a/x^2 + b/x)^2*x), x]
```

fricas [B] time = 1.28, size = 635, normalized size = 5.57

$$\left[\frac{2\,ab^4 - 12\,a^2b^2c + 16\,a^3c^2 + \left(ab^3 - 6\,a^2bc + \left(b^3c - 6\,abc^2\right)x^2 + \left(b^4 - 6\,ab^2c\right)x\right)\sqrt{b^2 - 4\,ac}\,\log\left(\frac{2\,c^2x^2 + 2\,bcx + b^2 - 2\,ac + \sqrt{b^2 - 4\,ac}\,(2\,cx + b)}{cx^2 + bx + a}\right) + 2\left(b^5 - 7\,ab^3c + 12\,a^2bc^2\right)x + \left(ab^4 - 8\,a^2b^2c + 16\,a^3c^2 + \left(b^4c - 8\,ab^2c^2 + 16\,a^2c^3\right)x^2 + \left(b^5 - 8\,ab^3c + 16\,a^2bc^2\right)x\right)\log\left(cx^2 + bx + a\right)}{2\left(ab^4c^2 - 8\,a^2b^2c^3 + 16\,a^3c^4 + \left(b^4c^3 - 8\,ab^2c^4 + 16\,a^2c^5\right)x^2 + \left(b^5c^2 - 8\,ab^3c^3 + 16\,a^2bc^4\right)x\right)}, \frac{2\,ab^4 - 12\,a^2b^2c + 16\,a^3c^2 + 2\left(ab^3 - 6\,a^2bc + \left(b^3c - 6\,abc^2\right)x^2 + \left(b^4 - 6\,ab^2c\right)x\right)\sqrt{-b^2 + 4\,ac}\,\arctan\left(-\frac{\sqrt{-b^2 + 4\,ac}\,(2\,cx + b)}{b^2 - 4\,ac}\right) + 2\left(b^5 - 7\,ab^3c + 12\,a^2bc^2\right)x + \left(ab^4 - 8\,a^2b^2c + 16\,a^3c^2 + \left(b^4c - 8\,ab^2c^2 + 16\,a^2c^3\right)x^2 + \left(b^5 - 8\,ab^3c + 16\,a^2bc^2\right)x\right)\log\left(cx^2 + bx + a\right)}{2\left(ab^4c^2 - 8\,a^2b^2c^3 + 16\,a^3c^4 + \left(b^4c^3 - 8\,ab^2c^4 + 16\,a^2c^5\right)x^2 + \left(b^5c^2 - 8\,ab^3c^3 + 16\,a^2bc^4\right)x\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x^2+b/x)^2/x,x, algorithm="fricas")
```

```
[Out] [1/2*(2*a*b^4 - 12*a^2*b^2*c + 16*a^3*c^2 + (a*b^3 - 6*a^2*b*c + (b^3*c - 6
*a*b*c^2)*x^2 + (b^4 - 6*a*b^2*c)*x)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^2 + 2*b
*c*x + b^2 - 2*a*c + sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) + 2*
(b^5 - 7*a*b^3*c + 12*a^2*b*c^2)*x + (a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2 + (b
^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*x^2 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*x)*
log(c*x^2 + b*x + a))/(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4 + (b^4*c^3 -
8*a*b^2*c^4 + 16*a^2*c^5)*x^2 + (b^5*c^2 - 8*a*b^3*c^3 + 16*a^2*b*c^4)*x),
1/2*(2*a*b^4 - 12*a^2*b^2*c + 16*a^3*c^2 + 2*(a*b^3 - 6*a^2*b*c + (b^3*c -
6*a*b*c^2)*x^2 + (b^4 - 6*a*b^2*c)*x)*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b^2
+ 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) + 2*(b^5 - 7*a*b^3*c + 12*a^2*b*c^2)*x
+ (a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2 + (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*x^
2 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*x)*log(c*x^2 + b*x + a))/(a*b^4*c^2 -
8*a^2*b^2*c^3 + 16*a^3*c^4 + (b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)*x^2 + (b^
5*c^2 - 8*a*b^3*c^3 + 16*a^2*b*c^4)*x)]
```

giac [A] time = 0.36, size = 125, normalized size = 1.10

$$-\frac{\left(b^3 - 6\,abc\right)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{\left(b^2c^2 - 4\,ac^3\right)\sqrt{-b^2 + 4\,ac}} + \frac{\log\left(cx^2 + bx + a\right)}{2\,c^2} + \frac{ab^2 - 2\,a^2c + \left(b^3 - 3\,abc\right)x}{\left(cx^2 + bx + a\right)\left(b^2 - 4\,ac\right)c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x^2+b/x)^2/x,x, algorithm="giac")
```

```
[Out] -(b^3 - 6*a*b*c)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/((b^2*c^2 - 4*a*c^3
)*sqrt(-b^2 + 4*a*c)) + 1/2*log(c*x^2 + b*x + a)/c^2 + (a*b^2 - 2*a^2*c + (
b^3 - 3*a*b*c)*x)/((c*x^2 + b*x + a)*(b^2 - 4*a*c)*c^2)
```

maple [A] time = 0.01, size = 209, normalized size = 1.83

$$-\frac{6ab\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{\left(4ac-b^2\right)^{\frac{3}{2}}c} + \frac{b^3\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{\left(4ac-b^2\right)^{\frac{3}{2}}c^2} + \frac{2a\ln\left(c\,x^2+bx+a\right)}{\left(4ac-b^2\right)c} - \frac{b^2\ln\left(c\,x^2+bx+a\right)}{2\left(4ac-b^2\right)c^2} + \frac{\frac{\left(3ac-b^2\right)bx}{\left(4ac-b^2\right)c^2} + \frac{\left(2ac-b^2\right)a}{\left(4ac-b^2\right)c^2}}{c\,x^2+bx+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c+a/x^2+b/x)^2/x,x)
```

```
[Out] ((3*a*c-b^2)/(4*a*c-b^2)*b/c^2*x+(2*a*c-b^2)/(4*a*c-b^2)*a/c^2)/(c*x^2+b*x+
a)+2/c/(4*a*c-b^2)*ln(c*x^2+b*x+a)*a-1/2/c^2/(4*a*c-b^2)*ln(c*x^2+b*x+a)*b^
2-6/c/(4*a*c-b^2)^(3/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*a*b+1/c^2/(4*a*
c-b^2)^(3/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*b^3
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x^2+b/x)^2/x,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

mupad [B] time = 1.86, size = 279, normalized size = 2.45

$$\frac{\frac{a\left(2ac-b^2\right)}{c^2\left(4ac-b^2\right)}+\frac{bx\left(3ac-b^2\right)}{c^2\left(4ac-b^2\right)}}{cx^2+bx+a}-\frac{\ln\left(cx^2+bx+a\right)\left(-64a^3c^3+48a^2b^2c^2-12ab^4c+b^6\right)}{2\left(64a^3c^5-48a^2b^2c^4+12ab^4c^3-b^6c^2\right)}+\frac{b\,\mathrm{atan}\left(\frac{c^2\left(4ac-b^2\right)^{5/2}\left(\frac{2bx\left(6ac-b^2\right)}{c\left(4ac-b^2\right)^3}+\frac{b^2\left(4ac^2-b^2c\right)\left(6ac-b^2\right)}{c^3\left(4ac-b^2\right)^4}\right)}{b^3-6abc}\right)\left(6ac-b^2\right)}{c^2\left(4ac-b^2\right)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(c + a/x^2 + b/x)^2),x)
```

```
[Out] ((a*(2*a*c - b^2))/(c^2*(4*a*c - b^2)) + (b*x*(3*a*c - b^2))/(c^2*(4*a*c -
b^2)))/(a + b*x + c*x^2) - (log(a + b*x + c*x^2)*(b^6 - 64*a^3*c^3 + 48*a^2
*b^2*c^2 - 12*a*b^4*c))/(2*(64*a^3*c^5 - b^6*c^2 + 12*a*b^4*c^3 - 48*a^2*b^
2*c^4)) + (b*atan((c^2*(4*a*c - b^2)^(5/2)*((2*b*x*(6*a*c - b^2))/(c*(4*a*c
 - b^2)^3) + (b^2*(4*a*c^2 - b^2*c)*(6*a*c - b^2))/(c^3*(4*a*c - b^2)^4)))/
(b^3 - 6*a*b*c))*(6*a*c - b^2))/(c^2*(4*a*c - b^2)^(3/2))
```

sympy [B] time = 1.32, size = 729, normalized size = 6.39

$$\left(-\frac{b\sqrt{-\left(4ac-b^2\right)^3}\left(6ac-b^2\right)}{2c^2\left(64a^3c^3-48a^2b^2c^2+12ab^4c-b^6\right)}+\frac{1}{2c^2}\right)\log\left(x+\frac{-16a^2c^3\left(-\frac{b\sqrt{-(4ac-b^2)^3}(6ac-b^2)}{2c^2(64a^3c^3-48a^2b^2c^2+12ab^4c-b^6)}+\frac{1}{2c^2}\right)+8a^2c+8ab^2c^2\left(-\frac{b\sqrt{-(4ac-b^2)^3}(6ac-b^2)}{2c^2(64a^3c^3-48a^2b^2c^2+12ab^4c-b^6)}+\frac{1}{2c^2}\right)-ab^2-b^4c\left(-\frac{b\sqrt{-(4ac-b^2)^3}(6ac-b^2)}{2c^2(64a^3c^3-48a^2b^2c^2+12ab^4c-b^6)}+\frac{1}{2c^2}\right)}{6abc-b^3}\right)+\left(\frac{b\sqrt{-\left(4ac-b^2\right)^3}\left(6ac-b^2\right)}{2c^2\left(64a^3c^3-48a^2b^2c^2+12ab^4c-b^6\right)}+\frac{1}{2c^2}\right)\log\left(x+\frac{-16a^2c^3\left(\frac{b\sqrt{-(4ac-b^2)^3}(6ac-b^2)}{2c^2(64a^3c^3-48a^2b^2c^2+12ab^4c-b^6)}+\frac{1}{2c^2}\right)+8a^2c+8ab^2c^2\left(\frac{b\sqrt{-(4ac-b^2)^3}(6ac-b^2)}{2c^2(64a^3c^3-48a^2b^2c^2+12ab^4c-b^6)}+\frac{1}{2c^2}\right)-ab^2-b^4c\left(\frac{b\sqrt{-(4ac-b^2)^3}(6ac-b^2)}{2c^2(64a^3c^3-48a^2b^2c^2+12ab^4c-b^6)}+\frac{1}{2c^2}\right)}{6abc-b^3}\right)+\frac{2a^2c-ab^2+x\left(3abc-b^3\right)}{4a^2c^3-ab^2c^2+x^2\left(4ac^4-b^2c^3\right)+x\left(4abc^3-b^3c^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x**2+b/x)**2/x,x)
```

```
[Out] (-b*sqrt(-(4*a*c - b**2)**3)*(6*a*c - b**2)/(2*c**2*(64*a**3*c**3 - 48*a**2
*b**2*c**2 + 12*a*b**4*c - b**6)) + 1/(2*c**2))*log(x + (-16*a**2*c**3*(-b*
sqrt(-(4*a*c - b**2)**3)*(6*a*c - b**2)/(2*c**2*(64*a**3*c**3 - 48*a**2*b**
2*c**2 + 12*a*b**4*c - b**6)) + 1/(2*c**2)) + 8*a**2*c + 8*a*b**2*c**2*(-b*
sqrt(-(4*a*c - b**2)**3)*(6*a*c - b**2)/(2*c**2*(64*a**3*c**3 - 48*a**2*b**
2*c**2 + 12*a*b**4*c - b**6)) + 1/(2*c**2)) - a*b**2 - b**4*c*(-b*sqrt(-(4*
a*c - b**2)**3)*(6*a*c - b**2)/(2*c**2*(64*a**3*c**3 - 48*a**2*b**2*c**2 +
12*a*b**4*c - b**6)) + 1/(2*c**2)))/(6*a*b*c - b**3)) + (b*sqrt(-(4*a*c - b
**2)**3)*(6*a*c - b**2)/(2*c**2*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b*
*4*c - b**6)) + 1/(2*c**2))*log(x + (-16*a**2*c**3*(b*sqrt(-(4*a*c - b**2)*
*3)*(6*a*c - b**2)/(2*c**2*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c
- b**6)) + 1/(2*c**2)) + 8*a**2*c + 8*a*b**2*c**2*(b*sqrt(-(4*a*c - b**2)**
3)*(6*a*c - b**2)/(2*c**2*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c -
 b**6)) + 1/(2*c**2)) - a*b**2 - b**4*c*(b*sqrt(-(4*a*c - b**2)**3)*(6*a*c
- b**2)/(2*c**2*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)) +
1/(2*c**2)))/(6*a*b*c - b**3)) + (2*a**2*c - a*b**2 + x*(3*a*b*c - b**3))/(
4*a**2*c**3 - a*b**2*c**2 + x**2*(4*a*c**4 - b**2*c**3) + x*(4*a*b*c**3 - b
**3*c**2))
```

3.364 $\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2 x^2} \, dx$

Optimal. Leaf size=71

$$\frac{\frac{2a}{x}+b}{\left(b^2-4ac\right)\left(\frac{a}{x^2}+\frac{b}{x}+c\right)}-\frac{4a\tanh^{-1}\left(\frac{\frac{2a}{x}+b}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{3/2}}$$

Rubi [A] time = 0.04, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {1352, 614, 618, 206}

$$\frac{\frac{2a}{x}+b}{\left(b^2-4ac\right)\left(\frac{a}{x^2}+\frac{b}{x}+c\right)}-\frac{4a\tanh^{-1}\left(\frac{\frac{2a}{x}+b}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c + a/x^2 + b/x)^2*x^2),x]
```

```
[Out] (b + (2*a)/x)/((b^2 - 4*a*c)*(c + a/x^2 + b/x)) - (4*a*ArcTanh[(b + (2*a)/x
)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(3/2)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 614

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[(2*c*(2*p +
 3))/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
egerQ[4*p]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1352

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a,
b, c, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2 x^2}\,dx &= -\text{Subst}\left(\int \frac{1}{\left(c+bx+ax^2\right)^2}\,dx,x,\frac{1}{x}\right)\\ &= \frac{b+\frac{2a}{x}}{\left(b^2-4ac\right)\left(c+\frac{a}{x^2}+\frac{b}{x}\right)}+\frac{(2a)\,\text{Subst}\left(\int \frac{1}{c+bx+ax^2}\,dx,x,\frac{1}{x}\right)}{b^2-4ac}\\ &= \frac{b+\frac{2a}{x}}{\left(b^2-4ac\right)\left(c+\frac{a}{x^2}+\frac{b}{x}\right)}-\frac{(4a)\,\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx,x,b+\frac{2a}{x}\right)}{b^2-4ac}\\ &= \frac{b+\frac{2a}{x}}{\left(b^2-4ac\right)\left(c+\frac{a}{x^2}+\frac{b}{x}\right)}-\frac{4a\tanh^{-1}\left(\frac{b+\frac{2a}{x}}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{3/2}}\end{aligned}$$

Mathematica [A] time = 0.09, size = 81, normalized size = 1.14

$$\frac{a(b-2cx)+b^2x}{c\left(4ac-b^2\right)(a+x(b+cx))}+\frac{4a\tan^{-1}\left(\frac{b+2cx}{\sqrt{4ac-b^2}}\right)}{\left(4ac-b^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c + a/x^2 + b/x)^2*x^2),x]
[Out] (b^2*x + a*(b - 2*c*x))/(c*(-b^2 + 4*a*c)*(a + x*(b + c*x))) + (4*a*ArcTan[
(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/(-b^2 + 4*a*c)^(3/2)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2 x^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/((c + a/x^2 + b/x)^2*x^2),x]
[Out] IntegrateAlgebraic[1/((c + a/x^2 + b/x)^2*x^2), x]
```

fricas [B] time = 1.20, size = 387, normalized size = 5.45

$$\left[-\frac{ab^3-4a^2bc+2\left(ac^2x^2+abcx+a^2c\right)\sqrt{b^2-4ac}\log\left(\frac{2c^2x^2+2bcx+b^2-2ac-\sqrt{b^2-4ac}(2cx+b)}{cx^2+bx+a}\right)+\left(b^4-6ab^2c+8a^2c^2\right)x}{ab^4c-8a^2b^2c^2+16a^3c^3+\left(b^4c^2-8ab^2c^3+16a^2c^4\right)x^2+\left(b^5c-8ab^3c^2+16a^2bc^3\right)x},-\frac{ab^3-4a^2bc-4\left(ac^2x^2+abcx+a^2c\right)\sqrt{-b^2+4ac}\arctan\left(-\frac{\sqrt{-b^2+4ac}(2cx+b)}{b^2-4ac}\right)+\left(b^4-6ab^2c+8a^2c^2\right)x}{ab^4c-8a^2b^2c^2+16a^3c^3+\left(b^4c^2-8ab^2c^3+16a^2c^4\right)x^2+\left(b^5c-8ab^3c^2+16a^2bc^3\right)x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x^2+b/x)^2/x^2,x, algorithm="fricas")
[Out] [-(a*b^3 - 4*a^2*b*c + 2*(a*c^2*x^2 + a*b*c*x + a^2*c)*sqrt(b^2 - 4*a*c)*lo
g((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c - sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^
2 + b*x + a)) + (b^4 - 6*a*b^2*c + 8*a^2*c^2)*x)/(a*b^4*c - 8*a^2*b^2*c^2 +
 16*a^3*c^3 + (b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*x^2 + (b^5*c - 8*a*b^3*c
^2 + 16*a^2*b*c^3)*x), -(a*b^3 - 4*a^2*b*c - 4*(a*c^2*x^2 + a*b*c*x + a^2*c
)*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c))
+ (b^4 - 6*a*b^2*c + 8*a^2*c^2)*x)/(a*b^4*c - 8*a^2*b^2*c^2 + 16*a^3*c^3 +
```

```
(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*x^2 + (b^5*c - 8*a*b^3*c^2 + 16*a^2*b*
c^3)*x)]
```

giac [A] time = 0.38, size = 88, normalized size = 1.24

$$-\frac{4\,a\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{\left(b^2-4\,ac\right)\sqrt{-b^2+4\,ac}}-\frac{b^2x-2\,acx+ab}{\left(b^2c-4\,ac^2\right)\left(cx^2+bx+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x^2+b/x)^2/x^2,x, algorithm="giac")
```

```
[Out] -4*a*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/((b^2 - 4*a*c)*sqrt(-b^2 + 4*a*
c)) - (b^2*x - 2*a*c*x + a*b)/((b^2*c - 4*a*c^2)*(c*x^2 + b*x + a))
```

maple [A] time = 0.01, size = 97, normalized size = 1.37

$$\frac{4a\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{\left(4ac-b^2\right)^{\frac{3}{2}}}+\frac{\frac{ab}{\left(4ac-b^2\right)c}-\frac{\left(2ac-b^2\right)x}{\left(4ac-b^2\right)c}}{c\,x^2+bx+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c+a/x^2+b/x)^2/x^2,x)
```

```
[Out] (-(2*a*c-b^2)/(4*a*c-b^2)/c*x+1/c*a*b/(4*a*c-b^2))/(c*x^2+b*x+a)+4*a/(4*a*c
-b^2)^(3/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x^2+b/x)^2/x^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

mupad [B] time = 1.37, size = 135, normalized size = 1.90

$$-\frac{\frac{x\left(2\,a\,c-b^2\right)}{c\left(4\,a\,c-b^2\right)}-\frac{a\,b}{c\left(4\,a\,c-b^2\right)}}{c\,x^2+b\,x+a}-\frac{4\,a\,\mathrm{atan}\left(\frac{\left(\frac{2\,a\left(b^3-4\,a\,b\,c\right)}{\left(4\,a\,c-b^2\right)^{5/2}}-\frac{4\,a\,c\,x}{\left(4\,a\,c-b^2\right)^{3/2}}\right)\left(4\,a\,c-b^2\right)}{2\,a}\right)}{\left(4\,a\,c-b^2\right)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(c + a/x^2 + b/x)^2),x)
```

```
[Out] - ((x*(2*a*c - b^2))/(c*(4*a*c - b^2)) - (a*b)/(c*(4*a*c - b^2)))/(a + b*x
+ c*x^2) - (4*a*atan((((2*a*(b^3 - 4*a*b*c))/(4*a*c - b^2)^(5/2) - (4*a*c*x
)/(4*a*c - b^2)^(3/2))*(4*a*c - b^2))/(2*a)))/(4*a*c - b^2)^(3/2)
```

sympy [B] time = 0.60, size = 280, normalized size = 3.94

$$-2a\sqrt{-\frac{1}{\left(4ac-b^2\right)^3}}\log\left(x+\frac{-32a^3c^2\sqrt{-\frac{1}{\left(4ac-b^2\right)^3}}+16a^2b^2c\sqrt{-\frac{1}{\left(4ac-b^2\right)^3}}-2ab^4\sqrt{-\frac{1}{\left(4ac-b^2\right)^3}}+2ab}{4ac}\right)+2a\sqrt{-\frac{1}{\left(4ac-b^2\right)^3}}\log\left(x+\frac{32a^3c^2\sqrt{-\frac{1}{\left(4ac-b^2\right)^3}}-16a^2b^2c\sqrt{-\frac{1}{\left(4ac-b^2\right)^3}}+2ab^4\sqrt{-\frac{1}{\left(4ac-b^2\right)^3}}+2ab}{4ac}\right)+\frac{ab+x\left(-2ac+b^2\right)}{4a^2c^2-ab^2c+x^2\left(4ac^3-b^2c^2\right)+x\left(4abc^2-b^3c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x**2+b/x)**2/x**2,x)
```

```
[Out] -2*a*sqrt(-1/(4*a*c - b**2)**3)*log(x + (-32*a**3*c**2*sqrt(-1/(4*a*c - b**
2)**3) + 16*a**2*b**2*c*sqrt(-1/(4*a*c - b**2)**3) - 2*a*b**4*sqrt(-1/(4*a*
c - b**2)**3) + 2*a*b)/(4*a*c)) + 2*a*sqrt(-1/(4*a*c - b**2)**3)*log(x + (3
2*a**3*c**2*sqrt(-1/(4*a*c - b**2)**3) - 16*a**2*b**2*c*sqrt(-1/(4*a*c - b*
*2)**3) + 2*a*b**4*sqrt(-1/(4*a*c - b**2)**3) + 2*a*b)/(4*a*c)) + (a*b + x*
(-2*a*c + b**2))/(4*a**2*c**2 - a*b**2*c + x**2*(4*a*c**3 - b**2*c**2) + x*
(4*a*b*c**2 - b**3*c))
```

3.365 $\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2 x^3}\,dx$

Optimal. Leaf size=66

$$\frac{2a+bx}{(b^2-4ac)(a+bx+cx^2)}-\frac{2b\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}}$$

Rubi [A] time = 0.03, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {1354, 638, 618, 206}

$$\frac{2a+bx}{(b^2-4ac)(a+bx+cx^2)}-\frac{2b\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c + a/x^2 + b/x)^2*x^3),x]
```

```
[Out] (2*a + b*x)/((b^2 - 4*a*c)*(a + b*x + c*x^2)) - (2*b*ArcTanh[(b + 2*c*x)/Sq
rt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(3/2)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 638

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)*(a + b*x + c*x^2)^(p + 1))/((p +
 1)*(b^2 - 4*a*c)), x] - Dist[((2*p + 3)*(2*c*d - b*e))/((p + 1)*(b^2 - 4*a
*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &&
 NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 1354

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> Int[x^(m + 2*n*p)*(c + b/x^n + a/x^(2*n))^p, x] /; FreeQ[{a, b, c, m, n
}, x] && EqQ[n2, 2*n] && ILtQ[p, 0] && NegQ[n]
```

Rubi steps

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2 x^3}\,dx = \int \frac{x}{\left(a+bx+cx^2\right)^2}\,dx$$

$$= \frac{2a+bx}{\left(b^2-4ac\right)\left(a+bx+cx^2\right)} + \frac{b\int \frac{1}{a+bx+cx^2}\,dx}{b^2-4ac}$$

$$= \frac{2a+bx}{\left(b^2-4ac\right)\left(a+bx+cx^2\right)} - \frac{(2b)\,\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx, x, b+2cx\right)}{b^2-4ac}$$

$$= \frac{2a+bx}{\left(b^2-4ac\right)\left(a+bx+cx^2\right)} - \frac{2b\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{3/2}}$$

Mathematica [A] time = 0.06, size = 69, normalized size = 1.05

$$\frac{2a+bx}{\left(b^2-4ac\right)(a+x(b+cx))} - \frac{2b\tan^{-1}\left(\frac{b+2cx}{\sqrt{4ac-b^2}}\right)}{\left(4ac-b^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c + a/x^2 + b/x)^2*x^3),x]
```

```
[Out] (2*a + b*x)/((b^2 - 4*a*c)*(a + x*(b + c*x))) - (2*b*ArcTan[(b + 2*c*x)/Sqr
t[-b^2 + 4*a*c]])/(-b^2 + 4*a*c)^(3/2)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2 x^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/((c + a/x^2 + b/x)^2*x^3),x]
```

```
[Out] IntegrateAlgebraic[1/((c + a/x^2 + b/x)^2*x^3), x]
```

fricas [B] time = 1.17, size = 338, normalized size = 5.12

$$\left[\frac{2ab^2-8a^2c-\left(bcx^2+b^2x+ab\right)\sqrt{b^2-4ac}\log\left(\frac{2c^2x^2+2bcx+b^2-2ac+\sqrt{b^2-4ac}(2cx+b)}{cx^2+bx+a}\right)+\left(b^3-4abc\right)x}{ab^4-8a^2b^2c+16a^3c^2+\left(b^4c-8ab^2c^2+16a^2c^3\right)x^2+\left(b^5-8ab^3c+16a^2bc^2\right)x}, \frac{2ab^2-8a^2c-2\left(bcx^2+b^2x+ab\right)\sqrt{-b^2+4ac}\arctan\left(-\frac{\sqrt{-b^2+4ac}(2cx+b)}{b^2-4ac}\right)+\left(b^3-4abc\right)x}{ab^4-8a^2b^2c+16a^3c^2+\left(b^4c-8ab^2c^2+16a^2c^3\right)x^2+\left(b^5-8ab^3c+16a^2bc^2\right)x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x^2+b/x)^2/x^3,x, algorithm="fricas")
```

```
[Out] [(2*a*b^2 - 8*a^2*c - (b*c*x^2 + b^2*x + a*b)*sqrt(b^2 - 4*a*c)*log((2*c^2*
x^2 + 2*b*c*x + b^2 - 2*a*c + sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x +
 a)) + (b^3 - 4*a*b*c)*x)/(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2 + (b^4*c - 8*a*
b^2*c^2 + 16*a^2*c^3)*x^2 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*x), (2*a*b^2 -
 8*a^2*c - 2*(b*c*x^2 + b^2*x + a*b)*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b^2 +
 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) + (b^3 - 4*a*b*c)*x)/(a*b^4 - 8*a^2*b^2*
c + 16*a^3*c^2 + (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*x^2 + (b^5 - 8*a*b^3*c
+ 16*a^2*b*c^2)*x)]
```

giac [A] time = 0.37, size = 76, normalized size = 1.15

$$\frac{2\,b\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{\left(b^2-4\,ac\right)\sqrt{-b^2+4\,ac}}+\frac{bx+2\,a}{\left(cx^2+bx+a\right)\left(b^2-4\,ac\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x^2+b/x)^2/x^3,x, algorithm="giac")
```

```
[Out] 2*b*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/((b^2 - 4*a*c)*sqrt(-b^2 + 4*a*c
)) + (b*x + 2*a)/((c*x^2 + b*x + a)*(b^2 - 4*a*c))
```

maple [A] time = 0.00, size = 70, normalized size = 1.06

$$-\frac{2b\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{\left(4ac-b^2\right)^{\frac{3}{2}}}+\frac{-bx-2a}{\left(4ac-b^2\right)\left(c\,x^2+bx+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c+a/x^2+b/x)^2/x^3,x)
```

```
[Out] (-b*x-2*a)/(4*a*c-b^2)/(c*x^2+b*x+a)-2*b/(4*a*c-b^2)^(3/2)*arctan((2*c*x+b)
/(4*a*c-b^2)^(1/2))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x^2+b/x)^2/x^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

mupad [B] time = 1.37, size = 110, normalized size = 1.67

$$-\frac{\frac{2\,a}{4\,a\,c-b^2}+\frac{b\,x}{4\,a\,c-b^2}}{c\,x^2+b\,x+a}-\frac{2\,b\,\mathrm{atan}\left(\frac{\left(\frac{b^2}{{\left(4\,a\,c-b^2\right)}^{3/2}}+\frac{2\,b\,c\,x}{{\left(4\,a\,c-b^2\right)}^{3/2}}\right)\left(4\,a\,c-b^2\right)}{b}\right)}{{\left(4\,a\,c-b^2\right)}^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(c + a/x^2 + b/x)^2),x)
```

```
[Out] - ((2*a)/(4*a*c - b^2) + (b*x)/(4*a*c - b^2))/(a + b*x + c*x^2) - (2*b*atan
(((b^2/(4*a*c - b^2)^(3/2) + (2*b*c*x)/(4*a*c - b^2)^(3/2))*(4*a*c - b^2))/
b))/(4*a*c - b^2)^(3/2)
```

sympy [B] time = 0.56, size = 253, normalized size = 3.83

$$b\sqrt{-\frac{1}{\left(4ac-b^2\right)^3}}\log\left(x+\frac{-16a^2bc^2\sqrt{-\frac{1}{\left(4ac-b^2\right)^3}}+8ab^3c\sqrt{-\frac{1}{\left(4ac-b^2\right)^3}}-b^5\sqrt{-\frac{1}{\left(4ac-b^2\right)^3}}+b^2}{2bc}\right)-b\sqrt{-\frac{1}{\left(4ac-b^2\right)^3}}\log\left(x+\frac{16a^2bc^2\sqrt{-\frac{1}{\left(4ac-b^2\right)^3}}-8ab^3c\sqrt{-\frac{1}{\left(4ac-b^2\right)^3}}+b^5\sqrt{-\frac{1}{\left(4ac-b^2\right)^3}}+b^2}{2bc}\right)+\frac{-2a-bx}{4a^2c-ab^2+x^2\left(4ac^2-b^2c\right)+x\left(4abc-b^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x**2+b/x)**2/x**3,x)
```

```
[Out] b*sqrt(-1/(4*a*c - b**2)**3)*log(x + (-16*a**2*b*c**2*sqrt(-1/(4*a*c - b**2
)**3) + 8*a*b**3*c*sqrt(-1/(4*a*c - b**2)**3) - b**5*sqrt(-1/(4*a*c - b**2)
**3) + b**2)/(2*b*c)) - b*sqrt(-1/(4*a*c - b**2)**3)*log(x + (16*a**2*b*c**
2*sqrt(-1/(4*a*c - b**2)**3) - 8*a*b**3*c*sqrt(-1/(4*a*c - b**2)**3) + b**5
*sqrt(-1/(4*a*c - b**2)**3) + b**2)/(2*b*c)) + (-2*a - b*x)/(4*a**2*c - a*b
**2 + x**2*(4*a*c**2 - b**2*c) + x*(4*a*b*c - b**3))
```

3.366 $\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2 x^4} \, dx$

Optimal. Leaf size=66

$$\frac{4c\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{3/2}}-\frac{b+2cx}{\left(b^2-4ac\right)\left(a+bx+cx^2\right)}$$

Rubi [A] time = 0.03, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {1354, 614, 618, 206}

$$\frac{4c\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{3/2}}-\frac{b+2cx}{\left(b^2-4ac\right)\left(a+bx+cx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c + a/x^2 + b/x)^2*x^4),x]
```

```
[Out] -((b + 2*c*x)/((b^2 - 4*a*c)*(a + b*x + c*x^2))) + (4*c*ArcTanh[(b + 2*c*x)
/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(3/2)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 614

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[(2*c*(2*p +
 3))/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
egerQ[4*p]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1354

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> Int[x^(m + 2*n*p)*(c + b/x^n + a/x^(2*n))^p, x] /; FreeQ[{a, b, c, m, n
}, x] && EqQ[n2, 2*n] && ILtQ[p, 0] && NegQ[n]
```

Rubi steps

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2 x^4}\,dx = \int \frac{1}{\left(a+bx+cx^2\right)^2}\,dx$$

$$= -\frac{b+2cx}{\left(b^2-4ac\right)\left(a+bx+cx^2\right)} - \frac{(2c)\int \frac{1}{a+bx+cx^2}\,dx}{b^2-4ac}$$

$$= -\frac{b+2cx}{\left(b^2-4ac\right)\left(a+bx+cx^2\right)} + \frac{(4c)\,\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx, x, b+2cx\right)}{b^2-4ac}$$

$$= -\frac{b+2cx}{\left(b^2-4ac\right)\left(a+bx+cx^2\right)} + \frac{4c\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{3/2}}$$

Mathematica [A] time = 0.08, size = 70, normalized size = 1.06

$$-\frac{\frac{4c\tan^{-1}\left(\frac{b+2cx}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}} + \frac{b+2cx}{a+x(b+cx)}}{b^2-4ac}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c + a/x^2 + b/x)^2*x^4),x]
```

```
[Out] -(((b + 2*c*x)/(a + x*(b + c*x)) + (4*c*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*
c]])/Sqrt[-b^2 + 4*a*c])/(b^2 - 4*a*c))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2 x^4}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/((c + a/x^2 + b/x)^2*x^4),x]
```

```
[Out] IntegrateAlgebraic[1/((c + a/x^2 + b/x)^2*x^4), x]
```

fricas [B] time = 1.26, size = 341, normalized size = 5.17

$$\left[-\frac{b^3-4abc+2\left(c^2x^2+bcx+ac\right)\sqrt{b^2-4ac}\log\left(\frac{2c^2x^2+2bcx+b^2-2ac-\sqrt{b^2-4ac}(2cx+b)}{cx^2+bx+a}\right)+2\left(b^2c-4ac^2\right)x}{ab^4-8a^2b^2c+16a^3c^2+\left(b^4c-8ab^2c^2+16a^2c^3\right)x^2+\left(b^5-8ab^3c+16a^2bc^2\right)x}, -\frac{b^3-4abc-4\left(c^2x^2+bcx+ac\right)\sqrt{-b^2+4ac}\arctan\left(-\frac{\sqrt{-b^2+4ac}(2cx+b)}{b^2-4ac}\right)+2\left(b^2c-4ac^2\right)x}{ab^4-8a^2b^2c+16a^3c^2+\left(b^4c-8ab^2c^2+16a^2c^3\right)x^2+\left(b^5-8ab^3c+16a^2bc^2\right)x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x^2+b/x)^2/x^4,x, algorithm="fricas")
```

```
[Out] [-(b^3 - 4*a*b*c + 2*(c^2*x^2 + b*c*x + a*c)*sqrt(b^2 - 4*a*c)*log((2*c^2*x
^2 + 2*b*c*x + b^2 - 2*a*c - sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x +
a)) + 2*(b^2*c - 4*a*c^2)*x)/(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2 + (b^4*c - 8
*a*b^2*c^2 + 16*a^2*c^3)*x^2 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*x), -(b^3 -
 4*a*b*c - 4*(c^2*x^2 + b*c*x + a*c)*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b^2 +
 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) + 2*(b^2*c - 4*a*c^2)*x)/(a*b^4 - 8*a^2*
b^2*c + 16*a^3*c^2 + (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*x^2 + (b^5 - 8*a*b^
3*c + 16*a^2*b*c^2)*x)]
```

giac [A] time = 0.39, size = 76, normalized size = 1.15

$$-\frac{4\,c\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{\left(b^2-4\,ac\right)\sqrt{-b^2+4\,ac}}-\frac{2\,cx+b}{\left(cx^2+bx+a\right)\left(b^2-4\,ac\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x^2+b/x)^2/x^4,x, algorithm="giac")
```

```
[Out] -4*c*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/((b^2 - 4*a*c)*sqrt(-b^2 + 4*a*
c)) - (2*c*x + b)/((c*x^2 + b*x + a)*(b^2 - 4*a*c))
```

maple [A] time = 0.00, size = 68, normalized size = 1.03

$$\frac{4c\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{\left(4ac-b^2\right)^{\frac{3}{2}}}+\frac{2cx+b}{\left(4ac-b^2\right)\left(c\,x^2+bx+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c+a/x^2+b/x)^2/x^4,x)
```

```
[Out] (2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)+4*c/(4*a*c-b^2)^(3/2)*arctan((2*c*x+b)/
(4*a*c-b^2)^(1/2))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x^2+b/x)^2/x^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

mupad [B] time = 0.08, size = 119, normalized size = 1.80

$$\frac{\frac{b}{4\,a\,c-b^2}+\frac{2\,c\,x}{4\,a\,c-b^2}}{c\,x^2+b\,x+a}-\frac{4\,c\,\mathrm{atan}\left(\frac{\left(\frac{2\,c\,\left(b^3-4\,a\,b\,c\right)}{{\left(4\,a\,c-b^2\right)}^{5/2}}-\frac{4\,c^2\,x}{{\left(4\,a\,c-b^2\right)}^{3/2}}\right)\,\left(4\,a\,c-b^2\right)}{2\,c}\right)}{{\left(4\,a\,c-b^2\right)}^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(c + a/x^2 + b/x)^2),x)
```

```
[Out] (b/(4*a*c - b^2) + (2*c*x)/(4*a*c - b^2))/(a + b*x + c*x^2) - (4*c*atan((((
2*c*(b^3 - 4*a*b*c))/(4*a*c - b^2)^(5/2) - (4*c^2*x)/(4*a*c - b^2)^(3/2))*(
4*a*c - b^2))/(2*c)))/(4*a*c - b^2)^(3/2)
```

sympy [B] time = 0.58, size = 265, normalized size = 4.02

$$-2c\sqrt{-\frac{1}{\left(4ac-b^2\right)^3}}\log\left(x+\frac{-32a^2c^3\sqrt{-\frac{1}{\left(4ac-b^2\right)^3}}+16ab^2c^2\sqrt{-\frac{1}{\left(4ac-b^2\right)^3}}-2b^4c\sqrt{-\frac{1}{\left(4ac-b^2\right)^3}}+2bc}{4c^2}\right)+2c\sqrt{-\frac{1}{\left(4ac-b^2\right)^3}}\log\left(x+\frac{32a^2c^3\sqrt{-\frac{1}{\left(4ac-b^2\right)^3}}-16ab^2c^2\sqrt{-\frac{1}{\left(4ac-b^2\right)^3}}+2b^4c\sqrt{-\frac{1}{\left(4ac-b^2\right)^3}}+2bc}{4c^2}\right)+\frac{b+2cx}{4a^2c-ab^2+x^2\left(4ac^2-b^2c\right)+x\left(4abc-b^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x**2+b/x)**2/x**4,x)
```

```
[Out] -2*c*sqrt(-1/(4*a*c - b**2)**3)*log(x + (-32*a**2*c**3*sqrt(-1/(4*a*c - b**
2)**3) + 16*a*b**2*c**2*sqrt(-1/(4*a*c - b**2)**3) - 2*b**4*c*sqrt(-1/(4*a*
c - b**2)**3) + 2*b*c)/(4*c**2)) + 2*c*sqrt(-1/(4*a*c - b**2)**3)*log(x + (
32*a**2*c**3*sqrt(-1/(4*a*c - b**2)**3) - 16*a*b**2*c**2*sqrt(-1/(4*a*c - b
**2)**3) + 2*b**4*c*sqrt(-1/(4*a*c - b**2)**3) + 2*b*c)/(4*c**2)) + (b + 2*
c*x)/(4*a**2*c - a*b**2 + x**2*(4*a*c**2 - b**2*c) + x*(4*a*b*c - b**3))
```

3.367 $\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2 x^5} \, dx$

Optimal. Leaf size=108

$$\frac{b\left(b^2-6ac\right)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a^2\left(b^2-4ac\right)^{3/2}}-\frac{\log\left(a+bx+cx^2\right)}{2a^2}+\frac{\log(x)}{a^2}+\frac{-2ac+b^2+bcx}{a\left(b^2-4ac\right)\left(a+bx+cx^2\right)}$$

Rubi [A] time = 0.14, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389, Rules used = {1354, 740, 800, 634, 618, 206, 628}

$$\frac{b\left(b^2-6ac\right)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a^2\left(b^2-4ac\right)^{3/2}}-\frac{\log\left(a+bx+cx^2\right)}{2a^2}+\frac{\log(x)}{a^2}+\frac{-2ac+b^2+bcx}{a\left(b^2-4ac\right)\left(a+bx+cx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c + a/x^2 + b/x)^2*x^5),x]
```

```
[Out] (b^2 - 2*a*c + b*c*x)/(a*(b^2 - 4*a*c)*(a + b*x + c*x^2)) + (b*(b^2 - 6*a*c
)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(a^2*(b^2 - 4*a*c)^(3/2)) + Log[x
]/a^2 - Log[a + b*x + c*x^2]/(2*a^2)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 740

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e
)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e
^2)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d +
 e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p +
 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4
```

```
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p,
-1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 1354

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> Int[x^(m + 2*n*p)*(c + b/x^n + a/x^(2*n))^p, x] /; FreeQ[{a, b, c, m, n
}, x] && EqQ[n2, 2*n] && ILtQ[p, 0] && NegQ[n]
```

Rubi steps

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2 x^5}\,dx=\int \frac{1}{x\left(a+bx+cx^2\right)^2}\,dx$$

$$=\frac{b^2-2ac+bcx}{a\left(b^2-4ac\right)\left(a+bx+cx^2\right)}-\frac{\int \frac{-b^2+4ac-bcx}{x\left(a+bx+cx^2\right)}\,dx}{a\left(b^2-4ac\right)}$$

$$=\frac{b^2-2ac+bcx}{a\left(b^2-4ac\right)\left(a+bx+cx^2\right)}-\frac{\int \left(\frac{-b^2+4ac}{ax}+\frac{b\left(b^2-5ac\right)+c\left(b^2-4ac\right)x}{a\left(a+bx+cx^2\right)}\right)dx}{a\left(b^2-4ac\right)}$$

$$=\frac{b^2-2ac+bcx}{a\left(b^2-4ac\right)\left(a+bx+cx^2\right)}+\frac{\log(x)}{a^2}-\frac{\int \frac{b\left(b^2-5ac\right)+c\left(b^2-4ac\right)x}{a+bx+cx^2}\,dx}{a^2\left(b^2-4ac\right)}$$

$$=\frac{b^2-2ac+bcx}{a\left(b^2-4ac\right)\left(a+bx+cx^2\right)}+\frac{\log(x)}{a^2}-\frac{\int \frac{b+2cx}{a+bx+cx^2}\,dx}{2a^2}-\frac{\left(b\left(b^2-6ac\right)\right)\int \frac{1}{a+bx+cx^2}\,dx}{2a^2\left(b^2-4ac\right)}$$

$$=\frac{b^2-2ac+bcx}{a\left(b^2-4ac\right)\left(a+bx+cx^2\right)}+\frac{\log(x)}{a^2}-\frac{\log\left(a+bx+cx^2\right)}{2a^2}+\frac{\left(b\left(b^2-6ac\right)\right)\text{Subst}\left(\int \frac{}{b^2-}\right.}{a^2\left(b^2-\right.}$$

$$=\frac{b^2-2ac+bcx}{a\left(b^2-4ac\right)\left(a+bx+cx^2\right)}+\frac{b\left(b^2-6ac\right)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a^2\left(b^2-4ac\right)^{3/2}}+\frac{\log(x)}{a^2}-\frac{\log\left(a+bx+c\right.}{2a^2}$$

Mathematica [A] time = 0.19, size = 107, normalized size = 0.99

$$\frac{\frac{2a\left(-2ac+b^2+bcx\right)}{\left(b^2-4ac\right)(a+x(b+cx))}+\frac{2b\left(b^2-6ac\right)\tan^{-1}\left(\frac{b+2cx}{\sqrt{4ac-b^2}}\right)}{\left(4ac-b^2\right)^{3/2}}-\log(a+x(b+cx))+2\log(x)}{2a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c + a/x^2 + b/x)^2*x^5),x]
```

```
[Out] ((2*a*(b^2 - 2*a*c + b*c*x))/((b^2 - 4*a*c)*(a + x*(b + c*x))) + (2*b*(b^2
- 6*a*c)*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/(-b^2 + 4*a*c)^(3/2) + 2*L
og[x] - Log[a + x*(b + c*x)])/(2*a^2)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2 x^5}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/((c + a/x^2 + b/x)^2*x^5),x]
```

```
[Out] IntegrateAlgebraic[1/((c + a/x^2 + b/x)^2*x^5), x]
```

fricas [B] time = 1.49, size = 781, normalized size = 7.23

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x^2+b/x)^2/x^5,x, algorithm="fricas")
```

```
[Out] [1/2*(2*a*b^4 - 12*a^2*b^2*c + 16*a^3*c^2 + (a*b^3 - 6*a^2*b*c + (b^3*c - 6
*a*b*c^2)*x^2 + (b^4 - 6*a*b^2*c)*x)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^2 + 2*b
*c*x + b^2 - 2*a*c + sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) + 2*
(a*b^3*c - 4*a^2*b*c^2)*x - (a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2 + (b^4*c - 8*
a*b^2*c^2 + 16*a^2*c^3)*x^2 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*x)*log(c*x^2
 + b*x + a) + 2*(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2 + (b^4*c - 8*a*b^2*c^2 +
16*a^2*c^3)*x^2 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*x)*log(x))/(a^3*b^4 - 8*
a^4*b^2*c + 16*a^5*c^2 + (a^2*b^4*c - 8*a^3*b^2*c^2 + 16*a^4*c^3)*x^2 + (a^
2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*x), 1/2*(2*a*b^4 - 12*a^2*b^2*c + 16*a^
3*c^2 + 2*(a*b^3 - 6*a^2*b*c + (b^3*c - 6*a*b*c^2)*x^2 + (b^4 - 6*a*b^2*c)*
x)*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c))
 + 2*(a*b^3*c - 4*a^2*b*c^2)*x - (a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2 + (b^4*c
 - 8*a*b^2*c^2 + 16*a^2*c^3)*x^2 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*x)*log(
c*x^2 + b*x + a) + 2*(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2 + (b^4*c - 8*a*b^2*c
^2 + 16*a^2*c^3)*x^2 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*x)*log(x))/(a^3*b^4
 - 8*a^4*b^2*c + 16*a^5*c^2 + (a^2*b^4*c - 8*a^3*b^2*c^2 + 16*a^4*c^3)*x^2
+ (a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*x)]
```

giac [A] time = 0.36, size = 126, normalized size = 1.17

$$-\frac{\left(b^3-6\,abc\right)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{\left(a^2b^2-4\,a^3c\right)\sqrt{-b^2+4\,ac}}-\frac{\log\left(cx^2+bx+a\right)}{2\,a^2}+\frac{\log\left(|x|\right)}{a^2}+\frac{abcx+ab^2-2\,a^2c}{\left(cx^2+bx+a\right)\left(b^2-4\,ac\right)a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x^2+b/x)^2/x^5,x, algorithm="giac")
```

```
[Out] -(b^3 - 6*a*b*c)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/((a^2*b^2 - 4*a^3*c
)*sqrt(-b^2 + 4*a*c)) - 1/2*log(c*x^2 + b*x + a)/a^2 + log(abs(x))/a^2 + (a
*b*c*x + a*b^2 - 2*a^2*c)/((c*x^2 + b*x + a)*(b^2 - 4*a*c)*a^2)
```

maple [B] time = 0.01, size = 237, normalized size = 2.19

$$-\frac{bcx}{\left(cx^2+bx+a\right)\left(4ac-b^2\right)a}-\frac{6bc\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{\left(4ac-b^2\right)^{\frac{3}{2}}a}+\frac{b^3\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{\left(4ac-b^2\right)^{\frac{3}{2}}a^2}-\frac{b^2}{\left(cx^2+bx+a\right)\left(4ac-b^2\right)a}-\frac{2c\ln\left(cx^2+bx+a\right)}{\left(4ac-b^2\right)a}+\frac{b^2\ln\left(cx^2+bx+a\right)}{2\left(4ac-b^2\right)a^2}+\frac{2c}{\left(cx^2+bx+a\right)\left(4ac-b^2\right)}+\frac{\ln(x)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c+a/x^2+b/x)^2/x^5,x)
```

```
[Out] ln(x)/a^2-1/a/(c*x^2+b*x+a)*b*c/(4*a*c-b^2)*x+2/(c*x^2+b*x+a)/(4*a*c-b^2)*c
-1/a/(c*x^2+b*x+a)/(4*a*c-b^2)*b^2-2/a/(4*a*c-b^2)*c*ln(c*x^2+b*x+a)+1/2/a^
```

```
2/(4*a*c-b^2)*ln(c*x^2+b*x+a)*b^2-6/a/(4*a*c-b^2)^(3/2)*arctan((2*c*x+b)/(4
*a*c-b^2)^(1/2))*b*c+1/a^2/(4*a*c-b^2)^(3/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(
1/2))*b^3
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x^2+b/x)^2/x^5,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

mupad [B] time = 2.10, size = 620, normalized size = 5.74

$$\frac{\ln(x)}{a^2}+\frac{\frac{2ac-b^2}{a\left(4ac-b^2\right)}-\frac{bcx}{a\left(4ac-b^2\right)}}{cx^2+bx+a}+\frac{\ln\left(2ab^6+2b^7x-96a^4c^3+2ab^3\sqrt{-{\left(4ac-b^2\right)}^3}-23a^2b^4c+2b^4x\sqrt{-{\left(4ac-b^2\right)}^3}+84a^3b^2c^2+94a^2b^3c^2x+12a^2c^2x\sqrt{-{\left(4ac-b^2\right)}^3}-24ab^5cx-9a^2bc\sqrt{-{\left(4ac-b^2\right)}^3}-120a^3bc^3x-12ab^2cx\sqrt{-{\left(4ac-b^2\right)}^3}\right)\left(b^6-64a^3c^3+b^3\sqrt{-{\left(4ac-b^2\right)}^3}+48a^2b^2c^2-12ab^4c-6abc\sqrt{-{\left(4ac-b^2\right)}^3}\right)}{2a^2{\left(4ac-b^2\right)}^3}+\frac{\ln\left(96a^4c^3-2b^7x-2ab^6+2ab^3\sqrt{-{\left(4ac-b^2\right)}^3}+23a^2b^4c+2b^4x\sqrt{-{\left(4ac-b^2\right)}^3}-84a^3b^2c^2-94a^2b^3c^2x+12a^2c^2x\sqrt{-{\left(4ac-b^2\right)}^3}+24ab^5cx-9a^2bc\sqrt{-{\left(4ac-b^2\right)}^3}+120a^3bc^3x-12ab^2cx\sqrt{-{\left(4ac-b^2\right)}^3}\right)\left(b^6-64a^3c^3-b^3\sqrt{-{\left(4ac-b^2\right)}^3}+48a^2b^2c^2-12ab^4c+6abc\sqrt{-{\left(4ac-b^2\right)}^3}\right)}{2a^2{\left(4ac-b^2\right)}^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^5*(c + a/x^2 + b/x)^2),x)
```

```
[Out] log(x)/a^2 + ((2*a*c - b^2)/(a*(4*a*c - b^2)) - (b*c*x)/(a*(4*a*c - b^2)))/
(a + b*x + c*x^2) + (log(2*a*b^6 + 2*b^7*x - 96*a^4*c^3 + 2*a*b^3*(-(4*a*c
- b^2)^3)^(1/2) - 23*a^2*b^4*c + 2*b^4*x*(-(4*a*c - b^2)^3)^(1/2) + 84*a^3*
b^2*c^2 + 94*a^2*b^3*c^2*x + 12*a^2*c^2*x*(-(4*a*c - b^2)^3)^(1/2) - 24*a*b
^5*c*x - 9*a^2*b*c*(-(4*a*c - b^2)^3)^(1/2) - 120*a^3*b*c^3*x - 12*a*b^2*c*
x*(-(4*a*c - b^2)^3)^(1/2))*(b^6 - 64*a^3*c^3 + b^3*(-(4*a*c - b^2)^3)^(1/2
) + 48*a^2*b^2*c^2 - 12*a*b^4*c - 6*a*b*c*(-(4*a*c - b^2)^3)^(1/2)))/(2*a^2
*(4*a*c - b^2)^3) + (log(96*a^4*c^3 - 2*b^7*x - 2*a*b^6 + 2*a*b^3*(-(4*a*c
- b^2)^3)^(1/2) + 23*a^2*b^4*c + 2*b^4*x*(-(4*a*c - b^2)^3)^(1/2) - 84*a^3*
b^2*c^2 - 94*a^2*b^3*c^2*x + 12*a^2*c^2*x*(-(4*a*c - b^2)^3)^(1/2) + 24*a*b
^5*c*x - 9*a^2*b*c*(-(4*a*c - b^2)^3)^(1/2) + 120*a^3*b*c^3*x - 12*a*b^2*c*
x*(-(4*a*c - b^2)^3)^(1/2))*(b^6 - 64*a^3*c^3 - b^3*(-(4*a*c - b^2)^3)^(1/2
) + 48*a^2*b^2*c^2 - 12*a*b^4*c + 6*a*b*c*(-(4*a*c - b^2)^3)^(1/2)))/(2*a^2
*(4*a*c - b^2)^3)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x**2+b/x)**2/x**5,x)
```

```
[Out] Timed out
```

3.368 $\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2 x^6}\,dx$

Optimal. Leaf size=148

$$\frac{b\log\left(a+bx+cx^2\right)}{a^3}-\frac{2b\log(x)}{a^3}-\frac{2\left(b^2-3ac\right)}{a^2x\left(b^2-4ac\right)}-\frac{2\left(6a^2c^2-6ab^2c+b^4\right)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a^3\left(b^2-4ac\right)^{3/2}}+\frac{-2ac+b^2+}{ax\left(b^2-4ac\right)\left(a+\right.}$$

Rubi [A] time = 0.18, antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389, Rules used = {1354, 740, 800, 634, 618, 206, 628}

$$-\frac{2\left(6a^2c^2-6ab^2c+b^4\right)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a^3\left(b^2-4ac\right)^{3/2}}-\frac{2\left(b^2-3ac\right)}{a^2x\left(b^2-4ac\right)}+\frac{b\log\left(a+bx+cx^2\right)}{a^3}-\frac{2b\log(x)}{a^3}+\frac{-2ac+b^2+bcx}{ax\left(b^2-4ac\right)\left(a+bx+cx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c + a/x^2 + b/x)^2*x^6),x]
```

```
[Out] (-2*(b^2 - 3*a*c))/(a^2*(b^2 - 4*a*c)*x) + (b^2 - 2*a*c + b*c*x)/(a*(b^2 -
4*a*c)*x*(a + b*x + c*x^2)) - (2*(b^4 - 6*a*b^2*c + 6*a^2*c^2)*ArcTanh[(b +
 2*c*x)/Sqrt[b^2 - 4*a*c]])/(a^3*(b^2 - 4*a*c)^(3/2)) - (2*b*Log[x])/a^3 +
(b*Log[a + b*x + c*x^2])/a^3
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 740

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e
)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e
^2)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d +
 e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p +
 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4
```

```
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p,
-1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 1354

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> Int[x^(m + 2*n*p)*(c + b/x^n + a/x^(2*n))^p, x] /; FreeQ[{a, b, c, m, n
}, x] && EqQ[n2, 2*n] && ILtQ[p, 0] && NegQ[n]
```

Rubi steps

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2 x^6}\,dx = \int \frac{1}{x^2\left(a+bx+cx^2\right)^2}\,dx$$

$$= \frac{b^2-2ac+bcx}{a\left(b^2-4ac\right)x\left(a+bx+cx^2\right)} - \frac{\int \frac{-2\left(b^2-3ac\right)-2bcx}{x^2\left(a+bx+cx^2\right)}\,dx}{a\left(b^2-4ac\right)}$$

$$= \frac{b^2-2ac+bcx}{a\left(b^2-4ac\right)x\left(a+bx+cx^2\right)} - \frac{\int \left(\frac{2\left(-b^2+3ac\right)}{ax^2} - \frac{2b\left(-b^2+4ac\right)}{a^2x} + \frac{2\left(-b^4+5ab^2c-3a^2c^2-bc\left(b^2-4ac\right)x\right)}{a^2\left(a+bx+cx^2\right)}\right) \ldots}{a\left(b^2-4ac\right)}$$

$$= -\frac{2\left(b^2-3ac\right)}{a^2\left(b^2-4ac\right)x} + \frac{b^2-2ac+bcx}{a\left(b^2-4ac\right)x\left(a+bx+cx^2\right)} - \frac{2b\log(x)}{a^3} - \frac{2\int \frac{-b^4+5ab^2c-3a^2c^2-bc\left(b^2-\ldots\right)}{a+bx+cx^2}}{a^3\left(b^2-4ac\right)}$$

$$= -\frac{2\left(b^2-3ac\right)}{a^2\left(b^2-4ac\right)x} + \frac{b^2-2ac+bcx}{a\left(b^2-4ac\right)x\left(a+bx+cx^2\right)} - \frac{2b\log(x)}{a^3} + \frac{b\int \frac{b+2cx}{a+bx+cx^2}\,dx}{a^3} + \frac{\left(b^4-\ldots\right)}{\ldots}$$

$$= -\frac{2\left(b^2-3ac\right)}{a^2\left(b^2-4ac\right)x} + \frac{b^2-2ac+bcx}{a\left(b^2-4ac\right)x\left(a+bx+cx^2\right)} - \frac{2b\log(x)}{a^3} + \frac{b\log\left(a+bx+cx^2\right)}{a^3} - \ldots$$

$$= -\frac{2\left(b^2-3ac\right)}{a^2\left(b^2-4ac\right)x} + \frac{b^2-2ac+bcx}{a\left(b^2-4ac\right)x\left(a+bx+cx^2\right)} - \frac{2\left(b^4-6ab^2c+6a^2c^2\right)\tanh^{-1}\left(\frac{b+2c\ldots}{\sqrt{b^2-4\ldots}}\right)}{a^3\left(b^2-4ac\right)^{3/2}}$$

Mathematica [A] time = 0.27, size = 131, normalized size = 0.89

$$-\frac{\frac{2\left(6a^2c^2-6ab^2c+b^4\right)\tan^{-1}\left(\frac{b+2cx}{\sqrt{4ac-b^2}}\right)}{\left(4ac-b^2\right)^{3/2}} + \frac{a\left(-3abc-2ac^2x+b^3+b^2cx\right)}{\left(b^2-4ac\right)(a+x(b+cx))} - b\log(a+x(b+cx)) + \frac{a}{x} + 2b\log(x)}{a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c + a/x^2 + b/x)^2*x^6),x]
```

```
[Out] -((a/x + (a*(b^3 - 3*a*b*c + b^2*c*x - 2*a*c^2*x))/((b^2 - 4*a*c)*(a + x*(b
 + c*x))) + (2*(b^4 - 6*a*b^2*c + 6*a^2*c^2)*ArcTan[(b + 2*c*x)/Sqrt[-b^2 +
 4*a*c]])/(-b^2 + 4*a*c)^(3/2) + 2*b*Log[x] - b*Log[a + x*(b + c*x)])/a^3)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2 x^6}\, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/((c + a/x^2 + b/x)^2*x^6),x]
```

```
[Out] IntegrateAlgebraic[1/((c + a/x^2 + b/x)^2*x^6), x]
```

fricas [B] time = 1.79, size = 975, normalized size = 6.59

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x^2+b/x)^2/x^6,x, algorithm="fricas")
```

```
[Out] [-(a^2*b^4 - 8*a^3*b^2*c + 16*a^4*c^2 + 2*(a*b^4*c - 7*a^2*b^2*c^2 + 12*a^3
*c^3)*x^2 + ((b^4*c - 6*a*b^2*c^2 + 6*a^2*c^3)*x^3 + (b^5 - 6*a*b^3*c + 6*a
^2*b*c^2)*x^2 + (a*b^4 - 6*a^2*b^2*c + 6*a^3*c^2)*x)*sqrt(b^2 - 4*a*c)*log(
(2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c + sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2
+ b*x + a)) + (2*a*b^5 - 15*a^2*b^3*c + 28*a^3*b*c^2)*x - ((b^5*c - 8*a*b^3
*c^2 + 16*a^2*b*c^3)*x^3 + (b^6 - 8*a*b^4*c + 16*a^2*b^2*c^2)*x^2 + (a*b^5
- 8*a^2*b^3*c + 16*a^3*b*c^2)*x)*log(c*x^2 + b*x + a) + 2*((b^5*c - 8*a*b^3
*c^2 + 16*a^2*b*c^3)*x^3 + (b^6 - 8*a*b^4*c + 16*a^2*b^2*c^2)*x^2 + (a*b^5
- 8*a^2*b^3*c + 16*a^3*b*c^2)*x)*log(x))/((a^3*b^4*c - 8*a^4*b^2*c^2 + 16*a
^5*c^3)*x^3 + (a^3*b^5 - 8*a^4*b^3*c + 16*a^5*b*c^2)*x^2 + (a^4*b^4 - 8*a^5
*b^2*c + 16*a^6*c^2)*x), -(a^2*b^4 - 8*a^3*b^2*c + 16*a^4*c^2 + 2*(a*b^4*c
- 7*a^2*b^2*c^2 + 12*a^3*c^3)*x^2 + 2*((b^4*c - 6*a*b^2*c^2 + 6*a^2*c^3)*x^
3 + (b^5 - 6*a*b^3*c + 6*a^2*b*c^2)*x^2 + (a*b^4 - 6*a^2*b^2*c + 6*a^3*c^2)
*x)*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)
) + (2*a*b^5 - 15*a^2*b^3*c + 28*a^3*b*c^2)*x - ((b^5*c - 8*a*b^3*c^2 + 16*
a^2*b*c^3)*x^3 + (b^6 - 8*a*b^4*c + 16*a^2*b^2*c^2)*x^2 + (a*b^5 - 8*a^2*b^
3*c + 16*a^3*b*c^2)*x)*log(c*x^2 + b*x + a) + 2*((b^5*c - 8*a*b^3*c^2 + 16*
a^2*b*c^3)*x^3 + (b^6 - 8*a*b^4*c + 16*a^2*b^2*c^2)*x^2 + (a*b^5 - 8*a^2*b^
3*c + 16*a^3*b*c^2)*x)*log(x))/((a^3*b^4*c - 8*a^4*b^2*c^2 + 16*a^5*c^3)*x^
3 + (a^3*b^5 - 8*a^4*b^3*c + 16*a^5*b*c^2)*x^2 + (a^4*b^4 - 8*a^5*b^2*c + 1
6*a^6*c^2)*x)]
```

giac [A] time = 0.42, size = 171, normalized size = 1.16

$$\frac{2\left(b^4-6\,ab^2c+6\,a^2c^2\right)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{\left(a^3b^2-4\,a^4c\right)\sqrt{-b^2+4\,ac}}-\frac{2\,b^2cx^2-6\,ac^2x^2+2\,b^3x-7\,abcx+ab^2-4\,a^2c}{\left(a^2b^2-4\,a^3c\right)\left(cx^3+bx^2+ax\right)}+\frac{b\log\left(cx^2+bx+a\right)}{a^3}-\frac{2\,b\log\left(|x|\right)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x^2+b/x)^2/x^6,x, algorithm="giac")
```

```
[Out] 2*(b^4 - 6*a*b^2*c + 6*a^2*c^2)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/((a^
3*b^2 - 4*a^4*c)*sqrt(-b^2 + 4*a*c)) - (2*b^2*c*x^2 - 6*a*c^2*x^2 + 2*b^3*x
 - 7*a*b*c*x + a*b^2 - 4*a^2*c)/((a^2*b^2 - 4*a^3*c)*(c*x^3 + b*x^2 + a*x))
 + b*log(c*x^2 + b*x + a)/a^3 - 2*b*log(abs(x))/a^3
```

maple [B] time = 0.01, size = 328, normalized size = 2.22

$$-\frac{2c^2x}{\left(c\,x^2+bx+a\right)\left(4ac-b^2\right)a}-\frac{12c^2\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{\left(4ac-b^2\right)^{\frac{3}{2}}a}+\frac{b^2cx}{\left(c\,x^2+bx+a\right)\left(4ac-b^2\right)a^2}+\frac{12b^2c\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{\left(4ac-b^2\right)^{\frac{3}{2}}a^2}-\frac{2b^4\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{\left(4ac-b^2\right)^{\frac{3}{2}}a^3}-\frac{3bc}{\left(c\,x^2+bx+a\right)\left(4ac-b^2\right)a}+\frac{b^3}{\left(c\,x^2+bx+a\right)\left(4ac-b^2\right)a^2}+\frac{4bc\ln\left(c\,x^2+bx+a\right)}{\left(4ac-b^2\right)a^2}-\frac{b^3\ln\left(c\,x^2+bx+a\right)}{\left(4ac-b^2\right)a^3}-\frac{2b\ln(x)}{a^3}-\frac{1}{a^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c+a/x^2+b/x)^2/x^6,x)
```

```
[Out] -1/a^2/x-2*b*ln(x)/a^3-2/a/(c*x^2+b*x+a)*c^2/(4*a*c-b^2)*x+1/a^2/(c*x^2+b*x
+a)*c/(4*a*c-b^2)*x*b^2-3/a/(c*x^2+b*x+a)*b/(4*a*c-b^2)*c+1/a^2/(c*x^2+b*x+
a)*b^3/(4*a*c-b^2)+4/a^2/(4*a*c-b^2)*c*ln(c*x^2+b*x+a)*b-1/a^3/(4*a*c-b^2)*
ln(c*x^2+b*x+a)*b^3-12/a/(4*a*c-b^2)^(3/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/
2))*c^2+12/a^2/(4*a*c-b^2)^(3/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*b^2*c-
2/a^3/(4*a*c-b^2)^(3/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*b^4
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x^2+b/x)^2/x^6,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

mupad [B] time = 2.13, size = 775, normalized size = 5.24

$$\ln\left(2ab^7+2b^8x+2ab^4\sqrt{-{\left(4ac-b^2\right)}^3}-23a^2b^5c-108a^4bc^3+24a^4c^4x+2b^5x\sqrt{-{\left(4ac-b^2\right)}^3}+87a^3b^3c^2+3a^3c^2\sqrt{-{\left(4ac-b^2\right)}^3}-9a^2b^2c\sqrt{-{\left(4ac-b^2\right)}^3}+97a^2b^4c^2x-138a^3b^2c^3x-24ab^6cx-12ab^3cx\sqrt{-{\left(4ac-b^2\right)}^3}+15a^2bc^2x\sqrt{-{\left(4ac-b^2\right)}^3}\right)\left(\frac{b^4\sqrt{-{\left(4ac-b^2\right)}^3}+6a^2c^2\sqrt{-{\left(4ac-b^2\right)}^3}-6ab^2c\sqrt{-{\left(4ac-b^2\right)}^3}}{-64a^6c^3+48a^5b^2c^2-12a^4b^4c+a^3b^6}+\frac{b}{a^3}\right)-\frac{\frac{1}{a}-\frac{x\left(2b^3-7abc\right)}{a^2\left(4ac-b^2\right)}+\frac{2cx^2\left(3ac-b^2\right)}{a^2\left(4ac-b^2\right)}}{cx^3+bx^2+ax}-\ln\left(2ab^4\sqrt{-{\left(4ac-b^2\right)}^3}-2b^8x-2ab^7+23a^2b^5c+108a^4bc^3-24a^4c^4x+2b^5x\sqrt{-{\left(4ac-b^2\right)}^3}-87a^3b^3c^2+3a^3c^2\sqrt{-{\left(4ac-b^2\right)}^3}-9a^2b^2c\sqrt{-{\left(4ac-b^2\right)}^3}-97a^2b^4c^2x+138a^3b^2c^3x+24ab^6cx-12ab^3cx\sqrt{-{\left(4ac-b^2\right)}^3}+15a^2bc^2x\sqrt{-{\left(4ac-b^2\right)}^3}\right)\left(\frac{b^4\sqrt{-{\left(4ac-b^2\right)}^3}+6a^2c^2\sqrt{-{\left(4ac-b^2\right)}^3}-6ab^2c\sqrt{-{\left(4ac-b^2\right)}^3}}{-64a^6c^3+48a^5b^2c^2-12a^4b^4c+a^3b^6}-\frac{b}{a^3}\right)-\frac{2b\ln\left(x\right)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^6*(c + a/x^2 + b/x)^2),x)
```

```
[Out] log(2*a*b^7 + 2*b^8*x + 2*a*b^4*(-(4*a*c - b^2)^3)^(1/2) - 23*a^2*b^5*c - 1
08*a^4*b*c^3 + 24*a^4*c^4*x + 2*b^5*x*(-(4*a*c - b^2)^3)^(1/2) + 87*a^3*b^3
*c^2 + 3*a^3*c^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a^2*b^2*c*(-(4*a*c - b^2)^3)^
(1/2) + 97*a^2*b^4*c^2*x - 138*a^3*b^2*c^3*x - 24*a*b^6*c*x - 12*a*b^3*c*x*
(-(4*a*c - b^2)^3)^(1/2) + 15*a^2*b*c^2*x*(-(4*a*c - b^2)^3)^(1/2))*((b^4*(
-(4*a*c - b^2)^3)^(1/2) + 6*a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c*(-
(4*a*c - b^2)^3)^(1/2))/(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*c
^2) + b/a^3) - (1/a - (x*(2*b^3 - 7*a*b*c))/(a^2*(4*a*c - b^2)) + (2*c*x^2*
(3*a*c - b^2))/(a^2*(4*a*c - b^2)))/(a*x + b*x^2 + c*x^3) - log(2*a*b^4*(-(
4*a*c - b^2)^3)^(1/2) - 2*b^8*x - 2*a*b^7 + 23*a^2*b^5*c + 108*a^4*b*c^3 -
24*a^4*c^4*x + 2*b^5*x*(-(4*a*c - b^2)^3)^(1/2) - 87*a^3*b^3*c^2 + 3*a^3*c^
2*(-(4*a*c - b^2)^3)^(1/2) - 9*a^2*b^2*c*(-(4*a*c - b^2)^3)^(1/2) - 97*a^2*
b^4*c^2*x + 138*a^3*b^2*c^3*x + 24*a*b^6*c*x - 12*a*b^3*c*x*(-(4*a*c - b^2)
^3)^(1/2) + 15*a^2*b*c^2*x*(-(4*a*c - b^2)^3)^(1/2))*((b^4*(-(4*a*c - b^2)^
3)^(1/2) + 6*a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c*(-(4*a*c - b^2)^3
)^(1/2))/(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*c^2) - b/a^3) -
(2*b*log(x))/a^3
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x**2+b/x)**2/x**6,x)
```

```
[Out] Timed out
```

3.369 $\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2 x^7}\,dx$

Optimal. Leaf size=202

$$-\frac{\left(3b^2-2ac\right)\log\left(a+bx+cx^2\right)}{2a^4}+\frac{\log(x)\left(3b^2-2ac\right)}{a^4}+\frac{b\left(3b^2-11ac\right)}{a^3x\left(b^2-4ac\right)}-\frac{3b^2-8ac}{2a^2x^2\left(b^2-4ac\right)}+\frac{b\left(30a^2c^2-20ab^2c+\right.}{a^4\left(b^2\right.}$$

Rubi [A] time = 0.24, antiderivative size = 202, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389, Rules used = {1354, 740, 800, 634, 618, 206, 628}

$$\frac{b\left(30a^2c^2-20ab^2c+3b^4\right)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a^4\left(b^2-4ac\right)^{3/2}}-\frac{3b^2-8ac}{2a^2x^2\left(b^2-4ac\right)}-\frac{\left(3b^2-2ac\right)\log\left(a+bx+cx^2\right)}{2a^4}+\frac{b\left(3b^2-11ac\right)}{a^3x\left(b^2-4ac\right)}+\frac{\log(x)\left(3b^2-2ac\right)}{a^4}+\frac{-2ac+b^2+bcx}{ax^2\left(b^2-4ac\right)\left(a+bx+cx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c + a/x^2 + b/x)^2*x^7),x]
```

```
[Out] -(3*b^2 - 8*a*c)/(2*a^2*(b^2 - 4*a*c)*x^2) + (b*(3*b^2 - 11*a*c))/(a^3*(b^2
 - 4*a*c)*x) + (b^2 - 2*a*c + b*c*x)/(a*(b^2 - 4*a*c)*x^2*(a + b*x + c*x^2)
) + (b*(3*b^4 - 20*a*b^2*c + 30*a^2*c^2)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a
*c]])/(a^4*(b^2 - 4*a*c)^(3/2)) + ((3*b^2 - 2*a*c)*Log[x])/a^4 - ((3*b^2 -
2*a*c)*Log[a + b*x + c*x^2])/(2*a^4)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 740

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e
)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e
^2)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d +
 e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p +
 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4
```

```
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p,
-1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 1354

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> Int[x^(m + 2*n*p)*(c + b/x^n + a/x^(2*n))^p, x] /; FreeQ[{a, b, c, m, n
}, x] && EqQ[n2, 2*n] && ILtQ[p, 0] && NegQ[n]
```

Rubi steps

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2 x^7}\,dx = \int \frac{1}{x^3\left(a+bx+cx^2\right)^2}\,dx$$

$$= \frac{b^2-2ac+bcx}{a\left(b^2-4ac\right)x^2\left(a+bx+cx^2\right)} - \frac{\int \frac{-3b^2+8ac-3bcx}{x^3\left(a+bx+cx^2\right)}\,dx}{a\left(b^2-4ac\right)}$$

$$= \frac{b^2-2ac+bcx}{a\left(b^2-4ac\right)x^2\left(a+bx+cx^2\right)} - \frac{\int \left(\frac{-3b^2+8ac}{ax^3}+\frac{3b^3-11abc}{a^2x^2}+\frac{\left(b^2-4ac\right)\left(-3b^2+2ac\right)}{a^3x}+\frac{b\left(3b^4-17ab^2c\right.}{}\right.}{a\left(b^2-4ac\right)}$$

$$= -\frac{3b^2-8ac}{2a^2\left(b^2-4ac\right)x^2} + \frac{b\left(3b^2-11ac\right)}{a^3\left(b^2-4ac\right)x} + \frac{b^2-2ac+bcx}{a\left(b^2-4ac\right)x^2\left(a+bx+cx^2\right)} + \frac{\left(3b^2-2ac\right)\log}{a^4}$$

$$= -\frac{3b^2-8ac}{2a^2\left(b^2-4ac\right)x^2} + \frac{b\left(3b^2-11ac\right)}{a^3\left(b^2-4ac\right)x} + \frac{b^2-2ac+bcx}{a\left(b^2-4ac\right)x^2\left(a+bx+cx^2\right)} + \frac{\left(3b^2-2ac\right)\log}{a^4}$$

$$= -\frac{3b^2-8ac}{2a^2\left(b^2-4ac\right)x^2} + \frac{b\left(3b^2-11ac\right)}{a^3\left(b^2-4ac\right)x} + \frac{b^2-2ac+bcx}{a\left(b^2-4ac\right)x^2\left(a+bx+cx^2\right)} + \frac{\left(3b^2-2ac\right)\log}{a^4}$$

$$= -\frac{3b^2-8ac}{2a^2\left(b^2-4ac\right)x^2} + \frac{b\left(3b^2-11ac\right)}{a^3\left(b^2-4ac\right)x} + \frac{b^2-2ac+bcx}{a\left(b^2-4ac\right)x^2\left(a+bx+cx^2\right)} + \frac{b\left(3b^4-20ab^2c\right.}{a}$$

Mathematica [A] time = 0.34, size = 175, normalized size = 0.87

$$\frac{\frac{2b\left(30a^2c^2-20ab^2c+3b^4\right)\tan^{-1}\left(\frac{b+2cx}{\sqrt{4ac-b^2}}\right)}{\left(4ac-b^2\right)^{3/2}}+\frac{2a\left(2a^2c^2-4ab^2c-3abc^2x+b^4+b^3cx\right)}{\left(b^2-4ac\right)\left(a+x(b+cx)\right)}-\frac{a^2}{x^2}+2\log(x)\left(3b^2-2ac\right)+\left(2ac-3b^2\right)\log(a+x(b+cx))+\frac{4ab}{x}}{2a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c + a/x^2 + b/x)^2*x^7),x]
```

```
[Out] (-(a^2/x^2) + (4*a*b)/x + (2*a*(b^4 - 4*a*b^2*c + 2*a^2*c^2 + b^3*c*x - 3*a
*b*c^2*x))/((b^2 - 4*a*c)*(a + x*(b + c*x))) + (2*b*(3*b^4 - 20*a*b^2*c + 3
0*a^2*c^2)*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/(-b^2 + 4*a*c)^(3/2) + 2
*(3*b^2 - 2*a*c)*Log[x] + (-3*b^2 + 2*a*c)*Log[a + x*(b + c*x)])/(2*a^4)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2 x^7}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/((c + a/x^2 + b/x)^2*x^7),x]
```

```
[Out] IntegrateAlgebraic[1/((c + a/x^2 + b/x)^2*x^7), x]
```

fricas [B] time = 2.00, size = 1226, normalized size = 6.07

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x^2+b/x)^2/x^7,x, algorithm="fricas")
```

```
[Out] [-1/2*(a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2 - 2*(3*a*b^5*c - 23*a^2*b^3*c^2 +
 44*a^3*b*c^3)*x^3 - (6*a*b^6 - 49*a^2*b^4*c + 108*a^3*b^2*c^2 - 32*a^4*c^3
)*x^2 + ((3*b^5*c - 20*a*b^3*c^2 + 30*a^2*b*c^3)*x^4 + (3*b^6 - 20*a*b^4*c
+ 30*a^2*b^2*c^2)*x^3 + (3*a*b^5 - 20*a^2*b^3*c + 30*a^3*b*c^2)*x^2)*sqrt(b
^2 - 4*a*c)*log((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c - sqrt(b^2 - 4*a*c)*(2*c
*x + b))/(c*x^2 + b*x + a)) - 3*(a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*x +
((3*b^6*c - 26*a*b^4*c^2 + 64*a^2*b^2*c^3 - 32*a^3*c^4)*x^4 + (3*b^7 - 26*a
*b^5*c + 64*a^2*b^3*c^2 - 32*a^3*b*c^3)*x^3 + (3*a*b^6 - 26*a^2*b^4*c + 64*
a^3*b^2*c^2 - 32*a^4*c^3)*x^2)*log(c*x^2 + b*x + a) - 2*((3*b^6*c - 26*a*b^
4*c^2 + 64*a^2*b^2*c^3 - 32*a^3*c^4)*x^4 + (3*b^7 - 26*a*b^5*c + 64*a^2*b^3
*c^2 - 32*a^3*b*c^3)*x^3 + (3*a*b^6 - 26*a^2*b^4*c + 64*a^3*b^2*c^2 - 32*a^
4*c^3)*x^2)*log(x))/((a^4*b^4*c - 8*a^5*b^2*c^2 + 16*a^6*c^3)*x^4 + (a^4*b^
5 - 8*a^5*b^3*c + 16*a^6*b*c^2)*x^3 + (a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*
x^2), -1/2*(a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2 - 2*(3*a*b^5*c - 23*a^2*b^3*
c^2 + 44*a^3*b*c^3)*x^3 - (6*a*b^6 - 49*a^2*b^4*c + 108*a^3*b^2*c^2 - 32*a^
4*c^3)*x^2 - 2*((3*b^5*c - 20*a*b^3*c^2 + 30*a^2*b*c^3)*x^4 + (3*b^6 - 20*a
*b^4*c + 30*a^2*b^2*c^2)*x^3 + (3*a*b^5 - 20*a^2*b^3*c + 30*a^3*b*c^2)*x^2)
*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) -
 3*(a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*x + ((3*b^6*c - 26*a*b^4*c^2 + 64
*a^2*b^2*c^3 - 32*a^3*c^4)*x^4 + (3*b^7 - 26*a*b^5*c + 64*a^2*b^3*c^2 - 32*
a^3*b*c^3)*x^3 + (3*a*b^6 - 26*a^2*b^4*c + 64*a^3*b^2*c^2 - 32*a^4*c^3)*x^2
)*log(c*x^2 + b*x + a) - 2*((3*b^6*c - 26*a*b^4*c^2 + 64*a^2*b^2*c^3 - 32*a
^3*c^4)*x^4 + (3*b^7 - 26*a*b^5*c + 64*a^2*b^3*c^2 - 32*a^3*b*c^3)*x^3 + (3
*a*b^6 - 26*a^2*b^4*c + 64*a^3*b^2*c^2 - 32*a^4*c^3)*x^2)*log(x))/((a^4*b^4
*c - 8*a^5*b^2*c^2 + 16*a^6*c^3)*x^4 + (a^4*b^5 - 8*a^5*b^3*c + 16*a^6*b*c^
2)*x^3 + (a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*x^2)]
```

giac [A] time = 0.33, size = 229, normalized size = 1.13

$$-\frac{\left(3\,b^5-20\,ab^3c+30\,a^2bc^2\right)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{\left(a^4b^2-4\,a^5c\right)\sqrt{-b^2+4\,ac}}-\frac{\left(3\,b^2-2\,ac\right)\log\left(cx^2+bx+a\right)}{2\,a^4}+\frac{\left(3\,b^2-2\,ac\right)\log\left(|x|\right)}{a^4}-\frac{a^3b^2-4\,a^4c-2\left(3\,ab^3c-11\,a^2bc^2\right)x^3-\left(6\,ab^4-25\,a^2b^2c+8\,a^3c^2\right)x^2-3\left(a^2b^3-4\,a^3bc\right)x}{2\left(cx^2+bx+a\right)\left(b^2-4\,ac\right)a^4x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x^2+b/x)^2/x^7,x, algorithm="giac")
```

```
[Out] -(3*b^5 - 20*a*b^3*c + 30*a^2*b*c^2)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))
/((a^4*b^2 - 4*a^5*c)*sqrt(-b^2 + 4*a*c)) - 1/2*(3*b^2 - 2*a*c)*log(c*x^2 +
 b*x + a)/a^4 + (3*b^2 - 2*a*c)*log(abs(x))/a^4 - 1/2*(a^3*b^2 - 4*a^4*c -
2*(3*a*b^3*c - 11*a^2*b*c^2)*x^3 - (6*a*b^4 - 25*a^2*b^2*c + 8*a^3*c^2)*x^2
 - 3*(a^2*b^3 - 4*a^3*b*c)*x)/((c*x^2 + b*x + a)*(b^2 - 4*a*c)*a^4*x^2)
```

maple [B] time = 0.02, size = 418, normalized size = 2.07

$$\frac{3b\,c^2x}{\left(c\,x^2+bx+a\right)\left(4ac-b^2\right)a^2}+\frac{30b\,c^2\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{\left(4ac-b^2\right)^{\frac{3}{2}}a^2}-\frac{b^3cx}{\left(c\,x^2+bx+a\right)\left(4ac-b^2\right)a^3}-\frac{20b^3c\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{\left(4ac-b^2\right)^{\frac{3}{2}}a^3}+\frac{3b^5\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{\left(4ac-b^2\right)^{\frac{3}{2}}a^4}-\frac{2c^2}{\left(c\,x^2+bx+a\right)\left(4ac-b^2\right)a}+\frac{4b^2c}{\left(c\,x^2+bx+a\right)\left(4ac-b^2\right)a^2}+\frac{4c^2\ln\left(c\,x^2+bx+a\right)}{\left(4ac-b^2\right)a^2}-\frac{b^4}{\left(c\,x^2+bx+a\right)\left(4ac-b^2\right)a^3}-\frac{7b^2c\ln\left(c\,x^2+bx+a\right)}{\left(4ac-b^2\right)a^3}+\frac{3b^4\ln\left(c\,x^2+bx+a\right)}{2\left(4ac-b^2\right)a^4}-\frac{2c\ln(x)}{a^3}+\frac{3b^2\ln(x)}{a^4}+\frac{2b}{a^3x}-\frac{1}{2a^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c+a/x^2+b/x)^2/x^7,x)
```

```
[Out] -1/2/a^2/x^2-2/a^3*ln(x)*c+3/a^4*ln(x)*b^2+2/a^3*b/x+3/a^2/(c*x^2+b*x+a)*b*
c^2/(4*a*c-b^2)*x-1/a^3/(c*x^2+b*x+a)*b^3*c/(4*a*c-b^2)*x-2/a/(c*x^2+b*x+a)
/(4*a*c-b^2)*c^2+4/a^2/(c*x^2+b*x+a)/(4*a*c-b^2)*b^2*c-1/a^3/(c*x^2+b*x+a)/
(4*a*c-b^2)*b^4+4/a^2/(4*a*c-b^2)*c^2*ln(c*x^2+b*x+a)-7/a^3/(4*a*c-b^2)*c*l
n(c*x^2+b*x+a)*b^2+3/2/a^4/(4*a*c-b^2)*ln(c*x^2+b*x+a)*b^4+30/a^2/(4*a*c-b^
2)^(3/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*b*c^2-20/a^3/(4*a*c-b^2)^(3/2)
*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*b^3*c+3/a^4/(4*a*c-b^2)^(3/2)*arctan((
2*c*x+b)/(4*a*c-b^2)^(1/2))*b^5
```

maxima [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x^2+b/x)^2/x^7,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

mupad [**B**] time = 2.30, size = 914, normalized size = 4.52

$$\frac{\ln\left(6ab^8+6b^9x+192a^5c^4-6ab^5\sqrt{-{\left(4ac-b^2\right)}^3}-73a^2b^6c-6b^6x\sqrt{-{\left(4ac-b^2\right)}^3}+307a^3b^4c^2-492a^4b^2c^3+31a^2b^3c\sqrt{-{\left(4ac-b^2\right)}^3}-27a^3bc^2\sqrt{-{\left(4ac-b^2\right)}^3}+339a^2b^5c^2x-602a^3b^3c^3x+24a^3c^3x\sqrt{-{\left(4ac-b^2\right)}^3}-76ab^7cx+312a^4bc^4x+40ab^4cx\sqrt{-{\left(4ac-b^2\right)}^3}-69a^2b^2c^2x\sqrt{-{\left(4ac-b^2\right)}^3}\right)\left(3b^8+128a^4c^4-3b^5\sqrt{-{\left(4ac-b^2\right)}^3}+168a^2b^4c^2-288a^3b^2c^3-38ab^6c-30a^2bc^2\sqrt{-{\left(4ac-b^2\right)}^3}+20ab^3c\sqrt{-{\left(4ac-b^2\right)}^3}\right)}{2a^4{\left(4ac-b^2\right)}^3}-\frac{\ln\left(x\right)\left(2ac-3b^2\right)}{a^4}-\frac{\frac{1}{2a}-\frac{3bx}{2a^2}+\frac{x^2\left(6b^4+8a^2c^2-25ab^2c\right)}{2a^3\left(4ac-b^2\right)}-\frac{bcx^3\left(11ac-3b^2\right)}{a^3\left(4ac-b^2\right)}}{cx^4+bx^3+ax^2}+\frac{\ln\left(6ab^8+6b^9x+192a^5c^4+6ab^5\sqrt{-{\left(4ac-b^2\right)}^3}-73a^2b^6c+6b^6x\sqrt{-{\left(4ac-b^2\right)}^3}+307a^3b^4c^2-492a^4b^2c^3-31a^2b^3c\sqrt{-{\left(4ac-b^2\right)}^3}+27a^3bc^2\sqrt{-{\left(4ac-b^2\right)}^3}+339a^2b^5c^2x-602a^3b^3c^3x-24a^3c^3x\sqrt{-{\left(4ac-b^2\right)}^3}-76ab^7cx+312a^4bc^4x-40ab^4cx\sqrt{-{\left(4ac-b^2\right)}^3}+69a^2b^2c^2x\sqrt{-{\left(4ac-b^2\right)}^3}\right)\left(3b^8+128a^4c^4+3b^5\sqrt{-{\left(4ac-b^2\right)}^3}+168a^2b^4c^2-288a^3b^2c^3-38ab^6c+30a^2bc^2\sqrt{-{\left(4ac-b^2\right)}^3}-20ab^3c\sqrt{-{\left(4ac-b^2\right)}^3}\right)}{2a^4{\left(4ac-b^2\right)}^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^7*(c + a/x^2 + b/x)^2),x)
```

```
[Out] (log(6*a*b^8 + 6*b^9*x + 192*a^5*c^4 - 6*a*b^5*(-(4*a*c - b^2)^3)^(1/2) - 7
3*a^2*b^6*c - 6*b^6*x*(-(4*a*c - b^2)^3)^(1/2) + 307*a^3*b^4*c^2 - 492*a^4*
b^2*c^3 + 31*a^2*b^3*c*(-(4*a*c - b^2)^3)^(1/2) - 27*a^3*b*c^2*(-(4*a*c - b
^2)^3)^(1/2) + 339*a^2*b^5*c^2*x - 602*a^3*b^3*c^3*x + 24*a^3*c^3*x*(-(4*a*
c - b^2)^3)^(1/2) - 76*a*b^7*c*x + 312*a^4*b*c^4*x + 40*a*b^4*c*x*(-(4*a*c
- b^2)^3)^(1/2) - 69*a^2*b^2*c^2*x*(-(4*a*c - b^2)^3)^(1/2))*(3*b^8 + 128*a
^4*c^4 - 3*b^5*(-(4*a*c - b^2)^3)^(1/2) + 168*a^2*b^4*c^2 - 288*a^3*b^2*c^3
 - 38*a*b^6*c - 30*a^2*b*c^2*(-(4*a*c - b^2)^3)^(1/2) + 20*a*b^3*c*(-(4*a*c
 - b^2)^3)^(1/2)))/(2*a^4*(4*a*c - b^2)^3) - (log(x)*(2*a*c - 3*b^2))/a^4 -
 (1/(2*a) - (3*b*x)/(2*a^2) + (x^2*(6*b^4 + 8*a^2*c^2 - 25*a*b^2*c))/(2*a^3
*(4*a*c - b^2)) - (b*c*x^3*(11*a*c - 3*b^2))/(a^3*(4*a*c - b^2)))/(a*x^2 +
b*x^3 + c*x^4) + (log(6*a*b^8 + 6*b^9*x + 192*a^5*c^4 + 6*a*b^5*(-(4*a*c -
b^2)^3)^(1/2) - 73*a^2*b^6*c + 6*b^6*x*(-(4*a*c - b^2)^3)^(1/2) + 307*a^3*b
^4*c^2 - 492*a^4*b^2*c^3 - 31*a^2*b^3*c*(-(4*a*c - b^2)^3)^(1/2) + 27*a^3*b
*c^2*(-(4*a*c - b^2)^3)^(1/2) + 339*a^2*b^5*c^2*x - 602*a^3*b^3*c^3*x - 24*
a^3*c^3*x*(-(4*a*c - b^2)^3)^(1/2) - 76*a*b^7*c*x + 312*a^4*b*c^4*x - 40*a*
b^4*c*x*(-(4*a*c - b^2)^3)^(1/2) + 69*a^2*b^2*c^2*x*(-(4*a*c - b^2)^3)^(1/2
))*(3*b^8 + 128*a^4*c^4 + 3*b^5*(-(4*a*c - b^2)^3)^(1/2) + 168*a^2*b^4*c^2
- 288*a^3*b^2*c^3 - 38*a*b^6*c + 30*a^2*b*c^2*(-(4*a*c - b^2)^3)^(1/2) - 20
*a*b^3*c*(-(4*a*c - b^2)^3)^(1/2)))/(2*a^4*(4*a*c - b^2)^3)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x**2+b/x)**2/x**7,x)
```

```
[Out] Timed out
```

3.370 $\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^3}\,dx$

Optimal. Leaf size=238

$$\frac{3x\left(10a^2c^2-7ab^2c+b^4\right)}{c^3\left(b^2-4ac\right)^2}-\frac{3\left(-20a^3c^3+30a^2b^2c^2-10ab^4c+b^6\right)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^4\left(b^2-4ac\right)^{5/2}}-\frac{3bx^2\left(b^2-6ac\right)}{2c^2\left(b^2-4ac\right)^2}+\frac{}{2\left(b^2-\right.}$$

Rubi [A] time = 0.29, antiderivative size = 238, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {1340, 738, 818, 800, 634, 618, 206, 628}

$$\frac{3x\left(10a^2c^2-7ab^2c+b^4\right)}{c^3\left(b^2-4ac\right)^2}-\frac{3\left(30a^2b^2c^2-20a^3c^3-10ab^4c+b^6\right)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^4\left(b^2-4ac\right)^{5/2}}-\frac{3bx^2\left(b^2-6ac\right)}{2c^2\left(b^2-4ac\right)^2}+\frac{x^5(2a+bx)}{2\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2}+\frac{x^3\left(bx\left(b^2-7ac\right)+a\left(b^2-10ac\right)\right)}{c\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)}-\frac{3b\log\left(a+bx+cx^2\right)}{2c^4}$$

Antiderivative was successfully verified.

```
[In] Int[(c + a/x^2 + b/x)^(-3),x]
```

```
[Out] (3*(b^4 - 7*a*b^2*c + 10*a^2*c^2)*x)/(c^3*(b^2 - 4*a*c)^2) - (3*b*(b^2 - 6*
a*c)*x^2)/(2*c^2*(b^2 - 4*a*c)^2) + (x^5*(2*a + b*x))/(2*(b^2 - 4*a*c)*(a +
 b*x + c*x^2)^2) + (x^3*(a*(b^2 - 10*a*c) + b*(b^2 - 7*a*c)*x))/(c*(b^2 - 4
*a*c)^2*(a + b*x + c*x^2)) - (3*(b^6 - 10*a*b^4*c + 30*a^2*b^2*c^2 - 20*a^3
*c^3)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(c^4*(b^2 - 4*a*c)^(5/2)) - (
3*b*Log[a + b*x + c*x^2])/(2*c^4)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 738

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m - 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*(a + b*x
+ c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*
c)), Int[(d + e*x)^(m - 2)*Simp[e*(2*a*e*(m - 1) + b*d*(2*p - m + 4)) - 2*c
*d^2*(2*p + 3) + e*(b*e - 2*d*c)*(m + 2*p + 2)*x, x]*(a + b*x + c*x^2)^(p +
 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^
```

```
2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p, -1] && GtQ[m, 1] &&
IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 818

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((d + e*x)^(m - 1)*(a + b*x + c*x^2)^
(p + 1)*(2*a*c*(e*f + d*g) - b*(c*d*f + a*e*g) - (2*c^2*d*f + b^2*e*g - c*(
b*e*f + b*d*g + 2*a*e*g))*x))/(c*(p + 1)*(b^2 - 4*a*c)), x] - Dist[1/(c*(p
+ 1)*(b^2 - 4*a*c)), Int[(d + e*x)^(m - 2)*(a + b*x + c*x^2)^(p + 1)*Simp[2
*c^2*d^2*f*(2*p + 3) + b*e*g*(a*e*(m - 1) + b*d*(p + 2)) - c*(2*a*e*(e*f*(m
 - 1) + d*g*m) + b*d*(d*g*(2*p + 3) - e*f*(m - 2*p - 4))) + e*(b^2*e*g*(m +
 p + 1) + 2*c^2*d*f*(m + 2*p + 2) - c*(2*a*e*g*m + b*(e*f + d*g)*(m + 2*p +
 2)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c,
0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] && ((EqQ[m,
2] && EqQ[p, -3] && RationalQ[a, b, c, d, e, f, g]) ||  !ILtQ[m + 2*p + 3,
0])
```

Rule 1340

```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[x^
(2*n*p)*(c + b/x^n + a/x^(2*n))^p, x] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n
] && LtQ[n, 0] && IntegerQ[p]
```

Rubi steps

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^3}\,dx=\int \frac{x^6}{\left(a+bx+cx^2\right)^3}\,dx$$

$$=\frac{x^5(2a+bx)}{2\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2}-\frac{\int \frac{x^4(10a+2bx)}{\left(a+bx+cx^2\right)^2}\,dx}{2\left(b^2-4ac\right)}$$

$$=\frac{x^5(2a+bx)}{2\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2}+\frac{x^3\left(a\left(b^2-10ac\right)+b\left(b^2-7ac\right)x\right)}{c\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)}-\frac{\int \frac{x^2\left(6a\left(b^2-10ac\right)+6b\left(b\right.\right.}{a+bx+cx^2}}{2c\left(b^2-4ac\right.}$$

$$=\frac{x^5(2a+bx)}{2\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2}+\frac{x^3\left(a\left(b^2-10ac\right)+b\left(b^2-7ac\right)x\right)}{c\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)}-\frac{\int\left(-\frac{6\left(b^4-7ab^2c+10a^2c\right.}{c^2}\right.}{}$$

$$=\frac{3\left(b^4-7ab^2c+10a^2c^2\right)x}{c^3\left(b^2-4ac\right)^2}-\frac{3b\left(b^2-6ac\right)x^2}{2c^2\left(b^2-4ac\right)^2}+\frac{x^5(2a+bx)}{2\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2}+\frac{x^3\left(a\left(b^2\right.\right.}{c\left(b^2\right.}$$

$$=\frac{3\left(b^4-7ab^2c+10a^2c^2\right)x}{c^3\left(b^2-4ac\right)^2}-\frac{3b\left(b^2-6ac\right)x^2}{2c^2\left(b^2-4ac\right)^2}+\frac{x^5(2a+bx)}{2\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2}+\frac{x^3\left(a\left(b^2\right.\right.}{c\left(b^2\right.}$$

$$=\frac{3\left(b^4-7ab^2c+10a^2c^2\right)x}{c^3\left(b^2-4ac\right)^2}-\frac{3b\left(b^2-6ac\right)x^2}{2c^2\left(b^2-4ac\right)^2}+\frac{x^5(2a+bx)}{2\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2}+\frac{x^3\left(a\left(b^2\right.\right.}{c\left(b^2\right.}$$

$$=\frac{3\left(b^4-7ab^2c+10a^2c^2\right)x}{c^3\left(b^2-4ac\right)^2}-\frac{3b\left(b^2-6ac\right)x^2}{2c^2\left(b^2-4ac\right)^2}+\frac{x^5(2a+bx)}{2\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2}+\frac{x^3\left(a\left(b^2\right.\right.}{c\left(b^2\right.}$$

Mathematica [A] time = 0.37, size = 260, normalized size = 1.09

$$\frac{\frac{a^3c^2(2cx-5b)+a^2b^2c(5b-9cx)-ab^4(b-6cx)+b^6(-x)}{\left(b^2-4ac\right)(a+x(b+cx))^2}+\frac{6c\left(-20a^3c^3+30a^2b^2c^2-10ab^4c+b^6\right)\tan^{-1}\left(\frac{b+2cx}{\sqrt{4ac-b^2}}\right)}{\left(4ac-b^2\right)^{5/2}}+\frac{-78a^3bc^3+36a^3c^4x+61a^2b^3c^2-102a^2b^2c^3x-14ab^5c+48ab^4c^2x+b^7-6b^6cx}{\left(b^2-4ac\right)^2(a+x(b+cx))}-3bc\log(a+x(b+cx))+2c^2x}{2c^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + a/x^2 + b/x)^(-3),x]
```

```
[Out] (2*c^2*x + (b^7 - 14*a*b^5*c + 61*a^2*b^3*c^2 - 78*a^3*b*c^3 - 6*b^6*c*x +
48*a*b^4*c^2*x - 102*a^2*b^2*c^3*x + 36*a^3*c^4*x)/((b^2 - 4*a*c)^2*(a + x*
(b + c*x))) + (-(b^6*x) + a^2*b^2*c*(5*b - 9*c*x) - a*b^4*(b - 6*c*x) + a^3
*c^2*(-5*b + 2*c*x))/((b^2 - 4*a*c)*(a + x*(b + c*x))^2) + (6*c*(b^6 - 10*a
*b^4*c + 30*a^2*b^2*c^2 - 20*a^3*c^3)*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]
])/(-b^2 + 4*a*c)^(5/2) - 3*b*c*Log[a + x*(b + c*x)])/(2*c^5)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + a/x^2 + b/x)^(-3),x]
```

```
[Out] IntegrateAlgebraic[(c + a/x^2 + b/x)^(-3), x]
```

fricas [B] time = 1.01, size = 1926, normalized size = 8.09

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x^2+b/x)^3,x, algorithm="fricas")
```

```
[Out] [-1/2*(5*a^2*b^7 - 56*a^3*b^5*c + 202*a^4*b^3*c^2 - 232*a^5*b*c^3 - 2*(b^6*
c^3 - 12*a*b^4*c^4 + 48*a^2*b^2*c^5 - 64*a^3*c^6)*x^5 - 4*(b^7*c^2 - 12*a*b
^5*c^3 + 48*a^2*b^3*c^4 - 64*a^3*b*c^5)*x^4 + 2*(2*b^8*c - 26*a*b^6*c^2 + 1
23*a^2*b^4*c^3 - 254*a^3*b^2*c^4 + 200*a^4*c^5)*x^3 + (5*b^9 - 58*a*b^7*c +
 225*a^2*b^5*c^2 - 314*a^3*b^3*c^3 + 88*a^4*b*c^4)*x^2 + 3*(a^2*b^6 - 10*a^
3*b^4*c + 30*a^4*b^2*c^2 - 20*a^5*c^3 + (b^6*c^2 - 10*a*b^4*c^3 + 30*a^2*b^
2*c^4 - 20*a^3*c^5)*x^4 + 2*(b^7*c - 10*a*b^5*c^2 + 30*a^2*b^3*c^3 - 20*a^3
*b*c^4)*x^3 + (b^8 - 8*a*b^6*c + 10*a^2*b^4*c^2 + 40*a^3*b^2*c^3 - 40*a^4*c
^4)*x^2 + 2*(a*b^7 - 10*a^2*b^5*c + 30*a^3*b^3*c^2 - 20*a^4*b*c^3)*x)*sqrt(
b^2 - 4*a*c)*log((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c + sqrt(b^2 - 4*a*c)*(2*
c*x + b))/(c*x^2 + b*x + a)) + 2*(5*a*b^8 - 59*a^2*b^6*c + 235*a^3*b^4*c^2
- 346*a^4*b^2*c^3 + 120*a^5*c^4)*x + 3*(a^2*b^7 - 12*a^3*b^5*c + 48*a^4*b^3
*c^2 - 64*a^5*b*c^3 + (b^7*c^2 - 12*a*b^5*c^3 + 48*a^2*b^3*c^4 - 64*a^3*b*c
^5)*x^4 + 2*(b^8*c - 12*a*b^6*c^2 + 48*a^2*b^4*c^3 - 64*a^3*b^2*c^4)*x^3 +
(b^9 - 10*a*b^7*c + 24*a^2*b^5*c^2 + 32*a^3*b^3*c^3 - 128*a^4*b*c^4)*x^2 +
2*(a*b^8 - 12*a^2*b^6*c + 48*a^3*b^4*c^2 - 64*a^4*b^2*c^3)*x)*log(c*x^2 + b
*x + a))/(a^2*b^6*c^4 - 12*a^3*b^4*c^5 + 48*a^4*b^2*c^6 - 64*a^5*c^7 + (b^6
*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64*a^3*c^9)*x^4 + 2*(b^7*c^5 - 12*a*
b^5*c^6 + 48*a^2*b^3*c^7 - 64*a^3*b*c^8)*x^3 + (b^8*c^4 - 10*a*b^6*c^5 + 24
*a^2*b^4*c^6 + 32*a^3*b^2*c^7 - 128*a^4*c^8)*x^2 + 2*(a*b^7*c^4 - 12*a^2*b^
5*c^5 + 48*a^3*b^3*c^6 - 64*a^4*b*c^7)*x), -1/2*(5*a^2*b^7 - 56*a^3*b^5*c +
 202*a^4*b^3*c^2 - 232*a^5*b*c^3 - 2*(b^6*c^3 - 12*a*b^4*c^4 + 48*a^2*b^2*c
^5 - 64*a^3*c^6)*x^5 - 4*(b^7*c^2 - 12*a*b^5*c^3 + 48*a^2*b^3*c^4 - 64*a^3*
b*c^5)*x^4 + 2*(2*b^8*c - 26*a*b^6*c^2 + 123*a^2*b^4*c^3 - 254*a^3*b^2*c^4
+ 200*a^4*c^5)*x^3 + (5*b^9 - 58*a*b^7*c + 225*a^2*b^5*c^2 - 314*a^3*b^3*c^
3 + 88*a^4*b*c^4)*x^2 + 6*(a^2*b^6 - 10*a^3*b^4*c + 30*a^4*b^2*c^2 - 20*a^5
*c^3 + (b^6*c^2 - 10*a*b^4*c^3 + 30*a^2*b^2*c^4 - 20*a^3*c^5)*x^4 + 2*(b^7*
c - 10*a*b^5*c^2 + 30*a^2*b^3*c^3 - 20*a^3*b*c^4)*x^3 + (b^8 - 8*a*b^6*c +
10*a^2*b^4*c^2 + 40*a^3*b^2*c^3 - 40*a^4*c^4)*x^2 + 2*(a*b^7 - 10*a^2*b^5*c
 + 30*a^3*b^3*c^2 - 20*a^4*b*c^3)*x)*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b^2 +
 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) + 2*(5*a*b^8 - 59*a^2*b^6*c + 235*a^3*b^
4*c^2 - 346*a^4*b^2*c^3 + 120*a^5*c^4)*x + 3*(a^2*b^7 - 12*a^3*b^5*c + 48*a
^4*b^3*c^2 - 64*a^5*b*c^3 + (b^7*c^2 - 12*a*b^5*c^3 + 48*a^2*b^3*c^4 - 64*a
^3*b*c^5)*x^4 + 2*(b^8*c - 12*a*b^6*c^2 + 48*a^2*b^4*c^3 - 64*a^3*b^2*c^4)*
x^3 + (b^9 - 10*a*b^7*c + 24*a^2*b^5*c^2 + 32*a^3*b^3*c^3 - 128*a^4*b*c^4)*
x^2 + 2*(a*b^8 - 12*a^2*b^6*c + 48*a^3*b^4*c^2 - 64*a^4*b^2*c^3)*x)*log(c*x
^2 + b*x + a))/(a^2*b^6*c^4 - 12*a^3*b^4*c^5 + 48*a^4*b^2*c^6 - 64*a^5*c^7
+ (b^6*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64*a^3*c^9)*x^4 + 2*(b^7*c^5 -
 12*a*b^5*c^6 + 48*a^2*b^3*c^7 - 64*a^3*b*c^8)*x^3 + (b^8*c^4 - 10*a*b^6*c^
5 + 24*a^2*b^4*c^6 + 32*a^3*b^2*c^7 - 128*a^4*c^8)*x^2 + 2*(a*b^7*c^4 - 12*
a^2*b^5*c^5 + 48*a^3*b^3*c^6 - 64*a^4*b*c^7)*x)]
```

giac [A] time = 0.38, size = 282, normalized size = 1.18

$$\frac{3\left(b^{6}-10\,ab^{4}c+30\,a^{2}b^{2}c^{2}-20\,a^{3}c^{3}\right)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^{2}+4\,ac}}\right)}{\left(b^{4}c^{4}-8\,ab^{2}c^{5}+16\,a^{2}c^{6}\right)\sqrt{-b^{2}+4\,ac}}+\frac{x}{c^{3}}-\frac{3\,b\log\left(cx^{2}+bx+a\right)}{2\,c^{4}}-\frac{5\,a^{2}b^{5}-36\,a^{3}b^{3}c+58\,a^{4}bc^{2}+6\left(b^{6}c-8\,ab^{4}c^{2}+17\,a^{2}b^{2}c^{3}-6\,a^{3}c^{4}\right)x^{3}+\left(5\,b^{7}-34\,ab^{5}c+41\,a^{2}b^{3}c^{2}+42\,a^{3}bc^{3}\right)x^{2}+2\left(5\,ab^{6}-38\,a^{2}b^{4}c+71\,a^{3}b^{2}c^{2}-14\,a^{4}c^{3}\right)x}{2\left(cx^{2}+bx+a\right)^{2}\left(b^{2}-4\,ac\right)^{2}c^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x^2+b/x)^3,x, algorithm="giac")
```

```
[Out] 3*(b^6 - 10*a*b^4*c + 30*a^2*b^2*c^2 - 20*a^3*c^3)*arctan((2*c*x + b)/sqrt(
-b^2 + 4*a*c))/((b^4*c^4 - 8*a*b^2*c^5 + 16*a^2*c^6)*sqrt(-b^2 + 4*a*c)) +
x/c^3 - 3/2*b*log(c*x^2 + b*x + a)/c^4 - 1/2*(5*a^2*b^5 - 36*a^3*b^3*c + 58
*a^4*b*c^2 + 6*(b^6*c - 8*a*b^4*c^2 + 17*a^2*b^2*c^3 - 6*a^3*c^4)*x^3 + (5*
```

```
b^7 - 34*a*b^5*c + 41*a^2*b^3*c^2 + 42*a^3*b*c^3)*x^2 + 2*(5*a*b^6 - 38*a^2
*b^4*c + 71*a^3*b^2*c^2 - 14*a^4*c^3)*x)/((c*x^2 + b*x + a)^2*(b^2 - 4*a*c)
^2*c^4)
```

maple [B] time = 0.02, size = 1040, normalized size = 4.37

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c+a/x^2+b/x)^3,x)
```

```
[Out] x/c^3+18/(c*x^2+b*x+a)^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x^3*a^3-51/c/(c*x^2+b*x
+a)^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x^3*a^2*b^2+24/c^2/(c*x^2+b*x+a)^2/(16*a^2
*c^2-8*a*b^2*c+b^4)*x^3*a*b^4-3/c^3/(c*x^2+b*x+a)^2/(16*a^2*c^2-8*a*b^2*c+b
^4)*x^3*b^6-21/c/(c*x^2+b*x+a)^2*b/(16*a^2*c^2-8*a*b^2*c+b^4)*x^2*a^3-41/2/
c^2/(c*x^2+b*x+a)^2*b^3/(16*a^2*c^2-8*a*b^2*c+b^4)*x^2*a^2+17/c^3/(c*x^2+b*
x+a)^2*b^5/(16*a^2*c^2-8*a*b^2*c+b^4)*x^2*a-5/2/c^4/(c*x^2+b*x+a)^2*b^7/(16
*a^2*c^2-8*a*b^2*c+b^4)*x^2+14/c/(c*x^2+b*x+a)^2*a^4/(16*a^2*c^2-8*a*b^2*c+
b^4)*x-71/c^2/(c*x^2+b*x+a)^2*a^3/(16*a^2*c^2-8*a*b^2*c+b^4)*x*b^2+38/c^3/(
c*x^2+b*x+a)^2*a^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x*b^4-5/c^4/(c*x^2+b*x+a)^2*a
/(16*a^2*c^2-8*a*b^2*c+b^4)*x*b^6-29/c^2/(c*x^2+b*x+a)^2*b*a^4/(16*a^2*c^2-
8*a*b^2*c+b^4)+18/c^3/(c*x^2+b*x+a)^2*b^3*a^3/(16*a^2*c^2-8*a*b^2*c+b^4)-5/
2/c^4/(c*x^2+b*x+a)^2*b^5*a^2/(16*a^2*c^2-8*a*b^2*c+b^4)-24/c^2/(16*a^2*c^2
-8*a*b^2*c+b^4)*ln(c*x^2+b*x+a)*a^2*b+12/c^3/(16*a^2*c^2-8*a*b^2*c+b^4)*ln(
c*x^2+b*x+a)*a*b^3-3/2/c^4/(16*a^2*c^2-8*a*b^2*c+b^4)*ln(c*x^2+b*x+a)*b^5-6
0/c/(16*a^2*c^2-8*a*b^2*c+b^4)/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^
2)^(1/2))*a^3+90/c^2/(16*a^2*c^2-8*a*b^2*c+b^4)/(4*a*c-b^2)^(1/2)*arctan((2
*c*x+b)/(4*a*c-b^2)^(1/2))*a^2*b^2-30/c^3/(16*a^2*c^2-8*a*b^2*c+b^4)/(4*a*c
-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*a*b^4+3/c^4/(16*a^2*c^2-8*a
*b^2*c+b^4)/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*b^6
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x^2+b/x)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

mupad [B] time = 2.00, size = 705, normalized size = 2.96

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c + a/x^2 + b/x)^3,x)
```

```
[Out] x/c^3 - ((3*x^3*(b^6 - 6*a^3*c^3 + 17*a^2*b^2*c^2 - 8*a*b^4*c))/(b^4 + 16*a
^2*c^2 - 8*a*b^2*c) + (x^2*(5*b^7 + 42*a^3*b*c^3 + 41*a^2*b^3*c^2 - 34*a*b^
5*c))/(2*c*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (a^2*(5*b^5 + 58*a^2*b*c^2 - 3
6*a*b^3*c))/(2*c*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (a*x*(5*b^6 - 14*a^3*c^3
 + 71*a^2*b^2*c^2 - 38*a*b^4*c))/(c*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)))/(a^2*c
^3 + c^5*x^4 + x^2*(2*a*c^4 + b^2*c^3) + 2*b*c^4*x^3 + 2*a*b*c^3*x) + (log(
a + b*x + c*x^2)*(3*b^11 - 3072*a^5*b*c^5 + 480*a^2*b^7*c^2 - 1920*a^3*b^5*
c^3 + 3840*a^4*b^3*c^4 - 60*a*b^9*c))/(2*(1024*a^5*c^9 - b^10*c^4 + 20*a*b^
8*c^5 - 160*a^2*b^6*c^6 + 640*a^3*b^4*c^7 - 1280*a^4*b^2*c^8)) + (3*atan(((
```

```
(3*x*(b^6 - 20*a^3*c^3 + 30*a^2*b^2*c^2 - 10*a*b^4*c))/(c^3*(4*a*c - b^2)^5
) + (3*(b^5*c^3 - 8*a*b^3*c^4 + 16*a^2*b*c^5)*(b^6 - 20*a^3*c^3 + 30*a^2*b^
2*c^2 - 10*a*b^4*c))/(2*c^7*(4*a*c - b^2)^5*(b^4 + 16*a^2*c^2 - 8*a*b^2*c))
)*(32*a^2*c^6*(4*a*c - b^2)^(5/2) + 2*b^4*c^4*(4*a*c - b^2)^(5/2) - 16*a*b^
2*c^5*(4*a*c - b^2)^(5/2)))/(3*b^6 - 60*a^3*c^3 + 90*a^2*b^2*c^2 - 30*a*b^4
*c))*(b^6 - 20*a^3*c^3 + 30*a^2*b^2*c^2 - 10*a*b^4*c))/(c^4*(4*a*c - b^2)^(
5/2))
```

sympy [B] time = 4.20, size = 1714, normalized size = 7.20

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x**2+b/x)**3,x)
```

```
[Out] (-3*b/(2*c**4) - 3*sqrt(-(4*a*c - b**2)**5)*(20*a**3*c**3 - 30*a**2*b**2*c*
*2 + 10*a*b**4*c - b**6)/(2*c**4*(1024*a**5*c**5 - 1280*a**4*b**2*c**4 + 64
0*a**3*b**4*c**3 - 160*a**2*b**6*c**2 + 20*a*b**8*c - b**10)))*log(x + (-66
*a**3*b*c**2 - 64*a**3*c**6*(-3*b/(2*c**4) - 3*sqrt(-(4*a*c - b**2)**5)*(20
*a**3*c**3 - 30*a**2*b**2*c**2 + 10*a*b**4*c - b**6)/(2*c**4*(1024*a**5*c**
5 - 1280*a**4*b**2*c**4 + 640*a**3*b**4*c**3 - 160*a**2*b**6*c**2 + 20*a*b*
*8*c - b**10))) + 27*a**2*b**3*c + 48*a**2*b**2*c**5*(-3*b/(2*c**4) - 3*sqr
t(-(4*a*c - b**2)**5)*(20*a**3*c**3 - 30*a**2*b**2*c**2 + 10*a*b**4*c - b**
6)/(2*c**4*(1024*a**5*c**5 - 1280*a**4*b**2*c**4 + 640*a**3*b**4*c**3 - 160
*a**2*b**6*c**2 + 20*a*b**8*c - b**10))) - 3*a*b**5 - 12*a*b**4*c**4*(-3*b/
(2*c**4) - 3*sqrt(-(4*a*c - b**2)**5)*(20*a**3*c**3 - 30*a**2*b**2*c**2 + 1
0*a*b**4*c - b**6)/(2*c**4*(1024*a**5*c**5 - 1280*a**4*b**2*c**4 + 640*a**3
*b**4*c**3 - 160*a**2*b**6*c**2 + 20*a*b**8*c - b**10))) + b**6*c**3*(-3*b/
(2*c**4) - 3*sqrt(-(4*a*c - b**2)**5)*(20*a**3*c**3 - 30*a**2*b**2*c**2 + 1
0*a*b**4*c - b**6)/(2*c**4*(1024*a**5*c**5 - 1280*a**4*b**2*c**4 + 640*a**3
*b**4*c**3 - 160*a**2*b**6*c**2 + 20*a*b**8*c - b**10))))/(60*a**3*c**3 - 9
0*a**2*b**2*c**2 + 30*a*b**4*c - 3*b**6)) + (-3*b/(2*c**4) + 3*sqrt(-(4*a*c
 - b**2)**5)*(20*a**3*c**3 - 30*a**2*b**2*c**2 + 10*a*b**4*c - b**6)/(2*c**
4*(1024*a**5*c**5 - 1280*a**4*b**2*c**4 + 640*a**3*b**4*c**3 - 160*a**2*b**
6*c**2 + 20*a*b**8*c - b**10)))*log(x + (-66*a**3*b*c**2 - 64*a**3*c**6*(-3
*b/(2*c**4) + 3*sqrt(-(4*a*c - b**2)**5)*(20*a**3*c**3 - 30*a**2*b**2*c**2
+ 10*a*b**4*c - b**6)/(2*c**4*(1024*a**5*c**5 - 1280*a**4*b**2*c**4 + 640*a
**3*b**4*c**3 - 160*a**2*b**6*c**2 + 20*a*b**8*c - b**10))) + 27*a**2*b**3*
c + 48*a**2*b**2*c**5*(-3*b/(2*c**4) + 3*sqrt(-(4*a*c - b**2)**5)*(20*a**3*
c**3 - 30*a**2*b**2*c**2 + 10*a*b**4*c - b**6)/(2*c**4*(1024*a**5*c**5 - 12
80*a**4*b**2*c**4 + 640*a**3*b**4*c**3 - 160*a**2*b**6*c**2 + 20*a*b**8*c -
 b**10))) - 3*a*b**5 - 12*a*b**4*c**4*(-3*b/(2*c**4) + 3*sqrt(-(4*a*c - b**
2)**5)*(20*a**3*c**3 - 30*a**2*b**2*c**2 + 10*a*b**4*c - b**6)/(2*c**4*(102
4*a**5*c**5 - 1280*a**4*b**2*c**4 + 640*a**3*b**4*c**3 - 160*a**2*b**6*c**2
 + 20*a*b**8*c - b**10))) + b**6*c**3*(-3*b/(2*c**4) + 3*sqrt(-(4*a*c - b**
2)**5)*(20*a**3*c**3 - 30*a**2*b**2*c**2 + 10*a*b**4*c - b**6)/(2*c**4*(102
4*a**5*c**5 - 1280*a**4*b**2*c**4 + 640*a**3*b**4*c**3 - 160*a**2*b**6*c**2
 + 20*a*b**8*c - b**10))))/(60*a**3*c**3 - 90*a**2*b**2*c**2 + 30*a*b**4*c
- 3*b**6)) + (-58*a**4*b*c**2 + 36*a**3*b**3*c - 5*a**2*b**5 + x**3*(36*a**
3*c**4 - 102*a**2*b**2*c**3 + 48*a*b**4*c**2 - 6*b**6*c) + x**2*(-42*a**3*b
*c**3 - 41*a**2*b**3*c**2 + 34*a*b**5*c - 5*b**7) + x*(28*a**4*c**3 - 142*a
**3*b**2*c**2 + 76*a**2*b**4*c - 10*a*b**6))/(32*a**4*c**6 - 16*a**3*b**2*c
**5 + 2*a**2*b**4*c**4 + x**4*(32*a**2*c**8 - 16*a*b**2*c**7 + 2*b**4*c**6)
 + x**3*(64*a**2*b*c**7 - 32*a*b**3*c**6 + 4*b**5*c**5) + x**2*(64*a**3*c**
7 - 12*a*b**4*c**5 + 2*b**6*c**4) + x*(64*a**3*b*c**6 - 32*a**2*b**3*c**5 +
 4*a*b**5*c**4)) + x/c**3
```

3.371 $\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^3 x}\,dx$

Optimal. Leaf size=190

$$\frac{b\left(30a^2c^2-10ab^2c+b^4\right)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^3\left(b^2-4ac\right)^{5/2}}-\frac{bx\left(b^2-7ac\right)}{c^2\left(b^2-4ac\right)^2}+\frac{x^2\left(bx\left(b^2-10ac\right)+a\left(b^2-16ac\right)\right)}{2c\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)}+\frac{x^4(2a}{2\left(b^2-4ac\right)(}$$

Rubi [A] time = 0.28, antiderivative size = 190, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {1354, 738, 818, 773, 634, 618, 206, 628}

$$\frac{b\left(30a^2c^2-10ab^2c+b^4\right)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^3\left(b^2-4ac\right)^{5/2}}-\frac{bx\left(b^2-7ac\right)}{c^2\left(b^2-4ac\right)^2}+\frac{x^4(2a+bx)}{2\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2}+\frac{x^2\left(bx\left(b^2-10ac\right)+a\left(b^2-16ac\right)\right)}{2c\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)}+\frac{\log\left(a+bx+cx^2\right)}{2c^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c + a/x^2 + b/x)^3*x),x]
```

```
[Out] -((b*(b^2 - 7*a*c)*x)/(c^2*(b^2 - 4*a*c)^2)) + (x^4*(2*a + b*x))/(2*(b^2 -
4*a*c)*(a + b*x + c*x^2)^2) + (x^2*(a*(b^2 - 16*a*c) + b*(b^2 - 10*a*c)*x))
/(2*c*(b^2 - 4*a*c)^2*(a + b*x + c*x^2)) + (b*(b^4 - 10*a*b^2*c + 30*a^2*c^
2)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(c^3*(b^2 - 4*a*c)^(5/2)) + Log[
a + b*x + c*x^2]/(2*c^3)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 738

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m - 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*(a + b*x
+ c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*
c)), Int[(d + e*x)^(m - 2)*Simp[e*(2*a*e*(m - 1) + b*d*(2*p - m + 4)) - 2*c
*d^2*(2*p + 3) + e*(b*e - 2*d*c)*(m + 2*p + 2)*x, x]*(a + b*x + c*x^2)^(p +
 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^
2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p, -1] && GtQ[m, 1] &&
```

```
IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 773

```
Int[(((d_.) + (e_.)*(x_))*((f_) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) + (c_.)*
(x_)^2), x_Symbol] :> Simp[(e*g*x)/c, x] + Dist[1/c, Int[(c*d*f - a*e*g + (
c*e*f + c*d*g - b*e*g)*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f, g}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 818

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((d + e*x)^(m - 1)*(a + b*x + c*x^2)^
(p + 1)*(2*a*c*(e*f + d*g) - b*(c*d*f + a*e*g) - (2*c^2*d*f + b^2*e*g - c*(
b*e*f + b*d*g + 2*a*e*g))*x))/(c*(p + 1)*(b^2 - 4*a*c)), x] - Dist[1/(c*(p
+ 1)*(b^2 - 4*a*c)), Int[(d + e*x)^(m - 2)*(a + b*x + c*x^2)^(p + 1)*Simp[2
*c^2*d^2*f*(2*p + 3) + b*e*g*(a*e*(m - 1) + b*d*(p + 2)) - c*(2*a*e*(e*f*(m
 - 1) + d*g*m) + b*d*(d*g*(2*p + 3) - e*f*(m - 2*p - 4))) + e*(b^2*e*g*(m +
 p + 1) + 2*c^2*d*f*(m + 2*p + 2) - c*(2*a*e*g*m + b*(e*f + d*g)*(m + 2*p +
 2)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c,
0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] && ((EqQ[m,
2] && EqQ[p, -3] && RationalQ[a, b, c, d, e, f, g]) ||  !ILtQ[m + 2*p + 3,
0])
```

Rule 1354

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> Int[x^(m + 2*n*p)*(c + b/x^n + a/x^(2*n))^p, x] /; FreeQ[{a, b, c, m, n
}, x] && EqQ[n2, 2*n] && ILtQ[p, 0] && NegQ[n]
```

Rubi steps

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^3 x}\,dx = \int \frac{x^5}{\left(a+bx+cx^2\right)^3}\,dx$$

$$= \frac{x^4(2a+bx)}{2\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2} - \frac{\int \frac{x^3(8a+bx)}{\left(a+bx+cx^2\right)^2}\,dx}{2\left(b^2-4ac\right)}$$

$$= \frac{x^4(2a+bx)}{2\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2} + \frac{x^2\left(a\left(b^2-16ac\right)+b\left(b^2-10ac\right)x\right)}{2c\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)} - \frac{\int \frac{x\left(2a\left(b^2-16ac\right)+2b\left(b^2 \cdots\right.\right.}{a+bx+cx^2}}{2c\left(b^2-4ac \cdots\right.}$$

$$= -\frac{b\left(b^2-7ac\right)x}{c^2\left(b^2-4ac\right)^2} + \frac{x^4(2a+bx)}{2\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2} + \frac{x^2\left(a\left(b^2-16ac\right)+b\left(b^2-10ac\right)x\right)}{2c\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)} - \cdots$$

$$= -\frac{b\left(b^2-7ac\right)x}{c^2\left(b^2-4ac\right)^2} + \frac{x^4(2a+bx)}{2\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2} + \frac{x^2\left(a\left(b^2-16ac\right)+b\left(b^2-10ac\right)x\right)}{2c\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)} + \cdots$$

$$= -\frac{b\left(b^2-7ac\right)x}{c^2\left(b^2-4ac\right)^2} + \frac{x^4(2a+bx)}{2\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2} + \frac{x^2\left(a\left(b^2-16ac\right)+b\left(b^2-10ac\right)x\right)}{2c\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)} + \cdots$$

$$= -\frac{b\left(b^2-7ac\right)x}{c^2\left(b^2-4ac\right)^2} + \frac{x^4(2a+bx)}{2\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2} + \frac{x^2\left(a\left(b^2-16ac\right)+b\left(b^2-10ac\right)x\right)}{2c\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)} + \cdots$$

Mathematica [A] time = 0.31, size = 221, normalized size = 1.16

$$\frac{-\frac{2bc\left(30a^2c^2-10ab^2c+b^4\right)\tan^{-1}\left(\frac{b+2cx}{\sqrt{4ac-b^2}}\right)}{\left(4ac-b^2\right)^{5/2}}+\frac{2a^3c^2+a^2bc(5cx-4b)+ab^3(b-5cx)+b^5x}{\left(b^2-4ac\right)(a+x(b+cx))^2}+\frac{32a^3c^3-39a^2b^2c^2+50a^2bc^3x+11ab^4c-30ab^3c^2x-b^6+4b^5cx}{\left(b^2-4ac\right)^2(a+x(b+cx))}+c\log(a+x(b+cx))}{2c^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c + a/x^2 + b/x)^3*x),x]
```

```
[Out] ((-b^6 + 11*a*b^4*c - 39*a^2*b^2*c^2 + 32*a^3*c^3 + 4*b^5*c*x - 30*a*b^3*c^
2*x + 50*a^2*b*c^3*x)/((b^2 - 4*a*c)^2*(a + x*(b + c*x))) + (2*a^3*c^2 + b^
5*x + a*b^3*(b - 5*c*x) + a^2*b*c*(-4*b + 5*c*x))/((b^2 - 4*a*c)*(a + x*(b
+ c*x))^2) - (2*b*c*(b^4 - 10*a*b^2*c + 30*a^2*c^2)*ArcTan[(b + 2*c*x)/Sqrt
[-b^2 + 4*a*c]])/(-b^2 + 4*a*c)^(5/2) + c*Log[a + x*(b + c*x)])/(2*c^4)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^3 x}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/((c + a/x^2 + b/x)^3*x),x]
```

```
[Out] IntegrateAlgebraic[1/((c + a/x^2 + b/x)^3*x), x]
```

fricas [B] time = 1.68, size = 1603, normalized size = 8.44

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x^2+b/x)^3/x,x, algorithm="fricas")
```

```
[Out] [1/2*(3*a^2*b^6 - 33*a^3*b^4*c + 108*a^4*b^2*c^2 - 96*a^5*c^3 + 2*(2*b^7*c
- 23*a*b^5*c^2 + 85*a^2*b^3*c^3 - 100*a^3*b*c^4)*x^3 + (3*b^8 - 31*a*b^6*c
+ 87*a^2*b^4*c^2 - 12*a^3*b^2*c^3 - 128*a^4*c^4)*x^2 + (a^2*b^5 - 10*a^3*b^
3*c + 30*a^4*b*c^2 + (b^5*c^2 - 10*a*b^3*c^3 + 30*a^2*b*c^4)*x^4 + 2*(b^6*c
 - 10*a*b^4*c^2 + 30*a^2*b^2*c^3)*x^3 + (b^7 - 8*a*b^5*c + 10*a^2*b^3*c^2 +
 60*a^3*b*c^3)*x^2 + 2*(a*b^6 - 10*a^2*b^4*c + 30*a^3*b^2*c^2)*x)*sqrt(b^2
- 4*a*c)*log((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c + sqrt(b^2 - 4*a*c)*(2*c*x
+ b))/(c*x^2 + b*x + a)) + 2*(3*a*b^7 - 34*a^2*b^5*c + 119*a^3*b^3*c^2 - 12
4*a^4*b*c^3)*x + (a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3 + (b
^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*x^4 + 2*(b^7*c - 12*a*
b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*x^3 + (b^8 - 10*a*b^6*c + 24*a^2*b
^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*x^2 + 2*(a*b^7 - 12*a^2*b^5*c + 48*a
^3*b^3*c^2 - 64*a^4*b*c^3)*x)*log(c*x^2 + b*x + a))/(a^2*b^6*c^3 - 12*a^3*b
^4*c^4 + 48*a^4*b^2*c^5 - 64*a^5*c^6 + (b^6*c^5 - 12*a*b^4*c^6 + 48*a^2*b^2
*c^7 - 64*a^3*c^8)*x^4 + 2*(b^7*c^4 - 12*a*b^5*c^5 + 48*a^2*b^3*c^6 - 64*a^
3*b*c^7)*x^3 + (b^8*c^3 - 10*a*b^6*c^4 + 24*a^2*b^4*c^5 + 32*a^3*b^2*c^6 -
128*a^4*c^7)*x^2 + 2*(a*b^7*c^3 - 12*a^2*b^5*c^4 + 48*a^3*b^3*c^5 - 64*a^4*
b*c^6)*x), 1/2*(3*a^2*b^6 - 33*a^3*b^4*c + 108*a^4*b^2*c^2 - 96*a^5*c^3 + 2
*(2*b^7*c - 23*a*b^5*c^2 + 85*a^2*b^3*c^3 - 100*a^3*b*c^4)*x^3 + (3*b^8 - 3
1*a*b^6*c + 87*a^2*b^4*c^2 - 12*a^3*b^2*c^3 - 128*a^4*c^4)*x^2 + 2*(a^2*b^5
 - 10*a^3*b^3*c + 30*a^4*b*c^2 + (b^5*c^2 - 10*a*b^3*c^3 + 30*a^2*b*c^4)*x^
4 + 2*(b^6*c - 10*a*b^4*c^2 + 30*a^2*b^2*c^3)*x^3 + (b^7 - 8*a*b^5*c + 10*a
^2*b^3*c^2 + 60*a^3*b*c^3)*x^2 + 2*(a*b^6 - 10*a^2*b^4*c + 30*a^3*b^2*c^2)*
x)*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c))
 + 2*(3*a*b^7 - 34*a^2*b^5*c + 119*a^3*b^3*c^2 - 124*a^4*b*c^3)*x + (a^2*b^
6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3 + (b^6*c^2 - 12*a*b^4*c^3 +
48*a^2*b^2*c^4 - 64*a^3*c^5)*x^4 + 2*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3
```

```
 - 64*a^3*b*c^4)*x^3 + (b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3
- 128*a^4*c^4)*x^2 + 2*(a*b^7 - 12*a^2*b^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^
3)*x)*log(c*x^2 + b*x + a))/(a^2*b^6*c^3 - 12*a^3*b^4*c^4 + 48*a^4*b^2*c^5
- 64*a^5*c^6 + (b^6*c^5 - 12*a*b^4*c^6 + 48*a^2*b^2*c^7 - 64*a^3*c^8)*x^4 +
 2*(b^7*c^4 - 12*a*b^5*c^5 + 48*a^2*b^3*c^6 - 64*a^3*b*c^7)*x^3 + (b^8*c^3
- 10*a*b^6*c^4 + 24*a^2*b^4*c^5 + 32*a^3*b^2*c^6 - 128*a^4*c^7)*x^2 + 2*(a*
b^7*c^3 - 12*a^2*b^5*c^4 + 48*a^3*b^3*c^5 - 64*a^4*b*c^6)*x)]
```

giac [A] time = 0.40, size = 245, normalized size = 1.29

$$-\frac{\left(b^5-10\,ab^3c+30\,a^2bc^2\right)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{\left(b^4c^3-8\,ab^2c^4+16\,a^2c^5\right)\sqrt{-b^2+4\,ac}}+\frac{\log\left(cx^2+bx+a\right)}{2\,c^3}+\frac{3\,a^2b^4-21\,a^3b^2c+24\,a^4c^2+2\left(2\,b^5c-15\,ab^3c^2+25\,a^2bc^3\right)x^3+\left(3\,b^6-19\,ab^4c+11\,a^2b^2c^2+32\,a^3c^3\right)x^2+2\left(3\,ab^5-22\,a^2b^3c+31\,a^3bc^2\right)x}{2\left(cx^2+bx+a\right)^2\left(b^2-4\,ac\right)^2c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x^2+b/x)^3/x,x, algorithm="giac")
```

```
[Out] -(b^5 - 10*a*b^3*c + 30*a^2*b*c^2)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/(
(b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)*sqrt(-b^2 + 4*a*c)) + 1/2*log(c*x^2 +
b*x + a)/c^3 + 1/2*(3*a^2*b^4 - 21*a^3*b^2*c + 24*a^4*c^2 + 2*(2*b^5*c - 15
*a*b^3*c^2 + 25*a^2*b*c^3)*x^3 + (3*b^6 - 19*a*b^4*c + 11*a^2*b^2*c^2 + 32*
a^3*c^3)*x^2 + 2*(3*a*b^5 - 22*a^2*b^3*c + 31*a^3*b*c^2)*x)/((c*x^2 + b*x +
 a)^2*(b^2 - 4*a*c)^2*c^3)
```

maple [B] time = 0.02, size = 530, normalized size = 2.79

$$-\frac{30a^2b\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{\left(16a^2c^2-8a\,b^2c+b^4\right)\sqrt{4ac-b^2}\;c}+\frac{10a\,b^3\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{\left(16a^2c^2-8a\,b^2c+b^4\right)\sqrt{4ac-b^2}\;c^2}-\frac{b^5\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{\left(16a^2c^2-8a\,b^2c+b^4\right)\sqrt{4ac-b^2}\;c^3}+\frac{8a^2\ln\left(c\,x^2+bx+a\right)}{\left(16a^2c^2-8a\,b^2c+b^4\right)c}-\frac{4a\,b^2\ln\left(c\,x^2+bx+a\right)}{\left(16a^2c^2-8a\,b^2c+b^4\right)c^2}+\frac{b^4\ln\left(c\,x^2+bx+a\right)}{2\left(16a^2c^2-8a\,b^2c+b^4\right)c^3}+\frac{\frac{\left(25a^2c^2-15a\,b^2c+2b^4\right)b\,x^3}{\left(16a^2c^2-8a\,b^2c+b^4\right)c^2}+\frac{\left(31a^2c^2-22a\,b^2c+3b^4\right)abx}{\left(16a^2c^2-8a\,b^2c+b^4\right)c^3}+\frac{3\left(8a^2c^2-7a\,b^2c+b^4\right)a^2}{2\left(16a^2c^2-8a\,b^2c+b^4\right)c^3}+\frac{\left(32a^3c^3+11a^2b^2c^2-19a\,b^4c+3b^6\right)x^2}{2\left(16a^2c^2-8a\,b^2c+b^4\right)c^3}}{\left(c\,x^2+bx+a\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c+a/x^2+b/x)^3/x,x)
```

```
[Out] (1/c^2*b*(25*a^2*c^2-15*a*b^2*c+2*b^4)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^3+1/2*(
32*a^3*c^3+11*a^2*b^2*c^2-19*a*b^4*c+3*b^6)/c^3/(16*a^2*c^2-8*a*b^2*c+b^4)*
x^2+a*b*(31*a^2*c^2-22*a*b^2*c+3*b^4)/(16*a^2*c^2-8*a*b^2*c+b^4)/c^3*x+3/2*
a^2*(8*a^2*c^2-7*a*b^2*c+b^4)/c^3/(16*a^2*c^2-8*a*b^2*c+b^4))/(c*x^2+b*x+a)
^2+8/c/(16*a^2*c^2-8*a*b^2*c+b^4)*ln(c*x^2+b*x+a)*a^2-4/c^2/(16*a^2*c^2-8*a
*b^2*c+b^4)*ln(c*x^2+b*x+a)*a*b^2+1/2/c^3/(16*a^2*c^2-8*a*b^2*c+b^4)*ln(c*x
^2+b*x+a)*b^4-30/c/(16*a^2*c^2-8*a*b^2*c+b^4)/(4*a*c-b^2)^(1/2)*arctan((2*c
*x+b)/(4*a*c-b^2)^(1/2))*a^2*b+10/c^2/(16*a^2*c^2-8*a*b^2*c+b^4)/(4*a*c-b^2
)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*a*b^3-1/c^3/(16*a^2*c^2-8*a*b^2
*c+b^4)/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*b^5
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x^2+b/x)^3/x,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

mupad [B] time = 2.20, size = 620, normalized size = 3.26

$$\frac{\frac{3\,a^2\left(8\,a^2\,c^2-7\,a\,b^2\,c+b^4\right)}{2\,c^3\left(16\,a^2\,c^2-8\,a\,b^2\,c+b^4\right)}+\frac{x^2\left(32\,a^3\,c^3+11\,a^2\,b^2\,c^2-19\,a\,b^4\,c+3\,b^6\right)}{2\,c^3\left(16\,a^2\,c^2-8\,a\,b^2\,c+b^4\right)}+\frac{b\,x^3\left(25\,a^2\,c^2-15\,a\,b^2\,c+2\,b^4\right)}{c^2\left(16\,a^2\,c^2-8\,a\,b^2\,c+b^4\right)}+\frac{a\,b\,x\left(31\,a^2\,c^2-22\,a\,b^2\,c+3\,b^4\right)}{c^3\left(16\,a^2\,c^2-8\,a\,b^2\,c+b^4\right)}}{x^2\left(b^2+2\,a\,c\right)+a^2+c^2\,x^4+2\,a\,b\,x+2\,b\,c\,x^3}-\frac{\ln\left(c\,x^2+b\,x+a\right)\left(-1024\,a^5\,c^5+1280\,a^4\,b^2\,c^4-640\,a^3\,b^4\,c^3+160\,a^2\,b^6\,c^2-20\,a\,b^8\,c+b^{10}\right)}{2\left(1024\,a^5\,c^8-1280\,a^4\,b^2\,c^7+640\,a^3\,b^4\,c^6-160\,a^2\,b^6\,c^5+20\,a\,b^8\,c^4-b^{10}\,c^3\right)}-\frac{b\,\mathrm{atan}\left(\frac{\left(\frac{b\,x\left(30\,a^2\,c^2-10\,a\,b^2\,c+b^4\right)}{c^2\left(4\,a\,c-b^2\right)^5}+\frac{b^2\left(16\,a^2\,c^4-8\,a\,b^2\,c^3+b^4\,c^2\right)\left(30\,a^2\,c^2-10\,a\,b^2\,c+b^4\right)}{2\,c^5\left(4\,a\,c-b^2\right)^5\left(16\,a^2\,c^2-8\,a\,b^2\,c+b^4\right)}\right)\left(32\,a^2\,c^5\left(4\,a\,c-b^2\right)^{5/2}+2\,b^4\,c^3\left(4\,a\,c-b^2\right)^{5/2}-16\,a\,b^2\,c^4\left(4\,a\,c-b^2\right)^{5/2}\right)}{30\,a^2\,b\,c^2-10\,a\,b^3\,c+b^5}\right)\left(30\,a^2\,c^2-10\,a\,b^2\,c+b^4\right)}{c^3\left(4\,a\,c-b^2\right)^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(c + a/x^2 + b/x)^3),x)
```

```
[Out] ((3*a^2*(b^4 + 8*a^2*c^2 - 7*a*b^2*c))/(2*c^3*(b^4 + 16*a^2*c^2 - 8*a*b^2*c
)) + (x^2*(3*b^6 + 32*a^3*c^3 + 11*a^2*b^2*c^2 - 19*a*b^4*c))/(2*c^3*(b^4 +
 16*a^2*c^2 - 8*a*b^2*c)) + (b*x^3*(2*b^4 + 25*a^2*c^2 - 15*a*b^2*c))/(c^2*
(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (a*b*x*(3*b^4 + 31*a^2*c^2 - 22*a*b^2*c))
/(c^3*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)))/(x^2*(2*a*c + b^2) + a^2 + c^2*x^4 +
 2*a*b*x + 2*b*c*x^3) - (log(a + b*x + c*x^2)*(b^10 - 1024*a^5*c^5 + 160*a^
2*b^6*c^2 - 640*a^3*b^4*c^3 + 1280*a^4*b^2*c^4 - 20*a*b^8*c))/(2*(1024*a^5*
c^8 - b^10*c^3 + 20*a*b^8*c^4 - 160*a^2*b^6*c^5 + 640*a^3*b^4*c^6 - 1280*a^
4*b^2*c^7)) - (b*atan((((b*x*(b^4 + 30*a^2*c^2 - 10*a*b^2*c))/(c^2*(4*a*c -
 b^2)^5) + (b^2*(16*a^2*c^4 + b^4*c^2 - 8*a*b^2*c^3)*(b^4 + 30*a^2*c^2 - 10
*a*b^2*c))/(2*c^5*(4*a*c - b^2)^5*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)))*(32*a^2*
c^5*(4*a*c - b^2)^(5/2) + 2*b^4*c^3*(4*a*c - b^2)^(5/2) - 16*a*b^2*c^4*(4*a
*c - b^2)^(5/2)))/(b^5 + 30*a^2*b*c^2 - 10*a*b^3*c))*(b^4 + 30*a^2*c^2 - 10
*a*b^2*c))/(c^3*(4*a*c - b^2)^(5/2))
```

sympy [B] time = 3.21, size = 1510, normalized size = 7.95

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x**2+b/x)**3/x,x)
```

```
[Out] (-b*sqrt(-(4*a*c - b**2)**5)*(30*a**2*c**2 - 10*a*b**2*c + b**4)/(2*c**3*(1
024*a**5*c**5 - 1280*a**4*b**2*c**4 + 640*a**3*b**4*c**3 - 160*a**2*b**6*c*
*2 + 20*a*b**8*c - b**10)) + 1/(2*c**3))*log(x + (-64*a**3*c**5*(-b*sqrt(-(
4*a*c - b**2)**5)*(30*a**2*c**2 - 10*a*b**2*c + b**4)/(2*c**3*(1024*a**5*c*
*5 - 1280*a**4*b**2*c**4 + 640*a**3*b**4*c**3 - 160*a**2*b**6*c**2 + 20*a*b
**8*c - b**10)) + 1/(2*c**3)) + 32*a**3*c**2 + 48*a**2*b**2*c**4*(-b*sqrt(-
(4*a*c - b**2)**5)*(30*a**2*c**2 - 10*a*b**2*c + b**4)/(2*c**3*(1024*a**5*c
**5 - 1280*a**4*b**2*c**4 + 640*a**3*b**4*c**3 - 160*a**2*b**6*c**2 + 20*a*
b**8*c - b**10)) + 1/(2*c**3)) - 9*a**2*b**2*c - 12*a*b**4*c**3*(-b*sqrt(-(
4*a*c - b**2)**5)*(30*a**2*c**2 - 10*a*b**2*c + b**4)/(2*c**3*(1024*a**5*c*
*5 - 1280*a**4*b**2*c**4 + 640*a**3*b**4*c**3 - 160*a**2*b**6*c**2 + 20*a*b
**8*c - b**10)) + 1/(2*c**3)) + a*b**4 + b**6*c**2*(-b*sqrt(-(4*a*c - b**2)
**5)*(30*a**2*c**2 - 10*a*b**2*c + b**4)/(2*c**3*(1024*a**5*c**5 - 1280*a**
4*b**2*c**4 + 640*a**3*b**4*c**3 - 160*a**2*b**6*c**2 + 20*a*b**8*c - b**10
)) + 1/(2*c**3)))/(30*a**2*b*c**2 - 10*a*b**3*c + b**5)) + (b*sqrt(-(4*a*c
- b**2)**5)*(30*a**2*c**2 - 10*a*b**2*c + b**4)/(2*c**3*(1024*a**5*c**5 - 1
280*a**4*b**2*c**4 + 640*a**3*b**4*c**3 - 160*a**2*b**6*c**2 + 20*a*b**8*c
- b**10)) + 1/(2*c**3))*log(x + (-64*a**3*c**5*(b*sqrt(-(4*a*c - b**2)**5)*
(30*a**2*c**2 - 10*a*b**2*c + b**4)/(2*c**3*(1024*a**5*c**5 - 1280*a**4*b**
2*c**4 + 640*a**3*b**4*c**3 - 160*a**2*b**6*c**2 + 20*a*b**8*c - b**10)) +
1/(2*c**3)) + 32*a**3*c**2 + 48*a**2*b**2*c**4*(b*sqrt(-(4*a*c - b**2)**5)*
(30*a**2*c**2 - 10*a*b**2*c + b**4)/(2*c**3*(1024*a**5*c**5 - 1280*a**4*b**
2*c**4 + 640*a**3*b**4*c**3 - 160*a**2*b**6*c**2 + 20*a*b**8*c - b**10)) +
1/(2*c**3)) - 9*a**2*b**2*c - 12*a*b**4*c**3*(b*sqrt(-(4*a*c - b**2)**5)*(3
0*a**2*c**2 - 10*a*b**2*c + b**4)/(2*c**3*(1024*a**5*c**5 - 1280*a**4*b**2*
c**4 + 640*a**3*b**4*c**3 - 160*a**2*b**6*c**2 + 20*a*b**8*c - b**10)) + 1/
(2*c**3)) + a*b**4 + b**6*c**2*(b*sqrt(-(4*a*c - b**2)**5)*(30*a**2*c**2 -
10*a*b**2*c + b**4)/(2*c**3*(1024*a**5*c**5 - 1280*a**4*b**2*c**4 + 640*a**
3*b**4*c**3 - 160*a**2*b**6*c**2 + 20*a*b**8*c - b**10)) + 1/(2*c**3)))/(30
*a**2*b*c**2 - 10*a*b**3*c + b**5)) + (24*a**4*c**2 - 21*a**3*b**2*c + 3*a*
*2*b**4 + x**3*(50*a**2*b*c**3 - 30*a*b**3*c**2 + 4*b**5*c) + x**2*(32*a**3
*c**3 + 11*a**2*b**2*c**2 - 19*a*b**4*c + 3*b**6) + x*(62*a**3*b*c**2 - 44*
a**2*b**3*c + 6*a*b**5))/(32*a**4*c**5 - 16*a**3*b**2*c**4 + 2*a**2*b**4*c*
*3 + x**4*(32*a**2*c**7 - 16*a*b**2*c**6 + 2*b**4*c**5) + x**3*(64*a**2*b*c
**6 - 32*a*b**3*c**5 + 4*b**5*c**4) + x**2*(64*a**3*c**6 - 12*a*b**4*c**4 +
 2*b**6*c**3) + x*(64*a**3*b*c**5 - 32*a**2*b**3*c**4 + 4*a*b**5*c**3))
```

3.372 $\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^3 x^2} \, dx$

Optimal. Leaf size=111

$$\frac{12a^2\tanh^{-1}\left(\frac{\frac{2a}{x}+b}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{5/2}}-\frac{3a\left(\frac{2a}{x}+b\right)}{\left(b^2-4ac\right)^2\left(\frac{a}{x^2}+\frac{b}{x}+c\right)}+\frac{\frac{2a}{x}+b}{2\left(b^2-4ac\right)\left(\frac{a}{x^2}+\frac{b}{x}+c\right)^2}$$

Rubi [A] time = 0.07, antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {1352, 614, 618, 206}

$$\frac{12a^2\tanh^{-1}\left(\frac{\frac{2a}{x}+b}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{5/2}}-\frac{3a\left(\frac{2a}{x}+b\right)}{\left(b^2-4ac\right)^2\left(\frac{a}{x^2}+\frac{b}{x}+c\right)}+\frac{\frac{2a}{x}+b}{2\left(b^2-4ac\right)\left(\frac{a}{x^2}+\frac{b}{x}+c\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c + a/x^2 + b/x)^3*x^2),x]
```

```
[Out] (b + (2*a)/x)/(2*(b^2 - 4*a*c)*(c + a/x^2 + b/x)^2) - (3*a*(b + (2*a)/x))/(
(b^2 - 4*a*c)^2*(c + a/x^2 + b/x)) + (12*a^2*ArcTanh[(b + (2*a)/x)/Sqrt[b^2
 - 4*a*c]])/(b^2 - 4*a*c)^(5/2)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 614

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[(2*c*(2*p +
 3))/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
egerQ[4*p]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1352

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a,
b, c, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

Rubi steps

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^3 x^2}\,dx = -\text{Subst}\left(\int \frac{1}{\left(c+bx+ax^2\right)^3}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{b+\frac{2a}{x}}{2\left(b^2-4ac\right)\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2} + \frac{(3a)\,\text{Subst}\left(\int \frac{1}{\left(c+bx+ax^2\right)^2}\,dx, x, \frac{1}{x}\right)}{b^2-4ac}$$

$$= \frac{b+\frac{2a}{x}}{2\left(b^2-4ac\right)\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2} - \frac{3a\left(b+\frac{2a}{x}\right)}{\left(b^2-4ac\right)^2\left(c+\frac{a}{x^2}+\frac{b}{x}\right)} - \frac{\left(6a^2\right)\text{Subst}\left(\int \frac{1}{c+bx+ax^2}\,dx, x \right.}{\left(b^2-4ac\right)^2}$$

$$= \frac{b+\frac{2a}{x}}{2\left(b^2-4ac\right)\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2} - \frac{3a\left(b+\frac{2a}{x}\right)}{\left(b^2-4ac\right)^2\left(c+\frac{a}{x^2}+\frac{b}{x}\right)} + \frac{\left(12a^2\right)\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx \right.}{\left(b^2-4ac\right)^2}$$

$$= \frac{b+\frac{2a}{x}}{2\left(b^2-4ac\right)\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2} - \frac{3a\left(b+\frac{2a}{x}\right)}{\left(b^2-4ac\right)^2\left(c+\frac{a}{x^2}+\frac{b}{x}\right)} + \frac{12a^2\tanh^{-1}\left(\frac{b+\frac{2a}{x}}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{5/2}}$$

Mathematica [A] time = 0.17, size = 174, normalized size = 1.57

$$\frac{1}{2}\left(\frac{24a^2\tan^{-1}\left(\frac{b+2cx}{\sqrt{4ac-b^2}}\right)}{\left(4ac-b^2\right)^{5/2}} + \frac{a^2c(2cx-3b)+ab^2(b-4cx)+b^4x}{c^3\left(4ac-b^2\right)(a+x(b+cx))^2} + \frac{22a^2bc^2-20a^2c^3x-8ab^3c+16ab^2c^2x+b^5-2b^4cx}{c^3\left(b^2-4ac\right)^2(a+x(b+cx))}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c + a/x^2 + b/x)^3*x^2),x]
```

```
[Out] ((b^5 - 8*a*b^3*c + 22*a^2*b*c^2 - 2*b^4*c*x + 16*a*b^2*c^2*x - 20*a^2*c^3*
x)/(c^3*(b^2 - 4*a*c)^2*(a + x*(b + c*x))) + (b^4*x + a*b^2*(b - 4*c*x) + a
^2*c*(-3*b + 2*c*x))/(c^3*(-b^2 + 4*a*c)*(a + x*(b + c*x))^2) + (24*a^2*Arc
Tan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/(-b^2 + 4*a*c)^(5/2))/2
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^3 x^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/((c + a/x^2 + b/x)^3*x^2),x]
```

```
[Out] IntegrateAlgebraic[1/((c + a/x^2 + b/x)^3*x^2), x]
```

fricas [B] time = 0.76, size = 953, normalized size = 8.59

$$\left[-\frac{a^2b^5 - 14a^3b^3c + 40a^4bc^2 + 2\left(b^6c - 12ab^4c^2 + 42a^2b^2c^3 - 40a^3c^4\right)x^3 + \left(b^7 - 12ab^5c + 30a^2b^3c^2 + 8a^3bc^3\right)x^2 - 12\left(a^2c^4x^4 + 2a^2bc^3x^3 + 2a^3bc^2x + a^4c^2 + \left(a^2b^2c^2 + 2a^3c^3\right)x^2\right)\sqrt{b^2-4ac}\,\log\left(\frac{2c^2x^2+2bcx+b^2-2ac-\sqrt{b^2-4ac}(2cx+b)}{cx^2+bx+a}\right) + 2\left(ab^6 - 14a^2b^4c + 46a^3b^2c^2 - 24a^4c^3\right)x}{2\left(a^2b^6c^2 - 12a^3b^4c^3 + 48a^4b^2c^4 - 64a^5c^5 + \left(b^6c^4 - 12ab^4c^5 + 48a^2b^2c^6 - 64a^3c^7\right)x^4 + 2\left(b^7c^3 - 12ab^5c^4 + 48a^2b^3c^5 - 64a^3bc^6\right)x^3 + \left(b^8c^2 - 10ab^6c^3 + 24a^2b^4c^4 + 32a^3b^2c^5 - 128a^4c^6\right)x^2 + 2\left(ab^7c^2 - 12a^2b^5c^3 + 48a^3b^3c^4 - 64a^4bc^5\right)x\right)}, -\frac{a^2b^5 - 14a^3b^3c + 40a^4bc^2 + 2\left(b^6c - 12ab^4c^2 + 42a^2b^2c^3 - 40a^3c^4\right)x^3 + \left(b^7 - 12ab^5c + 30a^2b^3c^2 + 8a^3bc^3\right)x^2 + 24\left(a^2c^4x^4 + 2a^2bc^3x^3 + 2a^3bc^2x + a^4c^2 + \left(a^2b^2c^2 + 2a^3c^3\right)x^2\right)\sqrt{-b^2+4ac}\,\arctan\left(-\frac{\sqrt{-b^2+4ac}(2cx+b)}{b^2-4ac}\right) + 2\left(ab^6 - 14a^2b^4c + 46a^3b^2c^2 - 24a^4c^3\right)x}{2\left(a^2b^6c^2 - 12a^3b^4c^3 + 48a^4b^2c^4 - 64a^5c^5 + \left(b^6c^4 - 12ab^4c^5 + 48a^2b^2c^6 - 64a^3c^7\right)x^4 + 2\left(b^7c^3 - 12ab^5c^4 + 48a^2b^3c^5 - 64a^3bc^6\right)x^3 + \left(b^8c^2 - 10ab^6c^3 + 24a^2b^4c^4 + 32a^3b^2c^5 - 128a^4c^6\right)x^2 + 2\left(ab^7c^2 - 12a^2b^5c^3 + 48a^3b^3c^4 - 64a^4bc^5\right)x\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x^2+b/x)^3/x^2,x, algorithm="fricas")
```

```
[Out] [-1/2*(a^2*b^5 - 14*a^3*b^3*c + 40*a^4*b*c^2 + 2*(b^6*c - 12*a*b^4*c^2 + 42
*a^2*b^2*c^3 - 40*a^3*c^4)*x^3 + (b^7 - 12*a*b^5*c + 30*a^2*b^3*c^2 + 8*a^3
```

```
*b*c^3)*x^2 - 12*(a^2*c^4*x^4 + 2*a^2*b*c^3*x^3 + 2*a^3*b*c^2*x + a^4*c^2 +
 (a^2*b^2*c^2 + 2*a^3*c^3)*x^2)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^2 + 2*b*c*x
+ b^2 - 2*a*c - sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) + 2*(a*b^
6 - 14*a^2*b^4*c + 46*a^3*b^2*c^2 - 24*a^4*c^3)*x)/(a^2*b^6*c^2 - 12*a^3*b^
4*c^3 + 48*a^4*b^2*c^4 - 64*a^5*c^5 + (b^6*c^4 - 12*a*b^4*c^5 + 48*a^2*b^2*
c^6 - 64*a^3*c^7)*x^4 + 2*(b^7*c^3 - 12*a*b^5*c^4 + 48*a^2*b^3*c^5 - 64*a^3
*b*c^6)*x^3 + (b^8*c^2 - 10*a*b^6*c^3 + 24*a^2*b^4*c^4 + 32*a^3*b^2*c^5 - 1
28*a^4*c^6)*x^2 + 2*(a*b^7*c^2 - 12*a^2*b^5*c^3 + 48*a^3*b^3*c^4 - 64*a^4*b
*c^5)*x), -1/2*(a^2*b^5 - 14*a^3*b^3*c + 40*a^4*b*c^2 + 2*(b^6*c - 12*a*b^4
*c^2 + 42*a^2*b^2*c^3 - 40*a^3*c^4)*x^3 + (b^7 - 12*a*b^5*c + 30*a^2*b^3*c^
2 + 8*a^3*b*c^3)*x^2 + 24*(a^2*c^4*x^4 + 2*a^2*b*c^3*x^3 + 2*a^3*b*c^2*x +
a^4*c^2 + (a^2*b^2*c^2 + 2*a^3*c^3)*x^2)*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b
^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) + 2*(a*b^6 - 14*a^2*b^4*c + 46*a^3*b
^2*c^2 - 24*a^4*c^3)*x)/(a^2*b^6*c^2 - 12*a^3*b^4*c^3 + 48*a^4*b^2*c^4 - 64
*a^5*c^5 + (b^6*c^4 - 12*a*b^4*c^5 + 48*a^2*b^2*c^6 - 64*a^3*c^7)*x^4 + 2*(
b^7*c^3 - 12*a*b^5*c^4 + 48*a^2*b^3*c^5 - 64*a^3*b*c^6)*x^3 + (b^8*c^2 - 10
*a*b^6*c^3 + 24*a^2*b^4*c^4 + 32*a^3*b^2*c^5 - 128*a^4*c^6)*x^2 + 2*(a*b^7*
c^2 - 12*a^2*b^5*c^3 + 48*a^3*b^3*c^4 - 64*a^4*b*c^5)*x)]
```

giac [A] time = 0.42, size = 202, normalized size = 1.82

$$\frac{12\,a^2\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{\left(b^4-8\,ab^2c+16\,a^2c^2\right)\sqrt{-b^2+4\,ac}}-\frac{2\,b^4cx^3-16\,ab^2c^2x^3+20\,a^2c^3x^3+b^5x^2-8\,ab^3cx^2-2\,a^2bc^2x^2+2\,ab^4x-20\,a^2b^2cx+12\,a^3c^2x+a^2b^3-10\,a^3bc}{2\left(b^4c^2-8\,ab^2c^3+16\,a^2c^4\right)\left(cx^2+bx+a\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x^2+b/x)^3/x^2,x, algorithm="giac")
```

```
[Out] 12*a^2*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/((b^4 - 8*a*b^2*c + 16*a^2*c^
2)*sqrt(-b^2 + 4*a*c)) - 1/2*(2*b^4*c*x^3 - 16*a*b^2*c^2*x^3 + 20*a^2*c^3*x
^3 + b^5*x^2 - 8*a*b^3*c*x^2 - 2*a^2*b*c^2*x^2 + 2*a*b^4*x - 20*a^2*b^2*c*x
 + 12*a^3*c^2*x + a^2*b^3 - 10*a^3*b*c)/((b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^
4)*(c*x^2 + b*x + a)^2)
```

maple [B] time = 0.01, size = 260, normalized size = 2.34

$$\frac{12a^2\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{\left(16a^2c^2-8a\,b^2c+b^4\right)\sqrt{4ac-b^2}}+\frac{-\frac{\left(10a^2c^2-8a\,b^2c+b^4\right)x^3}{\left(16a^2c^2-8a\,b^2c+b^4\right)c}+\frac{\left(10ac-b^2\right)a^2b}{2\left(16a^2c^2-8a\,b^2c+b^4\right)c^2}+\frac{\left(2a^2c^2+8a\,b^2c-b^4\right)b\,x^2}{2\left(16a^2c^2-8a\,b^2c+b^4\right)c^2}-\frac{\left(6a^2c^2-10a\,b^2c+b^4\right)ax}{\left(16a^2c^2-8a\,b^2c+b^4\right)c^2}}{\left(c\,x^2+bx+a\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c+a/x^2+b/x)^3/x^2,x)
```

```
[Out] (-1/c*(10*a^2*c^2-8*a*b^2*c+b^4)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^3+1/2*b*(2*a^
2*c^2+8*a*b^2*c-b^4)/c^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x^2-a*(6*a^2*c^2-10*a*b
^2*c+b^4)/(16*a^2*c^2-8*a*b^2*c+b^4)/c^2*x+1/2*a^2*b*(10*a*c-b^2)/c^2/(16*a
^2*c^2-8*a*b^2*c+b^4))/(c*x^2+b*x+a)^2+12*a^2/(16*a^2*c^2-8*a*b^2*c+b^4)/(4
*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x^2+b/x)^3/x^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

mupad [B] time = 0.19, size = 343, normalized size = 3.09

$$\frac{12\,a^2\,\mathrm{atan}\left(\frac{\left(\frac{6\,a^2\,\left(16\,a^2\,b\,c^2-8\,a\,b^3\,c+b^5\right)}{\left(4\,a\,c-b^2\right)^{5/2}\,\left(16\,a^2\,c^2-8\,a\,b^2\,c+b^4\right)}+\frac{12\,a^2\,c\,x}{\left(4\,a\,c-b^2\right)^{5/2}}\right)\left(16\,a^2\,c^2-8\,a\,b^2\,c+b^4\right)}{6\,a^2}\right)}{\left(4\,a\,c-b^2\right)^{5/2}}-\frac{\frac{x^3\,\left(10\,a^2\,c^2-8\,a\,b^2\,c+b^4\right)}{c\,\left(16\,a^2\,c^2-8\,a\,b^2\,c+b^4\right)}+\frac{a^2\,\left(b^3-10\,a\,b\,c\right)}{2\,c^2\,\left(16\,a^2\,c^2-8\,a\,b^2\,c+b^4\right)}-\frac{x^2\,\left(2\,a^2\,b\,c^2+8\,a\,b^3\,c-b^5\right)}{2\,c^2\,\left(16\,a^2\,c^2-8\,a\,b^2\,c+b^4\right)}+\frac{a\,x\,\left(6\,a^2\,c^2-10\,a\,b^2\,c+b^4\right)}{c^2\,\left(16\,a^2\,c^2-8\,a\,b^2\,c+b^4\right)}}{x^2\,\left(b^2+2\,a\,c\right)+a^2+c^2\,x^4+2\,a\,b\,x+2\,b\,c\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(c + a/x^2 + b/x)^3),x)
```

```
[Out] (12*a^2*atan((((6*a^2*(b^5 + 16*a^2*b*c^2 - 8*a*b^3*c))/((4*a*c - b^2)^(5/2
)*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (12*a^2*c*x)/(4*a*c - b^2)^(5/2))*(b^4
+ 16*a^2*c^2 - 8*a*b^2*c))/(6*a^2)))/(4*a*c - b^2)^(5/2) - ((x^3*(b^4 + 10*
a^2*c^2 - 8*a*b^2*c))/(c*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (a^2*(b^3 - 10*a
*b*c))/(2*c^2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) - (x^2*(2*a^2*b*c^2 - b^5 + 8
*a*b^3*c))/(2*c^2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (a*x*(b^4 + 6*a^2*c^2 -
 10*a*b^2*c))/(c^2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)))/(x^2*(2*a*c + b^2) + a^
2 + c^2*x^4 + 2*a*b*x + 2*b*c*x^3)
```

sympy [B] time = 1.41, size = 547, normalized size = 4.93

$$-6a^2\sqrt{-\frac{1}{(4ac-b^2)^5}}\log\left(x+\frac{-384a^5c^3\sqrt{-\frac{1}{(4ac-b^2)^5}}+288a^4b^2c^2\sqrt{-\frac{1}{(4ac-b^2)^5}}-72a^3b^4c\sqrt{-\frac{1}{(4ac-b^2)^5}}+6a^2b^6\sqrt{-\frac{1}{(4ac-b^2)^5}}+6a^2b}{12a^2c}\right)+6a^2\sqrt{-\frac{1}{(4ac-b^2)^5}}\log\left(x+\frac{384a^5c^3\sqrt{-\frac{1}{(4ac-b^2)^5}}-288a^4b^2c^2\sqrt{-\frac{1}{(4ac-b^2)^5}}+72a^3b^4c\sqrt{-\frac{1}{(4ac-b^2)^5}}-6a^2b^6\sqrt{-\frac{1}{(4ac-b^2)^5}}+6a^2b}{12a^2c}\right)+\frac{10a^3bc-a^2b^3+x^3(-20a^2c^3+16ab^2c^2-2b^4c)+x^2(2a^2bc^2+8ab^3c-b^5)+x(-12a^3c^2+20a^2b^2c-2ab^4)}{32a^4c^4-16a^3b^2c^3+2a^2b^4c^2+x^4(32a^2c^6-16ab^2c^5+2b^4c^4)+x^3(64a^2bc^5-32ab^3c^4+4b^5c^3)+x^2(64a^3c^5-12ab^4c^3+2b^6c^2)+x(64a^3bc^4-32a^2b^3c^3+4ab^5c^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x**2+b/x)**3/x**2,x)
```

```
[Out] -6*a**2*sqrt(-1/(4*a*c - b**2)**5)*log(x + (-384*a**5*c**3*sqrt(-1/(4*a*c -
 b**2)**5) + 288*a**4*b**2*c**2*sqrt(-1/(4*a*c - b**2)**5) - 72*a**3*b**4*c
*sqrt(-1/(4*a*c - b**2)**5) + 6*a**2*b**6*sqrt(-1/(4*a*c - b**2)**5) + 6*a*
*2*b)/(12*a**2*c)) + 6*a**2*sqrt(-1/(4*a*c - b**2)**5)*log(x + (384*a**5*c*
*3*sqrt(-1/(4*a*c - b**2)**5) - 288*a**4*b**2*c**2*sqrt(-1/(4*a*c - b**2)**
5) + 72*a**3*b**4*c*sqrt(-1/(4*a*c - b**2)**5) - 6*a**2*b**6*sqrt(-1/(4*a*c
 - b**2)**5) + 6*a**2*b)/(12*a**2*c)) + (10*a**3*b*c - a**2*b**3 + x**3*(-2
0*a**2*c**3 + 16*a*b**2*c**2 - 2*b**4*c) + x**2*(2*a**2*b*c**2 + 8*a*b**3*c
 - b**5) + x*(-12*a**3*c**2 + 20*a**2*b**2*c - 2*a*b**4))/(32*a**4*c**4 - 1
6*a**3*b**2*c**3 + 2*a**2*b**4*c**2 + x**4*(32*a**2*c**6 - 16*a*b**2*c**5 +
 2*b**4*c**4) + x**3*(64*a**2*b*c**5 - 32*a*b**3*c**4 + 4*b**5*c**3) + x**2
*(64*a**3*c**5 - 12*a*b**4*c**3 + 2*b**6*c**2) + x*(64*a**3*b*c**4 - 32*a**
2*b**3*c**3 + 4*a*b**5*c**2))
```

3.373 $\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^3 x^3} \, dx$

Optimal. Leaf size=107

$$\frac{3bx(2a+bx)}{2\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)}-\frac{x^3(b+2cx)}{2\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2}+\frac{6ab\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{5/2}}$$

Rubi [A] time = 0.05, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {1354, 728, 722, 618, 206}

$$-\frac{x^3(b+2cx)}{2\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2}+\frac{3bx(2a+bx)}{2\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)}+\frac{6ab\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c + a/x^2 + b/x)^3*x^3),x]
```

```
[Out] -(x^3*(b + 2*c*x))/(2*(b^2 - 4*a*c)*(a + b*x + c*x^2)^2) + (3*b*x*(2*a + b*
x))/(2*(b^2 - 4*a*c)^2*(a + b*x + c*x^2)) + (6*a*b*ArcTanh[(b + 2*c*x)/Sqrt
[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(5/2)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 722

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m - 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*(a + b*x
+ c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[(2*(2*p + 3)*(c*d^2 -
b*d*e + a*e^2))/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^(m - 2)*(a + b*x + c
*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] &
& NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && EqQ[m + 2*p + 2,
0] && LtQ[p, -1]
```

Rule 728

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^m*(b + 2*c*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)
*(b^2 - 4*a*c)), x] + Dist[(m*(2*c*d - b*e))/((p + 1)*(b^2 - 4*a*c)), Int[(
d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
 m, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2
*c*d - b*e, 0] && EqQ[m + 2*p + 3, 0] && LtQ[p, -1]
```

Rule 1354

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> Int[x^(m + 2*n*p)*(c + b/x^n + a/x^(2*n))^p, x] /; FreeQ[{a, b, c, m, n
```

```
}, x] && EqQ[n2, 2*n] && ILtQ[p, 0] && NegQ[n]
```

Rubi steps

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^3 x^3}\,dx=\int \frac{x^3}{\left(a+bx+cx^2\right)^3}\,dx$$

$$=-\frac{x^3(b+2cx)}{2\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2}+\frac{(3b)\int \frac{x^2}{\left(a+bx+cx^2\right)^2}\,dx}{2\left(b^2-4ac\right)}$$

$$=-\frac{x^3(b+2cx)}{2\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2}+\frac{3bx(2a+bx)}{2\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)}-\frac{(3ab)\int \frac{1}{a+bx+cx^2}\,dx}{\left(b^2-4ac\right)^2}$$

$$=-\frac{x^3(b+2cx)}{2\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2}+\frac{3bx(2a+bx)}{2\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)}+\frac{(6ab)\operatorname{Subst}\left(\int \frac{1}{b^2-4ac \ldots}\right.}{\left(b^2-4 \ldots\right.}$$

$$=-\frac{x^3(b+2cx)}{2\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2}+\frac{3bx(2a+bx)}{2\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)}+\frac{6ab\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{5/2}}$$

Mathematica [A] time = 0.19, size = 126, normalized size = 1.18

$$-\frac{8a^3c+a^2\left(b^2+10bcx+16c^2x^2\right)+abx\left(2b^2+bcx+6c^2x^2\right)+b^4x^2}{2c\left(b^2-4ac\right)^2(a+x(b+cx))^2}-\frac{6ab\tan^{-1}\left(\frac{b+2cx}{\sqrt{4ac-b^2}}\right)}{\left(4ac-b^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c + a/x^2 + b/x)^3*x^3),x]
```

```
[Out] -1/2*(8*a^3*c + b^4*x^2 + a*b*x*(2*b^2 + b*c*x + 6*c^2*x^2) + a^2*(b^2 + 10
*b*c*x + 16*c^2*x^2))/(c*(b^2 - 4*a*c)^2*(a + x*(b + c*x))^2) - (6*a*b*ArcT
an[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/(-b^2 + 4*a*c)^(5/2)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^3 x^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/((c + a/x^2 + b/x)^3*x^3),x]
```

```
[Out] IntegrateAlgebraic[1/((c + a/x^2 + b/x)^3*x^3), x]
```

fricas [B] time = 1.11, size = 872, normalized size = 8.15

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x^2+b/x)^3/x^3,x, algorithm="fricas")
```

```
[Out] [-1/2*(a^2*b^4 + 4*a^3*b^2*c - 32*a^4*c^2 + 6*(a*b^3*c^2 - 4*a^2*b*c^3)*x^3
 + (b^6 - 3*a*b^4*c + 12*a^2*b^2*c^2 - 64*a^3*c^3)*x^2 - 6*(a*b*c^3*x^4 + 2
```

```
*a*b^2*c^2*x^3 + 2*a^2*b^2*c*x + a^3*b*c + (a*b^3*c + 2*a^2*b*c^2)*x^2)*sqr
t(b^2 - 4*a*c)*log((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c + sqrt(b^2 - 4*a*c)*(
2*c*x + b))/(c*x^2 + b*x + a)) + 2*(a*b^5 + a^2*b^3*c - 20*a^3*b*c^2)*x)/(a
^2*b^6*c - 12*a^3*b^4*c^2 + 48*a^4*b^2*c^3 - 64*a^5*c^4 + (b^6*c^3 - 12*a*b
^4*c^4 + 48*a^2*b^2*c^5 - 64*a^3*c^6)*x^4 + 2*(b^7*c^2 - 12*a*b^5*c^3 + 48*
a^2*b^3*c^4 - 64*a^3*b*c^5)*x^3 + (b^8*c - 10*a*b^6*c^2 + 24*a^2*b^4*c^3 +
32*a^3*b^2*c^4 - 128*a^4*c^5)*x^2 + 2*(a*b^7*c - 12*a^2*b^5*c^2 + 48*a^3*b^
3*c^3 - 64*a^4*b*c^4)*x), -1/2*(a^2*b^4 + 4*a^3*b^2*c - 32*a^4*c^2 + 6*(a*b
^3*c^2 - 4*a^2*b*c^3)*x^3 + (b^6 - 3*a*b^4*c + 12*a^2*b^2*c^2 - 64*a^3*c^3)
*x^2 - 12*(a*b*c^3*x^4 + 2*a*b^2*c^2*x^3 + 2*a^2*b^2*c*x + a^3*b*c + (a*b^3
*c + 2*a^2*b*c^2)*x^2)*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x
 + b)/(b^2 - 4*a*c)) + 2*(a*b^5 + a^2*b^3*c - 20*a^3*b*c^2)*x)/(a^2*b^6*c -
 12*a^3*b^4*c^2 + 48*a^4*b^2*c^3 - 64*a^5*c^4 + (b^6*c^3 - 12*a*b^4*c^4 + 4
8*a^2*b^2*c^5 - 64*a^3*c^6)*x^4 + 2*(b^7*c^2 - 12*a*b^5*c^3 + 48*a^2*b^3*c^
4 - 64*a^3*b*c^5)*x^3 + (b^8*c - 10*a*b^6*c^2 + 24*a^2*b^4*c^3 + 32*a^3*b^2
*c^4 - 128*a^4*c^5)*x^2 + 2*(a*b^7*c - 12*a^2*b^5*c^2 + 48*a^3*b^3*c^3 - 64
*a^4*b*c^4)*x)]
```

giac [A] time = 0.31, size = 163, normalized size = 1.52

$$-\frac{6\,ab\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{\left(b^4-8\,ab^2c+16\,a^2c^2\right)\sqrt{-b^2+4\,ac}}-\frac{6\,abc^2x^3+b^4x^2+ab^2cx^2+16\,a^2c^2x^2+2\,ab^3x+10\,a^2bcx+a^2b^2+8\,a^3c}{2\left(b^4c-8\,ab^2c^2+16\,a^2c^3\right)\left(cx^2+bx+a\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x^2+b/x)^3/x^3,x, algorithm="giac")
```

```
[Out] -6*a*b*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/((b^4 - 8*a*b^2*c + 16*a^2*c^
2)*sqrt(-b^2 + 4*a*c)) - 1/2*(6*a*b*c^2*x^3 + b^4*x^2 + a*b^2*c*x^2 + 16*a^
2*c^2*x^2 + 2*a*b^3*x + 10*a^2*b*c*x + a^2*b^2 + 8*a^3*c)/((b^4*c - 8*a*b^2
*c^2 + 16*a^2*c^3)*(c*x^2 + b*x + a)^2)
```

maple [B] time = 0.01, size = 223, normalized size = 2.08

$$-\frac{6ab\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{\left(16a^2c^2-8a\,b^2c+b^4\right)\sqrt{4ac-b^2}}+\frac{-\frac{3abc\,x^3}{16a^2c^2-8a\,b^2c+b^4}-\frac{\left(5ac+b^2\right)abx}{\left(16a^2c^2-8a\,b^2c+b^4\right)c}-\frac{\left(8ac+b^2\right)a^2}{2\left(16a^2c^2-8a\,b^2c+b^4\right)c}-\frac{\left(16a^2c^2+a\,b^2c+b^4\right)x^2}{2\left(16a^2c^2-8a\,b^2c+b^4\right)c}}{\left(c\,x^2+bx+a\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c+a/x^2+b/x)^3/x^3,x)
```

```
[Out] (-3*a*b*c/(16*a^2*c^2-8*a*b^2*c+b^4)*x^3-1/2*(16*a^2*c^2+a*b^2*c+b^4)/c/(16
*a^2*c^2-8*a*b^2*c+b^4)*x^2-(5*a*c+b^2)*a*b/c/(16*a^2*c^2-8*a*b^2*c+b^4)*x-
1/2*a^2*(8*a*c+b^2)/c/(16*a^2*c^2-8*a*b^2*c+b^4))/(c*x^2+b*x+a)^2-6*a*b/(16
*a^2*c^2-8*a*b^2*c+b^4)/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2
))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x^2+b/x)^3/x^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

mupad [B] time = 1.43, size = 271, normalized size = 2.53

$$-\frac{\frac{x^2\left(16a^2c^2+ab^2c+b^4\right)}{2c\left(16a^2c^2-8ab^2c+b^4\right)}+\frac{a^2\left(b^2+8ac\right)}{2c\left(16a^2c^2-8ab^2c+b^4\right)}+\frac{3abcx^3}{16a^2c^2-8ab^2c+b^4}+\frac{abx\left(b^2+5ac\right)}{c\left(16a^2c^2-8ab^2c+b^4\right)}}{x^2\left(b^2+2ac\right)+a^2+c^2x^4+2abx+2bcx^3}-\frac{6ab\,\mathrm{atan}\left(\frac{\left(\frac{3ab^2}{\left(4ac-b^2\right)^{5/2}}+\frac{6abcx}{\left(4ac-b^2\right)^{5/2}}\right)\left(16a^2c^2-8ab^2c+b^4\right)}{3ab}\right)}{\left(4ac-b^2\right)^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(c + a/x^2 + b/x)^3),x)
```

```
[Out] - ((x^2*(b^4 + 16*a^2*c^2 + a*b^2*c))/(2*c*(b^4 + 16*a^2*c^2 - 8*a*b^2*c))
+ (a^2*(8*a*c + b^2))/(2*c*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (3*a*b*c*x^3)/
(b^4 + 16*a^2*c^2 - 8*a*b^2*c) + (a*b*x*(5*a*c + b^2))/(c*(b^4 + 16*a^2*c^2
 - 8*a*b^2*c)))/(x^2*(2*a*c + b^2) + a^2 + c^2*x^4 + 2*a*b*x + 2*b*c*x^3) -
 (6*a*b*atan((((3*a*b^2)/(4*a*c - b^2)^(5/2) + (6*a*b*c*x)/(4*a*c - b^2)^(5
/2))*(b^4 + 16*a^2*c^2 - 8*a*b^2*c))/(3*a*b)))/(4*a*c - b^2)^(5/2)
```

sympy [B] time = 1.21, size = 513, normalized size = 4.79

$$3ab\sqrt{-\frac{1}{\left(4ac-b^2\right)^5}}\log\left(x+\frac{-192a^4bc^3\sqrt{-\frac{1}{\left(4ac-b^2\right)^5}}+144a^3b^3c^2\sqrt{-\frac{1}{\left(4ac-b^2\right)^5}}-36a^2b^5c\sqrt{-\frac{1}{\left(4ac-b^2\right)^5}}+3ab^7\sqrt{-\frac{1}{\left(4ac-b^2\right)^5}}+3ab^2}{6abc}\right)-3ab\sqrt{-\frac{1}{\left(4ac-b^2\right)^5}}\log\left(x+\frac{192a^4bc^3\sqrt{-\frac{1}{\left(4ac-b^2\right)^5}}-144a^3b^3c^2\sqrt{-\frac{1}{\left(4ac-b^2\right)^5}}+36a^2b^5c\sqrt{-\frac{1}{\left(4ac-b^2\right)^5}}-3ab^7\sqrt{-\frac{1}{\left(4ac-b^2\right)^5}}+3ab^2}{6abc}\right)+\frac{-8a^3c-a^2b^2-6abc^2x^3+x^2\left(-16a^2c^2-ab^2c-b^4\right)+x\left(-10a^2bc-2ab^3\right)}{32a^4c^3-16a^3b^2c^2+2a^2b^4c+x^4\left(32a^2c^5-16ab^2c^4+2b^4c^3\right)+x^3\left(64a^2bc^4-32ab^3c^3+4b^5c^2\right)+x^2\left(64a^3c^4-12ab^4c^2+2b^6c\right)+x\left(64a^3bc^3-32a^2b^3c^2+4ab^5c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x**2+b/x)**3/x**3,x)
```

```
[Out] 3*a*b*sqrt(-1/(4*a*c - b**2)**5)*log(x + (-192*a**4*b*c**3*sqrt(-1/(4*a*c -
 b**2)**5) + 144*a**3*b**3*c**2*sqrt(-1/(4*a*c - b**2)**5) - 36*a**2*b**5*c
*sqrt(-1/(4*a*c - b**2)**5) + 3*a*b**7*sqrt(-1/(4*a*c - b**2)**5) + 3*a*b**
2)/(6*a*b*c)) - 3*a*b*sqrt(-1/(4*a*c - b**2)**5)*log(x + (192*a**4*b*c**3*s
qrt(-1/(4*a*c - b**2)**5) - 144*a**3*b**3*c**2*sqrt(-1/(4*a*c - b**2)**5) +
 36*a**2*b**5*c*sqrt(-1/(4*a*c - b**2)**5) - 3*a*b**7*sqrt(-1/(4*a*c - b**2
)**5) + 3*a*b**2)/(6*a*b*c)) + (-8*a**3*c - a**2*b**2 - 6*a*b*c**2*x**3 + x
**2*(-16*a**2*c**2 - a*b**2*c - b**4) + x*(-10*a**2*b*c - 2*a*b**3))/(32*a*
*4*c**3 - 16*a**3*b**2*c**2 + 2*a**2*b**4*c + x**4*(32*a**2*c**5 - 16*a*b**
2*c**4 + 2*b**4*c**3) + x**3*(64*a**2*b*c**4 - 32*a*b**3*c**3 + 4*b**5*c**2
) + x**2*(64*a**3*c**4 - 12*a*b**4*c**2 + 2*b**6*c) + x*(64*a**3*b*c**3 - 3
2*a**2*b**3*c**2 + 4*a*b**5*c))
```

3.374 $\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^3 x^4}\,dx$

Optimal. Leaf size=115

$$\frac{x(2a+bx)}{2\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2}+\frac{x\left(2ac+b^2\right)+3ab}{\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)}-\frac{2\left(2ac+b^2\right)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{5/2}}$$

Rubi [A] time = 0.07, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {1354, 738, 638, 618, 206}

$$\frac{x(2a+bx)}{2\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2}+\frac{x\left(2ac+b^2\right)+3ab}{\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)}-\frac{2\left(2ac+b^2\right)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c + a/x^2 + b/x)^3*x^4),x]
```

```
[Out] (x*(2*a + b*x))/(2*(b^2 - 4*a*c)*(a + b*x + c*x^2)^2) + (3*a*b + (b^2 + 2*a
*c)*x)/((b^2 - 4*a*c)^2*(a + b*x + c*x^2)) - (2*(b^2 + 2*a*c)*ArcTanh[(b +
2*c*x)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(5/2)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 638

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)*(a + b*x + c*x^2)^(p + 1))/((p +
 1)*(b^2 - 4*a*c)), x] - Dist[((2*p + 3)*(2*c*d - b*e))/((p + 1)*(b^2 - 4*a
*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &&
 NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 738

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m - 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*(a + b*x
+ c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*
c)), Int[(d + e*x)^(m - 2)*Simp[e*(2*a*e*(m - 1) + b*d*(2*p - m + 4)) - 2*c
*d^2*(2*p + 3) + e*(b*e - 2*d*c)*(m + 2*p + 2)*x, x]*(a + b*x + c*x^2)^(p +
 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^
2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p, -1] && GtQ[m, 1] &&
IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 1354

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> Int[x^(m + 2*n*p)*(c + b/x^n + a/x^(2*n))^p, x] /; FreeQ[{a, b, c, m, n
```

```
}, x] && EqQ[n2, 2*n] && ILtQ[p, 0] && NegQ[n]
```

Rubi steps

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^3 x^4}\,dx = \int \frac{x^2}{\left(a+bx+cx^2\right)^3}\,dx$$

$$= \frac{x(2a+bx)}{2\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2} - \frac{\int \frac{2a-2bx}{\left(a+bx+cx^2\right)^2}\,dx}{2\left(b^2-4ac\right)}$$

$$= \frac{x(2a+bx)}{2\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2} + \frac{3ab+\left(b^2+2ac\right)x}{\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)} + \frac{\left(b^2+2ac\right)\int \frac{1}{a+bx+cx^2}\,dx}{\left(b^2-4ac\right)^2}$$

$$= \frac{x(2a+bx)}{2\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2} + \frac{3ab+\left(b^2+2ac\right)x}{\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)} - \frac{\left(2\left(b^2+2ac\right)\right)\mathrm{Subst}\left(\int \right.}{\left(b^2-\right.}$$

$$= \frac{x(2a+bx)}{2\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2} + \frac{3ab+\left(b^2+2ac\right)x}{\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)} - \frac{2\left(b^2+2ac\right)\tanh^{-1}\left(\frac{b+}{\sqrt{b^2}}\right.}{\left(b^2-4ac\right)^{5/2}}$$

Mathematica [A] time = 0.14, size = 131, normalized size = 1.14

$$\frac{1}{2}\left(\frac{\left(2ac+b^2\right)(b+2cx)}{c\left(b^2-4ac\right)^2(a+x(b+cx))} + \frac{a(b-2cx)+b^2x}{c\left(4ac-b^2\right)(a+x(b+cx))^2} + \frac{4\left(2ac+b^2\right)\tan^{-1}\left(\frac{b+2cx}{\sqrt{4ac-b^2}}\right)}{\left(4ac-b^2\right)^{5/2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c + a/x^2 + b/x)^3*x^4),x]
```

```
[Out] ((b^2*x + a*(b - 2*c*x))/(c*(-b^2 + 4*a*c)*(a + x*(b + c*x))^2) + ((b^2 + 2
*a*c)*(b + 2*c*x))/(c*(b^2 - 4*a*c)^2*(a + x*(b + c*x))) + (4*(b^2 + 2*a*c)
*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/(-b^2 + 4*a*c)^(5/2))/2
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^3 x^4}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/((c + a/x^2 + b/x)^3*x^4),x]
```

```
[Out] IntegrateAlgebraic[1/((c + a/x^2 + b/x)^3*x^4), x]
```

fricas [B] time = 1.10, size = 887, normalized size = 7.71

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x^2+b/x)^3/x^4,x, algorithm="fricas")
```

```
[Out] [1/2*(6*a^2*b^3 - 24*a^3*b*c + 2*(b^4*c - 2*a*b^2*c^2 - 8*a^2*c^3)*x^3 + 3*
(b^5 - 2*a*b^3*c - 8*a^2*b*c^2)*x^2 + 2*((b^2*c^2 + 2*a*c^3)*x^4 + a^2*b^2
```

```
+ 2*a^3*c + 2*(b^3*c + 2*a*b*c^2)*x^3 + (b^4 + 4*a*b^2*c + 4*a^2*c^2)*x^2 +
 2*(a*b^3 + 2*a^2*b*c)*x)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^2 + 2*b*c*x + b^2
- 2*a*c - sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) + 2*(5*a*b^4 -
22*a^2*b^2*c + 8*a^3*c^2)*x)/(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*
a^5*c^3 + (b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*x^4 + 2*(b
^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*x^3 + (b^8 - 10*a*b^6*
c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*x^2 + 2*(a*b^7 - 12*a^2*
b^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3)*x), 1/2*(6*a^2*b^3 - 24*a^3*b*c + 2*
(b^4*c - 2*a*b^2*c^2 - 8*a^2*c^3)*x^3 + 3*(b^5 - 2*a*b^3*c - 8*a^2*b*c^2)*x
^2 - 4*((b^2*c^2 + 2*a*c^3)*x^4 + a^2*b^2 + 2*a^3*c + 2*(b^3*c + 2*a*b*c^2)
*x^3 + (b^4 + 4*a*b^2*c + 4*a^2*c^2)*x^2 + 2*(a*b^3 + 2*a^2*b*c)*x)*sqrt(-b
^2 + 4*a*c)*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) + 2*(5*a*
b^4 - 22*a^2*b^2*c + 8*a^3*c^2)*x)/(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2
 - 64*a^5*c^3 + (b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*x^4
+ 2*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*x^3 + (b^8 - 10*
a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*x^2 + 2*(a*b^7 - 1
2*a^2*b^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3)*x)]
```

giac [A] time = 0.44, size = 154, normalized size = 1.34

$$\frac{2\left(b^{2}+2\,ac\right)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^{2}+4\,ac}}\right)}{\left(b^{4}-8\,ab^{2}c+16\,a^{2}c^{2}\right)\sqrt{-b^{2}+4\,ac}}+\frac{2\,b^{2}cx^{3}+4\,ac^{2}x^{3}+3\,b^{3}x^{2}+6\,abcx^{2}+10\,ab^{2}x-4\,a^{2}cx+6\,a^{2}b}{2\left(b^{4}-8\,ab^{2}c+16\,a^{2}c^{2}\right)\left(cx^{2}+bx+a\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x^2+b/x)^3/x^4,x, algorithm="giac")
```

```
[Out] 2*(b^2 + 2*a*c)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/((b^4 - 8*a*b^2*c +
16*a^2*c^2)*sqrt(-b^2 + 4*a*c)) + 1/2*(2*b^2*c*x^3 + 4*a*c^2*x^3 + 3*b^3*x^
2 + 6*a*b*c*x^2 + 10*a*b^2*x - 4*a^2*c*x + 6*a^2*b)/((b^4 - 8*a*b^2*c + 16*
a^2*c^2)*(c*x^2 + b*x + a)^2)
```

maple [B] time = 0.01, size = 262, normalized size = 2.28

$$\frac{4ac\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^{2}}}\right)}{\left(16a^{2}c^{2}-8a\,b^{2}c+b^{4}\right)\sqrt{4ac-b^{2}}}+\frac{2b^{2}\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^{2}}}\right)}{\left(16a^{2}c^{2}-8a\,b^{2}c+b^{4}\right)\sqrt{4ac-b^{2}}}+\frac{\frac{\left(2ac+b^{2}\right)c\,x^{3}}{16a^{2}c^{2}-8a\,b^{2}c+b^{4}}+\frac{3a^{2}b}{16a^{2}c^{2}-8a\,b^{2}c+b^{4}}+\frac{3\left(2ac+b^{2}\right)b\,x^{2}}{2\left(16a^{2}c^{2}-8a\,b^{2}c+b^{4}\right)}-\frac{\left(2ac-5b^{2}\right)ax}{16a^{2}c^{2}-8a\,b^{2}c+b^{4}}}{\left(c\,x^{2}+bx+a\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c+a/x^2+b/x)^3/x^4,x)
```

```
[Out] (c*(2*a*c+b^2)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^3+3/2*b*(2*a*c+b^2)/(16*a^2*c^2
-8*a*b^2*c+b^4)*x^2-a*(2*a*c-5*b^2)/(16*a^2*c^2-8*a*b^2*c+b^4)*x+3*a^2*b/(1
6*a^2*c^2-8*a*b^2*c+b^4))/(c*x^2+b*x+a)^2+4/(16*a^2*c^2-8*a*b^2*c+b^4)/(4*a
*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*a*c+2/(16*a^2*c^2-8*a*b^2
*c+b^4)/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*b^2
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x^2+b/x)^3/x^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

mupad [B] time = 1.50, size = 313, normalized size = 2.72

$$\frac{\frac{3a^2b}{16a^2c^2-8ab^2c+b^4}-\frac{ax\left(2ac-5b^2\right)}{16a^2c^2-8ab^2c+b^4}+\frac{3bx^2\left(b^2+2ac\right)}{2\left(16a^2c^2-8ab^2c+b^4\right)}+\frac{cx^3\left(b^2+2ac\right)}{16a^2c^2-8ab^2c+b^4}}{x^2\left(b^2+2ac\right)+a^2+c^2x^4+2abx+2bcx^3}+\frac{2\,\mathrm{atan}\left(\frac{\left(\frac{\left(b^2+2ac\right)\left(16a^2bc^2-8ab^3c+b^5\right)}{\left(4ac-b^2\right)^{5/2}\left(16a^2c^2-8ab^2c+b^4\right)}+\frac{2cx\left(b^2+2ac\right)}{\left(4ac-b^2\right)^{5/2}}\right)\left(16a^2c^2-8ab^2c+b^4\right)}{b^2+2ac}\right)\left(b^2+2ac\right)}{\left(4ac-b^2\right)^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(c + a/x^2 + b/x)^3),x)
```

```
[Out] ((3*a^2*b)/(b^4 + 16*a^2*c^2 - 8*a*b^2*c) - (a*x*(2*a*c - 5*b^2))/(b^4 + 16
*a^2*c^2 - 8*a*b^2*c) + (3*b*x^2*(2*a*c + b^2))/(2*(b^4 + 16*a^2*c^2 - 8*a*
b^2*c)) + (c*x^3*(2*a*c + b^2))/(b^4 + 16*a^2*c^2 - 8*a*b^2*c))/(x^2*(2*a*c
 + b^2) + a^2 + c^2*x^4 + 2*a*b*x + 2*b*c*x^3) + (2*atan(((((2*a*c + b^2)*(
b^5 + 16*a^2*b*c^2 - 8*a*b^3*c))/((4*a*c - b^2)^(5/2)*(b^4 + 16*a^2*c^2 - 8
*a*b^2*c)) + (2*c*x*(2*a*c + b^2))/(4*a*c - b^2)^(5/2))*(b^4 + 16*a^2*c^2 -
 8*a*b^2*c))/(2*a*c + b^2))*(2*a*c + b^2))/(4*a*c - b^2)^(5/2)
```

sympy [B] time = 1.28, size = 570, normalized size = 4.96

$$-\sqrt{-\frac{1}{\left(4ac-b^2\right)^5}}\left(2ac+b^2\right)\log\left(x+\frac{-64a^3c^3\sqrt{-\frac{1}{\left(4ac-b^2\right)^5}}\left(2ac+b^2\right)+48a^2b^2c^2\sqrt{-\frac{1}{\left(4ac-b^2\right)^5}}\left(2ac+b^2\right)-12ab^4c\sqrt{-\frac{1}{\left(4ac-b^2\right)^5}}\left(2ac+b^2\right)+2abc+b^6\sqrt{-\frac{1}{\left(4ac-b^2\right)^5}}\left(2ac+b^2\right)+b^3}{4ac^2+2b^2c}\right)+\sqrt{-\frac{1}{\left(4ac-b^2\right)^5}}\left(2ac+b^2\right)\log\left(x+\frac{64a^3c^3\sqrt{-\frac{1}{\left(4ac-b^2\right)^5}}\left(2ac+b^2\right)-48a^2b^2c^2\sqrt{-\frac{1}{\left(4ac-b^2\right)^5}}\left(2ac+b^2\right)+12ab^4c\sqrt{-\frac{1}{\left(4ac-b^2\right)^5}}\left(2ac+b^2\right)+2abc-b^6\sqrt{-\frac{1}{\left(4ac-b^2\right)^5}}\left(2ac+b^2\right)+b^3}{4ac^2+2b^2c}\right)+\frac{6a^2b+x^3\left(4ac^2+2b^2c\right)+x^2\left(6abc+3b^3\right)+x\left(-4a^2c+10ab^2\right)}{32a^4c^2-16a^3b^2c+2a^2b^4+x^4\left(32a^2c^4-16ab^2c^3+2b^4c^2\right)+x^3\left(64a^2bc^3-32ab^3c^2+4b^5c\right)+x^2\left(64a^3c^3-12ab^4c+2b^6\right)+x\left(64a^3bc^2-32a^2b^3c+4ab^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x**2+b/x)**3/x**4,x)
```

```
[Out] -sqrt(-1/(4*a*c - b**2)**5)*(2*a*c + b**2)*log(x + (-64*a**3*c**3*sqrt(-1/(
4*a*c - b**2)**5)*(2*a*c + b**2) + 48*a**2*b**2*c**2*sqrt(-1/(4*a*c - b**2)
**5)*(2*a*c + b**2) - 12*a*b**4*c*sqrt(-1/(4*a*c - b**2)**5)*(2*a*c + b**2)
 + 2*a*b*c + b**6*sqrt(-1/(4*a*c - b**2)**5)*(2*a*c + b**2) + b**3)/(4*a*c*
*2 + 2*b**2*c)) + sqrt(-1/(4*a*c - b**2)**5)*(2*a*c + b**2)*log(x + (64*a**
3*c**3*sqrt(-1/(4*a*c - b**2)**5)*(2*a*c + b**2) - 48*a**2*b**2*c**2*sqrt(-
1/(4*a*c - b**2)**5)*(2*a*c + b**2) + 12*a*b**4*c*sqrt(-1/(4*a*c - b**2)**5
)*(2*a*c + b**2) + 2*a*b*c - b**6*sqrt(-1/(4*a*c - b**2)**5)*(2*a*c + b**2)
 + b**3)/(4*a*c**2 + 2*b**2*c)) + (6*a**2*b + x**3*(4*a*c**2 + 2*b**2*c) +
x**2*(6*a*b*c + 3*b**3) + x*(-4*a**2*c + 10*a*b**2))/(32*a**4*c**2 - 16*a**
3*b**2*c + 2*a**2*b**4 + x**4*(32*a**2*c**4 - 16*a*b**2*c**3 + 2*b**4*c**2)
 + x**3*(64*a**2*b*c**3 - 32*a*b**3*c**2 + 4*b**5*c) + x**2*(64*a**3*c**3 -
 12*a*b**4*c + 2*b**6) + x*(64*a**3*b*c**2 - 32*a**2*b**3*c + 4*a*b**5))
```

3.375 $\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^3 x^5} \, dx$

Optimal. Leaf size=103

$$\frac{2a+bx}{2\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2}-\frac{3b(b+2cx)}{2\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)}+\frac{6bc\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{5/2}}$$

Rubi [A] time = 0.04, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {1354, 638, 614, 618, 206}

$$\frac{2a+bx}{2\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2}-\frac{3b(b+2cx)}{2\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)}+\frac{6bc\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c + a/x^2 + b/x)^3*x^5),x]
```

```
[Out] (2*a + b*x)/(2*(b^2 - 4*a*c)*(a + b*x + c*x^2)^2) - (3*b*(b + 2*c*x))/(2*(b
^2 - 4*a*c)^2*(a + b*x + c*x^2)) + (6*b*c*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*
a*c]])/(b^2 - 4*a*c)^(5/2)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 614

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[(2*c*(2*p +
 3))/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
egerQ[4*p]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 638

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)*(a + b*x + c*x^2)^(p + 1))/((p +
 1)*(b^2 - 4*a*c)), x] - Dist[((2*p + 3)*(2*c*d - b*e))/((p + 1)*(b^2 - 4*a
*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &&
 NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 1354

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> Int[x^(m + 2*n*p)*(c + b/x^n + a/x^(2*n))^p, x] /; FreeQ[{a, b, c, m, n
}, x] && EqQ[n2, 2*n] && ILtQ[p, 0] && NegQ[n]
```

Rubi steps

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^3 x^5}\,dx = \int \frac{x}{\left(a+bx+cx^2\right)^3}\,dx$$

$$= \frac{2a+bx}{2\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2} + \frac{(3b)\int \frac{1}{\left(a+bx+cx^2\right)^2}\,dx}{2\left(b^2-4ac\right)}$$

$$= \frac{2a+bx}{2\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2} - \frac{3b(b+2cx)}{2\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)} - \frac{(3bc)\int \frac{1}{a+bx+cx^2}\,dx}{\left(b^2-4ac\right)^2}$$

$$= \frac{2a+bx}{2\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2} - \frac{3b(b+2cx)}{2\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)} + \frac{(6bc)\,\mathrm{Subst}\left(\int \frac{1}{b^2-4ac-x}\right.}{\left(b^2-4ac\right.}$$

$$= \frac{2a+bx}{2\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2} - \frac{3b(b+2cx)}{2\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)} + \frac{6bc\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{5/2}}$$

Mathematica [**A**] time = 0.10, size = 102, normalized size = 0.99

$$\frac{\frac{\left(b^2-4ac\right)(2a+bx)}{(a+x(b+cx))^2} - \frac{12bc\tan^{-1}\left(\frac{b+2cx}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}} - \frac{3b(b+2cx)}{a+x(b+cx)}}{2\left(b^2-4ac\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c + a/x^2 + b/x)^3*x^5),x]
```

```
[Out] (((b^2 - 4*a*c)*(2*a + b*x))/(a + x*(b + c*x))^2 - (3*b*(b + 2*c*x))/(a + x
*(b + c*x)) - (12*b*c*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4
*a*c])/(2*(b^2 - 4*a*c)^2)
```

IntegrateAlgebraic [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^3 x^5}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/((c + a/x^2 + b/x)^3*x^5),x]
```

```
[Out] IntegrateAlgebraic[1/((c + a/x^2 + b/x)^3*x^5), x]
```

fricas [**B**] time = 1.19, size = 788, normalized size = 7.65

$$\left[-\frac{ab^4+4a^2b^2c-32a^3c^2+6\left(b^3c^2-4abc^3\right)x^3+9\left(b^4c-4ab^2c^2\right)x^2-6\left(bc^3x^4+2b^2c^2x^3+2ab^2cx+a^2bc+\left(b^3c+2abc^2\right)x^2\right)\sqrt{b^2-4ac}\log\left(\frac{2c^2x^2+2bcx+b^2-2ac+\sqrt{b^2-4ac}(2cx+b)}{cx^2+bx+a}\right)+2\left(b^5+ab^3c-20a^2bc^2\right)x}{2\left(a^2b^6-12a^3b^4c+48a^4b^2c^2-64a^5c^3+\left(b^6c^2-12ab^4c^3+48a^2b^2c^4-64a^3c^5\right)x^4+2\left(b^7c-12ab^5c^2+48a^2b^3c^3-64a^3bc^4\right)x^3+\left(b^8-10ab^6c+24a^2b^4c^2+32a^3b^2c^3-128a^4c^4\right)x^2+2\left(ab^7-12a^2b^5c+48a^3b^3c^2-64a^4bc^3\right)x\right)}, -\frac{ab^4+4a^2b^2c-32a^3c^2+6\left(b^3c^2-4abc^3\right)x^3+9\left(b^4c-4ab^2c^2\right)x^2-12\left(bc^3x^4+2b^2c^2x^3+2ab^2cx+a^2bc+\left(b^3c+2abc^2\right)x^2\right)\sqrt{-b^2+4ac}\arctan\left(-\frac{\sqrt{-b^2+4ac}(2cx+b)}{b^2-4ac}\right)+2\left(b^5+ab^3c-20a^2bc^2\right)x}{2\left(a^2b^6-12a^3b^4c+48a^4b^2c^2-64a^5c^3+\left(b^6c^2-12ab^4c^3+48a^2b^2c^4-64a^3c^5\right)x^4+2\left(b^7c-12ab^5c^2+48a^2b^3c^3-64a^3bc^4\right)x^3+\left(b^8-10ab^6c+24a^2b^4c^2+32a^3b^2c^3-128a^4c^4\right)x^2+2\left(ab^7-12a^2b^5c+48a^3b^3c^2-64a^4bc^3\right)x\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x^2+b/x)^3/x^5,x, algorithm="fricas")
```

```
[Out] [-1/2*(a*b^4 + 4*a^2*b^2*c - 32*a^3*c^2 + 6*(b^3*c^2 - 4*a*b*c^3)*x^3 + 9*(
b^4*c - 4*a*b^2*c^2)*x^2 - 6*(b*c^3*x^4 + 2*b^2*c^2*x^3 + 2*a*b^2*c*x + a^2
*b*c + (b^3*c + 2*a*b*c^2)*x^2)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^2 + 2*b*c*x
+ b^2 - 2*a*c + sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) + 2*(b^5
```

```
+ a*b^3*c - 20*a^2*b*c^2)*x)/(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*
a^5*c^3 + (b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*x^4 + 2*(b
^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*x^3 + (b^8 - 10*a*b^6*
c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*x^2 + 2*(a*b^7 - 12*a^2*
b^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3)*x), -1/2*(a*b^4 + 4*a^2*b^2*c - 32*a
^3*c^2 + 6*(b^3*c^2 - 4*a*b*c^3)*x^3 + 9*(b^4*c - 4*a*b^2*c^2)*x^2 - 12*(b*
c^3*x^4 + 2*b^2*c^2*x^3 + 2*a*b^2*c*x + a^2*b*c + (b^3*c + 2*a*b*c^2)*x^2)*
sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) +
2*(b^5 + a*b^3*c - 20*a^2*b*c^2)*x)/(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^
2 - 64*a^5*c^3 + (b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*x^4
 + 2*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*x^3 + (b^8 - 10
*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*x^2 + 2*(a*b^7 -
12*a^2*b^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3)*x)]
```

giac [A] time = 0.41, size = 135, normalized size = 1.31

$$-\frac{6\,bc\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{\left(b^4-8\,ab^2c+16\,a^2c^2\right)\sqrt{-b^2+4\,ac}}-\frac{6\,bc^2x^3+9\,b^2cx^2+2\,b^3x+10\,abcx+ab^2+8\,a^2c}{2\left(b^4-8\,ab^2c+16\,a^2c^2\right)\left(cx^2+bx+a\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x^2+b/x)^3/x^5,x, algorithm="giac")
```

```
[Out] -6*b*c*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/((b^4 - 8*a*b^2*c + 16*a^2*c^
2)*sqrt(-b^2 + 4*a*c)) - 1/2*(6*b*c^2*x^3 + 9*b^2*c*x^2 + 2*b^3*x + 10*a*b*
c*x + a*b^2 + 8*a^2*c)/((b^4 - 8*a*b^2*c + 16*a^2*c^2)*(c*x^2 + b*x + a)^2)
```

maple [A] time = 0.00, size = 130, normalized size = 1.26

$$-\frac{3bcx}{\left(4ac-b^2\right)^2\left(c\,x^2+bx+a\right)}-\frac{6bc\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{\left(4ac-b^2\right)^{\frac{5}{2}}}-\frac{3b^2}{2\left(4ac-b^2\right)^2\left(c\,x^2+bx+a\right)}+\frac{-bx-2a}{2\left(4ac-b^2\right)\left(c\,x^2+bx+a\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c+a/x^2+b/x)^3/x^5,x)
```

```
[Out] 1/2*(-b*x-2*a)/(4*a*c-b^2)/(c*x^2+b*x+a)^2-3/(4*a*c-b^2)^2*b/(c*x^2+b*x+a)*
c*x-3/2/(4*a*c-b^2)^2*b^2/(c*x^2+b*x+a)-6/(4*a*c-b^2)^(5/2)*b*c*arctan((2*c
*x+b)/(4*a*c-b^2)^(1/2))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x^2+b/x)^3/x^5,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

mupad [B] time = 1.43, size = 253, normalized size = 2.46

$$-\frac{\frac{8\,c\,a^2+a\,b^2}{2\left(16\,a^2\,c^2-8\,a\,b^2\,c+b^4\right)}+\frac{9\,b^2\,c\,x^2}{2\left(16\,a^2\,c^2-8\,a\,b^2\,c+b^4\right)}+\frac{3\,b\,c^2\,x^3}{16\,a^2\,c^2-8\,a\,b^2\,c+b^4}+\frac{b\,x\left(b^2+5\,a\,c\right)}{16\,a^2\,c^2-8\,a\,b^2\,c+b^4}}{x^2\,\left(b^2+2\,a\,c\right)+a^2+c^2\,x^4+2\,a\,b\,x+2\,b\,c\,x^3}-\frac{6\,b\,c\,\mathrm{atan}\left(\frac{\left(\frac{3\,b^2\,c}{{\left(4\,a\,c-b^2\right)}^{5/2}}+\frac{6\,b\,c^2\,x}{{\left(4\,a\,c-b^2\right)}^{5/2}}\right)\left(16\,a^2\,c^2-8\,a\,b^2\,c+b^4\right)}{3\,b\,c}\right)}{{\left(4\,a\,c-b^2\right)}^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^5*(c + a/x^2 + b/x)^3),x)
```

```
[Out] - ((a*b^2 + 8*a^2*c)/(2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (9*b^2*c*x^2)/(2*
(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (3*b*c^2*x^3)/(b^4 + 16*a^2*c^2 - 8*a*b^2
*c) + (b*x*(5*a*c + b^2))/(b^4 + 16*a^2*c^2 - 8*a*b^2*c))/(x^2*(2*a*c + b^2
) + a^2 + c^2*x^4 + 2*a*b*x + 2*b*c*x^3) - (6*b*c*atan((((3*b^2*c)/(4*a*c -
 b^2)^(5/2) + (6*b*c^2*x)/(4*a*c - b^2)^(5/2))*(b^4 + 16*a^2*c^2 - 8*a*b^2*
c))/(3*b*c)))/(4*a*c - b^2)^(5/2)
```

sympy [B] time = 1.09, size = 481, normalized size = 4.67

$$3bc\sqrt{-\frac{1}{\left(4ac-b^2\right)^5}}\log\left(x+\frac{-192a^3bc^4\sqrt{-\frac{1}{\left(4ac-b^2\right)^5}}+144a^2b^3c^3\sqrt{-\frac{1}{\left(4ac-b^2\right)^5}}-36ab^5c^2\sqrt{-\frac{1}{\left(4ac-b^2\right)^5}}+3b^7c\sqrt{-\frac{1}{\left(4ac-b^2\right)^5}}+3b^2c}{6bc^2}\right)-3bc\sqrt{-\frac{1}{\left(4ac-b^2\right)^5}}\log\left(x+\frac{192a^3bc^4\sqrt{-\frac{1}{\left(4ac-b^2\right)^5}}-144a^2b^3c^3\sqrt{-\frac{1}{\left(4ac-b^2\right)^5}}+36ab^5c^2\sqrt{-\frac{1}{\left(4ac-b^2\right)^5}}-3b^7c\sqrt{-\frac{1}{\left(4ac-b^2\right)^5}}+3b^2c}{6bc^2}\right)+\frac{-8a^2c-ab^2-9b^2cx^2-6bc^2x^3+x\left(-10abc-2b^3\right)}{32a^4c^2-16a^3b^2c+2a^2b^4+x^4\left(32a^2c^4-16ab^2c^3+2b^4c^2\right)+x^3\left(64a^2bc^3-32ab^3c^2+4b^5c\right)+x^2\left(64a^3c^3-12ab^4c+2b^6\right)+x\left(64a^3bc^2-32a^2b^3c+4ab^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x**2+b/x)**3/x**5,x)
```

```
[Out] 3*b*c*sqrt(-1/(4*a*c - b**2)**5)*log(x + (-192*a**3*b*c**4*sqrt(-1/(4*a*c -
 b**2)**5) + 144*a**2*b**3*c**3*sqrt(-1/(4*a*c - b**2)**5) - 36*a*b**5*c**2
*sqrt(-1/(4*a*c - b**2)**5) + 3*b**7*c*sqrt(-1/(4*a*c - b**2)**5) + 3*b**2*
c)/(6*b*c**2)) - 3*b*c*sqrt(-1/(4*a*c - b**2)**5)*log(x + (192*a**3*b*c**4*
sqrt(-1/(4*a*c - b**2)**5) - 144*a**2*b**3*c**3*sqrt(-1/(4*a*c - b**2)**5)
+ 36*a*b**5*c**2*sqrt(-1/(4*a*c - b**2)**5) - 3*b**7*c*sqrt(-1/(4*a*c - b**
2)**5) + 3*b**2*c)/(6*b*c**2)) + (-8*a**2*c - a*b**2 - 9*b**2*c*x**2 - 6*b*
c**2*x**3 + x*(-10*a*b*c - 2*b**3))/(32*a**4*c**2 - 16*a**3*b**2*c + 2*a**2
*b**4 + x**4*(32*a**2*c**4 - 16*a*b**2*c**3 + 2*b**4*c**2) + x**3*(64*a**2*
b*c**3 - 32*a*b**3*c**2 + 4*b**5*c) + x**2*(64*a**3*c**3 - 12*a*b**4*c + 2*
b**6) + x*(64*a**3*b*c**2 - 32*a**2*b**3*c + 4*a*b**5))
```

3.376 $\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^3 x^6} \, dx$

Optimal. Leaf size=103

$$-\frac{12c^2\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{5/2}}+\frac{3c(b+2cx)}{\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)}+\frac{-b-2cx}{2\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2}$$

Rubi [A] time = 0.04, antiderivative size = 101, normalized size of antiderivative = 0.98, number of steps used = 5, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {1354, 614, 618, 206}

$$-\frac{12c^2\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{5/2}}+\frac{3c(b+2cx)}{\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)}-\frac{b+2cx}{2\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c + a/x^2 + b/x)^3*x^6),x]
```

```
[Out] -(b + 2*c*x)/(2*(b^2 - 4*a*c)*(a + b*x + c*x^2)^2) + (3*c*(b + 2*c*x))/((b^
2 - 4*a*c)^2*(a + b*x + c*x^2)) - (12*c^2*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*
a*c]])/(b^2 - 4*a*c)^(5/2)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 614

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[(2*c*(2*p +
 3))/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
egerQ[4*p]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1354

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> Int[x^(m + 2*n*p)*(c + b/x^n + a/x^(2*n))^p, x] /; FreeQ[{a, b, c, m, n
}, x] && EqQ[n2, 2*n] && ILtQ[p, 0] && NegQ[n]
```

Rubi steps

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^3 x^6}\,dx = \int \frac{1}{\left(a+bx+cx^2\right)^3}\,dx$$

$$= -\frac{b+2cx}{2\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2} - \frac{(3c)\int \frac{1}{\left(a+bx+cx^2\right)^2}\,dx}{b^2-4ac}$$

$$= -\frac{b+2cx}{2\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2} + \frac{3c(b+2cx)}{\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)} + \frac{\left(6c^2\right)\int \frac{1}{a+bx+cx^2}\,dx}{\left(b^2-4ac\right)^2}$$

$$= -\frac{b+2cx}{2\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2} + \frac{3c(b+2cx)}{\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)} - \frac{\left(12c^2\right)\operatorname{Subst}\left(\int \frac{1}{b^2-4ac-\cdots}\right.}{\left(b^2-4a\cdots\right.}$$

$$= -\frac{b+2cx}{2\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2} + \frac{3c(b+2cx)}{\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)} - \frac{12c^2\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{5/2}}$$

Mathematica [A] time = 0.10, size = 97, normalized size = 0.94

$$\frac{\frac{24c^2\tan^{-1}\left(\frac{b+2cx}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}} - \frac{(b+2cx)\left(-2c\left(5a+3cx^2\right)+b^2-6bcx\right)}{(a+x(b+cx))^2}}{2\left(b^2-4ac\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c + a/x^2 + b/x)^3*x^6),x]
```

```
[Out] (-(((b + 2*c*x)*(b^2 - 6*b*c*x - 2*c*(5*a + 3*c*x^2)))/(a + x*(b + c*x))^2)
 + (24*c^2*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c])/(2*(
b^2 - 4*a*c)^2)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^3 x^6}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/((c + a/x^2 + b/x)^3*x^6),x]
```

```
[Out] IntegrateAlgebraic[1/((c + a/x^2 + b/x)^3*x^6), x]
```

fricas [B] time = 1.70, size = 785, normalized size = 7.62

$$\left[-\frac{b^5 - 14\,ab^3c + 40\,a^2bc^2 - 12\left(b^2c^3 - 4\,ac^4\right)x^3 - 18\left(b^3c^2 - 4\,abc^3\right)x^2 - 12\left(c^4x^4 + 2\,bc^3x^3 + 2\,abc^2x + a^2c^2 + \left(b^2c^2 + 2\,ac^3\right)x^2\right)\sqrt{b^2-4\,ac}\,\log\left(\frac{2\,c^2x^2 + 2\,bcx + b^2 - 2\,ac - \sqrt{b^2-4\,ac}\,(2\,cx+b)}{cx^2+bx+a}\right) - 4\left(b^4c + ab^2c^2 - 20\,a^2c^3\right)x}{2\left(a^2b^6 - 12\,a^3b^4c + 48\,a^4b^2c^2 - 64\,a^5c^3 + \left(b^6c^2 - 12\,ab^4c^3 + 48\,a^2b^2c^4 - 64\,a^3c^5\right)x^4 + 2\left(b^7c - 12\,ab^5c^2 + 48\,a^2b^3c^3 - 64\,a^3bc^4\right)x^3 + \left(b^8 - 10\,ab^6c + 24\,a^2b^4c^2 + 32\,a^3b^2c^3 - 128\,a^4c^4\right)x^2 + 2\left(ab^7 - 12\,a^2b^5c + 48\,a^3b^3c^2 - 64\,a^4bc^3\right)x\right)},\ -\frac{b^5 - 14\,ab^3c + 40\,a^2bc^2 - 12\left(b^2c^3 - 4\,ac^4\right)x^3 - 18\left(b^3c^2 - 4\,abc^3\right)x^2 + 24\left(c^4x^4 + 2\,bc^3x^3 + 2\,abc^2x + a^2c^2 + \left(b^2c^2 + 2\,ac^3\right)x^2\right)\sqrt{-b^2+4\,ac}\,\arctan\left(-\frac{\sqrt{-b^2+4\,ac}\,(2\,cx+b)}{b^2-4\,ac}\right) - 4\left(b^4c + ab^2c^2 - 20\,a^2c^3\right)x}{2\left(a^2b^6 - 12\,a^3b^4c + 48\,a^4b^2c^2 - 64\,a^5c^3 + \left(b^6c^2 - 12\,ab^4c^3 + 48\,a^2b^2c^4 - 64\,a^3c^5\right)x^4 + 2\left(b^7c - 12\,ab^5c^2 + 48\,a^2b^3c^3 - 64\,a^3bc^4\right)x^3 + \left(b^8 - 10\,ab^6c + 24\,a^2b^4c^2 + 32\,a^3b^2c^3 - 128\,a^4c^4\right)x^2 + 2\left(ab^7 - 12\,a^2b^5c + 48\,a^3b^3c^2 - 64\,a^4bc^3\right)x\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x^2+b/x)^3/x^6,x, algorithm="fricas")
```

```
[Out] [-1/2*(b^5 - 14*a*b^3*c + 40*a^2*b*c^2 - 12*(b^2*c^3 - 4*a*c^4)*x^3 - 18*(b
^3*c^2 - 4*a*b*c^3)*x^2 - 12*(c^4*x^4 + 2*b*c^3*x^3 + 2*a*b*c^2*x + a^2*c^2
 + (b^2*c^2 + 2*a*c^3)*x^2)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^2 + 2*b*c*x + b^
2 - 2*a*c - sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) - 4*(b^4*c +
a*b^2*c^2 - 20*a^2*c^3)*x)/(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^
```

```
5*c^3 + (b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*x^4 + 2*(b^7
*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*x^3 + (b^8 - 10*a*b^6*c
+ 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*x^2 + 2*(a*b^7 - 12*a^2*b^
5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3)*x), -1/2*(b^5 - 14*a*b^3*c + 40*a^2*b*
c^2 - 12*(b^2*c^3 - 4*a*c^4)*x^3 - 18*(b^3*c^2 - 4*a*b*c^3)*x^2 + 24*(c^4*x
^4 + 2*b*c^3*x^3 + 2*a*b*c^2*x + a^2*c^2 + (b^2*c^2 + 2*a*c^3)*x^2)*sqrt(-b
^2 + 4*a*c)*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) - 4*(b^4*
c + a*b^2*c^2 - 20*a^2*c^3)*x)/(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 6
4*a^5*c^3 + (b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*x^4 + 2*
(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*x^3 + (b^8 - 10*a*b^
6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*x^2 + 2*(a*b^7 - 12*a^
2*b^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3)*x)]
```

giac [A] time = 0.34, size = 136, normalized size = 1.32

$$\frac{12\,c^{2}\arctan\left(\frac{2\,cx+b}{\sqrt{-b^{2}+4\,ac}}\right)}{\left(b^{4}-8\,ab^{2}c+16\,a^{2}c^{2}\right)\sqrt{-b^{2}+4\,ac}}+\frac{12\,c^{3}x^{3}+18\,bc^{2}x^{2}+4\,b^{2}cx+20\,ac^{2}x-b^{3}+10\,abc}{2\left(b^{4}-8\,ab^{2}c+16\,a^{2}c^{2}\right)\left(cx^{2}+bx+a\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x^2+b/x)^3/x^6,x, algorithm="giac")
```

```
[Out] 12*c^2*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/((b^4 - 8*a*b^2*c + 16*a^2*c^
2)*sqrt(-b^2 + 4*a*c)) + 1/2*(12*c^3*x^3 + 18*b*c^2*x^2 + 4*b^2*c*x + 20*a*
c^2*x - b^3 + 10*a*b*c)/((b^4 - 8*a*b^2*c + 16*a^2*c^2)*(c*x^2 + b*x + a)^2
)
```

maple [A] time = 0.00, size = 129, normalized size = 1.25

$$\frac{6c^{2}x}{\left(4ac-b^{2}\right)^{2}\left(c\,x^{2}+bx+a\right)}+\frac{12c^{2}\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^{2}}}\right)}{\left(4ac-b^{2}\right)^{\frac{5}{2}}}+\frac{3bc}{\left(4ac-b^{2}\right)^{2}\left(c\,x^{2}+bx+a\right)}+\frac{2cx+b}{2\left(4ac-b^{2}\right)\left(c\,x^{2}+bx+a\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c+a/x^2+b/x)^3/x^6,x)
```

```
[Out] 1/2*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^2+6*c^2/(4*a*c-b^2)^2/(c*x^2+b*x+a)
*x+3*c/(4*a*c-b^2)^2/(c*x^2+b*x+a)*b+12*c^2/(4*a*c-b^2)^(5/2)*arctan((2*c*x
+b)/(4*a*c-b^2)^(1/2))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x^2+b/x)^3/x^6,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

mupad [B] time = 1.42, size = 285, normalized size = 2.77

$$\frac{\frac{6\,c^{3}\,x^{3}}{16\,a^{2}\,c^{2}-8\,a\,b^{2}\,c+b^{4}}-\frac{b^{3}-10\,a\,b\,c}{2\,\left(16\,a^{2}\,c^{2}-8\,a\,b^{2}\,c+b^{4}\right)}+\frac{9\,b\,c^{2}\,x^{2}}{16\,a^{2}\,c^{2}-8\,a\,b^{2}\,c+b^{4}}+\frac{2\,c\,x\,\left(b^{2}+5\,a\,c\right)}{16\,a^{2}\,c^{2}-8\,a\,b^{2}\,c+b^{4}}}{x^{2}\,\left(b^{2}+2\,a\,c\right)+a^{2}+c^{2}\,x^{4}+2\,a\,b\,x+2\,b\,c\,x^{3}}+\frac{12\,c^{2}\,\mathrm{atan}\left(\frac{\left(\frac{12\,c^{3}\,x}{{\left(4\,a\,c-b^{2}\right)}^{5/2}}+\frac{6\,c^{2}\,\left(16\,a^{2}\,b\,c^{2}-8\,a\,b^{3}\,c+b^{5}\right)}{{\left(4\,a\,c-b^{2}\right)}^{5/2}\,\left(16\,a^{2}\,c^{2}-8\,a\,b^{2}\,c+b^{4}\right)}\right)\,\left(16\,a^{2}\,c^{2}-8\,a\,b^{2}\,c+b^{4}\right)}{6\,c^{2}}\right)}{{\left(4\,a\,c-b^{2}\right)}^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^6*(c + a/x^2 + b/x)^3),x)
```

```
[Out] ((6*c^3*x^3)/(b^4 + 16*a^2*c^2 - 8*a*b^2*c) - (b^3 - 10*a*b*c)/(2*(b^4 + 16
*a^2*c^2 - 8*a*b^2*c)) + (9*b*c^2*x^2)/(b^4 + 16*a^2*c^2 - 8*a*b^2*c) + (2*
c*x*(5*a*c + b^2))/(b^4 + 16*a^2*c^2 - 8*a*b^2*c))/(x^2*(2*a*c + b^2) + a^2
 + c^2*x^4 + 2*a*b*x + 2*b*c*x^3) + (12*c^2*atan((((12*c^3*x)/(4*a*c - b^2)
^(5/2) + (6*c^2*(b^5 + 16*a^2*b*c^2 - 8*a*b^3*c))/((4*a*c - b^2)^(5/2)*(b^4
 + 16*a^2*c^2 - 8*a*b^2*c)))*(b^4 + 16*a^2*c^2 - 8*a*b^2*c))/(6*c^2)))/(4*a
*c - b^2)^(5/2)
```

sympy [B] time = 1.11, size = 474, normalized size = 4.60

$$-6c^2\sqrt{-\frac{1}{\left(4ac-b^2\right)^5}}\log\left(x+\frac{-384a^3c^5\sqrt{-\frac{1}{(4ac-b^2)^5}}+288a^2b^2c^4\sqrt{-\frac{1}{(4ac-b^2)^5}}-72ab^4c^3\sqrt{-\frac{1}{(4ac-b^2)^5}}+6b^6c^2\sqrt{-\frac{1}{(4ac-b^2)^5}}+6bc^2}{12c^3}\right)+6c^2\sqrt{-\frac{1}{\left(4ac-b^2\right)^5}}\log\left(x+\frac{384a^3c^5\sqrt{-\frac{1}{(4ac-b^2)^5}}-288a^2b^2c^4\sqrt{-\frac{1}{(4ac-b^2)^5}}+72ab^4c^3\sqrt{-\frac{1}{(4ac-b^2)^5}}-6b^6c^2\sqrt{-\frac{1}{(4ac-b^2)^5}}+6bc^2}{12c^3}\right)+\frac{10abc-b^3+18bc^2x^2+12c^3x^3+x\left(20ac^2+4b^2c\right)}{32a^4c^2-16a^3b^2c+2a^2b^4+x^4\left(32a^2c^4-16ab^2c^3+2b^4c^2\right)+x^3\left(64a^2bc^3-32ab^3c^2+4b^5c\right)+x^2\left(64a^3c^3-12ab^4c+2b^6\right)+x\left(64a^3bc^2-32a^2b^3c+4ab^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x**2+b/x)**3/x**6,x)
```

```
[Out] -6*c**2*sqrt(-1/(4*a*c - b**2)**5)*log(x + (-384*a**3*c**5*sqrt(-1/(4*a*c -
 b**2)**5) + 288*a**2*b**2*c**4*sqrt(-1/(4*a*c - b**2)**5) - 72*a*b**4*c**3
*sqrt(-1/(4*a*c - b**2)**5) + 6*b**6*c**2*sqrt(-1/(4*a*c - b**2)**5) + 6*b*
c**2)/(12*c**3)) + 6*c**2*sqrt(-1/(4*a*c - b**2)**5)*log(x + (384*a**3*c**5
*sqrt(-1/(4*a*c - b**2)**5) - 288*a**2*b**2*c**4*sqrt(-1/(4*a*c - b**2)**5)
 + 72*a*b**4*c**3*sqrt(-1/(4*a*c - b**2)**5) - 6*b**6*c**2*sqrt(-1/(4*a*c -
 b**2)**5) + 6*b*c**2)/(12*c**3)) + (10*a*b*c - b**3 + 18*b*c**2*x**2 + 12*
c**3*x**3 + x*(20*a*c**2 + 4*b**2*c))/(32*a**4*c**2 - 16*a**3*b**2*c + 2*a*
*2*b**4 + x**4*(32*a**2*c**4 - 16*a*b**2*c**3 + 2*b**4*c**2) + x**3*(64*a**
2*b*c**3 - 32*a*b**3*c**2 + 4*b**5*c) + x**2*(64*a**3*c**3 - 12*a*b**4*c +
2*b**6) + x*(64*a**3*b*c**2 - 32*a**2*b**3*c + 4*a*b**5))
```

3.377 $\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^3 x^7} dx$

Optimal. Leaf size=185

$$-\frac{\log\left(a+bx+cx^2\right)}{2a^3}+\frac{\log(x)}{a^3}+\frac{16a^2c^2+2bcx\left(b^2-7ac\right)-15ab^2c+2b^4}{2a^2\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)}+\frac{b\left(30a^2c^2-10ab^2c+b^4\right)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a^3\left(b^2-4ac\right)^{5/2}}$$

Rubi [A] time = 0.22, antiderivative size = 185, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {1354, 740, 822, 800, 634, 618, 206, 628}

$$\frac{16a^2c^2+2bcx\left(b^2-7ac\right)-15ab^2c+2b^4}{2a^2\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)}+\frac{b\left(30a^2c^2-10ab^2c+b^4\right)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a^3\left(b^2-4ac\right)^{5/2}}-\frac{\log\left(a+bx+cx^2\right)}{2a^3}+\frac{\log(x)}{a^3}+\frac{-2ac+b^2+bcx}{2a\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c + a/x^2 + b/x)^3*x^7),x]
```

```
[Out] (b^2 - 2*a*c + b*c*x)/(2*a*(b^2 - 4*a*c)*(a + b*x + c*x^2)^2) + (2*b^4 - 15
*a*b^2*c + 16*a^2*c^2 + 2*b*c*(b^2 - 7*a*c)*x)/(2*a^2*(b^2 - 4*a*c)^2*(a +
b*x + c*x^2)) + (b*(b^4 - 10*a*b^2*c + 30*a^2*c^2)*ArcTanh[(b + 2*c*x)/Sqrt
[b^2 - 4*a*c]])/(a^3*(b^2 - 4*a*c)^(5/2)) + Log[x]/a^3 - Log[a + b*x + c*x^
2]/(2*a^3)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 740

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e
)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e
^2)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d +
 e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p +
 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4
```

```
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p,
-1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 822

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(f*(b*c*d - b^2*e +
2*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x)*(a
+ b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), x]
 + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m +
2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*m
 + b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) - f
*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1
] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 1354

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> Int[x^(m + 2*n*p)*(c + b/x^n + a/x^(2*n))^p, x] /; FreeQ[{a, b, c, m, n
}, x] && EqQ[n2, 2*n] && ILtQ[p, 0] && NegQ[n]
```

Rubi steps

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^3 x^7}\,dx = \int \frac{1}{x\left(a+bx+cx^2\right)^3}\,dx$$

$$= \frac{b^2-2ac+bcx}{2a\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2} - \frac{\int \frac{-2\left(b^2-4ac\right)-3bcx}{x\left(a+bx+cx^2\right)^2}\,dx}{2a\left(b^2-4ac\right)}$$

$$= \frac{b^2-2ac+bcx}{2a\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2} + \frac{2b^4-15ab^2c+16a^2c^2+2bc\left(b^2-7ac\right)x}{2a^2\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)} + \frac{\int \frac{2\left(b^2-4ac\right)^2 \cdots}{x(a+\cdots}}{2a^2\left(b\cdots\right.}$$

$$= \frac{b^2-2ac+bcx}{2a\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2} + \frac{2b^4-15ab^2c+16a^2c^2+2bc\left(b^2-7ac\right)x}{2a^2\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)} + \frac{\int \left(\frac{2\left(-b^2+4a\cdots\right.}{ax}\cdots\right.}{\cdots}$$

$$= \frac{b^2-2ac+bcx}{2a\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2} + \frac{2b^4-15ab^2c+16a^2c^2+2bc\left(b^2-7ac\right)x}{2a^2\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)} + \frac{\log(x)}{a^3} + \int \cdots$$

$$= \frac{b^2-2ac+bcx}{2a\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2} + \frac{2b^4-15ab^2c+16a^2c^2+2bc\left(b^2-7ac\right)x}{2a^2\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)} + \frac{\log(x)}{a^3} - \int \cdots$$

$$= \frac{b^2-2ac+bcx}{2a\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2} + \frac{2b^4-15ab^2c+16a^2c^2+2bc\left(b^2-7ac\right)x}{2a^2\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)} + \frac{\log(x)}{a^3} - \frac{\mathrm{lo}\cdots}{\cdots}$$

$$= \frac{b^2-2ac+bcx}{2a\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2} + \frac{2b^4-15ab^2c+16a^2c^2+2bc\left(b^2-7ac\right)x}{2a^2\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)} + \frac{b\left(b^4-10a\cdots\right.}{\cdots}$$

Mathematica [A] time = 0.35, size = 178, normalized size = 0.96

$$\frac{\frac{a^2\left(-2ac+b^2+bcx\right)}{\left(b^2-4ac\right)(a+x(b+cx))^2} - \frac{2b\left(30a^2c^2-10ab^2c+b^4\right)\tan^{-1}\left(\frac{b+2cx}{\sqrt{4ac-b^2}}\right)}{\left(4ac-b^2\right)^{5/2}} + \frac{a\left(16a^2c^2-15ab^2c-14abc^2x+2b^4+2b^3cx\right)}{\left(b^2-4ac\right)^2(a+x(b+cx))} - \log(a+x(b+cx)) + 2\log(x)}{2a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c + a/x^2 + b/x)^3*x^7),x]
```

```
[Out] ((a^2*(b^2 - 2*a*c + b*c*x))/((b^2 - 4*a*c)*(a + x*(b + c*x))^2) + (a*(2*b^
4 - 15*a*b^2*c + 16*a^2*c^2 + 2*b^3*c*x - 14*a*b*c^2*x))/((b^2 - 4*a*c)^2*(
a + x*(b + c*x))) - (2*b*(b^4 - 10*a*b^2*c + 30*a^2*c^2)*ArcTan[(b + 2*c*x)
/Sqrt[-b^2 + 4*a*c]])/(-b^2 + 4*a*c)^(5/2) + 2*Log[x] - Log[a + x*(b + c*x)
])/(2*a^3)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^3 x^7}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/((c + a/x^2 + b/x)^3*x^7),x]
```

```
[Out] IntegrateAlgebraic[1/((c + a/x^2 + b/x)^3*x^7), x]
```

fricas [B] time = 3.24, size = 1985, normalized size = 10.73

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x^2+b/x)^3/x^7,x, algorithm="fricas")
```

```
[Out] [1/2*(3*a^2*b^6 - 33*a^3*b^4*c + 108*a^4*b^2*c^2 - 96*a^5*c^3 + 2*(a*b^5*c^
2 - 11*a^2*b^3*c^3 + 28*a^3*b*c^4)*x^3 + (4*a*b^6*c - 45*a^2*b^4*c^2 + 132*
a^3*b^2*c^3 - 64*a^4*c^4)*x^2 + (a^2*b^5 - 10*a^3*b^3*c + 30*a^4*b*c^2 + (b
^5*c^2 - 10*a*b^3*c^3 + 30*a^2*b*c^4)*x^4 + 2*(b^6*c - 10*a*b^4*c^2 + 30*a^
2*b^2*c^3)*x^3 + (b^7 - 8*a*b^5*c + 10*a^2*b^3*c^2 + 60*a^3*b*c^3)*x^2 + 2*
(a*b^6 - 10*a^2*b^4*c + 30*a^3*b^2*c^2)*x)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^2
 + 2*b*c*x + b^2 - 2*a*c + sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)
) + 2*(a*b^7 - 10*a^2*b^5*c + 23*a^3*b^3*c^2 + 4*a^4*b*c^3)*x - (a^2*b^6 -
12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3 + (b^6*c^2 - 12*a*b^4*c^3 + 48*a
^2*b^2*c^4 - 64*a^3*c^5)*x^4 + 2*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 6
4*a^3*b*c^4)*x^3 + (b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 12
8*a^4*c^4)*x^2 + 2*(a*b^7 - 12*a^2*b^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3)*x
)*log(c*x^2 + b*x + a) + 2*(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^
5*c^3 + (b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*x^4 + 2*(b^7
*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*x^3 + (b^8 - 10*a*b^6*c
+ 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*x^2 + 2*(a*b^7 - 12*a^2*b^
5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3)*x)*log(x))/(a^5*b^6 - 12*a^6*b^4*c + 4
8*a^7*b^2*c^2 - 64*a^8*c^3 + (a^3*b^6*c^2 - 12*a^4*b^4*c^3 + 48*a^5*b^2*c^4
 - 64*a^6*c^5)*x^4 + 2*(a^3*b^7*c - 12*a^4*b^5*c^2 + 48*a^5*b^3*c^3 - 64*a^
6*b*c^4)*x^3 + (a^3*b^8 - 10*a^4*b^6*c + 24*a^5*b^4*c^2 + 32*a^6*b^2*c^3 -
128*a^7*c^4)*x^2 + 2*(a^4*b^7 - 12*a^5*b^5*c + 48*a^6*b^3*c^2 - 64*a^7*b*c^
3)*x), 1/2*(3*a^2*b^6 - 33*a^3*b^4*c + 108*a^4*b^2*c^2 - 96*a^5*c^3 + 2*(a*
b^5*c^2 - 11*a^2*b^3*c^3 + 28*a^3*b*c^4)*x^3 + (4*a*b^6*c - 45*a^2*b^4*c^2
+ 132*a^3*b^2*c^3 - 64*a^4*c^4)*x^2 + 2*(a^2*b^5 - 10*a^3*b^3*c + 30*a^4*b*
c^2 + (b^5*c^2 - 10*a*b^3*c^3 + 30*a^2*b*c^4)*x^4 + 2*(b^6*c - 10*a*b^4*c^2
 + 30*a^2*b^2*c^3)*x^3 + (b^7 - 8*a*b^5*c + 10*a^2*b^3*c^2 + 60*a^3*b*c^3)*
x^2 + 2*(a*b^6 - 10*a^2*b^4*c + 30*a^3*b^2*c^2)*x)*sqrt(-b^2 + 4*a*c)*arcta
n(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) + 2*(a*b^7 - 10*a^2*b^5*c
+ 23*a^3*b^3*c^2 + 4*a^4*b*c^3)*x - (a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^
2 - 64*a^5*c^3 + (b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*x^4
 + 2*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*x^3 + (b^8 - 10
*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*x^2 + 2*(a*b^7 -
12*a^2*b^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3)*x)*log(c*x^2 + b*x + a) + 2*(
a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3 + (b^6*c^2 - 12*a*b^4*
c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*x^4 + 2*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b
^3*c^3 - 64*a^3*b*c^4)*x^3 + (b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^
2*c^3 - 128*a^4*c^4)*x^2 + 2*(a*b^7 - 12*a^2*b^5*c + 48*a^3*b^3*c^2 - 64*a^
4*b*c^3)*x)*log(x))/(a^5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2*c^2 - 64*a^8*c^3 +
 (a^3*b^6*c^2 - 12*a^4*b^4*c^3 + 48*a^5*b^2*c^4 - 64*a^6*c^5)*x^4 + 2*(a^3*
b^7*c - 12*a^4*b^5*c^2 + 48*a^5*b^3*c^3 - 64*a^6*b*c^4)*x^3 + (a^3*b^8 - 10
*a^4*b^6*c + 24*a^5*b^4*c^2 + 32*a^6*b^2*c^3 - 128*a^7*c^4)*x^2 + 2*(a^4*b^
7 - 12*a^5*b^5*c + 48*a^6*b^3*c^2 - 64*a^7*b*c^3)*x)]
```

giac [A] time = 0.32, size = 239, normalized size = 1.29

$$-\frac{\left(b^{5}-10\,ab^{3}c+30\,a^{2}bc^{2}\right)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^{2}+4\,ac}}\right)}{\left(a^{3}b^{4}-8\,a^{4}b^{2}c+16\,a^{5}c^{2}\right)\sqrt{-b^{2}+4\,ac}}-\frac{\log\left(cx^{2}+bx+a\right)}{2\,a^{3}}+\frac{\log\left(|x|\right)}{a^{3}}+\frac{3\,a^{2}b^{4}-21\,a^{3}b^{2}c+24\,a^{4}c^{2}+2\left(ab^{3}c^{2}-7\,a^{2}bc^{3}\right)x^{3}+\left(4\,ab^{4}c-29\,a^{2}b^{2}c^{2}+16\,a^{3}c^{3}\right)x^{2}+2\left(ab^{5}-6\,a^{2}b^{3}c-a^{3}bc^{2}\right)x}{2\left(cx^{2}+bx+a\right)^{2}\left(b^{2}-4\,ac\right)^{2}a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x^2+b/x)^3/x^7,x, algorithm="giac")
```

```
[Out] -(b^5 - 10*a*b^3*c + 30*a^2*b*c^2)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/(
(a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*sqrt(-b^2 + 4*a*c)) - 1/2*log(c*x^2 +
```

```
b*x + a)/a^3 + log(abs(x))/a^3 + 1/2*(3*a^2*b^4 - 21*a^3*b^2*c + 24*a^4*c^2
 + 2*(a*b^3*c^2 - 7*a^2*b*c^3)*x^3 + (4*a*b^4*c - 29*a^2*b^2*c^2 + 16*a^3*c
^3)*x^2 + 2*(a*b^5 - 6*a^2*b^3*c - a^3*b*c^2)*x)/((c*x^2 + b*x + a)^2*(b^2
- 4*a*c)^2*a^3)
```

maple [B] time = 0.02, size = 781, normalized size = 4.22

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c+a/x^2+b/x)^3/x^7,x)
```

```
[Out] ln(x)/a^3-7/a/(c*x^2+b*x+a)^2*b*c^3/(16*a^2*c^2-8*a*b^2*c+b^4)*x^3+1/a^2/(c
*x^2+b*x+a)^2*b^3*c^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x^3+8/(c*x^2+b*x+a)^2*c^3/
(16*a^2*c^2-8*a*b^2*c+b^4)*x^2-29/2/a/(c*x^2+b*x+a)^2*c^2/(16*a^2*c^2-8*a*b
^2*c+b^4)*x^2*b^2+2/a^2/(c*x^2+b*x+a)^2*c/(16*a^2*c^2-8*a*b^2*c+b^4)*x^2*b^
4-1/(c*x^2+b*x+a)^2*b/(16*a^2*c^2-8*a*b^2*c+b^4)*x*c^2-6/a/(c*x^2+b*x+a)^2*
b^3/(16*a^2*c^2-8*a*b^2*c+b^4)*x*c+1/a^2/(c*x^2+b*x+a)^2*b^5/(16*a^2*c^2-8*
a*b^2*c+b^4)*x+12*a/(c*x^2+b*x+a)^2/(16*a^2*c^2-8*a*b^2*c+b^4)*c^2-21/2/(c*
x^2+b*x+a)^2/(16*a^2*c^2-8*a*b^2*c+b^4)*b^2*c+3/2/a/(c*x^2+b*x+a)^2/(16*a^2
*c^2-8*a*b^2*c+b^4)*b^4-8/a/(16*a^2*c^2-8*a*b^2*c+b^4)*c^2*ln(c*x^2+b*x+a)+
4/a^2/(16*a^2*c^2-8*a*b^2*c+b^4)*c*ln(c*x^2+b*x+a)*b^2-1/2/a^3/(16*a^2*c^2-
8*a*b^2*c+b^4)*ln(c*x^2+b*x+a)*b^4-30/a/(16*a^2*c^2-8*a*b^2*c+b^4)/(4*a*c-b
^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*b*c^2+10/a^2/(16*a^2*c^2-8*a*
b^2*c+b^4)/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*b^3*c-1/a^
3/(16*a^2*c^2-8*a*b^2*c+b^4)/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)
^(1/2))*b^5
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x^2+b/x)^3/x^7,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

mupad [B] time = 2.46, size = 1089, normalized size = 5.89

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^7*(c + a/x^2 + b/x)^3),x)
```

```
[Out] log(x)/a^3 + ((3*(b^4 + 8*a^2*c^2 - 7*a*b^2*c))/(2*a*(b^4 + 16*a^2*c^2 - 8*
a*b^2*c)) + (x^2*(4*b^4*c + 16*a^2*c^3 - 29*a*b^2*c^2))/(2*a^2*(b^4 + 16*a^
2*c^2 - 8*a*b^2*c)) - (b*x*(a^2*c^2 - b^4 + 6*a*b^2*c))/(a^2*(b^4 + 16*a^2*
c^2 - 8*a*b^2*c)) - (b*c^2*x^3*(7*a*c - b^2))/(a^2*(b^4 + 16*a^2*c^2 - 8*a*
b^2*c)))/(x^2*(2*a*c + b^2) + a^2 + c^2*x^4 + 2*a*b*x + 2*b*c*x^3) - (log(1
536*a^6*c^5 - 2*b^11*x - 2*a*b^10 + 2*a*b^5*(-(4*a*c - b^2)^5)^(1/2) + 39*a
^2*b^8*c + 2*b^6*x*(-(4*a*c - b^2)^5)^(1/2) - 303*a^3*b^6*c^2 + 1160*a^4*b^
4*c^3 - 2160*a^5*b^2*c^4 - 17*a^2*b^3*c*(-(4*a*c - b^2)^5)^(1/2) + 39*a^3*b
*c^2*(-(4*a*c - b^2)^5)^(1/2) - 321*a^2*b^7*c^2*x + 1286*a^3*b^5*c^3*x - 25
60*a^4*b^3*c^4*x - 48*a^3*c^3*x*(-(4*a*c - b^2)^5)^(1/2) + 40*a*b^9*c*x + 2
016*a^5*b*c^5*x - 20*a*b^4*c*x*(-(4*a*c - b^2)^5)^(1/2) + 63*a^2*b^2*c^2*x*
(-(4*a*c - b^2)^5)^(1/2))*(1024*a^5*c^5 - b^10 + b^5*(-(4*a*c - b^2)^5)^(1/
```

```
2) - 160*a^2*b^6*c^2 + 640*a^3*b^4*c^3 - 1280*a^4*b^2*c^4 + 20*a*b^8*c + 30
*a^2*b*c^2*(-(4*a*c - b^2)^5)^(1/2) - 10*a*b^3*c*(-(4*a*c - b^2)^5)^(1/2)))
/(2*a^3*(4*a*c - b^2)^5) + (log(2*a*b^10 + 2*b^11*x - 1536*a^6*c^5 + 2*a*b^
5*(-(4*a*c - b^2)^5)^(1/2) - 39*a^2*b^8*c + 2*b^6*x*(-(4*a*c - b^2)^5)^(1/2
) + 303*a^3*b^6*c^2 - 1160*a^4*b^4*c^3 + 2160*a^5*b^2*c^4 - 17*a^2*b^3*c*(-
(4*a*c - b^2)^5)^(1/2) + 39*a^3*b*c^2*(-(4*a*c - b^2)^5)^(1/2) + 321*a^2*b^
7*c^2*x - 1286*a^3*b^5*c^3*x + 2560*a^4*b^3*c^4*x - 48*a^3*c^3*x*(-(4*a*c -
 b^2)^5)^(1/2) - 40*a*b^9*c*x - 2016*a^5*b*c^5*x - 20*a*b^4*c*x*(-(4*a*c -
b^2)^5)^(1/2) + 63*a^2*b^2*c^2*x*(-(4*a*c - b^2)^5)^(1/2))*(b^10 - 1024*a^5
*c^5 + b^5*(-(4*a*c - b^2)^5)^(1/2) + 160*a^2*b^6*c^2 - 640*a^3*b^4*c^3 + 1
280*a^4*b^2*c^4 - 20*a*b^8*c + 30*a^2*b*c^2*(-(4*a*c - b^2)^5)^(1/2) - 10*a
*b^3*c*(-(4*a*c - b^2)^5)^(1/2)))/(2*a^3*(4*a*c - b^2)^5)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x**2+b/x)**3/x**7,x)
[Out] Timed out
```

3.378 $\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^3 x^8}\,dx$

Optimal. Leaf size=239

$$\frac{3b\log\left(a+bx+cx^2\right)}{2a^4}-\frac{3b\log(x)}{a^4}-\frac{3\left(b^2-5ac\right)\left(b^2-2ac\right)}{a^3x\left(b^2-4ac\right)^2}+\frac{20a^2c^2+3bcx\left(b^2-6ac\right)-20ab^2c+3b^4}{2a^2x\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)}-\frac{3\left(-20a^3c^3\right.}{}$$

Rubi [A] time = 0.28, antiderivative size = 239, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {1354, 740, 822, 800, 634, 618, 206, 628}

$$\frac{20a^2c^2+3bcx\left(b^2-6ac\right)-20ab^2c+3b^4}{2a^2x\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)}-\frac{3\left(30a^2b^2c^2-20a^3c^3-10ab^4c+b^6\right)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a^4\left(b^2-4ac\right)^{5/2}}-\frac{3\left(b^2-5ac\right)\left(b^2-2ac\right)}{a^3x\left(b^2-4ac\right)^2}+\frac{3b\log\left(a+bx+cx^2\right)}{2a^4}-\frac{3b\log(x)}{a^4}+\frac{-2ac+b^2+bcx}{2ax\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c + a/x^2 + b/x)^3*x^8),x]
```

```
[Out] (-3*(b^2 - 5*a*c)*(b^2 - 2*a*c))/(a^3*(b^2 - 4*a*c)^2*x) + (b^2 - 2*a*c + b
*c*x)/(2*a*(b^2 - 4*a*c)*x*(a + b*x + c*x^2)^2) + (3*b^4 - 20*a*b^2*c + 20*
a^2*c^2 + 3*b*c*(b^2 - 6*a*c)*x)/(2*a^2*(b^2 - 4*a*c)^2*x*(a + b*x + c*x^2)
) - (3*(b^6 - 10*a*b^4*c + 30*a^2*b^2*c^2 - 20*a^3*c^3)*ArcTanh[(b + 2*c*x)
/Sqrt[b^2 - 4*a*c]])/(a^4*(b^2 - 4*a*c)^(5/2)) - (3*b*Log[x])/a^4 + (3*b*Lo
g[a + b*x + c*x^2])/(2*a^4)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 740

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e
)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e
^2)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d +
 e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p +
 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
```

```
b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p,
-1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 822

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(f*(b*c*d - b^2*e +
2*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x)*(a
+ b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), x]
 + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m +
2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*m
 + b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) - f
*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1
] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 1354

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> Int[x^(m + 2*n*p)*(c + b/x^n + a/x^(2*n))^p, x] /; FreeQ[{a, b, c, m, n
}, x] && EqQ[n2, 2*n] && ILtQ[p, 0] && NegQ[n]
```

Rubi steps

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^3 x^8}\,dx = \int \frac{1}{x^2\left(a+bx+cx^2\right)^3}\,dx$$

$$= \frac{b^2-2ac+bcx}{2a\left(b^2-4ac\right)x\left(a+bx+cx^2\right)^2} - \frac{\int \frac{-3b^2+10ac-4bcx}{x^2\left(a+bx+cx^2\right)^2}\,dx}{2a\left(b^2-4ac\right)}$$

$$= \frac{b^2-2ac+bcx}{2a\left(b^2-4ac\right)x\left(a+bx+cx^2\right)^2} + \frac{3b^4-20ab^2c+20a^2c^2+3bc\left(b^2-6ac\right)x}{2a^2\left(b^2-4ac\right)^2 x\left(a+bx+cx^2\right)} + \frac{\int \frac{6\left(b^2-5ac\right.}{}}{2}$$

$$= \frac{b^2-2ac+bcx}{2a\left(b^2-4ac\right)x\left(a+bx+cx^2\right)^2} + \frac{3b^4-20ab^2c+20a^2c^2+3bc\left(b^2-6ac\right)x}{2a^2\left(b^2-4ac\right)^2 x\left(a+bx+cx^2\right)} + \frac{\int\left(\frac{6\left(b^2-5a\right.}{}\right.}{}$$

$$= -\frac{3\left(b^2-5ac\right)\left(b^2-2ac\right)}{a^3\left(b^2-4ac\right)^2 x} + \frac{b^2-2ac+bcx}{2a\left(b^2-4ac\right)x\left(a+bx+cx^2\right)^2} + \frac{3b^4-20ab^2c+20a^2c^2+3}{2a^2\left(b^2-4ac\right)^2 x\left(a+\right.}$$

$$= -\frac{3\left(b^2-5ac\right)\left(b^2-2ac\right)}{a^3\left(b^2-4ac\right)^2 x} + \frac{b^2-2ac+bcx}{2a\left(b^2-4ac\right)x\left(a+bx+cx^2\right)^2} + \frac{3b^4-20ab^2c+20a^2c^2+3}{2a^2\left(b^2-4ac\right)^2 x\left(a+\right.}$$

$$= -\frac{3\left(b^2-5ac\right)\left(b^2-2ac\right)}{a^3\left(b^2-4ac\right)^2 x} + \frac{b^2-2ac+bcx}{2a\left(b^2-4ac\right)x\left(a+bx+cx^2\right)^2} + \frac{3b^4-20ab^2c+20a^2c^2+3}{2a^2\left(b^2-4ac\right)^2 x\left(a+\right.}$$

$$= -\frac{3\left(b^2-5ac\right)\left(b^2-2ac\right)}{a^3\left(b^2-4ac\right)^2 x} + \frac{b^2-2ac+bcx}{2a\left(b^2-4ac\right)x\left(a+bx+cx^2\right)^2} + \frac{3b^4-20ab^2c+20a^2c^2+3}{2a^2\left(b^2-4ac\right)^2 x\left(a+\right.}$$

Mathematica [A] time = 0.44, size = 221, normalized size = 0.92

$$\frac{\frac{a^2\left(-3abc-2ac^2x+b^3+b^2cx\right)}{\left(4ac-b^2\right)(a+x(b+cx))^2} - \frac{a\left(46a^2bc^2+28a^2c^3x-29ab^3c-26ab^2c^2x+4b^5+4b^4cx\right)}{\left(b^2-4ac\right)^2(a+x(b+cx))} + \frac{6\left(-20a^3c^3+30a^2b^2c^2-10ab^4c+b^6\right)\tan^{-1}\left(\frac{b+2cx}{\sqrt{4ac-b^2}}\right)}{\left(4ac-b^2\right)^{5/2}} + 3b\log(a+x(b+cx)) - \frac{2a}{x} - 6b\log(x)}{2a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c + a/x^2 + b/x)^3*x^8),x]
```

```
[Out] ((-2*a)/x + (a^2*(b^3 - 3*a*b*c + b^2*c*x - 2*a*c^2*x))/((-b^2 + 4*a*c)*(a
+ x*(b + c*x))^2) - (a*(4*b^5 - 29*a*b^3*c + 46*a^2*b*c^2 + 4*b^4*c*x - 26*
a*b^2*c^2*x + 28*a^2*c^3*x))/((b^2 - 4*a*c)^2*(a + x*(b + c*x))) + (6*(b^6
- 10*a*b^4*c + 30*a^2*b^2*c^2 - 20*a^3*c^3)*ArcTan[(b + 2*c*x)/Sqrt[-b^2 +
4*a*c]])/(-b^2 + 4*a*c)^(5/2) - 6*b*Log[x] + 3*b*Log[a + x*(b + c*x)])/(2*a
^4)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^3 x^8}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/((c + a/x^2 + b/x)^3*x^8),x]
```

```
[Out] IntegrateAlgebraic[1/((c + a/x^2 + b/x)^3*x^8), x]
```

fricas [B] time = 3.19, size = 2280, normalized size = 9.54

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x^2+b/x)^3/x^8,x, algorithm="fricas")
```

```
[Out] [-1/2*(2*a^3*b^6 - 24*a^4*b^4*c + 96*a^5*b^2*c^2 - 128*a^6*c^3 + 6*(a*b^6*c
^2 - 11*a^2*b^4*c^3 + 38*a^3*b^2*c^4 - 40*a^4*c^5)*x^4 + 3*(4*a*b^7*c - 45*
a^2*b^5*c^2 + 162*a^3*b^3*c^3 - 184*a^4*b*c^4)*x^3 + 2*(3*a*b^8 - 30*a^2*b^
6*c + 79*a^3*b^4*c^2 + 22*a^4*b^2*c^3 - 200*a^5*c^4)*x^2 + 3*((b^6*c^2 - 10
*a*b^4*c^3 + 30*a^2*b^2*c^4 - 20*a^3*c^5)*x^5 + 2*(b^7*c - 10*a*b^5*c^2 + 3
0*a^2*b^3*c^3 - 20*a^3*b*c^4)*x^4 + (b^8 - 8*a*b^6*c + 10*a^2*b^4*c^2 + 40*
a^3*b^2*c^3 - 40*a^4*c^4)*x^3 + 2*(a*b^7 - 10*a^2*b^5*c + 30*a^3*b^3*c^2 -
20*a^4*b*c^3)*x^2 + (a^2*b^6 - 10*a^3*b^4*c + 30*a^4*b^2*c^2 - 20*a^5*c^3)*
x)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c + sqrt(b^2 - 4*
a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) + (9*a^2*b^7 - 104*a^3*b^5*c + 394*a^4
*b^3*c^2 - 488*a^5*b*c^3)*x - 3*((b^7*c^2 - 12*a*b^5*c^3 + 48*a^2*b^3*c^4 -
 64*a^3*b*c^5)*x^5 + 2*(b^8*c - 12*a*b^6*c^2 + 48*a^2*b^4*c^3 - 64*a^3*b^2*
c^4)*x^4 + (b^9 - 10*a*b^7*c + 24*a^2*b^5*c^2 + 32*a^3*b^3*c^3 - 128*a^4*b*
c^4)*x^3 + 2*(a*b^8 - 12*a^2*b^6*c + 48*a^3*b^4*c^2 - 64*a^4*b^2*c^3)*x^2 +
 (a^2*b^7 - 12*a^3*b^5*c + 48*a^4*b^3*c^2 - 64*a^5*b*c^3)*x)*log(c*x^2 + b*
x + a) + 6*((b^7*c^2 - 12*a*b^5*c^3 + 48*a^2*b^3*c^4 - 64*a^3*b*c^5)*x^5 +
2*(b^8*c - 12*a*b^6*c^2 + 48*a^2*b^4*c^3 - 64*a^3*b^2*c^4)*x^4 + (b^9 - 10*
a*b^7*c + 24*a^2*b^5*c^2 + 32*a^3*b^3*c^3 - 128*a^4*b*c^4)*x^3 + 2*(a*b^8 -
 12*a^2*b^6*c + 48*a^3*b^4*c^2 - 64*a^4*b^2*c^3)*x^2 + (a^2*b^7 - 12*a^3*b^
5*c + 48*a^4*b^3*c^2 - 64*a^5*b*c^3)*x)*log(x))/((a^4*b^6*c^2 - 12*a^5*b^4*
c^3 + 48*a^6*b^2*c^4 - 64*a^7*c^5)*x^5 + 2*(a^4*b^7*c - 12*a^5*b^5*c^2 + 48
*a^6*b^3*c^3 - 64*a^7*b*c^4)*x^4 + (a^4*b^8 - 10*a^5*b^6*c + 24*a^6*b^4*c^2
 + 32*a^7*b^2*c^3 - 128*a^8*c^4)*x^3 + 2*(a^5*b^7 - 12*a^6*b^5*c + 48*a^7*b
^3*c^2 - 64*a^8*b*c^3)*x^2 + (a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*
a^9*c^3)*x), -1/2*(2*a^3*b^6 - 24*a^4*b^4*c + 96*a^5*b^2*c^2 - 128*a^6*c^3
+ 6*(a*b^6*c^2 - 11*a^2*b^4*c^3 + 38*a^3*b^2*c^4 - 40*a^4*c^5)*x^4 + 3*(4*a
*b^7*c - 45*a^2*b^5*c^2 + 162*a^3*b^3*c^3 - 184*a^4*b*c^4)*x^3 + 2*(3*a*b^8
 - 30*a^2*b^6*c + 79*a^3*b^4*c^2 + 22*a^4*b^2*c^3 - 200*a^5*c^4)*x^2 + 6*((
b^6*c^2 - 10*a*b^4*c^3 + 30*a^2*b^2*c^4 - 20*a^3*c^5)*x^5 + 2*(b^7*c - 10*a
*b^5*c^2 + 30*a^2*b^3*c^3 - 20*a^3*b*c^4)*x^4 + (b^8 - 8*a*b^6*c + 10*a^2*b
^4*c^2 + 40*a^3*b^2*c^3 - 40*a^4*c^4)*x^3 + 2*(a*b^7 - 10*a^2*b^5*c + 30*a^
3*b^3*c^2 - 20*a^4*b*c^3)*x^2 + (a^2*b^6 - 10*a^3*b^4*c + 30*a^4*b^2*c^2 -
20*a^5*c^3)*x)*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b
^2 - 4*a*c)) + (9*a^2*b^7 - 104*a^3*b^5*c + 394*a^4*b^3*c^2 - 488*a^5*b*c^3
)*x - 3*((b^7*c^2 - 12*a*b^5*c^3 + 48*a^2*b^3*c^4 - 64*a^3*b*c^5)*x^5 + 2*(
b^8*c - 12*a*b^6*c^2 + 48*a^2*b^4*c^3 - 64*a^3*b^2*c^4)*x^4 + (b^9 - 10*a*b
^7*c + 24*a^2*b^5*c^2 + 32*a^3*b^3*c^3 - 128*a^4*b*c^4)*x^3 + 2*(a*b^8 - 12
*a^2*b^6*c + 48*a^3*b^4*c^2 - 64*a^4*b^2*c^3)*x^2 + (a^2*b^7 - 12*a^3*b^5*c
 + 48*a^4*b^3*c^2 - 64*a^5*b*c^3)*x)*log(c*x^2 + b*x + a) + 6*((b^7*c^2 - 1
2*a*b^5*c^3 + 48*a^2*b^3*c^4 - 64*a^3*b*c^5)*x^5 + 2*(b^8*c - 12*a*b^6*c^2
+ 48*a^2*b^4*c^3 - 64*a^3*b^2*c^4)*x^4 + (b^9 - 10*a*b^7*c + 24*a^2*b^5*c^2
 + 32*a^3*b^3*c^3 - 128*a^4*b*c^4)*x^3 + 2*(a*b^8 - 12*a^2*b^6*c + 48*a^3*b
^4*c^2 - 64*a^4*b^2*c^3)*x^2 + (a^2*b^7 - 12*a^3*b^5*c + 48*a^4*b^3*c^2 - 6
4*a^5*b*c^3)*x)*log(x))/((a^4*b^6*c^2 - 12*a^5*b^4*c^3 + 48*a^6*b^2*c^4 - 6
4*a^7*c^5)*x^5 + 2*(a^4*b^7*c - 12*a^5*b^5*c^2 + 48*a^6*b^3*c^3 - 64*a^7*b*
c^4)*x^4 + (a^4*b^8 - 10*a^5*b^6*c + 24*a^6*b^4*c^2 + 32*a^7*b^2*c^3 - 128*
a^8*c^4)*x^3 + 2*(a^5*b^7 - 12*a^6*b^5*c + 48*a^7*b^3*c^2 - 64*a^8*b*c^3)*x
^2 + (a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)*x)]
```

giac [A] time = 0.47, size = 309, normalized size = 1.29

$$\frac{3\left(b^{6}-10\,ab^{4}c+30\,a^{2}b^{2}c^{2}-20\,a^{3}c^{3}\right)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^{2}+4\,ac}}\right)}{\left(a^{4}b^{4}-8\,a^{5}b^{2}c+16\,a^{6}c^{2}\right)\sqrt{-b^{2}+4\,ac}}+\frac{3\,b\log\left(cx^{2}+bx+a\right)}{2\,a^{4}}-\frac{3\,b\log\left(|x|\right)}{a^{4}}-\frac{2\,a^{3}b^{4}-16\,a^{4}b^{2}c+32\,a^{5}c^{2}+6\left(ab^{4}c^{2}-7\,a^{2}b^{2}c^{3}+10\,a^{3}c^{4}\right)x^{4}+3\left(4\,ab^{5}c-29\,a^{2}b^{3}c^{2}+46\,a^{3}bc^{3}\right)x^{3}+2\left(3\,ab^{6}-18\,a^{2}b^{4}c+7\,a^{3}b^{2}c^{2}+50\,a^{4}c^{3}\right)x^{2}+\left(9\,a^{2}b^{5}-68\,a^{3}b^{3}c+122\,a^{4}bc^{2}\right)x}{2\left(cx^{2}+bx+a\right)^{2}\left(b^{2}-4\,ac\right)^{2}a^{4}x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x^2+b/x)^3/x^8,x, algorithm="giac")
```

```
[Out] 3*(b^6 - 10*a*b^4*c + 30*a^2*b^2*c^2 - 20*a^3*c^3)*arctan((2*c*x + b)/sqrt(
-b^2 + 4*a*c))/((a^4*b^4 - 8*a^5*b^2*c + 16*a^6*c^2)*sqrt(-b^2 + 4*a*c)) +
3/2*b*log(c*x^2 + b*x + a)/a^4 - 3*b*log(abs(x))/a^4 - 1/2*(2*a^3*b^4 - 16*
a^4*b^2*c + 32*a^5*c^2 + 6*(a*b^4*c^2 - 7*a^2*b^2*c^3 + 10*a^3*c^4)*x^4 + 3
*(4*a*b^5*c - 29*a^2*b^3*c^2 + 46*a^3*b*c^3)*x^3 + 2*(3*a*b^6 - 18*a^2*b^4*
c + 7*a^3*b^2*c^2 + 50*a^4*c^3)*x^2 + (9*a^2*b^5 - 68*a^3*b^3*c + 122*a^4*b
*c^2)*x)/((c*x^2 + b*x + a)^2*(b^2 - 4*a*c)^2*a^4*x)
```

maple [B] time = 0.02, size = 954, normalized size = 3.99

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c+a/x^2+b/x)^3/x^8,x)
```

```
[Out] -1/a^3/x-3*b*ln(x)/a^4-14/a/(c*x^2+b*x+a)^2*c^4/(16*a^2*c^2-8*a*b^2*c+b^4)*
x^3+13/a^2/(c*x^2+b*x+a)^2*c^3/(16*a^2*c^2-8*a*b^2*c+b^4)*x^3*b^2-2/a^3/(c*
x^2+b*x+a)^2*c^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x^3*b^4-37/a/(c*x^2+b*x+a)^2*b*
c^3/(16*a^2*c^2-8*a*b^2*c+b^4)*x^2+55/2/a^2/(c*x^2+b*x+a)^2*b^3*c^2/(16*a^2
*c^2-8*a*b^2*c+b^4)*x^2-4/a^3/(c*x^2+b*x+a)^2*b^5*c/(16*a^2*c^2-8*a*b^2*c+b
^4)*x^2-18/(c*x^2+b*x+a)^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x*c^3-7/a/(c*x^2+b*x+
a)^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x*b^2*c^2+12/a^2/(c*x^2+b*x+a)^2/(16*a^2*c^
2-8*a*b^2*c+b^4)*x*b^4*c-2/a^3/(c*x^2+b*x+a)^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x
*b^6-29/(c*x^2+b*x+a)^2/(16*a^2*c^2-8*a*b^2*c+b^4)*b*c^2+18/a/(c*x^2+b*x+a)
^2/(16*a^2*c^2-8*a*b^2*c+b^4)*b^3*c-5/2/a^2/(c*x^2+b*x+a)^2/(16*a^2*c^2-8*a
*b^2*c+b^4)*b^5+24/a^2/(16*a^2*c^2-8*a*b^2*c+b^4)*c^2*ln(c*x^2+b*x+a)*b-12/
a^3/(16*a^2*c^2-8*a*b^2*c+b^4)*c*ln(c*x^2+b*x+a)*b^3+3/2/a^4/(16*a^2*c^2-8*
a*b^2*c+b^4)*ln(c*x^2+b*x+a)*b^5-60/a/(16*a^2*c^2-8*a*b^2*c+b^4)/(4*a*c-b^2
)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*c^3+90/a^2/(16*a^2*c^2-8*a*b^2*
c+b^4)/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*b^2*c^2-30/a^3
/(16*a^2*c^2-8*a*b^2*c+b^4)/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^
(1/2))*b^4*c+3/a^4/(16*a^2*c^2-8*a*b^2*c+b^4)/(4*a*c-b^2)^(1/2)*arctan((2*c
*x+b)/(4*a*c-b^2)^(1/2))*b^6
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x^2+b/x)^3/x^8,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

mupad [B] time = 2.55, size = 1255, normalized size = 5.25

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^8*(c + a/x^2 + b/x)^3),x)
```

```
[Out] - (1/a + (x^2*(3*b^6 + 50*a^3*c^3 + 7*a^2*b^2*c^2 - 18*a*b^4*c))/(a^3*(b^4
+ 16*a^2*c^2 - 8*a*b^2*c)) + (x*(9*b^5 + 122*a^2*b*c^2 - 68*a*b^3*c))/(2*a^
2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (3*x^3*(4*b^5*c - 29*a*b^3*c^2 + 46*a^2
*b*c^3))/(2*a^3*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (3*c^2*x^4*(b^4 + 10*a^2*
c^2 - 7*a*b^2*c))/(a^3*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)))/(x^3*(2*a*c + b^2)
```

```
+ a^2*x + c^2*x^5 + 2*a*b*x^2 + 2*b*c*x^4) - (3*b*log(x))/a^4 - (3*log(2*a*
b^11 + 2*b^12*x + 2*a*b^6*(-(4*a*c - b^2)^5)^(1/2) - 39*a^2*b^9*c - 1696*a^
6*b*c^5 + 320*a^6*c^6*x + 2*b^7*x*(-(4*a*c - b^2)^5)^(1/2) + 303*a^3*b^7*c^
2 - 1170*a^4*b^5*c^3 + 2240*a^5*b^3*c^4 - 10*a^4*c^3*(-(4*a*c - b^2)^5)^(1/
2) - 17*a^2*b^4*c*(-(4*a*c - b^2)^5)^(1/2) + 321*a^2*b^8*c^2*x - 1296*a^3*b
^6*c^3*x + 2660*a^4*b^4*c^4*x - 2336*a^5*b^2*c^5*x - 40*a*b^10*c*x + 39*a^3
*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 20*a*b^5*c*x*(-(4*a*c - b^2)^5)^(1/2) -
 58*a^3*b*c^3*x*(-(4*a*c - b^2)^5)^(1/2) + 63*a^2*b^3*c^2*x*(-(4*a*c - b^2)
^5)^(1/2))*(b^11 + b^6*(-(4*a*c - b^2)^5)^(1/2) - 1024*a^5*b*c^5 + 160*a^2*
b^7*c^2 - 640*a^3*b^5*c^3 + 1280*a^4*b^3*c^4 - 20*a^3*c^3*(-(4*a*c - b^2)^5
)^(1/2) - 20*a*b^9*c + 30*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 10*a*b^4*c
*(-(4*a*c - b^2)^5)^(1/2)))/(2*a^4*(4*a*c - b^2)^5) - (3*log(2*a*b^11 + 2*b
^12*x - 2*a*b^6*(-(4*a*c - b^2)^5)^(1/2) - 39*a^2*b^9*c - 1696*a^6*b*c^5 +
320*a^6*c^6*x - 2*b^7*x*(-(4*a*c - b^2)^5)^(1/2) + 303*a^3*b^7*c^2 - 1170*a
^4*b^5*c^3 + 2240*a^5*b^3*c^4 + 10*a^4*c^3*(-(4*a*c - b^2)^5)^(1/2) + 17*a^
2*b^4*c*(-(4*a*c - b^2)^5)^(1/2) + 321*a^2*b^8*c^2*x - 1296*a^3*b^6*c^3*x +
 2660*a^4*b^4*c^4*x - 2336*a^5*b^2*c^5*x - 40*a*b^10*c*x - 39*a^3*b^2*c^2*(
-(4*a*c - b^2)^5)^(1/2) + 20*a*b^5*c*x*(-(4*a*c - b^2)^5)^(1/2) + 58*a^3*b*
c^3*x*(-(4*a*c - b^2)^5)^(1/2) - 63*a^2*b^3*c^2*x*(-(4*a*c - b^2)^5)^(1/2))
*(b^11 - b^6*(-(4*a*c - b^2)^5)^(1/2) - 1024*a^5*b*c^5 + 160*a^2*b^7*c^2 -
640*a^3*b^5*c^3 + 1280*a^4*b^3*c^4 + 20*a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) -
20*a*b^9*c - 30*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) + 10*a*b^4*c*(-(4*a*c
- b^2)^5)^(1/2)))/(2*a^4*(4*a*c - b^2)^5)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x**2+b/x)**3/x**8,x)
```

```
[Out] Timed out
```

3.379 $\int \frac{x^2}{15+\frac{2}{x^2}+\frac{13}{x}} dx$

Optimal. Leaf size=40

$$\frac{x^3}{45}-\frac{13x^2}{450}+\frac{139x}{3375}-\frac{16}{567}\log(3x+2)+\frac{\log(5x+1)}{4375}$$

Rubi [A] time = 0.02, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {1354, 701, 632, 31}

$$\frac{x^3}{45}-\frac{13x^2}{450}+\frac{139x}{3375}-\frac{16}{567}\log(3x+2)+\frac{\log(5x+1)}{4375}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(15 + 2/x^2 + 13/x),x]
```

```
[Out] (139*x)/3375 - (13*x^2)/450 + x^3/45 - (16*Log[2 + 3*x])/567 + Log[1 + 5*x]
/4375
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 632

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rule 701

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Int[PolynomialDivide[(d + e*x)^m, a + b*x + c*x^2, x], x] /; FreeQ[{a,
 b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
 NeQ[2*c*d - b*e, 0] && IGtQ[m, 1] && (NeQ[d, 0] || GtQ[m, 2])
```

Rule 1354

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> Int[x^(m + 2*n*p)*(c + b/x^n + a/x^(2*n))^p, x] /; FreeQ[{a, b, c, m, n
}, x] && EqQ[n2, 2*n] && ILtQ[p, 0] && NegQ[n]
```

Rubi steps

$$\int \frac{x^2}{15+\frac{2}{x^2}+\frac{13}{x}}\,dx = \int \frac{x^4}{2+13x+15x^2}\,dx$$
$$= \int \left(\frac{139}{3375}-\frac{13x}{225}+\frac{x^2}{15}-\frac{278+1417x}{3375\left(2+13x+15x^2\right)}\right)dx$$
$$= \frac{139x}{3375}-\frac{13x^2}{450}+\frac{x^3}{45}-\frac{\int \frac{278+1417x}{2+13x+15x^2}\,dx}{3375}$$
$$= \frac{139x}{3375}-\frac{13x^2}{450}+\frac{x^3}{45}+\frac{3}{875}\int \frac{1}{3+15x}\,dx-\frac{80}{189}\int \frac{1}{10+15x}\,dx$$
$$= \frac{139x}{3375}-\frac{13x^2}{450}+\frac{x^3}{45}-\frac{16}{567}\log(2+3x)+\frac{\log(1+5x)}{4375}$$

Mathematica [A] time = 0.01, size = 40, normalized size = 1.00

$$\frac{x^3}{45}-\frac{13x^2}{450}+\frac{139x}{3375}-\frac{16}{567}\log(3x+2)+\frac{\log(5x+1)}{4375}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(15 + 2/x^2 + 13/x),x]
```

```
[Out] (139*x)/3375 - (13*x^2)/450 + x^3/45 - (16*Log[2 + 3*x])/567 + Log[1 + 5*x]
/4375
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{15+\frac{2}{x^2}+\frac{13}{x}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^2/(15 + 2/x^2 + 13/x),x]
```

```
[Out] IntegrateAlgebraic[x^2/(15 + 2/x^2 + 13/x), x]
```

fricas [A] time = 1.72, size = 30, normalized size = 0.75

$$\frac{1}{45}\,x^3-\frac{13}{450}\,x^2+\frac{139}{3375}\,x+\frac{1}{4375}\,\log\left(5\,x+1\right)-\frac{16}{567}\,\log\left(3\,x+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(15+2/x^2+13/x),x, algorithm="fricas")
```

```
[Out] 1/45*x^3 - 13/450*x^2 + 139/3375*x + 1/4375*log(5*x + 1) - 16/567*log(3*x +
 2)
```

giac [A] time = 0.26, size = 32, normalized size = 0.80

$$\frac{1}{45}\,x^3-\frac{13}{450}\,x^2+\frac{139}{3375}\,x+\frac{1}{4375}\,\log\left(|5\,x+1|\right)-\frac{16}{567}\,\log\left(|3\,x+2|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(15+2/x^2+13/x),x, algorithm="giac")
```

```
[Out] 1/45*x^3 - 13/450*x^2 + 139/3375*x + 1/4375*log(abs(5*x + 1)) - 16/567*log(
abs(3*x + 2))
```

maple [A] time = 0.01, size = 31, normalized size = 0.78

$$\frac{x^3}{45} - \frac{13x^2}{450} + \frac{139x}{3375} + \frac{\ln(5x+1)}{4375} - \frac{16\ln(3x+2)}{567}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(15+2/x^2+13/x),x)
```

```
[Out] 139/3375*x-13/450*x^2+1/45*x^3-16/567*ln(2+3*x)+1/4375*ln(1+5*x)
```

maxima [A] time = 0.42, size = 30, normalized size = 0.75

$$\frac{1}{45}x^3 - \frac{13}{450}x^2 + \frac{139}{3375}x + \frac{1}{4375}\log(5x+1) - \frac{16}{567}\log(3x+2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(15+2/x^2+13/x),x, algorithm="maxima")
```

```
[Out] 1/45*x^3 - 13/450*x^2 + 139/3375*x + 1/4375*log(5*x + 1) - 16/567*log(3*x +
 2)
```

mupad [B] time = 0.05, size = 26, normalized size = 0.65

$$\frac{139x}{3375} - \frac{16\ln\left(x+\frac{2}{3}\right)}{567} + \frac{\ln\left(x+\frac{1}{5}\right)}{4375} - \frac{13x^2}{450} + \frac{x^3}{45}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(13/x + 2/x^2 + 15),x)
```

```
[Out] (139*x)/3375 - (16*log(x + 2/3))/567 + log(x + 1/5)/4375 - (13*x^2)/450 + x
^3/45
```

sympy [A] time = 0.12, size = 34, normalized size = 0.85

$$\frac{x^3}{45} - \frac{13x^2}{450} + \frac{139x}{3375} + \frac{\log\left(x+\frac{1}{5}\right)}{4375} - \frac{16\log\left(x+\frac{2}{3}\right)}{567}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(15+2/x**2+13/x),x)
```

```
[Out] x**3/45 - 13*x**2/450 + 139*x/3375 + log(x + 1/5)/4375 - 16*log(x + 2/3)/56
7
```

3.380 $\int \frac{x}{15+\frac{2}{x^2}+\frac{13}{x}} \, dx$

Optimal. Leaf size=33

$$\frac{x^2}{30} - \frac{13x}{225} + \frac{8}{189}\log(3x+2) - \frac{1}{875}\log(5x+1)$$

Rubi [A] time = 0.02, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1354, 701, 632, 31}

$$\frac{x^2}{30} - \frac{13x}{225} + \frac{8}{189}\log(3x+2) - \frac{1}{875}\log(5x+1)$$

Antiderivative was successfully verified.

```
[In] Int[x/(15 + 2/x^2 + 13/x),x]
```

```
[Out] (-13*x)/225 + x^2/30 + (8*Log[2 + 3*x])/189 - Log[1 + 5*x]/875
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 632

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rule 701

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Int[PolynomialDivide[(d + e*x)^m, a + b*x + c*x^2, x], x] /; FreeQ[{a,
 b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
 NeQ[2*c*d - b*e, 0] && IGtQ[m, 1] && (NeQ[d, 0] || GtQ[m, 2])
```

Rule 1354

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> Int[x^(m + 2*n*p)*(c + b/x^n + a/x^(2*n))^p, x] /; FreeQ[{a, b, c, m, n
}, x] && EqQ[n2, 2*n] && ILtQ[p, 0] && NegQ[n]
```

Rubi steps

$$\int \frac{x}{15+\frac{2}{x^2}+\frac{13}{x}}\,dx = \int \frac{x^3}{2+13x+15x^2}\,dx$$
$$= \int \left(-\frac{13}{225}+\frac{x}{15}+\frac{26+139x}{225\left(2+13x+15x^2\right)}\right)dx$$
$$= -\frac{13x}{225}+\frac{x^2}{30}+\frac{1}{225}\int \frac{26+139x}{2+13x+15x^2}\,dx$$
$$= -\frac{13x}{225}+\frac{x^2}{30}-\frac{3}{175}\int \frac{1}{3+15x}\,dx+\frac{40}{63}\int \frac{1}{10+15x}\,dx$$
$$= -\frac{13x}{225}+\frac{x^2}{30}+\frac{8}{189}\log(2+3x)-\frac{1}{875}\log(1+5x)$$

Mathematica [A] time = 0.00, size = 33, normalized size = 1.00

$$\frac{x^2}{30}-\frac{13x}{225}+\frac{8}{189}\log(3x+2)-\frac{1}{875}\log(5x+1)$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(15 + 2/x^2 + 13/x),x]
[Out] (-13*x)/225 + x^2/30 + (8*Log[2 + 3*x])/189 - Log[1 + 5*x]/875
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{15+\frac{2}{x^2}+\frac{13}{x}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x/(15 + 2/x^2 + 13/x),x]
[Out] IntegrateAlgebraic[x/(15 + 2/x^2 + 13/x), x]
```

fricas [A] time = 0.76, size = 25, normalized size = 0.76

$$\frac{1}{30}\,x^2-\frac{13}{225}\,x-\frac{1}{875}\,\log\left(5\,x+1\right)+\frac{8}{189}\,\log\left(3\,x+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(15+2/x^2+13/x),x, algorithm="fricas")
[Out] 1/30*x^2 - 13/225*x - 1/875*log(5*x + 1) + 8/189*log(3*x + 2)
```

giac [A] time = 0.28, size = 27, normalized size = 0.82

$$\frac{1}{30}\,x^2-\frac{13}{225}\,x-\frac{1}{875}\,\log\left(|5\,x+1|\right)+\frac{8}{189}\,\log\left(|3\,x+2|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(15+2/x^2+13/x),x, algorithm="giac")
[Out] 1/30*x^2 - 13/225*x - 1/875*log(abs(5*x + 1)) + 8/189*log(abs(3*x + 2))
```

maple [A] time = 0.00, size = 26, normalized size = 0.79

$$\frac{x^2}{30}-\frac{13x}{225}-\frac{\ln\left(5x+1\right)}{875}+\frac{8\ln\left(3x+2\right)}{189}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(15+2/x^2+13/x),x)
[Out] -13/225*x+1/30*x^2+8/189*ln(3*x+2)-1/875*ln(5*x+1)
```

maxima [A] time = 0.42, size = 25, normalized size = 0.76

$$\frac{1}{30}\,x^2 - \frac{13}{225}\,x - \frac{1}{875}\,\log\left(5\,x + 1\right) + \frac{8}{189}\,\log\left(3\,x + 2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(15+2/x^2+13/x),x, algorithm="maxima")
[Out] 1/30*x^2 - 13/225*x - 1/875*log(5*x + 1) + 8/189*log(3*x + 2)
```

mupad [B] time = 1.31, size = 21, normalized size = 0.64

$$\frac{8\,\ln\left(x+\frac{2}{3}\right)}{189} - \frac{13\,x}{225} - \frac{\ln\left(x+\frac{1}{5}\right)}{875} + \frac{x^2}{30}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(13/x + 2/x^2 + 15),x)
[Out] (8*log(x + 2/3))/189 - (13*x)/225 - log(x + 1/5)/875 + x^2/30
```

sympy [A] time = 0.12, size = 27, normalized size = 0.82

$$\frac{x^2}{30} - \frac{13x}{225} - \frac{\log\left(x+\frac{1}{5}\right)}{875} + \frac{8\log\left(x+\frac{2}{3}\right)}{189}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(15+2/x**2+13/x),x)
[Out] x**2/30 - 13*x/225 - log(x + 1/5)/875 + 8*log(x + 2/3)/189
```

3.381 $\int \frac{1}{15+\frac{2}{x^2}+\frac{13}{x}}\,dx$

Optimal. Leaf size=26

$$\frac{x}{15}-\frac{4}{63}\log(3x+2)+\frac{1}{175}\log(5x+1)$$

Rubi [A] time = 0.01, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {1340, 703, 632, 31}

$$\frac{x}{15}-\frac{4}{63}\log(3x+2)+\frac{1}{175}\log(5x+1)$$

Antiderivative was successfully verified.

```
[In] Int[(15 + 2/x^2 + 13/x)^(-1),x]
```

```
[Out] x/15 - (4*Log[2 + 3*x])/63 + Log[1 + 5*x]/175
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 632

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rule 703

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Simp[(e*(d + e*x)^(m - 1))/(c*(m - 1)), x] + Dist[1/c, Int[((d + e*x)^
(m - 2)*Simp[c*d^2 - a*e^2 + e*(2*c*d - b*e)*x, x])/(a + b*x + c*x^2), x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && GtQ[m, 1]
```

Rule 1340

```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[x^
(2*n*p)*(c + b/x^n + a/x^(2*n))^p, x] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n
] && LtQ[n, 0] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{15+\frac{2}{x^2}+\frac{13}{x}}\,dx &= \int \frac{x^2}{2+13x+15x^2}\,dx\\
&= \frac{x}{15}+\frac{1}{15}\int \frac{-2-13x}{2+13x+15x^2}\,dx\\
&= \frac{x}{15}+\frac{3}{35}\int \frac{1}{3+15x}\,dx-\frac{20}{21}\int \frac{1}{10+15x}\,dx\\
&= \frac{x}{15}-\frac{4}{63}\log(2+3x)+\frac{1}{175}\log(1+5x)
\end{aligned}$$

Mathematica [A] time = 0.00, size = 26, normalized size = 1.00

$$\frac{x}{15}-\frac{4}{63}\log(3x+2)+\frac{1}{175}\log(5x+1)$$

Antiderivative was successfully verified.

```
[In] Integrate[(15 + 2/x^2 + 13/x)^(-1),x]
[Out] x/15 - (4*Log[2 + 3*x])/63 + Log[1 + 5*x]/175
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{15+\frac{2}{x^2}+\frac{13}{x}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(15 + 2/x^2 + 13/x)^(-1),x]
[Out] IntegrateAlgebraic[(15 + 2/x^2 + 13/x)^(-1), x]
```

fricas [A] time = 1.22, size = 20, normalized size = 0.77

$$\frac{1}{15}\,x+\frac{1}{175}\,\log\left(5\,x+1\right)-\frac{4}{63}\,\log\left(3\,x+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(15+2/x^2+13/x),x, algorithm="fricas")
[Out] 1/15*x + 1/175*log(5*x + 1) - 4/63*log(3*x + 2)
```

giac [A] time = 0.28, size = 22, normalized size = 0.85

$$\frac{1}{15}\,x+\frac{1}{175}\,\log\left(|5\,x+1|\right)-\frac{4}{63}\,\log\left(|3\,x+2|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(15+2/x^2+13/x),x, algorithm="giac")
[Out] 1/15*x + 1/175*log(abs(5*x + 1)) - 4/63*log(abs(3*x + 2))
```

maple [A] time = 0.01, size = 21, normalized size = 0.81

$$\frac{x}{15}+\frac{\ln\left(5x+1\right)}{175}-\frac{4\ln\left(3x+2\right)}{63}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(15+2/x^2+13/x),x)
[Out] 1/15*x-4/63*ln(3*x+2)+1/175*ln(5*x+1)
```

maxima [A] time = 0.42, size = 20, normalized size = 0.77

$$\frac{1}{15}\,x+\frac{1}{175}\,\log\left(5\,x+1\right)-\frac{4}{63}\,\log\left(3\,x+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(15+2/x^2+13/x),x, algorithm="maxima")
[Out] 1/15*x + 1/175*log(5*x + 1) - 4/63*log(3*x + 2)
```

mupad [B] time = 0.08, size = 16, normalized size = 0.62

$$\frac{x}{15}-\frac{4\ \ln\left(x+\frac{2}{3}\right)}{63}+\frac{\ln\left(x+\frac{1}{5}\right)}{175}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(13/x + 2/x^2 + 15),x)
```

```
[Out] x/15 - (4*log(x + 2/3))/63 + log(x + 1/5)/175
```

sympy [A] time = 0.12, size = 20, normalized size = 0.77

$$\frac{x}{15}+\frac{\log\left(x+\frac{1}{5}\right)}{175}-\frac{4\log\left(x+\frac{2}{3}\right)}{63}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(15+2/x**2+13/x),x)
```

```
[Out] x/15 + log(x + 1/5)/175 - 4*log(x + 2/3)/63
```

3.382 $\int \frac{1}{\left(15+\frac{2}{x^2}+\frac{13}{x}\right)x} \, dx$

Optimal. Leaf size=21

$$\frac{2}{21}\log(3x+2)-\frac{1}{35}\log(5x+1)$$

Rubi [A] time = 0.01, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1354, 632, 31}

$$\frac{2}{21}\log(3x+2)-\frac{1}{35}\log(5x+1)$$

Antiderivative was successfully verified.

```
[In] Int[1/((15 + 2/x^2 + 13/x)*x),x]
```

```
[Out] (2*Log[2 + 3*x])/21 - Log[1 + 5*x]/35
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 632

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1354

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> Int[x^(m + 2*n*p)*(c + b/x^n + a/x^(2*n))^p, x] /; FreeQ[{a, b, c, m, n
}, x] && EqQ[n2, 2*n] && ILtQ[p, 0] && NegQ[n]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(15+\frac{2}{x^2}+\frac{13}{x}\right)x} \, dx &= \int \frac{x}{2+13x+15x^2} \, dx\\ &= -\left(\frac{3}{7}\int \frac{1}{3+15x} \, dx\right)+\frac{10}{7}\int \frac{1}{10+15x} \, dx\\ &= \frac{2}{21}\log(2+3x)-\frac{1}{35}\log(1+5x)\end{aligned}$$

Mathematica [A] time = 0.00, size = 21, normalized size = 1.00

$$\frac{2}{21}\log(3x+2)-\frac{1}{35}\log(5x+1)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((15 + 2/x^2 + 13/x)*x),x]
```

```
[Out] (2*Log[2 + 3*x])/21 - Log[1 + 5*x]/35
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(15+\frac{2}{x^2}+\frac{13}{x}\right)x}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/((15 + 2/x^2 + 13/x)*x),x]
```

```
[Out] IntegrateAlgebraic[1/((15 + 2/x^2 + 13/x)*x), x]
```

fricas [A] time = 1.40, size = 17, normalized size = 0.81

$$-\frac{1}{35}\,\log\left(5\,x+1\right)+\frac{2}{21}\,\log\left(3\,x+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(15+2/x^2+13/x)/x,x, algorithm="fricas")
```

```
[Out] -1/35*log(5*x + 1) + 2/21*log(3*x + 2)
```

giac [A] time = 0.28, size = 19, normalized size = 0.90

$$-\frac{1}{35}\,\log\left(|5\,x+1|\right)+\frac{2}{21}\,\log\left(|3\,x+2|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(15+2/x^2+13/x)/x,x, algorithm="giac")
```

```
[Out] -1/35*log(abs(5*x + 1)) + 2/21*log(abs(3*x + 2))
```

maple [A] time = 0.01, size = 18, normalized size = 0.86

$$-\frac{\ln\left(5x+1\right)}{35}+\frac{2\ln\left(3x+2\right)}{21}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(15+2/x^2+13/x)/x,x)
```

```
[Out] 2/21*ln(3*x+2)-1/35*ln(5*x+1)
```

maxima [A] time = 0.42, size = 17, normalized size = 0.81

$$-\frac{1}{35}\,\log\left(5\,x+1\right)+\frac{2}{21}\,\log\left(3\,x+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(15+2/x^2+13/x)/x,x, algorithm="maxima")
```

```
[Out] -1/35*log(5*x + 1) + 2/21*log(3*x + 2)
```

mupad [B] time = 0.07, size = 13, normalized size = 0.62

$$\frac{2\,\ln\left(x+\frac{2}{3}\right)}{21}-\frac{\ln\left(x+\frac{1}{5}\right)}{35}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(13/x + 2/x^2 + 15)),x)
```

```
[Out] (2*log(x + 2/3))/21 - log(x + 1/5)/35
```

sympy [A] time = 0.11, size = 17, normalized size = 0.81

$$-\frac{\log\left(x+\frac{1}{5}\right)}{35}+\frac{2\log\left(x+\frac{2}{3}\right)}{21}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(15+2/x**2+13/x)/x,x)
```

```
[Out] -log(x + 1/5)/35 + 2*log(x + 2/3)/21
```

3.383 $\int \frac{1}{\left(15+\frac{2}{x^2}+\frac{13}{x}\right)x^2}\,dx$

Optimal. Leaf size=23

$$\frac{1}{7}\log\left(\frac{1}{x}+5\right)-\frac{1}{7}\log\left(\frac{2}{x}+3\right)$$

Rubi [A] time = 0.01, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1352, 616, 31}

$$\frac{1}{7}\log\left(\frac{1}{x}+5\right)-\frac{1}{7}\log\left(\frac{2}{x}+3\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/((15 + 2/x^2 + 13/x)*x^2),x]
```

```
[Out] Log[5 + x^(-1)]/7 - Log[3 + 2/x]/7
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 616

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = Rt[b^2
 - 4*a*c, 2]}, Dist[c/q, Int[1/Simp[b/2 - q/2 + c*x, x], x], x] - Dist[c/q,
 Int[1/Simp[b/2 + q/2 + c*x, x], x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[b^2 - 4*a*c] && PerfectSquareQ[b^2 - 4*a*c]
```

Rule 1352

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a,
b, c, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\left(15+\frac{2}{x^2}+\frac{13}{x}\right)x^2}\,dx &= -\text{Subst}\left(\int \frac{1}{15+13x+2x^2}\,dx,x,\frac{1}{x}\right)\\
&= -\left(\frac{2}{7}\text{Subst}\left(\int \frac{1}{3+2x}\,dx,x,\frac{1}{x}\right)\right)+\frac{2}{7}\text{Subst}\left(\int \frac{1}{10+2x}\,dx,x,\frac{1}{x}\right)\\
&= \frac{1}{7}\log\left(5+\frac{1}{x}\right)-\frac{1}{7}\log\left(3+\frac{2}{x}\right)
\end{aligned}$$

Mathematica [A] time = 0.00, size = 21, normalized size = 0.91

$$\frac{1}{7}\log(5x+1)-\frac{1}{7}\log(3x+2)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((15 + 2/x^2 + 13/x)*x^2),x]
```

```
[Out] -1/7*Log[2 + 3*x] + Log[1 + 5*x]/7
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(15+\frac{2}{x^2}+\frac{13}{x}\right)x^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/((15 + 2/x^2 + 13/x)*x^2),x]
[Out] IntegrateAlgebraic[1/((15 + 2/x^2 + 13/x)*x^2), x]
```

fricas [A] time = 1.04, size = 17, normalized size = 0.74

$$\frac{1}{7}\,\log\left(5\,x+1\right)-\frac{1}{7}\,\log\left(3\,x+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(15+2/x^2+13/x)/x^2,x, algorithm="fricas")
[Out] 1/7*log(5*x + 1) - 1/7*log(3*x + 2)
```

giac [A] time = 0.22, size = 19, normalized size = 0.83

$$\frac{1}{7}\,\log\left(|5\,x+1|\right)-\frac{1}{7}\,\log\left(|3\,x+2|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(15+2/x^2+13/x)/x^2,x, algorithm="giac")
[Out] 1/7*log(abs(5*x + 1)) - 1/7*log(abs(3*x + 2))
```

maple [A] time = 0.01, size = 18, normalized size = 0.78

$$\frac{\ln\left(5x+1\right)}{7}-\frac{\ln\left(3x+2\right)}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(15+2/x^2+13/x)/x^2,x)
[Out] -1/7*ln(3*x+2)+1/7*ln(5*x+1)
```

maxima [A] time = 0.42, size = 17, normalized size = 0.74

$$\frac{1}{7}\,\log\left(5\,x+1\right)-\frac{1}{7}\,\log\left(3\,x+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(15+2/x^2+13/x)/x^2,x, algorithm="maxima")
[Out] 1/7*log(5*x + 1) - 1/7*log(3*x + 2)
```

mupad [B] time = 1.37, size = 8, normalized size = 0.35

$$-\frac{2\,\mathrm{atanh}\left(\frac{30\,x}{7}+\frac{13}{7}\right)}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(13/x + 2/x^2 + 15)),x)
```

```
[Out] -(2*atanh((30*x)/7 + 13/7))/7
```

sympy [**A**] time = 0.11, size = 15, normalized size = 0.65

$$\frac{\log\left(x+\frac{1}{5}\right)}{7}-\frac{\log\left(x+\frac{2}{3}\right)}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(15+2/x**2+13/x)/x**2,x)
```

```
[Out] log(x + 1/5)/7 - log(x + 2/3)/7
```

3.384 $\int \frac{1}{\left(15+\frac{2}{x^2}+\frac{13}{x}\right)x^3}\,dx$

Optimal. Leaf size=27

$$\frac{\log(x)}{2}+\frac{3}{14}\log(3x+2)-\frac{5}{7}\log(5x+1)$$

Rubi [A] time = 0.02, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {1354, 705, 29, 632, 31}

$$\frac{\log(x)}{2}+\frac{3}{14}\log(3x+2)-\frac{5}{7}\log(5x+1)$$

Antiderivative was successfully verified.

```
[In] Int[1/((15 + 2/x^2 + 13/x)*x^3),x]
```

```
[Out] Log[x]/2 + (3*Log[2 + 3*x])/14 - (5*Log[1 + 5*x])/7
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 632

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rule 705

```
Int[1/(((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)), x_Symbol]
 :> Dist[e^2/(c*d^2 - b*d*e + a*e^2), Int[1/(d + e*x), x], x] + Dist[1/(c*d
^2 - b*d*e + a*e^2), Int[(c*d - b*e - c*e*x)/(a + b*x + c*x^2), x], x] /; F
reeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^
2, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1354

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> Int[x^(m + 2*n*p)*(c + b/x^n + a/x^(2*n))^p, x] /; FreeQ[{a, b, c, m, n
}, x] && EqQ[n2, 2*n] && ILtQ[p, 0] && NegQ[n]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(15+\frac{2}{x^2}+\frac{13}{x}\right)x^3}\,dx &= \int \frac{1}{x\left(2+13x+15x^2\right)}\,dx\\ &= \frac{1}{2}\int \frac{1}{x}\,dx+\frac{1}{2}\int \frac{-13-15x}{2+13x+15x^2}\,dx\\ &= \frac{\log(x)}{2}+\frac{45}{14}\int \frac{1}{10+15x}\,dx-\frac{75}{7}\int \frac{1}{3+15x}\,dx\\ &= \frac{\log(x)}{2}+\frac{3}{14}\log(2+3x)-\frac{5}{7}\log(1+5x)\end{aligned}$$

Mathematica [A] time = 0.00, size = 27, normalized size = 1.00

$$\frac{\log(x)}{2}+\frac{3}{14}\log(3x+2)-\frac{5}{7}\log(5x+1)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((15 + 2/x^2 + 13/x)*x^3),x]
[Out] Log[x]/2 + (3*Log[2 + 3*x])/14 - (5*Log[1 + 5*x])/7
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(15+\frac{2}{x^2}+\frac{13}{x}\right)x^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/((15 + 2/x^2 + 13/x)*x^3),x]
[Out] IntegrateAlgebraic[1/((15 + 2/x^2 + 13/x)*x^3), x]
```

fricas [A] time = 1.30, size = 21, normalized size = 0.78

$$-\frac{5}{7}\log\left(5x+1\right)+\frac{3}{14}\log\left(3x+2\right)+\frac{1}{2}\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(15+2/x^2+13/x)/x^3,x, algorithm="fricas")
[Out] -5/7*log(5*x + 1) + 3/14*log(3*x + 2) + 1/2*log(x)
```

giac [A] time = 0.25, size = 24, normalized size = 0.89

$$-\frac{5}{7}\log\left(|5x+1|\right)+\frac{3}{14}\log\left(|3x+2|\right)+\frac{1}{2}\log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(15+2/x^2+13/x)/x^3,x, algorithm="giac")
[Out] -5/7*log(abs(5*x + 1)) + 3/14*log(abs(3*x + 2)) + 1/2*log(abs(x))
```

maple [A] time = 0.01, size = 22, normalized size = 0.81

$$\frac{\ln(x)}{2}-\frac{5\ln\left(5x+1\right)}{7}+\frac{3\ln\left(3x+2\right)}{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(15+2/x^2+13/x)/x^3,x)
[Out] 1/2*ln(x)+3/14*ln(3*x+2)-5/7*ln(5*x+1)
```

maxima [A] time = 0.42, size = 21, normalized size = 0.78

$$-\frac{5}{7}\log\left(5x+1\right)+\frac{3}{14}\log\left(3x+2\right)+\frac{1}{2}\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(15+2/x^2+13/x)/x^3,x, algorithm="maxima")
[Out] -5/7*log(5*x + 1) + 3/14*log(3*x + 2) + 1/2*log(x)
```

mupad [B] time = 1.39, size = 17, normalized size = 0.63

$$\frac{3\ln\left(x+\frac{2}{3}\right)}{14}-\frac{5\ln\left(x+\frac{1}{5}\right)}{7}+\frac{\ln(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(13/x + 2/x^2 + 15)),x)
[Out] (3*log(x + 2/3))/14 - (5*log(x + 1/5))/7 + log(x)/2
```

sympy [A] time = 0.15, size = 24, normalized size = 0.89

$$\frac{\log\left(x\right)}{2}-\frac{5\log\left(x+\frac{1}{5}\right)}{7}+\frac{3\log\left(x+\frac{2}{3}\right)}{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(15+2/x**2+13/x)/x**3,x)
[Out] log(x)/2 - 5*log(x + 1/5)/7 + 3*log(x + 2/3)/14
```

3.385 $\int \frac{1}{\left(15+\frac{2}{x^2}+\frac{13}{x}\right)x^4}\,dx$

Optimal. Leaf size=34

$$-\frac{1}{2x}-\frac{13\log(x)}{4}-\frac{9}{28}\log(3x+2)+\frac{25}{7}\log(5x+1)$$

Rubi [A] time = 0.03, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1354, 709, 800}

$$-\frac{1}{2x}-\frac{13\log(x)}{4}-\frac{9}{28}\log(3x+2)+\frac{25}{7}\log(5x+1)$$

Antiderivative was successfully verified.

```
[In] Int[1/((15 + 2/x^2 + 13/x)*x^4),x]
```

```
[Out] -1/(2*x) - (13*Log[x])/4 - (9*Log[2 + 3*x])/28 + (25*Log[1 + 5*x])/7
```

Rule 709

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Simp[(e*(d + e*x)^(m + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dis
t[1/(c*d^2 - b*d*e + a*e^2), Int[((d + e*x)^(m + 1)*Simp[c*d - b*e - c*e*x,
 x])/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 -
 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[m
, -1]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 1354

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> Int[x^(m + 2*n*p)*(c + b/x^n + a/x^(2*n))^p, x] /; FreeQ[{a, b, c, m, n
}, x] && EqQ[n2, 2*n] && ILtQ[p, 0] && NegQ[n]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\left(15+\frac{2}{x^2}+\frac{13}{x}\right)x^4}\,dx &= \int \frac{1}{x^2\left(2+13x+15x^2\right)}\,dx\\
&= -\frac{1}{2x}+\frac{1}{2}\int \frac{-13-15x}{x\left(2+13x+15x^2\right)}\,dx\\
&= -\frac{1}{2x}+\frac{1}{2}\int \left(-\frac{13}{2x}-\frac{27}{14(2+3x)}+\frac{250}{7(1+5x)}\right)dx\\
&= -\frac{1}{2x}-\frac{13\log(x)}{4}-\frac{9}{28}\log(2+3x)+\frac{25}{7}\log(1+5x)
\end{aligned}$$

Mathematica [A] time = 0.00, size = 34, normalized size = 1.00

$$-\frac{1}{2x}-\frac{13\log(x)}{4}-\frac{9}{28}\log(3x+2)+\frac{25}{7}\log(5x+1)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((15 + 2/x^2 + 13/x)*x^4),x]
[Out] -1/2*1/x - (13*Log[x])/4 - (9*Log[2 + 3*x])/28 + (25*Log[1 + 5*x])/7
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(15 + \frac{2}{x^2} + \frac{13}{x}\right) x^4} \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/((15 + 2/x^2 + 13/x)*x^4),x]
[Out] IntegrateAlgebraic[1/((15 + 2/x^2 + 13/x)*x^4), x]
```

fricas [A] time = 1.30, size = 30, normalized size = 0.88

$$\frac{100 \, x \log\left(5 \, x + 1\right) - 9 \, x \log\left(3 \, x + 2\right) - 91 \, x \log\left(x\right) - 14}{28 \, x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(15+2/x^2+13/x)/x^4,x, algorithm="fricas")
[Out] 1/28*(100*x*log(5*x + 1) - 9*x*log(3*x + 2) - 91*x*log(x) - 14)/x
```

giac [A] time = 0.31, size = 29, normalized size = 0.85

$$-\frac{1}{2 \, x} + \frac{25}{7} \, \log\left({\left| 5 \, x + 1 \right|}\right) - \frac{9}{28} \, \log\left({\left| 3 \, x + 2 \right|}\right) - \frac{13}{4} \, \log\left({\left| x \right|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(15+2/x^2+13/x)/x^4,x, algorithm="giac")
[Out] -1/2/x + 25/7*log(abs(5*x + 1)) - 9/28*log(abs(3*x + 2)) - 13/4*log(abs(x))
```

maple [A] time = 0.01, size = 27, normalized size = 0.79

$$-\frac{13 \ln\left(x \right)}{4}+\frac{25 \ln\left(5 x +1\right)}{7}-\frac{9 \ln\left(3 x +2\right)}{28}-\frac{1}{2 x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(15+2/x^2+13/x)/x^4,x)
[Out] -1/2/x-13/4*ln(x)-9/28*ln(3*x+2)+25/7*ln(5*x+1)
```

maxima [A] time = 0.43, size = 26, normalized size = 0.76

$$-\frac{1}{2 \, x} + \frac{25}{7} \, \log\left(5 \, x + 1\right) - \frac{9}{28} \, \log\left(3 \, x + 2\right) - \frac{13}{4} \, \log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(15+2/x^2+13/x)/x^4,x, algorithm="maxima")
[Out] -1/2/x + 25/7*log(5*x + 1) - 9/28*log(3*x + 2) - 13/4*log(x)
```

mupad [B] time = 0.04, size = 22, normalized size = 0.65

$$\frac{25\,\ln\left(x+\frac{1}{5}\right)}{7}-\frac{9\,\ln\left(x+\frac{2}{3}\right)}{28}-\frac{13\,\ln\left(x\right)}{4}-\frac{1}{2\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(13/x + 2/x^2 + 15)),x)
```

```
[Out] (25*log(x + 1/5))/7 - (9*log(x + 2/3))/28 - (13*log(x))/4 - 1/(2*x)
```

sympy [A] time = 0.16, size = 31, normalized size = 0.91

$$-\frac{13\log(x)}{4}+\frac{25\log\left(x+\frac{1}{5}\right)}{7}-\frac{9\log\left(x+\frac{2}{3}\right)}{28}-\frac{1}{2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(15+2/x**2+13/x)/x**4,x)
```

```
[Out] -13*log(x)/4 + 25*log(x + 1/5)/7 - 9*log(x + 2/3)/28 - 1/(2*x)
```

3.386 $\int \frac{1}{\left(15+\frac{2}{x^2}+\frac{13}{x}\right)x^5}\,dx$

Optimal. Leaf size=41

$$-\frac{1}{4x^2}+\frac{13}{4x}+\frac{139\log(x)}{8}+\frac{27}{56}\log(3x+2)-\frac{125}{7}\log(5x+1)$$

Rubi [A] time = 0.03, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1354, 709, 800}

$$-\frac{1}{4x^2}+\frac{13}{4x}+\frac{139\log(x)}{8}+\frac{27}{56}\log(3x+2)-\frac{125}{7}\log(5x+1)$$

Antiderivative was successfully verified.

```
[In] Int[1/((15 + 2/x^2 + 13/x)*x^5),x]
```

```
[Out] -1/(4*x^2) + 13/(4*x) + (139*Log[x])/8 + (27*Log[2 + 3*x])/56 - (125*Log[1
+ 5*x])/7
```

Rule 709

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Simp[(e*(d + e*x)^(m + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dis
t[1/(c*d^2 - b*d*e + a*e^2), Int[((d + e*x)^(m + 1)*Simp[c*d - b*e - c*e*x,
 x])/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 -
 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[m
, -1]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 1354

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> Int[x^(m + 2*n*p)*(c + b/x^n + a/x^(2*n))^p, x] /; FreeQ[{a, b, c, m, n
}, x] && EqQ[n2, 2*n] && ILtQ[p, 0] && NegQ[n]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\left(15+\frac{2}{x^2}+\frac{13}{x}\right)x^5}\,dx &= \int \frac{1}{x^3\left(2+13x+15x^2\right)}\,dx\\
&= -\frac{1}{4x^2}+\frac{1}{2}\int \frac{-13-15x}{x^2\left(2+13x+15x^2\right)}\,dx\\
&= -\frac{1}{4x^2}+\frac{1}{2}\int \left(-\frac{13}{2x^2}+\frac{139}{4x}+\frac{81}{28(2+3x)}-\frac{1250}{7(1+5x)}\right)dx\\
&= -\frac{1}{4x^2}+\frac{13}{4x}+\frac{139\log(x)}{8}+\frac{27}{56}\log(2+3x)-\frac{125}{7}\log(1+5x)
\end{aligned}$$

Mathematica [A] time = 0.00, size = 41, normalized size = 1.00

$$-\frac{1}{4x^2}+\frac{13}{4x}+\frac{139\log(x)}{8}+\frac{27}{56}\log(3x+2)-\frac{125}{7}\log(5x+1)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((15 + 2/x^2 + 13/x)*x^5),x]
```

```
[Out] -1/4*1/x^2 + 13/(4*x) + (139*Log[x])/8 + (27*Log[2 + 3*x])/56 - (125*Log[1
+ 5*x])/7
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\left(15+\frac{2}{x^2}+\frac{13}{x}\right)x^5}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/((15 + 2/x^2 + 13/x)*x^5),x]
```

```
[Out] IntegrateAlgebraic[1/((15 + 2/x^2 + 13/x)*x^5), x]
```

fricas [A] time = 1.30, size = 39, normalized size = 0.95

$$-\frac{1000\,x^2\log(5\,x+1)-27\,x^2\log(3\,x+2)-973\,x^2\log(x)-182\,x+14}{56\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(15+2/x^2+13/x)/x^5,x, algorithm="fricas")
```

```
[Out] -1/56*(1000*x^2*log(5*x + 1) - 27*x^2*log(3*x + 2) - 973*x^2*log(x) - 182*x
 + 14)/x^2
```

giac [A] time = 0.30, size = 34, normalized size = 0.83

$$\frac{13\,x-1}{4\,x^2}-\frac{125}{7}\log(|5\,x+1|)+\frac{27}{56}\log(|3\,x+2|)+\frac{139}{8}\log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(15+2/x^2+13/x)/x^5,x, algorithm="giac")
```

```
[Out] 1/4*(13*x - 1)/x^2 - 125/7*log(abs(5*x + 1)) + 27/56*log(abs(3*x + 2)) + 13
9/8*log(abs(x))
```

maple [A] time = 0.01, size = 32, normalized size = 0.78

$$\frac{139\ln(x)}{8}-\frac{125\ln(5x+1)}{7}+\frac{27\ln(3x+2)}{56}+\frac{13}{4x}-\frac{1}{4x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(15+2/x^2+13/x)/x^5,x)
```

```
[Out] -1/4/x^2+13/4/x+139/8*ln(x)+27/56*ln(3*x+2)-125/7*ln(5*x+1)
```

maxima [A] time = 0.42, size = 31, normalized size = 0.76

$$\frac{13\,x-1}{4\,x^2}-\frac{125}{7}\log(5\,x+1)+\frac{27}{56}\log(3\,x+2)+\frac{139}{8}\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(15+2/x^2+13/x)/x^5,x, algorithm="maxima")
```

```
[Out] 1/4*(13*x - 1)/x^2 - 125/7*log(5*x + 1) + 27/56*log(3*x + 2) + 139/8*log(x)
```

mupad [B] time = 1.31, size = 26, normalized size = 0.63

$$\frac{27\,\ln\left(x+\frac{2}{3}\right)}{56}-\frac{125\,\ln\left(x+\frac{1}{5}\right)}{7}+\frac{139\,\ln(x)}{8}+\frac{\frac{13\,x}{4}-\frac{1}{4}}{x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^5*(13/x + 2/x^2 + 15)),x)
```

```
[Out] (27*log(x + 2/3))/56 - (125*log(x + 1/5))/7 + (139*log(x))/8 + ((13*x)/4 -
1/4)/x^2
```

sympy [A] time = 0.17, size = 36, normalized size = 0.88

$$\frac{139\log(x)}{8}-\frac{125\log\left(x+\frac{1}{5}\right)}{7}+\frac{27\log\left(x+\frac{2}{3}\right)}{56}+\frac{13x-1}{4x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(15+2/x**2+13/x)/x**5,x)
```

```
[Out] 139*log(x)/8 - 125*log(x + 1/5)/7 + 27*log(x + 2/3)/56 + (13*x - 1)/(4*x**2
)
```

3.387 $\int \frac{1}{\left(15+\frac{2}{x^2}+\frac{13}{x}\right)x^6}\,dx$

Optimal. Leaf size=48

$$-\frac{1}{6x^3}+\frac{13}{8x^2}-\frac{139}{8x}-\frac{1417\log(x)}{16}-\frac{81}{112}\log(3x+2)+\frac{625}{7}\log(5x+1)$$

Rubi [A] time = 0.04, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1354, 709, 800}

$$\frac{13}{8x^2}-\frac{1}{6x^3}-\frac{139}{8x}-\frac{1417\log(x)}{16}-\frac{81}{112}\log(3x+2)+\frac{625}{7}\log(5x+1)$$

Antiderivative was successfully verified.

```
[In] Int[1/((15 + 2/x^2 + 13/x)*x^6),x]
```

```
[Out] -1/(6*x^3) + 13/(8*x^2) - 139/(8*x) - (1417*Log[x])/16 - (81*Log[2 + 3*x])/
112 + (625*Log[1 + 5*x])/7
```

Rule 709

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Simp[(e*(d + e*x)^(m + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dis
t[1/(c*d^2 - b*d*e + a*e^2), Int[((d + e*x)^(m + 1)*Simp[c*d - b*e - c*e*x,
 x])/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 -
 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[m
, -1]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 1354

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> Int[x^(m + 2*n*p)*(c + b/x^n + a/x^(2*n))^p, x] /; FreeQ[{a, b, c, m, n
}, x] && EqQ[n2, 2*n] && ILtQ[p, 0] && NegQ[n]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\left(15+\frac{2}{x^2}+\frac{13}{x}\right)x^6}\,dx &= \int \frac{1}{x^4\left(2+13x+15x^2\right)}\,dx\\
&= -\frac{1}{6x^3}+\frac{1}{2}\int \frac{-13-15x}{x^3\left(2+13x+15x^2\right)}\,dx\\
&= -\frac{1}{6x^3}+\frac{1}{2}\int \left(-\frac{13}{2x^3}+\frac{139}{4x^2}-\frac{1417}{8x}-\frac{243}{56(2+3x)}+\frac{6250}{7(1+5x)}\right)dx\\
&= -\frac{1}{6x^3}+\frac{13}{8x^2}-\frac{139}{8x}-\frac{1417\log(x)}{16}-\frac{81}{112}\log(2+3x)+\frac{625}{7}\log(1+5x)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 48, normalized size = 1.00

$$-\frac{1}{6x^3}+\frac{13}{8x^2}-\frac{139}{8x}-\frac{1417\log(x)}{16}-\frac{81}{112}\log(3x+2)+\frac{625}{7}\log(5x+1)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((15 + 2/x^2 + 13/x)*x^6),x]
```

```
[Out] -1/6*1/x^3 + 13/(8*x^2) - 139/(8*x) - (1417*Log[x])/16 - (81*Log[2 + 3*x])/
112 + (625*Log[1 + 5*x])/7
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(15+\frac{2}{x^2}+\frac{13}{x}\right)x^6}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/((15 + 2/x^2 + 13/x)*x^6),x]
```

```
[Out] IntegrateAlgebraic[1/((15 + 2/x^2 + 13/x)*x^6), x]
```

fricas [A] time = 1.14, size = 44, normalized size = 0.92

$$\frac{30000\,x^3\log\left(5\,x+1\right)-243\,x^3\log\left(3\,x+2\right)-29757\,x^3\log\left(x\right)-5838\,x^2+546\,x-56}{336\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(15+2/x^2+13/x)/x^6,x, algorithm="fricas")
```

```
[Out] 1/336*(30000*x^3*log(5*x + 1) - 243*x^3*log(3*x + 2) - 29757*x^3*log(x) - 5
838*x^2 + 546*x - 56)/x^3
```

giac [A] time = 0.29, size = 39, normalized size = 0.81

$$-\frac{417\,x^2-39\,x+4}{24\,x^3}+\frac{625}{7}\log\left(|5\,x+1|\right)-\frac{81}{112}\log\left(|3\,x+2|\right)-\frac{1417}{16}\log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(15+2/x^2+13/x)/x^6,x, algorithm="giac")
```

```
[Out] -1/24*(417*x^2 - 39*x + 4)/x^3 + 625/7*log(abs(5*x + 1)) - 81/112*log(abs(3
*x + 2)) - 1417/16*log(abs(x))
```

maple [A] time = 0.01, size = 37, normalized size = 0.77

$$-\frac{1417\ln(x)}{16}+\frac{625\ln\left(5x+1\right)}{7}-\frac{81\ln\left(3x+2\right)}{112}-\frac{139}{8x}+\frac{13}{8x^2}-\frac{1}{6x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(15+2/x^2+13/x)/x^6,x)
```

```
[Out] -1/6/x^3+13/8/x^2-139/8/x-1417/16*ln(x)-81/112*ln(3*x+2)+625/7*ln(5*x+1)
```

maxima [A] time = 0.65, size = 36, normalized size = 0.75

$$-\frac{417\,x^2-39\,x+4}{24\,x^3}+\frac{625}{7}\log\left(5\,x+1\right)-\frac{81}{112}\log\left(3\,x+2\right)-\frac{1417}{16}\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(15+2/x^2+13/x)/x^6,x, algorithm="maxima")
```

```
[Out] -1/24*(417*x^2 - 39*x + 4)/x^3 + 625/7*log(5*x + 1) - 81/112*log(3*x + 2) -
 1417/16*log(x)
```

mupad [B] time = 0.05, size = 32, normalized size = 0.67

$$\frac{625\ \ln\left(x+\frac{1}{5}\right)}{7}-\frac{81\ \ln\left(x+\frac{2}{3}\right)}{112}-\frac{1417\ \ln(x)}{16}-\frac{\frac{139\,x^2}{8}-\frac{13\,x}{8}+\frac{1}{6}}{x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^6*(13/x + 2/x^2 + 15)),x)
```

```
[Out] (625*log(x + 1/5))/7 - (81*log(x + 2/3))/112 - (1417*log(x))/16 - ((139*x^2
)/8 - (13*x)/8 + 1/6)/x^3
```

sympy [A] time = 0.18, size = 41, normalized size = 0.85

$$-\frac{1417\log\left(x\right)}{16}+\frac{625\log\left(x+\frac{1}{5}\right)}{7}-\frac{81\log\left(x+\frac{2}{3}\right)}{112}+\frac{-417x^2+39x-4}{24x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(15+2/x**2+13/x)/x**6,x)
```

```
[Out] -1417*log(x)/16 + 625*log(x + 1/5)/7 - 81*log(x + 2/3)/112 + (-417*x**2 + 3
9*x - 4)/(24*x**3)
```

3.388 $\int \left(a+\frac{c}{x^2}+\frac{b}{x}\right)^{5/2} dx$

Optimal. Leaf size=204

$$\frac{5}{2}a^{3/2}b\tanh^{-1}\left(\frac{2a+\frac{b}{x}}{2\sqrt{a}\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}\right)+\frac{5\left(-48a^2c^2-24ab^2c+b^4\right)\tanh^{-1}\left(\frac{b+\frac{2c}{x}}{2\sqrt{c}\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}\right)}{128c^{3/2}}-\frac{5\left(\frac{2c\left(12ac+b^2\right)}{x}+b\left(44a\right.\right.}{64}$$

Rubi [A] time = 0.23, antiderivative size = 204, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {1342, 732, 814, 843, 621, 206, 724}

$$\frac{5\left(-48a^2c^2-24ab^2c+b^4\right)\tanh^{-1}\left(\frac{b+\frac{2c}{x}}{2\sqrt{c}\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}\right)}{128c^{3/2}}+\frac{5}{2}a^{3/2}b\tanh^{-1}\left(\frac{2a+\frac{b}{x}}{2\sqrt{a}\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}\right)-\frac{5\left(\frac{2c\left(12ac+b^2\right)}{x}+b\left(44ac+b^2\right)\right)\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}{64c}+x\left(a+\frac{b}{x}+\frac{c}{x^2}\right)^{5/2}-\frac{5}{24}\left(7b+\frac{6c}{x}\right)\left(a+\frac{b}{x}+\frac{c}{x^2}\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + c/x^2 + b/x)^(5/2),x]
```

```
[Out] (-5*(a + c/x^2 + b/x)^(3/2)*(7*b + (6*c)/x))/24 - (5*Sqrt[a + c/x^2 + b/x]*
(b*(b^2 + 44*a*c) + (2*c*(b^2 + 12*a*c))/x))/(64*c) + (a + c/x^2 + b/x)^(5/
2)*x + (5*a^(3/2)*b*ArcTanh[(2*a + b/x)/(2*Sqrt[a]*Sqrt[a + c/x^2 + b/x])])
/2 + (5*(b^4 - 24*a*b^2*c - 48*a^2*c^2)*ArcTanh[(b + (2*c)/x)/(2*Sqrt[c]*Sq
rt[a + c/x^2 + b/x])])/(128*c^(3/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 732

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 1)), x] - Di
st[p/(e*(m + 1)), Int[(d + e*x)^(m + 1)*(b + 2*c*x)*(a + b*x + c*x^2)^(p -
1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*
d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && GtQ[p, 0] && (IntegerQ[p]
 || LtQ[m, -1]) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && IntQuadraticQ[a,
 b, c, d, e, m, p, x]
```

Rule 814

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2
) - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*(a + b*x + c*x^2)^p)
/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
|| IntegersQ[2*m, 2*p])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1342

```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[
Int[(a + b/x^n + c/x^(2*n))^p/x^2, x], x, 1/x] /; FreeQ[{a, b, c, p}, x] &&
 EqQ[n2, 2*n] && ILtQ[n, 0]
```

Rubi steps

$$\int \left(a+\frac{c}{x^2}+\frac{b}{x}\right)^{5/2} dx = -\text{Subst}\left(\int \frac{\left(a+bx+cx^2\right)^{5/2}}{x^2} dx, x, \frac{1}{x}\right)$$

$$= \left(a+\frac{c}{x^2}+\frac{b}{x}\right)^{5/2} x - \frac{5}{2}\text{Subst}\left(\int \frac{(b+2cx)\left(a+bx+cx^2\right)^{3/2}}{x} dx, x, \frac{1}{x}\right)$$

$$= -\frac{5}{24}\left(a+\frac{c}{x^2}+\frac{b}{x}\right)^{3/2}\left(7b+\frac{6c}{x}\right)+\left(a+\frac{c}{x^2}+\frac{b}{x}\right)^{5/2} x + \frac{5\,\text{Subst}\left(\int \frac{\left(-8abc-c\left(b^2+12ac\right)x\right)\sqrt{a+bx}}{x}\right.}{16c}$$

$$= -\frac{5}{24}\left(a+\frac{c}{x^2}+\frac{b}{x}\right)^{3/2}\left(7b+\frac{6c}{x}\right)-\frac{5\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\left(b\left(b^2+44ac\right)+\frac{2c\left(b^2+12ac\right)}{x}\right)}{64c}+\left(a+\frac{c}{x^2}\right.$$

$$= -\frac{5}{24}\left(a+\frac{c}{x^2}+\frac{b}{x}\right)^{3/2}\left(7b+\frac{6c}{x}\right)-\frac{5\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\left(b\left(b^2+44ac\right)+\frac{2c\left(b^2+12ac\right)}{x}\right)}{64c}+\left(a+\frac{c}{x^2}\right.$$

$$= -\frac{5}{24}\left(a+\frac{c}{x^2}+\frac{b}{x}\right)^{3/2}\left(7b+\frac{6c}{x}\right)-\frac{5\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\left(b\left(b^2+44ac\right)+\frac{2c\left(b^2+12ac\right)}{x}\right)}{64c}+\left(a+\frac{c}{x^2}\right.$$

$$= -\frac{5}{24}\left(a+\frac{c}{x^2}+\frac{b}{x}\right)^{3/2}\left(7b+\frac{6c}{x}\right)-\frac{5\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\left(b\left(b^2+44ac\right)+\frac{2c\left(b^2+12ac\right)}{x}\right)}{64c}+\left(a+\frac{c}{x^2}\right.$$

Mathematica [A] time = 0.52, size = 213, normalized size = 1.04

$$\frac{\sqrt{a+\frac{bx+c}{x^2}}\left(960a^{3/2}bc^{3/2}x^4\tanh^{-1}\left(\frac{2ax+b}{2\sqrt{a}\sqrt{x(ax+b)+c}}\right)+15x^4\left(-48a^2c^2-24ab^2c+b^4\right)\tanh^{-1}\left(\frac{bx+2c}{2\sqrt{c}\sqrt{x(ax+b)+c}}\right)-2\sqrt{c}\sqrt{x(ax+b)+c}\left(2cx^2\left(-96a^2x^2+278abx+59b^2\right)+8c^2x(27ax+17b)+15b^3x^3+48c^3\right)\right)}{384c^{3/2}x^3\sqrt{x(ax+b)+c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + c/x^2 + b/x)^(5/2),x]
```

```
[Out] (Sqrt[a + (c + b*x)/x^2]*(-2*Sqrt[c]*Sqrt[c + x*(b + a*x)]*(48*c^3 + 15*b^3
*x^3 + 8*c^2*x*(17*b + 27*a*x) + 2*c*x^2*(59*b^2 + 278*a*b*x - 96*a^2*x^2))
 + 960*a^(3/2)*b*c^(3/2)*x^4*ArcTanh[(b + 2*a*x)/(2*Sqrt[a]*Sqrt[c + x*(b +
 a*x)])] + 15*(b^4 - 24*a*b^2*c - 48*a^2*c^2)*x^4*ArcTanh[(2*c + b*x)/(2*Sq
rt[c]*Sqrt[c + x*(b + a*x)])]))/(384*c^(3/2)*x^3*Sqrt[c + x*(b + a*x)])
```

IntegrateAlgebraic [A] time = 8.56, size = 209, normalized size = 1.02

$$\frac{x^5\left(a+\frac{bx+c}{x^2}\right)^{5/2}\left(-\frac{5}{2}a^{3/2}b\log\left(-2\sqrt{a}\sqrt{ax^2+bx+c}+2ax+b\right)+\frac{5\left(-48a^2c^2-24ab^2c+b^4\right)\tanh^{-1}\left(\frac{\sqrt{ax^2+bx+c}-\sqrt{a}x}{\sqrt{c}}\right)}{64c^{3/2}}+\frac{\sqrt{ax^2+bx+c}\left(192a^2cx^4-556abcx^3-216ac^2x^2-15b^3x^3-118b^2cx^2-136bc^2x-48c^3\right)}{192cx^4}\right)}{(x(ax+b)+c)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + c/x^2 + b/x)^(5/2),x]
```

```
[Out] (x^5*(a + (c + b*x)/x^2)^(5/2)*((Sqrt[c + b*x + a*x^2]*(-48*c^3 - 136*b*c^2
*x - 118*b^2*c*x^2 - 216*a*c^2*x^2 - 15*b^3*x^3 - 556*a*b*c*x^3 + 192*a^2*c
*x^4))/(192*c*x^4) + (5*(b^4 - 24*a*b^2*c - 48*a^2*c^2)*ArcTanh[(-(Sqrt[a]*
x) + Sqrt[c + b*x + a*x^2])/Sqrt[c]])/(64*c^(3/2)) - (5*a^(3/2)*b*Log[b + 2
*a*x - 2*Sqrt[a]*Sqrt[c + b*x + a*x^2]])/2))/(c + x*(b + a*x))^(5/2)
```

fricas [A] time = 1.59, size = 959, normalized size = 4.70

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+c/x^2+b/x)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/768*(960*a^(3/2)*b*c^2*x^3*log(-8*a^2*x^2 - 8*a*b*x - b^2 - 4*a*c - 4*(2
*a*x^2 + b*x)*sqrt(a)*sqrt((a*x^2 + b*x + c)/x^2)) - 15*(b^4 - 24*a*b^2*c -
 48*a^2*c^2)*sqrt(c)*x^3*log(-(8*b*c*x + (b^2 + 4*a*c)*x^2 + 8*c^2 - 4*(b*x
^2 + 2*c*x)*sqrt(c)*sqrt((a*x^2 + b*x + c)/x^2))/x^2) + 4*(192*a^2*c^2*x^4
- 136*b*c^3*x - 48*c^4 - (15*b^3*c + 556*a*b*c^2)*x^3 - 2*(59*b^2*c^2 + 108
*a*c^3)*x^2)*sqrt((a*x^2 + b*x + c)/x^2))/(c^2*x^3), -1/768*(1920*sqrt(-a)*
a*b*c^2*x^3*arctan(1/2*(2*a*x^2 + b*x)*sqrt(-a)*sqrt((a*x^2 + b*x + c)/x^2)
/(a^2*x^2 + a*b*x + a*c)) + 15*(b^4 - 24*a*b^2*c - 48*a^2*c^2)*sqrt(c)*x^3*
log(-(8*b*c*x + (b^2 + 4*a*c)*x^2 + 8*c^2 - 4*(b*x^2 + 2*c*x)*sqrt(c)*sqrt(
(a*x^2 + b*x + c)/x^2))/x^2) - 4*(192*a^2*c^2*x^4 - 136*b*c^3*x - 48*c^4 -
(15*b^3*c + 556*a*b*c^2)*x^3 - 2*(59*b^2*c^2 + 108*a*c^3)*x^2)*sqrt((a*x^2
+ b*x + c)/x^2))/(c^2*x^3), 1/384*(480*a^(3/2)*b*c^2*x^3*log(-8*a^2*x^2 - 8
*a*b*x - b^2 - 4*a*c - 4*(2*a*x^2 + b*x)*sqrt(a)*sqrt((a*x^2 + b*x + c)/x^2
)) - 15*(b^4 - 24*a*b^2*c - 48*a^2*c^2)*sqrt(-c)*x^3*arctan(1/2*(b*x^2 + 2*
c*x)*sqrt(-c)*sqrt((a*x^2 + b*x + c)/x^2)/(a*c*x^2 + b*c*x + c^2)) + 2*(192
*a^2*c^2*x^4 - 136*b*c^3*x - 48*c^4 - (15*b^3*c + 556*a*b*c^2)*x^3 - 2*(59*
b^2*c^2 + 108*a*c^3)*x^2)*sqrt((a*x^2 + b*x + c)/x^2))/(c^2*x^3), -1/384*(9
60*sqrt(-a)*a*b*c^2*x^3*arctan(1/2*(2*a*x^2 + b*x)*sqrt(-a)*sqrt((a*x^2 + b
*x + c)/x^2)/(a^2*x^2 + a*b*x + a*c)) + 15*(b^4 - 24*a*b^2*c - 48*a^2*c^2)*
sqrt(-c)*x^3*arctan(1/2*(b*x^2 + 2*c*x)*sqrt(-c)*sqrt((a*x^2 + b*x + c)/x^2
)/(a*c*x^2 + b*c*x + c^2)) - 2*(192*a^2*c^2*x^4 - 136*b*c^3*x - 48*c^4 - (1
5*b^3*c + 556*a*b*c^2)*x^3 - 2*(59*b^2*c^2 + 108*a*c^3)*x^2)*sqrt((a*x^2 +
b*x + c)/x^2))/(c^2*x^3)]
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+c/x^2+b/x)^(5/2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 0.02, size = 701, normalized size = 3.44

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+c/x^2+b/x)^(5/2),x)
```

```
[Out] 1/384*((a*x^2+b*x+c)/x^2)^(5/2)*x*(720*a^(7/2)*(a*x^2+b*x+c)^(1/2)*x^4*c^4+
6*a^(3/2)*(a*x^2+b*x+c)^(7/2)*x^3*b^3-6*a^(3/2)*(a*x^2+b*x+c)^(5/2)*x^4*b^4
-720*ln((2*c+b*x+2*c^(1/2)*(a*x^2+b*x+c)^(1/2))/x)*c^(9/2)*a^(7/2)*x^4-6*a^
(5/2)*(a*x^2+b*x+c)^(5/2)*x^5*b^3+144*a^(7/2)*(a*x^2+b*x+c)^(5/2)*x^4*c^2-1
44*a^(5/2)*(a*x^2+b*x+c)^(7/2)*x^2*c^2+240*a^(7/2)*(a*x^2+b*x+c)^(3/2)*x^4*
c^3-360*ln((2*c+b*x+2*c^(1/2)*(a*x^2+b*x+c)^(1/2))/x)*c^(7/2)*a^(5/2)*x^4*b
^2+660*a^(5/2)*(a*x^2+b*x+c)^(1/2)*x^4*b^2*c^3+960*ln(1/2*(2*(a*x^2+b*x+c)^
(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*a^3*x^4*b*c^4+148*a^(5/2)*(a*x^2+b*x+c)^(5/
2)*x^4*b^2*c+280*a^(7/2)*(a*x^2+b*x+c)^(3/2)*x^5*b*c^2-10*a^(5/2)*(a*x^2+b*
x+c)^(3/2)*x^5*b^3*c+15*ln((2*c+b*x+2*c^(1/2)*(a*x^2+b*x+c)^(1/2))/x)*c^(5/
2)*a^(3/2)*x^4*b^4+260*a^(5/2)*(a*x^2+b*x+c)^(3/2)*x^4*b^2*c^2+600*a^(7/2)*
(a*x^2+b*x+c)^(1/2)*x^5*b*c^3-30*a^(5/2)*(a*x^2+b*x+c)^(1/2)*x^5*b^3*c^2+15
2*a^(7/2)*(a*x^2+b*x+c)^(5/2)*x^5*b*c-152*a^(5/2)*(a*x^2+b*x+c)^(7/2)*x^3*b
*c+4*a^(3/2)*(a*x^2+b*x+c)^(7/2)*x^2*b^2*c-10*a^(3/2)*(a*x^2+b*x+c)^(3/2)*x
^4*b^4*c+16*a^(3/2)*(a*x^2+b*x+c)^(7/2)*x*b*c^2-30*a^(3/2)*(a*x^2+b*x+c)^(1
/2)*x^4*b^4*c^2-96*(a*x^2+b*x+c)^(7/2)*c^3*a^(3/2))/(a*x^2+b*x+c)^(5/2)/c^4
/a^(3/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + \frac{b}{x} + \frac{c}{x^2}\right)^{\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+c/x^2+b/x)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((a + b/x + c/x^2)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int {\left(a+\frac{b}{x}+\frac{c}{x^2}\right)}^{5/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/x + c/x^2)^(5/2),x)
```

```
[Out] int((a + b/x + c/x^2)^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + \frac{b}{x} + \frac{c}{x^{2}}\right)^{\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+c/x**2+b/x)**(5/2),x)
```

```
[Out] Integral((a + b/x + c/x**2)**(5/2), x)
```

3.389 $\int \left(a+\frac{c}{x^2}+\frac{b}{x}\right)^{3/2} dx$

Optimal. Leaf size=145

$$-\frac{3\left(4ac+b^2\right)\tanh^{-1}\left(\frac{b+\frac{2c}{x}}{2\sqrt{c}\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}\right)}{8\sqrt{c}}+x\left(a+\frac{b}{x}+\frac{c}{x^2}\right)^{3/2}-\frac{3}{4}\left(3b+\frac{2c}{x}\right)\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}+\frac{3}{2}\sqrt{a}\,b\tanh^{-1}\left(\frac{2a+}{2\sqrt{a}\sqrt{a+}}\right.$$

Rubi [A] time = 0.13, antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {1342, 732, 814, 843, 621, 206, 724}

$$-\frac{3\left(4ac+b^2\right)\tanh^{-1}\left(\frac{b+\frac{2c}{x}}{2\sqrt{c}\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}\right)}{8\sqrt{c}}+x\left(a+\frac{b}{x}+\frac{c}{x^2}\right)^{3/2}-\frac{3}{4}\left(3b+\frac{2c}{x}\right)\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}+\frac{3}{2}\sqrt{a}\,b\tanh^{-1}\left(\frac{2a+\frac{b}{x}}{2\sqrt{a}\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(a + c/x^2 + b/x)^(3/2),x]
```

```
[Out] (-3*Sqrt[a + c/x^2 + b/x]*(3*b + (2*c)/x))/4 + (a + c/x^2 + b/x)^(3/2)*x +
(3*Sqrt[a]*b*ArcTanh[(2*a + b/x)/(2*Sqrt[a]*Sqrt[a + c/x^2 + b/x])])/2 - (3
*(b^2 + 4*a*c)*ArcTanh[(b + (2*c)/x)/(2*Sqrt[c]*Sqrt[a + c/x^2 + b/x])])/(8
*Sqrt[c])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 732

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 1)), x] - Di
st[p/(e*(m + 1)), Int[(d + e*x)^(m + 1)*(b + 2*c*x)*(a + b*x + c*x^2)^(p -
1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*
d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && GtQ[p, 0] && (IntegerQ[p]
 || LtQ[m, -1]) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && IntQuadraticQ[a,
 b, c, d, e, m, p, x]
```

Rule 814

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2
) - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*(a + b*x + c*x^2)^p)
/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
|| IntegersQ[2*m, 2*p])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1342

```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[
Int[(a + b/x^n + c/x^(2*n))^p/x^2, x], x, 1/x] /; FreeQ[{a, b, c, p}, x] &&
 EqQ[n2, 2*n] && ILtQ[n, 0]
```

Rubi steps

$$\int \left(a+\frac{c}{x^2}+\frac{b}{x}\right)^{3/2} dx = -\text{Subst}\left(\int \frac{\left(a+bx+cx^2\right)^{3/2}}{x^2} dx, x, \frac{1}{x}\right)$$

$$= \left(a+\frac{c}{x^2}+\frac{b}{x}\right)^{3/2} x - \frac{3}{2}\text{Subst}\left(\int \frac{(b+2cx)\sqrt{a+bx+cx^2}}{x} dx, x, \frac{1}{x}\right)$$

$$= -\frac{3}{4}\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\left(3b+\frac{2c}{x}\right)+\left(a+\frac{c}{x^2}+\frac{b}{x}\right)^{3/2} x + \frac{3\,\text{Subst}\left(\int \frac{-4abc-c\left(b^2+4ac\right)x}{x\sqrt{a+bx+cx^2}} dx, x, \frac{1}{x}\right.}{8c}$$

$$= -\frac{3}{4}\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\left(3b+\frac{2c}{x}\right)+\left(a+\frac{c}{x^2}+\frac{b}{x}\right)^{3/2} x - \frac{1}{2}(3ab)\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx+cx^2}}\right.$$

$$= -\frac{3}{4}\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\left(3b+\frac{2c}{x}\right)+\left(a+\frac{c}{x^2}+\frac{b}{x}\right)^{3/2} x + (3ab)\,\text{Subst}\left(\int \frac{1}{4a-x^2} dx, x, \frac{}{\sqrt{a}}\right.$$

$$= -\frac{3}{4}\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\left(3b+\frac{2c}{x}\right)+\left(a+\frac{c}{x^2}+\frac{b}{x}\right)^{3/2} x + \frac{3}{2}\sqrt{a}\,b\tanh^{-1}\left(\frac{2a+\frac{b}{x}}{2\sqrt{a}\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}}\right.$$

Mathematica [A] time = 0.25, size = 163, normalized size = 1.12

$$\frac{\sqrt{a+\frac{bx+c}{x^2}}\left(-3x^2\left(4ac+b^2\right)\tanh^{-1}\left(\frac{bx+2c}{2\sqrt{c}\sqrt{x(ax+b)+c}}\right)+12\sqrt{a}\,b\sqrt{c}\,x^2\tanh^{-1}\left(\frac{2ax+b}{2\sqrt{a}\sqrt{x(ax+b)+c}}\right)-2\sqrt{c}\,(x(5b-4ax)+2c)\sqrt{x(ax+b)+c}\right)}{8\sqrt{c}\,x\sqrt{x(ax+b)+c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + c/x^2 + b/x)^(3/2),x]
```

```
[Out] (Sqrt[a + (c + b*x)/x^2]*(-2*Sqrt[c]*(2*c + x*(5*b - 4*a*x))*Sqrt[c + x*(b
+ a*x)] + 12*Sqrt[a]*b*Sqrt[c]*x^2*ArcTanh[(b + 2*a*x)/(2*Sqrt[a]*Sqrt[c +
x*(b + a*x)])] - 3*(b^2 + 4*a*c)*x^2*ArcTanh[(2*c + b*x)/(2*Sqrt[c]*Sqrt[c
+ x*(b + a*x)])]))/(8*Sqrt[c]*x*Sqrt[c + x*(b + a*x)])
```

IntegrateAlgebraic [A] time = 7.17, size = 154, normalized size = 1.06

$$\frac{x^3\left(a+\frac{bx+c}{x^2}\right)^{3/2}\left(\frac{3\left(4ac+b^2\right)\tanh^{-1}\left(\frac{\sqrt{a}\,x-\sqrt{ax^2+bx+c}}{\sqrt{c}}\right)}{4\sqrt{c}}+\frac{\sqrt{ax^2+bx+c}\left(4ax^2-5bx-2c\right)}{4x^2}-\frac{3}{2}\sqrt{a}\,b\log\left(-2\sqrt{a}\sqrt{ax^2+bx+c}+2ax+b\right)\right)}{(x(ax+b)+c)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + c/x^2 + b/x)^(3/2),x]
```

```
[Out] (x^3*(a + (c + b*x)/x^2)^(3/2)*((Sqrt[c + b*x + a*x^2]*(-2*c - 5*b*x + 4*a*
x^2))/(4*x^2) + (3*(b^2 + 4*a*c)*ArcTanh[(Sqrt[a]*x - Sqrt[c + b*x + a*x^2]
)/Sqrt[c]])/(4*Sqrt[c]) - (3*Sqrt[a]*b*Log[b + 2*a*x - 2*Sqrt[a]*Sqrt[c + b
*x + a*x^2]])/2))/(c + x*(b + a*x))^(3/2)
```

fricas [A] time = 1.36, size = 709, normalized size = 4.89

$$\left[\frac{12\sqrt{a}bcx\log\left(-8a^2x^2-8abx-b^2-4ac-4\left(2ax^2+bx\right)\sqrt{a}\sqrt{\frac{ax^2+bx+c}{x^2}}\right)+3\left(b^2+4ac\right)\sqrt{c}x\log\left(-\frac{8bcx+\left(b^2+4ac\right)x^2+8c^2-4\left(bx^2+2cx\right)\sqrt{c}\sqrt{\frac{ax^2+bx+c}{x^2}}}{x^2}\right)+4\left(4acx^2-5bcx-2c^2\right)\sqrt{\frac{ax^2+bx+c}{x^2}}}{16cx},\ -\frac{24\sqrt{-a}bcx\arctan\left(\frac{\left(2ax^2+bx\right)\sqrt{-a}\sqrt{\frac{ax^2+bx+c}{x^2}}}{2\left(a^2x^2+abx+ac\right)}\right)-3\left(b^2+4ac\right)\sqrt{c}x\log\left(-\frac{8bcx+\left(b^2+4ac\right)x^2+8c^2-4\left(bx^2+2cx\right)\sqrt{c}\sqrt{\frac{ax^2+bx+c}{x^2}}}{x^2}\right)-4\left(4acx^2-5bcx-2c^2\right)\sqrt{\frac{ax^2+bx+c}{x^2}}}{16cx},\ \frac{6\sqrt{a}bcx\log\left(-8a^2x^2-8abx-b^2-4ac-4\left(2ax^2+bx\right)\sqrt{a}\sqrt{\frac{ax^2+bx+c}{x^2}}\right)+3\left(b^2+4ac\right)\sqrt{-c}x\arctan\left(\frac{\left(bx^2+2cx\right)\sqrt{-c}\sqrt{\frac{ax^2+bx+c}{x^2}}}{2\left(acx^2+bcx+c^2\right)}\right)+2\left(4acx^2-5bcx-2c^2\right)\sqrt{\frac{ax^2+bx+c}{x^2}}}{8cx},\ -\frac{12\sqrt{-a}bcx\arctan\left(\frac{\left(2ax^2+bx\right)\sqrt{-a}\sqrt{\frac{ax^2+bx+c}{x^2}}}{2\left(a^2x^2+abx+ac\right)}\right)-3\left(b^2+4ac\right)\sqrt{-c}x\arctan\left(\frac{\left(bx^2+2cx\right)\sqrt{-c}\sqrt{\frac{ax^2+bx+c}{x^2}}}{2\left(acx^2+bcx+c^2\right)}\right)-2\left(4acx^2-5bcx-2c^2\right)\sqrt{\frac{ax^2+bx+c}{x^2}}}{8cx}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+c/x^2+b/x)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/16*(12*sqrt(a)*b*c*x*log(-8*a^2*x^2 - 8*a*b*x - b^2 - 4*a*c - 4*(2*a*x^2
 + b*x)*sqrt(a)*sqrt((a*x^2 + b*x + c)/x^2)) + 3*(b^2 + 4*a*c)*sqrt(c)*x*lo
g(-(8*b*c*x + (b^2 + 4*a*c)*x^2 + 8*c^2 - 4*(b*x^2 + 2*c*x)*sqrt(c)*sqrt((a
*x^2 + b*x + c)/x^2))/x^2) + 4*(4*a*c*x^2 - 5*b*c*x - 2*c^2)*sqrt((a*x^2 +
b*x + c)/x^2))/(c*x), -1/16*(24*sqrt(-a)*b*c*x*arctan(1/2*(2*a*x^2 + b*x)*s
qrt(-a)*sqrt((a*x^2 + b*x + c)/x^2)/(a^2*x^2 + a*b*x + a*c)) - 3*(b^2 + 4*a
*c)*sqrt(c)*x*log(-(8*b*c*x + (b^2 + 4*a*c)*x^2 + 8*c^2 - 4*(b*x^2 + 2*c*x)
*sqrt(c)*sqrt((a*x^2 + b*x + c)/x^2))/x^2) - 4*(4*a*c*x^2 - 5*b*c*x - 2*c^2
)*sqrt((a*x^2 + b*x + c)/x^2))/(c*x), 1/8*(6*sqrt(a)*b*c*x*log(-8*a^2*x^2 -
 8*a*b*x - b^2 - 4*a*c - 4*(2*a*x^2 + b*x)*sqrt(a)*sqrt((a*x^2 + b*x + c)/x
^2)) + 3*(b^2 + 4*a*c)*sqrt(-c)*x*arctan(1/2*(b*x^2 + 2*c*x)*sqrt(-c)*sqrt(
(a*x^2 + b*x + c)/x^2)/(a*c*x^2 + b*c*x + c^2)) + 2*(4*a*c*x^2 - 5*b*c*x -
2*c^2)*sqrt((a*x^2 + b*x + c)/x^2))/(c*x), -1/8*(12*sqrt(-a)*b*c*x*arctan(1
/2*(2*a*x^2 + b*x)*sqrt(-a)*sqrt((a*x^2 + b*x + c)/x^2)/(a^2*x^2 + a*b*x +
a*c)) - 3*(b^2 + 4*a*c)*sqrt(-c)*x*arctan(1/2*(b*x^2 + 2*c*x)*sqrt(-c)*sqrt
((a*x^2 + b*x + c)/x^2)/(a*c*x^2 + b*c*x + c^2)) - 2*(4*a*c*x^2 - 5*b*c*x -
 2*c^2)*sqrt((a*x^2 + b*x + c)/x^2))/(c*x)]
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+c/x^2+b/x)^(3/2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 0.01, size = 334, normalized size = 2.30

$$-\frac{\left(\frac{ax^2+bx+c}{x^2}\right)^{\frac{3}{2}}\left(12a^{\frac{5}{2}}c^{\frac{5}{2}}x^2\ln\left(\frac{bx+2c+2\sqrt{ax^2+bx+c}\,\sqrt{c}}{x}\right)-12a^2bc^2x^2\ln\left(\frac{2ax+b+2\sqrt{ax^2+bx+c}\,\sqrt{a}}{2\sqrt{a}}\right)+3a^{\frac{3}{2}}b^2c^{\frac{3}{2}}x^2\ln\left(\frac{bx+2c+2\sqrt{ax^2+bx+c}\,\sqrt{c}}{x}\right)-6\sqrt{ax^2+bx+c}\,a^{\frac{5}{2}}bcx^3-2\left(ax^2+bx+c\right)^{\frac{3}{2}}a^{\frac{5}{2}}bx^3-12\sqrt{ax^2+bx+c}\,a^{\frac{5}{2}}c^2x^2-6\sqrt{ax^2+bx+c}\,a^{\frac{3}{2}}b^2cx^2-4\left(ax^2+bx+c\right)^{\frac{3}{2}}a^{\frac{5}{2}}cx^2-2\left(ax^2+bx+c\right)^{\frac{3}{2}}a^{\frac{3}{2}}b^2x^2+2\left(ax^2+bx+c\right)^{\frac{5}{2}}a^{\frac{3}{2}}bx+4\left(ax^2+bx+c\right)^{\frac{5}{2}}a^{\frac{3}{2}}c\right)x}{8\left(ax^2+bx+c\right)^{\frac{3}{2}}a^{\frac{3}{2}}c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+c/x^2+b/x)^(3/2),x)
```

```
[Out] -1/8*((a*x^2+b*x+c)/x^2)^(3/2)*x*(12*a^(5/2)*c^(5/2)*ln((b*x+2*c+2*(a*x^2+b
*x+c)^(1/2)*c^(1/2))/x)*x^2-2*a^(5/2)*(a*x^2+b*x+c)^(3/2)*x^3*b-4*a^(5/2)*(
a*x^2+b*x+c)^(3/2)*x^2*c-6*a^(5/2)*(a*x^2+b*x+c)^(1/2)*x^3*b*c+3*a^(3/2)*c^
(3/2)*ln((b*x+2*c+2*(a*x^2+b*x+c)^(1/2)*c^(1/2))/x)*x^2*b^2-12*a^(5/2)*(a*x
^2+b*x+c)^(1/2)*x^2*c^2+2*a^(3/2)*(a*x^2+b*x+c)^(5/2)*x*b-2*a^(3/2)*(a*x^2+
b*x+c)^(3/2)*x^2*b^2+4*(a*x^2+b*x+c)^(5/2)*c*a^(3/2)-6*a^(3/2)*(a*x^2+b*x+c
)^(1/2)*x^2*b^2*c-12*a^2*ln(1/2*(2*a*x+b+2*(a*x^2+b*x+c)^(1/2)*a^(1/2))/a^(
1/2))*x^2*b*c^2)/(a*x^2+b*x+c)^(3/2)/c^2/a^(3/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+\frac{b}{x}+\frac{c}{x^2}\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+c/x^2+b/x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a + b/x + c/x^2)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(a+\frac{b}{x}+\frac{c}{x^2}\right)}^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/x + c/x^2)^(3/2),x)
```

```
[Out] int((a + b/x + c/x^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+\frac{b}{x}+\frac{c}{x^2}\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+c/x**2+b/x)**(3/2),x)
```

```
[Out] Integral((a + b/x + c/x**2)**(3/2), x)
```

3.390 $\int \sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\,dx$

Optimal. Leaf size=105

$$x\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}+\frac{b\tanh^{-1}\left(\frac{2a+\frac{b}{x}}{2\sqrt{a}\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}\right)}{2\sqrt{a}}-\sqrt{c}\tanh^{-1}\left(\frac{b+\frac{2c}{x}}{2\sqrt{c}\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}\right)$$

Rubi [A] time = 0.08, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {1342, 732, 843, 621, 206, 724}

$$x\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}+\frac{b\tanh^{-1}\left(\frac{2a+\frac{b}{x}}{2\sqrt{a}\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}\right)}{2\sqrt{a}}-\sqrt{c}\tanh^{-1}\left(\frac{b+\frac{2c}{x}}{2\sqrt{c}\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + c/x^2 + b/x],x]
```

```
[Out] Sqrt[a + c/x^2 + b/x]*x + (b*ArcTanh[(2*a + b/x)/(2*Sqrt[a]*Sqrt[a + c/x^2
+ b/x])])/(2*Sqrt[a]) - Sqrt[c]*ArcTanh[(b + (2*c)/x)/(2*Sqrt[c]*Sqrt[a + c
/x^2 + b/x])]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 732

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 1)), x] - Di
st[p/(e*(m + 1)), Int[(d + e*x)^(m + 1)*(b + 2*c*x)*(a + b*x + c*x^2)^(p -
1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*
d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && GtQ[p, 0] && (IntegerQ[p]
 || LtQ[m, -1]) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && IntQuadraticQ[a,
 b, c, d, e, m, p, x]
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1342

```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[
Int[(a + b/x^n + c/x^(2*n))^p/x^2, x], x, 1/x] /; FreeQ[{a, b, c, p}, x] &&
 EqQ[n2, 2*n] && ILtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\int \sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\,dx &= -\text{Subst}\left(\int \frac{\sqrt{a+bx+cx^2}}{x^2}\,dx, x, \frac{1}{x}\right)\\
&= \sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\,x-\frac{1}{2}\text{Subst}\left(\int \frac{b+2cx}{x\sqrt{a+bx+cx^2}}\,dx, x, \frac{1}{x}\right)\\
&= \sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\,x-\frac{1}{2}b\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx+cx^2}}\,dx, x, \frac{1}{x}\right)-c\,\text{Subst}\left(\int \frac{1}{\sqrt{a+bx+cx^2}}\right.\\
&= \sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\,x+b\,\text{Subst}\left(\int \frac{1}{4a-x^2}\,dx, x, \frac{2a+\frac{b}{x}}{\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}}\right)-(2c)\,\text{Subst}\left(\int \frac{1}{4c-x^2}\,dx\right.\\
&= \sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\,x+\frac{b\tanh^{-1}\left(\frac{2a+\frac{b}{x}}{2\sqrt{a}\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}}\right)}{2\sqrt{a}}-\sqrt{c}\tanh^{-1}\left(\frac{b+\frac{2c}{x}}{2\sqrt{c}\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}}\right)
\end{aligned}$$

Mathematica [A] time = 0.08, size = 128, normalized size = 1.22

$$\frac{x\sqrt{a+\frac{bx+c}{x^2}}\left(b\tanh^{-1}\left(\frac{2ax+b}{2\sqrt{a}\sqrt{x(ax+b)+c}}\right)+2\sqrt{a}\left(\sqrt{x(ax+b)+c}-\sqrt{c}\tanh^{-1}\left(\frac{bx+2c}{2\sqrt{c}\sqrt{x(ax+b)+c}}\right)\right)\right)}{2\sqrt{a}\sqrt{x(ax+b)+c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + c/x^2 + b/x],x]
```

```
[Out] (x*Sqrt[a + (c + b*x)/x^2]*(b*ArcTanh[(b + 2*a*x)/(2*Sqrt[a]*Sqrt[c + x*(b
+ a*x)])] + 2*Sqrt[a]*(Sqrt[c + x*(b + a*x)] - Sqrt[c]*ArcTanh[(2*c + b*x)/
(2*Sqrt[c]*Sqrt[c + x*(b + a*x)])])))/(2*Sqrt[a]*Sqrt[c + x*(b + a*x)])
```

IntegrateAlgebraic [A] time = 5.60, size = 125, normalized size = 1.19

$$\frac{x\sqrt{a+\frac{bx+c}{x^2}}\left(\sqrt{ax^2+bx+c}-\frac{b\log\left(-2\sqrt{a}\sqrt{ax^2+bx+c}+2ax+b\right)}{2\sqrt{a}}+2\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{c}}-\frac{\sqrt{ax^2+bx+c}}{\sqrt{c}}\right)\right)}{\sqrt{x(ax+b)+c}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[a + c/x^2 + b/x],x]
```

```
[Out] (x*Sqrt[a + (c + b*x)/x^2]*(Sqrt[c + b*x + a*x^2] + 2*Sqrt[c]*ArcTanh[(Sqrt
[a]*x)/Sqrt[c] - Sqrt[c + b*x + a*x^2]/Sqrt[c]] - (b*Log[b + 2*a*x - 2*Sqrt
[a]*Sqrt[c + b*x + a*x^2]])/(2*Sqrt[a]))))/Sqrt[c + x*(b + a*x)]
```

fricas [A] time = 1.25, size = 590, normalized size = 5.62

$$\left[\frac{4\,ax\sqrt{\frac{ax^2+bx+c}{x^2}} + \sqrt{a}\,b\log\left(-8\,a^2x^2 - 8\,abx - b^2 - 4\,ac - 4\left(2\,ax^2 + bx\right)\sqrt{a}\sqrt{\frac{ax^2+bx+c}{x^2}}\right) + 2\,a\sqrt{c}\,\log\left(-\frac{8\,bcx + \left(b^2 + 4\,ac\right)x^2 + 8\,c^2 - 4\left(bx^2 + 2\,cx\right)\sqrt{c}\sqrt{\frac{ax^2+bx+c}{x^2}}}{x^2}\right)}{4\,a}, \frac{2\,ax\sqrt{\frac{ax^2+bx+c}{x^2}} - \sqrt{-a}\,b\arctan\left(\frac{\left(2\,ax^2+bx\right)\sqrt{-a}\sqrt{\frac{ax^2+bx+c}{x^2}}}{2\left(a^2x^2+abx+ac\right)}\right) + a\sqrt{c}\,\log\left(-\frac{8\,bcx + \left(b^2 + 4\,ac\right)x^2 + 8\,c^2 - 4\left(bx^2 + 2\,cx\right)\sqrt{c}\sqrt{\frac{ax^2+bx+c}{x^2}}}{x^2}\right)}{2\,a}, \frac{4\,ax\sqrt{\frac{ax^2+bx+c}{x^2}} + 4\,a\sqrt{-c}\,\arctan\left(\frac{\left(bx^2+2\,cx\right)\sqrt{-c}\sqrt{\frac{ax^2+bx+c}{x^2}}}{2\left(acx^2+bcx+c^2\right)}\right) + \sqrt{a}\,b\log\left(-8\,a^2x^2 - 8\,abx - b^2 - 4\,ac - 4\left(2\,ax^2 + bx\right)\sqrt{a}\sqrt{\frac{ax^2+bx+c}{x^2}}\right)}{4\,a}, \frac{2\,ax\sqrt{\frac{ax^2+bx+c}{x^2}} - \sqrt{-a}\,b\arctan\left(\frac{\left(2\,ax^2+bx\right)\sqrt{-a}\sqrt{\frac{ax^2+bx+c}{x^2}}}{2\left(a^2x^2+abx+ac\right)}\right) + 2\,a\sqrt{-c}\,\arctan\left(\frac{\left(bx^2+2\,cx\right)\sqrt{-c}\sqrt{\frac{ax^2+bx+c}{x^2}}}{2\left(acx^2+bcx+c^2\right)}\right)}{2\,a}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+c/x^2+b/x)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(4*a*x*sqrt((a*x^2 + b*x + c)/x^2) + sqrt(a)*b*log(-8*a^2*x^2 - 8*a*b*
x - b^2 - 4*a*c - 4*(2*a*x^2 + b*x)*sqrt(a)*sqrt((a*x^2 + b*x + c)/x^2)) +
2*a*sqrt(c)*log(-(8*b*c*x + (b^2 + 4*a*c)*x^2 + 8*c^2 - 4*(b*x^2 + 2*c*x)*s
qrt(c)*sqrt((a*x^2 + b*x + c)/x^2))/x^2))/a, 1/2*(2*a*x*sqrt((a*x^2 + b*x +
 c)/x^2) - sqrt(-a)*b*arctan(1/2*(2*a*x^2 + b*x)*sqrt(-a)*sqrt((a*x^2 + b*x
 + c)/x^2)/(a^2*x^2 + a*b*x + a*c)) + a*sqrt(c)*log(-(8*b*c*x + (b^2 + 4*a*
c)*x^2 + 8*c^2 - 4*(b*x^2 + 2*c*x)*sqrt(c)*sqrt((a*x^2 + b*x + c)/x^2))/x^2
))/a, 1/4*(4*a*x*sqrt((a*x^2 + b*x + c)/x^2) + 4*a*sqrt(-c)*arctan(1/2*(b*x
^2 + 2*c*x)*sqrt(-c)*sqrt((a*x^2 + b*x + c)/x^2)/(a*c*x^2 + b*c*x + c^2)) +
 sqrt(a)*b*log(-8*a^2*x^2 - 8*a*b*x - b^2 - 4*a*c - 4*(2*a*x^2 + b*x)*sqrt(
a)*sqrt((a*x^2 + b*x + c)/x^2)))/a, 1/2*(2*a*x*sqrt((a*x^2 + b*x + c)/x^2)
- sqrt(-a)*b*arctan(1/2*(2*a*x^2 + b*x)*sqrt(-a)*sqrt((a*x^2 + b*x + c)/x^2
)/(a^2*x^2 + a*b*x + a*c)) + 2*a*sqrt(-c)*arctan(1/2*(b*x^2 + 2*c*x)*sqrt(-
c)*sqrt((a*x^2 + b*x + c)/x^2)/(a*c*x^2 + b*c*x + c^2)))/a]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+c/x^2+b/x)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(x)]
index.cc index_m operator + Error: Bad Argument Value
```

maple [A] time = 0.01, size = 121, normalized size = 1.15

$$\frac{\sqrt{\frac{a\,x^2+bx+c}{x^2}}\,\left(-2\sqrt{a}\,\sqrt{c}\,\ln\left(\frac{bx+2c+2\sqrt{a\,x^2+bx+c}\,\sqrt{c}}{x}\right)+b\ln\left(\frac{2ax+b+2\sqrt{a\,x^2+bx+c}\,\sqrt{a}}{2\sqrt{a}}\right)+2\sqrt{a\,x^2+bx+c}\,\sqrt{a}\right)x}{2\sqrt{a\,x^2+bx+c}\,\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+c/x^2+b/x)^(1/2),x)
```

```
[Out] 1/2*((a*x^2+b*x+c)/x^2)^(1/2)*x*(-2*c^(1/2)*ln((b*x+2*c+2*(a*x^2+b*x+c)^(1/
2)*c^(1/2))/x)*a^(1/2)+b*ln(1/2*(2*a*x+b+2*(a*x^2+b*x+c)^(1/2)*a^(1/2))/a^(
1/2))+2*(a*x^2+b*x+c)^(1/2)*a^(1/2))/(a*x^2+b*x+c)^(1/2)/a^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a + \frac{b}{x} + \frac{c}{x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+c/x^2+b/x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a + b/x + c/x^2), x)
```

mupad [B] time = 0.13, size = 100, normalized size = 0.95

$$x\sqrt{\frac{1}{x^2}}\sqrt{a\,x^2+b\,x+c}-\sqrt{c}\,x\ln\left(\frac{2\,c+2\,\sqrt{c}\,\sqrt{a\,x^2+b\,x+c}+b\,x}{x}\right)\sqrt{\frac{1}{x^2}}+\frac{b\,x\ln\left(\frac{\frac{b}{2}+\sqrt{a}\,\sqrt{a\,x^2+b\,x+c}+a\,x}{\sqrt{a}}\right)\sqrt{\frac{1}{x^2}}}{2\,\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/x + c/x^2)^(1/2),x)
```

```
[Out] x*(1/x^2)^(1/2)*(c + b*x + a*x^2)^(1/2) - c^(1/2)*x*log((2*c + 2*c^(1/2)*(c
 + b*x + a*x^2)^(1/2) + b*x)/x)*(1/x^2)^(1/2) + (b*x*log((b/2 + a^(1/2)*(c
+ b*x + a*x^2)^(1/2) + a*x)/a^(1/2))*(1/x^2)^(1/2))/(2*a^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+c/x**2+b/x)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b/x + c/x**2), x)
```

3.391 $\int \frac{1}{\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}}\,dx$

Optimal. Leaf size=67

$$\frac{x\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}{a}-\frac{b\tanh^{-1}\left(\frac{2a+\frac{b}{x}}{2\sqrt{a}\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}\right)}{2a^{3/2}}$$

Rubi [A] time = 0.04, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1342, 730, 724, 206}

$$\frac{x\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}{a}-\frac{b\tanh^{-1}\left(\frac{2a+\frac{b}{x}}{2\sqrt{a}\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}\right)}{2a^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a + c/x^2 + b/x],x]
```

```
[Out] (Sqrt[a + c/x^2 + b/x]*x)/a - (b*ArcTanh[(2*a + b/x)/(2*Sqrt[a]*Sqrt[a + c/
x^2 + b/x])])/(2*a^(3/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 730

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 - b*d*e + a*e^2)), x] + Dist[(2*c*d - b*e)/(2*(c*d^2 - b*d*e + a*e^2)),
 Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e,
 m, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2
*c*d - b*e, 0] && EqQ[m + 2*p + 3, 0]
```

Rule 1342

```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[
Int[(a + b/x^n + c/x^(2*n))^p/x^2, x], x, 1/x] /; FreeQ[{a, b, c, p}, x] &&
 EqQ[n2, 2*n] && ILtQ[n, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}}\,dx = -\text{Subst}\left(\int \frac{1}{x^2\sqrt{a+bx+cx^2}}\,dx, x, \frac{1}{x}\right)$$
$$= \frac{\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\,x}{a} + \frac{b\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx+cx^2}}\,dx, x, \frac{1}{x}\right)}{2a}$$
$$= \frac{\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\,x}{a} - \frac{b\,\text{Subst}\left(\int \frac{1}{4a-x^2}\,dx, x, \frac{2a+\frac{b}{x}}{\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}}\right)}{a}$$
$$= \frac{\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\,x}{a} - \frac{b\tanh^{-1}\left(\frac{2a+\frac{b}{x}}{2\sqrt{a}\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}}\right)}{2a^{3/2}}$$

Mathematica [A] time = 0.06, size = 89, normalized size = 1.33

$$\frac{2\sqrt{a}\,(x(ax+b)+c) - b\sqrt{x(ax+b)+c}\,\tanh^{-1}\left(\frac{2ax+b}{2\sqrt{a}\,\sqrt{x(ax+b)+c}}\right)}{2a^{3/2}x\sqrt{a+\frac{bx+c}{x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a + c/x^2 + b/x],x]
```

```
[Out] (2*Sqrt[a]*(c + x*(b + a*x)) - b*Sqrt[c + x*(b + a*x)]*ArcTanh[(b + 2*a*x)/
(2*Sqrt[a]*Sqrt[c + x*(b + a*x)])])/(2*a^(3/2)*x*Sqrt[a + (c + b*x)/x^2])
```

IntegrateAlgebraic [A] time = 4.70, size = 93, normalized size = 1.39

$$\frac{\sqrt{x(ax+b)+c}\left(\frac{b\log\left(-2a^{3/2}\sqrt{ax^2+bx+c}+2a^2x+ab\right)}{2a^{3/2}}+\frac{\sqrt{ax^2+bx+c}}{a}\right)}{x\sqrt{a+\frac{bx+c}{x^2}}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/Sqrt[a + c/x^2 + b/x],x]
```

```
[Out] (Sqrt[c + x*(b + a*x)]*(Sqrt[c + b*x + a*x^2]/a + (b*Log[a*b + 2*a^2*x - 2*
a^(3/2)*Sqrt[c + b*x + a*x^2]])/(2*a^(3/2))))/(x*Sqrt[a + (c + b*x)/x^2])
```

fricas [A] time = 1.29, size = 171, normalized size = 2.55

$$\left[\frac{4\,ax\sqrt{\frac{ax^2+bx+c}{x^2}}+\sqrt{a}\,b\log\left(-8\,a^2x^2-8\,abx-b^2-4\,ac+4\left(2\,ax^2+bx\right)\sqrt{a}\,\sqrt{\frac{ax^2+bx+c}{x^2}}\right)}{4\,a^2}, \frac{2\,ax\sqrt{\frac{ax^2+bx+c}{x^2}}+\sqrt{-a}\,b\arctan\left(\frac{\left(2\,ax^2+bx\right)\sqrt{-a}\,\sqrt{\frac{ax^2+bx+c}{x^2}}}{2\left(a^2x^2+abx+ac\right)}\right)}{2\,a^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+c/x^2+b/x)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(4*a*x*sqrt((a*x^2 + b*x + c)/x^2) + sqrt(a)*b*log(-8*a^2*x^2 - 8*a*b*
x - b^2 - 4*a*c + 4*(2*a*x^2 + b*x)*sqrt(a)*sqrt((a*x^2 + b*x + c)/x^2)))/a
^2, 1/2*(2*a*x*sqrt((a*x^2 + b*x + c)/x^2) + sqrt(-a)*b*arctan(1/2*(2*a*x^2
 + b*x)*sqrt(-a)*sqrt((a*x^2 + b*x + c)/x^2)/(a^2*x^2 + a*b*x + a*c)))/a^2]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+c/x^2+b/x)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(x)]
index.cc index_m operator + Error: Bad Argument Value
```

maple [A] time = 0.00, size = 88, normalized size = 1.31

$$\frac{\sqrt{a\,x^{2}+bx+c}\,\left(-ab\ln\left(\frac{2ax+b+2\sqrt{a\,x^{2}+bx+c}\,\sqrt{a}}{2\sqrt{a}}\right)+2\sqrt{a\,x^{2}+bx+c}\,a^{\frac{3}{2}}\right)}{2\sqrt{\frac{a\,x^{2}+bx+c}{x^{2}}}\,a^{\frac{5}{2}}x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+c/x^2+b/x)^(1/2),x)
```

```
[Out] 1/2*(a*x^2+b*x+c)^(1/2)*(2*(a*x^2+b*x+c)^(1/2)*a^(3/2)-b*ln(1/2*(2*a*x+b+2*
(a*x^2+b*x+c)^(1/2)*a^(1/2))/a^(1/2))*a)/((a*x^2+b*x+c)/x^2)^(1/2)/x/a^(5/2
)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a+\frac{b}{x}+\frac{c}{x^{2}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+c/x^2+b/x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(a + b/x + c/x^2), x)
```

mupad [B] time = 1.45, size = 53, normalized size = 0.79

$$\frac{x\,\sqrt{a+\frac{b}{x}+\frac{c}{x^{2}}}}{a}-\frac{b\,\mathrm{atanh}\left(\frac{a+\frac{b}{2\,x}}{\sqrt{a}\,\sqrt{a+\frac{b}{x}+\frac{c}{x^{2}}}}\right)}{2\,a^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b/x + c/x^2)^(1/2),x)
```

```
[Out] (x*(a + b/x + c/x^2)^(1/2))/a - (b*atanh((a + b/(2*x))/(a^(1/2)*(a + b/x +
c/x^2)^(1/2))))/(2*a^(3/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a+\frac{b}{x}+\frac{c}{x^{2}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+c/x**2+b/x)**(1/2),x)
```

```
[Out] Integral(1/sqrt(a + b/x + c/x**2), x)
```

3.392 $\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)^{3/2}}\,dx$

Optimal. Leaf size=133

$$-\frac{3b\tanh^{-1}\left(\frac{2a+\frac{b}{x}}{2\sqrt{a}\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}\right)}{2a^{5/2}}+\frac{x\left(3b^2-8ac\right)\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}{a^2\left(b^2-4ac\right)}-\frac{2x\left(-2ac+b^2+\frac{bc}{x}\right)}{a\left(b^2-4ac\right)\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}$$

Rubi [A] time = 0.10, antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {1342, 740, 806, 724, 206}

$$\frac{x\left(3b^2-8ac\right)\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}{a^2\left(b^2-4ac\right)}-\frac{3b\tanh^{-1}\left(\frac{2a+\frac{b}{x}}{2\sqrt{a}\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}\right)}{2a^{5/2}}-\frac{2x\left(-2ac+b^2+\frac{bc}{x}\right)}{a\left(b^2-4ac\right)\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + c/x^2 + b/x)^(-3/2),x]
```

```
[Out] ((3*b^2 - 8*a*c)*Sqrt[a + c/x^2 + b/x]*x)/(a^2*(b^2 - 4*a*c)) - (2*(b^2 - 2
*a*c + (b*c)/x)*x)/(a*(b^2 - 4*a*c)*Sqrt[a + c/x^2 + b/x]) - (3*b*ArcTanh[(
2*a + b/x)/(2*Sqrt[a]*Sqrt[a + c/x^2 + b/x])])/(2*a^(5/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 740

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e
)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e
^2)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d +
 e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p +
 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p,
-1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 806

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b
*x + c*x^2)^(p + 1))/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[(b*(e*f
 + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m
```

```
+ 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &
& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m +
2*p + 3], 0]
```

Rule 1342

```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[
Int[(a + b/x^n + c/x^(2*n))^p/x^2, x], x, 1/x] /; FreeQ[{a, b, c, p}, x] &&
 EqQ[n2, 2*n] && ILtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)^{3/2}}\,dx &= -\text{Subst}\left(\int \frac{1}{x^2\left(a+bx+cx^2\right)^{3/2}}\,dx,x,\frac{1}{x}\right)\\
&= -\frac{2\left(b^2-2ac+\frac{bc}{x}\right)x}{a\left(b^2-4ac\right)\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}}+\frac{2\,\text{Subst}\left(\int \frac{\frac{1}{2}\left(-3b^2+8ac\right)-bcx}{x^2\sqrt{a+bx+cx^2}}\,dx,x,\frac{1}{x}\right)}{a\left(b^2-4ac\right)}\\
&= \frac{\left(3b^2-8ac\right)\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\,x}{a^2\left(b^2-4ac\right)}-\frac{2\left(b^2-2ac+\frac{bc}{x}\right)x}{a\left(b^2-4ac\right)\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}}+\frac{(3b)\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx+cx^2}}\,dx,x,\right.}{2a^2}\\
&= \frac{\left(3b^2-8ac\right)\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\,x}{a^2\left(b^2-4ac\right)}-\frac{2\left(b^2-2ac+\frac{bc}{x}\right)x}{a\left(b^2-4ac\right)\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}}-\frac{(3b)\,\text{Subst}\left(\int \frac{1}{4a-x^2}\,dx,x,\frac{2a+}{\sqrt{a+}}\right.}{a^2}\\
&= \frac{\left(3b^2-8ac\right)\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\,x}{a^2\left(b^2-4ac\right)}-\frac{2\left(b^2-2ac+\frac{bc}{x}\right)x}{a\left(b^2-4ac\right)\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}}-\frac{3b\tanh^{-1}\left(\frac{2a+\frac{b}{x}}{2\sqrt{a}\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}}\right)}{2a^{5/2}}
\end{aligned}$$

Mathematica [A] time = 0.17, size = 138, normalized size = 1.04

$$-\frac{3b\left(b^2-4ac\right)\sqrt{x(ax+b)+c}\,\tanh^{-1}\left(\frac{2ax+b}{2\sqrt{a}\sqrt{x(ax+b)+c}}\right)+2\sqrt{a}\left(-b^2\left(ax^2+3c\right)+10abcx+4ac\left(ax^2+2c\right)-3b^3x\right)}{2a^{5/2}x\left(b^2-4ac\right)\sqrt{a+\frac{bx+c}{x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + c/x^2 + b/x)^(-3/2),x]
```

```
[Out] -1/2*(2*Sqrt[a]*(-3*b^3*x + 10*a*b*c*x + 4*a*c*(2*c + a*x^2) - b^2*(3*c + a
*x^2)) + 3*b*(b^2 - 4*a*c)*Sqrt[c + x*(b + a*x)]*ArcTanh[(b + 2*a*x)/(2*Sqr
t[a]*Sqrt[c + x*(b + a*x)])])/(a^(5/2)*(b^2 - 4*a*c)*x*Sqrt[a + (c + b*x)/x
^2])
```

IntegrateAlgebraic [A] time = 5.93, size = 150, normalized size = 1.13

$$\frac{(x(ax+b)+c)^{3/2}\left(\frac{4a^2cx^2-ab^2x^2+10abcx+8ac^2-3b^3x-3b^2c}{a^2\left(4ac-b^2\right)\sqrt{ax^2+bx+c}}+\frac{3b\log\left(-2a^{5/2}\sqrt{ax^2+bx+c}+2a^3x+a^2b\right)}{2a^{5/2}}\right)}{x^3\left(a+\frac{bx+c}{x^2}\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + c/x^2 + b/x)^(-3/2),x]
```

```
[Out] ((c + x*(b + a*x))^(3/2)*((-3*b^2*c + 8*a*c^2 - 3*b^3*x + 10*a*b*c*x - a*b^
2*x^2 + 4*a^2*c*x^2)/(a^2*(-b^2 + 4*a*c)*Sqrt[c + b*x + a*x^2]) + (3*b*Log[
a^2*b + 2*a^3*x - 2*a^(5/2)*Sqrt[c + b*x + a*x^2]])/(2*a^(5/2))))/(x^3*(a +
 (c + b*x)/x^2)^(3/2))
```

fricas [A] time = 1.35, size = 465, normalized size = 3.50

$$\left[\frac{3\left(b^3c - 4abc^2 + (ab^3 - 4a^2bc)x^2 + (b^4 - 4ab^2c)x\right)\sqrt{a}\log\left(-8a^2x^2 - 8abx - b^2 - 4ac + 4(2ax^2 + bx)\sqrt{a}\sqrt{\frac{ax^2+bx+c}{x^2}}\right) + 4\left((a^2b^2 - 4a^3c)x^3 + (3ab^3 - 10a^2bc)x^2 + (3ab^2c - 8a^2c^2)x\right)\sqrt{\frac{ax^2+bx+c}{x^2}}}{4\left(a^3b^2c - 4a^4c^2 + (a^4b^2 - 4a^5c)x^2 + (a^3b^3 - 4a^4bc)x\right)}, \frac{3\left(b^3c - 4abc^2 + (ab^3 - 4a^2bc)x^2 + (b^4 - 4ab^2c)x\right)\sqrt{-a}\arctan\left(\frac{(2ax^2+bx)\sqrt{-a}\sqrt{\frac{ax^2+bx+c}{x^2}}}{2(a^2x^2+abx+ac)}\right) + 2\left((a^2b^2 - 4a^3c)x^3 + (3ab^3 - 10a^2bc)x^2 + (3ab^2c - 8a^2c^2)x\right)\sqrt{\frac{ax^2+bx+c}{x^2}}}{2\left(a^3b^2c - 4a^4c^2 + (a^4b^2 - 4a^5c)x^2 + (a^3b^3 - 4a^4bc)x\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+c/x^2+b/x)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/4*(3*(b^3*c - 4*a*b*c^2 + (a*b^3 - 4*a^2*b*c)*x^2 + (b^4 - 4*a*b^2*c)*x)
*sqrt(a)*log(-8*a^2*x^2 - 8*a*b*x - b^2 - 4*a*c + 4*(2*a*x^2 + b*x)*sqrt(a)
*sqrt((a*x^2 + b*x + c)/x^2)) + 4*((a^2*b^2 - 4*a^3*c)*x^3 + (3*a*b^3 - 10*
a^2*b*c)*x^2 + (3*a*b^2*c - 8*a^2*c^2)*x)*sqrt((a*x^2 + b*x + c)/x^2))/(a^3
*b^2*c - 4*a^4*c^2 + (a^4*b^2 - 4*a^5*c)*x^2 + (a^3*b^3 - 4*a^4*b*c)*x), 1/
2*(3*(b^3*c - 4*a*b*c^2 + (a*b^3 - 4*a^2*b*c)*x^2 + (b^4 - 4*a*b^2*c)*x)*sq
rt(-a)*arctan(1/2*(2*a*x^2 + b*x)*sqrt(-a)*sqrt((a*x^2 + b*x + c)/x^2)/(a^2
*x^2 + a*b*x + a*c)) + 2*((a^2*b^2 - 4*a^3*c)*x^3 + (3*a*b^3 - 10*a^2*b*c)*
x^2 + (3*a*b^2*c - 8*a^2*c^2)*x)*sqrt((a*x^2 + b*x + c)/x^2))/(a^3*b^2*c -
4*a^4*c^2 + (a^4*b^2 - 4*a^5*c)*x^2 + (a^3*b^3 - 4*a^4*b*c)*x)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+c/x^2+b/x)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(t_n
ostep)]index.cc index_m operator + Error: Bad Argument Value
```

maple [A] time = 0.01, size = 197, normalized size = 1.48

$$\frac{(ax^2+bx+c)\left(8a^{\frac{7}{2}}cx^2 - 2a^{\frac{5}{2}}b^2x^2 + 20a^{\frac{5}{2}}bcx - 6a^{\frac{3}{2}}b^3x - 12\sqrt{ax^2+bx+c}\,a^2bc\ln\left(\frac{2ax+b+2\sqrt{ax^2+bx+c}\,\sqrt{a}}{2\sqrt{a}}\right) + 3\sqrt{ax^2+bx+c}\,ab^3\ln\left(\frac{2ax+b+2\sqrt{ax^2+bx+c}\,\sqrt{a}}{2\sqrt{a}}\right) + 16a^{\frac{5}{2}}c^2 - 6a^{\frac{3}{2}}b^2c\right)}{2\left(\frac{ax^2+bx+c}{x^2}\right)^{\frac{3}{2}}(4ac-b^2)\,a^{\frac{7}{2}}x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+c/x^2+b/x)^(3/2),x)
```

```
[Out] 1/2*(a*x^2+b*x+c)/a^(7/2)*(8*a^(7/2)*x^2*c-2*a^(5/2)*x^2*b^2+20*a^(5/2)*x*b
*c-6*a^(3/2)*x*b^3+16*a^(5/2)*c^2-6*a^(3/2)*b^2*c-12*ln(1/2*(2*a*x+b+2*(a*x
^2+b*x+c)^(1/2)*a^(1/2))/a^(1/2))*(a*x^2+b*x+c)^(1/2)*a^2*b*c+3*ln(1/2*(2*a
*x+b+2*(a*x^2+b*x+c)^(1/2)*a^(1/2))/a^(1/2))*(a*x^2+b*x+c)^(1/2)*a*b^3)/((a
*x^2+b*x+c)/x^2)^(3/2)/x^3/(4*a*c-b^2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a + \frac{b}{x} + \frac{c}{x^2}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+c/x^2+b/x)^(3/2),x, algorithm="maxima")
[Out] integrate((a + b/x + c/x^2)^(-3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(a+\frac{b}{x}+\frac{c}{x^2}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b/x + c/x^2)^(3/2),x)
[Out] int(1/(a + b/x + c/x^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+\frac{b}{x}+\frac{c}{x^2}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+c/x**2+b/x)**(3/2),x)
[Out] Integral((a + b/x + c/x**2)**(-3/2), x)
```

3.393 $\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)^{5/2}}\,dx$

Optimal. Leaf size=220

$$-\frac{5b\tanh^{-1}\left(\frac{2a+\frac{b}{x}}{2\sqrt{a}\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}\right)}{2a^{7/2}}-\frac{2x\left(32a^2c^2+\frac{bc(5b^2-28ac)}{x}-32ab^2c+5b^4\right)}{3a^2\left(b^2-4ac\right)^2\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}+\frac{x\left(128a^2c^2-100ab^2c+15b^4\right)\sqrt{a+\frac{b}{x}}}{3a^3\left(b^2-4ac\right)^2}$$

Rubi [A] time = 0.19, antiderivative size = 220, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {1342, 740, 822, 806, 724, 206}

$$-\frac{2x\left(32a^2c^2+\frac{bc(5b^2-28ac)}{x}-32ab^2c+5b^4\right)}{3a^2\left(b^2-4ac\right)^2\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}+\frac{x\left(128a^2c^2-100ab^2c+15b^4\right)\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}{3a^3\left(b^2-4ac\right)^2}-\frac{5b\tanh^{-1}\left(\frac{2a+\frac{b}{x}}{2\sqrt{a}\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}\right)}{2a^{7/2}}-\frac{2x\left(-2ac+b^2+\frac{bc}{x}\right)}{3a\left(b^2-4ac\right)\left(a+\frac{b}{x}+\frac{c}{x^2}\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + c/x^2 + b/x)^(-5/2),x]
```

```
[Out] ((15*b^4 - 100*a*b^2*c + 128*a^2*c^2)*Sqrt[a + c/x^2 + b/x]*x)/(3*a^3*(b^2
- 4*a*c)^2) - (2*(b^2 - 2*a*c + (b*c)/x)*x)/(3*a*(b^2 - 4*a*c)*(a + c/x^2 +
 b/x)^(3/2)) - (2*(5*b^4 - 32*a*b^2*c + 32*a^2*c^2 + (b*c*(5*b^2 - 28*a*c))
/x)*x)/(3*a^2*(b^2 - 4*a*c)^2*Sqrt[a + c/x^2 + b/x]) - (5*b*ArcTanh[(2*a +
b/x)/(2*Sqrt[a]*Sqrt[a + c/x^2 + b/x])])/(2*a^(7/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 740

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e
)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e
^2)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d +
 e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p +
 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p,
-1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 806

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b
*x + c*x^2)^(p + 1))/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[(b*(e*f
```

```
 + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m
+ 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &
& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m +
2*p + 3], 0]
```

Rule 822

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(f*(b*c*d - b^2*e +
2*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x)*(a
+ b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), x]
 + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m +
2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*m
 + b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) - f
*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1
] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 1342

```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[
Int[(a + b/x^n + c/x^(2*n))^p/x^2, x], x, 1/x] /; FreeQ[{a, b, c, p}, x] &&
 EqQ[n2, 2*n] && ILtQ[n, 0]
```

Rubi steps

$$\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)^{5/2}}\,dx = -\text{Subst}\left(\int \frac{1}{x^2\left(a+bx+cx^2\right)^{5/2}}\,dx,x,\frac{1}{x}\right)$$

$$= -\frac{2\left(b^2-2ac+\frac{bc}{x}\right)x}{3a\left(b^2-4ac\right)\left(a+\frac{c}{x^2}+\frac{b}{x}\right)^{3/2}} + \frac{2\,\text{Subst}\left(\int \frac{\frac{1}{2}\left(-5b^2+16ac\right)-3bcx}{x^2\left(a+bx+cx^2\right)^{3/2}}\,dx,x,\frac{1}{x}\right)}{3a\left(b^2-4ac\right)}$$

$$= -\frac{2\left(b^2-2ac+\frac{bc}{x}\right)x}{3a\left(b^2-4ac\right)\left(a+\frac{c}{x^2}+\frac{b}{x}\right)^{3/2}} - \frac{2\left(5b^4-32ab^2c+32a^2c^2+\frac{bc\left(5b^2-28ac\right)}{x}\right)x}{3a^2\left(b^2-4ac\right)^2\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}} - \frac{4\,\text{Subst}\left(\int\right.}{}$$

$$= \frac{\left(15b^4-100ab^2c+128a^2c^2\right)\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\,x}{3a^3\left(b^2-4ac\right)^2} - \frac{2\left(b^2-2ac+\frac{bc}{x}\right)x}{3a\left(b^2-4ac\right)\left(a+\frac{c}{x^2}+\frac{b}{x}\right)^{3/2}} - \frac{2\left(5b^4-32\right.}{3a^2\left(\right.}$$

$$= \frac{\left(15b^4-100ab^2c+128a^2c^2\right)\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\,x}{3a^3\left(b^2-4ac\right)^2} - \frac{2\left(b^2-2ac+\frac{bc}{x}\right)x}{3a\left(b^2-4ac\right)\left(a+\frac{c}{x^2}+\frac{b}{x}\right)^{3/2}} - \frac{2\left(5b^4-32\right.}{3a^2\left(\right.}$$

$$= \frac{\left(15b^4-100ab^2c+128a^2c^2\right)\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\,x}{3a^3\left(b^2-4ac\right)^2} - \frac{2\left(b^2-2ac+\frac{bc}{x}\right)x}{3a\left(b^2-4ac\right)\left(a+\frac{c}{x^2}+\frac{b}{x}\right)^{3/2}} - \frac{2\left(5b^4-32\right.}{3a^2\left(\right.}$$

Mathematica [A] time = 0.38, size = 256, normalized size = 1.16

$$\frac{2\sqrt{a}\left(3b^4\left(a^2x^4-30acx^2+5c^2\right)-4ab^2c\left(6a^2x^4-12acx^2+25c^2\right)+8a^2bc^2x\left(32ax^2+39c\right)+16a^2c^2\left(3a^2x^4+12acx^2+8c^2\right)+10b^5\left(2ax^3+3cx\right)-2ab^3cx\left(74ax^2+105c\right)+15b^6x^2\right)-15b\left(b^2-4ac\right)^2\left(x(ax+b)+c\right)^{3/2}\tanh^{-1}\left(\frac{2ax+b}{2\sqrt{a}\sqrt{x(ax+b)+c}}\right)}{6a^{7/2}x\left(b^2-4ac\right)^2\left(x(ax+b)+c\right)\sqrt{a+\frac{bx+c}{x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + c/x^2 + b/x)^(-5/2),x]
```

```
[Out] (2*Sqrt[a]*(15*b^6*x^2 + 8*a^2*b*c^2*x*(39*c + 32*a*x^2) - 2*a*b^3*c*x*(105
*c + 74*a*x^2) + 10*b^5*(3*c*x + 2*a*x^3) + 3*b^4*(5*c^2 - 30*a*c*x^2 + a^2
*x^4) + 16*a^2*c^2*(8*c^2 + 12*a*c*x^2 + 3*a^2*x^4) - 4*a*b^2*c*(25*c^2 - 1
2*a*c*x^2 + 6*a^2*x^4)) - 15*b*(b^2 - 4*a*c)^2*(c + x*(b + a*x))^(3/2)*ArcT
anh[(b + 2*a*x)/(2*Sqrt[a]*Sqrt[c + x*(b + a*x)])])/(6*a^(7/2)*(b^2 - 4*a*c
)^2*x*(c + x*(b + a*x))*Sqrt[a + (c + b*x)/x^2])
```

IntegrateAlgebraic [A] time = 7.91, size = 273, normalized size = 1.24

$$\frac{(x(ax+b)+c)^{5/2}\left(\frac{5b\log\left(-2a^{7/2}\sqrt{ax^2+bx+c}+2a^4x+a^3b\right)}{2a^{7/2}}+\frac{48a^4c^2x^4-24a^3b^2cx^4+256a^3bc^2x^3+192a^3c^3x^2+3a^2b^4x^4-148a^2b^3cx^3+48a^2b^2c^2x^2+312a^2bc^3x+128a^2c^4+20ab^5x^3-90ab^4cx^2-210ab^3c^2x-100ab^2c^3+15b^6x^2+30b^5cx+15b^4c^2}{3a^3\left(4ac-b^2\right)^2\left(ax^2+bx+c\right)^{3/2}}\right)}{x^5\left(a+\frac{bx+c}{x^2}\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + c/x^2 + b/x)^(-5/2),x]
```

```
[Out] ((c + x*(b + a*x))^(5/2)*((15*b^4*c^2 - 100*a*b^2*c^3 + 128*a^2*c^4 + 30*b^
5*c*x - 210*a*b^3*c^2*x + 312*a^2*b*c^3*x + 15*b^6*x^2 - 90*a*b^4*c*x^2 + 4
8*a^2*b^2*c^2*x^2 + 192*a^3*c^3*x^2 + 20*a*b^5*x^3 - 148*a^2*b^3*c*x^3 + 25
6*a^3*b*c^2*x^3 + 3*a^2*b^4*x^4 - 24*a^3*b^2*c*x^4 + 48*a^4*c^2*x^4)/(3*a^3
*(-b^2 + 4*a*c)^2*(c + b*x + a*x^2)^(3/2)) + (5*b*Log[a^3*b + 2*a^4*x - 2*a
^(7/2)*Sqrt[c + b*x + a*x^2]])/(2*a^(7/2))))/(x^5*(a + (c + b*x)/x^2)^(5/2)
)
```

fricas [B] time = 1.86, size = 1081, normalized size = 4.91

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+c/x^2+b/x)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/12*(15*(b^5*c^2 - 8*a*b^3*c^3 + 16*a^2*b*c^4 + (a^2*b^5 - 8*a^3*b^3*c +
16*a^4*b*c^2)*x^4 + 2*(a*b^6 - 8*a^2*b^4*c + 16*a^3*b^2*c^2)*x^3 + (b^7 - 6
*a*b^5*c + 32*a^3*b*c^3)*x^2 + 2*(b^6*c - 8*a*b^4*c^2 + 16*a^2*b^2*c^3)*x)*
sqrt(a)*log(-8*a^2*x^2 - 8*a*b*x - b^2 - 4*a*c + 4*(2*a*x^2 + b*x)*sqrt(a)*
sqrt((a*x^2 + b*x + c)/x^2)) + 4*(3*(a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*x^
5 + 4*(5*a^2*b^5 - 37*a^3*b^3*c + 64*a^4*b*c^2)*x^4 + 3*(5*a*b^6 - 30*a^2*b
^4*c + 16*a^3*b^2*c^2 + 64*a^4*c^3)*x^3 + 6*(5*a*b^5*c - 35*a^2*b^3*c^2 + 5
2*a^3*b*c^3)*x^2 + (15*a*b^4*c^2 - 100*a^2*b^2*c^3 + 128*a^3*c^4)*x)*sqrt((
a*x^2 + b*x + c)/x^2))/(a^4*b^4*c^2 - 8*a^5*b^2*c^3 + 16*a^6*c^4 + (a^6*b^4
 - 8*a^7*b^2*c + 16*a^8*c^2)*x^4 + 2*(a^5*b^5 - 8*a^6*b^3*c + 16*a^7*b*c^2)
*x^3 + (a^4*b^6 - 6*a^5*b^4*c + 32*a^7*c^3)*x^2 + 2*(a^4*b^5*c - 8*a^5*b^3*
c^2 + 16*a^6*b*c^3)*x), 1/6*(15*(b^5*c^2 - 8*a*b^3*c^3 + 16*a^2*b*c^4 + (a^
2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*x^4 + 2*(a*b^6 - 8*a^2*b^4*c + 16*a^3*b
^2*c^2)*x^3 + (b^7 - 6*a*b^5*c + 32*a^3*b*c^3)*x^2 + 2*(b^6*c - 8*a*b^4*c^2
 + 16*a^2*b^2*c^3)*x)*sqrt(-a)*arctan(1/2*(2*a*x^2 + b*x)*sqrt(-a)*sqrt((a*
x^2 + b*x + c)/x^2)/(a^2*x^2 + a*b*x + a*c)) + 2*(3*(a^3*b^4 - 8*a^4*b^2*c
+ 16*a^5*c^2)*x^5 + 4*(5*a^2*b^5 - 37*a^3*b^3*c + 64*a^4*b*c^2)*x^4 + 3*(5*
a*b^6 - 30*a^2*b^4*c + 16*a^3*b^2*c^2 + 64*a^4*c^3)*x^3 + 6*(5*a*b^5*c - 35
*a^2*b^3*c^2 + 52*a^3*b*c^3)*x^2 + (15*a*b^4*c^2 - 100*a^2*b^2*c^3 + 128*a^
3*c^4)*x)*sqrt((a*x^2 + b*x + c)/x^2))/(a^4*b^4*c^2 - 8*a^5*b^2*c^3 + 16*a^
6*c^4 + (a^6*b^4 - 8*a^7*b^2*c + 16*a^8*c^2)*x^4 + 2*(a^5*b^5 - 8*a^6*b^3*c
```

```
 + 16*a^7*b*c^2)*x^3 + (a^4*b^6 - 6*a^5*b^4*c + 32*a^7*c^3)*x^2 + 2*(a^4*b^
5*c - 8*a^5*b^3*c^2 + 16*a^6*b*c^3)*x)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+c/x^2+b/x)^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(t_n
ostep)]index.cc index_m operator + Error: Bad Argument Value
```

maple [A] time = 0.01, size = 376, normalized size = 1.71

$$-\frac{\left(a x^2+b x+c\right)\left(-96a^{\frac{13}{2}}c^2x^4+48a^{\frac{11}{2}}b^2c x^4-6a^{\frac{9}{2}}b^4x^4-512a^{\frac{11}{2}}b c^2x^3+296a^{\frac{9}{2}}b^3c x^3-40a^{\frac{7}{2}}b^5x^3-384a^{\frac{11}{2}}c^3x^2-96a^{\frac{9}{2}}b^2c^2x^2+180a^{\frac{7}{2}}b^4c x^2-30a^{\frac{5}{2}}b^6x^2-624a^{\frac{9}{2}}b c^3x+420a^{\frac{7}{2}}b^3c^2x-60a^{\frac{5}{2}}b^5cx-256a^{\frac{9}{2}}c^4+200a^{\frac{7}{2}}b^2c^3-30a^{\frac{5}{2}}b^4c^2+240\left(a x^2+b x+c\right)^{\frac{3}{2}}a^4b c^2\ln\left(\frac{2ax+b+2\sqrt{a x^2+bx+c}\,\sqrt{a}}{2\sqrt{a}}\right)-120\left(a x^2+b x+c\right)^{\frac{3}{2}}a^3b^3c\ln\left(\frac{2ax+b+2\sqrt{a x^2+bx+c}\,\sqrt{a}}{2\sqrt{a}}\right)+15\left(a x^2+b x+c\right)^{\frac{3}{2}}a^2b^5\ln\left(\frac{2ax+b+2\sqrt{a x^2+bx+c}\,\sqrt{a}}{2\sqrt{a}}\right)\right)}{6\left(\frac{a x^2+bx+c}{x^2}\right)^{\frac{5}{2}}\left(4ac-b^2\right)^2a^{\frac{11}{2}}x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+c/x^2+b/x)^(5/2),x)
```

```
[Out] -1/6*(a*x^2+b*x+c)*(-96*a^(13/2)*x^4*c^2+48*a^(11/2)*x^4*b^2*c-512*a^(11/2)
*x^3*b*c^2-6*a^(9/2)*x^4*b^4-384*a^(11/2)*x^2*c^3+296*a^(9/2)*x^3*b^3*c-96*
a^(9/2)*x^2*b^2*c^2-40*a^(7/2)*x^3*b^5-624*a^(9/2)*x*b*c^3+180*a^(7/2)*x^2*
b^4*c-256*a^(9/2)*c^4+420*a^(7/2)*x*b^3*c^2-30*a^(5/2)*x^2*b^6+200*a^(7/2)*
b^2*c^3-60*a^(5/2)*x*b^5*c+240*ln(1/2*(2*a*x+b+2*(a*x^2+b*x+c)^(1/2)*a^(1/2
))/a^(1/2))*(a*x^2+b*x+c)^(3/2)*a^4*b*c^2-120*ln(1/2*(2*a*x+b+2*(a*x^2+b*x+
c)^(1/2)*a^(1/2))/a^(1/2))*(a*x^2+b*x+c)^(3/2)*a^3*b^3*c+15*ln(1/2*(2*a*x+b
+2*(a*x^2+b*x+c)^(1/2)*a^(1/2))/a^(1/2))*(a*x^2+b*x+c)^(3/2)*a^2*b^5-30*a^(
5/2)*b^4*c^2)/a^(11/2)/((a*x^2+b*x+c)/x^2)^(5/2)/x^5/(4*a*c-b^2)^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a + \frac{b}{x} + \frac{c}{x^2}\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+c/x^2+b/x)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((a + b/x + c/x^2)^(-5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{{\left(a + \frac{b}{x} + \frac{c}{x^2}\right)}^{5/2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b/x + c/x^2)^(5/2),x)
```

```
[Out] int(1/(a + b/x + c/x^2)^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a + \frac{b}{x} + \frac{c}{x^2}\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+c/x**2+b/x)**(5/2),x)
```

```
[Out] Integral((a + b/x + c/x**2)**(-5/2), x)
```

3.394 $\int \sqrt{a^2+\frac{b^2}{x^2}+\frac{2ab}{x}}\,dx$

Optimal. Leaf size=73

$$\frac{ax\sqrt{a^2+\frac{2ab}{x}+\frac{b^2}{x^2}}}{a+\frac{b}{x}}-\frac{b\log\left(\frac{1}{x}\right)\sqrt{a^2+\frac{2ab}{x}+\frac{b^2}{x^2}}}{a+\frac{b}{x}}$$

Rubi [A] time = 0.04, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {1342, 646, 43}

$$\frac{ax\sqrt{a^2+\frac{2ab}{x}+\frac{b^2}{x^2}}}{a+\frac{b}{x}}-\frac{b\log\left(\frac{1}{x}\right)\sqrt{a^2+\frac{2ab}{x}+\frac{b^2}{x^2}}}{a+\frac{b}{x}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a^2 + b^2/x^2 + (2*a*b)/x],x]
```

```
[Out] (a*Sqrt[a^2 + b^2/x^2 + (2*a*b)/x]*x)/(a + b/x) - (b*Sqrt[a^2 + b^2/x^2 + (
2*a*b)/x]*Log[x^(-1)])/(a + b/x)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 1342

```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[
Int[(a + b/x^n + c/x^(2*n))^p/x^2, x], x, 1/x] /; FreeQ[{a, b, c, p}, x] &&
 EqQ[n2, 2*n] && ILtQ[n, 0]
```

Rubi steps

$$\int \sqrt{a^2+\frac{b^2}{x^2}+\frac{2ab}{x}}\,dx = -\text{Subst}\left(\int \frac{\sqrt{a^2+2abx+b^2x^2}}{x^2}\,dx, x, \frac{1}{x}\right)$$
$$= -\frac{\sqrt{a^2+\frac{b^2}{x^2}+\frac{2ab}{x}}\,\text{Subst}\left(\int \frac{ab+b^2x}{x^2}\,dx, x, \frac{1}{x}\right)}{ab+\frac{b^2}{x}}$$
$$= -\frac{\sqrt{a^2+\frac{b^2}{x^2}+\frac{2ab}{x}}\,\text{Subst}\left(\int \left(\frac{ab}{x^2}+\frac{b^2}{x}\right)dx, x, \frac{1}{x}\right)}{ab+\frac{b^2}{x}}$$
$$= \frac{a\sqrt{a^2+\frac{b^2}{x^2}+\frac{2ab}{x}}\,x}{a+\frac{b}{x}} + \frac{b\sqrt{a^2+\frac{b^2}{x^2}+\frac{2ab}{x}}\,\log(x)}{a+\frac{b}{x}}$$

Mathematica [**A**] time = 0.02, size = 32, normalized size = 0.44

$$\frac{x\sqrt{\frac{(ax+b)^2}{x^2}}\,(ax+b\log(x))}{ax+b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a^2 + b^2/x^2 + (2*a*b)/x],x]
[Out] (x*Sqrt[(b + a*x)^2/x^2]*(a*x + b*Log[x]))/(b + a*x)
```

IntegrateAlgebraic [**A**] time = 4.98, size = 32, normalized size = 0.44

$$\frac{x\sqrt{\frac{(ax+b)^2}{x^2}}\,(ax+b\log(x))}{ax+b}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[a^2 + b^2/x^2 + (2*a*b)/x],x]
[Out] (x*Sqrt[(b + a*x)^2/x^2]*(a*x + b*Log[x]))/(b + a*x)
```

fricas [**A**] time = 1.60, size = 8, normalized size = 0.11

$$ax + b\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+b^2/x^2+2*a*b/x)^(1/2),x, algorithm="fricas")
[Out] a*x + b*log(x)
```

giac [**A**] time = 0.34, size = 29, normalized size = 0.40

$$ax\text{sgn}\left(ax^2+bx\right) + b\log\left(|x|\right)\text{sgn}\left(ax^2+bx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+b^2/x^2+2*a*b/x)^(1/2),x, algorithm="giac")
[Out] a*x*sgn(a*x^2 + b*x) + b*log(abs(x))*sgn(a*x^2 + b*x)
```

maple [A] time = 0.01, size = 40, normalized size = 0.55

$$\frac{\sqrt{\frac{a^2x^2+2axb+b^2}{x^2}}\,(ax+b\ln(x))\,x}{ax+b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2+b^2/x^2+2*a*b/x)^(1/2),x)
```

```
[Out] ((a^2*x^2+2*a*b*x+b^2)/x^2)^(1/2)/(a*x+b)*x*(a*x+b*ln(x))
```

maxima [A] time = 0.78, size = 8, normalized size = 0.11

$$ax+b\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+b^2/x^2+2*a*b/x)^(1/2),x, algorithm="maxima")
```

```
[Out] a*x + b*log(x)
```

mupad [B] time = 0.11, size = 134, normalized size = 1.84

$$x\sqrt{\frac{1}{x^2}}\,\sqrt{a^2\,x^2+2\,a\,b\,x+b^2}-x\,\ln\left(\frac{2\,\sqrt{b^2}\,\sqrt{a^2\,x^2+2\,a\,b\,x+b^2}+2\,b^2+2\,a\,b\,x}{x}\right)\sqrt{b^2}\,\sqrt{\frac{1}{x^2}}+\frac{a\,b\,x\,\ln\left(\frac{a\,b+\sqrt{a^2}\,\sqrt{a^2\,x^2+2\,a\,b\,x+b^2}+a^2\,x}{\sqrt{a^2}}\right)\sqrt{\frac{1}{x^2}}}{\sqrt{a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2/x^2 + (2*a*b)/x)^(1/2),x)
```

```
[Out] x*(1/x^2)^(1/2)*(b^2 + a^2*x^2 + 2*a*b*x)^(1/2) - x*log((2*(b^2)^(1/2)*(b^2
 + a^2*x^2 + 2*a*b*x)^(1/2) + 2*b^2 + 2*a*b*x)/x)*(b^2)^(1/2)*(1/x^2)^(1/2)
 + (a*b*x*log((a*b + (a^2)^(1/2)*(b^2 + a^2*x^2 + 2*a*b*x)^(1/2) + a^2*x)/(
a^2)^(1/2))*(1/x^2)^(1/2))/(a^2)^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{a^2+\frac{2ab}{x}+\frac{b^2}{x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2+b**2/x**2+2*a*b/x)**(1/2),x)
```

```
[Out] Integral(sqrt(a**2 + 2*a*b/x + b**2/x**2), x)
```

3.395 $\int \frac{1}{c+\frac{a}{x^4}+\frac{b}{x^2}}\,dx$

Optimal. Leaf size=179

$$-\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\,c^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\,c^{3/2}\sqrt{\sqrt{b^2-4ac}+b}}+\frac{x}{c}$$

Rubi [A] time = 0.29, antiderivative size = 179, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {1340, 1122, 1166, 205}

$$-\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\,c^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\,c^{3/2}\sqrt{\sqrt{b^2-4ac}+b}}+\frac{x}{c}$$

Antiderivative was successfully verified.

```
[In] Int[(c + a/x^4 + b/x^2)^(-1),x]
```

```
[Out] x/c - ((b - (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqr
t[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) -
((b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b +
Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1122

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Simp[(d^3*(d*x)^(m - 3)*(a + b*x^2 + c*x^4)^(p + 1))/(c*(m + 4*p + 1)),
 x] - Dist[d^4/(c*(m + 4*p + 1)), Int[(d*x)^(m - 4)*Simp[a*(m - 3) + b*(m +
 2*p - 1)*x^2, x]*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x
] && NeQ[b^2 - 4*a*c, 0] && GtQ[m, 3] && NeQ[m + 4*p + 1, 0] && IntegerQ[2*
p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1340

```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[x^
(2*n*p)*(c + b/x^n + a/x^(2*n))^p, x] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n
] && LtQ[n, 0] && IntegerQ[p]
```

Rubi steps

$$\int \frac{1}{c+\frac{a}{x^4}+\frac{b}{x^2}}\,dx = \int \frac{x^4}{a+bx^2+cx^4}\,dx$$

$$= \frac{x}{c} - \frac{\int \frac{a+bx^2}{a+bx^2+cx^4}\,dx}{c}$$

$$= \frac{x}{c} - \frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx}{2c} - \frac{\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx}{2c}$$

$$= \frac{x}{c} - \frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\,c^{3/2}\sqrt{b-\sqrt{b^2-4ac}}} - \frac{\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\,c^{3/2}\sqrt{b+\sqrt{b^2-4ac}}}$$

Mathematica [A] time = 0.12, size = 202, normalized size = 1.13

$$-\frac{\left(b\sqrt{b^2-4ac}+2ac-b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\,c^{3/2}\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}} - \frac{\left(b\sqrt{b^2-4ac}-2ac+b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\,c^{3/2}\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4ac}+b}} + \frac{x}{c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + a/x^4 + b/x^2)^(-1),x]
```

```
[Out] x/c - ((-b^2 + 2*a*c + b*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt
[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(3/2)*Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[
b^2 - 4*a*c]]) - ((b^2 - 2*a*c + b*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[
c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(3/2)*Sqrt[b^2 - 4*a*c]*Sqrt
[b + Sqrt[b^2 - 4*a*c]])
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{c+\frac{a}{x^4}+\frac{b}{x^2}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + a/x^4 + b/x^2)^(-1),x]
```

```
[Out] IntegrateAlgebraic[(c + a/x^4 + b/x^2)^(-1), x]
```

fricas [B] time = 0.96, size = 1059, normalized size = 5.92

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x^4+b/x^2),x, algorithm="fricas")
```

```
[Out] -1/2*(sqrt(1/2)*c*sqrt(-(b^3 - 3*a*b*c + (b^2*c^3 - 4*a*c^4)*sqrt((b^4 - 2*
a*b^2*c + a^2*c^2)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4))*log(-2*(a*b^2
 - a^2*c)*x + sqrt(1/2)*(b^4 - 5*a*b^2*c + 4*a^2*c^2 - (b^3*c^3 - 4*a*b*c^4
)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^2*c^6 - 4*a*c^7)))*sqrt(-(b^3 - 3*a*b
*c + (b^2*c^3 - 4*a*c^4)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^2*c^6 - 4*a*c^
7)))/(b^2*c^3 - 4*a*c^4))) - sqrt(1/2)*c*sqrt(-(b^3 - 3*a*b*c + (b^2*c^3 -
4*a*c^4)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 -
4*a*c^4))*log(-2*(a*b^2 - a^2*c)*x - sqrt(1/2)*(b^4 - 5*a*b^2*c + 4*a^2*c^2
```

```
 - (b^3*c^3 - 4*a*b*c^4)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^2*c^6 - 4*a*c^
7)))*sqrt(-(b^3 - 3*a*b*c + (b^2*c^3 - 4*a*c^4)*sqrt((b^4 - 2*a*b^2*c + a^2
*c^2)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4))) + sqrt(1/2)*c*sqrt(-(b^3
- 3*a*b*c - (b^2*c^3 - 4*a*c^4)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^2*c^6 -
 4*a*c^7)))/(b^2*c^3 - 4*a*c^4))*log(-2*(a*b^2 - a^2*c)*x + sqrt(1/2)*(b^4
- 5*a*b^2*c + 4*a^2*c^2 + (b^3*c^3 - 4*a*b*c^4)*sqrt((b^4 - 2*a*b^2*c + a^2
*c^2)/(b^2*c^6 - 4*a*c^7)))*sqrt(-(b^3 - 3*a*b*c - (b^2*c^3 - 4*a*c^4)*sqrt
((b^4 - 2*a*b^2*c + a^2*c^2)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4))) -
sqrt(1/2)*c*sqrt(-(b^3 - 3*a*b*c - (b^2*c^3 - 4*a*c^4)*sqrt((b^4 - 2*a*b^2*
c + a^2*c^2)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4))*log(-2*(a*b^2 - a^2
*c)*x - sqrt(1/2)*(b^4 - 5*a*b^2*c + 4*a^2*c^2 + (b^3*c^3 - 4*a*b*c^4)*sqrt
((b^4 - 2*a*b^2*c + a^2*c^2)/(b^2*c^6 - 4*a*c^7)))*sqrt(-(b^3 - 3*a*b*c - (
b^2*c^3 - 4*a*c^4)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^2*c^6 - 4*a*c^7)))/(
b^2*c^3 - 4*a*c^4))) - 2*x)/c
```

giac [B] time = 1.66, size = 2109, normalized size = 11.78

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x^4+b/x^2),x, algorithm="giac")
```

```
[Out] x/c + 1/8*(2*b^5*c^4 - 12*a*b^3*c^5 + 16*a^2*b*c^6 - sqrt(2)*sqrt(b^2 - 4*a
*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^5*c^2 + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*s
qrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^3 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt
(b*c - sqrt(b^2 - 4*a*c)*c)*b^4*c^3 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
- sqrt(b^2 - 4*a*c)*c)*a^2*b*c^4 - 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - s
qrt(b^2 - 4*a*c)*c)*a*b^2*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b
^2 - 4*a*c)*c)*b^3*c^4 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 -
4*a*c)*c)*a*b*c^5 - 2*(b^2 - 4*a*c)*b^3*c^4 + 4*(b^2 - 4*a*c)*a*b*c^5 - (2*
b^5*c^2 - 16*a*b^3*c^3 + 32*a^2*b*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
- sqrt(b^2 - 4*a*c)*c)*b^5 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^
2 - 4*a*c)*c)*a*b^3*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4
*a*c)*c)*b^4*c - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*
c)*a^2*b*c^2 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*
a*b^2*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c
^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^3 -
2*(b^2 - 4*a*c)*b^3*c^2 + 8*(b^2 - 4*a*c)*a*b*c^3)*c^2 - 2*(sqrt(2)*sqrt(b*
c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c^2 - 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)
*c)*a^2*b^2*c^3 - 2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^3 + 2*a
*b^4*c^3 + 16*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*c^4 + 8*sqrt(2)*s
qrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^4 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*
a*c)*c)*a*b^2*c^4 - 16*a^2*b^2*c^4 - 4*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)
*c)*a^2*c^5 + 32*a^3*c^5 - 2*(b^2 - 4*a*c)*a*b^2*c^3 + 8*(b^2 - 4*a*c)*a^2*
c^4)*abs(c))*arctan(2*sqrt(1/2)*x/sqrt((b*c + sqrt(b^2*c^2 - 4*a*c^3))/c^2)
)/((a*b^4*c^3 - 8*a^2*b^2*c^4 - 2*a*b^3*c^4 + 16*a^3*c^5 + 8*a^2*b*c^5 + a*
b^2*c^5 - 4*a^2*c^6)*c^2) - 1/8*(2*b^5*c^4 - 12*a*b^3*c^5 + 16*a^2*b*c^6 -
sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5*c^2 + 6*sqrt(
2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^3 + 2*sqrt(2)*
sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c^3 - 8*sqrt(2)*sqrt(
b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^4 - 4*sqrt(2)*sqrt(b^2
 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^4 - sqrt(2)*sqrt(b^2 - 4*
a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^4 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*
sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^5 - 2*(b^2 - 4*a*c)*b^3*c^4 + 4*(b^2
- 4*a*c)*a*b*c^5 - (2*b^5*c^2 - 16*a*b^3*c^3 + 32*a^2*b*c^4 - sqrt(2)*sqrt(
b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5 + 8*sqrt(2)*sqrt(b^2 - 4*a
*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*s
qrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*
c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^2 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c +
 sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt
```

```
(b^2 - 4*a*c)*c)*b^3*c^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
- 4*a*c)*c)*a*b*c^3 - 2*(b^2 - 4*a*c)*b^3*c^2 + 8*(b^2 - 4*a*c)*a*b*c^3)*c^
2 + 2*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c^2 - 8*sqrt(2)*sqrt(b
*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^3 - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a
*c)*c)*a*b^3*c^3 - 2*a*b^4*c^3 + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)
*a^3*c^4 + 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^4 + sqrt(2)*sq
rt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^4 + 16*a^2*b^2*c^4 - 4*sqrt(2)*sqrt(b
*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^5 - 32*a^3*c^5 + 2*(b^2 - 4*a*c)*a*b^2*c^3
- 8*(b^2 - 4*a*c)*a^2*c^4)*abs(c))*arctan(2*sqrt(1/2)*x/sqrt((b*c - sqrt(b^
2*c^2 - 4*a*c^3))/c^2))/((a*b^4*c^3 - 8*a^2*b^2*c^4 - 2*a*b^3*c^4 + 16*a^3*
c^5 + 8*a^2*b*c^5 + a*b^2*c^5 - 4*a^2*c^6)*c^2)
```

maple [B] time = 0.01, size = 343, normalized size = 1.92

$$\frac{\sqrt{2}\, a \operatorname{arctanh}\left(\frac{\sqrt{2}\, c x}{\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}\right)}{\sqrt{-4ac+b^2}\,\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}+\frac{\sqrt{2}\, a \arctan\left(\frac{\sqrt{2}\, c x}{\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}\right)}{\sqrt{-4ac+b^2}\,\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}-\frac{\sqrt{2}\, b^2 \operatorname{arctanh}\left(\frac{\sqrt{2}\, c x}{\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}\right)}{2\sqrt{-4ac+b^2}\,\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}\, c}-\frac{\sqrt{2}\, b^2 \arctan\left(\frac{\sqrt{2}\, c x}{\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}\right)}{2\sqrt{-4ac+b^2}\,\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}\, c}+\frac{\sqrt{2}\, b \operatorname{arctanh}\left(\frac{\sqrt{2}\, c x}{\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}\right)}{2\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}\, c}-\frac{\sqrt{2}\, b \arctan\left(\frac{\sqrt{2}\, c x}{\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}\right)}{2\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}\, c}+\frac{x}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c+a/x^4+b/x^2),x)
```

```
[Out] 1/c*x+1/2/c*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+
(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b+1/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a
*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c
*x)*a-1/2/(-4*a*c+b^2)^(1/2)/c*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*ar
ctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b^2-1/2/c*2^(1/2)/((b+
(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2
)*c*x)*b+1/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arct
an(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*a-1/2/(-4*a*c+b^2)^(1/2)/c
*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(
1/2))*c)^(1/2)*c*x)*b^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x^4+b/x^2),x, algorithm="maxima")
```

```
[Out] x/c - integrate((b*x^2 + a)/(c*x^4 + b*x^2 + a), x)/c
```

mupad [B] time = 2.08, size = 3026, normalized size = 16.91

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c + a/x^4 + b/x^2),x)
```

```
[Out] x/c - atan(((((16*a^2*c^3 - 4*a*b^2*c^2)/c - (2*x*(4*b^3*c^3 - 16*a*b*c^4)*
(-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4
*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2))/c)*(
-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*
a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2) - (2*x
*(b^4 + 2*a^2*c^2 - 4*a*b^2*c))/c)*(-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) +
12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5 + b
^4*c^3 - 8*a*b^2*c^4)))^(1/2)*1i - (((16*a^2*c^3 - 4*a*b^2*c^2)/c + (2*x*(4
*b^3*c^3 - 16*a*b*c^4)*(-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2
 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5 + b^4*c^3 - 8*a
*b^2*c^4)))^(1/2))/c)*(-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2
- 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*
b^2*c^4)))^(1/2) + (2*x*(b^4 + 2*a^2*c^2 - 4*a*b^2*c))/c)*(-(b^5 + b^2*(-(4
```

```
*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1
/2))/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2)*1i)/((((16*a^2*c^3 - 4
*a*b^2*c^2)/c - (2*x*(4*b^3*c^3 - 16*a*b*c^4)*(-(b^5 + b^2*(-(4*a*c - b^2)^
3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*
a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2))/c)*(-(b^5 + b^2*(-(4*a*c - b^2)^3
)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a
^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2) - (2*x*(b^4 + 2*a^2*c^2 - 4*a*b^2*c
))/c)*(-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*
c*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2)
 + (((16*a^2*c^3 - 4*a*b^2*c^2)/c + (2*x*(4*b^3*c^3 - 16*a*b*c^4)*(-(b^5 +
b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^
2)^3)^(1/2))/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2))/c)*(-(b^5 + b
^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2
)^3)^(1/2))/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2) + (2*x*(b^4 + 2
*a^2*c^2 - 4*a*b^2*c))/c)*(-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*
c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5 + b^4*c^3 -
8*a*b^2*c^4)))^(1/2) + (2*a^2*b)/c))*(-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2)
+ 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5 +
 b^4*c^3 - 8*a*b^2*c^4)))^(1/2)*2i - atan(((((16*a^2*c^3 - 4*a*b^2*c^2)/c -
 (2*x*(4*b^3*c^3 - 16*a*b*c^4)*(-(b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a
^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5 + b^4*c
^3 - 8*a*b^2*c^4)))^(1/2))/c)*(-(b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^
2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5 + b^4*c^
3 - 8*a*b^2*c^4)))^(1/2) - (2*x*(b^4 + 2*a^2*c^2 - 4*a*b^2*c))/c)*(-(b^5 -
b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^
2)^3)^(1/2))/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2)*1i - (((16*a^2
*c^3 - 4*a*b^2*c^2)/c + (2*x*(4*b^3*c^3 - 16*a*b*c^4)*(-(b^5 - b^2*(-(4*a*c
 - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2))
/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2))/c)*(-(b^5 - b^2*(-(4*a*c
- b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2))/
(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2) + (2*x*(b^4 + 2*a^2*c^2 - 4
*a*b^2*c))/c)*(-(b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^
3*c + a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)
))^(1/2)*1i)/((((16*a^2*c^3 - 4*a*b^2*c^2)/c - (2*x*(4*b^3*c^3 - 16*a*b*c^4
)*(-(b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-
(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2))/c)
*(-(b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(
4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2) - (2
*x*(b^4 + 2*a^2*c^2 - 4*a*b^2*c))/c)*(-(b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2)
+ 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5 +
 b^4*c^3 - 8*a*b^2*c^4)))^(1/2) + (((16*a^2*c^3 - 4*a*b^2*c^2)/c + (2*x*(4*
b^3*c^3 - 16*a*b*c^4)*(-(b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2
- 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*
b^2*c^4)))^(1/2))/c)*(-(b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 -
 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b
^2*c^4)))^(1/2) + (2*x*(b^4 + 2*a^2*c^2 - 4*a*b^2*c))/c)*(-(b^5 - b^2*(-(4*
a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/
2))/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2) + (2*a^2*b)/c))*(-(b^5
- b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c -
b^2)^3)^(1/2))/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2)*2i
```

sympy [A] time = 2.12, size = 129, normalized size = 0.72

$$\operatorname{RootSum}\left(t^4\left(256a^2c^5-128ab^2c^4+16b^4c^3\right)+t^2\left(48a^2bc^2-28ab^3c+4b^5\right)+a^3,\left(t\mapsto t\log\left(x+\frac{32t^3abc^4-8t^3b^3c^3-4ta^2c^2+8tab^2c-2tb^4}{a^2c-ab^2}\right)\right)\right)+\frac{x}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x**4+b/x**2),x)
```

```
[Out] RootSum(_t**4*(256*a**2*c**5 - 128*a*b**2*c**4 + 16*b**4*c**3) + _t**2*(48*
a**2*b*c**2 - 28*a*b**3*c + 4*b**5) + a**3, Lambda(_t, _t*log(x + (32*_t**3
```

```
*a*b*c**4 - 8*_t**3*b**3*c**3 - 4*_t*a**2*c**2 + 8*_t*a*b**2*c - 2*_t*b**4)
/(a**2*c - a*b**2)))) + x/c
```

3.396 $\int \frac{1}{c+\frac{a}{x^6}+\frac{b}{x^3}}\,dx$

Optimal. Leaf size=631

$$\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\log\left(-\sqrt[3]{2}\,\sqrt[3]{c}\,x\sqrt[3]{b-\sqrt{b^2-4ac}}+\left(b-\sqrt{b^2-4ac}\right)^{2/3}+2^{2/3}c^{2/3}x^2\right)}{6\sqrt[3]{2}\,c^{4/3}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}+\frac{\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\log\left(-\sqrt[3]{2}\,\sqrt[3]{c}\right.}{}$$

Rubi [A] time = 1.17, antiderivative size = 631, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 9, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.643, Rules used = {1340, 1367, 1422, 200, 31, 634, 617, 204, 628}

$$\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\log\left(-\sqrt[3]{2}\,\sqrt[3]{c}\,x\sqrt[3]{b-\sqrt{b^2-4ac}}+\left(b-\sqrt{b^2-4ac}\right)^{2/3}+2^{2/3}c^{2/3}x^2\right)}{6\sqrt[3]{2}\,c^{4/3}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}+\frac{\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\log\left(-\sqrt[3]{2}\,\sqrt[3]{c}\,x\sqrt[3]{\sqrt{b^2-4ac}+b}+\left(\sqrt{b^2-4ac}+b\right)^{2/3}+2^{2/3}c^{2/3}x^2\right)}{6\sqrt[3]{2}\,c^{4/3}\left(\sqrt{b^2-4ac}+b\right)^{2/3}}-\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\,\sqrt[3]{c}\,x\right)}{3\sqrt[3]{2}\,c^{4/3}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}-\frac{\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\log\left(\sqrt[3]{\sqrt{b^2-4ac}+b}+\sqrt[3]{2}\,\sqrt[3]{c}\,x\right)}{3\sqrt[3]{2}\,c^{4/3}\left(\sqrt{b^2-4ac}+b\right)^{2/3}}+\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\,\sqrt[3]{c}\,x}{\sqrt[3]{b-\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\,\sqrt{3}\,c^{4/3}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}+\frac{\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\,\sqrt[3]{c}\,x}{\sqrt[3]{\sqrt{b^2-4ac}+b}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\,\sqrt{3}\,c^{4/3}\left(\sqrt{b^2-4ac}+b\right)^{2/3}}+\frac{x}{c}$$

Antiderivative was successfully verified.

```
[In] Int[(c + a/x^6 + b/x^3)^(-1),x]
```

```
[Out] x/c + ((b - (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(1 - (2*2^(1/3)*c^(1/3)
*x)/(b - Sqrt[b^2 - 4*a*c])^(1/3))/Sqrt[3]])/(2^(1/3)*Sqrt[3]*c^(4/3)*(b -
Sqrt[b^2 - 4*a*c])^(2/3)) + ((b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(
1 - (2*2^(1/3)*c^(1/3)*x)/(b + Sqrt[b^2 - 4*a*c])^(1/3))/Sqrt[3]])/(2^(1/3)
*Sqrt[3]*c^(4/3)*(b + Sqrt[b^2 - 4*a*c])^(2/3)) - ((b - (b^2 - 2*a*c)/Sqrt[
b^2 - 4*a*c])*Log[(b - Sqrt[b^2 - 4*a*c])^(1/3) + 2^(1/3)*c^(1/3)*x])/(3*2^
(1/3)*c^(4/3)*(b - Sqrt[b^2 - 4*a*c])^(2/3)) - ((b + (b^2 - 2*a*c)/Sqrt[b^2
 - 4*a*c])*Log[(b + Sqrt[b^2 - 4*a*c])^(1/3) + 2^(1/3)*c^(1/3)*x])/(3*2^(1/
3)*c^(4/3)*(b + Sqrt[b^2 - 4*a*c])^(2/3)) + ((b - (b^2 - 2*a*c)/Sqrt[b^2 -
4*a*c])*Log[(b - Sqrt[b^2 - 4*a*c])^(2/3) - 2^(1/3)*c^(1/3)*(b - Sqrt[b^2 -
 4*a*c])^(1/3)*x + 2^(2/3)*c^(2/3)*x^2])/(6*2^(1/3)*c^(4/3)*(b - Sqrt[b^2 -
 4*a*c])^(2/3)) + ((b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*Log[(b + Sqrt[b^2
- 4*a*c])^(2/3) - 2^(1/3)*c^(1/3)*(b + Sqrt[b^2 - 4*a*c])^(1/3)*x + 2^(2/3)
*c^(2/3)*x^2])/(6*2^(1/3)*c^(4/3)*(b + Sqrt[b^2 - 4*a*c])^(2/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
```

```
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1340

```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[x^
(2*n*p)*(c + b/x^n + a/x^(2*n))^p, x] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n
] && LtQ[n, 0] && IntegerQ[p]
```

Rule 1367

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Simp[(d^(2*n - 1)*(d*x)^(m - 2*n + 1)*(a + b*x^n + c*x^(2*n))^(
p + 1))/(c*(m + 2*n*p + 1)), x] - Dist[d^(2*n)/(c*(m + 2*n*p + 1)), Int[(d*
x)^(m - 2*n)*Simp[a*(m - 2*n + 1) + b*(m + n*(p - 1) + 1)*x^n, x]*(a + b*x^
n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && EqQ[n2, 2*n] && Ne
Q[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GtQ[m, 2*n - 1] && NeQ[m + 2*n*p + 1, 0]
 && IntegerQ[p]
```

Rule 1422

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q),
Int[1/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(
b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (PosQ[b^2 - 4*a
*c] ||  !IGtQ[n/2, 0])
```

Rubi steps

$$\int \frac{1}{c+\frac{a}{x^6}+\frac{b}{x^3}}\,dx = \int \frac{x^6}{a+bx^3+cx^6}\,dx$$

$$= \frac{x}{c} - \frac{\int \frac{a+bx^3}{a+bx^3+cx^6}\,dx}{c}$$

$$= \frac{x}{c} - \frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^3}\,dx}{2c} - \frac{\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^3}\,dx}{2c}$$

$$= \frac{x}{c} - \frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\frac{\sqrt[3]{b-\sqrt{b^2-4ac}}}{\sqrt[3]{2}}+\sqrt[3]{c}\,x}\,dx}{3\sqrt[3]{2}\,c\left(b-\sqrt{b^2-4ac}\right)^{2/3}} - \frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\int \frac{2^{2/3}\sqrt[3]{b-\sqrt{b^2-4ac}}-\sqrt[3]{c}\,x}{\frac{\left(b-\sqrt{b^2-4ac}\right)^{2/3}}{2^{2/3}}-\frac{\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}\,x}{\sqrt[3]{2}}+c^{2/3}x^2 \cdots}}{3\sqrt[3]{2}\,c\left(b-\sqrt{b^2-4ac}\right)^{2/3}} \cdots$$

$$= \frac{x}{c} - \frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}\,x\right)}{3\sqrt[3]{2}\,c^{4/3}\left(b-\sqrt{b^2-4ac}\right)^{2/3}} - \frac{\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{b+\sqrt{b^2-4a\cdots}}\right.}{3\sqrt[3]{2}\,c^{4/3}\left(b+\sqrt{b^2-4ac\cdots}\right.} \cdots$$

$$= \frac{x}{c} - \frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}\,x\right)}{3\sqrt[3]{2}\,c^{4/3}\left(b-\sqrt{b^2-4ac}\right)^{2/3}} - \frac{\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{b+\sqrt{b^2-4a\cdots}}\right.}{3\sqrt[3]{2}\,c^{4/3}\left(b+\sqrt{b^2-4ac\cdots}\right.} \cdots$$

$$= \frac{x}{c} + \frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}\,x}{\sqrt[3]{b-\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}\,c^{4/3}\left(b-\sqrt{b^2-4ac}\right)^{2/3}} + \frac{\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}\,x}{\sqrt[3]{b+\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}\,c^{4/3}\left(b+\sqrt{b^2-4ac}\right)^{2/3}} - \frac{\left(b-\frac{b^2-\cdots}{\sqrt{b^2\cdots}}\right.\cdots}{\cdots}$$

Mathematica [C] time = 0.04, size = 70, normalized size = 0.11

$$\frac{x}{c} - \frac{\text{RootSum}\left[\#1^6c+\#1^3b+a\&,\frac{\#1^3b\log(x-\#1)+a\log(x-\#1)}{2\#1^5c+\#1^2b}\&\right]}{3c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + a/x^6 + b/x^3)^(-1),x]
```

```
[Out] x/c - RootSum[a + b*#1^3 + c*#1^6 & , (a*Log[x - #1] + b*Log[x - #1]*#1^3)/
(b*#1^2 + 2*c*#1^5) & ]/(3*c)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{c+\frac{a}{x^6}+\frac{b}{x^3}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + a/x^6 + b/x^3)^(-1),x]
```

```
[Out] IntegrateAlgebraic[(c + a/x^6 + b/x^3)^(-1), x]
```

fricas [B] time = 3.18, size = 5260, normalized size = 8.34

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x^6+b/x^3),x, algorithm="fricas")
```

```
[Out] 1/6*(4*sqrt(3)*(1/2)^(1/3)*c*(-(b^3 - 2*a*b*c + (b^2*c^4 - 4*a*c^5)*sqrt((b
^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(b^6*c^8 - 12
*a*b^4*c^9 + 48*a^2*b^2*c^10 - 64*a^3*c^11)))/(b^2*c^4 - 4*a*c^5))^(1/3)*ar
ctan(-1/6*(2*(1/2)^(2/3)*(sqrt(3)*(b^8*c^4 - 13*a*b^6*c^5 + 60*a^2*b^4*c^6
- 112*a^3*b^2*c^7 + 64*a^4*c^8)*x*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 -
16*a^3*b^2*c^3 + 4*a^4*c^4)/(b^6*c^8 - 12*a*b^4*c^9 + 48*a^2*b^2*c^10 - 64*
a^3*c^11)) - sqrt(3)*(b^9 - 11*a*b^7*c + 42*a^2*b^5*c^2 - 62*a^3*b^3*c^3 +
24*a^4*b*c^4)*x)*(-(b^3 - 2*a*b*c + (b^2*c^4 - 4*a*c^5)*sqrt((b^8 - 8*a*b^6
*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(b^6*c^8 - 12*a*b^4*c^9 +
 48*a^2*b^2*c^10 - 64*a^3*c^11)))/(b^2*c^4 - 4*a*c^5))^(2/3) - (1/2)^(1/6)*
(sqrt(3)*(b^8*c^4 - 13*a*b^6*c^5 + 60*a^2*b^4*c^6 - 112*a^3*b^2*c^7 + 64*a^
4*c^8)*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)
/(b^6*c^8 - 12*a*b^4*c^9 + 48*a^2*b^2*c^10 - 64*a^3*c^11)) - sqrt(3)*(b^9 -
 11*a*b^7*c + 42*a^2*b^5*c^2 - 62*a^3*b^3*c^3 + 24*a^4*b*c^4))*(-(b^3 - 2*a
*b*c + (b^2*c^4 - 4*a*c^5)*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*
b^2*c^3 + 4*a^4*c^4)/(b^6*c^8 - 12*a*b^4*c^9 + 48*a^2*b^2*c^10 - 64*a^3*c^1
1)))/(b^2*c^4 - 4*a*c^5))^(2/3)*sqrt((2*(a^2*b^4 - 4*a^3*b^2*c + 2*a^4*c^2)
*x^2 + (1/2)^(2/3)*(b^8 - 10*a*b^6*c + 34*a^2*b^4*c^2 - 44*a^3*b^2*c^3 + 16
*a^4*c^4 - (b^7*c^4 - 12*a*b^5*c^5 + 48*a^2*b^3*c^6 - 64*a^3*b*c^7)*sqrt((b
^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(b^6*c^8 - 12
*a*b^4*c^9 + 48*a^2*b^2*c^10 - 64*a^3*c^11)))*(-(b^3 - 2*a*b*c + (b^2*c^4 -
 4*a*c^5)*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c
^4)/(b^6*c^8 - 12*a*b^4*c^9 + 48*a^2*b^2*c^10 - 64*a^3*c^11)))/(b^2*c^4 - 4
*a*c^5))^(2/3) + (1/2)^(1/3)*((a*b^5*c^4 - 8*a^2*b^3*c^5 + 16*a^3*b*c^6)*x*
sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(b^6*c
^8 - 12*a*b^4*c^9 + 48*a^2*b^2*c^10 - 64*a^3*c^11)) - (a*b^6 - 8*a^2*b^4*c
+ 18*a^3*b^2*c^2 - 8*a^4*c^3)*x)*(-(b^3 - 2*a*b*c + (b^2*c^4 - 4*a*c^5)*sqr
t((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(b^6*c^8
- 12*a*b^4*c^9 + 48*a^2*b^2*c^10 - 64*a^3*c^11)))/(b^2*c^4 - 4*a*c^5))^(1/3
))/(a^2*b^4 - 4*a^3*b^2*c + 2*a^4*c^2)) + 2*sqrt(3)*(a^3*b^4 - 4*a^4*b^2*c
+ 2*a^5*c^2))/(a^3*b^4 - 4*a^4*b^2*c + 2*a^5*c^2)) - 4*sqrt(3)*(1/2)^(1/3)*
c*(-(b^3 - 2*a*b*c - (b^2*c^4 - 4*a*c^5)*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4
*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(b^6*c^8 - 12*a*b^4*c^9 + 48*a^2*b^2*c^1
0 - 64*a^3*c^11)))/(b^2*c^4 - 4*a*c^5))^(1/3)*arctan(-1/6*(2*(1/2)^(2/3)*(s
qrt(3)*(b^8*c^4 - 13*a*b^6*c^5 + 60*a^2*b^4*c^6 - 112*a^3*b^2*c^7 + 64*a^4*
c^8)*x*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)
/(b^6*c^8 - 12*a*b^4*c^9 + 48*a^2*b^2*c^10 - 64*a^3*c^11)) + sqrt(3)*(b^9 -
 11*a*b^7*c + 42*a^2*b^5*c^2 - 62*a^3*b^3*c^3 + 24*a^4*b*c^4)*x)*(-(b^3 - 2
*a*b*c - (b^2*c^4 - 4*a*c^5)*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^
3*b^2*c^3 + 4*a^4*c^4)/(b^6*c^8 - 12*a*b^4*c^9 + 48*a^2*b^2*c^10 - 64*a^3*c
^11)))/(b^2*c^4 - 4*a*c^5))^(2/3) - (1/2)^(1/6)*(sqrt(3)*(b^8*c^4 - 13*a*b^
6*c^5 + 60*a^2*b^4*c^6 - 112*a^3*b^2*c^7 + 64*a^4*c^8)*sqrt((b^8 - 8*a*b^6*
c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(b^6*c^8 - 12*a*b^4*c^9 +
48*a^2*b^2*c^10 - 64*a^3*c^11)) + sqrt(3)*(b^9 - 11*a*b^7*c + 42*a^2*b^5*c^
2 - 62*a^3*b^3*c^3 + 24*a^4*b*c^4))*(-(b^3 - 2*a*b*c - (b^2*c^4 - 4*a*c^5)*
sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(b^6*c
^8 - 12*a*b^4*c^9 + 48*a^2*b^2*c^10 - 64*a^3*c^11)))/(b^2*c^4 - 4*a*c^5))^(
2/3)*sqrt((2*(a^2*b^4 - 4*a^3*b^2*c + 2*a^4*c^2)*x^2 + (1/2)^(2/3)*(b^8 - 1
0*a*b^6*c + 34*a^2*b^4*c^2 - 44*a^3*b^2*c^3 + 16*a^4*c^4 + (b^7*c^4 - 12*a*
b^5*c^5 + 48*a^2*b^3*c^6 - 64*a^3*b*c^7)*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4
*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(b^6*c^8 - 12*a*b^4*c^9 + 48*a^2*b^2*c^1
0 - 64*a^3*c^11)))*(-(b^3 - 2*a*b*c - (b^2*c^4 - 4*a*c^5)*sqrt((b^8 - 8*a*b
^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(b^6*c^8 - 12*a*b^4*c^9
 + 48*a^2*b^2*c^10 - 64*a^3*c^11)))/(b^2*c^4 - 4*a*c^5))^(2/3) - (1/2)^(1/3
```

```
)*((a*b^5*c^4 - 8*a^2*b^3*c^5 + 16*a^3*b*c^6)*x*sqrt((b^8 - 8*a*b^6*c + 20*
a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(b^6*c^8 - 12*a*b^4*c^9 + 48*a^2*
b^2*c^10 - 64*a^3*c^11)) + (a*b^6 - 8*a^2*b^4*c + 18*a^3*b^2*c^2 - 8*a^4*c^
3)*x)*(-(b^3 - 2*a*b*c - (b^2*c^4 - 4*a*c^5)*sqrt((b^8 - 8*a*b^6*c + 20*a^2
*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(b^6*c^8 - 12*a*b^4*c^9 + 48*a^2*b^2
*c^10 - 64*a^3*c^11)))/(b^2*c^4 - 4*a*c^5))^(1/3))/(a^2*b^4 - 4*a^3*b^2*c +
 2*a^4*c^2)) - 2*sqrt(3)*(a^3*b^4 - 4*a^4*b^2*c + 2*a^5*c^2))/(a^3*b^4 - 4*
a^4*b^2*c + 2*a^5*c^2)) - (1/2)^(1/3)*c*(-(b^3 - 2*a*b*c + (b^2*c^4 - 4*a*c
^5)*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(b
^6*c^8 - 12*a*b^4*c^9 + 48*a^2*b^2*c^10 - 64*a^3*c^11)))/(b^2*c^4 - 4*a*c^5
))^(1/3)*log(2*(a^2*b^4 - 4*a^3*b^2*c + 2*a^4*c^2)*x^2 + (1/2)^(2/3)*(b^8 -
 10*a*b^6*c + 34*a^2*b^4*c^2 - 44*a^3*b^2*c^3 + 16*a^4*c^4 - (b^7*c^4 - 12*
a*b^5*c^5 + 48*a^2*b^3*c^6 - 64*a^3*b*c^7)*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b
^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(b^6*c^8 - 12*a*b^4*c^9 + 48*a^2*b^2*c
^10 - 64*a^3*c^11)))*(-(b^3 - 2*a*b*c + (b^2*c^4 - 4*a*c^5)*sqrt((b^8 - 8*a
*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(b^6*c^8 - 12*a*b^4*c
^9 + 48*a^2*b^2*c^10 - 64*a^3*c^11)))/(b^2*c^4 - 4*a*c^5))^(2/3) + (1/2)^(1
/3)*((a*b^5*c^4 - 8*a^2*b^3*c^5 + 16*a^3*b*c^6)*x*sqrt((b^8 - 8*a*b^6*c + 2
0*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(b^6*c^8 - 12*a*b^4*c^9 + 48*a^
2*b^2*c^10 - 64*a^3*c^11)) - (a*b^6 - 8*a^2*b^4*c + 18*a^3*b^2*c^2 - 8*a^4*
c^3)*x)*(-(b^3 - 2*a*b*c + (b^2*c^4 - 4*a*c^5)*sqrt((b^8 - 8*a*b^6*c + 20*a
^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(b^6*c^8 - 12*a*b^4*c^9 + 48*a^2*b
^2*c^10 - 64*a^3*c^11)))/(b^2*c^4 - 4*a*c^5))^(1/3)) - (1/2)^(1/3)*c*(-(b^3
 - 2*a*b*c - (b^2*c^4 - 4*a*c^5)*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 1
6*a^3*b^2*c^3 + 4*a^4*c^4)/(b^6*c^8 - 12*a*b^4*c^9 + 48*a^2*b^2*c^10 - 64*a
^3*c^11)))/(b^2*c^4 - 4*a*c^5))^(1/3)*log(2*(a^2*b^4 - 4*a^3*b^2*c + 2*a^4*
c^2)*x^2 + (1/2)^(2/3)*(b^8 - 10*a*b^6*c + 34*a^2*b^4*c^2 - 44*a^3*b^2*c^3
+ 16*a^4*c^4 + (b^7*c^4 - 12*a*b^5*c^5 + 48*a^2*b^3*c^6 - 64*a^3*b*c^7)*sqr
t((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(b^6*c^8
- 12*a*b^4*c^9 + 48*a^2*b^2*c^10 - 64*a^3*c^11)))*(-(b^3 - 2*a*b*c - (b^2*c
^4 - 4*a*c^5)*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a
^4*c^4)/(b^6*c^8 - 12*a*b^4*c^9 + 48*a^2*b^2*c^10 - 64*a^3*c^11)))/(b^2*c^4
 - 4*a*c^5))^(2/3) - (1/2)^(1/3)*((a*b^5*c^4 - 8*a^2*b^3*c^5 + 16*a^3*b*c^6
)*x*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(b
^6*c^8 - 12*a*b^4*c^9 + 48*a^2*b^2*c^10 - 64*a^3*c^11)) + (a*b^6 - 8*a^2*b^
4*c + 18*a^3*b^2*c^2 - 8*a^4*c^3)*x)*(-(b^3 - 2*a*b*c - (b^2*c^4 - 4*a*c^5)
*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(b^6*
c^8 - 12*a*b^4*c^9 + 48*a^2*b^2*c^10 - 64*a^3*c^11)))/(b^2*c^4 - 4*a*c^5))^
(1/3)) + 2*(1/2)^(1/3)*c*(-(b^3 - 2*a*b*c + (b^2*c^4 - 4*a*c^5)*sqrt((b^8 -
 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(b^6*c^8 - 12*a*b
^4*c^9 + 48*a^2*b^2*c^10 - 64*a^3*c^11)))/(b^2*c^4 - 4*a*c^5))^(1/3)*log(2*
(a*b^4 - 4*a^2*b^2*c + 2*a^3*c^2)*x + (1/2)^(1/3)*(b^6 - 8*a*b^4*c + 18*a^2
*b^2*c^2 - 8*a^3*c^3 - (b^5*c^4 - 8*a*b^3*c^5 + 16*a^2*b*c^6)*sqrt((b^8 - 8
*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(b^6*c^8 - 12*a*b^4
*c^9 + 48*a^2*b^2*c^10 - 64*a^3*c^11)))*(-(b^3 - 2*a*b*c + (b^2*c^4 - 4*a*c
^5)*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(b
^6*c^8 - 12*a*b^4*c^9 + 48*a^2*b^2*c^10 - 64*a^3*c^11)))/(b^2*c^4 - 4*a*c^5
))^(1/3)) + 2*(1/2)^(1/3)*c*(-(b^3 - 2*a*b*c - (b^2*c^4 - 4*a*c^5)*sqrt((b^
8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(b^6*c^8 - 12*
a*b^4*c^9 + 48*a^2*b^2*c^10 - 64*a^3*c^11)))/(b^2*c^4 - 4*a*c^5))^(1/3)*log
(2*(a*b^4 - 4*a^2*b^2*c + 2*a^3*c^2)*x + (1/2)^(1/3)*(b^6 - 8*a*b^4*c + 18*
a^2*b^2*c^2 - 8*a^3*c^3 + (b^5*c^4 - 8*a*b^3*c^5 + 16*a^2*b*c^6)*sqrt((b^8
- 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(b^6*c^8 - 12*a*
b^4*c^9 + 48*a^2*b^2*c^10 - 64*a^3*c^11)))*(-(b^3 - 2*a*b*c - (b^2*c^4 - 4*
a*c^5)*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)
/(b^6*c^8 - 12*a*b^4*c^9 + 48*a^2*b^2*c^10 - 64*a^3*c^11)))/(b^2*c^4 - 4*a*
c^5))^(1/3)) + 6*x)/c
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{c + \frac{b}{x^3} + \frac{a}{x^6}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x^6+b/x^3),x, algorithm="giac")
```

```
[Out] integrate(1/(c + b/x^3 + a/x^6), x)
```

maple [C] time = 0.01, size = 59, normalized size = 0.09

$$\frac{x}{c} + \frac{\left(-\operatorname{RootOf}\left(\_Z^6 c + \_Z^3 b + a\right)^3 b - a\right) \ln\left(-\operatorname{RootOf}\left(\_Z^6 c + \_Z^3 b + a\right) + x\right)}{3c\left(2\operatorname{RootOf}\left(\_Z^6 c + \_Z^3 b + a\right)^5 c + \operatorname{RootOf}\left(\_Z^6 c + \_Z^3 b + a\right)^2 b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c+a/x^6+b/x^3),x)
```

```
[Out] 1/c*x+1/3/c*sum((-_R^3*b-a)/(2*_R^5*c+_R^2*b)*ln(-_R+x),_R=RootOf(_Z^6*c+_Z
^3*b+a))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{x}{c} - \frac{\int \frac{bx^3+a}{cx^6+bx^3+a}\, dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x^6+b/x^3),x, algorithm="maxima")
```

```
[Out] x/c - integrate((b*x^3 + a)/(c*x^6 + b*x^3 + a), x)/c
```

mupad [B] time = 4.54, size = 2280, normalized size = 3.61

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c + a/x^6 + b/x^3),x)
```

```
[Out] log((3*a^2*x*(b^4 + 2*a^2*c^2 - 4*a*b^2*c))/c - (3*2^(2/3)*a*(-(b^4*(-(4*a*
c - b^2)^3)^(1/2) - b^7 + 32*a^3*b*c^3 - 32*a^2*b^3*c^2 + 2*a^2*c^2*(-(4*a*
c - b^2)^3)^(1/2) + 10*a*b^5*c - 4*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(c^4*(
4*a*c - b^2)^3))^(1/3)*(b^4 + 2*a^2*c^2 - 4*a*b^2*c)*(b*(-(4*a*c - b^2)^3)^
(1/2) + b^4 + 16*a^2*c^2 - 8*a*b^2*c))/(4*c*(4*a*c - b^2)))*(-(b^4*(-(4*a*c
 - b^2)^3)^(1/2) - b^7 + 32*a^3*b*c^3 - 32*a^2*b^3*c^2 + 2*a^2*c^2*(-(4*a*c
 - b^2)^3)^(1/2) + 10*a*b^5*c - 4*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(54*(64
*a^3*c^7 - b^6*c^4 + 12*a*b^4*c^5 - 48*a^2*b^2*c^6)))^(1/3) + x/c + log((3*
a^2*x*(b^4 + 2*a^2*c^2 - 4*a*b^2*c))/c + (3*2^(2/3)*a*((b^7 + b^4*(-(4*a*c
- b^2)^3)^(1/2) - 32*a^3*b*c^3 + 32*a^2*b^3*c^2 + 2*a^2*c^2*(-(4*a*c - b^2)
^3)^(1/2) - 10*a*b^5*c - 4*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(c^4*(4*a*c -
b^2)^3))^(1/3)*(b^4 + 2*a^2*c^2 - 4*a*b^2*c)*(b*(-(4*a*c - b^2)^3)^(1/2) -
b^4 - 16*a^2*c^2 + 8*a*b^2*c))/(4*c*(4*a*c - b^2)))*((b^7 + b^4*(-(4*a*c -
b^2)^3)^(1/2) - 32*a^3*b*c^3 + 32*a^2*b^3*c^2 + 2*a^2*c^2*(-(4*a*c - b^2)^3
)^(1/2) - 10*a*b^5*c - 4*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(54*(64*a^3*c^7
- b^6*c^4 + 12*a*b^4*c^5 - 48*a^2*b^2*c^6)))^(1/3) + log((3*a^2*x*(b^4 + 2*
a^2*c^2 - 4*a*b^2*c))/c + (3*2^(2/3)*a*(3^(1/2)*1i - 1)*((b^7 + b^4*(-(4*a*
c - b^2)^3)^(1/2) - 32*a^3*b*c^3 + 32*a^2*b^3*c^2 + 2*a^2*c^2*(-(4*a*c - b^
2)^3)^(1/2) - 10*a*b^5*c - 4*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(c^4*(4*a*c
```

```
- b^2)^3))^(1/3)*(b^4 + 2*a^2*c^2 - 4*a*b^2*c)*(b*(-(4*a*c - b^2)^3)^(1/2)
- b^4 - 16*a^2*c^2 + 8*a*b^2*c))/(8*c*(4*a*c - b^2)))*((3^(1/2)*1i)/2 - 1/2
)*((b^7 + b^4*(-(4*a*c - b^2)^3)^(1/2) - 32*a^3*b*c^3 + 32*a^2*b^3*c^2 + 2*
a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^5*c - 4*a*b^2*c*(-(4*a*c - b^2)^3
)^(1/2))/(54*(64*a^3*c^7 - b^6*c^4 + 12*a*b^4*c^5 - 48*a^2*b^2*c^6)))^(1/3)
 - log((3*a^2*x*(b^4 + 2*a^2*c^2 - 4*a*b^2*c))/c - (3*2^(2/3)*a*(3^(1/2)*1i
 + 1)*((b^7 + b^4*(-(4*a*c - b^2)^3)^(1/2) - 32*a^3*b*c^3 + 32*a^2*b^3*c^2
+ 2*a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^5*c - 4*a*b^2*c*(-(4*a*c - b^
2)^3)^(1/2))/(c^4*(4*a*c - b^2)^3))^(1/3)*(b^4 + 2*a^2*c^2 - 4*a*b^2*c)*(b*
(-(4*a*c - b^2)^3)^(1/2) - b^4 - 16*a^2*c^2 + 8*a*b^2*c))/(8*c*(4*a*c - b^2
)))*((3^(1/2)*1i)/2 + 1/2)*((b^7 + b^4*(-(4*a*c - b^2)^3)^(1/2) - 32*a^3*b*
c^3 + 32*a^2*b^3*c^2 + 2*a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^5*c - 4*
a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(54*(64*a^3*c^7 - b^6*c^4 + 12*a*b^4*c^5
- 48*a^2*b^2*c^6)))^(1/3) + log((3*a^2*x*(b^4 + 2*a^2*c^2 - 4*a*b^2*c))/c -
 (3*2^(2/3)*a*(3^(1/2)*1i - 1)*(-(b^4*(-(4*a*c - b^2)^3)^(1/2) - b^7 + 32*a
^3*b*c^3 - 32*a^2*b^3*c^2 + 2*a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) + 10*a*b^5*c
 - 4*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(c^4*(4*a*c - b^2)^3))^(1/3)*(b^4 +
2*a^2*c^2 - 4*a*b^2*c)*(b*(-(4*a*c - b^2)^3)^(1/2) + b^4 + 16*a^2*c^2 - 8*a
*b^2*c))/(8*c*(4*a*c - b^2)))*((3^(1/2)*1i)/2 - 1/2)*(-(b^4*(-(4*a*c - b^2)
^3)^(1/2) - b^7 + 32*a^3*b*c^3 - 32*a^2*b^3*c^2 + 2*a^2*c^2*(-(4*a*c - b^2)
^3)^(1/2) + 10*a*b^5*c - 4*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(54*(64*a^3*c^
7 - b^6*c^4 + 12*a*b^4*c^5 - 48*a^2*b^2*c^6)))^(1/3) - log((3*a^2*x*(b^4 +
2*a^2*c^2 - 4*a*b^2*c))/c + (3*2^(2/3)*a*(3^(1/2)*1i + 1)*(-(b^4*(-(4*a*c -
 b^2)^3)^(1/2) - b^7 + 32*a^3*b*c^3 - 32*a^2*b^3*c^2 + 2*a^2*c^2*(-(4*a*c -
 b^2)^3)^(1/2) + 10*a*b^5*c - 4*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(c^4*(4*a
*c - b^2)^3))^(1/3)*(b^4 + 2*a^2*c^2 - 4*a*b^2*c)*(b*(-(4*a*c - b^2)^3)^(1/
2) + b^4 + 16*a^2*c^2 - 8*a*b^2*c))/(8*c*(4*a*c - b^2)))*((3^(1/2)*1i)/2 +
1/2)*(-(b^4*(-(4*a*c - b^2)^3)^(1/2) - b^7 + 32*a^3*b*c^3 - 32*a^2*b^3*c^2
+ 2*a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) + 10*a*b^5*c - 4*a*b^2*c*(-(4*a*c - b^
2)^3)^(1/2))/(54*(64*a^3*c^7 - b^6*c^4 + 12*a*b^4*c^5 - 48*a^2*b^2*c^6)))^(
1/3)
```

sympy [A] time = 6.97, size = 196, normalized size = 0.31

$$\operatorname{RootSum}\left(t^6\left(46656a^3c^7-34992a^2b^2c^6+8748ab^4c^5-729b^6c^4\right)+t^3\left(864a^3bc^3-864a^2b^3c^2+270ab^5c-27b^7\right)+a^4,\left(t\mapsto t\log\left(x+\frac{1296t^4a^2bc^6-648t^4ab^3c^5+81t^4b^5c^4-12ta^3c^3+39ta^2b^2c^2-21tab^4c+3tb^6}{2a^3c^2-4a^2b^2c+ab^4}\right)\right)\right)+\frac{x}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x**6+b/x**3),x)
```

```
[Out] RootSum(_t**6*(46656*a**3*c**7 - 34992*a**2*b**2*c**6 + 8748*a*b**4*c**5 -
729*b**6*c**4) + _t**3*(864*a**3*b*c**3 - 864*a**2*b**3*c**2 + 270*a*b**5*c
 - 27*b**7) + a**4, Lambda(_t, _t*log(x + (1296*_t**4*a**2*b*c**6 - 648*_t*
*4*a*b**3*c**5 + 81*_t**4*b**5*c**4 - 12*_t*a**3*c**3 + 39*_t*a**2*b**2*c**
2 - 21*_t*a*b**4*c + 3*_t*b**6)/(2*a**3*c**2 - 4*a**2*b**2*c + a*b**4)))) +
 x/c
```

3.397 $\int \frac{1}{c+\frac{a}{x^8}+\frac{b}{x^4}}\,dx$

Optimal. Leaf size=376

$$\frac{\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\,x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2\sqrt[4]{2}\,c^{5/4}\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}+\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\,x}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{2\sqrt[4]{2}\,c^{5/4}\left(\sqrt{b^2-4ac}-b\right)^{3/4}}+\frac{\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\,x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2\sqrt[4]{2}\,c^{5/4}\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}+$$

Rubi [A] time = 0.67, antiderivative size = 376, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {1340, 1367, 1422, 212, 208, 205}

$$\frac{\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\,x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2\sqrt[4]{2}\,c^{5/4}\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}+\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\,x}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{2\sqrt[4]{2}\,c^{5/4}\left(\sqrt{b^2-4ac}-b\right)^{3/4}}+\frac{\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\,x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2\sqrt[4]{2}\,c^{5/4}\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}+\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\,x}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{2\sqrt[4]{2}\,c^{5/4}\left(\sqrt{b^2-4ac}-b\right)^{3/4}}+\frac{x}{c}$$

Antiderivative was successfully verified.

```
[In] Int[(c + a/x^8 + b/x^4)^(-1),x]
```

```
[Out] x/c + ((b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(2^(1/4)*c^(1/4)*x)/(-b
 - Sqrt[b^2 - 4*a*c])^(1/4)])/(2*2^(1/4)*c^(5/4)*(-b - Sqrt[b^2 - 4*a*c])^(
3/4)) + ((b - (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(2^(1/4)*c^(1/4)*x)/(
-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(2*2^(1/4)*c^(5/4)*(-b + Sqrt[b^2 - 4*a*c])
^(3/4)) + ((b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTanh[(2^(1/4)*c^(1/4)*x
)/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(2*2^(1/4)*c^(5/4)*(-b - Sqrt[b^2 - 4*a*
c])^(3/4)) + ((b - (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTanh[(2^(1/4)*c^(1/4
)*x)/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(2*2^(1/4)*c^(5/4)*(-b + Sqrt[b^2 - 4
*a*c])^(3/4))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 1340

```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[x^
(2*n*p)*(c + b/x^n + a/x^(2*n))^p, x] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n
] && LtQ[n, 0] && IntegerQ[p]
```

Rule 1367

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Simp[(d^(2*n - 1)*(d*x)^(m - 2*n + 1)*(a + b*x^n + c*x^(2*n))^(
p + 1))/(c*(m + 2*n*p + 1)), x] - Dist[d^(2*n)/(c*(m + 2*n*p + 1)), Int[(d*
```

```
x)^(m - 2*n)*Simp[a*(m - 2*n + 1) + b*(m + n*(p - 1) + 1)*x^n, x]*(a + b*x^
n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && EqQ[n2, 2*n] && Ne
Q[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GtQ[m, 2*n - 1] && NeQ[m + 2*n*p + 1, 0]
 && IntegerQ[p]
```

Rule 1422

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q),
Int[1/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(
b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (PosQ[b^2 - 4*a
*c] ||  !IGtQ[n/2, 0])
```

Rubi steps

$$\int \frac{1}{c+\frac{a}{x^8}+\frac{b}{x^4}}\,dx = \int \frac{x^8}{a+bx^4+cx^8}\,dx$$

$$= \frac{x}{c} - \frac{\int \frac{a+bx^4}{a+bx^4+cx^8}\,dx}{c}$$

$$= \frac{x}{c} - \frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^4}\,dx}{2c} - \frac{\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^4}\,dx}{2c}$$

$$= \frac{x}{c} + \frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\sqrt{-b+\sqrt{b^2-4ac}}-\sqrt{2}\sqrt{c}x^2}\,dx}{2c\sqrt{-b+\sqrt{b^2-4ac}}} + \frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\sqrt{-b+\sqrt{b^2-4ac}}+\sqrt{2}\sqrt{c}x^2}\,dx}{2c\sqrt{-b+\sqrt{b^2-4ac}}} - \cdots$$

$$= \frac{x}{c} + \frac{\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{2\sqrt[4]{2}c^{5/4}\left(-b-\sqrt{b^2-4ac}\right)^{3/4}} + \frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b+\sqrt{b^2-4ac}}}\right)}{2\sqrt[4]{2}c^{5/4}\left(-b+\sqrt{b^2-4ac}\right)^{3/4}} + \frac{\left(b+\frac{b^2\cdots}{\sqrt{b^2\cdots}}\right.}{2\sqrt[4]{2}\cdots}$$

Mathematica [C] time = 0.04, size = 70, normalized size = 0.19

$$\frac{x}{c} - \frac{\text{RootSum}\left[\#1^8c+\#1^4b+a\&,\frac{\#1^4b\log(x-\#1)+a\log(x-\#1)}{2\#1^7c+\#1^3b}\&\right]}{4c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + a/x^8 + b/x^4)^(-1),x]
```

```
[Out] x/c - RootSum[a + b*#1^4 + c*#1^8 & , (a*Log[x - #1] + b*Log[x - #1]*#1^4)/
(b*#1^3 + 2*c*#1^7) & ]/(4*c)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{c+\frac{a}{x^8}+\frac{b}{x^4}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + a/x^8 + b/x^4)^(-1),x]
```

```
[Out] IntegrateAlgebraic[(c + a/x^8 + b/x^4)^(-1), x]
```

fricas [B] time = 2.63, size = 5082, normalized size = 13.52

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x^8+b/x^4),x, algorithm="fricas")
```

```
[Out] -1/4*(4*c*sqrt(sqrt(1/2)*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 - (b^4*c^5 -
8*a*b^2*c^6 + 16*a^2*c^7)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^
2*c^3 + a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)
))/(b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)))*arctan(1/4*(2*sqrt(1/2)*((b^10*c^
5 - 16*a*b^8*c^6 + 98*a^2*b^6*c^7 - 280*a^3*b^4*c^8 + 352*a^4*b^2*c^9 - 128
*a^5*c^10)*x*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c
^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)) + (b^11 - 1
3*a*b^9*c + 63*a^2*b^7*c^2 - 138*a^3*b^5*c^3 + 128*a^4*b^3*c^4 - 32*a^5*b*c
^5)*x)*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 - (b^4*c^5 - 8*a*b^2*c^6 + 16*a
^2*c^7)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(
b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))/(b^4*c^5 - 8*a*
b^2*c^6 + 16*a^2*c^7)) - (b^11 - 13*a*b^9*c + 63*a^2*b^7*c^2 - 138*a^3*b^5*
c^3 + 128*a^4*b^3*c^4 - 32*a^5*b*c^5 + (b^10*c^5 - 16*a*b^8*c^6 + 98*a^2*b^
6*c^7 - 280*a^3*b^4*c^8 + 352*a^4*b^2*c^9 - 128*a^5*c^10)*sqrt((b^8 - 6*a*b
^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^11
+ 48*a^2*b^2*c^12 - 64*a^3*c^13)))*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 - (
b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2
- 6*a^3*b^2*c^3 + a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64
*a^3*c^13)))/(b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7))*sqrt((2*(a^2*b^4 - 3*a^3
*b^2*c + a^4*c^2)*x^2 + sqrt(1/2)*(b^8 - 9*a*b^6*c + 27*a^2*b^4*c^2 - 30*a^
3*b^2*c^3 + 8*a^4*c^4 + (b^7*c^5 - 12*a*b^5*c^6 + 48*a^2*b^3*c^7 - 64*a^3*b
*c^8)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^
6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))*sqrt(-(b^5 - 5*a*
b^3*c + 5*a^2*b*c^2 - (b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)*sqrt((b^8 - 6*a*
b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^11
 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))/(b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)))/
(a^2*b^4 - 3*a^3*b^2*c + a^4*c^2)))*sqrt(sqrt(1/2)*sqrt(-(b^5 - 5*a*b^3*c +
 5*a^2*b*c^2 - (b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)*sqrt((b^8 - 6*a*b^6*c +
 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a
^2*b^2*c^12 - 64*a^3*c^13)))/(b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)))/(a^4*b^
4 - 3*a^5*b^2*c + a^6*c^2)) - 4*c*sqrt(sqrt(1/2)*sqrt(-(b^5 - 5*a*b^3*c + 5
*a^2*b*c^2 + (b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)*sqrt((b^8 - 6*a*b^6*c + 1
1*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2
*b^2*c^12 - 64*a^3*c^13)))/(b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)))*arctan(1/
4*(2*sqrt(1/2)*((b^10*c^5 - 16*a*b^8*c^6 + 98*a^2*b^6*c^7 - 280*a^3*b^4*c^8
 + 352*a^4*b^2*c^9 - 128*a^5*c^10)*x*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2
 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 6
4*a^3*c^13)) - (b^11 - 13*a*b^9*c + 63*a^2*b^7*c^2 - 138*a^3*b^5*c^3 + 128*
a^4*b^3*c^4 - 32*a^5*b*c^5)*x)*sqrt(sqrt(1/2)*sqrt(-(b^5 - 5*a*b^3*c + 5*a^
2*b*c^2 + (b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)*sqrt((b^8 - 6*a*b^6*c + 11*a
^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^
2*c^12 - 64*a^3*c^13)))/(b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)))*sqrt(-(b^5 -
 5*a*b^3*c + 5*a^2*b*c^2 + (b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)*sqrt((b^8 -
 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^6*c^10 - 12*a*b^4
*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))/(b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^
7)) + (b^11 - 13*a*b^9*c + 63*a^2*b^7*c^2 - 138*a^3*b^5*c^3 + 128*a^4*b^3*c
^4 - 32*a^5*b*c^5 - (b^10*c^5 - 16*a*b^8*c^6 + 98*a^2*b^6*c^7 - 280*a^3*b^4
*c^8 + 352*a^4*b^2*c^9 - 128*a^5*c^10)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c
^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 -
 64*a^3*c^13)))*sqrt(sqrt(1/2)*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 + (b^4*
c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*
a^3*b^2*c^3 + a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3
*c^13)))/(b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)))*sqrt(-(b^5 - 5*a*b^3*c + 5*
```

```
a^2*b*c^2 + (b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)*sqrt((b^8 - 6*a*b^6*c + 11
*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*
b^2*c^12 - 64*a^3*c^13)))/(b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7))*sqrt((2*(a^
2*b^4 - 3*a^3*b^2*c + a^4*c^2)*x^2 + sqrt(1/2)*(b^8 - 9*a*b^6*c + 27*a^2*b^
4*c^2 - 30*a^3*b^2*c^3 + 8*a^4*c^4 - (b^7*c^5 - 12*a*b^5*c^6 + 48*a^2*b^3*c
^7 - 64*a^3*b*c^8)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 +
 a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))*sqrt
(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 + (b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)*sqr
t((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^6*c^10 -
12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))/(b^4*c^5 - 8*a*b^2*c^6 + 1
6*a^2*c^7)))/(a^2*b^4 - 3*a^3*b^2*c + a^4*c^2)))/(a^4*b^4 - 3*a^5*b^2*c + a
^6*c^2)) - c*sqrt(sqrt(1/2)*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 + (b^4*c^5
 - 8*a*b^2*c^6 + 16*a^2*c^7)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3
*b^2*c^3 + a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^
13)))/(b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)))*log((a*b^4 - 3*a^2*b^2*c + a^3
*c^2)*x + 1/2*(b^6 - 7*a*b^4*c + 13*a^2*b^2*c^2 - 4*a^3*c^3 - (b^5*c^5 - 8*
a*b^3*c^6 + 16*a^2*b*c^7)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^
2*c^3 + a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)
))*sqrt(sqrt(1/2)*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 + (b^4*c^5 - 8*a*b^2
*c^6 + 16*a^2*c^7)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 +
 a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))/(b^4
*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)))) + c*sqrt(sqrt(1/2)*sqrt(-(b^5 - 5*a*b^3
*c + 5*a^2*b*c^2 + (b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)*sqrt((b^8 - 6*a*b^6
*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^11 +
48*a^2*b^2*c^12 - 64*a^3*c^13)))/(b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)))*log
((a*b^4 - 3*a^2*b^2*c + a^3*c^2)*x - 1/2*(b^6 - 7*a*b^4*c + 13*a^2*b^2*c^2
- 4*a^3*c^3 - (b^5*c^5 - 8*a*b^3*c^6 + 16*a^2*b*c^7)*sqrt((b^8 - 6*a*b^6*c
+ 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*
a^2*b^2*c^12 - 64*a^3*c^13)))*sqrt(sqrt(1/2)*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2
*b*c^2 + (b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)*sqrt((b^8 - 6*a*b^6*c + 11*a^
2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2
*c^12 - 64*a^3*c^13)))/(b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)))) - c*sqrt(sqr
t(1/2)*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 - (b^4*c^5 - 8*a*b^2*c^6 + 16*a
^2*c^7)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(
b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))/(b^4*c^5 - 8*a*
b^2*c^6 + 16*a^2*c^7)))*log((a*b^4 - 3*a^2*b^2*c + a^3*c^2)*x + 1/2*(b^6 -
7*a*b^4*c + 13*a^2*b^2*c^2 - 4*a^3*c^3 + (b^5*c^5 - 8*a*b^3*c^6 + 16*a^2*b*
c^7)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^6
*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))*sqrt(sqrt(1/2)*sqr
t(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 - (b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)*sq
rt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^6*c^10 -
 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))/(b^4*c^5 - 8*a*b^2*c^6 +
16*a^2*c^7)))) + c*sqrt(sqrt(1/2)*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 - (b
^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 -
 6*a^3*b^2*c^3 + a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*
a^3*c^13)))/(b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)))*log((a*b^4 - 3*a^2*b^2*c
 + a^3*c^2)*x - 1/2*(b^6 - 7*a*b^4*c + 13*a^2*b^2*c^2 - 4*a^3*c^3 + (b^5*c^
5 - 8*a*b^3*c^6 + 16*a^2*b*c^7)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*
a^3*b^2*c^3 + a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3
*c^13)))*sqrt(sqrt(1/2)*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 - (b^4*c^5 - 8
*a*b^2*c^6 + 16*a^2*c^7)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2
*c^3 + a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13))
)/(b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)))) - 4*x)/c
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x^8+b/x^4),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Evaluation time: 6.96Unable to convert to re
al 1/4 Error: Bad Argument Value
```

maple [C] time = 0.00, size = 59, normalized size = 0.16

$$\frac{x}{c}+\frac{\left(-\operatorname{RootOf}\left(\_Z^{8}c+\_Z^{4}b+a\right)^{4}b-a\right)\ln\left(-\operatorname{RootOf}\left(\_Z^{8}c+\_Z^{4}b+a\right)+x\right)}{4c\left(2\operatorname{RootOf}\left(\_Z^{8}c+\_Z^{4}b+a\right)^{7}c+\operatorname{RootOf}\left(\_Z^{8}c+\_Z^{4}b+a\right)^{3}b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c+a/x^8+b/x^4),x)
```

```
[Out] 1/c*x+1/4/c*sum((-_R^4*b-a)/(2*_R^7*c+_R^3*b)*ln(-_R+x),_R=RootOf(_Z^8*c+_Z
^4*b+a))
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x^8+b/x^4),x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [B] time = 3.78, size = 10382, normalized size = 27.61

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c + a/x^8 + b/x^4),x)
```

```
[Out] atan(((((16*(a^3*b^6 - 4*a^6*c^3 - 7*a^4*b^4*c + 13*a^5*b^2*c^2))/c - (4*x*
(-(b^9 + b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120
*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c - 3*a*b^2*c*(-
(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2
*b^4*c^7 - 256*a^3*b^2*c^8)))^(3/4)*(4096*a^5*b*c^6 + 256*a^3*b^5*c^4 - 204
8*a^4*b^3*c^5))/c)*(-(b^9 + b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 6
1*a^2*b^5*c^2 - 120*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b
^7*c - 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16
*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4) - (4*x*(a^4*b^4 + 2*
a^6*c^2 - 4*a^5*b^2*c))/c)*(-(b^9 + b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b
*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^(1/2)
- 13*a*b^7*c - 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*
c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4)*1i - (((16*(
a^3*b^6 - 4*a^6*c^3 - 7*a^4*b^4*c + 13*a^5*b^2*c^2))/c + (4*x*(-(b^9 + b^4*
(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3
+ a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c - 3*a*b^2*c*(-(4*a*c - b^2)
^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 25
6*a^3*b^2*c^8)))^(3/4)*(4096*a^5*b*c^6 + 256*a^3*b^5*c^4 - 2048*a^4*b^3*c^5
))/c)*(-(b^9 + b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2
 - 120*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c - 3*a*b^
2*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 +
96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4) + (4*x*(a^4*b^4 + 2*a^6*c^2 - 4*a
^5*b^2*c))/c)*(-(b^9 + b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2
*b^5*c^2 - 120*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c
- 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^
6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4)*1i)/((((16*(a^3*b^6 - 4*a
^6*c^3 - 7*a^4*b^4*c + 13*a^5*b^2*c^2))/c - (4*x*(-(b^9 + b^4*(-(4*a*c - b^
2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 + a^2*c^2*(-(
```

```
4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c - 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(5
12*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8
)))^(3/4)*(4096*a^5*b*c^6 + 256*a^3*b^5*c^4 - 2048*a^4*b^3*c^5))/c)*(-(b^9
+ b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^
3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c - 3*a*b^2*c*(-(4*a*c
- b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^
7 - 256*a^3*b^2*c^8)))^(1/4) - (4*x*(a^4*b^4 + 2*a^6*c^2 - 4*a^5*b^2*c))/c)
*(-(b^9 + b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 12
0*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c - 3*a*b^2*c*(
-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^
2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4) + (((16*(a^3*b^6 - 4*a^6*c^3 - 7*a^4*b
^4*c + 13*a^5*b^2*c^2))/c + (4*x*(-(b^9 + b^4*(-(4*a*c - b^2)^5)^(1/2) + 80
*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^
(1/2) - 13*a*b^7*c - 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9
+ b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(3/4)*(4096*
a^5*b*c^6 + 256*a^3*b^5*c^4 - 2048*a^4*b^3*c^5))/c)*(-(b^9 + b^4*(-(4*a*c -
 b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 + a^2*c^2*
(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c - 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))
/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*
c^8)))^(1/4) + (4*x*(a^4*b^4 + 2*a^6*c^2 - 4*a^5*b^2*c))/c)*(-(b^9 + b^4*(-
(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 +
a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c - 3*a*b^2*c*(-(4*a*c - b^2)^5
)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*
a^3*b^2*c^8)))^(1/4)))*(-(b^9 + b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4
 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13
*a*b^7*c - 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5
- 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4)*2i + atan(((((16
*(a^3*b^6 - 4*a^6*c^3 - 7*a^4*b^4*c + 13*a^5*b^2*c^2))/c - (4*x*(-(b^9 - b^
4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^
3 - a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c + 3*a*b^2*c*(-(4*a*c - b^
2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 -
256*a^3*b^2*c^8)))^(3/4)*(4096*a^5*b*c^6 + 256*a^3*b^5*c^4 - 2048*a^4*b^3*c
^5))/c)*(-(b^9 - b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c
^2 - 120*a^3*b^3*c^3 - a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c + 3*a*
b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6
+ 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4) - (4*x*(a^4*b^4 + 2*a^6*c^2 - 4
*a^5*b^2*c))/c)*(-(b^9 - b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a
^2*b^5*c^2 - 120*a^3*b^3*c^3 - a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*
c + 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*
b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4)*1i - (((16*(a^3*b^6 - 4
*a^6*c^3 - 7*a^4*b^4*c + 13*a^5*b^2*c^2))/c + (4*x*(-(b^9 - b^4*(-(4*a*c -
b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 - a^2*c^2*(
-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c + 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/
(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c
^8)))^(3/4)*(4096*a^5*b*c^6 + 256*a^3*b^5*c^4 - 2048*a^4*b^3*c^5))/c)*(-(b^
9 - b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*
b^3*c^3 - a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c + 3*a*b^2*c*(-(4*a*
c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*
c^7 - 256*a^3*b^2*c^8)))^(1/4) + (4*x*(a^4*b^4 + 2*a^6*c^2 - 4*a^5*b^2*c))/
c)*(-(b^9 - b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 -
120*a^3*b^3*c^3 - a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c + 3*a*b^2*c
*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*
a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4)*1i)/((((16*(a^3*b^6 - 4*a^6*c^3 - 7*
a^4*b^4*c + 13*a^5*b^2*c^2))/c - (4*x*(-(b^9 - b^4*(-(4*a*c - b^2)^5)^(1/2)
 + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 - a^2*c^2*(-(4*a*c - b^2
)^5)^(1/2) - 13*a*b^7*c + 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4
*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(3/4)*(
4096*a^5*b*c^6 + 256*a^3*b^5*c^4 - 2048*a^4*b^3*c^5))/c)*(-(b^9 - b^4*(-(4*
a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 - a^2
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*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c + 3*a*b^2*c*(-(4*a*c - b^2)^5)^(
1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3
*b^2*c^8)))^(1/4) - (4*x*(a^4*b^4 + 2*a^6*c^2 - 4*a^5*b^2*c))/c)*(-(b^9 - b
^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c
^3 - a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c + 3*a*b^2*c*(-(4*a*c - b
^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 -
 256*a^3*b^2*c^8)))^(1/4) + (((16*(a^3*b^6 - 4*a^6*c^3 - 7*a^4*b^4*c + 13*a
^5*b^2*c^2))/c + (4*x*(-(b^9 - b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4
+ 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 - a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*
a*b^7*c + 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 -
 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(3/4)*(4096*a^5*b*c^6 +
 256*a^3*b^5*c^4 - 2048*a^4*b^3*c^5))/c)*(-(b^9 - b^4*(-(4*a*c - b^2)^5)^(1
/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 - a^2*c^2*(-(4*a*c -
b^2)^5)^(1/2) - 13*a*b^7*c + 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*
a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4
) + (4*x*(a^4*b^4 + 2*a^6*c^2 - 4*a^5*b^2*c))/c)*(-(b^9 - b^4*(-(4*a*c - b^
2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 - a^2*c^2*(-(
4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c + 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(5
12*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8
)))^(1/4)))*(-(b^9 - b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b
^5*c^2 - 120*a^3*b^3*c^3 - a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c +
3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*
c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4)*2i - 2*atan(((((16*(a^3*b^6
 - 4*a^6*c^3 - 7*a^4*b^4*c + 13*a^5*b^2*c^2))/c - (x*(-(b^9 + b^4*(-(4*a*c
- b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 + a^2*c^2
*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c - 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2)
)/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2
*c^8)))^(3/4)*(4096*a^5*b*c^6 + 256*a^3*b^5*c^4 - 2048*a^4*b^3*c^5)*4i)/c)*
(-(b^9 + b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120
*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c - 3*a*b^2*c*(-
(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2
*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4)*1i + (4*x*(a^4*b^4 + 2*a^6*c^2 - 4*a^5*
b^2*c))/c)*(-(b^9 + b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^
5*c^2 - 120*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c - 3
*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c
^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4) - (((16*(a^3*b^6 - 4*a^6*c^3
 - 7*a^4*b^4*c + 13*a^5*b^2*c^2))/c + (x*(-(b^9 + b^4*(-(4*a*c - b^2)^5)^(1
/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c -
b^2)^5)^(1/2) - 13*a*b^7*c - 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*
a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(3/4
)*(4096*a^5*b*c^6 + 256*a^3*b^5*c^4 - 2048*a^4*b^3*c^5)*4i)/c)*(-(b^9 + b^4
*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3
 + a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c - 3*a*b^2*c*(-(4*a*c - b^2
)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 2
56*a^3*b^2*c^8)))^(1/4)*1i - (4*x*(a^4*b^4 + 2*a^6*c^2 - 4*a^5*b^2*c))/c)*(
-(b^9 + b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*
a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c - 3*a*b^2*c*(-(
4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*
b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4))/((((16*(a^3*b^6 - 4*a^6*c^3 - 7*a^4*b^4
*c + 13*a^5*b^2*c^2))/c - (x*(-(b^9 + b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4
*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^(1/2
) - 13*a*b^7*c - 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^
8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(3/4)*(4096*a^5*
b*c^6 + 256*a^3*b^5*c^4 - 2048*a^4*b^3*c^5)*4i)/c)*(-(b^9 + b^4*(-(4*a*c -
b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 + a^2*c^2*(
-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c - 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/
(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c
^8)))^(1/4)*1i + (4*x*(a^4*b^4 + 2*a^6*c^2 - 4*a^5*b^2*c))/c)*(-(b^9 + b^4*
(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3
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+ a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c - 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4)*1i + (((16*(a^3*b^6 - 4*a^6*c^3 - 7*a^4*b^4*c + 13*a^5*b^2*c^2))/c + (x*(-(b^9 + b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c - 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(3/4)*(4096*a^5*b*c^6 + 256*a^3*b^5*c^4 - 2048*a^4*b^3*c^5)*4i)/c)*(-(b^9 + b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c - 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4)*1i - (4*x*(a^4*b^4 + 2*a^6*c^2 - 4*a^5*b^2*c))/c)*(-(b^9 + b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c - 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4)*1i))*(-(b^9 + b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c - 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4) - 2*atan(((((16*(a^3*b^6 - 4*a^6*c^3 - 7*a^4*b^4*c + 13*a^5*b^2*c^2))/c - (x*(-(b^9 - b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 - a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c + 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(3/4)*(4096*a^5*b*c^6 + 256*a^3*b^5*c^4 - 2048*a^4*b^3*c^5)*4i)/c)*(-(b^9 - b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 - a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c + 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4)*1i + (4*x*(a^4*b^4 + 2*a^6*c^2 - 4*a^5*b^2*c))/c)*(-(b^9 - b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 - a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c + 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4) - (((16*(a^3*b^6 - 4*a^6*c^3 - 7*a^4*b^4*c + 13*a^5*b^2*c^2))/c + (x*(-(b^9 - b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 - a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c + 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(3/4)*(4096*a^5*b*c^6 + 256*a^3*b^5*c^4 - 2048*a^4*b^3*c^5)*4i)/c)*(-(b^9 - b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 - a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c + 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4)*1i - (4*x*(a^4*b^4 + 2*a^6*c^2 - 4*a^5*b^2*c))/c)*(-(b^9 - b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 - a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c + 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4))/((((16*(a^3*b^6 - 4*a^6*c^3 - 7*a^4*b^4*c + 13*a^5*b^2*c^2))/c - (x*(-(b^9 - b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 - a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c + 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(3/4)*(4096*a^5*b*c^6 + 256*a^3*b^5*c^4 - 2048*a^4*b^3*c^5)*4i)/c)*(-(b^9 - b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 - a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c + 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4)*1i + (4*x*(a^4*b^4 + 2*a^6*c^2 - 4*a^5*b^2*c))/c)*(-(b^9 - b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 - a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c + 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4)*1i + (((16*(a^3*b^6 - 4*a^6*c^3 - 7*a^4*b^4*c + 13*a^5*b^2*c^2))/c + (x*(-(b^9 - b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^

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4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 - a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 1
3*a*b^7*c + 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5
 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(3/4)*(4096*a^5*b*c^6
 + 256*a^3*b^5*c^4 - 2048*a^4*b^3*c^5)*4i)/c)*(-(b^9 - b^4*(-(4*a*c - b^2)^
5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 - a^2*c^2*(-(4*a
*c - b^2)^5)^(1/2) - 13*a*b^7*c + 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(512*
(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))
^(1/4)*1i - (4*x*(a^4*b^4 + 2*a^6*c^2 - 4*a^5*b^2*c))/c)*(-(b^9 - b^4*(-(4*
a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 - a^2
*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c + 3*a*b^2*c*(-(4*a*c - b^2)^5)^(
1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3
*b^2*c^8)))^(1/4)*1i))*(-(b^9 - b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4
 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 - a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13
*a*b^7*c + 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5
- 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4) + x/c
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x**8+b/x**4),x)
```

```
[Out] Timed out
```

3.398 $\int \frac{\sqrt{a+b\sqrt{x}+cx}}{x}\,dx$

Optimal. Leaf size=106

$$2\sqrt{a+b\sqrt{x}+cx}-2\sqrt{a}\tanh^{-1}\left(\frac{2a+b\sqrt{x}}{2\sqrt{a}\sqrt{a+b\sqrt{x}+cx}}\right)+\frac{b\tanh^{-1}\left(\frac{b+2c\sqrt{x}}{2\sqrt{c}\sqrt{a+b\sqrt{x}+cx}}\right)}{\sqrt{c}}$$

Rubi [A] time = 0.09, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {1357, 734, 843, 621, 206, 724}

$$2\sqrt{a+b\sqrt{x}+cx}-2\sqrt{a}\tanh^{-1}\left(\frac{2a+b\sqrt{x}}{2\sqrt{a}\sqrt{a+b\sqrt{x}+cx}}\right)+\frac{b\tanh^{-1}\left(\frac{b+2c\sqrt{x}}{2\sqrt{c}\sqrt{a+b\sqrt{x}+cx}}\right)}{\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Sqrt[x] + c*x]/x,x]
```

```
[Out] 2*Sqrt[a + b*Sqrt[x] + c*x] - 2*Sqrt[a]*ArcTanh[(2*a + b*Sqrt[x])/(2*Sqrt[a
]*Sqrt[a + b*Sqrt[x] + c*x])] + (b*ArcTanh[(b + 2*c*Sqrt[x])/(2*Sqrt[c]*Sqr
t[a + b*Sqrt[x] + c*x])])/Sqrt[c]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 734

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 2*p + 1)), x
] - Dist[p/(e*(m + 2*p + 1)), Int[(d + e*x)^m*Simp[b*d - 2*a*e + (2*c*d - b
*e)*x, x]*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e
, 0] && GtQ[p, 0] && NeQ[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &
&  !ILtQ[m + 2*p, 0] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
```

```
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] && !IGtQ[m, 0]
```

Rule 1357

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{a+b\sqrt{x}+cx}}{x}\,dx &= 2\,\text{Subst}\left(\int \frac{\sqrt{a+bx+cx^2}}{x}\,dx, x, \sqrt{x}\right)\\ &= 2\sqrt{a+b\sqrt{x}+cx} - \text{Subst}\left(\int \frac{-2a-bx}{x\sqrt{a+bx+cx^2}}\,dx, x, \sqrt{x}\right)\\ &= 2\sqrt{a+b\sqrt{x}+cx} + (2a)\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx+cx^2}}\,dx, x, \sqrt{x}\right) + b\,\text{Subst}\left(\int \frac{1}{\sqrt{a+bx+\cdots}}\right.\\ &= 2\sqrt{a+b\sqrt{x}+cx} - (4a)\,\text{Subst}\left(\int \frac{1}{4a-x^2}\,dx, x, \frac{2a+b\sqrt{x}}{\sqrt{a+b\sqrt{x}+cx}}\right) + (2b)\,\text{Subst}\left(\int \frac{\cdots}{4c\cdots}\right.\\ &= 2\sqrt{a+b\sqrt{x}+cx} - 2\sqrt{a}\tanh^{-1}\left(\frac{2a+b\sqrt{x}}{2\sqrt{a}\sqrt{a+b\sqrt{x}+cx}}\right) + \frac{b\tanh^{-1}\left(\frac{b+2c\sqrt{x}}{2\sqrt{c}\sqrt{a+b\sqrt{x}+cx}}\right)}{\sqrt{c}}\end{aligned}$$

Mathematica [A] time = 0.06, size = 106, normalized size = 1.00

$$2\sqrt{a+b\sqrt{x}+cx} - 2\sqrt{a}\tanh^{-1}\left(\frac{2a+b\sqrt{x}}{2\sqrt{a}\sqrt{a+b\sqrt{x}+cx}}\right) + \frac{b\tanh^{-1}\left(\frac{b+2c\sqrt{x}}{2\sqrt{c}\sqrt{a+b\sqrt{x}+cx}}\right)}{\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*Sqrt[x] + c*x]/x,x]
```

```
[Out] 2*Sqrt[a + b*Sqrt[x] + c*x] - 2*Sqrt[a]*ArcTanh[(2*a + b*Sqrt[x])/(2*Sqrt[a
]*Sqrt[a + b*Sqrt[x] + c*x])] + (b*ArcTanh[(b + 2*c*Sqrt[x])/(2*Sqrt[c]*Sqr
t[a + b*Sqrt[x] + c*x])])/Sqrt[c]
```

IntegrateAlgebraic [A] time = 0.24, size = 109, normalized size = 1.03

$$2\sqrt{a+b\sqrt{x}+cx} - \frac{b\log\left(-2\sqrt{c}\sqrt{a+b\sqrt{x}+cx}+b+2c\sqrt{x}\right)}{\sqrt{c}} + 4\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{a}} - \frac{\sqrt{a+b\sqrt{x}+cx}}{\sqrt{a}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[a + b*Sqrt[x] + c*x]/x,x]
```

```
[Out] 2*Sqrt[a + b*Sqrt[x] + c*x] + 4*Sqrt[a]*ArcTanh[(Sqrt[c]*Sqrt[x])/Sqrt[a] -
 Sqrt[a + b*Sqrt[x] + c*x]/Sqrt[a]] - (b*Log[b + 2*c*Sqrt[x] - 2*Sqrt[c]*Sq
rt[a + b*Sqrt[x] + c*x]])/Sqrt[c]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+c*x+b*x^(1/2))^(1/2)/x,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+c*x+b*x^(1/2))^(1/2)/x,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:index.cc index_m operator + Error: Bad Argum
ent Value
```

maple [A] time = 0.01, size = 84, normalized size = 0.79

$$-2\sqrt{a}\,\ln\left(\frac{b\sqrt{x}+2a+2\sqrt{cx+b\sqrt{x}+a}\,\sqrt{a}}{\sqrt{x}}\right)+\frac{b\ln\left(\frac{c\sqrt{x}+\frac{b}{2}}{\sqrt{c}}+\sqrt{cx+b\sqrt{x}+a}\right)}{\sqrt{c}}+2\sqrt{cx+b\sqrt{x}+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+c*x+b*x^(1/2))^(1/2)/x,x)
```

```
[Out] 2*(a+c*x+b*x^(1/2))^(1/2)+b*ln((1/2*b+c*x^(1/2))/c^(1/2)+(a+c*x+b*x^(1/2))^
(1/2))/c^(1/2)-2*a^(1/2)*ln((2*a+b*x^(1/2)+2*a^(1/2)*(a+c*x+b*x^(1/2))^(1/2
))/x^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cx+b\sqrt{x}+a}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+c*x+b*x^(1/2))^(1/2)/x,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x + b*sqrt(x) + a)/x, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{a+c\,x+b\,\sqrt{x}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + c*x + b*x^(1/2))^(1/2)/x,x)
```

```
[Out] int((a + c*x + b*x^(1/2))^(1/2)/x, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a+b\sqrt{x}+cx}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+c*x+b*x**(1/2))**(1/2)/x,x)
```

```
[Out] Integral(sqrt(a + b*sqrt(x) + c*x)/x, x)
```

3.399 $\int \left(\frac{b^2}{4c}+b\sqrt{x}+cx\right)^2 dx$

Optimal. Leaf size=40

$$\frac{\left(b+2c\sqrt{x}\right)^6}{192c^4}-\frac{b\left(b+2c\sqrt{x}\right)^5}{160c^4}$$

Rubi [A] time = 0.02, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {28, 190, 43}

$$\frac{\left(b+2c\sqrt{x}\right)^6}{192c^4}-\frac{b\left(b+2c\sqrt{x}\right)^5}{160c^4}$$

Antiderivative was successfully verified.

```
[In] Int[(b^2/(4*c) + b*Sqrt[x] + c*x)^2,x]
```

```
[Out] -(b*(b + 2*c*Sqrt[x])^5)/(160*c^4) + (b + 2*c*Sqrt[x])^6/(192*c^4)
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 190

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[Int[x^(1/n
- 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, p}, x] && FractionQ[n] &&
 IntegerQ[1/n]
```

Rubi steps

$$\begin{aligned}
\int \left(\frac{b^2}{4c}+b\sqrt{x}+cx\right)^2 dx &= \frac{\int \left(\frac{b}{2}+c\sqrt{x}\right)^4 dx}{c^2}\\
&= \frac{2\,\text{Subst}\left(\int x\left(\frac{b}{2}+cx\right)^4 dx, x, \sqrt{x}\right)}{c^2}\\
&= \frac{2\,\text{Subst}\left(\int \left(-\frac{b\left(\frac{b}{2}+cx\right)^4}{2c}+\frac{\left(\frac{b}{2}+cx\right)^5}{c}\right) dx, x, \sqrt{x}\right)}{c^2}\\
&= -\frac{b\left(b+2c\sqrt{x}\right)^5}{160c^4}+\frac{\left(b+2c\sqrt{x}\right)^6}{192c^4}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 29, normalized size = 0.72

$$-\frac{\left(b-10c\sqrt{x}\right)\left(b+2c\sqrt{x}\right)^5}{960c^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b^2/(4*c) + b*Sqrt[x] + c*x)^2,x]
```

```
[Out] -1/960*((b - 10*c*Sqrt[x])*(b + 2*c*Sqrt[x])^5)/c^4
```

IntegrateAlgebraic [A] time = 0.03, size = 55, normalized size = 1.38

$$\frac{15b^4x+80b^3cx^{3/2}+180b^2c^2x^2+192bc^3x^{5/2}+80c^4x^3}{240c^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(b^2/(4*c) + b*Sqrt[x] + c*x)^2,x]
```

```
[Out] (15*b^4*x + 80*b^3*c*x^(3/2) + 180*b^2*c^2*x^2 + 192*b*c^3*x^(5/2) + 80*c^4
*x^3)/(240*c^2)
```

fricas [A] time = 1.07, size = 53, normalized size = 1.32

$$\frac{80\,c^4x^3+180\,b^2c^2x^2+15\,b^4x+16\left(12\,bc^3x^2+5\,b^3cx\right)\sqrt{x}}{240\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/4/c*b^2+c*x+b*x^(1/2))^2,x, algorithm="fricas")
```

```
[Out] 1/240*(80*c^4*x^3 + 180*b^2*c^2*x^2 + 15*b^4*x + 16*(12*b*c^3*x^2 + 5*b^3*c
*x)*sqrt(x))/c^2
```

giac [A] time = 0.33, size = 49, normalized size = 1.22

$$\frac{80\,c^4x^3+192\,bc^3x^{\frac{5}{2}}+180\,b^2c^2x^2+80\,b^3cx^{\frac{3}{2}}+15\,b^4x}{240\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/4/c*b^2+c*x+b*x^(1/2))^2,x, algorithm="giac")
```

```
[Out] 1/240*(80*c^4*x^3 + 192*b*c^3*x^(5/2) + 180*b^2*c^2*x^2 + 80*b^3*c*x^(3/2)
+ 15*b^4*x)/c^2
```

maple [A] time = 0.00, size = 52, normalized size = 1.30

$$\frac{b^2x^2}{2}+\frac{\left(\frac{8c^2x^{\frac{5}{2}}}{5}+\frac{2b^2x^{\frac{3}{2}}}{3}\right)b}{2c}+\frac{\left(cx+\frac{b^2}{4c}\right)^3}{3c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/4*b^2/c+c*x+b*x^(1/2))^2,x)
```

```
[Out] 1/2*b^2*x^2+1/2*b/c*(8/5*c^2*x^(5/2)+2/3*x^(3/2)*b^2)+1/3*(1/4*b^2/c+c*x)^3
/c
```

maxima [A] time = 0.87, size = 54, normalized size = 1.35

$$\frac{1}{3}\,c^2x^3+\frac{4}{5}\,bcx^{\frac{5}{2}}+\frac{1}{2}\,b^2x^2+\frac{b^4x}{16\,c^2}+\frac{\left(3\,cx^2+4\,bx^{\frac{3}{2}}\right)b^2}{12\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/4/c*b^2+c*x+b*x^(1/2))^2,x, algorithm="maxima")
```

```
[Out] 1/3*c^2*x^3 + 4/5*b*c*x^(5/2) + 1/2*b^2*x^2 + 1/16*b^4*x/c^2 + 1/12*(3*c*x^
2 + 4*b*x^(3/2))*b^2/c
```

mupad [B] time = 0.04, size = 44, normalized size = 1.10

$$\frac{3\,b^2\,x^2}{4}+\frac{c^2\,x^3}{3}+\frac{b^4\,x}{16\,c^2}+\frac{b^3\,x^{3/2}}{3\,c}+\frac{4\,b\,c\,x^{5/2}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x + b*x^(1/2) + b^2/(4*c))^2,x)
```

```
[Out] (3*b^2*x^2)/4 + (c^2*x^3)/3 + (b^4*x)/(16*c^2) + (b^3*x^(3/2))/(3*c) + (4*b
*c*x^(5/2))/5
```

sympy [A] time = 0.33, size = 51, normalized size = 1.28

$$\frac{b^4x}{16c^2}+\frac{b^3x^{\frac{3}{2}}}{3c}+\frac{3b^2x^2}{4}+\frac{4bcx^{\frac{5}{2}}}{5}+\frac{c^2x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/4/c*b**2+c*x+b*x**(1/2))**2,x)
```

```
[Out] b**4*x/(16*c**2) + b**3*x**(3/2)/(3*c) + 3*b**2*x**2/4 + 4*b*c*x**(5/2)/5 +
 c**2*x**3/3
```

3.400 $\int \frac{1}{\sqrt{a^2+2ab\sqrt{x}+b^2x}}\,dx$

Optimal. Leaf size=75

$$\frac{2\sqrt{a^2+2ab\sqrt{x}+b^2x}}{b^2}-\frac{2a\left(a+b\sqrt{x}\right)\log\left(a+b\sqrt{x}\right)}{b^2\sqrt{a^2+2ab\sqrt{x}+b^2x}}$$

Rubi [A] time = 0.04, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {1341, 640, 608, 31}

$$\frac{2\sqrt{a^2+2ab\sqrt{x}+b^2x}}{b^2}-\frac{2a\left(a+b\sqrt{x}\right)\log\left(a+b\sqrt{x}\right)}{b^2\sqrt{a^2+2ab\sqrt{x}+b^2x}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a^2 + 2*a*b*Sqrt[x] + b^2*x],x]
```

```
[Out] (2*Sqrt[a^2 + 2*a*b*Sqrt[x] + b^2*x])/b^2 - (2*a*(a + b*Sqrt[x])*Log[a + b*
Sqrt[x]])/(b^2*Sqrt[a^2 + 2*a*b*Sqrt[x] + b^2*x])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 608

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[(b/2 + c*x)
/Sqrt[a + b*x + c*x^2], Int[1/(b/2 + c*x), x], x] /; FreeQ[{a, b, c}, x] &&
 EqQ[b^2 - 4*a*c, 0]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1341

```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k
 = Denominator[n]}, Dist[k, Subst[Int[x^(k - 1)*(a + b*x^(k*n) + c*x^(2*k*n
))^p, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && EqQ[n2, 2*n] && Fra
ctionQ[n]
```

Rubi steps

$$\int \frac{1}{\sqrt{a^2+2ab\sqrt{x}+b^2x}}\,dx = 2\,\text{Subst}\left(\int \frac{x}{\sqrt{a^2+2abx+b^2x^2}}\,dx, x, \sqrt{x}\right)$$
$$= \frac{2\sqrt{a^2+2ab\sqrt{x}+b^2x}}{b^2} - \frac{(2a)\,\text{Subst}\left(\int \frac{1}{\sqrt{a^2+2abx+b^2x^2}}\,dx, x, \sqrt{x}\right)}{b}$$
$$= \frac{2\sqrt{a^2+2ab\sqrt{x}+b^2x}}{b^2} - \frac{\left(2a\left(a+b\sqrt{x}\right)\right)\text{Subst}\left(\int \frac{1}{ab+b^2x}\,dx, x, \sqrt{x}\right)}{\sqrt{a^2+2ab\sqrt{x}+b^2x}}$$
$$= \frac{2\sqrt{a^2+2ab\sqrt{x}+b^2x}}{b^2} - \frac{2a\left(a+b\sqrt{x}\right)\log\left(a+b\sqrt{x}\right)}{b^2\sqrt{a^2+2ab\sqrt{x}+b^2x}}$$

Mathematica [A] time = 0.03, size = 50, normalized size = 0.67

$$\frac{2\left(a+b\sqrt{x}\right)\left(b\sqrt{x}-a\log\left(a+b\sqrt{x}\right)\right)}{b^2\sqrt{\left(a+b\sqrt{x}\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a^2 + 2*a*b*Sqrt[x] + b^2*x],x]
```

```
[Out] (2*(a + b*Sqrt[x])*(b*Sqrt[x] - a*Log[a + b*Sqrt[x]]))/(b^2*Sqrt[(a + b*Sqr
t[x])^2])
```

IntegrateAlgebraic [B] time = 0.24, size = 151, normalized size = 2.01

$$\frac{\sqrt{a^2+2ab\sqrt{x}+b^2x}}{b^2} + \frac{a\left(\sqrt{b^2}+b\right)\log\left(\sqrt{a^2+2ab\sqrt{x}+b^2x}-a-\sqrt{b^2}\sqrt{x}\right)}{b^3} + \frac{a\left(\sqrt{b^2}-b\right)\log\left(\sqrt{a^2+2ab\sqrt{x}+b^2x}+a-\sqrt{b^2}\sqrt{x}\right)}{b^3} - \frac{\sqrt{x}}{\sqrt{b^2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/Sqrt[a^2 + 2*a*b*Sqrt[x] + b^2*x],x]
```

```
[Out] -(Sqrt[x]/Sqrt[b^2]) + Sqrt[a^2 + 2*a*b*Sqrt[x] + b^2*x]/b^2 + (a*(b + Sqrt
[b^2])*Log[-a - Sqrt[b^2]*Sqrt[x] + Sqrt[a^2 + 2*a*b*Sqrt[x] + b^2*x]])/b^3
 + (a*(-b + Sqrt[b^2])*Log[a - Sqrt[b^2]*Sqrt[x] + Sqrt[a^2 + 2*a*b*Sqrt[x]
 + b^2*x]])/b^3
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2+b^2*x+2*a*b*x^(1/2))^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [A] time = 0.40, size = 45, normalized size = 0.60

$$-\frac{2\,|a|\log\left(\left|\sqrt{b^2x}\,\text{sgn}(a)\text{sgn}(b)+|a|\right|\right)}{b^2} + \frac{2\,\sqrt{b^2x}}{b^2\text{sgn}(a)\text{sgn}(b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2+b^2*x+2*a*b*x^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] -2*abs(a)*log(abs(sqrt(b^2*x)*sgn(a)*sgn(b) + abs(a)))/b^2 + 2*sqrt(b^2*x)/
(b^2*sgn(a)*sgn(b))
```

maple [A] time = 0.01, size = 50, normalized size = 0.67

$$\frac{2\sqrt{b^2x+2ab\sqrt{x}+a^2}\,\left(-a\ln\left(b\sqrt{x}+a\right)+b\sqrt{x}\right)}{\left(b\sqrt{x}+a\right)b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a^2+b^2*x+2*a*b*x^(1/2))^(1/2),x)
```

```
[Out] 2*(a^2+b^2*x+2*a*b*x^(1/2))^(1/2)*(b*x^(1/2)-a*ln(a+b*x^(1/2)))/(a+b*x^(1/2
))/b^2
```

maxima [A] time = 0.96, size = 23, normalized size = 0.31

$$-\frac{2\,a\log\left(b\sqrt{x}+a\right)}{b^2}+\frac{2\,\sqrt{x}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2+b^2*x+2*a*b*x^(1/2))^(1/2),x, algorithm="maxima")
```

```
[Out] -2*a*log(b*sqrt(x) + a)/b^2 + 2*sqrt(x)/b
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{b^2\,x+a^2+2\,a\,b\,\sqrt{x}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b^2*x + a^2 + 2*a*b*x^(1/2))^(1/2),x)
```

```
[Out] int(1/(b^2*x + a^2 + 2*a*b*x^(1/2))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a^2+2ab\sqrt{x}+b^2x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a**2+b**2*x+2*a*b*x**(1/2))**(1/2),x)
```

```
[Out] Integral(1/sqrt(a**2 + 2*a*b*sqrt(x) + b**2*x), x)
```

3.401 $\int \left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^{7/2}\,dx$

Optimal. Leaf size=137

$$\frac{3\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}\left(a+b\sqrt[3]{x}\right)^9}{10b^3}-\frac{2a\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}\left(a+b\sqrt[3]{x}\right)^8}{3b^3}+\frac{3a^2\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}\left(a-\right.}{8b^3}$$

Rubi [A] time = 0.08, antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1341, 646, 43}

$$\frac{3\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}\left(a+b\sqrt[3]{x}\right)^9}{10b^3}-\frac{2a\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}\left(a+b\sqrt[3]{x}\right)^8}{3b^3}+\frac{3a^2\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}\left(a+b\sqrt[3]{x}\right)^7}{8b^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^(7/2),x]
```

```
[Out] (3*a^2*(a + b*x^(1/3))^7*Sqrt[a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3)])/(8*b^3) -
 (2*a*(a + b*x^(1/3))^8*Sqrt[a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3)])/(3*b^3) +
(3*(a + b*x^(1/3))^9*Sqrt[a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3)])/(10*b^3)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 1341

```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k
 = Denominator[n]}, Dist[k, Subst[Int[x^(k - 1)*(a + b*x^(k*n) + c*x^(2*k*n
))^p, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && EqQ[n2, 2*n] && Fra
ctionQ[n]
```

Rubi steps

$$\begin{aligned}
\int \left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^{7/2}\,dx &= 3\,\text{Subst}\left(\int x^2\left(a^2+2abx+b^2x^2\right)^{7/2}\,dx,x,\sqrt[3]{x}\right)\\
&= \frac{\left(3\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}\right)\text{Subst}\left(\int x^2\left(ab+b^2x\right)^7\,dx,x,\sqrt[3]{x}\right)}{b^7\left(a+b\sqrt[3]{x}\right)}\\
&= \frac{\left(3\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}\right)\text{Subst}\left(\int\left(\frac{a^2\left(ab+b^2x\right)^7}{b^2}-\frac{2a\left(ab+b^2x\right)^8}{b^3}+\frac{\left(ab+b^2x\right)^9}{b^4}\right)d\right.}{b^7\left(a+b\sqrt[3]{x}\right)}\\
&= \frac{3a^2\left(a+b\sqrt[3]{x}\right)^7\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}{8b^3}-\frac{2a\left(a+b\sqrt[3]{x}\right)^8\sqrt{a^2+2ab\sqrt[3]{x}+\right.}{3b^3}
\end{aligned}$$

Mathematica [A] time = 0.05, size = 56, normalized size = 0.41

$$\frac{\left(a+b\sqrt[3]{x}\right)^7\sqrt{\left(a+b\sqrt[3]{x}\right)^2}\left(a^2-8ab\sqrt[3]{x}+36b^2x^{2/3}\right)}{120b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^(7/2),x]
```

```
[Out] ((a + b*x^(1/3))^7*Sqrt[(a + b*x^(1/3))^2]*(a^2 - 8*a*b*x^(1/3) + 36*b^2*x^
(2/3)))/(120*b^3)
```

IntegrateAlgebraic [F] time = 2.56, size = 0, normalized size = 0.00

$$\int \left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^{7/2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^(7/2),x]
```

```
[Out] Defer[IntegrateAlgebraic][(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^(7/2), x]
```

fricas [A] time = 1.47, size = 84, normalized size = 0.61

$$\frac{7}{3}\,ab^6x^3+\frac{35}{2}\,a^4b^3x^2+a^7x+\frac{63}{40}\left(5\,a^2b^5x^2+8\,a^5b^2x\right)x^{\frac{2}{3}}+\frac{3}{20}\left(2\,b^7x^3+100\,a^3b^4x^2+35\,a^6bx\right)x^{\frac{1}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(7/2),x, algorithm="fricas")
```

```
[Out] 7/3*a*b^6*x^3 + 35/2*a^4*b^3*x^2 + a^7*x + 63/40*(5*a^2*b^5*x^2 + 8*a^5*b^2
*x)*x^(2/3) + 3/20*(2*b^7*x^3 + 100*a^3*b^4*x^2 + 35*a^6*b*x)*x^(1/3)
```

giac [A] time = 0.51, size = 140, normalized size = 1.02

$$\frac{3}{10}\,b^7x^{\frac{10}{3}}\mathrm{sgn}\left(bx^{\frac{1}{3}}+a\right)+\frac{7}{3}\,ab^6x^3\mathrm{sgn}\left(bx^{\frac{1}{3}}+a\right)+\frac{63}{8}\,a^2b^5x^{\frac{8}{3}}\mathrm{sgn}\left(bx^{\frac{1}{3}}+a\right)+15\,a^3b^4x^{\frac{7}{3}}\mathrm{sgn}\left(bx^{\frac{1}{3}}+a\right)+\frac{35}{2}\,a^4b^3x^2\mathrm{sgn}\left(bx^{\frac{1}{3}}+a\right)+\frac{63}{5}\,a^5b^2x^{\frac{5}{3}}\mathrm{sgn}\left(bx^{\frac{1}{3}}+a\right)+\frac{21}{4}\,a^6bx^{\frac{4}{3}}\mathrm{sgn}\left(bx^{\frac{1}{3}}+a\right)+a^7x\mathrm{sgn}\left(bx^{\frac{1}{3}}+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(7/2),x, algorithm="giac")
```

```
[Out] 3/10*b^7*x^(10/3)*sgn(b*x^(1/3) + a) + 7/3*a*b^6*x^3*sgn(b*x^(1/3) + a) + 6
3/8*a^2*b^5*x^(8/3)*sgn(b*x^(1/3) + a) + 15*a^3*b^4*x^(7/3)*sgn(b*x^(1/3) +
 a) + 35/2*a^4*b^3*x^2*sgn(b*x^(1/3) + a) + 63/5*a^5*b^2*x^(5/3)*sgn(b*x^(1
/3) + a) + 21/4*a^6*b*x^(4/3)*sgn(b*x^(1/3) + a) + a^7*x*sgn(b*x^(1/3) + a)
```

maple [A] time = 0.02, size = 109, normalized size = 0.80

$$\frac{\sqrt{b^2x^{\frac{2}{3}}+2ab\,x^{\frac{1}{3}}+a^2}\,\left(36b^7x^{\frac{10}{3}}+280a\,b^6x^3+945a^2b^5x^{\frac{8}{3}}+1800a^3b^4x^{\frac{7}{3}}+2100a^4b^3x^2+1512a^5b^2x^{\frac{5}{3}}+630a^6b\,x^{\frac{4}{3}}+120a^7x\right)}{120b\,x^{\frac{1}{3}}+120a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(7/2),x)
```

```
[Out] 1/120*(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(1/2)*(36*b^7*x^(10/3)+945*a^2*b^5*x^
(8/3)+1800*a^3*b^4*x^(7/3)+1512*a^5*b^2*x^(5/3)+630*a^6*b*x^(4/3)+280*a*b^6
*x^3+2100*a^4*b^3*x^2+120*a^7*x)/(a+b*x^(1/3))
```

maxima [A] time = 0.91, size = 114, normalized size = 0.83

$$\frac{3\left(b^2x^{\frac{2}{3}}+2\,abx^{\frac{1}{3}}+a^2\right)^{\frac{7}{2}}a^2x^{\frac{1}{3}}}{8\,b^2}+\frac{3\left(b^2x^{\frac{2}{3}}+2\,abx^{\frac{1}{3}}+a^2\right)^{\frac{7}{2}}a^3}{8\,b^3}+\frac{3\left(b^2x^{\frac{2}{3}}+2\,abx^{\frac{1}{3}}+a^2\right)^{\frac{9}{2}}x^{\frac{1}{3}}}{10\,b^2}-\frac{11\left(b^2x^{\frac{2}{3}}+2\,abx^{\frac{1}{3}}+a^2\right)^{\frac{9}{2}}a}{30\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(7/2),x, algorithm="maxima")
```

```
[Out] 3/8*(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^(7/2)*a^2*x^(1/3)/b^2 + 3/8*(b^2*x^
(2/3) + 2*a*b*x^(1/3) + a^2)^(7/2)*a^3/b^3 + 3/10*(b^2*x^(2/3) + 2*a*b*x^(1
/3) + a^2)^(9/2)*x^(1/3)/b^2 - 11/30*(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^(9
/2)*a/b^3
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(a^2+b^2\,x^{2/3}+2\,a\,b\,x^{1/3}\right)^{7/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^(2/3) + 2*a*b*x^(1/3))^(7/2),x)
```

```
[Out] int((a^2 + b^2*x^(2/3) + 2*a*b*x^(1/3))^(7/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a^2+2ab\sqrt[3]{x}+b^2x^{\frac{2}{3}}\right)^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2+2*a*b*x**(1/3)+b**2*x**(2/3))**(7/2),x)
```

```
[Out] Integral((a**2 + 2*a*b*x**(1/3) + b**2*x**(2/3))**(7/2), x)
```

3.402 $\int \left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^{5/2}\,dx$

Optimal. Leaf size=137

$$\frac{3\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}\left(a+b\sqrt[3]{x}\right)^7}{8b^3}-\frac{6a\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}\left(a+b\sqrt[3]{x}\right)^6}{7b^3}+\frac{a^2\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}\left(a+b\sqrt[3]{x}\right)^5}{2b^3}$$

Rubi [A] time = 0.07, antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1341, 646, 43}

$$\frac{3\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}\left(a+b\sqrt[3]{x}\right)^7}{8b^3}-\frac{6a\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}\left(a+b\sqrt[3]{x}\right)^6}{7b^3}+\frac{a^2\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}\left(a+b\sqrt[3]{x}\right)^5}{2b^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^(5/2),x]
```

```
[Out] (a^2*(a + b*x^(1/3))^5*Sqrt[a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3)])/(2*b^3) - (
6*a*(a + b*x^(1/3))^6*Sqrt[a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3)])/(7*b^3) + (3
*(a + b*x^(1/3))^7*Sqrt[a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3)])/(8*b^3)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 1341

```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k
 = Denominator[n]}, Dist[k, Subst[Int[x^(k - 1)*(a + b*x^(k*n) + c*x^(2*k*n
))^p, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && EqQ[n2, 2*n] && Fra
ctionQ[n]
```

Rubi steps

$$\begin{aligned}\int \left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^{5/2}dx &= 3\,\text{Subst}\left(\int x^2\left(a^2+2abx+b^2x^2\right)^{5/2}dx,x,\sqrt[3]{x}\right)\\ &= \frac{\left(3\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}\right)\text{Subst}\left(\int x^2\left(ab+b^2x\right)^5dx,x,\sqrt[3]{x}\right)}{b^5\left(a+b\sqrt[3]{x}\right)}\\ &= \frac{\left(3\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}\right)\text{Subst}\left(\int\left(\frac{a^2\left(ab+b^2x\right)^5}{b^2}-\frac{2a\left(ab+b^2x\right)^6}{b^3}+\frac{\left(ab+b^2x\right)^7}{b^4}\right)dx,x\right.}{b^5\left(a+b\sqrt[3]{x}\right)}\\ &= \frac{a^2\left(a+b\sqrt[3]{x}\right)^5\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}{2b^3}-\frac{6a\left(a+b\sqrt[3]{x}\right)^6\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/}}}{7b^3}\end{aligned}$$

Mathematica [A] time = 0.04, size = 56, normalized size = 0.41

$$\frac{\left(a+b\sqrt[3]{x}\right)^5\sqrt{\left(a+b\sqrt[3]{x}\right)^2}\left(a^2-6ab\sqrt[3]{x}+21b^2x^{2/3}\right)}{56b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^(5/2),x]
[Out] ((a + b*x^(1/3))^5*Sqrt[(a + b*x^(1/3))^2]*(a^2 - 6*a*b*x^(1/3) + 21*b^2*x^
(2/3)))/(56*b^3)
```

IntegrateAlgebraic [F] time = 2.40, size = 0, normalized size = 0.00

$$\int\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^{5/2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^(5/2),x]
[Out] Defer[IntegrateAlgebraic][(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^(5/2), x]
```

fricas [A] time = 1.12, size = 61, normalized size = 0.45

$$5\,a^2b^3x^2+a^5x+\frac{3}{8}\left(b^5x^2+16\,a^3b^2x\right)x^{\frac{2}{3}}+\frac{15}{28}\left(4\,ab^4x^2+7\,a^4bx\right)x^{\frac{1}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(5/2),x, algorithm="fricas")
[Out] 5*a^2*b^3*x^2 + a^5*x + 3/8*(b^5*x^2 + 16*a^3*b^2*x)*x^(2/3) + 15/28*(4*a*b
^4*x^2 + 7*a^4*b*x)*x^(1/3)
```

giac [A] time = 0.53, size = 102, normalized size = 0.74

$$\frac{3}{8}b^5x^{\frac{8}{3}}\mathrm{sgn}\left(bx^{\frac{1}{3}}+a\right)+\frac{15}{7}ab^4x^{\frac{7}{3}}\mathrm{sgn}\left(bx^{\frac{1}{3}}+a\right)+5\,a^2b^3x^2\mathrm{sgn}\left(bx^{\frac{1}{3}}+a\right)+6\,a^3b^2x^{\frac{5}{3}}\mathrm{sgn}\left(bx^{\frac{1}{3}}+a\right)+\frac{15}{4}a^4bx^{\frac{4}{3}}\mathrm{sgn}\left(bx^{\frac{1}{3}}+a\right)+a^5x\mathrm{sgn}\left(bx^{\frac{1}{3}}+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(5/2),x, algorithm="giac")
[Out] 3/8*b^5*x^(8/3)*sgn(b*x^(1/3) + a) + 15/7*a*b^4*x^(7/3)*sgn(b*x^(1/3) + a)
+ 5*a^2*b^3*x^2*sgn(b*x^(1/3) + a) + 6*a^3*b^2*x^(5/3)*sgn(b*x^(1/3) + a) +
 15/4*a^4*b*x^(4/3)*sgn(b*x^(1/3) + a) + a^5*x*sgn(b*x^(1/3) + a)
```

maple [A] time = 0.00, size = 87, normalized size = 0.64

$$\frac{\sqrt{b^2x^{\frac{2}{3}}+2ab\,x^{\frac{1}{3}}+a^2}\,\left(21b^5x^{\frac{8}{3}}+120a\,b^4x^{\frac{7}{3}}+280a^2b^3x^2+336a^3b^2x^{\frac{5}{3}}+210a^4b\,x^{\frac{4}{3}}+56a^5x\right)}{56b\,x^{\frac{1}{3}}+56a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^(2/3)+2*a*b*x^(1/3)+a^2)^(5/2),x)
[Out] 1/56*(b^2*x^(2/3)+2*a*b*x^(1/3)+a^2)^(1/2)*(21*b^5*x^(8/3)+120*a*b^4*x^(7/3
)+336*a^3*b^2*x^(5/3)+210*a^4*b*x^(4/3)+280*a^2*b^3*x^2+56*a^5*x)/(b*x^(1/3
)+a)
```

maxima [A]   time = 0.89, size = 114, normalized size = 0.83

$$\frac{\left(b^2x^{\frac{2}{3}}+2\,abx^{\frac{1}{3}}+a^2\right)^{\frac{5}{2}}a^2x^{\frac{1}{3}}}{2\,b^2}+\frac{\left(b^2x^{\frac{2}{3}}+2\,abx^{\frac{1}{3}}+a^2\right)^{\frac{5}{2}}a^3}{2\,b^3}+\frac{3\left(b^2x^{\frac{2}{3}}+2\,abx^{\frac{1}{3}}+a^2\right)^{\frac{7}{2}}x^{\frac{1}{3}}}{8\,b^2}-\frac{27\left(b^2x^{\frac{2}{3}}+2\,abx^{\frac{1}{3}}+a^2\right)^{\frac{7}{2}}a}{56\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(5/2),x, algorithm="maxima")
```

```
[Out] 1/2*(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^(5/2)*a^2*x^(1/3)/b^2 + 1/2*(b^2*x^
(2/3) + 2*a*b*x^(1/3) + a^2)^(5/2)*a^3/b^3 + 3/8*(b^2*x^(2/3) + 2*a*b*x^(1/
3) + a^2)^(7/2)*x^(1/3)/b^2 - 27/56*(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^(7/
2)*a/b^3
```

mupad [F]   time = 0.00, size = -1, normalized size = -0.01

$$\int \left(a^2+b^2\,x^{2/3}+2\,a\,b\,x^{1/3}\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^(2/3) + 2*a*b*x^(1/3))^(5/2),x)
```

```
[Out] int((a^2 + b^2*x^(2/3) + 2*a*b*x^(1/3))^(5/2), x)
```

sympy [F]   time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a^{2}+2ab\sqrt[3]{x}+b^{2}x^{\frac{2}{3}}\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2+2*a*b*x**(1/3)+b**2*x**(2/3))**(5/2),x)
```

```
[Out] Integral((a**2 + 2*a*b*x**(1/3) + b**2*x**(2/3))**(5/2), x)
```

3.403 $\int \left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^{3/2}\,dx$

Optimal. Leaf size=137

$$\frac{\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}\left(a+b\sqrt[3]{x}\right)^5}{2b^3}-\frac{6a\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}\left(a+b\sqrt[3]{x}\right)^4}{5b^3}+\frac{3a^2\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}\left(a+\right.}{4b^3}$$

Rubi [A] time = 0.06, antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1341, 645}

$$\frac{\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}\left(a+b\sqrt[3]{x}\right)^5}{2b^3}-\frac{6a\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}\left(a+b\sqrt[3]{x}\right)^4}{5b^3}+\frac{3a^2\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}\left(a+b\sqrt[3]{x}\right)^3}{4b^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^(3/2),x]
```

```
[Out] (3*a^2*(a + b*x^(1/3))^3*Sqrt[a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3)])/(4*b^3) -
 (6*a*(a + b*x^(1/3))^4*Sqrt[a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3)])/(5*b^3) +
((a + b*x^(1/3))^5*Sqrt[a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3)])/(2*b^3)
```

Rule 645

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[ExpandLinearProduct[(b/2 + c*x)^(2*p), (d + e*x)^m, b/2, c
, x], x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&
!IntegerQ[p] && NeQ[2*c*d - b*e, 0] && IGtQ[m, 0] && EqQ[m - 2*p + 1, 0]
```

Rule 1341

```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k
 = Denominator[n]}, Dist[k, Subst[Int[x^(k - 1)*(a + b*x^(k*n) + c*x^(2*k*n
))^p, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && EqQ[n2, 2*n] && Fra
ctionQ[n]
```

Rubi steps

$$\begin{aligned}\int \left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^{3/2}dx &= 3\,\text{Subst}\left(\int x^2\left(a^2+2abx+b^2x^2\right)^{3/2}dx,x,\sqrt[3]{x}\right)\\ &= \frac{\left(3\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}\right)\text{Subst}\left(\int\left(\frac{a^2\left(ab+b^2x\right)^3}{b^2}-\frac{2a\left(ab+b^2x\right)^4}{b^3}+\frac{\left(ab+b^2x\right)^5}{b^4}\right)d\right.}{b^3\left(a+b\sqrt[3]{x}\right)}\\ &= \frac{3a^2\left(a+b\sqrt[3]{x}\right)^3\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}{4b^3}-\frac{6a\left(a+b\sqrt[3]{x}\right)^4\sqrt{a^2+2ab\sqrt[3]{x}+\right.}{5b^3}\end{aligned}$$

Mathematica [A] time = 0.03, size = 65, normalized size = 0.47

$$\frac{x\sqrt{\left(a+b\sqrt[3]{x}\right)^2}\left(20a^3+45a^2b\sqrt[3]{x}+36ab^2x^{2/3}+10b^3x\right)}{20\left(a+b\sqrt[3]{x}\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^(3/2),x]
```

```
[Out] (Sqrt[(a + b*x^(1/3))^2]*x*(20*a^3 + 45*a^2*b*x^(1/3) + 36*a*b^2*x^(2/3) + 
10*b^3*x))/(20*(a + b*x^(1/3)))
```

IntegrateAlgebraic [F] time = 1.68, size = 0, normalized size = 0.00

$$\int \left(a^2 + 2ab\sqrt[3]{x} + b^2x^{2/3}\right)^{3/2} dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^(3/2),x]
```

```
[Out] Defer[IntegrateAlgebraic][(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^(3/2), x]
```

fricas [A] time = 1.40, size = 32, normalized size = 0.23

$$\frac{1}{2}\,b^3x^2 + \frac{9}{5}\,ab^2x^{\frac{5}{3}} + \frac{9}{4}\,a^2bx^{\frac{4}{3}} + a^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/2*b^3*x^2 + 9/5*a*b^2*x^(5/3) + 9/4*a^2*b*x^(4/3) + a^3*x
```

giac [A] time = 0.37, size = 64, normalized size = 0.47

$$\frac{1}{2}\,b^3x^2\mathrm{sgn}\left(bx^{\frac{1}{3}} + a\right) + \frac{9}{5}\,ab^2x^{\frac{5}{3}}\mathrm{sgn}\left(bx^{\frac{1}{3}} + a\right) + \frac{9}{4}\,a^2bx^{\frac{4}{3}}\mathrm{sgn}\left(bx^{\frac{1}{3}} + a\right) + a^3x\mathrm{sgn}\left(bx^{\frac{1}{3}} + a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(3/2),x, algorithm="giac")
```

```
[Out] 1/2*b^3*x^2*sgn(b*x^(1/3) + a) + 9/5*a*b^2*x^(5/3)*sgn(b*x^(1/3) + a) + 9/4
*a^2*b*x^(4/3)*sgn(b*x^(1/3) + a) + a^3*x*sgn(b*x^(1/3) + a)
```

maple [A] time = 0.00, size = 65, normalized size = 0.47

$$\frac{\sqrt{b^2x^{\frac{2}{3}} + 2ab\,x^{\frac{1}{3}} + a^2}\,\left(10b^3x^2 + 36a\,b^2x^{\frac{5}{3}} + 45a^2b\,x^{\frac{4}{3}} + 20a^3x\right)}{20b\,x^{\frac{1}{3}} + 20a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^(2/3)+2*a*b*x^(1/3)+a^2)^(3/2),x)
```

```
[Out] 1/20*(b^2*x^(2/3)+2*a*b*x^(1/3)+a^2)^(1/2)*(36*a*b^2*x^(5/3)+45*a^2*b*x^(4/
3)+10*b^3*x^2+20*a^3*x)/(b*x^(1/3)+a)
```

maxima [A] time = 0.91, size = 114, normalized size = 0.83

$$\frac{3\left(b^2x^{\frac{2}{3}} + 2\,abx^{\frac{1}{3}} + a^2\right)^{\frac{3}{2}}a^2x^{\frac{1}{3}}}{4\,b^2} + \frac{3\left(b^2x^{\frac{2}{3}} + 2\,abx^{\frac{1}{3}} + a^2\right)^{\frac{3}{2}}a^3}{4\,b^3} + \frac{\left(b^2x^{\frac{2}{3}} + 2\,abx^{\frac{1}{3}} + a^2\right)^{\frac{5}{2}}x^{\frac{1}{3}}}{2\,b^2} - \frac{7\left(b^2x^{\frac{2}{3}} + 2\,abx^{\frac{1}{3}} + a^2\right)^{\frac{5}{2}}a}{10\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(3/2),x, algorithm="maxima")
```

```
[Out] 3/4*(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^(3/2)*a^2*x^(1/3)/b^2 + 3/4*(b^2*x^
(2/3) + 2*a*b*x^(1/3) + a^2)^(3/2)*a^3/b^3 + 1/2*(b^2*x^(2/3) + 2*a*b*x^(1/
```

```
3) + a^2)^(5/2)*x^(1/3)/b^2 - 7/10*(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^(5/2
)*a/b^3
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(a^2 + b^2\, x^{2/3} + 2\, a\, b\, x^{1/3}\right)^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^(2/3) + 2*a*b*x^(1/3))^(3/2),x)
[Out] int((a^2 + b^2*x^(2/3) + 2*a*b*x^(1/3))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a^2 + 2ab\sqrt[3]{x} + b^2x^{\frac{2}{3}}\right)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2+2*a*b*x**(1/3)+b**2*x**(2/3))**(3/2),x)
[Out] Integral((a**2 + 2*a*b*x**(1/3) + b**2*x**(2/3))**(3/2), x)
```

3.404 $\int \sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}\,dx$

Optimal. Leaf size=88

$$\frac{3bx^{4/3}\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}{4\left(a+b\sqrt[3]{x}\right)}+\frac{ax\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}{a+b\sqrt[3]{x}}$$

Rubi [A] time = 0.04, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1341, 646, 43}

$$\frac{3bx^{4/3}\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}{4\left(a+b\sqrt[3]{x}\right)}+\frac{ax\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}{a+b\sqrt[3]{x}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3)],x]
```

```
[Out] (a*Sqrt[a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3)]*x)/(a + b*x^(1/3)) + (3*b*Sqrt[a
^2 + 2*a*b*x^(1/3) + b^2*x^(2/3)]*x^(4/3))/(4*(a + b*x^(1/3)))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 1341

```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k
 = Denominator[n]}, Dist[k, Subst[Int[x^(k - 1)*(a + b*x^(k*n) + c*x^(2*k*n
))^p, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && EqQ[n2, 2*n] && Fra
ctionQ[n]
```

Rubi steps

$$\int \sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}\,dx = 3\,\text{Subst}\left(\int x^2\sqrt{a^2+2abx+b^2x^2}\,dx, x, \sqrt[3]{x}\right)$$
$$= \frac{\left(3\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}\right)\text{Subst}\left(\int x^2\left(ab+b^2x\right)dx, x, \sqrt[3]{x}\right)}{b\left(a+b\sqrt[3]{x}\right)}$$
$$= \frac{\left(3\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}\right)\text{Subst}\left(\int \left(abx^2+b^2x^3\right)dx, x, \sqrt[3]{x}\right)}{b\left(a+b\sqrt[3]{x}\right)}$$
$$= \frac{a\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}\,x}{a+b\sqrt[3]{x}} + \frac{3b\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}\,x^{4/3}}{4\left(a+b\sqrt[3]{x}\right)}$$

Mathematica [A] time = 0.01, size = 43, normalized size = 0.49

$$\frac{\sqrt{\left(a+b\sqrt[3]{x}\right)^2}\left(4ax+3bx^{4/3}\right)}{4\left(a+b\sqrt[3]{x}\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3)],x]
[Out] (Sqrt[(a + b*x^(1/3))^2]*(4*a*x + 3*b*x^(4/3)))/(4*(a + b*x^(1/3)))
```

IntegrateAlgebraic [F] time = 0.82, size = 0, normalized size = 0.00

$$\int \sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[Sqrt[a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3)],x]
[Out] Defer[IntegrateAlgebraic][Sqrt[a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3)], x]
```

fricas [A] time = 0.95, size = 10, normalized size = 0.11

$$\frac{3}{4}bx^{\frac{4}{3}} + ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(1/2),x, algorithm="fricas")
[Out] 3/4*b*x^(4/3) + a*x
```

giac [A] time = 0.39, size = 26, normalized size = 0.30

$$\frac{3}{4}bx^{\frac{4}{3}}\text{sgn}\left(bx^{\frac{1}{3}}+a\right) + ax\text{sgn}\left(bx^{\frac{1}{3}}+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(1/2),x, algorithm="giac")
[Out] 3/4*b*x^(4/3)*sgn(b*x^(1/3) + a) + a*x*sgn(b*x^(1/3) + a)
```

maple [A] time = 0.00, size = 43, normalized size = 0.49

$$\frac{\sqrt{b^{2} x^{\frac{2}{3}}+2 a b\, x^{\frac{1}{3}}+a^{2}}\, \left(3 b\, x^{\frac{4}{3}}+4 a x \right)}{4 b\, x^{\frac{1}{3}}+4 a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^(2/3)+2*a*b*x^(1/3)+a^2)^(1/2),x)
```

```
[Out] 1/4*(b^2*x^(2/3)+2*a*b*x^(1/3)+a^2)^(1/2)*(3*b*x^(4/3)+4*a*x)/(b*x^(1/3)+a)
```

maxima [A] time = 0.88, size = 114, normalized size = 1.30

$$\frac{3 \sqrt{b^{2} x^{\frac{2}{3}}+2 \, a b x^{\frac{1}{3}}+a^{2}}\, a^{2} x^{\frac{1}{3}}}{2 \, b^{2}}+\frac{3 \sqrt{b^{2} x^{\frac{2}{3}}+2 \, a b x^{\frac{1}{3}}+a^{2}}\, a^{3}}{2 \, b^{3}}+\frac{3 \left(b^{2} x^{\frac{2}{3}}+2 \, a b x^{\frac{1}{3}}+a^{2}\right)^{\frac{3}{2}} x^{\frac{1}{3}}}{4 \, b^{2}}-\frac{5 \left(b^{2} x^{\frac{2}{3}}+2 \, a b x^{\frac{1}{3}}+a^{2}\right)^{\frac{3}{2}} a}{4 \, b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(1/2),x, algorithm="maxima")
```

```
[Out] 3/2*sqrt(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)*a^2*x^(1/3)/b^2 + 3/2*sqrt(b^2*
x^(2/3) + 2*a*b*x^(1/3) + a^2)*a^3/b^3 + 3/4*(b^2*x^(2/3) + 2*a*b*x^(1/3) +
 a^2)^(3/2)*x^(1/3)/b^2 - 5/4*(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^(3/2)*a/b
^3
```

mupad [B] time = 1.56, size = 71, normalized size = 0.81

$$\frac{\sqrt{a^2+b^2\,x^{2/3}+2\,a\,b\,x^{1/3}}\,\left(a^3-4\,a^2\,b\,x^{1/3}-5\,a\,b^2\,x^{2/3}+3\,b\,x^{1/3}\,\left(a^2+b^2\,x^{2/3}+2\,a\,b\,x^{1/3}\right)\right)}{4\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^(2/3) + 2*a*b*x^(1/3))^(1/2),x)
```

```
[Out] ((a^2 + b^2*x^(2/3) + 2*a*b*x^(1/3))^(1/2)*(a^3 - 4*a^2*b*x^(1/3) - 5*a*b^2
*x^(2/3) + 3*b*x^(1/3)*(a^2 + b^2*x^(2/3) + 2*a*b*x^(1/3))))/(4*b^3)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a^{2}+2 a b \sqrt[3]{x}+b^{2} x^{\frac{2}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2+2*a*b*x**(1/3)+b**2*x**(2/3))**(1/2),x)
```

```
[Out] Integral(sqrt(a**2 + 2*a*b*x**(1/3) + b**2*x**(2/3)), x)
```

3.405 $\int \frac{1}{\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}\,dx$

Optimal. Leaf size=147

$$-\frac{3a\sqrt[3]{x}\left(a+b\sqrt[3]{x}\right)}{b^2\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}+\frac{3x^{2/3}\left(a+b\sqrt[3]{x}\right)}{2b\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}+\frac{3a^2\left(a+b\sqrt[3]{x}\right)\log\left(a+b\sqrt[3]{x}\right)}{b^3\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}$$

Rubi [A] time = 0.07, antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1341, 646, 43}

$$-\frac{3a\sqrt[3]{x}\left(a+b\sqrt[3]{x}\right)}{b^2\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}+\frac{3x^{2/3}\left(a+b\sqrt[3]{x}\right)}{2b\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}+\frac{3a^2\left(a+b\sqrt[3]{x}\right)\log\left(a+b\sqrt[3]{x}\right)}{b^3\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3)],x]
```

```
[Out] (-3*a*(a + b*x^(1/3))*x^(1/3))/(b^2*Sqrt[a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3)]
) + (3*(a + b*x^(1/3))*x^(2/3))/(2*b*Sqrt[a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3)
]) + (3*a^2*(a + b*x^(1/3))*Log[a + b*x^(1/3)])/(b^3*Sqrt[a^2 + 2*a*b*x^(1/
3) + b^2*x^(2/3)])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 1341

```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k
 = Denominator[n]}, Dist[k, Subst[Int[x^(k - 1)*(a + b*x^(k*n) + c*x^(2*k*n
))^p, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && EqQ[n2, 2*n] && Fra
ctionQ[n]
```

Rubi steps

$$\int \frac{1}{\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}\,dx = 3\,\text{Subst}\left(\int \frac{x^2}{\sqrt{a^2+2abx+b^2x^2}}\,dx, x, \sqrt[3]{x}\right)$$

$$= \frac{\left(3b\left(a+b\sqrt[3]{x}\right)\right)\text{Subst}\left(\int \frac{x^2}{ab+b^2x}\,dx, x, \sqrt[3]{x}\right)}{\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}$$

$$= \frac{\left(3b\left(a+b\sqrt[3]{x}\right)\right)\text{Subst}\left(\int \left(-\frac{a}{b^3}+\frac{x}{b^2}+\frac{a^2}{b^3(a+bx)}\right)dx, x, \sqrt[3]{x}\right)}{\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}$$

$$= -\frac{3a\left(a+b\sqrt[3]{x}\right)\sqrt[3]{x}}{b^2\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}} + \frac{3\left(a+b\sqrt[3]{x}\right)x^{2/3}}{2b\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}} + \frac{3a^2\left(a+b\sqrt[3]{x}\right)\log\left(a\right.}{b^3\sqrt{a^2+2ab\sqrt[3]{x}+\right.}$$

Mathematica [A] time = 0.04, size = 65, normalized size = 0.44

$$\frac{3\left(a+b\sqrt[3]{x}\right)\left(2a^2\log\left(a+b\sqrt[3]{x}\right)+b\sqrt[3]{x}\left(b\sqrt[3]{x}-2a\right)\right)}{2b^3\sqrt{\left(a+b\sqrt[3]{x}\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3)],x]
```

```
[Out] (3*(a + b*x^(1/3))*(b*(-2*a + b*x^(1/3))*x^(1/3) + 2*a^2*Log[a + b*x^(1/3)]
))/(2*b^3*Sqrt[(a + b*x^(1/3))^2])
```

IntegrateAlgebraic [A] time = 0.36, size = 204, normalized size = 1.39

$$-\frac{3a^2\left(\sqrt{b^2}+b\right)\log\left(\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}-a-\sqrt{b^2}\sqrt[3]{x}\right)}{2b^4} - \frac{3a^2\left(\sqrt{b^2}-b\right)\log\left(\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}+a-\sqrt{b^2}\sqrt[3]{x}\right)}{2b^4} - \frac{3\left(3a-b\sqrt[3]{x}\right)\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}{4b^3} - \frac{3\left(bx^{2/3}-2a\sqrt[3]{x}\right)}{4b\sqrt{b^2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/Sqrt[a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3)],x]
```

```
[Out] (-3*(-2*a*x^(1/3) + b*x^(2/3)))/(4*b*Sqrt[b^2]) - (3*(3*a - b*x^(1/3))*Sqrt
[a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3)])/(4*b^3) - (3*a^2*(b + Sqrt[b^2])*Log[-
a + Sqrt[a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3)] - Sqrt[b^2]*x^(1/3)])/(2*b^4) -
 (3*a^2*(-b + Sqrt[b^2])*Log[a + Sqrt[a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3)] -
Sqrt[b^2]*x^(1/3)])/(2*b^4)
```

fricas [A] time = 1.37, size = 33, normalized size = 0.22

$$\frac{3\left(2\,a^2\log\left(bx^{\frac{1}{3}}+a\right)+b^2x^{\frac{2}{3}}-2\,abx^{\frac{1}{3}}\right)}{2\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(1/2),x, algorithm="fricas")
```

```
[Out] 3/2*(2*a^2*log(b*x^(1/3) + a) + b^2*x^(2/3) - 2*a*b*x^(1/3))/b^3
```

giac [A] time = 0.49, size = 61, normalized size = 0.41

$$\frac{3\left(bx^{\frac{2}{3}}\mathrm{sgn}\left(bx^{\frac{1}{3}}+a\right)-2\,ax^{\frac{1}{3}}\mathrm{sgn}\left(bx^{\frac{1}{3}}+a\right)\right)}{2\,b^2} + \frac{3\,a^2\log\left(\left|bx^{\frac{1}{3}}+a\right|\right)}{b^3\mathrm{sgn}\left(bx^{\frac{1}{3}}+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(1/2),x, algorithm="giac")
```

```
[Out] 3/2*(b*x^(2/3)*sgn(b*x^(1/3) + a) - 2*a*x^(1/3)*sgn(b*x^(1/3) + a))/b^2 + 3
*a^2*log(abs(b*x^(1/3) + a))/(b^3*sgn(b*x^(1/3) + a))
```

maple [A] time = 0.02, size = 103, normalized size = 0.70

$$\frac{\sqrt{b^2x^{\frac{2}{3}}+2abx^{\frac{1}{3}}+a^2}\left(4a^2\ln\left(bx^{\frac{1}{3}}+a\right)-2a^2\ln\left(b^2x^{\frac{2}{3}}-abx^{\frac{1}{3}}+a^2\right)+2a^2\ln\left(b^3x+a^3\right)+3b^2x^{\frac{2}{3}}-6abx^{\frac{1}{3}}\right)}{2\left(bx^{\frac{1}{3}}+a\right)b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b^2*x^(2/3)+2*a*b*x^(1/3)+a^2)^(1/2),x)
```

```
[Out] 1/2*(b^2*x^(2/3)+2*a*b*x^(1/3)+a^2)^(1/2)*(3*b^2*x^(2/3)-6*a*b*x^(1/3)+2*a^
2*ln(b^3*x+a^3)-2*a^2*ln(b^2*x^(2/3)-a*b*x^(1/3)+a^2)+4*a^2*ln(b*x^(1/3)+a)
)/(b*x^(1/3)+a)/b^3
```

maxima [A] time = 0.88, size = 36, normalized size = 0.24

$$\frac{3\,a^2\log\left(x^{\frac{1}{3}}+\frac{a}{b}\right)}{b^3}+\frac{3\,x^{\frac{2}{3}}}{2\,b}-\frac{3\,ax^{\frac{1}{3}}}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(1/2),x, algorithm="maxima")
```

```
[Out] 3*a^2*log(x^(1/3) + a/b)/b^3 + 3/2*x^(2/3)/b - 3*a*x^(1/3)/b^2
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{a^2+b^2\,x^{2/3}+2\,a\,b\,x^{1/3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a^2 + b^2*x^(2/3) + 2*a*b*x^(1/3))^(1/2),x)
```

```
[Out] int(1/(a^2 + b^2*x^(2/3) + 2*a*b*x^(1/3))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{\frac{2}{3}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a**2+2*a*b*x**(1/3)+b**2*x**(2/3))**(1/2),x)
```

```
[Out] Integral(1/sqrt(a**2 + 2*a*b*x**(1/3) + b**2*x**(2/3)), x)
```

3.406 $\int \frac{1}{\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^{3/2}}\,dx$

Optimal. Leaf size=130

$$-\frac{3a^2}{2b^3\left(a+b\sqrt[3]{x}\right)\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}+\frac{6a}{b^3\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}+\frac{3\left(a+b\sqrt[3]{x}\right)\log\left(a+b\sqrt[3]{x}\right)}{b^3\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}$$

Rubi [A] time = 0.07, antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1341, 646, 43}

$$-\frac{3a^2}{2b^3\left(a+b\sqrt[3]{x}\right)\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}+\frac{6a}{b^3\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}+\frac{3\left(a+b\sqrt[3]{x}\right)\log\left(a+b\sqrt[3]{x}\right)}{b^3\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^(-3/2),x]
```

```
[Out] (6*a)/(b^3*Sqrt[a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3)]) - (3*a^2)/(2*b^3*(a + b
*x^(1/3))*Sqrt[a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3)]) + (3*(a + b*x^(1/3))*Log
[a + b*x^(1/3)])/(b^3*Sqrt[a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3)])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 1341

```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k
 = Denominator[n]}, Dist[k, Subst[Int[x^(k - 1)*(a + b*x^(k*n) + c*x^(2*k*n
))^p, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && EqQ[n2, 2*n] && Fra
ctionQ[n]
```

Rubi steps

$$\int \frac{1}{\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^{3/2}}\,dx = 3\,\text{Subst}\left(\int \frac{x^2}{\left(a^2+2abx+b^2x^2\right)^{3/2}}\,dx, x, \sqrt[3]{x}\right)$$

$$= \frac{\left(3b^3\left(a+b\sqrt[3]{x}\right)\right)\text{Subst}\left(\int \frac{x^2}{\left(ab+b^2x\right)^3}\,dx, x, \sqrt[3]{x}\right)}{\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}$$

$$= \frac{\left(3b^3\left(a+b\sqrt[3]{x}\right)\right)\text{Subst}\left(\int \left(\frac{a^2}{b^5(a+bx)^3}-\frac{2a}{b^5(a+bx)^2}+\frac{1}{b^5(a+bx)}\right)dx, x, \sqrt[3]{x}\right)}{\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}$$

$$= \frac{6a}{b^3\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}} - \frac{3a^2}{2b^3\left(a+b\sqrt[3]{x}\right)\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}} + \frac{3\,(a}{b^3\sqrt{}}$$

Mathematica [A] time = 0.05, size = 72, normalized size = 0.55

$$\frac{3a\left(3a+4b\sqrt[3]{x}\right)+6\left(a+b\sqrt[3]{x}\right)^2\log\left(a+b\sqrt[3]{x}\right)}{2b^3\left(a+b\sqrt[3]{x}\right)\sqrt{\left(a+b\sqrt[3]{x}\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^(-3/2),x]
```

```
[Out] (3*a*(3*a + 4*b*x^(1/3)) + 6*(a + b*x^(1/3))^2*Log[a + b*x^(1/3)])/(2*b^3*(
a + b*x^(1/3))*Sqrt[(a + b*x^(1/3))^2])
```

IntegrateAlgebraic [B] time = 1.31, size = 1439, normalized size = 11.07

[illegible]

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^(-3/2),x]
```

```
[Out] ((12*a^3*Sqrt[a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3)])/b^3 - (36*a^3*x^(1/3))/(b
*Sqrt[b^2]) + (24*a^2*Sqrt[a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3)]*x^(1/3))/b^2
- (72*a^2*x^(2/3))/Sqrt[b^2] + (48*a*Sqrt[a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3)
]*x^(2/3))/b - (48*a*b*x)/Sqrt[b^2] + (24*a^2*x^(2/3)*ArcTanh[(Sqrt[a^2 + 2
*a*b*x^(1/3) + b^2*x^(2/3)] - Sqrt[b^2]*x^(1/3))/a]])/b - (24*a*Sqrt[a^2 + 2
*a*b*x^(1/3) + b^2*x^(2/3)]*x^(2/3)*ArcTanh[(Sqrt[a^2 + 2*a*b*x^(1/3) + b^2
*x^(2/3)] - Sqrt[b^2]*x^(1/3))/a])/Sqrt[b^2] + 48*a*x*ArcTanh[(Sqrt[a^2 + 2
*a*b*x^(1/3) + b^2*x^(2/3)] - Sqrt[b^2]*x^(1/3))/a] - (24*b*Sqrt[a^2 + 2*a*
b*x^(1/3) + b^2*x^(2/3)]*x*ArcTanh[(Sqrt[a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3)]
 - Sqrt[b^2]*x^(1/3))/a])/Sqrt[b^2] + 24*b*x^(4/3)*ArcTanh[(Sqrt[a^2 + 2*a*
b*x^(1/3) + b^2*x^(2/3)] - Sqrt[b^2]*x^(1/3))/a])/((-a + Sqrt[a^2 + 2*a*b*x
^(1/3) + b^2*x^(2/3)] - Sqrt[b^2]*x^(1/3))^2*(a + Sqrt[a^2 + 2*a*b*x^(1/3)
+ b^2*x^(2/3)] - Sqrt[b^2]*x^(1/3))^2) + ((12*a^4*Sqrt[b^2])/b^4 + (36*a^3*
Sqrt[b^2]*x^(1/3))/b^3 - (36*a^2*Sqrt[a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3)]*x^
(1/3))/b^2 + (36*a^2*(b^2)^(3/2)*x^(2/3))/b^4 - (12*a^2*(b^2)^(3/2)*x^(2/3)
*Log[-a + Sqrt[a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3)] - Sqrt[b^2]*x^(1/3)])/b^4
 + (12*a*Sqrt[a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3)]*x^(2/3)*Log[-a + Sqrt[a^2
+ 2*a*b*x^(1/3) + b^2*x^(2/3)] - Sqrt[b^2]*x^(1/3)])/b - (24*a*Sqrt[b^2]*x*
Log[-a + Sqrt[a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3)] - Sqrt[b^2]*x^(1/3)])/b +
12*Sqrt[a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3)]*x*Log[-a + Sqrt[a^2 + 2*a*b*x^(1
/3) + b^2*x^(2/3)] - Sqrt[b^2]*x^(1/3)] - 12*Sqrt[b^2]*x^(4/3)*Log[-a + Sqr
```

```
t[a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3)] - Sqrt[b^2]*x^(1/3)] - (12*a^2*(b^2)^(
3/2)*x^(2/3)*Log[a + Sqrt[a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3)] - Sqrt[b^2]*x^
(1/3)])/b^4 + (12*a*Sqrt[a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3)]*x^(2/3)*Log[a +
 Sqrt[a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3)] - Sqrt[b^2]*x^(1/3)])/b - (24*a*Sq
rt[b^2]*x*Log[a + Sqrt[a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3)] - Sqrt[b^2]*x^(1/
3)])/b + 12*Sqrt[a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3)]*x*Log[a + Sqrt[a^2 + 2*
a*b*x^(1/3) + b^2*x^(2/3)] - Sqrt[b^2]*x^(1/3)] - 12*Sqrt[b^2]*x^(4/3)*Log[
a + Sqrt[a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3)] - Sqrt[b^2]*x^(1/3)])/((-a + Sq
rt[a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3)] - Sqrt[b^2]*x^(1/3))^2*(a + Sqrt[a^2
+ 2*a*b*x^(1/3) + b^2*x^(2/3)] - Sqrt[b^2]*x^(1/3))^2)
```

fricas [A] time = 1.23, size = 113, normalized size = 0.87

$$\frac{3\left(6\,a^3b^3x+3\,a^6+2\left(b^6x^2+2\,a^3b^3x+a^6\right)\log\left(bx^{\frac{1}{3}}+a\right)+\left(4\,ab^5x+a^4b^2\right)x^{\frac{2}{3}}-\left(5\,a^2b^4x+2\,a^5b\right)x^{\frac{1}{3}}\right)}{2\left(b^9x^2+2\,a^3b^6x+a^6b^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(3/2),x, algorithm="fricas")
```

```
[Out] 3/2*(6*a^3*b^3*x + 3*a^6 + 2*(b^6*x^2 + 2*a^3*b^3*x + a^6)*log(b*x^(1/3) +
a) + (4*a*b^5*x + a^4*b^2)*x^(2/3) - (5*a^2*b^4*x + 2*a^5*b)*x^(1/3))/(b^9*
x^2 + 2*a^3*b^6*x + a^6*b^3)
```

giac [A] time = 0.53, size = 64, normalized size = 0.49

$$\frac{3\,\log\left(\left|bx^{\frac{1}{3}}+a\right|\right)}{b^3\mathrm{sgn}\left(bx^{\frac{1}{3}}+a\right)}+\frac{3\left(4\,ax^{\frac{1}{3}}+\frac{3\,a^2}{b}\right)}{2\left(bx^{\frac{1}{3}}+a\right)^2b^2\mathrm{sgn}\left(bx^{\frac{1}{3}}+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(3/2),x, algorithm="giac")
```

```
[Out] 3*log(abs(b*x^(1/3) + a))/(b^3*sgn(b*x^(1/3) + a)) + 3/2*(4*a*x^(1/3) + 3*a
^2/b)/((b*x^(1/3) + a)^2*b^2*sgn(b*x^(1/3) + a))
```

maple [A] time = 0.01, size = 92, normalized size = 0.71

$$\frac{3\sqrt{b^2x^{\frac{2}{3}}+2ab\,x^{\frac{1}{3}}+a^2}\,\left(2b^2x^{\frac{2}{3}}\ln\left(b\,x^{\frac{1}{3}}+a\right)+4ab\,x^{\frac{1}{3}}\ln\left(b\,x^{\frac{1}{3}}+a\right)+2a^2\ln\left(b\,x^{\frac{1}{3}}+a\right)+4ab\,x^{\frac{1}{3}}+3a^2\right)}{2\left(b\,x^{\frac{1}{3}}+a\right)^3b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b^2*x^(2/3)+2*a*b*x^(1/3)+a^2)^(3/2),x)
```

```
[Out] 3/2*(b^2*x^(2/3)+2*a*b*x^(1/3)+a^2)^(1/2)*(2*x^(2/3)*ln(b*x^(1/3)+a)*b^2+4*
x^(1/3)*ln(b*x^(1/3)+a)*a*b+4*a*b*x^(1/3)+2*a^2*ln(b*x^(1/3)+a)+3*a^2)/(b*x
^(1/3)+a)^3/b^3
```

maxima [A] time = 0.62, size = 55, normalized size = 0.42

$$\frac{3\,\log\left(x^{\frac{1}{3}}+\frac{a}{b}\right)}{b^3}+\frac{6\,ax^{\frac{1}{3}}}{b^4\left(x^{\frac{1}{3}}+\frac{a}{b}\right)^2}+\frac{9\,a^2}{2\,b^5\left(x^{\frac{1}{3}}+\frac{a}{b}\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(3/2),x, algorithm="maxima")
```

```
[Out] 3*log(x^(1/3) + a/b)/b^3 + 6*a*x^(1/3)/(b^4*(x^(1/3) + a/b)^2) + 9/2*a^2/(b
^5*(x^(1/3) + a/b)^2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(a^2+b^2\,x^{2/3}+2\,a\,b\,x^{1/3}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a^2 + b^2*x^(2/3) + 2*a*b*x^(1/3))^(3/2),x)
```

```
[Out] int(1/(a^2 + b^2*x^(2/3) + 2*a*b*x^(1/3))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a^2+2ab\sqrt[3]{x}+b^2x^{\frac{2}{3}}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a**2+2*a*b*x**(1/3)+b**2*x**(2/3))**(3/2),x)
```

```
[Out] Integral((a**2 + 2*a*b*x**(1/3) + b**2*x**(2/3))**(-3/2), x)
```

3.407 $\int \frac{1}{\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^{5/2}}\,dx$

Optimal. Leaf size=135

$$-\frac{3a^2}{4b^3\left(a+b\sqrt[3]{x}\right)^3\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}+\frac{2a}{b^3\left(a+b\sqrt[3]{x}\right)^2\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}-\frac{3}{2b^3\left(a+b\sqrt[3]{x}\right)\sqrt{a^2+2ab\sqrt[3]{x}}}$$

Rubi [A] time = 0.07, antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1341, 646, 43}

$$-\frac{3a^2}{4b^3\left(a+b\sqrt[3]{x}\right)^3\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}+\frac{2a}{b^3\left(a+b\sqrt[3]{x}\right)^2\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}-\frac{3}{2b^3\left(a+b\sqrt[3]{x}\right)\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^(-5/2),x]
```

```
[Out] (-3*a^2)/(4*b^3*(a + b*x^(1/3))^3*Sqrt[a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3)])
+ (2*a)/(b^3*(a + b*x^(1/3))^2*Sqrt[a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3)]) - 3
/(2*b^3*(a + b*x^(1/3))*Sqrt[a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3)])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 1341

```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k
 = Denominator[n]}, Dist[k, Subst[Int[x^(k - 1)*(a + b*x^(k*n) + c*x^(2*k*n
))^p, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && EqQ[n2, 2*n] && Fra
ctionQ[n]
```

Rubi steps

$$\int \frac{1}{\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^{5/2}}\,dx = 3\,\text{Subst}\left(\int \frac{x^2}{\left(a^2+2abx+b^2x^2\right)^{5/2}}\,dx, x, \sqrt[3]{x}\right)$$

$$= \frac{\left(3b^5\left(a+b\sqrt[3]{x}\right)\right)\text{Subst}\left(\int \frac{x^2}{(ab+b^2x)^5}\,dx, x, \sqrt[3]{x}\right)}{\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}$$

$$= \frac{\left(3b^5\left(a+b\sqrt[3]{x}\right)\right)\text{Subst}\left(\int \left(\frac{a^2}{b^7(a+bx)^5}-\frac{2a}{b^7(a+bx)^4}+\frac{1}{b^7(a+bx)^3}\right)dx, x, \sqrt[3]{x}\right)}{\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}$$

$$= -\frac{3a^2}{4b^3\left(a+b\sqrt[3]{x}\right)^3\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}} + \frac{2a}{b^3\left(a+b\sqrt[3]{x}\right)^2\sqrt{a^2+2ab\sqrt[3]{x}+}}$$

Mathematica [A] time = 0.04, size = 58, normalized size = 0.43

$$\frac{-a^2-4ab\sqrt[3]{x}-6b^2x^{2/3}}{4b^3\left(a+b\sqrt[3]{x}\right)^3\sqrt{\left(a+b\sqrt[3]{x}\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^(-5/2),x]
```

```
[Out] (-a^2 - 4*a*b*x^(1/3) - 6*b^2*x^(2/3))/(4*b^3*(a + b*x^(1/3))^3*Sqrt[(a + b
*x^(1/3))^2])
```

IntegrateAlgebraic [B] time = 0.77, size = 284, normalized size = 2.10

$$\frac{2\sqrt{b^2}\left(3a^6+a^2b^4x^{4/3}+4ab^5x^{5/3}+6b^6x^2\right)+2\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}\left(3a^5b-3a^4b^2\sqrt[3]{x}+3a^3b^3x^{2/3}-3a^2b^4x+2ab^5x^{4/3}-6b^6x^{5/3}\right)}{\sqrt{b^2}\,b^4x^{4/3}\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}\left(-8a^3b^3-24a^2b^4\sqrt[3]{x}-24ab^5x^{2/3}-8b^6x\right)+b^4x^{4/3}\left(8a^4b^4+32a^3b^5\sqrt[3]{x}+48a^2b^6x^{2/3}+32ab^7x+8b^8x^{4/3}\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^(-5/2),x]
```

```
[Out] (2*Sqrt[a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3)]*(3*a^5*b - 3*a^4*b^2*x^(1/3) + 3
*a^3*b^3*x^(2/3) - 3*a^2*b^4*x + 2*a*b^5*x^(4/3) - 6*b^6*x^(5/3)) + 2*Sqrt[
b^2]*(3*a^6 + a^2*b^4*x^(4/3) + 4*a*b^5*x^(5/3) + 6*b^6*x^2))/(b^4*Sqrt[b^2
]*Sqrt[a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3)]*x^(4/3)*(-8*a^3*b^3 - 24*a^2*b^4*
x^(1/3) - 24*a*b^5*x^(2/3) - 8*b^6*x) + b^4*x^(4/3)*(8*a^4*b^4 + 32*a^3*b^5
*x^(1/3) + 48*a^2*b^6*x^(2/3) + 32*a*b^7*x + 8*b^8*x^(4/3)))
```

fricas [A] time = 1.14, size = 136, normalized size = 1.01

$$\frac{20\,ab^9x^3-60\,a^4b^6x^2-a^{10}-9\left(5\,a^2b^8x^2-4\,a^5b^5x\right)x^{\frac{2}{3}}-3\left(2\,b^{10}x^3-20\,a^3b^7x^2+5\,a^6b^4x\right)x^{\frac{1}{3}}}{4\left(b^{15}x^4+4\,a^3b^{12}x^3+6\,a^6b^9x^2+4\,a^9b^6x+a^{12}b^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/4*(20*a*b^9*x^3 - 60*a^4*b^6*x^2 - a^10 - 9*(5*a^2*b^8*x^2 - 4*a^5*b^5*x)
*x^(2/3) - 3*(2*b^10*x^3 - 20*a^3*b^7*x^2 + 5*a^6*b^4*x)*x^(1/3))/(b^15*x^4
 + 4*a^3*b^12*x^3 + 6*a^6*b^9*x^2 + 4*a^9*b^6*x + a^12*b^3)
```

giac [A] time = 0.55, size = 43, normalized size = 0.32

$$-\frac{6\,b^2x^{\frac{2}{3}}+4\,abx^{\frac{1}{3}}+a^2}{4\left(bx^{\frac{1}{3}}+a\right)^4b^3\mathrm{sgn}\left(bx^{\frac{1}{3}}+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(5/2),x, algorithm="giac")
```

```
[Out] -1/4*(6*b^2*x^(2/3) + 4*a*b*x^(1/3) + a^2)/((b*x^(1/3) + a)^4*b^3*sgn(b*x^(
1/3) + a))
```

maple [A] time = 0.01, size = 54, normalized size = 0.40

$$-\frac{\sqrt{b^2x^{\frac{2}{3}}+2ab\,x^{\frac{1}{3}}+a^2}\,\left(6b^2x^{\frac{2}{3}}+4ab\,x^{\frac{1}{3}}+a^2\right)}{4\left(b\,x^{\frac{1}{3}}+a\right)^5b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b^2*x^(2/3)+2*a*b*x^(1/3)+a^2)^(5/2),x)
```

```
[Out] -1/4*(b^2*x^(2/3)+2*a*b*x^(1/3)+a^2)^(1/2)*(6*b^2*x^(2/3)+4*a*b*x^(1/3)+a^2
)/(b*x^(1/3)+a)^5/b^3
```

maxima [A] time = 0.72, size = 53, normalized size = 0.39

$$-\frac{3}{2\,b^5\left(x^{\frac{1}{3}}+\frac{a}{b}\right)^2}+\frac{2\,a}{b^6\left(x^{\frac{1}{3}}+\frac{a}{b}\right)^3}-\frac{3\,a^2}{4\,b^7\left(x^{\frac{1}{3}}+\frac{a}{b}\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(5/2),x, algorithm="maxima")
```

```
[Out] -3/2/(b^5*(x^(1/3) + a/b)^2) + 2*a/(b^6*(x^(1/3) + a/b)^3) - 3/4*a^2/(b^7*(
x^(1/3) + a/b)^4)
```

mupad [B] time = 2.80, size = 53, normalized size = 0.39

$$-\frac{\sqrt{a^2+b^2\,x^{2/3}+2\,a\,b\,x^{1/3}}\,\left(a^2+6\,b^2\,x^{2/3}+4\,a\,b\,x^{1/3}\right)}{4\,b^3\,\left(a+b\,x^{1/3}\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a^2 + b^2*x^(2/3) + 2*a*b*x^(1/3))^(5/2),x)
```

```
[Out] -((a^2 + b^2*x^(2/3) + 2*a*b*x^(1/3))^(1/2)*(a^2 + 6*b^2*x^(2/3) + 4*a*b*x^
(1/3)))/(4*b^3*(a + b*x^(1/3))^5)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\left(a^2+2ab\sqrt[3]{x}+b^2x^{\frac{2}{3}}\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a**2+2*a*b*x**(1/3)+b**2*x**(2/3))**(5/2),x)
```

```
[Out] Integral((a**2 + 2*a*b*x**(1/3) + b**2*x**(2/3))**(-5/2), x)
```

3.408 $\int \frac{1}{\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^{7/2}}\,dx$

Optimal. Leaf size=137

$$-\frac{a^2}{2b^3\left(a+b\sqrt[3]{x}\right)^5\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}+\frac{6a}{5b^3\left(a+b\sqrt[3]{x}\right)^4\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}-\frac{3}{4b^3\left(a+b\sqrt[3]{x}\right)^3\sqrt{a^2+2a\ldots}}$$

Rubi [A] time = 0.08, antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1341, 646, 43}

$$-\frac{a^2}{2b^3\left(a+b\sqrt[3]{x}\right)^5\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}+\frac{6a}{5b^3\left(a+b\sqrt[3]{x}\right)^4\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}-\frac{3}{4b^3\left(a+b\sqrt[3]{x}\right)^3\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^(-7/2),x]
```

```
[Out] -a^2/(2*b^3*(a + b*x^(1/3))^5*Sqrt[a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3)]) + (6
*a)/(5*b^3*(a + b*x^(1/3))^4*Sqrt[a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3)]) - 3/(
4*b^3*(a + b*x^(1/3))^3*Sqrt[a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3)])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 1341

```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k
 = Denominator[n]}, Dist[k, Subst[Int[x^(k - 1)*(a + b*x^(k*n) + c*x^(2*k*n
))^p, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && EqQ[n2, 2*n] && Fra
ctionQ[n]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^{7/2}}\,dx &= 3\,\text{Subst}\left(\int \frac{x^2}{\left(a^2+2abx+b^2x^2\right)^{7/2}}\,dx, x, \sqrt[3]{x}\right)\\
&= \frac{\left(3b^7\left(a+b\sqrt[3]{x}\right)\right)\text{Subst}\left(\int \frac{x^2}{\left(ab+b^2x\right)^7}\,dx, x, \sqrt[3]{x}\right)}{\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}\\
&= \frac{\left(3b^7\left(a+b\sqrt[3]{x}\right)\right)\text{Subst}\left(\int \left(\frac{a^2}{b^9(a+bx)^7}-\frac{2a}{b^9(a+bx)^6}+\frac{1}{b^9(a+bx)^5}\right)dx, x, \sqrt[3]{x}\right)}{\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}\\
&= -\frac{a^2}{2b^3\left(a+b\sqrt[3]{x}\right)^5\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}+\frac{6a}{5b^3\left(a+b\sqrt[3]{x}\right)^4\sqrt{a^2+2ab\sqrt[3]{x}+b}}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 58, normalized size = 0.42

$$\frac{-a^2-6ab\sqrt[3]{x}-15b^2x^{2/3}}{20b^3\left(a+b\sqrt[3]{x}\right)^5\sqrt{\left(a+b\sqrt[3]{x}\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^(-7/2),x]
```

```
[Out] (-a^2 - 6*a*b*x^(1/3) - 15*b^2*x^(2/3))/(20*b^3*(a + b*x^(1/3))^5*Sqrt[(a +
 b*x^(1/3))^2])
```

IntegrateAlgebraic [B] time = 0.89, size = 356, normalized size = 2.60

$$\frac{8\sqrt{b^2}\left(10a^8+a^2b^6x^2+6ab^7x^{7/3}+15b^8x^{8/3}\right)+8\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}\left(10a^7b-10a^6b^2\sqrt[3]{x}+10a^5b^3x^{2/3}-10a^4b^4x+10a^3b^5x^{4/3}-10a^2b^6x^{5/3}+9ab^7x^2-15b^8x^{7/3}\right)}{5\sqrt{b^2}\,b^4x^2\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}\left(-32a^5b^5-160a^4b^6\sqrt[3]{x}-320a^3b^7x^{2/3}-320a^2b^8x-160ab^9x^{4/3}-32b^{10}x^{5/3}\right)+5b^4x^2\left(32a^6b^6+192a^5b^7\sqrt[3]{x}+480a^4b^8x^{2/3}+640a^3b^9x+480a^2b^{10}x^{4/3}+192ab^{11}x^{5/3}+32b^{12}x^2\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^(-7/2),x]
```

```
[Out] (8*Sqrt[a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3)]*(10*a^7*b - 10*a^6*b^2*x^(1/3) +
 10*a^5*b^3*x^(2/3) - 10*a^4*b^4*x + 10*a^3*b^5*x^(4/3) - 10*a^2*b^6*x^(5/3
) + 9*a*b^7*x^2 - 15*b^8*x^(7/3)) + 8*Sqrt[b^2]*(10*a^8 + a^2*b^6*x^2 + 6*a
*b^7*x^(7/3) + 15*b^8*x^(8/3)))/(5*b^4*Sqrt[b^2]*Sqrt[a^2 + 2*a*b*x^(1/3) +
 b^2*x^(2/3)]*x^2*(-32*a^5*b^5 - 160*a^4*b^6*x^(1/3) - 320*a^3*b^7*x^(2/3)
- 320*a^2*b^8*x - 160*a*b^9*x^(4/3) - 32*b^10*x^(5/3)) + 5*b^4*x^2*(32*a^6*
b^6 + 192*a^5*b^7*x^(1/3) + 480*a^4*b^8*x^(2/3) + 640*a^3*b^9*x + 480*a^2*b
^10*x^(4/3) + 192*a*b^11*x^(5/3) + 32*b^12*x^2))
```

fricas [A] time = 1.31, size = 209, normalized size = 1.53

$$-\frac{280\,a^2b^{12}x^4-1400\,a^5b^9x^3+735\,a^8b^6x^2-14\,a^{11}b^3x+a^{14}+3\left(5\,b^{14}x^4-210\,a^3b^{11}x^3+483\,a^6b^8x^2-112\,a^9b^5x\right)x^{\frac{2}{3}}-3\left(28\,ab^{13}x^4-357\,a^4b^{10}x^3+390\,a^7b^7x^2-35\,a^{10}b^4x\right)x^{\frac{1}{3}}}{20\left(b^{21}x^6+6\,a^3b^{18}x^5+15\,a^6b^{15}x^4+20\,a^9b^{12}x^3+15\,a^{12}b^9x^2+6\,a^{15}b^6x+a^{18}b^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(7/2),x, algorithm="fricas")
```

```
[Out] -1/20*(280*a^2*b^12*x^4 - 1400*a^5*b^9*x^3 + 735*a^8*b^6*x^2 - 14*a^11*b^3*
x + a^14 + 3*(5*b^14*x^4 - 210*a^3*b^11*x^3 + 483*a^6*b^8*x^2 - 112*a^9*b^5
*x)*x^(2/3) - 3*(28*a*b^13*x^4 - 357*a^4*b^10*x^3 + 390*a^7*b^7*x^2 - 35*a^
10*b^4*x)*x^(1/3))/(b^21*x^6 + 6*a^3*b^18*x^5 + 15*a^6*b^15*x^4 + 20*a^9*b^
12*x^3 + 15*a^12*b^9*x^2 + 6*a^15*b^6*x + a^18*b^3)
```

giac [A] time = 0.59, size = 43, normalized size = 0.31

$$-\frac{15\,b^2x^{\frac{2}{3}} + 6\,abx^{\frac{1}{3}} + a^2}{20\left(bx^{\frac{1}{3}} + a\right)^6 b^3\mathrm{sgn}\left(bx^{\frac{1}{3}} + a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(7/2),x, algorithm="giac")
```

```
[Out] -1/20*(15*b^2*x^(2/3) + 6*a*b*x^(1/3) + a^2)/((b*x^(1/3) + a)^6*b^3*sgn(b*x
^(1/3) + a))
```

maple [A] time = 0.01, size = 54, normalized size = 0.39

$$-\frac{\sqrt{b^2x^{\frac{2}{3}} + 2ab\,x^{\frac{1}{3}} + a^2}\,\left(15b^2x^{\frac{2}{3}} + 6ab\,x^{\frac{1}{3}} + a^2\right)}{20\left(b\,x^{\frac{1}{3}} + a\right)^7 b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b^2*x^(2/3)+2*a*b*x^(1/3)+a^2)^(7/2),x)
```

```
[Out] -1/20*(b^2*x^(2/3)+2*a*b*x^(1/3)+a^2)^(1/2)*(15*b^2*x^(2/3)+6*a*b*x^(1/3)+a
^2)/(b*x^(1/3)+a)^7/b^3
```

maxima [A] time = 0.54, size = 53, normalized size = 0.39

$$-\frac{3}{4\,b^7\left(x^{\frac{1}{3}} + \frac{a}{b}\right)^4} + \frac{6\,a}{5\,b^8\left(x^{\frac{1}{3}} + \frac{a}{b}\right)^5} - \frac{a^2}{2\,b^9\left(x^{\frac{1}{3}} + \frac{a}{b}\right)^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(7/2),x, algorithm="maxima")
```

```
[Out] -3/4/(b^7*(x^(1/3) + a/b)^4) + 6/5*a/(b^8*(x^(1/3) + a/b)^5) - 1/2*a^2/(b^9
*(x^(1/3) + a/b)^6)
```

mupad [B] time = 3.23, size = 53, normalized size = 0.39

$$-\frac{\sqrt{a^2 + b^2\,x^{2/3} + 2\,a\,b\,x^{1/3}}\,\left(a^2 + 15\,b^2\,x^{2/3} + 6\,a\,b\,x^{1/3}\right)}{20\,b^3\,\left(a + b\,x^{1/3}\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a^2 + b^2*x^(2/3) + 2*a*b*x^(1/3))^(7/2),x)
```

```
[Out] -((a^2 + b^2*x^(2/3) + 2*a*b*x^(1/3))^(1/2)*(a^2 + 15*b^2*x^(2/3) + 6*a*b*x
^(1/3)))/(20*b^3*(a + b*x^(1/3))^7)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a^2 + 2ab\sqrt[3]{x} + b^2x^{\frac{2}{3}}\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a**2+2*a*b*x**(1/3)+b**2*x**(2/3))**(7/2),x)
```

```
[Out] Integral((a**2 + 2*a*b*x**(1/3) + b**2*x**(2/3))**(-7/2), x)
```

3.409 $\int \frac{1}{\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^{9/2}}\,dx$

Optimal. Leaf size=137

$$-\frac{3a^2}{8b^3\left(a+b\sqrt[3]{x}\right)^7\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}+\frac{6a}{7b^3\left(a+b\sqrt[3]{x}\right)^6\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}-\frac{1}{2b^3\left(a+b\sqrt[3]{x}\right)^5\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}$$

Rubi [A] time = 0.08, antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1341, 646, 43}

$$-\frac{3a^2}{8b^3\left(a+b\sqrt[3]{x}\right)^7\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}+\frac{6a}{7b^3\left(a+b\sqrt[3]{x}\right)^6\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}-\frac{1}{2b^3\left(a+b\sqrt[3]{x}\right)^5\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^(-9/2),x]
```

```
[Out] (-3*a^2)/(8*b^3*(a + b*x^(1/3))^7*Sqrt[a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3)])
+ (6*a)/(7*b^3*(a + b*x^(1/3))^6*Sqrt[a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3)]) -
 1/(2*b^3*(a + b*x^(1/3))^5*Sqrt[a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3)])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 1341

```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k
 = Denominator[n]}, Dist[k, Subst[Int[x^(k - 1)*(a + b*x^(k*n) + c*x^(2*k*n
))^p, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && EqQ[n2, 2*n] && Fra
ctionQ[n]
```

Rubi steps

$$\int \frac{1}{\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^{9/2}}\,dx = 3\,\text{Subst}\left(\int \frac{x^2}{\left(a^2+2abx+b^2x^2\right)^{9/2}}\,dx, x, \sqrt[3]{x}\right)$$

$$= \frac{\left(3b^9\left(a+b\sqrt[3]{x}\right)\right)\text{Subst}\left(\int \frac{x^2}{\left(ab+b^2x\right)^9}\,dx, x, \sqrt[3]{x}\right)}{\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}$$

$$= \frac{\left(3b^9\left(a+b\sqrt[3]{x}\right)\right)\text{Subst}\left(\int \left(\frac{a^2}{b^{11}(a+bx)^9}-\frac{2a}{b^{11}(a+bx)^8}+\frac{1}{b^{11}(a+bx)^7}\right)dx, x, \sqrt[3]{x}\right)}{\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}$$

$$= -\frac{3a^2}{8b^3\left(a+b\sqrt[3]{x}\right)^7\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}+\frac{6a}{7b^3\left(a+b\sqrt[3]{x}\right)^6\sqrt{a^2+2ab\sqrt[3]{x}}}$$

Mathematica [A] time = 0.04, size = 58, normalized size = 0.42

$$\frac{-a^2-8ab\sqrt[3]{x}-28b^2x^{2/3}}{56b^3\left(a+b\sqrt[3]{x}\right)^7\sqrt{\left(a+b\sqrt[3]{x}\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^(-9/2),x]
```

```
[Out] (-a^2 - 8*a*b*x^(1/3) - 28*b^2*x^(2/3))/(56*b^3*(a + b*x^(1/3))^7*Sqrt[(a +
 b*x^(1/3))^2])
```

IntegrateAlgebraic [B] time = 1.00, size = 434, normalized size = 3.17

$$\frac{16\sqrt{b^2}\left(21a^{10}+a^2b^8x^{8/3}+8ab^9x^3+28b^{10}x^{10/3}\right)+16\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}\left(21a^9b-21a^8b^2\sqrt[3]{x}+21a^7b^3x^{2/3}-21a^6b^4x+21a^5b^5x^{4/3}-21a^4b^6x^{5/3}+21a^3b^7x^2-21a^2b^8x^{7/3}+20ab^9x^{8/3}-28b^{10}x^3\right)}{7\sqrt{b^2}\,b^4x^{8/3}\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}\left(-128a^7b^7-896a^6b^8\sqrt[3]{x}-2688a^5b^9x^{2/3}-4480a^4b^{10}x-4480a^3b^{11}x^{4/3}-2688a^2b^{12}x^{5/3}-896ab^{13}x^2-128b^{14}x^{7/3}\right)+7b^4x^{8/3}\left(128a^8b^8+1024a^7b^9\sqrt[3]{x}+3584a^6b^{10}x^{2/3}+7168a^5b^{11}x+8960a^4b^{12}x^{4/3}+7168a^3b^{13}x^{5/3}+3584a^2b^{14}x^2+1024ab^{15}x^{7/3}+128b^{16}x^{8/3}\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^(-9/2),x]
```

```
[Out] (16*Sqrt[a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3)]*(21*a^9*b - 21*a^8*b^2*x^(1/3)
+ 21*a^7*b^3*x^(2/3) - 21*a^6*b^4*x + 21*a^5*b^5*x^(4/3) - 21*a^4*b^6*x^(5/
3) + 21*a^3*b^7*x^2 - 21*a^2*b^8*x^(7/3) + 20*a*b^9*x^(8/3) - 28*b^10*x^3)
+ 16*Sqrt[b^2]*(21*a^10 + a^2*b^8*x^(8/3) + 8*a*b^9*x^3 + 28*b^10*x^(10/3))
)/(7*b^4*Sqrt[b^2]*Sqrt[a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3)]*x^(8/3)*(-128*a^
7*b^7 - 896*a^6*b^8*x^(1/3) - 2688*a^5*b^9*x^(2/3) - 4480*a^4*b^10*x - 4480
*a^3*b^11*x^(4/3) - 2688*a^2*b^12*x^(5/3) - 896*a*b^13*x^2 - 128*b^14*x^(7/
3)) + 7*b^4*x^(8/3)*(128*a^8*b^8 + 1024*a^7*b^9*x^(1/3) + 3584*a^6*b^10*x^(
2/3) + 7168*a^5*b^11*x + 8960*a^4*b^12*x^(4/3) + 7168*a^3*b^13*x^(5/3) + 35
84*a^2*b^14*x^2 + 1024*a*b^15*x^(7/3) + 128*b^16*x^(8/3)))
```

fricas [B] time = 1.57, size = 275, normalized size = 2.01

$$-\frac{28b^{18}x^6-2856a^3b^{15}x^5+18186a^6b^{12}x^4-20608a^9b^9x^3+4200a^{12}b^6x^2-48a^{15}b^3x+a^{18}-27\left(8ab^{17}x^5-244a^4b^{14}x^4+840a^7b^{11}x^3-553a^{10}b^8x^2+56a^{13}b^5x\right)x^{\frac{2}{3}}+27\left(35a^2b^{16}x^5-448a^5b^{13}x^4+876a^8b^{10}x^3-328a^{11}b^7x^2+14a^{14}b^4x\right)x^{\frac{1}{3}}}{56\left(b^{27}x^8+8a^3b^{24}x^7+28a^6b^{21}x^6+56a^9b^{18}x^5+70a^{12}b^{15}x^4+56a^{15}b^{12}x^3+28a^{18}b^9x^2+8a^{21}b^6x+a^{24}b^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(9/2),x, algorithm="fricas")
```

```
[Out] -1/56*(28*b^18*x^6 - 2856*a^3*b^15*x^5 + 18186*a^6*b^12*x^4 - 20608*a^9*b^9
*x^3 + 4200*a^12*b^6*x^2 - 48*a^15*b^3*x + a^18 - 27*(8*a*b^17*x^5 - 244*a^
4*b^14*x^4 + 840*a^7*b^11*x^3 - 553*a^10*b^8*x^2 + 56*a^13*b^5*x)*x^(2/3) +
```

```
27*(35*a^2*b^16*x^5 - 448*a^5*b^13*x^4 + 876*a^8*b^10*x^3 - 328*a^11*b^7*x
^2 + 14*a^14*b^4*x)*x^(1/3))/(b^27*x^8 + 8*a^3*b^24*x^7 + 28*a^6*b^21*x^6 +
 56*a^9*b^18*x^5 + 70*a^12*b^15*x^4 + 56*a^15*b^12*x^3 + 28*a^18*b^9*x^2 +
8*a^21*b^6*x + a^24*b^3)
```

giac [A] time = 0.65, size = 43, normalized size = 0.31

$$-\frac{28\,b^2x^{\frac{2}{3}}+8\,abx^{\frac{1}{3}}+a^2}{56\left(bx^{\frac{1}{3}}+a\right)^8b^3\mathrm{sgn}\left(bx^{\frac{1}{3}}+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(9/2),x, algorithm="giac")
```

```
[Out] -1/56*(28*b^2*x^(2/3) + 8*a*b*x^(1/3) + a^2)/((b*x^(1/3) + a)^8*b^3*sgn(b*x
^(1/3) + a))
```

maple [A] time = 0.01, size = 54, normalized size = 0.39

$$-\frac{\sqrt{b^2x^{\frac{2}{3}}+2ab\,x^{\frac{1}{3}}+a^2}\,\left(28b^2x^{\frac{2}{3}}+8ab\,x^{\frac{1}{3}}+a^2\right)}{56\left(b\,x^{\frac{1}{3}}+a\right)^9b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b^2*x^(2/3)+2*a*b*x^(1/3)+a^2)^(9/2),x)
```

```
[Out] -1/56*(b^2*x^(2/3)+2*a*b*x^(1/3)+a^2)^(1/2)*(28*b^2*x^(2/3)+8*a*b*x^(1/3)+a
^2)/(b*x^(1/3)+a)^9/b^3
```

maxima [A] time = 0.72, size = 53, normalized size = 0.39

$$-\frac{1}{2\,b^9\left(x^{\frac{1}{3}}+\frac{a}{b}\right)^6}+\frac{6\,a}{7\,b^{10}\left(x^{\frac{1}{3}}+\frac{a}{b}\right)^7}-\frac{3\,a^2}{8\,b^{11}\left(x^{\frac{1}{3}}+\frac{a}{b}\right)^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(9/2),x, algorithm="maxima")
```

```
[Out] -1/2/(b^9*(x^(1/3) + a/b)^6) + 6/7*a/(b^10*(x^(1/3) + a/b)^7) - 3/8*a^2/(b^
11*(x^(1/3) + a/b)^8)
```

mupad [B] time = 3.65, size = 53, normalized size = 0.39

$$-\frac{\sqrt{a^2+b^2\,x^{2/3}+2\,a\,b\,x^{1/3}}\,\left(a^2+28\,b^2\,x^{2/3}+8\,a\,b\,x^{1/3}\right)}{56\,b^3\left(a+b\,x^{1/3}\right)^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a^2 + b^2*x^(2/3) + 2*a*b*x^(1/3))^(9/2),x)
```

```
[Out] -((a^2 + b^2*x^(2/3) + 2*a*b*x^(1/3))^(1/2)*(a^2 + 28*b^2*x^(2/3) + 8*a*b*x
^(1/3)))/(56*b^3*(a + b*x^(1/3))^9)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\left(a^2+2ab\sqrt[3]{x}+b^2x^{\frac{2}{3}}\right)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a**2+2*a*b*x**(1/3)+b**2*x**(2/3))**(9/2),x)
```

```
[Out] Integral((a**2 + 2*a*b*x**(1/3) + b**2*x**(2/3))**(-9/2), x)
```

3.410 $\int \frac{1}{\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^{11/2}}\,dx$

Optimal. Leaf size=137

$$-\frac{3a^2}{10b^3\left(a+b\sqrt[3]{x}\right)^9\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}+\frac{2a}{3b^3\left(a+b\sqrt[3]{x}\right)^8\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}-\frac{3}{8b^3\left(a+b\sqrt[3]{x}\right)^7\sqrt{a^2+2ab}}$$

Rubi [A] time = 0.08, antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1341, 646, 43}

$$-\frac{3a^2}{10b^3\left(a+b\sqrt[3]{x}\right)^9\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}+\frac{2a}{3b^3\left(a+b\sqrt[3]{x}\right)^8\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}-\frac{3}{8b^3\left(a+b\sqrt[3]{x}\right)^7\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^(-11/2),x]
```

```
[Out] (-3*a^2)/(10*b^3*(a + b*x^(1/3))^9*Sqrt[a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3)])
 + (2*a)/(3*b^3*(a + b*x^(1/3))^8*Sqrt[a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3)])
- 3/(8*b^3*(a + b*x^(1/3))^7*Sqrt[a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3)])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 1341

```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k
 = Denominator[n]}, Dist[k, Subst[Int[x^(k - 1)*(a + b*x^(k*n) + c*x^(2*k*n
))^p, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && EqQ[n2, 2*n] && Fra
ctionQ[n]
```

Rubi steps

$$\int \frac{1}{\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^{11/2}}\,dx = 3\,\text{Subst}\left(\int \frac{x^2}{\left(a^2+2abx+b^2x^2\right)^{11/2}}\,dx, x, \sqrt[3]{x}\right)$$

$$= \frac{\left(3b^{11}\left(a+b\sqrt[3]{x}\right)\right)\text{Subst}\left(\int \frac{x^2}{\left(ab+b^2x\right)^{11}}\,dx, x, \sqrt[3]{x}\right)}{\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}$$

$$= \frac{\left(3b^{11}\left(a+b\sqrt[3]{x}\right)\right)\text{Subst}\left(\int \left(\frac{a^2}{b^{13}(a+bx)^{11}} - \frac{2a}{b^{13}(a+bx)^{10}} + \frac{1}{b^{13}(a+bx)^{9}}\right) dx, x, \sqrt[3]{x}\right)}{\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}$$

$$= -\frac{3a^2}{10b^3\left(a+b\sqrt[3]{x}\right)^9\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}} + \frac{2a}{3b^3\left(a+b\sqrt[3]{x}\right)^8\sqrt{a^2+2ab\sqrt[3]{x}}}$$

Mathematica [A] time = 0.04, size = 58, normalized size = 0.42

$$\frac{-a^2-10ab\sqrt[3]{x}-45b^2x^{2/3}}{120b^3\left(a+b\sqrt[3]{x}\right)^9\sqrt{\left(a+b\sqrt[3]{x}\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^(-11/2),x]
```

```
[Out] (-a^2 - 10*a*b*x^(1/3) - 45*b^2*x^(2/3))/(120*b^3*(a + b*x^(1/3))^9*Sqrt[(a
 + b*x^(1/3))^2])
```

IntegrateAlgebraic [B] time = 1.22, size = 507, normalized size = 3.70

$$\frac{-64\left(-36a^{12}b-a^2b^{11}x^{10/3}-10ab^{12}x^{11/3}-45b^{13}x^4\right)-64\sqrt{b^2}\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}\left(-36a^{11}+36a^{10}b\sqrt[3]{x}-36a^9b^2x^{2/3}+36a^8b^3x-36a^7b^4x^{4/3}+36a^6b^5x^{5/3}-36a^5b^6x^2+36a^4b^7x^{7/3}-36a^3b^8x^{8/3}+36a^2b^9x^3-35ab^{10}x^{10/3}+45b^{11}x^{11/3}\right)}{15b^3x^{10/3}\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}\left(-512a^9b^{11}-4608a^8b^{12}\sqrt[3]{x}-18432a^7b^{13}x^{2/3}-43008a^6b^{14}x-64512a^5b^{15}x^{4/3}-64512a^4b^{16}x^{5/3}-43008a^3b^{17}x^2-18432a^2b^{18}x^{7/3}-4608ab^{19}x^{8/3}-512b^{20}x^3\right)+15b^3\sqrt{b^2}x^{10/3}\left(512a^{10}b^{10}+5120a^9b^{11}\sqrt[3]{x}+23040a^8b^{12}x^{2/3}+61440a^7b^{13}x+107520a^6b^{14}x^{4/3}+129024a^5b^{15}x^{5/3}+107520a^4b^{16}x^2+61440a^3b^{17}x^{7/3}+23040a^2b^{18}x^{8/3}+5120ab^{19}x^3+512b^{20}x^{10/3}\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^(-11/2),x]
```

```
[Out] (-64*Sqrt[b^2]*Sqrt[a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3)]*(-36*a^11 + 36*a^10*
b*x^(1/3) - 36*a^9*b^2*x^(2/3) + 36*a^8*b^3*x - 36*a^7*b^4*x^(4/3) + 36*a^6
*b^5*x^(5/3) - 36*a^5*b^6*x^2 + 36*a^4*b^7*x^(7/3) - 36*a^3*b^8*x^(8/3) + 3
6*a^2*b^9*x^3 - 35*a*b^10*x^(10/3) + 45*b^11*x^(11/3)) - 64*(-36*a^12*b - a
^2*b^11*x^(10/3) - 10*a*b^12*x^(11/3) - 45*b^13*x^4))/(15*b^3*Sqrt[a^2 + 2*
a*b*x^(1/3) + b^2*x^(2/3)]*x^(10/3)*(-512*a^9*b^11 - 4608*a^8*b^12*x^(1/3)
- 18432*a^7*b^13*x^(2/3) - 43008*a^6*b^14*x - 64512*a^5*b^15*x^(4/3) - 6451
2*a^4*b^16*x^(5/3) - 43008*a^3*b^17*x^2 - 18432*a^2*b^18*x^(7/3) - 4608*a*b
^19*x^(8/3) - 512*b^20*x^3) + 15*b^3*Sqrt[b^2]*x^(10/3)*(512*a^10*b^10 + 51
20*a^9*b^11*x^(1/3) + 23040*a^8*b^12*x^(2/3) + 61440*a^7*b^13*x + 107520*a^
6*b^14*x^(4/3) + 129024*a^5*b^15*x^(5/3) + 107520*a^4*b^16*x^2 + 61440*a^3*
b^17*x^(7/3) + 23040*a^2*b^18*x^(8/3) + 5120*a*b^19*x^3 + 512*b^20*x^(10/3)
))
```

fricas [B] time = 1.41, size = 343, normalized size = 2.50

$$\frac{440\,ab^{21}x^7-25630\,a^4b^{18}x^6+186252\,a^7b^{15}x^5-326150\,a^{10}b^{12}x^4+154000\,a^{13}b^9x^3-16005\,a^{16}b^6x^2+110\,a^{19}b^3x-a^{22}-27\left(88\,a^2b^{20}x^6-2200\,a^5b^{17}x^5+9625\,a^8b^{14}x^4-10910\,a^{11}b^{11}x^3+3245\,a^{14}b^8x^2-176\,a^{17}b^5x\right)x^{\frac{2}{3}}-9\left(5\,b^{22}x^7-990\,a^3b^{19}x^6+12705\,a^6b^{16}x^5-34760\,a^9b^{13}x^4+25542\,a^{12}b^{10}x^3-4620\,a^{15}b^7x^2+110\,a^{18}b^4x\right)x^{\frac{1}{3}}}{120\left(b^{33}x^{10}+10\,a^3b^{30}x^9+45\,a^6b^{27}x^8+120\,a^9b^{24}x^7+210\,a^{12}b^{21}x^6+252\,a^{15}b^{18}x^5+210\,a^{18}b^{15}x^4+120\,a^{21}b^{12}x^3+45\,a^{24}b^9x^2+10\,a^{27}b^6x+a^{30}b^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(11/2),x, algorithm="fricas")
```

```
[Out] 1/120*(440*a*b^21*x^7 - 25630*a^4*b^18*x^6 + 186252*a^7*b^15*x^5 - 326150*a
^10*b^12*x^4 + 154000*a^13*b^9*x^3 - 16005*a^16*b^6*x^2 + 110*a^19*b^3*x -
a^22 - 27*(88*a^2*b^20*x^6 - 2200*a^5*b^17*x^5 + 9625*a^8*b^14*x^4 - 10910*
a^11*b^11*x^3 + 3245*a^14*b^8*x^2 - 176*a^17*b^5*x)*x^(2/3) - 9*(5*b^22*x^7
 - 990*a^3*b^19*x^6 + 12705*a^6*b^16*x^5 - 34760*a^9*b^13*x^4 + 25542*a^12*
b^10*x^3 - 4620*a^15*b^7*x^2 + 110*a^18*b^4*x)*x^(1/3))/(b^33*x^10 + 10*a^3
*b^30*x^9 + 45*a^6*b^27*x^8 + 120*a^9*b^24*x^7 + 210*a^12*b^21*x^6 + 252*a^
15*b^18*x^5 + 210*a^18*b^15*x^4 + 120*a^21*b^12*x^3 + 45*a^24*b^9*x^2 + 10*
a^27*b^6*x + a^30*b^3)
```

giac [A] time = 0.74, size = 43, normalized size = 0.31

$$-\frac{45\,b^{2}x^{\frac{2}{3}}+10\,abx^{\frac{1}{3}}+a^{2}}{120\left(bx^{\frac{1}{3}}+a\right)^{10}b^{3}\mathrm{sgn}\left(bx^{\frac{1}{3}}+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(11/2),x, algorithm="giac")
```

```
[Out] -1/120*(45*b^2*x^(2/3) + 10*a*b*x^(1/3) + a^2)/((b*x^(1/3) + a)^10*b^3*sgn(
b*x^(1/3) + a))
```

maple [A] time = 0.01, size = 54, normalized size = 0.39

$$-\frac{\sqrt{b^{2}x^{\frac{2}{3}}+2ab\,x^{\frac{1}{3}}+a^{2}}\,\left(45b^{2}x^{\frac{2}{3}}+10ab\,x^{\frac{1}{3}}+a^{2}\right)}{120\left(b\,x^{\frac{1}{3}}+a\right)^{11}b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b^2*x^(2/3)+2*a*b*x^(1/3)+a^2)^(11/2),x)
```

```
[Out] -1/120*(b^2*x^(2/3)+2*a*b*x^(1/3)+a^2)^(1/2)*(45*b^2*x^(2/3)+10*a*b*x^(1/3)
+a^2)/(b*x^(1/3)+a)^11/b^3
```

maxima [A] time = 0.54, size = 53, normalized size = 0.39

$$-\frac{3}{8\,b^{11}\left(x^{\frac{1}{3}}+\frac{a}{b}\right)^{8}}+\frac{2\,a}{3\,b^{12}\left(x^{\frac{1}{3}}+\frac{a}{b}\right)^{9}}-\frac{3\,a^{2}}{10\,b^{13}\left(x^{\frac{1}{3}}+\frac{a}{b}\right)^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(11/2),x, algorithm="maxima")
```

```
[Out] -3/8/(b^11*(x^(1/3) + a/b)^8) + 2/3*a/(b^12*(x^(1/3) + a/b)^9) - 3/10*a^2/(
b^13*(x^(1/3) + a/b)^10)
```

mupad [B] time = 4.34, size = 53, normalized size = 0.39

$$-\frac{\sqrt{a^2+b^2\,x^{2/3}+2\,a\,b\,x^{1/3}}\,\left(a^2+45\,b^2\,x^{2/3}+10\,a\,b\,x^{1/3}\right)}{120\,b^3\,{\left(a+b\,x^{1/3}\right)}^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a^2 + b^2*x^(2/3) + 2*a*b*x^(1/3))^(11/2),x)
```

```
[Out] -((a^2 + b^2*x^(2/3) + 2*a*b*x^(1/3))^(1/2)*(a^2 + 45*b^2*x^(2/3) + 10*a*b*
x^(1/3)))/(120*b^3*(a + b*x^(1/3))^11)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a^2 + 2ab\sqrt[3]{x} + b^2x^{\frac{2}{3}}\right)^{\frac{11}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a**2+2*a*b*x**(1/3)+b**2*x**(2/3))**(11/2),x)
[Out] Integral((a**2 + 2*a*b*x**(1/3) + b**2*x**(2/3))**(-11/2), x)
```

3.411 $\int \left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p x^2\,dx$

Optimal. Leaf size=468

$$\frac{3a^9\left(\frac{b\sqrt[3]{x}}{a}+1\right)^9\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{b^9(2p+9)}-\frac{12a^9\left(\frac{b\sqrt[3]{x}}{a}+1\right)^8\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{b^9(p+4)}+\frac{84a^9\left(\frac{b\sqrt[3]{x}}{a}+1\right)^7\left(a^2+2ab\sqrt[3]{x}\right.}{b^9(2p+7)}$$

Rubi [A] time = 0.22, antiderivative size = 468, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {1356, 266, 43}

$$\frac{3a^9\left(\frac{b\sqrt[3]{x}}{a}+1\right)^9\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{b^9(2p+9)}-\frac{12a^9\left(\frac{b\sqrt[3]{x}}{a}+1\right)^8\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{b^9(p+4)}+\frac{84a^9\left(\frac{b\sqrt[3]{x}}{a}+1\right)^7\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{b^9(2p+7)}-\frac{84a^9\left(\frac{b\sqrt[3]{x}}{a}+1\right)^6\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{b^9(p+3)}+\frac{210a^9\left(\frac{b\sqrt[3]{x}}{a}+1\right)^5\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{b^9(2p+5)}-\frac{84a^9\left(\frac{b\sqrt[3]{x}}{a}+1\right)^4\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{b^9(p+2)}+\frac{84a^9\left(\frac{b\sqrt[3]{x}}{a}+1\right)^3\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{b^9(2p+3)}-\frac{12a^9\left(\frac{b\sqrt[3]{x}}{a}+1\right)^2\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{b^9(p+1)}+\frac{3a^9\left(\frac{b\sqrt[3]{x}}{a}+1\right)\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{b^9(2p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^p*x^2,x]
```

```
[Out] (3*a^9*(1 + (b*x^(1/3))/a)*(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^p)/(b^9*(1 +
 2*p)) - (12*a^9*(1 + (b*x^(1/3))/a)^2*(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^
p)/(b^9*(1 + p)) + (84*a^9*(1 + (b*x^(1/3))/a)^3*(a^2 + 2*a*b*x^(1/3) + b^2
*x^(2/3))^p)/(b^9*(3 + 2*p)) - (84*a^9*(1 + (b*x^(1/3))/a)^4*(a^2 + 2*a*b*x
^(1/3) + b^2*x^(2/3))^p)/(b^9*(2 + p)) + (210*a^9*(1 + (b*x^(1/3))/a)^5*(a^
2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^p)/(b^9*(5 + 2*p)) - (84*a^9*(1 + (b*x^(1/
3))/a)^6*(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^p)/(b^9*(3 + p)) + (84*a^9*(1
+ (b*x^(1/3))/a)^7*(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^p)/(b^9*(7 + 2*p)) -
 (12*a^9*(1 + (b*x^(1/3))/a)^8*(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^p)/(b^9*
(4 + p)) + (3*a^9*(1 + (b*x^(1/3))/a)^9*(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))
^p)/(b^9*(9 + 2*p))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1356

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n + c*x^(2*n))^FracPart[p])/(1 + (
2*c*x^n)/b)^(2*FracPart[p]), Int[(d*x)^m*(1 + (2*c*x^n)/b)^(2*p), x], x] /;
 FreeQ[{a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] &&
 !IntegerQ[2*p]
```

Rubi steps

$$\int \left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p x^2\,dx = \left(\left(1+\frac{b\sqrt[3]{x}}{a}\right)^{-2p}\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p\right)\int\left(1+\frac{b\sqrt[3]{x}}{a}\right)^{2p}x^2\,dx$$

$$= \left(3\left(1+\frac{b\sqrt[3]{x}}{a}\right)^{-2p}\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p\right)\text{Subst}\left(\int x^8\left(1+\frac{bx}{a}\right)^{2p}dx,x,\sqrt[3]{\right.$$

$$= \left(3\left(1+\frac{b\sqrt[3]{x}}{a}\right)^{-2p}\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p\right)\text{Subst}\left(\int\left(\frac{a^8\left(1+\frac{bx}{a}\right)^{2p}}{b^8}-\frac{8a^8}{\right.\right.$$

$$= \frac{3a^9\left(1+\frac{b\sqrt[3]{x}}{a}\right)\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{b^9(1+2p)} - \frac{12a^9\left(1+\frac{b\sqrt[3]{x}}{a}\right)^2\left(a^2+2ab\sqrt[3]{x}+\right.}{b^9(1+p)}$$

Mathematica [A] time = 0.22, size = 207, normalized size = 0.44

$$\frac{3\left(a+b\sqrt[3]{x}\right)\left(\frac{a^8}{2p+1}-\frac{4a^7\left(a+b\sqrt[3]{x}\right)}{p+1}+\frac{28a^6\left(a+b\sqrt[3]{x}\right)^2}{2p+3}-\frac{28a^5\left(a+b\sqrt[3]{x}\right)^3}{p+2}+\frac{70a^4\left(a+b\sqrt[3]{x}\right)^4}{2p+5}-\frac{28a^3\left(a+b\sqrt[3]{x}\right)^5}{p+3}+\frac{28a^2\left(a+b\sqrt[3]{x}\right)^6}{2p+7}-\frac{4a\left(a+b\sqrt[3]{x}\right)^7}{p+4}+\frac{\left(a+b\sqrt[3]{x}\right)^8}{2p+9}\right)\left(\left(a+b\sqrt[3]{x}\right)^2\right)^p}{b^9}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^p*x^2,x]
```

```
[Out] (3*(a^8/(1 + 2*p) - (4*a^7*(a + b*x^(1/3)))/(1 + p) + (28*a^6*(a + b*x^(1/3
))^2)/(3 + 2*p) - (28*a^5*(a + b*x^(1/3))^3)/(2 + p) + (70*a^4*(a + b*x^(1/
3))^4)/(5 + 2*p) - (28*a^3*(a + b*x^(1/3))^5)/(3 + p) + (28*a^2*(a + b*x^(1
/3))^6)/(7 + 2*p) - (4*a*(a + b*x^(1/3))^7)/(4 + p) + (a + b*x^(1/3))^8/(9
+ 2*p))*(a + b*x^(1/3))*((a + b*x^(1/3))^2)^p)/b^9
```

IntegrateAlgebraic [F] time = 0.54, size = 0, normalized size = 0.00

$$\int \left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p x^2\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^p*x^2,x]
```

```
[Out] Defer[IntegrateAlgebraic][(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^p*x^2, x]
```

fricas [A] time = 1.66, size = 579, normalized size = 1.24

$$\frac{3\left(2520a^9+\left(16b^9p^8+288b^9p^7+2184b^9p^6+9072b^9p^5+22449b^9p^4+33642b^9p^3+29531b^9p^2+13698b^9p+2520b^9\right)x^3+28\left(8a^3b^6p^6+60a^3b^6p^5+170a^3b^6p^4+225a^3b^6p^3+137a^3b^6p^2+30a^3b^6p\right)x^2-1680\left(2a^6b^3p^3+3a^6b^3p^2+a^6b^3p\right)x+\left(5040a^7b^2p^2+2520a^7b^2p+\left(16ab^8p^8+224ab^8p^7+1288ab^8p^6+3920ab^8p^5+6769ab^8p^4+6566ab^8p^3+3267ab^8p^2+630ab^8p\right)x^2-168\left(4a^4b^5p^5+20a^4b^5p^4+35a^4b^5p^3+25a^4b^5p^2+6a^4b^5p\right)x\right)x^{\frac{2}{3}}-4\left(1260a^8bp+2\left(8a^2b^7p^7+84a^2b^7p^6+350a^2b^7p^5+735a^2b^7p^4+812a^2b^7p^3+441a^2b^7p^2+90a^2b^7p\right)x^2-105\left(4a^5b^4p^4+12a^5b^4p^3+11a^5b^4p^2+3a^5b^4p\right)x\right)x^{\frac{1}{3}}\right)\left(b^2x^{\frac{2}{3}}+2abx^{\frac{1}{3}}+a^2\right)^p}{32b^9p^9+720b^9p^8+6960b^9p^7+37800b^9p^6+126546b^9p^5+269325b^9p^4+361840b^9p^3+293175b^9p^2+128322b^9p+22680b^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p*x^2,x, algorithm="fricas")
```

```
[Out] 3*(2520*a^9 + (16*b^9*p^8 + 288*b^9*p^7 + 2184*b^9*p^6 + 9072*b^9*p^5 + 224
49*b^9*p^4 + 33642*b^9*p^3 + 29531*b^9*p^2 + 13698*b^9*p + 2520*b^9)*x^3 +
28*(8*a^3*b^6*p^6 + 60*a^3*b^6*p^5 + 170*a^3*b^6*p^4 + 225*a^3*b^6*p^3 + 13
7*a^3*b^6*p^2 + 30*a^3*b^6*p)*x^2 - 1680*(2*a^6*b^3*p^3 + 3*a^6*b^3*p^2 + a
^6*b^3*p)*x + (5040*a^7*b^2*p^2 + 2520*a^7*b^2*p + (16*a*b^8*p^8 + 224*a*b^
8*p^7 + 1288*a*b^8*p^6 + 3920*a*b^8*p^5 + 6769*a*b^8*p^4 + 6566*a*b^8*p^3 +
 3267*a*b^8*p^2 + 630*a*b^8*p)*x^2 - 168*(4*a^4*b^5*p^5 + 20*a^4*b^5*p^4 +
35*a^4*b^5*p^3 + 25*a^4*b^5*p^2 + 6*a^4*b^5*p)*x)*x^(2/3) - 4*(1260*a^8*b*p
 + 2*(8*a^2*b^7*p^7 + 84*a^2*b^7*p^6 + 350*a^2*b^7*p^5 + 735*a^2*b^7*p^4 +
812*a^2*b^7*p^3 + 441*a^2*b^7*p^2 + 90*a^2*b^7*p)*x^2 - 105*(4*a^5*b^4*p^4
+ 12*a^5*b^4*p^3 + 11*a^5*b^4*p^2 + 3*a^5*b^4*p)*x)*x^(1/3))*(b^2*x^(2/3) +
```

```
 2*a*b*x^(1/3) + a^2)^p/(32*b^9*p^9 + 720*b^9*p^8 + 6960*b^9*p^7 + 37800*b^
9*p^6 + 126546*b^9*p^5 + 269325*b^9*p^4 + 361840*b^9*p^3 + 293175*b^9*p^2 +
 128322*b^9*p + 22680*b^9)
```

giac [B] time = 0.63, size = 1564, normalized size = 3.34

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p*x^2,x, algorithm="giac")
```

```
[Out] 3*(16*(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p*b^9*p^8*x^3 + 16*(b^2*x^(2/3) +
 2*a*b*x^(1/3) + a^2)^p*a*b^8*p^8*x^(8/3) + 288*(b^2*x^(2/3) + 2*a*b*x^(1/3
) + a^2)^p*b^9*p^7*x^3 + 224*(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p*a*b^8*p^
7*x^(8/3) - 64*(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p*a^2*b^7*p^7*x^(7/3) +
2184*(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p*b^9*p^6*x^3 + 1288*(b^2*x^(2/3)
+ 2*a*b*x^(1/3) + a^2)^p*a*b^8*p^6*x^(8/3) - 672*(b^2*x^(2/3) + 2*a*b*x^(1/
3) + a^2)^p*a^2*b^7*p^6*x^(7/3) + 224*(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p
*a^3*b^6*p^6*x^2 + 9072*(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p*b^9*p^5*x^3 +
 3920*(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p*a*b^8*p^5*x^(8/3) - 2800*(b^2*x
^(2/3) + 2*a*b*x^(1/3) + a^2)^p*a^2*b^7*p^5*x^(7/3) + 1680*(b^2*x^(2/3) + 2
*a*b*x^(1/3) + a^2)^p*a^3*b^6*p^5*x^2 + 22449*(b^2*x^(2/3) + 2*a*b*x^(1/3)
+ a^2)^p*b^9*p^4*x^3 - 672*(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p*a^4*b^5*p^
5*x^(5/3) + 6769*(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p*a*b^8*p^4*x^(8/3) -
5880*(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p*a^2*b^7*p^4*x^(7/3) + 4760*(b^2*
x^(2/3) + 2*a*b*x^(1/3) + a^2)^p*a^3*b^6*p^4*x^2 + 33642*(b^2*x^(2/3) + 2*a
*b*x^(1/3) + a^2)^p*b^9*p^3*x^3 - 3360*(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^
p*a^4*b^5*p^4*x^(5/3) + 6566*(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p*a*b^8*p^
3*x^(8/3) + 1680*(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p*a^5*b^4*p^4*x^(4/3)
- 6496*(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p*a^2*b^7*p^3*x^(7/3) + 6300*(b^
2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p*a^3*b^6*p^3*x^2 + 29531*(b^2*x^(2/3) + 2
*a*b*x^(1/3) + a^2)^p*b^9*p^2*x^3 - 5880*(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2
)^p*a^4*b^5*p^3*x^(5/3) + 3267*(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p*a*b^8*
p^2*x^(8/3) + 5040*(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p*a^5*b^4*p^3*x^(4/3
) - 3528*(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p*a^2*b^7*p^2*x^(7/3) - 3360*(
b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p*a^6*b^3*p^3*x + 3836*(b^2*x^(2/3) + 2*
a*b*x^(1/3) + a^2)^p*a^3*b^6*p^2*x^2 + 13698*(b^2*x^(2/3) + 2*a*b*x^(1/3) +
 a^2)^p*b^9*p*x^3 - 4200*(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p*a^4*b^5*p^2*
x^(5/3) + 630*(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p*a*b^8*p*x^(8/3) + 4620*
(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p*a^5*b^4*p^2*x^(4/3) - 720*(b^2*x^(2/3
) + 2*a*b*x^(1/3) + a^2)^p*a^2*b^7*p*x^(7/3) - 5040*(b^2*x^(2/3) + 2*a*b*x^
(1/3) + a^2)^p*a^6*b^3*p^2*x + 840*(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p*a^
3*b^6*p*x^2 + 2520*(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p*b^9*x^3 + 5040*(b^
2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p*a^7*b^2*p^2*x^(2/3) - 1008*(b^2*x^(2/3)
+ 2*a*b*x^(1/3) + a^2)^p*a^4*b^5*p*x^(5/3) + 1260*(b^2*x^(2/3) + 2*a*b*x^(1
/3) + a^2)^p*a^5*b^4*p*x^(4/3) - 1680*(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p
*a^6*b^3*p*x + 2520*(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p*a^7*b^2*p*x^(2/3)
 - 5040*(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p*a^8*b*p*x^(1/3) + 2520*(b^2*x
^(2/3) + 2*a*b*x^(1/3) + a^2)^p*a^9)/(32*b^9*p^9 + 720*b^9*p^8 + 6960*b^9*p
^7 + 37800*b^9*p^6 + 126546*b^9*p^5 + 269325*b^9*p^4 + 361840*b^9*p^3 + 293
175*b^9*p^2 + 128322*b^9*p + 22680*b^9)
```

maple [F] time = 0.01, size = 0, normalized size = 0.00

$$\int x^{2} \left(b^{2} x^{\frac{2}{3}}+2 a b \,x^{\frac{1}{3}}+a^{2}\right)^{p}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^(2/3)+2*a*b*x^(1/3)+a^2)^p*x^2,x)
```

```
[Out] int((b^2*x^(2/3)+2*a*b*x^(1/3)+a^2)^p*x^2,x)
```

maxima [A] time = 0.70, size = 362, normalized size = 0.77

$$\frac{3\left(\left(16p^8+288p^7+2184p^6+9072p^5+22449p^4+33642p^3+29531p^2+13698p+2520\right)b^9x^3+\left(16p^8+224p^7+1288p^6+3920p^5+6769p^4+6566p^3+3267p^2+630p\right)ab^8x^{\frac{8}{3}}-8\left(8p^7+84p^6+350p^5+735p^4+812p^3+441p^2+90p\right)a^2b^7x^{\frac{7}{3}}+28\left(8p^6+60p^5+170p^4+225p^3+137p^2+30p\right)a^3b^6x^2-168\left(4p^5+20p^4+35p^3+25p^2+6p\right)a^4b^5x^{\frac{5}{3}}+420\left(4p^4+12p^3+11p^2+3p\right)a^5b^4x^{\frac{4}{3}}-1680\left(2p^3+3p^2+p\right)a^6b^3x+2520\left(2p^2+p\right)a^7b^2x^{\frac{2}{3}}-5040a^8bpx^{\frac{1}{3}}+2520a^9\right)\left(bx^{\frac{1}{3}}+a\right)^{2p}}{\left(32p^9+720p^8+6960p^7+37800p^6+126546p^5+269325p^4+361840p^3+293175p^2+128322p+22680\right)b^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p*x^2,x, algorithm="maxima")
```

```
[Out] 3*((16*p^8 + 288*p^7 + 2184*p^6 + 9072*p^5 + 22449*p^4 + 33642*p^3 + 29531*
p^2 + 13698*p + 2520)*b^9*x^3 + (16*p^8 + 224*p^7 + 1288*p^6 + 3920*p^5 + 6
769*p^4 + 6566*p^3 + 3267*p^2 + 630*p)*a*b^8*x^(8/3) - 8*(8*p^7 + 84*p^6 +
350*p^5 + 735*p^4 + 812*p^3 + 441*p^2 + 90*p)*a^2*b^7*x^(7/3) + 28*(8*p^6 +
 60*p^5 + 170*p^4 + 225*p^3 + 137*p^2 + 30*p)*a^3*b^6*x^2 - 168*(4*p^5 + 20
*p^4 + 35*p^3 + 25*p^2 + 6*p)*a^4*b^5*x^(5/3) + 420*(4*p^4 + 12*p^3 + 11*p^
2 + 3*p)*a^5*b^4*x^(4/3) - 1680*(2*p^3 + 3*p^2 + p)*a^6*b^3*x + 2520*(2*p^2
 + p)*a^7*b^2*x^(2/3) - 5040*a^8*b*p*x^(1/3) + 2520*a^9)*(b*x^(1/3) + a)^(2
*p)/((32*p^9 + 720*p^8 + 6960*p^7 + 37800*p^6 + 126546*p^5 + 269325*p^4 + 3
61840*p^3 + 293175*p^2 + 128322*p + 22680)*b^9)
```

mupad [B] time = 3.52, size = 777, normalized size = 1.66

$$\left(a^2+b^2x^{2/3}+2abx^{1/3}\right)^p\left(\frac{3x^3\left(16p^8+288p^7+2184p^6+9072p^5+22449p^4+33642p^3+29531p^2+13698p+2520\right)}{32p^9+720p^8+6960p^7+37800p^6+126546p^5+269325p^4+361840p^3+293175p^2+128322p+22680}+\frac{7560a^9}{b^9\left(32p^9+720p^8+6960p^7+37800p^6+126546p^5+269325p^4+361840p^3+293175p^2+128322p+22680\right)}-\frac{15120a^8px^{1/3}}{b^8\left(32p^9+720p^8+6960p^7+37800p^6+126546p^5+269325p^4+361840p^3+293175p^2+128322p+22680\right)}+\frac{3apx^{8/3}\left(16p^7+224p^6+1288p^5+3920p^4+6769p^3+6566p^2+3267p+630\right)}{b\left(32p^9+720p^8+6960p^7+37800p^6+126546p^5+269325p^4+361840p^3+293175p^2+128322p+22680\right)}+\frac{84a^3px^2\left(8p^5+60p^4+170p^3+225p^2+137p+30\right)}{b^3\left(32p^9+720p^8+6960p^7+37800p^6+126546p^5+269325p^4+361840p^3+293175p^2+128322p+22680\right)}-\frac{5040a^6px\left(2p^2+3p+1\right)}{b^6\left(32p^9+720p^8+6960p^7+37800p^6+126546p^5+269325p^4+361840p^3+293175p^2+128322p+22680\right)}-\frac{24a^2px^{7/3}\left(8p^6+84p^5+350p^4+735p^3+812p^2+441p+90\right)}{b^2\left(32p^9+720p^8+6960p^7+37800p^6+126546p^5+269325p^4+361840p^3+293175p^2+128322p+22680\right)}+\frac{7560a^7px^{2/3}\left(2p+1\right)}{b^7\left(32p^9+720p^8+6960p^7+37800p^6+126546p^5+269325p^4+361840p^3+293175p^2+128322p+22680\right)}+\frac{1260a^5px^{4/3}\left(4p^3+12p^2+11p+3\right)}{b^5\left(32p^9+720p^8+6960p^7+37800p^6+126546p^5+269325p^4+361840p^3+293175p^2+128322p+22680\right)}-\frac{504a^4px^{5/3}\left(4p^4+20p^3+35p^2+25p+6\right)}{b^4\left(32p^9+720p^8+6960p^7+37800p^6+126546p^5+269325p^4+361840p^3+293175p^2+128322p+22680\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a^2 + b^2*x^(2/3) + 2*a*b*x^(1/3))^p,x)
```

```
[Out] (a^2 + b^2*x^(2/3) + 2*a*b*x^(1/3))^p*((3*x^3*(13698*p + 29531*p^2 + 33642*
p^3 + 22449*p^4 + 9072*p^5 + 2184*p^6 + 288*p^7 + 16*p^8 + 2520))/(128322*p
 + 293175*p^2 + 361840*p^3 + 269325*p^4 + 126546*p^5 + 37800*p^6 + 6960*p^7
 + 720*p^8 + 32*p^9 + 22680) + (7560*a^9)/(b^9*(128322*p + 293175*p^2 + 361
840*p^3 + 269325*p^4 + 126546*p^5 + 37800*p^6 + 6960*p^7 + 720*p^8 + 32*p^9
 + 22680)) - (15120*a^8*p*x^(1/3))/(b^8*(128322*p + 293175*p^2 + 361840*p^3
 + 269325*p^4 + 126546*p^5 + 37800*p^6 + 6960*p^7 + 720*p^8 + 32*p^9 + 2268
0)) + (3*a*p*x^(8/3)*(3267*p + 6566*p^2 + 6769*p^3 + 3920*p^4 + 1288*p^5 +
224*p^6 + 16*p^7 + 630))/(b*(128322*p + 293175*p^2 + 361840*p^3 + 269325*p^
4 + 126546*p^5 + 37800*p^6 + 6960*p^7 + 720*p^8 + 32*p^9 + 22680)) + (84*a^
3*p*x^2*(137*p + 225*p^2 + 170*p^3 + 60*p^4 + 8*p^5 + 30))/(b^3*(128322*p +
 293175*p^2 + 361840*p^3 + 269325*p^4 + 126546*p^5 + 37800*p^6 + 6960*p^7 +
 720*p^8 + 32*p^9 + 22680)) - (5040*a^6*p*x*(3*p + 2*p^2 + 1))/(b^6*(128322
*p + 293175*p^2 + 361840*p^3 + 269325*p^4 + 126546*p^5 + 37800*p^6 + 6960*p
^7 + 720*p^8 + 32*p^9 + 22680)) - (24*a^2*p*x^(7/3)*(441*p + 812*p^2 + 735*
p^3 + 350*p^4 + 84*p^5 + 8*p^6 + 90))/(b^2*(128322*p + 293175*p^2 + 361840*
p^3 + 269325*p^4 + 126546*p^5 + 37800*p^6 + 6960*p^7 + 720*p^8 + 32*p^9 + 2
2680)) + (7560*a^7*p*x^(2/3)*(2*p + 1))/(b^7*(128322*p + 293175*p^2 + 36184
0*p^3 + 269325*p^4 + 126546*p^5 + 37800*p^6 + 6960*p^7 + 720*p^8 + 32*p^9 +
 22680)) + (1260*a^5*p*x^(4/3)*(11*p + 12*p^2 + 4*p^3 + 3))/(b^5*(128322*p
+ 293175*p^2 + 361840*p^3 + 269325*p^4 + 126546*p^5 + 37800*p^6 + 6960*p^7
+ 720*p^8 + 32*p^9 + 22680)) - (504*a^4*p*x^(5/3)*(25*p + 35*p^2 + 20*p^3 +
 4*p^4 + 6))/(b^4*(128322*p + 293175*p^2 + 361840*p^3 + 269325*p^4 + 126546
*p^5 + 37800*p^6 + 6960*p^7 + 720*p^8 + 32*p^9 + 22680)))
```

sympy [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2+2*a*b*x**(1/3)+b**2*x**(2/3))**p*x**2,x)
```

```
[Out] Exception raised: HeuristicGCDFailed
```

3.412 $\int \left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p x\,dx$

Optimal. Leaf size=315

$$\frac{3a^6\left(\frac{b\sqrt[3]{x}}{a}+1\right)^6\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{2b^6(p+3)}-\frac{15a^6\left(\frac{b\sqrt[3]{x}}{a}+1\right)^5\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{b^6(2p+5)}+\frac{15a^6\left(\frac{b\sqrt[3]{x}}{a}+1\right)^4\left(a^2+2ab\sqrt[3]{x}\right.}{b^6(p+2)}$$

Rubi [A] time = 0.14, antiderivative size = 315, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1356, 266, 43}

$$\frac{3a^6\left(\frac{b\sqrt[3]{x}}{a}+1\right)^6\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{2b^6(p+3)}-\frac{15a^6\left(\frac{b\sqrt[3]{x}}{a}+1\right)^5\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{b^6(2p+5)}+\frac{15a^6\left(\frac{b\sqrt[3]{x}}{a}+1\right)^4\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{b^6(p+2)}-\frac{30a^6\left(\frac{b\sqrt[3]{x}}{a}+1\right)^3\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{b^6(2p+3)}+\frac{15a^6\left(\frac{b\sqrt[3]{x}}{a}+1\right)^2\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{2b^6(p+1)}-\frac{3a^6\left(\frac{b\sqrt[3]{x}}{a}+1\right)\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{b^6(2p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^p*x,x]
```

```
[Out] (-3*a^6*(1 + (b*x^(1/3))/a)*(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^p)/(b^6*(1
+ 2*p)) + (15*a^6*(1 + (b*x^(1/3))/a)^2*(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))
^p)/(2*b^6*(1 + p)) - (30*a^6*(1 + (b*x^(1/3))/a)^3*(a^2 + 2*a*b*x^(1/3) +
b^2*x^(2/3))^p)/(b^6*(3 + 2*p)) + (15*a^6*(1 + (b*x^(1/3))/a)^4*(a^2 + 2*a*
b*x^(1/3) + b^2*x^(2/3))^p)/(b^6*(2 + p)) - (15*a^6*(1 + (b*x^(1/3))/a)^5*(
a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^p)/(b^6*(5 + 2*p)) + (3*a^6*(1 + (b*x^(1
/3))/a)^6*(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^p)/(2*b^6*(3 + p))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1356

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n + c*x^(2*n))^FracPart[p])/(1 + (
2*c*x^n)/b)^(2*FracPart[p]), Int[(d*x)^m*(1 + (2*c*x^n)/b)^(2*p), x], x] /;
 FreeQ[{a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] &&
 !IntegerQ[2*p]
```

Rubi steps

$$\int \left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p x\,dx = \left(\left(1+\frac{b\sqrt[3]{x}}{a}\right)^{-2p}\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p\right)\int\left(1+\frac{b\sqrt[3]{x}}{a}\right)^{2p} x\,dx$$

$$= \left(3\left(1+\frac{b\sqrt[3]{x}}{a}\right)^{-2p}\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p\right)\text{Subst}\left(\int x^5\left(1+\frac{bx}{a}\right)^{2p}dx,x,\sqrt[3]{}\right.$$

$$= \left(3\left(1+\frac{b\sqrt[3]{x}}{a}\right)^{-2p}\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p\right)\text{Subst}\left(\int\left(-\frac{a^5\left(1+\frac{bx}{a}\right)^{2p}}{b^5}+\frac{5a^5}{}\right.\right.$$

$$= -\frac{3a^6\left(1+\frac{b\sqrt[3]{x}}{a}\right)\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{b^6(1+2p)}+\frac{15a^6\left(1+\frac{b\sqrt[3]{x}}{a}\right)^2\left(a^2+2ab\sqrt[3]{x}+\right.}{2b^6(1+p)}$$

Mathematica [**A**] time = 0.17, size = 143, normalized size = 0.45

$$\frac{3\left(a+b\sqrt[3]{x}\right)\left(-\frac{2a^5}{2p+1}+\frac{5a^4\left(a+b\sqrt[3]{x}\right)}{p+1}-\frac{20a^3\left(a+b\sqrt[3]{x}\right)^2}{2p+3}+\frac{10a^2\left(a+b\sqrt[3]{x}\right)^3}{p+2}-\frac{10a\left(a+b\sqrt[3]{x}\right)^4}{2p+5}+\frac{\left(a+b\sqrt[3]{x}\right)^5}{p+3}\right)\left(\left(a+b\sqrt[3]{x}\right)^2\right)^p}{2b^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^p*x,x]
```

```
[Out] (3*((-2*a^5)/(1 + 2*p) + (5*a^4*(a + b*x^(1/3)))/(1 + p) - (20*a^3*(a + b*x
^(1/3))^2)/(3 + 2*p) + (10*a^2*(a + b*x^(1/3))^3)/(2 + p) - (10*a*(a + b*x^
(1/3))^4)/(5 + 2*p) + (a + b*x^(1/3))^5/(3 + p))*(a + b*x^(1/3))*((a + b*x^
(1/3))^2)^p)/(2*b^6)
```

IntegrateAlgebraic [**F**] time = 0.19, size = 0, normalized size = 0.00

$$\int \left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p x\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^p*x,x]
```

```
[Out] Defer[IntegrateAlgebraic][(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^p*x, x]
```

fricas [**A**] time = 1.31, size = 297, normalized size = 0.94

$$-\frac{3\left(30a^6-\left(8b^6p^5+60b^6p^4+170b^6p^3+225b^6p^2+137b^6p+30b^6\right)x^2-20\left(2a^3b^3p^3+3a^3b^3p^2+a^3b^3p\right)x+2\left(30a^4b^2p^2+15a^4b^2p-\left(4ab^5p^5+20ab^5p^4+35ab^5p^3+25ab^5p^2+6ab^5p\right)x\right)x^{\frac{2}{3}}-5\left(12a^5bp-\left(4a^2b^4p^4+12a^2b^4p^3+11a^2b^4p^2+3a^2b^4p\right)x\right)x^{\frac{1}{3}}\right)\left(b^2x^{\frac{2}{3}}+2abx^{\frac{1}{3}}+a^2\right)^p}{2\left(8b^6p^6+84b^6p^5+350b^6p^4+735b^6p^3+812b^6p^2+441b^6p+90b^6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p*x,x, algorithm="fricas")
```

```
[Out] -3/2*(30*a^6 - (8*b^6*p^5 + 60*b^6*p^4 + 170*b^6*p^3 + 225*b^6*p^2 + 137*b^
6*p + 30*b^6)*x^2 - 20*(2*a^3*b^3*p^3 + 3*a^3*b^3*p^2 + a^3*b^3*p)*x + 2*(3
0*a^4*b^2*p^2 + 15*a^4*b^2*p - (4*a*b^5*p^5 + 20*a*b^5*p^4 + 35*a*b^5*p^3 +
 25*a*b^5*p^2 + 6*a*b^5*p)*x)*x^(2/3) - 5*(12*a^5*b*p - (4*a^2*b^4*p^4 + 12
*a^2*b^4*p^3 + 11*a^2*b^4*p^2 + 3*a^2*b^4*p)*x)*x^(1/3))*(b^2*x^(2/3) + 2*a
*b*x^(1/3) + a^2)^p/(8*b^6*p^6 + 84*b^6*p^5 + 350*b^6*p^4 + 735*b^6*p^3 + 8
12*b^6*p^2 + 441*b^6*p + 90*b^6)
```

giac [**B**] time = 0.48, size = 745, normalized size = 2.37

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p*x,x, algorithm="giac")
```

```
[Out] 3/2*(8*(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p*b^6*p^5*x^2 + 8*(b^2*x^(2/3) +
 2*a*b*x^(1/3) + a^2)^p*a*b^5*p^5*x^(5/3) + 60*(b^2*x^(2/3) + 2*a*b*x^(1/3)
 + a^2)^p*b^6*p^4*x^2 + 40*(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p*a*b^5*p^4*
x^(5/3) - 20*(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p*a^2*b^4*p^4*x^(4/3) + 17
0*(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p*b^6*p^3*x^2 + 70*(b^2*x^(2/3) + 2*a
*b*x^(1/3) + a^2)^p*a*b^5*p^3*x^(5/3) - 60*(b^2*x^(2/3) + 2*a*b*x^(1/3) + a
^2)^p*a^2*b^4*p^3*x^(4/3) + 40*(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p*a^3*b^
3*p^3*x + 225*(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p*b^6*p^2*x^2 + 50*(b^2*x
^(2/3) + 2*a*b*x^(1/3) + a^2)^p*a*b^5*p^2*x^(5/3) - 55*(b^2*x^(2/3) + 2*a*b
*x^(1/3) + a^2)^p*a^2*b^4*p^2*x^(4/3) + 60*(b^2*x^(2/3) + 2*a*b*x^(1/3) + a
^2)^p*a^3*b^3*p^2*x + 137*(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p*b^6*p*x^2 -
 60*(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p*a^4*b^2*p^2*x^(2/3) + 12*(b^2*x^(
2/3) + 2*a*b*x^(1/3) + a^2)^p*a*b^5*p*x^(5/3) - 15*(b^2*x^(2/3) + 2*a*b*x^(
1/3) + a^2)^p*a^2*b^4*p*x^(4/3) + 20*(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p*
a^3*b^3*p*x + 30*(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p*b^6*x^2 - 30*(b^2*x^
(2/3) + 2*a*b*x^(1/3) + a^2)^p*a^4*b^2*p*x^(2/3) + 60*(b^2*x^(2/3) + 2*a*b*
x^(1/3) + a^2)^p*a^5*b*p*x^(1/3) - 30*(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p
*a^6)/(8*b^6*p^6 + 84*b^6*p^5 + 350*b^6*p^4 + 735*b^6*p^3 + 812*b^6*p^2 + 4
41*b^6*p + 90*b^6)
```

maple [F] time = 0.01, size = 0, normalized size = 0.00

$$\int x \left(b^{2} x^{\frac{2}{3}} + 2 a b\, x^{\frac{1}{3}} + a^{2}\right)^{p}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^(2/3)+2*a*b*x^(1/3)+a^2)^p*x,x)
```

```
[Out] int((b^2*x^(2/3)+2*a*b*x^(1/3)+a^2)^p*x,x)
```

maxima [A] time = 0.64, size = 198, normalized size = 0.63

$$\frac{3\left(\left(8 p^{5} + 60 p^{4} + 170 p^{3} + 225 p^{2} + 137 p + 30\right) b^{6} x^{2} + 2\left(4 p^{5} + 20 p^{4} + 35 p^{3} + 25 p^{2} + 6 p\right) a b^{5} x^{\frac{5}{3}} - 5\left(4 p^{4} + 12 p^{3} + 11 p^{2} + 3 p\right) a^{2} b^{4} x^{\frac{4}{3}} + 20\left(2 p^{3} + 3 p^{2} + p\right) a^{3} b^{3} x - 30\left(2 p^{2} + p\right) a^{4} b^{2} x^{\frac{2}{3}} + 60 a^{5} b p x^{\frac{1}{3}} - 30 a^{6}\right)\left(b x^{\frac{1}{3}} + a\right)^{2 p}}{2\left(8 p^{6} + 84 p^{5} + 350 p^{4} + 735 p^{3} + 812 p^{2} + 441 p + 90\right) b^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p*x,x, algorithm="maxima")
```

```
[Out] 3/2*((8*p^5 + 60*p^4 + 170*p^3 + 225*p^2 + 137*p + 30)*b^6*x^2 + 2*(4*p^5 +
 20*p^4 + 35*p^3 + 25*p^2 + 6*p)*a*b^5*x^(5/3) - 5*(4*p^4 + 12*p^3 + 11*p^2
 + 3*p)*a^2*b^4*x^(4/3) + 20*(2*p^3 + 3*p^2 + p)*a^3*b^3*x - 30*(2*p^2 + p)
*a^4*b^2*x^(2/3) + 60*a^5*b*p*x^(1/3) - 30*a^6)*(b*x^(1/3) + a)^(2*p)/((8*p
^6 + 84*p^5 + 350*p^4 + 735*p^3 + 812*p^2 + 441*p + 90)*b^6)
```

mupad [B] time = 2.17, size = 390, normalized size = 1.24

$$\left(a^{2} + b^{2} x^{2/3} + 2 a b x^{1/3}\right)^{p} \left(\frac{3 x^{2} \left(8 p^{5} + 60 p^{4} + 170 p^{3} + 225 p^{2} + 137 p + 30\right)}{2\left(8 p^{6} + 84 p^{5} + 350 p^{4} + 735 p^{3} + 812 p^{2} + 441 p + 90\right)} - \frac{45 a^{6}}{b^{6}\left(8 p^{6} + 84 p^{5} + 350 p^{4} + 735 p^{3} + 812 p^{2} + 441 p + 90\right)} + \frac{90 a^{5} p x^{1/3}}{b^{5}\left(8 p^{6} + 84 p^{5} + 350 p^{4} + 735 p^{3} + 812 p^{2} + 441 p + 90\right)} - \frac{15 a^{2} p x^{4/3}\left(4 p^{3} + 12 p^{2} + 11 p + 3\right)}{2 b^{2}\left(8 p^{6} + 84 p^{5} + 350 p^{4} + 735 p^{3} + 812 p^{2} + 441 p + 90\right)} + \frac{30 a^{3} p x \left(2 p^{2} + 3 p + 1\right)}{b^{3}\left(8 p^{6} + 84 p^{5} + 350 p^{4} + 735 p^{3} + 812 p^{2} + 441 p + 90\right)} - \frac{45 a^{4} p x^{2/3}\left(2 p + 1\right)}{b^{4}\left(8 p^{6} + 84 p^{5} + 350 p^{4} + 735 p^{3} + 812 p^{2} + 441 p + 90\right)} + \frac{3 a p x^{5/3}\left(4 p^{4} + 20 p^{3} + 35 p^{2} + 25 p + 6\right)}{b\left(8 p^{6} + 84 p^{5} + 350 p^{4} + 735 p^{3} + 812 p^{2} + 441 p + 90\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a^2 + b^2*x^(2/3) + 2*a*b*x^(1/3))^p,x)
```

```
[Out] (a^2 + b^2*x^(2/3) + 2*a*b*x^(1/3))^p*((3*x^2*(137*p + 225*p^2 + 170*p^3 +
60*p^4 + 8*p^5 + 30))/(2*(441*p + 812*p^2 + 735*p^3 + 350*p^4 + 84*p^5 + 8*
p^6 + 90)) - (45*a^6)/(b^6*(441*p + 812*p^2 + 735*p^3 + 350*p^4 + 84*p^5 +
8*p^6 + 90)) + (90*a^5*p*x^(1/3))/(b^5*(441*p + 812*p^2 + 735*p^3 + 350*p^4
 + 84*p^5 + 8*p^6 + 90)) - (15*a^2*p*x^(4/3)*(11*p + 12*p^2 + 4*p^3 + 3))/(
```

```
2*b^2*(441*p + 812*p^2 + 735*p^3 + 350*p^4 + 84*p^5 + 8*p^6 + 90)) + (30*a^
3*p*x*(3*p + 2*p^2 + 1))/(b^3*(441*p + 812*p^2 + 735*p^3 + 350*p^4 + 84*p^5
 + 8*p^6 + 90)) - (45*a^4*p*x^(2/3)*(2*p + 1))/(b^4*(441*p + 812*p^2 + 735*
p^3 + 350*p^4 + 84*p^5 + 8*p^6 + 90)) + (3*a*p*x^(5/3)*(25*p + 35*p^2 + 20*
p^3 + 4*p^4 + 6))/(b*(441*p + 812*p^2 + 735*p^3 + 350*p^4 + 84*p^5 + 8*p^6
+ 90)))
```

sympy [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2+2*a*b*x**(1/3)+b**2*x**(2/3))**p*x,x)
```

```
[Out] Exception raised: HeuristicGCDFailed
```

3.413 $\int \left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p\,dx$

Optimal. Leaf size=142

$$\frac{3\left(a+b\sqrt[3]{x}\right)^3\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{b^3(2p+3)}-\frac{3a\left(a+b\sqrt[3]{x}\right)^2\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{b^3(p+1)}+\frac{3a^2\left(a+b\sqrt[3]{x}\right)\left(a^2+2ab\sqrt[3]{x}+b\right.}{b^3(2p+1)}$$

Rubi [A] time = 0.07, antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1341, 646, 43}

$$\frac{3\left(a+b\sqrt[3]{x}\right)^3\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{b^3(2p+3)}-\frac{3a\left(a+b\sqrt[3]{x}\right)^2\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{b^3(p+1)}+\frac{3a^2\left(a+b\sqrt[3]{x}\right)\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{b^3(2p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^p,x]
```

```
[Out] (3*a^2*(a + b*x^(1/3))*(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^p)/(b^3*(1 + 2*p
)) - (3*a*(a + b*x^(1/3))^2*(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^p)/(b^3*(1
+ p)) + (3*(a + b*x^(1/3))^3*(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^p)/(b^3*(3
 + 2*p))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 1341

```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k
 = Denominator[n]}, Dist[k, Subst[Int[x^(k - 1)*(a + b*x^(k*n) + c*x^(2*k*n
))^p, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && EqQ[n2, 2*n] && Fra
ctionQ[n]
```

Rubi steps

$$\begin{aligned}\int \left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p\,dx &= 3\,\text{Subst}\left(\int x^2\left(a^2+2abx+b^2x^2\right)^p\,dx,x,\sqrt[3]{x}\right)\\ &= \left(3\left(b\left(a+b\sqrt[3]{x}\right)\right)^{-2p}\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p\right)\text{Subst}\left(\int x^2\left(ab+b^2x\right)^{2p}\,dx,x,\sqrt[3]{\right.\\ &= \left(3\left(b\left(a+b\sqrt[3]{x}\right)\right)^{-2p}\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p\right)\text{Subst}\left(\int\left(\frac{a^2\left(ab+b^2x\right)^{2p}}{b^2}-\frac{2a(\right.\right.\\ &= \frac{3a^2\left(a+b\sqrt[3]{x}\right)\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{b^3(1+2p)}-\frac{3a\left(a+b\sqrt[3]{x}\right)^2\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)}{b^3(1+p)}\end{aligned}$$

Mathematica [A] time = 0.05, size = 83, normalized size = 0.58

$$\frac{3\left(a+b\sqrt[3]{x}\right)\left(\left(a+b\sqrt[3]{x}\right)^2\right)^p\left(a^2-ab(2p+1)\sqrt[3]{x}+b^2\left(2p^2+3p+1\right)x^{2/3}\right)}{b^3(p+1)(2p+1)(2p+3)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^p,x]
```

```
[Out] (3*(a + b*x^(1/3))*((a + b*x^(1/3))^2)^p*(a^2 - a*b*(1 + 2*p)*x^(1/3) + b^2
*(1 + 3*p + 2*p^2)*x^(2/3)))/(b^3*(1 + p)*(1 + 2*p)*(3 + 2*p))
```

IntegrateAlgebraic [F] time = 0.16, size = 0, normalized size = 0.00

$$\int\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^p,x]
```

```
[Out] Defer[IntegrateAlgebraic][(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^p, x]
```

fricas [A] time = 1.50, size = 110, normalized size = 0.77

$$-\frac{3\left(2\,a^2bpx^{\frac{1}{3}}-a^3-\left(2\,b^3p^2+3\,b^3p+b^3\right)x-\left(2\,ab^2p^2+ab^2p\right)x^{\frac{2}{3}}\right)\left(b^2x^{\frac{2}{3}}+2\,abx^{\frac{1}{3}}+a^2\right)^p}{4\,b^3p^3+12\,b^3p^2+11\,b^3p+3\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p,x, algorithm="fricas")
```

```
[Out] -3*(2*a^2*b*p*x^(1/3) - a^3 - (2*b^3*p^2 + 3*b^3*p + b^3)*x - (2*a*b^2*p^2
+ a*b^2*p)*x^(2/3))*(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p/(4*b^3*p^3 + 12*b
^3*p^2 + 11*b^3*p + 3*b^3)
```

giac [A] time = 0.48, size = 229, normalized size = 1.61

$$\frac{3\left(2\left(b^2x^{\frac{2}{3}}+2\,abx^{\frac{1}{3}}+a^2\right)^pb^3p^2x+2\left(b^2x^{\frac{2}{3}}+2\,abx^{\frac{1}{3}}+a^2\right)^pab^2p^2x^{\frac{2}{3}}+3\left(b^2x^{\frac{2}{3}}+2\,abx^{\frac{1}{3}}+a^2\right)^pb^3px+\left(b^2x^{\frac{2}{3}}+2\,abx^{\frac{1}{3}}+a^2\right)^pab^2px^{\frac{2}{3}}-2\left(b^2x^{\frac{2}{3}}+2\,abx^{\frac{1}{3}}+a^2\right)^pa^2bpx^{\frac{1}{3}}+\left(b^2x^{\frac{2}{3}}+2\,abx^{\frac{1}{3}}+a^2\right)^pb^3x+\left(b^2x^{\frac{2}{3}}+2\,abx^{\frac{1}{3}}+a^2\right)^pa^3\right)}{4\,b^3p^3+12\,b^3p^2+11\,b^3p+3\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p,x, algorithm="giac")
```

```
[Out] 3*(2*(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p*b^3*p^2*x + 2*(b^2*x^(2/3) + 2*a
*b*x^(1/3) + a^2)^p*a*b^2*p^2*x^(2/3) + 3*(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^
2)^p*b^3*p*x + (b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p*a*b^2*p*x^(2/3) - 2*(b
^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p*a^2*b*p*x^(1/3) + (b^2*x^(2/3) + 2*a*b*
x^(1/3) + a^2)^p*b^3*x + (b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p*a^3)/(4*b^3*
p^3 + 12*b^3*p^2 + 11*b^3*p + 3*b^3)
```

maple [F] time = 0.01, size = 0, normalized size = 0.00

$$\int\left(b^2x^{\frac{2}{3}}+2ab\,x^{\frac{1}{3}}+a^2\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^(2/3)+2*a*b*x^(1/3)+a^2)^p,x)
```

```
[Out] int((b^2*x^(2/3)+2*a*b*x^(1/3)+a^2)^p,x)
```

maxima [A] time = 0.80, size = 77, normalized size = 0.54

$$\frac{3\left(\left(2\,p^2+3\,p+1\right)b^3x+\left(2\,p^2+p\right)ab^2x^{\frac{2}{3}}-2\,a^2bpx^{\frac{1}{3}}+a^3\right)\left(bx^{\frac{1}{3}}+a\right)^{2\,p}}{\left(4\,p^3+12\,p^2+11\,p+3\right)b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p,x, algorithm="maxima")
```

```
[Out] 3*((2*p^2 + 3*p + 1)*b^3*x + (2*p^2 + p)*a*b^2*x^(2/3) - 2*a^2*b*p*x^(1/3)
+ a^3)*(b*x^(1/3) + a)^(2*p)/((4*p^3 + 12*p^2 + 11*p + 3)*b^3)
```

mupad [B] time = 1.54, size = 138, normalized size = 0.97

$$\left(a^2+b^2\,x^{2/3}+2\,a\,b\,x^{1/3}\right)^p\,\left(\frac{3\,x\,\left(2\,p^2+3\,p+1\right)}{4\,p^3+12\,p^2+11\,p+3}+\frac{3\,a^3}{b^3\,\left(4\,p^3+12\,p^2+11\,p+3\right)}-\frac{6\,a^2\,p\,x^{1/3}}{b^2\,\left(4\,p^3+12\,p^2+11\,p+3\right)}+\frac{3\,a\,p\,x^{2/3}\,\left(2\,p+1\right)}{b\,\left(4\,p^3+12\,p^2+11\,p+3\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^(2/3) + 2*a*b*x^(1/3))^p,x)
```

```
[Out] (a^2 + b^2*x^(2/3) + 2*a*b*x^(1/3))^p*((3*x*(3*p + 2*p^2 + 1))/(11*p + 12*p
^2 + 4*p^3 + 3) + (3*a^3)/(b^3*(11*p + 12*p^2 + 4*p^3 + 3)) - (6*a^2*p*x^(1
/3))/(b^2*(11*p + 12*p^2 + 4*p^3 + 3)) + (3*a*p*x^(2/3)*(2*p + 1))/(b*(11*p
 + 12*p^2 + 4*p^3 + 3)))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a^2+2ab\sqrt[3]{x}+b^2x^{\frac{2}{3}}\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2+2*a*b*x**(1/3)+b**2*x**(2/3))**p,x)
```

```
[Out] Integral((a**2 + 2*a*b*x**(1/3) + b**2*x**(2/3))**p, x)
```

3.414 $\int \left(\frac{\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{x^2}-\frac{2b^3(1-2p)(1-p)p\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{3a^3x}\right)dx$

Optimal. Leaf size=146

$$\frac{b(1-p)\left(a+b\sqrt[3]{x}\right)\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{a^2x^{2/3}}-\frac{\left(a+b\sqrt[3]{x}\right)\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{ax}-\frac{b^2(1-2p)(1-p)\left(a+b\sqrt[3]{x}\right.}{a^3\sqrt[3]{\ }}$$

Rubi [C] time = 0.10, antiderivative size = 162, normalized size of antiderivative = 1.11, number of steps used = 7, number of rules used = 3, integrand size = 77, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.039, Rules used = {1356, 266, 65}

$$\frac{2b^3(1-2p)(1-p)p\left(\frac{b\sqrt[3]{x}}{a}+1\right)\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p {}_2F_1\left(1,2p+1;2(p+1);\frac{\sqrt[3]{x}b}{a}+1\right)}{a^3(2p+1)}+\frac{3b^3\left(\frac{b\sqrt[3]{x}}{a}+1\right)\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p {}_2F_1\left(4,2p+1;2(p+1);\frac{\sqrt[3]{x}b}{a}+1\right)}{a^3(2p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^p/x^2 - (2*b^3*(1 - 2*p)*(1 - p)*p*
(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^p)/(3*a^3*x),x]
```

```
[Out] (2*b^3*(1 - 2*p)*(1 - p)*p*(1 + (b*x^(1/3))/a)*(a^2 + 2*a*b*x^(1/3) + b^2*x
^(2/3))^p*Hypergeometric2F1[1, 1 + 2*p, 2*(1 + p), 1 + (b*x^(1/3))/a])/(a^3
*(1 + 2*p)) + (3*b^3*(1 + (b*x^(1/3))/a)*(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3)
)^p*Hypergeometric2F1[4, 1 + 2*p, 2*(1 + p), 1 + (b*x^(1/3))/a])/(a^3*(1 +
2*p))
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1356

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n + c*x^(2*n))^FracPart[p])/(1 + (
2*c*x^n)/b)^(2*FracPart[p]), Int[(d*x)^m*(1 + (2*c*x^n)/b)^(2*p), x], x] /;
 FreeQ[{a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] &&
 !IntegerQ[2*p]
```

Rubi steps

$$\int \left(\frac{\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{x^2} - \frac{2b^3(1-2p)(1-p)p\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{3a^3x} \right) dx = -\frac{\left(2b^3(1-2p)(1-p)p\right)\int \frac{\left(a^2+2a\right.}{}}{3a^3}$$

$$= \left(\left(1+\frac{b\sqrt[3]{x}}{a}\right)^{-2p} \left(a^2+2ab\sqrt[3]{x}+\right.\right.$$

$$= \left(3\left(1+\frac{b\sqrt[3]{x}}{a}\right)^{-2p} \left(a^2+2ab\sqrt[3]{x}\right.\right.$$

$$= \frac{2b^3(1-2p)(1-p)p\left(1+\frac{b\sqrt[3]{x}}{a}\right)}{}$$

Mathematica [C] time = 0.08, size = 101, normalized size = 0.69

$$\frac{b^3\left(a+b\sqrt[3]{x}\right)\left(\left(a+b\sqrt[3]{x}\right)^2\right)^p\left(2p\left(2p^2-3p+1\right)\,{}_2F_1\left(1,2p+1;2(p+1);\frac{\sqrt[3]{x}b}{a}+1\right)+3\,{}_2F_1\left(4,2p+1;2(p+1);\frac{\sqrt[3]{x}b}{a}+1\right)\right)}{a^3(2ap+a)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^p/x^2 - (2*b^3*(1 - 2*p)*(1 -
 p)*p*(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^p)/(3*a^3*x),x]
```

```
[Out] (b^3*(a + b*x^(1/3))*((a + b*x^(1/3))^2)^p*(2*p*(1 - 3*p + 2*p^2)*Hypergeom
etric2F1[1, 1 + 2*p, 2*(1 + p), 1 + (b*x^(1/3))/a] + 3*Hypergeometric2F1[4,
 1 + 2*p, 2*(1 + p), 1 + (b*x^(1/3))/a]))/(a^3*(a + 2*a*p))
```

IntegrateAlgebraic [F] time = 5.79, size = 0, normalized size = 0.00

$$\int \left(\frac{\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{x^2} - \frac{2b^3(1-2p)(1-p)p\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{3a^3x} \right) dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^p/x^2 - (2*b^3*(1 -
2*p)*(1 - p)*p*(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^p)/(3*a^3*x),x]
```

```
[Out] Defer[IntegrateAlgebraic][(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^p/x^2 - (2*b^
3*(1 - 2*p)*(1 - p)*p*(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^p)/(3*a^3*x), x]
```

fricas [A] time = 1.67, size = 82, normalized size = 0.56

$$-\frac{\left(a^2bpx^{\frac{1}{3}}+a^3+\left(2\,b^3p^2-3\,b^3p+b^3\right)x+2\left(ab^2p^2-ab^2p\right)x^{\frac{2}{3}}\right)\left(b^2x^{\frac{2}{3}}+2\,abx^{\frac{1}{3}}+a^2\right)^p}{a^3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p/x^2-2/3*b^3*(1-2*p)*(1-p)*p*(a^
2+2*a*b*x^(1/3)+b^2*x^(2/3))^p/a^3/x,x, algorithm="fricas")
```

```
[Out] -(a^2*b*p*x^(1/3) + a^3 + (2*b^3*p^2 - 3*b^3*p + b^3)*x + 2*(a*b^2*p^2 - a*
b^2*p)*x^(2/3))*(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p/(a^3*x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{2\left(b^2x^{\frac{2}{3}}+2\,abx^{\frac{1}{3}}+a^2\right)^p b^3(2\,p-1)(p-1)p}{3\,a^3x}+\frac{\left(b^2x^{\frac{2}{3}}+2\,abx^{\frac{1}{3}}+a^2\right)^p}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p/x^2-2/3*b^3*(1-2*p)*(1-p)*p*(a^
2+2*a*b*x^(1/3)+b^2*x^(2/3))^p/a^3/x,x, algorithm="giac")
```

```
[Out] integrate(-2/3*(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p*b^3*(2*p - 1)*(p - 1)*
p/(a^3*x) + (b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p/x^2, x)
```

maple [F] time = 0.01, size = 0, normalized size = 0.00

$$\int -\frac{2(-2p+1)(-p+1)\,b^3p\left(b^2x^{\frac{2}{3}}+2ab\,x^{\frac{1}{3}}+a^2\right)^p}{3a^3x}+\frac{\left(b^2x^{\frac{2}{3}}+2ab\,x^{\frac{1}{3}}+a^2\right)^p}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2*(b^2*x^(2/3)+2*a*b*x^(1/3)+a^2)^p-2/3*b^3*(1-2*p)*(1-p)*p*(b^2*x^
(2/3)+2*a*b*x^(1/3)+a^2)^p/a^3/x,x)
```

```
[Out] int(1/x^2*(b^2*x^(2/3)+2*a*b*x^(1/3)+a^2)^p-2/3*b^3*(1-2*p)*(1-p)*p*(b^2*x^
(2/3)+2*a*b*x^(1/3)+a^2)^p/a^3/x,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{2\left(b^2x^{\frac{2}{3}}+2\,abx^{\frac{1}{3}}+a^2\right)^p b^3(2\,p-1)(p-1)p}{3\,a^3x}+\frac{\left(b^2x^{\frac{2}{3}}+2\,abx^{\frac{1}{3}}+a^2\right)^p}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p/x^2-2/3*b^3*(1-2*p)*(1-p)*p*(a^
2+2*a*b*x^(1/3)+b^2*x^(2/3))^p/a^3/x,x, algorithm="maxima")
```

```
[Out] integrate(-2/3*(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p*b^3*(2*p - 1)*(p - 1)*
p/(a^3*x) + (b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p/x^2, x)
```

mupad [B] time = 1.65, size = 69, normalized size = 0.47

$$-\frac{\left(a^2+b^2\,x^{2/3}+2\,a\,b\,x^{1/3}\right)^p\,\left(\frac{b^3\,x\,\left(2\,p^2-3\,p+1\right)}{a^3}+\frac{b\,p\,x^{1/3}}{a}+\frac{2\,b^2\,p\,x^{2/3}\,(p-1)}{a^2}+1\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^(2/3) + 2*a*b*x^(1/3))^p/x^2 - (2*b^3*p*(2*p - 1)*(p - 1)*
(a^2 + b^2*x^(2/3) + 2*a*b*x^(1/3))^p)/(3*a^3*x),x)
```

```
[Out] -((a^2 + b^2*x^(2/3) + 2*a*b*x^(1/3))^p*((b^3*x*(2*p^2 - 3*p + 1))/a^3 + (b
*p*x^(1/3))/a + (2*b^2*p*x^(2/3)*(p - 1))/a^2 + 1))/x
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int\left(-\frac{3a^3\left(a^2+2ab\sqrt[3]{x}+b^2x^{\frac{2}{3}}\right)^p}{x^2}\right)dx+\int\frac{2b^3p\left(a^2+2ab\sqrt[3]{x}+b^2x^{\frac{2}{3}}\right)^p}{x}\,dx+\int\left(-\frac{6b^3p^2\left(a^2+2ab\sqrt[3]{x}+b^2x^{\frac{2}{3}}\right)^p}{x}\right)dx+\int\frac{4b^3p^3\left(a^2+2ab\sqrt[3]{x}+b^2x^{\frac{2}{3}}\right)^p}{x}\,dx}{3a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2+2*a*b*x**(1/3)+b**2*x**(2/3))**p/x**2-2/3*b**3*(1-2*p)*(1-p
)*p*(a**2+2*a*b*x**(1/3)+b**2*x**(2/3))**p/a**3/x,x)
```

```
[Out] -(Integral(-3*a**3*(a**2 + 2*a*b*x**(1/3) + b**2*x**(2/3))**p/x**2, x) + In
tegral(2*b**3*p*(a**2 + 2*a*b*x**(1/3) + b**2*x**(2/3))**p/x, x) + Integral
(-6*b**3*p**2*(a**2 + 2*a*b*x**(1/3) + b**2*x**(2/3))**p/x, x) + Integral(4
*b**3*p**3*(a**2 + 2*a*b*x**(1/3) + b**2*x**(2/3))**p/x, x))/(3*a**3)
```

3.415 $\int \frac{1}{\left(a^2+2ab\sqrt[4]{x}+b^2\sqrt{x}\right)^{3/2}}\,dx$

Optimal. Leaf size=176

$$-\frac{12a^2}{b^4\sqrt{a^2+2ab\sqrt[4]{x}+b^2\sqrt{x}}}-\frac{12a\left(a+b\sqrt[4]{x}\right)\log\left(a+b\sqrt[4]{x}\right)}{b^4\sqrt{a^2+2ab\sqrt[4]{x}+b^2\sqrt{x}}}+\frac{4\sqrt[4]{x}\left(a+b\sqrt[4]{x}\right)}{b^3\sqrt{a^2+2ab\sqrt[4]{x}+b^2\sqrt{x}}}+\frac{2a^3}{b^4\left(a+b\sqrt[4]{x}\right)\sqrt{a^2+}}$$

Rubi [A] time = 0.10, antiderivative size = 176, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1341, 646, 43}

$$\frac{2a^3}{b^4\left(a+b\sqrt[4]{x}\right)\sqrt{a^2+2ab\sqrt[4]{x}+b^2\sqrt{x}}}-\frac{12a^2}{b^4\sqrt{a^2+2ab\sqrt[4]{x}+b^2\sqrt{x}}}+\frac{4\sqrt[4]{x}\left(a+b\sqrt[4]{x}\right)}{b^3\sqrt{a^2+2ab\sqrt[4]{x}+b^2\sqrt{x}}}-\frac{12a\left(a+b\sqrt[4]{x}\right)\log\left(a+b\sqrt[4]{x}\right)}{b^4\sqrt{a^2+2ab\sqrt[4]{x}+b^2\sqrt{x}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^(1/4) + b^2*Sqrt[x])^(-3/2),x]
```

```
[Out] (-12*a^2)/(b^4*Sqrt[a^2 + 2*a*b*x^(1/4) + b^2*Sqrt[x]]) + (2*a^3)/(b^4*(a +
 b*x^(1/4))*Sqrt[a^2 + 2*a*b*x^(1/4) + b^2*Sqrt[x]]) + (4*(a + b*x^(1/4))*x
^(1/4))/(b^3*Sqrt[a^2 + 2*a*b*x^(1/4) + b^2*Sqrt[x]]) - (12*a*(a + b*x^(1/4
))*Log[a + b*x^(1/4)])/(b^4*Sqrt[a^2 + 2*a*b*x^(1/4) + b^2*Sqrt[x]])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 1341

```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k
 = Denominator[n]}, Dist[k, Subst[Int[x^(k - 1)*(a + b*x^(k*n) + c*x^(2*k*n
))^p, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && EqQ[n2, 2*n] && Fra
ctionQ[n]
```

Rubi steps

$$\int \frac{1}{\left(a^2+2ab\sqrt[4]{x}+b^2\sqrt{x}\right)^{3/2}}\,dx = 4\,\text{Subst}\left(\int \frac{x^3}{\left(a^2+2abx+b^2x^2\right)^{3/2}}\,dx, x, \sqrt[4]{x}\right)$$
$$= \frac{\left(4b^3\left(a+b\sqrt[4]{x}\right)\right)\text{Subst}\left(\int \frac{x^3}{\left(ab+b^2x\right)^3}\,dx, x, \sqrt[4]{x}\right)}{\sqrt{a^2+2ab\sqrt[4]{x}+b^2\sqrt{x}}}$$
$$= \frac{\left(4b^3\left(a+b\sqrt[4]{x}\right)\right)\text{Subst}\left(\int \left(\frac{1}{b^6}-\frac{a^3}{b^6(a+bx)^3}+\frac{3a^2}{b^6(a+bx)^2}-\frac{3a}{b^6(a+bx)}\right)dx, x, \sqrt[4]{x}\right)}{\sqrt{a^2+2ab\sqrt[4]{x}+b^2\sqrt{x}}}$$
$$= -\frac{12a^2}{b^4\sqrt{a^2+2ab\sqrt[4]{x}+b^2\sqrt{x}}}+\frac{2a^3}{b^4\left(a+b\sqrt[4]{x}\right)\sqrt{a^2+2ab\sqrt[4]{x}+b^2\sqrt{x}}}+\frac{4(\ldots}{b^3\sqrt{a^2 \ldots}}$$

Mathematica [A] time = 0.07, size = 93, normalized size = 0.53

$$\frac{2\left(-5a^3-4a^2b\sqrt[4]{x}+4ab^2\sqrt{x}-6a\left(a+b\sqrt[4]{x}\right)^2\log\left(a+b\sqrt[4]{x}\right)+2b^3x^{3/4}\right)}{b^4\left(a+b\sqrt[4]{x}\right)\sqrt{\left(a+b\sqrt[4]{x}\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^(1/4) + b^2*Sqrt[x])^(-3/2),x]
```

```
[Out] (2*(-5*a^3 - 4*a^2*b*x^(1/4) + 4*a*b^2*Sqrt[x] + 2*b^3*x^(3/4) - 6*a*(a + b
*x^(1/4))^2*Log[a + b*x^(1/4)]))/(b^4*(a + b*x^(1/4))*Sqrt[(a + b*x^(1/4))^
2])
```

IntegrateAlgebraic [B] time = 1.26, size = 1582, normalized size = 8.99

result too large to display

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a^2 + 2*a*b*x^(1/4) + b^2*Sqrt[x])^(-3/2),x]
```

```
[Out] ((-16*a^4*Sqrt[a^2 + 2*a*b*x^(1/4) + b^2*Sqrt[x]])/b^4 + (64*a^4*Sqrt[b^2]*
x^(1/4))/b^4 - (48*a^3*Sqrt[a^2 + 2*a*b*x^(1/4) + b^2*Sqrt[x]]*x^(1/4))/b^3
 + (128*a^3*Sqrt[b^2]*Sqrt[x])/b^3 - (80*a^2*Sqrt[a^2 + 2*a*b*x^(1/4) + b^2
*Sqrt[x]]*Sqrt[x])/b^2 + (32*a^2*(b^2)^(3/2)*x^(3/4))/b^4 + (48*a*Sqrt[a^2
+ 2*a*b*x^(1/4) + b^2*Sqrt[x]]*x^(3/4))/b - (80*a*Sqrt[b^2]*x)/b + 32*Sqrt[
a^2 + 2*a*b*x^(1/4) + b^2*Sqrt[x]]*x - 32*Sqrt[b^2]*x^(5/4) - (96*a^3*Sqrt[
x]*ArcTanh[(Sqrt[a^2 + 2*a*b*x^(1/4) + b^2*Sqrt[x]] - Sqrt[b^2]*x^(1/4))/a]
)/b^2 + (96*a^2*Sqrt[b^2]*Sqrt[a^2 + 2*a*b*x^(1/4) + b^2*Sqrt[x]]*Sqrt[x]*A
rcTanh[(Sqrt[a^2 + 2*a*b*x^(1/4) + b^2*Sqrt[x]] - Sqrt[b^2]*x^(1/4))/a])/b^
3 - (192*a^2*x^(3/4)*ArcTanh[(Sqrt[a^2 + 2*a*b*x^(1/4) + b^2*Sqrt[x]] - Sqr
t[b^2]*x^(1/4))/a])/b + (96*a*(b^2)^(3/2)*Sqrt[a^2 + 2*a*b*x^(1/4) + b^2*Sq
rt[x]]*x^(3/4)*ArcTanh[(Sqrt[a^2 + 2*a*b*x^(1/4) + b^2*Sqrt[x]] - Sqrt[b^2]
*x^(1/4))/a])/b^4 - 96*a*x*ArcTanh[(Sqrt[a^2 + 2*a*b*x^(1/4) + b^2*Sqrt[x]]
 - Sqrt[b^2]*x^(1/4))/a])/((-a + Sqrt[a^2 + 2*a*b*x^(1/4) + b^2*Sqrt[x]] -
Sqrt[b^2]*x^(1/4))^2*(a + Sqrt[a^2 + 2*a*b*x^(1/4) + b^2*Sqrt[x]] - Sqrt[b^
2]*x^(1/4))^2) + ((-16*a^5)/(b^3*Sqrt[b^2]) - (64*a^4*x^(1/4))/(b^2)^(3/2)
+ (64*a^3*Sqrt[a^2 + 2*a*b*x^(1/4) + b^2*Sqrt[x]]*x^(1/4))/b^3 - (64*a^3*Sq
rt[x])/(b*Sqrt[b^2]) + (48*a^3*Sqrt[x]*Log[-a + Sqrt[a^2 + 2*a*b*x^(1/4) +
b^2*Sqrt[x]] - Sqrt[b^2]*x^(1/4)])/(b*Sqrt[b^2]) - (48*a^2*Sqrt[a^2 + 2*a*b
*x^(1/4) + b^2*Sqrt[x]]*Sqrt[x]*Log[-a + Sqrt[a^2 + 2*a*b*x^(1/4) + b^2*Sqr
t[x]] - Sqrt[b^2]*x^(1/4)])/b^2 + (96*a^2*x^(3/4)*Log[-a + Sqrt[a^2 + 2*a*b
```

```
*x^(1/4) + b^2*Sqrt[x]] - Sqrt[b^2]*x^(1/4)])/Sqrt[b^2] - (48*a*Sqrt[a^2 +
2*a*b*x^(1/4) + b^2*Sqrt[x]]*x^(3/4)*Log[-a + Sqrt[a^2 + 2*a*b*x^(1/4) + b^
2*Sqrt[x]] - Sqrt[b^2]*x^(1/4)])/b + (48*a*b*x*Log[-a + Sqrt[a^2 + 2*a*b*x^
(1/4) + b^2*Sqrt[x]] - Sqrt[b^2]*x^(1/4)])/Sqrt[b^2] + (48*a^3*Sqrt[x]*Log[
a + Sqrt[a^2 + 2*a*b*x^(1/4) + b^2*Sqrt[x]] - Sqrt[b^2]*x^(1/4)])/(b*Sqrt[b
^2]) - (48*a^2*Sqrt[a^2 + 2*a*b*x^(1/4) + b^2*Sqrt[x]]*Sqrt[x]*Log[a + Sqrt
[a^2 + 2*a*b*x^(1/4) + b^2*Sqrt[x]] - Sqrt[b^2]*x^(1/4)])/b^2 + (96*a^2*x^(
3/4)*Log[a + Sqrt[a^2 + 2*a*b*x^(1/4) + b^2*Sqrt[x]] - Sqrt[b^2]*x^(1/4)])/
Sqrt[b^2] - (48*a*Sqrt[a^2 + 2*a*b*x^(1/4) + b^2*Sqrt[x]]*x^(3/4)*Log[a + S
qrt[a^2 + 2*a*b*x^(1/4) + b^2*Sqrt[x]] - Sqrt[b^2]*x^(1/4)])/b + (48*a*b*x*
Log[a + Sqrt[a^2 + 2*a*b*x^(1/4) + b^2*Sqrt[x]] - Sqrt[b^2]*x^(1/4)])/Sqrt[
b^2])/((-a + Sqrt[a^2 + 2*a*b*x^(1/4) + b^2*Sqrt[x]] - Sqrt[b^2]*x^(1/4))^2
*(a + Sqrt[a^2 + 2*a*b*x^(1/4) + b^2*Sqrt[x]] - Sqrt[b^2]*x^(1/4))^2)
```

fricas [A] time = 11.03, size = 147, normalized size = 0.84

$$\frac{2\left(9\,a^5b^4x-5\,a^9-6\left(ab^8x^2-2\,a^5b^4x+a^9\right)\log\left(bx^{\frac14}+a\right)-2\left(3\,a^2b^7x-a^6b^3\right)x^{\frac34}+\left(7\,a^3b^6x-3\,a^7b^2\right)\sqrt{x}+2\left(b^9x^2-6\,a^4b^5x+3\,a^8b\right)x^{\frac14}\right)}{b^{12}x^2-2\,a^4b^8x+a^8b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2+2*a*b*x^(1/4)+b^2*x^(1/2))^(3/2),x, algorithm="fricas")
```

```
[Out] 2*(9*a^5*b^4*x - 5*a^9 - 6*(a*b^8*x^2 - 2*a^5*b^4*x + a^9)*log(b*x^(1/4) +
a) - 2*(3*a^2*b^7*x - a^6*b^3)*x^(3/4) + (7*a^3*b^6*x - 3*a^7*b^2)*sqrt(x)
+ 2*(b^9*x^2 - 6*a^4*b^5*x + 3*a^8*b)*x^(1/4))/(b^12*x^2 - 2*a^4*b^8*x + a^
8*b^4)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2+2*a*b*x^(1/4)+b^2*x^(1/2))^(3/2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.02, size = 114, normalized size = 0.65

$$\frac{2\sqrt{b^2\sqrt{x}+2ab\,x^{\frac14}+a^2}\,\left(-6a\,b^2\sqrt{x}\,\ln\left(b\,x^{\frac14}+a\right)-12a^2b\,x^{\frac14}\ln\left(b\,x^{\frac14}+a\right)-6a^3\ln\left(b\,x^{\frac14}+a\right)+2b^3x^{\frac34}+4a\,b^2\sqrt{x}-4a^2b\,x^{\frac14}-5a^3\right)}{\left(b\,x^{\frac14}+a\right)^3 b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a^2+2*a*b*x^(1/4)+b^2*x^(1/2))^(3/2),x)
```

```
[Out] 2*(a^2+2*a*b*x^(1/4)+b^2*x^(1/2))^(1/2)*(2*x^(3/4)*b^3-6*x^(1/2)*ln(a+b*x^(
1/4))*a*b^2+4*x^(1/2)*a*b^2-12*x^(1/4)*ln(a+b*x^(1/4))*a^2*b-4*x^(1/4)*a^2*
b-6*ln(a+b*x^(1/4))*a^3-5*a^3)/(a+b*x^(1/4))^3/b^4
```

maxima [A] time = 0.58, size = 114, normalized size = 0.65

$$\frac{4\sqrt{x}}{\sqrt{b^2\sqrt{x}+2\,abx^{\frac14}+a^2}\,b^2}-\frac{12\,a\log\left(x^{\frac14}+\frac{a}{b}\right)}{b^4}+\frac{8\,a^2}{\sqrt{b^2\sqrt{x}+2\,abx^{\frac14}+a^2}\,b^4}-\frac{24\,a^2x^{\frac14}}{b^5\left(x^{\frac14}+\frac{a}{b}\right)^2}-\frac{22\,a^3}{b^6\left(x^{\frac14}+\frac{a}{b}\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2+2*a*b*x^(1/4)+b^2*x^(1/2))^(3/2),x, algorithm="maxima")
```

```
[Out] 4*sqrt(x)/(sqrt(b^2*sqrt(x) + 2*a*b*x^(1/4) + a^2)*b^2) - 12*a*log(x^(1/4)
+ a/b)/b^4 + 8*a^2/(sqrt(b^2*sqrt(x) + 2*a*b*x^(1/4) + a^2)*b^4) - 24*a^2*x
^(1/4)/(b^5*(x^(1/4) + a/b)^2) - 22*a^3/(b^6*(x^(1/4) + a/b)^2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(a^2+b^2\,\sqrt{x}+2\,a\,b\,x^{1/4}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a^2 + b^2*x^(1/2) + 2*a*b*x^(1/4))^(3/2),x)
```

```
[Out] int(1/(a^2 + b^2*x^(1/2) + 2*a*b*x^(1/4))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a^{2}+2ab\sqrt[4]{x}+b^{2}\sqrt{x}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a**2+2*a*b*x**(1/4)+b**2*x**(1/2))**(3/2),x)
```

```
[Out] Integral((a**2 + 2*a*b*x**(1/4) + b**2*sqrt(x))**(-3/2), x)
```

3.416 $\int \frac{1}{\left(a^2+2ab\sqrt[6]{x}+b^2\sqrt[3]{x}\right)^{5/2}}\,dx$

Optimal. Leaf size=268

$$-\frac{60a^2}{b^6\sqrt{a^2+2ab\sqrt[6]{x}+b^2\sqrt[3]{x}}}-\frac{30a\left(a+b\sqrt[6]{x}\right)\log\left(a+b\sqrt[6]{x}\right)}{b^6\sqrt{a^2+2ab\sqrt[6]{x}+b^2\sqrt[3]{x}}}+\frac{6\sqrt[6]{x}\left(a+b\sqrt[6]{x}\right)}{b^5\sqrt{a^2+2ab\sqrt[6]{x}+b^2\sqrt[3]{x}}}+\frac{3a^5}{2b^6\left(a+b\sqrt[6]{x}\right)^3\sqrt{a^2}}$$

Rubi [A] time = 0.15, antiderivative size = 268, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1341, 646, 43}

$$\frac{3a^5}{2b^6\left(a+b\sqrt[6]{x}\right)^3\sqrt{a^2+2ab\sqrt[6]{x}+b^2\sqrt[3]{x}}}-\frac{10a^4}{b^6\left(a+b\sqrt[6]{x}\right)^2\sqrt{a^2+2ab\sqrt[6]{x}+b^2\sqrt[3]{x}}}+\frac{30a^3}{b^6\left(a+b\sqrt[6]{x}\right)\sqrt{a^2+2ab\sqrt[6]{x}+b^2\sqrt[3]{x}}}-\frac{60a^2}{b^6\sqrt{a^2+2ab\sqrt[6]{x}+b^2\sqrt[3]{x}}}+\frac{6\sqrt[6]{x}\left(a+b\sqrt[6]{x}\right)}{b^5\sqrt{a^2+2ab\sqrt[6]{x}+b^2\sqrt[3]{x}}}-\frac{30a\left(a+b\sqrt[6]{x}\right)\log\left(a+b\sqrt[6]{x}\right)}{b^6\sqrt{a^2+2ab\sqrt[6]{x}+b^2\sqrt[3]{x}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^(1/6) + b^2*x^(1/3))^(-5/2),x]
```

```
[Out] (-60*a^2)/(b^6*Sqrt[a^2 + 2*a*b*x^(1/6) + b^2*x^(1/3)]) + (3*a^5)/(2*b^6*(a
 + b*x^(1/6))^3*Sqrt[a^2 + 2*a*b*x^(1/6) + b^2*x^(1/3)]) - (10*a^4)/(b^6*(a
 + b*x^(1/6))^2*Sqrt[a^2 + 2*a*b*x^(1/6) + b^2*x^(1/3)]) + (30*a^3)/(b^6*(a
 + b*x^(1/6))*Sqrt[a^2 + 2*a*b*x^(1/6) + b^2*x^(1/3)]) + (6*(a + b*x^(1/6))
*x^(1/6))/(b^5*Sqrt[a^2 + 2*a*b*x^(1/6) + b^2*x^(1/3)]) - (30*a*(a + b*x^(1
/6))*Log[a + b*x^(1/6)])/(b^6*Sqrt[a^2 + 2*a*b*x^(1/6) + b^2*x^(1/3)])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 1341

```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k
 = Denominator[n]}, Dist[k, Subst[Int[x^(k - 1)*(a + b*x^(k*n) + c*x^(2*k*n
))^p, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && EqQ[n2, 2*n] && Fra
ctionQ[n]
```

Rubi steps

$$\int \frac{1}{\left(a^2+2ab\sqrt[6]{x}+b^2\sqrt[3]{x}\right)^{5/2}}\,dx = 6\,\text{Subst}\left(\int \frac{x^5}{\left(a^2+2abx+b^2x^2\right)^{5/2}}\,dx, x, \sqrt[6]{x}\right)$$

$$= \frac{\left(6b^5\left(a+b\sqrt[6]{x}\right)\right)\text{Subst}\left(\int \frac{x^5}{\left(ab+b^2x\right)^5}\,dx, x, \sqrt[6]{x}\right)}{\sqrt{a^2+2ab\sqrt[6]{x}+b^2\sqrt[3]{x}}}$$

$$= \frac{\left(6b^5\left(a+b\sqrt[6]{x}\right)\right)\text{Subst}\left(\int \left(\frac{1}{b^{10}} - \frac{a^5}{b^{10}(a+bx)^5} + \frac{5a^4}{b^{10}(a+bx)^4} - \frac{10a^3}{b^{10}(a+bx)^3} + \frac{10a^2}{b^{10}(a+bx)^2} -\right.\right.}{\sqrt{a^2+2ab\sqrt[6]{x}+b^2\sqrt[3]{x}}}$$

$$= -\frac{60a^2}{b^6\sqrt{a^2+2ab\sqrt[6]{x}+b^2\sqrt[3]{x}}} + \frac{3a^5}{2b^6\left(a+b\sqrt[6]{x}\right)^3\sqrt{a^2+2ab\sqrt[6]{x}+b^2\sqrt[3]{x}}} - \frac{}{b^6\left(a\right.}$$

Mathematica [A] time = 0.10, size = 121, normalized size = 0.45

$$\frac{-77a^5-248a^4b\sqrt[6]{x}-252a^3b^2\sqrt[3]{x}-48a^2b^3\sqrt{x}+48ab^4x^{2/3}-60a\left(a+b\sqrt[6]{x}\right)^4\log\left(a+b\sqrt[6]{x}\right)+12b^5x^{5/6}}{2b^6\left(a+b\sqrt[6]{x}\right)^3\sqrt{\left(a+b\sqrt[6]{x}\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^(1/6) + b^2*x^(1/3))^(-5/2),x]
```

```
[Out] (-77*a^5 - 248*a^4*b*x^(1/6) - 252*a^3*b^2*x^(1/3) - 48*a^2*b^3*Sqrt[x] + 4
8*a*b^4*x^(2/3) + 12*b^5*x^(5/6) - 60*a*(a + b*x^(1/6))^4*Log[a + b*x^(1/6)
])/(2*b^6*(a + b*x^(1/6))^3*Sqrt[(a + b*x^(1/6))^2])
```

IntegrateAlgebraic [B] time = 2.59, size = 2887, normalized size = 10.77

Result too large to show

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a^2 + 2*a*b*x^(1/6) + b^2*x^(1/3))^(-5/2),x]
```

```
[Out] ((-192*a^8*Sqrt[a^2 + 2*a*b*x^(1/6) + b^2*x^(1/3)])/b^6 + (1024*a^8*x^(1/6)
)/(b^4*Sqrt[b^2]) - (832*a^7*Sqrt[a^2 + 2*a*b*x^(1/6) + b^2*x^(1/3)]*x^(1/6
))/b^5 + (5056*a^7*x^(1/3))/(b^3*Sqrt[b^2]) - (4224*a^6*Sqrt[a^2 + 2*a*b*x^
(1/6) + b^2*x^(1/3)]*x^(1/3))/b^4 + (15744*a^6*Sqrt[x])/(b^2)^(3/2) - (1152
0*a^5*Sqrt[a^2 + 2*a*b*x^(1/6) + b^2*x^(1/3)]*Sqrt[x])/b^3 + (31616*a^5*x^(
2/3))/(b*Sqrt[b^2]) - (20096*a^4*Sqrt[a^2 + 2*a*b*x^(1/6) + b^2*x^(1/3)]*x^
(2/3))/b^2 + (37376*a^4*x^(5/6))/Sqrt[b^2] - (17280*a^3*Sqrt[a^2 + 2*a*b*x^
(1/6) + b^2*x^(1/3)]*x^(5/6))/b + (21504*a^3*b*x)/Sqrt[b^2] - 4224*a^2*Sqrt
[a^2 + 2*a*b*x^(1/6) + b^2*x^(1/3)]*x + 1536*a^2*Sqrt[b^2]*x^(7/6) + 2688*a
*b*Sqrt[a^2 + 2*a*b*x^(1/6) + b^2*x^(1/3)]*x^(7/6) - (3456*a*b^3*x^(4/3))/S
qrt[b^2] + 768*b^2*Sqrt[a^2 + 2*a*b*x^(1/6) + b^2*x^(1/3)]*x^(4/3) - (768*b
^4*x^(3/2))/Sqrt[b^2] - (3840*a^5*x^(2/3)*ArcTanh[(Sqrt[a^2 + 2*a*b*x^(1/6)
 + b^2*x^(1/3)] - Sqrt[b^2]*x^(1/6))/a])/b^2 + (3840*a^4*Sqrt[a^2 + 2*a*b*x
^(1/6) + b^2*x^(1/3)]*x^(2/3)*ArcTanh[(Sqrt[a^2 + 2*a*b*x^(1/6) + b^2*x^(1/
3)] - Sqrt[b^2]*x^(1/6))/a])/(b*Sqrt[b^2]) - (15360*a^4*x^(5/6)*ArcTanh[(Sq
rt[a^2 + 2*a*b*x^(1/6) + b^2*x^(1/3)] - Sqrt[b^2]*x^(1/6))/a])/b + (11520*a
^3*Sqrt[a^2 + 2*a*b*x^(1/6) + b^2*x^(1/3)]*x^(5/6)*ArcTanh[(Sqrt[a^2 + 2*a*
b*x^(1/6) + b^2*x^(1/3)] - Sqrt[b^2]*x^(1/6))/a])/Sqrt[b^2] - 23040*a^3*x*A
rcTanh[(Sqrt[a^2 + 2*a*b*x^(1/6) + b^2*x^(1/3)] - Sqrt[b^2]*x^(1/6))/a] + (
11520*a^2*b*Sqrt[a^2 + 2*a*b*x^(1/6) + b^2*x^(1/3)]*x*ArcTanh[(Sqrt[a^2 + 2
```

```
*a*b*x^(1/6) + b^2*x^(1/3)] - Sqrt[b^2]*x^(1/6))/a])/Sqrt[b^2] - 15360*a^2*
b*x^(7/6)*ArcTanh[(Sqrt[a^2 + 2*a*b*x^(1/6) + b^2*x^(1/3)] - Sqrt[b^2]*x^(1
/6))/a] + 3840*a*Sqrt[b^2]*Sqrt[a^2 + 2*a*b*x^(1/6) + b^2*x^(1/3)]*x^(7/6)*
ArcTanh[(Sqrt[a^2 + 2*a*b*x^(1/6) + b^2*x^(1/3)] - Sqrt[b^2]*x^(1/6))/a] -
3840*a*b^2*x^(4/3)*ArcTanh[(Sqrt[a^2 + 2*a*b*x^(1/6) + b^2*x^(1/3)] - Sqrt[
b^2]*x^(1/6))/a])/((-a + Sqrt[a^2 + 2*a*b*x^(1/6) + b^2*x^(1/3)] - Sqrt[b^2
]*x^(1/6))^4*(a + Sqrt[a^2 + 2*a*b*x^(1/6) + b^2*x^(1/3)] - Sqrt[b^2]*x^(1/
6))^4) + ((-192*a^9)/(b^5*Sqrt[b^2]) - (1024*a^8*x^(1/6))/(b^4*Sqrt[b^2]) +
 (1024*a^7*Sqrt[a^2 + 2*a*b*x^(1/6) + b^2*x^(1/3)]*x^(1/6))/b^5 - (5056*a^7
*x^(1/3))/(b^3*Sqrt[b^2]) + (4032*a^6*Sqrt[a^2 + 2*a*b*x^(1/6) + b^2*x^(1/3
)]*x^(1/3))/b^4 - (15744*a^6*Sqrt[x])/(b^2)^(3/2) + (11712*a^5*Sqrt[a^2 + 2
*a*b*x^(1/6) + b^2*x^(1/3)]*Sqrt[x])/b^3 - (27072*a^5*x^(2/3))/(b*Sqrt[b^2]
) + (15360*a^4*Sqrt[a^2 + 2*a*b*x^(1/6) + b^2*x^(1/3)]*x^(2/3))/b^2 - (2304
0*a^4*x^(5/6))/Sqrt[b^2] + (7680*a^3*Sqrt[a^2 + 2*a*b*x^(1/6) + b^2*x^(1/3)
]*x^(5/6))/b - (7680*a^3*b*x)/Sqrt[b^2] + (1920*a^5*x^(2/3)*Log[-a + Sqrt[a
^2 + 2*a*b*x^(1/6) + b^2*x^(1/3)] - Sqrt[b^2]*x^(1/6)])/(b*Sqrt[b^2]) - (19
20*a^4*Sqrt[a^2 + 2*a*b*x^(1/6) + b^2*x^(1/3)]*x^(2/3)*Log[-a + Sqrt[a^2 +
2*a*b*x^(1/6) + b^2*x^(1/3)] - Sqrt[b^2]*x^(1/6)])/b^2 + (7680*a^4*x^(5/6)*
Log[-a + Sqrt[a^2 + 2*a*b*x^(1/6) + b^2*x^(1/3)] - Sqrt[b^2]*x^(1/6)])/Sqrt
[b^2] - (5760*a^3*Sqrt[a^2 + 2*a*b*x^(1/6) + b^2*x^(1/3)]*x^(5/6)*Log[-a +
Sqrt[a^2 + 2*a*b*x^(1/6) + b^2*x^(1/3)] - Sqrt[b^2]*x^(1/6)])/b + (11520*a^
3*b*x*Log[-a + Sqrt[a^2 + 2*a*b*x^(1/6) + b^2*x^(1/3)] - Sqrt[b^2]*x^(1/6)]
)/Sqrt[b^2] - 5760*a^2*Sqrt[a^2 + 2*a*b*x^(1/6) + b^2*x^(1/3)]*x*Log[-a + S
qrt[a^2 + 2*a*b*x^(1/6) + b^2*x^(1/3)] - Sqrt[b^2]*x^(1/6)] + 7680*a^2*Sqrt
[b^2]*x^(7/6)*Log[-a + Sqrt[a^2 + 2*a*b*x^(1/6) + b^2*x^(1/3)] - Sqrt[b^2]*
x^(1/6)] - 1920*a*b*Sqrt[a^2 + 2*a*b*x^(1/6) + b^2*x^(1/3)]*x^(7/6)*Log[-a
+ Sqrt[a^2 + 2*a*b*x^(1/6) + b^2*x^(1/3)] - Sqrt[b^2]*x^(1/6)] + (1920*a*b^
3*x^(4/3)*Log[-a + Sqrt[a^2 + 2*a*b*x^(1/6) + b^2*x^(1/3)] - Sqrt[b^2]*x^(1
/6)])/Sqrt[b^2] + (1920*a^5*x^(2/3)*Log[a + Sqrt[a^2 + 2*a*b*x^(1/6) + b^2*
x^(1/3)] - Sqrt[b^2]*x^(1/6)])/(b*Sqrt[b^2]) - (1920*a^4*Sqrt[a^2 + 2*a*b*x
^(1/6) + b^2*x^(1/3)]*x^(2/3)*Log[a + Sqrt[a^2 + 2*a*b*x^(1/6) + b^2*x^(1/3
)] - Sqrt[b^2]*x^(1/6)])/b^2 + (7680*a^4*x^(5/6)*Log[a + Sqrt[a^2 + 2*a*b*x
^(1/6) + b^2*x^(1/3)] - Sqrt[b^2]*x^(1/6)])/Sqrt[b^2] - (5760*a^3*Sqrt[a^2
+ 2*a*b*x^(1/6) + b^2*x^(1/3)]*x^(5/6)*Log[a + Sqrt[a^2 + 2*a*b*x^(1/6) + b
^2*x^(1/3)] - Sqrt[b^2]*x^(1/6)])/b + (11520*a^3*b*x*Log[a + Sqrt[a^2 + 2*a
*b*x^(1/6) + b^2*x^(1/3)] - Sqrt[b^2]*x^(1/6)])/Sqrt[b^2] - 5760*a^2*Sqrt[a
^2 + 2*a*b*x^(1/6) + b^2*x^(1/3)]*x*Log[a + Sqrt[a^2 + 2*a*b*x^(1/6) + b^2*
x^(1/3)] - Sqrt[b^2]*x^(1/6)] + 7680*a^2*Sqrt[b^2]*x^(7/6)*Log[a + Sqrt[a^2
 + 2*a*b*x^(1/6) + b^2*x^(1/3)] - Sqrt[b^2]*x^(1/6)] - 1920*a*b*Sqrt[a^2 +
2*a*b*x^(1/6) + b^2*x^(1/3)]*x^(7/6)*Log[a + Sqrt[a^2 + 2*a*b*x^(1/6) + b^2
*x^(1/3)] - Sqrt[b^2]*x^(1/6)] + (1920*a*b^3*x^(4/3)*Log[a + Sqrt[a^2 + 2*a
*b*x^(1/6) + b^2*x^(1/3)] - Sqrt[b^2]*x^(1/6)])/Sqrt[b^2])/((-a + Sqrt[a^2
+ 2*a*b*x^(1/6) + b^2*x^(1/3)] - Sqrt[b^2]*x^(1/6))^4*(a + Sqrt[a^2 + 2*a*b
*x^(1/6) + b^2*x^(1/3)] - Sqrt[b^2]*x^(1/6))^4)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2+2*a*b*x^(1/6)+b^2*x^(1/3))^(5/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [A] time = 0.87, size = 237, normalized size = 0.88

$$\frac{3\,a^4|a|\log\left(\left|x^{\frac{1}{6}}|b|\mathrm{sgn}(a)\mathrm{sgn}(b)+|a|\right|\right)}{4\left(a^3b^5|a||b|\mathrm{sgn}(a)\mathrm{sgn}(b)-a^4b^6\right)}+\frac{3\left(24\,a^5b^2|b|\mathrm{sgn}(a)\mathrm{sgn}(b)-25\,a^4b^3|a|\right)\log\left(\left|bx^{\frac{1}{6}}+a\right|\right)}{4\left(a^3b^8|a||b|\mathrm{sgn}(a)\mathrm{sgn}(b)-a^4b^9\right)}+\frac{6\,x^{\frac{1}{6}}}{b^4|b|\mathrm{sgn}(a)\mathrm{sgn}(b)}+\frac{70\,a^5|b|\mathrm{sgn}(a)\mathrm{sgn}(b)-70\,a^4b|a|+93\left(a^3b^2|b|\mathrm{sgn}(a)\mathrm{sgn}(b)-a^2b^3|a|\right)x^{\frac{1}{3}}+159\left(a^4b|b|\mathrm{sgn}(a)\mathrm{sgn}(b)-a^3b^2|a|\right)x^{\frac{1}{6}}}{4\left(|a||b|\mathrm{sgn}(a)\mathrm{sgn}(b)-ab\right)\left(bx^{\frac{1}{6}}+a\right)^3b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2+2*a*b*x^(1/6)+b^2*x^(1/3))^(5/2),x, algorithm="giac")
```

```
[Out] 3/4*a^4*abs(a)*log(abs(x^(1/6)*abs(b)*sgn(a)*sgn(b) + abs(a)))/(a^3*b^5*abs
(a)*abs(b)*sgn(a)*sgn(b) - a^4*b^6) + 3/4*(24*a^5*b^2*abs(b)*sgn(a)*sgn(b)
- 25*a^4*b^3*abs(a))*log(abs(b*x^(1/6) + a))/(a^3*b^8*abs(a)*abs(b)*sgn(a)*
sgn(b) - a^4*b^9) + 6*x^(1/6)/(b^4*abs(b)*sgn(a)*sgn(b)) + 1/4*(70*a^5*abs(
b)*sgn(a)*sgn(b) - 70*a^4*b*abs(a) + 93*(a^3*b^2*abs(b)*sgn(a)*sgn(b) - a^2
*b^3*abs(a))*x^(1/3) + 159*(a^4*b*abs(b)*sgn(a)*sgn(b) - a^3*b^2*abs(a))*x^
(1/6))/((abs(a)*abs(b)*sgn(a)*sgn(b) - a*b)*(b*x^(1/6) + a)^3*b^6)
```

maple [A] time = 0.02, size = 174, normalized size = 0.65

$$\frac{\sqrt{b^{2} x^{\frac{1}{3}}+2 a b x^{\frac{1}{6}}+a^{2}}\, \left(-60 a b^{4} x^{\frac{2}{3}} \ln \left(b x^{\frac{1}{6}}+a \right)-240 a^{2} b^{3} \sqrt{x}\, \ln \left(b x^{\frac{1}{6}}+a \right)-360 a^{3} b^{2} x^{\frac{1}{3}} \ln \left(b x^{\frac{1}{6}}+a \right)-240 a^{4} b x^{\frac{1}{6}} \ln \left(b x^{\frac{1}{6}}+a \right)-60 a^{5} \ln \left(b x^{\frac{1}{6}}+a \right)+12 b^{5} x^{\frac{5}{6}}+48 a b^{4} x^{\frac{2}{3}}-48 a^{2} b^{3} \sqrt{x}-252 a^{3} b^{2} x^{\frac{1}{3}}-248 a^{4} b x^{\frac{1}{6}}-77 a^{5}\right)}{2 \left(b x^{\frac{1}{6}}+a \right)^{5} b^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a^2+2*a*b*x^(1/6)+b^2*x^(1/3))^(5/2),x)
```

```
[Out] 1/2*(a^2+2*a*b*x^(1/6)+b^2*x^(1/3))^(1/2)*(12*x^(5/6)*b^5-60*x^(2/3)*ln(a+b
*x^(1/6))*a*b^4+48*x^(2/3)*a*b^4-240*x^(1/2)*ln(a+b*x^(1/6))*a^2*b^3-48*x^(
1/2)*a^2*b^3-360*x^(1/3)*ln(a+b*x^(1/6))*a^3*b^2-252*x^(1/3)*a^3*b^2-240*x^
(1/6)*ln(a+b*x^(1/6))*a^4*b-248*x^(1/6)*a^4*b-60*ln(a+b*x^(1/6))*a^5-77*a^5
)/(a+b*x^(1/6))^5/b^6
```

maxima [A] time = 0.88, size = 119, normalized size = 0.44

$$\frac{12 \, b^{5} x^{\frac{5}{6}} + 48 \, a b^{4} x^{\frac{2}{3}} - 48 \, a^{2} b^{3} \sqrt{x} - 252 \, a^{3} b^{2} x^{\frac{1}{3}} - 248 \, a^{4} b x^{\frac{1}{6}} - 77 \, a^{5}}{2 \, {\left(b^{10} x^{\frac{2}{3}} + 4 \, a b^{9} \sqrt{x} + 6 \, a^{2} b^{8} x^{\frac{1}{3}} + 4 \, a^{3} b^{7} x^{\frac{1}{6}} + a^{4} b^{6}\right)}} - \frac{30 \, a \log\left(b x^{\frac{1}{6}} + a\right)}{b^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2+2*a*b*x^(1/6)+b^2*x^(1/3))^(5/2),x, algorithm="maxima")
```

```
[Out] 1/2*(12*b^5*x^(5/6) + 48*a*b^4*x^(2/3) - 48*a^2*b^3*sqrt(x) - 252*a^3*b^2*x
^(1/3) - 248*a^4*b*x^(1/6) - 77*a^5)/(b^10*x^(2/3) + 4*a*b^9*sqrt(x) + 6*a^
2*b^8*x^(1/3) + 4*a^3*b^7*x^(1/6) + a^4*b^6) - 30*a*log(b*x^(1/6) + a)/b^6
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{{\left(a^2+b^2\,x^{1/3}+2\,a\,b\,x^{1/6}\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a^2 + b^2*x^(1/3) + 2*a*b*x^(1/6))^(5/2),x)
```

```
[Out] int(1/(a^2 + b^2*x^(1/3) + 2*a*b*x^(1/6))^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a**2+2*a*b*x**(1/6)+b**2*x**(1/3))**(5/2),x)
```

```
[Out] Timed out
```

3.417 $\int \left(a^2+\frac{b^2}{x}+\frac{2ab}{\sqrt{x}}\right)^{3/2}\,dx$

Optimal. Leaf size=179

$$\frac{6a^2b\sqrt{x}\sqrt{a^2+\frac{2ab}{\sqrt{x}}+\frac{b^2}{x}}}{a+\frac{b}{\sqrt{x}}}+\frac{6ab^2\log\left(\sqrt{x}\right)\sqrt{a^2+\frac{2ab}{\sqrt{x}}+\frac{b^2}{x}}}{a+\frac{b}{\sqrt{x}}}-\frac{2b^3\sqrt{a^2+\frac{2ab}{\sqrt{x}}+\frac{b^2}{x}}}{\sqrt{x}\left(a+\frac{b}{\sqrt{x}}\right)}+\frac{a^3x\sqrt{a^2+\frac{2ab}{\sqrt{x}}+\frac{b^2}{x}}}{a+\frac{b}{\sqrt{x}}}$$

Rubi [A] time = 0.09, antiderivative size = 179, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1341, 1355, 263, 43}

$$\frac{a^3x\sqrt{a^2+\frac{2ab}{\sqrt{x}}+\frac{b^2}{x}}}{a+\frac{b}{\sqrt{x}}}+\frac{6a^2b\sqrt{x}\sqrt{a^2+\frac{2ab}{\sqrt{x}}+\frac{b^2}{x}}}{a+\frac{b}{\sqrt{x}}}-\frac{2b^3\sqrt{a^2+\frac{2ab}{\sqrt{x}}+\frac{b^2}{x}}}{\sqrt{x}\left(a+\frac{b}{\sqrt{x}}\right)}+\frac{6ab^2\log\left(\sqrt{x}\right)\sqrt{a^2+\frac{2ab}{\sqrt{x}}+\frac{b^2}{x}}}{a+\frac{b}{\sqrt{x}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + b^2/x + (2*a*b)/Sqrt[x])^(3/2),x]
```

```
[Out] (-2*b^3*Sqrt[a^2 + b^2/x + (2*a*b)/Sqrt[x]])/((a + b/Sqrt[x])*Sqrt[x]) + (6
*a^2*b*Sqrt[a^2 + b^2/x + (2*a*b)/Sqrt[x]]*Sqrt[x])/(a + b/Sqrt[x]) + (a^3*
Sqrt[a^2 + b^2/x + (2*a*b)/Sqrt[x]]*x)/(a + b/Sqrt[x]) + (6*a*b^2*Sqrt[a^2
+ b^2/x + (2*a*b)/Sqrt[x]]*Log[Sqrt[x]])/(a + b/Sqrt[x])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 1341

```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k
 = Denominator[n]}, Dist[k, Subst[Int[x^(k - 1)*(a + b*x^(k*n) + c*x^(2*k*n
))^p, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && EqQ[n2, 2*n] && Fra
ctionQ[n]
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\int \left(a^2+\frac{b^2}{x}+\frac{2ab}{\sqrt{x}}\right)^{3/2} dx = 2\,\text{Subst}\left(\int \left(a^2+\frac{b^2}{x^2}+\frac{2ab}{x}\right)^{3/2} x\,dx, x, \sqrt{x}\right)$$
$$= \frac{\left(2\sqrt{a^2+\frac{b^2}{x}+\frac{2ab}{\sqrt{x}}}\right)\text{Subst}\left(\int \left(ab+\frac{b^2}{x}\right)^3 x\,dx, x, \sqrt{x}\right)}{b^2\left(ab+\frac{b^2}{\sqrt{x}}\right)}$$
$$= \frac{\left(2\sqrt{a^2+\frac{b^2}{x}+\frac{2ab}{\sqrt{x}}}\right)\text{Subst}\left(\int \frac{(b^2+abx)^3}{x^2}dx, x, \sqrt{x}\right)}{b^2\left(ab+\frac{b^2}{\sqrt{x}}\right)}$$
$$= \frac{\left(2\sqrt{a^2+\frac{b^2}{x}+\frac{2ab}{\sqrt{x}}}\right)\text{Subst}\left(\int \left(3a^2b^4+\frac{b^6}{x^2}+\frac{3ab^5}{x}+a^3b^3x\right)dx, x, \sqrt{x}\right)}{b^2\left(ab+\frac{b^2}{\sqrt{x}}\right)}$$
$$= -\frac{2b^4\sqrt{a^2+\frac{b^2}{x}+\frac{2ab}{\sqrt{x}}}}{\left(ab+\frac{b^2}{\sqrt{x}}\right)\sqrt{x}} + \frac{6a^2b^2\sqrt{a^2+\frac{b^2}{x}+\frac{2ab}{\sqrt{x}}}\sqrt{x}}{ab+\frac{b^2}{\sqrt{x}}} + \frac{a^3\sqrt{a^2+\frac{b^2}{x}+\frac{2ab}{\sqrt{x}}}\,x}{a+\frac{b}{\sqrt{x}}} + \frac{3ab^3\sqrt{a^2}}{}$$

Mathematica [A] time = 0.03, size = 66, normalized size = 0.37

$$\frac{\sqrt{\frac{\left(a\sqrt{x}+b\right)^2}{x}}\left(a^3x^{3/2}+6a^2bx+3ab^2\sqrt{x}\log(x)-2b^3\right)}{a\sqrt{x}+b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + b^2/x + (2*a*b)/Sqrt[x])^(3/2),x]
```

```
[Out] (Sqrt[(b + a*Sqrt[x])^2/x]*(-2*b^3 + 6*a^2*b*x + a^3*x^(3/2) + 3*a*b^2*Sqrt
[x]*Log[x]))/(b + a*Sqrt[x])
```

IntegrateAlgebraic [A] time = 5.93, size = 77, normalized size = 0.43

$$\frac{\sqrt{x}\sqrt{\frac{\left(a\sqrt{x}+b\right)^2}{x}}\left(\frac{a^3x^{3/2}+6a^2bx-2b^3}{\sqrt{x}}+6ab^2\log\left(\sqrt{x}\right)\right)}{a\sqrt{x}+b}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a^2 + b^2/x + (2*a*b)/Sqrt[x])^(3/2),x]
```

```
[Out] (Sqrt[(b + a*Sqrt[x])^2/x]*Sqrt[x]*((-2*b^3 + 6*a^2*b*x + a^3*x^(3/2))/Sqrt
[x] + 6*a*b^2*Log[Sqrt[x]]))/(b + a*Sqrt[x])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+b^2/x+2*a*b/x^(1/2))^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [A] time = 0.39, size = 80, normalized size = 0.45

$$a^3x\text{sgn}\left(ax+b\sqrt{x}\right)\text{sgn}(x)+3\,ab^2\log\left(|x|\right)\text{sgn}\left(ax+b\sqrt{x}\right)\text{sgn}(x)+6\,a^2b\sqrt{x}\,\text{sgn}\left(ax+b\sqrt{x}\right)\text{sgn}(x)-\frac{2\,b^3\text{sgn}\left(ax+b\sqrt{x}\right)\text{sgn}(x)}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+b^2/x+2*a*b/x^(1/2))^(3/2),x, algorithm="giac")
```

```
[Out] a^3*x*sgn(a*x + b*sqrt(x))*sgn(x) + 3*a*b^2*log(abs(x))*sgn(a*x + b*sqrt(x)
)*sgn(x) + 6*a^2*b*sqrt(x)*sgn(a*x + b*sqrt(x))*sgn(x) - 2*b^3*sgn(a*x + b*
sqrt(x))*sgn(x)/sqrt(x)
```

maple [A] time = 0.03, size = 68, normalized size = 0.38

$$\frac{\sqrt{\frac{a^2x^{\frac{3}{2}}+2abx+b^2\sqrt{x}}{x^{\frac{3}{2}}}}\,\left(a^3x^{\frac{3}{2}}+3a\,b^2\sqrt{x}\,\ln(x)+6a^2bx-2b^3\right)}{a\sqrt{x}\,+b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2+b^2/x+2*a*b/x^(1/2))^(3/2),x)
```

```
[Out] ((a^2*x^(3/2)+b^2*x^(1/2)+2*a*x*b)/x^(3/2))^(1/2)*(3*a*b^2*ln(x)*x^(1/2)+6*
a^2*b*x+x^(3/2)*a^3-2*b^3)/(a*x^(1/2)+b)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$a^3x+3\,ab^2\int\frac{1}{x}\,dx+6\,a^2b\sqrt{x}-\frac{2\,b^3}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+b^2/x+2*a*b/x^(1/2))^(3/2),x, algorithm="maxima")
```

```
[Out] a^3*x + 3*a*b^2*integrate(1/x, x) + 6*a^2*b*sqrt(x) - 2*b^3/sqrt(x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int{\left(a^2+\frac{b^2}{x}+\frac{2\,a\,b}{\sqrt{x}}\right)}^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2/x + (2*a*b)/x^(1/2))^(3/2),x)
```

```
[Out] int((a^2 + b^2/x + (2*a*b)/x^(1/2))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a^2+\frac{2ab}{\sqrt{x}}+\frac{b^2}{x}\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2+b**2/x+2*a*b/x**(1/2))**(3/2),x)
```

```
[Out] Integral((a**2 + 2*a*b/sqrt(x) + b**2/x)**(3/2), x)
```

3.418 $\int \left(a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}\right)^{7/2} dx$

Optimal. Leaf size=391

$$-\frac{3b^7\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}}{4x^{4/3}\left(a+\frac{b}{\sqrt[3]{x}}\right)}-\frac{7ab^6\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}}{x\left(a+\frac{b}{\sqrt[3]{x}}\right)}-\frac{63a^2b^5\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}}{2x^{2/3}\left(a+\frac{b}{\sqrt[3]{x}}\right)}+\frac{a^7x\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}}{a+\frac{b}{\sqrt[3]{x}}}+\frac{21a^6bx^{2/3}\sqrt{a^2}}{2(a+}$$

Rubi [A] time = 0.19, antiderivative size = 391, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {1341, 1355, 263, 43}

$$\frac{a^7x\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}}{a+\frac{b}{\sqrt[3]{x}}}+\frac{21a^6bx^{2/3}\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}}{2\left(a+\frac{b}{\sqrt[3]{x}}\right)}+\frac{63a^5b^2\sqrt[3]{x}\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}}{a+\frac{b}{\sqrt[3]{x}}}-\frac{105a^3b^4\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}}{\sqrt[3]{x}\left(a+\frac{b}{\sqrt[3]{x}}\right)}-\frac{63a^2b^5\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}}{2x^{2/3}\left(a+\frac{b}{\sqrt[3]{x}}\right)}-\frac{7ab^6\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}}{x\left(a+\frac{b}{\sqrt[3]{x}}\right)}-\frac{3b^7\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}}{4x^{4/3}\left(a+\frac{b}{\sqrt[3]{x}}\right)}+\frac{105a^4b^3\log\left(\sqrt[3]{x}\right)\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}}{a+\frac{b}{\sqrt[3]{x}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3))^(7/2),x]
```

```
[Out] (-3*b^7*Sqrt[a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3)])/(4*(a + b/x^(1/3))*x^(4/
3)) - (7*a*b^6*Sqrt[a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3)])/((a + b/x^(1/3))*
x) - (63*a^2*b^5*Sqrt[a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3)])/(2*(a + b/x^(1/
3))*x^(2/3)) - (105*a^3*b^4*Sqrt[a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3)])/((a
+ b/x^(1/3))*x^(1/3)) + (63*a^5*b^2*Sqrt[a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3
)]*x^(1/3))/(a + b/x^(1/3)) + (21*a^6*b*Sqrt[a^2 + b^2/x^(2/3) + (2*a*b)/x^
(1/3)]*x^(2/3))/(2*(a + b/x^(1/3))) + (a^7*Sqrt[a^2 + b^2/x^(2/3) + (2*a*b)
/x^(1/3)]*x)/(a + b/x^(1/3)) + (105*a^4*b^3*Sqrt[a^2 + b^2/x^(2/3) + (2*a*b
)/x^(1/3)]*Log[x^(1/3)])/(a + b/x^(1/3))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 1341

```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k
 = Denominator[n]}, Dist[k, Subst[Int[x^(k - 1)*(a + b*x^(k*n) + c*x^(2*k*n
))^p, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && EqQ[n2, 2*n] && Fra
ctionQ[n]
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\int \left(a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}\right)^{7/2} dx = 3\,\text{Subst}\left(\int \left(a^2+\frac{b^2}{x^2}+\frac{2ab}{x}\right)^{7/2} x^2\,dx, x, \sqrt[3]{x}\right)$$
$$= \frac{\left(3\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}\right)\text{Subst}\left(\int \left(ab+\frac{b^2}{x}\right)^7 x^2\,dx, x, \sqrt[3]{x}\right)}{b^6\left(ab+\frac{b^2}{\sqrt[3]{x}}\right)}$$
$$= \frac{\left(3\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}\right)\text{Subst}\left(\int \frac{(b^2+abx)^7}{x^5}\,dx, x, \sqrt[3]{x}\right)}{b^6\left(ab+\frac{b^2}{\sqrt[3]{x}}\right)}$$
$$= \frac{\left(3\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}\right)\text{Subst}\left(\int \left(21a^5b^9+\frac{b^{14}}{x^5}+\frac{7ab^{13}}{x^4}+\frac{21a^2b^{12}}{x^3}+\frac{35a^3b^{11}}{x^2}+\frac{35a^4b^{10}}{x}\right.\right.}{b^6\left(ab+\frac{b^2}{\sqrt[3]{x}}\right)}$$
$$= -\frac{3b^8\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}}{4\left(ab+\frac{b^2}{\sqrt[3]{x}}\right)x^{4/3}} - \frac{7ab^7\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}}{\left(ab+\frac{b^2}{\sqrt[3]{x}}\right)x} - \frac{63a^2b^6\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}}{2\left(ab+\frac{b^2}{\sqrt[3]{x}}\right)x^{2/3}} - \cdots$$

Mathematica [A] time = 0.07, size = 125, normalized size = 0.32

$$\frac{\sqrt{\frac{\left(a\sqrt[3]{x}+b\right)^2}{x^{2/3}}}\left(4a^7x^{7/3}+42a^6bx^2+252a^5b^2x^{5/3}+140a^4b^3x^{4/3}\log(x)-420a^3b^4x-126a^2b^5x^{2/3}-28ab^6\sqrt[3]{x}-3b^7\right)}{4x\left(a\sqrt[3]{x}+b\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3))^(7/2),x]
```

```
[Out] (Sqrt[(b + a*x^(1/3))^2/x^(2/3)]*(-3*b^7 - 28*a*b^6*x^(1/3) - 126*a^2*b^5*x
^(2/3) - 420*a^3*b^4*x + 252*a^5*b^2*x^(5/3) + 42*a^6*b*x^2 + 4*a^7*x^(7/3)
 + 140*a^4*b^3*x^(4/3)*Log[x]))/(4*(b + a*x^(1/3))*x)
```

IntegrateAlgebraic [A] time = 6.54, size = 133, normalized size = 0.34

$$\frac{\sqrt[3]{x}\sqrt{\frac{\left(a\sqrt[3]{x}+b\right)^2}{x^{2/3}}}\left(105a^4b^3\log\left(\sqrt[3]{x}\right)+\frac{4a^7x^{7/3}+42a^6bx^2+252a^5b^2x^{5/3}-420a^3b^4x-126a^2b^5x^{2/3}-28ab^6\sqrt[3]{x}-3b^7}{4x^{4/3}}\right)}{a\sqrt[3]{x}+b}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3))^(7/2),x]
```

```
[Out] (Sqrt[(b + a*x^(1/3))^2/x^(2/3)]*x^(1/3)*((-3*b^7 - 28*a*b^6*x^(1/3) - 126*
a^2*b^5*x^(2/3) - 420*a^3*b^4*x + 252*a^5*b^2*x^(5/3) + 42*a^6*b*x^2 + 4*a^
7*x^(7/3))/(4*x^(4/3)) + 105*a^4*b^3*Log[x^(1/3)]))/(b + a*x^(1/3))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+b^2/x^(2/3)+2*a*b/x^(1/3))^(7/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [A] time = 0.54, size = 173, normalized size = 0.44

$$a^{7}x\operatorname{sgn}\left(ax+bx^{\frac{2}{3}}\right)\operatorname{sgn}(x)+35\,a^{4}b^{3}\log\left(|x|\right)\operatorname{sgn}\left(ax+bx^{\frac{2}{3}}\right)\operatorname{sgn}(x)+\frac{21}{2}\,a^{6}bx^{\frac{2}{3}}\operatorname{sgn}\left(ax+bx^{\frac{2}{3}}\right)\operatorname{sgn}(x)+63\,a^{5}b^{2}x^{\frac{1}{3}}\operatorname{sgn}\left(ax+bx^{\frac{2}{3}}\right)\operatorname{sgn}(x)-\frac{420\,a^{3}b^{4}x\operatorname{sgn}\left(ax+bx^{\frac{2}{3}}\right)\operatorname{sgn}(x)+126\,a^{2}b^{5}x^{\frac{2}{3}}\operatorname{sgn}\left(ax+bx^{\frac{2}{3}}\right)\operatorname{sgn}(x)+28\,ab^{6}x^{\frac{1}{3}}\operatorname{sgn}\left(ax+bx^{\frac{2}{3}}\right)\operatorname{sgn}(x)+3\,b^{7}\operatorname{sgn}\left(ax+bx^{\frac{2}{3}}\right)\operatorname{sgn}(x)}{4\,x^{\frac{4}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+b^2/x^(2/3)+2*a*b/x^(1/3))^(7/2),x, algorithm="giac")
```

```
[Out] a^7*x*sgn(a*x + b*x^(2/3))*sgn(x) + 35*a^4*b^3*log(abs(x))*sgn(a*x + b*x^(2
/3))*sgn(x) + 21/2*a^6*b*x^(2/3)*sgn(a*x + b*x^(2/3))*sgn(x) + 63*a^5*b^2*x
^(1/3)*sgn(a*x + b*x^(2/3))*sgn(x) - 1/4*(420*a^3*b^4*x*sgn(a*x + b*x^(2/3)
)*sgn(x) + 126*a^2*b^5*x^(2/3)*sgn(a*x + b*x^(2/3))*sgn(x) + 28*a*b^6*x^(1/
3)*sgn(a*x + b*x^(2/3))*sgn(x) + 3*b^7*sgn(a*x + b*x^(2/3))*sgn(x))/x^(4/3)
```

maple [A] time = 0.03, size = 115, normalized size = 0.29

$$\frac{\left(\frac{a^{2}x^{\frac{2}{3}}+2ab\,x^{\frac{1}{3}}+b^{2}}{x^{\frac{2}{3}}}\right)^{\frac{7}{2}}\left(4a^{7}x^{\frac{10}{3}}+140a^{4}b^{3}x^{\frac{7}{3}}\ln(x)+42a^{6}b\,x^{3}+252a^{5}b^{2}x^{\frac{8}{3}}-420a^{3}b^{4}x^{2}-126a^{2}b^{5}x^{\frac{5}{3}}-28a\,b^{6}x^{\frac{4}{3}}-3b^{7}x\right)}{4\left(a\,x^{\frac{1}{3}}+b\right)^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2+b^2/x^(2/3)+2*a*b/x^(1/3))^(7/2),x)
```

```
[Out] 1/4*((a^2*x^(2/3)+2*a*b*x^(1/3)+b^2)/x^(2/3))^(7/2)*(42*a^6*b*x^3+140*a^4*b
^3*ln(x)*x^(7/3)+252*a^5*b^2*x^(8/3)+4*a^7*x^(10/3)-28*a*b^6*x^(4/3)-420*a^
3*b^4*x^2-126*a^2*b^5*x^(5/3)-3*b^7*x)/(b+a*x^(1/3))^7
```

maxima [A] time = 0.60, size = 79, normalized size = 0.20

$$35\,a^{4}b^{3}\log(x)+\frac{4\,a^{7}x^{\frac{7}{3}}+42\,a^{6}bx^{2}+252\,a^{5}b^{2}x^{\frac{5}{3}}-420\,a^{3}b^{4}x-126\,a^{2}b^{5}x^{\frac{2}{3}}-28\,ab^{6}x^{\frac{1}{3}}-3\,b^{7}}{4\,x^{\frac{4}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+b^2/x^(2/3)+2*a*b/x^(1/3))^(7/2),x, algorithm="maxima")
```

```
[Out] 35*a^4*b^3*log(x) + 1/4*(4*a^7*x^(7/3) + 42*a^6*b*x^2 + 252*a^5*b^2*x^(5/3)
 - 420*a^3*b^4*x - 126*a^2*b^5*x^(2/3) - 28*a*b^6*x^(1/3) - 3*b^7)/x^(4/3)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int {\left(a^2+\frac{b^2}{x^{2/3}}+\frac{2\,a\,b}{x^{1/3}}\right)}^{7/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3))^(7/2),x)
```

```
[Out] int((a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3))^(7/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a^{2} + \frac{2ab}{\sqrt[3]{x}} + \frac{b^{2}}{x^{\frac{2}{3}}}\right)^{\frac{7}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2+b**2/x**(2/3)+2*a*b/x**(1/3))**(7/2),x)
```

```
[Out] Integral((a**2 + 2*a*b/x**(1/3) + b**2/x**(2/3))**(7/2), x)
```

3.419 $\int \left(a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}\right)^{5/2} dx$

Optimal. Leaf size=291

$$-\frac{3b^5\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}}{2x^{2/3}\left(a+\frac{b}{\sqrt[3]{x}}\right)}-\frac{15ab^4\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}}{\sqrt[3]{x}\left(a+\frac{b}{\sqrt[3]{x}}\right)}+\frac{30a^2b^3\log\left(\sqrt[3]{x}\right)\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}}{a+\frac{b}{\sqrt[3]{x}}}+\frac{a^5x\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}}{a+\frac{b}{\sqrt[3]{x}}}+$$

Rubi [A] time = 0.14, antiderivative size = 291, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {1341, 1355, 263, 43}

$$\frac{a^5x\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}}{a+\frac{b}{\sqrt[3]{x}}}+\frac{15a^4bx^{2/3}\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}}{2\left(a+\frac{b}{\sqrt[3]{x}}\right)}+\frac{30a^3b^2\sqrt[3]{x}\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}}{a+\frac{b}{\sqrt[3]{x}}}-\frac{15ab^4\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}}{\sqrt[3]{x}\left(a+\frac{b}{\sqrt[3]{x}}\right)}-\frac{3b^5\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}}{2x^{2/3}\left(a+\frac{b}{\sqrt[3]{x}}\right)}+\frac{30a^2b^3\log\left(\sqrt[3]{x}\right)\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}}{a+\frac{b}{\sqrt[3]{x}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3))^(5/2),x]
```

```
[Out] (-3*b^5*Sqrt[a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3)])/(2*(a + b/x^(1/3))*x^(2/
3)) - (15*a*b^4*Sqrt[a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3)])/((a + b/x^(1/3))
*x^(1/3)) + (30*a^3*b^2*Sqrt[a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3)]*x^(1/3))/
(a + b/x^(1/3)) + (15*a^4*b*Sqrt[a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3)]*x^(2/
3))/(2*(a + b/x^(1/3))) + (a^5*Sqrt[a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3)]*x)
/(a + b/x^(1/3)) + (30*a^2*b^3*Sqrt[a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3)]*Lo
g[x^(1/3)])/(a + b/x^(1/3))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 1341

```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k
 = Denominator[n]}, Dist[k, Subst[Int[x^(k - 1)*(a + b*x^(k*n) + c*x^(2*k*n
))^p, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && EqQ[n2, 2*n] && Fra
ctionQ[n]
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\int \left(a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}\right)^{5/2} dx = 3\,\text{Subst}\left(\int \left(a^2+\frac{b^2}{x^2}+\frac{2ab}{x}\right)^{5/2} x^2\,dx, x, \sqrt[3]{x}\right)$$

$$= \frac{\left(3\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}\right)\text{Subst}\left(\int \left(ab+\frac{b^2}{x}\right)^5 x^2\,dx, x, \sqrt[3]{x}\right)}{b^4\left(ab+\frac{b^2}{\sqrt[3]{x}}\right)}$$

$$= \frac{\left(3\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}\right)\text{Subst}\left(\int \frac{(b^2+abx)^5}{x^3}\,dx, x, \sqrt[3]{x}\right)}{b^4\left(ab+\frac{b^2}{\sqrt[3]{x}}\right)}$$

$$= \frac{\left(3\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}\right)\text{Subst}\left(\int \left(10a^3b^7+\frac{b^{10}}{x^3}+\frac{5ab^9}{x^2}+\frac{10a^2b^8}{x}+5a^4b^6x+a^5b^5x^2\right) dx\right.}{b^4\left(ab+\frac{b^2}{\sqrt[3]{x}}\right)}$$

$$= -\frac{3b^6\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}}{2\left(ab+\frac{b^2}{\sqrt[3]{x}}\right)x^{2/3}} - \frac{15ab^5\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}}{\left(ab+\frac{b^2}{\sqrt[3]{x}}\right)\sqrt[3]{x}} + \frac{30a^3b^3\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}\sqrt[3]{x}}{ab+\frac{b^2}{\sqrt[3]{x}}} +$$

Mathematica [A] time = 0.06, size = 99, normalized size = 0.34

$$\frac{\left(a\sqrt[3]{x}+b\right)\left(2a^5x^{5/3}+15a^4bx^{4/3}+60a^3b^2x+20a^2b^3x^{2/3}\log(x)-30ab^4\sqrt[3]{x}-3b^5\right)}{2x\sqrt{\frac{\left(a\sqrt[3]{x}+b\right)^2}{x^{2/3}}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3))^(5/2),x]
```

```
[Out] ((b + a*x^(1/3))*(-3*b^5 - 30*a*b^4*x^(1/3) + 60*a^3*b^2*x + 15*a^4*b*x^(4/
3) + 2*a^5*x^(5/3) + 20*a^2*b^3*x^(2/3)*Log[x]))/(2*Sqrt[(b + a*x^(1/3))^2/
x^(2/3)]*x)
```

IntegrateAlgebraic [A] time = 6.28, size = 109, normalized size = 0.37

$$\frac{\sqrt[3]{x}\sqrt{\frac{\left(a\sqrt[3]{x}+b\right)^2}{x^{2/3}}}\left(30a^2b^3\log\left(\sqrt[3]{x}\right)+\frac{2a^5x^{5/3}+15a^4bx^{4/3}+60a^3b^2x-30ab^4\sqrt[3]{x}-3b^5}{2x^{2/3}}\right)}{a\sqrt[3]{x}+b}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3))^(5/2),x]
```

```
[Out] (Sqrt[(b + a*x^(1/3))^2/x^(2/3)]*x^(1/3)*((-3*b^5 - 30*a*b^4*x^(1/3) + 60*a
^3*b^2*x + 15*a^4*b*x^(4/3) + 2*a^5*x^(5/3))/(2*x^(2/3)) + 30*a^2*b^3*Log[x
^(1/3)]))/(b + a*x^(1/3))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+b^2/x^(2/3)+2*a*b/x^(1/3))^(5/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [A] time = 0.51, size = 128, normalized size = 0.44

$$a^5 x\operatorname{sgn}\left(ax + bx^{\frac{2}{3}}\right)\operatorname{sgn}(x) + 10\,a^2b^3\log\left(|x|\right)\operatorname{sgn}\left(ax + bx^{\frac{2}{3}}\right)\operatorname{sgn}(x) + \frac{15}{2}\,a^4bx^{\frac{2}{3}}\operatorname{sgn}\left(ax + bx^{\frac{2}{3}}\right)\operatorname{sgn}(x) + 30\,a^3b^2x^{\frac{1}{3}}\operatorname{sgn}\left(ax + bx^{\frac{2}{3}}\right)\operatorname{sgn}(x) - \frac{3\left(10\,ab^4x^{\frac{1}{3}}\operatorname{sgn}\left(ax + bx^{\frac{2}{3}}\right)\operatorname{sgn}(x) + b^5\operatorname{sgn}\left(ax + bx^{\frac{2}{3}}\right)\operatorname{sgn}(x)\right)}{2\,x^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+b^2/x^(2/3)+2*a*b/x^(1/3))^(5/2),x, algorithm="giac")
```

```
[Out] a^5*x*sgn(a*x + b*x^(2/3))*sgn(x) + 10*a^2*b^3*log(abs(x))*sgn(a*x + b*x^(2
/3))*sgn(x) + 15/2*a^4*b*x^(2/3)*sgn(a*x + b*x^(2/3))*sgn(x) + 30*a^3*b^2*x
^(1/3)*sgn(a*x + b*x^(2/3))*sgn(x) - 3/2*(10*a*b^4*x^(1/3)*sgn(a*x + b*x^(2
/3))*sgn(x) + b^5*sgn(a*x + b*x^(2/3))*sgn(x))/x^(2/3)
```

maple [A] time = 0.01, size = 91, normalized size = 0.31

$$\frac{\left(\frac{a^2x^{\frac{2}{3}}+2ab\,x^{\frac{1}{3}}+b^2}{x^{\frac{2}{3}}}\right)^{\frac{5}{2}}\left(2a^5x^{\frac{5}{3}}+20a^2b^3x^{\frac{2}{3}}\ln(x)+15a^4b\,x^{\frac{4}{3}}+60a^3b^2x-30a\,b^4x^{\frac{1}{3}}-3b^5\right)x}{2\left(a\,x^{\frac{1}{3}}+b\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2+b^2/x^(2/3)+2*a*b/x^(1/3))^(5/2),x)
```

```
[Out] 1/2*((a^2*x^(2/3)+2*a*b*x^(1/3)+b^2)/x^(2/3))^(5/2)*x*(15*a^4*b*x^(4/3)+60*
a^3*b^2*x+20*a^2*b^3*ln(x)*x^(2/3)+2*a^5*x^(5/3)-30*a*b^4*x^(1/3)-3*b^5)/(a
*x^(1/3)+b)^5
```

maxima [A] time = 0.64, size = 57, normalized size = 0.20

$$10\,a^2b^3\log(x)+\frac{2\,a^5x^{\frac{5}{3}}+15\,a^4bx^{\frac{4}{3}}+60\,a^3b^2x-30\,ab^4x^{\frac{1}{3}}-3\,b^5}{2\,x^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+b^2/x^(2/3)+2*a*b/x^(1/3))^(5/2),x, algorithm="maxima")
```

```
[Out] 10*a^2*b^3*log(x) + 1/2*(2*a^5*x^(5/3) + 15*a^4*b*x^(4/3) + 60*a^3*b^2*x -
30*a*b^4*x^(1/3) - 3*b^5)/x^(2/3)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int {\left(a^2+\frac{b^2}{x^{2/3}}+\frac{2\,a\,b}{x^{1/3}}\right)}^{5/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3))^(5/2),x)
```

```
[Out] int((a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3))^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a^{2} + \frac{2ab}{\sqrt[3]{x}} + \frac{b^{2}}{x^{\frac{2}{3}}}\right)^{\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2+b**2/x**(2/3)+2*a*b/x**(1/3))**(5/2),x)
```

```
[Out] Integral((a**2 + 2*a*b/x**(1/3) + b**2/x**(2/3))**(5/2), x)
```

3.420 $\int \left(a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}\right)^{3/2} dx$

Optimal. Leaf size=189

$$\frac{9a^2bx^{2/3}\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}}{2\left(a+\frac{b}{\sqrt[3]{x}}\right)}+\frac{9ab^2\sqrt[3]{x}\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}}{a+\frac{b}{\sqrt[3]{x}}}+\frac{3b^3\log\left(\sqrt[3]{x}\right)\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}}{a+\frac{b}{\sqrt[3]{x}}}+\frac{a^3x\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}}{a+\frac{b}{\sqrt[3]{x}}}$$

Rubi [A] time = 0.09, antiderivative size = 189, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {1341, 1355, 263, 43}

$$\frac{a^3x\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}}{a+\frac{b}{\sqrt[3]{x}}}+\frac{9a^2bx^{2/3}\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}}{2\left(a+\frac{b}{\sqrt[3]{x}}\right)}+\frac{9ab^2\sqrt[3]{x}\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}}{a+\frac{b}{\sqrt[3]{x}}}+\frac{3b^3\log\left(\sqrt[3]{x}\right)\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}}{a+\frac{b}{\sqrt[3]{x}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3))^(3/2),x]
```

```
[Out] (9*a*b^2*Sqrt[a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3)]*x^(1/3))/(a + b/x^(1/3))
 + (9*a^2*b*Sqrt[a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3)]*x^(2/3))/(2*(a + b/x^
(1/3))) + (a^3*Sqrt[a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3)]*x)/(a + b/x^(1/3))
 + (3*b^3*Sqrt[a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3)]*Log[x^(1/3)])/(a + b/x^
(1/3))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 1341

```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k
 = Denominator[n]}, Dist[k, Subst[Int[x^(k - 1)*(a + b*x^(k*n) + c*x^(2*k*n
))^p, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && EqQ[n2, 2*n] && Fra
ctionQ[n]
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\int \left(a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}\right)^{3/2} dx = 3\,\text{Subst}\left(\int \left(a^2+\frac{b^2}{x^2}+\frac{2ab}{x}\right)^{3/2} x^2\,dx, x, \sqrt[3]{x}\right)$$
$$= \frac{\left(3\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}\right)\text{Subst}\left(\int \left(ab+\frac{b^2}{x}\right)^3 x^2\,dx, x, \sqrt[3]{x}\right)}{b^2\left(ab+\frac{b^2}{\sqrt[3]{x}}\right)}$$
$$= \frac{\left(3\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}\right)\text{Subst}\left(\int \frac{\left(b^2+abx\right)^3}{x}\,dx, x, \sqrt[3]{x}\right)}{b^2\left(ab+\frac{b^2}{\sqrt[3]{x}}\right)}$$
$$= \frac{\left(3\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}\right)\text{Subst}\left(\int \left(3ab^5+\frac{b^6}{x}+3a^2b^4x+a^3b^3x^2\right)dx, x, \sqrt[3]{x}\right)}{b^2\left(ab+\frac{b^2}{\sqrt[3]{x}}\right)}$$
$$= \frac{9ab^3\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}\sqrt[3]{x}}{ab+\frac{b^2}{\sqrt[3]{x}}} + \frac{9a^2b^2\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}\,x^{2/3}}{2\left(ab+\frac{b^2}{\sqrt[3]{x}}\right)} + \frac{a^3\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}\,x}{a+\frac{b}{\sqrt[3]{x}}}$$

Mathematica [A] time = 0.03, size = 77, normalized size = 0.41

$$\frac{\sqrt{\frac{\left(a\sqrt[3]{x}+b\right)^2}{x^{2/3}}}\left(2a^3x^{4/3}+9a^2bx+18ab^2x^{2/3}+2b^3\sqrt[3]{x}\log(x)\right)}{2\left(a\sqrt[3]{x}+b\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3))^(3/2),x]
```

```
[Out] (Sqrt[(b + a*x^(1/3))^2/x^(2/3)]*(18*a*b^2*x^(2/3) + 9*a^2*b*x + 2*a^3*x^(4
/3) + 2*b^3*x^(1/3)*Log[x]))/(2*(b + a*x^(1/3)))
```

IntegrateAlgebraic [A] time = 5.98, size = 85, normalized size = 0.45

$$\frac{\sqrt[3]{x}\sqrt{\frac{\left(a\sqrt[3]{x}+b\right)^2}{x^{2/3}}}\left(\frac{1}{2}a\sqrt[3]{x}\left(2a^2x^{2/3}+9ab\sqrt[3]{x}+18b^2\right)+3b^3\log\left(\sqrt[3]{x}\right)\right)}{a\sqrt[3]{x}+b}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3))^(3/2),x]
```

```
[Out] (Sqrt[(b + a*x^(1/3))^2/x^(2/3)]*x^(1/3)*((a*(18*b^2 + 9*a*b*x^(1/3) + 2*a^
2*x^(2/3))*x^(1/3))/2 + 3*b^3*Log[x^(1/3)]))/(b + a*x^(1/3))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+b^2/x^(2/3)+2*a*b/x^(1/3))^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [A] time = 0.41, size = 79, normalized size = 0.42

$$a^3x\mathrm{sgn}\left(ax+bx^{\frac{2}{3}}\right)\mathrm{sgn}(x)+b^3\log\left(|x|\right)\mathrm{sgn}\left(ax+bx^{\frac{2}{3}}\right)\mathrm{sgn}(x)+\frac{9}{2}a^2bx^{\frac{2}{3}}\mathrm{sgn}\left(ax+bx^{\frac{2}{3}}\right)\mathrm{sgn}(x)+9\,ab^2x^{\frac{1}{3}}\mathrm{sgn}\left(ax+bx^{\frac{2}{3}}\right)\mathrm{sgn}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+b^2/x^(2/3)+2*a*b/x^(1/3))^(3/2),x, algorithm="giac")
```

```
[Out] a^3*x*sgn(a*x + b*x^(2/3))*sgn(x) + b^3*log(abs(x))*sgn(a*x + b*x^(2/3))*sg
n(x) + 9/2*a^2*b*x^(2/3)*sgn(a*x + b*x^(2/3))*sgn(x) + 9*a*b^2*x^(1/3)*sgn(
a*x + b*x^(2/3))*sgn(x)
```

maple [A] time = 0.00, size = 69, normalized size = 0.37

$$\frac{\left(\frac{a^2x^{\frac{2}{3}}+2ab\,x^{\frac{1}{3}}+b^2}{x^{\frac{2}{3}}}\right)^{\frac{3}{2}}\left(2a^3x+2b^3\ln(x)+9a^2b\,x^{\frac{2}{3}}+18a\,b^2x^{\frac{1}{3}}\right)x}{2\left(a\,x^{\frac{1}{3}}+b\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2+b^2/x^(2/3)+2*a*b/x^(1/3))^(3/2),x)
```

```
[Out] 1/2*((a^2*x^(2/3)+2*a*b*x^(1/3)+b^2)/x^(2/3))^(3/2)*x*(9*a^2*b*x^(2/3)+18*a
*b^2*x^(1/3)+2*b^3*ln(x)+2*a^3*x)/(a*x^(1/3)+b)^3
```

maxima [A] time = 0.66, size = 30, normalized size = 0.16

$$a^3x+b^3\log(x)+\frac{9}{2}a^2bx^{\frac{2}{3}}+9\,ab^2x^{\frac{1}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+b^2/x^(2/3)+2*a*b/x^(1/3))^(3/2),x, algorithm="maxima")
```

```
[Out] a^3*x + b^3*log(x) + 9/2*a^2*b*x^(2/3) + 9*a*b^2*x^(1/3)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(a^2+\frac{b^2}{x^{2/3}}+\frac{2\,a\,b}{x^{1/3}}\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3))^(3/2),x)
```

```
[Out] int((a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{\frac{2}{3}}}\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2+b**2/x**(2/3)+2*a*b/x**(1/3))**(3/2),x)
```

```
[Out] Integral((a**2 + 2*a*b/x**(1/3) + b**2/x**(2/3))**(3/2), x)
```

3.421 $\int \sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}\,dx$

Optimal. Leaf size=88

$$\frac{3bx^{2/3}\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}}{2\left(a+\frac{b}{\sqrt[3]{x}}\right)}+\frac{ax\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}}{a+\frac{b}{\sqrt[3]{x}}}$$

Rubi [A] time = 0.05, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1341, 1355, 14}

$$\frac{3bx^{2/3}\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}}{2\left(a+\frac{b}{\sqrt[3]{x}}\right)}+\frac{ax\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}}{a+\frac{b}{\sqrt[3]{x}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3)],x]
```

```
[Out] (3*b*Sqrt[a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3)]*x^(2/3))/(2*(a + b/x^(1/3)))
 + (a*Sqrt[a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3)]*x)/(a + b/x^(1/3))
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 1341

```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k
 = Denominator[n]}, Dist[k, Subst[Int[x^(k - 1)*(a + b*x^(k*n) + c*x^(2*k*n
))^p, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && EqQ[n2, 2*n] && Fra
ctionQ[n]
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\int \sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}\,dx = 3\,\text{Subst}\left(\int \sqrt{a^2+\frac{b^2}{x^2}+\frac{2ab}{x}}\,x^2\,dx, x, \sqrt[3]{x}\right)$$
$$= \frac{\left(3\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}\right)\text{Subst}\left(\int \left(ab+\frac{b^2}{x}\right)x^2\,dx, x, \sqrt[3]{x}\right)}{ab+\frac{b^2}{\sqrt[3]{x}}}$$
$$= \frac{\left(3\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}\right)\text{Subst}\left(\int \left(b^2x+abx^2\right)dx, x, \sqrt[3]{x}\right)}{ab+\frac{b^2}{\sqrt[3]{x}}}$$
$$= \frac{3b^2\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}\,x^{2/3}}{2\left(ab+\frac{b^2}{\sqrt[3]{x}}\right)} + \frac{a\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}\,x}{a+\frac{b}{\sqrt[3]{x}}}$$

Mathematica [A] time = 0.01, size = 49, normalized size = 0.56

$$\frac{\sqrt{\frac{\left(a\sqrt[3]{x}+b\right)^2}{x^{2/3}}}\left(2ax^{4/3}+3bx\right)}{2\left(a\sqrt[3]{x}+b\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3)],x]
[Out] (Sqrt[(b + a*x^(1/3))^2/x^(2/3)]*(3*b*x + 2*a*x^(4/3)))/(2*(b + a*x^(1/3)))
```

IntegrateAlgebraic [A] time = 5.30, size = 54, normalized size = 0.61

$$\frac{\sqrt[3]{x}\sqrt{\frac{\left(a\sqrt[3]{x}+b\right)^2}{x^{2/3}}}\left(2ax+3bx^{2/3}\right)}{2\left(a\sqrt[3]{x}+b\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3)],x]
[Out] (Sqrt[(b + a*x^(1/3))^2/x^(2/3)]*x^(1/3)*(3*b*x^(2/3) + 2*a*x))/(2*(b + a*x
^(1/3)))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+b^2/x^(2/3)+2*a*b/x^(1/3))^(1/2),x, algorithm="fricas")
[Out] Timed out
```

giac [A] time = 0.40, size = 34, normalized size = 0.39

$$ax\text{sgn}\left(ax+bx^{\frac{2}{3}}\right)\text{sgn}(x)+\frac{3}{2}\,bx^{\frac{2}{3}}\text{sgn}\left(ax+bx^{\frac{2}{3}}\right)\text{sgn}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+b^2/x^(2/3)+2*a*b/x^(1/3))^(1/2),x, algorithm="giac")
[Out] a*x*sgn(a*x + b*x^(2/3))*sgn(x) + 3/2*b*x^(2/3)*sgn(a*x + b*x^(2/3))*sgn(x)
```

maple [A] time = 0.00, size = 50, normalized size = 0.57

$$\frac{\sqrt{\frac{a^{2} x^{\frac{2}{3}}+2 a b\, x^{\frac{1}{3}}+b^{2}}{x^{\frac{2}{3}}}}\, \left(2 a x+3 b\, x^{\frac{2}{3}}\right) x^{\frac{1}{3}}}{2 a\, x^{\frac{1}{3}}+2 b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2+b^2/x^(2/3)+2*a*b/x^(1/3))^(1/2),x)
[Out] 1/2*((a^2*x^(2/3)+2*a*b*x^(1/3)+b^2)/x^(2/3))^(1/2)*x^(1/3)*(3*x^(2/3)*b+2*
a*x)/(a*x^(1/3)+b)
```

maxima [A] time = 0.73, size = 10, normalized size = 0.11

$$a x + \frac{3}{2} \, b x^{\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+b^2/x^(2/3)+2*a*b/x^(1/3))^(1/2),x, algorithm="maxima")
[Out] a*x + 3/2*b*x^(2/3)
```

mupad [B] time = 1.43, size = 39, normalized size = 0.44

$$\frac{x\,\left(a+\frac{3\,b}{2\,x^{1/3}}\right)\,\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2\,a\,b}{x^{1/3}}}}{a+\frac{b}{x^{1/3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3))^(1/2),x)
[Out] (x*(a + (3*b)/(2*x^(1/3)))*(a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3))^(1/2))/(a
+ b/x^(1/3))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a^{2} + \frac{2 a b}{\sqrt[3]{x}} + \frac{b^{2}}{x^{\frac{2}{3}}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2+b**2/x**(2/3)+2*a*b/x**(1/3))**(1/2),x)
[Out] Integral(sqrt(a**2 + 2*a*b/x**(1/3) + b**2/x**(2/3)), x)
```

3.422 $\int \frac{1}{\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}}\,dx$

Optimal. Leaf size=190

$$-\frac{3bx^{2/3}\left(a+\frac{b}{\sqrt[3]{x}}\right)}{2a^2\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}}+\frac{x\left(a+\frac{b}{\sqrt[3]{x}}\right)}{a\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}}-\frac{3b^3\left(a+\frac{b}{\sqrt[3]{x}}\right)\log\left(a\sqrt[3]{x}+b\right)}{a^4\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}}+\frac{3b^2\sqrt[3]{x}\left(a+\frac{b}{\sqrt[3]{x}}\right)}{a^3\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}}$$

Rubi [A] time = 0.12, antiderivative size = 190, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {1341, 1355, 263, 43}

$$\frac{3b^2\sqrt[3]{x}\left(a+\frac{b}{\sqrt[3]{x}}\right)}{a^3\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}}-\frac{3bx^{2/3}\left(a+\frac{b}{\sqrt[3]{x}}\right)}{2a^2\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}}+\frac{x\left(a+\frac{b}{\sqrt[3]{x}}\right)}{a\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}}-\frac{3b^3\left(a+\frac{b}{\sqrt[3]{x}}\right)\log\left(a\sqrt[3]{x}+b\right)}{a^4\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3)],x]
```

```
[Out] (3*b^2*(a + b/x^(1/3))*x^(1/3))/(a^3*Sqrt[a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/
3)]) - (3*b*(a + b/x^(1/3))*x^(2/3))/(2*a^2*Sqrt[a^2 + b^2/x^(2/3) + (2*a*b
)/x^(1/3)]) + ((a + b/x^(1/3))*x)/(a*Sqrt[a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/
3)]) - (3*b^3*(a + b/x^(1/3))*Log[b + a*x^(1/3)])/(a^4*Sqrt[a^2 + b^2/x^(2/
3) + (2*a*b)/x^(1/3)])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 1341

```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k
 = Denominator[n]}, Dist[k, Subst[Int[x^(k - 1)*(a + b*x^(k*n) + c*x^(2*k*n
))^p, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && EqQ[n2, 2*n] && Fra
ctionQ[n]
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\int \frac{1}{\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}}\,dx = 3\,\text{Subst}\left(\int \frac{x^2}{\sqrt{a^2+\frac{b^2}{x^2}+\frac{2ab}{x}}}\,dx, x, \sqrt[3]{x}\right)$$

$$= \frac{\left(3\left(ab+\frac{b^2}{\sqrt[3]{x}}\right)\right)\text{Subst}\left(\int \frac{x^2}{ab+\frac{b^2}{x}}\,dx, x, \sqrt[3]{x}\right)}{\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}}$$

$$= \frac{\left(3\left(ab+\frac{b^2}{\sqrt[3]{x}}\right)\right)\text{Subst}\left(\int \frac{x^3}{b^2+abx}\,dx, x, \sqrt[3]{x}\right)}{\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}}$$

$$= \frac{\left(3\left(ab+\frac{b^2}{\sqrt[3]{x}}\right)\right)\text{Subst}\left(\int \left(\frac{b}{a^3}-\frac{x}{a^2}+\frac{x^2}{ab}-\frac{b^2}{a^3(b+ax)}\right)dx, x, \sqrt[3]{x}\right)}{\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}}$$

$$= \frac{3\left(ab^2+\frac{b^3}{\sqrt[3]{x}}\right)\sqrt[3]{x}}{a^3\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}} - \frac{3\left(ab+\frac{b^2}{\sqrt[3]{x}}\right)x^{2/3}}{2a^2\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}} + \frac{\left(a+\frac{b}{\sqrt[3]{x}}\right)x}{a\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}} - \frac{3\left(ab^3+\frac{b^4}{\sqrt[3]{x}}\right)\text{l}}{a^4\sqrt{a^2+}}$$

Mathematica [A] time = 0.04, size = 86, normalized size = 0.45

$$\frac{\left(a\sqrt[3]{x}+b\right)\left(2a^3x-3a^2bx^{2/3}-6b^3\log\left(a\sqrt[3]{x}+b\right)+6ab^2\sqrt[3]{x}\right)}{2a^4\sqrt[3]{x}\sqrt{\frac{\left(a\sqrt[3]{x}+b\right)^2}{x^{2/3}}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3)],x]
[Out] ((b + a*x^(1/3))*(6*a*b^2*x^(1/3) - 3*a^2*b*x^(2/3) + 2*a^3*x - 6*b^3*Log[b
 + a*x^(1/3)]))/(2*a^4*Sqrt[(b + a*x^(1/3))^2/x^(2/3)]*x^(1/3))
```

IntegrateAlgebraic [A] time = 4.80, size = 94, normalized size = 0.49

$$\frac{\sqrt[3]{x}\sqrt{\frac{\left(a\sqrt[3]{x}+b\right)^2}{x^{2/3}}}\left(\frac{\sqrt[3]{x}\left(2a^2x^{2/3}-3ab\sqrt[3]{x}+6b^2\right)}{2a^3}-\frac{3b^3\log\left(a\sqrt[3]{x}+b\right)}{a^4}\right)}{a\sqrt[3]{x}+b}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/Sqrt[a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3)],x]
[Out] (Sqrt[(b + a*x^(1/3))^2/x^(2/3)]*x^(1/3)*(((6*b^2 - 3*a*b*x^(1/3) + 2*a^2*x
^(2/3))*x^(1/3))/(2*a^3) - (3*b^3*Log[b + a*x^(1/3)])/a^4))/(b + a*x^(1/3))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2+b^2/x^(2/3)+2*a*b/x^(1/3))^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [A] time = 0.59, size = 77, normalized size = 0.41

$$-\frac{3\,b^3\log\left(\left|ax^{\frac{1}{3}}+b\right|\right)}{a^4\mathrm{sgn}\left(ax+bx^{\frac{2}{3}}\right)\mathrm{sgn}(x)}+\frac{2\,a^2x-3\,abx^{\frac{2}{3}}+6\,b^2x^{\frac{1}{3}}}{2\,a^3\mathrm{sgn}\left(ax+bx^{\frac{2}{3}}\right)\mathrm{sgn}(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2+b^2/x^(2/3)+2*a*b/x^(1/3))^(1/2),x, algorithm="giac")
```

```
[Out] -3*b^3*log(abs(a*x^(1/3) + b))/(a^4*sgn(a*x + b*x^(2/3))*sgn(x)) + 1/2*(2*a
^2*x - 3*a*b*x^(2/3) + 6*b^2*x^(1/3))/(a^3*sgn(a*x + b*x^(2/3))*sgn(x))
```

maple [A] time = 0.01, size = 78, normalized size = 0.41

$$-\frac{\left(a\,x^{\frac{1}{3}}+b\right)\left(-2a^3x+6b^3\ln\left(a\,x^{\frac{1}{3}}+b\right)+3a^2b\,x^{\frac{2}{3}}-6a\,b^2x^{\frac{1}{3}}\right)}{2\sqrt{\frac{a^2x^{\frac{2}{3}}+2ab\,x^{\frac{1}{3}}+b^2}{x^{\frac{2}{3}}}}\;a^4x^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a^2+b^2/x^(2/3)+2*a*b/x^(1/3))^(1/2),x)
```

```
[Out] -1/2/((a^2*x^(2/3)+2*a*b*x^(1/3)+b^2)/x^(2/3))^(1/2)/x^(1/3)*(a*x^(1/3)+b)*
(3*a^2*b*x^(2/3)-6*a*b^2*x^(1/3)+6*b^3*ln(a*x^(1/3)+b)-2*a^3*x)/a^4
```

maxima [A] time = 0.74, size = 44, normalized size = 0.23

$$-\frac{3\,b^3\log\left(ax^{\frac{1}{3}}+b\right)}{a^4}+\frac{2\,a^2x-3\,abx^{\frac{2}{3}}+6\,b^2x^{\frac{1}{3}}}{2\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2+b^2/x^(2/3)+2*a*b/x^(1/3))^(1/2),x, algorithm="maxima")
```

```
[Out] -3*b^3*log(a*x^(1/3) + b)/a^4 + 1/2*(2*a^2*x - 3*a*b*x^(2/3) + 6*b^2*x^(1/3
))/a^3
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2\,a\,b}{x^{1/3}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3))^(1/2),x)
```

```
[Out] int(1/(a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{\frac{2}{3}}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a**2+b**2/x**(2/3)+2*a*b/x**(1/3))**(1/2),x)
```

```
[Out] Integral(1/sqrt(a**2 + 2*a*b/x**(1/3) + b**2/x**(2/3)), x)
```

3.423 $\int \frac{1}{\left(a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}\right)^{3/2}}\,dx$

Optimal. Leaf size=300

$$\frac{3b^5\left(a+\frac{b}{\sqrt[3]{x}}\right)}{2a^6\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}\left(a\sqrt[3]{x}+b\right)^2}-\frac{15b^4\left(a+\frac{b}{\sqrt[3]{x}}\right)}{a^6\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}\left(a\sqrt[3]{x}+b\right)}-\frac{30b^3\left(a+\frac{b}{\sqrt[3]{x}}\right)\log\left(a\sqrt[3]{x}+b\right)}{a^6\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}}+\frac{18b^2\sqrt[3]{x}\,(}{a^5\sqrt{a^2+}}$$

Rubi [A] time = 0.19, antiderivative size = 300, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {1341, 1355, 263, 43}

$$\frac{3b^5\left(a+\frac{b}{\sqrt[3]{x}}\right)}{2a^6\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}\left(a\sqrt[3]{x}+b\right)^2}-\frac{15b^4\left(a+\frac{b}{\sqrt[3]{x}}\right)}{a^6\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}\left(a\sqrt[3]{x}+b\right)}+\frac{18b^2\sqrt[3]{x}\left(a+\frac{b}{\sqrt[3]{x}}\right)}{a^5\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}}-\frac{9bx^{2/3}\left(a+\frac{b}{\sqrt[3]{x}}\right)}{2a^4\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}}+\frac{x\left(a+\frac{b}{\sqrt[3]{x}}\right)}{a^3\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}}-\frac{30b^3\left(a+\frac{b}{\sqrt[3]{x}}\right)\log\left(a\sqrt[3]{x}+b\right)}{a^6\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3))^(-3/2),x]
```

```
[Out] (3*b^5*(a + b/x^(1/3)))/(2*a^6*Sqrt[a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3)]*(b
 + a*x^(1/3))^2) - (15*b^4*(a + b/x^(1/3)))/(a^6*Sqrt[a^2 + b^2/x^(2/3) + (
2*a*b)/x^(1/3)]*(b + a*x^(1/3))) + (18*b^2*(a + b/x^(1/3))*x^(1/3))/(a^5*Sq
rt[a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3)]) - (9*b*(a + b/x^(1/3))*x^(2/3))/(2
*a^4*Sqrt[a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3)]) + ((a + b/x^(1/3))*x)/(a^3*
Sqrt[a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3)]) - (30*b^3*(a + b/x^(1/3))*Log[b
+ a*x^(1/3)])/(a^6*Sqrt[a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3)])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 1341

```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k
 = Denominator[n]}, Dist[k, Subst[Int[x^(k - 1)*(a + b*x^(k*n) + c*x^(2*k*n
))^p, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && EqQ[n2, 2*n] && Fra
ctionQ[n]
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\int \frac{1}{\left(a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}\right)^{3/2}}\,dx = 3\,\text{Subst}\left(\int \frac{x^2}{\left(a^2+\frac{b^2}{x^2}+\frac{2ab}{x}\right)^{3/2}}\,dx, x, \sqrt[3]{x}\right)$$

$$= \frac{\left(3b^2\left(ab+\frac{b^2}{\sqrt[3]{x}}\right)\right)\text{Subst}\left(\int \frac{x^2}{\left(ab+\frac{b^2}{x}\right)^3}\,dx, x, \sqrt[3]{x}\right)}{\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}}$$

$$= \frac{\left(3b^2\left(ab+\frac{b^2}{\sqrt[3]{x}}\right)\right)\text{Subst}\left(\int \frac{x^5}{\left(b^2+abx\right)^3}\,dx, x, \sqrt[3]{x}\right)}{\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}}$$

$$= \frac{\left(3b^2\left(ab+\frac{b^2}{\sqrt[3]{x}}\right)\right)\text{Subst}\left(\int \left(\frac{6}{a^5b}-\frac{3x}{a^4b^2}+\frac{x^2}{a^3b^3}-\frac{b^2}{a^5(b+ax)^3}+\frac{5b}{a^5(b+ax)^2}-\frac{10}{a^5(b+ax)}\right)dx, x, \sqrt[3]{\ }\right.}{\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}}$$

$$= \frac{3\left(ab^5+\frac{b^6}{\sqrt[3]{x}}\right)}{2a^6\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}\left(b+a\sqrt[3]{x}\right)^2} - \frac{15\left(ab^4+\frac{b^5}{\sqrt[3]{x}}\right)}{a^6\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}\left(b+a\sqrt[3]{x}\right)} + \frac{18\left(ab^2+\frac{b^3}{\sqrt[3]{x}}\right.}{a^5\sqrt{a^2+\frac{b^2}{x^{2/3}}\ldots}}$$

Mathematica [A] time = 0.09, size = 126, normalized size = 0.42

$$\frac{\left(a\sqrt[3]{x}+b\right)\left(2a^5x^{5/3}-5a^4bx^{4/3}+20a^3b^2x+63a^2b^3x^{2/3}+6ab^4\sqrt[3]{x}-60b^3\left(a\sqrt[3]{x}+b\right)^2\log\left(a\sqrt[3]{x}+b\right)-27b^5\right)}{2a^6x\left(\frac{\left(a\sqrt[3]{x}+b\right)^2}{x^{2/3}}\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3))^(-3/2),x]
```

```
[Out] ((b + a*x^(1/3))*(-27*b^5 + 6*a*b^4*x^(1/3) + 63*a^2*b^3*x^(2/3) + 20*a^3*b
^2*x - 5*a^4*b*x^(4/3) + 2*a^5*x^(5/3) - 60*b^3*(b + a*x^(1/3))^2*Log[b + a
*x^(1/3)]))/(2*a^6*((b + a*x^(1/3))^2/x^(2/3))^(3/2)*x)
```

IntegrateAlgebraic [A] time = 6.44, size = 135, normalized size = 0.45

$$\frac{\sqrt[3]{x}\sqrt{\frac{\left(a\sqrt[3]{x}+b\right)^2}{x^{2/3}}}\left(\frac{2a^5x^{5/3}-5a^4bx^{4/3}+20a^3b^2x+63a^2b^3x^{2/3}+6ab^4\sqrt[3]{x}-27b^5}{2a^6\left(a\sqrt[3]{x}+b\right)^2}-\frac{30b^3\log\left(a\sqrt[3]{x}+b\right)}{a^6}\right)}{a\sqrt[3]{x}+b}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3))^(-3/2),x]
```

```
[Out] (Sqrt[(b + a*x^(1/3))^2/x^(2/3)]*x^(1/3)*((-27*b^5 + 6*a*b^4*x^(1/3) + 63*a
^2*b^3*x^(2/3) + 20*a^3*b^2*x - 5*a^4*b*x^(4/3) + 2*a^5*x^(5/3))/(2*a^6*(b
+ a*x^(1/3))^2) - (30*b^3*Log[b + a*x^(1/3)])/a^6))/(b + a*x^(1/3))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2+b^2/x^(2/3)+2*a*b/x^(1/3))^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [A] time = 0.66, size = 121, normalized size = 0.40

$$-\frac{30\,b^3\log\left(\left|ax^{\frac{1}{3}}+b\right|\right)}{a^6\mathrm{sgn}\left(ax^{\frac{2}{3}}+bx^{\frac{1}{3}}\right)}-\frac{3\left(10\,ab^4x^{\frac{1}{3}}+9\,b^5\right)}{2\left(ax^{\frac{1}{3}}+b\right)^2a^6\mathrm{sgn}\left(ax^{\frac{2}{3}}+bx^{\frac{1}{3}}\right)}+\frac{2\,a^6x-9\,a^5bx^{\frac{2}{3}}+36\,a^4b^2x^{\frac{1}{3}}}{2\,a^9\mathrm{sgn}\left(ax^{\frac{2}{3}}+bx^{\frac{1}{3}}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2+b^2/x^(2/3)+2*a*b/x^(1/3))^(3/2),x, algorithm="giac")
```

```
[Out] -30*b^3*log(abs(a*x^(1/3) + b))/(a^6*sgn(a*x^(2/3) + b*x^(1/3))) - 3/2*(10*
a*b^4*x^(1/3) + 9*b^5)/((a*x^(1/3) + b)^2*a^6*sgn(a*x^(2/3) + b*x^(1/3))) +
 1/2*(2*a^6*x - 9*a^5*b*x^(2/3) + 36*a^4*b^2*x^(1/3))/(a^9*sgn(a*x^(2/3) +
b*x^(1/3)))
```

maple [A] time = 0.01, size = 141, normalized size = 0.47

$$\frac{\left(2a^5x^{\frac{5}{3}}-60a^2b^3x^{\frac{2}{3}}\ln\left(a\,x^{\frac{1}{3}}+b\right)-5a^4b\,x^{\frac{4}{3}}-120a\,b^4x^{\frac{1}{3}}\ln\left(a\,x^{\frac{1}{3}}+b\right)+20a^3b^2x-60b^5\ln\left(a\,x^{\frac{1}{3}}+b\right)+63a^2b^3x^{\frac{2}{3}}+6a\,b^4x^{\frac{1}{3}}-27b^5\right)\left(a\,x^{\frac{1}{3}}+b\right)}{2\left(\frac{a^2x^{\frac{2}{3}}+2ab\,x^{\frac{1}{3}}+b^2}{x^{\frac{2}{3}}}\right)^{\frac{3}{2}}a^6x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a^2+b^2/x^(2/3)+2*a*b/x^(1/3))^(3/2),x)
```

```
[Out] 1/2/((a^2*x^(2/3)+2*a*b*x^(1/3)+b^2)/x^(2/3))^(3/2)/x*(2*a^5*x^(5/3)-5*a^4*
b*x^(4/3)-60*x^(2/3)*ln(a*x^(1/3)+b)*a^2*b^3+63*x^(2/3)*a^2*b^3-120*x^(1/3)
*ln(a*x^(1/3)+b)*a*b^4+6*a*b^4*x^(1/3)-60*ln(a*x^(1/3)+b)*b^5+20*a^3*b^2*x-
27*b^5)*(a*x^(1/3)+b)/a^6
```

maxima [A] time = 1.07, size = 97, normalized size = 0.32

$$\frac{2\,a^5x^{\frac{5}{3}}-5\,a^4bx^{\frac{4}{3}}+20\,a^3b^2x+63\,a^2b^3x^{\frac{2}{3}}+6\,ab^4x^{\frac{1}{3}}-27\,b^5}{2\left(a^8x^{\frac{2}{3}}+2\,a^7bx^{\frac{1}{3}}+a^6b^2\right)}-\frac{30\,b^3\log\left(ax^{\frac{1}{3}}+b\right)}{a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2+b^2/x^(2/3)+2*a*b/x^(1/3))^(3/2),x, algorithm="maxima")
```

```
[Out] 1/2*(2*a^5*x^(5/3) - 5*a^4*b*x^(4/3) + 20*a^3*b^2*x + 63*a^2*b^3*x^(2/3) +
6*a*b^4*x^(1/3) - 27*b^5)/(a^8*x^(2/3) + 2*a^7*b*x^(1/3) + a^6*b^2) - 30*b^
3*log(a*x^(1/3) + b)/a^6
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{{\left(a^2+\frac{b^2}{x^{2/3}}+\frac{2\,a\,b}{x^{1/3}}\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3))^(3/2),x)
```

```
[Out] int(1/(a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a^2 + \frac{2ab}{\sqrt[3]{x}} + \frac{b^2}{x^{\frac{2}{3}}}\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a**2+b**2/x**(2/3)+2*a*b/x**(1/3))**(3/2),x)
```

```
[Out] Integral((a**2 + 2*a*b/x**(1/3) + b**2/x**(2/3))**(-3/2), x)
```

3.424 $\int \frac{1}{\left(a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}\right)^{5/2}}\,dx$

Optimal. Leaf size=410

$$\frac{3b^7\left(a+\frac{b}{\sqrt[3]{x}}\right)}{4a^8\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}\left(a\sqrt[3]{x}+b\right)^4}-\frac{7b^6\left(a+\frac{b}{\sqrt[3]{x}}\right)}{a^8\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}\left(a\sqrt[3]{x}+b\right)^3}+\frac{63b^5\left(a+\frac{b}{\sqrt[3]{x}}\right)}{2a^8\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}\left(a\sqrt[3]{x}+b\right)^2}-\frac{\cdots}{a^8\sqrt{a\cdots}}$$

Rubi [A] time = 0.27, antiderivative size = 410, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {1341, 1355, 263, 43}

$$\frac{3b^7\left(a+\frac{b}{\sqrt[3]{x}}\right)}{4a^8\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}\left(a\sqrt[3]{x}+b\right)^4}-\frac{7b^6\left(a+\frac{b}{\sqrt[3]{x}}\right)}{a^8\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}\left(a\sqrt[3]{x}+b\right)^3}+\frac{63b^5\left(a+\frac{b}{\sqrt[3]{x}}\right)}{2a^8\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}\left(a\sqrt[3]{x}+b\right)^2}-\frac{105b^4\left(a+\frac{b}{\sqrt[3]{x}}\right)}{a^8\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}\left(a\sqrt[3]{x}+b\right)}+\frac{45b^2\sqrt[3]{x}\left(a+\frac{b}{\sqrt[3]{x}}\right)}{a^7\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}}-\frac{15bx^{2/3}\left(a+\frac{b}{\sqrt[3]{x}}\right)}{2a^6\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}}+\frac{x\left(a+\frac{b}{\sqrt[3]{x}}\right)}{a^5\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}}-\frac{105b^3\left(a+\frac{b}{\sqrt[3]{x}}\right)\log\left(a\sqrt[3]{x}+b\right)}{a^8\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3))^(-5/2),x]
```

```
[Out] (3*b^7*(a + b/x^(1/3)))/(4*a^8*Sqrt[a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3)]*(b
 + a*x^(1/3))^4) - (7*b^6*(a + b/x^(1/3)))/(a^8*Sqrt[a^2 + b^2/x^(2/3) + (2
*a*b)/x^(1/3)]*(b + a*x^(1/3))^3) + (63*b^5*(a + b/x^(1/3)))/(2*a^8*Sqrt[a^
2 + b^2/x^(2/3) + (2*a*b)/x^(1/3)]*(b + a*x^(1/3))^2) - (105*b^4*(a + b/x^(
1/3)))/(a^8*Sqrt[a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3)]*(b + a*x^(1/3))) + (4
5*b^2*(a + b/x^(1/3))*x^(1/3))/(a^7*Sqrt[a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3
)]) - (15*b*(a + b/x^(1/3))*x^(2/3))/(2*a^6*Sqrt[a^2 + b^2/x^(2/3) + (2*a*b
)/x^(1/3)]) + ((a + b/x^(1/3))*x)/(a^5*Sqrt[a^2 + b^2/x^(2/3) + (2*a*b)/x^(
1/3)]) - (105*b^3*(a + b/x^(1/3))*Log[b + a*x^(1/3)])/(a^8*Sqrt[a^2 + b^2/x
^(2/3) + (2*a*b)/x^(1/3)])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 1341

```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k
 = Denominator[n]}, Dist[k, Subst[Int[x^(k - 1)*(a + b*x^(k*n) + c*x^(2*k*n
))^p, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && EqQ[n2, 2*n] && Fra
ctionQ[n]
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\int \frac{1}{\left(a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}\right)^{5/2}}\,dx = 3\,\text{Subst}\left(\int \frac{x^2}{\left(a^2+\frac{b^2}{x^2}+\frac{2ab}{x}\right)^{5/2}}\,dx, x, \sqrt[3]{x}\right)$$

$$= \frac{\left(3b^4\left(ab+\frac{b^2}{\sqrt[3]{x}}\right)\right)\text{Subst}\left(\int \frac{x^2}{\left(ab+\frac{b^2}{x}\right)^5}\,dx, x, \sqrt[3]{x}\right)}{\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}}$$

$$= \frac{\left(3b^4\left(ab+\frac{b^2}{\sqrt[3]{x}}\right)\right)\text{Subst}\left(\int \frac{x^7}{\left(b^2+abx\right)^5}\,dx, x, \sqrt[3]{x}\right)}{\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}}$$

$$= \frac{\left(3b^4\left(ab+\frac{b^2}{\sqrt[3]{x}}\right)\right)\text{Subst}\left(\int \left(\frac{15}{a^7b^3}-\frac{5x}{a^6b^4}+\frac{x^2}{a^5b^5}-\frac{b^2}{a^7(b+ax)^5}+\frac{7b}{a^7(b+ax)^4}-\frac{21}{a^7(b+ax)^3}+\frac{3}{a^7b(b\cdots}\right.\right.}{\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}}$$

$$= \frac{3\left(ab^7+\frac{b^8}{\sqrt[3]{x}}\right)}{4a^8\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}\left(b+a\sqrt[3]{x}\right)^4} - \frac{7\left(ab^6+\frac{b^7}{\sqrt[3]{x}}\right)}{a^8\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}\left(b+a\sqrt[3]{x}\right)^3} + \frac{63\left(\cdots\right.}{2a^8\sqrt{a^2+\frac{b^2}{x^{2\cdots}}}}$$

Mathematica [A] time = 0.13, size = 152, normalized size = 0.37

$$\frac{\left(a\sqrt[3]{x}+b\right)\left(4a^7x^{7/3}-14a^6bx^2+84a^5b^2x^{5/3}+556a^4b^3x^{4/3}+544a^3b^4x-444a^2b^5x^{2/3}-856ab^6\sqrt[3]{x}-420b^3\left(a\sqrt[3]{x}+b\right)^4\log\left(a\sqrt[3]{x}+b\right)-319b^7\right)}{4a^8x^{5/3}\left(\frac{\left(a\sqrt[3]{x}+b\right)^2}{x^{2/3}}\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3))^(-5/2),x]
```

```
[Out] ((b + a*x^(1/3))*(-319*b^7 - 856*a*b^6*x^(1/3) - 444*a^2*b^5*x^(2/3) + 544*
a^3*b^4*x + 556*a^4*b^3*x^(4/3) + 84*a^5*b^2*x^(5/3) - 14*a^6*b*x^2 + 4*a^7
*x^(7/3) - 420*b^3*(b + a*x^(1/3))^4*Log[b + a*x^(1/3)]))/(4*a^8*((b + a*x^
(1/3))^2/x^(2/3))^(5/2)*x^(5/3))
```

IntegrateAlgebraic [A] time = 9.55, size = 159, normalized size = 0.39

$$\frac{\sqrt[3]{x}\sqrt{\frac{\left(a\sqrt[3]{x}+b\right)^2}{x^{2/3}}}\left(\frac{4a^7x^{7/3}-14a^6bx^2+84a^5b^2x^{5/3}+556a^4b^3x^{4/3}+544a^3b^4x-444a^2b^5x^{2/3}-856ab^6\sqrt[3]{x}-319b^7}{4a^8\left(a\sqrt[3]{x}+b\right)^4}-\frac{105b^3\log\left(a\sqrt[3]{x}+b\right)}{a^8}\right)}{a\sqrt[3]{x}+b}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3))^(-5/2),x]
```

```
[Out] (Sqrt[(b + a*x^(1/3))^2/x^(2/3)]*x^(1/3)*((-319*b^7 - 856*a*b^6*x^(1/3) - 4
44*a^2*b^5*x^(2/3) + 544*a^3*b^4*x + 556*a^4*b^3*x^(4/3) + 84*a^5*b^2*x^(5/
3) - 14*a^6*b*x^2 + 4*a^7*x^(7/3))/(4*a^8*(b + a*x^(1/3))^4) - (105*b^3*Log
[b + a*x^(1/3)])/a^8))/(b + a*x^(1/3))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2+b^2/x^(2/3)+2*a*b/x^(1/3))^(5/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [A] time = 0.72, size = 141, normalized size = 0.34

$$-\frac{105\,b^3\log\left(\left|ax^{\frac{1}{3}}+b\right|\right)}{a^8\mathrm{sgn}\left(ax^{\frac{2}{3}}+bx^{\frac{1}{3}}\right)}-\frac{420\,a^3b^4x+1134\,a^2b^5x^{\frac{2}{3}}+1036\,ab^6x^{\frac{1}{3}}+319\,b^7}{4\left(ax^{\frac{1}{3}}+b\right)^4a^8\mathrm{sgn}\left(ax^{\frac{2}{3}}+bx^{\frac{1}{3}}\right)}+\frac{2\,a^{10}x-15\,a^9bx^{\frac{2}{3}}+90\,a^8b^2x^{\frac{1}{3}}}{2\,a^{15}\mathrm{sgn}\left(ax^{\frac{2}{3}}+bx^{\frac{1}{3}}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2+b^2/x^(2/3)+2*a*b/x^(1/3))^(5/2),x, algorithm="giac")
```

```
[Out] -105*b^3*log(abs(a*x^(1/3) + b))/(a^8*sgn(a*x^(2/3) + b*x^(1/3))) - 1/4*(42
0*a^3*b^4*x + 1134*a^2*b^5*x^(2/3) + 1036*a*b^6*x^(1/3) + 319*b^7)/((a*x^(1
/3) + b)^4*a^8*sgn(a*x^(2/3) + b*x^(1/3))) + 1/2*(2*a^10*x - 15*a^9*b*x^(2/
3) + 90*a^8*b^2*x^(1/3))/(a^15*sgn(a*x^(2/3) + b*x^(1/3)))
```

maple [A] time = 0.01, size = 199, normalized size = 0.49

$$\frac{\left(4a^7x^{\frac{7}{3}}-420a^4b^3x^{\frac{4}{3}}\ln\left(ax^{\frac{1}{3}}+b\right)-14a^6bx^2-1680a^3b^4x\ln\left(ax^{\frac{1}{3}}+b\right)+84a^5b^2x^{\frac{5}{3}}-2520a^2b^5x^{\frac{2}{3}}\ln\left(ax^{\frac{1}{3}}+b\right)+556a^4b^3x^{\frac{4}{3}}-1680ab^6x^{\frac{1}{3}}\ln\left(ax^{\frac{1}{3}}+b\right)+544a^3b^4x-420b^7\ln\left(ax^{\frac{1}{3}}+b\right)-444a^2b^5x^{\frac{2}{3}}-856ab^6x^{\frac{1}{3}}-319b^7\right)\left(ax^{\frac{1}{3}}+b\right)}{4\left(\frac{a^2x^{\frac{2}{3}}+2abx^{\frac{1}{3}}+b^2}{x^{\frac{2}{3}}}\right)^{\frac{5}{2}}a^8x^{\frac{5}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a^2+b^2/x^(2/3)+2*a*b/x^(1/3))^(5/2),x)
```

```
[Out] 1/4/((a^2*x^(2/3)+2*a*b*x^(1/3)+b^2)/x^(2/3))^(5/2)/x^(5/3)*(4*x^(7/3)*a^7+
84*a^5*b^2*x^(5/3)-420*x^(4/3)*ln(a*x^(1/3)+b)*a^4*b^3+556*x^(4/3)*a^4*b^3-
2520*x^(2/3)*ln(a*x^(1/3)+b)*a^2*b^5-444*x^(2/3)*a^2*b^5-1680*x^(1/3)*ln(a*
x^(1/3)+b)*a*b^6-1680*x*ln(a*x^(1/3)+b)*a^3*b^4-14*a^6*b*x^2-856*x^(1/3)*a*
b^6-420*ln(a*x^(1/3)+b)*b^7+544*x*a^3*b^4-319*b^7)*(a*x^(1/3)+b)/a^8
```

maxima [A] time = 0.85, size = 139, normalized size = 0.34

$$\frac{4\,a^7x^{\frac{7}{3}}-14\,a^6bx^2+84\,a^5b^2x^{\frac{5}{3}}+556\,a^4b^3x^{\frac{4}{3}}+544\,a^3b^4x-444\,a^2b^5x^{\frac{2}{3}}-856\,ab^6x^{\frac{1}{3}}-319\,b^7}{4\left(a^{12}x^{\frac{4}{3}}+4\,a^{11}bx+6\,a^{10}b^2x^{\frac{2}{3}}+4\,a^9b^3x^{\frac{1}{3}}+a^8b^4\right)}-\frac{105\,b^3\log\left(ax^{\frac{1}{3}}+b\right)}{a^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2+b^2/x^(2/3)+2*a*b/x^(1/3))^(5/2),x, algorithm="maxima")
```

```
[Out] 1/4*(4*a^7*x^(7/3) - 14*a^6*b*x^2 + 84*a^5*b^2*x^(5/3) + 556*a^4*b^3*x^(4/3
) + 544*a^3*b^4*x - 444*a^2*b^5*x^(2/3) - 856*a*b^6*x^(1/3) - 319*b^7)/(a^1
2*x^(4/3) + 4*a^11*b*x + 6*a^10*b^2*x^(2/3) + 4*a^9*b^3*x^(1/3) + a^8*b^4)
- 105*b^3*log(a*x^(1/3) + b)/a^8
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{{\left(a^2+\frac{b^2}{x^{2/3}}+\frac{2\,a\,b}{x^{1/3}}\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3))^(5/2),x)
```

```
[Out] int(1/(a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3))^(5/2), x)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a^2 + \frac{2ab}{\sqrt[3]{x}} + \frac{b^2}{x^{\frac{2}{3}}}\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a**2+b**2/x**(2/3)+2*a*b/x**(1/3))**(5/2),x)
```

```
[Out] Integral((a**2 + 2*a*b/x**(1/3) + b**2/x**(2/3))**(-5/2), x)
```

3.425 $\int \left(a^2+\frac{b^2}{\sqrt{x}}+\frac{2ab}{\sqrt[4]{x}}\right)^{5/2}\,dx$

Optimal. Leaf size=289

$$-\frac{4b^5\sqrt{a^2+\frac{2ab}{\sqrt[4]{x}}+\frac{b^2}{\sqrt{x}}}}{\sqrt[4]{x}\left(a+\frac{b}{\sqrt[4]{x}}\right)}+\frac{20ab^4\log\left(\sqrt[4]{x}\right)\sqrt{a^2+\frac{2ab}{\sqrt[4]{x}}+\frac{b^2}{\sqrt{x}}}}{a+\frac{b}{\sqrt[4]{x}}}+\frac{40a^2b^3\sqrt[4]{x}\sqrt{a^2+\frac{2ab}{\sqrt[4]{x}}+\frac{b^2}{\sqrt{x}}}}{a+\frac{b}{\sqrt[4]{x}}}+\frac{a^5x\sqrt{a^2+\frac{2ab}{\sqrt[4]{x}}+\frac{b^2}{\sqrt{x}}}}{a+\frac{b}{\sqrt[4]{x}}}$$

Rubi [A] time = 0.14, antiderivative size = 289, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {1341, 1355, 263, 43}

$$\frac{20a^4bx^{3/4}\sqrt{a^2+\frac{2ab}{\sqrt[4]{x}}+\frac{b^2}{\sqrt{x}}}}{3\left(a+\frac{b}{\sqrt[4]{x}}\right)}+\frac{a^5x\sqrt{a^2+\frac{2ab}{\sqrt[4]{x}}+\frac{b^2}{\sqrt{x}}}}{a+\frac{b}{\sqrt[4]{x}}}+\frac{20a^3b^2\sqrt{x}\sqrt{a^2+\frac{2ab}{\sqrt[4]{x}}+\frac{b^2}{\sqrt{x}}}}{a+\frac{b}{\sqrt[4]{x}}}+\frac{40a^2b^3\sqrt[4]{x}\sqrt{a^2+\frac{2ab}{\sqrt[4]{x}}+\frac{b^2}{\sqrt{x}}}}{a+\frac{b}{\sqrt[4]{x}}}-\frac{4b^5\sqrt{a^2+\frac{2ab}{\sqrt[4]{x}}+\frac{b^2}{\sqrt{x}}}}{\sqrt[4]{x}\left(a+\frac{b}{\sqrt[4]{x}}\right)}+\frac{20ab^4\log\left(\sqrt[4]{x}\right)\sqrt{a^2+\frac{2ab}{\sqrt[4]{x}}+\frac{b^2}{\sqrt{x}}}}{a+\frac{b}{\sqrt[4]{x}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + b^2/Sqrt[x] + (2*a*b)/x^(1/4))^(5/2),x]
```

```
[Out] (-4*b^5*Sqrt[a^2 + b^2/Sqrt[x] + (2*a*b)/x^(1/4)])/((a + b/x^(1/4))*x^(1/4)
) + (40*a^2*b^3*Sqrt[a^2 + b^2/Sqrt[x] + (2*a*b)/x^(1/4)]*x^(1/4))/(a + b/x
^(1/4)) + (20*a^3*b^2*Sqrt[a^2 + b^2/Sqrt[x] + (2*a*b)/x^(1/4)]*Sqrt[x])/(a
 + b/x^(1/4)) + (20*a^4*b*Sqrt[a^2 + b^2/Sqrt[x] + (2*a*b)/x^(1/4)]*x^(3/4)
)/(3*(a + b/x^(1/4))) + (a^5*Sqrt[a^2 + b^2/Sqrt[x] + (2*a*b)/x^(1/4)]*x)/(
a + b/x^(1/4)) + (20*a*b^4*Sqrt[a^2 + b^2/Sqrt[x] + (2*a*b)/x^(1/4)]*Log[x^
(1/4)])/(a + b/x^(1/4))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 1341

```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k
 = Denominator[n]}, Dist[k, Subst[Int[x^(k - 1)*(a + b*x^(k*n) + c*x^(2*k*n
))^p, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && EqQ[n2, 2*n] && Fra
ctionQ[n]
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\int \left(a^2+\frac{b^2}{\sqrt{x}}+\frac{2ab}{\sqrt[4]{x}}\right)^{5/2}dx = 4\,\text{Subst}\left(\int \left(a^2+\frac{b^2}{x^2}+\frac{2ab}{x}\right)^{5/2}x^3\,dx, x, \sqrt[4]{x}\right)$$
$$= \frac{\left(4\sqrt{a^2+\frac{b^2}{\sqrt{x}}+\frac{2ab}{\sqrt[4]{x}}}\right)\text{Subst}\left(\int \left(ab+\frac{b^2}{x}\right)^5 x^3\,dx, x, \sqrt[4]{x}\right)}{b^4\left(ab+\frac{b^2}{\sqrt[4]{x}}\right)}$$
$$= \frac{\left(4\sqrt{a^2+\frac{b^2}{\sqrt{x}}+\frac{2ab}{\sqrt[4]{x}}}\right)\text{Subst}\left(\int \frac{(b^2+abx)^5}{x^2}\,dx, x, \sqrt[4]{x}\right)}{b^4\left(ab+\frac{b^2}{\sqrt[4]{x}}\right)}$$
$$= \frac{\left(4\sqrt{a^2+\frac{b^2}{\sqrt{x}}+\frac{2ab}{\sqrt[4]{x}}}\right)\text{Subst}\left(\int \left(10a^2b^8+\frac{b^{10}}{x^2}+\frac{5ab^9}{x}+10a^3b^7x+5a^4b^6x^2+a^5b^5x^3\right)\right.}{b^4\left(ab+\frac{b^2}{\sqrt[4]{x}}\right)}$$
$$= -\frac{4b^6\sqrt{a^2+\frac{b^2}{\sqrt{x}}+\frac{2ab}{\sqrt[4]{x}}}}{\left(ab+\frac{b^2}{\sqrt[4]{x}}\right)\sqrt[4]{x}}+\frac{40a^2b^4\sqrt{a^2+\frac{b^2}{\sqrt{x}}+\frac{2ab}{\sqrt[4]{x}}}\sqrt[4]{x}}{ab+\frac{b^2}{\sqrt[4]{x}}}+\frac{20a^3b^3\sqrt{a^2+\frac{b^2}{\sqrt{x}}+\frac{2ab}{\sqrt[4]{x}}}\sqrt{}}{ab+\frac{b^2}{\sqrt[4]{x}}}$$

Mathematica [A] time = 0.05, size = 98, normalized size = 0.34

$$\frac{\sqrt{\frac{\left(a\sqrt[4]{x}+b\right)^2}{\sqrt{x}}}\left(3a^5x^{5/4}+20a^4bx+60a^3b^2x^{3/4}+120a^2b^3\sqrt{x}+15ab^4\sqrt[4]{x}\log(x)-12b^5\right)}{3\left(a\sqrt[4]{x}+b\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + b^2/Sqrt[x] + (2*a*b)/x^(1/4))^(5/2),x]
```

```
[Out] (Sqrt[(b + a*x^(1/4))^2/Sqrt[x]]*(-12*b^5 + 120*a^2*b^3*Sqrt[x] + 60*a^3*b^
2*x^(3/4) + 20*a^4*b*x + 3*a^5*x^(5/4) + 15*a*b^4*x^(1/4)*Log[x]))/(3*(b +
a*x^(1/4)))
```

IntegrateAlgebraic [A] time = 6.28, size = 109, normalized size = 0.38

$$\frac{\sqrt[4]{x}\sqrt{\frac{\left(a\sqrt[4]{x}+b\right)^2}{\sqrt{x}}}\left(\frac{3a^5x^{5/4}+20a^4bx+60a^3b^2x^{3/4}+120a^2b^3\sqrt{x}-12b^5}{3\sqrt[4]{x}}+20ab^4\log\left(\sqrt[4]{x}\right)\right)}{a\sqrt[4]{x}+b}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a^2 + b^2/Sqrt[x] + (2*a*b)/x^(1/4))^(5/2),x]
```

```
[Out] (Sqrt[(b + a*x^(1/4))^2/Sqrt[x]]*x^(1/4)*((-12*b^5 + 120*a^2*b^3*Sqrt[x] +
60*a^3*b^2*x^(3/4) + 20*a^4*b*x + 3*a^5*x^(5/4))/(3*x^(1/4)) + 20*a*b^4*Log
[x^(1/4)]))/(b + a*x^(1/4))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+2*a*b/x^(1/4)+b^2/x^(1/2))^(5/2),x, algorithm="fricas")
```

[Out] Timed out

giac [A] time = 0.49, size = 126, normalized size = 0.44

$$a^5x\mathrm{sgn}\left(ax+bx^{\frac{3}{4}}\right)\mathrm{sgn}(x)+5\,ab^4\log(|x|)\,\mathrm{sgn}\left(ax+bx^{\frac{3}{4}}\right)\mathrm{sgn}(x)+\frac{20}{3}\,a^4bx^{\frac{3}{4}}\mathrm{sgn}\left(ax+bx^{\frac{3}{4}}\right)\mathrm{sgn}(x)+20\,a^3b^2\sqrt{x}\,\mathrm{sgn}\left(ax+bx^{\frac{3}{4}}\right)\mathrm{sgn}(x)+40\,a^2b^3x^{\frac{1}{4}}\mathrm{sgn}\left(ax+bx^{\frac{3}{4}}\right)\mathrm{sgn}(x)-\frac{4\,b^5\mathrm{sgn}\left(ax+bx^{\frac{3}{4}}\right)\mathrm{sgn}(x)}{x^{\frac{1}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+2*a*b/x^(1/4)+b^2/x^(1/2))^(5/2),x, algorithm="giac")
```

```
[Out] a^5*x*sgn(a*x + b*x^(3/4))*sgn(x) + 5*a*b^4*log(abs(x))*sgn(a*x + b*x^(3/4)
)*sgn(x) + 20/3*a^4*b*x^(3/4)*sgn(a*x + b*x^(3/4))*sgn(x) + 20*a^3*b^2*sqrt
(x)*sgn(a*x + b*x^(3/4))*sgn(x) + 40*a^2*b^3*x^(1/4)*sgn(a*x + b*x^(3/4))*s
gn(x) - 4*b^5*sgn(a*x + b*x^(3/4))*sgn(x)/x^(1/4)
```

maple [A] time = 0.04, size = 94, normalized size = 0.33

$$\frac{\sqrt{\frac{a^2x^{\frac{3}{4}}+2ab\sqrt{x}+b^2x^{\frac{1}{4}}}{x^{\frac{3}{4}}}}\left(3a^5x^{\frac{5}{4}}+15a\,b^4x^{\frac{1}{4}}\ln(x)+20a^4bx+60a^3b^2x^{\frac{3}{4}}+120a^2b^3\sqrt{x}-12b^5\right)}{3a\,x^{\frac{1}{4}}+3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2+2*a*b/x^(1/4)+b^2/x^(1/2))^(5/2),x)
```

```
[Out] 1/3*((a^2*x^(3/4)+b^2*x^(1/4)+2*a*b*x^(1/2))/x^(3/4))^(1/2)*(20*x*a^4*b+15*
ln(x)*x^(1/4)*a*b^4+120*a^2*b^3*x^(1/2)+60*x^(3/4)*a^3*b^2+3*x^(5/4)*a^5-12
*b^5)/(a*x^(1/4)+b)
```

maxima [A] time = 0.99, size = 57, normalized size = 0.20

$$5\,ab^4\log(x)+\frac{3\,a^5x^{\frac{5}{4}}+20\,a^4bx+60\,a^3b^2x^{\frac{3}{4}}+120\,a^2b^3\sqrt{x}-12\,b^5}{3\,x^{\frac{1}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+2*a*b/x^(1/4)+b^2/x^(1/2))^(5/2),x, algorithm="maxima")
```

```
[Out] 5*a*b^4*log(x) + 1/3*(3*a^5*x^(5/4) + 20*a^4*b*x + 60*a^3*b^2*x^(3/4) + 120
*a^2*b^3*sqrt(x) - 12*b^5)/x^(1/4)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int {\left(a^2+\frac{b^2}{\sqrt{x}}+\frac{2\,a\,b}{x^{1/4}}\right)}^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2/x^(1/2) + (2*a*b)/x^(1/4))^(5/2),x)
```

```
[Out] int((a^2 + b^2/x^(1/2) + (2*a*b)/x^(1/4))^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2+2*a*b/x**(1/4)+b**2/x**(1/2))**(5/2),x)
```

```
[Out] Timed out
```

3.426 $\int \left(a^2+\frac{b^2}{x^{2/5}}+\frac{2ab}{\sqrt[5]{x}}\right)^{5/2} dx$

Optimal. Leaf size=291

$$\frac{5b^5\log\left(\sqrt[5]{x}\right)\sqrt{a^2+\frac{2ab}{\sqrt[5]{x}}+\frac{b^2}{x^{2/5}}}}{a+\frac{b}{\sqrt[5]{x}}}+\frac{25ab^4\sqrt[5]{x}\sqrt{a^2+\frac{2ab}{\sqrt[5]{x}}+\frac{b^2}{x^{2/5}}}}{a+\frac{b}{\sqrt[5]{x}}}+\frac{25a^2b^3x^{2/5}\sqrt{a^2+\frac{2ab}{\sqrt[5]{x}}+\frac{b^2}{x^{2/5}}}}{a+\frac{b}{\sqrt[5]{x}}}+\frac{a^5x\sqrt{a^2+\frac{2ab}{\sqrt[5]{x}}+\frac{b^2}{x^{2/5}}}}{a+\frac{b}{\sqrt[5]{x}}}$$

Rubi [A] time = 0.14, antiderivative size = 291, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {1341, 1355, 263, 43}

$$\frac{a^5x\sqrt{a^2+\frac{2ab}{\sqrt[5]{x}}+\frac{b^2}{x^{2/5}}}}{a+\frac{b}{\sqrt[5]{x}}}+\frac{25a^4bx^{4/5}\sqrt{a^2+\frac{2ab}{\sqrt[5]{x}}+\frac{b^2}{x^{2/5}}}}{4\left(a+\frac{b}{\sqrt[5]{x}}\right)}+\frac{50a^3b^2x^{3/5}\sqrt{a^2+\frac{2ab}{\sqrt[5]{x}}+\frac{b^2}{x^{2/5}}}}{3\left(a+\frac{b}{\sqrt[5]{x}}\right)}+\frac{25a^2b^3x^{2/5}\sqrt{a^2+\frac{2ab}{\sqrt[5]{x}}+\frac{b^2}{x^{2/5}}}}{a+\frac{b}{\sqrt[5]{x}}}+\frac{25ab^4\sqrt[5]{x}\sqrt{a^2+\frac{2ab}{\sqrt[5]{x}}+\frac{b^2}{x^{2/5}}}}{a+\frac{b}{\sqrt[5]{x}}}+\frac{5b^5\log\left(\sqrt[5]{x}\right)\sqrt{a^2+\frac{2ab}{\sqrt[5]{x}}+\frac{b^2}{x^{2/5}}}}{a+\frac{b}{\sqrt[5]{x}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + b^2/x^(2/5) + (2*a*b)/x^(1/5))^(5/2),x]
```

```
[Out] (25*a*b^4*Sqrt[a^2 + b^2/x^(2/5) + (2*a*b)/x^(1/5)]*x^(1/5))/(a + b/x^(1/5)
) + (25*a^2*b^3*Sqrt[a^2 + b^2/x^(2/5) + (2*a*b)/x^(1/5)]*x^(2/5))/(a + b/x
^(1/5)) + (50*a^3*b^2*Sqrt[a^2 + b^2/x^(2/5) + (2*a*b)/x^(1/5)]*x^(3/5))/(3
*(a + b/x^(1/5))) + (25*a^4*b*Sqrt[a^2 + b^2/x^(2/5) + (2*a*b)/x^(1/5)]*x^(
4/5))/(4*(a + b/x^(1/5))) + (a^5*Sqrt[a^2 + b^2/x^(2/5) + (2*a*b)/x^(1/5)]*
x)/(a + b/x^(1/5)) + (5*b^5*Sqrt[a^2 + b^2/x^(2/5) + (2*a*b)/x^(1/5)]*Log[x
^(1/5)])/(a + b/x^(1/5))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 1341

```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k
 = Denominator[n]}, Dist[k, Subst[Int[x^(k - 1)*(a + b*x^(k*n) + c*x^(2*k*n
))^p, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && EqQ[n2, 2*n] && Fra
ctionQ[n]
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\int \left(a^2+\frac{b^2}{x^{2/5}}+\frac{2ab}{\sqrt[5]{x}}\right)^{5/2} dx = 5\,\text{Subst}\left(\int \left(a^2+\frac{b^2}{x^2}+\frac{2ab}{x}\right)^{5/2} x^4\,dx, x, \sqrt[5]{x}\right)$$
$$= \frac{\left(5\sqrt{a^2+\frac{b^2}{x^{2/5}}+\frac{2ab}{\sqrt[5]{x}}}\right)\text{Subst}\left(\int \left(ab+\frac{b^2}{x}\right)^5 x^4\,dx, x, \sqrt[5]{x}\right)}{b^4\left(ab+\frac{b^2}{\sqrt[5]{x}}\right)}$$
$$= \frac{\left(5\sqrt{a^2+\frac{b^2}{x^{2/5}}+\frac{2ab}{\sqrt[5]{x}}}\right)\text{Subst}\left(\int \frac{(b^2+abx)^5}{x}\,dx, x, \sqrt[5]{x}\right)}{b^4\left(ab+\frac{b^2}{\sqrt[5]{x}}\right)}$$
$$= \frac{\left(5\sqrt{a^2+\frac{b^2}{x^{2/5}}+\frac{2ab}{\sqrt[5]{x}}}\right)\text{Subst}\left(\int \left(5ab^9+\frac{b^{10}}{x}+10a^2b^8x+10a^3b^7x^2+5a^4b^6x^3+a\right.\right.}{b^4\left(ab+\frac{b^2}{\sqrt[5]{x}}\right)}$$
$$= \frac{25ab^5\sqrt{a^2+\frac{b^2}{x^{2/5}}+\frac{2ab}{\sqrt[5]{x}}}\sqrt[5]{x}}{ab+\frac{b^2}{\sqrt[5]{x}}} + \frac{25a^2b^4\sqrt{a^2+\frac{b^2}{x^{2/5}}+\frac{2ab}{\sqrt[5]{x}}}x^{2/5}}{ab+\frac{b^2}{\sqrt[5]{x}}} + \frac{50a^3b^3\sqrt{a^2+\frac{b^2}{x^{2/5}}}}{3\left(ab+\frac{b^2}{\sqrt[5]{x}}\right.}$$

Mathematica [A] time = 0.05, size = 103, normalized size = 0.35

$$\frac{\sqrt{\frac{\left(a\sqrt[5]{x}+b\right)^2}{x^{2/5}}}\left(12a^5x^{6/5}+75a^4bx+200a^3b^2x^{4/5}+300a^2b^3x^{3/5}+300ab^4x^{2/5}+12b^5\sqrt[5]{x}\log(x)\right)}{12\left(a\sqrt[5]{x}+b\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + b^2/x^(2/5) + (2*a*b)/x^(1/5))^(5/2),x]
```

```
[Out] (Sqrt[(b + a*x^(1/5))^2/x^(2/5)]*(300*a*b^4*x^(2/5) + 300*a^2*b^3*x^(3/5) +
 200*a^3*b^2*x^(4/5) + 75*a^4*b*x + 12*a^5*x^(6/5) + 12*b^5*x^(1/5)*Log[x])
)/(12*(b + a*x^(1/5)))
```

IntegrateAlgebraic [A] time = 6.30, size = 109, normalized size = 0.37

$$\frac{\sqrt[5]{x}\sqrt{\frac{\left(a\sqrt[5]{x}+b\right)^2}{x^{2/5}}}\left(\frac{1}{12}\left(12a^5x+75a^4bx^{4/5}+200a^3b^2x^{3/5}+300a^2b^3x^{2/5}+300ab^4\sqrt[5]{x}\right)+5b^5\log\left(\sqrt[5]{x}\right)\right)}{a\sqrt[5]{x}+b}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a^2 + b^2/x^(2/5) + (2*a*b)/x^(1/5))^(5/2),x]
```

```
[Out] (Sqrt[(b + a*x^(1/5))^2/x^(2/5)]*x^(1/5)*((300*a*b^4*x^(1/5) + 300*a^2*b^3*
x^(2/5) + 200*a^3*b^2*x^(3/5) + 75*a^4*b*x^(4/5) + 12*a^5*x)/12 + 5*b^5*Log
[x^(1/5)]))/(b + a*x^(1/5))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+b^2/x^(2/5)+2*a*b/x^(1/5))^(5/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [A] time = 0.48, size = 125, normalized size = 0.43

$$a^5x\operatorname{sgn}\left(ax+bx^{\frac{4}{5}}\right)\operatorname{sgn}(x)+b^5\log(|x|)\operatorname{sgn}\left(ax+bx^{\frac{4}{5}}\right)\operatorname{sgn}(x)+\frac{25}{4}a^4bx^{\frac{4}{5}}\operatorname{sgn}\left(ax+bx^{\frac{4}{5}}\right)\operatorname{sgn}(x)+\frac{50}{3}a^3b^2x^{\frac{3}{5}}\operatorname{sgn}\left(ax+bx^{\frac{4}{5}}\right)\operatorname{sgn}(x)+25\,a^2b^3x^{\frac{2}{5}}\operatorname{sgn}\left(ax+bx^{\frac{4}{5}}\right)\operatorname{sgn}(x)+25\,ab^4x^{\frac{1}{5}}\operatorname{sgn}\left(ax+bx^{\frac{4}{5}}\right)\operatorname{sgn}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+b^2/x^(2/5)+2*a*b/x^(1/5))^(5/2),x, algorithm="giac")
```

```
[Out] a^5*x*sgn(a*x + b*x^(4/5))*sgn(x) + b^5*log(abs(x))*sgn(a*x + b*x^(4/5))*sg
n(x) + 25/4*a^4*b*x^(4/5)*sgn(a*x + b*x^(4/5))*sgn(x) + 50/3*a^3*b^2*x^(3/5
)*sgn(a*x + b*x^(4/5))*sgn(x) + 25*a^2*b^3*x^(2/5)*sgn(a*x + b*x^(4/5))*sgn
(x) + 25*a*b^4*x^(1/5)*sgn(a*x + b*x^(4/5))*sgn(x)
```

maple [A] time = 0.03, size = 91, normalized size = 0.31

$$\frac{\left(\frac{a^2x^{\frac{2}{5}}+2ab\,x^{\frac{1}{5}}+b^2}{x^{\frac{2}{5}}}\right)^{\frac{5}{2}}\left(12a^5x+12b^5\ln(x)+75a^4b\,x^{\frac{4}{5}}+200a^3b^2x^{\frac{3}{5}}+300a^2b^3x^{\frac{2}{5}}+300a\,b^4x^{\frac{1}{5}}\right)x}{12\left(a\,x^{\frac{1}{5}}+b\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2+b^2/x^(2/5)+2*a*b/x^(1/5))^(5/2),x)
```

```
[Out] 1/12*((a^2*x^(2/5)+2*a*b*x^(1/5)+b^2)/x^(2/5))^(5/2)*x*(75*a^4*b*x^(4/5)+20
0*a^3*b^2*x^(3/5)+300*a^2*b^3*x^(2/5)+12*b^5*ln(x)+300*a*b^4*x^(1/5)+12*a^5
*x)/(x^(1/5)*a+b)^5
```

maxima [A] time = 0.94, size = 52, normalized size = 0.18

$$a^5x+b^5\log(x)+\frac{25}{4}\,a^4bx^{\frac{4}{5}}+\frac{50}{3}\,a^3b^2x^{\frac{3}{5}}+25\,a^2b^3x^{\frac{2}{5}}+25\,ab^4x^{\frac{1}{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+b^2/x^(2/5)+2*a*b/x^(1/5))^(5/2),x, algorithm="maxima")
```

```
[Out] a^5*x + b^5*log(x) + 25/4*a^4*b*x^(4/5) + 50/3*a^3*b^2*x^(3/5) + 25*a^2*b^3
*x^(2/5) + 25*a*b^4*x^(1/5)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int\left(a^2+\frac{b^2}{x^{2/5}}+\frac{2\,a\,b}{x^{1/5}}\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2/x^(2/5) + (2*a*b)/x^(1/5))^(5/2),x)
```

```
[Out] int((a^2 + b^2/x^(2/5) + (2*a*b)/x^(1/5))^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a^2+\frac{2ab}{\sqrt[5]{x}}+\frac{b^2}{x^{\frac{2}{5}}}\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2+b**2/x**(2/5)+2*a*b/x**(1/5))**(5/2),x)
```

```
[Out] Integral((a**2 + 2*a*b/x**(1/5) + b**2/x**(2/5))**(5/2), x)
```

3.427 $\int \frac{1}{\left(a^2+2ab\sqrt[5]{x}+b^2x^{2/5}\right)^{5/2}}\,dx$

Optimal. Leaf size=222

$$-\frac{15a^2}{b^5\left(a+b\sqrt[5]{x}\right)\sqrt{a^2+2ab\sqrt[5]{x}+b^2x^{2/5}}}+\frac{20a}{b^5\sqrt{a^2+2ab\sqrt[5]{x}+b^2x^{2/5}}}+\frac{5\left(a+b\sqrt[5]{x}\right)\log\left(a+b\sqrt[5]{x}\right)}{b^5\sqrt{a^2+2ab\sqrt[5]{x}+b^2x^{2/5}}}-\frac{}{4b^5\left(a+b\sqrt[5]{x}\right.}$$

Rubi [A] time = 0.13, antiderivative size = 222, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {1341, 646, 43}

$$-\frac{5a^4}{4b^5\left(a+b\sqrt[5]{x}\right)^3\sqrt{a^2+2ab\sqrt[5]{x}+b^2x^{2/5}}}+\frac{20a^3}{3b^5\left(a+b\sqrt[5]{x}\right)^2\sqrt{a^2+2ab\sqrt[5]{x}+b^2x^{2/5}}}-\frac{15a^2}{b^5\left(a+b\sqrt[5]{x}\right)\sqrt{a^2+2ab\sqrt[5]{x}+b^2x^{2/5}}}+\frac{20a}{b^5\sqrt{a^2+2ab\sqrt[5]{x}+b^2x^{2/5}}}+\frac{5\left(a+b\sqrt[5]{x}\right)\log\left(a+b\sqrt[5]{x}\right)}{b^5\sqrt{a^2+2ab\sqrt[5]{x}+b^2x^{2/5}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^(1/5) + b^2*x^(2/5))^(-5/2),x]
```

```
[Out] (20*a)/(b^5*Sqrt[a^2 + 2*a*b*x^(1/5) + b^2*x^(2/5)]) - (5*a^4)/(4*b^5*(a +
b*x^(1/5))^3*Sqrt[a^2 + 2*a*b*x^(1/5) + b^2*x^(2/5)]) + (20*a^3)/(3*b^5*(a
+ b*x^(1/5))^2*Sqrt[a^2 + 2*a*b*x^(1/5) + b^2*x^(2/5)]) - (15*a^2)/(b^5*(a
+ b*x^(1/5))*Sqrt[a^2 + 2*a*b*x^(1/5) + b^2*x^(2/5)]) + (5*(a + b*x^(1/5))*
Log[a + b*x^(1/5)])/(b^5*Sqrt[a^2 + 2*a*b*x^(1/5) + b^2*x^(2/5)])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 1341

```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k
 = Denominator[n]}, Dist[k, Subst[Int[x^(k - 1)*(a + b*x^(k*n) + c*x^(2*k*n
))^p, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && EqQ[n2, 2*n] && Fra
ctionQ[n]
```

Rubi steps

$$\int \frac{1}{\left(a^2+2ab\sqrt[5]{x}+b^2x^{2/5}\right)^{5/2}}\,dx = 5\,\text{Subst}\left(\int \frac{x^4}{\left(a^2+2abx+b^2x^2\right)^{5/2}}\,dx, x, \sqrt[5]{x}\right)$$

$$= \frac{\left(5b^5\left(a+b\sqrt[5]{x}\right)\right)\text{Subst}\left(\int \frac{x^4}{\left(ab+b^2x\right)^5}\,dx, x, \sqrt[5]{x}\right)}{\sqrt{a^2+2ab\sqrt[5]{x}+b^2x^{2/5}}}$$

$$= \frac{\left(5b^5\left(a+b\sqrt[5]{x}\right)\right)\text{Subst}\left(\int \left(\frac{a^4}{b^9(a+bx)^5}-\frac{4a^3}{b^9(a+bx)^4}+\frac{6a^2}{b^9(a+bx)^3}-\frac{4a}{b^9(a+bx)^2}+\frac{1}{b^9(a+bx)}\right.\right.}{\sqrt{a^2+2ab\sqrt[5]{x}+b^2x^{2/5}}}$$

$$= \frac{20a}{b^5\sqrt{a^2+2ab\sqrt[5]{x}+b^2x^{2/5}}} - \frac{5a^4}{4b^5\left(a+b\sqrt[5]{x}\right)^3\sqrt{a^2+2ab\sqrt[5]{x}+b^2x^{2/5}}} + \frac{}{3b^5\left(a\right.}$$

Mathematica [A] time = 0.09, size = 98, normalized size = 0.44

$$\frac{5a\left(25a^3+88a^2b\sqrt[5]{x}+108ab^2x^{2/5}+48b^3x^{3/5}\right)+60\left(a+b\sqrt[5]{x}\right)^4\log\left(a+b\sqrt[5]{x}\right)}{12b^5\left(a+b\sqrt[5]{x}\right)^3\sqrt{\left(a+b\sqrt[5]{x}\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^(1/5) + b^2*x^(2/5))^(-5/2),x]
```

```
[Out] (5*a*(25*a^3 + 88*a^2*b*x^(1/5) + 108*a*b^2*x^(2/5) + 48*b^3*x^(3/5)) + 60*
(a + b*x^(1/5))^4*Log[a + b*x^(1/5)])/(12*b^5*(a + b*x^(1/5))^3*Sqrt[(a + b
*x^(1/5))^2])
```

IntegrateAlgebraic [B] time = 2.81, size = 2765, normalized size = 12.45

Result too large to show

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a^2 + 2*a*b*x^(1/5) + b^2*x^(2/5))^(-5/2),x]
```

```
[Out] ((160*a^7*Sqrt[a^2 + 2*a*b*x^(1/5) + b^2*x^(2/5)])/b^5 - (2240*a^7*x^(1/5))
/(3*b^3*Sqrt[b^2]) + (1760*a^6*Sqrt[a^2 + 2*a*b*x^(1/5) + b^2*x^(2/5)]*x^(1
/5))/(3*b^4) - (9440*a^6*x^(2/5))/(3*(b^2)^(3/2)) + (2560*a^5*Sqrt[a^2 + 2*
a*b*x^(1/5) + b^2*x^(2/5)]*x^(2/5))/b^3 - (8000*a^5*x^(3/5))/(b*Sqrt[b^2])
+ (5440*a^4*Sqrt[a^2 + 2*a*b*x^(1/5) + b^2*x^(2/5)]*x^(3/5))/b^2 - (40000*a
^4*x^(4/5))/(3*Sqrt[b^2]) + (23680*a^3*Sqrt[a^2 + 2*a*b*x^(1/5) + b^2*x^(2/
5)]*x^(4/5))/(3*b) - (42880*a^3*b*x)/(3*Sqrt[b^2]) + 6400*a^2*Sqrt[a^2 + 2*
a*b*x^(1/5) + b^2*x^(2/5)]*x - 8960*a^2*Sqrt[b^2]*x^(6/5) + 2560*a*b*Sqrt[a
^2 + 2*a*b*x^(1/5) + b^2*x^(2/5)]*x^(6/5) - (2560*a*b^3*x^(7/5))/Sqrt[b^2]
+ (640*a^4*x^(4/5)*ArcTanh[(Sqrt[a^2 + 2*a*b*x^(1/5) + b^2*x^(2/5)] - Sqrt[
b^2]*x^(1/5))/a])/b - (640*a^3*Sqrt[a^2 + 2*a*b*x^(1/5) + b^2*x^(2/5)]*x^(4
/5)*ArcTanh[(Sqrt[a^2 + 2*a*b*x^(1/5) + b^2*x^(2/5)] - Sqrt[b^2]*x^(1/5))/a
])/Sqrt[b^2] + 2560*a^3*x*ArcTanh[(Sqrt[a^2 + 2*a*b*x^(1/5) + b^2*x^(2/5)]
- Sqrt[b^2]*x^(1/5))/a] - (1920*a^2*b*Sqrt[a^2 + 2*a*b*x^(1/5) + b^2*x^(2/5
)]*x*ArcTanh[(Sqrt[a^2 + 2*a*b*x^(1/5) + b^2*x^(2/5)] - Sqrt[b^2]*x^(1/5))/
a])/Sqrt[b^2] + 3840*a^2*b*x^(6/5)*ArcTanh[(Sqrt[a^2 + 2*a*b*x^(1/5) + b^2*
x^(2/5)] - Sqrt[b^2]*x^(1/5))/a] - 1920*a*Sqrt[b^2]*Sqrt[a^2 + 2*a*b*x^(1/5
) + b^2*x^(2/5)]*x^(6/5)*ArcTanh[(Sqrt[a^2 + 2*a*b*x^(1/5) + b^2*x^(2/5)] -
 Sqrt[b^2]*x^(1/5))/a] + 2560*a*b^2*x^(7/5)*ArcTanh[(Sqrt[a^2 + 2*a*b*x^(1/
5) + b^2*x^(2/5)] - Sqrt[b^2]*x^(1/5))/a] - (640*b^3*Sqrt[a^2 + 2*a*b*x^(1/
```

```
5) + b^2*x^(2/5)]*x^(7/5)*ArcTanh[(Sqrt[a^2 + 2*a*b*x^(1/5) + b^2*x^(2/5)]
 - Sqrt[b^2]*x^(1/5))/a])/Sqrt[b^2] + 640*b^3*x^(8/5)*ArcTanh[(Sqrt[a^2 + 2*
a*b*x^(1/5) + b^2*x^(2/5)] - Sqrt[b^2]*x^(1/5))/a])/((-a + Sqrt[a^2 + 2*a*b
*x^(1/5) + b^2*x^(2/5)] - Sqrt[b^2]*x^(1/5))^4*(a + Sqrt[a^2 + 2*a*b*x^(1/5
) + b^2*x^(2/5)] - Sqrt[b^2]*x^(1/5))^4) + ((160*a^8)/(b^4*Sqrt[b^2]) + (22
40*a^7*x^(1/5))/(3*b^3*Sqrt[b^2]) - (2240*a^6*Sqrt[a^2 + 2*a*b*x^(1/5) + b^
2*x^(2/5)]*x^(1/5))/(3*b^4) + (9440*a^6*x^(2/5))/(3*(b^2)^(3/2)) - (2400*a^
5*Sqrt[a^2 + 2*a*b*x^(1/5) + b^2*x^(2/5)]*x^(2/5))/b^3 + (8000*a^5*x^(3/5))
/(b*Sqrt[b^2]) - (5600*a^4*Sqrt[a^2 + 2*a*b*x^(1/5) + b^2*x^(2/5)]*x^(3/5))
/b^2 + (12000*a^4*x^(4/5))/Sqrt[b^2] - (6400*a^3*Sqrt[a^2 + 2*a*b*x^(1/5) +
 b^2*x^(2/5)]*x^(4/5))/b + (9600*a^3*b*x)/Sqrt[b^2] - 3200*a^2*Sqrt[a^2 + 2
*a*b*x^(1/5) + b^2*x^(2/5)]*x + 3200*a^2*Sqrt[b^2]*x^(6/5) - (320*a^4*x^(4/
5)*Log[-a + Sqrt[a^2 + 2*a*b*x^(1/5) + b^2*x^(2/5)] - Sqrt[b^2]*x^(1/5)])/S
qrt[b^2] + (320*a^3*Sqrt[a^2 + 2*a*b*x^(1/5) + b^2*x^(2/5)]*x^(4/5)*Log[-a
+ Sqrt[a^2 + 2*a*b*x^(1/5) + b^2*x^(2/5)] - Sqrt[b^2]*x^(1/5)])/b - (1280*a
^3*b*x*Log[-a + Sqrt[a^2 + 2*a*b*x^(1/5) + b^2*x^(2/5)] - Sqrt[b^2]*x^(1/5)
])/Sqrt[b^2] + 960*a^2*Sqrt[a^2 + 2*a*b*x^(1/5) + b^2*x^(2/5)]*x*Log[-a + S
qrt[a^2 + 2*a*b*x^(1/5) + b^2*x^(2/5)] - Sqrt[b^2]*x^(1/5)] - 1920*a^2*Sqrt
[b^2]*x^(6/5)*Log[-a + Sqrt[a^2 + 2*a*b*x^(1/5) + b^2*x^(2/5)] - Sqrt[b^2]*
x^(1/5)] + 960*a*b*Sqrt[a^2 + 2*a*b*x^(1/5) + b^2*x^(2/5)]*x^(6/5)*Log[-a +
 Sqrt[a^2 + 2*a*b*x^(1/5) + b^2*x^(2/5)] - Sqrt[b^2]*x^(1/5)] - (1280*a*b^3
*x^(7/5)*Log[-a + Sqrt[a^2 + 2*a*b*x^(1/5) + b^2*x^(2/5)] - Sqrt[b^2]*x^(1/
5)])/Sqrt[b^2] + 320*b^2*Sqrt[a^2 + 2*a*b*x^(1/5) + b^2*x^(2/5)]*x^(7/5)*Lo
g[-a + Sqrt[a^2 + 2*a*b*x^(1/5) + b^2*x^(2/5)] - Sqrt[b^2]*x^(1/5)] - (320*
b^4*x^(8/5)*Log[-a + Sqrt[a^2 + 2*a*b*x^(1/5) + b^2*x^(2/5)] - Sqrt[b^2]*x^
(1/5)])/Sqrt[b^2] - (320*a^4*x^(4/5)*Log[a + Sqrt[a^2 + 2*a*b*x^(1/5) + b^2
*x^(2/5)] - Sqrt[b^2]*x^(1/5)])/Sqrt[b^2] + (320*a^3*Sqrt[a^2 + 2*a*b*x^(1/
5) + b^2*x^(2/5)]*x^(4/5)*Log[a + Sqrt[a^2 + 2*a*b*x^(1/5) + b^2*x^(2/5)] -
 Sqrt[b^2]*x^(1/5)])/b - (1280*a^3*b*x*Log[a + Sqrt[a^2 + 2*a*b*x^(1/5) + b
^2*x^(2/5)] - Sqrt[b^2]*x^(1/5)])/Sqrt[b^2] + 960*a^2*Sqrt[a^2 + 2*a*b*x^(1
/5) + b^2*x^(2/5)]*x*Log[a + Sqrt[a^2 + 2*a*b*x^(1/5) + b^2*x^(2/5)] - Sqrt
[b^2]*x^(1/5)] - 1920*a^2*Sqrt[b^2]*x^(6/5)*Log[a + Sqrt[a^2 + 2*a*b*x^(1/5
) + b^2*x^(2/5)] - Sqrt[b^2]*x^(1/5)] + 960*a*b*Sqrt[a^2 + 2*a*b*x^(1/5) +
b^2*x^(2/5)]*x^(6/5)*Log[a + Sqrt[a^2 + 2*a*b*x^(1/5) + b^2*x^(2/5)] - Sqrt
[b^2]*x^(1/5)] - (1280*a*b^3*x^(7/5)*Log[a + Sqrt[a^2 + 2*a*b*x^(1/5) + b^2
*x^(2/5)] - Sqrt[b^2]*x^(1/5)])/Sqrt[b^2] + 320*b^2*Sqrt[a^2 + 2*a*b*x^(1/5
) + b^2*x^(2/5)]*x^(7/5)*Log[a + Sqrt[a^2 + 2*a*b*x^(1/5) + b^2*x^(2/5)] -
Sqrt[b^2]*x^(1/5)] - (320*b^4*x^(8/5)*Log[a + Sqrt[a^2 + 2*a*b*x^(1/5) + b^
2*x^(2/5)] - Sqrt[b^2]*x^(1/5)])/Sqrt[b^2])/((-a + Sqrt[a^2 + 2*a*b*x^(1/5)
 + b^2*x^(2/5)] - Sqrt[b^2]*x^(1/5))^4*(a + Sqrt[a^2 + 2*a*b*x^(1/5) + b^2*
x^(2/5)] - Sqrt[b^2]*x^(1/5))^4)
```

fricas [**A**] time = 0.91, size = 302, normalized size = 1.36

$$\frac{5\left(300\,a^{5}b^{15}x^{3}+100\,a^{15}b^{5}x+25\,a^{20}+12\left(b^{20}x^{4}+4\,a^{5}b^{15}x^{3}+6\,a^{10}b^{10}x^{2}+4\,a^{15}b^{5}x+a^{20}\right)\log\left(bx^{\frac{1}{5}}+a\right)+\left(48\,ab^{19}x^{3}-226\,a^{6}b^{14}x^{2}+104\,a^{11}b^{9}x+3\,a^{16}b^{4}\right)x^{\frac{4}{5}}-\left(84\,a^{2}b^{18}x^{3}-228\,a^{7}b^{13}x^{2}+67\,a^{12}b^{8}x+4\,a^{17}b^{3}\right)x^{\frac{3}{5}}+\left(136\,a^{3}b^{17}x^{3}-197\,a^{8}b^{12}x^{2}+48\,a^{13}b^{7}x+6\,a^{18}b^{2}\right)x^{\frac{2}{5}}-\left(207\,a^{4}b^{16}x^{3}-124\,a^{9}b^{11}x^{2}+56\,a^{14}b^{6}x+12\,a^{19}b\right)x^{\frac{1}{5}}\right)}{12\left(b^{25}x^{4}+4\,a^{5}b^{20}x^{3}+6\,a^{10}b^{15}x^{2}+4\,a^{15}b^{10}x+a^{20}b^{5}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2+2*a*b*x^(1/5)+b^2*x^(2/5))^(5/2),x, algorithm="fricas")
```

```
[Out] 5/12*(300*a^5*b^15*x^3 + 100*a^15*b^5*x + 25*a^20 + 12*(b^20*x^4 + 4*a^5*b^
15*x^3 + 6*a^10*b^10*x^2 + 4*a^15*b^5*x + a^20)*log(b*x^(1/5) + a) + (48*a*
b^19*x^3 - 226*a^6*b^14*x^2 + 104*a^11*b^9*x + 3*a^16*b^4)*x^(4/5) - (84*a^
2*b^18*x^3 - 228*a^7*b^13*x^2 + 67*a^12*b^8*x + 4*a^17*b^3)*x^(3/5) + (136*
a^3*b^17*x^3 - 197*a^8*b^12*x^2 + 48*a^13*b^7*x + 6*a^18*b^2)*x^(2/5) - (20
7*a^4*b^16*x^3 - 124*a^9*b^11*x^2 + 56*a^14*b^6*x + 12*a^19*b)*x^(1/5))/(b^
25*x^4 + 4*a^5*b^20*x^3 + 6*a^10*b^15*x^2 + 4*a^15*b^10*x + a^20*b^5)
```

giac [A] time = 0.59, size = 84, normalized size = 0.38

$$\frac{5\,\log\left(\left|bx^{\frac{1}{5}}+a\right|\right)}{b^{5}\mathrm{sgn}\left(bx^{\frac{1}{5}}+a\right)}+\frac{5\left(48\,ab^{2}x^{\frac{3}{5}}+108\,a^{2}bx^{\frac{2}{5}}+88\,a^{3}x^{\frac{1}{5}}+\frac{25\,a^{4}}{b}\right)}{12\left(bx^{\frac{1}{5}}+a\right)^{4}b^{4}\mathrm{sgn}\left(bx^{\frac{1}{5}}+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2+2*a*b*x^(1/5)+b^2*x^(2/5))^(5/2),x, algorithm="giac")
```

```
[Out] 5*log(abs(b*x^(1/5) + a))/(b^5*sgn(b*x^(1/5) + a)) + 5/12*(48*a*b^2*x^(3/5)
 + 108*a^2*b*x^(2/5) + 88*a^3*x^(1/5) + 25*a^4/b)/((b*x^(1/5) + a)^4*b^4*sg
n(b*x^(1/5) + a))
```

maple [A] time = 0.01, size = 152, normalized size = 0.68

$$\frac{5\sqrt{b^{2}x^{\frac{2}{5}}+2ab\,x^{\frac{1}{5}}+a^{2}}\,\left(12b^{4}x^{\frac{4}{5}}\ln\left(b\,x^{\frac{1}{5}}+a\right)+48a\,b^{3}x^{\frac{3}{5}}\ln\left(b\,x^{\frac{1}{5}}+a\right)+72a^{2}b^{2}x^{\frac{2}{5}}\ln\left(b\,x^{\frac{1}{5}}+a\right)+48a^{3}b\,x^{\frac{1}{5}}\ln\left(b\,x^{\frac{1}{5}}+a\right)+12a^{4}\ln\left(b\,x^{\frac{1}{5}}+a\right)+48a\,b^{3}x^{\frac{3}{5}}+108a^{2}b^{2}x^{\frac{2}{5}}+88a^{3}b\,x^{\frac{1}{5}}+25a^{4}\right)}{12\left(b\,x^{\frac{1}{5}}+a\right)^{5}b^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a^2+2*a*b*x^(1/5)+b^2*x^(2/5))^(5/2),x)
```

```
[Out] 5/12*(a^2+2*a*b*x^(1/5)+b^2*x^(2/5))^(1/2)*(12*x^(4/5)*ln(a+b*x^(1/5))*b^4+
48*x^(3/5)*ln(a+b*x^(1/5))*a*b^3+48*x^(3/5)*a*b^3+72*x^(2/5)*ln(a+b*x^(1/5)
)*a^2*b^2+108*x^(2/5)*a^2*b^2+48*x^(1/5)*ln(a+b*x^(1/5))*a^3*b+88*x^(1/5)*a
^3*b+12*ln(a+b*x^(1/5))*a^4+25*a^4)/(a+b*x^(1/5))^5/b^5
```

maxima [A] time = 1.02, size = 99, normalized size = 0.45

$$\frac{5\left(48\,ab^{3}x^{\frac{3}{5}}+108\,a^{2}b^{2}x^{\frac{2}{5}}+88\,a^{3}bx^{\frac{1}{5}}+25\,a^{4}\right)}{12\left(b^{9}x^{\frac{4}{5}}+4\,ab^{8}x^{\frac{3}{5}}+6\,a^{2}b^{7}x^{\frac{2}{5}}+4\,a^{3}b^{6}x^{\frac{1}{5}}+a^{4}b^{5}\right)}+\frac{5\,\log\left(bx^{\frac{1}{5}}+a\right)}{b^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2+2*a*b*x^(1/5)+b^2*x^(2/5))^(5/2),x, algorithm="maxima")
```

```
[Out] 5/12*(48*a*b^3*x^(3/5) + 108*a^2*b^2*x^(2/5) + 88*a^3*b*x^(1/5) + 25*a^4)/(
b^9*x^(4/5) + 4*a*b^8*x^(3/5) + 6*a^2*b^7*x^(2/5) + 4*a^3*b^6*x^(1/5) + a^4
*b^5) + 5*log(b*x^(1/5) + a)/b^5
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{{\left(a^2+b^2\,x^{2/5}+2\,a\,b\,x^{1/5}\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a^2 + b^2*x^(2/5) + 2*a*b*x^(1/5))^(5/2),x)
```

```
[Out] int(1/(a^2 + b^2*x^(2/5) + 2*a*b*x^(1/5))^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a^{2}+2ab\sqrt[5]{x}+b^{2}x^{\frac{2}{5}}\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a**2+2*a*b*x**(1/5)+b**2*x**(2/5))**(5/2),x)
```

```
[Out] Integral((a**2 + 2*a*b*x**(1/5) + b**2*x**(2/5))**(-5/2), x)
```

3.428 $\int \left(a^2+\frac{b^2}{\sqrt[3]{x}}+\frac{2ab}{\sqrt[6]{x}}\right)^{7/2} dx$

Optimal. Leaf size=391

$$-\frac{6b^7\sqrt{a^2+\frac{2ab}{\sqrt[6]{x}}+\frac{b^2}{\sqrt[3]{x}}}}{\sqrt[6]{x}\left(a+\frac{b}{\sqrt[6]{x}}\right)}+\frac{42ab^6\log\left(\sqrt[6]{x}\right)\sqrt{a^2+\frac{2ab}{\sqrt[6]{x}}+\frac{b^2}{\sqrt[3]{x}}}}{a+\frac{b}{\sqrt[6]{x}}}+\frac{126a^2b^5\sqrt[6]{x}\sqrt{a^2+\frac{2ab}{\sqrt[6]{x}}+\frac{b^2}{\sqrt[3]{x}}}}{a+\frac{b}{\sqrt[6]{x}}}+\frac{a^7x\sqrt{a^2+\frac{2ab}{\sqrt[6]{x}}+ \cdots}}{a+\frac{b}{\sqrt[6]{x}}}$$

Rubi [A] time = 0.18, antiderivative size = 391, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {1341, 1355, 263, 43}

$$\frac{42a^6bx^{5/6}\sqrt{a^2+\frac{2ab}{\sqrt[6]{x}}+\frac{b^2}{\sqrt[3]{x}}}}{5\left(a+\frac{b}{\sqrt[6]{x}}\right)}+\frac{63a^5b^2x^{2/3}\sqrt{a^2+\frac{2ab}{\sqrt[6]{x}}+\frac{b^2}{\sqrt[3]{x}}}}{2\left(a+\frac{b}{\sqrt[6]{x}}\right)}+\frac{a^7x\sqrt{a^2+\frac{2ab}{\sqrt[6]{x}}+\frac{b^2}{\sqrt[3]{x}}}}{a+\frac{b}{\sqrt[6]{x}}}+\frac{70a^4b^3\sqrt{x}\sqrt{a^2+\frac{2ab}{\sqrt[6]{x}}+\frac{b^2}{\sqrt[3]{x}}}}{a+\frac{b}{\sqrt[6]{x}}}+\frac{105a^3b^4\sqrt[3]{x}\sqrt{a^2+\frac{2ab}{\sqrt[6]{x}}+\frac{b^2}{\sqrt[3]{x}}}}{a+\frac{b}{\sqrt[6]{x}}}+\frac{126a^2b^5\sqrt[6]{x}\sqrt{a^2+\frac{2ab}{\sqrt[6]{x}}+\frac{b^2}{\sqrt[3]{x}}}}{a+\frac{b}{\sqrt[6]{x}}}-\frac{6b^7\sqrt{a^2+\frac{2ab}{\sqrt[6]{x}}+\frac{b^2}{\sqrt[3]{x}}}}{\sqrt[6]{x}\left(a+\frac{b}{\sqrt[6]{x}}\right)}+\frac{42ab^6\log\left(\sqrt[6]{x}\right)\sqrt{a^2+\frac{2ab}{\sqrt[6]{x}}+\frac{b^2}{\sqrt[3]{x}}}}{a+\frac{b}{\sqrt[6]{x}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + b^2/x^(1/3) + (2*a*b)/x^(1/6))^(7/2),x]
```

```
[Out] (-6*b^7*Sqrt[a^2 + b^2/x^(1/3) + (2*a*b)/x^(1/6)])/((a + b/x^(1/6))*x^(1/6)
) + (126*a^2*b^5*Sqrt[a^2 + b^2/x^(1/3) + (2*a*b)/x^(1/6)]*x^(1/6))/(a + b/
x^(1/6)) + (105*a^3*b^4*Sqrt[a^2 + b^2/x^(1/3) + (2*a*b)/x^(1/6)]*x^(1/3))/
(a + b/x^(1/6)) + (70*a^4*b^3*Sqrt[a^2 + b^2/x^(1/3) + (2*a*b)/x^(1/6)]*Sqr
t[x])/(a + b/x^(1/6)) + (63*a^5*b^2*Sqrt[a^2 + b^2/x^(1/3) + (2*a*b)/x^(1/6
)]*x^(2/3))/(2*(a + b/x^(1/6))) + (42*a^6*b*Sqrt[a^2 + b^2/x^(1/3) + (2*a*b
)/x^(1/6)]*x^(5/6))/(5*(a + b/x^(1/6))) + (a^7*Sqrt[a^2 + b^2/x^(1/3) + (2*
a*b)/x^(1/6)]*x)/(a + b/x^(1/6)) + (42*a*b^6*Sqrt[a^2 + b^2/x^(1/3) + (2*a*
b)/x^(1/6)]*Log[x^(1/6)])/(a + b/x^(1/6))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 1341

```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k
 = Denominator[n]}, Dist[k, Subst[Int[x^(k - 1)*(a + b*x^(k*n) + c*x^(2*k*n
))^p, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && EqQ[n2, 2*n] && Fra
ctionQ[n]
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\int \left(a^2+\frac{b^2}{\sqrt[3]{x}}+\frac{2ab}{\sqrt[6]{x}}\right)^{7/2} dx = 6\,\text{Subst}\left(\int \left(a^2+\frac{b^2}{x^2}+\frac{2ab}{x}\right)^{7/2} x^5\,dx, x, \sqrt[6]{x}\right)$$

$$= \frac{\left(6\sqrt{a^2+\frac{b^2}{\sqrt[3]{x}}+\frac{2ab}{\sqrt[6]{x}}}\right)\text{Subst}\left(\int \left(ab+\frac{b^2}{x}\right)^7 x^5\,dx, x, \sqrt[6]{x}\right)}{b^6\left(ab+\frac{b^2}{\sqrt[6]{x}}\right)}$$

$$= \frac{\left(6\sqrt{a^2+\frac{b^2}{\sqrt[3]{x}}+\frac{2ab}{\sqrt[6]{x}}}\right)\text{Subst}\left(\int \frac{\left(b^2+abx\right)^7}{x^2}\,dx, x, \sqrt[6]{x}\right)}{b^6\left(ab+\frac{b^2}{\sqrt[6]{x}}\right)}$$

$$= \frac{\left(6\sqrt{a^2+\frac{b^2}{\sqrt[3]{x}}+\frac{2ab}{\sqrt[6]{x}}}\right)\text{Subst}\left(\int \left(21a^2b^{12}+\frac{b^{14}}{x^2}+\frac{7ab^{13}}{x}+35a^3b^{11}x+35a^4b^{10}x^2+21\right.\right.}{b^6\left(ab+\frac{b^2}{\sqrt[6]{x}}\right)}$$

$$= -\frac{6b^8\sqrt{a^2+\frac{b^2}{\sqrt[3]{x}}+\frac{2ab}{\sqrt[6]{x}}}}{\left(ab+\frac{b^2}{\sqrt[6]{x}}\right)\sqrt[6]{x}}+\frac{126a^2b^6\sqrt{a^2+\frac{b^2}{\sqrt[3]{x}}+\frac{2ab}{\sqrt[6]{x}}}\sqrt[6]{x}}{ab+\frac{b^2}{\sqrt[6]{x}}}+\frac{105a^3b^5\sqrt{a^2+\frac{b^2}{\sqrt[3]{x}}+\frac{2ab}{\sqrt[6]{x}}}}{ab+\frac{b^2}{\sqrt[6]{x}}}$$

Mathematica [A] time = 0.07, size = 124, normalized size = 0.32

$$\frac{\sqrt{\frac{\left(a\sqrt[6]{x}+b\right)^2}{\sqrt[3]{x}}}\left(10a^7x^{7/6}+84a^6bx+315a^5b^2x^{5/6}+700a^4b^3x^{2/3}+1050a^3b^4\sqrt{x}+1260a^2b^5\sqrt[3]{x}+70ab^6\sqrt[6]{x}\log(x)-60b^7\right)}{10\left(a\sqrt[6]{x}+b\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + b^2/x^(1/3) + (2*a*b)/x^(1/6))^(7/2),x]
```

```
[Out] (Sqrt[(b + a*x^(1/6))^2/x^(1/3)]*(-60*b^7 + 1260*a^2*b^5*x^(1/3) + 1050*a^3
*b^4*Sqrt[x] + 700*a^4*b^3*x^(2/3) + 315*a^5*b^2*x^(5/6) + 84*a^6*b*x + 10*
a^7*x^(7/6) + 70*a*b^6*x^(1/6)*Log[x]))/(10*(b + a*x^(1/6)))
```

IntegrateAlgebraic [A] time = 6.78, size = 135, normalized size = 0.35

$$\frac{\sqrt[6]{x}\sqrt{\frac{\left(a\sqrt[6]{x}+b\right)^2}{\sqrt[3]{x}}}\left(\frac{10a^7x^{7/6}+84a^6bx+315a^5b^2x^{5/6}+700a^4b^3x^{2/3}+1050a^3b^4\sqrt{x}+1260a^2b^5\sqrt[3]{x}-60b^7}{10\sqrt[6]{x}}+42ab^6\log\left(\sqrt[6]{x}\right)\right)}{a\sqrt[6]{x}+b}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a^2 + b^2/x^(1/3) + (2*a*b)/x^(1/6))^(7/2),x]
```

```
[Out] (Sqrt[(b + a*x^(1/6))^2/x^(1/3)]*x^(1/6)*((-60*b^7 + 1260*a^2*b^5*x^(1/3) +
 1050*a^3*b^4*Sqrt[x] + 700*a^4*b^3*x^(2/3) + 315*a^5*b^2*x^(5/6) + 84*a^6*
b*x + 10*a^7*x^(7/6))/(10*x^(1/6)) + 42*a*b^6*Log[x^(1/6)]))/(b + a*x^(1/6)
)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+b^2/x^(1/3)+2*a*b/x^(1/6))^(7/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [A] time = 0.46, size = 172, normalized size = 0.44

$$a^7 x \operatorname{sgn}\left(ax + bx^{\frac{5}{6}}\right)\operatorname{sgn}(x) + 7\,ab^6 \log\left(|x|\right)\operatorname{sgn}\left(ax + bx^{\frac{5}{6}}\right)\operatorname{sgn}(x) + \frac{42}{5}\,a^6 b x^{\frac{5}{6}}\operatorname{sgn}\left(ax + bx^{\frac{5}{6}}\right)\operatorname{sgn}(x) + \frac{63}{2}\,a^5 b^2 x^{\frac{2}{3}}\operatorname{sgn}\left(ax + bx^{\frac{5}{6}}\right)\operatorname{sgn}(x) + 70\,a^4 b^3 \sqrt{x}\operatorname{sgn}\left(ax + bx^{\frac{5}{6}}\right)\operatorname{sgn}(x) + 105\,a^3 b^4 x^{\frac{1}{3}}\operatorname{sgn}\left(ax + bx^{\frac{5}{6}}\right)\operatorname{sgn}(x) + 126\,a^2 b^5 x^{\frac{1}{6}}\operatorname{sgn}\left(ax + bx^{\frac{5}{6}}\right)\operatorname{sgn}(x) - \frac{6\,b^7 \operatorname{sgn}\left(ax + bx^{\frac{5}{6}}\right)\operatorname{sgn}(x)}{x^{\frac{1}{6}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+b^2/x^(1/3)+2*a*b/x^(1/6))^(7/2),x, algorithm="giac")
```

```
[Out] a^7*x*sgn(a*x + b*x^(5/6))*sgn(x) + 7*a*b^6*log(abs(x))*sgn(a*x + b*x^(5/6)
)*sgn(x) + 42/5*a^6*b*x^(5/6)*sgn(a*x + b*x^(5/6))*sgn(x) + 63/2*a^5*b^2*x^
(2/3)*sgn(a*x + b*x^(5/6))*sgn(x) + 70*a^4*b^3*sqrt(x)*sgn(a*x + b*x^(5/6))
*sgn(x) + 105*a^3*b^4*x^(1/3)*sgn(a*x + b*x^(5/6))*sgn(x) + 126*a^2*b^5*x^(
1/6)*sgn(a*x + b*x^(5/6))*sgn(x) - 6*b^7*sgn(a*x + b*x^(5/6))*sgn(x)/x^(1/6
)
```

maple [A] time = 0.03, size = 116, normalized size = 0.30

$$\frac{\sqrt{\frac{a^2\sqrt{x}+2ab\,x^{\frac{1}{3}}+b^2 x^{\frac{1}{6}}}{\sqrt{x}}}\left(10a^7 x^{\frac{7}{6}}+70a\,b^6 x^{\frac{1}{6}}\ln(x)+84a^6 bx+315a^5 b^2 x^{\frac{5}{6}}+700a^4 b^3 x^{\frac{2}{3}}+1050a^3 b^4\sqrt{x}+1260a^2 b^5 x^{\frac{1}{3}}-60b^7\right)}{10a\,x^{\frac{1}{6}}+10b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2+b^2/x^(1/3)+2*a*b/x^(1/6))^(7/2),x)
```

```
[Out] 1/10*((a^2*x^(1/2)+2*a*b*x^(1/3)+b^2*x^(1/6))/x^(1/2))^(1/2)*(84*a^6*b*x+31
5*a^5*b^2*x^(5/6)+70*a*b^6*ln(x)*x^(1/6)+1050*a^3*b^4*x^(1/2)+1260*a^2*b^5*
x^(1/3)+700*a^4*b^3*x^(2/3)+10*a^7*x^(7/6)-60*b^7)/(a*x^(1/6)+b)
```

maxima [A] time = 0.95, size = 79, normalized size = 0.20

$$7\,ab^6\log(x)+\frac{10\,a^7 x^{\frac{7}{6}}+84\,a^6 bx+315\,a^5 b^2 x^{\frac{5}{6}}+700\,a^4 b^3 x^{\frac{2}{3}}+1050\,a^3 b^4\sqrt{x}+1260\,a^2 b^5 x^{\frac{1}{3}}-60\,b^7}{10\,x^{\frac{1}{6}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+b^2/x^(1/3)+2*a*b/x^(1/6))^(7/2),x, algorithm="maxima")
```

```
[Out] 7*a*b^6*log(x) + 1/10*(10*a^7*x^(7/6) + 84*a^6*b*x + 315*a^5*b^2*x^(5/6) +
700*a^4*b^3*x^(2/3) + 1050*a^3*b^4*sqrt(x) + 1260*a^2*b^5*x^(1/3) - 60*b^7)
/x^(1/6)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int {\left(a^2+\frac{b^2}{x^{1/3}}+\frac{2\,a\,b}{x^{1/6}}\right)}^{7/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2/x^(1/3) + (2*a*b)/x^(1/6))^(7/2),x)
```

```
[Out] int((a^2 + b^2/x^(1/3) + (2*a*b)/x^(1/6))^(7/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2+b**2/x**(1/3)+2*a*b/x**(1/6))**(7/2),x)
```

```
[Out] Timed out
```

3.429 $\int \frac{x^{-1+4n}}{bx^n+cx^{2n}}\,dx$

Optimal. Leaf size=46

$$\frac{b^2\log\left(b+cx^n\right)}{c^3n}-\frac{bx^n}{c^2n}+\frac{x^{2n}}{2cn}$$

Rubi [A] time = 0.04, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {1584, 266, 43}

$$\frac{b^2\log\left(b+cx^n\right)}{c^3n}-\frac{bx^n}{c^2n}+\frac{x^{2n}}{2cn}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 + 4*n)/(b*x^n + c*x^(2*n)),x]
```

```
[Out] -((b*x^n)/(c^2*n)) + x^(2*n)/(2*c*n) + (b^2*Log[b + c*x^n])/(c^3*n)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \frac{x^{-1+4n}}{bx^n+cx^{2n}}\,dx&=\int \frac{x^{-1+3n}}{b+cx^n}\,dx\\&=\frac{\text{Subst}\left(\int \frac{x^2}{b+cx}\,dx,x,x^n\right)}{n}\\&=\frac{\text{Subst}\left(\int \left(-\frac{b}{c^2}+\frac{x}{c}+\frac{b^2}{c^2(b+cx)}\right)\,dx,x,x^n\right)}{n}\\&=-\frac{bx^n}{c^2n}+\frac{x^{2n}}{2cn}+\frac{b^2\log\left(b+cx^n\right)}{c^3n}\end{aligned}$$

Mathematica [A] time = 0.03, size = 38, normalized size = 0.83

$$\frac{2b^2\log\left(b+cx^n\right)+cx^n\left(cx^n-2b\right)}{2c^3n}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 + 4*n)/(b*x^n + c*x^(2*n)),x]
[Out] (c*x^n*(-2*b + c*x^n) + 2*b^2*Log[b + c*x^n])/(2*c^3*n)
```

IntegrateAlgebraic [A] time = 0.06, size = 41, normalized size = 0.89

$$\frac{b^2 \log(b+cx^n)}{c^3 n}+\frac{x^n (cx^n-2b)}{2c^2 n}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(-1 + 4*n)/(b*x^n + c*x^(2*n)),x]
[Out] (x^n*(-2*b + c*x^n))/(2*c^2*n) + (b^2*Log[b + c*x^n])/(c^3*n)
```

fricas [A] time = 1.64, size = 38, normalized size = 0.83

$$\frac{c^2 x^{2n}-2\,bcx^n+2\,b^2\log(cx^n+b)}{2\,c^3 n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+4*n)/(b*x^n+c*x^(2*n)),x, algorithm="fricas")
[Out] 1/2*(c^2*x^(2*n) - 2*b*c*x^n + 2*b^2*log(c*x^n + b))/(c^3*n)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{4n-1}}{cx^{2n}+bx^n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+4*n)/(b*x^n+c*x^(2*n)),x, algorithm="giac")
[Out] integrate(x^(4*n - 1)/(c*x^(2*n) + b*x^n), x)
```

maple [A] time = 0.03, size = 62, normalized size = 1.35

$$\left(-\frac{b\,\mathrm{e}^{2n\ln(x)}}{c^2 n}+\frac{\mathrm{e}^{3n\ln(x)}}{2cn}\right)\mathrm{e}^{-n\ln(x)}+\frac{b^2\ln\left(c\,\mathrm{e}^{n\ln(x)}+b\right)}{c^3 n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1+4*n)/(b*x^n+c*x^(2*n)),x)
[Out] (1/2/c/n*exp(n*ln(x))^3-b/c^2/n*exp(n*ln(x))^2)/exp(n*ln(x))+b^2/c^3/n*ln(c
*exp(n*ln(x))+b)
```

maxima [A] time = 0.93, size = 45, normalized size = 0.98

$$\frac{b^2\log\left(\frac{cx^n+b}{c}\right)}{c^3 n}+\frac{cx^{2n}-2\,bx^n}{2\,c^2 n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+4*n)/(b*x^n+c*x^(2*n)),x, algorithm="maxima")
[Out] b^2*log((c*x^n + b)/c)/(c^3*n) + 1/2*(c*x^(2*n) - 2*b*x^n)/(c^2*n)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^{4n-1}}{b\,x^n+c\,x^{2n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(4*n - 1)/(b*x^n + c*x^(2*n)),x)
```

```
[Out] int(x^(4*n - 1)/(b*x^n + c*x^(2*n)), x)
```

sympy [A] time = 19.83, size = 42, normalized size = 0.91

$$\frac{b^2 \left(\begin{cases} \frac{x^n}{b} & \text{for } c = 0 \\ \frac{\log(b+cx^n)}{c} & \text{otherwise} \end{cases}\right)}{c^2 n} - \frac{bx^n}{c^2 n} + \frac{x^{2n}}{2cn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+4*n)/(b*x**n+c*x**(2*n)),x)
```

```
[Out] b**2*Piecewise((x**n/b, Eq(c, 0)), (log(b + c*x**n)/c, True))/(c**2*n) - b*
x**n/(c**2*n) + x**(2*n)/(2*c*n)
```

3.430 $\int \frac{x^{-1+3n}}{bx^n+cx^{2n}}\,dx$

Optimal. Leaf size=28

$$\frac{x^n}{cn}-\frac{b\log\left(b+cx^n\right)}{c^2n}$$

Rubi [A] time = 0.03, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {1584, 266, 43}

$$\frac{x^n}{cn}-\frac{b\log\left(b+cx^n\right)}{c^2n}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 + 3*n)/(b*x^n + c*x^(2*n)),x]
```

```
[Out] x^n/(c*n) - (b*Log[b + c*x^n])/(c^2*n)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^{-1+3n}}{bx^n+cx^{2n}}\,dx &= \int \frac{x^{-1+2n}}{b+cx^n}\,dx\\
&= \frac{\text{Subst}\left(\int \frac{x}{b+cx}\,dx,x,x^n\right)}{n}\\
&= \frac{\text{Subst}\left(\int \left(\frac{1}{c}-\frac{b}{c(b+cx)}\right)dx,x,x^n\right)}{n}\\
&= \frac{x^n}{cn}-\frac{b\log\left(b+cx^n\right)}{c^2n}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 26, normalized size = 0.93

$$\frac{\frac{x^n}{c}-\frac{b\log(b+cx^n)}{c^2}}{n}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 + 3*n)/(b*x^n + c*x^(2*n)),x]
[Out] (x^n/c - (b*Log[b + c*x^n])/c^2)/n
```

IntegrateAlgebraic [A] time = 0.07, size = 34, normalized size = 1.21

$$\frac{x^n}{cn} - \frac{b\log\left(bcn + c^2nx^n\right)}{c^2n}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(-1 + 3*n)/(b*x^n + c*x^(2*n)),x]
[Out] x^n/(c*n) - (b*Log[b*c*n + c^2*n*x^n])/(c^2*n)
```

fricas [A] time = 2.06, size = 24, normalized size = 0.86

$$\frac{cx^n - b\log\left(cx^n + b\right)}{c^2n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+3*n)/(b*x^n+c*x^(2*n)),x, algorithm="fricas")
[Out] (c*x^n - b*log(c*x^n + b))/(c^2*n)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{3n-1}}{cx^{2n} + bx^n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+3*n)/(b*x^n+c*x^(2*n)),x, algorithm="giac")
[Out] integrate(x^(3*n - 1)/(c*x^(2*n) + b*x^n), x)
```

maple [A] time = 0.02, size = 33, normalized size = 1.18

$$-\frac{b\ln\left(c\,e^{n\ln(x)} + b\right)}{c^2n} + \frac{e^{n\ln(x)}}{cn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1+3*n)/(b*x^n+c*x^(2*n)),x)
[Out] 1/c/n*exp(n*ln(x))-b/c^2/n*ln(c*exp(n*ln(x))+b)
```

maxima [A] time = 0.87, size = 32, normalized size = 1.14

$$\frac{x^n}{cn} - \frac{b\log\left(\frac{cx^n+b}{c}\right)}{c^2n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+3*n)/(b*x^n+c*x^(2*n)),x, algorithm="maxima")
[Out] x^n/(c*n) - b*log((c*x^n + b)/c)/(c^2*n)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{x^{3n-1}}{b\,x^n + c\,x^{2n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3*n - 1)/(b*x^n + c*x^(2*n)),x)
```

```
[Out] int(x^(3*n - 1)/(b*x^n + c*x^(2*n)), x)
```

sympy **[A]** time = 13.89, size = 26, normalized size = 0.93

$$-\frac{b\left(\begin{cases}\frac{x^{n}}{b} & \text{for}\: c = 0\\\frac{\log\left(b+cx^{n}\right)}{c} & \text{otherwise}\end{cases}\right)}{cn}+\frac{x^{n}}{cn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+3*n)/(b*x**n+c*x**(2*n)),x)
```

```
[Out] -b*Piecewise((x**n/b, Eq(c, 0)), (log(b + c*x**n)/c, True))/(c*n) + x**n/(c
*n)
```

3.431 $\int \frac{x^{-1+2n}}{bx^n+cx^{2n}}\,dx$

Optimal. Leaf size=15

$$\frac{\log(b+cx^n)}{cn}$$

Rubi [A] time = 0.01, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {1584, 260}

$$\frac{\log(b+cx^n)}{cn}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 + 2*n)/(b*x^n + c*x^(2*n)),x]
[Out] Log[b + c*x^n]/(c*n)
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \frac{x^{-1+2n}}{bx^n+cx^{2n}}\,dx &= \int \frac{x^{-1+n}}{b+cx^n}\,dx\\ &= \frac{\log(b+cx^n)}{cn}\end{aligned}$$

Mathematica [A] time = 0.00, size = 15, normalized size = 1.00

$$\frac{\log(b+cx^n)}{cn}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 + 2*n)/(b*x^n + c*x^(2*n)),x]
[Out] Log[b + c*x^n]/(c*n)
```

IntegrateAlgebraic [A] time = 0.05, size = 18, normalized size = 1.20

$$\frac{\log(bn+cnx^n)}{cn}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(-1 + 2*n)/(b*x^n + c*x^(2*n)),x]
[Out] Log[b*n + c*n*x^n]/(c*n)
```

fricas [A] time = 1.13, size = 15, normalized size = 1.00

$$\frac{\log\left(cx^{n}+b\right)}{cn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+2*n)/(b*x^n+c*x^(2*n)),x, algorithm="fricas")
```

```
[Out] log(c*x^n + b)/(c*n)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{2\,n-1}}{cx^{2\,n}+bx^{n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+2*n)/(b*x^n+c*x^(2*n)),x, algorithm="giac")
```

```
[Out] integrate(x^(2*n - 1)/(c*x^(2*n) + b*x^n), x)
```

maple [A] time = 0.02, size = 18, normalized size = 1.20

$$\frac{\ln\left(c\,\mathrm{e}^{n\ln(x)}+b\right)}{cn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1+2*n)/(b*x^n+c*x^(2*n)),x)
```

```
[Out] 1/c/n*ln(c*exp(n*ln(x))+b)
```

maxima [A] time = 0.89, size = 19, normalized size = 1.27

$$\frac{\log\left(\frac{cx^{n}+b}{c}\right)}{cn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+2*n)/(b*x^n+c*x^(2*n)),x, algorithm="maxima")
```

```
[Out] log((c*x^n + b)/c)/(c*n)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.07

$$\int \frac{x^{2\,n-1}}{b\,x^{n}+c\,x^{2\,n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(2*n - 1)/(b*x^n + c*x^(2*n)),x)
```

```
[Out] int(x^(2*n - 1)/(b*x^n + c*x^(2*n)), x)
```

sympy [A] time = 11.28, size = 37, normalized size = 2.47

$$\begin{cases} \frac{\log\left(x\right)}{b} & \text{for}\: c = 0 \wedge n = 0 \\ \frac{\log\left(x\right)}{b+c} & \text{for}\: n = 0 \\ \frac{x^{n}}{bn} & \text{for}\: c = 0 \\ -\frac{\log\left(x\right)}{c} + \frac{\log\left(\frac{bx^{n}}{c}+x^{2n}\right)}{cn} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+2*n)/(b*x**n+c*x**(2*n)),x)
```

```
[Out] Piecewise((log(x)/b, Eq(c, 0) & Eq(n, 0)), (log(x)/(b + c), Eq(n, 0)), (x**
n/(b*n), Eq(c, 0)), (-log(x)/c + log(b*x**n/c + x**(2*n))/(c*n), True))
```

3.432 $\int \frac{x^{-1+n}}{bx^n+cx^{2n}}\,dx$

Optimal. Leaf size=23

$$\frac{\log(x)}{b}-\frac{\log\left(b+cx^n\right)}{bn}$$

Rubi [A] time = 0.02, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {1584, 266, 36, 29, 31}

$$\frac{\log(x)}{b}-\frac{\log\left(b+cx^n\right)}{bn}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 + n)/(b*x^n + c*x^(2*n)),x]
```

```
[Out] Log[x]/b - Log[b + c*x^n]/(b*n)
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^{-1+n}}{bx^n+cx^{2n}}\,dx &= \int \frac{1}{x\left(b+cx^n\right)}\,dx\\
&= \frac{\text{Subst}\left(\int \frac{1}{x(b+cx)}\,dx,x,x^n\right)}{n}\\
&= \frac{\text{Subst}\left(\int \frac{1}{x}\,dx,x,x^n\right)}{bn}-\frac{c\,\text{Subst}\left(\int \frac{1}{b+cx}\,dx,x,x^n\right)}{bn}\\
&= \frac{\log(x)}{b}-\frac{\log\left(b+cx^n\right)}{bn}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 22, normalized size = 0.96

$$\frac{n\log(x)-\log(b+cx^n)}{bn}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 + n)/(b*x^n + c*x^(2*n)),x]
[Out] (n*Log[x] - Log[b + c*x^n])/(b*n)
```

IntegrateAlgebraic [A] time = 0.05, size = 34, normalized size = 1.48

$$\frac{\log(x^n)}{bn}-\frac{\log\left(b^2n+bcnx^n\right)}{bn}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(-1 + n)/(b*x^n + c*x^(2*n)),x]
[Out] Log[x^n]/(b*n) - Log[b^2*n + b*c*n*x^n]/(b*n)
```

fricas [A] time = 1.46, size = 22, normalized size = 0.96

$$\frac{n\log(x)-\log(cx^n+b)}{bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n)/(b*x^n+c*x^(2*n)),x, algorithm="fricas")
[Out] (n*log(x) - log(c*x^n + b))/(b*n)
```

giac [A] time = 0.25, size = 25, normalized size = 1.09

$$\frac{\log(|x|)}{b}-\frac{\log(|cx^n+b|)}{bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n)/(b*x^n+c*x^(2*n)),x, algorithm="giac")
[Out] log(abs(x))/b - log(abs(c*x^n + b))/(b*n)
```

maple [A] time = 0.02, size = 26, normalized size = 1.13

$$\frac{\ln(x)}{b}-\frac{\ln\left(c\,\mathrm{e}^{n\ln(x)}+b\right)}{bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1+n)/(b*x^n+c*x^(2*n)),x)
[Out] ln(x)/b-1/b/n*ln(c*exp(n*ln(x))+b)
```

maxima [A] time = 0.90, size = 27, normalized size = 1.17

$$\frac{\log(x)}{b}-\frac{\log\left(\frac{cx^n+b}{c}\right)}{bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n)/(b*x^n+c*x^(2*n)),x, algorithm="maxima")
```

```
[Out] log(x)/b - log((c*x^n + b)/c)/(b*n)
```

mupad [B] time = 1.37, size = 20, normalized size = 0.87

$$-\frac{2\,\mathrm{atanh}\left(\frac{2\,c\,x^n}{b}+1\right)}{b\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(n - 1)/(b*x^n + c*x^(2*n)),x)
```

```
[Out] -(2*atanh((2*c*x^n)/b + 1))/(b*n)
```

sympy [A] time = 16.49, size = 66, normalized size = 2.87

$$\begin{cases} \tilde{\infty} \log(x) & \text{for } b = 0 \wedge c = 0 \wedge n = 0 \\ -\frac{x^{-n}}{cn} & \text{for } b = 0 \\ \frac{\log(x)}{b+c} & \text{for } n = 0 \\ \frac{\frac{n^2 \log(x)}{n^2-n} - \frac{n \log(x)}{n^2-n}}{b} & \text{for } c = 0 \\ \frac{2\log(x)}{b} - \frac{\log\left(\frac{bx^n}{c} + x^{2n}\right)}{bn} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+n)/(b*x**n+c*x**(2*n)),x)
```

```
[Out] Piecewise((zoo*log(x), Eq(b, 0) & Eq(c, 0) & Eq(n, 0)), (-x**(-n)/(c*n), Eq
(b, 0)), (log(x)/(b + c), Eq(n, 0)), ((n**2*log(x)/(n**2 - n) - n*log(x)/(n
**2 - n))/b, Eq(c, 0)), (2*log(x)/b - log(b*x**n/c + x**(2*n))/(b*n), True)
)
```

3.433 $\int \frac{x^{-1-n}}{bx^n+cx^{2n}}\,dx$

Optimal. Leaf size=57

$$-\frac{c^2\log\left(b+cx^n\right)}{b^3n}+\frac{c^2\log(x)}{b^3}+\frac{cx^{-n}}{b^2n}-\frac{x^{-2n}}{2bn}$$

Rubi [A] time = 0.04, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {1584, 266, 44}

$$-\frac{c^2\log\left(b+cx^n\right)}{b^3n}+\frac{c^2\log(x)}{b^3}+\frac{cx^{-n}}{b^2n}-\frac{x^{-2n}}{2bn}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 - n)/(b*x^n + c*x^(2*n)),x]
```

```
[Out] -1/(2*b*n*x^(2*n)) + c/(b^2*n*x^n) + (c^2*Log[x])/b^3 - (c^2*Log[b + c*x^n]
)/(b^3*n)
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \frac{x^{-1-n}}{bx^n+cx^{2n}}\,dx &= \int \frac{x^{-1-2n}}{b+cx^n}\,dx\\ &= \frac{\text{Subst}\left(\int \frac{1}{x^3(b+cx)}\,dx,x,x^n\right)}{n}\\ &= \frac{\text{Subst}\left(\int \left(\frac{1}{bx^3}-\frac{c}{b^2x^2}+\frac{c^2}{b^3x}-\frac{c^3}{b^3(b+cx)}\right)dx,x,x^n\right)}{n}\\ &= -\frac{x^{-2n}}{2bn}+\frac{cx^{-n}}{b^2n}+\frac{c^2\log(x)}{b^3}-\frac{c^2\log\left(b+cx^n\right)}{b^3n}\end{aligned}$$

Mathematica [A] time = 0.07, size = 49, normalized size = 0.86

$$\frac{-2c^2\log\left(b+cx^n\right)+bx^{-2n}\left(2cx^n-b\right)+2c^2n\log(x)}{2b^3n}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 - n)/(b*x^n + c*x^(2*n)),x]
```

```
[Out] ((b*(-b + 2*c*x^n))/x^(2*n) + 2*c^2*n*Log[x] - 2*c^2*Log[b + c*x^n])/(2*b^3
*n)
```

IntegrateAlgebraic [A] time = 0.07, size = 59, normalized size = 1.04

$$\frac{c^2\log\left(x^n\right)}{b^3n}-\frac{c^2\log\left(b+cx^n\right)}{b^3n}+\frac{x^{-2n}\left(2cx^n-b\right)}{2b^2n}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(-1 - n)/(b*x^n + c*x^(2*n)),x]
```

```
[Out] (-b + 2*c*x^n)/(2*b^2*n*x^(2*n)) + (c^2*Log[x^n])/(b^3*n) - (c^2*Log[b + c*
x^n])/(b^3*n)
```

fricas [A] time = 1.39, size = 59, normalized size = 1.04

$$\frac{2\,c^2nx^{2\,n}\log(x)-2\,c^2x^{2\,n}\log\left(cx^n+b\right)+2\,bcx^n-b^2}{2\,b^3nx^{2\,n}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-n)/(b*x^n+c*x^(2*n)),x, algorithm="fricas")
```

```
[Out] 1/2*(2*c^2*n*x^(2*n)*log(x) - 2*c^2*x^(2*n)*log(c*x^n + b) + 2*b*c*x^n - b^
2)/(b^3*n*x^(2*n))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^{-n-1}}{cx^{2\,n}+bx^n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-n)/(b*x^n+c*x^(2*n)),x, algorithm="giac")
```

```
[Out] integrate(x^(-n - 1)/(c*x^(2*n) + b*x^n), x)
```

maple [A] time = 0.02, size = 69, normalized size = 1.21

$$\left(\frac{c^2\mathrm{e}^{2n\ln(x)}\ln(x)}{b^3}+\frac{c\,\mathrm{e}^{n\ln(x)}}{b^2n}-\frac{1}{2bn}\right)\mathrm{e}^{-2n\ln(x)}-\frac{c^2\ln\left(c\,\mathrm{e}^{n\ln(x)}+b\right)}{b^3n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1-n)/(b*x^n+c*x^(2*n)),x)
```

```
[Out] (c/b^2/n*exp(n*ln(x))-1/2/b/n+c^2/b^3*ln(x)*exp(n*ln(x))^2)/exp(n*ln(x))^2-
c^2/b^3/n*ln(c*exp(n*ln(x))+b)
```

maxima [A] time = 0.91, size = 58, normalized size = 1.02

$$\frac{c^2\log(x)}{b^3}-\frac{c^2\log\left(\frac{cx^n+b}{c}\right)}{b^3n}+\frac{2\,cx^n-b}{2\,b^2nx^{2\,n}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-n)/(b*x^n+c*x^(2*n)),x, algorithm="maxima")
```

```
[Out] c^2*log(x)/b^3 - c^2*log((c*x^n + b)/c)/(b^3*n) + 1/2*(2*c*x^n - b)/(b^2*n*
x^(2*n))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{x^{n+1}\left(b\,x^n+c\,x^{2\,n}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(n + 1)*(b*x^n + c*x^(2*n))),x)
```

```
[Out] int(1/(x^(n + 1)*(b*x^n + c*x^(2*n))), x)
```

sympy [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1-n)/(b*x**n+c*x**(2*n)),x)
```

```
[Out] Exception raised: HeuristicGCDFailed
```

3.434 $\int \frac{x^{-1-2n}}{bx^n+cx^{2n}}\,dx$

Optimal. Leaf size=76

$$\frac{c^3\log(b+cx^n)}{b^4n}-\frac{c^3\log(x)}{b^4}-\frac{c^2x^{-n}}{b^3n}+\frac{cx^{-2n}}{2b^2n}-\frac{x^{-3n}}{3bn}$$

Rubi [A] time = 0.05, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {1584, 266, 44}

$$-\frac{c^2x^{-n}}{b^3n}+\frac{c^3\log(b+cx^n)}{b^4n}-\frac{c^3\log(x)}{b^4}+\frac{cx^{-2n}}{2b^2n}-\frac{x^{-3n}}{3bn}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 - 2*n)/(b*x^n + c*x^(2*n)),x]
```

```
[Out] -1/(3*b*n*x^(3*n)) + c/(2*b^2*n*x^(2*n)) - c^2/(b^3*n*x^n) - (c^3*Log[x])/b
^4 + (c^3*Log[b + c*x^n])/(b^4*n)
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^{-1-2n}}{bx^n+cx^{2n}}\,dx &= \int \frac{x^{-1-3n}}{b+cx^n}\,dx\\
&= \frac{\text{Subst}\left(\int \frac{1}{x^4(b+cx)}\,dx,x,x^n\right)}{n}\\
&= \frac{\text{Subst}\left(\int \left(\frac{1}{bx^4}-\frac{c}{b^2x^3}+\frac{c^2}{b^3x^2}-\frac{c^3}{b^4x}+\frac{c^4}{b^4(b+cx)}\right)dx,x,x^n\right)}{n}\\
&= -\frac{x^{-3n}}{3bn}+\frac{cx^{-2n}}{2b^2n}-\frac{c^2x^{-n}}{b^3n}-\frac{c^3\log(x)}{b^4}+\frac{c^3\log(b+cx^n)}{b^4n}
\end{aligned}$$

Mathematica [A] time = 0.08, size = 62, normalized size = 0.82

$$-\frac{bx^{-3n}\left(2b^2-3bcx^n+6c^2x^{2n}\right)-6c^3\log(b+cx^n)+6c^3n\log(x)}{6b^4n}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 - 2*n)/(b*x^n + c*x^(2*n)),x]
```

```
[Out] -1/6*((b*(2*b^2 - 3*b*c*x^n + 6*c^2*x^(2*n)))/x^(3*n) + 6*c^3*n*Log[x] - 6*
c^3*Log[b + c*x^n])/(b^4*n)
```

IntegrateAlgebraic [A] time = 0.10, size = 72, normalized size = 0.95

$$-\frac{c^3\log(x^n)}{b^4n}+\frac{c^3\log(b+cx^n)}{b^4n}+\frac{x^{-3n}\left(-2b^2+3bcx^n-6c^2x^{2n}\right)}{6b^3n}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(-1 - 2*n)/(b*x^n + c*x^(2*n)),x]
```

```
[Out] (-2*b^2 + 3*b*c*x^n - 6*c^2*x^(2*n))/(6*b^3*n*x^(3*n)) - (c^3*Log[x^n])/(b^
4*n) + (c^3*Log[b + c*x^n])/(b^4*n)
```

fricas [A] time = 1.47, size = 72, normalized size = 0.95

$$-\frac{6\,c^3nx^{3\,n}\log(x)-6\,c^3x^{3\,n}\log\left(cx^n+b\right)+6\,bc^2x^{2\,n}-3\,b^2cx^n+2\,b^3}{6\,b^4nx^{3\,n}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-2*n)/(b*x^n+c*x^(2*n)),x, algorithm="fricas")
```

```
[Out] -1/6*(6*c^3*n*x^(3*n)*log(x) - 6*c^3*x^(3*n)*log(c*x^n + b) + 6*b*c^2*x^(2*
n) - 3*b^2*c*x^n + 2*b^3)/(b^4*n*x^(3*n))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^{-2\,n-1}}{cx^{2\,n}+bx^n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-2*n)/(b*x^n+c*x^(2*n)),x, algorithm="giac")
```

```
[Out] integrate(x^(-2*n - 1)/(c*x^(2*n) + b*x^n), x)
```

maple [A] time = 0.03, size = 88, normalized size = 1.16

$$\left(-\frac{c^3\mathrm{e}^{3n\ln(x)}\ln(x)}{b^4}+\frac{c\,\mathrm{e}^{n\ln(x)}}{2b^2n}-\frac{c^2\mathrm{e}^{2n\ln(x)}}{b^3n}-\frac{1}{3bn}\right)\mathrm{e}^{-3n\ln(x)}+\frac{c^3\ln\left(c\,\mathrm{e}^{n\ln(x)}+b\right)}{b^4n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1-2*n)/(b*x^n+c*x^(2*n)),x)
```

```
[Out] (-1/3/b/n+1/2/b^2*c/n*exp(n*ln(x))-c^2/b^3/n*exp(n*ln(x))^2-c^3/b^4*ln(x)*e
xp(n*ln(x))^3)/exp(n*ln(x))^3+c^3/b^4/n*ln(c*exp(n*ln(x))+b)
```

maxima [A] time = 0.91, size = 71, normalized size = 0.93

$$-\frac{c^3\log(x)}{b^4}+\frac{c^3\log\left(\frac{cx^n+b}{c}\right)}{b^4n}-\frac{6\,c^2x^{2\,n}-3\,bcx^n+2\,b^2}{6\,b^3nx^{3\,n}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-2*n)/(b*x^n+c*x^(2*n)),x, algorithm="maxima")
```

```
[Out] -c^3*log(x)/b^4 + c^3*log((c*x^n + b)/c)/(b^4*n) - 1/6*(6*c^2*x^(2*n) - 3*b
*c*x^n + 2*b^2)/(b^3*n*x^(3*n))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^{2n+1}\left(b\,x^n+c\,x^{2n}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(2*n + 1)*(b*x^n + c*x^(2*n))),x)
```

```
[Out] int(1/(x^(2*n + 1)*(b*x^n + c*x^(2*n))), x)
```

sympy [A] time = 52.91, size = 73, normalized size = 0.96

$$-\frac{x^{-3n}}{3bn}+\frac{cx^{-2n}}{2b^2n}-\frac{c^2x^{-n}}{b^3n}+\frac{c^4\left(\begin{cases}\frac{x^n}{b} & \text{for}\ c=0\\ \frac{\log\left(b+cx^n\right)}{c} & \text{otherwise}\end{cases}\right)}{b^4n}-\frac{c^3\log\left(x^n\right)}{b^4n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1-2*n)/(b*x**n+c*x**(2*n)),x)
```

```
[Out] -x**(-3*n)/(3*b*n) + c*x**(-2*n)/(2*b**2*n) - c**2*x**(-n)/(b**3*n) + c**4*
Piecewise((x**n/b, Eq(c, 0)), (log(b + c*x**n)/c, True))/(b**4*n) - c**3*lo
g(x**n)/(b**4*n)
```

3.435 $\int \frac{x^{-1-3n}}{bx^n+cx^{2n}}\,dx$

Optimal. Leaf size=93

$$-\frac{c^4 \log(b+cx^n)}{b^5 n}+\frac{c^4 \log(x)}{b^5}+\frac{c^3 x^{-n}}{b^4 n}-\frac{c^2 x^{-2n}}{2b^3 n}+\frac{c x^{-3n}}{3b^2 n}-\frac{x^{-4n}}{4bn}$$

Rubi [A] time = 0.05, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {1584, 266, 44}

$$-\frac{c^2 x^{-2n}}{2b^3 n}+\frac{c^3 x^{-n}}{b^4 n}-\frac{c^4 \log(b+cx^n)}{b^5 n}+\frac{c^4 \log(x)}{b^5}+\frac{c x^{-3n}}{3b^2 n}-\frac{x^{-4n}}{4bn}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 - 3*n)/(b*x^n + c*x^(2*n)),x]
```

```
[Out] -1/(4*b*n*x^(4*n)) + c/(3*b^2*n*x^(3*n)) - c^2/(2*b^3*n*x^(2*n)) + c^3/(b^4
*n*x^n) + (c^4*Log[x])/b^5 - (c^4*Log[b + c*x^n])/(b^5*n)
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \frac{x^{-1-3n}}{bx^n+cx^{2n}}\,dx &= \int \frac{x^{-1-4n}}{b+cx^n}\,dx\\ &= \frac{\text{Subst}\left(\int \frac{1}{x^5(b+cx)}\,dx,x,x^n\right)}{n}\\ &= \frac{\text{Subst}\left(\int \left(\frac{1}{bx^5}-\frac{c}{b^2x^4}+\frac{c^2}{b^3x^3}-\frac{c^3}{b^4x^2}+\frac{c^4}{b^5x}-\frac{c^5}{b^5(b+cx)}\right)dx,x,x^n\right)}{n}\\ &= -\frac{x^{-4n}}{4bn}+\frac{cx^{-3n}}{3b^2n}-\frac{c^2x^{-2n}}{2b^3n}+\frac{c^3x^{-n}}{b^4n}+\frac{c^4\log(x)}{b^5}-\frac{c^4\log(b+cx^n)}{b^5n}\end{aligned}$$

Mathematica [A] time = 0.09, size = 75, normalized size = 0.81

$$-\frac{bx^{-4n}\left(3b^3-4b^2cx^n+6bc^2x^{2n}-12c^3x^{3n}\right)+12c^4\log(b+cx^n)-12c^4 n\log(x)}{12b^5n}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 - 3*n)/(b*x^n + c*x^(2*n)),x]
```

```
[Out] -1/12*((b*(3*b^3 - 4*b^2*c*x^n + 6*b*c^2*x^(2*n) - 12*c^3*x^(3*n)))/x^(4*n)
 - 12*c^4*n*Log[x] + 12*c^4*Log[b + c*x^n])/(b^5*n)
```

IntegrateAlgebraic [A] time = 0.12, size = 85, normalized size = 0.91

$$\frac{c^4\log(x^n)}{b^5n}-\frac{c^4\log(b+cx^n)}{b^5n}+\frac{x^{-4n}\left(-3b^3+4b^2cx^n-6bc^2x^{2n}+12c^3x^{3n}\right)}{12b^4n}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(-1 - 3*n)/(b*x^n + c*x^(2*n)),x]
```

```
[Out] (-3*b^3 + 4*b^2*c*x^n - 6*b*c^2*x^(2*n) + 12*c^3*x^(3*n))/(12*b^4*n*x^(4*n)
) + (c^4*Log[x^n])/(b^5*n) - (c^4*Log[b + c*x^n])/(b^5*n)
```

fricas [A] time = 1.22, size = 85, normalized size = 0.91

$$\frac{12\,c^4nx^{4\,n}\log(x)-12\,c^4x^{4\,n}\log(cx^n+b)+12\,bc^3x^{3\,n}-6\,b^2c^2x^{2\,n}+4\,b^3cx^n-3\,b^4}{12\,b^5nx^{4\,n}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-3*n)/(b*x^n+c*x^(2*n)),x, algorithm="fricas")
```

```
[Out] 1/12*(12*c^4*n*x^(4*n)*log(x) - 12*c^4*x^(4*n)*log(c*x^n + b) + 12*b*c^3*x^
(3*n) - 6*b^2*c^2*x^(2*n) + 4*b^3*c*x^n - 3*b^4)/(b^5*n*x^(4*n))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^{-3\,n-1}}{cx^{2\,n}+bx^n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-3*n)/(b*x^n+c*x^(2*n)),x, algorithm="giac")
```

```
[Out] integrate(x^(-3*n - 1)/(c*x^(2*n) + b*x^n), x)
```

maple [A] time = 0.03, size = 105, normalized size = 1.13

$$\left(\frac{c^4\mathrm{e}^{4n\ln(x)}\ln(x)}{b^5}+\frac{c\,\mathrm{e}^{n\ln(x)}}{3b^2n}-\frac{c^2\mathrm{e}^{2n\ln(x)}}{2b^3n}+\frac{c^3\mathrm{e}^{3n\ln(x)}}{b^4n}-\frac{1}{4bn}\right)\mathrm{e}^{-4n\ln(x)}-\frac{c^4\ln\left(c\,\mathrm{e}^{n\ln(x)}+b\right)}{b^5n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1-3*n)/(b*x^n+c*x^(2*n)),x)
```

```
[Out] (c^3/b^4/n*exp(n*ln(x))^3-1/4/b/n+1/3/b^2*c/n*exp(n*ln(x))-1/2*c^2/b^3/n*ex
p(n*ln(x))^2+c^4/b^5*ln(x)*exp(n*ln(x))^4)/exp(n*ln(x))^4-c^4/b^5/n*ln(c*ex
p(n*ln(x))+b)
```

maxima [A] time = 0.92, size = 84, normalized size = 0.90

$$\frac{c^4\log(x)}{b^5}-\frac{c^4\log\left(\frac{cx^n+b}{c}\right)}{b^5n}+\frac{12\,c^3x^{3\,n}-6\,bc^2x^{2\,n}+4\,b^2cx^n-3\,b^3}{12\,b^4nx^{4\,n}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-3*n)/(b*x^n+c*x^(2*n)),x, algorithm="maxima")
```

```
[Out] c^4*log(x)/b^5 - c^4*log((c*x^n + b)/c)/(b^5*n) + 1/12*(12*c^3*x^(3*n) - 6*
b*c^2*x^(2*n) + 4*b^2*c*x^n - 3*b^3)/(b^4*n*x^(4*n))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^{3n+1}\,\left(b\,x^n+c\,x^{2n}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(3*n + 1)*(b*x^n + c*x^(2*n))),x)
```

```
[Out] int(1/(x^(3*n + 1)*(b*x^n + c*x^(2*n))), x)
```

sympy [A] time = 105.73, size = 88, normalized size = 0.95

$$-\frac{x^{-4n}}{4bn}+\frac{cx^{-3n}}{3b^2n}-\frac{c^2x^{-2n}}{2b^3n}+\frac{c^3x^{-n}}{b^4n}-\frac{c^5\left(\begin{cases}\frac{x^n}{b} & \text{for}\ c=0\\ \frac{\log\left(b+cx^n\right)}{c} & \text{otherwise}\end{cases}\right)}{b^5n}+\frac{c^4\log\left(x^n\right)}{b^5n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1-3*n)/(b*x**n+c*x**(2*n)),x)
```

```
[Out] -x**(-4*n)/(4*b*n) + c*x**(-3*n)/(3*b**2*n) - c**2*x**(-2*n)/(2*b**3*n) + c
**3*x**(-n)/(b**4*n) - c**5*Piecewise((x**n/b, Eq(c, 0)), (log(b + c*x**n)/
c, True))/(b**5*n) + c**4*log(x**n)/(b**5*n)
```

3.436 $\int \frac{x^{-1+\frac{n}{4}}}{bx^n+cx^{2n}}\,dx$

Optimal. Leaf size=236

$$\frac{c^{3/4}\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\,x^{n/4}+\sqrt{b}+\sqrt{c}\,x^{n/2}\right)}{\sqrt{2}\,b^{7/4}n}-\frac{c^{3/4}\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\,x^{n/4}+\sqrt{b}+\sqrt{c}\,x^{n/2}\right)}{\sqrt{2}\,b^{7/4}n}+\frac{\sqrt{2}\,c^{3/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}}{\sqrt[4]{b}}\right.}{b^{7/4}n}$$

Rubi [A] time = 0.21, antiderivative size = 236, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.360, Rules used = {1584, 362, 345, 211, 1165, 628, 1162, 617, 204}

$$\frac{c^{3/4}\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\,x^{n/4}+\sqrt{b}+\sqrt{c}\,x^{n/2}\right)}{\sqrt{2}\,b^{7/4}n}-\frac{c^{3/4}\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\,x^{n/4}+\sqrt{b}+\sqrt{c}\,x^{n/2}\right)}{\sqrt{2}\,b^{7/4}n}+\frac{\sqrt{2}\,c^{3/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\,x^{n/4}}{\sqrt[4]{b}}\right)}{b^{7/4}n}-\frac{\sqrt{2}\,c^{3/4}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\,x^{n/4}}{\sqrt[4]{b}}+1\right)}{b^{7/4}n}-\frac{4x^{-3n/4}}{3bn}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 + n/4)/(b*x^n + c*x^(2*n)),x]
```

```
[Out] -4/(3*b*n*x^((3*n)/4)) + (Sqrt[2]*c^(3/4)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x^(n/
4))/b^(1/4)])/(b^(7/4)*n) - (Sqrt[2]*c^(3/4)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x^
(n/4))/b^(1/4)])/(b^(7/4)*n) + (c^(3/4)*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/
4)*x^(n/4) + Sqrt[c]*x^(n/2)])/(Sqrt[2]*b^(7/4)*n) - (c^(3/4)*Log[Sqrt[b] +
 Sqrt[2]*b^(1/4)*c^(1/4)*x^(n/4) + Sqrt[c]*x^(n/2)])/(Sqrt[2]*b^(7/4)*n)
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 345

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/(m + 1),
Subst[Int[(a + b*x^Simplify[n/(m + 1)])^p, x], x, x^(m + 1)], x] /; FreeQ[{
a, b, m, n, p}, x] && IntegerQ[Simplify[n/(m + 1)]] &&  !IntegerQ[n]
```

Rule 362

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[x^(m + 1)/(a*(m +
 1)), x] - Dist[b/a, Int[x^Simplify[m + n]/(a + b*x^n), x], x] /; FreeQ[{a,
 b, m, n}, x] && FractionQ[Simplify[(m + 1)/n]] && SumSimplerQ[m, n]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int \frac{x^{-1+\frac{n}{4}}}{bx^n+cx^{2n}}\,dx = \int \frac{x^{-1-\frac{3n}{4}}}{b+cx^n}\,dx$$

$$= -\frac{4x^{-3n/4}}{3bn} - \frac{c\int \frac{x^{\frac{1}{4}(-4+n)}}{b+cx^n}\,dx}{b}$$

$$= -\frac{4x^{-3n/4}}{3bn} - \frac{(4c)\,\text{Subst}\left(\int \frac{1}{b+cx^4}\,dx, x, x^{1+\frac{1}{4}(-4+n)}\right)}{bn}$$

$$= -\frac{4x^{-3n/4}}{3bn} - \frac{(2c)\,\text{Subst}\left(\int \frac{\sqrt{b}-\sqrt{c}\,x^2}{b+cx^4}\,dx, x, x^{1+\frac{1}{4}(-4+n)}\right)}{b^{3/2}n} - \frac{(2c)\,\text{Subst}\left(\int \frac{\sqrt{b}+\sqrt{c}\,x^2}{b+cx^4}\,dx, x, x^{1+\frac{1}{4}(-4+n}\right.}{b^{3/2}n}$$

$$= -\frac{4x^{-3n/4}}{3bn} - \frac{\sqrt{c}\,\text{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{c}}+x^2}\,dx, x, x^{1+\frac{1}{4}(-4+n)}\right)}{b^{3/2}n} - \frac{\sqrt{c}\,\text{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}}+\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{c}}+x^2}\,dx, x,\right.}{b^{3/2}n}$$

$$= -\frac{4x^{-3n/4}}{3bn} + \frac{c^{3/4}\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\,x^{n/4}+\sqrt{c}\,x^{n/2}\right)}{\sqrt{2}\,b^{7/4}n} - \frac{c^{3/4}\log\left(\sqrt{b}+\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\,x^{n/4}+\sqrt{c}\,x^{n}\right.}{\sqrt{2}\,b^{7/4}n}$$

$$= -\frac{4x^{-3n/4}}{3bn} + \frac{\sqrt{2}\,c^{3/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\,x^{n/4}}{\sqrt[4]{b}}\right)}{b^{7/4}n} - \frac{\sqrt{2}\,c^{3/4}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{c}\,x^{n/4}}{\sqrt[4]{b}}\right)}{b^{7/4}n} + \frac{c^{3/4}\log\left(\sqrt{b}-\sqrt{2}\right.}{\sqrt{}}$$

Mathematica [C] time = 0.01, size = 34, normalized size = 0.14

$$-\frac{4x^{-3n/4}\,{}_2F_1\left(-\frac{3}{4},1;\frac{1}{4};-\frac{cx^n}{b}\right)}{3bn}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 + n/4)/(b*x^n + c*x^(2*n)),x]
```

```
[Out] (-4*Hypergeometric2F1[-3/4, 1, 1/4, -((c*x^n)/b)])/(3*b*n*x^((3*n)/4))
```

IntegrateAlgebraic [F] time = 0.11, size = 0, normalized size = 0.00

$$\int \frac{x^{-1+\frac{n}{4}}}{bx^n+cx^{2n}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^(-1 + n/4)/(b*x^n + c*x^(2*n)),x]
```

```
[Out] Defer[IntegrateAlgebraic][x^(-1 + n/4)/(b*x^n + c*x^(2*n)), x]
```

fricas [A] time = 0.92, size = 272, normalized size = 1.15

$$-\frac{12\,bnx^3x^{\frac{3}{4}n-3}\left(-\frac{c^3}{b^7n^4}\right)^{\frac{1}{4}}\arctan\left(-\frac{b^5cn^3xx^{\frac{1}{4}n-1}\left(-\frac{c^3}{b^7n^4}\right)^{\frac{3}{4}}-b^5n^3x\sqrt{\frac{b^4n^2\sqrt{-\frac{c^3}{b^7n^4}}+c^2x^2x^{\frac{1}{2}n-2}}{x^2}}\left(-\frac{c^3}{b^7n^4}\right)^{\frac{3}{4}}}{c^3}\right)+3\,bnx^3x^{\frac{3}{4}n-3}\left(-\frac{c^3}{b^7n^4}\right)^{\frac{1}{4}}\log\left(\frac{b^2n\left(-\frac{c^3}{b^7n^4}\right)^{\frac{1}{4}}+cxx^{\frac{1}{4}n-1}}{x}\right)-3\,bnx^3x^{\frac{3}{4}n-3}\left(-\frac{c^3}{b^7n^4}\right)^{\frac{1}{4}}\log\left(-\frac{b^2n\left(-\frac{c^3}{b^7n^4}\right)^{\frac{1}{4}}-cxx^{\frac{1}{4}n-1}}{x}\right)+4}{3\,bnx^3x^{\frac{3}{4}n-3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+1/4*n)/(b*x^n+c*x^(2*n)),x, algorithm="fricas")
```

```
[Out] -1/3*(12*b*n*x^3*x^(3/4*n - 3)*(-c^3/(b^7*n^4))^(1/4)*arctan(-(b^5*c*n^3*x*
x^(1/4*n - 1)*(-c^3/(b^7*n^4))^(3/4) - b^5*n^3*x*sqrt((b^4*n^2*sqrt(-c^3/(b
^7*n^4)) + c^2*x^2*x^(1/2*n - 2))/x^2)*(-c^3/(b^7*n^4))^(3/4))/c^3) + 3*b*n
*x^3*x^(3/4*n - 3)*(-c^3/(b^7*n^4))^(1/4)*log((b^2*n*(-c^3/(b^7*n^4))^(1/4)
 + c*x*x^(1/4*n - 1))/x) - 3*b*n*x^3*x^(3/4*n - 3)*(-c^3/(b^7*n^4))^(1/4)*l
og(-(b^2*n*(-c^3/(b^7*n^4))^(1/4) - c*x*x^(1/4*n - 1))/x) + 4)/(b*n*x^3*x^(
3/4*n - 3))
```

giac [A] time = 0.35, size = 203, normalized size = 0.86

$$-\frac{\frac{6\sqrt{2}\left(bc^3\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\left(x^n\right)^{\frac{1}{4}}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{b^2}+\frac{6\sqrt{2}\left(bc^3\right)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}-2\left(x^n\right)^{\frac{1}{4}}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{b^2}+\frac{3\sqrt{2}\left(bc^3\right)^{\frac{1}{4}}\log\left(x^{\frac{1}{2}n}+\sqrt{2}\left(x^n\right)^{\frac{1}{4}}\left(\frac{b}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{b}{c}}\right)}{b^2}-\frac{3\sqrt{2}\left(bc^3\right)^{\frac{1}{4}}\log\left(x^{\frac{1}{2}n}-\sqrt{2}\left(x^n\right)^{\frac{1}{4}}\left(\frac{b}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{b}{c}}\right)}{b^2}+\frac{8}{bx^{\frac{3}{4}n}}}{6\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+1/4*n)/(b*x^n+c*x^(2*n)),x, algorithm="giac")
```

```
[Out] -1/6*(6*sqrt(2)*(b*c^3)^(1/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) + 2*(
x^n)^(1/4))/(b/c)^(1/4))/b^2 + 6*sqrt(2)*(b*c^3)^(1/4)*arctan(-1/2*sqrt(2)*
(sqrt(2)*(b/c)^(1/4) - 2*(x^n)^(1/4))/(b/c)^(1/4))/b^2 + 3*sqrt(2)*(b*c^3)^
(1/4)*log(x^(1/2*n) + sqrt(2)*(x^n)^(1/4)*(b/c)^(1/4) + sqrt(b/c))/b^2 - 3*
sqrt(2)*(b*c^3)^(1/4)*log(x^(1/2*n) - sqrt(2)*(x^n)^(1/4)*(b/c)^(1/4) + sqr
t(b/c))/b^2 + 8/(b*x^(3/4*n)))/n
```

maple [C] time = 0.12, size = 54, normalized size = 0.23

$$\mathrm{RootOf}\left(b^7n^4\_Z^4+c^3\right)\ln\left(-\frac{\mathrm{RootOf}\left(b^7n^4\_Z^4+c^3\right)b^2n}{c}+x^{\frac{n}{4}}\right)-\frac{4x^{-\frac{3n}{4}}}{3bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1+1/4*n)/(b*x^n+c*x^(2*n)),x)
```

```
[Out] -4/3/b/n/(x^(1/4*n))^3+sum(_R*ln(x^(1/4*n)-b^2*n/c*_R),_R=RootOf(_Z^4*b^7*n
^4+c^3))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-c\int \frac{x^{\frac{1}{4}n}}{bcxx^n + b^2x}\,dx - \frac{4}{3\,bnx^{\frac{3}{4}n}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+1/4*n)/(b*x^n+c*x^(2*n)),x, algorithm="maxima")
```

```
[Out] -c*integrate(x^(1/4*n)/(b*c*x*x^n + b^2*x), x) - 4/3/(b*n*x^(3/4*n))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^{\frac{n}{4}-1}}{b\,x^n + c\,x^{2\,n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(n/4 - 1)/(b*x^n + c*x^(2*n)),x)
```

```
[Out] int(x^(n/4 - 1)/(b*x^n + c*x^(2*n)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{-n}x^{\frac{n}{4}-1}}{b + cx^n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+1/4*n)/(b*x**n+c*x**(2*n)),x)
```

```
[Out] Integral(x**(-n)*x**(n/4 - 1)/(b + c*x**n), x)
```

3.437 $\int \frac{x^{-1+\frac{n}{3}}}{bx^n+cx^{2n}}\,dx$

Optimal. Leaf size=160

$$-\frac{c^{2/3}\log\left(\sqrt[3]{b}+\sqrt[3]{c}\,x^{n/3}\right)}{b^{5/3}n}+\frac{c^{2/3}\log\left(b^{2/3}-\sqrt[3]{b}\,\sqrt[3]{c}\,x^{n/3}+c^{2/3}x^{2n/3}\right)}{2b^{5/3}n}+\frac{\sqrt{3}\,c^{2/3}\tan^{-1}\left(\frac{\sqrt[3]{b}-2\sqrt[3]{c}\,x^{n/3}}{\sqrt{3}\,\sqrt[3]{b}}\right)}{b^{5/3}n}-\frac{3x^{-2n/3}}{2bn}$$

Rubi [A] time = 0.13, antiderivative size = 160, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.360, Rules used = {1584, 362, 345, 200, 31, 634, 617, 204, 628}

$$-\frac{c^{2/3}\log\left(\sqrt[3]{b}+\sqrt[3]{c}\,x^{n/3}\right)}{b^{5/3}n}+\frac{c^{2/3}\log\left(b^{2/3}-\sqrt[3]{b}\,\sqrt[3]{c}\,x^{n/3}+c^{2/3}x^{2n/3}\right)}{2b^{5/3}n}+\frac{\sqrt{3}\,c^{2/3}\tan^{-1}\left(\frac{\sqrt[3]{b}-2\sqrt[3]{c}\,x^{n/3}}{\sqrt{3}\,\sqrt[3]{b}}\right)}{b^{5/3}n}-\frac{3x^{-2n/3}}{2bn}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 + n/3)/(b*x^n + c*x^(2*n)),x]
```

```
[Out] -3/(2*b*n*x^((2*n)/3)) + (Sqrt[3]*c^(2/3)*ArcTan[(b^(1/3) - 2*c^(1/3)*x^(n/
3))/(Sqrt[3]*b^(1/3))])/(b^(5/3)*n) - (c^(2/3)*Log[b^(1/3) + c^(1/3)*x^(n/3
)])/(b^(5/3)*n) + (c^(2/3)*Log[b^(2/3) - b^(1/3)*c^(1/3)*x^(n/3) + c^(2/3)*
x^((2*n)/3)])/(2*b^(5/3)*n)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 345

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/(m + 1),
Subst[Int[(a + b*x^Simplify[n/(m + 1)])^p, x], x, x^(m + 1)], x] /; FreeQ[{
a, b, m, n, p}, x] && IntegerQ[Simplify[n/(m + 1)]] && !IntegerQ[n]
```

Rule 362

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[x^(m + 1)/(a*(m +
 1)), x] - Dist[b/a, Int[x^Simplify[m + n]/(a + b*x^n), x], x] /; FreeQ[{a,
 b, m, n}, x] && FractionQ[Simplify[(m + 1)/n]] && SumSimplerQ[m, n]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; Free
```

```
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^{-1+\frac{n}{3}}}{bx^n+cx^{2n}}\,dx &= \int \frac{x^{-1-\frac{2n}{3}}}{b+cx^n}\,dx \\
&= -\frac{3x^{-2n/3}}{2bn}-\frac{c\int \frac{x^{\frac{1}{3}(-3+n)}}{b+cx^n}\,dx}{b} \\
&= -\frac{3x^{-2n/3}}{2bn}-\frac{(3c)\,\text{Subst}\left(\int \frac{1}{b+cx^3}\,dx,x,x^{1+\frac{1}{3}(-3+n)}\right)}{bn} \\
&= -\frac{3x^{-2n/3}}{2bn}-\frac{c\,\text{Subst}\left(\int \frac{1}{\sqrt[3]{b}+\sqrt[3]{c}x}\,dx,x,x^{1+\frac{1}{3}(-3+n)}\right)}{b^{5/3}n}-\frac{c\,\text{Subst}\left(\int \frac{2\sqrt[3]{b}-\sqrt[3]{c}x}{b^{2/3}-\sqrt[3]{b}\sqrt[3]{c}x+c^{2/3}x^2}\,dx,x,x^{1+\frac{1}{3}(-3}\right.}{b^{5/3}n} \\
&= -\frac{3x^{-2n/3}}{2bn}-\frac{c^{2/3}\log\left(\sqrt[3]{b}+\sqrt[3]{c}x^{n/3}\right)}{b^{5/3}n}+\frac{c^{2/3}\,\text{Subst}\left(\int \frac{-\sqrt[3]{b}\sqrt[3]{c}+2c^{2/3}x}{b^{2/3}-\sqrt[3]{b}\sqrt[3]{c}x+c^{2/3}x^2}\,dx,x,x^{1+\frac{1}{3}(-3+n)}\right)}{2b^{5/3}n}-\frac{(3c)}{} \\
&= -\frac{3x^{-2n/3}}{2bn}-\frac{c^{2/3}\log\left(\sqrt[3]{b}+\sqrt[3]{c}x^{n/3}\right)}{b^{5/3}n}+\frac{c^{2/3}\log\left(b^{2/3}-\sqrt[3]{b}\sqrt[3]{c}x^{n/3}+c^{2/3}x^{2n/3}\right)}{2b^{5/3}n}-\frac{\left(3c^{2/3}\right)\text{Subst}}{} \\
&= -\frac{3x^{-2n/3}}{2bn}+\frac{\sqrt{3}c^{2/3}\tan^{-1}\left(\frac{\sqrt[3]{b}-2\sqrt[3]{c}x^{n/3}}{\sqrt{3}\sqrt[3]{b}}\right)}{b^{5/3}n}-\frac{c^{2/3}\log\left(\sqrt[3]{b}+\sqrt[3]{c}x^{n/3}\right)}{b^{5/3}n}+\frac{c^{2/3}\log\left(b^{2/3}-\sqrt[3]{b}\sqrt[3]{c}x^{n/}\right.}{2b^{5/3}n}
\end{aligned}$$

Mathematica [C] time = 0.01, size = 34, normalized size = 0.21

$$-\frac{3x^{-2n/3}\,{}_2F_1\left(-\frac{2}{3},1;\frac{1}{3};-\frac{cx^n}{b}\right)}{2bn}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 + n/3)/(b*x^n + c*x^(2*n)),x]
```

```
[Out] (-3*Hypergeometric2F1[-2/3, 1, 1/3, -((c*x^n)/b)])/(2*b*n*x^((2*n)/3))
```

IntegrateAlgebraic [F] time = 0.10, size = 0, normalized size = 0.00

$$\int \frac{x^{-1+\frac{n}{3}}}{bx^n + cx^{2n}}\, dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x^(-1 + n/3)/(b*x^n + c*x^(2*n)),x]

[Out] Defer[IntegrateAlgebraic][x^(-1 + n/3)/(b*x^n + c*x^(2*n)), x]

fricas [A] time = 1.33, size = 212, normalized size = 1.32

$$\frac{2\sqrt{3}x^2x^{\frac{2}{3}n-2}\left(-\frac{c^2}{b^2}\right)^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}bxx^{\frac{1}{3}n-1}\left(-\frac{c^2}{b^2}\right)^{\frac{2}{3}}-\sqrt{3}c}{3c}\right)+2x^2x^{\frac{2}{3}n-2}\left(-\frac{c^2}{b^2}\right)^{\frac{1}{3}}\log\left(\frac{cxx^{\frac{1}{3}n-1}-b\left(-\frac{c^2}{b^2}\right)^{\frac{1}{3}}}{x}\right)-x^2x^{\frac{2}{3}n-2}\left(-\frac{c^2}{b^2}\right)^{\frac{1}{3}}\log\left(\frac{c^2x^2x^{\frac{2}{3}n-2}+bcxx^{\frac{1}{3}n-1}\left(-\frac{c^2}{b^2}\right)^{\frac{1}{3}}+b^2\left(-\frac{c^2}{b^2}\right)^{\frac{2}{3}}}{x^2}\right)-3}{2bnx^2x^{\frac{2}{3}n-2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(-1+1/3*n)/(b*x^n+c*x^(2*n)),x, algorithm="fricas")

[Out] 1/2*(2*sqrt(3)*x^2*x^(2/3*n - 2)*(-c^2/b^2)^(1/3)*arctan(1/3*(2*sqrt(3)*b*x
*x^(1/3*n - 1)*(-c^2/b^2)^(2/3) - sqrt(3)*c)/c) + 2*x^2*x^(2/3*n - 2)*(-c^2
/b^2)^(1/3)*log((c*x*x^(1/3*n - 1) - b*(-c^2/b^2)^(1/3))/x) - x^2*x^(2/3*n
- 2)*(-c^2/b^2)^(1/3)*log((c^2*x^2*x^(2/3*n - 2) + b*c*x*x^(1/3*n - 1)*(-c^
2/b^2)^(1/3) + b^2*(-c^2/b^2)^(2/3))/x^2) - 3)/(b*n*x^2*x^(2/3*n - 2))

giac [A] time = 0.42, size = 136, normalized size = 0.85

$$\frac{\frac{2c\left(-\frac{b}{c}\right)^{\frac{1}{3}}\log\left(\left|x^{\frac{1}{3}n}-\left(-\frac{b}{c}\right)^{\frac{1}{3}}\right|\right)}{b^2}-\frac{2\sqrt{3}\left(-bc^2\right)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2x^{\frac{1}{3}n}+\left(-\frac{b}{c}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{b}{c}\right)^{\frac{1}{3}}}\right)}{b^2}-\frac{\left(-bc^2\right)^{\frac{1}{3}}\log\left(x^{\frac{1}{3}n}\left(-\frac{b}{c}\right)^{\frac{1}{3}}+\left(x^n\right)^{\frac{2}{3}}+\left(-\frac{b}{c}\right)^{\frac{2}{3}}\right)}{b^2}-\frac{3}{b\left(x^n\right)^{\frac{2}{3}}}}{2n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(-1+1/3*n)/(b*x^n+c*x^(2*n)),x, algorithm="giac")

[Out] 1/2*(2*c*(-b/c)^(1/3)*log(abs(x^(1/3*n) - (-b/c)^(1/3)))/b^2 - 2*sqrt(3)*(-
b*c^2)^(1/3)*arctan(1/3*sqrt(3)*(2*x^(1/3*n) + (-b/c)^(1/3))/(-b/c)^(1/3))/
b^2 - (-b*c^2)^(1/3)*log(x^(1/3*n)*(-b/c)^(1/3) + (x^n)^(2/3) + (-b/c)^(2/3
))/b^2 - 3/(b*(x^n)^(2/3)))/n

maple [C] time = 0.07, size = 54, normalized size = 0.34

$$\mathrm{RootOf}\left(b^5n^3\_Z^3+c^2\right)\ln\left(-\frac{\mathrm{RootOf}\left(b^5n^3\_Z^3+c^2\right)b^2n}{c}+x^{\frac{n}{3}}\right)-\frac{3x^{-\frac{2n}{3}}}{2bn}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(-1+1/3*n)/(b*x^n+c*x^(2*n)),x)

[Out] -3/2/b/n/(x^(1/3*n))^2+sum(_R*ln(x^(1/3*n)-_R*b^2/c*n),_R=RootOf(_Z^3*b^5*n
^3+c^2))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-c\int \frac{x^{\frac{1}{3}n}}{bcxx^n + b^2x}\, dx - \frac{3}{2\,bnx^{\frac{2}{3}n}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+1/3*n)/(b*x^n+c*x^(2*n)),x, algorithm="maxima")
[Out] -c*integrate(x^(1/3*n)/(b*c*x*x^n + b^2*x), x) - 3/2/(b*n*x^(2/3*n))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{\frac{n}{3}-1}}{b\,x^n+c\,x^{2\,n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(n/3 - 1)/(b*x^n + c*x^(2*n)),x)
[Out] int(x^(n/3 - 1)/(b*x^n + c*x^(2*n)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+1/3*n)/(b*x**n+c*x**(2*n)),x)
[Out] Timed out
```

3.438 $\int \frac{x^{-1+\frac{n}{2}}}{bx^n+cx^{2n}}\,dx$

Optimal. Leaf size=50

$$\frac{2\sqrt{c}\tan^{-1}\left(\frac{\sqrt{b}\,x^{-n/2}}{\sqrt{c}}\right)}{b^{3/2}n}-\frac{2x^{-n/2}}{bn}$$

Rubi [A] time = 0.03, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1584, 345, 193, 321, 205}

$$\frac{2\sqrt{c}\tan^{-1}\left(\frac{\sqrt{b}\,x^{-n/2}}{\sqrt{c}}\right)}{b^{3/2}n}-\frac{2x^{-n/2}}{bn}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 + n/2)/(b*x^n + c*x^(2*n)),x]
```

```
[Out] -2/(b*n*x^(n/2)) + (2*Sqrt[c]*ArcTan[Sqrt[b]/(Sqrt[c]*x^(n/2))])/(b^(3/2)*n
)
```

Rule 193

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(n*p)*(b + a/x^n)^p,
x] /; FreeQ[{a, b}, x] && LtQ[n, 0] && IntegerQ[p]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 345

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/(m + 1),
Subst[Int[(a + b*x^Simplify[n/(m + 1)])^p, x], x, x^(m + 1)], x] /; FreeQ[{
a, b, m, n, p}, x] && IntegerQ[Simplify[n/(m + 1)]] &&  !IntegerQ[n]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int \frac{x^{-1+\frac{n}{2}}}{bx^n+cx^{2n}}\,dx = \int \frac{x^{-1-\frac{n}{2}}}{b+cx^n}\,dx$$
$$= -\frac{2\,\text{Subst}\left(\int \frac{1}{b+\frac{c}{x^2}}\,dx, x, x^{-n/2}\right)}{n}$$
$$= -\frac{2\,\text{Subst}\left(\int \frac{x^2}{c+bx^2}\,dx, x, x^{-n/2}\right)}{n}$$
$$= -\frac{2x^{-n/2}}{bn} + \frac{(2c)\,\text{Subst}\left(\int \frac{1}{c+bx^2}\,dx, x, x^{-n/2}\right)}{bn}$$
$$= -\frac{2x^{-n/2}}{bn} + \frac{2\sqrt{c}\,\tan^{-1}\left(\frac{\sqrt{b}\,x^{-n/2}}{\sqrt{c}}\right)}{b^{3/2}n}$$

Mathematica [C] time = 0.01, size = 32, normalized size = 0.64

$$-\frac{2x^{-n/2}\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};-\frac{cx^n}{b}\right)}{bn}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 + n/2)/(b*x^n + c*x^(2*n)),x]
[Out] (-2*Hypergeometric2F1[-1/2, 1, 1/2, -((c*x^n)/b)])/(b*n*x^(n/2))
```

IntegrateAlgebraic [F] time = 0.09, size = 0, normalized size = 0.00

$$\int \frac{x^{-1+\frac{n}{2}}}{bx^n+cx^{2n}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^(-1 + n/2)/(b*x^n + c*x^(2*n)),x]
[Out] Defer[IntegrateAlgebraic][x^(-1 + n/2)/(b*x^n + c*x^(2*n)), x]
```

fricas [A] time = 1.28, size = 151, normalized size = 3.02

$$\left[\frac{xx^{\frac{1}{2}n-1}\sqrt{-\frac{c}{b}}\log\left(\frac{cx^2x^{n-2}-2\,bxx^{\frac{1}{2}n-1}\sqrt{-\frac{c}{b}}-b}{cx^2x^{n-2}+b}\right)-2}{bnxx^{\frac{1}{2}n-1}}, \frac{2\left(xx^{\frac{1}{2}n-1}\sqrt{\frac{c}{b}}\arctan\left(\frac{b\sqrt{\frac{c}{b}}}{cxx^{\frac{1}{2}n-1}}\right)-1\right)}{bnxx^{\frac{1}{2}n-1}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+1/2*n)/(b*x^n+c*x^(2*n)),x, algorithm="fricas")
[Out] [(x*x^(1/2*n - 1)*sqrt(-c/b)*log((c*x^2*x^(n - 2) - 2*b*x*x^(1/2*n - 1)*sqr
t(-c/b) - b)/(c*x^2*x^(n - 2) + b)) - 2)/(b*n*x*x^(1/2*n - 1)), 2*(x*x^(1/2
*n - 1)*sqrt(c/b)*arctan(b*sqrt(c/b)/(c*x*x^(1/2*n - 1))) - 1)/(b*n*x*x^(1/
2*n - 1))]
```

giac [A] time = 0.38, size = 38, normalized size = 0.76

$$-\frac{2\left(\frac{c\arctan\left(\frac{c\sqrt{x^n}}{\sqrt{bc}}\right)}{\sqrt{bc}\,b}+\frac{1}{b\sqrt{x^n}}\right)}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+1/2*n)/(b*x^n+c*x^(2*n)),x, algorithm="giac")
[Out] -2*(c*arctan(c*sqrt(x^n)/sqrt(b*c))/(sqrt(b*c)*b) + 1/(b*sqrt(x^n)))/n
```

maple [A] time = 0.07, size = 79, normalized size = 1.58

$$-\frac{2x^{-\frac{n}{2}}}{bn}+\frac{\sqrt{-bc}\,\ln\left(x^{\frac{n}{2}}-\frac{\sqrt{-bc}}{c}\right)}{b^2n}-\frac{\sqrt{-bc}\,\ln\left(x^{\frac{n}{2}}+\frac{\sqrt{-bc}}{c}\right)}{b^2n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1+1/2*n)/(b*x^n+c*x^(2*n)),x)
[Out] -2/b/n/(x^(1/2*n))+1/b^2*(-b*c)^(1/2)/n*ln(x^(1/2*n)-(-b*c)^(1/2)/c)-1/b^2*
(-b*c)^(1/2)/n*ln(x^(1/2*n)+(-b*c)^(1/2)/c)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-c\int\frac{x^{\frac{1}{2}n}}{bcxx^n+b^2x}\,dx-\frac{2}{bnx^{\frac{1}{2}n}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+1/2*n)/(b*x^n+c*x^(2*n)),x, algorithm="maxima")
[Out] -c*integrate(x^(1/2*n)/(b*c*x*x^n + b^2*x), x) - 2/(b*n*x^(1/2*n))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int\frac{x^{\frac{n}{2}-1}}{b\,x^n+c\,x^{2\,n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(n/2 - 1)/(b*x^n + c*x^(2*n)),x)
[Out] int(x^(n/2 - 1)/(b*x^n + c*x^(2*n)), x)
```

sympy [A] time = 13.00, size = 36, normalized size = 0.72

$$-\frac{2\operatorname{atan}\left(\frac{x^{\frac{n}{2}}}{\sqrt{\frac{b}{c}}}\right)}{bn\sqrt{\frac{b}{c}}}-\frac{2x^{-\frac{n}{2}}}{bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+1/2*n)/(b*x**n+c*x**(2*n)),x)
[Out] -2*atan(x**(n/2)/sqrt(b/c))/(b*n*sqrt(b/c)) - 2*x**(-n/2)/(b*n)
```

3.439 $\int \frac{x^{-1-\frac{n}{2}}}{bx^n+cx^{2n}}\,dx$

Optimal. Leaf size=68

$$-\frac{2c^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\,x^{-n/2}}{\sqrt{c}}\right)}{b^{5/2}n}+\frac{2cx^{-n/2}}{b^2n}-\frac{2x^{-3n/2}}{3bn}$$

Rubi [A] time = 0.04, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {1584, 362, 345, 193, 321, 205}

$$-\frac{2c^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\,x^{-n/2}}{\sqrt{c}}\right)}{b^{5/2}n}+\frac{2cx^{-n/2}}{b^2n}-\frac{2x^{-3n/2}}{3bn}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 - n/2)/(b*x^n + c*x^(2*n)),x]
```

```
[Out] -2/(3*b*n*x^((3*n)/2)) + (2*c)/(b^2*n*x^(n/2)) - (2*c^(3/2)*ArcTan[Sqrt[b]/
(Sqrt[c]*x^(n/2))])/(b^(5/2)*n)
```

Rule 193

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(n*p)*(b + a/x^n)^p,
x] /; FreeQ[{a, b}, x] && LtQ[n, 0] && IntegerQ[p]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 345

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/(m + 1),
Subst[Int[(a + b*x^Simplify[n/(m + 1)])^p, x], x, x^(m + 1)], x] /; FreeQ[{
a, b, m, n, p}, x] && IntegerQ[Simplify[n/(m + 1)]] &&  !IntegerQ[n]
```

Rule 362

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[x^(m + 1)/(a*(m +
 1)), x] - Dist[b/a, Int[x^Simplify[m + n]/(a + b*x^n), x], x] /; FreeQ[{a,
 b, m, n}, x] && FractionQ[Simplify[(m + 1)/n]] && SumSimplerQ[m, n]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \frac{x^{-1-\frac{n}{2}}}{bx^n+cx^{2n}}\,dx &= \int \frac{x^{-1-\frac{3n}{2}}}{b+cx^n}\,dx\\ &= -\frac{2x^{-3n/2}}{3bn}-\frac{c\int \frac{x^{-1-\frac{n}{2}}}{b+cx^n}\,dx}{b}\\ &= -\frac{2x^{-3n/2}}{3bn}+\frac{(2c)\,\text{Subst}\left(\int \frac{1}{b+\frac{c}{x^2}}\,dx,x,x^{-n/2}\right)}{bn}\\ &= -\frac{2x^{-3n/2}}{3bn}+\frac{(2c)\,\text{Subst}\left(\int \frac{x^2}{c+bx^2}\,dx,x,x^{-n/2}\right)}{bn}\\ &= -\frac{2x^{-3n/2}}{3bn}+\frac{2cx^{-n/2}}{b^2n}-\frac{\left(2c^2\right)\text{Subst}\left(\int \frac{1}{c+bx^2}\,dx,x,x^{-n/2}\right)}{b^2n}\\ &= -\frac{2x^{-3n/2}}{3bn}+\frac{2cx^{-n/2}}{b^2n}-\frac{2c^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\,x^{-n/2}}{\sqrt{c}}\right)}{b^{5/2}n}\end{aligned}$$

Mathematica [C] time = 0.01, size = 34, normalized size = 0.50

$$-\frac{2x^{-3n/2}\,{}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};-\frac{cx^n}{b}\right)}{3bn}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 - n/2)/(b*x^n + c*x^(2*n)),x]
[Out] (-2*Hypergeometric2F1[-3/2, 1, -1/2, -((c*x^n)/b)])/(3*b*n*x^((3*n)/2))
```

IntegrateAlgebraic [F] time = 0.12, size = 0, normalized size = 0.00

$$\int \frac{x^{-1-\frac{n}{2}}}{bx^n+cx^{2n}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^(-1 - n/2)/(b*x^n + c*x^(2*n)),x]
[Out] Defer[IntegrateAlgebraic][x^(-1 - n/2)/(b*x^n + c*x^(2*n)), x]
```

fricas [A] time = 1.39, size = 161, normalized size = 2.37

$$\left[-\frac{2\,bx^3x^{-\frac{3}{2}n-3}-6\,cxx^{-\frac{1}{2}n-1}-3\,c\sqrt{-\frac{c}{b}}\log\left(\frac{bx^2x^{-n-2}-2\,bxx^{-\frac{1}{2}n-1}\sqrt{-\frac{c}{b}}-c}{bx^2x^{-n-2}+c}\right)}{3\,b^2n},-\frac{2\left(bx^3x^{-\frac{3}{2}n-3}-3\,cxx^{-\frac{1}{2}n-1}-3\,c\sqrt{\frac{c}{b}}\arctan\left(\frac{\sqrt{\frac{c}{b}}}{xx^{-\frac{1}{2}n-1}}\right)\right)}{3\,b^2n}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-1/2*n)/(b*x^n+c*x^(2*n)),x, algorithm="fricas")
[Out] [-1/3*(2*b*x^3*x^(-3/2*n - 3) - 6*c*x*x^(-1/2*n - 1) - 3*c*sqrt(-c/b)*log((
b*x^2*x^(-n - 2) - 2*b*x*x^(-1/2*n - 1)*sqrt(-c/b) - c)/(b*x^2*x^(-n - 2) +
 c)))/(b^2*n), -2/3*(b*x^3*x^(-3/2*n - 3) - 3*c*x*x^(-1/2*n - 1) - 3*c*sqrt
(c/b)*arctan(sqrt(c/b)/(x*x^(-1/2*n - 1))))/(b^2*n)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{-\frac{1}{2}n-1}}{cx^{2n}+bx^n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-1/2*n)/(b*x^n+c*x^(2*n)),x, algorithm="giac")
```

```
[Out] integrate(x^(-1/2*n - 1)/(c*x^(2*n) + b*x^n), x)
```

maple [A] time = 0.10, size = 97, normalized size = 1.43

$$-\frac{2x^{-\frac{3n}{2}}}{3bn}+\frac{2c\,x^{-\frac{n}{2}}}{b^2n}-\frac{\sqrt{-bc}\,c\ln\left(x^{\frac{n}{2}}-\frac{\sqrt{-bc}}{c}\right)}{b^3n}+\frac{\sqrt{-bc}\,c\ln\left(x^{\frac{n}{2}}+\frac{\sqrt{-bc}}{c}\right)}{b^3n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1-1/2*n)/(b*x^n+c*x^(2*n)),x)
```

```
[Out] 2*c/b^2/n/(x^(1/2*n))-2/3/b/n/(x^(1/2*n))^3+1/b^3*(-b*c)^(1/2)*c/n*ln(x^(1/
2*n)+(-b*c)^(1/2)/c)-1/b^3*(-b*c)^(1/2)*c/n*ln(x^(1/2*n)-(-b*c)^(1/2)/c)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$c^2\int \frac{x^{\frac{1}{2}n}}{b^2cxx^n+b^3x}\,dx+\frac{2\,(3\,cx^n-b)}{3\,b^2nx^{\frac{3}{2}n}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-1/2*n)/(b*x^n+c*x^(2*n)),x, algorithm="maxima")
```

```
[Out] c^2*integrate(x^(1/2*n)/(b^2*c*x*x^n + b^3*x), x) + 2/3*(3*c*x^n - b)/(b^2*
n*x^(3/2*n))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^{\frac{n}{2}+1}\,\left(b\,x^n+c\,x^{2\,n}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(n/2 + 1)*(b*x^n + c*x^(2*n))),x)
```

```
[Out] int(1/(x^(n/2 + 1)*(b*x^n + c*x^(2*n))), x)
```

sympy [A] time = 24.92, size = 58, normalized size = 0.85

$$-\frac{2x^{-\frac{3n}{2}}}{3bn}+\frac{2cx^{-\frac{n}{2}}}{b^2n}-\frac{2c^2\operatorname{atan}\left(\frac{x^{-\frac{n}{2}}}{\sqrt{\frac{c}{b}}}\right)}{b^3n\sqrt{\frac{c}{b}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1-1/2*n)/(b*x**n+c*x**(2*n)),x)
```

```
[Out] -2*x**(-3*n/2)/(3*b*n) + 2*c*x**(-n/2)/(b**2*n) - 2*c**2*atan(x**(-n/2)/sqr
t(c/b))/(b**3*n*sqrt(c/b))
```

3.440 $\int \frac{x^{-1-\frac{n}{3}}}{bx^n+cx^{2n}}\,dx$

Optimal. Leaf size=176

$$-\frac{c^{4/3}\log\left(\sqrt[3]{b}\,x^{-n/3}+\sqrt[3]{c}\right)}{b^{7/3}n}+\frac{c^{4/3}\log\left(b^{2/3}x^{-2n/3}-\sqrt[3]{b}\,\sqrt[3]{c}\,x^{-n/3}+c^{2/3}\right)}{2b^{7/3}n}+\frac{\sqrt{3}\,c^{4/3}\tan^{-1}\left(\frac{\sqrt[3]{c}-2\sqrt[3]{b}\,x^{-n/3}}{\sqrt{3}\,\sqrt[3]{c}}\right)}{b^{7/3}n}+\frac{3cx^{-n/3}}{b^2n}-\cdots$$

Rubi [A] time = 0.14, antiderivative size = 176, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 11, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.440, Rules used = {1584, 362, 345, 193, 321, 200, 31, 634, 617, 204, 628}

$$-\frac{c^{4/3}\log\left(\sqrt[3]{b}\,x^{-n/3}+\sqrt[3]{c}\right)}{b^{7/3}n}+\frac{c^{4/3}\log\left(b^{2/3}x^{-2n/3}-\sqrt[3]{b}\,\sqrt[3]{c}\,x^{-n/3}+c^{2/3}\right)}{2b^{7/3}n}+\frac{\sqrt{3}\,c^{4/3}\tan^{-1}\left(\frac{\sqrt[3]{c}-2\sqrt[3]{b}\,x^{-n/3}}{\sqrt{3}\,\sqrt[3]{c}}\right)}{b^{7/3}n}+\frac{3cx^{-n/3}}{b^2n}-\frac{3x^{-4n/3}}{4bn}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 - n/3)/(b*x^n + c*x^(2*n)),x]
```

```
[Out] -3/(4*b*n*x^((4*n)/3)) + (3*c)/(b^2*n*x^(n/3)) + (Sqrt[3]*c^(4/3)*ArcTan[(c
^(1/3) - (2*b^(1/3))/x^(n/3))/(Sqrt[3]*c^(1/3))])/(b^(7/3)*n) - (c^(4/3)*Lo
g[c^(1/3) + b^(1/3)/x^(n/3)])/(b^(7/3)*n) + (c^(4/3)*Log[c^(2/3) + b^(2/3)/
x^((2*n)/3) - (b^(1/3)*c^(1/3))/x^(n/3)])/(2*b^(7/3)*n)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 193

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(n*p)*(b + a/x^n)^p,
x] /; FreeQ[{a, b}, x] && LtQ[n, 0] && IntegerQ[p]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 345

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/(m + 1),
Subst[Int[(a + b*x^Simplify[n/(m + 1)])^p, x], x, x^(m + 1)], x] /; FreeQ[{
```

```
a, b, m, n, p}, x] && IntegerQ[Simplify[n/(m + 1)]] &&  !IntegerQ[n]
```

Rule 362

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[x^(m + 1)/(a*(m +
 1)), x] - Dist[b/a, Int[x^Simplify[m + n]/(a + b*x^n), x], x] /; FreeQ[{a,
 b, m, n}, x] && FractionQ[Simplify[(m + 1)/n]] && SumSimplerQ[m, n]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^{-1-\frac{n}{3}}}{bx^n+cx^{2n}}\,dx &= \int \frac{x^{-1-\frac{4n}{3}}}{b+cx^n}\,dx \\
&= -\frac{3x^{-4n/3}}{4bn} - \frac{c\int \frac{x^{-1-\frac{n}{3}}}{b+cx^n}\,dx}{b} \\
&= -\frac{3x^{-4n/3}}{4bn} + \frac{(3c)\,\mathrm{Subst}\left(\int \frac{1}{b+\frac{c}{x^3}}\,dx, x, x^{-n/3}\right)}{bn} \\
&= -\frac{3x^{-4n/3}}{4bn} + \frac{(3c)\,\mathrm{Subst}\left(\int \frac{x^3}{c+bx^3}\,dx, x, x^{-n/3}\right)}{bn} \\
&= -\frac{3x^{-4n/3}}{4bn} + \frac{3cx^{-n/3}}{b^2n} - \frac{\left(3c^2\right)\mathrm{Subst}\left(\int \frac{1}{c+bx^3}\,dx, x, x^{-n/3}\right)}{b^2n} \\
&= -\frac{3x^{-4n/3}}{4bn} + \frac{3cx^{-n/3}}{b^2n} - \frac{c^{4/3}\,\mathrm{Subst}\left(\int \frac{1}{\sqrt[3]{c}+\sqrt[3]{b}\,x}\,dx, x, x^{-n/3}\right)}{b^2n} - \frac{c^{4/3}\,\mathrm{Subst}\left(\int \frac{2\sqrt[3]{c}-\sqrt[3]{b}\,x}{c^{2/3}-\sqrt[3]{b}\,\sqrt[3]{c}\,x+b^{2/3}x^2}\right.}{b^2n} \\
&= -\frac{3x^{-4n/3}}{4bn} + \frac{3cx^{-n/3}}{b^2n} - \frac{c^{4/3}\log\left(\sqrt[3]{c}+\sqrt[3]{b}\,x^{-n/3}\right)}{b^{7/3}n} + \frac{c^{4/3}\,\mathrm{Subst}\left(\int \frac{-\sqrt[3]{b}\,\sqrt[3]{c}+2b^{2/3}x}{c^{2/3}-\sqrt[3]{b}\,\sqrt[3]{c}\,x+b^{2/3}x^2}\,dx, x, x^{-n}\right.}{2b^{7/3}n} \\
&= -\frac{3x^{-4n/3}}{4bn} + \frac{3cx^{-n/3}}{b^2n} - \frac{c^{4/3}\log\left(\sqrt[3]{c}+\sqrt[3]{b}\,x^{-n/3}\right)}{b^{7/3}n} + \frac{c^{4/3}\log\left(c^{2/3}+b^{2/3}x^{-2n/3}-\sqrt[3]{b}\,\sqrt[3]{c}\,x^{-n/3}\right)}{2b^{7/3}n} \\
&= -\frac{3x^{-4n/3}}{4bn} + \frac{3cx^{-n/3}}{b^2n} + \frac{\sqrt{3}\,c^{4/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,x^{-n/3}}{\sqrt[3]{c}}}{\sqrt{3}}\right)}{b^{7/3}n} - \frac{c^{4/3}\log\left(\sqrt[3]{c}+\sqrt[3]{b}\,x^{-n/3}\right)}{b^{7/3}n} + \frac{c^{4/3}\log\left(c^{2/\cdots\right.}{}
\end{aligned}
$$

Mathematica [C] time = 0.01, size = 34, normalized size = 0.19

$$
-\frac{3x^{-4n/3}\,{}_2F_1\left(-\frac{4}{3},1;-\frac{1}{3};-\frac{cx^n}{b}\right)}{4bn}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 - n/3)/(b*x^n + c*x^(2*n)),x]
```

```
[Out] (-3*Hypergeometric2F1[-4/3, 1, -1/3, -((c*x^n)/b)])/(4*b*n*x^((4*n)/3))
```

IntegrateAlgebraic [F] time = 0.10, size = 0, normalized size = 0.00

$$
\int \frac{x^{-1-\frac{n}{3}}}{bx^n+cx^{2n}}\,dx
$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^(-1 - n/3)/(b*x^n + c*x^(2*n)),x]
```

```
[Out] Defer[IntegrateAlgebraic][x^(-1 - n/3)/(b*x^n + c*x^(2*n)), x]
```

fricas [A] time = 1.42, size = 171, normalized size = 0.97

$$
-\frac{3\,bx^4x^{-\frac{4}{3}n-4} - 12\,cxx^{-\frac{1}{3}n-1} - 4\sqrt{3}\,c\left(-\frac{c}{b}\right)^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}\,bxx^{-\frac{1}{3}n-1}\left(-\frac{c}{b}\right)^{\frac{2}{3}}-\sqrt{3}\,c}{3\,c}\right) - 4\,c\left(-\frac{c}{b}\right)^{\frac{1}{3}}\log\left(\frac{xx^{-\frac{1}{3}n-1}-\left(-\frac{c}{b}\right)^{\frac{1}{3}}}{x}\right) + 2\,c\left(-\frac{c}{b}\right)^{\frac{1}{3}}\log\left(\frac{x^2x^{-\frac{2}{3}n-2}+xx^{-\frac{1}{3}n-1}\left(-\frac{c}{b}\right)^{\frac{1}{3}}+\left(-\frac{c}{b}\right)^{\frac{2}{3}}}{x^2}\right)}{4\,b^2n}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-1/3*n)/(b*x^n+c*x^(2*n)),x, algorithm="fricas")
```

```
[Out] -1/4*(3*b*x^4*x^(-4/3*n - 4) - 12*c*x*x^(-1/3*n - 1) - 4*sqrt(3)*c*(-c/b)^(
1/3)*arctan(1/3*(2*sqrt(3)*b*x*x^(-1/3*n - 1)*(-c/b)^(2/3) - sqrt(3)*c)/c)
- 4*c*(-c/b)^(1/3)*log((x*x^(-1/3*n - 1) - (-c/b)^(1/3))/x) + 2*c*(-c/b)^(1
/3)*log((x^2*x^(-2/3*n - 2) + x*x^(-1/3*n - 1)*(-c/b)^(1/3) + (-c/b)^(2/3))
/x^2))/(b^2*n)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{-\frac{1}{3}n-1}}{cx^{2n}+bx^{n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-1/3*n)/(b*x^n+c*x^(2*n)),x, algorithm="giac")
```

```
[Out] integrate(x^(-1/3*n - 1)/(c*x^(2*n) + b*x^n), x)
```

maple [C] time = 0.08, size = 73, normalized size = 0.41

$$\mathrm{RootOf}\left(b^{7}n^{3}\_Z^{3}+c^{4}\right)\ln\left(\frac{\mathrm{RootOf}\left(b^{7}n^{3}\_Z^{3}+c^{4}\right)^{2}b^{5}n^{2}}{c^{3}}+x^{\frac{n}{3}}\right)-\frac{3x^{-\frac{4n}{3}}}{4bn}+\frac{3c\,x^{-\frac{n}{3}}}{b^{2}n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1-1/3*n)/(b*x^n+c*x^(2*n)),x)
```

```
[Out] 3*c/b^2/n/(x^(1/3*n))-3/4/b/n/(x^(1/3*n))^4+sum(_R*ln(x^(1/3*n)+b^5*n^2/c^3
*_R^2),_R=RootOf(_Z^3*b^7*n^3+c^4))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$c^{2}\int \frac{x^{\frac{2}{3}n}}{b^{2}cxx^{n}+b^{3}x}\,dx+\frac{3\,(4\,cx^{n}-b)}{4\,b^{2}nx^{\frac{4}{3}n}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-1/3*n)/(b*x^n+c*x^(2*n)),x, algorithm="maxima")
```

```
[Out] c^2*integrate(x^(2/3*n)/(b^2*c*x*x^n + b^3*x), x) + 3/4*(4*c*x^n - b)/(b^2*
n*x^(4/3*n))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^{\frac{n}{3}+1}\,\left(b\,x^{n}+c\,x^{2\,n}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(n/3 + 1)*(b*x^n + c*x^(2*n))),x)
```

```
[Out] int(1/(x^(n/3 + 1)*(b*x^n + c*x^(2*n))), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1-1/3*n)/(b*x**n+c*x**(2*n)),x)
```

```
[Out] Timed out
```

3.441 $\int \frac{x^{-1-\frac{n}{4}}}{bx^n+cx^{2n}}\,dx$

Optimal. Leaf size=252

$$\frac{c^{5/4}\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\,x^{-n/4}+\sqrt{b}\,x^{-n/2}+\sqrt{c}\right)}{\sqrt{2}\,b^{9/4}n}-\frac{c^{5/4}\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\,x^{-n/4}+\sqrt{b}\,x^{-n/2}+\sqrt{c}\right)}{\sqrt{2}\,b^{9/4}n}+\frac{\sqrt{2}\,c^{5/4}\tan^{-1}\left(1-\right.}{b^{9/4}n}$$

Rubi [A] time = 0.22, antiderivative size = 252, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 11, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.440, Rules used = {1584, 362, 345, 193, 321, 211, 1165, 628, 1162, 617, 204}

$$\frac{c^{5/4}\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\,x^{-n/4}+\sqrt{b}\,x^{-n/2}+\sqrt{c}\right)}{\sqrt{2}\,b^{9/4}n}-\frac{c^{5/4}\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\,x^{-n/4}+\sqrt{b}\,x^{-n/2}+\sqrt{c}\right)}{\sqrt{2}\,b^{9/4}n}+\frac{\sqrt{2}\,c^{5/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\,x^{-n/4}}{\sqrt[4]{c}}\right)}{b^{9/4}n}-\frac{\sqrt{2}\,c^{5/4}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\,x^{-n/4}}{\sqrt[4]{c}}+1\right)}{b^{9/4}n}+\frac{4cx^{-n/4}}{b^2n}-\frac{4x^{-5n/4}}{5bn}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 - n/4)/(b*x^n + c*x^(2*n)),x]
```

```
[Out] -4/(5*b*n*x^((5*n)/4)) + (4*c)/(b^2*n*x^(n/4)) + (Sqrt[2]*c^(5/4)*ArcTan[1
- (Sqrt[2]*b^(1/4))/(c^(1/4)*x^(n/4))])/(b^(9/4)*n) - (Sqrt[2]*c^(5/4)*ArcT
an[1 + (Sqrt[2]*b^(1/4))/(c^(1/4)*x^(n/4))])/(b^(9/4)*n) + (c^(5/4)*Log[Sqr
t[c] + Sqrt[b]/x^(n/2) - (Sqrt[2]*b^(1/4)*c^(1/4))/x^(n/4)])/(Sqrt[2]*b^(9/
4)*n) - (c^(5/4)*Log[Sqrt[c] + Sqrt[b]/x^(n/2) + (Sqrt[2]*b^(1/4)*c^(1/4))/
x^(n/4)])/(Sqrt[2]*b^(9/4)*n)
```

Rule 193

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(n*p)*(b + a/x^n)^p,
x] /; FreeQ[{a, b}, x] && LtQ[n, 0] && IntegerQ[p]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 345

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/(m + 1),
Subst[Int[(a + b*x^Simplify[n/(m + 1)])^p, x], x, x^(m + 1)], x] /; FreeQ[{
a, b, m, n, p}, x] && IntegerQ[Simplify[n/(m + 1)]] &&  !IntegerQ[n]
```

Rule 362

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[x^(m + 1)/(a*(m +
 1)), x] - Dist[b/a, Int[x^Simplify[m + n]/(a + b*x^n), x], x] /; FreeQ[{a,
 b, m, n}, x] && FractionQ[Simplify[(m + 1)/n]] && SumSimplerQ[m, n]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^{-1-\frac{n}{4}}}{bx^n+cx^{2n}}\,dx &= \int \frac{x^{-1-\frac{5n}{4}}}{b+cx^n}\,dx \\
&= -\frac{4x^{-5n/4}}{5bn} - \frac{c\int \frac{x^{-1-\frac{n}{4}}}{b+cx^n}\,dx}{b} \\
&= -\frac{4x^{-5n/4}}{5bn} + \frac{(4c)\,\mathrm{Subst}\left(\int \frac{1}{b+\frac{c}{x^4}}\,dx, x, x^{-n/4}\right)}{bn} \\
&= -\frac{4x^{-5n/4}}{5bn} + \frac{(4c)\,\mathrm{Subst}\left(\int \frac{x^4}{c+bx^4}\,dx, x, x^{-n/4}\right)}{bn} \\
&= -\frac{4x^{-5n/4}}{5bn} + \frac{4cx^{-n/4}}{b^2n} - \frac{\left(4c^2\right)\mathrm{Subst}\left(\int \frac{1}{c+bx^4}\,dx, x, x^{-n/4}\right)}{b^2n} \\
&= -\frac{4x^{-5n/4}}{5bn} + \frac{4cx^{-n/4}}{b^2n} - \frac{\left(2c^{3/2}\right)\mathrm{Subst}\left(\int \frac{\sqrt{c}-\sqrt{b}\,x^2}{c+bx^4}\,dx, x, x^{-n/4}\right)}{b^2n} - \frac{\left(2c^{3/2}\right)\mathrm{Subst}\left(\int \frac{\sqrt{c}+\sqrt{b}\,x^2}{c+bx^4}\,d\right.}{b^2n} \\
&= -\frac{4x^{-5n/4}}{5bn} + \frac{4cx^{-n/4}}{b^2n} + \frac{c^{5/4}\,\mathrm{Subst}\left(\int \frac{\frac{\sqrt{2}\sqrt[4]{c}}{\sqrt[4]{b}}+2x}{-\frac{\sqrt{c}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{c}\,x}{\sqrt[4]{b}}-x^2}\,dx, x, x^{-n/4}\right)}{\sqrt{2}\,b^{9/4}n} + \frac{c^{5/4}\,\mathrm{Subst}\left(\int \frac{\frac{\sqrt{2}\sqrt[4]{c}}{\sqrt[4]{b}}-2}{-\frac{\sqrt{c}}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{c}}{\sqrt[4]{b}}}\right.}{\sqrt{2}\,b^{9/4}n} \\
&= -\frac{4x^{-5n/4}}{5bn} + \frac{4cx^{-n/4}}{b^2n} + \frac{c^{5/4}\log\left(\sqrt{c}+\sqrt{b}\,x^{-n/2}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\,x^{-n/4}\right)}{\sqrt{2}\,b^{9/4}n} - \frac{c^{5/4}\log\left(\sqrt{c}+\sqrt{b}\,x^{-n}\right.}{\sqrt{2}\,b^{9}} \\
&= -\frac{4x^{-5n/4}}{5bn} + \frac{4cx^{-n/4}}{b^2n} + \frac{\sqrt{2}\,c^{5/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\,x^{-n/4}}{\sqrt[4]{c}}\right)}{b^{9/4}n} - \frac{\sqrt{2}\,c^{5/4}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\,x^{-n/4}}{\sqrt[4]{c}}\right)}{b^{9/4}n} + \frac{c^{5}}{}
\end{aligned}
$$

Mathematica [C] time = 0.01, size = 34, normalized size = 0.13

$$
-\frac{4x^{-5n/4}\,{}_2F_1\left(-\frac{5}{4},1;-\frac{1}{4};-\frac{cx^n}{b}\right)}{5bn}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 - n/4)/(b*x^n + c*x^(2*n)),x]
```

```
[Out] (-4*Hypergeometric2F1[-5/4, 1, -1/4, -((c*x^n)/b)])/(5*b*n*x^((5*n)/4))
```

IntegrateAlgebraic [F] time = 0.12, size = 0, normalized size = 0.00

$$
\int \frac{x^{-1-\frac{n}{4}}}{bx^n+cx^{2n}}\,dx
$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^(-1 - n/4)/(b*x^n + c*x^(2*n)),x]
```

```
[Out] Defer[IntegrateAlgebraic][x^(-1 - n/4)/(b*x^n + c*x^(2*n)), x]
```

fricas [A] time = 1.38, size = 259, normalized size = 1.03

$$
-\frac{4\,bx^5x^{-\frac{5}{4}n-5} + 20\,b^2n\left(-\frac{c^5}{b^9n^4}\right)^{\frac{1}{4}}\arctan\left(-\frac{b^7cn^3xx^{-\frac{1}{4}n-1}\left(-\frac{c^5}{b^9n^4}\right)^{\frac{3}{4}} - b^7n^3x\sqrt{\frac{b^4n^2\sqrt{-\frac{c^5}{b^9n^4}}+c^2x^2x^{-\frac{1}{2}n-2}}{x^2}}\left(-\frac{c^5}{b^9n^4}\right)^{\frac{3}{4}}}{c^5}\right) + 5\,b^2n\left(-\frac{c^5}{b^9n^4}\right)^{\frac{1}{4}}\log\left(\frac{b^2n\left(-\frac{c^5}{b^9n^4}\right)^{\frac{1}{4}}+cxx^{-\frac{1}{4}n-1}}{x}\right) - 5\,b^2n\left(-\frac{c^5}{b^9n^4}\right)^{\frac{1}{4}}\log\left(-\frac{b^2n\left(-\frac{c^5}{b^9n^4}\right)^{\frac{1}{4}}-cxx^{-\frac{1}{4}n-1}}{x}\right) - 20\,cxx^{-\frac{1}{4}n-1}}{5\,b^2n}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-1/4*n)/(b*x^n+c*x^(2*n)),x, algorithm="fricas")
```

```
[Out] -1/5*(4*b*x^5*x^(-5/4*n - 5) + 20*b^2*n*(-c^5/(b^9*n^4))^(1/4)*arctan(-(b^7
*c*n^3*x*x^(-1/4*n - 1)*(-c^5/(b^9*n^4))^(3/4) - b^7*n^3*x*sqrt((b^4*n^2*sq
rt(-c^5/(b^9*n^4)) + c^2*x^2*x^(-1/2*n - 2))/x^2)*(-c^5/(b^9*n^4))^(3/4))/c
^5) + 5*b^2*n*(-c^5/(b^9*n^4))^(1/4)*log((b^2*n*(-c^5/(b^9*n^4))^(1/4) + c*
x*x^(-1/4*n - 1))/x) - 5*b^2*n*(-c^5/(b^9*n^4))^(1/4)*log(-(b^2*n*(-c^5/(b^
9*n^4))^(1/4) - c*x*x^(-1/4*n - 1))/x) - 20*c*x*x^(-1/4*n - 1))/(b^2*n)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{-\frac{1}{4}n-1}}{cx^{2n}+bx^{n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-1/4*n)/(b*x^n+c*x^(2*n)),x, algorithm="giac")
```

```
[Out] integrate(x^(-1/4*n - 1)/(c*x^(2*n) + b*x^n), x)
```

maple [C] time = 0.09, size = 73, normalized size = 0.29

$$\mathrm{RootOf}\left(b^{9}n^{4}\_Z^{4}+c^{5}\right)\ln\left(\frac{\mathrm{RootOf}\left(b^{9}n^{4}\_Z^{4}+c^{5}\right)^{3}b^{7}n^{3}}{c^{4}}+x^{\frac{n}{4}}\right)-\frac{4x^{-\frac{5n}{4}}}{5bn}+\frac{4c\,x^{-\frac{n}{4}}}{b^{2}n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1-1/4*n)/(b*x^n+c*x^(2*n)),x)
```

```
[Out] 4*c/b^2/n/(x^(1/4*n))-4/5/b/n/(x^(1/4*n))^5+sum(_R*ln(x^(1/4*n)+b^7*n^3/c^4
*_R^3),_R=RootOf(_Z^4*b^9*n^4+c^5))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$c^{2}\int \frac{x^{\frac{3}{4}n}}{b^{2}cxx^{n}+b^{3}x}\,dx+\frac{4\,(5\,cx^{n}-b)}{5\,b^{2}nx^{\frac{5}{4}n}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-1/4*n)/(b*x^n+c*x^(2*n)),x, algorithm="maxima")
```

```
[Out] c^2*integrate(x^(3/4*n)/(b^2*c*x*x^n + b^3*x), x) + 4/5*(5*c*x^n - b)/(b^2*
n*x^(5/4*n))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^{\frac{n}{4}+1}\left(b\,x^{n}+c\,x^{2\,n}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(n/4 + 1)*(b*x^n + c*x^(2*n))),x)
```

```
[Out] int(1/(x^(n/4 + 1)*(b*x^n + c*x^(2*n))), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1-1/4*n)/(b*x**n+c*x**(2*n)),x)
```

```
[Out] Timed out
```

3.442 $\int x^{-1-n(-1+p)}\left(bx^n+cx^{2n}\right)^p\,dx$

Optimal. Leaf size=37

$$\frac{x^{-n(p+1)}\left(bx^n+cx^{2n}\right)^{p+1}}{cn(p+1)}$$

Rubi [A] time = 0.04, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.038, Rules used = {2014}

$$\frac{x^{-n(p+1)}\left(bx^n+cx^{2n}\right)^{p+1}}{cn(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 - n*(-1 + p))*(b*x^n + c*x^(2*n))^p,x]
```

```
[Out] (b*x^n + c*x^(2*n))^(1 + p)/(c*n*(1 + p)*x^(n*(1 + p)))
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rubi steps

$$\int x^{-1-n(-1+p)}\left(bx^n+cx^{2n}\right)^p\,dx=\frac{x^{-n(1+p)}\left(bx^n+cx^{2n}\right)^{1+p}}{cn(1+p)}$$

Mathematica [A] time = 0.02, size = 38, normalized size = 1.03

$$\frac{x^{-np}\left(b+cx^n\right)\left(x^n\left(b+cx^n\right)\right)^p}{cn(p+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 - n*(-1 + p))*(b*x^n + c*x^(2*n))^p,x]
```

```
[Out] ((b + c*x^n)*(x^n*(b + c*x^n))^p)/(c*n*(1 + p)*x^(n*p))
```

IntegrateAlgebraic [F] time = 0.16, size = 0, normalized size = 0.00

$$\int x^{-1-n(-1+p)}\left(bx^n+cx^{2n}\right)^p\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^(-1 - n*(-1 + p))*(b*x^n + c*x^(2*n))^p,x]
```

```
[Out] Defer[IntegrateAlgebraic][x^(-1 - n*(-1 + p))*(b*x^n + c*x^(2*n))^p, x]
```

fricas [A] time = 1.34, size = 59, normalized size = 1.59

$$\frac{\left(cxx^{-np+n-1}x^n+bxx^{-np+n-1}\right)\left(cx^{2\,n}+bx^n\right)^p}{\left(cnp+cn\right)x^n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-n*(-1+p))*(b*x^n+c*x^(2*n))^p,x, algorithm="fricas")
```

```
[Out] (c*x*x^(-n*p + n - 1)*x^n + b*x*x^(-n*p + n - 1))*(c*x^(2*n) + b*x^n)^p/((c
*n*p + c*n)*x^n)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(cx^{2n} + bx^n\right)^p x^{-n(p-1)-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-n*(-1+p))*(b*x^n+c*x^(2*n))^p,x, algorithm="giac")
```

```
[Out] integrate((c*x^(2*n) + b*x^n)^p*x^(-n*(p - 1) - 1), x)
```

maple [F] time = 0.10, size = 0, normalized size = 0.00

$$\int x^{-(p-1)n-1}\left(b\,x^n + c\,x^{2n}\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1-n*(-1+p))*(b*x^n+c*x^(2*n))^p,x)
```

```
[Out] int(x^(-1-n*(-1+p))*(b*x^n+c*x^(2*n))^p,x)
```

maxima [A] time = 1.11, size = 43, normalized size = 1.16

$$\frac{(cx^n + b)e^{\left(-np\log(x)+p\log(cx^n+b)+p\log(x^n)\right)}}{cn\left(p+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-n*(-1+p))*(b*x^n+c*x^(2*n))^p,x, algorithm="maxima")
```

```
[Out] (c*x^n + b)*e^(-n*p*log(x) + p*log(c*x^n + b) + p*log(x^n))/(c*n*(p + 1))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\left(b\,x^n + c\,x^{2\,n}\right)^p}{x^{n\,(p-1)+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^n + c*x^(2*n))^p/x^(n*(p - 1) + 1),x)
```

```
[Out] int((b*x^n + c*x^(2*n))^p/x^(n*(p - 1) + 1), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1-n*(-1+p))*(b*x**n+c*x**(2*n))**p,x)
```

```
[Out] Timed out
```

3.443 $\int x^{-1-n(1+2p)}\left(bx^n+cx^{2n}\right)^p\,dx$

Optimal. Leaf size=38

$$-\frac{x^{-2n(p+1)}\left(bx^n+cx^{2n}\right)^{p+1}}{bn(p+1)}$$

Rubi [A] time = 0.04, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.036, Rules used = {2014}

$$-\frac{x^{-2n(p+1)}\left(bx^n+cx^{2n}\right)^{p+1}}{bn(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 - n*(1 + 2*p))*(b*x^n + c*x^(2*n))^p,x]
[Out] -((b*x^n + c*x^(2*n))^(1 + p)/(b*n*(1 + p)*x^(2*n*(1 + p))))
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rubi steps

$$\int x^{-1-n(1+2p)}\left(bx^n+cx^{2n}\right)^p\,dx=-\frac{x^{-2n(1+p)}\left(bx^n+cx^{2n}\right)^{1+p}}{bn(1+p)}$$

Mathematica [A] time = 0.02, size = 43, normalized size = 1.13

$$-\frac{x^{-n(2p+1)}\left(b+cx^n\right)\left(x^n\left(b+cx^n\right)\right)^p}{bn(p+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 - n*(1 + 2*p))*(b*x^n + c*x^(2*n))^p,x]
[Out] -(((b + c*x^n)*(x^n*(b + c*x^n))^p)/(b*n*(1 + p)*x^(n*(1 + 2*p))))
```

IntegrateAlgebraic [F] time = 0.15, size = 0, normalized size = 0.00

$$\int x^{-1-n(1+2p)}\left(bx^n+cx^{2n}\right)^p\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^(-1 - n*(1 + 2*p))*(b*x^n + c*x^(2*n))^p,x]
[Out] Defer[IntegrateAlgebraic][x^(-1 - n*(1 + 2*p))*(b*x^n + c*x^(2*n))^p, x]
```

fricas [A] time = 1.15, size = 59, normalized size = 1.55

$$-\frac{\left(cxx^{-2\,np-n-1}x^n+bxx^{-2\,np-n-1}\right)\left(cx^{2\,n}+bx^n\right)^p}{bnp+bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-n*(1+2*p))*(b*x^n+c*x^(2*n))^p,x, algorithm="fricas")
```

```
[Out] -(c*x*x^(-2*n*p - n - 1)*x^n + b*x*x^(-2*n*p - n - 1))*(c*x^(2*n) + b*x^n)^
p/(b*n*p + b*n)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(cx^{2n} + bx^{n}\right)^{p} x^{-n(2p+1)-1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-n*(1+2*p))*(b*x^n+c*x^(2*n))^p,x, algorithm="giac")
```

```
[Out] integrate((c*x^(2*n) + b*x^n)^p*x^(-n*(2*p + 1) - 1), x)
```

maple [F] time = 0.07, size = 0, normalized size = 0.00

$$\int x^{-(2p+1)n-1} \left(b\,x^{n} + c\,x^{2n}\right)^{p}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1-n*(2*p+1))*(b*x^n+c*x^(2*n))^p,x)
```

```
[Out] int(x^(-1-n*(2*p+1))*(b*x^n+c*x^(2*n))^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(cx^{2n} + bx^{n}\right)^{p} x^{-n(2p+1)-1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-n*(1+2*p))*(b*x^n+c*x^(2*n))^p,x, algorithm="maxima")
```

```
[Out] integrate((c*x^(2*n) + b*x^n)^p*x^(-n*(2*p + 1) - 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\left(b\,x^{n} + c\,x^{2n}\right)^{p}}{x^{n(2p+1)+1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^n + c*x^(2*n))^p/x^(n*(2*p + 1) + 1),x)
```

```
[Out] int((b*x^n + c*x^(2*n))^p/x^(n*(2*p + 1) + 1), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1-n*(1+2*p))*(b*x**n+c*x**(2*n))**p,x)
```

```
[Out] Timed out
```

3.444 $\int x^{-1+2n}\left(a^2+2abx^n+b^2x^{2n}\right)^{5/2}\,dx$

Optimal. Leaf size=112

$$\frac{(a+bx^n)^7\sqrt{a^2+2abx^n+b^2x^{2n}}}{7n\left(ab^2+b^3x^n\right)}-\frac{a\,(a+bx^n)^6\sqrt{a^2+2abx^n+b^2x^{2n}}}{6n\left(ab^2+b^3x^n\right)}$$

Rubi [A] time = 0.04, antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.094, Rules used = {1355, 266, 43}

$$\frac{(a+bx^n)^7\sqrt{a^2+2abx^n+b^2x^{2n}}}{7n\left(ab^2+b^3x^n\right)}-\frac{a\,(a+bx^n)^6\sqrt{a^2+2abx^n+b^2x^{2n}}}{6n\left(ab^2+b^3x^n\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 + 2*n)*(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(5/2),x]
```

```
[Out] -(a*(a + b*x^n)^6*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])/(6*n*(a*b^2 + b^3*x^
n)) + ((a + b*x^n)^7*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])/(7*n*(a*b^2 + b^3
*x^n))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\begin{aligned}
\int x^{-1+2n}\left(a^2+2abx^n+b^2x^{2n}\right)^{5/2}\,dx &= \frac{\sqrt{a^2+2abx^n+b^2x^{2n}}\int x^{-1+2n}\left(ab+b^2x^n\right)^5\,dx}{b^4\left(ab+b^2x^n\right)}\\
&= \frac{\sqrt{a^2+2abx^n+b^2x^{2n}}\,\text{Subst}\left(\int x\left(ab+b^2x\right)^5\,dx,x,x^n\right)}{b^4n\left(ab+b^2x^n\right)}\\
&= \frac{\sqrt{a^2+2abx^n+b^2x^{2n}}\,\text{Subst}\left(\int\left(-\frac{a\left(ab+b^2x\right)^5}{b}+\frac{\left(ab+b^2x\right)^6}{b^2}\right)dx,x,x^n\right)}{b^4n\left(ab+b^2x^n\right)}\\
&= -\frac{a\,(a+bx^n)^6\sqrt{a^2+2abx^n+b^2x^{2n}}}{6n\left(ab^2+b^3x^n\right)}+\frac{(a+bx^n)^7\sqrt{a^2+2abx^n+b^2x^{2n}}}{7n\left(ab^2+b^3x^n\right)}
\end{aligned}$$

Mathematica [A] time = 0.06, size = 40, normalized size = 0.36

$$-\frac{(a-6bx^n)(a+bx^n)^5\sqrt{(a+bx^n)^2}}{42b^2n}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 + 2*n)*(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(5/2),x]
```

```
[Out] -1/42*((a - 6*b*x^n)*(a + b*x^n)^5*Sqrt[(a + b*x^n)^2])/(b^2*n)
```

IntegrateAlgebraic [A] time = 0.07, size = 96, normalized size = 0.86

$$\frac{x^{2n}\sqrt{(a+bx^n)^2}\left(21a^5+70a^4bx^n+105a^3b^2x^{2n}+84a^2b^3x^{3n}+35ab^4x^{4n}+6b^5x^{5n}\right)}{42n(a+bx^n)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(-1 + 2*n)*(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(5/2),x]
```

```
[Out] (x^(2*n)*Sqrt[(a + b*x^n)^2]*(21*a^5 + 70*a^4*b*x^n + 105*a^3*b^2*x^(2*n) +
 84*a^2*b^3*x^(3*n) + 35*a*b^4*x^(4*n) + 6*b^5*x^(5*n)))/(42*n*(a + b*x^n))
```

fricas [A] time = 1.30, size = 74, normalized size = 0.66

$$\frac{6\,b^5x^{7\,n}+35\,ab^4x^{6\,n}+84\,a^2b^3x^{5\,n}+105\,a^3b^2x^{4\,n}+70\,a^4bx^{3\,n}+21\,a^5x^{2\,n}}{42\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+2*n)*(a^2+2*a*b*x^n+b^2*x^(2*n))^(5/2),x, algorithm="fricas
")
```

```
[Out] 1/42*(6*b^5*x^(7*n) + 35*a*b^4*x^(6*n) + 84*a^2*b^3*x^(5*n) + 105*a^3*b^2*x
^(4*n) + 70*a^4*b*x^(3*n) + 21*a^5*x^(2*n))/n
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b^2x^{2\,n}+2\,abx^n+a^2\right)^{\frac{5}{2}}x^{2\,n-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+2*n)*(a^2+2*a*b*x^n+b^2*x^(2*n))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b^2*x^(2*n) + 2*a*b*x^n + a^2)^(5/2)*x^(2*n - 1), x)
```

maple [A] time = 0.05, size = 208, normalized size = 1.86

$$\frac{\sqrt{(b\,x^n+a)^2}\;a^5x^{2n}}{2\,(b\,x^n+a)\,n}+\frac{5\sqrt{(b\,x^n+a)^2}\;a^4b\,x^{3n}}{3\,(b\,x^n+a)\,n}+\frac{5\sqrt{(b\,x^n+a)^2}\;a^3b^2x^{4n}}{2\,(b\,x^n+a)\,n}+\frac{2\sqrt{(b\,x^n+a)^2}\;a^2b^3x^{5n}}{(b\,x^n+a)\,n}+\frac{5\sqrt{(b\,x^n+a)^2}\;a\,b^4x^{6n}}{6\,(b\,x^n+a)\,n}+\frac{\sqrt{(b\,x^n+a)^2}\;b^5x^{7n}}{7\,(b\,x^n+a)\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1+2*n)*(a^2+2*a*b*x^n+b^2*x^(2*n))^(5/2),x)
```

```
[Out] 1/7*((a+b*x^n)^2)^(1/2)/(a+b*x^n)*b^5/n*(x^n)^7+5/6*((a+b*x^n)^2)^(1/2)/(a+
b*x^n)*a*b^4/n*(x^n)^6+2*((a+b*x^n)^2)^(1/2)/(a+b*x^n)*a^2*b^3/n*(x^n)^5+5/
2*((a+b*x^n)^2)^(1/2)/(a+b*x^n)*a^3*b^2/n*(x^n)^4+5/3*((a+b*x^n)^2)^(1/2)/(
a+b*x^n)*a^4*b/n*(x^n)^3+1/2*((a+b*x^n)^2)^(1/2)/(a+b*x^n)*a^5/n*(x^n)^2
```

maxima [A] time = 0.92, size = 74, normalized size = 0.66

$$\frac{6\,b^5x^{7\,n}+35\,ab^4x^{6\,n}+84\,a^2b^3x^{5\,n}+105\,a^3b^2x^{4\,n}+70\,a^4bx^{3\,n}+21\,a^5x^{2\,n}}{42\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+2*n)*(a^2+2*a*b*x^n+b^2*x^(2*n))^(5/2),x, algorithm="maxima
")
```

```
[Out] 1/42*(6*b^5*x^(7*n) + 35*a*b^4*x^(6*n) + 84*a^2*b^3*x^(5*n) + 105*a^3*b^2*x
^(4*n) + 70*a^4*b*x^(3*n) + 21*a^5*x^(2*n))/n
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^{2\,n-1}\,{\left(a^2+b^2\,x^{2\,n}+2\,a\,b\,x^n\right)}^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(2*n - 1)*(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(5/2),x)
```

```
[Out] int(x^(2*n - 1)*(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+2*n)*(a**2+2*a*b*x**n+b**2*x**(2*n))**(5/2),x)
```

```
[Out] Timed out
```

3.445 $\int x^{-1+2n}\left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}\,dx$

Optimal. Leaf size=112

$$\frac{(a+bx^n)^5\sqrt{a^2+2abx^n+b^2x^{2n}}}{5n\left(ab^2+b^3x^n\right)}-\frac{a\,(a+bx^n)^4\sqrt{a^2+2abx^n+b^2x^{2n}}}{4n\left(ab^2+b^3x^n\right)}$$

Rubi [A] time = 0.04, antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.094, Rules used = {1355, 266, 43}

$$\frac{(a+bx^n)^5\sqrt{a^2+2abx^n+b^2x^{2n}}}{5n\left(ab^2+b^3x^n\right)}-\frac{a\,(a+bx^n)^4\sqrt{a^2+2abx^n+b^2x^{2n}}}{4n\left(ab^2+b^3x^n\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 + 2*n)*(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(3/2),x]
```

```
[Out] -(a*(a + b*x^n)^4*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])/(4*n*(a*b^2 + b^3*x^
n)) + ((a + b*x^n)^5*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])/(5*n*(a*b^2 + b^3
*x^n))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\begin{aligned}\int x^{-1+2n}\left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}\,dx&=\frac{\sqrt{a^2+2abx^n+b^2x^{2n}}\int x^{-1+2n}\left(ab+b^2x^n\right)^3\,dx}{b^2\left(ab+b^2x^n\right)}\\&=\frac{\sqrt{a^2+2abx^n+b^2x^{2n}}\,\text{Subst}\left(\int x\left(ab+b^2x\right)^3\,dx,x,x^n\right)}{b^2n\left(ab+b^2x^n\right)}\\&=\frac{\sqrt{a^2+2abx^n+b^2x^{2n}}\,\text{Subst}\left(\int\left(-\frac{a\left(ab+b^2x\right)^3}{b}+\frac{\left(ab+b^2x\right)^4}{b^2}\right)dx,x,x^n\right)}{b^2n\left(ab+b^2x^n\right)}\\&=-\frac{a\,(a+bx^n)^4\sqrt{a^2+2abx^n+b^2x^{2n}}}{4n\left(ab^2+b^3x^n\right)}+\frac{(a+bx^n)^5\sqrt{a^2+2abx^n+b^2x^{2n}}}{5n\left(ab^2+b^3x^n\right)}\end{aligned}$$

Mathematica [A] time = 0.04, size = 40, normalized size = 0.36

$$-\frac{(a-4bx^n)(a+bx^n)^3\sqrt{(a+bx^n)^2}}{20b^2n}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 + 2*n)*(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(3/2),x]
```

```
[Out] -1/20*((a - 4*b*x^n)*(a + b*x^n)^3*Sqrt[(a + b*x^n)^2])/(b^2*n)
```

IntegrateAlgebraic [A] time = 0.06, size = 70, normalized size = 0.62

$$\frac{x^{2n}\sqrt{(a+bx^n)^2}\,\left(10a^3+20a^2bx^n+15ab^2x^{2n}+4b^3x^{3n}\right)}{20n\,(a+bx^n)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(-1 + 2*n)*(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(3/2),x]
```

```
[Out] (x^(2*n)*Sqrt[(a + b*x^n)^2]*(10*a^3 + 20*a^2*b*x^n + 15*a*b^2*x^(2*n) + 4*
b^3*x^(3*n)))/(20*n*(a + b*x^n))
```

fricas [A] time = 1.33, size = 48, normalized size = 0.43

$$\frac{4\,b^3x^{5\,n}+15\,ab^2x^{4\,n}+20\,a^2bx^{3\,n}+10\,a^3x^{2\,n}}{20\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+2*n)*(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x, algorithm="fricas
")
```

```
[Out] 1/20*(4*b^3*x^(5*n) + 15*a*b^2*x^(4*n) + 20*a^2*b*x^(3*n) + 10*a^3*x^(2*n))
/n
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b^2x^{2\,n}+2\,abx^n+a^2\right)^{\frac{3}{2}}x^{2\,n-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+2*n)*(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b^2*x^(2*n) + 2*a*b*x^n + a^2)^(3/2)*x^(2*n - 1), x)
```

maple [A] time = 0.03, size = 135, normalized size = 1.21

$$\frac{\sqrt{(b\,x^n+a)^2}\;a^3x^{2n}}{2\,(b\,x^n+a)\,n}+\frac{\sqrt{(b\,x^n+a)^2}\;a^2b\,x^{3n}}{(b\,x^n+a)\,n}+\frac{3\sqrt{(b\,x^n+a)^2}\;a\,b^2x^{4n}}{4\,(b\,x^n+a)\,n}+\frac{\sqrt{(b\,x^n+a)^2}\;b^3x^{5n}}{5\,(b\,x^n+a)\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1+2*n)*(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x)
```

```
[Out] 1/5*((b*x^n+a)^2)^(1/2)/(b*x^n+a)*b^3/n*(x^n)^5+3/4*((b*x^n+a)^2)^(1/2)/(b*
x^n+a)*a*b^2/n*(x^n)^4+((b*x^n+a)^2)^(1/2)/(b*x^n+a)*a^2*b/n*(x^n)^3+1/2*((
b*x^n+a)^2)^(1/2)/(b*x^n+a)*a^3/n*(x^n)^2
```

maxima [**A**] time = 0.92, size = 48, normalized size = 0.43

$$\frac{4\,b^3x^{5\,n} + 15\,ab^2x^{4\,n} + 20\,a^2bx^{3\,n} + 10\,a^3x^{2\,n}}{20\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+2*n)*(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x, algorithm="maxima
")
```

```
[Out] 1/20*(4*b^3*x^(5*n) + 15*a*b^2*x^(4*n) + 20*a^2*b*x^(3*n) + 10*a^3*x^(2*n))
/n
```

mupad [**F**] time = 0.00, size = -1, normalized size = -0.01

$$\int x^{2\,n-1}\,\left(a^2 + b^2\,x^{2\,n} + 2\,a\,b\,x^n\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(2*n - 1)*(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(3/2),x)
```

```
[Out] int(x^(2*n - 1)*(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(3/2), x)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+2*n)*(a**2+2*a*b*x**n+b**2*x**(2*n))**(3/2),x)
```

```
[Out] Timed out
```

3.446 $\int x^{-1+2n}\sqrt{a^2+2abx^n+b^2x^{2n}}\,dx$

Optimal. Leaf size=99

$$\frac{ax^{2n}\sqrt{a^2+2abx^n+b^2x^{2n}}}{2n\,(a+bx^n)}+\frac{b^2x^{3n}\sqrt{a^2+2abx^n+b^2x^{2n}}}{3n\left(ab+b^2x^n\right)}$$

Rubi [A] time = 0.03, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {1355, 14}

$$\frac{ax^{2n}\sqrt{a^2+2abx^n+b^2x^{2n}}}{2n\,(a+bx^n)}+\frac{b^2x^{3n}\sqrt{a^2+2abx^n+b^2x^{2n}}}{3n\left(ab+b^2x^n\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 + 2*n)*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)],x]
```

```
[Out] (a*x^(2*n)*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])/(2*n*(a + b*x^n)) + (b^2*x^
(3*n)*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])/(3*n*(a*b + b^2*x^n))
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\begin{aligned}\int x^{-1+2n}\sqrt{a^2+2abx^n+b^2x^{2n}}\,dx &= \frac{\sqrt{a^2+2abx^n+b^2x^{2n}}\int x^{-1+2n}\left(ab+b^2x^n\right)dx}{ab+b^2x^n}\\ &= \frac{\sqrt{a^2+2abx^n+b^2x^{2n}}\int\left(abx^{-1+2n}+b^2x^{-1+3n}\right)dx}{ab+b^2x^n}\\ &= \frac{ax^{2n}\sqrt{a^2+2abx^n+b^2x^{2n}}}{2n\,(a+bx^n)}+\frac{b^2x^{3n}\sqrt{a^2+2abx^n+b^2x^{2n}}}{3n\left(ab+b^2x^n\right)}\end{aligned}$$

Mathematica [A] time = 0.02, size = 44, normalized size = 0.44

$$\frac{x^{2n}\sqrt{(a+bx^n)^2}\,(3a+2bx^n)}{6n\,(a+bx^n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 + 2*n)*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)],x]
```

```
[Out] (x^(2*n)*Sqrt[(a + b*x^n)^2]*(3*a + 2*b*x^n))/(6*n*(a + b*x^n))
```

IntegrateAlgebraic [A] time = 0.05, size = 44, normalized size = 0.44

$$\frac{x^{2n}\sqrt{(a+bx^n)^2}\,(3a+2bx^n)}{6n\,(a+bx^n)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(-1 + 2*n)*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)],x]
[Out] (x^(2*n)*Sqrt[(a + b*x^n)^2]*(3*a + 2*b*x^n))/(6*n*(a + b*x^n))
```

fricas [A] time = 1.25, size = 22, normalized size = 0.22

$$\frac{2\,bx^{3\,n}+3\,ax^{2\,n}}{6\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+2*n)*(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x, algorithm="fricas
")
[Out] 1/6*(2*b*x^(3*n) + 3*a*x^(2*n))/n
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b^2x^{2\,n}+2\,abx^n+a^2}\,x^{2\,n-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+2*n)*(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(b^2*x^(2*n) + 2*a*b*x^n + a^2)*x^(2*n - 1), x)
```

maple [A] time = 0.02, size = 64, normalized size = 0.65

$$\frac{\sqrt{(b\,x^n+a)^2}\;a\,x^{2n}}{2\,(b\,x^n+a)\,n}+\frac{\sqrt{(b\,x^n+a)^2}\;b\,x^{3n}}{3\,(b\,x^n+a)\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1+2*n)*(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x)
[Out] 1/3*((b*x^n+a)^2)^(1/2)/(b*x^n+a)*b/n*(x^n)^3+1/2*((b*x^n+a)^2)^(1/2)/(b*x^
n+a)*a/n*(x^n)^2
```

maxima [A] time = 0.89, size = 22, normalized size = 0.22

$$\frac{2\,bx^{3\,n}+3\,ax^{2\,n}}{6\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+2*n)*(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x, algorithm="maxima
")
[Out] 1/6*(2*b*x^(3*n) + 3*a*x^(2*n))/n
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^{2\,n-1}\,\sqrt{a^2+b^2\,x^{2\,n}+2\,a\,b\,x^n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(2*n - 1)*(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(1/2),x)
```

```
[Out] int(x^(2*n - 1)*(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^{2n-1}\sqrt{(a+bx^n)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+2*n)*(a**2+2*a*b*x**n+b**2*x**(2*n))**(1/2),x)
```

```
[Out] Integral(x**(2*n - 1)*sqrt((a + b*x**n)**2), x)
```

3.447 $\int \frac{x^{-1+2n}}{\sqrt{a^2+2abx^n+b^2x^{2n}}}\,dx$

Optimal. Leaf size=90

$$\frac{x^n(a+bx^n)}{bn\sqrt{a^2+2abx^n+b^2x^{2n}}}-\frac{a(a+bx^n)\log(a+bx^n)}{b^2n\sqrt{a^2+2abx^n+b^2x^{2n}}}$$

Rubi [A] time = 0.04, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.094, Rules used = {1355, 266, 43}

$$\frac{x^n(a+bx^n)}{bn\sqrt{a^2+2abx^n+b^2x^{2n}}}-\frac{a(a+bx^n)\log(a+bx^n)}{b^2n\sqrt{a^2+2abx^n+b^2x^{2n}}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 + 2*n)/Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)],x]
```

```
[Out] (x^n*(a + b*x^n))/(b*n*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)]) - (a*(a + b*x^n
)*Log[a + b*x^n])/(b^2*n*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^{-1+2n}}{\sqrt{a^2+2abx^n+b^2x^{2n}}}\,dx &= \frac{\left(ab+b^2x^n\right)\int \frac{x^{-1+2n}}{ab+b^2x^n}\,dx}{\sqrt{a^2+2abx^n+b^2x^{2n}}}\\
&= \frac{\left(ab+b^2x^n\right)\operatorname{Subst}\left(\int \frac{x}{ab+b^2x}\,dx,x,x^n\right)}{n\sqrt{a^2+2abx^n+b^2x^{2n}}}\\
&= \frac{\left(ab+b^2x^n\right)\operatorname{Subst}\left(\int \left(\frac{1}{b^2}-\frac{a}{b^2(a+bx)}\right)dx,x,x^n\right)}{n\sqrt{a^2+2abx^n+b^2x^{2n}}}\\
&= \frac{x^n(a+bx^n)}{bn\sqrt{a^2+2abx^n+b^2x^{2n}}}-\frac{a(a+bx^n)\log(a+bx^n)}{b^2n\sqrt{a^2+2abx^n+b^2x^{2n}}}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 46, normalized size = 0.51

$$\frac{(a+bx^n)\left(\frac{x^n}{b}-\frac{a\log(a+bx^n)}{b^2}\right)}{n\sqrt{(a+bx^n)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 + 2*n)/Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)],x]
```

```
[Out] ((a + b*x^n)*(x^n/b - (a*Log[a + b*x^n])/b^2))/(n*Sqrt[(a + b*x^n)^2])
```

IntegrateAlgebraic [A] time = 0.07, size = 55, normalized size = 0.61

$$\frac{(a+bx^n)\left(\frac{x^n}{bn}-\frac{a\log\left(abn+b^2nx^n\right)}{b^2n}\right)}{\sqrt{(a+bx^n)^2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(-1 + 2*n)/Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)],x]
```

```
[Out] ((a + b*x^n)*(x^n/(b*n) - (a*Log[a*b*n + b^2*n*x^n])/(b^2*n)))/Sqrt[(a + b*
x^n)^2]
```

fricas [A] time = 1.31, size = 24, normalized size = 0.27

$$\frac{bx^n-a\log\left(bx^n+a\right)}{b^2n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+2*n)/(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x, algorithm="fricas
")
```

```
[Out] (b*x^n - a*log(b*x^n + a))/(b^2*n)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^{2\,n-1}}{\sqrt{b^2x^{2\,n}+2\,abx^n+a^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+2*n)/(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^(2*n - 1)/sqrt(b^2*x^(2*n) + 2*a*b*x^n + a^2), x)
```

maple [A] time = 0.03, size = 71, normalized size = 0.79

$$-\frac{\sqrt{(b\,x^n+a)^2}\;a\ln\left(x^n+\frac{a}{b}\right)}{(b\,x^n+a)\,b^2n}+\frac{\sqrt{(b\,x^n+a)^2}\;x^n}{(b\,x^n+a)\,bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1+2*n)/(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x)
```

```
[Out] ((b*x^n+a)^2)^(1/2)/(b*x^n+a)/b/n*x^n-((b*x^n+a)^2)^(1/2)/(b*x^n+a)*a/b^2/n
*ln(x^n+a/b)
```

maxima [A] time = 0.89, size = 32, normalized size = 0.36

$$\frac{x^n}{bn} - \frac{a \log\left(\frac{bx^n+a}{b}\right)}{b^2n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+2*n)/(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x, algorithm="maxima
")
```

```
[Out] x^n/(b*n) - a*log((b*x^n + a)/b)/(b^2*n)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{2n-1}}{\sqrt{a^2 + b^2 x^{2n} + 2 a b x^n}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(2*n - 1)/(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(1/2),x)
```

```
[Out] int(x^(2*n - 1)/(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{2n-1}}{\sqrt{(a + bx^n)^2}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+2*n)/(a**2+2*a*b*x**n+b**2*x**(2*n))**(1/2),x)
```

```
[Out] Integral(x**(2*n - 1)/sqrt((a + b*x**n)**2), x)
```

3.448 $\int \frac{x^{-1+2n}}{\left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}}\,dx$

Optimal. Leaf size=48

$$\frac{x^{2n}}{2an\,(a+bx^n)\sqrt{a^2+2abx^n+b^2x^{2n}}}$$

Rubi [A] time = 0.03, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {1355, 264}

$$\frac{x^{2n}}{2an\,(a+bx^n)\sqrt{a^2+2abx^n+b^2x^{2n}}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 + 2*n)/(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(3/2),x]
```

```
[Out] x^(2*n)/(2*a*n*(a + b*x^n)*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\begin{aligned}\int \frac{x^{-1+2n}}{\left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}}\,dx &= \frac{\left(b^2\left(ab+b^2x^n\right)\right)\int \frac{x^{-1+2n}}{\left(ab+b^2x^n\right)^3}\,dx}{\sqrt{a^2+2abx^n+b^2x^{2n}}}\\ &= \frac{x^{2n}}{2an\,(a+bx^n)\sqrt{a^2+2abx^n+b^2x^{2n}}}\end{aligned}$$

Mathematica [A] time = 0.02, size = 35, normalized size = 0.73

$$\frac{x^{2n}\,(a+bx^n)}{2an\left((a+bx^n)^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 + 2*n)/(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(3/2),x]
```

```
[Out] (x^(2*n)*(a + b*x^n))/(2*a*n*((a + b*x^n)^2)^(3/2))
```

IntegrateAlgebraic [A] time = 0.06, size = 42, normalized size = 0.88

$$\frac{-a-2bx^n}{2b^2n\left(a+bx^n\right)\sqrt{\left(a+bx^n\right)^2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(-1 + 2*n)/(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(3/2),x]
```

```
[Out] (-a - 2*b*x^n)/(2*b^2*n*(a + b*x^n)*Sqrt[(a + b*x^n)^2])
```

fricas [A] time = 1.30, size = 41, normalized size = 0.85

$$-\frac{2\,bx^n+a}{2\left(b^4nx^{2\,n}+2\,ab^3nx^n+a^2b^2n\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+2*n)/(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x, algorithm="fricas
")
```

```
[Out] -1/2*(2*b*x^n + a)/(b^4*n*x^(2*n) + 2*a*b^3*n*x^n + a^2*b^2*n)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{2\,n-1}}{\left(b^2x^{2\,n}+2\,abx^n+a^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+2*n)/(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x^(2*n - 1)/(b^2*x^(2*n) + 2*a*b*x^n + a^2)^(3/2), x)
```

maple [A] time = 0.03, size = 37, normalized size = 0.77

$$-\frac{\sqrt{\left(b\,x^n+a\right)^2}\,\left(2b\,x^n+a\right)}{2\left(b\,x^n+a\right)^3b^2n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1+2*n)/(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x)
```

```
[Out] -1/2*((b*x^n+a)^2)^(1/2)/(b*x^n+a)^3*(2*b*x^n+a)/b^2/n
```

maxima [A] time = 0.92, size = 41, normalized size = 0.85

$$-\frac{2\,bx^n+a}{2\left(b^4nx^{2\,n}+2\,ab^3nx^n+a^2b^2n\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+2*n)/(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x, algorithm="maxima
")
```

```
[Out] -1/2*(2*b*x^n + a)/(b^4*n*x^(2*n) + 2*a*b^3*n*x^n + a^2*b^2*n)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^{2\,n-1}}{\left(a^2+b^2\,x^{2\,n}+2\,a\,b\,x^n\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(2*n - 1)/(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(3/2),x)
[Out] int(x^(2*n - 1)/(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(3/2), x)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{2n-1}}{\left((a+bx^n)^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+2*n)/(a**2+2*a*b*x**n+b**2*x**(2*n))**(3/2),x)
[Out] Integral(x**(2*n - 1)/((a + b*x**n)**2)**(3/2), x)
```

3.449 $\int \frac{x^{-1+2n}}{\left(a^2+2abx^n+b^2x^{2n}\right)^{5/2}}\,dx$

Optimal. Leaf size=88

$$\frac{a}{4b^2n\,(a+bx^n)^3\,\sqrt{a^2+2abx^n+b^2x^{2n}}}-\frac{1}{3b^2n\,(a+bx^n)^2\,\sqrt{a^2+2abx^n+b^2x^{2n}}}$$

Rubi [A] time = 0.05, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.094, Rules used = {1355, 266, 43}

$$\frac{a}{4b^2n\,(a+bx^n)^3\,\sqrt{a^2+2abx^n+b^2x^{2n}}}-\frac{1}{3b^2n\,(a+bx^n)^2\,\sqrt{a^2+2abx^n+b^2x^{2n}}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 + 2*n)/(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(5/2),x]
```

```
[Out] a/(4*b^2*n*(a + b*x^n)^3*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)]) - 1/(3*b^2*n*
(a + b*x^n)^2*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^{-1+2n}}{\left(a^2+2abx^n+b^2x^{2n}\right)^{5/2}}\,dx &= \frac{\left(b^4\left(ab+b^2x^n\right)\right)\int \frac{x^{-1+2n}}{\left(ab+b^2x^n\right)^5}\,dx}{\sqrt{a^2+2abx^n+b^2x^{2n}}}\\
&= \frac{\left(b^4\left(ab+b^2x^n\right)\right)\operatorname{Subst}\left(\int \frac{x}{\left(ab+b^2x\right)^5}\,dx,x,x^n\right)}{n\sqrt{a^2+2abx^n+b^2x^{2n}}}\\
&= \frac{\left(b^4\left(ab+b^2x^n\right)\right)\operatorname{Subst}\left(\int \left(-\frac{a}{b^6(a+bx)^5}+\frac{1}{b^6(a+bx)^4}\right)dx,x,x^n\right)}{n\sqrt{a^2+2abx^n+b^2x^{2n}}}\\
&= \frac{a}{4b^2n\,(a+bx^n)^3\,\sqrt{a^2+2abx^n+b^2x^{2n}}}-\frac{1}{3b^2n\,(a+bx^n)^2\,\sqrt{a^2+2abx^n+b^2x^{2n}}}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 40, normalized size = 0.45

$$-\frac{a+4bx^n}{12b^2n\left(a+bx^n\right)^3\sqrt{\left(a+bx^n\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 + 2*n)/(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(5/2),x]
```

```
[Out] -1/12*(a + 4*b*x^n)/(b^2*n*(a + b*x^n)^3*Sqrt[(a + b*x^n)^2])
```

IntegrateAlgebraic [A] time = 0.14, size = 42, normalized size = 0.48

$$\frac{-a-4bx^n}{12b^2n\left(a+bx^n\right)^3\sqrt{\left(a+bx^n\right)^2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(-1 + 2*n)/(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(5/2),x]
```

```
[Out] (-a - 4*b*x^n)/(12*b^2*n*(a + b*x^n)^3*Sqrt[(a + b*x^n)^2])
```

fricas [A] time = 1.18, size = 69, normalized size = 0.78

$$-\frac{4\,bx^n+a}{12\left(b^6nx^{4\,n}+4\,ab^5nx^{3\,n}+6\,a^2b^4nx^{2\,n}+4\,a^3b^3nx^n+a^4b^2n\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+2*n)/(a^2+2*a*b*x^n+b^2*x^(2*n))^(5/2),x, algorithm="fricas
")
```

```
[Out] -1/12*(4*b*x^n + a)/(b^6*n*x^(4*n) + 4*a*b^5*n*x^(3*n) + 6*a^2*b^4*n*x^(2*n
) + 4*a^3*b^3*n*x^n + a^4*b^2*n)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^{2\,n-1}}{\left(b^2x^{2\,n}+2\,abx^n+a^2\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+2*n)/(a^2+2*a*b*x^n+b^2*x^(2*n))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(x^(2*n - 1)/(b^2*x^(2*n) + 2*a*b*x^n + a^2)^(5/2), x)
```

maple [A] time = 0.03, size = 37, normalized size = 0.42

$$-\frac{\sqrt{\left(b\,x^n+a\right)^2}\;\left(4b\,x^n+a\right)}{12\left(b\,x^n+a\right)^5b^2n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1+2*n)/(a^2+2*a*b*x^n+b^2*x^(2*n))^(5/2),x)
```

```
[Out] -1/12*((b*x^n+a)^2)^(1/2)/(b*x^n+a)^5*(4*b*x^n+a)/b^2/n
```

maxima [A] time = 0.95, size = 69, normalized size = 0.78

$$-\frac{4\,bx^n+a}{12\left(b^6nx^{4\,n}+4\,ab^5nx^{3\,n}+6\,a^2b^4nx^{2\,n}+4\,a^3b^3nx^n+a^4b^2n\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+2*n)/(a^2+2*a*b*x^n+b^2*x^(2*n))^(5/2),x, algorithm="maxima
")
```

```
[Out] -1/12*(4*b*x^n + a)/(b^6*n*x^(4*n) + 4*a*b^5*n*x^(3*n) + 6*a^2*b^4*n*x^(2*n
) + 4*a^3*b^3*n*x^n + a^4*b^2*n)
```

mupad [**F**] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{2\,n-1}}{{\left(a^2+b^2\,x^{2\,n}+2\,a\,b\,x^n\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(2*n - 1)/(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(5/2),x)
```

```
[Out] int(x^(2*n - 1)/(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(5/2), x)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+2*n)/(a**2+2*a*b*x**n+b**2*x**(2*n))**(5/2),x)
```

```
[Out] Timed out
```

3.450 $\int \frac{x^{-1+2n}}{\left(a^2+2abx^n+b^2x^{2n}\right)^{7/2}}\,dx$

Optimal. Leaf size=88

$$\frac{a}{6b^2n\left(a+bx^n\right)^5\sqrt{a^2+2abx^n+b^2x^{2n}}}-\frac{1}{5b^2n\left(a+bx^n\right)^4\sqrt{a^2+2abx^n+b^2x^{2n}}}$$

Rubi [A] time = 0.05, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.094, Rules used = {1355, 266, 43}

$$\frac{a}{6b^2n\left(a+bx^n\right)^5\sqrt{a^2+2abx^n+b^2x^{2n}}}-\frac{1}{5b^2n\left(a+bx^n\right)^4\sqrt{a^2+2abx^n+b^2x^{2n}}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 + 2*n)/(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(7/2),x]
```

```
[Out] a/(6*b^2*n*(a + b*x^n)^5*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)]) - 1/(5*b^2*n*
(a + b*x^n)^4*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^{-1+2n}}{\left(a^2+2abx^n+b^2x^{2n}\right)^{7/2}}\,dx &= \frac{\left(b^6\left(ab+b^2x^n\right)\right)\int \frac{x^{-1+2n}}{\left(ab+b^2x^n\right)^7}\,dx}{\sqrt{a^2+2abx^n+b^2x^{2n}}}\\
&= \frac{\left(b^6\left(ab+b^2x^n\right)\right)\text{Subst}\left(\int \frac{x}{\left(ab+b^2x\right)^7}\,dx,x,x^n\right)}{n\sqrt{a^2+2abx^n+b^2x^{2n}}}\\
&= \frac{\left(b^6\left(ab+b^2x^n\right)\right)\text{Subst}\left(\int \left(-\frac{a}{b^8(a+bx)^7}+\frac{1}{b^8(a+bx)^6}\right)dx,x,x^n\right)}{n\sqrt{a^2+2abx^n+b^2x^{2n}}}\\
&= \frac{a}{6b^2n\left(a+bx^n\right)^5\sqrt{a^2+2abx^n+b^2x^{2n}}}-\frac{1}{5b^2n\left(a+bx^n\right)^4\sqrt{a^2+2abx^n+b^2x^{2n}}}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 40, normalized size = 0.45

$$-\frac{a+6bx^n}{30b^2n(a+bx^n)^5\sqrt{(a+bx^n)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 + 2*n)/(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(7/2),x]
```

```
[Out] -1/30*(a + 6*b*x^n)/(b^2*n*(a + b*x^n)^5*Sqrt[(a + b*x^n)^2])
```

IntegrateAlgebraic [A] time = 0.15, size = 42, normalized size = 0.48

$$\frac{-a-6bx^n}{30b^2n(a+bx^n)^5\sqrt{(a+bx^n)^2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(-1 + 2*n)/(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(7/2),x]
```

```
[Out] (-a - 6*b*x^n)/(30*b^2*n*(a + b*x^n)^5*Sqrt[(a + b*x^n)^2])
```

fricas [A] time = 1.21, size = 97, normalized size = 1.10

$$-\frac{6\,bx^n+a}{30\left(b^8nx^{6\,n}+6\,ab^7nx^{5\,n}+15\,a^2b^6nx^{4\,n}+20\,a^3b^5nx^{3\,n}+15\,a^4b^4nx^{2\,n}+6\,a^5b^3nx^n+a^6b^2n\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+2*n)/(a^2+2*a*b*x^n+b^2*x^(2*n))^(7/2),x, algorithm="fricas
")
```

```
[Out] -1/30*(6*b*x^n + a)/(b^8*n*x^(6*n) + 6*a*b^7*n*x^(5*n) + 15*a^2*b^6*n*x^(4*
n) + 20*a^3*b^5*n*x^(3*n) + 15*a^4*b^4*n*x^(2*n) + 6*a^5*b^3*n*x^n + a^6*b^
2*n)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^{2\,n-1}}{\left(b^2x^{2\,n}+2\,abx^n+a^2\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+2*n)/(a^2+2*a*b*x^n+b^2*x^(2*n))^(7/2),x, algorithm="giac")
```

```
[Out] integrate(x^(2*n - 1)/(b^2*x^(2*n) + 2*a*b*x^n + a^2)^(7/2), x)
```

maple [A] time = 0.03, size = 37, normalized size = 0.42

$$-\frac{\sqrt{(b\,x^n+a)^2}\,\left(6b\,x^n+a\right)}{30\,(b\,x^n+a)^7\,b^2n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1+2*n)/(a^2+2*a*b*x^n+b^2*x^(2*n))^(7/2),x)
```

```
[Out] -1/30*((b*x^n+a)^2)^(1/2)/(b*x^n+a)^7*(6*b*x^n+a)/b^2/n
```

maxima [A] time = 0.97, size = 97, normalized size = 1.10

$$-\frac{6\,bx^n+a}{30\left(b^8nx^{6\,n}+6\,ab^7nx^{5\,n}+15\,a^2b^6nx^{4\,n}+20\,a^3b^5nx^{3\,n}+15\,a^4b^4nx^{2\,n}+6\,a^5b^3nx^n+a^6b^2n\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+2*n)/(a^2+2*a*b*x^n+b^2*x^(2*n))^(7/2),x, algorithm="maxima
")
```

```
[Out] -1/30*(6*b*x^n + a)/(b^8*n*x^(6*n) + 6*a*b^7*n*x^(5*n) + 15*a^2*b^6*n*x^(4*
n) + 20*a^3*b^5*n*x^(3*n) + 15*a^4*b^4*n*x^(2*n) + 6*a^5*b^3*n*x^n + a^6*b^
2*n)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{2n-1}}{\left(a^2+b^2x^{2n}+2abx^n\right)^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(2*n - 1)/(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(7/2),x)
```

```
[Out] int(x^(2*n - 1)/(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(7/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+2*n)/(a**2+2*a*b*x**n+b**2*x**(2*n))**(7/2),x)
```

```
[Out] Timed out
```

3.451 $\int (dx)^m \sqrt{a^2+2abx^n+b^2x^{2n}}\,dx$

Optimal. Leaf size=108

$$\frac{b^2x^{n+1}(dx)^m\sqrt{a^2+2abx^n+b^2x^{2n}}}{(m+n+1)\left(ab+b^2x^n\right)}+\frac{a(dx)^{m+1}\sqrt{a^2+2abx^n+b^2x^{2n}}}{d(m+1)\left(a+bx^n\right)}$$

Rubi [A] time = 0.04, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {1355, 14, 20, 30}

$$\frac{b^2x^{n+1}(dx)^m\sqrt{a^2+2abx^n+b^2x^{2n}}}{(m+n+1)\left(ab+b^2x^n\right)}+\frac{a(dx)^{m+1}\sqrt{a^2+2abx^n+b^2x^{2n}}}{d(m+1)\left(a+bx^n\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^m*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)],x]
```

```
[Out] (a*(d*x)^(1 + m)*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])/(d*(1 + m)*(a + b*x^n
)) + (b^2*x^(1 + n)*(d*x)^m*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])/((1 + m +
n)*(a*b + b^2*x^n))
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(b^IntPart
[n]*(b*v)^FracPart[n])/(a^IntPart[n]*(a*v)^FracPart[n]), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !
IntegerQ[m + n]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\int (dx)^m \sqrt{a^2+2abx^n+b^2x^{2n}}\,dx = \frac{\sqrt{a^2+2abx^n+b^2x^{2n}}\int (dx)^m \left(ab+b^2x^n\right)dx}{ab+b^2x^n}$$
$$= \frac{\sqrt{a^2+2abx^n+b^2x^{2n}}\int \left(ab(dx)^m+b^2x^n(dx)^m\right)dx}{ab+b^2x^n}$$
$$= \frac{a(dx)^{1+m}\sqrt{a^2+2abx^n+b^2x^{2n}}}{d(1+m)\left(a+bx^n\right)} + \frac{\left(b^2\sqrt{a^2+2abx^n+b^2x^{2n}}\right)\int x^n(dx)^m\,dx}{ab+b^2x^n}$$
$$= \frac{a(dx)^{1+m}\sqrt{a^2+2abx^n+b^2x^{2n}}}{d(1+m)\left(a+bx^n\right)} + \frac{\left(b^2x^{-m}(dx)^m\sqrt{a^2+2abx^n+b^2x^{2n}}\right)\int x^{m+n}}{ab+b^2x^n}$$
$$= \frac{a(dx)^{1+m}\sqrt{a^2+2abx^n+b^2x^{2n}}}{d(1+m)\left(a+bx^n\right)} + \frac{b^2x^{1+n}(dx)^m\sqrt{a^2+2abx^n+b^2x^{2n}}}{(1+m+n)\left(ab+b^2x^n\right)}$$

Mathematica [A] time = 0.03, size = 55, normalized size = 0.51

$$\frac{x(dx)^m\sqrt{\left(a+bx^n\right)^2}\left(a(m+n+1)+b(m+1)x^n\right)}{(m+1)(m+n+1)\left(a+bx^n\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^m*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)],x]
```

```
[Out] (x*(d*x)^m*Sqrt[(a + b*x^n)^2]*(a*(1 + m + n) + b*(1 + m)*x^n))/((1 + m)*(1
 + m + n)*(a + b*x^n))
```

IntegrateAlgebraic [F] time = 0.19, size = 0, normalized size = 0.00

$$\int (dx)^m\sqrt{a^2+2abx^n+b^2x^{2n}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(d*x)^m*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)],x]
```

```
[Out] Defer[IntegrateAlgebraic][(d*x)^m*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)], x]
```

fricas [A] time = 1.45, size = 57, normalized size = 0.53

$$\frac{(bm+b)xx^ne^{\left(m\log(d)+m\log(x)\right)}+(am+an+a)xe^{\left(m\log(d)+m\log(x)\right)}}{m^2+(m+1)n+2\,m+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x, algorithm="fricas")
```

```
[Out] ((b*m + b)*x*x^n*e^(m*log(d) + m*log(x)) + (a*m + a*n + a)*x*e^(m*log(d) +
m*log(x)))/(m^2 + (m + 1)*n + 2*m + 1)
```

giac [A] time = 0.35, size = 173, normalized size = 1.60

$$\frac{bmxx^ne^{\left(m\log(d)+m\log(x)\right)}\mathrm{sgn}\left(bx^n+a\right)+amxe^{\left(m\log(d)+m\log(x)\right)}\mathrm{sgn}\left(bx^n+a\right)+bmxe^{\left(m\log(d)+m\log(x)\right)}\mathrm{sgn}\left(bx^n+a\right)+anxe^{\left(m\log(d)+m\log(x)\right)}\mathrm{sgn}\left(bx^n+a\right)+bxx^ne^{\left(m\log(d)+m\log(x)\right)}\mathrm{sgn}\left(bx^n+a\right)+axe^{\left(m\log(d)+m\log(x)\right)}\mathrm{sgn}\left(bx^n+a\right)+bxe^{\left(m\log(d)+m\log(x)\right)}\mathrm{sgn}\left(bx^n+a\right)}{m^2+mn+2\,m+n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x, algorithm="giac")
```

```
[Out] (b*m*x*x^n*e^(m*log(d) + m*log(x))*sgn(b*x^n + a) + a*m*x*e^(m*log(d) + m*l
og(x))*sgn(b*x^n + a) + b*m*x*e^(m*log(d) + m*log(x))*sgn(b*x^n + a) + a*n*
```

```
x*e^(m*log(d) + m*log(x))*sgn(b*x^n + a) + b*x*x^n*e^(m*log(d) + m*log(x))*
sgn(b*x^n + a) + a*x*e^(m*log(d) + m*log(x))*sgn(b*x^n + a) + b*x*e^(m*log(
d) + m*log(x))*sgn(b*x^n + a))/(m^2 + m*n + 2*m + n + 1)
```

maple [C] time = 0.04, size = 132, normalized size = 1.22

$$\frac{\sqrt{(b\,x^{n}+a)^{2}}\,\,(bm\,x^{n}+am+an+b\,x^{n}+a)\,x\,\mathrm{e}^{\frac{\left(-i\pi\,\mathrm{csgn}(id)\mathrm{csgn}(ix)\mathrm{csgn}(idx)+i\pi\,\mathrm{csgn}(id)\mathrm{csgn}(idx)^{2}+i\pi\,\mathrm{csgn}(ix)\mathrm{csgn}(idx)^{2}-i\pi\mathrm{csgn}(idx)^{3}+2\ln(d)+2\ln(x)\right)m}{2}}}{(b\,x^{n}+a)\,(m+1)\,(m+n+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x)
```

```
[Out] ((b*x^n+a)^2)^(1/2)/(b*x^n+a)*x*(m*b*x^n+a*m+a*n+b*x^n+a)/(m+1)/(1+m+n)*exp
(1/2*m*(-I*Pi*csgn(I*d*x)^3+I*Pi*csgn(I*d*x)^2*csgn(I*d)+I*Pi*csgn(I*d*x)^2
*csgn(I*x)-I*Pi*csgn(I*d*x)*csgn(I*d)*csgn(I*x)+2*ln(d)+2*ln(x)))
```

maxima [A] time = 0.94, size = 47, normalized size = 0.44

$$\frac{ad^{m}(m+n+1)xx^{m}+bd^{m}(m+1)xe^{\left(m\log(x)+n\log(x)\right)}}{m^{2}+m(n+2)+n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x, algorithm="maxima")
```

```
[Out] (a*d^m*(m + n + 1)*x*x^m + b*d^m*(m + 1)*x*e^(m*log(x) + n*log(x)))/(m^2 +
m*(n + 2) + n + 1)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (d\,x)^{m}\,\sqrt{a^{2}+b^{2}\,x^{2\,n}+2\,a\,b\,x^{n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(1/2),x)
```

```
[Out] int((d*x)^m*(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx)^{m}\sqrt{(a+bx^{n})^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(a**2+2*a*b*x**n+b**2*x**(2*n))**(1/2),x)
```

```
[Out] Integral((d*x)**m*sqrt((a + b*x**n)**2), x)
```

3.452 $\int x^2\sqrt{a^2+2abx^n+b^2x^{2n}}\,dx$

Optimal. Leaf size=93

$$\frac{b^2x^{n+3}\sqrt{a^2+2abx^n+b^2x^{2n}}}{(n+3)\left(ab+b^2x^n\right)}+\frac{ax^3\sqrt{a^2+2abx^n+b^2x^{2n}}}{3\left(a+bx^n\right)}$$

Rubi [A] time = 0.03, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {1355, 14}

$$\frac{b^2x^{n+3}\sqrt{a^2+2abx^n+b^2x^{2n}}}{(n+3)\left(ab+b^2x^n\right)}+\frac{ax^3\sqrt{a^2+2abx^n+b^2x^{2n}}}{3\left(a+bx^n\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)],x]

[Out] (a*x^3*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])/(3*(a + b*x^n)) + (b^2*x^(3 + n
)*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])/((3 + n)*(a*b + b^2*x^n))
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\begin{aligned}\int x^2\sqrt{a^2+2abx^n+b^2x^{2n}}\,dx&=\frac{\sqrt{a^2+2abx^n+b^2x^{2n}}\int x^2\left(ab+b^2x^n\right)dx}{ab+b^2x^n}\\&=\frac{\sqrt{a^2+2abx^n+b^2x^{2n}}\int\left(abx^2+b^2x^{2+n}\right)dx}{ab+b^2x^n}\\&=\frac{ax^3\sqrt{a^2+2abx^n+b^2x^{2n}}}{3\left(a+bx^n\right)}+\frac{b^2x^{3+n}\sqrt{a^2+2abx^n+b^2x^{2n}}}{(3+n)\left(ab+b^2x^n\right)}\end{aligned}$$

Mathematica [A] time = 0.02, size = 46, normalized size = 0.49

$$\frac{x^3\sqrt{\left(a+bx^n\right)^2}\,\left(a(n+3)+3bx^n\right)}{3(n+3)\left(a+bx^n\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)],x]

[Out] (x^3*Sqrt[(a + b*x^n)^2]*(a*(3 + n) + 3*b*x^n))/(3*(3 + n)*(a + b*x^n))
```

IntegrateAlgebraic [F] time = 0.18, size = 0, normalized size = 0.00

$$\int x^2\sqrt{a^2+2abx^n+b^2x^{2n}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^2*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)],x]
```

```
[Out] Defer[IntegrateAlgebraic][x^2*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)], x]
```

fricas [A] time = 1.11, size = 28, normalized size = 0.30

$$\frac{3\,bx^3x^n+(an+3\,a)x^3}{3\,(n+3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/3*(3*b*x^3*x^n + (a*n + 3*a)*x^3)/(n + 3)
```

giac [A] time = 0.26, size = 53, normalized size = 0.57

$$\frac{3\,bx^3x^n\mathrm{sgn}\,(bx^n+a)+anx^3\mathrm{sgn}\,(bx^n+a)+3\,ax^3\mathrm{sgn}\,(bx^n+a)}{3\,(n+3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x, algorithm="giac")
```

```
[Out] 1/3*(3*b*x^3*x^n*sgn(b*x^n + a) + a*n*x^3*sgn(b*x^n + a) + 3*a*x^3*sgn(b*x^
n + a))/(n + 3)
```

maple [A] time = 0.02, size = 61, normalized size = 0.66

$$\frac{\sqrt{(b\,x^n+a)^2}\;b\,x^3x^n}{(b\,x^n+a)\,(n+3)}+\frac{\sqrt{(b\,x^n+a)^2}\;a\,x^3}{3b\,x^n+3a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x)
```

```
[Out] 1/3*((b*x^n+a)^2)^(1/2)/(b*x^n+a)*a*x^3+((b*x^n+a)^2)^(1/2)/(b*x^n+a)*b/(3+
n)*x^3*x^n
```

maxima [A] time = 0.95, size = 25, normalized size = 0.27

$$\frac{3\,bx^3x^n+a(n+3)x^3}{3\,(n+3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x, algorithm="maxima")
```

```
[Out] 1/3*(3*b*x^3*x^n + a*(n + 3)*x^3)/(n + 3)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^2\,\sqrt{a^2+b^2\,x^{2\,n}+2\,a\,b\,x^n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(1/2),x)
```

```
[Out] int(x^2*(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(1/2), x)
```

sympy [F]   time = 0.00, size = 0, normalized size = 0.00

$$\int x^2\sqrt{(a+bx^n)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a**2+2*a*b*x**n+b**2*x**(2*n))**(1/2),x)
```

```
[Out] Integral(x**2*sqrt((a + b*x**n)**2), x)
```

3.453 $\int x\sqrt{a^2+2abx^n+b^2x^{2n}}\,dx$

Optimal. Leaf size=93

$$\frac{b^2x^{n+2}\sqrt{a^2+2abx^n+b^2x^{2n}}}{(n+2)\left(ab+b^2x^n\right)}+\frac{ax^2\sqrt{a^2+2abx^n+b^2x^{2n}}}{2\left(a+bx^n\right)}$$

Rubi [A] time = 0.02, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1355, 14}

$$\frac{b^2x^{n+2}\sqrt{a^2+2abx^n+b^2x^{2n}}}{(n+2)\left(ab+b^2x^n\right)}+\frac{ax^2\sqrt{a^2+2abx^n+b^2x^{2n}}}{2\left(a+bx^n\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)],x]
```

```
[Out] (a*x^2*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])/(2*(a + b*x^n)) + (b^2*x^(2 + n
)*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])/((2 + n)*(a*b + b^2*x^n))
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\begin{aligned}\int x\sqrt{a^2+2abx^n+b^2x^{2n}}\,dx &= \frac{\sqrt{a^2+2abx^n+b^2x^{2n}}\int x\left(ab+b^2x^n\right)dx}{ab+b^2x^n}\\ &= \frac{\sqrt{a^2+2abx^n+b^2x^{2n}}\int\left(abx+b^2x^{1+n}\right)dx}{ab+b^2x^n}\\ &= \frac{ax^2\sqrt{a^2+2abx^n+b^2x^{2n}}}{2\left(a+bx^n\right)}+\frac{b^2x^{2+n}\sqrt{a^2+2abx^n+b^2x^{2n}}}{(2+n)\left(ab+b^2x^n\right)}\end{aligned}$$

Mathematica [A] time = 0.02, size = 46, normalized size = 0.49

$$\frac{x^2\sqrt{\left(a+bx^n\right)^2}\left(a(n+2)+2bx^n\right)}{2(n+2)\left(a+bx^n\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)],x]
```

```
[Out] (x^2*Sqrt[(a + b*x^n)^2]*(a*(2 + n) + 2*b*x^n))/(2*(2 + n)*(a + b*x^n))
```

IntegrateAlgebraic [F] time = 0.11, size = 0, normalized size = 0.00

$$\int x\sqrt{a^2 + 2abx^n + b^2x^{2n}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)],x]
```

```
[Out] Defer[IntegrateAlgebraic][x*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)], x]
```

fricas [A] time = 1.15, size = 28, normalized size = 0.30

$$\frac{2\,bx^2x^n + (an + 2\,a)x^2}{2\,(n+2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*(2*b*x^2*x^n + (a*n + 2*a)*x^2)/(n + 2)
```

giac [A] time = 0.29, size = 53, normalized size = 0.57

$$\frac{2\,bx^2x^n\mathrm{sgn}\left(bx^n + a\right) + anx^2\mathrm{sgn}\left(bx^n + a\right) + 2\,ax^2\mathrm{sgn}\left(bx^n + a\right)}{2\,(n+2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*(2*b*x^2*x^n*sgn(b*x^n + a) + a*n*x^2*sgn(b*x^n + a) + 2*a*x^2*sgn(b*x^
n + a))/(n + 2)
```

maple [A] time = 0.02, size = 61, normalized size = 0.66

$$\frac{\sqrt{\left(b\,x^n + a\right)^2}\;b\,x^2x^n}{\left(b\,x^n + a\right)(n+2)} + \frac{\sqrt{\left(b\,x^n + a\right)^2}\;a\,x^2}{2b\,x^n + 2a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x)
```

```
[Out] 1/2*((b*x^n+a)^2)^(1/2)/(b*x^n+a)*a*x^2+((b*x^n+a)^2)^(1/2)/(b*x^n+a)*b/(2+
n)*x^2*x^n
```

maxima [A] time = 0.88, size = 25, normalized size = 0.27

$$\frac{2\,bx^2x^n + a(n+2)x^2}{2\,(n+2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*(2*b*x^2*x^n + a*(n + 2)*x^2)/(n + 2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x\,\sqrt{a^2 + b^2\,x^{2\,n} + 2\,a\,b\,x^n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(1/2),x)
```

```
[Out] int(x*(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\sqrt{(a+bx^n)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a**2+2*a*b*x**n+b**2*x**(2*n))**(1/2),x)
```

```
[Out] Integral(x*sqrt((a + b*x**n)**2), x)
```

3.454 $\int \sqrt{a^2+2abx^n+b^2x^{2n}}\,dx$

Optimal. Leaf size=88

$$\frac{b^2x^{n+1}\sqrt{a^2+2abx^n+b^2x^{2n}}}{(n+1)\left(ab+b^2x^n\right)}+\frac{ax\sqrt{a^2+2abx^n+b^2x^{2n}}}{a+bx^n}$$

Rubi [A] time = 0.02, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {1343}

$$\frac{b^2x^{n+1}\sqrt{a^2+2abx^n+b^2x^{2n}}}{(n+1)\left(ab+b^2x^n\right)}+\frac{ax\sqrt{a^2+2abx^n+b^2x^{2n}}}{a+bx^n}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)],x]
```

```
[Out] (a*x*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])/(a + b*x^n) + (b^2*x^(1 + n)*Sqrt
[a^2 + 2*a*b*x^n + b^2*x^(2*n)])/((1 + n)*(a*b + b^2*x^n))
```

Rule 1343

```
Int[((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_), x_Symbol] :> Dist[(
a + b*x^n + c*x^(2*n))^p/(b + 2*c*x^n)^(2*p), Int[(b + 2*c*x^n)^(2*p), x],
x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}\int \sqrt{a^2+2abx^n+b^2x^{2n}}\,dx &= \frac{\sqrt{a^2+2abx^n+b^2x^{2n}}\int \left(2ab+2b^2x^n\right)dx}{2ab+2b^2x^n}\\ &= \frac{ax\sqrt{a^2+2abx^n+b^2x^{2n}}}{a+bx^n}+\frac{b^2x^{1+n}\sqrt{a^2+2abx^n+b^2x^{2n}}}{(1+n)\left(ab+b^2x^n\right)}\end{aligned}$$

Mathematica [A] time = 0.01, size = 39, normalized size = 0.44

$$\frac{x\sqrt{(a+bx^n)^2}\,(an+a+bx^n)}{(n+1)(a+bx^n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)],x]
```

```
[Out] (x*Sqrt[(a + b*x^n)^2]*(a + a*n + b*x^n))/((1 + n)*(a + b*x^n))
```

IntegrateAlgebraic [F] time = 0.09, size = 0, normalized size = 0.00

$$\int \sqrt{a^2+2abx^n+b^2x^{2n}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)],x]
```

```
[Out] Defer[IntegrateAlgebraic][Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)], x]
```

fricas [A] time = 1.32, size = 20, normalized size = 0.23

$$\frac{bxx^n + (an + a)x}{n + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x, algorithm="fricas")
[Out] (b*x*x^n + (a*n + a)*x)/(n + 1)
```

giac [A] time = 0.22, size = 25, normalized size = 0.28

$$\left(ax + \frac{bx^{n+1}}{n + 1}\right)\text{sgn}\left(bx^n + a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x, algorithm="giac")
[Out] (a*x + b*x^(n + 1)/(n + 1))*sgn(b*x^n + a)
```

maple [A] time = 0.02, size = 56, normalized size = 0.64

$$\frac{\sqrt{\left(b\,x^{n} + a\right)^{2}}\;bx\,x^{n}}{\left(b\,x^{n} + a\right)\left(n + 1\right)} + \frac{\sqrt{\left(b\,x^{n} + a\right)^{2}}\;ax}{b\,x^{n} + a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x)
[Out] ((b*x^n+a)^2)^(1/2)/(b*x^n+a)*a*x+((b*x^n+a)^2)^(1/2)/(b*x^n+a)*b/(1+n)*x*x
^n
```

maxima [A] time = 1.04, size = 19, normalized size = 0.22

$$\frac{a(n + 1)x + bxx^n}{n + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x, algorithm="maxima")
[Out] (a*(n + 1)*x + b*x*x^n)/(n + 1)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{a^2 + b^2\,x^{2\,n} + 2\,a\,b\,x^n}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(1/2),x)
[Out] int((a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a^2 + 2abx^n + b^2x^{2n}}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2+2*a*b*x**n+b**2*x**(2*n))**(1/2),x)
[Out] Integral(sqrt(a**2 + 2*a*b*x**n + b**2*x**(2*n)), x)
```

3.455 $\int \frac{\sqrt{a^2+2abx^n+b^2x^{2n}}}{x}\,dx$

Optimal. Leaf size=85

$$\frac{b^2x^n\sqrt{a^2+2abx^n+b^2x^{2n}}}{n\left(ab+b^2x^n\right)}+\frac{a\log(x)\sqrt{a^2+2abx^n+b^2x^{2n}}}{a+bx^n}$$

Rubi [A] time = 0.03, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {1355, 14}

$$\frac{b^2x^n\sqrt{a^2+2abx^n+b^2x^{2n}}}{n\left(ab+b^2x^n\right)}+\frac{a\log(x)\sqrt{a^2+2abx^n+b^2x^{2n}}}{a+bx^n}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)]/x,x]
```

```
[Out] (b^2*x^n*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])/(n*(a*b + b^2*x^n)) + (a*Sqrt
[a^2 + 2*a*b*x^n + b^2*x^(2*n)]*Log[x])/(a + b*x^n)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{a^2+2abx^n+b^2x^{2n}}}{x}\,dx &= \frac{\sqrt{a^2+2abx^n+b^2x^{2n}}\int \frac{ab+b^2x^n}{x}\,dx}{ab+b^2x^n}\\
&= \frac{\sqrt{a^2+2abx^n+b^2x^{2n}}\int \left(\frac{ab}{x}+b^2x^{-1+n}\right)dx}{ab+b^2x^n}\\
&= \frac{b^2x^n\sqrt{a^2+2abx^n+b^2x^{2n}}}{n\left(ab+b^2x^n\right)}+\frac{a\sqrt{a^2+2abx^n+b^2x^{2n}}\log(x)}{a+bx^n}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 37, normalized size = 0.44

$$\frac{\sqrt{(a+bx^n)^2}\left(an\log(x)+bx^n\right)}{n\left(a+bx^n\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)]/x,x]
```

```
[Out] (Sqrt[(a + b*x^n)^2]*(b*x^n + a*n*Log[x]))/(n*(a + b*x^n))
```

IntegrateAlgebraic [A] time = 0.04, size = 41, normalized size = 0.48

$$\frac{\sqrt{(a+bx^n)^2}\left(\frac{a\log(x^n)}{n}+\frac{bx^n}{n}\right)}{a+bx^n}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)]/x,x]
[Out] (Sqrt[(a + b*x^n)^2]*((b*x^n)/n + (a*Log[x^n])/n))/(a + b*x^n)
```

fricas [A] time = 1.36, size = 15, normalized size = 0.18

$$\frac{an\log(x)+bx^n}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2)/x,x, algorithm="fricas")
[Out] (a*n*log(x) + b*x^n)/n
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{b^2x^{2n}+2\,abx^n+a^2}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2)/x,x, algorithm="giac")
[Out] integrate(sqrt(b^2*x^(2*n) + 2*a*b*x^n + a^2)/x, x)
```

maple [A] time = 0.02, size = 54, normalized size = 0.64

$$\frac{\sqrt{(b\,x^n+a)^2}\;a\ln(x)}{b\,x^n+a}+\frac{\sqrt{(b\,x^n+a)^2}\;b\,x^n}{(b\,x^n+a)\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2)/x,x)
[Out] ((b*x^n+a)^2)^(1/2)/(b*x^n+a)*a*ln(x)+((b*x^n+a)^2)^(1/2)/(b*x^n+a)*b/n*x^n
```

maxima [A] time = 0.80, size = 13, normalized size = 0.15

$$a\log(x)+\frac{bx^n}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2)/x,x, algorithm="maxima")
[Out] a*log(x) + b*x^n/n
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{\sqrt{a^2+b^2\,x^{2\,n}+2\,a\,b\,x^n}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(1/2)/x,x)
```

```
[Out] int((a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(1/2)/x, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{(a + bx^n)^2}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2+2*a*b*x**n+b**2*x**(2*n))**(1/2)/x,x)
```

```
[Out] Integral(sqrt((a + b*x**n)**2)/x, x)
```

3.456 $\int \frac{\sqrt{a^2+2abx^n+b^2x^{2n}}}{x^2}\,dx$

Optimal. Leaf size=94

$$-\frac{b^2x^{n-1}\sqrt{a^2+2abx^n+b^2x^{2n}}}{(1-n)\left(ab+b^2x^n\right)}-\frac{a\sqrt{a^2+2abx^n+b^2x^{2n}}}{x\,(a+bx^n)}$$

Rubi [A] time = 0.03, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {1355, 14}

$$-\frac{b^2x^{n-1}\sqrt{a^2+2abx^n+b^2x^{2n}}}{(1-n)\left(ab+b^2x^n\right)}-\frac{a\sqrt{a^2+2abx^n+b^2x^{2n}}}{x\,(a+bx^n)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)]/x^2,x]
```

```
[Out] -((a*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])/(x*(a + b*x^n))) - (b^2*x^(-1 + n
)*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])/((1 - n)*(a*b + b^2*x^n))
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{a^2+2abx^n+b^2x^{2n}}}{x^2}\,dx &= \frac{\sqrt{a^2+2abx^n+b^2x^{2n}}\int \frac{ab+b^2x^n}{x^2}\,dx}{ab+b^2x^n}\\
&= \frac{\sqrt{a^2+2abx^n+b^2x^{2n}}\int \left(\frac{ab}{x^2}+b^2x^{-2+n}\right)dx}{ab+b^2x^n}\\
&= -\frac{a\sqrt{a^2+2abx^n+b^2x^{2n}}}{x\,(a+bx^n)}-\frac{b^2x^{-1+n}\sqrt{a^2+2abx^n+b^2x^{2n}}}{(1-n)\left(ab+b^2x^n\right)}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 42, normalized size = 0.45

$$\frac{\sqrt{(a+bx^n)^2}\,(-an+a+bx^n)}{(n-1)x\,(a+bx^n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)]/x^2,x]
```

```
[Out] (Sqrt[(a + b*x^n)^2]*(a - a*n + b*x^n))/((-1 + n)*x*(a + b*x^n))
```

IntegrateAlgebraic [F] time = 0.21, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a^2 + 2abx^n + b^2x^{2n}}}{x^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)]/x^2,x]
[Out] Defer[IntegrateAlgebraic][Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)]/x^2, x]
```

fricas [A] time = 1.29, size = 23, normalized size = 0.24

$$-\frac{an - bx^n - a}{(n-1)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2)/x^2,x, algorithm="fricas")
[Out] -(a*n - b*x^n - a)/((n - 1)*x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{b^2x^{2n} + 2\,abx^n + a^2}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2)/x^2,x, algorithm="giac")
[Out] integrate(sqrt(b^2*x^(2*n) + 2*a*b*x^n + a^2)/x^2, x)
```

maple [A] time = 0.02, size = 61, normalized size = 0.65

$$\frac{\sqrt{(b\,x^n + a)^2}\;b\,x^n}{(b\,x^n + a)\,(n-1)\,x} - \frac{\sqrt{(b\,x^n + a)^2}\;a}{(b\,x^n + a)\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2)/x^2,x)
[Out] -((b*x^n+a)^2)^(1/2)/(b*x^n+a)*a/x+((b*x^n+a)^2)^(1/2)/(b*x^n+a)/(-1+n)*b/x
*x^n
```

maxima [A] time = 0.67, size = 22, normalized size = 0.23

$$-\frac{a(n-1) - bx^n}{(n-1)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2)/x^2,x, algorithm="maxima")
[Out] -(a*(n - 1) - b*x^n)/((n - 1)*x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{a^2 + b^2\,x^{2\,n} + 2\,a\,b\,x^n}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(1/2)/x^2,x)
```

```
[Out] int((a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(1/2)/x^2, x)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{(a+bx^n)^2}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2+2*a*b*x**n+b**2*x**(2*n))**(1/2)/x**2,x)
```

```
[Out] Integral(sqrt((a + b*x**n)**2)/x**2, x)
```

3.457 $\int \frac{\sqrt{a^2+2abx^n+b^2x^{2n}}}{x^3}\,dx$

Optimal. Leaf size=96

$$-\frac{b^2x^{n-2}\sqrt{a^2+2abx^n+b^2x^{2n}}}{(2-n)\left(ab+b^2x^n\right)}-\frac{a\sqrt{a^2+2abx^n+b^2x^{2n}}}{2x^2\left(a+bx^n\right)}$$

Rubi [A] time = 0.03, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {1355, 14}

$$-\frac{b^2x^{n-2}\sqrt{a^2+2abx^n+b^2x^{2n}}}{(2-n)\left(ab+b^2x^n\right)}-\frac{a\sqrt{a^2+2abx^n+b^2x^{2n}}}{2x^2\left(a+bx^n\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)]/x^3,x]
```

```
[Out] -(a*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])/(2*x^2*(a + b*x^n)) - (b^2*x^(-2 +
 n)*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])/((2 - n)*(a*b + b^2*x^n))
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{a^2+2abx^n+b^2x^{2n}}}{x^3}\,dx &= \frac{\sqrt{a^2+2abx^n+b^2x^{2n}}\int \frac{ab+b^2x^n}{x^3}\,dx}{ab+b^2x^n}\\
&= \frac{\sqrt{a^2+2abx^n+b^2x^{2n}}\int \left(\frac{ab}{x^3}+b^2x^{-3+n}\right)dx}{ab+b^2x^n}\\
&= -\frac{a\sqrt{a^2+2abx^n+b^2x^{2n}}}{2x^2\left(a+bx^n\right)}-\frac{b^2x^{-2+n}\sqrt{a^2+2abx^n+b^2x^{2n}}}{(2-n)\left(ab+b^2x^n\right)}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 47, normalized size = 0.49

$$\frac{\sqrt{\left(a+bx^n\right)^2}\left(2bx^n-a(n-2)\right)}{2(n-2)x^2\left(a+bx^n\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)]/x^3,x]
```

```
[Out] (Sqrt[(a + b*x^n)^2]*(-(a*(-2 + n)) + 2*b*x^n))/(2*(-2 + n)*x^2*(a + b*x^n)
)
```

IntegrateAlgebraic [F] time = 0.24, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a^2 + 2abx^n + b^2x^{2n}}}{x^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)]/x^3,x]
[Out] Defer[IntegrateAlgebraic][Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)]/x^3, x]
```

fricas [A] time = 1.33, size = 23, normalized size = 0.24

$$-\frac{an - 2\,bx^n - 2\,a}{2\,(n-2)x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2)/x^3,x, algorithm="fricas")
[Out] -1/2*(a*n - 2*b*x^n - 2*a)/((n - 2)*x^2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{b^2x^{2\,n} + 2\,abx^n + a^2}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2)/x^3,x, algorithm="giac")
[Out] integrate(sqrt(b^2*x^(2*n) + 2*a*b*x^n + a^2)/x^3, x)
```

maple [A] time = 0.02, size = 61, normalized size = 0.64

$$\frac{\sqrt{(b\,x^n + a)^2}\;b\,x^n}{(b\,x^n + a)\,(n-2)\,x^2} - \frac{\sqrt{(b\,x^n + a)^2}\;a}{2\,(b\,x^n + a)\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2)/x^3,x)
[Out] -1/2*((b*x^n+a)^2)^(1/2)/(b*x^n+a)*a/x^2+((b*x^n+a)^2)^(1/2)/(b*x^n+a)/(-2+
n)*b/x^2*x^n
```

maxima [A] time = 0.91, size = 22, normalized size = 0.23

$$-\frac{a(n-2) - 2\,bx^n}{2\,(n-2)x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2)/x^3,x, algorithm="maxima")
[Out] -1/2*(a*(n - 2) - 2*b*x^n)/((n - 2)*x^2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{a^2 + b^2\,x^{2\,n} + 2\,a\,b\,x^n}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(1/2)/x^3,x)
```

```
[Out] int((a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(1/2)/x^3, x)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{(a+bx^n)^2}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2+2*a*b*x**n+b**2*x**(2*n))**(1/2)/x**3,x)
```

```
[Out] Integral(sqrt((a + b*x**n)**2)/x**3, x)
```

3.458 $\int (dx)^m \left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}\,dx$

Optimal. Leaf size=238

$$\frac{3a^2b^2x^{n+1}(dx)^m\sqrt{a^2+2abx^n+b^2x^{2n}}}{(m+n+1)\left(ab+b^2x^n\right)}+\frac{b^4x^{3n+1}(dx)^m\sqrt{a^2+2abx^n+b^2x^{2n}}}{(m+3n+1)\left(ab+b^2x^n\right)}+\frac{3ab^3x^{2n+1}(dx)^m\sqrt{a^2+2abx^n+b^2}}{(m+2n+1)\left(ab+b^2x^n\right)}$$

Rubi [A] time = 0.10, antiderivative size = 238, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {1355, 270, 20, 30}

$$\frac{3a^2b^2x^{n+1}(dx)^m\sqrt{a^2+2abx^n+b^2x^{2n}}}{(m+n+1)\left(ab+b^2x^n\right)}+\frac{3ab^3x^{2n+1}(dx)^m\sqrt{a^2+2abx^n+b^2x^{2n}}}{(m+2n+1)\left(ab+b^2x^n\right)}+\frac{b^4x^{3n+1}(dx)^m\sqrt{a^2+2abx^n+b^2x^{2n}}}{(m+3n+1)\left(ab+b^2x^n\right)}+\frac{a^3(dx)^{m+1}\sqrt{a^2+2abx^n+b^2x^{2n}}}{d(m+1)\left(a+bx^n\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^m*(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(3/2),x]
```

```
[Out] (a^3*(d*x)^(1 + m)*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])/(d*(1 + m)*(a + b*x
^n)) + (3*a^2*b^2*x^(1 + n)*(d*x)^m*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])/((
1 + m + n)*(a*b + b^2*x^n)) + (3*a*b^3*x^(1 + 2*n)*(d*x)^m*Sqrt[a^2 + 2*a*b
*x^n + b^2*x^(2*n)])/((1 + m + 2*n)*(a*b + b^2*x^n)) + (b^4*x^(1 + 3*n)*(d*
x)^m*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])/((1 + m + 3*n)*(a*b + b^2*x^n))
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(b^IntPart
[n]*(b*v)^FracPart[n])/(a^IntPart[n]*(a*v)^FracPart[n]), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !
IntegerQ[m + n]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\int (dx)^m \left(a^2+2abx^n+b^2x^{2n}\right)^{3/2} dx = \frac{\sqrt{a^2+2abx^n+b^2x^{2n}} \int (dx)^m \left(ab+b^2x^n\right)^3 dx}{b^2\left(ab+b^2x^n\right)}$$

$$= \frac{\sqrt{a^2+2abx^n+b^2x^{2n}} \int \left(a^3b^3(dx)^m+3a^2b^4x^n(dx)^m+3ab^5x^{2n}(dx)^m+b^6x\right.}{b^2\left(ab+b^2x^n\right)}$$

$$= \frac{a^3(dx)^{1+m}\sqrt{a^2+2abx^n+b^2x^{2n}}}{d(1+m)(a+bx^n)} + \frac{\left(3a^2b^2\sqrt{a^2+2abx^n+b^2x^{2n}}\right)\int x^n(dx)^n}{ab+b^2x^n}$$

$$= \frac{a^3(dx)^{1+m}\sqrt{a^2+2abx^n+b^2x^{2n}}}{d(1+m)(a+bx^n)} + \frac{\left(3a^2b^2x^{-m}(dx)^m\sqrt{a^2+2abx^n+b^2x^{2n}}\right)}{ab+b^2x^n}$$

$$= \frac{a^3(dx)^{1+m}\sqrt{a^2+2abx^n+b^2x^{2n}}}{d(1+m)(a+bx^n)} + \frac{3a^2b^2x^{1+n}(dx)^m\sqrt{a^2+2abx^n+b^2x^{2n}}}{(1+m+n)\left(ab+b^2x^n\right)} +$$

Mathematica [A] time = 0.11, size = 90, normalized size = 0.38

$$\frac{x(dx)^m\left((a+bx^n)^2\right)^{3/2}\left(\frac{a^3}{m+1}+\frac{3a^2bx^n}{m+n+1}+\frac{3ab^2x^{2n}}{m+2n+1}+\frac{b^3x^{3n}}{m+3n+1}\right)}{(a+bx^n)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^m*(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(3/2),x]
```

```
[Out] (x*(d*x)^m*((a + b*x^n)^2)^(3/2)*(a^3/(1 + m) + (3*a^2*b*x^n)/(1 + m + n) +
 (3*a*b^2*x^(2*n))/(1 + m + 2*n) + (b^3*x^(3*n))/(1 + m + 3*n)))/(a + b*x^n
)^3
```

IntegrateAlgebraic [F] time = 0.22, size = 0, normalized size = 0.00

$$\int (dx)^m \left(a^2+2abx^n+b^2x^{2n}\right)^{3/2} dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(d*x)^m*(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(3/2),x]
```

```
[Out] Defer[IntegrateAlgebraic][(d*x)^m*(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(3/2), x]
```

fricas [A] time = 1.01, size = 390, normalized size = 1.64

$$\frac{\left(b^3m^3+3b^3m^2+3b^3m+b^3+2\left(b^3m+b^3\right)n^2+3\left(b^3m^2+2b^3m+b^3\right)n\right)xx^{3n}e^{\left(m\log(d)+m\log(x)\right)}+3\left(ab^2m^3+3ab^2m^2+3ab^2m+ab^2+3\left(ab^2m+ab^2\right)n^2+4\left(ab^2m^2+2ab^2m+ab^2\right)n\right)xx^{2n}e^{\left(m\log(d)+m\log(x)\right)}+3\left(a^2bm^3+3a^2bm^2+3a^2bm+a^2b+6\left(a^2bm+a^2b\right)n^2+5\left(a^2bm^2+2a^2bm+a^2b\right)n\right)xx^ne^{\left(m\log(d)+m\log(x)\right)}+\left(a^3m^3+6a^3n^3+3a^3m^2+3a^3m+a^3+11\left(a^3m+a^3\right)n^2+6\left(a^3m^2+2a^3m+a^3\right)n\right)xe^{\left(m\log(d)+m\log(x)\right)}}{m^4+6(m+1)n^3+4m^3+11\left(m^2+2m+1\right)n^2+6m^2+6\left(m^3+3m^2+3m+1\right)n+4m+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x, algorithm="fricas")
```

```
[Out] ((b^3*m^3 + 3*b^3*m^2 + 3*b^3*m + b^3 + 2*(b^3*m + b^3)*n^2 + 3*(b^3*m^2 +
2*b^3*m + b^3)*n)*x*x^(3*n)*e^(m*log(d) + m*log(x)) + 3*(a*b^2*m^3 + 3*a*b^
2*m^2 + 3*a*b^2*m + a*b^2 + 3*(a*b^2*m + a*b^2)*n^2 + 4*(a*b^2*m^2 + 2*a*b^
2*m + a*b^2)*n)*x*x^(2*n)*e^(m*log(d) + m*log(x)) + 3*(a^2*b*m^3 + 3*a^2*b*
m^2 + 3*a^2*b*m + a^2*b + 6*(a^2*b*m + a^2*b)*n^2 + 5*(a^2*b*m^2 + 2*a^2*b*
m + a^2*b)*n)*x*x^n*e^(m*log(d) + m*log(x)) + (a^3*m^3 + 6*a^3*n^3 + 3*a^3*
m^2 + 3*a^3*m + a^3 + 11*(a^3*m + a^3)*n^2 + 6*(a^3*m^2 + 2*a^3*m + a^3)*n)
*x*e^(m*log(d) + m*log(x)))/(m^4 + 6*(m + 1)*n^3 + 4*m^3 + 11*(m^2 + 2*m +
1)*n^2 + 6*m^2 + 6*(m^3 + 3*m^2 + 3*m + 1)*n + 4*m + 1)
```

giac [B] time = 0.95, size = 2719, normalized size = 11.42

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x, algorithm="giac")
```

```
[Out] (b^3*m^3*x*x^(3*n)*e^(m*log(d) + m*log(x))*sgn(b*x^n + a) + 3*b^3*m^2*n*x*x
^(3*n)*e^(m*log(d) + m*log(x))*sgn(b*x^n + a) + 2*b^3*m*n^2*x*x^(3*n)*e^(m*
log(d) + m*log(x))*sgn(b*x^n + a) + 3*a*b^2*m^3*x*x^(2*n)*e^(m*log(d) + m*l
og(x))*sgn(b*x^n + a) + b^3*m^3*x*x^(2*n)*e^(m*log(d) + m*log(x))*sgn(b*x^n
 + a) + 12*a*b^2*m^2*n*x*x^(2*n)*e^(m*log(d) + m*log(x))*sgn(b*x^n + a) + 3
*b^3*m^2*n*x*x^(2*n)*e^(m*log(d) + m*log(x))*sgn(b*x^n + a) + 9*a*b^2*m*n^2
*x*x^(2*n)*e^(m*log(d) + m*log(x))*sgn(b*x^n + a) + 2*b^3*m*n^2*x*x^(2*n)*e
^(m*log(d) + m*log(x))*sgn(b*x^n + a) + 3*a^2*b*m^3*x*x^n*e^(m*log(d) + m*l
og(x))*sgn(b*x^n + a) + 3*a*b^2*m^3*x*x^n*e^(m*log(d) + m*log(x))*sgn(b*x^n
 + a) + b^3*m^3*x*x^n*e^(m*log(d) + m*log(x))*sgn(b*x^n + a) + 15*a^2*b*m^2
*n*x*x^n*e^(m*log(d) + m*log(x))*sgn(b*x^n + a) + 12*a*b^2*m^2*n*x*x^n*e^(m
*log(d) + m*log(x))*sgn(b*x^n + a) + 3*b^3*m^2*n*x*x^n*e^(m*log(d) + m*log(
x))*sgn(b*x^n + a) + 18*a^2*b*m*n^2*x*x^n*e^(m*log(d) + m*log(x))*sgn(b*x^n
 + a) + 9*a*b^2*m*n^2*x*x^n*e^(m*log(d) + m*log(x))*sgn(b*x^n + a) + 2*b^3*
m*n^2*x*x^n*e^(m*log(d) + m*log(x))*sgn(b*x^n + a) + a^3*m^3*x*e^(m*log(d)
+ m*log(x))*sgn(b*x^n + a) + 3*a^2*b*m^3*x*e^(m*log(d) + m*log(x))*sgn(b*x^
n + a) + 3*a*b^2*m^3*x*e^(m*log(d) + m*log(x))*sgn(b*x^n + a) + b^3*m^3*x*e
^(m*log(d) + m*log(x))*sgn(b*x^n + a) + 6*a^3*m^2*n*x*e^(m*log(d) + m*log(x
))*sgn(b*x^n + a) + 15*a^2*b*m^2*n*x*e^(m*log(d) + m*log(x))*sgn(b*x^n + a)
 + 12*a*b^2*m^2*n*x*e^(m*log(d) + m*log(x))*sgn(b*x^n + a) + 3*b^3*m^2*n*x*
e^(m*log(d) + m*log(x))*sgn(b*x^n + a) + 11*a^3*m*n^2*x*e^(m*log(d) + m*log
(x))*sgn(b*x^n + a) + 18*a^2*b*m*n^2*x*e^(m*log(d) + m*log(x))*sgn(b*x^n +
a) + 9*a*b^2*m*n^2*x*e^(m*log(d) + m*log(x))*sgn(b*x^n + a) + 2*b^3*m*n^2*x
*e^(m*log(d) + m*log(x))*sgn(b*x^n + a) + 6*a^3*n^3*x*e^(m*log(d) + m*log(x
))*sgn(b*x^n + a) + 3*b^3*m^2*x*x^(3*n)*e^(m*log(d) + m*log(x))*sgn(b*x^n +
 a) + 6*b^3*m*n*x*x^(3*n)*e^(m*log(d) + m*log(x))*sgn(b*x^n + a) + 2*b^3*n^
2*x*x^(3*n)*e^(m*log(d) + m*log(x))*sgn(b*x^n + a) + 9*a*b^2*m^2*x*x^(2*n)*
e^(m*log(d) + m*log(x))*sgn(b*x^n + a) + 3*b^3*m^2*x*x^(2*n)*e^(m*log(d) +
m*log(x))*sgn(b*x^n + a) + 24*a*b^2*m*n*x*x^(2*n)*e^(m*log(d) + m*log(x))*s
gn(b*x^n + a) + 6*b^3*m*n*x*x^(2*n)*e^(m*log(d) + m*log(x))*sgn(b*x^n + a)
+ 9*a*b^2*n^2*x*x^(2*n)*e^(m*log(d) + m*log(x))*sgn(b*x^n + a) + 2*b^3*n^2*
x*x^(2*n)*e^(m*log(d) + m*log(x))*sgn(b*x^n + a) + 9*a^2*b*m^2*x*x^n*e^(m*l
og(d) + m*log(x))*sgn(b*x^n + a) + 9*a*b^2*m^2*x*x^n*e^(m*log(d) + m*log(x)
)*sgn(b*x^n + a) + 3*b^3*m^2*x*x^n*e^(m*log(d) + m*log(x))*sgn(b*x^n + a) +
 30*a^2*b*m*n*x*x^n*e^(m*log(d) + m*log(x))*sgn(b*x^n + a) + 24*a*b^2*m*n*x
*x^n*e^(m*log(d) + m*log(x))*sgn(b*x^n + a) + 6*b^3*m*n*x*x^n*e^(m*log(d) +
 m*log(x))*sgn(b*x^n + a) + 18*a^2*b*n^2*x*x^n*e^(m*log(d) + m*log(x))*sgn(
b*x^n + a) + 9*a*b^2*n^2*x*x^n*e^(m*log(d) + m*log(x))*sgn(b*x^n + a) + 2*b
^3*n^2*x*x^n*e^(m*log(d) + m*log(x))*sgn(b*x^n + a) + 3*a^3*m^2*x*e^(m*log(
d) + m*log(x))*sgn(b*x^n + a) + 9*a^2*b*m^2*x*e^(m*log(d) + m*log(x))*sgn(b
*x^n + a) + 9*a*b^2*m^2*x*e^(m*log(d) + m*log(x))*sgn(b*x^n + a) + 3*b^3*m^
2*x*e^(m*log(d) + m*log(x))*sgn(b*x^n + a) + 12*a^3*m*n*x*e^(m*log(d) + m*l
og(x))*sgn(b*x^n + a) + 30*a^2*b*m*n*x*e^(m*log(d) + m*log(x))*sgn(b*x^n +
a) + 24*a*b^2*m*n*x*e^(m*log(d) + m*log(x))*sgn(b*x^n + a) + 6*b^3*m*n*x*e^
(m*log(d) + m*log(x))*sgn(b*x^n + a) + 11*a^3*n^2*x*e^(m*log(d) + m*log(x))
*sgn(b*x^n + a) + 18*a^2*b*n^2*x*e^(m*log(d) + m*log(x))*sgn(b*x^n + a) + 9
*a*b^2*n^2*x*e^(m*log(d) + m*log(x))*sgn(b*x^n + a) + 2*b^3*n^2*x*e^(m*log(
d) + m*log(x))*sgn(b*x^n + a) + 3*b^3*m*x*x^(3*n)*e^(m*log(d) + m*log(x))*s
gn(b*x^n + a) + 3*b^3*n*x*x^(3*n)*e^(m*log(d) + m*log(x))*sgn(b*x^n + a) +
9*a*b^2*m*x*x^(2*n)*e^(m*log(d) + m*log(x))*sgn(b*x^n + a) + 3*b^3*m*x*x^(2
*n)*e^(m*log(d) + m*log(x))*sgn(b*x^n + a) + 12*a*b^2*n*x*x^(2*n)*e^(m*log(
d) + m*log(x))*sgn(b*x^n + a) + 3*b^3*n*x*x^(2*n)*e^(m*log(d) + m*log(x))*s
gn(b*x^n + a) + 9*a^2*b*m*x*x^n*e^(m*log(d) + m*log(x))*sgn(b*x^n + a) + 9*
a*b^2*m*x*x^n*e^(m*log(d) + m*log(x))*sgn(b*x^n + a) + 3*b^3*m*x*x^n*e^(m*l
```

```
og(d) + m*log(x))*sgn(b*x^n + a) + 15*a^2*b*n*x*x^n*e^(m*log(d) + m*log(x))
*sgn(b*x^n + a) + 12*a*b^2*n*x*x^n*e^(m*log(d) + m*log(x))*sgn(b*x^n + a) +
 3*b^3*n*x*x^n*e^(m*log(d) + m*log(x))*sgn(b*x^n + a) + 3*a^3*m*x*e^(m*log(
d) + m*log(x))*sgn(b*x^n + a) + 9*a^2*b*m*x*e^(m*log(d) + m*log(x))*sgn(b*x
^n + a) + 9*a*b^2*m*x*e^(m*log(d) + m*log(x))*sgn(b*x^n + a) + 3*b^3*m*x*e^
(m*log(d) + m*log(x))*sgn(b*x^n + a) + 6*a^3*n*x*e^(m*log(d) + m*log(x))*sg
n(b*x^n + a) + 15*a^2*b*n*x*e^(m*log(d) + m*log(x))*sgn(b*x^n + a) + 12*a*b
^2*n*x*e^(m*log(d) + m*log(x))*sgn(b*x^n + a) + 3*b^3*n*x*e^(m*log(d) + m*l
og(x))*sgn(b*x^n + a) + b^3*x*x^(3*n)*e^(m*log(d) + m*log(x))*sgn(b*x^n + a
) + 3*a*b^2*x*x^(2*n)*e^(m*log(d) + m*log(x))*sgn(b*x^n + a) + b^3*x*x^(2*n
)*e^(m*log(d) + m*log(x))*sgn(b*x^n + a) + 3*a^2*b*x*x^n*e^(m*log(d) + m*lo
g(x))*sgn(b*x^n + a) + 3*a*b^2*x*x^n*e^(m*log(d) + m*log(x))*sgn(b*x^n + a)
 + b^3*x*x^n*e^(m*log(d) + m*log(x))*sgn(b*x^n + a) + a^3*x*e^(m*log(d) + m
*log(x))*sgn(b*x^n + a) + 3*a^2*b*x*e^(m*log(d) + m*log(x))*sgn(b*x^n + a)
+ 3*a*b^2*x*e^(m*log(d) + m*log(x))*sgn(b*x^n + a) + b^3*x*e^(m*log(d) + m*
log(x))*sgn(b*x^n + a))/(m^4 + 6*m^3*n + 11*m^2*n^2 + 6*m*n^3 + 4*m^3 + 18*
m^2*n + 22*m*n^2 + 6*n^3 + 6*m^2 + 18*m*n + 11*n^2 + 4*m + 6*n + 1)
```

maple [C] time = 0.06, size = 532, normalized size = 2.24

$$\frac{\sqrt{(bx^n+a)^2}\left(3a^2bm^3x^n+15a^2bm^2nx^n+18a^2bmn^2x^n+3ab^2m^3x^{2n}+12ab^2m^2nx^{2n}+9ab^2mn^2x^{2n}+b^3m^3x^{3n}+3b^3m^2nx^{3n}+2b^3mn^2x^{3n}+a^3m^3+6a^3m^2n+11a^3mn^2+6a^3n^3+9a^2bm^2x^n+30a^2bmnx^n+18a^2bn^2x^n+9ab^2m^2x^{2n}+24ab^2mnx^{2n}+9ab^2n^2x^{2n}+3b^3m^2x^{3n}+6b^3mnx^{3n}+2b^3n^2x^{3n}+3a^3m^2+12a^3mn+11a^3n^2+9a^2bmx^n+15a^2bnx^n+9ab^2mx^{2n}+12ab^2nx^{2n}+3b^3mx^{3n}+3b^3nx^{3n}+3a^3m+6a^3n+3a^2bx^n+3ab^2x^{2n}+b^3x^{3n}+a^3\right)x\,e^{\frac{\left(-i\pi\,\mathrm{csgn}(id)\mathrm{csgn}(ix)\mathrm{csgn}(idx)+i\pi\,\mathrm{csgn}(id)\mathrm{csgn}(idx)^2+i\pi\,\mathrm{csgn}(ix)\mathrm{csgn}(idx)^2-i\pi\,\mathrm{csgn}(idx)^3+2\ln(d)+2\ln(x)\right)m}{2}}}{(bx^n+a)(m+1)(m+n+1)(m+2n+1)(m+3n+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x)
```

```
[Out] ((b*x^n+a)^2)^(1/2)/(b*x^n+a)*x*(b^3*(x^n)^3+a^3*m^3+3*a^3*m^2+11*a^3*n^2+6
*a^3*n+6*a^3*n^3+a^3+15*a^2*b*n*x^n+3*b^3*m^2*n*(x^n)^3+2*b^3*m*n^2*(x^n)^3
+3*a*b^2*m^3*(x^n)^2+6*b^3*m*n*(x^n)^3+3*a^2*b*m^3*x^n+9*a*b^2*m^2*(x^n)^2+
9*a*b^2*n^2*(x^n)^2+9*a^2*b*m^2*x^n+18*a^2*b*n^2*x^n+9*m*a*b^2*(x^n)^2+12*a
*b^2*(x^n)^2*n+9*m*a^2*b*x^n+12*a*b^2*m^2*n*(x^n)^2+b^3*m^3*(x^n)^3+3*b^3*m
^2*(x^n)^3+2*b^3*n^2*(x^n)^3+3*m*b^3*(x^n)^3+3*b^3*(x^n)^3*n+3*a^2*b*x^n+3*
(x^n)^2*a*b^2+6*a^3*m^2*n+11*a^3*m*n^2+12*a^3*m*n+9*a*b^2*m*n^2*(x^n)^2+15*
a^2*b*m^2*n*x^n+18*a^2*b*m*n^2*x^n+3*m*a^3+24*a*b^2*m*n*(x^n)^2+30*a^2*b*m*
n*x^n)/(m+1)/(m+n+1)/(1+m+2*n)/(1+m+3*n)*exp(1/2*(-I*Pi*csgn(I*d)*csgn(I*x)
*csgn(I*d*x)+I*Pi*csgn(I*d)*csgn(I*d*x)^2+I*Pi*csgn(I*x)*csgn(I*d*x)^2-I*Pi
*csgn(I*d*x)^3+2*ln(d)+2*ln(x))*m)
```

maxima [A] time = 0.99, size = 276, normalized size = 1.16

$$\frac{\left(m^3+3m^2(2n+1)+6n^3+\left(11n^2+12n+3\right)m+11n^2+6n+1\right)a^3d^mxx^m+\left(m^3+3m^2(n+1)+\left(2n^2+6n+3\right)m+2n^2+3n+1\right)b^3d^mxe^{\left(m\log(x)+3n\log(x)\right)}+3\left(m^3+m^2(4n+3)+\left(3n^2+8n+3\right)m+3n^2+4n+1\right)ab^2d^mxe^{\left(m\log(x)+2n\log(x)\right)}+3\left(m^3+m^2(5n+3)+\left(6n^2+10n+3\right)m+6n^2+5n+1\right)a^2bd^mxe^{\left(m\log(x)+n\log(x)\right)}}{m^4+2m^3(3n+2)+\left(11n^2+18n+6\right)m^2+6n^3+2\left(3n^3+11n^2+9n+2\right)m+11n^2+6n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x, algorithm="maxima")
```

```
[Out] ((m^3 + 3*m^2*(2*n + 1) + 6*n^3 + (11*n^2 + 12*n + 3)*m + 11*n^2 + 6*n + 1)
*a^3*d^m*x*x^m + (m^3 + 3*m^2*(n + 1) + (2*n^2 + 6*n + 3)*m + 2*n^2 + 3*n +
 1)*b^3*d^m*x*e^(m*log(x) + 3*n*log(x)) + 3*(m^3 + m^2*(4*n + 3) + (3*n^2 +
 8*n + 3)*m + 3*n^2 + 4*n + 1)*a*b^2*d^m*x*e^(m*log(x) + 2*n*log(x)) + 3*(m
^3 + m^2*(5*n + 3) + (6*n^2 + 10*n + 3)*m + 6*n^2 + 5*n + 1)*a^2*b*d^m*x*e^
(m*log(x) + n*log(x)))/(m^4 + 2*m^3*(3*n + 2) + (11*n^2 + 18*n + 6)*m^2 + 6
*n^3 + 2*(3*n^3 + 11*n^2 + 9*n + 2)*m + 11*n^2 + 6*n + 1)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int (d\,x)^m\,\left(a^2+b^2\,x^{2\,n}+2\,a\,b\,x^n\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(3/2),x)
```

```
[Out] int((d*x)^m*(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx)^m \left((a+bx^n)^2\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(a**2+2*a*b*x**n+b**2*x**(2*n))**(3/2),x)
[Out] Integral((d*x)**m*((a + b*x**n)**2)**(3/2), x)
```

3.459 $\int x^2 \left(a^2+2abx^n+b^2x^{2n}\right)^{3/2} dx$

Optimal. Leaf size=212

$$\frac{3a^2b^2x^{n+3}\sqrt{a^2+2abx^n+b^2x^{2n}}}{(n+3)\left(ab+b^2x^n\right)}+\frac{b^4x^{3(n+1)}\sqrt{a^2+2abx^n+b^2x^{2n}}}{3(n+1)\left(ab+b^2x^n\right)}+\frac{3ab^3x^{2n+3}\sqrt{a^2+2abx^n+b^2x^{2n}}}{(2n+3)\left(ab+b^2x^n\right)}+\frac{a^3x^3\sqrt{a^2+2a}}{3(a+}$$

Rubi [A] time = 0.06, antiderivative size = 212, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {1355, 270}

$$\frac{b^4x^{3(n+1)}\sqrt{a^2+2abx^n+b^2x^{2n}}}{3(n+1)\left(ab+b^2x^n\right)}+\frac{3a^2b^2x^{n+3}\sqrt{a^2+2abx^n+b^2x^{2n}}}{(n+3)\left(ab+b^2x^n\right)}+\frac{3ab^3x^{2n+3}\sqrt{a^2+2abx^n+b^2x^{2n}}}{(2n+3)\left(ab+b^2x^n\right)}+\frac{a^3x^3\sqrt{a^2+2abx^n+b^2x^{2n}}}{3\left(a+bx^n\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(3/2),x]
```

```
[Out] (a^3*x^3*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])/(3*(a + b*x^n)) + (b^4*x^(3*(
1 + n))*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])/(3*(1 + n)*(a*b + b^2*x^n)) +
(3*a^2*b^2*x^(3 + n)*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])/((3 + n)*(a*b + b
^2*x^n)) + (3*a*b^3*x^(3 + 2*n)*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])/((3 +
2*n)*(a*b + b^2*x^n))
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\begin{aligned}\int x^2\left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}dx&=\frac{\sqrt{a^2+2abx^n+b^2x^{2n}}\int x^2\left(ab+b^2x^n\right)^3dx}{b^2\left(ab+b^2x^n\right)}\\&=\frac{\sqrt{a^2+2abx^n+b^2x^{2n}}\int\left(a^3b^3x^2+3ab^5x^{2(1+n)}+3a^2b^4x^{2+n}+b^6x^{2+3n}\right)dx}{b^2\left(ab+b^2x^n\right)}\\&=\frac{a^3x^3\sqrt{a^2+2abx^n+b^2x^{2n}}}{3\left(a+bx^n\right)}+\frac{b^4x^{3(1+n)}\sqrt{a^2+2abx^n+b^2x^{2n}}}{3(1+n)\left(ab+b^2x^n\right)}+\frac{3a^2b^2x^{3+n}\sqrt{a^2}}{(3+n)(}\end{aligned}$$

Mathematica [A] time = 0.08, size = 123, normalized size = 0.58

$$\frac{x^3\sqrt{\left(a+bx^n\right)^2}\left(a^3\left(2n^3+11n^2+18n+9\right)+9a^2b\left(2n^2+5n+3\right)x^n+9ab^2\left(n^2+4n+3\right)x^{2n}+b^3\left(2n^2+9n+9\right)x^{3n}\right)}{3(n+1)(n+3)(2n+3)\left(a+bx^n\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(3/2),x]
```

```
[Out] (x^3*Sqrt[(a + b*x^n)^2]*(a^3*(9 + 18*n + 11*n^2 + 2*n^3) + 9*a^2*b*(3 + 5*
n + 2*n^2)*x^n + 9*a*b^2*(3 + 4*n + n^2)*x^(2*n) + b^3*(9 + 9*n + 2*n^2)*x^
(3*n)))/(3*(1 + n)*(3 + n)*(3 + 2*n)*(a + b*x^n))
```

IntegrateAlgebraic [F] time = 0.15, size = 0, normalized size = 0.00

$$\int x^2 \left(a^2 + 2abx^n + b^2x^{2n}\right)^{3/2} dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^2*(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(3/2),x]
```

```
[Out] Defer[IntegrateAlgebraic][x^2*(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(3/2), x]
```

fricas [A] time = 1.10, size = 144, normalized size = 0.68

$$\frac{\left(2\,b^{3}n^{2}+9\,b^{3}n+9\,b^{3}\right)x^{3}x^{3\,n}+9\,\left(ab^{2}n^{2}+4\,ab^{2}n+3\,ab^{2}\right)x^{3}x^{2\,n}+9\,\left(2\,a^{2}bn^{2}+5\,a^{2}bn+3\,a^{2}b\right)x^{3}x^{n}+\left(2\,a^{3}n^{3}+11\,a^{3}n^{2}+18\,a^{3}n+9\,a^{3}\right)x^{3}}{3\,\left(2\,n^{3}+11\,n^{2}+18\,n+9\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/3*((2*b^3*n^2 + 9*b^3*n + 9*b^3)*x^3*x^(3*n) + 9*(a*b^2*n^2 + 4*a*b^2*n +
 3*a*b^2)*x^3*x^(2*n) + 9*(2*a^2*b*n^2 + 5*a^2*b*n + 3*a^2*b)*x^3*x^n + (2*
a^3*n^3 + 11*a^3*n^2 + 18*a^3*n + 9*a^3)*x^3)/(2*n^3 + 11*n^2 + 18*n + 9)
```

giac [A] time = 0.43, size = 292, normalized size = 1.38

$$\frac{2\,b^{3}n^{2}x^{3}x^{3\,n}\mathrm{sgn}\left(bx^{n}+a\right)+9\,ab^{2}n^{2}x^{3}x^{2\,n}\mathrm{sgn}\left(bx^{n}+a\right)+18\,a^{2}bn^{2}x^{3}x^{n}\mathrm{sgn}\left(bx^{n}+a\right)+2\,a^{3}n^{3}x^{3}\mathrm{sgn}\left(bx^{n}+a\right)+9\,b^{3}nx^{3}x^{3\,n}\mathrm{sgn}\left(bx^{n}+a\right)+36\,ab^{2}nx^{3}x^{2\,n}\mathrm{sgn}\left(bx^{n}+a\right)+45\,a^{2}bnx^{3}x^{n}\mathrm{sgn}\left(bx^{n}+a\right)+11\,a^{3}n^{2}x^{3}\mathrm{sgn}\left(bx^{n}+a\right)+9\,b^{3}x^{3}x^{3\,n}\mathrm{sgn}\left(bx^{n}+a\right)+27\,ab^{2}x^{3}x^{2\,n}\mathrm{sgn}\left(bx^{n}+a\right)+27\,a^{2}bx^{3}x^{n}\mathrm{sgn}\left(bx^{n}+a\right)+18\,a^{3}nx^{3}\mathrm{sgn}\left(bx^{n}+a\right)+9\,a^{3}x^{3}\mathrm{sgn}\left(bx^{n}+a\right)}{3\,\left(2\,n^{3}+11\,n^{2}+18\,n+9\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x, algorithm="giac")
```

```
[Out] 1/3*(2*b^3*n^2*x^3*x^(3*n)*sgn(b*x^n + a) + 9*a*b^2*n^2*x^3*x^(2*n)*sgn(b*x
^n + a) + 18*a^2*b*n^2*x^3*x^n*sgn(b*x^n + a) + 2*a^3*n^3*x^3*sgn(b*x^n + a
) + 9*b^3*n*x^3*x^(3*n)*sgn(b*x^n + a) + 36*a*b^2*n*x^3*x^(2*n)*sgn(b*x^n +
 a) + 45*a^2*b*n*x^3*x^n*sgn(b*x^n + a) + 11*a^3*n^2*x^3*sgn(b*x^n + a) + 9
*b^3*x^3*x^(3*n)*sgn(b*x^n + a) + 27*a*b^2*x^3*x^(2*n)*sgn(b*x^n + a) + 27*
a^2*b*x^3*x^n*sgn(b*x^n + a) + 18*a^3*n*x^3*sgn(b*x^n + a) + 9*a^3*x^3*sgn(
b*x^n + a))/(2*n^3 + 11*n^2 + 18*n + 9)
```

maple [A] time = 0.02, size = 146, normalized size = 0.69

$$\frac{3\sqrt{\left(b\,x^{n}+a\right)^{2}}\;a^{2}b\,x^{3}x^{n}}{\left(b\,x^{n}+a\right)\left(n+3\right)}+\frac{3\sqrt{\left(b\,x^{n}+a\right)^{2}}\;a\,b^{2}x^{3}x^{2n}}{\left(b\,x^{n}+a\right)\left(2n+3\right)}+\frac{\sqrt{\left(b\,x^{n}+a\right)^{2}}\;b^{3}x^{3}x^{3n}}{3\left(b\,x^{n}+a\right)\left(n+1\right)}+\frac{\sqrt{\left(b\,x^{n}+a\right)^{2}}\;a^{3}x^{3}}{3b\,x^{n}+3a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x)
```

```
[Out] 1/3*((b*x^n+a)^2)^(1/2)/(b*x^n+a)*x^3*a^3+1/3*((b*x^n+a)^2)^(1/2)/(b*x^n+a)
*b^3*x^3/(n+1)*(x^n)^3+3*((b*x^n+a)^2)^(1/2)/(b*x^n+a)*a*b^2/(3+2*n)*x^3*(x
^n)^2+3*((b*x^n+a)^2)^(1/2)/(b*x^n+a)*a^2*b/(n+3)*x^3*x^n
```

maxima [A] time = 0.93, size = 108, normalized size = 0.51

$$\frac{\left(2\,n^{2}+9\,n+9\right)b^{3}x^{3}x^{3\,n}+9\,\left(n^{2}+4\,n+3\right)ab^{2}x^{3}x^{2\,n}+9\,\left(2\,n^{2}+5\,n+3\right)a^{2}bx^{3}x^{n}+\left(2\,n^{3}+11\,n^{2}+18\,n+9\right)a^{3}x^{3}}{3\,\left(2\,n^{3}+11\,n^{2}+18\,n+9\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x, algorithm="maxima")
```

```
[Out] 1/3*((2*n^2 + 9*n + 9)*b^3*x^3*x^(3*n) + 9*(n^2 + 4*n + 3)*a*b^2*x^3*x^(2*n
) + 9*(2*n^2 + 5*n + 3)*a^2*b*x^3*x^n + (2*n^3 + 11*n^2 + 18*n + 9)*a^3*x^3
)/(2*n^3 + 11*n^2 + 18*n + 9)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^2 \left(a^2 + b^2 x^{2n} + 2 a b x^n\right)^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(3/2),x)
```

```
[Out] int(x^2*(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \left((a + bx^n)^2\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a**2+2*a*b*x**n+b**2*x**(2*n))**(3/2),x)
```

```
[Out] Integral(x**2*((a + b*x**n)**2)**(3/2), x)
```

3.460 $\int x \left(a^2+2abx^n+b^2x^{2n}\right)^{3/2} dx$

Optimal. Leaf size=211

$$\frac{3a^2b^2x^{n+2}\sqrt{a^2+2abx^n+b^2x^{2n}}}{(n+2)\left(ab+b^2x^n\right)}+\frac{b^4x^{3n+2}\sqrt{a^2+2abx^n+b^2x^{2n}}}{(3n+2)\left(ab+b^2x^n\right)}+\frac{3ab^3x^{2(n+1)}\sqrt{a^2+2abx^n+b^2x^{2n}}}{2(n+1)\left(ab+b^2x^n\right)}+\frac{a^3x^2\sqrt{a^2 -}}{2(}$$

Rubi [A] time = 0.06, antiderivative size = 211, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1355, 270}

$$\frac{3ab^3x^{2(n+1)}\sqrt{a^2+2abx^n+b^2x^{2n}}}{2(n+1)\left(ab+b^2x^n\right)}+\frac{3a^2b^2x^{n+2}\sqrt{a^2+2abx^n+b^2x^{2n}}}{(n+2)\left(ab+b^2x^n\right)}+\frac{b^4x^{3n+2}\sqrt{a^2+2abx^n+b^2x^{2n}}}{(3n+2)\left(ab+b^2x^n\right)}+\frac{a^3x^2\sqrt{a^2+2abx^n+b^2x^{2n}}}{2\left(a+bx^n\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(3/2),x]
```

```
[Out] (a^3*x^2*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])/(2*(a + b*x^n)) + (3*a*b^3*x^
(2*(1 + n))*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])/(2*(1 + n)*(a*b + b^2*x^n)
) + (3*a^2*b^2*x^(2 + n)*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])/((2 + n)*(a*b
 + b^2*x^n)) + (b^4*x^(2 + 3*n)*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])/((2 +
3*n)*(a*b + b^2*x^n))
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\begin{aligned}\int x\left(a^2+2abx^n+b^2x^{2n}\right)^{3/2} dx &= \frac{\sqrt{a^2+2abx^n+b^2x^{2n}}\int x\left(ab+b^2x^n\right)^3 dx}{b^2\left(ab+b^2x^n\right)}\\ &= \frac{\sqrt{a^2+2abx^n+b^2x^{2n}}\int\left(a^3b^3x+3a^2b^4x^{1+n}+3ab^5x^{1+2n}+b^6x^{1+3n}\right)dx}{b^2\left(ab+b^2x^n\right)}\\ &= \frac{a^3x^2\sqrt{a^2+2abx^n+b^2x^{2n}}}{2\left(a+bx^n\right)}+\frac{3ab^3x^{2(1+n)}\sqrt{a^2+2abx^n+b^2x^{2n}}}{2(1+n)\left(ab+b^2x^n\right)}+\frac{3a^2b^2x^{2+n}}{(2+}\end{aligned}$$

Mathematica [A] time = 0.07, size = 124, normalized size = 0.59

$$\frac{x^2\sqrt{\left(a+bx^n\right)^2}\left(a^3\left(3n^3+11n^2+12n+4\right)+6a^2b\left(3n^2+5n+2\right)x^n+3ab^2\left(3n^2+8n+4\right)x^{2n}+2b^3\left(n^2+3n+2\right)x^{3n}\right)}{2(n+1)(n+2)(3n+2)\left(a+bx^n\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(3/2),x]
```

```
[Out] (x^2*Sqrt[(a + b*x^n)^2]*(a^3*(4 + 12*n + 11*n^2 + 3*n^3) + 6*a^2*b*(2 + 5*
n + 3*n^2)*x^n + 3*a*b^2*(4 + 8*n + 3*n^2)*x^(2*n) + 2*b^3*(2 + 3*n + n^2)*
x^(3*n)))/(2*(1 + n)*(2 + n)*(2 + 3*n)*(a + b*x^n))
```

IntegrateAlgebraic [F] time = 0.12, size = 0, normalized size = 0.00

$$\int x \left(a^2 + 2abx^n + b^2x^{2n}\right)^{3/2} dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x*(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(3/2),x]
```

```
[Out] Defer[IntegrateAlgebraic][x*(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(3/2), x]
```

fricas [A] time = 1.32, size = 145, normalized size = 0.69

$$\frac{2\left(b^3n^2 + 3\,b^3n + 2\,b^3\right)x^2x^{3\,n} + 3\left(3\,ab^2n^2 + 8\,ab^2n + 4\,ab^2\right)x^2x^{2\,n} + 6\left(3\,a^2bn^2 + 5\,a^2bn + 2\,a^2b\right)x^2x^n + \left(3\,a^3n^3 + 11\,a^3n^2 + 12\,a^3n + 4\,a^3\right)x^2}{2\left(3\,n^3 + 11\,n^2 + 12\,n + 4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/2*(2*(b^3*n^2 + 3*b^3*n + 2*b^3)*x^2*x^(3*n) + 3*(3*a*b^2*n^2 + 8*a*b^2*n
 + 4*a*b^2)*x^2*x^(2*n) + 6*(3*a^2*b*n^2 + 5*a^2*b*n + 2*a^2*b)*x^2*x^n + (
3*a^3*n^3 + 11*a^3*n^2 + 12*a^3*n + 4*a^3)*x^2)/(3*n^3 + 11*n^2 + 12*n + 4)
```

giac [A] time = 0.37, size = 292, normalized size = 1.38

$$\frac{2\,b^3n^2x^2x^{3\,n}\mathrm{sgn}\left(bx^n + a\right) + 9\,ab^2n^2x^2x^{2\,n}\mathrm{sgn}\left(bx^n + a\right) + 18\,a^2bn^2x^2x^n\mathrm{sgn}\left(bx^n + a\right) + 3\,a^3n^3x^2\mathrm{sgn}\left(bx^n + a\right) + 6\,b^3nx^2x^{3\,n}\mathrm{sgn}\left(bx^n + a\right) + 24\,ab^2nx^2x^{2\,n}\mathrm{sgn}\left(bx^n + a\right) + 30\,a^2bnx^2x^n\mathrm{sgn}\left(bx^n + a\right) + 11\,a^3n^2x^2\mathrm{sgn}\left(bx^n + a\right) + 4\,b^3x^2x^{3\,n}\mathrm{sgn}\left(bx^n + a\right) + 12\,ab^2x^2x^{2\,n}\mathrm{sgn}\left(bx^n + a\right) + 12\,a^2bx^2x^n\mathrm{sgn}\left(bx^n + a\right) + 12\,a^3nx^2\mathrm{sgn}\left(bx^n + a\right) + 4\,a^3x^2\mathrm{sgn}\left(bx^n + a\right)}{2\left(3\,n^3 + 11\,n^2 + 12\,n + 4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x, algorithm="giac")
```

```
[Out] 1/2*(2*b^3*n^2*x^2*x^(3*n)*sgn(b*x^n + a) + 9*a*b^2*n^2*x^2*x^(2*n)*sgn(b*x
^n + a) + 18*a^2*b*n^2*x^2*x^n*sgn(b*x^n + a) + 3*a^3*n^3*x^2*sgn(b*x^n + a
) + 6*b^3*n*x^2*x^(3*n)*sgn(b*x^n + a) + 24*a*b^2*n*x^2*x^(2*n)*sgn(b*x^n +
 a) + 30*a^2*b*n*x^2*x^n*sgn(b*x^n + a) + 11*a^3*n^2*x^2*sgn(b*x^n + a) + 4
*b^3*x^2*x^(3*n)*sgn(b*x^n + a) + 12*a*b^2*x^2*x^(2*n)*sgn(b*x^n + a) + 12*
a^2*b*x^2*x^n*sgn(b*x^n + a) + 12*a^3*n*x^2*sgn(b*x^n + a) + 4*a^3*x^2*sgn(
b*x^n + a))/(3*n^3 + 11*n^2 + 12*n + 4)
```

maple [A] time = 0.02, size = 145, normalized size = 0.69

$$\frac{3\sqrt{\left(b\,x^n + a\right)^2}\;a^2b\,x^2x^n}{\left(b\,x^n + a\right)\left(n + 2\right)} + \frac{3\sqrt{\left(b\,x^n + a\right)^2}\;a\,b^2x^2x^{2n}}{2\left(b\,x^n + a\right)\left(n + 1\right)} + \frac{\sqrt{\left(b\,x^n + a\right)^2}\;b^3x^2x^{3n}}{\left(b\,x^n + a\right)\left(3n + 2\right)} + \frac{\sqrt{\left(b\,x^n + a\right)^2}\;a^3x^2}{2b\,x^n + 2a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x)
```

```
[Out] 1/2*((b*x^n+a)^2)^(1/2)/(b*x^n+a)*x^2*a^3+((b*x^n+a)^2)^(1/2)/(b*x^n+a)*b^3
/(2+3*n)*x^2*(x^n)^3+3/2*((b*x^n+a)^2)^(1/2)/(b*x^n+a)*a*b^2*x^2/(n+1)*(x^n
)^2+3*((b*x^n+a)^2)^(1/2)/(b*x^n+a)*a^2*b/(n+2)*x^2*x^n
```

maxima [A] time = 0.94, size = 109, normalized size = 0.52

$$\frac{2\left(n^2 + 3\,n + 2\right)b^3x^2x^{3\,n} + 3\left(3\,n^2 + 8\,n + 4\right)ab^2x^2x^{2\,n} + 6\left(3\,n^2 + 5\,n + 2\right)a^2bx^2x^n + \left(3\,n^3 + 11\,n^2 + 12\,n + 4\right)a^3x^2}{2\left(3\,n^3 + 11\,n^2 + 12\,n + 4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x, algorithm="maxima")
```

```
[Out] 1/2*(2*(n^2 + 3*n + 2)*b^3*x^2*x^(3*n) + 3*(3*n^2 + 8*n + 4)*a*b^2*x^2*x^(2
*n) + 6*(3*n^2 + 5*n + 2)*a^2*b*x^2*x^n + (3*n^3 + 11*n^2 + 12*n + 4)*a^3*x
^2)/(3*n^3 + 11*n^2 + 12*n + 4)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x\left(a^2+b^2\,x^{2\,n}+2\,a\,b\,x^n\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(3/2),x)
```

```
[Out] int(x*(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\left((a+bx^n)^2\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a**2+2*a*b*x**n+b**2*x**(2*n))**(3/2),x)
```

```
[Out] Integral(x*((a + b*x**n)**2)**(3/2), x)
```

3.461 $\int \left(a^2+2abx^n+b^2x^{2n}\right)^{3/2} dx$

Optimal. Leaf size=206

$$\frac{b^6x^{3n+1}\left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}}{(3n+1)\left(ab+b^2x^n\right)^3}+\frac{3ab^5x^{2n+1}\left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}}{(2n+1)\left(ab+b^2x^n\right)^3}+\frac{3a^2b^4x^{n+1}\left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}}{(n+1)\left(ab+b^2x^n\right)^3}+\frac{a^3x\left(a^2\right.}{}$$

Rubi [A] time = 0.05, antiderivative size = 206, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1343, 244}

$$\frac{3a^2b^4x^{n+1}\left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}}{(n+1)\left(ab+b^2x^n\right)^3}+\frac{3ab^5x^{2n+1}\left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}}{(2n+1)\left(ab+b^2x^n\right)^3}+\frac{b^6x^{3n+1}\left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}}{(3n+1)\left(ab+b^2x^n\right)^3}+\frac{a^3x\left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}}{(a+bx^n)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(3/2),x]
```

```
[Out] (a^3*x*(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(3/2))/(a + b*x^n)^3 + (3*a^2*b^4*x^
(1 + n)*(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(3/2))/((1 + n)*(a*b + b^2*x^n)^3)
+ (3*a*b^5*x^(1 + 2*n)*(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(3/2))/((1 + 2*n)*(a
*b + b^2*x^n)^3) + (b^6*x^(1 + 3*n)*(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(3/2))/
((1 + 3*n)*(a*b + b^2*x^n)^3)
```

Rule 244

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && IGtQ[p, 0]
```

Rule 1343

```
Int[((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_), x_Symbol] :> Dist[(
a + b*x^n + c*x^(2*n))^p/(b + 2*c*x^n)^(2*p), Int[(b + 2*c*x^n)^(2*p), x],
x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}\int \left(a^2+2abx^n+b^2x^{2n}\right)^{3/2} dx &= \frac{\left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}\int \left(2ab+2b^2x^n\right)^3 dx}{\left(2ab+2b^2x^n\right)^3}\\ &= \frac{\left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}\int \left(8a^3b^3+24a^2b^4x^n+24ab^5x^{2n}+8b^6x^{3n}\right) dx}{\left(2ab+2b^2x^n\right)^3}\\ &= \frac{a^3x\left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}}{(a+bx^n)^3}+\frac{3a^2b^4x^{1+n}\left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}}{(1+n)\left(ab+b^2x^n\right)^3}+\frac{3ab^5x^{1+2n}\left(\right.}{(1+2}\end{aligned}$$

Mathematica [A] time = 0.08, size = 122, normalized size = 0.59

$$\frac{x\sqrt{(a+bx^n)^2}\left(a^3\left(6n^3+11n^2+6n+1\right)+3a^2b\left(6n^2+5n+1\right)x^n+3ab^2\left(3n^2+4n+1\right)x^{2n}+b^3\left(2n^2+3n+1\right)x^{3n}\right)}{(n+1)(2n+1)(3n+1)(a+bx^n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(3/2),x]
```

```
[Out] (x*Sqrt[(a + b*x^n)^2]*(a^3*(1 + 6*n + 11*n^2 + 6*n^3) + 3*a^2*b*(1 + 5*n +
 6*n^2)*x^n + 3*a*b^2*(1 + 4*n + 3*n^2)*x^(2*n) + b^3*(1 + 3*n + 2*n^2)*x^(
3*n)))/((1 + n)*(1 + 2*n)*(1 + 3*n)*(a + b*x^n))
```

IntegrateAlgebraic [F] time = 0.09, size = 0, normalized size = 0.00

$$\int \left(a^2 + 2abx^n + b^2x^{2n}\right)^{3/2} dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(3/2),x]
```

```
[Out] Defer[IntegrateAlgebraic][(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(3/2), x]
```

fricas [A] time = 1.36, size = 130, normalized size = 0.63

$$\frac{\left(2\,b^3n^2 + 3\,b^3n + b^3\right)xx^{3\,n} + 3\left(3\,ab^2n^2 + 4\,ab^2n + ab^2\right)xx^{2\,n} + 3\left(6\,a^2bn^2 + 5\,a^2bn + a^2b\right)xx^n + \left(6\,a^3n^3 + 11\,a^3n^2 + 6\,a^3n + a^3\right)x}{6\,n^3 + 11\,n^2 + 6\,n + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x, algorithm="fricas")
```

```
[Out] ((2*b^3*n^2 + 3*b^3*n + b^3)*x*x^(3*n) + 3*(3*a*b^2*n^2 + 4*a*b^2*n + a*b^2
)*x*x^(2*n) + 3*(6*a^2*b*n^2 + 5*a^2*b*n + a^2*b)*x*x^n + (6*a^3*n^3 + 11*a
^3*n^2 + 6*a^3*n + a^3)*x)/(6*n^3 + 11*n^2 + 6*n + 1)
```

giac [A] time = 0.49, size = 263, normalized size = 1.28

$$\frac{6\,a^3n^3x\mathrm{sgn}\left(bx^n + a\right) + 2\,b^3n^2xx^{3\,n}\mathrm{sgn}\left(bx^n + a\right) + 9\,ab^2n^2xx^{2\,n}\mathrm{sgn}\left(bx^n + a\right) + 18\,a^2bn^2xx^n\mathrm{sgn}\left(bx^n + a\right) + 11\,a^3n^2x\mathrm{sgn}\left(bx^n + a\right) + 3\,b^3nxx^{3\,n}\mathrm{sgn}\left(bx^n + a\right) + 12\,ab^2nxx^{2\,n}\mathrm{sgn}\left(bx^n + a\right) + 15\,a^2bnxx^n\mathrm{sgn}\left(bx^n + a\right) + 6\,a^3nx\mathrm{sgn}\left(bx^n + a\right) + b^3xx^{3\,n}\mathrm{sgn}\left(bx^n + a\right) + 3\,ab^2xx^{2\,n}\mathrm{sgn}\left(bx^n + a\right) + 3\,a^2bxx^n\mathrm{sgn}\left(bx^n + a\right) + a^3x\mathrm{sgn}\left(bx^n + a\right)}{6\,n^3 + 11\,n^2 + 6\,n + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x, algorithm="giac")
```

```
[Out] (6*a^3*n^3*x*sgn(b*x^n + a) + 2*b^3*n^2*x*x^(3*n)*sgn(b*x^n + a) + 9*a*b^2*
n^2*x*x^(2*n)*sgn(b*x^n + a) + 18*a^2*b*n^2*x*x^n*sgn(b*x^n + a) + 11*a^3*n
^2*x*sgn(b*x^n + a) + 3*b^3*n*x*x^(3*n)*sgn(b*x^n + a) + 12*a*b^2*n*x*x^(2*
n)*sgn(b*x^n + a) + 15*a^2*b*n*x*x^n*sgn(b*x^n + a) + 6*a^3*n*x*sgn(b*x^n +
 a) + b^3*x*x^(3*n)*sgn(b*x^n + a) + 3*a*b^2*x*x^(2*n)*sgn(b*x^n + a) + 3*a
^2*b*x*x^n*sgn(b*x^n + a) + a^3*x*sgn(b*x^n + a))/(6*n^3 + 11*n^2 + 6*n + 1
)
```

maple [A] time = 0.02, size = 138, normalized size = 0.67

$$\frac{3\sqrt{\left(b\,x^n + a\right)^2}\;a^2bx\,x^n}{\left(b\,x^n + a\right)\left(n + 1\right)} + \frac{3\sqrt{\left(b\,x^n + a\right)^2}\;a\,b^2x\,x^{2n}}{\left(b\,x^n + a\right)\left(2n + 1\right)} + \frac{\sqrt{\left(b\,x^n + a\right)^2}\;b^3x\,x^{3n}}{\left(b\,x^n + a\right)\left(3n + 1\right)} + \frac{\sqrt{\left(b\,x^n + a\right)^2}\;a^3x}{b\,x^n + a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x)
```

```
[Out] ((b*x^n+a)^2)^(1/2)/(b*x^n+a)*a^3*x+((b*x^n+a)^2)^(1/2)/(b*x^n+a)*b^3/(1+3*
n)*x*(x^n)^3+3*((b*x^n+a)^2)^(1/2)/(b*x^n+a)*a*b^2/(1+2*n)*x*(x^n)^2+3*((b*
x^n+a)^2)^(1/2)/(b*x^n+a)*a^2*b/(n+1)*x*x^n
```

maxima [A] time = 0.95, size = 101, normalized size = 0.49

$$\frac{\left(2\,n^2 + 3\,n + 1\right)b^3xx^{3\,n} + 3\left(3\,n^2 + 4\,n + 1\right)ab^2xx^{2\,n} + 3\left(6\,n^2 + 5\,n + 1\right)a^2bxx^n + \left(6\,n^3 + 11\,n^2 + 6\,n + 1\right)a^3x}{6\,n^3 + 11\,n^2 + 6\,n + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x, algorithm="maxima")
```

```
[Out] ((2*n^2 + 3*n + 1)*b^3*x*x^(3*n) + 3*(3*n^2 + 4*n + 1)*a*b^2*x*x^(2*n) + 3*
(6*n^2 + 5*n + 1)*a^2*b*x*x^n + (6*n^3 + 11*n^2 + 6*n + 1)*a^3*x)/(6*n^3 +
11*n^2 + 6*n + 1)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \left(a^2+b^2\,x^{2\,n}+2\,a\,b\,x^n\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(3/2),x)
```

```
[Out] int((a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a^{2} + 2abx^{n} + b^{2}x^{2n}\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2+2*a*b*x**n+b**2*x**(2*n))**(3/2),x)
```

```
[Out] Integral((a**2 + 2*a*b*x**n + b**2*x**(2*n))**(3/2), x)
```

3.462 $\int \frac{\left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}}{x}\,dx$

Optimal. Leaf size=196

$$\frac{3a^2b^2x^n\sqrt{a^2+2abx^n+b^2x^{2n}}}{n\left(ab+b^2x^n\right)}+\frac{b^4x^{3n}\sqrt{a^2+2abx^n+b^2x^{2n}}}{3n\left(ab+b^2x^n\right)}+\frac{3ab^3x^{2n}\sqrt{a^2+2abx^n+b^2x^{2n}}}{2n\left(ab+b^2x^n\right)}+\frac{a^3\log(x)\sqrt{a^2+2\ldots}}{a+bx\ldots}$$

Rubi [A] time = 0.05, antiderivative size = 196, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.107$, Rules used = {1355, 266, 43}

$$\frac{3a^2b^2x^n\sqrt{a^2+2abx^n+b^2x^{2n}}}{n\left(ab+b^2x^n\right)}+\frac{3ab^3x^{2n}\sqrt{a^2+2abx^n+b^2x^{2n}}}{2n\left(ab+b^2x^n\right)}+\frac{b^4x^{3n}\sqrt{a^2+2abx^n+b^2x^{2n}}}{3n\left(ab+b^2x^n\right)}+\frac{a^3\log(x)\sqrt{a^2+2abx^n+b^2x^{2n}}}{a+bx^n}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(3/2)/x,x]
```

```
[Out] (3*a^2*b^2*x^n*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])/(n*(a*b + b^2*x^n)) + (
3*a*b^3*x^(2*n)*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])/(2*n*(a*b + b^2*x^n))
+ (b^4*x^(3*n)*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])/(3*n*(a*b + b^2*x^n)) +
 (a^3*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)]*Log[x])/(a + b*x^n)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\int \frac{\left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}}{x}\,dx = \frac{\sqrt{a^2+2abx^n+b^2x^{2n}}\int \frac{\left(ab+b^2x^n\right)^3}{x}\,dx}{b^2\left(ab+b^2x^n\right)}$$
$$= \frac{\sqrt{a^2+2abx^n+b^2x^{2n}}\,\text{Subst}\left(\int \frac{\left(ab+b^2x\right)^3}{x}\,dx,x,x^n\right)}{b^2n\left(ab+b^2x^n\right)}$$
$$= \frac{\sqrt{a^2+2abx^n+b^2x^{2n}}\,\text{Subst}\left(\int \left(3a^2b^4+\frac{a^3b^3}{x}+3ab^5x+b^6x^2\right)dx,x,x^n\right)}{b^2n\left(ab+b^2x^n\right)}$$
$$= \frac{3a^2b^2x^n\sqrt{a^2+2abx^n+b^2x^{2n}}}{n\left(ab+b^2x^n\right)}+\frac{3ab^3x^{2n}\sqrt{a^2+2abx^n+b^2x^{2n}}}{2n\left(ab+b^2x^n\right)}+\frac{b^4x^{3n}\sqrt{a^2+2a}}{3n\left(ab+\right.}$$

Mathematica [A] time = 0.04, size = 68, normalized size = 0.35

$$\frac{\left((a+bx^n)^2\right)^{3/2}\left(a^3n\log(x)+3a^2bx^n+\frac{3}{2}ab^2x^{2n}+\frac{1}{3}b^3x^{3n}\right)}{n\left(a+bx^n\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(3/2)/x,x]
```

```
[Out] (((a + b*x^n)^2)^(3/2)*(3*a^2*b*x^n + (3*a*b^2*x^(2*n))/2 + (b^3*x^(3*n))/3
 + a^3*n*Log[x]))/(n*(a + b*x^n)^3)
```

IntegrateAlgebraic [A] time = 0.04, size = 69, normalized size = 0.35

$$\frac{\sqrt{(a+bx^n)^2}\left(\frac{a^3\log(x^n)}{n}+\frac{bx^n\left(18a^2+9abx^n+2b^2x^{2n}\right)}{6n}\right)}{a+bx^n}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(3/2)/x,x]
```

```
[Out] (Sqrt[(a + b*x^n)^2]*((b*x^n*(18*a^2 + 9*a*b*x^n + 2*b^2*x^(2*n)))/(6*n) +
(a^3*Log[x^n])/n))/(a + b*x^n)
```

fricas [A] time = 0.80, size = 44, normalized size = 0.22

$$\frac{6\,a^3n\log(x)+2\,b^3x^{3\,n}+9\,ab^2x^{2\,n}+18\,a^2bx^n}{6\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2)/x,x, algorithm="fricas")
```

```
[Out] 1/6*(6*a^3*n*log(x) + 2*b^3*x^(3*n) + 9*a*b^2*x^(2*n) + 18*a^2*b*x^n)/n
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b^2x^{2\,n}+2\,abx^n+a^2\right)^{\frac{3}{2}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2)/x,x, algorithm="giac")
[Out] integrate((b^2*x^(2*n) + 2*a*b*x^n + a^2)^(3/2)/x, x)
```

maple [A] time = 0.02, size = 127, normalized size = 0.65

$$\frac{\sqrt{(b\,x^n+a)^2}\;a^3\ln(x)}{b\,x^n+a}+\frac{3\sqrt{(b\,x^n+a)^2}\;a^2b\,x^n}{(b\,x^n+a)\,n}+\frac{3\sqrt{(b\,x^n+a)^2}\;a\,b^2x^{2n}}{2\,(b\,x^n+a)\,n}+\frac{\sqrt{(b\,x^n+a)^2}\;b^3x^{3n}}{3\,(b\,x^n+a)\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2)/x,x)
[Out] ((b*x^n+a)^2)^(1/2)/(b*x^n+a)*a^3*ln(x)+1/3*((b*x^n+a)^2)^(1/2)/(b*x^n+a)*b
^3/n*(x^n)^3+3/2*((b*x^n+a)^2)^(1/2)/(b*x^n+a)*a*b^2/n*(x^n)^2+3*((b*x^n+a)
^2)^(1/2)/(b*x^n+a)*a^2*b/n*x^n
```

maxima [A] time = 0.90, size = 43, normalized size = 0.22

$$a^3\log(x)+\frac{2\,b^3x^{3\,n}+9\,ab^2x^{2\,n}+18\,a^2bx^n}{6\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2)/x,x, algorithm="maxima")
[Out] a^3*log(x) + 1/6*(2*b^3*x^(3*n) + 9*a*b^2*x^(2*n) + 18*a^2*b*x^n)/n
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a^2+b^2\,x^{2\,n}+2\,a\,b\,x^n\right)^{3/2}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(3/2)/x,x)
[Out] int((a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(3/2)/x, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left((a+bx^n)^2\right)^{\frac{3}{2}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2+2*a*b*x**n+b**2*x**(2*n))**(3/2)/x,x)
[Out] Integral(((a + b*x**n)**2)**(3/2)/x, x)
```

3.463 $\int \frac{\left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}}{x^2}\,dx$

Optimal. Leaf size=212

$$-\frac{3a^2b^2x^{n-1}\sqrt{a^2+2abx^n+b^2x^{2n}}}{(1-n)\left(ab+b^2x^n\right)}-\frac{b^4x^{3n-1}\sqrt{a^2+2abx^n+b^2x^{2n}}}{(1-3n)\left(ab+b^2x^n\right)}-\frac{3ab^3x^{2n-1}\sqrt{a^2+2abx^n+b^2x^{2n}}}{(1-2n)\left(ab+b^2x^n\right)}-\frac{a^3\sqrt{a^2+2ab}}{x(a+b}$$

Rubi [A] time = 0.07, antiderivative size = 212, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {1355, 270}

$$-\frac{3a^2b^2x^{n-1}\sqrt{a^2+2abx^n+b^2x^{2n}}}{(1-n)\left(ab+b^2x^n\right)}-\frac{3ab^3x^{2n-1}\sqrt{a^2+2abx^n+b^2x^{2n}}}{(1-2n)\left(ab+b^2x^n\right)}-\frac{b^4x^{3n-1}\sqrt{a^2+2abx^n+b^2x^{2n}}}{(1-3n)\left(ab+b^2x^n\right)}-\frac{a^3\sqrt{a^2+2abx^n+b^2x^{2n}}}{x\left(a+bx^n\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(3/2)/x^2,x]
```

```
[Out] -((a^3*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])/(x*(a + b*x^n))) - (3*a^2*b^2*x
^(-1 + n)*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])/((1 - n)*(a*b + b^2*x^n)) -
(3*a*b^3*x^(-1 + 2*n)*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])/((1 - 2*n)*(a*b
+ b^2*x^n)) - (b^4*x^(-1 + 3*n)*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])/((1 -
3*n)*(a*b + b^2*x^n))
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}}{x^2}\,dx &= \frac{\sqrt{a^2+2abx^n+b^2x^{2n}}\int \frac{\left(ab+b^2x^n\right)^3}{x^2}\,dx}{b^2\left(ab+b^2x^n\right)}\\ &= \frac{\sqrt{a^2+2abx^n+b^2x^{2n}}\int \left(\frac{a^3b^3}{x^2}+3a^2b^4x^{-2+n}+3ab^5x^{2(-1+n)}+b^6x^{-2+3n}\right)dx}{b^2\left(ab+b^2x^n\right)}\\ &= -\frac{a^3\sqrt{a^2+2abx^n+b^2x^{2n}}}{x\left(a+bx^n\right)}-\frac{3a^2b^2x^{-1+n}\sqrt{a^2+2abx^n+b^2x^{2n}}}{(1-n)\left(ab+b^2x^n\right)}-\frac{3ab^3x^{-1+2n}\sqrt{a^2}}{(1-2n)\left(\right.}\end{aligned}$$

Mathematica [A] time = 0.09, size = 124, normalized size = 0.58

$$\frac{\sqrt{\left(a+bx^n\right)^2}\left(a^3\left(-6n^3+11n^2-6n+1\right)+3a^2b\left(6n^2-5n+1\right)x^n+3ab^2\left(3n^2-4n+1\right)x^{2n}+b^3\left(2n^2-3n+1\right)x^{3n}\right)}{(n-1)(2n-1)(3n-1)x\left(a+bx^n\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(3/2)/x^2,x]
```

```
[Out] (Sqrt[(a + b*x^n)^2]*(a^3*(1 - 6*n + 11*n^2 - 6*n^3) + 3*a^2*b*(1 - 5*n + 6
*n^2)*x^n + 3*a*b^2*(1 - 4*n + 3*n^2)*x^(2*n) + b^3*(1 - 3*n + 2*n^2)*x^(3*
n)))/((-1 + n)*(-1 + 2*n)*(-1 + 3*n)*x*(a + b*x^n))
```

IntegrateAlgebraic [F] time = 0.19, size = 0, normalized size = 0.00

$$\int \frac{\left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}}{x^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(3/2)/x^2,x]
```

```
[Out] Defer[IntegrateAlgebraic][(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(3/2)/x^2, x]
```

fricas [A] time = 1.34, size = 131, normalized size = 0.62

$$-\frac{6\,a^3n^3-11\,a^3n^2+6\,a^3n-a^3-\left(2\,b^3n^2-3\,b^3n+b^3\right)x^{3\,n}-3\left(3\,ab^2n^2-4\,ab^2n+ab^2\right)x^{2\,n}-3\left(6\,a^2bn^2-5\,a^2bn+a^2b\right)x^n}{\left(6\,n^3-11\,n^2+6\,n-1\right)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2)/x^2,x, algorithm="fricas")
```

```
[Out] -(6*a^3*n^3 - 11*a^3*n^2 + 6*a^3*n - a^3 - (2*b^3*n^2 - 3*b^3*n + b^3)*x^(3
*n) - 3*(3*a*b^2*n^2 - 4*a*b^2*n + a*b^2)*x^(2*n) - 3*(6*a^2*b*n^2 - 5*a^2*
b*n + a^2*b)*x^n)/((6*n^3 - 11*n^2 + 6*n - 1)*x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b^2x^{2\,n}+2\,abx^n+a^2\right)^{\frac{3}{2}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2)/x^2,x, algorithm="giac")
```

```
[Out] integrate((b^2*x^(2*n) + 2*a*b*x^n + a^2)^(3/2)/x^2, x)
```

maple [A] time = 0.02, size = 147, normalized size = 0.69

$$\frac{3\sqrt{(b\,x^n+a)^2}\;a^2b\,x^n}{(b\,x^n+a)\,(n-1)\,x}+\frac{3\sqrt{(b\,x^n+a)^2}\;a\,b^2x^{2n}}{(b\,x^n+a)\,(2n-1)\,x}+\frac{\sqrt{(b\,x^n+a)^2}\;b^3x^{3n}}{(b\,x^n+a)\,(3n-1)\,x}-\frac{\sqrt{(b\,x^n+a)^2}\;a^3}{(b\,x^n+a)\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2)/x^2,x)
```

```
[Out] -((b*x^n+a)^2)^(1/2)/(b*x^n+a)*a^3/x+((b*x^n+a)^2)^(1/2)/(b*x^n+a)/(-1+3*n)
*b^3/x*(x^n)^3+3*((b*x^n+a)^2)^(1/2)/(b*x^n+a)/(-1+2*n)*a*b^2/x*(x^n)^2+3*(
(b*x^n+a)^2)^(1/2)/(b*x^n+a)/(n-1)*a^2*b/x*x^n
```

maxima [A] time = 0.94, size = 101, normalized size = 0.48

$$\frac{\left(2\,n^2-3\,n+1\right)b^3x^{3\,n}+3\left(3\,n^2-4\,n+1\right)ab^2x^{2\,n}+3\left(6\,n^2-5\,n+1\right)a^2bx^n-\left(6\,n^3-11\,n^2+6\,n-1\right)a^3}{\left(6\,n^3-11\,n^2+6\,n-1\right)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2)/x^2,x, algorithm="maxima")
```

```
[Out] ((2*n^2 - 3*n + 1)*b^3*x^(3*n) + 3*(3*n^2 - 4*n + 1)*a*b^2*x^(2*n) + 3*(6*n
^2 - 5*n + 1)*a^2*b*x^n - (6*n^3 - 11*n^2 + 6*n - 1)*a^3)/((6*n^3 - 11*n^2
+ 6*n - 1)*x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a^2+b^2\,x^{2\,n}+2\,a\,b\,x^n\right)^{3/2}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(3/2)/x^2,x)
```

```
[Out] int((a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(3/2)/x^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\left(a+bx^n\right)^2\right)^{\frac{3}{2}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2+2*a*b*x**n+b**2*x**(2*n))**(3/2)/x**2,x)
```

```
[Out] Integral(((a + b*x**n)**2)**(3/2)/x**2, x)
```

3.464 $\int \frac{(a^2+2abx^n+b^2x^{2n})^{3/2}}{x^3}\,dx$

Optimal. Leaf size=218

$$-\frac{3a^2b^2x^{n-2}\sqrt{a^2+2abx^n+b^2x^{2n}}}{(2-n)\left(ab+b^2x^n\right)}-\frac{b^4x^{3n-2}\sqrt{a^2+2abx^n+b^2x^{2n}}}{(2-3n)\left(ab+b^2x^n\right)}-\frac{3ab^3x^{-2(1-n)}\sqrt{a^2+2abx^n+b^2x^{2n}}}{2(1-n)\left(ab+b^2x^n\right)}-\frac{a^3\sqrt{a^2+ \cdots}}{2x^{\cdots}}$$

Rubi [A] time = 0.07, antiderivative size = 218, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {1355, 270}

$$-\frac{3ab^3x^{-2(1-n)}\sqrt{a^2+2abx^n+b^2x^{2n}}}{2(1-n)\left(ab+b^2x^n\right)}-\frac{3a^2b^2x^{n-2}\sqrt{a^2+2abx^n+b^2x^{2n}}}{(2-n)\left(ab+b^2x^n\right)}-\frac{b^4x^{3n-2}\sqrt{a^2+2abx^n+b^2x^{2n}}}{(2-3n)\left(ab+b^2x^n\right)}-\frac{a^3\sqrt{a^2+2abx^n+b^2x^{2n}}}{2x^2\left(a+bx^n\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(3/2)/x^3,x]
```

```
[Out] -(a^3*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])/(2*x^2*(a + b*x^n)) - (3*a*b^3*S
qrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])/(2*(1 - n)*x^(2*(1 - n))*(a*b + b^2*x^n
)) - (3*a^2*b^2*x^(-2 + n)*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])/((2 - n)*(a
*b + b^2*x^n)) - (b^4*x^(-2 + 3*n)*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])/((2
 - 3*n)*(a*b + b^2*x^n))
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}}{x^3}\,dx &= \frac{\sqrt{a^2+2abx^n+b^2x^{2n}}\int \frac{\left(ab+b^2x^n\right)^3}{x^3}\,dx}{b^2\left(ab+b^2x^n\right)}\\
&= \frac{\sqrt{a^2+2abx^n+b^2x^{2n}}\int \left(\frac{a^3b^3}{x^3}+3a^2b^4x^{-3+n}+b^6x^{3(-1+n)}+3ab^5x^{-3+2n}\right)dx}{b^2\left(ab+b^2x^n\right)}\\
&= -\frac{a^3\sqrt{a^2+2abx^n+b^2x^{2n}}}{2x^2\left(a+bx^n\right)}-\frac{3ab^3x^{-2(1-n)}\sqrt{a^2+2abx^n+b^2x^{2n}}}{2(1-n)\left(ab+b^2x^n\right)}-\frac{3a^2b^2x^{-2+n}\cdots}{(2-\cdots}
\end{aligned}$$

Mathematica [A] time = 0.09, size = 124, normalized size = 0.57

$$\frac{\sqrt{\left(a+bx^n\right)^2}\left(a^3\left(-3n^3+11n^2-12n+4\right)+6a^2b\left(3n^2-5n+2\right)x^n+3ab^2\left(3n^2-8n+4\right)x^{2n}+2b^3\left(n^2-3n+2\right)x^{3n}\right)}{2(n-2)(n-1)(3n-2)x^2\left(a+bx^n\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(3/2)/x^3,x]
```

```
[Out] (Sqrt[(a + b*x^n)^2]*(a^3*(4 - 12*n + 11*n^2 - 3*n^3) + 6*a^2*b*(2 - 5*n +
3*n^2)*x^n + 3*a*b^2*(4 - 8*n + 3*n^2)*x^(2*n) + 2*b^3*(2 - 3*n + n^2)*x^(3
*n)))/(2*(-2 + n)*(-1 + n)*(-2 + 3*n)*x^2*(a + b*x^n))
```

IntegrateAlgebraic [F] time = 0.16, size = 0, normalized size = 0.00

$$\int \frac{\left(a^2 + 2abx^n + b^2x^{2n}\right)^{3/2}}{x^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(3/2)/x^3,x]
```

```
[Out] Defer[IntegrateAlgebraic][(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(3/2)/x^3, x]
```

fricas [A] time = 1.33, size = 134, normalized size = 0.61

$$-\frac{3\,a^3n^3 - 11\,a^3n^2 + 12\,a^3n - 4\,a^3 - 2\left(b^3n^2 - 3\,b^3n + 2\,b^3\right)x^{3\,n} - 3\left(3\,ab^2n^2 - 8\,ab^2n + 4\,ab^2\right)x^{2\,n} - 6\left(3\,a^2bn^2 - 5\,a^2bn + 2\,a^2b\right)x^n}{2\left(3\,n^3 - 11\,n^2 + 12\,n - 4\right)x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2)/x^3,x, algorithm="fricas")
```

```
[Out] -1/2*(3*a^3*n^3 - 11*a^3*n^2 + 12*a^3*n - 4*a^3 - 2*(b^3*n^2 - 3*b^3*n + 2*
b^3)*x^(3*n) - 3*(3*a*b^2*n^2 - 8*a*b^2*n + 4*a*b^2)*x^(2*n) - 6*(3*a^2*b*n
^2 - 5*a^2*b*n + 2*a^2*b)*x^n)/((3*n^3 - 11*n^2 + 12*n - 4)*x^2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b^2x^{2\,n} + 2\,abx^n + a^2\right)^{\frac{3}{2}}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2)/x^3,x, algorithm="giac")
```

```
[Out] integrate((b^2*x^(2*n) + 2*a*b*x^n + a^2)^(3/2)/x^3, x)
```

maple [A] time = 0.03, size = 145, normalized size = 0.67

$$\frac{3\sqrt{\left(b\,x^n + a\right)^2}\;a^2b\,x^n}{\left(b\,x^n + a\right)\left(n - 2\right)x^2} + \frac{3\sqrt{\left(b\,x^n + a\right)^2}\;a\,b^2x^{2n}}{2\left(b\,x^n + a\right)\left(n - 1\right)x^2} + \frac{\sqrt{\left(b\,x^n + a\right)^2}\;b^3x^{3n}}{\left(b\,x^n + a\right)\left(3n - 2\right)x^2} - \frac{\sqrt{\left(b\,x^n + a\right)^2}\;a^3}{2\left(b\,x^n + a\right)x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2)/x^3,x)
```

```
[Out] -1/2*((b*x^n+a)^2)^(1/2)/(b*x^n+a)*a^3/x^2+((b*x^n+a)^2)^(1/2)/(b*x^n+a)/(-
2+3*n)*b^3/x^2*(x^n)^3+3/2*((b*x^n+a)^2)^(1/2)/(b*x^n+a)/(n-1)*a*b^2/x^2*(x
^n)^2+3*((b*x^n+a)^2)^(1/2)/(b*x^n+a)/(n-2)*a^2*b/x^2*x^n
```

maxima [A] time = 1.17, size = 101, normalized size = 0.46

$$\frac{2\left(n^2 - 3\,n + 2\right)b^3x^{3\,n} + 3\left(3\,n^2 - 8\,n + 4\right)ab^2x^{2\,n} + 6\left(3\,n^2 - 5\,n + 2\right)a^2bx^n - \left(3\,n^3 - 11\,n^2 + 12\,n - 4\right)a^3}{2\left(3\,n^3 - 11\,n^2 + 12\,n - 4\right)x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2)/x^3,x, algorithm="maxima")
```

```
[Out] 1/2*(2*(n^2 - 3*n + 2)*b^3*x^(3*n) + 3*(3*n^2 - 8*n + 4)*a*b^2*x^(2*n) + 6*
(3*n^2 - 5*n + 2)*a^2*b*x^n - (3*n^3 - 11*n^2 + 12*n - 4)*a^3)/((3*n^3 - 11
*n^2 + 12*n - 4)*x^2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a^2+b^2\,x^{2\,n}+2\,a\,b\,x^n\right)^{3/2}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(3/2)/x^3,x)
```

```
[Out] int((a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(3/2)/x^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\left(a+bx^n\right)^2\right)^{\frac{3}{2}}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2+2*a*b*x**n+b**2*x**(2*n))**(3/2)/x**3,x)
```

```
[Out] Integral(((a + b*x**n)**2)**(3/2)/x**3, x)
```

3.465 $\int \frac{1}{x\sqrt{a^2+2abx^n+b^2x^{2n}}}\,dx$

Optimal. Leaf size=85

$$\frac{\log(x)\,(a+bx^n)}{a\sqrt{a^2+2abx^n+b^2x^{2n}}}-\frac{(a+bx^n)\log\left(a+bx^n\right)}{an\sqrt{a^2+2abx^n+b^2x^{2n}}}$$

Rubi [A] time = 0.04, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {1355, 266, 36, 29, 31}

$$\frac{\log(x)\,(a+bx^n)}{a\sqrt{a^2+2abx^n+b^2x^{2n}}}-\frac{(a+bx^n)\log\left(a+bx^n\right)}{an\sqrt{a^2+2abx^n+b^2x^{2n}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)]),x]
```

```
[Out] ((a + b*x^n)*Log[x])/(a*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)]) - ((a + b*x^n)
*Log[a + b*x^n])/(a*n*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\int \frac{1}{x\sqrt{a^2+2abx^n+b^2x^{2n}}}\,dx = \frac{\left(ab+b^2x^n\right)\int \frac{1}{x\left(ab+b^2x^n\right)}\,dx}{\sqrt{a^2+2abx^n+b^2x^{2n}}}$$
$$= \frac{\left(ab+b^2x^n\right)\text{Subst}\left(\int \frac{1}{x\left(ab+b^2x\right)}\,dx,x,x^n\right)}{n\sqrt{a^2+2abx^n+b^2x^{2n}}}$$
$$= \frac{\left(ab+b^2x^n\right)\text{Subst}\left(\int \frac{1}{x}\,dx,x,x^n\right)}{abn\sqrt{a^2+2abx^n+b^2x^{2n}}} - \frac{\left(b\left(ab+b^2x^n\right)\right)\text{Subst}\left(\int \frac{1}{ab+b^2x}\,dx,x,x^n\right)}{an\sqrt{a^2+2abx^n+b^2x^{2n}}}$$
$$= \frac{(a+bx^n)\log(x)}{a\sqrt{a^2+2abx^n+b^2x^{2n}}} - \frac{(a+bx^n)\log\left(a+bx^n\right)}{an\sqrt{a^2+2abx^n+b^2x^{2n}}}$$

Mathematica [A] time = 0.01, size = 42, normalized size = 0.49

$$\frac{(a+bx^n)\left(n\log(x)-\log\left(a+bx^n\right)\right)}{an\sqrt{(a+bx^n)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)]),x]
[Out] ((a + b*x^n)*(n*Log[x] - Log[a + b*x^n]))/(a*n*Sqrt[(a + b*x^n)^2])
```

IntegrateAlgebraic [A] time = 0.04, size = 55, normalized size = 0.65

$$\frac{(a+bx^n)\left(\frac{\log(x^n)}{an}-\frac{\log\left(a^2n+abnx^n\right)}{an}\right)}{\sqrt{(a+bx^n)^2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)]),x]
[Out] ((a + b*x^n)*(Log[x^n]/(a*n) - Log[a^2*n + a*b*n*x^n]/(a*n)))/Sqrt[(a + b*x
^n)^2]
```

fricas [A] time = 1.22, size = 22, normalized size = 0.26

$$\frac{n\log(x)-\log\left(bx^n+a\right)}{an}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x, algorithm="fricas")
[Out] (n*log(x) - log(b*x^n + a))/(a*n)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{b^2x^{2\,n}+2\,abx^n+a^2}\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(b^2*x^(2*n) + 2*a*b*x^n + a^2)*x), x)
```

maple [A] time = 0.02, size = 66, normalized size = 0.78

$$\frac{\sqrt{(b\,x^{n}+a)^{2}}\,\ln(x)}{(b\,x^{n}+a)\,a}-\frac{\sqrt{(b\,x^{n}+a)^{2}}\,\ln\left(x^{n}+\frac{a}{b}\right)}{(b\,x^{n}+a)\,an}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(2*a*b*x^n+b^2*x^(2*n)+a^2)^(1/2),x)
```

```
[Out] ((b*x^n+a)^2)^(1/2)/(b*x^n+a)/a*ln(x)-((b*x^n+a)^2)^(1/2)/(b*x^n+a)/a/n*ln(
x^n+a/b)
```

maxima [A] time = 1.07, size = 27, normalized size = 0.32

$$\frac{\log(x)}{a}-\frac{\log\left(\frac{bx^{n}+a}{b}\right)}{an}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x, algorithm="maxima")
```

```
[Out] log(x)/a - log((b*x^n + a)/b)/(a*n)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x\,\sqrt{a^2+b^2\,x^{2\,n}+2\,a\,b\,x^n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(1/2)),x)
```

```
[Out] int(1/(x*(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\sqrt{(a+bx^{n})^{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a**2+2*a*b*x**n+b**2*x**(2*n))**(1/2),x)
```

```
[Out] Integral(1/(x*sqrt((a + b*x**n)**2)), x)
```

3.466 $\int \frac{1}{x\left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}}\,dx$

Optimal. Leaf size=159

$$\frac{1}{a^2n\sqrt{a^2+2abx^n+b^2x^{2n}}}+\frac{1}{2an\left(a+bx^n\right)\sqrt{a^2+2abx^n+b^2x^{2n}}}+\frac{\log(x)\left(a+bx^n\right)}{a^3\sqrt{a^2+2abx^n+b^2x^{2n}}}-\frac{\left(a+bx^n\right)\log\left(a+\right.}{a^3n\sqrt{a^2+2abx^n+\right.}$$

Rubi [A] time = 0.08, antiderivative size = 159, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {1355, 266, 44}

$$\frac{1}{a^2n\sqrt{a^2+2abx^n+b^2x^{2n}}}+\frac{1}{2an\left(a+bx^n\right)\sqrt{a^2+2abx^n+b^2x^{2n}}}+\frac{\log(x)\left(a+bx^n\right)}{a^3\sqrt{a^2+2abx^n+b^2x^{2n}}}-\frac{\left(a+bx^n\right)\log\left(a+bx^n\right)}{a^3n\sqrt{a^2+2abx^n+b^2x^{2n}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(3/2)),x]
```

```
[Out] 1/(a^2*n*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)]) + 1/(2*a*n*(a + b*x^n)*Sqrt[a
^2 + 2*a*b*x^n + b^2*x^(2*n)]) + ((a + b*x^n)*Log[x])/(a^3*Sqrt[a^2 + 2*a*b
*x^n + b^2*x^(2*n)]) - ((a + b*x^n)*Log[a + b*x^n])/(a^3*n*Sqrt[a^2 + 2*a*b
*x^n + b^2*x^(2*n)])
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x\left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}}\,dx &= \frac{\left(b^2\left(ab+b^2x^n\right)\right)\int \frac{1}{x\left(ab+b^2x^n\right)^3}\,dx}{\sqrt{a^2+2abx^n+b^2x^{2n}}}\\
&= \frac{\left(b^2\left(ab+b^2x^n\right)\right)\text{Subst}\left(\int \frac{1}{x\left(ab+b^2x\right)^3}\,dx,x,x^n\right)}{n\sqrt{a^2+2abx^n+b^2x^{2n}}}\\
&= \frac{\left(b^2\left(ab+b^2x^n\right)\right)\text{Subst}\left(\int \left(\frac{1}{a^3b^3x}-\frac{1}{ab^2(a+bx)^3}-\frac{1}{a^2b^2(a+bx)^2}-\frac{1}{a^3b^2(a+bx)}\right)dx,x,\right.}{n\sqrt{a^2+2abx^n+b^2x^{2n}}}\\
&= \frac{1}{a^2n\sqrt{a^2+2abx^n+b^2x^{2n}}}+\frac{1}{2an\left(a+bx^n\right)\sqrt{a^2+2abx^n+b^2x^{2n}}}+\frac{(a+}{a^3\sqrt{a^2+}}
\end{aligned}$$

Mathematica [A] time = 0.06, size = 76, normalized size = 0.48

$$\frac{(a+bx^n)^3\left(-\frac{\log(a+bx^n)}{a^3}+\frac{n\log(x)}{a^3}+\frac{1}{a^2(a+bx^n)}+\frac{1}{2a(a+bx^n)^2}\right)}{n\left((a+bx^n)^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(3/2)),x]
```

```
[Out] ((a + b*x^n)^3*(1/(2*a*(a + b*x^n)^2) + 1/(a^2*(a + b*x^n)) + (n*Log[x])/a^
3 - Log[a + b*x^n]/a^3))/(n*((a + b*x^n)^2)^(3/2))
```

IntegrateAlgebraic [A] time = 0.05, size = 78, normalized size = 0.49

$$\frac{(a+bx^n)\left(-\frac{\log(a+bx^n)}{a^3n}+\frac{\log(x^n)}{a^3n}+\frac{3a+2bx^n}{2a^2n(a+bx^n)^2}\right)}{\sqrt{(a+bx^n)^2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x*(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(3/2)),x]
```

```
[Out] ((a + b*x^n)*((3*a + 2*b*x^n)/(2*a^2*n*(a + b*x^n)^2) + Log[x^n]/(a^3*n) -
Log[a + b*x^n]/(a^3*n)))/Sqrt[(a + b*x^n)^2]
```

fricas [A] time = 1.49, size = 106, normalized size = 0.67

$$\frac{2\,b^2nx^{2\,n}\log(x)+2\,a^2n\log(x)+3\,a^2+2\left(2\,abn\log(x)+ab\right)x^n-2\left(b^2x^{2\,n}+2\,abx^n+a^2\right)\log\left(bx^n+a\right)}{2\left(a^3b^2nx^{2\,n}+2\,a^4bnx^n+a^5n\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/2*(2*b^2*n*x^(2*n)*log(x) + 2*a^2*n*log(x) + 3*a^2 + 2*(2*a*b*n*log(x) +
a*b)*x^n - 2*(b^2*x^(2*n) + 2*a*b*x^n + a^2)*log(b*x^n + a))/(a^3*b^2*n*x^(
2*n) + 2*a^4*b*n*x^n + a^5*n)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\left(b^2x^{2\,n}+2\,abx^n+a^2\right)^{\frac{3}{2}}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b^2*x^(2*n) + 2*a*b*x^n + a^2)^(3/2)*x), x)
```

maple [A] time = 0.02, size = 104, normalized size = 0.65

$$\frac{\sqrt{(b\,x^n+a)^2}\;\ln(x)}{(b\,x^n+a)\,a^3}+\frac{\sqrt{(b\,x^n+a)^2}\;(2b\,x^n+3a)}{2\,(b\,x^n+a)^3\,a^2n}-\frac{\sqrt{(b\,x^n+a)^2}\;\ln\left(x^n+\frac{a}{b}\right)}{(b\,x^n+a)\,a^3n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(2*a*b*x^n+b^2*x^(2*n)+a^2)^(3/2),x)
```

```
[Out] ((b*x^n+a)^2)^(1/2)/(b*x^n+a)/a^3*ln(x)+1/2*((b*x^n+a)^2)^(1/2)/(b*x^n+a)^3
*(2*b*x^n+3*a)/a^2/n-((b*x^n+a)^2)^(1/2)/(b*x^n+a)/a^3/n*ln(x^n+a/b)
```

maxima [A] time = 1.16, size = 70, normalized size = 0.44

$$\frac{2\,bx^{n}+3\,a}{2\,\left(a^{2}b^{2}nx^{2\,n}+2\,a^{3}bnx^{n}+a^{4}n\right)}+\frac{\log\left(x\right)}{a^{3}}-\frac{\log\left(\frac{bx^{n}+a}{b}\right)}{a^{3}n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x, algorithm="maxima")
```

```
[Out] 1/2*(2*b*x^n + 3*a)/(a^2*b^2*n*x^(2*n) + 2*a^3*b*n*x^n + a^4*n) + log(x)/a^
3 - log((b*x^n + a)/b)/(a^3*n)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x\,{\left(a^2+b^2\,x^{2\,n}+2\,a\,b\,x^n\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(3/2)),x)
```

```
[Out] int(1/(x*(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(\left(a+bx^{n}\right)^{2}\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a**2+2*a*b*x**n+b**2*x**(2*n))**(3/2),x)
```

```
[Out] Integral(1/(x*((a + b*x**n)**2)**(3/2)), x)
```

3.467 $\int \left(a^2+b^2 x^{-\frac{2}{1+2p}}+2abx^{-\frac{1}{1+2p}}\right)^p\,dx$

Optimal. Leaf size=52

$$\frac{x\left(a+bx^{\frac{1}{-2p-1}}\right)\left(a^2+2abx^{\frac{1}{-2p-1}}+b^2x^{-\frac{2}{2p+1}}\right)^p}{a}$$

Rubi [A] time = 0.02, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {1343, 191}

$$\frac{x\left(a+bx^{\frac{1}{-2p-1}}\right)\left(a^2+2abx^{\frac{1}{-2p-1}}+b^2x^{-\frac{2}{2p+1}}\right)^p}{a}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + b^2/x^(2/(1 + 2*p)) + (2*a*b)/x^(1 + 2*p)^(-1))^p,x]
```

```
[Out] (x*(a + b*x^(-1 - 2*p)^(-1))*(a^2 + 2*a*b*x^(-1 - 2*p)^(-1) + b^2/x^(2/(1 +
 2*p)))^p)/a
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 1343

```
Int[((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_), x_Symbol] :> Dist[(
a + b*x^n + c*x^(2*n))^p/(b + 2*c*x^n)^(2*p), Int[(b + 2*c*x^n)^(2*p), x],
x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \left(a^2+b^2x^{-\frac{2}{1+2p}}+2abx^{-\frac{1}{1+2p}}\right)^p dx = \left(\left(a^2+b^2x^{-\frac{2}{1+2p}}+2abx^{-\frac{1}{1+2p}}\right)^p\left(2ab+2b^2x^{-\frac{1}{1+2p}}\right)^{-2p}\right)\int \left(2ab+2b^2x^{-\frac{1}{1+2p}}\right)$$
$$= \frac{x\left(a+bx^{\frac{1}{-1-2p}}\right)\left(a^2+2abx^{\frac{1}{-1-2p}}+b^2x^{-\frac{2}{1+2p}}\right)^p}{a}$$

Mathematica [A] time = 0.02, size = 58, normalized size = 1.12

$$\frac{x^{\frac{2p}{2p+1}}\left(ax^{\frac{1}{2p+1}}+b\right)\left(x^{-\frac{2}{2p+1}}\left(ax^{\frac{1}{2p+1}}+b\right)^2\right)^p}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + b^2/x^(2/(1 + 2*p)) + (2*a*b)/x^(1 + 2*p)^(-1))^p,x]
```

```
[Out] (x^((2*p)/(1 + 2*p))*(b + a*x^(1 + 2*p)^(-1))*((b + a*x^(1 + 2*p)^(-1))^2/x
^(2/(1 + 2*p)))^p)/a
```

IntegrateAlgebraic [F] time = 0.14, size = 0, normalized size = 0.00

$$\int \left(a^2+b^2x^{-\frac{2}{1+2p}}+2abx^{-\frac{1}{1+2p}}\right)^p\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a^2 + b^2/x^(2/(1 + 2*p)) + (2*a*b)/x^(1 + 2*p)^(-1))^p
,x]
```

```
[Out] Defer[IntegrateAlgebraic][(a^2 + b^2/x^(2/(1 + 2*p)) + (2*a*b)/x^(1 + 2*p)^
(-1))^p, x]
```

fricas [A] time = 1.09, size = 79, normalized size = 1.52

$$\frac{\left(axx^{\left(\frac{1}{2p+1}\right)}+bx\right)\left(\frac{a^2x^{\frac{2}{2p+1}}+2\,abx^{\left(\frac{1}{2p+1}\right)}+b^2}{x^{\frac{2}{2p+1}}}\right)^p}{ax^{\left(\frac{1}{2p+1}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+b^2/(x^(2/(1+2*p)))+2*a*b/(x^(1/(1+2*p))))^p,x, algorithm="f
ricas")
```

```
[Out] (a*x*x^(1/(2*p + 1)) + b*x)*((a^2*x^(2/(2*p + 1)) + 2*a*b*x^(1/(2*p + 1)) +
 b^2)/x^(2/(2*p + 1)))^p/(a*x^(1/(2*p + 1)))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a^2+\frac{b^2}{x^{\frac{2}{2p+1}}}+\frac{2\,ab}{x^{\left(\frac{1}{2p+1}\right)}}\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+b^2/(x^(2/(1+2*p)))+2*a*b/(x^(1/(1+2*p))))^p,x, algorithm="g
iac")
```

```
[Out] integrate((a^2 + b^2/x^(2/(2*p + 1)) + 2*a*b/x^(1/(2*p + 1)))^p, x)
```

maple [F] time = 0.28, size = 0, normalized size = 0.00

$$\int \left(2ab\,x^{-\frac{1}{2p+1}}+b^2x^{-\frac{2}{2p+1}}+a^2\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2+b^2/(x^(2/(2*p+1)))+2*a*b/(x^(1/(2*p+1))))^p,x)
```

```
[Out] int((a^2+b^2/(x^(2/(2*p+1)))+2*a*b/(x^(1/(2*p+1))))^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a^2+\frac{b^2}{x^{\frac{2}{2p+1}}}+\frac{2\,ab}{x^{\left(\frac{1}{2p+1}\right)}}\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+b^2/(x^(2/(1+2*p)))+2*a*b/(x^(1/(1+2*p))))^p,x, algorithm="m
axima")
```

```
[Out] integrate((a^2 + b^2/x^(2/(2*p + 1)) + 2*a*b/x^(1/(2*p + 1)))^p, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \left(a^2 + \frac{b^2}{x^{\frac{2}{2p+1}}} + \frac{2\,a\,b}{x^{\frac{1}{2p+1}}}\right)^p \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2/x^(2/(2*p + 1)) + (2*a*b)/x^(1/(2*p + 1)))^p,x)
[Out] int((a^2 + b^2/x^(2/(2*p + 1)) + (2*a*b)/x^(1/(2*p + 1)))^p, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2+b**2/(x**(2/(1+2*p)))+2*a*b/(x**(1/(1+2*p))))**p,x)
[Out] Timed out
```

3.468 $\int \left(a^2+2abx^n+b^2x^{2n}\right)^{\frac{-1-n}{2n}}\,dx$

Optimal. Leaf size=43

$$\frac{x\left(a+bx^n\right)\left(a^2+2abx^n+b^2x^{2n}\right)^{-\frac{n+1}{2n}}}{a}$$

Rubi [A] time = 0.01, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {1343, 191}

$$\frac{x\left(a+bx^n\right)\left(a^2+2abx^n+b^2x^{2n}\right)^{-\frac{n+1}{2n}}}{a}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(-(1 + n)/(2*n)),x]
```

```
[Out] (x*(a + b*x^n))/(a*(a^2 + 2*a*b*x^n + b^2*x^(2*n))^((1 + n)/(2*n)))
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 1343

```
Int[((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_), x_Symbol] :> Dist[(
a + b*x^n + c*x^(2*n))^p/(b + 2*c*x^n)^(2*p), Int[(b + 2*c*x^n)^(2*p), x],
x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}\int \left(a^2+2abx^n+b^2x^{2n}\right)^{\frac{-1-n}{2n}}\,dx &= \left(\left(2ab+2b^2x^n\right)^{-\frac{-1-n}{n}}\left(a^2+2abx^n+b^2x^{2n}\right)^{\frac{-1-n}{2n}}\right)\int \left(2ab+2b^2x^n\right)^{\frac{-1-n}{n}}\,dx\\ &= \frac{x\left(a+bx^n\right)\left(a^2+2abx^n+b^2x^{2n}\right)^{-\frac{1+n}{2n}}}{a}\end{aligned}$$

Mathematica [A] time = 0.06, size = 32, normalized size = 0.74

$$\frac{x\left(a+bx^n\right)\left(\left(a+bx^n\right)^2\right)^{-\frac{n+1}{2n}}}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^n + b^2*x^(2*n))^((-1 - n)/(2*n)),x]
```

```
[Out] (x*(a + b*x^n))/(a*((a + b*x^n)^2)^((1 + n)/(2*n)))
```

IntegrateAlgebraic [F] time = 0.09, size = 0, normalized size = 0.00

$$\int \left(a^2+2abx^n+b^2x^{2n}\right)^{\frac{-1-n}{2n}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a^2 + 2*a*b*x^n + b^2*x^(2*n))^((-1 - n)/(2*n)),x]
[Out] Defer[IntegrateAlgebraic][(a^2 + 2*a*b*x^n + b^2*x^(2*n))^((-1 - n)/(2*n)),
 x]
```

fricas [A] time = 1.28, size = 45, normalized size = 1.05

$$\frac{bxx^n + ax}{\left(b^2x^{2n} + 2\,abx^n + a^2\right)^{\frac{n+1}{2n}}\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2*(1+n)/n)),x, algorithm="frica
s")
[Out] (b*x*x^n + a*x)/((b^2*x^(2*n) + 2*a*b*x^n + a^2)^(1/2*(n + 1)/n)*a)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b^2x^{2n} + 2\,abx^n + a^2\right)^{\frac{n+1}{2n}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2*(1+n)/n)),x, algorithm="giac"
)
[Out] integrate(1/((b^2*x^(2*n) + 2*a*b*x^n + a^2)^(1/2*(n + 1)/n)), x)
```

maple [A] time = 0.04, size = 51, normalized size = 1.19

$$\left(\frac{bx\,e^{n\ln(x)}}{a} + x\right)e^{-\frac{(n+1)\ln\left(2ab\,e^{n\ln(x)}+b^2e^{2n\ln(x)}+a^2\right)}{2n}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((2*a*b*x^n+b^2*x^(2*n)+a^2)^(1/2*(n+1)/n)),x)
[Out] (x+1/a*b*x*exp(n*ln(x)))/exp(1/2*(n+1)/n*ln(a^2+2*a*b*exp(n*ln(x))+b^2*exp(
n*ln(x))^2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b^2x^{2n} + 2\,abx^n + a^2\right)^{\frac{n+1}{2n}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2*(1+n)/n)),x, algorithm="maxim
a")
[Out] integrate(1/((b^2*x^(2*n) + 2*a*b*x^n + a^2)^(1/2*(n + 1)/n)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\left(a^2 + b^2\,x^{2\,n} + 2\,a\,b\,x^n\right)^{\frac{\frac{n}{2}+\frac{1}{2}}{n}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^((n/2 + 1/2)/n),x)
[Out] int(1/(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^((n/2 + 1/2)/n), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a^2 + 2abx^n + b^2x^{2n}\right)^{-\frac{\frac{n}{2}+\frac{1}{2}}{n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a**2+2*a*b*x**n+b**2*x**(2*n))**(1/2*(1+n)/n)),x)
[Out] Integral((a**2 + 2*a*b*x**n + b**2*x**(2*n))**(-(n/2 + 1/2)/n), x)
```

3.469 $\int \left(a^2+b^2x^{-\frac{1}{1+p}}+2abx^{-\frac{1}{2(1+p)}}\right)^p\,dx$

Optimal. Leaf size=130

$$\frac{2(p+1)x\left(a+bx^{-\frac{1}{2(p+1)}}\right)\left(a^2+2abx^{-\frac{1}{2(p+1)}}+b^2x^{-\frac{1}{p+1}}\right)^p}{a(2p+1)}-\frac{x\left(a+bx^{-\frac{1}{2(p+1)}}\right)^2\left(a^2+2abx^{-\frac{1}{2(p+1)}}+b^2x^{-\frac{1}{p+1}}\right)^p}{a^2(2p+1)}$$

Rubi [A] time = 0.06, antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.088, Rules used = {1343, 192, 191}

$$\frac{2(p+1)x\left(a+bx^{-\frac{1}{2(p+1)}}\right)\left(a^2+2abx^{-\frac{1}{2(p+1)}}+b^2x^{-\frac{1}{p+1}}\right)^p}{a(2p+1)}-\frac{x\left(a+bx^{-\frac{1}{2(p+1)}}\right)^2\left(a^2+2abx^{-\frac{1}{2(p+1)}}+b^2x^{-\frac{1}{p+1}}\right)^p}{a^2(2p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + b^2/x^(1 + p)^(-1) + (2*a*b)/x^(1/(2*(1 + p))))^p,x]
```

```
[Out] (2*(1 + p)*x*(a + b/x^(1/(2*(1 + p))))*(a^2 + b^2/x^(1 + p)^(-1) + (2*a*b)/
x^(1/(2*(1 + p))))^p)/(a*(1 + 2*p)) - (x*(a + b/x^(1/(2*(1 + p))))^2*(a^2 +
 b^2/x^(1 + p)^(-1) + (2*a*b)/x^(1/(2*(1 + p))))^p)/(a^2*(1 + 2*p))
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rule 1343

```
Int[((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_), x_Symbol] :> Dist[(
a + b*x^n + c*x^(2*n))^p/(b + 2*c*x^n)^(2*p), Int[(b + 2*c*x^n)^(2*p), x],
x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \left(a^2+b^2x^{-\frac{1}{1+p}}+2abx^{-\frac{1}{2(1+p)}}\right)^p dx=\left(\left(a^2+b^2x^{-\frac{1}{1+p}}+2abx^{-\frac{1}{2(1+p)}}\right)^p\left(2ab+2b^2x^{-\frac{1}{2(1+p)}}\right)^{-2p}\right)\int \left(2ab+2b^2x^{-\frac{1}{2(1+p)}}\right.$$

$$=\frac{2(1+p)x\left(a+bx^{-\frac{1}{2(1+p)}}\right)\left(a^2+b^2x^{-\frac{1}{1+p}}+2abx^{-\frac{1}{2(1+p)}}\right)^p}{a(1+2p)}-\frac{\left(\left(a^2+b^2x^{-\frac{1}{1+p}}+\right.\right.}{}$$

$$=\frac{2(1+p)x\left(a+bx^{-\frac{1}{2(1+p)}}\right)\left(a^2+b^2x^{-\frac{1}{1+p}}+2abx^{-\frac{1}{2(1+p)}}\right)^p}{a(1+2p)}-\frac{x\left(a+bx^{-\frac{1}{2(1+p)}}\right)^2}{}$$

Mathematica [A] time = 0.06, size = 80, normalized size = 0.62

$$\frac{x^{\frac{p}{p+1}}\left(ax^{\frac{1}{2p+2}}+b\right)\left(x^{-\frac{1}{p+1}}\left(ax^{\frac{1}{2p+2}}+b\right)^2\right)^p\left(a(2p+1)x^{\frac{1}{2p+2}}-b\right)}{a^2(2p+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + b^2/x^(1 + p)^(-1) + (2*a*b)/x^(1/(2*(1 + p))))^p,x]
```

```
[Out] (x^(p/(1 + p))*(b + a*x^(2 + 2*p)^(-1))*((b + a*x^(2 + 2*p)^(-1))^2/x^(1 +
p)^(-1))^p*(-b + a*(1 + 2*p)*x^(2 + 2*p)^(-1)))/(a^2*(1 + 2*p))
```

IntegrateAlgebraic [F] time = 0.14, size = 0, normalized size = 0.00

$$\int\left(a^2+b^2x^{-\frac{1}{1+p}}+2abx^{-\frac{1}{2(1+p)}}\right)^p\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a^2 + b^2/x^(1 + p)^(-1) + (2*a*b)/x^(1/(2*(1 + p))))^p
,x]
```

```
[Out] Defer[IntegrateAlgebraic][(a^2 + b^2/x^(1 + p)^(-1) + (2*a*b)/x^(1/(2*(1 +
p))))^p, x]
```

fricas [A] time = 0.93, size = 103, normalized size = 0.79

$$\frac{\left(2\,abpxx^{\frac{1}{2\,(p+1)}}-b^2x+\left(2\,a^2p+a^2\right)xx^{\left(\frac{1}{p+1}\right)}\right)\left(\frac{2\,abx^{\frac{1}{2\,(p+1)}}+a^2x^{\left(\frac{1}{p+1}\right)}+b^2}{x^{\left(\frac{1}{p+1}\right)}}\right)^p}{\left(2\,a^2p+a^2\right)x^{\left(\frac{1}{p+1}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+b^2/(x^(1/(1+p)))+2*a*b/(x^(1/2/(1+p))))^p,x, algorithm="fri
cas")
```

```
[Out] (2*a*b*p*x*x^(1/2/(p + 1)) - b^2*x + (2*a^2*p + a^2)*x*x^(1/(p + 1)))*((2*a
*b*x^(1/2/(p + 1)) + a^2*x^(1/(p + 1)) + b^2)/x^(1/(p + 1)))^p/((2*a^2*p +
a^2)*x^(1/(p + 1)))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a^2+\frac{2\,ab}{x^{\frac{1}{2\,(p+1)}}}+\frac{b^2}{x^{\left(\frac{1}{p+1}\right)}}\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+b^2/(x^(1/(1+p)))+2*a*b/(x^(1/2/(1+p))))^p,x, algorithm="gia
c")
```

```
[Out] integrate((a^2 + 2*a*b/x^(1/2/(p + 1)) + b^2/x^(1/(p + 1)))^p, x)
```

maple [F] time = 0.28, size = 0, normalized size = 0.00

$$\int\left(2ab\,x^{-\frac{1}{2(p+1)}}+b^2x^{-\frac{1}{p+1}}+a^2\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2+b^2/(x^(1/(p+1)))+2*a*b/(x^(1/2/(p+1))))^p,x)
```

```
[Out] int((a^2+b^2/(x^(1/(p+1)))+2*a*b/(x^(1/2/(p+1))))^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a^2 + \frac{2\,ab}{x^{\frac{1}{2\,(p+1)}}} + \frac{b^2}{x^{\left(\frac{1}{p+1}\right)}}\right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+b^2/(x^(1/(1+p)))+2*a*b/(x^(1/2/(1+p))))^p,x, algorithm="max
ima")
```

```
[Out] integrate((a^2 + 2*a*b/x^(1/2/(p + 1)) + b^2/x^(1/(p + 1)))^p, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(\frac{b^2}{x^{\frac{1}{p+1}}} + a^2 + \frac{2\,a\,b}{x^{\frac{1}{2\,(p+1)}}}\right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2/x^(1/(p + 1)) + a^2 + (2*a*b)/x^(1/(2*(p + 1))))^p,x)
```

```
[Out] int((b^2/x^(1/(p + 1)) + a^2 + (2*a*b)/x^(1/(2*(p + 1))))^p, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2+b**2/(x**(1/(1+p)))+2*a*b/(x**(1/2/(1+p))))**p,x)
```

```
[Out] Timed out
```

3.470 $\int \left(a^2+2abx^n+b^2x^{2n}\right)^{\frac{-1-2n}{2n}}\,dx$

Optimal. Leaf size=102

$$\frac{nx(a+bx^n)^2\left(a^2+2abx^n+b^2x^{2n}\right)^{\frac{1}{2}\left(-\frac{1}{n}-2\right)}}{a^2(n+1)}+\frac{x(a+bx^n)\left(a^2+2abx^n+b^2x^{2n}\right)^{\frac{1}{2}\left(-\frac{1}{n}-2\right)}}{a(n+1)}$$

Rubi [A] time = 0.04, antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1343, 192, 191}

$$\frac{nx(a+bx^n)^2\left(a^2+2abx^n+b^2x^{2n}\right)^{\frac{1}{2}\left(-\frac{1}{n}-2\right)}}{a^2(n+1)}+\frac{x(a+bx^n)\left(a^2+2abx^n+b^2x^{2n}\right)^{\frac{1}{2}\left(-\frac{1}{n}-2\right)}}{a(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(-(1 + 2*n)/(2*n)),x]
```

```
[Out] (x*(a + b*x^n)*(a^2 + 2*a*b*x^n + b^2*x^(2*n))^((-2 - n^(-1))/2))/(a*(1 + n
)) + (n*x*(a + b*x^n)^2*(a^2 + 2*a*b*x^n + b^2*x^(2*n))^((-2 - n^(-1))/2))/
(a^2*(1 + n))
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rule 1343

```
Int[((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_), x_Symbol] :> Dist[(
a + b*x^n + c*x^(2*n))^p/(b + 2*c*x^n)^(2*p), Int[(b + 2*c*x^n)^(2*p), x],
x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \left(a^2+2abx^n+b^2x^{2n}\right)^{\frac{-1-2n}{2n}}\,dx = \left(\left(2ab+2b^2x^n\right)^{-\frac{-1-2n}{n}}\left(a^2+2abx^n+b^2x^{2n}\right)^{\frac{-1-2n}{2n}}\right)\int \left(2ab+2b^2x^n\right)^{\frac{-1-2n}{n}}\,dx$$

$$= \frac{x(a+bx^n)\left(a^2+2abx^n+b^2x^{2n}\right)^{\frac{1}{2}\left(-2-\frac{1}{n}\right)}}{a(1+n)}+\frac{\left(n\left(2ab+2b^2x^n\right)^{-\frac{-1-2n}{n}}\left(a^2+2a\right.\right.}{}$$

$$= \frac{x(a+bx^n)\left(a^2+2abx^n+b^2x^{2n}\right)^{\frac{1}{2}\left(-2-\frac{1}{n}\right)}}{a(1+n)}+\frac{nx(a+bx^n)^2\left(a^2+2abx^n+b^2x\right.}{a^2(1+n)}$$

Mathematica [C] time = 0.04, size = 59, normalized size = 0.58

$$\frac{x\left((a+bx^n)^2\right)^{-\frac{1}{2}/n}\left(\frac{bx^n}{a}+1\right)^{\frac{1}{n}}\,{}_2F_1\left(2+\frac{1}{n},\frac{1}{n};1+\frac{1}{n};-\frac{bx^n}{a}\right)}{a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^n + b^2*x^(2*n))^((-1 - 2*n)/(2*n)),x]
```

```
[Out] (x*(1 + (b*x^n)/a)^n^(-1)*Hypergeometric2F1[2 + n^(-1), n^(-1), 1 + n^(-1),
 -((b*x^n)/a)])/(a^2*((a + b*x^n)^2)^(1/(2*n)))
```

IntegrateAlgebraic [F] time = 0.07, size = 0, normalized size = 0.00

$$\int \left(a^2+2abx^n+b^2x^{2n}\right)^{\frac{-1-2n}{2n}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a^2 + 2*a*b*x^n + b^2*x^(2*n))^((-1 - 2*n)/(2*n)),x]
```

```
[Out] Defer[IntegrateAlgebraic][(a^2 + 2*a*b*x^n + b^2*x^(2*n))^((-1 - 2*n)/(2*n)
), x]
```

fricas [A] time = 1.13, size = 82, normalized size = 0.80

$$\frac{b^2nxx^{2n}+(2\,abn+ab)xx^n+\left(a^2n+a^2\right)x}{\left(a^2n+a^2\right)\left(b^2x^{2n}+2\,abx^n+a^2\right)^{\frac{2n+1}{2n}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2*(1+2*n)/n)),x, algorithm="fri
cas")
```

```
[Out] (b^2*n*x*x^(2*n) + (2*a*b*n + a*b)*x*x^n + (a^2*n + a^2)*x)/((a^2*n + a^2)*
(b^2*x^(2*n) + 2*a*b*x^n + a^2)^(1/2*(2*n + 1)/n))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b^2x^{2n}+2\,abx^n+a^2\right)^{\frac{2n+1}{2n}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2*(1+2*n)/n)),x, algorithm="gia
c")
```

```
[Out] integrate(1/((b^2*x^(2*n) + 2*a*b*x^n + a^2)^(1/2*(2*n + 1)/n)), x)
```

maple [F] time = 0.11, size = 0, normalized size = 0.00

$$\int \left(2ab\,x^n+b^2x^{2n}+a^2\right)^{-\frac{2n+1}{2n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((2*a*b*x^n+b^2*x^(2*n)+a^2)^(1/2*(2*n+1)/n)),x)
```

```
[Out] int(1/((2*a*b*x^n+b^2*x^(2*n)+a^2)^(1/2*(2*n+1)/n)),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b^2x^{2n}+2\,abx^n+a^2\right)^{\frac{2n+1}{2n}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2*(1+2*n)/n)),x, algorithm="max
ima")
```

```
[Out] integrate(1/((b^2*x^(2*n) + 2*a*b*x^n + a^2)^(1/2*(2*n + 1)/n)), x)
```

mupad [F]   time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(a^2+b^2\,x^{2\,n}+2\,a\,b\,x^n\right)^{\frac{n+\frac{1}{2}}{n}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^((n + 1/2)/n),x)
```

```
[Out] int(1/(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^((n + 1/2)/n), x)
```

sympy [F(-1)]   time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a**2+2*a*b*x**n+b**2*x**(2*n))**(1/2*(1+2*n)/n)),x)
```

```
[Out] Timed out
```

3.471 $\int (dx)^{-1-2n(1+p)} \left(a^2+2abx^n+b^2x^{2n}\right)^p \, dx$

Optimal. Leaf size=117

$$\frac{(dx)^{-2n(p+1)}\left(a^2+2abx^n+b^2x^{2n}\right)^{p+1}}{2a^2dn(p+1)(2p+1)}-\frac{(a+bx^n)\,(dx)^{-2n(p+1)}\left(a^2+2abx^n+b^2x^{2n}\right)^p}{adn(2p+1)}$$

Rubi [A] time = 0.06, antiderivative size = 124, normalized size of antiderivative = 1.06, number of steps used = 3, number of rules used = 3, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.086, Rules used = {1356, 273, 264}

$$\frac{\left(\frac{bx^n}{a}+1\right)^2(dx)^{-2n(p+1)}\left(a^2+2abx^n+b^2x^{2n}\right)^p}{2dn\left(2p^2+3p+1\right)}-\frac{\left(\frac{bx^n}{a}+1\right)(dx)^{-2n(p+1)}\left(a^2+2abx^n+b^2x^{2n}\right)^p}{dn(2p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^(-1 - 2*n*(1 + p))*(a^2 + 2*a*b*x^n + b^2*x^(2*n))^p,x]
```

```
[Out] -(((1 + (b*x^n)/a)*(a^2 + 2*a*b*x^n + b^2*x^(2*n))^p)/(d*n*(1 + 2*p)*(d*x)^
(2*n*(1 + p)))) + ((1 + (b*x^n)/a)^2*(a^2 + 2*a*b*x^n + b^2*x^(2*n))^p)/(2*
d*n*(1 + 3*p + 2*p^2)*(d*x)^(2*n*(1 + p)))
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 273

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m, n, p}, x] && ILtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[p, -1]
```

Rule 1356

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n + c*x^(2*n))^FracPart[p])/(1 + (
2*c*x^n)/b)^(2*FracPart[p]), Int[(d*x)^m*(1 + (2*c*x^n)/b)^(2*p), x], x] /;
 FreeQ[{a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] &&
 !IntegerQ[2*p]
```

Rubi steps

$$\int (dx)^{-1-2n(1+p)}\left(a^2+2abx^n+b^2x^{2n}\right)^p\,dx=\left(\left(1+\frac{bx^n}{a}\right)^{-2p}\left(a^2+2abx^n+b^2x^{2n}\right)^p\right)\int (dx)^{-1-2n(1+p)}\left(1+\frac{bx^n}{a}\right)^{2p}$$

$$=-\frac{(dx)^{-2n(1+p)}\left(1+\frac{bx^n}{a}\right)\left(a^2+2abx^n+b^2x^{2n}\right)^p}{dn(1+2p)}+\frac{\left((-2n(1+p)+n(1\right.}{}$$

$$=-\frac{(dx)^{-2n(1+p)}\left(1+\frac{bx^n}{a}\right)\left(a^2+2abx^n+b^2x^{2n}\right)^p}{dn(1+2p)}+\frac{(dx)^{-2n(1+p)}\left(1+\frac{bx^n}{a}\right.}{2dn\left(1\right.}$$

Mathematica [C] time = 0.03, size = 75, normalized size = 0.64

$$-\frac{x(dx)^{-2n(p+1)-1}\left((a+bx^n)^2\right)^p\left(\frac{bx^n}{a}+1\right)^{-2p}{}_2F_1\left(-2p,-2(p+1);1-2(p+1);-\frac{bx^n}{a}\right)}{2n(p+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^(-1 - 2*n*(1 + p))*(a^2 + 2*a*b*x^n + b^2*x^(2*n))^p,x]
```

```
[Out] -1/2*(x*(d*x)^(-1 - 2*n*(1 + p))*((a + b*x^n)^2)^p*Hypergeometric2F1[-2*p,
-2*(1 + p), 1 - 2*(1 + p), -((b*x^n)/a)])/(n*(1 + p)*(1 + (b*x^n)/a)^(2*p))
```

IntegrateAlgebraic [F] time = 0.22, size = 0, normalized size = 0.00

$$\int (dx)^{-1-2n(1+p)}\left(a^2+2abx^n+b^2x^{2n}\right)^p\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(d*x)^(-1 - 2*n*(1 + p))*(a^2 + 2*a*b*x^n + b^2*x^(2*n))
^p,x]
```

```
[Out] Defer[IntegrateAlgebraic][(d*x)^(-1 - 2*n*(1 + p))*(a^2 + 2*a*b*x^n + b^2*x
^(2*n))^p, x]
```

fricas [A] time = 1.08, size = 165, normalized size = 1.41

$$-\frac{\left(2\,abpxx^ne^{\left(-\left(2\,np+2\,n+1\right)\log(d)-\left(2\,np+2\,n+1\right)\log(x)\right)}-b^2xx^{2\,n}e^{\left(-\left(2\,np+2\,n+1\right)\log(d)-\left(2\,np+2\,n+1\right)\log(x)\right)}+\left(2\,a^2p+a^2\right)xe^{\left(-\left(2\,np+2\,n+1\right)\log(d)-\left(2\,np+2\,n+1\right)\log(x)\right)}\right)\left(b^2x^{2\,n}+2\,abx^n+a^2\right)^p}{2\left(2\,a^2np^2+3\,a^2np+a^2n\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(-1-2*n*(1+p))*(a^2+2*a*b*x^n+b^2*x^(2*n))^p,x, algorithm="
fricas")
```

```
[Out] -1/2*(2*a*b*p*x*x^n*e^(-(2*n*p + 2*n + 1)*log(d) - (2*n*p + 2*n + 1)*log(x)
) - b^2*x*x^(2*n)*e^(-(2*n*p + 2*n + 1)*log(d) - (2*n*p + 2*n + 1)*log(x))
+ (2*a^2*p + a^2)*x*e^(-(2*n*p + 2*n + 1)*log(d) - (2*n*p + 2*n + 1)*log(x)
))*(b^2*x^(2*n) + 2*a*b*x^n + a^2)^p/(2*a^2*n*p^2 + 3*a^2*n*p + a^2*n)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b^2x^{2\,n}+2\,abx^n+a^2\right)^p(dx)^{-2\,n\left(p+1\right)-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(-1-2*n*(1+p))*(a^2+2*a*b*x^n+b^2*x^(2*n))^p,x, algorithm="
giac")
```

```
[Out] integrate((b^2*x^(2*n) + 2*a*b*x^n + a^2)^p*(d*x)^(-2*n*(p + 1) - 1), x)
```

maple [F] time = 0.05, size = 0, normalized size = 0.00

$$\int (dx)^{-2\left(p+1\right)n-1}\left(2ab\,x^n+b^2x^{2n}+a^2\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(-1-2*n*(p+1))*(2*a*b*x^n+b^2*x^(2*n)+a^2)^p,x)
```

```
[Out] int((d*x)^(-1-2*n*(p+1))*(2*a*b*x^n+b^2*x^(2*n)+a^2)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b^2x^{2\,n} + 2\,abx^n + a^2\right)^p \,(dx)^{-2\,n(p+1)-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(-1-2*n*(1+p))*(a^2+2*a*b*x^n+b^2*x^(2*n))^p,x, algorithm="
maxima")
```

```
[Out] integrate((b^2*x^(2*n) + 2*a*b*x^n + a^2)^p*(d*x)^(-2*n*(p + 1) - 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a^2 + b^2\,x^{2\,n} + 2\,a\,b\,x^n\right)^p}{(d\,x)^{2\,n\,(p+1)+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^(2*n) + 2*a*b*x^n)^p/(d*x)^(2*n*(p + 1) + 1),x)
```

```
[Out] int((a^2 + b^2*x^(2*n) + 2*a*b*x^n)^p/(d*x)^(2*n*(p + 1) + 1), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(-1-2*n*(1+p))*(a**2+2*a*b*x**n+b**2*x**(2*n))**p,x)
```

```
[Out] Timed out
```

3.472 $\int x^{-1+2n} \left(a^2 + 2abx^n + b^2x^{2n}\right)^p \, dx$

Optimal. Leaf size=103

$$\frac{a^2\left(\frac{bx^n}{a}+1\right)^2\left(a^2+2abx^n+b^2x^{2n}\right)^p}{2b^2n(p+1)}-\frac{a^2\left(\frac{bx^n}{a}+1\right)\left(a^2+2abx^n+b^2x^{2n}\right)^p}{b^2n(2p+1)}$$

Rubi [A] time = 0.06, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {1356, 266, 43}

$$\frac{a^2\left(\frac{bx^n}{a}+1\right)^2\left(a^2+2abx^n+b^2x^{2n}\right)^p}{2b^2n(p+1)}-\frac{a^2\left(\frac{bx^n}{a}+1\right)\left(a^2+2abx^n+b^2x^{2n}\right)^p}{b^2n(2p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 + 2*n)*(a^2 + 2*a*b*x^n + b^2*x^(2*n))^p,x]
```

```
[Out] -((a^2*(1 + (b*x^n)/a)*(a^2 + 2*a*b*x^n + b^2*x^(2*n))^p)/(b^2*n*(1 + 2*p))
) + (a^2*(1 + (b*x^n)/a)^2*(a^2 + 2*a*b*x^n + b^2*x^(2*n))^p)/(2*b^2*n*(1 +
 p))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1356

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n + c*x^(2*n))^FracPart[p])/(1 + (
2*c*x^n)/b)^(2*FracPart[p]), Int[(d*x)^m*(1 + (2*c*x^n)/b)^(2*p), x], x] /;
 FreeQ[{a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] &&
 !IntegerQ[2*p]
```

Rubi steps

$$\int x^{-1+2n}\left(a^2+2abx^n+b^2x^{2n}\right)^p\,dx=\left(\left(1+\frac{bx^n}{a}\right)^{-2p}\left(a^2+2abx^n+b^2x^{2n}\right)^p\right)\int x^{-1+2n}\left(1+\frac{bx^n}{a}\right)^{2p}dx$$

$$=\frac{\left(\left(1+\frac{bx^n}{a}\right)^{-2p}\left(a^2+2abx^n+b^2x^{2n}\right)^p\right)\text{Subst}\left(\int x\left(1+\frac{bx}{a}\right)^{2p}dx,x,x^n\right)}{n}$$

$$=\frac{\left(\left(1+\frac{bx^n}{a}\right)^{-2p}\left(a^2+2abx^n+b^2x^{2n}\right)^p\right)\text{Subst}\left(\int\left(-\frac{a\left(1+\frac{bx}{a}\right)^{2p}}{b}+\frac{a\left(1+\frac{bx}{a}\right)^{1+2p}}{b}\right)d\right.}{n}$$

$$=-\frac{a^2\left(1+\frac{bx^n}{a}\right)\left(a^2+2abx^n+b^2x^{2n}\right)^p}{b^2n(1+2p)}+\frac{a^2\left(1+\frac{bx^n}{a}\right)^2\left(a^2+2abx^n+b^2x^{2n}\right)^p}{2b^2n(1+p)}$$

Mathematica [A] time = 0.03, size = 54, normalized size = 0.52

$$\frac{(a+bx^n)\left((a+bx^n)^2\right)^p\left(b(2p+1)x^n-a\right)}{2b^2n(p+1)(2p+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 + 2*n)*(a^2 + 2*a*b*x^n + b^2*x^(2*n))^p,x]
```

```
[Out] ((a + b*x^n)*((a + b*x^n)^2)^p*(-a + b*(1 + 2*p)*x^n))/(2*b^2*n*(1 + p)*(1
+ 2*p))
```

IntegrateAlgebraic [F] time = 0.23, size = 0, normalized size = 0.00

$$\int x^{-1+2n}\left(a^2+2abx^n+b^2x^{2n}\right)^p\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^(-1 + 2*n)*(a^2 + 2*a*b*x^n + b^2*x^(2*n))^p,x]
```

```
[Out] Defer[IntegrateAlgebraic][x^(-1 + 2*n)*(a^2 + 2*a*b*x^n + b^2*x^(2*n))^p, x
]
```

fricas [A] time = 1.08, size = 78, normalized size = 0.76

$$\frac{\left(2\,abpx^n-a^2+\left(2\,b^2p+b^2\right)x^{2\,n}\right)\left(b^2x^{2\,n}+2\,abx^n+a^2\right)^p}{2\left(2\,b^2np^2+3\,b^2np+b^2n\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+2*n)*(a^2+2*a*b*x^n+b^2*x^(2*n))^p,x, algorithm="fricas")
```

```
[Out] 1/2*(2*a*b*p*x^n - a^2 + (2*b^2*p + b^2)*x^(2*n))*(b^2*x^(2*n) + 2*a*b*x^n
+ a^2)^p/(2*b^2*n*p^2 + 3*b^2*n*p + b^2*n)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b^2x^{2\,n}+2\,abx^n+a^2\right)^p x^{2\,n-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+2*n)*(a^2+2*a*b*x^n+b^2*x^(2*n))^p,x, algorithm="giac")
```

[Out] integrate((b^2*x^(2*n) + 2*a*b*x^n + a^2)^p*x^(2*n - 1), x)

maple [C] time = 0.07, size = 148, normalized size = 1.44

$$-\frac{\left(-2abp\,x^{n}-2b^{2}p\,x^{2n}-b^{2}x^{2n}+a^{2}\right)\mathrm{e}^{\frac{\left(-i\pi\mathrm{csgn}\left(i\left(b\,x^{n}+a\right)\right)^{2}\mathrm{csgn}\left(i\left(b\,x^{n}+a\right)^{2}\right)+2i\pi\,\mathrm{csgn}\left(i\left(b\,x^{n}+a\right)\right)\mathrm{csgn}\left(i\left(b\,x^{n}+a\right)^{2}\right)^{2}-i\pi\mathrm{csgn}\left(i\left(b\,x^{n}+a\right)^{2}\right)^{3}+4\ln\left(b\,x^{n}+a\right)\right)p}{2}}}{2\left(2p+1\right)\left(p+1\right)b^{2}n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(2*n-1)*(2*a*b*x^n+b^2*x^(2*n)+a^2)^p,x)

[Out] -1/2*(-2*b^2*p*(x^n)^2-2*a*p*x^n*b-b^2*(x^n)^2+a^2)/(2*p+1)/(p+1)/n/b^2*exp(1/2*p*(-I*Pi*csgn(I*(b*x^n+a)^2)^3+2*I*Pi*csgn(I*(b*x^n+a)^2)^2*csgn(I*(b*x^n+a))-I*Pi*csgn(I*(b*x^n+a)^2)*csgn(I*(b*x^n+a))^2+4*ln(b*x^n+a)))

maxima [A] time = 1.23, size = 59, normalized size = 0.57

$$\frac{\left(b^{2}\left(2\,p+1\right)x^{2\,n}+2\,abpx^{n}-a^{2}\right)\left(bx^{n}+a\right)^{2\,p}}{2\,\left(2\,p^{2}+3\,p+1\right)b^{2}n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(-1+2*n)*(a^2+2*a*b*x^n+b^2*x^(2*n))^p,x, algorithm="maxima")

[Out] 1/2*(b^2*(2*p + 1)*x^(2*n) + 2*a*b*p*x^n - a^2)*(b*x^n + a)^(2*p)/((2*p^2 + 3*p + 1)*b^2*n)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^{2\,n-1}\,\left(a^2+b^2\,x^{2\,n}+2\,a\,b\,x^n\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(2*n - 1)*(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^p,x)

[Out] int(x^(2*n - 1)*(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^p, x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**(-1+2*n)*(a**2+2*a*b*x**n+b**2*x**(2*n))**p,x)

[Out] Timed out

3.473 $\int \frac{x^{-1+4n}}{a+bx^n+cx^{2n}}\,dx$

Optimal. Leaf size=111

$$\frac{b\left(b^2-3ac\right)\tanh^{-1}\left(\frac{b+2cx^n}{\sqrt{b^2-4ac}}\right)}{c^3n\sqrt{b^2-4ac}}+\frac{\left(b^2-ac\right)\log\left(a+bx^n+cx^{2n}\right)}{2c^3n}-\frac{bx^n}{c^2n}+\frac{x^{2n}}{2cn}$$

Rubi [A] time = 0.12, antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1357, 701, 634, 618, 206, 628}

$$\frac{\left(b^2-ac\right)\log\left(a+bx^n+cx^{2n}\right)}{2c^3n}+\frac{b\left(b^2-3ac\right)\tanh^{-1}\left(\frac{b+2cx^n}{\sqrt{b^2-4ac}}\right)}{c^3n\sqrt{b^2-4ac}}-\frac{bx^n}{c^2n}+\frac{x^{2n}}{2cn}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 + 4*n)/(a + b*x^n + c*x^(2*n)),x]
```

```
[Out] -((b*x^n)/(c^2*n)) + x^(2*n)/(2*c*n) + (b*(b^2 - 3*a*c)*ArcTanh[(b + 2*c*x^
n)/Sqrt[b^2 - 4*a*c]])/(c^3*Sqrt[b^2 - 4*a*c]*n) + ((b^2 - a*c)*Log[a + b*x
^n + c*x^(2*n)])/(2*c^3*n)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 701

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Int[PolynomialDivide[(d + e*x)^m, a + b*x + c*x^2, x], x] /; FreeQ[{a,
 b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
 NeQ[2*c*d - b*e, 0] && IGtQ[m, 1] && (NeQ[d, 0] || GtQ[m, 2])
```

Rule 1357

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
```

```
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^{-1+4n}}{a+bx^n+cx^{2n}}\,dx = \frac{\text{Subst}\left(\int \frac{x^3}{a+bx+cx^2}\,dx, x, x^n\right)}{n}$$
$$= \frac{\text{Subst}\left(\int \left(-\frac{b}{c^2}+\frac{x}{c}+\frac{ab+\left(b^2-ac\right)x}{c^2\left(a+bx+cx^2\right)}\right)dx, x, x^n\right)}{n}$$
$$= -\frac{bx^n}{c^2n}+\frac{x^{2n}}{2cn}+\frac{\text{Subst}\left(\int \frac{ab+\left(b^2-ac\right)x}{a+bx+cx^2}\,dx, x, x^n\right)}{c^2n}$$
$$= -\frac{bx^n}{c^2n}+\frac{x^{2n}}{2cn}-\frac{\left(b\left(b^2-3ac\right)\right)\text{Subst}\left(\int \frac{1}{a+bx+cx^2}\,dx, x, x^n\right)}{2c^3n}+\frac{\left(b^2-ac\right)\text{Subst}\left(\int \frac{b+2cx}{a+bx+cx}\right.}{2c^3n}$$
$$= -\frac{bx^n}{c^2n}+\frac{x^{2n}}{2cn}+\frac{\left(b^2-ac\right)\log\left(a+bx^n+cx^{2n}\right)}{2c^3n}+\frac{\left(b\left(b^2-3ac\right)\right)\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx, x\right.}{c^3n}$$
$$= -\frac{bx^n}{c^2n}+\frac{x^{2n}}{2cn}+\frac{b\left(b^2-3ac\right)\tanh^{-1}\left(\frac{b+2cx^n}{\sqrt{b^2-4ac}}\right)}{c^3\sqrt{b^2-4ac}\,n}+\frac{\left(b^2-ac\right)\log\left(a+bx^n+cx^{2n}\right)}{2c^3n}$$

Mathematica [A] time = 0.19, size = 93, normalized size = 0.84

$$\frac{\left(b^2-ac\right)\log\left(a+x^n\left(b+cx^n\right)\right)+\frac{2b\left(b^2-3ac\right)\tanh^{-1}\left(\frac{b+2cx^n}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}}+cx^n\left(cx^n-2b\right)}{2c^3n}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 + 4*n)/(a + b*x^n + c*x^(2*n)),x]
```

```
[Out] (c*x^n*(-2*b + c*x^n) + (2*b*(b^2 - 3*a*c)*ArcTanh[(b + 2*c*x^n)/Sqrt[b^2 -
 4*a*c]])/Sqrt[b^2 - 4*a*c] + (b^2 - a*c)*Log[a + x^n*(b + c*x^n)])/(2*c^3*
n)
```

IntegrateAlgebraic [A] time = 0.25, size = 126, normalized size = 1.14

$$\frac{\left(b^2-ac\right)\log\left(a+bx^n+cx^{2n}\right)}{2c^3n}+\frac{\left(3abc-b^3\right)\tan^{-1}\left(\frac{2cx^n}{\sqrt{4ac-b^2}}+\frac{b}{\sqrt{4ac-b^2}}\right)}{c^3n\sqrt{4ac-b^2}}+\frac{x^n\left(cx^n-2b\right)}{2c^2n}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(-1 + 4*n)/(a + b*x^n + c*x^(2*n)),x]
```

```
[Out] (x^n*(-2*b + c*x^n))/(2*c^2*n) + ((-b^3 + 3*a*b*c)*ArcTan[b/Sqrt[-b^2 + 4*a
*c] + (2*c*x^n)/Sqrt[-b^2 + 4*a*c]])/(c^3*Sqrt[-b^2 + 4*a*c]*n) + ((b^2 - a
*c)*Log[a + b*x^n + c*x^(2*n)])/(2*c^3*n)
```

fricas [A] time = 1.18, size = 353, normalized size = 3.18

$$\left[-\frac{\left(b^3-3abc\right)\sqrt{b^2-4ac}\log\left(\frac{2c^2x^{2n}+b^2-2ac+2\left(bc-\sqrt{b^2-4ac}\,c\right)x^n-\sqrt{b^2-4ac}\,b}{cx^{2n}+bx^n+a}\right)-\left(b^2c^2-4ac^3\right)x^{2n}+2\left(b^3c-4abc^2\right)x^n-\left(b^4-5ab^2c+4a^2c^2\right)\log\left(cx^{2n}+bx^n+a\right)}{2\left(b^2c^3-4ac^4\right)n}, \frac{2\left(b^3-3abc\right)\sqrt{-b^2+4ac}\arctan\left(-\frac{2\sqrt{-b^2+4ac}\,cx^n+\sqrt{-b^2+4ac}\,b}{b^2-4ac}\right)+\left(b^2c^2-4ac^3\right)x^{2n}-2\left(b^3c-4abc^2\right)x^n+\left(b^4-5ab^2c+4a^2c^2\right)\log\left(cx^{2n}+bx^n+a\right)}{2\left(b^2c^3-4ac^4\right)n}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+4*n)/(a+b*x^n+c*x^(2*n)),x, algorithm="fricas")
```

```
[Out] [-1/2*((b^3 - 3*a*b*c)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^(2*n) + b^2 - 2*a*c +
 2*(b*c - sqrt(b^2 - 4*a*c)*c)*x^n - sqrt(b^2 - 4*a*c)*b)/(c*x^(2*n) + b*x^
n + a)) - (b^2*c^2 - 4*a*c^3)*x^(2*n) + 2*(b^3*c - 4*a*b*c^2)*x^n - (b^4 -
5*a*b^2*c + 4*a^2*c^2)*log(c*x^(2*n) + b*x^n + a))/((b^2*c^3 - 4*a*c^4)*n),
 1/2*(2*(b^3 - 3*a*b*c)*sqrt(-b^2 + 4*a*c)*arctan(-(2*sqrt(-b^2 + 4*a*c)*c*
x^n + sqrt(-b^2 + 4*a*c)*b)/(b^2 - 4*a*c)) + (b^2*c^2 - 4*a*c^3)*x^(2*n) -
2*(b^3*c - 4*a*b*c^2)*x^n + (b^4 - 5*a*b^2*c + 4*a^2*c^2)*log(c*x^(2*n) + b
*x^n + a))/((b^2*c^3 - 4*a*c^4)*n)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{4n-1}}{cx^{2n} + bx^n + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+4*n)/(a+b*x^n+c*x^(2*n)),x, algorithm="giac")
```

```
[Out] integrate(x^(4*n - 1)/(c*x^(2*n) + b*x^n + a), x)
```

maple [B] time = 0.16, size = 973, normalized size = 8.77

$$\frac{4a^2c^2n^2\ln(x)}{4ac^4n^2-b^2c^3n^2}-\frac{5ab^2cn^2\ln(x)}{4ac^4n^2-b^2c^3n^2}+\frac{b^4n^2\ln(x)}{4ac^4n^2-b^2c^3n^2}-\frac{2a^2\ln\left(x^n-\frac{-3ab^2c+b^4+\sqrt{-36a^3b^2c^3+33a^2b^4c^2-10ab^6c+b^8}}{2(3ac-b^2)bc}\right)}{(4ac-b^2)cn}-\frac{2a^2\ln\left(x^n+\frac{3ab^2c-b^4+\sqrt{-36a^3b^2c^3+33a^2b^4c^2-10ab^6c+b^8}}{2(3ac-b^2)bc}\right)}{(4ac-b^2)cn}+\frac{5ab^2\ln\left(x^n-\frac{-3ab^2c+b^4+\sqrt{-36a^3b^2c^3+33a^2b^4c^2-10ab^6c+b^8}}{2(3ac-b^2)bc}\right)}{2(4ac-b^2)c^2n}+\frac{5ab^2\ln\left(x^n+\frac{3ab^2c-b^4+\sqrt{-36a^3b^2c^3+33a^2b^4c^2-10ab^6c+b^8}}{2(3ac-b^2)bc}\right)}{2(4ac-b^2)c^2n}-\frac{b^4\ln\left(x^n-\frac{-3ab^2c+b^4+\sqrt{-36a^3b^2c^3+33a^2b^4c^2-10ab^6c+b^8}}{2(3ac-b^2)bc}\right)}{2(4ac-b^2)c^3n}-\frac{b^4\ln\left(x^n+\frac{3ab^2c-b^4+\sqrt{-36a^3b^2c^3+33a^2b^4c^2-10ab^6c+b^8}}{2(3ac-b^2)bc}\right)}{2(4ac-b^2)c^3n}-\frac{a\ln(x)}{c^2}+\frac{b^2\ln(x)}{c^3}-\frac{bx^n}{c^2n}+\frac{x^{2n}}{2cn}-\frac{\sqrt{-36a^3b^2c^3+33a^2b^4c^2-10ab^6c+b^8}\,\ln\left(x^n-\frac{-3ab^2c+b^4+\sqrt{-36a^3b^2c^3+33a^2b^4c^2-10ab^6c+b^8}}{2(3ac-b^2)bc}\right)}{2(4ac-b^2)c^3n}+\frac{\sqrt{-36a^3b^2c^3+33a^2b^4c^2-10ab^6c+b^8}\,\ln\left(x^n+\frac{3ab^2c-b^4+\sqrt{-36a^3b^2c^3+33a^2b^4c^2-10ab^6c+b^8}}{2(3ac-b^2)bc}\right)}{2(4ac-b^2)c^3n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1+4*n)/(a+b*x^n+c*x^(2*n)),x)
```

```
[Out] -1/c^2*ln(x)*a+1/c^3*ln(x)*b^2+1/2/c/n*(x^n)^2-b*x^n/c^2/n+4/(4*a*c^4*n^2-b
^2*c^3*n^2)*n^2*ln(x)*a^2*c^2-5/(4*a*c^4*n^2-b^2*c^3*n^2)*n^2*ln(x)*a*b^2*c
+1/(4*a*c^4*n^2-b^2*c^3*n^2)*n^2*ln(x)*b^4-2/c/(4*a*c-b^2)/n*ln(x^n+1/2*(3*
a*b^2*c-b^4+(-36*a^3*b^2*c^3+33*a^2*b^4*c^2-10*a*b^6*c+b^8)^(1/2))/c/b/(3*a
*c-b^2))*a^2+5/2/c^2/(4*a*c-b^2)/n*ln(x^n+1/2*(3*a*b^2*c-b^4+(-36*a^3*b^2*c
^3+33*a^2*b^4*c^2-10*a*b^6*c+b^8)^(1/2))/c/b/(3*a*c-b^2))*a*b^2-1/2/c^3/(4*
a*c-b^2)/n*ln(x^n+1/2*(3*a*b^2*c-b^4+(-36*a^3*b^2*c^3+33*a^2*b^4*c^2-10*a*b
^6*c+b^8)^(1/2))/c/b/(3*a*c-b^2))*b^4+1/2/c^3/(4*a*c-b^2)/n*ln(x^n+1/2*(3*a
*b^2*c-b^4+(-36*a^3*b^2*c^3+33*a^2*b^4*c^2-10*a*b^6*c+b^8)^(1/2))/c/b/(3*a*
c-b^2))*(-36*a^3*b^2*c^3+33*a^2*b^4*c^2-10*a*b^6*c+b^8)^(1/2)-2/c/(4*a*c-b^
2)/n*ln(x^n-1/2*(-3*a*b^2*c+b^4+(-36*a^3*b^2*c^3+33*a^2*b^4*c^2-10*a*b^6*c+
b^8)^(1/2))/c/b/(3*a*c-b^2))*a^2+5/2/c^2/(4*a*c-b^2)/n*ln(x^n-1/2*(-3*a*b^2
*c+b^4+(-36*a^3*b^2*c^3+33*a^2*b^4*c^2-10*a*b^6*c+b^8)^(1/2))/c/b/(3*a*c-b^
2))*a*b^2-1/2/c^3/(4*a*c-b^2)/n*ln(x^n-1/2*(-3*a*b^2*c+b^4+(-36*a^3*b^2*c^3
+33*a^2*b^4*c^2-10*a*b^6*c+b^8)^(1/2))/c/b/(3*a*c-b^2))*b^4-1/2/c^3/(4*a*c-
b^2)/n*ln(x^n-1/2*(-3*a*b^2*c+b^4+(-36*a^3*b^2*c^3+33*a^2*b^4*c^2-10*a*b^6*
c+b^8)^(1/2))/c/b/(3*a*c-b^2))*(-36*a^3*b^2*c^3+33*a^2*b^4*c^2-10*a*b^6*c+b
^8)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\left(b^2 - ac\right)\log(x)}{c^3} + \frac{cx^{2n} - 2\,bx^n}{2\,c^2n} + \int -\frac{ab^2 - a^2c + \left(b^3 - 2\,abc\right)x^n}{c^4xx^{2n} + bc^3xx^n + ac^3x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+4*n)/(a+b*x^n+c*x^(2*n)),x, algorithm="maxima")
```

```
[Out] (b^2 - a*c)*log(x)/c^3 + 1/2*(c*x^(2*n) - 2*b*x^n)/(c^2*n) + integrate(-(a*
b^2 - a^2*c + (b^3 - 2*a*b*c)*x^n)/(c^4*x*x^(2*n) + b*c^3*x*x^n + a*c^3*x),
 x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{4\,n-1}}{a+b\,x^n+c\,x^{2\,n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(4*n - 1)/(a + b*x^n + c*x^(2*n)),x)
```

```
[Out] int(x^(4*n - 1)/(a + b*x^n + c*x^(2*n)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+4*n)/(a+b*x**n+c*x**(2*n)),x)
```

```
[Out] Timed out
```

3.474 $\int \frac{x^{-1+3n}}{a+bx^n+cx^{2n}}\,dx$

Optimal. Leaf size=87

$$-\frac{(b^2-2ac)\tanh^{-1}\left(\frac{b+2cx^n}{\sqrt{b^2-4ac}}\right)}{c^2n\sqrt{b^2-4ac}}-\frac{b\log\left(a+bx^n+cx^{2n}\right)}{2c^2n}+\frac{x^n}{cn}$$

Rubi [A] time = 0.07, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1357, 703, 634, 618, 206, 628}

$$-\frac{(b^2-2ac)\tanh^{-1}\left(\frac{b+2cx^n}{\sqrt{b^2-4ac}}\right)}{c^2n\sqrt{b^2-4ac}}-\frac{b\log\left(a+bx^n+cx^{2n}\right)}{2c^2n}+\frac{x^n}{cn}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 + 3*n)/(a + b*x^n + c*x^(2*n)),x]
```

```
[Out] x^n/(c*n) - ((b^2 - 2*a*c)*ArcTanh[(b + 2*c*x^n)/Sqrt[b^2 - 4*a*c]])/(c^2*S
qrt[b^2 - 4*a*c]*n) - (b*Log[a + b*x^n + c*x^(2*n)])/(2*c^2*n)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 703

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Simp[(e*(d + e*x)^(m - 1))/(c*(m - 1)), x] + Dist[1/c, Int[((d + e*x)^
(m - 2)*Simp[c*d^2 - a*e^2 + e*(2*c*d - b*e)*x, x])/(a + b*x + c*x^2), x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && GtQ[m, 1]
```

Rule 1357

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
```

```
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^{-1+3n}}{a+bx^n+cx^{2n}}\,dx &= \frac{\text{Subst}\left(\int \frac{x^2}{a+bx+cx^2}\,dx,x,x^n\right)}{n}\\
&= \frac{x^n}{cn}+\frac{\text{Subst}\left(\int \frac{-a-bx}{a+bx+cx^2}\,dx,x,x^n\right)}{cn}\\
&= \frac{x^n}{cn}-\frac{b\,\text{Subst}\left(\int \frac{b+2cx}{a+bx+cx^2}\,dx,x,x^n\right)}{2c^2n}+\frac{\left(b^2-2ac\right)\text{Subst}\left(\int \frac{1}{a+bx+cx^2}\,dx,x,x^n\right)}{2c^2n}\\
&= \frac{x^n}{cn}-\frac{b\log\left(a+bx^n+cx^{2n}\right)}{2c^2n}-\frac{\left(b^2-2ac\right)\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx,x,b+2cx^n\right)}{c^2n}\\
&= \frac{x^n}{cn}-\frac{\left(b^2-2ac\right)\tanh^{-1}\left(\frac{b+2cx^n}{\sqrt{b^2-4ac}}\right)}{c^2\sqrt{b^2-4ac}\,n}-\frac{b\log\left(a+bx^n+cx^{2n}\right)}{2c^2n}
\end{aligned}
$$

Mathematica [A] time = 0.13, size = 80, normalized size = 0.92

$$
\frac{-\frac{\left(b^2-2ac\right)\tanh^{-1}\left(\frac{b+2cx^n}{\sqrt{b^2-4ac}}\right)}{c\sqrt{b^2-4ac}}-\frac{b\log(a+x^n(b+cx^n))}{2c}+x^n}{cn}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 + 3*n)/(a + b*x^n + c*x^(2*n)),x]
```

```
[Out] (x^n - ((b^2 - 2*a*c)*ArcTanh[(b + 2*c*x^n)/Sqrt[b^2 - 4*a*c]])/(c*Sqrt[b^2
 - 4*a*c]) - (b*Log[a + x^n*(b + c*x^n)])/(2*c))/(c*n)
```

IntegrateAlgebraic [A] time = 0.16, size = 104, normalized size = 1.20

$$
\frac{\left(b^2-2ac\right)\tan^{-1}\left(\frac{2cx^n}{\sqrt{4ac-b^2}}+\frac{b}{\sqrt{4ac-b^2}}\right)}{c^2n\sqrt{4ac-b^2}}-\frac{b\log\left(a+bx^n+cx^{2n}\right)}{2c^2n}+\frac{x^n}{cn}
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(-1 + 3*n)/(a + b*x^n + c*x^(2*n)),x]
```

```
[Out] x^n/(c*n) + ((b^2 - 2*a*c)*ArcTan[b/Sqrt[-b^2 + 4*a*c] + (2*c*x^n)/Sqrt[-b^
2 + 4*a*c]])/(c^2*Sqrt[-b^2 + 4*a*c]*n) - (b*Log[a + b*x^n + c*x^(2*n)])/(2
*c^2*n)
```

fricas [A] time = 1.08, size = 285, normalized size = 3.28

$$
\left[-\frac{\left(b^2-2\,ac\right)\sqrt{b^2-4\,ac}\,\log\left(\frac{2\,c^2x^{2\,n}+b^2-2\,ac+2\left(bc+\sqrt{b^2-4\,ac}\,c\right)x^n+\sqrt{b^2-4\,ac}\,b}{cx^{2\,n}+bx^n+a}\right)-2\left(b^2c-4\,ac^2\right)x^n+\left(b^3-4\,abc\right)\log\left(cx^{2\,n}+bx^n+a\right)}{2\left(b^2c^2-4\,ac^3\right)n},\ -\frac{2\left(b^2-2\,ac\right)\sqrt{-b^2+4\,ac}\,\arctan\left(-\frac{2\sqrt{-b^2+4\,ac}\,cx^n+\sqrt{-b^2+4\,ac}\,b}{b^2-4\,ac}\right)-2\left(b^2c-4\,ac^2\right)x^n+\left(b^3-4\,abc\right)\log\left(cx^{2\,n}+bx^n+a\right)}{2\left(b^2c^2-4\,ac^3\right)n}\right]
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+3*n)/(a+b*x^n+c*x^(2*n)),x, algorithm="fricas")
```

```
[Out] [-1/2*((b^2 - 2*a*c)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^(2*n) + b^2 - 2*a*c + 2
*(b*c + sqrt(b^2 - 4*a*c)*c)*x^n + sqrt(b^2 - 4*a*c)*b)/(c*x^(2*n) + b*x^n
```

```
+ a)) - 2*(b^2*c - 4*a*c^2)*x^n + (b^3 - 4*a*b*c)*log(c*x^(2*n) + b*x^n + a
))/((b^2*c^2 - 4*a*c^3)*n), -1/2*(2*(b^2 - 2*a*c)*sqrt(-b^2 + 4*a*c)*arctan
(-(2*sqrt(-b^2 + 4*a*c)*c*x^n + sqrt(-b^2 + 4*a*c)*b)/(b^2 - 4*a*c)) - 2*(b
^2*c - 4*a*c^2)*x^n + (b^3 - 4*a*b*c)*log(c*x^(2*n) + b*x^n + a))/((b^2*c^2
 - 4*a*c^3)*n)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{3\,n-1}}{cx^{2\,n} + bx^{n} + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+3*n)/(a+b*x^n+c*x^(2*n)),x, algorithm="giac")
```

```
[Out] integrate(x^(3*n - 1)/(c*x^(2*n) + b*x^n + a), x)
```

maple [B] time = 0.13, size = 664, normalized size = 7.63

$$\frac{4abc\,n^{2}\ln(x)}{4a\,c^{3}n^{2}-b^{2}c^{2}n^{2}}-\frac{b^{3}n^{2}\ln(x)}{4a\,c^{3}n^{2}-b^{2}c^{2}n^{2}}-\frac{2ab\ln\left(x^{n}-\frac{-2abc+b^{3}+\sqrt{-16a^{3}c^{3}+20b^{2}a^{2}c^{2}-8b^{4}ac+b^{6}}}{2\left(2ac-b^{2}\right)c}\right)}{\left(4ac-b^{2}\right)cn}-\frac{2ab\ln\left(x^{n}+\frac{2abc-b^{3}+\sqrt{-16a^{3}c^{3}+20b^{2}a^{2}c^{2}-8b^{4}ac+b^{6}}}{2\left(2ac-b^{2}\right)c}\right)}{\left(4ac-b^{2}\right)cn}+\frac{b^{3}\ln\left(x^{n}-\frac{-2abc+b^{3}+\sqrt{-16a^{3}c^{3}+20b^{2}a^{2}c^{2}-8b^{4}ac+b^{6}}}{2\left(2ac-b^{2}\right)c}\right)}{2\left(4ac-b^{2}\right)c^{2}n}+\frac{b^{3}\ln\left(x^{n}+\frac{2abc-b^{3}+\sqrt{-16a^{3}c^{3}+20b^{2}a^{2}c^{2}-8b^{4}ac+b^{6}}}{2\left(2ac-b^{2}\right)c}\right)}{2\left(4ac-b^{2}\right)c^{2}n}-\frac{b\ln(x)}{c^{2}}+\frac{x^{n}}{cn}+\frac{\sqrt{-16a^{3}c^{3}+20b^{2}a^{2}c^{2}-8b^{4}ac+b^{6}}\,\ln\left(x^{n}-\frac{-2abc+b^{3}+\sqrt{-16a^{3}c^{3}+20b^{2}a^{2}c^{2}-8b^{4}ac+b^{6}}}{2\left(2ac-b^{2}\right)c}\right)}{2\left(4ac-b^{2}\right)c^{2}n}-\frac{\sqrt{-16a^{3}c^{3}+20b^{2}a^{2}c^{2}-8b^{4}ac+b^{6}}\,\ln\left(x^{n}+\frac{2abc-b^{3}+\sqrt{-16a^{3}c^{3}+20b^{2}a^{2}c^{2}-8b^{4}ac+b^{6}}}{2\left(2ac-b^{2}\right)c}\right)}{2\left(4ac-b^{2}\right)c^{2}n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3*n-1)/(a+b*x^n+c*x^(2*n)),x)
```

```
[Out] -b/c^2*ln(x)+x^n/c/n+4/(4*a*c^3*n^2-b^2*c^2*n^2)*n^2*ln(x)*a*b*c-1/(4*a*c^3
*n^2-b^2*c^2*n^2)*n^2*ln(x)*b^3-2/c/(4*a*c-b^2)/n*ln(x^n-1/2*(-2*a*b*c+b^3+
(-16*a^3*c^3+20*a^2*b^2*c^2-8*a*b^4*c+b^6)^(1/2))/c/(2*a*c-b^2))*a*b+1/2/c^
2/(4*a*c-b^2)/n*ln(x^n-1/2*(-2*a*b*c+b^3+(-16*a^3*c^3+20*a^2*b^2*c^2-8*a*b^
4*c+b^6)^(1/2))/c/(2*a*c-b^2))*b^3+1/2/c^2/(4*a*c-b^2)/n*ln(x^n-1/2*(-2*a*b
*c+b^3+(-16*a^3*c^3+20*a^2*b^2*c^2-8*a*b^4*c+b^6)^(1/2))/c/(2*a*c-b^2))*(-1
6*a^3*c^3+20*a^2*b^2*c^2-8*a*b^4*c+b^6)^(1/2)-2/c/(4*a*c-b^2)/n*ln(x^n+1/2*
(2*a*b*c-b^3+(-16*a^3*c^3+20*a^2*b^2*c^2-8*a*b^4*c+b^6)^(1/2))/c/(2*a*c-b^2
))*a*b+1/2/c^2/(4*a*c-b^2)/n*ln(x^n+1/2*(2*a*b*c-b^3+(-16*a^3*c^3+20*a^2*b^
2*c^2-8*a*b^4*c+b^6)^(1/2))/c/(2*a*c-b^2))*b^3-1/2/c^2/(4*a*c-b^2)/n*ln(x^n
+1/2*(2*a*b*c-b^3+(-16*a^3*c^3+20*a^2*b^2*c^2-8*a*b^4*c+b^6)^(1/2))/c/(2*a*
c-b^2))*(-16*a^3*c^3+20*a^2*b^2*c^2-8*a*b^4*c+b^6)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{b\log(x)}{c^{2}} + \frac{x^{n}}{cn} - \int -\frac{ab + \left(b^{2} - ac\right)x^{n}}{c^{3}xx^{2\,n} + bc^{2}xx^{n} + ac^{2}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+3*n)/(a+b*x^n+c*x^(2*n)),x, algorithm="maxima")
```

```
[Out] -b*log(x)/c^2 + x^n/(c*n) - integrate(-(a*b + (b^2 - a*c)*x^n)/(c^3*x*x^(2*
n) + b*c^2*x*x^n + a*c^2*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{3\,n-1}}{a+b\,x^{n}+c\,x^{2\,n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3*n - 1)/(a + b*x^n + c*x^(2*n)),x)
```

```
[Out] int(x^(3*n - 1)/(a + b*x^n + c*x^(2*n)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+3*n)/(a+b*x**n+c*x**(2*n)),x)
```

```
[Out] Timed out
```

3.475 $\int \frac{x^{-1+2n}}{a+bx^n+cx^{2n}}\,dx$

Optimal. Leaf size=68

$$\frac{b\tanh^{-1}\left(\frac{b+2cx^n}{\sqrt{b^2-4ac}}\right)}{cn\sqrt{b^2-4ac}}+\frac{\log\left(a+bx^n+cx^{2n}\right)}{2cn}$$

Rubi [A] time = 0.05, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {1357, 634, 618, 206, 628}

$$\frac{b\tanh^{-1}\left(\frac{b+2cx^n}{\sqrt{b^2-4ac}}\right)}{cn\sqrt{b^2-4ac}}+\frac{\log\left(a+bx^n+cx^{2n}\right)}{2cn}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 + 2*n)/(a + b*x^n + c*x^(2*n)),x]
```

```
[Out] (b*ArcTanh[(b + 2*c*x^n)/Sqrt[b^2 - 4*a*c]])/(c*Sqrt[b^2 - 4*a*c]*n) + Log[
a + b*x^n + c*x^(2*n)]/(2*c*n)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1357

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^{-1+2n}}{a+bx^n+cx^{2n}}\,dx = \frac{\text{Subst}\left(\int \frac{x}{a+bx+cx^2}\,dx,x,x^n\right)}{n}$$
$$= \frac{\text{Subst}\left(\int \frac{b+2cx}{a+bx+cx^2}\,dx,x,x^n\right)}{2cn} - \frac{b\,\text{Subst}\left(\int \frac{1}{a+bx+cx^2}\,dx,x,x^n\right)}{2cn}$$
$$= \frac{\log\left(a+bx^n+cx^{2n}\right)}{2cn} + \frac{b\,\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx,x,b+2cx^n\right)}{cn}$$
$$= \frac{b\tanh^{-1}\left(\frac{b+2cx^n}{\sqrt{b^2-4ac}}\right)}{c\sqrt{b^2-4ac}\,n} + \frac{\log\left(a+bx^n+cx^{2n}\right)}{2cn}$$

Mathematica [A] time = 0.07, size = 62, normalized size = 0.91

$$\frac{\frac{2b\tanh^{-1}\left(\frac{b+2cx^n}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}} + \log\left(a+x^n\left(b+cx^n\right)\right)}{2cn}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 + 2*n)/(a + b*x^n + c*x^(2*n)),x]
```

```
[Out] ((2*b*ArcTanh[(b + 2*c*x^n)/Sqrt[b^2 - 4*a*c]])/Sqrt[b^2 - 4*a*c] + Log[a +
 x^n*(b + c*x^n)])/(2*c*n)
```

IntegrateAlgebraic [A] time = 0.12, size = 87, normalized size = 1.28

$$\frac{\log\left(a+bx^n+cx^{2n}\right)}{2cn} - \frac{b\tan^{-1}\left(\frac{2cx^n}{\sqrt{4ac-b^2}} + \frac{b}{\sqrt{4ac-b^2}}\right)}{cn\sqrt{4ac-b^2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(-1 + 2*n)/(a + b*x^n + c*x^(2*n)),x]
```

```
[Out] -((b*ArcTan[b/Sqrt[-b^2 + 4*a*c] + (2*c*x^n)/Sqrt[-b^2 + 4*a*c]])/(c*Sqrt[-
b^2 + 4*a*c]*n)) + Log[a + b*x^n + c*x^(2*n)]/(2*c*n)
```

fricas [A] time = 1.58, size = 231, normalized size = 3.40

$$\left[\frac{\sqrt{b^2-4\,ac}\,b\log\left(\frac{2\,c^2x^{2\,n}+b^2-2\,ac+2\,\left(bc+\sqrt{b^2-4\,ac}\,c\right)x^n+\sqrt{b^2-4\,ac}\,b}{cx^{2\,n}+bx^n+a}\right) + \left(b^2-4\,ac\right)\log\left(cx^{2\,n}+bx^n+a\right)}{2\,\left(b^2c-4\,ac^2\right)n}, \frac{2\,\sqrt{-b^2+4\,ac}\,b\arctan\left(-\frac{2\,\sqrt{-b^2+4\,ac}\,cx^n+\sqrt{-b^2+4\,ac}\,b}{b^2-4\,ac}\right) + \left(b^2-4\,ac\right)\log\left(cx^{2\,n}+bx^n+a\right)}{2\,\left(b^2c-4\,ac^2\right)n}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+2*n)/(a+b*x^n+c*x^(2*n)),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(b^2 - 4*a*c)*b*log((2*c^2*x^(2*n) + b^2 - 2*a*c + 2*(b*c + sqrt(
b^2 - 4*a*c)*c)*x^n + sqrt(b^2 - 4*a*c)*b)/(c*x^(2*n) + b*x^n + a)) + (b^2
- 4*a*c)*log(c*x^(2*n) + b*x^n + a))/((b^2*c - 4*a*c^2)*n), 1/2*(2*sqrt(-b^
2 + 4*a*c)*b*arctan(-(2*sqrt(-b^2 + 4*a*c)*c*x^n + sqrt(-b^2 + 4*a*c)*b)/(b
^2 - 4*a*c)) + (b^2 - 4*a*c)*log(c*x^(2*n) + b*x^n + a))/((b^2*c - 4*a*c^2)
*n)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{2\,n-1}}{cx^{2\,n}+bx^n+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+2*n)/(a+b*x^n+c*x^(2*n)),x, algorithm="giac")
```

```
[Out] integrate(x^(2*n - 1)/(c*x^(2*n) + b*x^n + a), x)
```

maple [B] time = 0.10, size = 402, normalized size = 5.91

$$-\frac{4ac\,n^2\ln(x)}{4a\,c^2n^2-b^2c\,n^2}+\frac{b^2n^2\ln(x)}{4a\,c^2n^2-b^2c\,n^2}+\frac{2a\ln\left(x^n-\frac{-b^2+\sqrt{-4a\,b^2c+b^4}}{2bc}\right)}{\left(4ac-b^2\right)n}+\frac{2a\ln\left(x^n+\frac{b^2+\sqrt{-4a\,b^2c+b^4}}{2bc}\right)}{\left(4ac-b^2\right)n}-\frac{b^2\ln\left(x^n-\frac{-b^2+\sqrt{-4a\,b^2c+b^4}}{2bc}\right)}{2\left(4ac-b^2\right)cn}-\frac{b^2\ln\left(x^n+\frac{b^2+\sqrt{-4a\,b^2c+b^4}}{2bc}\right)}{2\left(4ac-b^2\right)cn}+\frac{\ln(x)}{c}+\frac{\sqrt{-4a\,b^2c+b^4}\,\ln\left(x^n-\frac{-b^2+\sqrt{-4a\,b^2c+b^4}}{2bc}\right)}{2\left(4ac-b^2\right)cn}-\frac{\sqrt{-4a\,b^2c+b^4}\,\ln\left(x^n+\frac{b^2+\sqrt{-4a\,b^2c+b^4}}{2bc}\right)}{2\left(4ac-b^2\right)cn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(2*n-1)/(a+b*x^n+c*x^(2*n)),x)
```

```
[Out] 1/c*ln(x)-4/(4*a*c^2*n^2-b^2*c*n^2)*n^2*ln(x)*a*c+1/(4*a*c^2*n^2-b^2*c*n^2)
*n^2*ln(x)*b^2+2/(4*a*c-b^2)/n*ln(x^n-1/2*(-b^2+(-4*a*b^2*c+b^4)^(1/2))/b/c
)*a-1/2/(4*a*c-b^2)/c/n*ln(x^n-1/2*(-b^2+(-4*a*b^2*c+b^4)^(1/2))/b/c)*b^2+1
/2/(4*a*c-b^2)/c/n*ln(x^n-1/2*(-b^2+(-4*a*b^2*c+b^4)^(1/2))/b/c)*(-4*a*b^2*
c+b^4)^(1/2)+2/(4*a*c-b^2)/n*ln(x^n+1/2*(b^2+(-4*a*b^2*c+b^4)^(1/2))/b/c)*a
-1/2/(4*a*c-b^2)/c/n*ln(x^n+1/2*(b^2+(-4*a*b^2*c+b^4)^(1/2))/b/c)*b^2-1/2/(
4*a*c-b^2)/c/n*ln(x^n+1/2*(b^2+(-4*a*b^2*c+b^4)^(1/2))/b/c)*(-4*a*b^2*c+b^4
)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\log(x)}{c}-\int\frac{bx^n+a}{c^2xx^{2\,n}+bcxx^n+acx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+2*n)/(a+b*x^n+c*x^(2*n)),x, algorithm="maxima")
```

```
[Out] log(x)/c - integrate((b*x^n + a)/(c^2*x*x^(2*n) + b*c*x*x^n + a*c*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{x^{2\,n-1}}{a+b\,x^n+c\,x^{2\,n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(2*n - 1)/(a + b*x^n + c*x^(2*n)),x)
```

```
[Out] int(x^(2*n - 1)/(a + b*x^n + c*x^(2*n)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+2*n)/(a+b*x**n+c*x**(2*n)),x)
```

```
[Out] Timed out
```

3.476 $\int \frac{x^{-1+n}}{a+bx^n+cx^{2n}} dx$

Optimal. Leaf size=39

$$-\frac{2\tanh^{-1}\left(\frac{b+2cx^n}{\sqrt{b^2-4ac}}\right)}{n\sqrt{b^2-4ac}}$$

Rubi [A] time = 0.03, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {1352, 618, 206}

$$-\frac{2\tanh^{-1}\left(\frac{b+2cx^n}{\sqrt{b^2-4ac}}\right)}{n\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 + n)/(a + b*x^n + c*x^(2*n)),x]
```

```
[Out] (-2*ArcTanh[(b + 2*c*x^n)/Sqrt[b^2 - 4*a*c]])/(Sqrt[b^2 - 4*a*c]*n)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1352

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a,
b, c, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

Rubi steps

$$\begin{aligned} \int \frac{x^{-1+n}}{a+bx^n+cx^{2n}} dx &= \frac{\text{Subst}\left(\int \frac{1}{a+bx+cx^2} dx, x, x^n\right)}{n} \\ &= -\frac{2\,\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2} dx, x, b+2cx^n\right)}{n} \\ &= -\frac{2\tanh^{-1}\left(\frac{b+2cx^n}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}\,n} \end{aligned}$$

Mathematica [A] time = 0.06, size = 39, normalized size = 1.00

$$-\frac{2\tanh^{-1}\left(\frac{b+2cx^n}{\sqrt{b^2-4ac}}\right)}{n\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 + n)/(a + b*x^n + c*x^(2*n)),x]
```

```
[Out] (-2*ArcTanh[(b + 2*c*x^n)/Sqrt[b^2 - 4*a*c]])/(Sqrt[b^2 - 4*a*c]*n)
```

IntegrateAlgebraic [A] time = 0.08, size = 57, normalized size = 1.46

$$\frac{2\tan^{-1}\left(\frac{2cx^n}{\sqrt{4ac-b^2}}+\frac{b}{\sqrt{4ac-b^2}}\right)}{n\sqrt{4ac-b^2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(-1 + n)/(a + b*x^n + c*x^(2*n)),x]
```

```
[Out] (2*ArcTan[b/Sqrt[-b^2 + 4*a*c] + (2*c*x^n)/Sqrt[-b^2 + 4*a*c]])/(Sqrt[-b^2
+ 4*a*c]*n)
```

fricas [B] time = 1.22, size = 159, normalized size = 4.08

$$\left[\frac{\log\left(\frac{2\,c^2x^{2\,n}+b^2-2\,ac+2\left(bc-\sqrt{b^2-4\,ac}\,c\right)x^n-\sqrt{b^2-4\,ac}\,b}{cx^{2\,n}+bx^n+a}\right)}{\sqrt{b^2-4\,ac}\,n}, -\frac{2\,\sqrt{-b^2+4\,ac}\,\arctan\left(-\frac{2\,\sqrt{-b^2+4\,ac}\,cx^n+\sqrt{-b^2+4\,ac}\,b}{b^2-4\,ac}\right)}{\left(b^2-4\,ac\right)n}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n)/(a+b*x^n+c*x^(2*n)),x, algorithm="fricas")
```

```
[Out] [log((2*c^2*x^(2*n) + b^2 - 2*a*c + 2*(b*c - sqrt(b^2 - 4*a*c)*c)*x^n - sqr
t(b^2 - 4*a*c)*b)/(c*x^(2*n) + b*x^n + a))/(sqrt(b^2 - 4*a*c)*n), -2*sqrt(-
b^2 + 4*a*c)*arctan(-(2*sqrt(-b^2 + 4*a*c)*c*x^n + sqrt(-b^2 + 4*a*c)*b)/(b
^2 - 4*a*c))/((b^2 - 4*a*c)*n)]
```

giac [A] time = 0.41, size = 39, normalized size = 1.00

$$\frac{2\,\arctan\left(\frac{2\,cx^n+b}{\sqrt{-b^2+4\,ac}}\right)}{\sqrt{-b^2+4\,ac}\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n)/(a+b*x^n+c*x^(2*n)),x, algorithm="giac")
```

```
[Out] 2*arctan((2*c*x^n + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*n)
```

maple [B] time = 0.06, size = 113, normalized size = 2.90

$$-\frac{\ln\left(x^n+\frac{-4ac+b^2+\sqrt{-4ac+b^2}\,b}{2\sqrt{-4ac+b^2}\,c}\right)}{\sqrt{-4ac+b^2}\,n}+\frac{\ln\left(x^n+\frac{4ac-b^2+\sqrt{-4ac+b^2}\,b}{2\sqrt{-4ac+b^2}\,c}\right)}{\sqrt{-4ac+b^2}\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(n-1)/(a+b*x^n+c*x^(2*n)),x)
```

```
[Out] -1/(-4*a*c+b^2)^(1/2)/n*ln(x^n+1/2*((-4*a*c+b^2)^(1/2)*b-4*a*c+b^2)/c/(-4*a
*c+b^2)^(1/2))+1/(-4*a*c+b^2)^(1/2)/n*ln(x^n+1/2*((-4*a*c+b^2)^(1/2)*b+4*a*
c-b^2)/c/(-4*a*c+b^2)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{n-1}}{cx^{2\,n}+bx^n+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n)/(a+b*x^n+c*x^(2*n)),x, algorithm="maxima")
```

```
[Out] integrate(x^(n - 1)/(c*x^(2*n) + b*x^n + a), x)
```

mupad [B] time = 1.47, size = 39, normalized size = 1.00

$$\frac{2\,\mathrm{atan}\left(\frac{b+2\,c\,x^n}{\sqrt{4\,a\,c-b^2}}\right)}{n\,\sqrt{4\,a\,c-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(n - 1)/(a + b*x^n + c*x^(2*n)),x)
```

```
[Out] (2*atan((b + 2*c*x^n)/(4*a*c - b^2)^(1/2)))/(n*(4*a*c - b^2)^(1/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+n)/(a+b*x**n+c*x**(2*n)),x)
```

```
[Out] Timed out
```

3.477 $\int \frac{x^{-1-n}}{a+bx^n+cx^{2n}}\,dx$

Optimal. Leaf size=98

$$-\frac{(b^2-2ac)\tanh^{-1}\left(\frac{b+2cx^n}{\sqrt{b^2-4ac}}\right)}{a^2n\sqrt{b^2-4ac}}+\frac{b\log\left(a+bx^n+cx^{2n}\right)}{2a^2n}-\frac{b\log(x)}{a^2}-\frac{x^{-n}}{an}$$

Rubi [A] time = 0.13, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {1357, 709, 800, 634, 618, 206, 628}

$$-\frac{(b^2-2ac)\tanh^{-1}\left(\frac{b+2cx^n}{\sqrt{b^2-4ac}}\right)}{a^2n\sqrt{b^2-4ac}}+\frac{b\log\left(a+bx^n+cx^{2n}\right)}{2a^2n}-\frac{b\log(x)}{a^2}-\frac{x^{-n}}{an}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 - n)/(a + b*x^n + c*x^(2*n)),x]
```

```
[Out] -(1/(a*n*x^n)) - ((b^2 - 2*a*c)*ArcTanh[(b + 2*c*x^n)/Sqrt[b^2 - 4*a*c]])/(
a^2*Sqrt[b^2 - 4*a*c]*n) - (b*Log[x])/a^2 + (b*Log[a + b*x^n + c*x^(2*n)])/
(2*a^2*n)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 709

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Simp[(e*(d + e*x)^(m + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dis
t[1/(c*d^2 - b*d*e + a*e^2), Int[((d + e*x)^(m + 1)*Simp[c*d - b*e - c*e*x,
 x])/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 -
 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[m
, -1]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 1357

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^{-1-n}}{a+bx^n+cx^{2n}}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{x^2(a+bx+cx^2)}\,dx,x,x^n\right)}{n}\\
&= -\frac{x^{-n}}{an}+\frac{\text{Subst}\left(\int \frac{-b-cx}{x(a+bx+cx^2)}\,dx,x,x^n\right)}{an}\\
&= -\frac{x^{-n}}{an}+\frac{\text{Subst}\left(\int \left(-\frac{b}{ax}+\frac{b^2-ac+bcx}{a(a+bx+cx^2)}\right)dx,x,x^n\right)}{an}\\
&= -\frac{x^{-n}}{an}-\frac{b\log(x)}{a^2}+\frac{\text{Subst}\left(\int \frac{b^2-ac+bcx}{a+bx+cx^2}\,dx,x,x^n\right)}{a^2n}\\
&= -\frac{x^{-n}}{an}-\frac{b\log(x)}{a^2}+\frac{b\,\text{Subst}\left(\int \frac{b+2cx}{a+bx+cx^2}\,dx,x,x^n\right)}{2a^2n}+\frac{(b^2-2ac)\,\text{Subst}\left(\int \frac{1}{a+bx+cx^2}\,dx,x,\right.}{2a^2n}\\
&= -\frac{x^{-n}}{an}-\frac{b\log(x)}{a^2}+\frac{b\log\left(a+bx^n+cx^{2n}\right)}{2a^2n}-\frac{(b^2-2ac)\,\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx,x,b+2c\right.}{a^2n}\\
&= -\frac{x^{-n}}{an}-\frac{(b^2-2ac)\tanh^{-1}\left(\frac{b+2cx^n}{\sqrt{b^2-4ac}}\right)}{a^2\sqrt{b^2-4ac}\,n}-\frac{b\log(x)}{a^2}+\frac{b\log\left(a+bx^n+cx^{2n}\right)}{2a^2n}
\end{aligned}
$$

Mathematica [A] time = 0.67, size = 135, normalized size = 1.38

$$
-\frac{-\frac{4c^2\log\left(x^{-n}\left(b-\sqrt{b^2-4ac}\right)+2c\right)}{\sqrt{b^2-4ac}\left(b-\sqrt{b^2-4ac}\right)^2}+\frac{4c^2\log\left(x^{-n}\left(\sqrt{b^2-4ac}+b\right)+2c\right)}{\sqrt{b^2-4ac}\left(\sqrt{b^2-4ac}+b\right)^2}+\frac{x^{-n}}{a}}{n}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 - n)/(a + b*x^n + c*x^(2*n)),x]
```

```
[Out] -((1/(a*x^n) - (4*c^2*Log[2*c + (b - Sqrt[b^2 - 4*a*c])/x^n])/(Sqrt[b^2 - 4
*a*c]*(b - Sqrt[b^2 - 4*a*c])^2) + (4*c^2*Log[2*c + (b + Sqrt[b^2 - 4*a*c])
/x^n])/(Sqrt[b^2 - 4*a*c]*(b + Sqrt[b^2 - 4*a*c])^2))/n)
```

IntegrateAlgebraic [A] time = 0.19, size = 120, normalized size = 1.22

$$
\frac{(b^2-2ac)\tan^{-1}\left(\frac{2cx^n}{\sqrt{4ac-b^2}}+\frac{b}{\sqrt{4ac-b^2}}\right)}{a^2n\sqrt{4ac-b^2}}+\frac{b\log\left(a+bx^n+cx^{2n}\right)}{2a^2n}-\frac{b\log\left(x^n\right)}{a^2n}-\frac{x^{-n}}{an}
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(-1 - n)/(a + b*x^n + c*x^(2*n)),x]
```

```
[Out] -(1/(a*n*x^n)) + ((b^2 - 2*a*c)*ArcTan[b/Sqrt[-b^2 + 4*a*c] + (2*c*x^n)/Sqr
t[-b^2 + 4*a*c]])/(a^2*Sqrt[-b^2 + 4*a*c]*n) - (b*Log[x^n])/(a^2*n) + (b*Lo
g[a + b*x^n + c*x^(2*n)])/(2*a^2*n)
```

fricas [A] time = 1.41, size = 333, normalized size = 3.40

$$\left[-\frac{2\left(b^{3}-4\,abc\right)nx^{n}\log(x)+\left(b^{2}-2\,ac\right)\sqrt{b^{2}-4\,ac}\,x^{n}\log\left(\frac{2\,c^{2}x^{2\,n}+b^{2}-2\,ac+2\left(bc+\sqrt{b^{2}-4\,ac}\,c\right)x^{n}+\sqrt{b^{2}-4\,ac}\,b}{cx^{2\,n}+bx^{n}+a}\right)+2\,ab^{2}-8\,a^{2}c-\left(b^{3}-4\,abc\right)x^{n}\log\left(cx^{2\,n}+bx^{n}+a\right)}{2\left(a^{2}b^{2}-4\,a^{3}c\right)nx^{n}}, -\frac{2\left(b^{3}-4\,abc\right)nx^{n}\log(x)+2\left(b^{2}-2\,ac\right)\sqrt{-b^{2}+4\,ac}\,x^{n}\arctan\left(-\frac{2\sqrt{-b^{2}+4\,ac}\,cx^{n}+\sqrt{-b^{2}+4\,ac}\,b}{b^{2}-4\,ac}\right)+2\,ab^{2}-8\,a^{2}c-\left(b^{3}-4\,abc\right)x^{n}\log\left(cx^{2\,n}+bx^{n}+a\right)}{2\left(a^{2}b^{2}-4\,a^{3}c\right)nx^{n}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-n)/(a+b*x^n+c*x^(2*n)),x, algorithm="fricas")
```

```
[Out] [-1/2*(2*(b^3 - 4*a*b*c)*n*x^n*log(x) + (b^2 - 2*a*c)*sqrt(b^2 - 4*a*c)*x^n
*log((2*c^2*x^(2*n) + b^2 - 2*a*c + 2*(b*c + sqrt(b^2 - 4*a*c)*c)*x^n + sqr
t(b^2 - 4*a*c)*b)/(c*x^(2*n) + b*x^n + a)) + 2*a*b^2 - 8*a^2*c - (b^3 - 4*a
*b*c)*x^n*log(c*x^(2*n) + b*x^n + a))/((a^2*b^2 - 4*a^3*c)*n*x^n), -1/2*(2*
(b^3 - 4*a*b*c)*n*x^n*log(x) + 2*(b^2 - 2*a*c)*sqrt(-b^2 + 4*a*c)*x^n*arcta
n(-(2*sqrt(-b^2 + 4*a*c)*c*x^n + sqrt(-b^2 + 4*a*c)*b)/(b^2 - 4*a*c)) + 2*a
*b^2 - 8*a^2*c - (b^3 - 4*a*b*c)*x^n*log(c*x^(2*n) + b*x^n + a))/((a^2*b^2
- 4*a^3*c)*n*x^n)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{-n-1}}{cx^{2\,n}+bx^{n}+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-n)/(a+b*x^n+c*x^(2*n)),x, algorithm="giac")
```

```
[Out] integrate(x^(-n - 1)/(c*x^(2*n) + b*x^n + a), x)
```

maple [B] time = 0.15, size = 658, normalized size = 6.71

$$-\frac{4abc\,n^{2}\ln(x)}{4a^{3}c\,n^{2}-a^{2}b^{2}n^{2}}+\frac{b^{3}n^{2}\ln(x)}{4a^{3}c\,n^{2}-a^{2}b^{2}n^{2}}+\frac{2bc\ln\left(x^{n}-\frac{-2abc+b^{3}+\sqrt{-16a^{3}c^{3}+20b^{2}a^{2}c^{2}-8b^{4}ac+b^{6}}}{2\left(2ac-b^{2}\right)c}\right)}{\left(4ac-b^{2}\right)an}+\frac{2bc\ln\left(x^{n}+\frac{2abc-b^{3}+\sqrt{-16a^{3}c^{3}+20b^{2}a^{2}c^{2}-8b^{4}ac+b^{6}}}{2\left(2ac-b^{2}\right)c}\right)}{\left(4ac-b^{2}\right)an}-\frac{b^{3}\ln\left(x^{n}-\frac{-2abc+b^{3}+\sqrt{-16a^{3}c^{3}+20b^{2}a^{2}c^{2}-8b^{4}ac+b^{6}}}{2\left(2ac-b^{2}\right)c}\right)}{2\left(4ac-b^{2}\right)a^{2}n}-\frac{b^{3}\ln\left(x^{n}+\frac{2abc-b^{3}+\sqrt{-16a^{3}c^{3}+20b^{2}a^{2}c^{2}-8b^{4}ac+b^{6}}}{2\left(2ac-b^{2}\right)c}\right)}{2\left(4ac-b^{2}\right)a^{2}n}-\frac{x^{-n}}{an}+\frac{\sqrt{-16a^{3}c^{3}+20b^{2}a^{2}c^{2}-8b^{4}ac+b^{6}}\,\ln\left(x^{n}-\frac{-2abc+b^{3}+\sqrt{-16a^{3}c^{3}+20b^{2}a^{2}c^{2}-8b^{4}ac+b^{6}}}{2\left(2ac-b^{2}\right)c}\right)}{2\left(4ac-b^{2}\right)a^{2}n}-\frac{\sqrt{-16a^{3}c^{3}+20b^{2}a^{2}c^{2}-8b^{4}ac+b^{6}}\,\ln\left(x^{n}+\frac{2abc-b^{3}+\sqrt{-16a^{3}c^{3}+20b^{2}a^{2}c^{2}-8b^{4}ac+b^{6}}}{2\left(2ac-b^{2}\right)c}\right)}{2\left(4ac-b^{2}\right)a^{2}n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1-n)/(a+b*x^n+c*x^(2*n)),x)
```

```
[Out] -1/a/n/(x^n)-4/(4*a^3*c*n^2-a^2*b^2*n^2)*n^2*ln(x)*a*b*c+1/(4*a^3*c*n^2-a^2
*b^2*n^2)*n^2*ln(x)*b^3+2/a/(4*a*c-b^2)/n*ln(x^n-1/2*(-2*a*b*c+b^3+(-16*a^3
*c^3+20*a^2*b^2*c^2-8*a*b^4*c+b^6)^(1/2))/(2*a*c-b^2)/c)*b*c-1/2/a^2/(4*a*c
-b^2)/n*ln(x^n-1/2*(-2*a*b*c+b^3+(-16*a^3*c^3+20*a^2*b^2*c^2-8*a*b^4*c+b^6)
^(1/2))/(2*a*c-b^2)/c)*b^3+1/2/a^2/(4*a*c-b^2)/n*ln(x^n-1/2*(-2*a*b*c+b^3+(
-16*a^3*c^3+20*a^2*b^2*c^2-8*a*b^4*c+b^6)^(1/2))/(2*a*c-b^2)/c)*(-16*a^3*c^
3+20*a^2*b^2*c^2-8*a*b^4*c+b^6)^(1/2)+2/a/(4*a*c-b^2)/n*ln(x^n+1/2*(2*a*b*c
-b^3+(-16*a^3*c^3+20*a^2*b^2*c^2-8*a*b^4*c+b^6)^(1/2))/(2*a*c-b^2)/c)*b*c-1
/2/a^2/(4*a*c-b^2)/n*ln(x^n+1/2*(2*a*b*c-b^3+(-16*a^3*c^3+20*a^2*b^2*c^2-8*
a*b^4*c+b^6)^(1/2))/(2*a*c-b^2)/c)*b^3-1/2/a^2/(4*a*c-b^2)/n*ln(x^n+1/2*(2*
a*b*c-b^3+(-16*a^3*c^3+20*a^2*b^2*c^2-8*a*b^4*c+b^6)^(1/2))/(2*a*c-b^2)/c)*
(-16*a^3*c^3+20*a^2*b^2*c^2-8*a*b^4*c+b^6)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{anx^{n}}-\int \frac{cx^{n}+b}{acxx^{2\,n}+abxx^{n}+a^{2}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-n)/(a+b*x^n+c*x^(2*n)),x, algorithm="maxima")
```

```
[Out] -1/(a*n*x^n) - integrate((c*x^n + b)/(a*c*x*x^(2*n) + a*b*x*x^n + a^2*x), x
)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^{n+1}\,\left(a+b\,x^n+c\,x^{2\,n}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(n + 1)*(a + b*x^n + c*x^(2*n))),x)
```

```
[Out] int(1/(x^(n + 1)*(a + b*x^n + c*x^(2*n))), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1-n)/(a+b*x**n+c*x**(2*n)),x)
```

```
[Out] Timed out
```

3.478 $\int \frac{x^{-1-2n}}{a+bx^n+cx^{2n}}\,dx$

Optimal. Leaf size=126

$$\frac{b\left(b^2-3ac\right)\tanh^{-1}\left(\frac{b+2cx^n}{\sqrt{b^2-4ac}}\right)}{a^3n\sqrt{b^2-4ac}}-\frac{\left(b^2-ac\right)\log\left(a+bx^n+cx^{2n}\right)}{2a^3n}+\frac{\log(x)\left(b^2-ac\right)}{a^3}+\frac{bx^{-n}}{a^2n}-\frac{x^{-2n}}{2an}$$

Rubi [A] time = 0.17, antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {1357, 709, 800, 634, 618, 206, 628}

$$-\frac{\left(b^2-ac\right)\log\left(a+bx^n+cx^{2n}\right)}{2a^3n}+\frac{b\left(b^2-3ac\right)\tanh^{-1}\left(\frac{b+2cx^n}{\sqrt{b^2-4ac}}\right)}{a^3n\sqrt{b^2-4ac}}+\frac{\log(x)\left(b^2-ac\right)}{a^3}+\frac{bx^{-n}}{a^2n}-\frac{x^{-2n}}{2an}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 - 2*n)/(a + b*x^n + c*x^(2*n)),x]
```

```
[Out] -1/(2*a*n*x^(2*n)) + b/(a^2*n*x^n) + (b*(b^2 - 3*a*c)*ArcTanh[(b + 2*c*x^n)
/Sqrt[b^2 - 4*a*c]])/(a^3*Sqrt[b^2 - 4*a*c]*n) + ((b^2 - a*c)*Log[x])/a^3 -
 ((b^2 - a*c)*Log[a + b*x^n + c*x^(2*n)])/(2*a^3*n)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 709

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Simp[(e*(d + e*x)^(m + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dis
t[1/(c*d^2 - b*d*e + a*e^2), Int[((d + e*x)^(m + 1)*Simp[c*d - b*e - c*e*x,
 x])/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 -
 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[m
, -1]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 1357

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^{-1-2n}}{a+bx^n+cx^{2n}}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{x^3\left(a+bx+cx^2\right)}\,dx, x, x^n\right)}{n}\\
&= -\frac{x^{-2n}}{2an}+\frac{\text{Subst}\left(\int \frac{-b-cx}{x^2\left(a+bx+cx^2\right)}\,dx, x, x^n\right)}{an}\\
&= -\frac{x^{-2n}}{2an}+\frac{\text{Subst}\left(\int \left(-\frac{b}{ax^2}+\frac{b^2-ac}{a^2x}+\frac{-b\left(b^2-2ac\right)-c\left(b^2-ac\right)x}{a^2\left(a+bx+cx^2\right)}\right)dx, x, x^n\right)}{an}\\
&= -\frac{x^{-2n}}{2an}+\frac{bx^{-n}}{a^2n}+\frac{\left(b^2-ac\right)\log(x)}{a^3}+\frac{\text{Subst}\left(\int \frac{-b\left(b^2-2ac\right)-c\left(b^2-ac\right)x}{a+bx+cx^2}\,dx, x, x^n\right)}{a^3n}\\
&= -\frac{x^{-2n}}{2an}+\frac{bx^{-n}}{a^2n}+\frac{\left(b^2-ac\right)\log(x)}{a^3}-\frac{\left(b\left(b^2-3ac\right)\right)\text{Subst}\left(\int \frac{1}{a+bx+cx^2}\,dx, x, x^n\right)}{2a^3n}-\frac{\left(b^2-\right.}{}\\
&= -\frac{x^{-2n}}{2an}+\frac{bx^{-n}}{a^2n}+\frac{\left(b^2-ac\right)\log(x)}{a^3}-\frac{\left(b^2-ac\right)\log\left(a+bx^n+cx^{2n}\right)}{2a^3n}+\frac{\left(b\left(b^2-3ac\right)\right)\text{Su}}{}\\
&= -\frac{x^{-2n}}{2an}+\frac{bx^{-n}}{a^2n}+\frac{b\left(b^2-3ac\right)\tanh^{-1}\left(\frac{b+2cx^n}{\sqrt{b^2-4ac}}\right)}{a^3\sqrt{b^2-4ac}\,n}+\frac{\left(b^2-ac\right)\log(x)}{a^3}-\frac{\left(b^2-ac\right)\log\left(a+\right.}{2a^3n}
\end{aligned}$$

Mathematica [A] time = 0.33, size = 112, normalized size = 0.89

$$\frac{-a^2x^{-2n}-\left(b^2-ac\right)\log\left(a+x^n\left(b+cx^n\right)\right)+\frac{2b\left(b^2-3ac\right)\tanh^{-1}\left(\frac{b+2cx^n}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}}+2n\log(x)\left(b^2-ac\right)+2abx^{-n}}{2a^3n}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 - 2*n)/(a + b*x^n + c*x^(2*n)),x]
```

```
[Out] (-(a^2/x^(2*n)) + (2*a*b)/x^n + (2*b*(b^2 - 3*a*c)*ArcTanh[(b + 2*c*x^n)/Sq
rt[b^2 - 4*a*c]])/Sqrt[b^2 - 4*a*c] + 2*(b^2 - a*c)*n*Log[x] - (b^2 - a*c)*
Log[a + x^n*(b + c*x^n)])/(2*a^3*n)
```

IntegrateAlgebraic [A] time = 0.33, size = 149, normalized size = 1.18

$$\frac{\left(b^2-ac\right)\log\left(x^n\right)}{a^3n}+\frac{\left(ac-b^2\right)\log\left(a+bx^n+cx^{2n}\right)}{2a^3n}+\frac{\left(3abc-b^3\right)\tan^{-1}\left(\frac{2cx^n}{\sqrt{4ac-b^2}}+\frac{b}{\sqrt{4ac-b^2}}\right)}{a^3n\sqrt{4ac-b^2}}+\frac{x^{-2n}\left(2bx^n-a\right)}{2a^2n}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(-1 - 2*n)/(a + b*x^n + c*x^(2*n)),x]
```

```
[Out] (-a + 2*b*x^n)/(2*a^2*n*x^(2*n)) + ((-b^3 + 3*a*b*c)*ArcTan[b/Sqrt[-b^2 + 4
*a*c] + (2*c*x^n)/Sqrt[-b^2 + 4*a*c]])/(a^3*Sqrt[-b^2 + 4*a*c]*n) + ((b^2 -
 a*c)*Log[x^n])/(a^3*n) + ((-b^2 + a*c)*Log[a + b*x^n + c*x^(2*n)])/(2*a^3*
n)
```

fricas [A] time = 1.41, size = 429, normalized size = 3.40

$$\left[-\frac{a^{2} b^{2} - 4 a^{3} c - 2 \left(b^{4} - 5 a b^{2} c + 4 a^{2} c^{2}\right) n x^{2 n} \log(x) + \left(b^{3} - 3 a b c\right) \sqrt{b^{2} - 4 a c}\, x^{2 n} \log\left(\frac{2 c^{2} x^{2 n} + b^{2} - 2 a c + 2 \left(b c - \sqrt{b^{2} - 4 a c}\, c\right) x^{n} - \sqrt{b^{2} - 4 a c}\, b}{c x^{2 n} + b x^{n} + a}\right) + \left(b^{4} - 5 a b^{2} c + 4 a^{2} c^{2}\right) x^{2 n} \log\left(c x^{2 n} + b x^{n} + a\right) - 2 \left(a b^{3} - 4 a^{2} b c\right) x^{n}}{2 \left(a^{3} b^{2} - 4 a^{4} c\right) n x^{2 n}}, -\frac{a^{2} b^{2} - 4 a^{3} c - 2 \left(b^{4} - 5 a b^{2} c + 4 a^{2} c^{2}\right) n x^{2 n} \log(x) - 2 \left(b^{3} - 3 a b c\right) \sqrt{-b^{2} + 4 a c}\, x^{2 n} \arctan\left(-\frac{2 \sqrt{-b^{2} + 4 a c}\, c x^{n} + \sqrt{-b^{2} + 4 a c}\, b}{b^{2} - 4 a c}\right) + \left(b^{4} - 5 a b^{2} c + 4 a^{2} c^{2}\right) x^{2 n} \log\left(c x^{2 n} + b x^{n} + a\right) - 2 \left(a b^{3} - 4 a^{2} b c\right) x^{n}}{2 \left(a^{3} b^{2} - 4 a^{4} c\right) n x^{2 n}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-2*n)/(a+b*x^n+c*x^(2*n)),x, algorithm="fricas")
```

```
[Out] [-1/2*(a^2*b^2 - 4*a^3*c - 2*(b^4 - 5*a*b^2*c + 4*a^2*c^2)*n*x^(2*n)*log(x)
 + (b^3 - 3*a*b*c)*sqrt(b^2 - 4*a*c)*x^(2*n)*log((2*c^2*x^(2*n) + b^2 - 2*a
*c + 2*(b*c - sqrt(b^2 - 4*a*c)*c)*x^n - sqrt(b^2 - 4*a*c)*b)/(c*x^(2*n) +
b*x^n + a)) + (b^4 - 5*a*b^2*c + 4*a^2*c^2)*x^(2*n)*log(c*x^(2*n) + b*x^n +
 a) - 2*(a*b^3 - 4*a^2*b*c)*x^n)/((a^3*b^2 - 4*a^4*c)*n*x^(2*n)), -1/2*(a^2
*b^2 - 4*a^3*c - 2*(b^4 - 5*a*b^2*c + 4*a^2*c^2)*n*x^(2*n)*log(x) - 2*(b^3
- 3*a*b*c)*sqrt(-b^2 + 4*a*c)*x^(2*n)*arctan(-(2*sqrt(-b^2 + 4*a*c)*c*x^n +
 sqrt(-b^2 + 4*a*c)*b)/(b^2 - 4*a*c)) + (b^4 - 5*a*b^2*c + 4*a^2*c^2)*x^(2*
n)*log(c*x^(2*n) + b*x^n + a) - 2*(a*b^3 - 4*a^2*b*c)*x^n)/((a^3*b^2 - 4*a^
4*c)*n*x^(2*n))]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{-2 n-1}}{c x^{2 n} + b x^{n} + a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-2*n)/(a+b*x^n+c*x^(2*n)),x, algorithm="giac")
```

```
[Out] integrate(x^(-2*n - 1)/(c*x^(2*n) + b*x^n + a), x)
```

maple [B] time = 0.17, size = 958, normalized size = 7.60

$$-\frac{4 a^{2} c^{2} n^{2} \ln(x)}{4 a^{4} c n^{2} - a^{3} b^{2} n^{2}} + \frac{5 a b^{2} c n^{2} \ln(x)}{4 a^{4} c n^{2} - a^{3} b^{2} n^{2}} - \frac{b^{4} n^{2} \ln(x)}{4 a^{4} c n^{2} - a^{3} b^{2} n^{2}} + \frac{2 c^{2} \ln\left(x^{n} - \frac{-3 a b^{2} c + b^{4} + \sqrt{-36 a^{3} b^{2} c^{3} + 33 a^{2} b^{4} c^{2} - 10 a b^{6} c + b^{8}}}{2 \left(3 a c - b^{2}\right) b c}\right)}{\left(4 a c - b^{2}\right) a n} + \frac{2 c^{2} \ln\left(x^{n} + \frac{3 a b^{2} c - b^{4} + \sqrt{-36 a^{3} b^{2} c^{3} + 33 a^{2} b^{4} c^{2} - 10 a b^{6} c + b^{8}}}{2 \left(3 a c - b^{2}\right) b c}\right)}{\left(4 a c - b^{2}\right) a n} - \frac{5 b^{2} c \ln\left(x^{n} - \frac{-3 a b^{2} c + b^{4} + \sqrt{-36 a^{3} b^{2} c^{3} + 33 a^{2} b^{4} c^{2} - 10 a b^{6} c + b^{8}}}{2 \left(3 a c - b^{2}\right) b c}\right)}{2 \left(4 a c - b^{2}\right) a^{2} n} - \frac{5 b^{2} c \ln\left(x^{n} + \frac{3 a b^{2} c - b^{4} + \sqrt{-36 a^{3} b^{2} c^{3} + 33 a^{2} b^{4} c^{2} - 10 a b^{6} c + b^{8}}}{2 \left(3 a c - b^{2}\right) b c}\right)}{2 \left(4 a c - b^{2}\right) a^{2} n} + \frac{b^{4} \ln\left(x^{n} - \frac{-3 a b^{2} c + b^{4} + \sqrt{-36 a^{3} b^{2} c^{3} + 33 a^{2} b^{4} c^{2} - 10 a b^{6} c + b^{8}}}{2 \left(3 a c - b^{2}\right) b c}\right)}{2 \left(4 a c - b^{2}\right) a^{3} n} + \frac{b^{4} \ln\left(x^{n} + \frac{3 a b^{2} c - b^{4} + \sqrt{-36 a^{3} b^{2} c^{3} + 33 a^{2} b^{4} c^{2} - 10 a b^{6} c + b^{8}}}{2 \left(3 a c - b^{2}\right) b c}\right)}{2 \left(4 a c - b^{2}\right) a^{3} n} - \frac{x^{-2 n}}{2 a n} + \frac{b x^{-n}}{a^{2} n} - \frac{\sqrt{-36 a^{3} b^{2} c^{3} + 33 a^{2} b^{4} c^{2} - 10 a b^{6} c + b^{8}}\, \ln\left(x^{n} - \frac{-3 a b^{2} c + b^{4} + \sqrt{-36 a^{3} b^{2} c^{3} + 33 a^{2} b^{4} c^{2} - 10 a b^{6} c + b^{8}}}{2 \left(3 a c - b^{2}\right) b c}\right)}{2 \left(4 a c - b^{2}\right) a^{3} n} + \frac{\sqrt{-36 a^{3} b^{2} c^{3} + 33 a^{2} b^{4} c^{2} - 10 a b^{6} c + b^{8}}\, \ln\left(x^{n} + \frac{3 a b^{2} c - b^{4} + \sqrt{-36 a^{3} b^{2} c^{3} + 33 a^{2} b^{4} c^{2} - 10 a b^{6} c + b^{8}}}{2 \left(3 a c - b^{2}\right) b c}\right)}{2 \left(4 a c - b^{2}\right) a^{3} n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1-2*n)/(a+b*x^n+c*x^(2*n)),x)
```

```
[Out] b/a^2/n/(x^n)-1/2/a/n/(x^n)^2-4/(4*a^4*c*n^2-a^3*b^2*n^2)*n^2*ln(x)*a^2*c^2
+5/(4*a^4*c*n^2-a^3*b^2*n^2)*n^2*ln(x)*a*b^2*c-1/(4*a^4*c*n^2-a^3*b^2*n^2)*
n^2*ln(x)*b^4+2/a/(4*a*c-b^2)/n*ln(x^n+1/2*(3*a*b^2*c-b^4+(-36*a^3*b^2*c^3+
33*a^2*b^4*c^2-10*a*b^6*c+b^8)^(1/2))/(3*a*c-b^2)/b/c)*c^2-5/2/a^2/(4*a*c-b
^2)/n*ln(x^n+1/2*(3*a*b^2*c-b^4+(-36*a^3*b^2*c^3+33*a^2*b^4*c^2-10*a*b^6*c+
b^8)^(1/2))/(3*a*c-b^2)/b/c)*b^2*c+1/2/a^3/(4*a*c-b^2)/n*ln(x^n+1/2*(3*a*b^
2*c-b^4+(-36*a^3*b^2*c^3+33*a^2*b^4*c^2-10*a*b^6*c+b^8)^(1/2))/(3*a*c-b^2)/
b/c)*b^4+1/2/a^3/(4*a*c-b^2)/n*ln(x^n+1/2*(3*a*b^2*c-b^4+(-36*a^3*b^2*c^3+3
3*a^2*b^4*c^2-10*a*b^6*c+b^8)^(1/2))/(3*a*c-b^2)/b/c)*(-36*a^3*b^2*c^3+33*a
^2*b^4*c^2-10*a*b^6*c+b^8)^(1/2)+2/a/(4*a*c-b^2)/n*ln(x^n-1/2*(-3*a*b^2*c+b
^4+(-36*a^3*b^2*c^3+33*a^2*b^4*c^2-10*a*b^6*c+b^8)^(1/2))/(3*a*c-b^2)/b/c)*
c^2-5/2/a^2/(4*a*c-b^2)/n*ln(x^n-1/2*(-3*a*b^2*c+b^4+(-36*a^3*b^2*c^3+33*a^
2*b^4*c^2-10*a*b^6*c+b^8)^(1/2))/(3*a*c-b^2)/b/c)*b^2*c+1/2/a^3/(4*a*c-b^2)
/n*ln(x^n-1/2*(-3*a*b^2*c+b^4+(-36*a^3*b^2*c^3+33*a^2*b^4*c^2-10*a*b^6*c+b^
8)^(1/2))/(3*a*c-b^2)/b/c)*b^4-1/2/a^3/(4*a*c-b^2)/n*ln(x^n-1/2*(-3*a*b^2*c
+b^4+(-36*a^3*b^2*c^3+33*a^2*b^4*c^2-10*a*b^6*c+b^8)^(1/2))/(3*a*c-b^2)/b/c
)*(-36*a^3*b^2*c^3+33*a^2*b^4*c^2-10*a*b^6*c+b^8)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{2\,bx^n - a}{2\,a^2nx^{2\,n}} + \int \frac{bcx^n + b^2 - ac}{a^2cxx^{2\,n} + a^2bxx^n + a^3x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-2*n)/(a+b*x^n+c*x^(2*n)),x, algorithm="maxima")
```

```
[Out] 1/2*(2*b*x^n - a)/(a^2*n*x^(2*n)) + integrate((b*c*x^n + b^2 - a*c)/(a^2*c*
x*x^(2*n) + a^2*b*x*x^n + a^3*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^{2\,n+1}\,\left(a+b\,x^n+c\,x^{2\,n}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(2*n + 1)*(a + b*x^n + c*x^(2*n))),x)
```

```
[Out] int(1/(x^(2*n + 1)*(a + b*x^n + c*x^(2*n))), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1-2*n)/(a+b*x**n+c*x**(2*n)),x)
```

```
[Out] Timed out
```

3.479 $\int \frac{x^{-1-3n}}{a+bx^n+cx^{2n}} dx$

Optimal. Leaf size=164

$$\frac{b\left(b^2-2ac\right)\log\left(a+bx^n+cx^{2n}\right)}{2a^4n}-\frac{b\log(x)\left(b^2-2ac\right)}{a^4}-\frac{x^{-n}\left(b^2-ac\right)}{a^3n}+\frac{bx^{-2n}}{2a^2n}-\frac{\left(2a^2c^2-4ab^2c+b^4\right)\tanh^{-1}\left(\frac{b+2cx^n}{\sqrt{b^2-4ac}}\right)}{a^4n\sqrt{b^2-4ac}}$$

Rubi [A] time = 0.23, antiderivative size = 164, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {1357, 709, 800, 634, 618, 206, 628}

$$-\frac{\left(2a^2c^2-4ab^2c+b^4\right)\tanh^{-1}\left(\frac{b+2cx^n}{\sqrt{b^2-4ac}}\right)}{a^4n\sqrt{b^2-4ac}}-\frac{x^{-n}\left(b^2-ac\right)}{a^3n}+\frac{b\left(b^2-2ac\right)\log\left(a+bx^n+cx^{2n}\right)}{2a^4n}-\frac{b\log(x)\left(b^2-2ac\right)}{a^4}+\frac{bx^{-2n}}{2a^2n}-\frac{x^{-3n}}{3an}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 - 3*n)/(a + b*x^n + c*x^(2*n)),x]
```

```
[Out] -1/(3*a*n*x^(3*n)) + b/(2*a^2*n*x^(2*n)) - (b^2 - a*c)/(a^3*n*x^n) - ((b^4
- 4*a*b^2*c + 2*a^2*c^2)*ArcTanh[(b + 2*c*x^n)/Sqrt[b^2 - 4*a*c]])/(a^4*Sqr
t[b^2 - 4*a*c]*n) - (b*(b^2 - 2*a*c)*Log[x])/a^4 + (b*(b^2 - 2*a*c)*Log[a +
 b*x^n + c*x^(2*n)])/(2*a^4*n)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 709

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Simp[(e*(d + e*x)^(m + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dis
t[1/(c*d^2 - b*d*e + a*e^2), Int[((d + e*x)^(m + 1)*Simp[c*d - b*e - c*e*x,
 x])/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 -
 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[m
, -1]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 1357

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^{-1-3n}}{a+bx^n+cx^{2n}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x^4(a+bx+cx^2)}\,dx,x,x^n\right)}{n}$$
$$= -\frac{x^{-3n}}{3an} + \frac{\text{Subst}\left(\int \frac{-b-cx}{x^3(a+bx+cx^2)}\,dx,x,x^n\right)}{an}$$
$$= -\frac{x^{-3n}}{3an} + \frac{\text{Subst}\left(\int \left(-\frac{b}{ax^3}+\frac{b^2-ac}{a^2x^2}+\frac{-b^3+2abc}{a^3x}+\frac{b^4-3ab^2c+a^2c^2+bc(b^2-2ac)x}{a^3(a+bx+cx^2)}\right)dx,x,x^n\right)}{an}$$
$$= -\frac{x^{-3n}}{3an} + \frac{bx^{-2n}}{2a^2n} - \frac{(b^2-ac)x^{-n}}{a^3n} - \frac{b(b^2-2ac)\log(x)}{a^4} + \frac{\text{Subst}\left(\int \frac{b^4-3ab^2c+a^2c^2+bc(b^2-2ac}{a+bx+cx^2}\right.}{a^4n}$$
$$= -\frac{x^{-3n}}{3an} + \frac{bx^{-2n}}{2a^2n} - \frac{(b^2-ac)x^{-n}}{a^3n} - \frac{b(b^2-2ac)\log(x)}{a^4} + \frac{(b(b^2-2ac))\text{Subst}\left(\int \frac{b+2cx}{a+bx+c}\right.}{2a^4n}$$
$$= -\frac{x^{-3n}}{3an} + \frac{bx^{-2n}}{2a^2n} - \frac{(b^2-ac)x^{-n}}{a^3n} - \frac{b(b^2-2ac)\log(x)}{a^4} + \frac{b(b^2-2ac)\log(a+bx^n+cx^2}{2a^4n}$$
$$= -\frac{x^{-3n}}{3an} + \frac{bx^{-2n}}{2a^2n} - \frac{(b^2-ac)x^{-n}}{a^3n} - \frac{(b^4-4ab^2c+2a^2c^2)\tanh^{-1}\left(\frac{b+2cx^n}{\sqrt{b^2-4ac}}\right)}{a^4\sqrt{b^2-4ac}\,n} - \frac{b(b^2-2ac}{a^4}$$

Mathematica [A] time = 0.44, size = 143, normalized size = 0.87

$$\frac{-2a^3x^{-3n}-\frac{6(2a^2c^2-4ab^2c+b^4)\tanh^{-1}\left(\frac{b+2cx^n}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}}+3a^2bx^{-2n}+6ax^{-n}(ac-b^2)+3b(b^2-2ac)\log(a+x^n(b+cx^n))-6bn\log(x)(b^2-2ac)}{6a^4n}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 - 3*n)/(a + b*x^n + c*x^(2*n)),x]
```

```
[Out] ((-2*a^3)/x^(3*n) + (3*a^2*b)/x^(2*n) + (6*a*(-b^2 + a*c))/x^n - (6*(b^4 -
4*a*b^2*c + 2*a^2*c^2)*ArcTanh[(b + 2*c*x^n)/Sqrt[b^2 - 4*a*c]])/Sqrt[b^2 -
 4*a*c] - 6*b*(b^2 - 2*a*c)*n*Log[x] + 3*b*(b^2 - 2*a*c)*Log[a + x^n*(b + c
*x^n)])/(6*a^4*n)
```

IntegrateAlgebraic [A] time = 0.32, size = 182, normalized size = 1.11

$$\frac{(2abc-b^3)\log(x^n)}{a^4n}+\frac{(b^3-2abc)\log(a+bx^n+cx^{2n})}{2a^4n}+\frac{(2a^2c^2-4ab^2c+b^4)\tan^{-1}\left(\frac{2cx^n}{\sqrt{4ac-b^2}}+\frac{b}{\sqrt{4ac-b^2}}\right)}{a^4n\sqrt{4ac-b^2}}+\frac{x^{-3n}(-2a^2+3abx^n+6acx^{2n}-6b^2x^{2n})}{6a^3n}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^(-1 - 3*n)/(a + b*x^n + c*x^(2*n)),x]

[Out] (-2*a^2 + 3*a*b*x^n - 6*b^2*x^(2*n) + 6*a*c*x^(2*n))/(6*a^3*n*x^(3*n)) + ((b^4 - 4*a*b^2*c + 2*a^2*c^2)*ArcTan[b/Sqrt[-b^2 + 4*a*c] + (2*c*x^n)/Sqrt[-b^2 + 4*a*c]])/(a^4*Sqrt[-b^2 + 4*a*c]*n) + ((-b^3 + 2*a*b*c)*Log[x^n])/(a^4*n) + ((b^3 - 2*a*b*c)*Log[a + b*x^n + c*x^(2*n)])/(2*a^4*n)

fricas [A] time = 1.36, size = 522, normalized size = 3.18

$$\left[-\frac{2\,a^{3}b^{2}-8\,a^{4}c+6\left(b^{5}-6\,ab^{3}c+8\,a^{2}bc^{2}\right)nx^{3\,n}\log(x)-3\left(b^{4}-4\,ab^{2}c+2\,a^{2}c^{2}\right)\sqrt{b^{2}-4\,ac}\,x^{3\,n}\log\left(\frac{2\,c^{2}x^{2\,n}+b^{2}-2\,ac+2\left(bc-\sqrt{b^{2}-4\,ac}\,c\right)x^{n}-\sqrt{b^{2}-4\,ac}\,b}{cx^{2\,n}+bx^{n}+a}\right)-3\left(b^{5}-6\,ab^{3}c+8\,a^{2}bc^{2}\right)x^{3\,n}\log\left(cx^{2\,n}+bx^{n}+a\right)+6\left(ab^{4}-5\,a^{2}b^{2}c+4\,a^{3}c^{2}\right)x^{2\,n}-3\left(a^{2}b^{3}-4\,a^{3}bc\right)x^{n}}{6\left(a^{4}b^{2}-4\,a^{5}c\right)nx^{3\,n}},\ -\frac{2\,a^{3}b^{2}-8\,a^{4}c+6\left(b^{5}-6\,ab^{3}c+8\,a^{2}bc^{2}\right)nx^{3\,n}\log(x)+6\left(b^{4}-4\,ab^{2}c+2\,a^{2}c^{2}\right)\sqrt{-b^{2}+4\,ac}\,x^{3\,n}\arctan\left(-\frac{2\,\sqrt{-b^{2}+4\,ac}\,cx^{n}+\sqrt{-b^{2}+4\,ac}\,b}{b^{2}-4\,ac}\right)-3\left(b^{5}-6\,ab^{3}c+8\,a^{2}bc^{2}\right)x^{3\,n}\log\left(cx^{2\,n}+bx^{n}+a\right)+6\left(ab^{4}-5\,a^{2}b^{2}c+4\,a^{3}c^{2}\right)x^{2\,n}-3\left(a^{2}b^{3}-4\,a^{3}bc\right)x^{n}}{6\left(a^{4}b^{2}-4\,a^{5}c\right)nx^{3\,n}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(-1-3*n)/(a+b*x^n+c*x^(2*n)),x, algorithm="fricas")

[Out] [-1/6*(2*a^3*b^2 - 8*a^4*c + 6*(b^5 - 6*a*b^3*c + 8*a^2*b*c^2)*n*x^(3*n)*log(x) - 3*(b^4 - 4*a*b^2*c + 2*a^2*c^2)*sqrt(b^2 - 4*a*c)*x^(3*n)*log((2*c^2*x^(2*n) + b^2 - 2*a*c + 2*(b*c - sqrt(b^2 - 4*a*c)*c)*x^n - sqrt(b^2 - 4*a*c)*b)/(c*x^(2*n) + b*x^n + a)) - 3*(b^5 - 6*a*b^3*c + 8*a^2*b*c^2)*x^(3*n)*log(c*x^(2*n) + b*x^n + a) + 6*(a*b^4 - 5*a^2*b^2*c + 4*a^3*c^2)*x^(2*n) - 3*(a^2*b^3 - 4*a^3*b*c)*x^n)/((a^4*b^2 - 4*a^5*c)*n*x^(3*n)), -1/6*(2*a^3*b^2 - 8*a^4*c + 6*(b^5 - 6*a*b^3*c + 8*a^2*b*c^2)*n*x^(3*n)*log(x) + 6*(b^4 - 4*a*b^2*c + 2*a^2*c^2)*sqrt(-b^2 + 4*a*c)*x^(3*n)*arctan(-(2*sqrt(-b^2 + 4*a*c)*c*x^n + sqrt(-b^2 + 4*a*c)*b)/(b^2 - 4*a*c)) - 3*(b^5 - 6*a*b^3*c + 8*a^2*b*c^2)*x^(3*n)*log(c*x^(2*n) + b*x^n + a) + 6*(a*b^4 - 5*a^2*b^2*c + 4*a^3*c^2)*x^(2*n) - 3*(a^2*b^3 - 4*a^3*b*c)*x^n)/((a^4*b^2 - 4*a^5*c)*n*x^(3*n))]

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{-3\,n-1}}{cx^{2\,n}+bx^{n}+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(-1-3*n)/(a+b*x^n+c*x^(2*n)),x, algorithm="giac")

[Out] integrate(x^(-3*n - 1)/(c*x^(2*n) + b*x^n + a), x)

maple [B] time = 0.20, size = 1300, normalized size = 7.93

$$\frac{8a^{2}bc^{2}n^{2}\ln(x)}{4a^{5}cn^{2}-a^{4}b^{2}n^{2}}-\frac{6ab^{3}cn^{2}\ln(x)}{4a^{5}cn^{2}-a^{4}b^{2}n^{2}}+\frac{b^{5}n^{2}\ln(x)}{4a^{5}cn^{2}-a^{4}b^{2}n^{2}}-\frac{4bc^{2}\ln\left(x^{n}-\frac{-2a^{2}bc^{2}+4ab^{3}c-b^{5}+\sqrt{-16a^{5}c^{5}+68a^{4}b^{2}c^{4}-96a^{3}b^{4}c^{3}+52a^{2}b^{6}c^{2}-12ab^{8}c+b^{10}}}{2\left(2a^{2}c^{2}-4ab^{2}c+b^{4}\right)c}\right)}{\left(4ac-b^{2}\right)a^{2}n}-\frac{4bc^{2}\ln\left(x^{n}+\frac{2a^{2}bc^{2}-4ab^{3}c+b^{5}+\sqrt{-16a^{5}c^{5}+68a^{4}b^{2}c^{4}-96a^{3}b^{4}c^{3}+52a^{2}b^{6}c^{2}-12ab^{8}c+b^{10}}}{2\left(2a^{2}c^{2}-4ab^{2}c+b^{4}\right)c}\right)}{\left(4ac-b^{2}\right)a^{2}n}+\frac{3b^{3}c\ln\left(x^{n}-\frac{-2a^{2}bc^{2}+4ab^{3}c-b^{5}+\sqrt{-16a^{5}c^{5}+68a^{4}b^{2}c^{4}-96a^{3}b^{4}c^{3}+52a^{2}b^{6}c^{2}-12ab^{8}c+b^{10}}}{2\left(2a^{2}c^{2}-4ab^{2}c+b^{4}\right)c}\right)}{\left(4ac-b^{2}\right)a^{3}n}+\frac{3b^{3}c\ln\left(x^{n}+\frac{2a^{2}bc^{2}-4ab^{3}c+b^{5}+\sqrt{-16a^{5}c^{5}+68a^{4}b^{2}c^{4}-96a^{3}b^{4}c^{3}+52a^{2}b^{6}c^{2}-12ab^{8}c+b^{10}}}{2\left(2a^{2}c^{2}-4ab^{2}c+b^{4}\right)c}\right)}{\left(4ac-b^{2}\right)a^{3}n}-\frac{b^{5}\ln\left(x^{n}-\frac{-2a^{2}bc^{2}+4ab^{3}c-b^{5}+\sqrt{-16a^{5}c^{5}+68a^{4}b^{2}c^{4}-96a^{3}b^{4}c^{3}+52a^{2}b^{6}c^{2}-12ab^{8}c+b^{10}}}{2\left(2a^{2}c^{2}-4ab^{2}c+b^{4}\right)c}\right)}{2\left(4ac-b^{2}\right)a^{4}n}-\frac{b^{5}\ln\left(x^{n}+\frac{2a^{2}bc^{2}-4ab^{3}c+b^{5}+\sqrt{-16a^{5}c^{5}+68a^{4}b^{2}c^{4}-96a^{3}b^{4}c^{3}+52a^{2}b^{6}c^{2}-12ab^{8}c+b^{10}}}{2\left(2a^{2}c^{2}-4ab^{2}c+b^{4}\right)c}\right)}{2\left(4ac-b^{2}\right)a^{4}n}-\frac{x^{-3n}}{3an}+\frac{bx^{-2n}}{2a^{2}n}+\frac{cx^{-n}}{a^{2}n}-\frac{b^{2}x^{-n}}{a^{3}n}-\frac{\sqrt{-16a^{5}c^{5}+68a^{4}b^{2}c^{4}-96a^{3}b^{4}c^{3}+52a^{2}b^{6}c^{2}-12ab^{8}c+b^{10}}\,\ln\left(x^{n}-\frac{-2a^{2}bc^{2}+4ab^{3}c-b^{5}+\sqrt{-16a^{5}c^{5}+68a^{4}b^{2}c^{4}-96a^{3}b^{4}c^{3}+52a^{2}b^{6}c^{2}-12ab^{8}c+b^{10}}}{2\left(2a^{2}c^{2}-4ab^{2}c+b^{4}\right)c}\right)}{2\left(4ac-b^{2}\right)a^{4}n}+\frac{\sqrt{-16a^{5}c^{5}+68a^{4}b^{2}c^{4}-96a^{3}b^{4}c^{3}+52a^{2}b^{6}c^{2}-12ab^{8}c+b^{10}}\,\ln\left(x^{n}+\frac{2a^{2}bc^{2}-4ab^{3}c+b^{5}+\sqrt{-16a^{5}c^{5}+68a^{4}b^{2}c^{4}-96a^{3}b^{4}c^{3}+52a^{2}b^{6}c^{2}-12ab^{8}c+b^{10}}}{2\left(2a^{2}c^{2}-4ab^{2}c+b^{4}\right)c}\right)}{2\left(4ac-b^{2}\right)a^{4}n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(-1-3*n)/(a+b*x^n+c*x^(2*n)),x)

[Out] 1/a^2/n/(x^n)*c-1/a^3/n/(x^n)*b^2+1/2/a^2*b/n/(x^n)^2-1/3/a/n/(x^n)^3+8/(4*a^5*c*n^2-a^4*b^2*n^2)*n^2*ln(x)*a^2*b*c^2-6/(4*a^5*c*n^2-a^4*b^2*n^2)*n^2*ln(x)*a*b^3*c+1/(4*a^5*c*n^2-a^4*b^2*n^2)*n^2*ln(x)*b^5-4/a^2/(4*a*c-b^2)/n*ln(x^n+1/2*(2*a^2*b*c^2-4*a*b^3*c+b^5+(-16*a^5*c^5+68*a^4*b^2*c^4-96*a^3*b^4*c^3+52*a^2*b^6*c^2-12*a*b^8*c+b^10)^(1/2))/c/(2*a^2*c^2-4*a*b^2*c+b^4))*b*c^2+3/a^3/(4*a*c-b^2)/n*ln(x^n+1/2*(2*a^2*b*c^2-4*a*b^3*c+b^5+(-16*a^5*c^5+68*a^4*b^2*c^4-96*a^3*b^4*c^3+52*a^2*b^6*c^2-12*a*b^8*c+b^10)^(1/2))/c/(2*a^2*c^2-4*a*b^2*c+b^4))*b^3*c-1/2/a^4/(4*a*c-b^2)/n*ln(x^n+1/2*(2*a^2*b*c^2-4*a*b^3*c+b^5+(-16*a^5*c^5+68*a^4*b^2*c^4-96*a^3*b^4*c^3+52*a^2*b^6*c^2-12*a*b^8*c+b^10)^(1/2))/c/(2*a^2*c^2-4*a*b^2*c+b^4))*b^5+1/2/a^4/(4*a*c-b^2)/n*ln(x^n+1/2*(2*a^2*b*c^2-4*a*b^3*c+b^5+(-16*a^5*c^5+68*a^4*b^2*c^4-96*a^3*b^4*c^3+52*a^2*b^6*c^2-12*a*b^8*c+b^10)^(1/2))/c/(2*a^2*c^2-4*a*b^2*c+b^4))*(-16*a^5*c^5+68*a^4*b^2*c^4-96*a^3*b^4*c^3+52*a^2*b^6*c^2-12*a*b^8*c+b^10)^(1/2)-4/a^2/(4*a*c-b^2)/n*ln(x^n-1/2*(-2*a^2*b*c^2+4*a*b^3*c-b^5+(-16*a^5*c^5+68*a^4*b^2*c^4-96*a^3*b^4*c^3+52*a^2*b^6*c^2-12*a*b^8*c+b^10)^(1/2))/c/(2*a^2*c^2-4*a*b^2*c+b^4))*b*c^2+3/a^3/(4*a*c-b^2)/n*ln(x^n-1/2*(-2*a^2*b*

```
c^2+4*a*b^3*c-b^5+(-16*a^5*c^5+68*a^4*b^2*c^4-96*a^3*b^4*c^3+52*a^2*b^6*c^2
-12*a*b^8*c+b^10)^(1/2))/c/(2*a^2*c^2-4*a*b^2*c+b^4))*b^3*c-1/2/a^4/(4*a*c-
b^2)/n*ln(x^n-1/2*(-2*a^2*b*c^2+4*a*b^3*c-b^5+(-16*a^5*c^5+68*a^4*b^2*c^4-9
6*a^3*b^4*c^3+52*a^2*b^6*c^2-12*a*b^8*c+b^10)^(1/2))/c/(2*a^2*c^2-4*a*b^2*c
+b^4))*b^5-1/2/a^4/(4*a*c-b^2)/n*ln(x^n-1/2*(-2*a^2*b*c^2+4*a*b^3*c-b^5+(-1
6*a^5*c^5+68*a^4*b^2*c^4-96*a^3*b^4*c^3+52*a^2*b^6*c^2-12*a*b^8*c+b^10)^(1/
2))/c/(2*a^2*c^2-4*a*b^2*c+b^4))*(-16*a^5*c^5+68*a^4*b^2*c^4-96*a^3*b^4*c^3
+52*a^2*b^6*c^2-12*a*b^8*c+b^10)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{3\,abx^n - 2\,a^2 - 6\left(b^2 - ac\right)x^{2\,n}}{6\,a^3nx^{3\,n}} + \int -\frac{b^3 - 2\,abc + \left(b^2c - ac^2\right)x^n}{a^3cxx^{2\,n} + a^3bxx^n + a^4x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-3*n)/(a+b*x^n+c*x^(2*n)),x, algorithm="maxima")
```

```
[Out] 1/6*(3*a*b*x^n - 2*a^2 - 6*(b^2 - a*c)*x^(2*n))/(a^3*n*x^(3*n)) + integrate
(-(b^3 - 2*a*b*c + (b^2*c - a*c^2)*x^n)/(a^3*c*x*x^(2*n) + a^3*b*x*x^n + a^
4*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^{3\,n+1}\,\left(a + b\,x^n + c\,x^{2\,n}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(3*n + 1)*(a + b*x^n + c*x^(2*n))),x)
```

```
[Out] int(1/(x^(3*n + 1)*(a + b*x^n + c*x^(2*n))), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1-3*n)/(a+b*x**n+c*x**(2*n)),x)
```

```
[Out] Timed out
```

3.480 $\int \frac{x^{-1+\frac{n}{4}}}{a+bx^n+cx^{2n}}\,dx$

Optimal. Leaf size=353

$$\frac{2\,2^{3/4}c^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{2}\,\sqrt[4]{c}\,x^{n/4}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{n\sqrt{b^2-4ac}\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}-\frac{2\,2^{3/4}c^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{2}\,\sqrt[4]{c}\,x^{n/4}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{n\sqrt{b^2-4ac}\left(\sqrt{b^2-4ac}-b\right)^{3/4}}+\frac{2\,2^{3/4}c^{3/4}\tanh^{-1}\left(\frac{\sqrt[4]{2}\,\sqrt[4]{c}\,x^{n/4}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{n\sqrt{b^2-4ac}\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}-\frac{2\,2^{3/4}c^{3/4}}{n\sqrt{b^2-4a}}$$

Rubi [A] time = 0.63, antiderivative size = 353, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {1381, 1347, 212, 208, 205}

$$\frac{2\,2^{3/4}c^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{2}\,\sqrt[4]{c}\,x^{n/4}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{n\sqrt{b^2-4ac}\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}-\frac{2\,2^{3/4}c^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{2}\,\sqrt[4]{c}\,x^{n/4}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{n\sqrt{b^2-4ac}\left(\sqrt{b^2-4ac}-b\right)^{3/4}}+\frac{2\,2^{3/4}c^{3/4}\tanh^{-1}\left(\frac{\sqrt[4]{2}\,\sqrt[4]{c}\,x^{n/4}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{n\sqrt{b^2-4ac}\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}-\frac{2\,2^{3/4}c^{3/4}\tanh^{-1}\left(\frac{\sqrt[4]{2}\,\sqrt[4]{c}\,x^{n/4}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{n\sqrt{b^2-4ac}\left(\sqrt{b^2-4ac}-b\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 + n/4)/(a + b*x^n + c*x^(2*n)),x]
```

```
[Out] (2*2^(3/4)*c^(3/4)*ArcTan[(2^(1/4)*c^(1/4)*x^(n/4))/(-b - Sqrt[b^2 - 4*a*c]
)^(1/4)])/(Sqrt[b^2 - 4*a*c]*(-b - Sqrt[b^2 - 4*a*c])^(3/4)*n) - (2*2^(3/4)
*c^(3/4)*ArcTan[(2^(1/4)*c^(1/4)*x^(n/4))/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/
(Sqrt[b^2 - 4*a*c]*(-b + Sqrt[b^2 - 4*a*c])^(3/4)*n) + (2*2^(3/4)*c^(3/4)*A
rcTanh[(2^(1/4)*c^(1/4)*x^(n/4))/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(Sqrt[b^2
 - 4*a*c]*(-b - Sqrt[b^2 - 4*a*c])^(3/4)*n) - (2*2^(3/4)*c^(3/4)*ArcTanh[(2
^(1/4)*c^(1/4)*x^(n/4))/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(Sqrt[b^2 - 4*a*c]
*(-b + Sqrt[b^2 - 4*a*c])^(3/4)*n)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 1347

```
Int[((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^(-1), x_Symbol] :> With[{q
= Rt[b^2 - 4*a*c, 2]}, Dist[c/q, Int[1/(b/2 - q/2 + c*x^n), x], x] - Dist[c
/q, Int[1/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*
n] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1381

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> Dist[1/(m + 1), Subst[Int[(a + b*x^Simplify[n/(m + 1)] + c*x^Simplify[(
2*n)/(m + 1)])^p, x], x, x^(m + 1)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&
```

```
EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IntegerQ[Simplify[n/(m + 1)]] &&  !I
ntegerQ[n]
```

Rubi steps

$$\int \frac{x^{-1+\frac{n}{4}}}{a+bx^n+cx^{2n}}\,dx = \frac{4\,\text{Subst}\left(\int \frac{1}{a+bx^4+cx^8}\,dx, x, x^{n/4}\right)}{n}$$

$$= \frac{(4c)\,\text{Subst}\left(\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^4}\,dx, x, x^{n/4}\right)}{\sqrt{b^2-4ac}\,n} - \frac{(4c)\,\text{Subst}\left(\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^4}\,dx, x, x^{n/4}\right)}{\sqrt{b^2-4ac}\,n}$$

$$= \frac{(4c)\,\text{Subst}\left(\int \frac{1}{\sqrt{-b-\sqrt{b^2-4ac}}-\sqrt{2}\,\sqrt{c}\,x^2}\,dx, x, x^{n/4}\right)}{\sqrt{b^2-4ac}\,\sqrt{-b-\sqrt{b^2-4ac}}\,n} + \frac{(4c)\,\text{Subst}\left(\int \frac{1}{\sqrt{-b-\sqrt{b^2-4ac}}+\sqrt{2}\,\sqrt{c}\,x^2}\,d\right.}{\sqrt{b^2-4ac}\,\sqrt{-b-\sqrt{b^2-4ac}}}$$

$$= \frac{2\,2^{3/4}c^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{2}\,\sqrt[4]{c}\,x^{n/4}}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\,\left(-b-\sqrt{b^2-4ac}\right)^{3/4}n} - \frac{2\,2^{3/4}c^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{2}\,\sqrt[4]{c}\,x^{n/4}}{\sqrt[4]{-b+\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\,\left(-b+\sqrt{b^2-4ac}\right)^{3/4}n} + \frac{2\,2^{3/4}c^{3/4}\tan}{\sqrt{b^2-4ac}\,(-}$$

Mathematica [A] time = 0.90, size = 340, normalized size = 0.96

$$\frac{2\,2^{3/4}c^{3/4}\left(-\frac{\sqrt[4]{-\sqrt{b^2-4ac}-b}\,\tan^{-1}\left(\frac{\sqrt[4]{2}\,\sqrt[4]{c}\,x^{n/4}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{b\sqrt{b^2-4ac}-4ac+b^2}-\frac{\tan^{-1}\left(\frac{\sqrt[4]{2}\,\sqrt[4]{c}\,x^{n/4}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{\sqrt{b^2-4ac}\left(\sqrt{b^2-4ac}-b\right)^{3/4}}-\frac{\sqrt[4]{-\sqrt{b^2-4ac}-b}\,\tanh^{-1}\left(\frac{\sqrt[4]{2}\,\sqrt[4]{c}\,x^{n/4}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{b\sqrt{b^2-4ac}-4ac+b^2}-\frac{\tanh^{-1}\left(\frac{\sqrt[4]{2}\,\sqrt[4]{c}\,x^{n/4}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{\sqrt{b^2-4ac}\left(\sqrt{b^2-4ac}-b\right)^{3/4}}\right)}{n}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 + n/4)/(a + b*x^n + c*x^(2*n)),x]
```

```
[Out] (2*2^(3/4)*c^(3/4)*(-(((-b - Sqrt[b^2 - 4*a*c])^(1/4)*ArcTan[(2^(1/4)*c^(1/
4)*x^(n/4))/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(b^2 - 4*a*c + b*Sqrt[b^2 - 4*
a*c])) - ArcTan[(2^(1/4)*c^(1/4)*x^(n/4))/(-b + Sqrt[b^2 - 4*a*c])^(1/4)]/(
Sqrt[b^2 - 4*a*c]*(-b + Sqrt[b^2 - 4*a*c])^(3/4)) - ((-b - Sqrt[b^2 - 4*a*c
])^(1/4)*ArcTanh[(2^(1/4)*c^(1/4)*x^(n/4))/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])
/(b^2 - 4*a*c + b*Sqrt[b^2 - 4*a*c]) - ArcTanh[(2^(1/4)*c^(1/4)*x^(n/4))/(-
b + Sqrt[b^2 - 4*a*c])^(1/4)]/(Sqrt[b^2 - 4*a*c]*(-b + Sqrt[b^2 - 4*a*c])^(
3/4))))/n
```

IntegrateAlgebraic [F] time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{x^{-1+\frac{n}{4}}}{a+bx^n+cx^{2n}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^(-1 + n/4)/(a + b*x^n + c*x^(2*n)),x]
```

```
[Out] Defer[IntegrateAlgebraic][x^(-1 + n/4)/(a + b*x^n + c*x^(2*n)), x]
```

fricas [B] time = 2.56, size = 4426, normalized size = 12.54

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(-1+1/4*n)/(a+b*x^n+c*x^(2*n)),x, algorithm="fricas")

[Out] -2*sqrt(2)*sqrt(sqrt(2)*sqrt(-((a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*n^4*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/((a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)*n^8)) + b^3 - 3*a*b*c)/((a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*n^4)))*arctan(1/16*sqrt(2)*(2*sqrt(2)*((a^3*b^10*c - 15*a^4*b^8*c^2 + 86*a^5*b^6*c^3 - 232*a^6*b^4*c^4 + 288*a^7*b^2*c^5 - 128*a^8*c^6)*n^7*x*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/((a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)*n^8)) - (b^9*c - 10*a*b^7*c^2 + 33*a^2*b^5*c^3 - 40*a^3*b^3*c^4 + 16*a^4*b*c^5)*n^3*x)*x^(1/4*n - 1)*sqrt(-((a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*n^4*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/((a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)*n^8)) + b^3 - 3*a*b*c)/((a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*n^4)) + sqrt(2)*((a^3*b^8 - 14*a^4*b^6*c + 72*a^5*b^4*c^2 - 160*a^6*b^2*c^3 + 128*a^7*c^4)*n^7*x*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/((a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)*n^8)) - (b^7 - 9*a*b^5*c + 24*a^2*b^3*c^2 - 16*a^3*b*c^3)*n^3*x)*sqrt((4*(b^4*c^2 - 2*a*b^2*c^3 + a^2*c^4)*x^2*x^(1/2*n - 2) - sqrt(2)*((a^3*b^9 - 13*a^4*b^7*c + 60*a^5*b^5*c^2 - 112*a^6*b^3*c^3 + 64*a^7*b*c^4)*n^6*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/((a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)*n^8)) - (b^8 - 8*a*b^6*c + 21*a^2*b^4*c^2 - 22*a^3*b^2*c^3 + 8*a^4*c^4)*n^2)*sqrt(-((a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*n^4*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/((a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)*n^8)) + b^3 - 3*a*b*c)/((a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*n^4)))/x^2)*sqrt(-((a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*n^4*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/((a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)*n^8)) + b^3 - 3*a*b*c)/((a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*n^4)))*sqrt(sqrt(2)*sqrt(-((a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*n^4*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/((a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)*n^8)) + b^3 - 3*a*b*c)/((a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*n^4)))/(b^4*c^3 - 2*a*b^2*c^4 + a^2*c^5)) + 2*sqrt(2)*sqrt(sqrt(2)*sqrt(((a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*n^4*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/((a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)*n^8)) - b^3 + 3*a*b*c)/((a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*n^4)))*arctan(1/8*(2*((a^3*b^10*c - 15*a^4*b^8*c^2 + 86*a^5*b^6*c^3 - 232*a^6*b^4*c^4 + 288*a^7*b^2*c^5 - 128*a^8*c^6)*n^7*x*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/((a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)*n^8)) + (b^9*c - 10*a*b^7*c^2 + 33*a^2*b^5*c^3 - 40*a^3*b^3*c^4 + 16*a^4*b*c^5)*n^3*x)*x^(1/4*n - 1)*sqrt(sqrt(2)*sqrt(((a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*n^4*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/((a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)*n^8)) - b^3 + 3*a*b*c)/((a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*n^4)))*sqrt(((a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*n^4*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/((a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)*n^8)) - b^3 + 3*a*b*c)/((a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*n^4)) + ((a^3*b^8 - 14*a^4*b^6*c + 72*a^5*b^4*c^2 - 160*a^6*b^2*c^3 + 128*a^7*c^4)*n^7*x*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/((a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)*n^8)) + (b^7 - 9*a*b^5*c + 24*a^2*b^3*c^2 - 16*a^3*b*c^3)*n^3*x)*sqrt(sqrt(2)*sqrt(((a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*n^4*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/((a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)*n^8)) - b^3 + 3*a*b*c)/((a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*n^4)))*sqrt((4*(b^4*c^2 - 2*a*b^2*c^3 + a^2*c^4)*x^2*x^(1/2*n - 2) + sqrt(2)*((a^3*b^9 - 13*a^4*b^7*c + 60*a^5*b^5*c^2 - 112*a^6*b^3*c^3 + 64*a^7*b*c^4)*n^6*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/((a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)*n^8)) + (b^8 - 8*a*b^6*c + 21*a^2*b^4*c^2 - 22*a^3*b^2*c^3 + 8*a^4*c^4)*n^2)*sqrt(((a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*n^4*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/((a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)*n^8)) - b^3 + 3*a*b*c)/((a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*n^4)))/x^2)*sqrt(((a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*n^4*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/((a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)*n^8)) - b^3 + 3*a*b*c)/((a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*n^4)))/(b^4*c^3 - 2*a*b^2*c^4 + a^2*c^5)) + 1/2*sqrt(2)*sqrt(sqrt(2)*sqrt(-((a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*n^4*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/((a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)*n^8)) + b^3 - 3*a*b*c)/(

```
(a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*n^4)))*log(-(4*(b^2*c - a*c^2)*x*x^(1/
4*n - 1) + sqrt(2)*((a^3*b^5 - 8*a^4*b^3*c + 16*a^5*b*c^2)*n^5*sqrt((b^4 -
2*a*b^2*c + a^2*c^2)/((a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3
)*n^8)) - (b^4 - 5*a*b^2*c + 4*a^2*c^2)*n)*sqrt(sqrt(2)*sqrt(-((a^3*b^4 - 8
*a^4*b^2*c + 16*a^5*c^2)*n^4*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/((a^6*b^6 - 1
2*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)*n^8)) + b^3 - 3*a*b*c)/((a^3*b^4
 - 8*a^4*b^2*c + 16*a^5*c^2)*n^4))))/x) - 1/2*sqrt(2)*sqrt(sqrt(2)*sqrt(-((
a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*n^4*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/((
a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)*n^8)) + b^3 - 3*a*b*c
)/((a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*n^4)))*log(-(4*(b^2*c - a*c^2)*x*x^
(1/4*n - 1) - sqrt(2)*((a^3*b^5 - 8*a^4*b^3*c + 16*a^5*b*c^2)*n^5*sqrt((b^4
 - 2*a*b^2*c + a^2*c^2)/((a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*
c^3)*n^8)) - (b^4 - 5*a*b^2*c + 4*a^2*c^2)*n)*sqrt(sqrt(2)*sqrt(-((a^3*b^4
- 8*a^4*b^2*c + 16*a^5*c^2)*n^4*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/((a^6*b^6
- 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)*n^8)) + b^3 - 3*a*b*c)/((a^3*
b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*n^4))))/x) - 1/2*sqrt(2)*sqrt(sqrt(2)*sqrt(
((a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*n^4*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/
((a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)*n^8)) - b^3 + 3*a*b
*c)/((a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*n^4)))*log(-(4*(b^2*c - a*c^2)*x*
x^(1/4*n - 1) + sqrt(2)*((a^3*b^5 - 8*a^4*b^3*c + 16*a^5*b*c^2)*n^5*sqrt((b
^4 - 2*a*b^2*c + a^2*c^2)/((a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^
9*c^3)*n^8)) + (b^4 - 5*a*b^2*c + 4*a^2*c^2)*n)*sqrt(sqrt(2)*sqrt(((a^3*b^4
 - 8*a^4*b^2*c + 16*a^5*c^2)*n^4*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/((a^6*b^6
 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)*n^8)) - b^3 + 3*a*b*c)/((a^3
*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*n^4))))/x) + 1/2*sqrt(2)*sqrt(sqrt(2)*sqrt
(((a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*n^4*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)
/((a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)*n^8)) - b^3 + 3*a*
b*c)/((a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*n^4)))*log(-(4*(b^2*c - a*c^2)*x
*x^(1/4*n - 1) - sqrt(2)*((a^3*b^5 - 8*a^4*b^3*c + 16*a^5*b*c^2)*n^5*sqrt((
b^4 - 2*a*b^2*c + a^2*c^2)/((a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a
^9*c^3)*n^8)) + (b^4 - 5*a*b^2*c + 4*a^2*c^2)*n)*sqrt(sqrt(2)*sqrt(((a^3*b^
4 - 8*a^4*b^2*c + 16*a^5*c^2)*n^4*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/((a^6*b^
6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)*n^8)) - b^3 + 3*a*b*c)/((a^
3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*n^4))))/x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{\frac{1}{4}n-1}}{cx^{2n}+bx^{n}+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+1/4*n)/(a+b*x^n+c*x^(2*n)),x, algorithm="giac")
```

```
[Out] integrate(x^(1/4*n - 1)/(c*x^(2*n) + b*x^n + a), x)
```

maple [C] time = 0.75, size = 280, normalized size = 0.79

$$\mathrm{RootOf}\left(\left(256a^{7}c^{4}n^{8}-256a^{6}b^{2}c^{3}n^{8}+96a^{5}b^{4}c^{2}n^{8}-16a^{4}b^{6}c\,n^{8}+a^{3}b^{8}n^{8}\right)\_Z^{8}+\left(-48a^{3}b\,c^{3}n^{4}+40a^{2}b^{3}c^{2}n^{4}-11a\,b^{5}c\,n^{4}+b^{7}n^{4}\right)\_Z^{4}+c^{3}\right)\ln\left(\left(\frac{16a^{5}b\,c^{2}n^{5}}{a\,c^{2}-b^{2}c}-\frac{8a^{4}b^{3}c\,n^{5}}{a\,c^{2}-b^{2}c}+\frac{a^{3}b^{5}n^{5}}{a\,c^{2}-b^{2}c}\right)\mathrm{RootOf}\left(\left(256a^{7}c^{4}n^{8}-256a^{6}b^{2}c^{3}n^{8}+96a^{5}b^{4}c^{2}n^{8}-16a^{4}b^{6}c\,n^{8}+a^{3}b^{8}n^{8}\right)\_Z^{8}+\left(-48a^{3}b\,c^{3}n^{4}+40a^{2}b^{3}c^{2}n^{4}-11a\,b^{5}c\,n^{4}+b^{7}n^{4}\right)\_Z^{4}+c^{3}\right)^{5}+\left(\frac{2a^{2}c^{2}n}{a\,c^{2}-b^{2}c}-\frac{4a\,b^{2}cn}{a\,c^{2}-b^{2}c}+\frac{b^{4}n}{a\,c^{2}-b^{2}c}\right)\mathrm{RootOf}\left(\left(256a^{7}c^{4}n^{8}-256a^{6}b^{2}c^{3}n^{8}+96a^{5}b^{4}c^{2}n^{8}-16a^{4}b^{6}c\,n^{8}+a^{3}b^{8}n^{8}\right)\_Z^{8}+\left(-48a^{3}b\,c^{3}n^{4}+40a^{2}b^{3}c^{2}n^{4}-11a\,b^{5}c\,n^{4}+b^{7}n^{4}\right)\_Z^{4}+c^{3}\right)+x^{\frac{n}{4}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1+1/4*n)/(a+b*x^n+c*x^(2*n)),x)
```

```
[Out] sum(_R*ln(x^(1/4*n)+(16/(a*c^2-b^2*c)*n^5*b*a^5*c^2-8/(a*c^2-b^2*c)*n^5*b^3
*a^4*c+1/(a*c^2-b^2*c)*n^5*b^5*a^3)*_R^5+(2/(a*c^2-b^2*c)*n*a^2*c^2-4/(a*c^
2-b^2*c)*n*b^2*a*c+1/(a*c^2-b^2*c)*n*b^4)*_R),_R=RootOf((256*a^7*c^4*n^8-25
6*a^6*b^2*c^3*n^8+96*a^5*b^4*c^2*n^8-16*a^4*b^6*c*n^8+a^3*b^8*n^8)*_Z^8+(-4
8*a^3*b*c^3*n^4+40*a^2*b^3*c^2*n^4-11*a*b^5*c*n^4+b^7*n^4)*_Z^4+c^3))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{\frac{1}{4}n-1}}{cx^{2n} + bx^n + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+1/4*n)/(a+b*x^n+c*x^(2*n)),x, algorithm="maxima")
```

```
[Out] integrate(x^(1/4*n - 1)/(c*x^(2*n) + b*x^n + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^{\frac{n}{4}-1}}{a + b\,x^n + c\,x^{2\,n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(n/4 - 1)/(a + b*x^n + c*x^(2*n)),x)
```

```
[Out] int(x^(n/4 - 1)/(a + b*x^n + c*x^(2*n)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{\frac{n}{4}-1}}{a + bx^n + cx^{2n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+1/4*n)/(a+b*x**n+c*x**(2*n)),x)
```

```
[Out] Integral(x**(n/4 - 1)/(a + b*x**n + c*x**(2*n)), x)
```

3.481 $\int \frac{x^{-1+\frac{n}{3}}}{a+bx^n+cx^{2n}}\,dx$

Optimal. Leaf size=610

$$\frac{2^{2/3}c^{2/3}\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}\,x^{n/3}\right)}{n\sqrt{b^2-4ac}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}-\frac{2^{2/3}c^{2/3}\log\left(\sqrt[3]{\sqrt{b^2-4ac}+b}+\sqrt[3]{2}\sqrt[3]{c}\,x^{n/3}\right)}{n\sqrt{b^2-4ac}\left(\sqrt{b^2-4ac}+b\right)^{2/3}}-\frac{c^{2/3}\log\left(-\sqrt[3]{2}\sqrt[3]{c}\right.}{}$$

Rubi [A] time = 1.15, antiderivative size = 610, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 8, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {1381, 1347, 200, 31, 634, 617, 204, 628}

$$\frac{2^{2/3}c^{2/3}\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}x^{n/3}\right)}{n\sqrt{b^2-4ac}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}-\frac{2^{2/3}c^{2/3}\log\left(\sqrt[3]{\sqrt{b^2-4ac}+b}+\sqrt[3]{2}\sqrt[3]{c}x^{n/3}\right)}{n\sqrt{b^2-4ac}\left(\sqrt{b^2-4ac}+b\right)^{2/3}}-\frac{c^{2/3}\log\left(-\sqrt[3]{2}\sqrt[3]{c}x^{n/3}\sqrt[3]{b-\sqrt{b^2-4ac}}+\left(b-\sqrt{b^2-4ac}\right)^{2/3}+2^{2/3}c^{2/3}x^{2n/3}\right)}{\sqrt[3]{2}n\sqrt{b^2-4ac}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}+\frac{c^{2/3}\log\left(-\sqrt[3]{2}\sqrt[3]{c}x^{n/3}\sqrt[3]{\sqrt{b^2-4ac}+b}+\left(\sqrt{b^2-4ac}+b\right)^{2/3}+2^{2/3}c^{2/3}x^{2n/3}\right)}{\sqrt[3]{2}n\sqrt{b^2-4ac}\left(\sqrt{b^2-4ac}+b\right)^{2/3}}-\frac{2^{2/3}\sqrt{3}c^{2/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x^{n/3}}{\sqrt[3]{b-\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{n\sqrt{b^2-4ac}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}+\frac{2^{2/3}\sqrt{3}c^{2/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x^{n/3}}{\sqrt[3]{\sqrt{b^2-4ac}+b}}}{\sqrt{3}}\right)}{n\sqrt{b^2-4ac}\left(\sqrt{b^2-4ac}+b\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 + n/3)/(a + b*x^n + c*x^(2*n)),x]
```

```
[Out] -((2^(2/3)*Sqrt[3]*c^(2/3)*ArcTan[(1 - (2*2^(1/3)*c^(1/3)*x^(n/3))/(b - Sqr
t[b^2 - 4*a*c])^(1/3))/Sqrt[3]])/(Sqrt[b^2 - 4*a*c]*(b - Sqrt[b^2 - 4*a*c])
^(2/3)*n)) + (2^(2/3)*Sqrt[3]*c^(2/3)*ArcTan[(1 - (2*2^(1/3)*c^(1/3)*x^(n/3
))/(b + Sqrt[b^2 - 4*a*c])^(1/3))/Sqrt[3]])/(Sqrt[b^2 - 4*a*c]*(b + Sqrt[b^
2 - 4*a*c])^(2/3)*n) + (2^(2/3)*c^(2/3)*Log[(b - Sqrt[b^2 - 4*a*c])^(1/3) +
 2^(1/3)*c^(1/3)*x^(n/3)])/(Sqrt[b^2 - 4*a*c]*(b - Sqrt[b^2 - 4*a*c])^(2/3)
*n) - (2^(2/3)*c^(2/3)*Log[(b + Sqrt[b^2 - 4*a*c])^(1/3) + 2^(1/3)*c^(1/3)*
x^(n/3)])/(Sqrt[b^2 - 4*a*c]*(b + Sqrt[b^2 - 4*a*c])^(2/3)*n) - (c^(2/3)*Lo
g[(b - Sqrt[b^2 - 4*a*c])^(2/3) - 2^(1/3)*c^(1/3)*(b - Sqrt[b^2 - 4*a*c])^(
1/3)*x^(n/3) + 2^(2/3)*c^(2/3)*x^((2*n)/3)])/(2^(1/3)*Sqrt[b^2 - 4*a*c]*(b
- Sqrt[b^2 - 4*a*c])^(2/3)*n) + (c^(2/3)*Log[(b + Sqrt[b^2 - 4*a*c])^(2/3)
- 2^(1/3)*c^(1/3)*(b + Sqrt[b^2 - 4*a*c])^(1/3)*x^(n/3) + 2^(2/3)*c^(2/3)*x
^((2*n)/3)])/(2^(1/3)*Sqrt[b^2 - 4*a*c]*(b + Sqrt[b^2 - 4*a*c])^(2/3)*n)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
```

```
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1347

```
Int[((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^(-1), x_Symbol] :> With[{q
= Rt[b^2 - 4*a*c, 2]}, Dist[c/q, Int[1/(b/2 - q/2 + c*x^n), x], x] - Dist[c
/q, Int[1/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*
n] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1381

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> Dist[1/(m + 1), Subst[Int[(a + b*x^Simplify[n/(m + 1)] + c*x^Simplify[(
2*n)/(m + 1)])^p, x], x, x^(m + 1)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&
EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IntegerQ[Simplify[n/(m + 1)]] &&  !I
ntegerQ[n]
```

Rubi steps

$$\int \frac{x^{-1+\frac{n}{3}}}{a+bx^n+cx^{2n}}\,dx = \frac{3\,\text{Subst}\left(\int \frac{1}{a+bx^3+cx^6}\,dx, x, x^{n/3}\right)}{n}$$

$$= \frac{(3c)\,\text{Subst}\left(\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^3}\,dx, x, x^{n/3}\right)}{\sqrt{b^2-4ac}\,n} - \frac{(3c)\,\text{Subst}\left(\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^3}\,dx, x, x^{n/3}\right)}{\sqrt{b^2-4ac}\,n}$$

$$= \frac{\left(2^{2/3}c\right)\text{Subst}\left(\int \frac{1}{\frac{\sqrt[3]{b-\sqrt{b^2-4ac}}}{\sqrt[3]{2}}+\sqrt[3]{c}\,x}\,dx, x, x^{n/3}\right)}{\sqrt{b^2-4ac}\left(b-\sqrt{b^2-4ac}\right)^{2/3}n} + \frac{\left(2^{2/3}c\right)\text{Subst}\left(\int \frac{2^{2/3}\sqrt[3]{b-\sqrt{b^2-4ac}} - \cdots}{\frac{\left(b-\sqrt{b^2-4ac}\right)^{2/3}}{2^{2/3}} - \frac{\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-\cdots}}}{\sqrt[3]{2}}\cdots}\cdots\right)}{\sqrt{b^2-4ac}\left(b-\sqrt{b^2-4a\cdots}\right.\cdots}$$

$$= \frac{2^{2/3}c^{2/3}\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}\,x^{n/3}\right)}{\sqrt{b^2-4ac}\left(b-\sqrt{b^2-4ac}\right)^{2/3}n} - \frac{2^{2/3}c^{2/3}\log\left(\sqrt[3]{b+\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{\cdots}\right.}{\sqrt{b^2-4ac}\left(b+\sqrt{b^2-4ac}\right)^{2/3}n}\cdots$$

$$= \frac{2^{2/3}c^{2/3}\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}\,x^{n/3}\right)}{\sqrt{b^2-4ac}\left(b-\sqrt{b^2-4ac}\right)^{2/3}n} - \frac{2^{2/3}c^{2/3}\log\left(\sqrt[3]{b+\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{\cdots}\right.}{\sqrt{b^2-4ac}\left(b+\sqrt{b^2-4ac}\right)^{2/3}n}\cdots$$

$$= -\frac{2^{2/3}\sqrt{3}\,c^{2/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}\,x^{n/3}}{\sqrt[3]{b-\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{\sqrt{b^2-4ac}\left(b-\sqrt{b^2-4ac}\right)^{2/3}n} + \frac{2^{2/3}\sqrt{3}\,c^{2/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}\,x^{n/3}}{\sqrt[3]{b+\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{\sqrt{b^2-4ac}\left(b+\sqrt{b^2-4ac}\right)^{2/3}n} + \frac{2^{2/3}c^{2/3}\log\left(\sqrt[3]{\cdots}\right.}{\sqrt{b^2-4\cdots}}\cdots$$

<u>**Mathematica**</u> **[A]** time = 0.76, size = 526, normalized size = 0.86

$$\frac{c^{2/3}\left(-\left(\sqrt{b^2-4ac}+b\right)^{2/3}\log\left(-\sqrt[3]{2}\sqrt[3]{c}\,x^{n/3}\sqrt[3]{b-\sqrt{b^2-4ac}}+\left(b-\sqrt{b^2-4ac}\right)^{2/3}+2^{2/3}c^{2/3}x^{2n/3}\right)+\left(b-\sqrt{b^2-4ac}\right)^{2/3}\log\left(-\sqrt[3]{2}\sqrt[3]{c}\,x^{n/3}\sqrt[3]{\sqrt{b^2-4ac}+b}+\left(\sqrt{b^2-4ac}+b\right)^{2/3}+2^{2/3}c^{2/3}x^{2n/3}\right)+2\left(\sqrt{b^2-4ac}+b\right)^{2/3}\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}\,x^{n/3}\right)-2\left(b-\sqrt{b^2-4ac}\right)^{2/3}\log\left(\sqrt[3]{\sqrt{b^2-4ac}+b}+\sqrt[3]{2}\sqrt[3]{c}\,x^{n/3}\right)-2\sqrt{3}\left(\sqrt{b^2-4ac}+b\right)^{2/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}\,x^{n/3}}{\sqrt[3]{b-\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)+2\sqrt{3}\left(b-\sqrt{b^2-4ac}\right)^{2/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}\,x^{n/3}}{\sqrt[3]{\sqrt{b^2-4ac}+b}}}{\sqrt{3}}\right)\right)}{\sqrt[3]{2}\,n\sqrt{b^2-4ac}\left(b-\sqrt{b^2-4ac}\right)^{2/3}\left(\sqrt{b^2-4ac}+b\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 + n/3)/(a + b*x^n + c*x^(2*n)),x]
```

```
[Out] (c^(2/3)*(-2*Sqrt[3]*(b + Sqrt[b^2 - 4*a*c])^(2/3)*ArcTan[(1 - (2*2^(1/3)*c
^(1/3)*x^(n/3))/(b - Sqrt[b^2 - 4*a*c])^(1/3))/Sqrt[3]] + 2*Sqrt[3]*(b - Sq
rt[b^2 - 4*a*c])^(2/3)*ArcTan[(1 - (2*2^(1/3)*c^(1/3)*x^(n/3))/(b + Sqrt[b^
2 - 4*a*c])^(1/3))/Sqrt[3]] + 2*(b + Sqrt[b^2 - 4*a*c])^(2/3)*Log[(b - Sqrt
[b^2 - 4*a*c])^(1/3) + 2^(1/3)*c^(1/3)*x^(n/3)] - 2*(b - Sqrt[b^2 - 4*a*c])
^(2/3)*Log[(b + Sqrt[b^2 - 4*a*c])^(1/3) + 2^(1/3)*c^(1/3)*x^(n/3)] - (b +
Sqrt[b^2 - 4*a*c])^(2/3)*Log[(b - Sqrt[b^2 - 4*a*c])^(2/3) - 2^(1/3)*c^(1/3
)*(b - Sqrt[b^2 - 4*a*c])^(1/3)*x^(n/3) + 2^(2/3)*c^(2/3)*x^((2*n)/3)] + (b
 - Sqrt[b^2 - 4*a*c])^(2/3)*Log[(b + Sqrt[b^2 - 4*a*c])^(2/3) - 2^(1/3)*c^(
1/3)*(b + Sqrt[b^2 - 4*a*c])^(1/3)*x^(n/3) + 2^(2/3)*c^(2/3)*x^((2*n)/3)]))
/(2^(1/3)*Sqrt[b^2 - 4*a*c]*(b - Sqrt[b^2 - 4*a*c])^(2/3)*(b + Sqrt[b^2 - 4
*a*c])^(2/3)*n)
```

<u>**IntegrateAlgebraic**</u> **[F]** time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{x^{-1+\frac{n}{3}}}{a+bx^n+cx^{2n}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^(-1 + n/3)/(a + b*x^n + c*x^(2*n)),x]
```

```
[Out] Defer[IntegrateAlgebraic][x^(-1 + n/3)/(a + b*x^n + c*x^(2*n)), x]
```

fricas [B] time = 1.98, size = 4699, normalized size = 7.70

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+1/3*n)/(a+b*x^n+c*x^(2*n)),x, algorithm="fricas")
```

```
[Out] 2*sqrt(3)*(1/2)^(1/3)*(((a^2*b^2 - 4*a^3*c)*n^3*sqrt((b^4 - 4*a*b^2*c + 4*a
^2*c^2)/((a^4*b^6 - 12*a^5*b^4*c + 48*a^6*b^2*c^2 - 64*a^7*c^3)*n^6)) + b)/
((a^2*b^2 - 4*a^3*c)*n^3))^(1/3)*arctan(-1/6*(2*(1/2)^(2/3)*(sqrt(3)*(a^2*b
^8*c - 14*a^3*b^6*c^2 + 72*a^4*b^4*c^3 - 160*a^5*b^2*c^4 + 128*a^6*c^5)*n^5
*x*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/((a^4*b^6 - 12*a^5*b^4*c + 48*a^6*b^2
*c^2 - 64*a^7*c^3)*n^6)) - sqrt(3)*(b^7*c - 8*a*b^5*c^2 + 20*a^2*b^3*c^3 -
16*a^3*b*c^4)*n^2*x)*x^(1/3*n - 1)*(((a^2*b^2 - 4*a^3*c)*n^3*sqrt((b^4 - 4*
a*b^2*c + 4*a^2*c^2)/((a^4*b^6 - 12*a^5*b^4*c + 48*a^6*b^2*c^2 - 64*a^7*c^3
)*n^6)) + b)/((a^2*b^2 - 4*a^3*c)*n^3))^(2/3) + sqrt(2)*(1/2)^(2/3)*(sqrt(3
)*(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3)*n^5*x*sqrt((b^4 -
4*a*b^2*c + 4*a^2*c^2)/((a^4*b^6 - 12*a^5*b^4*c + 48*a^6*b^2*c^2 - 64*a^7*c
^3)*n^6)) - sqrt(3)*(b^5 - 6*a*b^3*c + 8*a^2*b*c^2)*n^2*x)*sqrt((2*(b^4*c^2
 - 4*a*b^2*c^3 + 4*a^2*c^4)*x^2*x^(2/3*n - 2) - (1/2)^(1/3)*((a^2*b^7*c - 1
0*a^3*b^5*c^2 + 32*a^4*b^3*c^3 - 32*a^5*b*c^4)*n^4*x*sqrt((b^4 - 4*a*b^2*c
+ 4*a^2*c^2)/((a^4*b^6 - 12*a^5*b^4*c + 48*a^6*b^2*c^2 - 64*a^7*c^3)*n^6))
- (b^6*c - 8*a*b^4*c^2 + 20*a^2*b^2*c^3 - 16*a^3*c^4)*n*x)*x^(1/3*n - 1)*((
(a^2*b^2 - 4*a^3*c)*n^3*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/((a^4*b^6 - 12*a
^5*b^4*c + 48*a^6*b^2*c^2 - 64*a^7*c^3)*n^6)) + b)/((a^2*b^2 - 4*a^3*c)*n^3
))^(1/3) - (1/2)^(2/3)*((a^2*b^9 - 14*a^3*b^7*c + 72*a^4*b^5*c^2 - 160*a^5*
b^3*c^3 + 128*a^6*b*c^4)*n^5*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/((a^4*b^6 -
 12*a^5*b^4*c + 48*a^6*b^2*c^2 - 64*a^7*c^3)*n^6)) - (b^8 - 10*a*b^6*c + 36
*a^2*b^4*c^2 - 56*a^3*b^2*c^3 + 32*a^4*c^4)*n^2)*(((a^2*b^2 - 4*a^3*c)*n^3*
sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/((a^4*b^6 - 12*a^5*b^4*c + 48*a^6*b^2*c^
2 - 64*a^7*c^3)*n^6)) + b)/((a^2*b^2 - 4*a^3*c)*n^3))^(2/3))/x^2)*(((a^2*b^
2 - 4*a^3*c)*n^3*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/((a^4*b^6 - 12*a^5*b^4*
c + 48*a^6*b^2*c^2 - 64*a^7*c^3)*n^6)) + b)/((a^2*b^2 - 4*a^3*c)*n^3))^(2/3
) + 2*sqrt(3)*(b^4*c^2 - 4*a*b^2*c^3 + 4*a^2*c^4))/(b^4*c^2 - 4*a*b^2*c^3 +
 4*a^2*c^4)) - 2*sqrt(3)*(1/2)^(1/3)*(-((a^2*b^2 - 4*a^3*c)*n^3*sqrt((b^4 -
 4*a*b^2*c + 4*a^2*c^2)/((a^4*b^6 - 12*a^5*b^4*c + 48*a^6*b^2*c^2 - 64*a^7*
c^3)*n^6)) - b)/((a^2*b^2 - 4*a^3*c)*n^3))^(1/3)*arctan(-1/6*(2*(1/2)^(2/3)
*(sqrt(3)*(a^2*b^8*c - 14*a^3*b^6*c^2 + 72*a^4*b^4*c^3 - 160*a^5*b^2*c^4 +
128*a^6*c^5)*n^5*x*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/((a^4*b^6 - 12*a^5*b^
4*c + 48*a^6*b^2*c^2 - 64*a^7*c^3)*n^6)) + sqrt(3)*(b^7*c - 8*a*b^5*c^2 + 2
0*a^2*b^3*c^3 - 16*a^3*b*c^4)*n^2*x)*x^(1/3*n - 1)*(-((a^2*b^2 - 4*a^3*c)*n
^3*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/((a^4*b^6 - 12*a^5*b^4*c + 48*a^6*b^2
*c^2 - 64*a^7*c^3)*n^6)) - b)/((a^2*b^2 - 4*a^3*c)*n^3))^(2/3) + sqrt(2)*(1
/2)^(2/3)*(sqrt(3)*(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3)*n
^5*x*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/((a^4*b^6 - 12*a^5*b^4*c + 48*a^6*b
^2*c^2 - 64*a^7*c^3)*n^6)) + sqrt(3)*(b^5 - 6*a*b^3*c + 8*a^2*b*c^2)*n^2*x)
*sqrt((2*(b^4*c^2 - 4*a*b^2*c^3 + 4*a^2*c^4)*x^2*x^(2/3*n - 2) + (1/2)^(1/3
)*((a^2*b^7*c - 10*a^3*b^5*c^2 + 32*a^4*b^3*c^3 - 32*a^5*b*c^4)*n^4*x*sqrt(
(b^4 - 4*a*b^2*c + 4*a^2*c^2)/((a^4*b^6 - 12*a^5*b^4*c + 48*a^6*b^2*c^2 - 6
4*a^7*c^3)*n^6)) + (b^6*c - 8*a*b^4*c^2 + 20*a^2*b^2*c^3 - 16*a^3*c^4)*n*x)
*x^(1/3*n - 1)*(-((a^2*b^2 - 4*a^3*c)*n^3*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2
)/((a^4*b^6 - 12*a^5*b^4*c + 48*a^6*b^2*c^2 - 64*a^7*c^3)*n^6)) - b)/((a^2*
b^2 - 4*a^3*c)*n^3))^(1/3) + (1/2)^(2/3)*((a^2*b^9 - 14*a^3*b^7*c + 72*a^4*
b^5*c^2 - 160*a^5*b^3*c^3 + 128*a^6*b*c^4)*n^5*sqrt((b^4 - 4*a*b^2*c + 4*a^
2*c^2)/((a^4*b^6 - 12*a^5*b^4*c + 48*a^6*b^2*c^2 - 64*a^7*c^3)*n^6)) + (b^8
```

```
 - 10*a*b^6*c + 36*a^2*b^4*c^2 - 56*a^3*b^2*c^3 + 32*a^4*c^4)*n^2)*(-((a^2*
b^2 - 4*a^3*c)*n^3*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/((a^4*b^6 - 12*a^5*b^
4*c + 48*a^6*b^2*c^2 - 64*a^7*c^3)*n^6)) - b)/((a^2*b^2 - 4*a^3*c)*n^3))^(2
/3))/x^2)*(-((a^2*b^2 - 4*a^3*c)*n^3*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/((a
^4*b^6 - 12*a^5*b^4*c + 48*a^6*b^2*c^2 - 64*a^7*c^3)*n^6)) - b)/((a^2*b^2 -
 4*a^3*c)*n^3))^(2/3) - 2*sqrt(3)*(b^4*c^2 - 4*a*b^2*c^3 + 4*a^2*c^4))/(b^4
*c^2 - 4*a*b^2*c^3 + 4*a^2*c^4)) + (1/2)^(1/3)*(((a^2*b^2 - 4*a^3*c)*n^3*sq
rt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/((a^4*b^6 - 12*a^5*b^4*c + 48*a^6*b^2*c^2
- 64*a^7*c^3)*n^6)) + b)/((a^2*b^2 - 4*a^3*c)*n^3))^(1/3)*log(-(2*(b^2*c -
2*a*c^2)*x*x^(1/3*n - 1) + (1/2)^(1/3)*((a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c
^2)*n^4*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/((a^4*b^6 - 12*a^5*b^4*c + 48*a^
6*b^2*c^2 - 64*a^7*c^3)*n^6)) - (b^4 - 6*a*b^2*c + 8*a^2*c^2)*n)*(((a^2*b^2
 - 4*a^3*c)*n^3*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/((a^4*b^6 - 12*a^5*b^4*c
 + 48*a^6*b^2*c^2 - 64*a^7*c^3)*n^6)) + b)/((a^2*b^2 - 4*a^3*c)*n^3))^(1/3)
)/x) + (1/2)^(1/3)*(-((a^2*b^2 - 4*a^3*c)*n^3*sqrt((b^4 - 4*a*b^2*c + 4*a^2
*c^2)/((a^4*b^6 - 12*a^5*b^4*c + 48*a^6*b^2*c^2 - 64*a^7*c^3)*n^6)) - b)/((
a^2*b^2 - 4*a^3*c)*n^3))^(1/3)*log(-(2*(b^2*c - 2*a*c^2)*x*x^(1/3*n - 1) -
(1/2)^(1/3)*((a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*n^4*sqrt((b^4 - 4*a*b^2
*c + 4*a^2*c^2)/((a^4*b^6 - 12*a^5*b^4*c + 48*a^6*b^2*c^2 - 64*a^7*c^3)*n^6
)) + (b^4 - 6*a*b^2*c + 8*a^2*c^2)*n)*(-((a^2*b^2 - 4*a^3*c)*n^3*sqrt((b^4
- 4*a*b^2*c + 4*a^2*c^2)/((a^4*b^6 - 12*a^5*b^4*c + 48*a^6*b^2*c^2 - 64*a^7
*c^3)*n^6)) - b)/((a^2*b^2 - 4*a^3*c)*n^3))^(1/3))/x) - 1/2*(1/2)^(1/3)*(((
a^2*b^2 - 4*a^3*c)*n^3*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/((a^4*b^6 - 12*a^
5*b^4*c + 48*a^6*b^2*c^2 - 64*a^7*c^3)*n^6)) + b)/((a^2*b^2 - 4*a^3*c)*n^3)
)^(1/3)*log(8*(2*(b^4*c^2 - 4*a*b^2*c^3 + 4*a^2*c^4)*x^2*x^(2/3*n - 2) - (1
/2)^(1/3)*((a^2*b^7*c - 10*a^3*b^5*c^2 + 32*a^4*b^3*c^3 - 32*a^5*b*c^4)*n^4
*x*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/((a^4*b^6 - 12*a^5*b^4*c + 48*a^6*b^2
*c^2 - 64*a^7*c^3)*n^6)) - (b^6*c - 8*a*b^4*c^2 + 20*a^2*b^2*c^3 - 16*a^3*c
^4)*n*x)*x^(1/3*n - 1)*(((a^2*b^2 - 4*a^3*c)*n^3*sqrt((b^4 - 4*a*b^2*c + 4*
a^2*c^2)/((a^4*b^6 - 12*a^5*b^4*c + 48*a^6*b^2*c^2 - 64*a^7*c^3)*n^6)) + b)
/((a^2*b^2 - 4*a^3*c)*n^3))^(1/3) - (1/2)^(2/3)*((a^2*b^9 - 14*a^3*b^7*c +
72*a^4*b^5*c^2 - 160*a^5*b^3*c^3 + 128*a^6*b*c^4)*n^5*sqrt((b^4 - 4*a*b^2*c
 + 4*a^2*c^2)/((a^4*b^6 - 12*a^5*b^4*c + 48*a^6*b^2*c^2 - 64*a^7*c^3)*n^6))
 - (b^8 - 10*a*b^6*c + 36*a^2*b^4*c^2 - 56*a^3*b^2*c^3 + 32*a^4*c^4)*n^2)*(
((a^2*b^2 - 4*a^3*c)*n^3*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/((a^4*b^6 - 12*
a^5*b^4*c + 48*a^6*b^2*c^2 - 64*a^7*c^3)*n^6)) + b)/((a^2*b^2 - 4*a^3*c)*n^
3))^(2/3))/x^2) - 1/2*(1/2)^(1/3)*(-((a^2*b^2 - 4*a^3*c)*n^3*sqrt((b^4 - 4*
a*b^2*c + 4*a^2*c^2)/((a^4*b^6 - 12*a^5*b^4*c + 48*a^6*b^2*c^2 - 64*a^7*c^3
)*n^6)) - b)/((a^2*b^2 - 4*a^3*c)*n^3))^(1/3)*log(8*(2*(b^4*c^2 - 4*a*b^2*c
^3 + 4*a^2*c^4)*x^2*x^(2/3*n - 2) + (1/2)^(1/3)*((a^2*b^7*c - 10*a^3*b^5*c^
2 + 32*a^4*b^3*c^3 - 32*a^5*b*c^4)*n^4*x*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)
/((a^4*b^6 - 12*a^5*b^4*c + 48*a^6*b^2*c^2 - 64*a^7*c^3)*n^6)) + (b^6*c - 8
*a*b^4*c^2 + 20*a^2*b^2*c^3 - 16*a^3*c^4)*n*x)*x^(1/3*n - 1)*(-((a^2*b^2 -
4*a^3*c)*n^3*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/((a^4*b^6 - 12*a^5*b^4*c +
48*a^6*b^2*c^2 - 64*a^7*c^3)*n^6)) - b)/((a^2*b^2 - 4*a^3*c)*n^3))^(1/3) +
(1/2)^(2/3)*((a^2*b^9 - 14*a^3*b^7*c + 72*a^4*b^5*c^2 - 160*a^5*b^3*c^3 + 1
28*a^6*b*c^4)*n^5*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/((a^4*b^6 - 12*a^5*b^4
*c + 48*a^6*b^2*c^2 - 64*a^7*c^3)*n^6)) + (b^8 - 10*a*b^6*c + 36*a^2*b^4*c^
2 - 56*a^3*b^2*c^3 + 32*a^4*c^4)*n^2)*(-((a^2*b^2 - 4*a^3*c)*n^3*sqrt((b^4
- 4*a*b^2*c + 4*a^2*c^2)/((a^4*b^6 - 12*a^5*b^4*c + 48*a^6*b^2*c^2 - 64*a^7
*c^3)*n^6)) - b)/((a^2*b^2 - 4*a^3*c)*n^3))^(2/3))/x^2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{\frac{1}{3}n-1}}{cx^{2n} + bx^n + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+1/3*n)/(a+b*x^n+c*x^(2*n)),x, algorithm="giac")
```

[Out] integrate(x^(1/3*n - 1)/(c*x^(2*n) + b*x^n + a), x)

maple [C] time = 0.46, size = 260, normalized size = 0.43

$$\mathrm{RootOf}\left(\left(64a^5c^3n^6-48a^4b^2c^2n^6+12a^3b^4c\,n^6-a^2b^6n^6\right)\_Z^6+\left(16a^2b\,c^2n^3-8a\,b^3c\,n^3+b^5n^3\right)\_Z^3+c^2\right)\ln\left(\left(-\frac{16a^4b\,c^2n^4}{2a\,c^2-b^2c}+\frac{8a^3b^3c\,n^4}{2a\,c^2-b^2c}-\frac{a^2b^5n^4}{2a\,c^2-b^2c}\right)\mathrm{RootOf}\left(\left(64a^5c^3n^6-48a^4b^2c^2n^6+12a^3b^4c\,n^6-a^2b^6n^6\right)\_Z^6+\left(16a^2b\,c^2n^3-8a\,b^3c\,n^3+b^5n^3\right)\_Z^3+c^2\right)^4+\left(\frac{4a^2c^2n}{2a\,c^2-b^2c}-\frac{5a\,b^2cn}{2a\,c^2-b^2c}+\frac{b^4n}{2a\,c^2-b^2c}\right)\mathrm{RootOf}\left(\left(64a^5c^3n^6-48a^4b^2c^2n^6+12a^3b^4c\,n^6-a^2b^6n^6\right)\_Z^6+\left(16a^2b\,c^2n^3-8a\,b^3c\,n^3+b^5n^3\right)\_Z^3+c^2\right)+x^{\frac{n}{3}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(-1+1/3*n)/(a+b*x^n+c*x^(2*n)),x)

[Out] sum(_R*ln(x^(1/3*n)+(-16/(2*a*c^2-b^2*c)*n^4*b*a^4*c^2+8/(2*a*c^2-b^2*c)*n^4*b^3*a^3*c-1/(2*a*c^2-b^2*c)*n^4*b^5*a^2)*_R^4+(4/(2*a*c^2-b^2*c)*n*a^2*c^2-5/(2*a*c^2-b^2*c)*n*b^2*a*c+1/(2*a*c^2-b^2*c)*n*b^4)*_R),_R=RootOf((64*a^5*c^3*n^6-48*a^4*b^2*c^2*n^6+12*a^3*b^4*c*n^6-a^2*b^6*n^6)*_Z^6+(16*a^2*b*c^2*n^3-8*a*b^3*c*n^3+b^5*n^3)*_Z^3+c^2))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{\frac{1}{3}\,n-1}}{cx^{2\,n} + bx^{n} + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(-1+1/3*n)/(a+b*x^n+c*x^(2*n)),x, algorithm="maxima")

[Out] integrate(x^(1/3*n - 1)/(c*x^(2*n) + b*x^n + a), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^{\frac{n}{3}-1}}{a+b\,x^n+c\,x^{2\,n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(n/3 - 1)/(a + b*x^n + c*x^(2*n)),x)

[Out] int(x^(n/3 - 1)/(a + b*x^n + c*x^(2*n)), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**(-1+1/3*n)/(a+b*x**n+c*x**(2*n)),x)

[Out] Timed out

3.482 $\int \frac{x^{-1+\frac{n}{2}}}{a+bx^n+cx^{2n}}\,dx$

Optimal. Leaf size=169

$$\frac{2\sqrt{2}\sqrt{c}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x^{n/2}}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{n\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{2\sqrt{2}\sqrt{c}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x^{n/2}}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{n\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4ac}+b}}$$

Rubi [A] time = 0.19, antiderivative size = 169, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1381, 1093, 205}

$$\frac{2\sqrt{2}\sqrt{c}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x^{n/2}}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{n\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{2\sqrt{2}\sqrt{c}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x^{n/2}}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{n\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4ac}+b}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 + n/2)/(a + b*x^n + c*x^(2*n)),x]
```

```
[Out] (2*Sqrt[2]*Sqrt[c]*ArcTan[(Sqrt[2]*Sqrt[c]*x^(n/2))/Sqrt[b - Sqrt[b^2 - 4*a
*c]]])/(Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]*n) - (2*Sqrt[2]*Sqrt[
c]*ArcTan[(Sqrt[2]*Sqrt[c]*x^(n/2))/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2
 - 4*a*c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]*n)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1093

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[b^
2 - 4*a*c, 2]}, Dist[c/q, Int[1/(b/2 - q/2 + c*x^2), x], x] - Dist[c/q, Int
[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
0] && PosQ[b^2 - 4*a*c]
```

Rule 1381

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> Dist[1/(m + 1), Subst[Int[(a + b*x^Simplify[n/(m + 1)] + c*x^Simplify[(
2*n)/(m + 1)])^p, x], x, x^(m + 1)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&
EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IntegerQ[Simplify[n/(m + 1)]] &&  !I
ntegerQ[n]
```

Rubi steps

$$\int \frac{x^{-1+\frac{n}{2}}}{a+bx^n+cx^{2n}}\,dx = \frac{2\,\text{Subst}\left(\int \frac{1}{a+bx^2+cx^4}\,dx,x,x^{n/2}\right)}{n}$$
$$= \frac{(2c)\,\text{Subst}\left(\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx,x,x^{n/2}\right)}{\sqrt{b^2-4ac}\,n} - \frac{(2c)\,\text{Subst}\left(\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx,x,x^{n/2}\right)}{\sqrt{b^2-4ac}\,n}$$
$$= \frac{2\sqrt{2}\,\sqrt{c}\,\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt{c}\,x^{n/2}}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\,\sqrt{b-\sqrt{b^2-4ac}}\,n} - \frac{2\sqrt{2}\,\sqrt{c}\,\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt{c}\,x^{n/2}}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\,\sqrt{b+\sqrt{b^2-4ac}}\,n}$$

Mathematica [A] time = 0.26, size = 145, normalized size = 0.86

$$\frac{2\sqrt{2}\,\sqrt{c}\left(\frac{\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt{c}\,x^{n/2}}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b-\sqrt{b^2-4ac}}} - \frac{\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt{c}\,x^{n/2}}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{n\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 + n/2)/(a + b*x^n + c*x^(2*n)),x]
```

```
[Out] (2*Sqrt[2]*Sqrt[c]*(ArcTan[(Sqrt[2]*Sqrt[c]*x^(n/2))/Sqrt[b - Sqrt[b^2 - 4*
a*c]]]/Sqrt[b - Sqrt[b^2 - 4*a*c]] - ArcTan[(Sqrt[2]*Sqrt[c]*x^(n/2))/Sqrt[
b + Sqrt[b^2 - 4*a*c]]]/Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(Sqrt[b^2 - 4*a*c]*n)
```

IntegrateAlgebraic [F] time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{x^{-1+\frac{n}{2}}}{a+bx^n+cx^{2n}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^(-1 + n/2)/(a + b*x^n + c*x^(2*n)),x]
```

```
[Out] Defer[IntegrateAlgebraic][x^(-1 + n/2)/(a + b*x^n + c*x^(2*n)), x]
```

fricas [B] time = 1.36, size = 801, normalized size = 4.74

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+1/2*n)/(a+b*x^n+c*x^(2*n)),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(2)*sqrt(-((a*b^2 - 4*a^2*c)*n^2*sqrt(1/((a^2*b^2 - 4*a^3*c)*n^4))
+ b)/((a*b^2 - 4*a^2*c)*n^2))*log((4*c*x*x^(1/2*n - 1) + sqrt(2)*((a*b^3 -
4*a^2*b*c)*n^3*sqrt(1/((a^2*b^2 - 4*a^3*c)*n^4)) - (b^2 - 4*a*c)*n)*sqrt(-(
(a*b^2 - 4*a^2*c)*n^2*sqrt(1/((a^2*b^2 - 4*a^3*c)*n^4)) + b)/((a*b^2 - 4*a^
2*c)*n^2)))/x) - 1/2*sqrt(2)*sqrt(-((a*b^2 - 4*a^2*c)*n^2*sqrt(1/((a^2*b^2
- 4*a^3*c)*n^4)) + b)/((a*b^2 - 4*a^2*c)*n^2))*log((4*c*x*x^(1/2*n - 1) - s
qrt(2)*((a*b^3 - 4*a^2*b*c)*n^3*sqrt(1/((a^2*b^2 - 4*a^3*c)*n^4)) - (b^2 -
4*a*c)*n)*sqrt(-((a*b^2 - 4*a^2*c)*n^2*sqrt(1/((a^2*b^2 - 4*a^3*c)*n^4)) +
b)/((a*b^2 - 4*a^2*c)*n^2)))/x) - 1/2*sqrt(2)*sqrt(((a*b^2 - 4*a^2*c)*n^2*s
qrt(1/((a^2*b^2 - 4*a^3*c)*n^4)) - b)/((a*b^2 - 4*a^2*c)*n^2))*log((4*c*x*x
^(1/2*n - 1) + sqrt(2)*((a*b^3 - 4*a^2*b*c)*n^3*sqrt(1/((a^2*b^2 - 4*a^3*c)
```

```
*n^4)) + (b^2 - 4*a*c)*n)*sqrt(((a*b^2 - 4*a^2*c)*n^2*sqrt(1/((a^2*b^2 - 4*
a^3*c)*n^4)) - b)/((a*b^2 - 4*a^2*c)*n^2)))/x) + 1/2*sqrt(2)*sqrt(((a*b^2 -
 4*a^2*c)*n^2*sqrt(1/((a^2*b^2 - 4*a^3*c)*n^4)) - b)/((a*b^2 - 4*a^2*c)*n^2
))*log((4*c*x*x^(1/2*n - 1) - sqrt(2)*((a*b^3 - 4*a^2*b*c)*n^3*sqrt(1/((a^2
*b^2 - 4*a^3*c)*n^4)) + (b^2 - 4*a*c)*n)*sqrt(((a*b^2 - 4*a^2*c)*n^2*sqrt(1
/((a^2*b^2 - 4*a^3*c)*n^4)) - b)/((a*b^2 - 4*a^2*c)*n^2)))/x)
```

giac [**B**] time = 1.85, size = 1035, normalized size = 6.12

$$\frac{\frac{\left(\sqrt{2}\sqrt{bc+\sqrt{b^2-4ac}\,c}\,b^4-8\sqrt{2}\sqrt{bc+\sqrt{b^2-4ac}\,c}\,ab^2c-2\sqrt{2}\sqrt{bc+\sqrt{b^2-4ac}\,c}\,b^3c-2b^4c+16\sqrt{2}\sqrt{bc+\sqrt{b^2-4ac}\,c}\,a^2c^2+8\sqrt{2}\sqrt{bc+\sqrt{b^2-4ac}\,c}\,abc^2+\sqrt{2}\sqrt{bc+\sqrt{b^2-4ac}\,c}\,b^2c^2+16ab^2c^2-2b^3c^2-4\sqrt{2}\sqrt{bc+\sqrt{b^2-4ac}\,c}\,ac^3-32a^2c^3+8abc^3+\sqrt{2}\sqrt{b^2-4ac}\sqrt{bc+\sqrt{b^2-4ac}\,c}\,b^3-4\sqrt{2}\sqrt{b^2-4ac}\sqrt{bc+\sqrt{b^2-4ac}\,c}\,abc-2\sqrt{2}\sqrt{b^2-4ac}\sqrt{bc+\sqrt{b^2-4ac}\,c}\,b^2c+\sqrt{2}\sqrt{b^2-4ac}\sqrt{bc+\sqrt{b^2-4ac}\,c}\,bc^2+2(b^2-4ac)b^2c-8(b^2-4ac)ac^2+2(b^2-4ac)bc^2\right)\arctan\left(\frac{2\sqrt{\frac{1}{2}}\sqrt{x^n}}{\sqrt{\frac{b+\sqrt{b^2-4ac}}{c}}}\right)}{(ab^4-8a^2b^2c-2ab^3c+16a^3c^2+8a^2bc^2+ab^2c^2-4a^2c^3)|c|}+\frac{\left(\sqrt{2}\sqrt{bc-\sqrt{b^2-4ac}\,c}\,b^4-8\sqrt{2}\sqrt{bc-\sqrt{b^2-4ac}\,c}\,ab^2c-2\sqrt{2}\sqrt{bc-\sqrt{b^2-4ac}\,c}\,b^3c+2b^4c+16\sqrt{2}\sqrt{bc-\sqrt{b^2-4ac}\,c}\,a^2c^2+8\sqrt{2}\sqrt{bc-\sqrt{b^2-4ac}\,c}\,abc^2+\sqrt{2}\sqrt{bc-\sqrt{b^2-4ac}\,c}\,b^2c^2-16ab^2c^2-2b^3c^2-4\sqrt{2}\sqrt{bc-\sqrt{b^2-4ac}\,c}\,ac^3+32a^2c^3+8abc^3+\sqrt{2}\sqrt{b^2-4ac}\sqrt{bc-\sqrt{b^2-4ac}\,c}\,b^3-4\sqrt{2}\sqrt{b^2-4ac}\sqrt{bc-\sqrt{b^2-4ac}\,c}\,abc-2\sqrt{2}\sqrt{b^2-4ac}\sqrt{bc-\sqrt{b^2-4ac}\,c}\,b^2c+\sqrt{2}\sqrt{b^2-4ac}\sqrt{bc-\sqrt{b^2-4ac}\,c}\,bc^2-2(b^2-4ac)b^2c+8(b^2-4ac)ac^2+2(b^2-4ac)bc^2\right)\arctan\left(\frac{2\sqrt{\frac{1}{2}}\sqrt{x^n}}{\sqrt{\frac{b-\sqrt{b^2-4ac}}{c}}}\right)}{(ab^4-8a^2b^2c-2ab^3c+16a^3c^2+8a^2bc^2+ab^2c^2-4a^2c^3)|c|}}{2n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+1/2*n)/(a+b*x^n+c*x^(2*n)),x, algorithm="giac")
```

```
[Out] 1/2*((sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4 - 8*sqrt(2)*sqrt(b*c + sq
rt(b^2 - 4*a*c)*c)*a*b^2*c - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*
c - 2*b^4*c + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^2 + 8*sqrt(2
)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^2 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4
*a*c)*c)*b^2*c^2 + 16*a*b^2*c^2 - 2*b^3*c^2 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2
 - 4*a*c)*c)*a*c^3 - 32*a^2*c^3 + 8*a*b*c^3 + sqrt(2)*sqrt(b^2 - 4*a*c)*sqr
t(b*c + sqrt(b^2 - 4*a*c)*c)*b^3 - 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + s
qrt(b^2 - 4*a*c)*c)*a*b*c - 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
 - 4*a*c)*c)*b^2*c + sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)
*c)*b*c^2 + 2*(b^2 - 4*a*c)*b^2*c - 8*(b^2 - 4*a*c)*a*c^2 + 2*(b^2 - 4*a*c)
*b*c^2)*arctan(2*sqrt(1/2)*sqrt(x^n)/sqrt((b + sqrt(b^2 - 4*a*c))/c))/((a*b
^4 - 8*a^2*b^2*c - 2*a*b^3*c + 16*a^3*c^2 + 8*a^2*b*c^2 + a*b^2*c^2 - 4*a^2
*c^3)*abs(c)) + (sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4 - 8*sqrt(2)*sq
rt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c - 2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a
*c)*c)*b^3*c + 2*b^4*c + 16*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*c^2
 + 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^2 + sqrt(2)*sqrt(b*c - s
qrt(b^2 - 4*a*c)*c)*b^2*c^2 - 16*a*b^2*c^2 - 2*b^3*c^2 - 4*sqrt(2)*sqrt(b*c
 - sqrt(b^2 - 4*a*c)*c)*a*c^3 + 32*a^2*c^3 + 8*a*b*c^3 + sqrt(2)*sqrt(b^2 -
 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3 - 4*sqrt(2)*sqrt(b^2 - 4*a*c)*s
qrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c - 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
 - sqrt(b^2 - 4*a*c)*c)*b^2*c + sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b
^2 - 4*a*c)*c)*b*c^2 - 2*(b^2 - 4*a*c)*b^2*c + 8*(b^2 - 4*a*c)*a*c^2 + 2*(b
^2 - 4*a*c)*b*c^2)*arctan(2*sqrt(1/2)*sqrt(x^n)/sqrt((b - sqrt(b^2 - 4*a*c)
)/c))/((a*b^4 - 8*a^2*b^2*c - 2*a*b^3*c + 16*a^3*c^2 + 8*a^2*b*c^2 + a*b^2*
c^2 - 4*a^2*c^3)*abs(c)))/n
```

maple [**C**] time = 0.19, size = 114, normalized size = 0.67

$$\mathrm{RootOf}\left(\left(16a^3c^2n^4-8a^2b^2c\,n^4+a\,b^4n^4\right)\_Z^4+\left(-4abc\,n^2+b^3n^2\right)\_Z^2+c\right)\ln\left(\left(4a^2b\,n^3-\frac{a\,b^3n^3}{c}\right)\mathrm{RootOf}\left(\left(16a^3c^2n^4-8a^2b^2c\,n^4+a\,b^4n^4\right)\_Z^4+\left(-4abc\,n^2+b^3n^2\right)\_Z^2+c\right)^3+\left(2an-\frac{b^2n}{c}\right)\mathrm{RootOf}\left(\left(16a^3c^2n^4-8a^2b^2c\,n^4+a\,b^4n^4\right)\_Z^4+\left(-4abc\,n^2+b^3n^2\right)\_Z^2+c\right)+x^{\frac{n}{2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1+1/2*n)/(a+b*x^n+c*x^(2*n)),x)
```

```
[Out] sum(_R*ln(x^(1/2*n)+(4*n^3*b*a^2-1/c*n^3*b^3*a)*_R^3+(2*a*n-1/c*n*b^2)*_R),
_R=RootOf((16*a^3*c^2*n^4-8*a^2*b^2*c*n^4+a*b^4*n^4)*_Z^4+(-4*a*b*c*n^2+b^3
*n^2)*_Z^2+c))
```

maxima [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{\frac{1}{2}n-1}}{cx^{2n}+bx^n+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+1/2*n)/(a+b*x^n+c*x^(2*n)),x, algorithm="maxima")
```

```
[Out] integrate(x^(1/2*n - 1)/(c*x^(2*n) + b*x^n + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{\frac{n}{2}-1}}{a+b\,x^n+c\,x^{2\,n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(n/2 - 1)/(a + b*x^n + c*x^(2*n)),x)
[Out] int(x^(n/2 - 1)/(a + b*x^n + c*x^(2*n)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{\frac{n}{2}-1}}{a+bx^n+cx^{2n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+1/2*n)/(a+b*x**n+c*x**(2*n)),x)
[Out] Integral(x**(n/2 - 1)/(a + b*x**n + c*x**(2*n)), x)
```

3.483 $\int \frac{x^{-1-\frac{n}{2}}}{a+bx^n+cx^{2n}}\,dx$

Optimal. Leaf size=205

$$\frac{\sqrt{2}\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{a}\,x^{-n/2}}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{a^{3/2}n\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{2}\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{a}\,x^{-n/2}}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{a^{3/2}n\sqrt{\sqrt{b^2-4ac}+b}}-\frac{2x^{-n/2}}{an}$$

Rubi [A] time = 0.40, antiderivative size = 205, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {1381, 1340, 1122, 1166, 205}

$$\frac{\sqrt{2}\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{a}\,x^{-n/2}}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{a^{3/2}n\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{2}\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{a}\,x^{-n/2}}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{a^{3/2}n\sqrt{\sqrt{b^2-4ac}+b}}-\frac{2x^{-n/2}}{an}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 - n/2)/(a + b*x^n + c*x^(2*n)),x]
```

```
[Out] -2/(a*n*x^(n/2)) + (Sqrt[2]*(b - (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(S
qrt[2]*Sqrt[a])/(Sqrt[b - Sqrt[b^2 - 4*a*c]]*x^(n/2))])/(a^(3/2)*Sqrt[b - S
qrt[b^2 - 4*a*c]]*n) + (Sqrt[2]*(b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTa
n[(Sqrt[2]*Sqrt[a])/(Sqrt[b + Sqrt[b^2 - 4*a*c]]*x^(n/2))])/(a^(3/2)*Sqrt[b
 + Sqrt[b^2 - 4*a*c]]*n)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1122

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Simp[(d^3*(d*x)^(m - 3)*(a + b*x^2 + c*x^4)^(p + 1))/(c*(m + 4*p + 1)),
 x] - Dist[d^4/(c*(m + 4*p + 1)), Int[(d*x)^(m - 4)*Simp[a*(m - 3) + b*(m +
 2*p - 1)*x^2, x]*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x
] && NeQ[b^2 - 4*a*c, 0] && GtQ[m, 3] && NeQ[m + 4*p + 1, 0] && IntegerQ[2*
p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1340

```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[x^
(2*n*p)*(c + b/x^n + a/x^(2*n))^p, x] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n
] && LtQ[n, 0] && IntegerQ[p]
```

Rule 1381

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> Dist[1/(m + 1), Subst[Int[(a + b*x^Simplify[n/(m + 1)] + c*x^Simplify[(
2*n)/(m + 1)])^p, x], x, x^(m + 1)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&
EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IntegerQ[Simplify[n/(m + 1)]] &&  !I
ntegerQ[n]
```

Rubi steps

$$\begin{aligned}\int \frac{x^{-1-\frac{n}{2}}}{a+bx^n+cx^{2n}}\,dx &= -\frac{2\,\text{Subst}\left(\int \frac{1}{a+\frac{c}{x^4}+\frac{b}{x^2}}\,dx, x, x^{-n/2}\right)}{n}\\ &= -\frac{2\,\text{Subst}\left(\int \frac{x^4}{c+bx^2+ax^4}\,dx, x, x^{-n/2}\right)}{n}\\ &= -\frac{2x^{-n/2}}{an}+\frac{2\,\text{Subst}\left(\int \frac{c+bx^2}{c+bx^2+ax^4}\,dx, x, x^{-n/2}\right)}{an}\\ &= -\frac{2x^{-n/2}}{an}+\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\text{Subst}\left(\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+ax^2}\,dx, x, x^{-n/2}\right)}{an}+\frac{\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\text{Subst}\left(\int\right.}{}\\ &= -\frac{2x^{-n/2}}{an}+\frac{\sqrt{2}\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{a}\,x^{-n/2}}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{a^{3/2}\sqrt{b-\sqrt{b^2-4ac}}\,n}+\frac{\sqrt{2}\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{a}\,x^{-n/2}}{\sqrt{b+\sqrt{b^2-4ac}}}\right.}{a^{3/2}\sqrt{b+\sqrt{b^2-4ac}}\,n}\end{aligned}$$

Mathematica [C] time = 0.18, size = 127, normalized size = 0.62

$$\frac{4cx^{-n/2}\left(\frac{{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{2cx^n}{\sqrt{b^2-4ac}-b}\right)}{-b\sqrt{b^2-4ac}-4ac+b^2}+\frac{{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{b\sqrt{b^2-4ac}-4ac+b^2}\right)}{n}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 - n/2)/(a + b*x^n + c*x^(2*n)),x]
```

```
[Out] (4*c*(Hypergeometric2F1[-1/2, 1, 1/2, (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])]/(
b^2 - 4*a*c - b*Sqrt[b^2 - 4*a*c]) + Hypergeometric2F1[-1/2, 1, 1/2, (-2*c*
x^n)/(b + Sqrt[b^2 - 4*a*c])]/(b^2 - 4*a*c + b*Sqrt[b^2 - 4*a*c])))/(n*x^(n
/2))
```

IntegrateAlgebraic [F] time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{x^{-1-\frac{n}{2}}}{a+bx^n+cx^{2n}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^(-1 - n/2)/(a + b*x^n + c*x^(2*n)),x]
```

```
[Out] Defer[IntegrateAlgebraic][x^(-1 - n/2)/(a + b*x^n + c*x^(2*n)), x]
```

fricas [B] time = 1.25, size = 1229, normalized size = 6.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-1/2*n)/(a+b*x^n+c*x^(2*n)),x, algorithm="fricas")
```

```
[Out] 1/2*(sqrt(2)*a*n*sqrt(-((a^3*b^2 - 4*a^4*c)*n^2*sqrt((b^4 - 2*a*b^2*c + a^2
*c^2)/((a^6*b^2 - 4*a^7*c)*n^4)) + b^3 - 3*a*b*c)/((a^3*b^2 - 4*a^4*c)*n^2)
)*log(-(4*(b^2*c - a*c^2)*x*x^(-1/2*n - 1) + sqrt(2)*((a^3*b^3 - 4*a^4*b*c)
*n^3*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/((a^6*b^2 - 4*a^7*c)*n^4)) - (b^4 - 5
*a*b^2*c + 4*a^2*c^2)*n)*sqrt(-((a^3*b^2 - 4*a^4*c)*n^2*sqrt((b^4 - 2*a*b^2
*c + a^2*c^2)/((a^6*b^2 - 4*a^7*c)*n^4)) + b^3 - 3*a*b*c)/((a^3*b^2 - 4*a^4
*c)*n^2)))/x) - sqrt(2)*a*n*sqrt(-((a^3*b^2 - 4*a^4*c)*n^2*sqrt((b^4 - 2*a*
b^2*c + a^2*c^2)/((a^6*b^2 - 4*a^7*c)*n^4)) + b^3 - 3*a*b*c)/((a^3*b^2 - 4*
a^4*c)*n^2))*log(-(4*(b^2*c - a*c^2)*x*x^(-1/2*n - 1) - sqrt(2)*((a^3*b^3 -
 4*a^4*b*c)*n^3*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/((a^6*b^2 - 4*a^7*c)*n^4))
 - (b^4 - 5*a*b^2*c + 4*a^2*c^2)*n)*sqrt(-((a^3*b^2 - 4*a^4*c)*n^2*sqrt((b^
4 - 2*a*b^2*c + a^2*c^2)/((a^6*b^2 - 4*a^7*c)*n^4)) + b^3 - 3*a*b*c)/((a^3*
b^2 - 4*a^4*c)*n^2)))/x) - sqrt(2)*a*n*sqrt(((a^3*b^2 - 4*a^4*c)*n^2*sqrt((
b^4 - 2*a*b^2*c + a^2*c^2)/((a^6*b^2 - 4*a^7*c)*n^4)) - b^3 + 3*a*b*c)/((a^
3*b^2 - 4*a^4*c)*n^2))*log(-(4*(b^2*c - a*c^2)*x*x^(-1/2*n - 1) + sqrt(2)*(
(a^3*b^3 - 4*a^4*b*c)*n^3*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/((a^6*b^2 - 4*a^
7*c)*n^4)) + (b^4 - 5*a*b^2*c + 4*a^2*c^2)*n)*sqrt(((a^3*b^2 - 4*a^4*c)*n^2
*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/((a^6*b^2 - 4*a^7*c)*n^4)) - b^3 + 3*a*b*
c)/((a^3*b^2 - 4*a^4*c)*n^2)))/x) + sqrt(2)*a*n*sqrt(((a^3*b^2 - 4*a^4*c)*n
^2*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/((a^6*b^2 - 4*a^7*c)*n^4)) - b^3 + 3*a*
b*c)/((a^3*b^2 - 4*a^4*c)*n^2))*log(-(4*(b^2*c - a*c^2)*x*x^(-1/2*n - 1) -
sqrt(2)*((a^3*b^3 - 4*a^4*b*c)*n^3*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/((a^6*b
^2 - 4*a^7*c)*n^4)) + (b^4 - 5*a*b^2*c + 4*a^2*c^2)*n)*sqrt(((a^3*b^2 - 4*a
^4*c)*n^2*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/((a^6*b^2 - 4*a^7*c)*n^4)) - b^3
 + 3*a*b*c)/((a^3*b^2 - 4*a^4*c)*n^2)))/x) - 4*x*x^(-1/2*n - 1))/(a*n)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{-\frac{1}{2}n-1}}{cx^{2n}+bx^n+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-1/2*n)/(a+b*x^n+c*x^(2*n)),x, algorithm="giac")
```

```
[Out] integrate(x^(-1/2*n - 1)/(c*x^(2*n) + b*x^n + a), x)
```

maple [C] time = 0.35, size = 268, normalized size = 1.31

$$\mathrm{RootOf}\left(\left(16a^5c^2n^4-8a^4b^2cn^4+a^3b^4n^4\right)\_Z^4+c^3+\left(12a^2bc^2n^2-7ab^3cn^2+b^5n^2\right)\_Z^2\right)\ln\left(\left(-\frac{8a^5c^2n^3}{ac^3-b^2c^2}+\frac{6a^4b^2cn^3}{ac^3-b^2c^2}-\frac{a^3b^4n^3}{ac^3-b^2c^2}\right)\mathrm{RootOf}\left(\left(16a^5c^2n^4-8a^4b^2cn^4+a^3b^4n^4\right)\_Z^4+c^3+\left(12a^2bc^2n^2-7ab^3cn^2+b^5n^2\right)\_Z^2\right)^3+\left(-\frac{5a^2bc^2n}{ac^3-b^2c^2}+\frac{5ab^3cn}{ac^3-b^2c^2}-\frac{b^5n}{ac^3-b^2c^2}\right)\mathrm{RootOf}\left(\left(16a^5c^2n^4-8a^4b^2cn^4+a^3b^4n^4\right)\_Z^4+c^3+\left(12a^2bc^2n^2-7ab^3cn^2+b^5n^2\right)\_Z^2\right)+x^{\frac{n}{2}}\right)-\frac{2x^{-\frac{n}{2}}}{an}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1-1/2*n)/(a+b*x^n+c*x^(2*n)),x)
```

```
[Out] -2/a/n/(x^(1/2*n))+sum(_R*ln(x^(1/2*n)+(-8/(a*c^3-b^2*c^2)*n^3*a^5*c^2+6/(a
*c^3-b^2*c^2)*n^3*b^2*a^4*c-1/(a*c^3-b^2*c^2)*n^3*b^4*a^3)*_R^3+(-5/(a*c^3-
b^2*c^2)*n*b*a^2*c^2+5/(a*c^3-b^2*c^2)*n*b^3*a*c-1/(a*c^3-b^2*c^2)*n*b^5)*_
R),_R=RootOf((16*a^5*c^2*n^4-8*a^4*b^2*c*n^4+a^3*b^4*n^4)*_Z^4+(12*a^2*b*c^
2*n^2-7*a*b^3*c*n^2+b^5*n^2)*_Z^2+c^3))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{2}{anx^{\frac{1}{2}n}}-\int \frac{cx^{\frac{3}{2}n}+bx^{\frac{1}{2}n}}{acx x^{2n}+abx x^{n}+a^{2}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-1/2*n)/(a+b*x^n+c*x^(2*n)),x, algorithm="maxima")
```

```
[Out] -2/(a*n*x^(1/2*n)) - integrate((c*x^(3/2*n) + b*x^(1/2*n))/(a*c*x*x^(2*n) +
 a*b*x*x^n + a^2*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^{\frac{n}{2}+1}\,\left(a+b\,x^n+c\,x^{2\,n}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(n/2 + 1)*(a + b*x^n + c*x^(2*n))),x)
```

```
[Out] int(1/(x^(n/2 + 1)*(a + b*x^n + c*x^(2*n))), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{-\frac{n}{2}-1}}{a+bx^n+cx^{2n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1-1/2*n)/(a+b*x**n+c*x**(2*n)),x)
```

```
[Out] Integral(x**(-n/2 - 1)/(a + b*x**n + c*x**(2*n)), x)
```

3.484 $\int \frac{x^{-1-\frac{n}{3}}}{a+bx^n+cx^{2n}}\,dx$

Optimal. Leaf size=699

$$\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{a}\,x^{-n/3}\right)}{\sqrt[3]{2}\,a^{4/3}n\left(b-\sqrt{b^2-4ac}\right)^{2/3}}+\frac{\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\log\left(\sqrt[3]{\sqrt{b^2-4ac}+b}+\sqrt[3]{2}\sqrt[3]{a}\,x^{-n/3}\right)}{\sqrt[3]{2}\,a^{4/3}n\left(\sqrt{b^2-4ac}+b\right)^{2/3}}$$

Rubi [A] time = 1.49, antiderivative size = 699, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 10, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {1381, 1340, 1367, 1422, 200, 31, 634, 617, 204, 628}

$$\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{a}\,x^{-n/3}\right)}{\sqrt[3]{2}\,a^{4/3}n\left(b-\sqrt{b^2-4ac}\right)^{2/3}}+\frac{\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\log\left(\sqrt[3]{\sqrt{b^2-4ac}+b}+\sqrt[3]{2}\sqrt[3]{a}\,x^{-n/3}\right)}{\sqrt[3]{2}\,a^{4/3}n\left(\sqrt{b^2-4ac}+b\right)^{2/3}}-\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\log\left(2^{2/3}a^{2/3}x^{-2n/3}-\sqrt[3]{2}\sqrt[3]{a}\,x^{-n/3}\sqrt[3]{b-\sqrt{b^2-4ac}}+\left(b-\sqrt{b^2-4ac}\right)^{2/3}\right)}{2\sqrt[3]{2}\,a^{4/3}n\left(b-\sqrt{b^2-4ac}\right)^{2/3}}-\frac{\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\log\left(2^{2/3}a^{2/3}x^{-2n/3}-\sqrt[3]{2}\sqrt[3]{a}\,x^{-n/3}\sqrt[3]{\sqrt{b^2-4ac}+b}+\left(\sqrt{b^2-4ac}+b\right)^{2/3}\right)}{2\sqrt[3]{2}\,a^{4/3}n\left(\sqrt{b^2-4ac}+b\right)^{2/3}}-\frac{\sqrt{3}\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{a}\,x^{-n/3}}{\sqrt[3]{b-\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\,a^{4/3}n\left(b-\sqrt{b^2-4ac}\right)^{2/3}}-\frac{\sqrt{3}\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{a}\,x^{-n/3}}{\sqrt[3]{\sqrt{b^2-4ac}+b}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\,a^{4/3}n\left(\sqrt{b^2-4ac}+b\right)^{2/3}}-\frac{3x^{-n/3}}{an}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 - n/3)/(a + b*x^n + c*x^(2*n)),x]
```

```
[Out] -3/(a*n*x^(n/3)) - (Sqrt[3]*(b - (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(1
 - (2*2^(1/3)*a^(1/3))/((b - Sqrt[b^2 - 4*a*c])^(1/3)*x^(n/3)))/Sqrt[3]])/(
2^(1/3)*a^(4/3)*(b - Sqrt[b^2 - 4*a*c])^(2/3)*n) - (Sqrt[3]*(b + (b^2 - 2*a
*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(1 - (2*2^(1/3)*a^(1/3))/((b + Sqrt[b^2 - 4*a
*c])^(1/3)*x^(n/3)))/Sqrt[3]])/(2^(1/3)*a^(4/3)*(b + Sqrt[b^2 - 4*a*c])^(2/
3)*n) + ((b - (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*Log[(b - Sqrt[b^2 - 4*a*c])^
(1/3) + (2^(1/3)*a^(1/3))/x^(n/3)])/(2^(1/3)*a^(4/3)*(b - Sqrt[b^2 - 4*a*c]
)^(2/3)*n) + ((b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*Log[(b + Sqrt[b^2 - 4*a
*c])^(1/3) + (2^(1/3)*a^(1/3))/x^(n/3)])/(2^(1/3)*a^(4/3)*(b + Sqrt[b^2 - 4
*a*c])^(2/3)*n) - ((b - (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*Log[(b - Sqrt[b^2
- 4*a*c])^(2/3) + (2^(2/3)*a^(2/3))/x^((2*n)/3) - (2^(1/3)*a^(1/3)*(b - Sqr
t[b^2 - 4*a*c])^(1/3))/x^(n/3)])/(2*2^(1/3)*a^(4/3)*(b - Sqrt[b^2 - 4*a*c])
^(2/3)*n) - ((b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*Log[(b + Sqrt[b^2 - 4*a*
c])^(2/3) + (2^(2/3)*a^(2/3))/x^((2*n)/3) - (2^(1/3)*a^(1/3)*(b + Sqrt[b^2
- 4*a*c])^(1/3))/x^(n/3)])/(2*2^(1/3)*a^(4/3)*(b + Sqrt[b^2 - 4*a*c])^(2/3)
*n)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
```

```
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1340

```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[x^
(2*n*p)*(c + b/x^n + a/x^(2*n))^p, x] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n
] && LtQ[n, 0] && IntegerQ[p]
```

Rule 1367

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Simp[(d^(2*n - 1)*(d*x)^(m - 2*n + 1)*(a + b*x^n + c*x^(2*n))^(
p + 1))/(c*(m + 2*n*p + 1)), x] - Dist[d^(2*n)/(c*(m + 2*n*p + 1)), Int[(d*
x)^(m - 2*n)*Simp[a*(m - 2*n + 1) + b*(m + n*(p - 1) + 1)*x^n, x]*(a + b*x^
n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && EqQ[n2, 2*n] && Ne
Q[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GtQ[m, 2*n - 1] && NeQ[m + 2*n*p + 1, 0]
 && IntegerQ[p]
```

Rule 1381

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> Dist[1/(m + 1), Subst[Int[(a + b*x^Simplify[n/(m + 1)] + c*x^Simplify[(
2*n)/(m + 1)])^p, x], x, x^(m + 1)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&
EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IntegerQ[Simplify[n/(m + 1)]] &&  !I
ntegerQ[n]
```

Rule 1422

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q),
Int[1/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(
b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (PosQ[b^2 - 4*a
*c] ||  !IGtQ[n/2, 0])
```

Rubi steps

$$\int \frac{x^{-1-\frac{n}{3}}}{a+bx^n+cx^{2n}}\,dx = -\frac{3\,\mathrm{Subst}\left(\int \frac{1}{a+\frac{c}{x^6}+\frac{b}{x^3}}\,dx, x, x^{-n/3}\right)}{n}$$

$$= -\frac{3\,\mathrm{Subst}\left(\int \frac{x^6}{c+bx^3+ax^6}\,dx, x, x^{-n/3}\right)}{n}$$

$$= -\frac{3x^{-n/3}}{an} + \frac{3\,\mathrm{Subst}\left(\int \frac{c+bx^3}{c+bx^3+ax^6}\,dx, x, x^{-n/3}\right)}{an}$$

$$= -\frac{3x^{-n/3}}{an} + \frac{\left(3\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\right)\mathrm{Subst}\left(\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+ax^3}\,dx, x, x^{-n/3}\right)}{2an} + \frac{\left(3\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\right.}{}$$

$$= -\frac{3x^{-n/3}}{an} + \frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\mathrm{Subst}\left(\int \frac{1}{\frac{\sqrt[3]{b-\sqrt{b^2-4ac}}}{\sqrt[3]{2}}+\sqrt[3]{a}\,x}\,dx, x, x^{-n/3}\right)}{\sqrt[3]{2}\,a\left(b-\sqrt{b^2-4ac}\right)^{2/3}n} + \frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\mathrm{Subst}}{}$$

$$= -\frac{3x^{-n/3}}{an} + \frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{a}\,x^{-n/3}\right)}{\sqrt[3]{2}\,a^{4/3}\left(b-\sqrt{b^2-4ac}\right)^{2/3}n} + \frac{\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\log\left(\right.}{\sqrt[3]{2}\,a^{4/3}}$$

$$= -\frac{3x^{-n/3}}{an} + \frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{a}\,x^{-n/3}\right)}{\sqrt[3]{2}\,a^{4/3}\left(b-\sqrt{b^2-4ac}\right)^{2/3}n} + \frac{\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\log\left(\right.}{\sqrt[3]{2}\,a^{4/3}}$$

$$= -\frac{3x^{-n/3}}{an} - \frac{\sqrt{3}\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{a}\,x^{-n/3}}{\sqrt[3]{b-\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\,a^{4/3}\left(b-\sqrt{b^2-4ac}\right)^{2/3}n} - \frac{\sqrt{3}\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{a}}{\sqrt[3]{b+\sqrt{b}}}}{\sqrt{3}}\right.}{\sqrt[3]{2}\,a^{4/3}\left(b+\sqrt{b^2-4ac}\right)^{2/3}n}$$

Mathematica [C] time = 0.13, size = 127, normalized size = 0.18

$$\frac{6cx^{-n/3}\left(\frac{{}_2F_1\left(-\frac{1}{3},1;\frac{2}{3};\frac{2cx^n}{\sqrt{b^2-4ac}-b}\right)}{-b\sqrt{b^2-4ac}-4ac+b^2}+\frac{{}_2F_1\left(-\frac{1}{3},1;\frac{2}{3};-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{b\sqrt{b^2-4ac}-4ac+b^2}\right)}{n}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 - n/3)/(a + b*x^n + c*x^(2*n)),x]
```

```
[Out] (6*c*(Hypergeometric2F1[-1/3, 1, 2/3, (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])]/(
b^2 - 4*a*c - b*Sqrt[b^2 - 4*a*c]) + Hypergeometric2F1[-1/3, 1, 2/3, (-2*c*
x^n)/(b + Sqrt[b^2 - 4*a*c])]/(b^2 - 4*a*c + b*Sqrt[b^2 - 4*a*c])))/(n*x^(n
/3))
```

IntegrateAlgebraic [F] time = 0.10, size = 0, normalized size = 0.00

$$\int \frac{x^{-1-\frac{n}{3}}}{a+bx^n+cx^{2n}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^(-1 - n/3)/(a + b*x^n + c*x^(2*n)),x]
```

```
[Out] Defer[IntegrateAlgebraic][x^(-1 - n/3)/(a + b*x^n + c*x^(2*n)), x]
```

fricas [B] time = 7.49, size = 6279, normalized size = 8.98

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-1/3*n)/(a+b*x^n+c*x^(2*n)),x, algorithm="fricas")
```

```
[Out] 1/2*(4*sqrt(3)*(1/2)^(1/3)*a*n*(((a^4*b^2 - 4*a^5*c)*n^3*sqrt((b^8 - 8*a*b^
6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/((a^8*b^6 - 12*a^9*b^4*c
 + 48*a^10*b^2*c^2 - 64*a^11*c^3)*n^6)) + b^3 - 2*a*b*c)/((a^4*b^2 - 4*a^5*
c)*n^3))^(1/3)*arctan(-1/6*(2*(1/2)^(2/3)*(sqrt(3)*(a^4*b^12*c - 17*a^5*b^1
0*c^2 + 114*a^6*b^8*c^3 - 378*a^7*b^6*c^4 + 632*a^8*b^4*c^5 - 480*a^9*b^2*c
^6 + 128*a^10*c^7)*n^5*x*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^
2*c^3 + 4*a^4*c^4)/((a^8*b^6 - 12*a^9*b^4*c + 48*a^10*b^2*c^2 - 64*a^11*c^3
)*n^6)) - sqrt(3)*(b^13*c - 15*a*b^11*c^2 + 88*a^2*b^9*c^3 - 252*a^3*b^7*c^
4 + 356*a^4*b^5*c^5 - 220*a^5*b^3*c^6 + 48*a^6*b*c^7)*n^2*x)*x^(-1/3*n - 1)
*(((a^4*b^2 - 4*a^5*c)*n^3*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*
b^2*c^3 + 4*a^4*c^4)/((a^8*b^6 - 12*a^9*b^4*c + 48*a^10*b^2*c^2 - 64*a^11*c
^3)*n^6)) + b^3 - 2*a*b*c)/((a^4*b^2 - 4*a^5*c)*n^3))^(2/3) - sqrt(2)*(1/2)
^(2/3)*(sqrt(3)*(a^4*b^8 - 13*a^5*b^6*c + 60*a^6*b^4*c^2 - 112*a^7*b^2*c^3
+ 64*a^8*c^4)*n^5*x*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3
 + 4*a^4*c^4)/((a^8*b^6 - 12*a^9*b^4*c + 48*a^10*b^2*c^2 - 64*a^11*c^3)*n^6
)) - sqrt(3)*(b^9 - 11*a*b^7*c + 42*a^2*b^5*c^2 - 62*a^3*b^3*c^3 + 24*a^4*b
*c^4)*n^2*x)*sqrt((2*(b^8*c^2 - 8*a*b^6*c^3 + 20*a^2*b^4*c^4 - 16*a^3*b^2*c
^5 + 4*a^4*c^6)*x^2*x^(-2/3*n - 2) - (1/2)^(1/3)*((a^4*b^9*c - 12*a^5*b^7*c
^2 + 50*a^6*b^5*c^3 - 80*a^7*b^3*c^4 + 32*a^8*b*c^5)*n^4*x*sqrt((b^8 - 8*a*
b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/((a^8*b^6 - 12*a^9*b^4
*c + 48*a^10*b^2*c^2 - 64*a^11*c^3)*n^6)) - (b^10*c - 12*a*b^8*c^2 + 52*a^2
*b^6*c^3 - 96*a^3*b^4*c^4 + 68*a^4*b^2*c^5 - 16*a^5*c^6)*n*x)*x^(-1/3*n - 1
)*(((a^4*b^2 - 4*a^5*c)*n^3*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3
*b^2*c^3 + 4*a^4*c^4)/((a^8*b^6 - 12*a^9*b^4*c + 48*a^10*b^2*c^2 - 64*a^11*
c^3)*n^6)) + b^3 - 2*a*b*c)/((a^4*b^2 - 4*a^5*c)*n^3))^(1/3) - (1/2)^(2/3)*
((a^4*b^11 - 16*a^5*b^9*c + 98*a^6*b^7*c^2 - 280*a^7*b^5*c^3 + 352*a^8*b^3*
c^4 - 128*a^9*b*c^5)*n^5*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^
2*c^3 + 4*a^4*c^4)/((a^8*b^6 - 12*a^9*b^4*c + 48*a^10*b^2*c^2 - 64*a^11*c^3
)*n^6)) - (b^12 - 14*a*b^10*c + 76*a^2*b^8*c^2 - 200*a^3*b^6*c^3 + 260*a^4*
b^4*c^4 - 152*a^5*b^2*c^5 + 32*a^6*c^6)*n^2)*(((a^4*b^2 - 4*a^5*c)*n^3*sqrt
((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/((a^8*b^6
- 12*a^9*b^4*c + 48*a^10*b^2*c^2 - 64*a^11*c^3)*n^6)) + b^3 - 2*a*b*c)/((a^
4*b^2 - 4*a^5*c)*n^3))^(2/3))/x^2)*(((a^4*b^2 - 4*a^5*c)*n^3*sqrt((b^8 - 8*
a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/((a^8*b^6 - 12*a^9*b
^4*c + 48*a^10*b^2*c^2 - 64*a^11*c^3)*n^6)) + b^3 - 2*a*b*c)/((a^4*b^2 - 4*
a^5*c)*n^3))^(2/3) + 2*sqrt(3)*(b^8*c^4 - 8*a*b^6*c^5 + 20*a^2*b^4*c^6 - 16
*a^3*b^2*c^7 + 4*a^4*c^8))/(b^8*c^4 - 8*a*b^6*c^5 + 20*a^2*b^4*c^6 - 16*a^3
*b^2*c^7 + 4*a^4*c^8)) - 4*sqrt(3)*(1/2)^(1/3)*a*n*(-((a^4*b^2 - 4*a^5*c)*n
^3*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/((a
^8*b^6 - 12*a^9*b^4*c + 48*a^10*b^2*c^2 - 64*a^11*c^3)*n^6)) - b^3 + 2*a*b*
c)/((a^4*b^2 - 4*a^5*c)*n^3))^(1/3)*arctan(-1/6*(2*(1/2)^(2/3)*(sqrt(3)*(a^
4*b^12*c - 17*a^5*b^10*c^2 + 114*a^6*b^8*c^3 - 378*a^7*b^6*c^4 + 632*a^8*b^
4*c^5 - 480*a^9*b^2*c^6 + 128*a^10*c^7)*n^5*x*sqrt((b^8 - 8*a*b^6*c + 20*a^
2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/((a^8*b^6 - 12*a^9*b^4*c + 48*a^10*
b^2*c^2 - 64*a^11*c^3)*n^6)) + sqrt(3)*(b^13*c - 15*a*b^11*c^2 + 88*a^2*b^9
*c^3 - 252*a^3*b^7*c^4 + 356*a^4*b^5*c^5 - 220*a^5*b^3*c^6 + 48*a^6*b*c^7)*
n^2*x)*x^(-1/3*n - 1)*(-((a^4*b^2 - 4*a^5*c)*n^3*sqrt((b^8 - 8*a*b^6*c + 20
*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/((a^8*b^6 - 12*a^9*b^4*c + 48*a^
```

```
10*b^2*c^2 - 64*a^11*c^3)*n^6)) - b^3 + 2*a*b*c)/((a^4*b^2 - 4*a^5*c)*n^3))
^(2/3) - sqrt(2)*(1/2)^(2/3)*(sqrt(3)*(a^4*b^8 - 13*a^5*b^6*c + 60*a^6*b^4*
c^2 - 112*a^7*b^2*c^3 + 64*a^8*c^4)*n^5*x*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^
4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/((a^8*b^6 - 12*a^9*b^4*c + 48*a^10*b^2*
c^2 - 64*a^11*c^3)*n^6)) + sqrt(3)*(b^9 - 11*a*b^7*c + 42*a^2*b^5*c^2 - 62*
a^3*b^3*c^3 + 24*a^4*b*c^4)*n^2*x)*sqrt((2*(b^8*c^2 - 8*a*b^6*c^3 + 20*a^2*
b^4*c^4 - 16*a^3*b^2*c^5 + 4*a^4*c^6)*x^2*x^(-2/3*n - 2) + (1/2)^(1/3)*((a^
4*b^9*c - 12*a^5*b^7*c^2 + 50*a^6*b^5*c^3 - 80*a^7*b^3*c^4 + 32*a^8*b*c^5)*
n^4*x*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/
((a^8*b^6 - 12*a^9*b^4*c + 48*a^10*b^2*c^2 - 64*a^11*c^3)*n^6)) + (b^10*c -
 12*a*b^8*c^2 + 52*a^2*b^6*c^3 - 96*a^3*b^4*c^4 + 68*a^4*b^2*c^5 - 16*a^5*c
^6)*n*x)*x^(-1/3*n - 1)*(-((a^4*b^2 - 4*a^5*c)*n^3*sqrt((b^8 - 8*a*b^6*c +
20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/((a^8*b^6 - 12*a^9*b^4*c + 48*
a^10*b^2*c^2 - 64*a^11*c^3)*n^6)) - b^3 + 2*a*b*c)/((a^4*b^2 - 4*a^5*c)*n^3
))^(1/3) + (1/2)^(2/3)*((a^4*b^11 - 16*a^5*b^9*c + 98*a^6*b^7*c^2 - 280*a^7
*b^5*c^3 + 352*a^8*b^3*c^4 - 128*a^9*b*c^5)*n^5*sqrt((b^8 - 8*a*b^6*c + 20*
a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/((a^8*b^6 - 12*a^9*b^4*c + 48*a^1
0*b^2*c^2 - 64*a^11*c^3)*n^6)) + (b^12 - 14*a*b^10*c + 76*a^2*b^8*c^2 - 200
*a^3*b^6*c^3 + 260*a^4*b^4*c^4 - 152*a^5*b^2*c^5 + 32*a^6*c^6)*n^2)*(-((a^4
*b^2 - 4*a^5*c)*n^3*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3
 + 4*a^4*c^4)/((a^8*b^6 - 12*a^9*b^4*c + 48*a^10*b^2*c^2 - 64*a^11*c^3)*n^6
)) - b^3 + 2*a*b*c)/((a^4*b^2 - 4*a^5*c)*n^3))^(2/3))/x^2)*(-((a^4*b^2 - 4*
a^5*c)*n^3*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*
c^4)/((a^8*b^6 - 12*a^9*b^4*c + 48*a^10*b^2*c^2 - 64*a^11*c^3)*n^6)) - b^3
+ 2*a*b*c)/((a^4*b^2 - 4*a^5*c)*n^3))^(2/3) - 2*sqrt(3)*(b^8*c^4 - 8*a*b^6*
c^5 + 20*a^2*b^4*c^6 - 16*a^3*b^2*c^7 + 4*a^4*c^8))/(b^8*c^4 - 8*a*b^6*c^5
+ 20*a^2*b^4*c^6 - 16*a^3*b^2*c^7 + 4*a^4*c^8)) + 2*(1/2)^(1/3)*a*n*(((a^4*
b^2 - 4*a^5*c)*n^3*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3
+ 4*a^4*c^4)/((a^8*b^6 - 12*a^9*b^4*c + 48*a^10*b^2*c^2 - 64*a^11*c^3)*n^6)
) + b^3 - 2*a*b*c)/((a^4*b^2 - 4*a^5*c)*n^3))^(1/3)*log((2*(b^4*c - 4*a*b^2
*c^2 + 2*a^2*c^3)*x*x^(-1/3*n - 1) + (1/2)^(1/3)*((a^4*b^5 - 8*a^5*b^3*c +
16*a^6*b*c^2)*n^4*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 +
 4*a^4*c^4)/((a^8*b^6 - 12*a^9*b^4*c + 48*a^10*b^2*c^2 - 64*a^11*c^3)*n^6))
 - (b^6 - 8*a*b^4*c + 18*a^2*b^2*c^2 - 8*a^3*c^3)*n)*(((a^4*b^2 - 4*a^5*c)*
n^3*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/((
a^8*b^6 - 12*a^9*b^4*c + 48*a^10*b^2*c^2 - 64*a^11*c^3)*n^6)) + b^3 - 2*a*b
*c)/((a^4*b^2 - 4*a^5*c)*n^3))^(1/3))/x) + 2*(1/2)^(1/3)*a*n*(-((a^4*b^2 -
4*a^5*c)*n^3*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^
4*c^4)/((a^8*b^6 - 12*a^9*b^4*c + 48*a^10*b^2*c^2 - 64*a^11*c^3)*n^6)) - b^
3 + 2*a*b*c)/((a^4*b^2 - 4*a^5*c)*n^3))^(1/3)*log((2*(b^4*c - 4*a*b^2*c^2 +
 2*a^2*c^3)*x*x^(-1/3*n - 1) - (1/2)^(1/3)*((a^4*b^5 - 8*a^5*b^3*c + 16*a^6
*b*c^2)*n^4*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4
*c^4)/((a^8*b^6 - 12*a^9*b^4*c + 48*a^10*b^2*c^2 - 64*a^11*c^3)*n^6)) + (b^
6 - 8*a*b^4*c + 18*a^2*b^2*c^2 - 8*a^3*c^3)*n)*(-((a^4*b^2 - 4*a^5*c)*n^3*s
qrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/((a^8*b
^6 - 12*a^9*b^4*c + 48*a^10*b^2*c^2 - 64*a^11*c^3)*n^6)) - b^3 + 2*a*b*c)/(
(a^4*b^2 - 4*a^5*c)*n^3))^(1/3))/x) - (1/2)^(1/3)*a*n*(((a^4*b^2 - 4*a^5*c)
*n^3*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(
(a^8*b^6 - 12*a^9*b^4*c + 48*a^10*b^2*c^2 - 64*a^11*c^3)*n^6)) + b^3 - 2*a*
b*c)/((a^4*b^2 - 4*a^5*c)*n^3))^(1/3)*log(8*(2*(b^8*c^2 - 8*a*b^6*c^3 + 20*
a^2*b^4*c^4 - 16*a^3*b^2*c^5 + 4*a^4*c^6)*x^2*x^(-2/3*n - 2) - (1/2)^(1/3)*
((a^4*b^9*c - 12*a^5*b^7*c^2 + 50*a^6*b^5*c^3 - 80*a^7*b^3*c^4 + 32*a^8*b*c
^5)*n^4*x*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c
^4)/((a^8*b^6 - 12*a^9*b^4*c + 48*a^10*b^2*c^2 - 64*a^11*c^3)*n^6)) - (b^10
*c - 12*a*b^8*c^2 + 52*a^2*b^6*c^3 - 96*a^3*b^4*c^4 + 68*a^4*b^2*c^5 - 16*a
^5*c^6)*n*x)*x^(-1/3*n - 1)*(((a^4*b^2 - 4*a^5*c)*n^3*sqrt((b^8 - 8*a*b^6*c
 + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/((a^8*b^6 - 12*a^9*b^4*c +
48*a^10*b^2*c^2 - 64*a^11*c^3)*n^6)) + b^3 - 2*a*b*c)/((a^4*b^2 - 4*a^5*c)*
n^3))^(1/3) - (1/2)^(2/3)*((a^4*b^11 - 16*a^5*b^9*c + 98*a^6*b^7*c^2 - 280*
```

```
a^7*b^5*c^3 + 352*a^8*b^3*c^4 - 128*a^9*b*c^5)*n^5*sqrt((b^8 - 8*a*b^6*c +
20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/((a^8*b^6 - 12*a^9*b^4*c + 48*
a^10*b^2*c^2 - 64*a^11*c^3)*n^6)) - (b^12 - 14*a*b^10*c + 76*a^2*b^8*c^2 -
200*a^3*b^6*c^3 + 260*a^4*b^4*c^4 - 152*a^5*b^2*c^5 + 32*a^6*c^6)*n^2)*(((a
^4*b^2 - 4*a^5*c)*n^3*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c
^3 + 4*a^4*c^4)/((a^8*b^6 - 12*a^9*b^4*c + 48*a^10*b^2*c^2 - 64*a^11*c^3)*n
^6)) + b^3 - 2*a*b*c)/((a^4*b^2 - 4*a^5*c)*n^3))^(2/3))/x^2) - (1/2)^(1/3)*
a*n*(-((a^4*b^2 - 4*a^5*c)*n^3*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*
a^3*b^2*c^3 + 4*a^4*c^4)/((a^8*b^6 - 12*a^9*b^4*c + 48*a^10*b^2*c^2 - 64*a^
11*c^3)*n^6)) - b^3 + 2*a*b*c)/((a^4*b^2 - 4*a^5*c)*n^3))^(1/3)*log(8*(2*(b
^8*c^2 - 8*a*b^6*c^3 + 20*a^2*b^4*c^4 - 16*a^3*b^2*c^5 + 4*a^4*c^6)*x^2*x^(
-2/3*n - 2) + (1/2)^(1/3)*((a^4*b^9*c - 12*a^5*b^7*c^2 + 50*a^6*b^5*c^3 - 8
0*a^7*b^3*c^4 + 32*a^8*b*c^5)*n^4*x*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2
- 16*a^3*b^2*c^3 + 4*a^4*c^4)/((a^8*b^6 - 12*a^9*b^4*c + 48*a^10*b^2*c^2 -
64*a^11*c^3)*n^6)) + (b^10*c - 12*a*b^8*c^2 + 52*a^2*b^6*c^3 - 96*a^3*b^4*c
^4 + 68*a^4*b^2*c^5 - 16*a^5*c^6)*n*x)*x^(-1/3*n - 1)*(-((a^4*b^2 - 4*a^5*c
)*n^3*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/
((a^8*b^6 - 12*a^9*b^4*c + 48*a^10*b^2*c^2 - 64*a^11*c^3)*n^6)) - b^3 + 2*a
*b*c)/((a^4*b^2 - 4*a^5*c)*n^3))^(1/3) + (1/2)^(2/3)*((a^4*b^11 - 16*a^5*b^
9*c + 98*a^6*b^7*c^2 - 280*a^7*b^5*c^3 + 352*a^8*b^3*c^4 - 128*a^9*b*c^5)*n
^5*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/((a
^8*b^6 - 12*a^9*b^4*c + 48*a^10*b^2*c^2 - 64*a^11*c^3)*n^6)) + (b^12 - 14*a
*b^10*c + 76*a^2*b^8*c^2 - 200*a^3*b^6*c^3 + 260*a^4*b^4*c^4 - 152*a^5*b^2*
c^5 + 32*a^6*c^6)*n^2)*(-((a^4*b^2 - 4*a^5*c)*n^3*sqrt((b^8 - 8*a*b^6*c + 2
0*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/((a^8*b^6 - 12*a^9*b^4*c + 48*a
^10*b^2*c^2 - 64*a^11*c^3)*n^6)) - b^3 + 2*a*b*c)/((a^4*b^2 - 4*a^5*c)*n^3)
)^(2/3))/x^2) - 6*x*x^(-1/3*n - 1))/(a*n)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{-\frac{1}{3}n-1}}{cx^{2n} + bx^{n} + a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-1/3*n)/(a+b*x^n+c*x^(2*n)),x, algorithm="giac")
```

```
[Out] integrate(x^(-1/3*n - 1)/(c*x^(2*n) + b*x^n + a), x)
```

maple [C] time = 0.75, size = 534, normalized size = 0.76

$$\mathrm{RootOf}\left(\left(64a^7c^3n^6-48a^6b^2c^2n^6+12a^5b^4cn^6-a^4b^6n^6\right)\_Z^6+c^4+\left(-32a^3bc^3n^3+32a^2b^3c^2n^3-10ab^5cn^3+b^7n^3\right)\_Z^3\right)\ln\left(\left(-\frac{64a^8c^4n^5}{2a^2c^5-4ab^2c^4+b^4c^3}+\frac{112a^7b^2c^3n^5}{2a^2c^5-4ab^2c^4+b^4c^3}-\frac{60a^6b^4c^2n^5}{2a^2c^5-4ab^2c^4+b^4c^3}+\frac{13a^5b^6cn^5}{2a^2c^5-4ab^2c^4+b^4c^3}-\frac{a^4b^8n^5}{2a^2c^5-4ab^2c^4+b^4c^3}\right)\mathrm{RootOf}\left(\left(64a^7c^3n^6-48a^6b^2c^2n^6+12a^5b^4cn^6-a^4b^6n^6\right)\_Z^6+c^4+\left(-32a^3bc^3n^3+32a^2b^3c^2n^3-10ab^5cn^3+b^7n^3\right)\_Z^3\right)^5+\left(\frac{28a^4bc^4n^2}{2a^2c^5-4ab^2c^4+b^4c^3}-\frac{63a^3b^3c^3n^2}{2a^2c^5-4ab^2c^4+b^4c^3}+\frac{42a^2b^5c^2n^2}{2a^2c^5-4ab^2c^4+b^4c^3}-\frac{11ab^7cn^2}{2a^2c^5-4ab^2c^4+b^4c^3}+\frac{b^9n^2}{2a^2c^5-4ab^2c^4+b^4c^3}\right)\mathrm{RootOf}\left(\left(64a^7c^3n^6-48a^6b^2c^2n^6+12a^5b^4cn^6-a^4b^6n^6\right)\_Z^6+c^4+\left(-32a^3bc^3n^3+32a^2b^3c^2n^3-10ab^5cn^3+b^7n^3\right)\_Z^3\right)^2+x^{\frac{n}{3}}\right)-\frac{3x^{-\frac{n}{3}}}{an}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1-1/3*n)/(a+b*x^n+c*x^(2*n)),x)
```

```
[Out] -3/a/n/(x^(1/3*n))+sum(_R*ln(x^(1/3*n)+(-64/(2*a^2*c^5-4*a*b^2*c^4+b^4*c^3)
*n^5*a^8*c^4+112/(2*a^2*c^5-4*a*b^2*c^4+b^4*c^3)*n^5*b^2*a^7*c^3-60/(2*a^2*
c^5-4*a*b^2*c^4+b^4*c^3)*n^5*b^4*a^6*c^2+13/(2*a^2*c^5-4*a*b^2*c^4+b^4*c^3)
*n^5*b^6*a^5*c-1/(2*a^2*c^5-4*a*b^2*c^4+b^4*c^3)*n^5*b^8*a^4)*_R^5+(28/(2*a
^2*c^5-4*a*b^2*c^4+b^4*c^3)*n^2*b*a^4*c^4-63/(2*a^2*c^5-4*a*b^2*c^4+b^4*c^3
)*n^2*b^3*a^3*c^3+42/(2*a^2*c^5-4*a*b^2*c^4+b^4*c^3)*n^2*b^5*a^2*c^2-11/(2*
a^2*c^5-4*a*b^2*c^4+b^4*c^3)*n^2*b^7*a*c+1/(2*a^2*c^5-4*a*b^2*c^4+b^4*c^3)*
n^2*b^9)*_R^2),_R=RootOf((64*a^7*c^3*n^6-48*a^6*b^2*c^2*n^6+12*a^5*b^4*c*n^
6-a^4*b^6*n^6)*_Z^6+(-32*a^3*b*c^3*n^3+32*a^2*b^3*c^2*n^3-10*a*b^5*c*n^3+b^
7*n^3)*_Z^3+c^4))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{3}{anx^{\frac{1}{3}n}} - \int \frac{cx^{\frac{5}{3}n} + bx^{\frac{2}{3}n}}{acxx^{2n} + abxx^{n} + a^2x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-1/3*n)/(a+b*x^n+c*x^(2*n)),x, algorithm="maxima")
```

```
[Out] -3/(a*n*x^(1/3*n)) - integrate((c*x^(5/3*n) + b*x^(2/3*n))/(a*c*x*x^(2*n) +
 a*b*x*x^n + a^2*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^{\frac{n}{3}+1}\,\left(a+b\,x^n+c\,x^{2\,n}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(n/3 + 1)*(a + b*x^n + c*x^(2*n))),x)
```

```
[Out] int(1/(x^(n/3 + 1)*(a + b*x^n + c*x^(2*n))), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1-1/3*n)/(a+b*x**n+c*x**(2*n)),x)
```

```
[Out] Timed out
```

3.485 $\int \frac{x^{-1-\frac{n}{4}}}{a+bx^n+cx^{2n}}\,dx$

Optimal. Leaf size=414

$$-\frac{2^{3/4}\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{a}\,x^{-n/4}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{a^{5/4}n\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}-\frac{2^{3/4}\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{a}\,x^{-n/4}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{a^{5/4}n\left(\sqrt{b^2-4ac}-b\right)^{3/4}}-\frac{2^{3/4}\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\tanh^{-1}\left(\cdots\right)}{a^{5/4}n\left(-\sqrt{b^2-4ac}-b\right.\cdots}$$

Rubi [A] time = 0.79, antiderivative size = 414, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {1381, 1340, 1367, 1422, 212, 208, 205}

$$-\frac{2^{3/4}\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{a}\,x^{-n/4}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{a^{5/4}n\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}-\frac{2^{3/4}\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{a}\,x^{-n/4}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{a^{5/4}n\left(\sqrt{b^2-4ac}-b\right)^{3/4}}-\frac{2^{3/4}\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{a}\,x^{-n/4}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{a^{5/4}n\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}-\frac{2^{3/4}\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{a}\,x^{-n/4}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{a^{5/4}n\left(\sqrt{b^2-4ac}-b\right)^{3/4}}-\frac{4x^{-n/4}}{an}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 - n/4)/(a + b*x^n + c*x^(2*n)),x]
```

```
[Out] -4/(a*n*x^(n/4)) - (2^(3/4)*(b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(2
^(1/4)*a^(1/4))/((-b - Sqrt[b^2 - 4*a*c])^(1/4)*x^(n/4))])/(a^(5/4)*(-b - S
qrt[b^2 - 4*a*c])^(3/4)*n) - (2^(3/4)*(b - (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])
*ArcTan[(2^(1/4)*a^(1/4))/((-b + Sqrt[b^2 - 4*a*c])^(1/4)*x^(n/4))])/(a^(5/
4)*(-b + Sqrt[b^2 - 4*a*c])^(3/4)*n) - (2^(3/4)*(b + (b^2 - 2*a*c)/Sqrt[b^2
 - 4*a*c])*ArcTanh[(2^(1/4)*a^(1/4))/((-b - Sqrt[b^2 - 4*a*c])^(1/4)*x^(n/4
))])/(a^(5/4)*(-b - Sqrt[b^2 - 4*a*c])^(3/4)*n) - (2^(3/4)*(b - (b^2 - 2*a*
c)/Sqrt[b^2 - 4*a*c])*ArcTanh[(2^(1/4)*a^(1/4))/((-b + Sqrt[b^2 - 4*a*c])^(
1/4)*x^(n/4))])/(a^(5/4)*(-b + Sqrt[b^2 - 4*a*c])^(3/4)*n)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 1340

```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[x^
(2*n*p)*(c + b/x^n + a/x^(2*n))^p, x] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n
] && LtQ[n, 0] && IntegerQ[p]
```

Rule 1367

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Simp[(d^(2*n - 1)*(d*x)^(m - 2*n + 1)*(a + b*x^n + c*x^(2*n))^(
p + 1))/(c*(m + 2*n*p + 1)), x] - Dist[d^(2*n)/(c*(m + 2*n*p + 1)), Int[(d*
```

```
x)^(m - 2*n)*Simp[a*(m - 2*n + 1) + b*(m + n*(p - 1) + 1)*x^n, x]*(a + b*x^
n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && EqQ[n2, 2*n] && Ne
Q[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GtQ[m, 2*n - 1] && NeQ[m + 2*n*p + 1, 0]
 && IntegerQ[p]
```

Rule 1381

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> Dist[1/(m + 1), Subst[Int[(a + b*x^Simplify[n/(m + 1)] + c*x^Simplify[(
2*n)/(m + 1)])^p, x], x, x^(m + 1)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&
EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IntegerQ[Simplify[n/(m + 1)]] &&  !I
ntegerQ[n]
```

Rule 1422

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q),
Int[1/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(
b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (PosQ[b^2 - 4*a
*c] ||  !IGtQ[n/2, 0])
```

Rubi steps

$$\int \frac{x^{-1-\frac{n}{4}}}{a+bx^n+cx^{2n}}\,dx = -\frac{4\,\text{Subst}\left(\int \frac{1}{a+\frac{c}{x^8}+\frac{b}{x^4}}\,dx, x, x^{-n/4}\right)}{n}$$

$$= -\frac{4\,\text{Subst}\left(\int \frac{x^8}{c+bx^4+ax^8}\,dx, x, x^{-n/4}\right)}{n}$$

$$= -\frac{4x^{-n/4}}{an} + \frac{4\,\text{Subst}\left(\int \frac{c+bx^4}{c+bx^4+ax^8}\,dx, x, x^{-n/4}\right)}{an}$$

$$= -\frac{4x^{-n/4}}{an} + \frac{\left(2\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\right)\text{Subst}\left(\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+ax^4}\,dx, x, x^{-n/4}\right)}{an} + \frac{\left(2\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\right.}{}$$

$$= -\frac{4x^{-n/4}}{an} - \frac{\left(2\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\right)\text{Subst}\left(\int \frac{1}{\sqrt{-b+\sqrt{b^2-4ac}}-\sqrt{2}\sqrt{a}\,x^2}\,dx, x, x^{-n/4}\right)}{a\sqrt{-b+\sqrt{b^2-4ac}}\;n} - \frac{\left(2\left(b-\frac{b^2-}{\sqrt{b^2}}\right.\right.}{}$$

$$= -\frac{4x^{-n/4}}{an} - \frac{2^{3/4}\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{a}\,x^{-n/4}}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{a^{5/4}\left(-b-\sqrt{b^2-4ac}\right)^{3/4}n} - \frac{2^{3/4}\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{a}}{\sqrt[4]{-b+\sqrt{b}}}\right.}{a^{5/4}\left(-b+\sqrt{b^2-4ac}\right)^{3/4}n}$$

Mathematica [C] time = 0.12, size = 127, normalized size = 0.31

$$\frac{8cx^{-n/4}\left(\frac{{}_2F_1\left(-\frac{1}{4},1;\frac{3}{4};\frac{2cx^n}{\sqrt{b^2-4ac}-b}\right)}{-b\sqrt{b^2-4ac}-4ac+b^2}+\frac{{}_2F_1\left(-\frac{1}{4},1;\frac{3}{4};-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{b\sqrt{b^2-4ac}-4ac+b^2}\right)}{n}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 - n/4)/(a + b*x^n + c*x^(2*n)),x]
```

```
[Out] (8*c*(Hypergeometric2F1[-1/4, 1, 3/4, (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])]/(
b^2 - 4*a*c - b*Sqrt[b^2 - 4*a*c]) + Hypergeometric2F1[-1/4, 1, 3/4, (-2*c*
x^n)/(b + Sqrt[b^2 - 4*a*c])]/(b^2 - 4*a*c + b*Sqrt[b^2 - 4*a*c])))/(n*x^(n
/4))
```

IntegrateAlgebraic [F] time = 0.07, size = 0, normalized size = 0.00

$$\int \frac{x^{-1-\frac{n}{4}}}{a+bx^n+cx^{2n}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^(-1 - n/4)/(a + b*x^n + c*x^(2*n)),x]
```

```
[Out] Defer[IntegrateAlgebraic][x^(-1 - n/4)/(a + b*x^n + c*x^(2*n)), x]
```

fricas [B] time = 4.18, size = 5712, normalized size = 13.80

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-1/4*n)/(a+b*x^n+c*x^(2*n)),x, algorithm="fricas")
```

```
[Out] 1/2*(4*sqrt(2)*a*n*sqrt(sqrt(2)*sqrt(((a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*
n^4*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/((a^1
0*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)*n^8)) - b^5 + 5*a*b^
3*c - 5*a^2*b*c^2)/((a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*n^4)))*arctan(1/16
*sqrt(2)*(2*sqrt(2)*((a^5*b^14*c - 19*a^6*b^12*c^2 + 147*a^7*b^10*c^3 - 590
*a^8*b^8*c^4 + 1290*a^9*b^6*c^5 - 1464*a^10*b^4*c^6 + 736*a^11*b^2*c^7 - 12
8*a^12*c^8)*n^7*x*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 +
a^4*c^4)/((a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)*n^8))
+ (b^15*c - 16*a*b^13*c^2 + 103*a^2*b^11*c^3 - 340*a^3*b^9*c^4 + 605*a^4*b^
7*c^5 - 554*a^5*b^5*c^6 + 224*a^6*b^3*c^7 - 32*a^7*b*c^8)*n^3*x)*x^(-1/4*n
- 1)*sqrt(((a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*n^4*sqrt((b^8 - 6*a*b^6*c +
 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/((a^10*b^6 - 12*a^11*b^4*c + 48*
a^12*b^2*c^2 - 64*a^13*c^3)*n^8)) - b^5 + 5*a*b^3*c - 5*a^2*b*c^2)/((a^5*b^
4 - 8*a^6*b^2*c + 16*a^7*c^2)*n^4)) - sqrt(2)*((a^5*b^10 - 16*a^6*b^8*c + 9
8*a^7*b^6*c^2 - 280*a^8*b^4*c^3 + 352*a^9*b^2*c^4 - 128*a^10*c^5)*n^7*x*sqr
t((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/((a^10*b^6 -
 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)*n^8)) + (b^11 - 13*a*b^9*c
+ 63*a^2*b^7*c^2 - 138*a^3*b^5*c^3 + 128*a^4*b^3*c^4 - 32*a^5*b*c^5)*n^3*x)
*sqrt((4*(b^8*c^2 - 6*a*b^6*c^3 + 11*a^2*b^4*c^4 - 6*a^3*b^2*c^5 + a^4*c^6)
*x^2*x^(-1/2*n - 2) + sqrt(2)*((a^5*b^11 - 15*a^6*b^9*c + 85*a^7*b^7*c^2 -
220*a^8*b^5*c^3 + 240*a^9*b^3*c^4 - 64*a^10*b*c^5)*n^6*sqrt((b^8 - 6*a*b^6*
c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/((a^10*b^6 - 12*a^11*b^4*c +
48*a^12*b^2*c^2 - 64*a^13*c^3)*n^8)) + (b^12 - 12*a*b^10*c + 55*a^2*b^8*c^2
 - 120*a^3*b^6*c^3 + 125*a^4*b^4*c^4 - 54*a^5*b^2*c^5 + 8*a^6*c^6)*n^2)*sqr
t(((a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*n^4*sqrt((b^8 - 6*a*b^6*c + 11*a^2*
b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/((a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2
*c^2 - 64*a^13*c^3)*n^8)) - b^5 + 5*a*b^3*c - 5*a^2*b*c^2)/((a^5*b^4 - 8*a^
6*b^2*c + 16*a^7*c^2)*n^4)))/x^2)*sqrt(((a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2
)*n^4*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/((a
^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)*n^8)) - b^5 + 5*a*
b^3*c - 5*a^2*b*c^2)/((a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*n^4)))*sqrt(sqrt
(2)*sqrt(((a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*n^4*sqrt((b^8 - 6*a*b^6*c +
11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/((a^10*b^6 - 12*a^11*b^4*c + 48*a
^12*b^2*c^2 - 64*a^13*c^3)*n^8)) - b^5 + 5*a*b^3*c - 5*a^2*b*c^2)/((a^5*b^4
 - 8*a^6*b^2*c + 16*a^7*c^2)*n^4)))/(b^8*c^5 - 6*a*b^6*c^6 + 11*a^2*b^4*c^7
 - 6*a^3*b^2*c^8 + a^4*c^9)) - 4*sqrt(2)*a*n*sqrt(sqrt(2)*sqrt(-((a^5*b^4 -
 8*a^6*b^2*c + 16*a^7*c^2)*n^4*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a
^3*b^2*c^3 + a^4*c^4)/((a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^1
```

```
3*c^3)*n^8)) + b^5 - 5*a*b^3*c + 5*a^2*b*c^2)/((a^5*b^4 - 8*a^6*b^2*c + 16*
a^7*c^2)*n^4)))*arctan(1/8*(2*((a^5*b^14*c - 19*a^6*b^12*c^2 + 147*a^7*b^10
*c^3 - 590*a^8*b^8*c^4 + 1290*a^9*b^6*c^5 - 1464*a^10*b^4*c^6 + 736*a^11*b^
2*c^7 - 128*a^12*c^8)*n^7*x*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*
b^2*c^3 + a^4*c^4)/((a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c
^3)*n^8)) - (b^15*c - 16*a*b^13*c^2 + 103*a^2*b^11*c^3 - 340*a^3*b^9*c^4 +
605*a^4*b^7*c^5 - 554*a^5*b^5*c^6 + 224*a^6*b^3*c^7 - 32*a^7*b*c^8)*n^3*x)*
x^(-1/4*n - 1)*sqrt(sqrt(2)*sqrt(-((a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*n^4
*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/((a^10*b
^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)*n^8)) + b^5 - 5*a*b^3*c
 + 5*a^2*b*c^2)/((a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*n^4)))*sqrt(-((a^5*b^
4 - 8*a^6*b^2*c + 16*a^7*c^2)*n^4*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 -
6*a^3*b^2*c^3 + a^4*c^4)/((a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*
a^13*c^3)*n^8)) + b^5 - 5*a*b^3*c + 5*a^2*b*c^2)/((a^5*b^4 - 8*a^6*b^2*c +
16*a^7*c^2)*n^4)) - ((a^5*b^10 - 16*a^6*b^8*c + 98*a^7*b^6*c^2 - 280*a^8*b^
4*c^3 + 352*a^9*b^2*c^4 - 128*a^10*c^5)*n^7*x*sqrt((b^8 - 6*a*b^6*c + 11*a^
2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/((a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b
^2*c^2 - 64*a^13*c^3)*n^8)) - (b^11 - 13*a*b^9*c + 63*a^2*b^7*c^2 - 138*a^3
*b^5*c^3 + 128*a^4*b^3*c^4 - 32*a^5*b*c^5)*n^3*x)*sqrt(sqrt(2)*sqrt(-((a^5*
b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*n^4*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2
- 6*a^3*b^2*c^3 + a^4*c^4)/((a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 6
4*a^13*c^3)*n^8)) + b^5 - 5*a*b^3*c + 5*a^2*b*c^2)/((a^5*b^4 - 8*a^6*b^2*c
+ 16*a^7*c^2)*n^4)))*sqrt((4*(b^8*c^2 - 6*a*b^6*c^3 + 11*a^2*b^4*c^4 - 6*a^
3*b^2*c^5 + a^4*c^6)*x^2*x^(-1/2*n - 2) - sqrt(2)*((a^5*b^11 - 15*a^6*b^9*c
 + 85*a^7*b^7*c^2 - 220*a^8*b^5*c^3 + 240*a^9*b^3*c^4 - 64*a^10*b*c^5)*n^6*
sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/((a^10*b^
6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)*n^8)) - (b^12 - 12*a*b^1
0*c + 55*a^2*b^8*c^2 - 120*a^3*b^6*c^3 + 125*a^4*b^4*c^4 - 54*a^5*b^2*c^5 +
 8*a^6*c^6)*n^2)*sqrt(-((a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*n^4*sqrt((b^8
- 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/((a^10*b^6 - 12*a^1
1*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)*n^8)) + b^5 - 5*a*b^3*c + 5*a^2*b*
c^2)/((a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*n^4)))/x^2)*sqrt(-((a^5*b^4 - 8*
a^6*b^2*c + 16*a^7*c^2)*n^4*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*
b^2*c^3 + a^4*c^4)/((a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c
^3)*n^8)) + b^5 - 5*a*b^3*c + 5*a^2*b*c^2)/((a^5*b^4 - 8*a^6*b^2*c + 16*a^7
*c^2)*n^4)))/(b^8*c^5 - 6*a*b^6*c^6 + 11*a^2*b^4*c^7 - 6*a^3*b^2*c^8 + a^4*
c^9)) + sqrt(2)*a*n*sqrt(sqrt(2)*sqrt(-((a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2
)*n^4*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/((a
^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)*n^8)) + b^5 - 5*a*
b^3*c + 5*a^2*b*c^2)/((a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*n^4)))*log((4*(b
^4*c - 3*a*b^2*c^2 + a^2*c^3)*x*x^(-1/4*n - 1) + sqrt(2)*((a^5*b^5 - 8*a^6*
b^3*c + 16*a^7*b*c^2)*n^5*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^
2*c^3 + a^4*c^4)/((a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3
)*n^8)) - (b^6 - 7*a*b^4*c + 13*a^2*b^2*c^2 - 4*a^3*c^3)*n)*sqrt(sqrt(2)*sq
rt(-((a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*n^4*sqrt((b^8 - 6*a*b^6*c + 11*a^
2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/((a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b
^2*c^2 - 64*a^13*c^3)*n^8)) + b^5 - 5*a*b^3*c + 5*a^2*b*c^2)/((a^5*b^4 - 8*
a^6*b^2*c + 16*a^7*c^2)*n^4))))/x) - sqrt(2)*a*n*sqrt(sqrt(2)*sqrt(-((a^5*b
^4 - 8*a^6*b^2*c + 16*a^7*c^2)*n^4*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 -
 6*a^3*b^2*c^3 + a^4*c^4)/((a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64
*a^13*c^3)*n^8)) + b^5 - 5*a*b^3*c + 5*a^2*b*c^2)/((a^5*b^4 - 8*a^6*b^2*c +
 16*a^7*c^2)*n^4)))*log((4*(b^4*c - 3*a*b^2*c^2 + a^2*c^3)*x*x^(-1/4*n - 1)
 - sqrt(2)*((a^5*b^5 - 8*a^6*b^3*c + 16*a^7*b*c^2)*n^5*sqrt((b^8 - 6*a*b^6*
c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/((a^10*b^6 - 12*a^11*b^4*c +
48*a^12*b^2*c^2 - 64*a^13*c^3)*n^8)) - (b^6 - 7*a*b^4*c + 13*a^2*b^2*c^2 -
4*a^3*c^3)*n)*sqrt(sqrt(2)*sqrt(-((a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*n^4*
sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/((a^10*b^
6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)*n^8)) + b^5 - 5*a*b^3*c
+ 5*a^2*b*c^2)/((a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*n^4))))/x) - sqrt(2)*a
```

```
*n*sqrt(sqrt(2)*sqrt(((a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*n^4*sqrt((b^8 -
6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/((a^10*b^6 - 12*a^11*
b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)*n^8)) - b^5 + 5*a*b^3*c - 5*a^2*b*c^
2)/((a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*n^4)))*log((4*(b^4*c - 3*a*b^2*c^2
 + a^2*c^3)*x*x^(-1/4*n - 1) + sqrt(2)*((a^5*b^5 - 8*a^6*b^3*c + 16*a^7*b*c
^2)*n^5*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(
(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)*n^8)) + (b^6 - 7
*a*b^4*c + 13*a^2*b^2*c^2 - 4*a^3*c^3)*n)*sqrt(sqrt(2)*sqrt(((a^5*b^4 - 8*a
^6*b^2*c + 16*a^7*c^2)*n^4*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b
^2*c^3 + a^4*c^4)/((a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^
3)*n^8)) - b^5 + 5*a*b^3*c - 5*a^2*b*c^2)/((a^5*b^4 - 8*a^6*b^2*c + 16*a^7*
c^2)*n^4))))/x) + sqrt(2)*a*n*sqrt(sqrt(2)*sqrt(((a^5*b^4 - 8*a^6*b^2*c + 1
6*a^7*c^2)*n^4*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4
*c^4)/((a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)*n^8)) - b
^5 + 5*a*b^3*c - 5*a^2*b*c^2)/((a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*n^4)))*
log((4*(b^4*c - 3*a*b^2*c^2 + a^2*c^3)*x*x^(-1/4*n - 1) - sqrt(2)*((a^5*b^5
 - 8*a^6*b^3*c + 16*a^7*b*c^2)*n^5*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 -
 6*a^3*b^2*c^3 + a^4*c^4)/((a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64
*a^13*c^3)*n^8)) + (b^6 - 7*a*b^4*c + 13*a^2*b^2*c^2 - 4*a^3*c^3)*n)*sqrt(s
qrt(2)*sqrt(((a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*n^4*sqrt((b^8 - 6*a*b^6*c
 + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/((a^10*b^6 - 12*a^11*b^4*c + 4
8*a^12*b^2*c^2 - 64*a^13*c^3)*n^8)) - b^5 + 5*a*b^3*c - 5*a^2*b*c^2)/((a^5*
b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*n^4))))/x) - 8*x*x^(-1/4*n - 1))/(a*n)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{-\frac{1}{4}n-1}}{cx^{2n} + bx^n + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-1/4*n)/(a+b*x^n+c*x^(2*n)),x, algorithm="giac")
```

```
[Out] integrate(x^(-1/4*n - 1)/(c*x^(2*n) + b*x^n + a), x)
```

maple [C] time = 1.22, size = 630, normalized size = 1.52

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1-1/4*n)/(a+b*x^n+c*x^(2*n)),x)
```

```
[Out] -4/a/n/(x^(1/4*n))+sum(_R*ln(x^(1/4*n)+(-128/(a^2*c^6-3*a*b^2*c^5+b^4*c^4)*
n^7*a^10*c^5+352/(a^2*c^6-3*a*b^2*c^5+b^4*c^4)*n^7*b^2*a^9*c^4-280/(a^2*c^6
-3*a*b^2*c^5+b^4*c^4)*n^7*b^4*a^8*c^3+98/(a^2*c^6-3*a*b^2*c^5+b^4*c^4)*n^7*
b^6*a^7*c^2-16/(a^2*c^6-3*a*b^2*c^5+b^4*c^4)*n^7*b^8*a^6*c+1/(a^2*c^6-3*a*b
^2*c^5+b^4*c^4)*n^7*b^10*a^5)*_R^7+(-36/(a^2*c^6-3*a*b^2*c^5+b^4*c^4)*n^3*b
*a^5*c^5+129/(a^2*c^6-3*a*b^2*c^5+b^4*c^4)*n^3*b^3*a^4*c^4-138/(a^2*c^6-3*a
*b^2*c^5+b^4*c^4)*n^3*b^5*a^3*c^3+63/(a^2*c^6-3*a*b^2*c^5+b^4*c^4)*n^3*b^7*
a^2*c^2-13/(a^2*c^6-3*a*b^2*c^5+b^4*c^4)*n^3*b^9*a*c+1/(a^2*c^6-3*a*b^2*c^5
+b^4*c^4)*n^3*b^11)*_R^3),_R=RootOf((256*a^9*c^4*n^8-256*a^8*b^2*c^3*n^8+96
*a^7*b^4*c^2*n^8-16*a^6*b^6*c*n^8+a^5*b^8*n^8)*_Z^8+(80*a^4*b*c^4*n^4-120*a
^3*b^3*c^3*n^4+61*a^2*b^5*c^2*n^4-13*a*b^7*c*n^4+b^9*n^4)*_Z^4+c^5))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{4}{anx^{\frac{1}{4}n}} - \int \frac{cx^{\frac{7}{4}n} + bx^{\frac{3}{4}n}}{acxx^{2n} + abxx^n + a^2x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-1/4*n)/(a+b*x^n+c*x^(2*n)),x, algorithm="maxima")
```

```
[Out] -4/(a*n*x^(1/4*n)) - integrate((c*x^(7/4*n) + b*x^(3/4*n))/(a*c*x*x^(2*n) +
 a*b*x*x^n + a^2*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^{\frac{n}{4}+1}\,\left(a+b\,x^n+c\,x^{2\,n}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(n/4 + 1)*(a + b*x^n + c*x^(2*n))),x)
```

```
[Out] int(1/(x^(n/4 + 1)*(a + b*x^n + c*x^(2*n))), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1-1/4*n)/(a+b*x**n+c*x**(2*n)),x)
```

```
[Out] Timed out
```

3.486 $\int \frac{1}{x(a+bx^n+cx^{2n})}\,dx$

Optimal. Leaf size=74

$$\frac{b\tanh^{-1}\left(\frac{b+2cx^n}{\sqrt{b^2-4ac}}\right)}{an\sqrt{b^2-4ac}}-\frac{\log\left(a+bx^n+cx^{2n}\right)}{2an}+\frac{\log(x)}{a}$$

Rubi [A] time = 0.07, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.350, Rules used = {1357, 705, 29, 634, 618, 206, 628}

$$\frac{b\tanh^{-1}\left(\frac{b+2cx^n}{\sqrt{b^2-4ac}}\right)}{an\sqrt{b^2-4ac}}-\frac{\log\left(a+bx^n+cx^{2n}\right)}{2an}+\frac{\log(x)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a + b*x^n + c*x^(2*n))),x]
```

```
[Out] (b*ArcTanh[(b + 2*c*x^n)/Sqrt[b^2 - 4*a*c]])/(a*Sqrt[b^2 - 4*a*c]*n) + Log[
x]/a - Log[a + b*x^n + c*x^(2*n)]/(2*a*n)
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 705

```
Int[1/(((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)), x_Symbol]
 :> Dist[e^2/(c*d^2 - b*d*e + a*e^2), Int[1/(d + e*x), x], x] + Dist[1/(c*d
^2 - b*d*e + a*e^2), Int[(c*d - b*e - c*e*x)/(a + b*x + c*x^2), x], x] /; F
reeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^
2, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1357

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x\left(a+bx^n+cx^{2n}\right)}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{x\left(a+bx+cx^2\right)}\,dx,x,x^n\right)}{n}\\
&= \frac{\text{Subst}\left(\int \frac{1}{x}\,dx,x,x^n\right)}{an}+\frac{\text{Subst}\left(\int \frac{-b-cx}{a+bx+cx^2}\,dx,x,x^n\right)}{an}\\
&= \frac{\log(x)}{a}-\frac{\text{Subst}\left(\int \frac{b+2cx}{a+bx+cx^2}\,dx,x,x^n\right)}{2an}-\frac{b\,\text{Subst}\left(\int \frac{1}{a+bx+cx^2}\,dx,x,x^n\right)}{2an}\\
&= \frac{\log(x)}{a}-\frac{\log\left(a+bx^n+cx^{2n}\right)}{2an}+\frac{b\,\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx,x,b+2cx^n\right)}{an}\\
&= \frac{b\tanh^{-1}\left(\frac{b+2cx^n}{\sqrt{b^2-4ac}}\right)}{a\sqrt{b^2-4ac}\,n}+\frac{\log(x)}{a}-\frac{\log\left(a+bx^n+cx^{2n}\right)}{2an}
\end{aligned}
$$

Mathematica [A] time = 0.14, size = 74, normalized size = 1.00

$$
-\frac{\frac{2b\tan^{-1}\left(\frac{b+2cx^n}{\sqrt{4ac-b^2}}\right)}{n\sqrt{4ac-b^2}}+\frac{\log(a+x^n(b+cx^n))}{n}-2\log(x)}{2a}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a + b*x^n + c*x^(2*n))),x]
```

```
[Out] -1/2*((2*b*ArcTan[(b + 2*c*x^n)/Sqrt[-b^2 + 4*a*c]])/(Sqrt[-b^2 + 4*a*c]*n)
 - 2*Log[x] + Log[a + x^n*(b + c*x^n)]/n)/a
```

IntegrateAlgebraic [A] time = 0.15, size = 98, normalized size = 1.32

$$
-\frac{b\tan^{-1}\left(\frac{2cx^n}{\sqrt{4ac-b^2}}+\frac{b}{\sqrt{4ac-b^2}}\right)}{an\sqrt{4ac-b^2}}-\frac{\log\left(a+bx^n+cx^{2n}\right)}{2an}+\frac{\log\left(x^n\right)}{an}
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x*(a + b*x^n + c*x^(2*n))),x]
```

```
[Out] -((b*ArcTan[b/Sqrt[-b^2 + 4*a*c] + (2*c*x^n)/Sqrt[-b^2 + 4*a*c]])/(a*Sqrt[-
b^2 + 4*a*c]*n)) + Log[x^n]/(a*n) - Log[a + b*x^n + c*x^(2*n)]/(2*a*n)
```

fricas [A] time = 1.01, size = 259, normalized size = 3.50

$$
\left[\frac{2\left(b^2-4ac\right)n\log(x)+\sqrt{b^2-4ac}\,b\log\left(\frac{2c^2x^{2n}+b^2-2ac+2\left(bc+\sqrt{b^2-4ac}\,c\right)x^n+\sqrt{b^2-4ac}\,b}{cx^{2n}+bx^n+a}\right)-\left(b^2-4ac\right)\log\left(cx^{2n}+bx^n+a\right)}{2\left(ab^2-4a^2c\right)n},\ \frac{2\left(b^2-4ac\right)n\log(x)+2\sqrt{-b^2+4ac}\,b\arctan\left(-\frac{2\sqrt{-b^2+4ac}\,cx^n+\sqrt{-b^2+4ac}\,b}{b^2-4ac}\right)-\left(b^2-4ac\right)\log\left(cx^{2n}+bx^n+a\right)}{2\left(ab^2-4a^2c\right)n}\right]
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*x^n+c*x^(2*n)),x, algorithm="fricas")
```

```
[Out] [1/2*(2*(b^2 - 4*a*c)*n*log(x) + sqrt(b^2 - 4*a*c)*b*log((2*c^2*x^(2*n) + b
^2 - 2*a*c + 2*(b*c + sqrt(b^2 - 4*a*c)*c)*x^n + sqrt(b^2 - 4*a*c)*b)/(c*x^
(2*n) + b*x^n + a)) - (b^2 - 4*a*c)*log(c*x^(2*n) + b*x^n + a))/((a*b^2 - 4
*a^2*c)*n), 1/2*(2*(b^2 - 4*a*c)*n*log(x) + 2*sqrt(-b^2 + 4*a*c)*b*arctan(-
(2*sqrt(-b^2 + 4*a*c)*c*x^n + sqrt(-b^2 + 4*a*c)*b)/(b^2 - 4*a*c)) - (b^2 -
 4*a*c)*log(c*x^(2*n) + b*x^n + a))/((a*b^2 - 4*a^2*c)*n)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(cx^{2n} + bx^{n} + a\right)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*x^n+c*x^(2*n)),x, algorithm="giac")
```

```
[Out] integrate(1/((c*x^(2*n) + b*x^n + a)*x), x)
```

maple [B] time = 0.10, size = 397, normalized size = 5.36

$$\frac{4ac\,n^{2}\ln(x)}{4a^{2}c\,n^{2}-a\,b^{2}n^{2}}-\frac{b^{2}n^{2}\ln(x)}{4a^{2}c\,n^{2}-a\,b^{2}n^{2}}+\frac{b^{2}\ln\left(x^{n}-\frac{-b^{2}+\sqrt{-4a\,b^{2}c+b^{4}}}{2bc}\right)}{2\left(4ac-b^{2}\right)an}+\frac{b^{2}\ln\left(x^{n}+\frac{b^{2}+\sqrt{-4a\,b^{2}c+b^{4}}}{2bc}\right)}{2\left(4ac-b^{2}\right)an}-\frac{2c\ln\left(x^{n}-\frac{-b^{2}+\sqrt{-4a\,b^{2}c+b^{4}}}{2bc}\right)}{\left(4ac-b^{2}\right)n}-\frac{2c\ln\left(x^{n}+\frac{b^{2}+\sqrt{-4a\,b^{2}c+b^{4}}}{2bc}\right)}{\left(4ac-b^{2}\right)n}+\frac{\sqrt{-4a\,b^{2}c+b^{4}}\,\ln\left(x^{n}-\frac{-b^{2}+\sqrt{-4a\,b^{2}c+b^{4}}}{2bc}\right)}{2\left(4ac-b^{2}\right)an}-\frac{\sqrt{-4a\,b^{2}c+b^{4}}\,\ln\left(x^{n}+\frac{b^{2}+\sqrt{-4a\,b^{2}c+b^{4}}}{2bc}\right)}{2\left(4ac-b^{2}\right)an}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x^n+c*x^(2*n)+a),x)
```

```
[Out] 4/(4*a^2*c*n^2-a*b^2*n^2)*n^2*ln(x)*a*c-1/(4*a^2*c*n^2-a*b^2*n^2)*n^2*ln(x)
*b^2-2/(4*a*c-b^2)/n*ln(x^n-1/2*(-b^2+(-4*a*b^2*c+b^4)^(1/2))/b/c)*c+1/2/a/
(4*a*c-b^2)/n*ln(x^n-1/2*(-b^2+(-4*a*b^2*c+b^4)^(1/2))/b/c)*b^2+1/2/a/(4*a*
c-b^2)/n*ln(x^n-1/2*(-b^2+(-4*a*b^2*c+b^4)^(1/2))/b/c)*(-4*a*b^2*c+b^4)^(1/
2)-2/(4*a*c-b^2)/n*ln(x^n+1/2*(b^2+(-4*a*b^2*c+b^4)^(1/2))/b/c)*c+1/2/a/(4*
a*c-b^2)/n*ln(x^n+1/2*(b^2+(-4*a*b^2*c+b^4)^(1/2))/b/c)*b^2-1/2/a/(4*a*c-b^
2)/n*ln(x^n+1/2*(b^2+(-4*a*b^2*c+b^4)^(1/2))/b/c)*(-4*a*b^2*c+b^4)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(cx^{2n} + bx^{n} + a\right)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*x^n+c*x^(2*n)),x, algorithm="maxima")
```

```
[Out] integrate(1/((c*x^(2*n) + b*x^n + a)*x), x)
```

mupad [B] time = 1.61, size = 224, normalized size = 3.03

$$\frac{\ln\left(-\frac{1}{c\,x}-\frac{\left(2\,a\,n+b\,n\,x^{n}\right)\left(4\,a\,c+b\,\sqrt{b^{2}-4\,a\,c}-b^{2}\right)}{2\,c\,x\left(a\,b^{2}\,n-4\,a^{2}\,c\,n\right)}\right)\left(4\,a\,c+b\,\sqrt{b^{2}-4\,a\,c}-b^{2}\right)}{2\left(a\,b^{2}\,n-4\,a^{2}\,c\,n\right)}-\frac{\ln\left(\frac{\left(2\,a\,n+b\,n\,x^{n}\right)\left(b\,\sqrt{b^{2}-4\,a\,c}-4\,a\,c+b^{2}\right)}{2\,c\,x\left(a\,b^{2}\,n-4\,a^{2}\,c\,n\right)}-\frac{1}{c\,x}\right)\left(b\,\sqrt{b^{2}-4\,a\,c}-4\,a\,c+b^{2}\right)}{2\left(a\,b^{2}\,n-4\,a^{2}\,c\,n\right)}+\frac{\ln(x)\,\left(n-1\right)}{a\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b*x^n + c*x^(2*n))),x)
```

```
[Out] (log(- 1/(c*x) - ((2*a*n + b*n*x^n)*(4*a*c + b*(b^2 - 4*a*c)^(1/2) - b^2))/
(2*c*x*(a*b^2*n - 4*a^2*c*n)))*(4*a*c + b*(b^2 - 4*a*c)^(1/2) - b^2))/(2*(a
*b^2*n - 4*a^2*c*n)) - (log(((2*a*n + b*n*x^n)*(b*(b^2 - 4*a*c)^(1/2) - 4*a
*c + b^2))/(2*c*x*(a*b^2*n - 4*a^2*c*n)) - 1/(c*x))*(b*(b^2 - 4*a*c)^(1/2)
- 4*a*c + b^2))/(2*(a*b^2*n - 4*a^2*c*n)) + (log(x)*(n - 1))/(a*n)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*x**n+c*x**(2*n)),x)
```

```
[Out] Timed out
```

3.487 $\int \frac{\sqrt{a+bx^n+cx^{2n}}}{x} dx$

Optimal. Leaf size=119

$$\frac{\sqrt{a+bx^n+cx^{2n}}}{n}-\frac{\sqrt{a}\tanh^{-1}\left(\frac{2a+bx^n}{2\sqrt{a}\sqrt{a+bx^n+cx^{2n}}}\right)}{n}+\frac{b\tanh^{-1}\left(\frac{b+2cx^n}{2\sqrt{c}\sqrt{a+bx^n+cx^{2n}}}\right)}{2\sqrt{c}\,n}$$

Rubi [A] time = 0.10, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {1357, 734, 843, 621, 206, 724}

$$\frac{\sqrt{a+bx^n+cx^{2n}}}{n}-\frac{\sqrt{a}\tanh^{-1}\left(\frac{2a+bx^n}{2\sqrt{a}\sqrt{a+bx^n+cx^{2n}}}\right)}{n}+\frac{b\tanh^{-1}\left(\frac{b+2cx^n}{2\sqrt{c}\sqrt{a+bx^n+cx^{2n}}}\right)}{2\sqrt{c}\,n}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x^n + c*x^(2*n)]/x,x]
```

```
[Out] Sqrt[a + b*x^n + c*x^(2*n)]/n - (Sqrt[a]*ArcTanh[(2*a + b*x^n)/(2*Sqrt[a]*S
qrt[a + b*x^n + c*x^(2*n)])])/n + (b*ArcTanh[(b + 2*c*x^n)/(2*Sqrt[c]*Sqrt[
a + b*x^n + c*x^(2*n)])])/(2*Sqrt[c]*n)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 734

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 2*p + 1)), x
] - Dist[p/(e*(m + 2*p + 1)), Int[(d + e*x)^m*Simp[b*d - 2*a*e + (2*c*d - b
*e)*x, x]*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e
, 0] && GtQ[p, 0] && NeQ[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &
&  !ILtQ[m + 2*p, 0] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
```

```
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1357

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{\sqrt{a+bx^n+cx^{2n}}}{x}\,dx = \frac{\text{Subst}\left(\int \frac{\sqrt{a+bx+cx^2}}{x}\,dx, x, x^n\right)}{n}$$
$$= \frac{\sqrt{a+bx^n+cx^{2n}}}{n} - \frac{\text{Subst}\left(\int \frac{-2a-bx}{x\sqrt{a+bx+cx^2}}\,dx, x, x^n\right)}{2n}$$
$$= \frac{\sqrt{a+bx^n+cx^{2n}}}{n} + \frac{a\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx+cx^2}}\,dx, x, x^n\right)}{n} + \frac{b\,\text{Subst}\left(\int \frac{1}{\sqrt{a+bx+cx^2}}\,dx, x, x\right)}{2n}$$
$$= \frac{\sqrt{a+bx^n+cx^{2n}}}{n} - \frac{(2a)\,\text{Subst}\left(\int \frac{1}{4a-x^2}\,dx, x, \frac{2a+bx^n}{\sqrt{a+bx^n+cx^{2n}}}\right)}{n} + \frac{b\,\text{Subst}\left(\int \frac{1}{4c-x^2}\,dx, x,\right)}{n}$$
$$= \frac{\sqrt{a+bx^n+cx^{2n}}}{n} - \frac{\sqrt{a}\,\tanh^{-1}\left(\frac{2a+bx^n}{2\sqrt{a}\sqrt{a+bx^n+cx^{2n}}}\right)}{n} + \frac{b\tanh^{-1}\left(\frac{b+2cx^n}{2\sqrt{c}\sqrt{a+bx^n+cx^{2n}}}\right)}{2\sqrt{c}\,n}$$

Mathematica [A] time = 0.15, size = 110, normalized size = 0.92

$$\frac{\sqrt{a+x^n(b+cx^n)} - \sqrt{a}\,\tanh^{-1}\left(\frac{2a+bx^n}{2\sqrt{a}\sqrt{a+x^n(b+cx^n)}}\right) + \frac{b\tanh^{-1}\left(\frac{b+2cx^n}{2\sqrt{c}\sqrt{a+x^n(b+cx^n)}}\right)}{2\sqrt{c}}}{n}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x^n + c*x^(2*n)]/x,x]
```

```
[Out] (Sqrt[a + x^n*(b + c*x^n)] - Sqrt[a]*ArcTanh[(2*a + b*x^n)/(2*Sqrt[a]*Sqrt[
a + x^n*(b + c*x^n)])] + (b*ArcTanh[(b + 2*c*x^n)/(2*Sqrt[c]*Sqrt[a + x^n*(
b + c*x^n)])])/(2*Sqrt[c]))/n
```

IntegrateAlgebraic [A] time = 0.35, size = 125, normalized size = 1.05

$$\frac{\sqrt{a+bx^n+cx^{2n}}}{n} - \frac{b\log\left(-2\sqrt{c}\,n\sqrt{a+bx^n+cx^{2n}} + bn + 2cnx^n\right)}{2\sqrt{c}\,n} + \frac{2\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{c}\,x^n}{\sqrt{a}} - \frac{\sqrt{a+bx^n+cx^{2n}}}{\sqrt{a}}\right)}{n}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[a + b*x^n + c*x^(2*n)]/x,x]
```

```
[Out] Sqrt[a + b*x^n + c*x^(2*n)]/n + (2*Sqrt[a]*ArcTanh[(Sqrt[c]*x^n)/Sqrt[a] -
Sqrt[a + b*x^n + c*x^(2*n)]/Sqrt[a]])/n - (b*Log[b*n + 2*c*n*x^n - 2*Sqrt[c
]*n*Sqrt[a + b*x^n + c*x^(2*n)]])/(2*Sqrt[c]*n)
```

fricas [A] time = 1.61, size = 658, normalized size = 5.53

$$\left[\frac{b\sqrt{c}\log\left(-8c^2x^{2n} - 8bcx^n - b^2 - 4ac - 4\left(2c^{\frac{3}{2}}x^n + b\sqrt{c}\right)\sqrt{cx^{2n}+bx^n+a}\right) + 2\sqrt{a}\,c\log\left(-\frac{8abx^n + 8a^2 + (b^2+4ac)x^{2n} - 4\left(\sqrt{a}\,bx^n + 2a^{\frac{3}{2}}\right)\sqrt{cx^{2n}+bx^n+a}}{x^{2n}}\right) + 4\sqrt{cx^{2n}+bx^n+a}\,c}{4cn}, -\frac{b\sqrt{-c}\arctan\left(\frac{\left(2\sqrt{-c}\,cx^n + b\sqrt{-c}\right)\sqrt{cx^{2n}+bx^n+a}}{2\left(c^2x^{2n}+bcx^n+ac\right)}\right) - \sqrt{a}\,c\log\left(-\frac{8abx^n + 8a^2 + (b^2+4ac)x^{2n} - 4\left(\sqrt{a}\,bx^n + 2a^{\frac{3}{2}}\right)\sqrt{cx^{2n}+bx^n+a}}{x^{2n}}\right) - 2\sqrt{cx^{2n}+bx^n+a}\,c}{2cn}, \frac{4\sqrt{-a}\,c\arctan\left(\frac{\left(\sqrt{-a}\,bx^n + 2\sqrt{-a}\,a\right)\sqrt{cx^{2n}+bx^n+a}}{2\left(acx^{2n}+abx^n+a^2\right)}\right) + b\sqrt{c}\log\left(-8c^2x^{2n} - 8bcx^n - b^2 - 4ac - 4\left(2c^{\frac{3}{2}}x^n + b\sqrt{c}\right)\sqrt{cx^{2n}+bx^n+a}\right) + 4\sqrt{cx^{2n}+bx^n+a}\,c}{4cn}, \frac{2\sqrt{-a}\,c\arctan\left(\frac{\left(\sqrt{-a}\,bx^n + 2\sqrt{-a}\,a\right)\sqrt{cx^{2n}+bx^n+a}}{2\left(acx^{2n}+abx^n+a^2\right)}\right) - b\sqrt{-c}\arctan\left(\frac{\left(2\sqrt{-c}\,cx^n + b\sqrt{-c}\right)\sqrt{cx^{2n}+bx^n+a}}{2\left(c^2x^{2n}+bcx^n+ac\right)}\right) + 2\sqrt{cx^{2n}+bx^n+a}\,c}{2cn}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n+c*x^(2*n))^(1/2)/x,x, algorithm="fricas")
```

```
[Out] [1/4*(b*sqrt(c)*log(-8*c^2*x^(2*n) - 8*b*c*x^n - b^2 - 4*a*c - 4*(2*c^(3/2)
*x^n + b*sqrt(c))*sqrt(c*x^(2*n) + b*x^n + a)) + 2*sqrt(a)*c*log(-(8*a*b*x^
n + 8*a^2 + (b^2 + 4*a*c)*x^(2*n) - 4*(sqrt(a)*b*x^n + 2*a^(3/2))*sqrt(c*x^
(2*n) + b*x^n + a))/x^(2*n)) + 4*sqrt(c*x^(2*n) + b*x^n + a)*c)/(c*n), -1/2
*(b*sqrt(-c)*arctan(1/2*(2*sqrt(-c)*c*x^n + b*sqrt(-c))*sqrt(c*x^(2*n) + b*
x^n + a)/(c^2*x^(2*n) + b*c*x^n + a*c)) - sqrt(a)*c*log(-(8*a*b*x^n + 8*a^2
 + (b^2 + 4*a*c)*x^(2*n) - 4*(sqrt(a)*b*x^n + 2*a^(3/2))*sqrt(c*x^(2*n) + b
*x^n + a))/x^(2*n)) - 2*sqrt(c*x^(2*n) + b*x^n + a)*c)/(c*n), 1/4*(4*sqrt(-
a)*c*arctan(1/2*(sqrt(-a)*b*x^n + 2*sqrt(-a)*a)*sqrt(c*x^(2*n) + b*x^n + a)
/(a*c*x^(2*n) + a*b*x^n + a^2)) + b*sqrt(c)*log(-8*c^2*x^(2*n) - 8*b*c*x^n
- b^2 - 4*a*c - 4*(2*c^(3/2)*x^n + b*sqrt(c))*sqrt(c*x^(2*n) + b*x^n + a))
+ 4*sqrt(c*x^(2*n) + b*x^n + a)*c)/(c*n), 1/2*(2*sqrt(-a)*c*arctan(1/2*(sqr
t(-a)*b*x^n + 2*sqrt(-a)*a)*sqrt(c*x^(2*n) + b*x^n + a)/(a*c*x^(2*n) + a*b*
x^n + a^2)) - b*sqrt(-c)*arctan(1/2*(2*sqrt(-c)*c*x^n + b*sqrt(-c))*sqrt(c*
x^(2*n) + b*x^n + a)/(c^2*x^(2*n) + b*c*x^n + a*c)) + 2*sqrt(c*x^(2*n) + b*
x^n + a)*c)/(c*n)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cx^{2n} + bx^n + a}}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n+c*x^(2*n))^(1/2)/x,x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x^(2*n) + b*x^n + a)/x, x)
```

maple [A] time = 0.12, size = 125, normalized size = 1.05

$$-\frac{\sqrt{a}\, \ln\left(\left(b\,{\mathrm e}^{n \ln(x)}+2 a+2 \sqrt{b\,{\mathrm e}^{n \ln(x)}+c\,{\mathrm e}^{2 n \ln(x)}+a}\, \sqrt{a}\right) {\mathrm e}^{-n \ln(x)}\right)}{n}+\frac{b \ln\left(\frac{c\,{\mathrm e}^{n \ln(x)}+\frac{b}{2}}{\sqrt{c}}+\sqrt{b\,{\mathrm e}^{n \ln(x)}+c\,{\mathrm e}^{2 n \ln(x)}+a}\right)}{2 \sqrt{c}\, n}+\frac{\sqrt{b\,{\mathrm e}^{n \ln(x)}+c\,{\mathrm e}^{2 n \ln(x)}+a}}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^n+c*x^(2*n)+a)^(1/2)/x,x)
```

```
[Out] 1/n*(a+b*exp(n*ln(x))+c*exp(n*ln(x))^2)^(1/2)+1/2/n*b*ln((1/2*b+c*exp(n*ln(
x)))/c^(1/2)+(a+b*exp(n*ln(x))+c*exp(n*ln(x))^2)^(1/2))/c^(1/2)-1/n*a^(1/2)
*ln((2*a+b*exp(n*ln(x))+2*a^(1/2)*(a+b*exp(n*ln(x))+c*exp(n*ln(x))^2)^(1/2)
)/exp(n*ln(x)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cx^{2n} + bx^n + a}}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n+c*x^(2*n))^(1/2)/x,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^(2*n) + b*x^n + a)/x, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{a + b\,x^n + c\,x^{2\,n}}}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^n + c*x^(2*n))^(1/2)/x,x)
```

```
[Out] int((a + b*x^n + c*x^(2*n))^(1/2)/x, x)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a + bx^n + cx^{2n}}}{x} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x**n+c*x**(2*n))**(1/2)/x,x)
```

```
[Out] Integral(sqrt(a + b*x**n + c*x**(2*n))/x, x)
```

3.488 $\int \frac{\left(a+bx^n+cx^{2n}\right)^{3/2}}{x}\,dx$

Optimal. Leaf size=173

$$-\frac{a^{3/2}\tanh^{-1}\left(\frac{2a+bx^n}{2\sqrt{a}\sqrt{a+bx^n+cx^{2n}}}\right)}{n}-\frac{b\left(b^2-12ac\right)\tanh^{-1}\left(\frac{b+2cx^n}{2\sqrt{c}\sqrt{a+bx^n+cx^{2n}}}\right)}{16c^{3/2}n}+\frac{\left(8ac+b^2+2bcx^n\right)\sqrt{a+bx^n+cx^{2n}}}{8cn}+\frac{(a}{}$$

Rubi [A] time = 0.16, antiderivative size = 173, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {1357, 734, 814, 843, 621, 206, 724}

$$-\frac{a^{3/2}\tanh^{-1}\left(\frac{2a+bx^n}{2\sqrt{a}\sqrt{a+bx^n+cx^{2n}}}\right)}{n}-\frac{b\left(b^2-12ac\right)\tanh^{-1}\left(\frac{b+2cx^n}{2\sqrt{c}\sqrt{a+bx^n+cx^{2n}}}\right)}{16c^{3/2}n}+\frac{\left(8ac+b^2+2bcx^n\right)\sqrt{a+bx^n+cx^{2n}}}{8cn}+\frac{\left(a+bx^n+cx^{2n}\right)^{3/2}}{3n}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^n + c*x^(2*n))^(3/2)/x,x]
```

```
[Out] ((b^2 + 8*a*c + 2*b*c*x^n)*Sqrt[a + b*x^n + c*x^(2*n)])/(8*c*n) + (a + b*x^
n + c*x^(2*n))^(3/2)/(3*n) - (a^(3/2)*ArcTanh[(2*a + b*x^n)/(2*Sqrt[a]*Sqrt
[a + b*x^n + c*x^(2*n)])])/n - (b*(b^2 - 12*a*c)*ArcTanh[(b + 2*c*x^n)/(2*S
qrt[c]*Sqrt[a + b*x^n + c*x^(2*n)])])/(16*c^(3/2)*n)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 734

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 2*p + 1)), x
] - Dist[p/(e*(m + 2*p + 1)), Int[(d + e*x)^m*Simp[b*d - 2*a*e + (2*c*d - b
*e)*x, x]*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e
, 0] && GtQ[p, 0] && NeQ[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &
&  !ILtQ[m + 2*p, 0] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 814

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2
) - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*(a + b*x + c*x^2)^p)
/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
```

```
 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
|| IntegersQ[2*m, 2*p])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1357

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{\left(a+bx^n+cx^{2n}\right)^{3/2}}{x}\,dx = \frac{\text{Subst}\left(\int \frac{\left(a+bx+cx^2\right)^{3/2}}{x}\,dx,x,x^n\right)}{n}$$
$$= \frac{\left(a+bx^n+cx^{2n}\right)^{3/2}}{3n} - \frac{\text{Subst}\left(\int \frac{(-2a-bx)\sqrt{a+bx+cx^2}}{x}\,dx,x,x^n\right)}{2n}$$
$$= \frac{\left(b^2+8ac+2bcx^n\right)\sqrt{a+bx^n+cx^{2n}}}{8cn} + \frac{\left(a+bx^n+cx^{2n}\right)^{3/2}}{3n} + \frac{\text{Subst}\left(\int \frac{8a^2c-\frac{1}{2}b\left(b^2-\right.}{x\sqrt{a+bx+}}\right.}{8cn}$$
$$= \frac{\left(b^2+8ac+2bcx^n\right)\sqrt{a+bx^n+cx^{2n}}}{8cn} + \frac{\left(a+bx^n+cx^{2n}\right)^{3/2}}{3n} + \frac{a^2\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx+}}\right.}{n}$$
$$= \frac{\left(b^2+8ac+2bcx^n\right)\sqrt{a+bx^n+cx^{2n}}}{8cn} + \frac{\left(a+bx^n+cx^{2n}\right)^{3/2}}{3n} - \frac{\left(2a^2\right)\text{Subst}\left(\int \frac{1}{4a-x}\right.}{n}$$
$$= \frac{\left(b^2+8ac+2bcx^n\right)\sqrt{a+bx^n+cx^{2n}}}{8cn} + \frac{\left(a+bx^n+cx^{2n}\right)^{3/2}}{3n} - \frac{a^{3/2}\tanh^{-1}\left(\frac{2a-}{2\sqrt{a}\sqrt{a+}}\right.}{n}$$

Mathematica [A] time = 0.28, size = 158, normalized size = 0.91

$$\frac{-48a^{3/2}c^{3/2}\tanh^{-1}\left(\frac{2a+bx^n}{2\sqrt{a}\sqrt{a+x^n(b+cx^n)}}\right)-3b\left(b^2-12ac\right)\tanh^{-1}\left(\frac{b+2cx^n}{2\sqrt{c}\sqrt{a+x^n(b+cx^n)}}\right)+2\sqrt{c}\sqrt{a+x^n\left(b+cx^n\right)}\left(8c\left(4a+cx^{2n}\right)+3b^2+14bcx^n\right)}{48c^{3/2}n}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^n + c*x^(2*n))^(3/2)/x,x]
```

```
[Out] (2*Sqrt[c]*Sqrt[a + x^n*(b + c*x^n)]*(3*b^2 + 14*b*c*x^n + 8*c*(4*a + c*x^(
2*n))) - 48*a^(3/2)*c^(3/2)*ArcTanh[(2*a + b*x^n)/(2*Sqrt[a]*Sqrt[a + x^n*(
```

```
b + c*x^n)])] - 3*b*(b^2 - 12*a*c)*ArcTanh[(b + 2*c*x^n)/(2*Sqrt[c]*Sqrt[a
+ x^n*(b + c*x^n)])])/(48*c^(3/2)*n)
```

IntegrateAlgebraic [A] time = 0.89, size = 169, normalized size = 0.98

$$\frac{2a^{3/2}\tanh^{-1}\left(\frac{\sqrt{c}\,x^n}{\sqrt{a}}-\frac{\sqrt{a+bx^n+cx^{2n}}}{\sqrt{a}}\right)}{n}+\frac{\left(b^3-12abc\right)\log\left(-2c^{3/2}n\sqrt{a+bx^n+cx^{2n}}+bcn+2c^2nx^n\right)}{16c^{3/2}n}+\frac{\sqrt{a+bx^n+cx^{2n}}\left(32ac+3b^2+14bcx^n+8c^2x^{2n}\right)}{24cn}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x^n + c*x^(2*n))^(3/2)/x,x]
```

```
[Out] (Sqrt[a + b*x^n + c*x^(2*n)]*(3*b^2 + 32*a*c + 14*b*c*x^n + 8*c^2*x^(2*n)))
/(24*c*n) + (2*a^(3/2)*ArcTanh[(Sqrt[c]*x^n)/Sqrt[a] - Sqrt[a + b*x^n + c*x
^(2*n)]/Sqrt[a]])/n + ((b^3 - 12*a*b*c)*Log[b*c*n + 2*c^2*n*x^n - 2*c^(3/2)
*n*Sqrt[a + b*x^n + c*x^(2*n)]])/(16*c^(3/2)*n)
```

fricas [A] time = 1.80, size = 827, normalized size = 4.78

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n+c*x^(2*n))^(3/2)/x,x, algorithm="fricas")
```

```
[Out] [1/96*(48*a^(3/2)*c^2*log(-(8*a*b*x^n + 8*a^2 + (b^2 + 4*a*c)*x^(2*n) - 4*(
sqrt(a)*b*x^n + 2*a^(3/2))*sqrt(c*x^(2*n) + b*x^n + a))/x^(2*n)) - 3*(b^3 -
 12*a*b*c)*sqrt(c)*log(-8*c^2*x^(2*n) - 8*b*c*x^n - b^2 - 4*a*c - 4*(2*c^(3
/2)*x^n + b*sqrt(c))*sqrt(c*x^(2*n) + b*x^n + a)) + 4*(8*c^3*x^(2*n) + 14*b
*c^2*x^n + 3*b^2*c + 32*a*c^2)*sqrt(c*x^(2*n) + b*x^n + a))/(c^2*n), 1/48*(
24*a^(3/2)*c^2*log(-(8*a*b*x^n + 8*a^2 + (b^2 + 4*a*c)*x^(2*n) - 4*(sqrt(a)
*b*x^n + 2*a^(3/2))*sqrt(c*x^(2*n) + b*x^n + a))/x^(2*n)) + 3*(b^3 - 12*a*b
*c)*sqrt(-c)*arctan(1/2*(2*sqrt(-c)*c*x^n + b*sqrt(-c))*sqrt(c*x^(2*n) + b*
x^n + a)/(c^2*x^(2*n) + b*c*x^n + a*c)) + 2*(8*c^3*x^(2*n) + 14*b*c^2*x^n +
 3*b^2*c + 32*a*c^2)*sqrt(c*x^(2*n) + b*x^n + a))/(c^2*n), 1/96*(96*sqrt(-a
)*a*c^2*arctan(1/2*(sqrt(-a)*b*x^n + 2*sqrt(-a)*a)*sqrt(c*x^(2*n) + b*x^n +
 a)/(a*c*x^(2*n) + a*b*x^n + a^2)) - 3*(b^3 - 12*a*b*c)*sqrt(c)*log(-8*c^2*
x^(2*n) - 8*b*c*x^n - b^2 - 4*a*c - 4*(2*c^(3/2)*x^n + b*sqrt(c))*sqrt(c*x^
(2*n) + b*x^n + a)) + 4*(8*c^3*x^(2*n) + 14*b*c^2*x^n + 3*b^2*c + 32*a*c^2)
*sqrt(c*x^(2*n) + b*x^n + a))/(c^2*n), 1/48*(48*sqrt(-a)*a*c^2*arctan(1/2*(
sqrt(-a)*b*x^n + 2*sqrt(-a)*a)*sqrt(c*x^(2*n) + b*x^n + a)/(a*c*x^(2*n) + a
*b*x^n + a^2)) + 3*(b^3 - 12*a*b*c)*sqrt(-c)*arctan(1/2*(2*sqrt(-c)*c*x^n +
 b*sqrt(-c))*sqrt(c*x^(2*n) + b*x^n + a)/(c^2*x^(2*n) + b*c*x^n + a*c)) + 2
*(8*c^3*x^(2*n) + 14*b*c^2*x^n + 3*b^2*c + 32*a*c^2)*sqrt(c*x^(2*n) + b*x^n
 + a))/(c^2*n)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cx^{2n}+bx^n+a\right)^{\frac{3}{2}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n+c*x^(2*n))^(3/2)/x,x, algorithm="giac")
```

```
[Out] integrate((c*x^(2*n) + b*x^n + a)^(3/2)/x, x)
```

maple [A] time = 0.05, size = 209, normalized size = 1.21

$$-\frac{a^{\frac{3}{2}}\ln\left(\left(b\,e^{n\ln(x)}+2a+2\sqrt{b\,e^{n\ln(x)}+c\,e^{2n\ln(x)}+a}\,\sqrt{a}\right)e^{-n\ln(x)}\right)}{n}+\frac{3ab\ln\left(\frac{c\,e^{n\ln(x)}+\frac{b}{2}}{\sqrt{c}}+\sqrt{b\,e^{n\ln(x)}+c\,e^{2n\ln(x)}+a}\right)}{4\sqrt{c}\,n}-\frac{b^3\ln\left(\frac{c\,e^{n\ln(x)}+\frac{b}{2}}{\sqrt{c}}+\sqrt{b\,e^{n\ln(x)}+c\,e^{2n\ln(x)}+a}\right)}{16c^{\frac{3}{2}}n}+\frac{\left(14bc\,e^{n\ln(x)}+8c^2e^{2n\ln(x)}+32ac+3b^2\right)\sqrt{b\,e^{n\ln(x)}+c\,e^{2n\ln(x)}+a}}{24cn}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x^n+c*x^(2*n)+a)^(3/2)/x,x)

[Out] 1/24*(8*c^2*exp(n*ln(x))^2+14*b*c*exp(n*ln(x))+32*a*c+3*b^2)*(a+b*exp(n*ln(x))+c*exp(n*ln(x))^2)^(1/2)/c/n+3/4/c^(1/2)/n*a*b*ln((c*exp(n*ln(x))+1/2*b)/c^(1/2)+(a+b*exp(n*ln(x))+c*exp(n*ln(x))^2)^(1/2))-1/16/c^(3/2)/n*b^3*ln((c*exp(n*ln(x))+1/2*b)/c^(1/2)+(a+b*exp(n*ln(x))+c*exp(n*ln(x))^2)^(1/2))-1/n*a^(3/2)*ln((2*a+b*exp(n*ln(x))+2*a^(1/2)*(a+b*exp(n*ln(x))+c*exp(n*ln(x))^2)^(1/2))/exp(n*ln(x)))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cx^{2n} + bx^n + a\right)^{\frac{3}{2}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*x^n+c*x^(2*n))^(3/2)/x,x, algorithm="maxima")

[Out] integrate((c*x^(2*n) + b*x^n + a)^(3/2)/x, x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a + b\,x^n + c\,x^{2\,n}\right)^{3/2}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b*x^n + c*x^(2*n))^(3/2)/x,x)

[Out] int((a + b*x^n + c*x^(2*n))^(3/2)/x, x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a + bx^n + cx^{2n}\right)^{\frac{3}{2}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*x**n+c*x**(2*n))**(3/2)/x,x)

[Out] Integral((a + b*x**n + c*x**(2*n))**(3/2)/x, x)

3.489 $\int \frac{1}{x\sqrt{a+bx^n+cx^{2n}}}\,dx$

Optimal. Leaf size=47

$$-\frac{\tanh^{-1}\left(\frac{2a+bx^n}{2\sqrt{a}\sqrt{a+bx^n+cx^{2n}}}\right)}{\sqrt{a}\,n}$$

Rubi [A] time = 0.03, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {1357, 724, 206}

$$-\frac{\tanh^{-1}\left(\frac{2a+bx^n}{2\sqrt{a}\sqrt{a+bx^n+cx^{2n}}}\right)}{\sqrt{a}\,n}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Sqrt[a + b*x^n + c*x^(2*n)]),x]
```

```
[Out] -(ArcTanh[(2*a + b*x^n)/(2*Sqrt[a]*Sqrt[a + b*x^n + c*x^(2*n)])]/(Sqrt[a]*n
))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1357

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x\sqrt{a+bx^n+cx^{2n}}}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx+cx^2}}\,dx, x, x^n\right)}{n}\\ &= -\frac{2\,\text{Subst}\left(\int \frac{1}{4a-x^2}\,dx, x, \frac{2a+bx^n}{\sqrt{a+bx^n+cx^{2n}}}\right)}{n}\\ &= -\frac{\tanh^{-1}\left(\frac{2a+bx^n}{2\sqrt{a}\sqrt{a+bx^n+cx^{2n}}}\right)}{\sqrt{a}\,n}\end{aligned}$$

Mathematica [A] time = 0.06, size = 47, normalized size = 1.00

$$-\frac{\tanh^{-1}\left(\frac{2a+bx^n}{2\sqrt{a}\sqrt{a+bx^n+cx^{2n}}}\right)}{\sqrt{a}n}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Sqrt[a + b*x^n + c*x^(2*n)]),x]
```

```
[Out] -(ArcTanh[(2*a + b*x^n)/(2*Sqrt[a]*Sqrt[a + b*x^n + c*x^(2*n)])]/(Sqrt[a]*n
))
```

IntegrateAlgebraic [A] time = 0.17, size = 51, normalized size = 1.09

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{c}x^n}{\sqrt{a}}-\frac{\sqrt{a+bx^n+cx^{2n}}}{\sqrt{a}}\right)}{\sqrt{a}n}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x*Sqrt[a + b*x^n + c*x^(2*n)]),x]
```

```
[Out] (2*ArcTanh[(Sqrt[c]*x^n)/Sqrt[a] - Sqrt[a + b*x^n + c*x^(2*n)]/Sqrt[a]])/(S
qrt[a]*n)
```

fricas [A] time = 1.34, size = 148, normalized size = 3.15

$$\left[\frac{\log\left(-\frac{8\,abx^n+8\,a^2+\left(b^2+4\,ac\right)x^{2\,n}-4\left(\sqrt{a}\,bx^n+2\,a^{\frac{3}{2}}\right)\sqrt{cx^{2\,n}+bx^n+a}}{x^{2\,n}}\right)}{2\,\sqrt{a}\,n}, \frac{\sqrt{-a}\,\arctan\left(\frac{\left(\sqrt{-a}\,bx^n+2\,\sqrt{-a}\,a\right)\sqrt{cx^{2\,n}+bx^n+a}}{2\left(acx^{2\,n}+abx^n+a^2\right)}\right)}{an}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*x^n+c*x^(2*n))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*log(-(8*a*b*x^n + 8*a^2 + (b^2 + 4*a*c)*x^(2*n) - 4*(sqrt(a)*b*x^n + 2
*a^(3/2))*sqrt(c*x^(2*n) + b*x^n + a))/x^(2*n))/(sqrt(a)*n), sqrt(-a)*arcta
n(1/2*(sqrt(-a)*b*x^n + 2*sqrt(-a)*a)*sqrt(c*x^(2*n) + b*x^n + a)/(a*c*x^(2
*n) + a*b*x^n + a^2))/(a*n)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{cx^{2\,n}+bx^n+a}\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*x^n+c*x^(2*n))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(c*x^(2*n) + b*x^n + a)*x), x)
```

maple [F] time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{b\,x^n+c\,x^{2n}+a}\;x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x^n+c*x^(2*n)+a)^(1/2),x)
```

```
[Out] int(1/x/(b*x^n+c*x^(2*n)+a)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{cx^{2n} + bx^n + a}\, x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*x^n+c*x^(2*n))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(c*x^(2*n) + b*x^n + a)*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{x\,\sqrt{a + b\,x^n + c\,x^{2\,n}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b*x^n + c*x^(2*n))^(1/2)),x)
```

```
[Out] int(1/(x*(a + b*x^n + c*x^(2*n))^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\sqrt{a + bx^n + cx^{2n}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*x**n+c*x**(2*n))**(1/2),x)
```

```
[Out] Integral(1/(x*sqrt(a + b*x**n + c*x**(2*n))), x)
```

3.490 $\int \frac{1}{x\left(a+bx^n+cx^{2n}\right)^{3/2}}\,dx$

Optimal. Leaf size=98

$$\frac{2\left(-2ac+b^2+bcx^n\right)}{an\left(b^2-4ac\right)\sqrt{a+bx^n+cx^{2n}}}-\frac{\tanh^{-1}\left(\frac{2a+bx^n}{2\sqrt{a}\sqrt{a+bx^n+cx^{2n}}}\right)}{a^{3/2}n}$$

Rubi [A] time = 0.07, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {1357, 740, 12, 724, 206}

$$\frac{2\left(-2ac+b^2+bcx^n\right)}{an\left(b^2-4ac\right)\sqrt{a+bx^n+cx^{2n}}}-\frac{\tanh^{-1}\left(\frac{2a+bx^n}{2\sqrt{a}\sqrt{a+bx^n+cx^{2n}}}\right)}{a^{3/2}n}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a + b*x^n + c*x^(2*n))^(3/2)),x]

[Out] (2*(b^2 - 2*a*c + b*c*x^n))/(a*(b^2 - 4*a*c)*n*Sqrt[a + b*x^n + c*x^(2*n)])
 - ArcTanh[(2*a + b*x^n)/(2*Sqrt[a]*Sqrt[a + b*x^n + c*x^(2*n)])]/(a^(3/2)*
n)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 740

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e
)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e
^2)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d +
 e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p +
 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p,
-1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 1357

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
```

```
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x\left(a+bx^n+cx^{2n}\right)^{3/2}}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{x\left(a+bx+cx^2\right)^{3/2}}\,dx, x, x^n\right)}{n} \\
&= \frac{2\left(b^2-2ac+bcx^n\right)}{a\left(b^2-4ac\right)n\sqrt{a+bx^n+cx^{2n}}} - \frac{2\,\text{Subst}\left(\int \frac{-\frac{b^2}{2}+2ac}{x\sqrt{a+bx+cx^2}}\,dx, x, x^n\right)}{a\left(b^2-4ac\right)n} \\
&= \frac{2\left(b^2-2ac+bcx^n\right)}{a\left(b^2-4ac\right)n\sqrt{a+bx^n+cx^{2n}}} + \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx+cx^2}}\,dx, x, x^n\right)}{an} \\
&= \frac{2\left(b^2-2ac+bcx^n\right)}{a\left(b^2-4ac\right)n\sqrt{a+bx^n+cx^{2n}}} - \frac{2\,\text{Subst}\left(\int \frac{1}{4a-x^2}\,dx, x, \frac{2a+bx^n}{\sqrt{a+bx^n+cx^{2n}}}\right)}{an} \\
&= \frac{2\left(b^2-2ac+bcx^n\right)}{a\left(b^2-4ac\right)n\sqrt{a+bx^n+cx^{2n}}} - \frac{\tanh^{-1}\left(\frac{2a+bx^n}{2\sqrt{a}\sqrt{a+bx^n+cx^{2n}}}\right)}{a^{3/2}n}
\end{aligned}
$$

Mathematica [A] time = 0.31, size = 94, normalized size = 0.96

$$
\frac{\frac{2\left(-2ac+b^2+bcx^n\right)}{a\left(b^2-4ac\right)\sqrt{a+x^n\left(b+cx^n\right)}} - \frac{\tanh^{-1}\left(\frac{2a+bx^n}{2\sqrt{a}\sqrt{a+x^n\left(b+cx^n\right)}}\right)}{a^{3/2}}}{n}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a + b*x^n + c*x^(2*n))^(3/2)),x]
```

```
[Out] ((2*(b^2 - 2*a*c + b*c*x^n))/(a*(b^2 - 4*a*c)*Sqrt[a + x^n*(b + c*x^n)]) -
ArcTanh[(2*a + b*x^n)/(2*Sqrt[a]*Sqrt[a + x^n*(b + c*x^n)])]/a^(3/2))/n
```

IntegrateAlgebraic [A] time = 0.59, size = 107, normalized size = 1.09

$$
\frac{2\tanh^{-1}\left(\frac{\sqrt{c}\,x^n}{\sqrt{a}} - \frac{\sqrt{a+bx^n+cx^{2n}}}{\sqrt{a}}\right)}{a^{3/2}n} + \frac{2\left(2ac-b^2-bcx^n\right)}{an\left(4ac-b^2\right)\sqrt{a+bx^n+cx^{2n}}}
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x*(a + b*x^n + c*x^(2*n))^(3/2)),x]
```

```
[Out] (2*(-b^2 + 2*a*c - b*c*x^n))/(a*(-b^2 + 4*a*c)*n*Sqrt[a + b*x^n + c*x^(2*n)
]) + (2*ArcTanh[(Sqrt[c]*x^n)/Sqrt[a] - Sqrt[a + b*x^n + c*x^(2*n)]/Sqrt[a]
])/(a^(3/2)*n)
```

fricas [B] time = 1.70, size = 449, normalized size = 4.58

$$
\left[\frac{\left(\left(b^2c-4ac^2\right)\sqrt{a}\,x^{2n}+\left(b^3-4abc\right)\sqrt{a}\,x^n+\left(ab^2-4a^2c\right)\sqrt{a}\right)\log\left(-\frac{8abx^n+8a^2+\left(b^2+4ac\right)x^{2n}-4\left(\sqrt{a}\,bx^n+2a^{\frac{3}{2}}\right)\sqrt{cx^{2n}+bx^n+a}}{x^{2n}}\right)+4\left(abcx^n+ab^2-2a^2c\right)\sqrt{cx^{2n}+bx^n+a}}{2\left(\left(a^2b^2c-4a^3c^2\right)nx^{2n}+\left(a^2b^3-4a^3bc\right)nx^n+\left(a^3b^2-4a^4c\right)n\right)}, \frac{\left(\left(b^2c-4ac^2\right)\sqrt{-a}\,x^{2n}+\left(b^3-4abc\right)\sqrt{-a}\,x^n+\left(ab^2-4a^2c\right)\sqrt{-a}\right)\arctan\left(\frac{\left(\sqrt{-a}\,bx^n+2\sqrt{-a}\,a\right)\sqrt{cx^{2n}+bx^n+a}}{2\left(acx^{2n}+abx^n+a^2\right)}\right)+2\left(abcx^n+ab^2-2a^2c\right)\sqrt{cx^{2n}+bx^n+a}}{\left(a^2b^2c-4a^3c^2\right)nx^{2n}+\left(a^2b^3-4a^3bc\right)nx^n+\left(a^3b^2-4a^4c\right)n}\right]
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*x^n+c*x^(2*n))^(3/2),x, algorithm="fricas")
```

```
[Out] [1/2*(((b^2*c - 4*a*c^2)*sqrt(a)*x^(2*n) + (b^3 - 4*a*b*c)*sqrt(a)*x^n + (a
*b^2 - 4*a^2*c)*sqrt(a))*log(-(8*a*b*x^n + 8*a^2 + (b^2 + 4*a*c)*x^(2*n) -
4*(sqrt(a)*b*x^n + 2*a^(3/2))*sqrt(c*x^(2*n) + b*x^n + a))/x^(2*n)) + 4*(a*
b*c*x^n + a*b^2 - 2*a^2*c)*sqrt(c*x^(2*n) + b*x^n + a))/((a^2*b^2*c - 4*a^3
*c^2)*n*x^(2*n) + (a^2*b^3 - 4*a^3*b*c)*n*x^n + (a^3*b^2 - 4*a^4*c)*n), (((
b^2*c - 4*a*c^2)*sqrt(-a)*x^(2*n) + (b^3 - 4*a*b*c)*sqrt(-a)*x^n + (a*b^2 -
 4*a^2*c)*sqrt(-a))*arctan(1/2*(sqrt(-a)*b*x^n + 2*sqrt(-a)*a)*sqrt(c*x^(2*
n) + b*x^n + a)/(a*c*x^(2*n) + a*b*x^n + a^2)) + 2*(a*b*c*x^n + a*b^2 - 2*a
^2*c)*sqrt(c*x^(2*n) + b*x^n + a))/((a^2*b^2*c - 4*a^3*c^2)*n*x^(2*n) + (a^
2*b^3 - 4*a^3*b*c)*n*x^n + (a^3*b^2 - 4*a^4*c)*n)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(cx^{2n} + bx^n + a\right)^{\frac{3}{2}} x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*x^n+c*x^(2*n))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((c*x^(2*n) + b*x^n + a)^(3/2)*x), x)
```

maple [F] time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\,x^n + c\,x^{2n} + a\right)^{\frac{3}{2}} x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x^n+c*x^(2*n)+a)^(3/2),x)
```

```
[Out] int(1/x/(b*x^n+c*x^(2*n)+a)^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(cx^{2n} + bx^n + a\right)^{\frac{3}{2}} x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*x^n+c*x^(2*n))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((c*x^(2*n) + b*x^n + a)^(3/2)*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x\left(a + b\,x^n + c\,x^{2n}\right)^{3/2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b*x^n + c*x^(2*n))^(3/2)),x)
```

```
[Out] int(1/(x*(a + b*x^n + c*x^(2*n))^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(a + bx^n + cx^{2n}\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*x**n+c*x**(2*n))**(3/2),x)
```

```
[Out] Integral(1/(x*(a + b*x**n + c*x**(2*n))**(3/2)), x)
```

3.491 $\int (dx)^m \left(a+bx^n+cx^{2n}\right)^3 \, dx$

Optimal. Leaf size=182

$$\frac{a^3(dx)^{m+1}}{d(m+1)}+\frac{3a^2bx^{n+1}(dx)^m}{m+n+1}+\frac{3ax^{2n+1}\left(ac+b^2\right)(dx)^m}{m+2n+1}+\frac{bx^{3n+1}\left(6ac+b^2\right)(dx)^m}{m+3n+1}+\frac{3cx^{4n+1}\left(ac+b^2\right)(dx)^m}{m+4n+1}+\frac{3bc^2\ldots}{m\ldots}$$

Rubi [A] time = 0.16, antiderivative size = 182, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {1353, 20, 30}

$$\frac{3a^2bx^{n+1}(dx)^m}{m+n+1}+\frac{a^3(dx)^{m+1}}{d(m+1)}+\frac{3ax^{2n+1}\left(ac+b^2\right)(dx)^m}{m+2n+1}+\frac{bx^{3n+1}\left(6ac+b^2\right)(dx)^m}{m+3n+1}+\frac{3cx^{4n+1}\left(ac+b^2\right)(dx)^m}{m+4n+1}+\frac{3bc^2x^{5n+1}(dx)^m}{m+5n+1}+\frac{c^3x^{6n+1}(dx)^m}{m+6n+1}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^m*(a + b*x^n + c*x^(2*n))^3,x]
```

```
[Out] (3*a^2*b*x^(1 + n)*(d*x)^m)/(1 + m + n) + (3*a*(b^2 + a*c)*x^(1 + 2*n)*(d*x
)^m)/(1 + m + 2*n) + (b*(b^2 + 6*a*c)*x^(1 + 3*n)*(d*x)^m)/(1 + m + 3*n) +
(3*c*(b^2 + a*c)*x^(1 + 4*n)*(d*x)^m)/(1 + m + 4*n) + (3*b*c^2*x^(1 + 5*n)*
(d*x)^m)/(1 + m + 5*n) + (c^3*x^(1 + 6*n)*(d*x)^m)/(1 + m + 6*n) + (a^3*(d*
x)^(1 + m))/(d*(1 + m))
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(b^IntPart
[n]*(b*v)^FracPart[n])/(a^IntPart[n]*(a*v)^FracPart[n]), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !
IntegerQ[m + n]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 1353

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.),
x_Symbol] :> Int[ExpandIntegrand[(d*x)^m*(a + b*x^n + c*x^(2*n))^p, x], x]
/; FreeQ[{a, b, c, d, m, n}, x] && EqQ[n2, 2*n] && IGtQ[p, 0] && !IntegerQ
[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int (dx)^m\left(a+bx^n+cx^{2n}\right)^3 dx &= \int \left(a^3(dx)^m+3a^2bx^n(dx)^m+3ab^2\left(1+\frac{ac}{b^2}\right)x^{2n}(dx)^m+b^3\left(1+\frac{6ac}{b^2}\right)x^{3n}(dx)^{\ldots}\right.\\ &= \frac{a^3(dx)^{1+m}}{d(1+m)}+\left(3a^2b\right)\int x^n(dx)^m\,dx+\left(3bc^2\right)\int x^{5n}(dx)^m\,dx+c^3\int x^{6n}(dx)^m\,d\ldots\\ &= \frac{a^3(dx)^{1+m}}{d(1+m)}+\left(3a^2bx^{-m}(dx)^m\right)\int x^{m+n}\,dx+\left(3bc^2x^{-m}(dx)^m\right)\int x^{m+5n}\,dx+\left(c^3\ldots\right.\\ &= \frac{3a^2bx^{1+n}(dx)^m}{1+m+n}+\frac{3a\left(b^2+ac\right)x^{1+2n}(dx)^m}{1+m+2n}+\frac{b\left(b^2+6ac\right)x^{1+3n}(dx)^m}{1+m+3n}+\frac{3c\left(b^2\ldots\right.}{\ldots}\end{aligned}$$

Mathematica [A] time = 0.35, size = 137, normalized size = 0.75

$$x(dx)^m\left(\frac{a^3}{m+1}+\frac{3a^2bx^n}{m+n+1}+\frac{3ax^{2n}\left(ac+b^2\right)}{m+2n+1}+\frac{bx^{3n}\left(6ac+b^2\right)}{m+3n+1}+\frac{3cx^{4n}\left(ac+b^2\right)}{m+4n+1}+\frac{3bc^2x^{5n}}{m+5n+1}+\frac{c^3x^{6n}}{m+6n+1}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^m*(a + b*x^n + c*x^(2*n))^3,x]
```

```
[Out] x*(d*x)^m*(a^3/(1 + m) + (3*a^2*b*x^n)/(1 + m + n) + (3*a*(b^2 + a*c)*x^(2*
n))/(1 + m + 2*n) + (b*(b^2 + 6*a*c)*x^(3*n))/(1 + m + 3*n) + (3*c*(b^2 + a
*c)*x^(4*n))/(1 + m + 4*n) + (3*b*c^2*x^(5*n))/(1 + m + 5*n) + (c^3*x^(6*n)
)/(1 + m + 6*n))
```

IntegrateAlgebraic [F] time = 0.18, size = 0, normalized size = 0.00

$$\int (dx)^m \left(a + bx^n + cx^{2n}\right)^3 dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(d*x)^m*(a + b*x^n + c*x^(2*n))^3,x]
```

```
[Out] Defer[IntegrateAlgebraic][(d*x)^m*(a + b*x^n + c*x^(2*n))^3, x]
```

fricas [B] time = 1.39, size = 2303, normalized size = 12.65

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*x^n+c*x^(2*n))^3,x, algorithm="fricas")
```

```
[Out] ((c^3*m^6 + 6*c^3*m^5 + 15*c^3*m^4 + 20*c^3*m^3 + 120*(c^3*m + c^3)*n^5 + 1
5*c^3*m^2 + 274*(c^3*m^2 + 2*c^3*m + c^3)*n^4 + 6*c^3*m + 225*(c^3*m^3 + 3*
c^3*m^2 + 3*c^3*m + c^3)*n^3 + c^3 + 85*(c^3*m^4 + 4*c^3*m^3 + 6*c^3*m^2 +
4*c^3*m + c^3)*n^2 + 15*(c^3*m^5 + 5*c^3*m^4 + 10*c^3*m^3 + 10*c^3*m^2 + 5*
c^3*m + c^3)*n)*x*x^(6*n)*e^(m*log(d) + m*log(x)) + 3*(b*c^2*m^6 + 6*b*c^2*
m^5 + 15*b*c^2*m^4 + 20*b*c^2*m^3 + 144*(b*c^2*m + b*c^2)*n^5 + 15*b*c^2*m^
2 + 324*(b*c^2*m^2 + 2*b*c^2*m + b*c^2)*n^4 + 6*b*c^2*m + 260*(b*c^2*m^3 +
3*b*c^2*m^2 + 3*b*c^2*m + b*c^2)*n^3 + b*c^2 + 95*(b*c^2*m^4 + 4*b*c^2*m^3
+ 6*b*c^2*m^2 + 4*b*c^2*m + b*c^2)*n^2 + 16*(b*c^2*m^5 + 5*b*c^2*m^4 + 10*b
*c^2*m^3 + 10*b*c^2*m^2 + 5*b*c^2*m + b*c^2)*n)*x*x^(5*n)*e^(m*log(d) + m*l
og(x)) + 3*((b^2*c + a*c^2)*m^6 + 6*(b^2*c + a*c^2)*m^5 + 180*(b^2*c + a*c^
2 + (b^2*c + a*c^2)*m)*n^5 + 15*(b^2*c + a*c^2)*m^4 + 396*(b^2*c + a*c^2 +
(b^2*c + a*c^2)*m^2 + 2*(b^2*c + a*c^2)*m)*n^4 + 20*(b^2*c + a*c^2)*m^3 + 3
07*((b^2*c + a*c^2)*m^3 + b^2*c + a*c^2 + 3*(b^2*c + a*c^2)*m^2 + 3*(b^2*c
+ a*c^2)*m)*n^3 + b^2*c + a*c^2 + 15*(b^2*c + a*c^2)*m^2 + 107*((b^2*c + a*
c^2)*m^4 + 4*(b^2*c + a*c^2)*m^3 + b^2*c + a*c^2 + 6*(b^2*c + a*c^2)*m^2 +
4*(b^2*c + a*c^2)*m)*n^2 + 6*(b^2*c + a*c^2)*m + 17*((b^2*c + a*c^2)*m^5 +
5*(b^2*c + a*c^2)*m^4 + 10*(b^2*c + a*c^2)*m^3 + b^2*c + a*c^2 + 10*(b^2*c
+ a*c^2)*m^2 + 5*(b^2*c + a*c^2)*m)*n)*x*x^(4*n)*e^(m*log(d) + m*log(x)) +
((b^3 + 6*a*b*c)*m^6 + 6*(b^3 + 6*a*b*c)*m^5 + 240*(b^3 + 6*a*b*c + (b^3 +
6*a*b*c)*m)*n^5 + 15*(b^3 + 6*a*b*c)*m^4 + 508*(b^3 + 6*a*b*c + (b^3 + 6*a*
b*c)*m^2 + 2*(b^3 + 6*a*b*c)*m)*n^4 + 20*(b^3 + 6*a*b*c)*m^3 + 372*((b^3 +
6*a*b*c)*m^3 + b^3 + 6*a*b*c + 3*(b^3 + 6*a*b*c)*m^2 + 3*(b^3 + 6*a*b*c)*m)
*n^3 + b^3 + 6*a*b*c + 15*(b^3 + 6*a*b*c)*m^2 + 121*((b^3 + 6*a*b*c)*m^4 +
4*(b^3 + 6*a*b*c)*m^3 + b^3 + 6*a*b*c + 6*(b^3 + 6*a*b*c)*m^2 + 4*(b^3 + 6*
a*b*c)*m)*n^2 + 6*(b^3 + 6*a*b*c)*m + 18*((b^3 + 6*a*b*c)*m^5 + 5*(b^3 + 6*
a*b*c)*m^4 + 10*(b^3 + 6*a*b*c)*m^3 + b^3 + 6*a*b*c + 10*(b^3 + 6*a*b*c)*m^
2 + 5*(b^3 + 6*a*b*c)*m)*n)*x*x^(3*n)*e^(m*log(d) + m*log(x)) + 3*((a*b^2 +
 a^2*c)*m^6 + 6*(a*b^2 + a^2*c)*m^5 + 360*(a*b^2 + a^2*c + (a*b^2 + a^2*c)*
m)*n^5 + 15*(a*b^2 + a^2*c)*m^4 + 702*(a*b^2 + a^2*c + (a*b^2 + a^2*c)*m^2
+ 2*(a*b^2 + a^2*c)*m)*n^4 + 20*(a*b^2 + a^2*c)*m^3 + 461*((a*b^2 + a^2*c)*
m^3 + a*b^2 + a^2*c + 3*(a*b^2 + a^2*c)*m^2 + 3*(a*b^2 + a^2*c)*m)*n^3 + a*
b^2 + a^2*c + 15*(a*b^2 + a^2*c)*m^2 + 137*((a*b^2 + a^2*c)*m^4 + 4*(a*b^2
+ a^2*c)*m^3 + a*b^2 + a^2*c + 6*(a*b^2 + a^2*c)*m^2 + 4*(a*b^2 + a^2*c)*m)
*n^2 + 6*(a*b^2 + a^2*c)*m + 19*((a*b^2 + a^2*c)*m^5 + 5*(a*b^2 + a^2*c)*m^
```

```
4 + 10*(a*b^2 + a^2*c)*m^3 + a*b^2 + a^2*c + 10*(a*b^2 + a^2*c)*m^2 + 5*(a*
b^2 + a^2*c)*m)*n)*x*x^(2*n)*e^(m*log(d) + m*log(x)) + 3*(a^2*b*m^6 + 6*a^2
*b*m^5 + 15*a^2*b*m^4 + 20*a^2*b*m^3 + 720*(a^2*b*m + a^2*b)*n^5 + 15*a^2*b
*m^2 + 1044*(a^2*b*m^2 + 2*a^2*b*m + a^2*b)*n^4 + 6*a^2*b*m + 580*(a^2*b*m^
3 + 3*a^2*b*m^2 + 3*a^2*b*m + a^2*b)*n^3 + a^2*b + 155*(a^2*b*m^4 + 4*a^2*b
*m^3 + 6*a^2*b*m^2 + 4*a^2*b*m + a^2*b)*n^2 + 20*(a^2*b*m^5 + 5*a^2*b*m^4 +
 10*a^2*b*m^3 + 10*a^2*b*m^2 + 5*a^2*b*m + a^2*b)*n)*x*x^n*e^(m*log(d) + m*
log(x)) + (a^3*m^6 + 720*a^3*n^6 + 6*a^3*m^5 + 15*a^3*m^4 + 20*a^3*m^3 + 17
64*(a^3*m + a^3)*n^5 + 15*a^3*m^2 + 1624*(a^3*m^2 + 2*a^3*m + a^3)*n^4 + 6*
a^3*m + 735*(a^3*m^3 + 3*a^3*m^2 + 3*a^3*m + a^3)*n^3 + a^3 + 175*(a^3*m^4
+ 4*a^3*m^3 + 6*a^3*m^2 + 4*a^3*m + a^3)*n^2 + 21*(a^3*m^5 + 5*a^3*m^4 + 10
*a^3*m^3 + 10*a^3*m^2 + 5*a^3*m + a^3)*n)*x*e^(m*log(d) + m*log(x)))/(m^7 +
 720*(m + 1)*n^6 + 7*m^6 + 1764*(m^2 + 2*m + 1)*n^5 + 21*m^5 + 1624*(m^3 +
3*m^2 + 3*m + 1)*n^4 + 35*m^4 + 735*(m^4 + 4*m^3 + 6*m^2 + 4*m + 1)*n^3 + 3
5*m^3 + 175*(m^5 + 5*m^4 + 10*m^3 + 10*m^2 + 5*m + 1)*n^2 + 21*m^2 + 21*(m^
6 + 6*m^5 + 15*m^4 + 20*m^3 + 15*m^2 + 6*m + 1)*n + 7*m + 1)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*x^n+c*x^(2*n))^3,x, algorithm="giac")
```

```
[Out] Timed out
```

maple [C] time = 0.11, size = 3798, normalized size = 20.87

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(b*x^n+c*x^(2*n)+a)^3,x)
```

```
[Out] x*(c^3*(x^n)^6+a^3*m^6+6*a^3*m^5+1764*a^3*n^5+15*a^3*m^4+1624*a^3*n^4+720*a
^3*n^6+b^3*(x^n)^3+20*a^3*m^3+15*a^3*m^2+175*a^3*n^2+21*a^3*n+735*a^3*n^3+a
^3+972*b*c^2*n^4*(x^n)^5+45*b*c^2*m^4*(x^n)^5+240*b^3*m*n^5*(x^n)^3+18*b^2*
c*m^5*(x^n)^4+540*b^2*c*n^5*(x^n)^4+150*c^3*m^3*n*(x^n)^6+510*c^3*m^2*n^2*(
x^n)^6+675*c^3*m*n^3*(x^n)^6+18*a*c^2*m^5*(x^n)^4+540*a*c^2*n^5*(x^n)^4+18*
b^3*m^5*n*(x^n)^3+508*b^3*m^2*n^4*(x^n)^3+15*c^3*m^5*n*(x^n)^6+85*c^3*m^4*n
^2*(x^n)^6+225*c^3*m^3*n^3*(x^n)^6+274*c^3*m^2*n^4*(x^n)^6+120*c^3*m*n^5*(x
^n)^6+3*b*c^2*m^6*(x^n)^5+75*c^3*m^4*n*(x^n)^6+340*c^3*m^3*n^2*(x^n)^6+675*
c^3*m^2*n^3*(x^n)^6+548*c^3*m*n^4*(x^n)^6+3*a*c^2*m^6*(x^n)^4+3*b^2*c*m^6*(
x^n)^4+18*b*c^2*m^5*(x^n)^5+432*b*c^2*n^5*(x^n)^5+1116*b^3*m^2*n^3*(x^n)^3+
1016*b^3*m*n^4*(x^n)^3+45*b^2*c*m^4*(x^n)^4+1188*b^2*c*n^4*(x^n)^4+60*b*c^2
*m^3*(x^n)^5+780*b*c^2*n^3*(x^n)^5+75*c^3*m*n*(x^n)^6+3*a^2*b*m^6*x^n+18*a^
2*c*m^5*(x^n)^2+1080*a^2*c*n^5*(x^n)^2+18*a*b^2*m^5*(x^n)^2+1080*a*b^2*n^5*
(x^n)^2+60*a*c^2*m^3*(x^n)^4+921*a*c^2*n^3*(x^n)^4+18*m*b*c^2*(x^n)^5+48*b*
c^2*(x^n)^5*n+45*a^2*b*m^4*x^n+3132*a^2*b*n^4*x^n+121*b^3*m^4*n^2*(x^n)^3+3
72*b^3*m^3*n^3*(x^n)^3+340*c^3*m*n^2*(x^n)^6+3*a^2*c*m^6*(x^n)^2+3*a*b^2*m^
6*(x^n)^2+45*a*c^2*m^4*(x^n)^4+1188*a*c^2*n^4*(x^n)^4+90*b^3*m^4*n*(x^n)^3+
484*b^3*m^3*n^2*(x^n)^3+45*b*c^2*m^2*(x^n)^5+285*b*c^2*n^2*(x^n)^5+18*a^2*b
*m^5*x^n+2160*a^2*b*n^5*x^n+45*a^2*c*m^4*(x^n)^2+2106*a^2*c*n^4*(x^n)^2+45*
a*b^2*m^4*(x^n)^2+2106*a*b^2*n^4*(x^n)^2+45*a*c^2*m^2*(x^n)^4+321*a*c^2*n^2
*(x^n)^4+45*b^2*c*m^2*(x^n)^4+321*b^2*c*n^2*(x^n)^4+18*b^2*c*(x^n)^4*m+51*b
^2*c*(x^n)^4*n+1740*a^2*b*n^3*x^n+45*a^2*c*m^2*(x^n)^2+411*a^2*c*n^2*(x^n)^
2+180*b^3*m^3*n*(x^n)^3+726*b^3*m^2*n^2*(x^n)^3+1116*b^3*m*n^3*(x^n)^3+60*b
^2*c*m^3*(x^n)^4+921*b^2*c*n^3*(x^n)^4+18*a^2*c*(x^n)^2*m+57*a^2*c*(x^n)^2*
n+6*a*b*c*(x^n)^3+60*a^2*c*m^3*(x^n)^2+1383*a^2*c*n^3*(x^n)^2+1383*a*b^2*n^
3*(x^n)^2+18*a*c^2*(x^n)^4*m+51*a*c^2*(x^n)^4*n+60*a^2*b*n*x^n+180*b^3*m^2*
```

```
n*(x^n)^3+484*b^3*m*n^2*(x^n)^3+60*a*b^2*m^3*(x^n)^2+90*b^3*m*n*(x^n)^3+60*
a^2*b*m^3*x^n+45*a*b^2*m^2*(x^n)^2+411*a*b^2*n^2*(x^n)^2+45*a^2*b*m^2*x^n+4
65*a^2*b*n^2*x^n+18*m*a*b^2*(x^n)^2+57*a*b^2*(x^n)^2*n+18*a^2*b*m*x^n+1080*
a*b*c*m^3*n*(x^n)^3+570*a*b^2*m^2*n*(x^n)^2+2904*a*b*c*m*n^2*(x^n)^3+540*a*
b*c*m*n*(x^n)^3+240*b^3*n^5*(x^n)^3+15*c^3*m^2*(x^n)^6+85*c^3*n^2*(x^n)^6+1
5*b^3*m^4*(x^n)^3+1284*b^2*c*m*n^2*(x^n)^4+240*b*c^2*m*n*(x^n)^5+300*a^2*b*
m^4*n*x^n+1860*a^2*b*m^3*n^2*x^n+5220*a^2*b*m^2*n^3*x^n+6264*a^2*b*m*n^4*x^
n+570*a^2*c*m^3*n*(x^n)^2+2466*a^2*c*m^2*n^2*(x^n)^2+4149*a^2*c*m*n^3*(x^n)
^2+570*a*b^2*m^3*n*(x^n)^2+2466*a*b^2*m^2*n^2*(x^n)^2+4149*a*b^2*m*n^3*(x^n
)^2+120*a*b*c*m^3*(x^n)^3+2232*a*b*c*n^3*(x^n)^3+6696*a*b*c*m^2*n^3*(x^n)^3
+150*c^3*m^2*n*(x^n)^6+105*a^3*m^4*n+700*a^3*m^3*n^2+2205*a^3*m^2*n^3+3248*
a^3*m*n^4+210*a^3*m^3*n+1050*a^3*m^2*n^2+2205*a^3*m*n^3+20*b^3*m^3*(x^n)^3+
15*b^3*m^2*(x^n)^3+121*b^3*n^2*(x^n)^3+6*m*b^3*(x^n)^3+18*b^3*(x^n)^3*n+3*a
^2*b*x^n+3*(x^n)^2*a*b^2+57*a*b^2*m^5*n*(x^n)^2+411*a*b^2*m^4*n^2*(x^n)^2+1
383*a*b^2*m^3*n^3*(x^n)^2+2106*a*b^2*m^2*n^4*(x^n)^2+1080*a*b^2*m*n^5*(x^n)
^2+36*a*b*c*m^5*(x^n)^3+1440*a*b*c*n^5*(x^n)^3+510*a*c^2*m^3*n*(x^n)^4+1926
*a*c^2*m^2*n^2*(x^n)^4+2763*a*c^2*m*n^3*(x^n)^4+510*b^2*c*m^3*n*(x^n)^4+192
6*b^2*c*m^2*n^2*(x^n)^4+2763*b^2*c*m*n^3*(x^n)^4+480*b*c^2*m^2*n*(x^n)^5+43
56*a*b*c*m^2*n^2*(x^n)^3+6696*a*b*c*m*n^3*(x^n)^3+1080*a*b*c*m^2*n*(x^n)^3+
1140*b*c^2*m*n^2*(x^n)^5+60*a^2*b*m^5*n*x^n+465*a^2*b*m^4*n^2*x^n+1740*a^2*
b*m^3*n^3*x^n+3132*a^2*b*m^2*n^4*x^n+2160*a^2*b*m*n^5*x^n+285*a^2*c*m^4*n*(
x^n)^2+1644*a^2*c*m^3*n^2*(x^n)^2+4149*a^2*c*m^2*n^3*(x^n)^2+4212*a^2*c*m*n
^4*(x^n)^2+285*a*b^2*m^4*n*(x^n)^2+1644*a*b^2*m^3*n^2*(x^n)^2+4149*a*b^2*m^
2*n^3*(x^n)^2+4212*a*b^2*m*n^4*(x^n)^2+3*(x^n)^5*b*c^2+108*a*b*c*m^5*n*(x^n
)^3+90*a*b*c*m^4*(x^n)^3+3048*a*b*c*n^4*(x^n)^3+510*a*c^2*m^2*n*(x^n)^4+128
4*a*c^2*m*n^2*(x^n)^4+510*b^2*c*m^2*n*(x^n)^4+210*a^3*m^2*n+700*a^3*m*n^2+1
05*a^3*m*n+1644*a*b^2*m*n^2*(x^n)^2+600*a^2*b*m^2*n*x^n+1860*a^2*b*m*n^2*x^
n+6*a^3*m+1440*a*b*c*m*n^5*(x^n)^3+2904*a*b*c*m^3*n^2*(x^n)^3+48*b*c^2*m^5*
n*(x^n)^5+285*b*c^2*m^4*n^2*(x^n)^5+780*b*c^2*m^3*n^3*(x^n)^5+972*b*c^2*m^2
*n^4*(x^n)^5+432*b*c^2*m*n^5*(x^n)^5+51*a*c^2*m^5*n*(x^n)^4+321*a*c^2*m^4*n
^2*(x^n)^4+21*a^3*m^5*n+175*a^3*m^4*n^2+735*a^3*m^3*n^3+1624*a^3*m^2*n^4+17
64*a^3*m*n^5+540*a*b*c*m^4*n*(x^n)^3+c^3*m^6*(x^n)^6+6*c^3*m^5*(x^n)^6+120*
c^3*n^5*(x^n)^6+15*c^3*m^4*(x^n)^6+274*c^3*n^4*(x^n)^6+b^3*m^6*(x^n)^3+20*c
^3*m^3*(x^n)^6+225*c^3*n^3*(x^n)^6+6*b^3*m^5*(x^n)^3+255*a*c^2*m*n*(x^n)^4+
255*b^2*c*m*n*(x^n)^4+600*a^2*b*m^3*n*x^n+2790*a^2*b*m^2*n^2*x^n+5220*a^2*b
*m*n^3*x^n+570*a^2*c*m^2*n*(x^n)^2+1644*a^2*c*m*n^2*(x^n)^2+90*a*b*c*m^2*(x
^n)^3+726*a*b*c*n^2*(x^n)^3+285*a^2*c*m*n*(x^n)^2+36*a*b*c*(x^n)^3*m+108*a*
b*c*(x^n)^3*n+1284*a*c^2*m^3*n^2*(x^n)^4+2763*a*c^2*m^2*n^3*(x^n)^4+2376*a*
c^2*m*n^4*(x^n)^4+255*b^2*c*m^4*n*(x^n)^4+1284*b^2*c*m^3*n^2*(x^n)^4+2763*b
^2*c*m^2*n^3*(x^n)^4+2376*b^2*c*m*n^4*(x^n)^4+480*b*c^2*m^3*n*(x^n)^5+1710*
b*c^2*m^2*n^2*(x^n)^5+2340*b*c^2*m*n^3*(x^n)^5+57*a^2*c*m^5*n*(x^n)^2+411*a
^2*c*m^4*n^2*(x^n)^2+1383*a^2*c*m^3*n^3*(x^n)^2+2106*a^2*c*m^2*n^4*(x^n)^2+
1080*a^2*c*m*n^5*(x^n)^2+372*b^3*n^3*(x^n)^3+3*(x^n)^2*a^2*c+3*(x^n)^4*a*c^
2+3*(x^n)^4*b^2*c+6096*a*b*c*m*n^4*(x^n)^3+508*b^3*n^4*(x^n)^3+6*m*c^3*(x^n
)^6+15*c^3*(x^n)^6*n+726*a*b*c*m^4*n^2*(x^n)^3+2232*a*b*c*m^3*n^3*(x^n)^3+3
048*a*b*c*m^2*n^4*(x^n)^3+921*a*c^2*m^3*n^3*(x^n)^4+1188*a*c^2*m^2*n^4*(x^n
)^4+540*a*c^2*m*n^5*(x^n)^4+51*b^2*c*m^5*n*(x^n)^4+321*b^2*c*m^4*n^2*(x^n)^
4+921*b^2*c*m^3*n^3*(x^n)^4+1188*b^2*c*m^2*n^4*(x^n)^4+540*b^2*c*m*n^5*(x^n
)^4+240*b*c^2*m^4*n*(x^n)^5+1140*b*c^2*m^3*n^2*(x^n)^5+2340*b*c^2*m^2*n^3*(
x^n)^5+1944*b*c^2*m*n^4*(x^n)^5+6*a*b*c*m^6*(x^n)^3+255*a*c^2*m^4*n*(x^n)^4
+285*a*b^2*m*n*(x^n)^2+300*a^2*b*m*n*x^n)/(m+1)/(m+n+1)/(m+2*n+1)/(m+3*n+1)
/(1+m+4*n)/(1+m+5*n)/(1+m+6*n)*exp(1/2*(-I*Pi*csgn(I*d)*csgn(I*x)*csgn(I*d*
x)+I*Pi*csgn(I*d)*csgn(I*d*x)^2+I*Pi*csgn(I*x)*csgn(I*d*x)^2-I*Pi*csgn(I*d*
x)^3+2*ln(d)+2*ln(x))*m)
```

maxima [A] time = 1.30, size = 273, normalized size = 1.50

$$\frac{c^3 d^m x e^{\left(m \log(x)+6 n \log(x)\right)}}{m+6n+1} + \frac{3 bc^2 d^m x e^{\left(m \log(x)+5 n \log(x)\right)}}{m+5n+1} + \frac{3 b^2 c d^m x e^{\left(m \log(x)+4 n \log(x)\right)}}{m+4n+1} + \frac{3 ac^2 d^m x e^{\left(m \log(x)+4 n \log(x)\right)}}{m+4n+1} + \frac{b^3 d^m x e^{\left(m \log(x)+3 n \log(x)\right)}}{m+3n+1} + \frac{6 abcd^m x e^{\left(m \log(x)+3 n \log(x)\right)}}{m+3n+1} + \frac{3 ab^2 d^m x e^{\left(m \log(x)+2 n \log(x)\right)}}{m+2n+1} + \frac{3 a^2 cd^m x e^{\left(m \log(x)+2 n \log(x)\right)}}{m+2n+1} + \frac{3 a^2 bd^m x e^{\left(m \log(x)+n \log(x)\right)}}{m+n+1} + \frac{(dx)^{m+1} a^3}{d(m+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*x^n+c*x^(2*n))^3,x, algorithm="maxima")
```

```
[Out] c^3*d^m*x*e^(m*log(x) + 6*n*log(x))/(m + 6*n + 1) + 3*b*c^2*d^m*x*e^(m*log(
x) + 5*n*log(x))/(m + 5*n + 1) + 3*b^2*c*d^m*x*e^(m*log(x) + 4*n*log(x))/(m
 + 4*n + 1) + 3*a*c^2*d^m*x*e^(m*log(x) + 4*n*log(x))/(m + 4*n + 1) + b^3*d
^m*x*e^(m*log(x) + 3*n*log(x))/(m + 3*n + 1) + 6*a*b*c*d^m*x*e^(m*log(x) +
3*n*log(x))/(m + 3*n + 1) + 3*a*b^2*d^m*x*e^(m*log(x) + 2*n*log(x))/(m + 2*
n + 1) + 3*a^2*c*d^m*x*e^(m*log(x) + 2*n*log(x))/(m + 2*n + 1) + 3*a^2*b*d^
m*x*e^(m*log(x) + n*log(x))/(m + n + 1) + (d*x)^(m + 1)*a^3/(d*(m + 1))
```

mupad [B] time = 2.16, size = 1734, normalized size = 9.53

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(a + b*x^n + c*x^(2*n))^3,x)
```

```
[Out] (a^3*x*(d*x)^m)/(m + 1) + (c^3*x*x^(6*n)*(d*x)^m*(5*m + 15*n + 60*m*n + 255
*m*n^2 + 90*m^2*n + 450*m*n^3 + 60*m^3*n + 274*m*n^4 + 15*m^4*n + 10*m^2 +
10*m^3 + 5*m^4 + m^5 + 85*n^2 + 225*n^3 + 274*n^4 + 120*n^5 + 255*m^2*n^2 +
 225*m^2*n^3 + 85*m^3*n^2 + 1))/(6*m + 21*n + 105*m*n + 700*m*n^2 + 210*m^2
*n + 2205*m*n^3 + 210*m^3*n + 3248*m*n^4 + 105*m^4*n + 1764*m*n^5 + 21*m^5*
n + 15*m^2 + 20*m^3 + 15*m^4 + 6*m^5 + m^6 + 175*n^2 + 735*n^3 + 1624*n^4 +
 1764*n^5 + 720*n^6 + 1050*m^2*n^2 + 2205*m^2*n^3 + 700*m^3*n^2 + 1624*m^2*
n^4 + 735*m^3*n^3 + 175*m^4*n^2 + 1) + (3*a*x*x^(2*n)*(d*x)^m*(a*c + b^2)*(
5*m + 19*n + 76*m*n + 411*m*n^2 + 114*m^2*n + 922*m*n^3 + 76*m^3*n + 702*m*
n^4 + 19*m^4*n + 10*m^2 + 10*m^3 + 5*m^4 + m^5 + 137*n^2 + 461*n^3 + 702*n^
4 + 360*n^5 + 411*m^2*n^2 + 461*m^2*n^3 + 137*m^3*n^2 + 1))/(6*m + 21*n + 1
05*m*n + 700*m*n^2 + 210*m^2*n + 2205*m*n^3 + 210*m^3*n + 3248*m*n^4 + 105*
m^4*n + 1764*m*n^5 + 21*m^5*n + 15*m^2 + 20*m^3 + 15*m^4 + 6*m^5 + m^6 + 17
5*n^2 + 735*n^3 + 1624*n^4 + 1764*n^5 + 720*n^6 + 1050*m^2*n^2 + 2205*m^2*n
^3 + 700*m^3*n^2 + 1624*m^2*n^4 + 735*m^3*n^3 + 175*m^4*n^2 + 1) + (b*x*x^(
3*n)*(d*x)^m*(6*a*c + b^2)*(5*m + 18*n + 72*m*n + 363*m*n^2 + 108*m^2*n + 7
44*m*n^3 + 72*m^3*n + 508*m*n^4 + 18*m^4*n + 10*m^2 + 10*m^3 + 5*m^4 + m^5
+ 121*n^2 + 372*n^3 + 508*n^4 + 240*n^5 + 363*m^2*n^2 + 372*m^2*n^3 + 121*m
^3*n^2 + 1))/(6*m + 21*n + 105*m*n + 700*m*n^2 + 210*m^2*n + 2205*m*n^3 + 2
10*m^3*n + 3248*m*n^4 + 105*m^4*n + 1764*m*n^5 + 21*m^5*n + 15*m^2 + 20*m^3
 + 15*m^4 + 6*m^5 + m^6 + 175*n^2 + 735*n^3 + 1624*n^4 + 1764*n^5 + 720*n^6
 + 1050*m^2*n^2 + 2205*m^2*n^3 + 700*m^3*n^2 + 1624*m^2*n^4 + 735*m^3*n^3 +
 175*m^4*n^2 + 1) + (3*c*x*x^(4*n)*(d*x)^m*(a*c + b^2)*(5*m + 17*n + 68*m*n
 + 321*m*n^2 + 102*m^2*n + 614*m*n^3 + 68*m^3*n + 396*m*n^4 + 17*m^4*n + 10
*m^2 + 10*m^3 + 5*m^4 + m^5 + 107*n^2 + 307*n^3 + 396*n^4 + 180*n^5 + 321*m
^2*n^2 + 307*m^2*n^3 + 107*m^3*n^2 + 1))/(6*m + 21*n + 105*m*n + 700*m*n^2
+ 210*m^2*n + 2205*m*n^3 + 210*m^3*n + 3248*m*n^4 + 105*m^4*n + 1764*m*n^5
+ 21*m^5*n + 15*m^2 + 20*m^3 + 15*m^4 + 6*m^5 + m^6 + 175*n^2 + 735*n^3 + 1
624*n^4 + 1764*n^5 + 720*n^6 + 1050*m^2*n^2 + 2205*m^2*n^3 + 700*m^3*n^2 +
1624*m^2*n^4 + 735*m^3*n^3 + 175*m^4*n^2 + 1) + (3*a^2*b*x*x^n*(d*x)^m*(5*m
 + 20*n + 80*m*n + 465*m*n^2 + 120*m^2*n + 1160*m*n^3 + 80*m^3*n + 1044*m*n
^4 + 20*m^4*n + 10*m^2 + 10*m^3 + 5*m^4 + m^5 + 155*n^2 + 580*n^3 + 1044*n^
4 + 720*n^5 + 465*m^2*n^2 + 580*m^2*n^3 + 155*m^3*n^2 + 1))/(6*m + 21*n + 1
05*m*n + 700*m*n^2 + 210*m^2*n + 2205*m*n^3 + 210*m^3*n + 3248*m*n^4 + 105*
m^4*n + 1764*m*n^5 + 21*m^5*n + 15*m^2 + 20*m^3 + 15*m^4 + 6*m^5 + m^6 + 17
5*n^2 + 735*n^3 + 1624*n^4 + 1764*n^5 + 720*n^6 + 1050*m^2*n^2 + 2205*m^2*n
^3 + 700*m^3*n^2 + 1624*m^2*n^4 + 735*m^3*n^3 + 175*m^4*n^2 + 1) + (3*b*c^2
*x*x^(5*n)*(d*x)^m*(5*m + 16*n + 64*m*n + 285*m*n^2 + 96*m^2*n + 520*m*n^3
+ 64*m^3*n + 324*m*n^4 + 16*m^4*n + 10*m^2 + 10*m^3 + 5*m^4 + m^5 + 95*n^2
+ 260*n^3 + 324*n^4 + 144*n^5 + 285*m^2*n^2 + 260*m^2*n^3 + 95*m^3*n^2 + 1)
)/(6*m + 21*n + 105*m*n + 700*m*n^2 + 210*m^2*n + 2205*m*n^3 + 210*m^3*n +
3248*m*n^4 + 105*m^4*n + 1764*m*n^5 + 21*m^5*n + 15*m^2 + 20*m^3 + 15*m^4 +
 6*m^5 + m^6 + 175*n^2 + 735*n^3 + 1624*n^4 + 1764*n^5 + 720*n^6 + 1050*m^2
*n^2 + 2205*m^2*n^3 + 700*m^3*n^2 + 1624*m^2*n^4 + 735*m^3*n^3 + 175*m^4*n^
```

```
2 + 1)
```

sympy [F(-1)]   time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(a+b*x**n+c*x**(2*n))**3,x)
[Out] Timed out
```

3.492 $\int (dx)^m \left(a+bx^n+cx^{2n}\right)^2 \, dx$

Optimal. Leaf size=117

$$\frac{a^2(dx)^{m+1}}{d(m+1)}+\frac{x^{2n+1}\left(2ac+b^2\right)(dx)^m}{m+2n+1}+\frac{2abx^{n+1}(dx)^m}{m+n+1}+\frac{2bcx^{3n+1}(dx)^m}{m+3n+1}+\frac{c^2x^{4n+1}(dx)^m}{m+4n+1}$$

Rubi [A] time = 0.07, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {1353, 20, 30}

$$\frac{a^2(dx)^{m+1}}{d(m+1)}+\frac{x^{2n+1}\left(2ac+b^2\right)(dx)^m}{m+2n+1}+\frac{2abx^{n+1}(dx)^m}{m+n+1}+\frac{2bcx^{3n+1}(dx)^m}{m+3n+1}+\frac{c^2x^{4n+1}(dx)^m}{m+4n+1}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^m*(a + b*x^n + c*x^(2*n))^2,x]
```

```
[Out] (2*a*b*x^(1 + n)*(d*x)^m)/(1 + m + n) + ((b^2 + 2*a*c)*x^(1 + 2*n)*(d*x)^m)
/(1 + m + 2*n) + (2*b*c*x^(1 + 3*n)*(d*x)^m)/(1 + m + 3*n) + (c^2*x^(1 + 4*
n)*(d*x)^m)/(1 + m + 4*n) + (a^2*(d*x)^(1 + m))/(d*(1 + m))
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(b^IntPart
[n]*(b*v)^FracPart[n])/(a^IntPart[n]*(a*v)^FracPart[n]), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !
IntegerQ[m + n]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 1353

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.),
x_Symbol] :> Int[ExpandIntegrand[(d*x)^m*(a + b*x^n + c*x^(2*n))^p, x], x]
/; FreeQ[{a, b, c, d, m, n}, x] && EqQ[n2, 2*n] && IGtQ[p, 0] && !IntegerQ
[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int (dx)^m \left(a+bx^n+cx^{2n}\right)^2 \, dx &= \int \left(a^2(dx)^m+2abx^n(dx)^m+b^2\left(1+\frac{2ac}{b^2}\right)x^{2n}(dx)^m+2bcx^{3n}(dx)^m+c^2x^{4n}(dx\\ &= \frac{a^2(dx)^{1+m}}{d(1+m)}+(2ab)\int x^n(dx)^m\,dx+(2bc)\int x^{3n}(dx)^m\,dx+c^2\int x^{4n}(dx)^m\,dx+\\ &= \frac{a^2(dx)^{1+m}}{d(1+m)}+\left(2abx^{-m}(dx)^m\right)\int x^{m+n}\,dx+\left(2bcx^{-m}(dx)^m\right)\int x^{m+3n}\,dx+\left(c^2x^{-\right.\\ &= \frac{2abx^{1+n}(dx)^m}{1+m+n}+\frac{\left(b^2+2ac\right)x^{1+2n}(dx)^m}{1+m+2n}+\frac{2bcx^{1+3n}(dx)^m}{1+m+3n}+\frac{c^2x^{1+4n}(dx)^m}{1+m+4n}+\end{aligned}$$

Mathematica [A] time = 0.15, size = 86, normalized size = 0.74

$$x(dx)^m\left(\frac{a^2}{m+1}+\frac{x^{2n}\left(2ac+b^2\right)}{m+2n+1}+\frac{2abx^n}{m+n+1}+\frac{2bcx^{3n}}{m+3n+1}+\frac{c^2x^{4n}}{m+4n+1}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^m*(a + b*x^n + c*x^(2*n))^2,x]
```

```
[Out] x*(d*x)^m*(a^2/(1 + m) + (2*a*b*x^n)/(1 + m + n) + ((b^2 + 2*a*c)*x^(2*n))/
(1 + m + 2*n) + (2*b*c*x^(3*n))/(1 + m + 3*n) + (c^2*x^(4*n))/(1 + m + 4*n)
)
```

IntegrateAlgebraic [F] time = 0.14, size = 0, normalized size = 0.00

$$\int (dx)^m \left(a + bx^n + cx^{2n}\right)^2 \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(d*x)^m*(a + b*x^n + c*x^(2*n))^2,x]
```

```
[Out] Defer[IntegrateAlgebraic][(d*x)^m*(a + b*x^n + c*x^(2*n))^2, x]
```

fricas [B] time = 1.25, size = 706, normalized size = 6.03

$$\frac{\left(c^2m^4 + 4c^2m^3 + 6c^2m^2 + 6\left(c^2m + c^2\right)n^3 + 4c^2m + 11\left(c^2m^2 + 2c^2m + c^2\right)n^2 + c^2 + 6\left(c^2m^3 + 3c^2m^2 + 3c^2m + c^2\right)n\right)xx^{4n}e^{\left(m\log(d) + m\log(x)\right)} + 2\left(bcm^4 + 4bcm^3 + 6bcm^2 + 8\left(bcm + bc\right)n^3 + 4bcm + 14\left(bcm^2 + 2bcm + bc\right)n^2 + bc + 7\left(bcm^3 + 3bcm^2 + 3bcm + bc\right)n\right)xx^{3n}e^{\left(m\log(d) + m\log(x)\right)} + \left(\left(b^2 + 2ac\right)m^4 + 4\left(b^2 + 2ac\right)m^3 + 12\left(b^2 + 2ac + \left(b^2 + 2ac\right)m\right)n^3 + 6\left(b^2 + 2ac\right)m^2 + 19\left(\left(b^2 + 2ac\right)m^2 + b^2 + 2ac + 2\left(b^2 + 2ac\right)m\right)n^2 + b^2 + 2ac + 4\left(b^2 + 2ac\right)m + 8\left(\left(b^2 + 2ac\right)m^3 + 3\left(b^2 + 2ac\right)m^2 + b^2 + 2ac + 3\left(b^2 + 2ac\right)m\right)n\right)xx^{2n}e^{\left(m\log(d) + m\log(x)\right)} + 2\left(abm^4 + 4abm^3 + 6abm^2 + 24\left(abm + ab\right)n^3 + 4abm + 26\left(abm^2 + 2abm + ab\right)n^2 + ab + 9\left(abm^3 + 3abm^2 + 3abm + ab\right)n\right)xx^ne^{\left(m\log(d) + m\log(x)\right)} + \left(a^2m^4 + 24a^2n^4 + 4a^2m^3 + 6a^2m^2 + 50\left(a^2m + a^2\right)n^3 + 4a^2m + 35\left(a^2m^2 + 2a^2m + a^2\right)n^2 + a^2 + 10\left(a^2m^3 + 3a^2m^2 + 3a^2m + a^2\right)n\right)xe^{\left(m\log(d) + m\log(x)\right)}}{m^5 + 24\left(m + 1\right)n^4 + 5m^4 + 50\left(m^2 + 2m + 1\right)n^3 + 10m^3 + 35\left(m^3 + 3m^2 + 3m + 1\right)n^2 + 10m^2 + 10\left(m^4 + 4m^3 + 6m^2 + 4m + 1\right)n + 5m + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*x^n+c*x^(2*n))^2,x, algorithm="fricas")
```

```
[Out] ((c^2*m^4 + 4*c^2*m^3 + 6*c^2*m^2 + 6*(c^2*m + c^2)*n^3 + 4*c^2*m + 11*(c^2
*m^2 + 2*c^2*m + c^2)*n^2 + c^2 + 6*(c^2*m^3 + 3*c^2*m^2 + 3*c^2*m + c^2)*n
)*x*x^(4*n)*e^(m*log(d) + m*log(x)) + 2*(b*c*m^4 + 4*b*c*m^3 + 6*b*c*m^2 +
8*(b*c*m + b*c)*n^3 + 4*b*c*m + 14*(b*c*m^2 + 2*b*c*m + b*c)*n^2 + b*c + 7*
(b*c*m^3 + 3*b*c*m^2 + 3*b*c*m + b*c)*n)*x*x^(3*n)*e^(m*log(d) + m*log(x))
+ ((b^2 + 2*a*c)*m^4 + 4*(b^2 + 2*a*c)*m^3 + 12*(b^2 + 2*a*c + (b^2 + 2*a*c
)*m)*n^3 + 6*(b^2 + 2*a*c)*m^2 + 19*((b^2 + 2*a*c)*m^2 + b^2 + 2*a*c + 2*(b
^2 + 2*a*c)*m)*n^2 + b^2 + 2*a*c + 4*(b^2 + 2*a*c)*m + 8*((b^2 + 2*a*c)*m^3
 + 3*(b^2 + 2*a*c)*m^2 + b^2 + 2*a*c + 3*(b^2 + 2*a*c)*m)*n)*x*x^(2*n)*e^(m
*log(d) + m*log(x)) + 2*(a*b*m^4 + 4*a*b*m^3 + 6*a*b*m^2 + 24*(a*b*m + a*b)
*n^3 + 4*a*b*m + 26*(a*b*m^2 + 2*a*b*m + a*b)*n^2 + a*b + 9*(a*b*m^3 + 3*a*
b*m^2 + 3*a*b*m + a*b)*n)*x*x^n*e^(m*log(d) + m*log(x)) + (a^2*m^4 + 24*a^2
*n^4 + 4*a^2*m^3 + 6*a^2*m^2 + 50*(a^2*m + a^2)*n^3 + 4*a^2*m + 35*(a^2*m^2
 + 2*a^2*m + a^2)*n^2 + a^2 + 10*(a^2*m^3 + 3*a^2*m^2 + 3*a^2*m + a^2)*n)*x
*e^(m*log(d) + m*log(x)))/(m^5 + 24*(m + 1)*n^4 + 5*m^4 + 50*(m^2 + 2*m + 1
)*n^3 + 10*m^3 + 35*(m^3 + 3*m^2 + 3*m + 1)*n^2 + 10*m^2 + 10*(m^4 + 4*m^3
+ 6*m^2 + 4*m + 1)*n + 5*m + 1)
```

giac [B] time = 0.80, size = 5454, normalized size = 46.62

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*x^n+c*x^(2*n))^2,x, algorithm="giac")
```

```
[Out] (c^2*m^4*x*x^(4*n)*e^(m*log(d) + m*log(x)) + 6*c^2*m^3*n*x*x^(4*n)*e^(m*log
(d) + m*log(x)) + 11*c^2*m^2*n^2*x*x^(4*n)*e^(m*log(d) + m*log(x)) + 6*c^2*
m*n^3*x*x^(4*n)*e^(m*log(d) + m*log(x)) + 2*b*c*m^4*x*x^(3*n)*e^(m*log(d) +
 m*log(x)) + c^2*m^4*x*x^(3*n)*e^(m*log(d) + m*log(x)) + 14*b*c*m^3*n*x*x^(
3*n)*e^(m*log(d) + m*log(x)) + 6*c^2*m^3*n*x*x^(3*n)*e^(m*log(d) + m*log(x)
) + 28*b*c*m^2*n^2*x*x^(3*n)*e^(m*log(d) + m*log(x)) + 11*c^2*m^2*n^2*x*x^(
3*n)*e^(m*log(d) + m*log(x)) + 16*b*c*m*n^3*x*x^(3*n)*e^(m*log(d) + m*log(x
)) + 6*c^2*m*n^3*x*x^(3*n)*e^(m*log(d) + m*log(x)) + b^2*m^4*x*x^(2*n)*e^(m
*log(d) + m*log(x)) + 2*a*c*m^4*x*x^(2*n)*e^(m*log(d) + m*log(x)) + 2*b*c*m
^4*x*x^(2*n)*e^(m*log(d) + m*log(x)) + c^2*m^4*x*x^(2*n)*e^(m*log(d) + m*lo
g(x)) + 8*b^2*m^3*n*x*x^(2*n)*e^(m*log(d) + m*log(x)) + 16*a*c*m^3*n*x*x^(2
```

*n)*e^(m*log(d) + m*log(x)) + 14*b*c*m^3*n*x*x^(2*n)*e^(m*log(d) + m*log(x)
) + 6*c^2*m^3*n*x*x^(2*n)*e^(m*log(d) + m*log(x)) + 19*b^2*m^2*n^2*x*x^(2*n
)*e^(m*log(d) + m*log(x)) + 38*a*c*m^2*n^2*x*x^(2*n)*e^(m*log(d) + m*log(x)
) + 28*b*c*m^2*n^2*x*x^(2*n)*e^(m*log(d) + m*log(x)) + 11*c^2*m^2*n^2*x*x^(
2*n)*e^(m*log(d) + m*log(x)) + 12*b^2*m*n^3*x*x^(2*n)*e^(m*log(d) + m*log(x
)) + 24*a*c*m*n^3*x*x^(2*n)*e^(m*log(d) + m*log(x)) + 16*b*c*m*n^3*x*x^(2*n
)*e^(m*log(d) + m*log(x)) + 6*c^2*m*n^3*x*x^(2*n)*e^(m*log(d) + m*log(x)) +
 2*a*b*m^4*x*x^n*e^(m*log(d) + m*log(x)) + b^2*m^4*x*x^n*e^(m*log(d) + m*lo
g(x)) + 2*a*c*m^4*x*x^n*e^(m*log(d) + m*log(x)) + 2*b*c*m^4*x*x^n*e^(m*log(
d) + m*log(x)) + c^2*m^4*x*x^n*e^(m*log(d) + m*log(x)) + 18*a*b*m^3*n*x*x^n
*e^(m*log(d) + m*log(x)) + 8*b^2*m^3*n*x*x^n*e^(m*log(d) + m*log(x)) + 16*a
*c*m^3*n*x*x^n*e^(m*log(d) + m*log(x)) + 14*b*c*m^3*n*x*x^n*e^(m*log(d) + m
*log(x)) + 6*c^2*m^3*n*x*x^n*e^(m*log(d) + m*log(x)) + 52*a*b*m^2*n^2*x*x^n
*e^(m*log(d) + m*log(x)) + 19*b^2*m^2*n^2*x*x^n*e^(m*log(d) + m*log(x)) + 3
8*a*c*m^2*n^2*x*x^n*e^(m*log(d) + m*log(x)) + 28*b*c*m^2*n^2*x*x^n*e^(m*log
(d) + m*log(x)) + 11*c^2*m^2*n^2*x*x^n*e^(m*log(d) + m*log(x)) + 48*a*b*m*n
^3*x*x^n*e^(m*log(d) + m*log(x)) + 12*b^2*m*n^3*x*x^n*e^(m*log(d) + m*log(x
)) + 24*a*c*m*n^3*x*x^n*e^(m*log(d) + m*log(x)) + 16*b*c*m*n^3*x*x^n*e^(m*l
og(d) + m*log(x)) + 6*c^2*m*n^3*x*x^n*e^(m*log(d) + m*log(x)) + a^2*m^4*x*e
^(m*log(d) + m*log(x)) + 2*a*b*m^4*x*e^(m*log(d) + m*log(x)) + b^2*m^4*x*e^
(m*log(d) + m*log(x)) + 2*a*c*m^4*x*e^(m*log(d) + m*log(x)) + 2*b*c*m^4*x*e
^(m*log(d) + m*log(x)) + c^2*m^4*x*e^(m*log(d) + m*log(x)) + 10*a^2*m^3*n*x
*e^(m*log(d) + m*log(x)) + 18*a*b*m^3*n*x*e^(m*log(d) + m*log(x)) + 8*b^2*m
^3*n*x*e^(m*log(d) + m*log(x)) + 16*a*c*m^3*n*x*e^(m*log(d) + m*log(x)) + 1
4*b*c*m^3*n*x*e^(m*log(d) + m*log(x)) + 6*c^2*m^3*n*x*e^(m*log(d) + m*log(x
)) + 35*a^2*m^2*n^2*x*e^(m*log(d) + m*log(x)) + 52*a*b*m^2*n^2*x*e^(m*log(d
) + m*log(x)) + 19*b^2*m^2*n^2*x*e^(m*log(d) + m*log(x)) + 38*a*c*m^2*n^2*x
*e^(m*log(d) + m*log(x)) + 28*b*c*m^2*n^2*x*e^(m*log(d) + m*log(x)) + 11*c^
2*m^2*n^2*x*e^(m*log(d) + m*log(x)) + 50*a^2*m*n^3*x*e^(m*log(d) + m*log(x)
) + 48*a*b*m*n^3*x*e^(m*log(d) + m*log(x)) + 12*b^2*m*n^3*x*e^(m*log(d) + m
*log(x)) + 24*a*c*m*n^3*x*e^(m*log(d) + m*log(x)) + 16*b*c*m*n^3*x*e^(m*log
(d) + m*log(x)) + 6*c^2*m*n^3*x*e^(m*log(d) + m*log(x)) + 24*a^2*n^4*x*e^(m
*log(d) + m*log(x)) + 4*c^2*m^3*x*x^(4*n)*e^(m*log(d) + m*log(x)) + 18*c^2*
m^2*n*x*x^(4*n)*e^(m*log(d) + m*log(x)) + 22*c^2*m*n^2*x*x^(4*n)*e^(m*log(d
) + m*log(x)) + 6*c^2*n^3*x*x^(4*n)*e^(m*log(d) + m*log(x)) + 8*b*c*m^3*x*x
^(3*n)*e^(m*log(d) + m*log(x)) + 4*c^2*m^3*x*x^(3*n)*e^(m*log(d) + m*log(x)
) + 42*b*c*m^2*n*x*x^(3*n)*e^(m*log(d) + m*log(x)) + 18*c^2*m^2*n*x*x^(3*n)
*e^(m*log(d) + m*log(x)) + 56*b*c*m*n^2*x*x^(3*n)*e^(m*log(d) + m*log(x)) +
 22*c^2*m*n^2*x*x^(3*n)*e^(m*log(d) + m*log(x)) + 16*b*c*n^3*x*x^(3*n)*e^(m
*log(d) + m*log(x)) + 6*c^2*n^3*x*x^(3*n)*e^(m*log(d) + m*log(x)) + 4*b^2*m
^3*x*x^(2*n)*e^(m*log(d) + m*log(x)) + 8*a*c*m^3*x*x^(2*n)*e^(m*log(d) + m*
log(x)) + 8*b*c*m^3*x*x^(2*n)*e^(m*log(d) + m*log(x)) + 4*c^2*m^3*x*x^(2*n)
*e^(m*log(d) + m*log(x)) + 24*b^2*m^2*n*x*x^(2*n)*e^(m*log(d) + m*log(x)) +
 48*a*c*m^2*n*x*x^(2*n)*e^(m*log(d) + m*log(x)) + 42*b*c*m^2*n*x*x^(2*n)*e^
(m*log(d) + m*log(x)) + 18*c^2*m^2*n*x*x^(2*n)*e^(m*log(d) + m*log(x)) + 38
*b^2*m*n^2*x*x^(2*n)*e^(m*log(d) + m*log(x)) + 76*a*c*m*n^2*x*x^(2*n)*e^(m*
log(d) + m*log(x)) + 56*b*c*m*n^2*x*x^(2*n)*e^(m*log(d) + m*log(x)) + 22*c^
2*m*n^2*x*x^(2*n)*e^(m*log(d) + m*log(x)) + 12*b^2*n^3*x*x^(2*n)*e^(m*log(d
) + m*log(x)) + 24*a*c*n^3*x*x^(2*n)*e^(m*log(d) + m*log(x)) + 16*b*c*n^3*x
*x^(2*n)*e^(m*log(d) + m*log(x)) + 6*c^2*n^3*x*x^(2*n)*e^(m*log(d) + m*log(
x)) + 8*a*b*m^3*x*x^n*e^(m*log(d) + m*log(x)) + 4*b^2*m^3*x*x^n*e^(m*log(d)
 + m*log(x)) + 8*a*c*m^3*x*x^n*e^(m*log(d) + m*log(x)) + 8*b*c*m^3*x*x^n*e^
(m*log(d) + m*log(x)) + 4*c^2*m^3*x*x^n*e^(m*log(d) + m*log(x)) + 54*a*b*m^
2*n*x*x^n*e^(m*log(d) + m*log(x)) + 24*b^2*m^2*n*x*x^n*e^(m*log(d) + m*log(
x)) + 48*a*c*m^2*n*x*x^n*e^(m*log(d) + m*log(x)) + 42*b*c*m^2*n*x*x^n*e^(m*
log(d) + m*log(x)) + 18*c^2*m^2*n*x*x^n*e^(m*log(d) + m*log(x)) + 104*a*b*m
*n^2*x*x^n*e^(m*log(d) + m*log(x)) + 38*b^2*m*n^2*x*x^n*e^(m*log(d) + m*log
(x)) + 76*a*c*m*n^2*x*x^n*e^(m*log(d) + m*log(x)) + 56*b*c*m*n^2*x*x^n*e^(m
*log(d) + m*log(x)) + 22*c^2*m*n^2*x*x^n*e^(m*log(d) + m*log(x)) + 48*a*b*n

```
^3*x*x^n*e^(m*log(d) + m*log(x)) + 12*b^2*n^3*x*x^n*e^(m*log(d) + m*log(x))
 + 24*a*c*n^3*x*x^n*e^(m*log(d) + m*log(x)) + 16*b*c*n^3*x*x^n*e^(m*log(d)
+ m*log(x)) + 6*c^2*n^3*x*x^n*e^(m*log(d) + m*log(x)) + 4*a^2*m^3*x*e^(m*lo
g(d) + m*log(x)) + 8*a*b*m^3*x*e^(m*log(d) + m*log(x)) + 4*b^2*m^3*x*e^(m*l
og(d) + m*log(x)) + 8*a*c*m^3*x*e^(m*log(d) + m*log(x)) + 8*b*c*m^3*x*e^(m*
log(d) + m*log(x)) + 4*c^2*m^3*x*e^(m*log(d) + m*log(x)) + 30*a^2*m^2*n*x*e
^(m*log(d) + m*log(x)) + 54*a*b*m^2*n*x*e^(m*log(d) + m*log(x)) + 24*b^2*m^
2*n*x*e^(m*log(d) + m*log(x)) + 48*a*c*m^2*n*x*e^(m*log(d) + m*log(x)) + 42
*b*c*m^2*n*x*e^(m*log(d) + m*log(x)) + 18*c^2*m^2*n*x*e^(m*log(d) + m*log(x
)) + 70*a^2*m*n^2*x*e^(m*log(d) + m*log(x)) + 104*a*b*m*n^2*x*e^(m*log(d) +
 m*log(x)) + 38*b^2*m*n^2*x*e^(m*log(d) + m*log(x)) + 76*a*c*m*n^2*x*e^(m*l
og(d) + m*log(x)) + 56*b*c*m*n^2*x*e^(m*log(d) + m*log(x)) + 22*c^2*m*n^2*x
*e^(m*log(d) + m*log(x)) + 50*a^2*n^3*x*e^(m*log(d) + m*log(x)) + 48*a*b*n^
3*x*e^(m*log(d) + m*log(x)) + 12*b^2*n^3*x*e^(m*log(d) + m*log(x)) + 24*a*c
*n^3*x*e^(m*log(d) + m*log(x)) + 16*b*c*n^3*x*e^(m*log(d) + m*log(x)) + 6*c
^2*n^3*x*e^(m*log(d) + m*log(x)) + 6*c^2*m^2*x*x^(4*n)*e^(m*log(d) + m*log(
x)) + 18*c^2*m*n*x*x^(4*n)*e^(m*log(d) + m*log(x)) + 11*c^2*n^2*x*x^(4*n)*e
^(m*log(d) + m*log(x)) + 12*b*c*m^2*x*x^(3*n)*e^(m*log(d) + m*log(x)) + 6*c
^2*m^2*x*x^(3*n)*e^(m*log(d) + m*log(x)) + 42*b*c*m*n*x*x^(3*n)*e^(m*log(d)
 + m*log(x)) + 18*c^2*m*n*x*x^(3*n)*e^(m*log(d) + m*log(x)) + 28*b*c*n^2*x*
x^(3*n)*e^(m*log(d) + m*log(x)) + 11*c^2*n^2*x*x^(3*n)*e^(m*log(d) + m*log(
x)) + 6*b^2*m^2*x*x^(2*n)*e^(m*log(d) + m*log(x)) + 12*a*c*m^2*x*x^(2*n)*e^
(m*log(d) + m*log(x)) + 12*b*c*m^2*x*x^(2*n)*e^(m*log(d) + m*log(x)) + 6*c^
2*m^2*x*x^(2*n)*e^(m*log(d) + m*log(x)) + 24*b^2*m*n*x*x^(2*n)*e^(m*log(d)
+ m*log(x)) + 48*a*c*m*n*x*x^(2*n)*e^(m*log(d) + m*log(x)) + 42*b*c*m*n*x*x
^(2*n)*e^(m*log(d) + m*log(x)) + 18*c^2*m*n*x*x^(2*n)*e^(m*log(d) + m*log(x
)) + 19*b^2*n^2*x*x^(2*n)*e^(m*log(d) + m*log(x)) + 38*a*c*n^2*x*x^(2*n)*e^
(m*log(d) + m*log(x)) + 28*b*c*n^2*x*x^(2*n)*e^(m*log(d) + m*log(x)) + 11*c
^2*n^2*x*x^(2*n)*e^(m*log(d) + m*log(x)) + 12*a*b*m^2*x*x^n*e^(m*log(d) + m
*log(x)) + 6*b^2*m^2*x*x^n*e^(m*log(d) + m*log(x)) + 12*a*c*m^2*x*x^n*e^(m*
log(d) + m*log(x)) + 12*b*c*m^2*x*x^n*e^(m*log(d) + m*log(x)) + 6*c^2*m^2*x
*x^n*e^(m*log(d) + m*log(x)) + 54*a*b*m*n*x*x^n*e^(m*log(d) + m*log(x)) + 2
4*b^2*m*n*x*x^n*e^(m*log(d) + m*log(x)) + 48*a*c*m*n*x*x^n*e^(m*log(d) + m*
log(x)) + 42*b*c*m*n*x*x^n*e^(m*log(d) + m*log(x)) + 18*c^2*m*n*x*x^n*e^(m*
log(d) + m*log(x)) + 52*a*b*n^2*x*x^n*e^(m*log(d) + m*log(x)) + 19*b^2*n^2*
x*x^n*e^(m*log(d) + m*log(x)) + 38*a*c*n^2*x*x^n*e^(m*log(d) + m*log(x)) +
28*b*c*n^2*x*x^n*e^(m*log(d) + m*log(x)) + 11*c^2*n^2*x*x^n*e^(m*log(d) + m
*log(x)) + 6*a^2*m^2*x*e^(m*log(d) + m*log(x)) + 12*a*b*m^2*x*e^(m*log(d) +
 m*log(x)) + 6*b^2*m^2*x*e^(m*log(d) + m*log(x)) + 12*a*c*m^2*x*e^(m*log(d)
 + m*log(x)) + 12*b*c*m^2*x*e^(m*log(d) + m*log(x)) + 6*c^2*m^2*x*e^(m*log(
d) + m*log(x)) + 30*a^2*m*n*x*e^(m*log(d) + m*log(x)) + 54*a*b*m*n*x*e^(m*l
og(d) + m*log(x)) + 24*b^2*m*n*x*e^(m*log(d) + m*log(x)) + 48*a*c*m*n*x*e^(
m*log(d) + m*log(x)) + 42*b*c*m*n*x*e^(m*log(d) + m*log(x)) + 18*c^2*m*n*x*
e^(m*log(d) + m*log(x)) + 35*a^2*n^2*x*e^(m*log(d) + m*log(x)) + 52*a*b*n^2
*x*e^(m*log(d) + m*log(x)) + 19*b^2*n^2*x*e^(m*log(d) + m*log(x)) + 38*a*c*
n^2*x*e^(m*log(d) + m*log(x)) + 28*b*c*n^2*x*e^(m*log(d) + m*log(x)) + 11*c
^2*n^2*x*e^(m*log(d) + m*log(x)) + 4*c^2*m*x*x^(4*n)*e^(m*log(d) + m*log(x)
) + 6*c^2*n*x*x^(4*n)*e^(m*log(d) + m*log(x)) + 8*b*c*m*x*x^(3*n)*e^(m*log(
d) + m*log(x)) + 4*c^2*m*x*x^(3*n)*e^(m*log(d) + m*log(x)) + 14*b*c*n*x*x^(
3*n)*e^(m*log(d) + m*log(x)) + 6*c^2*n*x*x^(3*n)*e^(m*log(d) + m*log(x)) +
4*b^2*m*x*x^(2*n)*e^(m*log(d) + m*log(x)) + 8*a*c*m*x*x^(2*n)*e^(m*log(d) +
 m*log(x)) + 8*b*c*m*x*x^(2*n)*e^(m*log(d) + m*log(x)) + 4*c^2*m*x*x^(2*n)*
e^(m*log(d) + m*log(x)) + 8*b^2*n*x*x^(2*n)*e^(m*log(d) + m*log(x)) + 16*a*
c*n*x*x^(2*n)*e^(m*log(d) + m*log(x)) + 14*b*c*n*x*x^(2*n)*e^(m*log(d) + m*
log(x)) + 6*c^2*n*x*x^(2*n)*e^(m*log(d) + m*log(x)) + 8*a*b*m*x*x^n*e^(m*lo
g(d) + m*log(x)) + 4*b^2*m*x*x^n*e^(m*log(d) + m*log(x)) + 8*a*c*m*x*x^n*e^
(m*log(d) + m*log(x)) + 8*b*c*m*x*x^n*e^(m*log(d) + m*log(x)) + 4*c^2*m*x*x
^n*e^(m*log(d) + m*log(x)) + 18*a*b*n*x*x^n*e^(m*log(d) + m*log(x)) + 8*b^2
*n*x*x^n*e^(m*log(d) + m*log(x)) + 16*a*c*n*x*x^n*e^(m*log(d) + m*log(x)) +
```

```
14*b*c*n*x*x^n*e^(m*log(d) + m*log(x)) + 6*c^2*n*x*x^n*e^(m*log(d) + m*log
(x)) + 4*a^2*m*x*e^(m*log(d) + m*log(x)) + 8*a*b*m*x*e^(m*log(d) + m*log(x)
) + 4*b^2*m*x*e^(m*log(d) + m*log(x)) + 8*a*c*m*x*e^(m*log(d) + m*log(x)) +
 8*b*c*m*x*e^(m*log(d) + m*log(x)) + 4*c^2*m*x*e^(m*log(d) + m*log(x)) + 10
*a^2*n*x*e^(m*log(d) + m*log(x)) + 18*a*b*n*x*e^(m*log(d) + m*log(x)) + 8*b
^2*n*x*e^(m*log(d) + m*log(x)) + 16*a*c*n*x*e^(m*log(d) + m*log(x)) + 14*b*
c*n*x*e^(m*log(d) + m*log(x)) + 6*c^2*n*x*e^(m*log(d) + m*log(x)) + c^2*x*x
^(4*n)*e^(m*log(d) + m*log(x)) + 2*b*c*x*x^(3*n)*e^(m*log(d) + m*log(x)) +
c^2*x*x^(3*n)*e^(m*log(d) + m*log(x)) + b^2*x*x^(2*n)*e^(m*log(d) + m*log(x
)) + 2*a*c*x*x^(2*n)*e^(m*log(d) + m*log(x)) + 2*b*c*x*x^(2*n)*e^(m*log(d)
+ m*log(x)) + c^2*x*x^(2*n)*e^(m*log(d) + m*log(x)) + 2*a*b*x*x^n*e^(m*log(
d) + m*log(x)) + b^2*x*x^n*e^(m*log(d) + m*log(x)) + 2*a*c*x*x^n*e^(m*log(d
) + m*log(x)) + 2*b*c*x*x^n*e^(m*log(d) + m*log(x)) + c^2*x*x^n*e^(m*log(d)
 + m*log(x)) + a^2*x*e^(m*log(d) + m*log(x)) + 2*a*b*x*e^(m*log(d) + m*log(
x)) + b^2*x*e^(m*log(d) + m*log(x)) + 2*a*c*x*e^(m*log(d) + m*log(x)) + 2*b
*c*x*e^(m*log(d) + m*log(x)) + c^2*x*e^(m*log(d) + m*log(x)))/(m^5 + 10*m^4
*n + 35*m^3*n^2 + 50*m^2*n^3 + 24*m*n^4 + 5*m^4 + 40*m^3*n + 105*m^2*n^2 +
100*m*n^3 + 24*n^4 + 10*m^3 + 60*m^2*n + 105*m*n^2 + 50*n^3 + 10*m^2 + 40*m
*n + 35*n^2 + 5*m + 10*n + 1)
```

maple [C] time = 0.07, size = 1065, normalized size = 9.10

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(b*x^n+c*x^(2*n)+a)^2,x)
```

```
[Out] x*(2*(x^n)^3*b*c+a^2+c^2*(x^n)^4+a^2*m^4+4*a^2*m^3+50*a^2*n^3+6*a^2*m^2+35*
a^2*n^2+24*a^2*n^4+4*a^2*m+10*a^2*n+b^2*(x^n)^2+6*c^2*m^3*n*(x^n)^4+11*c^2*
m^2*n^2*(x^n)^4+6*c^2*m*n^3*(x^n)^4+2*b*c*m^4*(x^n)^3+18*c^2*m^2*n*(x^n)^4+
22*c^2*m*n^2*(x^n)^4+52*a*b*n^2*x^n+8*a*c*(x^n)^2*m+12*a*b*m^2*x^n+12*a*c*m
^2*(x^n)^2+38*a*c*n^2*(x^n)^2+24*b^2*m*n*(x^n)^2+8*m*b*c*(x^n)^3+14*b*c*(x^
n)^3*n+18*c^2*m*n*(x^n)^4+2*a*b*m^4*x^n+8*a*c*m^3*(x^n)^2+24*a*c*n^3*(x^n)^
2+24*b^2*m^2*n*(x^n)^2+38*b^2*m*n^2*(x^n)^2+12*b*c*m^2*(x^n)^3+28*b*c*n^2*(
x^n)^3+8*a*b*m^3*x^n+48*a*b*n^3*x^n+16*a*c*(x^n)^2*n+8*a*b*x^n*m+18*a*b*x^n
*n+2*a*c*m^4*(x^n)^2+8*b^2*m^3*n*(x^n)^2+19*b^2*m^2*n^2*(x^n)^2+12*b^2*m*n^
3*(x^n)^2+8*b*c*m^3*(x^n)^3+16*b*c*n^3*(x^n)^3+2*(x^n)^2*a*c+c^2*m^4*(x^n)^
4+4*c^2*m^3*(x^n)^4+56*b*c*m*n^2*(x^n)^3+18*a*b*m^3*n*x^n+52*a*b*m^2*n^2*x^
n+48*a*b*m*n^3*x^n+48*a*c*m^2*n*(x^n)^2+76*a*c*m*n^2*(x^n)^2+42*b*c*m*n*(x^
n)^3+54*a*b*m^2*n*x^n+104*a*b*m*n^2*x^n+48*a*c*m*n*(x^n)^2+54*a*b*m*n*x^n+6
*c^2*n^3*(x^n)^4+b^2*m^4*(x^n)^2+6*c^2*m^2*(x^n)^4+11*c^2*n^2*(x^n)^4+4*b^2
*m^3*(x^n)^2+12*b^2*n^3*(x^n)^2+4*m*c^2*(x^n)^4+6*c^2*(x^n)^4*n+6*b^2*m^2*(
x^n)^2+14*b*c*m^3*n*(x^n)^3+28*b*c*m^2*n^2*(x^n)^3+16*b*c*m*n^3*(x^n)^3+16*
a*c*m^3*n*(x^n)^2+38*a*c*m^2*n^2*(x^n)^2+24*a*c*m*n^3*(x^n)^2+42*b*c*m^2*n*
(x^n)^3+19*b^2*n^2*(x^n)^2+4*b^2*(x^n)^2*m+8*b^2*(x^n)^2*n+30*a^2*m*n+2*a*b
*x^n+10*a^2*m^3*n+35*a^2*m^2*n^2+50*a^2*m*n^3+30*a^2*m^2*n+70*a^2*m*n^2)/(m
+1)/(m+n+1)/(m+2*n+1)/(m+3*n+1)/(1+m+4*n)*exp(1/2*(-I*Pi*csgn(I*d)*csgn(I*x
)*csgn(I*d*x)+I*Pi*csgn(I*d)*csgn(I*d*x)^2+I*Pi*csgn(I*x)*csgn(I*d*x)^2-I*P
i*csgn(I*d*x)^3+2*ln(d)+2*ln(x))*m)
```

maxima [A] time = 1.27, size = 152, normalized size = 1.30

$$\frac{c^{2} d^{m} x e^{\left(m \log\left(x\right) + 4 \, n \log\left(x\right)\right)}}{m + 4 \, n + 1} + \frac{2 \, b c d^{m} x e^{\left(m \log\left(x\right) + 3 \, n \log\left(x\right)\right)}}{m + 3 \, n + 1} + \frac{b^{2} d^{m} x e^{\left(m \log\left(x\right) + 2 \, n \log\left(x\right)\right)}}{m + 2 \, n + 1} + \frac{2 \, a c d^{m} x e^{\left(m \log\left(x\right) + 2 \, n \log\left(x\right)\right)}}{m + 2 \, n + 1} + \frac{2 \, a b d^{m} x e^{\left(m \log\left(x\right) + n \log\left(x\right)\right)}}{m + n + 1} + \frac{\left(d x\right)^{m + 1} a^{2}}{d {\left(m + 1\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*x^n+c*x^(2*n))^2,x, algorithm="maxima")
```

```
[Out] c^2*d^m*x*e^(m*log(x) + 4*n*log(x))/(m + 4*n + 1) + 2*b*c*d^m*x*e^(m*log(x)
 + 3*n*log(x))/(m + 3*n + 1) + b^2*d^m*x*e^(m*log(x) + 2*n*log(x))/(m + 2*n
```

```
 + 1) + 2*a*c*d^m*x*e^(m*log(x) + 2*n*log(x))/(m + 2*n + 1) + 2*a*b*d^m*x*e
^(m*log(x) + n*log(x))/(m + n + 1) + (d*x)^(m + 1)*a^2/(d*(m + 1))
```

mupad [B] time = 1.62, size = 543, normalized size = 4.64

$$\frac{a^2\,x\,{\left(d\,x\right)}^m}{m+1}+\frac{x\,x^{2\,n}\,{\left(d\,x\right)}^m\,\left(b^2+2\,a\,c\right)\,\left(m^3+8\,m^2\,n+3\,m^2+19\,m\,n^2+16\,m\,n+3\,m+12\,n^3+19\,n^2+8\,n+1\right)}{m^4+10\,m^3\,n+4\,m^3+35\,m^2\,n^2+30\,m^2\,n+6\,m^2+50\,m\,n^3+70\,m\,n^2+30\,m\,n+4\,m+24\,n^4+50\,n^3+35\,n^2+10\,n+1}+\frac{c^2\,x\,x^{4\,n}\,{\left(d\,x\right)}^m\,\left(m^3+6\,m^2\,n+3\,m^2+11\,m\,n^2+12\,m\,n+3\,m+6\,n^3+11\,n^2+6\,n+1\right)}{m^4+10\,m^3\,n+4\,m^3+35\,m^2\,n^2+30\,m^2\,n+6\,m^2+50\,m\,n^3+70\,m\,n^2+30\,m\,n+4\,m+24\,n^4+50\,n^3+35\,n^2+10\,n+1}+\frac{2\,a\,b\,x\,x^n\,{\left(d\,x\right)}^m\,\left(m^3+9\,m^2\,n+3\,m^2+26\,m\,n^2+18\,m\,n+3\,m+24\,n^3+26\,n^2+9\,n+1\right)}{m^4+10\,m^3\,n+4\,m^3+35\,m^2\,n^2+30\,m^2\,n+6\,m^2+50\,m\,n^3+70\,m\,n^2+30\,m\,n+4\,m+24\,n^4+50\,n^3+35\,n^2+10\,n+1}+\frac{2\,b\,c\,x\,x^{3\,n}\,{\left(d\,x\right)}^m\,\left(m^3+7\,m^2\,n+3\,m^2+14\,m\,n^2+14\,m\,n+3\,m+8\,n^3+14\,n^2+7\,n+1\right)}{m^4+10\,m^3\,n+4\,m^3+35\,m^2\,n^2+30\,m^2\,n+6\,m^2+50\,m\,n^3+70\,m\,n^2+30\,m\,n+4\,m+24\,n^4+50\,n^3+35\,n^2+10\,n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(a + b*x^n + c*x^(2*n))^2,x)
```

```
[Out] (a^2*x*(d*x)^m)/(m + 1) + (x*x^(2*n)*(d*x)^m*(2*a*c + b^2)*(3*m + 8*n + 16*
m*n + 19*m*n^2 + 8*m^2*n + 3*m^2 + m^3 + 19*n^2 + 12*n^3 + 1))/(4*m + 10*n
+ 30*m*n + 70*m*n^2 + 30*m^2*n + 50*m*n^3 + 10*m^3*n + 6*m^2 + 4*m^3 + m^4
+ 35*n^2 + 50*n^3 + 24*n^4 + 35*m^2*n^2 + 1) + (c^2*x*x^(4*n)*(d*x)^m*(3*m
+ 6*n + 12*m*n + 11*m*n^2 + 6*m^2*n + 3*m^2 + m^3 + 11*n^2 + 6*n^3 + 1))/(4
*m + 10*n + 30*m*n + 70*m*n^2 + 30*m^2*n + 50*m*n^3 + 10*m^3*n + 6*m^2 + 4*
m^3 + m^4 + 35*n^2 + 50*n^3 + 24*n^4 + 35*m^2*n^2 + 1) + (2*a*b*x*x^n*(d*x)
^m*(3*m + 9*n + 18*m*n + 26*m*n^2 + 9*m^2*n + 3*m^2 + m^3 + 26*n^2 + 24*n^3
 + 1))/(4*m + 10*n + 30*m*n + 70*m*n^2 + 30*m^2*n + 50*m*n^3 + 10*m^3*n + 6
*m^2 + 4*m^3 + m^4 + 35*n^2 + 50*n^3 + 24*n^4 + 35*m^2*n^2 + 1) + (2*b*c*x*
x^(3*n)*(d*x)^m*(3*m + 7*n + 14*m*n + 14*m*n^2 + 7*m^2*n + 3*m^2 + m^3 + 14
*n^2 + 8*n^3 + 1))/(4*m + 10*n + 30*m*n + 70*m*n^2 + 30*m^2*n + 50*m*n^3 +
10*m^3*n + 6*m^2 + 4*m^3 + m^4 + 35*n^2 + 50*n^3 + 24*n^4 + 35*m^2*n^2 + 1)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(a+b*x**n+c*x**(2*n))**2,x)
```

```
[Out] Timed out
```

3.493 $\int (dx)^m \left(a+bx^n+cx^{2n}\right) dx$

Optimal. Leaf size=58

$$\frac{a(dx)^{m+1}}{d(m+1)}+\frac{bx^{n+1}(dx)^m}{m+n+1}+\frac{cx^{2n+1}(dx)^m}{m+2n+1}$$

Rubi [A] time = 0.02, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {14, 20, 30}

$$\frac{a(dx)^{m+1}}{d(m+1)}+\frac{bx^{n+1}(dx)^m}{m+n+1}+\frac{cx^{2n+1}(dx)^m}{m+2n+1}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^m*(a + b*x^n + c*x^(2*n)),x]
```

```
[Out] (b*x^(1 + n)*(d*x)^m)/(1 + m + n) + (c*x^(1 + 2*n)*(d*x)^m)/(1 + m + 2*n) +
 (a*(d*x)^(1 + m))/(d*(1 + m))
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(b^IntPart
[n]*(b*v)^FracPart[n])/(a^IntPart[n]*(a*v)^FracPart[n]), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !
IntegerQ[m + n]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int (dx)^m \left(a+bx^n+cx^{2n}\right) dx &= \int \left(a(dx)^m+bx^n(dx)^m+cx^{2n}(dx)^m\right) dx\\
&= \frac{a(dx)^{1+m}}{d(1+m)}+b\int x^n(dx)^m\, dx+c\int x^{2n}(dx)^m\, dx\\
&= \frac{a(dx)^{1+m}}{d(1+m)}+\left(bx^{-m}(dx)^m\right)\int x^{m+n}\, dx+\left(cx^{-m}(dx)^m\right)\int x^{m+2n}\, dx\\
&= \frac{bx^{1+n}(dx)^m}{1+m+n}+\frac{cx^{1+2n}(dx)^m}{1+m+2n}+\frac{a(dx)^{1+m}}{d(1+m)}
\end{aligned}$$

Mathematica [A] time = 0.07, size = 41, normalized size = 0.71

$$x(dx)^m\left(\frac{a}{m+1}+x^n\left(\frac{b}{m+n+1}+\frac{cx^n}{m+2n+1}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^m*(a + b*x^n + c*x^(2*n)),x]
```

```
[Out] x*(d*x)^m*(a/(1 + m) + x^n*(b/(1 + m + n) + (c*x^n)/(1 + m + 2*n)))
```

IntegrateAlgebraic [F] time = 0.08, size = 0, normalized size = 0.00

$$\int (dx)^m \left(a+bx^n+cx^{2n}\right)\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(d*x)^m*(a + b*x^n + c*x^(2*n)),x]
```

```
[Out] Defer[IntegrateAlgebraic][(d*x)^m*(a + b*x^n + c*x^(2*n)), x]
```

fricas [B] time = 1.28, size = 142, normalized size = 2.45

$$\frac{\left(cm^2+2\,cm+(cm+c)n+c\right)xx^{2\,n}e^{\left(m\log(d)+m\log(x)\right)}+\left(bm^2+2\,bm+2\,(bm+b)n+b\right)xx^{n}e^{\left(m\log(d)+m\log(x)\right)}+\left(am^2+2\,an^2+2\,am+3\,(am+a)n+a\right)xe^{\left(m\log(d)+m\log(x)\right)}}{m^3+2\,(m+1)n^2+3\,m^2+3\left(m^2+2\,m+1\right)n+3\,m+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*x^n+c*x^(2*n)),x, algorithm="fricas")
```

```
[Out] ((c*m^2 + 2*c*m + (c*m + c)*n + c)*x*x^(2*n)*e^(m*log(d) + m*log(x)) + (b*m
^2 + 2*b*m + 2*(b*m + b)*n + b)*x*x^n*e^(m*log(d) + m*log(x)) + (a*m^2 + 2*
a*n^2 + 2*a*m + 3*(a*m + a)*n + a)*x*e^(m*log(d) + m*log(x)))/(m^3 + 2*(m +
 1)*n^2 + 3*m^2 + 3*(m^2 + 2*m + 1)*n + 3*m + 1)
```

giac [B] time = 0.40, size = 557, normalized size = 9.60

$$\frac{cm^2xx^{2n}e^{\left(m\log(d)+m\log(x)\right)}+cmnxx^{2n}e^{\left(m\log(d)+m\log(x)\right)}+bm^2xx^ne^{\left(m\log(d)+m\log(x)\right)}+cm^2xx^ne^{\left(m\log(d)+m\log(x)\right)}+2\,bmnxx^ne^{\left(m\log(d)+m\log(x)\right)}+cmnxx^ne^{\left(m\log(d)+m\log(x)\right)}+am^2xe^{\left(m\log(d)+m\log(x)\right)}+bm^2xe^{\left(m\log(d)+m\log(x)\right)}+cm^2xe^{\left(m\log(d)+m\log(x)\right)}+3\,amnxe^{\left(m\log(d)+m\log(x)\right)}+2\,bmnxe^{\left(m\log(d)+m\log(x)\right)}+cmnxe^{\left(m\log(d)+m\log(x)\right)}+2\,an^2xe^{\left(m\log(d)+m\log(x)\right)}+2\,cmxx^{2n}e^{\left(m\log(d)+m\log(x)\right)}+cnxx^{2n}e^{\left(m\log(d)+m\log(x)\right)}+2\,bmxx^ne^{\left(m\log(d)+m\log(x)\right)}+2\,cmxx^ne^{\left(m\log(d)+m\log(x)\right)}+2\,bnxx^ne^{\left(m\log(d)+m\log(x)\right)}+cnxx^ne^{\left(m\log(d)+m\log(x)\right)}+2\,amxe^{\left(m\log(d)+m\log(x)\right)}+2\,bmxe^{\left(m\log(d)+m\log(x)\right)}+2\,cmxe^{\left(m\log(d)+m\log(x)\right)}+3\,anxe^{\left(m\log(d)+m\log(x)\right)}+2\,bnxe^{\left(m\log(d)+m\log(x)\right)}+cnxe^{\left(m\log(d)+m\log(x)\right)}+cxx^{2n}e^{\left(m\log(d)+m\log(x)\right)}+bxx^ne^{\left(m\log(d)+m\log(x)\right)}+cxx^ne^{\left(m\log(d)+m\log(x)\right)}+axe^{\left(m\log(d)+m\log(x)\right)}+bxe^{\left(m\log(d)+m\log(x)\right)}+cxe^{\left(m\log(d)+m\log(x)\right)}}{m^3+3\,m^2n+2\,mn^2+3\,m^2+6\,mn+2\,n^2+3\,m+3\,n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*x^n+c*x^(2*n)),x, algorithm="giac")
```

```
[Out] (c*m^2*x*x^(2*n)*e^(m*log(d) + m*log(x)) + c*m*n*x*x^(2*n)*e^(m*log(d) + m*
log(x)) + b*m^2*x*x^n*e^(m*log(d) + m*log(x)) + c*m^2*x*x^n*e^(m*log(d) + m
*log(x)) + 2*b*m*n*x*x^n*e^(m*log(d) + m*log(x)) + c*m*n*x*x^n*e^(m*log(d)
+ m*log(x)) + a*m^2*x*e^(m*log(d) + m*log(x)) + b*m^2*x*e^(m*log(d) + m*log
(x)) + c*m^2*x*e^(m*log(d) + m*log(x)) + 3*a*m*n*x*e^(m*log(d) + m*log(x))
+ 2*b*m*n*x*e^(m*log(d) + m*log(x)) + c*m*n*x*e^(m*log(d) + m*log(x)) + 2*a
*n^2*x*e^(m*log(d) + m*log(x)) + 2*c*m*x*x^(2*n)*e^(m*log(d) + m*log(x)) +
c*n*x*x^(2*n)*e^(m*log(d) + m*log(x)) + 2*b*m*x*x^n*e^(m*log(d) + m*log(x))
 + 2*c*m*x*x^n*e^(m*log(d) + m*log(x)) + 2*b*n*x*x^n*e^(m*log(d) + m*log(x)
) + c*n*x*x^n*e^(m*log(d) + m*log(x)) + 2*a*m*x*e^(m*log(d) + m*log(x)) + 2
*b*m*x*e^(m*log(d) + m*log(x)) + 2*c*m*x*e^(m*log(d) + m*log(x)) + 3*a*n*x*
e^(m*log(d) + m*log(x)) + 2*b*n*x*e^(m*log(d) + m*log(x)) + c*n*x*e^(m*log(
d) + m*log(x)) + c*x*x^(2*n)*e^(m*log(d) + m*log(x)) + b*x*x^n*e^(m*log(d)
+ m*log(x)) + c*x*x^n*e^(m*log(d) + m*log(x)) + a*x*e^(m*log(d) + m*log(x))
 + b*x*e^(m*log(d) + m*log(x)) + c*x*e^(m*log(d) + m*log(x)))/(m^3 + 3*m^2*
n + 2*m*n^2 + 3*m^2 + 6*m*n + 2*n^2 + 3*m + 3*n + 1)
```

maple [C] time = 0.05, size = 205, normalized size = 3.53

$$\frac{\left(b\,m^2x^n+2bmn\,x^n+c\,m^2x^{2n}+cmn\,x^{2n}+a\,m^2+3amn+2a\,n^2+2bm\,x^n+2bn\,x^n+2cm\,x^{2n}+cn\,x^{2n}+2am+3an+b\,x^n+c\,x^{2n}+a\right)x\,\mathrm{e}^{\frac{\left(-i\pi\,\mathrm{csgn}(id)\mathrm{csgn}(ix)\mathrm{csgn}(idx)+i\pi\,\mathrm{csgn}(id)\mathrm{csgn}(idx)^2+i\pi\,\mathrm{csgn}(ix)\mathrm{csgn}(idx)^2-i\pi\mathrm{csgn}(idx)^3+2\ln(d)+2\ln(x)\right)m}{2}}}{(m+1)\,(m+n+1)\,(m+2n+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(b*x^n+c*x^(2*n)+a),x)
```

```
[Out] x*(c*m^2*(x^n)^2+c*m*n*(x^n)^2+b*m^2*x^n+2*b*m*n*x^n+2*m*c*(x^n)^2+c*(x^n)^
2*n+a*m^2+3*a*m*n+2*a*n^2+2*b*m*x^n+2*b*x^n*n+c*(x^n)^2+2*a*m+3*a*n+b*x^n+a
```

```
)/(m+1)/(m+n+1)/(m+2*n+1)*exp(1/2*(-I*Pi*csgn(I*d)*csgn(I*x)*csgn(I*d*x)+I*
Pi*csgn(I*d)*csgn(I*d*x)^2+I*Pi*csgn(I*x)*csgn(I*d*x)^2-I*Pi*csgn(I*d*x)^3+
2*ln(d)+2*ln(x))*m)
```

maxima [A] time = 1.12, size = 65, normalized size = 1.12

$$\frac{cd^m xe^{\left(m\log(x)+2\,n\log(x)\right)}}{m+2\,n+1}+\frac{bd^m xe^{\left(m\log(x)+n\log(x)\right)}}{m+n+1}+\frac{(dx)^{m+1}\,a}{d(m+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*x^n+c*x^(2*n)),x, algorithm="maxima")
```

```
[Out] c*d^m*x*e^(m*log(x) + 2*n*log(x))/(m + 2*n + 1) + b*d^m*x*e^(m*log(x) + n*l
og(x))/(m + n + 1) + (d*x)^(m + 1)*a/(d*(m + 1))
```

mupad [B] time = 1.41, size = 83, normalized size = 1.43

$$(d\,x)^m\,\left(\frac{a\,x}{m+1}+\frac{b\,x\,x^n\,\left(m+2\,n+1\right)}{m^2+3\,m\,n+2\,m+2\,n^2+3\,n+1}+\frac{c\,x\,x^{2\,n}\,\left(m+n+1\right)}{m^2+3\,m\,n+2\,m+2\,n^2+3\,n+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(a + b*x^n + c*x^(2*n)),x)
```

```
[Out] (d*x)^m*((a*x)/(m + 1) + (b*x*x^n*(m + 2*n + 1))/(2*m + 3*n + 3*m*n + m^2 +
 2*n^2 + 1) + (c*x*x^(2*n)*(m + n + 1))/(2*m + 3*n + 3*m*n + m^2 + 2*n^2 +
1))
```

sympy [A] time = 43.00, size = 1239, normalized size = 21.36

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(a+b*x**n+c*x**(2*n)),x)
```

```
[Out] Piecewise(((a + b + c)*log(x)/d, Eq(m, -1) & Eq(n, 0)), ((a*log(x) + b*x**n
/n + c*x**(2*n)/(2*n))/d, Eq(m, -1)), (a*Piecewise((log(x), Eq(n, 0)), (-x*
*(-2*n)*(0**(1/n))**(-2*n)/(2*n), Eq(d, 0**(1/n))), (-d**(-2*n)*x**(-2*n)/(
2*n), True))/d + b*Piecewise((log(x), Eq(n, 0)), (-x**n/(2*0**(1/n)*zoo**(1
/n)*n*x**(2*n)*(0**(1/n))**(2*n) - n*x**(2*n)*(0**(1/n))**(2*n)), Eq(d, 0**
(1/n))), (-d**(-2*n)*x**(-n)/n, True))/d + c*Piecewise((d**(-2*n)*log(x), A
bs(x) < 1), (-d**(-2*n)*log(1/x), 1/Abs(x) < 1), (-d**(-2*n)*meijerg(((), (
1, 1)), ((0, 0), ()), x) + d**(-2*n)*meijerg(((1, 1), ()), ((), (0, 0)), x)
, True))/d, Eq(m, -2*n - 1)), (a*Piecewise((log(x), Eq(n, 0)), (-x**(-n)*(0
**(1/n))**(-n)/n, Eq(d, 0**(1/n))), (-d**(-n)*x**(-n)/n, True))/d + b*Piece
wise((d**(-n)*log(x), Abs(x) < 1), (-d**(-n)*log(1/x), 1/Abs(x) < 1), (-d**
(-n)*meijerg(((), (1, 1)), ((0, 0), ()), x) + d**(-n)*meijerg(((1, 1), ()),
 ((), (0, 0)), x), True))/d + c*Piecewise((log(x), Eq(n, 0)), (-x**(2*n)/(0
**(1/n)*zoo**(1/n)*n*x**n*(0**(1/n))**n - 2*n*x**n*(0**(1/n))**n), Eq(d, 0*
*(1/n))), (d**(-n)*x**n/n, True))/d, Eq(m, -n - 1)), (a*d**m*m**2*x*x**m/(m
**3 + 3*m**2*n + 3*m**2 + 2*m*n**2 + 6*m*n + 3*m + 2*n**2 + 3*n + 1) + 3*a*
d**m*m*n*x*x**m/(m**3 + 3*m**2*n + 3*m**2 + 2*m*n**2 + 6*m*n + 3*m + 2*n**2
 + 3*n + 1) + 2*a*d**m*m*x*x**m/(m**3 + 3*m**2*n + 3*m**2 + 2*m*n**2 + 6*m*
n + 3*m + 2*n**2 + 3*n + 1) + 2*a*d**m*n**2*x*x**m/(m**3 + 3*m**2*n + 3*m**
2 + 2*m*n**2 + 6*m*n + 3*m + 2*n**2 + 3*n + 1) + 3*a*d**m*n*x*x**m/(m**3 +
3*m**2*n + 3*m**2 + 2*m*n**2 + 6*m*n + 3*m + 2*n**2 + 3*n + 1) + a*d**m*x*x
**m/(m**3 + 3*m**2*n + 3*m**2 + 2*m*n**2 + 6*m*n + 3*m + 2*n**2 + 3*n + 1)
+ b*d**m*m**2*x*x**m*x**n/(m**3 + 3*m**2*n + 3*m**2 + 2*m*n**2 + 6*m*n + 3*
m + 2*n**2 + 3*n + 1) + 2*b*d**m*m*n*x*x**m*x**n/(m**3 + 3*m**2*n + 3*m**2
+ 2*m*n**2 + 6*m*n + 3*m + 2*n**2 + 3*n + 1) + 2*b*d**m*m*x*x**m*x**n/(m**3
 + 3*m**2*n + 3*m**2 + 2*m*n**2 + 6*m*n + 3*m + 2*n**2 + 3*n + 1) + 2*b*d**
```

```
m*n*x*x**m*x**n/(m**3 + 3*m**2*n + 3*m**2 + 2*m*n**2 + 6*m*n + 3*m + 2*n**2
 + 3*n + 1) + b*d**m*x*x**m*x**n/(m**3 + 3*m**2*n + 3*m**2 + 2*m*n**2 + 6*m
*n + 3*m + 2*n**2 + 3*n + 1) + c*d**m*m**2*x*x**m*x**(2*n)/(m**3 + 3*m**2*n
 + 3*m**2 + 2*m*n**2 + 6*m*n + 3*m + 2*n**2 + 3*n + 1) + c*d**m*m*n*x*x**m*
x**(2*n)/(m**3 + 3*m**2*n + 3*m**2 + 2*m*n**2 + 6*m*n + 3*m + 2*n**2 + 3*n
+ 1) + 2*c*d**m*m*x*x**m*x**(2*n)/(m**3 + 3*m**2*n + 3*m**2 + 2*m*n**2 + 6*
m*n + 3*m + 2*n**2 + 3*n + 1) + c*d**m*n*x*x**m*x**(2*n)/(m**3 + 3*m**2*n +
 3*m**2 + 2*m*n**2 + 6*m*n + 3*m + 2*n**2 + 3*n + 1) + c*d**m*x*x**m*x**(2*
n)/(m**3 + 3*m**2*n + 3*m**2 + 2*m*n**2 + 6*m*n + 3*m + 2*n**2 + 3*n + 1),
True))
```

3.494 $\int (d+ex)^3 \left(a+b(d+ex)^2+c(d+ex)^4\right) \, dx$

Optimal. Leaf size=46

$$\frac{a(d+ex)^4}{4e}+\frac{b(d+ex)^6}{6e}+\frac{c(d+ex)^8}{8e}$$

Rubi [A] time = 0.05, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {1142, 14}

$$\frac{a(d+ex)^4}{4e}+\frac{b(d+ex)^6}{6e}+\frac{c(d+ex)^8}{8e}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^3*(a + b*(d + e*x)^2 + c*(d + e*x)^4),x]
```

```
[Out] (a*(d + e*x)^4)/(4*e) + (b*(d + e*x)^6)/(6*e) + (c*(d + e*x)^8)/(8*e)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 1142

```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Di
st[u^m/(Coefficient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p,
 x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

Rubi steps

$$\begin{aligned}
\int (d+ex)^3 \left(a+b(d+ex)^2+c(d+ex)^4\right) \, dx &= \frac{\text{Subst}\left(\int x^3 \left(a+bx^2+cx^4\right) \, dx,x,d+ex\right)}{e}\\
&= \frac{\text{Subst}\left(\int \left(ax^3+bx^5+cx^7\right) \, dx,x,d+ex\right)}{e}\\
&= \frac{a(d+ex)^4}{4e}+\frac{b(d+ex)^6}{6e}+\frac{c(d+ex)^8}{8e}
\end{aligned}$$

Mathematica [B] time = 0.04, size = 150, normalized size = 3.26

$$\frac{1}{4}e^3x^4\left(a+10bd^2+35cd^4\right)+\frac{1}{3}de^2x^3\left(3a+10bd^2+21cd^4\right)+\frac{1}{2}d^2ex^2\left(3a+5bd^2+7cd^4\right)+d^3x\left(a+bd^2+cd^4\right)+\frac{1}{6}e^5x^6\left(b+21cd^2\right)+de^4x^5\left(b+7cd^2\right)+cde^6x^7+\frac{1}{8}ce^7x^8$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^3*(a + b*(d + e*x)^2 + c*(d + e*x)^4),x]
```

```
[Out] d^3*(a + b*d^2 + c*d^4)*x + (d^2*(3*a + 5*b*d^2 + 7*c*d^4)*e*x^2)/2 + (d*(3
*a + 10*b*d^2 + 21*c*d^4)*e^2*x^3)/3 + ((a + 10*b*d^2 + 35*c*d^4)*e^3*x^4)/
4 + d*(b + 7*c*d^2)*e^4*x^5 + ((b + 21*c*d^2)*e^5*x^6)/6 + c*d*e^6*x^7 + (c
*e^7*x^8)/8
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (d+ex)^3 \left(a+b(d+ex)^2+c(d+ex)^4\right) \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(d + e*x)^3*(a + b*(d + e*x)^2 + c*(d + e*x)^4),x]
```

```
[Out] IntegrateAlgebraic[(d + e*x)^3*(a + b*(d + e*x)^2 + c*(d + e*x)^4), x]
```

fricas [B] time = 1.16, size = 175, normalized size = 3.80

$$\frac{1}{8}x^8e^7c + x^7e^6dc + \frac{7}{2}x^6e^5d^2c + 7x^5e^4d^3c + \frac{35}{4}x^4e^3d^4c + \frac{1}{6}x^6e^5b + 7x^3e^2d^5c + x^5e^4db + \frac{7}{2}x^2ed^6c + \frac{5}{2}x^4e^3d^2b + xd^7c + \frac{10}{3}x^3e^2d^3b + \frac{5}{2}x^2ed^4b + \frac{1}{4}x^4e^3a + xd^5b + x^3e^2da + \frac{3}{2}x^2ed^2a + xd^3a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="fricas")
```

```
[Out] 1/8*x^8*e^7*c + x^7*e^6*d*c + 7/2*x^6*e^5*d^2*c + 7*x^5*e^4*d^3*c + 35/4*x^
4*e^3*d^4*c + 1/6*x^6*e^5*b + 7*x^3*e^2*d^5*c + x^5*e^4*d*b + 7/2*x^2*e*d^6
*c + 5/2*x^4*e^3*d^2*b + x*d^7*c + 10/3*x^3*e^2*d^3*b + 5/2*x^2*e*d^4*b + 1
/4*x^4*e^3*a + x*d^5*b + x^3*e^2*d*a + 3/2*x^2*e*d^2*a + x*d^3*a
```

giac [B] time = 0.41, size = 169, normalized size = 3.67

$$\frac{1}{2}\left(x^2e + 2\,dx\right)cd^6 + \frac{3}{4}\left(x^2e + 2\,dx\right)^2cd^4e + \frac{1}{2}\left(x^2e + 2\,dx\right)^3cd^2e^2 + \frac{1}{2}\left(x^2e + 2\,dx\right)bd^4 + \frac{1}{8}\left(x^2e + 2\,dx\right)^4ce^3 + \frac{1}{2}\left(x^2e + 2\,dx\right)^2bd^2e + \frac{1}{6}\left(x^2e + 2\,dx\right)^3be^2 + \frac{1}{2}\left(x^2e + 2\,dx\right)ad^2 + \frac{1}{4}\left(x^2e + 2\,dx\right)^2ae$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="giac")
```

```
[Out] 1/2*(x^2*e + 2*d*x)*c*d^6 + 3/4*(x^2*e + 2*d*x)^2*c*d^4*e + 1/2*(x^2*e + 2*
d*x)^3*c*d^2*e^2 + 1/2*(x^2*e + 2*d*x)*b*d^4 + 1/8*(x^2*e + 2*d*x)^4*c*e^3
+ 1/2*(x^2*e + 2*d*x)^2*b*d^2*e + 1/6*(x^2*e + 2*d*x)^3*b*e^2 + 1/2*(x^2*e
+ 2*d*x)*a*d^2 + 1/4*(x^2*e + 2*d*x)^2*a*e
```

maple [B] time = 0.00, size = 298, normalized size = 6.48

$$\frac{c\,e^7x^8}{8} + cd\,e^6x^7 + \frac{\left(15c\,d^2e^5 + \left(6c\,d^2e^2 + b\,e^2\right)e^3\right)x^6}{6} + \frac{\left(13c\,d^3e^4 + 3\left(6c\,d^2e^2 + b\,e^2\right)d\,e^2 + \left(4c\,d^3e + 2deb\right)e^3\right)x^5}{5} + \left(c\,d^4 + b\,d^2 + a\right)d^3x + \frac{\left(4c\,d^4e^3 + 3\left(6c\,d^2e^2 + b\,e^2\right)d^2e + 3\left(4c\,d^3e + 2deb\right)d\,e^2 + \left(c\,d^4 + b\,d^2 + a\right)e^3\right)x^4}{4} + \frac{\left(\left(6c\,d^2e^2 + b\,e^2\right)d^3 + 3\left(4c\,d^3e + 2deb\right)d^2e + 3\left(c\,d^4 + b\,d^2 + a\right)d\,e^2\right)x^3}{3} + \frac{\left(\left(4c\,d^3e + 2deb\right)d^3 + 3\left(c\,d^4 + b\,d^2 + a\right)d^2e\right)x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3*(a+b*(e*x+d)^2+c*(e*x+d)^4),x)
```

```
[Out] 1/8*e^7*c*x^8+d*e^6*c*x^7+1/6*(15*d^2*e^5*c+e^3*(6*c*d^2*e^2+b*e^2))*x^6+1/
5*(13*d^3*c*e^4+3*d*e^2*(6*c*d^2*e^2+b*e^2)+e^3*(4*c*d^3*e+2*b*d*e))*x^5+1/
4*(4*d^4*c*e^3+3*d^2*e*(6*c*d^2*e^2+b*e^2)+3*d*e^2*(4*c*d^3*e+2*b*d*e)+e^3*
(c*d^4+b*d^2+a))*x^4+1/3*(d^3*(6*c*d^2*e^2+b*e^2)+3*d^2*e*(4*c*d^3*e+2*b*d*
e)+3*d*e^2*(c*d^4+b*d^2+a))*x^3+1/2*(d^3*(4*c*d^3*e+2*b*d*e)+3*d^2*e*(c*d^4
+b*d^2+a))*x^2+d^3*(c*d^4+b*d^2+a)*x
```

maxima [B] time = 1.02, size = 142, normalized size = 3.09

$$\frac{1}{8}\,ce^7x^8 + cde^6x^7 + \frac{1}{6}\left(21\,cd^2 + b\right)e^5x^6 + \left(7\,cd^3 + bd\right)e^4x^5 + \frac{1}{4}\left(35\,cd^4 + 10\,bd^2 + a\right)e^3x^4 + \frac{1}{3}\left(21\,cd^5 + 10\,bd^3 + 3\,ad\right)e^2x^3 + \frac{1}{2}\left(7\,cd^6 + 5\,bd^4 + 3\,ad^2\right)ex^2 + \left(cd^7 + bd^5 + ad^3\right)x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="maxima")
```

```
[Out] 1/8*c*e^7*x^8 + c*d*e^6*x^7 + 1/6*(21*c*d^2 + b)*e^5*x^6 + (7*c*d^3 + b*d)*
e^4*x^5 + 1/4*(35*c*d^4 + 10*b*d^2 + a)*e^3*x^4 + 1/3*(21*c*d^5 + 10*b*d^3
+ 3*a*d)*e^2*x^3 + 1/2*(7*c*d^6 + 5*b*d^4 + 3*a*d^2)*e*x^2 + (c*d^7 + b*d^5
 + a*d^3)*x
```

mupad [B] time = 0.08, size = 141, normalized size = 3.07

$$x\left(c\,d^7 + b\,d^5 + a\,d^3\right) + \frac{e^5\,x^6\,\left(21\,c\,d^2 + b\right)}{6} + \frac{c\,e^7\,x^8}{8} + \frac{e^3\,x^4\,\left(35\,c\,d^4 + 10\,b\,d^2 + a\right)}{4} + \frac{d^2\,e\,x^2\,\left(7\,c\,d^4 + 5\,b\,d^2 + 3\,a\right)}{2} + \frac{d\,e^2\,x^3\,\left(21\,c\,d^4 + 10\,b\,d^2 + 3\,a\right)}{3} + d\,e^4\,x^5\,\left(7\,c\,d^2 + b\right) + c\,d\,e^6\,x^7$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^3*(a + b*(d + e*x)^2 + c*(d + e*x)^4),x)
```

```
[Out] x*(a*d^3 + b*d^5 + c*d^7) + (e^5*x^6*(b + 21*c*d^2))/6 + (c*e^7*x^8)/8 + (e
^3*x^4*(a + 10*b*d^2 + 35*c*d^4))/4 + (d^2*e*x^2*(3*a + 5*b*d^2 + 7*c*d^4))
/2 + (d*e^2*x^3*(3*a + 10*b*d^2 + 21*c*d^4))/3 + d*e^4*x^5*(b + 7*c*d^2) +
c*d*e^6*x^7
```

sympy [B] time = 0.11, size = 178, normalized size = 3.87

$$cde^6x^7 + \frac{ce^7x^8}{8} + x^6\left(\frac{be^5}{6} + \frac{7cd^2e^5}{2}\right) + x^5\left(bde^4 + 7cd^3e^4\right) + x^4\left(\frac{ae^3}{4} + \frac{5bd^2e^3}{2} + \frac{35cd^4e^3}{4}\right) + x^3\left(ade^2 + \frac{10bd^3e^2}{3} + 7cd^5e^2\right) + x^2\left(\frac{3ad^2e}{2} + \frac{5bd^4e}{2} + \frac{7cd^6e}{2}\right) + x\left(ad^3 + bd^5 + cd^7\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3*(a+b*(e*x+d)**2+c*(e*x+d)**4),x)
```

```
[Out] c*d*e**6*x**7 + c*e**7*x**8/8 + x**6*(b*e**5/6 + 7*c*d**2*e**5/2) + x**5*(b
*d*e**4 + 7*c*d**3*e**4) + x**4*(a*e**3/4 + 5*b*d**2*e**3/2 + 35*c*d**4*e**
3/4) + x**3*(a*d*e**2 + 10*b*d**3*e**2/3 + 7*c*d**5*e**2) + x**2*(3*a*d**2*
e/2 + 5*b*d**4*e/2 + 7*c*d**6*e/2) + x*(a*d**3 + b*d**5 + c*d**7)
```

3.495 $\int (d+ex)^3 \left(a+b(d+ex)^2+c(d+ex)^4\right)^2 \, dx$

Optimal. Leaf size=89

$$\frac{a^2(d+ex)^4}{4e}+\frac{\left(2ac+b^2\right)(d+ex)^8}{8e}+\frac{ab(d+ex)^6}{3e}+\frac{bc(d+ex)^{10}}{5e}+\frac{c^2(d+ex)^{12}}{12e}$$

Rubi [A] time = 0.19, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {1142, 1114, 631}

$$\frac{a^2(d+ex)^4}{4e}+\frac{\left(2ac+b^2\right)(d+ex)^8}{8e}+\frac{ab(d+ex)^6}{3e}+\frac{bc(d+ex)^{10}}{5e}+\frac{c^2(d+ex)^{12}}{12e}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^3*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2,x]
```

```
[Out] (a^2*(d + e*x)^4)/(4*e) + (a*b*(d + e*x)^6)/(3*e) + ((b^2 + 2*a*c)*(d + e*x
)^8)/(8*e) + (b*c*(d + e*x)^10)/(5*e) + (c^2*(d + e*x)^12)/(12*e)
```

Rule 631

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbo
l] :> Int[ExpandIntegrand[(d + e*x)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a
, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || Eq
Q[a, 0])
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 1142

```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Di
st[u^m/(Coefficient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p,
 x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

Rubi steps

$$\begin{aligned}\int (d+ex)^3 \left(a+b(d+ex)^2+c(d+ex)^4\right)^2 \, dx &= \frac{\text{Subst}\left(\int x^3 \left(a+bx^2+cx^4\right)^2 \, dx, x, d+ex\right)}{e}\\ &= \frac{\text{Subst}\left(\int x \left(a+bx+cx^2\right)^2 \, dx, x, (d+ex)^2\right)}{2e}\\ &= \frac{\text{Subst}\left(\int \left(a^2x+2abx^2+\left(b^2+2ac\right)x^3+2bcx^4+c^2x^5\right) dx, x, (d\right.}{2e}\\ &= \frac{a^2(d+ex)^4}{4e}+\frac{ab(d+ex)^6}{3e}+\frac{\left(b^2+2ac\right)(d+ex)^8}{8e}+\frac{bc(d+ex)^{10}}{5e}\end{aligned}$$

Mathematica [B] time = 0.11, size = 401, normalized size = 4.51

$$\frac{1}{4}e^3x^4\left(a^2+20abd^2+70acd^4+35b^2d^4+168bcd^6+165c^2d^8\right)+\frac{1}{3}de^2x^3\left(3a^2+20abd^2+42acd^4+21b^2d^4+72bcd^6+55c^2d^8\right)+\frac{1}{2}d^2ex^2\left(3a^2+10abd^2+14acd^4+7b^2d^4+18bcd^6+11c^2d^8\right)+\frac{1}{8}e^7x^8\left(2ac+b^2+72bcd^2+330c^2d^4\right)+de^6x^7\left(2ac+b^2+24bcd^2+66c^2d^4\right)+\frac{1}{6}e^5x^6\left(2ab+42acd^2+21b^2d^2+252bcd^4+462c^2d^6\right)+\frac{1}{5}de^4x^5\left(10ab+70acd^2+35b^2d^2+252bcd^4+330c^2d^6\right)+d^3x\left(a+bd^2+cd^4\right)^2+\frac{1}{10}ce^9x^{10}\left(2b+55cd^2\right)+\frac{1}{3}cde^8x^9\left(6b+55cd^2\right)+c^2de^{10}x^{11}+\frac{1}{12}c^2e^{11}x^{12}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^3*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2,x]
```

```
[Out] d^3*(a + b*d^2 + c*d^4)^2*x + (d^2*(3*a^2 + 10*a*b*d^2 + 7*b^2*d^4 + 14*a*c
*d^4 + 18*b*c*d^6 + 11*c^2*d^8)*e*x^2)/2 + (d*(3*a^2 + 20*a*b*d^2 + 21*b^2*
d^4 + 42*a*c*d^4 + 72*b*c*d^6 + 55*c^2*d^8)*e^2*x^3)/3 + ((a^2 + 20*a*b*d^2
 + 35*b^2*d^4 + 70*a*c*d^4 + 168*b*c*d^6 + 165*c^2*d^8)*e^3*x^4)/4 + (d*(10
*a*b + 35*b^2*d^2 + 70*a*c*d^2 + 252*b*c*d^4 + 330*c^2*d^6)*e^4*x^5)/5 + ((
2*a*b + 21*b^2*d^2 + 42*a*c*d^2 + 252*b*c*d^4 + 462*c^2*d^6)*e^5*x^6)/6 + d
*(b^2 + 2*a*c + 24*b*c*d^2 + 66*c^2*d^4)*e^6*x^7 + ((b^2 + 2*a*c + 72*b*c*d
^2 + 330*c^2*d^4)*e^7*x^8)/8 + (c*d*(6*b + 55*c*d^2)*e^8*x^9)/3 + (c*(2*b +
 55*c*d^2)*e^9*x^10)/10 + c^2*d*e^10*x^11 + (c^2*e^11*x^12)/12
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (d+ex)^3\left(a+b(d+ex)^2+c(d+ex)^4\right)^2\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(d + e*x)^3*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2,x]
```

```
[Out] IntegrateAlgebraic[(d + e*x)^3*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2, x]
```

fricas [B] time = 1.14, size = 571, normalized size = 6.42

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="fricas")
```

```
[Out] 1/12*x^12*e^11*c^2 + x^11*e^10*d*c^2 + 11/2*x^10*e^9*d^2*c^2 + 55/3*x^9*e^8
*d^3*c^2 + 165/4*x^8*e^7*d^4*c^2 + 1/5*x^10*e^9*c*b + 66*x^7*e^6*d^5*c^2 +
2*x^9*e^8*d*c*b + 77*x^6*e^5*d^6*c^2 + 9*x^8*e^7*d^2*c*b + 66*x^5*e^4*d^7*c
^2 + 24*x^7*e^6*d^3*c*b + 165/4*x^4*e^3*d^8*c^2 + 42*x^6*e^5*d^4*c*b + 1/8*
x^8*e^7*b^2 + 1/4*x^8*e^7*c*a + 55/3*x^3*e^2*d^9*c^2 + 252/5*x^5*e^4*d^5*c*
b + x^7*e^6*d*b^2 + 2*x^7*e^6*d*c*a + 11/2*x^2*e*d^10*c^2 + 42*x^4*e^3*d^6*
c*b + 7/2*x^6*e^5*d^2*b^2 + 7*x^6*e^5*d^2*c*a + x*d^11*c^2 + 24*x^3*e^2*d^7
*c*b + 7*x^5*e^4*d^3*b^2 + 14*x^5*e^4*d^3*c*a + 9*x^2*e*d^8*c*b + 35/4*x^4*
e^3*d^4*b^2 + 35/2*x^4*e^3*d^4*c*a + 1/3*x^6*e^5*b*a + 2*x*d^9*c*b + 7*x^3*
e^2*d^5*b^2 + 14*x^3*e^2*d^5*c*a + 2*x^5*e^4*d*b*a + 7/2*x^2*e*d^6*b^2 + 7*
x^2*e*d^6*c*a + 5*x^4*e^3*d^2*b*a + x*d^7*b^2 + 2*x*d^7*c*a + 20/3*x^3*e^2*
d^3*b*a + 5*x^2*e*d^4*b*a + 1/4*x^4*e^3*a^2 + 2*x*d^5*b*a + x^3*e^2*d*a^2 +
 3/2*x^2*e*d^2*a^2 + x*d^3*a^2
```

giac [B] time = 0.41, size = 493, normalized size = 5.54

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="giac")
```

```
[Out] 1/2*(x^2*e + 2*d*x)*c^2*d^10 + 5/4*(x^2*e + 2*d*x)^2*c^2*d^8*e + 5/3*(x^2*e
 + 2*d*x)^3*c^2*d^6*e^2 + (x^2*e + 2*d*x)*b*c*d^8 + 5/4*(x^2*e + 2*d*x)^4*c
^2*d^4*e^3 + 2*(x^2*e + 2*d*x)^2*b*c*d^6*e + 1/2*(x^2*e + 2*d*x)^5*c^2*d^2*
e^4 + 2*(x^2*e + 2*d*x)^3*b*c*d^4*e^2 + 1/2*(x^2*e + 2*d*x)*b^2*d^6 + (x^2*
e + 2*d*x)*a*c*d^6 + 1/12*(x^2*e + 2*d*x)^6*c^2*e^5 + (x^2*e + 2*d*x)^4*b*c
*d^2*e^3 + 3/4*(x^2*e + 2*d*x)^2*b^2*d^4*e + 3/2*(x^2*e + 2*d*x)^2*a*c*d^4*
e + 1/5*(x^2*e + 2*d*x)^5*b*c*e^4 + 1/2*(x^2*e + 2*d*x)^3*b^2*d^2*e^2 + (x^
2*e + 2*d*x)^3*a*c*d^2*e^2 + (x^2*e + 2*d*x)*a*b*d^4 + 1/8*(x^2*e + 2*d*x)^
```

```
4*b^2*e^3 + 1/4*(x^2*e + 2*d*x)^4*a*c*e^3 + (x^2*e + 2*d*x)^2*a*b*d^2*e + 1
/3*(x^2*e + 2*d*x)^3*a*b*e^2 + 1/2*(x^2*e + 2*d*x)*a^2*d^2 + 1/4*(x^2*e + 2
*d*x)^2*a^2*e
```

maple [B] time = 0.00, size = 1314, normalized size = 14.76

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3*(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x)
```

```
[Out] 1/12*e^11*c^2*x^12+d*e^10*c^2*x^11+1/10*(27*d^2*e^9*c^2+e^3*(2*(6*c*d^2*e^2
+b*e^2)*c*e^4+16*c^2*d^2*e^6))*x^10+1/9*(25*d^3*c^2*e^8+3*d*e^2*(2*(6*c*d^2
*e^2+b*e^2)*c*e^4+16*c^2*d^2*e^6)+e^3*(2*(4*c*d^3*e+2*b*d*e)*c*e^4+8*(6*c*d
^2*e^2+b*e^2)*c*d*e^3))*x^9+1/8*(8*d^4*c^2*e^7+3*d^2*e*(2*(6*c*d^2*e^2+b*e^
2)*c*e^4+16*c^2*d^2*e^6)+3*d*e^2*(2*(4*c*d^3*e+2*b*d*e)*c*e^4+8*(6*c*d^2*e^
2+b*e^2)*c*d*e^3)+e^3*(2*(c*d^4+b*d^2+a)*c*e^4+8*(4*c*d^3*e+2*b*d*e)*c*d*e^
3+(6*c*d^2*e^2+b*e^2)^2))*x^8+1/7*(d^3*(2*(6*c*d^2*e^2+b*e^2)*c*e^4+16*c^2*
d^2*e^6)+3*d^2*e*(2*(4*c*d^3*e+2*b*d*e)*c*e^4+8*(6*c*d^2*e^2+b*e^2)*c*d*e^3
)+3*d*e^2*(2*(c*d^4+b*d^2+a)*c*e^4+8*(4*c*d^3*e+2*b*d*e)*c*d*e^3+(6*c*d^2*e
^2+b*e^2)^2)+e^3*(8*(c*d^4+b*d^2+a)*c*d*e^3+2*(4*c*d^3*e+2*b*d*e)*(6*c*d^2*
e^2+b*e^2)))*x^7+1/6*(d^3*(2*(4*c*d^3*e+2*b*d*e)*c*e^4+8*(6*c*d^2*e^2+b*e^2
)*c*d*e^3)+3*d^2*e*(2*(c*d^4+b*d^2+a)*c*e^4+8*(4*c*d^3*e+2*b*d*e)*c*d*e^3+(
6*c*d^2*e^2+b*e^2)^2)+3*d*e^2*(8*(c*d^4+b*d^2+a)*c*d*e^3+2*(4*c*d^3*e+2*b*d
*e)*(6*c*d^2*e^2+b*e^2))+e^3*(2*(c*d^4+b*d^2+a)*(6*c*d^2*e^2+b*e^2)+(4*c*d^
3*e+2*b*d*e)^2))*x^6+1/5*(d^3*(2*(c*d^4+b*d^2+a)*c*e^4+8*(4*c*d^3*e+2*b*d*e
)*c*d*e^3+(6*c*d^2*e^2+b*e^2)^2)+3*d^2*e*(8*(c*d^4+b*d^2+a)*c*d*e^3+2*(4*c*
d^3*e+2*b*d*e)*(6*c*d^2*e^2+b*e^2))+3*d*e^2*(2*(c*d^4+b*d^2+a)*(6*c*d^2*e^2
+b*e^2)+(4*c*d^3*e+2*b*d*e)^2)+2*e^3*(c*d^4+b*d^2+a)*(4*c*d^3*e+2*b*d*e))*x
^5+1/4*(d^3*(8*(c*d^4+b*d^2+a)*c*d*e^3+2*(4*c*d^3*e+2*b*d*e)*(6*c*d^2*e^2+b
*e^2))+3*d^2*e*(2*(c*d^4+b*d^2+a)*(6*c*d^2*e^2+b*e^2)+(4*c*d^3*e+2*b*d*e)^2
)+6*d*e^2*(c*d^4+b*d^2+a)*(4*c*d^3*e+2*b*d*e)+e^3*(c*d^4+b*d^2+a)^2)*x^4+1/
3*(d^3*(2*(c*d^4+b*d^2+a)*(6*c*d^2*e^2+b*e^2)+(4*c*d^3*e+2*b*d*e)^2)+6*d^2*
e*(c*d^4+b*d^2+a)*(4*c*d^3*e+2*b*d*e)+3*d*e^2*(c*d^4+b*d^2+a)^2)*x^3+1/2*(2
*d^3*(c*d^4+b*d^2+a)*(4*c*d^3*e+2*b*d*e)+3*d^2*e*(c*d^4+b*d^2+a)^2)*x^2+d^3
*(c*d^4+b*d^2+a)^2*x
```

maxima [B] time = 1.06, size = 403, normalized size = 4.53

$$\frac{1}{12}c^{2}e^{11}x^{12} + c^{2}de^{10}x^{11} + \frac{1}{10}\left(55c^{2}d^{2} + 2bc\right)e^{9}x^{10} + \frac{1}{3}\left(55c^{2}d^{3} + 6bcd\right)e^{8}x^{9} + \frac{1}{8}\left(330c^{2}d^{4} + 72bcd^{2} + b^{2} + 2ac\right)e^{7}x^{8} + \left(66c^{2}d^{5} + 24bcd^{3} + \left(b^{2} + 2ac\right)d\right)e^{6}x^{7} + \frac{1}{6}\left(462c^{2}d^{6} + 252bcd^{4} + 21\left(b^{2} + 2ac\right)d^{2} + 2ab\right)e^{5}x^{6} + \frac{1}{5}\left(330c^{2}d^{7} + 252bcd^{5} + 35\left(b^{2} + 2ac\right)d^{3} + 10abd\right)e^{4}x^{5} + \frac{1}{4}\left(165c^{2}d^{8} + 168bcd^{6} + 35\left(b^{2} + 2ac\right)d^{4} + 20abd^{2} + a^{2}\right)e^{3}x^{4} + \frac{1}{3}\left(55c^{2}d^{9} + 72bcd^{7} + 21\left(b^{2} + 2ac\right)d^{5} + 20abd^{3} + 3a^{2}d\right)e^{2}x^{3} + \frac{1}{2}\left(11c^{2}d^{10} + 18bcd^{8} + 7\left(b^{2} + 2ac\right)d^{6} + 10abd^{4} + 3a^{2}d^{2}\right)ex^{2} + \left(c^{2}d^{11} + 2bcd^{9} + \left(b^{2} + 2ac\right)d^{7} + 2abd^{5} + a^{2}d^{3}\right)x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="maxima")
```

```
[Out] 1/12*c^2*e^11*x^12 + c^2*d*e^10*x^11 + 1/10*(55*c^2*d^2 + 2*b*c)*e^9*x^10 +
 1/3*(55*c^2*d^3 + 6*b*c*d)*e^8*x^9 + 1/8*(330*c^2*d^4 + 72*b*c*d^2 + b^2 +
 2*a*c)*e^7*x^8 + (66*c^2*d^5 + 24*b*c*d^3 + (b^2 + 2*a*c)*d)*e^6*x^7 + 1/6
*(462*c^2*d^6 + 252*b*c*d^4 + 21*(b^2 + 2*a*c)*d^2 + 2*a*b)*e^5*x^6 + 1/5*(
330*c^2*d^7 + 252*b*c*d^5 + 35*(b^2 + 2*a*c)*d^3 + 10*a*b*d)*e^4*x^5 + 1/4*
(165*c^2*d^8 + 168*b*c*d^6 + 35*(b^2 + 2*a*c)*d^4 + 20*a*b*d^2 + a^2)*e^3*x
^4 + 1/3*(55*c^2*d^9 + 72*b*c*d^7 + 21*(b^2 + 2*a*c)*d^5 + 20*a*b*d^3 + 3*a
^2*d)*e^2*x^3 + 1/2*(11*c^2*d^10 + 18*b*c*d^8 + 7*(b^2 + 2*a*c)*d^6 + 10*a*
b*d^4 + 3*a^2*d^2)*e*x^2 + (c^2*d^11 + 2*b*c*d^9 + (b^2 + 2*a*c)*d^7 + 2*a*
b*d^5 + a^2*d^3)*x
```

mupad [B] time = 1.48, size = 383, normalized size = 4.30

$$\frac{e^{7}x^{8}\left(b^{2}+72bcd^{2}+330c^{2}d^{4}+2ac\right)}{8}+\frac{e^{5}x^{6}\left(21b^{2}d^{2}+252bcd^{4}+2ab+462c^{2}d^{6}+42acd^{2}\right)}{6}+\frac{e^{3}x^{4}\left(a^{2}+20abd^{2}+70acd^{4}+35b^{2}d^{4}+168bcd^{6}+165c^{2}d^{8}\right)}{4}+\frac{c^{2}e^{11}x^{12}}{12}+d^{3}x\left(cd^{4}+bd^{2}+a\right)^{2}+\frac{ce^{9}x^{10}\left(55cd^{2}+2b\right)}{10}+c^{2}de^{10}x^{11}+\frac{d^{2}ex^{2}\left(3a^{2}+10abd^{2}+14acd^{4}+7b^{2}d^{4}+18bcd^{6}+11c^{2}d^{8}\right)}{2}+\frac{de^{2}x^{3}\left(3a^{2}+20abd^{2}+42acd^{4}+21b^{2}d^{4}+72bcd^{6}+55c^{2}d^{8}\right)}{3}+de^{6}x^{7}\left(b^{2}+24bcd^{2}+66c^{2}d^{4}+2ac\right)+\frac{de^{4}x^{5}\left(35b^{2}d^{2}+252bcd^{4}+10ab+330c^{2}d^{6}+70acd^{2}\right)}{5}+\frac{cde^{8}x^{9}\left(55cd^{2}+6b\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^3*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2,x)
```

```
[Out] (e^7*x^8*(2*a*c + b^2 + 330*c^2*d^4 + 72*b*c*d^2))/8 + (e^5*x^6*(2*a*b + 21
*b^2*d^2 + 462*c^2*d^6 + 42*a*c*d^2 + 252*b*c*d^4))/6 + (e^3*x^4*(a^2 + 35*
b^2*d^4 + 165*c^2*d^8 + 20*a*b*d^2 + 70*a*c*d^4 + 168*b*c*d^6))/4 + (c^2*e^
11*x^12)/12 + d^3*x*(a + b*d^2 + c*d^4)^2 + (c*e^9*x^10*(2*b + 55*c*d^2))/1
0 + c^2*d*e^10*x^11 + (d^2*e*x^2*(3*a^2 + 7*b^2*d^4 + 11*c^2*d^8 + 10*a*b*d
^2 + 14*a*c*d^4 + 18*b*c*d^6))/2 + (d*e^2*x^3*(3*a^2 + 21*b^2*d^4 + 55*c^2*
d^8 + 20*a*b*d^2 + 42*a*c*d^4 + 72*b*c*d^6))/3 + d*e^6*x^7*(2*a*c + b^2 + 6
6*c^2*d^4 + 24*b*c*d^2) + (d*e^4*x^5*(10*a*b + 35*b^2*d^2 + 330*c^2*d^6 + 7
0*a*c*d^2 + 252*b*c*d^4))/5 + (c*d*e^8*x^9*(6*b + 55*c*d^2))/3
```

sympy [B] time = 0.19, size = 559, normalized size = 6.28

$$c^2de^{10}x^{11} + \frac{c^2e^{11}x^{12}}{12} + x^{10}\left(\frac{bce^9}{5} + \frac{11c^2d^2e^9}{2}\right) + x^9\left(2bcde^8 + \frac{55c^2d^3e^8}{3}\right) + x^8\left(\frac{ace^7}{4} + \frac{b^2e^7}{8} + 9bcd^2e^7 + \frac{165c^2d^4e^7}{4}\right) + x^7\left(2acde^6 + b^2de^6 + 24bcd^3e^6 + 66c^2d^5e^6\right) + x^6\left(\frac{abe^5}{3} + 7acd^2e^5 + \frac{7b^2d^2e^5}{2} + 42bcd^4e^5 + 77c^2d^6e^5\right) + x^5\left(2abde^4 + 14acd^3e^4 + 7b^2d^3e^4 + \frac{252bcd^5e^4}{5} + 66c^2d^7e^4\right) + x^4\left(\frac{a^2e^3}{4} + 5abd^2e^3 + \frac{35acd^4e^3}{2} + \frac{35b^2d^4e^3}{4} + 42bcd^6e^3 + \frac{165c^2d^8e^3}{4}\right) + x^3\left(a^2de^2 + \frac{20abd^3e^2}{3} + 14acd^5e^2 + 7b^2d^5e^2 + 24bcd^7e^2 + \frac{55c^2d^9e^2}{3}\right) + x^2\left(\frac{3a^2d^2e}{2} + 5abd^4e + 7acd^6e + \frac{7b^2d^6e}{2} + 9bcd^8e + \frac{11c^2d^{10}e}{2}\right) + x\left(a^2d^3 + 2abd^5 + 2acd^7 + b^2d^7 + 2bcd^9 + c^2d^{11}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3*(a+b*(e*x+d)**2+c*(e*x+d)**4)**2,x)
```

```
[Out] c**2*d*e**10*x**11 + c**2*e**11*x**12/12 + x**10*(b*c*e**9/5 + 11*c**2*d**2
*e**9/2) + x**9*(2*b*c*d*e**8 + 55*c**2*d**3*e**8/3) + x**8*(a*c*e**7/4 + b
**2*e**7/8 + 9*b*c*d**2*e**7 + 165*c**2*d**4*e**7/4) + x**7*(2*a*c*d*e**6 +
 b**2*d*e**6 + 24*b*c*d**3*e**6 + 66*c**2*d**5*e**6) + x**6*(a*b*e**5/3 + 7
*a*c*d**2*e**5 + 7*b**2*d**2*e**5/2 + 42*b*c*d**4*e**5 + 77*c**2*d**6*e**5)
 + x**5*(2*a*b*d*e**4 + 14*a*c*d**3*e**4 + 7*b**2*d**3*e**4 + 252*b*c*d**5*
e**4/5 + 66*c**2*d**7*e**4) + x**4*(a**2*e**3/4 + 5*a*b*d**2*e**3 + 35*a*c*
d**4*e**3/2 + 35*b**2*d**4*e**3/4 + 42*b*c*d**6*e**3 + 165*c**2*d**8*e**3/4
) + x**3*(a**2*d*e**2 + 20*a*b*d**3*e**2/3 + 14*a*c*d**5*e**2 + 7*b**2*d**5
*e**2 + 24*b*c*d**7*e**2 + 55*c**2*d**9*e**2/3) + x**2*(3*a**2*d**2*e/2 + 5
*a*b*d**4*e + 7*a*c*d**6*e + 7*b**2*d**6*e/2 + 9*b*c*d**8*e + 11*c**2*d**10
*e/2) + x*(a**2*d**3 + 2*a*b*d**5 + 2*a*c*d**7 + b**2*d**7 + 2*b*c*d**9 + c
**2*d**11)
```

3.496 $\int (d+ex)^3 \left(a+b(d+ex)^2+c(d+ex)^4\right)^3 dx$

Optimal. Leaf size=138

$$\frac{a^3(d+ex)^4}{4e}+\frac{a^2b(d+ex)^6}{2e}+\frac{c\left(ac+b^2\right)(d+ex)^{12}}{4e}+\frac{b\left(6ac+b^2\right)(d+ex)^{10}}{10e}+\frac{3a\left(ac+b^2\right)(d+ex)^8}{8e}+\frac{3bc^2(d+ex)^{14}}{14e}$$

Rubi [A] time = 0.37, antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {1142, 1114, 631}

$$\frac{a^2b(d+ex)^6}{2e}+\frac{a^3(d+ex)^4}{4e}+\frac{c\left(ac+b^2\right)(d+ex)^{12}}{4e}+\frac{b\left(6ac+b^2\right)(d+ex)^{10}}{10e}+\frac{3a\left(ac+b^2\right)(d+ex)^8}{8e}+\frac{3bc^2(d+ex)^{14}}{14e}+\frac{c^3(d+ex)^{16}}{16e}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^3*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3,x]
```

```
[Out] (a^3*(d + e*x)^4)/(4*e) + (a^2*b*(d + e*x)^6)/(2*e) + (3*a*(b^2 + a*c)*(d +
 e*x)^8)/(8*e) + (b*(b^2 + 6*a*c)*(d + e*x)^10)/(10*e) + (c*(b^2 + a*c)*(d
+ e*x)^12)/(4*e) + (3*b*c^2*(d + e*x)^14)/(14*e) + (c^3*(d + e*x)^16)/(16*e
)
```

Rule 631

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbo
l] :> Int[ExpandIntegrand[(d + e*x)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a
, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || Eq
Q[a, 0])
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 1142

```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Di
st[u^m/(Coefficient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p,
 x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

Rubi steps

$$\begin{aligned}\int (d+ex)^3 \left(a+b(d+ex)^2+c(d+ex)^4\right)^3 dx &= \frac{\text{Subst}\left(\int x^3 \left(a+bx^2+cx^4\right)^3 dx, x, d+ex\right)}{e}\\ &= \frac{\text{Subst}\left(\int x \left(a+bx+cx^2\right)^3 dx, x, (d+ex)^2\right)}{2e}\\ &= \frac{\text{Subst}\left(\int \left(a^3x+3a^2bx^2+3a\left(b^2+ac\right)x^3+b\left(b^2+6ac\right)x^4+3c\left(\right.\right.}{2e}\\ &= \frac{a^3(d+ex)^4}{4e}+\frac{a^2b(d+ex)^6}{2e}+\frac{3a\left(b^2+ac\right)(d+ex)^8}{8e}+\frac{b\left(b^2+6a\right.}{1}\end{aligned}$$

Mathematica [B] time = 0.28, size = 797, normalized size = 5.78

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^3*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3,x]
```

```
[Out] d^3*(a + b*d^2 + c*d^4)^3*x + (3*d^2*(a + b*d^2 + c*d^4)^2*(a + 3*b*d^2 + 5
*c*d^4)*e*x^2)/2 + d*(a^3 + 10*a^2*b*d^2 + 21*a*b^2*d^4 + 21*a^2*c*d^4 + 12
*b^3*d^6 + 72*a*b*c*d^6 + 55*b^2*c*d^8 + 55*a*c^2*d^8 + 78*b*c^2*d^10 + 35*
c^3*d^12)*e^2*x^3 + ((a^3 + 30*a^2*b*d^2 + 105*a*b^2*d^4 + 105*a^2*c*d^4 +
84*b^3*d^6 + 504*a*b*c*d^6 + 495*b^2*c*d^8 + 495*a*c^2*d^8 + 858*b*c^2*d^10
 + 455*c^3*d^12)*e^3*x^4)/4 + (3*d*(5*a^2*b + 35*a*b^2*d^2 + 35*a^2*c*d^2 +
 42*b^3*d^4 + 252*a*b*c*d^4 + 330*b^2*c*d^6 + 330*a*c^2*d^6 + 715*b*c^2*d^8
 + 455*c^3*d^10)*e^4*x^5)/5 + ((a^2*b + 21*a*b^2*d^2 + 21*a^2*c*d^2 + 42*b^
3*d^4 + 252*a*b*c*d^4 + 462*b^2*c*d^6 + 462*a*c^2*d^6 + 1287*b*c^2*d^8 + 10
01*c^3*d^10)*e^5*x^6)/2 + (d*(21*a*b^2 + 21*a^2*c + 84*b^3*d^2 + 504*a*b*c*
d^2 + 1386*b^2*c*d^4 + 1386*a*c^2*d^4 + 5148*b*c^2*d^6 + 5005*c^3*d^8)*e^6*
x^7)/7 + (3*(a*b^2 + a^2*c + 12*b^3*d^2 + 72*a*b*c*d^2 + 330*b^2*c*d^4 + 33
0*a*c^2*d^4 + 1716*b*c^2*d^6 + 2145*c^3*d^8)*e^7*x^8)/8 + d*(b^3 + 6*a*b*c
+ 55*b^2*c*d^2 + 55*a*c^2*d^2 + 429*b*c^2*d^4 + 715*c^3*d^6)*e^8*x^9 + ((b^
3 + 6*a*b*c + 165*b^2*c*d^2 + 165*a*c^2*d^2 + 2145*b*c^2*d^4 + 5005*c^3*d^6
)*e^9*x^10)/10 + 3*c*d*(b^2 + a*c + 26*b*c*d^2 + 91*c^2*d^4)*e^10*x^11 + (c
*(b^2 + a*c + 78*b*c*d^2 + 455*c^2*d^4)*e^11*x^12)/4 + c^2*d*(3*b + 35*c*d^
2)*e^12*x^13 + (3*c^2*(b + 35*c*d^2)*e^13*x^14)/14 + c^3*d*e^14*x^15 + (c^3
*e^15*x^16)/16
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (d+ex)^3\left(a+b(d+ex)^2+c(d+ex)^4\right)^3\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(d + e*x)^3*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3,x]
```

```
[Out] IntegrateAlgebraic[(d + e*x)^3*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3, x]
```

fricas [B] time = 1.24, size = 1335, normalized size = 9.67

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="fricas")
```

```
[Out] 1/16*x^16*e^15*c^3 + x^15*e^14*d*c^3 + 15/2*x^14*e^13*d^2*c^3 + 35*x^13*e^1
2*d^3*c^3 + 455/4*x^12*e^11*d^4*c^3 + 3/14*x^14*e^13*c^2*b + 273*x^11*e^10*
d^5*c^3 + 3*x^13*e^12*d*c^2*b + 1001/2*x^10*e^9*d^6*c^3 + 39/2*x^12*e^11*d^
2*c^2*b + 715*x^9*e^8*d^7*c^3 + 78*x^11*e^10*d^3*c^2*b + 6435/8*x^8*e^7*d^8
*c^3 + 429/2*x^10*e^9*d^4*c^2*b + 1/4*x^12*e^11*c*b^2 + 1/4*x^12*e^11*c^2*a
 + 715*x^7*e^6*d^9*c^3 + 429*x^9*e^8*d^5*c^2*b + 3*x^11*e^10*d*c*b^2 + 3*x^
11*e^10*d*c^2*a + 1001/2*x^6*e^5*d^10*c^3 + 1287/2*x^8*e^7*d^6*c^2*b + 33/2
*x^10*e^9*d^2*c*b^2 + 33/2*x^10*e^9*d^2*c^2*a + 273*x^5*e^4*d^11*c^3 + 5148
/7*x^7*e^6*d^7*c^2*b + 55*x^9*e^8*d^3*c*b^2 + 55*x^9*e^8*d^3*c^2*a + 455/4*
x^4*e^3*d^12*c^3 + 1287/2*x^6*e^5*d^8*c^2*b + 495/4*x^8*e^7*d^4*c*b^2 + 1/1
0*x^10*e^9*b^3 + 495/4*x^8*e^7*d^4*c^2*a + 3/5*x^10*e^9*c*b*a + 35*x^3*e^2*
d^13*c^3 + 429*x^5*e^4*d^9*c^2*b + 198*x^7*e^6*d^5*c*b^2 + x^9*e^8*d*b^3 +
198*x^7*e^6*d^5*c^2*a + 6*x^9*e^8*d*c*b*a + 15/2*x^2*e*d^14*c^3 + 429/2*x^4
*e^3*d^10*c^2*b + 231*x^6*e^5*d^6*c*b^2 + 9/2*x^8*e^7*d^2*b^3 + 231*x^6*e^5
*d^6*c^2*a + 27*x^8*e^7*d^2*c*b*a + x*d^15*c^3 + 78*x^3*e^2*d^11*c^2*b + 19
8*x^5*e^4*d^7*c*b^2 + 12*x^7*e^6*d^3*b^3 + 198*x^5*e^4*d^7*c^2*a + 72*x^7*e
^6*d^3*c*b*a + 39/2*x^2*e*d^12*c^2*b + 495/4*x^4*e^3*d^8*c*b^2 + 21*x^6*e^5
*d^4*b^3 + 495/4*x^4*e^3*d^8*c^2*a + 126*x^6*e^5*d^4*c*b*a + 3/8*x^8*e^7*b^
2*a + 3/8*x^8*e^7*c*a^2 + 3*x*d^13*c^2*b + 55*x^3*e^2*d^9*c*b^2 + 126/5*x^5
*e^4*d^5*b^3 + 55*x^3*e^2*d^9*c^2*a + 756/5*x^5*e^4*d^5*c*b*a + 3*x^7*e^6*d
```

```
*b^2*a + 3*x^7*e^6*d*c*a^2 + 33/2*x^2*e*d^10*c*b^2 + 21*x^4*e^3*d^6*b^3 + 3
3/2*x^2*e*d^10*c^2*a + 126*x^4*e^3*d^6*c*b*a + 21/2*x^6*e^5*d^2*b^2*a + 21/
2*x^6*e^5*d^2*c*a^2 + 3*x*d^11*c*b^2 + 12*x^3*e^2*d^7*b^3 + 3*x*d^11*c^2*a
+ 72*x^3*e^2*d^7*c*b*a + 21*x^5*e^4*d^3*b^2*a + 21*x^5*e^4*d^3*c*a^2 + 9/2*
x^2*e*d^8*b^3 + 27*x^2*e*d^8*c*b*a + 105/4*x^4*e^3*d^4*b^2*a + 105/4*x^4*e^
3*d^4*c*a^2 + 1/2*x^6*e^5*b*a^2 + x*d^9*b^3 + 6*x*d^9*c*b*a + 21*x^3*e^2*d^
5*b^2*a + 21*x^3*e^2*d^5*c*a^2 + 3*x^5*e^4*d*b*a^2 + 21/2*x^2*e*d^6*b^2*a +
 21/2*x^2*e*d^6*c*a^2 + 15/2*x^4*e^3*d^2*b*a^2 + 3*x*d^7*b^2*a + 3*x*d^7*c*
a^2 + 10*x^3*e^2*d^3*b*a^2 + 15/2*x^2*e*d^4*b*a^2 + 1/4*x^4*e^3*a^3 + 3*x*d
^5*b*a^2 + x^3*e^2*d*a^3 + 3/2*x^2*e*d^2*a^3 + x*d^3*a^3
```

giac [B] time = 0.62, size = 1109, normalized size = 8.04

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="giac")
```

```
[Out] 1/2*(x^2*e + 2*d*x)*c^3*d^14 + 7/4*(x^2*e + 2*d*x)^2*c^3*d^12*e + 7/2*(x^2*
e + 2*d*x)^3*c^3*d^10*e^2 + 3/2*(x^2*e + 2*d*x)*b*c^2*d^12 + 35/8*(x^2*e +
2*d*x)^4*c^3*d^8*e^3 + 9/2*(x^2*e + 2*d*x)^2*b*c^2*d^10*e + 7/2*(x^2*e + 2*
d*x)^5*c^3*d^6*e^4 + 15/2*(x^2*e + 2*d*x)^3*b*c^2*d^8*e^2 + 3/2*(x^2*e + 2*
d*x)*b^2*c*d^10 + 3/2*(x^2*e + 2*d*x)*a*c^2*d^10 + 7/4*(x^2*e + 2*d*x)^6*c^
3*d^4*e^5 + 15/2*(x^2*e + 2*d*x)^4*b*c^2*d^6*e^3 + 15/4*(x^2*e + 2*d*x)^2*b
^2*c*d^8*e + 15/4*(x^2*e + 2*d*x)^2*a*c^2*d^8*e + 1/2*(x^2*e + 2*d*x)^7*c^3
*d^2*e^6 + 9/2*(x^2*e + 2*d*x)^5*b*c^2*d^4*e^4 + 5*(x^2*e + 2*d*x)^3*b^2*c*
d^6*e^2 + 5*(x^2*e + 2*d*x)^3*a*c^2*d^6*e^2 + 1/2*(x^2*e + 2*d*x)*b^3*d^8 +
 3*(x^2*e + 2*d*x)*a*b*c*d^8 + 1/16*(x^2*e + 2*d*x)^8*c^3*e^7 + 3/2*(x^2*e
+ 2*d*x)^6*b*c^2*d^2*e^5 + 15/4*(x^2*e + 2*d*x)^4*b^2*c*d^4*e^3 + 15/4*(x^2
*e + 2*d*x)^4*a*c^2*d^4*e^3 + (x^2*e + 2*d*x)^2*b^3*d^6*e + 6*(x^2*e + 2*d*
x)^2*a*b*c*d^6*e + 3/14*(x^2*e + 2*d*x)^7*b*c^2*e^6 + 3/2*(x^2*e + 2*d*x)^5
*b^2*c*d^2*e^4 + 3/2*(x^2*e + 2*d*x)^5*a*c^2*d^2*e^4 + (x^2*e + 2*d*x)^3*b^
3*d^4*e^2 + 6*(x^2*e + 2*d*x)^3*a*b*c*d^4*e^2 + 3/2*(x^2*e + 2*d*x)*a*b^2*d
^6 + 3/2*(x^2*e + 2*d*x)*a^2*c*d^6 + 1/4*(x^2*e + 2*d*x)^6*b^2*c*e^5 + 1/4*
(x^2*e + 2*d*x)^6*a*c^2*e^5 + 1/2*(x^2*e + 2*d*x)^4*b^3*d^2*e^3 + 3*(x^2*e
+ 2*d*x)^4*a*b*c*d^2*e^3 + 9/4*(x^2*e + 2*d*x)^2*a*b^2*d^4*e + 9/4*(x^2*e +
 2*d*x)^2*a^2*c*d^4*e + 1/10*(x^2*e + 2*d*x)^5*b^3*e^4 + 3/5*(x^2*e + 2*d*x
)^5*a*b*c*e^4 + 3/2*(x^2*e + 2*d*x)^3*a*b^2*d^2*e^2 + 3/2*(x^2*e + 2*d*x)^3
*a^2*c*d^2*e^2 + 3/2*(x^2*e + 2*d*x)*a^2*b*d^4 + 3/8*(x^2*e + 2*d*x)^4*a*b^
2*e^3 + 3/8*(x^2*e + 2*d*x)^4*a^2*c*e^3 + 3/2*(x^2*e + 2*d*x)^2*a^2*b*d^2*e
 + 1/2*(x^2*e + 2*d*x)^3*a^2*b*e^2 + 1/2*(x^2*e + 2*d*x)*a^3*d^2 + 1/4*(x^2
*e + 2*d*x)^2*a^3*e
```

maple [B] time = 0.00, size = 7550, normalized size = 54.71

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3*(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x)
```

```
[Out] result too large to display
```

maxima [B] time = 1.13, size = 872, normalized size = 6.32

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="maxima")
```

```
[Out] 1/16*c^3*e^15*x^16 + c^3*d*e^14*x^15 + 3/14*(35*c^3*d^2 + b*c^2)*e^13*x^14
+ (35*c^3*d^3 + 3*b*c^2*d)*e^12*x^13 + 1/4*(455*c^3*d^4 + 78*b*c^2*d^2 + b^
```

```
2*c + a*c^2)*e^11*x^12 + 3*(91*c^3*d^5 + 26*b*c^2*d^3 + (b^2*c + a*c^2)*d)*
e^10*x^11 + 1/10*(5005*c^3*d^6 + 2145*b*c^2*d^4 + b^3 + 6*a*b*c + 165*(b^2*
c + a*c^2)*d^2)*e^9*x^10 + (715*c^3*d^7 + 429*b*c^2*d^5 + 55*(b^2*c + a*c^2
)*d^3 + (b^3 + 6*a*b*c)*d)*e^8*x^9 + 3/8*(2145*c^3*d^8 + 1716*b*c^2*d^6 + 3
30*(b^2*c + a*c^2)*d^4 + a*b^2 + a^2*c + 12*(b^3 + 6*a*b*c)*d^2)*e^7*x^8 +
1/7*(5005*c^3*d^9 + 5148*b*c^2*d^7 + 1386*(b^2*c + a*c^2)*d^5 + 84*(b^3 + 6
*a*b*c)*d^3 + 21*(a*b^2 + a^2*c)*d)*e^6*x^7 + 1/2*(1001*c^3*d^10 + 1287*b*c
^2*d^8 + 462*(b^2*c + a*c^2)*d^6 + 42*(b^3 + 6*a*b*c)*d^4 + a^2*b + 21*(a*b
^2 + a^2*c)*d^2)*e^5*x^6 + 3/5*(455*c^3*d^11 + 715*b*c^2*d^9 + 330*(b^2*c +
 a*c^2)*d^7 + 42*(b^3 + 6*a*b*c)*d^5 + 5*a^2*b*d + 35*(a*b^2 + a^2*c)*d^3)*
e^4*x^5 + 1/4*(455*c^3*d^12 + 858*b*c^2*d^10 + 495*(b^2*c + a*c^2)*d^8 + 84
*(b^3 + 6*a*b*c)*d^6 + 30*a^2*b*d^2 + 105*(a*b^2 + a^2*c)*d^4 + a^3)*e^3*x^
4 + (35*c^3*d^13 + 78*b*c^2*d^11 + 55*(b^2*c + a*c^2)*d^9 + 12*(b^3 + 6*a*b
*c)*d^7 + 10*a^2*b*d^3 + 21*(a*b^2 + a^2*c)*d^5 + a^3*d)*e^2*x^3 + 3/2*(5*c
^3*d^14 + 13*b*c^2*d^12 + 11*(b^2*c + a*c^2)*d^10 + 3*(b^3 + 6*a*b*c)*d^8 +
 5*a^2*b*d^4 + 7*(a*b^2 + a^2*c)*d^6 + a^3*d^2)*e*x^2 + (c^3*d^15 + 3*b*c^2
*d^13 + 3*(b^2*c + a*c^2)*d^11 + (b^3 + 6*a*b*c)*d^9 + 3*a^2*b*d^5 + 3*(a*b
^2 + a^2*c)*d^7 + a^3*d^3)*x
```

mupad [B] time = 1.66, size = 777, normalized size = 5.63

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^3*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3,x)
```

```
[Out] (3*e^7*x^8*(a*b^2 + a^2*c + 12*b^3*d^2 + 2145*c^3*d^8 + 330*a*c^2*d^4 + 330
*b^2*c*d^4 + 1716*b*c^2*d^6 + 72*a*b*c*d^2))/8 + (e^5*x^6*(a^2*b + 42*b^3*d
^4 + 1001*c^3*d^10 + 21*a*b^2*d^2 + 21*a^2*c*d^2 + 462*a*c^2*d^6 + 462*b^2*
c*d^6 + 1287*b*c^2*d^8 + 252*a*b*c*d^4))/2 + (e^9*x^10*(b^3 + 5005*c^3*d^6
+ 165*a*c^2*d^2 + 165*b^2*c*d^2 + 2145*b*c^2*d^4 + 6*a*b*c))/10 + (c^3*e^15
*x^16)/16 + d^3*x*(a + b*d^2 + c*d^4)^3 + (e^3*x^4*(a^3 + 84*b^3*d^6 + 455*
c^3*d^12 + 30*a^2*b*d^2 + 105*a*b^2*d^4 + 105*a^2*c*d^4 + 495*a*c^2*d^8 + 4
95*b^2*c*d^8 + 858*b*c^2*d^10 + 504*a*b*c*d^6))/4 + (3*c^2*e^13*x^14*(b + 3
5*c*d^2))/14 + c^3*d*e^14*x^15 + d*e^2*x^3*(a^3 + 12*b^3*d^6 + 35*c^3*d^12
+ 10*a^2*b*d^2 + 21*a*b^2*d^4 + 21*a^2*c*d^4 + 55*a*c^2*d^8 + 55*b^2*c*d^8
+ 78*b*c^2*d^10 + 72*a*b*c*d^6) + (c*e^11*x^12*(a*c + b^2 + 455*c^2*d^4 + 7
8*b*c*d^2))/4 + (d*e^6*x^7*(21*a*b^2 + 21*a^2*c + 84*b^3*d^2 + 5005*c^3*d^8
 + 1386*a*c^2*d^4 + 1386*b^2*c*d^4 + 5148*b*c^2*d^6 + 504*a*b*c*d^2))/7 + (
3*d*e^4*x^5*(5*a^2*b + 42*b^3*d^4 + 455*c^3*d^10 + 35*a*b^2*d^2 + 35*a^2*c*
d^2 + 330*a*c^2*d^6 + 330*b^2*c*d^6 + 715*b*c^2*d^8 + 252*a*b*c*d^4))/5 + d
*e^8*x^9*(b^3 + 715*c^3*d^6 + 55*a*c^2*d^2 + 55*b^2*c*d^2 + 429*b*c^2*d^4 +
 6*a*b*c) + (3*d^2*e*x^2*(a + b*d^2 + c*d^4)^2*(a + 3*b*d^2 + 5*c*d^4))/2 +
 c^2*d*e^12*x^13*(3*b + 35*c*d^2) + 3*c*d*e^10*x^11*(a*c + b^2 + 91*c^2*d^4
 + 26*b*c*d^2)
```

sympy [B] time = 0.36, size = 1314, normalized size = 9.52

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3*(a+b*(e*x+d)**2+c*(e*x+d)**4)**3,x)
```

```
[Out] c**3*d*e**14*x**15 + c**3*e**15*x**16/16 + x**14*(3*b*c**2*e**13/14 + 15*c*
*3*d**2*e**13/2) + x**13*(3*b*c**2*d*e**12 + 35*c**3*d**3*e**12) + x**12*(a
*c**2*e**11/4 + b**2*c*e**11/4 + 39*b*c**2*d**2*e**11/2 + 455*c**3*d**4*e**
11/4) + x**11*(3*a*c**2*d*e**10 + 3*b**2*c*d*e**10 + 78*b*c**2*d**3*e**10 +
 273*c**3*d**5*e**10) + x**10*(3*a*b*c*e**9/5 + 33*a*c**2*d**2*e**9/2 + b**
3*e**9/10 + 33*b**2*c*d**2*e**9/2 + 429*b*c**2*d**4*e**9/2 + 1001*c**3*d**6
```

```
*e**9/2) + x**9*(6*a*b*c*d*e**8 + 55*a*c**2*d**3*e**8 + b**3*d*e**8 + 55*b*
*2*c*d**3*e**8 + 429*b*c**2*d**5*e**8 + 715*c**3*d**7*e**8) + x**8*(3*a**2*
c*e**7/8 + 3*a*b**2*e**7/8 + 27*a*b*c*d**2*e**7 + 495*a*c**2*d**4*e**7/4 +
9*b**3*d**2*e**7/2 + 495*b**2*c*d**4*e**7/4 + 1287*b*c**2*d**6*e**7/2 + 643
5*c**3*d**8*e**7/8) + x**7*(3*a**2*c*d*e**6 + 3*a*b**2*d*e**6 + 72*a*b*c*d*
*3*e**6 + 198*a*c**2*d**5*e**6 + 12*b**3*d**3*e**6 + 198*b**2*c*d**5*e**6 +
 5148*b*c**2*d**7*e**6/7 + 715*c**3*d**9*e**6) + x**6*(a**2*b*e**5/2 + 21*a
**2*c*d**2*e**5/2 + 21*a*b**2*d**2*e**5/2 + 126*a*b*c*d**4*e**5 + 231*a*c**
2*d**6*e**5 + 21*b**3*d**4*e**5 + 231*b**2*c*d**6*e**5 + 1287*b*c**2*d**8*e
**5/2 + 1001*c**3*d**10*e**5/2) + x**5*(3*a**2*b*d*e**4 + 21*a**2*c*d**3*e*
*4 + 21*a*b**2*d**3*e**4 + 756*a*b*c*d**5*e**4/5 + 198*a*c**2*d**7*e**4 + 1
26*b**3*d**5*e**4/5 + 198*b**2*c*d**7*e**4 + 429*b*c**2*d**9*e**4 + 273*c**
3*d**11*e**4) + x**4*(a**3*e**3/4 + 15*a**2*b*d**2*e**3/2 + 105*a**2*c*d**4
*e**3/4 + 105*a*b**2*d**4*e**3/4 + 126*a*b*c*d**6*e**3 + 495*a*c**2*d**8*e*
*3/4 + 21*b**3*d**6*e**3 + 495*b**2*c*d**8*e**3/4 + 429*b*c**2*d**10*e**3/2
 + 455*c**3*d**12*e**3/4) + x**3*(a**3*d*e**2 + 10*a**2*b*d**3*e**2 + 21*a*
*2*c*d**5*e**2 + 21*a*b**2*d**5*e**2 + 72*a*b*c*d**7*e**2 + 55*a*c**2*d**9*
e**2 + 12*b**3*d**7*e**2 + 55*b**2*c*d**9*e**2 + 78*b*c**2*d**11*e**2 + 35*
c**3*d**13*e**2) + x**2*(3*a**3*d**2*e/2 + 15*a**2*b*d**4*e/2 + 21*a**2*c*d
**6*e/2 + 21*a*b**2*d**6*e/2 + 27*a*b*c*d**8*e + 33*a*c**2*d**10*e/2 + 9*b*
*3*d**8*e/2 + 33*b**2*c*d**10*e/2 + 39*b*c**2*d**12*e/2 + 15*c**3*d**14*e/2
) + x*(a**3*d**3 + 3*a**2*b*d**5 + 3*a**2*c*d**7 + 3*a*b**2*d**7 + 6*a*b*c*
d**9 + 3*a*c**2*d**11 + b**3*d**9 + 3*b**2*c*d**11 + 3*b*c**2*d**13 + c**3*
d**15)
```

3.497 $\int (df+efx)^3 \left(a+b(d+ex)^2+c(d+ex)^4\right) \, dx$

Optimal. Leaf size=55

$$\frac{af^3(d+ex)^4}{4e}+\frac{bf^3(d+ex)^6}{6e}+\frac{cf^3(d+ex)^8}{8e}$$

Rubi [A] time = 0.05, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {1142, 14}

$$\frac{af^3(d+ex)^4}{4e}+\frac{bf^3(d+ex)^6}{6e}+\frac{cf^3(d+ex)^8}{8e}$$

Antiderivative was successfully verified.

```
[In] Int[(d*f + e*f*x)^3*(a + b*(d + e*x)^2 + c*(d + e*x)^4),x]

[Out] (a*f^3*(d + e*x)^4)/(4*e) + (b*f^3*(d + e*x)^6)/(6*e) + (c*f^3*(d + e*x)^8)
/(8*e)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 1142

```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Di
st[u^m/(Coefficient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p,
 x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

Rubi steps

$$\begin{aligned}
\int (df+efx)^3 \left(a+b(d+ex)^2+c(d+ex)^4\right) \, dx &= \frac{f^3 \operatorname{Subst}\left(\int x^3 \left(a+bx^2+cx^4\right) \, dx,x,d+ex\right)}{e}\\
&= \frac{f^3 \operatorname{Subst}\left(\int \left(ax^3+bx^5+cx^7\right) \, dx,x,d+ex\right)}{e}\\
&= \frac{af^3(d+ex)^4}{4e}+\frac{bf^3(d+ex)^6}{6e}+\frac{cf^3(d+ex)^8}{8e}
\end{aligned}$$

Mathematica [B] time = 0.01, size = 154, normalized size = 2.80

$$f^3 \left(\frac{1}{4}e^3x^4\left(a+10bd^2+35cd^4\right)+\frac{1}{3}de^2x^3\left(3a+10bd^2+21cd^4\right)+\frac{1}{2}d^2ex^2\left(3a+5bd^2+7cd^4\right)+d^3x\left(a+bd^2+cd^4\right)+\frac{1}{6}e^5x^6\left(b+21cd^2\right)+de^4x^5\left(b+7cd^2\right)+cde^6x^7+\frac{1}{8}ce^7x^8\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*f + e*f*x)^3*(a + b*(d + e*x)^2 + c*(d + e*x)^4),x]

[Out] f^3*(d^3*(a + b*d^2 + c*d^4)*x + (d^2*(3*a + 5*b*d^2 + 7*c*d^4)*e*x^2)/2 +
(d*(3*a + 10*b*d^2 + 21*c*d^4)*e^2*x^3)/3 + ((a + 10*b*d^2 + 35*c*d^4)*e^3*
x^4)/4 + d*(b + 7*c*d^2)*e^4*x^5 + ((b + 21*c*d^2)*e^5*x^6)/6 + c*d*e^6*x^7
 + (c*e^7*x^8)/8)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (df+efx)^3 \left(a+b(d+ex)^2+c(d+ex)^4\right) \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(d*f + e*f*x)^3*(a + b*(d + e*x)^2 + c*(d + e*x)^4),x]
```

```
[Out] IntegrateAlgebraic[(d*f + e*f*x)^3*(a + b*(d + e*x)^2 + c*(d + e*x)^4), x]
```

fricas [B] time = 1.09, size = 229, normalized size = 4.16

$$\frac{1}{8}x^8f^3e^7c + x^7f^3e^6dc + \frac{7}{2}x^6f^3e^5d^2c + 7x^5f^3e^4d^3c + \frac{35}{4}x^4f^3e^3d^4c + \frac{1}{6}x^6f^3e^5b + 7x^3f^3e^2d^5c + x^5f^3e^4db + \frac{7}{2}x^2f^3ed^6c + \frac{5}{2}x^4f^3e^3d^2b + xf^3d^7c + \frac{10}{3}x^3f^3e^2d^3b + \frac{5}{2}x^2f^3ed^4b + \frac{1}{4}x^4f^3e^3a + xf^3d^5b + x^3f^3e^2da + \frac{3}{2}x^2f^3ed^2a + xf^3d^3a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*f*x+d*f)^3*(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="fricas")
```

```
[Out] 1/8*x^8*f^3*e^7*c + x^7*f^3*e^6*d*c + 7/2*x^6*f^3*e^5*d^2*c + 7*x^5*f^3*e^4
*d^3*c + 35/4*x^4*f^3*e^3*d^4*c + 1/6*x^6*f^3*e^5*b + 7*x^3*f^3*e^2*d^5*c +
 x^5*f^3*e^4*d*b + 7/2*x^2*f^3*e*d^6*c + 5/2*x^4*f^3*e^3*d^2*b + x*f^3*d^7*
c + 10/3*x^3*f^3*e^2*d^3*b + 5/2*x^2*f^3*e*d^4*b + 1/4*x^4*f^3*e^3*a + x*f^
3*d^5*b + x^3*f^3*e^2*d*a + 3/2*x^2*f^3*e*d^2*a + x*f^3*d^3*a
```

giac [B] time = 0.30, size = 213, normalized size = 3.87

$$\frac{1}{2}\left(fx^2e + 2dfx\right)cd^6f^2 + \frac{1}{2}\left(fx^2e + 2dfx\right)bd^4f^2 + \frac{1}{2}\left(fx^2e + 2dfx\right)ad^2f^2 + \frac{18\left(fx^2e + 2dfx\right)^2cd^4f^2e + 12\left(fx^2e + 2dfx\right)^3cd^2fe^2 + 12\left(fx^2e + 2dfx\right)^2bd^2f^2e + 3\left(fx^2e + 2dfx\right)^4ce^3 + 4\left(fx^2e + 2dfx\right)^3bfe^2 + 6\left(fx^2e + 2dfx\right)^2af^2e}{24f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*f*x+d*f)^3*(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="giac")
```

```
[Out] 1/2*(f*x^2*e + 2*d*f*x)*c*d^6*f^2 + 1/2*(f*x^2*e + 2*d*f*x)*b*d^4*f^2 + 1/2
*(f*x^2*e + 2*d*f*x)*a*d^2*f^2 + 1/24*(18*(f*x^2*e + 2*d*f*x)^2*c*d^4*f^2*e
 + 12*(f*x^2*e + 2*d*f*x)^3*c*d^2*f*e^2 + 12*(f*x^2*e + 2*d*f*x)^2*b*d^2*f^
2*e + 3*(f*x^2*e + 2*d*f*x)^4*c*e^3 + 4*(f*x^2*e + 2*d*f*x)^3*b*f*e^2 + 6*(
f*x^2*e + 2*d*f*x)^2*a*f^2*e)/f
```

maple [B] time = 0.00, size = 349, normalized size = 6.35

$$\frac{ce^7f^3x^8}{8} + cde^6f^3x^7 + \left(cd^4 + bd^2 + a\right)d^3f^3x + \frac{\left(15cd^2e^5f^3 + \left(6cd^2e^2 + be^2\right)e^3f^3\right)x^6}{6} + \frac{\left(13cd^3e^4f^3 + 3\left(6cd^2e^2 + be^2\right)de^2f^3 + \left(4cd^3e + 2deb\right)e^3f^3\right)x^5}{5} + \frac{\left(4cd^4e^3f^3 + 3\left(6cd^2e^2 + be^2\right)d^2ef^3 + 3\left(4cd^3e + 2deb\right)de^2f^3 + \left(cd^4 + bd^2 + a\right)e^3f^3\right)x^4}{4} + \frac{\left(\left(6cd^2e^2 + be^2\right)d^3f^3 + 3\left(4cd^3e + 2deb\right)d^2ef^3 + 3\left(cd^4 + bd^2 + a\right)de^2f^3\right)x^3}{3} + \frac{\left(\left(4cd^3e + 2deb\right)d^3f^3 + 3\left(cd^4 + bd^2 + a\right)d^2ef^3\right)x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*f*x+d*f)^3*(a+b*(e*x+d)^2+c*(e*x+d)^4),x)
```

```
[Out] 1/8*e^7*f^3*c*x^8+d*f^3*e^6*c*x^7+1/6*(15*d^2*f^3*e^5*c+e^3*f^3*(6*c*d^2*e^
2+b*e^2))*x^6+1/5*(13*d^3*f^3*c*e^4+3*d*f^3*e^2*(6*c*d^2*e^2+b*e^2)+e^3*f^3
*(4*c*d^3*e+2*b*d*e))*x^5+1/4*(4*d^4*f^3*c*e^3+3*d^2*f^3*e*(6*c*d^2*e^2+b*e
^2)+3*d*f^3*e^2*(4*c*d^3*e+2*b*d*e)+e^3*f^3*(c*d^4+b*d^2+a))*x^4+1/3*(d^3*f
^3*(6*c*d^2*e^2+b*e^2)+3*d^2*f^3*e*(4*c*d^3*e+2*b*d*e)+3*d*f^3*e^2*(c*d^4+b
*d^2+a))*x^3+1/2*(d^3*f^3*(4*c*d^3*e+2*b*d*e)+3*d^2*f^3*e*(c*d^4+b*d^2+a))*
x^2+d^3*f^3*(c*d^4+b*d^2+a)*x
```

maxima [B] time = 1.08, size = 166, normalized size = 3.02

$$\frac{1}{8}ce^7f^3x^8 + cde^6f^3x^7 + \frac{1}{6}\left(21cd^2 + b\right)e^5f^3x^6 + \left(7cd^3 + bd\right)e^4f^3x^5 + \frac{1}{4}\left(35cd^4 + 10bd^2 + a\right)e^3f^3x^4 + \frac{1}{3}\left(21cd^5 + 10bd^3 + 3ad\right)e^2f^3x^3 + \frac{1}{2}\left(7cd^6 + 5bd^4 + 3ad^2\right)ef^3x^2 + \left(cd^7 + bd^5 + ad^3\right)f^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*f*x+d*f)^3*(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="maxima")
```

```
[Out] 1/8*c*e^7*f^3*x^8 + c*d*e^6*f^3*x^7 + 1/6*(21*c*d^2 + b)*e^5*f^3*x^6 + (7*c
*d^3 + b*d)*e^4*f^3*x^5 + 1/4*(35*c*d^4 + 10*b*d^2 + a)*e^3*f^3*x^4 + 1/3*(
21*c*d^5 + 10*b*d^3 + 3*a*d)*e^2*f^3*x^3 + 1/2*(7*c*d^6 + 5*b*d^4 + 3*a*d^2
)*e*f^3*x^2 + (c*d^7 + b*d^5 + a*d^3)*f^3*x
```

mupad [B] time = 0.08, size = 164, normalized size = 2.98

$$\frac{e^5\,f^3\,x^6\,\left(21\,c\,d^2+b\right)}{6}+\frac{c\,e^7\,f^3\,x^8}{8}+d^3\,f^3\,x\,\left(c\,d^4+b\,d^2+a\right)+\frac{e^3\,f^3\,x^4\,\left(35\,c\,d^4+10\,b\,d^2+a\right)}{4}+\frac{d^2\,e\,f^3\,x^2\,\left(7\,c\,d^4+5\,b\,d^2+3\,a\right)}{2}+\frac{d\,e^2\,f^3\,x^3\,\left(21\,c\,d^4+10\,b\,d^2+3\,a\right)}{3}+d\,e^4\,f^3\,x^5\,\left(7\,c\,d^2+b\right)+c\,d\,e^6\,f^3\,x^7$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*f + e*f*x)^3*(a + b*(d + e*x)^2 + c*(d + e*x)^4),x)
```

```
[Out] (e^5*f^3*x^6*(b + 21*c*d^2))/6 + (c*e^7*f^3*x^8)/8 + d^3*f^3*x*(a + b*d^2 +
 c*d^4) + (e^3*f^3*x^4*(a + 10*b*d^2 + 35*c*d^4))/4 + (d^2*e*f^3*x^2*(3*a +
 5*b*d^2 + 7*c*d^4))/2 + (d*e^2*f^3*x^3*(3*a + 10*b*d^2 + 21*c*d^4))/3 + d*
e^4*f^3*x^5*(b + 7*c*d^2) + c*d*e^6*f^3*x^7
```

sympy [B] time = 0.11, size = 240, normalized size = 4.36

$$cde^6f^3x^7+\frac{ce^7f^3x^8}{8}+x^6\left(\frac{be^5f^3}{6}+\frac{7cd^2e^5f^3}{2}\right)+x^5\left(bde^4f^3+7cd^3e^4f^3\right)+x^4\left(\frac{ae^3f^3}{4}+\frac{5bd^2e^3f^3}{2}+\frac{35cd^4e^3f^3}{4}\right)+x^3\left(ade^2f^3+\frac{10bd^3e^2f^3}{3}+7cd^5e^2f^3\right)+x^2\left(\frac{3ad^2ef^3}{2}+\frac{5bd^4ef^3}{2}+\frac{7cd^6ef^3}{2}\right)+x\left(ad^3f^3+bd^5f^3+cd^7f^3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*f*x+d*f)**3*(a+b*(e*x+d)**2+c*(e*x+d)**4),x)
```

```
[Out] c*d*e**6*f**3*x**7 + c*e**7*f**3*x**8/8 + x**6*(b*e**5*f**3/6 + 7*c*d**2*e*
*5*f**3/2) + x**5*(b*d*e**4*f**3 + 7*c*d**3*e**4*f**3) + x**4*(a*e**3*f**3/
4 + 5*b*d**2*e**3*f**3/2 + 35*c*d**4*e**3*f**3/4) + x**3*(a*d*e**2*f**3 + 1
0*b*d**3*e**2*f**3/3 + 7*c*d**5*e**2*f**3) + x**2*(3*a*d**2*e*f**3/2 + 5*b*
d**4*e*f**3/2 + 7*c*d**6*e*f**3/2) + x*(a*d**3*f**3 + b*d**5*f**3 + c*d**7*
f**3)
```

3.498 $\int (df+efx)^3 \left(a+b(d+ex)^2+c(d+ex)^4\right)^2 \, dx$

Optimal. Leaf size=104

$$\frac{a^2 f^3 (d+ex)^4}{4e}+\frac{f^3 \left(2ac+b^2\right) (d+ex)^8}{8e}+\frac{abf^3 (d+ex)^6}{3e}+\frac{bcf^3 (d+ex)^{10}}{5e}+\frac{c^2 f^3 (d+ex)^{12}}{12e}$$

Rubi [A] time = 0.16, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1142, 1114, 631}

$$\frac{a^2 f^3 (d+ex)^4}{4e}+\frac{f^3 \left(2ac+b^2\right) (d+ex)^8}{8e}+\frac{abf^3 (d+ex)^6}{3e}+\frac{bcf^3 (d+ex)^{10}}{5e}+\frac{c^2 f^3 (d+ex)^{12}}{12e}$$

Antiderivative was successfully verified.

```
[In] Int[(d*f + e*f*x)^3*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2,x]
```

```
[Out] (a^2*f^3*(d + e*x)^4)/(4*e) + (a*b*f^3*(d + e*x)^6)/(3*e) + ((b^2 + 2*a*c)*
f^3*(d + e*x)^8)/(8*e) + (b*c*f^3*(d + e*x)^10)/(5*e) + (c^2*f^3*(d + e*x)^
12)/(12*e)
```

Rule 631

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbo
l] :> Int[ExpandIntegrand[(d + e*x)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a
, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || Eq
Q[a, 0])
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 1142

```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Di
st[u^m/(Coefficient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p,
 x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

Rubi steps

$$\begin{aligned}
\int (df+efx)^3 \left(a+b(d+ex)^2+c(d+ex)^4\right)^2 \, dx &= \frac{f^3 \operatorname{Subst}\left(\int x^3 \left(a+bx^2+cx^4\right)^2 \, dx, x, d+ex\right)}{e}\\
&= \frac{f^3 \operatorname{Subst}\left(\int x \left(a+bx+cx^2\right)^2 \, dx, x, (d+ex)^2\right)}{2e}\\
&= \frac{f^3 \operatorname{Subst}\left(\int \left(a^2x+2abx^2+\left(b^2+2ac\right)x^3+2bcx^4+c^2x^5\right) dx,\right.}{2e}\\
&= \frac{a^2 f^3 (d+ex)^4}{4e}+\frac{abf^3 (d+ex)^6}{3e}+\frac{\left(b^2+2ac\right) f^3 (d+ex)^8}{8e}+\frac{bc}{}
\end{aligned}$$

Mathematica [B] time = 0.07, size = 405, normalized size = 3.89

$$f^3 \left(\frac{1}{4}e^3x^4 \left(a^2+20abd^2+70acd^4+35b^2d^4+168bcd^6+165c^2d^8\right)+\frac{1}{3}de^2x^3 \left(3a^2+20abd^2+42acd^4+21b^2d^4+72bcd^6+55c^2d^8\right)+\frac{1}{2}d^2ex^2 \left(3a^2+10abd^2+14acd^4+7b^2d^4+18bcd^6+11c^2d^8\right)+\frac{1}{8}e^7x^8 \left(2ac+b^2+72bcd^2+330c^2d^4\right)+de^6x^7 \left(2ac+b^2+24bcd^2+66c^2d^4\right)+\frac{1}{6}e^5x^6 \left(2ab+42acd^2+21b^2d^2+252bcd^4+462c^2d^6\right)+\frac{1}{5}de^4x^5 \left(10ab+70acd^2+35b^2d^2+252bcd^4+330c^2d^6\right)+d^3x \left(a+bd^2+cd^4\right)^2+\frac{1}{10}ce^9x^{10} \left(2b+55cd^2\right)+\frac{1}{3}cde^8x^9 \left(6b+55cd^2\right)+c^2de^{10}x^{11}+\frac{1}{12}c^2e^{11}x^{12}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*f + e*f*x)^3*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2,x]
```

```
[Out] f^3*(d^3*(a + b*d^2 + c*d^4)^2*x + (d^2*(3*a^2 + 10*a*b*d^2 + 7*b^2*d^4 + 1
4*a*c*d^4 + 18*b*c*d^6 + 11*c^2*d^8)*e*x^2)/2 + (d*(3*a^2 + 20*a*b*d^2 + 21
*b^2*d^4 + 42*a*c*d^4 + 72*b*c*d^6 + 55*c^2*d^8)*e^2*x^3)/3 + ((a^2 + 20*a*
b*d^2 + 35*b^2*d^4 + 70*a*c*d^4 + 168*b*c*d^6 + 165*c^2*d^8)*e^3*x^4)/4 + (
d*(10*a*b + 35*b^2*d^2 + 70*a*c*d^2 + 252*b*c*d^4 + 330*c^2*d^6)*e^4*x^5)/5
 + ((2*a*b + 21*b^2*d^2 + 42*a*c*d^2 + 252*b*c*d^4 + 462*c^2*d^6)*e^5*x^6)/
6 + d*(b^2 + 2*a*c + 24*b*c*d^2 + 66*c^2*d^4)*e^6*x^7 + ((b^2 + 2*a*c + 72*
b*c*d^2 + 330*c^2*d^4)*e^7*x^8)/8 + (c*d*(6*b + 55*c*d^2)*e^8*x^9)/3 + (c*(
2*b + 55*c*d^2)*e^9*x^10)/10 + c^2*d*e^10*x^11 + (c^2*e^11*x^12)/12)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (df + efx)^3 \left(a + b(d + ex)^2 + c(d + ex)^4\right)^2 \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(d*f + e*f*x)^3*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2,x]
```

```
[Out] IntegrateAlgebraic[(d*f + e*f*x)^3*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2, x
]
```

fricas [B] time = 1.02, size = 715, normalized size = 6.88

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*f*x+d*f)^3*(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="fricas"
)
```

```
[Out] 1/12*x^12*f^3*e^11*c^2 + x^11*f^3*e^10*d*c^2 + 11/2*x^10*f^3*e^9*d^2*c^2 +
55/3*x^9*f^3*e^8*d^3*c^2 + 165/4*x^8*f^3*e^7*d^4*c^2 + 1/5*x^10*f^3*e^9*c*b
 + 66*x^7*f^3*e^6*d^5*c^2 + 2*x^9*f^3*e^8*d*c*b + 77*x^6*f^3*e^5*d^6*c^2 +
9*x^8*f^3*e^7*d^2*c*b + 66*x^5*f^3*e^4*d^7*c^2 + 24*x^7*f^3*e^6*d^3*c*b + 1
65/4*x^4*f^3*e^3*d^8*c^2 + 42*x^6*f^3*e^5*d^4*c*b + 1/8*x^8*f^3*e^7*b^2 + 1
/4*x^8*f^3*e^7*c*a + 55/3*x^3*f^3*e^2*d^9*c^2 + 252/5*x^5*f^3*e^4*d^5*c*b +
 x^7*f^3*e^6*d*b^2 + 2*x^7*f^3*e^6*d*c*a + 11/2*x^2*f^3*e*d^10*c^2 + 42*x^4
*f^3*e^3*d^6*c*b + 7/2*x^6*f^3*e^5*d^2*b^2 + 7*x^6*f^3*e^5*d^2*c*a + x*f^3*
d^11*c^2 + 24*x^3*f^3*e^2*d^7*c*b + 7*x^5*f^3*e^4*d^3*b^2 + 14*x^5*f^3*e^4*
d^3*c*a + 9*x^2*f^3*e*d^8*c*b + 35/4*x^4*f^3*e^3*d^4*b^2 + 35/2*x^4*f^3*e^3
*d^4*c*a + 1/3*x^6*f^3*e^5*b*a + 2*x*f^3*d^9*c*b + 7*x^3*f^3*e^2*d^5*b^2 +
14*x^3*f^3*e^2*d^5*c*a + 2*x^5*f^3*e^4*d*b*a + 7/2*x^2*f^3*e*d^6*b^2 + 7*x^
2*f^3*e*d^6*c*a + 5*x^4*f^3*e^3*d^2*b*a + x*f^3*d^7*b^2 + 2*x*f^3*d^7*c*a +
 20/3*x^3*f^3*e^2*d^3*b*a + 5*x^2*f^3*e*d^4*b*a + 1/4*x^4*f^3*e^3*a^2 + 2*x
*f^3*d^5*b*a + x^3*f^3*e^2*d*a^2 + 3/2*x^2*f^3*e*d^2*a^2 + x*f^3*d^3*a^2
```

giac [B] time = 0.43, size = 615, normalized size = 5.91

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*f*x+d*f)^3*(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="giac")
```

```
[Out] 1/2*(f*x^2*e + 2*d*f*x)*c^2*d^10*f^2 + (f*x^2*e + 2*d*f*x)*b*c*d^8*f^2 + 1/
2*(f*x^2*e + 2*d*f*x)*b^2*d^6*f^2 + (f*x^2*e + 2*d*f*x)*a*c*d^6*f^2 + (f*x^
2*e + 2*d*f*x)*a*b*d^4*f^2 + 1/2*(f*x^2*e + 2*d*f*x)*a^2*d^2*f^2 + 1/120*(1
50*(f*x^2*e + 2*d*f*x)^2*c^2*d^8*f^4*e + 200*(f*x^2*e + 2*d*f*x)^3*c^2*d^6*
```

```
f^3*e^2 + 240*(f*x^2*e + 2*d*f*x)^2*b*c*d^6*f^4*e + 150*(f*x^2*e + 2*d*f*x)
^4*c^2*d^4*f^2*e^3 + 240*(f*x^2*e + 2*d*f*x)^3*b*c*d^4*f^3*e^2 + 90*(f*x^2*
e + 2*d*f*x)^2*b^2*d^4*f^4*e + 180*(f*x^2*e + 2*d*f*x)^2*a*c*d^4*f^4*e + 60
*(f*x^2*e + 2*d*f*x)^5*c^2*d^2*f*e^4 + 120*(f*x^2*e + 2*d*f*x)^4*b*c*d^2*f^
2*e^3 + 60*(f*x^2*e + 2*d*f*x)^3*b^2*d^2*f^3*e^2 + 120*(f*x^2*e + 2*d*f*x)^
3*a*c*d^2*f^3*e^2 + 120*(f*x^2*e + 2*d*f*x)^2*a*b*d^2*f^4*e + 10*(f*x^2*e +
 2*d*f*x)^6*c^2*e^5 + 24*(f*x^2*e + 2*d*f*x)^5*b*c*f*e^4 + 15*(f*x^2*e + 2*
d*f*x)^4*b^2*f^2*e^3 + 30*(f*x^2*e + 2*d*f*x)^4*a*c*f^2*e^3 + 40*(f*x^2*e +
 2*d*f*x)^3*a*b*f^3*e^2 + 30*(f*x^2*e + 2*d*f*x)^2*a^2*f^4*e)/f^3
```

maple [**B**] time = 0.00, size = 1413, normalized size = 13.59

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*f*x+d*f)^3*(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x)
```

```
[Out] 1/12*e^11*f^3*c^2*x^12+d*f^3*e^10*c^2*x^11+1/10*(27*d^2*f^3*e^9*c^2+e^3*f^3
*(16*c^2*d^2*e^6+2*(6*c*d^2*e^2+b*e^2)*c*e^4))*x^10+1/9*(25*d^3*f^3*c^2*e^8
+3*d*f^3*e^2*(16*c^2*d^2*e^6+2*(6*c*d^2*e^2+b*e^2)*c*e^4)+e^3*f^3*(8*(6*c*d
^2*e^2+b*e^2)*c*d*e^3+2*(4*c*d^3*e+2*b*d*e)*c*e^4))*x^9+1/8*(8*d^4*f^3*c^2*
e^7+3*d^2*f^3*e*(16*c^2*d^2*e^6+2*(6*c*d^2*e^2+b*e^2)*c*e^4)+3*d*f^3*e^2*(8
*(6*c*d^2*e^2+b*e^2)*c*d*e^3+2*(4*c*d^3*e+2*b*d*e)*c*e^4)+e^3*f^3*(8*(4*c*d
^3*e+2*b*d*e)*c*d*e^3+2*(c*d^4+b*d^2+a)*c*e^4+(6*c*d^2*e^2+b*e^2)^2))*x^8+1
/7*(d^3*f^3*(16*c^2*d^2*e^6+2*(6*c*d^2*e^2+b*e^2)*c*e^4)+3*d^2*f^3*e*(8*(6*
c*d^2*e^2+b*e^2)*c*d*e^3+2*(4*c*d^3*e+2*b*d*e)*c*e^4)+3*d*f^3*e^2*(8*(4*c*d
^3*e+2*b*d*e)*c*d*e^3+2*(c*d^4+b*d^2+a)*c*e^4+(6*c*d^2*e^2+b*e^2)^2)+e^3*f^
3*(8*(c*d^4+b*d^2+a)*c*d*e^3+2*(4*c*d^3*e+2*b*d*e)*(6*c*d^2*e^2+b*e^2)))*x^
7+1/6*(d^3*f^3*(8*(6*c*d^2*e^2+b*e^2)*c*d*e^3+2*(4*c*d^3*e+2*b*d*e)*c*e^4)+
3*d^2*f^3*e*(8*(4*c*d^3*e+2*b*d*e)*c*d*e^3+2*(c*d^4+b*d^2+a)*c*e^4+(6*c*d^2
*e^2+b*e^2)^2)+3*d*f^3*e^2*(8*(c*d^4+b*d^2+a)*c*d*e^3+2*(4*c*d^3*e+2*b*d*e)
*(6*c*d^2*e^2+b*e^2))+e^3*f^3*(2*(c*d^4+b*d^2+a)*(6*c*d^2*e^2+b*e^2)+(4*c*d
^3*e+2*b*d*e)^2))*x^6+1/5*(d^3*f^3*(8*(4*c*d^3*e+2*b*d*e)*c*d*e^3+2*(c*d^4+
b*d^2+a)*c*e^4+(6*c*d^2*e^2+b*e^2)^2)+3*d^2*f^3*e*(8*(c*d^4+b*d^2+a)*c*d*e^
3+2*(4*c*d^3*e+2*b*d*e)*(6*c*d^2*e^2+b*e^2))+3*d*f^3*e^2*(2*(c*d^4+b*d^2+a)
*(6*c*d^2*e^2+b*e^2)+(4*c*d^3*e+2*b*d*e)^2)+2*e^3*f^3*(c*d^4+b*d^2+a)*(4*c*
d^3*e+2*b*d*e))*x^5+1/4*(d^3*f^3*(8*(c*d^4+b*d^2+a)*c*d*e^3+2*(4*c*d^3*e+2*
b*d*e)*(6*c*d^2*e^2+b*e^2))+3*d^2*f^3*e*(2*(c*d^4+b*d^2+a)*(6*c*d^2*e^2+b*e
^2)+(4*c*d^3*e+2*b*d*e)^2)+6*d*f^3*e^2*(c*d^4+b*d^2+a)*(4*c*d^3*e+2*b*d*e)+
e^3*f^3*(c*d^4+b*d^2+a)^2)*x^4+1/3*(d^3*f^3*(2*(c*d^4+b*d^2+a)*(6*c*d^2*e^2
+b*e^2)+(4*c*d^3*e+2*b*d*e)^2)+6*d^2*f^3*e*(c*d^4+b*d^2+a)*(4*c*d^3*e+2*b*d
*e)+3*d*f^3*e^2*(c*d^4+b*d^2+a)^2)*x^3+1/2*(2*d^3*f^3*(c*d^4+b*d^2+a)*(4*c*
d^3*e+2*b*d*e)+3*d^2*f^3*e*(c*d^4+b*d^2+a)^2)*x^2+d^3*f^3*(c*d^4+b*d^2+a)^2
*x
```

maxima [**B**] time = 1.03, size = 439, normalized size = 4.22

$$\frac{1}{12}c^2e^{11}f^3x^{12}+c^2de^{10}f^3x^{11}+\frac{1}{10}\left(55c^2d^2+2bc\right)e^9f^3x^{10}+\frac{1}{3}\left(55c^2d^3+6bcd\right)e^8f^3x^9+\frac{1}{8}\left(330c^2d^4+72bcd^2+b^2+2ac\right)e^7f^3x^8+\left(66c^2d^5+24bcd^3+\left(b^2+2ac\right)d\right)e^6f^3x^7+\frac{1}{6}\left(462c^2d^6+252bcd^4+21\left(b^2+2ac\right)d^2+2ab\right)e^5f^3x^6+\frac{1}{5}\left(330c^2d^7+252bcd^5+35\left(b^2+2ac\right)d^3+10abd\right)e^4f^3x^5+\frac{1}{4}\left(165c^2d^8+168bcd^6+35\left(b^2+2ac\right)d^4+20abd^2+a^2\right)e^3f^3x^4+\frac{1}{3}\left(55c^2d^9+72bcd^7+21\left(b^2+2ac\right)d^5+20abd^3+3a^2d\right)e^2f^3x^3+\frac{1}{2}\left(11c^2d^{10}+18bcd^8+7\left(b^2+2ac\right)d^6+10abd^4+3a^2d^2\right)ef^3x^2+\left(c^2d^{11}+2bcd^9+\left(b^2+2ac\right)d^7+2abd^5+a^2d^3\right)f^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*f*x+d*f)^3*(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="maxima"
)
```

```
[Out] 1/12*c^2*e^11*f^3*x^12 + c^2*d*e^10*f^3*x^11 + 1/10*(55*c^2*d^2 + 2*b*c)*e^
9*f^3*x^10 + 1/3*(55*c^2*d^3 + 6*b*c*d)*e^8*f^3*x^9 + 1/8*(330*c^2*d^4 + 72
*b*c*d^2 + b^2 + 2*a*c)*e^7*f^3*x^8 + (66*c^2*d^5 + 24*b*c*d^3 + (b^2 + 2*a
*c)*d)*e^6*f^3*x^7 + 1/6*(462*c^2*d^6 + 252*b*c*d^4 + 21*(b^2 + 2*a*c)*d^2
+ 2*a*b)*e^5*f^3*x^6 + 1/5*(330*c^2*d^7 + 252*b*c*d^5 + 35*(b^2 + 2*a*c)*d^
3 + 10*a*b*d)*e^4*f^3*x^5 + 1/4*(165*c^2*d^8 + 168*b*c*d^6 + 35*(b^2 + 2*a*
c)*d^4 + 20*a*b*d^2 + a^2)*e^3*f^3*x^4 + 1/3*(55*c^2*d^9 + 72*b*c*d^7 + 21*
(b^2 + 2*a*c)*d^5 + 20*a*b*d^3 + 3*a^2*d)*e^2*f^3*x^3 + 1/2*(11*c^2*d^10 +
```

```
18*b*c*d^8 + 7*(b^2 + 2*a*c)*d^6 + 10*a*b*d^4 + 3*a^2*d^2)*e*f^3*x^2 + (c^2
*d^11 + 2*b*c*d^9 + (b^2 + 2*a*c)*d^7 + 2*a*b*d^5 + a^2*d^3)*f^3*x
```

mupad [B] time = 1.47, size = 419, normalized size = 4.03

$$\frac{e^3 f^3 x^4 \left(a^2+20\,a\,b\,d^2+70\,a\,c\,d^4+35\,b^2 d^4+168\,b\,c\,d^6+165\,c^2 d^8\right)}{4}+\frac{c^2 e^{11} f^3 x^{12}}{12}+d^3 f^3 x \left(c\,d^4+b\,d^2+a\right)^2+\frac{e^7 f^3 x^8 \left(b^2+72\,b\,c\,d^2+330\,c^2 d^4+2\,a\,c\right)}{8}+\frac{e^5 f^3 x^6 \left(21\,b^2 d^2+252\,b\,c\,d^4+2\,a\,b+462\,c^2 d^6+42\,a\,c\,d^2\right)}{6}+\frac{d^2 e f^3 x^2 \left(3\,a^2+10\,a\,b\,d^2+14\,a\,c\,d^4+7\,b^2 d^4+18\,b\,c\,d^6+11\,c^2 d^8\right)}{2}+\frac{d\,e^2 f^3 x^3 \left(3\,a^2+20\,a\,b\,d^2+42\,a\,c\,d^4+21\,b^2 d^4+72\,b\,c\,d^6+55\,c^2 d^8\right)}{3}+d\,e^6 f^3 x^7 \left(b^2+24\,b\,c\,d^2+66\,c^2 d^4+2\,a\,c\right)+\frac{d\,e^4 f^3 x^5 \left(35\,b^2 d^2+252\,b\,c\,d^4+10\,a\,b+330\,c^2 d^6+70\,a\,c\,d^2\right)}{5}+\frac{c\,e^9 f^3 x^{10} \left(55\,c\,d^2+2\,b\right)}{10}+c^2 d\,e^{10} f^3 x^{11}+\frac{c\,d\,e^8 f^3 x^9 \left(55\,c\,d^2+6\,b\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*f + e*f*x)^3*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2,x)
```

```
[Out] (e^3*f^3*x^4*(a^2 + 35*b^2*d^4 + 165*c^2*d^8 + 20*a*b*d^2 + 70*a*c*d^4 + 16
8*b*c*d^6))/4 + (c^2*e^11*f^3*x^12)/12 + d^3*f^3*x*(a + b*d^2 + c*d^4)^2 +
(e^7*f^3*x^8*(2*a*c + b^2 + 330*c^2*d^4 + 72*b*c*d^2))/8 + (e^5*f^3*x^6*(2*
a*b + 21*b^2*d^2 + 462*c^2*d^6 + 42*a*c*d^2 + 252*b*c*d^4))/6 + (d^2*e*f^3*
x^2*(3*a^2 + 7*b^2*d^4 + 11*c^2*d^8 + 10*a*b*d^2 + 14*a*c*d^4 + 18*b*c*d^6)
)/2 + (d*e^2*f^3*x^3*(3*a^2 + 21*b^2*d^4 + 55*c^2*d^8 + 20*a*b*d^2 + 42*a*c
*d^4 + 72*b*c*d^6))/3 + d*e^6*f^3*x^7*(2*a*c + b^2 + 66*c^2*d^4 + 24*b*c*d^
2) + (d*e^4*f^3*x^5*(10*a*b + 35*b^2*d^2 + 330*c^2*d^6 + 70*a*c*d^2 + 252*b
*c*d^4))/5 + (c*e^9*f^3*x^10*(2*b + 55*c*d^2))/10 + c^2*d*e^10*f^3*x^11 + (
c*d*e^8*f^3*x^9*(6*b + 55*c*d^2))/3
```

sympy [B] time = 0.21, size = 722, normalized size = 6.94

$$c^{2} d e^{10} f^{3} x^{11} + \frac{c^{2} e^{11} f^{3} x^{12}}{12} + x^{10} \left(\frac{b c e^{9} f^{3}}{5} + \frac{11 c^{2} d^{2} e^{9} f^{3}}{2}\right) + x^{9} \left(2 b c d e^{8} f^{3} + \frac{55 c^{2} d^{3} e^{8} f^{3}}{3}\right) + x^{8} \left(\frac{a c e^{7} f^{3}}{4} + \frac{b^{2} e^{7} f^{3}}{8} + 9 b c d^{2} e^{7} f^{3} + \frac{165 c^{2} d^{4} e^{7} f^{3}}{4}\right) + x^{7} \left(2 a c d e^{6} f^{3} + b^{2} d e^{6} f^{3} + 24 b c d^{3} e^{6} f^{3} + 66 c^{2} d^{5} e^{6} f^{3}\right) + x^{6} \left(\frac{a b e^{5} f^{3}}{3} + 7 a c d^{2} e^{5} f^{3} + \frac{7 b^{2} d^{2} e^{5} f^{3}}{2} + 42 b c d^{4} e^{5} f^{3} + 77 c^{2} d^{6} e^{5} f^{3}\right) + x^{5} \left(2 a b d e^{4} f^{3} + 14 a c d^{3} e^{4} f^{3} + 7 b^{2} d^{3} e^{4} f^{3} + \frac{252 b c d^{5} e^{4} f^{3}}{5} + 66 c^{2} d^{7} e^{4} f^{3}\right) + x^{4} \left(\frac{a^{2} e^{3} f^{3}}{4} + 5 a b d^{2} e^{3} f^{3} + \frac{35 a c d^{4} e^{3} f^{3}}{2} + \frac{35 b^{2} d^{4} e^{3} f^{3}}{4} + 42 b c d^{6} e^{3} f^{3} + \frac{165 c^{2} d^{8} e^{3} f^{3}}{4}\right) + x^{3} \left(a^{2} d e^{2} f^{3} + \frac{20 a b d^{3} e^{2} f^{3}}{3} + 14 a c d^{5} e^{2} f^{3} + 7 b^{2} d^{5} e^{2} f^{3} + 24 b c d^{7} e^{2} f^{3} + \frac{55 c^{2} d^{9} e^{2} f^{3}}{3}\right) + x^{2} \left(\frac{3 a^{2} d^{2} e f^{3}}{2} + 5 a b d^{4} e f^{3} + 7 a c d^{6} e f^{3} + \frac{7 b^{2} d^{6} e f^{3}}{2} + 9 b c d^{8} e f^{3} + \frac{11 c^{2} d^{10} e f^{3}}{2}\right) + x \left(a^{2} d^{3} f^{3} + 2 a b d^{5} f^{3} + 2 a c d^{7} f^{3} + b^{2} d^{7} f^{3} + 2 b c d^{9} f^{3} + c^{2} d^{11} f^{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*f*x+d*f)**3*(a+b*(e*x+d)**2+c*(e*x+d)**4)**2,x)
```

```
[Out] c**2*d*e**10*f**3*x**11 + c**2*e**11*f**3*x**12/12 + x**10*(b*c*e**9*f**3/5
 + 11*c**2*d**2*e**9*f**3/2) + x**9*(2*b*c*d*e**8*f**3 + 55*c**2*d**3*e**8*
f**3/3) + x**8*(a*c*e**7*f**3/4 + b**2*e**7*f**3/8 + 9*b*c*d**2*e**7*f**3 +
 165*c**2*d**4*e**7*f**3/4) + x**7*(2*a*c*d*e**6*f**3 + b**2*d*e**6*f**3 +
24*b*c*d**3*e**6*f**3 + 66*c**2*d**5*e**6*f**3) + x**6*(a*b*e**5*f**3/3 + 7
*a*c*d**2*e**5*f**3 + 7*b**2*d**2*e**5*f**3/2 + 42*b*c*d**4*e**5*f**3 + 77*
c**2*d**6*e**5*f**3) + x**5*(2*a*b*d*e**4*f**3 + 14*a*c*d**3*e**4*f**3 + 7*
b**2*d**3*e**4*f**3 + 252*b*c*d**5*e**4*f**3/5 + 66*c**2*d**7*e**4*f**3) +
x**4*(a**2*e**3*f**3/4 + 5*a*b*d**2*e**3*f**3 + 35*a*c*d**4*e**3*f**3/2 + 3
5*b**2*d**4*e**3*f**3/4 + 42*b*c*d**6*e**3*f**3 + 165*c**2*d**8*e**3*f**3/4
) + x**3*(a**2*d*e**2*f**3 + 20*a*b*d**3*e**2*f**3/3 + 14*a*c*d**5*e**2*f**
3 + 7*b**2*d**5*e**2*f**3 + 24*b*c*d**7*e**2*f**3 + 55*c**2*d**9*e**2*f**3/
3) + x**2*(3*a**2*d**2*e*f**3/2 + 5*a*b*d**4*e*f**3 + 7*a*c*d**6*e*f**3 + 7
*b**2*d**6*e*f**3/2 + 9*b*c*d**8*e*f**3 + 11*c**2*d**10*e*f**3/2) + x*(a**2
*d**3*f**3 + 2*a*b*d**5*f**3 + 2*a*c*d**7*f**3 + b**2*d**7*f**3 + 2*b*c*d**
9*f**3 + c**2*d**11*f**3)
```

3.499 $\int (df+efx)^3 \left(a+b(d+ex)^2+c(d+ex)^4\right)^3 \, dx$

Optimal. Leaf size=159

$$\frac{a^3f^3(d+ex)^4}{4e}+\frac{a^2bf^3(d+ex)^6}{2e}+\frac{cf^3\left(ac+b^2\right)(d+ex)^{12}}{4e}+\frac{bf^3\left(6ac+b^2\right)(d+ex)^{10}}{10e}+\frac{3af^3\left(ac+b^2\right)(d+ex)^8}{8e}+\frac{3l}{}$$

Rubi [A] time = 0.32, antiderivative size = 159, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1142, 1114, 631}

$$\frac{a^2bf^3(d+ex)^6}{2e}+\frac{a^3f^3(d+ex)^4}{4e}+\frac{cf^3\left(ac+b^2\right)(d+ex)^{12}}{4e}+\frac{bf^3\left(6ac+b^2\right)(d+ex)^{10}}{10e}+\frac{3af^3\left(ac+b^2\right)(d+ex)^8}{8e}+\frac{3bc^2f^3(d+ex)^{14}}{14e}+\frac{c^3f^3(d+ex)^{16}}{16e}$$

Antiderivative was successfully verified.

```
[In] Int[(d*f + e*f*x)^3*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3,x]
```

```
[Out] (a^3*f^3*(d + e*x)^4)/(4*e) + (a^2*b*f^3*(d + e*x)^6)/(2*e) + (3*a*(b^2 + a
*c)*f^3*(d + e*x)^8)/(8*e) + (b*(b^2 + 6*a*c)*f^3*(d + e*x)^10)/(10*e) + (c
*(b^2 + a*c)*f^3*(d + e*x)^12)/(4*e) + (3*b*c^2*f^3*(d + e*x)^14)/(14*e) +
(c^3*f^3*(d + e*x)^16)/(16*e)
```

Rule 631

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbo
l] :> Int[ExpandIntegrand[(d + e*x)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a
, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || Eq
Q[a, 0])
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 1142

```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Di
st[u^m/(Coefficient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p,
 x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

Rubi steps

$$\begin{aligned}\int (df+efx)^3\left(a+b(d+ex)^2+c(d+ex)^4\right)^3 dx &= \frac{f^3\,\text{Subst}\left(\int x^3\left(a+bx^2+cx^4\right)^3 dx, x, d+ex\right)}{e}\\ &= \frac{f^3\,\text{Subst}\left(\int x\left(a+bx+cx^2\right)^3 dx, x, (d+ex)^2\right)}{2e}\\ &= \frac{f^3\,\text{Subst}\left(\int\left(a^3x+3a^2bx^2+3a\left(b^2+ac\right)x^3+b\left(b^2+6ac\right)x^4\right.\right.}{2e}\\ &= \frac{a^3f^3(d+ex)^4}{4e}+\frac{a^2bf^3(d+ex)^6}{2e}+\frac{3a\left(b^2+ac\right)f^3(d+ex)^8}{8e}+\end{aligned}$$

Mathematica [B] time = 0.04, size = 801, normalized size = 5.04

Antiderivative was successfully verified.

```
[In] Integrate[(d*f + e*f*x)^3*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3,x]
```

```
[Out] f^3*(d^3*(a + b*d^2 + c*d^4)^3*x + (3*d^2*(a + b*d^2 + c*d^4)^2*(a + 3*b*d^
2 + 5*c*d^4)*e*x^2)/2 + d*(a^3 + 10*a^2*b*d^2 + 21*a*b^2*d^4 + 21*a^2*c*d^4
 + 12*b^3*d^6 + 72*a*b*c*d^6 + 55*b^2*c*d^8 + 55*a*c^2*d^8 + 78*b*c^2*d^10
+ 35*c^3*d^12)*e^2*x^3 + ((a^3 + 30*a^2*b*d^2 + 105*a*b^2*d^4 + 105*a^2*c*d
^4 + 84*b^3*d^6 + 504*a*b*c*d^6 + 495*b^2*c*d^8 + 495*a*c^2*d^8 + 858*b*c^2
*d^10 + 455*c^3*d^12)*e^3*x^4)/4 + (3*d*(5*a^2*b + 35*a*b^2*d^2 + 35*a^2*c*
d^2 + 42*b^3*d^4 + 252*a*b*c*d^4 + 330*b^2*c*d^6 + 330*a*c^2*d^6 + 715*b*c^
2*d^8 + 455*c^3*d^10)*e^4*x^5)/5 + ((a^2*b + 21*a*b^2*d^2 + 21*a^2*c*d^2 +
42*b^3*d^4 + 252*a*b*c*d^4 + 462*b^2*c*d^6 + 462*a*c^2*d^6 + 1287*b*c^2*d^8
 + 1001*c^3*d^10)*e^5*x^6)/2 + (d*(21*a*b^2 + 21*a^2*c + 84*b^3*d^2 + 504*a
*b*c*d^2 + 1386*b^2*c*d^4 + 1386*a*c^2*d^4 + 5148*b*c^2*d^6 + 5005*c^3*d^8)
*e^6*x^7)/7 + (3*(a*b^2 + a^2*c + 12*b^3*d^2 + 72*a*b*c*d^2 + 330*b^2*c*d^4
 + 330*a*c^2*d^4 + 1716*b*c^2*d^6 + 2145*c^3*d^8)*e^7*x^8)/8 + d*(b^3 + 6*a
*b*c + 55*b^2*c*d^2 + 55*a*c^2*d^2 + 429*b*c^2*d^4 + 715*c^3*d^6)*e^8*x^9 +
 ((b^3 + 6*a*b*c + 165*b^2*c*d^2 + 165*a*c^2*d^2 + 2145*b*c^2*d^4 + 5005*c^
3*d^6)*e^9*x^10)/10 + 3*c*d*(b^2 + a*c + 26*b*c*d^2 + 91*c^2*d^4)*e^10*x^11
 + (c*(b^2 + a*c + 78*b*c*d^2 + 455*c^2*d^4)*e^11*x^12)/4 + c^2*d*(3*b + 35
*c*d^2)*e^12*x^13 + (3*c^2*(b + 35*c*d^2)*e^13*x^14)/14 + c^3*d*e^14*x^15 +
 (c^3*e^15*x^16)/16)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (df + efx)^3 \left(a + b(d + ex)^2 + c(d + ex)^4\right)^3 \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(d*f + e*f*x)^3*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3,x]
```

```
[Out] IntegrateAlgebraic[(d*f + e*f*x)^3*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3, x
]
```

fricas [B] time = 0.73, size = 1635, normalized size = 10.28

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*f*x+d*f)^3*(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="fricas"
)
```

```
[Out] 1/16*x^16*f^3*e^15*c^3 + x^15*f^3*e^14*d*c^3 + 15/2*x^14*f^3*e^13*d^2*c^3 +
 35*x^13*f^3*e^12*d^3*c^3 + 455/4*x^12*f^3*e^11*d^4*c^3 + 3/14*x^14*f^3*e^1
3*c^2*b + 273*x^11*f^3*e^10*d^5*c^3 + 3*x^13*f^3*e^12*d*c^2*b + 1001/2*x^10
*f^3*e^9*d^6*c^3 + 39/2*x^12*f^3*e^11*d^2*c^2*b + 715*x^9*f^3*e^8*d^7*c^3 +
 78*x^11*f^3*e^10*d^3*c^2*b + 6435/8*x^8*f^3*e^7*d^8*c^3 + 429/2*x^10*f^3*e
^9*d^4*c^2*b + 1/4*x^12*f^3*e^11*c*b^2 + 1/4*x^12*f^3*e^11*c^2*a + 715*x^7*
f^3*e^6*d^9*c^3 + 429*x^9*f^3*e^8*d^5*c^2*b + 3*x^11*f^3*e^10*d*c*b^2 + 3*x
^11*f^3*e^10*d*c^2*a + 1001/2*x^6*f^3*e^5*d^10*c^3 + 1287/2*x^8*f^3*e^7*d^6
*c^2*b + 33/2*x^10*f^3*e^9*d^2*c*b^2 + 33/2*x^10*f^3*e^9*d^2*c^2*a + 273*x^
5*f^3*e^4*d^11*c^3 + 5148/7*x^7*f^3*e^6*d^7*c^2*b + 55*x^9*f^3*e^8*d^3*c*b^
2 + 55*x^9*f^3*e^8*d^3*c^2*a + 455/4*x^4*f^3*e^3*d^12*c^3 + 1287/2*x^6*f^3*
e^5*d^8*c^2*b + 495/4*x^8*f^3*e^7*d^4*c*b^2 + 1/10*x^10*f^3*e^9*b^3 + 495/4
*x^8*f^3*e^7*d^4*c^2*a + 3/5*x^10*f^3*e^9*c*b*a + 35*x^3*f^3*e^2*d^13*c^3 +
 429*x^5*f^3*e^4*d^9*c^2*b + 198*x^7*f^3*e^6*d^5*c*b^2 + x^9*f^3*e^8*d*b^3
+ 198*x^7*f^3*e^6*d^5*c^2*a + 6*x^9*f^3*e^8*d*c*b*a + 15/2*x^2*f^3*e*d^14*c
^3 + 429/2*x^4*f^3*e^3*d^10*c^2*b + 231*x^6*f^3*e^5*d^6*c*b^2 + 9/2*x^8*f^3
*e^7*d^2*b^3 + 231*x^6*f^3*e^5*d^6*c^2*a + 27*x^8*f^3*e^7*d^2*c*b*a + x*f^3
*d^15*c^3 + 78*x^3*f^3*e^2*d^11*c^2*b + 198*x^5*f^3*e^4*d^7*c*b^2 + 12*x^7*
```

```
f^3*e^6*d^3*b^3 + 198*x^5*f^3*e^4*d^7*c^2*a + 72*x^7*f^3*e^6*d^3*c*b*a + 39
/2*x^2*f^3*e*d^12*c^2*b + 495/4*x^4*f^3*e^3*d^8*c*b^2 + 21*x^6*f^3*e^5*d^4*
b^3 + 495/4*x^4*f^3*e^3*d^8*c^2*a + 126*x^6*f^3*e^5*d^4*c*b*a + 3/8*x^8*f^3
*e^7*b^2*a + 3/8*x^8*f^3*e^7*c*a^2 + 3*x*f^3*d^13*c^2*b + 55*x^3*f^3*e^2*d^
9*c*b^2 + 126/5*x^5*f^3*e^4*d^5*b^3 + 55*x^3*f^3*e^2*d^9*c^2*a + 756/5*x^5*
f^3*e^4*d^5*c*b*a + 3*x^7*f^3*e^6*d*b^2*a + 3*x^7*f^3*e^6*d*c*a^2 + 33/2*x^
2*f^3*e*d^10*c*b^2 + 21*x^4*f^3*e^3*d^6*b^3 + 33/2*x^2*f^3*e*d^10*c^2*a + 1
26*x^4*f^3*e^3*d^6*c*b*a + 21/2*x^6*f^3*e^5*d^2*b^2*a + 21/2*x^6*f^3*e^5*d^
2*c*a^2 + 3*x*f^3*d^11*c*b^2 + 12*x^3*f^3*e^2*d^7*b^3 + 3*x*f^3*d^11*c^2*a
+ 72*x^3*f^3*e^2*d^7*c*b*a + 21*x^5*f^3*e^4*d^3*b^2*a + 21*x^5*f^3*e^4*d^3*
c*a^2 + 9/2*x^2*f^3*e*d^8*b^3 + 27*x^2*f^3*e*d^8*c*b*a + 105/4*x^4*f^3*e^3*
d^4*b^2*a + 105/4*x^4*f^3*e^3*d^4*c*a^2 + 1/2*x^6*f^3*e^5*b*a^2 + x*f^3*d^9
*b^3 + 6*x*f^3*d^9*c*b*a + 21*x^3*f^3*e^2*d^5*b^2*a + 21*x^3*f^3*e^2*d^5*c*
a^2 + 3*x^5*f^3*e^4*d*b*a^2 + 21/2*x^2*f^3*e*d^6*b^2*a + 21/2*x^2*f^3*e*d^6
*c*a^2 + 15/2*x^4*f^3*e^3*d^2*b*a^2 + 3*x*f^3*d^7*b^2*a + 3*x*f^3*d^7*c*a^2
 + 10*x^3*f^3*e^2*d^3*b*a^2 + 15/2*x^2*f^3*e*d^4*b*a^2 + 1/4*x^4*f^3*e^3*a^
3 + 3*x*f^3*d^5*b*a^2 + x^3*f^3*e^2*d*a^3 + 3/2*x^2*f^3*e*d^2*a^3 + x*f^3*d
^3*a^3
```

giac [B] time = 0.51, size = 1360, normalized size = 8.55

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*f*x+d*f)^3*(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="giac")
```

```
[Out] 1/2*(f*x^2*e + 2*d*f*x)*c^3*d^14*f^2 + 3/2*(f*x^2*e + 2*d*f*x)*b*c^2*d^12*f
^2 + 3/2*(f*x^2*e + 2*d*f*x)*b^2*c*d^10*f^2 + 3/2*(f*x^2*e + 2*d*f*x)*a*c^2
*d^10*f^2 + 1/2*(f*x^2*e + 2*d*f*x)*b^3*d^8*f^2 + 3*(f*x^2*e + 2*d*f*x)*a*b
*c*d^8*f^2 + 3/2*(f*x^2*e + 2*d*f*x)*a*b^2*d^6*f^2 + 3/2*(f*x^2*e + 2*d*f*x
)*a^2*c*d^6*f^2 + 3/2*(f*x^2*e + 2*d*f*x)*a^2*b*d^4*f^2 + 1/2*(f*x^2*e + 2*
d*f*x)*a^3*d^2*f^2 + 1/560*(980*(f*x^2*e + 2*d*f*x)^2*c^3*d^12*f^6*e + 1960
*(f*x^2*e + 2*d*f*x)^3*c^3*d^10*f^5*e^2 + 2520*(f*x^2*e + 2*d*f*x)^2*b*c^2*
d^10*f^6*e + 2450*(f*x^2*e + 2*d*f*x)^4*c^3*d^8*f^4*e^3 + 4200*(f*x^2*e + 2
*d*f*x)^3*b*c^2*d^8*f^5*e^2 + 2100*(f*x^2*e + 2*d*f*x)^2*b^2*c*d^8*f^6*e +
2100*(f*x^2*e + 2*d*f*x)^2*a*c^2*d^8*f^6*e + 1960*(f*x^2*e + 2*d*f*x)^5*c^3
*d^6*f^3*e^4 + 4200*(f*x^2*e + 2*d*f*x)^4*b*c^2*d^6*f^4*e^3 + 2800*(f*x^2*e
 + 2*d*f*x)^3*b^2*c*d^6*f^5*e^2 + 2800*(f*x^2*e + 2*d*f*x)^3*a*c^2*d^6*f^5*
e^2 + 560*(f*x^2*e + 2*d*f*x)^2*b^3*d^6*f^6*e + 3360*(f*x^2*e + 2*d*f*x)^2*
a*b*c*d^6*f^6*e + 980*(f*x^2*e + 2*d*f*x)^6*c^3*d^4*f^2*e^5 + 2520*(f*x^2*e
 + 2*d*f*x)^5*b*c^2*d^4*f^3*e^4 + 2100*(f*x^2*e + 2*d*f*x)^4*b^2*c*d^4*f^4*
e^3 + 2100*(f*x^2*e + 2*d*f*x)^4*a*c^2*d^4*f^4*e^3 + 560*(f*x^2*e + 2*d*f*x
)^3*b^3*d^4*f^5*e^2 + 3360*(f*x^2*e + 2*d*f*x)^3*a*b*c*d^4*f^5*e^2 + 1260*(
f*x^2*e + 2*d*f*x)^2*a*b^2*d^4*f^6*e + 1260*(f*x^2*e + 2*d*f*x)^2*a^2*c*d^4
*f^6*e + 280*(f*x^2*e + 2*d*f*x)^7*c^3*d^2*f*e^6 + 840*(f*x^2*e + 2*d*f*x)^
6*b*c^2*d^2*f^2*e^5 + 840*(f*x^2*e + 2*d*f*x)^5*b^2*c*d^2*f^3*e^4 + 840*(f*
x^2*e + 2*d*f*x)^5*a*c^2*d^2*f^3*e^4 + 280*(f*x^2*e + 2*d*f*x)^4*b^3*d^2*f^
4*e^3 + 1680*(f*x^2*e + 2*d*f*x)^4*a*b*c*d^2*f^4*e^3 + 840*(f*x^2*e + 2*d*f
*x)^3*a*b^2*d^2*f^5*e^2 + 840*(f*x^2*e + 2*d*f*x)^3*a^2*c*d^2*f^5*e^2 + 840
*(f*x^2*e + 2*d*f*x)^2*a^2*b*d^2*f^6*e + 35*(f*x^2*e + 2*d*f*x)^8*c^3*e^7 +
 120*(f*x^2*e + 2*d*f*x)^7*b*c^2*f*e^6 + 140*(f*x^2*e + 2*d*f*x)^6*b^2*c*f^
2*e^5 + 140*(f*x^2*e + 2*d*f*x)^6*a*c^2*f^2*e^5 + 56*(f*x^2*e + 2*d*f*x)^5*
b^3*f^3*e^4 + 336*(f*x^2*e + 2*d*f*x)^5*a*b*c*f^3*e^4 + 210*(f*x^2*e + 2*d*
f*x)^4*a*b^2*f^4*e^3 + 210*(f*x^2*e + 2*d*f*x)^4*a^2*c*f^4*e^3 + 280*(f*x^2
*e + 2*d*f*x)^3*a^2*b*f^5*e^2 + 140*(f*x^2*e + 2*d*f*x)^2*a^3*f^6*e)/f^5
```

maple [B] time = 0.00, size = 7697, normalized size = 48.41

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*f*x+d*f)^3*(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x)
```

```
[Out] result too large to display
```

maxima [B] time = 1.16, size = 920, normalized size = 5.79

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*f*x+d*f)^3*(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="maxima"
)
```

```
[Out] 1/16*c^3*e^15*f^3*x^16 + c^3*d*e^14*f^3*x^15 + 3/14*(35*c^3*d^2 + b*c^2)*e^
13*f^3*x^14 + (35*c^3*d^3 + 3*b*c^2*d)*e^12*f^3*x^13 + 1/4*(455*c^3*d^4 + 7
8*b*c^2*d^2 + b^2*c + a*c^2)*e^11*f^3*x^12 + 3*(91*c^3*d^5 + 26*b*c^2*d^3 +
 (b^2*c + a*c^2)*d)*e^10*f^3*x^11 + 1/10*(5005*c^3*d^6 + 2145*b*c^2*d^4 + b
^3 + 6*a*b*c + 165*(b^2*c + a*c^2)*d^2)*e^9*f^3*x^10 + (715*c^3*d^7 + 429*b
*c^2*d^5 + 55*(b^2*c + a*c^2)*d^3 + (b^3 + 6*a*b*c)*d)*e^8*f^3*x^9 + 3/8*(2
145*c^3*d^8 + 1716*b*c^2*d^6 + 330*(b^2*c + a*c^2)*d^4 + a*b^2 + a^2*c + 12
*(b^3 + 6*a*b*c)*d^2)*e^7*f^3*x^8 + 1/7*(5005*c^3*d^9 + 5148*b*c^2*d^7 + 13
86*(b^2*c + a*c^2)*d^5 + 84*(b^3 + 6*a*b*c)*d^3 + 21*(a*b^2 + a^2*c)*d)*e^6
*f^3*x^7 + 1/2*(1001*c^3*d^10 + 1287*b*c^2*d^8 + 462*(b^2*c + a*c^2)*d^6 +
42*(b^3 + 6*a*b*c)*d^4 + a^2*b + 21*(a*b^2 + a^2*c)*d^2)*e^5*f^3*x^6 + 3/5*
(455*c^3*d^11 + 715*b*c^2*d^9 + 330*(b^2*c + a*c^2)*d^7 + 42*(b^3 + 6*a*b*c
)*d^5 + 5*a^2*b*d + 35*(a*b^2 + a^2*c)*d^3)*e^4*f^3*x^5 + 1/4*(455*c^3*d^12
 + 858*b*c^2*d^10 + 495*(b^2*c + a*c^2)*d^8 + 84*(b^3 + 6*a*b*c)*d^6 + 30*a
^2*b*d^2 + 105*(a*b^2 + a^2*c)*d^4 + a^3)*e^3*f^3*x^4 + (35*c^3*d^13 + 78*b
*c^2*d^11 + 55*(b^2*c + a*c^2)*d^9 + 12*(b^3 + 6*a*b*c)*d^7 + 10*a^2*b*d^3
+ 21*(a*b^2 + a^2*c)*d^5 + a^3*d)*e^2*f^3*x^3 + 3/2*(5*c^3*d^14 + 13*b*c^2*
d^12 + 11*(b^2*c + a*c^2)*d^10 + 3*(b^3 + 6*a*b*c)*d^8 + 5*a^2*b*d^4 + 7*(a
*b^2 + a^2*c)*d^6 + a^3*d^2)*e*f^3*x^2 + (c^3*d^15 + 3*b*c^2*d^13 + 3*(b^2*
c + a*c^2)*d^11 + (b^3 + 6*a*b*c)*d^9 + 3*a^2*b*d^5 + 3*(a*b^2 + a^2*c)*d^7
 + a^3*d^3)*f^3*x
```

mupad [B] time = 1.65, size = 825, normalized size = 5.19

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*f + e*f*x)^3*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3,x)
```

```
[Out] (3*e^7*f^3*x^8*(a*b^2 + a^2*c + 12*b^3*d^2 + 2145*c^3*d^8 + 330*a*c^2*d^4 +
 330*b^2*c*d^4 + 1716*b*c^2*d^6 + 72*a*b*c*d^2))/8 + (e^5*f^3*x^6*(a^2*b +
42*b^3*d^4 + 1001*c^3*d^10 + 21*a*b^2*d^2 + 21*a^2*c*d^2 + 462*a*c^2*d^6 +
462*b^2*c*d^6 + 1287*b*c^2*d^8 + 252*a*b*c*d^4))/2 + (e^9*f^3*x^10*(b^3 + 5
005*c^3*d^6 + 165*a*c^2*d^2 + 165*b^2*c*d^2 + 2145*b*c^2*d^4 + 6*a*b*c))/10
 + (c^3*e^15*f^3*x^16)/16 + d^3*f^3*x*(a + b*d^2 + c*d^4)^3 + (e^3*f^3*x^4*
(a^3 + 84*b^3*d^6 + 455*c^3*d^12 + 30*a^2*b*d^2 + 105*a*b^2*d^4 + 105*a^2*c
*d^4 + 495*a*c^2*d^8 + 495*b^2*c*d^8 + 858*b*c^2*d^10 + 504*a*b*c*d^6))/4 +
 (c*e^11*f^3*x^12*(a*c + b^2 + 455*c^2*d^4 + 78*b*c*d^2))/4 + (d*e^6*f^3*x^
7*(21*a*b^2 + 21*a^2*c + 84*b^3*d^2 + 5005*c^3*d^8 + 1386*a*c^2*d^4 + 1386*
b^2*c*d^4 + 5148*b*c^2*d^6 + 504*a*b*c*d^2))/7 + (3*d*e^4*f^3*x^5*(5*a^2*b
+ 42*b^3*d^4 + 455*c^3*d^10 + 35*a*b^2*d^2 + 35*a^2*c*d^2 + 330*a*c^2*d^6 +
 330*b^2*c*d^6 + 715*b*c^2*d^8 + 252*a*b*c*d^4))/5 + d*e^8*f^3*x^9*(b^3 + 7
15*c^3*d^6 + 55*a*c^2*d^2 + 55*b^2*c*d^2 + 429*b*c^2*d^4 + 6*a*b*c) + (3*c^
2*e^13*f^3*x^14*(b + 35*c*d^2))/14 + c^3*d*e^14*f^3*x^15 + d*e^2*f^3*x^3*(a
^3 + 12*b^3*d^6 + 35*c^3*d^12 + 10*a^2*b*d^2 + 21*a*b^2*d^4 + 21*a^2*c*d^4
+ 55*a*c^2*d^8 + 55*b^2*c*d^8 + 78*b*c^2*d^10 + 72*a*b*c*d^6) + (3*d^2*e*f^
3*x^2*(a + b*d^2 + c*d^4)^2*(a + 3*b*d^2 + 5*c*d^4))/2 + c^2*d*e^12*f^3*x^1
```

```
3*(3*b + 35*c*d^2) + 3*c*d*e^10*f^3*x^11*(a*c + b^2 + 91*c^2*d^4 + 26*b*c*d
^2)
```

sympy [B] time = 0.39, size = 1654, normalized size = 10.40

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*f*x+d*f)**3*(a+b*(e*x+d)**2+c*(e*x+d)**4)**3,x)
```

```
[Out] c**3*d*e**14*f**3*x**15 + c**3*e**15*f**3*x**16/16 + x**14*(3*b*c**2*e**13*
f**3/14 + 15*c**3*d**2*e**13*f**3/2) + x**13*(3*b*c**2*d*e**12*f**3 + 35*c*
*3*d**3*e**12*f**3) + x**12*(a*c**2*e**11*f**3/4 + b**2*c*e**11*f**3/4 + 39
*b*c**2*d**2*e**11*f**3/2 + 455*c**3*d**4*e**11*f**3/4) + x**11*(3*a*c**2*d
*e**10*f**3 + 3*b**2*c*d*e**10*f**3 + 78*b*c**2*d**3*e**10*f**3 + 273*c**3*
d**5*e**10*f**3) + x**10*(3*a*b*c*e**9*f**3/5 + 33*a*c**2*d**2*e**9*f**3/2
+ b**3*e**9*f**3/10 + 33*b**2*c*d**2*e**9*f**3/2 + 429*b*c**2*d**4*e**9*f**
3/2 + 1001*c**3*d**6*e**9*f**3/2) + x**9*(6*a*b*c*d*e**8*f**3 + 55*a*c**2*d
**3*e**8*f**3 + b**3*d*e**8*f**3 + 55*b**2*c*d**3*e**8*f**3 + 429*b*c**2*d*
*5*e**8*f**3 + 715*c**3*d**7*e**8*f**3) + x**8*(3*a**2*c*e**7*f**3/8 + 3*a*
b**2*e**7*f**3/8 + 27*a*b*c*d**2*e**7*f**3 + 495*a*c**2*d**4*e**7*f**3/4 +
9*b**3*d**2*e**7*f**3/2 + 495*b**2*c*d**4*e**7*f**3/4 + 1287*b*c**2*d**6*e*
*7*f**3/2 + 6435*c**3*d**8*e**7*f**3/8) + x**7*(3*a**2*c*d*e**6*f**3 + 3*a*
b**2*d*e**6*f**3 + 72*a*b*c*d**3*e**6*f**3 + 198*a*c**2*d**5*e**6*f**3 + 12
*b**3*d**3*e**6*f**3 + 198*b**2*c*d**5*e**6*f**3 + 5148*b*c**2*d**7*e**6*f*
*3/7 + 715*c**3*d**9*e**6*f**3) + x**6*(a**2*b*e**5*f**3/2 + 21*a**2*c*d**2
*e**5*f**3/2 + 21*a*b**2*d**2*e**5*f**3/2 + 126*a*b*c*d**4*e**5*f**3 + 231*
a*c**2*d**6*e**5*f**3 + 21*b**3*d**4*e**5*f**3 + 231*b**2*c*d**6*e**5*f**3
+ 1287*b*c**2*d**8*e**5*f**3/2 + 1001*c**3*d**10*e**5*f**3/2) + x**5*(3*a**
2*b*d*e**4*f**3 + 21*a**2*c*d**3*e**4*f**3 + 21*a*b**2*d**3*e**4*f**3 + 756
*a*b*c*d**5*e**4*f**3/5 + 198*a*c**2*d**7*e**4*f**3 + 126*b**3*d**5*e**4*f*
*3/5 + 198*b**2*c*d**7*e**4*f**3 + 429*b*c**2*d**9*e**4*f**3 + 273*c**3*d**
11*e**4*f**3) + x**4*(a**3*e**3*f**3/4 + 15*a**2*b*d**2*e**3*f**3/2 + 105*a
**2*c*d**4*e**3*f**3/4 + 105*a*b**2*d**4*e**3*f**3/4 + 126*a*b*c*d**6*e**3*
f**3 + 495*a*c**2*d**8*e**3*f**3/4 + 21*b**3*d**6*e**3*f**3 + 495*b**2*c*d*
*8*e**3*f**3/4 + 429*b*c**2*d**10*e**3*f**3/2 + 455*c**3*d**12*e**3*f**3/4)
 + x**3*(a**3*d*e**2*f**3 + 10*a**2*b*d**3*e**2*f**3 + 21*a**2*c*d**5*e**2*
f**3 + 21*a*b**2*d**5*e**2*f**3 + 72*a*b*c*d**7*e**2*f**3 + 55*a*c**2*d**9*
e**2*f**3 + 12*b**3*d**7*e**2*f**3 + 55*b**2*c*d**9*e**2*f**3 + 78*b*c**2*d
**11*e**2*f**3 + 35*c**3*d**13*e**2*f**3) + x**2*(3*a**3*d**2*e*f**3/2 + 15
*a**2*b*d**4*e*f**3/2 + 21*a**2*c*d**6*e*f**3/2 + 21*a*b**2*d**6*e*f**3/2 +
 27*a*b*c*d**8*e*f**3 + 33*a*c**2*d**10*e*f**3/2 + 9*b**3*d**8*e*f**3/2 + 3
3*b**2*c*d**10*e*f**3/2 + 39*b*c**2*d**12*e*f**3/2 + 15*c**3*d**14*e*f**3/2
) + x*(a**3*d**3*f**3 + 3*a**2*b*d**5*f**3 + 3*a**2*c*d**7*f**3 + 3*a*b**2*
d**7*f**3 + 6*a*b*c*d**9*f**3 + 3*a*c**2*d**11*f**3 + b**3*d**9*f**3 + 3*b*
*2*c*d**11*f**3 + 3*b*c**2*d**13*f**3 + c**3*d**15*f**3)
```

3.500 $\int \frac{(d+ex)^4}{a+b(d+ex)^2+c(d+ex)^4}\,dx$

Optimal. Leaf size=193

$$-\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{3/2}e\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}c^{3/2}e\sqrt{\sqrt{b^2-4ac}+b}}+\frac{x}{c}$$

Rubi [A] time = 0.44, antiderivative size = 193, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {1142, 1122, 1166, 205}

$$-\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{3/2}e\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}c^{3/2}e\sqrt{\sqrt{b^2-4ac}+b}}+\frac{x}{c}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^4/(a + b*(d + e*x)^2 + c*(d + e*x)^4),x]
```

```
[Out] x/c - ((b - (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e
*x))/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a
*c]]*e) - ((b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d
 + e*x))/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(3/2)*Sqrt[b + Sqrt[b^2 -
 4*a*c]]*e)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1122

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Simp[(d^3*(d*x)^(m - 3)*(a + b*x^2 + c*x^4)^(p + 1))/(c*(m + 4*p + 1)),
 x] - Dist[d^4/(c*(m + 4*p + 1)), Int[(d*x)^(m - 4)*Simp[a*(m - 3) + b*(m +
 2*p - 1)*x^2, x]*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x
] && NeQ[b^2 - 4*a*c, 0] && GtQ[m, 3] && NeQ[m + 4*p + 1, 0] && IntegerQ[2*
p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1142

```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Di
st[u^m/(Coefficient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p,
 x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{(d+ex)^4}{a+b(d+ex)^2+c(d+ex)^4}\,dx = \frac{\text{Subst}\left(\int \frac{x^4}{a+bx^2+cx^4}\,dx, x, d+ex\right)}{e}$$
$$= \frac{x}{c} - \frac{\text{Subst}\left(\int \frac{a+bx^2}{a+bx^2+cx^4}\,dx, x, d+ex\right)}{ce}$$
$$= \frac{x}{c} - \frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\text{Subst}\left(\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx, x, d+ex\right)}{2ce} - \frac{\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\text{Su}}{}$$
$$= \frac{x}{c} - \frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{3/2}\sqrt{b-\sqrt{b^2-4ac}}\,e} - \frac{\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{3/2}\sqrt{b+\sqrt{b^2-4ac}}\,e}$$

Mathematica [A] time = 0.14, size = 219, normalized size = 1.13

$$\frac{-\frac{\sqrt{2}\left(b\sqrt{b^2-4ac}+2ac-b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}} - \frac{\sqrt{2}\left(b\sqrt{b^2-4ac}-2ac+b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4ac}+b}} + 2\sqrt{c}(d+ex)}{2c^{3/2}e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^4/(a + b*(d + e*x)^2 + c*(d + e*x)^4),x]
```

```
[Out] (2*Sqrt[c]*(d + e*x) - (Sqrt[2]*(-b^2 + 2*a*c + b*Sqrt[b^2 - 4*a*c])*ArcTan
[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2 - 4*a*
c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - (Sqrt[2]*(b^2 - 2*a*c + b*Sqrt[b^2 - 4*a*
c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[
b^2 - 4*a*c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(2*c^(3/2)*e)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(d+ex)^4}{a+b(d+ex)^2+c(d+ex)^4}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(d + e*x)^4/(a + b*(d + e*x)^2 + c*(d + e*x)^4),x]
```

```
[Out] IntegrateAlgebraic[(d + e*x)^4/(a + b*(d + e*x)^2 + c*(d + e*x)^4), x]
```

fricas [B] time = 2.18, size = 1231, normalized size = 6.38

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^4/(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="fricas")
```

```
[Out] 1/2*(sqrt(1/2)*c*sqrt(-((b^2*c^3 - 4*a*c^4)*e^2*sqrt((b^4 - 2*a*b^2*c + a^2
*c^2)/((b^2*c^6 - 4*a*c^7)*e^4)) + b^3 - 3*a*b*c)/((b^2*c^3 - 4*a*c^4)*e^2)
)*log(-2*(a*b^2 - a^2*c)*e*x - 2*(a*b^2 - a^2*c)*d + sqrt(1/2)*((b^3*c^3 -
4*a*b*c^4)*e^3*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/((b^2*c^6 - 4*a*c^7)*e^4))
- (b^4 - 5*a*b^2*c + 4*a^2*c^2)*e)*sqrt(-((b^2*c^3 - 4*a*c^4)*e^2*sqrt((b^4
 - 2*a*b^2*c + a^2*c^2)/((b^2*c^6 - 4*a*c^7)*e^4)) + b^3 - 3*a*b*c)/((b^2*c
^3 - 4*a*c^4)*e^2))) - sqrt(1/2)*c*sqrt(-((b^2*c^3 - 4*a*c^4)*e^2*sqrt((b^4
 - 2*a*b^2*c + a^2*c^2)/((b^2*c^6 - 4*a*c^7)*e^4)) + b^3 - 3*a*b*c)/((b^2*c
```

```
^3 - 4*a*c^4)*e^2))*log(-2*(a*b^2 - a^2*c)*e*x - 2*(a*b^2 - a^2*c)*d - sqrt
(1/2)*((b^3*c^3 - 4*a*b*c^4)*e^3*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/((b^2*c^6
 - 4*a*c^7)*e^4)) - (b^4 - 5*a*b^2*c + 4*a^2*c^2)*e)*sqrt(-((b^2*c^3 - 4*a*
c^4)*e^2*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/((b^2*c^6 - 4*a*c^7)*e^4)) + b^3
- 3*a*b*c)/((b^2*c^3 - 4*a*c^4)*e^2))) - sqrt(1/2)*c*sqrt(((b^2*c^3 - 4*a*c
^4)*e^2*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/((b^2*c^6 - 4*a*c^7)*e^4)) - b^3 +
 3*a*b*c)/((b^2*c^3 - 4*a*c^4)*e^2))*log(-2*(a*b^2 - a^2*c)*e*x - 2*(a*b^2
- a^2*c)*d + sqrt(1/2)*((b^3*c^3 - 4*a*b*c^4)*e^3*sqrt((b^4 - 2*a*b^2*c + a
^2*c^2)/((b^2*c^6 - 4*a*c^7)*e^4)) + (b^4 - 5*a*b^2*c + 4*a^2*c^2)*e)*sqrt(
((b^2*c^3 - 4*a*c^4)*e^2*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/((b^2*c^6 - 4*a*c
^7)*e^4)) - b^3 + 3*a*b*c)/((b^2*c^3 - 4*a*c^4)*e^2))) + sqrt(1/2)*c*sqrt((
(b^2*c^3 - 4*a*c^4)*e^2*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/((b^2*c^6 - 4*a*c^
7)*e^4)) - b^3 + 3*a*b*c)/((b^2*c^3 - 4*a*c^4)*e^2))*log(-2*(a*b^2 - a^2*c)
*e*x - 2*(a*b^2 - a^2*c)*d - sqrt(1/2)*((b^3*c^3 - 4*a*b*c^4)*e^3*sqrt((b^4
 - 2*a*b^2*c + a^2*c^2)/((b^2*c^6 - 4*a*c^7)*e^4)) + (b^4 - 5*a*b^2*c + 4*a
^2*c^2)*e)*sqrt(((b^2*c^3 - 4*a*c^4)*e^2*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(
(b^2*c^6 - 4*a*c^7)*e^4)) - b^3 + 3*a*b*c)/((b^2*c^3 - 4*a*c^4)*e^2))) + 2*
x)/c
```

giac [B] time = 0.43, size = 1194, normalized size = 6.19

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^4/(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="giac")
```

```
[Out] 1/2*(((d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c)
)^2*b*e^6 - 2*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e
^(-4)/c))*b*d*e^5 + b*d^2*e^4 + a*e^4)*log(d*e^(-1) + x + sqrt(1/2)*sqrt(-(
b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))/(2*(d*e^(-1) + sqrt(1/2)*sqrt(-(b
*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^3*c*e^4 - 6*(d*e^(-1) + sqrt(1/2)*
sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^2*c*d*e^3 - 2*c*d^3*e - b*
d*e + (6*c*d^2*e^2 + b*e^2)*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 -
 4*a*c)*e^2)*e^(-4)/c))) + ((d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 -
 4*a*c)*e^2)*e^(-4)/c))^2*b*e^6 - 2*(d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 + sq
rt(b^2 - 4*a*c)*e^2)*e^(-4)/c))*b*d*e^5 + b*d^2*e^4 + a*e^4)*log(d*e^(-1) +
 x - sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))/(2*(d*e^(-1
) - sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^3*c*e^4 - 6*
(d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^2*c*
d*e^3 - 2*c*d^3*e - b*d*e + (6*c*d^2*e^2 + b*e^2)*(d*e^(-1) - sqrt(1/2)*sqr
t(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))) + ((d*e^(-1) + sqrt(1/2)*sqr
t(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^2*b*e^6 - 2*(d*e^(-1) + sqrt(
1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))*b*d*e^5 + b*d^2*e^4 +
 a*e^4)*log(d*e^(-1) + x + sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*
e^(-4)/c))/(2*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e
^(-4)/c))^3*c*e^4 - 6*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c
)*e^2)*e^(-4)/c))^2*c*d*e^3 - 2*c*d^3*e - b*d*e + (6*c*d^2*e^2 + b*e^2)*(d*
e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))) + ((d*
e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^2*b*e^6
 - 2*(d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))
*b*d*e^5 + b*d^2*e^4 + a*e^4)*log(d*e^(-1) + x - sqrt(1/2)*sqrt(-(b*e^2 - s
qrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))/(2*(d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 - sq
rt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^3*c*e^4 - 6*(d*e^(-1) - sqrt(1/2)*sqrt(-(b*
e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^2*c*d*e^3 - 2*c*d^3*e - b*d*e + (6*
c*d^2*e^2 + b*e^2)*(d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e
^2)*e^(-4)/c))))*e^(-4)/c + x/c
```

maple [C] time = 0.07, size = 158, normalized size = 0.82

$$\frac{x}{c}+\frac{\left(-\mathrm{RootOf}\left(\_Z^{4} c\,e^{4}+4\_Z^{3} c d\,e^{3}+c\,d^{4}+b\,d^{2}+\left(6 c\,d^{2} e^{2}+b\,e^{2}\right)\_Z^{2}+\left(4 c\,d^{3} e+2 d e b\right)\_Z+a\right)^{2} b\,e^{2}-2\,\mathrm{RootOf}\left(\_Z^{4} c\,e^{4}+4\_Z^{3} c d\,e^{3}+c\,d^{4}+b\,d^{2}+\left(6 c\,d^{2} e^{2}+b\,e^{2}\right)\_Z^{2}+\left(4 c\,d^{3} e+2 d e b\right)\_Z+a\right) b d e-b\,d^{2}-a\right) \ln\left(-\mathrm{RootOf}\left(\_Z^{4} c\,e^{4}+4\_Z^{3} c d\,e^{3}+c\,d^{4}+b\,d^{2}+\left(6 c\,d^{2} e^{2}+b\,e^{2}\right)\_Z^{2}+\left(4 c\,d^{3} e+2 d e b\right)\_Z+a\right)+x\right)}{2 c e \left(2 c\,e^{3}\,\mathrm{RootOf}\left(\_Z^{4} c\,e^{4}+4\_Z^{3} c d\,e^{3}+c\,d^{4}+b\,d^{2}+\left(6 c\,d^{2} e^{2}+b\,e^{2}\right)\_Z^{2}+\left(4 c\,d^{3} e+2 d e b\right)\_Z+a\right)^{3}+6 c d\,e^{2}\,\mathrm{RootOf}\left(\_Z^{4} c\,e^{4}+4\_Z^{3} c d\,e^{3}+c\,d^{4}+b\,d^{2}+\left(6 c\,d^{2} e^{2}+b\,e^{2}\right)\_Z^{2}+\left(4 c\,d^{3} e+2 d e b\right)\_Z+a\right)^{2}+6 e c\,d^{2}\,\mathrm{RootOf}\left(\_Z^{4} c\,e^{4}+4\_Z^{3} c d\,e^{3}+c\,d^{4}+b\,d^{2}+\left(6 c\,d^{2} e^{2}+b\,e^{2}\right)\_Z^{2}+\left(4 c\,d^{3} e+2 d e b\right)\_Z+a\right)+2 c\,d^{3}+b e\,\mathrm{RootOf}\left(\_Z^{4} c\,e^{4}+4\_Z^{3} c d\,e^{3}+c\,d^{4}+b\,d^{2}+\left(6 c\,d^{2} e^{2}+b\,e^{2}\right)\_Z^{2}+\left(4 c\,d^{3} e+2 d e b\right)\_Z+a\right)+b d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e*x+d)^4/(a+b*(e*x+d)^2+c*(e*x+d)^4),x)

[Out] 1/c*x+1/2/c/e*sum((-_R^2*b*e^2-2*_R*b*d*e-b*d^2-a)/(2*_R^3*c*e^3+6*_R^2*c*d*e^2+6*_R*c*d^2*e+2*c*d^3+_R*b*e+b*d)*ln(-_R+x),_R=RootOf(c*e^4*_Z^4+4*c*d*e^3*_Z^3+(6*c*d^2*e^2+b*e^2)*_Z^2+(4*c*d^3*e+2*b*d*e)*_Z+c*d^4+b*d^2+a))

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x+d)^4/(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="maxima")

[Out] Timed out

mupad [B] time = 2.32, size = 3988, normalized size = 20.66

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d + e*x)^4/(a + b*(d + e*x)^2 + c*(d + e*x)^4),x)

[Out] atan(((-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5*e^2 + b^4*c^3*e^2 - 8*a*b^2*c^4*e^2)))^(1/2)*(((16*a^2*c^3*e^12 - 4*a*b^2*c^2*e^12)/c + ((8*b^3*c^3*d*e^13 - 32*a*b*c^4*d*e^13)/c + (2*x*(4*b^3*c^3*e^14 - 16*a*b*c^4*e^14))/c)*(-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5*e^2 + b^4*c^3*e^2 - 8*a*b^2*c^4*e^2)))^(1/2))*(-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5*e^2 + b^4*c^3*e^2 - 8*a*b^2*c^4*e^2)))^(1/2) + (2*b^4*d*e^11 + 4*a^2*c^2*d*e^11 - 8*a*b^2*c*d*e^11)/c + (2*x*(b^4*e^12 + 2*a^2*c^2*e^12 - 4*a*b^2*c*e^12))/c)*1i + (-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5*e^2 + b^4*c^3*e^2 - 8*a*b^2*c^4*e^2)))^(1/2)*((2*b^4*d*e^11 + 4*a^2*c^2*d*e^11 - 8*a*b^2*c*d*e^11)/c - ((16*a^2*c^3*e^12 - 4*a*b^2*c^2*e^12)/c - ((8*b^3*c^3*d*e^13 - 32*a*b*c^4*d*e^13)/c + (2*x*(4*b^3*c^3*e^14 - 16*a*b*c^4*e^14))/c)*(-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5*e^2 + b^4*c^3*e^2 - 8*a*b^2*c^4*e^2)))^(1/2))*(-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5*e^2 + b^4*c^3*e^2 - 8*a*b^2*c^4*e^2)))^(1/2) + (2*x*(b^4*e^12 + 2*a^2*c^2*e^12 - 4*a*b^2*c*e^12))/c)*1i)/((-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5*e^2 + b^4*c^3*e^2 - 8*a*b^2*c^4*e^2)))^(1/2)*(((16*a^2*c^3*e^12 - 4*a*b^2*c^2*e^12)/c + ((8*b^3*c^3*d*e^13 - 32*a*b*c^4*d*e^13)/c + (2*x*(4*b^3*c^3*e^14 - 16*a*b*c^4*e^14))/c)*(-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5*e^2 + b^4*c^3*e^2 - 8*a*b^2*c^4*e^2)))^(1/2))*(-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5*e^2 + b^4*c^3*e^2 - 8*a*b^2*c^4*e^2)))^(1/2) + (2*b^4*d*e^11 + 4*a^2*c^2*d*e^11 - 8*a*b^2*c*d*e^11)/c + (2*x*(b^4*e^12 + 2*a^2*c^2*e^12 - 4*a*b^2*c*e^12))/c) - (-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5*e^2 + b^4*c^3*e^2 - 8*a*b^2*c^4*e^2)))^(1/2)*((2*b^4*d*e^11 + 4*a^2*c^2*d*e^11 - 8*a*b^2*c*d*e^11)/c - ((16*a^2*c^3*e^12 - 4*a*b^2*c^2*e^12)/c - ((8*b^3*c^3*d*e^13 - 32*a*b*c^4*d*e^13)/c + (2*x*(4*b^3*c^3*e^14 - 16*a*b*c^4*e^14))/c)*(-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5*e^2 + b^4*c^3*e^2 - 8*a*b^2*c^4*e^2)))^(1/2))*(-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^

```
2*c^5*e^2 + b^4*c^3*e^2 - 8*a*b^2*c^4*e^2)))^(1/2) + (2*x*(b^4*e^12 + 2*a^2
*c^2*e^12 - 4*a*b^2*c*e^12))/c) + (2*a^2*b*e^10)/c))*(-(b^5 + b^2*(-(4*a*c
- b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/
(8*(16*a^2*c^5*e^2 + b^4*c^3*e^2 - 8*a*b^2*c^4*e^2)))^(1/2)*2i + atan(((-(b
^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c
 - b^2)^3)^(1/2))/(8*(16*a^2*c^5*e^2 + b^4*c^3*e^2 - 8*a*b^2*c^4*e^2)))^(1/
2)*(((16*a^2*c^3*e^12 - 4*a*b^2*c^2*e^12)/c + ((8*b^3*c^3*d*e^13 - 32*a*b*c
^4*d*e^13)/c + (2*x*(4*b^3*c^3*e^14 - 16*a*b*c^4*e^14))/c)*(-(b^5 - b^2*(-(
4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(
1/2))/(8*(16*a^2*c^5*e^2 + b^4*c^3*e^2 - 8*a*b^2*c^4*e^2)))^(1/2))*(-(b^5 -
 b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b
^2)^3)^(1/2))/(8*(16*a^2*c^5*e^2 + b^4*c^3*e^2 - 8*a*b^2*c^4*e^2)))^(1/2) +
 (2*b^4*d*e^11 + 4*a^2*c^2*d*e^11 - 8*a*b^2*c*d*e^11)/c + (2*x*(b^4*e^12 +
2*a^2*c^2*e^12 - 4*a*b^2*c*e^12))/c)*1i + (-(b^5 - b^2*(-(4*a*c - b^2)^3)^(
1/2) + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*
c^5*e^2 + b^4*c^3*e^2 - 8*a*b^2*c^4*e^2)))^(1/2)*((2*b^4*d*e^11 + 4*a^2*c^2
*d*e^11 - 8*a*b^2*c*d*e^11)/c - ((16*a^2*c^3*e^12 - 4*a*b^2*c^2*e^12)/c - (
(8*b^3*c^3*d*e^13 - 32*a*b*c^4*d*e^13)/c + (2*x*(4*b^3*c^3*e^14 - 16*a*b*c^
4*e^14))/c)*(-(b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*
c + a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5*e^2 + b^4*c^3*e^2 - 8*a*b^
2*c^4*e^2)))^(1/2))*(-(b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 -
7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5*e^2 + b^4*c^3*e^2
- 8*a*b^2*c^4*e^2)))^(1/2) + (2*x*(b^4*e^12 + 2*a^2*c^2*e^12 - 4*a*b^2*c*e^
12))/c)*1i)/((-(b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3
*c + a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5*e^2 + b^4*c^3*e^2 - 8*a*b
^2*c^4*e^2)))^(1/2)*(((16*a^2*c^3*e^12 - 4*a*b^2*c^2*e^12)/c + ((8*b^3*c^3*
d*e^13 - 32*a*b*c^4*d*e^13)/c + (2*x*(4*b^3*c^3*e^14 - 16*a*b*c^4*e^14))/c)
*(-(b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(
4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5*e^2 + b^4*c^3*e^2 - 8*a*b^2*c^4*e^2))
)^(1/2))*(-(b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c +
 a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5*e^2 + b^4*c^3*e^2 - 8*a*b^2*c
^4*e^2)))^(1/2) + (2*b^4*d*e^11 + 4*a^2*c^2*d*e^11 - 8*a*b^2*c*d*e^11)/c +
(2*x*(b^4*e^12 + 2*a^2*c^2*e^12 - 4*a*b^2*c*e^12))/c) - (-(b^5 - b^2*(-(4*a
*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2
))/(8*(16*a^2*c^5*e^2 + b^4*c^3*e^2 - 8*a*b^2*c^4*e^2)))^(1/2)*((2*b^4*d*e^
11 + 4*a^2*c^2*d*e^11 - 8*a*b^2*c*d*e^11)/c - ((16*a^2*c^3*e^12 - 4*a*b^2*c
^2*e^12)/c - ((8*b^3*c^3*d*e^13 - 32*a*b*c^4*d*e^13)/c + (2*x*(4*b^3*c^3*e^
14 - 16*a*b*c^4*e^14))/c)*(-(b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*
c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5*e^2 + b^4*c^
3*e^2 - 8*a*b^2*c^4*e^2)))^(1/2))*(-(b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 1
2*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5*e^2
+ b^4*c^3*e^2 - 8*a*b^2*c^4*e^2)))^(1/2) + (2*x*(b^4*e^12 + 2*a^2*c^2*e^12
- 4*a*b^2*c*e^12))/c) + (2*a^2*b*e^10)/c))*(-(b^5 - b^2*(-(4*a*c - b^2)^3)^
(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2
*c^5*e^2 + b^4*c^3*e^2 - 8*a*b^2*c^4*e^2)))^(1/2)*2i + x/c
```

sympy [A] time = 3.07, size = 178, normalized size = 0.92

$$\operatorname{RootSum}\left(t^4\left(256a^2c^5e^4-128ab^2c^4e^4+16b^4c^3e^4\right)+t^2\left(48a^2bc^2e^2-28ab^3ce^2+4b^5e^2\right)+a^3,\left(t\mapsto t\log\left(x+\frac{32t^3abc^4e^3-8t^3b^3c^3e^3-4ta^2c^2e+8tab^2ce-2tb^4e+a^2cd-ab^2d}{a^2ce-ab^2e}\right)\right)\right)+\frac{x}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**4/(a+b*(e*x+d)**2+c*(e*x+d)**4),x)
```

```
[Out] RootSum(_t**4*(256*a**2*c**5*e**4 - 128*a*b**2*c**4*e**4 + 16*b**4*c**3*e**
4) + _t**2*(48*a**2*b*c**2*e**2 - 28*a*b**3*c*e**2 + 4*b**5*e**2) + a**3, L
ambda(_t, _t*log(x + (32*_t**3*a*b*c**4*e**3 - 8*_t**3*b**3*c**3*e**3 - 4*_
t*a**2*c**2*e + 8*_t*a*b**2*c*e - 2*_t*b**4*e + a**2*c*d - a*b**2*d)/(a**2*
c*e - a*b**2*e)))) + x/c
```

3.501 $\int \frac{(d+ex)^3}{a+b(d+ex)^2+c(d+ex)^4} \, dx$

Optimal. Leaf size=81

$$\frac{b \tanh^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{2ce\sqrt{b^2-4ac}}+\frac{\log\left(a+b(d+ex)^2+c(d+ex)^4\right)}{4ce}$$

Rubi [A] time = 0.13, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1142, 1114, 634, 618, 206, 628}

$$\frac{b \tanh^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{2ce\sqrt{b^2-4ac}}+\frac{\log\left(a+b(d+ex)^2+c(d+ex)^4\right)}{4ce}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^3/(a + b*(d + e*x)^2 + c*(d + e*x)^4),x]
```

```
[Out] (b*ArcTanh[(b + 2*c*(d + e*x)^2)/Sqrt[b^2 - 4*a*c]])/(2*c*Sqrt[b^2 - 4*a*c]
*e) + Log[a + b*(d + e*x)^2 + c*(d + e*x)^4]/(4*c*e)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 1142

```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Di
st[u^m/(Coefficient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p,
 x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

Rubi steps

$$\begin{aligned}\int \frac{(d+ex)^3}{a+b(d+ex)^2+c(d+ex)^4}\,dx &= \frac{\text{Subst}\left(\int \frac{x^3}{a+bx^2+cx^4}\,dx, x, d+ex\right)}{e}\\ &= \frac{\text{Subst}\left(\int \frac{x}{a+bx+cx^2}\,dx, x, (d+ex)^2\right)}{2e}\\ &= \frac{\text{Subst}\left(\int \frac{b+2cx}{a+bx+cx^2}\,dx, x, (d+ex)^2\right)}{4ce} - \frac{b\,\text{Subst}\left(\int \frac{1}{a+bx+cx^2}\,dx, x, (d+ex)^2\right)}{4ce}\\ &= \frac{\log\left(a+b(d+ex)^2+c(d+ex)^4\right)}{4ce} + \frac{b\,\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx, x, b+2c(d+e\right.}{2ce}\\ &= \frac{b\tanh^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{2c\sqrt{b^2-4ac}\,e} + \frac{\log\left(a+b(d+ex)^2+c(d+ex)^4\right)}{4ce}\end{aligned}$$

Mathematica [A] time = 0.04, size = 77, normalized size = 0.95

$$\frac{\log\left(a+b(d+ex)^2+c(d+ex)^4\right) - \frac{2b\tan^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}}{4ce}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^3/(a + b*(d + e*x)^2 + c*(d + e*x)^4),x]
```

```
[Out] ((-2*b*ArcTan[(b + 2*c*(d + e*x)^2)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c]
 + Log[a + b*(d + e*x)^2 + c*(d + e*x)^4])/(4*c*e)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(d+ex)^3}{a+b(d+ex)^2+c(d+ex)^4}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(d + e*x)^3/(a + b*(d + e*x)^2 + c*(d + e*x)^4),x]
```

```
[Out] IntegrateAlgebraic[(d + e*x)^3/(a + b*(d + e*x)^2 + c*(d + e*x)^4), x]
```

fricas [B] time = 1.01, size = 434, normalized size = 5.36

$$\left[\frac{\sqrt{b^2-4ac}\,b\log\left(\frac{2c^2e^4x^4+8c^2de^3x^3+2c^2d^4+2(6c^2d^2+bc)e^2x^2+2bcd^2+4(2c^2d^3+bcd)ex+b^2-2ac+(2ce^2x^2+4cdex+2cd^2+b)\sqrt{b^2-4ac}}{ce^4x^4+4cde^3x^3+cd^4+(6cd^2+b)e^2x^2+bd^2+2(2cd^3+bd)ex+a}\right)+(b^2-4ac)\log\left(ce^4x^4+4cde^3x^3+cd^4+(6cd^2+b)e^2x^2+bd^2+2(2cd^3+bd)ex+a\right)}{4(b^2c-4ac^2)e}, \frac{2\sqrt{-b^2+4ac}\,b\arctan\left(-\frac{(2ce^2x^2+4cdex+2cd^2+b)\sqrt{-b^2+4ac}}{b^2-4ac}\right)+(b^2-4ac)\log\left(ce^4x^4+4cde^3x^3+cd^4+(6cd^2+b)e^2x^2+bd^2+2(2cd^3+bd)ex+a\right)}{4(b^2c-4ac^2)e}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="fricas")
```

```
[Out] [1/4*(sqrt(b^2 - 4*a*c)*b*log((2*c^2*e^4*x^4 + 8*c^2*d*e^3*x^3 + 2*c^2*d^4
+ 2*(6*c^2*d^2 + b*c)*e^2*x^2 + 2*b*c*d^2 + 4*(2*c^2*d^3 + b*c*d)*e*x + b^2
 - 2*a*c + (2*c*e^2*x^2 + 4*c*d*e*x + 2*c*d^2 + b)*sqrt(b^2 - 4*a*c))/(c*e^
4*x^4 + 4*c*d*e^3*x^3 + c*d^4 + (6*c*d^2 + b)*e^2*x^2 + b*d^2 + 2*(2*c*d^3
+ b*d)*e*x + a)) + (b^2 - 4*a*c)*log(c*e^4*x^4 + 4*c*d*e^3*x^3 + c*d^4 + (6
*c*d^2 + b)*e^2*x^2 + b*d^2 + 2*(2*c*d^3 + b*d)*e*x + a))/((b^2*c - 4*a*c^2
)*e), 1/4*(2*sqrt(-b^2 + 4*a*c)*b*arctan(-(2*c*e^2*x^2 + 4*c*d*e*x + 2*c*d^
2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) + (b^2 - 4*a*c)*log(c*e^4*x^4 + 4*
c*d*e^3*x^3 + c*d^4 + (6*c*d^2 + b)*e^2*x^2 + b*d^2 + 2*(2*c*d^3 + b*d)*e*x
 + a))/((b^2*c - 4*a*c^2)*e)]
```

giac [A] time = 0.40, size = 130, normalized size = 1.60

$$-\frac{b \arctan\left(\frac{2\, c d^{2} + 2\, \left(x^{2} e + 2\, d x\right) c e + b}{\sqrt{-b^{2} + 4\, a c}}\right) e^{(-1)}}{2\, \sqrt{-b^{2} + 4\, a c}\, c} + \frac{e^{(-1)} \log\left(c d^{4} + 2\, \left(x^{2} e + 2\, d x\right) c d^{2} e + \left(x^{2} e + 2\, d x\right)^{2} c e^{2} + b d^{2} + \left(x^{2} e + 2\, d x\right) b e + a\right)}{4\, c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="giac")
```

```
[Out] -1/2*b*arctan((2*c*d^2 + 2*(x^2*e + 2*d*x)*c*e + b)/sqrt(-b^2 + 4*a*c))*e^(
-1)/(sqrt(-b^2 + 4*a*c)*c) + 1/4*e^(-1)*log(c*d^4 + 2*(x^2*e + 2*d*x)*c*d^2
*e + (x^2*e + 2*d*x)^2*c*e^2 + b*d^2 + (x^2*e + 2*d*x)*b*e + a)/c
```

maple [C] time = 0.00, size = 151, normalized size = 1.86

$$\frac{\left(\mathrm{RootOf}\left(\_Z^{4} c e^{4} + 4\_Z^{3} c d e^{3} + c d^{4} + b d^{2} + \left(6 c d^{2} e^{2} + b e^{2}\right)\_Z^{2} + \left(4 c d^{3} e + 2 d e b\right)\_Z + a\right)^{3} e^{3} + 3\,\mathrm{RootOf}\left(\_Z^{4} c e^{4} + 4\_Z^{3} c d e^{3} + c d^{4} + b d^{2} + \left(6 c d^{2} e^{2} + b e^{2}\right)\_Z^{2} + \left(4 c d^{3} e + 2 d e b\right)\_Z + a\right)^{2} d e^{2} + 3\,\mathrm{RootOf}\left(\_Z^{4} c e^{4} + 4\_Z^{3} c d e^{3} + c d^{4} + b d^{2} + \left(6 c d^{2} e^{2} + b e^{2}\right)\_Z^{2} + \left(4 c d^{3} e + 2 d e b\right)\_Z + a\right) d^{2} e + d^{3}\right) \ln\left(-\mathrm{RootOf}\left(\_Z^{4} c e^{4} + 4\_Z^{3} c d e^{3} + c d^{4} + b d^{2} + \left(6 c d^{2} e^{2} + b e^{2}\right)\_Z^{2} + \left(4 c d^{3} e + 2 d e b\right)\_Z + a\right) + x\right)}{2 e \left(2 c e^{3}\,\mathrm{RootOf}\left(\_Z^{4} c e^{4} + 4\_Z^{3} c d e^{3} + c d^{4} + b d^{2} + \left(6 c d^{2} e^{2} + b e^{2}\right)\_Z^{2} + \left(4 c d^{3} e + 2 d e b\right)\_Z + a\right)^{3} + 6 c d e^{2}\,\mathrm{RootOf}\left(\_Z^{4} c e^{4} + 4\_Z^{3} c d e^{3} + c d^{4} + b d^{2} + \left(6 c d^{2} e^{2} + b e^{2}\right)\_Z^{2} + \left(4 c d^{3} e + 2 d e b\right)\_Z + a\right)^{2} + 6 e c d^{2}\,\mathrm{RootOf}\left(\_Z^{4} c e^{4} + 4\_Z^{3} c d e^{3} + c d^{4} + b d^{2} + \left(6 c d^{2} e^{2} + b e^{2}\right)\_Z^{2} + \left(4 c d^{3} e + 2 d e b\right)\_Z + a\right) + 2 c d^{3} + b e\,\mathrm{RootOf}\left(\_Z^{4} c e^{4} + 4\_Z^{3} c d e^{3} + c d^{4} + b d^{2} + \left(6 c d^{2} e^{2} + b e^{2}\right)\_Z^{2} + \left(4 c d^{3} e + 2 d e b\right)\_Z + a\right) + b d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4),x)
```

```
[Out] 1/2/e*sum((_R^3*e^3+3*_R^2*d*e^2+3*_R*d^2*e+d^3)/(2*_R^3*c*e^3+6*_R^2*c*d*e
^2+6*_R*c*d^2*e+2*c*d^3+_R*b*e+b*d)*ln(-_R+x),_R=RootOf(_Z^4*c*e^4+4*_Z^3*c
*d*e^3+c*d^4+b*d^2+(6*c*d^2*e^2+b*e^2)*_Z^2+(4*c*d^3*e+2*b*d*e)*_Z+a))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex+d)^{3}}{(ex+d)^{4}c + (ex+d)^{2}b + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)^3/((e*x + d)^4*c + (e*x + d)^2*b + a), x)
```

mupad [B] time = 1.76, size = 278, normalized size = 3.43

$$\frac{4\,a\,c\,e\,\ln\left(c\,d^4+4\,c\,d^3\,e\,x+6\,c\,d^2\,e^2\,x^2+b\,d^2+4\,c\,d\,e^3\,x^3+2\,b\,d\,e\,x+c\,e^4\,x^4+b\,e^2\,x^2+a\right)}{16\,a\,c^2\,e^2-4\,b^2\,c\,e^2}-\frac{b^2\,e\,\ln\left(c\,d^4+4\,c\,d^3\,e\,x+6\,c\,d^2\,e^2\,x^2+b\,d^2+4\,c\,d\,e^3\,x^3+2\,b\,d\,e\,x+c\,e^4\,x^4+b\,e^2\,x^2+a\right)}{16\,a\,c^2\,e^2-4\,b^2\,c\,e^2}-\frac{b\,\mathrm{atan}\left(\frac{b}{\sqrt{4\,a\,c-b^2}}+\frac{2\,c\,d^2}{\sqrt{4\,a\,c-b^2}}+\frac{2\,c\,e^2\,x^2}{\sqrt{4\,a\,c-b^2}}+\frac{4\,c\,d\,e\,x}{\sqrt{4\,a\,c-b^2}}\right)}{2\,c\,e\,\sqrt{4\,a\,c-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^3/(a + b*(d + e*x)^2 + c*(d + e*x)^4),x)
```

```
[Out] (4*a*c*e*log(a + b*d^2 + c*d^4 + b*e^2*x^2 + c*e^4*x^4 + 2*b*d*e*x + 6*c*d^
2*e^2*x^2 + 4*c*d^3*e*x + 4*c*d*e^3*x^3))/(16*a*c^2*e^2 - 4*b^2*c*e^2) - (b
^2*e*log(a + b*d^2 + c*d^4 + b*e^2*x^2 + c*e^4*x^4 + 2*b*d*e*x + 6*c*d^2*e^
2*x^2 + 4*c*d^3*e*x + 4*c*d*e^3*x^3))/(16*a*c^2*e^2 - 4*b^2*c*e^2) - (b*ata
n(b/(4*a*c - b^2)^(1/2) + (2*c*d^2)/(4*a*c - b^2)^(1/2) + (2*c*e^2*x^2)/(4*
a*c - b^2)^(1/2) + (4*c*d*e*x)/(4*a*c - b^2)^(1/2)))/(2*c*e*(4*a*c - b^2)^(
1/2))
```

sympy [B] time = 1.64, size = 280, normalized size = 3.46

$$\left(- \frac{b \sqrt{- 4 a c + b^{2}}}{4 c e \left(4 a c - b^{2}\right)} + \frac{1}{4 c e}\right) \log{\left(\frac{2 d x}{e} + x^{2} + \frac{- 8 a c e \left(- \frac{b \sqrt{- 4 a c + b^{2}}}{4 c e \left(4 a c - b^{2}\right)} + \frac{1}{4 c e}\right) + 2 a + 2 b^{2} e \left(- \frac{b \sqrt{- 4 a c + b^{2}}}{4 c e \left(4 a c - b^{2}\right)} + \frac{1}{4 c e}\right) + b d^{2}}{b e^{2}} \right)} + \left(\frac{b \sqrt{- 4 a c + b^{2}}}{4 c e \left(4 a c - b^{2}\right)} + \frac{1}{4 c e}\right) \log{\left(\frac{2 d x}{e} + x^{2} + \frac{- 8 a c e \left(\frac{b \sqrt{- 4 a c + b^{2}}}{4 c e \left(4 a c - b^{2}\right)} + \frac{1}{4 c e}\right) + 2 a + 2 b^{2} e \left(\frac{b \sqrt{- 4 a c + b^{2}}}{4 c e \left(4 a c - b^{2}\right)} + \frac{1}{4 c e}\right) + b d^{2}}{b e^{2}} \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3/(a+b*(e*x+d)**2+c*(e*x+d)**4),x)
```

```
[Out] (-b*sqrt(-4*a*c + b**2)/(4*c*e*(4*a*c - b**2)) + 1/(4*c*e))*log(2*d*x/e + x
**2 + (-8*a*c*e*(-b*sqrt(-4*a*c + b**2)/(4*c*e*(4*a*c - b**2)) + 1/(4*c*e))
```

```
 + 2*a + 2*b**2*e*(-b*sqrt(-4*a*c + b**2)/(4*c*e*(4*a*c - b**2)) + 1/(4*c*e
)) + b*d**2)/(b*e**2)) + (b*sqrt(-4*a*c + b**2)/(4*c*e*(4*a*c - b**2)) + 1/
(4*c*e))*log(2*d*x/e + x**2 + (-8*a*c*e*(b*sqrt(-4*a*c + b**2)/(4*c*e*(4*a*
c - b**2)) + 1/(4*c*e)) + 2*a + 2*b**2*e*(b*sqrt(-4*a*c + b**2)/(4*c*e*(4*a
*c - b**2)) + 1/(4*c*e)) + b*d**2)/(b*e**2))
```

3.502 $\int \frac{(d+ex)^2}{a+b(d+ex)^2+c(d+ex)^4}\,dx$

Optimal. Leaf size=164

$$\frac{\sqrt{\sqrt{b^2-4ac}+b}\,\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\sqrt{c}\,e\sqrt{b^2-4ac}}-\frac{\sqrt{b-\sqrt{b^2-4ac}}\,\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\,e\sqrt{b^2-4ac}}$$

Rubi [A] time = 0.15, antiderivative size = 164, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {1142, 1130, 205}

$$\frac{\sqrt{\sqrt{b^2-4ac}+b}\,\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\sqrt{c}\,e\sqrt{b^2-4ac}}-\frac{\sqrt{b-\sqrt{b^2-4ac}}\,\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\,e\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^2/(a + b*(d + e*x)^2 + c*(d + e*x)^4),x]
```

```
[Out] -((Sqrt[b - Sqrt[b^2 - 4*a*c]]*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b -
Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b^2 - 4*a*c]*e)) + (Sqrt[b + Sqr
t[b^2 - 4*a*c]]*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b + Sqrt[b^2 - 4*a*
c]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b^2 - 4*a*c]*e)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1130

```
Int[((d_.)*(x_))^(m_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Wi
th[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(d^2*(b/q + 1))/2, Int[(d*x)^(m - 2)/(b/2
 + q/2 + c*x^2), x], x] - Dist[(d^2*(b/q - 1))/2, Int[(d*x)^(m - 2)/(b/2 -
q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && G
eQ[m, 2]
```

Rule 1142

```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Di
st[u^m/(Coefficient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p,
 x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

Rubi steps

$$\int \frac{(d+ex)^2}{a+b(d+ex)^2+c(d+ex)^4}\,dx=\frac{\text{Subst}\left(\int \frac{x^2}{a+bx^2+cx^4}\,dx,x,d+ex\right)}{e}$$

$$=\frac{\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\text{Subst}\left(\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx,x,d+ex\right)}{2e}+\frac{\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)\text{Subst}\left(\right.}{}$$

$$=-\frac{\sqrt{b-\sqrt{b^2-4ac}}\,\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}\,e}+\frac{\sqrt{b+\sqrt{b^2-4ac}}\,\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}\,e}$$

Mathematica [A] time = 0.09, size = 175, normalized size = 1.07

$$\frac{\left(\sqrt{b^2-4ac}-b\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)+\sqrt{b-\sqrt{b^2-4ac}}\sqrt{\sqrt{b^2-4ac}+b}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\sqrt{c}e\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^2/(a + b*(d + e*x)^2 + c*(d + e*x)^4),x]
```

```
[Out] ((-b + Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b - Sqrt[
b^2 - 4*a*c]]] + Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[b + Sqrt[b^2 - 4*a*c]]*Ar
cTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqr
t[c]*Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]*e)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(d+ex)^2}{a+b(d+ex)^2+c(d+ex)^4}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(d + e*x)^2/(a + b*(d + e*x)^2 + c*(d + e*x)^4),x]
```

```
[Out] IntegrateAlgebraic[(d + e*x)^2/(a + b*(d + e*x)^2 + c*(d + e*x)^4), x]
```

fricas [B] time = 1.25, size = 703, normalized size = 4.29

$$\frac{1}{2}\sqrt{\frac{1}{2}}\sqrt{-\frac{\left(b^2c-4ac^2\right)e^2\sqrt{\frac{1}{\left(b^2c^2-4ac^3\right)e^4}}+b}{\left(b^2c-4ac^2\right)e^2}}\log\left(\sqrt{\frac{1}{2}}\left(b^2c-4ac^2\right)e^3\sqrt{-\frac{\left(b^2c-4ac^2\right)e^2\sqrt{\frac{1}{\left(b^2c^2-4ac^3\right)e^4}}+b}{\left(b^2c-4ac^2\right)e^2}}\sqrt{\frac{1}{\left(b^2c^2-4ac^3\right)e^4}}+ex+d\right)-\frac{1}{2}\sqrt{\frac{1}{2}}\sqrt{-\frac{\left(b^2c-4ac^2\right)e^2\sqrt{\frac{1}{\left(b^2c^2-4ac^3\right)e^4}}+b}{\left(b^2c-4ac^2\right)e^2}}\log\left(-\sqrt{\frac{1}{2}}\left(b^2c-4ac^2\right)e^3\sqrt{-\frac{\left(b^2c-4ac^2\right)e^2\sqrt{\frac{1}{\left(b^2c^2-4ac^3\right)e^4}}+b}{\left(b^2c-4ac^2\right)e^2}}\sqrt{\frac{1}{\left(b^2c^2-4ac^3\right)e^4}}+ex+d\right)-\frac{1}{2}\sqrt{\frac{1}{2}}\sqrt{\frac{\left(b^2c-4ac^2\right)e^2\sqrt{\frac{1}{\left(b^2c^2-4ac^3\right)e^4}}-b}{\left(b^2c-4ac^2\right)e^2}}\log\left(\sqrt{\frac{1}{2}}\left(b^2c-4ac^2\right)e^3\sqrt{\frac{\left(b^2c-4ac^2\right)e^2\sqrt{\frac{1}{\left(b^2c^2-4ac^3\right)e^4}}-b}{\left(b^2c-4ac^2\right)e^2}}\sqrt{\frac{1}{\left(b^2c^2-4ac^3\right)e^4}}+ex+d\right)+\frac{1}{2}\sqrt{\frac{1}{2}}\sqrt{\frac{\left(b^2c-4ac^2\right)e^2\sqrt{\frac{1}{\left(b^2c^2-4ac^3\right)e^4}}-b}{\left(b^2c-4ac^2\right)e^2}}\log\left(-\sqrt{\frac{1}{2}}\left(b^2c-4ac^2\right)e^3\sqrt{\frac{\left(b^2c-4ac^2\right)e^2\sqrt{\frac{1}{\left(b^2c^2-4ac^3\right)e^4}}-b}{\left(b^2c-4ac^2\right)e^2}}\sqrt{\frac{1}{\left(b^2c^2-4ac^3\right)e^4}}+ex+d\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(1/2)*sqrt(-((b^2*c - 4*a*c^2)*e^2*sqrt(1/((b^2*c^2 - 4*a*c^3)*e^4)
) + b)/((b^2*c - 4*a*c^2)*e^2))*log(sqrt(1/2)*(b^2*c - 4*a*c^2)*e^3*sqrt(-(
(b^2*c - 4*a*c^2)*e^2*sqrt(1/((b^2*c^2 - 4*a*c^3)*e^4)) + b)/((b^2*c - 4*a*
c^2)*e^2))*sqrt(1/((b^2*c^2 - 4*a*c^3)*e^4)) + e*x + d) - 1/2*sqrt(1/2)*sqr
t(-((b^2*c - 4*a*c^2)*e^2*sqrt(1/((b^2*c^2 - 4*a*c^3)*e^4)) + b)/((b^2*c -
4*a*c^2)*e^2))*log(-sqrt(1/2)*(b^2*c - 4*a*c^2)*e^3*sqrt(-((b^2*c - 4*a*c^2
)*e^2*sqrt(1/((b^2*c^2 - 4*a*c^3)*e^4)) + b)/((b^2*c - 4*a*c^2)*e^2))*sqrt(
1/((b^2*c^2 - 4*a*c^3)*e^4)) + e*x + d) - 1/2*sqrt(1/2)*sqrt(((b^2*c - 4*a*
c^2)*e^2*sqrt(1/((b^2*c^2 - 4*a*c^3)*e^4)) - b)/((b^2*c - 4*a*c^2)*e^2))*lo
g(sqrt(1/2)*(b^2*c - 4*a*c^2)*e^3*sqrt(((b^2*c - 4*a*c^2)*e^2*sqrt(1/((b^2*
c^2 - 4*a*c^3)*e^4)) - b)/((b^2*c - 4*a*c^2)*e^2))*sqrt(1/((b^2*c^2 - 4*a*c
^3)*e^4)) + e*x + d) + 1/2*sqrt(1/2)*sqrt(((b^2*c - 4*a*c^2)*e^2*sqrt(1/((b
^2*c^2 - 4*a*c^3)*e^4)) - b)/((b^2*c - 4*a*c^2)*e^2))*log(-sqrt(1/2)*(b^2*c
 - 4*a*c^2)*e^3*sqrt(((b^2*c - 4*a*c^2)*e^2*sqrt(1/((b^2*c^2 - 4*a*c^3)*e^4
)) - b)/((b^2*c - 4*a*c^2)*e^2))*sqrt(1/((b^2*c^2 - 4*a*c^3)*e^4)) + e*x +
d)
```

giac [B] time = 0.45, size = 1285, normalized size = 7.84

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="giac")
```

```
[Out] -1/2*((d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c)
)^2*e^2 - 2*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(
-4)/c))*d*e + d^2)*log(d*e^(-1) + x + sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4
```

```
*a*c)*e^2)*e^(-4)/c))/(2*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*
a*c)*e^2)*e^(-4)/c))^3*c*e^4 - 6*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 + sqrt(
b^2 - 4*a*c)*e^2)*e^(-4)/c))^2*c*d*e^3 + 6*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e
^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))*c*d^2*e^2 - 2*c*d^3*e + (d*e^(-1) +
sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))*b*e^2 - b*d*e) -
 1/2*((d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c)
)^2*e^2 - 2*(d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(
-4)/c))*d*e + d^2)*log(d*e^(-1) + x - sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4
*a*c)*e^2)*e^(-4)/c))/(2*(d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*
a*c)*e^2)*e^(-4)/c))^3*c*e^4 - 6*(d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 + sqrt(
b^2 - 4*a*c)*e^2)*e^(-4)/c))^2*c*d*e^3 + 6*(d*e^(-1) - sqrt(1/2)*sqrt(-(b*e
^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))*c*d^2*e^2 - 2*c*d^3*e + (d*e^(-1) -
sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))*b*e^2 - b*d*e) -
 1/2*((d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c)
)^2*e^2 - 2*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(
-4)/c))*d*e + d^2)*log(d*e^(-1) + x + sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4
*a*c)*e^2)*e^(-4)/c))/(2*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*
a*c)*e^2)*e^(-4)/c))^3*c*e^4 - 6*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 - sqrt(
b^2 - 4*a*c)*e^2)*e^(-4)/c))^2*c*d*e^3 + 6*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e
^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))*c*d^2*e^2 - 2*c*d^3*e + (d*e^(-1) +
sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))*b*e^2 - b*d*e) -
 1/2*((d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c)
)^2*e^2 - 2*(d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(
-4)/c))*d*e + d^2)*log(d*e^(-1) + x - sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4
*a*c)*e^2)*e^(-4)/c))/(2*(d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*
a*c)*e^2)*e^(-4)/c))^3*c*e^4 - 6*(d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 - sqrt(
b^2 - 4*a*c)*e^2)*e^(-4)/c))^2*c*d*e^3 + 6*(d*e^(-1) - sqrt(1/2)*sqrt(-(b*e
^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))*c*d^2*e^2 - 2*c*d^3*e + (d*e^(-1) -
sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))*b*e^2 - b*d*e)
```

maple [C] time = 0.01, size = 140, normalized size = 0.85

$$\frac{\left(\mathrm{RootOf}\left(\_Z^4ce^4+4\_Z^3cde^3+cd^4+bd^2+\left(6cd^2e^2+be^2\right)\_Z^2+\left(4cd^3e+2deb\right)\_Z+a\right)^2e^2+2\,\mathrm{RootOf}\left(\_Z^4ce^4+4\_Z^3cde^3+cd^4+bd^2+\left(6cd^2e^2+be^2\right)\_Z^2+\left(4cd^3e+2deb\right)\_Z+a\right)de+d^2\right)\ln\left(-\mathrm{RootOf}\left(\_Z^4ce^4+4\_Z^3cde^3+cd^4+bd^2+\left(6cd^2e^2+be^2\right)\_Z^2+\left(4cd^3e+2deb\right)\_Z+a\right)+x\right)}{2e\left(2ce^3\,\mathrm{RootOf}\left(\_Z^4ce^4+4\_Z^3cde^3+cd^4+bd^2+\left(6cd^2e^2+be^2\right)\_Z^2+\left(4cd^3e+2deb\right)\_Z+a\right)^3+6cde^2\,\mathrm{RootOf}\left(\_Z^4ce^4+4\_Z^3cde^3+cd^4+bd^2+\left(6cd^2e^2+be^2\right)\_Z^2+\left(4cd^3e+2deb\right)\_Z+a\right)^2+6ecd^2\,\mathrm{RootOf}\left(\_Z^4ce^4+4\_Z^3cde^3+cd^4+bd^2+\left(6cd^2e^2+be^2\right)\_Z^2+\left(4cd^3e+2deb\right)\_Z+a\right)+2cd^3+be\,\mathrm{RootOf}\left(\_Z^4ce^4+4\_Z^3cde^3+cd^4+bd^2+\left(6cd^2e^2+be^2\right)\_Z^2+\left(4cd^3e+2deb\right)\_Z+a\right)+bd\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4),x)
```

```
[Out] 1/2/e*sum((_R^2*e^2+2*_R*d*e+d^2)/(2*_R^3*c*e^3+6*_R^2*c*d*e^2+6*_R*c*d^2*e
+2*c*d^3+_R*b*e+b*d)*ln(-_R+x),_R=RootOf(_Z^4*c*e^4+4*_Z^3*c*d*e^3+c*d^4+b*
d^2+(6*c*d^2*e^2+b*e^2)*_Z^2+(4*c*d^3*e+2*b*d*e)*_Z+a))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex+d)^2}{(ex+d)^4c+(ex+d)^2b+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)^2/((e*x + d)^4*c + (e*x + d)^2*b + a), x)
```

mupad [B] time = 1.74, size = 590, normalized size = 3.60

$$-2\,\mathrm{atanh}\left(\frac{\sqrt{-\frac{b^3+\sqrt{-\left(4ac-b^2\right)^3}-4abc}{8\left(16a^2c^3e^2-8ab^2c^2e^2+b^4ce^2\right)}}\left(x\left(4ac^2e^{12}-2b^2ce^{12}\right)+\frac{\left(x\left(8b^3c^2e^{14}-32abc^3e^{14}\right)+8b^3c^2de^{13}-32abc^3de^{13}\right)\left(b^3+\sqrt{-\left(4ac-b^2\right)^3}-4abc\right)}{8\left(16a^2c^3e^2-8ab^2c^2e^2+b^4ce^2\right)}+4ac^2de^{11}-2b^2cde^{11}\right)}{ace^{10}}\right)\sqrt{-\frac{b^3+\sqrt{-\left(4ac-b^2\right)^3}-4abc}{8\left(16a^2c^3e^2-8ab^2c^2e^2+b^4ce^2\right)}}-2\,\mathrm{atanh}\left(\frac{\sqrt{\frac{\sqrt{-\left(4ac-b^2\right)^3}-b^3+4abc}{8\left(16a^2c^3e^2-8ab^2c^2e^2+b^4ce^2\right)}}\left(x\left(4ac^2e^{12}-2b^2ce^{12}\right)-\frac{\left(x\left(8b^3c^2e^{14}-32abc^3e^{14}\right)+8b^3c^2de^{13}-32abc^3de^{13}\right)\left(\sqrt{-\left(4ac-b^2\right)^3}-b^3+4abc\right)}{8\left(16a^2c^3e^2-8ab^2c^2e^2+b^4ce^2\right)}+4ac^2de^{11}-2b^2cde^{11}\right)}{ace^{10}}\right)\sqrt{\frac{\sqrt{-\left(4ac-b^2\right)^3}-b^3+4abc}{8\left(16a^2c^3e^2-8ab^2c^2e^2+b^4ce^2\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^2/(a + b*(d + e*x)^2 + c*(d + e*x)^4),x)
```

```
[Out] - 2*atanh(((-(b^3 + (-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c)/(8*(b^4*c*e^2 + 16*
a^2*c^3*e^2 - 8*a*b^2*c^2*e^2)))^(1/2)*(x*(4*a*c^2*e^12 - 2*b^2*c*e^12) + (
(x*(8*b^3*c^2*e^14 - 32*a*b*c^3*e^14) + 8*b^3*c^2*d*e^13 - 32*a*b*c^3*d*e^1
3)*(b^3 + (-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c))/(8*(b^4*c*e^2 + 16*a^2*c^3*e
^2 - 8*a*b^2*c^2*e^2)) + 4*a*c^2*d*e^11 - 2*b^2*c*d*e^11))/(a*c*e^10))*(-(b
^3 + (-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c)/(8*(b^4*c*e^2 + 16*a^2*c^3*e^2 - 8
*a*b^2*c^2*e^2)))^(1/2) - 2*atanh(((((-(4*a*c - b^2)^3)^(1/2) - b^3 + 4*a*b
*c)/(8*(b^4*c*e^2 + 16*a^2*c^3*e^2 - 8*a*b^2*c^2*e^2)))^(1/2)*(x*(4*a*c^2*e
^12 - 2*b^2*c*e^12) - ((x*(8*b^3*c^2*e^14 - 32*a*b*c^3*e^14) + 8*b^3*c^2*d*
e^13 - 32*a*b*c^3*d*e^13)*((-(4*a*c - b^2)^3)^(1/2) - b^3 + 4*a*b*c))/(8*(b
^4*c*e^2 + 16*a^2*c^3*e^2 - 8*a*b^2*c^2*e^2)) + 4*a*c^2*d*e^11 - 2*b^2*c*d*
e^11))/(a*c*e^10))*(((-(4*a*c - b^2)^3)^(1/2) - b^3 + 4*a*b*c)/(8*(b^4*c*e^
2 + 16*a^2*c^3*e^2 - 8*a*b^2*c^2*e^2)))^(1/2)
```

sympy [A] time = 1.35, size = 104, normalized size = 0.63

$$\operatorname{RootSum}\left(t^4\left(256a^2c^3e^4-128ab^2c^2e^4+16b^4ce^4\right)+t^2\left(-16abce^2+4b^3e^2\right)+a,\left(t\mapsto t\log\left(x+\frac{64t^3ac^2e^3-16t^3b^2ce^3-2tbe+d}{e}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2/(a+b*(e*x+d)**2+c*(e*x+d)**4),x)
```

```
[Out] RootSum(_t**4*(256*a**2*c**3*e**4 - 128*a*b**2*c**2*e**4 + 16*b**4*c*e**4)
+ _t**2*(-16*a*b*c*e**2 + 4*b**3*e**2) + a, Lambda(_t, _t*log(x + (64*_t**3
*a*c**2*e**3 - 16*_t**3*b**2*c*e**3 - 2*_t*b*e + d)/e)))
```

3.503 $\int \frac{d+ex}{a+b(d+ex)^2+c(d+ex)^4}\,dx$

Optimal. Leaf size=43

$$-\frac{\tanh^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{e\sqrt{b^2-4ac}}$$

Rubi [A] time = 0.06, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {1142, 1107, 618, 206}

$$-\frac{\tanh^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{e\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)/(a + b*(d + e*x)^2 + c*(d + e*x)^4),x]
```

```
[Out] -(ArcTanh[(b + 2*c*(d + e*x)^2)/Sqrt[b^2 - 4*a*c]]/(Sqrt[b^2 - 4*a*c]*e))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1107

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rule 1142

```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Di
st[u^m/(Coefficient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p,
 x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{d+ex}{a+b(d+ex)^2+c(d+ex)^4}\,dx &= \frac{\text{Subst}\left(\int \frac{x}{a+bx^2+cx^4}\,dx,x,d+ex\right)}{e}\\
&= \frac{\text{Subst}\left(\int \frac{1}{a+bx+cx^2}\,dx,x,(d+ex)^2\right)}{2e}\\
&= -\frac{\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx,x,b+2c(d+ex)^2\right)}{e}\\
&= -\frac{\tanh^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}\,e}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 46, normalized size = 1.07

$$\frac{\tan^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{4ac-b^2}}\right)}{e\sqrt{4ac-b^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)/(a + b*(d + e*x)^2 + c*(d + e*x)^4),x]
[Out] ArcTan[(b + 2*c*(d + e*x)^2)/Sqrt[-b^2 + 4*a*c]]/(Sqrt[-b^2 + 4*a*c]*e)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{d+ex}{a+b(d+ex)^2+c(d+ex)^4}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(d + e*x)/(a + b*(d + e*x)^2 + c*(d + e*x)^4),x]
[Out] IntegrateAlgebraic[(d + e*x)/(a + b*(d + e*x)^2 + c*(d + e*x)^4), x]
```

fricas [A] time = 1.30, size = 272, normalized size = 6.33

$$\left[\frac{\log\left(\frac{2c^2e^4x^4+8c^2de^3x^3+2c^2d^4+2\left(6c^2d^2+bc\right)e^2x^2+2bcd^2+4\left(2c^2d^3+bcd\right)ex+b^2-2ac-\left(2ce^2x^2+4cdex+2cd^2+b\right)\sqrt{b^2-4ac}}{ce^4x^4+4cde^3x^3+cd^4+\left(6cd^2+b\right)e^2x^2+bd^2+2\left(2cd^3+bd\right)ex+a}\right)}{2\sqrt{b^2-4ac}\,e}, -\frac{\sqrt{-b^2+4ac}\arctan\left(-\frac{\left(2ce^2x^2+4cdex+2cd^2+b\right)\sqrt{-b^2+4ac}}{b^2-4ac}\right)}{\left(b^2-4ac\right)e}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="fricas")
[Out] [1/2*log((2*c^2*e^4*x^4 + 8*c^2*d*e^3*x^3 + 2*c^2*d^4 + 2*(6*c^2*d^2 + b*c)
*e^2*x^2 + 2*b*c*d^2 + 4*(2*c^2*d^3 + b*c*d)*e*x + b^2 - 2*a*c - (2*c*e^2*x
^2 + 4*c*d*e*x + 2*c*d^2 + b)*sqrt(b^2 - 4*a*c))/(c*e^4*x^4 + 4*c*d*e^3*x^3
 + c*d^4 + (6*c*d^2 + b)*e^2*x^2 + b*d^2 + 2*(2*c*d^3 + b*d)*e*x + a))/(sqr
t(b^2 - 4*a*c)*e), -sqrt(-b^2 + 4*a*c)*arctan(-(2*c*e^2*x^2 + 4*c*d*e*x + 2
*c*d^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c))/((b^2 - 4*a*c)*e)]
```

giac [A] time = 0.45, size = 53, normalized size = 1.23

$$\frac{\arctan\left(\frac{2\,cd^2+2\left(x^2e+2\,dx\right)ce+b}{\sqrt{-b^2+4\,ac}}\right)e^{(-1)}}{\sqrt{-b^2+4\,ac}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="giac")
[Out] arctan((2*c*d^2 + 2*(x^2*e + 2*d*x)*c*e + b)/sqrt(-b^2 + 4*a*c))*e^(-1)/sqr
t(-b^2 + 4*a*c)
```

maple [C] time = 0.01, size = 129, normalized size = 3.00

$$\frac{\left(\mathrm{RootOf}\left(\_Z^4ce^4+4\_Z^3cde^3+cd^4+bd^2+\left(6cd^2e^2+be^2\right)\_Z^2+\left(4cd^3e+2deb\right)\_Z+a\right)e+d\right)\ln\left(-\mathrm{RootOf}\left(\_Z^4ce^4+4\_Z^3cde^3+cd^4+bd^2+\left(6cd^2e^2+be^2\right)\_Z^2+\left(4cd^3e+2deb\right)\_Z+a\right)+x\right)}{2e\left(2ce^3\,\mathrm{RootOf}\left(\_Z^4ce^4+4\_Z^3cde^3+cd^4+bd^2+\left(6cd^2e^2+be^2\right)\_Z^2+\left(4cd^3e+2deb\right)\_Z+a\right)^3+6cde^2\,\mathrm{RootOf}\left(\_Z^4ce^4+4\_Z^3cde^3+cd^4+bd^2+\left(6cd^2e^2+be^2\right)\_Z^2+\left(4cd^3e+2deb\right)\_Z+a\right)^2+6ecd^2\,\mathrm{RootOf}\left(\_Z^4ce^4+4\_Z^3cde^3+cd^4+bd^2+\left(6cd^2e^2+be^2\right)\_Z^2+\left(4cd^3e+2deb\right)\_Z+a\right)+2cd^3+be\,\mathrm{RootOf}\left(\_Z^4ce^4+4\_Z^3cde^3+cd^4+bd^2+\left(6cd^2e^2+be^2\right)\_Z^2+\left(4cd^3e+2deb\right)\_Z+a\right)+bd\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)/(a+b*(e*x+d)^2+c*(e*x+d)^4),x)
[Out] 1/2/e*sum((_R*e+d)/(2*_R^3*c*e^3+6*_R^2*c*d*e^2+6*_R*c*d^2*e+2*c*d^3+_R*b*e
+b*d)*ln(-_R+x),_R=RootOf(_Z^4*c*e^4+4*_Z^3*c*d*e^3+c*d^4+b*d^2+(6*c*d^2*e^
2+b*e^2)*_Z^2+(4*c*d^3*e+2*b*d*e)*_Z+a))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ex+d}{(ex+d)^4c+(ex+d)^2b+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)/((e*x + d)^4*c + (e*x + d)^2*b + a), x)
```

mupad [B] time = 0.09, size = 61, normalized size = 1.42

$$\frac{\operatorname{atan}\left(\frac{2\,a\,c\,d^2+4\,a\,c\,d\,e\,x+2\,a\,c\,e^2\,x^2+a\,b}{a\,\sqrt{4\,a\,c-b^2}}\right)}{e\,\sqrt{4\,a\,c-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)/(a + b*(d + e*x)^2 + c*(d + e*x)^4),x)
```

```
[Out] atan((a*b + 2*a*c*d^2 + 2*a*c*e^2*x^2 + 4*a*c*d*e*x)/(a*(4*a*c - b^2)^(1/2)
))/(e*(4*a*c - b^2)^(1/2))
```

sympy [B] time = 1.04, size = 168, normalized size = 3.91

$$-\frac{\sqrt{-\frac{1}{4ac-b^2}}\log\left(\frac{2dx}{e}+x^2+\frac{-4ac\sqrt{-\frac{1}{4ac-b^2}}+b^2\sqrt{-\frac{1}{4ac-b^2}}+b+2cd^2}{2ce^2}\right)}{2e}+\frac{\sqrt{-\frac{1}{4ac-b^2}}\log\left(\frac{2dx}{e}+x^2+\frac{4ac\sqrt{-\frac{1}{4ac-b^2}}-b^2\sqrt{-\frac{1}{4ac-b^2}}+b+2cd^2}{2ce^2}\right)}{2e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(a+b*(e*x+d)**2+c*(e*x+d)**4),x)
```

```
[Out] -sqrt(-1/(4*a*c - b**2))*log(2*d*x/e + x**2 + (-4*a*c*sqrt(-1/(4*a*c - b**2
)) + b**2*sqrt(-1/(4*a*c - b**2)) + b + 2*c*d**2)/(2*c*e**2))/(2*e) + sqrt(
-1/(4*a*c - b**2))*log(2*d*x/e + x**2 + (4*a*c*sqrt(-1/(4*a*c - b**2)) - b*
*2*sqrt(-1/(4*a*c - b**2)) + b + 2*c*d**2)/(2*c*e**2))/(2*e)
```

3.504 $\int \frac{1}{(d+ex)\left(a+b(d+ex)^2+c(d+ex)^4\right)} dx$

Optimal. Leaf size=94

$$\frac{b \tanh^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{2ae\sqrt{b^2-4ac}}-\frac{\log\left(a+b(d+ex)^2+c(d+ex)^4\right)}{4ae}+\frac{\log(d+ex)}{ae}$$

Rubi [A] time = 0.13, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {1142, 1114, 705, 29, 634, 618, 206, 628}

$$\frac{b \tanh^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{2ae\sqrt{b^2-4ac}}-\frac{\log\left(a+b(d+ex)^2+c(d+ex)^4\right)}{4ae}+\frac{\log(d+ex)}{ae}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)*(a + b*(d + e*x)^2 + c*(d + e*x)^4)),x]
```

```
[Out] (b*ArcTanh[(b + 2*c*(d + e*x)^2)/Sqrt[b^2 - 4*a*c]])/(2*a*Sqrt[b^2 - 4*a*c]
*e) + Log[d + e*x]/(a*e) - Log[a + b*(d + e*x)^2 + c*(d + e*x)^4]/(4*a*e)
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 705

```
Int[1/(((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)), x_Symbol]
 :> Dist[e^2/(c*d^2 - b*d*e + a*e^2), Int[1/(d + e*x), x], x] + Dist[1/(c*d
^2 - b*d*e + a*e^2), Int[(c*d - b*e - c*e*x)/(a + b*x + c*x^2), x], x] /; F
reeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^
2, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 1142

```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Di
st[u^m/(Coefficient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p,
 x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

Rubi steps

$$\int \frac{1}{(d+ex)\left(a+b(d+ex)^2+c(d+ex)^4\right)}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x\left(a+bx^2+cx^4\right)}\,dx, x, d+ex\right)}{e}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{x\left(a+bx+cx^2\right)}\,dx, x, (d+ex)^2\right)}{2e}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{x}\,dx, x, (d+ex)^2\right)}{2ae} + \frac{\text{Subst}\left(\int \frac{-b-cx}{a+bx+cx^2}\,dx, x, (d+ex)^2\right)}{2ae}$$

$$= \frac{\log(d+ex)}{ae} - \frac{\text{Subst}\left(\int \frac{b+2cx}{a+bx+cx^2}\,dx, x, (d+ex)^2\right)}{4ae} - \frac{b\,\text{Subst}\left(\int \frac{}{a+b}\right.}{}$$

$$= \frac{\log(d+ex)}{ae} - \frac{\log\left(a+b(d+ex)^2+c(d+ex)^4\right)}{4ae} + \frac{b\,\text{Subst}\left(\int \frac{1}{b^2-4a}\right.}{}$$

$$= \frac{b\tanh^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{2a\sqrt{b^2-4ac}\,e} + \frac{\log(d+ex)}{ae} - \frac{\log\left(a+b(d+ex)^2+c(d+\right.}{4ae}$$

Mathematica [A] time = 0.08, size = 128, normalized size = 1.36

$$\frac{4\sqrt{b^2-4ac}\log(d+ex)-\left(\sqrt{b^2-4ac}+b\right)\log\left(-\sqrt{b^2-4ac}+b+2c(d+ex)^2\right)+\left(b-\sqrt{b^2-4ac}\right)\log\left(\sqrt{b^2-4ac}+b+2c(d+ex)^2\right)}{4ae\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)*(a + b*(d + e*x)^2 + c*(d + e*x)^4)),x]
```

```
[Out] (4*Sqrt[b^2 - 4*a*c]*Log[d + e*x] - (b + Sqrt[b^2 - 4*a*c])*Log[b - Sqrt[b^
2 - 4*a*c] + 2*c*(d + e*x)^2] + (b - Sqrt[b^2 - 4*a*c])*Log[b + Sqrt[b^2 -
4*a*c] + 2*c*(d + e*x)^2])/(4*a*Sqrt[b^2 - 4*a*c]*e)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(d+ex)\left(a+b(d+ex)^2+c(d+ex)^4\right)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/((d + e*x)*(a + b*(d + e*x)^2 + c*(d + e*x)^4)),x]
```

```
[Out] IntegrateAlgebraic[1/((d + e*x)*(a + b*(d + e*x)^2 + c*(d + e*x)^4)), x]
```

fricas [A] time = 1.30, size = 468, normalized size = 4.98

$$\left[\frac{\sqrt{b^2-4ac}\,b\log\left(\frac{2c^2e^4x^4+8c^2de^3x^3+2c^2d^4+2(6c^2d^2+bc)e^2x^2+2bcd^2+4(2c^2d^3+bcd)ex+b^2-2ac+(2ce^2x^2+4cdex+2cd^2+b)\sqrt{b^2-4ac}}{ce^4x^4+4cde^3x^3+cd^4+(6cd^2+b)e^2x^2+bd^2+2(2cd^3+bd)ex+a}\right)-(b^2-4ac)\log\left(ce^4x^4+4cde^3x^3+cd^4+(6cd^2+b)e^2x^2+bd^2+2(2cd^3+bd)ex+a\right)+4(b^2-4ac)\log(ex+d)}{4(ab^2-4a^2c)e},\ \frac{2\sqrt{-b^2+4ac}\,b\arctan\left(-\frac{(2ce^2x^2+4cdex+2cd^2+b)\sqrt{-b^2+4ac}}{b^2-4ac}\right)-(b^2-4ac)\log\left(ce^4x^4+4cde^3x^3+cd^4+(6cd^2+b)e^2x^2+bd^2+2(2cd^3+bd)ex+a\right)+4(b^2-4ac)\log(ex+d)}{4(ab^2-4a^2c)e}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="fricas")
```

```
[Out] [1/4*(sqrt(b^2 - 4*a*c)*b*log((2*c^2*e^4*x^4 + 8*c^2*d*e^3*x^3 + 2*c^2*d^4
+ 2*(6*c^2*d^2 + b*c)*e^2*x^2 + 2*b*c*d^2 + 4*(2*c^2*d^3 + b*c*d)*e*x + b^2
 - 2*a*c + (2*c*e^2*x^2 + 4*c*d*e*x + 2*c*d^2 + b)*sqrt(b^2 - 4*a*c))/(c*e^
4*x^4 + 4*c*d*e^3*x^3 + c*d^4 + (6*c*d^2 + b)*e^2*x^2 + b*d^2 + 2*(2*c*d^3
+ b*d)*e*x + a)) - (b^2 - 4*a*c)*log(c*e^4*x^4 + 4*c*d*e^3*x^3 + c*d^4 + (6
*c*d^2 + b)*e^2*x^2 + b*d^2 + 2*(2*c*d^3 + b*d)*e*x + a) + 4*(b^2 - 4*a*c)*
log(e*x + d))/((a*b^2 - 4*a^2*c)*e), 1/4*(2*sqrt(-b^2 + 4*a*c)*b*arctan(-(2
*c*e^2*x^2 + 4*c*d*e*x + 2*c*d^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) - (
b^2 - 4*a*c)*log(c*e^4*x^4 + 4*c*d*e^3*x^3 + c*d^4 + (6*c*d^2 + b)*e^2*x^2
+ b*d^2 + 2*(2*c*d^3 + b*d)*e*x + a) + 4*(b^2 - 4*a*c)*log(e*x + d))/((a*b^
2 - 4*a^2*c)*e)]
```

giac [B] time = 1.15, size = 274, normalized size = 2.91

$$-\frac{e^{(-1)}\log\left(\left|cx^4e^4+4cdx^3e^3+6cd^2x^2e^2+4cd^3xe+cd^4+bx^2e^2+2bdxe+bd^2+a\right|\right)}{4a}+\frac{e^{(-1)}\log\left(\left|xe+d\right|\right)}{a}-\frac{\left(\frac{abce^3\log\left(\left|bx^2e^2+2bdxe+\sqrt{b^2-4ac}\,x^2e^2+2\sqrt{b^2-4ac}\,dxe+bd^2+\sqrt{b^2-4ac}\,d^2+2a\right|\right)}{\sqrt{b^2-4ac}}-\frac{abce^3\log\left(\left|-bx^2e^2-2bdxe+\sqrt{b^2-4ac}\,x^2e^2+2\sqrt{b^2-4ac}\,dxe-bd^2+\sqrt{b^2-4ac}\,d^2-2a\right|\right)}{\sqrt{b^2-4ac}}\right)e^{(-4)}}{4a^2c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="giac")
```

```
[Out] -1/4*e^(-1)*log(abs(c*x^4*e^4 + 4*c*d*x^3*e^3 + 6*c*d^2*x^2*e^2 + 4*c*d^3*x
*e + c*d^4 + b*x^2*e^2 + 2*b*d*x*e + b*d^2 + a))/a + e^(-1)*log(abs(x*e + d
))/a - 1/4*(a*b*c*e^3*log(abs(b*x^2*e^2 + 2*b*d*x*e + sqrt(b^2 - 4*a*c)*x^2
*e^2 + 2*sqrt(b^2 - 4*a*c)*d*x*e + b*d^2 + sqrt(b^2 - 4*a*c)*d^2 + 2*a))/sq
rt(b^2 - 4*a*c) - a*b*c*e^3*log(abs(-b*x^2*e^2 - 2*b*d*x*e + sqrt(b^2 - 4*a
*c)*x^2*e^2 + 2*sqrt(b^2 - 4*a*c)*d*x*e - b*d^2 + sqrt(b^2 - 4*a*c)*d^2 - 2
*a))/sqrt(b^2 - 4*a*c))*e^(-4)/(a^2*c)
```

maple [C] time = 0.01, size = 184, normalized size = 1.96

$$\frac{\ln(ex+d)}{ae}+\sum_{\_R=\mathrm{RootOf}\left(\_Z^4ce^4+4\_Z^3cde^3+cd^4+bd^2+(6cd^2e^2+be^2)\_Z^2+(4cd^3e+2bde)\_Z+a\right)}\frac{\left(-ce^3\_R^3-3cde^2\_R^2+(-3cd^2-b)e\,\_R-cd^3-bd\right)\ln(-\_R+x)}{2ae\left(2ce^3\_R^3+6cde^2\_R^2+6cd^2e\,\_R+2cd^3+be\,\_R+bd\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)/(a+b*(e*x+d)^2+c*(e*x+d)^4),x)
```

```
[Out] 1/2/a/e*sum((-c*e^3*_R^3-3*c*d*e^2*_R^2+e*(-3*c*d^2-b)*_R-c*d^3-b*d)/(2*_R^
3*c*e^3+6*_R^2*c*d*e^2+6*_R*c*d^2*e+2*c*d^3+_R*b*e+b*d)*ln(-_R+x),_R=RootOf
(_Z^4*c*e^4+4*_Z^3*c*d*e^3+c*d^4+b*d^2+(6*c*d^2*e^2+b*e^2)*_Z^2+(4*c*d^3*e+
2*b*d*e)*_Z+a))+ln(e*x+d)/a/e
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [B] time = 2.50, size = 2173, normalized size = 23.12

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((d + e*x)*(a + b*(d + e*x)^2 + c*(d + e*x)^4)),x)
```

```
[Out] log(d + e*x)/(a*e) - (log(a + b*d^2 + c*d^4 + b*e^2*x^2 + c*e^4*x^4 + 2*b*d
*e*x + 6*c*d^2*e^2*x^2 + 4*c*d^3*e*x + 4*c*d*e^3*x^3)*(2*b^2*e - 8*a*c*e))/
(2*(4*a*b^2*e^2 - 16*a^2*c*e^2)) - (b*atan((16*a^3*x^2*(((3*b^3 - 8*a*b*c)*
((b^2*(10*b*c^3*e^18 + ((2*b^2*e - 8*a*c*e)*(12*b^3*c^2*e^19 - 40*a*b*c^3*e
^19))/(2*(4*a*b^2*e^2 - 16*a^2*c*e^2))))/(16*a^2*e^2*(4*a*c - b^2)) - ((2*b
^2*e - 8*a*c*e)^2*(10*b*c^3*e^18 + ((2*b^2*e - 8*a*c*e)*(12*b^3*c^2*e^19 -
40*a*b*c^3*e^19))/(2*(4*a*b^2*e^2 - 16*a^2*c*e^2))))/(4*(4*a*b^2*e^2 - 16*a
^2*c*e^2)^2) + (b^2*(2*b^2*e - 8*a*c*e)*(12*b^3*c^2*e^19 - 40*a*b*c^3*e^19)
)/(16*a^2*e^2*(4*a*c - b^2)*(4*a*b^2*e^2 - 16*a^2*c*e^2))))/(8*a^3*c^2*(25*
a*c - 6*b^2)) - (((b*(2*b^2*e - 8*a*c*e)^2*(12*b^3*c^2*e^19 - 40*a*b*c^3*e^
19))/(16*a*e*(4*a*c - b^2)^(1/2)*(4*a*b^2*e^2 - 16*a^2*c*e^2)^2) - (b^3*(12
*b^3*c^2*e^19 - 40*a*b*c^3*e^19))/(64*a^3*e^3*(4*a*c - b^2)^(3/2)) + (b*(2*
b^2*e - 8*a*c*e)*(10*b*c^3*e^18 + ((2*b^2*e - 8*a*c*e)*(12*b^3*c^2*e^19 - 4
0*a*b*c^3*e^19))/(2*(4*a*b^2*e^2 - 16*a^2*c*e^2))))/(4*a*e*(4*a*c - b^2)^(1
/2)*(4*a*b^2*e^2 - 16*a^2*c*e^2)))*(3*b^4 + 10*a^2*c^2 - 14*a*b^2*c))/(8*a^
3*c^2*(4*a*c - b^2)^(1/2)*(25*a*c - 6*b^2)))*(4*a*c - b^2)^(3/2))/(b^2*c^2*
e^14) + (2*(3*b^3 - 8*a*b*c)*(4*a*c - b^2)^(3/2)*((b^2*(((2*b^2*e - 8*a*c*e
)*(4*a*b^2*c^2*e^17 + 12*b^3*c^2*d^2*e^17 - 40*a*b*c^3*d^2*e^17))/(2*(4*a*b
^2*e^2 - 16*a^2*c*e^2)) + 4*b^2*c^2*e^16 + 10*b*c^3*d^2*e^16))/(16*a^2*e^2*
(4*a*c - b^2)) - ((2*b^2*e - 8*a*c*e)^2*(((2*b^2*e - 8*a*c*e)*(4*a*b^2*c^2*
e^17 + 12*b^3*c^2*d^2*e^17 - 40*a*b*c^3*d^2*e^17))/(2*(4*a*b^2*e^2 - 16*a^2
*c*e^2)) + 4*b^2*c^2*e^16 + 10*b*c^3*d^2*e^16))/(4*(4*a*b^2*e^2 - 16*a^2*c*
e^2)^2) + (b^2*(2*b^2*e - 8*a*c*e)*(4*a*b^2*c^2*e^17 + 12*b^3*c^2*d^2*e^17
 - 40*a*b*c^3*d^2*e^17))/(16*a^2*e^2*(4*a*c - b^2)*(4*a*b^2*e^2 - 16*a^2*c*e
^2))))/(b^2*c^4*e^14*(25*a*c - 6*b^2)) - (2*(4*a*c - b^2)*(3*b^4 + 10*a^2*c
^2 - 14*a*b^2*c)*((b*(2*b^2*e - 8*a*c*e)*(((2*b^2*e - 8*a*c*e)*(4*a*b^2*c^2
*e^17 + 12*b^3*c^2*d^2*e^17 - 40*a*b*c^3*d^2*e^17))/(2*(4*a*b^2*e^2 - 16*a^
2*c*e^2)) + 4*b^2*c^2*e^16 + 10*b*c^3*d^2*e^16))/(4*a*e*(4*a*c - b^2)^(1/2)
*(4*a*b^2*e^2 - 16*a^2*c*e^2)) - (b^3*(4*a*b^2*c^2*e^17 + 12*b^3*c^2*d^2*e^
17 - 40*a*b*c^3*d^2*e^17))/(64*a^3*e^3*(4*a*c - b^2)^(3/2)) + (b*(2*b^2*e -
 8*a*c*e)^2*(4*a*b^2*c^2*e^17 + 12*b^3*c^2*d^2*e^17 - 40*a*b*c^3*d^2*e^17))
/(16*a*e*(4*a*c - b^2)^(1/2)*(4*a*b^2*e^2 - 16*a^2*c*e^2)^2)))/(b^2*c^4*e^1
4*(25*a*c - 6*b^2)) + (16*a^3*x*(((3*b^3 - 8*a*b*c)*((b^2*(((2*b^2*e - 8*a*
c*e)*(24*b^3*c^2*d*e^18 - 80*a*b*c^3*d*e^18))/(2*(4*a*b^2*e^2 - 16*a^2*c*e^
2)) + 20*b*c^3*d*e^17))/(16*a^2*e^2*(4*a*c - b^2)) - ((2*b^2*e - 8*a*c*e)^2
*(((2*b^2*e - 8*a*c*e)*(24*b^3*c^2*d*e^18 - 80*a*b*c^3*d*e^18))/(2*(4*a*b^2
*e^2 - 16*a^2*c*e^2)) + 20*b*c^3*d*e^17))/(4*(4*a*b^2*e^2 - 16*a^2*c*e^2)^2
) + (b^2*(2*b^2*e - 8*a*c*e)*(24*b^3*c^2*d*e^18 - 80*a*b*c^3*d*e^18))/(16*a
^2*e^2*(4*a*c - b^2)*(4*a*b^2*e^2 - 16*a^2*c*e^2))))/(8*a^3*c^2*(25*a*c - 6
*b^2)) - ((3*b^4 + 10*a^2*c^2 - 14*a*b^2*c)*((b*(2*b^2*e - 8*a*c*e)*(((2*b^
2*e - 8*a*c*e)*(24*b^3*c^2*d*e^18 - 80*a*b*c^3*d*e^18))/(2*(4*a*b^2*e^2 - 1
6*a^2*c*e^2)) + 20*b*c^3*d*e^17))/(4*a*e*(4*a*c - b^2)^(1/2)*(4*a*b^2*e^2 -
 16*a^2*c*e^2)) - (b^3*(24*b^3*c^2*d*e^18 - 80*a*b*c^3*d*e^18))/(64*a^3*e^3
*(4*a*c - b^2)^(3/2)) + (b*(2*b^2*e - 8*a*c*e)^2*(24*b^3*c^2*d*e^18 - 80*a*
b*c^3*d*e^18))/(16*a*e*(4*a*c - b^2)^(1/2)*(4*a*b^2*e^2 - 16*a^2*c*e^2)^2))
)/(8*a^3*c^2*(4*a*c - b^2)^(1/2)*(25*a*c - 6*b^2)))*(4*a*c - b^2)^(3/2))/(b
^2*c^2*e^14)))/(2*a*e*(4*a*c - b^2)^(1/2))
```

sympy [B] time = 6.04, size = 320, normalized size = 3.40

$$\left(- \frac{b \sqrt{-4ac + b^{2}}}{4ae \left(4ac - b^{2}\right)} - \frac{1}{4ae}\right) \log{\left(\frac{2dx}{e} + x^{2} + \frac{-8a^{2}ce \left(- \frac{b \sqrt{-4ac+b^{2}}}{4ae(4ac-b^{2})} - \frac{1}{4ae}\right) + 2ab^{2}e \left(- \frac{b \sqrt{-4ac+b^{2}}}{4ae(4ac-b^{2})} - \frac{1}{4ae}\right) - 2ac + b^{2} + bcd^{2}}{bce^{2}} \right)} + \left(\frac{b \sqrt{-4ac + b^{2}}}{4ae \left(4ac - b^{2}\right)} - \frac{1}{4ae}\right) \log{\left(\frac{2dx}{e} + x^{2} + \frac{-8a^{2}ce \left(\frac{b \sqrt{-4ac+b^{2}}}{4ae(4ac-b^{2})} - \frac{1}{4ae}\right) + 2ab^{2}e \left(\frac{b \sqrt{-4ac+b^{2}}}{4ae(4ac-b^{2})} - \frac{1}{4ae}\right) - 2ac + b^{2} + bcd^{2}}{bce^{2}} \right)} + \frac{\log{\left(\frac{d}{e} + x \right)}}{ae}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(a+b*(e*x+d)**2+c*(e*x+d)**4),x)
```

```
[Out] (-b*sqrt(-4*a*c + b**2)/(4*a*e*(4*a*c - b**2)) - 1/(4*a*e))*log(2*d*x/e + x
**2 + (-8*a**2*c*e*(-b*sqrt(-4*a*c + b**2)/(4*a*e*(4*a*c - b**2)) - 1/(4*a*
e)) + 2*a*b**2*e*(-b*sqrt(-4*a*c + b**2)/(4*a*e*(4*a*c - b**2)) - 1/(4*a*e)
) - 2*a*c + b**2 + b*c*d**2)/(b*c*e**2)) + (b*sqrt(-4*a*c + b**2)/(4*a*e*(4
*a*c - b**2)) - 1/(4*a*e))*log(2*d*x/e + x**2 + (-8*a**2*c*e*(b*sqrt(-4*a*c
 + b**2)/(4*a*e*(4*a*c - b**2)) - 1/(4*a*e)) + 2*a*b**2*e*(b*sqrt(-4*a*c +
b**2)/(4*a*e*(4*a*c - b**2)) - 1/(4*a*e)) - 2*a*c + b**2 + b*c*d**2)/(b*c*e
**2)) + log(d/e + x)/(a*e)
```

3.505 $\int \frac{1}{(d+ex)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)}\,dx$

Optimal. Leaf size=195

$$-\frac{\sqrt{c}\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\,ae\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{c}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\,ae\sqrt{\sqrt{b^2-4ac}+b}}-\frac{1}{ae(d+ex)}$$

Rubi [A] time = 0.29, antiderivative size = 195, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {1142, 1123, 1166, 205}

$$-\frac{\sqrt{c}\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\,ae\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{c}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\,ae\sqrt{\sqrt{b^2-4ac}+b}}-\frac{1}{ae(d+ex)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)^2*(a + b*(d + e*x)^2 + c*(d + e*x)^4)),x]
```

```
[Out] -(1/(a*e*(d + e*x))) - (Sqrt[c]*(1 + b/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*S
qrt[c]*(d + e*x))/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*a*Sqrt[b - Sqrt[b^
2 - 4*a*c]]*e) - (Sqrt[c]*(1 - b/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]
*(d + e*x))/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*a*Sqrt[b + Sqrt[b^2 - 4*
a*c]]*e)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1123

```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*x^2 + c*x^4)^(p + 1))/(a*d*(m + 1)), x] - Dis
t[1/(a*d^2*(m + 1)), Int[(d*x)^(m + 2)*(b*(m + 2*p + 3) + c*(m + 4*p + 5)*x
^2)*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b^2 -
 4*a*c, 0] && LtQ[m, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1142

```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Di
st[u^m/(Coefficient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p,
 x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{1}{(d+ex)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)}dx = \frac{\text{Subst}\left(\int \frac{1}{x^2(a+bx^2+cx^4)}dx, x, d+ex\right)}{e}$$

$$= -\frac{1}{ae(d+ex)} + \frac{\text{Subst}\left(\int \frac{-b-cx^2}{a+bx^2+cx^4}dx, x, d+ex\right)}{ae}$$

$$= -\frac{1}{ae(d+ex)} - \frac{\left(c\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\right)\text{Subst}\left(\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^2}dx, x\right.}{2ae}$$

$$= -\frac{1}{ae(d+ex)} - \frac{\sqrt{c}\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}a\sqrt{b-\sqrt{b^2-4ac}}\,e} - \frac{\sqrt{c}\left(1\right.}{}$$

Mathematica [A] time = 0.36, size = 206, normalized size = 1.06

$$-\frac{\frac{\sqrt{2}\sqrt{c}\left(\sqrt{b^2-4ac}+b\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{2}\sqrt{c}\left(\sqrt{b^2-4ac}-b\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4ac}+b}}+\frac{2}{d+ex}}{2ae}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)^2*(a + b*(d + e*x)^2 + c*(d + e*x)^4)),x]
```

```
[Out] -1/2*(2/(d + e*x) + (Sqrt[2]*Sqrt[c]*(b + Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2
]*Sqrt[c]*(d + e*x))/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2 - 4*a*c]*Sqrt[
b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[2]*Sqrt[c]*(-b + Sqrt[b^2 - 4*a*c])*ArcTan[
(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2 - 4*a*c
]*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(a*e)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(d+ex)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)}dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/((d + e*x)^2*(a + b*(d + e*x)^2 + c*(d + e*x)^4)),x]
```

```
[Out] IntegrateAlgebraic[1/((d + e*x)^2*(a + b*(d + e*x)^2 + c*(d + e*x)^4)), x]
```

fricas [B] time = 1.36, size = 1339, normalized size = 6.87

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="fricas")
```

```
[Out] 1/2*(sqrt(1/2)*(a*e^2*x + a*d*e)*sqrt(-((a^3*b^2 - 4*a^4*c)*e^2*sqrt((b^4 -
 2*a*b^2*c + a^2*c^2)/((a^6*b^2 - 4*a^7*c)*e^4)) + b^3 - 3*a*b*c)/((a^3*b^2
 - 4*a^4*c)*e^2))*log(-2*(b^2*c^2 - a*c^3)*e*x - 2*(b^2*c^2 - a*c^3)*d + sq
rt(1/2)*((a^3*b^4 - 6*a^4*b^2*c + 8*a^5*c^2)*e^3*sqrt((b^4 - 2*a*b^2*c + a^
2*c^2)/((a^6*b^2 - 4*a^7*c)*e^4)) - (b^5 - 5*a*b^3*c + 4*a^2*b*c^2)*e)*sqrt
(-((a^3*b^2 - 4*a^4*c)*e^2*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/((a^6*b^2 - 4*a
^7*c)*e^4)) + b^3 - 3*a*b*c)/((a^3*b^2 - 4*a^4*c)*e^2))) - sqrt(1/2)*(a*e^2
```

```
*x + a*d*e)*sqrt(-((a^3*b^2 - 4*a^4*c)*e^2*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)
/((a^6*b^2 - 4*a^7*c)*e^4)) + b^3 - 3*a*b*c)/((a^3*b^2 - 4*a^4*c)*e^2))*log
(-2*(b^2*c^2 - a*c^3)*e*x - 2*(b^2*c^2 - a*c^3)*d - sqrt(1/2)*((a^3*b^4 - 6
*a^4*b^2*c + 8*a^5*c^2)*e^3*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/((a^6*b^2 - 4*
a^7*c)*e^4)) - (b^5 - 5*a*b^3*c + 4*a^2*b*c^2)*e)*sqrt(-((a^3*b^2 - 4*a^4*c
)*e^2*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/((a^6*b^2 - 4*a^7*c)*e^4)) + b^3 - 3
*a*b*c)/((a^3*b^2 - 4*a^4*c)*e^2))) - sqrt(1/2)*(a*e^2*x + a*d*e)*sqrt(((a^
3*b^2 - 4*a^4*c)*e^2*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/((a^6*b^2 - 4*a^7*c)*
e^4)) - b^3 + 3*a*b*c)/((a^3*b^2 - 4*a^4*c)*e^2))*log(-2*(b^2*c^2 - a*c^3)*
e*x - 2*(b^2*c^2 - a*c^3)*d + sqrt(1/2)*((a^3*b^4 - 6*a^4*b^2*c + 8*a^5*c^2
)*e^3*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/((a^6*b^2 - 4*a^7*c)*e^4)) + (b^5 -
5*a*b^3*c + 4*a^2*b*c^2)*e)*sqrt(((a^3*b^2 - 4*a^4*c)*e^2*sqrt((b^4 - 2*a*b
^2*c + a^2*c^2)/((a^6*b^2 - 4*a^7*c)*e^4)) - b^3 + 3*a*b*c)/((a^3*b^2 - 4*a
^4*c)*e^2))) + sqrt(1/2)*(a*e^2*x + a*d*e)*sqrt(((a^3*b^2 - 4*a^4*c)*e^2*sq
rt((b^4 - 2*a*b^2*c + a^2*c^2)/((a^6*b^2 - 4*a^7*c)*e^4)) - b^3 + 3*a*b*c)/
((a^3*b^2 - 4*a^4*c)*e^2))*log(-2*(b^2*c^2 - a*c^3)*e*x - 2*(b^2*c^2 - a*c^
3)*d - sqrt(1/2)*((a^3*b^4 - 6*a^4*b^2*c + 8*a^5*c^2)*e^3*sqrt((b^4 - 2*a*b
^2*c + a^2*c^2)/((a^6*b^2 - 4*a^7*c)*e^4)) + (b^5 - 5*a*b^3*c + 4*a^2*b*c^2
)*e)*sqrt(((a^3*b^2 - 4*a^4*c)*e^2*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/((a^6*b
^2 - 4*a^7*c)*e^4)) - b^3 + 3*a*b*c)/((a^3*b^2 - 4*a^4*c)*e^2))) - 2)/(a*e^
2*x + a*d*e)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Error index.cc index_gcd Error: Bad Argument
 ValueError index.cc index_gcd Error: Bad Argument ValueDone
```

maple [C] time = 0.01, size = 168, normalized size = 0.86

$$\frac{\left(-\mathrm{RootOf}\left(\_Z^4 c e^4+4\_Z^3 c d e^3+c d^4+b d^2+\left(6 c d^2 e^2+b e^2\right)\_Z^2+\left(4 c d^3 e+2 d e b\right)\_Z+a\right)^2 c e^2-2\,\mathrm{RootOf}\left(\_Z^4 c e^4+4\_Z^3 c d e^3+c d^4+b d^2+\left(6 c d^2 e^2+b e^2\right)\_Z^2+\left(4 c d^3 e+2 d e b\right)\_Z+a\right) c d e-c d^2-b\right)\ln\left(-\mathrm{RootOf}\left(\_Z^4 c e^4+4\_Z^3 c d e^3+c d^4+b d^2+\left(6 c d^2 e^2+b e^2\right)\_Z^2+\left(4 c d^3 e+2 d e b\right)\_Z+a\right)+x\right)}{2 a e\left(2 c e^3\,\mathrm{RootOf}\left(\_Z^4 c e^4+4\_Z^3 c d e^3+c d^4+b d^2+\left(6 c d^2 e^2+b e^2\right)\_Z^2+\left(4 c d^3 e+2 d e b\right)\_Z+a\right)^3+6 c d e^2\,\mathrm{RootOf}\left(\_Z^4 c e^4+4\_Z^3 c d e^3+c d^4+b d^2+\left(6 c d^2 e^2+b e^2\right)\_Z^2+\left(4 c d^3 e+2 d e b\right)\_Z+a\right)^2+6 e c d^2\,\mathrm{RootOf}\left(\_Z^4 c e^4+4\_Z^3 c d e^3+c d^4+b d^2+\left(6 c d^2 e^2+b e^2\right)\_Z^2+\left(4 c d^3 e+2 d e b\right)\_Z+a\right)+2 c d^3+b e\,\mathrm{RootOf}\left(\_Z^4 c e^4+4\_Z^3 c d e^3+c d^4+b d^2+\left(6 c d^2 e^2+b e^2\right)\_Z^2+\left(4 c d^3 e+2 d e b\right)\_Z+a\right)+b d\right)}-\frac{1}{(e x+d) a e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4),x)
```

```
[Out] 1/2/a/e*sum((-_R^2*c*e^2-2*_R*c*d*e-c*d^2-b)/(2*_R^3*c*e^3+6*_R^2*c*d*e^2+6
*_R*c*d^2*e+2*c*d^3+_R*b*e+b*d)*ln(-_R+x),_R=RootOf(_Z^4*c*e^4+4*_Z^3*c*d*e
^3+c*d^4+b*d^2+(6*c*d^2*e^2+b*e^2)*_Z^2+(4*c*d^3*e+2*b*d*e)*_Z+a))-1/a/e/(e
*x+d)
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [B] time = 2.39, size = 3844, normalized size = 19.71

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((d + e*x)^2*(a + b*(d + e*x)^2 + c*(d + e*x)^4)),x)
```

```
[Out] - atan(((-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c -
a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4*e^2 + 16*a^5*c^2*e^2 - 8*a^4*b^2*
c*e^2)))^(1/2)*(x*(4*a^4*c^4*e^12 - 2*a^3*b^2*c^3*e^12) + ((x*(32*a^6*b*c^3
*e^14 - 8*a^5*b^3*c^2*e^14) + 32*a^6*b*c^3*d*e^13 - 8*a^5*b^3*c^2*d*e^13)*(
-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*
a*c - b^2)^3)^(1/2))/(8*(a^3*b^4*e^2 + 16*a^5*c^2*e^2 - 8*a^4*b^2*c*e^2)))^
(1/2) - 16*a^5*b*c^3*e^12 + 4*a^4*b^3*c^2*e^12)*(-(b^5 + b^2*(-(4*a*c - b^2
)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a
^3*b^4*e^2 + 16*a^5*c^2*e^2 - 8*a^4*b^2*c*e^2)))^(1/2) + 4*a^4*c^4*d*e^11 -
 2*a^3*b^2*c^3*d*e^11)*1i + (-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*
b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4*e^2 + 16*a^5*
c^2*e^2 - 8*a^4*b^2*c*e^2)))^(1/2)*(x*(4*a^4*c^4*e^12 - 2*a^3*b^2*c^3*e^12)
 + ((x*(32*a^6*b*c^3*e^14 - 8*a^5*b^3*c^2*e^14) + 32*a^6*b*c^3*d*e^13 - 8*a
^5*b^3*c^2*d*e^13)*(-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7
*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4*e^2 + 16*a^5*c^2*e^2 -
 8*a^4*b^2*c*e^2)))^(1/2) + 16*a^5*b*c^3*e^12 - 4*a^4*b^3*c^2*e^12)*(-(b^5
+ b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c -
b^2)^3)^(1/2))/(8*(a^3*b^4*e^2 + 16*a^5*c^2*e^2 - 8*a^4*b^2*c*e^2)))^(1/2)
+ 4*a^4*c^4*d*e^11 - 2*a^3*b^2*c^3*d*e^11)*1i)/((-(b^5 + b^2*(-(4*a*c - b^2
)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a
^3*b^4*e^2 + 16*a^5*c^2*e^2 - 8*a^4*b^2*c*e^2)))^(1/2)*(x*(4*a^4*c^4*e^12 -
 2*a^3*b^2*c^3*e^12) + ((x*(32*a^6*b*c^3*e^14 - 8*a^5*b^3*c^2*e^14) + 32*a^
6*b*c^3*d*e^13 - 8*a^5*b^3*c^2*d*e^13)*(-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2
) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4*e^
2 + 16*a^5*c^2*e^2 - 8*a^4*b^2*c*e^2)))^(1/2) + 16*a^5*b*c^3*e^12 - 4*a^4*b
^3*c^2*e^12)*(-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3
*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4*e^2 + 16*a^5*c^2*e^2 - 8*a^4
*b^2*c*e^2)))^(1/2) + 4*a^4*c^4*d*e^11 - 2*a^3*b^2*c^3*d*e^11) - (-(b^5 + b
^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2
)^3)^(1/2))/(8*(a^3*b^4*e^2 + 16*a^5*c^2*e^2 - 8*a^4*b^2*c*e^2)))^(1/2)*(x*
(4*a^4*c^4*e^12 - 2*a^3*b^2*c^3*e^12) + ((x*(32*a^6*b*c^3*e^14 - 8*a^5*b^3*
c^2*e^14) + 32*a^6*b*c^3*d*e^13 - 8*a^5*b^3*c^2*d*e^13)*(-(b^5 + b^2*(-(4*a
*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2
))/(8*(a^3*b^4*e^2 + 16*a^5*c^2*e^2 - 8*a^4*b^2*c*e^2)))^(1/2) - 16*a^5*b*c
^3*e^12 + 4*a^4*b^3*c^2*e^12)*(-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^
2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4*e^2 + 16*a^
5*c^2*e^2 - 8*a^4*b^2*c*e^2)))^(1/2) + 4*a^4*c^4*d*e^11 - 2*a^3*b^2*c^3*d*e
^11) + 2*a^3*c^4*e^10))*(-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^
2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4*e^2 + 16*a^5*c^2*
e^2 - 8*a^4*b^2*c*e^2)))^(1/2)*2i - atan(((-(b^5 - b^2*(-(4*a*c - b^2)^3)^(
1/2) + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4
*e^2 + 16*a^5*c^2*e^2 - 8*a^4*b^2*c*e^2)))^(1/2)*(x*(4*a^4*c^4*e^12 - 2*a^3
*b^2*c^3*e^12) + ((x*(32*a^6*b*c^3*e^14 - 8*a^5*b^3*c^2*e^14) + 32*a^6*b*c^
3*d*e^13 - 8*a^5*b^3*c^2*d*e^13)*(-(b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 12
*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4*e^2 + 16
*a^5*c^2*e^2 - 8*a^4*b^2*c*e^2)))^(1/2) - 16*a^5*b*c^3*e^12 + 4*a^4*b^3*c^2
*e^12)*(-(b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c + a
*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4*e^2 + 16*a^5*c^2*e^2 - 8*a^4*b^2*c
*e^2)))^(1/2) + 4*a^4*c^4*d*e^11 - 2*a^3*b^2*c^3*d*e^11)*1i + (-(b^5 - b^2*
(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3
)^(1/2))/(8*(a^3*b^4*e^2 + 16*a^5*c^2*e^2 - 8*a^4*b^2*c*e^2)))^(1/2)*(x*(4*
a^4*c^4*e^12 - 2*a^3*b^2*c^3*e^12) + ((x*(32*a^6*b*c^3*e^14 - 8*a^5*b^3*c^2
*e^14) + 32*a^6*b*c^3*d*e^13 - 8*a^5*b^3*c^2*d*e^13)*(-(b^5 - b^2*(-(4*a*c
- b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2))/
(8*(a^3*b^4*e^2 + 16*a^5*c^2*e^2 - 8*a^4*b^2*c*e^2)))^(1/2) + 16*a^5*b*c^3*
e^12 - 4*a^4*b^3*c^2*e^12)*(-(b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b
*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4*e^2 + 16*a^5*c
^2*e^2 - 8*a^4*b^2*c*e^2)))^(1/2) + 4*a^4*c^4*d*e^11 - 2*a^3*b^2*c^3*d*e^11
)*1i)/((-(b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c + a
```

```
*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4*e^2 + 16*a^5*c^2*e^2 - 8*a^4*b^2*c
*e^2)))^(1/2)*(x*(4*a^4*c^4*e^12 - 2*a^3*b^2*c^3*e^12) + ((x*(32*a^6*b*c^3*
e^14 - 8*a^5*b^3*c^2*e^14) + 32*a^6*b*c^3*d*e^13 - 8*a^5*b^3*c^2*d*e^13)*(-
(b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a
*c - b^2)^3)^(1/2))/(8*(a^3*b^4*e^2 + 16*a^5*c^2*e^2 - 8*a^4*b^2*c*e^2)))^(
1/2) + 16*a^5*b*c^3*e^12 - 4*a^4*b^3*c^2*e^12)*(-(b^5 - b^2*(-(4*a*c - b^2)
^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^
3*b^4*e^2 + 16*a^5*c^2*e^2 - 8*a^4*b^2*c*e^2)))^(1/2) + 4*a^4*c^4*d*e^11 -
2*a^3*b^2*c^3*d*e^11) - (-(b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^
2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4*e^2 + 16*a^5*c^2*
e^2 - 8*a^4*b^2*c*e^2)))^(1/2)*(x*(4*a^4*c^4*e^12 - 2*a^3*b^2*c^3*e^12) + (
(x*(32*a^6*b*c^3*e^14 - 8*a^5*b^3*c^2*e^14) + 32*a^6*b*c^3*d*e^13 - 8*a^5*b
^3*c^2*d*e^13)*(-(b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b
^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4*e^2 + 16*a^5*c^2*e^2 - 8*a
^4*b^2*c*e^2)))^(1/2) - 16*a^5*b*c^3*e^12 + 4*a^4*b^3*c^2*e^12)*(-(b^5 - b^
2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)
^3)^(1/2))/(8*(a^3*b^4*e^2 + 16*a^5*c^2*e^2 - 8*a^4*b^2*c*e^2)))^(1/2) + 4*
a^4*c^4*d*e^11 - 2*a^3*b^2*c^3*d*e^11) + 2*a^3*c^4*e^10))*(-(b^5 - b^2*(-(4
*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1
/2))/(8*(a^3*b^4*e^2 + 16*a^5*c^2*e^2 - 8*a^4*b^2*c*e^2)))^(1/2)*2i - 1/(a*
e*(d + e*x))
```

sympy [A] time = 4.27, size = 211, normalized size = 1.08

$$\operatorname{RootSum}\left(t^4\left(256a^5c^2e^4-128a^4b^2ce^4+16a^3b^4e^4\right)+t^2\left(48a^2bc^2e^2-28ab^3ce^2+4b^5e^2\right)+c^3,\left(t\mapsto t\log\left(x+\frac{-64t^3a^5c^2e^3+48t^3a^4b^2ce^3-8t^3a^3b^4e^3-10ta^2bc^2e+10tab^3ce-2tb^5e+ac^3d-b^2c^2d}{ac^3e-b^2c^2e}\right)\right)\right)-\frac{1}{ade+ae^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)**2/(a+b*(e*x+d)**2+c*(e*x+d)**4),x)
```

```
[Out] RootSum(_t**4*(256*a**5*c**2*e**4 - 128*a**4*b**2*c*e**4 + 16*a**3*b**4*e**
4) + _t**2*(48*a**2*b*c**2*e**2 - 28*a*b**3*c*e**2 + 4*b**5*e**2) + c**3, L
ambda(_t, _t*log(x + (-64*_t**3*a**5*c**2*e**3 + 48*_t**3*a**4*b**2*c*e**3
- 8*_t**3*a**3*b**4*e**3 - 10*_t*a**2*b*c**2*e + 10*_t*a*b**3*c*e - 2*_t*b*
*5*e + a*c**3*d - b**2*c**2*d)/(a*c**3*e - b**2*c**2*e)))) - 1/(a*d*e + a*e
**2*x)
```

3.506 $\int \frac{1}{(d+ex)^3\left(a+b(d+ex)^2+c(d+ex)^4\right)} dx$

Optimal. Leaf size=121

$$-\frac{(b^2-2ac)\tanh^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{2a^2e\sqrt{b^2-4ac}}+\frac{b\log\left(a+b(d+ex)^2+c(d+ex)^4\right)}{4a^2e}-\frac{b\log(d+ex)}{a^2e}-\frac{1}{2ae(d+ex)^2}$$

Rubi [A] time = 0.20, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {1142, 1114, 709, 800, 634, 618, 206, 628}

$$-\frac{(b^2-2ac)\tanh^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{2a^2e\sqrt{b^2-4ac}}+\frac{b\log\left(a+b(d+ex)^2+c(d+ex)^4\right)}{4a^2e}-\frac{b\log(d+ex)}{a^2e}-\frac{1}{2ae(d+ex)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)^3*(a + b*(d + e*x)^2 + c*(d + e*x)^4)),x]
```

```
[Out] -1/(2*a*e*(d + e*x)^2) - ((b^2 - 2*a*c)*ArcTanh[(b + 2*c*(d + e*x)^2)/Sqrt[
b^2 - 4*a*c]])/(2*a^2*Sqrt[b^2 - 4*a*c]*e) - (b*Log[d + e*x])/(a^2*e) + (b*
Log[a + b*(d + e*x)^2 + c*(d + e*x)^4])/(4*a^2*e)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 709

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Simp[(e*(d + e*x)^(m + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dis
t[1/(c*d^2 - b*d*e + a*e^2), Int[((d + e*x)^(m + 1)*Simp[c*d - b*e - c*e*x,
 x])/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 -
 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[m
, -1]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 1142

```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Di
st[u^m/(Coefficient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p,
 x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(d+ex)^3\left(a+b(d+ex)^2+c(d+ex)^4\right)}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{x^3\left(a+bx^2+cx^4\right)}\,dx,x,d+ex\right)}{e}\\
&= \frac{\text{Subst}\left(\int \frac{1}{x^2\left(a+bx+cx^2\right)}\,dx,x,(d+ex)^2\right)}{2e}\\
&= -\frac{1}{2ae(d+ex)^2}+\frac{\text{Subst}\left(\int \frac{-b-cx}{x\left(a+bx+cx^2\right)}\,dx,x,(d+ex)^2\right)}{2ae}\\
&= -\frac{1}{2ae(d+ex)^2}+\frac{\text{Subst}\left(\int \left(-\frac{b}{ax}+\frac{b^2-ac+bcx}{a\left(a+bx+cx^2\right)}\right)dx,x,(d+ex)^2\right)}{2ae}\\
&= -\frac{1}{2ae(d+ex)^2}-\frac{b\log(d+ex)}{a^2e}+\frac{\text{Subst}\left(\int \frac{b^2-ac+bcx}{a+bx+cx^2}\,dx,x,(d+ex\right.}{2a^2e}\\
&= -\frac{1}{2ae(d+ex)^2}-\frac{b\log(d+ex)}{a^2e}+\frac{b\,\text{Subst}\left(\int \frac{b+2cx}{a+bx+cx^2}\,dx,x,(d+e\right.}{4a^2e}\\
&= -\frac{1}{2ae(d+ex)^2}-\frac{b\log(d+ex)}{a^2e}+\frac{b\log\left(a+b(d+ex)^2+c(d+ex\right.}{4a^2e}\\
&= -\frac{1}{2ae(d+ex)^2}-\frac{\left(b^2-2ac\right)\tanh^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{2a^2\sqrt{b^2-4ac}\,e}-\frac{b\log(d+ex)}{a^2e}+
\end{aligned}$$

Mathematica [A] time = 0.13, size = 154, normalized size = 1.27

$$\frac{\frac{\left(b\sqrt{b^2-4ac}-2ac+b^2\right)\log\left(-\sqrt{b^2-4ac}+b+2c(d+ex)^2\right)}{\sqrt{b^2-4ac}}+\frac{\left(b\sqrt{b^2-4ac}+2ac-b^2\right)\log\left(\sqrt{b^2-4ac}+b+2c(d+ex)^2\right)}{\sqrt{b^2-4ac}}-\frac{2a}{(d+ex)^2}-4b\log(d+ex)}{4a^2e}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)^3*(a + b*(d + e*x)^2 + c*(d + e*x)^4)),x]
```

```
[Out] ((-2*a)/(d + e*x)^2 - 4*b*Log[d + e*x] + ((b^2 - 2*a*c + b*Sqrt[b^2 - 4*a*c
])*Log[b - Sqrt[b^2 - 4*a*c] + 2*c*(d + e*x)^2])/Sqrt[b^2 - 4*a*c] + ((-b^2
```

```
 + 2*a*c + b*Sqrt[b^2 - 4*a*c])*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*(d + e*x)^2
])/Sqrt[b^2 - 4*a*c])/(4*a^2*e)
```

IntegrateAlgebraic [F]   time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(d+ex)^3\left(a+b(d+ex)^2+c(d+ex)^4\right)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/((d + e*x)^3*(a + b*(d + e*x)^2 + c*(d + e*x)^4)),x]
```

```
[Out] IntegrateAlgebraic[1/((d + e*x)^3*(a + b*(d + e*x)^2 + c*(d + e*x)^4)), x]
```

fricas [B]   time = 1.10, size = 810, normalized size = 6.69

$$\left[-\frac{2ab^2-8a^2c+\left(\left(b^2-2ac\right)e^2x^2+2\left(b^2-2ac\right)dex+\left(b^2-2ac\right)d^2\right)\sqrt{b^2-4ac}\,\log\left(\frac{2c^2e^4x^4+8c^2de^3x^3+2c^2d^4+2\left(6c^2d^2+bc\right)e^2x^2+2bcd^2+4\left(2c^2d^3+bcd\right)ex+b^2-2ac+\left(2ce^2x^2+4cdex+2cd^2+b\right)\sqrt{b^2-4ac}}{ce^4x^4+4cde^3x^3+cd^4+\left(6cd^2+b\right)e^2x^2+bd^2+2\left(2cd^3+bd\right)ex+a}\right)-\left(\left(b^3-4abc\right)e^2x^2+2\left(b^3-4abc\right)dex+\left(b^3-4abc\right)d^2\right)\log\left(ce^4x^4+4cde^3x^3+cd^4+\left(6cd^2+b\right)e^2x^2+bd^2+2\left(2cd^3+bd\right)ex+a\right)+4\left(\left(b^3-4abc\right)e^2x^2+2\left(b^3-4abc\right)dex+\left(b^3-4abc\right)d^2\right)\log\left(ex+d\right)}{4\left(\left(a^2b^2-4a^3c\right)e^3x^2+2\left(a^2b^2-4a^3c\right)de^2x+\left(a^2b^2-4a^3c\right)d^2e\right)},\ -\frac{2ab^2-8a^2c+2\left(\left(b^2-2ac\right)e^2x^2+2\left(b^2-2ac\right)dex+\left(b^2-2ac\right)d^2\right)\sqrt{-b^2+4ac}\,\arctan\left(-\frac{\left(2ce^2x^2+4cdex+2cd^2+b\right)\sqrt{-b^2+4ac}}{b^2-4ac}\right)-\left(\left(b^3-4abc\right)e^2x^2+2\left(b^3-4abc\right)dex+\left(b^3-4abc\right)d^2\right)\log\left(ce^4x^4+4cde^3x^3+cd^4+\left(6cd^2+b\right)e^2x^2+bd^2+2\left(2cd^3+bd\right)ex+a\right)+4\left(\left(b^3-4abc\right)e^2x^2+2\left(b^3-4abc\right)dex+\left(b^3-4abc\right)d^2\right)\log\left(ex+d\right)}{4\left(\left(a^2b^2-4a^3c\right)e^3x^2+2\left(a^2b^2-4a^3c\right)de^2x+\left(a^2b^2-4a^3c\right)d^2e\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="fricas")
```

```
[Out] [-1/4*(2*a*b^2 - 8*a^2*c + ((b^2 - 2*a*c)*e^2*x^2 + 2*(b^2 - 2*a*c)*d*e*x +
 (b^2 - 2*a*c)*d^2)*sqrt(b^2 - 4*a*c)*log((2*c^2*e^4*x^4 + 8*c^2*d*e^3*x^3
+ 2*c^2*d^4 + 2*(6*c^2*d^2 + b*c)*e^2*x^2 + 2*b*c*d^2 + 4*(2*c^2*d^3 + b*c*
d)*e*x + b^2 - 2*a*c + (2*c*e^2*x^2 + 4*c*d*e*x + 2*c*d^2 + b)*sqrt(b^2 - 4
*a*c))/(c*e^4*x^4 + 4*c*d*e^3*x^3 + c*d^4 + (6*c*d^2 + b)*e^2*x^2 + b*d^2 +
 2*(2*c*d^3 + b*d)*e*x + a)) - ((b^3 - 4*a*b*c)*e^2*x^2 + 2*(b^3 - 4*a*b*c)
*d*e*x + (b^3 - 4*a*b*c)*d^2)*log(c*e^4*x^4 + 4*c*d*e^3*x^3 + c*d^4 + (6*c*
d^2 + b)*e^2*x^2 + b*d^2 + 2*(2*c*d^3 + b*d)*e*x + a) + 4*((b^3 - 4*a*b*c)*
e^2*x^2 + 2*(b^3 - 4*a*b*c)*d*e*x + (b^3 - 4*a*b*c)*d^2)*log(e*x + d))/((a^
2*b^2 - 4*a^3*c)*e^3*x^2 + 2*(a^2*b^2 - 4*a^3*c)*d*e^2*x + (a^2*b^2 - 4*a^3
*c)*d^2*e), -1/4*(2*a*b^2 - 8*a^2*c + 2*((b^2 - 2*a*c)*e^2*x^2 + 2*(b^2 - 2
*a*c)*d*e*x + (b^2 - 2*a*c)*d^2)*sqrt(-b^2 + 4*a*c)*arctan(-(2*c*e^2*x^2 +
4*c*d*e*x + 2*c*d^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) - ((b^3 - 4*a*b*
c)*e^2*x^2 + 2*(b^3 - 4*a*b*c)*d*e*x + (b^3 - 4*a*b*c)*d^2)*log(c*e^4*x^4 +
 4*c*d*e^3*x^3 + c*d^4 + (6*c*d^2 + b)*e^2*x^2 + b*d^2 + 2*(2*c*d^3 + b*d)*
e*x + a) + 4*((b^3 - 4*a*b*c)*e^2*x^2 + 2*(b^3 - 4*a*b*c)*d*e*x + (b^3 - 4*
a*b*c)*d^2)*log(e*x + d))/((a^2*b^2 - 4*a^3*c)*e^3*x^2 + 2*(a^2*b^2 - 4*a^3
*c)*d*e^2*x + (a^2*b^2 - 4*a^3*c)*d^2*e)]
```

giac [A]   time = 0.40, size = 102, normalized size = 0.84

$$\frac{be^{(-1)}\log\left(c+\frac{b}{(xe+d)^2}+\frac{a}{(xe+d)^4}\right)}{4\,a^2}+\frac{\left(b^2-2\,ac\right)\arctan\left(-\frac{b+\frac{2\,a}{(xe+d)^2}}{\sqrt{-b^2+4\,ac}}\right)e^{(-1)}}{2\,\sqrt{-b^2+4\,ac}\,a^2}-\frac{e^{(-1)}}{2\,(xe+d)^2a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="giac")
```

```
[Out] 1/4*b*e^(-1)*log(c + b/(x*e + d)^2 + a/(x*e + d)^4)/a^2 + 1/2*(b^2 - 2*a*c)
*arctan(-(b + 2*a/(x*e + d)^2)/sqrt(-b^2 + 4*a*c))*e^(-1)/(sqrt(-b^2 + 4*a*
c)*a^2) - 1/2*e^(-1)/((x*e + d)^2*a)
```

maple [C]   time = 0.01, size = 213, normalized size = 1.76

$$-\frac{b\ln\left(ex+d\right)}{a^2e}-\frac{1}{2\left(ex+d\right)^2ae}+\frac{\left(\mathrm{RootOf}\left(\_Z^4ce^4+4\_Z^3cde^3+cd^4+bd^2+\left(6cd^2e^2+be^2\right)\_Z^2+\left(4cd^3e+2deb\right)\_Z+a\right)^3bce^3+3\,\mathrm{RootOf}\left(\_Z^4ce^4+4\_Z^3cde^3+cd^4+bd^2+\left(6cd^2e^2+be^2\right)\_Z^2+\left(4cd^3e+2deb\right)\_Z+a\right)^2bcde^2+bcd^3-acd+b^2d+\left(3bcd^2-ac+b^2\right)\mathrm{RootOf}\left(\_Z^4ce^4+4\_Z^3cde^3+cd^4+bd^2+\left(6cd^2e^2+be^2\right)\_Z^2+\left(4cd^3e+2deb\right)\_Z+a\right)e\right)\ln\left(-\mathrm{RootOf}\left(\_Z^4ce^4+4\_Z^3cde^3+cd^4+bd^2+\left(6cd^2e^2+be^2\right)\_Z^2+\left(4cd^3e+2deb\right)\_Z+a\right)+x\right)}{2a^2e\left(2ce^3\,\mathrm{RootOf}\left(\_Z^4ce^4+4\_Z^3cde^3+cd^4+bd^2+\left(6cd^2e^2+be^2\right)\_Z^2+\left(4cd^3e+2deb\right)\_Z+a\right)^3+6cde^2\,\mathrm{RootOf}\left(\_Z^4ce^4+4\_Z^3cde^3+cd^4+bd^2+\left(6cd^2e^2+be^2\right)\_Z^2+\left(4cd^3e+2deb\right)\_Z+a\right)^2+6ecd^2\,\mathrm{RootOf}\left(\_Z^4ce^4+4\_Z^3cde^3+cd^4+bd^2+\left(6cd^2e^2+be^2\right)\_Z^2+\left(4cd^3e+2deb\right)\_Z+a\right)+2cd^3+be\,\mathrm{RootOf}\left(\_Z^4ce^4+4\_Z^3cde^3+cd^4+bd^2+\left(6cd^2e^2+be^2\right)\_Z^2+\left(4cd^3e+2deb\right)\_Z+a\right)+bd\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4),x)
```

```
[Out] 1/2/a^2/e*sum((_R^3*b*c*e^3+3*_R^2*b*c*d*e^2+e*(3*b*c*d^2-a*c+b^2)*_R+b*c*d
^3-a*c*d+b^2*d)/(2*_R^3*c*e^3+6*_R^2*c*d*e^2+6*_R*c*d^2*e+2*c*d^3+_R*b*e+b*
d)*ln(-_R+x),_R=RootOf(_Z^4*c*e^4+4*_Z^3*c*d*e^3+c*d^4+b*d^2+(6*c*d^2*e^2+b
*e^2)*_Z^2+(4*c*d^3*e+2*b*d*e)*_Z+a))-1/2/a/e/(e*x+d)^2-b*ln(e*x+d)/a^2/e
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [B] time = 5.86, size = 4950, normalized size = 40.91

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((d + e*x)^3*(a + b*(d + e*x)^2 + c*(d + e*x)^4)),x)
```

```
[Out] (atan((16*a^6*x^2*(4*a*c - b^2)^(3/2)*(((3*b^4 + a^2*c^2 - 9*a*b^2*c)*(((((
(20*a^3*c^4*e^18 + 2*a^2*b^2*c^3*e^18)/a^3 + ((2*b^3*e - 8*a*b*c*e)*(40*a^4
*b*c^3*e^19 - 12*a^3*b^3*c^2*e^19))/(2*a^3*(16*a^3*c*e^2 - 4*a^2*b^2*e^2)))
*(2*b^3*e - 8*a*b*c*e))/(2*(16*a^3*c*e^2 - 4*a^2*b^2*e^2)) + (6*b*c^4*e^17)
/a^2)*(2*b^3*e - 8*a*b*c*e))/(2*(16*a^3*c*e^2 - 4*a^2*b^2*e^2)) + (c^5*e^16
)/a^3 - (((((20*a^3*c^4*e^18 + 2*a^2*b^2*c^3*e^18)/a^3 + ((2*b^3*e - 8*a*b*
c*e)*(40*a^4*b*c^3*e^19 - 12*a^3*b^3*c^2*e^19))/(2*a^3*(16*a^3*c*e^2 - 4*a^
2*b^2*e^2)))*(2*a*c - b^2))/(4*a^2*e*(4*a*c - b^2)^(1/2)) + ((2*a*c - b^2)*
(2*b^3*e - 8*a*b*c*e)*(40*a^4*b*c^3*e^19 - 12*a^3*b^3*c^2*e^19))/(8*a^5*e*(
16*a^3*c*e^2 - 4*a^2*b^2*e^2)*(4*a*c - b^2)^(1/2)))*(2*a*c - b^2))/(4*a^2*e
*(4*a*c - b^2)^(1/2)) - ((2*a*c - b^2)^2*(2*b^3*e - 8*a*b*c*e)*(40*a^4*b*c^
3*e^19 - 12*a^3*b^3*c^2*e^19))/(32*a^7*e^2*(16*a^3*c*e^2 - 4*a^2*b^2*e^2)*(
4*a*c - b^2))))/(8*a^3*c^2*(a^2*c^2 - 6*b^4 + 24*a*b^2*c)) + ((((((20*a^3*
c^4*e^18 + 2*a^2*b^2*c^3*e^18)/a^3 + ((2*b^3*e - 8*a*b*c*e)*(40*a^4*b*c^3*e
^19 - 12*a^3*b^3*c^2*e^19))/(2*a^3*(16*a^3*c*e^2 - 4*a^2*b^2*e^2)))*(2*a*c
- b^2))/(4*a^2*e*(4*a*c - b^2)^(1/2)) + ((2*a*c - b^2)*(2*b^3*e - 8*a*b*c*e
)*(40*a^4*b*c^3*e^19 - 12*a^3*b^3*c^2*e^19))/(8*a^5*e*(16*a^3*c*e^2 - 4*a^2
*b^2*e^2)*(4*a*c - b^2)^(1/2)))*(2*b^3*e - 8*a*b*c*e))/(2*(16*a^3*c*e^2 - 4
*a^2*b^2*e^2)) + (((((20*a^3*c^4*e^18 + 2*a^2*b^2*c^3*e^18)/a^3 + ((2*b^3*e
 - 8*a*b*c*e)*(40*a^4*b*c^3*e^19 - 12*a^3*b^3*c^2*e^19))/(2*a^3*(16*a^3*c*e
^2 - 4*a^2*b^2*e^2)))*(2*b^3*e - 8*a*b*c*e))/(2*(16*a^3*c*e^2 - 4*a^2*b^2*e
^2)) + (6*b*c^4*e^17)/a^2)*(2*a*c - b^2))/(4*a^2*e*(4*a*c - b^2)^(1/2)) - (
(2*a*c - b^2)^3*(40*a^4*b*c^3*e^19 - 12*a^3*b^3*c^2*e^19))/(64*a^9*e^3*(4*a
*c - b^2)^(3/2)))*(3*b^5 + 13*a^2*b*c^2 - 15*a*b^3*c))/(8*a^3*c^2*(4*a*c -
b^2)^(1/2)*(a^2*c^2 - 6*b^4 + 24*a*b^2*c))))/(4*a^2*c^4*e^14 + b^4*c^2*e^14
 - 4*a*b^2*c^3*e^14) + (16*a^6*x*(((3*b^4 + a^2*c^2 - 9*a*b^2*c)*(((((2*b^3
*e - 8*a*b*c*e)*((2*(20*a^3*c^4*d*e^17 + 2*a^2*b^2*c^3*d*e^17))/a^3 + ((40*
a^4*b*c^3*d*e^18 - 12*a^3*b^3*c^2*d*e^18)*(2*b^3*e - 8*a*b*c*e))/(a^3*(16*a
^3*c*e^2 - 4*a^2*b^2*e^2))))/(2*(16*a^3*c*e^2 - 4*a^2*b^2*e^2)) + (12*b*c^4
*d*e^16)/a^2)*(2*b^3*e - 8*a*b*c*e))/(2*(16*a^3*c*e^2 - 4*a^2*b^2*e^2)) + (
2*c^5*d*e^15)/a^3 - ((((2*a*c - b^2)*((2*(20*a^3*c^4*d*e^17 + 2*a^2*b^2*c^3
*d*e^17))/a^3 + ((40*a^4*b*c^3*d*e^18 - 12*a^3*b^3*c^2*d*e^18)*(2*b^3*e - 8
*a*b*c*e))/(a^3*(16*a^3*c*e^2 - 4*a^2*b^2*e^2))))/(4*a^2*e*(4*a*c - b^2)^(1
/2)) + ((40*a^4*b*c^3*d*e^18 - 12*a^3*b^3*c^2*d*e^18)*(2*a*c - b^2)*(2*b^3*
e - 8*a*b*c*e))/(4*a^5*e*(16*a^3*c*e^2 - 4*a^2*b^2*e^2)*(4*a*c - b^2)^(1/2)
))*(2*a*c - b^2))/(4*a^2*e*(4*a*c - b^2)^(1/2)) - ((40*a^4*b*c^3*d*e^18 - 1
2*a^3*b^3*c^2*d*e^18)*(2*a*c - b^2)^2*(2*b^3*e - 8*a*b*c*e))/(16*a^7*e^2*(1
6*a^3*c*e^2 - 4*a^2*b^2*e^2)*(4*a*c - b^2))))/(8*a^3*c^2*(a^2*c^2 - 6*b^4 +
```

```
24*a*b^2*c)) + ((((((2*a*c - b^2)*((2*(20*a^3*c^4*d*e^17 + 2*a^2*b^2*c^3*d
*e^17))/a^3 + ((40*a^4*b*c^3*d*e^18 - 12*a^3*b^3*c^2*d*e^18)*(2*b^3*e - 8*a
*b*c*e))/(a^3*(16*a^3*c*e^2 - 4*a^2*b^2*e^2))))/(4*a^2*e*(4*a*c - b^2)^(1/2
)) + ((40*a^4*b*c^3*d*e^18 - 12*a^3*b^3*c^2*d*e^18)*(2*a*c - b^2)*(2*b^3*e
- 8*a*b*c*e))/(4*a^5*e*(16*a^3*c*e^2 - 4*a^2*b^2*e^2)*(4*a*c - b^2)^(1/2)))
*(2*b^3*e - 8*a*b*c*e))/(2*(16*a^3*c*e^2 - 4*a^2*b^2*e^2)) + ((((2*b^3*e -
8*a*b*c*e)*((2*(20*a^3*c^4*d*e^17 + 2*a^2*b^2*c^3*d*e^17))/a^3 + ((40*a^4*b
*c^3*d*e^18 - 12*a^3*b^3*c^2*d*e^18)*(2*b^3*e - 8*a*b*c*e))/(a^3*(16*a^3*c*
e^2 - 4*a^2*b^2*e^2))))/(2*(16*a^3*c*e^2 - 4*a^2*b^2*e^2)) + (12*b*c^4*d*e^
16)/a^2)*(2*a*c - b^2))/(4*a^2*e*(4*a*c - b^2)^(1/2)) - ((40*a^4*b*c^3*d*e^
18 - 12*a^3*b^3*c^2*d*e^18)*(2*a*c - b^2)^3)/(32*a^9*e^3*(4*a*c - b^2)^(3/2
)))*(3*b^5 + 13*a^2*b*c^2 - 15*a*b^3*c))/(8*a^3*c^2*(4*a*c - b^2)^(1/2)*(a^
2*c^2 - 6*b^4 + 24*a*b^2*c)))*(4*a*c - b^2)^(3/2))/(4*a^2*c^4*e^14 + b^4*c^
2*e^14 - 4*a*b^2*c^3*e^14) + (2*a^3*(4*a*c - b^2)^(3/2)*(3*b^4 + a^2*c^2 -
9*a*b^2*c)*((b*c^4*e^14 + c^5*d^2*e^14)/a^3 + (((4*a*b^2*c^3*e^15 - a^2*c^4
*e^15 + 6*a*b*c^4*d^2*e^15)/a^3 + (((4*a^2*b^3*c^2*e^16 - 4*a^3*b*c^3*e^16
+ 20*a^3*c^4*d^2*e^16 + 2*a^2*b^2*c^3*d^2*e^16)/a^3 - ((2*b^3*e - 8*a*b*c*e
)*(4*a^4*b^2*c^2*e^17 + 12*a^3*b^3*c^2*d^2*e^17 - 40*a^4*b*c^3*d^2*e^17))/(
2*a^3*(16*a^3*c*e^2 - 4*a^2*b^2*e^2)))*(2*b^3*e - 8*a*b*c*e))/(2*(16*a^3*c*
e^2 - 4*a^2*b^2*e^2)))*(2*b^3*e - 8*a*b*c*e))/(2*(16*a^3*c*e^2 - 4*a^2*b^2*
e^2)) - ((2*a*c - b^2)*((((4*a^2*b^3*c^2*e^16 - 4*a^3*b*c^3*e^16 + 20*a^3*c
^4*d^2*e^16 + 2*a^2*b^2*c^3*d^2*e^16)/a^3 - ((2*b^3*e - 8*a*b*c*e)*(4*a^4*b
^2*c^2*e^17 + 12*a^3*b^3*c^2*d^2*e^17 - 40*a^4*b*c^3*d^2*e^17))/(2*a^3*(16*
a^3*c*e^2 - 4*a^2*b^2*e^2)))*(2*a*c - b^2))/(4*a^2*e*(4*a*c - b^2)^(1/2)) -
 ((2*a*c - b^2)*(2*b^3*e - 8*a*b*c*e)*(4*a^4*b^2*c^2*e^17 + 12*a^3*b^3*c^2*
d^2*e^17 - 40*a^4*b*c^3*d^2*e^17))/(8*a^5*e*(16*a^3*c*e^2 - 4*a^2*b^2*e^2)*
(4*a*c - b^2)^(1/2))))/(4*a^2*e*(4*a*c - b^2)^(1/2)) + ((2*a*c - b^2)^2*(2*
b^3*e - 8*a*b*c*e)*(4*a^4*b^2*c^2*e^17 + 12*a^3*b^3*c^2*d^2*e^17 - 40*a^4*b
*c^3*d^2*e^17))/(32*a^7*e^2*(16*a^3*c*e^2 - 4*a^2*b^2*e^2)*(4*a*c - b^2))))
/(c^2*(a^2*c^2 - 6*b^4 + 24*a*b^2*c)*(4*a^2*c^4*e^14 + b^4*c^2*e^14 - 4*a*b
^2*c^3*e^14)) + (2*a^3*(4*a*c - b^2)*(((2*b^3*e - 8*a*b*c*e)*((((4*a^2*b^3*
c^2*e^16 - 4*a^3*b*c^3*e^16 + 20*a^3*c^4*d^2*e^16 + 2*a^2*b^2*c^3*d^2*e^16)
/a^3 - ((2*b^3*e - 8*a*b*c*e)*(4*a^4*b^2*c^2*e^17 + 12*a^3*b^3*c^2*d^2*e^17
 - 40*a^4*b*c^3*d^2*e^17))/(2*a^3*(16*a^3*c*e^2 - 4*a^2*b^2*e^2)))*(2*a*c -
 b^2))/(4*a^2*e*(4*a*c - b^2)^(1/2)) - ((2*a*c - b^2)*(2*b^3*e - 8*a*b*c*e)
*(4*a^4*b^2*c^2*e^17 + 12*a^3*b^3*c^2*d^2*e^17 - 40*a^4*b*c^3*d^2*e^17))/(8
*a^5*e*(16*a^3*c*e^2 - 4*a^2*b^2*e^2)*(4*a*c - b^2)^(1/2))))/(2*(16*a^3*c*e
^2 - 4*a^2*b^2*e^2)) + (((4*a*b^2*c^3*e^15 - a^2*c^4*e^15 + 6*a*b*c^4*d^2*e
^15)/a^3 + (((4*a^2*b^3*c^2*e^16 - 4*a^3*b*c^3*e^16 + 20*a^3*c^4*d^2*e^16 +
 2*a^2*b^2*c^3*d^2*e^16)/a^3 - ((2*b^3*e - 8*a*b*c*e)*(4*a^4*b^2*c^2*e^17 +
 12*a^3*b^3*c^2*d^2*e^17 - 40*a^4*b*c^3*d^2*e^17))/(2*a^3*(16*a^3*c*e^2 - 4
*a^2*b^2*e^2)))*(2*b^3*e - 8*a*b*c*e))/(2*(16*a^3*c*e^2 - 4*a^2*b^2*e^2)))*
(2*a*c - b^2))/(4*a^2*e*(4*a*c - b^2)^(1/2)) + ((2*a*c - b^2)^3*(4*a^4*b^2*
c^2*e^17 + 12*a^3*b^3*c^2*d^2*e^17 - 40*a^4*b*c^3*d^2*e^17))/(64*a^9*e^3*(4
*a*c - b^2)^(3/2)))*(3*b^5 + 13*a^2*b*c^2 - 15*a*b^3*c))/(c^2*(a^2*c^2 - 6*
b^4 + 24*a*b^2*c)*(4*a^2*c^4*e^14 + b^4*c^2*e^14 - 4*a*b^2*c^3*e^14)))*(2*a
*c - b^2))/(2*a^2*e*(4*a*c - b^2)^(1/2)) - (log(((c^5*e^16*x^2)/a^3 - ((b +
 a^2*e*(-(2*a*c - b^2)^2/(a^4*e^2*(4*a*c - b^2)))^(1/2))*((c^3*e^15*(4*b^2
- a*c + 6*b*c*d^2))/a^2 - ((b + a^2*e*(-(2*a*c - b^2)^2/(a^4*e^2*(4*a*c - b
^2)))^(1/2))*((2*c^2*e^16*(2*b^3 + 10*a*c^2*d^2 + b^2*c*d^2 - 2*a*b*c))/a +
 (2*c^3*e^18*x^2*(10*a*c + b^2))/a + (b*c^2*e^16*(b + a^2*e*(-(2*a*c - b^2)
^2/(a^4*e^2*(4*a*c - b^2)))^(1/2))*(a*b + 3*b^2*d^2 + 3*b^2*e^2*x^2 - 10*a*
c*d^2 + 6*b^2*d*e*x - 10*a*c*e^2*x^2 - 20*a*c*d*e*x))/a^2 + (4*c^3*d*e^17*x
*(10*a*c + b^2))/a))/(4*a^2*e) + (6*b*c^4*e^17*x^2)/a^2 + (12*b*c^4*d*e^16*
x)/a^2))/(4*a^2*e) + (c^4*e^14*(b + c*d^2))/a^3 + (2*c^5*d*e^15*x)/a^3)*((c
^5*e^16*x^2)/a^3 - ((b - a^2*e*(-(2*a*c - b^2)^2/(a^4*e^2*(4*a*c - b^2)))^(
1/2))*((c^3*e^15*(4*b^2 - a*c + 6*b*c*d^2))/a^2 - ((b - a^2*e*(-(2*a*c - b^
2)^2/(a^4*e^2*(4*a*c - b^2)))^(1/2))*((2*c^2*e^16*(2*b^3 + 10*a*c^2*d^2 + b
^2*c*d^2 - 2*a*b*c))/a + (2*c^3*e^18*x^2*(10*a*c + b^2))/a + (b*c^2*e^16*(b
```

```
 - a^2*e*(-(2*a*c - b^2)^2/(a^4*e^2*(4*a*c - b^2)))^(1/2))*(a*b + 3*b^2*d^2
 + 3*b^2*e^2*x^2 - 10*a*c*d^2 + 6*b^2*d*e*x - 10*a*c*e^2*x^2 - 20*a*c*d*e*x
))/a^2 + (4*c^3*d*e^17*x*(10*a*c + b^2))/a))/(4*a^2*e) + (6*b*c^4*e^17*x^2)
/a^2 + (12*b*c^4*d*e^16*x)/a^2))/(4*a^2*e) + (c^4*e^14*(b + c*d^2))/a^3 + (
2*c^5*d*e^15*x)/a^3))*(2*b^3*e - 8*a*b*c*e))/(2*(16*a^3*c*e^2 - 4*a^2*b^2*e
^2)) - (b*log(d + e*x))/(a^2*e) - 1/(2*a*e*(d^2 + e^2*x^2 + 2*d*e*x))
```

sympy [B] time = 146.46, size = 464, normalized size = 3.83

$$\left(\frac{b}{4a^2e} - \frac{\sqrt{-4ac+b^2}\left(2ac-b^2\right)}{4a^2e\left(4ac-b^2\right)}\right)\log\left(\frac{2dx}{e} + x^2 + \frac{-8a^3ce\left(\frac{b}{4a^2e} - \frac{\sqrt{-4ac+b^2}\left(2ac-b^2\right)}{4a^2e\left(4ac-b^2\right)}\right) + 2a^2b^2e\left(\frac{b}{4a^2e} - \frac{\sqrt{-4ac+b^2}\left(2ac-b^2\right)}{4a^2e\left(4ac-b^2\right)}\right) + 3abc + 2ac^2d^2 - b^3 - b^2cd^2}{2ac^2e^2 - b^2ce^2}\right) + \left(\frac{b}{4a^2e} + \frac{\sqrt{-4ac+b^2}\left(2ac-b^2\right)}{4a^2e\left(4ac-b^2\right)}\right)\log\left(\frac{2dx}{e} + x^2 + \frac{-8a^3ce\left(\frac{b}{4a^2e} + \frac{\sqrt{-4ac+b^2}\left(2ac-b^2\right)}{4a^2e\left(4ac-b^2\right)}\right) + 2a^2b^2e\left(\frac{b}{4a^2e} + \frac{\sqrt{-4ac+b^2}\left(2ac-b^2\right)}{4a^2e\left(4ac-b^2\right)}\right) + 3abc + 2ac^2d^2 - b^3 - b^2cd^2}{2ac^2e^2 - b^2ce^2}\right) - \frac{1}{2ad^2e + 4ade^2x + 2ae^3x^2} - \frac{b\log\left(\frac{d}{e} + x\right)}{a^2e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)**3/(a+b*(e*x+d)**2+c*(e*x+d)**4),x)
```

```
[Out] (b/(4*a**2*e) - sqrt(-4*a*c + b**2)*(2*a*c - b**2)/(4*a**2*e*(4*a*c - b**2)
))*log(2*d*x/e + x**2 + (-8*a**3*c*e*(b/(4*a**2*e) - sqrt(-4*a*c + b**2)*(2
*a*c - b**2)/(4*a**2*e*(4*a*c - b**2))) + 2*a**2*b**2*e*(b/(4*a**2*e) - sqr
t(-4*a*c + b**2)*(2*a*c - b**2)/(4*a**2*e*(4*a*c - b**2))) + 3*a*b*c + 2*a*
c**2*d**2 - b**3 - b**2*c*d**2)/(2*a*c**2*e**2 - b**2*c*e**2)) + (b/(4*a**2
*e) + sqrt(-4*a*c + b**2)*(2*a*c - b**2)/(4*a**2*e*(4*a*c - b**2)))*log(2*d
*x/e + x**2 + (-8*a**3*c*e*(b/(4*a**2*e) + sqrt(-4*a*c + b**2)*(2*a*c - b**
2)/(4*a**2*e*(4*a*c - b**2))) + 2*a**2*b**2*e*(b/(4*a**2*e) + sqrt(-4*a*c +
 b**2)*(2*a*c - b**2)/(4*a**2*e*(4*a*c - b**2))) + 3*a*b*c + 2*a*c**2*d**2
- b**3 - b**2*c*d**2)/(2*a*c**2*e**2 - b**2*c*e**2)) - 1/(2*a*d**2*e + 4*a*
d*e**2*x + 2*a*e**3*x**2) - b*log(d/e + x)/(a**2*e)
```

3.507 $\int \frac{1}{(d+ex)^4\left(a+b(d+ex)^2+c(d+ex)^4\right)}\,dx$

Optimal. Leaf size=224

$$\frac{\sqrt{c}\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\,a^2e\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{c}\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\,a^2e\sqrt{\sqrt{b^2-4ac}+b}}+\frac{b}{a^2e(d+ex)}-\frac{1}{3ae(d+ex)^3}$$

Rubi [A] time = 0.50, antiderivative size = 224, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1142, 1123, 1281, 1166, 205}

$$\frac{\sqrt{c}\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\,a^2e\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{c}\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\,a^2e\sqrt{\sqrt{b^2-4ac}+b}}+\frac{b}{a^2e(d+ex)}-\frac{1}{3ae(d+ex)^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)^4*(a + b*(d + e*x)^2 + c*(d + e*x)^4)),x]
```

```
[Out] -1/(3*a*e*(d + e*x)^3) + b/(a^2*e*(d + e*x)) + (Sqrt[c]*(b + (b^2 - 2*a*c)/
Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b - Sqrt[b^2 - 4
*a*c]]])/(Sqrt[2]*a^2*Sqrt[b - Sqrt[b^2 - 4*a*c]]*e) + (Sqrt[c]*(b - (b^2 -
 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b + Sqrt
[b^2 - 4*a*c]]])/(Sqrt[2]*a^2*Sqrt[b + Sqrt[b^2 - 4*a*c]]*e)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1123

```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*x^2 + c*x^4)^(p + 1))/(a*d*(m + 1)), x] - Dis
t[1/(a*d^2*(m + 1)), Int[(d*x)^(m + 2)*(b*(m + 2*p + 3) + c*(m + 4*p + 5)*x
^2)*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b^2 -
 4*a*c, 0] && LtQ[m, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1142

```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Di
st[u^m/(Coefficient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p,
 x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1281

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[(d*(f*x)^(m + 1)*(a + b*x^2 + c*x^4)^(p + 1)
)/(a*f*(m + 1)), x] + Dist[1/(a*f^2*(m + 1)), Int[(f*x)^(m + 2)*(a + b*x^2
```

```
+ c*x^4)^p*Simp[a*e*(m + 1) - b*d*(m + 2*p + 3) - c*d*(m + 4*p + 5)*x^2, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[m
, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rubi steps

$$\int \frac{1}{(d+ex)^4\left(a+b(d+ex)^2+c(d+ex)^4\right)}dx = \frac{\text{Subst}\left(\int \frac{1}{x^4(a+bx^2+cx^4)}dx, x, d+ex\right)}{e}$$

$$= -\frac{1}{3ae(d+ex)^3}+\frac{\text{Subst}\left(\int \frac{-3b-3cx^2}{x^2(a+bx^2+cx^4)}dx, x, d+ex\right)}{3ae}$$

$$= -\frac{1}{3ae(d+ex)^3}+\frac{b}{a^2e(d+ex)}-\frac{\text{Subst}\left(\int \frac{-3(b^2-ac)-3bcx^2}{a+bx^2+cx^4}dx, x, d+e\right.}{3a^2e}$$

$$= -\frac{1}{3ae(d+ex)^3}+\frac{b}{a^2e(d+ex)}+\frac{\left(c\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\right)\text{Subst}\left(\int \frac{}{\frac{b}{2}+\frac{1}{2}\sqrt{b}}\right.}{2a^2e}$$

$$= -\frac{1}{3ae(d+ex)^3}+\frac{b}{a^2e(d+ex)}+\frac{\sqrt{c}\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+}{\sqrt{b-\sqrt{b^2-4}}}\right.}{\sqrt{2}a^2\sqrt{b-\sqrt{b^2-4ac}}\,e}$$

Mathematica [A] time = 0.21, size = 235, normalized size = 1.05

$$\frac{\frac{3\sqrt{2}\sqrt{c}\left(b\sqrt{b^2-4ac}-2ac+b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{3\sqrt{2}\sqrt{c}\left(b\sqrt{b^2-4ac}+2ac-b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4ac}+b}}-\frac{2a}{(d+ex)^3}+\frac{6b}{d+ex}}{6a^2e}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)^4*(a + b*(d + e*x)^2 + c*(d + e*x)^4)),x]
```

```
[Out] ((-2*a)/(d + e*x)^3 + (6*b)/(d + e*x) + (3*Sqrt[2]*Sqrt[c]*(b^2 - 2*a*c + b
*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b - Sqrt[b^2 -
4*a*c]]])/(Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (3*Sqrt[2]*Sqrt
[c]*(-b^2 + 2*a*c + b*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))
/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2 - 4*a*c]*Sqrt[b + Sqrt[b^2 - 4*a*c
]]))/(6*a^2*e)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(d+ex)^4\left(a+b(d+ex)^2+c(d+ex)^4\right)}dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/((d + e*x)^4*(a + b*(d + e*x)^2 + c*(d + e*x)^4)),x]
```

```
[Out] IntegrateAlgebraic[1/((d + e*x)^4*(a + b*(d + e*x)^2 + c*(d + e*x)^4)), x]
```

fricas [B] time = 1.22, size = 2044, normalized size = 9.12

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^4/(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="fricas")
```

```
[Out] 1/6*(6*b*e^2*x^2 + 12*b*d*e*x + 6*b*d^2 + 3*sqrt(1/2)*(a^2*e^4*x^3 + 3*a^2*
d*e^3*x^2 + 3*a^2*d^2*e^2*x + a^2*d^3*e)*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c
^2 + (a^5*b^2 - 4*a^6*c)*e^2*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3
*b^2*c^3 + a^4*c^4)/((a^10*b^2 - 4*a^11*c)*e^4)))/((a^5*b^2 - 4*a^6*c)*e^2)
)*log(2*(b^4*c^3 - 3*a*b^2*c^4 + a^2*c^5)*e*x + 2*(b^4*c^3 - 3*a*b^2*c^4 +
a^2*c^5)*d + sqrt(1/2)*((a^5*b^5 - 7*a^6*b^3*c + 12*a^7*b*c^2)*e^3*sqrt((b^
8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/((a^10*b^2 - 4*a^
11*c)*e^4)) - (b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 17*a^3*b^2*c^3 + 4*a^4*c^
4)*e)*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 + (a^5*b^2 - 4*a^6*c)*e^2*sqrt((
b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/((a^10*b^2 - 4*
a^11*c)*e^4)))/((a^5*b^2 - 4*a^6*c)*e^2))) - 3*sqrt(1/2)*(a^2*e^4*x^3 + 3*a
^2*d*e^3*x^2 + 3*a^2*d^2*e^2*x + a^2*d^3*e)*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*
b*c^2 + (a^5*b^2 - 4*a^6*c)*e^2*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*
a^3*b^2*c^3 + a^4*c^4)/((a^10*b^2 - 4*a^11*c)*e^4)))/((a^5*b^2 - 4*a^6*c)*e
^2))*log(2*(b^4*c^3 - 3*a*b^2*c^4 + a^2*c^5)*e*x + 2*(b^4*c^3 - 3*a*b^2*c^4
 + a^2*c^5)*d - sqrt(1/2)*((a^5*b^5 - 7*a^6*b^3*c + 12*a^7*b*c^2)*e^3*sqrt(
(b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/((a^10*b^2 - 4
*a^11*c)*e^4)) - (b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 17*a^3*b^2*c^3 + 4*a^4
*c^4)*e)*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 + (a^5*b^2 - 4*a^6*c)*e^2*sqr
t((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/((a^10*b^2 -
 4*a^11*c)*e^4)))/((a^5*b^2 - 4*a^6*c)*e^2))) - 3*sqrt(1/2)*(a^2*e^4*x^3 +
3*a^2*d*e^3*x^2 + 3*a^2*d^2*e^2*x + a^2*d^3*e)*sqrt(-(b^5 - 5*a*b^3*c + 5*a
^2*b*c^2 - (a^5*b^2 - 4*a^6*c)*e^2*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 -
 6*a^3*b^2*c^3 + a^4*c^4)/((a^10*b^2 - 4*a^11*c)*e^4)))/((a^5*b^2 - 4*a^6*c
)*e^2))*log(2*(b^4*c^3 - 3*a*b^2*c^4 + a^2*c^5)*e*x + 2*(b^4*c^3 - 3*a*b^2*
c^4 + a^2*c^5)*d + sqrt(1/2)*((a^5*b^5 - 7*a^6*b^3*c + 12*a^7*b*c^2)*e^3*sq
rt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/((a^10*b^2
- 4*a^11*c)*e^4)) + (b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 17*a^3*b^2*c^3 + 4*
a^4*c^4)*e)*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 - (a^5*b^2 - 4*a^6*c)*e^2*
sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/((a^10*b^
2 - 4*a^11*c)*e^4)))/((a^5*b^2 - 4*a^6*c)*e^2))) + 3*sqrt(1/2)*(a^2*e^4*x^3
 + 3*a^2*d*e^3*x^2 + 3*a^2*d^2*e^2*x + a^2*d^3*e)*sqrt(-(b^5 - 5*a*b^3*c +
5*a^2*b*c^2 - (a^5*b^2 - 4*a^6*c)*e^2*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^
2 - 6*a^3*b^2*c^3 + a^4*c^4)/((a^10*b^2 - 4*a^11*c)*e^4)))/((a^5*b^2 - 4*a^
6*c)*e^2))*log(2*(b^4*c^3 - 3*a*b^2*c^4 + a^2*c^5)*e*x + 2*(b^4*c^3 - 3*a*b
^2*c^4 + a^2*c^5)*d - sqrt(1/2)*((a^5*b^5 - 7*a^6*b^3*c + 12*a^7*b*c^2)*e^3
*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/((a^10*b
^2 - 4*a^11*c)*e^4)) + (b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 17*a^3*b^2*c^3 +
 4*a^4*c^4)*e)*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 - (a^5*b^2 - 4*a^6*c)*e
^2*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/((a^10
*b^2 - 4*a^11*c)*e^4)))/((a^5*b^2 - 4*a^6*c)*e^2))) - 2*a)/(a^2*e^4*x^3 + 3
*a^2*d*e^3*x^2 + 3*a^2*d^2*e^2*x + a^2*d^3*e)
```

giac [B] time = 0.49, size = 1243, normalized size = 5.55

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^4/(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="giac")
```

```
[Out] -1/2*(((d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c
))^2*b*c*e^2 - 2*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2
)*e^(-4)/c))*b*c*d*e + b*c*d^2 + b^2 - a*c)*log(d*e^(-1) + x + sqrt(1/2)*sq
rt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))/(2*(d*e^(-1) + sqrt(1/2)*sqr
t(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^3*c*e^4 - 6*(d*e^(-1) + sqrt(
1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^2*c*d*e^3 - 2*c*d^3*e
 - b*d*e + (6*c*d^2*e^2 + b*e^2)*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 + sqrt(
```

```
b^2 - 4*a*c)*e^2)*e^(-4)/c))) + ((d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 + sqrt(
b^2 - 4*a*c)*e^2)*e^(-4)/c))^2*b*c*e^2 - 2*(d*e^(-1) - sqrt(1/2)*sqrt(-(b*e
^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))*b*c*d*e + b*c*d^2 + b^2 - a*c)*log(d
*e^(-1) + x - sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))/(2
*(d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^3*c
*e^4 - 6*(d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)
/c))^2*c*d*e^3 - 2*c*d^3*e - b*d*e + (6*c*d^2*e^2 + b*e^2)*(d*e^(-1) - sqrt
(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))) + ((d*e^(-1) + sqrt
(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^2*b*c*e^2 - 2*(d*e^(
-1) + sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))*b*c*d*e +
b*c*d^2 + b^2 - a*c)*log(d*e^(-1) + x + sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 -
 4*a*c)*e^2)*e^(-4)/c))/(2*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 -
4*a*c)*e^2)*e^(-4)/c))^3*c*e^4 - 6*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 - sqr
t(b^2 - 4*a*c)*e^2)*e^(-4)/c))^2*c*d*e^3 - 2*c*d^3*e - b*d*e + (6*c*d^2*e^2
 + b*e^2)*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4
)/c))) + ((d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4
)/c))^2*b*c*e^2 - 2*(d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*
e^2)*e^(-4)/c))*b*c*d*e + b*c*d^2 + b^2 - a*c)*log(d*e^(-1) + x - sqrt(1/2)
*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))/(2*(d*e^(-1) - sqrt(1/2)*
sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^3*c*e^4 - 6*(d*e^(-1) - sq
rt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^2*c*d*e^3 - 2*c*d^
3*e - b*d*e + (6*c*d^2*e^2 + b*e^2)*(d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 - sq
rt(b^2 - 4*a*c)*e^2)*e^(-4)/c))))/a^2 + 1/3*(3*b*x^2*e^2 + 6*b*d*x*e + 3*b*
d^2 - a)*e^(-1)/((x*e + d)^3*a^2)
```

maple [C] time = 0.01, size = 188, normalized size = 0.84

$$-\frac{1}{3\left(ex+d\right)^{3}ae}+\frac{b}{\left(ex+d\right)a^{2}e}+\frac{\left(\mathrm{RootOf}\left(\_Z^{4}ce^{4}+4\_Z^{3}cde^{3}+cd^{4}+bd^{2}+\left(6cd^{2}e^{2}+be^{2}\right)\_Z^{2}+\left(4cd^{3}e+2deb\right)\_Z+a\right)^{2}bce^{2}+2\,\mathrm{RootOf}\left(\_Z^{4}ce^{4}+4\_Z^{3}cde^{3}+cd^{4}+bd^{2}+\left(6cd^{2}e^{2}+be^{2}\right)\_Z^{2}+\left(4cd^{3}e+2deb\right)\_Z+a\right)bcde+bcd^{2}-ac+b^{2}\right)\ln\left(-\mathrm{RootOf}\left(\_Z^{4}ce^{4}+4\_Z^{3}cde^{3}+cd^{4}+bd^{2}+\left(6cd^{2}e^{2}+be^{2}\right)\_Z^{2}+\left(4cd^{3}e+2deb\right)\_Z+a\right)+x\right)}{2a^{2}e\left(2ce^{3}\mathrm{RootOf}\left(\_Z^{4}ce^{4}+4\_Z^{3}cde^{3}+cd^{4}+bd^{2}+\left(6cd^{2}e^{2}+be^{2}\right)\_Z^{2}+\left(4cd^{3}e+2deb\right)\_Z+a\right)^{3}+6cde^{2}\mathrm{RootOf}\left(\_Z^{4}ce^{4}+4\_Z^{3}cde^{3}+cd^{4}+bd^{2}+\left(6cd^{2}e^{2}+be^{2}\right)\_Z^{2}+\left(4cd^{3}e+2deb\right)\_Z+a\right)^{2}+6ecd^{2}\mathrm{RootOf}\left(\_Z^{4}ce^{4}+4\_Z^{3}cde^{3}+cd^{4}+bd^{2}+\left(6cd^{2}e^{2}+be^{2}\right)\_Z^{2}+\left(4cd^{3}e+2deb\right)\_Z+a\right)+2cd^{3}+be\,\mathrm{RootOf}\left(\_Z^{4}ce^{4}+4\_Z^{3}cde^{3}+cd^{4}+bd^{2}+\left(6cd^{2}e^{2}+be^{2}\right)\_Z^{2}+\left(4cd^{3}e+2deb\right)\_Z+a\right)+bd\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)^4/(a+b*(e*x+d)^2+c*(e*x+d)^4),x)
```

```
[Out] 1/2/a^2/e*sum((_R^2*b*c*e^2+2*_R*b*c*d*e+b*c*d^2-a*c+b^2)/(2*_R^3*c*e^3+6*_
R^2*c*d*e^2+6*_R*c*d^2*e+2*c*d^3+_R*b*e+b*d)*ln(-_R+x),_R=RootOf(_Z^4*c*e^4
+4*_Z^3*c*d*e^3+c*d^4+b*d^2+(6*c*d^2*e^2+b*e^2)*_Z^2+(4*c*d^3*e+2*b*d*e)*_Z
+a))-1/3/a/e/(e*x+d)^3+b/a^2/e/(e*x+d)
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^4/(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [B] time = 2.83, size = 5214, normalized size = 23.28

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((d + e*x)^4*(a + b*(d + e*x)^2 + c*(d + e*x)^4)),x)
```

```
[Out] ((2*b*d*x)/a^2 - (a - 3*b*d^2)/(3*a^2*e) + (b*e*x^2)/a^2)/(d^3 + e^3*x^3 +
3*d*e^2*x^2 + 3*d^2*e*x) - atan((((b^4*(-(4*a*c - b^2)^3)^(1/2) - b^7 + 20*
a^3*b*c^3 - 25*a^2*b^3*c^2 + a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) + 9*a*b^5*c -
 3*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4*e^2 + 16*a^7*c^2*e^2 - 8*a
^6*b^2*c*e^2)))^(1/2)*(x*(4*a^8*c^5*e^12 + 2*a^6*b^4*c^3*e^12 - 8*a^7*b^2*c
^4*e^12) - ((b^4*(-(4*a*c - b^2)^3)^(1/2) - b^7 + 20*a^3*b*c^3 - 25*a^2*b^3
*c^2 + a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) + 9*a*b^5*c - 3*a*b^2*c*(-(4*a*c -
```

```
b^2)^3)^(1/2))/(8*(a^5*b^4*e^2 + 16*a^7*c^2*e^2 - 8*a^6*b^2*c*e^2)))^(1/2)*
((x*(32*a^11*b*c^3*e^14 - 8*a^10*b^3*c^2*e^14) + 32*a^11*b*c^3*d*e^13 - 8*a
^10*b^3*c^2*d*e^13)*((b^4*(-(4*a*c - b^2)^3)^(1/2) - b^7 + 20*a^3*b*c^3 - 2
5*a^2*b^3*c^2 + a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) + 9*a*b^5*c - 3*a*b^2*c*(-
(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4*e^2 + 16*a^7*c^2*e^2 - 8*a^6*b^2*c*e^2)
))^(1/2) - 16*a^10*c^4*e^12 - 4*a^8*b^4*c^2*e^12 + 20*a^9*b^2*c^3*e^12) + 4
*a^8*c^5*d*e^11 + 2*a^6*b^4*c^3*d*e^11 - 8*a^7*b^2*c^4*d*e^11)*1i + ((b^4*(
-(4*a*c - b^2)^3)^(1/2) - b^7 + 20*a^3*b*c^3 - 25*a^2*b^3*c^2 + a^2*c^2*(-(
4*a*c - b^2)^3)^(1/2) + 9*a*b^5*c - 3*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(8*
(a^5*b^4*e^2 + 16*a^7*c^2*e^2 - 8*a^6*b^2*c*e^2)))^(1/2)*(x*(4*a^8*c^5*e^12
 + 2*a^6*b^4*c^3*e^12 - 8*a^7*b^2*c^4*e^12) - ((b^4*(-(4*a*c - b^2)^3)^(1/2
) - b^7 + 20*a^3*b*c^3 - 25*a^2*b^3*c^2 + a^2*c^2*(-(4*a*c - b^2)^3)^(1/2)
+ 9*a*b^5*c - 3*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4*e^2 + 16*a^7*
c^2*e^2 - 8*a^6*b^2*c*e^2)))^(1/2)*((x*(32*a^11*b*c^3*e^14 - 8*a^10*b^3*c^2
*e^14) + 32*a^11*b*c^3*d*e^13 - 8*a^10*b^3*c^2*d*e^13)*((b^4*(-(4*a*c - b^2
)^3)^(1/2) - b^7 + 20*a^3*b*c^3 - 25*a^2*b^3*c^2 + a^2*c^2*(-(4*a*c - b^2)^
3)^(1/2) + 9*a*b^5*c - 3*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4*e^2
+ 16*a^7*c^2*e^2 - 8*a^6*b^2*c*e^2)))^(1/2) + 16*a^10*c^4*e^12 + 4*a^8*b^4*
c^2*e^12 - 20*a^9*b^2*c^3*e^12) + 4*a^8*c^5*d*e^11 + 2*a^6*b^4*c^3*d*e^11 -
 8*a^7*b^2*c^4*d*e^11)*1i)/(((b^4*(-(4*a*c - b^2)^3)^(1/2) - b^7 + 20*a^3*b
*c^3 - 25*a^2*b^3*c^2 + a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) + 9*a*b^5*c - 3*a*
b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4*e^2 + 16*a^7*c^2*e^2 - 8*a^6*b^
2*c*e^2)))^(1/2)*(x*(4*a^8*c^5*e^12 + 2*a^6*b^4*c^3*e^12 - 8*a^7*b^2*c^4*e^
12) - ((b^4*(-(4*a*c - b^2)^3)^(1/2) - b^7 + 20*a^3*b*c^3 - 25*a^2*b^3*c^2
+ a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) + 9*a*b^5*c - 3*a*b^2*c*(-(4*a*c - b^2)^
3)^(1/2))/(8*(a^5*b^4*e^2 + 16*a^7*c^2*e^2 - 8*a^6*b^2*c*e^2)))^(1/2)*((x*(
32*a^11*b*c^3*e^14 - 8*a^10*b^3*c^2*e^14) + 32*a^11*b*c^3*d*e^13 - 8*a^10*b
^3*c^2*d*e^13)*((b^4*(-(4*a*c - b^2)^3)^(1/2) - b^7 + 20*a^3*b*c^3 - 25*a^2
*b^3*c^2 + a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) + 9*a*b^5*c - 3*a*b^2*c*(-(4*a*
c - b^2)^3)^(1/2))/(8*(a^5*b^4*e^2 + 16*a^7*c^2*e^2 - 8*a^6*b^2*c*e^2)))^(1
/2) + 16*a^10*c^4*e^12 + 4*a^8*b^4*c^2*e^12 - 20*a^9*b^2*c^3*e^12) + 4*a^8*
c^5*d*e^11 + 2*a^6*b^4*c^3*d*e^11 - 8*a^7*b^2*c^4*d*e^11) - ((b^4*(-(4*a*c
- b^2)^3)^(1/2) - b^7 + 20*a^3*b*c^3 - 25*a^2*b^3*c^2 + a^2*c^2*(-(4*a*c -
b^2)^3)^(1/2) + 9*a*b^5*c - 3*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4
*e^2 + 16*a^7*c^2*e^2 - 8*a^6*b^2*c*e^2)))^(1/2)*(x*(4*a^8*c^5*e^12 + 2*a^6
*b^4*c^3*e^12 - 8*a^7*b^2*c^4*e^12) - ((b^4*(-(4*a*c - b^2)^3)^(1/2) - b^7
+ 20*a^3*b*c^3 - 25*a^2*b^3*c^2 + a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) + 9*a*b^
5*c - 3*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4*e^2 + 16*a^7*c^2*e^2
- 8*a^6*b^2*c*e^2)))^(1/2)*((x*(32*a^11*b*c^3*e^14 - 8*a^10*b^3*c^2*e^14) +
 32*a^11*b*c^3*d*e^13 - 8*a^10*b^3*c^2*d*e^13)*((b^4*(-(4*a*c - b^2)^3)^(1/
2) - b^7 + 20*a^3*b*c^3 - 25*a^2*b^3*c^2 + a^2*c^2*(-(4*a*c - b^2)^3)^(1/2)
 + 9*a*b^5*c - 3*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4*e^2 + 16*a^7
*c^2*e^2 - 8*a^6*b^2*c*e^2)))^(1/2) - 16*a^10*c^4*e^12 - 4*a^8*b^4*c^2*e^12
 + 20*a^9*b^2*c^3*e^12) + 4*a^8*c^5*d*e^11 + 2*a^6*b^4*c^3*d*e^11 - 8*a^7*b
^2*c^4*d*e^11) + 2*a^6*b*c^5*e^10))*((b^4*(-(4*a*c - b^2)^3)^(1/2) - b^7 +
20*a^3*b*c^3 - 25*a^2*b^3*c^2 + a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) + 9*a*b^5*
c - 3*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4*e^2 + 16*a^7*c^2*e^2 -
8*a^6*b^2*c*e^2)))^(1/2)*2i - atan(((-(b^7 + b^4*(-(4*a*c - b^2)^3)^(1/2) -
 20*a^3*b*c^3 + 25*a^2*b^3*c^2 + a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5
*c - 3*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4*e^2 + 16*a^7*c^2*e^2 -
 8*a^6*b^2*c*e^2)))^(1/2)*(x*(4*a^8*c^5*e^12 + 2*a^6*b^4*c^3*e^12 - 8*a^7*b
^2*c^4*e^12) - (-(b^7 + b^4*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3 + 25*a^
2*b^3*c^2 + a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c - 3*a*b^2*c*(-(4*a
*c - b^2)^3)^(1/2))/(8*(a^5*b^4*e^2 + 16*a^7*c^2*e^2 - 8*a^6*b^2*c*e^2)))^(
1/2)*((x*(32*a^11*b*c^3*e^14 - 8*a^10*b^3*c^2*e^14) + 32*a^11*b*c^3*d*e^13
- 8*a^10*b^3*c^2*d*e^13)*(-(b^7 + b^4*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c
^3 + 25*a^2*b^3*c^2 + a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c - 3*a*b^
2*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4*e^2 + 16*a^7*c^2*e^2 - 8*a^6*b^2*
c*e^2)))^(1/2) - 16*a^10*c^4*e^12 - 4*a^8*b^4*c^2*e^12 + 20*a^9*b^2*c^3*e^1
```

```
2) + 4*a^8*c^5*d*e^11 + 2*a^6*b^4*c^3*d*e^11 - 8*a^7*b^2*c^4*d*e^11)*1i + (
-(b^7 + b^4*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3 + 25*a^2*b^3*c^2 + a^2*
c^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c - 3*a*b^2*c*(-(4*a*c - b^2)^3)^(1/
2))/(8*(a^5*b^4*e^2 + 16*a^7*c^2*e^2 - 8*a^6*b^2*c*e^2)))^(1/2)*(x*(4*a^8*c
^5*e^12 + 2*a^6*b^4*c^3*e^12 - 8*a^7*b^2*c^4*e^12) - (-(b^7 + b^4*(-(4*a*c
- b^2)^3)^(1/2) - 20*a^3*b*c^3 + 25*a^2*b^3*c^2 + a^2*c^2*(-(4*a*c - b^2)^3
)^(1/2) - 9*a*b^5*c - 3*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4*e^2 +
 16*a^7*c^2*e^2 - 8*a^6*b^2*c*e^2)))^(1/2)*((x*(32*a^11*b*c^3*e^14 - 8*a^10
*b^3*c^2*e^14) + 32*a^11*b*c^3*d*e^13 - 8*a^10*b^3*c^2*d*e^13)*(-(b^7 + b^4
*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3 + 25*a^2*b^3*c^2 + a^2*c^2*(-(4*a*
c - b^2)^3)^(1/2) - 9*a*b^5*c - 3*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5
*b^4*e^2 + 16*a^7*c^2*e^2 - 8*a^6*b^2*c*e^2)))^(1/2) + 16*a^10*c^4*e^12 + 4
*a^8*b^4*c^2*e^12 - 20*a^9*b^2*c^3*e^12) + 4*a^8*c^5*d*e^11 + 2*a^6*b^4*c^3
*d*e^11 - 8*a^7*b^2*c^4*d*e^11)*1i)/((-(b^7 + b^4*(-(4*a*c - b^2)^3)^(1/2)
- 20*a^3*b*c^3 + 25*a^2*b^3*c^2 + a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^
5*c - 3*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4*e^2 + 16*a^7*c^2*e^2
- 8*a^6*b^2*c*e^2)))^(1/2)*(x*(4*a^8*c^5*e^12 + 2*a^6*b^4*c^3*e^12 - 8*a^7*
b^2*c^4*e^12) - (-(b^7 + b^4*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3 + 25*a
^2*b^3*c^2 + a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c - 3*a*b^2*c*(-(4*
a*c - b^2)^3)^(1/2))/(8*(a^5*b^4*e^2 + 16*a^7*c^2*e^2 - 8*a^6*b^2*c*e^2)))^
(1/2)*((x*(32*a^11*b*c^3*e^14 - 8*a^10*b^3*c^2*e^14) + 32*a^11*b*c^3*d*e^13
 - 8*a^10*b^3*c^2*d*e^13)*(-(b^7 + b^4*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*
c^3 + 25*a^2*b^3*c^2 + a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c - 3*a*b
^2*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4*e^2 + 16*a^7*c^2*e^2 - 8*a^6*b^2
*c*e^2)))^(1/2) + 16*a^10*c^4*e^12 + 4*a^8*b^4*c^2*e^12 - 20*a^9*b^2*c^3*e^
12) + 4*a^8*c^5*d*e^11 + 2*a^6*b^4*c^3*d*e^11 - 8*a^7*b^2*c^4*d*e^11) - (-(
b^7 + b^4*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3 + 25*a^2*b^3*c^2 + a^2*c^
2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c - 3*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2)
)/(8*(a^5*b^4*e^2 + 16*a^7*c^2*e^2 - 8*a^6*b^2*c*e^2)))^(1/2)*(x*(4*a^8*c^5
*e^12 + 2*a^6*b^4*c^3*e^12 - 8*a^7*b^2*c^4*e^12) - (-(b^7 + b^4*(-(4*a*c -
b^2)^3)^(1/2) - 20*a^3*b*c^3 + 25*a^2*b^3*c^2 + a^2*c^2*(-(4*a*c - b^2)^3)^
(1/2) - 9*a*b^5*c - 3*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4*e^2 + 1
6*a^7*c^2*e^2 - 8*a^6*b^2*c*e^2)))^(1/2)*((x*(32*a^11*b*c^3*e^14 - 8*a^10*b
^3*c^2*e^14) + 32*a^11*b*c^3*d*e^13 - 8*a^10*b^3*c^2*d*e^13)*(-(b^7 + b^4*(
-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3 + 25*a^2*b^3*c^2 + a^2*c^2*(-(4*a*c
- b^2)^3)^(1/2) - 9*a*b^5*c - 3*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b
^4*e^2 + 16*a^7*c^2*e^2 - 8*a^6*b^2*c*e^2)))^(1/2) - 16*a^10*c^4*e^12 - 4*a
^8*b^4*c^2*e^12 + 20*a^9*b^2*c^3*e^12) + 4*a^8*c^5*d*e^11 + 2*a^6*b^4*c^3*d
*e^11 - 8*a^7*b^2*c^4*d*e^11) + 2*a^6*b*c^5*e^10))*(-(b^7 + b^4*(-(4*a*c -
b^2)^3)^(1/2) - 20*a^3*b*c^3 + 25*a^2*b^3*c^2 + a^2*c^2*(-(4*a*c - b^2)^3)^
(1/2) - 9*a*b^5*c - 3*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4*e^2 + 1
6*a^7*c^2*e^2 - 8*a^6*b^2*c*e^2)))^(1/2)*2i
```

sympy [A] time = 12.77, size = 347, normalized size = 1.55

$$\frac{-a+3bd^2+6bdex+3be^2x^2}{3a^2d^3e+9a^2d^2e^2x+9a^2de^3x^2+3a^2e^4x^3}+\operatorname{RootSum}\left(t^4\left(256a^7c^2e^4-128a^6b^2ce^4+16a^5b^4e^4\right)+t^2\left(-80a^3bc^3e^2+100a^2b^3c^2e^2-36ab^5ce^2+4b^7e^2\right)+c^5,\left(t\mapsto t\log\left(x+\frac{-96t^3a^7bc^2e^3+56t^3a^6b^3ce^3-8t^3a^5b^5e^3-4ta^4c^4e+32ta^3b^2c^3e-40ta^2b^4c^2e+16tab^6ce-2tb^8e+a^2c^5d-3ab^2c^4d+b^4c^3d}{a^2c^5e-3ab^2c^4e+b^4c^3e}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)**4/(a+b*(e*x+d)**2+c*(e*x+d)**4),x)
```

```
[Out] (-a + 3*b*d**2 + 6*b*d*e*x + 3*b*e**2*x**2)/(3*a**2*d**3*e + 9*a**2*d**2*e*
*2*x + 9*a**2*d*e**3*x**2 + 3*a**2*e**4*x**3) + RootSum(_t**4*(256*a**7*c**
2*e**4 - 128*a**6*b**2*c*e**4 + 16*a**5*b**4*e**4) + _t**2*(-80*a**3*b*c**3
*e**2 + 100*a**2*b**3*c**2*e**2 - 36*a*b**5*c*e**2 + 4*b**7*e**2) + c**5, L
ambda(_t, _t*log(x + (-96*_t**3*a**7*b*c**2*e**3 + 56*_t**3*a**6*b**3*c*e**
3 - 8*_t**3*a**5*b**5*e**3 - 4*_t*a**4*c**4*e + 32*_t*a**3*b**2*c**3*e - 40
*_t*a**2*b**4*c**2*e + 16*_t*a*b**6*c*e - 2*_t*b**8*e + a**2*c**5*d - 3*a*b
**2*c**4*d + b**4*c**3*d)/(a**2*c**5*e - 3*a*b**2*c**4*e + b**4*c**3*e))))
```

3.508 $\int \frac{(d+ex)^4}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx$

Optimal. Leaf size=270

$$\frac{(d+ex)\left(2a+b(d+ex)^2\right)}{2e\left(b^2-4ac\right)\left(a+b(d+ex)^2+c(d+ex)^4\right)}+\frac{\left(b-\frac{4ac+b^2}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\sqrt{c}\,e\left(b^2-4ac\right)\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\left(b\sqrt{b^2-4ac}+4ac+b^2\right)\mathrm{t}}{2\sqrt{2}\sqrt{c}\,e\left(b^2-4ac\right)^{3/2}}$$

Rubi [A] time = 0.58, antiderivative size = 270, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {1142, 1120, 1166, 205}

$$\frac{(d+ex)\left(2a+b(d+ex)^2\right)}{2e\left(b^2-4ac\right)\left(a+b(d+ex)^2+c(d+ex)^4\right)}+\frac{\left(b-\frac{4ac+b^2}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\sqrt{c}\,e\left(b^2-4ac\right)\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\left(b\sqrt{b^2-4ac}+4ac+b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,(d+ex)}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{2\sqrt{2}\sqrt{c}\,e\left(b^2-4ac\right)^{3/2}\sqrt{\sqrt{b^2-4ac}+b}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^4/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2,x]
```

```
[Out] ((d + e*x)*(2*a + b*(d + e*x)^2))/(2*(b^2 - 4*a*c)*e*(a + b*(d + e*x)^2 + c
*(d + e*x)^4)) + ((b - (b^2 + 4*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqr
t[c]*(d + e*x))/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*Sqrt[c]*(b^2 - 4*a
*c)*Sqrt[b - Sqrt[b^2 - 4*a*c]]*e) + ((b^2 + 4*a*c + b*Sqrt[b^2 - 4*a*c])*A
rcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*
Sqrt[c]*(b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]*e)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1120

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> -Simp[(d^3*(d*x)^(m - 3)*(2*a + b*x^2)*(a + b*x^2 + c*x^4)^(p + 1))/(2*
(p + 1)*(b^2 - 4*a*c)), x] + Dist[d^4/(2*(p + 1)*(b^2 - 4*a*c)), Int[(d*x)^
(m - 4)*(2*a*(m - 3) + b*(m + 4*p + 3)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && GtQ[
m, 3] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1142

```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Di
st[u^m/(Coefficient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p,
 x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{(d+ex)^4}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{x^4}{\left(a+bx^2+cx^4\right)^2}\,dx, x, d+ex\right)}{e}$$

$$= \frac{(d+ex)\left(2a+b(d+ex)^2\right)}{2\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)} - \frac{\text{Subst}\left(\int \frac{2a-bx^2}{a+bx^2+cx^4}\,dx, x, d+\right.}{2\left(b^2-4ac\right)e}$$

$$= \frac{(d+ex)\left(2a+b(d+ex)^2\right)}{2\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)} - \frac{\left(b^2+4ac-b\sqrt{b^2-4ac}\right)\text{Su}}{4\left(b\right.}$$

$$= \frac{(d+ex)\left(2a+b(d+ex)^2\right)}{2\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)} - \frac{\left(b^2+4ac-b\sqrt{b^2-4ac}\right)\text{ta}}{2\sqrt{2}\sqrt{c}\left(b^2-4ac\right)^{3/2}\sqrt{b}}$$

Mathematica [A] time = 0.47, size = 263, normalized size = 0.97

$$\frac{-\frac{2\left(-2a(d+ex)-b(d+ex)^3\right)}{\left(b^2-4ac\right)\left(a+b(d+ex)^2+c(d+ex)^4\right)}+\frac{\sqrt{2}\left(b\sqrt{b^2-4ac}-4ac-b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{c}\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{2}\left(b\sqrt{b^2-4ac}+4ac+b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{c}\left(b^2-4ac\right)^{3/2}\sqrt{\sqrt{b^2-4ac}+b}}}{4e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^4/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2,x]
```

```
[Out] ((-2*(-2*a*(d + e*x) - b*(d + e*x)^3))/((b^2 - 4*a*c)*(a + b*(d + e*x)^2 +
c*(d + e*x)^4)) + (Sqrt[2]*(-b^2 - 4*a*c + b*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqr
t[2]*Sqrt[c]*(d + e*x))/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[c]*(b^2 - 4*a*c
)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[2]*(b^2 + 4*a*c + b*Sqrt[b^2 -
 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(
Sqrt[c]*(b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(4*e)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(d+ex)^4}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(d + e*x)^4/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2,x]
```

```
[Out] IntegrateAlgebraic[(d + e*x)^4/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2, x]
```

fricas [B] time = 1.11, size = 2454, normalized size = 9.09

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^4/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="fricas")
```

```
[Out] 1/4*(2*b*e^3*x^3 + 6*b*d*e^2*x^2 + 2*b*d^3 + 2*(3*b*d^2 + 2*a)*e*x + sqrt(1
/2)*((b^2*c - 4*a*c^2)*e^5*x^4 + 4*(b^2*c - 4*a*c^2)*d*e^4*x^3 + (b^3 - 4*a
*b*c + 6*(b^2*c - 4*a*c^2)*d^2)*e^3*x^2 + 2*(2*(b^2*c - 4*a*c^2)*d^3 + (b^3
 - 4*a*b*c)*d)*e^2*x + ((b^2*c - 4*a*c^2)*d^4 + a*b^2 - 4*a^2*c + (b^3 - 4*
a*b*c)*d^2)*e)*sqrt(-((b^6*c - 12*a*b^4*c^2 + 48*a^2*b^2*c^3 - 64*a^3*c^4)*
```

```
e^2*sqrt(1/((b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*e^4)) +
b^3 + 12*a*b*c)/((b^6*c - 12*a*b^4*c^2 + 48*a^2*b^2*c^3 - 64*a^3*c^4)*e^2))
*log((3*b^2 + 4*a*c)*e*x + (3*b^2 + 4*a*c)*d + sqrt(1/2)*(2*(b^7*c - 12*a*b
^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*e^3*sqrt(1/((b^6*c^2 - 12*a*b^4*c^3
 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*e^4)) + (b^4 - 8*a*b^2*c + 16*a^2*c^2)*e)*s
qrt(-((b^6*c - 12*a*b^4*c^2 + 48*a^2*b^2*c^3 - 64*a^3*c^4)*e^2*sqrt(1/((b^6
*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*e^4)) + b^3 + 12*a*b*c)/
((b^6*c - 12*a*b^4*c^2 + 48*a^2*b^2*c^3 - 64*a^3*c^4)*e^2))) - sqrt(1/2)*((
b^2*c - 4*a*c^2)*e^5*x^4 + 4*(b^2*c - 4*a*c^2)*d*e^4*x^3 + (b^3 - 4*a*b*c +
 6*(b^2*c - 4*a*c^2)*d^2)*e^3*x^2 + 2*(2*(b^2*c - 4*a*c^2)*d^3 + (b^3 - 4*a
*b*c)*d)*e^2*x + ((b^2*c - 4*a*c^2)*d^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)
*d^2)*e)*sqrt(-((b^6*c - 12*a*b^4*c^2 + 48*a^2*b^2*c^3 - 64*a^3*c^4)*e^2*sq
rt(1/((b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*e^4)) + b^3 +
12*a*b*c)/((b^6*c - 12*a*b^4*c^2 + 48*a^2*b^2*c^3 - 64*a^3*c^4)*e^2))*log((
3*b^2 + 4*a*c)*e*x + (3*b^2 + 4*a*c)*d - sqrt(1/2)*(2*(b^7*c - 12*a*b^5*c^2
 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*e^3*sqrt(1/((b^6*c^2 - 12*a*b^4*c^3 + 48*
a^2*b^2*c^4 - 64*a^3*c^5)*e^4)) + (b^4 - 8*a*b^2*c + 16*a^2*c^2)*e)*sqrt(-(
(b^6*c - 12*a*b^4*c^2 + 48*a^2*b^2*c^3 - 64*a^3*c^4)*e^2*sqrt(1/((b^6*c^2 -
 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*e^4)) + b^3 + 12*a*b*c)/((b^6*
c - 12*a*b^4*c^2 + 48*a^2*b^2*c^3 - 64*a^3*c^4)*e^2))) - sqrt(1/2)*((b^2*c
- 4*a*c^2)*e^5*x^4 + 4*(b^2*c - 4*a*c^2)*d*e^4*x^3 + (b^3 - 4*a*b*c + 6*(b^
2*c - 4*a*c^2)*d^2)*e^3*x^2 + 2*(2*(b^2*c - 4*a*c^2)*d^3 + (b^3 - 4*a*b*c)*
d)*e^2*x + ((b^2*c - 4*a*c^2)*d^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*d^2)*
e)*sqrt(((b^6*c - 12*a*b^4*c^2 + 48*a^2*b^2*c^3 - 64*a^3*c^4)*e^2*sqrt(1/((
b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*e^4)) - b^3 - 12*a*b*
c)/((b^6*c - 12*a*b^4*c^2 + 48*a^2*b^2*c^3 - 64*a^3*c^4)*e^2))*log((3*b^2 +
 4*a*c)*e*x + (3*b^2 + 4*a*c)*d + sqrt(1/2)*(2*(b^7*c - 12*a*b^5*c^2 + 48*a
^2*b^3*c^3 - 64*a^3*b*c^4)*e^3*sqrt(1/((b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2
*c^4 - 64*a^3*c^5)*e^4)) - (b^4 - 8*a*b^2*c + 16*a^2*c^2)*e)*sqrt(((b^6*c -
 12*a*b^4*c^2 + 48*a^2*b^2*c^3 - 64*a^3*c^4)*e^2*sqrt(1/((b^6*c^2 - 12*a*b^
4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*e^4)) - b^3 - 12*a*b*c)/((b^6*c - 12*a
*b^4*c^2 + 48*a^2*b^2*c^3 - 64*a^3*c^4)*e^2))) + sqrt(1/2)*((b^2*c - 4*a*c^
2)*e^5*x^4 + 4*(b^2*c - 4*a*c^2)*d*e^4*x^3 + (b^3 - 4*a*b*c + 6*(b^2*c - 4*
a*c^2)*d^2)*e^3*x^2 + 2*(2*(b^2*c - 4*a*c^2)*d^3 + (b^3 - 4*a*b*c)*d)*e^2*x
 + ((b^2*c - 4*a*c^2)*d^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*d^2)*e)*sqrt(
((b^6*c - 12*a*b^4*c^2 + 48*a^2*b^2*c^3 - 64*a^3*c^4)*e^2*sqrt(1/((b^6*c^2
- 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*e^4)) - b^3 - 12*a*b*c)/((b^6
*c - 12*a*b^4*c^2 + 48*a^2*b^2*c^3 - 64*a^3*c^4)*e^2))*log((3*b^2 + 4*a*c)*
e*x + (3*b^2 + 4*a*c)*d - sqrt(1/2)*(2*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c
^3 - 64*a^3*b*c^4)*e^3*sqrt(1/((b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 6
4*a^3*c^5)*e^4)) - (b^4 - 8*a*b^2*c + 16*a^2*c^2)*e)*sqrt(((b^6*c - 12*a*b^
4*c^2 + 48*a^2*b^2*c^3 - 64*a^3*c^4)*e^2*sqrt(1/((b^6*c^2 - 12*a*b^4*c^3 +
48*a^2*b^2*c^4 - 64*a^3*c^5)*e^4)) - b^3 - 12*a*b*c)/((b^6*c - 12*a*b^4*c^2
 + 48*a^2*b^2*c^3 - 64*a^3*c^4)*e^2))) + 4*a*d)/((b^2*c - 4*a*c^2)*e^5*x^4
+ 4*(b^2*c - 4*a*c^2)*d*e^4*x^3 + (b^3 - 4*a*b*c + 6*(b^2*c - 4*a*c^2)*d^2)
*e^3*x^2 + 2*(2*(b^2*c - 4*a*c^2)*d^3 + (b^3 - 4*a*b*c)*d)*e^2*x + ((b^2*c
- 4*a*c^2)*d^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*d^2)*e)
```

giac [B] time = 0.52, size = 1304, normalized size = 4.83

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^4/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="giac")
```

```
[Out] -1/4*(((d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c
))^2*b*e^2 - 2*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*
e^(-4)/c))*b*d*e + b*d^2 - 2*a)*log(d*e^(-1) + x + sqrt(1/2)*sqrt(-(b*e^2 +
 sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))/(2*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 +
sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^3*c*e^4 - 6*(d*e^(-1) + sqrt(1/2)*sqrt(-(
```

```
b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^2*c*d*e^3 - 2*c*d^3*e - b*d*e + (
6*c*d^2*e^2 + b*e^2)*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)
*e^2)*e^(-4)/c))) + ((d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)
*e^2)*e^(-4)/c))^2*b*e^2 - 2*(d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2
- 4*a*c)*e^2)*e^(-4)/c))*b*d*e + b*d^2 - 2*a)*log(d*e^(-1) + x - sqrt(1/2)*
sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))/(2*(d*e^(-1) - sqrt(1/2)*s
qrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^3*c*e^4 - 6*(d*e^(-1) - sqr
t(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^2*c*d*e^3 - 2*c*d^3
*e - b*d*e + (6*c*d^2*e^2 + b*e^2)*(d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 + sqr
t(b^2 - 4*a*c)*e^2)*e^(-4)/c))) + ((d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 - sqr
t(b^2 - 4*a*c)*e^2)*e^(-4)/c))^2*b*e^2 - 2*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e
^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))*b*d*e + b*d^2 - 2*a)*log(d*e^(-1) +
x + sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))/(2*(d*e^(-1)
 + sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^3*c*e^4 - 6*(
d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^2*c*d
*e^3 - 2*c*d^3*e - b*d*e + (6*c*d^2*e^2 + b*e^2)*(d*e^(-1) + sqrt(1/2)*sqrt
(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))) + ((d*e^(-1) - sqrt(1/2)*sqrt
(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^2*b*e^2 - 2*(d*e^(-1) - sqrt(1
/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))*b*d*e + b*d^2 - 2*a)*l
og(d*e^(-1) + x - sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c)
)/(2*(d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))
^3*c*e^4 - 6*(d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^
(-4)/c))^2*c*d*e^3 - 2*c*d^3*e - b*d*e + (6*c*d^2*e^2 + b*e^2)*(d*e^(-1) -
sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))))/(b^2 - 4*a*c)
+ 1/2*(b*x^3*e^3 + 3*b*d*x^2*e^2 + 3*b*d^2*x*e + b*d^3 + 2*a*x*e + 2*a*d)/(
(c*x^4*e^4 + 4*c*d*x^3*e^3 + 6*c*d^2*x^2*e^2 + 4*c*d^3*x*e + c*d^4 + b*x^2*
e^2 + 2*b*d*x*e + b*d^2 + a)*(b^2*e - 4*a*c*e))
```

maple [C] time = 0.02, size = 323, normalized size = 1.20

$$\frac{\left(-\mathrm{RootOf}\left(\_Z^4ce^4+4\_Z^3cde^3+cd^4+bd^2+\left(6cd^2e^2+be^2\right)\_Z^2+\left(4cd^3e+2deb\right)\_Z+a\right)^2be^2-2\,\mathrm{RootOf}\left(\_Z^4ce^4+4\_Z^3cde^3+cd^4+bd^2+\left(6cd^2e^2+be^2\right)\_Z^2+\left(4cd^3e+2deb\right)\_Z+a\right)bde-bd^2+2a\right)\ln\left(-\mathrm{RootOf}\left(\_Z^4ce^4+4\_Z^3cde^3+cd^4+bd^2+\left(6cd^2e^2+be^2\right)\_Z^2+\left(4cd^3e+2deb\right)\_Z+a\right)+x\right)}{4\left(4ac-b^2\right)e\left(2ce^3\,\mathrm{RootOf}\left(\_Z^4ce^4+4\_Z^3cde^3+cd^4+bd^2+\left(6cd^2e^2+be^2\right)\_Z^2+\left(4cd^3e+2deb\right)\_Z+a\right)^3+6cde^2\,\mathrm{RootOf}\left(\_Z^4ce^4+4\_Z^3cde^3+cd^4+bd^2+\left(6cd^2e^2+be^2\right)\_Z^2+\left(4cd^3e+2deb\right)\_Z+a\right)^2+6ecd^2\,\mathrm{RootOf}\left(\_Z^4ce^4+4\_Z^3cde^3+cd^4+bd^2+\left(6cd^2e^2+be^2\right)\_Z^2+\left(4cd^3e+2deb\right)\_Z+a\right)+2cd^3+be\,\mathrm{RootOf}\left(\_Z^4ce^4+4\_Z^3cde^3+cd^4+bd^2+\left(6cd^2e^2+be^2\right)\_Z^2+\left(4cd^3e+2deb\right)\_Z+a\right)+bd\right)}+\frac{-\frac{be^2x^3}{2\left(4ac-b^2\right)}-\frac{3bdex^2}{2\left(4ac-b^2\right)}-\frac{\left(3bd^2+2a\right)x}{2\left(4ac-b^2\right)}-\frac{\left(bd^2+2a\right)d}{2\left(4ac-b^2\right)e}}{ce^4x^4+4cde^3x^3+6cd^2e^2x^2+4cd^3ex+be^2x^2+cd^4+2bdex+bd^2+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^4/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x)
```

```
[Out] (-1/2*b*e^2/(4*a*c-b^2)*x^3-3/2*d*b*e/(4*a*c-b^2)*x^2-1/2*(3*b*d^2+2*a)/(4*
a*c-b^2)*x-1/2*d/e*(b*d^2+2*a)/(4*a*c-b^2))/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^
2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)+1/4/(4*a*c-b^2)/e*
sum((-_R^2*b*e^2-2*_R*b*d*e-b*d^2+2*a)/(2*_R^3*c*e^3+6*_R^2*c*d*e^2+6*_R*c*
d^2*e+2*c*d^3+_R*b*e+b*d)*ln(-_R+x),_R=RootOf(_Z^4*c*e^4+4*_Z^3*c*d*e^3+c*d
^4+b*d^2+(6*c*d^2*e^2+b*e^2)*_Z^2+(4*c*d^3*e+2*b*d*e)*_Z+a))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{be^3x^3+3bde^2x^2+bd^3+\left(3bd^2+2a\right)ex+2ad}{2\left(\left(b^2c-4ac^2\right)e^5x^4+4\left(b^2c-4ac^2\right)de^4x^3+\left(b^3-4abc+6\left(b^2c-4ac^2\right)d^2\right)e^3x^2+2\left(2\left(b^2c-4ac^2\right)d^3+\left(b^3-4abc\right)d\right)e^2x+\left(\left(b^2c-4ac^2\right)d^4+ab^2-4a^2c+\left(b^3-4abc\right)d^2\right)e\right)}-\frac{1}{2}\int-\frac{be^2x^2+2bdex+bd^2-2a}{\left(b^2c-4ac^2\right)e^4x^4+4\left(b^2c-4ac^2\right)de^3x^3+\left(b^2c-4ac^2\right)d^4+\left(b^3-4abc+6\left(b^2c-4ac^2\right)d^2\right)e^2x^2+ab^2-4a^2c+\left(b^3-4abc\right)d^2+2\left(2\left(b^2c-4ac^2\right)d^3+\left(b^3-4abc\right)d\right)ex}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^4/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(b*e^3*x^3 + 3*b*d*e^2*x^2 + b*d^3 + (3*b*d^2 + 2*a)*e*x + 2*a*d)/((b^2
*c - 4*a*c^2)*e^5*x^4 + 4*(b^2*c - 4*a*c^2)*d*e^4*x^3 + (b^3 - 4*a*b*c + 6*
(b^2*c - 4*a*c^2)*d^2)*e^3*x^2 + 2*(2*(b^2*c - 4*a*c^2)*d^3 + (b^3 - 4*a*b*
c)*d)*e^2*x + ((b^2*c - 4*a*c^2)*d^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*d^
2)*e) - 1/2*integrate(-(b*e^2*x^2 + 2*b*d*e*x + b*d^2 - 2*a)/((b^2*c - 4*a*
c^2)*e^4*x^4 + 4*(b^2*c - 4*a*c^2)*d*e^3*x^3 + (b^2*c - 4*a*c^2)*d^4 + (b^3
 - 4*a*b*c + 6*(b^2*c - 4*a*c^2)*d^2)*e^2*x^2 + a*b^2 - 4*a^2*c + (b^3 - 4*
a*b*c)*d^2 + 2*(2*(b^2*c - 4*a*c^2)*d^3 + (b^3 - 4*a*b*c)*d)*e*x), x)
```

mupad [B] time = 4.76, size = 7327, normalized size = 27.14

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^4/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2,x)
```

```
[Out] atan(-(((((64*b^9*c^2*d*e^13 - 1024*a*b^7*c^3*d*e^13 + 16384*a^4*b*c^6*d*e^
13 + 6144*a^2*b^5*c^4*d*e^13 - 16384*a^3*b^3*c^5*d*e^13)/(8*(b^6 - 64*a^3*c
^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) + (x*(16*b^7*c^2*e^14 - 192*a*b^5*c^3*e^
14 - 1024*a^3*b*c^5*e^14 + 768*a^2*b^3*c^4*e^14))/(2*(b^4 + 16*a^2*c^2 - 8*
a*b^2*c)))*(-(b^9 + (-(4*a*c - b^2)^9)^(1/2) - 768*a^4*b*c^4 - 96*a^2*b^5*c
^2 + 512*a^3*b^3*c^3)/(32*(b^12*c*e^2 + 4096*a^6*c^7*e^2 - 24*a*b^10*c^2*e^
2 + 240*a^2*b^8*c^3*e^2 - 1280*a^3*b^6*c^4*e^2 + 3840*a^4*b^4*c^5*e^2 - 614
4*a^5*b^2*c^6*e^2)))^(1/2) - (2048*a^4*c^5*e^12 - 32*a*b^6*c^2*e^12 + 384*a
^2*b^4*c^3*e^12 - 1536*a^3*b^2*c^4*e^12)/(8*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*
c^2 - 12*a*b^4*c)))*(-(b^9 + (-(4*a*c - b^2)^9)^(1/2) - 768*a^4*b*c^4 - 96*
a^2*b^5*c^2 + 512*a^3*b^3*c^3)/(32*(b^12*c*e^2 + 4096*a^6*c^7*e^2 - 24*a*b^
10*c^2*e^2 + 240*a^2*b^8*c^3*e^2 - 1280*a^3*b^6*c^4*e^2 + 3840*a^4*b^4*c^5*
e^2 - 6144*a^5*b^2*c^6*e^2)))^(1/2) - (128*a^3*c^4*d*e^11 - 4*b^6*c*d*e^11
+ 8*a*b^4*c^2*d*e^11)/(8*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c))
+ (x*(b^4*c*e^12 + 8*a^2*c^3*e^12 + 2*a*b^2*c^2*e^12))/(2*(b^4 + 16*a^2*c^2
 - 8*a*b^2*c)))*(-(b^9 + (-(4*a*c - b^2)^9)^(1/2) - 768*a^4*b*c^4 - 96*a^2*
b^5*c^2 + 512*a^3*b^3*c^3)/(32*(b^12*c*e^2 + 4096*a^6*c^7*e^2 - 24*a*b^10*c
^2*e^2 + 240*a^2*b^8*c^3*e^2 - 1280*a^3*b^6*c^4*e^2 + 3840*a^4*b^4*c^5*e^2
- 6144*a^5*b^2*c^6*e^2)))^(1/2)*1i + ((((64*b^9*c^2*d*e^13 - 1024*a*b^7*c^3
*d*e^13 + 16384*a^4*b*c^6*d*e^13 + 6144*a^2*b^5*c^4*d*e^13 - 16384*a^3*b^3*
c^5*d*e^13)/(8*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) + (x*(16*b
^7*c^2*e^14 - 192*a*b^5*c^3*e^14 - 1024*a^3*b*c^5*e^14 + 768*a^2*b^3*c^4*e^
14))/(2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)))*(-(b^9 + (-(4*a*c - b^2)^9)^(1/2)
- 768*a^4*b*c^4 - 96*a^2*b^5*c^2 + 512*a^3*b^3*c^3)/(32*(b^12*c*e^2 + 4096*
a^6*c^7*e^2 - 24*a*b^10*c^2*e^2 + 240*a^2*b^8*c^3*e^2 - 1280*a^3*b^6*c^4*e^
2 + 3840*a^4*b^4*c^5*e^2 - 6144*a^5*b^2*c^6*e^2)))^(1/2) + (2048*a^4*c^5*e^
12 - 32*a*b^6*c^2*e^12 + 384*a^2*b^4*c^3*e^12 - 1536*a^3*b^2*c^4*e^12)/(8*(
b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)))*(-(b^9 + (-(4*a*c - b^2)^
9)^(1/2) - 768*a^4*b*c^4 - 96*a^2*b^5*c^2 + 512*a^3*b^3*c^3)/(32*(b^12*c*e^
2 + 4096*a^6*c^7*e^2 - 24*a*b^10*c^2*e^2 + 240*a^2*b^8*c^3*e^2 - 1280*a^3*b
^6*c^4*e^2 + 3840*a^4*b^4*c^5*e^2 - 6144*a^5*b^2*c^6*e^2)))^(1/2) - (128*a^
3*c^4*d*e^11 - 4*b^6*c*d*e^11 + 8*a*b^4*c^2*d*e^11)/(8*(b^6 - 64*a^3*c^3 +
48*a^2*b^2*c^2 - 12*a*b^4*c)) + (x*(b^4*c*e^12 + 8*a^2*c^3*e^12 + 2*a*b^2*c
^2*e^12))/(2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)))*(-(b^9 + (-(4*a*c - b^2)^9)^(
1/2) - 768*a^4*b*c^4 - 96*a^2*b^5*c^2 + 512*a^3*b^3*c^3)/(32*(b^12*c*e^2 +
4096*a^6*c^7*e^2 - 24*a*b^10*c^2*e^2 + 240*a^2*b^8*c^3*e^2 - 1280*a^3*b^6*c
^4*e^2 + 3840*a^4*b^4*c^5*e^2 - 6144*a^5*b^2*c^6*e^2)))^(1/2)*1i)/(((((64*b
^9*c^2*d*e^13 - 1024*a*b^7*c^3*d*e^13 + 16384*a^4*b*c^6*d*e^13 + 6144*a^2*b
^5*c^4*d*e^13 - 16384*a^3*b^3*c^5*d*e^13)/(8*(b^6 - 64*a^3*c^3 + 48*a^2*b^2
*c^2 - 12*a*b^4*c)) + (x*(16*b^7*c^2*e^14 - 192*a*b^5*c^3*e^14 - 1024*a^3*b
*c^5*e^14 + 768*a^2*b^3*c^4*e^14))/(2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)))*(-(b
^9 + (-(4*a*c - b^2)^9)^(1/2) - 768*a^4*b*c^4 - 96*a^2*b^5*c^2 + 512*a^3*b^
3*c^3)/(32*(b^12*c*e^2 + 4096*a^6*c^7*e^2 - 24*a*b^10*c^2*e^2 + 240*a^2*b^8
*c^3*e^2 - 1280*a^3*b^6*c^4*e^2 + 3840*a^4*b^4*c^5*e^2 - 6144*a^5*b^2*c^6*e
^2)))^(1/2) - (2048*a^4*c^5*e^12 - 32*a*b^6*c^2*e^12 + 384*a^2*b^4*c^3*e^12
 - 1536*a^3*b^2*c^4*e^12)/(8*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*
c)))*(-(b^9 + (-(4*a*c - b^2)^9)^(1/2) - 768*a^4*b*c^4 - 96*a^2*b^5*c^2 + 5
12*a^3*b^3*c^3)/(32*(b^12*c*e^2 + 4096*a^6*c^7*e^2 - 24*a*b^10*c^2*e^2 + 24
0*a^2*b^8*c^3*e^2 - 1280*a^3*b^6*c^4*e^2 + 3840*a^4*b^4*c^5*e^2 - 6144*a^5*
b^2*c^6*e^2)))^(1/2) - (128*a^3*c^4*d*e^11 - 4*b^6*c*d*e^11 + 8*a*b^4*c^2*d
*e^11)/(8*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) + (x*(b^4*c*e^1
2 + 8*a^2*c^3*e^12 + 2*a*b^2*c^2*e^12))/(2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)))
*(-(b^9 + (-(4*a*c - b^2)^9)^(1/2) - 768*a^4*b*c^4 - 96*a^2*b^5*c^2 + 512*a
^3*b^3*c^3)/(32*(b^12*c*e^2 + 4096*a^6*c^7*e^2 - 24*a*b^10*c^2*e^2 + 240*a^
2*b^8*c^3*e^2 - 1280*a^3*b^6*c^4*e^2 + 3840*a^4*b^4*c^5*e^2 - 6144*a^5*b^2*
c^6*e^2)))^(1/2) - ((((64*b^9*c^2*d*e^13 - 1024*a*b^7*c^3*d*e^13 + 16384*a^
```

4*b*c^6*d*e^13 + 6144*a^2*b^5*c^4*d*e^13 - 16384*a^3*b^3*c^5*d*e^13)/(8*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) + (x*(16*b^7*c^2*e^14 - 192*a*b^5*c^3*e^14 - 1024*a^3*b*c^5*e^14 + 768*a^2*b^3*c^4*e^14))/(2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)))*(-(b^9 + (-(4*a*c - b^2)^9)^(1/2) - 768*a^4*b*c^4 - 96*a^2*b^5*c^2 + 512*a^3*b^3*c^3)/(32*(b^12*c*e^2 + 4096*a^6*c^7*e^2 - 24*a*b^10*c^2*e^2 + 240*a^2*b^8*c^3*e^2 - 1280*a^3*b^6*c^4*e^2 + 3840*a^4*b^4*c^5*e^2 - 6144*a^5*b^2*c^6*e^2)))^(1/2) + (2048*a^4*c^5*e^12 - 32*a*b^6*c^2*e^12 + 384*a^2*b^4*c^3*e^12 - 1536*a^3*b^2*c^4*e^12)/(8*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)))*(-(b^9 + (-(4*a*c - b^2)^9)^(1/2) - 768*a^4*b*c^4 - 96*a^2*b^5*c^2 + 512*a^3*b^3*c^3)/(32*(b^12*c*e^2 + 4096*a^6*c^7*e^2 - 24*a*b^10*c^2*e^2 + 240*a^2*b^8*c^3*e^2 - 1280*a^3*b^6*c^4*e^2 + 3840*a^4*b^4*c^5*e^2 - 6144*a^5*b^2*c^6*e^2)))^(1/2) - (128*a^3*c^4*d*e^11 - 4*b^6*c*d*e^11 + 8*a*b^4*c^2*d*e^11)/(8*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) + (x*(b^4*c*e^12 + 8*a^2*c^3*e^12 + 2*a*b^2*c^2*e^12))/(2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)))*(-(b^9 + (-(4*a*c - b^2)^9)^(1/2) - 768*a^4*b*c^4 - 96*a^2*b^5*c^2 + 512*a^3*b^3*c^3)/(32*(b^12*c*e^2 + 4096*a^6*c^7*e^2 - 24*a*b^10*c^2*e^2 + 240*a^2*b^8*c^3*e^2 - 1280*a^3*b^6*c^4*e^2 + 3840*a^4*b^4*c^5*e^2 - 6144*a^5*b^2*c^6*e^2)))^(1/2) + (4*a^2*b*c^2*e^10 + 3*a*b^3*c*e^10)/(4*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c))))*(-(b^9 + (-(4*a*c - b^2)^9)^(1/2) - 768*a^4*b*c^4 - 96*a^2*b^5*c^2 + 512*a^3*b^3*c^3)/(32*(b^12*c*e^2 + 4096*a^6*c^7*e^2 - 24*a*b^10*c^2*e^2 + 240*a^2*b^8*c^3*e^2 - 1280*a^3*b^6*c^4*e^2 + 3840*a^4*b^4*c^5*e^2 - 6144*a^5*b^2*c^6*e^2)))^(1/2)*2i - ((2*a*d + b*d^3)/(2*e*(4*a*c - b^2)) + (x*(2*a + 3*b*d^2))/(2*(4*a*c - b^2)) + (b*e^2*x^3)/(2*(4*a*c - b^2)) + (3*b*d*e*x^2)/(2*(4*a*c - b^2)))/(a + x^2*(b*e^2 + 6*c*d^2*e^2) + b*d^2 + c*d^4 + x*(2*b*d*e + 4*c*d^3*e) + c*e^4*x^4 + 4*c*d*e^3*x^3) + atan(((((-(4*a*c - b^2)^9)^(1/2) - b^9 + 768*a^4*b*c^4 + 96*a^2*b^5*c^2 - 512*a^3*b^3*c^3)/(32*(b^12*c*e^2 + 4096*a^6*c^7*e^2 - 24*a*b^10*c^2*e^2 + 240*a^2*b^8*c^3*e^2 - 1280*a^3*b^6*c^4*e^2 + 3840*a^4*b^4*c^5*e^2 - 6144*a^5*b^2*c^6*e^2)))^(1/2)*(((2048*a^4*c^5*e^12 - 32*a*b^6*c^2*e^12 + 384*a^2*b^4*c^3*e^12 - 1536*a^3*b^2*c^4*e^12)/(8*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) + ((64*b^9*c^2*d*e^13 - 1024*a*b^7*c^3*d*e^13 + 16384*a^4*b*c^6*d*e^13 + 6144*a^2*b^5*c^4*d*e^13 - 16384*a^3*b^3*c^5*d*e^13)/(8*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) + (x*(16*b^7*c^2*e^14 - 192*a*b^5*c^3*e^14 - 1024*a^3*b*c^5*e^14 + 768*a^2*b^3*c^4*e^14))/(2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)))*(((-(4*a*c - b^2)^9)^(1/2) - b^9 + 768*a^4*b*c^4 + 96*a^2*b^5*c^2 - 512*a^3*b^3*c^3)/(32*(b^12*c*e^2 + 4096*a^6*c^7*e^2 - 24*a*b^10*c^2*e^2 + 240*a^2*b^8*c^3*e^2 - 1280*a^3*b^6*c^4*e^2 + 3840*a^4*b^4*c^5*e^2 - 6144*a^5*b^2*c^6*e^2)))^(1/2))*(((-(4*a*c - b^2)^9)^(1/2) - b^9 + 768*a^4*b*c^4 + 96*a^2*b^5*c^2 - 512*a^3*b^3*c^3)/(32*(b^12*c*e^2 + 4096*a^6*c^7*e^2 - 24*a*b^10*c^2*e^2 + 240*a^2*b^8*c^3*e^2 - 1280*a^3*b^6*c^4*e^2 + 3840*a^4*b^4*c^5*e^2 - 6144*a^5*b^2*c^6*e^2)))^(1/2) - (128*a^3*c^4*d*e^11 - 4*b^6*c*d*e^11 + 8*a*b^4*c^2*d*e^11)/(8*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) + (x*(b^4*c*e^12 + 8*a^2*c^3*e^12 + 2*a*b^2*c^2*e^12))/(2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)))*1i - (((-(4*a*c - b^2)^9)^(1/2) - b^9 + 768*a^4*b*c^4 + 96*a^2*b^5*c^2 - 512*a^3*b^3*c^3)/(32*(b^12*c*e^2 + 4096*a^6*c^7*e^2 - 24*a*b^10*c^2*e^2 + 240*a^2*b^8*c^3*e^2 - 1280*a^3*b^6*c^4*e^2 + 3840*a^4*b^4*c^5*e^2 - 6144*a^5*b^2*c^6*e^2)))^(1/2)*(((2048*a^4*c^5*e^12 - 32*a*b^6*c^2*e^12 + 384*a^2*b^4*c^3*e^12 - 1536*a^3*b^2*c^4*e^12)/(8*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) - ((64*b^9*c^2*d*e^13 - 1024*a*b^7*c^3*d*e^13 + 16384*a^4*b*c^6*d*e^13 + 6144*a^2*b^5*c^4*d*e^13 - 16384*a^3*b^3*c^5*d*e^13)/(8*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) + (x*(16*b^7*c^2*e^14 - 192*a*b^5*c^3*e^14 - 1024*a^3*b*c^5*e^14 + 768*a^2*b^3*c^4*e^14))/(2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)))*(((-(4*a*c - b^2)^9)^(1/2) - b^9 + 768*a^4*b*c^4 + 96*a^2*b^5*c^2 - 512*a^3*b^3*c^3)/(32*(b^12*c*e^2 + 4096*a^6*c^7*e^2 - 24*a*b^10*c^2*e^2 + 240*a^2*b^8*c^3*e^2 - 1280*a^3*b^6*c^4*e^2 + 3840*a^4*b^4*c^5*e^2 - 6144*a^5*b^2*c^6*e^2)))^(1/2))*(((-(4*a*c - b^2)^9)^(1/2) - b^9 + 768*a^4*b*c^4 + 96*a^2*b^5*c^2 - 512*a^3*b^3*c^3)/(32*(b^12*c*e^2 + 4096*a^6*c^7*e^2 - 24*a*b^10*c^2*e^2 + 240*a^2*b^8*c^3*e^2 - 1280*a^3*b^6*c^4*e^2 + 3840*a^4*b^4*c^5*e^2 -

```
6144*a^5*b^2*c^6*e^2)))^(1/2) + (128*a^3*c^4*d*e^11 - 4*b^6*c*d*e^11 + 8*a
*b^4*c^2*d*e^11)/(8*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) - (x*
(b^4*c*e^12 + 8*a^2*c^3*e^12 + 2*a*b^2*c^2*e^12))/(2*(b^4 + 16*a^2*c^2 - 8*
a*b^2*c)))*1i)/((((-(4*a*c - b^2)^9)^(1/2) - b^9 + 768*a^4*b*c^4 + 96*a^2*b
^5*c^2 - 512*a^3*b^3*c^3)/(32*(b^12*c*e^2 + 4096*a^6*c^7*e^2 - 24*a*b^10*c^
2*e^2 + 240*a^2*b^8*c^3*e^2 - 1280*a^3*b^6*c^4*e^2 + 3840*a^4*b^4*c^5*e^2 -
 6144*a^5*b^2*c^6*e^2)))^(1/2)*(((2048*a^4*c^5*e^12 - 32*a*b^6*c^2*e^12 + 3
84*a^2*b^4*c^3*e^12 - 1536*a^3*b^2*c^4*e^12)/(8*(b^6 - 64*a^3*c^3 + 48*a^2*
b^2*c^2 - 12*a*b^4*c)) + ((64*b^9*c^2*d*e^13 - 1024*a*b^7*c^3*d*e^13 + 1638
4*a^4*b*c^6*d*e^13 + 6144*a^2*b^5*c^4*d*e^13 - 16384*a^3*b^3*c^5*d*e^13)/(8
*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) + (x*(16*b^7*c^2*e^14 -
192*a*b^5*c^3*e^14 - 1024*a^3*b*c^5*e^14 + 768*a^2*b^3*c^4*e^14))/(2*(b^4 +
 16*a^2*c^2 - 8*a*b^2*c)))*(((-(4*a*c - b^2)^9)^(1/2) - b^9 + 768*a^4*b*c^4
 + 96*a^2*b^5*c^2 - 512*a^3*b^3*c^3)/(32*(b^12*c*e^2 + 4096*a^6*c^7*e^2 - 2
4*a*b^10*c^2*e^2 + 240*a^2*b^8*c^3*e^2 - 1280*a^3*b^6*c^4*e^2 + 3840*a^4*b^
4*c^5*e^2 - 6144*a^5*b^2*c^6*e^2)))^(1/2))*(((-(4*a*c - b^2)^9)^(1/2) - b^9
 + 768*a^4*b*c^4 + 96*a^2*b^5*c^2 - 512*a^3*b^3*c^3)/(32*(b^12*c*e^2 + 4096
*a^6*c^7*e^2 - 24*a*b^10*c^2*e^2 + 240*a^2*b^8*c^3*e^2 - 1280*a^3*b^6*c^4*e
^2 + 3840*a^4*b^4*c^5*e^2 - 6144*a^5*b^2*c^6*e^2)))^(1/2) - (128*a^3*c^4*d*
e^11 - 4*b^6*c*d*e^11 + 8*a*b^4*c^2*d*e^11)/(8*(b^6 - 64*a^3*c^3 + 48*a^2*b
^2*c^2 - 12*a*b^4*c)) + (x*(b^4*c*e^12 + 8*a^2*c^3*e^12 + 2*a*b^2*c^2*e^12)
)/(2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c))) + (((-(4*a*c - b^2)^9)^(1/2) - b^9 +
768*a^4*b*c^4 + 96*a^2*b^5*c^2 - 512*a^3*b^3*c^3)/(32*(b^12*c*e^2 + 4096*a^
6*c^7*e^2 - 24*a*b^10*c^2*e^2 + 240*a^2*b^8*c^3*e^2 - 1280*a^3*b^6*c^4*e^2
+ 3840*a^4*b^4*c^5*e^2 - 6144*a^5*b^2*c^6*e^2)))^(1/2)*(((2048*a^4*c^5*e^12
 - 32*a*b^6*c^2*e^12 + 384*a^2*b^4*c^3*e^12 - 1536*a^3*b^2*c^4*e^12)/(8*(b^
6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) - ((64*b^9*c^2*d*e^13 - 1024
*a*b^7*c^3*d*e^13 + 16384*a^4*b*c^6*d*e^13 + 6144*a^2*b^5*c^4*d*e^13 - 1638
4*a^3*b^3*c^5*d*e^13)/(8*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c))
+ (x*(16*b^7*c^2*e^14 - 192*a*b^5*c^3*e^14 - 1024*a^3*b*c^5*e^14 + 768*a^2*
b^3*c^4*e^14))/(2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)))*(((-(4*a*c - b^2)^9)^(1/
2) - b^9 + 768*a^4*b*c^4 + 96*a^2*b^5*c^2 - 512*a^3*b^3*c^3)/(32*(b^12*c*e^
2 + 4096*a^6*c^7*e^2 - 24*a*b^10*c^2*e^2 + 240*a^2*b^8*c^3*e^2 - 1280*a^3*b
^6*c^4*e^2 + 3840*a^4*b^4*c^5*e^2 - 6144*a^5*b^2*c^6*e^2)))^(1/2))*(((-(4*a
*c - b^2)^9)^(1/2) - b^9 + 768*a^4*b*c^4 + 96*a^2*b^5*c^2 - 512*a^3*b^3*c^3
)/(32*(b^12*c*e^2 + 4096*a^6*c^7*e^2 - 24*a*b^10*c^2*e^2 + 240*a^2*b^8*c^3*
e^2 - 1280*a^3*b^6*c^4*e^2 + 3840*a^4*b^4*c^5*e^2 - 6144*a^5*b^2*c^6*e^2)))
^(1/2) + (128*a^3*c^4*d*e^11 - 4*b^6*c*d*e^11 + 8*a*b^4*c^2*d*e^11)/(8*(b^6
 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) - (x*(b^4*c*e^12 + 8*a^2*c^3*
e^12 + 2*a*b^2*c^2*e^12))/(2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c))) - (4*a^2*b*c^
2*e^10 + 3*a*b^3*c*e^10)/(4*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c
))))*(((-(4*a*c - b^2)^9)^(1/2) - b^9 + 768*a^4*b*c^4 + 96*a^2*b^5*c^2 - 51
2*a^3*b^3*c^3)/(32*(b^12*c*e^2 + 4096*a^6*c^7*e^2 - 24*a*b^10*c^2*e^2 + 240
*a^2*b^8*c^3*e^2 - 1280*a^3*b^6*c^4*e^2 + 3840*a^4*b^4*c^5*e^2 - 6144*a^5*b
^2*c^6*e^2)))^(1/2)*2i
```

sympy [B] time = 8.97, size = 573, normalized size = 2.12

$$\frac{-2ad - bd^3 - 3bde^2x^2 - be^3x^3 + x\left(-2ae - 3bd^2e\right)}{8a^2ce - 2ab^2e + 8abcd^2e + 8ac^2d^4e - 2b^3d^2e - 2b^2cd^4e + x^4\left(8ac^2e^5 - 2b^2ce^5\right) + x^3\left(32ac^2de^4 - 8b^2cde^4\right) + x^2\left(8abce^3 + 48ac^2d^2e^3 - 2b^3e^3 - 12b^2cd^2e^3\right) + x\left(16abcde^2 + 32ac^2d^3e^2 - 4b^3de^2 - 8b^2cd^3e^2\right)} + \operatorname{RootSum}\left(t^4\left(1048576a^6c^7e^4 - 1572864a^5b^2c^6e^4 + 983040a^4b^4c^5e^4 - 327680a^3b^6c^4e^4 + 61440a^2b^8c^3e^4 - 6144ab^{10}c^2e^4 + 256b^{12}ce^4\right) + t^2\left(-12288a^4bc^4e^2 + 8192a^3b^3c^3e^2 - 1536a^2b^5c^2e^2 + 16b^9e^2\right) + 16a^3c^2 + 24a^2b^2c + 9ab^4, \left(t \mapsto t\log\left(x + \frac{16384t^3a^3bc^4e^3 - 12288t^3a^2b^3c^3e^3 + 3072t^3ab^5c^2e^3 - 256t^3b^7ce^3 + 64ta^2c^2e - 128tab^2ce - 4tb^4e + 4acd + 3b^2d}{4ace + 3b^2e}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**4/(a+b*(e*x+d)**2+c*(e*x+d)**4)**2,x)
```

```
[Out] (-2*a*d - b*d**3 - 3*b*d*e**2*x**2 - b*e**3*x**3 + x*(-2*a*e - 3*b*d**2*e))
/(8*a**2*c*e - 2*a*b**2*e + 8*a*b*c*d**2*e + 8*a*c**2*d**4*e - 2*b**3*d**2*
e - 2*b**2*c*d**4*e + x**4*(8*a*c**2*e**5 - 2*b**2*c*e**5) + x**3*(32*a*c**
2*d*e**4 - 8*b**2*c*d*e**4) + x**2*(8*a*b*c*e**3 + 48*a*c**2*d**2*e**3 - 2*
b**3*e**3 - 12*b**2*c*d**2*e**3) + x*(16*a*b*c*d*e**2 + 32*a*c**2*d**3*e**2
 - 4*b**3*d*e**2 - 8*b**2*c*d**3*e**2)) + RootSum(_t**4*(1048576*a**6*c**7*
e**4 - 1572864*a**5*b**2*c**6*e**4 + 983040*a**4*b**4*c**5*e**4 - 327680*a*
```

```
*3*b**6*c**4*e**4 + 61440*a**2*b**8*c**3*e**4 - 6144*a*b**10*c**2*e**4 + 25
6*b**12*c*e**4) + _t**2*(-12288*a**4*b*c**4*e**2 + 8192*a**3*b**3*c**3*e**2
 - 1536*a**2*b**5*c**2*e**2 + 16*b**9*e**2) + 16*a**3*c**2 + 24*a**2*b**2*c
 + 9*a*b**4, Lambda(_t, _t*log(x + (16384*_t**3*a**3*b*c**4*e**3 - 12288*_t
**3*a**2*b**3*c**3*e**3 + 3072*_t**3*a*b**5*c**2*e**3 - 256*_t**3*b**7*c*e*
*3 + 64*_t*a**2*c**2*e - 128*_t*a*b**2*c*e - 4*_t*b**4*e + 4*a*c*d + 3*b**2
*d)/(4*a*c*e + 3*b**2*e))))
```

3.509 $\int \frac{(d+ex)^3}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^2} dx$

Optimal. Leaf size=97

$$\frac{2a+b(d+ex)^2}{2e\left(b^2-4ac\right)\left(a+b(d+ex)^2+c(d+ex)^4\right)}-\frac{b\tanh^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{e\left(b^2-4ac\right)^{3/2}}$$

Rubi [A] time = 0.14, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1142, 1114, 638, 618, 206}

$$\frac{2a+b(d+ex)^2}{2e\left(b^2-4ac\right)\left(a+b(d+ex)^2+c(d+ex)^4\right)}-\frac{b\tanh^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{e\left(b^2-4ac\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^3/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2,x]

[Out] (2*a + b*(d + e*x)^2)/(2*(b^2 - 4*a*c)*e*(a + b*(d + e*x)^2 + c*(d + e*x)^4
)) - (b*ArcTanh[(b + 2*c*(d + e*x)^2)/Sqrt[b^2 - 4*a*c]])/((b^2 - 4*a*c)^(3
/2)*e)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 638

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)*(a + b*x + c*x^2)^(p + 1))/((p +
 1)*(b^2 - 4*a*c)), x] - Dist[((2*p + 3)*(2*c*d - b*e))/((p + 1)*(b^2 - 4*a
*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &&
 NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 1142

```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Di
st[u^m/(Coefficient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p,
 x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

Rubi steps

$$\int \frac{(d+ex)^3}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{x^3}{\left(a+bx^2+cx^4\right)^2}\,dx, x, d+ex\right)}{e}$$

$$= \frac{\text{Subst}\left(\int \frac{x}{\left(a+bx+cx^2\right)^2}\,dx, x, (d+ex)^2\right)}{2e}$$

$$= \frac{2a+b(d+ex)^2}{2\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)} + \frac{b\,\text{Subst}\left(\int \frac{1}{a+bx+cx^2}\,dx, x, (d\right.}{2\left(b^2-4ac\right)e}$$

$$= \frac{2a+b(d+ex)^2}{2\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)} - \frac{b\,\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx, x, b\right.}{\left(b^2-4ac\right)e}$$

$$= \frac{2a+b(d+ex)^2}{2\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)} - \frac{b\tanh^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{3/2}e}$$

Mathematica [A] time = 0.13, size = 100, normalized size = 1.03

$$\frac{\frac{2a+b(d+ex)^2}{\left(b^2-4ac\right)\left(a+(d+ex)^2\left(b+c(d+ex)^2\right)\right)} - \frac{2b\tan^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{4ac-b^2}}\right)}{\left(4ac-b^2\right)^{3/2}}}{2e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^3/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2,x]
```

```
[Out] ((2*a + b*(d + e*x)^2)/((b^2 - 4*a*c)*(a + (d + e*x)^2*(b + c*(d + e*x)^2))
) - (2*b*ArcTan[(b + 2*c*(d + e*x)^2)/Sqrt[-b^2 + 4*a*c]])/(-b^2 + 4*a*c)^(
3/2))/(2*e)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(d+ex)^3}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(d + e*x)^3/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2,x]
```

```
[Out] IntegrateAlgebraic[(d + e*x)^3/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2, x]
```

fricas [B] time = 1.03, size = 1021, normalized size = 10.53

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="fricas")
```

```
[Out] [1/2*((b^3 - 4*a*b*c)*e^2*x^2 + 2*(b^3 - 4*a*b*c)*d*e*x + 2*a*b^2 - 8*a^2*c
 + (b^3 - 4*a*b*c)*d^2 - (b*c*e^4*x^4 + 4*b*c*d*e^3*x^3 + b*c*d^4 + (6*b*c*
d^2 + b^2)*e^2*x^2 + b^2*d^2 + 2*(2*b*c*d^3 + b^2*d)*e*x + a*b)*sqrt(b^2 -
4*a*c)*log((2*c^2*e^4*x^4 + 8*c^2*d*e^3*x^3 + 2*c^2*d^4 + 2*(6*c^2*d^2 + b*
c)*e^2*x^2 + 2*b*c*d^2 + 4*(2*c^2*d^3 + b*c*d)*e*x + b^2 - 2*a*c + (2*c*e^2
```

```
*x^2 + 4*c*d*e*x + 2*c*d^2 + b)*sqrt(b^2 - 4*a*c))/(c*e^4*x^4 + 4*c*d*e^3*x
^3 + c*d^4 + (6*c*d^2 + b)*e^2*x^2 + b*d^2 + 2*(2*c*d^3 + b*d)*e*x + a)))/(
(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*e^5*x^4 + 4*(b^4*c - 8*a*b^2*c^2 + 16*a^
2*c^3)*d*e^4*x^3 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2 + 6*(b^4*c - 8*a*b^2*c^2
 + 16*a^2*c^3)*d^2)*e^3*x^2 + 2*(2*(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*d^3 +
 (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*d)*e^2*x + (a*b^4 - 8*a^2*b^2*c + 16*a^3*
c^2 + (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*d^4 + (b^5 - 8*a*b^3*c + 16*a^2*b*
c^2)*d^2)*e), 1/2*((b^3 - 4*a*b*c)*e^2*x^2 + 2*(b^3 - 4*a*b*c)*d*e*x + 2*a*
b^2 - 8*a^2*c + (b^3 - 4*a*b*c)*d^2 - 2*(b*c*e^4*x^4 + 4*b*c*d*e^3*x^3 + b*
c*d^4 + (6*b*c*d^2 + b^2)*e^2*x^2 + b^2*d^2 + 2*(2*b*c*d^3 + b^2*d)*e*x + a
*b)*sqrt(-b^2 + 4*a*c)*arctan(-(2*c*e^2*x^2 + 4*c*d*e*x + 2*c*d^2 + b)*sqrt
(-b^2 + 4*a*c)/(b^2 - 4*a*c)))/((b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*e^5*x^4
+ 4*(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*d*e^4*x^3 + (b^5 - 8*a*b^3*c + 16*a^
2*b*c^2 + 6*(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*d^2)*e^3*x^2 + 2*(2*(b^4*c -
 8*a*b^2*c^2 + 16*a^2*c^3)*d^3 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*d)*e^2*x
+ (a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2 + (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*d^
4 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*d^2)*e)]
```

giac [A] time = 0.57, size = 171, normalized size = 1.76

$$\frac{b\arctan\left(\frac{2\,cd^2+2\left(x^2e+2\,dx\right)ce+b}{\sqrt{-b^2+4\,ac}}\right)e^{(-1)}}{\left(b^2-4\,ac\right)\sqrt{-b^2+4\,ac}}+\frac{bd^2+\left(x^2e+2\,dx\right)be+2\,a}{2\left(cd^4+2\left(x^2e+2\,dx\right)cd^2e+\left(x^2e+2\,dx\right)^2ce^2+bd^2+\left(x^2e+2\,dx\right)be+a\right)\left(b^2e-4\,ace\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="giac")
```

```
[Out] b*arctan((2*c*d^2 + 2*(x^2*e + 2*d*x)*c*e + b)/sqrt(-b^2 + 4*a*c))*e^(-1)/(
(b^2 - 4*a*c)*sqrt(-b^2 + 4*a*c)) + 1/2*(b*d^2 + (x^2*e + 2*d*x)*b*e + 2*a)
/((c*d^4 + 2*(x^2*e + 2*d*x)*c*d^2*e + (x^2*e + 2*d*x)^2*c*e^2 + b*d^2 + (x
^2*e + 2*d*x)*b*e + a)*(b^2*e - 4*a*c*e))
```

maple [C] time = 0.02, size = 276, normalized size = 2.85

$$\frac{b\left(-\mathrm{RootOf}\left(\_Z^4ce^4+4\_Z^3cde^3+cd^4+bd^2+\left(6cd^2e^2+be^2\right)\_Z^2+\left(4cd^3e+2deb\right)\_Z+a\right)e-d\right)\ln\left(-\mathrm{RootOf}\left(\_Z^4ce^4+4\_Z^3cde^3+cd^4+bd^2+\left(6cd^2e^2+be^2\right)\_Z^2+\left(4cd^3e+2deb\right)\_Z+a\right)+x\right)}{2\left(4ac-b^2\right)e\left(2ce^3\mathrm{RootOf}\left(\_Z^4ce^4+4\_Z^3cde^3+cd^4+bd^2+\left(6cd^2e^2+be^2\right)\_Z^2+\left(4cd^3e+2deb\right)\_Z+a\right)^3+6cde^2\mathrm{RootOf}\left(\_Z^4ce^4+4\_Z^3cde^3+cd^4+bd^2+\left(6cd^2e^2+be^2\right)\_Z^2+\left(4cd^3e+2deb\right)\_Z+a\right)^2+6ecd^2\mathrm{RootOf}\left(\_Z^4ce^4+4\_Z^3cde^3+cd^4+bd^2+\left(6cd^2e^2+be^2\right)\_Z^2+\left(4cd^3e+2deb\right)\_Z+a\right)+2cd^3+be\,\mathrm{RootOf}\left(\_Z^4ce^4+4\_Z^3cde^3+cd^4+bd^2+\left(6cd^2e^2+be^2\right)\_Z^2+\left(4cd^3e+2deb\right)\_Z+a\right)+bd\right)}+\frac{-\frac{bex^2}{2\left(4ac-b^2\right)}-\frac{bdx}{4ac-b^2}-\frac{bd^2+2a}{2\left(4ac-b^2\right)e}}{ce^4x^4+4cde^3x^3+6cd^2e^2x^2+4cd^3ex+be^2x^2+cd^4+2bdex+bd^2+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x)
```

```
[Out] (-1/2*b*e/(4*a*c-b^2)*x^2-b*d/(4*a*c-b^2)*x-1/2/e*(b*d^2+2*a)/(4*a*c-b^2))/
(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*
e*x+b*d^2+a)+1/2/(4*a*c-b^2)*b/e*sum((-_R*e-d)/(2*_R^3*c*e^3+6*_R^2*c*d*e^2
+6*_R*c*d^2*e+2*c*d^3+_R*b*e+b*d)*ln(-_R+x),_R=RootOf(_Z^4*c*e^4+4*_Z^3*c*d
*e^3+c*d^4+b*d^2+(6*c*d^2*e^2+b*e^2)*_Z^2+(4*c*d^3*e+2*b*d*e)*_Z+a))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-b\int-\frac{ex+d}{\left(b^2c-4\,ac^2\right)e^4x^4+4\left(b^2c-4\,ac^2\right)de^3x^3+\left(b^2c-4\,ac^2\right)d^4+\left(b^3-4\,abc+6\left(b^2c-4\,ac^2\right)d^2\right)e^2x^2+ab^2-4\,a^2c+\left(b^3-4\,abc\right)d^2+2\left(2\left(b^2c-4\,ac^2\right)d^3+\left(b^3-4\,abc\right)d\right)ex}\,dx+\frac{be^2x^2+2\,bdex+bd^2+2\,a}{2\left(\left(b^2c-4\,ac^2\right)e^5x^4+4\left(b^2c-4\,ac^2\right)de^4x^3+\left(b^3-4\,abc+6\left(b^2c-4\,ac^2\right)d^2\right)e^3x^2+2\left(2\left(b^2c-4\,ac^2\right)d^3+\left(b^3-4\,abc\right)d\right)e^2x+\left(\left(b^2c-4\,ac^2\right)d^4+ab^2-4\,a^2c+\left(b^3-4\,abc\right)d^2\right)e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="maxima")
```

```
[Out] -b*integrate(-(e*x + d)/((b^2*c - 4*a*c^2)*e^4*x^4 + 4*(b^2*c - 4*a*c^2)*d*
e^3*x^3 + (b^2*c - 4*a*c^2)*d^4 + (b^3 - 4*a*b*c + 6*(b^2*c - 4*a*c^2)*d^2)
*e^2*x^2 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*d^2 + 2*(2*(b^2*c - 4*a*c^2)*d
^3 + (b^3 - 4*a*b*c)*d)*e*x), x) + 1/2*(b*e^2*x^2 + 2*b*d*e*x + b*d^2 + 2*a
)/((b^2*c - 4*a*c^2)*e^5*x^4 + 4*(b^2*c - 4*a*c^2)*d*e^4*x^3 + (b^3 - 4*a*b
*c + 6*(b^2*c - 4*a*c^2)*d^2)*e^3*x^2 + 2*(2*(b^2*c - 4*a*c^2)*d^3 + (b^3 -
 4*a*b*c)*d)*e^2*x + ((b^2*c - 4*a*c^2)*d^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*
b*c)*d^2)*e)
```

mupad [B] time = 1.77, size = 427, normalized size = 4.40

$$\frac{b\operatorname{atan}\left(\frac{(4ac-b^2)^4\left(x\left(\frac{b^3(2b^3c^2de^9-8abc^3de^9)}{ae^2(4ac-b^2)^{11/2}}-\frac{2b^2c^2de^7}{a(4ac-b^2)^{7/2}}\right)+x^2\left(\frac{b^3(2b^3c^2e^{10}-8abc^3e^{10})}{2ae^2(4ac-b^2)^{11/2}}-\frac{b^2c^2e^8}{a(4ac-b^2)^{7/2}}\right)-\frac{b^3(16a^2c^3e^8-4ab^2c^2e^8+8abc^3d^2e^8-2b^3c^2d^2e^8)}{2ae^2(4ac-b^2)^{11/2}}-\frac{b^2c^2d^2e^6}{a(4ac-b^2)^{7/2}}\right)}{2b^2c^2e^6}\right)}{e\left(4ac-b^2\right)^{3/2}}-\frac{\frac{bd^2+2a}{2e(4ac-b^2)}+\frac{bex^2}{2(4ac-b^2)}+\frac{bdx}{4ac-b^2}}{a+x^2\left(6cd^2e^2+be^2\right)+bd^2+cd^4+x\left(4ced^3+2bed\right)+ce^4x^4+4cde^3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^3/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2,x)
```

```
[Out] (b*atan(((4*a*c - b^2)^4*(x*((b^3*(2*b^3*c^2*d*e^9 - 8*a*b*c^3*d*e^9))/(a*e
^2*(4*a*c - b^2)^(11/2)) - (2*b^2*c^2*d*e^7)/(a*(4*a*c - b^2)^(7/2))) + x^2
*((b^3*(2*b^3*c^2*e^10 - 8*a*b*c^3*e^10))/(2*a*e^2*(4*a*c - b^2)^(11/2)) -
(b^2*c^2*e^8)/(a*(4*a*c - b^2)^(7/2))) - (b^3*(16*a^2*c^3*e^8 - 4*a*b^2*c^2
*e^8 - 2*b^3*c^2*d^2*e^8 + 8*a*b*c^3*d^2*e^8))/(2*a*e^2*(4*a*c - b^2)^(11/2
)) - (b^2*c^2*d^2*e^6)/(a*(4*a*c - b^2)^(7/2))))/(2*b^2*c^2*e^6)))/(e*(4*a*
c - b^2)^(3/2)) - ((2*a + b*d^2)/(2*e*(4*a*c - b^2)) + (b*e*x^2)/(2*(4*a*c
- b^2)) + (b*d*x)/(4*a*c - b^2))/(a + x^2*(b*e^2 + 6*c*d^2*e^2) + b*d^2 + c
*d^4 + x*(2*b*d*e + 4*c*d^3*e) + c*e^4*x^4 + 4*c*d*e^3*x^3)
```

sympy [B] time = 4.88, size = 495, normalized size = 5.10

$$\frac{b\sqrt{-\frac{1}{(4ac-b^2)^3}}\log\left(\frac{2dx}{e}+x^2+\frac{-16a^2bc^2\sqrt{-\frac{1}{(4ac-b^2)^3}}+8ab^3c\sqrt{-\frac{1}{(4ac-b^2)^3}}-b^5\sqrt{-\frac{1}{(4ac-b^2)^3}}+b^2+2bcd^2}{2bce^2}\right)}{2e}-\frac{b\sqrt{-\frac{1}{(4ac-b^2)^3}}\log\left(\frac{2dx}{e}+x^2+\frac{16a^2bc^2\sqrt{-\frac{1}{(4ac-b^2)^3}}-8ab^3c\sqrt{-\frac{1}{(4ac-b^2)^3}}+b^5\sqrt{-\frac{1}{(4ac-b^2)^3}}+b^2+2bcd^2}{2bce^2}\right)}{2e}+\frac{-2a-bd^2-2bdex-be^2x^2}{8a^2ce-2ab^2e+8abcd^2e+8ac^2d^4e-2b^3d^2e-2b^2cd^4e+x^4\left(8ac^2e^5-2b^2ce^5\right)+x^3\left(32ac^2de^4-8b^2cde^4\right)+x^2\left(8abce^3+48ac^2d^2e^3-2b^3e^3-12b^2cd^2e^3\right)+x\left(16abcde^2+32ac^2d^3e^2-4b^3de^2-8b^2cd^3e^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3/(a+b*(e*x+d)**2+c*(e*x+d)**4)**2,x)
```

```
[Out] b*sqrt(-1/(4*a*c - b**2)**3)*log(2*d*x/e + x**2 + (-16*a**2*b*c**2*sqrt(-1/
(4*a*c - b**2)**3) + 8*a*b**3*c*sqrt(-1/(4*a*c - b**2)**3) - b**5*sqrt(-1/(
4*a*c - b**2)**3) + b**2 + 2*b*c*d**2)/(2*b*c*e**2))/(2*e) - b*sqrt(-1/(4*a
*c - b**2)**3)*log(2*d*x/e + x**2 + (16*a**2*b*c**2*sqrt(-1/(4*a*c - b**2)*
*3) - 8*a*b**3*c*sqrt(-1/(4*a*c - b**2)**3) + b**5*sqrt(-1/(4*a*c - b**2)**
3) + b**2 + 2*b*c*d**2)/(2*b*c*e**2))/(2*e) + (-2*a - b*d**2 - 2*b*d*e*x -
b*e**2*x**2)/(8*a**2*c*e - 2*a*b**2*e + 8*a*b*c*d**2*e + 8*a*c**2*d**4*e -
2*b**3*d**2*e - 2*b**2*c*d**4*e + x**4*(8*a*c**2*e**5 - 2*b**2*c*e**5) + x*
*3*(32*a*c**2*d*e**4 - 8*b**2*c*d*e**4) + x**2*(8*a*b*c*e**3 + 48*a*c**2*d*
*2*e**3 - 2*b**3*e**3 - 12*b**2*c*d**2*e**3) + x*(16*a*b*c*d*e**2 + 32*a*c*
*2*d**3*e**2 - 4*b**3*d*e**2 - 8*b**2*c*d**3*e**2))
```

3.510 $\int \frac{(d+ex)^2}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^2} dx$

Optimal. Leaf size=254

$$-\frac{(d+ex)\left(b+2c(d+ex)^2\right)}{2e\left(b^2-4ac\right)\left(a+b(d+ex)^2+c(d+ex)^4\right)}+\frac{\sqrt{c}\left(2b-\sqrt{b^2-4ac}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}e\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{c}\left(\sqrt{b^2-4ac}+2b\right.}{\sqrt{2}e\left(b^2-4ac\right)^{3/2}}$$

Rubi [A] time = 0.39, antiderivative size = 254, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {1142, 1119, 1166, 205}

$$-\frac{(d+ex)\left(b+2c(d+ex)^2\right)}{2e\left(b^2-4ac\right)\left(a+b(d+ex)^2+c(d+ex)^4\right)}+\frac{\sqrt{c}\left(2b-\sqrt{b^2-4ac}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}e\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{c}\left(\sqrt{b^2-4ac}+2b\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}e\left(b^2-4ac\right)^{3/2}\sqrt{\sqrt{b^2-4ac}+b}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^2/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2,x]
```

```
[Out] -((d + e*x)*(b + 2*c*(d + e*x)^2))/(2*(b^2 - 4*a*c)*e*(a + b*(d + e*x)^2 +
c*(d + e*x)^4)) + (Sqrt[c]*(2*b - Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c
]*(d + e*x))/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*(b^2 - 4*a*c)^(3/2)*Sqr
t[b - Sqrt[b^2 - 4*a*c]]*e) - (Sqrt[c]*(2*b + Sqrt[b^2 - 4*a*c])*ArcTan[(Sq
rt[2]*Sqrt[c]*(d + e*x))/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*(b^2 - 4*a*
c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]*e)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1119

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Simp[(d*(d*x)^(m - 1)*(b + 2*c*x^2)*(a + b*x^2 + c*x^4)^(p + 1))/(2*(p
+ 1)*(b^2 - 4*a*c)), x] - Dist[d^2/(2*(p + 1)*(b^2 - 4*a*c)), Int[(d*x)^(m
- 2)*(b*(m - 1) + 2*c*(m + 4*p + 5)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x
] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && GtQ[m,
1] && LeQ[m, 3] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1142

```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Di
st[u^m/(Coefficient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p,
 x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{(d+ex)^2}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{x^2}{\left(a+bx^2+cx^4\right)^2}\,dx, x, d+ex\right)}{e}$$

$$= -\frac{(d+ex)\left(b+2c(d+ex)^2\right)}{2\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)} + \frac{\text{Subst}\left(\int \frac{b-2cx^2}{a+bx^2+cx^4}\,dx, x, d\right.}{2\left(b^2-4ac\right)e}$$

$$= -\frac{(d+ex)\left(b+2c(d+ex)^2\right)}{2\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)} + \frac{\left(c\left(2b-\sqrt{b^2-4ac}\right)\right)\text{Subs}}{2\left(b^2\right.}$$

$$= -\frac{(d+ex)\left(b+2c(d+ex)^2\right)}{2\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)} + \frac{\sqrt{c}\left(2b-\sqrt{b^2-4ac}\right)\tan^{-}}{\sqrt{2}\left(b^2-4ac\right)^{3/2}\sqrt{b-}}$$

Mathematica [**A**] time = 0.98, size = 247, normalized size = 0.97

$$-\frac{\frac{b(d+ex)+2c(d+ex)^3}{\left(b^2-4ac\right)\left(a+b(d+ex)^2+c(d+ex)^4\right)} + \frac{\sqrt{2}\sqrt{c}\left(\sqrt{b^2-4ac}-2b\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}} + \frac{\sqrt{2}\sqrt{c}\left(\sqrt{b^2-4ac}+2b\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\left(b^2-4ac\right)^{3/2}\sqrt{\sqrt{b^2-4ac}+b}}}{2e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^2/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2,x]
```

```
[Out] -1/2*((b*(d + e*x) + 2*c*(d + e*x)^3)/((b^2 - 4*a*c)*(a + b*(d + e*x)^2 + c
*(d + e*x)^4)) + (Sqrt[2]*Sqrt[c]*(-2*b + Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2
]*Sqrt[c]*(d + e*x))/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(3/2)*Sqr
t[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[2]*Sqrt[c]*(2*b + Sqrt[b^2 - 4*a*c])*ArcT
an[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)
^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/e
```

IntegrateAlgebraic [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(d+ex)^2}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(d + e*x)^2/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2,x]
```

```
[Out] IntegrateAlgebraic[(d + e*x)^2/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2, x]
```

fricas [**B**] time = 1.32, size = 2474, normalized size = 9.74

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="fricas")
```

```
[Out] -1/4*(4*c*e^3*x^3 + 12*c*d*e^2*x^2 + 4*c*d^3 + 2*(6*c*d^2 + b)*e*x - sqrt(1
/2)*((b^2*c - 4*a*c^2)*e^5*x^4 + 4*(b^2*c - 4*a*c^2)*d*e^4*x^3 + (b^3 - 4*a
*b*c + 6*(b^2*c - 4*a*c^2)*d^2)*e^3*x^2 + 2*(2*(b^2*c - 4*a*c^2)*d^3 + (b^3
 - 4*a*b*c)*d)*e^2*x + ((b^2*c - 4*a*c^2)*d^4 + a*b^2 - 4*a^2*c + (b^3 - 4*
```

```
a*b*c)*d^2)*e)*sqrt(-((a*b^6 - 12*a^2*b^4*c + 48*a^3*b^2*c^2 - 64*a^4*c^3)*
e^2*sqrt(1/((a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3)*e^4)) +
b^3 + 12*a*b*c)/((a*b^6 - 12*a^2*b^4*c + 48*a^3*b^2*c^2 - 64*a^4*c^3)*e^2))
*log((3*b^2*c + 4*a*c^2)*e*x + (3*b^2*c + 4*a*c^2)*d + 1/2*sqrt(1/2)*((a*b^
8 - 8*a^2*b^6*c + 128*a^4*b^2*c^3 - 256*a^5*c^4)*e^3*sqrt(1/((a^2*b^6 - 12*
a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3)*e^4)) - (b^5 - 8*a*b^3*c + 16*a^2*
b*c^2)*e)*sqrt(-((a*b^6 - 12*a^2*b^4*c + 48*a^3*b^2*c^2 - 64*a^4*c^3)*e^2*s
qrt(1/((a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3)*e^4)) + b^3 +
 12*a*b*c)/((a*b^6 - 12*a^2*b^4*c + 48*a^3*b^2*c^2 - 64*a^4*c^3)*e^2))) + s
qrt(1/2)*((b^2*c - 4*a*c^2)*e^5*x^4 + 4*(b^2*c - 4*a*c^2)*d*e^4*x^3 + (b^3
- 4*a*b*c + 6*(b^2*c - 4*a*c^2)*d^2)*e^3*x^2 + 2*(2*(b^2*c - 4*a*c^2)*d^3 +
 (b^3 - 4*a*b*c)*d)*e^2*x + ((b^2*c - 4*a*c^2)*d^4 + a*b^2 - 4*a^2*c + (b^3
 - 4*a*b*c)*d^2)*e)*sqrt(-((a*b^6 - 12*a^2*b^4*c + 48*a^3*b^2*c^2 - 64*a^4*
c^3)*e^2*sqrt(1/((a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3)*e^4
)) + b^3 + 12*a*b*c)/((a*b^6 - 12*a^2*b^4*c + 48*a^3*b^2*c^2 - 64*a^4*c^3)*
e^2))*log((3*b^2*c + 4*a*c^2)*e*x + (3*b^2*c + 4*a*c^2)*d - 1/2*sqrt(1/2)*(
(a*b^8 - 8*a^2*b^6*c + 128*a^4*b^2*c^3 - 256*a^5*c^4)*e^3*sqrt(1/((a^2*b^6
- 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3)*e^4)) - (b^5 - 8*a*b^3*c + 16
*a^2*b*c^2)*e)*sqrt(-((a*b^6 - 12*a^2*b^4*c + 48*a^3*b^2*c^2 - 64*a^4*c^3)*
e^2*sqrt(1/((a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3)*e^4)) +
b^3 + 12*a*b*c)/((a*b^6 - 12*a^2*b^4*c + 48*a^3*b^2*c^2 - 64*a^4*c^3)*e^2))
) + sqrt(1/2)*((b^2*c - 4*a*c^2)*e^5*x^4 + 4*(b^2*c - 4*a*c^2)*d*e^4*x^3 +
(b^3 - 4*a*b*c + 6*(b^2*c - 4*a*c^2)*d^2)*e^3*x^2 + 2*(2*(b^2*c - 4*a*c^2)*
d^3 + (b^3 - 4*a*b*c)*d)*e^2*x + ((b^2*c - 4*a*c^2)*d^4 + a*b^2 - 4*a^2*c +
 (b^3 - 4*a*b*c)*d^2)*e)*sqrt(((a*b^6 - 12*a^2*b^4*c + 48*a^3*b^2*c^2 - 64*
a^4*c^3)*e^2*sqrt(1/((a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3)
*e^4)) - b^3 - 12*a*b*c)/((a*b^6 - 12*a^2*b^4*c + 48*a^3*b^2*c^2 - 64*a^4*c
^3)*e^2))*log((3*b^2*c + 4*a*c^2)*e*x + (3*b^2*c + 4*a*c^2)*d + 1/2*sqrt(1/
2)*((a*b^8 - 8*a^2*b^6*c + 128*a^4*b^2*c^3 - 256*a^5*c^4)*e^3*sqrt(1/((a^2*
b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3)*e^4)) + (b^5 - 8*a*b^3*c
+ 16*a^2*b*c^2)*e)*sqrt(((a*b^6 - 12*a^2*b^4*c + 48*a^3*b^2*c^2 - 64*a^4*c^
3)*e^2*sqrt(1/((a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3)*e^4))
 - b^3 - 12*a*b*c)/((a*b^6 - 12*a^2*b^4*c + 48*a^3*b^2*c^2 - 64*a^4*c^3)*e^
2))) - sqrt(1/2)*((b^2*c - 4*a*c^2)*e^5*x^4 + 4*(b^2*c - 4*a*c^2)*d*e^4*x^3
 + (b^3 - 4*a*b*c + 6*(b^2*c - 4*a*c^2)*d^2)*e^3*x^2 + 2*(2*(b^2*c - 4*a*c^
2)*d^3 + (b^3 - 4*a*b*c)*d)*e^2*x + ((b^2*c - 4*a*c^2)*d^4 + a*b^2 - 4*a^2*
c + (b^3 - 4*a*b*c)*d^2)*e)*sqrt(((a*b^6 - 12*a^2*b^4*c + 48*a^3*b^2*c^2 -
64*a^4*c^3)*e^2*sqrt(1/((a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c
^3)*e^4)) - b^3 - 12*a*b*c)/((a*b^6 - 12*a^2*b^4*c + 48*a^3*b^2*c^2 - 64*a^
4*c^3)*e^2))*log((3*b^2*c + 4*a*c^2)*e*x + (3*b^2*c + 4*a*c^2)*d - 1/2*sqrt
(1/2)*((a*b^8 - 8*a^2*b^6*c + 128*a^4*b^2*c^3 - 256*a^5*c^4)*e^3*sqrt(1/((a
^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3)*e^4)) + (b^5 - 8*a*b^3
*c + 16*a^2*b*c^2)*e)*sqrt(((a*b^6 - 12*a^2*b^4*c + 48*a^3*b^2*c^2 - 64*a^4
*c^3)*e^2*sqrt(1/((a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3)*e^
4)) - b^3 - 12*a*b*c)/((a*b^6 - 12*a^2*b^4*c + 48*a^3*b^2*c^2 - 64*a^4*c^3)
*e^2))) + 2*b*d)/((b^2*c - 4*a*c^2)*e^5*x^4 + 4*(b^2*c - 4*a*c^2)*d*e^4*x^3
 + (b^3 - 4*a*b*c + 6*(b^2*c - 4*a*c^2)*d^2)*e^3*x^2 + 2*(2*(b^2*c - 4*a*c^
2)*d^3 + (b^3 - 4*a*b*c)*d)*e^2*x + ((b^2*c - 4*a*c^2)*d^4 + a*b^2 - 4*a^2*
c + (b^3 - 4*a*b*c)*d^2)*e)
```

giac [B] time = 0.53, size = 1312, normalized size = 5.17

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="giac")
```

```
[Out] 1/4*((2*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/
c))^2*c*e^2 - 4*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)
*e^(-4)/c))*c*d*e + 2*c*d^2 - b)*log(d*e^(-1) + x + sqrt(1/2)*sqrt(-(b*e^2
```

```
+ sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))/(2*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 +
 sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^3*c*e^4 - 6*(d*e^(-1) + sqrt(1/2)*sqrt(-
(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^2*c*d*e^3 - 2*c*d^3*e - b*d*e +
(6*c*d^2*e^2 + b*e^2)*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c
)*e^2)*e^(-4)/c))) + (2*(d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a
*c)*e^2)*e^(-4)/c))^2*c*e^2 - 4*(d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b
^2 - 4*a*c)*e^2)*e^(-4)/c))*c*d*e + 2*c*d^2 - b)*log(d*e^(-1) + x - sqrt(1/
2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))/(2*(d*e^(-1) - sqrt(1/2
)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^3*c*e^4 - 6*(d*e^(-1) -
sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^2*c*d*e^3 - 2*c*
d^3*e - b*d*e + (6*c*d^2*e^2 + b*e^2)*(d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 +
sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))) + (2*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2
- sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^2*c*e^2 - 4*(d*e^(-1) + sqrt(1/2)*sqrt(
-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))*c*d*e + 2*c*d^2 - b)*log(d*e^(-
1) + x + sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))/(2*(d*e
^(-1) + sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^3*c*e^4
- 6*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^
2*c*d*e^3 - 2*c*d^3*e - b*d*e + (6*c*d^2*e^2 + b*e^2)*(d*e^(-1) + sqrt(1/2)
*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))) + (2*(d*e^(-1) - sqrt(1/
2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^2*c*e^2 - 4*(d*e^(-1) -
 sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))*c*d*e + 2*c*d^2
 - b)*log(d*e^(-1) + x - sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^
(-4)/c))/(2*(d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(
-4)/c))^3*c*e^4 - 6*(d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*
e^2)*e^(-4)/c))^2*c*d*e^3 - 2*c*d^3*e - b*d*e + (6*c*d^2*e^2 + b*e^2)*(d*e^
(-1) - sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))))/(b^2 -
4*a*c) - 1/2*(2*c*x^3*e^3 + 6*c*d*x^2*e^2 + 6*c*d^2*x*e + 2*c*d^3 + b*x*e +
 b*d)/((c*x^4*e^4 + 4*c*d*x^3*e^3 + 6*c*d^2*x^2*e^2 + 4*c*d^3*x*e + c*d^4 +
 b*x^2*e^2 + 2*b*d*x*e + b*d^2 + a)*(b^2*e - 4*a*c*e))
```

maple [C] time = 0.02, size = 319, normalized size = 1.26

$$\frac{\left(2\,\mathrm{RootOf}\left(\_Z^4 c e^4+4\_Z^3 c d e^3+c d^4+b d^2+\left(6 c d^2 e^2+b e^2\right)\_Z^2+\left(4 c d^3 e+2 d e b\right)\_Z+a\right)^2 c e^2+4\,\mathrm{RootOf}\left(\_Z^4 c e^4+4\_Z^3 c d e^3+c d^4+b d^2+\left(6 c d^2 e^2+b e^2\right)\_Z^2+\left(4 c d^3 e+2 d e b\right)\_Z+a\right) c d e+2 c d^2-b\right)\ln\left(-\mathrm{RootOf}\left(\_Z^4 c e^4+4\_Z^3 c d e^3+c d^4+b d^2+\left(6 c d^2 e^2+b e^2\right)\_Z^2+\left(4 c d^3 e+2 d e b\right)\_Z+a\right)+x\right)}{4\left(4 a c-b^2\right) e\left(2 c e^3\,\mathrm{RootOf}\left(\_Z^4 c e^4+4\_Z^3 c d e^3+c d^4+b d^2+\left(6 c d^2 e^2+b e^2\right)\_Z^2+\left(4 c d^3 e+2 d e b\right)\_Z+a\right)^3+6 c d e^2\,\mathrm{RootOf}\left(\_Z^4 c e^4+4\_Z^3 c d e^3+c d^4+b d^2+\left(6 c d^2 e^2+b e^2\right)\_Z^2+\left(4 c d^3 e+2 d e b\right)\_Z+a\right)^2+6 e c d^2\,\mathrm{RootOf}\left(\_Z^4 c e^4+4\_Z^3 c d e^3+c d^4+b d^2+\left(6 c d^2 e^2+b e^2\right)\_Z^2+\left(4 c d^3 e+2 d e b\right)\_Z+a\right)+2 c d^3+b e\,\mathrm{RootOf}\left(\_Z^4 c e^4+4\_Z^3 c d e^3+c d^4+b d^2+\left(6 c d^2 e^2+b e^2\right)\_Z^2+\left(4 c d^3 e+2 d e b\right)\_Z+a\right)+b d\right)}+\frac{\frac{c e^2 x^3}{4 a c-b^2}+\frac{3 c d e x^2}{4 a c-b^2}+\frac{\left(6 c d^2+b\right) x}{8 a c-2 b^2}+\frac{\left(2 c d^2+b\right) d}{2\left(4 a c-b^2\right) e}}{c e^4 x^4+4 c d e^3 x^3+6 c d^2 e^2 x^2+4 c d^3 e x+b e^2 x^2+c d^4+2 b d e x+b d^2+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x)
```

```
[Out] (c*e^2/(4*a*c-b^2)*x^3+3*d*c*e/(4*a*c-b^2)*x^2+1/2*(6*c*d^2+b)/(4*a*c-b^2)*
x+1/2*d/e*(2*c*d^2+b)/(4*a*c-b^2))/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2
+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)+1/4/(4*a*c-b^2)/e*sum((2*_R
^2*c*e^2+4*_R*c*d*e+2*c*d^2-b)/(2*_R^3*c*e^3+6*_R^2*c*d*e^2+6*_R*c*d^2*e+2*
c*d^3+_R*b*e+b*d)*ln(-_R+x),_R=RootOf(_Z^4*c*e^4+4*_Z^3*c*d*e^3+c*d^4+b*d^2
+(6*c*d^2*e^2+b*e^2)*_Z^2+(4*c*d^3*e+2*b*d*e)*_Z+a))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{2\,c e^3 x^3+6\,c d e^2 x^2+2\,c d^3+\left(6\,c d^2+b\right) e x+b d}{2\left(\left(b^2 c-4\,a c^2\right) e^5 x^4+4\left(b^2 c-4\,a c^2\right) d e^4 x^3+\left(b^3-4\,a b c+6\left(b^2 c-4\,a c^2\right) d^2\right) e^3 x^2+2\left(2\left(b^2 c-4\,a c^2\right) d^3+\left(b^3-4\,a b c\right) d\right) e^2 x+\left(\left(b^2 c-4\,a c^2\right) d^4+a b^2-4\,a^2 c+\left(b^3-4\,a b c\right) d^2\right) e\right)}+\frac{1}{2}\int-\frac{2\,c e^2 x^2+4\,c d e x+2\,c d^2-b}{\left(b^2 c-4\,a c^2\right) e^4 x^4+4\left(b^2 c-4\,a c^2\right) d e^3 x^3+\left(b^2 c-4\,a c^2\right) d^4+\left(b^3-4\,a b c+6\left(b^2 c-4\,a c^2\right) d^2\right) e^2 x^2+a b^2-4\,a^2 c+\left(b^3-4\,a b c\right) d^2+2\left(2\left(b^2 c-4\,a c^2\right) d^3+\left(b^3-4\,a b c\right) d\right) e x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(2*c*e^3*x^3 + 6*c*d*e^2*x^2 + 2*c*d^3 + (6*c*d^2 + b)*e*x + b*d)/((b^
2*c - 4*a*c^2)*e^5*x^4 + 4*(b^2*c - 4*a*c^2)*d*e^4*x^3 + (b^3 - 4*a*b*c + 6
*(b^2*c - 4*a*c^2)*d^2)*e^3*x^2 + 2*(2*(b^2*c - 4*a*c^2)*d^3 + (b^3 - 4*a*b
*c)*d)*e^2*x + ((b^2*c - 4*a*c^2)*d^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*d
^2)*e) + 1/2*integrate(-(2*c*e^2*x^2 + 4*c*d*e*x + 2*c*d^2 - b)/((b^2*c - 4
*a*c^2)*e^4*x^4 + 4*(b^2*c - 4*a*c^2)*d*e^3*x^3 + (b^2*c - 4*a*c^2)*d^4 + (
b^3 - 4*a*b*c + 6*(b^2*c - 4*a*c^2)*d^2)*e^2*x^2 + a*b^2 - 4*a^2*c + (b^3 -
 4*a*b*c)*d^2 + 2*(2*(b^2*c - 4*a*c^2)*d^3 + (b^3 - 4*a*b*c)*d)*e*x), x)
```

mupad [B] time = 3.95, size = 7200, normalized size = 28.35

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^2/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2,x)
```

```
[Out] atan(((((-(4*a*c - b^2)^9)^(1/2) - b^9 + 768*a^4*b*c^4 + 96*a^2*b^5*c^2 - 5
12*a^3*b^3*c^3)/(32*(a*b^12*e^2 + 4096*a^7*c^6*e^2 - 24*a^2*b^10*c*e^2 + 24
0*a^3*b^8*c^2*e^2 - 1280*a^4*b^6*c^3*e^2 + 3840*a^5*b^4*c^4*e^2 - 6144*a^6*
b^2*c^5*e^2)))^(1/2)*((64*a^2*c^5*d*e^11 + 20*b^4*c^3*d*e^11 - 96*a*b^2*c^4
*d*e^11)/(4*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) + (((32*b^9*c
^2*d*e^13 - 512*a*b^7*c^3*d*e^13 + 8192*a^4*b*c^6*d*e^13 + 3072*a^2*b^5*c^4
*d*e^13 - 8192*a^3*b^3*c^5*d*e^13)/(4*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 -
12*a*b^4*c)) + (x*(8*b^7*c^2*e^14 - 96*a*b^5*c^3*e^14 - 512*a^3*b*c^5*e^14
+ 384*a^2*b^3*c^4*e^14))/(b^4 + 16*a^2*c^2 - 8*a*b^2*c))*(((-(4*a*c - b^2)^
9)^(1/2) - b^9 + 768*a^4*b*c^4 + 96*a^2*b^5*c^2 - 512*a^3*b^3*c^3)/(32*(a*b
^12*e^2 + 4096*a^7*c^6*e^2 - 24*a^2*b^10*c*e^2 + 240*a^3*b^8*c^2*e^2 - 1280
*a^4*b^6*c^3*e^2 + 3840*a^5*b^4*c^4*e^2 - 6144*a^6*b^2*c^5*e^2)))^(1/2) - (
8*b^7*c^2*e^12 - 96*a*b^5*c^3*e^12 - 512*a^3*b*c^5*e^12 + 384*a^2*b^3*c^4*e
^12)/(4*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)))*(((-(4*a*c - b^2
)^9)^(1/2) - b^9 + 768*a^4*b*c^4 + 96*a^2*b^5*c^2 - 512*a^3*b^3*c^3)/(32*(a
*b^12*e^2 + 4096*a^7*c^6*e^2 - 24*a^2*b^10*c*e^2 + 240*a^3*b^8*c^2*e^2 - 12
80*a^4*b^6*c^3*e^2 + 3840*a^5*b^4*c^4*e^2 - 6144*a^6*b^2*c^5*e^2)))^(1/2) -
 (x*(4*a*c^4*e^12 - 5*b^2*c^3*e^12))/(b^4 + 16*a^2*c^2 - 8*a*b^2*c))*1i + (
((-(4*a*c - b^2)^9)^(1/2) - b^9 + 768*a^4*b*c^4 + 96*a^2*b^5*c^2 - 512*a^3*
b^3*c^3)/(32*(a*b^12*e^2 + 4096*a^7*c^6*e^2 - 24*a^2*b^10*c*e^2 + 240*a^3*b
^8*c^2*e^2 - 1280*a^4*b^6*c^3*e^2 + 3840*a^5*b^4*c^4*e^2 - 6144*a^6*b^2*c^5
*e^2)))^(1/2)*((64*a^2*c^5*d*e^11 + 20*b^4*c^3*d*e^11 - 96*a*b^2*c^4*d*e^11
)/(4*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) + (((32*b^9*c^2*d*e^
13 - 512*a*b^7*c^3*d*e^13 + 8192*a^4*b*c^6*d*e^13 + 3072*a^2*b^5*c^4*d*e^13
 - 8192*a^3*b^3*c^5*d*e^13)/(4*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^
4*c)) + (x*(8*b^7*c^2*e^14 - 96*a*b^5*c^3*e^14 - 512*a^3*b*c^5*e^14 + 384*a
^2*b^3*c^4*e^14))/(b^4 + 16*a^2*c^2 - 8*a*b^2*c))*(((-(4*a*c - b^2)^9)^(1/2
) - b^9 + 768*a^4*b*c^4 + 96*a^2*b^5*c^2 - 512*a^3*b^3*c^3)/(32*(a*b^12*e^2
 + 4096*a^7*c^6*e^2 - 24*a^2*b^10*c*e^2 + 240*a^3*b^8*c^2*e^2 - 1280*a^4*b^
6*c^3*e^2 + 3840*a^5*b^4*c^4*e^2 - 6144*a^6*b^2*c^5*e^2)))^(1/2) + (8*b^7*c
^2*e^12 - 96*a*b^5*c^3*e^12 - 512*a^3*b*c^5*e^12 + 384*a^2*b^3*c^4*e^12)/(4
*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)))*(((-(4*a*c - b^2)^9)^(1
/2) - b^9 + 768*a^4*b*c^4 + 96*a^2*b^5*c^2 - 512*a^3*b^3*c^3)/(32*(a*b^12*e
^2 + 4096*a^7*c^6*e^2 - 24*a^2*b^10*c*e^2 + 240*a^3*b^8*c^2*e^2 - 1280*a^4*
b^6*c^3*e^2 + 3840*a^5*b^4*c^4*e^2 - 6144*a^6*b^2*c^5*e^2)))^(1/2) - (x*(4*
a*c^4*e^12 - 5*b^2*c^3*e^12))/(b^4 + 16*a^2*c^2 - 8*a*b^2*c))*1i)/((((-(4*a
*c - b^2)^9)^(1/2) - b^9 + 768*a^4*b*c^4 + 96*a^2*b^5*c^2 - 512*a^3*b^3*c^3
)/(32*(a*b^12*e^2 + 4096*a^7*c^6*e^2 - 24*a^2*b^10*c*e^2 + 240*a^3*b^8*c^2*
e^2 - 1280*a^4*b^6*c^3*e^2 + 3840*a^5*b^4*c^4*e^2 - 6144*a^6*b^2*c^5*e^2)))
^(1/2)*((64*a^2*c^5*d*e^11 + 20*b^4*c^3*d*e^11 - 96*a*b^2*c^4*d*e^11)/(4*(b
^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) + (((32*b^9*c^2*d*e^13 - 51
2*a*b^7*c^3*d*e^13 + 8192*a^4*b*c^6*d*e^13 + 3072*a^2*b^5*c^4*d*e^13 - 8192
*a^3*b^3*c^5*d*e^13)/(4*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) +
 (x*(8*b^7*c^2*e^14 - 96*a*b^5*c^3*e^14 - 512*a^3*b*c^5*e^14 + 384*a^2*b^3*
c^4*e^14))/(b^4 + 16*a^2*c^2 - 8*a*b^2*c))*(((-(4*a*c - b^2)^9)^(1/2) - b^9
 + 768*a^4*b*c^4 + 96*a^2*b^5*c^2 - 512*a^3*b^3*c^3)/(32*(a*b^12*e^2 + 4096
*a^7*c^6*e^2 - 24*a^2*b^10*c*e^2 + 240*a^3*b^8*c^2*e^2 - 1280*a^4*b^6*c^3*e
^2 + 3840*a^5*b^4*c^4*e^2 - 6144*a^6*b^2*c^5*e^2)))^(1/2) + (8*b^7*c^2*e^12
 - 96*a*b^5*c^3*e^12 - 512*a^3*b*c^5*e^12 + 384*a^2*b^3*c^4*e^12)/(4*(b^6 -
 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)))*(((-(4*a*c - b^2)^9)^(1/2) - b
^9 + 768*a^4*b*c^4 + 96*a^2*b^5*c^2 - 512*a^3*b^3*c^3)/(32*(a*b^12*e^2 + 40
96*a^7*c^6*e^2 - 24*a^2*b^10*c*e^2 + 240*a^3*b^8*c^2*e^2 - 1280*a^4*b^6*c^3
*e^2 + 3840*a^5*b^4*c^4*e^2 - 6144*a^6*b^2*c^5*e^2)))^(1/2) - (x*(4*a*c^4*e
```

```
^12 - 5*b^2*c^3*e^12))/(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) - (((-(4*a*c - b^2)^
9)^(1/2) - b^9 + 768*a^4*b*c^4 + 96*a^2*b^5*c^2 - 512*a^3*b^3*c^3)/(32*(a*b
^12*e^2 + 4096*a^7*c^6*e^2 - 24*a^2*b^10*c*e^2 + 240*a^3*b^8*c^2*e^2 - 1280
*a^4*b^6*c^3*e^2 + 3840*a^5*b^4*c^4*e^2 - 6144*a^6*b^2*c^5*e^2)))^(1/2)*((6
4*a^2*c^5*d*e^11 + 20*b^4*c^3*d*e^11 - 96*a*b^2*c^4*d*e^11)/(4*(b^6 - 64*a^
3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) + (((32*b^9*c^2*d*e^13 - 512*a*b^7*c^
3*d*e^13 + 8192*a^4*b*c^6*d*e^13 + 3072*a^2*b^5*c^4*d*e^13 - 8192*a^3*b^3*c
^5*d*e^13)/(4*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) + (x*(8*b^7
*c^2*e^14 - 96*a*b^5*c^3*e^14 - 512*a^3*b*c^5*e^14 + 384*a^2*b^3*c^4*e^14))
/(b^4 + 16*a^2*c^2 - 8*a*b^2*c))*(((-(4*a*c - b^2)^9)^(1/2) - b^9 + 768*a^4
*b*c^4 + 96*a^2*b^5*c^2 - 512*a^3*b^3*c^3)/(32*(a*b^12*e^2 + 4096*a^7*c^6*e
^2 - 24*a^2*b^10*c*e^2 + 240*a^3*b^8*c^2*e^2 - 1280*a^4*b^6*c^3*e^2 + 3840*
a^5*b^4*c^4*e^2 - 6144*a^6*b^2*c^5*e^2)))^(1/2) - (8*b^7*c^2*e^12 - 96*a*b^
5*c^3*e^12 - 512*a^3*b*c^5*e^12 + 384*a^2*b^3*c^4*e^12)/(4*(b^6 - 64*a^3*c^
3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)))*(((-(4*a*c - b^2)^9)^(1/2) - b^9 + 768*a
^4*b*c^4 + 96*a^2*b^5*c^2 - 512*a^3*b^3*c^3)/(32*(a*b^12*e^2 + 4096*a^7*c^6
*e^2 - 24*a^2*b^10*c*e^2 + 240*a^3*b^8*c^2*e^2 - 1280*a^4*b^6*c^3*e^2 + 384
0*a^5*b^4*c^4*e^2 - 6144*a^6*b^2*c^5*e^2)))^(1/2) - (x*(4*a*c^4*e^12 - 5*b^
2*c^3*e^12))/(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (4*a*c^4*e^10 + 3*b^2*c^3*e^
10)/(2*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c))))*(((-(4*a*c - b^2
)^9)^(1/2) - b^9 + 768*a^4*b*c^4 + 96*a^2*b^5*c^2 - 512*a^3*b^3*c^3)/(32*(a
*b^12*e^2 + 4096*a^7*c^6*e^2 - 24*a^2*b^10*c*e^2 + 240*a^3*b^8*c^2*e^2 - 12
80*a^4*b^6*c^3*e^2 + 3840*a^5*b^4*c^4*e^2 - 6144*a^6*b^2*c^5*e^2)))^(1/2)*2
i - atan(-(((((32*b^9*c^2*d*e^13 - 512*a*b^7*c^3*d*e^13 + 8192*a^4*b*c^6*d*
e^13 + 3072*a^2*b^5*c^4*d*e^13 - 8192*a^3*b^3*c^5*d*e^13)/(4*(b^6 - 64*a^3*
c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) + (x*(8*b^7*c^2*e^14 - 96*a*b^5*c^3*e^1
4 - 512*a^3*b*c^5*e^14 + 384*a^2*b^3*c^4*e^14))/(b^4 + 16*a^2*c^2 - 8*a*b^2
*c))*(-(b^9 + (-(4*a*c - b^2)^9)^(1/2) - 768*a^4*b*c^4 - 96*a^2*b^5*c^2 + 5
12*a^3*b^3*c^3)/(32*(a*b^12*e^2 + 4096*a^7*c^6*e^2 - 24*a^2*b^10*c*e^2 + 24
0*a^3*b^8*c^2*e^2 - 1280*a^4*b^6*c^3*e^2 + 3840*a^5*b^4*c^4*e^2 - 6144*a^6*
b^2*c^5*e^2)))^(1/2) - (8*b^7*c^2*e^12 - 96*a*b^5*c^3*e^12 - 512*a^3*b*c^5*
e^12 + 384*a^2*b^3*c^4*e^12)/(4*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b
^4*c)))*(-(b^9 + (-(4*a*c - b^2)^9)^(1/2) - 768*a^4*b*c^4 - 96*a^2*b^5*c^2
+ 512*a^3*b^3*c^3)/(32*(a*b^12*e^2 + 4096*a^7*c^6*e^2 - 24*a^2*b^10*c*e^2 +
 240*a^3*b^8*c^2*e^2 - 1280*a^4*b^6*c^3*e^2 + 3840*a^5*b^4*c^4*e^2 - 6144*a
^6*b^2*c^5*e^2)))^(1/2) + (64*a^2*c^5*d*e^11 + 20*b^4*c^3*d*e^11 - 96*a*b^2
*c^4*d*e^11)/(4*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) - (x*(4*a
*c^4*e^12 - 5*b^2*c^3*e^12))/(b^4 + 16*a^2*c^2 - 8*a*b^2*c))*(-(b^9 + (-(4*
a*c - b^2)^9)^(1/2) - 768*a^4*b*c^4 - 96*a^2*b^5*c^2 + 512*a^3*b^3*c^3)/(32
*(a*b^12*e^2 + 4096*a^7*c^6*e^2 - 24*a^2*b^10*c*e^2 + 240*a^3*b^8*c^2*e^2 -
 1280*a^4*b^6*c^3*e^2 + 3840*a^5*b^4*c^4*e^2 - 6144*a^6*b^2*c^5*e^2)))^(1/2
)*1i + ((((32*b^9*c^2*d*e^13 - 512*a*b^7*c^3*d*e^13 + 8192*a^4*b*c^6*d*e^13
 + 3072*a^2*b^5*c^4*d*e^13 - 8192*a^3*b^3*c^5*d*e^13)/(4*(b^6 - 64*a^3*c^3
+ 48*a^2*b^2*c^2 - 12*a*b^4*c)) + (x*(8*b^7*c^2*e^14 - 96*a*b^5*c^3*e^14 -
512*a^3*b*c^5*e^14 + 384*a^2*b^3*c^4*e^14))/(b^4 + 16*a^2*c^2 - 8*a*b^2*c))
*(-(b^9 + (-(4*a*c - b^2)^9)^(1/2) - 768*a^4*b*c^4 - 96*a^2*b^5*c^2 + 512*a
^3*b^3*c^3)/(32*(a*b^12*e^2 + 4096*a^7*c^6*e^2 - 24*a^2*b^10*c*e^2 + 240*a^
3*b^8*c^2*e^2 - 1280*a^4*b^6*c^3*e^2 + 3840*a^5*b^4*c^4*e^2 - 6144*a^6*b^2*
c^5*e^2)))^(1/2) + (8*b^7*c^2*e^12 - 96*a*b^5*c^3*e^12 - 512*a^3*b*c^5*e^12
 + 384*a^2*b^3*c^4*e^12)/(4*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c
)))*(-(b^9 + (-(4*a*c - b^2)^9)^(1/2) - 768*a^4*b*c^4 - 96*a^2*b^5*c^2 + 51
2*a^3*b^3*c^3)/(32*(a*b^12*e^2 + 4096*a^7*c^6*e^2 - 24*a^2*b^10*c*e^2 + 240
*a^3*b^8*c^2*e^2 - 1280*a^4*b^6*c^3*e^2 + 3840*a^5*b^4*c^4*e^2 - 6144*a^6*b
^2*c^5*e^2)))^(1/2) + (64*a^2*c^5*d*e^11 + 20*b^4*c^3*d*e^11 - 96*a*b^2*c^4
*d*e^11)/(4*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) - (x*(4*a*c^4
*e^12 - 5*b^2*c^3*e^12))/(b^4 + 16*a^2*c^2 - 8*a*b^2*c))*(-(b^9 + (-(4*a*c
- b^2)^9)^(1/2) - 768*a^4*b*c^4 - 96*a^2*b^5*c^2 + 512*a^3*b^3*c^3)/(32*(a*
b^12*e^2 + 4096*a^7*c^6*e^2 - 24*a^2*b^10*c*e^2 + 240*a^3*b^8*c^2*e^2 - 128
0*a^4*b^6*c^3*e^2 + 3840*a^5*b^4*c^4*e^2 - 6144*a^6*b^2*c^5*e^2)))^(1/2)*1i
```

```
)/((4*a*c^4*e^10 + 3*b^2*c^3*e^10)/(2*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 -
12*a*b^4*c)) - ((((32*b^9*c^2*d*e^13 - 512*a*b^7*c^3*d*e^13 + 8192*a^4*b*c^
6*d*e^13 + 3072*a^2*b^5*c^4*d*e^13 - 8192*a^3*b^3*c^5*d*e^13)/(4*(b^6 - 64*
a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) + (x*(8*b^7*c^2*e^14 - 96*a*b^5*c^3
*e^14 - 512*a^3*b*c^5*e^14 + 384*a^2*b^3*c^4*e^14))/(b^4 + 16*a^2*c^2 - 8*a
*b^2*c))*(-(b^9 + (-(4*a*c - b^2)^9)^(1/2) - 768*a^4*b*c^4 - 96*a^2*b^5*c^2
 + 512*a^3*b^3*c^3)/(32*(a*b^12*e^2 + 4096*a^7*c^6*e^2 - 24*a^2*b^10*c*e^2
+ 240*a^3*b^8*c^2*e^2 - 1280*a^4*b^6*c^3*e^2 + 3840*a^5*b^4*c^4*e^2 - 6144*
a^6*b^2*c^5*e^2)))^(1/2) - (8*b^7*c^2*e^12 - 96*a*b^5*c^3*e^12 - 512*a^3*b*
c^5*e^12 + 384*a^2*b^3*c^4*e^12)/(4*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12
*a*b^4*c)))*(-(b^9 + (-(4*a*c - b^2)^9)^(1/2) - 768*a^4*b*c^4 - 96*a^2*b^5*
c^2 + 512*a^3*b^3*c^3)/(32*(a*b^12*e^2 + 4096*a^7*c^6*e^2 - 24*a^2*b^10*c*e
^2 + 240*a^3*b^8*c^2*e^2 - 1280*a^4*b^6*c^3*e^2 + 3840*a^5*b^4*c^4*e^2 - 61
44*a^6*b^2*c^5*e^2)))^(1/2) + (64*a^2*c^5*d*e^11 + 20*b^4*c^3*d*e^11 - 96*a
*b^2*c^4*d*e^11)/(4*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) - (x*
(4*a*c^4*e^12 - 5*b^2*c^3*e^12))/(b^4 + 16*a^2*c^2 - 8*a*b^2*c))*(-(b^9 + (
-(4*a*c - b^2)^9)^(1/2) - 768*a^4*b*c^4 - 96*a^2*b^5*c^2 + 512*a^3*b^3*c^3)
/(32*(a*b^12*e^2 + 4096*a^7*c^6*e^2 - 24*a^2*b^10*c*e^2 + 240*a^3*b^8*c^2*e
^2 - 1280*a^4*b^6*c^3*e^2 + 3840*a^5*b^4*c^4*e^2 - 6144*a^6*b^2*c^5*e^2)))^
(1/2) + ((((32*b^9*c^2*d*e^13 - 512*a*b^7*c^3*d*e^13 + 8192*a^4*b*c^6*d*e^1
3 + 3072*a^2*b^5*c^4*d*e^13 - 8192*a^3*b^3*c^5*d*e^13)/(4*(b^6 - 64*a^3*c^3
 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) + (x*(8*b^7*c^2*e^14 - 96*a*b^5*c^3*e^14 -
 512*a^3*b*c^5*e^14 + 384*a^2*b^3*c^4*e^14))/(b^4 + 16*a^2*c^2 - 8*a*b^2*c)
)*(-(b^9 + (-(4*a*c - b^2)^9)^(1/2) - 768*a^4*b*c^4 - 96*a^2*b^5*c^2 + 512*
a^3*b^3*c^3)/(32*(a*b^12*e^2 + 4096*a^7*c^6*e^2 - 24*a^2*b^10*c*e^2 + 240*a
^3*b^8*c^2*e^2 - 1280*a^4*b^6*c^3*e^2 + 3840*a^5*b^4*c^4*e^2 - 6144*a^6*b^2
*c^5*e^2)))^(1/2) + (8*b^7*c^2*e^12 - 96*a*b^5*c^3*e^12 - 512*a^3*b*c^5*e^1
2 + 384*a^2*b^3*c^4*e^12)/(4*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*
c)))*(-(b^9 + (-(4*a*c - b^2)^9)^(1/2) - 768*a^4*b*c^4 - 96*a^2*b^5*c^2 + 5
12*a^3*b^3*c^3)/(32*(a*b^12*e^2 + 4096*a^7*c^6*e^2 - 24*a^2*b^10*c*e^2 + 24
0*a^3*b^8*c^2*e^2 - 1280*a^4*b^6*c^3*e^2 + 3840*a^5*b^4*c^4*e^2 - 6144*a^6*
b^2*c^5*e^2)))^(1/2) + (64*a^2*c^5*d*e^11 + 20*b^4*c^3*d*e^11 - 96*a*b^2*c^
4*d*e^11)/(4*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) - (x*(4*a*c^
4*e^12 - 5*b^2*c^3*e^12))/(b^4 + 16*a^2*c^2 - 8*a*b^2*c))*(-(b^9 + (-(4*a*c
 - b^2)^9)^(1/2) - 768*a^4*b*c^4 - 96*a^2*b^5*c^2 + 512*a^3*b^3*c^3)/(32*(a
*b^12*e^2 + 4096*a^7*c^6*e^2 - 24*a^2*b^10*c*e^2 + 240*a^3*b^8*c^2*e^2 - 12
80*a^4*b^6*c^3*e^2 + 3840*a^5*b^4*c^4*e^2 - 6144*a^6*b^2*c^5*e^2)))^(1/2)))
*(-(b^9 + (-(4*a*c - b^2)^9)^(1/2) - 768*a^4*b*c^4 - 96*a^2*b^5*c^2 + 512*a
^3*b^3*c^3)/(32*(a*b^12*e^2 + 4096*a^7*c^6*e^2 - 24*a^2*b^10*c*e^2 + 240*a^
3*b^8*c^2*e^2 - 1280*a^4*b^6*c^3*e^2 + 3840*a^5*b^4*c^4*e^2 - 6144*a^6*b^2*
c^5*e^2)))^(1/2)*2i + ((x*(b + 6*c*d^2))/(2*(4*a*c - b^2)) + (b*d + 2*c*d^3
)/(2*e*(4*a*c - b^2)) + (c*e^2*x^3)/(4*a*c - b^2) + (3*c*d*e*x^2)/(4*a*c -
b^2))/(a + x^2*(b*e^2 + 6*c*d^2*e^2) + b*d^2 + c*d^4 + x*(2*b*d*e + 4*c*d^3
*e) + c*e^4*x^4 + 4*c*d*e^3*x^3)
```

sympy [B] time = 18.53, size = 578, normalized size = 2.28

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2/(a+b*(e*x+d)**2+c*(e*x+d)**4)**2,x)
```

```
[Out] (b*d + 2*c*d**3 + 6*c*d*e**2*x**2 + 2*c*e**3*x**3 + x*(b*e + 6*c*d**2*e))/(
8*a**2*c*e - 2*a*b**2*e + 8*a*b*c*d**2*e + 8*a*c**2*d**4*e - 2*b**3*d**2*e
- 2*b**2*c*d**4*e + x**4*(8*a*c**2*e**5 - 2*b**2*c*e**5) + x**3*(32*a*c**2*
d*e**4 - 8*b**2*c*d*e**4) + x**2*(8*a*b*c*e**3 + 48*a*c**2*d**2*e**3 - 2*b*
*3*e**3 - 12*b**2*c*d**2*e**3) + x*(16*a*b*c*d*e**2 + 32*a*c**2*d**3*e**2 -
 4*b**3*d*e**2 - 8*b**2*c*d**3*e**2)) + RootSum(_t**4*(1048576*a**7*c**6*e*
*4 - 1572864*a**6*b**2*c**5*e**4 + 983040*a**5*b**4*c**4*e**4 - 327680*a**4
*b**6*c**3*e**4 + 61440*a**3*b**8*c**2*e**4 - 6144*a**2*b**10*c*e**4 + 256*
```

```
a*b**12*e**4) + _t**2*(-12288*a**4*b*c**4*e**2 + 8192*a**3*b**3*c**3*e**2 -
 1536*a**2*b**5*c**2*e**2 + 16*b**9*e**2) + 16*a**2*c**3 + 24*a*b**2*c**2 +
 9*b**4*c, Lambda(_t, _t*log(x + (16384*_t**3*a**5*c**4*e**3 - 8192*_t**3*a
**4*b**2*c**3*e**3 + 512*_t**3*a**2*b**6*c*e**3 - 64*_t**3*a*b**8*e**3 - 12
8*_t*a**2*b*c**2*e - 16*_t*a*b**3*c*e - 4*_t*b**5*e + 4*a*c**2*d + 3*b**2*c
*d)/(4*a*c**2*e + 3*b**2*c*e))))
```

3.511 $\int \frac{d+ex}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx$

Optimal. Leaf size=98

$$\frac{-b-2c(d+ex)^2}{2e\left(b^2-4ac\right)\left(a+b(d+ex)^2+c(d+ex)^4\right)}+\frac{2c\tanh^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{e\left(b^2-4ac\right)^{3/2}}$$

Rubi [A] time = 0.12, antiderivative size = 96, normalized size of antiderivative = 0.98, number of steps used = 5, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {1142, 1107, 614, 618, 206}

$$\frac{2c\tanh^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{e\left(b^2-4ac\right)^{3/2}}-\frac{b+2c(d+ex)^2}{2e\left(b^2-4ac\right)\left(a+b(d+ex)^2+c(d+ex)^4\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2,x]
```

```
[Out] -(b + 2*c*(d + e*x)^2)/(2*(b^2 - 4*a*c)*e*(a + b*(d + e*x)^2 + c*(d + e*x)^
4)) + (2*c*ArcTanh[(b + 2*c*(d + e*x)^2)/Sqrt[b^2 - 4*a*c]])/((b^2 - 4*a*c)
^(3/2)*e)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 614

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[(2*c*(2*p +
 3))/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
egerQ[4*p]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1107

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rule 1142

```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Di
st[u^m/(Coefficient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p,
 x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

Rubi steps

$$\int \frac{d+ex}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx=\frac{\text{Subst}\left(\int \frac{x}{\left(a+bx^2+cx^4\right)^2}\,dx,x,d+ex\right)}{e}$$
$$=\frac{\text{Subst}\left(\int \frac{1}{\left(a+bx+cx^2\right)^2}\,dx,x,(d+ex)^2\right)}{2e}$$
$$=-\frac{b+2c(d+ex)^2}{2\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)}-\frac{c\,\text{Subst}\left(\int \frac{1}{a+bx+cx^2}\,dx,x,(\right.}{\left(b^2-4ac\right)e}$$
$$=-\frac{b+2c(d+ex)^2}{2\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)}+\frac{(2c)\,\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx,\right.}{\left(b^2-4ac\right.}$$
$$=-\frac{b+2c(d+ex)^2}{2\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)}+\frac{2c\tanh^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{3/2}e}$$

Mathematica [A] time = 0.12, size = 98, normalized size = 1.00

$$-\frac{\frac{4c\tan^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}+\frac{b+2c(d+ex)^2}{a+b(d+ex)^2+c(d+ex)^4}}{2e\left(b^2-4ac\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2,x]
```

```
[Out] -1/2*((b + 2*c*(d + e*x)^2)/(a + b*(d + e*x)^2 + c*(d + e*x)^4) + (4*c*ArcT
an[(b + 2*c*(d + e*x)^2)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c])/((b^2 - 4
*a*c)*e)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{d+ex}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(d + e*x)/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2,x]
```

```
[Out] IntegrateAlgebraic[(d + e*x)/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2, x]
```

fricas [B] time = 0.87, size = 1042, normalized size = 10.63

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="fricas")
```

```
[Out] [-1/2*(2*(b^2*c - 4*a*c^2)*e^2*x^2 + 4*(b^2*c - 4*a*c^2)*d*e*x + b^3 - 4*a*
b*c + 2*(b^2*c - 4*a*c^2)*d^2 + 2*(c^2*e^4*x^4 + 4*c^2*d*e^3*x^3 + c^2*d^4
+ (6*c^2*d^2 + b*c)*e^2*x^2 + b*c*d^2 + 2*(2*c^2*d^3 + b*c*d)*e*x + a*c)*sq
rt(b^2 - 4*a*c)*log((2*c^2*e^4*x^4 + 8*c^2*d*e^3*x^3 + 2*c^2*d^4 + 2*(6*c^2
*d^2 + b*c)*e^2*x^2 + 2*b*c*d^2 + 4*(2*c^2*d^3 + b*c*d)*e*x + b^2 - 2*a*c -
```

```
(2*c*e^2*x^2 + 4*c*d*e*x + 2*c*d^2 + b)*sqrt(b^2 - 4*a*c))/(c*e^4*x^4 + 4*
c*d*e^3*x^3 + c*d^4 + (6*c*d^2 + b)*e^2*x^2 + b*d^2 + 2*(2*c*d^3 + b*d)*e*x
 + a)))/((b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*e^5*x^4 + 4*(b^4*c - 8*a*b^2*c^
2 + 16*a^2*c^3)*d*e^4*x^3 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2 + 6*(b^4*c - 8*
a*b^2*c^2 + 16*a^2*c^3)*d^2)*e^3*x^2 + 2*(2*(b^4*c - 8*a*b^2*c^2 + 16*a^2*c
^3)*d^3 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*d)*e^2*x + (a*b^4 - 8*a^2*b^2*c
+ 16*a^3*c^2 + (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*d^4 + (b^5 - 8*a*b^3*c +
16*a^2*b*c^2)*d^2)*e), -1/2*(2*(b^2*c - 4*a*c^2)*e^2*x^2 + 4*(b^2*c - 4*a*c
^2)*d*e*x + b^3 - 4*a*b*c + 2*(b^2*c - 4*a*c^2)*d^2 - 4*(c^2*e^4*x^4 + 4*c^
2*d*e^3*x^3 + c^2*d^4 + (6*c^2*d^2 + b*c)*e^2*x^2 + b*c*d^2 + 2*(2*c^2*d^3
+ b*c*d)*e*x + a*c)*sqrt(-b^2 + 4*a*c)*arctan(-(2*c*e^2*x^2 + 4*c*d*e*x + 2
*c*d^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)))/((b^4*c - 8*a*b^2*c^2 + 16*a
^2*c^3)*e^5*x^4 + 4*(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*d*e^4*x^3 + (b^5 - 8
*a*b^3*c + 16*a^2*b*c^2 + 6*(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*d^2)*e^3*x^2
 + 2*(2*(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*d^3 + (b^5 - 8*a*b^3*c + 16*a^2*
b*c^2)*d)*e^2*x + (a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2 + (b^4*c - 8*a*b^2*c^2
+ 16*a^2*c^3)*d^4 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*d^2)*e)]
```

giac [A] time = 0.46, size = 172, normalized size = 1.76

$$-\frac{2\,c\arctan\left(\frac{2\,cd^2+2\left(x^2e+2\,dx\right)ce+b}{\sqrt{-b^2+4\,ac}}\right)e^{(-1)}}{\left(b^2-4\,ac\right)\sqrt{-b^2+4\,ac}}-\frac{2\,cd^2+2\left(x^2e+2\,dx\right)ce+b}{2\left(cd^4+2\left(x^2e+2\,dx\right)cd^2e+\left(x^2e+2\,dx\right)^2ce^2+bd^2+\left(x^2e+2\,dx\right)be+a\right)\left(b^2e-4\,ace\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="giac")
```

```
[Out] -2*c*arctan((2*c*d^2 + 2*(x^2*e + 2*d*x)*c*e + b)/sqrt(-b^2 + 4*a*c))*e^(-1
)/((b^2 - 4*a*c)*sqrt(-b^2 + 4*a*c)) - 1/2*(2*c*d^2 + 2*(x^2*e + 2*d*x)*c*e
 + b)/((c*d^4 + 2*(x^2*e + 2*d*x)*c*d^2*e + (x^2*e + 2*d*x)^2*c*e^2 + b*d^2
 + (x^2*e + 2*d*x)*b*e + a)*(b^2*e - 4*a*c*e))
```

maple [C] time = 0.02, size = 270, normalized size = 2.76

$$\frac{c\left(\mathrm{RootOf}\left(\_Z^4ce^4+4\_Z^3cde^3+cd^4+bd^2+\left(6cd^2e^2+be^2\right)\_Z^2+\left(4cd^3e+2deb\right)\_Z+a\right)e+d\right)\ln\left(-\mathrm{RootOf}\left(\_Z^4ce^4+4\_Z^3cde^3+cd^4+bd^2+\left(6cd^2e^2+be^2\right)\_Z^2+\left(4cd^3e+2deb\right)\_Z+a\right)+x\right)}{\left(4ac-b^2\right)e\left(2ce^3\mathrm{RootOf}\left(\_Z^4ce^4+4\_Z^3cde^3+cd^4+bd^2+\left(6cd^2e^2+be^2\right)\_Z^2+\left(4cd^3e+2deb\right)\_Z+a\right)^3+6cde^2\mathrm{RootOf}\left(\_Z^4ce^4+4\_Z^3cde^3+cd^4+bd^2+\left(6cd^2e^2+be^2\right)\_Z^2+\left(4cd^3e+2deb\right)\_Z+a\right)^2+6ecd^2\mathrm{RootOf}\left(\_Z^4ce^4+4\_Z^3cde^3+cd^4+bd^2+\left(6cd^2e^2+be^2\right)\_Z^2+\left(4cd^3e+2deb\right)\_Z+a\right)+2cd^3+be\,\mathrm{RootOf}\left(\_Z^4ce^4+4\_Z^3cde^3+cd^4+bd^2+\left(6cd^2e^2+be^2\right)\_Z^2+\left(4cd^3e+2deb\right)\_Z+a\right)+bd\right)}+\frac{\frac{ce\,x^2}{4ac-b^2}+\frac{2cdx}{4ac-b^2}+\frac{2cd^2+b}{2\left(4ac-b^2\right)e}}{ce^4x^4+4cde^3x^3+6cd^2e^2x^2+4cd^3ex+be^2x^2+cd^4+2bdex+bd^2+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x)
```

```
[Out] (c*e/(4*a*c-b^2)*x^2+2*c*d/(4*a*c-b^2)*x+1/2/e*(2*c*d^2+b)/(4*a*c-b^2))/(c*
e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x
+b*d^2+a)+c/(4*a*c-b^2)/e*sum((_R*e+d)/(2*_R^3*c*e^3+6*_R^2*c*d*e^2+6*_R*c*
d^2*e+2*c*d^3+_R*b*e+b*d)*ln(-_R+x),_R=RootOf(_Z^4*c*e^4+4*_Z^3*c*d*e^3+c*d
^4+b*d^2+(6*c*d^2*e^2+b*e^2)*_Z^2+(4*c*d^3*e+2*b*d*e)*_Z+a))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$2\,c\int-\frac{ex+d}{\left(b^2c-4\,ac^2\right)e^4x^4+4\left(b^2c-4\,ac^2\right)de^3x^3+\left(b^2c-4\,ac^2\right)d^4+\left(b^3-4\,abc+6\left(b^2c-4\,ac^2\right)d^2\right)e^2x^2+ab^2-4\,a^2c+\left(b^3-4\,abc\right)d^2+2\left(2\left(b^2c-4\,ac^2\right)d^3+\left(b^3-4\,abc\right)d\right)ex}\,dx-\frac{2\,ce^2x^2+4\,cdex+2\,cd^2+b}{2\left(\left(b^2c-4\,ac^2\right)e^5x^4+4\left(b^2c-4\,ac^2\right)de^4x^3+\left(b^3-4\,abc+6\left(b^2c-4\,ac^2\right)d^2\right)e^3x^2+2\left(2\left(b^2c-4\,ac^2\right)d^3+\left(b^3-4\,abc\right)d\right)e^2x+\left(\left(b^2c-4\,ac^2\right)d^4+ab^2-4\,a^2c+\left(b^3-4\,abc\right)d^2\right)e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="maxima")
```

```
[Out] 2*c*integrate(-(e*x + d)/((b^2*c - 4*a*c^2)*e^4*x^4 + 4*(b^2*c - 4*a*c^2)*d
*e^3*x^3 + (b^2*c - 4*a*c^2)*d^4 + (b^3 - 4*a*b*c + 6*(b^2*c - 4*a*c^2)*d^2
)*e^2*x^2 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*d^2 + 2*(2*(b^2*c - 4*a*c^2)*
d^3 + (b^3 - 4*a*b*c)*d)*e*x), x) - 1/2*(2*c*e^2*x^2 + 4*c*d*e*x + 2*c*d^2
+ b)/((b^2*c - 4*a*c^2)*e^5*x^4 + 4*(b^2*c - 4*a*c^2)*d*e^4*x^3 + (b^3 - 4*
a*b*c + 6*(b^2*c - 4*a*c^2)*d^2)*e^3*x^2 + 2*(2*(b^2*c - 4*a*c^2)*d^3 + (b^
3 - 4*a*b*c)*d)*e^2*x + ((b^2*c - 4*a*c^2)*d^4 + a*b^2 - 4*a^2*c + (b^3 - 4
*a*b*c)*d^2)*e)
```

mupad [B] time = 1.72, size = 417, normalized size = 4.26

$$\frac{\frac{2cd^2+b}{2e(4ac-b^2)}+\frac{cex^2}{4ac-b^2}+\frac{2cdx}{4ac-b^2}}{a+x^2\left(6cd^2e^2+be^2\right)+bd^2+cd^4+x\left(4ced^3+2bed\right)+ce^4x^4+4cde^3x^3}+\frac{2c\,\mathrm{atan}\left(\frac{\left(4ac-b^2\right)^4\left(x\left(\frac{8c^4de^7}{a\left(4ac-b^2\right)^{7/2}}-\frac{8bc^2\left(b^3c^2de^9-4abc^3de^9\right)}{ae^2\left(4ac-b^2\right)^{11/2}}\right)+x^2\left(\frac{4c^4e^8}{a\left(4ac-b^2\right)^{7/2}}-\frac{4bc^2\left(b^3c^2e^{10}-4abc^3e^{10}\right)}{ae^2\left(4ac-b^2\right)^{11/2}}\right)+\frac{4c^4d^2e^6}{a\left(4ac-b^2\right)^{7/2}}+\frac{4bc^2\left(8a^2c^3e^8-2ab^2c^2e^8+4abc^3d^2e^8-b^3c^2d^2e^8\right)}{ae^2\left(4ac-b^2\right)^{11/2}}\right)}{8c^4e^6}\right)}{e\left(4ac-b^2\right)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2,x)
```

```
[Out] ((b + 2*c*d^2)/(2*e*(4*a*c - b^2)) + (c*e*x^2)/(4*a*c - b^2) + (2*c*d*x)/(4
*a*c - b^2))/(a + x^2*(b*e^2 + 6*c*d^2*e^2) + b*d^2 + c*d^4 + x*(2*b*d*e +
4*c*d^3*e) + c*e^4*x^4 + 4*c*d*e^3*x^3) + (2*c*atan(((4*a*c - b^2)^4*(x*((8
*c^4*d*e^7)/(a*(4*a*c - b^2)^(7/2)) - (8*b*c^2*(b^3*c^2*d*e^9 - 4*a*b*c^3*d
*e^9))/(a*e^2*(4*a*c - b^2)^(11/2))) + x^2*((4*c^4*e^8)/(a*(4*a*c - b^2)^(7
/2)) - (4*b*c^2*(b^3*c^2*e^10 - 4*a*b*c^3*e^10))/(a*e^2*(4*a*c - b^2)^(11/2
))) + (4*c^4*d^2*e^6)/(a*(4*a*c - b^2)^(7/2)) + (4*b*c^2*(8*a^2*c^3*e^8 - 2
*a*b^2*c^2*e^8 - b^3*c^2*d^2*e^8 + 4*a*b*c^3*d^2*e^8))/(a*e^2*(4*a*c - b^2)
^(11/2))))/(8*c^4*e^6)))/(e*(4*a*c - b^2)^(3/2))
```

sympy [B] time = 4.68, size = 495, normalized size = 5.05

$$-\frac{c\sqrt{-\frac{1}{\left(4ac-b^2\right)^3}}\log\left(\frac{2dx}{e}+x^2+\frac{-16a^2c^3\sqrt{-\frac{1}{\left(4ac-b^2\right)^3}}+8ab^2c^2\sqrt{-\frac{1}{\left(4ac-b^2\right)^3}}-b^4c\sqrt{-\frac{1}{\left(4ac-b^2\right)^3}}+bc+2c^2d^2}{2c^2e^2}\right)}{e}+\frac{c\sqrt{-\frac{1}{\left(4ac-b^2\right)^3}}\log\left(\frac{2dx}{e}+x^2+\frac{16a^2c^3\sqrt{-\frac{1}{\left(4ac-b^2\right)^3}}-8ab^2c^2\sqrt{-\frac{1}{\left(4ac-b^2\right)^3}}+b^4c\sqrt{-\frac{1}{\left(4ac-b^2\right)^3}}+bc+2c^2d^2}{2c^2e^2}\right)}{e}+\frac{b+2cd^2+4cdex+2ce^2x^2}{8a^2ce-2ab^2e+8abcd^2e+8ac^2d^4e-2b^3d^2e-2b^2cd^4e+x^4\left(8ac^2e^5-2b^2ce^5\right)+x^3\left(32ac^2de^4-8b^2cde^4\right)+x^2\left(8abce^3+48ac^2d^2e^3-2b^3e^3-12b^2cd^2e^3\right)+x\left(16abcde^2+32ac^2d^3e^2-4b^3de^2-8b^2cd^3e^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(a+b*(e*x+d)**2+c*(e*x+d)**4)**2,x)
```

```
[Out] -c*sqrt(-1/(4*a*c - b**2)**3)*log(2*d*x/e + x**2 + (-16*a**2*c**3*sqrt(-1/(
4*a*c - b**2)**3) + 8*a*b**2*c**2*sqrt(-1/(4*a*c - b**2)**3) - b**4*c*sqrt(
-1/(4*a*c - b**2)**3) + b*c + 2*c**2*d**2)/(2*c**2*e**2))/e + c*sqrt(-1/(4*
a*c - b**2)**3)*log(2*d*x/e + x**2 + (16*a**2*c**3*sqrt(-1/(4*a*c - b**2)**
3) - 8*a*b**2*c**2*sqrt(-1/(4*a*c - b**2)**3) + b**4*c*sqrt(-1/(4*a*c - b**
2)**3) + b*c + 2*c**2*d**2)/(2*c**2*e**2))/e + (b + 2*c*d**2 + 4*c*d*e*x +
2*c*e**2*x**2)/(8*a**2*c*e - 2*a*b**2*e + 8*a*b*c*d**2*e + 8*a*c**2*d**4*e
- 2*b**3*d**2*e - 2*b**2*c*d**4*e + x**4*(8*a*c**2*e**5 - 2*b**2*c*e**5) +
x**3*(32*a*c**2*d*e**4 - 8*b**2*c*d*e**4) + x**2*(8*a*b*c*e**3 + 48*a*c**2*
d**2*e**3 - 2*b**3*e**3 - 12*b**2*c*d**2*e**3) + x*(16*a*b*c*d*e**2 + 32*a*
c**2*d**3*e**2 - 4*b**3*d*e**2 - 8*b**2*c*d**3*e**2))
```

3.512 $\int \frac{1}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx$

Optimal. Leaf size=299

$$\frac{\left(\frac{d}{e}+x\right)\left(-2ac+b^2+bce^2\left(\frac{d}{e}+x\right)^2\right)}{2a\left(b^2-4ac\right)\left(a+be^2\left(\frac{d}{e}+x\right)^2+ce^4\left(\frac{d}{e}+x\right)^4\right)}+\frac{\sqrt{c}\left(b\sqrt{b^2-4ac}-12ac+b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\,ae\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{c}\left(-b\sqrt{\right.}{2\sqrt{}}$$

Rubi [A] time = 0.70, antiderivative size = 299, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {1106, 1092, 1166, 205}

$$\frac{\left(\frac{d}{e}+x\right)\left(-2ac+b^2+bce^2\left(\frac{d}{e}+x\right)^2\right)}{2a\left(b^2-4ac\right)\left(a+be^2\left(\frac{d}{e}+x\right)^2+ce^4\left(\frac{d}{e}+x\right)^4\right)}+\frac{\sqrt{c}\left(b\sqrt{b^2-4ac}-12ac+b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\,ae\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{c}\left(-b\sqrt{b^2-4ac}-12ac+b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{2\sqrt{2}\,ae\left(b^2-4ac\right)^{3/2}\sqrt{\sqrt{b^2-4ac}+b}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*(d + e*x)^2 + c*(d + e*x)^4)^(-2),x]
```

```
[Out] ((d/e + x)*(b^2 - 2*a*c + b*c*e^2*(d/e + x)^2))/(2*a*(b^2 - 4*a*c)*(a + b*e
^2*(d/e + x)^2 + c*e^4*(d/e + x)^4)) + (Sqrt[c]*(b^2 - 12*a*c + b*Sqrt[b^2
- 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/
(2*Sqrt[2]*a*(b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]*e) - (Sqrt[c]*
(b^2 - 12*a*c - b*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqr
t[b + Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a*(b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b
^2 - 4*a*c]]*e)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1092

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> -Simp[(x*(b^2 -
 2*a*c + b*c*x^2)*(a + b*x^2 + c*x^4)^(p + 1))/(2*a*(p + 1)*(b^2 - 4*a*c)),
 x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(b^2 - 2*a*c + 2*(p + 1)*(b^2
 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ
[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1106

```
Int[(P4_)^(p_), x_Symbol] :> With[{a = Coeff[P4, x, 0], b = Coeff[P4, x, 1]
, c = Coeff[P4, x, 2], d = Coeff[P4, x, 3], e = Coeff[P4, x, 4]}, Subst[Int
[SimplifyIntegrand[(a + d^4/(256*e^3) - (b*d)/(8*e) + (c - (3*d^2)/(8*e))*x
^2 + e*x^4)^p, x], x], x, d/(4*e) + x] /; EqQ[d^3 - 4*c*d*e + 8*b*e^2, 0] &
& NeQ[d, 0]] /; FreeQ[p, x] && PolyQ[P4, x, 4] && NeQ[p, 2] && NeQ[p, 3]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{1}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx = \text{Subst}\left(\int \frac{1}{\left(a+be^2x^2+ce^4x^4\right)^2}\,dx, x, \frac{d}{e}+x\right)$$

$$= \frac{\left(\frac{d}{e}+x\right)\left(b^2-2ac+bce^2\left(\frac{d}{e}+x\right)^2\right)}{2a\left(b^2-4ac\right)\left(a+be^2\left(\frac{d}{e}+x\right)^2+ce^4\left(\frac{d}{e}+x\right)^4\right)} - \frac{\text{Subst}\left(\int \frac{b^2e^4-2ace^4-2\left(b^2e^{4}\cdots\right.}{a+be^2x^2\cdots}\cdots\right.}{2a\left(b^2\cdots\right.}$$

$$= \frac{\left(\frac{d}{e}+x\right)\left(b^2-2ac+bce^2\left(\frac{d}{e}+x\right)^2\right)}{2a\left(b^2-4ac\right)\left(a+be^2\left(\frac{d}{e}+x\right)^2+ce^4\left(\frac{d}{e}+x\right)^4\right)} - \frac{\left(c\left(b^2-12ac-b\sqrt{b^2-\cdots}\right.\right.}{\cdots}$$

$$= \frac{\left(\frac{d}{e}+x\right)\left(b^2-2ac+bce^2\left(\frac{d}{e}+x\right)^2\right)}{2a\left(b^2-4ac\right)\left(a+be^2\left(\frac{d}{e}+x\right)^2+ce^4\left(\frac{d}{e}+x\right)^4\right)} + \frac{\sqrt{c}\left(b^2-12ac+b\sqrt{b^2-\cdots}\right.}{2\sqrt{2}\,a\left(b^2-4ac\right)^{3\cdots}}$$

Mathematica [A] time = 0.89, size = 271, normalized size = 0.91

$$\frac{\frac{2(d+ex)\left(-2ac+b^2+bc(d+ex)^2\right)}{\left(b^2-4ac\right)\left(a+(d+ex)^2\left(b+c(d+ex)^2\right)\right)}+\frac{\sqrt{2}\sqrt{c}\left(b\sqrt{b^2-4ac}-12ac+b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{2}\sqrt{c}\left(b\sqrt{b^2-4ac}+12ac-b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\left(b^2-4ac\right)^{3/2}\sqrt{\sqrt{b^2-4ac}+b}}}{4ae}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*(d + e*x)^2 + c*(d + e*x)^4)^(-2),x]
```

```
[Out] ((2*(d + e*x)*(b^2 - 2*a*c + b*c*(d + e*x)^2))/((b^2 - 4*a*c)*(a + (d + e*x
)^2*(b + c*(d + e*x)^2))) + (Sqrt[2]*Sqrt[c]*(b^2 - 12*a*c + b*Sqrt[b^2 - 4
*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/((b
^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[2]*Sqrt[c]*(-b^2 + 1
2*a*c + b*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b + Sq
rt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(4*a*
e)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*(d + e*x)^2 + c*(d + e*x)^4)^(-2),x]
```

```
[Out] IntegrateAlgebraic[(a + b*(d + e*x)^2 + c*(d + e*x)^4)^(-2), x]
```

fricas [B] time = 1.73, size = 3228, normalized size = 10.80

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="fricas")
```

```
[Out] 1/4*(2*b*c*e^3*x^3 + 6*b*c*d*e^2*x^2 + 2*b*c*d^3 + 2*(3*b*c*d^2 + b^2 - 2*a
*c)*e*x - sqrt(1/2)*((a*b^2*c - 4*a^2*c^2)*e^5*x^4 + 4*(a*b^2*c - 4*a^2*c^2
)*d*e^4*x^3 + (a*b^3 - 4*a^2*b*c + 6*(a*b^2*c - 4*a^2*c^2)*d^2)*e^3*x^2 + 2
*(2*(a*b^2*c - 4*a^2*c^2)*d^3 + (a*b^3 - 4*a^2*b*c)*d)*e^2*x + ((a*b^2*c -
4*a^2*c^2)*d^4 + a^2*b^2 - 4*a^3*c + (a*b^3 - 4*a^2*b*c)*d^2)*e)*sqrt(-(b^5
 - 15*a*b^3*c + 60*a^2*b*c^2 + (a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 - 6
4*a^6*c^3)*e^2*sqrt((b^4 - 18*a*b^2*c + 81*a^2*c^2)/((a^6*b^6 - 12*a^7*b^4*
c + 48*a^8*b^2*c^2 - 64*a^9*c^3)*e^4)))/((a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b
^2*c^2 - 64*a^6*c^3)*e^2))*log((5*b^4*c^2 - 81*a*b^2*c^3 + 324*a^2*c^4)*e*x
 + (5*b^4*c^2 - 81*a*b^2*c^3 + 324*a^2*c^4)*d + 1/2*sqrt(1/2)*((a^3*b^9 - 2
0*a^4*b^7*c + 144*a^5*b^5*c^2 - 448*a^6*b^3*c^3 + 512*a^7*b*c^4)*e^3*sqrt((
b^4 - 18*a*b^2*c + 81*a^2*c^2)/((a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 -
64*a^9*c^3)*e^4)) - (b^8 - 23*a*b^6*c + 190*a^2*b^4*c^2 - 672*a^3*b^2*c^3 +
 864*a^4*c^4)*e)*sqrt(-(b^5 - 15*a*b^3*c + 60*a^2*b*c^2 + (a^3*b^6 - 12*a^4
*b^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3)*e^2*sqrt((b^4 - 18*a*b^2*c + 81*a^2*c
^2)/((a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)*e^4)))/((a^3*b^
6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3)*e^2))) + sqrt(1/2)*((a*b^2*
c - 4*a^2*c^2)*e^5*x^4 + 4*(a*b^2*c - 4*a^2*c^2)*d*e^4*x^3 + (a*b^3 - 4*a^2
*b*c + 6*(a*b^2*c - 4*a^2*c^2)*d^2)*e^3*x^2 + 2*(2*(a*b^2*c - 4*a^2*c^2)*d^
3 + (a*b^3 - 4*a^2*b*c)*d)*e^2*x + ((a*b^2*c - 4*a^2*c^2)*d^4 + a^2*b^2 - 4
*a^3*c + (a*b^3 - 4*a^2*b*c)*d^2)*e)*sqrt(-(b^5 - 15*a*b^3*c + 60*a^2*b*c^2
 + (a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3)*e^2*sqrt((b^4 - 1
8*a*b^2*c + 81*a^2*c^2)/((a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*
c^3)*e^4)))/((a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3)*e^2))*l
og((5*b^4*c^2 - 81*a*b^2*c^3 + 324*a^2*c^4)*e*x + (5*b^4*c^2 - 81*a*b^2*c^3
 + 324*a^2*c^4)*d - 1/2*sqrt(1/2)*((a^3*b^9 - 20*a^4*b^7*c + 144*a^5*b^5*c^
2 - 448*a^6*b^3*c^3 + 512*a^7*b*c^4)*e^3*sqrt((b^4 - 18*a*b^2*c + 81*a^2*c^
2)/((a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)*e^4)) - (b^8 - 2
3*a*b^6*c + 190*a^2*b^4*c^2 - 672*a^3*b^2*c^3 + 864*a^4*c^4)*e)*sqrt(-(b^5
- 15*a*b^3*c + 60*a^2*b*c^2 + (a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 - 64
*a^6*c^3)*e^2*sqrt((b^4 - 18*a*b^2*c + 81*a^2*c^2)/((a^6*b^6 - 12*a^7*b^4*c
 + 48*a^8*b^2*c^2 - 64*a^9*c^3)*e^4)))/((a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b^
2*c^2 - 64*a^6*c^3)*e^2))) + sqrt(1/2)*((a*b^2*c - 4*a^2*c^2)*e^5*x^4 + 4*(
a*b^2*c - 4*a^2*c^2)*d*e^4*x^3 + (a*b^3 - 4*a^2*b*c + 6*(a*b^2*c - 4*a^2*c^
2)*d^2)*e^3*x^2 + 2*(2*(a*b^2*c - 4*a^2*c^2)*d^3 + (a*b^3 - 4*a^2*b*c)*d)*e
^2*x + ((a*b^2*c - 4*a^2*c^2)*d^4 + a^2*b^2 - 4*a^3*c + (a*b^3 - 4*a^2*b*c)
*d^2)*e)*sqrt(-(b^5 - 15*a*b^3*c + 60*a^2*b*c^2 - (a^3*b^6 - 12*a^4*b^4*c +
 48*a^5*b^2*c^2 - 64*a^6*c^3)*e^2*sqrt((b^4 - 18*a*b^2*c + 81*a^2*c^2)/((a^
6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)*e^4)))/((a^3*b^6 - 12*a
^4*b^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3)*e^2))*log((5*b^4*c^2 - 81*a*b^2*c^3
 + 324*a^2*c^4)*e*x + (5*b^4*c^2 - 81*a*b^2*c^3 + 324*a^2*c^4)*d + 1/2*sqrt
(1/2)*((a^3*b^9 - 20*a^4*b^7*c + 144*a^5*b^5*c^2 - 448*a^6*b^3*c^3 + 512*a^
7*b*c^4)*e^3*sqrt((b^4 - 18*a*b^2*c + 81*a^2*c^2)/((a^6*b^6 - 12*a^7*b^4*c
+ 48*a^8*b^2*c^2 - 64*a^9*c^3)*e^4)) + (b^8 - 23*a*b^6*c + 190*a^2*b^4*c^2
- 672*a^3*b^2*c^3 + 864*a^4*c^4)*e)*sqrt(-(b^5 - 15*a*b^3*c + 60*a^2*b*c^2
- (a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3)*e^2*sqrt((b^4 - 18
*a*b^2*c + 81*a^2*c^2)/((a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c
^3)*e^4)))/((a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3)*e^2))) -
 sqrt(1/2)*((a*b^2*c - 4*a^2*c^2)*e^5*x^4 + 4*(a*b^2*c - 4*a^2*c^2)*d*e^4*x
^3 + (a*b^3 - 4*a^2*b*c + 6*(a*b^2*c - 4*a^2*c^2)*d^2)*e^3*x^2 + 2*(2*(a*b^
2*c - 4*a^2*c^2)*d^3 + (a*b^3 - 4*a^2*b*c)*d)*e^2*x + ((a*b^2*c - 4*a^2*c^2
)*d^4 + a^2*b^2 - 4*a^3*c + (a*b^3 - 4*a^2*b*c)*d^2)*e)*sqrt(-(b^5 - 15*a*b
^3*c + 60*a^2*b*c^2 - (a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3
)*e^2*sqrt((b^4 - 18*a*b^2*c + 81*a^2*c^2)/((a^6*b^6 - 12*a^7*b^4*c + 48*a^
8*b^2*c^2 - 64*a^9*c^3)*e^4)))/((a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 -
64*a^6*c^3)*e^2))*log((5*b^4*c^2 - 81*a*b^2*c^3 + 324*a^2*c^4)*e*x + (5*b^4
*c^2 - 81*a*b^2*c^3 + 324*a^2*c^4)*d - 1/2*sqrt(1/2)*((a^3*b^9 - 20*a^4*b^7
*c + 144*a^5*b^5*c^2 - 448*a^6*b^3*c^3 + 512*a^7*b*c^4)*e^3*sqrt((b^4 - 18*
a*b^2*c + 81*a^2*c^2)/((a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^
```

```
3)*e^4)) + (b^8 - 23*a*b^6*c + 190*a^2*b^4*c^2 - 672*a^3*b^2*c^3 + 864*a^4*
c^4)*e)*sqrt(-(b^5 - 15*a*b^3*c + 60*a^2*b*c^2 - (a^3*b^6 - 12*a^4*b^4*c +
48*a^5*b^2*c^2 - 64*a^6*c^3)*e^2*sqrt((b^4 - 18*a*b^2*c + 81*a^2*c^2)/((a^6
*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)*e^4)))/((a^3*b^6 - 12*a^
4*b^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3)*e^2))) + 2*(b^2 - 2*a*c)*d)/((a*b^2*
c - 4*a^2*c^2)*e^5*x^4 + 4*(a*b^2*c - 4*a^2*c^2)*d*e^4*x^3 + (a*b^3 - 4*a^2
*b*c + 6*(a*b^2*c - 4*a^2*c^2)*d^2)*e^3*x^2 + 2*(2*(a*b^2*c - 4*a^2*c^2)*d^
3 + (a*b^3 - 4*a^2*b*c)*d)*e^2*x + ((a*b^2*c - 4*a^2*c^2)*d^4 + a^2*b^2 - 4
*a^3*c + (a*b^3 - 4*a^2*b*c)*d^2)*e)
```

giac [B] time = 0.46, size = 1357, normalized size = 4.54

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="giac")
```

```
[Out] -1/4*(((d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c
))^2*b*c*e^2 - 2*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2
)*e^(-4)/c))*b*c*d*e + b*c*d^2 + b^2 - 6*a*c)*log(d*e^(-1) + x + sqrt(1/2)*
sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))/(2*(d*e^(-1) + sqrt(1/2)*s
qrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^3*c*e^4 - 6*(d*e^(-1) + sqr
t(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^2*c*d*e^3 - 2*c*d^3
*e - b*d*e + (6*c*d^2*e^2 + b*e^2)*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 + sqr
t(b^2 - 4*a*c)*e^2)*e^(-4)/c))) + ((d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 + sqr
t(b^2 - 4*a*c)*e^2)*e^(-4)/c))^2*b*c*e^2 - 2*(d*e^(-1) - sqrt(1/2)*sqrt(-(b
*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))*b*c*d*e + b*c*d^2 + b^2 - 6*a*c)*l
og(d*e^(-1) + x - sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c)
)/(2*(d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))
^3*c*e^4 - 6*(d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^
(-4)/c))^2*c*d*e^3 - 2*c*d^3*e - b*d*e + (6*c*d^2*e^2 + b*e^2)*(d*e^(-1) -
sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))) + ((d*e^(-1) +
sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^2*b*c*e^2 - 2*(d
*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))*b*c*d*
e + b*c*d^2 + b^2 - 6*a*c)*log(d*e^(-1) + x + sqrt(1/2)*sqrt(-(b*e^2 - sqrt
(b^2 - 4*a*c)*e^2)*e^(-4)/c))/(2*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 - sqrt(
b^2 - 4*a*c)*e^2)*e^(-4)/c))^3*c*e^4 - 6*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2
 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^2*c*d*e^3 - 2*c*d^3*e - b*d*e + (6*c*d
^2*e^2 + b*e^2)*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)
*e^(-4)/c))) + ((d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)
*e^(-4)/c))^2*b*c*e^2 - 2*(d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4
*a*c)*e^2)*e^(-4)/c))*b*c*d*e + b*c*d^2 + b^2 - 6*a*c)*log(d*e^(-1) + x - s
qrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))/(2*(d*e^(-1) - sq
rt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^3*c*e^4 - 6*(d*e^(
-1) - sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^2*c*d*e^3
- 2*c*d^3*e - b*d*e + (6*c*d^2*e^2 + b*e^2)*(d*e^(-1) - sqrt(1/2)*sqrt(-(b*
e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))))/(a*b^2 - 4*a^2*c) + 1/2*(b*c*x^3*
e^3 + 3*b*c*d*x^2*e^2 + 3*b*c*d^2*x*e + b*c*d^3 + b^2*x*e - 2*a*c*x*e + b^2
*d - 2*a*c*d)/((c*x^4*e^4 + 4*c*d*x^3*e^3 + 6*c*d^2*x^2*e^2 + 4*c*d^3*x*e +
 c*d^4 + b*x^2*e^2 + 2*b*d*x*e + b*d^2 + a)*(a*b^2*e - 4*a^2*c*e))
```

maple [C] time = 0.02, size = 364, normalized size = 1.22

$$\frac{\left(-\mathrm{RootOf}\left(\_Z^4ce^4+4\_Z^3cde^3+cd^4+bd^2+\left(6cd^2e^2+be^2\right)\_Z^2+\left(4cd^3e+2deb\right)\_Z+a\right)^2bce^2-2\,\mathrm{RootOf}\left(\_Z^4ce^4+4\_Z^3cde^3+cd^4+bd^2+\left(6cd^2e^2+be^2\right)\_Z^2+\left(4cd^3e+2deb\right)\_Z+a\right)bcde-bcd^2+6ac-b^2\right)\ln\left(-\mathrm{RootOf}\left(\_Z^4ce^4+4\_Z^3cde^3+cd^4+bd^2+\left(6cd^2e^2+be^2\right)\_Z^2+\left(4cd^3e+2deb\right)\_Z+a\right)+x\right)}{4\left(4ac-b^2\right)ae\left(2ce^3\,\mathrm{RootOf}\left(\_Z^4ce^4+4\_Z^3cde^3+cd^4+bd^2+\left(6cd^2e^2+be^2\right)\_Z^2+\left(4cd^3e+2deb\right)\_Z+a\right)^3+6cde^2\,\mathrm{RootOf}\left(\_Z^4ce^4+4\_Z^3cde^3+cd^4+bd^2+\left(6cd^2e^2+be^2\right)\_Z^2+\left(4cd^3e+2deb\right)\_Z+a\right)^2+6ecd^2\,\mathrm{RootOf}\left(\_Z^4ce^4+4\_Z^3cde^3+cd^4+bd^2+\left(6cd^2e^2+be^2\right)\_Z^2+\left(4cd^3e+2deb\right)\_Z+a\right)+2cd^3+be\,\mathrm{RootOf}\left(\_Z^4ce^4+4\_Z^3cde^3+cd^4+bd^2+\left(6cd^2e^2+be^2\right)\_Z^2+\left(4cd^3e+2deb\right)\_Z+a\right)+bd\right)}+\frac{-\frac{bce^2x^3}{2\left(4ac-b^2\right)a}-\frac{3bcdex^2}{2\left(4ac-b^2\right)a}+\frac{\left(-3bcd^2+2ac-b^2\right)x}{2\left(4ac-b^2\right)a}+\frac{\left(-bcd^2+2ac-b^2\right)d}{2\left(4ac-b^2\right)ae}}{ce^4x^4+4cde^3x^3+6cd^2e^2x^2+4cd^3ex+be^2x^2+cd^4+2bdex+bd^2+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x)
```

```
[Out] (-1/2*b*c*e^2/a/(4*a*c-b^2)*x^3-3/2*d*b*c*e/a/(4*a*c-b^2)*x^2+1/2*(-3*b*c*d
^2+2*a*c-b^2)/a/(4*a*c-b^2)*x+1/2*d/e*(-b*c*d^2+2*a*c-b^2)/a/(4*a*c-b^2))/(
```

```
c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e
*x+b*d^2+a)+1/4/a/(4*a*c-b^2)/e*sum((-_R^2*b*c*e^2-2*_R*b*c*d*e-b*c*d^2+6*a
*c-b^2)/(2*_R^3*c*e^3+6*_R^2*c*d*e^2+6*_R*c*d^2*e+2*c*d^3+_R*b*e+b*d)*ln(-_
R+x),_R=RootOf(_Z^4*c*e^4+4*_Z^3*c*d*e^3+c*d^4+b*d^2+(6*c*d^2*e^2+b*e^2)*_Z
^2+(4*c*d^3*e+2*b*d*e)*_Z+a))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{bce^3x^3 + 3\,bcde^2x^2 + bcd^3 + \left(3\,bcd^2 + b^2 - 2\,ac\right)ex + \left(b^2 - 2\,ac\right)d}{2\left(\left(ab^2c - 4\,a^2c^2\right)e^5x^4 + 4\left(ab^2c - 4\,a^2c^2\right)de^4x^3 + \left(ab^3 - 4\,a^2bc + 6\left(ab^2c - 4\,a^2c^2\right)d^2\right)e^3x^2 + 2\left(2\left(ab^2c - 4\,a^2c^2\right)d^3 + \left(ab^3 - 4\,a^2bc\right)d\right)e^2x + \left(\left(ab^2c - 4\,a^2c^2\right)d^4 + a^2b^2 - 4\,a^3c + \left(ab^3 - 4\,a^2bc\right)d^2\right)e\right)} - \frac{-\int \frac{bce^2x^2+2\,bcdex+bcd^2+b^2-6\,ac}{\left(b^2c-4\,ac^2\right)e^4x^4+4\left(b^2c-4\,ac^2\right)de^3x^3+\left(b^2c-4\,ac^2\right)d^4+\left(b^3-4\,abc+6\left(b^2c-4\,ac^2\right)d^2\right)e^2x^2+ab^2-4\,a^2c+\left(b^3-4\,abc\right)d^2+2\left(2\left(b^2c-4\,ac^2\right)d^3+\left(b^3-4\,abc\right)d\right)ex}\,dx}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(b*c*e^3*x^3 + 3*b*c*d*e^2*x^2 + b*c*d^3 + (3*b*c*d^2 + b^2 - 2*a*c)*e*
x + (b^2 - 2*a*c)*d)/((a*b^2*c - 4*a^2*c^2)*e^5*x^4 + 4*(a*b^2*c - 4*a^2*c^
2)*d*e^4*x^3 + (a*b^3 - 4*a^2*b*c + 6*(a*b^2*c - 4*a^2*c^2)*d^2)*e^3*x^2 +
2*(2*(a*b^2*c - 4*a^2*c^2)*d^3 + (a*b^3 - 4*a^2*b*c)*d)*e^2*x + ((a*b^2*c -
 4*a^2*c^2)*d^4 + a^2*b^2 - 4*a^3*c + (a*b^3 - 4*a^2*b*c)*d^2)*e) - 1/2*int
egrate(-(b*c*e^2*x^2 + 2*b*c*d*e*x + b*c*d^2 + b^2 - 6*a*c)/((b^2*c - 4*a*c
^2)*e^4*x^4 + 4*(b^2*c - 4*a*c^2)*d*e^3*x^3 + (b^2*c - 4*a*c^2)*d^4 + (b^3
- 4*a*b*c + 6*(b^2*c - 4*a*c^2)*d^2)*e^2*x^2 + a*b^2 - 4*a^2*c + (b^3 - 4*a
*b*c)*d^2 + 2*(2*(b^2*c - 4*a*c^2)*d^3 + (b^3 - 4*a*b*c)*d)*e*x), x)/a
```

mupad [B] time = 4.85, size = 9056, normalized size = 30.29

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2,x)
```

```
[Out] atan(((-(b^11 + b^2*(-(4*a*c - b^2)^9)^(1/2) - 3840*a^5*b*c^5 + 288*a^2*b^7
*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 - 27*a*b^9*c - 9*a*c*(-(4*a*c -
b^2)^9)^(1/2))/(32*(a^3*b^12*e^2 + 4096*a^9*c^6*e^2 - 24*a^4*b^10*c*e^2 + 2
40*a^5*b^8*c^2*e^2 - 1280*a^6*b^6*c^3*e^2 + 3840*a^7*b^4*c^4*e^2 - 6144*a^8
*b^2*c^5*e^2)))^(1/2)*(((6144*a^5*c^6*e^12 + 16*a*b^8*c^2*e^12 - 288*a^2*b^
6*c^3*e^12 + 1920*a^3*b^4*c^4*e^12 - 5632*a^4*b^2*c^5*e^12)/(8*(a^2*b^6 - 6
4*a^5*c^3 - 12*a^3*b^4*c + 48*a^4*b^2*c^2)) + ((16384*a^6*b*c^6*d*e^13 + 64
*a^2*b^9*c^2*d*e^13 - 1024*a^3*b^7*c^3*d*e^13 + 6144*a^4*b^5*c^4*d*e^13 - 1
6384*a^5*b^3*c^5*d*e^13)/(8*(a^2*b^6 - 64*a^5*c^3 - 12*a^3*b^4*c + 48*a^4*b
^2*c^2)) - (x*(1024*a^5*b*c^5*e^14 - 16*a^2*b^7*c^2*e^14 + 192*a^3*b^5*c^3*
e^14 - 768*a^4*b^3*c^4*e^14))/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))*(-(
b^11 + b^2*(-(4*a*c - b^2)^9)^(1/2) - 3840*a^5*b*c^5 + 288*a^2*b^7*c^2 - 15
04*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 - 27*a*b^9*c - 9*a*c*(-(4*a*c - b^2)^9)^(
1/2))/(32*(a^3*b^12*e^2 + 4096*a^9*c^6*e^2 - 24*a^4*b^10*c*e^2 + 240*a^5*b^
8*c^2*e^2 - 1280*a^6*b^6*c^3*e^2 + 3840*a^7*b^4*c^4*e^2 - 6144*a^8*b^2*c^5*
e^2)))^(1/2))*(-(b^11 + b^2*(-(4*a*c - b^2)^9)^(1/2) - 3840*a^5*b*c^5 + 288
*a^2*b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 - 27*a*b^9*c - 9*a*c*(-(
4*a*c - b^2)^9)^(1/2))/(32*(a^3*b^12*e^2 + 4096*a^9*c^6*e^2 - 24*a^4*b^10*c
*e^2 + 240*a^5*b^8*c^2*e^2 - 1280*a^6*b^6*c^3*e^2 + 3840*a^7*b^4*c^4*e^2 -
6144*a^8*b^2*c^5*e^2)))^(1/2) - (1152*a^3*c^6*d*e^11 - 4*b^6*c^3*d*e^11 + 7
2*a*b^4*c^4*d*e^11 - 512*a^2*b^2*c^5*d*e^11)/(8*(a^2*b^6 - 64*a^5*c^3 - 12*
a^3*b^4*c + 48*a^4*b^2*c^2)) + (x*(72*a^2*c^5*e^12 + b^4*c^3*e^12 - 14*a*b^
2*c^4*e^12))/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))*1i - (-(b^11 + b^2*(
-(4*a*c - b^2)^9)^(1/2) - 3840*a^5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^3*b^5*c
^3 + 3840*a^4*b^3*c^4 - 27*a*b^9*c - 9*a*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a
^3*b^12*e^2 + 4096*a^9*c^6*e^2 - 24*a^4*b^10*c*e^2 + 240*a^5*b^8*c^2*e^2 -
1280*a^6*b^6*c^3*e^2 + 3840*a^7*b^4*c^4*e^2 - 6144*a^8*b^2*c^5*e^2)))^(1/2)
*(((6144*a^5*c^6*e^12 + 16*a*b^8*c^2*e^12 - 288*a^2*b^6*c^3*e^12 + 1920*a^3
*b^4*c^4*e^12 - 5632*a^4*b^2*c^5*e^12)/(8*(a^2*b^6 - 64*a^5*c^3 - 12*a^3*b^
4*c + 48*a^4*b^2*c^2)) - ((16384*a^6*b*c^6*d*e^13 + 64*a^2*b^9*c^2*d*e^13 -
 1024*a^3*b^7*c^3*d*e^13 + 6144*a^4*b^5*c^4*d*e^13 - 16384*a^5*b^3*c^5*d*e^
```

13)/(8*(a^2*b^6 - 64*a^5*c^3 - 12*a^3*b^4*c + 48*a^4*b^2*c^2)) - (x*(1024*a^5*b*c^5*e^14 - 16*a^2*b^7*c^2*e^14 + 192*a^3*b^5*c^3*e^14 - 768*a^4*b^3*c^4*e^14))/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))*(-(b^11 + b^2*(-(4*a*c - b^2)^9)^(1/2) - 3840*a^5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 - 27*a*b^9*c - 9*a*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^3*b^12*e^2 + 4096*a^9*c^6*e^2 - 24*a^4*b^10*c*e^2 + 240*a^5*b^8*c^2*e^2 - 1280*a^6*b^6*c^3*e^2 + 3840*a^7*b^4*c^4*e^2 - 6144*a^8*b^2*c^5*e^2)))^(1/2))*(-(b^11 + b^2*(-(4*a*c - b^2)^9)^(1/2) - 3840*a^5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 - 27*a*b^9*c - 9*a*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^3*b^12*e^2 + 4096*a^9*c^6*e^2 - 24*a^4*b^10*c*e^2 + 240*a^5*b^8*c^2*e^2 - 1280*a^6*b^6*c^3*e^2 + 3840*a^7*b^4*c^4*e^2 - 6144*a^8*b^2*c^5*e^2)))^(1/2) + (1152*a^3*c^6*d*e^11 - 4*b^6*c^3*d*e^11 + 72*a*b^4*c^4*d*e^11 - 512*a^2*b^2*c^5*d*e^11)/(8*(a^2*b^6 - 64*a^5*c^3 - 12*a^3*b^4*c + 48*a^4*b^2*c^2)) - (x*(72*a^2*c^5*e^12 + b^4*c^3*e^12 - 14*a*b^2*c^4*e^12))/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))*1i)/((-(b^11 + b^2*(-(4*a*c - b^2)^9)^(1/2) - 3840*a^5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 - 27*a*b^9*c - 9*a*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^3*b^12*e^2 + 4096*a^9*c^6*e^2 - 24*a^4*b^10*c*e^2 + 240*a^5*b^8*c^2*e^2 - 1280*a^6*b^6*c^3*e^2 + 3840*a^7*b^4*c^4*e^2 - 6144*a^8*b^2*c^5*e^2)))^(1/2)*(((6144*a^5*c^6*e^12 + 16*a*b^8*c^2*e^12 - 288*a^2*b^6*c^3*e^12 + 1920*a^3*b^4*c^4*e^12 - 5632*a^4*b^2*c^5*e^12)/(8*(a^2*b^6 - 64*a^5*c^3 - 12*a^3*b^4*c + 48*a^4*b^2*c^2)) + ((16384*a^6*b*c^6*d*e^13 + 64*a^2*b^9*c^2*d*e^13 - 1024*a^3*b^7*c^3*d*e^13 + 6144*a^4*b^5*c^4*d*e^13 - 16384*a^5*b^3*c^5*d*e^13)/(8*(a^2*b^6 - 64*a^5*c^3 - 12*a^3*b^4*c + 48*a^4*b^2*c^2)) - (x*(1024*a^5*b*c^5*e^14 - 16*a^2*b^7*c^2*e^14 + 192*a^3*b^5*c^3*e^14 - 768*a^4*b^3*c^4*e^14))/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))*(-(b^11 + b^2*(-(4*a*c - b^2)^9)^(1/2) - 3840*a^5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 - 27*a*b^9*c - 9*a*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^3*b^12*e^2 + 4096*a^9*c^6*e^2 - 24*a^4*b^10*c*e^2 + 240*a^5*b^8*c^2*e^2 - 1280*a^6*b^6*c^3*e^2 + 3840*a^7*b^4*c^4*e^2 - 6144*a^8*b^2*c^5*e^2)))^(1/2))*(-(b^11 + b^2*(-(4*a*c - b^2)^9)^(1/2) - 3840*a^5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 - 27*a*b^9*c - 9*a*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^3*b^12*e^2 + 4096*a^9*c^6*e^2 - 24*a^4*b^10*c*e^2 + 240*a^5*b^8*c^2*e^2 - 1280*a^6*b^6*c^3*e^2 + 3840*a^7*b^4*c^4*e^2 - 6144*a^8*b^2*c^5*e^2)))^(1/2) - (1152*a^3*c^6*d*e^11 - 4*b^6*c^3*d*e^11 + 72*a*b^4*c^4*d*e^11 - 512*a^2*b^2*c^5*d*e^11)/(8*(a^2*b^6 - 64*a^5*c^3 - 12*a^3*b^4*c + 48*a^4*b^2*c^2)) + (x*(72*a^2*c^5*e^12 + b^4*c^3*e^12 - 14*a*b^2*c^4*e^12))/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c))) + (-(b^11 + b^2*(-(4*a*c - b^2)^9)^(1/2) - 3840*a^5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 - 27*a*b^9*c - 9*a*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^3*b^12*e^2 + 4096*a^9*c^6*e^2 - 24*a^4*b^10*c*e^2 + 240*a^5*b^8*c^2*e^2 - 1280*a^6*b^6*c^3*e^2 + 3840*a^7*b^4*c^4*e^2 - 6144*a^8*b^2*c^5*e^2)))^(1/2)*(((6144*a^5*c^6*e^12 + 16*a*b^8*c^2*e^12 - 288*a^2*b^6*c^3*e^12 + 1920*a^3*b^4*c^4*e^12 - 5632*a^4*b^2*c^5*e^12)/(8*(a^2*b^6 - 64*a^5*c^3 - 12*a^3*b^4*c + 48*a^4*b^2*c^2)) - ((16384*a^6*b*c^6*d*e^13 + 64*a^2*b^9*c^2*d*e^13 - 1024*a^3*b^7*c^3*d*e^13 + 6144*a^4*b^5*c^4*d*e^13 - 16384*a^5*b^3*c^5*d*e^13)/(8*(a^2*b^6 - 64*a^5*c^3 - 12*a^3*b^4*c + 48*a^4*b^2*c^2)) - (x*(1024*a^5*b*c^5*e^14 - 16*a^2*b^7*c^2*e^14 + 192*a^3*b^5*c^3*e^14 - 768*a^4*b^3*c^4*e^14))/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))*(-(b^11 + b^2*(-(4*a*c - b^2)^9)^(1/2) - 3840*a^5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 - 27*a*b^9*c - 9*a*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^3*b^12*e^2 + 4096*a^9*c^6*e^2 - 24*a^4*b^10*c*e^2 + 240*a^5*b^8*c^2*e^2 - 1280*a^6*b^6*c^3*e^2 + 3840*a^7*b^4*c^4*e^2 - 6144*a^8*b^2*c^5*e^2)))^(1/2))*(-(b^11 + b^2*(-(4*a*c - b^2)^9)^(1/2) - 3840*a^5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 - 27*a*b^9*c - 9*a*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^3*b^12*e^2 + 4096*a^9*c^6*e^2 - 24*a^4*b^10*c*e^2 + 240*a^5*b^8*c^2*e^2 - 1280*a^6*b^6*c^3*e^2 + 3840*a^7*b^4*c^4*e^2 - 6144*a^8*b^2*c^5*e^2)))^(1/2) + (1152*a^3*c^6*d*e^11 - 4*b^6*c^3*d*e^11 + 72*a*b^4*c^4*d*e^11 - 512*a^2*b^2*c^5*d*e^11)/(8*(a^2*b^6 - 64*a^5*c^3 - 12*a^3*b^4*c + 48*a^4*b^2*c^2)) - (x*(72*a^2*c^5*e^12 + b^4*c^3*e^12

- 14*a*b^2*c^4*e^12))/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c))) + (5*b^3*c^
4*e^10 - 36*a*b*c^5*e^10)/(4*(a^2*b^6 - 64*a^5*c^3 - 12*a^3*b^4*c + 48*a^4*
b^2*c^2))))*(-(b^11 + b^2*(-(4*a*c - b^2)^9)^(1/2) - 3840*a^5*b*c^5 + 288*a
^2*b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 - 27*a*b^9*c - 9*a*c*(-(4*
a*c - b^2)^9)^(1/2))/(32*(a^3*b^12*e^2 + 4096*a^9*c^6*e^2 - 24*a^4*b^10*c*e
^2 + 240*a^5*b^8*c^2*e^2 - 1280*a^6*b^6*c^3*e^2 + 3840*a^7*b^4*c^4*e^2 - 61
44*a^8*b^2*c^5*e^2)))^(1/2)*2i + atan(((-(b^11 - b^2*(-(4*a*c - b^2)^9)^(1/
2) - 3840*a^5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4*b^3*c^4
 - 27*a*b^9*c + 9*a*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^3*b^12*e^2 + 4096*a^
9*c^6*e^2 - 24*a^4*b^10*c*e^2 + 240*a^5*b^8*c^2*e^2 - 1280*a^6*b^6*c^3*e^2
+ 3840*a^7*b^4*c^4*e^2 - 6144*a^8*b^2*c^5*e^2)))^(1/2)*(((6144*a^5*c^6*e^12
 + 16*a*b^8*c^2*e^12 - 288*a^2*b^6*c^3*e^12 + 1920*a^3*b^4*c^4*e^12 - 5632*
a^4*b^2*c^5*e^12)/(8*(a^2*b^6 - 64*a^5*c^3 - 12*a^3*b^4*c + 48*a^4*b^2*c^2)
) + ((16384*a^6*b*c^6*d*e^13 + 64*a^2*b^9*c^2*d*e^13 - 1024*a^3*b^7*c^3*d*e
^13 + 6144*a^4*b^5*c^4*d*e^13 - 16384*a^5*b^3*c^5*d*e^13)/(8*(a^2*b^6 - 64*
a^5*c^3 - 12*a^3*b^4*c + 48*a^4*b^2*c^2)) - (x*(1024*a^5*b*c^5*e^14 - 16*a^
2*b^7*c^2*e^14 + 192*a^3*b^5*c^3*e^14 - 768*a^4*b^3*c^4*e^14))/(2*(a^2*b^4
+ 16*a^4*c^2 - 8*a^3*b^2*c)))*(-(b^11 - b^2*(-(4*a*c - b^2)^9)^(1/2) - 3840
*a^5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 - 27*a*b
^9*c + 9*a*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^3*b^12*e^2 + 4096*a^9*c^6*e^2
 - 24*a^4*b^10*c*e^2 + 240*a^5*b^8*c^2*e^2 - 1280*a^6*b^6*c^3*e^2 + 3840*a^
7*b^4*c^4*e^2 - 6144*a^8*b^2*c^5*e^2)))^(1/2))*(-(b^11 - b^2*(-(4*a*c - b^2
)^9)^(1/2) - 3840*a^5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4
*b^3*c^4 - 27*a*b^9*c + 9*a*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^3*b^12*e^2 +
 4096*a^9*c^6*e^2 - 24*a^4*b^10*c*e^2 + 240*a^5*b^8*c^2*e^2 - 1280*a^6*b^6*
c^3*e^2 + 3840*a^7*b^4*c^4*e^2 - 6144*a^8*b^2*c^5*e^2)))^(1/2) - (1152*a^3*
c^6*d*e^11 - 4*b^6*c^3*d*e^11 + 72*a*b^4*c^4*d*e^11 - 512*a^2*b^2*c^5*d*e^1
1)/(8*(a^2*b^6 - 64*a^5*c^3 - 12*a^3*b^4*c + 48*a^4*b^2*c^2)) + (x*(72*a^2*
c^5*e^12 + b^4*c^3*e^12 - 14*a*b^2*c^4*e^12))/(2*(a^2*b^4 + 16*a^4*c^2 - 8*
a^3*b^2*c)))*1i - (-(b^11 - b^2*(-(4*a*c - b^2)^9)^(1/2) - 3840*a^5*b*c^5 +
 288*a^2*b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 - 27*a*b^9*c + 9*a*c
*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^3*b^12*e^2 + 4096*a^9*c^6*e^2 - 24*a^4*b^
10*c*e^2 + 240*a^5*b^8*c^2*e^2 - 1280*a^6*b^6*c^3*e^2 + 3840*a^7*b^4*c^4*e^
2 - 6144*a^8*b^2*c^5*e^2)))^(1/2)*(((6144*a^5*c^6*e^12 + 16*a*b^8*c^2*e^12
- 288*a^2*b^6*c^3*e^12 + 1920*a^3*b^4*c^4*e^12 - 5632*a^4*b^2*c^5*e^12)/(8*
(a^2*b^6 - 64*a^5*c^3 - 12*a^3*b^4*c + 48*a^4*b^2*c^2)) - ((16384*a^6*b*c^6
*d*e^13 + 64*a^2*b^9*c^2*d*e^13 - 1024*a^3*b^7*c^3*d*e^13 + 6144*a^4*b^5*c^
4*d*e^13 - 16384*a^5*b^3*c^5*d*e^13)/(8*(a^2*b^6 - 64*a^5*c^3 - 12*a^3*b^4*
c + 48*a^4*b^2*c^2)) - (x*(1024*a^5*b*c^5*e^14 - 16*a^2*b^7*c^2*e^14 + 192*
a^3*b^5*c^3*e^14 - 768*a^4*b^3*c^4*e^14))/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*
b^2*c)))*(-(b^11 - b^2*(-(4*a*c - b^2)^9)^(1/2) - 3840*a^5*b*c^5 + 288*a^2*
b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 - 27*a*b^9*c + 9*a*c*(-(4*a*c
 - b^2)^9)^(1/2))/(32*(a^3*b^12*e^2 + 4096*a^9*c^6*e^2 - 24*a^4*b^10*c*e^2
+ 240*a^5*b^8*c^2*e^2 - 1280*a^6*b^6*c^3*e^2 + 3840*a^7*b^4*c^4*e^2 - 6144*
a^8*b^2*c^5*e^2)))^(1/2))*(-(b^11 - b^2*(-(4*a*c - b^2)^9)^(1/2) - 3840*a^5
*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 - 27*a*b^9*c
 + 9*a*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^3*b^12*e^2 + 4096*a^9*c^6*e^2 - 2
4*a^4*b^10*c*e^2 + 240*a^5*b^8*c^2*e^2 - 1280*a^6*b^6*c^3*e^2 + 3840*a^7*b^
4*c^4*e^2 - 6144*a^8*b^2*c^5*e^2)))^(1/2) + (1152*a^3*c^6*d*e^11 - 4*b^6*c^
3*d*e^11 + 72*a*b^4*c^4*d*e^11 - 512*a^2*b^2*c^5*d*e^11)/(8*(a^2*b^6 - 64*a
^5*c^3 - 12*a^3*b^4*c + 48*a^4*b^2*c^2)) - (x*(72*a^2*c^5*e^12 + b^4*c^3*e^
12 - 14*a*b^2*c^4*e^12))/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))*1i)/((-(
b^11 - b^2*(-(4*a*c - b^2)^9)^(1/2) - 3840*a^5*b*c^5 + 288*a^2*b^7*c^2 - 15
04*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 - 27*a*b^9*c + 9*a*c*(-(4*a*c - b^2)^9)^(
1/2))/(32*(a^3*b^12*e^2 + 4096*a^9*c^6*e^2 - 24*a^4*b^10*c*e^2 + 240*a^5*b^
8*c^2*e^2 - 1280*a^6*b^6*c^3*e^2 + 3840*a^7*b^4*c^4*e^2 - 6144*a^8*b^2*c^5*
e^2)))^(1/2)*(((6144*a^5*c^6*e^12 + 16*a*b^8*c^2*e^12 - 288*a^2*b^6*c^3*e^1
2 + 1920*a^3*b^4*c^4*e^12 - 5632*a^4*b^2*c^5*e^12)/(8*(a^2*b^6 - 64*a^5*c^3
 - 12*a^3*b^4*c + 48*a^4*b^2*c^2)) + ((16384*a^6*b*c^6*d*e^13 + 64*a^2*b^9*

```
c^2*d*e^13 - 1024*a^3*b^7*c^3*d*e^13 + 6144*a^4*b^5*c^4*d*e^13 - 16384*a^5*
b^3*c^5*d*e^13)/(8*(a^2*b^6 - 64*a^5*c^3 - 12*a^3*b^4*c + 48*a^4*b^2*c^2))
- (x*(1024*a^5*b*c^5*e^14 - 16*a^2*b^7*c^2*e^14 + 192*a^3*b^5*c^3*e^14 - 76
8*a^4*b^3*c^4*e^14))/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))*(-(b^11 - b^
2*(-(4*a*c - b^2)^9)^(1/2) - 3840*a^5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^3*b^
5*c^3 + 3840*a^4*b^3*c^4 - 27*a*b^9*c + 9*a*c*(-(4*a*c - b^2)^9)^(1/2))/(32
*(a^3*b^12*e^2 + 4096*a^9*c^6*e^2 - 24*a^4*b^10*c*e^2 + 240*a^5*b^8*c^2*e^2
 - 1280*a^6*b^6*c^3*e^2 + 3840*a^7*b^4*c^4*e^2 - 6144*a^8*b^2*c^5*e^2)))^(1
/2))*(-(b^11 - b^2*(-(4*a*c - b^2)^9)^(1/2) - 3840*a^5*b*c^5 + 288*a^2*b^7*
c^2 - 1504*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 - 27*a*b^9*c + 9*a*c*(-(4*a*c - b
^2)^9)^(1/2))/(32*(a^3*b^12*e^2 + 4096*a^9*c^6*e^2 - 24*a^4*b^10*c*e^2 + 24
0*a^5*b^8*c^2*e^2 - 1280*a^6*b^6*c^3*e^2 + 3840*a^7*b^4*c^4*e^2 - 6144*a^8*
b^2*c^5*e^2)))^(1/2) - (1152*a^3*c^6*d*e^11 - 4*b^6*c^3*d*e^11 + 72*a*b^4*c
^4*d*e^11 - 512*a^2*b^2*c^5*d*e^11)/(8*(a^2*b^6 - 64*a^5*c^3 - 12*a^3*b^4*c
 + 48*a^4*b^2*c^2)) + (x*(72*a^2*c^5*e^12 + b^4*c^3*e^12 - 14*a*b^2*c^4*e^1
2))/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c))) + (-(b^11 - b^2*(-(4*a*c - b^
2)^9)^(1/2) - 3840*a^5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*a^
4*b^3*c^4 - 27*a*b^9*c + 9*a*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^3*b^12*e^2
+ 4096*a^9*c^6*e^2 - 24*a^4*b^10*c*e^2 + 240*a^5*b^8*c^2*e^2 - 1280*a^6*b^6
*c^3*e^2 + 3840*a^7*b^4*c^4*e^2 - 6144*a^8*b^2*c^5*e^2)))^(1/2)*(((6144*a^5
*c^6*e^12 + 16*a*b^8*c^2*e^12 - 288*a^2*b^6*c^3*e^12 + 1920*a^3*b^4*c^4*e^1
2 - 5632*a^4*b^2*c^5*e^12)/(8*(a^2*b^6 - 64*a^5*c^3 - 12*a^3*b^4*c + 48*a^4
*b^2*c^2)) - ((16384*a^6*b*c^6*d*e^13 + 64*a^2*b^9*c^2*d*e^13 - 1024*a^3*b^
7*c^3*d*e^13 + 6144*a^4*b^5*c^4*d*e^13 - 16384*a^5*b^3*c^5*d*e^13)/(8*(a^2*
b^6 - 64*a^5*c^3 - 12*a^3*b^4*c + 48*a^4*b^2*c^2)) - (x*(1024*a^5*b*c^5*e^1
4 - 16*a^2*b^7*c^2*e^14 + 192*a^3*b^5*c^3*e^14 - 768*a^4*b^3*c^4*e^14))/(2*
(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))*(-(b^11 - b^2*(-(4*a*c - b^2)^9)^(1/
2) - 3840*a^5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4*b^3*c^4
 - 27*a*b^9*c + 9*a*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^3*b^12*e^2 + 4096*a^
9*c^6*e^2 - 24*a^4*b^10*c*e^2 + 240*a^5*b^8*c^2*e^2 - 1280*a^6*b^6*c^3*e^2
+ 3840*a^7*b^4*c^4*e^2 - 6144*a^8*b^2*c^5*e^2)))^(1/2))*(-(b^11 - b^2*(-(4*
a*c - b^2)^9)^(1/2) - 3840*a^5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^3*b^5*c^3 +
 3840*a^4*b^3*c^4 - 27*a*b^9*c + 9*a*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^3*b
^12*e^2 + 4096*a^9*c^6*e^2 - 24*a^4*b^10*c*e^2 + 240*a^5*b^8*c^2*e^2 - 1280
*a^6*b^6*c^3*e^2 + 3840*a^7*b^4*c^4*e^2 - 6144*a^8*b^2*c^5*e^2)))^(1/2) + (
1152*a^3*c^6*d*e^11 - 4*b^6*c^3*d*e^11 + 72*a*b^4*c^4*d*e^11 - 512*a^2*b^2*
c^5*d*e^11)/(8*(a^2*b^6 - 64*a^5*c^3 - 12*a^3*b^4*c + 48*a^4*b^2*c^2)) - (x
*(72*a^2*c^5*e^12 + b^4*c^3*e^12 - 14*a*b^2*c^4*e^12))/(2*(a^2*b^4 + 16*a^4
*c^2 - 8*a^3*b^2*c))) + (5*b^3*c^4*e^10 - 36*a*b*c^5*e^10)/(4*(a^2*b^6 - 64
*a^5*c^3 - 12*a^3*b^4*c + 48*a^4*b^2*c^2))))*(-(b^11 - b^2*(-(4*a*c - b^2)^
9)^(1/2) - 3840*a^5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4*b
^3*c^4 - 27*a*b^9*c + 9*a*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^3*b^12*e^2 + 4
096*a^9*c^6*e^2 - 24*a^4*b^10*c*e^2 + 240*a^5*b^8*c^2*e^2 - 1280*a^6*b^6*c^
3*e^2 + 3840*a^7*b^4*c^4*e^2 - 6144*a^8*b^2*c^5*e^2)))^(1/2)*2i - ((b^2*d -
 2*a*c*d + b*c*d^3)/(2*a*e*(4*a*c - b^2)) + (x*(b^2 - 2*a*c + 3*b*c*d^2))/(
2*a*(4*a*c - b^2)) + (b*c*e^2*x^3)/(2*a*(4*a*c - b^2)) + (3*b*c*d*e*x^2)/(2
*a*(4*a*c - b^2)))/(a + x^2*(b*e^2 + 6*c*d^2*e^2) + b*d^2 + c*d^4 + x*(2*b*
d*e + 4*c*d^3*e) + c*e^4*x^4 + 4*c*d*e^3*x^3)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*(e*x+d)**2+c*(e*x+d)**4)**2,x)
[Out] Timed out
```

3.513 $\int \frac{1}{(d+ex)\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx$

Optimal. Leaf size=162

$$\frac{b\left(b^2-6ac\right)\tanh^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{2a^2e\left(b^2-4ac\right)^{3/2}}-\frac{\log\left(a+b(d+ex)^2+c(d+ex)^4\right)}{4a^2e}+\frac{\log(d+ex)}{a^2e}+\frac{-2ac+b^2+bc(d}{2ae\left(b^2-4ac\right)\left(a+b(d+ex\right.}$$

Rubi [A] time = 0.29, antiderivative size = 162, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {1142, 1114, 740, 800, 634, 618, 206, 628}

$$\frac{b\left(b^2-6ac\right)\tanh^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{2a^2e\left(b^2-4ac\right)^{3/2}}-\frac{\log\left(a+b(d+ex)^2+c(d+ex)^4\right)}{4a^2e}+\frac{\log(d+ex)}{a^2e}+\frac{-2ac+b^2+bc(d+ex)^2}{2ae\left(b^2-4ac\right)\left(a+b(d+ex)^2+c(d+ex)^4\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2),x]
```

```
[Out] (b^2 - 2*a*c + b*c*(d + e*x)^2)/(2*a*(b^2 - 4*a*c)*e*(a + b*(d + e*x)^2 + c
*(d + e*x)^4)) + (b*(b^2 - 6*a*c)*ArcTanh[(b + 2*c*(d + e*x)^2)/Sqrt[b^2 -
4*a*c]])/(2*a^2*(b^2 - 4*a*c)^(3/2)*e) + Log[d + e*x]/(a^2*e) - Log[a + b*(
d + e*x)^2 + c*(d + e*x)^4]/(4*a^2*e)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 740

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e
)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e
^2)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d +
 e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p +
 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p,
```

```
-1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 1142

```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Di
st[u^m/(Coefficient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p,
 x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

Rubi steps

$$\int \frac{1}{(d+ex)\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x\left(a+bx^2+cx^4\right)^2}\,dx, x, d+ex\right)}{e}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{x\left(a+bx+cx^2\right)^2}\,dx, x, (d+ex)^2\right)}{2e}$$

$$= \frac{b^2-2ac+bc(d+ex)^2}{2a\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)} - \frac{\text{Subst}\left(\int \frac{-b^2+4ac-b}{x(a+bx+cx}\right.}{2a\left(b^2 -\right.}$$

$$= \frac{b^2-2ac+bc(d+ex)^2}{2a\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)} - \frac{\text{Subst}\left(\int \left(\frac{-b^2+4ac}{ax}\right.\right.}{}$$

$$= \frac{b^2-2ac+bc(d+ex)^2}{2a\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)} + \frac{\log(d+ex)}{a^2e} - \frac{\text{Sub}}{}$$

$$= \frac{b^2-2ac+bc(d+ex)^2}{2a\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)} + \frac{\log(d+ex)}{a^2e} - \frac{\text{Sub}}{}$$

$$= \frac{b^2-2ac+bc(d+ex)^2}{2a\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)} + \frac{\log(d+ex)}{a^2e} - \frac{\log}{}$$

$$= \frac{b^2-2ac+bc(d+ex)^2}{2a\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)} + \frac{b\left(b^2-6ac\right)\tanh^{-}}{2a^2\left(b^2-4a\right.}$$

Mathematica [A] time = 0.47, size = 235, normalized size = 1.45

$$\frac{\frac{2a\left(-2ac+b^2+bc(d+ex)^2\right)}{\left(b^2-4ac\right)\left(a+(d+ex)^2\left(b+c(d+ex)^2\right)\right)} - \frac{\left(b^2\sqrt{b^2-4ac}-4ac\sqrt{b^2-4ac}-6abc+b^3\right)\log\left(-\sqrt{b^2-4ac}+b+2c(d+ex)^2\right)}{\left(b^2-4ac\right)^{3/2}} + \frac{\left(-b^2\sqrt{b^2-4ac}+4ac\sqrt{b^2-4ac}-6abc+b^3\right)\log\left(\sqrt{b^2-4ac}+b+2c(d+ex)^2\right)}{\left(b^2-4ac\right)^{3/2}} + 4\log(d+ex)}{4a^2e}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2),x]
```

```
[Out] ((2*a*(b^2 - 2*a*c + b*c*(d + e*x)^2))/((b^2 - 4*a*c)*(a + (d + e*x)^2*(b +
 c*(d + e*x)^2))) + 4*Log[d + e*x] - ((b^3 - 6*a*b*c + b^2*Sqrt[b^2 - 4*a*c
] - 4*a*c*Sqrt[b^2 - 4*a*c])*Log[b - Sqrt[b^2 - 4*a*c] + 2*c*(d + e*x)^2])/
(b^2 - 4*a*c)^(3/2) + ((b^3 - 6*a*b*c - b^2*Sqrt[b^2 - 4*a*c] + 4*a*c*Sqrt[
b^2 - 4*a*c])*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*(d + e*x)^2])/(b^2 - 4*a*c)^(
3/2))/(4*a^2*e)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(d+ex)\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/((d + e*x)*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2),x]
```

```
[Out] IntegrateAlgebraic[1/((d + e*x)*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2), x]
```

fricas [B] time = 1.79, size = 2476, normalized size = 15.28

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="fricas")
```

```
[Out] [1/4*(2*a*b^4 - 12*a^2*b^2*c + 16*a^3*c^2 + 2*(a*b^3*c - 4*a^2*b*c^2)*e^2*x
^2 + 4*(a*b^3*c - 4*a^2*b*c^2)*d*e*x + 2*(a*b^3*c - 4*a^2*b*c^2)*d^2 + ((b^
3*c - 6*a*b*c^2)*e^4*x^4 + 4*(b^3*c - 6*a*b*c^2)*d*e^3*x^3 + (b^3*c - 6*a*b
*c^2)*d^4 + (b^4 - 6*a*b^2*c + 6*(b^3*c - 6*a*b*c^2)*d^2)*e^2*x^2 + a*b^3 -
 6*a^2*b*c + (b^4 - 6*a*b^2*c)*d^2 + 2*(2*(b^3*c - 6*a*b*c^2)*d^3 + (b^4 -
6*a*b^2*c)*d)*e*x)*sqrt(b^2 - 4*a*c)*log((2*c^2*e^4*x^4 + 8*c^2*d*e^3*x^3 +
 2*c^2*d^4 + 2*(6*c^2*d^2 + b*c)*e^2*x^2 + 2*b*c*d^2 + 4*(2*c^2*d^3 + b*c*d
)*e*x + b^2 - 2*a*c + (2*c*e^2*x^2 + 4*c*d*e*x + 2*c*d^2 + b)*sqrt(b^2 - 4*
a*c))/(c*e^4*x^4 + 4*c*d*e^3*x^3 + c*d^4 + (6*c*d^2 + b)*e^2*x^2 + b*d^2 +
2*(2*c*d^3 + b*d)*e*x + a)) - ((b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*e^4*x^4 +
 4*(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*d*e^3*x^3 + a*b^4 - 8*a^2*b^2*c + 16*
a^3*c^2 + (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*d^4 + (b^5 - 8*a*b^3*c + 16*a^
2*b*c^2 + 6*(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*d^2)*e^2*x^2 + (b^5 - 8*a*b^
3*c + 16*a^2*b*c^2)*d^2 + 2*(2*(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*d^3 + (b^
5 - 8*a*b^3*c + 16*a^2*b*c^2)*d)*e*x)*log(c*e^4*x^4 + 4*c*d*e^3*x^3 + c*d^4
 + (6*c*d^2 + b)*e^2*x^2 + b*d^2 + 2*(2*c*d^3 + b*d)*e*x + a) + 4*((b^4*c -
 8*a*b^2*c^2 + 16*a^2*c^3)*e^4*x^4 + 4*(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*d
*e^3*x^3 + a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2 + (b^4*c - 8*a*b^2*c^2 + 16*a^2
*c^3)*d^4 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2 + 6*(b^4*c - 8*a*b^2*c^2 + 16*a
^2*c^3)*d^2)*e^2*x^2 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*d^2 + 2*(2*(b^4*c -
 8*a*b^2*c^2 + 16*a^2*c^3)*d^3 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*d)*e*x)*l
og(e*x + d))/((a^2*b^4*c - 8*a^3*b^2*c^2 + 16*a^4*c^3)*e^5*x^4 + 4*(a^2*b^4
*c - 8*a^3*b^2*c^2 + 16*a^4*c^3)*d*e^4*x^3 + (a^2*b^5 - 8*a^3*b^3*c + 16*a^
4*b*c^2 + 6*(a^2*b^4*c - 8*a^3*b^2*c^2 + 16*a^4*c^3)*d^2)*e^3*x^2 + 2*(2*(a
^2*b^4*c - 8*a^3*b^2*c^2 + 16*a^4*c^3)*d^3 + (a^2*b^5 - 8*a^3*b^3*c + 16*a^
4*b*c^2)*d)*e^2*x + (a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2 + (a^2*b^4*c - 8*a^
3*b^2*c^2 + 16*a^4*c^3)*d^4 + (a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*d^2)*e
), 1/4*(2*a*b^4 - 12*a^2*b^2*c + 16*a^3*c^2 + 2*(a*b^3*c - 4*a^2*b*c^2)*e^2
*x^2 + 4*(a*b^3*c - 4*a^2*b*c^2)*d*e*x + 2*(a*b^3*c - 4*a^2*b*c^2)*d^2 + 2*
((b^3*c - 6*a*b*c^2)*e^4*x^4 + 4*(b^3*c - 6*a*b*c^2)*d*e^3*x^3 + (b^3*c - 6
*a*b*c^2)*d^4 + (b^4 - 6*a*b^2*c + 6*(b^3*c - 6*a*b*c^2)*d^2)*e^2*x^2 + a*b
^3 - 6*a^2*b*c + (b^4 - 6*a*b^2*c)*d^2 + 2*(2*(b^3*c - 6*a*b*c^2)*d^3 + (b^
```

```
4 - 6*a*b^2*c)*d)*e*x)*sqrt(-b^2 + 4*a*c)*arctan(-(2*c*e^2*x^2 + 4*c*d*e*x
+ 2*c*d^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) - ((b^4*c - 8*a*b^2*c^2 +
16*a^2*c^3)*e^4*x^4 + 4*(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*d*e^3*x^3 + a*b^
4 - 8*a^2*b^2*c + 16*a^3*c^2 + (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*d^4 + (b^
5 - 8*a*b^3*c + 16*a^2*b*c^2 + 6*(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*d^2)*e^
2*x^2 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*d^2 + 2*(2*(b^4*c - 8*a*b^2*c^2 +
16*a^2*c^3)*d^3 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*d)*e*x)*log(c*e^4*x^4 +
4*c*d*e^3*x^3 + c*d^4 + (6*c*d^2 + b)*e^2*x^2 + b*d^2 + 2*(2*c*d^3 + b*d)*e
*x + a) + 4*((b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*e^4*x^4 + 4*(b^4*c - 8*a*b^
2*c^2 + 16*a^2*c^3)*d*e^3*x^3 + a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2 + (b^4*c -
 8*a*b^2*c^2 + 16*a^2*c^3)*d^4 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2 + 6*(b^4*c
 - 8*a*b^2*c^2 + 16*a^2*c^3)*d^2)*e^2*x^2 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2
)*d^2 + 2*(2*(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*d^3 + (b^5 - 8*a*b^3*c + 16
*a^2*b*c^2)*d)*e*x)*log(e*x + d))/((a^2*b^4*c - 8*a^3*b^2*c^2 + 16*a^4*c^3)
*e^5*x^4 + 4*(a^2*b^4*c - 8*a^3*b^2*c^2 + 16*a^4*c^3)*d*e^4*x^3 + (a^2*b^5
- 8*a^3*b^3*c + 16*a^4*b*c^2 + 6*(a^2*b^4*c - 8*a^3*b^2*c^2 + 16*a^4*c^3)*d
^2)*e^3*x^2 + 2*(2*(a^2*b^4*c - 8*a^3*b^2*c^2 + 16*a^4*c^3)*d^3 + (a^2*b^5
- 8*a^3*b^3*c + 16*a^4*b*c^2)*d)*e^2*x + (a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^
2 + (a^2*b^4*c - 8*a^3*b^2*c^2 + 16*a^4*c^3)*d^4 + (a^2*b^5 - 8*a^3*b^3*c +
 16*a^4*b*c^2)*d^2)*e)]
```

giac [B] time = 1.25, size = 454, normalized size = 2.80

$$-\frac{(a^2b^3ce^3 - 6a^3bc^2e^3)\sqrt{b^2-4ac}\log\left(\left|bx^2e^2 + 2bdxe + \sqrt{b^2-4ac}x^2e^2 + 2\sqrt{b^2-4ac}dxe + bd^2 + \sqrt{b^2-4ac}d^2 + 2a\right|\right) - (a^2b^3ce^3 - 6a^3bc^2e^3)\sqrt{b^2-4ac}\log\left(\left|-bx^2e^2 - 2bdxe + \sqrt{b^2-4ac}x^2e^2 + 2\sqrt{b^2-4ac}dxe - bd^2 + \sqrt{b^2-4ac}d^2 - 2a\right|\right)}{4(a^4b^4ce^4 - 8a^5b^2c^2e^4 + 16a^6c^3e^4)} - \frac{e^{(-1)}\log\left(\left|cx^4e^4 + 4cdx^3e^3 + 6cd^2x^2e^2 + 4cd^3xe + cd^4 + bx^2e^2 + 2bdxe + bd^2 + a\right|\right)}{4a^2} + \frac{e^{(-1)}\log(|xe+d|)}{a^2} + \frac{(abcx^2e^2 + 2abcdxe + abcd^2 + ab^2 - 2a^2c)e^{(-1)}}{2(cx^4e^4 + 4cdx^3e^3 + 6cd^2x^2e^2 + 4cd^3xe + cd^4 + bx^2e^2 + 2bdxe + bd^2 + a)(b^2-4ac)a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="giac")
```

```
[Out] -1/4*((a^2*b^3*c*e^3 - 6*a^3*b*c^2*e^3)*sqrt(b^2 - 4*a*c)*log(abs(b*x^2*e^2
 + 2*b*d*x*e + sqrt(b^2 - 4*a*c)*x^2*e^2 + 2*sqrt(b^2 - 4*a*c)*d*x*e + b*d^
2 + sqrt(b^2 - 4*a*c)*d^2 + 2*a)) - (a^2*b^3*c*e^3 - 6*a^3*b*c^2*e^3)*sqrt(
b^2 - 4*a*c)*log(abs(-b*x^2*e^2 - 2*b*d*x*e + sqrt(b^2 - 4*a*c)*x^2*e^2 + 2
*sqrt(b^2 - 4*a*c)*d*x*e - b*d^2 + sqrt(b^2 - 4*a*c)*d^2 - 2*a)))/(a^4*b^4*
c*e^4 - 8*a^5*b^2*c^2*e^4 + 16*a^6*c^3*e^4) - 1/4*e^(-1)*log(abs(c*x^4*e^4
+ 4*c*d*x^3*e^3 + 6*c*d^2*x^2*e^2 + 4*c*d^3*x*e + c*d^4 + b*x^2*e^2 + 2*b*d
*x*e + b*d^2 + a))/a^2 + e^(-1)*log(abs(x*e + d))/a^2 + 1/2*(a*b*c*x^2*e^2
+ 2*a*b*c*d*x*e + a*b*c*d^2 + a*b^2 - 2*a^2*c)*e^(-1)/((c*x^4*e^4 + 4*c*d*x
^3*e^3 + 6*c*d^2*x^2*e^2 + 4*c*d^3*x*e + c*d^4 + b*x^2*e^2 + 2*b*d*x*e + b*
d^2 + a)*(b^2 - 4*a*c)*a^2)
```

maple [C] time = 0.03, size = 693, normalized size = 4.28

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x)
```

```
[Out] -1/2/a/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4
+2*b*d*e*x+b*d^2+a)*b*c*e/(4*a*c-b^2)*x^2-1/a/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*
d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)*b*c*d/(4*a*c-b^2
)*x-1/2/a/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*
d^4+2*b*d*e*x+b*d^2+a)/e/(4*a*c-b^2)*b*c*d^2+1/(c*e^4*x^4+4*c*d*e^3*x^3+6*c
*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)/e/(4*a*c-b^2)*c
-1/2/a/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4
+2*b*d*e*x+b*d^2+a)/e/(4*a*c-b^2)*b^2-1/2/a^2/(4*a*c-b^2)/e*sum((c*e^3*(4*a
*c-b^2)*_R^3+3*c*d*e^2*(4*a*c-b^2)*_R^2+e*(12*a*c^2*d^2-3*b^2*c*d^2+5*a*b*c
-b^3)*_R+4*a*c^2*d^3-b^2*c*d^3+5*a*b*c*d-b^3*d)/(2*_R^3*c*e^3+6*_R^2*c*d*e^
2+6*_R*c*d^2*e+2*c*d^3+_R*b*e+b*d)*ln(-_R+x),_R=RootOf(_Z^4*c*e^4+4*_Z^3*c*
d*e^3+c*d^4+b*d^2+(6*c*d^2*e^2+b*e^2)*_Z^2+(4*c*d^3*e+2*b*d*e)*_Z+a))+ln(e*
x+d)/a^2/e
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [B] time = 11.35, size = 11072, normalized size = 68.35

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((d + e*x)*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2),x)
```

```
[Out] ((b^2 - 2*a*c + b*c*d^2)/(2*e*(a*b^2 - 4*a^2*c)) + (b*c*e*x^2)/(2*(a*b^2 -
4*a^2*c)) + (b*c*d*x)/(a*b^2 - 4*a^2*c))/(a + x^2*(b*e^2 + 6*c*d^2*e^2) + b
*d^2 + c*d^4 + x*(2*b*d*e + 4*c*d^3*e) + c*e^4*x^4 + 4*c*d*e^3*x^3) + log(d
 + e*x)/(a^2*e) - (log((((a^2*e*(-(b^2*(6*a*c - b^2)^2)/(a^4*e^2*(4*a*c - b
^2)^3))^(1/2) - 1)*(((a^2*e*(-(b^2*(6*a*c - b^2)^2)/(a^4*e^2*(4*a*c - b^2)^
3))^(1/2) - 1)*((b*c^2*e^16*(a^2*e*(-(b^2*(6*a*c - b^2)^2)/(a^4*e^2*(4*a*c
- b^2)^3))^(1/2) - 1)*(a*b + 3*b^2*d^2 + 3*b^2*e^2*x^2 - 10*a*c*d^2 + 6*b^2
*d*e*x - 10*a*c*e^2*x^2 - 20*a*c*d*e*x))/a^2 + (2*b*c^2*e^16*(2*b^3 - 10*a*
c^2*d^2 + b^2*c*d^2 - 10*a*b*c))/(a*(4*a*c - b^2)) - (2*b*c^3*e^18*x^2*(10*
a*c - b^2))/(a*(4*a*c - b^2)) - (4*b*c^3*d*e^17*x*(10*a*c - b^2))/(a*(4*a*c
 - b^2))))/(4*a^2*e) - (b*c^3*e^15*(4*b^3 - 20*a*c^2*d^2 + 6*b^2*c*d^2 - 17
*a*b*c))/(a^2*(4*a*c - b^2)^2) + (2*b*c^4*e^17*x^2*(10*a*c - 3*b^2))/(a^2*(
4*a*c - b^2)^2) + (4*b*c^4*d*e^16*x*(10*a*c - 3*b^2))/(a^2*(4*a*c - b^2)^2)
))/(4*a^2*e) + (b^3*c^5*e^16*x^2)/(a^3*(4*a*c - b^2)^3) + (b^2*c^4*e^14*(b^
2 - 4*a*c + b*c*d^2))/(a^3*(4*a*c - b^2)^3) + (2*b^3*c^5*d*e^15*x)/(a^3*(4*
a*c - b^2)^3))*((b^3*c^5*e^16*x^2)/(a^3*(4*a*c - b^2)^3) - ((a^2*e*(-(b^2*(
6*a*c - b^2)^2)/(a^4*e^2*(4*a*c - b^2)^3))^(1/2) + 1)*(((a^2*e*(-(b^2*(6*a*
c - b^2)^2)/(a^4*e^2*(4*a*c - b^2)^3))^(1/2) + 1)*((b*c^2*e^16*(a^2*e*(-(b^
2*(6*a*c - b^2)^2)/(a^4*e^2*(4*a*c - b^2)^3))^(1/2) + 1)*(a*b + 3*b^2*d^2 +
 3*b^2*e^2*x^2 - 10*a*c*d^2 + 6*b^2*d*e*x - 10*a*c*e^2*x^2 - 20*a*c*d*e*x))
/a^2 - (2*b*c^2*e^16*(2*b^3 - 10*a*c^2*d^2 + b^2*c*d^2 - 10*a*b*c))/(a*(4*a
*c - b^2)) + (2*b*c^3*e^18*x^2*(10*a*c - b^2))/(a*(4*a*c - b^2)) + (4*b*c^3
*d*e^17*x*(10*a*c - b^2))/(a*(4*a*c - b^2))))/(4*a^2*e) - (b*c^3*e^15*(4*b^
3 - 20*a*c^2*d^2 + 6*b^2*c*d^2 - 17*a*b*c))/(a^2*(4*a*c - b^2)^2) + (2*b*c^
4*e^17*x^2*(10*a*c - 3*b^2))/(a^2*(4*a*c - b^2)^2) + (4*b*c^4*d*e^16*x*(10*
a*c - 3*b^2))/(a^2*(4*a*c - b^2)^2)))/(4*a^2*e) + (b^2*c^4*e^14*(b^2 - 4*a*
c + b*c*d^2))/(a^3*(4*a*c - b^2)^3) + (2*b^3*c^5*d*e^15*x)/(a^3*(4*a*c - b^
2)^3)))*(2*b^6*e - 128*a^3*c^3*e + 96*a^2*b^2*c^2*e - 24*a*b^4*c*e))/(2*(4*
a^2*b^6*e^2 - 256*a^5*c^3*e^2 - 48*a^3*b^4*c*e^2 + 192*a^4*b^2*c^2*e^2)) +
(b*atan(((16*a^6*b^6*(4*a*c - b^2)^(9/2) - 1024*a^9*c^3*(4*a*c - b^2)^(9/2)
 - 192*a^7*b^4*c*(4*a*c - b^2)^(9/2) + 768*a^8*b^2*c^2*(4*a*c - b^2)^(9/2))
*(x^2*((((((b*((320*a^5*b*c^6*e^18 - 2*a^2*b^7*c^3*e^18 + 36*a^3*b^5*c^4*e^
18 - 192*a^4*b^3*c^5*e^18)/(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^
2*c^2) - ((2*b^6*e - 128*a^3*c^3*e + 96*a^2*b^2*c^2*e - 24*a*b^4*c*e)*(2560
*a^7*b*c^6*e^19 + 12*a^3*b^9*c^2*e^19 - 184*a^4*b^7*c^3*e^19 + 1056*a^5*b^5
*c^4*e^19 - 2688*a^6*b^3*c^5*e^19))/(2*(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c
 + 48*a^5*b^2*c^2)*(4*a^2*b^6*e^2 - 256*a^5*c^3*e^2 - 48*a^3*b^4*c*e^2 + 19
2*a^4*b^2*c^2*e^2)))*(6*a*c - b^2))/(4*a^2*e*(4*a*c - b^2)^(3/2)) - (b*(6*a
*c - b^2)*(2*b^6*e - 128*a^3*c^3*e + 96*a^2*b^2*c^2*e - 24*a*b^4*c*e)*(2560
*a^7*b*c^6*e^19 + 12*a^3*b^9*c^2*e^19 - 184*a^4*b^7*c^3*e^19 + 1056*a^5*b^5
*c^4*e^19 - 2688*a^6*b^3*c^5*e^19))/(8*a^2*e*(4*a*c - b^2)^(3/2)*(a^3*b^6 -
 64*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*c^2)*(4*a^2*b^6*e^2 - 256*a^5*c^3*e
^2 - 48*a^3*b^4*c*e^2 + 192*a^4*b^2*c^2*e^2)))*(2*b^6*e - 128*a^3*c^3*e + 9
6*a^2*b^2*c^2*e - 24*a*b^4*c*e))/(2*(4*a^2*b^6*e^2 - 256*a^5*c^3*e^2 - 48*a
```

^3*b^4*c*e^2 + 192*a^4*b^2*c^2*e^2)) + (b*(6*a*c - b^2)*((6*a*b^5*c^4*e^17
+ 80*a^3*b*c^6*e^17 - 44*a^2*b^3*c^5*e^17)/(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b
^4*c + 48*a^5*b^2*c^2) + (((320*a^5*b*c^6*e^18 - 2*a^2*b^7*c^3*e^18 + 36*a^
3*b^5*c^4*e^18 - 192*a^4*b^3*c^5*e^18)/(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c
 + 48*a^5*b^2*c^2) - ((2*b^6*e - 128*a^3*c^3*e + 96*a^2*b^2*c^2*e - 24*a*b^
4*c*e)*(2560*a^7*b*c^6*e^19 + 12*a^3*b^9*c^2*e^19 - 184*a^4*b^7*c^3*e^19 +
1056*a^5*b^5*c^4*e^19 - 2688*a^6*b^3*c^5*e^19))/(2*(a^3*b^6 - 64*a^6*c^3 -
12*a^4*b^4*c + 48*a^5*b^2*c^2)*(4*a^2*b^6*e^2 - 256*a^5*c^3*e^2 - 48*a^3*b^
4*c*e^2 + 192*a^4*b^2*c^2*e^2)))*(2*b^6*e - 128*a^3*c^3*e + 96*a^2*b^2*c^2*
e - 24*a*b^4*c*e))/(2*(4*a^2*b^6*e^2 - 256*a^5*c^3*e^2 - 48*a^3*b^4*c*e^2 +
 192*a^4*b^2*c^2*e^2))))/(4*a^2*e*(4*a*c - b^2)^(3/2)) + (b^3*(6*a*c - b^2)
^3*(2560*a^7*b*c^6*e^19 + 12*a^3*b^9*c^2*e^19 - 184*a^4*b^7*c^3*e^19 + 1056
*a^5*b^5*c^4*e^19 - 2688*a^6*b^3*c^5*e^19))/(64*a^6*e^3*(4*a*c - b^2)^(9/2)
*(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*c^2)))*(3*b^6 - 40*a^3*c
^3 + 69*a^2*b^2*c^2 - 27*a*b^4*c))/(8*a^3*c^2*(4*a*c - b^2)^(7/2)*(6*b^6 -
400*a^3*c^3 + 291*a^2*b^2*c^2 - 72*a*b^4*c)) + (3*b*(b^4 + 11*a^2*c^2 - 7*a
*b^2*c)*((((6*a*b^5*c^4*e^17 + 80*a^3*b*c^6*e^17 - 44*a^2*b^3*c^5*e^17)/(a^
3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*c^2) + (((320*a^5*b*c^6*e^18
 - 2*a^2*b^7*c^3*e^18 + 36*a^3*b^5*c^4*e^18 - 192*a^4*b^3*c^5*e^18)/(a^3*b^
6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*c^2) - ((2*b^6*e - 128*a^3*c^3*e
 + 96*a^2*b^2*c^2*e - 24*a*b^4*c*e)*(2560*a^7*b*c^6*e^19 + 12*a^3*b^9*c^2*e
^19 - 184*a^4*b^7*c^3*e^19 + 1056*a^5*b^5*c^4*e^19 - 2688*a^6*b^3*c^5*e^19)
)/(2*(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*c^2)*(4*a^2*b^6*e^2
- 256*a^5*c^3*e^2 - 48*a^3*b^4*c*e^2 + 192*a^4*b^2*c^2*e^2)))*(2*b^6*e - 12
8*a^3*c^3*e + 96*a^2*b^2*c^2*e - 24*a*b^4*c*e))/(2*(4*a^2*b^6*e^2 - 256*a^5
*c^3*e^2 - 48*a^3*b^4*c*e^2 + 192*a^4*b^2*c^2*e^2)))*(2*b^6*e - 128*a^3*c^3
*e + 96*a^2*b^2*c^2*e - 24*a*b^4*c*e))/(2*(4*a^2*b^6*e^2 - 256*a^5*c^3*e^2
- 48*a^3*b^4*c*e^2 + 192*a^4*b^2*c^2*e^2)) - (b^3*c^5*e^16)/(a^3*b^6 - 64*a
^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*c^2) - (b*((b*((320*a^5*b*c^6*e^18 - 2*a
^2*b^7*c^3*e^18 + 36*a^3*b^5*c^4*e^18 - 192*a^4*b^3*c^5*e^18)/(a^3*b^6 - 64
*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*c^2) - ((2*b^6*e - 128*a^3*c^3*e + 96*
a^2*b^2*c^2*e - 24*a*b^4*c*e)*(2560*a^7*b*c^6*e^19 + 12*a^3*b^9*c^2*e^19 -
184*a^4*b^7*c^3*e^19 + 1056*a^5*b^5*c^4*e^19 - 2688*a^6*b^3*c^5*e^19))/(2*(
a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*c^2)*(4*a^2*b^6*e^2 - 256*
a^5*c^3*e^2 - 48*a^3*b^4*c*e^2 + 192*a^4*b^2*c^2*e^2)))*(6*a*c - b^2))/(4*a
^2*e*(4*a*c - b^2)^(3/2)) - (b*(6*a*c - b^2)*(2*b^6*e - 128*a^3*c^3*e + 96*
a^2*b^2*c^2*e - 24*a*b^4*c*e)*(2560*a^7*b*c^6*e^19 + 12*a^3*b^9*c^2*e^19 -
184*a^4*b^7*c^3*e^19 + 1056*a^5*b^5*c^4*e^19 - 2688*a^6*b^3*c^5*e^19))/(8*a
^2*e*(4*a*c - b^2)^(3/2)*(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*
c^2)*(4*a^2*b^6*e^2 - 256*a^5*c^3*e^2 - 48*a^3*b^4*c*e^2 + 192*a^4*b^2*c^2*
e^2)))*(6*a*c - b^2))/(4*a^2*e*(4*a*c - b^2)^(3/2)) + (b^2*(6*a*c - b^2)^2*
(2*b^6*e - 128*a^3*c^3*e + 96*a^2*b^2*c^2*e - 24*a*b^4*c*e)*(2560*a^7*b*c^6
*e^19 + 12*a^3*b^9*c^2*e^19 - 184*a^4*b^7*c^3*e^19 + 1056*a^5*b^5*c^4*e^19
- 2688*a^6*b^3*c^5*e^19))/(32*a^4*e^2*(4*a*c - b^2)^3*(a^3*b^6 - 64*a^6*c^3
 - 12*a^4*b^4*c + 48*a^5*b^2*c^2)*(4*a^2*b^6*e^2 - 256*a^5*c^3*e^2 - 48*a^3
*b^4*c*e^2 + 192*a^4*b^2*c^2*e^2))))/(8*a^3*c^2*(4*a*c - b^2)^3*(6*b^6 - 40
0*a^3*c^3 + 291*a^2*b^2*c^2 - 72*a*b^4*c))) + x*((((((b*((2*(320*a^5*b*c^6*
d*e^17 - 2*a^2*b^7*c^3*d*e^17 + 36*a^3*b^5*c^4*d*e^17 - 192*a^4*b^3*c^5*d*e
^17))/(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*c^2) - ((2*b^6*e -
128*a^3*c^3*e + 96*a^2*b^2*c^2*e - 24*a*b^4*c*e)*(2560*a^7*b*c^6*d*e^18 + 1
2*a^3*b^9*c^2*d*e^18 - 184*a^4*b^7*c^3*d*e^18 + 1056*a^5*b^5*c^4*d*e^18 - 2
688*a^6*b^3*c^5*d*e^18))/((a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2
c^2)(4*a^2*b^6*e^2 - 256*a^5*c^3*e^2 - 48*a^3*b^4*c*e^2 + 192*a^4*b^2*c^2
e^2)))(6*a*c - b^2))/(4*a^2*e*(4*a*c - b^2)^(3/2)) - (b*(6*a*c - b^2)*(2*
b^6*e - 128*a^3*c^3*e + 96*a^2*b^2*c^2*e - 24*a*b^4*c*e)*(2560*a^7*b*c^6*d*
e^18 + 12*a^3*b^9*c^2*d*e^18 - 184*a^4*b^7*c^3*d*e^18 + 1056*a^5*b^5*c^4*d*
e^18 - 2688*a^6*b^3*c^5*d*e^18))/(4*a^2*e*(4*a*c - b^2)^(3/2)*(a^3*b^6 - 64
*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*c^2)*(4*a^2*b^6*e^2 - 256*a^5*c^3*e^2
- 48*a^3*b^4*c*e^2 + 192*a^4*b^2*c^2*e^2)))*(2*b^6*e - 128*a^3*c^3*e + 96*a

^2*b^2*c^2*e - 24*a*b^4*c*e))/(2*(4*a^2*b^6*e^2 - 256*a^5*c^3*e^2 - 48*a^3*
b^4*c*e^2 + 192*a^4*b^2*c^2*e^2)) + (b*(6*a*c - b^2)*((2*(6*a*b^5*c^4*d*e^1
6 + 80*a^3*b*c^6*d*e^16 - 44*a^2*b^3*c^5*d*e^16))/(a^3*b^6 - 64*a^6*c^3 - 1
2*a^4*b^4*c + 48*a^5*b^2*c^2) + (((2*(320*a^5*b*c^6*d*e^17 - 2*a^2*b^7*c^3*
d*e^17 + 36*a^3*b^5*c^4*d*e^17 - 192*a^4*b^3*c^5*d*e^17))/(a^3*b^6 - 64*a^6
*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*c^2) - ((2*b^6*e - 128*a^3*c^3*e + 96*a^2*
b^2*c^2*e - 24*a*b^4*c*e)*(2560*a^7*b*c^6*d*e^18 + 12*a^3*b^9*c^2*d*e^18 -
184*a^4*b^7*c^3*d*e^18 + 1056*a^5*b^5*c^4*d*e^18 - 2688*a^6*b^3*c^5*d*e^18)
)/((a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*c^2)*(4*a^2*b^6*e^2 -
256*a^5*c^3*e^2 - 48*a^3*b^4*c*e^2 + 192*a^4*b^2*c^2*e^2)))*(2*b^6*e - 128*
a^3*c^3*e + 96*a^2*b^2*c^2*e - 24*a*b^4*c*e))/(2*(4*a^2*b^6*e^2 - 256*a^5*c
^3*e^2 - 48*a^3*b^4*c*e^2 + 192*a^4*b^2*c^2*e^2))))/(4*a^2*e*(4*a*c - b^2)^
(3/2)) + (b^3*(6*a*c - b^2)^3*(2560*a^7*b*c^6*d*e^18 + 12*a^3*b^9*c^2*d*e^1
8 - 184*a^4*b^7*c^3*d*e^18 + 1056*a^5*b^5*c^4*d*e^18 - 2688*a^6*b^3*c^5*d*e
^18))/(32*a^6*e^3*(4*a*c - b^2)^(9/2)*(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c
+ 48*a^5*b^2*c^2)))*(3*b^6 - 40*a^3*c^3 + 69*a^2*b^2*c^2 - 27*a*b^4*c))/(8*
a^3*c^2*(4*a*c - b^2)^(7/2)*(6*b^6 - 400*a^3*c^3 + 291*a^2*b^2*c^2 - 72*a*b
^4*c)) + (3*b*(b^4 + 11*a^2*c^2 - 7*a*b^2*c)*((((2*(6*a*b^5*c^4*d*e^16 + 80
*a^3*b*c^6*d*e^16 - 44*a^2*b^3*c^5*d*e^16))/(a^3*b^6 - 64*a^6*c^3 - 12*a^4*
b^4*c + 48*a^5*b^2*c^2) + (((2*(320*a^5*b*c^6*d*e^17 - 2*a^2*b^7*c^3*d*e^17
 + 36*a^3*b^5*c^4*d*e^17 - 192*a^4*b^3*c^5*d*e^17))/(a^3*b^6 - 64*a^6*c^3 -
 12*a^4*b^4*c + 48*a^5*b^2*c^2) - ((2*b^6*e - 128*a^3*c^3*e + 96*a^2*b^2*c^
2*e - 24*a*b^4*c*e)*(2560*a^7*b*c^6*d*e^18 + 12*a^3*b^9*c^2*d*e^18 - 184*a^
4*b^7*c^3*d*e^18 + 1056*a^5*b^5*c^4*d*e^18 - 2688*a^6*b^3*c^5*d*e^18))/((a^
3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*c^2)*(4*a^2*b^6*e^2 - 256*a^
5*c^3*e^2 - 48*a^3*b^4*c*e^2 + 192*a^4*b^2*c^2*e^2)))*(2*b^6*e - 128*a^3*c^
3*e + 96*a^2*b^2*c^2*e - 24*a*b^4*c*e))/(2*(4*a^2*b^6*e^2 - 256*a^5*c^3*e^2
 - 48*a^3*b^4*c*e^2 + 192*a^4*b^2*c^2*e^2)))*(2*b^6*e - 128*a^3*c^3*e + 96*
a^2*b^2*c^2*e - 24*a*b^4*c*e))/(2*(4*a^2*b^6*e^2 - 256*a^5*c^3*e^2 - 48*a^3
*b^4*c*e^2 + 192*a^4*b^2*c^2*e^2)) - (2*b^3*c^5*d*e^15)/(a^3*b^6 - 64*a^6*c
^3 - 12*a^4*b^4*c + 48*a^5*b^2*c^2) - (b*(6*a*c - b^2)*((b*((2*(320*a^5*b*c
^6*d*e^17 - 2*a^2*b^7*c^3*d*e^17 + 36*a^3*b^5*c^4*d*e^17 - 192*a^4*b^3*c^5*
d*e^17))/(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*c^2) - ((2*b^6*e
 - 128*a^3*c^3*e + 96*a^2*b^2*c^2*e - 24*a*b^4*c*e)*(2560*a^7*b*c^6*d*e^18
+ 12*a^3*b^9*c^2*d*e^18 - 184*a^4*b^7*c^3*d*e^18 + 1056*a^5*b^5*c^4*d*e^18
- 2688*a^6*b^3*c^5*d*e^18))/((a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*
b^2*c^2)*(4*a^2*b^6*e^2 - 256*a^5*c^3*e^2 - 48*a^3*b^4*c*e^2 + 192*a^4*b^2*
c^2*e^2)))*(6*a*c - b^2))/(4*a^2*e*(4*a*c - b^2)^(3/2)) - (b*(6*a*c - b^2)*
(2*b^6*e - 128*a^3*c^3*e + 96*a^2*b^2*c^2*e - 24*a*b^4*c*e)*(2560*a^7*b*c^6
*d*e^18 + 12*a^3*b^9*c^2*d*e^18 - 184*a^4*b^7*c^3*d*e^18 + 1056*a^5*b^5*c^4
*d*e^18 - 2688*a^6*b^3*c^5*d*e^18))/(4*a^2*e*(4*a*c - b^2)^(3/2)*(a^3*b^6 -
 64*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*c^2)*(4*a^2*b^6*e^2 - 256*a^5*c^3*e
^2 - 48*a^3*b^4*c*e^2 + 192*a^4*b^2*c^2*e^2))))/(4*a^2*e*(4*a*c - b^2)^(3/2
)) + (b^2*(6*a*c - b^2)^2*(2*b^6*e - 128*a^3*c^3*e + 96*a^2*b^2*c^2*e - 24*
a*b^4*c*e)*(2560*a^7*b*c^6*d*e^18 + 12*a^3*b^9*c^2*d*e^18 - 184*a^4*b^7*c^3
*d*e^18 + 1056*a^5*b^5*c^4*d*e^18 - 2688*a^6*b^3*c^5*d*e^18))/(16*a^4*e^2*(
4*a*c - b^2)^3*(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*c^2)*(4*a^
2*b^6*e^2 - 256*a^5*c^3*e^2 - 48*a^3*b^4*c*e^2 + 192*a^4*b^2*c^2*e^2))))/(8
*a^3*c^2*(4*a*c - b^2)^3*(6*b^6 - 400*a^3*c^3 + 291*a^2*b^2*c^2 - 72*a*b^4*
c))) + (((b*((4*a*b^6*c^3*e^15 - 33*a^2*b^4*c^4*e^15 + 68*a^3*b^2*c^5*e^15
- 44*a^2*b^3*c^5*d^2*e^15 + 6*a*b^5*c^4*d^2*e^15 + 80*a^3*b*c^6*d^2*e^15)/(
a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*c^2) - (((4*a^2*b^8*c^2*e^
16 - 52*a^3*b^6*c^3*e^16 + 224*a^4*b^4*c^4*e^16 - 320*a^5*b^2*c^5*e^16 + 2*
a^2*b^7*c^3*d^2*e^16 - 36*a^3*b^5*c^4*d^2*e^16 + 192*a^4*b^3*c^5*d^2*e^16 -
 320*a^5*b*c^6*d^2*e^16)/(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*
c^2) + ((2*b^6*e - 128*a^3*c^3*e + 96*a^2*b^2*c^2*e - 24*a*b^4*c*e)*(4*a^4*
b^8*c^2*e^17 - 48*a^5*b^6*c^3*e^17 + 192*a^6*b^4*c^4*e^17 - 256*a^7*b^2*c^5
*e^17 + 12*a^3*b^9*c^2*d^2*e^17 - 184*a^4*b^7*c^3*d^2*e^17 + 1056*a^5*b^5*c
^4*d^2*e^17 - 2688*a^6*b^3*c^5*d^2*e^17 + 2560*a^7*b*c^6*d^2*e^17))/(2*(a^3

*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*c^2)*(4*a^2*b^6*e^2 - 256*a^5
*c^3*e^2 - 48*a^3*b^4*c*e^2 + 192*a^4*b^2*c^2*e^2)))*(2*b^6*e - 128*a^3*c^3
*e + 96*a^2*b^2*c^2*e - 24*a*b^4*c*e))/(2*(4*a^2*b^6*e^2 - 256*a^5*c^3*e^2
- 48*a^3*b^4*c*e^2 + 192*a^4*b^2*c^2*e^2)))*(6*a*c - b^2))/(4*a^2*e*(4*a*c
- b^2)^(3/2)) - (((b*(6*a*c - b^2)*((4*a^2*b^8*c^2*e^16 - 52*a^3*b^6*c^3*e^
16 + 224*a^4*b^4*c^4*e^16 - 320*a^5*b^2*c^5*e^16 + 2*a^2*b^7*c^3*d^2*e^16 -
36*a^3*b^5*c^4*d^2*e^16 + 192*a^4*b^3*c^5*d^2*e^16 - 320*a^5*b*c^6*d^2*e^1
6)/(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*c^2) + ((2*b^6*e - 128
*a^3*c^3*e + 96*a^2*b^2*c^2*e - 24*a*b^4*c*e)*(4*a^4*b^8*c^2*e^17 - 48*a^5*
b^6*c^3*e^17 + 192*a^6*b^4*c^4*e^17 - 256*a^7*b^2*c^5*e^17 + 12*a^3*b^9*c^2
*d^2*e^17 - 184*a^4*b^7*c^3*d^2*e^17 + 1056*a^5*b^5*c^4*d^2*e^17 - 2688*a^6
*b^3*c^5*d^2*e^17 + 2560*a^7*b*c^6*d^2*e^17))/(2*(a^3*b^6 - 64*a^6*c^3 - 12
*a^4*b^4*c + 48*a^5*b^2*c^2)*(4*a^2*b^6*e^2 - 256*a^5*c^3*e^2 - 48*a^3*b^4*
c*e^2 + 192*a^4*b^2*c^2*e^2))))/(4*a^2*e*(4*a*c - b^2)^(3/2)) + (b*(6*a*c -
b^2)*(2*b^6*e - 128*a^3*c^3*e + 96*a^2*b^2*c^2*e - 24*a*b^4*c*e)*(4*a^4*b^
8*c^2*e^17 - 48*a^5*b^6*c^3*e^17 + 192*a^6*b^4*c^4*e^17 - 256*a^7*b^2*c^5*e
^17 + 12*a^3*b^9*c^2*d^2*e^17 - 184*a^4*b^7*c^3*d^2*e^17 + 1056*a^5*b^5*c^4
*d^2*e^17 - 2688*a^6*b^3*c^5*d^2*e^17 + 2560*a^7*b*c^6*d^2*e^17))/(8*a^2*e*
(4*a*c - b^2)^(3/2)*(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*c^2)*
(4*a^2*b^6*e^2 - 256*a^5*c^3*e^2 - 48*a^3*b^4*c*e^2 + 192*a^4*b^2*c^2*e^2))
)*(2*b^6*e - 128*a^3*c^3*e + 96*a^2*b^2*c^2*e - 24*a*b^4*c*e))/(2*(4*a^2*b^
6*e^2 - 256*a^5*c^3*e^2 - 48*a^3*b^4*c*e^2 + 192*a^4*b^2*c^2*e^2)) + (b^3*(
6*a*c - b^2)^3*(4*a^4*b^8*c^2*e^17 - 48*a^5*b^6*c^3*e^17 + 192*a^6*b^4*c^4*
e^17 - 256*a^7*b^2*c^5*e^17 + 12*a^3*b^9*c^2*d^2*e^17 - 184*a^4*b^7*c^3*d^2
*e^17 + 1056*a^5*b^5*c^4*d^2*e^17 - 2688*a^6*b^3*c^5*d^2*e^17 + 2560*a^7*b*
c^6*d^2*e^17))/(64*a^6*e^3*(4*a*c - b^2)^(9/2)*(a^3*b^6 - 64*a^6*c^3 - 12*a
^4*b^4*c + 48*a^5*b^2*c^2)))*(3*b^6 - 40*a^3*c^3 + 69*a^2*b^2*c^2 - 27*a*b^
4*c))/(8*a^3*c^2*(4*a*c - b^2)^(7/2)*(6*b^6 - 400*a^3*c^3 + 291*a^2*b^2*c^2
- 72*a*b^4*c)) + (3*b*(b^4 + 11*a^2*c^2 - 7*a*b^2*c)*((((4*a*b^6*c^3*e^15
- 33*a^2*b^4*c^4*e^15 + 68*a^3*b^2*c^5*e^15 - 44*a^2*b^3*c^5*d^2*e^15 + 6*a
*b^5*c^4*d^2*e^15 + 80*a^3*b*c^6*d^2*e^15)/(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b
^4*c + 48*a^5*b^2*c^2) - (((4*a^2*b^8*c^2*e^16 - 52*a^3*b^6*c^3*e^16 + 224*
a^4*b^4*c^4*e^16 - 320*a^5*b^2*c^5*e^16 + 2*a^2*b^7*c^3*d^2*e^16 - 36*a^3*b
^5*c^4*d^2*e^16 + 192*a^4*b^3*c^5*d^2*e^16 - 320*a^5*b*c^6*d^2*e^16)/(a^3*b
^6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*c^2) + ((2*b^6*e - 128*a^3*c^3*
e + 96*a^2*b^2*c^2*e - 24*a*b^4*c*e)*(4*a^4*b^8*c^2*e^17 - 48*a^5*b^6*c^3*e
^17 + 192*a^6*b^4*c^4*e^17 - 256*a^7*b^2*c^5*e^17 + 12*a^3*b^9*c^2*d^2*e^17
- 184*a^4*b^7*c^3*d^2*e^17 + 1056*a^5*b^5*c^4*d^2*e^17 - 2688*a^6*b^3*c^5*
d^2*e^17 + 2560*a^7*b*c^6*d^2*e^17))/(2*(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*
c + 48*a^5*b^2*c^2)*(4*a^2*b^6*e^2 - 256*a^5*c^3*e^2 - 48*a^3*b^4*c*e^2 + 1
92*a^4*b^2*c^2*e^2)))*(2*b^6*e - 128*a^3*c^3*e + 96*a^2*b^2*c^2*e - 24*a*b^
4*c*e))/(2*(4*a^2*b^6*e^2 - 256*a^5*c^3*e^2 - 48*a^3*b^4*c*e^2 + 192*a^4*b^
2*c^2*e^2)))*(2*b^6*e - 128*a^3*c^3*e + 96*a^2*b^2*c^2*e - 24*a*b^4*c*e))/(
2*(4*a^2*b^6*e^2 - 256*a^5*c^3*e^2 - 48*a^3*b^4*c*e^2 + 192*a^4*b^2*c^2*e^2
)) - (b^4*c^4*e^14 - 4*a*b^2*c^5*e^14 + b^3*c^5*d^2*e^14)/(a^3*b^6 - 64*a^6
*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*c^2) + (b*(6*a*c - b^2)*((b*(6*a*c - b^2)*
((4*a^2*b^8*c^2*e^16 - 52*a^3*b^6*c^3*e^16 + 224*a^4*b^4*c^4*e^16 - 320*a^5
*b^2*c^5*e^16 + 2*a^2*b^7*c^3*d^2*e^16 - 36*a^3*b^5*c^4*d^2*e^16 + 192*a^4*
b^3*c^5*d^2*e^16 - 320*a^5*b*c^6*d^2*e^16)/(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b
^4*c + 48*a^5*b^2*c^2) + ((2*b^6*e - 128*a^3*c^3*e + 96*a^2*b^2*c^2*e - 24*
a*b^4*c*e)*(4*a^4*b^8*c^2*e^17 - 48*a^5*b^6*c^3*e^17 + 192*a^6*b^4*c^4*e^17
- 256*a^7*b^2*c^5*e^17 + 12*a^3*b^9*c^2*d^2*e^17 - 184*a^4*b^7*c^3*d^2*e^1
7 + 1056*a^5*b^5*c^4*d^2*e^17 - 2688*a^6*b^3*c^5*d^2*e^17 + 2560*a^7*b*c^6*
d^2*e^17))/(2*(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*c^2)*(4*a^2
*b^6*e^2 - 256*a^5*c^3*e^2 - 48*a^3*b^4*c*e^2 + 192*a^4*b^2*c^2*e^2))))/(4*
a^2*e*(4*a*c - b^2)^(3/2)) + (b*(6*a*c - b^2)*(2*b^6*e - 128*a^3*c^3*e + 96
*a^2*b^2*c^2*e - 24*a*b^4*c*e)*(4*a^4*b^8*c^2*e^17 - 48*a^5*b^6*c^3*e^17 +
192*a^6*b^4*c^4*e^17 - 256*a^7*b^2*c^5*e^17 + 12*a^3*b^9*c^2*d^2*e^17 - 184
*a^4*b^7*c^3*d^2*e^17 + 1056*a^5*b^5*c^4*d^2*e^17 - 2688*a^6*b^3*c^5*d^2*e^

```
17 + 2560*a^7*b*c^6*d^2*e^17))/(8*a^2*e*(4*a*c - b^2)^(3/2)*(a^3*b^6 - 64*a
^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*c^2)*(4*a^2*b^6*e^2 - 256*a^5*c^3*e^2 -
48*a^3*b^4*c*e^2 + 192*a^4*b^2*c^2*e^2))))/(4*a^2*e*(4*a*c - b^2)^(3/2)) +
(b^2*(6*a*c - b^2)^2*(2*b^6*e - 128*a^3*c^3*e + 96*a^2*b^2*c^2*e - 24*a*b^4
*c*e)*(4*a^4*b^8*c^2*e^17 - 48*a^5*b^6*c^3*e^17 + 192*a^6*b^4*c^4*e^17 - 25
6*a^7*b^2*c^5*e^17 + 12*a^3*b^9*c^2*d^2*e^17 - 184*a^4*b^7*c^3*d^2*e^17 + 1
056*a^5*b^5*c^4*d^2*e^17 - 2688*a^6*b^3*c^5*d^2*e^17 + 2560*a^7*b*c^6*d^2*e
^17))/(32*a^4*e^2*(4*a*c - b^2)^3*(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48
*a^5*b^2*c^2)*(4*a^2*b^6*e^2 - 256*a^5*c^3*e^2 - 48*a^3*b^4*c*e^2 + 192*a^4
*b^2*c^2*e^2))))/(8*a^3*c^2*(4*a*c - b^2)^3*(6*b^6 - 400*a^3*c^3 + 291*a^2*
b^2*c^2 - 72*a*b^4*c))))/(b^6*c^2*e^14 - 12*a*b^4*c^3*e^14 + 36*a^2*b^2*c^4
*e^14))*(6*a*c - b^2))/(2*a^2*e*(4*a*c - b^2)^(3/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(a+b*(e*x+d)**2+c*(e*x+d)**4)**2,x)
```

```
[Out] Timed out
```

3.514 $\int \frac{1}{(d+ex)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx$

Optimal. Leaf size=348

$$-\frac{3b^2-10ac}{2a^2e\left(b^2-4ac\right)(d+ex)}-\frac{\sqrt{c}\left(\left(3b^2-10ac\right)\sqrt{b^2-4ac}-16abc+3b^3\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\,a^2e\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{c}\left(-\left(3b^2-10ac\right)\right.}{2\sqrt{2}\,a}$$

Rubi [A] time = 1.65, antiderivative size = 348, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1142, 1121, 1281, 1166, 205}

$$-\frac{3b^2-10ac}{2a^2e\left(b^2-4ac\right)(d+ex)}-\frac{\sqrt{c}\left(\left(3b^2-10ac\right)\sqrt{b^2-4ac}-16abc+3b^3\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}a^2e\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{c}\left(-\left(3b^2-10ac\right)\sqrt{b^2-4ac}-16abc+3b^3\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{2\sqrt{2}a^2e\left(b^2-4ac\right)^{3/2}\sqrt{\sqrt{b^2-4ac}+b}}+\frac{-2ac+b^2+bc(d+ex)^2}{2ae\left(b^2-4ac\right)(d+ex)\left(a+b(d+ex)^2+c(d+ex)^4\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)^2*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2),x]
```

```
[Out] -(3*b^2 - 10*a*c)/(2*a^2*(b^2 - 4*a*c)*e*(d + e*x)) + (b^2 - 2*a*c + b*c*(d
 + e*x)^2)/(2*a*(b^2 - 4*a*c)*e*(d + e*x)*(a + b*(d + e*x)^2 + c*(d + e*x)^
4)) - (Sqrt[c]*(3*b^3 - 16*a*b*c + (3*b^2 - 10*a*c)*Sqrt[b^2 - 4*a*c])*ArcT
an[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a^2
*(b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]*e) + (Sqrt[c]*(3*b^3 - 16*
a*b*c - (3*b^2 - 10*a*c)*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*
x))/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a^2*(b^2 - 4*a*c)^(3/2)*Sqrt[b
 + Sqrt[b^2 - 4*a*c]]*e)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1121

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> -Simp[((d*x)^(m + 1)*(b^2 - 2*a*c + b*c*x^2)*(a + b*x^2 + c*x^4)^(p + 1
))/(2*a*d*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)),
Int[(d*x)^m*(a + b*x^2 + c*x^4)^(p + 1)*Simp[b^2*(m + 2*p + 3) - 2*a*c*(m +
 4*p + 5) + b*c*(m + 4*p + 7)*x^2, x], x], x] /; FreeQ[{a, b, c, d, m}, x]
&& NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p] && (IntegerQ[p] || In
tegerQ[m])
```

Rule 1142

```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Di
st[u^m/(Coefficient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p,
 x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1281

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[(d*(f*x)^(m + 1)*(a + b*x^2 + c*x^4)^(p + 1)
)/(a*f*(m + 1)), x] + Dist[1/(a*f^2*(m + 1)), Int[(f*x)^(m + 2)*(a + b*x^2
+ c*x^4)^p*Simp[a*e*(m + 1) - b*d*(m + 2*p + 3) - c*d*(m + 4*p + 5)*x^2, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[m
, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rubi steps

$$\int \frac{1}{(d+ex)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x^2\left(a+bx^2+cx^4\right)^2}\,dx, x, d+ex\right)}{e}$$

$$= \frac{b^2-2ac+bc(d+ex)^2}{2a\left(b^2-4ac\right)e(d+ex)\left(a+b(d+ex)^2+c(d+ex)^4\right)} - \frac{\text{Subst}}{}$$

$$= -\frac{3b^2-10ac}{2a^2\left(b^2-4ac\right)e(d+ex)} + \frac{b^2-2ac+bc(d+}{2a\left(b^2-4ac\right)e(d+ex)\left(a+b(d+\right.}$$

$$= -\frac{3b^2-10ac}{2a^2\left(b^2-4ac\right)e(d+ex)} + \frac{b^2-2ac+bc(d+}{2a\left(b^2-4ac\right)e(d+ex)\left(a+b(d+\right.}$$

$$= -\frac{3b^2-10ac}{2a^2\left(b^2-4ac\right)e(d+ex)} + \frac{b^2-2ac+bc(d+}{2a\left(b^2-4ac\right)e(d+ex)\left(a+b(d+\right.}$$

Mathematica [A] time = 1.63, size = 339, normalized size = 0.97

$$\frac{\frac{2(d+ex)\left(-3abc-2ac^2(d+ex)^2+b^3+b^2c(d+ex)^2\right)}{\left(4ac-b^2\right)\left(a+b(d+ex)^2+c(d+ex)^4\right)}+\frac{\sqrt{2}\sqrt{c}\left(-3b^2\sqrt{b^2-4ac}+10ac\sqrt{b^2-4ac}+16abc-3b^3\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{2}\sqrt{c}\left(-3b^2\sqrt{b^2-4ac}+10ac\sqrt{b^2-4ac}-16abc+3b^3\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\left(b^2-4ac\right)^{3/2}\sqrt{\sqrt{b^2-4ac}+b}}-\frac{4}{d+ex}}{4a^2e}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)^2*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2),x]
```

```
[Out] (-4/(d + e*x) + (2*(d + e*x)*(b^3 - 3*a*b*c + b^2*c*(d + e*x)^2 - 2*a*c^2*(
d + e*x)^2))/((-b^2 + 4*a*c)*(a + b*(d + e*x)^2 + c*(d + e*x)^4)) + (Sqrt[2
]*Sqrt[c]*(-3*b^3 + 16*a*b*c - 3*b^2*Sqrt[b^2 - 4*a*c] + 10*a*c*Sqrt[b^2 -
4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/((
b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[2]*Sqrt[c]*(3*b^3 -
 16*a*b*c - 3*b^2*Sqrt[b^2 - 4*a*c] + 10*a*c*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqr
t[2]*Sqrt[c]*(d + e*x))/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(3/2)*
Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(4*a^2*e)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(d+ex)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/((d + e*x)^2*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2),x]
```

```
[Out] IntegrateAlgebraic[1/((d + e*x)^2*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2), x
]
```

fricas [B] time = 1.86, size = 4330, normalized size = 12.44

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="fricas")
```

```
[Out] -1/4*(2*(3*b^2*c - 10*a*c^2)*e^4*x^4 + 8*(3*b^2*c - 10*a*c^2)*d*e^3*x^3 + 2
*(3*b^2*c - 10*a*c^2)*d^4 + 2*(3*b^3 - 11*a*b*c + 6*(3*b^2*c - 10*a*c^2)*d^
2)*e^2*x^2 + 4*a*b^2 - 16*a^2*c + 2*(3*b^3 - 11*a*b*c)*d^2 + 4*(2*(3*b^2*c
- 10*a*c^2)*d^3 + (3*b^3 - 11*a*b*c)*d)*e*x + sqrt(1/2)*((a^2*b^2*c - 4*a^3
*c^2)*e^6*x^5 + 5*(a^2*b^2*c - 4*a^3*c^2)*d*e^5*x^4 + (a^2*b^3 - 4*a^3*b*c
+ 10*(a^2*b^2*c - 4*a^3*c^2)*d^2)*e^4*x^3 + (10*(a^2*b^2*c - 4*a^3*c^2)*d^3
 + 3*(a^2*b^3 - 4*a^3*b*c)*d)*e^3*x^2 + (a^3*b^2 - 4*a^4*c + 5*(a^2*b^2*c -
 4*a^3*c^2)*d^4 + 3*(a^2*b^3 - 4*a^3*b*c)*d^2)*e^2*x + ((a^2*b^2*c - 4*a^3*
c^2)*d^5 + (a^2*b^3 - 4*a^3*b*c)*d^3 + (a^3*b^2 - 4*a^4*c)*d)*e)*sqrt(-(9*b
^7 - 105*a*b^5*c + 385*a^2*b^3*c^2 - 420*a^3*b*c^3 + (a^5*b^6 - 12*a^6*b^4*
c + 48*a^7*b^2*c^2 - 64*a^8*c^3)*e^2*sqrt((81*b^8 - 918*a*b^6*c + 3051*a^2*
b^4*c^2 - 2550*a^3*b^2*c^3 + 625*a^4*c^4)/((a^10*b^6 - 12*a^11*b^4*c + 48*a
^12*b^2*c^2 - 64*a^13*c^3)*e^4)))/((a^5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2*c^2
 - 64*a^8*c^3)*e^2))*log(-(189*b^6*c^3 - 1971*a*b^4*c^4 + 5625*a^2*b^2*c^5
- 2500*a^3*c^6)*e*x - (189*b^6*c^3 - 1971*a*b^4*c^4 + 5625*a^2*b^2*c^5 - 25
00*a^3*c^6)*d + 1/2*sqrt(1/2)*((3*a^5*b^10 - 55*a^6*b^8*c + 392*a^7*b^6*c^2
 - 1344*a^8*b^4*c^3 + 2176*a^9*b^2*c^4 - 1280*a^10*c^5)*e^3*sqrt((81*b^8 -
918*a*b^6*c + 3051*a^2*b^4*c^2 - 2550*a^3*b^2*c^3 + 625*a^4*c^4)/((a^10*b^6
 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)*e^4)) - (27*b^11 - 486*a*
b^9*c + 3330*a^2*b^7*c^2 - 10549*a^3*b^5*c^3 + 14408*a^4*b^3*c^4 - 5200*a^5
*b*c^5)*e)*sqrt(-(9*b^7 - 105*a*b^5*c + 385*a^2*b^3*c^2 - 420*a^3*b*c^3 + (
a^5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2*c^2 - 64*a^8*c^3)*e^2*sqrt((81*b^8 - 91
8*a*b^6*c + 3051*a^2*b^4*c^2 - 2550*a^3*b^2*c^3 + 625*a^4*c^4)/((a^10*b^6 -
 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)*e^4)))/((a^5*b^6 - 12*a^6*b
^4*c + 48*a^7*b^2*c^2 - 64*a^8*c^3)*e^2))) - sqrt(1/2)*((a^2*b^2*c - 4*a^3*
c^2)*e^6*x^5 + 5*(a^2*b^2*c - 4*a^3*c^2)*d*e^5*x^4 + (a^2*b^3 - 4*a^3*b*c +
 10*(a^2*b^2*c - 4*a^3*c^2)*d^2)*e^4*x^3 + (10*(a^2*b^2*c - 4*a^3*c^2)*d^3
+ 3*(a^2*b^3 - 4*a^3*b*c)*d)*e^3*x^2 + (a^3*b^2 - 4*a^4*c + 5*(a^2*b^2*c -
4*a^3*c^2)*d^4 + 3*(a^2*b^3 - 4*a^3*b*c)*d^2)*e^2*x + ((a^2*b^2*c - 4*a^3*c
^2)*d^5 + (a^2*b^3 - 4*a^3*b*c)*d^3 + (a^3*b^2 - 4*a^4*c)*d)*e)*sqrt(-(9*b^
7 - 105*a*b^5*c + 385*a^2*b^3*c^2 - 420*a^3*b*c^3 + (a^5*b^6 - 12*a^6*b^4*c
 + 48*a^7*b^2*c^2 - 64*a^8*c^3)*e^2*sqrt((81*b^8 - 918*a*b^6*c + 3051*a^2*b
^4*c^2 - 2550*a^3*b^2*c^3 + 625*a^4*c^4)/((a^10*b^6 - 12*a^11*b^4*c + 48*a^
12*b^2*c^2 - 64*a^13*c^3)*e^4)))/((a^5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2*c^2
- 64*a^8*c^3)*e^2))*log(-(189*b^6*c^3 - 1971*a*b^4*c^4 + 5625*a^2*b^2*c^5 -
 2500*a^3*c^6)*e*x - (189*b^6*c^3 - 1971*a*b^4*c^4 + 5625*a^2*b^2*c^5 - 250
0*a^3*c^6)*d - 1/2*sqrt(1/2)*((3*a^5*b^10 - 55*a^6*b^8*c + 392*a^7*b^6*c^2
- 1344*a^8*b^4*c^3 + 2176*a^9*b^2*c^4 - 1280*a^10*c^5)*e^3*sqrt((81*b^8 - 9
18*a*b^6*c + 3051*a^2*b^4*c^2 - 2550*a^3*b^2*c^3 + 625*a^4*c^4)/((a^10*b^6
- 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)*e^4)) - (27*b^11 - 486*a*b
^9*c + 3330*a^2*b^7*c^2 - 10549*a^3*b^5*c^3 + 14408*a^4*b^3*c^4 - 5200*a^5*
b*c^5)*e)*sqrt(-(9*b^7 - 105*a*b^5*c + 385*a^2*b^3*c^2 - 420*a^3*b*c^3 + (a
^5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2*c^2 - 64*a^8*c^3)*e^2*sqrt((81*b^8 - 918
*a*b^6*c + 3051*a^2*b^4*c^2 - 2550*a^3*b^2*c^3 + 625*a^4*c^4)/((a^10*b^6 -
12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)*e^4)))/((a^5*b^6 - 12*a^6*b^
4*c + 48*a^7*b^2*c^2 - 64*a^8*c^3)*e^2))) - sqrt(1/2)*((a^2*b^2*c - 4*a^3*c
^2)*e^6*x^5 + 5*(a^2*b^2*c - 4*a^3*c^2)*d*e^5*x^4 + (a^2*b^3 - 4*a^3*b*c +
10*(a^2*b^2*c - 4*a^3*c^2)*d^2)*e^4*x^3 + (10*(a^2*b^2*c - 4*a^3*c^2)*d^3 +
 3*(a^2*b^3 - 4*a^3*b*c)*d)*e^3*x^2 + (a^3*b^2 - 4*a^4*c + 5*(a^2*b^2*c - 4
*a^3*c^2)*d^4 + 3*(a^2*b^3 - 4*a^3*b*c)*d^2)*e^2*x + ((a^2*b^2*c - 4*a^3*c^
```

```
2)*d^5 + (a^2*b^3 - 4*a^3*b*c)*d^3 + (a^3*b^2 - 4*a^4*c)*d)*e)*sqrt(-(9*b^7
 - 105*a*b^5*c + 385*a^2*b^3*c^2 - 420*a^3*b*c^3 - (a^5*b^6 - 12*a^6*b^4*c
+ 48*a^7*b^2*c^2 - 64*a^8*c^3)*e^2*sqrt((81*b^8 - 918*a*b^6*c + 3051*a^2*b^
4*c^2 - 2550*a^3*b^2*c^3 + 625*a^4*c^4)/((a^10*b^6 - 12*a^11*b^4*c + 48*a^1
2*b^2*c^2 - 64*a^13*c^3)*e^4)))/((a^5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2*c^2 -
 64*a^8*c^3)*e^2))*log(-(189*b^6*c^3 - 1971*a*b^4*c^4 + 5625*a^2*b^2*c^5 -
2500*a^3*c^6)*e*x - (189*b^6*c^3 - 1971*a*b^4*c^4 + 5625*a^2*b^2*c^5 - 2500
*a^3*c^6)*d + 1/2*sqrt(1/2)*((3*a^5*b^10 - 55*a^6*b^8*c + 392*a^7*b^6*c^2 -
 1344*a^8*b^4*c^3 + 2176*a^9*b^2*c^4 - 1280*a^10*c^5)*e^3*sqrt((81*b^8 - 91
8*a*b^6*c + 3051*a^2*b^4*c^2 - 2550*a^3*b^2*c^3 + 625*a^4*c^4)/((a^10*b^6 -
 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)*e^4)) + (27*b^11 - 486*a*b^
9*c + 3330*a^2*b^7*c^2 - 10549*a^3*b^5*c^3 + 14408*a^4*b^3*c^4 - 5200*a^5*b
*c^5)*e)*sqrt(-(9*b^7 - 105*a*b^5*c + 385*a^2*b^3*c^2 - 420*a^3*b*c^3 - (a^
5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2*c^2 - 64*a^8*c^3)*e^2*sqrt((81*b^8 - 918*
a*b^6*c + 3051*a^2*b^4*c^2 - 2550*a^3*b^2*c^3 + 625*a^4*c^4)/((a^10*b^6 - 1
2*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)*e^4)))/((a^5*b^6 - 12*a^6*b^4
*c + 48*a^7*b^2*c^2 - 64*a^8*c^3)*e^2))) + sqrt(1/2)*((a^2*b^2*c - 4*a^3*c^
2)*e^6*x^5 + 5*(a^2*b^2*c - 4*a^3*c^2)*d*e^5*x^4 + (a^2*b^3 - 4*a^3*b*c + 1
0*(a^2*b^2*c - 4*a^3*c^2)*d^2)*e^4*x^3 + (10*(a^2*b^2*c - 4*a^3*c^2)*d^3 +
3*(a^2*b^3 - 4*a^3*b*c)*d)*e^3*x^2 + (a^3*b^2 - 4*a^4*c + 5*(a^2*b^2*c - 4*
a^3*c^2)*d^4 + 3*(a^2*b^3 - 4*a^3*b*c)*d^2)*e^2*x + ((a^2*b^2*c - 4*a^3*c^2
)*d^5 + (a^2*b^3 - 4*a^3*b*c)*d^3 + (a^3*b^2 - 4*a^4*c)*d)*e)*sqrt(-(9*b^7
- 105*a*b^5*c + 385*a^2*b^3*c^2 - 420*a^3*b*c^3 - (a^5*b^6 - 12*a^6*b^4*c +
 48*a^7*b^2*c^2 - 64*a^8*c^3)*e^2*sqrt((81*b^8 - 918*a*b^6*c + 3051*a^2*b^4
*c^2 - 2550*a^3*b^2*c^3 + 625*a^4*c^4)/((a^10*b^6 - 12*a^11*b^4*c + 48*a^12
*b^2*c^2 - 64*a^13*c^3)*e^4)))/((a^5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2*c^2 -
64*a^8*c^3)*e^2))*log(-(189*b^6*c^3 - 1971*a*b^4*c^4 + 5625*a^2*b^2*c^5 - 2
500*a^3*c^6)*e*x - (189*b^6*c^3 - 1971*a*b^4*c^4 + 5625*a^2*b^2*c^5 - 2500*
a^3*c^6)*d - 1/2*sqrt(1/2)*((3*a^5*b^10 - 55*a^6*b^8*c + 392*a^7*b^6*c^2 -
1344*a^8*b^4*c^3 + 2176*a^9*b^2*c^4 - 1280*a^10*c^5)*e^3*sqrt((81*b^8 - 918
*a*b^6*c + 3051*a^2*b^4*c^2 - 2550*a^3*b^2*c^3 + 625*a^4*c^4)/((a^10*b^6 -
12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)*e^4)) + (27*b^11 - 486*a*b^9
*c + 3330*a^2*b^7*c^2 - 10549*a^3*b^5*c^3 + 14408*a^4*b^3*c^4 - 5200*a^5*b*
c^5)*e)*sqrt(-(9*b^7 - 105*a*b^5*c + 385*a^2*b^3*c^2 - 420*a^3*b*c^3 - (a^5
*b^6 - 12*a^6*b^4*c + 48*a^7*b^2*c^2 - 64*a^8*c^3)*e^2*sqrt((81*b^8 - 918*a
*b^6*c + 3051*a^2*b^4*c^2 - 2550*a^3*b^2*c^3 + 625*a^4*c^4)/((a^10*b^6 - 12
*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)*e^4)))/((a^5*b^6 - 12*a^6*b^4*
c + 48*a^7*b^2*c^2 - 64*a^8*c^3)*e^2))))/((a^2*b^2*c - 4*a^3*c^2)*e^6*x^5 +
 5*(a^2*b^2*c - 4*a^3*c^2)*d*e^5*x^4 + (a^2*b^3 - 4*a^3*b*c + 10*(a^2*b^2*c
 - 4*a^3*c^2)*d^2)*e^4*x^3 + (10*(a^2*b^2*c - 4*a^3*c^2)*d^3 + 3*(a^2*b^3 -
 4*a^3*b*c)*d)*e^3*x^2 + (a^3*b^2 - 4*a^4*c + 5*(a^2*b^2*c - 4*a^3*c^2)*d^4
 + 3*(a^2*b^3 - 4*a^3*b*c)*d^2)*e^2*x + ((a^2*b^2*c - 4*a^3*c^2)*d^5 + (a^2
*b^3 - 4*a^3*b*c)*d^3 + (a^3*b^2 - 4*a^4*c)*d)*e)
```

giac [B] time = 0.79, size = 847, normalized size = 2.43

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="giac")
```

```
[Out] 1/16*(2*(3*a^3*b^2*c - 10*a^4*c^2)*sqrt(2*a*b + 2*sqrt(b^2 - 4*a*c)*a)*sqrt
(b^2 - 4*a*c)*abs(a^2*b^2*e^2 - 4*a^3*c*e^2)*e^2 - (a^2*b^2*e^2 - 4*a^3*c*e
^2)^2*(3*b^3 - 13*a*b*c)*sqrt(2*a*b + 2*sqrt(b^2 - 4*a*c)*a) + (3*a^4*b^7 -
 31*a^5*b^5*c + 96*a^6*b^3*c^2 - 80*a^7*b*c^3)*sqrt(2*a*b + 2*sqrt(b^2 - 4*
a*c)*a)*e^4)*arctan(2*sqrt(1/2)*e^(-1)/((x*e + d)*sqrt((a^2*b^3*e^2 - 4*a^3
*b*c*e^2 + sqrt((a^2*b^3*e^2 - 4*a^3*b*c*e^2)^2 - 4*(a^3*b^2*e^4 - 4*a^4*c*
e^4)*(a^2*b^2*c - 4*a^3*c^2)))/(a^3*b^2*e^4 - 4*a^4*c*e^4))))*e^(-3)/((a^5*
b^2*c - 4*a^6*c^2)*sqrt(b^2 - 4*a*c)*abs(a^2*b^2*e^2 - 4*a^3*c*e^2)*abs(a))
 + 1/16*(2*(3*a^3*b^2*c - 10*a^4*c^2)*sqrt(2*a*b - 2*sqrt(b^2 - 4*a*c)*a)*s
```

```
qrt(b^2 - 4*a*c)*abs(a^2*b^2*e^2 - 4*a^3*c*e^2)*e^2 + (a^2*b^2*e^2 - 4*a^3*
c*e^2)^2*(3*b^3 - 13*a*b*c)*sqrt(2*a*b - 2*sqrt(b^2 - 4*a*c)*a) - (3*a^4*b^
7 - 31*a^5*b^5*c + 96*a^6*b^3*c^2 - 80*a^7*b*c^3)*sqrt(2*a*b - 2*sqrt(b^2 -
 4*a*c)*a)*e^4)*arctan(2*sqrt(1/2)*e^(-1)/((x*e + d)*sqrt((a^2*b^3*e^2 - 4*
a^3*b*c*e^2 - sqrt((a^2*b^3*e^2 - 4*a^3*b*c*e^2)^2 - 4*(a^3*b^2*e^4 - 4*a^4
*c*e^4)*(a^2*b^2*c - 4*a^3*c^2)))/(a^3*b^2*e^4 - 4*a^4*c*e^4))))*e^(-3)/((a
^5*b^2*c - 4*a^6*c^2)*sqrt(b^2 - 4*a*c)*abs(a^2*b^2*e^2 - 4*a^3*c*e^2)*abs(
a)) - 1/2*(b^2*c*e^(-1)/(x*e + d) - 2*a*c^2*e^(-1)/(x*e + d) + b^3*e^(-1)/(
x*e + d)^3 - 3*a*b*c*e^(-1)/(x*e + d)^3)/((a^2*b^2 - 4*a^3*c)*(c + b/(x*e +
 d)^2 + a/(x*e + d)^4)) - e^(-1)/((x*e + d)*a^2)
```

maple [C] time = 0.03, size = 1304, normalized size = 3.75

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x)
```

```
[Out] -1/a/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2
*b*d*e*x+b*d^2+a)*c^2*e^2/(4*a*c-b^2)*x^3+1/2/a^2/(c*e^4*x^4+4*c*d*e^3*x^3+
6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)*c*e^2/(4*a*c
-b^2)*x^3*b^2-3/a/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^
2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)*d*c^2*e/(4*a*c-b^2)*x^2+3/2/a^2/(c*e^4*x^4+4
*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)
*d*c*e/(4*a*c-b^2)*x^2*b^2-3/a/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c
*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)/(4*a*c-b^2)*x*c^2*d^2+3/2/a^2/(
c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e
*x+b*d^2+a)/(4*a*c-b^2)*x*b^2*c*d^2-3/2/a/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*
e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)/(4*a*c-b^2)*x*b*c+1/
2/a^2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+
2*b*d*e*x+b*d^2+a)/(4*a*c-b^2)*x*b^3-1/a/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e
^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)*d^3/e/(4*a*c-b^2)*c^2
+1/2/a^2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d
^4+2*b*d*e*x+b*d^2+a)*d^3/e/(4*a*c-b^2)*b^2*c-3/2/a/(c*e^4*x^4+4*c*d*e^3*x^
3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)*d/e/(4*a*c
-b^2)*b*c+1/2/a^2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^
2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)*d/e/(4*a*c-b^2)*b^3-1/4/a^2/(4*a*c-b^2)/e*su
m((c*e^2*(10*a*c-3*b^2)*_R^2+2*c*d*e*(10*a*c-3*b^2)*_R+10*a*c^2*d^2-3*b^2*c
*d^2+13*a*b*c-3*b^3)/(2*_R^3*c*e^3+6*_R^2*c*d*e^2+6*_R*c*d^2*e+2*c*d^3+_R*b
*e+b*d)*ln(-_R+x),_R=RootOf(_Z^4*c*e^4+4*_Z^3*c*d*e^3+c*d^4+b*d^2+(6*c*d^2*
e^2+b*e^2)*_Z^2+(4*c*d^3*e+2*b*d*e)*_Z+a))-1/a^2/e/(e*x+d)
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [B] time = 6.38, size = 10556, normalized size = 30.33

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((d + e*x)^2*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2),x)
```

```
[Out] - atan(((-(9*b^13 - 9*b^4*(-(4*a*c - b^2)^9)^(1/2) + 26880*a^6*b*c^6 + 2077
*a^2*b^9*c^2 - 10656*a^3*b^7*c^3 + 30240*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 -
25*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c + 51*a*b^2*c*(-(4*a*c -
```

b^2)^9)^(1/2))/(32*(a^5*b^12*e^2 + 4096*a^11*c^6*e^2 - 24*a^6*b^10*c*e^2 + 240*a^7*b^8*c^2*e^2 - 1280*a^8*b^6*c^3*e^2 + 3840*a^9*b^4*c^4*e^2 - 6144*a^10*b^2*c^5*e^2)))^(1/2)*(x*(204800*a^12*c^9*e^12 + 144*a^6*b^12*c^3*e^12 - 3264*a^7*b^10*c^4*e^12 + 30112*a^8*b^8*c^5*e^12 - 143360*a^9*b^6*c^6*e^12 + 365568*a^10*b^4*c^7*e^12 - 458752*a^11*b^2*c^8*e^12) + (-(9*b^13 - 9*b^4*(-(4*a*c - b^2)^9)^(1/2) + 26880*a^6*b*c^6 + 2077*a^2*b^9*c^2 - 10656*a^3*b^7*c^3 + 30240*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 - 25*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c + 51*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^5*b^12*e^2 + 4096*a^11*c^6*e^2 - 24*a^6*b^10*c*e^2 + 240*a^7*b^8*c^2*e^2 - 1280*a^8*b^6*c^3*e^2 + 3840*a^9*b^4*c^4*e^2 - 6144*a^10*b^2*c^5*e^2)))^(1/2)*((-(9*b^13 - 9*b^4*(-(4*a*c - b^2)^9)^(1/2) + 26880*a^6*b*c^6 + 2077*a^2*b^9*c^2 - 10656*a^3*b^7*c^3 + 30240*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 - 25*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c + 51*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^5*b^12*e^2 + 4096*a^11*c^6*e^2 - 24*a^6*b^10*c*e^2 + 240*a^7*b^8*c^2*e^2 - 1280*a^8*b^6*c^3*e^2 + 3840*a^9*b^4*c^4*e^2 - 6144*a^10*b^2*c^5*e^2)))^(1/2)*(x*(1048576*a^16*b*c^8*e^14 + 256*a^10*b^13*c^2*e^14 - 6144*a^11*b^11*c^3*e^14 + 61440*a^12*b^9*c^4*e^14 - 327680*a^13*b^7*c^5*e^14 + 983040*a^14*b^5*c^6*e^14 - 1572864*a^15*b^3*c^7*e^14) + 1048576*a^16*b*c^8*d*e^13 + 256*a^10*b^13*c^2*d*e^13 - 6144*a^11*b^11*c^3*d*e^13 + 61440*a^12*b^9*c^4*d*e^13 - 327680*a^13*b^7*c^5*d*e^13 + 983040*a^14*b^5*c^6*d*e^13 - 1572864*a^15*b^3*c^7*d*e^13) - 851968*a^14*b*c^8*e^12 - 192*a^8*b^13*c^2*e^12 + 4672*a^9*b^11*c^3*e^12 - 47360*a^10*b^9*c^4*e^12 + 256000*a^11*b^7*c^5*e^12 - 778240*a^12*b^5*c^6*e^12 + 1261568*a^13*b^3*c^7*e^12) + 204800*a^12*c^9*d*e^11 + 144*a^6*b^12*c^3*d*e^11 - 3264*a^7*b^10*c^4*d*e^11 + 30112*a^8*b^8*c^5*d*e^11 - 143360*a^9*b^6*c^6*d*e^11 + 365568*a^10*b^4*c^7*d*e^11 - 458752*a^11*b^2*c^8*d*e^11)*1i + (-(9*b^13 - 9*b^4*(-(4*a*c - b^2)^9)^(1/2) + 26880*a^6*b*c^6 + 2077*a^2*b^9*c^2 - 10656*a^3*b^7*c^3 + 30240*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 - 25*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c + 51*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^5*b^12*e^2 + 4096*a^11*c^6*e^2 - 24*a^6*b^10*c*e^2 + 240*a^7*b^8*c^2*e^2 - 1280*a^8*b^6*c^3*e^2 + 3840*a^9*b^4*c^4*e^2 - 6144*a^10*b^2*c^5*e^2)))^(1/2)*(x*(204800*a^12*c^9*e^12 + 144*a^6*b^12*c^3*e^12 - 3264*a^7*b^10*c^4*e^12 + 30112*a^8*b^8*c^5*e^12 - 143360*a^9*b^6*c^6*e^12 + 365568*a^10*b^4*c^7*e^12 - 458752*a^11*b^2*c^8*e^12) + (-(9*b^13 - 9*b^4*(-(4*a*c - b^2)^9)^(1/2) + 26880*a^6*b*c^6 + 2077*a^2*b^9*c^2 - 10656*a^3*b^7*c^3 + 30240*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 - 25*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c + 51*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^5*b^12*e^2 + 4096*a^11*c^6*e^2 - 24*a^6*b^10*c*e^2 + 240*a^7*b^8*c^2*e^2 - 1280*a^8*b^6*c^3*e^2 + 3840*a^9*b^4*c^4*e^2 - 6144*a^10*b^2*c^5*e^2)))^(1/2)*((-(9*b^13 - 9*b^4*(-(4*a*c - b^2)^9)^(1/2) + 26880*a^6*b*c^6 + 2077*a^2*b^9*c^2 - 10656*a^3*b^7*c^3 + 30240*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 - 25*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c + 51*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^5*b^12*e^2 + 4096*a^11*c^6*e^2 - 24*a^6*b^10*c*e^2 + 240*a^7*b^8*c^2*e^2 - 1280*a^8*b^6*c^3*e^2 + 3840*a^9*b^4*c^4*e^2 - 6144*a^10*b^2*c^5*e^2)))^(1/2)*(x*(1048576*a^16*b*c^8*e^14 + 256*a^10*b^13*c^2*e^14 - 6144*a^11*b^11*c^3*e^14 + 61440*a^12*b^9*c^4*e^14 - 327680*a^13*b^7*c^5*e^14 + 983040*a^14*b^5*c^6*e^14 - 1572864*a^15*b^3*c^7*e^14) + 1048576*a^16*b*c^8*d*e^13 + 256*a^10*b^13*c^2*d*e^13 - 6144*a^11*b^11*c^3*d*e^13 + 61440*a^12*b^9*c^4*d*e^13 - 327680*a^13*b^7*c^5*d*e^13 + 983040*a^14*b^5*c^6*d*e^13 - 1572864*a^15*b^3*c^7*d*e^13) + 851968*a^14*b*c^8*e^12 + 192*a^8*b^13*c^2*e^12 - 4672*a^9*b^11*c^3*e^12 + 47360*a^10*b^9*c^4*e^12 - 256000*a^11*b^7*c^5*e^12 + 778240*a^12*b^5*c^6*e^12 - 1261568*a^13*b^3*c^7*e^12) + 204800*a^12*c^9*d*e^11 + 144*a^6*b^12*c^3*d*e^11 - 3264*a^7*b^10*c^4*d*e^11 + 30112*a^8*b^8*c^5*d*e^11 - 143360*a^9*b^6*c^6*d*e^11 + 365568*a^10*b^4*c^7*d*e^11 - 458752*a^11*b^2*c^8*d*e^11)*1i)/((-(9*b^13 - 9*b^4*(-(4*a*c - b^2)^9)^(1/2) + 26880*a^6*b*c^6 + 2077*a^2*b^9*c^2 - 10656*a^3*b^7*c^3 + 30240*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 - 25*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c + 51*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^5*b^12*e^2 + 4096*a^11*c^6*e^2 - 24*a^6*b^10*c*e^2 + 240*a^7*b^8*c^2*e^2 - 1280*a^8*b^6*c^3*e^2 + 3840*a^9*b^4*c^4*e^2 - 6144*a^10*b^2*c^5*e^2)))^(1/2)*(x*

(204800*a^12*c^9*e^12 + 144*a^6*b^12*c^3*e^12 - 3264*a^7*b^10*c^4*e^12 + 30
112*a^8*b^8*c^5*e^12 - 143360*a^9*b^6*c^6*e^12 + 365568*a^10*b^4*c^7*e^12 -
 458752*a^11*b^2*c^8*e^12) + (-(9*b^13 - 9*b^4*(-(4*a*c - b^2)^9)^(1/2) + 2
6880*a^6*b*c^6 + 2077*a^2*b^9*c^2 - 10656*a^3*b^7*c^3 + 30240*a^4*b^5*c^4 -
 44800*a^5*b^3*c^5 - 25*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c + 5
1*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^5*b^12*e^2 + 4096*a^11*c^6*e^2 -
 24*a^6*b^10*c*e^2 + 240*a^7*b^8*c^2*e^2 - 1280*a^8*b^6*c^3*e^2 + 3840*a^9*
b^4*c^4*e^2 - 6144*a^10*b^2*c^5*e^2)))^(1/2)*((-(9*b^13 - 9*b^4*(-(4*a*c -
b^2)^9)^(1/2) + 26880*a^6*b*c^6 + 2077*a^2*b^9*c^2 - 10656*a^3*b^7*c^3 + 30
240*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 - 25*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) -
 213*a*b^11*c + 51*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^5*b^12*e^2 + 40
96*a^11*c^6*e^2 - 24*a^6*b^10*c*e^2 + 240*a^7*b^8*c^2*e^2 - 1280*a^8*b^6*c^
3*e^2 + 3840*a^9*b^4*c^4*e^2 - 6144*a^10*b^2*c^5*e^2)))^(1/2)*(x*(1048576*a
^16*b*c^8*e^14 + 256*a^10*b^13*c^2*e^14 - 6144*a^11*b^11*c^3*e^14 + 61440*a
^12*b^9*c^4*e^14 - 327680*a^13*b^7*c^5*e^14 + 983040*a^14*b^5*c^6*e^14 - 15
72864*a^15*b^3*c^7*e^14) + 1048576*a^16*b*c^8*d*e^13 + 256*a^10*b^13*c^2*d*
e^13 - 6144*a^11*b^11*c^3*d*e^13 + 61440*a^12*b^9*c^4*d*e^13 - 327680*a^13*
b^7*c^5*d*e^13 + 983040*a^14*b^5*c^6*d*e^13 - 1572864*a^15*b^3*c^7*d*e^13)
+ 851968*a^14*b*c^8*e^12 + 192*a^8*b^13*c^2*e^12 - 4672*a^9*b^11*c^3*e^12 +
 47360*a^10*b^9*c^4*e^12 - 256000*a^11*b^7*c^5*e^12 + 778240*a^12*b^5*c^6*e
^12 - 1261568*a^13*b^3*c^7*e^12) + 204800*a^12*c^9*d*e^11 + 144*a^6*b^12*c^
3*d*e^11 - 3264*a^7*b^10*c^4*d*e^11 + 30112*a^8*b^8*c^5*d*e^11 - 143360*a^9
*b^6*c^6*d*e^11 + 365568*a^10*b^4*c^7*d*e^11 - 458752*a^11*b^2*c^8*d*e^11)
- (-(9*b^13 - 9*b^4*(-(4*a*c - b^2)^9)^(1/2) + 26880*a^6*b*c^6 + 2077*a^2*b
^9*c^2 - 10656*a^3*b^7*c^3 + 30240*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 - 25*a^2
c^2(-(4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c + 51*a*b^2*c*(-(4*a*c - b^2)^9
)^(1/2))/(32*(a^5*b^12*e^2 + 4096*a^11*c^6*e^2 - 24*a^6*b^10*c*e^2 + 240*a^
7*b^8*c^2*e^2 - 1280*a^8*b^6*c^3*e^2 + 3840*a^9*b^4*c^4*e^2 - 6144*a^10*b^2
*c^5*e^2)))^(1/2)*(x*(204800*a^12*c^9*e^12 + 144*a^6*b^12*c^3*e^12 - 3264*a
^7*b^10*c^4*e^12 + 30112*a^8*b^8*c^5*e^12 - 143360*a^9*b^6*c^6*e^12 + 36556
8*a^10*b^4*c^7*e^12 - 458752*a^11*b^2*c^8*e^12) + (-(9*b^13 - 9*b^4*(-(4*a*
c - b^2)^9)^(1/2) + 26880*a^6*b*c^6 + 2077*a^2*b^9*c^2 - 10656*a^3*b^7*c^3
+ 30240*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 - 25*a^2*c^2*(-(4*a*c - b^2)^9)^(1/
2) - 213*a*b^11*c + 51*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^5*b^12*e^2
+ 4096*a^11*c^6*e^2 - 24*a^6*b^10*c*e^2 + 240*a^7*b^8*c^2*e^2 - 1280*a^8*b^
6*c^3*e^2 + 3840*a^9*b^4*c^4*e^2 - 6144*a^10*b^2*c^5*e^2)))^(1/2)*((-(9*b^1
3 - 9*b^4*(-(4*a*c - b^2)^9)^(1/2) + 26880*a^6*b*c^6 + 2077*a^2*b^9*c^2 - 1
0656*a^3*b^7*c^3 + 30240*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 - 25*a^2*c^2*(-(4*
a*c - b^2)^9)^(1/2) - 213*a*b^11*c + 51*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2))/(
32*(a^5*b^12*e^2 + 4096*a^11*c^6*e^2 - 24*a^6*b^10*c*e^2 + 240*a^7*b^8*c^2*
e^2 - 1280*a^8*b^6*c^3*e^2 + 3840*a^9*b^4*c^4*e^2 - 6144*a^10*b^2*c^5*e^2))
)^(1/2)*(x*(1048576*a^16*b*c^8*e^14 + 256*a^10*b^13*c^2*e^14 - 6144*a^11*b^
11*c^3*e^14 + 61440*a^12*b^9*c^4*e^14 - 327680*a^13*b^7*c^5*e^14 + 983040*a
^14*b^5*c^6*e^14 - 1572864*a^15*b^3*c^7*e^14) + 1048576*a^16*b*c^8*d*e^13 +
 256*a^10*b^13*c^2*d*e^13 - 6144*a^11*b^11*c^3*d*e^13 + 61440*a^12*b^9*c^4*
d*e^13 - 327680*a^13*b^7*c^5*d*e^13 + 983040*a^14*b^5*c^6*d*e^13 - 1572864*
a^15*b^3*c^7*d*e^13) - 851968*a^14*b*c^8*e^12 - 192*a^8*b^13*c^2*e^12 + 467
2*a^9*b^11*c^3*e^12 - 47360*a^10*b^9*c^4*e^12 + 256000*a^11*b^7*c^5*e^12 -
778240*a^12*b^5*c^6*e^12 + 1261568*a^13*b^3*c^7*e^12) + 204800*a^12*c^9*d*e
^11 + 144*a^6*b^12*c^3*d*e^11 - 3264*a^7*b^10*c^4*d*e^11 + 30112*a^8*b^8*c^
5*d*e^11 - 143360*a^9*b^6*c^6*d*e^11 + 365568*a^10*b^4*c^7*d*e^11 - 458752*
a^11*b^2*c^8*d*e^11) + 128000*a^10*c^9*e^10 + 504*a^6*b^8*c^5*e^10 - 8112*a
^7*b^6*c^6*e^10 + 48704*a^8*b^4*c^7*e^10 - 129280*a^9*b^2*c^8*e^10))*(-(9*b
^13 - 9*b^4*(-(4*a*c - b^2)^9)^(1/2) + 26880*a^6*b*c^6 + 2077*a^2*b^9*c^2 -
 10656*a^3*b^7*c^3 + 30240*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 - 25*a^2*c^2*(-(
4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c + 51*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2))
/(32*(a^5*b^12*e^2 + 4096*a^11*c^6*e^2 - 24*a^6*b^10*c*e^2 + 240*a^7*b^8*c^
2*e^2 - 1280*a^8*b^6*c^3*e^2 + 3840*a^9*b^4*c^4*e^2 - 6144*a^10*b^2*c^5*e^2
)))^(1/2)*2i - atan(((-(9*b^13 + 9*b^4*(-(4*a*c - b^2)^9)^(1/2) + 26880*a^6

```
*b*c^6 + 2077*a^2*b^9*c^2 - 10656*a^3*b^7*c^3 + 30240*a^4*b^5*c^4 - 44800*a
^5*b^3*c^5 + 25*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c - 51*a*b^2*
c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^5*b^12*e^2 + 4096*a^11*c^6*e^2 - 24*a^6*
b^10*c*e^2 + 240*a^7*b^8*c^2*e^2 - 1280*a^8*b^6*c^3*e^2 + 3840*a^9*b^4*c^4*
e^2 - 6144*a^10*b^2*c^5*e^2)))^(1/2)*(x*(204800*a^12*c^9*e^12 + 144*a^6*b^1
2*c^3*e^12 - 3264*a^7*b^10*c^4*e^12 + 30112*a^8*b^8*c^5*e^12 - 143360*a^9*b
^6*c^6*e^12 + 365568*a^10*b^4*c^7*e^12 - 458752*a^11*b^2*c^8*e^12) + (-(9*b
^13 + 9*b^4*(-(4*a*c - b^2)^9)^(1/2) + 26880*a^6*b*c^6 + 2077*a^2*b^9*c^2 -
 10656*a^3*b^7*c^3 + 30240*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 + 25*a^2*c^2*(-(
4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c - 51*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2))
/(32*(a^5*b^12*e^2 + 4096*a^11*c^6*e^2 - 24*a^6*b^10*c*e^2 + 240*a^7*b^8*c^
2*e^2 - 1280*a^8*b^6*c^3*e^2 + 3840*a^9*b^4*c^4*e^2 - 6144*a^10*b^2*c^5*e^2
)))^(1/2)*((-(9*b^13 + 9*b^4*(-(4*a*c - b^2)^9)^(1/2) + 26880*a^6*b*c^6 + 2
077*a^2*b^9*c^2 - 10656*a^3*b^7*c^3 + 30240*a^4*b^5*c^4 - 44800*a^5*b^3*c^5
 + 25*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c - 51*a*b^2*c*(-(4*a*c
 - b^2)^9)^(1/2))/(32*(a^5*b^12*e^2 + 4096*a^11*c^6*e^2 - 24*a^6*b^10*c*e^2
 + 240*a^7*b^8*c^2*e^2 - 1280*a^8*b^6*c^3*e^2 + 3840*a^9*b^4*c^4*e^2 - 6144
*a^10*b^2*c^5*e^2)))^(1/2)*(x*(1048576*a^16*b*c^8*e^14 + 256*a^10*b^13*c^2*
e^14 - 6144*a^11*b^11*c^3*e^14 + 61440*a^12*b^9*c^4*e^14 - 327680*a^13*b^7*
c^5*e^14 + 983040*a^14*b^5*c^6*e^14 - 1572864*a^15*b^3*c^7*e^14) + 1048576*
a^16*b*c^8*d*e^13 + 256*a^10*b^13*c^2*d*e^13 - 6144*a^11*b^11*c^3*d*e^13 +
61440*a^12*b^9*c^4*d*e^13 - 327680*a^13*b^7*c^5*d*e^13 + 983040*a^14*b^5*c^
6*d*e^13 - 1572864*a^15*b^3*c^7*d*e^13) - 851968*a^14*b*c^8*e^12 - 192*a^8*
b^13*c^2*e^12 + 4672*a^9*b^11*c^3*e^12 - 47360*a^10*b^9*c^4*e^12 + 256000*a
^11*b^7*c^5*e^12 - 778240*a^12*b^5*c^6*e^12 + 1261568*a^13*b^3*c^7*e^12) +
204800*a^12*c^9*d*e^11 + 144*a^6*b^12*c^3*d*e^11 - 3264*a^7*b^10*c^4*d*e^11
 + 30112*a^8*b^8*c^5*d*e^11 - 143360*a^9*b^6*c^6*d*e^11 + 365568*a^10*b^4*c
^7*d*e^11 - 458752*a^11*b^2*c^8*d*e^11)*1i + (-(9*b^13 + 9*b^4*(-(4*a*c - b
^2)^9)^(1/2) + 26880*a^6*b*c^6 + 2077*a^2*b^9*c^2 - 10656*a^3*b^7*c^3 + 302
40*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 + 25*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) -
213*a*b^11*c - 51*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^5*b^12*e^2 + 409
6*a^11*c^6*e^2 - 24*a^6*b^10*c*e^2 + 240*a^7*b^8*c^2*e^2 - 1280*a^8*b^6*c^3
*e^2 + 3840*a^9*b^4*c^4*e^2 - 6144*a^10*b^2*c^5*e^2)))^(1/2)*(x*(204800*a^1
2*c^9*e^12 + 144*a^6*b^12*c^3*e^12 - 3264*a^7*b^10*c^4*e^12 + 30112*a^8*b^8
*c^5*e^12 - 143360*a^9*b^6*c^6*e^12 + 365568*a^10*b^4*c^7*e^12 - 458752*a^1
1*b^2*c^8*e^12) + (-(9*b^13 + 9*b^4*(-(4*a*c - b^2)^9)^(1/2) + 26880*a^6*b*
c^6 + 2077*a^2*b^9*c^2 - 10656*a^3*b^7*c^3 + 30240*a^4*b^5*c^4 - 44800*a^5*
b^3*c^5 + 25*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c - 51*a*b^2*c*(
-(4*a*c - b^2)^9)^(1/2))/(32*(a^5*b^12*e^2 + 4096*a^11*c^6*e^2 - 24*a^6*b^1
0*c*e^2 + 240*a^7*b^8*c^2*e^2 - 1280*a^8*b^6*c^3*e^2 + 3840*a^9*b^4*c^4*e^2
 - 6144*a^10*b^2*c^5*e^2)))^(1/2)*((-(9*b^13 + 9*b^4*(-(4*a*c - b^2)^9)^(1/
2) + 26880*a^6*b*c^6 + 2077*a^2*b^9*c^2 - 10656*a^3*b^7*c^3 + 30240*a^4*b^5
*c^4 - 44800*a^5*b^3*c^5 + 25*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 213*a*b^11
*c - 51*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^5*b^12*e^2 + 4096*a^11*c^6
*e^2 - 24*a^6*b^10*c*e^2 + 240*a^7*b^8*c^2*e^2 - 1280*a^8*b^6*c^3*e^2 + 384
0*a^9*b^4*c^4*e^2 - 6144*a^10*b^2*c^5*e^2)))^(1/2)*(x*(1048576*a^16*b*c^8*e
^14 + 256*a^10*b^13*c^2*e^14 - 6144*a^11*b^11*c^3*e^14 + 61440*a^12*b^9*c^4
*e^14 - 327680*a^13*b^7*c^5*e^14 + 983040*a^14*b^5*c^6*e^14 - 1572864*a^15*
b^3*c^7*e^14) + 1048576*a^16*b*c^8*d*e^13 + 256*a^10*b^13*c^2*d*e^13 - 6144
*a^11*b^11*c^3*d*e^13 + 61440*a^12*b^9*c^4*d*e^13 - 327680*a^13*b^7*c^5*d*e
^13 + 983040*a^14*b^5*c^6*d*e^13 - 1572864*a^15*b^3*c^7*d*e^13) + 851968*a^
14*b*c^8*e^12 + 192*a^8*b^13*c^2*e^12 - 4672*a^9*b^11*c^3*e^12 + 47360*a^10
*b^9*c^4*e^12 - 256000*a^11*b^7*c^5*e^12 + 778240*a^12*b^5*c^6*e^12 - 12615
68*a^13*b^3*c^7*e^12) + 204800*a^12*c^9*d*e^11 + 144*a^6*b^12*c^3*d*e^11 -
3264*a^7*b^10*c^4*d*e^11 + 30112*a^8*b^8*c^5*d*e^11 - 143360*a^9*b^6*c^6*d*
e^11 + 365568*a^10*b^4*c^7*d*e^11 - 458752*a^11*b^2*c^8*d*e^11)*1i)/((-(9*b
^13 + 9*b^4*(-(4*a*c - b^2)^9)^(1/2) + 26880*a^6*b*c^6 + 2077*a^2*b^9*c^2 -
 10656*a^3*b^7*c^3 + 30240*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 + 25*a^2*c^2*(-(
4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c - 51*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2))
```

/(32*(a^5*b^12*e^2 + 4096*a^11*c^6*e^2 - 24*a^6*b^10*c*e^2 + 240*a^7*b^8*c^
2*e^2 - 1280*a^8*b^6*c^3*e^2 + 3840*a^9*b^4*c^4*e^2 - 6144*a^10*b^2*c^5*e^2
)))^(1/2)*(x*(204800*a^12*c^9*e^12 + 144*a^6*b^12*c^3*e^12 - 3264*a^7*b^10*
c^4*e^12 + 30112*a^8*b^8*c^5*e^12 - 143360*a^9*b^6*c^6*e^12 + 365568*a^10*b
^4*c^7*e^12 - 458752*a^11*b^2*c^8*e^12) + (-(9*b^13 + 9*b^4*(-(4*a*c - b^2)
^9)^(1/2) + 26880*a^6*b*c^6 + 2077*a^2*b^9*c^2 - 10656*a^3*b^7*c^3 + 30240*
a^4*b^5*c^4 - 44800*a^5*b^3*c^5 + 25*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 213
*a*b^11*c - 51*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^5*b^12*e^2 + 4096*a
^11*c^6*e^2 - 24*a^6*b^10*c*e^2 + 240*a^7*b^8*c^2*e^2 - 1280*a^8*b^6*c^3*e^
2 + 3840*a^9*b^4*c^4*e^2 - 6144*a^10*b^2*c^5*e^2)))^(1/2)*((-(9*b^13 + 9*b^
4*(-(4*a*c - b^2)^9)^(1/2) + 26880*a^6*b*c^6 + 2077*a^2*b^9*c^2 - 10656*a^3
*b^7*c^3 + 30240*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 + 25*a^2*c^2*(-(4*a*c - b^
2)^9)^(1/2) - 213*a*b^11*c - 51*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^5*
b^12*e^2 + 4096*a^11*c^6*e^2 - 24*a^6*b^10*c*e^2 + 240*a^7*b^8*c^2*e^2 - 12
80*a^8*b^6*c^3*e^2 + 3840*a^9*b^4*c^4*e^2 - 6144*a^10*b^2*c^5*e^2)))^(1/2)*
(x*(1048576*a^16*b*c^8*e^14 + 256*a^10*b^13*c^2*e^14 - 6144*a^11*b^11*c^3*e
^14 + 61440*a^12*b^9*c^4*e^14 - 327680*a^13*b^7*c^5*e^14 + 983040*a^14*b^5*
c^6*e^14 - 1572864*a^15*b^3*c^7*e^14) + 1048576*a^16*b*c^8*d*e^13 + 256*a^1
0*b^13*c^2*d*e^13 - 6144*a^11*b^11*c^3*d*e^13 + 61440*a^12*b^9*c^4*d*e^13 -
327680*a^13*b^7*c^5*d*e^13 + 983040*a^14*b^5*c^6*d*e^13 - 1572864*a^15*b^3
*c^7*d*e^13) + 851968*a^14*b*c^8*e^12 + 192*a^8*b^13*c^2*e^12 - 4672*a^9*b^
11*c^3*e^12 + 47360*a^10*b^9*c^4*e^12 - 256000*a^11*b^7*c^5*e^12 + 778240*a
^12*b^5*c^6*e^12 - 1261568*a^13*b^3*c^7*e^12) + 204800*a^12*c^9*d*e^11 + 14
4*a^6*b^12*c^3*d*e^11 - 3264*a^7*b^10*c^4*d*e^11 + 30112*a^8*b^8*c^5*d*e^11
- 143360*a^9*b^6*c^6*d*e^11 + 365568*a^10*b^4*c^7*d*e^11 - 458752*a^11*b^2
*c^8*d*e^11) - (-(9*b^13 + 9*b^4*(-(4*a*c - b^2)^9)^(1/2) + 26880*a^6*b*c^6
+ 2077*a^2*b^9*c^2 - 10656*a^3*b^7*c^3 + 30240*a^4*b^5*c^4 - 44800*a^5*b^3
*c^5 + 25*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c - 51*a*b^2*c*(-(4
*a*c - b^2)^9)^(1/2))/(32*(a^5*b^12*e^2 + 4096*a^11*c^6*e^2 - 24*a^6*b^10*c
*e^2 + 240*a^7*b^8*c^2*e^2 - 1280*a^8*b^6*c^3*e^2 + 3840*a^9*b^4*c^4*e^2 -
6144*a^10*b^2*c^5*e^2)))^(1/2)*(x*(204800*a^12*c^9*e^12 + 144*a^6*b^12*c^3*
e^12 - 3264*a^7*b^10*c^4*e^12 + 30112*a^8*b^8*c^5*e^12 - 143360*a^9*b^6*c^6
*e^12 + 365568*a^10*b^4*c^7*e^12 - 458752*a^11*b^2*c^8*e^12) + (-(9*b^13 +
9*b^4*(-(4*a*c - b^2)^9)^(1/2) + 26880*a^6*b*c^6 + 2077*a^2*b^9*c^2 - 10656
*a^3*b^7*c^3 + 30240*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 + 25*a^2*c^2*(-(4*a*c
- b^2)^9)^(1/2) - 213*a*b^11*c - 51*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(
a^5*b^12*e^2 + 4096*a^11*c^6*e^2 - 24*a^6*b^10*c*e^2 + 240*a^7*b^8*c^2*e^2
- 1280*a^8*b^6*c^3*e^2 + 3840*a^9*b^4*c^4*e^2 - 6144*a^10*b^2*c^5*e^2)))^(1
/2)*((-(9*b^13 + 9*b^4*(-(4*a*c - b^2)^9)^(1/2) + 26880*a^6*b*c^6 + 2077*a^
2*b^9*c^2 - 10656*a^3*b^7*c^3 + 30240*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 + 25*
a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c - 51*a*b^2*c*(-(4*a*c - b^2
)^9)^(1/2))/(32*(a^5*b^12*e^2 + 4096*a^11*c^6*e^2 - 24*a^6*b^10*c*e^2 + 240
*a^7*b^8*c^2*e^2 - 1280*a^8*b^6*c^3*e^2 + 3840*a^9*b^4*c^4*e^2 - 6144*a^10*
b^2*c^5*e^2)))^(1/2)*(x*(1048576*a^16*b*c^8*e^14 + 256*a^10*b^13*c^2*e^14 -
6144*a^11*b^11*c^3*e^14 + 61440*a^12*b^9*c^4*e^14 - 327680*a^13*b^7*c^5*e^
14 + 983040*a^14*b^5*c^6*e^14 - 1572864*a^15*b^3*c^7*e^14) + 1048576*a^16*b
*c^8*d*e^13 + 256*a^10*b^13*c^2*d*e^13 - 6144*a^11*b^11*c^3*d*e^13 + 61440*
a^12*b^9*c^4*d*e^13 - 327680*a^13*b^7*c^5*d*e^13 + 983040*a^14*b^5*c^6*d*e^
13 - 1572864*a^15*b^3*c^7*d*e^13) - 851968*a^14*b*c^8*e^12 - 192*a^8*b^13*c
^2*e^12 + 4672*a^9*b^11*c^3*e^12 - 47360*a^10*b^9*c^4*e^12 + 256000*a^11*b^
7*c^5*e^12 - 778240*a^12*b^5*c^6*e^12 + 1261568*a^13*b^3*c^7*e^12) + 204800
*a^12*c^9*d*e^11 + 144*a^6*b^12*c^3*d*e^11 - 3264*a^7*b^10*c^4*d*e^11 + 301
12*a^8*b^8*c^5*d*e^11 - 143360*a^9*b^6*c^6*d*e^11 + 365568*a^10*b^4*c^7*d*e
^11 - 458752*a^11*b^2*c^8*d*e^11) + 128000*a^10*c^9*e^10 + 504*a^6*b^8*c^5*
e^10 - 8112*a^7*b^6*c^6*e^10 + 48704*a^8*b^4*c^7*e^10 - 129280*a^9*b^2*c^8*
e^10))*(-(9*b^13 + 9*b^4*(-(4*a*c - b^2)^9)^(1/2) + 26880*a^6*b*c^6 + 2077*
a^2*b^9*c^2 - 10656*a^3*b^7*c^3 + 30240*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 + 2
5*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c - 51*a*b^2*c*(-(4*a*c - b
^2)^9)^(1/2))/(32*(a^5*b^12*e^2 + 4096*a^11*c^6*e^2 - 24*a^6*b^10*c*e^2 + 2

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40*a^7*b^8*c^2*e^2 - 1280*a^8*b^6*c^3*e^2 + 3840*a^9*b^4*c^4*e^2 - 6144*a^1
0*b^2*c^5*e^2)))^(1/2)*2i - ((x*(3*b^3*d - 20*a*c^2*d^3 + 6*b^2*c*d^3 - 11*
a*b*c*d))/(a*(a*b^2 - 4*a^2*c)) - (x^4*(10*a*c^2*e^3 - 3*b^2*c*e^3))/(2*a*(
a*b^2 - 4*a^2*c)) - (2*x^3*(10*a*c^2*d*e^2 - 3*b^2*c*d*e^2))/(a*(a*b^2 - 4*
a^2*c)) + (2*a*b^2 - 8*a^2*c + 3*b^3*d^2 - 10*a*c^2*d^4 + 3*b^2*c*d^4 - 11*
a*b*c*d^2)/(2*a*e*(a*b^2 - 4*a^2*c)) + (x^2*(3*b^3*e - 60*a*c^2*d^2*e + 18*
b^2*c*d^2*e - 11*a*b*c*e))/(2*a*(a*b^2 - 4*a^2*c)))/(a*d + x*(a*e + 3*b*d^2
*e + 5*c*d^4*e) + x^3*(b*e^3 + 10*c*d^2*e^3) + b*d^3 + c*d^5 + x^2*(10*c*d^
3*e^2 + 3*b*d*e^2) + c*e^5*x^5 + 5*c*d*e^4*x^4)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)**2/(a+b*(e*x+d)**2+c*(e*x+d)**4)**2,x)
```

```
[Out] Timed out
```

3.515 $\int \frac{1}{(d+ex)^3\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx$

Optimal. Leaf size=213

$$\frac{b\log\left(a+b(d+ex)^2+c(d+ex)^4\right)}{2a^3e}-\frac{2b\log(d+ex)}{a^3e}-\frac{b^2-3ac}{a^2e\left(b^2-4ac\right)(d+ex)^2}-\frac{\left(6a^2c^2-6ab^2c+b^4\right)\tanh^{-1}\left(\frac{b+2c(d}{\sqrt{b^2-}}\right.}{a^3e\left(b^2-4ac\right)^{3/2}}$$

Rubi [A] time = 0.39, antiderivative size = 213, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {1142, 1114, 740, 800, 634, 618, 206, 628}

$$-\frac{\left(6a^2c^2-6ab^2c+b^4\right)\tanh^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{a^3e\left(b^2-4ac\right)^{3/2}}-\frac{b^2-3ac}{a^2e\left(b^2-4ac\right)(d+ex)^2}+\frac{b\log\left(a+b(d+ex)^2+c(d+ex)^4\right)}{2a^3e}-\frac{2b\log(d+ex)}{a^3e}+\frac{-2ac+b^2+bc(d+ex)^2}{2ae\left(b^2-4ac\right)(d+ex)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)^3*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2),x]
```

```
[Out] -((b^2 - 3*a*c)/(a^2*(b^2 - 4*a*c)*e*(d + e*x)^2)) + (b^2 - 2*a*c + b*c*(d
+ e*x)^2)/(2*a*(b^2 - 4*a*c)*e*(d + e*x)^2*(a + b*(d + e*x)^2 + c*(d + e*x)
^4)) - ((b^4 - 6*a*b^2*c + 6*a^2*c^2)*ArcTanh[(b + 2*c*(d + e*x)^2)/Sqrt[b^
2 - 4*a*c]])/(a^3*(b^2 - 4*a*c)^(3/2)*e) - (2*b*Log[d + e*x])/(a^3*e) + (b*
Log[a + b*(d + e*x)^2 + c*(d + e*x)^4])/(2*a^3*e)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 740

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e
)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e
^2)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d +
 e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p +
 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4
```

```
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p,
-1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 1142

```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Di
st[u^m/(Coefficient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p,
 x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

Rubi steps

$$\int \frac{1}{(d+ex)^3\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x^3\left(a+bx^2+cx^4\right)^2}\,dx, x, d+ex\right)}{e}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{x^2\left(a+bx+cx^2\right)^2}\,dx, x, (d+ex)^2\right)}{2e}$$

$$= \frac{b^2-2ac+bc(d+ex)^2}{2a\left(b^2-4ac\right)e(d+ex)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)} - \text{Subs}$$

$$= \frac{b^2-2ac+bc(d+ex)^2}{2a\left(b^2-4ac\right)e(d+ex)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)} - \text{Subs}$$

$$= -\frac{b^2-3ac}{a^2\left(b^2-4ac\right)e(d+ex)^2} + \frac{b^2-2ac+bc(d+}{2a\left(b^2-4ac\right)e(d+ex)^2\left(a+b(d-\right.}$$

$$= -\frac{b^2-3ac}{a^2\left(b^2-4ac\right)e(d+ex)^2} + \frac{b^2-2ac+bc(d+}{2a\left(b^2-4ac\right)e(d+ex)^2\left(a+b(d-\right.}$$

$$= -\frac{b^2-3ac}{a^2\left(b^2-4ac\right)e(d+ex)^2} + \frac{b^2-2ac+bc(d+}{2a\left(b^2-4ac\right)e(d+ex)^2\left(a+b(d-\right.}$$

$$= -\frac{b^2-3ac}{a^2\left(b^2-4ac\right)e(d+ex)^2} + \frac{b^2-2ac+bc(d+}{2a\left(b^2-4ac\right)e(d+ex)^2\left(a+b(d-\right.}$$

Mathematica [A] time = 0.52, size = 284, normalized size = 1.33

$$\frac{\frac{\left(6a^2c^2-6ab^2c-4abc\sqrt{b^2-4ac}+b^3\sqrt{b^2-4ac}+b^4\right)\log\left(-\sqrt{b^2-4ac}+b+2c(d+ex)^2\right)}{(b^2-4ac)^{3/2}}+\frac{\left(-6a^2c^2+6ab^2c-4abc\sqrt{b^2-4ac}+b^3\sqrt{b^2-4ac}-b^4\right)\log\left(\sqrt{b^2-4ac}+b+2c(d+ex)^2\right)}{(b^2-4ac)^{3/2}}+\frac{a\left(-3abc-2ac^2(d+ex)^2+b^3+b^2c(d+ex)^2\right)}{(4ac-b^2)\left(a+b(d+ex)^2+c(d+ex)^4\right)}-\frac{a}{(d+ex)^2}-4b\log(d+ex)}{2a^3e}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)^3*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2),x]
```

```
[Out] (-(a/(d + e*x)^2) + (a*(b^3 - 3*a*b*c + b^2*c*(d + e*x)^2 - 2*a*c^2*(d + e*
x)^2))/((-b^2 + 4*a*c)*(a + b*(d + e*x)^2 + c*(d + e*x)^4)) - 4*b*Log[d + e
*x] + ((b^4 - 6*a*b^2*c + 6*a^2*c^2 + b^3*Sqrt[b^2 - 4*a*c] - 4*a*b*c*Sqrt[
b^2 - 4*a*c])*Log[b - Sqrt[b^2 - 4*a*c] + 2*c*(d + e*x)^2])/(b^2 - 4*a*c)^(
3/2) + ((-b^4 + 6*a*b^2*c - 6*a^2*c^2 + b^3*Sqrt[b^2 - 4*a*c] - 4*a*b*c*Sqr
t[b^2 - 4*a*c])*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*(d + e*x)^2])/(b^2 - 4*a*c)
^(3/2))/(2*a^3*e)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(d+ex)^3\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/((d + e*x)^3*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2),x]
```

```
[Out] IntegrateAlgebraic[1/((d + e*x)^3*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2), x
]
```

fricas [B] time = 2.51, size = 4562, normalized size = 21.42

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="fricas")
```

```
[Out] [-1/2*(2*(a*b^4*c - 7*a^2*b^2*c^2 + 12*a^3*c^3)*e^4*x^4 + 8*(a*b^4*c - 7*a^
2*b^2*c^2 + 12*a^3*c^3)*d*e^3*x^3 + a^2*b^4 - 8*a^3*b^2*c + 16*a^4*c^2 + 2*
(a*b^4*c - 7*a^2*b^2*c^2 + 12*a^3*c^3)*d^4 + (2*a*b^5 - 15*a^2*b^3*c + 28*a
^3*b*c^2 + 12*(a*b^4*c - 7*a^2*b^2*c^2 + 12*a^3*c^3)*d^2)*e^2*x^2 + (2*a*b^
5 - 15*a^2*b^3*c + 28*a^3*b*c^2)*d^2 + 2*(4*(a*b^4*c - 7*a^2*b^2*c^2 + 12*a
^3*c^3)*d^3 + (2*a*b^5 - 15*a^2*b^3*c + 28*a^3*b*c^2)*d)*e*x + ((b^4*c - 6*
a*b^2*c^2 + 6*a^2*c^3)*e^6*x^6 + 6*(b^4*c - 6*a*b^2*c^2 + 6*a^2*c^3)*d*e^5*
x^5 + (b^5 - 6*a*b^3*c + 6*a^2*b*c^2 + 15*(b^4*c - 6*a*b^2*c^2 + 6*a^2*c^3)
*d^2)*e^4*x^4 + (b^4*c - 6*a*b^2*c^2 + 6*a^2*c^3)*d^6 + 4*(5*(b^4*c - 6*a*b
^2*c^2 + 6*a^2*c^3)*d^3 + (b^5 - 6*a*b^3*c + 6*a^2*b*c^2)*d)*e^3*x^3 + (b^5
 - 6*a*b^3*c + 6*a^2*b*c^2)*d^4 + (a*b^4 - 6*a^2*b^2*c + 6*a^3*c^2 + 15*(b^
4*c - 6*a*b^2*c^2 + 6*a^2*c^3)*d^4 + 6*(b^5 - 6*a*b^3*c + 6*a^2*b*c^2)*d^2)
*e^2*x^2 + (a*b^4 - 6*a^2*b^2*c + 6*a^3*c^2)*d^2 + 2*(3*(b^4*c - 6*a*b^2*c^
2 + 6*a^2*c^3)*d^5 + 2*(b^5 - 6*a*b^3*c + 6*a^2*b*c^2)*d^3 + (a*b^4 - 6*a^2
*b^2*c + 6*a^3*c^2)*d)*e*x)*sqrt(b^2 - 4*a*c)*log((2*c^2*e^4*x^4 + 8*c^2*d*
e^3*x^3 + 2*c^2*d^4 + 2*(6*c^2*d^2 + b*c)*e^2*x^2 + 2*b*c*d^2 + 4*(2*c^2*d^
3 + b*c*d)*e*x + b^2 - 2*a*c + (2*c*e^2*x^2 + 4*c*d*e*x + 2*c*d^2 + b)*sqrt
(b^2 - 4*a*c))/(c*e^4*x^4 + 4*c*d*e^3*x^3 + c*d^4 + (6*c*d^2 + b)*e^2*x^2 +
 b*d^2 + 2*(2*c*d^3 + b*d)*e*x + a)) - ((b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3
)*e^6*x^6 + 6*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d*e^5*x^5 + (b^6 - 8*a*b
^4*c + 16*a^2*b^2*c^2 + 15*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^2)*e^4*x^
4 + (b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^6 + 4*(5*(b^5*c - 8*a*b^3*c^2 +
16*a^2*b*c^3)*d^3 + (b^6 - 8*a*b^4*c + 16*a^2*b^2*c^2)*d)*e^3*x^3 + (b^6 -
8*a*b^4*c + 16*a^2*b^2*c^2)*d^4 + (a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2 + 15*
(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^4 + 6*(b^6 - 8*a*b^4*c + 16*a^2*b^2*
c^2)*d^2)*e^2*x^2 + (a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*d^2 + 2*(3*(b^5*c
```

```
- 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^5 + 2*(b^6 - 8*a*b^4*c + 16*a^2*b^2*c^2)*d^
3 + (a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*d)*e*x)*log(c*e^4*x^4 + 4*c*d*e^3*
x^3 + c*d^4 + (6*c*d^2 + b)*e^2*x^2 + b*d^2 + 2*(2*c*d^3 + b*d)*e*x + a) +
4*((b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*e^6*x^6 + 6*(b^5*c - 8*a*b^3*c^2 +
16*a^2*b*c^3)*d*e^5*x^5 + (b^6 - 8*a*b^4*c + 16*a^2*b^2*c^2 + 15*(b^5*c - 8
*a*b^3*c^2 + 16*a^2*b*c^3)*d^2)*e^4*x^4 + (b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c
^3)*d^6 + 4*(5*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^3 + (b^6 - 8*a*b^4*c
+ 16*a^2*b^2*c^2)*d)*e^3*x^3 + (b^6 - 8*a*b^4*c + 16*a^2*b^2*c^2)*d^4 + (a*
b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2 + 15*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*
d^4 + 6*(b^6 - 8*a*b^4*c + 16*a^2*b^2*c^2)*d^2)*e^2*x^2 + (a*b^5 - 8*a^2*b^
3*c + 16*a^3*b*c^2)*d^2 + 2*(3*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^5 + 2
*(b^6 - 8*a*b^4*c + 16*a^2*b^2*c^2)*d^3 + (a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c
^2)*d)*e*x)*log(e*x + d))/((a^3*b^4*c - 8*a^4*b^2*c^2 + 16*a^5*c^3)*e^7*x^6
 + 6*(a^3*b^4*c - 8*a^4*b^2*c^2 + 16*a^5*c^3)*d*e^6*x^5 + (a^3*b^5 - 8*a^4*
b^3*c + 16*a^5*b*c^2 + 15*(a^3*b^4*c - 8*a^4*b^2*c^2 + 16*a^5*c^3)*d^2)*e^5
*x^4 + 4*(5*(a^3*b^4*c - 8*a^4*b^2*c^2 + 16*a^5*c^3)*d^3 + (a^3*b^5 - 8*a^4
*b^3*c + 16*a^5*b*c^2)*d)*e^4*x^3 + (a^4*b^4 - 8*a^5*b^2*c + 16*a^6*c^2 + 1
5*(a^3*b^4*c - 8*a^4*b^2*c^2 + 16*a^5*c^3)*d^4 + 6*(a^3*b^5 - 8*a^4*b^3*c +
 16*a^5*b*c^2)*d^2)*e^3*x^2 + 2*(3*(a^3*b^4*c - 8*a^4*b^2*c^2 + 16*a^5*c^3)
*d^5 + 2*(a^3*b^5 - 8*a^4*b^3*c + 16*a^5*b*c^2)*d^3 + (a^4*b^4 - 8*a^5*b^2*
c + 16*a^6*c^2)*d)*e^2*x + ((a^3*b^4*c - 8*a^4*b^2*c^2 + 16*a^5*c^3)*d^6 +
(a^3*b^5 - 8*a^4*b^3*c + 16*a^5*b*c^2)*d^4 + (a^4*b^4 - 8*a^5*b^2*c + 16*a^
6*c^2)*d^2)*e), -1/2*(2*(a*b^4*c - 7*a^2*b^2*c^2 + 12*a^3*c^3)*e^4*x^4 + 8*
(a*b^4*c - 7*a^2*b^2*c^2 + 12*a^3*c^3)*d*e^3*x^3 + a^2*b^4 - 8*a^3*b^2*c +
16*a^4*c^2 + 2*(a*b^4*c - 7*a^2*b^2*c^2 + 12*a^3*c^3)*d^4 + (2*a*b^5 - 15*a
^2*b^3*c + 28*a^3*b*c^2 + 12*(a*b^4*c - 7*a^2*b^2*c^2 + 12*a^3*c^3)*d^2)*e^
2*x^2 + (2*a*b^5 - 15*a^2*b^3*c + 28*a^3*b*c^2)*d^2 + 2*(4*(a*b^4*c - 7*a^2
*b^2*c^2 + 12*a^3*c^3)*d^3 + (2*a*b^5 - 15*a^2*b^3*c + 28*a^3*b*c^2)*d)*e*x
 + 2*((b^4*c - 6*a*b^2*c^2 + 6*a^2*c^3)*e^6*x^6 + 6*(b^4*c - 6*a*b^2*c^2 +
6*a^2*c^3)*d*e^5*x^5 + (b^5 - 6*a*b^3*c + 6*a^2*b*c^2 + 15*(b^4*c - 6*a*b^2
*c^2 + 6*a^2*c^3)*d^2)*e^4*x^4 + (b^4*c - 6*a*b^2*c^2 + 6*a^2*c^3)*d^6 + 4*
(5*(b^4*c - 6*a*b^2*c^2 + 6*a^2*c^3)*d^3 + (b^5 - 6*a*b^3*c + 6*a^2*b*c^2)*
d)*e^3*x^3 + (b^5 - 6*a*b^3*c + 6*a^2*b*c^2)*d^4 + (a*b^4 - 6*a^2*b^2*c + 6
*a^3*c^2 + 15*(b^4*c - 6*a*b^2*c^2 + 6*a^2*c^3)*d^4 + 6*(b^5 - 6*a*b^3*c +
6*a^2*b*c^2)*d^2)*e^2*x^2 + (a*b^4 - 6*a^2*b^2*c + 6*a^3*c^2)*d^2 + 2*(3*(b
^4*c - 6*a*b^2*c^2 + 6*a^2*c^3)*d^5 + 2*(b^5 - 6*a*b^3*c + 6*a^2*b*c^2)*d^3
 + (a*b^4 - 6*a^2*b^2*c + 6*a^3*c^2)*d)*e*x)*sqrt(-b^2 + 4*a*c)*arctan(-(2*
c*e^2*x^2 + 4*c*d*e*x + 2*c*d^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) - ((
b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*e^6*x^6 + 6*(b^5*c - 8*a*b^3*c^2 + 16*a
^2*b*c^3)*d*e^5*x^5 + (b^6 - 8*a*b^4*c + 16*a^2*b^2*c^2 + 15*(b^5*c - 8*a*b
^3*c^2 + 16*a^2*b*c^3)*d^2)*e^4*x^4 + (b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*
d^6 + 4*(5*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^3 + (b^6 - 8*a*b^4*c + 16
*a^2*b^2*c^2)*d)*e^3*x^3 + (b^6 - 8*a*b^4*c + 16*a^2*b^2*c^2)*d^4 + (a*b^5
- 8*a^2*b^3*c + 16*a^3*b*c^2 + 15*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^4
+ 6*(b^6 - 8*a*b^4*c + 16*a^2*b^2*c^2)*d^2)*e^2*x^2 + (a*b^5 - 8*a^2*b^3*c
+ 16*a^3*b*c^2)*d^2 + 2*(3*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^5 + 2*(b^
6 - 8*a*b^4*c + 16*a^2*b^2*c^2)*d^3 + (a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*
d)*e*x)*log(c*e^4*x^4 + 4*c*d*e^3*x^3 + c*d^4 + (6*c*d^2 + b)*e^2*x^2 + b*d
^2 + 2*(2*c*d^3 + b*d)*e*x + a) + 4*((b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*e
^6*x^6 + 6*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d*e^5*x^5 + (b^6 - 8*a*b^4*
c + 16*a^2*b^2*c^2 + 15*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^2)*e^4*x^4 +
 (b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^6 + 4*(5*(b^5*c - 8*a*b^3*c^2 + 16*
a^2*b*c^3)*d^3 + (b^6 - 8*a*b^4*c + 16*a^2*b^2*c^2)*d)*e^3*x^3 + (b^6 - 8*a
*b^4*c + 16*a^2*b^2*c^2)*d^4 + (a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2 + 15*(b^
5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^4 + 6*(b^6 - 8*a*b^4*c + 16*a^2*b^2*c^2
)*d^2)*e^2*x^2 + (a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*d^2 + 2*(3*(b^5*c - 8
*a*b^3*c^2 + 16*a^2*b*c^3)*d^5 + 2*(b^6 - 8*a*b^4*c + 16*a^2*b^2*c^2)*d^3 +
 (a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*d)*e*x)*log(e*x + d))/((a^3*b^4*c - 8
*a^4*b^2*c^2 + 16*a^5*c^3)*e^7*x^6 + 6*(a^3*b^4*c - 8*a^4*b^2*c^2 + 16*a^5*
```

```
c^3)*d*e^6*x^5 + (a^3*b^5 - 8*a^4*b^3*c + 16*a^5*b*c^2 + 15*(a^3*b^4*c - 8*
a^4*b^2*c^2 + 16*a^5*c^3)*d^2)*e^5*x^4 + 4*(5*(a^3*b^4*c - 8*a^4*b^2*c^2 +
16*a^5*c^3)*d^3 + (a^3*b^5 - 8*a^4*b^3*c + 16*a^5*b*c^2)*d)*e^4*x^3 + (a^4*
b^4 - 8*a^5*b^2*c + 16*a^6*c^2 + 15*(a^3*b^4*c - 8*a^4*b^2*c^2 + 16*a^5*c^3
)*d^4 + 6*(a^3*b^5 - 8*a^4*b^3*c + 16*a^5*b*c^2)*d^2)*e^3*x^2 + 2*(3*(a^3*b
^4*c - 8*a^4*b^2*c^2 + 16*a^5*c^3)*d^5 + 2*(a^3*b^5 - 8*a^4*b^3*c + 16*a^5*
b*c^2)*d^3 + (a^4*b^4 - 8*a^5*b^2*c + 16*a^6*c^2)*d)*e^2*x + ((a^3*b^4*c -
8*a^4*b^2*c^2 + 16*a^5*c^3)*d^6 + (a^3*b^5 - 8*a^4*b^3*c + 16*a^5*b*c^2)*d^
4 + (a^4*b^4 - 8*a^5*b^2*c + 16*a^6*c^2)*d^2)*e)]
```

giac [A] time = 0.43, size = 224, normalized size = 1.05

$$\frac{\left(b^{4}-6\,ab^{2}c+6\,a^{2}c^{2}\right)\arctan\left(-\frac{b+\frac{2\,a}{(xe+d)^{2}}}{\sqrt{-b^{2}+4\,ac}}\right)e^{(-1)}}{\left(a^{3}b^{2}-4\,a^{4}c\right)\sqrt{-b^{2}+4\,ac}}+\frac{be^{(-1)}\log\left(c+\frac{b}{(xe+d)^{2}}+\frac{a}{(xe+d)^{4}}\right)}{2\,a^{3}}+\frac{\left(\frac{b^{3}c-3\,abc^{2}}{a}+\frac{\left(b^{4}e-4\,ab^{2}ce+2\,a^{2}c^{2}e\right)e^{(-1)}}{(xe+d)^{2}a}\right)e^{(-1)}}{2\left(b^{2}-4\,ac\right)a^{2}\left(c+\frac{b}{(xe+d)^{2}}+\frac{a}{(xe+d)^{4}}\right)}-\frac{e^{(-1)}}{2\,(xe+d)^{2}a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="giac")
```

```
[Out] (b^4 - 6*a*b^2*c + 6*a^2*c^2)*arctan(-(b + 2*a/(x*e + d)^2)/sqrt(-b^2 + 4*a
*c))*e^(-1)/((a^3*b^2 - 4*a^4*c)*sqrt(-b^2 + 4*a*c)) + 1/2*b*e^(-1)*log(c +
 b/(x*e + d)^2 + a/(x*e + d)^4)/a^3 + 1/2*((b^3*c - 3*a*b*c^2)/a + (b^4*e -
 4*a*b^2*c*e + 2*a^2*c^2*e)*e^(-1)/((x*e + d)^2*a))*e^(-1)/((b^2 - 4*a*c)*a
^2*(c + b/(x*e + d)^2 + a/(x*e + d)^4)) - 1/2*e^(-1)/((x*e + d)^2*a^2)
```

maple [C] time = 0.04, size = 1014, normalized size = 4.76

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x)
```

```
[Out] -1/a/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2
*b*d*e*x+b*d^2+a)*c^2*e/(4*a*c-b^2)*x^2+1/2/a^2/(c*e^4*x^4+4*c*d*e^3*x^3+6*
c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)*c*e/(4*a*c-b^2
)*x^2*b^2-2/a/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^
2+c*d^4+2*b*d*e*x+b*d^2+a)*c^2*d/(4*a*c-b^2)*x+1/a^2/(c*e^4*x^4+4*c*d*e^3*x
^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)*c*d/(4*a*
c-b^2)*x*b^2-1/a/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2
*x^2+c*d^4+2*b*d*e*x+b*d^2+a)/e/(4*a*c-b^2)*c^2*d^2+1/2/a^2/(c*e^4*x^4+4*c*
d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)/e/
(4*a*c-b^2)*b^2*c*d^2-3/2/a/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^
3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)/e/(4*a*c-b^2)*b*c+1/2/a^2/(c*e^4*x
^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^
2+a)/e/(4*a*c-b^2)*b^3+1/a^3/(4*a*c-b^2)/e*sum((b*c*e^3*(4*a*c-b^2)*_R^3+3*
b*c*d*e^2*(4*a*c-b^2)*_R^2+e*(12*a*b*c^2*d^2-3*b^3*c*d^2-3*a^2*c^2+5*a*b^2*
c-b^4)*_R+4*a*b*c^2*d^3-b^3*c*d^3-3*a^2*c^2*d+5*a*b^2*c*d-b^4*d)/(2*_R^3*c*
e^3+6*_R^2*c*d*e^2+6*_R*c*d^2*e+2*c*d^3+_R*b*e+b*d)*ln(-_R+x),_R=RootOf(_Z^
4*c*e^4+4*_Z^3*c*d*e^3+c*d^4+b*d^2+(6*c*d^2*e^2+b*e^2)*_Z^2+(4*c*d^3*e+2*b*
d*e)*_Z+a))-1/2/a^2/e/(e*x+d)^2-2*b*ln(e*x+d)/a^3/e
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [B] time = 12.32, size = 12436, normalized size = 58.38

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((d + e*x)^3*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2),x)
```

```
[Out] ((x*(2*b^3*d - 12*a*c^2*d^3 + 4*b^2*c*d^3 - 7*a*b*c*d))/(4*a^3*c - a^2*b^2)
 - (x^4*(3*a*c^2*e^3 - b^2*c*e^3))/(4*a^3*c - a^2*b^2) - (4*x^3*(3*a*c^2*d*
e^2 - b^2*c*d*e^2))/(4*a^3*c - a^2*b^2) + (a*b^2 - 4*a^2*c + 2*b^3*d^2 - 6*
a*c^2*d^4 + 2*b^2*c*d^4 - 7*a*b*c*d^2)/(2*e*(4*a^3*c - a^2*b^2)) + (x^2*(2*
b^3*e - 36*a*c^2*d^2*e + 12*b^2*c*d^2*e - 7*a*b*c*e))/(2*(4*a^3*c - a^2*b^2
)))/(x^4*(b*e^4 + 15*c*d^2*e^4) + a*d^2 + b*d^4 + c*d^6 + x*(2*a*d*e + 4*b*
d^3*e + 6*c*d^5*e) + x^2*(a*e^2 + 6*b*d^2*e^2 + 15*c*d^4*e^2) + x^3*(20*c*d
^3*e^3 + 4*b*d*e^3) + c*e^6*x^6 + 6*c*d*e^5*x^5) + (log((((b + a^3*e*(-(b^4
 + 6*a^2*c^2 - 6*a*b^2*c)^2/(a^6*e^2*(4*a*c - b^2)^3))^(1/2))*(((b + a^3*e*
(-(b^4 + 6*a^2*c^2 - 6*a*b^2*c)^2/(a^6*e^2*(4*a*c - b^2)^3))^(1/2))*((4*c^2
*e^16*(2*b^5 + 6*a^2*b*c^2 + b^4*c*d^2 - 30*a^2*c^3*d^2 - 10*a*b^3*c + 2*a*
b^2*c^2*d^2))/(a^2*(4*a*c - b^2)) + (4*c^3*e^18*x^2*(b^4 - 30*a^2*c^2 + 2*a
*b^2*c))/(a^2*(4*a*c - b^2)) - (2*b*c^2*e^16*(b + a^3*e*(-(b^4 + 6*a^2*c^2
- 6*a*b^2*c)^2/(a^6*e^2*(4*a*c - b^2)^3))^(1/2))*(a*b + 3*b^2*d^2 + 3*b^2*e
^2*x^2 - 10*a*c*d^2 + 6*b^2*d*e*x - 10*a*c*e^2*x^2 - 20*a*c*d*e*x))/a^3 + (
8*c^3*d*e^17*x*(b^4 - 30*a^2*c^2 + 2*a*b^2*c))/(a^2*(4*a*c - b^2))))/(2*a^3
*e) - (4*c^3*e^15*(3*a*c - b^2)*(4*b^4 + 3*a^2*c^2 + 6*b^3*c*d^2 - 17*a*b^2
*c - 23*a*b*c^2*d^2))/(a^4*(4*a*c - b^2)^2) + (4*b*c^4*e^17*x^2*(6*b^4 + 69
*a^2*c^2 - 41*a*b^2*c))/(a^4*(4*a*c - b^2)^2) + (8*b*c^4*d*e^16*x*(6*b^4 +
69*a^2*c^2 - 41*a*b^2*c))/(a^4*(4*a*c - b^2)^2)))/(2*a^3*e) - (8*c^5*e^16*x
^2*(3*a*c - b^2)^3)/(a^6*(4*a*c - b^2)^3) + (8*c^4*e^14*(3*a*c - b^2)^2*(b^
3 - 3*a*c^2*d^2 + b^2*c*d^2 - 4*a*b*c))/(a^6*(4*a*c - b^2)^3) - (16*c^5*d*e
^15*x*(3*a*c - b^2)^3)/(a^6*(4*a*c - b^2)^3))*(((b - a^3*e*(-(b^4 + 6*a^2*c
^2 - 6*a*b^2*c)^2/(a^6*e^2*(4*a*c - b^2)^3))^(1/2))*(((b - a^3*e*(-(b^4 + 6
*a^2*c^2 - 6*a*b^2*c)^2/(a^6*e^2*(4*a*c - b^2)^3))^(1/2))*((4*c^2*e^16*(2*b
^5 + 6*a^2*b*c^2 + b^4*c*d^2 - 30*a^2*c^3*d^2 - 10*a*b^3*c + 2*a*b^2*c^2*d^
2))/(a^2*(4*a*c - b^2)) + (4*c^3*e^18*x^2*(b^4 - 30*a^2*c^2 + 2*a*b^2*c))/(
a^2*(4*a*c - b^2)) - (2*b*c^2*e^16*(b - a^3*e*(-(b^4 + 6*a^2*c^2 - 6*a*b^2*
c)^2/(a^6*e^2*(4*a*c - b^2)^3))^(1/2))*(a*b + 3*b^2*d^2 + 3*b^2*e^2*x^2 - 1
0*a*c*d^2 + 6*b^2*d*e*x - 10*a*c*e^2*x^2 - 20*a*c*d*e*x))/a^3 + (8*c^3*d*e^
17*x*(b^4 - 30*a^2*c^2 + 2*a*b^2*c))/(a^2*(4*a*c - b^2))))/(2*a^3*e) - (4*c
^3*e^15*(3*a*c - b^2)*(4*b^4 + 3*a^2*c^2 + 6*b^3*c*d^2 - 17*a*b^2*c - 23*a*
b*c^2*d^2))/(a^4*(4*a*c - b^2)^2) + (4*b*c^4*e^17*x^2*(6*b^4 + 69*a^2*c^2 -
 41*a*b^2*c))/(a^4*(4*a*c - b^2)^2) + (8*b*c^4*d*e^16*x*(6*b^4 + 69*a^2*c^2
 - 41*a*b^2*c))/(a^4*(4*a*c - b^2)^2)))/(2*a^3*e) - (8*c^5*e^16*x^2*(3*a*c
- b^2)^3)/(a^6*(4*a*c - b^2)^3) + (8*c^4*e^14*(3*a*c - b^2)^2*(b^3 - 3*a*c^
2*d^2 + b^2*c*d^2 - 4*a*b*c))/(a^6*(4*a*c - b^2)^3) - (16*c^5*d*e^15*x*(3*a
*c - b^2)^3)/(a^6*(4*a*c - b^2)^3)))*(b^7*e + 48*a^2*b^3*c^2*e - 12*a*b^5*c
*e - 64*a^3*b*c^3*e))/(2*(a^3*b^6*e^2 - 64*a^6*c^3*e^2 - 12*a^4*b^4*c*e^2 +
 48*a^5*b^2*c^2*e^2)) - (2*b*log(d + e*x))/(a^3*e) - (atan(((2*a^9*b^6*(4*a
*c - b^2)^(9/2) - 128*a^12*c^3*(4*a*c - b^2)^(9/2) - 24*a^10*b^4*c*(4*a*c -
 b^2)^(9/2) + 96*a^11*b^2*c^2*(4*a*c - b^2)^(9/2))*(x*((((8*(54*a^3*c^8*d*e
^15 - 2*b^6*c^5*d*e^15 + 18*a*b^4*c^6*d*e^15 - 54*a^2*b^2*c^7*d*e^15))/(a^6
*b^6 - 64*a^9*c^3 - 12*a^7*b^4*c + 48*a^8*b^2*c^2) - (((8*(276*a^5*b*c^7*d*
e^16 - 6*a^2*b^7*c^4*d*e^16 + 65*a^3*b^5*c^5*d*e^16 - 233*a^4*b^3*c^6*d*e^1
6))/(a^6*b^6 - 64*a^9*c^3 - 12*a^7*b^4*c + 48*a^8*b^2*c^2) - (((8*(480*a^8*
c^7*d*e^17 - a^4*b^8*c^3*d*e^17 + 6*a^5*b^6*c^4*d*e^17 + 30*a^6*b^4*c^5*d*e
^17 - 272*a^7*b^2*c^6*d*e^17))/(a^6*b^6 - 64*a^9*c^3 - 12*a^7*b^4*c + 48*a^
8*b^2*c^2) - (4*(b^7*e + 48*a^2*b^3*c^2*e - 12*a*b^5*c*e - 64*a^3*b*c^3*e)*
(640*a^10*b*c^6*d*e^18 + 3*a^6*b^9*c^2*d*e^18 - 46*a^7*b^7*c^3*d*e^18 + 264
*a^8*b^5*c^4*d*e^18 - 672*a^9*b^3*c^5*d*e^18))/((a^6*b^6 - 64*a^9*c^3 - 12*
a^7*b^4*c + 48*a^8*b^2*c^2)*(a^3*b^6*e^2 - 64*a^6*c^3*e^2 - 12*a^4*b^4*c*e^
2 + 48*a^5*b^2*c^2*e^2)))*(b^7*e + 48*a^2*b^3*c^2*e - 12*a*b^5*c*e - 64*a^3
*b*c^3*e))/(2*(a^3*b^6*e^2 - 64*a^6*c^3*e^2 - 12*a^4*b^4*c*e^2 + 48*a^5*b^2
```

```
*c^2*e^2)))*(b^7*e + 48*a^2*b^3*c^2*e - 12*a*b^5*c*e - 64*a^3*b*c^3*e))/(2*
(a^3*b^6*e^2 - 64*a^6*c^3*e^2 - 12*a^4*b^4*c*e^2 + 48*a^5*b^2*c^2*e^2)) - (
((((8*(480*a^8*c^7*d*e^17 - a^4*b^8*c^3*d*e^17 + 6*a^5*b^6*c^4*d*e^17 + 30*
a^6*b^4*c^5*d*e^17 - 272*a^7*b^2*c^6*d*e^17))/(a^6*b^6 - 64*a^9*c^3 - 12*a^
7*b^4*c + 48*a^8*b^2*c^2) - (4*(b^7*e + 48*a^2*b^3*c^2*e - 12*a*b^5*c*e - 6
4*a^3*b*c^3*e)*(640*a^10*b*c^6*d*e^18 + 3*a^6*b^9*c^2*d*e^18 - 46*a^7*b^7*c
^3*d*e^18 + 264*a^8*b^5*c^4*d*e^18 - 672*a^9*b^3*c^5*d*e^18))/((a^6*b^6 - 6
4*a^9*c^3 - 12*a^7*b^4*c + 48*a^8*b^2*c^2)*(a^3*b^6*e^2 - 64*a^6*c^3*e^2 -
12*a^4*b^4*c*e^2 + 48*a^5*b^2*c^2*e^2)))*(b^4 + 6*a^2*c^2 - 6*a*b^2*c))/(2*
a^3*e*(4*a*c - b^2)^(3/2)) - (2*(b^4 + 6*a^2*c^2 - 6*a*b^2*c)*(b^7*e + 48*a
^2*b^3*c^2*e - 12*a*b^5*c*e - 64*a^3*b*c^3*e)*(640*a^10*b*c^6*d*e^18 + 3*a^
6*b^9*c^2*d*e^18 - 46*a^7*b^7*c^3*d*e^18 + 264*a^8*b^5*c^4*d*e^18 - 672*a^9
*b^3*c^5*d*e^18))/(a^3*e*(4*a*c - b^2)^(3/2)*(a^6*b^6 - 64*a^9*c^3 - 12*a^7
*b^4*c + 48*a^8*b^2*c^2)*(a^3*b^6*e^2 - 64*a^6*c^3*e^2 - 12*a^4*b^4*c*e^2 +
 48*a^5*b^2*c^2*e^2)))*(b^4 + 6*a^2*c^2 - 6*a*b^2*c))/(2*a^3*e*(4*a*c - b^2
)^(3/2)) + ((b^4 + 6*a^2*c^2 - 6*a*b^2*c)^2*(b^7*e + 48*a^2*b^3*c^2*e - 12*
a*b^5*c*e - 64*a^3*b*c^3*e)*(640*a^10*b*c^6*d*e^18 + 3*a^6*b^9*c^2*d*e^18 -
 46*a^7*b^7*c^3*d*e^18 + 264*a^8*b^5*c^4*d*e^18 - 672*a^9*b^3*c^5*d*e^18))/
(a^6*e^2*(4*a*c - b^2)^3*(a^6*b^6 - 64*a^9*c^3 - 12*a^7*b^4*c + 48*a^8*b^2*
c^2)*(a^3*b^6*e^2 - 64*a^6*c^3*e^2 - 12*a^4*b^4*c*e^2 + 48*a^5*b^2*c^2*e^2)
))*(3*b^6 - 3*a^3*c^3 + 36*a^2*b^2*c^2 - 21*a*b^4*c))/(8*a^3*c^2*(4*a*c - b
^2)^3*(9*a^4*c^4 - 6*b^8 - 288*a^2*b^4*c^2 + 382*a^3*b^2*c^3 + 72*a*b^6*c))
 - (b*((((((8*(480*a^8*c^7*d*e^17 - a^4*b^8*c^3*d*e^17 + 6*a^5*b^6*c^4*d*e^
17 + 30*a^6*b^4*c^5*d*e^17 - 272*a^7*b^2*c^6*d*e^17))/(a^6*b^6 - 64*a^9*c^3
 - 12*a^7*b^4*c + 48*a^8*b^2*c^2) - (4*(b^7*e + 48*a^2*b^3*c^2*e - 12*a*b^5
*c*e - 64*a^3*b*c^3*e)*(640*a^10*b*c^6*d*e^18 + 3*a^6*b^9*c^2*d*e^18 - 46*a
^7*b^7*c^3*d*e^18 + 264*a^8*b^5*c^4*d*e^18 - 672*a^9*b^3*c^5*d*e^18))/((a^6
*b^6 - 64*a^9*c^3 - 12*a^7*b^4*c + 48*a^8*b^2*c^2)*(a^3*b^6*e^2 - 64*a^6*c^
3*e^2 - 12*a^4*b^4*c*e^2 + 48*a^5*b^2*c^2*e^2)))*(b^4 + 6*a^2*c^2 - 6*a*b^2
*c))/(2*a^3*e*(4*a*c - b^2)^(3/2)) - (2*(b^4 + 6*a^2*c^2 - 6*a*b^2*c)*(b^7*
e + 48*a^2*b^3*c^2*e - 12*a*b^5*c*e - 64*a^3*b*c^3*e)*(640*a^10*b*c^6*d*e^1
8 + 3*a^6*b^9*c^2*d*e^18 - 46*a^7*b^7*c^3*d*e^18 + 264*a^8*b^5*c^4*d*e^18 -
 672*a^9*b^3*c^5*d*e^18))/(a^3*e*(4*a*c - b^2)^(3/2)*(a^6*b^6 - 64*a^9*c^3
- 12*a^7*b^4*c + 48*a^8*b^2*c^2)*(a^3*b^6*e^2 - 64*a^6*c^3*e^2 - 12*a^4*b^4
*c*e^2 + 48*a^5*b^2*c^2*e^2)))*(b^7*e + 48*a^2*b^3*c^2*e - 12*a*b^5*c*e - 6
4*a^3*b*c^3*e))/(2*(a^3*b^6*e^2 - 64*a^6*c^3*e^2 - 12*a^4*b^4*c*e^2 + 48*a^
5*b^2*c^2*e^2)) - (((8*(276*a^5*b*c^7*d*e^16 - 6*a^2*b^7*c^4*d*e^16 + 65*a^
3*b^5*c^5*d*e^16 - 233*a^4*b^3*c^6*d*e^16))/(a^6*b^6 - 64*a^9*c^3 - 12*a^7*
b^4*c + 48*a^8*b^2*c^2) - (((8*(480*a^8*c^7*d*e^17 - a^4*b^8*c^3*d*e^17 + 6
*a^5*b^6*c^4*d*e^17 + 30*a^6*b^4*c^5*d*e^17 - 272*a^7*b^2*c^6*d*e^17))/(a^6
*b^6 - 64*a^9*c^3 - 12*a^7*b^4*c + 48*a^8*b^2*c^2) - (4*(b^7*e + 48*a^2*b^3
*c^2*e - 12*a*b^5*c*e - 64*a^3*b*c^3*e)*(640*a^10*b*c^6*d*e^18 + 3*a^6*b^9*
c^2*d*e^18 - 46*a^7*b^7*c^3*d*e^18 + 264*a^8*b^5*c^4*d*e^18 - 672*a^9*b^3*c
^5*d*e^18))/((a^6*b^6 - 64*a^9*c^3 - 12*a^7*b^4*c + 48*a^8*b^2*c^2)*(a^3*b^
6*e^2 - 64*a^6*c^3*e^2 - 12*a^4*b^4*c*e^2 + 48*a^5*b^2*c^2*e^2)))*(b^7*e +
48*a^2*b^3*c^2*e - 12*a*b^5*c*e - 64*a^3*b*c^3*e))/(2*(a^3*b^6*e^2 - 64*a^6
*c^3*e^2 - 12*a^4*b^4*c*e^2 + 48*a^5*b^2*c^2*e^2)))*(b^4 + 6*a^2*c^2 - 6*a*
b^2*c))/(2*a^3*e*(4*a*c - b^2)^(3/2)) + ((b^4 + 6*a^2*c^2 - 6*a*b^2*c)^3*(6
40*a^10*b*c^6*d*e^18 + 3*a^6*b^9*c^2*d*e^18 - 46*a^7*b^7*c^3*d*e^18 + 264*a
^8*b^5*c^4*d*e^18 - 672*a^9*b^3*c^5*d*e^18))/(a^9*e^3*(4*a*c - b^2)^(9/2)*(
a^6*b^6 - 64*a^9*c^3 - 12*a^7*b^4*c + 48*a^8*b^2*c^2)))*(3*b^6 - 49*a^3*c^3
 + 72*a^2*b^2*c^2 - 27*a*b^4*c))/(8*a^3*c^2*(4*a*c - b^2)^(7/2)*(9*a^4*c^4
- 6*b^8 - 288*a^2*b^4*c^2 + 382*a^3*b^2*c^3 + 72*a*b^6*c))) + x^2*((((4*(54
*a^3*c^8*e^16 - 2*b^6*c^5*e^16 + 18*a*b^4*c^6*e^16 - 54*a^2*b^2*c^7*e^16))/
(a^6*b^6 - 64*a^9*c^3 - 12*a^7*b^4*c + 48*a^8*b^2*c^2) - (((4*(276*a^5*b*c^
7*e^17 - 6*a^2*b^7*c^4*e^17 + 65*a^3*b^5*c^5*e^17 - 233*a^4*b^3*c^6*e^17))/
(a^6*b^6 - 64*a^9*c^3 - 12*a^7*b^4*c + 48*a^8*b^2*c^2) - (((4*(480*a^8*c^7*
e^18 - a^4*b^8*c^3*e^18 + 6*a^5*b^6*c^4*e^18 + 30*a^6*b^4*c^5*e^18 - 272*a^
7*b^2*c^6*e^18))/(a^6*b^6 - 64*a^9*c^3 - 12*a^7*b^4*c + 48*a^8*b^2*c^2) - (
```

2*(b^7*e + 48*a^2*b^3*c^2*e - 12*a*b^5*c*e - 64*a^3*b*c^3*e)*(640*a^10*b*c^
6*e^19 + 3*a^6*b^9*c^2*e^19 - 46*a^7*b^7*c^3*e^19 + 264*a^8*b^5*c^4*e^19 -
672*a^9*b^3*c^5*e^19))/((a^6*b^6 - 64*a^9*c^3 - 12*a^7*b^4*c + 48*a^8*b^2*c
^2)*(a^3*b^6*e^2 - 64*a^6*c^3*e^2 - 12*a^4*b^4*c*e^2 + 48*a^5*b^2*c^2*e^2))
)*(b^7*e + 48*a^2*b^3*c^2*e - 12*a*b^5*c*e - 64*a^3*b*c^3*e))/(2*(a^3*b^6*e
^2 - 64*a^6*c^3*e^2 - 12*a^4*b^4*c*e^2 + 48*a^5*b^2*c^2*e^2)))*(b^7*e + 48*
a^2*b^3*c^2*e - 12*a*b^5*c*e - 64*a^3*b*c^3*e))/(2*(a^3*b^6*e^2 - 64*a^6*c^
3*e^2 - 12*a^4*b^4*c*e^2 + 48*a^5*b^2*c^2*e^2)) - (((((4*(480*a^8*c^7*e^18
- a^4*b^8*c^3*e^18 + 6*a^5*b^6*c^4*e^18 + 30*a^6*b^4*c^5*e^18 - 272*a^7*b^2
*c^6*e^18))/(a^6*b^6 - 64*a^9*c^3 - 12*a^7*b^4*c + 48*a^8*b^2*c^2) - (2*(b^
7*e + 48*a^2*b^3*c^2*e - 12*a*b^5*c*e - 64*a^3*b*c^3*e)*(640*a^10*b*c^6*e^1
9 + 3*a^6*b^9*c^2*e^19 - 46*a^7*b^7*c^3*e^19 + 264*a^8*b^5*c^4*e^19 - 672*a
^9*b^3*c^5*e^19))/((a^6*b^6 - 64*a^9*c^3 - 12*a^7*b^4*c + 48*a^8*b^2*c^2)*(
a^3*b^6*e^2 - 64*a^6*c^3*e^2 - 12*a^4*b^4*c*e^2 + 48*a^5*b^2*c^2*e^2)))*(b^
4 + 6*a^2*c^2 - 6*a*b^2*c))/(2*a^3*e*(4*a*c - b^2)^(3/2)) - ((b^4 + 6*a^2*c
^2 - 6*a*b^2*c)*(b^7*e + 48*a^2*b^3*c^2*e - 12*a*b^5*c*e - 64*a^3*b*c^3*e)*
(640*a^10*b*c^6*e^19 + 3*a^6*b^9*c^2*e^19 - 46*a^7*b^7*c^3*e^19 + 264*a^8*b
^5*c^4*e^19 - 672*a^9*b^3*c^5*e^19))/(a^3*e*(4*a*c - b^2)^(3/2)*(a^6*b^6 -
64*a^9*c^3 - 12*a^7*b^4*c + 48*a^8*b^2*c^2)*(a^3*b^6*e^2 - 64*a^6*c^3*e^2 -
 12*a^4*b^4*c*e^2 + 48*a^5*b^2*c^2*e^2)))*(b^4 + 6*a^2*c^2 - 6*a*b^2*c))/(2
*a^3*e*(4*a*c - b^2)^(3/2)) + ((b^4 + 6*a^2*c^2 - 6*a*b^2*c)^2*(b^7*e + 48*
a^2*b^3*c^2*e - 12*a*b^5*c*e - 64*a^3*b*c^3*e)*(640*a^10*b*c^6*e^19 + 3*a^6
*b^9*c^2*e^19 - 46*a^7*b^7*c^3*e^19 + 264*a^8*b^5*c^4*e^19 - 672*a^9*b^3*c^
5*e^19))/(2*a^6*e^2*(4*a*c - b^2)^3*(a^6*b^6 - 64*a^9*c^3 - 12*a^7*b^4*c +
48*a^8*b^2*c^2)*(a^3*b^6*e^2 - 64*a^6*c^3*e^2 - 12*a^4*b^4*c*e^2 + 48*a^5*b
^2*c^2*e^2)))*(3*b^6 - 3*a^3*c^3 + 36*a^2*b^2*c^2 - 21*a*b^4*c))/(8*a^3*c^2
*(4*a*c - b^2)^3*(9*a^4*c^4 - 6*b^8 - 288*a^2*b^4*c^2 + 382*a^3*b^2*c^3 + 7
2*a*b^6*c)) - (b*((((((4*(480*a^8*c^7*e^18 - a^4*b^8*c^3*e^18 + 6*a^5*b^6*c
^4*e^18 + 30*a^6*b^4*c^5*e^18 - 272*a^7*b^2*c^6*e^18))/(a^6*b^6 - 64*a^9*c^
3 - 12*a^7*b^4*c + 48*a^8*b^2*c^2) - (2*(b^7*e + 48*a^2*b^3*c^2*e - 12*a*b^
5*c*e - 64*a^3*b*c^3*e)*(640*a^10*b*c^6*e^19 + 3*a^6*b^9*c^2*e^19 - 46*a^7*
b^7*c^3*e^19 + 264*a^8*b^5*c^4*e^19 - 672*a^9*b^3*c^5*e^19))/((a^6*b^6 - 64
*a^9*c^3 - 12*a^7*b^4*c + 48*a^8*b^2*c^2)*(a^3*b^6*e^2 - 64*a^6*c^3*e^2 - 1
2*a^4*b^4*c*e^2 + 48*a^5*b^2*c^2*e^2)))*(b^4 + 6*a^2*c^2 - 6*a*b^2*c))/(2*a
^3*e*(4*a*c - b^2)^(3/2)) - ((b^4 + 6*a^2*c^2 - 6*a*b^2*c)*(b^7*e + 48*a^2*
b^3*c^2*e - 12*a*b^5*c*e - 64*a^3*b*c^3*e)*(640*a^10*b*c^6*e^19 + 3*a^6*b^9
*c^2*e^19 - 46*a^7*b^7*c^3*e^19 + 264*a^8*b^5*c^4*e^19 - 672*a^9*b^3*c^5*e^
19))/(a^3*e*(4*a*c - b^2)^(3/2)*(a^6*b^6 - 64*a^9*c^3 - 12*a^7*b^4*c + 48*a
^8*b^2*c^2)*(a^3*b^6*e^2 - 64*a^6*c^3*e^2 - 12*a^4*b^4*c*e^2 + 48*a^5*b^2*c
^2*e^2)))*(b^7*e + 48*a^2*b^3*c^2*e - 12*a*b^5*c*e - 64*a^3*b*c^3*e))/(2*(a
^3*b^6*e^2 - 64*a^6*c^3*e^2 - 12*a^4*b^4*c*e^2 + 48*a^5*b^2*c^2*e^2)) - (((
4*(276*a^5*b*c^7*e^17 - 6*a^2*b^7*c^4*e^17 + 65*a^3*b^5*c^5*e^17 - 233*a^4*
b^3*c^6*e^17))/(a^6*b^6 - 64*a^9*c^3 - 12*a^7*b^4*c + 48*a^8*b^2*c^2) - (((
4*(480*a^8*c^7*e^18 - a^4*b^8*c^3*e^18 + 6*a^5*b^6*c^4*e^18 + 30*a^6*b^4*c^
5*e^18 - 272*a^7*b^2*c^6*e^18))/(a^6*b^6 - 64*a^9*c^3 - 12*a^7*b^4*c + 48*a
^8*b^2*c^2) - (2*(b^7*e + 48*a^2*b^3*c^2*e - 12*a*b^5*c*e - 64*a^3*b*c^3*e)
*(640*a^10*b*c^6*e^19 + 3*a^6*b^9*c^2*e^19 - 46*a^7*b^7*c^3*e^19 + 264*a^8*
b^5*c^4*e^19 - 672*a^9*b^3*c^5*e^19))/((a^6*b^6 - 64*a^9*c^3 - 12*a^7*b^4*c
 + 48*a^8*b^2*c^2)*(a^3*b^6*e^2 - 64*a^6*c^3*e^2 - 12*a^4*b^4*c*e^2 + 48*a^
5*b^2*c^2*e^2)))*(b^7*e + 48*a^2*b^3*c^2*e - 12*a*b^5*c*e - 64*a^3*b*c^3*e)
)/(2*(a^3*b^6*e^2 - 64*a^6*c^3*e^2 - 12*a^4*b^4*c*e^2 + 48*a^5*b^2*c^2*e^2)
))*(b^4 + 6*a^2*c^2 - 6*a*b^2*c))/(2*a^3*e*(4*a*c - b^2)^(3/2)) + ((b^4 + 6
*a^2*c^2 - 6*a*b^2*c)^3*(640*a^10*b*c^6*e^19 + 3*a^6*b^9*c^2*e^19 - 46*a^7*
b^7*c^3*e^19 + 264*a^8*b^5*c^4*e^19 - 672*a^9*b^3*c^5*e^19))/(2*a^9*e^3*(4*
a*c - b^2)^(9/2)*(a^6*b^6 - 64*a^9*c^3 - 12*a^7*b^4*c + 48*a^8*b^2*c^2)))*(
3*b^6 - 49*a^3*c^3 + 72*a^2*b^2*c^2 - 27*a*b^4*c))/(8*a^3*c^2*(4*a*c - b^2)
^(7/2)*(9*a^4*c^4 - 6*b^8 - 288*a^2*b^4*c^2 + 382*a^3*b^2*c^3 + 72*a*b^6*c)
)) + (((((4*(36*a^6*c^7*e^15 + 4*a^2*b^8*c^3*e^15 - 45*a^3*b^6*c^4*e^15 + 1
70*a^4*b^4*c^5*e^15 - 225*a^5*b^2*c^6*e^15 + 6*a^2*b^7*c^4*d^2*e^15 - 65*a^

3*b^5*c^5*d^2*e^15 + 233*a^4*b^3*c^6*d^2*e^15 - 276*a^5*b*c^7*d^2*e^15))/(a^6*b^6 - 64*a^9*c^3 - 12*a^7*b^4*c + 48*a^8*b^2*c^2) - (((4*(96*a^8*b*c^6*e^16 + 2*a^4*b^9*c^2*e^16 - 26*a^5*b^7*c^3*e^16 + 118*a^6*b^5*c^4*e^16 - 208*a^7*b^3*c^5*e^16 - 480*a^8*c^7*d^2*e^16 + a^4*b^8*c^3*d^2*e^16 - 6*a^5*b^6*c^4*d^2*e^16 - 30*a^6*b^4*c^5*d^2*e^16 + 272*a^7*b^2*c^6*d^2*e^16))/(a^6*b^6 - 64*a^9*c^3 - 12*a^7*b^4*c + 48*a^8*b^2*c^2) + (2*(b^7*e + 48*a^2*b^3*c^2*e - 12*a*b^5*c*e - 64*a^3*b*c^3*e)*(a^7*b^8*c^2*e^17 - 12*a^8*b^6*c^3*e^17 + 48*a^9*b^4*c^4*e^17 - 64*a^10*b^2*c^5*e^17 + 3*a^6*b^9*c^2*d^2*e^17 - 46*a^7*b^7*c^3*d^2*e^17 + 264*a^8*b^5*c^4*d^2*e^17 - 672*a^9*b^3*c^5*d^2*e^17 + 640*a^10*b*c^6*d^2*e^17))/((a^6*b^6 - 64*a^9*c^3 - 12*a^7*b^4*c + 48*a^8*b^2*c^2)*(a^3*b^6*e^2 - 64*a^6*c^3*e^2 - 12*a^4*b^4*c*e^2 + 48*a^5*b^2*c^2*e^2)))*(b^7*e + 48*a^2*b^3*c^2*e - 12*a*b^5*c*e - 64*a^3*b*c^3*e))/(2*(a^3*b^6*e^2 - 64*a^6*c^3*e^2 - 12*a^4*b^4*c*e^2 + 48*a^5*b^2*c^2*e^2)))*(b^7*e + 48*a^2*b^3*c^2*e - 12*a*b^5*c*e - 64*a^3*b*c^3*e))/(2*(a^3*b^6*e^2 - 64*a^6*c^3*e^2 - 12*a^4*b^4*c*e^2 + 48*a^5*b^2*c^2*e^2)) - (4*(2*b^7*c^4*e^14 - 20*a*b^5*c^5*e^14 - 72*a^3*b*c^7*e^14 + 66*a^2*b^3*c^6*e^14 - 54*a^3*c^8*d^2*e^14 + 2*b^6*c^5*d^2*e^14 + 54*a^2*b^2*c^7*d^2*e^14 - 18*a*b^4*c^6*d^2*e^14))/(a^6*b^6 - 64*a^9*c^3 - 12*a^7*b^4*c + 48*a^8*b^2*c^2) + (((((4*(96*a^8*b*c^6*e^16 + 2*a^4*b^9*c^2*e^16 - 26*a^5*b^7*c^3*e^16 + 118*a^6*b^5*c^4*e^16 - 208*a^7*b^3*c^5*e^16 - 480*a^8*c^7*d^2*e^16 + a^4*b^8*c^3*d^2*e^16 - 6*a^5*b^6*c^4*d^2*e^16 - 30*a^6*b^4*c^5*d^2*e^16 + 272*a^7*b^2*c^6*d^2*e^16))/(a^6*b^6 - 64*a^9*c^3 - 12*a^7*b^4*c + 48*a^8*b^2*c^2) + (2*(b^7*e + 48*a^2*b^3*c^2*e - 12*a*b^5*c*e - 64*a^3*b*c^3*e)*(a^7*b^8*c^2*e^17 - 12*a^8*b^6*c^3*e^17 + 48*a^9*b^4*c^4*e^17 - 64*a^10*b^2*c^5*e^17 + 3*a^6*b^9*c^2*d^2*e^17 - 46*a^7*b^7*c^3*d^2*e^17 + 264*a^8*b^5*c^4*d^2*e^17 - 672*a^9*b^3*c^5*d^2*e^17 + 640*a^10*b*c^6*d^2*e^17))/((a^6*b^6 - 64*a^9*c^3 - 12*a^7*b^4*c + 48*a^8*b^2*c^2)*(a^3*b^6*e^2 - 64*a^6*c^3*e^2 - 12*a^4*b^4*c*e^2 + 48*a^5*b^2*c^2*e^2)))*(b^4 + 6*a^2*c^2 - 6*a*b^2*c))/(2*a^3*e*(4*a*c - b^2)^(3/2)) + ((b^4 + 6*a^2*c^2 - 6*a*b^2*c)*(b^7*e + 48*a^2*b^3*c^2*e - 12*a*b^5*c*e - 64*a^3*b*c^3*e)*(a^7*b^8*c^2*e^17 - 12*a^8*b^6*c^3*e^17 + 48*a^9*b^4*c^4*e^17 - 64*a^10*b^2*c^5*e^17 + 3*a^6*b^9*c^2*d^2*e^17 - 46*a^7*b^7*c^3*d^2*e^17 + 264*a^8*b^5*c^4*d^2*e^17 - 672*a^9*b^3*c^5*d^2*e^17 + 640*a^10*b*c^6*d^2*e^17))/(a^3*e*(4*a*c - b^2)^(3/2)*(a^6*b^6 - 64*a^9*c^3 - 12*a^7*b^4*c + 48*a^8*b^2*c^2)*(a^3*b^6*e^2 - 64*a^6*c^3*e^2 - 12*a^4*b^4*c*e^2 + 48*a^5*b^2*c^2*e^2)))*(b^4 + 6*a^2*c^2 - 6*a*b^2*c))/(2*a^3*e*(4*a*c - b^2)^(3/2)) + ((b^4 + 6*a^2*c^2 - 6*a*b^2*c)^2*(b^7*e + 48*a^2*b^3*c^2*e - 12*a*b^5*c*e - 64*a^3*b*c^3*e)*(a^7*b^8*c^2*e^17 - 12*a^8*b^6*c^3*e^17 + 48*a^9*b^4*c^4*e^17 - 64*a^10*b^2*c^5*e^17 + 3*a^6*b^9*c^2*d^2*e^17 - 46*a^7*b^7*c^3*d^2*e^17 + 264*a^8*b^5*c^4*d^2*e^17 - 672*a^9*b^3*c^5*d^2*e^17 + 640*a^10*b*c^6*d^2*e^17))/(2*a^6*e^2*(4*a*c - b^2)^3*(a^6*b^6 - 64*a^9*c^3 - 12*a^7*b^4*c + 48*a^8*b^2*c^2)*(a^3*b^6*e^2 - 64*a^6*c^3*e^2 - 12*a^4*b^4*c*e^2 + 48*a^5*b^2*c^2*e^2)))*(3*b^6 - 3*a^3*c^3 + 36*a^2*b^2*c^2 - 21*a*b^4*c))/(8*a^3*c^2*(4*a*c - b^2)^3*(9*a^4*c^4 - 6*b^8 - 288*a^2*b^4*c^2 + 382*a^3*b^2*c^3 + 72*a*b^6*c)) - (b*((((4*(36*a^6*c^7*e^15 + 4*a^2*b^8*c^3*e^15 - 45*a^3*b^6*c^4*e^15 + 170*a^4*b^4*c^5*e^15 - 225*a^5*b^2*c^6*e^15 + 6*a^2*b^7*c^4*d^2*e^15 - 65*a^3*b^5*c^5*d^2*e^15 + 233*a^4*b^3*c^6*d^2*e^15 - 276*a^5*b*c^7*d^2*e^15))/(a^6*b^6 - 64*a^9*c^3 - 12*a^7*b^4*c + 48*a^8*b^2*c^2) - (((4*(96*a^8*b*c^6*e^16 + 2*a^4*b^9*c^2*e^16 - 26*a^5*b^7*c^3*e^16 + 118*a^6*b^5*c^4*e^16 - 208*a^7*b^3*c^5*e^16 - 480*a^8*c^7*d^2*e^16 + a^4*b^8*c^3*d^2*e^16 - 6*a^5*b^6*c^4*d^2*e^16 - 30*a^6*b^4*c^5*d^2*e^16 + 272*a^7*b^2*c^6*d^2*e^16))/(a^6*b^6 - 64*a^9*c^3 - 12*a^7*b^4*c + 48*a^8*b^2*c^2) + (2*(b^7*e + 48*a^2*b^3*c^2*e - 12*a*b^5*c*e - 64*a^3*b*c^3*e)*(a^7*b^8*c^2*e^17 - 12*a^8*b^6*c^3*e^17 + 48*a^9*b^4*c^4*e^17 - 64*a^10*b^2*c^5*e^17 + 3*a^6*b^9*c^2*d^2*e^17 - 46*a^7*b^7*c^3*d^2*e^17 + 264*a^8*b^5*c^4*d^2*e^17 - 672*a^9*b^3*c^5*d^2*e^17 + 640*a^10*b*c^6*d^2*e^17))/((a^6*b^6 - 64*a^9*c^3 - 12*a^7*b^4*c + 48*a^8*b^2*c^2)*(a^3*b^6*e^2 - 64*a^6*c^3*e^2 - 12*a^4*b^4*c*e^2 + 48*a^5*b^2*c^2*e^2)))*(b^7*e + 48*a^2*b^3*c^2*e - 12*a*b^5*c*e - 64*a^3*b*c^3*e))/(2*(a^3*b^6*e^2 - 64*a^6*c^3*e^2 - 12*a^4*b^4*c*e^2 + 48*a^5*b^2*c^2*e^2)))*(b^4 + 6*a^2*c^2 - 6*a*b^2*c))/(2*a^3*e*(4*a*c - b^2)^(

```
3/2)) - (((((4*(96*a^8*b*c^6*e^16 + 2*a^4*b^9*c^2*e^16 - 26*a^5*b^7*c^3*e^1
6 + 118*a^6*b^5*c^4*e^16 - 208*a^7*b^3*c^5*e^16 - 480*a^8*c^7*d^2*e^16 + a^
4*b^8*c^3*d^2*e^16 - 6*a^5*b^6*c^4*d^2*e^16 - 30*a^6*b^4*c^5*d^2*e^16 + 272
*a^7*b^2*c^6*d^2*e^16))/(a^6*b^6 - 64*a^9*c^3 - 12*a^7*b^4*c + 48*a^8*b^2*c
^2) + (2*(b^7*e + 48*a^2*b^3*c^2*e - 12*a*b^5*c*e - 64*a^3*b*c^3*e)*(a^7*b^
8*c^2*e^17 - 12*a^8*b^6*c^3*e^17 + 48*a^9*b^4*c^4*e^17 - 64*a^10*b^2*c^5*e^
17 + 3*a^6*b^9*c^2*d^2*e^17 - 46*a^7*b^7*c^3*d^2*e^17 + 264*a^8*b^5*c^4*d^2
*e^17 - 672*a^9*b^3*c^5*d^2*e^17 + 640*a^10*b*c^6*d^2*e^17))/((a^6*b^6 - 64
*a^9*c^3 - 12*a^7*b^4*c + 48*a^8*b^2*c^2)*(a^3*b^6*e^2 - 64*a^6*c^3*e^2 - 1
2*a^4*b^4*c*e^2 + 48*a^5*b^2*c^2*e^2)))*(b^4 + 6*a^2*c^2 - 6*a*b^2*c))/(2*a
^3*e*(4*a*c - b^2)^(3/2)) + ((b^4 + 6*a^2*c^2 - 6*a*b^2*c)*(b^7*e + 48*a^2*
b^3*c^2*e - 12*a*b^5*c*e - 64*a^3*b*c^3*e)*(a^7*b^8*c^2*e^17 - 12*a^8*b^6*c
^3*e^17 + 48*a^9*b^4*c^4*e^17 - 64*a^10*b^2*c^5*e^17 + 3*a^6*b^9*c^2*d^2*e^
17 - 46*a^7*b^7*c^3*d^2*e^17 + 264*a^8*b^5*c^4*d^2*e^17 - 672*a^9*b^3*c^5*d
^2*e^17 + 640*a^10*b*c^6*d^2*e^17))/(a^3*e*(4*a*c - b^2)^(3/2)*(a^6*b^6 - 6
4*a^9*c^3 - 12*a^7*b^4*c + 48*a^8*b^2*c^2)*(a^3*b^6*e^2 - 64*a^6*c^3*e^2 -
12*a^4*b^4*c*e^2 + 48*a^5*b^2*c^2*e^2)))*(b^7*e + 48*a^2*b^3*c^2*e - 12*a*b
^5*c*e - 64*a^3*b*c^3*e))/(2*(a^3*b^6*e^2 - 64*a^6*c^3*e^2 - 12*a^4*b^4*c*e
^2 + 48*a^5*b^2*c^2*e^2)) + ((b^4 + 6*a^2*c^2 - 6*a*b^2*c)^3*(a^7*b^8*c^2*e
^17 - 12*a^8*b^6*c^3*e^17 + 48*a^9*b^4*c^4*e^17 - 64*a^10*b^2*c^5*e^17 + 3*
a^6*b^9*c^2*d^2*e^17 - 46*a^7*b^7*c^3*d^2*e^17 + 264*a^8*b^5*c^4*d^2*e^17 -
 672*a^9*b^3*c^5*d^2*e^17 + 640*a^10*b*c^6*d^2*e^17))/(2*a^9*e^3*(4*a*c - b
^2)^(9/2)*(a^6*b^6 - 64*a^9*c^3 - 12*a^7*b^4*c + 48*a^8*b^2*c^2)))*(3*b^6 -
 49*a^3*c^3 + 72*a^2*b^2*c^2 - 27*a*b^4*c))/(8*a^3*c^2*(4*a*c - b^2)^(7/2)*
(9*a^4*c^4 - 6*b^8 - 288*a^2*b^4*c^2 + 382*a^3*b^2*c^3 + 72*a*b^6*c))))/(36
*a^4*c^6*e^14 + b^8*c^2*e^14 - 12*a*b^6*c^3*e^14 + 48*a^2*b^4*c^4*e^14 - 72
*a^3*b^2*c^5*e^14))*(b^4 + 6*a^2*c^2 - 6*a*b^2*c))/(a^3*e*(4*a*c - b^2)^(3/
2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)**3/(a+b*(e*x+d)**2+c*(e*x+d)**4)**2,x)
```

```
[Out] Timed out
```

3.516 $\int \frac{1}{(d+ex)^4\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx$

Optimal. Leaf size=408

$$\frac{b\left(5b^2-19ac\right)}{2a^3e\left(b^2-4ac\right)(d+ex)}-\frac{5b^2-14ac}{6a^2e\left(b^2-4ac\right)(d+ex)^3}+\frac{\sqrt{c}\left(28a^2c^2-29ab^2c+b\left(5b^2-19ac\right)\sqrt{b^2-4ac}+5b^4\right)\tan^{-1}}{2\sqrt{2}\,a^3e\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}$$

Rubi [A] time = 3.68, antiderivative size = 408, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1142, 1121, 1281, 1166, 205}

$$\frac{\sqrt{c}\left(28a^2c^2-29ab^2c+b\left(5b^2-19ac\right)\sqrt{b^2-4ac}+5b^4\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\,a^3e\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{c}\left(28a^2c^2-29ab^2c-b\left(5b^2-19ac\right)\sqrt{b^2-4ac}+5b^4\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{2\sqrt{2}\,a^3e\left(b^2-4ac\right)^{3/2}\sqrt{\sqrt{b^2-4ac}+b}}+\frac{b\left(5b^2-19ac\right)}{2a^3e\left(b^2-4ac\right)(d+ex)}-\frac{5b^2-14ac}{6a^2e\left(b^2-4ac\right)(d+ex)^3}+\frac{-2ac+b^2+bc(d+ex)^2}{2ae\left(b^2-4ac\right)(d+ex)^3\left(a+b(d+ex)^2+c(d+ex)^4\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)^4*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2),x]
```

```
[Out] -(5*b^2 - 14*a*c)/(6*a^2*(b^2 - 4*a*c)*e*(d + e*x)^3) + (b*(5*b^2 - 19*a*c)
)/(2*a^3*(b^2 - 4*a*c)*e*(d + e*x)) + (b^2 - 2*a*c + b*c*(d + e*x)^2)/(2*a*
(b^2 - 4*a*c)*e*(d + e*x)^3*(a + b*(d + e*x)^2 + c*(d + e*x)^4)) + (Sqrt[c]
*(5*b^4 - 29*a*b^2*c + 28*a^2*c^2 + b*(5*b^2 - 19*a*c)*Sqrt[b^2 - 4*a*c])*A
rcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*
a^3*(b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]*e) - (Sqrt[c]*(5*b^4 -
29*a*b^2*c + 28*a^2*c^2 - b*(5*b^2 - 19*a*c)*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqr
t[2]*Sqrt[c]*(d + e*x))/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a^3*(b^2 -
 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]*e)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1121

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> -Simp[((d*x)^(m + 1)*(b^2 - 2*a*c + b*c*x^2)*(a + b*x^2 + c*x^4)^(p + 1
))/(2*a*d*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)),
Int[(d*x)^m*(a + b*x^2 + c*x^4)^(p + 1)*Simp[b^2*(m + 2*p + 3) - 2*a*c*(m +
 4*p + 5) + b*c*(m + 4*p + 7)*x^2, x], x], x] /; FreeQ[{a, b, c, d, m}, x]
&& NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p] && (IntegerQ[p] || In
tegerQ[m])
```

Rule 1142

```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Di
st[u^m/(Coefficient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p,
 x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1281

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[(d*(f*x)^(m + 1)*(a + b*x^2 + c*x^4)^(p + 1)
)/(a*f*(m + 1)), x] + Dist[1/(a*f^2*(m + 1)), Int[(f*x)^(m + 2)*(a + b*x^2
+ c*x^4)^p*Simp[a*e*(m + 1) - b*d*(m + 2*p + 3) - c*d*(m + 4*p + 5)*x^2, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[m
, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rubi steps

$$\int \frac{1}{(d+ex)^4\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x^4\left(a+bx^2+cx^4\right)^2}\,dx, x, d+ex\right)}{e}$$

$$= \frac{b^2-2ac+bc(d+ex)^2}{2a\left(b^2-4ac\right)e(d+ex)^3\left(a+b(d+ex)^2+c(d+ex)^4\right)} - \frac{\text{Subs}}{}$$

$$= -\frac{5b^2-14ac}{6a^2\left(b^2-4ac\right)e(d+ex)^3} + \frac{b^2-2ac+bc(d+}{2a\left(b^2-4ac\right)e(d+ex)^3\left(a+b(d\right.}$$

$$= -\frac{5b^2-14ac}{6a^2\left(b^2-4ac\right)e(d+ex)^3} + \frac{b\left(5b^2-19ac\right)}{2a^3\left(b^2-4ac\right)e(d+ex)} + \frac{}{2a\left(b\right.}$$

$$= -\frac{5b^2-14ac}{6a^2\left(b^2-4ac\right)e(d+ex)^3} + \frac{b\left(5b^2-19ac\right)}{2a^3\left(b^2-4ac\right)e(d+ex)} + \frac{}{2a\left(b\right.}$$

$$= -\frac{5b^2-14ac}{6a^2\left(b^2-4ac\right)e(d+ex)^3} + \frac{b\left(5b^2-19ac\right)}{2a^3\left(b^2-4ac\right)e(d+ex)} + \frac{}{2a\left(b\right.}$$

Mathematica [A] time = 2.98, size = 384, normalized size = 0.94

$$\frac{\frac{6(d+ex)\left(2a^2c^2-4ab^2c-3abc^2(d+ex)^2+b^4+b^3c(d+ex)^2\right)}{\left(b^2-4ac\right)\left(a+(d+ex)^2\left(b+c(d+ex)^2\right)\right)}+\frac{3\sqrt{2}\sqrt{c}\left(28a^2c^2-29ab^2c-19abc\sqrt{b^2-4ac}+5b^3\sqrt{b^2-4ac}+5b^4\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{3\sqrt{2}\sqrt{c}\left(-28a^2c^2+29ab^2c-19abc\sqrt{b^2-4ac}+5b^3\sqrt{b^2-4ac}-5b^4\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\left(b^2-4ac\right)^{3/2}\sqrt{\sqrt{b^2-4ac}+b}}-\frac{4a}{(d+ex)^3}+\frac{24b}{d+ex}}{12a^3e}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)^4*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2),x]
```

```
[Out] ((-4*a)/(d + e*x)^3 + (24*b)/(d + e*x) + (6*(d + e*x)*(b^4 - 4*a*b^2*c + 2*
a^2*c^2 + b^3*c*(d + e*x)^2 - 3*a*b*c^2*(d + e*x)^2))/((b^2 - 4*a*c)*(a + (
d + e*x)^2*(b + c*(d + e*x)^2))) + (3*Sqrt[2]*Sqrt[c]*(5*b^4 - 29*a*b^2*c +
 28*a^2*c^2 + 5*b^3*Sqrt[b^2 - 4*a*c] - 19*a*b*c*Sqrt[b^2 - 4*a*c])*ArcTan[
(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(3
/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (3*Sqrt[2]*Sqrt[c]*(-5*b^4 + 29*a*b^2*c
- 28*a^2*c^2 + 5*b^3*Sqrt[b^2 - 4*a*c] - 19*a*b*c*Sqrt[b^2 - 4*a*c])*ArcTan
[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(
3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(12*a^3*e)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(d+ex)^4 \left(a+b(d+ex)^2+c(d+ex)^4\right)^2} \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/((d + e*x)^4*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2),x]
```

```
[Out] IntegrateAlgebraic[1/((d + e*x)^4*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2), x
]
```

fricas [B] time = 2.38, size = 5734, normalized size = 14.05

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^4/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="fricas")
```

```
[Out] 1/12*(6*(5*b^3*c - 19*a*b*c^2)*e^6*x^6 + 36*(5*b^3*c - 19*a*b*c^2)*d*e^5*x^
5 + 2*(15*b^4 - 62*a*b^2*c + 14*a^2*c^2 + 45*(5*b^3*c - 19*a*b*c^2)*d^2)*e^
4*x^4 + 6*(5*b^3*c - 19*a*b*c^2)*d^6 + 8*(15*(5*b^3*c - 19*a*b*c^2)*d^3 + (
15*b^4 - 62*a*b^2*c + 14*a^2*c^2)*d)*e^3*x^3 + 2*(15*b^4 - 62*a*b^2*c + 14*
a^2*c^2)*d^4 + 2*(45*(5*b^3*c - 19*a*b*c^2)*d^4 + 10*a*b^3 - 40*a^2*b*c + 6
*(15*b^4 - 62*a*b^2*c + 14*a^2*c^2)*d^2)*e^2*x^2 - 4*a^2*b^2 + 16*a^3*c + 2
0*(a*b^3 - 4*a^2*b*c)*d^2 + 4*(9*(5*b^3*c - 19*a*b*c^2)*d^5 + 2*(15*b^4 - 6
2*a*b^2*c + 14*a^2*c^2)*d^3 + 10*(a*b^3 - 4*a^2*b*c)*d)*e*x - 3*sqrt(1/2)*(
(a^3*b^2*c - 4*a^4*c^2)*e^8*x^7 + 7*(a^3*b^2*c - 4*a^4*c^2)*d*e^7*x^6 + (a^
3*b^3 - 4*a^4*b*c + 21*(a^3*b^2*c - 4*a^4*c^2)*d^2)*e^6*x^5 + 5*(7*(a^3*b^2
*c - 4*a^4*c^2)*d^3 + (a^3*b^3 - 4*a^4*b*c)*d)*e^5*x^4 + (a^4*b^2 - 4*a^5*c
 + 35*(a^3*b^2*c - 4*a^4*c^2)*d^4 + 10*(a^3*b^3 - 4*a^4*b*c)*d^2)*e^4*x^3 +
 (21*(a^3*b^2*c - 4*a^4*c^2)*d^5 + 10*(a^3*b^3 - 4*a^4*b*c)*d^3 + 3*(a^4*b^
2 - 4*a^5*c)*d)*e^3*x^2 + (7*(a^3*b^2*c - 4*a^4*c^2)*d^6 + 5*(a^3*b^3 - 4*a
^4*b*c)*d^4 + 3*(a^4*b^2 - 4*a^5*c)*d^2)*e^2*x + ((a^3*b^2*c - 4*a^4*c^2)*d
^7 + (a^3*b^3 - 4*a^4*b*c)*d^5 + (a^4*b^2 - 4*a^5*c)*d^3)*e)*sqrt(-(25*b^9
- 315*a*b^7*c + 1386*a^2*b^5*c^2 - 2415*a^3*b^3*c^3 + 1260*a^4*b*c^4 + (a^7
*b^6 - 12*a^8*b^4*c + 48*a^9*b^2*c^2 - 64*a^10*c^3)*e^2*sqrt((625*b^12 - 82
50*a*b^10*c + 39525*a^2*b^8*c^2 - 83630*a^3*b^6*c^3 + 76686*a^4*b^4*c^4 - 2
4108*a^5*b^2*c^5 + 2401*a^6*c^6)/((a^14*b^6 - 12*a^15*b^4*c + 48*a^16*b^2*c
^2 - 64*a^17*c^3)*e^4)))/((a^7*b^6 - 12*a^8*b^4*c + 48*a^9*b^2*c^2 - 64*a^1
0*c^3)*e^2))*log((1125*b^8*c^4 - 12325*a*b^6*c^5 + 43410*a^2*b^4*c^6 - 5042
1*a^3*b^2*c^7 + 9604*a^4*c^8)*e*x + (1125*b^8*c^4 - 12325*a*b^6*c^5 + 43410
*a^2*b^4*c^6 - 50421*a^3*b^2*c^7 + 9604*a^4*c^8)*d + 1/2*sqrt(1/2)*((5*a^7*
b^11 - 94*a^8*b^9*c + 700*a^9*b^7*c^2 - 2576*a^10*b^5*c^3 + 4672*a^11*b^3*c
^4 - 3328*a^12*b*c^5)*e^3*sqrt((625*b^12 - 8250*a*b^10*c + 39525*a^2*b^8*c^
2 - 83630*a^3*b^6*c^3 + 76686*a^4*b^4*c^4 - 24108*a^5*b^2*c^5 + 2401*a^6*c^
6)/((a^14*b^6 - 12*a^15*b^4*c + 48*a^16*b^2*c^2 - 64*a^17*c^3)*e^4)) - (125
*b^14 - 2425*a*b^12*c + 18940*a^2*b^10*c^2 - 75579*a^3*b^8*c^3 + 160932*a^4
*b^6*c^4 - 172990*a^5*b^4*c^5 + 79408*a^6*b^2*c^6 - 10976*a^7*c^7)*e)*sqrt(
-(25*b^9 - 315*a*b^7*c + 1386*a^2*b^5*c^2 - 2415*a^3*b^3*c^3 + 1260*a^4*b*c
^4 + (a^7*b^6 - 12*a^8*b^4*c + 48*a^9*b^2*c^2 - 64*a^10*c^3)*e^2*sqrt((625*
b^12 - 8250*a*b^10*c + 39525*a^2*b^8*c^2 - 83630*a^3*b^6*c^3 + 76686*a^4*b^
4*c^4 - 24108*a^5*b^2*c^5 + 2401*a^6*c^6)/((a^14*b^6 - 12*a^15*b^4*c + 48*a
^16*b^2*c^2 - 64*a^17*c^3)*e^4)))/((a^7*b^6 - 12*a^8*b^4*c + 48*a^9*b^2*c^2
 - 64*a^10*c^3)*e^2))) + 3*sqrt(1/2)*((a^3*b^2*c - 4*a^4*c^2)*e^8*x^7 + 7*(
a^3*b^2*c - 4*a^4*c^2)*d*e^7*x^6 + (a^3*b^3 - 4*a^4*b*c + 21*(a^3*b^2*c - 4
*a^4*c^2)*d^2)*e^6*x^5 + 5*(7*(a^3*b^2*c - 4*a^4*c^2)*d^3 + (a^3*b^3 - 4*a^
4*b*c)*d)*e^5*x^4 + (a^4*b^2 - 4*a^5*c + 35*(a^3*b^2*c - 4*a^4*c^2)*d^4 + 1
0*(a^3*b^3 - 4*a^4*b*c)*d^2)*e^4*x^3 + (21*(a^3*b^2*c - 4*a^4*c^2)*d^5 + 10
*(a^3*b^3 - 4*a^4*b*c)*d^3 + 3*(a^4*b^2 - 4*a^5*c)*d)*e^3*x^2 + (7*(a^3*b^2
```

```
*c - 4*a^4*c^2)*d^6 + 5*(a^3*b^3 - 4*a^4*b*c)*d^4 + 3*(a^4*b^2 - 4*a^5*c)*d
^2)*e^2*x + ((a^3*b^2*c - 4*a^4*c^2)*d^7 + (a^3*b^3 - 4*a^4*b*c)*d^5 + (a^4
*b^2 - 4*a^5*c)*d^3)*e)*sqrt(-(25*b^9 - 315*a*b^7*c + 1386*a^2*b^5*c^2 - 24
15*a^3*b^3*c^3 + 1260*a^4*b*c^4 + (a^7*b^6 - 12*a^8*b^4*c + 48*a^9*b^2*c^2
- 64*a^10*c^3)*e^2*sqrt((625*b^12 - 8250*a*b^10*c + 39525*a^2*b^8*c^2 - 836
30*a^3*b^6*c^3 + 76686*a^4*b^4*c^4 - 24108*a^5*b^2*c^5 + 2401*a^6*c^6)/((a^
14*b^6 - 12*a^15*b^4*c + 48*a^16*b^2*c^2 - 64*a^17*c^3)*e^4)))/((a^7*b^6 -
12*a^8*b^4*c + 48*a^9*b^2*c^2 - 64*a^10*c^3)*e^2))*log((1125*b^8*c^4 - 1232
5*a*b^6*c^5 + 43410*a^2*b^4*c^6 - 50421*a^3*b^2*c^7 + 9604*a^4*c^8)*e*x + (
1125*b^8*c^4 - 12325*a*b^6*c^5 + 43410*a^2*b^4*c^6 - 50421*a^3*b^2*c^7 + 96
04*a^4*c^8)*d - 1/2*sqrt(1/2)*((5*a^7*b^11 - 94*a^8*b^9*c + 700*a^9*b^7*c^2
 - 2576*a^10*b^5*c^3 + 4672*a^11*b^3*c^4 - 3328*a^12*b*c^5)*e^3*sqrt((625*b
^12 - 8250*a*b^10*c + 39525*a^2*b^8*c^2 - 83630*a^3*b^6*c^3 + 76686*a^4*b^4
*c^4 - 24108*a^5*b^2*c^5 + 2401*a^6*c^6)/((a^14*b^6 - 12*a^15*b^4*c + 48*a^
16*b^2*c^2 - 64*a^17*c^3)*e^4)) - (125*b^14 - 2425*a*b^12*c + 18940*a^2*b^1
0*c^2 - 75579*a^3*b^8*c^3 + 160932*a^4*b^6*c^4 - 172990*a^5*b^4*c^5 + 79408
*a^6*b^2*c^6 - 10976*a^7*c^7)*e)*sqrt(-(25*b^9 - 315*a*b^7*c + 1386*a^2*b^5
*c^2 - 2415*a^3*b^3*c^3 + 1260*a^4*b*c^4 + (a^7*b^6 - 12*a^8*b^4*c + 48*a^9
*b^2*c^2 - 64*a^10*c^3)*e^2*sqrt((625*b^12 - 8250*a*b^10*c + 39525*a^2*b^8*
c^2 - 83630*a^3*b^6*c^3 + 76686*a^4*b^4*c^4 - 24108*a^5*b^2*c^5 + 2401*a^6*
c^6)/((a^14*b^6 - 12*a^15*b^4*c + 48*a^16*b^2*c^2 - 64*a^17*c^3)*e^4)))/((a
^7*b^6 - 12*a^8*b^4*c + 48*a^9*b^2*c^2 - 64*a^10*c^3)*e^2))) + 3*sqrt(1/2)*
((a^3*b^2*c - 4*a^4*c^2)*e^8*x^7 + 7*(a^3*b^2*c - 4*a^4*c^2)*d*e^7*x^6 + (a
^3*b^3 - 4*a^4*b*c + 21*(a^3*b^2*c - 4*a^4*c^2)*d^2)*e^6*x^5 + 5*(7*(a^3*b^
2*c - 4*a^4*c^2)*d^3 + (a^3*b^3 - 4*a^4*b*c)*d)*e^5*x^4 + (a^4*b^2 - 4*a^5*
c + 35*(a^3*b^2*c - 4*a^4*c^2)*d^4 + 10*(a^3*b^3 - 4*a^4*b*c)*d^2)*e^4*x^3
+ (21*(a^3*b^2*c - 4*a^4*c^2)*d^5 + 10*(a^3*b^3 - 4*a^4*b*c)*d^3 + 3*(a^4*b
^2 - 4*a^5*c)*d)*e^3*x^2 + (7*(a^3*b^2*c - 4*a^4*c^2)*d^6 + 5*(a^3*b^3 - 4*
a^4*b*c)*d^4 + 3*(a^4*b^2 - 4*a^5*c)*d^2)*e^2*x + ((a^3*b^2*c - 4*a^4*c^2)*
d^7 + (a^3*b^3 - 4*a^4*b*c)*d^5 + (a^4*b^2 - 4*a^5*c)*d^3)*e)*sqrt(-(25*b^9
 - 315*a*b^7*c + 1386*a^2*b^5*c^2 - 2415*a^3*b^3*c^3 + 1260*a^4*b*c^4 - (a^
7*b^6 - 12*a^8*b^4*c + 48*a^9*b^2*c^2 - 64*a^10*c^3)*e^2*sqrt((625*b^12 - 8
250*a*b^10*c + 39525*a^2*b^8*c^2 - 83630*a^3*b^6*c^3 + 76686*a^4*b^4*c^4 -
24108*a^5*b^2*c^5 + 2401*a^6*c^6)/((a^14*b^6 - 12*a^15*b^4*c + 48*a^16*b^2*
c^2 - 64*a^17*c^3)*e^4)))/((a^7*b^6 - 12*a^8*b^4*c + 48*a^9*b^2*c^2 - 64*a^
10*c^3)*e^2))*log((1125*b^8*c^4 - 12325*a*b^6*c^5 + 43410*a^2*b^4*c^6 - 504
21*a^3*b^2*c^7 + 9604*a^4*c^8)*e*x + (1125*b^8*c^4 - 12325*a*b^6*c^5 + 4341
0*a^2*b^4*c^6 - 50421*a^3*b^2*c^7 + 9604*a^4*c^8)*d + 1/2*sqrt(1/2)*((5*a^7
*b^11 - 94*a^8*b^9*c + 700*a^9*b^7*c^2 - 2576*a^10*b^5*c^3 + 4672*a^11*b^3*
c^4 - 3328*a^12*b*c^5)*e^3*sqrt((625*b^12 - 8250*a*b^10*c + 39525*a^2*b^8*c
^2 - 83630*a^3*b^6*c^3 + 76686*a^4*b^4*c^4 - 24108*a^5*b^2*c^5 + 2401*a^6*c
^6)/((a^14*b^6 - 12*a^15*b^4*c + 48*a^16*b^2*c^2 - 64*a^17*c^3)*e^4)) + (12
5*b^14 - 2425*a*b^12*c + 18940*a^2*b^10*c^2 - 75579*a^3*b^8*c^3 + 160932*a^
4*b^6*c^4 - 172990*a^5*b^4*c^5 + 79408*a^6*b^2*c^6 - 10976*a^7*c^7)*e)*sqrt
(-(25*b^9 - 315*a*b^7*c + 1386*a^2*b^5*c^2 - 2415*a^3*b^3*c^3 + 1260*a^4*b*
c^4 - (a^7*b^6 - 12*a^8*b^4*c + 48*a^9*b^2*c^2 - 64*a^10*c^3)*e^2*sqrt((625
*b^12 - 8250*a*b^10*c + 39525*a^2*b^8*c^2 - 83630*a^3*b^6*c^3 + 76686*a^4*b
^4*c^4 - 24108*a^5*b^2*c^5 + 2401*a^6*c^6)/((a^14*b^6 - 12*a^15*b^4*c + 48*
a^16*b^2*c^2 - 64*a^17*c^3)*e^4)))/((a^7*b^6 - 12*a^8*b^4*c + 48*a^9*b^2*c^
2 - 64*a^10*c^3)*e^2))) - 3*sqrt(1/2)*((a^3*b^2*c - 4*a^4*c^2)*e^8*x^7 + 7*
(a^3*b^2*c - 4*a^4*c^2)*d*e^7*x^6 + (a^3*b^3 - 4*a^4*b*c + 21*(a^3*b^2*c -
4*a^4*c^2)*d^2)*e^6*x^5 + 5*(7*(a^3*b^2*c - 4*a^4*c^2)*d^3 + (a^3*b^3 - 4*a
^4*b*c)*d)*e^5*x^4 + (a^4*b^2 - 4*a^5*c + 35*(a^3*b^2*c - 4*a^4*c^2)*d^4 +
10*(a^3*b^3 - 4*a^4*b*c)*d^2)*e^4*x^3 + (21*(a^3*b^2*c - 4*a^4*c^2)*d^5 + 1
0*(a^3*b^3 - 4*a^4*b*c)*d^3 + 3*(a^4*b^2 - 4*a^5*c)*d)*e^3*x^2 + (7*(a^3*b^
2*c - 4*a^4*c^2)*d^6 + 5*(a^3*b^3 - 4*a^4*b*c)*d^4 + 3*(a^4*b^2 - 4*a^5*c)*
d^2)*e^2*x + ((a^3*b^2*c - 4*a^4*c^2)*d^7 + (a^3*b^3 - 4*a^4*b*c)*d^5 + (a^
4*b^2 - 4*a^5*c)*d^3)*e)*sqrt(-(25*b^9 - 315*a*b^7*c + 1386*a^2*b^5*c^2 - 2
415*a^3*b^3*c^3 + 1260*a^4*b*c^4 - (a^7*b^6 - 12*a^8*b^4*c + 48*a^9*b^2*c^2
```

```
 - 64*a^10*c^3)*e^2*sqrt((625*b^12 - 8250*a*b^10*c + 39525*a^2*b^8*c^2 - 83
630*a^3*b^6*c^3 + 76686*a^4*b^4*c^4 - 24108*a^5*b^2*c^5 + 2401*a^6*c^6)/((a
^14*b^6 - 12*a^15*b^4*c + 48*a^16*b^2*c^2 - 64*a^17*c^3)*e^4)))/((a^7*b^6 -
 12*a^8*b^4*c + 48*a^9*b^2*c^2 - 64*a^10*c^3)*e^2))*log((1125*b^8*c^4 - 123
25*a*b^6*c^5 + 43410*a^2*b^4*c^6 - 50421*a^3*b^2*c^7 + 9604*a^4*c^8)*e*x +
(1125*b^8*c^4 - 12325*a*b^6*c^5 + 43410*a^2*b^4*c^6 - 50421*a^3*b^2*c^7 + 9
604*a^4*c^8)*d - 1/2*sqrt(1/2)*((5*a^7*b^11 - 94*a^8*b^9*c + 700*a^9*b^7*c^
2 - 2576*a^10*b^5*c^3 + 4672*a^11*b^3*c^4 - 3328*a^12*b*c^5)*e^3*sqrt((625*
b^12 - 8250*a*b^10*c + 39525*a^2*b^8*c^2 - 83630*a^3*b^6*c^3 + 76686*a^4*b^
4*c^4 - 24108*a^5*b^2*c^5 + 2401*a^6*c^6)/((a^14*b^6 - 12*a^15*b^4*c + 48*a
^16*b^2*c^2 - 64*a^17*c^3)*e^4)) + (125*b^14 - 2425*a*b^12*c + 18940*a^2*b^
10*c^2 - 75579*a^3*b^8*c^3 + 160932*a^4*b^6*c^4 - 172990*a^5*b^4*c^5 + 7940
8*a^6*b^2*c^6 - 10976*a^7*c^7)*e)*sqrt(-(25*b^9 - 315*a*b^7*c + 1386*a^2*b^
5*c^2 - 2415*a^3*b^3*c^3 + 1260*a^4*b*c^4 - (a^7*b^6 - 12*a^8*b^4*c + 48*a^
9*b^2*c^2 - 64*a^10*c^3)*e^2*sqrt((625*b^12 - 8250*a*b^10*c + 39525*a^2*b^8
*c^2 - 83630*a^3*b^6*c^3 + 76686*a^4*b^4*c^4 - 24108*a^5*b^2*c^5 + 2401*a^6
*c^6)/((a^14*b^6 - 12*a^15*b^4*c + 48*a^16*b^2*c^2 - 64*a^17*c^3)*e^4)))/((
a^7*b^6 - 12*a^8*b^4*c + 48*a^9*b^2*c^2 - 64*a^10*c^3)*e^2))))/((a^3*b^2*c
 - 4*a^4*c^2)*e^8*x^7 + 7*(a^3*b^2*c - 4*a^4*c^2)*d*e^7*x^6 + (a^3*b^3 - 4*a
^4*b*c + 21*(a^3*b^2*c - 4*a^4*c^2)*d^2)*e^6*x^5 + 5*(7*(a^3*b^2*c - 4*a^4*
c^2)*d^3 + (a^3*b^3 - 4*a^4*b*c)*d)*e^5*x^4 + (a^4*b^2 - 4*a^5*c + 35*(a^3*
b^2*c - 4*a^4*c^2)*d^4 + 10*(a^3*b^3 - 4*a^4*b*c)*d^2)*e^4*x^3 + (21*(a^3*b
^2*c - 4*a^4*c^2)*d^5 + 10*(a^3*b^3 - 4*a^4*b*c)*d^3 + 3*(a^4*b^2 - 4*a^5*c
)*d)*e^3*x^2 + (7*(a^3*b^2*c - 4*a^4*c^2)*d^6 + 5*(a^3*b^3 - 4*a^4*b*c)*d^4
 + 3*(a^4*b^2 - 4*a^5*c)*d^2)*e^2*x + ((a^3*b^2*c - 4*a^4*c^2)*d^7 + (a^3*b
^3 - 4*a^4*b*c)*d^5 + (a^4*b^2 - 4*a^5*c)*d^3)*e)
```

giac [B] time = 0.55, size = 1987, normalized size = 4.87

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^4/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="giac")
```

```
[Out] -1/4*((5*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)
/c))^2*b^3*c*e^2 - 19*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c
)*e^2)*e^(-4)/c))^2*a*b*c^2*e^2 - 10*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 + s
qrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))*b^3*c*d*e + 38*(d*e^(-1) + sqrt(1/2)*sqrt(
-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))*a*b*c^2*d*e + 5*b^3*c*d^2 - 19*
a*b*c^2*d^2 + 5*b^4 - 24*a*b^2*c + 14*a^2*c^2)*log(d*e^(-1) + x + sqrt(1/2)
*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))/(2*(d*e^(-1) + sqrt(1/2)*
sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^3*c*e^4 - 6*(d*e^(-1) + sq
rt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^2*c*d*e^3 - 2*c*d^
3*e - b*d*e + (6*c*d^2*e^2 + b*e^2)*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 + sq
rt(b^2 - 4*a*c)*e^2)*e^(-4)/c))) + (5*(d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 +
sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^2*b^3*c*e^2 - 19*(d*e^(-1) - sqrt(1/2)*sq
rt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^2*a*b*c^2*e^2 - 10*(d*e^(-1)
 - sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))*b^3*c*d*e + 3
8*(d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))*a*
b*c^2*d*e + 5*b^3*c*d^2 - 19*a*b*c^2*d^2 + 5*b^4 - 24*a*b^2*c + 14*a^2*c^2)
*log(d*e^(-1) + x - sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/
c))/(2*(d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c
))^3*c*e^4 - 6*(d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*
e^(-4)/c))^2*c*d*e^3 - 2*c*d^3*e - b*d*e + (6*c*d^2*e^2 + b*e^2)*(d*e^(-1)
- sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))) + (5*(d*e^(-1
) + sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^2*b^3*c*e^2
- 19*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))
^2*a*b*c^2*e^2 - 10*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*
e^2)*e^(-4)/c))*b^3*c*d*e + 38*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^
2 - 4*a*c)*e^2)*e^(-4)/c))*a*b*c^2*d*e + 5*b^3*c*d^2 - 19*a*b*c^2*d^2 + 5*b
```

```
^4 - 24*a*b^2*c + 14*a^2*c^2)*log(d*e^(-1) + x + sqrt(1/2)*sqrt(-(b*e^2 - s
qrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))/(2*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 - sq
rt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^3*c*e^4 - 6*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*
e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^2*c*d*e^3 - 2*c*d^3*e - b*d*e + (6*
c*d^2*e^2 + b*e^2)*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e
^2)*e^(-4)/c))) + (5*(d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)
*e^2)*e^(-4)/c))^2*b^3*c*e^2 - 19*(d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 - sqrt
(b^2 - 4*a*c)*e^2)*e^(-4)/c))^2*a*b*c^2*e^2 - 10*(d*e^(-1) - sqrt(1/2)*sqrt
(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))*b^3*c*d*e + 38*(d*e^(-1) - sqr
t(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))*a*b*c^2*d*e + 5*b^3
*c*d^2 - 19*a*b*c^2*d^2 + 5*b^4 - 24*a*b^2*c + 14*a^2*c^2)*log(d*e^(-1) + x
 - sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))/(2*(d*e^(-1)
- sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^3*c*e^4 - 6*(d
*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^2*c*d*
e^3 - 2*c*d^3*e - b*d*e + (6*c*d^2*e^2 + b*e^2)*(d*e^(-1) - sqrt(1/2)*sqrt(
-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))))/(a^3*b^2 - 4*a^4*c) + 1/2*(b^
3*c*x^3*e^3 - 3*a*b*c^2*x^3*e^3 + 3*b^3*c*d*x^2*e^2 - 9*a*b*c^2*d*x^2*e^2 +
 3*b^3*c*d^2*x*e - 9*a*b*c^2*d^2*x*e + b^3*c*d^3 - 3*a*b*c^2*d^3 + b^4*x*e
- 4*a*b^2*c*x*e + 2*a^2*c^2*x*e + b^4*d - 4*a*b^2*c*d + 2*a^2*c^2*d)/((c*x^
4*e^4 + 4*c*d*x^3*e^3 + 6*c*d^2*x^2*e^2 + 4*c*d^3*x*e + c*d^4 + b*x^2*e^2 +
 2*b*d*x*e + b*d^2 + a)*(a^3*b^2*e - 4*a^4*c*e)) + 1/3*(6*b*x^2*e^2 + 12*b*
d*x*e + 6*b*d^2 - a)*e^(-1)/((x*e + d)^3*a^3)
```

maple [C] time = 0.04, size = 1518, normalized size = 3.72

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)^4/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x)
```

```
[Out] 3/2/a^2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^
4+2*b*d*e*x+b*d^2+a)*b*c^2*e^2/(4*a*c-b^2)*x^3-1/2/a^3/(c*e^4*x^4+4*c*d*e^3
*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)*b^3*c*e
^2/(4*a*c-b^2)*x^3+9/2/a^2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3
*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)*d*b*c^2*e/(4*a*c-b^2)*x^2-3/2/a^3/(
c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e
*x+b*d^2+a)*d*b^3*c*e/(4*a*c-b^2)*x^2+9/2/a^2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*
d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)/(4*a*c-b^2)*x*b*
c^2*d^2-3/2/a^3/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*
x^2+c*d^4+2*b*d*e*x+b*d^2+a)/(4*a*c-b^2)*x*b^3*c*d^2-1/a/(c*e^4*x^4+4*c*d*e
^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)/(4*a*
c-b^2)*x*c^2+2/a^2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e
^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)/(4*a*c-b^2)*x*b^2*c-1/2/a^3/(c*e^4*x^4+4*c*
d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)/(4
*a*c-b^2)*x*b^4+3/2/a^2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*
x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)*d^3/e/(4*a*c-b^2)*b*c^2-1/2/a^3/(c*e^4
*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*
d^2+a)*d^3/e/(4*a*c-b^2)*b^3*c-1/a/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2
+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)*d/e/(4*a*c-b^2)*c^2+2/a^2/(
c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e
*x+b*d^2+a)*d/e/(4*a*c-b^2)*b^2*c-1/2/a^3/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*
e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)*d/e/(4*a*c-b^2)*b^4+
1/4/a^3/(4*a*c-b^2)/e*sum((b*c*e^2*(19*a*c-5*b^2)*_R^2+2*d*e*b*c*(19*a*c-5*
b^2)*_R+19*a*b*c^2*d^2-5*b^3*c*d^2-14*a^2*c^2+24*a*b^2*c-5*b^4)/(2*_R^3*c*e
^3+6*_R^2*c*d*e^2+6*_R*c*d^2*e+2*c*d^3+_R*b*e+b*d)*ln(-_R+x),_R=RootOf(_Z^4
*c*e^4+4*_Z^3*c*d*e^3+c*d^4+b*d^2+(6*c*d^2*e^2+b*e^2)*_Z^2+(4*c*d^3*e+2*b*d
*e)*_Z+a))-1/3/a^2/e/(e*x+d)^3+2/a^3*b/e/(e*x+d)
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^4/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [B]   time = 8.72, size = 12239, normalized size = 30.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((d + e*x)^4*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2),x)
```

```
[Out] atan(((-(25*b^15 - 25*b^6*(-(4*a*c - b^2)^9)^(1/2) - 80640*a^7*b*c^7 + 6366
*a^2*b^11*c^2 - 35767*a^3*b^9*c^3 + 116928*a^4*b^7*c^4 - 219744*a^5*b^5*c^5
 + 215040*a^6*b^3*c^6 + 49*a^3*c^3*(-(4*a*c - b^2)^9)^(1/2) - 615*a*b^13*c
- 246*a^2*b^2*c^2*(-(4*a*c - b^2)^9)^(1/2) + 165*a*b^4*c*(-(4*a*c - b^2)^9)
^(1/2))/(32*(a^7*b^12*e^2 + 4096*a^13*c^6*e^2 - 24*a^8*b^10*c*e^2 + 240*a^9
*b^8*c^2*e^2 - 1280*a^10*b^6*c^3*e^2 + 3840*a^11*b^4*c^4*e^2 - 6144*a^12*b^
2*c^5*e^2)))^(1/2)*((-(25*b^15 - 25*b^6*(-(4*a*c - b^2)^9)^(1/2) - 80640*a^
7*b*c^7 + 6366*a^2*b^11*c^2 - 35767*a^3*b^9*c^3 + 116928*a^4*b^7*c^4 - 2197
44*a^5*b^5*c^5 + 215040*a^6*b^3*c^6 + 49*a^3*c^3*(-(4*a*c - b^2)^9)^(1/2) -
 615*a*b^13*c - 246*a^2*b^2*c^2*(-(4*a*c - b^2)^9)^(1/2) + 165*a*b^4*c*(-(4
*a*c - b^2)^9)^(1/2))/(32*(a^7*b^12*e^2 + 4096*a^13*c^6*e^2 - 24*a^8*b^10*c
*e^2 + 240*a^9*b^8*c^2*e^2 - 1280*a^10*b^6*c^3*e^2 + 3840*a^11*b^4*c^4*e^2
- 6144*a^12*b^2*c^5*e^2)))^(1/2)*((-(25*b^15 - 25*b^6*(-(4*a*c - b^2)^9)^(1
/2) - 80640*a^7*b*c^7 + 6366*a^2*b^11*c^2 - 35767*a^3*b^9*c^3 + 116928*a^4*
b^7*c^4 - 219744*a^5*b^5*c^5 + 215040*a^6*b^3*c^6 + 49*a^3*c^3*(-(4*a*c - b
^2)^9)^(1/2) - 615*a*b^13*c - 246*a^2*b^2*c^2*(-(4*a*c - b^2)^9)^(1/2) + 16
5*a*b^4*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^7*b^12*e^2 + 4096*a^13*c^6*e^2 -
 24*a^8*b^10*c*e^2 + 240*a^9*b^8*c^2*e^2 - 1280*a^10*b^6*c^3*e^2 + 3840*a^1
1*b^4*c^4*e^2 - 6144*a^12*b^2*c^5*e^2)))^(1/2)*(x*(1048576*a^21*b*c^8*e^14
+ 256*a^15*b^13*c^2*e^14 - 6144*a^16*b^11*c^3*e^14 + 61440*a^17*b^9*c^4*e^1
4 - 327680*a^18*b^7*c^5*e^14 + 983040*a^19*b^5*c^6*e^14 - 1572864*a^20*b^3*
c^7*e^14) + 1048576*a^21*b*c^8*d*e^13 + 256*a^15*b^13*c^2*d*e^13 - 6144*a^1
6*b^11*c^3*d*e^13 + 61440*a^17*b^9*c^4*d*e^13 - 327680*a^18*b^7*c^5*d*e^13
+ 983040*a^19*b^5*c^6*d*e^13 - 1572864*a^20*b^3*c^7*d*e^13) - 917504*a^19*c
^9*e^12 + 320*a^12*b^14*c^2*e^12 - 7936*a^13*b^12*c^3*e^12 + 82816*a^14*b^1
0*c^4*e^12 - 468480*a^15*b^8*c^5*e^12 + 1536000*a^16*b^6*c^6*e^12 - 2867200
*a^17*b^4*c^7*e^12 + 2719744*a^18*b^2*c^8*e^12) - x*(401408*a^16*c^10*e^12
- 400*a^9*b^14*c^3*e^12 + 9440*a^10*b^12*c^4*e^12 - 92816*a^11*b^10*c^5*e^1
2 + 488096*a^12*b^8*c^6*e^12 - 1458688*a^13*b^6*c^7*e^12 + 2401280*a^14*b^4
*c^8*e^12 - 1871872*a^15*b^2*c^9*e^12) - 401408*a^16*c^10*d*e^11 + 400*a^9*
b^14*c^3*d*e^11 - 9440*a^10*b^12*c^4*d*e^11 + 92816*a^11*b^10*c^5*d*e^11 -
488096*a^12*b^8*c^6*d*e^11 + 1458688*a^13*b^6*c^7*d*e^11 - 2401280*a^14*b^4
*c^8*d*e^11 + 1871872*a^15*b^2*c^9*d*e^11)*1i + (-(25*b^15 - 25*b^6*(-(4*a*
c - b^2)^9)^(1/2) - 80640*a^7*b*c^7 + 6366*a^2*b^11*c^2 - 35767*a^3*b^9*c^3
 + 116928*a^4*b^7*c^4 - 219744*a^5*b^5*c^5 + 215040*a^6*b^3*c^6 + 49*a^3*c^
3*(-(4*a*c - b^2)^9)^(1/2) - 615*a*b^13*c - 246*a^2*b^2*c^2*(-(4*a*c - b^2)
^9)^(1/2) + 165*a*b^4*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^7*b^12*e^2 + 4096*
a^13*c^6*e^2 - 24*a^8*b^10*c*e^2 + 240*a^9*b^8*c^2*e^2 - 1280*a^10*b^6*c^3*
e^2 + 3840*a^11*b^4*c^4*e^2 - 6144*a^12*b^2*c^5*e^2)))^(1/2)*((-(25*b^15 -
25*b^6*(-(4*a*c - b^2)^9)^(1/2) - 80640*a^7*b*c^7 + 6366*a^2*b^11*c^2 - 357
67*a^3*b^9*c^3 + 116928*a^4*b^7*c^4 - 219744*a^5*b^5*c^5 + 215040*a^6*b^3*c
^6 + 49*a^3*c^3*(-(4*a*c - b^2)^9)^(1/2) - 615*a*b^13*c - 246*a^2*b^2*c^2*(
-(4*a*c - b^2)^9)^(1/2) + 165*a*b^4*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^7*b^
12*e^2 + 4096*a^13*c^6*e^2 - 24*a^8*b^10*c*e^2 + 240*a^9*b^8*c^2*e^2 - 1280
*a^10*b^6*c^3*e^2 + 3840*a^11*b^4*c^4*e^2 - 6144*a^12*b^2*c^5*e^2)))^(1/2)*
((-(25*b^15 - 25*b^6*(-(4*a*c - b^2)^9)^(1/2) - 80640*a^7*b*c^7 + 6366*a^2*
b^11*c^2 - 35767*a^3*b^9*c^3 + 116928*a^4*b^7*c^4 - 219744*a^5*b^5*c^5 + 21
5040*a^6*b^3*c^6 + 49*a^3*c^3*(-(4*a*c - b^2)^9)^(1/2) - 615*a*b^13*c - 246
```

*a^2*b^2*c^2*(-(4*a*c - b^2)^9)^(1/2) + 165*a*b^4*c*(-(4*a*c - b^2)^9)^(1/2
))/(32*(a^7*b^12*e^2 + 4096*a^13*c^6*e^2 - 24*a^8*b^10*c*e^2 + 240*a^9*b^8*
c^2*e^2 - 1280*a^10*b^6*c^3*e^2 + 3840*a^11*b^4*c^4*e^2 - 6144*a^12*b^2*c^5
e^2)))^(1/2)(x*(1048576*a^21*b*c^8*e^14 + 256*a^15*b^13*c^2*e^14 - 6144*a
^16*b^11*c^3*e^14 + 61440*a^17*b^9*c^4*e^14 - 327680*a^18*b^7*c^5*e^14 + 98
3040*a^19*b^5*c^6*e^14 - 1572864*a^20*b^3*c^7*e^14) + 1048576*a^21*b*c^8*d*
e^13 + 256*a^15*b^13*c^2*d*e^13 - 6144*a^16*b^11*c^3*d*e^13 + 61440*a^17*b^
9*c^4*d*e^13 - 327680*a^18*b^7*c^5*d*e^13 + 983040*a^19*b^5*c^6*d*e^13 - 15
72864*a^20*b^3*c^7*d*e^13) + 917504*a^19*c^9*e^12 - 320*a^12*b^14*c^2*e^12
+ 7936*a^13*b^12*c^3*e^12 - 82816*a^14*b^10*c^4*e^12 + 468480*a^15*b^8*c^5*
e^12 - 1536000*a^16*b^6*c^6*e^12 + 2867200*a^17*b^4*c^7*e^12 - 2719744*a^18
*b^2*c^8*e^12) - x*(401408*a^16*c^10*e^12 - 400*a^9*b^14*c^3*e^12 + 9440*a^
10*b^12*c^4*e^12 - 92816*a^11*b^10*c^5*e^12 + 488096*a^12*b^8*c^6*e^12 - 14
58688*a^13*b^6*c^7*e^12 + 2401280*a^14*b^4*c^8*e^12 - 1871872*a^15*b^2*c^9*
e^12) - 401408*a^16*c^10*d*e^11 + 400*a^9*b^14*c^3*d*e^11 - 9440*a^10*b^12*
c^4*d*e^11 + 92816*a^11*b^10*c^5*d*e^11 - 488096*a^12*b^8*c^6*d*e^11 + 1458
688*a^13*b^6*c^7*d*e^11 - 2401280*a^14*b^4*c^8*d*e^11 + 1871872*a^15*b^2*c^
9*d*e^11)*1i)/((-(25*b^15 - 25*b^6*(-(4*a*c - b^2)^9)^(1/2) - 80640*a^7*b*c
^7 + 6366*a^2*b^11*c^2 - 35767*a^3*b^9*c^3 + 116928*a^4*b^7*c^4 - 219744*a^
5*b^5*c^5 + 215040*a^6*b^3*c^6 + 49*a^3*c^3*(-(4*a*c - b^2)^9)^(1/2) - 615*
a*b^13*c - 246*a^2*b^2*c^2*(-(4*a*c - b^2)^9)^(1/2) + 165*a*b^4*c*(-(4*a*c
- b^2)^9)^(1/2))/(32*(a^7*b^12*e^2 + 4096*a^13*c^6*e^2 - 24*a^8*b^10*c*e^2
+ 240*a^9*b^8*c^2*e^2 - 1280*a^10*b^6*c^3*e^2 + 3840*a^11*b^4*c^4*e^2 - 614
4*a^12*b^2*c^5*e^2)))^(1/2)*((-(25*b^15 - 25*b^6*(-(4*a*c - b^2)^9)^(1/2) -
80640*a^7*b*c^7 + 6366*a^2*b^11*c^2 - 35767*a^3*b^9*c^3 + 116928*a^4*b^7*c
^4 - 219744*a^5*b^5*c^5 + 215040*a^6*b^3*c^6 + 49*a^3*c^3*(-(4*a*c - b^2)^9
)^(1/2) - 615*a*b^13*c - 246*a^2*b^2*c^2*(-(4*a*c - b^2)^9)^(1/2) + 165*a*b
^4*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^7*b^12*e^2 + 4096*a^13*c^6*e^2 - 24*a
^8*b^10*c*e^2 + 240*a^9*b^8*c^2*e^2 - 1280*a^10*b^6*c^3*e^2 + 3840*a^11*b^4
*c^4*e^2 - 6144*a^12*b^2*c^5*e^2)))^(1/2)*((-(25*b^15 - 25*b^6*(-(4*a*c - b
^2)^9)^(1/2) - 80640*a^7*b*c^7 + 6366*a^2*b^11*c^2 - 35767*a^3*b^9*c^3 + 11
6928*a^4*b^7*c^4 - 219744*a^5*b^5*c^5 + 215040*a^6*b^3*c^6 + 49*a^3*c^3*(-(
4*a*c - b^2)^9)^(1/2) - 615*a*b^13*c - 246*a^2*b^2*c^2*(-(4*a*c - b^2)^9)^(
1/2) + 165*a*b^4*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^7*b^12*e^2 + 4096*a^13*
c^6*e^2 - 24*a^8*b^10*c*e^2 + 240*a^9*b^8*c^2*e^2 - 1280*a^10*b^6*c^3*e^2 +
3840*a^11*b^4*c^4*e^2 - 6144*a^12*b^2*c^5*e^2)))^(1/2)*(x*(1048576*a^21*b*
c^8*e^14 + 256*a^15*b^13*c^2*e^14 - 6144*a^16*b^11*c^3*e^14 + 61440*a^17*b^
9*c^4*e^14 - 327680*a^18*b^7*c^5*e^14 + 983040*a^19*b^5*c^6*e^14 - 1572864*
a^20*b^3*c^7*e^14) + 1048576*a^21*b*c^8*d*e^13 + 256*a^15*b^13*c^2*d*e^13 -
6144*a^16*b^11*c^3*d*e^13 + 61440*a^17*b^9*c^4*d*e^13 - 327680*a^18*b^7*c^
5*d*e^13 + 983040*a^19*b^5*c^6*d*e^13 - 1572864*a^20*b^3*c^7*d*e^13) - 9175
04*a^19*c^9*e^12 + 320*a^12*b^14*c^2*e^12 - 7936*a^13*b^12*c^3*e^12 + 82816
*a^14*b^10*c^4*e^12 - 468480*a^15*b^8*c^5*e^12 + 1536000*a^16*b^6*c^6*e^12
- 2867200*a^17*b^4*c^7*e^12 + 2719744*a^18*b^2*c^8*e^12) - x*(401408*a^16*c
^10*e^12 - 400*a^9*b^14*c^3*e^12 + 9440*a^10*b^12*c^4*e^12 - 92816*a^11*b^1
0*c^5*e^12 + 488096*a^12*b^8*c^6*e^12 - 1458688*a^13*b^6*c^7*e^12 + 2401280
*a^14*b^4*c^8*e^12 - 1871872*a^15*b^2*c^9*e^12) - 401408*a^16*c^10*d*e^11 +
400*a^9*b^14*c^3*d*e^11 - 9440*a^10*b^12*c^4*d*e^11 + 92816*a^11*b^10*c^5*
d*e^11 - 488096*a^12*b^8*c^6*d*e^11 + 1458688*a^13*b^6*c^7*d*e^11 - 2401280
*a^14*b^4*c^8*d*e^11 + 1871872*a^15*b^2*c^9*d*e^11) - (-(25*b^15 - 25*b^6*(
-(4*a*c - b^2)^9)^(1/2) - 80640*a^7*b*c^7 + 6366*a^2*b^11*c^2 - 35767*a^3*b
^9*c^3 + 116928*a^4*b^7*c^4 - 219744*a^5*b^5*c^5 + 215040*a^6*b^3*c^6 + 49*
a^3*c^3*(-(4*a*c - b^2)^9)^(1/2) - 615*a*b^13*c - 246*a^2*b^2*c^2*(-(4*a*c
- b^2)^9)^(1/2) + 165*a*b^4*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^7*b^12*e^2 +
4096*a^13*c^6*e^2 - 24*a^8*b^10*c*e^2 + 240*a^9*b^8*c^2*e^2 - 1280*a^10*b^
6*c^3*e^2 + 3840*a^11*b^4*c^4*e^2 - 6144*a^12*b^2*c^5*e^2)))^(1/2)*((-(25*b
^15 - 25*b^6*(-(4*a*c - b^2)^9)^(1/2) - 80640*a^7*b*c^7 + 6366*a^2*b^11*c^2
- 35767*a^3*b^9*c^3 + 116928*a^4*b^7*c^4 - 219744*a^5*b^5*c^5 + 215040*a^6
*b^3*c^6 + 49*a^3*c^3*(-(4*a*c - b^2)^9)^(1/2) - 615*a*b^13*c - 246*a^2*b^2

```
*c^2*(-(4*a*c - b^2)^9)^(1/2) + 165*a*b^4*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(
a^7*b^12*e^2 + 4096*a^13*c^6*e^2 - 24*a^8*b^10*c*e^2 + 240*a^9*b^8*c^2*e^2
 - 1280*a^10*b^6*c^3*e^2 + 3840*a^11*b^4*c^4*e^2 - 6144*a^12*b^2*c^5*e^2)))^
(1/2)*((-(25*b^15 - 25*b^6*(-(4*a*c - b^2)^9)^(1/2) - 80640*a^7*b*c^7 + 636
6*a^2*b^11*c^2 - 35767*a^3*b^9*c^3 + 116928*a^4*b^7*c^4 - 219744*a^5*b^5*c^
5 + 215040*a^6*b^3*c^6 + 49*a^3*c^3*(-(4*a*c - b^2)^9)^(1/2) - 615*a*b^13*c
 - 246*a^2*b^2*c^2*(-(4*a*c - b^2)^9)^(1/2) + 165*a*b^4*c*(-(4*a*c - b^2)^9
)^(1/2))/(32*(a^7*b^12*e^2 + 4096*a^13*c^6*e^2 - 24*a^8*b^10*c*e^2 + 240*a^
9*b^8*c^2*e^2 - 1280*a^10*b^6*c^3*e^2 + 3840*a^11*b^4*c^4*e^2 - 6144*a^12*b
^2*c^5*e^2)))^(1/2)*(x*(1048576*a^21*b*c^8*e^14 + 256*a^15*b^13*c^2*e^14 -
6144*a^16*b^11*c^3*e^14 + 61440*a^17*b^9*c^4*e^14 - 327680*a^18*b^7*c^5*e^1
4 + 983040*a^19*b^5*c^6*e^14 - 1572864*a^20*b^3*c^7*e^14) + 1048576*a^21*b*
c^8*d*e^13 + 256*a^15*b^13*c^2*d*e^13 - 6144*a^16*b^11*c^3*d*e^13 + 61440*a
^17*b^9*c^4*d*e^13 - 327680*a^18*b^7*c^5*d*e^13 + 983040*a^19*b^5*c^6*d*e^1
3 - 1572864*a^20*b^3*c^7*d*e^13) + 917504*a^19*c^9*e^12 - 320*a^12*b^14*c^2
*e^12 + 7936*a^13*b^12*c^3*e^12 - 82816*a^14*b^10*c^4*e^12 + 468480*a^15*b^
8*c^5*e^12 - 1536000*a^16*b^6*c^6*e^12 + 2867200*a^17*b^4*c^7*e^12 - 271974
4*a^18*b^2*c^8*e^12) - x*(401408*a^16*c^10*e^12 - 400*a^9*b^14*c^3*e^12 + 9
440*a^10*b^12*c^4*e^12 - 92816*a^11*b^10*c^5*e^12 + 488096*a^12*b^8*c^6*e^1
2 - 1458688*a^13*b^6*c^7*e^12 + 2401280*a^14*b^4*c^8*e^12 - 1871872*a^15*b^
2*c^9*e^12) - 401408*a^16*c^10*d*e^11 + 400*a^9*b^14*c^3*d*e^11 - 9440*a^10
*b^12*c^4*d*e^11 + 92816*a^11*b^10*c^5*d*e^11 - 488096*a^12*b^8*c^6*d*e^11
 + 1458688*a^13*b^6*c^7*d*e^11 - 2401280*a^14*b^4*c^8*d*e^11 + 1871872*a^15*
b^2*c^9*d*e^11) + 476672*a^13*b*c^10*e^10 + 1800*a^9*b^9*c^6*e^10 - 29080*a
^10*b^7*c^7*e^10 + 176032*a^11*b^5*c^8*e^10 - 473216*a^12*b^3*c^9*e^10))*(-
(25*b^15 - 25*b^6*(-(4*a*c - b^2)^9)^(1/2) - 80640*a^7*b*c^7 + 6366*a^2*b^1
1*c^2 - 35767*a^3*b^9*c^3 + 116928*a^4*b^7*c^4 - 219744*a^5*b^5*c^5 + 21504
0*a^6*b^3*c^6 + 49*a^3*c^3*(-(4*a*c - b^2)^9)^(1/2) - 615*a*b^13*c - 246*a^
2*b^2*c^2*(-(4*a*c - b^2)^9)^(1/2) + 165*a*b^4*c*(-(4*a*c - b^2)^9)^(1/2))/
(32*(a^7*b^12*e^2 + 4096*a^13*c^6*e^2 - 24*a^8*b^10*c*e^2 + 240*a^9*b^8*c^2
*e^2 - 1280*a^10*b^6*c^3*e^2 + 3840*a^11*b^4*c^4*e^2 - 6144*a^12*b^2*c^5*e^
2)))^(1/2)*2i - ((x^4*(15*b^4*e^3 + 14*a^2*c^2*e^3 + 225*b^3*c*d^2*e^3 - 62
*a*b^2*c*e^3 - 855*a*b*c^2*d^2*e^3))/(6*a*(4*a^3*c - a^2*b^2)) + (3*x^5*(5*
b^3*c*d*e^4 - 19*a*b*c^2*d*e^4))/(a*(4*a^3*c - a^2*b^2)) + (2*x^3*(15*b^4*d
*e^2 + 14*a^2*c^2*d*e^2 + 75*b^3*c*d^3*e^2 - 62*a*b^2*c*d*e^2 - 285*a*b*c^2
*d^3*e^2))/(3*a*(4*a^3*c - a^2*b^2)) + (x*(30*b^4*d^3 + 45*b^3*c*d^5 + 28*a
^2*c^2*d^3 + 10*a*b^3*d - 40*a^2*b*c*d - 124*a*b^2*c*d^3 - 171*a*b*c^2*d^5)
)/(3*a*(4*a^3*c - a^2*b^2)) + (x^6*(5*b^3*c*e^5 - 19*a*b*c^2*e^5))/(2*a*(4*
a^3*c - a^2*b^2)) + (x^2*(90*b^4*d^2*e + 10*a*b^3*e + 84*a^2*c^2*d^2*e - 40
*a^2*b*c*e + 225*b^3*c*d^4*e - 372*a*b^2*c*d^2*e - 855*a*b*c^2*d^4*e))/(6*a
*(4*a^3*c - a^2*b^2)) + (8*a^3*c - 2*a^2*b^2 + 15*b^4*d^4 + 10*a*b^3*d^2 +
15*b^3*c*d^6 + 14*a^2*c^2*d^4 - 40*a^2*b*c*d^2 - 62*a*b^2*c*d^4 - 57*a*b*c^
2*d^6)/(6*a*e*(4*a^3*c - a^2*b^2)))/(x^2*(10*b*d^3*e^2 + 21*c*d^5*e^2 + 3*a
*d*e^2) + x^5*(b*e^5 + 21*c*d^2*e^5) + a*d^3 + b*d^5 + c*d^7 + x^3*(a*e^3 +
 10*b*d^2*e^3 + 35*c*d^4*e^3) + x^4*(35*c*d^3*e^4 + 5*b*d*e^4) + x*(3*a*d^2
*e + 5*b*d^4*e + 7*c*d^6*e) + c*e^7*x^7 + 7*c*d*e^6*x^6) + atan(((-(25*b^15
 + 25*b^6*(-(4*a*c - b^2)^9)^(1/2) - 80640*a^7*b*c^7 + 6366*a^2*b^11*c^2 -
35767*a^3*b^9*c^3 + 116928*a^4*b^7*c^4 - 219744*a^5*b^5*c^5 + 215040*a^6*b^
3*c^6 - 49*a^3*c^3*(-(4*a*c - b^2)^9)^(1/2) - 615*a*b^13*c + 246*a^2*b^2*c^
2*(-(4*a*c - b^2)^9)^(1/2) - 165*a*b^4*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^7
*b^12*e^2 + 4096*a^13*c^6*e^2 - 24*a^8*b^10*c*e^2 + 240*a^9*b^8*c^2*e^2 - 1
280*a^10*b^6*c^3*e^2 + 3840*a^11*b^4*c^4*e^2 - 6144*a^12*b^2*c^5*e^2)))^(1/
2)*((-(25*b^15 + 25*b^6*(-(4*a*c - b^2)^9)^(1/2) - 80640*a^7*b*c^7 + 6366*a
^2*b^11*c^2 - 35767*a^3*b^9*c^3 + 116928*a^4*b^7*c^4 - 219744*a^5*b^5*c^5 +
 215040*a^6*b^3*c^6 - 49*a^3*c^3*(-(4*a*c - b^2)^9)^(1/2) - 615*a*b^13*c +
246*a^2*b^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 165*a*b^4*c*(-(4*a*c - b^2)^9)^(
1/2))/(32*(a^7*b^12*e^2 + 4096*a^13*c^6*e^2 - 24*a^8*b^10*c*e^2 + 240*a^9*b
^8*c^2*e^2 - 1280*a^10*b^6*c^3*e^2 + 3840*a^11*b^4*c^4*e^2 - 6144*a^12*b^2*
c^5*e^2)))^(1/2)*((-(25*b^15 + 25*b^6*(-(4*a*c - b^2)^9)^(1/2) - 80640*a^7*
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b*c^7 + 6366*a^2*b^11*c^2 - 35767*a^3*b^9*c^3 + 116928*a^4*b^7*c^4 - 219744
*a^5*b^5*c^5 + 215040*a^6*b^3*c^6 - 49*a^3*c^3*(-(4*a*c - b^2)^9)^(1/2) - 6
15*a*b^13*c + 246*a^2*b^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 165*a*b^4*c*(-(4*a
*c - b^2)^9)^(1/2))/(32*(a^7*b^12*e^2 + 4096*a^13*c^6*e^2 - 24*a^8*b^10*c*e
^2 + 240*a^9*b^8*c^2*e^2 - 1280*a^10*b^6*c^3*e^2 + 3840*a^11*b^4*c^4*e^2 -
6144*a^12*b^2*c^5*e^2)))^(1/2)*(x*(1048576*a^21*b*c^8*e^14 + 256*a^15*b^13*
c^2*e^14 - 6144*a^16*b^11*c^3*e^14 + 61440*a^17*b^9*c^4*e^14 - 327680*a^18*
b^7*c^5*e^14 + 983040*a^19*b^5*c^6*e^14 - 1572864*a^20*b^3*c^7*e^14) + 1048
576*a^21*b*c^8*d*e^13 + 256*a^15*b^13*c^2*d*e^13 - 6144*a^16*b^11*c^3*d*e^1
3 + 61440*a^17*b^9*c^4*d*e^13 - 327680*a^18*b^7*c^5*d*e^13 + 983040*a^19*b^
5*c^6*d*e^13 - 1572864*a^20*b^3*c^7*d*e^13) - 917504*a^19*c^9*e^12 + 320*a^
12*b^14*c^2*e^12 - 7936*a^13*b^12*c^3*e^12 + 82816*a^14*b^10*c^4*e^12 - 468
480*a^15*b^8*c^5*e^12 + 1536000*a^16*b^6*c^6*e^12 - 2867200*a^17*b^4*c^7*e^
12 + 2719744*a^18*b^2*c^8*e^12) - x*(401408*a^16*c^10*e^12 - 400*a^9*b^14*c
^3*e^12 + 9440*a^10*b^12*c^4*e^12 - 92816*a^11*b^10*c^5*e^12 + 488096*a^12*
b^8*c^6*e^12 - 1458688*a^13*b^6*c^7*e^12 + 2401280*a^14*b^4*c^8*e^12 - 1871
872*a^15*b^2*c^9*e^12) - 401408*a^16*c^10*d*e^11 + 400*a^9*b^14*c^3*d*e^11
- 9440*a^10*b^12*c^4*d*e^11 + 92816*a^11*b^10*c^5*d*e^11 - 488096*a^12*b^8*
c^6*d*e^11 + 1458688*a^13*b^6*c^7*d*e^11 - 2401280*a^14*b^4*c^8*d*e^11 + 18
71872*a^15*b^2*c^9*d*e^11)*1i + (-(25*b^15 + 25*b^6*(-(4*a*c - b^2)^9)^(1/2
) - 80640*a^7*b*c^7 + 6366*a^2*b^11*c^2 - 35767*a^3*b^9*c^3 + 116928*a^4*b^
7*c^4 - 219744*a^5*b^5*c^5 + 215040*a^6*b^3*c^6 - 49*a^3*c^3*(-(4*a*c - b^2
)^9)^(1/2) - 615*a*b^13*c + 246*a^2*b^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 165*
a*b^4*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^7*b^12*e^2 + 4096*a^13*c^6*e^2 - 2
4*a^8*b^10*c*e^2 + 240*a^9*b^8*c^2*e^2 - 1280*a^10*b^6*c^3*e^2 + 3840*a^11*
b^4*c^4*e^2 - 6144*a^12*b^2*c^5*e^2)))^(1/2)*((-(25*b^15 + 25*b^6*(-(4*a*c
- b^2)^9)^(1/2) - 80640*a^7*b*c^7 + 6366*a^2*b^11*c^2 - 35767*a^3*b^9*c^3 +
 116928*a^4*b^7*c^4 - 219744*a^5*b^5*c^5 + 215040*a^6*b^3*c^6 - 49*a^3*c^3*
(-(4*a*c - b^2)^9)^(1/2) - 615*a*b^13*c + 246*a^2*b^2*c^2*(-(4*a*c - b^2)^9
)^(1/2) - 165*a*b^4*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^7*b^12*e^2 + 4096*a^
13*c^6*e^2 - 24*a^8*b^10*c*e^2 + 240*a^9*b^8*c^2*e^2 - 1280*a^10*b^6*c^3*e^
2 + 3840*a^11*b^4*c^4*e^2 - 6144*a^12*b^2*c^5*e^2)))^(1/2)*((-(25*b^15 + 25
*b^6*(-(4*a*c - b^2)^9)^(1/2) - 80640*a^7*b*c^7 + 6366*a^2*b^11*c^2 - 35767
*a^3*b^9*c^3 + 116928*a^4*b^7*c^4 - 219744*a^5*b^5*c^5 + 215040*a^6*b^3*c^6
 - 49*a^3*c^3*(-(4*a*c - b^2)^9)^(1/2) - 615*a*b^13*c + 246*a^2*b^2*c^2*(-(
4*a*c - b^2)^9)^(1/2) - 165*a*b^4*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^7*b^12
*e^2 + 4096*a^13*c^6*e^2 - 24*a^8*b^10*c*e^2 + 240*a^9*b^8*c^2*e^2 - 1280*a
^10*b^6*c^3*e^2 + 3840*a^11*b^4*c^4*e^2 - 6144*a^12*b^2*c^5*e^2)))^(1/2)*(x
*(1048576*a^21*b*c^8*e^14 + 256*a^15*b^13*c^2*e^14 - 6144*a^16*b^11*c^3*e^1
4 + 61440*a^17*b^9*c^4*e^14 - 327680*a^18*b^7*c^5*e^14 + 983040*a^19*b^5*c^
6*e^14 - 1572864*a^20*b^3*c^7*e^14) + 1048576*a^21*b*c^8*d*e^13 + 256*a^15*
b^13*c^2*d*e^13 - 6144*a^16*b^11*c^3*d*e^13 + 61440*a^17*b^9*c^4*d*e^13 - 3
27680*a^18*b^7*c^5*d*e^13 + 983040*a^19*b^5*c^6*d*e^13 - 1572864*a^20*b^3*c
^7*d*e^13) + 917504*a^19*c^9*e^12 - 320*a^12*b^14*c^2*e^12 + 7936*a^13*b^12
*c^3*e^12 - 82816*a^14*b^10*c^4*e^12 + 468480*a^15*b^8*c^5*e^12 - 1536000*a
^16*b^6*c^6*e^12 + 2867200*a^17*b^4*c^7*e^12 - 2719744*a^18*b^2*c^8*e^12) -
 x*(401408*a^16*c^10*e^12 - 400*a^9*b^14*c^3*e^12 + 9440*a^10*b^12*c^4*e^12
 - 92816*a^11*b^10*c^5*e^12 + 488096*a^12*b^8*c^6*e^12 - 1458688*a^13*b^6*c
^7*e^12 + 2401280*a^14*b^4*c^8*e^12 - 1871872*a^15*b^2*c^9*e^12) - 401408*a
^16*c^10*d*e^11 + 400*a^9*b^14*c^3*d*e^11 - 9440*a^10*b^12*c^4*d*e^11 + 928
16*a^11*b^10*c^5*d*e^11 - 488096*a^12*b^8*c^6*d*e^11 + 1458688*a^13*b^6*c^7
*d*e^11 - 2401280*a^14*b^4*c^8*d*e^11 + 1871872*a^15*b^2*c^9*d*e^11)*1i)/((
-(25*b^15 + 25*b^6*(-(4*a*c - b^2)^9)^(1/2) - 80640*a^7*b*c^7 + 6366*a^2*b^
11*c^2 - 35767*a^3*b^9*c^3 + 116928*a^4*b^7*c^4 - 219744*a^5*b^5*c^5 + 2150
40*a^6*b^3*c^6 - 49*a^3*c^3*(-(4*a*c - b^2)^9)^(1/2) - 615*a*b^13*c + 246*a
^2*b^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 165*a*b^4*c*(-(4*a*c - b^2)^9)^(1/2))
/(32*(a^7*b^12*e^2 + 4096*a^13*c^6*e^2 - 24*a^8*b^10*c*e^2 + 240*a^9*b^8*c^
2*e^2 - 1280*a^10*b^6*c^3*e^2 + 3840*a^11*b^4*c^4*e^2 - 6144*a^12*b^2*c^5*e
^2)))^(1/2)*((-(25*b^15 + 25*b^6*(-(4*a*c - b^2)^9)^(1/2) - 80640*a^7*b*c^7
```

+ 6366*a^2*b^11*c^2 - 35767*a^3*b^9*c^3 + 116928*a^4*b^7*c^4 - 219744*a^5*b^5*c^5 + 215040*a^6*b^3*c^6 - 49*a^3*c^3*(-(4*a*c - b^2)^9)^(1/2) - 615*a*b^13*c + 246*a^2*b^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 165*a*b^4*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^7*b^12*e^2 + 4096*a^13*c^6*e^2 - 24*a^8*b^10*c*e^2 + 240*a^9*b^8*c^2*e^2 - 1280*a^10*b^6*c^3*e^2 + 3840*a^11*b^4*c^4*e^2 - 6144*a^12*b^2*c^5*e^2)))^(1/2)*((-(25*b^15 + 25*b^6*(-(4*a*c - b^2)^9)^(1/2) - 80640*a^7*b*c^7 + 6366*a^2*b^11*c^2 - 35767*a^3*b^9*c^3 + 116928*a^4*b^7*c^4 - 219744*a^5*b^5*c^5 + 215040*a^6*b^3*c^6 - 49*a^3*c^3*(-(4*a*c - b^2)^9)^(1/2) - 615*a*b^13*c + 246*a^2*b^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 165*a*b^4*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^7*b^12*e^2 + 4096*a^13*c^6*e^2 - 24*a^8*b^10*c*e^2 + 240*a^9*b^8*c^2*e^2 - 1280*a^10*b^6*c^3*e^2 + 3840*a^11*b^4*c^4*e^2 - 6144*a^12*b^2*c^5*e^2)))^(1/2)*(x*(1048576*a^21*b*c^8*e^14 + 256*a^15*b^13*c^2*e^14 - 6144*a^16*b^11*c^3*e^14 + 61440*a^17*b^9*c^4*e^14 - 327680*a^18*b^7*c^5*e^14 + 983040*a^19*b^5*c^6*e^14 - 1572864*a^20*b^3*c^7*e^14) + 1048576*a^21*b*c^8*d*e^13 + 256*a^15*b^13*c^2*d*e^13 - 6144*a^16*b^11*c^3*d*e^13 + 61440*a^17*b^9*c^4*d*e^13 - 327680*a^18*b^7*c^5*d*e^13 + 983040*a^19*b^5*c^6*d*e^13 - 1572864*a^20*b^3*c^7*d*e^13) - 917504*a^19*c^9*e^12 + 320*a^12*b^14*c^2*e^12 - 7936*a^13*b^12*c^3*e^12 + 82816*a^14*b^10*c^4*e^12 - 468480*a^15*b^8*c^5*e^12 + 1536000*a^16*b^6*c^6*e^12 - 2867200*a^17*b^4*c^7*e^12 + 2719744*a^18*b^2*c^8*e^12) - x*(401408*a^16*c^10*e^12 - 400*a^9*b^14*c^3*e^12 + 9440*a^10*b^12*c^4*e^12 - 92816*a^11*b^10*c^5*e^12 + 488096*a^12*b^8*c^6*e^12 - 1458688*a^13*b^6*c^7*e^12 + 2401280*a^14*b^4*c^8*e^12 - 1871872*a^15*b^2*c^9*e^12) - 401408*a^16*c^10*d*e^11 + 400*a^9*b^14*c^3*d*e^11 - 9440*a^10*b^12*c^4*d*e^11 + 92816*a^11*b^10*c^5*d*e^11 - 488096*a^12*b^8*c^6*d*e^11 + 1458688*a^13*b^6*c^7*d*e^11 - 2401280*a^14*b^4*c^8*d*e^11 + 1871872*a^15*b^2*c^9*d*e^11) - (-(25*b^15 + 25*b^6*(-(4*a*c - b^2)^9)^(1/2) - 80640*a^7*b*c^7 + 6366*a^2*b^11*c^2 - 35767*a^3*b^9*c^3 + 116928*a^4*b^7*c^4 - 219744*a^5*b^5*c^5 + 215040*a^6*b^3*c^6 - 49*a^3*c^3*(-(4*a*c - b^2)^9)^(1/2) - 615*a*b^13*c + 246*a^2*b^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 165*a*b^4*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^7*b^12*e^2 + 4096*a^13*c^6*e^2 - 24*a^8*b^10*c*e^2 + 240*a^9*b^8*c^2*e^2 - 1280*a^10*b^6*c^3*e^2 + 3840*a^11*b^4*c^4*e^2 - 6144*a^12*b^2*c^5*e^2)))^(1/2)*((-(25*b^15 + 25*b^6*(-(4*a*c - b^2)^9)^(1/2) - 80640*a^7*b*c^7 + 6366*a^2*b^11*c^2 - 35767*a^3*b^9*c^3 + 116928*a^4*b^7*c^4 - 219744*a^5*b^5*c^5 + 215040*a^6*b^3*c^6 - 49*a^3*c^3*(-(4*a*c - b^2)^9)^(1/2) - 615*a*b^13*c + 246*a^2*b^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 165*a*b^4*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^7*b^12*e^2 + 4096*a^13*c^6*e^2 - 24*a^8*b^10*c*e^2 + 240*a^9*b^8*c^2*e^2 - 1280*a^10*b^6*c^3*e^2 + 3840*a^11*b^4*c^4*e^2 - 6144*a^12*b^2*c^5*e^2)))^(1/2)*((-(25*b^15 + 25*b^6*(-(4*a*c - b^2)^9)^(1/2) - 80640*a^7*b*c^7 + 6366*a^2*b^11*c^2 - 35767*a^3*b^9*c^3 + 116928*a^4*b^7*c^4 - 219744*a^5*b^5*c^5 + 215040*a^6*b^3*c^6 - 49*a^3*c^3*(-(4*a*c - b^2)^9)^(1/2) - 615*a*b^13*c + 246*a^2*b^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 165*a*b^4*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^7*b^12*e^2 + 4096*a^13*c^6*e^2 - 24*a^8*b^10*c*e^2 + 240*a^9*b^8*c^2*e^2 - 1280*a^10*b^6*c^3*e^2 + 3840*a^11*b^4*c^4*e^2 - 6144*a^12*b^2*c^5*e^2)))^(1/2)*(x*(1048576*a^21*b*c^8*e^14 + 256*a^15*b^13*c^2*e^14 - 6144*a^16*b^11*c^3*e^14 + 61440*a^17*b^9*c^4*e^14 - 327680*a^18*b^7*c^5*e^14 + 983040*a^19*b^5*c^6*e^14 - 1572864*a^20*b^3*c^7*e^14) + 1048576*a^21*b*c^8*d*e^13 + 256*a^15*b^13*c^2*d*e^13 - 6144*a^16*b^11*c^3*d*e^13 + 61440*a^17*b^9*c^4*d*e^13 - 327680*a^18*b^7*c^5*d*e^13 + 983040*a^19*b^5*c^6*d*e^13 - 1572864*a^20*b^3*c^7*d*e^13) + 917504*a^19*c^9*e^12 - 320*a^12*b^14*c^2*e^12 + 7936*a^13*b^12*c^3*e^12 - 82816*a^14*b^10*c^4*e^12 + 468480*a^15*b^8*c^5*e^12 - 1536000*a^16*b^6*c^6*e^12 + 2867200*a^17*b^4*c^7*e^12 - 2719744*a^18*b^2*c^8*e^12) - x*(401408*a^16*c^10*e^12 - 400*a^9*b^14*c^3*e^12 + 9440*a^10*b^12*c^4*e^12 - 92816*a^11*b^10*c^5*e^12 + 488096*a^12*b^8*c^6*e^12 - 1458688*a^13*b^6*c^7*e^12 + 2401280*a^14*b^4*c^8*e^12 - 1871872*a^15*b^2*c^9*e^12) - 401408*a^16*c^10*d*e^11 + 400*a^9*b^14*c^3*d*e^11 - 9440*a^10*b^12*c^4*d*e^11 + 92816*a^11*b^10*c^5*d*e^11 - 488096*a^12*b^8*c^6*d*e^11 + 1458688*a^13*b^6*c^7*d*e^11 - 2401280*a^14*b^4*c^8*d*e^11 + 1871872*a^15*b^2*c^9*d*e^11) + 476672*a^13*b*c^10*e^10 + 1800*a^9*b^9*c^6*e^10 - 29080*a^10*b^7*c^7*e^10

```
 + 176032*a^11*b^5*c^8*e^10 - 473216*a^12*b^3*c^9*e^10))*(-(25*b^15 + 25*b^
6*(-(4*a*c - b^2)^9)^(1/2) - 80640*a^7*b*c^7 + 6366*a^2*b^11*c^2 - 35767*a^
3*b^9*c^3 + 116928*a^4*b^7*c^4 - 219744*a^5*b^5*c^5 + 215040*a^6*b^3*c^6 -
49*a^3*c^3*(-(4*a*c - b^2)^9)^(1/2) - 615*a*b^13*c + 246*a^2*b^2*c^2*(-(4*a
*c - b^2)^9)^(1/2) - 165*a*b^4*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^7*b^12*e^
2 + 4096*a^13*c^6*e^2 - 24*a^8*b^10*c*e^2 + 240*a^9*b^8*c^2*e^2 - 1280*a^10
*b^6*c^3*e^2 + 3840*a^11*b^4*c^4*e^2 - 6144*a^12*b^2*c^5*e^2)))^(1/2)*2i
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)**4/(a+b*(e*x+d)**2+c*(e*x+d)**4)**2,x)
```

```
[Out] Timed out
```

3.517 $\int \frac{(d+ex)^4}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^3}\,dx$

Optimal. Leaf size=341

$$\frac{(d+ex)\left(2a+b(d+ex)^2\right)}{4e\left(b^2-4ac\right)\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}-\frac{(d+ex)\left(-4ac+7b^2+12bc(d+ex)^2\right)}{8e\left(b^2-4ac\right)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)}+\frac{3\sqrt{c}\left(-2b\sqrt{b^2-4ac}+\right.}{4\sqrt{2}\,e\left(b^2-4\right.}$$

Rubi [A] time = 0.95, antiderivative size = 341, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1142, 1120, 1178, 1166, 205}

$$\frac{(d+ex)\left(2a+b(d+ex)^2\right)}{4e\left(b^2-4ac\right)\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}-\frac{(d+ex)\left(-4ac+7b^2+12bc(d+ex)^2\right)}{8e\left(b^2-4ac\right)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)}+\frac{3\sqrt{c}\left(-2b\sqrt{b^2-4ac}+4ac+3b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{4\sqrt{2}e\left(b^2-4ac\right)^{5/2}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{3\sqrt{c}\left(2b\sqrt{b^2-4ac}+4ac+3b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{4\sqrt{2}e\left(b^2-4ac\right)^{5/2}\sqrt{\sqrt{b^2-4ac}+b}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^4/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3,x]
```

```
[Out] ((d + e*x)*(2*a + b*(d + e*x)^2))/(4*(b^2 - 4*a*c)*e*(a + b*(d + e*x)^2 + c
*(d + e*x)^4)^2) - ((d + e*x)*(7*b^2 - 4*a*c + 12*b*c*(d + e*x)^2))/(8*(b^2
 - 4*a*c)^2*e*(a + b*(d + e*x)^2 + c*(d + e*x)^4)) + (3*Sqrt[c]*(3*b^2 + 4*
a*c - 2*b*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b - Sq
rt[b^2 - 4*a*c]]])/(4*Sqrt[2]*(b^2 - 4*a*c)^(5/2)*Sqrt[b - Sqrt[b^2 - 4*a*c
]]*e) - (3*Sqrt[c]*(3*b^2 + 4*a*c + 2*b*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*
Sqrt[c]*(d + e*x))/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(4*Sqrt[2]*(b^2 - 4*a*c)^(
5/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]*e)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1120

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> -Simp[(d^3*(d*x)^(m - 3)*(2*a + b*x^2)*(a + b*x^2 + c*x^4)^(p + 1))/(2*
(p + 1)*(b^2 - 4*a*c)), x] + Dist[d^4/(2*(p + 1)*(b^2 - 4*a*c)), Int[(d*x)^
(m - 4)*(2*a*(m - 3) + b*(m + 4*p + 3)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && GtQ[
m, 3] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1142

```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Di
st[u^m/(Coefficient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p,
 x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1178

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[(x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*(a + b*x^2 +
 c*x^4)^(p + 1))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2
 - 4*a*c)), Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p + 7
)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
LtQ[p, -1] && IntegerQ[2*p]
```

Rubi steps

$$\int \frac{(d+ex)^4}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^3}\,dx = \frac{\text{Subst}\left(\int \frac{x^4}{\left(a+bx^2+cx^4\right)^3}\,dx, x, d+ex\right)}{e}$$

$$= \frac{(d+ex)\left(2a+b(d+ex)^2\right)}{4\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)^2} - \frac{\text{Subst}\left(\int \frac{2a-5bx^2}{\left(a+bx^2+cx^4\right)^2}\,dx\right.}{4\left(b^2-4ac\right)e}$$

$$= \frac{(d+ex)\left(2a+b(d+ex)^2\right)}{4\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)^2} - \frac{(d+ex)\left(7b^2-4ac\right.}{8\left(b^2-4ac\right)^2e\left(a+b(d\right.}$$

$$= \frac{(d+ex)\left(2a+b(d+ex)^2\right)}{4\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)^2} - \frac{(d+ex)\left(7b^2-4ac\right.}{8\left(b^2-4ac\right)^2e\left(a+b(d\right.}$$

$$= \frac{(d+ex)\left(2a+b(d+ex)^2\right)}{4\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)^2} - \frac{(d+ex)\left(7b^2-4ac\right.}{8\left(b^2-4ac\right)^2e\left(a+b(d\right.}$$

Mathematica [A] time = 4.43, size = 328, normalized size = 0.96

$$\frac{\frac{(d+ex)\left(4ac-7b^2-12bc(d+ex)^2\right)}{\left(b^2-4ac\right)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)} - \frac{2\left(-2a(d+ex)-b(d+ex)^3\right)}{\left(b^2-4ac\right)\left(a+b(d+ex)^2+c(d+ex)^4\right)^2} + \frac{3\sqrt{2}\sqrt{c}\left(-2b\sqrt{b^2-4ac}+4ac+3b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\left(b^2-4ac\right)^{5/2}\sqrt{b-\sqrt{b^2-4ac}}} - \frac{3\sqrt{2}\sqrt{c}\left(2b\sqrt{b^2-4ac}+4ac+3b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\left(b^2-4ac\right)^{5/2}\sqrt{\sqrt{b^2-4ac}+b}}}{8e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^4/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3,x]
```

```
[Out] ((-2*(-2*a*(d + e*x) - b*(d + e*x)^3))/((b^2 - 4*a*c)*(a + b*(d + e*x)^2 +
c*(d + e*x)^4)^2) + ((d + e*x)*(-7*b^2 + 4*a*c - 12*b*c*(d + e*x)^2))/((b^2
 - 4*a*c)^2*(a + b*(d + e*x)^2 + c*(d + e*x)^4)) + (3*Sqrt[2]*Sqrt[c]*(3*b^
2 + 4*a*c - 2*b*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[
b - Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(5/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]])
- (3*Sqrt[2]*Sqrt[c]*(3*b^2 + 4*a*c + 2*b*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2
]*Sqrt[c]*(d + e*x))/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(5/2)*Sqr
t[b + Sqrt[b^2 - 4*a*c]]))/(8*e)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(d+ex)^4}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(d + e*x)^4/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3,x]
```

```
[Out] IntegrateAlgebraic[(d + e*x)^4/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3, x]
```

fricas [B] time = 2.04, size = 6633, normalized size = 19.45

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^4/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="fricas")
```

```
[Out] -1/16*(24*b*c^2*e^7*x^7 + 168*b*c^2*d*e^6*x^6 + 2*(252*b*c^2*d^2 + 19*b^2*c
 - 4*a*c^2)*e^5*x^5 + 24*b*c^2*d^7 + 10*(84*b*c^2*d^3 + (19*b^2*c - 4*a*c^2
)*d)*e^4*x^4 + 2*(420*b*c^2*d^4 + 5*b^3 + 16*a*b*c + 10*(19*b^2*c - 4*a*c^2
)*d^2)*e^3*x^3 + 2*(19*b^2*c - 4*a*c^2)*d^5 + 2*(252*b*c^2*d^5 + 10*(19*b^2
*c - 4*a*c^2)*d^3 + 3*(5*b^3 + 16*a*b*c)*d)*e^2*x^2 + 2*(5*b^3 + 16*a*b*c)*
d^3 + 2*(84*b*c^2*d^6 + 5*(19*b^2*c - 4*a*c^2)*d^4 + 3*a*b^2 + 12*a^2*c + 3
*(5*b^3 + 16*a*b*c)*d^2)*e*x - 3*sqrt(1/2)*((b^4*c^2 - 8*a*b^2*c^3 + 16*a^2
*c^4)*e^9*x^8 + 8*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d*e^8*x^7 + 2*(b^5*c
 - 8*a*b^3*c^2 + 16*a^2*b*c^3 + 14*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^2
)*e^7*x^6 + 4*(14*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^3 + 3*(b^5*c - 8*a
*b^3*c^2 + 16*a^2*b*c^3)*d)*e^6*x^5 + (b^6 - 6*a*b^4*c + 32*a^3*c^3 + 70*(b
^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^4 + 30*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b
*c^3)*d^2)*e^5*x^4 + 4*(14*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^5 + 10*(b
^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^3 + (b^6 - 6*a*b^4*c + 32*a^3*c^3)*d)*
e^4*x^3 + 2*(14*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^6 + a*b^5 - 8*a^2*b^
3*c + 16*a^3*b*c^2 + 15*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^4 + 3*(b^6 -
 6*a*b^4*c + 32*a^3*c^3)*d^2)*e^3*x^2 + 4*(2*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^
2*c^4)*d^7 + 3*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^5 + (b^6 - 6*a*b^4*c
+ 32*a^3*c^3)*d^3 + (a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*d)*e^2*x + ((b^4*c
^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^8 + 2*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)
*d^6 + a^2*b^4 - 8*a^3*b^2*c + 16*a^4*c^2 + (b^6 - 6*a*b^4*c + 32*a^3*c^3)*
d^4 + 2*(a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*d^2)*e)*sqrt(-(b^5 + 40*a*b^3*
c + 80*a^2*b*c^2 + (a*b^10 - 20*a^2*b^8*c + 160*a^3*b^6*c^2 - 640*a^4*b^4*c
^3 + 1280*a^5*b^2*c^4 - 1024*a^6*c^5)*e^2*sqrt(1/((a^2*b^10 - 20*a^3*b^8*c
+ 160*a^4*b^6*c^2 - 640*a^5*b^4*c^3 + 1280*a^6*b^2*c^4 - 1024*a^7*c^5)*e^4)
))/((a*b^10 - 20*a^2*b^8*c + 160*a^3*b^6*c^2 - 640*a^4*b^4*c^3 + 1280*a^5*b
^2*c^4 - 1024*a^6*c^5)*e^2))*log(3*(5*b^4*c + 40*a*b^2*c^2 + 16*a^2*c^3)*e*
x + 3*(5*b^4*c + 40*a*b^2*c^2 + 16*a^2*c^3)*d + 3/2*sqrt(1/2)*((a*b^13 - 8*
a^2*b^11*c - 80*a^3*b^9*c^2 + 1280*a^4*b^7*c^3 - 6400*a^5*b^5*c^4 + 14336*a
^6*b^3*c^5 - 12288*a^7*b*c^6)*e^3*sqrt(1/((a^2*b^10 - 20*a^3*b^8*c + 160*a^
4*b^6*c^2 - 640*a^5*b^4*c^3 + 1280*a^6*b^2*c^4 - 1024*a^7*c^5)*e^4)) - (b^8
 - 8*a*b^6*c + 128*a^3*b^2*c^3 - 256*a^4*c^4)*e)*sqrt(-(b^5 + 40*a*b^3*c +
80*a^2*b*c^2 + (a*b^10 - 20*a^2*b^8*c + 160*a^3*b^6*c^2 - 640*a^4*b^4*c^3 +
 1280*a^5*b^2*c^4 - 1024*a^6*c^5)*e^2*sqrt(1/((a^2*b^10 - 20*a^3*b^8*c + 16
0*a^4*b^6*c^2 - 640*a^5*b^4*c^3 + 1280*a^6*b^2*c^4 - 1024*a^7*c^5)*e^4)))/(
(a*b^10 - 20*a^2*b^8*c + 160*a^3*b^6*c^2 - 640*a^4*b^4*c^3 + 1280*a^5*b^2*c
^4 - 1024*a^6*c^5)*e^2))) + 3*sqrt(1/2)*((b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^
4)*e^9*x^8 + 8*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d*e^8*x^7 + 2*(b^5*c -
8*a*b^3*c^2 + 16*a^2*b*c^3 + 14*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^2)*e
^7*x^6 + 4*(14*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^3 + 3*(b^5*c - 8*a*b^
3*c^2 + 16*a^2*b*c^3)*d)*e^6*x^5 + (b^6 - 6*a*b^4*c + 32*a^3*c^3 + 70*(b^4*
c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^4 + 30*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^
3)*d^2)*e^5*x^4 + 4*(14*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^5 + 10*(b^5*
c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^3 + (b^6 - 6*a*b^4*c + 32*a^3*c^3)*d)*e^4
*x^3 + 2*(14*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^6 + a*b^5 - 8*a^2*b^3*c
 + 16*a^3*b*c^2 + 15*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^4 + 3*(b^6 - 6*
a*b^4*c + 32*a^3*c^3)*d^2)*e^3*x^2 + 4*(2*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c
^4)*d^7 + 3*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^5 + (b^6 - 6*a*b^4*c + 3
2*a^3*c^3)*d^3 + (a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*d)*e^2*x + ((b^4*c^2
- 8*a*b^2*c^3 + 16*a^2*c^4)*d^8 + 2*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^
```

6 + a^2*b^4 - 8*a^3*b^2*c + 16*a^4*c^2 + (b^6 - 6*a*b^4*c + 32*a^3*c^3)*d^4
 + 2*(a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*d^2)*e)*sqrt(-(b^5 + 40*a*b^3*c +
 80*a^2*b*c^2 + (a*b^10 - 20*a^2*b^8*c + 160*a^3*b^6*c^2 - 640*a^4*b^4*c^3
+ 1280*a^5*b^2*c^4 - 1024*a^6*c^5)*e^2*sqrt(1/((a^2*b^10 - 20*a^3*b^8*c + 1
60*a^4*b^6*c^2 - 640*a^5*b^4*c^3 + 1280*a^6*b^2*c^4 - 1024*a^7*c^5)*e^4)))/
((a*b^10 - 20*a^2*b^8*c + 160*a^3*b^6*c^2 - 640*a^4*b^4*c^3 + 1280*a^5*b^2*
c^4 - 1024*a^6*c^5)*e^2))*log(3*(5*b^4*c + 40*a*b^2*c^2 + 16*a^2*c^3)*e*x +
 3*(5*b^4*c + 40*a*b^2*c^2 + 16*a^2*c^3)*d - 3/2*sqrt(1/2)*((a*b^13 - 8*a^2
*b^11*c - 80*a^3*b^9*c^2 + 1280*a^4*b^7*c^3 - 6400*a^5*b^5*c^4 + 14336*a^6*
b^3*c^5 - 12288*a^7*b*c^6)*e^3*sqrt(1/((a^2*b^10 - 20*a^3*b^8*c + 160*a^4*b
^6*c^2 - 640*a^5*b^4*c^3 + 1280*a^6*b^2*c^4 - 1024*a^7*c^5)*e^4)) - (b^8 -
8*a*b^6*c + 128*a^3*b^2*c^3 - 256*a^4*c^4)*e)*sqrt(-(b^5 + 40*a*b^3*c + 80*
a^2*b*c^2 + (a*b^10 - 20*a^2*b^8*c + 160*a^3*b^6*c^2 - 640*a^4*b^4*c^3 + 12
80*a^5*b^2*c^4 - 1024*a^6*c^5)*e^2*sqrt(1/((a^2*b^10 - 20*a^3*b^8*c + 160*a
^4*b^6*c^2 - 640*a^5*b^4*c^3 + 1280*a^6*b^2*c^4 - 1024*a^7*c^5)*e^4)))/((a*
b^10 - 20*a^2*b^8*c + 160*a^3*b^6*c^2 - 640*a^4*b^4*c^3 + 1280*a^5*b^2*c^4
- 1024*a^6*c^5)*e^2))) + 3*sqrt(1/2)*((b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*
e^9*x^8 + 8*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d*e^8*x^7 + 2*(b^5*c - 8*a
*b^3*c^2 + 16*a^2*b*c^3 + 14*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^2)*e^7*
x^6 + 4*(14*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^3 + 3*(b^5*c - 8*a*b^3*c
^2 + 16*a^2*b*c^3)*d)*e^6*x^5 + (b^6 - 6*a*b^4*c + 32*a^3*c^3 + 70*(b^4*c^2
 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^4 + 30*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*
d^2)*e^5*x^4 + 4*(14*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^5 + 10*(b^5*c -
 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^3 + (b^6 - 6*a*b^4*c + 32*a^3*c^3)*d)*e^4*x^
3 + 2*(14*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^6 + a*b^5 - 8*a^2*b^3*c +
16*a^3*b*c^2 + 15*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^4 + 3*(b^6 - 6*a*b
^4*c + 32*a^3*c^3)*d^2)*e^3*x^2 + 4*(2*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)
d^7 + 3(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^5 + (b^6 - 6*a*b^4*c + 32*a
^3*c^3)*d^3 + (a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*d)*e^2*x + ((b^4*c^2 - 8
*a*b^2*c^3 + 16*a^2*c^4)*d^8 + 2*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^6 +
 a^2*b^4 - 8*a^3*b^2*c + 16*a^4*c^2 + (b^6 - 6*a*b^4*c + 32*a^3*c^3)*d^4 +
2*(a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*d^2)*e)*sqrt(-(b^5 + 40*a*b^3*c + 80
*a^2*b*c^2 - (a*b^10 - 20*a^2*b^8*c + 160*a^3*b^6*c^2 - 640*a^4*b^4*c^3 + 1
280*a^5*b^2*c^4 - 1024*a^6*c^5)*e^2*sqrt(1/((a^2*b^10 - 20*a^3*b^8*c + 160*
a^4*b^6*c^2 - 640*a^5*b^4*c^3 + 1280*a^6*b^2*c^4 - 1024*a^7*c^5)*e^4)))/((a
*b^10 - 20*a^2*b^8*c + 160*a^3*b^6*c^2 - 640*a^4*b^4*c^3 + 1280*a^5*b^2*c^4
 - 1024*a^6*c^5)*e^2))*log(3*(5*b^4*c + 40*a*b^2*c^2 + 16*a^2*c^3)*e*x + 3*
(5*b^4*c + 40*a*b^2*c^2 + 16*a^2*c^3)*d + 3/2*sqrt(1/2)*((a*b^13 - 8*a^2*b^
11*c - 80*a^3*b^9*c^2 + 1280*a^4*b^7*c^3 - 6400*a^5*b^5*c^4 + 14336*a^6*b^3
*c^5 - 12288*a^7*b*c^6)*e^3*sqrt(1/((a^2*b^10 - 20*a^3*b^8*c + 160*a^4*b^6*
c^2 - 640*a^5*b^4*c^3 + 1280*a^6*b^2*c^4 - 1024*a^7*c^5)*e^4)) + (b^8 - 8*a
*b^6*c + 128*a^3*b^2*c^3 - 256*a^4*c^4)*e)*sqrt(-(b^5 + 40*a*b^3*c + 80*a^2
*b*c^2 - (a*b^10 - 20*a^2*b^8*c + 160*a^3*b^6*c^2 - 640*a^4*b^4*c^3 + 1280*
a^5*b^2*c^4 - 1024*a^6*c^5)*e^2*sqrt(1/((a^2*b^10 - 20*a^3*b^8*c + 160*a^4*
b^6*c^2 - 640*a^5*b^4*c^3 + 1280*a^6*b^2*c^4 - 1024*a^7*c^5)*e^4)))/((a*b^1
0 - 20*a^2*b^8*c + 160*a^3*b^6*c^2 - 640*a^4*b^4*c^3 + 1280*a^5*b^2*c^4 - 1
024*a^6*c^5)*e^2))) - 3*sqrt(1/2)*((b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*e^9
x^8 + 8(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d*e^8*x^7 + 2*(b^5*c - 8*a*b^
3*c^2 + 16*a^2*b*c^3 + 14*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^2)*e^7*x^6
 + 4*(14*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^3 + 3*(b^5*c - 8*a*b^3*c^2
+ 16*a^2*b*c^3)*d)*e^6*x^5 + (b^6 - 6*a*b^4*c + 32*a^3*c^3 + 70*(b^4*c^2 -
8*a*b^2*c^3 + 16*a^2*c^4)*d^4 + 30*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^2
)*e^5*x^4 + 4*(14*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^5 + 10*(b^5*c - 8*
a*b^3*c^2 + 16*a^2*b*c^3)*d^3 + (b^6 - 6*a*b^4*c + 32*a^3*c^3)*d)*e^4*x^3 +
 2*(14*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^6 + a*b^5 - 8*a^2*b^3*c + 16*
a^3*b*c^2 + 15*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^4 + 3*(b^6 - 6*a*b^4*
c + 32*a^3*c^3)*d^2)*e^3*x^2 + 4*(2*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^
7 + 3*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^5 + (b^6 - 6*a*b^4*c + 32*a^3*
c^3)*d^3 + (a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*d)*e^2*x + ((b^4*c^2 - 8*a*

```
b^2*c^3 + 16*a^2*c^4)*d^8 + 2*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^6 + a^
2*b^4 - 8*a^3*b^2*c + 16*a^4*c^2 + (b^6 - 6*a*b^4*c + 32*a^3*c^3)*d^4 + 2*(
a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*d^2)*e)*sqrt(-(b^5 + 40*a*b^3*c + 80*a^
2*b*c^2 - (a*b^10 - 20*a^2*b^8*c + 160*a^3*b^6*c^2 - 640*a^4*b^4*c^3 + 1280
*a^5*b^2*c^4 - 1024*a^6*c^5)*e^2*sqrt(1/((a^2*b^10 - 20*a^3*b^8*c + 160*a^4
*b^6*c^2 - 640*a^5*b^4*c^3 + 1280*a^6*b^2*c^4 - 1024*a^7*c^5)*e^4)))/((a*b^
10 - 20*a^2*b^8*c + 160*a^3*b^6*c^2 - 640*a^4*b^4*c^3 + 1280*a^5*b^2*c^4 -
1024*a^6*c^5)*e^2))*log(3*(5*b^4*c + 40*a*b^2*c^2 + 16*a^2*c^3)*e*x + 3*(5*
b^4*c + 40*a*b^2*c^2 + 16*a^2*c^3)*d - 3/2*sqrt(1/2)*((a*b^13 - 8*a^2*b^11*
c - 80*a^3*b^9*c^2 + 1280*a^4*b^7*c^3 - 6400*a^5*b^5*c^4 + 14336*a^6*b^3*c^
5 - 12288*a^7*b*c^6)*e^3*sqrt(1/((a^2*b^10 - 20*a^3*b^8*c + 160*a^4*b^6*c^2
 - 640*a^5*b^4*c^3 + 1280*a^6*b^2*c^4 - 1024*a^7*c^5)*e^4)) + (b^8 - 8*a*b^
6*c + 128*a^3*b^2*c^3 - 256*a^4*c^4)*e)*sqrt(-(b^5 + 40*a*b^3*c + 80*a^2*b*
c^2 - (a*b^10 - 20*a^2*b^8*c + 160*a^3*b^6*c^2 - 640*a^4*b^4*c^3 + 1280*a^5
*b^2*c^4 - 1024*a^6*c^5)*e^2*sqrt(1/((a^2*b^10 - 20*a^3*b^8*c + 160*a^4*b^6
*c^2 - 640*a^5*b^4*c^3 + 1280*a^6*b^2*c^4 - 1024*a^7*c^5)*e^4)))/((a*b^10 -
 20*a^2*b^8*c + 160*a^3*b^6*c^2 - 640*a^4*b^4*c^3 + 1280*a^5*b^2*c^4 - 1024
*a^6*c^5)*e^2))) + 6*(a*b^2 + 4*a^2*c)*d)/((b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*
c^4)*e^9*x^8 + 8*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d*e^8*x^7 + 2*(b^5*c
- 8*a*b^3*c^2 + 16*a^2*b*c^3 + 14*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^2)
*e^7*x^6 + 4*(14*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^3 + 3*(b^5*c - 8*a*
b^3*c^2 + 16*a^2*b*c^3)*d)*e^6*x^5 + (b^6 - 6*a*b^4*c + 32*a^3*c^3 + 70*(b^
4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^4 + 30*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*
c^3)*d^2)*e^5*x^4 + 4*(14*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^5 + 10*(b^
5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^3 + (b^6 - 6*a*b^4*c + 32*a^3*c^3)*d)*e
^4*x^3 + 2*(14*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^6 + a*b^5 - 8*a^2*b^3
*c + 16*a^3*b*c^2 + 15*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^4 + 3*(b^6 -
6*a*b^4*c + 32*a^3*c^3)*d^2)*e^3*x^2 + 4*(2*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2
*c^4)*d^7 + 3*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^5 + (b^6 - 6*a*b^4*c +
 32*a^3*c^3)*d^3 + (a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*d)*e^2*x + ((b^4*c^
2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^8 + 2*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*
d^6 + a^2*b^4 - 8*a^3*b^2*c + 16*a^4*c^2 + (b^6 - 6*a*b^4*c + 32*a^3*c^3)*d
^4 + 2*(a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*d^2)*e)
```

giac [B] time = 0.75, size = 1688, normalized size = 4.95

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^4/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="giac")
```

```
[Out] 3/16*((4*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)
/c))^2*b*c*e^2 - 8*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e
^2)*e^(-4)/c))*b*c*d*e + 4*b*c*d^2 - b^2 - 4*a*c)*log(d*e^(-1) + x + sqrt(1
/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))/(2*(d*e^(-1) + sqrt(1/
2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^3*c*e^4 - 6*(d*e^(-1) +
 sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^2*c*d*e^3 - 2*c
*d^3*e - b*d*e + (6*c*d^2*e^2 + b*e^2)*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 +
 sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))) + (4*(d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2
 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^2*b*c*e^2 - 8*(d*e^(-1) - sqrt(1/2)*sq
rt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))*b*c*d*e + 4*b*c*d^2 - b^2 -
4*a*c)*log(d*e^(-1) + x - sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e
^(-4)/c))/(2*(d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^
(-4)/c))^3*c*e^4 - 6*(d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)
*e^2)*e^(-4)/c))^2*c*d*e^3 - 2*c*d^3*e - b*d*e + (6*c*d^2*e^2 + b*e^2)*(d*e
^(-1) - sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))) + (4*(d
*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^2*b*c*
e^2 - 8*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/
c))*b*c*d*e + 4*b*c*d^2 - b^2 - 4*a*c)*log(d*e^(-1) + x + sqrt(1/2)*sqrt(-(
b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))/(2*(d*e^(-1) + sqrt(1/2)*sqrt(-(b
```

```
*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^3*c*e^4 - 6*(d*e^(-1) + sqrt(1/2)*
sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^2*c*d*e^3 - 2*c*d^3*e - b*
d*e + (6*c*d^2*e^2 + b*e^2)*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 -
 4*a*c)*e^2)*e^(-4)/c))) + (4*(d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2
 - 4*a*c)*e^2)*e^(-4)/c))^2*b*c*e^2 - 8*(d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2
- sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))*b*c*d*e + 4*b*c*d^2 - b^2 - 4*a*c)*log(
d*e^(-1) + x - sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))/(
2*(d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^3*
c*e^4 - 6*(d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4
)/c))^2*c*d*e^3 - 2*c*d^3*e - b*d*e + (6*c*d^2*e^2 + b*e^2)*(d*e^(-1) - sqr
t(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))))/(b^4 - 8*a*b^2*c
+ 16*a^2*c^2) - 1/8*(12*b*c^2*x^7*e^7 + 84*b*c^2*d*x^6*e^6 + 252*b*c^2*d^2*
x^5*e^5 + 420*b*c^2*d^3*x^4*e^4 + 420*b*c^2*d^4*x^3*e^3 + 252*b*c^2*d^5*x^2
*e^2 + 84*b*c^2*d^6*x*e + 12*b*c^2*d^7 + 19*b^2*c*x^5*e^5 - 4*a*c^2*x^5*e^5
 + 95*b^2*c*d*x^4*e^4 - 20*a*c^2*d*x^4*e^4 + 190*b^2*c*d^2*x^3*e^3 - 40*a*c
^2*d^2*x^3*e^3 + 190*b^2*c*d^3*x^2*e^2 - 40*a*c^2*d^3*x^2*e^2 + 95*b^2*c*d^
4*x*e - 20*a*c^2*d^4*x*e + 19*b^2*c*d^5 - 4*a*c^2*d^5 + 5*b^3*x^3*e^3 + 16*
a*b*c*x^3*e^3 + 15*b^3*d*x^2*e^2 + 48*a*b*c*d*x^2*e^2 + 15*b^3*d^2*x*e + 48
*a*b*c*d^2*x*e + 5*b^3*d^3 + 16*a*b*c*d^3 + 3*a*b^2*x*e + 12*a^2*c*x*e + 3*
a*b^2*d + 12*a^2*c*d)/((c*x^4*e^4 + 4*c*d*x^3*e^3 + 6*c*d^2*x^2*e^2 + 4*c*d
^3*x*e + c*d^4 + b*x^2*e^2 + 2*b*d*x*e + b*d^2 + a)^2*(b^4*e - 8*a*b^2*c*e
+ 16*a^2*c^2*e))
```

maple [C] time = 0.05, size = 704, normalized size = 2.06

$$\frac{3\left(-4 \operatorname{RootOf}\left(\_Z^4 c e^4+4\_Z^3 c d e^3+c d^4+b d^2+\left(6 c d^2 e^2+b e^2\right)\_Z^2+\left(4 c d^3 e+2 d e b\right)\_Z+a\right)^2 b c e^2-8 \operatorname{RootOf}\left(\_Z^4 c e^4+4\_Z^3 c d e^3+c d^4+b d^2+\left(6 c d^2 e^2+b e^2\right)\_Z^2+\left(4 c d^3 e+2 d e b\right)\_Z+a\right) b c d e-4 b c d^2+4 a c+b^2\right) \ln\left(-\operatorname{RootOf}\left(\_Z^4 c e^4+4\_Z^3 c d e^3+c d^4+b d^2+\left(6 c d^2 e^2+b e^2\right)\_Z^2+\left(4 c d^3 e+2 d e b\right)\_Z+a\right)+x\right)}{16\left(16 a^2 c^2-8 a b^2 c+b^4\right) e\left(2 c e^3 \operatorname{RootOf}\left(\_Z^4 c e^4+4\_Z^3 c d e^3+c d^4+b d^2+\left(6 c d^2 e^2+b e^2\right)\_Z^2+\left(4 c d^3 e+2 d e b\right)\_Z+a\right)^3+6 c d e^2 \operatorname{RootOf}\left(\_Z^4 c e^4+4\_Z^3 c d e^3+c d^4+b d^2+\left(6 c d^2 e^2+b e^2\right)\_Z^2+\left(4 c d^3 e+2 d e b\right)\_Z+a\right)^2+6 e c d^2 \operatorname{RootOf}\left(\_Z^4 c e^4+4\_Z^3 c d e^3+c d^4+b d^2+\left(6 c d^2 e^2+b e^2\right)\_Z^2+\left(4 c d^3 e+2 d e b\right)\_Z+a\right)+2 c d^3+b e \operatorname{RootOf}\left(\_Z^4 c e^4+4\_Z^3 c d e^3+c d^4+b d^2+\left(6 c d^2 e^2+b e^2\right)\_Z^2+\left(4 c d^3 e+2 d e b\right)\_Z+a\right)+b d\right)}+\frac{-\frac{3 b c^2 e^6 x^7}{2\left(16 a^2 c^2-8 a b^2 c+b^4\right)}-\frac{21 b c^2 d e^5 x^6}{2\left(16 a^2 c^2-8 a b^2 c+b^4\right)}+\frac{\left(-252 b c d^2+4 a c-19 b^2\right) c e^4 x^5}{128 a^2 c^2-64 a b^2 c+8 b^4}+\frac{5\left(-84 b c d^2+4 a c-19 b^2\right) c d e^3 x^4}{8\left(16 a^2 c^2-8 a b^2 c+b^4\right)}-\frac{\left(420 b c^2 d^4-40 a c^2 d^2+190 b^2 c d^2+16 a b c+5 b^3\right) e^2 x^3}{8\left(16 a^2 c^2-8 a b^2 c+b^4\right)}-\frac{\left(252 b c^2 d^4-40 a c^2 d^2+190 b^2 c d^2+48 a b c+15 b^3\right) d e x^2}{8\left(16 a^2 c^2-8 a b^2 c+b^4\right)}-\frac{\left(84 b c^2 d^6-20 a c^2 d^4+95 b^2 c d^4+48 a b c d^2+15 b^3 d^2+12 a^2 c+3 a b^2\right) x}{8\left(16 a^2 c^2-8 a b^2 c+b^4\right)}-\frac{\left(12 b c^2 d^6-4 a c^2 d^4+19 b^2 c d^4+16 a b c d^2+5 b^3 d^2+12 a^2 c+3 a b^2\right) d}{8\left(16 a^2 c^2-8 a b^2 c+b^4\right) e}}{\left(c e^4 x^4+4 c d e^3 x^3+6 c d^2 e^2 x^2+4 c d^3 e x+b e^2 x^2+c d^4+2 b d e x+b d^2+a\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^4/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x)
```

```
[Out] (-3/2*c^2*e^6*b/(16*a^2*c^2-8*a*b^2*c+b^4)*x^7-21/2*c^2*d*e^5*b/(16*a^2*c^2
-8*a*b^2*c+b^4)*x^6+1/8*(-252*b*c*d^2+4*a*c-19*b^2)*c*e^4/(16*a^2*c^2-8*a*b
^2*c+b^4)*x^5+5/8*c*d*e^3*(-84*b*c*d^2+4*a*c-19*b^2)/(16*a^2*c^2-8*a*b^2*c+
b^4)*x^4-1/8*e^2*(420*b*c^2*d^4-40*a*c^2*d^2+190*b^2*c*d^2+16*a*b*c+5*b^3)/
(16*a^2*c^2-8*a*b^2*c+b^4)*x^3-1/8*d*e*(252*b*c^2*d^4-40*a*c^2*d^2+190*b^2*
c*d^2+48*a*b*c+15*b^3)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^2-1/8*(84*b*c^2*d^6-20*
a*c^2*d^4+95*b^2*c*d^4+48*a*b*c*d^2+15*b^3*d^2+12*a^2*c+3*a*b^2)/(16*a^2*c^
2-8*a*b^2*c+b^4)*x-1/8*d/e*(12*b*c^2*d^6-4*a*c^2*d^4+19*b^2*c*d^4+16*a*b*c*
d^2+5*b^3*d^2+12*a^2*c+3*a*b^2)/(16*a^2*c^2-8*a*b^2*c+b^4))/(c*e^4*x^4+4*c*
d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2+
3/16/(16*a^2*c^2-8*a*b^2*c+b^4)/e*sum((-4*_R^2*b*c*e^2-8*_R*b*c*d*e-4*b*c*d
^2+4*a*c+b^2)/(2*_R^3*c*e^3+6*_R^2*c*d*e^2+6*_R*c*d^2*e+2*c*d^3+_R*b*e+b*d)
*ln(-_R+x),_R=RootOf(_Z^4*c*e^4+4*_Z^3*c*d*e^3+c*d^4+b*d^2+(6*c*d^2*e^2+b*e
^2)*_Z^2+(4*c*d^3*e+2*b*d*e)*_Z+a))
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^4/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [B] time = 7.02, size = 12677, normalized size = 37.18

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^4/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3,x)
```

```
[Out] atan(((((786432*a^6*c^8*e^12 - 192*b^12*c^2*e^12 + 3072*a*b^10*c^3*e^12 - 1
5360*a^2*b^8*c^4*e^12 + 245760*a^4*b^4*c^6*e^12 - 786432*a^5*b^2*c^7*e^12)/
(128*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b
^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c)) + ((1024*b^15*c^2*d*e^13 - 28672*
a*b^13*c^3*d*e^13 - 16777216*a^7*b*c^9*d*e^13 + 344064*a^2*b^11*c^4*d*e^13
- 2293760*a^3*b^9*c^5*d*e^13 + 9175040*a^4*b^7*c^6*d*e^13 - 22020096*a^5*b^
5*c^7*d*e^13 + 29360128*a^6*b^3*c^8*d*e^13)/(128*(b^12 + 4096*a^6*c^6 + 240
*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*
a*b^10*c)) + (x*(128*b^11*c^2*e^14 - 2560*a*b^9*c^3*e^14 - 131072*a^5*b*c^7
*e^14 + 20480*a^2*b^7*c^4*e^14 - 81920*a^3*b^5*c^5*e^14 + 163840*a^4*b^3*c^
6*e^14))/(16*(b^8 + 256*a^4*c^4 + 96*a^2*b^4*c^2 - 256*a^3*b^2*c^3 - 16*a*b
^6*c)))*((9*((-(4*a*c - b^2)^15)^(1/2) - b^15 + 81920*a^7*b*c^7 + 560*a^2*b
^11*c^2 - 4160*a^3*b^9*c^3 + 11520*a^4*b^7*c^4 + 1024*a^5*b^5*c^5 - 61440*a
^6*b^3*c^6 - 20*a*b^13*c))/(512*(a*b^20*e^2 + 1048576*a^11*c^10*e^2 - 40*a^
2*b^18*c*e^2 + 720*a^3*b^16*c^2*e^2 - 7680*a^4*b^14*c^3*e^2 + 53760*a^5*b^1
2*c^4*e^2 - 258048*a^6*b^10*c^5*e^2 + 860160*a^7*b^8*c^6*e^2 - 1966080*a^8*
b^6*c^7*e^2 + 2949120*a^9*b^4*c^8*e^2 - 2621440*a^10*b^2*c^9*e^2)))^(1/2))*
((9*((-(4*a*c - b^2)^15)^(1/2) - b^15 + 81920*a^7*b*c^7 + 560*a^2*b^11*c^2
- 4160*a^3*b^9*c^3 + 11520*a^4*b^7*c^4 + 1024*a^5*b^5*c^5 - 61440*a^6*b^3*c
^6 - 20*a*b^13*c))/(512*(a*b^20*e^2 + 1048576*a^11*c^10*e^2 - 40*a^2*b^18*c
*e^2 + 720*a^3*b^16*c^2*e^2 - 7680*a^4*b^14*c^3*e^2 + 53760*a^5*b^12*c^4*e^
2 - 258048*a^6*b^10*c^5*e^2 + 860160*a^7*b^8*c^6*e^2 - 1966080*a^8*b^6*c^7*
e^2 + 2949120*a^9*b^4*c^8*e^2 - 2621440*a^10*b^2*c^9*e^2)))^(1/2) + (18432*
a^4*c^7*d*e^11 + 936*b^8*c^3*d*e^11 - 6912*a*b^6*c^4*d*e^11 + 11520*a^2*b^4
*c^5*d*e^11)/(128*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3
 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c)) + (x*(144*a^2*c^5*e^
12 + 117*b^4*c^3*e^12 + 72*a*b^2*c^4*e^12))/(16*(b^8 + 256*a^4*c^4 + 96*a^2
*b^4*c^2 - 256*a^3*b^2*c^3 - 16*a*b^6*c)))*((9*((-(4*a*c - b^2)^15)^(1/2) -
 b^15 + 81920*a^7*b*c^7 + 560*a^2*b^11*c^2 - 4160*a^3*b^9*c^3 + 11520*a^4*b
^7*c^4 + 1024*a^5*b^5*c^5 - 61440*a^6*b^3*c^6 - 20*a*b^13*c))/(512*(a*b^20*
e^2 + 1048576*a^11*c^10*e^2 - 40*a^2*b^18*c*e^2 + 720*a^3*b^16*c^2*e^2 - 76
80*a^4*b^14*c^3*e^2 + 53760*a^5*b^12*c^4*e^2 - 258048*a^6*b^10*c^5*e^2 + 86
0160*a^7*b^8*c^6*e^2 - 1966080*a^8*b^6*c^7*e^2 + 2949120*a^9*b^4*c^8*e^2 -
2621440*a^10*b^2*c^9*e^2)))^(1/2)*1i + ((18432*a^4*c^7*d*e^11 + 936*b^8*c^3
*d*e^11 - 6912*a*b^6*c^4*d*e^11 + 11520*a^2*b^4*c^5*d*e^11)/(128*(b^12 + 40
96*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a
^5*b^2*c^5 - 24*a*b^10*c)) - ((786432*a^6*c^8*e^12 - 192*b^12*c^2*e^12 + 30
72*a*b^10*c^3*e^12 - 15360*a^2*b^8*c^4*e^12 + 245760*a^4*b^4*c^6*e^12 - 786
432*a^5*b^2*c^7*e^12)/(128*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^
3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c)) - ((1024*b^
15*c^2*d*e^13 - 28672*a*b^13*c^3*d*e^13 - 16777216*a^7*b*c^9*d*e^13 + 34406
4*a^2*b^11*c^4*d*e^13 - 2293760*a^3*b^9*c^5*d*e^13 + 9175040*a^4*b^7*c^6*d*
e^13 - 22020096*a^5*b^5*c^7*d*e^13 + 29360128*a^6*b^3*c^8*d*e^13)/(128*(b^1
2 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 -
6144*a^5*b^2*c^5 - 24*a*b^10*c)) + (x*(128*b^11*c^2*e^14 - 2560*a*b^9*c^3*e
^14 - 131072*a^5*b*c^7*e^14 + 20480*a^2*b^7*c^4*e^14 - 81920*a^3*b^5*c^5*e^
14 + 163840*a^4*b^3*c^6*e^14))/(16*(b^8 + 256*a^4*c^4 + 96*a^2*b^4*c^2 - 25
6*a^3*b^2*c^3 - 16*a*b^6*c)))*((9*((-(4*a*c - b^2)^15)^(1/2) - b^15 + 81920
*a^7*b*c^7 + 560*a^2*b^11*c^2 - 4160*a^3*b^9*c^3 + 11520*a^4*b^7*c^4 + 1024
*a^5*b^5*c^5 - 61440*a^6*b^3*c^6 - 20*a*b^13*c))/(512*(a*b^20*e^2 + 1048576
*a^11*c^10*e^2 - 40*a^2*b^18*c*e^2 + 720*a^3*b^16*c^2*e^2 - 7680*a^4*b^14*c
^3*e^2 + 53760*a^5*b^12*c^4*e^2 - 258048*a^6*b^10*c^5*e^2 + 860160*a^7*b^8*
c^6*e^2 - 1966080*a^8*b^6*c^7*e^2 + 2949120*a^9*b^4*c^8*e^2 - 2621440*a^10*
b^2*c^9*e^2)))^(1/2))*((9*((-(4*a*c - b^2)^15)^(1/2) - b^15 + 81920*a^7*b*c
^7 + 560*a^2*b^11*c^2 - 4160*a^3*b^9*c^3 + 11520*a^4*b^7*c^4 + 1024*a^5*b^5
*c^5 - 61440*a^6*b^3*c^6 - 20*a*b^13*c))/(512*(a*b^20*e^2 + 1048576*a^11*c^
10*e^2 - 40*a^2*b^18*c*e^2 + 720*a^3*b^16*c^2*e^2 - 7680*a^4*b^14*c^3*e^2 +
 53760*a^5*b^12*c^4*e^2 - 258048*a^6*b^10*c^5*e^2 + 860160*a^7*b^8*c^6*e^2
- 1966080*a^8*b^6*c^7*e^2 + 2949120*a^9*b^4*c^8*e^2 - 2621440*a^10*b^2*c^9*
```

e^2)))^(1/2) + (x*(144*a^2*c^5*e^12 + 117*b^4*c^3*e^12 + 72*a*b^2*c^4*e^12)
)/(16*(b^8 + 256*a^4*c^4 + 96*a^2*b^4*c^2 - 256*a^3*b^2*c^3 - 16*a*b^6*c)))
((9((-(4*a*c - b^2)^15)^(1/2) - b^15 + 81920*a^7*b*c^7 + 560*a^2*b^11*c^2
 - 4160*a^3*b^9*c^3 + 11520*a^4*b^7*c^4 + 1024*a^5*b^5*c^5 - 61440*a^6*b^3*
c^6 - 20*a*b^13*c))/(512*(a*b^20*e^2 + 1048576*a^11*c^10*e^2 - 40*a^2*b^18*
c*e^2 + 720*a^3*b^16*c^2*e^2 - 7680*a^4*b^14*c^3*e^2 + 53760*a^5*b^12*c^4*e
^2 - 258048*a^6*b^10*c^5*e^2 + 860160*a^7*b^8*c^6*e^2 - 1966080*a^8*b^6*c^7
*e^2 + 2949120*a^9*b^4*c^8*e^2 - 2621440*a^10*b^2*c^9*e^2)))^(1/2)*1i)/((13
5*b^5*c^3*e^10 + 1080*a*b^3*c^4*e^10 + 432*a^2*b*c^5*e^10)/(64*(b^12 + 4096
*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5
*b^2*c^5 - 24*a*b^10*c)) - (((786432*a^6*c^8*e^12 - 192*b^12*c^2*e^12 + 307
2*a*b^10*c^3*e^12 - 15360*a^2*b^8*c^4*e^12 + 245760*a^4*b^4*c^6*e^12 - 7864
32*a^5*b^2*c^7*e^12)/(128*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3
*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c)) + ((1024*b^1
5*c^2*d*e^13 - 28672*a*b^13*c^3*d*e^13 - 16777216*a^7*b*c^9*d*e^13 + 344064
*a^2*b^11*c^4*d*e^13 - 2293760*a^3*b^9*c^5*d*e^13 + 9175040*a^4*b^7*c^6*d*e
^13 - 22020096*a^5*b^5*c^7*d*e^13 + 29360128*a^6*b^3*c^8*d*e^13)/(128*(b^12
 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6
144*a^5*b^2*c^5 - 24*a*b^10*c)) + (x*(128*b^11*c^2*e^14 - 2560*a*b^9*c^3*e^
14 - 131072*a^5*b*c^7*e^14 + 20480*a^2*b^7*c^4*e^14 - 81920*a^3*b^5*c^5*e^1
4 + 163840*a^4*b^3*c^6*e^14))/(16*(b^8 + 256*a^4*c^4 + 96*a^2*b^4*c^2 - 256
*a^3*b^2*c^3 - 16*a*b^6*c)))*((9*((-(4*a*c - b^2)^15)^(1/2) - b^15 + 81920*
a^7*b*c^7 + 560*a^2*b^11*c^2 - 4160*a^3*b^9*c^3 + 11520*a^4*b^7*c^4 + 1024*
a^5*b^5*c^5 - 61440*a^6*b^3*c^6 - 20*a*b^13*c))/(512*(a*b^20*e^2 + 1048576*
a^11*c^10*e^2 - 40*a^2*b^18*c*e^2 + 720*a^3*b^16*c^2*e^2 - 7680*a^4*b^14*c^
3*e^2 + 53760*a^5*b^12*c^4*e^2 - 258048*a^6*b^10*c^5*e^2 + 860160*a^7*b^8*c
^6*e^2 - 1966080*a^8*b^6*c^7*e^2 + 2949120*a^9*b^4*c^8*e^2 - 2621440*a^10*b
^2*c^9*e^2)))^(1/2))*((9*((-(4*a*c - b^2)^15)^(1/2) - b^15 + 81920*a^7*b*c^
7 + 560*a^2*b^11*c^2 - 4160*a^3*b^9*c^3 + 11520*a^4*b^7*c^4 + 1024*a^5*b^5*
c^5 - 61440*a^6*b^3*c^6 - 20*a*b^13*c))/(512*(a*b^20*e^2 + 1048576*a^11*c^1
0*e^2 - 40*a^2*b^18*c*e^2 + 720*a^3*b^16*c^2*e^2 - 7680*a^4*b^14*c^3*e^2 +
53760*a^5*b^12*c^4*e^2 - 258048*a^6*b^10*c^5*e^2 + 860160*a^7*b^8*c^6*e^2 -
 1966080*a^8*b^6*c^7*e^2 + 2949120*a^9*b^4*c^8*e^2 - 2621440*a^10*b^2*c^9*e
^2)))^(1/2) + (18432*a^4*c^7*d*e^11 + 936*b^8*c^3*d*e^11 - 6912*a*b^6*c^4*d
*e^11 + 11520*a^2*b^4*c^5*d*e^11)/(128*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c
^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c))
 + (x*(144*a^2*c^5*e^12 + 117*b^4*c^3*e^12 + 72*a*b^2*c^4*e^12))/(16*(b^8 +
 256*a^4*c^4 + 96*a^2*b^4*c^2 - 256*a^3*b^2*c^3 - 16*a*b^6*c)))*((9*((-(4*a
*c - b^2)^15)^(1/2) - b^15 + 81920*a^7*b*c^7 + 560*a^2*b^11*c^2 - 4160*a^3*
b^9*c^3 + 11520*a^4*b^7*c^4 + 1024*a^5*b^5*c^5 - 61440*a^6*b^3*c^6 - 20*a*b
^13*c))/(512*(a*b^20*e^2 + 1048576*a^11*c^10*e^2 - 40*a^2*b^18*c*e^2 + 720*
a^3*b^16*c^2*e^2 - 7680*a^4*b^14*c^3*e^2 + 53760*a^5*b^12*c^4*e^2 - 258048*
a^6*b^10*c^5*e^2 + 860160*a^7*b^8*c^6*e^2 - 1966080*a^8*b^6*c^7*e^2 + 29491
20*a^9*b^4*c^8*e^2 - 2621440*a^10*b^2*c^9*e^2)))^(1/2) + ((18432*a^4*c^7*d*
e^11 + 936*b^8*c^3*d*e^11 - 6912*a*b^6*c^4*d*e^11 + 11520*a^2*b^4*c^5*d*e^1
1)/(128*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^
4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c)) - ((786432*a^6*c^8*e^12 - 192*
b^12*c^2*e^12 + 3072*a*b^10*c^3*e^12 - 15360*a^2*b^8*c^4*e^12 + 245760*a^4*
b^4*c^6*e^12 - 786432*a^5*b^2*c^7*e^12)/(128*(b^12 + 4096*a^6*c^6 + 240*a^2
*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^
10*c)) - ((1024*b^15*c^2*d*e^13 - 28672*a*b^13*c^3*d*e^13 - 16777216*a^7*b*
c^9*d*e^13 + 344064*a^2*b^11*c^4*d*e^13 - 2293760*a^3*b^9*c^5*d*e^13 + 9175
040*a^4*b^7*c^6*d*e^13 - 22020096*a^5*b^5*c^7*d*e^13 + 29360128*a^6*b^3*c^8
*d*e^13)/(128*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3
840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c)) + (x*(128*b^11*c^2*e^14
 - 2560*a*b^9*c^3*e^14 - 131072*a^5*b*c^7*e^14 + 20480*a^2*b^7*c^4*e^14 - 81
920*a^3*b^5*c^5*e^14 + 163840*a^4*b^3*c^6*e^14))/(16*(b^8 + 256*a^4*c^4 + 9
6*a^2*b^4*c^2 - 256*a^3*b^2*c^3 - 16*a*b^6*c)))*((9*((-(4*a*c - b^2)^15)^(1
/2) - b^15 + 81920*a^7*b*c^7 + 560*a^2*b^11*c^2 - 4160*a^3*b^9*c^3 + 11520*

a^4*b^7*c^4 + 1024*a^5*b^5*c^5 - 61440*a^6*b^3*c^6 - 20*a*b^13*c))/(512*(a*
b^20*e^2 + 1048576*a^11*c^10*e^2 - 40*a^2*b^18*c*e^2 + 720*a^3*b^16*c^2*e^2
- 7680*a^4*b^14*c^3*e^2 + 53760*a^5*b^12*c^4*e^2 - 258048*a^6*b^10*c^5*e^2
+ 860160*a^7*b^8*c^6*e^2 - 1966080*a^8*b^6*c^7*e^2 + 2949120*a^9*b^4*c^8*e
^2 - 2621440*a^10*b^2*c^9*e^2)))^(1/2))*((9*((-(4*a*c - b^2)^15)^(1/2) - b^
15 + 81920*a^7*b*c^7 + 560*a^2*b^11*c^2 - 4160*a^3*b^9*c^3 + 11520*a^4*b^7*
c^4 + 1024*a^5*b^5*c^5 - 61440*a^6*b^3*c^6 - 20*a*b^13*c))/(512*(a*b^20*e^2
+ 1048576*a^11*c^10*e^2 - 40*a^2*b^18*c*e^2 + 720*a^3*b^16*c^2*e^2 - 7680*
a^4*b^14*c^3*e^2 + 53760*a^5*b^12*c^4*e^2 - 258048*a^6*b^10*c^5*e^2 + 86016
0*a^7*b^8*c^6*e^2 - 1966080*a^8*b^6*c^7*e^2 + 2949120*a^9*b^4*c^8*e^2 - 262
1440*a^10*b^2*c^9*e^2)))^(1/2) + (x*(144*a^2*c^5*e^12 + 117*b^4*c^3*e^12 +
72*a*b^2*c^4*e^12))/(16*(b^8 + 256*a^4*c^4 + 96*a^2*b^4*c^2 - 256*a^3*b^2*c
^3 - 16*a*b^6*c)))*((9*((-(4*a*c - b^2)^15)^(1/2) - b^15 + 81920*a^7*b*c^7
+ 560*a^2*b^11*c^2 - 4160*a^3*b^9*c^3 + 11520*a^4*b^7*c^4 + 1024*a^5*b^5*c^
5 - 61440*a^6*b^3*c^6 - 20*a*b^13*c))/(512*(a*b^20*e^2 + 1048576*a^11*c^10*
e^2 - 40*a^2*b^18*c*e^2 + 720*a^3*b^16*c^2*e^2 - 7680*a^4*b^14*c^3*e^2 + 53
760*a^5*b^12*c^4*e^2 - 258048*a^6*b^10*c^5*e^2 + 860160*a^7*b^8*c^6*e^2 - 1
966080*a^8*b^6*c^7*e^2 + 2949120*a^9*b^4*c^8*e^2 - 2621440*a^10*b^2*c^9*e^2
)))^(1/2)))*((9*((-(4*a*c - b^2)^15)^(1/2) - b^15 + 81920*a^7*b*c^7 + 560*a
^2*b^11*c^2 - 4160*a^3*b^9*c^3 + 11520*a^4*b^7*c^4 + 1024*a^5*b^5*c^5 - 614
40*a^6*b^3*c^6 - 20*a*b^13*c))/(512*(a*b^20*e^2 + 1048576*a^11*c^10*e^2 - 4
0*a^2*b^18*c*e^2 + 720*a^3*b^16*c^2*e^2 - 7680*a^4*b^14*c^3*e^2 + 53760*a^5
*b^12*c^4*e^2 - 258048*a^6*b^10*c^5*e^2 + 860160*a^7*b^8*c^6*e^2 - 1966080*
a^8*b^6*c^7*e^2 + 2949120*a^9*b^4*c^8*e^2 - 2621440*a^10*b^2*c^9*e^2)))^(1/
2)*2i + atan(((((786432*a^6*c^8*e^12 - 192*b^12*c^2*e^12 + 3072*a*b^10*c^3*
e^12 - 15360*a^2*b^8*c^4*e^12 + 245760*a^4*b^4*c^6*e^12 - 786432*a^5*b^2*c^
7*e^12)/(128*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 38
40*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c)) + ((1024*b^15*c^2*d*e^13
- 28672*a*b^13*c^3*d*e^13 - 16777216*a^7*b*c^9*d*e^13 + 344064*a^2*b^11*c^4
*d*e^13 - 2293760*a^3*b^9*c^5*d*e^13 + 9175040*a^4*b^7*c^6*d*e^13 - 2202009
6*a^5*b^5*c^7*d*e^13 + 29360128*a^6*b^3*c^8*d*e^13)/(128*(b^12 + 4096*a^6*c
^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c
^5 - 24*a*b^10*c)) + (x*(128*b^11*c^2*e^14 - 2560*a*b^9*c^3*e^14 - 131072*a
^5*b*c^7*e^14 + 20480*a^2*b^7*c^4*e^14 - 81920*a^3*b^5*c^5*e^14 + 163840*a^
4*b^3*c^6*e^14))/(16*(b^8 + 256*a^4*c^4 + 96*a^2*b^4*c^2 - 256*a^3*b^2*c^3
- 16*a*b^6*c)))*(-(9*(b^15 + (-(4*a*c - b^2)^15)^(1/2) - 81920*a^7*b*c^7 -
560*a^2*b^11*c^2 + 4160*a^3*b^9*c^3 - 11520*a^4*b^7*c^4 - 1024*a^5*b^5*c^5
+ 61440*a^6*b^3*c^6 + 20*a*b^13*c))/(512*(a*b^20*e^2 + 1048576*a^11*c^10*e^
2 - 40*a^2*b^18*c*e^2 + 720*a^3*b^16*c^2*e^2 - 7680*a^4*b^14*c^3*e^2 + 5376
0*a^5*b^12*c^4*e^2 - 258048*a^6*b^10*c^5*e^2 + 860160*a^7*b^8*c^6*e^2 - 196
6080*a^8*b^6*c^7*e^2 + 2949120*a^9*b^4*c^8*e^2 - 2621440*a^10*b^2*c^9*e^2))
)^(1/2))*(-(9*(b^15 + (-(4*a*c - b^2)^15)^(1/2) - 81920*a^7*b*c^7 - 560*a^2
*b^11*c^2 + 4160*a^3*b^9*c^3 - 11520*a^4*b^7*c^4 - 1024*a^5*b^5*c^5 + 61440
*a^6*b^3*c^6 + 20*a*b^13*c))/(512*(a*b^20*e^2 + 1048576*a^11*c^10*e^2 - 40*
a^2*b^18*c*e^2 + 720*a^3*b^16*c^2*e^2 - 7680*a^4*b^14*c^3*e^2 + 53760*a^5*b
^12*c^4*e^2 - 258048*a^6*b^10*c^5*e^2 + 860160*a^7*b^8*c^6*e^2 - 1966080*a^
8*b^6*c^7*e^2 + 2949120*a^9*b^4*c^8*e^2 - 2621440*a^10*b^2*c^9*e^2)))^(1/2)
+ (18432*a^4*c^7*d*e^11 + 936*b^8*c^3*d*e^11 - 6912*a*b^6*c^4*d*e^11 + 115
20*a^2*b^4*c^5*d*e^11)/(128*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a
^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c)) + (x*(144*
a^2*c^5*e^12 + 117*b^4*c^3*e^12 + 72*a*b^2*c^4*e^12))/(16*(b^8 + 256*a^4*c^
4 + 96*a^2*b^4*c^2 - 256*a^3*b^2*c^3 - 16*a*b^6*c)))*(-(9*(b^15 + (-(4*a*c
- b^2)^15)^(1/2) - 81920*a^7*b*c^7 - 560*a^2*b^11*c^2 + 4160*a^3*b^9*c^3 -
11520*a^4*b^7*c^4 - 1024*a^5*b^5*c^5 + 61440*a^6*b^3*c^6 + 20*a*b^13*c))/(5
12*(a*b^20*e^2 + 1048576*a^11*c^10*e^2 - 40*a^2*b^18*c*e^2 + 720*a^3*b^16*c
^2*e^2 - 7680*a^4*b^14*c^3*e^2 + 53760*a^5*b^12*c^4*e^2 - 258048*a^6*b^10*c
^5*e^2 + 860160*a^7*b^8*c^6*e^2 - 1966080*a^8*b^6*c^7*e^2 + 2949120*a^9*b^4
*c^8*e^2 - 2621440*a^10*b^2*c^9*e^2)))^(1/2)*1i + ((18432*a^4*c^7*d*e^11 +
936*b^8*c^3*d*e^11 - 6912*a*b^6*c^4*d*e^11 + 11520*a^2*b^4*c^5*d*e^11)/(128

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*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c
^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c)) - ((786432*a^6*c^8*e^12 - 192*b^12*c^
2*e^12 + 3072*a*b^10*c^3*e^12 - 15360*a^2*b^8*c^4*e^12 + 245760*a^4*b^4*c^6
*e^12 - 786432*a^5*b^2*c^7*e^12)/(128*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^
2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c))
- ((1024*b^15*c^2*d*e^13 - 28672*a*b^13*c^3*d*e^13 - 16777216*a^7*b*c^9*d*e
^13 + 344064*a^2*b^11*c^4*d*e^13 - 2293760*a^3*b^9*c^5*d*e^13 + 9175040*a^4
*b^7*c^6*d*e^13 - 22020096*a^5*b^5*c^7*d*e^13 + 29360128*a^6*b^3*c^8*d*e^13
)/(128*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4
*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c)) + (x*(128*b^11*c^2*e^14 - 2560*
a*b^9*c^3*e^14 - 131072*a^5*b*c^7*e^14 + 20480*a^2*b^7*c^4*e^14 - 81920*a^3
*b^5*c^5*e^14 + 163840*a^4*b^3*c^6*e^14))/(16*(b^8 + 256*a^4*c^4 + 96*a^2*b
^4*c^2 - 256*a^3*b^2*c^3 - 16*a*b^6*c)))*(-(9*(b^15 + (-(4*a*c - b^2)^15)^(
1/2) - 81920*a^7*b*c^7 - 560*a^2*b^11*c^2 + 4160*a^3*b^9*c^3 - 11520*a^4*b^
7*c^4 - 1024*a^5*b^5*c^5 + 61440*a^6*b^3*c^6 + 20*a*b^13*c))/(512*(a*b^20*e
^2 + 1048576*a^11*c^10*e^2 - 40*a^2*b^18*c*e^2 + 720*a^3*b^16*c^2*e^2 - 768
0*a^4*b^14*c^3*e^2 + 53760*a^5*b^12*c^4*e^2 - 258048*a^6*b^10*c^5*e^2 + 860
160*a^7*b^8*c^6*e^2 - 1966080*a^8*b^6*c^7*e^2 + 2949120*a^9*b^4*c^8*e^2 - 2
621440*a^10*b^2*c^9*e^2)))^(1/2))*(-(9*(b^15 + (-(4*a*c - b^2)^15)^(1/2) -
81920*a^7*b*c^7 - 560*a^2*b^11*c^2 + 4160*a^3*b^9*c^3 - 11520*a^4*b^7*c^4 -
 1024*a^5*b^5*c^5 + 61440*a^6*b^3*c^6 + 20*a*b^13*c))/(512*(a*b^20*e^2 + 10
48576*a^11*c^10*e^2 - 40*a^2*b^18*c*e^2 + 720*a^3*b^16*c^2*e^2 - 7680*a^4*b
^14*c^3*e^2 + 53760*a^5*b^12*c^4*e^2 - 258048*a^6*b^10*c^5*e^2 + 860160*a^7
*b^8*c^6*e^2 - 1966080*a^8*b^6*c^7*e^2 + 2949120*a^9*b^4*c^8*e^2 - 2621440*
a^10*b^2*c^9*e^2)))^(1/2) + (x*(144*a^2*c^5*e^12 + 117*b^4*c^3*e^12 + 72*a*
b^2*c^4*e^12))/(16*(b^8 + 256*a^4*c^4 + 96*a^2*b^4*c^2 - 256*a^3*b^2*c^3 -
16*a*b^6*c)))*(-(9*(b^15 + (-(4*a*c - b^2)^15)^(1/2) - 81920*a^7*b*c^7 - 56
0*a^2*b^11*c^2 + 4160*a^3*b^9*c^3 - 11520*a^4*b^7*c^4 - 1024*a^5*b^5*c^5 +
61440*a^6*b^3*c^6 + 20*a*b^13*c))/(512*(a*b^20*e^2 + 1048576*a^11*c^10*e^2
- 40*a^2*b^18*c*e^2 + 720*a^3*b^16*c^2*e^2 - 7680*a^4*b^14*c^3*e^2 + 53760*
a^5*b^12*c^4*e^2 - 258048*a^6*b^10*c^5*e^2 + 860160*a^7*b^8*c^6*e^2 - 19660
80*a^8*b^6*c^7*e^2 + 2949120*a^9*b^4*c^8*e^2 - 2621440*a^10*b^2*c^9*e^2)))^
(1/2)*1i)/((135*b^5*c^3*e^10 + 1080*a*b^3*c^4*e^10 + 432*a^2*b*c^5*e^10)/(6
4*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*
c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c)) - (((786432*a^6*c^8*e^12 - 192*b^12*
c^2*e^12 + 3072*a*b^10*c^3*e^12 - 15360*a^2*b^8*c^4*e^12 + 245760*a^4*b^4*c
^6*e^12 - 786432*a^5*b^2*c^7*e^12)/(128*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*
c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c)
) + ((1024*b^15*c^2*d*e^13 - 28672*a*b^13*c^3*d*e^13 - 16777216*a^7*b*c^9*d
*e^13 + 344064*a^2*b^11*c^4*d*e^13 - 2293760*a^3*b^9*c^5*d*e^13 + 9175040*a
^4*b^7*c^6*d*e^13 - 22020096*a^5*b^5*c^7*d*e^13 + 29360128*a^6*b^3*c^8*d*e^
13)/(128*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a
^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c)) + (x*(128*b^11*c^2*e^14 - 256
0*a*b^9*c^3*e^14 - 131072*a^5*b*c^7*e^14 + 20480*a^2*b^7*c^4*e^14 - 81920*a
^3*b^5*c^5*e^14 + 163840*a^4*b^3*c^6*e^14))/(16*(b^8 + 256*a^4*c^4 + 96*a^2
*b^4*c^2 - 256*a^3*b^2*c^3 - 16*a*b^6*c)))*(-(9*(b^15 + (-(4*a*c - b^2)^15)
^(1/2) - 81920*a^7*b*c^7 - 560*a^2*b^11*c^2 + 4160*a^3*b^9*c^3 - 11520*a^4*
b^7*c^4 - 1024*a^5*b^5*c^5 + 61440*a^6*b^3*c^6 + 20*a*b^13*c))/(512*(a*b^20
*e^2 + 1048576*a^11*c^10*e^2 - 40*a^2*b^18*c*e^2 + 720*a^3*b^16*c^2*e^2 - 7
680*a^4*b^14*c^3*e^2 + 53760*a^5*b^12*c^4*e^2 - 258048*a^6*b^10*c^5*e^2 + 8
60160*a^7*b^8*c^6*e^2 - 1966080*a^8*b^6*c^7*e^2 + 2949120*a^9*b^4*c^8*e^2 -
 2621440*a^10*b^2*c^9*e^2)))^(1/2))*(-(9*(b^15 + (-(4*a*c - b^2)^15)^(1/2)
- 81920*a^7*b*c^7 - 560*a^2*b^11*c^2 + 4160*a^3*b^9*c^3 - 11520*a^4*b^7*c^4
 - 1024*a^5*b^5*c^5 + 61440*a^6*b^3*c^6 + 20*a*b^13*c))/(512*(a*b^20*e^2 +
1048576*a^11*c^10*e^2 - 40*a^2*b^18*c*e^2 + 720*a^3*b^16*c^2*e^2 - 7680*a^4
*b^14*c^3*e^2 + 53760*a^5*b^12*c^4*e^2 - 258048*a^6*b^10*c^5*e^2 + 860160*a
^7*b^8*c^6*e^2 - 1966080*a^8*b^6*c^7*e^2 + 2949120*a^9*b^4*c^8*e^2 - 262144
0*a^10*b^2*c^9*e^2)))^(1/2) + (18432*a^4*c^7*d*e^11 + 936*b^8*c^3*d*e^11 -
6912*a*b^6*c^4*d*e^11 + 11520*a^2*b^4*c^5*d*e^11)/(128*(b^12 + 4096*a^6*c^6
```

+ 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c)) + (x*(144*a^2*c^5*e^12 + 117*b^4*c^3*e^12 + 72*a*b^2*c^4*e^12))/(16*(b^8 + 256*a^4*c^4 + 96*a^2*b^4*c^2 - 256*a^3*b^2*c^3 - 16*a*b^6*c)))*(-(9*(b^15 + (-(4*a*c - b^2)^15)^(1/2) - 81920*a^7*b*c^7 - 560*a^2*b^11*c^2 + 4160*a^3*b^9*c^3 - 11520*a^4*b^7*c^4 - 1024*a^5*b^5*c^5 + 61440*a^6*b^3*c^6 + 20*a*b^13*c))/(512*(a*b^20*e^2 + 1048576*a^11*c^10*e^2 - 40*a^2*b^18*c*e^2 + 720*a^3*b^16*c^2*e^2 - 7680*a^4*b^14*c^3*e^2 + 53760*a^5*b^12*c^4*e^2 - 258048*a^6*b^10*c^5*e^2 + 860160*a^7*b^8*c^6*e^2 - 1966080*a^8*b^6*c^7*e^2 + 2949120*a^9*b^4*c^8*e^2 - 2621440*a^10*b^2*c^9*e^2)))^(1/2) + ((18432*a^4*c^7*d*e^11 + 936*b^8*c^3*d*e^11 - 6912*a*b^6*c^4*d*e^11 + 11520*a^2*b^4*c^5*d*e^11)/(128*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c)) - ((786432*a^6*c^8*e^12 - 192*b^12*c^2*e^12 + 3072*a*b^10*c^3*e^12 - 15360*a^2*b^8*c^4*e^12 + 245760*a^4*b^4*c^6*e^12 - 786432*a^5*b^2*c^7*e^12)/(128*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c)) - ((1024*b^15*c^2*d*e^13 - 28672*a*b^13*c^3*d*e^13 - 16777216*a^7*b*c^9*d*e^13 + 344064*a^2*b^11*c^4*d*e^13 - 2293760*a^3*b^9*c^5*d*e^13 + 9175040*a^4*b^7*c^6*d*e^13 - 22020096*a^5*b^5*c^7*d*e^13 + 29360128*a^6*b^3*c^8*d*e^13)/(128*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c)) + (x*(128*b^11*c^2*e^14 - 2560*a*b^9*c^3*e^14 - 131072*a^5*b*c^7*e^14 + 20480*a^2*b^7*c^4*e^14 - 81920*a^3*b^5*c^5*e^14 + 163840*a^4*b^3*c^6*e^14))/(16*(b^8 + 256*a^4*c^4 + 96*a^2*b^4*c^2 - 256*a^3*b^2*c^3 - 16*a*b^6*c)))*(-(9*(b^15 + (-(4*a*c - b^2)^15)^(1/2) - 81920*a^7*b*c^7 - 560*a^2*b^11*c^2 + 4160*a^3*b^9*c^3 - 11520*a^4*b^7*c^4 - 1024*a^5*b^5*c^5 + 61440*a^6*b^3*c^6 + 20*a*b^13*c))/(512*(a*b^20*e^2 + 1048576*a^11*c^10*e^2 - 40*a^2*b^18*c*e^2 + 720*a^3*b^16*c^2*e^2 - 7680*a^4*b^14*c^3*e^2 + 53760*a^5*b^12*c^4*e^2 - 258048*a^6*b^10*c^5*e^2 + 860160*a^7*b^8*c^6*e^2 - 1966080*a^8*b^6*c^7*e^2 + 2949120*a^9*b^4*c^8*e^2 - 2621440*a^10*b^2*c^9*e^2)))^(1/2))*(-(9*(b^15 + (-(4*a*c - b^2)^15)^(1/2) - 81920*a^7*b*c^7 - 560*a^2*b^11*c^2 + 4160*a^3*b^9*c^3 - 11520*a^4*b^7*c^4 - 1024*a^5*b^5*c^5 + 61440*a^6*b^3*c^6 + 20*a*b^13*c))/(512*(a*b^20*e^2 + 1048576*a^11*c^10*e^2 - 40*a^2*b^18*c*e^2 + 720*a^3*b^16*c^2*e^2 - 7680*a^4*b^14*c^3*e^2 + 53760*a^5*b^12*c^4*e^2 - 258048*a^6*b^10*c^5*e^2 + 860160*a^7*b^8*c^6*e^2 - 1966080*a^8*b^6*c^7*e^2 + 2949120*a^9*b^4*c^8*e^2 - 2621440*a^10*b^2*c^9*e^2)))^(1/2) + (x*(144*a^2*c^5*e^12 + 117*b^4*c^3*e^12 + 72*a*b^2*c^4*e^12))/(16*(b^8 + 256*a^4*c^4 + 96*a^2*b^4*c^2 - 256*a^3*b^2*c^3 - 16*a*b^6*c)))*(-(9*(b^15 + (-(4*a*c - b^2)^15)^(1/2) - 81920*a^7*b*c^7 - 560*a^2*b^11*c^2 + 4160*a^3*b^9*c^3 - 11520*a^4*b^7*c^4 - 1024*a^5*b^5*c^5 + 61440*a^6*b^3*c^6 + 20*a*b^13*c))/(512*(a*b^20*e^2 + 1048576*a^11*c^10*e^2 - 40*a^2*b^18*c*e^2 + 720*a^3*b^16*c^2*e^2 - 7680*a^4*b^14*c^3*e^2 + 53760*a^5*b^12*c^4*e^2 - 258048*a^6*b^10*c^5*e^2 + 860160*a^7*b^8*c^6*e^2 - 1966080*a^8*b^6*c^7*e^2 + 2949120*a^9*b^4*c^8*e^2 - 2621440*a^10*b^2*c^9*e^2)))^(1/2)))*(-(9*(b^15 + (-(4*a*c - b^2)^15)^(1/2) - 81920*a^7*b*c^7 - 560*a^2*b^11*c^2 + 4160*a^3*b^9*c^3 - 11520*a^4*b^7*c^4 - 1024*a^5*b^5*c^5 + 61440*a^6*b^3*c^6 + 20*a*b^13*c))/(512*(a*b^20*e^2 + 1048576*a^11*c^10*e^2 - 40*a^2*b^18*c*e^2 + 720*a^3*b^16*c^2*e^2 - 7680*a^4*b^14*c^3*e^2 + 53760*a^5*b^12*c^4*e^2 - 258048*a^6*b^10*c^5*e^2 + 860160*a^7*b^8*c^6*e^2 - 1966080*a^8*b^6*c^7*e^2 + 2949120*a^9*b^4*c^8*e^2 - 2621440*a^10*b^2*c^9*e^2)))^(1/2)*2i - ((x^2*(15*b^3*d*e - 40*a*c^2*d^3*e + 190*b^2*c*d^3*e + 252*b*c^2*d^5*e + 48*a*b*c*d*e))/(8*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (x^3*(5*b^3*e^2 - 40*a*c^2*d^2*e^2 + 190*b^2*c*d^2*e^2 + 420*b*c^2*d^4*e^2 + 16*a*b*c*e^2))/(8*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (5*x^4*(84*b*c^2*d^3*e^3 - 4*a*c^2*d*e^3 + 19*b^2*c*d*e^3))/(8*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (x^5*(19*b^2*c*e^4 - 4*a*c^2*e^4 + 252*b*c^2*d^2*e^4))/(8*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (x*(3*a*b^2 + 12*a^2*c + 15*b^3*d^2 - 20*a*c^2*d^4 + 95*b^2*c*d^4 + 84*b*c^2*d^6 + 48*a*b*c*d^2))/(8*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (5*b^3*d^3 - 4*a*c^2*d^5 + 19*b^2*c*d^5 + 12*b*c^2*d^7 + 3*a*b^2*d + 12*a^2*c*d + 16*a*b*c*d^3)/(8*e*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (3*b*c^2*e^6*x^7)/(2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (21*b*c^2*d*e^5*x^6)/(2*(b^4

```
 + 16*a^2*c^2 - 8*a*b^2*c)))/(x^2*(6*b^2*d^2*e^2 + 28*c^2*d^6*e^2 + 2*a*b*e
^2 + 12*a*c*d^2*e^2 + 30*b*c*d^4*e^2) + x^6*(28*c^2*d^2*e^6 + 2*b*c*e^6) +
x*(4*b^2*d^3*e + 8*c^2*d^7*e + 8*a*c*d^3*e + 12*b*c*d^5*e + 4*a*b*d*e) + x^
3*(4*b^2*d*e^3 + 56*c^2*d^5*e^3 + 8*a*c*d*e^3 + 40*b*c*d^3*e^3) + x^5*(56*c
^2*d^3*e^5 + 12*b*c*d*e^5) + x^4*(b^2*e^4 + 70*c^2*d^4*e^4 + 2*a*c*e^4 + 30
*b*c*d^2*e^4) + a^2 + b^2*d^4 + c^2*d^8 + c^2*e^8*x^8 + 2*a*b*d^2 + 2*a*c*d
^4 + 2*b*c*d^6 + 8*c^2*d*e^7*x^7)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**4/(a+b*(e*x+d)**2+c*(e*x+d)**4)**3,x)
```

```
[Out] Timed out
```

3.518 $\int \frac{(d+ex)^3}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^3} \, dx$

Optimal. Leaf size=150

$$\frac{2a+b(d+ex)^2}{4e(b^2-4ac)\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}-\frac{3b\left(b+2c(d+ex)^2\right)}{4e(b^2-4ac)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)}+\frac{3bc\tanh^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{e(b^2-4ac)^{5/2}}$$

Rubi [A] time = 0.20, antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1142, 1114, 638, 614, 618, 206}

$$\frac{2a+b(d+ex)^2}{4e(b^2-4ac)\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}-\frac{3b\left(b+2c(d+ex)^2\right)}{4e(b^2-4ac)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)}+\frac{3bc\tanh^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{e(b^2-4ac)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^3/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3,x]
```

```
[Out] (2*a + b*(d + e*x)^2)/(4*(b^2 - 4*a*c)*e*(a + b*(d + e*x)^2 + c*(d + e*x)^4
)^2) - (3*b*(b + 2*c*(d + e*x)^2))/(4*(b^2 - 4*a*c)^2*e*(a + b*(d + e*x)^2
+ c*(d + e*x)^4)) + (3*b*c*ArcTanh[(b + 2*c*(d + e*x)^2)/Sqrt[b^2 - 4*a*c]]
)/((b^2 - 4*a*c)^(5/2)*e)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 614

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[(2*c*(2*p +
 3))/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
egerQ[4*p]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 638

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)*(a + b*x + c*x^2)^(p + 1))/((p +
 1)*(b^2 - 4*a*c)), x] - Dist[((2*p + 3)*(2*c*d - b*e))/((p + 1)*(b^2 - 4*a
*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &&
 NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 1142

```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Di
st[u^m/(Coefficient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p,
 x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

Rubi steps

$$\int \frac{(d+ex)^3}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^3}\,dx = \frac{\text{Subst}\left(\int \frac{x^3}{\left(a+bx^2+cx^4\right)^3}\,dx, x, d+ex\right)}{e}$$

$$= \frac{\text{Subst}\left(\int \frac{x}{\left(a+bx+cx^2\right)^3}\,dx, x, (d+ex)^2\right)}{2e}$$

$$= \frac{2a+b(d+ex)^2}{4\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)^2} + \frac{(3b)\,\text{Subst}\left(\int \frac{1}{\left(a+bx+cx^2\right)^2}\right.}{4\left(b^2-4a\right.}$$

$$= \frac{2a+b(d+ex)^2}{4\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)^2} - \frac{3b\left(b+2c(d\right.}{4\left(b^2-4ac\right)^2e\left(a+b(d\right.}$$

$$= \frac{2a+b(d+ex)^2}{4\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)^2} - \frac{3b\left(b+2c(d\right.}{4\left(b^2-4ac\right)^2e\left(a+b(d\right.}$$

$$= \frac{2a+b(d+ex)^2}{4\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)^2} - \frac{3b\left(b+2c(d\right.}{4\left(b^2-4ac\right)^2e\left(a+b(d\right.}$$

Mathematica [A] time = 0.21, size = 146, normalized size = 0.97

$$\frac{\frac{\left(b^2-4ac\right)\left(2a+b(d+ex)^2\right)}{\left(a+(d+ex)^2\left(b+c(d+ex)^2\right)\right)^2} - \frac{12bc\tan^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}} - \frac{3b\left(b+2c(d+ex)^2\right)}{a+b(d+ex)^2+c(d+ex)^4}}{4e\left(b^2-4ac\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^3/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3,x]
```

```
[Out] ((-3*b*(b + 2*c*(d + e*x)^2))/(a + b*(d + e*x)^2 + c*(d + e*x)^4) + ((b^2 -
 4*a*c)*(2*a + b*(d + e*x)^2))/(a + (d + e*x)^2*(b + c*(d + e*x)^2))^2 - (1
2*b*c*ArcTan[(b + 2*c*(d + e*x)^2)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c])
/(4*(b^2 - 4*a*c)^2*e)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(d+ex)^3}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(d + e*x)^3/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3,x]
```

```
[Out] IntegrateAlgebraic[(d + e*x)^3/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3, x]
```

fricas [B] time = 1.53, size = 3739, normalized size = 24.93

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="fricas")
```

```
[Out] [-1/4*(6*(b^3*c^2 - 4*a*b*c^3)*e^6*x^6 + 36*(b^3*c^2 - 4*a*b*c^3)*d*e^5*x^5
 + 9*(b^4*c - 4*a*b^2*c^2 + 10*(b^3*c^2 - 4*a*b*c^3)*d^2)*e^4*x^4 + 6*(b^3*
c^2 - 4*a*b*c^3)*d^6 + 12*(10*(b^3*c^2 - 4*a*b*c^3)*d^3 + 3*(b^4*c - 4*a*b^
2*c^2)*d)*e^3*x^3 + a*b^4 + 4*a^2*b^2*c - 32*a^3*c^2 + 9*(b^4*c - 4*a*b^2*c
^2)*d^4 + 2*(b^5 + a*b^3*c - 20*a^2*b*c^2 + 45*(b^3*c^2 - 4*a*b*c^3)*d^4 +
27*(b^4*c - 4*a*b^2*c^2)*d^2)*e^2*x^2 + 2*(b^5 + a*b^3*c - 20*a^2*b*c^2)*d^
2 + 4*(9*(b^3*c^2 - 4*a*b*c^3)*d^5 + 9*(b^4*c - 4*a*b^2*c^2)*d^3 + (b^5 + a
*b^3*c - 20*a^2*b*c^2)*d)*e*x - 6*(b*c^3*e^8*x^8 + 8*b*c^3*d*e^7*x^7 + 2*(1
4*b*c^3*d^2 + b^2*c^2)*e^6*x^6 + b*c^3*d^8 + 4*(14*b*c^3*d^3 + 3*b^2*c^2*d)
*e^5*x^5 + 2*b^2*c^2*d^6 + (70*b*c^3*d^4 + 30*b^2*c^2*d^2 + b^3*c + 2*a*b*c
^2)*e^4*x^4 + 4*(14*b*c^3*d^5 + 10*b^2*c^2*d^3 + (b^3*c + 2*a*b*c^2)*d)*e^3
*x^3 + 2*a*b^2*c*d^2 + (b^3*c + 2*a*b*c^2)*d^4 + 2*(14*b*c^3*d^6 + 15*b^2*c
^2*d^4 + a*b^2*c + 3*(b^3*c + 2*a*b*c^2)*d^2)*e^2*x^2 + a^2*b*c + 4*(2*b*c^
3*d^7 + 3*b^2*c^2*d^5 + a*b^2*c*d + (b^3*c + 2*a*b*c^2)*d^3)*e*x)*sqrt(b^2
- 4*a*c)*log((2*c^2*e^4*x^4 + 8*c^2*d*e^3*x^3 + 2*c^2*d^4 + 2*(6*c^2*d^2 +
b*c)*e^2*x^2 + 2*b*c*d^2 + 4*(2*c^2*d^3 + b*c*d)*e*x + b^2 - 2*a*c + (2*c*e
^2*x^2 + 4*c*d*e*x + 2*c*d^2 + b)*sqrt(b^2 - 4*a*c))/(c*e^4*x^4 + 4*c*d*e^3
*x^3 + c*d^4 + (6*c*d^2 + b)*e^2*x^2 + b*d^2 + 2*(2*c*d^3 + b*d)*e*x + a)))
/((b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*e^9*x^8 + 8*(b^6*c
^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d*e^8*x^7 + 2*(b^7*c - 12*
a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4 + 14*(b^6*c^2 - 12*a*b^4*c^3 + 48
*a^2*b^2*c^4 - 64*a^3*c^5)*d^2)*e^7*x^6 + 4*(14*(b^6*c^2 - 12*a*b^4*c^3 + 4
8*a^2*b^2*c^4 - 64*a^3*c^5)*d^3 + 3*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3
- 64*a^3*b*c^4)*d)*e^6*x^5 + (b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^
2*c^3 - 128*a^4*c^4 + 70*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*
c^5)*d^4 + 30*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*d^2)*e
^5*x^4 + 4*(14*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^5 +
 10*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*d^3 + (b^8 - 10*
a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*d)*e^4*x^3 + 2*(a*
b^7 - 12*a^2*b^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3 + 14*(b^6*c^2 - 12*a*b^4
*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^6 + 15*(b^7*c - 12*a*b^5*c^2 + 48*a^2
*b^3*c^3 - 64*a^3*b*c^4)*d^4 + 3*(b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^
3*b^2*c^3 - 128*a^4*c^4)*d^2)*e^3*x^2 + 4*(2*(b^6*c^2 - 12*a*b^4*c^3 + 48*a
^2*b^2*c^4 - 64*a^3*c^5)*d^7 + 3*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 6
4*a^3*b*c^4)*d^5 + (b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 12
8*a^4*c^4)*d^3 + (a*b^7 - 12*a^2*b^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3)*d)*
e^2*x + ((b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^8 + a^2*b
^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3 + 2*(b^7*c - 12*a*b^5*c^2 +
 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*d^6 + (b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 +
32*a^3*b^2*c^3 - 128*a^4*c^4)*d^4 + 2*(a*b^7 - 12*a^2*b^5*c + 48*a^3*b^3*c^
2 - 64*a^4*b*c^3)*d^2)*e), -1/4*(6*(b^3*c^2 - 4*a*b*c^3)*e^6*x^6 + 36*(b^3*
c^2 - 4*a*b*c^3)*d*e^5*x^5 + 9*(b^4*c - 4*a*b^2*c^2 + 10*(b^3*c^2 - 4*a*b*c
^3)*d^2)*e^4*x^4 + 6*(b^3*c^2 - 4*a*b*c^3)*d^6 + 12*(10*(b^3*c^2 - 4*a*b*c^
3)*d^3 + 3*(b^4*c - 4*a*b^2*c^2)*d)*e^3*x^3 + a*b^4 + 4*a^2*b^2*c - 32*a^3*
c^2 + 9*(b^4*c - 4*a*b^2*c^2)*d^4 + 2*(b^5 + a*b^3*c - 20*a^2*b*c^2 + 45*(b
^3*c^2 - 4*a*b*c^3)*d^4 + 27*(b^4*c - 4*a*b^2*c^2)*d^2)*e^2*x^2 + 2*(b^5 +
a*b^3*c - 20*a^2*b*c^2)*d^2 + 4*(9*(b^3*c^2 - 4*a*b*c^3)*d^5 + 9*(b^4*c - 4
*a*b^2*c^2)*d^3 + (b^5 + a*b^3*c - 20*a^2*b*c^2)*d)*e*x - 12*(b*c^3*e^8*x^8
 + 8*b*c^3*d*e^7*x^7 + 2*(14*b*c^3*d^2 + b^2*c^2)*e^6*x^6 + b*c^3*d^8 + 4*(
14*b*c^3*d^3 + 3*b^2*c^2*d)*e^5*x^5 + 2*b^2*c^2*d^6 + (70*b*c^3*d^4 + 30*b^
2*c^2*d^2 + b^3*c + 2*a*b*c^2)*e^4*x^4 + 4*(14*b*c^3*d^5 + 10*b^2*c^2*d^3 +
```

```
(b^3*c + 2*a*b*c^2)*d)*e^3*x^3 + 2*a*b^2*c*d^2 + (b^3*c + 2*a*b*c^2)*d^4 +
 2*(14*b*c^3*d^6 + 15*b^2*c^2*d^4 + a*b^2*c + 3*(b^3*c + 2*a*b*c^2)*d^2)*e^
2*x^2 + a^2*b*c + 4*(2*b*c^3*d^7 + 3*b^2*c^2*d^5 + a*b^2*c*d + (b^3*c + 2*a
*b*c^2)*d^3)*e*x)*sqrt(-b^2 + 4*a*c)*arctan(-(2*c*e^2*x^2 + 4*c*d*e*x + 2*c
*d^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)))/((b^6*c^2 - 12*a*b^4*c^3 + 48*
a^2*b^2*c^4 - 64*a^3*c^5)*e^9*x^8 + 8*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*
c^4 - 64*a^3*c^5)*d*e^8*x^7 + 2*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64
*a^3*b*c^4 + 14*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^2)
*e^7*x^6 + 4*(14*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^3
 + 3*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*d)*e^6*x^5 + (b
^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4 + 70*(b^6*c
^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^4 + 30*(b^7*c - 12*a*b^5
*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*d^2)*e^5*x^4 + 4*(14*(b^6*c^2 - 12*a*
b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^5 + 10*(b^7*c - 12*a*b^5*c^2 + 48*
a^2*b^3*c^3 - 64*a^3*b*c^4)*d^3 + (b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a
^3*b^2*c^3 - 128*a^4*c^4)*d)*e^4*x^3 + 2*(a*b^7 - 12*a^2*b^5*c + 48*a^3*b^3
*c^2 - 64*a^4*b*c^3 + 14*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*
c^5)*d^6 + 15*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*d^4 +
3*(b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*d^2)*e
^3*x^2 + 4*(2*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^7 +
3*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*d^5 + (b^8 - 10*a*
b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*d^3 + (a*b^7 - 12*a^
2*b^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3)*d)*e^2*x + ((b^6*c^2 - 12*a*b^4*c^
3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^8 + a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*
c^2 - 64*a^5*c^3 + 2*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)
*d^6 + (b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*d
^4 + 2*(a*b^7 - 12*a^2*b^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3)*d^2)*e)]
```

giac [B] time = 0.69, size = 365, normalized size = 2.43

$$-\frac{3\,bc\arctan\left(\frac{2\,cd^{2}+2\,(x^{2}e+2\,dx)ce+b}{\sqrt{-b^{2}+4\,ac}}\right)e^{(-1)}}{(b^{4}-8\,ab^{2}c+16\,a^{2}c^{2})\sqrt{-b^{2}+4\,ac}}-\frac{6\,bc^{2}d^{6}+18\,(x^{2}e+2\,dx)bc^{2}d^{4}e+18\,(x^{2}e+2\,dx)^{2}bc^{2}d^{2}e^{2}+9\,b^{2}cd^{4}+6\,(x^{2}e+2\,dx)^{3}bc^{2}e^{3}+18\,(x^{2}e+2\,dx)b^{2}cd^{2}e+9\,(x^{2}e+2\,dx)^{2}b^{2}ce^{2}+2\,b^{3}d^{2}+10\,abcd^{2}+2\,(x^{2}e+2\,dx)b^{3}e+10\,(x^{2}e+2\,dx)abce+ab^{2}+8\,a^{2}c}{4\left(cd^{4}+2\,(x^{2}e+2\,dx)cd^{2}e+(x^{2}e+2\,dx)^{2}ce^{2}+bd^{2}+(x^{2}e+2\,dx)be+a\right)^{2}(b^{4}e-8\,ab^{2}ce+16\,a^{2}c^{2}e)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="giac")
```

```
[Out] -3*b*c*arctan((2*c*d^2 + 2*(x^2*e + 2*d*x)*c*e + b)/sqrt(-b^2 + 4*a*c))*e^(
-1)/((b^4 - 8*a*b^2*c + 16*a^2*c^2)*sqrt(-b^2 + 4*a*c)) - 1/4*(6*b*c^2*d^6
+ 18*(x^2*e + 2*d*x)*b*c^2*d^4*e + 18*(x^2*e + 2*d*x)^2*b*c^2*d^2*e^2 + 9*b
^2*c*d^4 + 6*(x^2*e + 2*d*x)^3*b*c^2*e^3 + 18*(x^2*e + 2*d*x)*b^2*c*d^2*e +
 9*(x^2*e + 2*d*x)^2*b^2*c*e^2 + 2*b^3*d^2 + 10*a*b*c*d^2 + 2*(x^2*e + 2*d*
x)*b^3*e + 10*(x^2*e + 2*d*x)*a*b*c*e + a*b^2 + 8*a^2*c)/((c*d^4 + 2*(x^2*e
 + 2*d*x)*c*d^2*e + (x^2*e + 2*d*x)^2*c*e^2 + b*d^2 + (x^2*e + 2*d*x)*b*e +
 a)^2*(b^4*e - 8*a*b^2*c*e + 16*a^2*c^2*e))
```

maple [C] time = 0.05, size = 544, normalized size = 3.63

$$\frac{3bc\left(-\mathrm{RootOf}\left(\_Z^{4}ce^{4}+4\_Z^{3}cde^{3}+cd^{4}+bd^{2}+(6cd^{2}e^{2}+be^{2})\_Z^{2}+(4cd^{3}e+2deb)\_Z+a\right)e-d\right)\ln\left(-\mathrm{RootOf}\left(\_Z^{4}ce^{4}+4\_Z^{3}cde^{3}+cd^{4}+bd^{2}+(6cd^{2}e^{2}+be^{2})\_Z^{2}+(4cd^{3}e+2deb)\_Z+a\right)+x\right)}{2(16a^{2}c^{2}-8ab^{2}c+b^{4})e\left(2ce^{3}\mathrm{RootOf}\left(\_Z^{4}ce^{4}+4\_Z^{3}cde^{3}+cd^{4}+bd^{2}+(6cd^{2}e^{2}+be^{2})\_Z^{2}+(4cd^{3}e+2deb)\_Z+a\right)^{3}+6cde^{2}\mathrm{RootOf}\left(\_Z^{4}ce^{4}+4\_Z^{3}cde^{3}+cd^{4}+bd^{2}+(6cd^{2}e^{2}+be^{2})\_Z^{2}+(4cd^{3}e+2deb)\_Z+a\right)^{2}+6ecd^{2}\mathrm{RootOf}\left(\_Z^{4}ce^{4}+4\_Z^{3}cde^{3}+cd^{4}+bd^{2}+(6cd^{2}e^{2}+be^{2})\_Z^{2}+(4cd^{3}e+2deb)\_Z+a\right)+2cd^{3}+be\,\mathrm{RootOf}\left(\_Z^{4}ce^{4}+4\_Z^{3}cde^{3}+cd^{4}+bd^{2}+(6cd^{2}e^{2}+be^{2})\_Z^{2}+(4cd^{3}e+2deb)\_Z+a\right)+bd\right)}+\frac{-\frac{3bc^{2}e^{5}x^{6}}{2(16a^{2}c^{2}-8ab^{2}c+b^{4})}-\frac{9bc^{2}de^{4}x^{5}}{16a^{2}c^{2}-8ab^{2}c+b^{4}}-\frac{9(10cd^{2}+b)bce^{3}x^{4}}{4(16a^{2}c^{2}-8ab^{2}c+b^{4})}-\frac{3(10cd^{2}+3b)bcde^{2}x^{3}}{16a^{2}c^{2}-8ab^{2}c+b^{4}}-\frac{(45c^{2}d^{4}+27bcd^{2}+5ac+b^{2})bex^{2}}{2(16a^{2}c^{2}-8ab^{2}c+b^{4})}-\frac{(9c^{2}d^{4}+9bcd^{2}+5ac+b^{2})bdx}{16a^{2}c^{2}-8ab^{2}c+b^{4}}-\frac{6bc^{2}d^{6}+9b^{2}cd^{4}+10abcd^{2}+2b^{3}d^{2}+8a^{2}c+ab^{2}}{4(16a^{2}c^{2}-8ab^{2}c+b^{4})e}}{(ce^{4}x^{4}+4cde^{3}x^{3}+6cd^{2}e^{2}x^{2}+4cd^{3}ex+be^{2}x^{2}+cd^{4}+2bdex+bd^{2}+a)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x)
```

```
[Out] (-3/2*c^2*e^5*b/(16*a^2*c^2-8*a*b^2*c+b^4)*x^6-9*e^4*b*c^2*d/(16*a^2*c^2-8*
a*b^2*c+b^4)*x^5-9/4*b*c*e^3*(10*c*d^2+b)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^4-3*
c*d*e^2*b*(10*c*d^2+3*b)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^3-1/2*b*e*(45*c^2*d^4
+27*b*c*d^2+5*a*c+b^2)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^2-d*b*(9*c^2*d^4+9*b*c*
d^2+5*a*c+b^2)/(16*a^2*c^2-8*a*b^2*c+b^4)*x-1/4/e*(6*b*c^2*d^6+9*b^2*c*d^4+
10*a*b*c*d^2+2*b^3*d^2+8*a^2*c+a*b^2)/(16*a^2*c^2-8*a*b^2*c+b^4))/(c*e^4*x^
4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2
+a)^2+3/2*b*c/(16*a^2*c^2-8*a*b^2*c+b^4)/e*sum((-_R*e-d)/(2*_R^3*c*e^3+6*_R
```

```
^2*c*d*e^2+6*_R*c*d^2*e+2*c*d^3+_R*b*e+b*d)*ln(-_R+x),_R=RootOf(_Z^4*c*e^4+
4*_Z^3*c*d*e^3+c*d^4+b*d^2+(6*c*d^2*e^2+b*e^2)*_Z^2+(4*c*d^3*e+2*b*d*e)*_Z+
a))
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [B] time = 3.85, size = 1182, normalized size = 7.88

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^3/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3,x)
```

```
[Out] - ((9*x^4*(b^2*c*e^3 + 10*b*c^2*d^2*e^3))/(4*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)
) + (a*b^2 + 8*a^2*c + 2*b^3*d^2 + 9*b^2*c*d^4 + 6*b*c^2*d^6 + 10*a*b*c*d^2
)/(4*e*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (x^2*(b^3*e + 27*b^2*c*d^2*e + 45*
b*c^2*d^4*e + 5*a*b*c*e))/(2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (3*d*x^3*(3*
b^2*c*e^2 + 10*b*c^2*d^2*e^2))/(b^4 + 16*a^2*c^2 - 8*a*b^2*c) + (d*x*(b^3 +
 9*b^2*c*d^2 + 9*b*c^2*d^4 + 5*a*b*c))/(b^4 + 16*a^2*c^2 - 8*a*b^2*c) + (3*
b*c^2*e^5*x^6)/(2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (9*b*c^2*d*e^4*x^5)/(b^
4 + 16*a^2*c^2 - 8*a*b^2*c))/(x^2*(6*b^2*d^2*e^2 + 28*c^2*d^6*e^2 + 2*a*b*e
^2 + 12*a*c*d^2*e^2 + 30*b*c*d^4*e^2) + x^6*(28*c^2*d^2*e^6 + 2*b*c*e^6) +
x*(4*b^2*d^3*e + 8*c^2*d^7*e + 8*a*c*d^3*e + 12*b*c*d^5*e + 4*a*b*d*e) + x^
3*(4*b^2*d*e^3 + 56*c^2*d^5*e^3 + 8*a*c*d*e^3 + 40*b*c*d^3*e^3) + x^5*(56*c
^2*d^3*e^5 + 12*b*c*d*e^5) + x^4*(b^2*e^4 + 70*c^2*d^4*e^4 + 2*a*c*e^4 + 30
*b*c*d^2*e^4) + a^2 + b^2*d^4 + c^2*d^8 + c^2*e^8*x^8 + 2*a*b*d^2 + 2*a*c*d
^4 + 2*b*c*d^6 + 8*c^2*d*e^7*x^7) - (3*b*c*atan(((b^4*(4*a*c - b^2)^5 + 16*
a^2*c^2*(4*a*c - b^2)^5 - 8*a*b^2*c*(4*a*c - b^2)^5)*(x^2*((9*b^2*c^4*e^8)/
(a*(4*a*c - b^2)^(9/2)*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (9*b^3*c^2*(2*b^5*
c^2*e^10 - 16*a*b^3*c^3*e^10 + 32*a^2*b*c^4*e^10))/(2*a*e^2*(4*a*c - b^2)^(
15/2)*(b^4 + 16*a^2*c^2 - 8*a*b^2*c))) + x*((18*b^2*c^4*d*e^7)/(a*(4*a*c -
b^2)^(9/2)*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (9*b^3*c^2*(2*b^5*c^2*d*e^9 -
16*a*b^3*c^3*d*e^9 + 32*a^2*b*c^4*d*e^9))/(a*e^2*(4*a*c - b^2)^(15/2)*(b^4
+ 16*a^2*c^2 - 8*a*b^2*c))) + (9*b^3*c^2*(64*a^3*c^4*e^8 + 4*a*b^4*c^2*e^8
- 32*a^2*b^2*c^3*e^8 + 2*b^5*c^2*d^2*e^8 - 16*a*b^3*c^3*d^2*e^8 + 32*a^2*b*
c^4*d^2*e^8))/(2*a*e^2*(4*a*c - b^2)^(15/2)*(b^4 + 16*a^2*c^2 - 8*a*b^2*c))
 + (9*b^2*c^4*d^2*e^6)/(a*(4*a*c - b^2)^(9/2)*(b^4 + 16*a^2*c^2 - 8*a*b^2*c
))))/(18*b^2*c^4*e^6)))/(e*(4*a*c - b^2)^(5/2))
```

sympy [B] time = 14.45, size = 1671, normalized size = 11.14

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3/(a+b*(e*x+d)**2+c*(e*x+d)**4)**3,x)
```

```
[Out] 3*b*c*sqrt(-1/(4*a*c - b**2)**5)*log(2*d*x/e + x**2 + (-192*a**3*b*c**4*sqr
t(-1/(4*a*c - b**2)**5) + 144*a**2*b**3*c**3*sqrt(-1/(4*a*c - b**2)**5) - 3
6*a*b**5*c**2*sqrt(-1/(4*a*c - b**2)**5) + 3*b**7*c*sqrt(-1/(4*a*c - b**2)*
*5) + 3*b**2*c + 6*b*c**2*d**2)/(6*b*c**2*e**2))/(2*e) - 3*b*c*sqrt(-1/(4*a
*c - b**2)**5)*log(2*d*x/e + x**2 + (192*a**3*b*c**4*sqrt(-1/(4*a*c - b**2)
**5) - 144*a**2*b**3*c**3*sqrt(-1/(4*a*c - b**2)**5) + 36*a*b**5*c**2*sqrt(
-1/(4*a*c - b**2)**5) - 3*b**7*c*sqrt(-1/(4*a*c - b**2)**5) + 3*b**2*c + 6*
```

```
b*c**2*d**2)/(6*b*c**2*e**2))/(2*e) + (-8*a**2*c - a*b**2 - 10*a*b*c*d**2 -
 2*b**3*d**2 - 9*b**2*c*d**4 - 6*b*c**2*d**6 - 36*b*c**2*d*e**5*x**5 - 6*b*
c**2*e**6*x**6 + x**4*(-9*b**2*c*e**4 - 90*b*c**2*d**2*e**4) + x**3*(-36*b*
*2*c*d*e**3 - 120*b*c**2*d**3*e**3) + x**2*(-10*a*b*c*e**2 - 2*b**3*e**2 -
54*b**2*c*d**2*e**2 - 90*b*c**2*d**4*e**2) + x*(-20*a*b*c*d*e - 4*b**3*d*e
- 36*b**2*c*d**3*e - 36*b*c**2*d**5*e))/(64*a**4*c**2*e - 32*a**3*b**2*c*e
+ 128*a**3*b*c**2*d**2*e + 128*a**3*c**3*d**4*e + 4*a**2*b**4*e - 64*a**2*b
**3*c*d**2*e + 128*a**2*b*c**3*d**6*e + 64*a**2*c**4*d**8*e + 8*a*b**5*d**2
*e - 24*a*b**4*c*d**4*e - 64*a*b**3*c**2*d**6*e - 32*a*b**2*c**3*d**8*e + 4
*b**6*d**4*e + 8*b**5*c*d**6*e + 4*b**4*c**2*d**8*e + x**8*(64*a**2*c**4*e*
*9 - 32*a*b**2*c**3*e**9 + 4*b**4*c**2*e**9) + x**7*(512*a**2*c**4*d*e**8 -
 256*a*b**2*c**3*d*e**8 + 32*b**4*c**2*d*e**8) + x**6*(128*a**2*b*c**3*e**7
 + 1792*a**2*c**4*d**2*e**7 - 64*a*b**3*c**2*e**7 - 896*a*b**2*c**3*d**2*e*
*7 + 8*b**5*c*e**7 + 112*b**4*c**2*d**2*e**7) + x**5*(768*a**2*b*c**3*d*e**
6 + 3584*a**2*c**4*d**3*e**6 - 384*a*b**3*c**2*d*e**6 - 1792*a*b**2*c**3*d*
*3*e**6 + 48*b**5*c*d*e**6 + 224*b**4*c**2*d**3*e**6) + x**4*(128*a**3*c**3
*e**5 + 1920*a**2*b*c**3*d**2*e**5 + 4480*a**2*c**4*d**4*e**5 - 24*a*b**4*c
*e**5 - 960*a*b**3*c**2*d**2*e**5 - 2240*a*b**2*c**3*d**4*e**5 + 4*b**6*e**
5 + 120*b**5*c*d**2*e**5 + 280*b**4*c**2*d**4*e**5) + x**3*(512*a**3*c**3*d
*e**4 + 2560*a**2*b*c**3*d**3*e**4 + 3584*a**2*c**4*d**5*e**4 - 96*a*b**4*c
*d*e**4 - 1280*a*b**3*c**2*d**3*e**4 - 1792*a*b**2*c**3*d**5*e**4 + 16*b**6
*d*e**4 + 160*b**5*c*d**3*e**4 + 224*b**4*c**2*d**5*e**4) + x**2*(128*a**3*
b*c**2*e**3 + 768*a**3*c**3*d**2*e**3 - 64*a**2*b**3*c*e**3 + 1920*a**2*b*c
**3*d**4*e**3 + 1792*a**2*c**4*d**6*e**3 + 8*a*b**5*e**3 - 144*a*b**4*c*d**
2*e**3 - 960*a*b**3*c**2*d**4*e**3 - 896*a*b**2*c**3*d**6*e**3 + 24*b**6*d*
*2*e**3 + 120*b**5*c*d**4*e**3 + 112*b**4*c**2*d**6*e**3) + x*(256*a**3*b*c
**2*d*e**2 + 512*a**3*c**3*d**3*e**2 - 128*a**2*b**3*c*d*e**2 + 768*a**2*b*
c**3*d**5*e**2 + 512*a**2*c**4*d**7*e**2 + 16*a*b**5*d*e**2 - 96*a*b**4*c*d
**3*e**2 - 384*a*b**3*c**2*d**5*e**2 - 256*a*b**2*c**3*d**7*e**2 + 16*b**6*
d**3*e**2 + 48*b**5*c*d**5*e**2 + 32*b**4*c**2*d**7*e**2))
```

3.519 $\int \frac{(d+ex)^2}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^3}\,dx$

Optimal. Leaf size=363

$$-\frac{(d+ex)\left(b+2c(d+ex)^2\right)}{4e\left(b^2-4ac\right)\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}+\frac{(d+ex)\left(c\left(20ac+b^2\right)(d+ex)^2+b\left(8ac+b^2\right)\right)}{8ae\left(b^2-4ac\right)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)}+\frac{\sqrt{c}\left(\frac{b\left(b^2-52ac\right)}{\sqrt{b^2-4ac}}+\right.}{8\sqrt{2}\,ae\left(b^2\right.}$$

Rubi [A] time = 1.04, antiderivative size = 363, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1142, 1119, 1178, 1166, 205}

$$-\frac{(d+ex)\left(b+2c(d+ex)^2\right)}{4e\left(b^2-4ac\right)\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}+\frac{(d+ex)\left(c\left(20ac+b^2\right)(d+ex)^2+b\left(8ac+b^2\right)\right)}{8ae\left(b^2-4ac\right)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)}+\frac{\sqrt{c}\left(\frac{b\left(b^2-52ac\right)}{\sqrt{b^2-4ac}}+20ac+b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{8\sqrt{2}\,ae\left(b^2-4ac\right)^2\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{c}\left(-\frac{b\left(b^2-52ac\right)}{\sqrt{b^2-4ac}}+20ac+b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{8\sqrt{2}\,ae\left(b^2-4ac\right)^2\sqrt{\sqrt{b^2-4ac}+b}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^2/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3,x]
```

```
[Out] -((d + e*x)*(b + 2*c*(d + e*x)^2))/(4*(b^2 - 4*a*c)*e*(a + b*(d + e*x)^2 +
c*(d + e*x)^4)^2) + ((d + e*x)*(b*(b^2 + 8*a*c) + c*(b^2 + 20*a*c)*(d + e*x
)^2))/(8*a*(b^2 - 4*a*c)^2*e*(a + b*(d + e*x)^2 + c*(d + e*x)^4)) + (Sqrt[c
]*(b^2 + 20*a*c + (b*(b^2 - 52*a*c))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqr
t[c]*(d + e*x))/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(8*Sqrt[2]*a*(b^2 - 4*a*c)^2*
Sqrt[b - Sqrt[b^2 - 4*a*c]]*e) + (Sqrt[c]*(b^2 + 20*a*c - (b*(b^2 - 52*a*c)
)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b + Sqrt[b^2 -
 4*a*c]]])/(8*Sqrt[2]*a*(b^2 - 4*a*c)^2*Sqrt[b + Sqrt[b^2 - 4*a*c]]*e)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1119

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Simp[(d*(d*x)^(m - 1)*(b + 2*c*x^2)*(a + b*x^2 + c*x^4)^(p + 1))/(2*(p
+ 1)*(b^2 - 4*a*c)), x] - Dist[d^2/(2*(p + 1)*(b^2 - 4*a*c)), Int[(d*x)^(m
- 2)*(b*(m - 1) + 2*c*(m + 4*p + 5)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x
] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && GtQ[m,
1] && LeQ[m, 3] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1142

```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Di
st[u^m/(Coefficient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p,
 x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1178

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[(x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*(a + b*x^2 +
 c*x^4)^(p + 1))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2
 - 4*a*c)), Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p + 7
)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
LtQ[p, -1] && IntegerQ[2*p]
```

Rubi steps

$$\int \frac{(d+ex)^2}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^3}\,dx = \frac{\text{Subst}\left(\int \frac{x^2}{\left(a+bx^2+cx^4\right)^3}\,dx, x, d+ex\right)}{e}$$

$$= -\frac{(d+ex)\left(b+2c(d+ex)^2\right)}{4\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)^2} + \frac{\text{Subst}\left(\int \frac{b-10cx^2}{\left(a+bx^2+cx^4\right)^2}\,d\right.}{4\left(b^2-4ac\right)}$$

$$= -\frac{(d+ex)\left(b+2c(d+ex)^2\right)}{4\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)^2} + \frac{(d+ex)\left(b\left(b^2+8ac\right)\right.}{8a\left(b^2-4ac\right)^2e\left(a+\right.}$$

$$= -\frac{(d+ex)\left(b+2c(d+ex)^2\right)}{4\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)^2} + \frac{(d+ex)\left(b\left(b^2+8ac\right)\right.}{8a\left(b^2-4ac\right)^2e\left(a+\right.}$$

$$= -\frac{(d+ex)\left(b+2c(d+ex)^2\right)}{4\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)^2} + \frac{(d+ex)\left(b\left(b^2+8ac\right)\right.}{8a\left(b^2-4ac\right)^2e\left(a+\right.}$$

Mathematica [A] time = 4.86, size = 382, normalized size = 1.05

$$\frac{-\frac{4\left(b(d+ex)+2c(d+ex)^3\right)}{\left(b^2-4ac\right)\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}+\frac{2(d+ex)\left(8abc+20ac^2(d+ex)^2+b^3+b^2c(d+ex)^2\right)}{a\left(b^2-4ac\right)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)}+\frac{\sqrt{2}\sqrt{c}\left(b^2\sqrt{b^2-4ac}+20ac\sqrt{b^2-4ac}-52abc+b^3\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{a\left(b^2-4ac\right)^{5/2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{2}\sqrt{c}\left(b^2\sqrt{b^2-4ac}+20ac\sqrt{b^2-4ac}+52abc-b^3\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{a\left(b^2-4ac\right)^{5/2}\sqrt{\sqrt{b^2-4ac}+b}}}{16e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^2/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3,x]
```

```
[Out] ((-4*(b*(d + e*x) + 2*c*(d + e*x)^3))/((b^2 - 4*a*c)*(a + b*(d + e*x)^2 + c
*(d + e*x)^4)^2) + (2*(d + e*x)*(b^3 + 8*a*b*c + b^2*c*(d + e*x)^2 + 20*a*c
^2*(d + e*x)^2))/(a*(b^2 - 4*a*c)^2*(a + b*(d + e*x)^2 + c*(d + e*x)^4)) +
(Sqrt[2]*Sqrt[c]*(b^3 - 52*a*b*c + b^2*Sqrt[b^2 - 4*a*c] + 20*a*c*Sqrt[b^2
- 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/
(a*(b^2 - 4*a*c)^(5/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[2]*Sqrt[c]*(-b^
3 + 52*a*b*c + b^2*Sqrt[b^2 - 4*a*c] + 20*a*c*Sqrt[b^2 - 4*a*c])*ArcTan[(Sq
rt[2]*Sqrt[c]*(d + e*x))/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(a*(b^2 - 4*a*c)^(5/
2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(16*e)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(d+ex)^2}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(d + e*x)^2/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3,x]
[Out] IntegrateAlgebraic[(d + e*x)^2/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3, x]
```

fricas [B] time = 2.26, size = 7701, normalized size = 21.21

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="fricas")
[Out] 1/16*(2*(b^2*c^2 + 20*a*c^3)*e^7*x^7 + 14*(b^2*c^2 + 20*a*c^3)*d*e^6*x^6 +
2*(2*b^3*c + 28*a*b*c^2 + 21*(b^2*c^2 + 20*a*c^3)*d^2)*e^5*x^5 + 10*(7*(b^2
*c^2 + 20*a*c^3)*d^3 + 2*(b^3*c + 14*a*b*c^2)*d)*e^4*x^4 + 2*(b^2*c^2 + 20*
a*c^3)*d^7 + 2*(35*(b^2*c^2 + 20*a*c^3)*d^4 + b^4 + 5*a*b^2*c + 36*a^2*c^2
+ 20*(b^3*c + 14*a*b*c^2)*d^2)*e^3*x^3 + 4*(b^3*c + 14*a*b*c^2)*d^5 + 2*(21
*(b^2*c^2 + 20*a*c^3)*d^5 + 20*(b^3*c + 14*a*b*c^2)*d^3 + 3*(b^4 + 5*a*b^2*
c + 36*a^2*c^2)*d)*e^2*x^2 + 2*(b^4 + 5*a*b^2*c + 36*a^2*c^2)*d^3 + 2*(7*(b
^2*c^2 + 20*a*c^3)*d^6 + 10*(b^3*c + 14*a*b*c^2)*d^4 - a*b^3 + 16*a^2*b*c +
 3*(b^4 + 5*a*b^2*c + 36*a^2*c^2)*d^2)*e*x - sqrt(1/2)*((a*b^4*c^2 - 8*a^2*
b^2*c^3 + 16*a^3*c^4)*e^9*x^8 + 8*(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*
d*e^8*x^7 + 2*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3 + 14*(a*b^4*c^2 - 8*a
^2*b^2*c^3 + 16*a^3*c^4)*d^2)*e^7*x^6 + 4*(14*(a*b^4*c^2 - 8*a^2*b^2*c^3 +
16*a^3*c^4)*d^3 + 3*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*d)*e^6*x^5 + (
a*b^6 - 6*a^2*b^4*c + 32*a^4*c^3 + 70*(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c
^4)*d^4 + 30*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*d^2)*e^5*x^4 + 4*(14*
(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*d^5 + 10*(a*b^5*c - 8*a^2*b^3*c^2
+ 16*a^3*b*c^3)*d^3 + (a*b^6 - 6*a^2*b^4*c + 32*a^4*c^3)*d)*e^4*x^3 + 2*(a^
2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2 + 14*(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3
*c^4)*d^6 + 15*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*d^4 + 3*(a*b^6 - 6*
a^2*b^4*c + 32*a^4*c^3)*d^2)*e^3*x^2 + 4*(2*(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16
*a^3*c^4)*d^7 + 3*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*d^5 + (a*b^6 - 6
*a^2*b^4*c + 32*a^4*c^3)*d^3 + (a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*d)*e^
2*x + ((a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*d^8 + a^3*b^4 - 8*a^4*b^2*c
 + 16*a^5*c^2 + 2*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*d^6 + (a*b^6 - 6
*a^2*b^4*c + 32*a^4*c^3)*d^4 + 2*(a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*d^2
)*e)*sqrt(-(b^7 - 35*a*b^5*c + 280*a^2*b^3*c^2 + 1680*a^3*b*c^3 + (a^3*b^10
 - 20*a^4*b^8*c + 160*a^5*b^6*c^2 - 640*a^6*b^4*c^3 + 1280*a^7*b^2*c^4 - 10
24*a^8*c^5)*e^2*sqrt((b^4 - 50*a*b^2*c + 625*a^2*c^2)/((a^6*b^10 - 20*a^7*b
^8*c + 160*a^8*b^6*c^2 - 640*a^9*b^4*c^3 + 1280*a^10*b^2*c^4 - 1024*a^11*c^
5)*e^4)))/((a^3*b^10 - 20*a^4*b^8*c + 160*a^5*b^6*c^2 - 640*a^6*b^4*c^3 + 1
280*a^7*b^2*c^4 - 1024*a^8*c^5)*e^2))*log((35*b^6*c^2 - 1491*a*b^4*c^3 + 15
000*a^2*b^2*c^4 + 10000*a^3*c^5)*e*x + (35*b^6*c^2 - 1491*a*b^4*c^3 + 15000
*a^2*b^2*c^4 + 10000*a^3*c^5)*d + 1/2*sqrt(1/2)*((a^3*b^14 - 38*a^4*b^12*c
+ 480*a^5*b^10*c^2 - 2720*a^6*b^8*c^3 + 6400*a^7*b^6*c^4 + 1536*a^8*b^4*c^5
 - 32768*a^9*b^2*c^6 + 40960*a^10*c^7)*e^3*sqrt((b^4 - 50*a*b^2*c + 625*a^2
*c^2)/((a^6*b^10 - 20*a^7*b^8*c + 160*a^8*b^6*c^2 - 640*a^9*b^4*c^3 + 1280*
a^10*b^2*c^4 - 1024*a^11*c^5)*e^4)) - (b^11 - 53*a*b^9*c + 940*a^2*b^7*c^2
- 6832*a^3*b^5*c^3 + 21824*a^4*b^3*c^4 - 25600*a^5*b*c^5)*e)*sqrt(-(b^7 - 3
5*a*b^5*c + 280*a^2*b^3*c^2 + 1680*a^3*b*c^3 + (a^3*b^10 - 20*a^4*b^8*c + 1
60*a^5*b^6*c^2 - 640*a^6*b^4*c^3 + 1280*a^7*b^2*c^4 - 1024*a^8*c^5)*e^2*sqr
t((b^4 - 50*a*b^2*c + 625*a^2*c^2)/((a^6*b^10 - 20*a^7*b^8*c + 160*a^8*b^6*
c^2 - 640*a^9*b^4*c^3 + 1280*a^10*b^2*c^4 - 1024*a^11*c^5)*e^4)))/((a^3*b^1
0 - 20*a^4*b^8*c + 160*a^5*b^6*c^2 - 640*a^6*b^4*c^3 + 1280*a^7*b^2*c^4 - 1
024*a^8*c^5)*e^2))) + sqrt(1/2)*((a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*e
^9*x^8 + 8*(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*d*e^8*x^7 + 2*(a*b^5*c
- 8*a^2*b^3*c^2 + 16*a^3*b*c^3 + 14*(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4
)*d^2)*e^7*x^6 + 4*(14*(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*d^3 + 3*(a*
b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*d)*e^6*x^5 + (a*b^6 - 6*a^2*b^4*c + 3
```

```
2*a^4*c^3 + 70*(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*d^4 + 30*(a*b^5*c -
 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*d^2)*e^5*x^4 + 4*(14*(a*b^4*c^2 - 8*a^2*b^2*
c^3 + 16*a^3*c^4)*d^5 + 10*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*d^3 + (
a*b^6 - 6*a^2*b^4*c + 32*a^4*c^3)*d)*e^4*x^3 + 2*(a^2*b^5 - 8*a^3*b^3*c + 1
6*a^4*b*c^2 + 14*(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*d^6 + 15*(a*b^5*c
 - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*d^4 + 3*(a*b^6 - 6*a^2*b^4*c + 32*a^4*c^3)
*d^2)*e^3*x^2 + 4*(2*(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*d^7 + 3*(a*b^
5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*d^5 + (a*b^6 - 6*a^2*b^4*c + 32*a^4*c^3
)*d^3 + (a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*d)*e^2*x + ((a*b^4*c^2 - 8*a
^2*b^2*c^3 + 16*a^3*c^4)*d^8 + a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2 + 2*(a*b^
5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*d^6 + (a*b^6 - 6*a^2*b^4*c + 32*a^4*c^3
)*d^4 + 2*(a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*d^2)*e)*sqrt(-(b^7 - 35*a*
b^5*c + 280*a^2*b^3*c^2 + 1680*a^3*b*c^3 + (a^3*b^10 - 20*a^4*b^8*c + 160*a
^5*b^6*c^2 - 640*a^6*b^4*c^3 + 1280*a^7*b^2*c^4 - 1024*a^8*c^5)*e^2*sqrt((b
^4 - 50*a*b^2*c + 625*a^2*c^2)/((a^6*b^10 - 20*a^7*b^8*c + 160*a^8*b^6*c^2
 - 640*a^9*b^4*c^3 + 1280*a^10*b^2*c^4 - 1024*a^11*c^5)*e^4)))/((a^3*b^10 -
20*a^4*b^8*c + 160*a^5*b^6*c^2 - 640*a^6*b^4*c^3 + 1280*a^7*b^2*c^4 - 1024*
a^8*c^5)*e^2))*log((35*b^6*c^2 - 1491*a*b^4*c^3 + 15000*a^2*b^2*c^4 + 10000
*a^3*c^5)*e*x + (35*b^6*c^2 - 1491*a*b^4*c^3 + 15000*a^2*b^2*c^4 + 10000*a^
3*c^5)*d - 1/2*sqrt(1/2)*((a^3*b^14 - 38*a^4*b^12*c + 480*a^5*b^10*c^2 - 27
20*a^6*b^8*c^3 + 6400*a^7*b^6*c^4 + 1536*a^8*b^4*c^5 - 32768*a^9*b^2*c^6 +
40960*a^10*c^7)*e^3*sqrt((b^4 - 50*a*b^2*c + 625*a^2*c^2)/((a^6*b^10 - 20*a
^7*b^8*c + 160*a^8*b^6*c^2 - 640*a^9*b^4*c^3 + 1280*a^10*b^2*c^4 - 1024*a^1
1*c^5)*e^4)) - (b^11 - 53*a*b^9*c + 940*a^2*b^7*c^2 - 6832*a^3*b^5*c^3 + 21
824*a^4*b^3*c^4 - 25600*a^5*b*c^5)*e)*sqrt(-(b^7 - 35*a*b^5*c + 280*a^2*b^3
*c^2 + 1680*a^3*b*c^3 + (a^3*b^10 - 20*a^4*b^8*c + 160*a^5*b^6*c^2 - 640*a^
6*b^4*c^3 + 1280*a^7*b^2*c^4 - 1024*a^8*c^5)*e^2*sqrt((b^4 - 50*a*b^2*c + 6
25*a^2*c^2)/((a^6*b^10 - 20*a^7*b^8*c + 160*a^8*b^6*c^2 - 640*a^9*b^4*c^3 +
 1280*a^10*b^2*c^4 - 1024*a^11*c^5)*e^4)))/((a^3*b^10 - 20*a^4*b^8*c + 160*
a^5*b^6*c^2 - 640*a^6*b^4*c^3 + 1280*a^7*b^2*c^4 - 1024*a^8*c^5)*e^2))) + s
qrt(1/2)*((a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*e^9*x^8 + 8*(a*b^4*c^2 -
 8*a^2*b^2*c^3 + 16*a^3*c^4)*d*e^8*x^7 + 2*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^
3*b*c^3 + 14*(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*d^2)*e^7*x^6 + 4*(14*
(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*d^3 + 3*(a*b^5*c - 8*a^2*b^3*c^2 +
 16*a^3*b*c^3)*d)*e^6*x^5 + (a*b^6 - 6*a^2*b^4*c + 32*a^4*c^3 + 70*(a*b^4*c
^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*d^4 + 30*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3
*b*c^3)*d^2)*e^5*x^4 + 4*(14*(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*d^5 +
 10*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*d^3 + (a*b^6 - 6*a^2*b^4*c + 3
2*a^4*c^3)*d)*e^4*x^3 + 2*(a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2 + 14*(a*b^4
*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*d^6 + 15*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a
^3*b*c^3)*d^4 + 3*(a*b^6 - 6*a^2*b^4*c + 32*a^4*c^3)*d^2)*e^3*x^2 + 4*(2*(a
*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*d^7 + 3*(a*b^5*c - 8*a^2*b^3*c^2 + 1
6*a^3*b*c^3)*d^5 + (a*b^6 - 6*a^2*b^4*c + 32*a^4*c^3)*d^3 + (a^2*b^5 - 8*a^
3*b^3*c + 16*a^4*b*c^2)*d)*e^2*x + ((a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4
)*d^8 + a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2 + 2*(a*b^5*c - 8*a^2*b^3*c^2 + 1
6*a^3*b*c^3)*d^6 + (a*b^6 - 6*a^2*b^4*c + 32*a^4*c^3)*d^4 + 2*(a^2*b^5 - 8*
a^3*b^3*c + 16*a^4*b*c^2)*d^2)*e)*sqrt(-(b^7 - 35*a*b^5*c + 280*a^2*b^3*c^2
 + 1680*a^3*b*c^3 - (a^3*b^10 - 20*a^4*b^8*c + 160*a^5*b^6*c^2 - 640*a^6*b^
4*c^3 + 1280*a^7*b^2*c^4 - 1024*a^8*c^5)*e^2*sqrt((b^4 - 50*a*b^2*c + 625*a
^2*c^2)/((a^6*b^10 - 20*a^7*b^8*c + 160*a^8*b^6*c^2 - 640*a^9*b^4*c^3 + 128
0*a^10*b^2*c^4 - 1024*a^11*c^5)*e^4)))/((a^3*b^10 - 20*a^4*b^8*c + 160*a^5*
b^6*c^2 - 640*a^6*b^4*c^3 + 1280*a^7*b^2*c^4 - 1024*a^8*c^5)*e^2))*log((35*
b^6*c^2 - 1491*a*b^4*c^3 + 15000*a^2*b^2*c^4 + 10000*a^3*c^5)*e*x + (35*b^6
*c^2 - 1491*a*b^4*c^3 + 15000*a^2*b^2*c^4 + 10000*a^3*c^5)*d + 1/2*sqrt(1/2
)*((a^3*b^14 - 38*a^4*b^12*c + 480*a^5*b^10*c^2 - 2720*a^6*b^8*c^3 + 6400*a
^7*b^6*c^4 + 1536*a^8*b^4*c^5 - 32768*a^9*b^2*c^6 + 40960*a^10*c^7)*e^3*sqr
t((b^4 - 50*a*b^2*c + 625*a^2*c^2)/((a^6*b^10 - 20*a^7*b^8*c + 160*a^8*b^6*
c^2 - 640*a^9*b^4*c^3 + 1280*a^10*b^2*c^4 - 1024*a^11*c^5)*e^4)) + (b^11 -
53*a*b^9*c + 940*a^2*b^7*c^2 - 6832*a^3*b^5*c^3 + 21824*a^4*b^3*c^4 - 25600
```

```
*a^5*b*c^5)*e)*sqrt(-(b^7 - 35*a*b^5*c + 280*a^2*b^3*c^2 + 1680*a^3*b*c^3 -
 (a^3*b^10 - 20*a^4*b^8*c + 160*a^5*b^6*c^2 - 640*a^6*b^4*c^3 + 1280*a^7*b^
2*c^4 - 1024*a^8*c^5)*e^2*sqrt((b^4 - 50*a*b^2*c + 625*a^2*c^2)/((a^6*b^10
- 20*a^7*b^8*c + 160*a^8*b^6*c^2 - 640*a^9*b^4*c^3 + 1280*a^10*b^2*c^4 - 10
24*a^11*c^5)*e^4)))/((a^3*b^10 - 20*a^4*b^8*c + 160*a^5*b^6*c^2 - 640*a^6*b
^4*c^3 + 1280*a^7*b^2*c^4 - 1024*a^8*c^5)*e^2))) - sqrt(1/2)*((a*b^4*c^2 -
8*a^2*b^2*c^3 + 16*a^3*c^4)*e^9*x^8 + 8*(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3
*c^4)*d*e^8*x^7 + 2*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3 + 14*(a*b^4*c^2
 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*d^2)*e^7*x^6 + 4*(14*(a*b^4*c^2 - 8*a^2*b^2*
c^3 + 16*a^3*c^4)*d^3 + 3*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*d)*e^6*x
^5 + (a*b^6 - 6*a^2*b^4*c + 32*a^4*c^3 + 70*(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16
*a^3*c^4)*d^4 + 30*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*d^2)*e^5*x^4 +
4*(14*(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*d^5 + 10*(a*b^5*c - 8*a^2*b^
3*c^2 + 16*a^3*b*c^3)*d^3 + (a*b^6 - 6*a^2*b^4*c + 32*a^4*c^3)*d)*e^4*x^3 +
 2*(a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2 + 14*(a*b^4*c^2 - 8*a^2*b^2*c^3 +
16*a^3*c^4)*d^6 + 15*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*d^4 + 3*(a*b^
6 - 6*a^2*b^4*c + 32*a^4*c^3)*d^2)*e^3*x^2 + 4*(2*(a*b^4*c^2 - 8*a^2*b^2*c^
3 + 16*a^3*c^4)*d^7 + 3*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*d^5 + (a*b
^6 - 6*a^2*b^4*c + 32*a^4*c^3)*d^3 + (a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)
*d)*e^2*x + ((a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*d^8 + a^3*b^4 - 8*a^4
*b^2*c + 16*a^5*c^2 + 2*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*d^6 + (a*b
^6 - 6*a^2*b^4*c + 32*a^4*c^3)*d^4 + 2*(a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^
2)*d^2)*e)*sqrt(-(b^7 - 35*a*b^5*c + 280*a^2*b^3*c^2 + 1680*a^3*b*c^3 - (a^
3*b^10 - 20*a^4*b^8*c + 160*a^5*b^6*c^2 - 640*a^6*b^4*c^3 + 1280*a^7*b^2*c^
4 - 1024*a^8*c^5)*e^2*sqrt((b^4 - 50*a*b^2*c + 625*a^2*c^2)/((a^6*b^10 - 20
*a^7*b^8*c + 160*a^8*b^6*c^2 - 640*a^9*b^4*c^3 + 1280*a^10*b^2*c^4 - 1024*a
^11*c^5)*e^4)))/((a^3*b^10 - 20*a^4*b^8*c + 160*a^5*b^6*c^2 - 640*a^6*b^4*c
^3 + 1280*a^7*b^2*c^4 - 1024*a^8*c^5)*e^2))*log((35*b^6*c^2 - 1491*a*b^4*c^
3 + 15000*a^2*b^2*c^4 + 10000*a^3*c^5)*e*x + (35*b^6*c^2 - 1491*a*b^4*c^3 +
 15000*a^2*b^2*c^4 + 10000*a^3*c^5)*d - 1/2*sqrt(1/2)*((a^3*b^14 - 38*a^4*b
^12*c + 480*a^5*b^10*c^2 - 2720*a^6*b^8*c^3 + 6400*a^7*b^6*c^4 + 1536*a^8*b
^4*c^5 - 32768*a^9*b^2*c^6 + 40960*a^10*c^7)*e^3*sqrt((b^4 - 50*a*b^2*c + 6
25*a^2*c^2)/((a^6*b^10 - 20*a^7*b^8*c + 160*a^8*b^6*c^2 - 640*a^9*b^4*c^3 +
 1280*a^10*b^2*c^4 - 1024*a^11*c^5)*e^4)) + (b^11 - 53*a*b^9*c + 940*a^2*b^
7*c^2 - 6832*a^3*b^5*c^3 + 21824*a^4*b^3*c^4 - 25600*a^5*b*c^5)*e)*sqrt(-(b
^7 - 35*a*b^5*c + 280*a^2*b^3*c^2 + 1680*a^3*b*c^3 - (a^3*b^10 - 20*a^4*b^8
*c + 160*a^5*b^6*c^2 - 640*a^6*b^4*c^3 + 1280*a^7*b^2*c^4 - 1024*a^8*c^5)*e
^2*sqrt((b^4 - 50*a*b^2*c + 625*a^2*c^2)/((a^6*b^10 - 20*a^7*b^8*c + 160*a^
8*b^6*c^2 - 640*a^9*b^4*c^3 + 1280*a^10*b^2*c^4 - 1024*a^11*c^5)*e^4)))/((a
^3*b^10 - 20*a^4*b^8*c + 160*a^5*b^6*c^2 - 640*a^6*b^4*c^3 + 1280*a^7*b^2*c
^4 - 1024*a^8*c^5)*e^2))) - 2*(a*b^3 - 16*a^2*b*c)*d)/((a*b^4*c^2 - 8*a^2*b
^2*c^3 + 16*a^3*c^4)*e^9*x^8 + 8*(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*d
*e^8*x^7 + 2*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3 + 14*(a*b^4*c^2 - 8*a^
2*b^2*c^3 + 16*a^3*c^4)*d^2)*e^7*x^6 + 4*(14*(a*b^4*c^2 - 8*a^2*b^2*c^3 + 1
6*a^3*c^4)*d^3 + 3*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*d)*e^6*x^5 + (a
*b^6 - 6*a^2*b^4*c + 32*a^4*c^3 + 70*(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^
4)*d^4 + 30*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*d^2)*e^5*x^4 + 4*(14*(
a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*d^5 + 10*(a*b^5*c - 8*a^2*b^3*c^2 +
 16*a^3*b*c^3)*d^3 + (a*b^6 - 6*a^2*b^4*c + 32*a^4*c^3)*d)*e^4*x^3 + 2*(a^2
*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2 + 14*(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*
c^4)*d^6 + 15*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*d^4 + 3*(a*b^6 - 6*a
^2*b^4*c + 32*a^4*c^3)*d^2)*e^3*x^2 + 4*(2*(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*
a^3*c^4)*d^7 + 3*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*d^5 + (a*b^6 - 6*
a^2*b^4*c + 32*a^4*c^3)*d^3 + (a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*d)*e^2
*x + ((a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*d^8 + a^3*b^4 - 8*a^4*b^2*c
+ 16*a^5*c^2 + 2*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*d^6 + (a*b^6 - 6*
a^2*b^4*c + 32*a^4*c^3)*d^4 + 2*(a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*d^2)
*e)
```

giac [B] time = 0.87, size = 2295, normalized size = 6.32

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="giac")
```

```
[Out] -1/16*(((d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/
c))^2*b^2*c*e^2 + 20*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)
*e^2)*e^(-4)/c))^2*a*c^2*e^2 - 2*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 + sqrt(
b^2 - 4*a*c)*e^2)*e^(-4)/c))*b^2*c*d*e - 40*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*
e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))*a*c^2*d*e + b^2*c*d^2 + 20*a*c^2*d^
2 + b^3 - 16*a*b*c)*log(d*e^(-1) + x + sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 -
4*a*c)*e^2)*e^(-4)/c))/(2*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4
*a*c)*e^2)*e^(-4)/c))^3*c*e^4 - 6*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 + sqrt
(b^2 - 4*a*c)*e^2)*e^(-4)/c))^2*c*d*e^3 - 2*c*d^3*e - b*d*e + (6*c*d^2*e^2
+ b*e^2)*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)
/c))) + ((d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)
/c))^2*b^2*c*e^2 + 20*(d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c
)*e^2)*e^(-4)/c))^2*a*c^2*e^2 - 2*(d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 + sqrt
(b^2 - 4*a*c)*e^2)*e^(-4)/c))*b^2*c*d*e - 40*(d*e^(-1) - sqrt(1/2)*sqrt(-(b
*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))*a*c^2*d*e + b^2*c*d^2 + 20*a*c^2*d
^2 + b^3 - 16*a*b*c)*log(d*e^(-1) + x - sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 -
 4*a*c)*e^2)*e^(-4)/c))/(2*(d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 -
4*a*c)*e^2)*e^(-4)/c))^3*c*e^4 - 6*(d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 + sqr
t(b^2 - 4*a*c)*e^2)*e^(-4)/c))^2*c*d*e^3 - 2*c*d^3*e - b*d*e + (6*c*d^2*e^2
 + b*e^2)*(d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4
)/c))) + ((d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4
)/c))^2*b^2*c*e^2 + 20*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*
c)*e^2)*e^(-4)/c))^2*a*c^2*e^2 - 2*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 - sqr
t(b^2 - 4*a*c)*e^2)*e^(-4)/c))*b^2*c*d*e - 40*(d*e^(-1) + sqrt(1/2)*sqrt(-(
b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))*a*c^2*d*e + b^2*c*d^2 + 20*a*c^2*
d^2 + b^3 - 16*a*b*c)*log(d*e^(-1) + x + sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2
- 4*a*c)*e^2)*e^(-4)/c))/(2*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 -
 4*a*c)*e^2)*e^(-4)/c))^3*c*e^4 - 6*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 - sq
rt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^2*c*d*e^3 - 2*c*d^3*e - b*d*e + (6*c*d^2*e^
2 + b*e^2)*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-
4)/c))) + ((d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-
4)/c))^2*b^2*c*e^2 + 20*(d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a
*c)*e^2)*e^(-4)/c))^2*a*c^2*e^2 - 2*(d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 - sq
rt(b^2 - 4*a*c)*e^2)*e^(-4)/c))*b^2*c*d*e - 40*(d*e^(-1) - sqrt(1/2)*sqrt(-
(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))*a*c^2*d*e + b^2*c*d^2 + 20*a*c^2
*d^2 + b^3 - 16*a*b*c)*log(d*e^(-1) + x - sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2
 - 4*a*c)*e^2)*e^(-4)/c))/(2*(d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2
- 4*a*c)*e^2)*e^(-4)/c))^3*c*e^4 - 6*(d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 - s
qrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^2*c*d*e^3 - 2*c*d^3*e - b*d*e + (6*c*d^2*e
^2 + b*e^2)*(d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(
-4)/c))))/(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2) + 1/8*(b^2*c^2*x^7*e^7 + 20*a*
c^3*x^7*e^7 + 7*b^2*c^2*d*x^6*e^6 + 140*a*c^3*d*x^6*e^6 + 21*b^2*c^2*d^2*x^
5*e^5 + 420*a*c^3*d^2*x^5*e^5 + 35*b^2*c^2*d^3*x^4*e^4 + 700*a*c^3*d^3*x^4*
e^4 + 35*b^2*c^2*d^4*x^3*e^3 + 700*a*c^3*d^4*x^3*e^3 + 21*b^2*c^2*d^5*x^2*e
^2 + 420*a*c^3*d^5*x^2*e^2 + 7*b^2*c^2*d^6*x*e + 140*a*c^3*d^6*x*e + b^2*c^
2*d^7 + 20*a*c^3*d^7 + 2*b^3*c*x^5*e^5 + 28*a*b*c^2*x^5*e^5 + 10*b^3*c*d*x^
4*e^4 + 140*a*b*c^2*d*x^4*e^4 + 20*b^3*c*d^2*x^3*e^3 + 280*a*b*c^2*d^2*x^3*
e^3 + 20*b^3*c*d^3*x^2*e^2 + 280*a*b*c^2*d^3*x^2*e^2 + 10*b^3*c*d^4*x*e + 1
40*a*b*c^2*d^4*x*e + 2*b^3*c*d^5 + 28*a*b*c^2*d^5 + b^4*x^3*e^3 + 5*a*b^2*c
*x^3*e^3 + 36*a^2*c^2*x^3*e^3 + 3*b^4*d*x^2*e^2 + 15*a*b^2*c*d*x^2*e^2 + 10
8*a^2*c^2*d*x^2*e^2 + 3*b^4*d^2*x*e + 15*a*b^2*c*d^2*x*e + 108*a^2*c^2*d^2*
x*e + b^4*d^3 + 5*a*b^2*c*d^3 + 36*a^2*c^2*d^3 - a*b^3*x*e + 16*a^2*b*c*x*e
 - a*b^3*d + 16*a^2*b*c*d)/((c*x^4*e^4 + 4*c*d*x^3*e^3 + 6*c*d^2*x^2*e^2 +
4*c*d^3*x*e + c*d^4 + b*x^2*e^2 + 2*b*d*x*e + b*d^2 + a)^2*(a*b^4*e - 8*a^2
```

```
*b^2*c*e + 16*a^3*c^2*e))
```

maple [C] time = 0.05, size = 885, normalized size = 2.44

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x)
```

```
[Out] (1/8*c^2*e^6*(20*a*c+b^2)/(16*a^2*c^2-8*a*b^2*c+b^4)/a*x^7+7/8*c^2*d*e^5*(2
0*a*c+b^2)/(16*a^2*c^2-8*a*b^2*c+b^4)/a*x^6+1/8*(420*a*c^2*d^2+21*b^2*c*d^2
+28*a*b*c+2*b^3)*c*e^4/(16*a^2*c^2-8*a*b^2*c+b^4)/a*x^5+5/8*c*d*e^3*(140*a*
c^2*d^2+7*b^2*c*d^2+28*a*b*c+2*b^3)/(16*a^2*c^2-8*a*b^2*c+b^4)/a*x^4+1/8*e^
2*(700*a*c^3*d^4+35*b^2*c^2*d^4+280*a*b*c^2*d^2+20*b^3*c*d^2+36*a^2*c^2+5*a
*b^2*c+b^4)/(16*a^2*c^2-8*a*b^2*c+b^4)/a*x^3+1/8*d*e*(420*a*c^3*d^4+21*b^2*
c^2*d^4+280*a*b*c^2*d^2+20*b^3*c*d^2+108*a^2*c^2+15*a*b^2*c+3*b^4)/(16*a^2*
c^2-8*a*b^2*c+b^4)/a*x^2+1/8*(140*a*c^3*d^6+7*b^2*c^2*d^6+140*a*b*c^2*d^4+1
0*b^3*c*d^4+108*a^2*c^2*d^2+15*a*b^2*c*d^2+3*b^4*d^2+16*a^2*b*c-a*b^3)/(16*
a^2*c^2-8*a*b^2*c+b^4)/a*x+1/8*d/e*(20*a*c^3*d^6+b^2*c^2*d^6+28*a*b*c^2*d^4
+2*b^3*c*d^4+36*a^2*c^2*d^2+5*a*b^2*c*d^2+b^4*d^2+16*a^2*b*c-a*b^3)/(16*a^2
*c^2-8*a*b^2*c+b^4)/a)/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x
+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2+1/16/(16*a^2*c^2-8*a*b^2*c+b^4)/a/e*s
um((c*e^2*(20*a*c+b^2)*_R^2+2*c*d*e*(20*a*c+b^2)*_R+20*a*c^2*d^2+b^2*c*d^2-
16*a*b*c+b^3)/(2*_R^3*c*e^3+6*_R^2*c*d*e^2+6*_R*c*d^2*e+2*c*d^3+_R*b*e+b*d)
*ln(-_R+x),_R=RootOf(_Z^4*c*e^4+4*_Z^3*c*d*e^3+c*d^4+b*d^2+(6*c*d^2*e^2+b*e
^2)*_Z^2+(4*c*d^3*e+2*b*d*e)*_Z+a))
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [B] time = 7.43, size = 14584, normalized size = 40.18

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^2/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3,x)
```

```
[Out] ((x^5*(2*b^3*c*e^4 + 420*a*c^3*d^2*e^4 + 21*b^2*c^2*d^2*e^4 + 28*a*b*c^2*e^
4))/(8*a*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (x^2*(3*b^4*d*e + 21*b^2*c^2*d^5
*e + 108*a^2*c^2*d*e + 420*a*c^3*d^5*e + 20*b^3*c*d^3*e + 280*a*b*c^2*d^3*e
 + 15*a*b^2*c*d*e))/(8*a*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (7*x^6*(b^2*c^2*
d*e^5 + 20*a*c^3*d*e^5))/(8*a*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (x^7*(20*a*
c^3*e^6 + b^2*c^2*e^6))/(8*a*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (x*(3*b^4*d^
2 - a*b^3 + 140*a*c^3*d^6 + 10*b^3*c*d^4 + 108*a^2*c^2*d^2 + 7*b^2*c^2*d^6
+ 16*a^2*b*c + 15*a*b^2*c*d^2 + 140*a*b*c^2*d^4))/(8*a*(b^4 + 16*a^2*c^2 -
8*a*b^2*c)) + (x^3*(b^4*e^2 + 36*a^2*c^2*e^2 + 700*a*c^3*d^4*e^2 + 20*b^3*c
*d^2*e^2 + 35*b^2*c^2*d^4*e^2 + 5*a*b^2*c*e^2 + 280*a*b*c^2*d^2*e^2))/(8*a*
(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (b^4*d^3 + 20*a*c^3*d^7 + 2*b^3*c*d^5 + 3
6*a^2*c^2*d^3 + b^2*c^2*d^7 - a*b^3*d + 16*a^2*b*c*d + 5*a*b^2*c*d^3 + 28*a
*b*c^2*d^5)/(8*a*e*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (5*x^4*(140*a*c^3*d^3*
e^3 + 7*b^2*c^2*d^3*e^3 + 2*b^3*c*d*e^3 + 28*a*b*c^2*d*e^3))/(8*a*(b^4 + 16
*a^2*c^2 - 8*a*b^2*c)))/(x^2*(6*b^2*d^2*e^2 + 28*c^2*d^6*e^2 + 2*a*b*e^2 +
12*a*c*d^2*e^2 + 30*b*c*d^4*e^2) + x^6*(28*c^2*d^2*e^6 + 2*b*c*e^6) + x*(4*
b^2*d^3*e + 8*c^2*d^7*e + 8*a*c*d^3*e + 12*b*c*d^5*e + 4*a*b*d*e) + x^3*(4*
b^2*d*e^3 + 56*c^2*d^5*e^3 + 8*a*c*d*e^3 + 40*b*c*d^3*e^3) + x^5*(56*c^2*d^
```

3*e^5 + 12*b*c*d*e^5) + x^4*(b^2*e^4 + 70*c^2*d^4*e^4 + 2*a*c*e^4 + 30*b*c*d^2*e^4) + a^2 + b^2*d^4 + c^2*d^8 + c^2*e^8*x^8 + 2*a*b*d^2 + 2*a*c*d^4 + 2*b*c*d^6 + 8*c^2*d*e^7*x^7) + atan(((((256*a*b^13*c^2*e^12 + 4194304*a^7*b*c^8*e^12 - 9216*a^2*b^11*c^3*e^12 + 122880*a^3*b^9*c^4*e^12 - 819200*a^4*b^7*c^5*e^12 + 2949120*a^5*b^5*c^6*e^12 - 5505024*a^6*b^3*c^7*e^12)/(512*(a^2*b^12 + 4096*a^8*c^6 - 24*a^3*b^10*c + 240*a^4*b^8*c^2 - 1280*a^5*b^6*c^3 + 3840*a^6*b^4*c^4 - 6144*a^7*b^2*c^5)) + ((67108864*a^9*b*c^9*d*e^13 - 4096*a^2*b^15*c^2*d*e^13 + 114688*a^3*b^13*c^3*d*e^13 - 1376256*a^4*b^11*c^4*d*e^13 + 9175040*a^5*b^9*c^5*d*e^13 - 36700160*a^6*b^7*c^6*d*e^13 + 88080384*a^7*b^5*c^7*d*e^13 - 117440512*a^8*b^3*c^8*d*e^13)/(512*(a^2*b^12 + 4096*a^8*c^6 - 24*a^3*b^10*c + 240*a^4*b^8*c^2 - 1280*a^5*b^6*c^3 + 3840*a^6*b^4*c^4 - 6144*a^7*b^2*c^5)) + (x*(262144*a^7*b*c^7*e^14 - 256*a^2*b^11*c^2*e^14 + 5120*a^3*b^9*c^3*e^14 - 40960*a^4*b^7*c^4*e^14 + 163840*a^5*b^5*c^5*e^14 - 327680*a^6*b^3*c^6*e^14))/(32*(a^2*b^8 + 256*a^6*c^4 - 16*a^3*b^6*c + 96*a^4*b^4*c^2 - 256*a^5*b^2*c^3)))*(-(b^17 + b^2*(-(4*a*c - b^2)^15)^(1/2) - 1720320*a^8*b*c^8 + 1140*a^2*b^13*c^2 - 10160*a^3*b^11*c^3 + 34880*a^4*b^9*c^4 + 43776*a^5*b^7*c^5 - 680960*a^6*b^5*c^6 + 1863680*a^7*b^3*c^7 - 55*a*b^15*c - 25*a*c*(-(4*a*c - b^2)^15)^(1/2))/(512*(a^3*b^20*e^2 + 1048576*a^13*c^10*e^2 - 40*a^4*b^18*c*e^2 + 720*a^5*b^16*c^2*e^2 - 7680*a^6*b^14*c^3*e^2 + 53760*a^7*b^12*c^4*e^2 - 258048*a^8*b^10*c^5*e^2 + 860160*a^9*b^8*c^6*e^2 - 1966080*a^10*b^6*c^7*e^2 + 2949120*a^11*b^4*c^8*e^2 - 2621440*a^12*b^2*c^9*e^2)))^(1/2))*(-(b^17 + b^2*(-(4*a*c - b^2)^15)^(1/2) - 1720320*a^8*b*c^8 + 1140*a^2*b^13*c^2 - 10160*a^3*b^11*c^3 + 34880*a^4*b^9*c^4 + 43776*a^5*b^7*c^5 - 680960*a^6*b^5*c^6 + 1863680*a^7*b^3*c^7 - 55*a*b^15*c - 25*a*c*(-(4*a*c - b^2)^15)^(1/2))/(512*(a^3*b^20*e^2 + 1048576*a^13*c^10*e^2 - 40*a^4*b^18*c*e^2 + 720*a^5*b^16*c^2*e^2 - 7680*a^6*b^14*c^3*e^2 + 53760*a^7*b^12*c^4*e^2 - 258048*a^8*b^10*c^5*e^2 + 860160*a^9*b^8*c^6*e^2 - 1966080*a^10*b^6*c^7*e^2 + 2949120*a^11*b^4*c^8*e^2 - 2621440*a^12*b^2*c^9*e^2)))^(1/2) + (204800*a^5*c^8*d*e^11 - 16*b^10*c^3*d*e^11 + 672*a*b^8*c^4*d*e^11 - 28160*a^2*b^6*c^5*d*e^11 + 209920*a^3*b^4*c^6*d*e^11 - 479232*a^4*b^2*c^7*d*e^11)/(512*(a^2*b^12 + 4096*a^8*c^6 - 24*a^3*b^10*c + 240*a^4*b^8*c^2 - 1280*a^5*b^6*c^3 + 3840*a^6*b^4*c^4 - 6144*a^7*b^2*c^5)) + (x*(800*a^3*c^6*e^12 - b^6*c^3*e^12 + 34*a*b^4*c^4*e^12 - 1472*a^2*b^2*c^5*e^12))/(32*(a^2*b^8 + 256*a^6*c^4 - 16*a^3*b^6*c + 96*a^4*b^4*c^2 - 256*a^5*b^2*c^3)))*(-(b^17 + b^2*(-(4*a*c - b^2)^15)^(1/2) - 1720320*a^8*b*c^8 + 1140*a^2*b^13*c^2 - 10160*a^3*b^11*c^3 + 34880*a^4*b^9*c^4 + 43776*a^5*b^7*c^5 - 680960*a^6*b^5*c^6 + 1863680*a^7*b^3*c^7 - 55*a*b^15*c - 25*a*c*(-(4*a*c - b^2)^15)^(1/2))/(512*(a^3*b^20*e^2 + 1048576*a^13*c^10*e^2 - 40*a^4*b^18*c*e^2 + 720*a^5*b^16*c^2*e^2 - 7680*a^6*b^14*c^3*e^2 + 53760*a^7*b^12*c^4*e^2 - 258048*a^8*b^10*c^5*e^2 + 860160*a^9*b^8*c^6*e^2 - 1966080*a^10*b^6*c^7*e^2 + 2949120*a^11*b^4*c^8*e^2 - 2621440*a^12*b^2*c^9*e^2)))^(1/2)*1i + ((204800*a^5*c^8*d*e^11 - 16*b^10*c^3*d*e^11 + 672*a*b^8*c^4*d*e^11 - 28160*a^2*b^6*c^5*d*e^11 + 209920*a^3*b^4*c^6*d*e^11 - 479232*a^4*b^2*c^7*d*e^11)/(512*(a^2*b^12 + 4096*a^8*c^6 - 24*a^3*b^10*c + 240*a^4*b^8*c^2 - 1280*a^5*b^6*c^3 + 3840*a^6*b^4*c^4 - 6144*a^7*b^2*c^5)) - ((256*a*b^13*c^2*e^12 + 4194304*a^7*b*c^8*e^12 - 9216*a^2*b^11*c^3*e^12 + 122880*a^3*b^9*c^4*e^12 - 819200*a^4*b^7*c^5*e^12 + 2949120*a^5*b^5*c^6*e^12 - 5505024*a^6*b^3*c^7*e^12)/(512*(a^2*b^12 + 4096*a^8*c^6 - 24*a^3*b^10*c + 240*a^4*b^8*c^2 - 1280*a^5*b^6*c^3 + 3840*a^6*b^4*c^4 - 6144*a^7*b^2*c^5)) - ((67108864*a^9*b*c^9*d*e^13 - 4096*a^2*b^15*c^2*d*e^13 + 114688*a^3*b^13*c^3*d*e^13 - 1376256*a^4*b^11*c^4*d*e^13 + 9175040*a^5*b^9*c^5*d*e^13 - 36700160*a^6*b^7*c^6*d*e^13 + 88080384*a^7*b^5*c^7*d*e^13 - 117440512*a^8*b^3*c^8*d*e^13)/(512*(a^2*b^12 + 4096*a^8*c^6 - 24*a^3*b^10*c + 240*a^4*b^8*c^2 - 1280*a^5*b^6*c^3 + 3840*a^6*b^4*c^4 - 6144*a^7*b^2*c^5)) + (x*(262144*a^7*b*c^7*e^14 - 256*a^2*b^11*c^2*e^14 + 5120*a^3*b^9*c^3*e^14 - 40960*a^4*b^7*c^4*e^14 + 163840*a^5*b^5*c^5*e^14 - 327680*a^6*b^3*c^6*e^14))/(32*(a^2*b^8 + 256*a^6*c^4 - 16*a^3*b^6*c + 96*a^4*b^4*c^2 - 256*a^5*b^2*c^3)))*(-(b^17 + b^2*(-(4*a*c - b^2)^15)^(1/2) - 1720320*a^8*b*c^8 + 1140*a^2*b^13*c^2 - 10160*a^3*b^11*c^3 + 34880*a^4*b^9*c^4 + 43776*a^5*b^7*c^5 - 680960*a^6*b^5*c^6 + 1863680*a^7*b^3*c^7 - 55*a

```
*b^15*c - 25*a*c*(-(4*a*c - b^2)^15)^(1/2))/(512*(a^3*b^20*e^2 + 1048576*a^
13*c^10*e^2 - 40*a^4*b^18*c*e^2 + 720*a^5*b^16*c^2*e^2 - 7680*a^6*b^14*c^3*
e^2 + 53760*a^7*b^12*c^4*e^2 - 258048*a^8*b^10*c^5*e^2 + 860160*a^9*b^8*c^6
*e^2 - 1966080*a^10*b^6*c^7*e^2 + 2949120*a^11*b^4*c^8*e^2 - 2621440*a^12*b
^2*c^9*e^2)))^(1/2))*(-(b^17 + b^2*(-(4*a*c - b^2)^15)^(1/2) - 1720320*a^8*
b*c^8 + 1140*a^2*b^13*c^2 - 10160*a^3*b^11*c^3 + 34880*a^4*b^9*c^4 + 43776*
a^5*b^7*c^5 - 680960*a^6*b^5*c^6 + 1863680*a^7*b^3*c^7 - 55*a*b^15*c - 25*a
*c*(-(4*a*c - b^2)^15)^(1/2))/(512*(a^3*b^20*e^2 + 1048576*a^13*c^10*e^2 -
40*a^4*b^18*c*e^2 + 720*a^5*b^16*c^2*e^2 - 7680*a^6*b^14*c^3*e^2 + 53760*a^
7*b^12*c^4*e^2 - 258048*a^8*b^10*c^5*e^2 + 860160*a^9*b^8*c^6*e^2 - 1966080
*a^10*b^6*c^7*e^2 + 2949120*a^11*b^4*c^8*e^2 - 2621440*a^12*b^2*c^9*e^2)))^
(1/2) + (x*(800*a^3*c^6*e^12 - b^6*c^3*e^12 + 34*a*b^4*c^4*e^12 - 1472*a^2*
b^2*c^5*e^12))/(32*(a^2*b^8 + 256*a^6*c^4 - 16*a^3*b^6*c + 96*a^4*b^4*c^2 -
 256*a^5*b^2*c^3)))*(-(b^17 + b^2*(-(4*a*c - b^2)^15)^(1/2) - 1720320*a^8*b
*c^8 + 1140*a^2*b^13*c^2 - 10160*a^3*b^11*c^3 + 34880*a^4*b^9*c^4 + 43776*a
^5*b^7*c^5 - 680960*a^6*b^5*c^6 + 1863680*a^7*b^3*c^7 - 55*a*b^15*c - 25*a*
c*(-(4*a*c - b^2)^15)^(1/2))/(512*(a^3*b^20*e^2 + 1048576*a^13*c^10*e^2 - 4
0*a^4*b^18*c*e^2 + 720*a^5*b^16*c^2*e^2 - 7680*a^6*b^14*c^3*e^2 + 53760*a^7
*b^12*c^4*e^2 - 258048*a^8*b^10*c^5*e^2 + 860160*a^9*b^8*c^6*e^2 - 1966080*
a^10*b^6*c^7*e^2 + 2949120*a^11*b^4*c^8*e^2 - 2621440*a^12*b^2*c^9*e^2)))^(
1/2)*1i)/((8000*a^3*c^7*e^10 - 35*b^6*c^4*e^10 - 84*a*b^4*c^5*e^10 + 12720*
a^2*b^2*c^6*e^10)/(256*(a^2*b^12 + 4096*a^8*c^6 - 24*a^3*b^10*c + 240*a^4*b
^8*c^2 - 1280*a^5*b^6*c^3 + 3840*a^6*b^4*c^4 - 6144*a^7*b^2*c^5)) - (((256*
a*b^13*c^2*e^12 + 4194304*a^7*b*c^8*e^12 - 9216*a^2*b^11*c^3*e^12 + 122880*
a^3*b^9*c^4*e^12 - 819200*a^4*b^7*c^5*e^12 + 2949120*a^5*b^5*c^6*e^12 - 550
5024*a^6*b^3*c^7*e^12)/(512*(a^2*b^12 + 4096*a^8*c^6 - 24*a^3*b^10*c + 240*
a^4*b^8*c^2 - 1280*a^5*b^6*c^3 + 3840*a^6*b^4*c^4 - 6144*a^7*b^2*c^5)) + ((
67108864*a^9*b*c^9*d*e^13 - 4096*a^2*b^15*c^2*d*e^13 + 114688*a^3*b^13*c^3*
d*e^13 - 1376256*a^4*b^11*c^4*d*e^13 + 9175040*a^5*b^9*c^5*d*e^13 - 3670016
0*a^6*b^7*c^6*d*e^13 + 88080384*a^7*b^5*c^7*d*e^13 - 117440512*a^8*b^3*c^8*
d*e^13)/(512*(a^2*b^12 + 4096*a^8*c^6 - 24*a^3*b^10*c + 240*a^4*b^8*c^2 - 1
280*a^5*b^6*c^3 + 3840*a^6*b^4*c^4 - 6144*a^7*b^2*c^5)) + (x*(262144*a^7*b*
c^7*e^14 - 256*a^2*b^11*c^2*e^14 + 5120*a^3*b^9*c^3*e^14 - 40960*a^4*b^7*c^
4*e^14 + 163840*a^5*b^5*c^5*e^14 - 327680*a^6*b^3*c^6*e^14))/(32*(a^2*b^8 +
 256*a^6*c^4 - 16*a^3*b^6*c + 96*a^4*b^4*c^2 - 256*a^5*b^2*c^3)))*(-(b^17 +
 b^2*(-(4*a*c - b^2)^15)^(1/2) - 1720320*a^8*b*c^8 + 1140*a^2*b^13*c^2 - 10
160*a^3*b^11*c^3 + 34880*a^4*b^9*c^4 + 43776*a^5*b^7*c^5 - 680960*a^6*b^5*c
^6 + 1863680*a^7*b^3*c^7 - 55*a*b^15*c - 25*a*c*(-(4*a*c - b^2)^15)^(1/2))/
(512*(a^3*b^20*e^2 + 1048576*a^13*c^10*e^2 - 40*a^4*b^18*c*e^2 + 720*a^5*b^
16*c^2*e^2 - 7680*a^6*b^14*c^3*e^2 + 53760*a^7*b^12*c^4*e^2 - 258048*a^8*b^
10*c^5*e^2 + 860160*a^9*b^8*c^6*e^2 - 1966080*a^10*b^6*c^7*e^2 + 2949120*a^
11*b^4*c^8*e^2 - 2621440*a^12*b^2*c^9*e^2)))^(1/2))*(-(b^17 + b^2*(-(4*a*c
- b^2)^15)^(1/2) - 1720320*a^8*b*c^8 + 1140*a^2*b^13*c^2 - 10160*a^3*b^11*c
^3 + 34880*a^4*b^9*c^4 + 43776*a^5*b^7*c^5 - 680960*a^6*b^5*c^6 + 1863680*a
^7*b^3*c^7 - 55*a*b^15*c - 25*a*c*(-(4*a*c - b^2)^15)^(1/2))/(512*(a^3*b^20
*e^2 + 1048576*a^13*c^10*e^2 - 40*a^4*b^18*c*e^2 + 720*a^5*b^16*c^2*e^2 - 7
680*a^6*b^14*c^3*e^2 + 53760*a^7*b^12*c^4*e^2 - 258048*a^8*b^10*c^5*e^2 + 8
60160*a^9*b^8*c^6*e^2 - 1966080*a^10*b^6*c^7*e^2 + 2949120*a^11*b^4*c^8*e^2
 - 2621440*a^12*b^2*c^9*e^2)))^(1/2) + (204800*a^5*c^8*d*e^11 - 16*b^10*c^3
*d*e^11 + 672*a*b^8*c^4*d*e^11 - 28160*a^2*b^6*c^5*d*e^11 + 209920*a^3*b^4*
c^6*d*e^11 - 479232*a^4*b^2*c^7*d*e^11)/(512*(a^2*b^12 + 4096*a^8*c^6 - 24*
a^3*b^10*c + 240*a^4*b^8*c^2 - 1280*a^5*b^6*c^3 + 3840*a^6*b^4*c^4 - 6144*a
^7*b^2*c^5)) + (x*(800*a^3*c^6*e^12 - b^6*c^3*e^12 + 34*a*b^4*c^4*e^12 - 14
72*a^2*b^2*c^5*e^12))/(32*(a^2*b^8 + 256*a^6*c^4 - 16*a^3*b^6*c + 96*a^4*b^
4*c^2 - 256*a^5*b^2*c^3)))*(-(b^17 + b^2*(-(4*a*c - b^2)^15)^(1/2) - 172032
0*a^8*b*c^8 + 1140*a^2*b^13*c^2 - 10160*a^3*b^11*c^3 + 34880*a^4*b^9*c^4 +
43776*a^5*b^7*c^5 - 680960*a^6*b^5*c^6 + 1863680*a^7*b^3*c^7 - 55*a*b^15*c
- 25*a*c*(-(4*a*c - b^2)^15)^(1/2))/(512*(a^3*b^20*e^2 + 1048576*a^13*c^10*
e^2 - 40*a^4*b^18*c*e^2 + 720*a^5*b^16*c^2*e^2 - 7680*a^6*b^14*c^3*e^2 + 53
```

760*a^7*b^12*c^4*e^2 - 258048*a^8*b^10*c^5*e^2 + 860160*a^9*b^8*c^6*e^2 - 1
966080*a^10*b^6*c^7*e^2 + 2949120*a^11*b^4*c^8*e^2 - 2621440*a^12*b^2*c^9*e
^2)))^(1/2) + ((204800*a^5*c^8*d*e^11 - 16*b^10*c^3*d*e^11 + 672*a*b^8*c^4*
d*e^11 - 28160*a^2*b^6*c^5*d*e^11 + 209920*a^3*b^4*c^6*d*e^11 - 479232*a^4*
b^2*c^7*d*e^11)/(512*(a^2*b^12 + 4096*a^8*c^6 - 24*a^3*b^10*c + 240*a^4*b^8
*c^2 - 1280*a^5*b^6*c^3 + 3840*a^6*b^4*c^4 - 6144*a^7*b^2*c^5)) - ((256*a*b
^13*c^2*e^12 + 4194304*a^7*b*c^8*e^12 - 9216*a^2*b^11*c^3*e^12 + 122880*a^3
*b^9*c^4*e^12 - 819200*a^4*b^7*c^5*e^12 + 2949120*a^5*b^5*c^6*e^12 - 550502
4*a^6*b^3*c^7*e^12)/(512*(a^2*b^12 + 4096*a^8*c^6 - 24*a^3*b^10*c + 240*a^4
*b^8*c^2 - 1280*a^5*b^6*c^3 + 3840*a^6*b^4*c^4 - 6144*a^7*b^2*c^5)) - ((671
08864*a^9*b*c^9*d*e^13 - 4096*a^2*b^15*c^2*d*e^13 + 114688*a^3*b^13*c^3*d*e
^13 - 1376256*a^4*b^11*c^4*d*e^13 + 9175040*a^5*b^9*c^5*d*e^13 - 36700160*a
^6*b^7*c^6*d*e^13 + 88080384*a^7*b^5*c^7*d*e^13 - 117440512*a^8*b^3*c^8*d*e
^13)/(512*(a^2*b^12 + 4096*a^8*c^6 - 24*a^3*b^10*c + 240*a^4*b^8*c^2 - 1280
*a^5*b^6*c^3 + 3840*a^6*b^4*c^4 - 6144*a^7*b^2*c^5)) + (x*(262144*a^7*b*c^7
*e^14 - 256*a^2*b^11*c^2*e^14 + 5120*a^3*b^9*c^3*e^14 - 40960*a^4*b^7*c^4*e
^14 + 163840*a^5*b^5*c^5*e^14 - 327680*a^6*b^3*c^6*e^14))/(32*(a^2*b^8 + 25
6*a^6*c^4 - 16*a^3*b^6*c + 96*a^4*b^4*c^2 - 256*a^5*b^2*c^3)))*(-(b^17 + b^
2*(-(4*a*c - b^2)^15)^(1/2) - 1720320*a^8*b*c^8 + 1140*a^2*b^13*c^2 - 10160
*a^3*b^11*c^3 + 34880*a^4*b^9*c^4 + 43776*a^5*b^7*c^5 - 680960*a^6*b^5*c^6
+ 1863680*a^7*b^3*c^7 - 55*a*b^15*c - 25*a*c*(-(4*a*c - b^2)^15)^(1/2))/(51
2*(a^3*b^20*e^2 + 1048576*a^13*c^10*e^2 - 40*a^4*b^18*c*e^2 + 720*a^5*b^16*
c^2*e^2 - 7680*a^6*b^14*c^3*e^2 + 53760*a^7*b^12*c^4*e^2 - 258048*a^8*b^10*
c^5*e^2 + 860160*a^9*b^8*c^6*e^2 - 1966080*a^10*b^6*c^7*e^2 + 2949120*a^11*
b^4*c^8*e^2 - 2621440*a^12*b^2*c^9*e^2)))^(1/2))*(-(b^17 + b^2*(-(4*a*c - b
^2)^15)^(1/2) - 1720320*a^8*b*c^8 + 1140*a^2*b^13*c^2 - 10160*a^3*b^11*c^3
+ 34880*a^4*b^9*c^4 + 43776*a^5*b^7*c^5 - 680960*a^6*b^5*c^6 + 1863680*a^7*
b^3*c^7 - 55*a*b^15*c - 25*a*c*(-(4*a*c - b^2)^15)^(1/2))/(512*(a^3*b^20*e^
2 + 1048576*a^13*c^10*e^2 - 40*a^4*b^18*c*e^2 + 720*a^5*b^16*c^2*e^2 - 7680
*a^6*b^14*c^3*e^2 + 53760*a^7*b^12*c^4*e^2 - 258048*a^8*b^10*c^5*e^2 + 8601
60*a^9*b^8*c^6*e^2 - 1966080*a^10*b^6*c^7*e^2 + 2949120*a^11*b^4*c^8*e^2 -
2621440*a^12*b^2*c^9*e^2)))^(1/2) + (x*(800*a^3*c^6*e^12 - b^6*c^3*e^12 + 3
4*a*b^4*c^4*e^12 - 1472*a^2*b^2*c^5*e^12))/(32*(a^2*b^8 + 256*a^6*c^4 - 16*
a^3*b^6*c + 96*a^4*b^4*c^2 - 256*a^5*b^2*c^3)))*(-(b^17 + b^2*(-(4*a*c - b^
2)^15)^(1/2) - 1720320*a^8*b*c^8 + 1140*a^2*b^13*c^2 - 10160*a^3*b^11*c^3 +
34880*a^4*b^9*c^4 + 43776*a^5*b^7*c^5 - 680960*a^6*b^5*c^6 + 1863680*a^7*b
^3*c^7 - 55*a*b^15*c - 25*a*c*(-(4*a*c - b^2)^15)^(1/2))/(512*(a^3*b^20*e^2
+ 1048576*a^13*c^10*e^2 - 40*a^4*b^18*c*e^2 + 720*a^5*b^16*c^2*e^2 - 7680*
a^6*b^14*c^3*e^2 + 53760*a^7*b^12*c^4*e^2 - 258048*a^8*b^10*c^5*e^2 + 86016
0*a^9*b^8*c^6*e^2 - 1966080*a^10*b^6*c^7*e^2 + 2949120*a^11*b^4*c^8*e^2 - 2
621440*a^12*b^2*c^9*e^2)))^(1/2)))*(-(b^17 + b^2*(-(4*a*c - b^2)^15)^(1/2)
- 1720320*a^8*b*c^8 + 1140*a^2*b^13*c^2 - 10160*a^3*b^11*c^3 + 34880*a^4*b^
9*c^4 + 43776*a^5*b^7*c^5 - 680960*a^6*b^5*c^6 + 1863680*a^7*b^3*c^7 - 55*a
*b^15*c - 25*a*c*(-(4*a*c - b^2)^15)^(1/2))/(512*(a^3*b^20*e^2 + 1048576*a^
13*c^10*e^2 - 40*a^4*b^18*c*e^2 + 720*a^5*b^16*c^2*e^2 - 7680*a^6*b^14*c^3*
e^2 + 53760*a^7*b^12*c^4*e^2 - 258048*a^8*b^10*c^5*e^2 + 860160*a^9*b^8*c^6
*e^2 - 1966080*a^10*b^6*c^7*e^2 + 2949120*a^11*b^4*c^8*e^2 - 2621440*a^12*b
^2*c^9*e^2)))^(1/2)*2i + atan(((((256*a*b^13*c^2*e^12 + 4194304*a^7*b*c^8*e
^12 - 9216*a^2*b^11*c^3*e^12 + 122880*a^3*b^9*c^4*e^12 - 819200*a^4*b^7*c^5
*e^12 + 2949120*a^5*b^5*c^6*e^12 - 5505024*a^6*b^3*c^7*e^12)/(512*(a^2*b^12
+ 4096*a^8*c^6 - 24*a^3*b^10*c + 240*a^4*b^8*c^2 - 1280*a^5*b^6*c^3 + 3840
*a^6*b^4*c^4 - 6144*a^7*b^2*c^5)) + ((67108864*a^9*b*c^9*d*e^13 - 4096*a^2*
b^15*c^2*d*e^13 + 114688*a^3*b^13*c^3*d*e^13 - 1376256*a^4*b^11*c^4*d*e^13
+ 9175040*a^5*b^9*c^5*d*e^13 - 36700160*a^6*b^7*c^6*d*e^13 + 88080384*a^7*b
^5*c^7*d*e^13 - 117440512*a^8*b^3*c^8*d*e^13)/(512*(a^2*b^12 + 4096*a^8*c^6
- 24*a^3*b^10*c + 240*a^4*b^8*c^2 - 1280*a^5*b^6*c^3 + 3840*a^6*b^4*c^4 -
6144*a^7*b^2*c^5)) + (x*(262144*a^7*b*c^7*e^14 - 256*a^2*b^11*c^2*e^14 + 51
20*a^3*b^9*c^3*e^14 - 40960*a^4*b^7*c^4*e^14 + 163840*a^5*b^5*c^5*e^14 - 32
7680*a^6*b^3*c^6*e^14))/(32*(a^2*b^8 + 256*a^6*c^4 - 16*a^3*b^6*c + 96*a^4*

b^4*c^2 - 256*a^5*b^2*c^3)))*(-(b^17 - b^2*(-(4*a*c - b^2)^15)^(1/2) - 1720
320*a^8*b*c^8 + 1140*a^2*b^13*c^2 - 10160*a^3*b^11*c^3 + 34880*a^4*b^9*c^4
+ 43776*a^5*b^7*c^5 - 680960*a^6*b^5*c^6 + 1863680*a^7*b^3*c^7 - 55*a*b^15*
c + 25*a*c*(-(4*a*c - b^2)^15)^(1/2))/(512*(a^3*b^20*e^2 + 1048576*a^13*c^1
0*e^2 - 40*a^4*b^18*c*e^2 + 720*a^5*b^16*c^2*e^2 - 7680*a^6*b^14*c^3*e^2 +
53760*a^7*b^12*c^4*e^2 - 258048*a^8*b^10*c^5*e^2 + 860160*a^9*b^8*c^6*e^2 -
1966080*a^10*b^6*c^7*e^2 + 2949120*a^11*b^4*c^8*e^2 - 2621440*a^12*b^2*c^9
e^2)))^(1/2))(-(b^17 - b^2*(-(4*a*c - b^2)^15)^(1/2) - 1720320*a^8*b*c^8
+ 1140*a^2*b^13*c^2 - 10160*a^3*b^11*c^3 + 34880*a^4*b^9*c^4 + 43776*a^5*b^
7*c^5 - 680960*a^6*b^5*c^6 + 1863680*a^7*b^3*c^7 - 55*a*b^15*c + 25*a*c*(-(
4*a*c - b^2)^15)^(1/2))/(512*(a^3*b^20*e^2 + 1048576*a^13*c^10*e^2 - 40*a^4
*b^18*c*e^2 + 720*a^5*b^16*c^2*e^2 - 7680*a^6*b^14*c^3*e^2 + 53760*a^7*b^12
*c^4*e^2 - 258048*a^8*b^10*c^5*e^2 + 860160*a^9*b^8*c^6*e^2 - 1966080*a^10*
b^6*c^7*e^2 + 2949120*a^11*b^4*c^8*e^2 - 2621440*a^12*b^2*c^9*e^2)))^(1/2)
+ (204800*a^5*c^8*d*e^11 - 16*b^10*c^3*d*e^11 + 672*a*b^8*c^4*d*e^11 - 2816
0*a^2*b^6*c^5*d*e^11 + 209920*a^3*b^4*c^6*d*e^11 - 479232*a^4*b^2*c^7*d*e^1
1)/(512*(a^2*b^12 + 4096*a^8*c^6 - 24*a^3*b^10*c + 240*a^4*b^8*c^2 - 1280*a
^5*b^6*c^3 + 3840*a^6*b^4*c^4 - 6144*a^7*b^2*c^5)) + (x*(800*a^3*c^6*e^12 -
b^6*c^3*e^12 + 34*a*b^4*c^4*e^12 - 1472*a^2*b^2*c^5*e^12))/(32*(a^2*b^8 +
256*a^6*c^4 - 16*a^3*b^6*c + 96*a^4*b^4*c^2 - 256*a^5*b^2*c^3)))*(-(b^17 -
b^2*(-(4*a*c - b^2)^15)^(1/2) - 1720320*a^8*b*c^8 + 1140*a^2*b^13*c^2 - 101
60*a^3*b^11*c^3 + 34880*a^4*b^9*c^4 + 43776*a^5*b^7*c^5 - 680960*a^6*b^5*c^
6 + 1863680*a^7*b^3*c^7 - 55*a*b^15*c + 25*a*c*(-(4*a*c - b^2)^15)^(1/2))/(
512*(a^3*b^20*e^2 + 1048576*a^13*c^10*e^2 - 40*a^4*b^18*c*e^2 + 720*a^5*b^1
6*c^2*e^2 - 7680*a^6*b^14*c^3*e^2 + 53760*a^7*b^12*c^4*e^2 - 258048*a^8*b^1
0*c^5*e^2 + 860160*a^9*b^8*c^6*e^2 - 1966080*a^10*b^6*c^7*e^2 + 2949120*a^1
1*b^4*c^8*e^2 - 2621440*a^12*b^2*c^9*e^2)))^(1/2)*1i + ((204800*a^5*c^8*d*e
^11 - 16*b^10*c^3*d*e^11 + 672*a*b^8*c^4*d*e^11 - 28160*a^2*b^6*c^5*d*e^11
+ 209920*a^3*b^4*c^6*d*e^11 - 479232*a^4*b^2*c^7*d*e^11)/(512*(a^2*b^12 + 4
096*a^8*c^6 - 24*a^3*b^10*c + 240*a^4*b^8*c^2 - 1280*a^5*b^6*c^3 + 3840*a^6
*b^4*c^4 - 6144*a^7*b^2*c^5)) - ((256*a*b^13*c^2*e^12 + 4194304*a^7*b*c^8*e
^12 - 9216*a^2*b^11*c^3*e^12 + 122880*a^3*b^9*c^4*e^12 - 819200*a^4*b^7*c^5
*e^12 + 2949120*a^5*b^5*c^6*e^12 - 5505024*a^6*b^3*c^7*e^12)/(512*(a^2*b^12
+ 4096*a^8*c^6 - 24*a^3*b^10*c + 240*a^4*b^8*c^2 - 1280*a^5*b^6*c^3 + 3840
*a^6*b^4*c^4 - 6144*a^7*b^2*c^5)) - ((67108864*a^9*b*c^9*d*e^13 - 4096*a^2*
b^15*c^2*d*e^13 + 114688*a^3*b^13*c^3*d*e^13 - 1376256*a^4*b^11*c^4*d*e^13
+ 9175040*a^5*b^9*c^5*d*e^13 - 36700160*a^6*b^7*c^6*d*e^13 + 88080384*a^7*b
^5*c^7*d*e^13 - 117440512*a^8*b^3*c^8*d*e^13)/(512*(a^2*b^12 + 4096*a^8*c^6
- 24*a^3*b^10*c + 240*a^4*b^8*c^2 - 1280*a^5*b^6*c^3 + 3840*a^6*b^4*c^4 -
6144*a^7*b^2*c^5)) + (x*(262144*a^7*b*c^7*e^14 - 256*a^2*b^11*c^2*e^14 + 51
20*a^3*b^9*c^3*e^14 - 40960*a^4*b^7*c^4*e^14 + 163840*a^5*b^5*c^5*e^14 - 32
7680*a^6*b^3*c^6*e^14))/(32*(a^2*b^8 + 256*a^6*c^4 - 16*a^3*b^6*c + 96*a^4*
b^4*c^2 - 256*a^5*b^2*c^3)))*(-(b^17 - b^2*(-(4*a*c - b^2)^15)^(1/2) - 1720
320*a^8*b*c^8 + 1140*a^2*b^13*c^2 - 10160*a^3*b^11*c^3 + 34880*a^4*b^9*c^4
+ 43776*a^5*b^7*c^5 - 680960*a^6*b^5*c^6 + 1863680*a^7*b^3*c^7 - 55*a*b^15*
c + 25*a*c*(-(4*a*c - b^2)^15)^(1/2))/(512*(a^3*b^20*e^2 + 1048576*a^13*c^1
0*e^2 - 40*a^4*b^18*c*e^2 + 720*a^5*b^16*c^2*e^2 - 7680*a^6*b^14*c^3*e^2 +
53760*a^7*b^12*c^4*e^2 - 258048*a^8*b^10*c^5*e^2 + 860160*a^9*b^8*c^6*e^2 -
1966080*a^10*b^6*c^7*e^2 + 2949120*a^11*b^4*c^8*e^2 - 2621440*a^12*b^2*c^9
e^2)))^(1/2))(-(b^17 - b^2*(-(4*a*c - b^2)^15)^(1/2) - 1720320*a^8*b*c^8
+ 1140*a^2*b^13*c^2 - 10160*a^3*b^11*c^3 + 34880*a^4*b^9*c^4 + 43776*a^5*b^
7*c^5 - 680960*a^6*b^5*c^6 + 1863680*a^7*b^3*c^7 - 55*a*b^15*c + 25*a*c*(-(
4*a*c - b^2)^15)^(1/2))/(512*(a^3*b^20*e^2 + 1048576*a^13*c^10*e^2 - 40*a^4
*b^18*c*e^2 + 720*a^5*b^16*c^2*e^2 - 7680*a^6*b^14*c^3*e^2 + 53760*a^7*b^12
*c^4*e^2 - 258048*a^8*b^10*c^5*e^2 + 860160*a^9*b^8*c^6*e^2 - 1966080*a^10*
b^6*c^7*e^2 + 2949120*a^11*b^4*c^8*e^2 - 2621440*a^12*b^2*c^9*e^2)))^(1/2)
+ (x*(800*a^3*c^6*e^12 - b^6*c^3*e^12 + 34*a*b^4*c^4*e^12 - 1472*a^2*b^2*c^
5*e^12))/(32*(a^2*b^8 + 256*a^6*c^4 - 16*a^3*b^6*c + 96*a^4*b^4*c^2 - 256*a
^5*b^2*c^3)))*(-(b^17 - b^2*(-(4*a*c - b^2)^15)^(1/2) - 1720320*a^8*b*c^8 +

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 1140*a^2*b^13*c^2 - 10160*a^3*b^11*c^3 + 34880*a^4*b^9*c^4 + 43776*a^5*b^7
*c^5 - 680960*a^6*b^5*c^6 + 1863680*a^7*b^3*c^7 - 55*a*b^15*c + 25*a*c*(-(4
*a*c - b^2)^15)^(1/2))/(512*(a^3*b^20*e^2 + 1048576*a^13*c^10*e^2 - 40*a^4*
b^18*c*e^2 + 720*a^5*b^16*c^2*e^2 - 7680*a^6*b^14*c^3*e^2 + 53760*a^7*b^12*
c^4*e^2 - 258048*a^8*b^10*c^5*e^2 + 860160*a^9*b^8*c^6*e^2 - 1966080*a^10*b
^6*c^7*e^2 + 2949120*a^11*b^4*c^8*e^2 - 2621440*a^12*b^2*c^9*e^2)))^(1/2)*1
i)/((8000*a^3*c^7*e^10 - 35*b^6*c^4*e^10 - 84*a*b^4*c^5*e^10 + 12720*a^2*b^
2*c^6*e^10)/(256*(a^2*b^12 + 4096*a^8*c^6 - 24*a^3*b^10*c + 240*a^4*b^8*c^2
 - 1280*a^5*b^6*c^3 + 3840*a^6*b^4*c^4 - 6144*a^7*b^2*c^5)) - (((256*a*b^13
*c^2*e^12 + 4194304*a^7*b*c^8*e^12 - 9216*a^2*b^11*c^3*e^12 + 122880*a^3*b^
9*c^4*e^12 - 819200*a^4*b^7*c^5*e^12 + 2949120*a^5*b^5*c^6*e^12 - 5505024*a
^6*b^3*c^7*e^12)/(512*(a^2*b^12 + 4096*a^8*c^6 - 24*a^3*b^10*c + 240*a^4*b^
8*c^2 - 1280*a^5*b^6*c^3 + 3840*a^6*b^4*c^4 - 6144*a^7*b^2*c^5)) + ((671088
64*a^9*b*c^9*d*e^13 - 4096*a^2*b^15*c^2*d*e^13 + 114688*a^3*b^13*c^3*d*e^13
 - 1376256*a^4*b^11*c^4*d*e^13 + 9175040*a^5*b^9*c^5*d*e^13 - 36700160*a^6*
b^7*c^6*d*e^13 + 88080384*a^7*b^5*c^7*d*e^13 - 117440512*a^8*b^3*c^8*d*e^13
)/(512*(a^2*b^12 + 4096*a^8*c^6 - 24*a^3*b^10*c + 240*a^4*b^8*c^2 - 1280*a^
5*b^6*c^3 + 3840*a^6*b^4*c^4 - 6144*a^7*b^2*c^5)) + (x*(262144*a^7*b*c^7*e^
14 - 256*a^2*b^11*c^2*e^14 + 5120*a^3*b^9*c^3*e^14 - 40960*a^4*b^7*c^4*e^14
 + 163840*a^5*b^5*c^5*e^14 - 327680*a^6*b^3*c^6*e^14))/(32*(a^2*b^8 + 256*a
^6*c^4 - 16*a^3*b^6*c + 96*a^4*b^4*c^2 - 256*a^5*b^2*c^3)))*(-(b^17 - b^2*(
-(4*a*c - b^2)^15)^(1/2) - 1720320*a^8*b*c^8 + 1140*a^2*b^13*c^2 - 10160*a^
3*b^11*c^3 + 34880*a^4*b^9*c^4 + 43776*a^5*b^7*c^5 - 680960*a^6*b^5*c^6 + 1
863680*a^7*b^3*c^7 - 55*a*b^15*c + 25*a*c*(-(4*a*c - b^2)^15)^(1/2))/(512*(
a^3*b^20*e^2 + 1048576*a^13*c^10*e^2 - 40*a^4*b^18*c*e^2 + 720*a^5*b^16*c^2
*e^2 - 7680*a^6*b^14*c^3*e^2 + 53760*a^7*b^12*c^4*e^2 - 258048*a^8*b^10*c^5
*e^2 + 860160*a^9*b^8*c^6*e^2 - 1966080*a^10*b^6*c^7*e^2 + 2949120*a^11*b^4
*c^8*e^2 - 2621440*a^12*b^2*c^9*e^2)))^(1/2))*(-(b^17 - b^2*(-(4*a*c - b^2)
^15)^(1/2) - 1720320*a^8*b*c^8 + 1140*a^2*b^13*c^2 - 10160*a^3*b^11*c^3 + 3
4880*a^4*b^9*c^4 + 43776*a^5*b^7*c^5 - 680960*a^6*b^5*c^6 + 1863680*a^7*b^3
*c^7 - 55*a*b^15*c + 25*a*c*(-(4*a*c - b^2)^15)^(1/2))/(512*(a^3*b^20*e^2 +
 1048576*a^13*c^10*e^2 - 40*a^4*b^18*c*e^2 + 720*a^5*b^16*c^2*e^2 - 7680*a^
6*b^14*c^3*e^2 + 53760*a^7*b^12*c^4*e^2 - 258048*a^8*b^10*c^5*e^2 + 860160*
a^9*b^8*c^6*e^2 - 1966080*a^10*b^6*c^7*e^2 + 2949120*a^11*b^4*c^8*e^2 - 262
1440*a^12*b^2*c^9*e^2)))^(1/2) + (204800*a^5*c^8*d*e^11 - 16*b^10*c^3*d*e^1
1 + 672*a*b^8*c^4*d*e^11 - 28160*a^2*b^6*c^5*d*e^11 + 209920*a^3*b^4*c^6*d*
e^11 - 479232*a^4*b^2*c^7*d*e^11)/(512*(a^2*b^12 + 4096*a^8*c^6 - 24*a^3*b^
10*c + 240*a^4*b^8*c^2 - 1280*a^5*b^6*c^3 + 3840*a^6*b^4*c^4 - 6144*a^7*b^2
*c^5)) + (x*(800*a^3*c^6*e^12 - b^6*c^3*e^12 + 34*a*b^4*c^4*e^12 - 1472*a^2
*b^2*c^5*e^12))/(32*(a^2*b^8 + 256*a^6*c^4 - 16*a^3*b^6*c + 96*a^4*b^4*c^2
 - 256*a^5*b^2*c^3)))*(-(b^17 - b^2*(-(4*a*c - b^2)^15)^(1/2) - 1720320*a^8*
b*c^8 + 1140*a^2*b^13*c^2 - 10160*a^3*b^11*c^3 + 34880*a^4*b^9*c^4 + 43776*
a^5*b^7*c^5 - 680960*a^6*b^5*c^6 + 1863680*a^7*b^3*c^7 - 55*a*b^15*c + 25*a
*c*(-(4*a*c - b^2)^15)^(1/2))/(512*(a^3*b^20*e^2 + 1048576*a^13*c^10*e^2 -
40*a^4*b^18*c*e^2 + 720*a^5*b^16*c^2*e^2 - 7680*a^6*b^14*c^3*e^2 + 53760*a^
7*b^12*c^4*e^2 - 258048*a^8*b^10*c^5*e^2 + 860160*a^9*b^8*c^6*e^2 - 1966080
*a^10*b^6*c^7*e^2 + 2949120*a^11*b^4*c^8*e^2 - 2621440*a^12*b^2*c^9*e^2)))^
(1/2) + ((204800*a^5*c^8*d*e^11 - 16*b^10*c^3*d*e^11 + 672*a*b^8*c^4*d*e^11
 - 28160*a^2*b^6*c^5*d*e^11 + 209920*a^3*b^4*c^6*d*e^11 - 479232*a^4*b^2*c^
7*d*e^11)/(512*(a^2*b^12 + 4096*a^8*c^6 - 24*a^3*b^10*c + 240*a^4*b^8*c^2 -
 1280*a^5*b^6*c^3 + 3840*a^6*b^4*c^4 - 6144*a^7*b^2*c^5)) - ((256*a*b^13*c^
2*e^12 + 4194304*a^7*b*c^8*e^12 - 9216*a^2*b^11*c^3*e^12 + 122880*a^3*b^9*c
^4*e^12 - 819200*a^4*b^7*c^5*e^12 + 2949120*a^5*b^5*c^6*e^12 - 5505024*a^6*
b^3*c^7*e^12)/(512*(a^2*b^12 + 4096*a^8*c^6 - 24*a^3*b^10*c + 240*a^4*b^8*c
^2 - 1280*a^5*b^6*c^3 + 3840*a^6*b^4*c^4 - 6144*a^7*b^2*c^5)) - ((67108864*
a^9*b*c^9*d*e^13 - 4096*a^2*b^15*c^2*d*e^13 + 114688*a^3*b^13*c^3*d*e^13 -
1376256*a^4*b^11*c^4*d*e^13 + 9175040*a^5*b^9*c^5*d*e^13 - 36700160*a^6*b^7
*c^6*d*e^13 + 88080384*a^7*b^5*c^7*d*e^13 - 117440512*a^8*b^3*c^8*d*e^13)/(
512*(a^2*b^12 + 4096*a^8*c^6 - 24*a^3*b^10*c + 240*a^4*b^8*c^2 - 1280*a^5*b
```

```
^6*c^3 + 3840*a^6*b^4*c^4 - 6144*a^7*b^2*c^5)) + (x*(262144*a^7*b*c^7*e^14
- 256*a^2*b^11*c^2*e^14 + 5120*a^3*b^9*c^3*e^14 - 40960*a^4*b^7*c^4*e^14 +
163840*a^5*b^5*c^5*e^14 - 327680*a^6*b^3*c^6*e^14))/(32*(a^2*b^8 + 256*a^6*
c^4 - 16*a^3*b^6*c + 96*a^4*b^4*c^2 - 256*a^5*b^2*c^3)))*(-(b^17 - b^2*(-(4
*a*c - b^2)^15)^(1/2) - 1720320*a^8*b*c^8 + 1140*a^2*b^13*c^2 - 10160*a^3*b
^11*c^3 + 34880*a^4*b^9*c^4 + 43776*a^5*b^7*c^5 - 680960*a^6*b^5*c^6 + 1863
680*a^7*b^3*c^7 - 55*a*b^15*c + 25*a*c*(-(4*a*c - b^2)^15)^(1/2))/(512*(a^3
*b^20*e^2 + 1048576*a^13*c^10*e^2 - 40*a^4*b^18*c*e^2 + 720*a^5*b^16*c^2*e^
2 - 7680*a^6*b^14*c^3*e^2 + 53760*a^7*b^12*c^4*e^2 - 258048*a^8*b^10*c^5*e^
2 + 860160*a^9*b^8*c^6*e^2 - 1966080*a^10*b^6*c^7*e^2 + 2949120*a^11*b^4*c^
8*e^2 - 2621440*a^12*b^2*c^9*e^2)))^(1/2))*(-(b^17 - b^2*(-(4*a*c - b^2)^15
)^(1/2) - 1720320*a^8*b*c^8 + 1140*a^2*b^13*c^2 - 10160*a^3*b^11*c^3 + 3488
0*a^4*b^9*c^4 + 43776*a^5*b^7*c^5 - 680960*a^6*b^5*c^6 + 1863680*a^7*b^3*c^
7 - 55*a*b^15*c + 25*a*c*(-(4*a*c - b^2)^15)^(1/2))/(512*(a^3*b^20*e^2 + 10
48576*a^13*c^10*e^2 - 40*a^4*b^18*c*e^2 + 720*a^5*b^16*c^2*e^2 - 7680*a^6*b
^14*c^3*e^2 + 53760*a^7*b^12*c^4*e^2 - 258048*a^8*b^10*c^5*e^2 + 860160*a^9
*b^8*c^6*e^2 - 1966080*a^10*b^6*c^7*e^2 + 2949120*a^11*b^4*c^8*e^2 - 262144
0*a^12*b^2*c^9*e^2)))^(1/2) + (x*(800*a^3*c^6*e^12 - b^6*c^3*e^12 + 34*a*b^
4*c^4*e^12 - 1472*a^2*b^2*c^5*e^12))/(32*(a^2*b^8 + 256*a^6*c^4 - 16*a^3*b^
6*c + 96*a^4*b^4*c^2 - 256*a^5*b^2*c^3)))*(-(b^17 - b^2*(-(4*a*c - b^2)^15)
^(1/2) - 1720320*a^8*b*c^8 + 1140*a^2*b^13*c^2 - 10160*a^3*b^11*c^3 + 34880
*a^4*b^9*c^4 + 43776*a^5*b^7*c^5 - 680960*a^6*b^5*c^6 + 1863680*a^7*b^3*c^7
 - 55*a*b^15*c + 25*a*c*(-(4*a*c - b^2)^15)^(1/2))/(512*(a^3*b^20*e^2 + 104
8576*a^13*c^10*e^2 - 40*a^4*b^18*c*e^2 + 720*a^5*b^16*c^2*e^2 - 7680*a^6*b^
14*c^3*e^2 + 53760*a^7*b^12*c^4*e^2 - 258048*a^8*b^10*c^5*e^2 + 860160*a^9*
b^8*c^6*e^2 - 1966080*a^10*b^6*c^7*e^2 + 2949120*a^11*b^4*c^8*e^2 - 2621440
*a^12*b^2*c^9*e^2)))^(1/2)))*(-(b^17 - b^2*(-(4*a*c - b^2)^15)^(1/2) - 1720
320*a^8*b*c^8 + 1140*a^2*b^13*c^2 - 10160*a^3*b^11*c^3 + 34880*a^4*b^9*c^4
+ 43776*a^5*b^7*c^5 - 680960*a^6*b^5*c^6 + 1863680*a^7*b^3*c^7 - 55*a*b^15*
c + 25*a*c*(-(4*a*c - b^2)^15)^(1/2))/(512*(a^3*b^20*e^2 + 1048576*a^13*c^1
0*e^2 - 40*a^4*b^18*c*e^2 + 720*a^5*b^16*c^2*e^2 - 7680*a^6*b^14*c^3*e^2 +
53760*a^7*b^12*c^4*e^2 - 258048*a^8*b^10*c^5*e^2 + 860160*a^9*b^8*c^6*e^2 -
 1966080*a^10*b^6*c^7*e^2 + 2949120*a^11*b^4*c^8*e^2 - 2621440*a^12*b^2*c^9
*e^2)))^(1/2)*2i
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2/(a+b*(e*x+d)**2+c*(e*x+d)**4)**3,x)
```

```
[Out] Timed out
```

3.520 $\int \frac{d+ex}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^3}\,dx$

Optimal. Leaf size=152

$$-\frac{6c^2\tanh^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{e\left(b^2-4ac\right)^{5/2}}+\frac{3c\left(b+2c(d+ex)^2\right)}{2e\left(b^2-4ac\right)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)}+\frac{-b-2c(d+ex)^2}{4e\left(b^2-4ac\right)\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}$$

Rubi [A] time = 0.18, antiderivative size = 150, normalized size of antiderivative = 0.99, number of steps used = 6, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {1142, 1107, 614, 618, 206}

$$-\frac{6c^2\tanh^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{e\left(b^2-4ac\right)^{5/2}}+\frac{3c\left(b+2c(d+ex)^2\right)}{2e\left(b^2-4ac\right)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)}-\frac{b+2c(d+ex)^2}{4e\left(b^2-4ac\right)\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3,x]
```

```
[Out] -(b + 2*c*(d + e*x)^2)/(4*(b^2 - 4*a*c)*e*(a + b*(d + e*x)^2 + c*(d + e*x)^
4)^2) + (3*c*(b + 2*c*(d + e*x)^2))/(2*(b^2 - 4*a*c)^2*e*(a + b*(d + e*x)^2
 + c*(d + e*x)^4)) - (6*c^2*ArcTanh[(b + 2*c*(d + e*x)^2)/Sqrt[b^2 - 4*a*c]
])/((b^2 - 4*a*c)^(5/2)*e)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 614

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[(2*c*(2*p +
 3))/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
egerQ[4*p]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1107

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rule 1142

```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Di
st[u^m/(Coefficient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p,
 x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

Rule steps

$$\int \frac{d+ex}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^3}\,dx = \frac{\text{Subst}\left(\int \frac{x}{\left(a+bx^2+cx^4\right)^3}\,dx, x, d+ex\right)}{e}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{\left(a+bx+cx^2\right)^3}\,dx, x, (d+ex)^2\right)}{2e}$$

$$= -\frac{b+2c(d+ex)^2}{4\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)^2} - \frac{(3c)\,\text{Subst}\left(\int \frac{1}{\left(a+bx+cx^2\right)^2}\, d\right.}{2\left(b^2-4ac\right)}$$

$$= -\frac{b+2c(d+ex)^2}{4\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)^2} + \frac{3c\left(b+2c(d+\right.}{2\left(b^2-4ac\right)^2 e\left(a+b(d+\right.}$$

$$= -\frac{b+2c(d+ex)^2}{4\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)^2} + \frac{3c\left(b+2c(d+\right.}{2\left(b^2-4ac\right)^2 e\left(a+b(d+\right.}$$

$$= -\frac{b+2c(d+ex)^2}{4\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)^2} + \frac{3c\left(b+2c(d+\right.}{2\left(b^2-4ac\right)^2 e\left(a+b(d+\right.}$$

Mathematica [A] time = 0.18, size = 147, normalized size = 0.97

$$\frac{\frac{24c^2\tan^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}} + \frac{\left(b^2-4ac\right)\left(-b-2c(d+ex)^2\right)}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^2} + \frac{6c\left(b+2c(d+ex)^2\right)}{a+b(d+ex)^2+c(d+ex)^4}}{4e\left(b^2-4ac\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3,x]
```

```
[Out] (((b^2 - 4*a*c)*(-b - 2*c*(d + e*x)^2))/(a + b*(d + e*x)^2 + c*(d + e*x)^4)
^2 + (6*c*(b + 2*c*(d + e*x)^2))/(a + b*(d + e*x)^2 + c*(d + e*x)^4) + (24*
c^2*ArcTan[(b + 2*c*(d + e*x)^2)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c])/(
4*(b^2 - 4*a*c)^2*e)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{d+ex}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(d + e*x)/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3,x]
```

```
[Out] IntegrateAlgebraic[(d + e*x)/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3, x]
```

fricas [B] time = 1.47, size = 3708, normalized size = 24.39

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="fricas")
```

```
[Out] [1/4*(12*(b^2*c^3 - 4*a*c^4)*e^6*x^6 + 72*(b^2*c^3 - 4*a*c^4)*d*e^5*x^5 + 1
8*(b^3*c^2 - 4*a*b*c^3 + 10*(b^2*c^3 - 4*a*c^4)*d^2)*e^4*x^4 + 12*(b^2*c^3
- 4*a*c^4)*d^6 + 24*(10*(b^2*c^3 - 4*a*c^4)*d^3 + 3*(b^3*c^2 - 4*a*b*c^3)*d
)*e^3*x^3 - b^5 + 14*a*b^3*c - 40*a^2*b*c^2 + 18*(b^3*c^2 - 4*a*b*c^3)*d^4
+ 4*(b^4*c + a*b^2*c^2 - 20*a^2*c^3 + 45*(b^2*c^3 - 4*a*c^4)*d^4 + 27*(b^3*
c^2 - 4*a*b*c^3)*d^2)*e^2*x^2 + 4*(b^4*c + a*b^2*c^2 - 20*a^2*c^3)*d^2 + 8*
(9*(b^2*c^3 - 4*a*c^4)*d^5 + 9*(b^3*c^2 - 4*a*b*c^3)*d^3 + (b^4*c + a*b^2*c
^2 - 20*a^2*c^3)*d)*e*x + 12*(c^4*e^8*x^8 + 8*c^4*d*e^7*x^7 + 2*(14*c^4*d^2
 + b*c^3)*e^6*x^6 + c^4*d^8 + 4*(14*c^4*d^3 + 3*b*c^3*d)*e^5*x^5 + 2*b*c^3*
d^6 + (70*c^4*d^4 + 30*b*c^3*d^2 + b^2*c^2 + 2*a*c^3)*e^4*x^4 + 4*(14*c^4*d
^5 + 10*b*c^3*d^3 + (b^2*c^2 + 2*a*c^3)*d)*e^3*x^3 + 2*a*b*c^2*d^2 + (b^2*c
^2 + 2*a*c^3)*d^4 + 2*(14*c^4*d^6 + 15*b*c^3*d^4 + a*b*c^2 + 3*(b^2*c^2 + 2
*a*c^3)*d^2)*e^2*x^2 + a^2*c^2 + 4*(2*c^4*d^7 + 3*b*c^3*d^5 + a*b*c^2*d + (
b^2*c^2 + 2*a*c^3)*d^3)*e*x)*sqrt(b^2 - 4*a*c)*log((2*c^2*e^4*x^4 + 8*c^2*d
*e^3*x^3 + 2*c^2*d^4 + 2*(6*c^2*d^2 + b*c)*e^2*x^2 + 2*b*c*d^2 + 4*(2*c^2*d
^3 + b*c*d)*e*x + b^2 - 2*a*c - (2*c*e^2*x^2 + 4*c*d*e*x + 2*c*d^2 + b)*sqr
t(b^2 - 4*a*c))/(c*e^4*x^4 + 4*c*d*e^3*x^3 + c*d^4 + (6*c*d^2 + b)*e^2*x^2
+ b*d^2 + 2*(2*c*d^3 + b*d)*e*x + a)))/((b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^
2*c^4 - 64*a^3*c^5)*e^9*x^8 + 8*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 -
64*a^3*c^5)*d*e^8*x^7 + 2*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b
*c^4 + 14*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^2)*e^7*x
^6 + 4*(14*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^3 + 3*(
b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*d)*e^6*x^5 + (b^8 - 1
0*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4 + 70*(b^6*c^2 - 1
2*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^4 + 30*(b^7*c - 12*a*b^5*c^2 +
 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*d^2)*e^5*x^4 + 4*(14*(b^6*c^2 - 12*a*b^4*c^
3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^5 + 10*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^
3*c^3 - 64*a^3*b*c^4)*d^3 + (b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2
*c^3 - 128*a^4*c^4)*d)*e^4*x^3 + 2*(a*b^7 - 12*a^2*b^5*c + 48*a^3*b^3*c^2 -
 64*a^4*b*c^3 + 14*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d
^6 + 15*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*d^4 + 3*(b^8
 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*d^2)*e^3*x^2
 + 4*(2*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^7 + 3*(b^7
*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*d^5 + (b^8 - 10*a*b^6*c
+ 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*d^3 + (a*b^7 - 12*a^2*b^5*
c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3)*d)*e^2*x + ((b^6*c^2 - 12*a*b^4*c^3 + 48
*a^2*b^2*c^4 - 64*a^3*c^5)*d^8 + a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 -
64*a^5*c^3 + 2*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*d^6 +
 (b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*d^4 + 2
*(a*b^7 - 12*a^2*b^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3)*d^2)*e), 1/4*(12*(b
^2*c^3 - 4*a*c^4)*e^6*x^6 + 72*(b^2*c^3 - 4*a*c^4)*d*e^5*x^5 + 18*(b^3*c^2
- 4*a*b*c^3 + 10*(b^2*c^3 - 4*a*c^4)*d^2)*e^4*x^4 + 12*(b^2*c^3 - 4*a*c^4)*
d^6 + 24*(10*(b^2*c^3 - 4*a*c^4)*d^3 + 3*(b^3*c^2 - 4*a*b*c^3)*d)*e^3*x^3 -
 b^5 + 14*a*b^3*c - 40*a^2*b*c^2 + 18*(b^3*c^2 - 4*a*b*c^3)*d^4 + 4*(b^4*c
+ a*b^2*c^2 - 20*a^2*c^3 + 45*(b^2*c^3 - 4*a*c^4)*d^4 + 27*(b^3*c^2 - 4*a*b
*c^3)*d^2)*e^2*x^2 + 4*(b^4*c + a*b^2*c^2 - 20*a^2*c^3)*d^2 + 8*(9*(b^2*c^3
 - 4*a*c^4)*d^5 + 9*(b^3*c^2 - 4*a*b*c^3)*d^3 + (b^4*c + a*b^2*c^2 - 20*a^2
*c^3)*d)*e*x - 24*(c^4*e^8*x^8 + 8*c^4*d*e^7*x^7 + 2*(14*c^4*d^2 + b*c^3)*e
^6*x^6 + c^4*d^8 + 4*(14*c^4*d^3 + 3*b*c^3*d)*e^5*x^5 + 2*b*c^3*d^6 + (70*c
^4*d^4 + 30*b*c^3*d^2 + b^2*c^2 + 2*a*c^3)*e^4*x^4 + 4*(14*c^4*d^5 + 10*b*c
^3*d^3 + (b^2*c^2 + 2*a*c^3)*d)*e^3*x^3 + 2*a*b*c^2*d^2 + (b^2*c^2 + 2*a*c^
3)*d^4 + 2*(14*c^4*d^6 + 15*b*c^3*d^4 + a*b*c^2 + 3*(b^2*c^2 + 2*a*c^3)*d^2
)*e^2*x^2 + a^2*c^2 + 4*(2*c^4*d^7 + 3*b*c^3*d^5 + a*b*c^2*d + (b^2*c^2 + 2
*a*c^3)*d^3)*e*x)*sqrt(-b^2 + 4*a*c)*arctan(-(2*c*e^2*x^2 + 4*c*d*e*x + 2*c
*d^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)))/((b^6*c^2 - 12*a*b^4*c^3 + 48*
a^2*b^2*c^4 - 64*a^3*c^5)*e^9*x^8 + 8*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*
c^4 - 64*a^3*c^5)*d*e^8*x^7 + 2*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64
*a^3*b*c^4 + 14*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^2)
*e^7*x^6 + 4*(14*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^3
```

```
 + 3*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*d)*e^6*x^5 + (b
^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4 + 70*(b^6*c
^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^4 + 30*(b^7*c - 12*a*b^5
*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*d^2)*e^5*x^4 + 4*(14*(b^6*c^2 - 12*a*
b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^5 + 10*(b^7*c - 12*a*b^5*c^2 + 48*
a^2*b^3*c^3 - 64*a^3*b*c^4)*d^3 + (b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a
^3*b^2*c^3 - 128*a^4*c^4)*d)*e^4*x^3 + 2*(a*b^7 - 12*a^2*b^5*c + 48*a^3*b^3
*c^2 - 64*a^4*b*c^3 + 14*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*
c^5)*d^6 + 15*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*d^4 +
3*(b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*d^2)*e
^3*x^2 + 4*(2*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^7 +
3*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*d^5 + (b^8 - 10*a*
b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*d^3 + (a*b^7 - 12*a^
2*b^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3)*d)*e^2*x + ((b^6*c^2 - 12*a*b^4*c^
3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^8 + a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*
c^2 - 64*a^5*c^3 + 2*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)
*d^6 + (b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*d
^4 + 2*(a*b^7 - 12*a^2*b^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3)*d^2)*e)]
```

giac [B] time = 0.63, size = 365, normalized size = 2.40

$$\frac{6\,c^{2}\arctan\left(\frac{2\,cd^{2}+2\,\left(x^{2}e+2\,dx\right)ce+b}{\sqrt{-b^{2}+4\,ac}}\right)e^{(-1)}}{\left(b^{4}-8\,ab^{2}c+16\,a^{2}c^{2}\right)\sqrt{-b^{2}+4\,ac}}+\frac{12\,c^{3}d^{6}+36\,\left(x^{2}e+2\,dx\right)c^{3}d^{4}e+36\,\left(x^{2}e+2\,dx\right)^{2}c^{3}d^{2}e^{2}+18\,bc^{2}d^{4}+12\,\left(x^{2}e+2\,dx\right)^{3}c^{3}e^{3}+36\,\left(x^{2}e+2\,dx\right)bc^{2}d^{2}e+18\,\left(x^{2}e+2\,dx\right)^{2}bc^{2}e^{2}+4\,b^{2}cd^{2}+20\,ac^{2}d^{2}+4\,\left(x^{2}e+2\,dx\right)b^{2}ce+20\,\left(x^{2}e+2\,dx\right)ac^{2}e-b^{3}+10\,abc}{4\,\left(cd^{4}+2\,\left(x^{2}e+2\,dx\right)cd^{2}e+\left(x^{2}e+2\,dx\right)^{2}ce^{2}+bd^{2}+\left(x^{2}e+2\,dx\right)be+a\right)^{2}\left(b^{4}e-8\,ab^{2}ce+16\,a^{2}c^{2}e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="giac")
```

```
[Out] 6*c^2*arctan((2*c*d^2 + 2*(x^2*e + 2*d*x)*c*e + b)/sqrt(-b^2 + 4*a*c))*e^(-
1)/((b^4 - 8*a*b^2*c + 16*a^2*c^2)*sqrt(-b^2 + 4*a*c)) + 1/4*(12*c^3*d^6 +
36*(x^2*e + 2*d*x)*c^3*d^4*e + 36*(x^2*e + 2*d*x)^2*c^3*d^2*e^2 + 18*b*c^2*
d^4 + 12*(x^2*e + 2*d*x)^3*c^3*e^3 + 36*(x^2*e + 2*d*x)*b*c^2*d^2*e + 18*(x
^2*e + 2*d*x)^2*b*c^2*e^2 + 4*b^2*c*d^2 + 20*a*c^2*d^2 + 4*(x^2*e + 2*d*x)*
b^2*c*e + 20*(x^2*e + 2*d*x)*a*c^2*e - b^3 + 10*a*b*c)/((c*d^4 + 2*(x^2*e +
 2*d*x)*c*d^2*e + (x^2*e + 2*d*x)^2*c*e^2 + b*d^2 + (x^2*e + 2*d*x)*b*e + a
)^2*(b^4*e - 8*a*b^2*c*e + 16*a^2*c^2*e))
```

maple [C] time = 0.05, size = 541, normalized size = 3.56

$$\frac{3c^{2}\left(\mathrm{RootOf}\left(\_Z^{4}ce^{4}+4\_Z^{3}cde^{3}+cd^{4}+bd^{2}+\left(6cd^{2}e^{2}+be^{2}\right)\_Z^{2}+\left(4cd^{3}e+2deb\right)\_Z+a\right)e+d\right)\ln\left(-\mathrm{RootOf}\left(\_Z^{4}ce^{4}+4\_Z^{3}cde^{3}+cd^{4}+bd^{2}+\left(6cd^{2}e^{2}+be^{2}\right)\_Z^{2}+\left(4cd^{3}e+2deb\right)\_Z+a\right)+x\right)}{\left(16a^{2}c^{2}-8ab^{2}c+b^{4}\right)e\left(2ce^{3}\mathrm{RootOf}\left(\_Z^{4}ce^{4}+4\_Z^{3}cde^{3}+cd^{4}+bd^{2}+\left(6cd^{2}e^{2}+be^{2}\right)\_Z^{2}+\left(4cd^{3}e+2deb\right)\_Z+a\right)^{3}+6cde^{2}\mathrm{RootOf}\left(\_Z^{4}ce^{4}+4\_Z^{3}cde^{3}+cd^{4}+bd^{2}+\left(6cd^{2}e^{2}+be^{2}\right)\_Z^{2}+\left(4cd^{3}e+2deb\right)\_Z+a\right)^{2}+6ecd^{2}\mathrm{RootOf}\left(\_Z^{4}ce^{4}+4\_Z^{3}cde^{3}+cd^{4}+bd^{2}+\left(6cd^{2}e^{2}+be^{2}\right)\_Z^{2}+\left(4cd^{3}e+2deb\right)\_Z+a\right)+2cd^{3}+be\,\mathrm{RootOf}\left(\_Z^{4}ce^{4}+4\_Z^{3}cde^{3}+cd^{4}+bd^{2}+\left(6cd^{2}e^{2}+be^{2}\right)\_Z^{2}+\left(4cd^{3}e+2deb\right)\_Z+a\right)+bd\right)}+\frac{\frac{3c^{3}e^{5}x^{6}}{16a^{2}c^{2}-8ab^{2}c+b^{4}}+\frac{18c^{3}de^{4}x^{5}}{16a^{2}c^{2}-8ab^{2}c+b^{4}}+\frac{9\left(10cd^{2}+b\right)c^{2}e^{3}x^{4}}{2\left(16a^{2}c^{2}-8ab^{2}c+b^{4}\right)}+\frac{6\left(10cd^{2}+3b\right)c^{2}de^{2}x^{3}}{16a^{2}c^{2}-8ab^{2}c+b^{4}}+\frac{\left(45c^{2}d^{4}+27bcd^{2}+5ac+b^{2}\right)cex^{2}}{16a^{2}c^{2}-8ab^{2}c+b^{4}}+\frac{2\left(9c^{2}d^{4}+9bcd^{2}+5ac+b^{2}\right)cdx}{16a^{2}c^{2}-8ab^{2}c+b^{4}}+\frac{12c^{3}d^{6}+18bc^{2}d^{4}+20ac^{2}d^{2}+4b^{2}cd^{2}+10abc-b^{3}}{4\left(16a^{2}c^{2}-8ab^{2}c+b^{4}\right)e}}{\left(ce^{4}x^{4}+4cde^{3}x^{3}+6cd^{2}e^{2}x^{2}+4cd^{3}ex+be^{2}x^{2}+cd^{4}+2bdex+bd^{2}+a\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x)
```

```
[Out] (3*c^3*e^5/(16*a^2*c^2-8*a*b^2*c+b^4)*x^6+18*e^4*c^3*d/(16*a^2*c^2-8*a*b^2*
c+b^4)*x^5+9/2*c^2*e^3*(10*c*d^2+b)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^4+6*c^2*d*
e^2*(10*c*d^2+3*b)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^3+c*e*(45*c^2*d^4+27*b*c*d^
2+5*a*c+b^2)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^2+2*c*d*(9*c^2*d^4+9*b*c*d^2+5*a*
c+b^2)/(16*a^2*c^2-8*a*b^2*c+b^4)*x+1/4/e*(12*c^3*d^6+18*b*c^2*d^4+20*a*c^2
*d^2+4*b^2*c*d^2+10*a*b*c-b^3)/(16*a^2*c^2-8*a*b^2*c+b^4))/(c*e^4*x^4+4*c*d
*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2+3
*c^2/(16*a^2*c^2-8*a*b^2*c+b^4)/e*sum((_R*e+d)/(2*_R^3*c*e^3+6*_R^2*c*d*e^2
+6*_R*c*d^2*e+2*c*d^3+_R*b*e+b*d)*ln(-_R+x),_R=RootOf(_Z^4*c*e^4+4*_Z^3*c*d
*e^3+c*d^4+b*d^2+(6*c*d^2*e^2+b*e^2)*_Z^2+(4*c*d^3*e+2*b*d*e)*_Z+a))
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="maxima")
```

[Out] Timed out

mupad [B] time = 3.80, size = 1157, normalized size = 7.61

$$\frac{\frac{-b^3+4b^2cd^2+18bc^2d^4+10abc+12c^3d^6+20ac^2d^2}{4e\left(16a^2c^2-8ab^2c+b^4\right)}+\frac{x^2\left(eb^2c+27ebc^2d^2+45ec^3d^4+5aec^2\right)}{16a^2c^2-8ab^2c+b^4}+\frac{9x^4\left(10c^3d^2e^3+bc^2e^3\right)}{2\left(16a^2c^2-8ab^2c+b^4\right)}+\frac{3c^3e^5x^6}{16a^2c^2-8ab^2c+b^4}+\frac{2dx\left(b^2c+9bc^2d^2+9c^3d^4+5ac^2\right)}{16a^2c^2-8ab^2c+b^4}+\frac{6dx^3\left(10c^3d^2e^2+3bc^2e^2\right)}{16a^2c^2-8ab^2c+b^4}+\frac{18c^3de^4x^5}{16a^2c^2-8ab^2c+b^4}}{x^2\left(6b^2d^2e^2+30bcd^4e^2+2abe^2+28c^2d^6e^2+12acd^2e^2\right)+x^6\left(28c^2d^2e^6+2bce^6\right)+x\left(4eb^2d^3+12ebcd^5+4aebd+8ec^2d^7+8aecd^3\right)+x^3\left(4b^2de^3+40bcd^3e^3+56c^2d^5e^3+8acde^3\right)+x^5\left(56c^2d^3e^5+12bcde^5\right)+x^4\left(b^2e^4+30bcd^2e^4+70c^2d^4e^4+2ace^4\right)+a^2+b^2d^4+c^2d^8+c^2e^8x^8+2abd^2+2acd^4+2bcd^6+8c^2de^7x^7}+\frac{6c^2\,\mathrm{atan}\left(\frac{\left(b^4\left(4ac-b^2\right)^5+16a^2c^2\left(4ac-b^2\right)^5-8ab^2c\left(4ac-b^2\right)^5\right)\left(x\left(\frac{72c^6de^7}{a\left(4ac-b^2\right)^{9/2}\left(16a^2c^2-8ab^2c+b^4\right)}+\frac{72bc^4\left(16da^2bc^4e^9-8dab^3c^3e^9+db^5c^2e^9\right)}{ae^2\left(4ac-b^2\right)^{15/2}\left(16a^2c^2-8ab^2c+b^4\right)}\right)+x^2\left(\frac{36c^6e^8}{a\left(4ac-b^2\right)^{9/2}\left(16a^2c^2-8ab^2c+b^4\right)}+\frac{36bc^4\left(16a^2bc^4e^{10}-8ab^3c^3e^{10}+b^5c^2e^{10}\right)}{ae^2\left(4ac-b^2\right)^{15/2}\left(16a^2c^2-8ab^2c+b^4\right)}\right)+\frac{36c^6d^2e^6}{a\left(4ac-b^2\right)^{9/2}\left(16a^2c^2-8ab^2c+b^4\right)}+\frac{36bc^4\left(32a^3c^4e^8-16a^2b^2c^3e^8+16a^2bc^4d^2e^8+2ab^4c^2e^8-8ab^3c^3d^2e^8+b^5c^2d^2e^8\right)}{ae^2\left(4ac-b^2\right)^{15/2}\left(16a^2c^2-8ab^2c+b^4\right)}\right)}{72c^6e^6}\right)}{e\left(4ac-b^2\right)^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3,x)
```

```
[Out] ((12*c^3*d^6 - b^3 + 20*a*c^2*d^2 + 4*b^2*c*d^2 + 18*b*c^2*d^4 + 10*a*b*c)/
(4*e*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (x^2*(45*c^3*d^4*e + 5*a*c^2*e + b^2
*c*e + 27*b*c^2*d^2*e))/(b^4 + 16*a^2*c^2 - 8*a*b^2*c) + (9*x^4*(b*c^2*e^3
+ 10*c^3*d^2*e^3))/(2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (3*c^3*e^5*x^6)/(b^
4 + 16*a^2*c^2 - 8*a*b^2*c) + (2*d*x*(5*a*c^2 + b^2*c + 9*c^3*d^4 + 9*b*c^2
*d^2))/(b^4 + 16*a^2*c^2 - 8*a*b^2*c) + (6*d*x^3*(3*b*c^2*e^2 + 10*c^3*d^2*
e^2))/(b^4 + 16*a^2*c^2 - 8*a*b^2*c) + (18*c^3*d*e^4*x^5)/(b^4 + 16*a^2*c^2
 - 8*a*b^2*c))/(x^2*(6*b^2*d^2*e^2 + 28*c^2*d^6*e^2 + 2*a*b*e^2 + 12*a*c*d^
2*e^2 + 30*b*c*d^4*e^2) + x^6*(28*c^2*d^2*e^6 + 2*b*c*e^6) + x*(4*b^2*d^3*e
 + 8*c^2*d^7*e + 8*a*c*d^3*e + 12*b*c*d^5*e + 4*a*b*d*e) + x^3*(4*b^2*d*e^3
 + 56*c^2*d^5*e^3 + 8*a*c*d*e^3 + 40*b*c*d^3*e^3) + x^5*(56*c^2*d^3*e^5 + 1
2*b*c*d*e^5) + x^4*(b^2*e^4 + 70*c^2*d^4*e^4 + 2*a*c*e^4 + 30*b*c*d^2*e^4)
+ a^2 + b^2*d^4 + c^2*d^8 + c^2*e^8*x^8 + 2*a*b*d^2 + 2*a*c*d^4 + 2*b*c*d^6
 + 8*c^2*d*e^7*x^7) + (6*c^2*atan(((b^4*(4*a*c - b^2)^5 + 16*a^2*c^2*(4*a*c
 - b^2)^5 - 8*a*b^2*c*(4*a*c - b^2)^5)*(x*((72*c^6*d*e^7)/(a*(4*a*c - b^2)^
(9/2)*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (72*b*c^4*(b^5*c^2*d*e^9 - 8*a*b^3*
c^3*d*e^9 + 16*a^2*b*c^4*d*e^9))/(a*e^2*(4*a*c - b^2)^(15/2)*(b^4 + 16*a^2*
c^2 - 8*a*b^2*c))) + x^2*((36*c^6*e^8)/(a*(4*a*c - b^2)^(9/2)*(b^4 + 16*a^2
*c^2 - 8*a*b^2*c)) + (36*b*c^4*(b^5*c^2*e^10 - 8*a*b^3*c^3*e^10 + 16*a^2*b*
c^4*e^10))/(a*e^2*(4*a*c - b^2)^(15/2)*(b^4 + 16*a^2*c^2 - 8*a*b^2*c))) + (
36*c^6*d^2*e^6)/(a*(4*a*c - b^2)^(9/2)*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (3
6*b*c^4*(32*a^3*c^4*e^8 + 2*a*b^4*c^2*e^8 - 16*a^2*b^2*c^3*e^8 + b^5*c^2*d^
2*e^8 - 8*a*b^3*c^3*d^2*e^8 + 16*a^2*b*c^4*d^2*e^8))/(a*e^2*(4*a*c - b^2)^(
15/2)*(b^4 + 16*a^2*c^2 - 8*a*b^2*c))))/(72*c^6*e^6)))/(e*(4*a*c - b^2)^(5/
2))
```

sympy [B] time = 13.93, size = 1646, normalized size = 10.83

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(a+b*(e*x+d)**2+c*(e*x+d)**4)**3,x)
```

```
[Out] -3*c**2*sqrt(-1/(4*a*c - b**2)**5)*log(2*d*x/e + x**2 + (-192*a**3*c**5*sqr
t(-1/(4*a*c - b**2)**5) + 144*a**2*b**2*c**4*sqrt(-1/(4*a*c - b**2)**5) - 3
6*a*b**4*c**3*sqrt(-1/(4*a*c - b**2)**5) + 3*b**6*c**2*sqrt(-1/(4*a*c - b**
2)**5) + 3*b*c**2 + 6*c**3*d**2)/(6*c**3*e**2))/e + 3*c**2*sqrt(-1/(4*a*c -
 b**2)**5)*log(2*d*x/e + x**2 + (192*a**3*c**5*sqrt(-1/(4*a*c - b**2)**5) -
 144*a**2*b**2*c**4*sqrt(-1/(4*a*c - b**2)**5) + 36*a*b**4*c**3*sqrt(-1/(4*
a*c - b**2)**5) - 3*b**6*c**2*sqrt(-1/(4*a*c - b**2)**5) + 3*b*c**2 + 6*c**
3*d**2)/(6*c**3*e**2))/e + (10*a*b*c + 20*a*c**2*d**2 - b**3 + 4*b**2*c*d**
2 + 18*b*c**2*d**4 + 12*c**3*d**6 + 72*c**3*d*e**5*x**5 + 12*c**3*e**6*x**6
 + x**4*(18*b*c**2*e**4 + 180*c**3*d**2*e**4) + x**3*(72*b*c**2*d*e**3 + 24
0*c**3*d**3*e**3) + x**2*(20*a*c**2*e**2 + 4*b**2*c*e**2 + 108*b*c**2*d**2*
e**2 + 180*c**3*d**4*e**2) + x*(40*a*c**2*d*e + 8*b**2*c*d*e + 72*b*c**2*d*
*3*e + 72*c**3*d**5*e))/(64*a**4*c**2*e - 32*a**3*b**2*c*e + 128*a**3*b*c**
2*d**2*e + 128*a**3*c**3*d**4*e + 4*a**2*b**4*e - 64*a**2*b**3*c*d**2*e + 1
28*a**2*b*c**3*d**6*e + 64*a**2*c**4*d**8*e + 8*a*b**5*d**2*e - 24*a*b**4*c
*d**4*e - 64*a*b**3*c**2*d**6*e - 32*a*b**2*c**3*d**8*e + 4*b**6*d**4*e + 8
*b**5*c*d**6*e + 4*b**4*c**2*d**8*e + x**8*(64*a**2*c**4*e**9 - 32*a*b**2*c
**3*e**9 + 4*b**4*c**2*e**9) + x**7*(512*a**2*c**4*d*e**8 - 256*a*b**2*c**3
```

```
*d*e**8 + 32*b**4*c**2*d*e**8) + x**6*(128*a**2*b*c**3*e**7 + 1792*a**2*c**
4*d**2*e**7 - 64*a*b**3*c**2*e**7 - 896*a*b**2*c**3*d**2*e**7 + 8*b**5*c*e*
*7 + 112*b**4*c**2*d**2*e**7) + x**5*(768*a**2*b*c**3*d*e**6 + 3584*a**2*c*
*4*d**3*e**6 - 384*a*b**3*c**2*d*e**6 - 1792*a*b**2*c**3*d**3*e**6 + 48*b**
5*c*d*e**6 + 224*b**4*c**2*d**3*e**6) + x**4*(128*a**3*c**3*e**5 + 1920*a**
2*b*c**3*d**2*e**5 + 4480*a**2*c**4*d**4*e**5 - 24*a*b**4*c*e**5 - 960*a*b*
*3*c**2*d**2*e**5 - 2240*a*b**2*c**3*d**4*e**5 + 4*b**6*e**5 + 120*b**5*c*d
**2*e**5 + 280*b**4*c**2*d**4*e**5) + x**3*(512*a**3*c**3*d*e**4 + 2560*a**
2*b*c**3*d**3*e**4 + 3584*a**2*c**4*d**5*e**4 - 96*a*b**4*c*d*e**4 - 1280*a
*b**3*c**2*d**3*e**4 - 1792*a*b**2*c**3*d**5*e**4 + 16*b**6*d*e**4 + 160*b*
*5*c*d**3*e**4 + 224*b**4*c**2*d**5*e**4) + x**2*(128*a**3*b*c**2*e**3 + 76
8*a**3*c**3*d**2*e**3 - 64*a**2*b**3*c*e**3 + 1920*a**2*b*c**3*d**4*e**3 +
1792*a**2*c**4*d**6*e**3 + 8*a*b**5*e**3 - 144*a*b**4*c*d**2*e**3 - 960*a*b
**3*c**2*d**4*e**3 - 896*a*b**2*c**3*d**6*e**3 + 24*b**6*d**2*e**3 + 120*b*
*5*c*d**4*e**3 + 112*b**4*c**2*d**6*e**3) + x*(256*a**3*b*c**2*d*e**2 + 512
*a**3*c**3*d**3*e**2 - 128*a**2*b**3*c*d*e**2 + 768*a**2*b*c**3*d**5*e**2 +
 512*a**2*c**4*d**7*e**2 + 16*a*b**5*d*e**2 - 96*a*b**4*c*d**3*e**2 - 384*a
*b**3*c**2*d**5*e**2 - 256*a*b**2*c**3*d**7*e**2 + 16*b**6*d**3*e**2 + 48*b
**5*c*d**5*e**2 + 32*b**4*c**2*d**7*e**2))
```

3.521 $\int \frac{1}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^3}\,dx$

Optimal. Leaf size=437

$$\frac{\left(\frac{d}{e}+x\right)\left(3bce^2\left(b^2-8ac\right)\left(\frac{d}{e}+x\right)^2+\left(b^2-7ac\right)\left(3b^2-4ac\right)\right)}{8a^2\left(b^2-4ac\right)^2\left(a+be^2\left(\frac{d}{e}+x\right)^2+ce^4\left(\frac{d}{e}+x\right)^4\right)}+\frac{3\sqrt{c}\left(56a^2c^2-10ab^2c+b\left(b^2-8ac\right)\sqrt{b^2-4ac}\right.}{8\sqrt{2}\,a^2e\left(b^2-4ac\right)^{5/2}\sqrt{b-\sqrt{}}}$$

Rubi [A] time = 5.36, antiderivative size = 437, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {1106, 1092, 1178, 1166, 205}

$$\frac{3\sqrt{c}\left(56a^2c^2-10ab^2c+b\left(b^2-8ac\right)\sqrt{b^2-4ac}+b^4\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{8\sqrt{2}\,a^2e\left(b^2-4ac\right)^{5/2}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{3\sqrt{c}\left(56a^2c^2-10ab^2c-b\left(b^2-8ac\right)\sqrt{b^2-4ac}+b^4\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{8\sqrt{2}\,a^2e\left(b^2-4ac\right)^{5/2}\sqrt{\sqrt{b^2-4ac}+b}}+\frac{\left(\frac{d}{e}+x\right)\left(3bce^2\left(b^2-8ac\right)\left(\frac{d}{e}+x\right)^2+\left(b^2-7ac\right)\left(3b^2-4ac\right)\right)}{8a^2\left(b^2-4ac\right)^2\left(a+be^2\left(\frac{d}{e}+x\right)^2+ce^4\left(\frac{d}{e}+x\right)^4\right)}+\frac{\left(\frac{d}{e}+x\right)\left(-2ac+b^2+bce^2\left(\frac{d}{e}+x\right)^2\right)}{4a\left(b^2-4ac\right)\left(a+be^2\left(\frac{d}{e}+x\right)^2+ce^4\left(\frac{d}{e}+x\right)^4\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*(d + e*x)^2 + c*(d + e*x)^4)^(-3),x]
```

```
[Out] ((d/e + x)*(b^2 - 2*a*c + b*c*e^2*(d/e + x)^2))/(4*a*(b^2 - 4*a*c)*(a + b*e
^2*(d/e + x)^2 + c*e^4*(d/e + x)^4)^2) + ((d/e + x)*((b^2 - 7*a*c)*(3*b^2 -
 4*a*c) + 3*b*c*(b^2 - 8*a*c)*e^2*(d/e + x)^2))/(8*a^2*(b^2 - 4*a*c)^2*(a +
 b*e^2*(d/e + x)^2 + c*e^4*(d/e + x)^4)) + (3*Sqrt[c]*(b^4 - 10*a*b^2*c + 5
6*a^2*c^2 + b*(b^2 - 8*a*c)*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d +
 e*x))/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(8*Sqrt[2]*a^2*(b^2 - 4*a*c)^(5/2)*Sqr
t[b - Sqrt[b^2 - 4*a*c]]*e) - (3*Sqrt[c]*(b^4 - 10*a*b^2*c + 56*a^2*c^2 - b
*(b^2 - 8*a*c)*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b
 + Sqrt[b^2 - 4*a*c]]])/(8*Sqrt[2]*a^2*(b^2 - 4*a*c)^(5/2)*Sqrt[b + Sqrt[b^
2 - 4*a*c]]*e)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1092

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> -Simp[(x*(b^2 -
 2*a*c + b*c*x^2)*(a + b*x^2 + c*x^4)^(p + 1))/(2*a*(p + 1)*(b^2 - 4*a*c)),
 x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(b^2 - 2*a*c + 2*(p + 1)*(b^2
 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ
[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1106

```
Int[(P4_)^(p_), x_Symbol] :> With[{a = Coeff[P4, x, 0], b = Coeff[P4, x, 1]
, c = Coeff[P4, x, 2], d = Coeff[P4, x, 3], e = Coeff[P4, x, 4]}, Subst[Int
[SimplifyIntegrand[(a + d^4/(256*e^3) - (b*d)/(8*e) + (c - (3*d^2)/(8*e))*x
^2 + e*x^4)^p, x], x], x, d/(4*e) + x] /; EqQ[d^3 - 4*c*d*e + 8*b*e^2, 0] &
& NeQ[d, 0]] /; FreeQ[p, x] && PolyQ[P4, x, 4] && NeQ[p, 2] && NeQ[p, 3]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
```

```
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1178

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[(x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*(a + b*x^2 +
 c*x^4)^(p + 1))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2
 - 4*a*c)), Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p + 7
)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
LtQ[p, -1] && IntegerQ[2*p]
```

Rubi steps

$$\int \frac{1}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^3}\,dx = \text{Subst}\left(\int \frac{1}{\left(a+be^2x^2+ce^4x^4\right)^3}\,dx, x, \frac{d}{e}+x\right)$$

$$= \frac{\left(\frac{d}{e}+x\right)\left(b^2-2ac+bce^2\left(\frac{d}{e}+x\right)^2\right)}{4a\left(b^2-4ac\right)\left(a+be^2\left(\frac{d}{e}+x\right)^2+ce^4\left(\frac{d}{e}+x\right)^4\right)^2} - \frac{\text{Subst}\left(\int \frac{b^2e^4-2ace^4-4(b^2 \ldots}{(a+be^2x \ldots} \ldots\right.}{4a\,(b^2 \ldots}$$

$$= \frac{\left(\frac{d}{e}+x\right)\left(b^2-2ac+bce^2\left(\frac{d}{e}+x\right)^2\right)}{4a\left(b^2-4ac\right)\left(a+be^2\left(\frac{d}{e}+x\right)^2+ce^4\left(\frac{d}{e}+x\right)^4\right)^2} + \frac{(d+ex)\left(\left(b^2-7ac\right)(3 \ldots\right.}{8a^2\left(b^2-4ac\right)^2 e \ldots}$$

$$= \frac{\left(\frac{d}{e}+x\right)\left(b^2-2ac+bce^2\left(\frac{d}{e}+x\right)^2\right)}{4a\left(b^2-4ac\right)\left(a+be^2\left(\frac{d}{e}+x\right)^2+ce^4\left(\frac{d}{e}+x\right)^4\right)^2} + \frac{(d+ex)\left(\left(b^2-7ac\right)(3 \ldots\right.}{8a^2\left(b^2-4ac\right)^2 e \ldots}$$

$$= \frac{\left(\frac{d}{e}+x\right)\left(b^2-2ac+bce^2\left(\frac{d}{e}+x\right)^2\right)}{4a\left(b^2-4ac\right)\left(a+be^2\left(\frac{d}{e}+x\right)^2+ce^4\left(\frac{d}{e}+x\right)^4\right)^2} + \frac{(d+ex)\left(\left(b^2-7ac\right)(3 \ldots\right.}{8a^2\left(b^2-4ac\right)^2 e \ldots}$$

Mathematica [A] time = 6.17, size = 463, normalized size = 1.06

$$\frac{28a^2c^2(d+ex)-25ab^2c(d+ex)-24abc^2(d+ex)^3+3b^4(d+ex)+3b^3c(d+ex)^3}{8a^2e\left(4ac-b^2\right)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)}+\frac{3\sqrt{c}\left(56a^2c^2-10ab^2c-8abc\sqrt{b^2-4ac}+b^3\sqrt{b^2-4ac}+b^4\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{8\sqrt{2}a^2e\left(b^2-4ac\right)^{5/2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{3\sqrt{c}\left(-56a^2c^2+10ab^2c-8abc\sqrt{b^2-4ac}+b^3\sqrt{b^2-4ac}-b^4\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{8\sqrt{2}a^2e\left(b^2-4ac\right)^{5/2}\sqrt{\sqrt{b^2-4ac}+b}}+\frac{2ac(d+ex)-b^2(d+ex)-bc(d+ex)^3}{4ae\left(4ac-b^2\right)\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*(d + e*x)^2 + c*(d + e*x)^4)^(-3),x]
```

```
[Out] (-(b^2*(d + e*x)) + 2*a*c*(d + e*x) - b*c*(d + e*x)^3)/(4*a*(-b^2 + 4*a*c)*
e*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2) + (3*b^4*(d + e*x) - 25*a*b^2*c*(d
 + e*x) + 28*a^2*c^2*(d + e*x) + 3*b^3*c*(d + e*x)^3 - 24*a*b*c^2*(d + e*x)
^3)/(8*a^2*(-b^2 + 4*a*c)^2*e*(a + b*(d + e*x)^2 + c*(d + e*x)^4)) + (3*Sqr
t[c]*(b^4 - 10*a*b^2*c + 56*a^2*c^2 + b^3*Sqrt[b^2 - 4*a*c] - 8*a*b*c*Sqrt[
b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b - Sqrt[b^2 - 4*a*c]
]])/(8*Sqrt[2]*a^2*(b^2 - 4*a*c)^(5/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]*e) + (3*
Sqrt[c]*(-b^4 + 10*a*b^2*c - 56*a^2*c^2 + b^3*Sqrt[b^2 - 4*a*c] - 8*a*b*c*S
qrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b + Sqrt[b^2 - 4*
a*c]]])/(8*Sqrt[2]*a^2*(b^2 - 4*a*c)^(5/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]*e)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*(d + e*x)^2 + c*(d + e*x)^4)^(-3),x]
```

```
[Out] IntegrateAlgebraic[(a + b*(d + e*x)^2 + c*(d + e*x)^4)^(-3), x]
```

fricas [B] time = 2.55, size = 8554, normalized size = 19.57

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="fricas")
```

```
[Out] 1/16*(6*(b^3*c^2 - 8*a*b*c^3)*e^7*x^7 + 42*(b^3*c^2 - 8*a*b*c^3)*d*e^6*x^6
+ 2*(6*b^4*c - 49*a*b^2*c^2 + 28*a^2*c^3 + 63*(b^3*c^2 - 8*a*b*c^3)*d^2)*e^
5*x^5 + 10*(21*(b^3*c^2 - 8*a*b*c^3)*d^3 + (6*b^4*c - 49*a*b^2*c^2 + 28*a^2
*c^3)*d)*e^4*x^4 + 6*(b^3*c^2 - 8*a*b*c^3)*d^7 + 2*(3*b^5 - 20*a*b^3*c - 4*
a^2*b*c^2 + 105*(b^3*c^2 - 8*a*b*c^3)*d^4 + 10*(6*b^4*c - 49*a*b^2*c^2 + 28
*a^2*c^3)*d^2)*e^3*x^3 + 2*(6*b^4*c - 49*a*b^2*c^2 + 28*a^2*c^3)*d^5 + 2*(6
3*(b^3*c^2 - 8*a*b*c^3)*d^5 + 10*(6*b^4*c - 49*a*b^2*c^2 + 28*a^2*c^3)*d^3
+ 3*(3*b^5 - 20*a*b^3*c - 4*a^2*b*c^2)*d)*e^2*x^2 + 2*(3*b^5 - 20*a*b^3*c -
 4*a^2*b*c^2)*d^3 + 2*(21*(b^3*c^2 - 8*a*b*c^3)*d^6 + 5*a*b^4 - 37*a^2*b^2*
c + 44*a^3*c^2 + 5*(6*b^4*c - 49*a*b^2*c^2 + 28*a^2*c^3)*d^4 + 3*(3*b^5 - 2
0*a*b^3*c - 4*a^2*b*c^2)*d^2)*e*x + 3*sqrt(1/2)*((a^2*b^4*c^2 - 8*a^3*b^2*c
^3 + 16*a^4*c^4)*e^9*x^8 + 8*(a^2*b^4*c^2 - 8*a^3*b^2*c^3 + 16*a^4*c^4)*d*e
^8*x^7 + 2*(a^2*b^5*c - 8*a^3*b^3*c^2 + 16*a^4*b*c^3 + 14*(a^2*b^4*c^2 - 8*
a^3*b^2*c^3 + 16*a^4*c^4)*d^2)*e^7*x^6 + 4*(14*(a^2*b^4*c^2 - 8*a^3*b^2*c^3
 + 16*a^4*c^4)*d^3 + 3*(a^2*b^5*c - 8*a^3*b^3*c^2 + 16*a^4*b*c^3)*d)*e^6*x^
5 + (a^2*b^6 - 6*a^3*b^4*c + 32*a^5*c^3 + 70*(a^2*b^4*c^2 - 8*a^3*b^2*c^3 +
 16*a^4*c^4)*d^4 + 30*(a^2*b^5*c - 8*a^3*b^3*c^2 + 16*a^4*b*c^3)*d^2)*e^5*x
^4 + 4*(14*(a^2*b^4*c^2 - 8*a^3*b^2*c^3 + 16*a^4*c^4)*d^5 + 10*(a^2*b^5*c -
 8*a^3*b^3*c^2 + 16*a^4*b*c^3)*d^3 + (a^2*b^6 - 6*a^3*b^4*c + 32*a^5*c^3)*d
)*e^4*x^3 + 2*(a^3*b^5 - 8*a^4*b^3*c + 16*a^5*b*c^2 + 14*(a^2*b^4*c^2 - 8*a
^3*b^2*c^3 + 16*a^4*c^4)*d^6 + 15*(a^2*b^5*c - 8*a^3*b^3*c^2 + 16*a^4*b*c^3
)*d^4 + 3*(a^2*b^6 - 6*a^3*b^4*c + 32*a^5*c^3)*d^2)*e^3*x^2 + 4*(2*(a^2*b^4
*c^2 - 8*a^3*b^2*c^3 + 16*a^4*c^4)*d^7 + 3*(a^2*b^5*c - 8*a^3*b^3*c^2 + 16*
a^4*b*c^3)*d^5 + (a^2*b^6 - 6*a^3*b^4*c + 32*a^5*c^3)*d^3 + (a^3*b^5 - 8*a^
4*b^3*c + 16*a^5*b*c^2)*d)*e^2*x + ((a^2*b^4*c^2 - 8*a^3*b^2*c^3 + 16*a^4*c
^4)*d^8 + a^4*b^4 - 8*a^5*b^2*c + 16*a^6*c^2 + 2*(a^2*b^5*c - 8*a^3*b^3*c^2
 + 16*a^4*b*c^3)*d^6 + (a^2*b^6 - 6*a^3*b^4*c + 32*a^5*c^3)*d^4 + 2*(a^3*b^
5 - 8*a^4*b^3*c + 16*a^5*b*c^2)*d^2)*e)*sqrt(-(b^9 - 21*a*b^7*c + 189*a^2*b
^5*c^2 - 840*a^3*b^3*c^3 + 1680*a^4*b*c^4 + (a^5*b^10 - 20*a^6*b^8*c + 160*
a^7*b^6*c^2 - 640*a^8*b^4*c^3 + 1280*a^9*b^2*c^4 - 1024*a^10*c^5)*e^2*sqrt(
(b^8 - 22*a*b^6*c + 219*a^2*b^4*c^2 - 1078*a^3*b^2*c^3 + 2401*a^4*c^4)/((a^
10*b^10 - 20*a^11*b^8*c + 160*a^12*b^6*c^2 - 640*a^13*b^4*c^3 + 1280*a^14*b
^2*c^4 - 1024*a^15*c^5)*e^4)))/((a^5*b^10 - 20*a^6*b^8*c + 160*a^7*b^6*c^2
- 640*a^8*b^4*c^3 + 1280*a^9*b^2*c^4 - 1024*a^10*c^5)*e^2))*log(27*(21*b^8*
c^3 - 447*a*b^6*c^4 + 4189*a^2*b^4*c^5 - 19208*a^3*b^2*c^6 + 38416*a^4*c^7)
*e*x + 27*(21*b^8*c^3 - 447*a*b^6*c^4 + 4189*a^2*b^4*c^5 - 19208*a^3*b^2*c^
6 + 38416*a^4*c^7)*d + 27/2*sqrt(1/2)*((a^5*b^15 - 31*a^6*b^13*c + 424*a^7*
b^11*c^2 - 3280*a^8*b^9*c^3 + 15360*a^9*b^7*c^4 - 43264*a^10*b^5*c^5 + 6758
4*a^11*b^3*c^6 - 45056*a^12*b*c^7)*e^3*sqrt((b^8 - 22*a*b^6*c + 219*a^2*b^4
*c^2 - 1078*a^3*b^2*c^3 + 2401*a^4*c^4)/((a^10*b^10 - 20*a^11*b^8*c + 160*a
^12*b^6*c^2 - 640*a^13*b^4*c^3 + 1280*a^14*b^2*c^4 - 1024*a^15*c^5)*e^4)) -
 (b^14 - 32*a*b^12*c + 464*a^2*b^10*c^2 - 3885*a^3*b^8*c^3 + 20088*a^4*b^6*
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c^4 - 63680*a^5*b^4*c^5 + 113792*a^6*b^2*c^6 - 87808*a^7*c^7)*e)*sqrt(-(b^9 - 21*a*b^7*c + 189*a^2*b^5*c^2 - 840*a^3*b^3*c^3 + 1680*a^4*b*c^4 + (a^5*b^10 - 20*a^6*b^8*c + 160*a^7*b^6*c^2 - 640*a^8*b^4*c^3 + 1280*a^9*b^2*c^4 - 1024*a^10*c^5)*e^2*sqrt((b^8 - 22*a*b^6*c + 219*a^2*b^4*c^2 - 1078*a^3*b^2*c^3 + 2401*a^4*c^4)/((a^10*b^10 - 20*a^11*b^8*c + 160*a^12*b^6*c^2 - 640*a^13*b^4*c^3 + 1280*a^14*b^2*c^4 - 1024*a^15*c^5)*e^4)))/((a^5*b^10 - 20*a^6*b^8*c + 160*a^7*b^6*c^2 - 640*a^8*b^4*c^3 + 1280*a^9*b^2*c^4 - 1024*a^10*c^5)*e^2))) - 3*sqrt(1/2)*((a^2*b^4*c^2 - 8*a^3*b^2*c^3 + 16*a^4*c^4)*e^9*x^8 + 8*(a^2*b^4*c^2 - 8*a^3*b^2*c^3 + 16*a^4*c^4)*d*e^8*x^7 + 2*(a^2*b^5*c - 8*a^3*b^3*c^2 + 16*a^4*b*c^3 + 14*(a^2*b^4*c^2 - 8*a^3*b^2*c^3 + 16*a^4*c^4)*d^2)*e^7*x^6 + 4*(14*(a^2*b^4*c^2 - 8*a^3*b^2*c^3 + 16*a^4*c^4)*d^3 + 3*(a^2*b^5*c - 8*a^3*b^3*c^2 + 16*a^4*b*c^3)*d)*e^6*x^5 + (a^2*b^6 - 6*a^3*b^4*c + 32*a^5*c^3 + 70*(a^2*b^4*c^2 - 8*a^3*b^2*c^3 + 16*a^4*c^4)*d^4 + 30*(a^2*b^5*c - 8*a^3*b^3*c^2 + 16*a^4*b*c^3)*d^2)*e^5*x^4 + 4*(14*(a^2*b^4*c^2 - 8*a^3*b^2*c^3 + 16*a^4*c^4)*d^5 + 10*(a^2*b^5*c - 8*a^3*b^3*c^2 + 16*a^4*b*c^3)*d^3 + (a^2*b^6 - 6*a^3*b^4*c + 32*a^5*c^3)*d)*e^4*x^3 + 2*(a^3*b^5 - 8*a^4*b^3*c + 16*a^5*b*c^2 + 14*(a^2*b^4*c^2 - 8*a^3*b^2*c^3 + 16*a^4*c^4)*d^6 + 15*(a^2*b^5*c - 8*a^3*b^3*c^2 + 16*a^4*b*c^3)*d^4 + 3*(a^2*b^6 - 6*a^3*b^4*c + 32*a^5*c^3)*d^2)*e^3*x^2 + 4*(2*(a^2*b^4*c^2 - 8*a^3*b^2*c^3 + 16*a^4*c^4)*d^7 + 3*(a^2*b^5*c - 8*a^3*b^3*c^2 + 16*a^4*b*c^3)*d^5 + (a^2*b^6 - 6*a^3*b^4*c + 32*a^5*c^3)*d^3 + (a^3*b^5 - 8*a^4*b^3*c + 16*a^5*b*c^2)*d)*e^2*x + ((a^2*b^4*c^2 - 8*a^3*b^2*c^3 + 16*a^4*c^4)*d^8 + a^4*b^4 - 8*a^5*b^2*c + 16*a^6*c^2 + 2*(a^2*b^5*c - 8*a^3*b^3*c^2 + 16*a^4*b*c^3)*d^6 + (a^2*b^6 - 6*a^3*b^4*c + 32*a^5*c^3)*d^4 + 2*(a^3*b^5 - 8*a^4*b^3*c + 16*a^5*b*c^2)*d^2)*e)*sqrt(-(b^9 - 21*a*b^7*c + 189*a^2*b^5*c^2 - 840*a^3*b^3*c^3 + 1680*a^4*b*c^4 + (a^5*b^10 - 20*a^6*b^8*c + 160*a^7*b^6*c^2 - 640*a^8*b^4*c^3 + 1280*a^9*b^2*c^4 - 1024*a^10*c^5)*e^2*sqrt((b^8 - 22*a*b^6*c + 219*a^2*b^4*c^2 - 1078*a^3*b^2*c^3 + 2401*a^4*c^4)/((a^10*b^10 - 20*a^11*b^8*c + 160*a^12*b^6*c^2 - 640*a^13*b^4*c^3 + 1280*a^14*b^2*c^4 - 1024*a^15*c^5)*e^4)))/((a^5*b^10 - 20*a^6*b^8*c + 160*a^7*b^6*c^2 - 640*a^8*b^4*c^3 + 1280*a^9*b^2*c^4 - 1024*a^10*c^5)*e^2))*log(27*(21*b^8*c^3 - 447*a*b^6*c^4 + 4189*a^2*b^4*c^5 - 19208*a^3*b^2*c^6 + 38416*a^4*c^7)*e*x + 27*(21*b^8*c^3 - 447*a*b^6*c^4 + 4189*a^2*b^4*c^5 - 19208*a^3*b^2*c^6 + 38416*a^4*c^7)*d - 27/2*sqrt(1/2)*((a^5*b^15 - 31*a^6*b^13*c + 424*a^7*b^11*c^2 - 3280*a^8*b^9*c^3 + 15360*a^9*b^7*c^4 - 43264*a^10*b^5*c^5 + 67584*a^11*b^3*c^6 - 45056*a^12*b*c^7)*e^3*sqrt((b^8 - 22*a*b^6*c + 219*a^2*b^4*c^2 - 1078*a^3*b^2*c^3 + 2401*a^4*c^4)/((a^10*b^10 - 20*a^11*b^8*c + 160*a^12*b^6*c^2 - 640*a^13*b^4*c^3 + 1280*a^14*b^2*c^4 - 1024*a^15*c^5)*e^4)) - (b^14 - 32*a*b^12*c + 464*a^2*b^10*c^2 - 3885*a^3*b^8*c^3 + 20088*a^4*b^6*c^4 - 63680*a^5*b^4*c^5 + 113792*a^6*b^2*c^6 - 87808*a^7*c^7)*e)*sqrt(-(b^9 - 21*a*b^7*c + 189*a^2*b^5*c^2 - 840*a^3*b^3*c^3 + 1680*a^4*b*c^4 + (a^5*b^10 - 20*a^6*b^8*c + 160*a^7*b^6*c^2 - 640*a^8*b^4*c^3 + 1280*a^9*b^2*c^4 - 1024*a^10*c^5)*e^2*sqrt((b^8 - 22*a*b^6*c + 219*a^2*b^4*c^2 - 1078*a^3*b^2*c^3 + 2401*a^4*c^4)/((a^10*b^10 - 20*a^11*b^8*c + 160*a^12*b^6*c^2 - 640*a^13*b^4*c^3 + 1280*a^14*b^2*c^4 - 1024*a^15*c^5)*e^4)))/((a^5*b^10 - 20*a^6*b^8*c + 160*a^7*b^6*c^2 - 640*a^8*b^4*c^3 + 1280*a^9*b^2*c^4 - 1024*a^10*c^5)*e^2))) - 3*sqrt(1/2)*((a^2*b^4*c^2 - 8*a^3*b^2*c^3 + 16*a^4*c^4)*e^9*x^8 + 8*(a^2*b^4*c^2 - 8*a^3*b^2*c^3 + 16*a^4*c^4)*d*e^8*x^7 + 2*(a^2*b^5*c - 8*a^3*b^3*c^2 + 16*a^4*b*c^3 + 14*(a^2*b^4*c^2 - 8*a^3*b^2*c^3 + 16*a^4*c^4)*d^2)*e^7*x^6 + 4*(14*(a^2*b^4*c^2 - 8*a^3*b^2*c^3 + 16*a^4*c^4)*d^3 + 3*(a^2*b^5*c - 8*a^3*b^3*c^2 + 16*a^4*b*c^3)*d)*e^6*x^5 + (a^2*b^6 - 6*a^3*b^4*c + 32*a^5*c^3 + 70*(a^2*b^4*c^2 - 8*a^3*b^2*c^3 + 16*a^4*c^4)*d^4 + 30*(a^2*b^5*c - 8*a^3*b^3*c^2 + 16*a^4*b*c^3)*d^2)*e^5*x^4 + 4*(14*(a^2*b^4*c^2 - 8*a^3*b^2*c^3 + 16*a^4*c^4)*d^5 + 10*(a^2*b^5*c - 8*a^3*b^3*c^2 + 16*a^4*b*c^3)*d^3 + (a^2*b^6 - 6*a^3*b^4*c + 32*a^5*c^3)*d)*e^4*x^3 + 2*(a^3*b^5 - 8*a^4*b^3*c + 16*a^5*b*c^2 + 14*(a^2*b^4*c^2 - 8*a^3*b^2*c^3 + 16*a^4*c^4)*d^6 + 15*(a^2*b^5*c - 8*a^3*b^3*c^2 + 16*a^4*b*c^3)*d^4 + 3*(a^2*b^6 - 6*a^3*b^4*c + 32*a^5*c^3)*d^2)*e^3*x^2 + 4*(2*(a^2*b^4*c^2 - 8*a^3*b^2*c^3 + 16*a^4*c^4)*d^7 + 3*(a^2*b^5*c - 8*a^3*b^3*c^2 + 16*a^4*b*c^3)*d^5 + (a^2*b^6 - 6*a^3*b^4*c + 32*a

^5*c^3)*d^3 + (a^3*b^5 - 8*a^4*b^3*c + 16*a^5*b*c^2)*d)*e^2*x + ((a^2*b^4*c
^2 - 8*a^3*b^2*c^3 + 16*a^4*c^4)*d^8 + a^4*b^4 - 8*a^5*b^2*c + 16*a^6*c^2 +
2*(a^2*b^5*c - 8*a^3*b^3*c^2 + 16*a^4*b*c^3)*d^6 + (a^2*b^6 - 6*a^3*b^4*c
+ 32*a^5*c^3)*d^4 + 2*(a^3*b^5 - 8*a^4*b^3*c + 16*a^5*b*c^2)*d^2)*e)*sqrt(-
(b^9 - 21*a*b^7*c + 189*a^2*b^5*c^2 - 840*a^3*b^3*c^3 + 1680*a^4*b*c^4 - (a
^5*b^10 - 20*a^6*b^8*c + 160*a^7*b^6*c^2 - 640*a^8*b^4*c^3 + 1280*a^9*b^2*c
^4 - 1024*a^10*c^5)*e^2*sqrt((b^8 - 22*a*b^6*c + 219*a^2*b^4*c^2 - 1078*a^3
*b^2*c^3 + 2401*a^4*c^4)/((a^10*b^10 - 20*a^11*b^8*c + 160*a^12*b^6*c^2 - 6
40*a^13*b^4*c^3 + 1280*a^14*b^2*c^4 - 1024*a^15*c^5)*e^4)))/((a^5*b^10 - 20
*a^6*b^8*c + 160*a^7*b^6*c^2 - 640*a^8*b^4*c^3 + 1280*a^9*b^2*c^4 - 1024*a^
10*c^5)*e^2))*log(27*(21*b^8*c^3 - 447*a*b^6*c^4 + 4189*a^2*b^4*c^5 - 19208
*a^3*b^2*c^6 + 38416*a^4*c^7)*e*x + 27*(21*b^8*c^3 - 447*a*b^6*c^4 + 4189*a
^2*b^4*c^5 - 19208*a^3*b^2*c^6 + 38416*a^4*c^7)*d + 27/2*sqrt(1/2)*((a^5*b^
15 - 31*a^6*b^13*c + 424*a^7*b^11*c^2 - 3280*a^8*b^9*c^3 + 15360*a^9*b^7*c^
4 - 43264*a^10*b^5*c^5 + 67584*a^11*b^3*c^6 - 45056*a^12*b*c^7)*e^3*sqrt((b
^8 - 22*a*b^6*c + 219*a^2*b^4*c^2 - 1078*a^3*b^2*c^3 + 2401*a^4*c^4)/((a^10
*b^10 - 20*a^11*b^8*c + 160*a^12*b^6*c^2 - 640*a^13*b^4*c^3 + 1280*a^14*b^2
*c^4 - 1024*a^15*c^5)*e^4)) + (b^14 - 32*a*b^12*c + 464*a^2*b^10*c^2 - 3885
*a^3*b^8*c^3 + 20088*a^4*b^6*c^4 - 63680*a^5*b^4*c^5 + 113792*a^6*b^2*c^6 -
87808*a^7*c^7)*e)*sqrt(-(b^9 - 21*a*b^7*c + 189*a^2*b^5*c^2 - 840*a^3*b^3*
c^3 + 1680*a^4*b*c^4 - (a^5*b^10 - 20*a^6*b^8*c + 160*a^7*b^6*c^2 - 640*a^8
*b^4*c^3 + 1280*a^9*b^2*c^4 - 1024*a^10*c^5)*e^2*sqrt((b^8 - 22*a*b^6*c + 2
19*a^2*b^4*c^2 - 1078*a^3*b^2*c^3 + 2401*a^4*c^4)/((a^10*b^10 - 20*a^11*b^8
*c + 160*a^12*b^6*c^2 - 640*a^13*b^4*c^3 + 1280*a^14*b^2*c^4 - 1024*a^15*c^
5)*e^4)))/((a^5*b^10 - 20*a^6*b^8*c + 160*a^7*b^6*c^2 - 640*a^8*b^4*c^3 + 1
280*a^9*b^2*c^4 - 1024*a^10*c^5)*e^2))) + 3*sqrt(1/2)*((a^2*b^4*c^2 - 8*a^3
*b^2*c^3 + 16*a^4*c^4)*e^9*x^8 + 8*(a^2*b^4*c^2 - 8*a^3*b^2*c^3 + 16*a^4*c^
4)*d*e^8*x^7 + 2*(a^2*b^5*c - 8*a^3*b^3*c^2 + 16*a^4*b*c^3 + 14*(a^2*b^4*c^
2 - 8*a^3*b^2*c^3 + 16*a^4*c^4)*d^2)*e^7*x^6 + 4*(14*(a^2*b^4*c^2 - 8*a^3*b
^2*c^3 + 16*a^4*c^4)*d^3 + 3*(a^2*b^5*c - 8*a^3*b^3*c^2 + 16*a^4*b*c^3)*d)*
e^6*x^5 + (a^2*b^6 - 6*a^3*b^4*c + 32*a^5*c^3 + 70*(a^2*b^4*c^2 - 8*a^3*b^2
*c^3 + 16*a^4*c^4)*d^4 + 30*(a^2*b^5*c - 8*a^3*b^3*c^2 + 16*a^4*b*c^3)*d^2)
*e^5*x^4 + 4*(14*(a^2*b^4*c^2 - 8*a^3*b^2*c^3 + 16*a^4*c^4)*d^5 + 10*(a^2*b
^5*c - 8*a^3*b^3*c^2 + 16*a^4*b*c^3)*d^3 + (a^2*b^6 - 6*a^3*b^4*c + 32*a^5*
c^3)*d)*e^4*x^3 + 2*(a^3*b^5 - 8*a^4*b^3*c + 16*a^5*b*c^2 + 14*(a^2*b^4*c^2
- 8*a^3*b^2*c^3 + 16*a^4*c^4)*d^6 + 15*(a^2*b^5*c - 8*a^3*b^3*c^2 + 16*a^4
*b*c^3)*d^4 + 3*(a^2*b^6 - 6*a^3*b^4*c + 32*a^5*c^3)*d^2)*e^3*x^2 + 4*(2*(a
^2*b^4*c^2 - 8*a^3*b^2*c^3 + 16*a^4*c^4)*d^7 + 3*(a^2*b^5*c - 8*a^3*b^3*c^2
+ 16*a^4*b*c^3)*d^5 + (a^2*b^6 - 6*a^3*b^4*c + 32*a^5*c^3)*d^3 + (a^3*b^5
- 8*a^4*b^3*c + 16*a^5*b*c^2)*d)*e^2*x + ((a^2*b^4*c^2 - 8*a^3*b^2*c^3 + 16
*a^4*c^4)*d^8 + a^4*b^4 - 8*a^5*b^2*c + 16*a^6*c^2 + 2*(a^2*b^5*c - 8*a^3*b
^3*c^2 + 16*a^4*b*c^3)*d^6 + (a^2*b^6 - 6*a^3*b^4*c + 32*a^5*c^3)*d^4 + 2*(
a^3*b^5 - 8*a^4*b^3*c + 16*a^5*b*c^2)*d^2)*e)*sqrt(-(b^9 - 21*a*b^7*c + 189
*a^2*b^5*c^2 - 840*a^3*b^3*c^3 + 1680*a^4*b*c^4 - (a^5*b^10 - 20*a^6*b^8*c
+ 160*a^7*b^6*c^2 - 640*a^8*b^4*c^3 + 1280*a^9*b^2*c^4 - 1024*a^10*c^5)*e^2
*sqrt((b^8 - 22*a*b^6*c + 219*a^2*b^4*c^2 - 1078*a^3*b^2*c^3 + 2401*a^4*c^4
)/((a^10*b^10 - 20*a^11*b^8*c + 160*a^12*b^6*c^2 - 640*a^13*b^4*c^3 + 1280*
a^14*b^2*c^4 - 1024*a^15*c^5)*e^4)))/((a^5*b^10 - 20*a^6*b^8*c + 160*a^7*b^
6*c^2 - 640*a^8*b^4*c^3 + 1280*a^9*b^2*c^4 - 1024*a^10*c^5)*e^2))*log(27*(2
1*b^8*c^3 - 447*a*b^6*c^4 + 4189*a^2*b^4*c^5 - 19208*a^3*b^2*c^6 + 38416*a^
4*c^7)*e*x + 27*(21*b^8*c^3 - 447*a*b^6*c^4 + 4189*a^2*b^4*c^5 - 19208*a^3*
b^2*c^6 + 38416*a^4*c^7)*d - 27/2*sqrt(1/2)*((a^5*b^15 - 31*a^6*b^13*c + 42
4*a^7*b^11*c^2 - 3280*a^8*b^9*c^3 + 15360*a^9*b^7*c^4 - 43264*a^10*b^5*c^5
+ 67584*a^11*b^3*c^6 - 45056*a^12*b*c^7)*e^3*sqrt((b^8 - 22*a*b^6*c + 219*a
^2*b^4*c^2 - 1078*a^3*b^2*c^3 + 2401*a^4*c^4)/((a^10*b^10 - 20*a^11*b^8*c +
160*a^12*b^6*c^2 - 640*a^13*b^4*c^3 + 1280*a^14*b^2*c^4 - 1024*a^15*c^5)*e
^4)) + (b^14 - 32*a*b^12*c + 464*a^2*b^10*c^2 - 3885*a^3*b^8*c^3 + 20088*a^
4*b^6*c^4 - 63680*a^5*b^4*c^5 + 113792*a^6*b^2*c^6 - 87808*a^7*c^7)*e)*sqrt
(-(b^9 - 21*a*b^7*c + 189*a^2*b^5*c^2 - 840*a^3*b^3*c^3 + 1680*a^4*b*c^4 -

```
(a^5*b^10 - 20*a^6*b^8*c + 160*a^7*b^6*c^2 - 640*a^8*b^4*c^3 + 1280*a^9*b^2
*c^4 - 1024*a^10*c^5)*e^2*sqrt((b^8 - 22*a*b^6*c + 219*a^2*b^4*c^2 - 1078*a
^3*b^2*c^3 + 2401*a^4*c^4)/((a^10*b^10 - 20*a^11*b^8*c + 160*a^12*b^6*c^2 -
 640*a^13*b^4*c^3 + 1280*a^14*b^2*c^4 - 1024*a^15*c^5)*e^4)))/((a^5*b^10 -
20*a^6*b^8*c + 160*a^7*b^6*c^2 - 640*a^8*b^4*c^3 + 1280*a^9*b^2*c^4 - 1024*
a^10*c^5)*e^2))) + 2*(5*a*b^4 - 37*a^2*b^2*c + 44*a^3*c^2)*d)/((a^2*b^4*c^2
 - 8*a^3*b^2*c^3 + 16*a^4*c^4)*e^9*x^8 + 8*(a^2*b^4*c^2 - 8*a^3*b^2*c^3 + 1
6*a^4*c^4)*d*e^8*x^7 + 2*(a^2*b^5*c - 8*a^3*b^3*c^2 + 16*a^4*b*c^3 + 14*(a^
2*b^4*c^2 - 8*a^3*b^2*c^3 + 16*a^4*c^4)*d^2)*e^7*x^6 + 4*(14*(a^2*b^4*c^2 -
 8*a^3*b^2*c^3 + 16*a^4*c^4)*d^3 + 3*(a^2*b^5*c - 8*a^3*b^3*c^2 + 16*a^4*b*
c^3)*d)*e^6*x^5 + (a^2*b^6 - 6*a^3*b^4*c + 32*a^5*c^3 + 70*(a^2*b^4*c^2 - 8
*a^3*b^2*c^3 + 16*a^4*c^4)*d^4 + 30*(a^2*b^5*c - 8*a^3*b^3*c^2 + 16*a^4*b*c
^3)*d^2)*e^5*x^4 + 4*(14*(a^2*b^4*c^2 - 8*a^3*b^2*c^3 + 16*a^4*c^4)*d^5 + 1
0*(a^2*b^5*c - 8*a^3*b^3*c^2 + 16*a^4*b*c^3)*d^3 + (a^2*b^6 - 6*a^3*b^4*c +
 32*a^5*c^3)*d)*e^4*x^3 + 2*(a^3*b^5 - 8*a^4*b^3*c + 16*a^5*b*c^2 + 14*(a^2
*b^4*c^2 - 8*a^3*b^2*c^3 + 16*a^4*c^4)*d^6 + 15*(a^2*b^5*c - 8*a^3*b^3*c^2
+ 16*a^4*b*c^3)*d^4 + 3*(a^2*b^6 - 6*a^3*b^4*c + 32*a^5*c^3)*d^2)*e^3*x^2 +
 4*(2*(a^2*b^4*c^2 - 8*a^3*b^2*c^3 + 16*a^4*c^4)*d^7 + 3*(a^2*b^5*c - 8*a^3
*b^3*c^2 + 16*a^4*b*c^3)*d^5 + (a^2*b^6 - 6*a^3*b^4*c + 32*a^5*c^3)*d^3 + (
a^3*b^5 - 8*a^4*b^3*c + 16*a^5*b*c^2)*d)*e^2*x + ((a^2*b^4*c^2 - 8*a^3*b^2*
c^3 + 16*a^4*c^4)*d^8 + a^4*b^4 - 8*a^5*b^2*c + 16*a^6*c^2 + 2*(a^2*b^5*c -
 8*a^3*b^3*c^2 + 16*a^4*b*c^3)*d^6 + (a^2*b^6 - 6*a^3*b^4*c + 32*a^5*c^3)*d
^4 + 2*(a^3*b^5 - 8*a^4*b^3*c + 16*a^5*b*c^2)*d^2)*e)
```

giac [B] time = 0.56, size = 2487, normalized size = 5.69

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="giac")
```

```
[Out] -3/16*(((d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/
c))^2*b^3*c*e^2 - 8*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*
e^2)*e^(-4)/c))^2*a*b*c^2*e^2 - 2*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 + sqrt
(b^2 - 4*a*c)*e^2)*e^(-4)/c))*b^3*c*d*e + 16*(d*e^(-1) + sqrt(1/2)*sqrt(-(b
*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))*a*b*c^2*d*e + b^3*c*d^2 - 8*a*b*c^
2*d^2 + b^4 - 9*a*b^2*c + 28*a^2*c^2)*log(d*e^(-1) + x + sqrt(1/2)*sqrt(-(b
*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))/(2*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*
e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^3*c*e^4 - 6*(d*e^(-1) + sqrt(1/2)*s
qrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^2*c*d*e^3 - 2*c*d^3*e - b*d
*e + (6*c*d^2*e^2 + b*e^2)*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 -
4*a*c)*e^2)*e^(-4)/c))) + ((d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 -
4*a*c)*e^2)*e^(-4)/c))^2*b^3*c*e^2 - 8*(d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 +
 sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^2*a*b*c^2*e^2 - 2*(d*e^(-1) - sqrt(1/2)*
sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))*b^3*c*d*e + 16*(d*e^(-1) -
 sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))*a*b*c^2*d*e + b
^3*c*d^2 - 8*a*b*c^2*d^2 + b^4 - 9*a*b^2*c + 28*a^2*c^2)*log(d*e^(-1) + x -
 sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))/(2*(d*e^(-1) -
sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^3*c*e^4 - 6*(d*e
^(-1) - sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^2*c*d*e^
3 - 2*c*d^3*e - b*d*e + (6*c*d^2*e^2 + b*e^2)*(d*e^(-1) - sqrt(1/2)*sqrt(-(
b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))) + ((d*e^(-1) + sqrt(1/2)*sqrt(-(
b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^2*b^3*c*e^2 - 8*(d*e^(-1) + sqrt(
1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^2*a*b*c^2*e^2 - 2*(d*
e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))*b^3*c*d
*e + 16*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/
c))*a*b*c^2*d*e + b^3*c*d^2 - 8*a*b*c^2*d^2 + b^4 - 9*a*b^2*c + 28*a^2*c^2)
*log(d*e^(-1) + x + sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/
c))/(2*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c
))^3*c*e^4 - 6*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*
```

```
e^(-4)/c))^2*c*d*e^3 - 2*c*d^3*e - b*d*e + (6*c*d^2*e^2 + b*e^2)*(d*e^(-1)
+ sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))) + ((d*e^(-1)
- sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^2*b^3*c*e^2 -
8*(d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^2*
a*b*c^2*e^2 - 2*(d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)
*e^(-4)/c))*b^3*c*d*e + 16*(d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 -
4*a*c)*e^2)*e^(-4)/c))*a*b*c^2*d*e + b^3*c*d^2 - 8*a*b*c^2*d^2 + b^4 - 9*a*
b^2*c + 28*a^2*c^2)*log(d*e^(-1) + x - sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 -
4*a*c)*e^2)*e^(-4)/c))/(2*(d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4
*a*c)*e^2)*e^(-4)/c))^3*c*e^4 - 6*(d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 - sqrt
(b^2 - 4*a*c)*e^2)*e^(-4)/c))^2*c*d*e^3 - 2*c*d^3*e - b*d*e + (6*c*d^2*e^2
+ b*e^2)*(d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)
/c))))/(a^2*b^4 - 8*a^3*b^2*c + 16*a^4*c^2) + 1/8*(3*b^3*c^2*x^7*e^7 - 24*a
*b*c^3*x^7*e^7 + 21*b^3*c^2*d*x^6*e^6 - 168*a*b*c^3*d*x^6*e^6 + 63*b^3*c^2*
d^2*x^5*e^5 - 504*a*b*c^3*d^2*x^5*e^5 + 105*b^3*c^2*d^3*x^4*e^4 - 840*a*b*c
^3*d^3*x^4*e^4 + 105*b^3*c^2*d^4*x^3*e^3 - 840*a*b*c^3*d^4*x^3*e^3 + 63*b^3
*c^2*d^5*x^2*e^2 - 504*a*b*c^3*d^5*x^2*e^2 + 21*b^3*c^2*d^6*x*e - 168*a*b*c
^3*d^6*x*e + 3*b^3*c^2*d^7 - 24*a*b*c^3*d^7 + 6*b^4*c*x^5*e^5 - 49*a*b^2*c^
2*x^5*e^5 + 28*a^2*c^3*x^5*e^5 + 30*b^4*c*d*x^4*e^4 - 245*a*b^2*c^2*d*x^4*e
^4 + 140*a^2*c^3*d*x^4*e^4 + 60*b^4*c*d^2*x^3*e^3 - 490*a*b^2*c^2*d^2*x^3*e
^3 + 280*a^2*c^3*d^2*x^3*e^3 + 60*b^4*c*d^3*x^2*e^2 - 490*a*b^2*c^2*d^3*x^2
*e^2 + 280*a^2*c^3*d^3*x^2*e^2 + 30*b^4*c*d^4*x*e - 245*a*b^2*c^2*d^4*x*e +
 140*a^2*c^3*d^4*x*e + 6*b^4*c*d^5 - 49*a*b^2*c^2*d^5 + 28*a^2*c^3*d^5 + 3*
b^5*x^3*e^3 - 20*a*b^3*c*x^3*e^3 - 4*a^2*b*c^2*x^3*e^3 + 9*b^5*d*x^2*e^2 -
60*a*b^3*c*d*x^2*e^2 - 12*a^2*b*c^2*d*x^2*e^2 + 9*b^5*d^2*x*e - 60*a*b^3*c*
d^2*x*e - 12*a^2*b*c^2*d^2*x*e + 3*b^5*d^3 - 20*a*b^3*c*d^3 - 4*a^2*b*c^2*d
^3 + 5*a*b^4*x*e - 37*a^2*b^2*c*x*e + 44*a^3*c^2*x*e + 5*a*b^4*d - 37*a^2*b
^2*c*d + 44*a^3*c^2*d)/((a^2*b^4*e - 8*a^3*b^2*c*e + 16*a^4*c^2*e)*(c*x^4*e
^4 + 4*c*d*x^3*e^3 + 6*c*d^2*x^2*e^2 + 4*c*d^3*x*e + c*d^4 + b*x^2*e^2 + 2*
b*d*x*e + b*d^2 + a)^2)
```

maple [C] time = 0.05, size = 1010, normalized size = 2.31

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x)
```

```
[Out] (-3/8*c^2*e^6*b*(8*a*c-b^2)/(16*a^2*c^2-8*a*b^2*c+b^4)/a^2*x^7-21/8*c^2*d*e
^5*b*(8*a*c-b^2)/(16*a^2*c^2-8*a*b^2*c+b^4)/a^2*x^6+1/8*(-504*a*b*c^2*d^2+6
3*b^3*c*d^2+28*a^2*c^2-49*a*b^2*c+6*b^4)*c*e^4/(16*a^2*c^2-8*a*b^2*c+b^4)/a
^2*x^5+5/8*c*d*e^3*(-168*a*b*c^2*d^2+21*b^3*c*d^2+28*a^2*c^2-49*a*b^2*c+6*b
^4)/(16*a^2*c^2-8*a*b^2*c+b^4)/a^2*x^4-1/8*e^2*(840*a*b*c^3*d^4-105*b^3*c^2
*d^4-280*a^2*c^3*d^2+490*a*b^2*c^2*d^2-60*b^4*c*d^2+4*a^2*b*c^2+20*a*b^3*c-
3*b^5)/a^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x^3-1/8*d*e*(504*a*b*c^3*d^4-63*b^3*c
^2*d^4-280*a^2*c^3*d^2+490*a*b^2*c^2*d^2-60*b^4*c*d^2+12*a^2*b*c^2+60*a*b^3
*c-9*b^5)/(16*a^2*c^2-8*a*b^2*c+b^4)/a^2*x^2+1/8*(-168*a*b*c^3*d^6+21*b^3*c
^2*d^6+140*a^2*c^3*d^4-245*a*b^2*c^2*d^4+30*b^4*c*d^4-12*a^2*b*c^2*d^2-60*a
*b^3*c*d^2+9*b^5*d^2+44*a^3*c^2-37*a^2*b^2*c+5*a*b^4)/(16*a^2*c^2-8*a*b^2*c
+b^4)/a^2*x+1/8*d/e*(-24*a*b*c^3*d^6+3*b^3*c^2*d^6+28*a^2*c^3*d^4-49*a*b^2*
c^2*d^4+6*b^4*c*d^4-4*a^2*b*c^2*d^2-20*a*b^3*c*d^2+3*b^5*d^2+44*a^3*c^2-37*
a^2*b^2*c+5*a*b^4)/(16*a^2*c^2-8*a*b^2*c+b^4)/a^2)/(c*e^4*x^4+4*c*d*e^3*x^3
+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2+3/16/(16*
a^2*c^2-8*a*b^2*c+b^4)/a^2/e*sum((b*c*e^2*(-8*a*c+b^2)*_R^2+2*b*c*d*e*(-8*a
*c+b^2)*_R-8*a*b*c^2*d^2+b^3*c*d^2+28*a^2*c^2-9*a*b^2*c+b^4)/(2*_R^3*c*e^3+
6*_R^2*c*d*e^2+6*_R*c*d^2*e+2*c*d^3+_R*b*e+b*d)*ln(-_R+x),_R=RootOf(_Z^4*c*
e^4+4*_Z^3*c*d*e^3+c*d^4+b*d^2+(6*c*d^2*e^2+b*e^2)*_Z^2+(4*c*d^3*e+2*b*d*e)
*_Z+a))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="maxima")
```

```
[Out] 1/8*(3*(b^3*c^2 - 8*a*b*c^3)*e^7*x^7 + 21*(b^3*c^2 - 8*a*b*c^3)*d*e^6*x^6 +
 (6*b^4*c - 49*a*b^2*c^2 + 28*a^2*c^3 + 63*(b^3*c^2 - 8*a*b*c^3)*d^2)*e^5*x
^5 + 5*(21*(b^3*c^2 - 8*a*b*c^3)*d^3 + (6*b^4*c - 49*a*b^2*c^2 + 28*a^2*c^3
)*d)*e^4*x^4 + 3*(b^3*c^2 - 8*a*b*c^3)*d^7 + (3*b^5 - 20*a*b^3*c - 4*a^2*b*
c^2 + 105*(b^3*c^2 - 8*a*b*c^3)*d^4 + 10*(6*b^4*c - 49*a*b^2*c^2 + 28*a^2*c
^3)*d^2)*e^3*x^3 + (6*b^4*c - 49*a*b^2*c^2 + 28*a^2*c^3)*d^5 + (63*(b^3*c^2
 - 8*a*b*c^3)*d^5 + 10*(6*b^4*c - 49*a*b^2*c^2 + 28*a^2*c^3)*d^3 + 3*(3*b^5
 - 20*a*b^3*c - 4*a^2*b*c^2)*d)*e^2*x^2 + (3*b^5 - 20*a*b^3*c - 4*a^2*b*c^2
)*d^3 + (21*(b^3*c^2 - 8*a*b*c^3)*d^6 + 5*a*b^4 - 37*a^2*b^2*c + 44*a^3*c^2
 + 5*(6*b^4*c - 49*a*b^2*c^2 + 28*a^2*c^3)*d^4 + 3*(3*b^5 - 20*a*b^3*c - 4*
a^2*b*c^2)*d^2)*e*x + (5*a*b^4 - 37*a^2*b^2*c + 44*a^3*c^2)*d)/((a^2*b^4*c^
2 - 8*a^3*b^2*c^3 + 16*a^4*c^4)*e^9*x^8 + 8*(a^2*b^4*c^2 - 8*a^3*b^2*c^3 +
16*a^4*c^4)*d*e^8*x^7 + 2*(a^2*b^5*c - 8*a^3*b^3*c^2 + 16*a^4*b*c^3 + 14*(a
^2*b^4*c^2 - 8*a^3*b^2*c^3 + 16*a^4*c^4)*d^2)*e^7*x^6 + 4*(14*(a^2*b^4*c^2
- 8*a^3*b^2*c^3 + 16*a^4*c^4)*d^3 + 3*(a^2*b^5*c - 8*a^3*b^3*c^2 + 16*a^4*b
*c^3)*d)*e^6*x^5 + (a^2*b^6 - 6*a^3*b^4*c + 32*a^5*c^3 + 70*(a^2*b^4*c^2 -
8*a^3*b^2*c^3 + 16*a^4*c^4)*d^4 + 30*(a^2*b^5*c - 8*a^3*b^3*c^2 + 16*a^4*b*
c^3)*d^2)*e^5*x^4 + 4*(14*(a^2*b^4*c^2 - 8*a^3*b^2*c^3 + 16*a^4*c^4)*d^5 +
10*(a^2*b^5*c - 8*a^3*b^3*c^2 + 16*a^4*b*c^3)*d^3 + (a^2*b^6 - 6*a^3*b^4*c
+ 32*a^5*c^3)*d)*e^4*x^3 + 2*(a^3*b^5 - 8*a^4*b^3*c + 16*a^5*b*c^2 + 14*(a^
2*b^4*c^2 - 8*a^3*b^2*c^3 + 16*a^4*c^4)*d^6 + 15*(a^2*b^5*c - 8*a^3*b^3*c^2
 + 16*a^4*b*c^3)*d^4 + 3*(a^2*b^6 - 6*a^3*b^4*c + 32*a^5*c^3)*d^2)*e^3*x^2
+ 4*(2*(a^2*b^4*c^2 - 8*a^3*b^2*c^3 + 16*a^4*c^4)*d^7 + 3*(a^2*b^5*c - 8*a^
3*b^3*c^2 + 16*a^4*b*c^3)*d^5 + (a^2*b^6 - 6*a^3*b^4*c + 32*a^5*c^3)*d^3 +
(a^3*b^5 - 8*a^4*b^3*c + 16*a^5*b*c^2)*d)*e^2*x + ((a^2*b^4*c^2 - 8*a^3*b^2
*c^3 + 16*a^4*c^4)*d^8 + a^4*b^4 - 8*a^5*b^2*c + 16*a^6*c^2 + 2*(a^2*b^5*c
- 8*a^3*b^3*c^2 + 16*a^4*b*c^3)*d^6 + (a^2*b^6 - 6*a^3*b^4*c + 32*a^5*c^3)*
d^4 + 2*(a^3*b^5 - 8*a^4*b^3*c + 16*a^5*b*c^2)*d^2)*e) - 3/8*integrate(-((b
^3*c - 8*a*b*c^2)*e^2*x^2 + b^4 - 9*a*b^2*c + 28*a^2*c^2 + 2*(b^3*c - 8*a*b
*c^2)*d*e*x + (b^3*c - 8*a*b*c^2)*d^2)/(c*e^4*x^4 + 4*c*d*e^3*x^3 + c*d^4 +
 (6*c*d^2 + b)*e^2*x^2 + b*d^2 + 2*(2*c*d^3 + b*d)*e*x + a), x)/(a^2*b^4 -
8*a^3*b^2*c + 16*a^4*c^2)
```

mupad [B] time = 7.80, size = 16086, normalized size = 36.81

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3,x)
```

```
[Out] ((3*b^5*d^3 + 44*a^3*c^2*d + 6*b^4*c*d^5 + 28*a^2*c^3*d^5 + 3*b^3*c^2*d^7 +
 5*a*b^4*d - 4*a^2*b*c^2*d^3 - 49*a*b^2*c^2*d^5 - 37*a^2*b^2*c*d - 20*a*b^3
*c*d^3 - 24*a*b*c^3*d^7)/(8*a^2*e*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (x^3*(3
*b^5*e^2 - 4*a^2*b*c^2*e^2 + 60*b^4*c*d^2*e^2 + 280*a^2*c^3*d^2*e^2 + 105*b
^3*c^2*d^4*e^2 - 20*a*b^3*c*e^2 - 840*a*b*c^3*d^4*e^2 - 490*a*b^2*c^2*d^2*e
^2))/(8*a^2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (x^5*(6*b^4*c*e^4 + 28*a^2*c^
3*e^4 - 49*a*b^2*c^2*e^4 + 63*b^3*c^2*d^2*e^4 - 504*a*b*c^3*d^2*e^4))/(8*a^
2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (x^2*(9*b^5*d*e + 280*a^2*c^3*d^3*e + 6
3*b^3*c^2*d^5*e + 60*b^4*c*d^3*e - 12*a^2*b*c^2*d*e - 504*a*b*c^3*d^5*e - 4
90*a*b^2*c^2*d^3*e - 60*a*b^3*c*d*e))/(8*a^2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)
) + (5*x^4*(28*a^2*c^3*d*e^3 + 21*b^3*c^2*d^3*e^3 + 6*b^4*c*d*e^3 - 49*a*b^
2*c^2*d*e^3 - 168*a*b*c^3*d^3*e^3))/(8*a^2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c))
+ (21*x^6*(b^3*c^2*d*e^5 - 8*a*b*c^3*d*e^5))/(8*a^2*(b^4 + 16*a^2*c^2 - 8*a
*b^2*c)) + (x*(5*a*b^4 + 44*a^3*c^2 + 9*b^5*d^2 - 37*a^2*b^2*c + 30*b^4*c*d
^4 + 140*a^2*c^3*d^4 + 21*b^3*c^2*d^6 - 12*a^2*b*c^2*d^2 - 245*a*b^2*c^2*d^
```

4 - 60*a*b^3*c*d^2 - 168*a*b*c^3*d^6))/(8*a^2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c
)) + (3*x^7*(b^3*c^2*e^6 - 8*a*b*c^3*e^6))/(8*a^2*(b^4 + 16*a^2*c^2 - 8*a*b
^2*c)))/(x^2*(6*b^2*d^2*e^2 + 28*c^2*d^6*e^2 + 2*a*b*e^2 + 12*a*c*d^2*e^2 +
 30*b*c*d^4*e^2) + x^6*(28*c^2*d^2*e^6 + 2*b*c*e^6) + x*(4*b^2*d^3*e + 8*c^
2*d^7*e + 8*a*c*d^3*e + 12*b*c*d^5*e + 4*a*b*d*e) + x^3*(4*b^2*d*e^3 + 56*c
^2*d^5*e^3 + 8*a*c*d*e^3 + 40*b*c*d^3*e^3) + x^5*(56*c^2*d^3*e^5 + 12*b*c*d
e^5) + x^4(b^2*e^4 + 70*c^2*d^4*e^4 + 2*a*c*e^4 + 30*b*c*d^2*e^4) + a^2 +
 b^2*d^4 + c^2*d^8 + c^2*e^8*x^8 + 2*a*b*d^2 + 2*a*c*d^4 + 2*b*c*d^6 + 8*c^
2*d*e^7*x^7) - atan((((3612672*a^6*c^9*d*e^11 + 144*b^12*c^3*d*e^11 - 4032*
a*b^10*c^4*d*e^11 + 49824*a^2*b^8*c^5*d*e^11 - 340992*a^3*b^6*c^6*d*e^11 +
1410048*a^4*b^4*c^7*d*e^11 - 3391488*a^5*b^2*c^8*d*e^11)/(512*(a^4*b^12 + 4
096*a^10*c^6 - 24*a^5*b^10*c + 240*a^6*b^8*c^2 - 1280*a^7*b^6*c^3 + 3840*a^
8*b^4*c^4 - 6144*a^9*b^2*c^5)) - (((67108864*a^11*b*c^9*d*e^13 - 4096*a^4*b
^15*c^2*d*e^13 + 114688*a^5*b^13*c^3*d*e^13 - 1376256*a^6*b^11*c^4*d*e^13 +
 9175040*a^7*b^9*c^5*d*e^13 - 36700160*a^8*b^7*c^6*d*e^13 + 88080384*a^9*b^
5*c^7*d*e^13 - 117440512*a^10*b^3*c^8*d*e^13)/(512*(a^4*b^12 + 4096*a^10*c^
6 - 24*a^5*b^10*c + 240*a^6*b^8*c^2 - 1280*a^7*b^6*c^3 + 3840*a^8*b^4*c^4 -
 6144*a^9*b^2*c^5)) + (x*(262144*a^9*b*c^7*e^14 - 256*a^4*b^11*c^2*e^14 + 5
120*a^5*b^9*c^3*e^14 - 40960*a^6*b^7*c^4*e^14 + 163840*a^7*b^5*c^5*e^14 - 3
27680*a^8*b^3*c^6*e^14))/(32*(a^4*b^8 + 256*a^8*c^4 - 16*a^5*b^6*c + 96*a^6
*b^4*c^2 - 256*a^7*b^2*c^3)))*(-(9*(b^19 + b^4*(-(4*a*c - b^2)^15)^(1/2) -
1720320*a^9*b*c^9 + 769*a^2*b^15*c^2 - 8620*a^3*b^13*c^3 + 63440*a^4*b^11*c
^4 - 316864*a^5*b^9*c^5 + 1069824*a^6*b^7*c^6 - 2343936*a^7*b^5*c^7 + 30105
60*a^8*b^3*c^8 + 49*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 41*a*b^17*c - 11*a*
b^2*c*(-(4*a*c - b^2)^15)^(1/2)))/(512*(a^5*b^20*e^2 + 1048576*a^15*c^10*e^
2 - 40*a^6*b^18*c*e^2 + 720*a^7*b^16*c^2*e^2 - 7680*a^8*b^14*c^3*e^2 + 5376
0*a^9*b^12*c^4*e^2 - 258048*a^10*b^10*c^5*e^2 + 860160*a^11*b^8*c^6*e^2 - 1
966080*a^12*b^6*c^7*e^2 + 2949120*a^13*b^4*c^8*e^2 - 2621440*a^14*b^2*c^9*e
^2)))^(1/2) - (22020096*a^9*c^9*e^12 - 768*a^2*b^14*c^2*e^12 + 22272*a^3*b^
12*c^3*e^12 - 282624*a^4*b^10*c^4*e^12 + 2027520*a^5*b^8*c^5*e^12 - 8847360
*a^6*b^6*c^6*e^12 + 23396352*a^7*b^4*c^7*e^12 - 34603008*a^8*b^2*c^8*e^12)/
(512*(a^4*b^12 + 4096*a^10*c^6 - 24*a^5*b^10*c + 240*a^6*b^8*c^2 - 1280*a^7
*b^6*c^3 + 3840*a^8*b^4*c^4 - 6144*a^9*b^2*c^5)))*(-(9*(b^19 + b^4*(-(4*a*c
 - b^2)^15)^(1/2) - 1720320*a^9*b*c^9 + 769*a^2*b^15*c^2 - 8620*a^3*b^13*c^
3 + 63440*a^4*b^11*c^4 - 316864*a^5*b^9*c^5 + 1069824*a^6*b^7*c^6 - 2343936
*a^7*b^5*c^7 + 3010560*a^8*b^3*c^8 + 49*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) -
 41*a*b^17*c - 11*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2)))/(512*(a^5*b^20*e^2 +
1048576*a^15*c^10*e^2 - 40*a^6*b^18*c*e^2 + 720*a^7*b^16*c^2*e^2 - 7680*a^8
*b^14*c^3*e^2 + 53760*a^9*b^12*c^4*e^2 - 258048*a^10*b^10*c^5*e^2 + 860160*
a^11*b^8*c^6*e^2 - 1966080*a^12*b^6*c^7*e^2 + 2949120*a^13*b^4*c^8*e^2 - 26
21440*a^14*b^2*c^9*e^2)))^(1/2) + (x*(14112*a^4*c^7*e^12 + 9*b^8*c^3*e^12 -
 180*a*b^6*c^4*e^12 + 1530*a^2*b^4*c^5*e^12 - 6192*a^3*b^2*c^6*e^12))/(32*(
a^4*b^8 + 256*a^8*c^4 - 16*a^5*b^6*c + 96*a^6*b^4*c^2 - 256*a^7*b^2*c^3)))*
(-(9*(b^19 + b^4*(-(4*a*c - b^2)^15)^(1/2) - 1720320*a^9*b*c^9 + 769*a^2*b^
15*c^2 - 8620*a^3*b^13*c^3 + 63440*a^4*b^11*c^4 - 316864*a^5*b^9*c^5 + 1069
824*a^6*b^7*c^6 - 2343936*a^7*b^5*c^7 + 3010560*a^8*b^3*c^8 + 49*a^2*c^2*(-
(4*a*c - b^2)^15)^(1/2) - 41*a*b^17*c - 11*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2
)))/(512*(a^5*b^20*e^2 + 1048576*a^15*c^10*e^2 - 40*a^6*b^18*c*e^2 + 720*a^
7*b^16*c^2*e^2 - 7680*a^8*b^14*c^3*e^2 + 53760*a^9*b^12*c^4*e^2 - 258048*a^
10*b^10*c^5*e^2 + 860160*a^11*b^8*c^6*e^2 - 1966080*a^12*b^6*c^7*e^2 + 2949
120*a^13*b^4*c^8*e^2 - 2621440*a^14*b^2*c^9*e^2)))^(1/2)*1i + ((3612672*a^6
*c^9*d*e^11 + 144*b^12*c^3*d*e^11 - 4032*a*b^10*c^4*d*e^11 + 49824*a^2*b^8*
c^5*d*e^11 - 340992*a^3*b^6*c^6*d*e^11 + 1410048*a^4*b^4*c^7*d*e^11 - 33914
88*a^5*b^2*c^8*d*e^11)/(512*(a^4*b^12 + 4096*a^10*c^6 - 24*a^5*b^10*c + 240
*a^6*b^8*c^2 - 1280*a^7*b^6*c^3 + 3840*a^8*b^4*c^4 - 6144*a^9*b^2*c^5)) - (
((67108864*a^11*b*c^9*d*e^13 - 4096*a^4*b^15*c^2*d*e^13 + 114688*a^5*b^13*c
^3*d*e^13 - 1376256*a^6*b^11*c^4*d*e^13 + 9175040*a^7*b^9*c^5*d*e^13 - 3670
0160*a^8*b^7*c^6*d*e^13 + 88080384*a^9*b^5*c^7*d*e^13 - 117440512*a^10*b^3*
c^8*d*e^13)/(512*(a^4*b^12 + 4096*a^10*c^6 - 24*a^5*b^10*c + 240*a^6*b^8*c^

2 - 1280*a^7*b^6*c^3 + 3840*a^8*b^4*c^4 - 6144*a^9*b^2*c^5)) + (x*(262144*a
^9*b*c^7*e^14 - 256*a^4*b^11*c^2*e^14 + 5120*a^5*b^9*c^3*e^14 - 40960*a^6*b
^7*c^4*e^14 + 163840*a^7*b^5*c^5*e^14 - 327680*a^8*b^3*c^6*e^14))/(32*(a^4*
b^8 + 256*a^8*c^4 - 16*a^5*b^6*c + 96*a^6*b^4*c^2 - 256*a^7*b^2*c^3)))*(-(9
(b^19 + b^4(-(4*a*c - b^2)^15)^(1/2) - 1720320*a^9*b*c^9 + 769*a^2*b^15*c
^2 - 8620*a^3*b^13*c^3 + 63440*a^4*b^11*c^4 - 316864*a^5*b^9*c^5 + 1069824*
a^6*b^7*c^6 - 2343936*a^7*b^5*c^7 + 3010560*a^8*b^3*c^8 + 49*a^2*c^2*(-(4*a
*c - b^2)^15)^(1/2) - 41*a*b^17*c - 11*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2)))/
(512*(a^5*b^20*e^2 + 1048576*a^15*c^10*e^2 - 40*a^6*b^18*c*e^2 + 720*a^7*b^
16*c^2*e^2 - 7680*a^8*b^14*c^3*e^2 + 53760*a^9*b^12*c^4*e^2 - 258048*a^10*b
^10*c^5*e^2 + 860160*a^11*b^8*c^6*e^2 - 1966080*a^12*b^6*c^7*e^2 + 2949120*
a^13*b^4*c^8*e^2 - 2621440*a^14*b^2*c^9*e^2)))^(1/2) + (22020096*a^9*c^9*e^
12 - 768*a^2*b^14*c^2*e^12 + 22272*a^3*b^12*c^3*e^12 - 282624*a^4*b^10*c^4*
e^12 + 2027520*a^5*b^8*c^5*e^12 - 8847360*a^6*b^6*c^6*e^12 + 23396352*a^7*b
^4*c^7*e^12 - 34603008*a^8*b^2*c^8*e^12)/(512*(a^4*b^12 + 4096*a^10*c^6 - 2
4*a^5*b^10*c + 240*a^6*b^8*c^2 - 1280*a^7*b^6*c^3 + 3840*a^8*b^4*c^4 - 6144
*a^9*b^2*c^5)))*(-(9*(b^19 + b^4*(-(4*a*c - b^2)^15)^(1/2) - 1720320*a^9*b*
c^9 + 769*a^2*b^15*c^2 - 8620*a^3*b^13*c^3 + 63440*a^4*b^11*c^4 - 316864*a^
5*b^9*c^5 + 1069824*a^6*b^7*c^6 - 2343936*a^7*b^5*c^7 + 3010560*a^8*b^3*c^8
 + 49*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 41*a*b^17*c - 11*a*b^2*c*(-(4*a*c
 - b^2)^15)^(1/2)))/(512*(a^5*b^20*e^2 + 1048576*a^15*c^10*e^2 - 40*a^6*b^1
8*c*e^2 + 720*a^7*b^16*c^2*e^2 - 7680*a^8*b^14*c^3*e^2 + 53760*a^9*b^12*c^4
*e^2 - 258048*a^10*b^10*c^5*e^2 + 860160*a^11*b^8*c^6*e^2 - 1966080*a^12*b^
6*c^7*e^2 + 2949120*a^13*b^4*c^8*e^2 - 2621440*a^14*b^2*c^9*e^2)))^(1/2) +
(x*(14112*a^4*c^7*e^12 + 9*b^8*c^3*e^12 - 180*a*b^6*c^4*e^12 + 1530*a^2*b^4
*c^5*e^12 - 6192*a^3*b^2*c^6*e^12))/(32*(a^4*b^8 + 256*a^8*c^4 - 16*a^5*b^6
*c + 96*a^6*b^4*c^2 - 256*a^7*b^2*c^3)))*(-(9*(b^19 + b^4*(-(4*a*c - b^2)^1
5)^(1/2) - 1720320*a^9*b*c^9 + 769*a^2*b^15*c^2 - 8620*a^3*b^13*c^3 + 63440
*a^4*b^11*c^4 - 316864*a^5*b^9*c^5 + 1069824*a^6*b^7*c^6 - 2343936*a^7*b^5*
c^7 + 3010560*a^8*b^3*c^8 + 49*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 41*a*b^1
7*c - 11*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2)))/(512*(a^5*b^20*e^2 + 1048576*a
^15*c^10*e^2 - 40*a^6*b^18*c*e^2 + 720*a^7*b^16*c^2*e^2 - 7680*a^8*b^14*c^3
*e^2 + 53760*a^9*b^12*c^4*e^2 - 258048*a^10*b^10*c^5*e^2 + 860160*a^11*b^8*
c^6*e^2 - 1966080*a^12*b^6*c^7*e^2 + 2949120*a^13*b^4*c^8*e^2 - 2621440*a^1
4*b^2*c^9*e^2)))^(1/2)*1i)/((567*b^7*c^5*e^10 - 10368*a*b^5*c^6*e^10 - 1693
44*a^3*b*c^8*e^10 + 67824*a^2*b^3*c^7*e^10)/(256*(a^4*b^12 + 4096*a^10*c^6
- 24*a^5*b^10*c + 240*a^6*b^8*c^2 - 1280*a^7*b^6*c^3 + 3840*a^8*b^4*c^4 - 6
144*a^9*b^2*c^5)) + ((3612672*a^6*c^9*d*e^11 + 144*b^12*c^3*d*e^11 - 4032*a
*b^10*c^4*d*e^11 + 49824*a^2*b^8*c^5*d*e^11 - 340992*a^3*b^6*c^6*d*e^11 + 1
410048*a^4*b^4*c^7*d*e^11 - 3391488*a^5*b^2*c^8*d*e^11)/(512*(a^4*b^12 + 40
96*a^10*c^6 - 24*a^5*b^10*c + 240*a^6*b^8*c^2 - 1280*a^7*b^6*c^3 + 3840*a^8
*b^4*c^4 - 6144*a^9*b^2*c^5)) - (((67108864*a^11*b*c^9*d*e^13 - 4096*a^4*b^
15*c^2*d*e^13 + 114688*a^5*b^13*c^3*d*e^13 - 1376256*a^6*b^11*c^4*d*e^13 +
9175040*a^7*b^9*c^5*d*e^13 - 36700160*a^8*b^7*c^6*d*e^13 + 88080384*a^9*b^5
*c^7*d*e^13 - 117440512*a^10*b^3*c^8*d*e^13)/(512*(a^4*b^12 + 4096*a^10*c^6
 - 24*a^5*b^10*c + 240*a^6*b^8*c^2 - 1280*a^7*b^6*c^3 + 3840*a^8*b^4*c^4 -
6144*a^9*b^2*c^5)) + (x*(262144*a^9*b*c^7*e^14 - 256*a^4*b^11*c^2*e^14 + 51
20*a^5*b^9*c^3*e^14 - 40960*a^6*b^7*c^4*e^14 + 163840*a^7*b^5*c^5*e^14 - 32
7680*a^8*b^3*c^6*e^14))/(32*(a^4*b^8 + 256*a^8*c^4 - 16*a^5*b^6*c + 96*a^6*
b^4*c^2 - 256*a^7*b^2*c^3)))*(-(9*(b^19 + b^4*(-(4*a*c - b^2)^15)^(1/2) - 1
720320*a^9*b*c^9 + 769*a^2*b^15*c^2 - 8620*a^3*b^13*c^3 + 63440*a^4*b^11*c^
4 - 316864*a^5*b^9*c^5 + 1069824*a^6*b^7*c^6 - 2343936*a^7*b^5*c^7 + 301056
0*a^8*b^3*c^8 + 49*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 41*a*b^17*c - 11*a*b
^2*c*(-(4*a*c - b^2)^15)^(1/2)))/(512*(a^5*b^20*e^2 + 1048576*a^15*c^10*e^2
 - 40*a^6*b^18*c*e^2 + 720*a^7*b^16*c^2*e^2 - 7680*a^8*b^14*c^3*e^2 + 53760
*a^9*b^12*c^4*e^2 - 258048*a^10*b^10*c^5*e^2 + 860160*a^11*b^8*c^6*e^2 - 19
66080*a^12*b^6*c^7*e^2 + 2949120*a^13*b^4*c^8*e^2 - 2621440*a^14*b^2*c^9*e^
2)))^(1/2) - (22020096*a^9*c^9*e^12 - 768*a^2*b^14*c^2*e^12 + 22272*a^3*b^1
2*c^3*e^12 - 282624*a^4*b^10*c^4*e^12 + 2027520*a^5*b^8*c^5*e^12 - 8847360*

a^6*b^6*c^6*e^12 + 23396352*a^7*b^4*c^7*e^12 - 34603008*a^8*b^2*c^8*e^12)/(
512*(a^4*b^12 + 4096*a^10*c^6 - 24*a^5*b^10*c + 240*a^6*b^8*c^2 - 1280*a^7*
b^6*c^3 + 3840*a^8*b^4*c^4 - 6144*a^9*b^2*c^5)))*(-(9*(b^19 + b^4*(-(4*a*c
- b^2)^15)^(1/2) - 1720320*a^9*b*c^9 + 769*a^2*b^15*c^2 - 8620*a^3*b^13*c^3
+ 63440*a^4*b^11*c^4 - 316864*a^5*b^9*c^5 + 1069824*a^6*b^7*c^6 - 2343936*
a^7*b^5*c^7 + 3010560*a^8*b^3*c^8 + 49*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) -
41*a*b^17*c - 11*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2)))/(512*(a^5*b^20*e^2 + 1
048576*a^15*c^10*e^2 - 40*a^6*b^18*c*e^2 + 720*a^7*b^16*c^2*e^2 - 7680*a^8*
b^14*c^3*e^2 + 53760*a^9*b^12*c^4*e^2 - 258048*a^10*b^10*c^5*e^2 + 860160*a
^11*b^8*c^6*e^2 - 1966080*a^12*b^6*c^7*e^2 + 2949120*a^13*b^4*c^8*e^2 - 262
1440*a^14*b^2*c^9*e^2)))^(1/2) + (x*(14112*a^4*c^7*e^12 + 9*b^8*c^3*e^12 -
180*a*b^6*c^4*e^12 + 1530*a^2*b^4*c^5*e^12 - 6192*a^3*b^2*c^6*e^12))/(32*(a
^4*b^8 + 256*a^8*c^4 - 16*a^5*b^6*c + 96*a^6*b^4*c^2 - 256*a^7*b^2*c^3)))*(
-(9*(b^19 + b^4*(-(4*a*c - b^2)^15)^(1/2) - 1720320*a^9*b*c^9 + 769*a^2*b^1
5*c^2 - 8620*a^3*b^13*c^3 + 63440*a^4*b^11*c^4 - 316864*a^5*b^9*c^5 + 10698
24*a^6*b^7*c^6 - 2343936*a^7*b^5*c^7 + 3010560*a^8*b^3*c^8 + 49*a^2*c^2*(-(
4*a*c - b^2)^15)^(1/2) - 41*a*b^17*c - 11*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2)
))/(512*(a^5*b^20*e^2 + 1048576*a^15*c^10*e^2 - 40*a^6*b^18*c*e^2 + 720*a^7
*b^16*c^2*e^2 - 7680*a^8*b^14*c^3*e^2 + 53760*a^9*b^12*c^4*e^2 - 258048*a^1
0*b^10*c^5*e^2 + 860160*a^11*b^8*c^6*e^2 - 1966080*a^12*b^6*c^7*e^2 + 29491
20*a^13*b^4*c^8*e^2 - 2621440*a^14*b^2*c^9*e^2)))^(1/2) - ((3612672*a^6*c^9
*d*e^11 + 144*b^12*c^3*d*e^11 - 4032*a*b^10*c^4*d*e^11 + 49824*a^2*b^8*c^5*
d*e^11 - 340992*a^3*b^6*c^6*d*e^11 + 1410048*a^4*b^4*c^7*d*e^11 - 3391488*a
^5*b^2*c^8*d*e^11)/(512*(a^4*b^12 + 4096*a^10*c^6 - 24*a^5*b^10*c + 240*a^6
*b^8*c^2 - 1280*a^7*b^6*c^3 + 3840*a^8*b^4*c^4 - 6144*a^9*b^2*c^5)) - (((67
108864*a^11*b*c^9*d*e^13 - 4096*a^4*b^15*c^2*d*e^13 + 114688*a^5*b^13*c^3*d
*e^13 - 1376256*a^6*b^11*c^4*d*e^13 + 9175040*a^7*b^9*c^5*d*e^13 - 36700160
*a^8*b^7*c^6*d*e^13 + 88080384*a^9*b^5*c^7*d*e^13 - 117440512*a^10*b^3*c^8*
d*e^13)/(512*(a^4*b^12 + 4096*a^10*c^6 - 24*a^5*b^10*c + 240*a^6*b^8*c^2 -
1280*a^7*b^6*c^3 + 3840*a^8*b^4*c^4 - 6144*a^9*b^2*c^5)) + (x*(262144*a^9*b
*c^7*e^14 - 256*a^4*b^11*c^2*e^14 + 5120*a^5*b^9*c^3*e^14 - 40960*a^6*b^7*c
^4*e^14 + 163840*a^7*b^5*c^5*e^14 - 327680*a^8*b^3*c^6*e^14))/(32*(a^4*b^8
+ 256*a^8*c^4 - 16*a^5*b^6*c + 96*a^6*b^4*c^2 - 256*a^7*b^2*c^3)))*(-(9*(b^
19 + b^4*(-(4*a*c - b^2)^15)^(1/2) - 1720320*a^9*b*c^9 + 769*a^2*b^15*c^2 -
8620*a^3*b^13*c^3 + 63440*a^4*b^11*c^4 - 316864*a^5*b^9*c^5 + 1069824*a^6*
b^7*c^6 - 2343936*a^7*b^5*c^7 + 3010560*a^8*b^3*c^8 + 49*a^2*c^2*(-(4*a*c -
b^2)^15)^(1/2) - 41*a*b^17*c - 11*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2)))/(512
*(a^5*b^20*e^2 + 1048576*a^15*c^10*e^2 - 40*a^6*b^18*c*e^2 + 720*a^7*b^16*c
^2*e^2 - 7680*a^8*b^14*c^3*e^2 + 53760*a^9*b^12*c^4*e^2 - 258048*a^10*b^10*
c^5*e^2 + 860160*a^11*b^8*c^6*e^2 - 1966080*a^12*b^6*c^7*e^2 + 2949120*a^13
*b^4*c^8*e^2 - 2621440*a^14*b^2*c^9*e^2)))^(1/2) + (22020096*a^9*c^9*e^12 -
768*a^2*b^14*c^2*e^12 + 22272*a^3*b^12*c^3*e^12 - 282624*a^4*b^10*c^4*e^12
+ 2027520*a^5*b^8*c^5*e^12 - 8847360*a^6*b^6*c^6*e^12 + 23396352*a^7*b^4*c
^7*e^12 - 34603008*a^8*b^2*c^8*e^12)/(512*(a^4*b^12 + 4096*a^10*c^6 - 24*a^
5*b^10*c + 240*a^6*b^8*c^2 - 1280*a^7*b^6*c^3 + 3840*a^8*b^4*c^4 - 6144*a^9
*b^2*c^5)))*(-(9*(b^19 + b^4*(-(4*a*c - b^2)^15)^(1/2) - 1720320*a^9*b*c^9
+ 769*a^2*b^15*c^2 - 8620*a^3*b^13*c^3 + 63440*a^4*b^11*c^4 - 316864*a^5*b^
9*c^5 + 1069824*a^6*b^7*c^6 - 2343936*a^7*b^5*c^7 + 3010560*a^8*b^3*c^8 + 4
9*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 41*a*b^17*c - 11*a*b^2*c*(-(4*a*c - b
^2)^15)^(1/2)))/(512*(a^5*b^20*e^2 + 1048576*a^15*c^10*e^2 - 40*a^6*b^18*c*
e^2 + 720*a^7*b^16*c^2*e^2 - 7680*a^8*b^14*c^3*e^2 + 53760*a^9*b^12*c^4*e^2
- 258048*a^10*b^10*c^5*e^2 + 860160*a^11*b^8*c^6*e^2 - 1966080*a^12*b^6*c^
7*e^2 + 2949120*a^13*b^4*c^8*e^2 - 2621440*a^14*b^2*c^9*e^2)))^(1/2) + (x*(
14112*a^4*c^7*e^12 + 9*b^8*c^3*e^12 - 180*a*b^6*c^4*e^12 + 1530*a^2*b^4*c^5
*e^12 - 6192*a^3*b^2*c^6*e^12))/(32*(a^4*b^8 + 256*a^8*c^4 - 16*a^5*b^6*c +
96*a^6*b^4*c^2 - 256*a^7*b^2*c^3)))*(-(9*(b^19 + b^4*(-(4*a*c - b^2)^15)^(
1/2) - 1720320*a^9*b*c^9 + 769*a^2*b^15*c^2 - 8620*a^3*b^13*c^3 + 63440*a^4
*b^11*c^4 - 316864*a^5*b^9*c^5 + 1069824*a^6*b^7*c^6 - 2343936*a^7*b^5*c^7
+ 3010560*a^8*b^3*c^8 + 49*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 41*a*b^17*c

- 11*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2)))/(512*(a^5*b^20*e^2 + 1048576*a^15*c^10*e^2 - 40*a^6*b^18*c*e^2 + 720*a^7*b^16*c^2*e^2 - 7680*a^8*b^14*c^3*e^2 + 53760*a^9*b^12*c^4*e^2 - 258048*a^10*b^10*c^5*e^2 + 860160*a^11*b^8*c^6*e^2 - 1966080*a^12*b^6*c^7*e^2 + 2949120*a^13*b^4*c^8*e^2 - 2621440*a^14*b^2*c^9*e^2)))^(1/2)))*(-(9*(b^19 + b^4*(-(4*a*c - b^2)^15)^(1/2) - 1720320*a^9*b*c^9 + 769*a^2*b^15*c^2 - 8620*a^3*b^13*c^3 + 63440*a^4*b^11*c^4 - 316864*a^5*b^9*c^5 + 1069824*a^6*b^7*c^6 - 2343936*a^7*b^5*c^7 + 3010560*a^8*b^3*c^8 + 49*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 41*a*b^17*c - 11*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2)))/(512*(a^5*b^20*e^2 + 1048576*a^15*c^10*e^2 - 40*a^6*b^18*c*e^2 + 720*a^7*b^16*c^2*e^2 - 7680*a^8*b^14*c^3*e^2 + 53760*a^9*b^12*c^4*e^2 - 258048*a^10*b^10*c^5*e^2 + 860160*a^11*b^8*c^6*e^2 - 1966080*a^12*b^6*c^7*e^2 + 2949120*a^13*b^4*c^8*e^2 - 2621440*a^14*b^2*c^9*e^2)))^(1/2)*2i - atan((((9*(b^4*(-(4*a*c - b^2)^15)^(1/2) - b^19 + 1720320*a^9*b*c^9 - 769*a^2*b^15*c^2 + 8620*a^3*b^13*c^3 - 63440*a^4*b^11*c^4 + 316864*a^5*b^9*c^5 - 1069824*a^6*b^7*c^6 + 2343936*a^7*b^5*c^7 - 3010560*a^8*b^3*c^8 + 49*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 41*a*b^17*c - 11*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2)))/(512*(a^5*b^20*e^2 + 1048576*a^15*c^10*e^2 - 40*a^6*b^18*c*e^2 + 720*a^7*b^16*c^2*e^2 - 7680*a^8*b^14*c^3*e^2 + 53760*a^9*b^12*c^4*e^2 - 258048*a^10*b^10*c^5*e^2 + 860160*a^11*b^8*c^6*e^2 - 1966080*a^12*b^6*c^7*e^2 + 2949120*a^13*b^4*c^8*e^2 - 2621440*a^14*b^2*c^9*e^2)))^(1/2)*(((22020096*a^9*c^9*e^12 - 768*a^2*b^14*c^2*e^12 + 22272*a^3*b^12*c^3*e^12 - 282624*a^4*b^10*c^4*e^12 + 2027520*a^5*b^8*c^5*e^12 - 8847360*a^6*b^6*c^6*e^12 + 23396352*a^7*b^4*c^7*e^12 - 34603008*a^8*b^2*c^8*e^12)/(512*(a^4*b^12 + 4096*a^10*c^6 - 24*a^5*b^10*c + 240*a^6*b^8*c^2 - 1280*a^7*b^6*c^3 + 3840*a^8*b^4*c^4 - 6144*a^9*b^2*c^5)) - ((67108864*a^11*b*c^9*d*e^13 - 4096*a^4*b^15*c^2*d*e^13 + 114688*a^5*b^13*c^3*d*e^13 - 1376256*a^6*b^11*c^4*d*e^13 + 9175040*a^7*b^9*c^5*d*e^13 - 36700160*a^8*b^7*c^6*d*e^13 + 88080384*a^9*b^5*c^7*d*e^13 - 117440512*a^10*b^3*c^8*d*e^13)/(512*(a^4*b^12 + 4096*a^10*c^6 - 24*a^5*b^10*c + 240*a^6*b^8*c^2 - 1280*a^7*b^6*c^3 + 3840*a^8*b^4*c^4 - 6144*a^9*b^2*c^5)) + (x*(262144*a^9*b*c^7*e^14 - 256*a^4*b^11*c^2*e^14 + 5120*a^5*b^9*c^3*e^14 - 40960*a^6*b^7*c^4*e^14 + 163840*a^7*b^5*c^5*e^14 - 327680*a^8*b^3*c^6*e^14))/(32*(a^4*b^8 + 256*a^8*c^4 - 16*a^5*b^6*c + 96*a^6*b^4*c^2 - 256*a^7*b^2*c^3)))*((9*(b^4*(-(4*a*c - b^2)^15)^(1/2) - b^19 + 1720320*a^9*b*c^9 - 769*a^2*b^15*c^2 + 8620*a^3*b^13*c^3 - 63440*a^4*b^11*c^4 + 316864*a^5*b^9*c^5 - 1069824*a^6*b^7*c^6 + 2343936*a^7*b^5*c^7 - 3010560*a^8*b^3*c^8 + 49*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 41*a*b^17*c - 11*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2)))/(512*(a^5*b^20*e^2 + 1048576*a^15*c^10*e^2 - 40*a^6*b^18*c*e^2 + 720*a^7*b^16*c^2*e^2 - 7680*a^8*b^14*c^3*e^2 + 53760*a^9*b^12*c^4*e^2 - 258048*a^10*b^10*c^5*e^2 + 860160*a^11*b^8*c^6*e^2 - 1966080*a^12*b^6*c^7*e^2 + 2949120*a^13*b^4*c^8*e^2 - 2621440*a^14*b^2*c^9*e^2)))^(1/2))*((9*(b^4*(-(4*a*c - b^2)^15)^(1/2) - b^19 + 1720320*a^9*b*c^9 - 769*a^2*b^15*c^2 + 8620*a^3*b^13*c^3 - 63440*a^4*b^11*c^4 + 316864*a^5*b^9*c^5 - 1069824*a^6*b^7*c^6 + 2343936*a^7*b^5*c^7 - 3010560*a^8*b^3*c^8 + 49*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 41*a*b^17*c - 11*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2)))/(512*(a^5*b^20*e^2 + 1048576*a^15*c^10*e^2 - 40*a^6*b^18*c*e^2 + 720*a^7*b^16*c^2*e^2 - 7680*a^8*b^14*c^3*e^2 + 53760*a^9*b^12*c^4*e^2 - 258048*a^10*b^10*c^5*e^2 + 860160*a^11*b^8*c^6*e^2 - 1966080*a^12*b^6*c^7*e^2 + 2949120*a^13*b^4*c^8*e^2 - 2621440*a^14*b^2*c^9*e^2)))^(1/2) + (3612672*a^6*c^9*d*e^11 + 144*b^12*c^3*d*e^11 - 4032*a*b^10*c^4*d*e^11 + 49824*a^2*b^8*c^5*d*e^11 - 340992*a^3*b^6*c^6*d*e^11 + 1410048*a^4*b^4*c^7*d*e^11 - 3391488*a^5*b^2*c^8*d*e^11)/(512*(a^4*b^12 + 4096*a^10*c^6 - 24*a^5*b^10*c + 240*a^6*b^8*c^2 - 1280*a^7*b^6*c^3 + 3840*a^8*b^4*c^4 - 6144*a^9*b^2*c^5)) + (x*(14112*a^4*c^7*e^12 + 9*b^8*c^3*e^12 - 180*a*b^6*c^4*e^12 + 1530*a^2*b^4*c^5*e^12 - 6192*a^3*b^2*c^6*e^12))/(32*(a^4*b^8 + 256*a^8*c^4 - 16*a^5*b^6*c + 96*a^6*b^4*c^2 - 256*a^7*b^2*c^3)))*1i + ((9*(b^4*(-(4*a*c - b^2)^15)^(1/2) - b^19 + 1720320*a^9*b*c^9 - 769*a^2*b^15*c^2 + 8620*a^3*b^13*c^3 - 63440*a^4*b^11*c^4 + 316864*a^5*b^9*c^5 - 1069824*a^6*b^7*c^6 + 2343936*a^7*b^5*c^7 - 3010560*a^8*b^3*c^8 + 49*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 41*a*b^17*c - 11*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2)))/(512*(a^5*b^20*e^2 +

1048576*a^15*c^10*e^2 - 40*a^6*b^18*c*e^2 + 720*a^7*b^16*c^2*e^2 - 7680*a^8*b^14*c^3*e^2 + 53760*a^9*b^12*c^4*e^2 - 258048*a^10*b^10*c^5*e^2 + 860160*a^11*b^8*c^6*e^2 - 1966080*a^12*b^6*c^7*e^2 + 2949120*a^13*b^4*c^8*e^2 - 2621440*a^14*b^2*c^9*e^2)))^(1/2)*((3612672*a^6*c^9*d*e^11 + 144*b^12*c^3*d*e^11 - 4032*a*b^10*c^4*d*e^11 + 49824*a^2*b^8*c^5*d*e^11 - 340992*a^3*b^6*c^6*d*e^11 + 1410048*a^4*b^4*c^7*d*e^11 - 3391488*a^5*b^2*c^8*d*e^11)/(512*(a^4*b^12 + 4096*a^10*c^6 - 24*a^5*b^10*c + 240*a^6*b^8*c^2 - 1280*a^7*b^6*c^3 + 3840*a^8*b^4*c^4 - 6144*a^9*b^2*c^5)) - ((22020096*a^9*c^9*e^12 - 768*a^2*b^14*c^2*e^12 + 22272*a^3*b^12*c^3*e^12 - 282624*a^4*b^10*c^4*e^12 + 2027520*a^5*b^8*c^5*e^12 - 8847360*a^6*b^6*c^6*e^12 + 23396352*a^7*b^4*c^7*e^12 - 34603008*a^8*b^2*c^8*e^12)/(512*(a^4*b^12 + 4096*a^10*c^6 - 24*a^5*b^10*c + 240*a^6*b^8*c^2 - 1280*a^7*b^6*c^3 + 3840*a^8*b^4*c^4 - 6144*a^9*b^2*c^5)) + ((67108864*a^11*b*c^9*d*e^13 - 4096*a^4*b^15*c^2*d*e^13 + 114688*a^5*b^13*c^3*d*e^13 - 1376256*a^6*b^11*c^4*d*e^13 + 9175040*a^7*b^9*c^5*d*e^13 - 36700160*a^8*b^7*c^6*d*e^13 + 88080384*a^9*b^5*c^7*d*e^13 - 117440512*a^10*b^3*c^8*d*e^13)/(512*(a^4*b^12 + 4096*a^10*c^6 - 24*a^5*b^10*c + 240*a^6*b^8*c^2 - 1280*a^7*b^6*c^3 + 3840*a^8*b^4*c^4 - 6144*a^9*b^2*c^5)) + (x*(262144*a^9*b*c^7*e^14 - 256*a^4*b^11*c^2*e^14 + 5120*a^5*b^9*c^3*e^14 - 40960*a^6*b^7*c^4*e^14 + 163840*a^7*b^5*c^5*e^14 - 327680*a^8*b^3*c^6*e^14))/(32*(a^4*b^8 + 256*a^8*c^4 - 16*a^5*b^6*c + 96*a^6*b^4*c^2 - 256*a^7*b^2*c^3)))*((9*(b^4*(-(4*a*c - b^2)^15)^(1/2) - b^19 + 1720320*a^9*b*c^9 - 769*a^2*b^15*c^2 + 8620*a^3*b^13*c^3 - 63440*a^4*b^11*c^4 + 316864*a^5*b^9*c^5 - 1069824*a^6*b^7*c^6 + 2343936*a^7*b^5*c^7 - 3010560*a^8*b^3*c^8 + 49*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 41*a*b^17*c - 11*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2)))/(512*(a^5*b^20*e^2 + 1048576*a^15*c^10*e^2 - 40*a^6*b^18*c*e^2 + 720*a^7*b^16*c^2*e^2 - 7680*a^8*b^14*c^3*e^2 + 53760*a^9*b^12*c^4*e^2 - 258048*a^10*b^10*c^5*e^2 + 860160*a^11*b^8*c^6*e^2 - 1966080*a^12*b^6*c^7*e^2 + 2949120*a^13*b^4*c^8*e^2 - 2621440*a^14*b^2*c^9*e^2)))^(1/2))*((9*(b^4*(-(4*a*c - b^2)^15)^(1/2) - b^19 + 1720320*a^9*b*c^9 - 769*a^2*b^15*c^2 + 8620*a^3*b^13*c^3 - 63440*a^4*b^11*c^4 + 316864*a^5*b^9*c^5 - 1069824*a^6*b^7*c^6 + 2343936*a^7*b^5*c^7 - 3010560*a^8*b^3*c^8 + 49*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 41*a*b^17*c - 11*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2)))/(512*(a^5*b^20*e^2 + 1048576*a^15*c^10*e^2 - 40*a^6*b^18*c*e^2 + 720*a^7*b^16*c^2*e^2 - 7680*a^8*b^14*c^3*e^2 + 53760*a^9*b^12*c^4*e^2 - 258048*a^10*b^10*c^5*e^2 + 860160*a^11*b^8*c^6*e^2 - 1966080*a^12*b^6*c^7*e^2 + 2949120*a^13*b^4*c^8*e^2 - 2621440*a^14*b^2*c^9*e^2)))^(1/2) + (x*(14112*a^4*c^7*e^12 + 9*b^8*c^3*e^12 - 180*a*b^6*c^4*e^12 + 1530*a^2*b^4*c^5*e^12 - 6192*a^3*b^2*c^6*e^12))/(32*(a^4*b^8 + 256*a^8*c^4 - 16*a^5*b^6*c + 96*a^6*b^4*c^2 - 256*a^7*b^2*c^3)))*1i)/((567*b^7*c^5*e^10 - 10368*a*b^5*c^6*e^10 - 169344*a^3*b*c^8*e^10 + 67824*a^2*b^3*c^7*e^10)/(256*(a^4*b^12 + 4096*a^10*c^6 - 24*a^5*b^10*c + 240*a^6*b^8*c^2 - 1280*a^7*b^6*c^3 + 3840*a^8*b^4*c^4 - 6144*a^9*b^2*c^5)) + ((9*(b^4*(-(4*a*c - b^2)^15)^(1/2) - b^19 + 1720320*a^9*b*c^9 - 769*a^2*b^15*c^2 + 8620*a^3*b^13*c^3 - 63440*a^4*b^11*c^4 + 316864*a^5*b^9*c^5 - 1069824*a^6*b^7*c^6 + 2343936*a^7*b^5*c^7 - 3010560*a^8*b^3*c^8 + 49*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 41*a*b^17*c - 11*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2)))/(512*(a^5*b^20*e^2 + 1048576*a^15*c^10*e^2 - 40*a^6*b^18*c*e^2 + 720*a^7*b^16*c^2*e^2 - 7680*a^8*b^14*c^3*e^2 + 53760*a^9*b^12*c^4*e^2 - 258048*a^10*b^10*c^5*e^2 + 860160*a^11*b^8*c^6*e^2 - 1966080*a^12*b^6*c^7*e^2 + 2949120*a^13*b^4*c^8*e^2 - 2621440*a^14*b^2*c^9*e^2)))^(1/2)*(((22020096*a^9*c^9*e^12 - 768*a^2*b^14*c^2*e^12 + 22272*a^3*b^12*c^3*e^12 - 282624*a^4*b^10*c^4*e^12 + 2027520*a^5*b^8*c^5*e^12 - 8847360*a^6*b^6*c^6*e^12 + 23396352*a^7*b^4*c^7*e^12 - 34603008*a^8*b^2*c^8*e^12)/(512*(a^4*b^12 + 4096*a^10*c^6 - 24*a^5*b^10*c + 240*a^6*b^8*c^2 - 1280*a^7*b^6*c^3 + 3840*a^8*b^4*c^4 - 6144*a^9*b^2*c^5)) - ((67108864*a^11*b*c^9*d*e^13 - 4096*a^4*b^15*c^2*d*e^13 + 114688*a^5*b^13*c^3*d*e^13 - 1376256*a^6*b^11*c^4*d*e^13 + 9175040*a^7*b^9*c^5*d*e^13 - 36700160*a^8*b^7*c^6*d*e^13 + 88080384*a^9*b^5*c^7*d*e^13 - 117440512*a^10*b^3*c^8*d*e^13)/(512*(a^4*b^12 + 4096*a^10*c^6 - 24*a^5*b^10*c + 240*a^6*b^8*c^2 - 1280*a^7*b^6*c^3 + 3840*a^8*b^4*c^4 - 6144*a^9*b^2*c^5)) + (x*(262144*a^9*b*c^7*e^14 - 256*a^4*b^11*c^2*e^14 + 5120*a^5

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*b^9*c^3*e^14 - 40960*a^6*b^7*c^4*e^14 + 163840*a^7*b^5*c^5*e^14 - 327680*a
^8*b^3*c^6*e^14))/(32*(a^4*b^8 + 256*a^8*c^4 - 16*a^5*b^6*c + 96*a^6*b^4*c^
2 - 256*a^7*b^2*c^3)))*((9*(b^4*(-(4*a*c - b^2)^15)^(1/2) - b^19 + 1720320*
a^9*b*c^9 - 769*a^2*b^15*c^2 + 8620*a^3*b^13*c^3 - 63440*a^4*b^11*c^4 + 316
864*a^5*b^9*c^5 - 1069824*a^6*b^7*c^6 + 2343936*a^7*b^5*c^7 - 3010560*a^8*b
^3*c^8 + 49*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 41*a*b^17*c - 11*a*b^2*c*(-
(4*a*c - b^2)^15)^(1/2)))/(512*(a^5*b^20*e^2 + 1048576*a^15*c^10*e^2 - 40*a
^6*b^18*c*e^2 + 720*a^7*b^16*c^2*e^2 - 7680*a^8*b^14*c^3*e^2 + 53760*a^9*b^
12*c^4*e^2 - 258048*a^10*b^10*c^5*e^2 + 860160*a^11*b^8*c^6*e^2 - 1966080*a
^12*b^6*c^7*e^2 + 2949120*a^13*b^4*c^8*e^2 - 2621440*a^14*b^2*c^9*e^2)))^(1
/2))*((9*(b^4*(-(4*a*c - b^2)^15)^(1/2) - b^19 + 1720320*a^9*b*c^9 - 769*a^
2*b^15*c^2 + 8620*a^3*b^13*c^3 - 63440*a^4*b^11*c^4 + 316864*a^5*b^9*c^5 -
1069824*a^6*b^7*c^6 + 2343936*a^7*b^5*c^7 - 3010560*a^8*b^3*c^8 + 49*a^2*c^
2*(-(4*a*c - b^2)^15)^(1/2) + 41*a*b^17*c - 11*a*b^2*c*(-(4*a*c - b^2)^15)^
(1/2)))/(512*(a^5*b^20*e^2 + 1048576*a^15*c^10*e^2 - 40*a^6*b^18*c*e^2 + 72
0*a^7*b^16*c^2*e^2 - 7680*a^8*b^14*c^3*e^2 + 53760*a^9*b^12*c^4*e^2 - 25804
8*a^10*b^10*c^5*e^2 + 860160*a^11*b^8*c^6*e^2 - 1966080*a^12*b^6*c^7*e^2 +
2949120*a^13*b^4*c^8*e^2 - 2621440*a^14*b^2*c^9*e^2)))^(1/2) + (3612672*a^6
*c^9*d*e^11 + 144*b^12*c^3*d*e^11 - 4032*a*b^10*c^4*d*e^11 + 49824*a^2*b^8*
c^5*d*e^11 - 340992*a^3*b^6*c^6*d*e^11 + 1410048*a^4*b^4*c^7*d*e^11 - 33914
88*a^5*b^2*c^8*d*e^11)/(512*(a^4*b^12 + 4096*a^10*c^6 - 24*a^5*b^10*c + 240
*a^6*b^8*c^2 - 1280*a^7*b^6*c^3 + 3840*a^8*b^4*c^4 - 6144*a^9*b^2*c^5)) + (
x*(14112*a^4*c^7*e^12 + 9*b^8*c^3*e^12 - 180*a*b^6*c^4*e^12 + 1530*a^2*b^4*
c^5*e^12 - 6192*a^3*b^2*c^6*e^12))/(32*(a^4*b^8 + 256*a^8*c^4 - 16*a^5*b^6*
c + 96*a^6*b^4*c^2 - 256*a^7*b^2*c^3))) - ((9*(b^4*(-(4*a*c - b^2)^15)^(1/2
) - b^19 + 1720320*a^9*b*c^9 - 769*a^2*b^15*c^2 + 8620*a^3*b^13*c^3 - 63440
*a^4*b^11*c^4 + 316864*a^5*b^9*c^5 - 1069824*a^6*b^7*c^6 + 2343936*a^7*b^5*
c^7 - 3010560*a^8*b^3*c^8 + 49*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 41*a*b^1
7*c - 11*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2)))/(512*(a^5*b^20*e^2 + 1048576*a
^15*c^10*e^2 - 40*a^6*b^18*c*e^2 + 720*a^7*b^16*c^2*e^2 - 7680*a^8*b^14*c^3
*e^2 + 53760*a^9*b^12*c^4*e^2 - 258048*a^10*b^10*c^5*e^2 + 860160*a^11*b^8*
c^6*e^2 - 1966080*a^12*b^6*c^7*e^2 + 2949120*a^13*b^4*c^8*e^2 - 2621440*a^1
4*b^2*c^9*e^2)))^(1/2)*((3612672*a^6*c^9*d*e^11 + 144*b^12*c^3*d*e^11 - 403
2*a*b^10*c^4*d*e^11 + 49824*a^2*b^8*c^5*d*e^11 - 340992*a^3*b^6*c^6*d*e^11
+ 1410048*a^4*b^4*c^7*d*e^11 - 3391488*a^5*b^2*c^8*d*e^11)/(512*(a^4*b^12 +
 4096*a^10*c^6 - 24*a^5*b^10*c + 240*a^6*b^8*c^2 - 1280*a^7*b^6*c^3 + 3840*
a^8*b^4*c^4 - 6144*a^9*b^2*c^5)) - ((22020096*a^9*c^9*e^12 - 768*a^2*b^14*c
^2*e^12 + 22272*a^3*b^12*c^3*e^12 - 282624*a^4*b^10*c^4*e^12 + 2027520*a^5*
b^8*c^5*e^12 - 8847360*a^6*b^6*c^6*e^12 + 23396352*a^7*b^4*c^7*e^12 - 34603
008*a^8*b^2*c^8*e^12)/(512*(a^4*b^12 + 4096*a^10*c^6 - 24*a^5*b^10*c + 240*
a^6*b^8*c^2 - 1280*a^7*b^6*c^3 + 3840*a^8*b^4*c^4 - 6144*a^9*b^2*c^5)) + ((
67108864*a^11*b*c^9*d*e^13 - 4096*a^4*b^15*c^2*d*e^13 + 114688*a^5*b^13*c^3
*d*e^13 - 1376256*a^6*b^11*c^4*d*e^13 + 9175040*a^7*b^9*c^5*d*e^13 - 367001
60*a^8*b^7*c^6*d*e^13 + 88080384*a^9*b^5*c^7*d*e^13 - 117440512*a^10*b^3*c^
8*d*e^13)/(512*(a^4*b^12 + 4096*a^10*c^6 - 24*a^5*b^10*c + 240*a^6*b^8*c^2
- 1280*a^7*b^6*c^3 + 3840*a^8*b^4*c^4 - 6144*a^9*b^2*c^5)) + (x*(262144*a^9
*b*c^7*e^14 - 256*a^4*b^11*c^2*e^14 + 5120*a^5*b^9*c^3*e^14 - 40960*a^6*b^7
*c^4*e^14 + 163840*a^7*b^5*c^5*e^14 - 327680*a^8*b^3*c^6*e^14))/(32*(a^4*b^
8 + 256*a^8*c^4 - 16*a^5*b^6*c + 96*a^6*b^4*c^2 - 256*a^7*b^2*c^3)))*((9*(b
^4*(-(4*a*c - b^2)^15)^(1/2) - b^19 + 1720320*a^9*b*c^9 - 769*a^2*b^15*c^2
+ 8620*a^3*b^13*c^3 - 63440*a^4*b^11*c^4 + 316864*a^5*b^9*c^5 - 1069824*a^6
*b^7*c^6 + 2343936*a^7*b^5*c^7 - 3010560*a^8*b^3*c^8 + 49*a^2*c^2*(-(4*a*c
- b^2)^15)^(1/2) + 41*a*b^17*c - 11*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2)))/(51
2*(a^5*b^20*e^2 + 1048576*a^15*c^10*e^2 - 40*a^6*b^18*c*e^2 + 720*a^7*b^16*
c^2*e^2 - 7680*a^8*b^14*c^3*e^2 + 53760*a^9*b^12*c^4*e^2 - 258048*a^10*b^10
*c^5*e^2 + 860160*a^11*b^8*c^6*e^2 - 1966080*a^12*b^6*c^7*e^2 + 2949120*a^1
3*b^4*c^8*e^2 - 2621440*a^14*b^2*c^9*e^2)))^(1/2))*((9*(b^4*(-(4*a*c - b^2)
^15)^(1/2) - b^19 + 1720320*a^9*b*c^9 - 769*a^2*b^15*c^2 + 8620*a^3*b^13*c^
3 - 63440*a^4*b^11*c^4 + 316864*a^5*b^9*c^5 - 1069824*a^6*b^7*c^6 + 2343936
```

```
*a^7*b^5*c^7 - 3010560*a^8*b^3*c^8 + 49*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) +
 41*a*b^17*c - 11*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2)))/(512*(a^5*b^20*e^2 +
1048576*a^15*c^10*e^2 - 40*a^6*b^18*c*e^2 + 720*a^7*b^16*c^2*e^2 - 7680*a^8
*b^14*c^3*e^2 + 53760*a^9*b^12*c^4*e^2 - 258048*a^10*b^10*c^5*e^2 + 860160*
a^11*b^8*c^6*e^2 - 1966080*a^12*b^6*c^7*e^2 + 2949120*a^13*b^4*c^8*e^2 - 26
21440*a^14*b^2*c^9*e^2)))^(1/2) + (x*(14112*a^4*c^7*e^12 + 9*b^8*c^3*e^12 -
 180*a*b^6*c^4*e^12 + 1530*a^2*b^4*c^5*e^12 - 6192*a^3*b^2*c^6*e^12))/(32*(
a^4*b^8 + 256*a^8*c^4 - 16*a^5*b^6*c + 96*a^6*b^4*c^2 - 256*a^7*b^2*c^3))))
)*((9*(b^4*(-(4*a*c - b^2)^15)^(1/2) - b^19 + 1720320*a^9*b*c^9 - 769*a^2*b
^15*c^2 + 8620*a^3*b^13*c^3 - 63440*a^4*b^11*c^4 + 316864*a^5*b^9*c^5 - 106
9824*a^6*b^7*c^6 + 2343936*a^7*b^5*c^7 - 3010560*a^8*b^3*c^8 + 49*a^2*c^2*(
-(4*a*c - b^2)^15)^(1/2) + 41*a*b^17*c - 11*a*b^2*c*(-(4*a*c - b^2)^15)^(1/
2)))/(512*(a^5*b^20*e^2 + 1048576*a^15*c^10*e^2 - 40*a^6*b^18*c*e^2 + 720*a
^7*b^16*c^2*e^2 - 7680*a^8*b^14*c^3*e^2 + 53760*a^9*b^12*c^4*e^2 - 258048*a
^10*b^10*c^5*e^2 + 860160*a^11*b^8*c^6*e^2 - 1966080*a^12*b^6*c^7*e^2 + 294
9120*a^13*b^4*c^8*e^2 - 2621440*a^14*b^2*c^9*e^2)))^(1/2)*2i
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*(e*x+d)**2+c*(e*x+d)**4)**3,x)
[Out] Timed out
```

3.522 $\int \frac{1}{(d+ex)\left(a+b(d+ex)^2+c(d+ex)^4\right)^3}\,dx$

Optimal. Leaf size=255

$$-\frac{\log\left(a+b(d+ex)^2+c(d+ex)^4\right)}{4a^3e}+\frac{\log(d+ex)}{a^3e}+\frac{16a^2c^2+2bc\left(b^2-7ac\right)(d+ex)^2-15ab^2c+2b^4}{4a^2e\left(b^2-4ac\right)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)}+\frac{b\left(30a^2c^2-\right.}{}$$

Rubi [A] time = 0.49, antiderivative size = 255, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {1142, 1114, 740, 822, 800, 634, 618, 206, 628}

$$\frac{16a^2c^2+2bc\left(b^2-7ac\right)(d+ex)^2-15ab^2c+2b^4}{4a^2e\left(b^2-4ac\right)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)}+\frac{b\left(30a^2c^2-10ab^2c+b^4\right)\tanh^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{2a^3e\left(b^2-4ac\right)^{5/2}}-\frac{\log\left(a+b(d+ex)^2+c(d+ex)^4\right)}{4a^3e}+\frac{\log(d+ex)}{a^3e}+\frac{-2ac+b^2+bc(d+ex)^2}{4ae\left(b^2-4ac\right)\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3),x]
```

```
[Out] (b^2 - 2*a*c + b*c*(d + e*x)^2)/(4*a*(b^2 - 4*a*c)*e*(a + b*(d + e*x)^2 + c
*(d + e*x)^4)^2) + (2*b^4 - 15*a*b^2*c + 16*a^2*c^2 + 2*b*c*(b^2 - 7*a*c)*(
d + e*x)^2)/(4*a^2*(b^2 - 4*a*c)^2*e*(a + b*(d + e*x)^2 + c*(d + e*x)^4)) +
 (b*(b^4 - 10*a*b^2*c + 30*a^2*c^2)*ArcTanh[(b + 2*c*(d + e*x)^2)/Sqrt[b^2
- 4*a*c]])/(2*a^3*(b^2 - 4*a*c)^(5/2)*e) + Log[d + e*x]/(a^3*e) - Log[a + b
*(d + e*x)^2 + c*(d + e*x)^4]/(4*a^3*e)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 740

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e
)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e
^2)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d +
 e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p +
 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
```

```
b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p,
-1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 822

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(f*(b*c*d - b^2*e +
2*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x)*(a
+ b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), x]
 + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m +
2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*m
 + b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) - f
*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1
] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 1142

```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Di
st[u^m/(Coefficient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p,
 x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

Rubi steps

$$\int \frac{1}{(d+ex)\left(a+b(d+ex)^2+c(d+ex)^4\right)^3}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x\left(a+bx^2+cx^4\right)^3}\,dx, x, d+ex\right)}{e}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{x\left(a+bx+cx^2\right)^3}\,dx, x, (d+ex)^2\right)}{2e}$$

$$= \frac{b^2-2ac+bc(d+ex)^2}{4a\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)^2} - \frac{\text{Subst}\left(\int \frac{-2\left(b^2-4a\right.}{x\left(a+bx+\right.}\right.}{4a\left(b\right.}$$

$$= \frac{b^2-2ac+bc(d+ex)^2}{4a\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)^2} + \frac{2b^4-15ab^2c+16}{4a^2\left(b^2-4ac\right)^2 e}$$

$$= \frac{b^2-2ac+bc(d+ex)^2}{4a\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)^2} + \frac{2b^4-15ab^2c+16}{4a^2\left(b^2-4ac\right)^2 e}$$

$$= \frac{b^2-2ac+bc(d+ex)^2}{4a\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)^2} + \frac{2b^4-15ab^2c+16}{4a^2\left(b^2-4ac\right)^2 e}$$

$$= \frac{b^2-2ac+bc(d+ex)^2}{4a\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)^2} + \frac{2b^4-15ab^2c+16}{4a^2\left(b^2-4ac\right)^2 e}$$

$$= \frac{b^2-2ac+bc(d+ex)^2}{4a\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)^2} + \frac{2b^4-15ab^2c+16}{4a^2\left(b^2-4ac\right)^2 e}$$

$$= \frac{b^2-2ac+bc(d+ex)^2}{4a\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)^2} + \frac{2b^4-15ab^2c+16}{4a^2\left(b^2-4ac\right)^2 e}$$

Mathematica [A] time = 3.95, size = 391, normalized size = 1.53

$$\frac{\frac{a^2\left(2ac-b^2-bc(d+ex)^2\right)}{\left(4ac-b^2\right)\left(a+b(d+ex)^2+c(d+ex)^4\right)^2} + \frac{a\left(16a^2c^2-15ab^2c-14abc^2(d+ex)^2+2b^4+2b^3c(d+ex)^2\right)}{\left(b^2-4ac\right)^2\left(a+(d+ex)^2\left(b+c(d+ex)^2\right)\right)} - \frac{\left(16a^2c^2\sqrt{b^2-4ac}+30a^2bc^2-10ab^3c-8ab^2c\sqrt{b^2-4ac}+b^4\sqrt{b^2-4ac}+b^5\right)\log\left(-\sqrt{b^2-4ac}+b+2c(d+ex)^2\right)}{\left(b^2-4ac\right)^{5/2}} + \frac{\left(-16a^2c^2\sqrt{b^2-4ac}+30a^2bc^2-10ab^3c+8ab^2c\sqrt{b^2-4ac}-b^4\sqrt{b^2-4ac}+b^5\right)\log\left(\sqrt{b^2-4ac}+b+2c(d+ex)^2\right)}{\left(b^2-4ac\right)^{5/2}} + 4\log(d+ex)}{4a^3e}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3),x]
```

```
[Out] ((a^2*(-b^2 + 2*a*c - b*c*(d + e*x)^2))/((-b^2 + 4*a*c)*(a + b*(d + e*x)^2
+ c*(d + e*x)^4)^2) + (a*(2*b^4 - 15*a*b^2*c + 16*a^2*c^2 + 2*b^3*c*(d + e*
x)^2 - 14*a*b*c^2*(d + e*x)^2))/((b^2 - 4*a*c)^2*(a + (d + e*x)^2*(b + c*(d
 + e*x)^2))) + 4*Log[d + e*x] - ((b^5 - 10*a*b^3*c + 30*a^2*b*c^2 + b^4*Sqr
t[b^2 - 4*a*c] - 8*a*b^2*c*Sqrt[b^2 - 4*a*c] + 16*a^2*c^2*Sqrt[b^2 - 4*a*c]
)*Log[b - Sqrt[b^2 - 4*a*c] + 2*c*(d + e*x)^2])/(b^2 - 4*a*c)^(5/2) + ((b^5
 - 10*a*b^3*c + 30*a^2*b*c^2 - b^4*Sqrt[b^2 - 4*a*c] + 8*a*b^2*c*Sqrt[b^2 -
 4*a*c] - 16*a^2*c^2*Sqrt[b^2 - 4*a*c])*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*(d
+ e*x)^2])/(b^2 - 4*a*c)^(5/2))/(4*a^3*e)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(d+ex)\left(a+b(d+ex)^2+c(d+ex)^4\right)^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/((d + e*x)*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3),x]
```

```
[Out] IntegrateAlgebraic[1/((d + e*x)*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3), x]
```

fricas [B] time = 4.11, size = 9908, normalized size = 38.85

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="fricas")
```

```
[Out] [1/4*(2*(a*b^5*c^2 - 11*a^2*b^3*c^3 + 28*a^3*b*c^4)*e^6*x^6 + 12*(a*b^5*c^2
 - 11*a^2*b^3*c^3 + 28*a^3*b*c^4)*d*e^5*x^5 + (4*a*b^6*c - 45*a^2*b^4*c^2 +
 132*a^3*b^2*c^3 - 64*a^4*c^4 + 30*(a*b^5*c^2 - 11*a^2*b^3*c^3 + 28*a^3*b*c
^4)*d^2)*e^4*x^4 + 3*a^2*b^6 - 33*a^3*b^4*c + 108*a^4*b^2*c^2 - 96*a^5*c^3
+ 2*(a*b^5*c^2 - 11*a^2*b^3*c^3 + 28*a^3*b*c^4)*d^6 + 4*(10*(a*b^5*c^2 - 11
*a^2*b^3*c^3 + 28*a^3*b*c^4)*d^3 + (4*a*b^6*c - 45*a^2*b^4*c^2 + 132*a^3*b^
2*c^3 - 64*a^4*c^4)*d)*e^3*x^3 + (4*a*b^6*c - 45*a^2*b^4*c^2 + 132*a^3*b^2*
c^3 - 64*a^4*c^4)*d^4 + 2*(a*b^7 - 10*a^2*b^5*c + 23*a^3*b^3*c^2 + 4*a^4*b*
c^3 + 15*(a*b^5*c^2 - 11*a^2*b^3*c^3 + 28*a^3*b*c^4)*d^4 + 3*(4*a*b^6*c - 4
5*a^2*b^4*c^2 + 132*a^3*b^2*c^3 - 64*a^4*c^4)*d^2)*e^2*x^2 + 2*(a*b^7 - 10*
a^2*b^5*c + 23*a^3*b^3*c^2 + 4*a^4*b*c^3)*d^2 + 4*(3*(a*b^5*c^2 - 11*a^2*b^
3*c^3 + 28*a^3*b*c^4)*d^5 + (4*a*b^6*c - 45*a^2*b^4*c^2 + 132*a^3*b^2*c^3 -
 64*a^4*c^4)*d^3 + (a*b^7 - 10*a^2*b^5*c + 23*a^3*b^3*c^2 + 4*a^4*b*c^3)*d)
*e*x + ((b^5*c^2 - 10*a*b^3*c^3 + 30*a^2*b*c^4)*e^8*x^8 + 8*(b^5*c^2 - 10*a
*b^3*c^3 + 30*a^2*b*c^4)*d*e^7*x^7 + 2*(b^6*c - 10*a*b^4*c^2 + 30*a^2*b^2*c
^3 + 14*(b^5*c^2 - 10*a*b^3*c^3 + 30*a^2*b*c^4)*d^2)*e^6*x^6 + 4*(14*(b^5*c
^2 - 10*a*b^3*c^3 + 30*a^2*b*c^4)*d^3 + 3*(b^6*c - 10*a*b^4*c^2 + 30*a^2*b^
2*c^3)*d)*e^5*x^5 + (b^5*c^2 - 10*a*b^3*c^3 + 30*a^2*b*c^4)*d^8 + (b^7 - 8*
a*b^5*c + 10*a^2*b^3*c^2 + 60*a^3*b*c^3 + 70*(b^5*c^2 - 10*a*b^3*c^3 + 30*a
^2*b*c^4)*d^4 + 30*(b^6*c - 10*a*b^4*c^2 + 30*a^2*b^2*c^3)*d^2)*e^4*x^4 + a
^2*b^5 - 10*a^3*b^3*c + 30*a^4*b*c^2 + 2*(b^6*c - 10*a*b^4*c^2 + 30*a^2*b^2
*c^3)*d^6 + 4*(14*(b^5*c^2 - 10*a*b^3*c^3 + 30*a^2*b*c^4)*d^5 + 10*(b^6*c -
 10*a*b^4*c^2 + 30*a^2*b^2*c^3)*d^3 + (b^7 - 8*a*b^5*c + 10*a^2*b^3*c^2 + 6
0*a^3*b*c^3)*d)*e^3*x^3 + (b^7 - 8*a*b^5*c + 10*a^2*b^3*c^2 + 60*a^3*b*c^3)
*d^4 + 2*(a*b^6 - 10*a^2*b^4*c + 30*a^3*b^2*c^2 + 14*(b^5*c^2 - 10*a*b^3*c^
3 + 30*a^2*b*c^4)*d^6 + 15*(b^6*c - 10*a*b^4*c^2 + 30*a^2*b^2*c^3)*d^4 + 3*
(b^7 - 8*a*b^5*c + 10*a^2*b^3*c^2 + 60*a^3*b*c^3)*d^2)*e^2*x^2 + 2*(a*b^6 -
 10*a^2*b^4*c + 30*a^3*b^2*c^2)*d^2 + 4*(2*(b^5*c^2 - 10*a*b^3*c^3 + 30*a^2
*b*c^4)*d^7 + 3*(b^6*c - 10*a*b^4*c^2 + 30*a^2*b^2*c^3)*d^5 + (b^7 - 8*a*b^
5*c + 10*a^2*b^3*c^2 + 60*a^3*b*c^3)*d^3 + (a*b^6 - 10*a^2*b^4*c + 30*a^3*b
^2*c^2)*d)*e*x)*sqrt(b^2 - 4*a*c)*log((2*c^2*e^4*x^4 + 8*c^2*d*e^3*x^3 + 2*
c^2*d^4 + 2*(6*c^2*d^2 + b*c)*e^2*x^2 + 2*b*c*d^2 + 4*(2*c^2*d^3 + b*c*d)*e
*x + b^2 - 2*a*c + (2*c*e^2*x^2 + 4*c*d*e*x + 2*c*d^2 + b)*sqrt(b^2 - 4*a*c
))/(c*e^4*x^4 + 4*c*d*e^3*x^3 + c*d^4 + (6*c*d^2 + b)*e^2*x^2 + b*d^2 + 2*(
2*c*d^3 + b*d)*e*x + a)) - ((b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a
^3*c^5)*e^8*x^8 + 8*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*
d*e^7*x^7 + 2*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4 + 14*(b
^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^2)*e^6*x^6 + 4*(14*(
b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^3 + 3*(b^7*c - 12*a
*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*d)*e^5*x^5 + (b^6*c^2 - 12*a*b^4*
c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^8 + (b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2
 + 32*a^3*b^2*c^3 - 128*a^4*c^4 + 70*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c
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^4 - 64*a^3*c^5)*d^4 + 30*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b
*c^4)*d^2)*e^4*x^4 + a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3 +
 2*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*d^6 + 4*(14*(b^6*
c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^5 + 10*(b^7*c - 12*a*b^
5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*d^3 + (b^8 - 10*a*b^6*c + 24*a^2*b^4
*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*d)*e^3*x^3 + (b^8 - 10*a*b^6*c + 24*a^
2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*d^4 + 2*(a*b^7 - 12*a^2*b^5*c + 4
8*a^3*b^3*c^2 - 64*a^4*b*c^3 + 14*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4
 - 64*a^3*c^5)*d^6 + 15*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^
4)*d^4 + 3*(b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^
4)*d^2)*e^2*x^2 + 2*(a*b^7 - 12*a^2*b^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3)*
d^2 + 4*(2*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^7 + 3*(
b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*d^5 + (b^8 - 10*a*b^6
*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*d^3 + (a*b^7 - 12*a^2*b
^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3)*d)*e*x)*log(c*e^4*x^4 + 4*c*d*e^3*x^3
 + c*d^4 + (6*c*d^2 + b)*e^2*x^2 + b*d^2 + 2*(2*c*d^3 + b*d)*e*x + a) + 4*(
(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*e^8*x^8 + 8*(b^6*c^2
 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d*e^7*x^7 + 2*(b^7*c - 12*a*
b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4 + 14*(b^6*c^2 - 12*a*b^4*c^3 + 48*a
^2*b^2*c^4 - 64*a^3*c^5)*d^2)*e^6*x^6 + 4*(14*(b^6*c^2 - 12*a*b^4*c^3 + 48*
a^2*b^2*c^4 - 64*a^3*c^5)*d^3 + 3*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 -
64*a^3*b*c^4)*d)*e^5*x^5 + (b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^
3*c^5)*d^8 + (b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*
c^4 + 70*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^4 + 30*(b
^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*d^2)*e^4*x^4 + a^2*b^6
 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3 + 2*(b^7*c - 12*a*b^5*c^2 + 4
8*a^2*b^3*c^3 - 64*a^3*b*c^4)*d^6 + 4*(14*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*
b^2*c^4 - 64*a^3*c^5)*d^5 + 10*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*
a^3*b*c^4)*d^3 + (b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*
a^4*c^4)*d)*e^3*x^3 + (b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 -
 128*a^4*c^4)*d^4 + 2*(a*b^7 - 12*a^2*b^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3
 + 14*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^6 + 15*(b^7*
c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*d^4 + 3*(b^8 - 10*a*b^6*c
 + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*d^2)*e^2*x^2 + 2*(a*b^7 -
 12*a^2*b^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3)*d^2 + 4*(2*(b^6*c^2 - 12*a*b
^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^7 + 3*(b^7*c - 12*a*b^5*c^2 + 48*a^
2*b^3*c^3 - 64*a^3*b*c^4)*d^5 + (b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3
*b^2*c^3 - 128*a^4*c^4)*d^3 + (a*b^7 - 12*a^2*b^5*c + 48*a^3*b^3*c^2 - 64*a
^4*b*c^3)*d)*e*x)*log(e*x + d))/((a^3*b^6*c^2 - 12*a^4*b^4*c^3 + 48*a^5*b^2
*c^4 - 64*a^6*c^5)*e^9*x^8 + 8*(a^3*b^6*c^2 - 12*a^4*b^4*c^3 + 48*a^5*b^2*c
^4 - 64*a^6*c^5)*d*e^8*x^7 + 2*(a^3*b^7*c - 12*a^4*b^5*c^2 + 48*a^5*b^3*c^3
 - 64*a^6*b*c^4 + 14*(a^3*b^6*c^2 - 12*a^4*b^4*c^3 + 48*a^5*b^2*c^4 - 64*a^
6*c^5)*d^2)*e^7*x^6 + 4*(14*(a^3*b^6*c^2 - 12*a^4*b^4*c^3 + 48*a^5*b^2*c^4
 - 64*a^6*c^5)*d^3 + 3*(a^3*b^7*c - 12*a^4*b^5*c^2 + 48*a^5*b^3*c^3 - 64*a^6
*b*c^4)*d)*e^6*x^5 + (a^3*b^8 - 10*a^4*b^6*c + 24*a^5*b^4*c^2 + 32*a^6*b^2*
c^3 - 128*a^7*c^4 + 70*(a^3*b^6*c^2 - 12*a^4*b^4*c^3 + 48*a^5*b^2*c^4 - 64*
a^6*c^5)*d^4 + 30*(a^3*b^7*c - 12*a^4*b^5*c^2 + 48*a^5*b^3*c^3 - 64*a^6*b*c
^4)*d^2)*e^5*x^4 + 4*(14*(a^3*b^6*c^2 - 12*a^4*b^4*c^3 + 48*a^5*b^2*c^4 - 6
4*a^6*c^5)*d^5 + 10*(a^3*b^7*c - 12*a^4*b^5*c^2 + 48*a^5*b^3*c^3 - 64*a^6*b
*c^4)*d^3 + (a^3*b^8 - 10*a^4*b^6*c + 24*a^5*b^4*c^2 + 32*a^6*b^2*c^3 - 128
*a^7*c^4)*d)*e^4*x^3 + 2*(a^4*b^7 - 12*a^5*b^5*c + 48*a^6*b^3*c^2 - 64*a^7*
b*c^3 + 14*(a^3*b^6*c^2 - 12*a^4*b^4*c^3 + 48*a^5*b^2*c^4 - 64*a^6*c^5)*d^6
 + 15*(a^3*b^7*c - 12*a^4*b^5*c^2 + 48*a^5*b^3*c^3 - 64*a^6*b*c^4)*d^4 + 3*
(a^3*b^8 - 10*a^4*b^6*c + 24*a^5*b^4*c^2 + 32*a^6*b^2*c^3 - 128*a^7*c^4)*d^
2)*e^3*x^2 + 4*(2*(a^3*b^6*c^2 - 12*a^4*b^4*c^3 + 48*a^5*b^2*c^4 - 64*a^6*c
^5)*d^7 + 3*(a^3*b^7*c - 12*a^4*b^5*c^2 + 48*a^5*b^3*c^3 - 64*a^6*b*c^4)*d^
5 + (a^3*b^8 - 10*a^4*b^6*c + 24*a^5*b^4*c^2 + 32*a^6*b^2*c^3 - 128*a^7*c^4
)*d^3 + (a^4*b^7 - 12*a^5*b^5*c + 48*a^6*b^3*c^2 - 64*a^7*b*c^3)*d)*e^2*x +
 (a^5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2*c^2 - 64*a^8*c^3 + (a^3*b^6*c^2 - 12*
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a^4*b^4*c^3 + 48*a^5*b^2*c^4 - 64*a^6*c^5)*d^8 + 2*(a^3*b^7*c - 12*a^4*b^5*
c^2 + 48*a^5*b^3*c^3 - 64*a^6*b*c^4)*d^6 + (a^3*b^8 - 10*a^4*b^6*c + 24*a^5
*b^4*c^2 + 32*a^6*b^2*c^3 - 128*a^7*c^4)*d^4 + 2*(a^4*b^7 - 12*a^5*b^5*c +
48*a^6*b^3*c^2 - 64*a^7*b*c^3)*d^2)*e), 1/4*(2*(a*b^5*c^2 - 11*a^2*b^3*c^3
+ 28*a^3*b*c^4)*e^6*x^6 + 12*(a*b^5*c^2 - 11*a^2*b^3*c^3 + 28*a^3*b*c^4)*d*
e^5*x^5 + (4*a*b^6*c - 45*a^2*b^4*c^2 + 132*a^3*b^2*c^3 - 64*a^4*c^4 + 30*(
a*b^5*c^2 - 11*a^2*b^3*c^3 + 28*a^3*b*c^4)*d^2)*e^4*x^4 + 3*a^2*b^6 - 33*a^
3*b^4*c + 108*a^4*b^2*c^2 - 96*a^5*c^3 + 2*(a*b^5*c^2 - 11*a^2*b^3*c^3 + 28
*a^3*b*c^4)*d^6 + 4*(10*(a*b^5*c^2 - 11*a^2*b^3*c^3 + 28*a^3*b*c^4)*d^3 + (
4*a*b^6*c - 45*a^2*b^4*c^2 + 132*a^3*b^2*c^3 - 64*a^4*c^4)*d)*e^3*x^3 + (4*
a*b^6*c - 45*a^2*b^4*c^2 + 132*a^3*b^2*c^3 - 64*a^4*c^4)*d^4 + 2*(a*b^7 - 1
0*a^2*b^5*c + 23*a^3*b^3*c^2 + 4*a^4*b*c^3 + 15*(a*b^5*c^2 - 11*a^2*b^3*c^3
 + 28*a^3*b*c^4)*d^4 + 3*(4*a*b^6*c - 45*a^2*b^4*c^2 + 132*a^3*b^2*c^3 - 64
*a^4*c^4)*d^2)*e^2*x^2 + 2*(a*b^7 - 10*a^2*b^5*c + 23*a^3*b^3*c^2 + 4*a^4*b
*c^3)*d^2 + 4*(3*(a*b^5*c^2 - 11*a^2*b^3*c^3 + 28*a^3*b*c^4)*d^5 + (4*a*b^6
*c - 45*a^2*b^4*c^2 + 132*a^3*b^2*c^3 - 64*a^4*c^4)*d^3 + (a*b^7 - 10*a^2*b
^5*c + 23*a^3*b^3*c^2 + 4*a^4*b*c^3)*d)*e*x + 2*((b^5*c^2 - 10*a*b^3*c^3 +
30*a^2*b*c^4)*e^8*x^8 + 8*(b^5*c^2 - 10*a*b^3*c^3 + 30*a^2*b*c^4)*d*e^7*x^7
 + 2*(b^6*c - 10*a*b^4*c^2 + 30*a^2*b^2*c^3 + 14*(b^5*c^2 - 10*a*b^3*c^3 +
30*a^2*b*c^4)*d^2)*e^6*x^6 + 4*(14*(b^5*c^2 - 10*a*b^3*c^3 + 30*a^2*b*c^4)*
d^3 + 3*(b^6*c - 10*a*b^4*c^2 + 30*a^2*b^2*c^3)*d)*e^5*x^5 + (b^5*c^2 - 10*
a*b^3*c^3 + 30*a^2*b*c^4)*d^8 + (b^7 - 8*a*b^5*c + 10*a^2*b^3*c^2 + 60*a^3*
b*c^3 + 70*(b^5*c^2 - 10*a*b^3*c^3 + 30*a^2*b*c^4)*d^4 + 30*(b^6*c - 10*a*b
^4*c^2 + 30*a^2*b^2*c^3)*d^2)*e^4*x^4 + a^2*b^5 - 10*a^3*b^3*c + 30*a^4*b*c
^2 + 2*(b^6*c - 10*a*b^4*c^2 + 30*a^2*b^2*c^3)*d^6 + 4*(14*(b^5*c^2 - 10*a*
b^3*c^3 + 30*a^2*b*c^4)*d^5 + 10*(b^6*c - 10*a*b^4*c^2 + 30*a^2*b^2*c^3)*d^
3 + (b^7 - 8*a*b^5*c + 10*a^2*b^3*c^2 + 60*a^3*b*c^3)*d)*e^3*x^3 + (b^7 - 8
*a*b^5*c + 10*a^2*b^3*c^2 + 60*a^3*b*c^3)*d^4 + 2*(a*b^6 - 10*a^2*b^4*c + 3
0*a^3*b^2*c^2 + 14*(b^5*c^2 - 10*a*b^3*c^3 + 30*a^2*b*c^4)*d^6 + 15*(b^6*c
- 10*a*b^4*c^2 + 30*a^2*b^2*c^3)*d^4 + 3*(b^7 - 8*a*b^5*c + 10*a^2*b^3*c^2
+ 60*a^3*b*c^3)*d^2)*e^2*x^2 + 2*(a*b^6 - 10*a^2*b^4*c + 30*a^3*b^2*c^2)*d^
2 + 4*(2*(b^5*c^2 - 10*a*b^3*c^3 + 30*a^2*b*c^4)*d^7 + 3*(b^6*c - 10*a*b^4*
c^2 + 30*a^2*b^2*c^3)*d^5 + (b^7 - 8*a*b^5*c + 10*a^2*b^3*c^2 + 60*a^3*b*c^
3)*d^3 + (a*b^6 - 10*a^2*b^4*c + 30*a^3*b^2*c^2)*d)*e*x)*sqrt(-b^2 + 4*a*c)
*arctan(-(2*c*e^2*x^2 + 4*c*d*e*x + 2*c*d^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 -
4*a*c)) - ((b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*e^8*x^8 +
 8*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d*e^7*x^7 + 2*(b^
7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4 + 14*(b^6*c^2 - 12*a*b^4
*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^2)*e^6*x^6 + 4*(14*(b^6*c^2 - 12*a*b^
4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^3 + 3*(b^7*c - 12*a*b^5*c^2 + 48*a^2
*b^3*c^3 - 64*a^3*b*c^4)*d)*e^5*x^5 + (b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*
c^4 - 64*a^3*c^5)*d^8 + (b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3
 - 128*a^4*c^4 + 70*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*
d^4 + 30*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*d^2)*e^4*x^
4 + a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3 + 2*(b^7*c - 12*a*
b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*d^6 + 4*(14*(b^6*c^2 - 12*a*b^4*c^
3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^5 + 10*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^
3*c^3 - 64*a^3*b*c^4)*d^3 + (b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2
*c^3 - 128*a^4*c^4)*d)*e^3*x^3 + (b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^
3*b^2*c^3 - 128*a^4*c^4)*d^4 + 2*(a*b^7 - 12*a^2*b^5*c + 48*a^3*b^3*c^2 - 6
4*a^4*b*c^3 + 14*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^6
 + 15*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*d^4 + 3*(b^8 -
 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*d^2)*e^2*x^2 +
 2*(a*b^7 - 12*a^2*b^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3)*d^2 + 4*(2*(b^6*c
^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^7 + 3*(b^7*c - 12*a*b^5*
c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*d^5 + (b^8 - 10*a*b^6*c + 24*a^2*b^4*c
^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*d^3 + (a*b^7 - 12*a^2*b^5*c + 48*a^3*b^3
*c^2 - 64*a^4*b*c^3)*d)*e*x)*log(c*e^4*x^4 + 4*c*d*e^3*x^3 + c*d^4 + (6*c*d
^2 + b)*e^2*x^2 + b*d^2 + 2*(2*c*d^3 + b*d)*e*x + a) + 4*((b^6*c^2 - 12*a*b
```

```
^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*e^8*x^8 + 8*(b^6*c^2 - 12*a*b^4*c^3 +
 48*a^2*b^2*c^4 - 64*a^3*c^5)*d*e^7*x^7 + 2*(b^7*c - 12*a*b^5*c^2 + 48*a^2*
b^3*c^3 - 64*a^3*b*c^4 + 14*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a
^3*c^5)*d^2)*e^6*x^6 + 4*(14*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*
a^3*c^5)*d^3 + 3*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*d)*
e^5*x^5 + (b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^8 + (b^8
 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4 + 70*(b^6*c^2
 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^4 + 30*(b^7*c - 12*a*b^5*c
^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*d^2)*e^4*x^4 + a^2*b^6 - 12*a^3*b^4*c +
 48*a^4*b^2*c^2 - 64*a^5*c^3 + 2*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 6
4*a^3*b*c^4)*d^6 + 4*(14*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*
c^5)*d^5 + 10*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*d^3 +
(b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*d)*e^3*x
^3 + (b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*d^4
 + 2*(a*b^7 - 12*a^2*b^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3 + 14*(b^6*c^2 -
12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^6 + 15*(b^7*c - 12*a*b^5*c^2
+ 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*d^4 + 3*(b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2
 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*d^2)*e^2*x^2 + 2*(a*b^7 - 12*a^2*b^5*c + 4
8*a^3*b^3*c^2 - 64*a^4*b*c^3)*d^2 + 4*(2*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b
^2*c^4 - 64*a^3*c^5)*d^7 + 3*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^
3*b*c^4)*d^5 + (b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^
4*c^4)*d^3 + (a*b^7 - 12*a^2*b^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3)*d)*e*x)
*log(e*x + d))/((a^3*b^6*c^2 - 12*a^4*b^4*c^3 + 48*a^5*b^2*c^4 - 64*a^6*c^5
)*e^9*x^8 + 8*(a^3*b^6*c^2 - 12*a^4*b^4*c^3 + 48*a^5*b^2*c^4 - 64*a^6*c^5)*
d*e^8*x^7 + 2*(a^3*b^7*c - 12*a^4*b^5*c^2 + 48*a^5*b^3*c^3 - 64*a^6*b*c^4 +
 14*(a^3*b^6*c^2 - 12*a^4*b^4*c^3 + 48*a^5*b^2*c^4 - 64*a^6*c^5)*d^2)*e^7*x
^6 + 4*(14*(a^3*b^6*c^2 - 12*a^4*b^4*c^3 + 48*a^5*b^2*c^4 - 64*a^6*c^5)*d^3
 + 3*(a^3*b^7*c - 12*a^4*b^5*c^2 + 48*a^5*b^3*c^3 - 64*a^6*b*c^4)*d)*e^6*x^
5 + (a^3*b^8 - 10*a^4*b^6*c + 24*a^5*b^4*c^2 + 32*a^6*b^2*c^3 - 128*a^7*c^4
 + 70*(a^3*b^6*c^2 - 12*a^4*b^4*c^3 + 48*a^5*b^2*c^4 - 64*a^6*c^5)*d^4 + 30
*(a^3*b^7*c - 12*a^4*b^5*c^2 + 48*a^5*b^3*c^3 - 64*a^6*b*c^4)*d^2)*e^5*x^4
+ 4*(14*(a^3*b^6*c^2 - 12*a^4*b^4*c^3 + 48*a^5*b^2*c^4 - 64*a^6*c^5)*d^5 +
10*(a^3*b^7*c - 12*a^4*b^5*c^2 + 48*a^5*b^3*c^3 - 64*a^6*b*c^4)*d^3 + (a^3*
b^8 - 10*a^4*b^6*c + 24*a^5*b^4*c^2 + 32*a^6*b^2*c^3 - 128*a^7*c^4)*d)*e^4*
x^3 + 2*(a^4*b^7 - 12*a^5*b^5*c + 48*a^6*b^3*c^2 - 64*a^7*b*c^3 + 14*(a^3*b
^6*c^2 - 12*a^4*b^4*c^3 + 48*a^5*b^2*c^4 - 64*a^6*c^5)*d^6 + 15*(a^3*b^7*c
- 12*a^4*b^5*c^2 + 48*a^5*b^3*c^3 - 64*a^6*b*c^4)*d^4 + 3*(a^3*b^8 - 10*a^4
*b^6*c + 24*a^5*b^4*c^2 + 32*a^6*b^2*c^3 - 128*a^7*c^4)*d^2)*e^3*x^2 + 4*(2
*(a^3*b^6*c^2 - 12*a^4*b^4*c^3 + 48*a^5*b^2*c^4 - 64*a^6*c^5)*d^7 + 3*(a^3*
b^7*c - 12*a^4*b^5*c^2 + 48*a^5*b^3*c^3 - 64*a^6*b*c^4)*d^5 + (a^3*b^8 - 10
*a^4*b^6*c + 24*a^5*b^4*c^2 + 32*a^6*b^2*c^3 - 128*a^7*c^4)*d^3 + (a^4*b^7
- 12*a^5*b^5*c + 48*a^6*b^3*c^2 - 64*a^7*b*c^3)*d)*e^2*x + (a^5*b^6 - 12*a^
6*b^4*c + 48*a^7*b^2*c^2 - 64*a^8*c^3 + (a^3*b^6*c^2 - 12*a^4*b^4*c^3 + 48*
a^5*b^2*c^4 - 64*a^6*c^5)*d^8 + 2*(a^3*b^7*c - 12*a^4*b^5*c^2 + 48*a^5*b^3*
c^3 - 64*a^6*b*c^4)*d^6 + (a^3*b^8 - 10*a^4*b^6*c + 24*a^5*b^4*c^2 + 32*a^6
*b^2*c^3 - 128*a^7*c^4)*d^4 + 2*(a^4*b^7 - 12*a^5*b^5*c + 48*a^6*b^3*c^2 -
64*a^7*b*c^3)*d^2)*e)]
```

giac [B] time = 1.72, size = 1012, normalized size = 3.97

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="giac")

[Out] -1/4*((a^3*b^7*c*e^3 - 14*a^4*b^5*c^2*e^3 + 70*a^5*b^3*c^3*e^3 - 120*a^6*b*
c^4*e^3)*sqrt(b^2 - 4*a*c)*log(abs(b*x^2*e^2 + 2*b*d*x*e + sqrt(b^2 - 4*a*c
)*x^2*e^2 + 2*sqrt(b^2 - 4*a*c)*d*x*e + b*d^2 + sqrt(b^2 - 4*a*c)*d^2 + 2*a
)) - (a^3*b^7*c*e^3 - 14*a^4*b^5*c^2*e^3 + 70*a^5*b^3*c^3*e^3 - 120*a^6*b*c
^4*e^3)*sqrt(b^2 - 4*a*c)*log(abs(-b*x^2*e^2 - 2*b*d*x*e + sqrt(b^2 - 4*a*c
```

```
)*x^2*e^2 + 2*sqrt(b^2 - 4*a*c)*d*x*e - b*d^2 + sqrt(b^2 - 4*a*c)*d^2 - 2*a
)))/(a^6*b^8*c*e^4 - 16*a^7*b^6*c^2*e^4 + 96*a^8*b^4*c^3*e^4 - 256*a^9*b^2*
c^4*e^4 + 256*a^10*c^5*e^4) - 1/4*e^(-1)*log(abs(c*x^4*e^4 + 4*c*d*x^3*e^3
+ 6*c*d^2*x^2*e^2 + 4*c*d^3*x*e + c*d^4 + b*x^2*e^2 + 2*b*d*x*e + b*d^2 + a
))/a^3 + e^(-1)*log(abs(x*e + d))/a^3 + 1/4*(2*a*b^3*c^2*d^6 - 14*a^2*b*c^3
*d^6 + 4*a*b^4*c*d^4 - 29*a^2*b^2*c^2*d^4 + 16*a^3*c^3*d^4 + 2*a*b^5*d^2 -
12*a^2*b^3*c*d^2 - 2*a^3*b*c^2*d^2 + 2*(a*b^3*c^2*e^6 - 7*a^2*b*c^3*e^6)*x^
6 + 3*a^2*b^4 - 21*a^3*b^2*c + 24*a^4*c^2 + 12*(a*b^3*c^2*d*e^5 - 7*a^2*b*c
^3*d*e^5)*x^5 + (30*a*b^3*c^2*d^2*e^4 - 210*a^2*b*c^3*d^2*e^4 + 4*a*b^4*c*e
^4 - 29*a^2*b^2*c^2*e^4 + 16*a^3*c^3*e^4)*x^4 + 4*(10*a*b^3*c^2*d^3*e^3 - 7
0*a^2*b*c^3*d^3*e^3 + 4*a*b^4*c*d*e^3 - 29*a^2*b^2*c^2*d*e^3 + 16*a^3*c^3*d
*e^3)*x^3 + 2*(15*a*b^3*c^2*d^4*e^2 - 105*a^2*b*c^3*d^4*e^2 + 12*a*b^4*c*d^
2*e^2 - 87*a^2*b^2*c^2*d^2*e^2 + 48*a^3*c^3*d^2*e^2 + a*b^5*e^2 - 6*a^2*b^3
*c*e^2 - a^3*b*c^2*e^2)*x^2 + 4*(3*a*b^3*c^2*d^5*e - 21*a^2*b*c^3*d^5*e + 4
*a*b^4*c*d^3*e - 29*a^2*b^2*c^2*d^3*e + 16*a^3*c^3*d^3*e + a*b^5*d*e - 6*a^
2*b^3*c*d*e - a^3*b*c^2*d*e)*x)*e^(-1)/((c*x^4*e^4 + 4*c*d*x^3*e^3 + 6*c*d^
2*x^2*e^2 + 4*c*d^3*x*e + c*d^4 + b*x^2*e^2 + 2*b*d*x*e + b*d^2 + a)^2*(b^2
 - 4*a*c)^2*a^3)
```

maple [C] time = 0.08, size = 4477, normalized size = 17.56

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x)
```

```
[Out] -1/2/a^3/(16*a^2*c^2-8*a*b^2*c+b^4)/e*sum((c*e^3*(16*a^2*c^2-8*a*b^2*c+b^4)
*_R^3+3*c*d*e^2*(16*a^2*c^2-8*a*b^2*c+b^4)*_R^2+e*(48*a^2*c^3*d^2-24*a*b^2*
c^2*d^2+3*b^4*c*d^2+23*a^2*b*c^2-9*a*b^3*c+b^5)*_R+16*a^2*c^3*d^3-8*a*b^2*c
^2*d^3+b^4*c*d^3+23*a^2*b*c^2*d-9*a*b^3*c*d+b^5*d)/(2*_R^3*c*e^3+6*_R^2*c*d
*e^2+6*_R*c*d^2*e+2*c*d^3+_R*b*e+b*d)*ln(-_R+x),_R=RootOf(_Z^4*c*e^4+4*_Z^3
*c*d*e^3+c*d^4+b*d^2+(6*c*d^2*e^2+b*e^2)*_Z^2+(4*c*d^3*e+2*b*d*e)*_Z+a))-3/
a/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*
d*e*x+b*d^2+a)^2*e/(16*a^2*c^2-8*a*b^2*c+b^4)*x^2*b^3*c-21/a/(c*e^4*x^4+4*c
*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2
*d^5/(16*a^2*c^2-8*a*b^2*c+b^4)*x*b*c^3+3/a^2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*
d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*d^5/(16*a^2*c^
2-8*a*b^2*c+b^4)*x*b^3*c^2-29/a/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*
c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*d^3/(16*a^2*c^2-8*a*b^2*c+b^
4)*x*b^2*c^2+4/a^2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e
^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*d^3/(16*a^2*c^2-8*a*b^2*c+b^4)*x*b^4*c-6/
a/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*
d*e*x+b*d^2+a)^2*d/(16*a^2*c^2-8*a*b^2*c+b^4)*x*b^3*c-7/2/a/(c*e^4*x^4+4*c*
d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2/
e/(16*a^2*c^2-8*a*b^2*c+b^4)*b*c^3*d^6+1/2/a^2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c
*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2/e/(16*a^2*c^2
-8*a*b^2*c+b^4)*b^3*c^2*d^6-29/4/a/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2
+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2/e/(16*a^2*c^2-8*a*b^2*c+b
^4)*b^2*c^2*d^4+1/a^2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+
b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2/e/(16*a^2*c^2-8*a*b^2*c+b^4)*b^4*c*d^4
-3/a/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2
*b*d*e*x+b*d^2+a)^2/e/(16*a^2*c^2-8*a*b^2*c+b^4)*b^3*c*d^2-29/4/a/(c*e^4*x^
4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2
+a)^2*e^3*c^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x^4*b^2-7/2/a/(c*e^4*x^4+4*c*d*e^3
*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*c^3*e
^5*b/(16*a^2*c^2-8*a*b^2*c+b^4)*x^6+1/2/a^2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^
2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*c^2*e^5*b^3/(16*
a^2*c^2-8*a*b^2*c+b^4)*x^6-1/2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c
*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*e/(16*a^2*c^2-8*a*b^2*c+b^4)*
x^2*b*c^2-1/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+
```

```
c*d^4+2*b*d*e*x+b*d^2+a)^2*d/(16*a^2*c^2-8*a*b^2*c+b^4)*x*b*c^2-1/2/(c*e^4*
x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d
^2+a)^2/e/(16*a^2*c^2-8*a*b^2*c+b^4)*b*c^2*d^2+1/2/a^2/(c*e^4*x^4+4*c*d*e^3
*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*e/(16
*a^2*c^2-8*a*b^2*c+b^4)*x^2*b^5+1/a^2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*
x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*d/(16*a^2*c^2-8*a*b^2*
c+b^4)*x*b^5+1/2/a^2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b
*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2/e/(16*a^2*c^2-8*a*b^2*c+b^4)*b^5*d^2+1/
a^2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*
b*d*e*x+b*d^2+a)^2*e^3*c/(16*a^2*c^2-8*a*b^2*c+b^4)*x^4*b^4+16/(c*e^4*x^4+4
*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)
^2*c^3*d*e^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x^3+24/(c*e^4*x^4+4*c*d*e^3*x^3+6*c
*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*e/(16*a^2*c^2
-8*a*b^2*c+b^4)*x^2*c^3*d^2+6*a/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*
c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2/e/(16*a^2*c^2-8*a*b^2*c+b^4)
*c^2+3/4/a/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c
*d^4+2*b*d*e*x+b*d^2+a)^2/e/(16*a^2*c^2-8*a*b^2*c+b^4)*b^4+4/(c*e^4*x^4+4*c
*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2
*e^3*c^3/(16*a^2*c^2-8*a*b^2*c+b^4)*x^4+16/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2
*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*d^3/(16*a^2*c^2-8
*a*b^2*c+b^4)*x*c^3+4/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+
b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2/e/(16*a^2*c^2-8*a*b^2*c+b^4)*c^3*d^4-2
1/4/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*
b*d*e*x+b*d^2+a)^2/e/(16*a^2*c^2-8*a*b^2*c+b^4)*b^2*c+ln(e*x+d)/a^3/e-21/a/
(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*
e*x+b*d^2+a)^2*b*c^3*d*e^4/(16*a^2*c^2-8*a*b^2*c+b^4)*x^5+3/a^2/(c*e^4*x^4+
4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a
)^2*b^3*c^2*d*e^4/(16*a^2*c^2-8*a*b^2*c+b^4)*x^5-105/2/a/(c*e^4*x^4+4*c*d*e
^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*e^3
*c^3/(16*a^2*c^2-8*a*b^2*c+b^4)*x^4*b*d^2+15/2/a^2/(c*e^4*x^4+4*c*d*e^3*x^3
+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*e^3*c^2/(
16*a^2*c^2-8*a*b^2*c+b^4)*x^4*b^3*d^2-70/a/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2
*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*c^3*d^3*e^2/(16*a
^2*c^2-8*a*b^2*c+b^4)*x^3*b+10/a^2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2
+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*c^2*d^3*e^2/(16*a^2*c^2-8
*a*b^2*c+b^4)*x^3*b^3-29/a/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3
*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*c^2*d*e^2/(16*a^2*c^2-8*a*b^2*c+b
^4)*x^3*b^2+4/a^2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^
2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*c*d*e^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x^3*b^4
-105/2/a/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d
^4+2*b*d*e*x+b*d^2+a)^2*e/(16*a^2*c^2-8*a*b^2*c+b^4)*x^2*b*c^3*d^4+15/2/a^2
/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d
*e*x+b*d^2+a)^2*e/(16*a^2*c^2-8*a*b^2*c+b^4)*x^2*b^3*c^2*d^4-87/2/a/(c*e^4*
x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d
^2+a)^2*e/(16*a^2*c^2-8*a*b^2*c+b^4)*x^2*b^2*c^2*d^2+6/a^2/(c*e^4*x^4+4*c*d
*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*e
/(16*a^2*c^2-8*a*b^2*c+b^4)*x^2*b^4*c*d^2
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [B] time = 17.98, size = 19440, normalized size = 76.24

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((d + e*x)*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3),x)
```

```
[Out] ((x^2*(b^5*e + 48*a^2*c^3*d^2*e + 15*b^3*c^2*d^4*e - 6*a*b^3*c*e - a^2*b*c^
2*e + 12*b^4*c*d^2*e - 105*a*b*c^3*d^4*e - 87*a*b^2*c^2*d^2*e))/(2*(a^2*b^4
 + 16*a^4*c^2 - 8*a^3*b^2*c)) + (x^4*(4*b^4*c*e^3 + 16*a^2*c^3*e^3 - 29*a*b
^2*c^2*e^3 + 30*b^3*c^2*d^2*e^3 - 210*a*b*c^3*d^2*e^3))/(4*(a^2*b^4 + 16*a^
4*c^2 - 8*a^3*b^2*c)) + (x^3*(16*a^2*c^3*d*e^2 + 10*b^3*c^2*d^3*e^2 + 4*b^4
*c*d*e^2 - 29*a*b^2*c^2*d*e^2 - 70*a*b*c^3*d^3*e^2))/(a^2*b^4 + 16*a^4*c^2
- 8*a^3*b^2*c) + (3*x^5*(b^3*c^2*d*e^4 - 7*a*b*c^3*d*e^4))/(a^2*b^4 + 16*a^
4*c^2 - 8*a^3*b^2*c) + (x^6*(b^3*c^2*e^5 - 7*a*b*c^3*e^5))/(2*(a^2*b^4 + 16
*a^4*c^2 - 8*a^3*b^2*c)) + (x*(b^5*d + 4*b^4*c*d^3 + 16*a^2*c^3*d^3 + 3*b^3
*c^2*d^5 - 29*a*b^2*c^2*d^3 - 6*a*b^3*c*d - a^2*b*c^2*d - 21*a*b*c^3*d^5))/
(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c) + (3*a*b^4 + 24*a^3*c^2 + 2*b^5*d^2 -
21*a^2*b^2*c + 4*b^4*c*d^4 + 16*a^2*c^3*d^4 + 2*b^3*c^2*d^6 - 2*a^2*b*c^2*d
^2 - 29*a*b^2*c^2*d^4 - 12*a*b^3*c*d^2 - 14*a*b*c^3*d^6)/(4*e*(a^2*b^4 + 16
*a^4*c^2 - 8*a^3*b^2*c)))/(x^2*(6*b^2*d^2*e^2 + 28*c^2*d^6*e^2 + 2*a*b*e^2
+ 12*a*c*d^2*e^2 + 30*b*c*d^4*e^2) + x^6*(28*c^2*d^2*e^6 + 2*b*c*e^6) + x*(
4*b^2*d^3*e + 8*c^2*d^7*e + 8*a*c*d^3*e + 12*b*c*d^5*e + 4*a*b*d*e) + x^3*(
4*b^2*d*e^3 + 56*c^2*d^5*e^3 + 8*a*c*d*e^3 + 40*b*c*d^3*e^3) + x^5*(56*c^2*
d^3*e^5 + 12*b*c*d*e^5) + x^4*(b^2*e^4 + 70*c^2*d^4*e^4 + 2*a*c*e^4 + 30*b*
c*d^2*e^4) + a^2 + b^2*d^4 + c^2*d^8 + c^2*e^8*x^8 + 2*a*b*d^2 + 2*a*c*d^4
+ 2*b*c*d^6 + 8*c^2*d*e^7*x^7) + log(d + e*x)/(a^3*e) - (log((((a^3*e*(-(b^
2*(b^4 + 30*a^2*c^2 - 10*a*b^2*c)^2)/(a^6*e^2*(4*a*c - b^2)^5))^(1/2) + 1)*
(((a^3*e*(-(b^2*(b^4 + 30*a^2*c^2 - 10*a*b^2*c)^2)/(a^6*e^2*(4*a*c - b^2)^5
))^(1/2) + 1)*((2*b*c^2*e^16*(2*b^5 + 46*a^2*b*c^2 + b^4*c*d^2 + 10*a^2*c^3
*d^2 - 18*a*b^3*c - 2*a*b^2*c^2*d^2))/(a^2*(4*a*c - b^2)^2) + (b*c^2*e^16*(
a^3*e*(-(b^2*(b^4 + 30*a^2*c^2 - 10*a*b^2*c)^2)/(a^6*e^2*(4*a*c - b^2)^5))^
(1/2) + 1)*(a*b + 3*b^2*d^2 + 3*b^2*e^2*x^2 - 10*a*c*d^2 + 6*b^2*d*e*x - 10
*a*c*e^2*x^2 - 20*a*c*d*e*x))/a^3 + (2*b*c^3*e^18*x^2*(b^4 + 10*a^2*c^2 - 2
*a*b^2*c))/(a^2*(4*a*c - b^2)^2) + (4*b*c^3*d*e^17*x*(b^4 + 10*a^2*c^2 - 2*
a*b^2*c))/(a^2*(4*a*c - b^2)^2)))/(4*a^3*e) + (b*c^3*e^15*(7*a*c - b^2)*(4*
b^5 + 71*a^2*b*c^2 + 6*b^4*c*d^2 + 80*a^2*c^3*d^2 - 33*a*b^3*c - 47*a*b^2*c
^2*d^2))/(a^4*(4*a*c - b^2)^4) - (b*c^4*e^17*x^2*(6*b^6 - 560*a^3*c^3 + 409
*a^2*b^2*c^2 - 89*a*b^4*c))/(a^4*(4*a*c - b^2)^4) - (2*b*c^4*d*e^16*x*(6*b^
6 - 560*a^3*c^3 + 409*a^2*b^2*c^2 - 89*a*b^4*c))/(a^4*(4*a*c - b^2)^4)))/(4
*a^3*e) - (b^3*c^5*e^16*x^2*(7*a*c - b^2)^3)/(a^6*(4*a*c - b^2)^6) + (b^2*c
^4*e^14*(7*a*c - b^2)^2*(b^4 + 16*a^2*c^2 + b^3*c*d^2 - 8*a*b^2*c - 7*a*b*c
^2*d^2))/(a^6*(4*a*c - b^2)^6) - (2*b^3*c^5*d*e^15*x*(7*a*c - b^2)^3)/(a^6*
(4*a*c - b^2)^6))*(((a^3*e*(-(b^2*(b^4 + 30*a^2*c^2 - 10*a*b^2*c)^2)/(a^6*e
^2*(4*a*c - b^2)^5))^(1/2) - 1)*(((a^3*e*(-(b^2*(b^4 + 30*a^2*c^2 - 10*a*b^
2*c)^2)/(a^6*e^2*(4*a*c - b^2)^5))^(1/2) - 1)*((2*b*c^2*e^16*(2*b^5 + 46*a^
2*b*c^2 + b^4*c*d^2 + 10*a^2*c^3*d^2 - 18*a*b^3*c - 2*a*b^2*c^2*d^2))/(a^2*
(4*a*c - b^2)^2) - (b*c^2*e^16*(a^3*e*(-(b^2*(b^4 + 30*a^2*c^2 - 10*a*b^2*c
)^2)/(a^6*e^2*(4*a*c - b^2)^5))^(1/2) - 1)*(a*b + 3*b^2*d^2 + 3*b^2*e^2*x^2
 - 10*a*c*d^2 + 6*b^2*d*e*x - 10*a*c*e^2*x^2 - 20*a*c*d*e*x))/a^3 + (2*b*c^
3*e^18*x^2*(b^4 + 10*a^2*c^2 - 2*a*b^2*c))/(a^2*(4*a*c - b^2)^2) + (4*b*c^3
*d*e^17*x*(b^4 + 10*a^2*c^2 - 2*a*b^2*c))/(a^2*(4*a*c - b^2)^2)))/(4*a^3*e)
 - (b*c^3*e^15*(7*a*c - b^2)*(4*b^5 + 71*a^2*b*c^2 + 6*b^4*c*d^2 + 80*a^2*c
^3*d^2 - 33*a*b^3*c - 47*a*b^2*c^2*d^2))/(a^4*(4*a*c - b^2)^4) + (b*c^4*e^1
7*x^2*(6*b^6 - 560*a^3*c^3 + 409*a^2*b^2*c^2 - 89*a*b^4*c))/(a^4*(4*a*c - b
^2)^4) + (2*b*c^4*d*e^16*x*(6*b^6 - 560*a^3*c^3 + 409*a^2*b^2*c^2 - 89*a*b^
4*c))/(a^4*(4*a*c - b^2)^4)))/(4*a^3*e) - (b^3*c^5*e^16*x^2*(7*a*c - b^2)^3
)/(a^6*(4*a*c - b^2)^6) + (b^2*c^4*e^14*(7*a*c - b^2)^2*(b^4 + 16*a^2*c^2 +
 b^3*c*d^2 - 8*a*b^2*c - 7*a*b*c^2*d^2))/(a^6*(4*a*c - b^2)^6) - (2*b^3*c^5
*d*e^15*x*(7*a*c - b^2)^3)/(a^6*(4*a*c - b^2)^6)))*(2*b^10*e - 2048*a^5*c^5
*e + 320*a^2*b^6*c^2*e - 1280*a^3*b^4*c^3*e + 2560*a^4*b^2*c^4*e - 40*a*b^8
*c*e))/(2*(4*a^3*b^10*e^2 - 4096*a^8*c^5*e^2 - 80*a^4*b^8*c*e^2 + 640*a^5*b
^6*c^2*e^2 - 2560*a^6*b^4*c^3*e^2 + 5120*a^7*b^2*c^4*e^2)) - (b*atan((x*(((
```

(((b*((2*(5120*a^10*b*c^9*d*e^17 + 2*a^4*b^13*c^3*d*e^17 - 36*a^5*b^11*c^4*d*e^17 + 276*a^6*b^9*c^5*d*e^17 - 1216*a^7*b^7*c^6*d*e^17 + 3456*a^8*b^5*c^7*d*e^17 - 6144*a^9*b^3*c^8*d*e^17))/(a^6*b^12 + 4096*a^12*c^6 - 24*a^7*b^10*c + 240*a^8*b^8*c^2 - 1280*a^9*b^6*c^3 + 3840*a^10*b^4*c^4 - 6144*a^11*b^2*c^5) - ((2*b^10*e - 2048*a^5*c^5*e + 320*a^2*b^6*c^2*e - 1280*a^3*b^4*c^3*e + 2560*a^4*b^2*c^4*e - 40*a*b^8*c*e)*(163840*a^13*b*c^9*d*e^18 - 12*a^6*b^15*c^2*d*e^18 + 328*a^7*b^13*c^3*d*e^18 - 3840*a^8*b^11*c^4*d*e^18 + 24960*a^9*b^9*c^5*d*e^18 - 97280*a^10*b^7*c^6*d*e^18 + 227328*a^11*b^5*c^7*d*e^18 - 294912*a^12*b^3*c^8*d*e^18))/((4*a^3*b^10*e^2 - 4096*a^8*c^5*e^2 - 80*a^4*b^8*c*e^2 + 640*a^5*b^6*c^2*e^2 - 2560*a^6*b^4*c^3*e^2 + 5120*a^7*b^2*c^4*e^2)*(a^6*b^12 + 4096*a^12*c^6 - 24*a^7*b^10*c + 240*a^8*b^8*c^2 - 1280*a^9*b^6*c^3 + 3840*a^10*b^4*c^4 - 6144*a^11*b^2*c^5)))*(b^4 + 30*a^2*c^2 - 10*a*b^2*c))/(4*a^3*e*(4*a*c - b^2)^(5/2)) - (b*(b^4 + 30*a^2*c^2 - 10*a*b^2*c)*(2*b^10*e - 2048*a^5*c^5*e + 320*a^2*b^6*c^2*e - 1280*a^3*b^4*c^3*e + 2560*a^4*b^2*c^4*e - 40*a*b^8*c*e)*(163840*a^13*b*c^9*d*e^18 - 12*a^6*b^15*c^2*d*e^18 + 328*a^7*b^13*c^3*d*e^18 - 3840*a^8*b^11*c^4*d*e^18 + 24960*a^9*b^9*c^5*d*e^18 - 97280*a^10*b^7*c^6*d*e^18 + 227328*a^11*b^5*c^7*d*e^18 - 294912*a^12*b^3*c^8*d*e^18))/(4*a^3*e*(4*a*c - b^2)^(5/2)*(4*a^3*b^10*e^2 - 4096*a^8*c^5*e^2 - 80*a^4*b^8*c*e^2 + 640*a^5*b^6*c^2*e^2 - 2560*a^6*b^4*c^3*e^2 + 5120*a^7*b^2*c^4*e^2)*(a^6*b^12 + 4096*a^12*c^6 - 24*a^7*b^10*c + 240*a^8*b^8*c^2 - 1280*a^9*b^6*c^3 + 3840*a^10*b^4*c^4 - 6144*a^11*b^2*c^5)))*(2*b^10*e - 2048*a^5*c^5*e + 320*a^2*b^6*c^2*e - 1280*a^3*b^4*c^3*e + 2560*a^4*b^2*c^4*e - 40*a*b^8*c*e))/(2*(4*a^3*b^10*e^2 - 4096*a^8*c^5*e^2 - 80*a^4*b^8*c*e^2 + 640*a^5*b^6*c^2*e^2 - 2560*a^6*b^4*c^3*e^2 + 5120*a^7*b^2*c^4*e^2)) + (b*((2*(8960*a^7*b*c^9*d*e^16 - 6*a^2*b^11*c^4*d*e^16 + 137*a^3*b^9*c^5*d*e^16 - 1217*a^4*b^7*c^6*d*e^16 + 5256*a^5*b^5*c^7*d*e^16 - 11024*a^6*b^3*c^8*d*e^16))/(a^6*b^12 + 4096*a^12*c^6 - 24*a^7*b^10*c + 240*a^8*b^8*c^2 - 1280*a^9*b^6*c^3 + 3840*a^10*b^4*c^4 - 6144*a^11*b^2*c^5) + (((2*(5120*a^10*b*c^9*d*e^17 + 2*a^4*b^13*c^3*d*e^17 - 36*a^5*b^11*c^4*d*e^17 + 276*a^6*b^9*c^5*d*e^17 - 1216*a^7*b^7*c^6*d*e^17 + 3456*a^8*b^5*c^7*d*e^17 - 6144*a^9*b^3*c^8*d*e^17))/(a^6*b^12 + 4096*a^12*c^6 - 24*a^7*b^10*c + 240*a^8*b^8*c^2 - 1280*a^9*b^6*c^3 + 3840*a^10*b^4*c^4 - 6144*a^11*b^2*c^5) - ((2*b^10*e - 2048*a^5*c^5*e + 320*a^2*b^6*c^2*e - 1280*a^3*b^4*c^3*e + 2560*a^4*b^2*c^4*e - 40*a*b^8*c*e)*(163840*a^13*b*c^9*d*e^18 - 12*a^6*b^15*c^2*d*e^18 + 328*a^7*b^13*c^3*d*e^18 - 3840*a^8*b^11*c^4*d*e^18 + 24960*a^9*b^9*c^5*d*e^18 - 97280*a^10*b^7*c^6*d*e^18 + 227328*a^11*b^5*c^7*d*e^18 - 294912*a^12*b^3*c^8*d*e^18))/((4*a^3*b^10*e^2 - 4096*a^8*c^5*e^2 - 80*a^4*b^8*c*e^2 + 640*a^5*b^6*c^2*e^2 - 2560*a^6*b^4*c^3*e^2 + 5120*a^7*b^2*c^4*e^2)*(a^6*b^12 + 4096*a^12*c^6 - 24*a^7*b^10*c + 240*a^8*b^8*c^2 - 1280*a^9*b^6*c^3 + 3840*a^10*b^4*c^4 - 6144*a^11*b^2*c^5)))*(2*b^10*e - 2048*a^5*c^5*e + 320*a^2*b^6*c^2*e - 1280*a^3*b^4*c^3*e + 2560*a^4*b^2*c^4*e - 40*a*b^8*c*e))/(2*(4*a^3*b^10*e^2 - 4096*a^8*c^5*e^2 - 80*a^4*b^8*c*e^2 + 640*a^5*b^6*c^2*e^2 - 2560*a^6*b^4*c^3*e^2 + 5120*a^7*b^2*c^4*e^2)))*(b^4 + 30*a^2*c^2 - 10*a*b^2*c))/(4*a^3*e*(4*a*c - b^2)^(5/2)) + (b^3*(b^4 + 30*a^2*c^2 - 10*a*b^2*c)^3*(163840*a^13*b*c^9*d*e^18 - 12*a^6*b^15*c^2*d*e^18 + 328*a^7*b^13*c^3*d*e^18 - 3840*a^8*b^11*c^4*d*e^18 + 24960*a^9*b^9*c^5*d*e^18 - 97280*a^10*b^7*c^6*d*e^18 + 227328*a^11*b^5*c^7*d*e^18 - 294912*a^12*b^3*c^8*d*e^18))/(32*a^9*e^3*(4*a*c - b^2)^(15/2)*(a^6*b^12 + 4096*a^12*c^6 - 24*a^7*b^10*c + 240*a^8*b^8*c^2 - 1280*a^9*b^6*c^3 + 3840*a^10*b^4*c^4 - 6144*a^11*b^2*c^5)))*(3*b^8 + 160*a^4*c^4 + 180*a^2*b^4*c^2 - 325*a^3*b^2*c^3 - 39*a*b^6*c))/(8*a^3*c^2*(4*a*c - b^2)^(13/2)*(6*b^10 - 6400*a^5*c^5 + 960*a^2*b^6*c^2 - 3850*a^3*b^4*c^3 + 7775*a^4*b^2*c^4 - 120*a*b^8*c)) + (3*b*((2*(b^9*c^5*d*e^15 - 21*a*b^7*c^6*d*e^15 + 147*a^2*b^5*c^7*d*e^15 - 343*a^3*b^3*c^8*d*e^15))/(a^6*b^12 + 4096*a^12*c^6 - 24*a^7*b^10*c + 240*a^8*b^8*c^2 - 1280*a^9*b^6*c^3 + 3840*a^10*b^4*c^4 - 6144*a^11*b^2*c^5) + (((2*(8960*a^7*b*c^9*d*e^16 - 6*a^2*b^11*c^4*d*e^16 + 137*a^3*b^9*c^5*d*e^16 - 1217*a^4*b^7*c^6*d*e^16 + 5256*a^5*b^5*c^7*d*e^16 - 11024*a^6*b^3*c^8*d*e^16))/(a^6*b^12 + 4096*a^12*c^6 - 24*a^7*b^10*c + 240*a^8*b^8*c^2 - 1280*a^9*b^6*c^3 + 3840*a^10*b^4*c^4 - 6144*a^11*b^2*c^5) + (((2*(5120*a^10*b*c^9*d*e^17 + 2*a^4*b^1

3*c^3*d*e^17 - 36*a^5*b^11*c^4*d*e^17 + 276*a^6*b^9*c^5*d*e^17 - 1216*a^7*b^7*c^6*d*e^17 + 3456*a^8*b^5*c^7*d*e^17 - 6144*a^9*b^3*c^8*d*e^17))/(a^6*b^12 + 4096*a^12*c^6 - 24*a^7*b^10*c + 240*a^8*b^8*c^2 - 1280*a^9*b^6*c^3 + 3840*a^10*b^4*c^4 - 6144*a^11*b^2*c^5) - ((2*b^10*e - 2048*a^5*c^5*e + 320*a^2*b^6*c^2*e - 1280*a^3*b^4*c^3*e + 2560*a^4*b^2*c^4*e - 40*a*b^8*c*e)*(163840*a^13*b*c^9*d*e^18 - 12*a^6*b^15*c^2*d*e^18 + 328*a^7*b^13*c^3*d*e^18 - 3840*a^8*b^11*c^4*d*e^18 + 24960*a^9*b^9*c^5*d*e^18 - 97280*a^10*b^7*c^6*d*e^18 + 227328*a^11*b^5*c^7*d*e^18 - 294912*a^12*b^3*c^8*d*e^18))/((4*a^3*b^10*e^2 - 4096*a^8*c^5*e^2 - 80*a^4*b^8*c*e^2 + 640*a^5*b^6*c^2*e^2 - 2560*a^6*b^4*c^3*e^2 + 5120*a^7*b^2*c^4*e^2)*(a^6*b^12 + 4096*a^12*c^6 - 24*a^7*b^10*c + 240*a^8*b^8*c^2 - 1280*a^9*b^6*c^3 + 3840*a^10*b^4*c^4 - 6144*a^11*b^2*c^5)))*(2*b^10*e - 2048*a^5*c^5*e + 320*a^2*b^6*c^2*e - 1280*a^3*b^4*c^3*e + 2560*a^4*b^2*c^4*e - 40*a*b^8*c*e))/(2*(4*a^3*b^10*e^2 - 4096*a^8*c^5*e^2 - 80*a^4*b^8*c*e^2 + 640*a^5*b^6*c^2*e^2 - 2560*a^6*b^4*c^3*e^2 + 5120*a^7*b^2*c^4*e^2)))*(2*b^10*e - 2048*a^5*c^5*e + 320*a^2*b^6*c^2*e - 1280*a^3*b^4*c^3*e + 2560*a^4*b^2*c^4*e - 40*a*b^8*c*e))/(2*(4*a^3*b^10*e^2 - 4096*a^8*c^5*e^2 - 80*a^4*b^8*c*e^2 + 640*a^5*b^6*c^2*e^2 - 2560*a^6*b^4*c^3*e^2 + 5120*a^7*b^2*c^4*e^2)) - (b*((b*((2*(5120*a^10*b*c^9*d*e^17 + 2*a^4*b^13*c^3*d*e^17 - 36*a^5*b^11*c^4*d*e^17 + 276*a^6*b^9*c^5*d*e^17 - 1216*a^7*b^7*c^6*d*e^17 + 3456*a^8*b^5*c^7*d*e^17 - 6144*a^9*b^3*c^8*d*e^17))/(a^6*b^12 + 4096*a^12*c^6 - 24*a^7*b^10*c + 240*a^8*b^8*c^2 - 1280*a^9*b^6*c^3 + 3840*a^10*b^4*c^4 - 6144*a^11*b^2*c^5) - ((2*b^10*e - 2048*a^5*c^5*e + 320*a^2*b^6*c^2*e - 1280*a^3*b^4*c^3*e + 2560*a^4*b^2*c^4*e - 40*a*b^8*c*e)*(163840*a^13*b*c^9*d*e^18 - 12*a^6*b^15*c^2*d*e^18 + 328*a^7*b^13*c^3*d*e^18 - 3840*a^8*b^11*c^4*d*e^18 + 24960*a^9*b^9*c^5*d*e^18 - 97280*a^10*b^7*c^6*d*e^18 + 227328*a^11*b^5*c^7*d*e^18 - 294912*a^12*b^3*c^8*d*e^18))/((4*a^3*b^10*e^2 - 4096*a^8*c^5*e^2 - 80*a^4*b^8*c*e^2 + 640*a^5*b^6*c^2*e^2 - 2560*a^6*b^4*c^3*e^2 + 5120*a^7*b^2*c^4*e^2)*(a^6*b^12 + 4096*a^12*c^6 - 24*a^7*b^10*c + 240*a^8*b^8*c^2 - 1280*a^9*b^6*c^3 + 3840*a^10*b^4*c^4 - 6144*a^11*b^2*c^5)))*(b^4 + 30*a^2*c^2 - 10*a*b^2*c))/(4*a^3*e*(4*a*c - b^2)^(5/2)) - (b*(b^4 + 30*a^2*c^2 - 10*a*b^2*c)*(2*b^10*e - 2048*a^5*c^5*e + 320*a^2*b^6*c^2*e - 1280*a^3*b^4*c^3*e + 2560*a^4*b^2*c^4*e - 40*a*b^8*c*e)*(163840*a^13*b*c^9*d*e^18 - 12*a^6*b^15*c^2*d*e^18 + 328*a^7*b^13*c^3*d*e^18 - 3840*a^8*b^11*c^4*d*e^18 + 24960*a^9*b^9*c^5*d*e^18 - 97280*a^10*b^7*c^6*d*e^18 + 227328*a^11*b^5*c^7*d*e^18 - 294912*a^12*b^3*c^8*d*e^18))/(4*a^3*e*(4*a*c - b^2)^(5/2)*(4*a^3*b^10*e^2 - 4096*a^8*c^5*e^2 - 80*a^4*b^8*c*e^2 + 640*a^5*b^6*c^2*e^2 - 2560*a^6*b^4*c^3*e^2 + 5120*a^7*b^2*c^4*e^2)*(a^6*b^12 + 4096*a^12*c^6 - 24*a^7*b^10*c + 240*a^8*b^8*c^2 - 1280*a^9*b^6*c^3 + 3840*a^10*b^4*c^4 - 6144*a^11*b^2*c^5)))*(b^4 + 30*a^2*c^2 - 10*a*b^2*c))/(4*a^3*e*(4*a*c - b^2)^(5/2)) + (b^2*(b^4 + 30*a^2*c^2 - 10*a*b^2*c)^2*(2*b^10*e - 2048*a^5*c^5*e + 320*a^2*b^6*c^2*e - 1280*a^3*b^4*c^3*e + 2560*a^4*b^2*c^4*e - 40*a*b^8*c*e)*(163840*a^13*b*c^9*d*e^18 - 12*a^6*b^15*c^2*d*e^18 + 328*a^7*b^13*c^3*d*e^18 - 3840*a^8*b^11*c^4*d*e^18 + 24960*a^9*b^9*c^5*d*e^18 - 97280*a^10*b^7*c^6*d*e^18 + 227328*a^11*b^5*c^7*d*e^18 - 294912*a^12*b^3*c^8*d*e^18))/(16*a^6*e^2*(4*a*c - b^2)^5*(4*a^3*b^10*e^2 - 4096*a^8*c^5*e^2 - 80*a^4*b^8*c*e^2 + 640*a^5*b^6*c^2*e^2 - 2560*a^6*b^4*c^3*e^2 + 5120*a^7*b^2*c^4*e^2)*(a^6*b^12 + 4096*a^12*c^6 - 24*a^7*b^10*c + 240*a^8*b^8*c^2 - 1280*a^9*b^6*c^3 + 3840*a^10*b^4*c^4 - 6144*a^11*b^2*c^5)))*(b^6 - 45*a^3*c^3 + 40*a^2*b^2*c^2 - 11*a*b^4*c))/(8*a^3*c^2*(4*a*c - b^2)^6*(6*b^10 - 6400*a^5*c^5 + 960*a^2*b^6*c^2 - 3850*a^3*b^4*c^3 + 7775*a^4*b^2*c^4 - 120*a*b^8*c)))*(16*a^9*b^12*(4*a*c - b^2)^(15/2) + 65536*a^15*c^6*(4*a*c - b^2)^(15/2) - 384*a^10*b^10*c*(4*a*c - b^2)^(15/2) + 3840*a^11*b^8*c^2*(4*a*c - b^2)^(15/2) - 20480*a^12*b^6*c^3*(4*a*c - b^2)^(15/2) + 61440*a^13*b^4*c^4*(4*a*c - b^2)^(15/2) - 98304*a^14*b^2*c^5*(4*a*c - b^2)^(15/2)))/(b^10*c^2*e^14 - 20*a*b^8*c^3*e^14 + 160*a^2*b^6*c^4*e^14 - 600*a^3*b^4*c^5*e^14 + 900*a^4*b^2*c^6*e^14) + (x^2*((((((b*((5120*a^10*b*c^9*e^18 + 2*a^4*b^13*c^3*e^18 - 36*a^5*b^11*c^4*e^18 + 276*a^6*b^9*c^5*e^18 - 1216*a^7*b^7*c^6*e^18 + 3456*a^8*b^5*c^7*e^18 - 6144*a^9*b^3*c^8*e^18)/(a^6*b^12 + 4096*a^12*c^6 - 24*a^7*b^10*c + 240*a^8*b^8*c^2 - 1280*a^9*b^6*c^3 + 3840*a^10*b^4*c^4 - 6

144*a^11*b^2*c^5) - ((2*b^10*e - 2048*a^5*c^5*e + 320*a^2*b^6*c^2*e - 1280*
a^3*b^4*c^3*e + 2560*a^4*b^2*c^4*e - 40*a*b^8*c*e)*(163840*a^13*b*c^9*e^19
- 12*a^6*b^15*c^2*e^19 + 328*a^7*b^13*c^3*e^19 - 3840*a^8*b^11*c^4*e^19 + 2
4960*a^9*b^9*c^5*e^19 - 97280*a^10*b^7*c^6*e^19 + 227328*a^11*b^5*c^7*e^19
- 294912*a^12*b^3*c^8*e^19))/(2*(4*a^3*b^10*e^2 - 4096*a^8*c^5*e^2 - 80*a^4
*b^8*c*e^2 + 640*a^5*b^6*c^2*e^2 - 2560*a^6*b^4*c^3*e^2 + 5120*a^7*b^2*c^4*
e^2)*(a^6*b^12 + 4096*a^12*c^6 - 24*a^7*b^10*c + 240*a^8*b^8*c^2 - 1280*a^9
*b^6*c^3 + 3840*a^10*b^4*c^4 - 6144*a^11*b^2*c^5)))*(b^4 + 30*a^2*c^2 - 10*
a*b^2*c))/(4*a^3*e*(4*a*c - b^2)^(5/2)) - (b*(b^4 + 30*a^2*c^2 - 10*a*b^2*c
)*(2*b^10*e - 2048*a^5*c^5*e + 320*a^2*b^6*c^2*e - 1280*a^3*b^4*c^3*e + 256
0*a^4*b^2*c^4*e - 40*a*b^8*c*e)*(163840*a^13*b*c^9*e^19 - 12*a^6*b^15*c^2*e
^19 + 328*a^7*b^13*c^3*e^19 - 3840*a^8*b^11*c^4*e^19 + 24960*a^9*b^9*c^5*e^
19 - 97280*a^10*b^7*c^6*e^19 + 227328*a^11*b^5*c^7*e^19 - 294912*a^12*b^3*c
^8*e^19))/(8*a^3*e*(4*a*c - b^2)^(5/2)*(4*a^3*b^10*e^2 - 4096*a^8*c^5*e^2 -
 80*a^4*b^8*c*e^2 + 640*a^5*b^6*c^2*e^2 - 2560*a^6*b^4*c^3*e^2 + 5120*a^7*b
^2*c^4*e^2)*(a^6*b^12 + 4096*a^12*c^6 - 24*a^7*b^10*c + 240*a^8*b^8*c^2 - 1
280*a^9*b^6*c^3 + 3840*a^10*b^4*c^4 - 6144*a^11*b^2*c^5)))*(2*b^10*e - 2048
*a^5*c^5*e + 320*a^2*b^6*c^2*e - 1280*a^3*b^4*c^3*e + 2560*a^4*b^2*c^4*e -
40*a*b^8*c*e))/(2*(4*a^3*b^10*e^2 - 4096*a^8*c^5*e^2 - 80*a^4*b^8*c*e^2 + 6
40*a^5*b^6*c^2*e^2 - 2560*a^6*b^4*c^3*e^2 + 5120*a^7*b^2*c^4*e^2)) + (b*((8
960*a^7*b*c^9*e^17 - 6*a^2*b^11*c^4*e^17 + 137*a^3*b^9*c^5*e^17 - 1217*a^4*
b^7*c^6*e^17 + 5256*a^5*b^5*c^7*e^17 - 11024*a^6*b^3*c^8*e^17)/(a^6*b^12 +
4096*a^12*c^6 - 24*a^7*b^10*c + 240*a^8*b^8*c^2 - 1280*a^9*b^6*c^3 + 3840*a
^10*b^4*c^4 - 6144*a^11*b^2*c^5) + (((5120*a^10*b*c^9*e^18 + 2*a^4*b^13*c^3
*e^18 - 36*a^5*b^11*c^4*e^18 + 276*a^6*b^9*c^5*e^18 - 1216*a^7*b^7*c^6*e^18
 + 3456*a^8*b^5*c^7*e^18 - 6144*a^9*b^3*c^8*e^18)/(a^6*b^12 + 4096*a^12*c^6
 - 24*a^7*b^10*c + 240*a^8*b^8*c^2 - 1280*a^9*b^6*c^3 + 3840*a^10*b^4*c^4 -
 6144*a^11*b^2*c^5) - ((2*b^10*e - 2048*a^5*c^5*e + 320*a^2*b^6*c^2*e - 128
0*a^3*b^4*c^3*e + 2560*a^4*b^2*c^4*e - 40*a*b^8*c*e)*(163840*a^13*b*c^9*e^1
9 - 12*a^6*b^15*c^2*e^19 + 328*a^7*b^13*c^3*e^19 - 3840*a^8*b^11*c^4*e^19 +
 24960*a^9*b^9*c^5*e^19 - 97280*a^10*b^7*c^6*e^19 + 227328*a^11*b^5*c^7*e^1
9 - 294912*a^12*b^3*c^8*e^19))/(2*(4*a^3*b^10*e^2 - 4096*a^8*c^5*e^2 - 80*a
^4*b^8*c*e^2 + 640*a^5*b^6*c^2*e^2 - 2560*a^6*b^4*c^3*e^2 + 5120*a^7*b^2*c^
4*e^2)*(a^6*b^12 + 4096*a^12*c^6 - 24*a^7*b^10*c + 240*a^8*b^8*c^2 - 1280*a
^9*b^6*c^3 + 3840*a^10*b^4*c^4 - 6144*a^11*b^2*c^5)))*(2*b^10*e - 2048*a^5*
c^5*e + 320*a^2*b^6*c^2*e - 1280*a^3*b^4*c^3*e + 2560*a^4*b^2*c^4*e - 40*a*
b^8*c*e))/(2*(4*a^3*b^10*e^2 - 4096*a^8*c^5*e^2 - 80*a^4*b^8*c*e^2 + 640*a^
5*b^6*c^2*e^2 - 2560*a^6*b^4*c^3*e^2 + 5120*a^7*b^2*c^4*e^2)))*(b^4 + 30*a^
2*c^2 - 10*a*b^2*c))/(4*a^3*e*(4*a*c - b^2)^(5/2)) + (b^3*(b^4 + 30*a^2*c^2
 - 10*a*b^2*c)^3*(163840*a^13*b*c^9*e^19 - 12*a^6*b^15*c^2*e^19 + 328*a^7*b
^13*c^3*e^19 - 3840*a^8*b^11*c^4*e^19 + 24960*a^9*b^9*c^5*e^19 - 97280*a^10
*b^7*c^6*e^19 + 227328*a^11*b^5*c^7*e^19 - 294912*a^12*b^3*c^8*e^19))/(64*a
^9*e^3*(4*a*c - b^2)^(15/2)*(a^6*b^12 + 4096*a^12*c^6 - 24*a^7*b^10*c + 240
*a^8*b^8*c^2 - 1280*a^9*b^6*c^3 + 3840*a^10*b^4*c^4 - 6144*a^11*b^2*c^5)))*
(3*b^8 + 160*a^4*c^4 + 180*a^2*b^4*c^2 - 325*a^3*b^2*c^3 - 39*a*b^6*c))/(8*
a^3*c^2*(4*a*c - b^2)^(13/2)*(6*b^10 - 6400*a^5*c^5 + 960*a^2*b^6*c^2 - 385
0*a^3*b^4*c^3 + 7775*a^4*b^2*c^4 - 120*a*b^8*c)) + (3*b*((b^9*c^5*e^16 - 21
*a*b^7*c^6*e^16 + 147*a^2*b^5*c^7*e^16 - 343*a^3*b^3*c^8*e^16)/(a^6*b^12 +
4096*a^12*c^6 - 24*a^7*b^10*c + 240*a^8*b^8*c^2 - 1280*a^9*b^6*c^3 + 3840*a
^10*b^4*c^4 - 6144*a^11*b^2*c^5) + (((8960*a^7*b*c^9*e^17 - 6*a^2*b^11*c^4*
e^17 + 137*a^3*b^9*c^5*e^17 - 1217*a^4*b^7*c^6*e^17 + 5256*a^5*b^5*c^7*e^17
 - 11024*a^6*b^3*c^8*e^17)/(a^6*b^12 + 4096*a^12*c^6 - 24*a^7*b^10*c + 240*
a^8*b^8*c^2 - 1280*a^9*b^6*c^3 + 3840*a^10*b^4*c^4 - 6144*a^11*b^2*c^5) + (
((5120*a^10*b*c^9*e^18 + 2*a^4*b^13*c^3*e^18 - 36*a^5*b^11*c^4*e^18 + 276*a
^6*b^9*c^5*e^18 - 1216*a^7*b^7*c^6*e^18 + 3456*a^8*b^5*c^7*e^18 - 6144*a^9*
b^3*c^8*e^18)/(a^6*b^12 + 4096*a^12*c^6 - 24*a^7*b^10*c + 240*a^8*b^8*c^2 -
 1280*a^9*b^6*c^3 + 3840*a^10*b^4*c^4 - 6144*a^11*b^2*c^5) - ((2*b^10*e - 2
048*a^5*c^5*e + 320*a^2*b^6*c^2*e - 1280*a^3*b^4*c^3*e + 2560*a^4*b^2*c^4*e
 - 40*a*b^8*c*e)*(163840*a^13*b*c^9*e^19 - 12*a^6*b^15*c^2*e^19 + 328*a^7*b

^13*c^3*e^19 - 3840*a^8*b^11*c^4*e^19 + 24960*a^9*b^9*c^5*e^19 - 97280*a^10
*b^7*c^6*e^19 + 227328*a^11*b^5*c^7*e^19 - 294912*a^12*b^3*c^8*e^19))/(2*(4
*a^3*b^10*e^2 - 4096*a^8*c^5*e^2 - 80*a^4*b^8*c*e^2 + 640*a^5*b^6*c^2*e^2 -
2560*a^6*b^4*c^3*e^2 + 5120*a^7*b^2*c^4*e^2)*(a^6*b^12 + 4096*a^12*c^6 - 2
4*a^7*b^10*c + 240*a^8*b^8*c^2 - 1280*a^9*b^6*c^3 + 3840*a^10*b^4*c^4 - 614
4*a^11*b^2*c^5)))*(2*b^10*e - 2048*a^5*c^5*e + 320*a^2*b^6*c^2*e - 1280*a^3
*b^4*c^3*e + 2560*a^4*b^2*c^4*e - 40*a*b^8*c*e))/(2*(4*a^3*b^10*e^2 - 4096*
a^8*c^5*e^2 - 80*a^4*b^8*c*e^2 + 640*a^5*b^6*c^2*e^2 - 2560*a^6*b^4*c^3*e^2
+ 5120*a^7*b^2*c^4*e^2)))*(2*b^10*e - 2048*a^5*c^5*e + 320*a^2*b^6*c^2*e -
1280*a^3*b^4*c^3*e + 2560*a^4*b^2*c^4*e - 40*a*b^8*c*e))/(2*(4*a^3*b^10*e^
2 - 4096*a^8*c^5*e^2 - 80*a^4*b^8*c*e^2 + 640*a^5*b^6*c^2*e^2 - 2560*a^6*b^
4*c^3*e^2 + 5120*a^7*b^2*c^4*e^2)) - (b*((b*((5120*a^10*b*c^9*e^18 + 2*a^4*
b^13*c^3*e^18 - 36*a^5*b^11*c^4*e^18 + 276*a^6*b^9*c^5*e^18 - 1216*a^7*b^7*
c^6*e^18 + 3456*a^8*b^5*c^7*e^18 - 6144*a^9*b^3*c^8*e^18)/(a^6*b^12 + 4096*
a^12*c^6 - 24*a^7*b^10*c + 240*a^8*b^8*c^2 - 1280*a^9*b^6*c^3 + 3840*a^10*b
^4*c^4 - 6144*a^11*b^2*c^5) - ((2*b^10*e - 2048*a^5*c^5*e + 320*a^2*b^6*c^2
*e - 1280*a^3*b^4*c^3*e + 2560*a^4*b^2*c^4*e - 40*a*b^8*c*e)*(163840*a^13*b
*c^9*e^19 - 12*a^6*b^15*c^2*e^19 + 328*a^7*b^13*c^3*e^19 - 3840*a^8*b^11*c^
4*e^19 + 24960*a^9*b^9*c^5*e^19 - 97280*a^10*b^7*c^6*e^19 + 227328*a^11*b^5
*c^7*e^19 - 294912*a^12*b^3*c^8*e^19))/(2*(4*a^3*b^10*e^2 - 4096*a^8*c^5*e^
2 - 80*a^4*b^8*c*e^2 + 640*a^5*b^6*c^2*e^2 - 2560*a^6*b^4*c^3*e^2 + 5120*a^
7*b^2*c^4*e^2)*(a^6*b^12 + 4096*a^12*c^6 - 24*a^7*b^10*c + 240*a^8*b^8*c^2
- 1280*a^9*b^6*c^3 + 3840*a^10*b^4*c^4 - 6144*a^11*b^2*c^5)))*(b^4 + 30*a^2
*c^2 - 10*a*b^2*c))/(4*a^3*e*(4*a*c - b^2)^(5/2)) - (b*(b^4 + 30*a^2*c^2 -
10*a*b^2*c)*(2*b^10*e - 2048*a^5*c^5*e + 320*a^2*b^6*c^2*e - 1280*a^3*b^4*c
^3*e + 2560*a^4*b^2*c^4*e - 40*a*b^8*c*e)*(163840*a^13*b*c^9*e^19 - 12*a^6*
b^15*c^2*e^19 + 328*a^7*b^13*c^3*e^19 - 3840*a^8*b^11*c^4*e^19 + 24960*a^9*
b^9*c^5*e^19 - 97280*a^10*b^7*c^6*e^19 + 227328*a^11*b^5*c^7*e^19 - 294912*
a^12*b^3*c^8*e^19))/(8*a^3*e*(4*a*c - b^2)^(5/2)*(4*a^3*b^10*e^2 - 4096*a^8
*c^5*e^2 - 80*a^4*b^8*c*e^2 + 640*a^5*b^6*c^2*e^2 - 2560*a^6*b^4*c^3*e^2 +
5120*a^7*b^2*c^4*e^2)*(a^6*b^12 + 4096*a^12*c^6 - 24*a^7*b^10*c + 240*a^8*b
^8*c^2 - 1280*a^9*b^6*c^3 + 3840*a^10*b^4*c^4 - 6144*a^11*b^2*c^5)))*(b^4 +
30*a^2*c^2 - 10*a*b^2*c))/(4*a^3*e*(4*a*c - b^2)^(5/2)) + (b^2*(b^4 + 30*a
^2*c^2 - 10*a*b^2*c)^2*(2*b^10*e - 2048*a^5*c^5*e + 320*a^2*b^6*c^2*e - 128
0*a^3*b^4*c^3*e + 2560*a^4*b^2*c^4*e - 40*a*b^8*c*e)*(163840*a^13*b*c^9*e^1
9 - 12*a^6*b^15*c^2*e^19 + 328*a^7*b^13*c^3*e^19 - 3840*a^8*b^11*c^4*e^19 +
24960*a^9*b^9*c^5*e^19 - 97280*a^10*b^7*c^6*e^19 + 227328*a^11*b^5*c^7*e^1
9 - 294912*a^12*b^3*c^8*e^19))/(32*a^6*e^2*(4*a*c - b^2)^5*(4*a^3*b^10*e^2
- 4096*a^8*c^5*e^2 - 80*a^4*b^8*c*e^2 + 640*a^5*b^6*c^2*e^2 - 2560*a^6*b^4*
c^3*e^2 + 5120*a^7*b^2*c^4*e^2)*(a^6*b^12 + 4096*a^12*c^6 - 24*a^7*b^10*c +
240*a^8*b^8*c^2 - 1280*a^9*b^6*c^3 + 3840*a^10*b^4*c^4 - 6144*a^11*b^2*c^5
)))*(b^6 - 45*a^3*c^3 + 40*a^2*b^2*c^2 - 11*a*b^4*c))/(8*a^3*c^2*(4*a*c - b
^2)^6*(6*b^10 - 6400*a^5*c^5 + 960*a^2*b^6*c^2 - 3850*a^3*b^4*c^3 + 7775*a^
4*b^2*c^4 - 120*a*b^8*c)))*(16*a^9*b^12*(4*a*c - b^2)^(15/2) + 65536*a^15*c
^6*(4*a*c - b^2)^(15/2) - 384*a^10*b^10*c*(4*a*c - b^2)^(15/2) + 3840*a^11*
b^8*c^2*(4*a*c - b^2)^(15/2) - 20480*a^12*b^6*c^3*(4*a*c - b^2)^(15/2) + 61
440*a^13*b^4*c^4*(4*a*c - b^2)^(15/2) - 98304*a^14*b^2*c^5*(4*a*c - b^2)^(1
5/2)))/(b^10*c^2*e^14 - 20*a*b^8*c^3*e^14 + 160*a^2*b^6*c^4*e^14 - 600*a^3*
b^4*c^5*e^14 + 900*a^4*b^2*c^6*e^14) - (((b*((4*a^2*b^12*c^3*e^15 - 93*a^3*
b^10*c^4*e^15 + 854*a^4*b^8*c^5*e^15 - 3889*a^5*b^6*c^6*e^15 + 8808*a^6*b^4
*c^7*e^15 - 7952*a^7*b^2*c^8*e^15 + 6*a^2*b^11*c^4*d^2*e^15 - 137*a^3*b^9*c
^5*d^2*e^15 + 1217*a^4*b^7*c^6*d^2*e^15 - 5256*a^5*b^5*c^7*d^2*e^15 + 11024
*a^6*b^3*c^8*d^2*e^15 - 8960*a^7*b*c^9*d^2*e^15)/(a^6*b^12 + 4096*a^12*c^6
- 24*a^7*b^10*c + 240*a^8*b^8*c^2 - 1280*a^9*b^6*c^3 + 3840*a^10*b^4*c^4 -
6144*a^11*b^2*c^5) - (((4*a^4*b^14*c^2*e^16 - 100*a^5*b^12*c^3*e^16 + 1052*
a^6*b^10*c^4*e^16 - 5952*a^7*b^8*c^5*e^16 + 19072*a^8*b^6*c^6*e^16 - 32768*
a^9*b^4*c^7*e^16 + 23552*a^10*b^2*c^8*e^16 + 2*a^4*b^13*c^3*d^2*e^16 - 36*a
^5*b^11*c^4*d^2*e^16 + 276*a^6*b^9*c^5*d^2*e^16 - 1216*a^7*b^7*c^6*d^2*e^16
+ 3456*a^8*b^5*c^7*d^2*e^16 - 6144*a^9*b^3*c^8*d^2*e^16 + 5120*a^10*b*c^9*

d^2*e^16)/(a^6*b^12 + 4096*a^12*c^6 - 24*a^7*b^10*c + 240*a^8*b^8*c^2 - 128
0*a^9*b^6*c^3 + 3840*a^10*b^4*c^4 - 6144*a^11*b^2*c^5) + ((2*b^10*e - 2048*
a^5*c^5*e + 320*a^2*b^6*c^2*e - 1280*a^3*b^4*c^3*e + 2560*a^4*b^2*c^4*e - 4
0*a*b^8*c*e)*(4*a^7*b^14*c^2*e^17 - 96*a^8*b^12*c^3*e^17 + 960*a^9*b^10*c^4
*e^17 - 5120*a^10*b^8*c^5*e^17 + 15360*a^11*b^6*c^6*e^17 - 24576*a^12*b^4*c
^7*e^17 + 16384*a^13*b^2*c^8*e^17 + 12*a^6*b^15*c^2*d^2*e^17 - 328*a^7*b^13
*c^3*d^2*e^17 + 3840*a^8*b^11*c^4*d^2*e^17 - 24960*a^9*b^9*c^5*d^2*e^17 + 9
7280*a^10*b^7*c^6*d^2*e^17 - 227328*a^11*b^5*c^7*d^2*e^17 + 294912*a^12*b^3
*c^8*d^2*e^17 - 163840*a^13*b*c^9*d^2*e^17))/(2*(4*a^3*b^10*e^2 - 4096*a^8*
c^5*e^2 - 80*a^4*b^8*c*e^2 + 640*a^5*b^6*c^2*e^2 - 2560*a^6*b^4*c^3*e^2 + 5
120*a^7*b^2*c^4*e^2)*(a^6*b^12 + 4096*a^12*c^6 - 24*a^7*b^10*c + 240*a^8*b^
8*c^2 - 1280*a^9*b^6*c^3 + 3840*a^10*b^4*c^4 - 6144*a^11*b^2*c^5)))*(2*b^10
*e - 2048*a^5*c^5*e + 320*a^2*b^6*c^2*e - 1280*a^3*b^4*c^3*e + 2560*a^4*b^2
*c^4*e - 40*a*b^8*c*e))/(2*(4*a^3*b^10*e^2 - 4096*a^8*c^5*e^2 - 80*a^4*b^8*
c*e^2 + 640*a^5*b^6*c^2*e^2 - 2560*a^6*b^4*c^3*e^2 + 5120*a^7*b^2*c^4*e^2))
)*(b^4 + 30*a^2*c^2 - 10*a*b^2*c))/(4*a^3*e*(4*a*c - b^2)^(5/2)) - (((b*((4
*a^4*b^14*c^2*e^16 - 100*a^5*b^12*c^3*e^16 + 1052*a^6*b^10*c^4*e^16 - 5952*
a^7*b^8*c^5*e^16 + 19072*a^8*b^6*c^6*e^16 - 32768*a^9*b^4*c^7*e^16 + 23552*
a^10*b^2*c^8*e^16 + 2*a^4*b^13*c^3*d^2*e^16 - 36*a^5*b^11*c^4*d^2*e^16 + 27
6*a^6*b^9*c^5*d^2*e^16 - 1216*a^7*b^7*c^6*d^2*e^16 + 3456*a^8*b^5*c^7*d^2*e
^16 - 6144*a^9*b^3*c^8*d^2*e^16 + 5120*a^10*b*c^9*d^2*e^16)/(a^6*b^12 + 409
6*a^12*c^6 - 24*a^7*b^10*c + 240*a^8*b^8*c^2 - 1280*a^9*b^6*c^3 + 3840*a^10
*b^4*c^4 - 6144*a^11*b^2*c^5) + ((2*b^10*e - 2048*a^5*c^5*e + 320*a^2*b^6*c
^2*e - 1280*a^3*b^4*c^3*e + 2560*a^4*b^2*c^4*e - 40*a*b^8*c*e)*(4*a^7*b^14*
c^2*e^17 - 96*a^8*b^12*c^3*e^17 + 960*a^9*b^10*c^4*e^17 - 5120*a^10*b^8*c^5
*e^17 + 15360*a^11*b^6*c^6*e^17 - 24576*a^12*b^4*c^7*e^17 + 16384*a^13*b^2*
c^8*e^17 + 12*a^6*b^15*c^2*d^2*e^17 - 328*a^7*b^13*c^3*d^2*e^17 + 3840*a^8*
b^11*c^4*d^2*e^17 - 24960*a^9*b^9*c^5*d^2*e^17 + 97280*a^10*b^7*c^6*d^2*e^1
7 - 227328*a^11*b^5*c^7*d^2*e^17 + 294912*a^12*b^3*c^8*d^2*e^17 - 163840*a^
13*b*c^9*d^2*e^17))/(2*(4*a^3*b^10*e^2 - 4096*a^8*c^5*e^2 - 80*a^4*b^8*c*e^
2 + 640*a^5*b^6*c^2*e^2 - 2560*a^6*b^4*c^3*e^2 + 5120*a^7*b^2*c^4*e^2)*(a^6
*b^12 + 4096*a^12*c^6 - 24*a^7*b^10*c + 240*a^8*b^8*c^2 - 1280*a^9*b^6*c^3
+ 3840*a^10*b^4*c^4 - 6144*a^11*b^2*c^5)))*(b^4 + 30*a^2*c^2 - 10*a*b^2*c))
/(4*a^3*e*(4*a*c - b^2)^(5/2)) + (b*(b^4 + 30*a^2*c^2 - 10*a*b^2*c)*(2*b^10
*e - 2048*a^5*c^5*e + 320*a^2*b^6*c^2*e - 1280*a^3*b^4*c^3*e + 2560*a^4*b^2
*c^4*e - 40*a*b^8*c*e)*(4*a^7*b^14*c^2*e^17 - 96*a^8*b^12*c^3*e^17 + 960*a^
9*b^10*c^4*e^17 - 5120*a^10*b^8*c^5*e^17 + 15360*a^11*b^6*c^6*e^17 - 24576*
a^12*b^4*c^7*e^17 + 16384*a^13*b^2*c^8*e^17 + 12*a^6*b^15*c^2*d^2*e^17 - 32
8*a^7*b^13*c^3*d^2*e^17 + 3840*a^8*b^11*c^4*d^2*e^17 - 24960*a^9*b^9*c^5*d^
2*e^17 + 97280*a^10*b^7*c^6*d^2*e^17 - 227328*a^11*b^5*c^7*d^2*e^17 + 29491
2*a^12*b^3*c^8*d^2*e^17 - 163840*a^13*b*c^9*d^2*e^17))/(8*a^3*e*(4*a*c - b^
2)^(5/2)*(4*a^3*b^10*e^2 - 4096*a^8*c^5*e^2 - 80*a^4*b^8*c*e^2 + 640*a^5*b^
6*c^2*e^2 - 2560*a^6*b^4*c^3*e^2 + 5120*a^7*b^2*c^4*e^2)*(a^6*b^12 + 4096*a
^12*c^6 - 24*a^7*b^10*c + 240*a^8*b^8*c^2 - 1280*a^9*b^6*c^3 + 3840*a^10*b^
4*c^4 - 6144*a^11*b^2*c^5)))*(2*b^10*e - 2048*a^5*c^5*e + 320*a^2*b^6*c^2*e
 - 1280*a^3*b^4*c^3*e + 2560*a^4*b^2*c^4*e - 40*a*b^8*c*e))/(2*(4*a^3*b^10*
e^2 - 4096*a^8*c^5*e^2 - 80*a^4*b^8*c*e^2 + 640*a^5*b^6*c^2*e^2 - 2560*a^6*
b^4*c^3*e^2 + 5120*a^7*b^2*c^4*e^2)) + (b^3*(b^4 + 30*a^2*c^2 - 10*a*b^2*c)
^3*(4*a^7*b^14*c^2*e^17 - 96*a^8*b^12*c^3*e^17 + 960*a^9*b^10*c^4*e^17 - 51
20*a^10*b^8*c^5*e^17 + 15360*a^11*b^6*c^6*e^17 - 24576*a^12*b^4*c^7*e^17 +
16384*a^13*b^2*c^8*e^17 + 12*a^6*b^15*c^2*d^2*e^17 - 328*a^7*b^13*c^3*d^2*e
^17 + 3840*a^8*b^11*c^4*d^2*e^17 - 24960*a^9*b^9*c^5*d^2*e^17 + 97280*a^10*
b^7*c^6*d^2*e^17 - 227328*a^11*b^5*c^7*d^2*e^17 + 294912*a^12*b^3*c^8*d^2*e
^17 - 163840*a^13*b*c^9*d^2*e^17))/(64*a^9*e^3*(4*a*c - b^2)^(15/2)*(a^6*b^
12 + 4096*a^12*c^6 - 24*a^7*b^10*c + 240*a^8*b^8*c^2 - 1280*a^9*b^6*c^3 + 3
840*a^10*b^4*c^4 - 6144*a^11*b^2*c^5)))*(3*b^8 + 160*a^4*c^4 + 180*a^2*b^4*
c^2 - 325*a^3*b^2*c^3 - 39*a*b^6*c)*(16*a^9*b^12*(4*a*c - b^2)^(15/2) + 655
36*a^15*c^6*(4*a*c - b^2)^(15/2) - 384*a^10*b^10*c*(4*a*c - b^2)^(15/2) + 3
840*a^11*b^8*c^2*(4*a*c - b^2)^(15/2) - 20480*a^12*b^6*c^3*(4*a*c - b^2)^(1

5/2) + 61440*a^13*b^4*c^4*(4*a*c - b^2)^(15/2) - 98304*a^14*b^2*c^5*(4*a*c - b^2)^(15/2)))/(8*a^3*c^2*(4*a*c - b^2)^(13/2)*(b^10*c^2*e^14 - 20*a*b^8*c^3*e^14 + 160*a^2*b^6*c^4*e^14 - 600*a^3*b^4*c^5*e^14 + 900*a^4*b^2*c^6*e^14)*(6*b^10 - 6400*a^5*c^5 + 960*a^2*b^6*c^2 - 3850*a^3*b^4*c^3 + 7775*a^4*b^2*c^4 - 120*a*b^8*c)) - (3*b*((((4*a^2*b^12*c^3*e^15 - 93*a^3*b^10*c^4*e^15 + 854*a^4*b^8*c^5*e^15 - 3889*a^5*b^6*c^6*e^15 + 8808*a^6*b^4*c^7*e^15 - 7952*a^7*b^2*c^8*e^15 + 6*a^2*b^11*c^4*d^2*e^15 - 137*a^3*b^9*c^5*d^2*e^15 + 1217*a^4*b^7*c^6*d^2*e^15 - 5256*a^5*b^5*c^7*d^2*e^15 + 11024*a^6*b^3*c^8*d^2*e^15 - 8960*a^7*b*c^9*d^2*e^15)/(a^6*b^12 + 4096*a^12*c^6 - 24*a^7*b^10*c + 240*a^8*b^8*c^2 - 1280*a^9*b^6*c^3 + 3840*a^10*b^4*c^4 - 6144*a^11*b^2*c^5) - (((4*a^4*b^14*c^2*e^16 - 100*a^5*b^12*c^3*e^16 + 1052*a^6*b^10*c^4*e^16 - 5952*a^7*b^8*c^5*e^16 + 19072*a^8*b^6*c^6*e^16 - 32768*a^9*b^4*c^7*e^16 + 23552*a^10*b^2*c^8*e^16 + 2*a^4*b^13*c^3*d^2*e^16 - 36*a^5*b^11*c^4*d^2*e^16 + 276*a^6*b^9*c^5*d^2*e^16 - 1216*a^7*b^7*c^6*d^2*e^16 + 3456*a^8*b^5*c^7*d^2*e^16 - 6144*a^9*b^3*c^8*d^2*e^16 + 5120*a^10*b*c^9*d^2*e^16)/(a^6*b^12 + 4096*a^12*c^6 - 24*a^7*b^10*c + 240*a^8*b^8*c^2 - 1280*a^9*b^6*c^3 + 3840*a^10*b^4*c^4 - 6144*a^11*b^2*c^5) + ((2*b^10*e - 2048*a^5*c^5*e + 320*a^2*b^6*c^2*e - 1280*a^3*b^4*c^3*e + 2560*a^4*b^2*c^4*e - 40*a*b^8*c*e)*(4*a^7*b^14*c^2*e^17 - 96*a^8*b^12*c^3*e^17 + 960*a^9*b^10*c^4*e^17 - 5120*a^10*b^8*c^5*e^17 + 15360*a^11*b^6*c^6*e^17 - 24576*a^12*b^4*c^7*e^17 + 16384*a^13*b^2*c^8*e^17 + 12*a^6*b^15*c^2*d^2*e^17 - 328*a^7*b^13*c^3*d^2*e^17 + 3840*a^8*b^11*c^4*d^2*e^17 - 24960*a^9*b^9*c^5*d^2*e^17 + 97280*a^10*b^7*c^6*d^2*e^17 - 227328*a^11*b^5*c^7*d^2*e^17 + 294912*a^12*b^3*c^8*d^2*e^17 - 163840*a^13*b*c^9*d^2*e^17))/(2*(4*a^3*b^10*e^2 - 4096*a^8*c^5*e^2 - 80*a^4*b^8*c*e^2 + 640*a^5*b^6*c^2*e^2 - 2560*a^6*b^4*c^3*e^2 + 5120*a^7*b^2*c^4*e^2)*(a^6*b^12 + 4096*a^12*c^6 - 24*a^7*b^10*c + 240*a^8*b^8*c^2 - 1280*a^9*b^6*c^3 + 3840*a^10*b^4*c^4 - 6144*a^11*b^2*c^5)))*(2*b^10*e - 2048*a^5*c^5*e + 320*a^2*b^6*c^2*e - 1280*a^3*b^4*c^3*e + 2560*a^4*b^2*c^4*e - 40*a*b^8*c*e))/(2*(4*a^3*b^10*e^2 - 4096*a^8*c^5*e^2 - 80*a^4*b^8*c*e^2 + 640*a^5*b^6*c^2*e^2 - 2560*a^6*b^4*c^3*e^2 + 5120*a^7*b^2*c^4*e^2)))*(2*b^10*e - 2048*a^5*c^5*e + 320*a^2*b^6*c^2*e - 1280*a^3*b^4*c^3*e + 2560*a^4*b^2*c^4*e - 40*a*b^8*c*e))/(2*(4*a^3*b^10*e^2 - 4096*a^8*c^5*e^2 - 80*a^4*b^8*c*e^2 + 640*a^5*b^6*c^2*e^2 - 2560*a^6*b^4*c^3*e^2 + 5120*a^7*b^2*c^4*e^2)) - (b^10*c^4*e^14 - 22*a*b^8*c^5*e^14 + 177*a^2*b^6*c^6*e^14 - 616*a^3*b^4*c^7*e^14 + 784*a^4*b^2*c^8*e^14 + b^9*c^5*d^2*e^14 + 147*a^2*b^5*c^7*d^2*e^14 - 343*a^3*b^3*c^8*d^2*e^14 - 21*a*b^7*c^6*d^2*e^14)/(a^6*b^12 + 4096*a^12*c^6 - 24*a^7*b^10*c + 240*a^8*b^8*c^2 - 1280*a^9*b^6*c^3 + 3840*a^10*b^4*c^4 - 6144*a^11*b^2*c^5) + (b*((b*((4*a^4*b^14*c^2*e^16 - 100*a^5*b^12*c^3*e^16 + 1052*a^6*b^10*c^4*e^16 - 5952*a^7*b^8*c^5*e^16 + 19072*a^8*b^6*c^6*e^16 - 32768*a^9*b^4*c^7*e^16 + 23552*a^10*b^2*c^8*e^16 + 2*a^4*b^13*c^3*d^2*e^16 - 36*a^5*b^11*c^4*d^2*e^16 + 276*a^6*b^9*c^5*d^2*e^16 - 1216*a^7*b^7*c^6*d^2*e^16 + 3456*a^8*b^5*c^7*d^2*e^16 - 6144*a^9*b^3*c^8*d^2*e^16 + 5120*a^10*b*c^9*d^2*e^16)/(a^6*b^12 + 4096*a^12*c^6 - 24*a^7*b^10*c + 240*a^8*b^8*c^2 - 1280*a^9*b^6*c^3 + 3840*a^10*b^4*c^4 - 6144*a^11*b^2*c^5) + ((2*b^10*e - 2048*a^5*c^5*e + 320*a^2*b^6*c^2*e - 1280*a^3*b^4*c^3*e + 2560*a^4*b^2*c^4*e - 40*a*b^8*c*e)*(4*a^7*b^14*c^2*e^17 - 96*a^8*b^12*c^3*e^17 + 960*a^9*b^10*c^4*e^17 - 5120*a^10*b^8*c^5*e^17 + 15360*a^11*b^6*c^6*e^17 - 24576*a^12*b^4*c^7*e^17 + 16384*a^13*b^2*c^8*e^17 + 12*a^6*b^15*c^2*d^2*e^17 - 328*a^7*b^13*c^3*d^2*e^17 + 3840*a^8*b^11*c^4*d^2*e^17 - 24960*a^9*b^9*c^5*d^2*e^17 + 97280*a^10*b^7*c^6*d^2*e^17 - 227328*a^11*b^5*c^7*d^2*e^17 + 294912*a^12*b^3*c^8*d^2*e^17 - 163840*a^13*b*c^9*d^2*e^17))/(2*(4*a^3*b^10*e^2 - 4096*a^8*c^5*e^2 - 80*a^4*b^8*c*e^2 + 640*a^5*b^6*c^2*e^2 - 2560*a^6*b^4*c^3*e^2 + 5120*a^7*b^2*c^4*e^2)*(a^6*b^12 + 4096*a^12*c^6 - 24*a^7*b^10*c + 240*a^8*b^8*c^2 - 1280*a^9*b^6*c^3 + 3840*a^10*b^4*c^4 - 6144*a^11*b^2*c^5)))*(b^4 + 30*a^2*c^2 - 10*a*b^2*c))/(4*a^3*e*(4*a*c - b^2)^(5/2)) + (b*(b^4 + 30*a^2*c^2 - 10*a*b^2*c)*(2*b^10*e - 2048*a^5*c^5*e + 320*a^2*b^6*c^2*e - 1280*a^3*b^4*c^3*e + 2560*a^4*b^2*c^4*e - 40*a*b^8*c*e)*(4*a^7*b^14*c^2*e^17 - 96*a^8*b^12*c^3*e^17 + 960*a^9*b^10*c^4*e^17 - 5120*a^10*b^8*c^5*e^17 + 15360*a^11*b^6*c^6*e^17 - 24576*a^12*b^4*c^7*e^17 + 16384*a^13*b^2*c^8*e^

```
17 + 12*a^6*b^15*c^2*d^2*e^17 - 328*a^7*b^13*c^3*d^2*e^17 + 3840*a^8*b^11*c
^4*d^2*e^17 - 24960*a^9*b^9*c^5*d^2*e^17 + 97280*a^10*b^7*c^6*d^2*e^17 - 22
7328*a^11*b^5*c^7*d^2*e^17 + 294912*a^12*b^3*c^8*d^2*e^17 - 163840*a^13*b*c
^9*d^2*e^17))/(8*a^3*e*(4*a*c - b^2)^(5/2)*(4*a^3*b^10*e^2 - 4096*a^8*c^5*e
^2 - 80*a^4*b^8*c*e^2 + 640*a^5*b^6*c^2*e^2 - 2560*a^6*b^4*c^3*e^2 + 5120*a
^7*b^2*c^4*e^2)*(a^6*b^12 + 4096*a^12*c^6 - 24*a^7*b^10*c + 240*a^8*b^8*c^2
 - 1280*a^9*b^6*c^3 + 3840*a^10*b^4*c^4 - 6144*a^11*b^2*c^5)))*(b^4 + 30*a^
2*c^2 - 10*a*b^2*c))/(4*a^3*e*(4*a*c - b^2)^(5/2)) + (b^2*(b^4 + 30*a^2*c^2
 - 10*a*b^2*c)^2*(2*b^10*e - 2048*a^5*c^5*e + 320*a^2*b^6*c^2*e - 1280*a^3*
b^4*c^3*e + 2560*a^4*b^2*c^4*e - 40*a*b^8*c*e)*(4*a^7*b^14*c^2*e^17 - 96*a^
8*b^12*c^3*e^17 + 960*a^9*b^10*c^4*e^17 - 5120*a^10*b^8*c^5*e^17 + 15360*a^
11*b^6*c^6*e^17 - 24576*a^12*b^4*c^7*e^17 + 16384*a^13*b^2*c^8*e^17 + 12*a^
6*b^15*c^2*d^2*e^17 - 328*a^7*b^13*c^3*d^2*e^17 + 3840*a^8*b^11*c^4*d^2*e^1
7 - 24960*a^9*b^9*c^5*d^2*e^17 + 97280*a^10*b^7*c^6*d^2*e^17 - 227328*a^11*
b^5*c^7*d^2*e^17 + 294912*a^12*b^3*c^8*d^2*e^17 - 163840*a^13*b*c^9*d^2*e^1
7))/(32*a^6*e^2*(4*a*c - b^2)^5*(4*a^3*b^10*e^2 - 4096*a^8*c^5*e^2 - 80*a^4
*b^8*c*e^2 + 640*a^5*b^6*c^2*e^2 - 2560*a^6*b^4*c^3*e^2 + 5120*a^7*b^2*c^4*
e^2)*(a^6*b^12 + 4096*a^12*c^6 - 24*a^7*b^10*c + 240*a^8*b^8*c^2 - 1280*a^9
*b^6*c^3 + 3840*a^10*b^4*c^4 - 6144*a^11*b^2*c^5)))*(b^6 - 45*a^3*c^3 + 40*
a^2*b^2*c^2 - 11*a*b^4*c)*(16*a^9*b^12*(4*a*c - b^2)^(15/2) + 65536*a^15*c^
6*(4*a*c - b^2)^(15/2) - 384*a^10*b^10*c*(4*a*c - b^2)^(15/2) + 3840*a^11*b
^8*c^2*(4*a*c - b^2)^(15/2) - 20480*a^12*b^6*c^3*(4*a*c - b^2)^(15/2) + 614
40*a^13*b^4*c^4*(4*a*c - b^2)^(15/2) - 98304*a^14*b^2*c^5*(4*a*c - b^2)^(15
/2)))/(8*a^3*c^2*(4*a*c - b^2)^6*(b^10*c^2*e^14 - 20*a*b^8*c^3*e^14 + 160*a
^2*b^6*c^4*e^14 - 600*a^3*b^4*c^5*e^14 + 900*a^4*b^2*c^6*e^14)*(6*b^10 - 64
00*a^5*c^5 + 960*a^2*b^6*c^2 - 3850*a^3*b^4*c^3 + 7775*a^4*b^2*c^4 - 120*a*
b^8*c)))*(b^4 + 30*a^2*c^2 - 10*a*b^2*c))/(2*a^3*e*(4*a*c - b^2)^(5/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(a+b*(e*x+d)**2+c*(e*x+d)**4)**3,x)
```

```
[Out] Timed out
```

3.523 $\int \frac{1}{(d+ex)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)^3}\,dx$

Optimal. Leaf size=484

$$-\frac{3\left(5b^2-12ac\right)\left(b^2-5ac\right)}{8a^3e\left(b^2-4ac\right)^2(d+ex)}+\frac{36a^2c^2+bc\left(5b^2-32ac\right)(d+ex)^2-35ab^2c+5b^4}{8a^2e\left(b^2-4ac\right)^2(d+ex)\left(a+b(d+ex)^2+c(d+ex)^4\right)}-\frac{3\sqrt{c}\left(\frac{b\left(124a^2c^2-47ab^2c+5b^4\right)}{\sqrt{b^2-4ac}}\right.}{8\sqrt{2}\,a^3e}$$

Rubi [A] time = 1.23, antiderivative size = 484, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1142, 1121, 1277, 1281, 1166, 205}

$$\frac{36a^2c^2+bc\left(5b^2-32ac\right)(d+ex)^2-35ab^2c+5b^4}{8a^2e\left(b^2-4ac\right)^2(d+ex)\left(a+b(d+ex)^2+c(d+ex)^4\right)}-\frac{3\sqrt{c}\left(\frac{b\left(124a^2c^2-47ab^2c+5b^4\right)}{\sqrt{b^2-4ac}}+\left(5b^2-12ac\right)\left(b^2-5ac\right)\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{8\sqrt{2}\,a^3e\left(b^2-4ac\right)^2\sqrt{b-\sqrt{b^2-4ac}}}-\frac{3\sqrt{c}\left(\left(5b^2-12ac\right)\left(b^2-5ac\right)-\frac{124a^2bc^2-47ab^3c+5b^5}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{8\sqrt{2}\,a^3e\left(b^2-4ac\right)^2\sqrt{\sqrt{b^2-4ac}+b}}-\frac{3\left(5b^2-12ac\right)\left(b^2-5ac\right)}{8a^3e\left(b^2-4ac\right)^2(d+ex)}+\frac{-2ac+b^2+bc(d+ex)^2}{4ae\left(b^2-4ac\right)(d+ex)\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)^2*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3),x]
```

```
[Out] (-3*(5*b^2 - 12*a*c)*(b^2 - 5*a*c))/(8*a^3*(b^2 - 4*a*c)^2*e*(d + e*x)) + (
b^2 - 2*a*c + b*c*(d + e*x)^2)/(4*a*(b^2 - 4*a*c)*e*(d + e*x)*(a + b*(d + e
*x)^2 + c*(d + e*x)^4)^2) + (5*b^4 - 35*a*b^2*c + 36*a^2*c^2 + b*c*(5*b^2 -
 32*a*c)*(d + e*x)^2)/(8*a^2*(b^2 - 4*a*c)^2*e*(d + e*x)*(a + b*(d + e*x)^2
 + c*(d + e*x)^4)) - (3*Sqrt[c]*((5*b^2 - 12*a*c)*(b^2 - 5*a*c) + (b*(5*b^4
 - 47*a*b^2*c + 124*a^2*c^2))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d
 + e*x))/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(8*Sqrt[2]*a^3*(b^2 - 4*a*c)^2*Sqrt[
b - Sqrt[b^2 - 4*a*c]]*e) - (3*Sqrt[c]*((5*b^2 - 12*a*c)*(b^2 - 5*a*c) - (5
*b^5 - 47*a*b^3*c + 124*a^2*b*c^2)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[
c]*(d + e*x))/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(8*Sqrt[2]*a^3*(b^2 - 4*a*c)^2*
Sqrt[b + Sqrt[b^2 - 4*a*c]]*e)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1121

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> -Simp[((d*x)^(m + 1)*(b^2 - 2*a*c + b*c*x^2)*(a + b*x^2 + c*x^4)^(p + 1
))/(2*a*d*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)),
Int[(d*x)^m*(a + b*x^2 + c*x^4)^(p + 1)*Simp[b^2*(m + 2*p + 3) - 2*a*c*(m +
 4*p + 5) + b*c*(m + 4*p + 7)*x^2, x], x], x] /; FreeQ[{a, b, c, d, m}, x]
&& NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p] && (IntegerQ[p] || In
tegerQ[m])
```

Rule 1142

```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Di
st[u^m/(Coefficient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p,
 x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
```

```
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1277

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> -Simp[((f*x)^(m + 1)*(a + b*x^2 + c*x^4)^(p + 1)*
(d*(b^2 - 2*a*c) - a*b*e + (b*d - 2*a*e)*c*x^2))/(2*a*f*(p + 1)*(b^2 - 4*a*
c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(f*x)^m*(a + b*x^2 + c*x^
4)^(p + 1)*Simp[d*(b^2*(m + 2*(p + 1) + 1) - 2*a*c*(m + 4*(p + 1) + 1)) - a
*b*e*(m + 1) + c*(m + 2*(2*p + 3) + 1)*(b*d - 2*a*e)*x^2, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, m}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && Intege
rQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1281

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[(d*(f*x)^(m + 1)*(a + b*x^2 + c*x^4)^(p + 1)
)/(a*f*(m + 1)), x] + Dist[1/(a*f^2*(m + 1)), Int[(f*x)^(m + 2)*(a + b*x^2
+ c*x^4)^p*Simp[a*e*(m + 1) - b*d*(m + 2*p + 3) - c*d*(m + 4*p + 5)*x^2, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[m
, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rubi steps

$$\int \frac{1}{(d+ex)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)^3}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x^2\left(a+bx^2+cx^4\right)^3}\,dx, x, d+ex\right)}{e}$$

$$= \frac{b^2-2ac+bc(d+ex)^2}{4a\left(b^2-4ac\right)e(d+ex)\left(a+b(d+ex)^2+c(d+ex)^4\right)^2} - \frac{\text{Subst}\Big(\int$$

$$= \frac{b^2-2ac+bc(d+ex)^2}{4a\left(b^2-4ac\right)e(d+ex)\left(a+b(d+ex)^2+c(d+ex)^4\right)^2} + \frac{5b^4-}{8a^2\left(b^2-\right.}$$

$$= -\frac{3\left(5b^2-12ac\right)\left(b^2-5ac\right)}{8a^3\left(b^2-4ac\right)^2e(d+ex)} + \frac{b^2-2ac+bc(d+e}{4a\left(b^2-4ac\right)e(d+ex)\left(a+b(d+e\right.}$$

$$= -\frac{3\left(5b^2-12ac\right)\left(b^2-5ac\right)}{8a^3\left(b^2-4ac\right)^2e(d+ex)} + \frac{b^2-2ac+bc(d+e}{4a\left(b^2-4ac\right)e(d+ex)\left(a+b(d+e\right.}$$

$$= -\frac{3\left(5b^2-12ac\right)\left(b^2-5ac\right)}{8a^3\left(b^2-4ac\right)^2e(d+ex)} + \frac{b^2-2ac+bc(d+e}{4a\left(b^2-4ac\right)e(d+ex)\left(a+b(d+e\right.}$$

Mathematica [A] time = 6.24, size = 560, normalized size = 1.16

$$-\frac{1}{a^3e(d+ex)}+\frac{-3abc(d+ex)-2ac^2(d+ex)^3+b^3(d+ex)+b^2c(d+ex)^3}{4a^2e\left(4ac-b^2\right)\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}-\frac{3\sqrt{c}\left(60a^2c^2\sqrt{b^2-4ac}+124a^2bc^2-47ab^3c-37ab^2c\sqrt{b^2-4ac}+5b^4\sqrt{b^2-4ac}+5b^5\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{8\sqrt{2}a^3e\left(b^2-4ac\right)^{5/2}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{3\sqrt{c}\left(60a^2c^2\sqrt{b^2-4ac}-124a^2bc^2+47ab^3c-37ab^2c\sqrt{b^2-4ac}+5b^4\sqrt{b^2-4ac}-5b^5\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{8\sqrt{2}a^3e\left(b^2-4ac\right)^{5/2}\sqrt{\sqrt{b^2-4ac}+b}}+\frac{-84a^2bc^2(d+ex)-52a^2c^3(d+ex)^3+52ab^3c(d+ex)+47ab^2c^2(d+ex)^3-7b^5(d+ex)-7b^4c(d+ex)^3}{8a^3e\left(4ac-b^2\right)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)^2*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3),x]
```

```
[Out] -(1/(a^3*e*(d + e*x))) + (b^3*(d + e*x) - 3*a*b*c*(d + e*x) + b^2*c*(d + e*
x)^3 - 2*a*c^2*(d + e*x)^3)/(4*a^2*(-b^2 + 4*a*c)*e*(a + b*(d + e*x)^2 + c*
(d + e*x)^4)^2) + (-7*b^5*(d + e*x) + 52*a*b^3*c*(d + e*x) - 84*a^2*b*c^2*(
d + e*x) - 7*b^4*c*(d + e*x)^3 + 47*a*b^2*c^2*(d + e*x)^3 - 52*a^2*c^3*(d +
 e*x)^3)/(8*a^3*(-b^2 + 4*a*c)^2*e*(a + b*(d + e*x)^2 + c*(d + e*x)^4)) - (
3*Sqrt[c]*(5*b^5 - 47*a*b^3*c + 124*a^2*b*c^2 + 5*b^4*Sqrt[b^2 - 4*a*c] - 3
7*a*b^2*c*Sqrt[b^2 - 4*a*c] + 60*a^2*c^2*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]
*Sqrt[c]*(d + e*x))/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(8*Sqrt[2]*a^3*(b^2 - 4*a
*c)^(5/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]*e) - (3*Sqrt[c]*(-5*b^5 + 47*a*b^3*c
- 124*a^2*b*c^2 + 5*b^4*Sqrt[b^2 - 4*a*c] - 37*a*b^2*c*Sqrt[b^2 - 4*a*c] +
60*a^2*c^2*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b + S
qrt[b^2 - 4*a*c]]])/(8*Sqrt[2]*a^3*(b^2 - 4*a*c)^(5/2)*Sqrt[b + Sqrt[b^2 -
4*a*c]]*e)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(d+ex)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/((d + e*x)^2*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3),x]
```

```
[Out] IntegrateAlgebraic[1/((d + e*x)^2*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3), x
]
```

fricas [B] time = 3.92, size = 10260, normalized size = 21.20

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="fricas")
```

```
[Out] -1/16*(6*(5*b^4*c^2 - 37*a*b^2*c^3 + 60*a^2*c^4)*e^8*x^8 + 48*(5*b^4*c^2 -
37*a*b^2*c^3 + 60*a^2*c^4)*d*e^7*x^7 + 2*(30*b^5*c - 227*a*b^3*c^2 + 392*a^
2*b*c^3 + 84*(5*b^4*c^2 - 37*a*b^2*c^3 + 60*a^2*c^4)*d^2)*e^6*x^6 + 12*(28*
(5*b^4*c^2 - 37*a*b^2*c^3 + 60*a^2*c^4)*d^3 + (30*b^5*c - 227*a*b^3*c^2 + 3
92*a^2*b*c^3)*d)*e^5*x^5 + 6*(5*b^4*c^2 - 37*a*b^2*c^3 + 60*a^2*c^4)*d^8 +
2*(15*b^6 - 91*a*b^4*c + 25*a^2*b^2*c^2 + 324*a^3*c^3 + 210*(5*b^4*c^2 - 37
*a*b^2*c^3 + 60*a^2*c^4)*d^4 + 15*(30*b^5*c - 227*a*b^3*c^2 + 392*a^2*b*c^3
)*d^2)*e^4*x^4 + 2*(30*b^5*c - 227*a*b^3*c^2 + 392*a^2*b*c^3)*d^6 + 8*(42*(
5*b^4*c^2 - 37*a*b^2*c^3 + 60*a^2*c^4)*d^5 + 5*(30*b^5*c - 227*a*b^3*c^2 +
392*a^2*b*c^3)*d^3 + (15*b^6 - 91*a*b^4*c + 25*a^2*b^2*c^2 + 324*a^3*c^3)*d
)*e^3*x^3 + 16*a^2*b^4 - 128*a^3*b^2*c + 256*a^4*c^2 + 2*(15*b^6 - 91*a*b^4
*c + 25*a^2*b^2*c^2 + 324*a^3*c^3)*d^4 + 2*(84*(5*b^4*c^2 - 37*a*b^2*c^3 +
60*a^2*c^4)*d^6 + 25*a*b^5 - 194*a^2*b^3*c + 364*a^3*b*c^2 + 15*(30*b^5*c -
 227*a*b^3*c^2 + 392*a^2*b*c^3)*d^4 + 6*(15*b^6 - 91*a*b^4*c + 25*a^2*b^2*c
^2 + 324*a^3*c^3)*d^2)*e^2*x^2 + 2*(25*a*b^5 - 194*a^2*b^3*c + 364*a^3*b*c^
2)*d^2 + 4*(12*(5*b^4*c^2 - 37*a*b^2*c^3 + 60*a^2*c^4)*d^7 + 3*(30*b^5*c -
227*a*b^3*c^2 + 392*a^2*b*c^3)*d^5 + 2*(15*b^6 - 91*a*b^4*c + 25*a^2*b^2*c^
2 + 324*a^3*c^3)*d^3 + (25*a*b^5 - 194*a^2*b^3*c + 364*a^3*b*c^2)*d)*e*x -
3*sqrt(1/2)*((a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*e^10*x^9 + 9*(a^3*b
^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d*e^9*x^8 + 2*(a^3*b^5*c - 8*a^4*b^3*c
^2 + 16*a^5*b*c^3 + 18*(a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d^2)*e^8*
x^7 + 14*(6*(a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d^3 + (a^3*b^5*c - 8
*a^4*b^3*c^2 + 16*a^5*b*c^3)*d)*e^7*x^6 + (a^3*b^6 - 6*a^4*b^4*c + 32*a^6*c
^3 + 126*(a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d^4 + 42*(a^3*b^5*c - 8
*a^4*b^3*c^2 + 16*a^5*b*c^3)*d^2)*e^6*x^5 + (126*(a^3*b^4*c^2 - 8*a^4*b^2*c
^3 + 16*a^5*c^4)*d^5 + 70*(a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*d^3 +
5*(a^3*b^6 - 6*a^4*b^4*c + 32*a^6*c^3)*d)*e^5*x^4 + 2*(a^4*b^5 - 8*a^5*b^3*
c + 16*a^6*b*c^2 + 42*(a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d^6 + 35*(
```

```
a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*d^4 + 5*(a^3*b^6 - 6*a^4*b^4*c +
32*a^6*c^3)*d^2)*e^4*x^3 + 2*(18*(a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)
*d^7 + 21*(a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*d^5 + 5*(a^3*b^6 - 6*a
^4*b^4*c + 32*a^6*c^3)*d^3 + 3*(a^4*b^5 - 8*a^5*b^3*c + 16*a^6*b*c^2)*d)*e^
3*x^2 + (a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2 + 9*(a^3*b^4*c^2 - 8*a^4*b^2*c^
3 + 16*a^5*c^4)*d^8 + 14*(a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*d^6 + 5
*(a^3*b^6 - 6*a^4*b^4*c + 32*a^6*c^3)*d^4 + 6*(a^4*b^5 - 8*a^5*b^3*c + 16*a
^6*b*c^2)*d^2)*e^2*x + ((a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d^9 + 2*
(a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*d^7 + (a^3*b^6 - 6*a^4*b^4*c + 3
2*a^6*c^3)*d^5 + 2*(a^4*b^5 - 8*a^5*b^3*c + 16*a^6*b*c^2)*d^3 + (a^5*b^4 -
8*a^6*b^2*c + 16*a^7*c^2)*d)*e)*sqrt(-(25*b^11 - 495*a*b^9*c + 3894*a^2*b^7
*c^2 - 15015*a^3*b^5*c^3 + 27720*a^4*b^3*c^4 - 18480*a^5*b*c^5 + (a^7*b^10
- 20*a^8*b^8*c + 160*a^9*b^6*c^2 - 640*a^10*b^4*c^3 + 1280*a^11*b^2*c^4 - 1
024*a^12*c^5)*e^2*sqrt((625*b^12 - 12250*a*b^10*c + 94725*a^2*b^8*c^2 - 351
310*a^3*b^6*c^3 + 591886*a^4*b^4*c^4 - 312300*a^5*b^2*c^5 + 50625*a^6*c^6)/
((a^14*b^10 - 20*a^15*b^8*c + 160*a^16*b^6*c^2 - 640*a^17*b^4*c^3 + 1280*a^
18*b^2*c^4 - 1024*a^19*c^5)*e^4)))/((a^7*b^10 - 20*a^8*b^8*c + 160*a^9*b^6*
c^2 - 640*a^10*b^4*c^3 + 1280*a^11*b^2*c^4 - 1024*a^12*c^5)*e^2))*log(-27*(
4125*b^10*c^4 - 77825*a*b^8*c^5 + 571030*a^2*b^6*c^6 - 1957349*a^3*b^4*c^7
+ 2835000*a^4*b^2*c^8 - 810000*a^5*c^9)*e*x - 27*(4125*b^10*c^4 - 77825*a*b
^8*c^5 + 571030*a^2*b^6*c^6 - 1957349*a^3*b^4*c^7 + 2835000*a^4*b^2*c^8 - 8
10000*a^5*c^9)*d + 27/2*sqrt(1/2)*((5*a^7*b^16 - 152*a^8*b^14*c + 2006*a^9*
b^12*c^2 - 14960*a^10*b^10*c^3 + 68640*a^11*b^8*c^4 - 197120*a^12*b^6*c^5 +
 342528*a^13*b^4*c^6 - 323584*a^14*b^2*c^7 + 122880*a^15*c^8)*e^3*sqrt((625
*b^12 - 12250*a*b^10*c + 94725*a^2*b^8*c^2 - 351310*a^3*b^6*c^3 + 591886*a^
4*b^4*c^4 - 312300*a^5*b^2*c^5 + 50625*a^6*c^6)/((a^14*b^10 - 20*a^15*b^8*c
 + 160*a^16*b^6*c^2 - 640*a^17*b^4*c^3 + 1280*a^18*b^2*c^4 - 1024*a^19*c^5)
*e^4)) - (125*b^17 - 3775*a*b^15*c + 49360*a^2*b^13*c^2 - 362733*a^3*b^11*c
^3 + 1623534*a^4*b^9*c^4 - 4463140*a^5*b^7*c^5 + 7146736*a^6*b^5*c^6 - 5684
672*a^7*b^3*c^7 + 1324800*a^8*b*c^8)*e)*sqrt(-(25*b^11 - 495*a*b^9*c + 3894
*a^2*b^7*c^2 - 15015*a^3*b^5*c^3 + 27720*a^4*b^3*c^4 - 18480*a^5*b*c^5 + (a
^7*b^10 - 20*a^8*b^8*c + 160*a^9*b^6*c^2 - 640*a^10*b^4*c^3 + 1280*a^11*b^2
*c^4 - 1024*a^12*c^5)*e^2*sqrt((625*b^12 - 12250*a*b^10*c + 94725*a^2*b^8*c
^2 - 351310*a^3*b^6*c^3 + 591886*a^4*b^4*c^4 - 312300*a^5*b^2*c^5 + 50625*a
^6*c^6)/((a^14*b^10 - 20*a^15*b^8*c + 160*a^16*b^6*c^2 - 640*a^17*b^4*c^3 +
 1280*a^18*b^2*c^4 - 1024*a^19*c^5)*e^4)))/((a^7*b^10 - 20*a^8*b^8*c + 160*
a^9*b^6*c^2 - 640*a^10*b^4*c^3 + 1280*a^11*b^2*c^4 - 1024*a^12*c^5)*e^2)))
+ 3*sqrt(1/2)*((a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*e^10*x^9 + 9*(a^3
*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d*e^9*x^8 + 2*(a^3*b^5*c - 8*a^4*b^3
*c^2 + 16*a^5*b*c^3 + 18*(a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d^2)*e^
8*x^7 + 14*(6*(a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d^3 + (a^3*b^5*c -
 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*d)*e^7*x^6 + (a^3*b^6 - 6*a^4*b^4*c + 32*a^6
*c^3 + 126*(a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d^4 + 42*(a^3*b^5*c -
 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*d^2)*e^6*x^5 + (126*(a^3*b^4*c^2 - 8*a^4*b^2
*c^3 + 16*a^5*c^4)*d^5 + 70*(a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*d^3
+ 5*(a^3*b^6 - 6*a^4*b^4*c + 32*a^6*c^3)*d)*e^5*x^4 + 2*(a^4*b^5 - 8*a^5*b^
3*c + 16*a^6*b*c^2 + 42*(a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d^6 + 35
*(a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*d^4 + 5*(a^3*b^6 - 6*a^4*b^4*c
+ 32*a^6*c^3)*d^2)*e^4*x^3 + 2*(18*(a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^
4)*d^7 + 21*(a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*d^5 + 5*(a^3*b^6 - 6
*a^4*b^4*c + 32*a^6*c^3)*d^3 + 3*(a^4*b^5 - 8*a^5*b^3*c + 16*a^6*b*c^2)*d)*
e^3*x^2 + (a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2 + 9*(a^3*b^4*c^2 - 8*a^4*b^2*
c^3 + 16*a^5*c^4)*d^8 + 14*(a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*d^6 +
 5*(a^3*b^6 - 6*a^4*b^4*c + 32*a^6*c^3)*d^4 + 6*(a^4*b^5 - 8*a^5*b^3*c + 16
*a^6*b*c^2)*d^2)*e^2*x + ((a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d^9 +
2*(a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*d^7 + (a^3*b^6 - 6*a^4*b^4*c +
 32*a^6*c^3)*d^5 + 2*(a^4*b^5 - 8*a^5*b^3*c + 16*a^6*b*c^2)*d^3 + (a^5*b^4
- 8*a^6*b^2*c + 16*a^7*c^2)*d)*e)*sqrt(-(25*b^11 - 495*a*b^9*c + 3894*a^2*b
^7*c^2 - 15015*a^3*b^5*c^3 + 27720*a^4*b^3*c^4 - 18480*a^5*b*c^5 + (a^7*b^1
```

0 - 20*a^8*b^8*c + 160*a^9*b^6*c^2 - 640*a^10*b^4*c^3 + 1280*a^11*b^2*c^4 -
 1024*a^12*c^5)*e^2*sqrt((625*b^12 - 12250*a*b^10*c + 94725*a^2*b^8*c^2 - 3
51310*a^3*b^6*c^3 + 591886*a^4*b^4*c^4 - 312300*a^5*b^2*c^5 + 50625*a^6*c^6
)/((a^14*b^10 - 20*a^15*b^8*c + 160*a^16*b^6*c^2 - 640*a^17*b^4*c^3 + 1280*
a^18*b^2*c^4 - 1024*a^19*c^5)*e^4)))/((a^7*b^10 - 20*a^8*b^8*c + 160*a^9*b^
6*c^2 - 640*a^10*b^4*c^3 + 1280*a^11*b^2*c^4 - 1024*a^12*c^5)*e^2))*log(-27
*(4125*b^10*c^4 - 77825*a*b^8*c^5 + 571030*a^2*b^6*c^6 - 1957349*a^3*b^4*c^
7 + 2835000*a^4*b^2*c^8 - 810000*a^5*c^9)*e*x - 27*(4125*b^10*c^4 - 77825*a
*b^8*c^5 + 571030*a^2*b^6*c^6 - 1957349*a^3*b^4*c^7 + 2835000*a^4*b^2*c^8 -
 810000*a^5*c^9)*d - 27/2*sqrt(1/2)*((5*a^7*b^16 - 152*a^8*b^14*c + 2006*a^
9*b^12*c^2 - 14960*a^10*b^10*c^3 + 68640*a^11*b^8*c^4 - 197120*a^12*b^6*c^5
 + 342528*a^13*b^4*c^6 - 323584*a^14*b^2*c^7 + 122880*a^15*c^8)*e^3*sqrt((6
25*b^12 - 12250*a*b^10*c + 94725*a^2*b^8*c^2 - 351310*a^3*b^6*c^3 + 591886*
a^4*b^4*c^4 - 312300*a^5*b^2*c^5 + 50625*a^6*c^6)/((a^14*b^10 - 20*a^15*b^8
*c + 160*a^16*b^6*c^2 - 640*a^17*b^4*c^3 + 1280*a^18*b^2*c^4 - 1024*a^19*c^
5)*e^4)) - (125*b^17 - 3775*a*b^15*c + 49360*a^2*b^13*c^2 - 362733*a^3*b^11
*c^3 + 1623534*a^4*b^9*c^4 - 4463140*a^5*b^7*c^5 + 7146736*a^6*b^5*c^6 - 56
84672*a^7*b^3*c^7 + 1324800*a^8*b*c^8)*e)*sqrt(-(25*b^11 - 495*a*b^9*c + 38
94*a^2*b^7*c^2 - 15015*a^3*b^5*c^3 + 27720*a^4*b^3*c^4 - 18480*a^5*b*c^5 +
(a^7*b^10 - 20*a^8*b^8*c + 160*a^9*b^6*c^2 - 640*a^10*b^4*c^3 + 1280*a^11*b
^2*c^4 - 1024*a^12*c^5)*e^2*sqrt((625*b^12 - 12250*a*b^10*c + 94725*a^2*b^8
*c^2 - 351310*a^3*b^6*c^3 + 591886*a^4*b^4*c^4 - 312300*a^5*b^2*c^5 + 50625
*a^6*c^6)/((a^14*b^10 - 20*a^15*b^8*c + 160*a^16*b^6*c^2 - 640*a^17*b^4*c^3
 + 1280*a^18*b^2*c^4 - 1024*a^19*c^5)*e^4)))/((a^7*b^10 - 20*a^8*b^8*c + 16
0*a^9*b^6*c^2 - 640*a^10*b^4*c^3 + 1280*a^11*b^2*c^4 - 1024*a^12*c^5)*e^2))
) + 3*sqrt(1/2)*((a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*e^10*x^9 + 9*(a
^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d*e^9*x^8 + 2*(a^3*b^5*c - 8*a^4*b
^3*c^2 + 16*a^5*b*c^3 + 18*(a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d^2)*
e^8*x^7 + 14*(6*(a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d^3 + (a^3*b^5*c
 - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*d)*e^7*x^6 + (a^3*b^6 - 6*a^4*b^4*c + 32*a
^6*c^3 + 126*(a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d^4 + 42*(a^3*b^5*c
 - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*d^2)*e^6*x^5 + (126*(a^3*b^4*c^2 - 8*a^4*b
^2*c^3 + 16*a^5*c^4)*d^5 + 70*(a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*d^
3 + 5*(a^3*b^6 - 6*a^4*b^4*c + 32*a^6*c^3)*d)*e^5*x^4 + 2*(a^4*b^5 - 8*a^5*
b^3*c + 16*a^6*b*c^2 + 42*(a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d^6 +
35*(a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*d^4 + 5*(a^3*b^6 - 6*a^4*b^4*
c + 32*a^6*c^3)*d^2)*e^4*x^3 + 2*(18*(a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*
c^4)*d^7 + 21*(a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*d^5 + 5*(a^3*b^6 -
 6*a^4*b^4*c + 32*a^6*c^3)*d^3 + 3*(a^4*b^5 - 8*a^5*b^3*c + 16*a^6*b*c^2)*d
)*e^3*x^2 + (a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2 + 9*(a^3*b^4*c^2 - 8*a^4*b^
2*c^3 + 16*a^5*c^4)*d^8 + 14*(a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*d^6
 + 5*(a^3*b^6 - 6*a^4*b^4*c + 32*a^6*c^3)*d^4 + 6*(a^4*b^5 - 8*a^5*b^3*c +
16*a^6*b*c^2)*d^2)*e^2*x + ((a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d^9
+ 2*(a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*d^7 + (a^3*b^6 - 6*a^4*b^4*c
 + 32*a^6*c^3)*d^5 + 2*(a^4*b^5 - 8*a^5*b^3*c + 16*a^6*b*c^2)*d^3 + (a^5*b^
4 - 8*a^6*b^2*c + 16*a^7*c^2)*d)*e)*sqrt(-(25*b^11 - 495*a*b^9*c + 3894*a^2
*b^7*c^2 - 15015*a^3*b^5*c^3 + 27720*a^4*b^3*c^4 - 18480*a^5*b*c^5 - (a^7*b
^10 - 20*a^8*b^8*c + 160*a^9*b^6*c^2 - 640*a^10*b^4*c^3 + 1280*a^11*b^2*c^4
 - 1024*a^12*c^5)*e^2*sqrt((625*b^12 - 12250*a*b^10*c + 94725*a^2*b^8*c^2 -
 351310*a^3*b^6*c^3 + 591886*a^4*b^4*c^4 - 312300*a^5*b^2*c^5 + 50625*a^6*c
^6)/((a^14*b^10 - 20*a^15*b^8*c + 160*a^16*b^6*c^2 - 640*a^17*b^4*c^3 + 128
0*a^18*b^2*c^4 - 1024*a^19*c^5)*e^4)))/((a^7*b^10 - 20*a^8*b^8*c + 160*a^9*
b^6*c^2 - 640*a^10*b^4*c^3 + 1280*a^11*b^2*c^4 - 1024*a^12*c^5)*e^2))*log(-
27*(4125*b^10*c^4 - 77825*a*b^8*c^5 + 571030*a^2*b^6*c^6 - 1957349*a^3*b^4*
c^7 + 2835000*a^4*b^2*c^8 - 810000*a^5*c^9)*e*x - 27*(4125*b^10*c^4 - 77825
*a*b^8*c^5 + 571030*a^2*b^6*c^6 - 1957349*a^3*b^4*c^7 + 2835000*a^4*b^2*c^8
 - 810000*a^5*c^9)*d + 27/2*sqrt(1/2)*((5*a^7*b^16 - 152*a^8*b^14*c + 2006*
a^9*b^12*c^2 - 14960*a^10*b^10*c^3 + 68640*a^11*b^8*c^4 - 197120*a^12*b^6*c
^5 + 342528*a^13*b^4*c^6 - 323584*a^14*b^2*c^7 + 122880*a^15*c^8)*e^3*sqrt(

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(625*b^12 - 12250*a*b^10*c + 94725*a^2*b^8*c^2 - 351310*a^3*b^6*c^3 + 59188
6*a^4*b^4*c^4 - 312300*a^5*b^2*c^5 + 50625*a^6*c^6)/((a^14*b^10 - 20*a^15*b
^8*c + 160*a^16*b^6*c^2 - 640*a^17*b^4*c^3 + 1280*a^18*b^2*c^4 - 1024*a^19*
c^5)*e^4)) + (125*b^17 - 3775*a*b^15*c + 49360*a^2*b^13*c^2 - 362733*a^3*b^
11*c^3 + 1623534*a^4*b^9*c^4 - 4463140*a^5*b^7*c^5 + 7146736*a^6*b^5*c^6 -
5684672*a^7*b^3*c^7 + 1324800*a^8*b*c^8)*e)*sqrt(-(25*b^11 - 495*a*b^9*c +
3894*a^2*b^7*c^2 - 15015*a^3*b^5*c^3 + 27720*a^4*b^3*c^4 - 18480*a^5*b*c^5
- (a^7*b^10 - 20*a^8*b^8*c + 160*a^9*b^6*c^2 - 640*a^10*b^4*c^3 + 1280*a^11
*b^2*c^4 - 1024*a^12*c^5)*e^2*sqrt((625*b^12 - 12250*a*b^10*c + 94725*a^2*b
^8*c^2 - 351310*a^3*b^6*c^3 + 591886*a^4*b^4*c^4 - 312300*a^5*b^2*c^5 + 506
25*a^6*c^6)/((a^14*b^10 - 20*a^15*b^8*c + 160*a^16*b^6*c^2 - 640*a^17*b^4*c
^3 + 1280*a^18*b^2*c^4 - 1024*a^19*c^5)*e^4)))/((a^7*b^10 - 20*a^8*b^8*c +
160*a^9*b^6*c^2 - 640*a^10*b^4*c^3 + 1280*a^11*b^2*c^4 - 1024*a^12*c^5)*e^2
))) - 3*sqrt(1/2)*((a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*e^10*x^9 + 9*
(a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d*e^9*x^8 + 2*(a^3*b^5*c - 8*a^4
*b^3*c^2 + 16*a^5*b*c^3 + 18*(a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d^2
)*e^8*x^7 + 14*(6*(a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d^3 + (a^3*b^5
*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*d)*e^7*x^6 + (a^3*b^6 - 6*a^4*b^4*c + 32
*a^6*c^3 + 126*(a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d^4 + 42*(a^3*b^5
*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*d^2)*e^6*x^5 + (126*(a^3*b^4*c^2 - 8*a^4
*b^2*c^3 + 16*a^5*c^4)*d^5 + 70*(a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*
d^3 + 5*(a^3*b^6 - 6*a^4*b^4*c + 32*a^6*c^3)*d)*e^5*x^4 + 2*(a^4*b^5 - 8*a^
5*b^3*c + 16*a^6*b*c^2 + 42*(a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d^6
+ 35*(a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*d^4 + 5*(a^3*b^6 - 6*a^4*b^
4*c + 32*a^6*c^3)*d^2)*e^4*x^3 + 2*(18*(a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^
5*c^4)*d^7 + 21*(a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*d^5 + 5*(a^3*b^6
 - 6*a^4*b^4*c + 32*a^6*c^3)*d^3 + 3*(a^4*b^5 - 8*a^5*b^3*c + 16*a^6*b*c^2)
*d)*e^3*x^2 + (a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2 + 9*(a^3*b^4*c^2 - 8*a^4*
b^2*c^3 + 16*a^5*c^4)*d^8 + 14*(a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*d
^6 + 5*(a^3*b^6 - 6*a^4*b^4*c + 32*a^6*c^3)*d^4 + 6*(a^4*b^5 - 8*a^5*b^3*c
+ 16*a^6*b*c^2)*d^2)*e^2*x + ((a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d^
9 + 2*(a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*d^7 + (a^3*b^6 - 6*a^4*b^4
*c + 32*a^6*c^3)*d^5 + 2*(a^4*b^5 - 8*a^5*b^3*c + 16*a^6*b*c^2)*d^3 + (a^5*
b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*d)*e)*sqrt(-(25*b^11 - 495*a*b^9*c + 3894*a
^2*b^7*c^2 - 15015*a^3*b^5*c^3 + 27720*a^4*b^3*c^4 - 18480*a^5*b*c^5 - (a^7
*b^10 - 20*a^8*b^8*c + 160*a^9*b^6*c^2 - 640*a^10*b^4*c^3 + 1280*a^11*b^2*c
^4 - 1024*a^12*c^5)*e^2*sqrt((625*b^12 - 12250*a*b^10*c + 94725*a^2*b^8*c^2
 - 351310*a^3*b^6*c^3 + 591886*a^4*b^4*c^4 - 312300*a^5*b^2*c^5 + 50625*a^6
*c^6)/((a^14*b^10 - 20*a^15*b^8*c + 160*a^16*b^6*c^2 - 640*a^17*b^4*c^3 + 1
280*a^18*b^2*c^4 - 1024*a^19*c^5)*e^4)))/((a^7*b^10 - 20*a^8*b^8*c + 160*a^
9*b^6*c^2 - 640*a^10*b^4*c^3 + 1280*a^11*b^2*c^4 - 1024*a^12*c^5)*e^2))*log
(-27*(4125*b^10*c^4 - 77825*a*b^8*c^5 + 571030*a^2*b^6*c^6 - 1957349*a^3*b^
4*c^7 + 2835000*a^4*b^2*c^8 - 810000*a^5*c^9)*e*x - 27*(4125*b^10*c^4 - 778
25*a*b^8*c^5 + 571030*a^2*b^6*c^6 - 1957349*a^3*b^4*c^7 + 2835000*a^4*b^2*c
^8 - 810000*a^5*c^9)*d - 27/2*sqrt(1/2)*((5*a^7*b^16 - 152*a^8*b^14*c + 200
6*a^9*b^12*c^2 - 14960*a^10*b^10*c^3 + 68640*a^11*b^8*c^4 - 197120*a^12*b^6
*c^5 + 342528*a^13*b^4*c^6 - 323584*a^14*b^2*c^7 + 122880*a^15*c^8)*e^3*sqr
t((625*b^12 - 12250*a*b^10*c + 94725*a^2*b^8*c^2 - 351310*a^3*b^6*c^3 + 591
886*a^4*b^4*c^4 - 312300*a^5*b^2*c^5 + 50625*a^6*c^6)/((a^14*b^10 - 20*a^15
*b^8*c + 160*a^16*b^6*c^2 - 640*a^17*b^4*c^3 + 1280*a^18*b^2*c^4 - 1024*a^1
9*c^5)*e^4)) + (125*b^17 - 3775*a*b^15*c + 49360*a^2*b^13*c^2 - 362733*a^3*
b^11*c^3 + 1623534*a^4*b^9*c^4 - 4463140*a^5*b^7*c^5 + 7146736*a^6*b^5*c^6
- 5684672*a^7*b^3*c^7 + 1324800*a^8*b*c^8)*e)*sqrt(-(25*b^11 - 495*a*b^9*c
+ 3894*a^2*b^7*c^2 - 15015*a^3*b^5*c^3 + 27720*a^4*b^3*c^4 - 18480*a^5*b*c^
5 - (a^7*b^10 - 20*a^8*b^8*c + 160*a^9*b^6*c^2 - 640*a^10*b^4*c^3 + 1280*a^
11*b^2*c^4 - 1024*a^12*c^5)*e^2*sqrt((625*b^12 - 12250*a*b^10*c + 94725*a^2
*b^8*c^2 - 351310*a^3*b^6*c^3 + 591886*a^4*b^4*c^4 - 312300*a^5*b^2*c^5 + 5
0625*a^6*c^6)/((a^14*b^10 - 20*a^15*b^8*c + 160*a^16*b^6*c^2 - 640*a^17*b^4
*c^3 + 1280*a^18*b^2*c^4 - 1024*a^19*c^5)*e^4)))/((a^7*b^10 - 20*a^8*b^8*c
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+ 160*a^9*b^6*c^2 - 640*a^10*b^4*c^3 + 1280*a^11*b^2*c^4 - 1024*a^12*c^5)*e
^2))))/((a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*e^10*x^9 + 9*(a^3*b^4*c^
2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d*e^9*x^8 + 2*(a^3*b^5*c - 8*a^4*b^3*c^2 +
16*a^5*b*c^3 + 18*(a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d^2)*e^8*x^7 +
 14*(6*(a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d^3 + (a^3*b^5*c - 8*a^4*
b^3*c^2 + 16*a^5*b*c^3)*d)*e^7*x^6 + (a^3*b^6 - 6*a^4*b^4*c + 32*a^6*c^3 +
126*(a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d^4 + 42*(a^3*b^5*c - 8*a^4*
b^3*c^2 + 16*a^5*b*c^3)*d^2)*e^6*x^5 + (126*(a^3*b^4*c^2 - 8*a^4*b^2*c^3 +
16*a^5*c^4)*d^5 + 70*(a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*d^3 + 5*(a^
3*b^6 - 6*a^4*b^4*c + 32*a^6*c^3)*d)*e^5*x^4 + 2*(a^4*b^5 - 8*a^5*b^3*c + 1
6*a^6*b*c^2 + 42*(a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d^6 + 35*(a^3*b
^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*d^4 + 5*(a^3*b^6 - 6*a^4*b^4*c + 32*a^
6*c^3)*d^2)*e^4*x^3 + 2*(18*(a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d^7
+ 21*(a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*d^5 + 5*(a^3*b^6 - 6*a^4*b^
4*c + 32*a^6*c^3)*d^3 + 3*(a^4*b^5 - 8*a^5*b^3*c + 16*a^6*b*c^2)*d)*e^3*x^2
 + (a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2 + 9*(a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 1
6*a^5*c^4)*d^8 + 14*(a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*d^6 + 5*(a^3
*b^6 - 6*a^4*b^4*c + 32*a^6*c^3)*d^4 + 6*(a^4*b^5 - 8*a^5*b^3*c + 16*a^6*b*
c^2)*d^2)*e^2*x + ((a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d^9 + 2*(a^3*
b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*d^7 + (a^3*b^6 - 6*a^4*b^4*c + 32*a^6
*c^3)*d^5 + 2*(a^4*b^5 - 8*a^5*b^3*c + 16*a^6*b*c^2)*d^3 + (a^5*b^4 - 8*a^6
*b^2*c + 16*a^7*c^2)*d)*e)
```

giac [B] time = 1.22, size = 1412, normalized size = 2.92

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="giac")
```

```
[Out] 3/64*(2*(5*a^4*b^6*c - 57*a^5*b^4*c^2 + 208*a^6*b^2*c^3 - 240*a^7*c^4)*sqrt
(2*a*b + 2*sqrt(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*abs(a^3*b^4*e^2 - 8*a^4*b
^2*c*e^2 + 16*a^5*c^2*e^2)*e^2 - (a^3*b^4*e^2 - 8*a^4*b^2*c*e^2 + 16*a^5*c^
2*e^2)^2*(5*b^5 - 42*a*b^3*c + 92*a^2*b*c^2)*sqrt(2*a*b + 2*sqrt(b^2 - 4*a*
c)*a) + (5*a^6*b^13 - 112*a^7*b^11*c + 1030*a^8*b^9*c^2 - 4928*a^9*b^7*c^3
+ 12736*a^10*b^5*c^4 - 16384*a^11*b^3*c^5 + 7680*a^12*b*c^6)*sqrt(2*a*b + 2
*sqrt(b^2 - 4*a*c)*a)*e^4)*arctan(2*sqrt(1/2)*e^(-1)/((x*e + d)*sqrt((a^3*b
^5*e^2 - 8*a^4*b^3*c*e^2 + 16*a^5*b*c^2*e^2 + sqrt((a^3*b^5*e^2 - 8*a^4*b^3
*c*e^2 + 16*a^5*b*c^2*e^2)^2 - 4*(a^4*b^4*e^4 - 8*a^5*b^2*c*e^4 + 16*a^6*c^
2*e^4)*(a^3*b^4*c - 8*a^4*b^2*c^2 + 16*a^5*c^3)))/(a^4*b^4*e^4 - 8*a^5*b^2*
c*e^4 + 16*a^6*c^2*e^4))))*e^(-3)/((a^7*b^6*c - 12*a^8*b^4*c^2 + 48*a^9*b^2
*c^3 - 64*a^10*c^4)*sqrt(b^2 - 4*a*c)*abs(a^3*b^4*e^2 - 8*a^4*b^2*c*e^2 + 1
6*a^5*c^2*e^2)*abs(a)) + 3/64*(2*(5*a^4*b^6*c - 57*a^5*b^4*c^2 + 208*a^6*b^
2*c^3 - 240*a^7*c^4)*sqrt(2*a*b - 2*sqrt(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*
abs(a^3*b^4*e^2 - 8*a^4*b^2*c*e^2 + 16*a^5*c^2*e^2)*e^2 + (a^3*b^4*e^2 - 8*
a^4*b^2*c*e^2 + 16*a^5*c^2*e^2)^2*(5*b^5 - 42*a*b^3*c + 92*a^2*b*c^2)*sqrt(
2*a*b - 2*sqrt(b^2 - 4*a*c)*a) - (5*a^6*b^13 - 112*a^7*b^11*c + 1030*a^8*b^
9*c^2 - 4928*a^9*b^7*c^3 + 12736*a^10*b^5*c^4 - 16384*a^11*b^3*c^5 + 7680*a
^12*b*c^6)*sqrt(2*a*b - 2*sqrt(b^2 - 4*a*c)*a)*e^4)*arctan(2*sqrt(1/2)*e^(-
1)/((x*e + d)*sqrt((a^3*b^5*e^2 - 8*a^4*b^3*c*e^2 + 16*a^5*b*c^2*e^2 - sqrt
((a^3*b^5*e^2 - 8*a^4*b^3*c*e^2 + 16*a^5*b*c^2*e^2)^2 - 4*(a^4*b^4*e^4 - 8*
a^5*b^2*c*e^4 + 16*a^6*c^2*e^4)*(a^3*b^4*c - 8*a^4*b^2*c^2 + 16*a^5*c^3)))/
(a^4*b^4*e^4 - 8*a^5*b^2*c*e^4 + 16*a^6*c^2*e^4))))*e^(-3)/((a^7*b^6*c - 12
*a^8*b^4*c^2 + 48*a^9*b^2*c^3 - 64*a^10*c^4)*sqrt(b^2 - 4*a*c)*abs(a^3*b^4*
e^2 - 8*a^4*b^2*c*e^2 + 16*a^5*c^2*e^2)*abs(a)) - 1/8*(7*b^4*c^2*e^(-1)/(x*
e + d) - 47*a*b^2*c^3*e^(-1)/(x*e + d) + 52*a^2*c^4*e^(-1)/(x*e + d) + 14*b
^5*c*e^(-1)/(x*e + d)^3 - 99*a*b^3*c^2*e^(-1)/(x*e + d)^3 + 136*a^2*b*c^3*e
^(-1)/(x*e + d)^3 + 7*b^6*e^(-1)/(x*e + d)^5 - 43*a*b^4*c*e^(-1)/(x*e + d)^
5 + 25*a^2*b^2*c^2*e^(-1)/(x*e + d)^5 + 68*a^3*c^3*e^(-1)/(x*e + d)^5 + 9*a
*b^5*e^(-1)/(x*e + d)^7 - 66*a^2*b^3*c*e^(-1)/(x*e + d)^7 + 108*a^3*b*c^2*e
```

```
^(-1)/(x*e + d)^7)/((a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*(c + b/(x*e + d)^2
 + a/(x*e + d)^4)^2) - e^(-1)/((x*e + d)*a^3)
```

maple [C] time = 0.07, size = 6821, normalized size = 14.09

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x)
```

```
[Out] result too large to display
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [B] time = 14.38, size = 18112, normalized size = 37.42

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((d + e*x)^2*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3),x)
```

```
[Out] - ((x^4*(15*b^6*e^3 + 324*a^3*c^3*e^3 + 450*b^5*c*d^2*e^3 + 25*a^2*b^2*c^2*
e^3 + 12600*a^2*c^4*d^4*e^3 + 1050*b^4*c^2*d^4*e^3 - 91*a*b^4*c*e^3 - 3405*
a*b^3*c^2*d^2*e^3 + 5880*a^2*b*c^3*d^2*e^3 - 7770*a*b^2*c^3*d^4*e^3))/(8*a*
(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)) + (x^6*(30*b^5*c*e^5 - 227*a*b^3*c^2*
e^5 + 392*a^2*b*c^3*e^5 + 5040*a^2*c^4*d^2*e^5 + 420*b^4*c^2*d^2*e^5 - 3108
*a*b^2*c^3*d^2*e^5))/(8*a*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)) + (x*(30*b^
6*d^3 + 90*b^5*c*d^5 + 648*a^3*c^3*d^3 + 720*a^2*c^4*d^7 + 60*b^4*c^2*d^7 +
 25*a*b^5*d - 681*a*b^3*c^2*d^5 + 1176*a^2*b*c^3*d^5 - 444*a*b^2*c^3*d^7 +
50*a^2*b^2*c^2*d^3 - 194*a^2*b^3*c*d + 364*a^3*b*c^2*d - 182*a*b^4*c*d^3))/
(4*a*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)) + (3*x^5*(1680*a^2*c^4*d^3*e^4 +
 140*b^4*c^2*d^3*e^4 + 30*b^5*c*d*e^4 - 227*a*b^3*c^2*d*e^4 + 392*a^2*b*c^3
*d*e^4 - 1036*a*b^2*c^3*d^3*e^4))/(4*a*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)
) + (3*x^8*(60*a^2*c^4*e^7 + 5*b^4*c^2*e^7 - 37*a*b^2*c^3*e^7))/(8*a*(a^2*b
^4 + 16*a^4*c^2 - 8*a^3*b^2*c)) + (x^2*(90*b^6*d^2*e + 25*a*b^5*e + 1944*a^
3*c^3*d^2*e + 5040*a^2*c^4*d^6*e + 420*b^4*c^2*d^6*e - 194*a^2*b^3*c*e + 36
4*a^3*b*c^2*e + 450*b^5*c*d^4*e - 546*a*b^4*c*d^2*e - 3405*a*b^3*c^2*d^4*e
+ 5880*a^2*b*c^3*d^4*e - 3108*a*b^2*c^3*d^6*e + 150*a^2*b^2*c^2*d^2*e))/(8*
a*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)) + (x^3*(15*b^6*d*e^2 + 324*a^3*c^3*
d*e^2 + 150*b^5*c*d^3*e^2 + 2520*a^2*c^4*d^5*e^2 + 210*b^4*c^2*d^5*e^2 - 91
*a*b^4*c*d*e^2 + 25*a^2*b^2*c^2*d*e^2 - 1135*a*b^3*c^2*d^3*e^2 + 1960*a^2*b
*c^3*d^3*e^2 - 1554*a*b^2*c^3*d^5*e^2))/(2*a*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*
b^2*c)) + (3*x^7*(60*a^2*c^4*d*e^6 + 5*b^4*c^2*d*e^6 - 37*a*b^2*c^3*d*e^6))
/(a*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)) + (8*a^2*b^4 + 128*a^4*c^2 + 15*b
^6*d^4 - 64*a^3*b^2*c + 25*a*b^5*d^2 + 30*b^5*c*d^6 + 324*a^3*c^3*d^4 + 180
*a^2*c^4*d^8 + 15*b^4*c^2*d^8 - 194*a^2*b^3*c*d^2 + 364*a^3*b*c^2*d^2 - 227
*a*b^3*c^2*d^6 + 392*a^2*b*c^3*d^6 - 111*a*b^2*c^3*d^8 + 25*a^2*b^2*c^2*d^4
 - 91*a*b^4*c*d^4)/(8*a*e*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))/(x^3*(10*b
^2*d^2*e^3 + 84*c^2*d^6*e^3 + 2*a*b*e^3 + 20*a*c*d^2*e^3 + 70*b*c*d^4*e^3)
+ x^7*(36*c^2*d^2*e^7 + 2*b*c*e^7) + x*(a^2*e + 5*b^2*d^4*e + 9*c^2*d^8*e +
 6*a*b*d^2*e + 10*a*c*d^4*e + 14*b*c*d^6*e) + x^4*(5*b^2*d*e^4 + 126*c^2*d^
5*e^4 + 10*a*c*d*e^4 + 70*b*c*d^3*e^4) + a^2*d + x^2*(10*b^2*d^3*e^2 + 36*c
^2*d^7*e^2 + 6*a*b*d*e^2 + 20*a*c*d^3*e^2 + 42*b*c*d^5*e^2) + x^6*(84*c^2*d
```

^3*e^6 + 14*b*c*d*e^6) + x^5*(b^2*e^5 + 126*c^2*d^4*e^5 + 2*a*c*e^5 + 42*b*c*d^2*e^5) + b^2*d^5 + c^2*d^9 + c^2*e^9*x^9 + 2*a*b*d^3 + 2*a*c*d^5 + 2*b*c*d^7 + 9*c^2*d*e^8*x^8) - atan(((-(9*(25*b^21 - 25*b^6*(-(4*a*c - b^2)^15)^(1/2) + 18923520*a^10*b*c^10 + 17794*a^2*b^17*c^2 - 188095*a^3*b^15*c^3 + 1299860*a^4*b^13*c^4 - 6126640*a^5*b^11*c^5 + 19905600*a^6*b^9*c^6 - 43904256*a^7*b^7*c^7 + 62684160*a^8*b^5*c^8 - 52039680*a^9*b^3*c^9 + 225*a^3*c^3*(-(4*a*c - b^2)^15)^(1/2) - 995*a*b^19*c - 694*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 245*a*b^4*c*(-(4*a*c - b^2)^15)^(1/2)))/(512*(a^7*b^20*e^2 + 1048576*a^17*c^10*e^2 - 40*a^8*b^18*c*e^2 + 720*a^9*b^16*c^2*e^2 - 7680*a^10*b^14*c^3*e^2 + 53760*a^11*b^12*c^4*e^2 - 258048*a^12*b^10*c^5*e^2 + 860160*a^13*b^8*c^6*e^2 - 1966080*a^14*b^6*c^7*e^2 + 2949120*a^15*b^4*c^8*e^2 - 2621440*a^16*b^2*c^9*e^2)))^(1/2)*((-(9*(25*b^21 - 25*b^6*(-(4*a*c - b^2)^15)^(1/2) + 18923520*a^10*b*c^10 + 17794*a^2*b^17*c^2 - 188095*a^3*b^15*c^3 + 1299860*a^4*b^13*c^4 - 6126640*a^5*b^11*c^5 + 19905600*a^6*b^9*c^6 - 43904256*a^7*b^7*c^7 + 62684160*a^8*b^5*c^8 - 52039680*a^9*b^3*c^9 + 225*a^3*c^3*(-(4*a*c - b^2)^15)^(1/2) - 995*a*b^19*c - 694*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 245*a*b^4*c*(-(4*a*c - b^2)^15)^(1/2)))/(512*(a^7*b^20*e^2 + 1048576*a^17*c^10*e^2 - 40*a^8*b^18*c*e^2 + 720*a^9*b^16*c^2*e^2 - 7680*a^10*b^14*c^3*e^2 + 53760*a^11*b^12*c^4*e^2 - 258048*a^12*b^10*c^5*e^2 + 860160*a^13*b^8*c^6*e^2 - 1966080*a^14*b^6*c^7*e^2 + 2949120*a^15*b^4*c^8*e^2 - 2621440*a^16*b^2*c^9*e^2)))^(1/2)*((-(9*(25*b^21 - 25*b^6*(-(4*a*c - b^2)^15)^(1/2) + 18923520*a^10*b*c^10 + 17794*a^2*b^17*c^2 - 188095*a^3*b^15*c^3 + 1299860*a^4*b^13*c^4 - 6126640*a^5*b^11*c^5 + 19905600*a^6*b^9*c^6 - 43904256*a^7*b^7*c^7 + 62684160*a^8*b^5*c^8 - 52039680*a^9*b^3*c^9 + 225*a^3*c^3*(-(4*a*c - b^2)^15)^(1/2) - 995*a*b^19*c - 694*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 245*a*b^4*c*(-(4*a*c - b^2)^15)^(1/2)))/(512*(a^7*b^20*e^2 + 1048576*a^17*c^10*e^2 - 40*a^8*b^18*c*e^2 + 720*a^9*b^16*c^2*e^2 - 7680*a^10*b^14*c^3*e^2 + 53760*a^11*b^12*c^4*e^2 - 258048*a^12*b^10*c^5*e^2 + 860160*a^13*b^8*c^6*e^2 - 1966080*a^14*b^6*c^7*e^2 + 2949120*a^15*b^4*c^8*e^2 - 2621440*a^16*b^2*c^9*e^2)))^(1/2)*(x*(1099511627776*a^26*b*c^13*e^14 - 262144*a^15*b^23*c^2*e^14 + 11534336*a^16*b^21*c^3*e^14 - 230686720*a^17*b^19*c^4*e^14 + 2768240640*a^18*b^17*c^5*e^14 - 22145925120*a^19*b^15*c^6*e^14 + 124017180672*a^20*b^13*c^7*e^14 - 496068722688*a^21*b^11*c^8*e^14 + 1417339207680*a^22*b^9*c^9*e^14 - 2834678415360*a^23*b^7*c^10*e^14 + 3779571220480*a^24*b^5*c^11*e^14 - 3023656976384*a^25*b^3*c^12*e^14) + 1099511627776*a^26*b*c^13*d*e^13 - 262144*a^15*b^23*c^2*d*e^13 + 11534336*a^16*b^21*c^3*d*e^13 - 230686720*a^17*b^19*c^4*d*e^13 + 2768240640*a^18*b^17*c^5*d*e^13 - 22145925120*a^19*b^15*c^6*d*e^13 + 124017180672*a^20*b^13*c^7*d*e^13 - 496068722688*a^21*b^11*c^8*d*e^13 + 1417339207680*a^22*b^9*c^9*d*e^13 - 2834678415360*a^23*b^7*c^10*d*e^13 + 3779571220480*a^24*b^5*c^11*d*e^13 - 3023656976384*a^25*b^3*c^12*d*e^13) - 1185410973696*a^23*b*c^13*e^12 + 245760*a^12*b^23*c^2*e^12 - 10911744*a^13*b^21*c^3*e^12 + 220397568*a^14*b^19*c^4*e^12 - 2673082368*a^15*b^17*c^5*e^12 + 21630025728*a^16*b^15*c^6*e^12 - 122607894528*a^17*b^13*c^7*e^12 + 496773365760*a^18*b^11*c^8*e^12 - 1438679826432*a^19*b^9*c^9*e^12 + 2918430277632*a^20*b^7*c^10*e^12 - 3949222428672*a^21*b^5*c^11*e^12 + 3208340570112*a^22*b^3*c^12*e^12) + x*(271790899200*a^20*c^14*e^12 - 230400*a^9*b^22*c^3*e^12 + 9861120*a^10*b^20*c^4*e^12 - 191038464*a^11*b^18*c^5*e^12 + 2207803392*a^12*b^16*c^6*e^12 - 16878108672*a^13*b^14*c^7*e^12 + 89374851072*a^14*b^12*c^8*e^12 - 333226967040*a^15*b^10*c^9*e^12 + 869815812096*a^16*b^8*c^10*e^12 - 1543847804928*a^17*b^6*c^11*e^12 + 1747313491968*a^18*b^4*c^12*e^12 - 1101055131648*a^19*b^2*c^13*e^12) + 271790899200*a^20*c^14*d*e^11 - 230400*a^9*b^22*c^3*d*e^11 + 9861120*a^10*b^20*c^4*d*e^11 - 191038464*a^11*b^18*c^5*d*e^11 + 2207803392*a^12*b^16*c^6*d*e^11 - 16878108672*a^13*b^14*c^7*d*e^11 + 89374851072*a^14*b^12*c^8*d*e^11 - 333226967040*a^15*b^10*c^9*d*e^11 + 869815812096*a^16*b^8*c^10*d*e^11 - 1543847804928*a^17*b^6*c^11*d*e^11 + 1747313491968*a^18*b^4*c^12*d*e^11 - 1101055131648*a^19*b^2*c^13*d*e^11)*1i + (-(9*(25*b^21 - 25*b^6*(-(4*a*c - b^2)^15)^(1/2) + 18923520*a^10*b*c^10 + 17794*a^2*b^17*c^2 - 188095*a^3*b^15*c^3 + 1299860*a^4*b^13*c^4 - 6126640*a^5*b^11*c^5 + 19905600*a^6*b^9*c^6 - 43904256*a^

7*b^7*c^7 + 62684160*a^8*b^5*c^8 - 52039680*a^9*b^3*c^9 + 225*a^3*c^3*(-(4*a*c - b^2)^15)^(1/2) - 995*a*b^19*c - 694*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 245*a*b^4*c*(-(4*a*c - b^2)^15)^(1/2)))/(512*(a^7*b^20*e^2 + 1048576*a^17*c^10*e^2 - 40*a^8*b^18*c*e^2 + 720*a^9*b^16*c^2*e^2 - 7680*a^10*b^14*c^3*e^2 + 53760*a^11*b^12*c^4*e^2 - 258048*a^12*b^10*c^5*e^2 + 860160*a^13*b^8*c^6*e^2 - 1966080*a^14*b^6*c^7*e^2 + 2949120*a^15*b^4*c^8*e^2 - 2621440*a^16*b^2*c^9*e^2)))^(1/2)*((-(9*(25*b^21 - 25*b^6*(-(4*a*c - b^2)^15)^(1/2) + 18923520*a^10*b*c^10 + 17794*a^2*b^17*c^2 - 188095*a^3*b^15*c^3 + 1299860*a^4*b^13*c^4 - 6126640*a^5*b^11*c^5 + 19905600*a^6*b^9*c^6 - 43904256*a^7*b^7*c^7 + 62684160*a^8*b^5*c^8 - 52039680*a^9*b^3*c^9 + 225*a^3*c^3*(-(4*a*c - b^2)^15)^(1/2) - 995*a*b^19*c - 694*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 245*a*b^4*c*(-(4*a*c - b^2)^15)^(1/2)))/(512*(a^7*b^20*e^2 + 1048576*a^17*c^10*e^2 - 40*a^8*b^18*c*e^2 + 720*a^9*b^16*c^2*e^2 - 7680*a^10*b^14*c^3*e^2 + 53760*a^11*b^12*c^4*e^2 - 258048*a^12*b^10*c^5*e^2 + 860160*a^13*b^8*c^6*e^2 - 1966080*a^14*b^6*c^7*e^2 + 2949120*a^15*b^4*c^8*e^2 - 2621440*a^16*b^2*c^9*e^2)))^(1/2)*((-(9*(25*b^21 - 25*b^6*(-(4*a*c - b^2)^15)^(1/2) + 18923520*a^10*b*c^10 + 17794*a^2*b^17*c^2 - 188095*a^3*b^15*c^3 + 1299860*a^4*b^13*c^4 - 6126640*a^5*b^11*c^5 + 19905600*a^6*b^9*c^6 - 43904256*a^7*b^7*c^7 + 62684160*a^8*b^5*c^8 - 52039680*a^9*b^3*c^9 + 225*a^3*c^3*(-(4*a*c - b^2)^15)^(1/2) - 995*a*b^19*c - 694*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 245*a*b^4*c*(-(4*a*c - b^2)^15)^(1/2)))/(512*(a^7*b^20*e^2 + 1048576*a^17*c^10*e^2 - 40*a^8*b^18*c*e^2 + 720*a^9*b^16*c^2*e^2 - 7680*a^10*b^14*c^3*e^2 + 53760*a^11*b^12*c^4*e^2 - 258048*a^12*b^10*c^5*e^2 + 860160*a^13*b^8*c^6*e^2 - 1966080*a^14*b^6*c^7*e^2 + 2949120*a^15*b^4*c^8*e^2 - 2621440*a^16*b^2*c^9*e^2)))^(1/2)*(x*(1099511627776*a^26*b*c^13*e^14 - 262144*a^15*b^23*c^2*e^14 + 11534336*a^16*b^21*c^3*e^14 - 230686720*a^17*b^19*c^4*e^14 + 2768240640*a^18*b^17*c^5*e^14 - 22145925120*a^19*b^15*c^6*e^14 + 124017180672*a^20*b^13*c^7*e^14 - 496068722688*a^21*b^11*c^8*e^14 + 1417339207680*a^22*b^9*c^9*e^14 - 2834678415360*a^23*b^7*c^10*e^14 + 3779571220480*a^24*b^5*c^11*e^14 - 3023656976384*a^25*b^3*c^12*e^14) + 1099511627776*a^26*b*c^13*d*e^13 - 262144*a^15*b^23*c^2*d*e^13 + 11534336*a^16*b^21*c^3*d*e^13 - 230686720*a^17*b^19*c^4*d*e^13 + 2768240640*a^18*b^17*c^5*d*e^13 - 22145925120*a^19*b^15*c^6*d*e^13 + 124017180672*a^20*b^13*c^7*d*e^13 - 496068722688*a^21*b^11*c^8*d*e^13 + 1417339207680*a^22*b^9*c^9*d*e^13 - 2834678415360*a^23*b^7*c^10*d*e^13 + 3779571220480*a^24*b^5*c^11*d*e^13 - 3023656976384*a^25*b^3*c^12*d*e^13) + 1185410973696*a^23*b*c^13*e^12 - 245760*a^12*b^23*c^2*e^12 + 10911744*a^13*b^21*c^3*e^12 - 220397568*a^14*b^19*c^4*e^12 + 2673082368*a^15*b^17*c^5*e^12 - 21630025728*a^16*b^15*c^6*e^12 + 122607894528*a^17*b^13*c^7*e^12 - 496773365760*a^18*b^11*c^8*e^12 + 1438679826432*a^19*b^9*c^9*e^12 - 2918430277632*a^20*b^7*c^10*e^12 + 3949222428672*a^21*b^5*c^11*e^12 - 3208340570112*a^22*b^3*c^12*e^12) + x*(271790899200*a^20*c^14*e^12 - 230400*a^9*b^22*c^3*e^12 + 9861120*a^10*b^20*c^4*e^12 - 191038464*a^11*b^18*c^5*e^12 + 2207803392*a^12*b^16*c^6*e^12 - 16878108672*a^13*b^14*c^7*e^12 + 89374851072*a^14*b^12*c^8*e^12 - 333226967040*a^15*b^10*c^9*e^12 + 869815812096*a^16*b^8*c^10*e^12 - 1543847804928*a^17*b^6*c^11*e^12 + 1747313491968*a^18*b^4*c^12*e^12 - 1101055131648*a^19*b^2*c^13*e^12) + 271790899200*a^20*c^14*d*e^11 - 230400*a^9*b^22*c^3*d*e^11 + 9861120*a^10*b^20*c^4*d*e^11 - 191038464*a^11*b^18*c^5*d*e^11 + 2207803392*a^12*b^16*c^6*d*e^11 - 16878108672*a^13*b^14*c^7*d*e^11 + 89374851072*a^14*b^12*c^8*d*e^11 - 333226967040*a^15*b^10*c^9*d*e^11 + 869815812096*a^16*b^8*c^10*d*e^11 - 1543847804928*a^17*b^6*c^11*d*e^11 + 1747313491968*a^18*b^4*c^12*d*e^11 - 1101055131648*a^19*b^2*c^13*d*e^11)*1i)/((-(9*(25*b^21 - 25*b^6*(-(4*a*c - b^2)^15)^(1/2) + 18923520*a^10*b*c^10 + 17794*a^2*b^17*c^2 - 188095*a^3*b^15*c^3 + 1299860*a^4*b^13*c^4 - 6126640*a^5*b^11*c^5 + 19905600*a^6*b^9*c^6 - 43904256*a^7*b^7*c^7 + 62684160*a^8*b^5*c^8 - 52039680*a^9*b^3*c^9 + 225*a^3*c^3*(-(4*a*c - b^2)^15)^(1/2) - 995*a*b^19*c - 694*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 245*a*b^4*c*(-(4*a*c - b^2)^15)^(1/2)))/(512*(a^7*b^20*e^2 + 1048576*a^17*c^10*e^2 - 40*a^8*b^18*c*e^2 + 720*a^9*b^16*c^2*e^2 - 7680*a^10*b^14*c^3*e^2 + 53760*a^11*b^12*c^4*e^2 - 258048*a^12*b^10*c^5*e^2 + 860160*a^13*b^8*c

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^6*e^2 - 1966080*a^14*b^6*c^7*e^2 + 2949120*a^15*b^4*c^8*e^2 - 2621440*a^16
*b^2*c^9*e^2)))^(1/2)*((-(9*(25*b^21 - 25*b^6*(-(4*a*c - b^2)^15)^(1/2) + 1
8923520*a^10*b*c^10 + 17794*a^2*b^17*c^2 - 188095*a^3*b^15*c^3 + 1299860*a^
4*b^13*c^4 - 6126640*a^5*b^11*c^5 + 19905600*a^6*b^9*c^6 - 43904256*a^7*b^7
*c^7 + 62684160*a^8*b^5*c^8 - 52039680*a^9*b^3*c^9 + 225*a^3*c^3*(-(4*a*c -
 b^2)^15)^(1/2) - 995*a*b^19*c - 694*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2)
+ 245*a*b^4*c*(-(4*a*c - b^2)^15)^(1/2)))/(512*(a^7*b^20*e^2 + 1048576*a^17
*c^10*e^2 - 40*a^8*b^18*c*e^2 + 720*a^9*b^16*c^2*e^2 - 7680*a^10*b^14*c^3*e
^2 + 53760*a^11*b^12*c^4*e^2 - 258048*a^12*b^10*c^5*e^2 + 860160*a^13*b^8*c
^6*e^2 - 1966080*a^14*b^6*c^7*e^2 + 2949120*a^15*b^4*c^8*e^2 - 2621440*a^16
*b^2*c^9*e^2)))^(1/2)*((-(9*(25*b^21 - 25*b^6*(-(4*a*c - b^2)^15)^(1/2) + 1
8923520*a^10*b*c^10 + 17794*a^2*b^17*c^2 - 188095*a^3*b^15*c^3 + 1299860*a^
4*b^13*c^4 - 6126640*a^5*b^11*c^5 + 19905600*a^6*b^9*c^6 - 43904256*a^7*b^7
*c^7 + 62684160*a^8*b^5*c^8 - 52039680*a^9*b^3*c^9 + 225*a^3*c^3*(-(4*a*c -
 b^2)^15)^(1/2) - 995*a*b^19*c - 694*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2)
+ 245*a*b^4*c*(-(4*a*c - b^2)^15)^(1/2)))/(512*(a^7*b^20*e^2 + 1048576*a^17
*c^10*e^2 - 40*a^8*b^18*c*e^2 + 720*a^9*b^16*c^2*e^2 - 7680*a^10*b^14*c^3*e
^2 + 53760*a^11*b^12*c^4*e^2 - 258048*a^12*b^10*c^5*e^2 + 860160*a^13*b^8*c
^6*e^2 - 1966080*a^14*b^6*c^7*e^2 + 2949120*a^15*b^4*c^8*e^2 - 2621440*a^16
*b^2*c^9*e^2)))^(1/2)*(x*(1099511627776*a^26*b*c^13*e^14 - 262144*a^15*b^23
*c^2*e^14 + 11534336*a^16*b^21*c^3*e^14 - 230686720*a^17*b^19*c^4*e^14 + 27
68240640*a^18*b^17*c^5*e^14 - 22145925120*a^19*b^15*c^6*e^14 + 124017180672
*a^20*b^13*c^7*e^14 - 496068722688*a^21*b^11*c^8*e^14 + 1417339207680*a^22*
b^9*c^9*e^14 - 2834678415360*a^23*b^7*c^10*e^14 + 3779571220480*a^24*b^5*c^
11*e^14 - 3023656976384*a^25*b^3*c^12*e^14) + 1099511627776*a^26*b*c^13*d*e
^13 - 262144*a^15*b^23*c^2*d*e^13 + 11534336*a^16*b^21*c^3*d*e^13 - 2306867
20*a^17*b^19*c^4*d*e^13 + 2768240640*a^18*b^17*c^5*d*e^13 - 22145925120*a^1
9*b^15*c^6*d*e^13 + 124017180672*a^20*b^13*c^7*d*e^13 - 496068722688*a^21*b
^11*c^8*d*e^13 + 1417339207680*a^22*b^9*c^9*d*e^13 - 2834678415360*a^23*b^7
*c^10*d*e^13 + 3779571220480*a^24*b^5*c^11*d*e^13 - 3023656976384*a^25*b^3*
c^12*d*e^13) + 1185410973696*a^23*b*c^13*e^12 - 245760*a^12*b^23*c^2*e^12 +
 10911744*a^13*b^21*c^3*e^12 - 220397568*a^14*b^19*c^4*e^12 + 2673082368*a^
15*b^17*c^5*e^12 - 21630025728*a^16*b^15*c^6*e^12 + 122607894528*a^17*b^13*
c^7*e^12 - 496773365760*a^18*b^11*c^8*e^12 + 1438679826432*a^19*b^9*c^9*e^1
2 - 2918430277632*a^20*b^7*c^10*e^12 + 3949222428672*a^21*b^5*c^11*e^12 - 3
208340570112*a^22*b^3*c^12*e^12) + x*(271790899200*a^20*c^14*e^12 - 230400*
a^9*b^22*c^3*e^12 + 9861120*a^10*b^20*c^4*e^12 - 191038464*a^11*b^18*c^5*e^
12 + 2207803392*a^12*b^16*c^6*e^12 - 16878108672*a^13*b^14*c^7*e^12 + 89374
851072*a^14*b^12*c^8*e^12 - 333226967040*a^15*b^10*c^9*e^12 + 869815812096*
a^16*b^8*c^10*e^12 - 1543847804928*a^17*b^6*c^11*e^12 + 1747313491968*a^18*
b^4*c^12*e^12 - 1101055131648*a^19*b^2*c^13*e^12) + 271790899200*a^20*c^14*
d*e^11 - 230400*a^9*b^22*c^3*d*e^11 + 9861120*a^10*b^20*c^4*d*e^11 - 191038
464*a^11*b^18*c^5*d*e^11 + 2207803392*a^12*b^16*c^6*d*e^11 - 16878108672*a^
13*b^14*c^7*d*e^11 + 89374851072*a^14*b^12*c^8*d*e^11 - 333226967040*a^15*b
^10*c^9*d*e^11 + 869815812096*a^16*b^8*c^10*d*e^11 - 1543847804928*a^17*b^6
*c^11*d*e^11 + 1747313491968*a^18*b^4*c^12*d*e^11 - 1101055131648*a^19*b^2*
c^13*d*e^11) - (-(9*(25*b^21 - 25*b^6*(-(4*a*c - b^2)^15)^(1/2) + 18923520*
a^10*b*c^10 + 17794*a^2*b^17*c^2 - 188095*a^3*b^15*c^3 + 1299860*a^4*b^13*c
^4 - 6126640*a^5*b^11*c^5 + 19905600*a^6*b^9*c^6 - 43904256*a^7*b^7*c^7 + 6
2684160*a^8*b^5*c^8 - 52039680*a^9*b^3*c^9 + 225*a^3*c^3*(-(4*a*c - b^2)^15
)^(1/2) - 995*a*b^19*c - 694*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 245*a*
b^4*c*(-(4*a*c - b^2)^15)^(1/2)))/(512*(a^7*b^20*e^2 + 1048576*a^17*c^10*e^
2 - 40*a^8*b^18*c*e^2 + 720*a^9*b^16*c^2*e^2 - 7680*a^10*b^14*c^3*e^2 + 537
60*a^11*b^12*c^4*e^2 - 258048*a^12*b^10*c^5*e^2 + 860160*a^13*b^8*c^6*e^2 -
 1966080*a^14*b^6*c^7*e^2 + 2949120*a^15*b^4*c^8*e^2 - 2621440*a^16*b^2*c^9
*e^2)))^(1/2)*((-(9*(25*b^21 - 25*b^6*(-(4*a*c - b^2)^15)^(1/2) + 18923520*
a^10*b*c^10 + 17794*a^2*b^17*c^2 - 188095*a^3*b^15*c^3 + 1299860*a^4*b^13*c
^4 - 6126640*a^5*b^11*c^5 + 19905600*a^6*b^9*c^6 - 43904256*a^7*b^7*c^7 + 6
2684160*a^8*b^5*c^8 - 52039680*a^9*b^3*c^9 + 225*a^3*c^3*(-(4*a*c - b^2)^15
```

)^(1/2) - 995*a*b^19*c - 694*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 245*a*
b^4*c*(-(4*a*c - b^2)^15)^(1/2)))/(512*(a^7*b^20*e^2 + 1048576*a^17*c^10*e^
2 - 40*a^8*b^18*c*e^2 + 720*a^9*b^16*c^2*e^2 - 7680*a^10*b^14*c^3*e^2 + 537
60*a^11*b^12*c^4*e^2 - 258048*a^12*b^10*c^5*e^2 + 860160*a^13*b^8*c^6*e^2 -
 1966080*a^14*b^6*c^7*e^2 + 2949120*a^15*b^4*c^8*e^2 - 2621440*a^16*b^2*c^9
e^2)))^(1/2)((-(9*(25*b^21 - 25*b^6*(-(4*a*c - b^2)^15)^(1/2) + 18923520*
a^10*b*c^10 + 17794*a^2*b^17*c^2 - 188095*a^3*b^15*c^3 + 1299860*a^4*b^13*c
^4 - 6126640*a^5*b^11*c^5 + 19905600*a^6*b^9*c^6 - 43904256*a^7*b^7*c^7 + 6
2684160*a^8*b^5*c^8 - 52039680*a^9*b^3*c^9 + 225*a^3*c^3*(-(4*a*c - b^2)^15
)^(1/2) - 995*a*b^19*c - 694*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 245*a*
b^4*c*(-(4*a*c - b^2)^15)^(1/2)))/(512*(a^7*b^20*e^2 + 1048576*a^17*c^10*e^
2 - 40*a^8*b^18*c*e^2 + 720*a^9*b^16*c^2*e^2 - 7680*a^10*b^14*c^3*e^2 + 537
60*a^11*b^12*c^4*e^2 - 258048*a^12*b^10*c^5*e^2 + 860160*a^13*b^8*c^6*e^2 -
 1966080*a^14*b^6*c^7*e^2 + 2949120*a^15*b^4*c^8*e^2 - 2621440*a^16*b^2*c^9
e^2)))^(1/2)(x*(1099511627776*a^26*b*c^13*e^14 - 262144*a^15*b^23*c^2*e^1
4 + 11534336*a^16*b^21*c^3*e^14 - 230686720*a^17*b^19*c^4*e^14 + 2768240640
*a^18*b^17*c^5*e^14 - 22145925120*a^19*b^15*c^6*e^14 + 124017180672*a^20*b^
13*c^7*e^14 - 496068722688*a^21*b^11*c^8*e^14 + 1417339207680*a^22*b^9*c^9*
e^14 - 2834678415360*a^23*b^7*c^10*e^14 + 3779571220480*a^24*b^5*c^11*e^14
- 3023656976384*a^25*b^3*c^12*e^14) + 1099511627776*a^26*b*c^13*d*e^13 - 26
2144*a^15*b^23*c^2*d*e^13 + 11534336*a^16*b^21*c^3*d*e^13 - 230686720*a^17*
b^19*c^4*d*e^13 + 2768240640*a^18*b^17*c^5*d*e^13 - 22145925120*a^19*b^15*c
^6*d*e^13 + 124017180672*a^20*b^13*c^7*d*e^13 - 496068722688*a^21*b^11*c^8*
d*e^13 + 1417339207680*a^22*b^9*c^9*d*e^13 - 2834678415360*a^23*b^7*c^10*d*
e^13 + 3779571220480*a^24*b^5*c^11*d*e^13 - 3023656976384*a^25*b^3*c^12*d*e
^13) - 1185410973696*a^23*b*c^13*e^12 + 245760*a^12*b^23*c^2*e^12 - 1091174
4*a^13*b^21*c^3*e^12 + 220397568*a^14*b^19*c^4*e^12 - 2673082368*a^15*b^17*
c^5*e^12 + 21630025728*a^16*b^15*c^6*e^12 - 122607894528*a^17*b^13*c^7*e^12
 + 496773365760*a^18*b^11*c^8*e^12 - 1438679826432*a^19*b^9*c^9*e^12 + 2918
430277632*a^20*b^7*c^10*e^12 - 3949222428672*a^21*b^5*c^11*e^12 + 320834057
0112*a^22*b^3*c^12*e^12) + x*(271790899200*a^20*c^14*e^12 - 230400*a^9*b^22
*c^3*e^12 + 9861120*a^10*b^20*c^4*e^12 - 191038464*a^11*b^18*c^5*e^12 + 220
7803392*a^12*b^16*c^6*e^12 - 16878108672*a^13*b^14*c^7*e^12 + 89374851072*a
^14*b^12*c^8*e^12 - 333226967040*a^15*b^10*c^9*e^12 + 869815812096*a^16*b^8
*c^10*e^12 - 1543847804928*a^17*b^6*c^11*e^12 + 1747313491968*a^18*b^4*c^12
*e^12 - 1101055131648*a^19*b^2*c^13*e^12) + 271790899200*a^20*c^14*d*e^11 -
 230400*a^9*b^22*c^3*d*e^11 + 9861120*a^10*b^20*c^4*d*e^11 - 191038464*a^11
*b^18*c^5*d*e^11 + 2207803392*a^12*b^16*c^6*d*e^11 - 16878108672*a^13*b^14*
c^7*d*e^11 + 89374851072*a^14*b^12*c^8*d*e^11 - 333226967040*a^15*b^10*c^9*
d*e^11 + 869815812096*a^16*b^8*c^10*d*e^11 - 1543847804928*a^17*b^6*c^11*d*
e^11 + 1747313491968*a^18*b^4*c^12*d*e^11 - 1101055131648*a^19*b^2*c^13*d*e
^11) + 191102976000*a^17*c^14*e^10 + 2851200*a^9*b^16*c^6*e^10 - 92568960*a
^10*b^14*c^7*e^10 + 1312630272*a^11*b^12*c^8*e^10 - 10611136512*a^12*b^10*c
^9*e^10 + 53445353472*a^13*b^8*c^10*e^10 - 171591892992*a^14*b^6*c^11*e^10
+ 342580396032*a^15*b^4*c^12*e^10 - 388363714560*a^16*b^2*c^13*e^10))*(-(9*
(25*b^21 - 25*b^6*(-(4*a*c - b^2)^15)^(1/2) + 18923520*a^10*b*c^10 + 17794*
a^2*b^17*c^2 - 188095*a^3*b^15*c^3 + 1299860*a^4*b^13*c^4 - 6126640*a^5*b^1
1*c^5 + 19905600*a^6*b^9*c^6 - 43904256*a^7*b^7*c^7 + 62684160*a^8*b^5*c^8
- 52039680*a^9*b^3*c^9 + 225*a^3*c^3*(-(4*a*c - b^2)^15)^(1/2) - 995*a*b^19
*c - 694*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 245*a*b^4*c*(-(4*a*c - b^2
)^15)^(1/2)))/(512*(a^7*b^20*e^2 + 1048576*a^17*c^10*e^2 - 40*a^8*b^18*c*e^
2 + 720*a^9*b^16*c^2*e^2 - 7680*a^10*b^14*c^3*e^2 + 53760*a^11*b^12*c^4*e^2
 - 258048*a^12*b^10*c^5*e^2 + 860160*a^13*b^8*c^6*e^2 - 1966080*a^14*b^6*c^
7*e^2 + 2949120*a^15*b^4*c^8*e^2 - 2621440*a^16*b^2*c^9*e^2)))^(1/2)*2i - a
tan(((-(9*(25*b^21 + 25*b^6*(-(4*a*c - b^2)^15)^(1/2) + 18923520*a^10*b*c^1
0 + 17794*a^2*b^17*c^2 - 188095*a^3*b^15*c^3 + 1299860*a^4*b^13*c^4 - 61266
40*a^5*b^11*c^5 + 19905600*a^6*b^9*c^6 - 43904256*a^7*b^7*c^7 + 62684160*a^
8*b^5*c^8 - 52039680*a^9*b^3*c^9 - 225*a^3*c^3*(-(4*a*c - b^2)^15)^(1/2) -
995*a*b^19*c + 694*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 245*a*b^4*c*(-(4

*a*c - b^2)^15)^(1/2)))/(512*(a^7*b^20*e^2 + 1048576*a^17*c^10*e^2 - 40*a^8*b^18*c*e^2 + 720*a^9*b^16*c^2*e^2 - 7680*a^10*b^14*c^3*e^2 + 53760*a^11*b^12*c^4*e^2 - 258048*a^12*b^10*c^5*e^2 + 860160*a^13*b^8*c^6*e^2 - 1966080*a^14*b^6*c^7*e^2 + 2949120*a^15*b^4*c^8*e^2 - 2621440*a^16*b^2*c^9*e^2)))^(1/2)*((-(9*(25*b^21 + 25*b^6*(-(4*a*c - b^2)^15)^(1/2) + 18923520*a^10*b*c^10 + 17794*a^2*b^17*c^2 - 188095*a^3*b^15*c^3 + 1299860*a^4*b^13*c^4 - 6126640*a^5*b^11*c^5 + 19905600*a^6*b^9*c^6 - 43904256*a^7*b^7*c^7 + 62684160*a^8*b^5*c^8 - 52039680*a^9*b^3*c^9 - 225*a^3*c^3*(-(4*a*c - b^2)^15)^(1/2) - 995*a*b^19*c + 694*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 245*a*b^4*c*(-(4*a*c - b^2)^15)^(1/2)))/(512*(a^7*b^20*e^2 + 1048576*a^17*c^10*e^2 - 40*a^8*b^18*c*e^2 + 720*a^9*b^16*c^2*e^2 - 7680*a^10*b^14*c^3*e^2 + 53760*a^11*b^12*c^4*e^2 - 258048*a^12*b^10*c^5*e^2 + 860160*a^13*b^8*c^6*e^2 - 1966080*a^14*b^6*c^7*e^2 + 2949120*a^15*b^4*c^8*e^2 - 2621440*a^16*b^2*c^9*e^2)))^(1/2)*((-(9*(25*b^21 + 25*b^6*(-(4*a*c - b^2)^15)^(1/2) + 18923520*a^10*b*c^10 + 17794*a^2*b^17*c^2 - 188095*a^3*b^15*c^3 + 1299860*a^4*b^13*c^4 - 6126640*a^5*b^11*c^5 + 19905600*a^6*b^9*c^6 - 43904256*a^7*b^7*c^7 + 62684160*a^8*b^5*c^8 - 52039680*a^9*b^3*c^9 - 225*a^3*c^3*(-(4*a*c - b^2)^15)^(1/2) - 995*a*b^19*c + 694*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 245*a*b^4*c*(-(4*a*c - b^2)^15)^(1/2)))/(512*(a^7*b^20*e^2 + 1048576*a^17*c^10*e^2 - 40*a^8*b^18*c*e^2 + 720*a^9*b^16*c^2*e^2 - 7680*a^10*b^14*c^3*e^2 + 53760*a^11*b^12*c^4*e^2 - 258048*a^12*b^10*c^5*e^2 + 860160*a^13*b^8*c^6*e^2 - 1966080*a^14*b^6*c^7*e^2 + 2949120*a^15*b^4*c^8*e^2 - 2621440*a^16*b^2*c^9*e^2)))^(1/2)*(x*(1099511627776*a^26*b*c^13*e^14 - 262144*a^15*b^23*c^2*e^14 + 11534336*a^16*b^21*c^3*e^14 - 230686720*a^17*b^19*c^4*e^14 + 2768240640*a^18*b^17*c^5*e^14 - 22145925120*a^19*b^15*c^6*e^14 + 124017180672*a^20*b^13*c^7*e^14 - 496068722688*a^21*b^11*c^8*e^14 + 1417339207680*a^22*b^9*c^9*e^14 - 2834678415360*a^23*b^7*c^10*e^14 + 3779571220480*a^24*b^5*c^11*e^14 - 3023656976384*a^25*b^3*c^12*e^14) + 1099511627776*a^26*b*c^13*d*e^13 - 262144*a^15*b^23*c^2*d*e^13 + 11534336*a^16*b^21*c^3*d*e^13 - 230686720*a^17*b^19*c^4*d*e^13 + 2768240640*a^18*b^17*c^5*d*e^13 - 22145925120*a^19*b^15*c^6*d*e^13 + 124017180672*a^20*b^13*c^7*d*e^13 - 496068722688*a^21*b^11*c^8*d*e^13 + 1417339207680*a^22*b^9*c^9*d*e^13 - 2834678415360*a^23*b^7*c^10*d*e^13 + 3779571220480*a^24*b^5*c^11*d*e^13 - 3023656976384*a^25*b^3*c^12*d*e^13) - 1185410973696*a^23*b*c^13*e^12 + 245760*a^12*b^23*c^2*e^12 - 10911744*a^13*b^21*c^3*e^12 + 220397568*a^14*b^19*c^4*e^12 - 2673082368*a^15*b^17*c^5*e^12 + 21630025728*a^16*b^15*c^6*e^12 - 122607894528*a^17*b^13*c^7*e^12 + 496773365760*a^18*b^11*c^8*e^12 - 1438679826432*a^19*b^9*c^9*e^12 + 2918430277632*a^20*b^7*c^10*e^12 - 3949222428672*a^21*b^5*c^11*e^12 + 3208340570112*a^22*b^3*c^12*e^12) + x*(271790899200*a^20*c^14*e^12 - 230400*a^9*b^22*c^3*e^12 + 9861120*a^10*b^20*c^4*e^12 - 191038464*a^11*b^18*c^5*e^12 + 2207803392*a^12*b^16*c^6*e^12 - 16878108672*a^13*b^14*c^7*e^12 + 89374851072*a^14*b^12*c^8*e^12 - 333226967040*a^15*b^10*c^9*e^12 + 869815812096*a^16*b^8*c^10*e^12 - 1543847804928*a^17*b^6*c^11*e^12 + 1747313491968*a^18*b^4*c^12*e^12 - 1101055131648*a^19*b^2*c^13*e^12) + 271790899200*a^20*c^14*d*e^11 - 230400*a^9*b^22*c^3*d*e^11 + 9861120*a^10*b^20*c^4*d*e^11 - 191038464*a^11*b^18*c^5*d*e^11 + 2207803392*a^12*b^16*c^6*d*e^11 - 16878108672*a^13*b^14*c^7*d*e^11 + 89374851072*a^14*b^12*c^8*d*e^11 - 333226967040*a^15*b^10*c^9*d*e^11 + 869815812096*a^16*b^8*c^10*d*e^11 - 1543847804928*a^17*b^6*c^11*d*e^11 + 1747313491968*a^18*b^4*c^12*d*e^11 - 1101055131648*a^19*b^2*c^13*d*e^11)*1i + (-(9*(25*b^21 + 25*b^6*(-(4*a*c - b^2)^15)^(1/2) + 18923520*a^10*b*c^10 + 17794*a^2*b^17*c^2 - 188095*a^3*b^15*c^3 + 1299860*a^4*b^13*c^4 - 6126640*a^5*b^11*c^5 + 19905600*a^6*b^9*c^6 - 43904256*a^7*b^7*c^7 + 62684160*a^8*b^5*c^8 - 52039680*a^9*b^3*c^9 - 225*a^3*c^3*(-(4*a*c - b^2)^15)^(1/2) - 995*a*b^19*c + 694*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 245*a*b^4*c*(-(4*a*c - b^2)^15)^(1/2)))/(512*(a^7*b^20*e^2 + 1048576*a^17*c^10*e^2 - 40*a^8*b^18*c*e^2 + 720*a^9*b^16*c^2*e^2 - 7680*a^10*b^14*c^3*e^2 + 53760*a^11*b^12*c^4*e^2 - 258048*a^12*b^10*c^5*e^2 + 860160*a^13*b^8*c^6*e^2 - 1966080*a^14*b^6*c^7*e^2 + 2949120*a^15*b^4*c^8*e^2 - 2621440*a^16*b^2*c^9*e^2)))^(1/2)*((-(9*(25*b^21 + 25*b^6*(-(4*a*c - b^2)^15)^(1/2) + 18923520*a^10*b*c^10 + 1

7794*a^2*b^17*c^2 - 188095*a^3*b^15*c^3 + 1299860*a^4*b^13*c^4 - 6126640*a^5*b^11*c^5 + 19905600*a^6*b^9*c^6 - 43904256*a^7*b^7*c^7 + 62684160*a^8*b^5*c^8 - 52039680*a^9*b^3*c^9 - 225*a^3*c^3*(-(4*a*c - b^2)^15)^(1/2) - 995*a*b^19*c + 694*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 245*a*b^4*c*(-(4*a*c - b^2)^15)^(1/2)))/(512*(a^7*b^20*e^2 + 1048576*a^17*c^10*e^2 - 40*a^8*b^18*c*e^2 + 720*a^9*b^16*c^2*e^2 - 7680*a^10*b^14*c^3*e^2 + 53760*a^11*b^12*c^4*e^2 - 258048*a^12*b^10*c^5*e^2 + 860160*a^13*b^8*c^6*e^2 - 1966080*a^14*b^6*c^7*e^2 + 2949120*a^15*b^4*c^8*e^2 - 2621440*a^16*b^2*c^9*e^2)))^(1/2)*((-(9*(25*b^21 + 25*b^6*(-(4*a*c - b^2)^15)^(1/2) + 18923520*a^10*b*c^10 + 17794*a^2*b^17*c^2 - 188095*a^3*b^15*c^3 + 1299860*a^4*b^13*c^4 - 6126640*a^5*b^11*c^5 + 19905600*a^6*b^9*c^6 - 43904256*a^7*b^7*c^7 + 62684160*a^8*b^5*c^8 - 52039680*a^9*b^3*c^9 - 225*a^3*c^3*(-(4*a*c - b^2)^15)^(1/2) - 995*a*b^19*c + 694*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 245*a*b^4*c*(-(4*a*c - b^2)^15)^(1/2)))/(512*(a^7*b^20*e^2 + 1048576*a^17*c^10*e^2 - 40*a^8*b^18*c*e^2 + 720*a^9*b^16*c^2*e^2 - 7680*a^10*b^14*c^3*e^2 + 53760*a^11*b^12*c^4*e^2 - 258048*a^12*b^10*c^5*e^2 + 860160*a^13*b^8*c^6*e^2 - 1966080*a^14*b^6*c^7*e^2 + 2949120*a^15*b^4*c^8*e^2 - 2621440*a^16*b^2*c^9*e^2)))^(1/2)*(x*(1099511627776*a^26*b*c^13*e^14 - 262144*a^15*b^23*c^2*e^14 + 11534336*a^16*b^21*c^3*e^14 - 230686720*a^17*b^19*c^4*e^14 + 2768240640*a^18*b^17*c^5*e^14 - 22145925120*a^19*b^15*c^6*e^14 + 124017180672*a^20*b^13*c^7*e^14 - 496068722688*a^21*b^11*c^8*e^14 + 1417339207680*a^22*b^9*c^9*e^14 - 2834678415360*a^23*b^7*c^10*e^14 + 3779571220480*a^24*b^5*c^11*e^14 - 3023656976384*a^25*b^3*c^12*e^14) + 1099511627776*a^26*b*c^13*d*e^13 - 262144*a^15*b^23*c^2*d*e^13 + 11534336*a^16*b^21*c^3*d*e^13 - 230686720*a^17*b^19*c^4*d*e^13 + 2768240640*a^18*b^17*c^5*d*e^13 - 22145925120*a^19*b^15*c^6*d*e^13 + 124017180672*a^20*b^13*c^7*d*e^13 - 496068722688*a^21*b^11*c^8*d*e^13 + 1417339207680*a^22*b^9*c^9*d*e^13 - 2834678415360*a^23*b^7*c^10*d*e^13 + 3779571220480*a^24*b^5*c^11*d*e^13 - 3023656976384*a^25*b^3*c^12*d*e^13) + 1185410973696*a^23*b*c^13*e^12 - 245760*a^12*b^23*c^2*e^12 + 10911744*a^13*b^21*c^3*e^12 - 220397568*a^14*b^19*c^4*e^12 + 2673082368*a^15*b^17*c^5*e^12 - 21630025728*a^16*b^15*c^6*e^12 + 122607894528*a^17*b^13*c^7*e^12 - 496773365760*a^18*b^11*c^8*e^12 + 1438679826432*a^19*b^9*c^9*e^12 - 2918430277632*a^20*b^7*c^10*e^12 + 3949222428672*a^21*b^5*c^11*e^12 - 3208340570112*a^22*b^3*c^12*e^12) + x*(271790899200*a^20*c^14*e^12 - 230400*a^9*b^22*c^3*e^12 + 9861120*a^10*b^20*c^4*e^12 - 191038464*a^11*b^18*c^5*e^12 + 2207803392*a^12*b^16*c^6*e^12 - 16878108672*a^13*b^14*c^7*e^12 + 89374851072*a^14*b^12*c^8*e^12 - 333226967040*a^15*b^10*c^9*e^12 + 869815812096*a^16*b^8*c^10*e^12 - 1543847804928*a^17*b^6*c^11*e^12 + 1747313491968*a^18*b^4*c^12*e^12 - 1101055131648*a^19*b^2*c^13*e^12) + 271790899200*a^20*c^14*d*e^11 - 230400*a^9*b^22*c^3*d*e^11 + 9861120*a^10*b^20*c^4*d*e^11 - 191038464*a^11*b^18*c^5*d*e^11 + 2207803392*a^12*b^16*c^6*d*e^11 - 16878108672*a^13*b^14*c^7*d*e^11 + 89374851072*a^14*b^12*c^8*d*e^11 - 333226967040*a^15*b^10*c^9*d*e^11 + 869815812096*a^16*b^8*c^10*d*e^11 - 1543847804928*a^17*b^6*c^11*d*e^11 + 1747313491968*a^18*b^4*c^12*d*e^11 - 1101055131648*a^19*b^2*c^13*d*e^11)*1i)/((-(9*(25*b^21 + 25*b^6*(-(4*a*c - b^2)^15)^(1/2) + 18923520*a^10*b*c^10 + 17794*a^2*b^17*c^2 - 188095*a^3*b^15*c^3 + 1299860*a^4*b^13*c^4 - 6126640*a^5*b^11*c^5 + 19905600*a^6*b^9*c^6 - 43904256*a^7*b^7*c^7 + 62684160*a^8*b^5*c^8 - 52039680*a^9*b^3*c^9 - 225*a^3*c^3*(-(4*a*c - b^2)^15)^(1/2) - 995*a*b^19*c + 694*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 245*a*b^4*c*(-(4*a*c - b^2)^15)^(1/2)))/(512*(a^7*b^20*e^2 + 1048576*a^17*c^10*e^2 - 40*a^8*b^18*c*e^2 + 720*a^9*b^16*c^2*e^2 - 7680*a^10*b^14*c^3*e^2 + 53760*a^11*b^12*c^4*e^2 - 258048*a^12*b^10*c^5*e^2 + 860160*a^13*b^8*c^6*e^2 - 1966080*a^14*b^6*c^7*e^2 + 2949120*a^15*b^4*c^8*e^2 - 2621440*a^16*b^2*c^9*e^2)))^(1/2)*((-(9*(25*b^21 + 25*b^6*(-(4*a*c - b^2)^15)^(1/2) + 18923520*a^10*b*c^10 + 17794*a^2*b^17*c^2 - 188095*a^3*b^15*c^3 + 1299860*a^4*b^13*c^4 - 6126640*a^5*b^11*c^5 + 19905600*a^6*b^9*c^6 - 43904256*a^7*b^7*c^7 + 62684160*a^8*b^5*c^8 - 52039680*a^9*b^3*c^9 - 225*a^3*c^3*(-(4*a*c - b^2)^15)^(1/2) - 995*a*b^19*c + 694*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 245*a*b^4*c*(-(4*a*c - b^2)^15)^(1/2)))/(512*(a^7*b^20*e^2 + 1048576*a^17*c^10*e^2 - 40*a^8*b^18*c*e^

2 + 720*a^9*b^16*c^2*e^2 - 7680*a^10*b^14*c^3*e^2 + 53760*a^11*b^12*c^4*e^2
 - 258048*a^12*b^10*c^5*e^2 + 860160*a^13*b^8*c^6*e^2 - 1966080*a^14*b^6*c^
7*e^2 + 2949120*a^15*b^4*c^8*e^2 - 2621440*a^16*b^2*c^9*e^2)))^(1/2)*((-(9*
(25*b^21 + 25*b^6*(-(4*a*c - b^2)^15)^(1/2) + 18923520*a^10*b*c^10 + 17794*
a^2*b^17*c^2 - 188095*a^3*b^15*c^3 + 1299860*a^4*b^13*c^4 - 6126640*a^5*b^1
1*c^5 + 19905600*a^6*b^9*c^6 - 43904256*a^7*b^7*c^7 + 62684160*a^8*b^5*c^8
 - 52039680*a^9*b^3*c^9 - 225*a^3*c^3*(-(4*a*c - b^2)^15)^(1/2) - 995*a*b^19
*c + 694*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 245*a*b^4*c*(-(4*a*c - b^2
)^15)^(1/2)))/(512*(a^7*b^20*e^2 + 1048576*a^17*c^10*e^2 - 40*a^8*b^18*c*e^
2 + 720*a^9*b^16*c^2*e^2 - 7680*a^10*b^14*c^3*e^2 + 53760*a^11*b^12*c^4*e^2
 - 258048*a^12*b^10*c^5*e^2 + 860160*a^13*b^8*c^6*e^2 - 1966080*a^14*b^6*c^
7*e^2 + 2949120*a^15*b^4*c^8*e^2 - 2621440*a^16*b^2*c^9*e^2)))^(1/2)*(x*(10
99511627776*a^26*b*c^13*e^14 - 262144*a^15*b^23*c^2*e^14 + 11534336*a^16*b^
21*c^3*e^14 - 230686720*a^17*b^19*c^4*e^14 + 2768240640*a^18*b^17*c^5*e^14
 - 22145925120*a^19*b^15*c^6*e^14 + 124017180672*a^20*b^13*c^7*e^14 - 496068
722688*a^21*b^11*c^8*e^14 + 1417339207680*a^22*b^9*c^9*e^14 - 2834678415360
*a^23*b^7*c^10*e^14 + 3779571220480*a^24*b^5*c^11*e^14 - 3023656976384*a^25
*b^3*c^12*e^14) + 1099511627776*a^26*b*c^13*d*e^13 - 262144*a^15*b^23*c^2*d
*e^13 + 11534336*a^16*b^21*c^3*d*e^13 - 230686720*a^17*b^19*c^4*d*e^13 + 27
68240640*a^18*b^17*c^5*d*e^13 - 22145925120*a^19*b^15*c^6*d*e^13 + 12401718
0672*a^20*b^13*c^7*d*e^13 - 496068722688*a^21*b^11*c^8*d*e^13 + 14173392076
80*a^22*b^9*c^9*d*e^13 - 2834678415360*a^23*b^7*c^10*d*e^13 + 3779571220480
*a^24*b^5*c^11*d*e^13 - 3023656976384*a^25*b^3*c^12*d*e^13) + 1185410973696
*a^23*b*c^13*e^12 - 245760*a^12*b^23*c^2*e^12 + 10911744*a^13*b^21*c^3*e^12
 - 220397568*a^14*b^19*c^4*e^12 + 2673082368*a^15*b^17*c^5*e^12 - 216300257
28*a^16*b^15*c^6*e^12 + 122607894528*a^17*b^13*c^7*e^12 - 496773365760*a^18
*b^11*c^8*e^12 + 1438679826432*a^19*b^9*c^9*e^12 - 2918430277632*a^20*b^7*c
^10*e^12 + 3949222428672*a^21*b^5*c^11*e^12 - 3208340570112*a^22*b^3*c^12*e
^12) + x*(271790899200*a^20*c^14*e^12 - 230400*a^9*b^22*c^3*e^12 + 9861120*
a^10*b^20*c^4*e^12 - 191038464*a^11*b^18*c^5*e^12 + 2207803392*a^12*b^16*c^
6*e^12 - 16878108672*a^13*b^14*c^7*e^12 + 89374851072*a^14*b^12*c^8*e^12 -
333226967040*a^15*b^10*c^9*e^12 + 869815812096*a^16*b^8*c^10*e^12 - 1543847
804928*a^17*b^6*c^11*e^12 + 1747313491968*a^18*b^4*c^12*e^12 - 110105513164
8*a^19*b^2*c^13*e^12) + 271790899200*a^20*c^14*d*e^11 - 230400*a^9*b^22*c^3
*d*e^11 + 9861120*a^10*b^20*c^4*d*e^11 - 191038464*a^11*b^18*c^5*d*e^11 + 2
207803392*a^12*b^16*c^6*d*e^11 - 16878108672*a^13*b^14*c^7*d*e^11 + 8937485
1072*a^14*b^12*c^8*d*e^11 - 333226967040*a^15*b^10*c^9*d*e^11 + 86981581209
6*a^16*b^8*c^10*d*e^11 - 1543847804928*a^17*b^6*c^11*d*e^11 + 1747313491968
*a^18*b^4*c^12*d*e^11 - 1101055131648*a^19*b^2*c^13*d*e^11) - (-(9*(25*b^21
 + 25*b^6*(-(4*a*c - b^2)^15)^(1/2) + 18923520*a^10*b*c^10 + 17794*a^2*b^17
*c^2 - 188095*a^3*b^15*c^3 + 1299860*a^4*b^13*c^4 - 6126640*a^5*b^11*c^5 +
19905600*a^6*b^9*c^6 - 43904256*a^7*b^7*c^7 + 62684160*a^8*b^5*c^8 - 520396
80*a^9*b^3*c^9 - 225*a^3*c^3*(-(4*a*c - b^2)^15)^(1/2) - 995*a*b^19*c + 694
*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 245*a*b^4*c*(-(4*a*c - b^2)^15)^(1
/2)))/(512*(a^7*b^20*e^2 + 1048576*a^17*c^10*e^2 - 40*a^8*b^18*c*e^2 + 720*
a^9*b^16*c^2*e^2 - 7680*a^10*b^14*c^3*e^2 + 53760*a^11*b^12*c^4*e^2 - 25804
8*a^12*b^10*c^5*e^2 + 860160*a^13*b^8*c^6*e^2 - 1966080*a^14*b^6*c^7*e^2 +
2949120*a^15*b^4*c^8*e^2 - 2621440*a^16*b^2*c^9*e^2)))^(1/2)*((-(9*(25*b^21
 + 25*b^6*(-(4*a*c - b^2)^15)^(1/2) + 18923520*a^10*b*c^10 + 17794*a^2*b^17
*c^2 - 188095*a^3*b^15*c^3 + 1299860*a^4*b^13*c^4 - 6126640*a^5*b^11*c^5 +
19905600*a^6*b^9*c^6 - 43904256*a^7*b^7*c^7 + 62684160*a^8*b^5*c^8 - 520396
80*a^9*b^3*c^9 - 225*a^3*c^3*(-(4*a*c - b^2)^15)^(1/2) - 995*a*b^19*c + 694
*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 245*a*b^4*c*(-(4*a*c - b^2)^15)^(1
/2)))/(512*(a^7*b^20*e^2 + 1048576*a^17*c^10*e^2 - 40*a^8*b^18*c*e^2 + 720*
a^9*b^16*c^2*e^2 - 7680*a^10*b^14*c^3*e^2 + 53760*a^11*b^12*c^4*e^2 - 25804
8*a^12*b^10*c^5*e^2 + 860160*a^13*b^8*c^6*e^2 - 1966080*a^14*b^6*c^7*e^2 +
2949120*a^15*b^4*c^8*e^2 - 2621440*a^16*b^2*c^9*e^2)))^(1/2)*((-(9*(25*b^21
 + 25*b^6*(-(4*a*c - b^2)^15)^(1/2) + 18923520*a^10*b*c^10 + 17794*a^2*b^17
*c^2 - 188095*a^3*b^15*c^3 + 1299860*a^4*b^13*c^4 - 6126640*a^5*b^11*c^5 +

```
19905600*a^6*b^9*c^6 - 43904256*a^7*b^7*c^7 + 62684160*a^8*b^5*c^8 - 520396
80*a^9*b^3*c^9 - 225*a^3*c^3*(-(4*a*c - b^2)^15)^(1/2) - 995*a*b^19*c + 694
*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 245*a*b^4*c*(-(4*a*c - b^2)^15)^(1
/2)))/(512*(a^7*b^20*e^2 + 1048576*a^17*c^10*e^2 - 40*a^8*b^18*c*e^2 + 720*
a^9*b^16*c^2*e^2 - 7680*a^10*b^14*c^3*e^2 + 53760*a^11*b^12*c^4*e^2 - 25804
8*a^12*b^10*c^5*e^2 + 860160*a^13*b^8*c^6*e^2 - 1966080*a^14*b^6*c^7*e^2 +
2949120*a^15*b^4*c^8*e^2 - 2621440*a^16*b^2*c^9*e^2)))^(1/2)*(x*(1099511627
776*a^26*b*c^13*e^14 - 262144*a^15*b^23*c^2*e^14 + 11534336*a^16*b^21*c^3*e
^14 - 230686720*a^17*b^19*c^4*e^14 + 2768240640*a^18*b^17*c^5*e^14 - 221459
25120*a^19*b^15*c^6*e^14 + 124017180672*a^20*b^13*c^7*e^14 - 496068722688*a
^21*b^11*c^8*e^14 + 1417339207680*a^22*b^9*c^9*e^14 - 2834678415360*a^23*b^
7*c^10*e^14 + 3779571220480*a^24*b^5*c^11*e^14 - 3023656976384*a^25*b^3*c^1
2*e^14) + 1099511627776*a^26*b*c^13*d*e^13 - 262144*a^15*b^23*c^2*d*e^13 +
11534336*a^16*b^21*c^3*d*e^13 - 230686720*a^17*b^19*c^4*d*e^13 + 2768240640
*a^18*b^17*c^5*d*e^13 - 22145925120*a^19*b^15*c^6*d*e^13 + 124017180672*a^2
0*b^13*c^7*d*e^13 - 496068722688*a^21*b^11*c^8*d*e^13 + 1417339207680*a^22*
b^9*c^9*d*e^13 - 2834678415360*a^23*b^7*c^10*d*e^13 + 3779571220480*a^24*b^
5*c^11*d*e^13 - 3023656976384*a^25*b^3*c^12*d*e^13) - 1185410973696*a^23*b*
c^13*e^12 + 245760*a^12*b^23*c^2*e^12 - 10911744*a^13*b^21*c^3*e^12 + 22039
7568*a^14*b^19*c^4*e^12 - 2673082368*a^15*b^17*c^5*e^12 + 21630025728*a^16*
b^15*c^6*e^12 - 122607894528*a^17*b^13*c^7*e^12 + 496773365760*a^18*b^11*c^
8*e^12 - 1438679826432*a^19*b^9*c^9*e^12 + 2918430277632*a^20*b^7*c^10*e^12
 - 3949222428672*a^21*b^5*c^11*e^12 + 3208340570112*a^22*b^3*c^12*e^12) + x
*(271790899200*a^20*c^14*e^12 - 230400*a^9*b^22*c^3*e^12 + 9861120*a^10*b^2
0*c^4*e^12 - 191038464*a^11*b^18*c^5*e^12 + 2207803392*a^12*b^16*c^6*e^12 -
 16878108672*a^13*b^14*c^7*e^12 + 89374851072*a^14*b^12*c^8*e^12 - 33322696
7040*a^15*b^10*c^9*e^12 + 869815812096*a^16*b^8*c^10*e^12 - 1543847804928*a
^17*b^6*c^11*e^12 + 1747313491968*a^18*b^4*c^12*e^12 - 1101055131648*a^19*b
^2*c^13*e^12) + 271790899200*a^20*c^14*d*e^11 - 230400*a^9*b^22*c^3*d*e^11
+ 9861120*a^10*b^20*c^4*d*e^11 - 191038464*a^11*b^18*c^5*d*e^11 + 220780339
2*a^12*b^16*c^6*d*e^11 - 16878108672*a^13*b^14*c^7*d*e^11 + 89374851072*a^1
4*b^12*c^8*d*e^11 - 333226967040*a^15*b^10*c^9*d*e^11 + 869815812096*a^16*b
^8*c^10*d*e^11 - 1543847804928*a^17*b^6*c^11*d*e^11 + 1747313491968*a^18*b^
4*c^12*d*e^11 - 1101055131648*a^19*b^2*c^13*d*e^11) + 191102976000*a^17*c^1
4*e^10 + 2851200*a^9*b^16*c^6*e^10 - 92568960*a^10*b^14*c^7*e^10 + 13126302
72*a^11*b^12*c^8*e^10 - 10611136512*a^12*b^10*c^9*e^10 + 53445353472*a^13*b
^8*c^10*e^10 - 171591892992*a^14*b^6*c^11*e^10 + 342580396032*a^15*b^4*c^12
*e^10 - 388363714560*a^16*b^2*c^13*e^10))*(-(9*(25*b^21 + 25*b^6*(-(4*a*c -
 b^2)^15)^(1/2) + 18923520*a^10*b*c^10 + 17794*a^2*b^17*c^2 - 188095*a^3*b^
15*c^3 + 1299860*a^4*b^13*c^4 - 6126640*a^5*b^11*c^5 + 19905600*a^6*b^9*c^6
 - 43904256*a^7*b^7*c^7 + 62684160*a^8*b^5*c^8 - 52039680*a^9*b^3*c^9 - 225
*a^3*c^3*(-(4*a*c - b^2)^15)^(1/2) - 995*a*b^19*c + 694*a^2*b^2*c^2*(-(4*a*
c - b^2)^15)^(1/2) - 245*a*b^4*c*(-(4*a*c - b^2)^15)^(1/2)))/(512*(a^7*b^20
*e^2 + 1048576*a^17*c^10*e^2 - 40*a^8*b^18*c*e^2 + 720*a^9*b^16*c^2*e^2 - 7
680*a^10*b^14*c^3*e^2 + 53760*a^11*b^12*c^4*e^2 - 258048*a^12*b^10*c^5*e^2
+ 860160*a^13*b^8*c^6*e^2 - 1966080*a^14*b^6*c^7*e^2 + 2949120*a^15*b^4*c^8
*e^2 - 2621440*a^16*b^2*c^9*e^2)))^(1/2)*2i
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)**2/(a+b*(e*x+d)**2+c*(e*x+d)**4)**3,x)
```

```
[Out] Timed out
```

3.524 $\int \frac{1}{(d+ex)^3\left(a+b(d+ex)^2+c(d+ex)^4\right)^3}\,dx$

Optimal. Leaf size=325

$$\frac{3b\log\left(a+b(d+ex)^2+c(d+ex)^4\right)}{4a^4e}-\frac{3b\log(d+ex)}{a^4e}-\frac{3\left(b^2-5ac\right)\left(b^2-2ac\right)}{2a^3e\left(b^2-4ac\right)^2(d+ex)^2}+\frac{20a^2c^2+3bc\left(b^2-6ac\right)(}{4a^2e\left(b^2-4ac\right)^2(d+ex)^2\,(a}$$

Rubi [A] time = 0.58, antiderivative size = 325, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {1142, 1114, 740, 822, 800, 634, 618, 206, 628}

$$\frac{20a^2c^2+3bc\left(b^2-6ac\right)(d+ex)^2-20ab^2c+3b^4}{4a^2e\left(b^2-4ac\right)^2(d+ex)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)}-\frac{3\left(30a^2b^2c^2-20a^3c^3-10ab^4c+b^6\right)\tanh^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{2a^4e\left(b^2-4ac\right)^{5/2}}-\frac{3\left(b^2-5ac\right)\left(b^2-2ac\right)}{2a^3e\left(b^2-4ac\right)^2(d+ex)^2}+\frac{3b\log\left(a+b(d+ex)^2+c(d+ex)^4\right)}{4a^4e}-\frac{3b\log(d+ex)}{a^4e}+\frac{-2ac+b^2+bc(d+ex)^2}{4ae\left(b^2-4ac\right)(d+ex)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)^3*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3),x]
```

```
[Out] (-3*(b^2 - 5*a*c)*(b^2 - 2*a*c))/(2*a^3*(b^2 - 4*a*c)^2*e*(d + e*x)^2) + (b
^2 - 2*a*c + b*c*(d + e*x)^2)/(4*a*(b^2 - 4*a*c)*e*(d + e*x)^2*(a + b*(d +
e*x)^2 + c*(d + e*x)^4)^2) + (3*b^4 - 20*a*b^2*c + 20*a^2*c^2 + 3*b*c*(b^2
- 6*a*c)*(d + e*x)^2)/(4*a^2*(b^2 - 4*a*c)^2*e*(d + e*x)^2*(a + b*(d + e*x)
^2 + c*(d + e*x)^4)) - (3*(b^6 - 10*a*b^4*c + 30*a^2*b^2*c^2 - 20*a^3*c^3)*
ArcTanh[(b + 2*c*(d + e*x)^2)/Sqrt[b^2 - 4*a*c]])/(2*a^4*(b^2 - 4*a*c)^(5/2
)*e) - (3*b*Log[d + e*x])/(a^4*e) + (3*b*Log[a + b*(d + e*x)^2 + c*(d + e*x
)^4])/(4*a^4*e)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 740

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e
)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e
^2)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d +
 e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p +
```

```
 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p,
-1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 822

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(f*(b*c*d - b^2*e +
2*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x)*(a
+ b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), x]
 + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m +
2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*m
 + b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) - f
*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1
] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 1142

```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Di
st[u^m/(Coefficient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p,
 x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

Rubi steps

$$\int \frac{1}{(d+ex)^3\left(a+b(d+ex)^2+c(d+ex)^4\right)^3}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x^3\left(a+bx^2+cx^4\right)^3}\,dx, x, d+ex\right)}{e}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{x^2\left(a+bx+cx^2\right)^3}\,dx, x, (d+ex)^2\right)}{2e}$$

$$= \frac{b^2-2ac+bc(d+ex)^2}{4a\left(b^2-4ac\right)e(d+ex)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)^2} - \text{Subs}\cdots$$

$$= \frac{b^2-2ac+bc(d+ex)^2}{4a\left(b^2-4ac\right)e(d+ex)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)^2} + \frac{3\cdots}{4a^2\cdots}$$

$$= \frac{b^2-2ac+bc(d+ex)^2}{4a\left(b^2-4ac\right)e(d+ex)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)^2} + \frac{3\cdots}{4a^2\cdots}$$

$$= -\frac{3\left(b^2-5ac\right)\left(b^2-2ac\right)}{2a^3\left(b^2-4ac\right)^2 e(d+ex)^2} + \frac{b^2-2ac+bc(d\cdots}{4a\left(b^2-4ac\right)e(d+ex)^2\left(a+b(\cdots\right.}$$

$$= -\frac{3\left(b^2-5ac\right)\left(b^2-2ac\right)}{2a^3\left(b^2-4ac\right)^2 e(d+ex)^2} + \frac{b^2-2ac+bc(d\cdots}{4a\left(b^2-4ac\right)e(d+ex)^2\left(a+b(\cdots\right.}$$

$$= -\frac{3\left(b^2-5ac\right)\left(b^2-2ac\right)}{2a^3\left(b^2-4ac\right)^2 e(d+ex)^2} + \frac{b^2-2ac+bc(d\cdots}{4a\left(b^2-4ac\right)e(d+ex)^2\left(a+b(\cdots\right.}$$

$$= -\frac{3\left(b^2-5ac\right)\left(b^2-2ac\right)}{2a^3\left(b^2-4ac\right)^2 e(d+ex)^2} + \frac{b^2-2ac+bc(d\cdots}{4a\left(b^2-4ac\right)e(d+ex)^2\left(a+b(\cdots\right.}$$

Mathematica [A] time = 6.18, size = 491, normalized size = 1.51

$$-\frac{3b\log(d+ex)}{a^4e}-\frac{1}{2a^3e(d+ex)^2}+\frac{-3abc-2ac^2(d+ex)^2+b^3+b^2c(d+ex)^2}{4a^2e\left(4ac-b^2\right)\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}+\frac{-46a^2bc^2-28a^2c^3(d+ex)^2+29ab^3c+26ab^2c^2(d+ex)^2-4b^5-4b^4c(d+ex)^2}{4a^3e\left(4ac-b^2\right)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)}+\frac{3\left(-20a^3c^3+30a^2b^2c^2+16a^2bc^2\sqrt{b^2-4ac}-10ab^4c+b^5\sqrt{b^2-4ac}-8ab^3c\sqrt{b^2-4ac}+b^6\right)\log\left(-\sqrt{b^2-4ac}+b+2c(d+ex)^2\right)}{4a^4e\left(b^2-4ac\right)^{5/2}}+\frac{3\left(20a^3c^3-30a^2b^2c^2+16a^2bc^2\sqrt{b^2-4ac}+10ab^4c+b^5\sqrt{b^2-4ac}-8ab^3c\sqrt{b^2-4ac}-b^6\right)\log\left(\sqrt{b^2-4ac}+b+2c(d+ex)^2\right)}{4a^4e\left(b^2-4ac\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)^3*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3),x]
```

```
[Out] -1/2*1/(a^3*e*(d + e*x)^2) + (b^3 - 3*a*b*c + b^2*c*(d + e*x)^2 - 2*a*c^2*(
d + e*x)^2)/(4*a^2*(-b^2 + 4*a*c)*e*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2)
+ (-4*b^5 + 29*a*b^3*c - 46*a^2*b*c^2 - 4*b^4*c*(d + e*x)^2 + 26*a*b^2*c^2*
(d + e*x)^2 - 28*a^2*c^3*(d + e*x)^2)/(4*a^3*(-b^2 + 4*a*c)^2*e*(a + b*(d +
 e*x)^2 + c*(d + e*x)^4)) - (3*b*Log[d + e*x])/(a^4*e) + (3*(b^6 - 10*a*b^4
*c + 30*a^2*b^2*c^2 - 20*a^3*c^3 + b^5*Sqrt[b^2 - 4*a*c] - 8*a*b^3*c*Sqrt[b
^2 - 4*a*c] + 16*a^2*b*c^2*Sqrt[b^2 - 4*a*c])*Log[b - Sqrt[b^2 - 4*a*c] + 2
*c*(d + e*x)^2])/(4*a^4*(b^2 - 4*a*c)^(5/2)*e) + (3*(-b^6 + 10*a*b^4*c - 30
*a^2*b^2*c^2 + 20*a^3*c^3 + b^5*Sqrt[b^2 - 4*a*c] - 8*a*b^3*c*Sqrt[b^2 - 4*
a*c] + 16*a^2*b*c^2*Sqrt[b^2 - 4*a*c])*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*(d +
 e*x)^2])/(4*a^4*(b^2 - 4*a*c)^(5/2)*e)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(d+ex)^3\left(a+b(d+ex)^2+c(d+ex)^4\right)^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/((d + e*x)^3*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3),x]
```

```
[Out] IntegrateAlgebraic[1/((d + e*x)^3*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3), x
]
```

fricas [B] time = 7.53, size = 15165, normalized size = 46.66

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="fricas")
```

```
[Out] [-1/4*(6*(a*b^6*c^2 - 11*a^2*b^4*c^3 + 38*a^3*b^2*c^4 - 40*a^4*c^5)*e^8*x^8
 + 48*(a*b^6*c^2 - 11*a^2*b^4*c^3 + 38*a^3*b^2*c^4 - 40*a^4*c^5)*d*e^7*x^7
+ 3*(4*a*b^7*c - 45*a^2*b^5*c^2 + 162*a^3*b^3*c^3 - 184*a^4*b*c^4 + 56*(a*b
^6*c^2 - 11*a^2*b^4*c^3 + 38*a^3*b^2*c^4 - 40*a^4*c^5)*d^2)*e^6*x^6 + 6*(56
*(a*b^6*c^2 - 11*a^2*b^4*c^3 + 38*a^3*b^2*c^4 - 40*a^4*c^5)*d^3 + 3*(4*a*b^
7*c - 45*a^2*b^5*c^2 + 162*a^3*b^3*c^3 - 184*a^4*b*c^4)*d)*e^5*x^5 + 2*a^3*
b^6 - 24*a^4*b^4*c + 96*a^5*b^2*c^2 - 128*a^6*c^3 + 6*(a*b^6*c^2 - 11*a^2*b
^4*c^3 + 38*a^3*b^2*c^4 - 40*a^4*c^5)*d^8 + (6*a*b^8 - 60*a^2*b^6*c + 158*a
^3*b^4*c^2 + 44*a^4*b^2*c^3 - 400*a^5*c^4 + 420*(a*b^6*c^2 - 11*a^2*b^4*c^3
 + 38*a^3*b^2*c^4 - 40*a^4*c^5)*d^4 + 45*(4*a*b^7*c - 45*a^2*b^5*c^2 + 162*
a^3*b^3*c^3 - 184*a^4*b*c^4)*d^2)*e^4*x^4 + 3*(4*a*b^7*c - 45*a^2*b^5*c^2 +
 162*a^3*b^3*c^3 - 184*a^4*b*c^4)*d^6 + 4*(84*(a*b^6*c^2 - 11*a^2*b^4*c^3 +
 38*a^3*b^2*c^4 - 40*a^4*c^5)*d^5 + 15*(4*a*b^7*c - 45*a^2*b^5*c^2 + 162*a^
3*b^3*c^3 - 184*a^4*b*c^4)*d^3 + 2*(3*a*b^8 - 30*a^2*b^6*c + 79*a^3*b^4*c^2
 + 22*a^4*b^2*c^3 - 200*a^5*c^4)*d)*e^3*x^3 + 2*(3*a*b^8 - 30*a^2*b^6*c + 7
9*a^3*b^4*c^2 + 22*a^4*b^2*c^3 - 200*a^5*c^4)*d^4 + (9*a^2*b^7 - 104*a^3*b^
5*c + 394*a^4*b^3*c^2 - 488*a^5*b*c^3 + 168*(a*b^6*c^2 - 11*a^2*b^4*c^3 + 3
8*a^3*b^2*c^4 - 40*a^4*c^5)*d^6 + 45*(4*a*b^7*c - 45*a^2*b^5*c^2 + 162*a^3*
b^3*c^3 - 184*a^4*b*c^4)*d^4 + 12*(3*a*b^8 - 30*a^2*b^6*c + 79*a^3*b^4*c^2
+ 22*a^4*b^2*c^3 - 200*a^5*c^4)*d^2)*e^2*x^2 + (9*a^2*b^7 - 104*a^3*b^5*c +
 394*a^4*b^3*c^2 - 488*a^5*b*c^3)*d^2 + 2*(24*(a*b^6*c^2 - 11*a^2*b^4*c^3 +
 38*a^3*b^2*c^4 - 40*a^4*c^5)*d^7 + 9*(4*a*b^7*c - 45*a^2*b^5*c^2 + 162*a^3
*b^3*c^3 - 184*a^4*b*c^4)*d^5 + 4*(3*a*b^8 - 30*a^2*b^6*c + 79*a^3*b^4*c^2
+ 22*a^4*b^2*c^3 - 200*a^5*c^4)*d^3 + (9*a^2*b^7 - 104*a^3*b^5*c + 394*a^4*
b^3*c^2 - 488*a^5*b*c^3)*d)*e*x + 3*((b^6*c^2 - 10*a*b^4*c^3 + 30*a^2*b^2*c
^4 - 20*a^3*c^5)*e^10*x^10 + 10*(b^6*c^2 - 10*a*b^4*c^3 + 30*a^2*b^2*c^4 -
20*a^3*c^5)*d*e^9*x^9 + (2*b^7*c - 20*a*b^5*c^2 + 60*a^2*b^3*c^3 - 40*a^3*b
*c^4 + 45*(b^6*c^2 - 10*a*b^4*c^3 + 30*a^2*b^2*c^4 - 20*a^3*c^5)*d^2)*e^8*x
^8 + 8*(15*(b^6*c^2 - 10*a*b^4*c^3 + 30*a^2*b^2*c^4 - 20*a^3*c^5)*d^3 + 2*(
b^7*c - 10*a*b^5*c^2 + 30*a^2*b^3*c^3 - 20*a^3*b*c^4)*d)*e^7*x^7 + (b^8 - 8
*a*b^6*c + 10*a^2*b^4*c^2 + 40*a^3*b^2*c^3 - 40*a^4*c^4 + 210*(b^6*c^2 - 10
*a*b^4*c^3 + 30*a^2*b^2*c^4 - 20*a^3*c^5)*d^4 + 56*(b^7*c - 10*a*b^5*c^2 +
30*a^2*b^3*c^3 - 20*a^3*b*c^4)*d^2)*e^6*x^6 + (b^6*c^2 - 10*a*b^4*c^3 + 30*
a^2*b^2*c^4 - 20*a^3*c^5)*d^10 + 2*(126*(b^6*c^2 - 10*a*b^4*c^3 + 30*a^2*b^
2*c^4 - 20*a^3*c^5)*d^5 + 56*(b^7*c - 10*a*b^5*c^2 + 30*a^2*b^3*c^3 - 20*a^
3*b*c^4)*d^3 + 3*(b^8 - 8*a*b^6*c + 10*a^2*b^4*c^2 + 40*a^3*b^2*c^3 - 40*a^
4*c^4)*d)*e^5*x^5 + 2*(b^7*c - 10*a*b^5*c^2 + 30*a^2*b^3*c^3 - 20*a^3*b*c^4
)*d^8 + (2*a*b^7 - 20*a^2*b^5*c + 60*a^3*b^3*c^2 - 40*a^4*b*c^3 + 210*(b^6*
c^2 - 10*a*b^4*c^3 + 30*a^2*b^2*c^4 - 20*a^3*c^5)*d^6 + 140*(b^7*c - 10*a*b
^5*c^2 + 30*a^2*b^3*c^3 - 20*a^3*b*c^4)*d^4 + 15*(b^8 - 8*a*b^6*c + 10*a^2*
b^4*c^2 + 40*a^3*b^2*c^3 - 40*a^4*c^4)*d^2)*e^4*x^4 + (b^8 - 8*a*b^6*c + 10
```

```
*a^2*b^4*c^2 + 40*a^3*b^2*c^3 - 40*a^4*c^4)*d^6 + 4*(30*(b^6*c^2 - 10*a*b^4
*c^3 + 30*a^2*b^2*c^4 - 20*a^3*c^5)*d^7 + 28*(b^7*c - 10*a*b^5*c^2 + 30*a^2
*b^3*c^3 - 20*a^3*b*c^4)*d^5 + 5*(b^8 - 8*a*b^6*c + 10*a^2*b^4*c^2 + 40*a^3
*b^2*c^3 - 40*a^4*c^4)*d^3 + 2*(a*b^7 - 10*a^2*b^5*c + 30*a^3*b^3*c^2 - 20*
a^4*b*c^3)*d)*e^3*x^3 + 2*(a*b^7 - 10*a^2*b^5*c + 30*a^3*b^3*c^2 - 20*a^4*b
*c^3)*d^4 + (45*(b^6*c^2 - 10*a*b^4*c^3 + 30*a^2*b^2*c^4 - 20*a^3*c^5)*d^8
+ a^2*b^6 - 10*a^3*b^4*c + 30*a^4*b^2*c^2 - 20*a^5*c^3 + 56*(b^7*c - 10*a*b
^5*c^2 + 30*a^2*b^3*c^3 - 20*a^3*b*c^4)*d^6 + 15*(b^8 - 8*a*b^6*c + 10*a^2*
b^4*c^2 + 40*a^3*b^2*c^3 - 40*a^4*c^4)*d^4 + 12*(a*b^7 - 10*a^2*b^5*c + 30*
a^3*b^3*c^2 - 20*a^4*b*c^3)*d^2)*e^2*x^2 + (a^2*b^6 - 10*a^3*b^4*c + 30*a^4
*b^2*c^2 - 20*a^5*c^3)*d^2 + 2*(5*(b^6*c^2 - 10*a*b^4*c^3 + 30*a^2*b^2*c^4
- 20*a^3*c^5)*d^9 + 8*(b^7*c - 10*a*b^5*c^2 + 30*a^2*b^3*c^3 - 20*a^3*b*c^4
)*d^7 + 3*(b^8 - 8*a*b^6*c + 10*a^2*b^4*c^2 + 40*a^3*b^2*c^3 - 40*a^4*c^4)*
d^5 + 4*(a*b^7 - 10*a^2*b^5*c + 30*a^3*b^3*c^2 - 20*a^4*b*c^3)*d^3 + (a^2*b
^6 - 10*a^3*b^4*c + 30*a^4*b^2*c^2 - 20*a^5*c^3)*d)*e*x)*sqrt(b^2 - 4*a*c)*
log((2*c^2*e^4*x^4 + 8*c^2*d*e^3*x^3 + 2*c^2*d^4 + 2*(6*c^2*d^2 + b*c)*e^2*
x^2 + 2*b*c*d^2 + 4*(2*c^2*d^3 + b*c*d)*e*x + b^2 - 2*a*c + (2*c*e^2*x^2 +
4*c*d*e*x + 2*c*d^2 + b)*sqrt(b^2 - 4*a*c))/(c*e^4*x^4 + 4*c*d*e^3*x^3 + c*
d^4 + (6*c*d^2 + b)*e^2*x^2 + b*d^2 + 2*(2*c*d^3 + b*d)*e*x + a)) - 3*((b^7
*c^2 - 12*a*b^5*c^3 + 48*a^2*b^3*c^4 - 64*a^3*b*c^5)*e^10*x^10 + 10*(b^7*c^
2 - 12*a*b^5*c^3 + 48*a^2*b^3*c^4 - 64*a^3*b*c^5)*d*e^9*x^9 + (2*b^8*c - 24
*a*b^6*c^2 + 96*a^2*b^4*c^3 - 128*a^3*b^2*c^4 + 45*(b^7*c^2 - 12*a*b^5*c^3
+ 48*a^2*b^3*c^4 - 64*a^3*b*c^5)*d^2)*e^8*x^8 + 8*(15*(b^7*c^2 - 12*a*b^5*c
^3 + 48*a^2*b^3*c^4 - 64*a^3*b*c^5)*d^3 + 2*(b^8*c - 12*a*b^6*c^2 + 48*a^2*
b^4*c^3 - 64*a^3*b^2*c^4)*d)*e^7*x^7 + (b^9 - 10*a*b^7*c + 24*a^2*b^5*c^2 +
 32*a^3*b^3*c^3 - 128*a^4*b*c^4 + 210*(b^7*c^2 - 12*a*b^5*c^3 + 48*a^2*b^3*
c^4 - 64*a^3*b*c^5)*d^4 + 56*(b^8*c - 12*a*b^6*c^2 + 48*a^2*b^4*c^3 - 64*a^
3*b^2*c^4)*d^2)*e^6*x^6 + (b^7*c^2 - 12*a*b^5*c^3 + 48*a^2*b^3*c^4 - 64*a^3
*b*c^5)*d^10 + 2*(126*(b^7*c^2 - 12*a*b^5*c^3 + 48*a^2*b^3*c^4 - 64*a^3*b*c
^5)*d^5 + 56*(b^8*c - 12*a*b^6*c^2 + 48*a^2*b^4*c^3 - 64*a^3*b^2*c^4)*d^3 +
 3*(b^9 - 10*a*b^7*c + 24*a^2*b^5*c^2 + 32*a^3*b^3*c^3 - 128*a^4*b*c^4)*d)*
e^5*x^5 + 2*(b^8*c - 12*a*b^6*c^2 + 48*a^2*b^4*c^3 - 64*a^3*b^2*c^4)*d^8 +
(2*a*b^8 - 24*a^2*b^6*c + 96*a^3*b^4*c^2 - 128*a^4*b^2*c^3 + 210*(b^7*c^2 -
 12*a*b^5*c^3 + 48*a^2*b^3*c^4 - 64*a^3*b*c^5)*d^6 + 140*(b^8*c - 12*a*b^6*
c^2 + 48*a^2*b^4*c^3 - 64*a^3*b^2*c^4)*d^4 + 15*(b^9 - 10*a*b^7*c + 24*a^2*
b^5*c^2 + 32*a^3*b^3*c^3 - 128*a^4*b*c^4)*d^2)*e^4*x^4 + (b^9 - 10*a*b^7*c
+ 24*a^2*b^5*c^2 + 32*a^3*b^3*c^3 - 128*a^4*b*c^4)*d^6 + 4*(30*(b^7*c^2 - 1
2*a*b^5*c^3 + 48*a^2*b^3*c^4 - 64*a^3*b*c^5)*d^7 + 28*(b^8*c - 12*a*b^6*c^2
 + 48*a^2*b^4*c^3 - 64*a^3*b^2*c^4)*d^5 + 5*(b^9 - 10*a*b^7*c + 24*a^2*b^5*
c^2 + 32*a^3*b^3*c^3 - 128*a^4*b*c^4)*d^3 + 2*(a*b^8 - 12*a^2*b^6*c + 48*a^
3*b^4*c^2 - 64*a^4*b^2*c^3)*d)*e^3*x^3 + 2*(a*b^8 - 12*a^2*b^6*c + 48*a^3*b
^4*c^2 - 64*a^4*b^2*c^3)*d^4 + (a^2*b^7 - 12*a^3*b^5*c + 48*a^4*b^3*c^2 - 6
4*a^5*b*c^3 + 45*(b^7*c^2 - 12*a*b^5*c^3 + 48*a^2*b^3*c^4 - 64*a^3*b*c^5)*d
^8 + 56*(b^8*c - 12*a*b^6*c^2 + 48*a^2*b^4*c^3 - 64*a^3*b^2*c^4)*d^6 + 15*(
b^9 - 10*a*b^7*c + 24*a^2*b^5*c^2 + 32*a^3*b^3*c^3 - 128*a^4*b*c^4)*d^4 + 1
2*(a*b^8 - 12*a^2*b^6*c + 48*a^3*b^4*c^2 - 64*a^4*b^2*c^3)*d^2)*e^2*x^2 + (
a^2*b^7 - 12*a^3*b^5*c + 48*a^4*b^3*c^2 - 64*a^5*b*c^3)*d^2 + 2*(5*(b^7*c^2
 - 12*a*b^5*c^3 + 48*a^2*b^3*c^4 - 64*a^3*b*c^5)*d^9 + 8*(b^8*c - 12*a*b^6*
c^2 + 48*a^2*b^4*c^3 - 64*a^3*b^2*c^4)*d^7 + 3*(b^9 - 10*a*b^7*c + 24*a^2*b
^5*c^2 + 32*a^3*b^3*c^3 - 128*a^4*b*c^4)*d^5 + 4*(a*b^8 - 12*a^2*b^6*c + 48
*a^3*b^4*c^2 - 64*a^4*b^2*c^3)*d^3 + (a^2*b^7 - 12*a^3*b^5*c + 48*a^4*b^3*c
^2 - 64*a^5*b*c^3)*d)*e*x)*log(c*e^4*x^4 + 4*c*d*e^3*x^3 + c*d^4 + (6*c*d^2
 + b)*e^2*x^2 + b*d^2 + 2*(2*c*d^3 + b*d)*e*x + a) + 12*((b^7*c^2 - 12*a*b^
5*c^3 + 48*a^2*b^3*c^4 - 64*a^3*b*c^5)*e^10*x^10 + 10*(b^7*c^2 - 12*a*b^5*c
^3 + 48*a^2*b^3*c^4 - 64*a^3*b*c^5)*d*e^9*x^9 + (2*b^8*c - 24*a*b^6*c^2 + 9
6*a^2*b^4*c^3 - 128*a^3*b^2*c^4 + 45*(b^7*c^2 - 12*a*b^5*c^3 + 48*a^2*b^3*c
^4 - 64*a^3*b*c^5)*d^2)*e^8*x^8 + 8*(15*(b^7*c^2 - 12*a*b^5*c^3 + 48*a^2*b^
3*c^4 - 64*a^3*b*c^5)*d^3 + 2*(b^8*c - 12*a*b^6*c^2 + 48*a^2*b^4*c^3 - 64*a
^3*b^2*c^4)*d)*e^7*x^7 + (b^9 - 10*a*b^7*c + 24*a^2*b^5*c^2 + 32*a^3*b^3*c^
```

```
3 - 128*a^4*b*c^4 + 210*(b^7*c^2 - 12*a*b^5*c^3 + 48*a^2*b^3*c^4 - 64*a^3*b
*c^5)*d^4 + 56*(b^8*c - 12*a*b^6*c^2 + 48*a^2*b^4*c^3 - 64*a^3*b^2*c^4)*d^2
)*e^6*x^6 + (b^7*c^2 - 12*a*b^5*c^3 + 48*a^2*b^3*c^4 - 64*a^3*b*c^5)*d^10 +
 2*(126*(b^7*c^2 - 12*a*b^5*c^3 + 48*a^2*b^3*c^4 - 64*a^3*b*c^5)*d^5 + 56*(
b^8*c - 12*a*b^6*c^2 + 48*a^2*b^4*c^3 - 64*a^3*b^2*c^4)*d^3 + 3*(b^9 - 10*a
*b^7*c + 24*a^2*b^5*c^2 + 32*a^3*b^3*c^3 - 128*a^4*b*c^4)*d)*e^5*x^5 + 2*(b
^8*c - 12*a*b^6*c^2 + 48*a^2*b^4*c^3 - 64*a^3*b^2*c^4)*d^8 + (2*a*b^8 - 24*
a^2*b^6*c + 96*a^3*b^4*c^2 - 128*a^4*b^2*c^3 + 210*(b^7*c^2 - 12*a*b^5*c^3
+ 48*a^2*b^3*c^4 - 64*a^3*b*c^5)*d^6 + 140*(b^8*c - 12*a*b^6*c^2 + 48*a^2*b
^4*c^3 - 64*a^3*b^2*c^4)*d^4 + 15*(b^9 - 10*a*b^7*c + 24*a^2*b^5*c^2 + 32*a
^3*b^3*c^3 - 128*a^4*b*c^4)*d^2)*e^4*x^4 + (b^9 - 10*a*b^7*c + 24*a^2*b^5*c
^2 + 32*a^3*b^3*c^3 - 128*a^4*b*c^4)*d^6 + 4*(30*(b^7*c^2 - 12*a*b^5*c^3 +
48*a^2*b^3*c^4 - 64*a^3*b*c^5)*d^7 + 28*(b^8*c - 12*a*b^6*c^2 + 48*a^2*b^4*
c^3 - 64*a^3*b^2*c^4)*d^5 + 5*(b^9 - 10*a*b^7*c + 24*a^2*b^5*c^2 + 32*a^3*b
^3*c^3 - 128*a^4*b*c^4)*d^3 + 2*(a*b^8 - 12*a^2*b^6*c + 48*a^3*b^4*c^2 - 64
*a^4*b^2*c^3)*d)*e^3*x^3 + 2*(a*b^8 - 12*a^2*b^6*c + 48*a^3*b^4*c^2 - 64*a^
4*b^2*c^3)*d^4 + (a^2*b^7 - 12*a^3*b^5*c + 48*a^4*b^3*c^2 - 64*a^5*b*c^3 +
45*(b^7*c^2 - 12*a*b^5*c^3 + 48*a^2*b^3*c^4 - 64*a^3*b*c^5)*d^8 + 56*(b^8*c
 - 12*a*b^6*c^2 + 48*a^2*b^4*c^3 - 64*a^3*b^2*c^4)*d^6 + 15*(b^9 - 10*a*b^7
*c + 24*a^2*b^5*c^2 + 32*a^3*b^3*c^3 - 128*a^4*b*c^4)*d^4 + 12*(a*b^8 - 12*
a^2*b^6*c + 48*a^3*b^4*c^2 - 64*a^4*b^2*c^3)*d^2)*e^2*x^2 + (a^2*b^7 - 12*a
^3*b^5*c + 48*a^4*b^3*c^2 - 64*a^5*b*c^3)*d^2 + 2*(5*(b^7*c^2 - 12*a*b^5*c^
3 + 48*a^2*b^3*c^4 - 64*a^3*b*c^5)*d^9 + 8*(b^8*c - 12*a*b^6*c^2 + 48*a^2*b
^4*c^3 - 64*a^3*b^2*c^4)*d^7 + 3*(b^9 - 10*a*b^7*c + 24*a^2*b^5*c^2 + 32*a^
3*b^3*c^3 - 128*a^4*b*c^4)*d^5 + 4*(a*b^8 - 12*a^2*b^6*c + 48*a^3*b^4*c^2 -
 64*a^4*b^2*c^3)*d^3 + (a^2*b^7 - 12*a^3*b^5*c + 48*a^4*b^3*c^2 - 64*a^5*b*
c^3)*d)*e*x)*log(e*x + d))/((a^4*b^6*c^2 - 12*a^5*b^4*c^3 + 48*a^6*b^2*c^4
- 64*a^7*c^5)*e^11*x^10 + 10*(a^4*b^6*c^2 - 12*a^5*b^4*c^3 + 48*a^6*b^2*c^4
 - 64*a^7*c^5)*d*e^10*x^9 + (2*a^4*b^7*c - 24*a^5*b^5*c^2 + 96*a^6*b^3*c^3
- 128*a^7*b*c^4 + 45*(a^4*b^6*c^2 - 12*a^5*b^4*c^3 + 48*a^6*b^2*c^4 - 64*a^
7*c^5)*d^2)*e^9*x^8 + 8*(15*(a^4*b^6*c^2 - 12*a^5*b^4*c^3 + 48*a^6*b^2*c^4
- 64*a^7*c^5)*d^3 + 2*(a^4*b^7*c - 12*a^5*b^5*c^2 + 48*a^6*b^3*c^3 - 64*a^7
*b*c^4)*d)*e^8*x^7 + (a^4*b^8 - 10*a^5*b^6*c + 24*a^6*b^4*c^2 + 32*a^7*b^2*
c^3 - 128*a^8*c^4 + 210*(a^4*b^6*c^2 - 12*a^5*b^4*c^3 + 48*a^6*b^2*c^4 - 64
*a^7*c^5)*d^4 + 56*(a^4*b^7*c - 12*a^5*b^5*c^2 + 48*a^6*b^3*c^3 - 64*a^7*b*
c^4)*d^2)*e^7*x^6 + 2*(126*(a^4*b^6*c^2 - 12*a^5*b^4*c^3 + 48*a^6*b^2*c^4 -
 64*a^7*c^5)*d^5 + 56*(a^4*b^7*c - 12*a^5*b^5*c^2 + 48*a^6*b^3*c^3 - 64*a^7
*b*c^4)*d^3 + 3*(a^4*b^8 - 10*a^5*b^6*c + 24*a^6*b^4*c^2 + 32*a^7*b^2*c^3 -
 128*a^8*c^4)*d)*e^6*x^5 + (2*a^5*b^7 - 24*a^6*b^5*c + 96*a^7*b^3*c^2 - 128
*a^8*b*c^3 + 210*(a^4*b^6*c^2 - 12*a^5*b^4*c^3 + 48*a^6*b^2*c^4 - 64*a^7*c^
5)*d^6 + 140*(a^4*b^7*c - 12*a^5*b^5*c^2 + 48*a^6*b^3*c^3 - 64*a^7*b*c^4)*d
^4 + 15*(a^4*b^8 - 10*a^5*b^6*c + 24*a^6*b^4*c^2 + 32*a^7*b^2*c^3 - 128*a^8
*c^4)*d^2)*e^5*x^4 + 4*(30*(a^4*b^6*c^2 - 12*a^5*b^4*c^3 + 48*a^6*b^2*c^4 -
 64*a^7*c^5)*d^7 + 28*(a^4*b^7*c - 12*a^5*b^5*c^2 + 48*a^6*b^3*c^3 - 64*a^7
*b*c^4)*d^5 + 5*(a^4*b^8 - 10*a^5*b^6*c + 24*a^6*b^4*c^2 + 32*a^7*b^2*c^3 -
 128*a^8*c^4)*d^3 + 2*(a^5*b^7 - 12*a^6*b^5*c + 48*a^7*b^3*c^2 - 64*a^8*b*c
^3)*d)*e^4*x^3 + (a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3 + 45
*(a^4*b^6*c^2 - 12*a^5*b^4*c^3 + 48*a^6*b^2*c^4 - 64*a^7*c^5)*d^8 + 56*(a^4
*b^7*c - 12*a^5*b^5*c^2 + 48*a^6*b^3*c^3 - 64*a^7*b*c^4)*d^6 + 15*(a^4*b^8
- 10*a^5*b^6*c + 24*a^6*b^4*c^2 + 32*a^7*b^2*c^3 - 128*a^8*c^4)*d^4 + 12*(a
^5*b^7 - 12*a^6*b^5*c + 48*a^7*b^3*c^2 - 64*a^8*b*c^3)*d^2)*e^3*x^2 + 2*(5*
(a^4*b^6*c^2 - 12*a^5*b^4*c^3 + 48*a^6*b^2*c^4 - 64*a^7*c^5)*d^9 + 8*(a^4*b
^7*c - 12*a^5*b^5*c^2 + 48*a^6*b^3*c^3 - 64*a^7*b*c^4)*d^7 + 3*(a^4*b^8 - 1
0*a^5*b^6*c + 24*a^6*b^4*c^2 + 32*a^7*b^2*c^3 - 128*a^8*c^4)*d^5 + 4*(a^5*b
^7 - 12*a^6*b^5*c + 48*a^7*b^3*c^2 - 64*a^8*b*c^3)*d^3 + (a^6*b^6 - 12*a^7*
b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)*d)*e^2*x + ((a^4*b^6*c^2 - 12*a^5*b^4*
c^3 + 48*a^6*b^2*c^4 - 64*a^7*c^5)*d^10 + 2*(a^4*b^7*c - 12*a^5*b^5*c^2 + 4
8*a^6*b^3*c^3 - 64*a^7*b*c^4)*d^8 + (a^4*b^8 - 10*a^5*b^6*c + 24*a^6*b^4*c^
2 + 32*a^7*b^2*c^3 - 128*a^8*c^4)*d^6 + 2*(a^5*b^7 - 12*a^6*b^5*c + 48*a^7*
```

b^3*c^2 - 64*a^8*b*c^3)*d^4 + (a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64
*a^9*c^3)*d^2)*e), -1/4*(6*(a*b^6*c^2 - 11*a^2*b^4*c^3 + 38*a^3*b^2*c^4 - 4
0*a^4*c^5)*e^8*x^8 + 48*(a*b^6*c^2 - 11*a^2*b^4*c^3 + 38*a^3*b^2*c^4 - 40*a
^4*c^5)*d*e^7*x^7 + 3*(4*a*b^7*c - 45*a^2*b^5*c^2 + 162*a^3*b^3*c^3 - 184*a
^4*b*c^4 + 56*(a*b^6*c^2 - 11*a^2*b^4*c^3 + 38*a^3*b^2*c^4 - 40*a^4*c^5)*d^
2)*e^6*x^6 + 6*(56*(a*b^6*c^2 - 11*a^2*b^4*c^3 + 38*a^3*b^2*c^4 - 40*a^4*c^
5)*d^3 + 3*(4*a*b^7*c - 45*a^2*b^5*c^2 + 162*a^3*b^3*c^3 - 184*a^4*b*c^4)*d
)*e^5*x^5 + 2*a^3*b^6 - 24*a^4*b^4*c + 96*a^5*b^2*c^2 - 128*a^6*c^3 + 6*(a*
b^6*c^2 - 11*a^2*b^4*c^3 + 38*a^3*b^2*c^4 - 40*a^4*c^5)*d^8 + (6*a*b^8 - 60
*a^2*b^6*c + 158*a^3*b^4*c^2 + 44*a^4*b^2*c^3 - 400*a^5*c^4 + 420*(a*b^6*c^
2 - 11*a^2*b^4*c^3 + 38*a^3*b^2*c^4 - 40*a^4*c^5)*d^4 + 45*(4*a*b^7*c - 45*
a^2*b^5*c^2 + 162*a^3*b^3*c^3 - 184*a^4*b*c^4)*d^2)*e^4*x^4 + 3*(4*a*b^7*c
- 45*a^2*b^5*c^2 + 162*a^3*b^3*c^3 - 184*a^4*b*c^4)*d^6 + 4*(84*(a*b^6*c^2
- 11*a^2*b^4*c^3 + 38*a^3*b^2*c^4 - 40*a^4*c^5)*d^5 + 15*(4*a*b^7*c - 45*a^
2*b^5*c^2 + 162*a^3*b^3*c^3 - 184*a^4*b*c^4)*d^3 + 2*(3*a*b^8 - 30*a^2*b^6*
c + 79*a^3*b^4*c^2 + 22*a^4*b^2*c^3 - 200*a^5*c^4)*d)*e^3*x^3 + 2*(3*a*b^8
- 30*a^2*b^6*c + 79*a^3*b^4*c^2 + 22*a^4*b^2*c^3 - 200*a^5*c^4)*d^4 + (9*a^
2*b^7 - 104*a^3*b^5*c + 394*a^4*b^3*c^2 - 488*a^5*b*c^3 + 168*(a*b^6*c^2 -
11*a^2*b^4*c^3 + 38*a^3*b^2*c^4 - 40*a^4*c^5)*d^6 + 45*(4*a*b^7*c - 45*a^2*
b^5*c^2 + 162*a^3*b^3*c^3 - 184*a^4*b*c^4)*d^4 + 12*(3*a*b^8 - 30*a^2*b^6*c
 + 79*a^3*b^4*c^2 + 22*a^4*b^2*c^3 - 200*a^5*c^4)*d^2)*e^2*x^2 + (9*a^2*b^7
 - 104*a^3*b^5*c + 394*a^4*b^3*c^2 - 488*a^5*b*c^3)*d^2 + 2*(24*(a*b^6*c^2
- 11*a^2*b^4*c^3 + 38*a^3*b^2*c^4 - 40*a^4*c^5)*d^7 + 9*(4*a*b^7*c - 45*a^2
*b^5*c^2 + 162*a^3*b^3*c^3 - 184*a^4*b*c^4)*d^5 + 4*(3*a*b^8 - 30*a^2*b^6*c
 + 79*a^3*b^4*c^2 + 22*a^4*b^2*c^3 - 200*a^5*c^4)*d^3 + (9*a^2*b^7 - 104*a^
3*b^5*c + 394*a^4*b^3*c^2 - 488*a^5*b*c^3)*d)*e*x + 6*((b^6*c^2 - 10*a*b^4*
c^3 + 30*a^2*b^2*c^4 - 20*a^3*c^5)*e^10*x^10 + 10*(b^6*c^2 - 10*a*b^4*c^3 +
 30*a^2*b^2*c^4 - 20*a^3*c^5)*d*e^9*x^9 + (2*b^7*c - 20*a*b^5*c^2 + 60*a^2*
b^3*c^3 - 40*a^3*b*c^4 + 45*(b^6*c^2 - 10*a*b^4*c^3 + 30*a^2*b^2*c^4 - 20*a
^3*c^5)*d^2)*e^8*x^8 + 8*(15*(b^6*c^2 - 10*a*b^4*c^3 + 30*a^2*b^2*c^4 - 20*
a^3*c^5)*d^3 + 2*(b^7*c - 10*a*b^5*c^2 + 30*a^2*b^3*c^3 - 20*a^3*b*c^4)*d)*
e^7*x^7 + (b^8 - 8*a*b^6*c + 10*a^2*b^4*c^2 + 40*a^3*b^2*c^3 - 40*a^4*c^4 +
 210*(b^6*c^2 - 10*a*b^4*c^3 + 30*a^2*b^2*c^4 - 20*a^3*c^5)*d^4 + 56*(b^7*c
 - 10*a*b^5*c^2 + 30*a^2*b^3*c^3 - 20*a^3*b*c^4)*d^2)*e^6*x^6 + (b^6*c^2 -
10*a*b^4*c^3 + 30*a^2*b^2*c^4 - 20*a^3*c^5)*d^10 + 2*(126*(b^6*c^2 - 10*a*b
^4*c^3 + 30*a^2*b^2*c^4 - 20*a^3*c^5)*d^5 + 56*(b^7*c - 10*a*b^5*c^2 + 30*a
^2*b^3*c^3 - 20*a^3*b*c^4)*d^3 + 3*(b^8 - 8*a*b^6*c + 10*a^2*b^4*c^2 + 40*a
^3*b^2*c^3 - 40*a^4*c^4)*d)*e^5*x^5 + 2*(b^7*c - 10*a*b^5*c^2 + 30*a^2*b^3*
c^3 - 20*a^3*b*c^4)*d^8 + (2*a*b^7 - 20*a^2*b^5*c + 60*a^3*b^3*c^2 - 40*a^4
*b*c^3 + 210*(b^6*c^2 - 10*a*b^4*c^3 + 30*a^2*b^2*c^4 - 20*a^3*c^5)*d^6 + 1
40*(b^7*c - 10*a*b^5*c^2 + 30*a^2*b^3*c^3 - 20*a^3*b*c^4)*d^4 + 15*(b^8 - 8
*a*b^6*c + 10*a^2*b^4*c^2 + 40*a^3*b^2*c^3 - 40*a^4*c^4)*d^2)*e^4*x^4 + (b^
8 - 8*a*b^6*c + 10*a^2*b^4*c^2 + 40*a^3*b^2*c^3 - 40*a^4*c^4)*d^6 + 4*(30*(
b^6*c^2 - 10*a*b^4*c^3 + 30*a^2*b^2*c^4 - 20*a^3*c^5)*d^7 + 28*(b^7*c - 10*
a*b^5*c^2 + 30*a^2*b^3*c^3 - 20*a^3*b*c^4)*d^5 + 5*(b^8 - 8*a*b^6*c + 10*a^
2*b^4*c^2 + 40*a^3*b^2*c^3 - 40*a^4*c^4)*d^3 + 2*(a*b^7 - 10*a^2*b^5*c + 30
*a^3*b^3*c^2 - 20*a^4*b*c^3)*d)*e^3*x^3 + 2*(a*b^7 - 10*a^2*b^5*c + 30*a^3*
b^3*c^2 - 20*a^4*b*c^3)*d^4 + (45*(b^6*c^2 - 10*a*b^4*c^3 + 30*a^2*b^2*c^4
- 20*a^3*c^5)*d^8 + a^2*b^6 - 10*a^3*b^4*c + 30*a^4*b^2*c^2 - 20*a^5*c^3 +
56*(b^7*c - 10*a*b^5*c^2 + 30*a^2*b^3*c^3 - 20*a^3*b*c^4)*d^6 + 15*(b^8 - 8
*a*b^6*c + 10*a^2*b^4*c^2 + 40*a^3*b^2*c^3 - 40*a^4*c^4)*d^4 + 12*(a*b^7 -
10*a^2*b^5*c + 30*a^3*b^3*c^2 - 20*a^4*b*c^3)*d^2)*e^2*x^2 + (a^2*b^6 - 10*
a^3*b^4*c + 30*a^4*b^2*c^2 - 20*a^5*c^3)*d^2 + 2*(5*(b^6*c^2 - 10*a*b^4*c^3
 + 30*a^2*b^2*c^4 - 20*a^3*c^5)*d^9 + 8*(b^7*c - 10*a*b^5*c^2 + 30*a^2*b^3*
c^3 - 20*a^3*b*c^4)*d^7 + 3*(b^8 - 8*a*b^6*c + 10*a^2*b^4*c^2 + 40*a^3*b^2*
c^3 - 40*a^4*c^4)*d^5 + 4*(a*b^7 - 10*a^2*b^5*c + 30*a^3*b^3*c^2 - 20*a^4*b
*c^3)*d^3 + (a^2*b^6 - 10*a^3*b^4*c + 30*a^4*b^2*c^2 - 20*a^5*c^3)*d)*e*x)*
sqrt(-b^2 + 4*a*c)*arctan(-(2*c*e^2*x^2 + 4*c*d*e*x + 2*c*d^2 + b)*sqrt(-b^
2 + 4*a*c)/(b^2 - 4*a*c)) - 3*((b^7*c^2 - 12*a*b^5*c^3 + 48*a^2*b^3*c^4 - 6

4*a^3*b*c^5)*e^10*x^10 + 10*(b^7*c^2 - 12*a*b^5*c^3 + 48*a^2*b^3*c^4 - 64*a ^3*b*c^5)*d*e^9*x^9 + (2*b^8*c - 24*a*b^6*c^2 + 96*a^2*b^4*c^3 - 128*a^3*b^ 2*c^4 + 45*(b^7*c^2 - 12*a*b^5*c^3 + 48*a^2*b^3*c^4 - 64*a^3*b*c^5)*d^2)*e^ 8*x^8 + 8*(15*(b^7*c^2 - 12*a*b^5*c^3 + 48*a^2*b^3*c^4 - 64*a^3*b*c^5)*d^3 + 2*(b^8*c - 12*a*b^6*c^2 + 48*a^2*b^4*c^3 - 64*a^3*b^2*c^4)*d)*e^7*x^7 + (b^9 - 10*a*b^7*c + 24*a^2*b^5*c^2 + 32*a^3*b^3*c^3 - 128*a^4*b*c^4 + 210*(b ^7*c^2 - 12*a*b^5*c^3 + 48*a^2*b^3*c^4 - 64*a^3*b*c^5)*d^4 + 56*(b^8*c - 12 *a*b^6*c^2 + 48*a^2*b^4*c^3 - 64*a^3*b^2*c^4)*d^2)*e^6*x^6 + (b^7*c^2 - 12* a*b^5*c^3 + 48*a^2*b^3*c^4 - 64*a^3*b*c^5)*d^10 + 2*(126*(b^7*c^2 - 12*a*b^ 5*c^3 + 48*a^2*b^3*c^4 - 64*a^3*b*c^5)*d^5 + 56*(b^8*c - 12*a*b^6*c^2 + 48* a^2*b^4*c^3 - 64*a^3*b^2*c^4)*d^3 + 3*(b^9 - 10*a*b^7*c + 24*a^2*b^5*c^2 + 32*a^3*b^3*c^3 - 128*a^4*b*c^4)*d)*e^5*x^5 + 2*(b^8*c - 12*a*b^6*c^2 + 48*a ^2*b^4*c^3 - 64*a^3*b^2*c^4)*d^8 + (2*a*b^8 - 24*a^2*b^6*c + 96*a^3*b^4*c^2 - 128*a^4*b^2*c^3 + 210*(b^7*c^2 - 12*a*b^5*c^3 + 48*a^2*b^3*c^4 - 64*a^3* b*c^5)*d^6 + 140*(b^8*c - 12*a*b^6*c^2 + 48*a^2*b^4*c^3 - 64*a^3*b^2*c^4)*d ^4 + 15*(b^9 - 10*a*b^7*c + 24*a^2*b^5*c^2 + 32*a^3*b^3*c^3 - 128*a^4*b*c^4)*d^2)*e^4*x^4 + (b^9 - 10*a*b^7*c + 24*a^2*b^5*c^2 + 32*a^3*b^3*c^3 - 128* a^4*b*c^4)*d^6 + 4*(30*(b^7*c^2 - 12*a*b^5*c^3 + 48*a^2*b^3*c^4 - 64*a^3*b* c^5)*d^7 + 28*(b^8*c - 12*a*b^6*c^2 + 48*a^2*b^4*c^3 - 64*a^3*b^2*c^4)*d^5 + 5*(b^9 - 10*a*b^7*c + 24*a^2*b^5*c^2 + 32*a^3*b^3*c^3 - 128*a^4*b*c^4)*d^ 3 + 2*(a*b^8 - 12*a^2*b^6*c + 48*a^3*b^4*c^2 - 64*a^4*b^2*c^3)*d)*e^3*x^3 + 2*(a*b^8 - 12*a^2*b^6*c + 48*a^3*b^4*c^2 - 64*a^4*b^2*c^3)*d^4 + (a^2*b^7 - 12*a^3*b^5*c + 48*a^4*b^3*c^2 - 64*a^5*b*c^3 + 45*(b^7*c^2 - 12*a*b^5*c^3 + 48*a^2*b^3*c^4 - 64*a^3*b*c^5)*d^8 + 56*(b^8*c - 12*a*b^6*c^2 + 48*a^2*b ^4*c^3 - 64*a^3*b^2*c^4)*d^6 + 15*(b^9 - 10*a*b^7*c + 24*a^2*b^5*c^2 + 32*a ^3*b^3*c^3 - 128*a^4*b*c^4)*d^4 + 12*(a*b^8 - 12*a^2*b^6*c + 48*a^3*b^4*c^2 - 64*a^4*b^2*c^3)*d^2)*e^2*x^2 + (a^2*b^7 - 12*a^3*b^5*c + 48*a^4*b^3*c^2 - 64*a^5*b*c^3)*d^2 + 2*(5*(b^7*c^2 - 12*a*b^5*c^3 + 48*a^2*b^3*c^4 - 64*a^ 3*b*c^5)*d^9 + 8*(b^8*c - 12*a*b^6*c^2 + 48*a^2*b^4*c^3 - 64*a^3*b^2*c^4)*d ^7 + 3*(b^9 - 10*a*b^7*c + 24*a^2*b^5*c^2 + 32*a^3*b^3*c^3 - 128*a^4*b*c^4) *d^5 + 4*(a*b^8 - 12*a^2*b^6*c + 48*a^3*b^4*c^2 - 64*a^4*b^2*c^3)*d^3 + (a^ 2*b^7 - 12*a^3*b^5*c + 48*a^4*b^3*c^2 - 64*a^5*b*c^3)*d)*e*x)*log(c*e^4*x^4 + 4*c*d*e^3*x^3 + c*d^4 + (6*c*d^2 + b)*e^2*x^2 + b*d^2 + 2*(2*c*d^3 + b*d)*e*x + a) + 12*((b^7*c^2 - 12*a*b^5*c^3 + 48*a^2*b^3*c^4 - 64*a^3*b*c^5)*e ^10*x^10 + 10*(b^7*c^2 - 12*a*b^5*c^3 + 48*a^2*b^3*c^4 - 64*a^3*b*c^5)*d*e^ 9*x^9 + (2*b^8*c - 24*a*b^6*c^2 + 96*a^2*b^4*c^3 - 128*a^3*b^2*c^4 + 45*(b^ 7*c^2 - 12*a*b^5*c^3 + 48*a^2*b^3*c^4 - 64*a^3*b*c^5)*d^2)*e^8*x^8 + 8*(15* (b^7*c^2 - 12*a*b^5*c^3 + 48*a^2*b^3*c^4 - 64*a^3*b*c^5)*d^3 + 2*(b^8*c - 1 2*a*b^6*c^2 + 48*a^2*b^4*c^3 - 64*a^3*b^2*c^4)*d)*e^7*x^7 + (b^9 - 10*a*b^7 *c + 24*a^2*b^5*c^2 + 32*a^3*b^3*c^3 - 128*a^4*b*c^4 + 210*(b^7*c^2 - 12*a* b^5*c^3 + 48*a^2*b^3*c^4 - 64*a^3*b*c^5)*d^4 + 56*(b^8*c - 12*a*b^6*c^2 + 4 8*a^2*b^4*c^3 - 64*a^3*b^2*c^4)*d^2)*e^6*x^6 + (b^7*c^2 - 12*a*b^5*c^3 + 48 *a^2*b^3*c^4 - 64*a^3*b*c^5)*d^10 + 2*(126*(b^7*c^2 - 12*a*b^5*c^3 + 48*a^2 *b^3*c^4 - 64*a^3*b*c^5)*d^5 + 56*(b^8*c - 12*a*b^6*c^2 + 48*a^2*b^4*c^3 - 64*a^3*b^2*c^4)*d^3 + 3*(b^9 - 10*a*b^7*c + 24*a^2*b^5*c^2 + 32*a^3*b^3*c^3 - 128*a^4*b*c^4)*d)*e^5*x^5 + 2*(b^8*c - 12*a*b^6*c^2 + 48*a^2*b^4*c^3 - 6 4*a^3*b^2*c^4)*d^8 + (2*a*b^8 - 24*a^2*b^6*c + 96*a^3*b^4*c^2 - 128*a^4*b^2 *c^3 + 210*(b^7*c^2 - 12*a*b^5*c^3 + 48*a^2*b^3*c^4 - 64*a^3*b*c^5)*d^6 + 1 40*(b^8*c - 12*a*b^6*c^2 + 48*a^2*b^4*c^3 - 64*a^3*b^2*c^4)*d^4 + 15*(b^9 - 10*a*b^7*c + 24*a^2*b^5*c^2 + 32*a^3*b^3*c^3 - 128*a^4*b*c^4)*d^2)*e^4*x^4 + (b^9 - 10*a*b^7*c + 24*a^2*b^5*c^2 + 32*a^3*b^3*c^3 - 128*a^4*b*c^4)*d^6 + 4*(30*(b^7*c^2 - 12*a*b^5*c^3 + 48*a^2*b^3*c^4 - 64*a^3*b*c^5)*d^7 + 28* (b^8*c - 12*a*b^6*c^2 + 48*a^2*b^4*c^3 - 64*a^3*b^2*c^4)*d^5 + 5*(b^9 - 10* a*b^7*c + 24*a^2*b^5*c^2 + 32*a^3*b^3*c^3 - 128*a^4*b*c^4)*d^3 + 2*(a*b^8 - 12*a^2*b^6*c + 48*a^3*b^4*c^2 - 64*a^4*b^2*c^3)*d)*e^3*x^3 + 2*(a*b^8 - 12 *a^2*b^6*c + 48*a^3*b^4*c^2 - 64*a^4*b^2*c^3)*d^4 + (a^2*b^7 - 12*a^3*b^5*c + 48*a^4*b^3*c^2 - 64*a^5*b*c^3 + 45*(b^7*c^2 - 12*a*b^5*c^3 + 48*a^2*b^3* c^4 - 64*a^3*b*c^5)*d^8 + 56*(b^8*c - 12*a*b^6*c^2 + 48*a^2*b^4*c^3 - 64*a^ 3*b^2*c^4)*d^6 + 15*(b^9 - 10*a*b^7*c + 24*a^2*b^5*c^2 + 32*a^3*b^3*c^3 - 1

```
28*a^4*b*c^4)*d^4 + 12*(a*b^8 - 12*a^2*b^6*c + 48*a^3*b^4*c^2 - 64*a^4*b^2*
c^3)*d^2)*e^2*x^2 + (a^2*b^7 - 12*a^3*b^5*c + 48*a^4*b^3*c^2 - 64*a^5*b*c^3
)*d^2 + 2*(5*(b^7*c^2 - 12*a*b^5*c^3 + 48*a^2*b^3*c^4 - 64*a^3*b*c^5)*d^9 +
 8*(b^8*c - 12*a*b^6*c^2 + 48*a^2*b^4*c^3 - 64*a^3*b^2*c^4)*d^7 + 3*(b^9 -
10*a*b^7*c + 24*a^2*b^5*c^2 + 32*a^3*b^3*c^3 - 128*a^4*b*c^4)*d^5 + 4*(a*b^
8 - 12*a^2*b^6*c + 48*a^3*b^4*c^2 - 64*a^4*b^2*c^3)*d^3 + (a^2*b^7 - 12*a^3
*b^5*c + 48*a^4*b^3*c^2 - 64*a^5*b*c^3)*d)*e*x)*log(e*x + d))/((a^4*b^6*c^2
 - 12*a^5*b^4*c^3 + 48*a^6*b^2*c^4 - 64*a^7*c^5)*e^11*x^10 + 10*(a^4*b^6*c^
2 - 12*a^5*b^4*c^3 + 48*a^6*b^2*c^4 - 64*a^7*c^5)*d*e^10*x^9 + (2*a^4*b^7*c
 - 24*a^5*b^5*c^2 + 96*a^6*b^3*c^3 - 128*a^7*b*c^4 + 45*(a^4*b^6*c^2 - 12*a
^5*b^4*c^3 + 48*a^6*b^2*c^4 - 64*a^7*c^5)*d^2)*e^9*x^8 + 8*(15*(a^4*b^6*c^2
 - 12*a^5*b^4*c^3 + 48*a^6*b^2*c^4 - 64*a^7*c^5)*d^3 + 2*(a^4*b^7*c - 12*a^
5*b^5*c^2 + 48*a^6*b^3*c^3 - 64*a^7*b*c^4)*d)*e^8*x^7 + (a^4*b^8 - 10*a^5*b
^6*c + 24*a^6*b^4*c^2 + 32*a^7*b^2*c^3 - 128*a^8*c^4 + 210*(a^4*b^6*c^2 - 1
2*a^5*b^4*c^3 + 48*a^6*b^2*c^4 - 64*a^7*c^5)*d^4 + 56*(a^4*b^7*c - 12*a^5*b
^5*c^2 + 48*a^6*b^3*c^3 - 64*a^7*b*c^4)*d^2)*e^7*x^6 + 2*(126*(a^4*b^6*c^2
- 12*a^5*b^4*c^3 + 48*a^6*b^2*c^4 - 64*a^7*c^5)*d^5 + 56*(a^4*b^7*c - 12*a^
5*b^5*c^2 + 48*a^6*b^3*c^3 - 64*a^7*b*c^4)*d^3 + 3*(a^4*b^8 - 10*a^5*b^6*c
+ 24*a^6*b^4*c^2 + 32*a^7*b^2*c^3 - 128*a^8*c^4)*d)*e^6*x^5 + (2*a^5*b^7 -
24*a^6*b^5*c + 96*a^7*b^3*c^2 - 128*a^8*b*c^3 + 210*(a^4*b^6*c^2 - 12*a^5*b
^4*c^3 + 48*a^6*b^2*c^4 - 64*a^7*c^5)*d^6 + 140*(a^4*b^7*c - 12*a^5*b^5*c^2
 + 48*a^6*b^3*c^3 - 64*a^7*b*c^4)*d^4 + 15*(a^4*b^8 - 10*a^5*b^6*c + 24*a^6
*b^4*c^2 + 32*a^7*b^2*c^3 - 128*a^8*c^4)*d^2)*e^5*x^4 + 4*(30*(a^4*b^6*c^2
- 12*a^5*b^4*c^3 + 48*a^6*b^2*c^4 - 64*a^7*c^5)*d^7 + 28*(a^4*b^7*c - 12*a^
5*b^5*c^2 + 48*a^6*b^3*c^3 - 64*a^7*b*c^4)*d^5 + 5*(a^4*b^8 - 10*a^5*b^6*c
+ 24*a^6*b^4*c^2 + 32*a^7*b^2*c^3 - 128*a^8*c^4)*d^3 + 2*(a^5*b^7 - 12*a^6*
b^5*c + 48*a^7*b^3*c^2 - 64*a^8*b*c^3)*d)*e^4*x^3 + (a^6*b^6 - 12*a^7*b^4*c
 + 48*a^8*b^2*c^2 - 64*a^9*c^3 + 45*(a^4*b^6*c^2 - 12*a^5*b^4*c^3 + 48*a^6*
b^2*c^4 - 64*a^7*c^5)*d^8 + 56*(a^4*b^7*c - 12*a^5*b^5*c^2 + 48*a^6*b^3*c^3
 - 64*a^7*b*c^4)*d^6 + 15*(a^4*b^8 - 10*a^5*b^6*c + 24*a^6*b^4*c^2 + 32*a^7
*b^2*c^3 - 128*a^8*c^4)*d^4 + 12*(a^5*b^7 - 12*a^6*b^5*c + 48*a^7*b^3*c^2 -
 64*a^8*b*c^3)*d^2)*e^3*x^2 + 2*(5*(a^4*b^6*c^2 - 12*a^5*b^4*c^3 + 48*a^6*b
^2*c^4 - 64*a^7*c^5)*d^9 + 8*(a^4*b^7*c - 12*a^5*b^5*c^2 + 48*a^6*b^3*c^3 -
 64*a^7*b*c^4)*d^7 + 3*(a^4*b^8 - 10*a^5*b^6*c + 24*a^6*b^4*c^2 + 32*a^7*b^
2*c^3 - 128*a^8*c^4)*d^5 + 4*(a^5*b^7 - 12*a^6*b^5*c + 48*a^7*b^3*c^2 - 64*
a^8*b*c^3)*d^3 + (a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)*d)*
e^2*x + ((a^4*b^6*c^2 - 12*a^5*b^4*c^3 + 48*a^6*b^2*c^4 - 64*a^7*c^5)*d^10
+ 2*(a^4*b^7*c - 12*a^5*b^5*c^2 + 48*a^6*b^3*c^3 - 64*a^7*b*c^4)*d^8 + (a^4
*b^8 - 10*a^5*b^6*c + 24*a^6*b^4*c^2 + 32*a^7*b^2*c^3 - 128*a^8*c^4)*d^6 +
2*(a^5*b^7 - 12*a^6*b^5*c + 48*a^7*b^3*c^2 - 64*a^8*b*c^3)*d^4 + (a^6*b^6 -
 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)*d^2)*e)]
```

giac [A] time = 0.64, size = 377, normalized size = 1.16

$$\frac{3\left(b^{6}-10\,ab^{4}c+30\,a^{2}b^{2}c^{2}-20\,a^{3}c^{3}\right)\arctan\left(-\frac{b+\frac{2\,a}{(xe+d)^{2}}}{\sqrt{-b^{2}+4\,ac}}\right)e^{(-1)}}{2\left(a^{4}b^{4}-8\,a^{5}b^{2}c+16\,a^{6}c^{2}\right)\sqrt{-b^{2}+4\,ac}}+\frac{3\,be^{(-1)}\log\left(c+\frac{b}{(xe+d)^{2}}+\frac{a}{(xe+d)^{4}}\right)}{4\,a^{4}}-\frac{e^{(-1)}}{2\,(xe+d)^{2}a^{3}}+\frac{\left(5\,b^{5}c^{2}-36\,ab^{3}c^{3}+58\,a^{2}bc^{4}+\frac{2\left(5\,b^{6}ce-38\,ab^{4}c^{2}e+71\,a^{2}b^{2}c^{3}e-14\,a^{3}c^{4}e\right)e^{(-1)}}{(xe+d)^{2}}+\frac{\left(5\,b^{7}e^{2}-34\,ab^{5}ce^{2}+41\,a^{2}b^{3}c^{2}e^{2}+42\,a^{3}bc^{3}e^{2}\right)e^{(-2)}}{(xe+d)^{4}}+\frac{6\left(ab^{6}e^{3}-8\,a^{2}b^{4}ce^{3}+17\,a^{3}b^{2}c^{2}e^{3}-6\,a^{4}c^{3}e^{3}\right)e^{(-3)}}{(xe+d)^{6}}\right)e^{(-1)}}{4\left(b^{2}-4\,ac\right)^{2}a^{4}\left(c+\frac{b}{(xe+d)^{2}}+\frac{a}{(xe+d)^{4}}\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="giac")
```

```
[Out] 3/2*(b^6 - 10*a*b^4*c + 30*a^2*b^2*c^2 - 20*a^3*c^3)*arctan(-(b + 2*a/(x*e
+ d)^2)/sqrt(-b^2 + 4*a*c))*e^(-1)/((a^4*b^4 - 8*a^5*b^2*c + 16*a^6*c^2)*sq
rt(-b^2 + 4*a*c)) + 3/4*b*e^(-1)*log(c + b/(x*e + d)^2 + a/(x*e + d)^4)/a^4
 - 1/2*e^(-1)/((x*e + d)^2*a^3) + 1/4*(5*b^5*c^2 - 36*a*b^3*c^3 + 58*a^2*b*
c^4 + 2*(5*b^6*c*e - 38*a*b^4*c^2*e + 71*a^2*b^2*c^3*e - 14*a^3*c^4*e)*e^(-
1)/(x*e + d)^2 + (5*b^7*e^2 - 34*a*b^5*c*e^2 + 41*a^2*b^3*c^2*e^2 + 42*a^3*
b*c^3*e^2)*e^(-2)/(x*e + d)^4 + 6*(a*b^6*e^3 - 8*a^2*b^4*c*e^3 + 17*a^3*b^2
*c^2*e^3 - 6*a^4*c^3*e^3)*e^(-3)/(x*e + d)^6)*e^(-1)/((b^2 - 4*a*c)^2*a^4*(
c + b/(x*e + d)^2 + a/(x*e + d)^4)^2)
```

maple [C] time = 0.09, size = 5575, normalized size = 17.15

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x)
[Out] result too large to display
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="maxima")
[Out] Timed out
```

mupad [B] time = 22.45, size = 21465, normalized size = 66.05

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((d + e*x)^3*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3),x)
[Out] (log(((27*c^4*e^14*(b^4 + 10*a^2*c^2 - 7*a*b^2*c)^2*(b^5 + 16*a^2*b*c^2 + b
^4*c*d^2 + 10*a^2*c^3*d^2 - 8*a*b^3*c - 7*a*b^2*c^2*d^2))/(a^9*(4*a*c - b^2
)^6) - ((3*b - 3*a^4*e*(-(b^6 - 20*a^3*c^3 + 30*a^2*b^2*c^2 - 10*a*b^4*c)^2
/(a^8*e^2*(4*a*c - b^2)^5))^(1/2))*((9*c^3*e^15*(b^4 + 10*a^2*c^2 - 7*a*b^2
*c)*(4*b^6 - 10*a^3*c^3 + 6*b^5*c*d^2 + 71*a^2*b^2*c^2 - 33*a*b^4*c - 47*a*
b^3*c^2*d^2 + 90*a^2*b*c^3*d^2))/(a^6*(4*a*c - b^2)^4) - ((3*b - 3*a^4*e*(-
(b^6 - 20*a^3*c^3 + 30*a^2*b^2*c^2 - 10*a*b^4*c)^2/(a^8*e^2*(4*a*c - b^2)^5
))^(1/2))*((6*c^2*e^16*(2*b^7 - 20*a^3*b*c^3 + b^6*c*d^2 + 46*a^2*b^3*c^2 +
 100*a^3*c^4*d^2 - 18*a*b^5*c - 2*a*b^4*c^2*d^2 - 30*a^2*b^2*c^3*d^2))/(a^3
*(4*a*c - b^2)^2) + (6*c^3*e^18*x^2*(b^6 + 100*a^3*c^3 - 30*a^2*b^2*c^2 - 2
*a*b^4*c))/(a^3*(4*a*c - b^2)^2) + (b*c^2*e^16*(3*b - 3*a^4*e*(-(b^6 - 20*a
^3*c^3 + 30*a^2*b^2*c^2 - 10*a*b^4*c)^2/(a^8*e^2*(4*a*c - b^2)^5))^(1/2))*(
a*b + 3*b^2*d^2 + 3*b^2*e^2*x^2 - 10*a*c*d^2 + 6*b^2*d*e*x - 10*a*c*e^2*x^2
 - 20*a*c*d*e*x))/a^4 + (12*c^3*d*e^17*x*(b^6 + 100*a^3*c^3 - 30*a^2*b^2*c^
2 - 2*a*b^4*c))/(a^3*(4*a*c - b^2)^2)))/(4*a^4*e) + (9*b*c^4*e^17*x^2*(6*b^
8 + 900*a^4*c^4 + 479*a^2*b^4*c^2 - 1100*a^3*b^2*c^3 - 89*a*b^6*c))/(a^6*(4
*a*c - b^2)^4) + (18*b*c^4*d*e^16*x*(6*b^8 + 900*a^4*c^4 + 479*a^2*b^4*c^2
- 1100*a^3*b^2*c^3 - 89*a*b^6*c))/(a^6*(4*a*c - b^2)^4)))/(4*a^4*e) + (27*c
^5*e^16*x^2*(b^4 + 10*a^2*c^2 - 7*a*b^2*c)^3)/(a^9*(4*a*c - b^2)^6) + (54*c
^5*d*e^15*x*(b^4 + 10*a^2*c^2 - 7*a*b^2*c)^3)/(a^9*(4*a*c - b^2)^6))*((27*c
^4*e^14*(b^4 + 10*a^2*c^2 - 7*a*b^2*c)^2*(b^5 + 16*a^2*b*c^2 + b^4*c*d^2 +
10*a^2*c^3*d^2 - 8*a*b^3*c - 7*a*b^2*c^2*d^2))/(a^9*(4*a*c - b^2)^6) - ((3*
b + 3*a^4*e*(-(b^6 - 20*a^3*c^3 + 30*a^2*b^2*c^2 - 10*a*b^4*c)^2/(a^8*e^2*(
4*a*c - b^2)^5))^(1/2))*((9*c^3*e^15*(b^4 + 10*a^2*c^2 - 7*a*b^2*c)*(4*b^6
- 10*a^3*c^3 + 6*b^5*c*d^2 + 71*a^2*b^2*c^2 - 33*a*b^4*c - 47*a*b^3*c^2*d^2
 + 90*a^2*b*c^3*d^2))/(a^6*(4*a*c - b^2)^4) - ((3*b + 3*a^4*e*(-(b^6 - 20*a
^3*c^3 + 30*a^2*b^2*c^2 - 10*a*b^4*c)^2/(a^8*e^2*(4*a*c - b^2)^5))^(1/2))*(
(6*c^2*e^16*(2*b^7 - 20*a^3*b*c^3 + b^6*c*d^2 + 46*a^2*b^3*c^2 + 100*a^3*c^
4*d^2 - 18*a*b^5*c - 2*a*b^4*c^2*d^2 - 30*a^2*b^2*c^3*d^2))/(a^3*(4*a*c - b
^2)^2) + (6*c^3*e^18*x^2*(b^6 + 100*a^3*c^3 - 30*a^2*b^2*c^2 - 2*a*b^4*c))/
(a^3*(4*a*c - b^2)^2) + (b*c^2*e^16*(3*b + 3*a^4*e*(-(b^6 - 20*a^3*c^3 + 30
*a^2*b^2*c^2 - 10*a*b^4*c)^2/(a^8*e^2*(4*a*c - b^2)^5))^(1/2))*(a*b + 3*b^2
*d^2 + 3*b^2*e^2*x^2 - 10*a*c*d^2 + 6*b^2*d*e*x - 10*a*c*e^2*x^2 - 20*a*c*d
*e*x))/a^4 + (12*c^3*d*e^17*x*(b^6 + 100*a^3*c^3 - 30*a^2*b^2*c^2 - 2*a*b^4
*c))/(a^3*(4*a*c - b^2)^2)))/(4*a^4*e) + (9*b*c^4*e^17*x^2*(6*b^8 + 900*a^4
```

*c^4 + 479*a^2*b^4*c^2 - 1100*a^3*b^2*c^3 - 89*a*b^6*c))/(a^6*(4*a*c - b^2)
^4) + (18*b*c^4*d*e^16*x*(6*b^8 + 900*a^4*c^4 + 479*a^2*b^4*c^2 - 1100*a^3*
b^2*c^3 - 89*a*b^6*c))/(a^6*(4*a*c - b^2)^4)))/(4*a^4*e) + (27*c^5*e^16*x^2
*(b^4 + 10*a^2*c^2 - 7*a*b^2*c)^3)/(a^9*(4*a*c - b^2)^6) + (54*c^5*d*e^15*x
*(b^4 + 10*a^2*c^2 - 7*a*b^2*c)^3)/(a^9*(4*a*c - b^2)^6)))*(6*b^11*e + 960*
a^2*b^7*c^2*e - 3840*a^3*b^5*c^3*e + 7680*a^4*b^3*c^4*e - 120*a*b^9*c*e - 6
144*a^5*b*c^5*e))/(2*(4*a^4*b^10*e^2 - 4096*a^9*c^5*e^2 - 80*a^5*b^8*c*e^2
+ 640*a^6*b^6*c^2*e^2 - 2560*a^7*b^4*c^3*e^2 + 5120*a^8*b^2*c^4*e^2)) - ((x
^4*(6*b^6*e^3 + 100*a^3*c^3*e^3 + 180*b^5*c*d^2*e^3 + 14*a^2*b^2*c^2*e^3 +
4200*a^2*c^4*d^4*e^3 + 420*b^4*c^2*d^4*e^3 - 36*a*b^4*c*e^3 - 1305*a*b^3*c^
2*d^2*e^3 + 2070*a^2*b*c^3*d^2*e^3 - 2940*a*b^2*c^3*d^4*e^3))/(4*(a^3*b^4 +
16*a^5*c^2 - 8*a^4*b^2*c)) + (3*x^6*(4*b^5*c*e^5 - 29*a*b^3*c^2*e^5 + 46*a
^2*b*c^3*e^5 + 560*a^2*c^4*d^2*e^5 + 56*b^4*c^2*d^2*e^5 - 392*a*b^2*c^3*d^2
e^5))/(4(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)) + (x*(12*b^6*d^3 + 36*b^5*c
*d^5 + 200*a^3*c^3*d^3 + 240*a^2*c^4*d^7 + 24*b^4*c^2*d^7 + 9*a*b^5*d - 261
*a*b^3*c^2*d^5 + 414*a^2*b*c^3*d^5 - 168*a*b^2*c^3*d^7 + 28*a^2*b^2*c^2*d^3
- 68*a^2*b^3*c*d + 122*a^3*b*c^2*d - 72*a*b^4*c*d^3))/(2*(a^3*b^4 + 16*a^5
*c^2 - 8*a^4*b^2*c)) + (3*x^5*(560*a^2*c^4*d^3*e^4 + 56*b^4*c^2*d^3*e^4 + 1
2*b^5*c*d*e^4 - 87*a*b^3*c^2*d*e^4 + 138*a^2*b*c^3*d*e^4 - 392*a*b^2*c^3*d^
3*e^4))/(2*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)) + (3*x^8*(10*a^2*c^4*e^7 +
b^4*c^2*e^7 - 7*a*b^2*c^3*e^7))/(2*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)) +
(x^2*(36*b^6*d^2*e + 9*a*b^5*e + 600*a^3*c^3*d^2*e + 1680*a^2*c^4*d^6*e +
168*b^4*c^2*d^6*e - 68*a^2*b^3*c*e + 122*a^3*b*c^2*e + 180*b^5*c*d^4*e - 21
6*a*b^4*c*d^2*e - 1305*a*b^3*c^2*d^4*e + 2070*a^2*b*c^3*d^4*e - 1176*a*b^2*
c^3*d^6*e + 84*a^2*b^2*c^2*d^2*e))/(4*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c))
+ (x^3*(6*b^6*d*e^2 + 100*a^3*c^3*d*e^2 + 60*b^5*c*d^3*e^2 + 840*a^2*c^4*d
^5*e^2 + 84*b^4*c^2*d^5*e^2 - 36*a*b^4*c*d*e^2 + 14*a^2*b^2*c^2*d*e^2 - 435
*a*b^3*c^2*d^3*e^2 + 690*a^2*b*c^3*d^3*e^2 - 588*a*b^2*c^3*d^5*e^2))/(a^3*b
^4 + 16*a^5*c^2 - 8*a^4*b^2*c) + (12*x^7*(10*a^2*c^4*d*e^6 + b^4*c^2*d*e^6
- 7*a*b^2*c^3*d*e^6))/(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c) + (2*a^2*b^4 + 3
2*a^4*c^2 + 6*b^6*d^4 - 16*a^3*b^2*c + 9*a*b^5*d^2 + 12*b^5*c*d^6 + 100*a^3
*c^3*d^4 + 60*a^2*c^4*d^8 + 6*b^4*c^2*d^8 - 68*a^2*b^3*c*d^2 + 122*a^3*b*c^
2*d^2 - 87*a*b^3*c^2*d^6 + 138*a^2*b*c^3*d^6 - 42*a*b^2*c^3*d^8 + 14*a^2*b^
2*c^2*d^4 - 36*a*b^4*c*d^4)/(4*e*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))/(x^
4*(15*b^2*d^2*e^4 + 210*c^2*d^6*e^4 + 2*a*b*e^4 + 30*a*c*d^2*e^4 + 140*b*c*
d^4*e^4) + x^8*(45*c^2*d^2*e^8 + 2*b*c*e^8) + x^5*(6*b^2*d*e^5 + 252*c^2*d^
5*e^5 + 12*a*c*d*e^5 + 112*b*c*d^3*e^5) + x^3*(20*b^2*d^3*e^3 + 120*c^2*d^7
*e^3 + 8*a*b*d*e^3 + 40*a*c*d^3*e^3 + 112*b*c*d^5*e^3) + x^7*(120*c^2*d^3*e
^7 + 16*b*c*d*e^7) + x*(6*b^2*d^5*e + 10*c^2*d^9*e + 2*a^2*d*e + 8*a*b*d^3*
e + 12*a*c*d^5*e + 16*b*c*d^7*e) + x^6*(b^2*e^6 + 210*c^2*d^4*e^6 + 2*a*c*e
^6 + 56*b*c*d^2*e^6) + x^2*(a^2*e^2 + 15*b^2*d^4*e^2 + 45*c^2*d^8*e^2 + 12*
a*b*d^2*e^2 + 30*a*c*d^4*e^2 + 56*b*c*d^6*e^2) + a^2*d^2 + b^2*d^6 + c^2*d^
10 + c^2*e^10*x^10 + 2*a*b*d^4 + 2*a*c*d^6 + 2*b*c*d^8 + 10*c^2*d*e^9*x^9)
- (3*b*log(d + e*x))/(a^4*e) - (3*atan((x^2*((((27000*a^6*c^11*e^16 + 27*b^
12*c^5*e^16 - 567*a*b^10*c^6*e^16 + 4779*a^2*b^8*c^7*e^16 - 20601*a^3*b^6*c
^8*e^16 + 47790*a^4*b^4*c^9*e^16 - 56700*a^5*b^2*c^10*e^16)/(a^9*b^12 + 409
6*a^15*c^6 - 24*a^10*b^10*c + 240*a^11*b^8*c^2 - 1280*a^12*b^6*c^3 + 3840*a
^13*b^4*c^4 - 6144*a^14*b^2*c^5) - (((129600*a^9*b*c^10*e^17 + 54*a^3*b^13*
c^4*e^17 - 1233*a^4*b^11*c^5*e^17 + 11583*a^5*b^9*c^6*e^17 - 57204*a^6*b^7*
c^7*e^17 + 156276*a^7*b^5*c^8*e^17 - 223200*a^8*b^3*c^9*e^17)/(a^9*b^12 + 4
096*a^15*c^6 - 24*a^10*b^10*c + 240*a^11*b^8*c^2 - 1280*a^12*b^6*c^3 + 3840
*a^13*b^4*c^4 - 6144*a^14*b^2*c^5) - (((153600*a^13*c^10*e^18 + 6*a^6*b^14*
c^3*e^18 - 108*a^7*b^12*c^4*e^18 + 588*a^8*b^10*c^5*e^18 + 792*a^9*b^8*c^6*
e^18 - 22272*a^10*b^6*c^7*e^18 + 100608*a^11*b^4*c^8*e^18 - 199680*a^12*b^2
*c^9*e^18)/(a^9*b^12 + 4096*a^15*c^6 - 24*a^10*b^10*c + 240*a^11*b^8*c^2 -
1280*a^12*b^6*c^3 + 3840*a^13*b^4*c^4 - 6144*a^14*b^2*c^5) - ((6*b^11*e + 9
60*a^2*b^7*c^2*e - 3840*a^3*b^5*c^3*e + 7680*a^4*b^3*c^4*e - 120*a*b^9*c*e
- 6144*a^5*b*c^5*e)*(163840*a^16*b*c^9*e^19 - 12*a^9*b^15*c^2*e^19 + 328*a^
10*b^13*c^3*e^19 - 3840*a^11*b^11*c^4*e^19 + 24960*a^12*b^9*c^5*e^19 - 9728

0*a^13*b^7*c^6*e^19 + 227328*a^14*b^5*c^7*e^19 - 294912*a^15*b^3*c^8*e^19))
/(2*(4*a^4*b^10*e^2 - 4096*a^9*c^5*e^2 - 80*a^5*b^8*c*e^2 + 640*a^6*b^6*c^2
*e^2 - 2560*a^7*b^4*c^3*e^2 + 5120*a^8*b^2*c^4*e^2)*(a^9*b^12 + 4096*a^15*c
^6 - 24*a^10*b^10*c + 240*a^11*b^8*c^2 - 1280*a^12*b^6*c^3 + 3840*a^13*b^4*
c^4 - 6144*a^14*b^2*c^5)))*(6*b^11*e + 960*a^2*b^7*c^2*e - 3840*a^3*b^5*c^3
*e + 7680*a^4*b^3*c^4*e - 120*a*b^9*c*e - 6144*a^5*b*c^5*e))/(2*(4*a^4*b^10
*e^2 - 4096*a^9*c^5*e^2 - 80*a^5*b^8*c*e^2 + 640*a^6*b^6*c^2*e^2 - 2560*a^7
*b^4*c^3*e^2 + 5120*a^8*b^2*c^4*e^2)))*(6*b^11*e + 960*a^2*b^7*c^2*e - 3840
*a^3*b^5*c^3*e + 7680*a^4*b^3*c^4*e - 120*a*b^9*c*e - 6144*a^5*b*c^5*e))/(2
*(4*a^4*b^10*e^2 - 4096*a^9*c^5*e^2 - 80*a^5*b^8*c*e^2 + 640*a^6*b^6*c^2*e^
2 - 2560*a^7*b^4*c^3*e^2 + 5120*a^8*b^2*c^4*e^2)) - (3*((3*((153600*a^13*c^
10*e^18 + 6*a^6*b^14*c^3*e^18 - 108*a^7*b^12*c^4*e^18 + 588*a^8*b^10*c^5*e^
18 + 792*a^9*b^8*c^6*e^18 - 22272*a^10*b^6*c^7*e^18 + 100608*a^11*b^4*c^8*e
^18 - 199680*a^12*b^2*c^9*e^18)/(a^9*b^12 + 4096*a^15*c^6 - 24*a^10*b^10*c
+ 240*a^11*b^8*c^2 - 1280*a^12*b^6*c^3 + 3840*a^13*b^4*c^4 - 6144*a^14*b^2*
c^5) - ((6*b^11*e + 960*a^2*b^7*c^2*e - 3840*a^3*b^5*c^3*e + 7680*a^4*b^3*c
^4*e - 120*a*b^9*c*e - 6144*a^5*b*c^5*e)*(163840*a^16*b*c^9*e^19 - 12*a^9*b
^15*c^2*e^19 + 328*a^10*b^13*c^3*e^19 - 3840*a^11*b^11*c^4*e^19 + 24960*a^1
2*b^9*c^5*e^19 - 97280*a^13*b^7*c^6*e^19 + 227328*a^14*b^5*c^7*e^19 - 29491
2*a^15*b^3*c^8*e^19))/(2*(4*a^4*b^10*e^2 - 4096*a^9*c^5*e^2 - 80*a^5*b^8*c*
e^2 + 640*a^6*b^6*c^2*e^2 - 2560*a^7*b^4*c^3*e^2 + 5120*a^8*b^2*c^4*e^2)*(a
^9*b^12 + 4096*a^15*c^6 - 24*a^10*b^10*c + 240*a^11*b^8*c^2 - 1280*a^12*b^6
*c^3 + 3840*a^13*b^4*c^4 - 6144*a^14*b^2*c^5)))*(b^6 - 20*a^3*c^3 + 30*a^2*
b^2*c^2 - 10*a*b^4*c))/(4*a^4*e*(4*a*c - b^2)^(5/2)) - (3*(b^6 - 20*a^3*c^3
 + 30*a^2*b^2*c^2 - 10*a*b^4*c)*(6*b^11*e + 960*a^2*b^7*c^2*e - 3840*a^3*b^
5*c^3*e + 7680*a^4*b^3*c^4*e - 120*a*b^9*c*e - 6144*a^5*b*c^5*e)*(163840*a^
16*b*c^9*e^19 - 12*a^9*b^15*c^2*e^19 + 328*a^10*b^13*c^3*e^19 - 3840*a^11*b
^11*c^4*e^19 + 24960*a^12*b^9*c^5*e^19 - 97280*a^13*b^7*c^6*e^19 + 227328*a
^14*b^5*c^7*e^19 - 294912*a^15*b^3*c^8*e^19))/(8*a^4*e*(4*a*c - b^2)^(5/2)*
(4*a^4*b^10*e^2 - 4096*a^9*c^5*e^2 - 80*a^5*b^8*c*e^2 + 640*a^6*b^6*c^2*e^2
 - 2560*a^7*b^4*c^3*e^2 + 5120*a^8*b^2*c^4*e^2)*(a^9*b^12 + 4096*a^15*c^6 -
 24*a^10*b^10*c + 240*a^11*b^8*c^2 - 1280*a^12*b^6*c^3 + 3840*a^13*b^4*c^4
- 6144*a^14*b^2*c^5)))*(b^6 - 20*a^3*c^3 + 30*a^2*b^2*c^2 - 10*a*b^4*c))/(4
*a^4*e*(4*a*c - b^2)^(5/2)) + (9*(b^6 - 20*a^3*c^3 + 30*a^2*b^2*c^2 - 10*a*
b^4*c)^2*(6*b^11*e + 960*a^2*b^7*c^2*e - 3840*a^3*b^5*c^3*e + 7680*a^4*b^3*
c^4*e - 120*a*b^9*c*e - 6144*a^5*b*c^5*e)*(163840*a^16*b*c^9*e^19 - 12*a^9*
b^15*c^2*e^19 + 328*a^10*b^13*c^3*e^19 - 3840*a^11*b^11*c^4*e^19 + 24960*a^
12*b^9*c^5*e^19 - 97280*a^13*b^7*c^6*e^19 + 227328*a^14*b^5*c^7*e^19 - 2949
12*a^15*b^3*c^8*e^19))/(32*a^8*e^2*(4*a*c - b^2)^5*(4*a^4*b^10*e^2 - 4096*a
^9*c^5*e^2 - 80*a^5*b^8*c*e^2 + 640*a^6*b^6*c^2*e^2 - 2560*a^7*b^4*c^3*e^2
+ 5120*a^8*b^2*c^4*e^2)*(a^9*b^12 + 4096*a^15*c^6 - 24*a^10*b^10*c + 240*a^
11*b^8*c^2 - 1280*a^12*b^6*c^3 + 3840*a^13*b^4*c^4 - 6144*a^14*b^2*c^5)))*(
3*b^8 + 10*a^4*c^4 + 120*a^2*b^4*c^2 - 145*a^3*b^2*c^3 - 33*a*b^6*c))/(8*a^
3*c^2*(4*a*c - b^2)^6*(100*a^6*c^6 - 6*b^12 - 960*a^2*b^8*c^2 + 3840*a^3*b^
6*c^3 - 7675*a^4*b^4*c^4 + 6100*a^5*b^2*c^5 + 120*a*b^10*c)) + (b*((((3*((1
53600*a^13*c^10*e^18 + 6*a^6*b^14*c^3*e^18 - 108*a^7*b^12*c^4*e^18 + 588*a^
8*b^10*c^5*e^18 + 792*a^9*b^8*c^6*e^18 - 22272*a^10*b^6*c^7*e^18 + 100608*a
^11*b^4*c^8*e^18 - 199680*a^12*b^2*c^9*e^18)/(a^9*b^12 + 4096*a^15*c^6 - 24
*a^10*b^10*c + 240*a^11*b^8*c^2 - 1280*a^12*b^6*c^3 + 3840*a^13*b^4*c^4 - 6
144*a^14*b^2*c^5) - ((6*b^11*e + 960*a^2*b^7*c^2*e - 3840*a^3*b^5*c^3*e + 7
680*a^4*b^3*c^4*e - 120*a*b^9*c*e - 6144*a^5*b*c^5*e)*(163840*a^16*b*c^9*e^
19 - 12*a^9*b^15*c^2*e^19 + 328*a^10*b^13*c^3*e^19 - 3840*a^11*b^11*c^4*e^1
9 + 24960*a^12*b^9*c^5*e^19 - 97280*a^13*b^7*c^6*e^19 + 227328*a^14*b^5*c^7
*e^19 - 294912*a^15*b^3*c^8*e^19))/(2*(4*a^4*b^10*e^2 - 4096*a^9*c^5*e^2 -
80*a^5*b^8*c*e^2 + 640*a^6*b^6*c^2*e^2 - 2560*a^7*b^4*c^3*e^2 + 5120*a^8*b^
2*c^4*e^2)*(a^9*b^12 + 4096*a^15*c^6 - 24*a^10*b^10*c + 240*a^11*b^8*c^2 -
1280*a^12*b^6*c^3 + 3840*a^13*b^4*c^4 - 6144*a^14*b^2*c^5)))*(b^6 - 20*a^3*
c^3 + 30*a^2*b^2*c^2 - 10*a*b^4*c))/(4*a^4*e*(4*a*c - b^2)^(5/2)) - (3*(b^6
 - 20*a^3*c^3 + 30*a^2*b^2*c^2 - 10*a*b^4*c)*(6*b^11*e + 960*a^2*b^7*c^2*e

- 3840*a^3*b^5*c^3*e + 7680*a^4*b^3*c^4*e - 120*a*b^9*c*e - 6144*a^5*b*c^5*e)*(163840*a^16*b*c^9*e^19 - 12*a^9*b^15*c^2*e^19 + 328*a^10*b^13*c^3*e^19 - 3840*a^11*b^11*c^4*e^19 + 24960*a^12*b^9*c^5*e^19 - 97280*a^13*b^7*c^6*e^19 + 227328*a^14*b^5*c^7*e^19 - 294912*a^15*b^3*c^8*e^19))/(8*a^4*e*(4*a*c - b^2)^(5/2)*(4*a^4*b^10*e^2 - 4096*a^9*c^5*e^2 - 80*a^5*b^8*c*e^2 + 640*a^6*b^6*c^2*e^2 - 2560*a^7*b^4*c^3*e^2 + 5120*a^8*b^2*c^4*e^2)*(a^9*b^12 + 4096*a^15*c^6 - 24*a^10*b^10*c + 240*a^11*b^8*c^2 - 1280*a^12*b^6*c^3 + 3840*a^13*b^4*c^4 - 6144*a^14*b^2*c^5)))*(6*b^11*e + 960*a^2*b^7*c^2*e - 3840*a^3*b^5*c^3*e + 7680*a^4*b^3*c^4*e - 120*a*b^9*c*e - 6144*a^5*b*c^5*e))/(2*(4*a^4*b^10*e^2 - 4096*a^9*c^5*e^2 - 80*a^5*b^8*c*e^2 + 640*a^6*b^6*c^2*e^2 - 2560*a^7*b^4*c^3*e^2 + 5120*a^8*b^2*c^4*e^2)) - (3*((129600*a^9*b*c^10*e^17 + 54*a^3*b^13*c^4*e^17 - 1233*a^4*b^11*c^5*e^17 + 11583*a^5*b^9*c^6*e^17 - 57204*a^6*b^7*c^7*e^17 + 156276*a^7*b^5*c^8*e^17 - 223200*a^8*b^3*c^9*e^17)/(a^9*b^12 + 4096*a^15*c^6 - 24*a^10*b^10*c + 240*a^11*b^8*c^2 - 1280*a^12*b^6*c^3 + 3840*a^13*b^4*c^4 - 6144*a^14*b^2*c^5) - (((153600*a^13*c^10*e^18 + 6*a^6*b^14*c^3*e^18 - 108*a^7*b^12*c^4*e^18 + 588*a^8*b^10*c^5*e^18 + 792*a^9*b^8*c^6*e^18 - 22272*a^10*b^6*c^7*e^18 + 100608*a^11*b^4*c^8*e^18 - 199680*a^12*b^2*c^9*e^18)/(a^9*b^12 + 4096*a^15*c^6 - 24*a^10*b^10*c + 240*a^11*b^8*c^2 - 1280*a^12*b^6*c^3 + 3840*a^13*b^4*c^4 - 6144*a^14*b^2*c^5) - ((6*b^11*e + 960*a^2*b^7*c^2*e - 3840*a^3*b^5*c^3*e + 7680*a^4*b^3*c^4*e - 120*a*b^9*c*e - 6144*a^5*b*c^5*e)*(163840*a^16*b*c^9*e^19 - 12*a^9*b^15*c^2*e^19 + 328*a^10*b^13*c^3*e^19 - 3840*a^11*b^11*c^4*e^19 + 24960*a^12*b^9*c^5*e^19 - 97280*a^13*b^7*c^6*e^19 + 227328*a^14*b^5*c^7*e^19 - 294912*a^15*b^3*c^8*e^19))/(2*(4*a^4*b^10*e^2 - 4096*a^9*c^5*e^2 - 80*a^5*b^8*c*e^2 + 640*a^6*b^6*c^2*e^2 - 2560*a^7*b^4*c^3*e^2 + 5120*a^8*b^2*c^4*e^2)*(a^9*b^12 + 4096*a^15*c^6 - 24*a^10*b^10*c + 240*a^11*b^8*c^2 - 1280*a^12*b^6*c^3 + 3840*a^13*b^4*c^4 - 6144*a^14*b^2*c^5)))*(6*b^11*e + 960*a^2*b^7*c^2*e - 3840*a^3*b^5*c^3*e + 7680*a^4*b^3*c^4*e - 120*a*b^9*c*e - 6144*a^5*b*c^5*e))/(2*(4*a^4*b^10*e^2 - 4096*a^9*c^5*e^2 - 80*a^5*b^8*c*e^2 + 640*a^6*b^6*c^2*e^2 - 2560*a^7*b^4*c^3*e^2 + 5120*a^8*b^2*c^4*e^2)))*(b^6 - 20*a^3*c^3 + 30*a^2*b^2*c^2 - 10*a*b^4*c))/(4*a^4*e*(4*a*c - b^2)^(5/2)) + (27*(b^6 - 20*a^3*c^3 + 30*a^2*b^2*c^2 - 10*a*b^4*c)^3*(163840*a^16*b*c^9*e^19 - 12*a^9*b^15*c^2*e^19 + 328*a^10*b^13*c^3*e^19 - 3840*a^11*b^11*c^4*e^19 + 24960*a^12*b^9*c^5*e^19 - 97280*a^13*b^7*c^6*e^19 + 227328*a^14*b^5*c^7*e^19 - 294912*a^15*b^3*c^8*e^19))/(64*a^12*e^3*(4*a*c - b^2)^(15/2)*(a^9*b^12 + 4096*a^15*c^6 - 24*a^10*b^10*c + 240*a^11*b^8*c^2 - 1280*a^12*b^6*c^3 + 3840*a^13*b^4*c^4 - 6144*a^14*b^2*c^5)))*(3*b^8 + 190*a^4*c^4 + 180*a^2*b^4*c^2 - 335*a^3*b^2*c^3 - 39*a*b^6*c))/(8*a^3*c^2*(4*a*c - b^2)^(13/2)*(100*a^6*c^6 - 6*b^12 - 960*a^2*b^8*c^2 + 3840*a^3*b^6*c^3 - 7675*a^4*b^4*c^4 + 6100*a^5*b^2*c^5 + 120*a*b^10*c)))*(16*a^12*b^12*(4*a*c - b^2)^(15/2) + 65536*a^18*c^6*(4*a*c - b^2)^(15/2) - 384*a^13*b^10*c*(4*a*c - b^2)^(15/2) + 3840*a^14*b^8*c^2*(4*a*c - b^2)^(15/2) - 20480*a^15*b^6*c^3*(4*a*c - b^2)^(15/2) + 61440*a^16*b^4*c^4*(4*a*c - b^2)^(15/2) - 98304*a^17*b^2*c^5*(4*a*c - b^2)^(15/2)))/(10800*a^6*c^8*e^14 + 27*b^12*c^2*e^14 - 540*a*b^10*c^3*e^14 + 4320*a^2*b^8*c^4*e^14 - 17280*a^3*b^6*c^5*e^14 + 35100*a^4*b^4*c^6*e^14 - 32400*a^5*b^2*c^7*e^14) + (x*((((2*(27000*a^6*c^11*d*e^15 + 27*b^12*c^5*d*e^15 - 567*a*b^10*c^6*d*e^15 + 4779*a^2*b^8*c^7*d*e^15 - 20601*a^3*b^6*c^8*d*e^15 + 47790*a^4*b^4*c^9*d*e^15 - 56700*a^5*b^2*c^10*d*e^15))/(a^9*b^12 + 4096*a^15*c^6 - 24*a^10*b^10*c + 240*a^11*b^8*c^2 - 1280*a^12*b^6*c^3 + 3840*a^13*b^4*c^4 - 6144*a^14*b^2*c^5) - (((2*(129600*a^9*b*c^10*d*e^16 + 54*a^3*b^13*c^4*d*e^16 - 1233*a^4*b^11*c^5*d*e^16 + 11583*a^5*b^9*c^6*d*e^16 - 57204*a^6*b^7*c^7*d*e^16 + 156276*a^7*b^5*c^8*d*e^16 - 223200*a^8*b^3*c^9*d*e^16))/(a^9*b^12 + 4096*a^15*c^6 - 24*a^10*b^10*c + 240*a^11*b^8*c^2 - 1280*a^12*b^6*c^3 + 3840*a^13*b^4*c^4 - 6144*a^14*b^2*c^5) - (((2*(153600*a^13*c^10*d*e^17 + 6*a^6*b^14*c^3*d*e^17 - 108*a^7*b^12*c^4*d*e^17 + 588*a^8*b^10*c^5*d*e^17 + 792*a^9*b^8*c^6*d*e^17 - 22272*a^10*b^6*c^7*d*e^17 + 100608*a^11*b^4*c^8*d*e^17 - 199680*a^12*b^2*c^9*d*e^17))/(a^9*b^12 + 4096*a^15*c^6 - 24*a^10*b^10*c + 240*a^11*b^8*c^2 - 1280*a^12*b^6*c^3 + 3840*a^13*b^4*c^4 - 6144*a^14*b^2*c^5) - ((6*b^11*e + 960*a^2*b^7*c^2*e - 3840*a^3*b^5*c^3*e +

7680*a^4*b^3*c^4*e - 120*a*b^9*c*e - 6144*a^5*b*c^5*e)*(163840*a^16*b*c^9*d
*e^18 - 12*a^9*b^15*c^2*d*e^18 + 328*a^10*b^13*c^3*d*e^18 - 3840*a^11*b^11*
c^4*d*e^18 + 24960*a^12*b^9*c^5*d*e^18 - 97280*a^13*b^7*c^6*d*e^18 + 227328
*a^14*b^5*c^7*d*e^18 - 294912*a^15*b^3*c^8*d*e^18))/((4*a^4*b^10*e^2 - 4096
*a^9*c^5*e^2 - 80*a^5*b^8*c*e^2 + 640*a^6*b^6*c^2*e^2 - 2560*a^7*b^4*c^3*e^
2 + 5120*a^8*b^2*c^4*e^2)*(a^9*b^12 + 4096*a^15*c^6 - 24*a^10*b^10*c + 240*
a^11*b^8*c^2 - 1280*a^12*b^6*c^3 + 3840*a^13*b^4*c^4 - 6144*a^14*b^2*c^5)))
*(6*b^11*e + 960*a^2*b^7*c^2*e - 3840*a^3*b^5*c^3*e + 7680*a^4*b^3*c^4*e -
120*a*b^9*c*e - 6144*a^5*b*c^5*e))/(2*(4*a^4*b^10*e^2 - 4096*a^9*c^5*e^2 -
80*a^5*b^8*c*e^2 + 640*a^6*b^6*c^2*e^2 - 2560*a^7*b^4*c^3*e^2 + 5120*a^8*b^
2*c^4*e^2)))*(6*b^11*e + 960*a^2*b^7*c^2*e - 3840*a^3*b^5*c^3*e + 7680*a^4*
b^3*c^4*e - 120*a*b^9*c*e - 6144*a^5*b*c^5*e))/(2*(4*a^4*b^10*e^2 - 4096*a^
9*c^5*e^2 - 80*a^5*b^8*c*e^2 + 640*a^6*b^6*c^2*e^2 - 2560*a^7*b^4*c^3*e^2 +
 5120*a^8*b^2*c^4*e^2)) - (3*((3*((2*(153600*a^13*c^10*d*e^17 + 6*a^6*b^14*
c^3*d*e^17 - 108*a^7*b^12*c^4*d*e^17 + 588*a^8*b^10*c^5*d*e^17 + 792*a^9*b^
8*c^6*d*e^17 - 22272*a^10*b^6*c^7*d*e^17 + 100608*a^11*b^4*c^8*d*e^17 - 199
680*a^12*b^2*c^9*d*e^17))/(a^9*b^12 + 4096*a^15*c^6 - 24*a^10*b^10*c + 240*
a^11*b^8*c^2 - 1280*a^12*b^6*c^3 + 3840*a^13*b^4*c^4 - 6144*a^14*b^2*c^5) -
 ((6*b^11*e + 960*a^2*b^7*c^2*e - 3840*a^3*b^5*c^3*e + 7680*a^4*b^3*c^4*e -
 120*a*b^9*c*e - 6144*a^5*b*c^5*e)*(163840*a^16*b*c^9*d*e^18 - 12*a^9*b^15*
c^2*d*e^18 + 328*a^10*b^13*c^3*d*e^18 - 3840*a^11*b^11*c^4*d*e^18 + 24960*a
^12*b^9*c^5*d*e^18 - 97280*a^13*b^7*c^6*d*e^18 + 227328*a^14*b^5*c^7*d*e^18
 - 294912*a^15*b^3*c^8*d*e^18))/((4*a^4*b^10*e^2 - 4096*a^9*c^5*e^2 - 80*a^
5*b^8*c*e^2 + 640*a^6*b^6*c^2*e^2 - 2560*a^7*b^4*c^3*e^2 + 5120*a^8*b^2*c^4
e^2)(a^9*b^12 + 4096*a^15*c^6 - 24*a^10*b^10*c + 240*a^11*b^8*c^2 - 1280*
a^12*b^6*c^3 + 3840*a^13*b^4*c^4 - 6144*a^14*b^2*c^5)))*(b^6 - 20*a^3*c^3 +
 30*a^2*b^2*c^2 - 10*a*b^4*c))/(4*a^4*e*(4*a*c - b^2)^(5/2)) - (3*(b^6 - 20
*a^3*c^3 + 30*a^2*b^2*c^2 - 10*a*b^4*c)*(6*b^11*e + 960*a^2*b^7*c^2*e - 384
0*a^3*b^5*c^3*e + 7680*a^4*b^3*c^4*e - 120*a*b^9*c*e - 6144*a^5*b*c^5*e)*(1
63840*a^16*b*c^9*d*e^18 - 12*a^9*b^15*c^2*d*e^18 + 328*a^10*b^13*c^3*d*e^18
 - 3840*a^11*b^11*c^4*d*e^18 + 24960*a^12*b^9*c^5*d*e^18 - 97280*a^13*b^7*c
^6*d*e^18 + 227328*a^14*b^5*c^7*d*e^18 - 294912*a^15*b^3*c^8*d*e^18))/(4*a^
4*e*(4*a*c - b^2)^(5/2)*(4*a^4*b^10*e^2 - 4096*a^9*c^5*e^2 - 80*a^5*b^8*c*e
^2 + 640*a^6*b^6*c^2*e^2 - 2560*a^7*b^4*c^3*e^2 + 5120*a^8*b^2*c^4*e^2)*(a^
9*b^12 + 4096*a^15*c^6 - 24*a^10*b^10*c + 240*a^11*b^8*c^2 - 1280*a^12*b^6*
c^3 + 3840*a^13*b^4*c^4 - 6144*a^14*b^2*c^5)))*(b^6 - 20*a^3*c^3 + 30*a^2*b
^2*c^2 - 10*a*b^4*c))/(4*a^4*e*(4*a*c - b^2)^(5/2)) + (9*(b^6 - 20*a^3*c^3
 + 30*a^2*b^2*c^2 - 10*a*b^4*c)^2*(6*b^11*e + 960*a^2*b^7*c^2*e - 3840*a^3*b
^5*c^3*e + 7680*a^4*b^3*c^4*e - 120*a*b^9*c*e - 6144*a^5*b*c^5*e)*(163840*a
^16*b*c^9*d*e^18 - 12*a^9*b^15*c^2*d*e^18 + 328*a^10*b^13*c^3*d*e^18 - 3840
*a^11*b^11*c^4*d*e^18 + 24960*a^12*b^9*c^5*d*e^18 - 97280*a^13*b^7*c^6*d*e^
18 + 227328*a^14*b^5*c^7*d*e^18 - 294912*a^15*b^3*c^8*d*e^18))/(16*a^8*e^2*
(4*a*c - b^2)^5*(4*a^4*b^10*e^2 - 4096*a^9*c^5*e^2 - 80*a^5*b^8*c*e^2 + 640
*a^6*b^6*c^2*e^2 - 2560*a^7*b^4*c^3*e^2 + 5120*a^8*b^2*c^4*e^2)*(a^9*b^12 +
 4096*a^15*c^6 - 24*a^10*b^10*c + 240*a^11*b^8*c^2 - 1280*a^12*b^6*c^3 + 38
40*a^13*b^4*c^4 - 6144*a^14*b^2*c^5)))*(3*b^8 + 10*a^4*c^4 + 120*a^2*b^4*c^
2 - 145*a^3*b^2*c^3 - 33*a*b^6*c))/(8*a^3*c^2*(4*a*c - b^2)^6*(100*a^6*c^6
 - 6*b^12 - 960*a^2*b^8*c^2 + 3840*a^3*b^6*c^3 - 7675*a^4*b^4*c^4 + 6100*a^5
*b^2*c^5 + 120*a*b^10*c)) + (b*((((3*((2*(153600*a^13*c^10*d*e^17 + 6*a^6*b
^14*c^3*d*e^17 - 108*a^7*b^12*c^4*d*e^17 + 588*a^8*b^10*c^5*d*e^17 + 792*a^
9*b^8*c^6*d*e^17 - 22272*a^10*b^6*c^7*d*e^17 + 100608*a^11*b^4*c^8*d*e^17 -
 199680*a^12*b^2*c^9*d*e^17))/(a^9*b^12 + 4096*a^15*c^6 - 24*a^10*b^10*c +
240*a^11*b^8*c^2 - 1280*a^12*b^6*c^3 + 3840*a^13*b^4*c^4 - 6144*a^14*b^2*c^
5) - ((6*b^11*e + 960*a^2*b^7*c^2*e - 3840*a^3*b^5*c^3*e + 7680*a^4*b^3*c^4
*e - 120*a*b^9*c*e - 6144*a^5*b*c^5*e)*(163840*a^16*b*c^9*d*e^18 - 12*a^9*b
^15*c^2*d*e^18 + 328*a^10*b^13*c^3*d*e^18 - 3840*a^11*b^11*c^4*d*e^18 + 249
60*a^12*b^9*c^5*d*e^18 - 97280*a^13*b^7*c^6*d*e^18 + 227328*a^14*b^5*c^7*d*
e^18 - 294912*a^15*b^3*c^8*d*e^18))/((4*a^4*b^10*e^2 - 4096*a^9*c^5*e^2 - 8
0*a^5*b^8*c*e^2 + 640*a^6*b^6*c^2*e^2 - 2560*a^7*b^4*c^3*e^2 + 5120*a^8*b^2

*c^4*e^2)*(a^9*b^12 + 4096*a^15*c^6 - 24*a^10*b^10*c + 240*a^11*b^8*c^2 - 1
280*a^12*b^6*c^3 + 3840*a^13*b^4*c^4 - 6144*a^14*b^2*c^5)))*(b^6 - 20*a^3*c
^3 + 30*a^2*b^2*c^2 - 10*a*b^4*c))/(4*a^4*e*(4*a*c - b^2)^(5/2)) - (3*(b^6
- 20*a^3*c^3 + 30*a^2*b^2*c^2 - 10*a*b^4*c)*(6*b^11*e + 960*a^2*b^7*c^2*e -
 3840*a^3*b^5*c^3*e + 7680*a^4*b^3*c^4*e - 120*a*b^9*c*e - 6144*a^5*b*c^5*e
)*(163840*a^16*b*c^9*d*e^18 - 12*a^9*b^15*c^2*d*e^18 + 328*a^10*b^13*c^3*d*
e^18 - 3840*a^11*b^11*c^4*d*e^18 + 24960*a^12*b^9*c^5*d*e^18 - 97280*a^13*b
^7*c^6*d*e^18 + 227328*a^14*b^5*c^7*d*e^18 - 294912*a^15*b^3*c^8*d*e^18))/(
4*a^4*e*(4*a*c - b^2)^(5/2)*(4*a^4*b^10*e^2 - 4096*a^9*c^5*e^2 - 80*a^5*b^8
*c*e^2 + 640*a^6*b^6*c^2*e^2 - 2560*a^7*b^4*c^3*e^2 + 5120*a^8*b^2*c^4*e^2)
*(a^9*b^12 + 4096*a^15*c^6 - 24*a^10*b^10*c + 240*a^11*b^8*c^2 - 1280*a^12*
b^6*c^3 + 3840*a^13*b^4*c^4 - 6144*a^14*b^2*c^5)))*(6*b^11*e + 960*a^2*b^7*
c^2*e - 3840*a^3*b^5*c^3*e + 7680*a^4*b^3*c^4*e - 120*a*b^9*c*e - 6144*a^5*
b*c^5*e))/(2*(4*a^4*b^10*e^2 - 4096*a^9*c^5*e^2 - 80*a^5*b^8*c*e^2 + 640*a^
6*b^6*c^2*e^2 - 2560*a^7*b^4*c^3*e^2 + 5120*a^8*b^2*c^4*e^2)) - (3*((2*(129
600*a^9*b*c^10*d*e^16 + 54*a^3*b^13*c^4*d*e^16 - 1233*a^4*b^11*c^5*d*e^16 +
 11583*a^5*b^9*c^6*d*e^16 - 57204*a^6*b^7*c^7*d*e^16 + 156276*a^7*b^5*c^8*d
*e^16 - 223200*a^8*b^3*c^9*d*e^16))/(a^9*b^12 + 4096*a^15*c^6 - 24*a^10*b^1
0*c + 240*a^11*b^8*c^2 - 1280*a^12*b^6*c^3 + 3840*a^13*b^4*c^4 - 6144*a^14*
b^2*c^5) - (((2*(153600*a^13*c^10*d*e^17 + 6*a^6*b^14*c^3*d*e^17 - 108*a^7*
b^12*c^4*d*e^17 + 588*a^8*b^10*c^5*d*e^17 + 792*a^9*b^8*c^6*d*e^17 - 22272*
a^10*b^6*c^7*d*e^17 + 100608*a^11*b^4*c^8*d*e^17 - 199680*a^12*b^2*c^9*d*e^
17))/(a^9*b^12 + 4096*a^15*c^6 - 24*a^10*b^10*c + 240*a^11*b^8*c^2 - 1280*a
^12*b^6*c^3 + 3840*a^13*b^4*c^4 - 6144*a^14*b^2*c^5) - ((6*b^11*e + 960*a^2
*b^7*c^2*e - 3840*a^3*b^5*c^3*e + 7680*a^4*b^3*c^4*e - 120*a*b^9*c*e - 6144
*a^5*b*c^5*e)*(163840*a^16*b*c^9*d*e^18 - 12*a^9*b^15*c^2*d*e^18 + 328*a^10
*b^13*c^3*d*e^18 - 3840*a^11*b^11*c^4*d*e^18 + 24960*a^12*b^9*c^5*d*e^18 -
97280*a^13*b^7*c^6*d*e^18 + 227328*a^14*b^5*c^7*d*e^18 - 294912*a^15*b^3*c^
8*d*e^18))/((4*a^4*b^10*e^2 - 4096*a^9*c^5*e^2 - 80*a^5*b^8*c*e^2 + 640*a^6
*b^6*c^2*e^2 - 2560*a^7*b^4*c^3*e^2 + 5120*a^8*b^2*c^4*e^2)*(a^9*b^12 + 409
6*a^15*c^6 - 24*a^10*b^10*c + 240*a^11*b^8*c^2 - 1280*a^12*b^6*c^3 + 3840*a
^13*b^4*c^4 - 6144*a^14*b^2*c^5)))*(6*b^11*e + 960*a^2*b^7*c^2*e - 3840*a^3
*b^5*c^3*e + 7680*a^4*b^3*c^4*e - 120*a*b^9*c*e - 6144*a^5*b*c^5*e))/(2*(4*
a^4*b^10*e^2 - 4096*a^9*c^5*e^2 - 80*a^5*b^8*c*e^2 + 640*a^6*b^6*c^2*e^2 -
2560*a^7*b^4*c^3*e^2 + 5120*a^8*b^2*c^4*e^2)))*(b^6 - 20*a^3*c^3 + 30*a^2*b
^2*c^2 - 10*a*b^4*c))/(4*a^4*e*(4*a*c - b^2)^(5/2)) + (27*(b^6 - 20*a^3*c^3
 + 30*a^2*b^2*c^2 - 10*a*b^4*c)^3*(163840*a^16*b*c^9*d*e^18 - 12*a^9*b^15*c
^2*d*e^18 + 328*a^10*b^13*c^3*d*e^18 - 3840*a^11*b^11*c^4*d*e^18 + 24960*a^
12*b^9*c^5*d*e^18 - 97280*a^13*b^7*c^6*d*e^18 + 227328*a^14*b^5*c^7*d*e^18
- 294912*a^15*b^3*c^8*d*e^18))/(32*a^12*e^3*(4*a*c - b^2)^(15/2)*(a^9*b^12
+ 4096*a^15*c^6 - 24*a^10*b^10*c + 240*a^11*b^8*c^2 - 1280*a^12*b^6*c^3 + 3
840*a^13*b^4*c^4 - 6144*a^14*b^2*c^5)))*(3*b^8 + 190*a^4*c^4 + 180*a^2*b^4*
c^2 - 335*a^3*b^2*c^3 - 39*a*b^6*c))/(8*a^3*c^2*(4*a*c - b^2)^(13/2)*(100*a
^6*c^6 - 6*b^12 - 960*a^2*b^8*c^2 + 3840*a^3*b^6*c^3 - 7675*a^4*b^4*c^4 + 6
100*a^5*b^2*c^5 + 120*a*b^10*c)))*(16*a^12*b^12*(4*a*c - b^2)^(15/2) + 6553
6*a^18*c^6*(4*a*c - b^2)^(15/2) - 384*a^13*b^10*c*(4*a*c - b^2)^(15/2) + 38
40*a^14*b^8*c^2*(4*a*c - b^2)^(15/2) - 20480*a^15*b^6*c^3*(4*a*c - b^2)^(15
/2) + 61440*a^16*b^4*c^4*(4*a*c - b^2)^(15/2) - 98304*a^17*b^2*c^5*(4*a*c -
 b^2)^(15/2)))/(10800*a^6*c^8*e^14 + 27*b^12*c^2*e^14 - 540*a*b^10*c^3*e^14
 + 4320*a^2*b^8*c^4*e^14 - 17280*a^3*b^6*c^5*e^14 + 35100*a^4*b^4*c^6*e^14
- 32400*a^5*b^2*c^7*e^14) - (((((36*a^3*b^14*c^3*e^15 - 14400*a^10*c^10*e^1
5 - 837*a^4*b^12*c^4*e^15 + 8046*a^5*b^10*c^5*e^15 - 40941*a^6*b^8*c^6*e^15
 + 116532*a^7*b^6*c^7*e^15 - 177588*a^8*b^4*c^8*e^15 + 119520*a^9*b^2*c^9*e
^15 + 54*a^3*b^13*c^4*d^2*e^15 - 1233*a^4*b^11*c^5*d^2*e^15 + 11583*a^5*b^9
*c^6*d^2*e^15 - 57204*a^6*b^7*c^7*d^2*e^15 + 156276*a^7*b^5*c^8*d^2*e^15 -
223200*a^8*b^3*c^9*d^2*e^15 + 129600*a^9*b*c^10*d^2*e^15)/(a^9*b^12 + 4096*
a^15*c^6 - 24*a^10*b^10*c + 240*a^11*b^8*c^2 - 1280*a^12*b^6*c^3 + 3840*a^1
3*b^4*c^4 - 6144*a^14*b^2*c^5) - (((12*a^6*b^15*c^2*e^16 - 30720*a^13*b*c^9
*e^16 - 300*a^7*b^13*c^3*e^16 + 3156*a^8*b^11*c^4*e^16 - 17976*a^9*b^9*c^5*

e^16 + 59136*a^10*b^7*c^6*e^16 - 109824*a^11*b^5*c^7*e^16 + 101376*a^12*b^3
*c^8*e^16 + 153600*a^13*c^10*d^2*e^16 + 6*a^6*b^14*c^3*d^2*e^16 - 108*a^7*b
^12*c^4*d^2*e^16 + 588*a^8*b^10*c^5*d^2*e^16 + 792*a^9*b^8*c^6*d^2*e^16 - 2
2272*a^10*b^6*c^7*d^2*e^16 + 100608*a^11*b^4*c^8*d^2*e^16 - 199680*a^12*b^2
*c^9*d^2*e^16)/(a^9*b^12 + 4096*a^15*c^6 - 24*a^10*b^10*c + 240*a^11*b^8*c^
2 - 1280*a^12*b^6*c^3 + 3840*a^13*b^4*c^4 - 6144*a^14*b^2*c^5) + ((6*b^11*e
 + 960*a^2*b^7*c^2*e - 3840*a^3*b^5*c^3*e + 7680*a^4*b^3*c^4*e - 120*a*b^9*
c*e - 6144*a^5*b*c^5*e)*(4*a^10*b^14*c^2*e^17 - 96*a^11*b^12*c^3*e^17 + 960
*a^12*b^10*c^4*e^17 - 5120*a^13*b^8*c^5*e^17 + 15360*a^14*b^6*c^6*e^17 - 24
576*a^15*b^4*c^7*e^17 + 16384*a^16*b^2*c^8*e^17 + 12*a^9*b^15*c^2*d^2*e^17
 - 328*a^10*b^13*c^3*d^2*e^17 + 3840*a^11*b^11*c^4*d^2*e^17 - 24960*a^12*b^9
*c^5*d^2*e^17 + 97280*a^13*b^7*c^6*d^2*e^17 - 227328*a^14*b^5*c^7*d^2*e^17
 + 294912*a^15*b^3*c^8*d^2*e^17 - 163840*a^16*b*c^9*d^2*e^17))/(2*(4*a^4*b^1
0*e^2 - 4096*a^9*c^5*e^2 - 80*a^5*b^8*c*e^2 + 640*a^6*b^6*c^2*e^2 - 2560*a^
7*b^4*c^3*e^2 + 5120*a^8*b^2*c^4*e^2)*(a^9*b^12 + 4096*a^15*c^6 - 24*a^10*b
^10*c + 240*a^11*b^8*c^2 - 1280*a^12*b^6*c^3 + 3840*a^13*b^4*c^4 - 6144*a^1
4*b^2*c^5)))*(6*b^11*e + 960*a^2*b^7*c^2*e - 3840*a^3*b^5*c^3*e + 7680*a^4*
b^3*c^4*e - 120*a*b^9*c*e - 6144*a^5*b*c^5*e))/(2*(4*a^4*b^10*e^2 - 4096*a^
9*c^5*e^2 - 80*a^5*b^8*c*e^2 + 640*a^6*b^6*c^2*e^2 - 2560*a^7*b^4*c^3*e^2 +
 5120*a^8*b^2*c^4*e^2)))*(6*b^11*e + 960*a^2*b^7*c^2*e - 3840*a^3*b^5*c^3*e
 + 7680*a^4*b^3*c^4*e - 120*a*b^9*c*e - 6144*a^5*b*c^5*e))/(2*(4*a^4*b^10*e
^2 - 4096*a^9*c^5*e^2 - 80*a^5*b^8*c*e^2 + 640*a^6*b^6*c^2*e^2 - 2560*a^7*b
^4*c^3*e^2 + 5120*a^8*b^2*c^4*e^2)) - (27*b^13*c^4*e^14 - 594*a*b^11*c^5*e^
14 + 43200*a^6*b*c^10*e^14 + 5319*a^2*b^9*c^6*e^14 - 24732*a^3*b^7*c^7*e^14
 + 62748*a^4*b^5*c^8*e^14 - 82080*a^5*b^3*c^9*e^14 + 27000*a^6*c^11*d^2*e^1
4 + 27*b^12*c^5*d^2*e^14 + 4779*a^2*b^8*c^7*d^2*e^14 - 20601*a^3*b^6*c^8*d^
2*e^14 + 47790*a^4*b^4*c^9*d^2*e^14 - 56700*a^5*b^2*c^10*d^2*e^14 - 567*a*b
^10*c^6*d^2*e^14)/(a^9*b^12 + 4096*a^15*c^6 - 24*a^10*b^10*c + 240*a^11*b^8
*c^2 - 1280*a^12*b^6*c^3 + 3840*a^13*b^4*c^4 - 6144*a^14*b^2*c^5) + (3*((3*
((12*a^6*b^15*c^2*e^16 - 30720*a^13*b*c^9*e^16 - 300*a^7*b^13*c^3*e^16 + 31
56*a^8*b^11*c^4*e^16 - 17976*a^9*b^9*c^5*e^16 + 59136*a^10*b^7*c^6*e^16 - 1
09824*a^11*b^5*c^7*e^16 + 101376*a^12*b^3*c^8*e^16 + 153600*a^13*c^10*d^2*e
^16 + 6*a^6*b^14*c^3*d^2*e^16 - 108*a^7*b^12*c^4*d^2*e^16 + 588*a^8*b^10*c^
5*d^2*e^16 + 792*a^9*b^8*c^6*d^2*e^16 - 22272*a^10*b^6*c^7*d^2*e^16 + 10060
8*a^11*b^4*c^8*d^2*e^16 - 199680*a^12*b^2*c^9*d^2*e^16)/(a^9*b^12 + 4096*a^
15*c^6 - 24*a^10*b^10*c + 240*a^11*b^8*c^2 - 1280*a^12*b^6*c^3 + 3840*a^13*
b^4*c^4 - 6144*a^14*b^2*c^5) + ((6*b^11*e + 960*a^2*b^7*c^2*e - 3840*a^3*b^
5*c^3*e + 7680*a^4*b^3*c^4*e - 120*a*b^9*c*e - 6144*a^5*b*c^5*e)*(4*a^10*b^
14*c^2*e^17 - 96*a^11*b^12*c^3*e^17 + 960*a^12*b^10*c^4*e^17 - 5120*a^13*b^
8*c^5*e^17 + 15360*a^14*b^6*c^6*e^17 - 24576*a^15*b^4*c^7*e^17 + 16384*a^16
*b^2*c^8*e^17 + 12*a^9*b^15*c^2*d^2*e^17 - 328*a^10*b^13*c^3*d^2*e^17 + 384
0*a^11*b^11*c^4*d^2*e^17 - 24960*a^12*b^9*c^5*d^2*e^17 + 97280*a^13*b^7*c^6
*d^2*e^17 - 227328*a^14*b^5*c^7*d^2*e^17 + 294912*a^15*b^3*c^8*d^2*e^17 - 1
63840*a^16*b*c^9*d^2*e^17))/(2*(4*a^4*b^10*e^2 - 4096*a^9*c^5*e^2 - 80*a^5*
b^8*c*e^2 + 640*a^6*b^6*c^2*e^2 - 2560*a^7*b^4*c^3*e^2 + 5120*a^8*b^2*c^4*e
^2)*(a^9*b^12 + 4096*a^15*c^6 - 24*a^10*b^10*c + 240*a^11*b^8*c^2 - 1280*a^
12*b^6*c^3 + 3840*a^13*b^4*c^4 - 6144*a^14*b^2*c^5)))*(b^6 - 20*a^3*c^3 + 3
0*a^2*b^2*c^2 - 10*a*b^4*c))/(4*a^4*e*(4*a*c - b^2)^(5/2)) + (3*(b^6 - 20*a
^3*c^3 + 30*a^2*b^2*c^2 - 10*a*b^4*c)*(6*b^11*e + 960*a^2*b^7*c^2*e - 3840*
a^3*b^5*c^3*e + 7680*a^4*b^3*c^4*e - 120*a*b^9*c*e - 6144*a^5*b*c^5*e)*(4*a
^10*b^14*c^2*e^17 - 96*a^11*b^12*c^3*e^17 + 960*a^12*b^10*c^4*e^17 - 5120*a
^13*b^8*c^5*e^17 + 15360*a^14*b^6*c^6*e^17 - 24576*a^15*b^4*c^7*e^17 + 1638
4*a^16*b^2*c^8*e^17 + 12*a^9*b^15*c^2*d^2*e^17 - 328*a^10*b^13*c^3*d^2*e^17
 + 3840*a^11*b^11*c^4*d^2*e^17 - 24960*a^12*b^9*c^5*d^2*e^17 + 97280*a^13*b
^7*c^6*d^2*e^17 - 227328*a^14*b^5*c^7*d^2*e^17 + 294912*a^15*b^3*c^8*d^2*e^
17 - 163840*a^16*b*c^9*d^2*e^17))/(8*a^4*e*(4*a*c - b^2)^(5/2)*(4*a^4*b^10*
e^2 - 4096*a^9*c^5*e^2 - 80*a^5*b^8*c*e^2 + 640*a^6*b^6*c^2*e^2 - 2560*a^7*
b^4*c^3*e^2 + 5120*a^8*b^2*c^4*e^2)*(a^9*b^12 + 4096*a^15*c^6 - 24*a^10*b^1
0*c + 240*a^11*b^8*c^2 - 1280*a^12*b^6*c^3 + 3840*a^13*b^4*c^4 - 6144*a^14*

b^2*c^5)))*(b^6 - 20*a^3*c^3 + 30*a^2*b^2*c^2 - 10*a*b^4*c))/(4*a^4*e*(4*a*
c - b^2)^(5/2)) + (9*(b^6 - 20*a^3*c^3 + 30*a^2*b^2*c^2 - 10*a*b^4*c)^2*(6*
b^11*e + 960*a^2*b^7*c^2*e - 3840*a^3*b^5*c^3*e + 7680*a^4*b^3*c^4*e - 120*
a*b^9*c*e - 6144*a^5*b*c^5*e)*(4*a^10*b^14*c^2*e^17 - 96*a^11*b^12*c^3*e^17
 + 960*a^12*b^10*c^4*e^17 - 5120*a^13*b^8*c^5*e^17 + 15360*a^14*b^6*c^6*e^1
7 - 24576*a^15*b^4*c^7*e^17 + 16384*a^16*b^2*c^8*e^17 + 12*a^9*b^15*c^2*d^2
*e^17 - 328*a^10*b^13*c^3*d^2*e^17 + 3840*a^11*b^11*c^4*d^2*e^17 - 24960*a^
12*b^9*c^5*d^2*e^17 + 97280*a^13*b^7*c^6*d^2*e^17 - 227328*a^14*b^5*c^7*d^2
*e^17 + 294912*a^15*b^3*c^8*d^2*e^17 - 163840*a^16*b*c^9*d^2*e^17))/(32*a^8
e^2(4*a*c - b^2)^5*(4*a^4*b^10*e^2 - 4096*a^9*c^5*e^2 - 80*a^5*b^8*c*e^2
+ 640*a^6*b^6*c^2*e^2 - 2560*a^7*b^4*c^3*e^2 + 5120*a^8*b^2*c^4*e^2)*(a^9*b
^12 + 4096*a^15*c^6 - 24*a^10*b^10*c + 240*a^11*b^8*c^2 - 1280*a^12*b^6*c^3
 + 3840*a^13*b^4*c^4 - 6144*a^14*b^2*c^5)))*(3*b^8 + 10*a^4*c^4 + 120*a^2*b
^4*c^2 - 145*a^3*b^2*c^3 - 33*a*b^6*c)*(16*a^12*b^12*(4*a*c - b^2)^(15/2) +
 65536*a^18*c^6*(4*a*c - b^2)^(15/2) - 384*a^13*b^10*c*(4*a*c - b^2)^(15/2)
 + 3840*a^14*b^8*c^2*(4*a*c - b^2)^(15/2) - 20480*a^15*b^6*c^3*(4*a*c - b^2
)^(15/2) + 61440*a^16*b^4*c^4*(4*a*c - b^2)^(15/2) - 98304*a^17*b^2*c^5*(4*
a*c - b^2)^(15/2)))/(8*a^3*c^2*(4*a*c - b^2)^6*(10800*a^6*c^8*e^14 + 27*b^1
2*c^2*e^14 - 540*a*b^10*c^3*e^14 + 4320*a^2*b^8*c^4*e^14 - 17280*a^3*b^6*c^
5*e^14 + 35100*a^4*b^4*c^6*e^14 - 32400*a^5*b^2*c^7*e^14)*(100*a^6*c^6 - 6*
b^12 - 960*a^2*b^8*c^2 + 3840*a^3*b^6*c^3 - 7675*a^4*b^4*c^4 + 6100*a^5*b^2
*c^5 + 120*a*b^10*c)) - (b*((3*((36*a^3*b^14*c^3*e^15 - 14400*a^10*c^10*e^1
5 - 837*a^4*b^12*c^4*e^15 + 8046*a^5*b^10*c^5*e^15 - 40941*a^6*b^8*c^6*e^15
 + 116532*a^7*b^6*c^7*e^15 - 177588*a^8*b^4*c^8*e^15 + 119520*a^9*b^2*c^9*e
^15 + 54*a^3*b^13*c^4*d^2*e^15 - 1233*a^4*b^11*c^5*d^2*e^15 + 11583*a^5*b^9
*c^6*d^2*e^15 - 57204*a^6*b^7*c^7*d^2*e^15 + 156276*a^7*b^5*c^8*d^2*e^15 -
223200*a^8*b^3*c^9*d^2*e^15 + 129600*a^9*b*c^10*d^2*e^15)/(a^9*b^12 + 4096*
a^15*c^6 - 24*a^10*b^10*c + 240*a^11*b^8*c^2 - 1280*a^12*b^6*c^3 + 3840*a^1
3*b^4*c^4 - 6144*a^14*b^2*c^5) - (((12*a^6*b^15*c^2*e^16 - 30720*a^13*b*c^9
*e^16 - 300*a^7*b^13*c^3*e^16 + 3156*a^8*b^11*c^4*e^16 - 17976*a^9*b^9*c^5*
e^16 + 59136*a^10*b^7*c^6*e^16 - 109824*a^11*b^5*c^7*e^16 + 101376*a^12*b^3
*c^8*e^16 + 153600*a^13*c^10*d^2*e^16 + 6*a^6*b^14*c^3*d^2*e^16 - 108*a^7*b
^12*c^4*d^2*e^16 + 588*a^8*b^10*c^5*d^2*e^16 + 792*a^9*b^8*c^6*d^2*e^16 - 2
2272*a^10*b^6*c^7*d^2*e^16 + 100608*a^11*b^4*c^8*d^2*e^16 - 199680*a^12*b^2
*c^9*d^2*e^16)/(a^9*b^12 + 4096*a^15*c^6 - 24*a^10*b^10*c + 240*a^11*b^8*c^
2 - 1280*a^12*b^6*c^3 + 3840*a^13*b^4*c^4 - 6144*a^14*b^2*c^5) + ((6*b^11*e
 + 960*a^2*b^7*c^2*e - 3840*a^3*b^5*c^3*e + 7680*a^4*b^3*c^4*e - 120*a*b^9*
c*e - 6144*a^5*b*c^5*e)*(4*a^10*b^14*c^2*e^17 - 96*a^11*b^12*c^3*e^17 + 960
*a^12*b^10*c^4*e^17 - 5120*a^13*b^8*c^5*e^17 + 15360*a^14*b^6*c^6*e^17 - 24
576*a^15*b^4*c^7*e^17 + 16384*a^16*b^2*c^8*e^17 + 12*a^9*b^15*c^2*d^2*e^17
- 328*a^10*b^13*c^3*d^2*e^17 + 3840*a^11*b^11*c^4*d^2*e^17 - 24960*a^12*b^9
*c^5*d^2*e^17 + 97280*a^13*b^7*c^6*d^2*e^17 - 227328*a^14*b^5*c^7*d^2*e^17
+ 294912*a^15*b^3*c^8*d^2*e^17 - 163840*a^16*b*c^9*d^2*e^17))/(2*(4*a^4*b^1
0*e^2 - 4096*a^9*c^5*e^2 - 80*a^5*b^8*c*e^2 + 640*a^6*b^6*c^2*e^2 - 2560*a^
7*b^4*c^3*e^2 + 5120*a^8*b^2*c^4*e^2)*(a^9*b^12 + 4096*a^15*c^6 - 24*a^10*b
^10*c + 240*a^11*b^8*c^2 - 1280*a^12*b^6*c^3 + 3840*a^13*b^4*c^4 - 6144*a^1
4*b^2*c^5)))*(6*b^11*e + 960*a^2*b^7*c^2*e - 3840*a^3*b^5*c^3*e + 7680*a^4*
b^3*c^4*e - 120*a*b^9*c*e - 6144*a^5*b*c^5*e))/(2*(4*a^4*b^10*e^2 - 4096*a^
9*c^5*e^2 - 80*a^5*b^8*c*e^2 + 640*a^6*b^6*c^2*e^2 - 2560*a^7*b^4*c^3*e^2 +
 5120*a^8*b^2*c^4*e^2)))*(b^6 - 20*a^3*c^3 + 30*a^2*b^2*c^2 - 10*a*b^4*c))/
(4*a^4*e*(4*a*c - b^2)^(5/2)) - (((3*((12*a^6*b^15*c^2*e^16 - 30720*a^13*b*
c^9*e^16 - 300*a^7*b^13*c^3*e^16 + 3156*a^8*b^11*c^4*e^16 - 17976*a^9*b^9*c
^5*e^16 + 59136*a^10*b^7*c^6*e^16 - 109824*a^11*b^5*c^7*e^16 + 101376*a^12*
b^3*c^8*e^16 + 153600*a^13*c^10*d^2*e^16 + 6*a^6*b^14*c^3*d^2*e^16 - 108*a^
7*b^12*c^4*d^2*e^16 + 588*a^8*b^10*c^5*d^2*e^16 + 792*a^9*b^8*c^6*d^2*e^16
- 22272*a^10*b^6*c^7*d^2*e^16 + 100608*a^11*b^4*c^8*d^2*e^16 - 199680*a^12*
b^2*c^9*d^2*e^16)/(a^9*b^12 + 4096*a^15*c^6 - 24*a^10*b^10*c + 240*a^11*b^8
*c^2 - 1280*a^12*b^6*c^3 + 3840*a^13*b^4*c^4 - 6144*a^14*b^2*c^5) + ((6*b^1
1*e + 960*a^2*b^7*c^2*e - 3840*a^3*b^5*c^3*e + 7680*a^4*b^3*c^4*e - 120*a*b

```
^9*c*e - 6144*a^5*b*c^5*e)*(4*a^10*b^14*c^2*e^17 - 96*a^11*b^12*c^3*e^17 +
960*a^12*b^10*c^4*e^17 - 5120*a^13*b^8*c^5*e^17 + 15360*a^14*b^6*c^6*e^17 -
 24576*a^15*b^4*c^7*e^17 + 16384*a^16*b^2*c^8*e^17 + 12*a^9*b^15*c^2*d^2*e^
17 - 328*a^10*b^13*c^3*d^2*e^17 + 3840*a^11*b^11*c^4*d^2*e^17 - 24960*a^12*
b^9*c^5*d^2*e^17 + 97280*a^13*b^7*c^6*d^2*e^17 - 227328*a^14*b^5*c^7*d^2*e^
17 + 294912*a^15*b^3*c^8*d^2*e^17 - 163840*a^16*b*c^9*d^2*e^17))/(2*(4*a^4*
b^10*e^2 - 4096*a^9*c^5*e^2 - 80*a^5*b^8*c*e^2 + 640*a^6*b^6*c^2*e^2 - 2560
*a^7*b^4*c^3*e^2 + 5120*a^8*b^2*c^4*e^2)*(a^9*b^12 + 4096*a^15*c^6 - 24*a^1
0*b^10*c + 240*a^11*b^8*c^2 - 1280*a^12*b^6*c^3 + 3840*a^13*b^4*c^4 - 6144*
a^14*b^2*c^5)))*(b^6 - 20*a^3*c^3 + 30*a^2*b^2*c^2 - 10*a*b^4*c))/(4*a^4*e*
(4*a*c - b^2)^(5/2)) + (3*(b^6 - 20*a^3*c^3 + 30*a^2*b^2*c^2 - 10*a*b^4*c)*
(6*b^11*e + 960*a^2*b^7*c^2*e - 3840*a^3*b^5*c^3*e + 7680*a^4*b^3*c^4*e - 1
20*a*b^9*c*e - 6144*a^5*b*c^5*e)*(4*a^10*b^14*c^2*e^17 - 96*a^11*b^12*c^3*e
^17 + 960*a^12*b^10*c^4*e^17 - 5120*a^13*b^8*c^5*e^17 + 15360*a^14*b^6*c^6*
e^17 - 24576*a^15*b^4*c^7*e^17 + 16384*a^16*b^2*c^8*e^17 + 12*a^9*b^15*c^2*
d^2*e^17 - 328*a^10*b^13*c^3*d^2*e^17 + 3840*a^11*b^11*c^4*d^2*e^17 - 24960
*a^12*b^9*c^5*d^2*e^17 + 97280*a^13*b^7*c^6*d^2*e^17 - 227328*a^14*b^5*c^7*
d^2*e^17 + 294912*a^15*b^3*c^8*d^2*e^17 - 163840*a^16*b*c^9*d^2*e^17))/(8*a
^4*e*(4*a*c - b^2)^(5/2)*(4*a^4*b^10*e^2 - 4096*a^9*c^5*e^2 - 80*a^5*b^8*c*
e^2 + 640*a^6*b^6*c^2*e^2 - 2560*a^7*b^4*c^3*e^2 + 5120*a^8*b^2*c^4*e^2)*(a
^9*b^12 + 4096*a^15*c^6 - 24*a^10*b^10*c + 240*a^11*b^8*c^2 - 1280*a^12*b^6
*c^3 + 3840*a^13*b^4*c^4 - 6144*a^14*b^2*c^5)))*(6*b^11*e + 960*a^2*b^7*c^2
*e - 3840*a^3*b^5*c^3*e + 7680*a^4*b^3*c^4*e - 120*a*b^9*c*e - 6144*a^5*b*c
^5*e))/(2*(4*a^4*b^10*e^2 - 4096*a^9*c^5*e^2 - 80*a^5*b^8*c*e^2 + 640*a^6*b
^6*c^2*e^2 - 2560*a^7*b^4*c^3*e^2 + 5120*a^8*b^2*c^4*e^2)) + (27*(b^6 - 20*
a^3*c^3 + 30*a^2*b^2*c^2 - 10*a*b^4*c)^3*(4*a^10*b^14*c^2*e^17 - 96*a^11*b^
12*c^3*e^17 + 960*a^12*b^10*c^4*e^17 - 5120*a^13*b^8*c^5*e^17 + 15360*a^14*
b^6*c^6*e^17 - 24576*a^15*b^4*c^7*e^17 + 16384*a^16*b^2*c^8*e^17 + 12*a^9*b
^15*c^2*d^2*e^17 - 328*a^10*b^13*c^3*d^2*e^17 + 3840*a^11*b^11*c^4*d^2*e^17
 - 24960*a^12*b^9*c^5*d^2*e^17 + 97280*a^13*b^7*c^6*d^2*e^17 - 227328*a^14*
b^5*c^7*d^2*e^17 + 294912*a^15*b^3*c^8*d^2*e^17 - 163840*a^16*b*c^9*d^2*e^1
7))/(64*a^12*e^3*(4*a*c - b^2)^(15/2)*(a^9*b^12 + 4096*a^15*c^6 - 24*a^10*b
^10*c + 240*a^11*b^8*c^2 - 1280*a^12*b^6*c^3 + 3840*a^13*b^4*c^4 - 6144*a^1
4*b^2*c^5)))*(3*b^8 + 190*a^4*c^4 + 180*a^2*b^4*c^2 - 335*a^3*b^2*c^3 - 39*
a*b^6*c)*(16*a^12*b^12*(4*a*c - b^2)^(15/2) + 65536*a^18*c^6*(4*a*c - b^2)^
(15/2) - 384*a^13*b^10*c*(4*a*c - b^2)^(15/2) + 3840*a^14*b^8*c^2*(4*a*c -
b^2)^(15/2) - 20480*a^15*b^6*c^3*(4*a*c - b^2)^(15/2) + 61440*a^16*b^4*c^4*
(4*a*c - b^2)^(15/2) - 98304*a^17*b^2*c^5*(4*a*c - b^2)^(15/2))))/(8*a^3*c^2
*(4*a*c - b^2)^(13/2)*(10800*a^6*c^8*e^14 + 27*b^12*c^2*e^14 - 540*a*b^10*c
^3*e^14 + 4320*a^2*b^8*c^4*e^14 - 17280*a^3*b^6*c^5*e^14 + 35100*a^4*b^4*c^
6*e^14 - 32400*a^5*b^2*c^7*e^14)*(100*a^6*c^6 - 6*b^12 - 960*a^2*b^8*c^2 +
3840*a^3*b^6*c^3 - 7675*a^4*b^4*c^4 + 6100*a^5*b^2*c^5 + 120*a*b^10*c)))*(b
^6 - 20*a^3*c^3 + 30*a^2*b^2*c^2 - 10*a*b^4*c))/(2*a^4*e*(4*a*c - b^2)^(5/2
))
```

sympy [F(-1)]   time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)**3/(a+b*(e*x+d)**2+c*(e*x+d)**4)**3,x)
```

```
[Out] Timed out
```

3.525 $\int \frac{(df+efx)^4}{a+b(d+ex)^2+c(d+ex)^4}\,dx$

Optimal. Leaf size=202

$$-\frac{f^4\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{3/2}e\sqrt{b-\sqrt{b^2-4ac}}}-\frac{f^4\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}c^{3/2}e\sqrt{\sqrt{b^2-4ac}+b}}+\frac{f^4x}{c}$$

Rubi [A] time = 0.36, antiderivative size = 202, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.121, Rules used = {1142, 1122, 1166, 205}

$$-\frac{f^4\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{3/2}e\sqrt{b-\sqrt{b^2-4ac}}}-\frac{f^4\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}c^{3/2}e\sqrt{\sqrt{b^2-4ac}+b}}+\frac{f^4x}{c}$$

Antiderivative was successfully verified.

```
[In] Int[(d*f + e*f*x)^4/(a + b*(d + e*x)^2 + c*(d + e*x)^4),x]
```

```
[Out] (f^4*x)/c - ((b - (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*f^4*ArcTan[(Sqrt[2]*Sqrt
[c]*(d + e*x))/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(3/2)*Sqrt[b - Sqrt
[b^2 - 4*a*c]]*e) - ((b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*f^4*ArcTan[(Sqrt
[2]*Sqrt[c]*(d + e*x))/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(3/2)*Sqrt[
b + Sqrt[b^2 - 4*a*c]]*e)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1122

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Simp[(d^3*(d*x)^(m - 3)*(a + b*x^2 + c*x^4)^(p + 1))/(c*(m + 4*p + 1)),
 x] - Dist[d^4/(c*(m + 4*p + 1)), Int[(d*x)^(m - 4)*Simp[a*(m - 3) + b*(m +
 2*p - 1)*x^2, x]*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x
] && NeQ[b^2 - 4*a*c, 0] && GtQ[m, 3] && NeQ[m + 4*p + 1, 0] && IntegerQ[2*
p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1142

```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Di
st[u^m/(Coefficient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p,
 x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{(df+efx)^4}{a+b(d+ex)^2+c(d+ex)^4}\,dx = \frac{f^4\,\text{Subst}\left(\int \frac{x^4}{a+bx^2+cx^4}\,dx, x, d+ex\right)}{e}$$
$$= \frac{f^4x}{c} - \frac{f^4\,\text{Subst}\left(\int \frac{a+bx^2}{a+bx^2+cx^4}\,dx, x, d+ex\right)}{ce}$$
$$= \frac{f^4x}{c} - \frac{\left(\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)f^4\right)\text{Subst}\left(\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx, x, d+ex\right)}{2ce} - \frac{\left(\left(b+\frac{b^2-}{\sqrt{b^2}}\right.\right.}{}$$
$$= \frac{f^4x}{c} - \frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)f^4\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\,c^{3/2}\sqrt{b-\sqrt{b^2-4ac}}\,e} - \frac{\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)f^4\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}}{\sqrt{b+\sqrt{}}}\right.}{\sqrt{2}\,c^{3/2}\sqrt{b+\sqrt{b^2-4ac}}}$$

Mathematica [A] time = 0.05, size = 222, normalized size = 1.10

$$\frac{f^4\left(-\frac{\sqrt{2}\left(b\sqrt{b^2-4ac}+2ac-b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}} - \frac{\sqrt{2}\left(b\sqrt{b^2-4ac}-2ac+b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4ac}+b}} + 2\sqrt{c}(d+ex)\right)}{2c^{3/2}e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*f + e*f*x)^4/(a + b*(d + e*x)^2 + c*(d + e*x)^4),x]
```

```
[Out] (f^4*(2*Sqrt[c]*(d + e*x) - (Sqrt[2]*(-b^2 + 2*a*c + b*Sqrt[b^2 - 4*a*c])*A
rcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2 -
 4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - (Sqrt[2]*(b^2 - 2*a*c + b*Sqrt[b^2 -
 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(
Sqrt[b^2 - 4*a*c]*Sqrt[b + Sqrt[b^2 - 4*a*c]])))/(2*c^(3/2)*e)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(df+efx)^4}{a+b(d+ex)^2+c(d+ex)^4}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(d*f + e*f*x)^4/(a + b*(d + e*x)^2 + c*(d + e*x)^4),x]
```

```
[Out] IntegrateAlgebraic[(d*f + e*f*x)^4/(a + b*(d + e*x)^2 + c*(d + e*x)^4), x]
```

fricas [B] time = 1.32, size = 1346, normalized size = 6.66

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*f*x+d*f)^4/(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="fricas")
```

```
[Out] 1/2*(2*f^4*x - sqrt(1/2)*c*sqrt(-((b^3 - 3*a*b*c)*f^8 + sqrt((b^4 - 2*a*b^2
*c + a^2*c^2)*f^16/((b^2*c^6 - 4*a*c^7)*e^4))*(b^2*c^3 - 4*a*c^4)*e^2)/((b^
2*c^3 - 4*a*c^4)*e^2))*log(-2*(a*b^2 - a^2*c)*e*f^12*x - 2*(a*b^2 - a^2*c)*
d*f^12 + sqrt(1/2)*((b^4 - 5*a*b^2*c + 4*a^2*c^2)*e*f^8 - sqrt((b^4 - 2*a*b
^2*c + a^2*c^2)*f^16/((b^2*c^6 - 4*a*c^7)*e^4))*(b^3*c^3 - 4*a*b*c^4)*e^3)*
sqrt(-((b^3 - 3*a*b*c)*f^8 + sqrt((b^4 - 2*a*b^2*c + a^2*c^2)*f^16/((b^2*c^
6 - 4*a*c^7)*e^4))*(b^2*c^3 - 4*a*c^4)*e^2)/((b^2*c^3 - 4*a*c^4)*e^2))) + s
qrt(1/2)*c*sqrt(-((b^3 - 3*a*b*c)*f^8 + sqrt((b^4 - 2*a*b^2*c + a^2*c^2)*f^
```

```
16/((b^2*c^6 - 4*a*c^7)*e^4))*(b^2*c^3 - 4*a*c^4)*e^2)/((b^2*c^3 - 4*a*c^4)
*e^2))*log(-2*(a*b^2 - a^2*c)*e*f^12*x - 2*(a*b^2 - a^2*c)*d*f^12 - sqrt(1/
2)*((b^4 - 5*a*b^2*c + 4*a^2*c^2)*e*f^8 - sqrt((b^4 - 2*a*b^2*c + a^2*c^2)*
f^16/((b^2*c^6 - 4*a*c^7)*e^4))*(b^3*c^3 - 4*a*b*c^4)*e^3)*sqrt(-((b^3 - 3*
a*b*c)*f^8 + sqrt((b^4 - 2*a*b^2*c + a^2*c^2)*f^16/((b^2*c^6 - 4*a*c^7)*e^4
))*(b^2*c^3 - 4*a*c^4)*e^2)/((b^2*c^3 - 4*a*c^4)*e^2))) - sqrt(1/2)*c*sqrt(
-((b^3 - 3*a*b*c)*f^8 - sqrt((b^4 - 2*a*b^2*c + a^2*c^2)*f^16/((b^2*c^6 - 4
*a*c^7)*e^4))*(b^2*c^3 - 4*a*c^4)*e^2)/((b^2*c^3 - 4*a*c^4)*e^2))*log(-2*(a
*b^2 - a^2*c)*e*f^12*x - 2*(a*b^2 - a^2*c)*d*f^12 + sqrt(1/2)*((b^4 - 5*a*b
^2*c + 4*a^2*c^2)*e*f^8 + sqrt((b^4 - 2*a*b^2*c + a^2*c^2)*f^16/((b^2*c^6 -
 4*a*c^7)*e^4))*(b^3*c^3 - 4*a*b*c^4)*e^3)*sqrt(-((b^3 - 3*a*b*c)*f^8 - sqr
t((b^4 - 2*a*b^2*c + a^2*c^2)*f^16/((b^2*c^6 - 4*a*c^7)*e^4))*(b^2*c^3 - 4*
a*c^4)*e^2)/((b^2*c^3 - 4*a*c^4)*e^2))) + sqrt(1/2)*c*sqrt(-((b^3 - 3*a*b*c
)*f^8 - sqrt((b^4 - 2*a*b^2*c + a^2*c^2)*f^16/((b^2*c^6 - 4*a*c^7)*e^4))*(b
^2*c^3 - 4*a*c^4)*e^2)/((b^2*c^3 - 4*a*c^4)*e^2))*log(-2*(a*b^2 - a^2*c)*e*
f^12*x - 2*(a*b^2 - a^2*c)*d*f^12 - sqrt(1/2)*((b^4 - 5*a*b^2*c + 4*a^2*c^2
)*e*f^8 + sqrt((b^4 - 2*a*b^2*c + a^2*c^2)*f^16/((b^2*c^6 - 4*a*c^7)*e^4))*
(b^3*c^3 - 4*a*b*c^4)*e^3)*sqrt(-((b^3 - 3*a*b*c)*f^8 - sqrt((b^4 - 2*a*b^2
*c + a^2*c^2)*f^16/((b^2*c^6 - 4*a*c^7)*e^4))*(b^2*c^3 - 4*a*c^4)*e^2)/((b^
2*c^3 - 4*a*c^4)*e^2))))/c
```

giac [B] time = 0.54, size = 1245, normalized size = 6.16

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*f*x+d*f)^4/(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="giac")
```

```
[Out] f^4*x/c + 1/2*(((d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)
*e^(-4)/c))^2*b*f^4*e^6 - 2*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 -
 4*a*c)*e^2)*e^(-4)/c))*b*d*f^4*e^5 + b*d^2*f^4*e^4 + a*f^4*e^4)*log(d*e^(-
1) + x + sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))/(2*(d*e
^(-1) + sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^3*c*e^4
- 6*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^
2*c*d*e^3 - 2*c*d^3*e - b*d*e + (6*c*d^2*e^2 + b*e^2)*(d*e^(-1) + sqrt(1/2)
*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))) + ((d*e^(-1) - sqrt(1/2)
*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^2*b*f^4*e^6 - 2*(d*e^(-1)
 - sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))*b*d*f^4*e^5 +
 b*d^2*f^4*e^4 + a*f^4*e^4)*log(d*e^(-1) + x - sqrt(1/2)*sqrt(-(b*e^2 + sqr
t(b^2 - 4*a*c)*e^2)*e^(-4)/c))/(2*(d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 + sqrt
(b^2 - 4*a*c)*e^2)*e^(-4)/c))^3*c*e^4 - 6*(d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^
2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^2*c*d*e^3 - 2*c*d^3*e - b*d*e + (6*c*
d^2*e^2 + b*e^2)*(d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2
)*e^(-4)/c))) + ((d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2
)*e^(-4)/c))^2*b*f^4*e^6 - 2*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2
- 4*a*c)*e^2)*e^(-4)/c))*b*d*f^4*e^5 + b*d^2*f^4*e^4 + a*f^4*e^4)*log(d*e^(
-1) + x + sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))/(2*(d*
e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^3*c*e^4
 - 6*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))
^2*c*d*e^3 - 2*c*d^3*e - b*d*e + (6*c*d^2*e^2 + b*e^2)*(d*e^(-1) + sqrt(1/2
)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))) + ((d*e^(-1) - sqrt(1/2
)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^2*b*f^4*e^6 - 2*(d*e^(-1
) - sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))*b*d*f^4*e^5
+ b*d^2*f^4*e^4 + a*f^4*e^4)*log(d*e^(-1) + x - sqrt(1/2)*sqrt(-(b*e^2 - sq
rt(b^2 - 4*a*c)*e^2)*e^(-4)/c))/(2*(d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 - sqr
t(b^2 - 4*a*c)*e^2)*e^(-4)/c))^3*c*e^4 - 6*(d*e^(-1) - sqrt(1/2)*sqrt(-(b*e
^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^2*c*d*e^3 - 2*c*d^3*e - b*d*e + (6*c
*d^2*e^2 + b*e^2)*(d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^
2)*e^(-4)/c))))*e^(-4)/c
```

maple [C] time = 0.00, size = 164, normalized size = 0.81

$$\frac{f^4 x}{c}+\frac{f^4\left(-\mathrm{RootOf}\left(\_Z^4 c e^4+4\_Z^3 c d e^3+c d^4+b d^2+\left(6 c d^2 e^2+b e^2\right)\_Z^2+\left(4 c d^3 e+2 d e b\right)\_Z+a\right)^2 b e^2-2\,\mathrm{RootOf}\left(\_Z^4 c e^4+4\_Z^3 c d e^3+c d^4+b d^2+\left(6 c d^2 e^2+b e^2\right)\_Z^2+\left(4 c d^3 e+2 d e b\right)\_Z+a\right) b d e-b d^2-a\right)\ln\left(-\mathrm{RootOf}\left(\_Z^4 c e^4+4\_Z^3 c d e^3+c d^4+b d^2+\left(6 c d^2 e^2+b e^2\right)\_Z^2+\left(4 c d^3 e+2 d e b\right)\_Z+a\right)+x\right)}{2 c e\left(2 c e^3\,\mathrm{RootOf}\left(\_Z^4 c e^4+4\_Z^3 c d e^3+c d^4+b d^2+\left(6 c d^2 e^2+b e^2\right)\_Z^2+\left(4 c d^3 e+2 d e b\right)\_Z+a\right)^3+6 c d e^2\,\mathrm{RootOf}\left(\_Z^4 c e^4+4\_Z^3 c d e^3+c d^4+b d^2+\left(6 c d^2 e^2+b e^2\right)\_Z^2+\left(4 c d^3 e+2 d e b\right)\_Z+a\right)^2+6 e c d^2\,\mathrm{RootOf}\left(\_Z^4 c e^4+4\_Z^3 c d e^3+c d^4+b d^2+\left(6 c d^2 e^2+b e^2\right)\_Z^2+\left(4 c d^3 e+2 d e b\right)\_Z+a\right)+2 c d^3+b e\,\mathrm{RootOf}\left(\_Z^4 c e^4+4\_Z^3 c d e^3+c d^4+b d^2+\left(6 c d^2 e^2+b e^2\right)\_Z^2+\left(4 c d^3 e+2 d e b\right)\_Z+a\right)+b d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*f*x+d*f)^4/(a+b*(e*x+d)^2+c*(e*x+d)^4),x)
```

```
[Out] f^4*x/c+1/2*f^4/c/e*sum((-_R^2*b*e^2-2*_R*b*d*e-b*d^2-a)/(2*_R^3*c*e^3+6*_R
^2*c*d*e^2+6*_R*c*d^2*e+2*c*d^3+_R*b*e+b*d)*ln(-_R+x),_R=RootOf(_Z^4*c*e^4+
4*_Z^3*c*d*e^3+c*d^4+b*d^2+(6*c*d^2*e^2+b*e^2)*_Z^2+(4*c*d^3*e+2*b*d*e)*_Z+
a))
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*f*x+d*f)^4/(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [B] time = 1.34, size = 4605, normalized size = 22.80

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*f + e*f*x)^4/(a + b*(d + e*x)^2 + c*(d + e*x)^4),x)
```

```
[Out] atan((((2*b^4*d*e^11*f^8 + 4*a^2*c^2*d*e^11*f^8 - 8*a*b^2*c*d*e^11*f^8)/c +
 ((16*a^2*c^3*e^12*f^4 - 4*a*b^2*c^2*e^12*f^4)/c + ((8*b^3*c^3*d*e^13 - 32*
a*b*c^4*d*e^13)/c + (2*x*(4*b^3*c^3*e^14 - 16*a*b*c^4*e^14))/c)*(-(b^5*f^8
+ b^2*f^8*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*f^8 - 7*a*b^3*c*f^8 - a*c
*f^8*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5*e^2 + b^4*c^3*e^2 - 8*a*b^2*c
^4*e^2)))^(1/2))*(-(b^5*f^8 + b^2*f^8*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c
^2*f^8 - 7*a*b^3*c*f^8 - a*c*f^8*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5*e
^2 + b^4*c^3*e^2 - 8*a*b^2*c^4*e^2)))^(1/2) + (2*x*(b^4*e^12*f^8 + 2*a^2*c^
2*e^12*f^8 - 4*a*b^2*c*e^12*f^8))/c)*(-(b^5*f^8 + b^2*f^8*(-(4*a*c - b^2)^3
)^(1/2) + 12*a^2*b*c^2*f^8 - 7*a*b^3*c*f^8 - a*c*f^8*(-(4*a*c - b^2)^3)^(1/
2))/(8*(16*a^2*c^5*e^2 + b^4*c^3*e^2 - 8*a*b^2*c^4*e^2)))^(1/2)*1i + ((2*b^
4*d*e^11*f^8 + 4*a^2*c^2*d*e^11*f^8 - 8*a*b^2*c*d*e^11*f^8)/c - ((16*a^2*c^
3*e^12*f^4 - 4*a*b^2*c^2*e^12*f^4)/c - ((8*b^3*c^3*d*e^13 - 32*a*b*c^4*d*e^
13)/c + (2*x*(4*b^3*c^3*e^14 - 16*a*b*c^4*e^14))/c)*(-(b^5*f^8 + b^2*f^8*(-
(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*f^8 - 7*a*b^3*c*f^8 - a*c*f^8*(-(4*a*
c - b^2)^3)^(1/2))/(8*(16*a^2*c^5*e^2 + b^4*c^3*e^2 - 8*a*b^2*c^4*e^2)))^(1
/2))*(-(b^5*f^8 + b^2*f^8*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*f^8 - 7*a
*b^3*c*f^8 - a*c*f^8*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5*e^2 + b^4*c^3
*e^2 - 8*a*b^2*c^4*e^2)))^(1/2) + (2*x*(b^4*e^12*f^8 + 2*a^2*c^2*e^12*f^8 -
 4*a*b^2*c*e^12*f^8))/c)*(-(b^5*f^8 + b^2*f^8*(-(4*a*c - b^2)^3)^(1/2) + 12
*a^2*b*c^2*f^8 - 7*a*b^3*c*f^8 - a*c*f^8*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a
^2*c^5*e^2 + b^4*c^3*e^2 - 8*a*b^2*c^4*e^2)))^(1/2)*1i)/(((2*b^4*d*e^11*f^8
 + 4*a^2*c^2*d*e^11*f^8 - 8*a*b^2*c*d*e^11*f^8)/c + ((16*a^2*c^3*e^12*f^4 -
 4*a*b^2*c^2*e^12*f^4)/c + ((8*b^3*c^3*d*e^13 - 32*a*b*c^4*d*e^13)/c + (2*x
*(4*b^3*c^3*e^14 - 16*a*b*c^4*e^14))/c)*(-(b^5*f^8 + b^2*f^8*(-(4*a*c - b^2
)^3)^(1/2) + 12*a^2*b*c^2*f^8 - 7*a*b^3*c*f^8 - a*c*f^8*(-(4*a*c - b^2)^3)^
(1/2))/(8*(16*a^2*c^5*e^2 + b^4*c^3*e^2 - 8*a*b^2*c^4*e^2)))^(1/2))*(-(b^5*
f^8 + b^2*f^8*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*f^8 - 7*a*b^3*c*f^8 -
 a*c*f^8*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5*e^2 + b^4*c^3*e^2 - 8*a*b
^2*c^4*e^2)))^(1/2) + (2*x*(b^4*e^12*f^8 + 2*a^2*c^2*e^12*f^8 - 4*a*b^2*c*e
^12*f^8))/c)*(-(b^5*f^8 + b^2*f^8*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*f
^8 - 7*a*b^3*c*f^8 - a*c*f^8*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5*e^2 +
```

b^4*c^3*e^2 - 8*a*b^2*c^4*e^2)))^(1/2) - ((2*b^4*d*e^11*f^8 + 4*a^2*c^2*d*
e^11*f^8 - 8*a*b^2*c*d*e^11*f^8)/c - ((16*a^2*c^3*e^12*f^4 - 4*a*b^2*c^2*e^
12*f^4)/c - ((8*b^3*c^3*d*e^13 - 32*a*b*c^4*d*e^13)/c + (2*x*(4*b^3*c^3*e^1
4 - 16*a*b*c^4*e^14))/c)*(-(b^5*f^8 + b^2*f^8*(-(4*a*c - b^2)^3)^(1/2) + 12
*a^2*b*c^2*f^8 - 7*a*b^3*c*f^8 - a*c*f^8*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a
^2*c^5*e^2 + b^4*c^3*e^2 - 8*a*b^2*c^4*e^2)))^(1/2))*(-(b^5*f^8 + b^2*f^8*(
-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*f^8 - 7*a*b^3*c*f^8 - a*c*f^8*(-(4*a
c - b^2)^3)^(1/2))/(8(16*a^2*c^5*e^2 + b^4*c^3*e^2 - 8*a*b^2*c^4*e^2)))^(
1/2) + (2*x*(b^4*e^12*f^8 + 2*a^2*c^2*e^12*f^8 - 4*a*b^2*c*e^12*f^8))/c)*(-
(b^5*f^8 + b^2*f^8*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*f^8 - 7*a*b^3*c*
f^8 - a*c*f^8*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5*e^2 + b^4*c^3*e^2 -
8*a*b^2*c^4*e^2)))^(1/2) + (2*a^2*b*e^10*f^12)/c))*(-(b^5*f^8 + b^2*f^8*(-(
4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*f^8 - 7*a*b^3*c*f^8 - a*c*f^8*(-(4*a*c
 - b^2)^3)^(1/2))/(8*(16*a^2*c^5*e^2 + b^4*c^3*e^2 - 8*a*b^2*c^4*e^2)))^(1/
2)*2i + atan((((2*b^4*d*e^11*f^8 + 4*a^2*c^2*d*e^11*f^8 - 8*a*b^2*c*d*e^11*
f^8)/c + ((16*a^2*c^3*e^12*f^4 - 4*a*b^2*c^2*e^12*f^4)/c + ((8*b^3*c^3*d*e^
13 - 32*a*b*c^4*d*e^13)/c + (2*x*(4*b^3*c^3*e^14 - 16*a*b*c^4*e^14))/c)*(-(
b^5*f^8 - b^2*f^8*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*f^8 - 7*a*b^3*c*f
^8 + a*c*f^8*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5*e^2 + b^4*c^3*e^2 - 8
*a*b^2*c^4*e^2)))^(1/2))*(-(b^5*f^8 - b^2*f^8*(-(4*a*c - b^2)^3)^(1/2) + 12
*a^2*b*c^2*f^8 - 7*a*b^3*c*f^8 + a*c*f^8*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a
^2*c^5*e^2 + b^4*c^3*e^2 - 8*a*b^2*c^4*e^2)))^(1/2) + (2*x*(b^4*e^12*f^8 +
2*a^2*c^2*e^12*f^8 - 4*a*b^2*c*e^12*f^8))/c)*(-(b^5*f^8 - b^2*f^8*(-(4*a*c
- b^2)^3)^(1/2) + 12*a^2*b*c^2*f^8 - 7*a*b^3*c*f^8 + a*c*f^8*(-(4*a*c - b^2
)^3)^(1/2))/(8*(16*a^2*c^5*e^2 + b^4*c^3*e^2 - 8*a*b^2*c^4*e^2)))^(1/2)*1i
+ ((2*b^4*d*e^11*f^8 + 4*a^2*c^2*d*e^11*f^8 - 8*a*b^2*c*d*e^11*f^8)/c - ((1
6*a^2*c^3*e^12*f^4 - 4*a*b^2*c^2*e^12*f^4)/c - ((8*b^3*c^3*d*e^13 - 32*a*b*
c^4*d*e^13)/c + (2*x*(4*b^3*c^3*e^14 - 16*a*b*c^4*e^14))/c)*(-(b^5*f^8 - b^
2*f^8*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*f^8 - 7*a*b^3*c*f^8 + a*c*f^8
*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5*e^2 + b^4*c^3*e^2 - 8*a*b^2*c^4*e
^2)))^(1/2))*(-(b^5*f^8 - b^2*f^8*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*f
^8 - 7*a*b^3*c*f^8 + a*c*f^8*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5*e^2 +
 b^4*c^3*e^2 - 8*a*b^2*c^4*e^2)))^(1/2) + (2*x*(b^4*e^12*f^8 + 2*a^2*c^2*e^
12*f^8 - 4*a*b^2*c*e^12*f^8))/c)*(-(b^5*f^8 - b^2*f^8*(-(4*a*c - b^2)^3)^(1
/2) + 12*a^2*b*c^2*f^8 - 7*a*b^3*c*f^8 + a*c*f^8*(-(4*a*c - b^2)^3)^(1/2))/
(8*(16*a^2*c^5*e^2 + b^4*c^3*e^2 - 8*a*b^2*c^4*e^2)))^(1/2)*1i)/(((2*b^4*d*
e^11*f^8 + 4*a^2*c^2*d*e^11*f^8 - 8*a*b^2*c*d*e^11*f^8)/c + ((16*a^2*c^3*e^
12*f^4 - 4*a*b^2*c^2*e^12*f^4)/c + ((8*b^3*c^3*d*e^13 - 32*a*b*c^4*d*e^13)/
c + (2*x*(4*b^3*c^3*e^14 - 16*a*b*c^4*e^14))/c)*(-(b^5*f^8 - b^2*f^8*(-(4*a
*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*f^8 - 7*a*b^3*c*f^8 + a*c*f^8*(-(4*a*c -
b^2)^3)^(1/2))/(8*(16*a^2*c^5*e^2 + b^4*c^3*e^2 - 8*a*b^2*c^4*e^2)))^(1/2))
*(-(b^5*f^8 - b^2*f^8*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*f^8 - 7*a*b^3
*c*f^8 + a*c*f^8*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5*e^2 + b^4*c^3*e^2
 - 8*a*b^2*c^4*e^2)))^(1/2) + (2*x*(b^4*e^12*f^8 + 2*a^2*c^2*e^12*f^8 - 4*a
*b^2*c*e^12*f^8))/c)*(-(b^5*f^8 - b^2*f^8*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2
*b*c^2*f^8 - 7*a*b^3*c*f^8 + a*c*f^8*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c
^5*e^2 + b^4*c^3*e^2 - 8*a*b^2*c^4*e^2)))^(1/2) - ((2*b^4*d*e^11*f^8 + 4*a^
2*c^2*d*e^11*f^8 - 8*a*b^2*c*d*e^11*f^8)/c - ((16*a^2*c^3*e^12*f^4 - 4*a*b^
2*c^2*e^12*f^4)/c - ((8*b^3*c^3*d*e^13 - 32*a*b*c^4*d*e^13)/c + (2*x*(4*b^3
*c^3*e^14 - 16*a*b*c^4*e^14))/c)*(-(b^5*f^8 - b^2*f^8*(-(4*a*c - b^2)^3)^(1
/2) + 12*a^2*b*c^2*f^8 - 7*a*b^3*c*f^8 + a*c*f^8*(-(4*a*c - b^2)^3)^(1/2))/
(8*(16*a^2*c^5*e^2 + b^4*c^3*e^2 - 8*a*b^2*c^4*e^2)))^(1/2))*(-(b^5*f^8 - b
^2*f^8*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*f^8 - 7*a*b^3*c*f^8 + a*c*f^
8*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5*e^2 + b^4*c^3*e^2 - 8*a*b^2*c^4*
e^2)))^(1/2) + (2*x*(b^4*e^12*f^8 + 2*a^2*c^2*e^12*f^8 - 4*a*b^2*c*e^12*f^8
))/c)*(-(b^5*f^8 - b^2*f^8*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*f^8 - 7*
a*b^3*c*f^8 + a*c*f^8*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5*e^2 + b^4*c^
3*e^2 - 8*a*b^2*c^4*e^2)))^(1/2) + (2*a^2*b*e^10*f^12)/c))*(-(b^5*f^8 - b^2
f^8(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*f^8 - 7*a*b^3*c*f^8 + a*c*f^8*

```
(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5*e^2 + b^4*c^3*e^2 - 8*a*b^2*c^4*e^
2)))^(1/2)*2i + (f^4*x)/c
```

sympy [A] time = 3.62, size = 219, normalized size = 1.08

$$\operatorname{RootSum}\left(t^{4}\left(256a^{2}c^{5}e^{4}-128ab^{2}c^{4}e^{4}+16b^{4}c^{3}e^{4}\right)+t^{2}\left(48a^{2}bc^{2}e^{2}f^{8}-28ab^{3}ce^{2}f^{8}+4b^{5}e^{2}f^{8}\right)+a^{3}f^{16},\left(t\mapsto t\log\left(x+\frac{32t^{3}abc^{4}e^{3}-8t^{3}b^{3}c^{3}e^{3}-4ta^{2}c^{2}ef^{8}+8tab^{2}cef^{8}-2tb^{4}ef^{8}+a^{2}cdf^{12}-ab^{2}df^{12}}{a^{2}cef^{12}-ab^{2}ef^{12}}\right)\right)\right)+\frac{f^{4}x}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*f*x+d*f)**4/(a+b*(e*x+d)**2+c*(e*x+d)**4),x)
```

```
[Out] RootSum(_t**4*(256*a**2*c**5*e**4 - 128*a*b**2*c**4*e**4 + 16*b**4*c**3*e**
4) + _t**2*(48*a**2*b*c**2*e**2*f**8 - 28*a*b**3*c*e**2*f**8 + 4*b**5*e**2*
f**8) + a**3*f**16, Lambda(_t, _t*log(x + (32*_t**3*a*b*c**4*e**3 - 8*_t**3
*b**3*c**3*e**3 - 4*_t*a**2*c**2*e*f**8 + 8*_t*a*b**2*c*e*f**8 - 2*_t*b**4*
e*f**8 + a**2*c*d*f**12 - a*b**2*d*f**12)/(a**2*c*e*f**12 - a*b**2*e*f**12)
))) + f**4*x/c
```

3.526 $\int \frac{(df+efx)^3}{a+b(d+ex)^2+c(d+ex)^4} \, dx$

Optimal. Leaf size=87

$$\frac{bf^3 \tanh^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{2ce\sqrt{b^2-4ac}}+\frac{f^3 \log\left(a+b(d+ex)^2+c(d+ex)^4\right)}{4ce}$$

Rubi [A] time = 0.13, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {1142, 1114, 634, 618, 206, 628}

$$\frac{bf^3 \tanh^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{2ce\sqrt{b^2-4ac}}+\frac{f^3 \log\left(a+b(d+ex)^2+c(d+ex)^4\right)}{4ce}$$

Antiderivative was successfully verified.

```
[In] Int[(d*f + e*f*x)^3/(a + b*(d + e*x)^2 + c*(d + e*x)^4),x]
```

```
[Out] (b*f^3*ArcTanh[(b + 2*c*(d + e*x)^2)/Sqrt[b^2 - 4*a*c]])/(2*c*Sqrt[b^2 - 4*
a*c]*e) + (f^3*Log[a + b*(d + e*x)^2 + c*(d + e*x)^4])/(4*c*e)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 1142

```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Di
st[u^m/(Coefficient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p,
 x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

Rubi steps

$$\begin{aligned}\int \frac{(df+efx)^3}{a+b(d+ex)^2+c(d+ex)^4}\,dx &= \frac{f^3\,\text{Subst}\left(\int \frac{x^3}{a+bx^2+cx^4}\,dx,x,d+ex\right)}{e}\\ &= \frac{f^3\,\text{Subst}\left(\int \frac{x}{a+bx+cx^2}\,dx,x,(d+ex)^2\right)}{2e}\\ &= \frac{f^3\,\text{Subst}\left(\int \frac{b+2cx}{a+bx+cx^2}\,dx,x,(d+ex)^2\right)}{4ce}-\frac{\left(bf^3\right)\text{Subst}\left(\int \frac{1}{a+bx+cx^2}\,dx,x,(d+ex\right.}{4ce}\\ &= \frac{f^3\log\left(a+b(d+ex)^2+c(d+ex)^4\right)}{4ce}+\frac{\left(bf^3\right)\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx,x,b+2c(\right.}{2ce}\\ &= \frac{bf^3\tanh^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{2c\sqrt{b^2-4ac}\,e}+\frac{f^3\log\left(a+b(d+ex)^2+c(d+ex)^4\right)}{4ce}\end{aligned}$$

Mathematica [A] time = 0.04, size = 80, normalized size = 0.92

$$\frac{f^3\left(\log\left(a+b(d+ex)^2+c(d+ex)^4\right)-\frac{2b\tan^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}\right)}{4ce}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*f + e*f*x)^3/(a + b*(d + e*x)^2 + c*(d + e*x)^4),x]
```

```
[Out] (f^3*((-2*b*ArcTan[(b + 2*c*(d + e*x)^2)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4
*a*c] + Log[a + b*(d + e*x)^2 + c*(d + e*x)^4]))/(4*c*e)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(df+efx)^3}{a+b(d+ex)^2+c(d+ex)^4}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(d*f + e*f*x)^3/(a + b*(d + e*x)^2 + c*(d + e*x)^4),x]
```

```
[Out] IntegrateAlgebraic[(d*f + e*f*x)^3/(a + b*(d + e*x)^2 + c*(d + e*x)^4), x]
```

fricas [A] time = 1.19, size = 446, normalized size = 5.13

$$\left[\frac{\sqrt{b^2-4\,ac}\,bf^3\log\left(\frac{2\,c^2e^4x^4+8\,c^2de^3x^3+2\,c^2d^4+2\left(6\,c^2d^2+bc\right)e^2x^2+2\,bcd^2+4\left(2\,c^2d^3+bcd\right)ex+b^2-2\,ac+\left(2\,ce^2x^2+4\,cdex+2\,cd^2+b\right)\sqrt{b^2-4\,ac}}{ce^4x^4+4\,cde^3x^3+cd^4+\left(6\,cd^2+b\right)e^2x^2+bd^2+2\left(2\,cd^3+bd\right)ex+a}\right)+\left(b^2-4\,ac\right)f^3\log\left(ce^4x^4+4\,cde^3x^3+cd^4+\left(6\,cd^2+b\right)e^2x^2+bd^2+2\left(2\,cd^3+bd\right)ex+a\right)}{4\left(b^2c-4\,ac^2\right)e},\ \frac{2\sqrt{-b^2+4\,ac}\,bf^3\arctan\left(-\frac{\left(2\,ce^2x^2+4\,cdex+2\,cd^2+b\right)\sqrt{-b^2+4\,ac}}{b^2-4\,ac}\right)+\left(b^2-4\,ac\right)f^3\log\left(ce^4x^4+4\,cde^3x^3+cd^4+\left(6\,cd^2+b\right)e^2x^2+bd^2+2\left(2\,cd^3+bd\right)ex+a\right)}{4\left(b^2c-4\,ac^2\right)e}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*f*x+d*f)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="fricas")
```

```
[Out] [1/4*(sqrt(b^2 - 4*a*c)*b*f^3*log((2*c^2*e^4*x^4 + 8*c^2*d*e^3*x^3 + 2*c^2*
d^4 + 2*(6*c^2*d^2 + b*c)*e^2*x^2 + 2*b*c*d^2 + 4*(2*c^2*d^3 + b*c*d)*e*x +
 b^2 - 2*a*c + (2*c*e^2*x^2 + 4*c*d*e*x + 2*c*d^2 + b)*sqrt(b^2 - 4*a*c))/(
c*e^4*x^4 + 4*c*d*e^3*x^3 + c*d^4 + (6*c*d^2 + b)*e^2*x^2 + b*d^2 + 2*(2*c*
d^3 + b*d)*e*x + a)) + (b^2 - 4*a*c)*f^3*log(c*e^4*x^4 + 4*c*d*e^3*x^3 + c*
d^4 + (6*c*d^2 + b)*e^2*x^2 + b*d^2 + 2*(2*c*d^3 + b*d)*e*x + a))/((b^2*c -
 4*a*c^2)*e), 1/4*(2*sqrt(-b^2 + 4*a*c)*b*f^3*arctan(-(2*c*e^2*x^2 + 4*c*d*
e*x + 2*c*d^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) + (b^2 - 4*a*c)*f^3*lo
```

g(c*e^4*x^4 + 4*c*d*e^3*x^3 + c*d^4 + (6*c*d^2 + b)*e^2*x^2 + b*d^2 + 2*(2*c*d^3 + b*d)*e*x + a))/((b^2*c - 4*a*c^2)*e)]

giac [B] time = 0.43, size = 162, normalized size = 1.86

$$-\frac{bf^3\arctan\left(\frac{2cd^2f+2\left(fx^2e+2dfx\right)ce+bf}{\sqrt{-b^2+4ac}\,f}\right)e^{(-1)}}{2\sqrt{-b^2+4ac}\,c}+\frac{f^3e^{(-1)}\log\left(cd^4f^2+2\left(fx^2e+2dfx\right)cd^2fe+bd^2f^2+\left(fx^2e+2dfx\right)^2ce^2+\left(fx^2e+2dfx\right)bfe+af^2\right)}{4c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*f*x+d*f)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="giac")

[Out] -1/2*b*f^3*arctan((2*c*d^2*f + 2*(f*x^2*e + 2*d*f*x)*c*e + b*f)/(sqrt(-b^2 + 4*a*c)*f))*e^(-1)/(sqrt(-b^2 + 4*a*c)*c) + 1/4*f^3*e^(-1)*log(c*d^4*f^2 + 2*(f*x^2*e + 2*d*f*x)*c*d^2*f*e + b*d^2*f^2 + (f*x^2*e + 2*d*f*x)^2*c*e^2 + (f*x^2*e + 2*d*f*x)*b*f*e + a*f^2)/c

maple [C] time = 0.00, size = 154, normalized size = 1.77

$$\frac{f^3\left(\operatorname{RootOf}\left(\_Z^4ce^4+4\_Z^3cde^3+cd^4+bd^2+\left(6cd^2e^2+be^2\right)\_Z^2+\left(4cd^3e+2deb\right)\_Z+a\right)^3e^3+3\operatorname{RootOf}\left(\_Z^4ce^4+4\_Z^3cde^3+cd^4+bd^2+\left(6cd^2e^2+be^2\right)\_Z^2+\left(4cd^3e+2deb\right)\_Z+a\right)^2de^2+3\operatorname{RootOf}\left(\_Z^4ce^4+4\_Z^3cde^3+cd^4+bd^2+\left(6cd^2e^2+be^2\right)\_Z^2+\left(4cd^3e+2deb\right)\_Z+a\right)d^2e+d^3\right)\ln\left(-\operatorname{RootOf}\left(\_Z^4ce^4+4\_Z^3cde^3+cd^4+bd^2+\left(6cd^2e^2+be^2\right)\_Z^2+\left(4cd^3e+2deb\right)\_Z+a\right)+x\right)}{2e\left(2ce^3\operatorname{RootOf}\left(\_Z^4ce^4+4\_Z^3cde^3+cd^4+bd^2+\left(6cd^2e^2+be^2\right)\_Z^2+\left(4cd^3e+2deb\right)\_Z+a\right)^3+6cde^2\operatorname{RootOf}\left(\_Z^4ce^4+4\_Z^3cde^3+cd^4+bd^2+\left(6cd^2e^2+be^2\right)\_Z^2+\left(4cd^3e+2deb\right)\_Z+a\right)^2+6ecd^2\operatorname{RootOf}\left(\_Z^4ce^4+4\_Z^3cde^3+cd^4+bd^2+\left(6cd^2e^2+be^2\right)\_Z^2+\left(4cd^3e+2deb\right)\_Z+a\right)+2cd^3+be\operatorname{RootOf}\left(\_Z^4ce^4+4\_Z^3cde^3+cd^4+bd^2+\left(6cd^2e^2+be^2\right)\_Z^2+\left(4cd^3e+2deb\right)\_Z+a\right)+bd\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e*f*x+d*f)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4),x)

[Out] 1/2*f^3/e*sum((_R^3*e^3+3*_R^2*d*e^2+3*_R*d^2*e+d^3)/(2*_R^3*c*e^3+6*_R^2*c*d*e^2+6*_R*c*d^2*e+2*c*d^3+_R*b*e+b*d)*ln(-_R+x),_R=RootOf(_Z^4*c*e^4+4*_Z^3*c*d*e^3+c*d^4+b*d^2+(6*c*d^2*e^2+b*e^2)*_Z^2+(4*c*d^3*e+2*b*d*e)*_Z+a))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(efx+df\right)^3}{(ex+d)^4c+(ex+d)^2b+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*f*x+d*f)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="maxima")

[Out] integrate((e*f*x + d*f)^3/((e*x + d)^4*c + (e*x + d)^2*b + a), x)

mupad [B] time = 0.44, size = 287, normalized size = 3.30

$$\frac{4acef^3\ln\left(cd^4+4cd^3ex+6cd^2e^2x^2+bd^2+4cde^3x^3+2bdex+ce^4x^4+be^2x^2+a\right)}{16ac^2e^2-4b^2ce^2}-\frac{bf^3\,\mathrm{atan}\left(\frac{b}{\sqrt{4ac-b^2}}+\frac{2cd^2}{\sqrt{4ac-b^2}}+\frac{2ce^2x^2}{\sqrt{4ac-b^2}}+\frac{4cdex}{\sqrt{4ac-b^2}}\right)}{2ce\sqrt{4ac-b^2}}-\frac{b^2ef^3\ln\left(cd^4+4cd^3ex+6cd^2e^2x^2+bd^2+4cde^3x^3+2bdex+ce^4x^4+be^2x^2+a\right)}{16ac^2e^2-4b^2ce^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*f + e*f*x)^3/(a + b*(d + e*x)^2 + c*(d + e*x)^4),x)

[Out] (4*a*c*e*f^3*log(a + b*d^2 + c*d^4 + b*e^2*x^2 + c*e^4*x^4 + 2*b*d*e*x + 6*c*d^2*e^2*x^2 + 4*c*d^3*e*x + 4*c*d*e^3*x^3))/(16*a*c^2*e^2 - 4*b^2*c*e^2) - (b*f^3*atan(b/(4*a*c - b^2)^(1/2) + (2*c*d^2)/(4*a*c - b^2)^(1/2) + (2*c*e^2*x^2)/(4*a*c - b^2)^(1/2) + (4*c*d*e*x)/(4*a*c - b^2)^(1/2)))/(2*c*e*(4*a*c - b^2)^(1/2)) - (b^2*e*f^3*log(a + b*d^2 + c*d^4 + b*e^2*x^2 + c*e^4*x^4 + 2*b*d*e*x + 6*c*d^2*e^2*x^2 + 4*c*d^3*e*x + 4*c*d*e^3*x^3))/(16*a*c^2*e^2 - 4*b^2*c*e^2)

sympy [B] time = 1.95, size = 332, normalized size = 3.82

$$\left(-\frac{bf^3\sqrt{-4ac+b^2}}{4ce\left(4ac-b^2\right)}+\frac{f^3}{4ce}\right)\log\left(\frac{2dx}{e}+x^2+\frac{-8ace\left(-\frac{bf^3\sqrt{-4ac+b^2}}{4ce(4ac-b^2)}+\frac{f^3}{4ce}\right)+2af^3+2b^2e\left(-\frac{bf^3\sqrt{-4ac+b^2}}{4ce(4ac-b^2)}+\frac{f^3}{4ce}\right)+bd^2f^3}{be^2f^3}\right)+\left(\frac{bf^3\sqrt{-4ac+b^2}}{4ce\left(4ac-b^2\right)}+\frac{f^3}{4ce}\right)\log\left(\frac{2dx}{e}+x^2+\frac{-8ace\left(\frac{bf^3\sqrt{-4ac+b^2}}{4ce(4ac-b^2)}+\frac{f^3}{4ce}\right)+2af^3+2b^2e\left(\frac{bf^3\sqrt{-4ac+b^2}}{4ce(4ac-b^2)}+\frac{f^3}{4ce}\right)+bd^2f^3}{be^2f^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*f*x+d*f)**3/(a+b*(e*x+d)**2+c*(e*x+d)**4),x)
```

```
[Out] (-b*f**3*sqrt(-4*a*c + b**2)/(4*c*e*(4*a*c - b**2)) + f**3/(4*c*e))*log(2*d
*x/e + x**2 + (-8*a*c*e*(-b*f**3*sqrt(-4*a*c + b**2)/(4*c*e*(4*a*c - b**2))
 + f**3/(4*c*e)) + 2*a*f**3 + 2*b**2*e*(-b*f**3*sqrt(-4*a*c + b**2)/(4*c*e*
(4*a*c - b**2)) + f**3/(4*c*e)) + b*d**2*f**3)/(b*e**2*f**3)) + (b*f**3*sqr
t(-4*a*c + b**2)/(4*c*e*(4*a*c - b**2)) + f**3/(4*c*e))*log(2*d*x/e + x**2
+ (-8*a*c*e*(b*f**3*sqrt(-4*a*c + b**2)/(4*c*e*(4*a*c - b**2)) + f**3/(4*c*
e)) + 2*a*f**3 + 2*b**2*e*(b*f**3*sqrt(-4*a*c + b**2)/(4*c*e*(4*a*c - b**2)
) + f**3/(4*c*e)) + b*d**2*f**3)/(b*e**2*f**3))
```

3.527 $\int \frac{(df+efx)^2}{a+b(d+ex)^2+c(d+ex)^4} \, dx$

Optimal. Leaf size=170

$$\frac{f^2\sqrt{\sqrt{b^2-4ac}+b}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\sqrt{c}e\sqrt{b^2-4ac}}-\frac{f^2\sqrt{b-\sqrt{b^2-4ac}}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}e\sqrt{b^2-4ac}}$$

Rubi [A] time = 0.15, antiderivative size = 170, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1142, 1130, 205}

$$\frac{f^2\sqrt{\sqrt{b^2-4ac}+b}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\sqrt{c}e\sqrt{b^2-4ac}}-\frac{f^2\sqrt{b-\sqrt{b^2-4ac}}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}e\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*f + e*f*x)^2/(a + b*(d + e*x)^2 + c*(d + e*x)^4),x]
```

```
[Out] -((Sqrt[b - Sqrt[b^2 - 4*a*c]]*f^2*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[
b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b^2 - 4*a*c]*e)) + (Sqrt[b +
 Sqrt[b^2 - 4*a*c]]*f^2*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b + Sqrt[b^
2 - 4*a*c]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b^2 - 4*a*c]*e)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1130

```
Int[((d_.)*(x_))^(m_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Wi
th[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(d^2*(b/q + 1))/2, Int[(d*x)^(m - 2)/(b/2
 + q/2 + c*x^2), x], x] - Dist[(d^2*(b/q - 1))/2, Int[(d*x)^(m - 2)/(b/2 -
q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && G
eQ[m, 2]
```

Rule 1142

```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Di
st[u^m/(Coefficient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p,
 x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

Rubi steps

$$\int \frac{(df+efx)^2}{a+b(d+ex)^2+c(d+ex)^4}\,dx=\frac{f^2\,\text{Subst}\left(\int \frac{x^2}{a+bx^2+cx^4}\,dx,x,d+ex\right)}{e}$$

$$=\frac{\left(\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)f^2\right)\text{Subst}\left(\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx,x,d+ex\right)}{2e}+\frac{\left(\left(1+\frac{b}{\sqrt{b^2-4ac}}\right.\right.}{}$$

$$=-\frac{\sqrt{b-\sqrt{b^2-4ac}}\,f^2\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}\,e}+\frac{\sqrt{b+\sqrt{b^2-4ac}}\,f^2\tan^{-1}\left(\frac{\sqrt{}}{\sqrt{}}\right.}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}\,e}$$

Mathematica [A] time = 0.10, size = 178, normalized size = 1.05

$$\frac{f^2\left(\left(\sqrt{b^2-4ac}-b\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)+\sqrt{b-\sqrt{b^2-4ac}}\sqrt{\sqrt{b^2-4ac}+b}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{\sqrt{b^2-4ac}+b}}\right)\right)}{\sqrt{2}\sqrt{c}e\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*f + e*f*x)^2/(a + b*(d + e*x)^2 + c*(d + e*x)^4),x]
```

```
[Out] (f^2*((-b + Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b -
Sqrt[b^2 - 4*a*c]]] + Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[b + Sqrt[b^2 - 4*a*c
]]*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b + Sqrt[b^2 - 4*a*c]]]))/(Sqrt[
2]*Sqrt[c]*Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]*e)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(df+efx)^2}{a+b(d+ex)^2+c(d+ex)^4}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(d*f + e*f*x)^2/(a + b*(d + e*x)^2 + c*(d + e*x)^4),x]
```

```
[Out] IntegrateAlgebraic[(d*f + e*f*x)^2/(a + b*(d + e*x)^2 + c*(d + e*x)^4), x]
```

fricas [B] time = 1.10, size = 799, normalized size = 4.70

$$\frac{1}{2}\sqrt{\frac{1}{2}}\sqrt{-\frac{bf^4+(b^2c-4ac^2)\sqrt{\frac{f^8}{(b^2c^2-4ac^3)e^4}}e^2}{(b^2c-4ac^2)e^2}}\log\left(ef^6x+df^6+\sqrt{\frac{1}{2}}(b^2c-4ac^2)\sqrt{\frac{f^8}{(b^2c^2-4ac^3)e^4}}e^3\sqrt{-\frac{bf^4+(b^2c-4ac^2)\sqrt{\frac{f^8}{(b^2c^2-4ac^3)e^4}}e^2}{(b^2c-4ac^2)e^2}}\right)-\frac{1}{2}\sqrt{\frac{1}{2}}\sqrt{-\frac{bf^4+(b^2c-4ac^2)\sqrt{\frac{f^8}{(b^2c^2-4ac^3)e^4}}e^2}{(b^2c-4ac^2)e^2}}\log\left(ef^6x+df^6-\sqrt{\frac{1}{2}}(b^2c-4ac^2)\sqrt{\frac{f^8}{(b^2c^2-4ac^3)e^4}}e^3\sqrt{-\frac{bf^4+(b^2c-4ac^2)\sqrt{\frac{f^8}{(b^2c^2-4ac^3)e^4}}e^2}{(b^2c-4ac^2)e^2}}\right)-\frac{1}{2}\sqrt{\frac{1}{2}}\sqrt{-\frac{bf^4-(b^2c-4ac^2)\sqrt{\frac{f^8}{(b^2c^2-4ac^3)e^4}}e^2}{(b^2c-4ac^2)e^2}}\log\left(ef^6x+df^6+\sqrt{\frac{1}{2}}(b^2c-4ac^2)\sqrt{\frac{f^8}{(b^2c^2-4ac^3)e^4}}e^3\sqrt{-\frac{bf^4-(b^2c-4ac^2)\sqrt{\frac{f^8}{(b^2c^2-4ac^3)e^4}}e^2}{(b^2c-4ac^2)e^2}}\right)+\frac{1}{2}\sqrt{\frac{1}{2}}\sqrt{-\frac{bf^4-(b^2c-4ac^2)\sqrt{\frac{f^8}{(b^2c^2-4ac^3)e^4}}e^2}{(b^2c-4ac^2)e^2}}\log\left(ef^6x+df^6-\sqrt{\frac{1}{2}}(b^2c-4ac^2)\sqrt{\frac{f^8}{(b^2c^2-4ac^3)e^4}}e^3\sqrt{-\frac{bf^4-(b^2c-4ac^2)\sqrt{\frac{f^8}{(b^2c^2-4ac^3)e^4}}e^2}{(b^2c-4ac^2)e^2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*f*x+d*f)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(1/2)*sqrt(-(b*f^4 + (b^2*c - 4*a*c^2)*sqrt(f^8/((b^2*c^2 - 4*a*c^3
)*e^4))*e^2)/((b^2*c - 4*a*c^2)*e^2))*log(e*f^6*x + d*f^6 + sqrt(1/2)*(b^2*
c - 4*a*c^2)*sqrt(f^8/((b^2*c^2 - 4*a*c^3)*e^4))*e^3*sqrt(-(b*f^4 + (b^2*c
- 4*a*c^2)*sqrt(f^8/((b^2*c^2 - 4*a*c^3)*e^4))*e^2)/((b^2*c - 4*a*c^2)*e^2)
)) - 1/2*sqrt(1/2)*sqrt(-(b*f^4 + (b^2*c - 4*a*c^2)*sqrt(f^8/((b^2*c^2 - 4*
a*c^3)*e^4))*e^2)/((b^2*c - 4*a*c^2)*e^2))*log(e*f^6*x + d*f^6 - sqrt(1/2)*
(b^2*c - 4*a*c^2)*sqrt(f^8/((b^2*c^2 - 4*a*c^3)*e^4))*e^3*sqrt(-(b*f^4 + (b
^2*c - 4*a*c^2)*sqrt(f^8/((b^2*c^2 - 4*a*c^3)*e^4))*e^2)/((b^2*c - 4*a*c^2)
*e^2))) - 1/2*sqrt(1/2)*sqrt(-(b*f^4 - (b^2*c - 4*a*c^2)*sqrt(f^8/((b^2*c^2
 - 4*a*c^3)*e^4))*e^2)/((b^2*c - 4*a*c^2)*e^2))*log(e*f^6*x + d*f^6 + sqrt(
1/2)*(b^2*c - 4*a*c^2)*sqrt(f^8/((b^2*c^2 - 4*a*c^3)*e^4))*e^3*sqrt(-(b*f^4
 - (b^2*c - 4*a*c^2)*sqrt(f^8/((b^2*c^2 - 4*a*c^3)*e^4))*e^2)/((b^2*c - 4*a
*c^2)*e^2))) + 1/2*sqrt(1/2)*sqrt(-(b*f^4 - (b^2*c - 4*a*c^2)*sqrt(f^8/((b^
2*c^2 - 4*a*c^3)*e^4))*e^2)/((b^2*c - 4*a*c^2)*e^2))*log(e*f^6*x + d*f^6 -
sqrt(1/2)*(b^2*c - 4*a*c^2)*sqrt(f^8/((b^2*c^2 - 4*a*c^3)*e^4))*e^3*sqrt(-(
b*f^4 - (b^2*c - 4*a*c^2)*sqrt(f^8/((b^2*c^2 - 4*a*c^3)*e^4))*e^2)/((b^2*c
- 4*a*c^2)*e^2)))
```

giac [B] time = 0.47, size = 1325, normalized size = 7.79

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*f*x+d*f)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="giac")
```

```
[Out] -1/2*((d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c)
)^2*f^2*e^2 - 2*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)
```

```
*e^(-4)/c))*d*f^2*e + d^2*f^2)*log(d*e^(-1) + x + sqrt(1/2)*sqrt(-(b*e^2 +
sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))/(2*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 + s
qrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^3*c*e^4 - 6*(d*e^(-1) + sqrt(1/2)*sqrt(-(b
*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^2*c*d*e^3 + 6*(d*e^(-1) + sqrt(1/2
)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))*c*d^2*e^2 - 2*c*d^3*e +
(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))*b*e^
2 - b*d*e) - 1/2*((d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^
2)*e^(-4)/c))^2*f^2*e^2 - 2*(d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 -
 4*a*c)*e^2)*e^(-4)/c))*d*f^2*e + d^2*f^2)*log(d*e^(-1) + x - sqrt(1/2)*sqr
t(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))/(2*(d*e^(-1) - sqrt(1/2)*sqrt
(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^3*c*e^4 - 6*(d*e^(-1) - sqrt(1
/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^2*c*d*e^3 + 6*(d*e^(-1
) - sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))*c*d^2*e^2 -
2*c*d^3*e + (d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(
-4)/c))*b*e^2 - b*d*e) - 1/2*((d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2
 - 4*a*c)*e^2)*e^(-4)/c))^2*f^2*e^2 - 2*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2
- sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))*d*f^2*e + d^2*f^2)*log(d*e^(-1) + x + s
qrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))/(2*(d*e^(-1) + sq
rt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^3*c*e^4 - 6*(d*e^(
-1) + sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^2*c*d*e^3
+ 6*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))*
c*d^2*e^2 - 2*c*d^3*e + (d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a
*c)*e^2)*e^(-4)/c))*b*e^2 - b*d*e) - 1/2*((d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^
2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^2*f^2*e^2 - 2*(d*e^(-1) - sqrt(1/2)*s
qrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))*d*f^2*e + d^2*f^2)*log(d*e^
(-1) + x - sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))/(2*(d
*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^3*c*e^
4 - 6*(d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c)
)^2*c*d*e^3 + 6*(d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)
*e^(-4)/c))*c*d^2*e^2 - 2*c*d^3*e + (d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 - sq
rt(b^2 - 4*a*c)*e^2)*e^(-4)/c))*b*e^2 - b*d*e)
```

maple [C] time = 0.00, size = 143, normalized size = 0.84

$$\frac{f^{2} \left(\mathrm{RootOf}\left(\_Z^{4} c\,e^{4}+4\_Z^{3} c d\,e^{3}+c\,d^{4}+b\,d^{2}+\left(6 c\,d^{2} e^{2}+b\,e^{2}\right)\_Z^{2}+\left(4 c\,d^{3} e+2 d e b\right)\_Z+a\right)^{2} e^{2}+2\,\mathrm{RootOf}\left(\_Z^{4} c\,e^{4}+4\_Z^{3} c d\,e^{3}+c\,d^{4}+b\,d^{2}+\left(6 c\,d^{2} e^{2}+b\,e^{2}\right)\_Z^{2}+\left(4 c\,d^{3} e+2 d e b\right)\_Z+a\right) d e+d^{2}\right) \ln\left(-\mathrm{RootOf}\left(\_Z^{4} c\,e^{4}+4\_Z^{3} c d\,e^{3}+c\,d^{4}+b\,d^{2}+\left(6 c\,d^{2} e^{2}+b\,e^{2}\right)\_Z^{2}+\left(4 c\,d^{3} e+2 d e b\right)\_Z+a\right)+x\right)}{2 e \left(2 c\,e^{3} \mathrm{RootOf}\left(\_Z^{4} c\,e^{4}+4\_Z^{3} c d\,e^{3}+c\,d^{4}+b\,d^{2}+\left(6 c\,d^{2} e^{2}+b\,e^{2}\right)\_Z^{2}+\left(4 c\,d^{3} e+2 d e b\right)\_Z+a\right)^{3}+6 c d\,e^{2} \mathrm{RootOf}\left(\_Z^{4} c\,e^{4}+4\_Z^{3} c d\,e^{3}+c\,d^{4}+b\,d^{2}+\left(6 c\,d^{2} e^{2}+b\,e^{2}\right)\_Z^{2}+\left(4 c\,d^{3} e+2 d e b\right)\_Z+a\right)^{2}+6 e c\,d^{2} \mathrm{RootOf}\left(\_Z^{4} c\,e^{4}+4\_Z^{3} c d\,e^{3}+c\,d^{4}+b\,d^{2}+\left(6 c\,d^{2} e^{2}+b\,e^{2}\right)\_Z^{2}+\left(4 c\,d^{3} e+2 d e b\right)\_Z+a\right)+2 c\,d^{3}+b e\,\mathrm{RootOf}\left(\_Z^{4} c\,e^{4}+4\_Z^{3} c d\,e^{3}+c\,d^{4}+b\,d^{2}+\left(6 c\,d^{2} e^{2}+b\,e^{2}\right)\_Z^{2}+\left(4 c\,d^{3} e+2 d e b\right)\_Z+a\right)+b d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*f*x+d*f)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4),x)
```

```
[Out] 1/2*f^2/e*sum((_R^2*e^2+2*_R*d*e+d^2)/(2*_R^3*c*e^3+6*_R^2*c*d*e^2+6*_R*c*d
^2*e+2*c*d^3+_R*b*e+b*d)*ln(-_R+x),_R=RootOf(_Z^4*c*e^4+4*_Z^3*c*d*e^3+c*d^
4+b*d^2+(6*c*d^2*e^2+b*e^2)*_Z^2+(4*c*d^3*e+2*b*d*e)*_Z+a))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(efx + df\right)^{2}}{(ex + d)^{4} c + (ex + d)^{2} b + a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*f*x+d*f)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="maxima")
```

```
[Out] integrate((e*f*x + d*f)^2/((e*x + d)^4*c + (e*x + d)^2*b + a), x)
```

mupad [B] time = 1.79, size = 683, normalized size = 4.02

$$-2\,\mathrm{atanh}\left(\frac{\sqrt{-\frac{b^3 f^4+f^4\sqrt{-{\left(4 a c-b^2\right)}^3}-4 a b c f^4}{8\left(16 a^2 c^3 e^2-8 a b^2 c^2 e^2+b^4 c e^2\right)}}\left(x\left(4 a c^2 e^{12} f^4-2 b^2 c e^{12} f^4\right)+\frac{\left(x\left(8 b^3 c^2 e^{14}-32 a b c^3 e^{14}\right)+8 b^3 c^2 d e^{13}-32 a b c^3 d e^{13}\right)\left(b^3 f^4+f^4\sqrt{-{\left(4 a c-b^2\right)}^3}-4 a b c f^4\right)}{8\left(16 a^2 c^3 e^2-8 a b^2 c^2 e^2+b^4 c e^2\right)}+4 a c^2 d e^{11} f^4-2 b^2 c d e^{11} f^4\right)}{a c e^{10} f^6}\right)\sqrt{-\frac{b^3 f^4+f^4\sqrt{-{\left(4 a c-b^2\right)}^3}-4 a b c f^4}{8\left(16 a^2 c^3 e^2-8 a b^2 c^2 e^2+b^4 c e^2\right)}}-2\,\mathrm{atanh}\left(\frac{\sqrt{\frac{f^4\sqrt{-{\left(4 a c-b^2\right)}^3}-b^3 f^4+4 a b c f^4}{8\left(16 a^2 c^3 e^2-8 a b^2 c^2 e^2+b^4 c e^2\right)}}\left(x\left(4 a c^2 e^{12} f^4-2 b^2 c e^{12} f^4\right)-\frac{\left(x\left(8 b^3 c^2 e^{14}-32 a b c^3 e^{14}\right)+8 b^3 c^2 d e^{13}-32 a b c^3 d e^{13}\right)\left(f^4\sqrt{-{\left(4 a c-b^2\right)}^3}-b^3 f^4+4 a b c f^4\right)}{8\left(16 a^2 c^3 e^2-8 a b^2 c^2 e^2+b^4 c e^2\right)}+4 a c^2 d e^{11} f^4-2 b^2 c d e^{11} f^4\right)}{a c e^{10} f^6}\right)\sqrt{\frac{f^4\sqrt{-{\left(4 a c-b^2\right)}^3}-b^3 f^4+4 a b c f^4}{8\left(16 a^2 c^3 e^2-8 a b^2 c^2 e^2+b^4 c e^2\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*f + e*f*x)^2/(a + b*(d + e*x)^2 + c*(d + e*x)^4),x)
```

```
[Out] - 2*atanh(((-(b^3*f^4 + f^4*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c*f^4)/(8*(b^4
*c*e^2 + 16*a^2*c^3*e^2 - 8*a*b^2*c^2*e^2)))^(1/2)*(x*(4*a*c^2*e^12*f^4 - 2
*b^2*c*e^12*f^4) + ((x*(8*b^3*c^2*e^14 - 32*a*b*c^3*e^14) + 8*b^3*c^2*d*e^1
3 - 32*a*b*c^3*d*e^13)*(b^3*f^4 + f^4*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c*f^
4))/(8*(b^4*c*e^2 + 16*a^2*c^3*e^2 - 8*a*b^2*c^2*e^2)) + 4*a*c^2*d*e^11*f^4
 - 2*b^2*c*d*e^11*f^4))/(a*c*e^10*f^6))*(-(b^3*f^4 + f^4*(-(4*a*c - b^2)^3)
^(1/2) - 4*a*b*c*f^4)/(8*(b^4*c*e^2 + 16*a^2*c^3*e^2 - 8*a*b^2*c^2*e^2)))^(
1/2) - 2*atanh((((f^4*(-(4*a*c - b^2)^3)^(1/2) - b^3*f^4 + 4*a*b*c*f^4)/(8*
(b^4*c*e^2 + 16*a^2*c^3*e^2 - 8*a*b^2*c^2*e^2)))^(1/2)*(x*(4*a*c^2*e^12*f^4
 - 2*b^2*c*e^12*f^4) - ((x*(8*b^3*c^2*e^14 - 32*a*b*c^3*e^14) + 8*b^3*c^2*d
*e^13 - 32*a*b*c^3*d*e^13)*(f^4*(-(4*a*c - b^2)^3)^(1/2) - b^3*f^4 + 4*a*b*
c*f^4))/(8*(b^4*c*e^2 + 16*a^2*c^3*e^2 - 8*a*b^2*c^2*e^2)) + 4*a*c^2*d*e^11
*f^4 - 2*b^2*c*d*e^11*f^4))/(a*c*e^10*f^6))*((f^4*(-(4*a*c - b^2)^3)^(1/2)
- b^3*f^4 + 4*a*b*c*f^4)/(8*(b^4*c*e^2 + 16*a^2*c^3*e^2 - 8*a*b^2*c^2*e^2))
)^(1/2)
```

sympy [A] time = 1.59, size = 124, normalized size = 0.73

$$\operatorname{RootSum}\left(t^{4}\left(256a^{2}c^{3}e^{4}-128ab^{2}c^{2}e^{4}+16b^{4}ce^{4}\right)+t^{2}\left(-16abce^{2}f^{4}+4b^{3}e^{2}f^{4}\right)+af^{8},\left(t\mapsto t\log\left(x+\frac{64t^{3}ac^{2}e^{3}-16t^{3}b^{2}ce^{3}-2tbef^{4}+df^{6}}{ef^{6}}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*f*x+d*f)**2/(a+b*(e*x+d)**2+c*(e*x+d)**4),x)
```

```
[Out] RootSum(_t**4*(256*a**2*c**3*e**4 - 128*a*b**2*c**2*e**4 + 16*b**4*c*e**4)
+ _t**2*(-16*a*b*c*e**2*f**4 + 4*b**3*e**2*f**4) + a*f**8, Lambda(_t, _t*lo
g(x + (64*_t**3*a*c**2*e**3 - 16*_t**3*b**2*c*e**3 - 2*_t*b*e*f**4 + d*f**6
)/(e*f**6))))
```

3.528 $\int \frac{df+efx}{a+b(d+ex)^2+c(d+ex)^4} dx$

Optimal. Leaf size=44

$$-\frac{f\tanh^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{e\sqrt{b^2-4ac}}$$

Rubi [A] time = 0.06, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129, Rules used = {1142, 1107, 618, 206}

$$-\frac{f\tanh^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{e\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*f + e*f*x)/(a + b*(d + e*x)^2 + c*(d + e*x)^4),x]
```

```
[Out] -((f*ArcTanh[(b + 2*c*(d + e*x)^2)/Sqrt[b^2 - 4*a*c]])/(Sqrt[b^2 - 4*a*c]*e
))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1107

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rule 1142

```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Di
st[u^m/(Coefficient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p,
 x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{df+efx}{a+b(d+ex)^2+c(d+ex)^4} dx &= \frac{f\,\text{Subst}\left(\int \frac{x}{a+bx^2+cx^4} dx, x, d+ex\right)}{e}\\
&= \frac{f\,\text{Subst}\left(\int \frac{1}{a+bx+cx^2} dx, x, (d+ex)^2\right)}{2e}\\
&= -\frac{f\,\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2} dx, x, b+2c(d+ex)^2\right)}{e}\\
&= -\frac{f\tanh^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}\,e}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 47, normalized size = 1.07

$$\frac{f\tan^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{4ac-b^2}}\right)}{e\sqrt{4ac-b^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*f + e*f*x)/(a + b*(d + e*x)^2 + c*(d + e*x)^4),x]
```

```
[Out] (f*ArcTan[(b + 2*c*(d + e*x)^2)/Sqrt[-b^2 + 4*a*c]])/(Sqrt[-b^2 + 4*a*c]*e)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{df+efx}{a+b(d+ex)^2+c(d+ex)^4}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(d*f + e*f*x)/(a + b*(d + e*x)^2 + c*(d + e*x)^4),x]
```

```
[Out] IntegrateAlgebraic[(d*f + e*f*x)/(a + b*(d + e*x)^2 + c*(d + e*x)^4), x]
```

fricas [A] time = 1.22, size = 274, normalized size = 6.23

$$\left[\frac{f\log\left(\frac{2c^2e^4x^4+8c^2de^3x^3+2c^2d^4+2\left(6c^2d^2+bc\right)e^2x^2+2bcd^2+4\left(2c^2d^3+bcd\right)ex+b^2-2ac-\left(2ce^2x^2+4cdex+2cd^2+b\right)\sqrt{b^2-4ac}}{ce^4x^4+4cde^3x^3+cd^4+\left(6cd^2+b\right)e^2x^2+bd^2+2\left(2cd^3+bd\right)ex+a}\right)}{2\sqrt{b^2-4ac}\,e}, -\frac{\sqrt{-b^2+4ac}\,f\arctan\left(-\frac{\left(2ce^2x^2+4cdex+2cd^2+b\right)\sqrt{-b^2+4ac}}{b^2-4ac}\right)}{\left(b^2-4ac\right)e}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*f*x+d*f)/(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="fricas")
```

```
[Out] [1/2*f*log((2*c^2*e^4*x^4 + 8*c^2*d*e^3*x^3 + 2*c^2*d^4 + 2*(6*c^2*d^2 + b*
c)*e^2*x^2 + 2*b*c*d^2 + 4*(2*c^2*d^3 + b*c*d)*e*x + b^2 - 2*a*c - (2*c*e^2
*x^2 + 4*c*d*e*x + 2*c*d^2 + b)*sqrt(b^2 - 4*a*c))/(c*e^4*x^4 + 4*c*d*e^3*x
^3 + c*d^4 + (6*c*d^2 + b)*e^2*x^2 + b*d^2 + 2*(2*c*d^3 + b*d)*e*x + a))/(s
qrt(b^2 - 4*a*c)*e), -sqrt(-b^2 + 4*a*c)*f*arctan(-(2*c*e^2*x^2 + 4*c*d*e*x
 + 2*c*d^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c))/((b^2 - 4*a*c)*e)]
```

giac [A] time = 0.40, size = 62, normalized size = 1.41

$$\frac{f\arctan\left(\frac{2\,cd^2f+2\left(fx^2e+2\,dfx\right)ce+bf}{\sqrt{-b^2+4\,ac}\,f}\right)e^{(-1)}}{\sqrt{-b^2+4\,ac}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*f*x+d*f)/(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="giac")
```

```
[Out] f*arctan((2*c*d^2*f + 2*(f*x^2*e + 2*d*f*x)*c*e + b*f)/(sqrt(-b^2 + 4*a*c)*
f))*e^(-1)/sqrt(-b^2 + 4*a*c)
```

maple [C] time = 0.00, size = 130, normalized size = 2.95

$$\frac{f\left(\mathrm{RootOf}\left(\_Z^4ce^4+4\_Z^3cde^3+cd^4+bd^2+\left(6cd^2e^2+be^2\right)\_Z^2+\left(4cd^3e+2deb\right)\_Z+a\right)e+d\right)\ln\left(-\mathrm{RootOf}\left(\_Z^4ce^4+4\_Z^3cde^3+cd^4+bd^2+\left(6cd^2e^2+be^2\right)\_Z^2+\left(4cd^3e+2deb\right)\_Z+a\right)+x\right)}{2e\left(2ce^3\mathrm{RootOf}\left(\_Z^4ce^4+4\_Z^3cde^3+cd^4+bd^2+\left(6cd^2e^2+be^2\right)\_Z^2+\left(4cd^3e+2deb\right)\_Z+a\right)^3+6cde^2\mathrm{RootOf}\left(\_Z^4ce^4+4\_Z^3cde^3+cd^4+bd^2+\left(6cd^2e^2+be^2\right)\_Z^2+\left(4cd^3e+2deb\right)\_Z+a\right)^2+6ecd^2\mathrm{RootOf}\left(\_Z^4ce^4+4\_Z^3cde^3+cd^4+bd^2+\left(6cd^2e^2+be^2\right)\_Z^2+\left(4cd^3e+2deb\right)\_Z+a\right)+2cd^3+be\,\mathrm{RootOf}\left(\_Z^4ce^4+4\_Z^3cde^3+cd^4+bd^2+\left(6cd^2e^2+be^2\right)\_Z^2+\left(4cd^3e+2deb\right)\_Z+a\right)+bd\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*f*x+d*f)/(a+b*(e*x+d)^2+c*(e*x+d)^4),x)
```

```
[Out] 1/2*f/e*sum((_R*e+d)/(2*_R^3*c*e^3+6*_R^2*c*d*e^2+6*_R*c*d^2*e+2*c*d^3+_R*b
*e+b*d)*ln(-_R+x),_R=RootOf(_Z^4*c*e^4+4*_Z^3*c*d*e^3+c*d^4+b*d^2+(6*c*d^2*
e^2+b*e^2)*_Z^2+(4*c*d^3*e+2*b*d*e)*_Z+a))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{efx+df}{(ex+d)^4c+(ex+d)^2b+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*f*x+d*f)/(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="maxima")
```

```
[Out] integrate((e*f*x + d*f)/((e*x + d)^4*c + (e*x + d)^2*b + a), x)
```

mupad [B] time = 1.62, size = 477, normalized size = 10.84

$$\frac{f\,\mathrm{atan}\left(\frac{\frac{f\left(4c^2d^2e^7f+4c^2e^9fx^2-\frac{f\left(8bc^2d^2e^8+16bc^2de^9x+8bc^2e^{10}x^2+16ac^2e^8\right)}{2e\sqrt{b^2-4ac}}+8c^2de^8fx\right)1i}{2e\sqrt{b^2-4ac}}+\frac{f\left(4c^2d^2e^7f+4c^2e^9fx^2+\frac{f\left(8bc^2d^2e^8+16bc^2de^9x+8bc^2e^{10}x^2+16ac^2e^8\right)}{2e\sqrt{b^2-4ac}}+8c^2de^8fx\right)1i}{2e\sqrt{b^2-4ac}}}{\frac{f\left(4c^2d^2e^7f+4c^2e^9fx^2-\frac{f\left(8bc^2d^2e^8+16bc^2de^9x+8bc^2e^{10}x^2+16ac^2e^8\right)}{2e\sqrt{b^2-4ac}}+8c^2de^8fx\right)}{2e\sqrt{b^2-4ac}}-\frac{f\left(4c^2d^2e^7f+4c^2e^9fx^2+\frac{f\left(8bc^2d^2e^8+16bc^2de^9x+8bc^2e^{10}x^2+16ac^2e^8\right)}{2e\sqrt{b^2-4ac}}+8c^2de^8fx\right)}{2e\sqrt{b^2-4ac}}}\right)1i}{e\sqrt{b^2-4ac}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*f + e*f*x)/(a + b*(d + e*x)^2 + c*(d + e*x)^4),x)
```

```
[Out] (f*atan(((f*(4*c^2*d^2*e^7*f + 4*c^2*e^9*f*x^2 - (f*(16*a*c^2*e^8 + 8*b*c^2
*d^2*e^8 + 8*b*c^2*e^10*x^2 + 16*b*c^2*d*e^9*x))/(2*e*(b^2 - 4*a*c)^(1/2))
+ 8*c^2*d*e^8*f*x)*1i)/(2*e*(b^2 - 4*a*c)^(1/2)) + (f*(4*c^2*d^2*e^7*f + 4*
c^2*e^9*f*x^2 + (f*(16*a*c^2*e^8 + 8*b*c^2*d^2*e^8 + 8*b*c^2*e^10*x^2 + 16*
b*c^2*d*e^9*x))/(2*e*(b^2 - 4*a*c)^(1/2)) + 8*c^2*d*e^8*f*x)*1i)/(2*e*(b^2
- 4*a*c)^(1/2)))/((f*(4*c^2*d^2*e^7*f + 4*c^2*e^9*f*x^2 - (f*(16*a*c^2*e^8
+ 8*b*c^2*d^2*e^8 + 8*b*c^2*e^10*x^2 + 16*b*c^2*d*e^9*x))/(2*e*(b^2 - 4*a*c
)^(1/2)) + 8*c^2*d*e^8*f*x))/(2*e*(b^2 - 4*a*c)^(1/2)) - (f*(4*c^2*d^2*e^7*
f + 4*c^2*e^9*f*x^2 + (f*(16*a*c^2*e^8 + 8*b*c^2*d^2*e^8 + 8*b*c^2*e^10*x^2
 + 16*b*c^2*d*e^9*x))/(2*e*(b^2 - 4*a*c)^(1/2)) + 8*c^2*d*e^8*f*x))/(2*e*(b
^2 - 4*a*c)^(1/2))))*1i)/(e*(b^2 - 4*a*c)^(1/2))
```

sympy [B] time = 1.19, size = 189, normalized size = 4.30

$$-\frac{f\sqrt{-\frac{1}{4ac-b^2}}\log\left(\frac{2dx}{e}+x^2+\frac{-4acf\sqrt{-\frac{1}{4ac-b^2}}+b^2f\sqrt{-\frac{1}{4ac-b^2}}+bf+2cd^2f}{2ce^2f}\right)}{2e}+\frac{f\sqrt{-\frac{1}{4ac-b^2}}\log\left(\frac{2dx}{e}+x^2+\frac{4acf\sqrt{-\frac{1}{4ac-b^2}}-b^2f\sqrt{-\frac{1}{4ac-b^2}}+bf+2cd^2f}{2ce^2f}\right)}{2e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*f*x+d*f)/(a+b*(e*x+d)**2+c*(e*x+d)**4),x)
```

```
[Out] -f*sqrt(-1/(4*a*c - b**2))*log(2*d*x/e + x**2 + (-4*a*c*f*sqrt(-1/(4*a*c -
b**2)) + b**2*f*sqrt(-1/(4*a*c - b**2)) + b*f + 2*c*d**2*f)/(2*c*e**2*f))/(
2*e) + f*sqrt(-1/(4*a*c - b**2))*log(2*d*x/e + x**2 + (4*a*c*f*sqrt(-1/(4*a
*c - b**2)) - b**2*f*sqrt(-1/(4*a*c - b**2)) + b*f + 2*c*d**2*f)/(2*c*e**2*
f))/(2*e)
```

3.529 $\int \frac{1}{(df+efx)\left(a+b(d+ex)^2+c(d+ex)^4\right)}\,dx$

Optimal. Leaf size=103

$$\frac{b\tanh^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{2aef\sqrt{b^2-4ac}}-\frac{\log\left(a+b(d+ex)^2+c(d+ex)^4\right)}{4aef}+\frac{\log(d+ex)}{aef}$$

Rubi [A] time = 0.14, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.242, Rules used = {1142, 1114, 705, 29, 634, 618, 206, 628}

$$\frac{b\tanh^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{2aef\sqrt{b^2-4ac}}-\frac{\log\left(a+b(d+ex)^2+c(d+ex)^4\right)}{4aef}+\frac{\log(d+ex)}{aef}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d*f + e*f*x)*(a + b*(d + e*x)^2 + c*(d + e*x)^4)),x]
```

```
[Out] (b*ArcTanh[(b + 2*c*(d + e*x)^2)/Sqrt[b^2 - 4*a*c]])/(2*a*Sqrt[b^2 - 4*a*c]
*e*f) + Log[d + e*x]/(a*e*f) - Log[a + b*(d + e*x)^2 + c*(d + e*x)^4]/(4*a*
e*f)
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 705

```
Int[1/(((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)), x_Symbol]
 :> Dist[e^2/(c*d^2 - b*d*e + a*e^2), Int[1/(d + e*x), x], x] + Dist[1/(c*d
^2 - b*d*e + a*e^2), Int[(c*d - b*e - c*e*x)/(a + b*x + c*x^2), x], x] /; F
reeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^
```

```
2, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 1142

```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Di
st[u^m/(Coefficient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p,
 x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

Rubi steps

$$\int \frac{1}{(df+efx)\left(a+b(d+ex)^2+c(d+ex)^4\right)}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x(a+bx^2+cx^4)}\,dx, x, d+ex\right)}{ef}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{x(a+bx+cx^2)}\,dx, x, (d+ex)^2\right)}{2ef}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{x}\,dx, x, (d+ex)^2\right)}{2aef} + \frac{\text{Subst}\left(\int \frac{-b-cx}{a+bx+cx^2}\,dx, x, (d+e$$

$$= \frac{\log(d+ex)}{aef} - \frac{\text{Subst}\left(\int \frac{b+2cx}{a+bx+cx^2}\,dx, x, (d+ex)^2\right)}{4aef} - \frac{b\,\text{Subst}\left(\right.}{}$$

$$= \frac{\log(d+ex)}{aef} - \frac{\log\left(a+b(d+ex)^2+c(d+ex)^4\right)}{4aef} + \frac{b\,\text{Subst}\left(\right.}{}$$

$$= \frac{b\tanh^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{2a\sqrt{b^2-4ac}\,ef} + \frac{\log(d+ex)}{aef} - \frac{\log\left(a+b(d+ex)^2+\right.}{4aef}$$

Mathematica [A] time = 0.07, size = 131, normalized size = 1.27

$$\frac{4\sqrt{b^2-4ac}\log(d+ex)-\left(\sqrt{b^2-4ac}+b\right)\log\left(-\sqrt{b^2-4ac}+b+2c(d+ex)^2\right)+\left(b-\sqrt{b^2-4ac}\right)\log\left(\sqrt{b^2-4ac}+b+2c(d+ex)^2\right)}{4aef\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d*f + e*f*x)*(a + b*(d + e*x)^2 + c*(d + e*x)^4)),x]
```

```
[Out] (4*Sqrt[b^2 - 4*a*c]*Log[d + e*x] - (b + Sqrt[b^2 - 4*a*c])*Log[b - Sqrt[b^
2 - 4*a*c] + 2*c*(d + e*x)^2] + (b - Sqrt[b^2 - 4*a*c])*Log[b + Sqrt[b^2 -
4*a*c] + 2*c*(d + e*x)^2])/(4*a*Sqrt[b^2 - 4*a*c]*e*f)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(df+efx)\left(a+b(d+ex)^2+c(d+ex)^4\right)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/((d*f + e*f*x)*(a + b*(d + e*x)^2 + c*(d + e*x)^4)),x]
```

```
[Out] IntegrateAlgebraic[1/((d*f + e*f*x)*(a + b*(d + e*x)^2 + c*(d + e*x)^4)), x
]
```

fricas [A] time = 1.18, size = 474, normalized size = 4.60

$$\left[\frac{\sqrt{b^2-4ac}\,b\log\left(\frac{2c^2e^4x^4+8c^2de^3x^3+2c^2d^4+2\left(6c^2d^2+bc\right)e^2x^2+2bcd^2+4\left(2c^2d^3+bcd\right)ex+b^2-2ac+\left(2ce^2x^2+4cdex+2cd^2+b\right)\sqrt{b^2-4ac}}{ce^4x^4+4cde^3x^3+cd^4+\left(6cd^2+b\right)e^2x^2+bd^2+2\left(2cd^3+bd\right)ex+a}\right)-\left(b^2-4ac\right)\log\left(ce^4x^4+4cde^3x^3+cd^4+\left(6cd^2+b\right)e^2x^2+bd^2+2\left(2cd^3+bd\right)ex+a\right)+4\left(b^2-4ac\right)\log\left(ex+d\right)}{4\left(ab^2-4a^2c\right)ef},\ \frac{2\sqrt{-b^2+4ac}\,b\arctan\left(-\frac{\left(2ce^2x^2+4cdex+2cd^2+b\right)\sqrt{-b^2+4ac}}{b^2-4ac}\right)-\left(b^2-4ac\right)\log\left(ce^4x^4+4cde^3x^3+cd^4+\left(6cd^2+b\right)e^2x^2+bd^2+2\left(2cd^3+bd\right)ex+a\right)+4\left(b^2-4ac\right)\log\left(ex+d\right)}{4\left(ab^2-4a^2c\right)ef}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*f*x+d*f)/(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="fricas")
```

```
[Out] [1/4*(sqrt(b^2 - 4*a*c)*b*log((2*c^2*e^4*x^4 + 8*c^2*d*e^3*x^3 + 2*c^2*d^4
+ 2*(6*c^2*d^2 + b*c)*e^2*x^2 + 2*b*c*d^2 + 4*(2*c^2*d^3 + b*c*d)*e*x + b^2
 - 2*a*c + (2*c*e^2*x^2 + 4*c*d*e*x + 2*c*d^2 + b)*sqrt(b^2 - 4*a*c))/(c*e^
4*x^4 + 4*c*d*e^3*x^3 + c*d^4 + (6*c*d^2 + b)*e^2*x^2 + b*d^2 + 2*(2*c*d^3
+ b*d)*e*x + a)) - (b^2 - 4*a*c)*log(c*e^4*x^4 + 4*c*d*e^3*x^3 + c*d^4 + (6
*c*d^2 + b)*e^2*x^2 + b*d^2 + 2*(2*c*d^3 + b*d)*e*x + a) + 4*(b^2 - 4*a*c)*
log(e*x + d))/((a*b^2 - 4*a^2*c)*e*f), 1/4*(2*sqrt(-b^2 + 4*a*c)*b*arctan(-
(2*c*e^2*x^2 + 4*c*d*e*x + 2*c*d^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) -
 (b^2 - 4*a*c)*log(c*e^4*x^4 + 4*c*d*e^3*x^3 + c*d^4 + (6*c*d^2 + b)*e^2*x^
2 + b*d^2 + 2*(2*c*d^3 + b*d)*e*x + a) + 4*(b^2 - 4*a*c)*log(e*x + d))/((a*
b^2 - 4*a^2*c)*e*f)]
```

giac [B] time = 1.19, size = 285, normalized size = 2.77

$$-\frac{e^{(-1)}\log\left(\left|cx^4e^4+4cdx^3e^3+6cd^2x^2e^2+4cd^3xe+cd^4+bx^2e^2+2bdxe+bd^2+a\right|\right)}{4af}+\frac{e^{(-1)}\log\left(\left|xe+d\right|\right)}{af}-\frac{\left(\frac{abcfe^3\log\left(\left|bx^2e^2+2bdxe+\sqrt{b^2-4ac}\,x^2e^2+2\sqrt{b^2-4ac}\,dxe+bd^2+\sqrt{b^2-4ac}\,d^2+2a\right|\right)}{\sqrt{b^2-4ac}}-\frac{abcfe^3\log\left(\left|-bx^2e^2-2bdxe+\sqrt{b^2-4ac}\,x^2e^2+2\sqrt{b^2-4ac}\,dxe-bd^2+\sqrt{b^2-4ac}\,d^2-2a\right|\right)}{\sqrt{b^2-4ac}}\right)e^{(-4)}}{4a^2cf^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*f*x+d*f)/(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="giac")
```

```
[Out] -1/4*e^(-1)*log(abs(c*x^4*e^4 + 4*c*d*x^3*e^3 + 6*c*d^2*x^2*e^2 + 4*c*d^3*x
*e + c*d^4 + b*x^2*e^2 + 2*b*d*x*e + b*d^2 + a))/(a*f) + e^(-1)*log(abs(x*e
 + d))/(a*f) - 1/4*(a*b*c*f*e^3*log(abs(b*x^2*e^2 + 2*b*d*x*e + sqrt(b^2 -
4*a*c)*x^2*e^2 + 2*sqrt(b^2 - 4*a*c)*d*x*e + b*d^2 + sqrt(b^2 - 4*a*c)*d^2
+ 2*a))/sqrt(b^2 - 4*a*c) - a*b*c*f*e^3*log(abs(-b*x^2*e^2 - 2*b*d*x*e + sq
rt(b^2 - 4*a*c)*x^2*e^2 + 2*sqrt(b^2 - 4*a*c)*d*x*e - b*d^2 + sqrt(b^2 - 4*
a*c)*d^2 - 2*a))/sqrt(b^2 - 4*a*c))*e^(-4)/(a^2*c*f^2)
```

maple [C] time = 0.01, size = 190, normalized size = 1.84

$$\frac{\ln\left(ex+d\right)}{aef}+\frac{\left(-ce^3\operatorname{RootOf}\left(\_Z^4ce^4+4\_Z^3cde^3+cd^4+bd^2+\left(6cd^2e^2+be^2\right)\_Z^2+\left(4cd^3e+2deb\right)\_Z+a\right)^3-3cde^2\operatorname{RootOf}\left(\_Z^4ce^4+4\_Z^3cde^3+cd^4+bd^2+\left(6cd^2e^2+be^2\right)\_Z^2+\left(4cd^3e+2deb\right)\_Z+a\right)^2-cd^3+\left(-3cd^2-b\right)\operatorname{RootOf}\left(\_Z^4ce^4+4\_Z^3cde^3+cd^4+bd^2+\left(6cd^2e^2+be^2\right)\_Z^2+\left(4cd^3e+2deb\right)\_Z+a\right)e-bd\right)\ln\left(-\operatorname{RootOf}\left(\_Z^4ce^4+4\_Z^3cde^3+cd^4+bd^2+\left(6cd^2e^2+be^2\right)\_Z^2+\left(4cd^3e+2deb\right)\_Z+a\right)+x\right)}{2aef\left(2ce^3\operatorname{RootOf}\left(\_Z^4ce^4+4\_Z^3cde^3+cd^4+bd^2+\left(6cd^2e^2+be^2\right)\_Z^2+\left(4cd^3e+2deb\right)\_Z+a\right)^3+6cde^2\operatorname{RootOf}\left(\_Z^4ce^4+4\_Z^3cde^3+cd^4+bd^2+\left(6cd^2e^2+be^2\right)\_Z^2+\left(4cd^3e+2deb\right)\_Z+a\right)^2+6ecd^2\operatorname{RootOf}\left(\_Z^4ce^4+4\_Z^3cde^3+cd^4+bd^2+\left(6cd^2e^2+be^2\right)\_Z^2+\left(4cd^3e+2deb\right)\_Z+a\right)+2cd^3+be\operatorname{RootOf}\left(\_Z^4ce^4+4\_Z^3cde^3+cd^4+bd^2+\left(6cd^2e^2+be^2\right)\_Z^2+\left(4cd^3e+2deb\right)\_Z+a\right)+bd\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*f*x+d*f)/(a+b*(e*x+d)^2+c*(e*x+d)^4),x)
```

```
[Out] 1/2/f/a/e*sum((-_R^3*c*e^3-3*_R^2*c*d*e^2-c*d^3+(-3*c*d^2-b)*_R*e-b*d)/(2*_
R^3*c*e^3+6*_R^2*c*d*e^2+6*_R*c*d^2*e+2*c*d^3+_R*b*e+b*d)*ln(-_R+x),_R=Root
Of(_Z^4*c*e^4+4*_Z^3*c*d*e^3+c*d^4+b*d^2+(6*c*d^2*e^2+b*e^2)*_Z^2+(4*c*d^3*
e+2*b*d*e)*_Z+a))+ln(e*x+d)/a/e/f
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*f*x+d*f)/(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [B] time = 3.46, size = 2520, normalized size = 24.47

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((d*f + e*f*x)*(a + b*(d + e*x)^2 + c*(d + e*x)^4)),x)
```

```
[Out] log(d + e*x)/(a*e*f) - (log(a + b*d^2 + c*d^4 + b*e^2*x^2 + c*e^4*x^4 + 2*b
*d*e*x + 6*c*d^2*e^2*x^2 + 4*c*d^3*e*x + 4*c*d*e^3*x^3)*(2*b^2*e*f - 8*a*c*
e*f))/(2*(4*a*b^2*e^2*f^2 - 16*a^2*c*e^2*f^2)) - (b*atan((16*a^3*f^3*x*(4*a
*c - b^2)^(3/2)*(((3*b^3 - 8*a*b*c)*((b^2*((2*(2*b^2*e*f - 8*a*c*e*f)*(6*b^
3*c^2*d*e^18*f - 20*a*b*c^3*d*e^18*f))/(f*(4*a*b^2*e^2*f^2 - 16*a^2*c*e^2*f
^2)) + (20*b*c^3*d*e^17)/f))/(16*a^2*e^2*f^2*(4*a*c - b^2)) - ((2*b^2*e*f -
 8*a*c*e*f)^2*((2*(2*b^2*e*f - 8*a*c*e*f)*(6*b^3*c^2*d*e^18*f - 20*a*b*c^3*
d*e^18*f))/(f*(4*a*b^2*e^2*f^2 - 16*a^2*c*e^2*f^2)) + (20*b*c^3*d*e^17)/f))
/(4*(4*a*b^2*e^2*f^2 - 16*a^2*c*e^2*f^2)^2) + (b^2*(2*b^2*e*f - 8*a*c*e*f)*
(6*b^3*c^2*d*e^18*f - 20*a*b*c^3*d*e^18*f))/(4*a^2*e^2*f^3*(4*a*b^2*e^2*f^2
 - 16*a^2*c*e^2*f^2)*(4*a*c - b^2))))/(8*a^3*c^2*(25*a*c - 6*b^2)) - (((b*(
2*b^2*e*f - 8*a*c*e*f)*((2*(2*b^2*e*f - 8*a*c*e*f)*(6*b^3*c^2*d*e^18*f - 20
*a*b*c^3*d*e^18*f))/(f*(4*a*b^2*e^2*f^2 - 16*a^2*c*e^2*f^2)) + (20*b*c^3*d*
e^17)/f))/(4*a*e*f*(4*a*b^2*e^2*f^2 - 16*a^2*c*e^2*f^2)*(4*a*c - b^2)^(1/2)
) - (b^3*(6*b^3*c^2*d*e^18*f - 20*a*b*c^3*d*e^18*f))/(16*a^3*e^3*f^4*(4*a*c
 - b^2)^(3/2)) + (b*(2*b^2*e*f - 8*a*c*e*f)^2*(6*b^3*c^2*d*e^18*f - 20*a*b*
c^3*d*e^18*f))/(4*a*e*f^2*(4*a*b^2*e^2*f^2 - 16*a^2*c*e^2*f^2)^2*(4*a*c - b
^2)^(1/2)))*(3*b^4 + 10*a^2*c^2 - 14*a*b^2*c))/(8*a^3*c^2*(4*a*c - b^2)^(1/
2)*(25*a*c - 6*b^2))))/(b^2*c^2*e^14) + (2*f^3*(3*b^3 - 8*a*b*c)*(4*a*c - b
^2)^(3/2)*((b^2*((2*(2*b^2*c^2*e^16 + 5*b*c^3*d^2*e^16))/f + ((2*b^2*e*f -
8*a*c*e*f)*(2*a*b^2*c^2*e^17*f + 6*b^3*c^2*d^2*e^17*f - 20*a*b*c^3*d^2*e^17
*f))/(f*(4*a*b^2*e^2*f^2 - 16*a^2*c*e^2*f^2))))/(16*a^2*e^2*f^2*(4*a*c - b^
2)) - ((2*b^2*e*f - 8*a*c*e*f)^2*((2*(2*b^2*c^2*e^16 + 5*b*c^3*d^2*e^16))/f
 + ((2*b^2*e*f - 8*a*c*e*f)*(2*a*b^2*c^2*e^17*f + 6*b^3*c^2*d^2*e^17*f - 20
*a*b*c^3*d^2*e^17*f))/(f*(4*a*b^2*e^2*f^2 - 16*a^2*c*e^2*f^2))))/(4*(4*a*b^
2*e^2*f^2 - 16*a^2*c*e^2*f^2)^2) + (b^2*(2*b^2*e*f - 8*a*c*e*f)*(2*a*b^2*c^
2*e^17*f + 6*b^3*c^2*d^2*e^17*f - 20*a*b*c^3*d^2*e^17*f))/(8*a^2*e^2*f^3*(4
*a*b^2*e^2*f^2 - 16*a^2*c*e^2*f^2)*(4*a*c - b^2))))/(b^2*c^4*e^14*(25*a*c -
 6*b^2)) + (16*a^3*f^3*x^2*(4*a*c - b^2)^(3/2)*(((3*b^3 - 8*a*b*c)*((b^2*((
10*b*c^3*e^18)/f + ((2*b^2*e*f - 8*a*c*e*f)*(6*b^3*c^2*e^19*f - 20*a*b*c^3*
e^19*f))/(f*(4*a*b^2*e^2*f^2 - 16*a^2*c*e^2*f^2))))/(16*a^2*e^2*f^2*(4*a*c
- b^2)) - (((10*b*c^3*e^18)/f + ((2*b^2*e*f - 8*a*c*e*f)*(6*b^3*c^2*e^19*f
- 20*a*b*c^3*e^19*f))/(f*(4*a*b^2*e^2*f^2 - 16*a^2*c*e^2*f^2)))*(2*b^2*e*f
- 8*a*c*e*f)^2)/(4*(4*a*b^2*e^2*f^2 - 16*a^2*c*e^2*f^2)^2) + (b^2*(2*b^2*e*
f - 8*a*c*e*f)*(6*b^3*c^2*e^19*f - 20*a*b*c^3*e^19*f))/(8*a^2*e^2*f^3*(4*a*
b^2*e^2*f^2 - 16*a^2*c*e^2*f^2)*(4*a*c - b^2))))/(8*a^3*c^2*(25*a*c - 6*b^2
)) - ((3*b^4 + 10*a^2*c^2 - 14*a*b^2*c)*((b*((10*b*c^3*e^18)/f + ((2*b^2*e*
f - 8*a*c*e*f)*(6*b^3*c^2*e^19*f - 20*a*b*c^3*e^19*f))/(f*(4*a*b^2*e^2*f^2
- 16*a^2*c*e^2*f^2)))*(2*b^2*e*f - 8*a*c*e*f))/(4*a*e*f*(4*a*b^2*e^2*f^2 -
16*a^2*c*e^2*f^2)*(4*a*c - b^2)^(1/2)) - (b^3*(6*b^3*c^2*e^19*f - 20*a*b*c^
3*e^19*f))/(32*a^3*e^3*f^4*(4*a*c - b^2)^(3/2)) + (b*(2*b^2*e*f - 8*a*c*e*f
)^2*(6*b^3*c^2*e^19*f - 20*a*b*c^3*e^19*f))/(8*a*e*f^2*(4*a*b^2*e^2*f^2 - 1
6*a^2*c*e^2*f^2)^2*(4*a*c - b^2)^(1/2))))/(8*a^3*c^2*(4*a*c - b^2)^(1/2)*(2
5*a*c - 6*b^2))))/(b^2*c^2*e^14) - (2*f^3*(4*a*c - b^2)*(3*b^4 + 10*a^2*c^2
 - 14*a*b^2*c)*((b*(2*b^2*e*f - 8*a*c*e*f)*((2*(2*b^2*c^2*e^16 + 5*b*c^3*d^
2*e^16))/f + ((2*b^2*e*f - 8*a*c*e*f)*(2*a*b^2*c^2*e^17*f + 6*b^3*c^2*d^2*e
^17*f - 20*a*b*c^3*d^2*e^17*f))/(f*(4*a*b^2*e^2*f^2 - 16*a^2*c*e^2*f^2))))/
(4*a*e*f*(4*a*b^2*e^2*f^2 - 16*a^2*c*e^2*f^2)*(4*a*c - b^2)^(1/2)) - (b^3*(
2*a*b^2*c^2*e^17*f + 6*b^3*c^2*d^2*e^17*f - 20*a*b*c^3*d^2*e^17*f))/(32*a^3
*e^3*f^4*(4*a*c - b^2)^(3/2)) + (b*(2*b^2*e*f - 8*a*c*e*f)^2*(2*a*b^2*c^2*e
^17*f + 6*b^3*c^2*d^2*e^17*f - 20*a*b*c^3*d^2*e^17*f))/(8*a*e*f^2*(4*a*b^2*
e^2*f^2 - 16*a^2*c*e^2*f^2)^2*(4*a*c - b^2)^(1/2))))/(b^2*c^4*e^14*(25*a*c
- 6*b^2))))/(2*a*e*f*(4*a*c - b^2)^(1/2))
```

sympy [B] time = 7.16, size = 348, normalized size = 3.38

$$\left(-\frac{b\sqrt{-4ac+b^2}}{4aef\left(4ac-b^2\right)}-\frac{1}{4aef}\right)\log\left(\frac{2dx}{e}+x^2+\frac{-8a^2cef\left(-\frac{b\sqrt{-4ac+b^2}}{4aef(4ac-b^2)}-\frac{1}{4aef}\right)+2ab^2ef\left(-\frac{b\sqrt{-4ac+b^2}}{4aef(4ac-b^2)}-\frac{1}{4aef}\right)-2ac+b^2+bcd^2}{bce^2}\right)+\left(\frac{b\sqrt{-4ac+b^2}}{4aef\left(4ac-b^2\right)}-\frac{1}{4aef}\right)\log\left(\frac{2dx}{e}+x^2+\frac{-8a^2cef\left(\frac{b\sqrt{-4ac+b^2}}{4aef(4ac-b^2)}-\frac{1}{4aef}\right)+2ab^2ef\left(\frac{b\sqrt{-4ac+b^2}}{4aef(4ac-b^2)}-\frac{1}{4aef}\right)-2ac+b^2+bcd^2}{bce^2}\right)+\frac{\log\left(\frac{d}{e}+x\right)}{aef}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*f*x+d*f)/(a+b*(e*x+d)**2+c*(e*x+d)**4),x)
```

```
[Out] (-b*sqrt(-4*a*c + b**2)/(4*a*e*f*(4*a*c - b**2)) - 1/(4*a*e*f))*log(2*d*x/e
 + x**2 + (-8*a**2*c*e*f*(-b*sqrt(-4*a*c + b**2)/(4*a*e*f*(4*a*c - b**2)) -
 1/(4*a*e*f)) + 2*a*b**2*e*f*(-b*sqrt(-4*a*c + b**2)/(4*a*e*f*(4*a*c - b**2
)) - 1/(4*a*e*f)) - 2*a*c + b**2 + b*c*d**2)/(b*c*e**2)) + (b*sqrt(-4*a*c +
 b**2)/(4*a*e*f*(4*a*c - b**2)) - 1/(4*a*e*f))*log(2*d*x/e + x**2 + (-8*a**
2*c*e*f*(b*sqrt(-4*a*c + b**2)/(4*a*e*f*(4*a*c - b**2)) - 1/(4*a*e*f)) + 2*
a*b**2*e*f*(b*sqrt(-4*a*c + b**2)/(4*a*e*f*(4*a*c - b**2)) - 1/(4*a*e*f)) -
 2*a*c + b**2 + b*c*d**2)/(b*c*e**2)) + log(d/e + x)/(a*e*f)
```

3.530 $\int \frac{1}{(df+efx)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)}\,dx$

Optimal. Leaf size=204

$$-\frac{\sqrt{c}\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\,aef^2\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{c}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,(d+ex)}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\,aef^2\sqrt{\sqrt{b^2-4ac}+b}}-\frac{1}{aef^2(d+ex)}$$

Rubi [A] time = 0.27, antiderivative size = 204, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.121, Rules used = {1142, 1123, 1166, 205}

$$-\frac{\sqrt{c}\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\,aef^2\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{c}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,(d+ex)}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\,aef^2\sqrt{\sqrt{b^2-4ac}+b}}-\frac{1}{aef^2(d+ex)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d*f + e*f*x)^2*(a + b*(d + e*x)^2 + c*(d + e*x)^4)),x]
```

```
[Out] -(1/(a*e*f^2*(d + e*x))) - (Sqrt[c]*(1 + b/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[
2]*Sqrt[c]*(d + e*x))/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*a*Sqrt[b - Sqr
t[b^2 - 4*a*c]]*e*f^2) - (Sqrt[c]*(1 - b/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]
*Sqrt[c]*(d + e*x))/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*a*Sqrt[b + Sqrt[
b^2 - 4*a*c]]*e*f^2)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1123

```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*x^2 + c*x^4)^(p + 1))/(a*d*(m + 1)), x] - Dis
t[1/(a*d^2*(m + 1)), Int[(d*x)^(m + 2)*(b*(m + 2*p + 3) + c*(m + 4*p + 5)*x
^2)*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b^2 -
 4*a*c, 0] && LtQ[m, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1142

```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Di
st[u^m/(Coefficient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p,
 x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{1}{(df+efx)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x^2\left(a+bx^2+cx^4\right)}\,dx,x,d+ex\right)}{ef^2}$$
$$= -\frac{1}{aef^2(d+ex)} + \frac{\text{Subst}\left(\int \frac{-b-cx^2}{a+bx^2+cx^4}\,dx,x,d+ex\right)}{aef^2}$$
$$= -\frac{1}{aef^2(d+ex)} - \frac{\left(c\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\right)\text{Subst}\left(\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx\right.}{2aef^2}$$
$$= -\frac{1}{aef^2(d+ex)} - \frac{\sqrt{c}\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\,a\sqrt{b-\sqrt{b^2-4ac}}\,ef^2} - \frac{\sqrt{c}\,(}{}$$

Mathematica [A] time = 0.34, size = 209, normalized size = 1.02

$$-\frac{\frac{\sqrt{2}\sqrt{c}\left(\sqrt{b^2-4ac}+b\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{2}\sqrt{c}\left(\sqrt{b^2-4ac}-b\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4ac}+b}}+\frac{2}{d+ex}}{2aef^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d*f + e*f*x)^2*(a + b*(d + e*x)^2 + c*(d + e*x)^4)),x]
```

```
[Out] -1/2*(2/(d + e*x) + (Sqrt[2]*Sqrt[c]*(b + Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2
]*Sqrt[c]*(d + e*x))/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2 - 4*a*c]*Sqrt[
b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[2]*Sqrt[c]*(-b + Sqrt[b^2 - 4*a*c])*ArcTan[
(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2 - 4*a*c
]*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(a*e*f^2)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(df+efx)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/((d*f + e*f*x)^2*(a + b*(d + e*x)^2 + c*(d + e*x)^4)),
x]
```

```
[Out] IntegrateAlgebraic[1/((d*f + e*f*x)^2*(a + b*(d + e*x)^2 + c*(d + e*x)^4)),
 x]
```

fricas [B] time = 1.45, size = 1477, normalized size = 7.24

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*f*x+d*f)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="fricas"
)
```

```
[Out] 1/2*(sqrt(1/2)*(a*e^2*f^2*x + a*d*e*f^2)*sqrt(-((a^3*b^2 - 4*a^4*c)*e^2*f^4
*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/((a^6*b^2 - 4*a^7*c)*e^4*f^8)) + b^3 - 3*
a*b*c)/((a^3*b^2 - 4*a^4*c)*e^2*f^4))*log(-2*(b^2*c^2 - a*c^3)*e*x - 2*(b^2
*c^2 - a*c^3)*d + sqrt(1/2)*((a^3*b^4 - 6*a^4*b^2*c + 8*a^5*c^2)*e^3*f^6*sq
```

```
rt((b^4 - 2*a*b^2*c + a^2*c^2)/((a^6*b^2 - 4*a^7*c)*e^4*f^8)) - (b^5 - 5*a*
b^3*c + 4*a^2*b*c^2)*e*f^2)*sqrt(-((a^3*b^2 - 4*a^4*c)*e^2*f^4*sqrt((b^4 -
2*a*b^2*c + a^2*c^2)/((a^6*b^2 - 4*a^7*c)*e^4*f^8)) + b^3 - 3*a*b*c)/((a^3*
b^2 - 4*a^4*c)*e^2*f^4))) - sqrt(1/2)*(a*e^2*f^2*x + a*d*e*f^2)*sqrt(-((a^3
*b^2 - 4*a^4*c)*e^2*f^4*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/((a^6*b^2 - 4*a^7*
c)*e^4*f^8)) + b^3 - 3*a*b*c)/((a^3*b^2 - 4*a^4*c)*e^2*f^4))*log(-2*(b^2*c^
2 - a*c^3)*e*x - 2*(b^2*c^2 - a*c^3)*d - sqrt(1/2)*((a^3*b^4 - 6*a^4*b^2*c
+ 8*a^5*c^2)*e^3*f^6*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/((a^6*b^2 - 4*a^7*c)*
e^4*f^8)) - (b^5 - 5*a*b^3*c + 4*a^2*b*c^2)*e*f^2)*sqrt(-((a^3*b^2 - 4*a^4*
c)*e^2*f^4*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/((a^6*b^2 - 4*a^7*c)*e^4*f^8))
+ b^3 - 3*a*b*c)/((a^3*b^2 - 4*a^4*c)*e^2*f^4))) - sqrt(1/2)*(a*e^2*f^2*x +
 a*d*e*f^2)*sqrt(((a^3*b^2 - 4*a^4*c)*e^2*f^4*sqrt((b^4 - 2*a*b^2*c + a^2*c
^2)/((a^6*b^2 - 4*a^7*c)*e^4*f^8)) - b^3 + 3*a*b*c)/((a^3*b^2 - 4*a^4*c)*e^
2*f^4))*log(-2*(b^2*c^2 - a*c^3)*e*x - 2*(b^2*c^2 - a*c^3)*d + sqrt(1/2)*((
a^3*b^4 - 6*a^4*b^2*c + 8*a^5*c^2)*e^3*f^6*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)
/((a^6*b^2 - 4*a^7*c)*e^4*f^8)) + (b^5 - 5*a*b^3*c + 4*a^2*b*c^2)*e*f^2)*sq
rt(((a^3*b^2 - 4*a^4*c)*e^2*f^4*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/((a^6*b^2
- 4*a^7*c)*e^4*f^8)) - b^3 + 3*a*b*c)/((a^3*b^2 - 4*a^4*c)*e^2*f^4))) + sqr
t(1/2)*(a*e^2*f^2*x + a*d*e*f^2)*sqrt(((a^3*b^2 - 4*a^4*c)*e^2*f^4*sqrt((b^
4 - 2*a*b^2*c + a^2*c^2)/((a^6*b^2 - 4*a^7*c)*e^4*f^8)) - b^3 + 3*a*b*c)/((
a^3*b^2 - 4*a^4*c)*e^2*f^4))*log(-2*(b^2*c^2 - a*c^3)*e*x - 2*(b^2*c^2 - a*
c^3)*d - sqrt(1/2)*((a^3*b^4 - 6*a^4*b^2*c + 8*a^5*c^2)*e^3*f^6*sqrt((b^4 -
 2*a*b^2*c + a^2*c^2)/((a^6*b^2 - 4*a^7*c)*e^4*f^8)) + (b^5 - 5*a*b^3*c + 4
*a^2*b*c^2)*e*f^2)*sqrt(((a^3*b^2 - 4*a^4*c)*e^2*f^4*sqrt((b^4 - 2*a*b^2*c
+ a^2*c^2)/((a^6*b^2 - 4*a^7*c)*e^4*f^8)) - b^3 + 3*a*b*c)/((a^3*b^2 - 4*a^
4*c)*e^2*f^4))) - 2)/(a*e^2*f^2*x + a*d*e*f^2)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*f*x+d*f)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Error index.cc index_gcd Error: Bad Argument
 ValueError index.cc index_gcd Error: Bad Argument ValueDone
```

maple [C] time = 0.01, size = 174, normalized size = 0.85

$$\frac{\left(-\mathrm{RootOf}\left(\_Z^4 c e^4+4\_Z^3 c d e^3+c d^4+b d^2+\left(6 c d^2 e^2+b e^2\right)\_Z^2+\left(4 c d^3 e+2 d e b\right)\_Z+a\right)^2 c e^2-2\,\mathrm{RootOf}\left(\_Z^4 c e^4+4\_Z^3 c d e^3+c d^4+b d^2+\left(6 c d^2 e^2+b e^2\right)\_Z^2+\left(4 c d^3 e+2 d e b\right)\_Z+a\right) c d e-c d^2-b\right)\ln\left(-\mathrm{RootOf}\left(\_Z^4 c e^4+4\_Z^3 c d e^3+c d^4+b d^2+\left(6 c d^2 e^2+b e^2\right)\_Z^2+\left(4 c d^3 e+2 d e b\right)\_Z+a\right)+x\right)}{2 a e f^2\left(2 c e^3\,\mathrm{RootOf}\left(\_Z^4 c e^4+4\_Z^3 c d e^3+c d^4+b d^2+\left(6 c d^2 e^2+b e^2\right)\_Z^2+\left(4 c d^3 e+2 d e b\right)\_Z+a\right)^3+6 c d e^2\,\mathrm{RootOf}\left(\_Z^4 c e^4+4\_Z^3 c d e^3+c d^4+b d^2+\left(6 c d^2 e^2+b e^2\right)\_Z^2+\left(4 c d^3 e+2 d e b\right)\_Z+a\right)^2+6 e c d^2\,\mathrm{RootOf}\left(\_Z^4 c e^4+4\_Z^3 c d e^3+c d^4+b d^2+\left(6 c d^2 e^2+b e^2\right)\_Z^2+\left(4 c d^3 e+2 d e b\right)\_Z+a\right)+2 c d^3+b e\,\mathrm{RootOf}\left(\_Z^4 c e^4+4\_Z^3 c d e^3+c d^4+b d^2+\left(6 c d^2 e^2+b e^2\right)\_Z^2+\left(4 c d^3 e+2 d e b\right)\_Z+a\right)+b d\right)}-\frac{1}{(e x+d) a e f^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*f*x+d*f)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4),x)
```

```
[Out] 1/2/f^2/a/e*sum((-_R^2*c*e^2-2*_R*c*d*e-c*d^2-b)/(2*_R^3*c*e^3+6*_R^2*c*d*e
^2+6*_R*c*d^2*e+2*c*d^3+_R*b*e+b*d)*ln(-_R+x),_R=RootOf(_Z^4*c*e^4+4*_Z^3*c
*d*e^3+c*d^4+b*d^2+(6*c*d^2*e^2+b*e^2)*_Z^2+(4*c*d^3*e+2*b*d*e)*_Z+a))-1/a/
e/f^2/(e*x+d)
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*f*x+d*f)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="maxima"
)
```

```
[Out] Timed out
```

mupad [B] time = 3.87, size = 4339, normalized size = 21.27

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((d*f + e*f*x)^2*(a + b*(d + e*x)^2 + c*(d + e*x)^4)),x)
```

```
[Out] - atan(((-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c -
a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4*e^2*f^4 + 16*a^5*c^2*e^2*f^4 - 8*
a^4*b^2*c*e^2*f^4)))^(1/2)*(x*(4*a^4*c^4*e^12*f^6 - 2*a^3*b^2*c^3*e^12*f^6)
 - ((x*(8*a^5*b^3*c^2*e^14*f^10 - 32*a^6*b*c^3*e^14*f^10) - 32*a^6*b*c^3*d*
e^13*f^10 + 8*a^5*b^3*c^2*d*e^13*f^10)*(-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2
) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4*e^
2*f^4 + 16*a^5*c^2*e^2*f^4 - 8*a^4*b^2*c*e^2*f^4)))^(1/2) - 4*a^4*b^3*c^2*e
^12*f^8 + 16*a^5*b*c^3*e^12*f^8)*(-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12
*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4*e^2*f^4
+ 16*a^5*c^2*e^2*f^4 - 8*a^4*b^2*c*e^2*f^4)))^(1/2) + 4*a^4*c^4*d*e^11*f^6
- 2*a^3*b^2*c^3*d*e^11*f^6)*1i + (-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12
*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4*e^2*f^4
+ 16*a^5*c^2*e^2*f^4 - 8*a^4*b^2*c*e^2*f^4)))^(1/2)*(x*(4*a^4*c^4*e^12*f^6
- 2*a^3*b^2*c^3*e^12*f^6) - ((x*(8*a^5*b^3*c^2*e^14*f^10 - 32*a^6*b*c^3*e^1
4*f^10) - 32*a^6*b*c^3*d*e^13*f^10 + 8*a^5*b^3*c^2*d*e^13*f^10)*(-(b^5 + b^
2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)
^3)^(1/2))/(8*(a^3*b^4*e^2*f^4 + 16*a^5*c^2*e^2*f^4 - 8*a^4*b^2*c*e^2*f^4))
)^(1/2) + 4*a^4*b^3*c^2*e^12*f^8 - 16*a^5*b*c^3*e^12*f^8)*(-(b^5 + b^2*(-(4
*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1
/2))/(8*(a^3*b^4*e^2*f^4 + 16*a^5*c^2*e^2*f^4 - 8*a^4*b^2*c*e^2*f^4)))^(1/2
) + 4*a^4*c^4*d*e^11*f^6 - 2*a^3*b^2*c^3*d*e^11*f^6)*1i)/((-(b^5 + b^2*(-(4
*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1
/2))/(8*(a^3*b^4*e^2*f^4 + 16*a^5*c^2*e^2*f^4 - 8*a^4*b^2*c*e^2*f^4)))^(1/2
)*(x*(4*a^4*c^4*e^12*f^6 - 2*a^3*b^2*c^3*e^12*f^6) - ((x*(8*a^5*b^3*c^2*e^1
4*f^10 - 32*a^6*b*c^3*e^14*f^10) - 32*a^6*b*c^3*d*e^13*f^10 + 8*a^5*b^3*c^2
*d*e^13*f^10)*(-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^
3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4*e^2*f^4 + 16*a^5*c^2*e^2*f^
4 - 8*a^4*b^2*c*e^2*f^4)))^(1/2) + 4*a^4*b^3*c^2*e^12*f^8 - 16*a^5*b*c^3*e^
12*f^8)*(-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c -
a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4*e^2*f^4 + 16*a^5*c^2*e^2*f^4 - 8*
a^4*b^2*c*e^2*f^4)))^(1/2) + 4*a^4*c^4*d*e^11*f^6 - 2*a^3*b^2*c^3*d*e^11*f^
6) - (-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c
*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4*e^2*f^4 + 16*a^5*c^2*e^2*f^4 - 8*a^4
*b^2*c*e^2*f^4)))^(1/2)*(x*(4*a^4*c^4*e^12*f^6 - 2*a^3*b^2*c^3*e^12*f^6) -
((x*(8*a^5*b^3*c^2*e^14*f^10 - 32*a^6*b*c^3*e^14*f^10) - 32*a^6*b*c^3*d*e^1
3*f^10 + 8*a^5*b^3*c^2*d*e^13*f^10)*(-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) +
 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4*e^2*f
^4 + 16*a^5*c^2*e^2*f^4 - 8*a^4*b^2*c*e^2*f^4)))^(1/2) - 4*a^4*b^3*c^2*e^12
*f^8 + 16*a^5*b*c^3*e^12*f^8)*(-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^
2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4*e^2*f^4 + 1
6*a^5*c^2*e^2*f^4 - 8*a^4*b^2*c*e^2*f^4)))^(1/2) + 4*a^4*c^4*d*e^11*f^6 - 2
*a^3*b^2*c^3*d*e^11*f^6) + 2*a^3*c^4*e^10*f^4))*(-(b^5 + b^2*(-(4*a*c - b^2
)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a
^3*b^4*e^2*f^4 + 16*a^5*c^2*e^2*f^4 - 8*a^4*b^2*c*e^2*f^4)))^(1/2)*2i - ata
n(((-(b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(
-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4*e^2*f^4 + 16*a^5*c^2*e^2*f^4 - 8*a^4*b
^2*c*e^2*f^4)))^(1/2)*(x*(4*a^4*c^4*e^12*f^6 - 2*a^3*b^2*c^3*e^12*f^6) - ((
x*(8*a^5*b^3*c^2*e^14*f^10 - 32*a^6*b*c^3*e^14*f^10) - 32*a^6*b*c^3*d*e^13*
f^10 + 8*a^5*b^3*c^2*d*e^13*f^10)*(-(b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 1
2*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4*e^2*f^4
 + 16*a^5*c^2*e^2*f^4 - 8*a^4*b^2*c*e^2*f^4)))^(1/2) - 4*a^4*b^3*c^2*e^12*f
^8 + 16*a^5*b*c^3*e^12*f^8)*(-(b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*
b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4*e^2*f^4 + 16*
```

```
a^5*c^2*e^2*f^4 - 8*a^4*b^2*c*e^2*f^4)))^(1/2) + 4*a^4*c^4*d*e^11*f^6 - 2*a
^3*b^2*c^3*d*e^11*f^6)*1i + (-(b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*
b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4*e^2*f^4 + 16*
a^5*c^2*e^2*f^4 - 8*a^4*b^2*c*e^2*f^4)))^(1/2)*(x*(4*a^4*c^4*e^12*f^6 - 2*a
^3*b^2*c^3*e^12*f^6) - ((x*(8*a^5*b^3*c^2*e^14*f^10 - 32*a^6*b*c^3*e^14*f^1
0) - 32*a^6*b*c^3*d*e^13*f^10 + 8*a^5*b^3*c^2*d*e^13*f^10)*(-(b^5 - b^2*(-(
4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(
1/2))/(8*(a^3*b^4*e^2*f^4 + 16*a^5*c^2*e^2*f^4 - 8*a^4*b^2*c*e^2*f^4)))^(1/
2) + 4*a^4*b^3*c^2*e^12*f^8 - 16*a^5*b*c^3*e^12*f^8)*(-(b^5 - b^2*(-(4*a*c
- b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2))/
(8*(a^3*b^4*e^2*f^4 + 16*a^5*c^2*e^2*f^4 - 8*a^4*b^2*c*e^2*f^4)))^(1/2) + 4
*a^4*c^4*d*e^11*f^6 - 2*a^3*b^2*c^3*d*e^11*f^6)*1i)/((-(b^5 - b^2*(-(4*a*c
- b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2))/
(8*(a^3*b^4*e^2*f^4 + 16*a^5*c^2*e^2*f^4 - 8*a^4*b^2*c*e^2*f^4)))^(1/2)*(x*
(4*a^4*c^4*e^12*f^6 - 2*a^3*b^2*c^3*e^12*f^6) - ((x*(8*a^5*b^3*c^2*e^14*f^1
0 - 32*a^6*b*c^3*e^14*f^10) - 32*a^6*b*c^3*d*e^13*f^10 + 8*a^5*b^3*c^2*d*e^
13*f^10)*(-(b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c +
 a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4*e^2*f^4 + 16*a^5*c^2*e^2*f^4 - 8
*a^4*b^2*c*e^2*f^4)))^(1/2) + 4*a^4*b^3*c^2*e^12*f^8 - 16*a^5*b*c^3*e^12*f^
8)*(-(b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(
-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4*e^2*f^4 + 16*a^5*c^2*e^2*f^4 - 8*a^4*b
^2*c*e^2*f^4)))^(1/2) + 4*a^4*c^4*d*e^11*f^6 - 2*a^3*b^2*c^3*d*e^11*f^6) -
(-(b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4
*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4*e^2*f^4 + 16*a^5*c^2*e^2*f^4 - 8*a^4*b^2*
c*e^2*f^4)))^(1/2)*(x*(4*a^4*c^4*e^12*f^6 - 2*a^3*b^2*c^3*e^12*f^6) - ((x*(
8*a^5*b^3*c^2*e^14*f^10 - 32*a^6*b*c^3*e^14*f^10) - 32*a^6*b*c^3*d*e^13*f^1
0 + 8*a^5*b^3*c^2*d*e^13*f^10)*(-(b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a
^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4*e^2*f^4 +
16*a^5*c^2*e^2*f^4 - 8*a^4*b^2*c*e^2*f^4)))^(1/2) - 4*a^4*b^3*c^2*e^12*f^8
+ 16*a^5*b*c^3*e^12*f^8)*(-(b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c
^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4*e^2*f^4 + 16*a^5
*c^2*e^2*f^4 - 8*a^4*b^2*c*e^2*f^4)))^(1/2) + 4*a^4*c^4*d*e^11*f^6 - 2*a^3*
b^2*c^3*d*e^11*f^6) + 2*a^3*c^4*e^10*f^4))*(-(b^5 - b^2*(-(4*a*c - b^2)^3)^
(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^
4*e^2*f^4 + 16*a^5*c^2*e^2*f^4 - 8*a^4*b^2*c*e^2*f^4)))^(1/2)*2i - 1/(a*e*(
d*f^2 + e*f^2*x))
```

sympy [A] time = 4.90, size = 258, normalized size = 1.26

$$\operatorname{RootSum}\left(t^4\left(256a^5c^2e^4f^8-128a^4b^2ce^4f^8+16a^3b^4e^4f^8\right)+t^2\left(48a^2bc^2e^2f^4-28ab^3ce^2f^4+4b^5e^2f^4\right)+c^3,\left(t\mapsto t\log\left(x+\frac{-64t^3a^5c^2e^3f^6+48t^3a^4b^2ce^3f^6-8t^3a^3b^4e^3f^6-10ta^2bc^2ef^2+10tab^3cef^2-2tb^5ef^2+ac^3d-b^2c^2d}{ac^3e-b^2c^2e}\right)\right)\right)-\frac{1}{adef^2+ae^2f^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*f*x+d*f)**2/(a+b*(e*x+d)**2+c*(e*x+d)**4),x)
```

```
[Out] RootSum(_t**4*(256*a**5*c**2*e**4*f**8 - 128*a**4*b**2*c*e**4*f**8 + 16*a**
3*b**4*e**4*f**8) + _t**2*(48*a**2*b*c**2*e**2*f**4 - 28*a*b**3*c*e**2*f**4
 + 4*b**5*e**2*f**4) + c**3, Lambda(_t, _t*log(x + (-64*_t**3*a**5*c**2*e**
3*f**6 + 48*_t**3*a**4*b**2*c*e**3*f**6 - 8*_t**3*a**3*b**4*e**3*f**6 - 10*
_t*a**2*b*c**2*e*f**2 + 10*_t*a*b**3*c*e*f**2 - 2*_t*b**5*e*f**2 + a*c**3*d
 - b**2*c**2*d)/(a*c**3*e - b**2*c**2*e)))) - 1/(a*d*e*f**2 + a*e**2*f**2*x
)
```

3.531 $\int \frac{1}{(df+efx)^3\left(a+b(d+ex)^2+c(d+ex)^4\right)}\,dx$

Optimal. Leaf size=133

$$-\frac{\left(b^2-2ac\right)\tanh^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{2a^2ef^3\sqrt{b^2-4ac}}+\frac{b\log\left(a+b(d+ex)^2+c(d+ex)^4\right)}{4a^2ef^3}-\frac{b\log(d+ex)}{a^2ef^3}-\frac{1}{2aef^3(d+ex)^2}$$

Rubi [A] time = 0.20, antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.242, Rules used = {1142, 1114, 709, 800, 634, 618, 206, 628}

$$-\frac{\left(b^2-2ac\right)\tanh^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{2a^2ef^3\sqrt{b^2-4ac}}+\frac{b\log\left(a+b(d+ex)^2+c(d+ex)^4\right)}{4a^2ef^3}-\frac{b\log(d+ex)}{a^2ef^3}-\frac{1}{2aef^3(d+ex)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d*f + e*f*x)^3*(a + b*(d + e*x)^2 + c*(d + e*x)^4)),x]
```

```
[Out] -1/(2*a*e*f^3*(d + e*x)^2) - ((b^2 - 2*a*c)*ArcTanh[(b + 2*c*(d + e*x)^2)/S
qrt[b^2 - 4*a*c]])/(2*a^2*Sqrt[b^2 - 4*a*c]*e*f^3) - (b*Log[d + e*x])/(a^2*
e*f^3) + (b*Log[a + b*(d + e*x)^2 + c*(d + e*x)^4])/(4*a^2*e*f^3)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 709

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Simp[(e*(d + e*x)^(m + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dis
t[1/(c*d^2 - b*d*e + a*e^2), Int[((d + e*x)^(m + 1)*Simp[c*d - b*e - c*e*x,
 x])/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 -
 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[m
, -1]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 1142

```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Di
st[u^m/(Coefficient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p,
 x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

Rubi steps

$$\int \frac{1}{(df+efx)^3\left(a+b(d+ex)^2+c(d+ex)^4\right)}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x^3\left(a+bx^2+cx^4\right)}\,dx, x, d+ex\right)}{ef^3}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{x^2\left(a+bx+cx^2\right)}\,dx, x, (d+ex)^2\right)}{2ef^3}$$

$$= -\frac{1}{2aef^3(d+ex)^2} + \frac{\text{Subst}\left(\int \frac{-b-cx}{x\left(a+bx+cx^2\right)}\,dx, x, (d+ex)^2\right)}{2aef^3}$$

$$= -\frac{1}{2aef^3(d+ex)^2} + \frac{\text{Subst}\left(\int \left(-\frac{b}{ax} + \frac{b^2-ac+bcx}{a\left(a+bx+cx^2\right)}\right) dx, x, (d+\right.}{2aef^3}$$

$$= -\frac{1}{2aef^3(d+ex)^2} - \frac{b\log(d+ex)}{a^2ef^3} + \frac{\text{Subst}\left(\int \frac{b^2-ac+bcx}{a+bx+cx^2}\,dx, x\right.}{2a^2ef^3}$$

$$= -\frac{1}{2aef^3(d+ex)^2} - \frac{b\log(d+ex)}{a^2ef^3} + \frac{b\,\text{Subst}\left(\int \frac{b+2cx}{a+bx+cx^2}\,dx,\right.}{4a^2ef^3}$$

$$= -\frac{1}{2aef^3(d+ex)^2} - \frac{b\log(d+ex)}{a^2ef^3} + \frac{b\log\left(a+b(d+ex)^2+\right.}{4a^2ef^3}$$

$$= -\frac{1}{2aef^3(d+ex)^2} - \frac{\left(b^2-2ac\right)\tanh^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{2a^2\sqrt{b^2-4ac}\,ef^3} - \frac{b\log(}{a^2e}$$

Mathematica [A] time = 0.14, size = 157, normalized size = 1.18

$$\frac{\frac{\left(b\sqrt{b^2-4ac}-2ac+b^2\right)\log\left(-\sqrt{b^2-4ac}+b+2c(d+ex)^2\right)}{\sqrt{b^2-4ac}} + \frac{\left(b\sqrt{b^2-4ac}+2ac-b^2\right)\log\left(\sqrt{b^2-4ac}+b+2c(d+ex)^2\right)}{\sqrt{b^2-4ac}} - \frac{2a}{(d+ex)^2} - 4b\log(d+ex)}{4a^2ef^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d*f + e*f*x)^3*(a + b*(d + e*x)^2 + c*(d + e*x)^4)),x]
```

```
[Out] ((-2*a)/(d + e*x)^2 - 4*b*Log[d + e*x] + ((b^2 - 2*a*c + b*Sqrt[b^2 - 4*a*c
])*Log[b - Sqrt[b^2 - 4*a*c] + 2*c*(d + e*x)^2])/Sqrt[b^2 - 4*a*c] + ((-b^2
 + 2*a*c + b*Sqrt[b^2 - 4*a*c])*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*(d + e*x)^2
])/Sqrt[b^2 - 4*a*c])/(4*a^2*e*f^3)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(df+efx)^3\left(a+b(d+ex)^2+c(d+ex)^4\right)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/((d*f + e*f*x)^3*(a + b*(d + e*x)^2 + c*(d + e*x)^4)),
x]
```

```
[Out] IntegrateAlgebraic[1/((d*f + e*f*x)^3*(a + b*(d + e*x)^2 + c*(d + e*x)^4)),
 x]
```

fricas [B] time = 1.93, size = 828, normalized size = 6.23

$$\left[-\frac{2ab^2-8a^2c+\left((b^2-2ac)e^2x^2+2(b^2-2ac)dex+(b^2-2ac)d^2\right)\sqrt{b^2-4ac}\log\left(\frac{2c^2e^4x^4+8c^2de^3x^3+2c^2d^4+2(6c^2d^2+bc)e^2x^2+2bcd^2+4(2c^2d^3+bcd)ex+b^2-2ac+(2ce^2x^2+4cdex+2cd^2+b)\sqrt{b^2-4ac}}{ce^4x^4+4cde^3x^3+cd^4+(6cd^2+b)e^2x^2+bd^2+2(2cd^3+bd)ex+a}\right)-\left((b^3-4abc)e^2x^2+2(b^3-4abc)dex+(b^3-4abc)d^2\right)\log\left(ce^4x^4+4cde^3x^3+cd^4+(6cd^2+b)e^2x^2+bd^2+2(2cd^3+bd)ex+a\right)+4\left((b^3-4abc)e^2x^2+2(b^3-4abc)dex+(b^3-4abc)d^2\right)\log(ex+d)}{4\left((a^2b^2-4a^3c)e^3f^3x^2+2(a^2b^2-4a^3c)de^2f^3x+(a^2b^2-4a^3c)d^2ef^3\right)},\ -\frac{2ab^2-8a^2c+2\left((b^2-2ac)e^2x^2+2(b^2-2ac)dex+(b^2-2ac)d^2\right)\sqrt{-b^2+4ac}\arctan\left(-\frac{(2ce^2x^2+4cdex+2cd^2+b)\sqrt{-b^2+4ac}}{b^2-4ac}\right)-\left((b^3-4abc)e^2x^2+2(b^3-4abc)dex+(b^3-4abc)d^2\right)\log\left(ce^4x^4+4cde^3x^3+cd^4+(6cd^2+b)e^2x^2+bd^2+2(2cd^3+bd)ex+a\right)+4\left((b^3-4abc)e^2x^2+2(b^3-4abc)dex+(b^3-4abc)d^2\right)\log(ex+d)}{4\left((a^2b^2-4a^3c)e^3f^3x^2+2(a^2b^2-4a^3c)de^2f^3x+(a^2b^2-4a^3c)d^2ef^3\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*f*x+d*f)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="fricas"
)
```

```
[Out] [-1/4*(2*a*b^2 - 8*a^2*c + ((b^2 - 2*a*c)*e^2*x^2 + 2*(b^2 - 2*a*c)*d*e*x +
 (b^2 - 2*a*c)*d^2)*sqrt(b^2 - 4*a*c)*log((2*c^2*e^4*x^4 + 8*c^2*d*e^3*x^3
+ 2*c^2*d^4 + 2*(6*c^2*d^2 + b*c)*e^2*x^2 + 2*b*c*d^2 + 4*(2*c^2*d^3 + b*c*
d)*e*x + b^2 - 2*a*c + (2*c*e^2*x^2 + 4*c*d*e*x + 2*c*d^2 + b)*sqrt(b^2 - 4
*a*c))/(c*e^4*x^4 + 4*c*d*e^3*x^3 + c*d^4 + (6*c*d^2 + b)*e^2*x^2 + b*d^2 +
 2*(2*c*d^3 + b*d)*e*x + a)) - ((b^3 - 4*a*b*c)*e^2*x^2 + 2*(b^3 - 4*a*b*c)
*d*e*x + (b^3 - 4*a*b*c)*d^2)*log(c*e^4*x^4 + 4*c*d*e^3*x^3 + c*d^4 + (6*c*
d^2 + b)*e^2*x^2 + b*d^2 + 2*(2*c*d^3 + b*d)*e*x + a) + 4*((b^3 - 4*a*b*c)*
e^2*x^2 + 2*(b^3 - 4*a*b*c)*d*e*x + (b^3 - 4*a*b*c)*d^2)*log(e*x + d))/((a^
2*b^2 - 4*a^3*c)*e^3*f^3*x^2 + 2*(a^2*b^2 - 4*a^3*c)*d*e^2*f^3*x + (a^2*b^2
 - 4*a^3*c)*d^2*e*f^3), -1/4*(2*a*b^2 - 8*a^2*c + 2*((b^2 - 2*a*c)*e^2*x^2
+ 2*(b^2 - 2*a*c)*d*e*x + (b^2 - 2*a*c)*d^2)*sqrt(-b^2 + 4*a*c)*arctan(-(2*
c*e^2*x^2 + 4*c*d*e*x + 2*c*d^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) - ((
b^3 - 4*a*b*c)*e^2*x^2 + 2*(b^3 - 4*a*b*c)*d*e*x + (b^3 - 4*a*b*c)*d^2)*log
(c*e^4*x^4 + 4*c*d*e^3*x^3 + c*d^4 + (6*c*d^2 + b)*e^2*x^2 + b*d^2 + 2*(2*c
*d^3 + b*d)*e*x + a) + 4*((b^3 - 4*a*b*c)*e^2*x^2 + 2*(b^3 - 4*a*b*c)*d*e*x
 + (b^3 - 4*a*b*c)*d^2)*log(e*x + d))/((a^2*b^2 - 4*a^3*c)*e^3*f^3*x^2 + 2*
(a^2*b^2 - 4*a^3*c)*d*e^2*f^3*x + (a^2*b^2 - 4*a^3*c)*d^2*e*f^3)]
```

giac [B] time = 1.11, size = 348, normalized size = 2.62

$$\frac{be^{(-1)}\log\left(\left|cx^4e^4+4cdx^3e^3+6cd^2x^2e^2+4cd^3xe+cd^4+bx^2e^2+2bdxe+bd^2+a\right|\right)}{4a^2f^3}-\frac{be^{(-1)}\log\left(|xe+d|\right)}{a^2f^3}-\frac{e^{(-1)}}{2(xe+d)^2af^3}+\frac{\left(\frac{(a^2b^2cf^3e^3-2a^3c^2f^3e^3)\log\left(\left|bx^2e^2+2bdxe+\sqrt{b^2-4ac}\,x^2e^2+2\sqrt{b^2-4ac}\,dxe+bd^2+\sqrt{b^2-4ac}\,d^2+2a\right|\right)}{\sqrt{b^2-4ac}}-\frac{(a^2b^2cf^3e^3-2a^3c^2f^3e^3)\log\left(\left|-bx^2e^2-2bdxe+\sqrt{b^2-4ac}\,x^2e^2+2\sqrt{b^2-4ac}\,dxe-bd^2+\sqrt{b^2-4ac}\,d^2-2a\right|\right)}{\sqrt{b^2-4ac}}\right)e^{(-4)}}{4a^4cf^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*f*x+d*f)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="giac")
```

```
[Out] 1/4*b*e^(-1)*log(abs(c*x^4*e^4 + 4*c*d*x^3*e^3 + 6*c*d^2*x^2*e^2 + 4*c*d^3*
x*e + c*d^4 + b*x^2*e^2 + 2*b*d*x*e + b*d^2 + a))/(a^2*f^3) - b*e^(-1)*log(
abs(x*e + d))/(a^2*f^3) - 1/2*e^(-1)/((x*e + d)^2*a*f^3) + 1/4*((a^2*b^2*c*
f^3*e^3 - 2*a^3*c^2*f^3*e^3)*log(abs(b*x^2*e^2 + 2*b*d*x*e + sqrt(b^2 - 4*a
*c)*x^2*e^2 + 2*sqrt(b^2 - 4*a*c)*d*x*e + b*d^2 + sqrt(b^2 - 4*a*c)*d^2 + 2
*a))/sqrt(b^2 - 4*a*c) - (a^2*b^2*c*f^3*e^3 - 2*a^3*c^2*f^3*e^3)*log(abs(-b
*x^2*e^2 - 2*b*d*x*e + sqrt(b^2 - 4*a*c)*x^2*e^2 + 2*sqrt(b^2 - 4*a*c)*d*x*
```

```
e - b*d^2 + sqrt(b^2 - 4*a*c)*d^2 - 2*a))/sqrt(b^2 - 4*a*c))*e^(-4)/(a^4*c*
f^6)
```

maple [C] time = 0.01, size = 222, normalized size = 1.67

$$-\frac{b\ln\left(ex+d\right)}{a^{2}e\,f^{3}}-\frac{1}{2\left(ex+d\right)^{2}ae\,f^{3}}+\frac{\left(\operatorname{RootOf}\left(\_Z^{4}c\,e^{4}+4\_Z^{3}cd\,e^{3}+c\,d^{4}+b\,d^{2}+\left(6c\,d^{2}e^{2}+b\,e^{2}\right)\_Z^{2}+\left(4c\,d^{3}e+2deb\right)\_Z+a\right)^{3}bc\,e^{3}+3\operatorname{RootOf}\left(\_Z^{4}c\,e^{4}+4\_Z^{3}cd\,e^{3}+c\,d^{4}+b\,d^{2}+\left(6c\,d^{2}e^{2}+b\,e^{2}\right)\_Z^{2}+\left(4c\,d^{3}e+2deb\right)\_Z+a\right)^{2}bcd\,e^{2}+bc\,d^{3}-acd+b^{2}d+\left(3bc\,d^{2}-ac+b^{2}\right)\operatorname{RootOf}\left(\_Z^{4}c\,e^{4}+4\_Z^{3}cd\,e^{3}+c\,d^{4}+b\,d^{2}+\left(6c\,d^{2}e^{2}+b\,e^{2}\right)\_Z^{2}+\left(4c\,d^{3}e+2deb\right)\_Z+a\right)e\right)\ln\left(-\operatorname{RootOf}\left(\_Z^{4}c\,e^{4}+4\_Z^{3}cd\,e^{3}+c\,d^{4}+b\,d^{2}+\left(6c\,d^{2}e^{2}+b\,e^{2}\right)\_Z^{2}+\left(4c\,d^{3}e+2deb\right)\_Z+a\right)+x\right)}{2a^{2}e\,f^{3}\left(2c\,e^{3}\operatorname{RootOf}\left(\_Z^{4}c\,e^{4}+4\_Z^{3}cd\,e^{3}+c\,d^{4}+b\,d^{2}+\left(6c\,d^{2}e^{2}+b\,e^{2}\right)\_Z^{2}+\left(4c\,d^{3}e+2deb\right)\_Z+a\right)^{3}+6cd\,e^{2}\operatorname{RootOf}\left(\_Z^{4}c\,e^{4}+4\_Z^{3}cd\,e^{3}+c\,d^{4}+b\,d^{2}+\left(6c\,d^{2}e^{2}+b\,e^{2}\right)\_Z^{2}+\left(4c\,d^{3}e+2deb\right)\_Z+a\right)^{2}+6ec\,d^{2}\operatorname{RootOf}\left(\_Z^{4}c\,e^{4}+4\_Z^{3}cd\,e^{3}+c\,d^{4}+b\,d^{2}+\left(6c\,d^{2}e^{2}+b\,e^{2}\right)\_Z^{2}+\left(4c\,d^{3}e+2deb\right)\_Z+a\right)+2c\,d^{3}+be\operatorname{RootOf}\left(\_Z^{4}c\,e^{4}+4\_Z^{3}cd\,e^{3}+c\,d^{4}+b\,d^{2}+\left(6c\,d^{2}e^{2}+b\,e^{2}\right)\_Z^{2}+\left(4c\,d^{3}e+2deb\right)\_Z+a\right)+bd\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*f*x+d*f)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4),x)
```

```
[Out] 1/2/f^3/a^2/e*sum((_R^3*b*c*e^3+3*_R^2*b*c*d*e^2+b*c*d^3-a*c*d+b^2*d+(3*b*c
*d^2-a*c+b^2)*_R*e)/(2*_R^3*c*e^3+6*_R^2*c*d*e^2+6*_R*c*d^2*e+2*c*d^3+_R*b*
e+b*d)*ln(-_R+x),_R=RootOf(_Z^4*c*e^4+4*_Z^3*c*d*e^3+c*d^4+b*d^2+(6*c*d^2*e
^2+b*e^2)*_Z^2+(4*c*d^3*e+2*b*d*e)*_Z+a))-1/2/a/e/f^3/(e*x+d)^2-b*ln(e*x+d)
/a^2/e/f^3
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*f*x+d*f)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="maxima"
)
```

```
[Out] Timed out
```

mupad [B] time = 6.98, size = 5947, normalized size = 44.71

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((d*f + e*f*x)^3*(a + b*(d + e*x)^2 + c*(d + e*x)^4)),x)
```

```
[Out] (atan((16*a^6*f^9*x*(((3*b^4 + a^2*c^2 - 9*a*b^2*c)*(((((2*b^3*e*f^3 - 8*a*
b*c*e*f^3)*((2*(20*a^3*c^4*d*e^17*f^6 + 2*a^2*b^2*c^3*d*e^17*f^6))/(a^3*f^9
) + ((40*a^4*b*c^3*d*e^18*f^9 - 12*a^3*b^3*c^2*d*e^18*f^9)*(2*b^3*e*f^3 - 8
*a*b*c*e*f^3))/(a^3*f^9*(16*a^3*c*e^2*f^6 - 4*a^2*b^2*e^2*f^6))))/(2*(16*a^
3*c*e^2*f^6 - 4*a^2*b^2*e^2*f^6)) + (12*b*c^4*d*e^16)/(a^2*f^6))*(2*b^3*e*f
^3 - 8*a*b*c*e*f^3))/(2*(16*a^3*c*e^2*f^6 - 4*a^2*b^2*e^2*f^6)) + (2*c^5*d*
e^15)/(a^3*f^9) - (((((2*(20*a^3*c^4*d*e^17*f^6 + 2*a^2*b^2*c^3*d*e^17*f^6)
)/(a^3*f^9) + ((40*a^4*b*c^3*d*e^18*f^9 - 12*a^3*b^3*c^2*d*e^18*f^9)*(2*b^3
*e*f^3 - 8*a*b*c*e*f^3))/(a^3*f^9*(16*a^3*c*e^2*f^6 - 4*a^2*b^2*e^2*f^6)))*
(2*a*c - b^2))/(4*a^2*e*f^3*(4*a*c - b^2)^(1/2)) + ((40*a^4*b*c^3*d*e^18*f^
9 - 12*a^3*b^3*c^2*d*e^18*f^9)*(2*b^3*e*f^3 - 8*a*b*c*e*f^3)*(2*a*c - b^2))
/(4*a^5*e*f^12*(4*a*c - b^2)^(1/2)*(16*a^3*c*e^2*f^6 - 4*a^2*b^2*e^2*f^6)))
*(2*a*c - b^2))/(4*a^2*e*f^3*(4*a*c - b^2)^(1/2)) - ((40*a^4*b*c^3*d*e^18*f
^9 - 12*a^3*b^3*c^2*d*e^18*f^9)*(2*b^3*e*f^3 - 8*a*b*c*e*f^3)*(2*a*c - b^2)
^2)/(16*a^7*e^2*f^15*(4*a*c - b^2)*(16*a^3*c*e^2*f^6 - 4*a^2*b^2*e^2*f^6)))
)/(8*a^3*c^2*(a^2*c^2 - 6*b^4 + 24*a*b^2*c)) + ((3*b^5 + 13*a^2*b*c^2 - 15*
a*b^3*c)*((((((2*(20*a^3*c^4*d*e^17*f^6 + 2*a^2*b^2*c^3*d*e^17*f^6))/(a^3*f
^9) + ((40*a^4*b*c^3*d*e^18*f^9 - 12*a^3*b^3*c^2*d*e^18*f^9)*(2*b^3*e*f^3 -
 8*a*b*c*e*f^3))/(a^3*f^9*(16*a^3*c*e^2*f^6 - 4*a^2*b^2*e^2*f^6)))*(2*a*c -
 b^2))/(4*a^2*e*f^3*(4*a*c - b^2)^(1/2)) + ((40*a^4*b*c^3*d*e^18*f^9 - 12*a
^3*b^3*c^2*d*e^18*f^9)*(2*b^3*e*f^3 - 8*a*b*c*e*f^3)*(2*a*c - b^2))/(4*a^5*
e*f^12*(4*a*c - b^2)^(1/2)*(16*a^3*c*e^2*f^6 - 4*a^2*b^2*e^2*f^6)))*(2*b^3*
e*f^3 - 8*a*b*c*e*f^3))/(2*(16*a^3*c*e^2*f^6 - 4*a^2*b^2*e^2*f^6)) + ((((2*
b^3*e*f^3 - 8*a*b*c*e*f^3)*((2*(20*a^3*c^4*d*e^17*f^6 + 2*a^2*b^2*c^3*d*e^1
7*f^6))/(a^3*f^9) + ((40*a^4*b*c^3*d*e^18*f^9 - 12*a^3*b^3*c^2*d*e^18*f^9)*
(2*b^3*e*f^3 - 8*a*b*c*e*f^3))/(a^3*f^9*(16*a^3*c*e^2*f^6 - 4*a^2*b^2*e^2*f
^6))))/(2*(16*a^3*c*e^2*f^6 - 4*a^2*b^2*e^2*f^6)) + (12*b*c^4*d*e^16)/(a^2*
```

f^6))*(2*a*c - b^2))/(4*a^2*e*f^3*(4*a*c - b^2)^(1/2)) - ((40*a^4*b*c^3*d*e
^18*f^9 - 12*a^3*b^3*c^2*d*e^18*f^9)*(2*a*c - b^2)^3)/(32*a^9*e^3*f^18*(4*a
*c - b^2)^(3/2))))/(8*a^3*c^2*(4*a*c - b^2)^(1/2)*(a^2*c^2 - 6*b^4 + 24*a*b
^2*c)))*(4*a*c - b^2)^(3/2))/(4*a^2*c^4*e^14 + b^4*c^2*e^14 - 4*a*b^2*c^3*e
^14) + (16*a^6*f^9*x^2*(((3*b^4 + a^2*c^2 - 9*a*b^2*c)*((((((20*a^3*c^4*e^1
8*f^6 + 2*a^2*b^2*c^3*e^18*f^6)/(a^3*f^9) - ((2*b^3*e*f^3 - 8*a*b*c*e*f^3)*
(12*a^3*b^3*c^2*e^19*f^9 - 40*a^4*b*c^3*e^19*f^9))/(2*a^3*f^9*(16*a^3*c*e^2
*f^6 - 4*a^2*b^2*e^2*f^6)))*(2*b^3*e*f^3 - 8*a*b*c*e*f^3))/(2*(16*a^3*c*e^2
*f^6 - 4*a^2*b^2*e^2*f^6)) + (6*b*c^4*e^17)/(a^2*f^6))*(2*b^3*e*f^3 - 8*a*b
*c*e*f^3))/(2*(16*a^3*c*e^2*f^6 - 4*a^2*b^2*e^2*f^6)) + (c^5*e^16)/(a^3*f^9
) - ((2*a*c - b^2)*((((20*a^3*c^4*e^18*f^6 + 2*a^2*b^2*c^3*e^18*f^6)/(a^3*f
^9) - ((2*b^3*e*f^3 - 8*a*b*c*e*f^3)*(12*a^3*b^3*c^2*e^19*f^9 - 40*a^4*b*c^
3*e^19*f^9))/(2*a^3*f^9*(16*a^3*c*e^2*f^6 - 4*a^2*b^2*e^2*f^6)))*(2*a*c - b
^2))/(4*a^2*e*f^3*(4*a*c - b^2)^(1/2)) - ((2*b^3*e*f^3 - 8*a*b*c*e*f^3)*(12
*a^3*b^3*c^2*e^19*f^9 - 40*a^4*b*c^3*e^19*f^9)*(2*a*c - b^2))/(8*a^5*e*f^12
*(4*a*c - b^2)^(1/2)*(16*a^3*c*e^2*f^6 - 4*a^2*b^2*e^2*f^6))))/(4*a^2*e*f^3
*(4*a*c - b^2)^(1/2)) + ((2*b^3*e*f^3 - 8*a*b*c*e*f^3)*(12*a^3*b^3*c^2*e^19
*f^9 - 40*a^4*b*c^3*e^19*f^9)*(2*a*c - b^2)^2)/(32*a^7*e^2*f^15*(4*a*c - b^
2)*(16*a^3*c*e^2*f^6 - 4*a^2*b^2*e^2*f^6))))/(8*a^3*c^2*(a^2*c^2 - 6*b^4 +
24*a*b^2*c)) + ((3*b^5 + 13*a^2*b*c^2 - 15*a*b^3*c)*(((2*b^3*e*f^3 - 8*a*b*
c*e*f^3)*((((20*a^3*c^4*e^18*f^6 + 2*a^2*b^2*c^3*e^18*f^6)/(a^3*f^9) - ((2*
b^3*e*f^3 - 8*a*b*c*e*f^3)*(12*a^3*b^3*c^2*e^19*f^9 - 40*a^4*b*c^3*e^19*f^9
))/(2*a^3*f^9*(16*a^3*c*e^2*f^6 - 4*a^2*b^2*e^2*f^6)))*(2*a*c - b^2))/(4*a^
2*e*f^3*(4*a*c - b^2)^(1/2)) - ((2*b^3*e*f^3 - 8*a*b*c*e*f^3)*(12*a^3*b^3*c
^2*e^19*f^9 - 40*a^4*b*c^3*e^19*f^9)*(2*a*c - b^2))/(8*a^5*e*f^12*(4*a*c -
b^2)^(1/2)*(16*a^3*c*e^2*f^6 - 4*a^2*b^2*e^2*f^6))))/(2*(16*a^3*c*e^2*f^6 -
4*a^2*b^2*e^2*f^6)) + ((12*a^3*b^3*c^2*e^19*f^9 - 40*a^4*b*c^3*e^19*f^9)*(
2*a*c - b^2)^3)/(64*a^9*e^3*f^18*(4*a*c - b^2)^(3/2)) + (((((20*a^3*c^4*e^1
8*f^6 + 2*a^2*b^2*c^3*e^18*f^6)/(a^3*f^9) - ((2*b^3*e*f^3 - 8*a*b*c*e*f^3)*
(12*a^3*b^3*c^2*e^19*f^9 - 40*a^4*b*c^3*e^19*f^9))/(2*a^3*f^9*(16*a^3*c*e^2
*f^6 - 4*a^2*b^2*e^2*f^6)))*(2*b^3*e*f^3 - 8*a*b*c*e*f^3))/(2*(16*a^3*c*e^2
*f^6 - 4*a^2*b^2*e^2*f^6)) + (6*b*c^4*e^17)/(a^2*f^6))*(2*a*c - b^2))/(4*a^
2*e*f^3*(4*a*c - b^2)^(1/2))))/(8*a^3*c^2*(4*a*c - b^2)^(1/2)*(a^2*c^2 - 6*
b^4 + 24*a*b^2*c)))*(4*a*c - b^2)^(3/2))/(4*a^2*c^4*e^14 + b^4*c^2*e^14 - 4
*a*b^2*c^3*e^14) + (2*a^3*f^9*(4*a*c - b^2)^(3/2)*(3*b^4 + a^2*c^2 - 9*a*b^
2*c)*((b*c^4*e^14 + c^5*d^2*e^14)/(a^3*f^9) + (((((4*a^2*b^3*c^2*e^16*f^6 +
20*a^3*c^4*d^2*e^16*f^6 - 4*a^3*b*c^3*e^16*f^6 + 2*a^2*b^2*c^3*d^2*e^16*f^
6)/(a^3*f^9) - ((2*b^3*e*f^3 - 8*a*b*c*e*f^3)*(4*a^4*b^2*c^2*e^17*f^9 - 40*
a^4*b*c^3*d^2*e^17*f^9 + 12*a^3*b^3*c^2*d^2*e^17*f^9))/(2*a^3*f^9*(16*a^3*c
*e^2*f^6 - 4*a^2*b^2*e^2*f^6)))*(2*b^3*e*f^3 - 8*a*b*c*e*f^3))/(2*(16*a^3*c
*e^2*f^6 - 4*a^2*b^2*e^2*f^6)) + (4*a*b^2*c^3*e^15*f^3 - a^2*c^4*e^15*f^3 +
6*a*b*c^4*d^2*e^15*f^3)/(a^3*f^9))*(2*b^3*e*f^3 - 8*a*b*c*e*f^3))/(2*(16*a
^3*c*e^2*f^6 - 4*a^2*b^2*e^2*f^6)) - ((2*a*c - b^2)*((((4*a^2*b^3*c^2*e^16*
f^6 + 20*a^3*c^4*d^2*e^16*f^6 - 4*a^3*b*c^3*e^16*f^6 + 2*a^2*b^2*c^3*d^2*e^
16*f^6)/(a^3*f^9) - ((2*b^3*e*f^3 - 8*a*b*c*e*f^3)*(4*a^4*b^2*c^2*e^17*f^9
- 40*a^4*b*c^3*d^2*e^17*f^9 + 12*a^3*b^3*c^2*d^2*e^17*f^9))/(2*a^3*f^9*(16*
a^3*c*e^2*f^6 - 4*a^2*b^2*e^2*f^6)))*(2*a*c - b^2))/(4*a^2*e*f^3*(4*a*c - b
^2)^(1/2)) - ((2*b^3*e*f^3 - 8*a*b*c*e*f^3)*(2*a*c - b^2)*(4*a^4*b^2*c^2*e^
17*f^9 - 40*a^4*b*c^3*d^2*e^17*f^9 + 12*a^3*b^3*c^2*d^2*e^17*f^9))/(8*a^5*e
f^12(4*a*c - b^2)^(1/2)*(16*a^3*c*e^2*f^6 - 4*a^2*b^2*e^2*f^6))))/(4*a^2*
e*f^3*(4*a*c - b^2)^(1/2)) + ((2*b^3*e*f^3 - 8*a*b*c*e*f^3)*(2*a*c - b^2)^2
*(4*a^4*b^2*c^2*e^17*f^9 - 40*a^4*b*c^3*d^2*e^17*f^9 + 12*a^3*b^3*c^2*d^2*e
^17*f^9))/(32*a^7*e^2*f^15*(4*a*c - b^2)*(16*a^3*c*e^2*f^6 - 4*a^2*b^2*e^2*
f^6))))/(c^2*(a^2*c^2 - 6*b^4 + 24*a*b^2*c)*(4*a^2*c^4*e^14 + b^4*c^2*e^14
- 4*a*b^2*c^3*e^14)) + (2*a^3*f^9*(4*a*c - b^2)*(((2*b^3*e*f^3 - 8*a*b*c*e*
f^3)*((((4*a^2*b^3*c^2*e^16*f^6 + 20*a^3*c^4*d^2*e^16*f^6 - 4*a^3*b*c^3*e^1
6*f^6 + 2*a^2*b^2*c^3*d^2*e^16*f^6)/(a^3*f^9) - ((2*b^3*e*f^3 - 8*a*b*c*e*f
^3)*(4*a^4*b^2*c^2*e^17*f^9 - 40*a^4*b*c^3*d^2*e^17*f^9 + 12*a^3*b^3*c^2*d^
2*e^17*f^9))/(2*a^3*f^9*(16*a^3*c*e^2*f^6 - 4*a^2*b^2*e^2*f^6)))*(2*a*c - b

^2))/(4*a^2*e*f^3*(4*a*c - b^2)^(1/2)) - ((2*b^3*e*f^3 - 8*a*b*c*e*f^3)*(2*
a*c - b^2)*(4*a^4*b^2*c^2*e^17*f^9 - 40*a^4*b*c^3*d^2*e^17*f^9 + 12*a^3*b^3
*c^2*d^2*e^17*f^9))/(8*a^5*e*f^12*(4*a*c - b^2)^(1/2)*(16*a^3*c*e^2*f^6 - 4
*a^2*b^2*e^2*f^6))))/(2*(16*a^3*c*e^2*f^6 - 4*a^2*b^2*e^2*f^6)) + (((((4*a^
2*b^3*c^2*e^16*f^6 + 20*a^3*c^4*d^2*e^16*f^6 - 4*a^3*b*c^3*e^16*f^6 + 2*a^2
*b^2*c^3*d^2*e^16*f^6)/(a^3*f^9) - ((2*b^3*e*f^3 - 8*a*b*c*e*f^3)*(4*a^4*b^
2*c^2*e^17*f^9 - 40*a^4*b*c^3*d^2*e^17*f^9 + 12*a^3*b^3*c^2*d^2*e^17*f^9))/
(2*a^3*f^9*(16*a^3*c*e^2*f^6 - 4*a^2*b^2*e^2*f^6)))*(2*b^3*e*f^3 - 8*a*b*c*
e*f^3))/(2*(16*a^3*c*e^2*f^6 - 4*a^2*b^2*e^2*f^6)) + (4*a*b^2*c^3*e^15*f^3
- a^2*c^4*e^15*f^3 + 6*a*b*c^4*d^2*e^15*f^3)/(a^3*f^9))*(2*a*c - b^2))/(4*a
^2*e*f^3*(4*a*c - b^2)^(1/2)) + ((2*a*c - b^2)^3*(4*a^4*b^2*c^2*e^17*f^9 -
40*a^4*b*c^3*d^2*e^17*f^9 + 12*a^3*b^3*c^2*d^2*e^17*f^9))/(64*a^9*e^3*f^18*
(4*a*c - b^2)^(3/2)))*(3*b^5 + 13*a^2*b*c^2 - 15*a*b^3*c))/(c^2*(a^2*c^2 -
6*b^4 + 24*a*b^2*c)*(4*a^2*c^4*e^14 + b^4*c^2*e^14 - 4*a*b^2*c^3*e^14)))*(2
*a*c - b^2))/(2*a^2*e*f^3*(4*a*c - b^2)^(1/2)) - 1/(2*a*e*(d^2*f^3 + e^2*f^
3*x^2 + 2*d*e*f^3*x)) - (b*log(d + e*x))/(a^2*e*f^3) - (log(((c^5*e^16*x^2)
/(a^3*f^9) - ((b + a^2*e*f^3*(-(2*a*c - b^2)^2/(a^4*e^2*f^6*(4*a*c - b^2)))
^(1/2))*((c^3*e^15*(4*b^2 - a*c + 6*b*c*d^2))/(a^2*f^6) - ((b + a^2*e*f^3*(
-(2*a*c - b^2)^2/(a^4*e^2*f^6*(4*a*c - b^2)))^(1/2))*((2*c^2*e^16*(2*b^3 +
10*a*c^2*d^2 + b^2*c*d^2 - 2*a*b*c))/(a*f^3) + (2*c^3*e^18*x^2*(10*a*c + b^
2))/(a*f^3) + (b*c^2*e^16*(b + a^2*e*f^3*(-(2*a*c - b^2)^2/(a^4*e^2*f^6*(4*
a*c - b^2)))^(1/2))*(a*b + 3*b^2*d^2 + 3*b^2*e^2*x^2 - 10*a*c*d^2 + 6*b^2*d
*e*x - 10*a*c*e^2*x^2 - 20*a*c*d*e*x))/(a^2*f^3) + (4*c^3*d*e^17*x*(10*a*c
+ b^2))/(a*f^3)))/(4*a^2*e*f^3) + (6*b*c^4*e^17*x^2)/(a^2*f^6) + (12*b*c^4*
d*e^16*x)/(a^2*f^6)))/(4*a^2*e*f^3) + (c^4*e^14*(b + c*d^2))/(a^3*f^9) + (2
*c^5*d*e^15*x)/(a^3*f^9))*((c^5*e^16*x^2)/(a^3*f^9) - ((b - a^2*e*f^3*(-(2*
a*c - b^2)^2/(a^4*e^2*f^6*(4*a*c - b^2)))^(1/2))*((c^3*e^15*(4*b^2 - a*c +
6*b*c*d^2))/(a^2*f^6) - ((b - a^2*e*f^3*(-(2*a*c - b^2)^2/(a^4*e^2*f^6*(4*a
c - b^2)))^(1/2))((2*c^2*e^16*(2*b^3 + 10*a*c^2*d^2 + b^2*c*d^2 - 2*a*b*c
))/(a*f^3) + (2*c^3*e^18*x^2*(10*a*c + b^2))/(a*f^3) + (b*c^2*e^16*(b - a^2
*e*f^3*(-(2*a*c - b^2)^2/(a^4*e^2*f^6*(4*a*c - b^2)))^(1/2))*(a*b + 3*b^2*d
^2 + 3*b^2*e^2*x^2 - 10*a*c*d^2 + 6*b^2*d*e*x - 10*a*c*e^2*x^2 - 20*a*c*d*e
*x))/(a^2*f^3) + (4*c^3*d*e^17*x*(10*a*c + b^2))/(a*f^3)))/(4*a^2*e*f^3) +
(6*b*c^4*e^17*x^2)/(a^2*f^6) + (12*b*c^4*d*e^16*x)/(a^2*f^6)))/(4*a^2*e*f^3
) + (c^4*e^14*(b + c*d^2))/(a^3*f^9) + (2*c^5*d*e^15*x)/(a^3*f^9)))*(2*b^3*
e*f^3 - 8*a*b*c*e*f^3))/(2*(16*a^3*c*e^2*f^6 - 4*a^2*b^2*e^2*f^6))

<u>**sympy**</u> **[B]** time = 156.63, size = 532, normalized size = 4.00

$$\left(\frac{b}{4a^2ef^3} - \frac{\sqrt{-4ac+b^2}\left(2ac-b^2\right)}{4a^2ef^3\left(4ac-b^2\right)}\right)\log\left(\frac{2dx}{e} + x^2 + \frac{-8a^3cef^3\left(\frac{b}{4a^2ef^3} - \frac{\sqrt{-4ac+b^2}\left(2ac-b^2\right)}{4a^2ef^3\left(4ac-b^2\right)}\right) + 2a^2b^2ef^3\left(\frac{b}{4a^2ef^3} - \frac{\sqrt{-4ac+b^2}\left(2ac-b^2\right)}{4a^2ef^3\left(4ac-b^2\right)}\right) + 3abc + 2ac^2d^2 - b^3 - b^2cd^2}{2ac^2e^2 - b^2ce^2}\right) + \left(\frac{b}{4a^2ef^3} + \frac{\sqrt{-4ac+b^2}\left(2ac-b^2\right)}{4a^2ef^3\left(4ac-b^2\right)}\right)\log\left(\frac{2dx}{e} + x^2 + \frac{-8a^3cef^3\left(\frac{b}{4a^2ef^3} + \frac{\sqrt{-4ac+b^2}\left(2ac-b^2\right)}{4a^2ef^3\left(4ac-b^2\right)}\right) + 2a^2b^2ef^3\left(\frac{b}{4a^2ef^3} + \frac{\sqrt{-4ac+b^2}\left(2ac-b^2\right)}{4a^2ef^3\left(4ac-b^2\right)}\right) + 3abc + 2ac^2d^2 - b^3 - b^2cd^2}{2ac^2e^2 - b^2ce^2}\right) - \frac{1}{2ad^2ef^3 + 4ade^2f^3x + 2ae^3f^3x^2} - \frac{b\log\left(\frac{d}{e} + x\right)}{a^2ef^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e*f*x+d*f)**3/(a+b*(e*x+d)**2+c*(e*x+d)**4),x)

[Out] (b/(4*a**2*e*f**3) - sqrt(-4*a*c + b**2)*(2*a*c - b**2)/(4*a**2*e*f**3*(4*a
*c - b**2)))*log(2*d*x/e + x**2 + (-8*a**3*c*e*f**3*(b/(4*a**2*e*f**3) - sq
rt(-4*a*c + b**2)*(2*a*c - b**2)/(4*a**2*e*f**3*(4*a*c - b**2))) + 2*a**2*b
2*e*f3*(b/(4*a**2*e*f**3) - sqrt(-4*a*c + b**2)*(2*a*c - b**2)/(4*a**2*
e*f**3*(4*a*c - b**2))) + 3*a*b*c + 2*a*c**2*d**2 - b**3 - b**2*c*d**2)/(2*
a*c**2*e**2 - b**2*c*e**2)) + (b/(4*a**2*e*f**3) + sqrt(-4*a*c + b**2)*(2*a
*c - b**2)/(4*a**2*e*f**3*(4*a*c - b**2)))*log(2*d*x/e + x**2 + (-8*a**3*c*
e*f**3*(b/(4*a**2*e*f**3) + sqrt(-4*a*c + b**2)*(2*a*c - b**2)/(4*a**2*e*f*
3(4*a*c - b**2))) + 2*a**2*b**2*e*f**3*(b/(4*a**2*e*f**3) + sqrt(-4*a*c +
 b**2)*(2*a*c - b**2)/(4*a**2*e*f**3*(4*a*c - b**2))) + 3*a*b*c + 2*a*c**2*
d**2 - b**3 - b**2*c*d**2)/(2*a*c**2*e**2 - b**2*c*e**2)) - 1/(2*a*d**2*e*f
3 + 4*a*d*e2*f**3*x + 2*a*e**3*f**3*x**2) - b*log(d/e + x)/(a**2*e*f**3
)

3.532 $\int \frac{1}{(df+efx)^4\left(a+b(d+ex)^2+c(d+ex)^4\right)}\,dx$

Optimal. Leaf size=236

$$\frac{\sqrt{c}\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\,a^2ef^4\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{c}\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,(d+ex)}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\,a^2ef^4\sqrt{\sqrt{b^2-4ac}+b}}+\frac{b}{a^2ef^4(d+ex)}-\frac{1}{3aef^4(d+ex)^3}$$

Rubi [A] time = 0.49, antiderivative size = 236, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {1142, 1123, 1281, 1166, 205}

$$\frac{\sqrt{c}\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\,a^2ef^4\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{c}\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,(d+ex)}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\,a^2ef^4\sqrt{\sqrt{b^2-4ac}+b}}+\frac{b}{a^2ef^4(d+ex)}-\frac{1}{3aef^4(d+ex)^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d*f + e*f*x)^4*(a + b*(d + e*x)^2 + c*(d + e*x)^4)),x]
```

```
[Out] -1/(3*a*e*f^4*(d + e*x)^3) + b/(a^2*e*f^4*(d + e*x)) + (Sqrt[c]*(b + (b^2 -
 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b - Sqrt
[b^2 - 4*a*c]]])/(Sqrt[2]*a^2*Sqrt[b - Sqrt[b^2 - 4*a*c]]*e*f^4) + (Sqrt[c]
*(b - (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/S
qrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*a^2*Sqrt[b + Sqrt[b^2 - 4*a*c]]*e*f^4
)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1123

```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*x^2 + c*x^4)^(p + 1))/(a*d*(m + 1)), x] - Dis
t[1/(a*d^2*(m + 1)), Int[(d*x)^(m + 2)*(b*(m + 2*p + 3) + c*(m + 4*p + 5)*x
^2)*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b^2 -
 4*a*c, 0] && LtQ[m, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1142

```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Di
st[u^m/(Coefficient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p,
 x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1281

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[(d*(f*x)^(m + 1)*(a + b*x^2 + c*x^4)^(p + 1)
```

```
)/(a*f*(m + 1)), x] + Dist[1/(a*f^2*(m + 1)), Int[(f*x)^(m + 2)*(a + b*x^2
+ c*x^4)^p*Simp[a*e*(m + 1) - b*d*(m + 2*p + 3) - c*d*(m + 4*p + 5)*x^2, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[m
, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rubi steps

$$\int \frac{1}{(df+efx)^4\left(a+b(d+ex)^2+c(d+ex)^4\right)}dx = \frac{\text{Subst}\left(\int \frac{1}{x^4(a+bx^2+cx^4)}dx,x,d+ex\right)}{ef^4}$$

$$= -\frac{1}{3aef^4(d+ex)^3}+\frac{\text{Subst}\left(\int \frac{-3b-3cx^2}{x^2(a+bx^2+cx^4)}dx,x,d+ex\right)}{3aef^4}$$

$$= -\frac{1}{3aef^4(d+ex)^3}+\frac{b}{a^2ef^4(d+ex)}-\frac{\text{Subst}\left(\int \frac{-3(b^2-ac)-3bcx^2}{a+bx^2+cx^4}\right.}{3a^2ef^4}$$

$$= -\frac{1}{3aef^4(d+ex)^3}+\frac{b}{a^2ef^4(d+ex)}+\frac{\left(c\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\right)\text{Subs}}{}$$

$$= -\frac{1}{3aef^4(d+ex)^3}+\frac{b}{a^2ef^4(d+ex)}+\frac{\sqrt{c}\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}}{\sqrt{2}\,a^2\sqrt{b-\sqrt{b^2-}}}$$

Mathematica [A] time = 0.10, size = 238, normalized size = 1.01

$$\frac{\frac{3\sqrt{2}\sqrt{c}\left(b\sqrt{b^2-4ac}-2ac+b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{3\sqrt{2}\sqrt{c}\left(b\sqrt{b^2-4ac}+2ac-b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4ac}+b}}-\frac{2a}{(d+ex)^3}+\frac{6b}{d+ex}}{6a^2ef^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d*f + e*f*x)^4*(a + b*(d + e*x)^2 + c*(d + e*x)^4)),x]
```

```
[Out] ((-2*a)/(d + e*x)^3 + (6*b)/(d + e*x) + (3*Sqrt[2]*Sqrt[c]*(b^2 - 2*a*c + b
*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b - Sqrt[b^2 -
4*a*c]]])/(Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (3*Sqrt[2]*Sqrt
[c]*(-b^2 + 2*a*c + b*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))
/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2 - 4*a*c]*Sqrt[b + Sqrt[b^2 - 4*a*c
]]))/(6*a^2*e*f^4)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(df+efx)^4\left(a+b(d+ex)^2+c(d+ex)^4\right)}dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/((d*f + e*f*x)^4*(a + b*(d + e*x)^2 + c*(d + e*x)^4)),
x]
```

```
[Out] IntegrateAlgebraic[1/((d*f + e*f*x)^4*(a + b*(d + e*x)^2 + c*(d + e*x)^4)),
 x]
```

fricas [B] time = 1.32, size = 2212, normalized size = 9.37

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*f*x+d*f)^4/(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="fricas"
)
```

```
[Out] 1/6*(6*b*e^2*x^2 + 12*b*d*e*x + 6*b*d^2 + 3*sqrt(1/2)*(a^2*e^4*f^4*x^3 + 3*
a^2*d*e^3*f^4*x^2 + 3*a^2*d^2*e^2*f^4*x + a^2*d^3*e*f^4)*sqrt(-((a^5*b^2 -
4*a^6*c)*e^2*f^8*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a
^4*c^4)/((a^10*b^2 - 4*a^11*c)*e^4*f^16)) + b^5 - 5*a*b^3*c + 5*a^2*b*c^2)/
((a^5*b^2 - 4*a^6*c)*e^2*f^8))*log(2*(b^4*c^3 - 3*a*b^2*c^4 + a^2*c^5)*e*x
+ 2*(b^4*c^3 - 3*a*b^2*c^4 + a^2*c^5)*d + sqrt(1/2)*((a^5*b^5 - 7*a^6*b^3*c
 + 12*a^7*b*c^2)*e^3*f^12*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^
2*c^3 + a^4*c^4)/((a^10*b^2 - 4*a^11*c)*e^4*f^16)) - (b^8 - 8*a*b^6*c + 20*
a^2*b^4*c^2 - 17*a^3*b^2*c^3 + 4*a^4*c^4)*e*f^4)*sqrt(-((a^5*b^2 - 4*a^6*c)
*e^2*f^8*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/
((a^10*b^2 - 4*a^11*c)*e^4*f^16)) + b^5 - 5*a*b^3*c + 5*a^2*b*c^2)/((a^5*b^
2 - 4*a^6*c)*e^2*f^8))) - 3*sqrt(1/2)*(a^2*e^4*f^4*x^3 + 3*a^2*d*e^3*f^4*x^
2 + 3*a^2*d^2*e^2*f^4*x + a^2*d^3*e*f^4)*sqrt(-((a^5*b^2 - 4*a^6*c)*e^2*f^8
*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/((a^10*b
^2 - 4*a^11*c)*e^4*f^16)) + b^5 - 5*a*b^3*c + 5*a^2*b*c^2)/((a^5*b^2 - 4*a^
6*c)*e^2*f^8))*log(2*(b^4*c^3 - 3*a*b^2*c^4 + a^2*c^5)*e*x + 2*(b^4*c^3 - 3
*a*b^2*c^4 + a^2*c^5)*d - sqrt(1/2)*((a^5*b^5 - 7*a^6*b^3*c + 12*a^7*b*c^2)
*e^3*f^12*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)
/((a^10*b^2 - 4*a^11*c)*e^4*f^16)) - (b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 17
*a^3*b^2*c^3 + 4*a^4*c^4)*e*f^4)*sqrt(-((a^5*b^2 - 4*a^6*c)*e^2*f^8*sqrt((b
^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/((a^10*b^2 - 4*a
^11*c)*e^4*f^16)) + b^5 - 5*a*b^3*c + 5*a^2*b*c^2)/((a^5*b^2 - 4*a^6*c)*e^2
*f^8))) - 3*sqrt(1/2)*(a^2*e^4*f^4*x^3 + 3*a^2*d*e^3*f^4*x^2 + 3*a^2*d^2*e^
2*f^4*x + a^2*d^3*e*f^4)*sqrt(((a^5*b^2 - 4*a^6*c)*e^2*f^8*sqrt((b^8 - 6*a*
b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/((a^10*b^2 - 4*a^11*c)*e^
4*f^16)) - b^5 + 5*a*b^3*c - 5*a^2*b*c^2)/((a^5*b^2 - 4*a^6*c)*e^2*f^8))*lo
g(2*(b^4*c^3 - 3*a*b^2*c^4 + a^2*c^5)*e*x + 2*(b^4*c^3 - 3*a*b^2*c^4 + a^2*
c^5)*d + sqrt(1/2)*((a^5*b^5 - 7*a^6*b^3*c + 12*a^7*b*c^2)*e^3*f^12*sqrt((b
^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/((a^10*b^2 - 4*a
^11*c)*e^4*f^16)) + (b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 17*a^3*b^2*c^3 + 4*
a^4*c^4)*e*f^4)*sqrt(((a^5*b^2 - 4*a^6*c)*e^2*f^8*sqrt((b^8 - 6*a*b^6*c + 1
1*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/((a^10*b^2 - 4*a^11*c)*e^4*f^16))
- b^5 + 5*a*b^3*c - 5*a^2*b*c^2)/((a^5*b^2 - 4*a^6*c)*e^2*f^8))) + 3*sqrt(1
/2)*(a^2*e^4*f^4*x^3 + 3*a^2*d*e^3*f^4*x^2 + 3*a^2*d^2*e^2*f^4*x + a^2*d^3*
e*f^4)*sqrt(((a^5*b^2 - 4*a^6*c)*e^2*f^8*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4
*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/((a^10*b^2 - 4*a^11*c)*e^4*f^16)) - b^5 + 5
*a*b^3*c - 5*a^2*b*c^2)/((a^5*b^2 - 4*a^6*c)*e^2*f^8))*log(2*(b^4*c^3 - 3*a
*b^2*c^4 + a^2*c^5)*e*x + 2*(b^4*c^3 - 3*a*b^2*c^4 + a^2*c^5)*d - sqrt(1/2)
*((a^5*b^5 - 7*a^6*b^3*c + 12*a^7*b*c^2)*e^3*f^12*sqrt((b^8 - 6*a*b^6*c + 1
1*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/((a^10*b^2 - 4*a^11*c)*e^4*f^16))
+ (b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 17*a^3*b^2*c^3 + 4*a^4*c^4)*e*f^4)*sq
rt(((a^5*b^2 - 4*a^6*c)*e^2*f^8*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*
a^3*b^2*c^3 + a^4*c^4)/((a^10*b^2 - 4*a^11*c)*e^4*f^16)) - b^5 + 5*a*b^3*c
- 5*a^2*b*c^2)/((a^5*b^2 - 4*a^6*c)*e^2*f^8))) - 2*a)/(a^2*e^4*f^4*x^3 + 3*
a^2*d*e^3*f^4*x^2 + 3*a^2*d^2*e^2*f^4*x + a^2*d^3*e*f^4)
```

giac [B] time = 0.57, size = 1249, normalized size = 5.29

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*f*x+d*f)^4/(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="giac")
```

```
[Out] -1/2*(((d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c
))^2*b*c*e^2 - 2*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2
)*e^(-4)/c))*b*c*d*e + b*c*d^2 + b^2 - a*c)*log(d*e^(-1) + x + sqrt(1/2)*sq
rt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))/(2*(d*e^(-1) + sqrt(1/2)*sqr
t(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^3*c*e^4 - 6*(d*e^(-1) + sqrt(
1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^2*c*d*e^3 - 2*c*d^3*e
 - b*d*e + (6*c*d^2*e^2 + b*e^2)*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 + sqrt(
b^2 - 4*a*c)*e^2)*e^(-4)/c))) + ((d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 + sqrt(
b^2 - 4*a*c)*e^2)*e^(-4)/c))^2*b*c*e^2 - 2*(d*e^(-1) - sqrt(1/2)*sqrt(-(b*e
^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))*b*c*d*e + b*c*d^2 + b^2 - a*c)*log(d
*e^(-1) + x - sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))/(2
*(d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^3*c
*e^4 - 6*(d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)
/c))^2*c*d*e^3 - 2*c*d^3*e - b*d*e + (6*c*d^2*e^2 + b*e^2)*(d*e^(-1) - sqrt
(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))) + ((d*e^(-1) + sqrt
(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^2*b*c*e^2 - 2*(d*e^(
-1) + sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))*b*c*d*e +
b*c*d^2 + b^2 - a*c)*log(d*e^(-1) + x + sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 -
 4*a*c)*e^2)*e^(-4)/c))/(2*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 -
4*a*c)*e^2)*e^(-4)/c))^3*c*e^4 - 6*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 - sqr
t(b^2 - 4*a*c)*e^2)*e^(-4)/c))^2*c*d*e^3 - 2*c*d^3*e - b*d*e + (6*c*d^2*e^2
 + b*e^2)*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4
)/c))) + ((d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4
)/c))^2*b*c*e^2 - 2*(d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*
e^2)*e^(-4)/c))*b*c*d*e + b*c*d^2 + b^2 - a*c)*log(d*e^(-1) + x - sqrt(1/2)
*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))/(2*(d*e^(-1) - sqrt(1/2)*
sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^3*c*e^4 - 6*(d*e^(-1) - sq
rt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^2*c*d*e^3 - 2*c*d^
3*e - b*d*e + (6*c*d^2*e^2 + b*e^2)*(d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 - sq
rt(b^2 - 4*a*c)*e^2)*e^(-4)/c))))/(a^2*f^4) + 1/3*(3*b*x^2*e^2 + 6*b*d*x*e
+ 3*b*d^2 - a)*e^(-1)/((x*e + d)^3*a^2*f^4)
```

maple [**C**] time = 0.01, size = 197, normalized size = 0.83

$$-\frac{1}{3\left(ex+d\right)^{3} ae f^{4}}+\frac{b}{\left(ex+d\right) a^{2}e f^{4}}+\frac{\left(\operatorname{RootOf}\left(\_Z^{4}c e^{4}+4\_Z^{3}cd e^{3}+c d^{4}+b d^{2}+\left(6c d^{2}e^{2}+b e^{2}\right)\_Z^{2}+\left(4c d^{3}e+2deb\right)\_Z+a\right)^{2} bc e^{2}+2\operatorname{RootOf}\left(\_Z^{4}c e^{4}+4\_Z^{3}cd e^{3}+c d^{4}+b d^{2}+\left(6c d^{2}e^{2}+b e^{2}\right)\_Z^{2}+\left(4c d^{3}e+2deb\right)\_Z+a\right) bcde+bc d^{2}-ac+b^{2}\right)\ln\left(-\operatorname{RootOf}\left(\_Z^{4}c e^{4}+4\_Z^{3}cd e^{3}+c d^{4}+b d^{2}+\left(6c d^{2}e^{2}+b e^{2}\right)\_Z^{2}+\left(4c d^{3}e+2deb\right)\_Z+a\right)+x\right)}{2a^{2}e f^{4}\left(2c e^{3}\operatorname{RootOf}\left(\_Z^{4}c e^{4}+4\_Z^{3}cd e^{3}+c d^{4}+b d^{2}+\left(6c d^{2}e^{2}+b e^{2}\right)\_Z^{2}+\left(4c d^{3}e+2deb\right)\_Z+a\right)^{3}+6cd e^{2}\operatorname{RootOf}\left(\_Z^{4}c e^{4}+4\_Z^{3}cd e^{3}+c d^{4}+b d^{2}+\left(6c d^{2}e^{2}+b e^{2}\right)\_Z^{2}+\left(4c d^{3}e+2deb\right)\_Z+a\right)^{2}+6ec d^{2}\operatorname{RootOf}\left(\_Z^{4}c e^{4}+4\_Z^{3}cd e^{3}+c d^{4}+b d^{2}+\left(6c d^{2}e^{2}+b e^{2}\right)\_Z^{2}+\left(4c d^{3}e+2deb\right)\_Z+a\right)+2c d^{3}+be\operatorname{RootOf}\left(\_Z^{4}c e^{4}+4\_Z^{3}cd e^{3}+c d^{4}+b d^{2}+\left(6c d^{2}e^{2}+b e^{2}\right)\_Z^{2}+\left(4c d^{3}e+2deb\right)\_Z+a\right)+bd\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*f*x+d*f)^4/(a+b*(e*x+d)^2+c*(e*x+d)^4),x)
```

```
[Out] 1/2/f^4/a^2/e*sum((_R^2*b*c*e^2+2*_R*b*c*d*e+b*c*d^2-a*c+b^2)/(2*_R^3*c*e^3
+6*_R^2*c*d*e^2+6*_R*c*d^2*e+2*c*d^3+_R*b*e+b*d)*ln(-_R+x),_R=RootOf(_Z^4*c
*e^4+4*_Z^3*c*d*e^3+c*d^4+b*d^2+(6*c*d^2*e^2+b*e^2)*_Z^2+(4*c*d^3*e+2*b*d*e
)*_Z+a))-1/3/a/e/f^4/(e*x+d)^3+b/a^2/e/f^4/(e*x+d)
```

maxima [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*f*x+d*f)^4/(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="maxima"
)
```

```
[Out] Timed out
```

mupad [**B**] time = 3.17, size = 5771, normalized size = 24.45

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((d*f + e*f*x)^4*(a + b*(d + e*x)^2 + c*(d + e*x)^4)),x)
```

[Out] ((2*b*d*x)/a^2 - (a - 3*b*d^2)/(3*a^2*e) + (b*e*x^2)/a^2)/(d^3*f^4 + e^3*f^
4*x^3 + 3*d*e^2*f^4*x^2 + 3*d^2*e*f^4*x) - atan((((b^4*(-(4*a*c - b^2)^3)^(
1/2) - b^7 + 20*a^3*b*c^3 - 25*a^2*b^3*c^2 + a^2*c^2*(-(4*a*c - b^2)^3)^(1/
2) + 9*a*b^5*c - 3*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4*e^2*f^8 +
16*a^7*c^2*e^2*f^8 - 8*a^6*b^2*c*e^2*f^8)))^(1/2)*(((b^4*(-(4*a*c - b^2)^3)
^(1/2) - b^7 + 20*a^3*b*c^3 - 25*a^2*b^3*c^2 + a^2*c^2*(-(4*a*c - b^2)^3)^(
1/2) + 9*a*b^5*c - 3*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4*e^2*f^8
 + 16*a^7*c^2*e^2*f^8 - 8*a^6*b^2*c*e^2*f^8)))^(1/2)*(((b^4*(-(4*a*c - b^2)^
3)^(1/2) - b^7 + 20*a^3*b*c^3 - 25*a^2*b^3*c^2 + a^2*c^2*(-(4*a*c - b^2)^3)
^(1/2) + 9*a*b^5*c - 3*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4*e^2*f^
8 + 16*a^7*c^2*e^2*f^8 - 8*a^6*b^2*c*e^2*f^8)))^(1/2)*(x*(8*a^10*b^3*c^2*e^
14*f^20 - 32*a^11*b*c^3*e^14*f^20) - 32*a^11*b*c^3*d*e^13*f^20 + 8*a^10*b^3
*c^2*d*e^13*f^20) - 16*a^10*c^4*e^12*f^16 - 4*a^8*b^4*c^2*e^12*f^16 + 20*a^
9*b^2*c^3*e^12*f^16) + x*(4*a^8*c^5*e^12*f^12 + 2*a^6*b^4*c^3*e^12*f^12 - 8
*a^7*b^2*c^4*e^12*f^12) + 4*a^8*c^5*d*e^11*f^12 + 2*a^6*b^4*c^3*d*e^11*f^12
 - 8*a^7*b^2*c^4*d*e^11*f^12)*1i + ((b^4*(-(4*a*c - b^2)^3)^(1/2) - b^7 + 2
0*a^3*b*c^3 - 25*a^2*b^3*c^2 + a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) + 9*a*b^5*c
 - 3*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4*e^2*f^8 + 16*a^7*c^2*e^2
*f^8 - 8*a^6*b^2*c*e^2*f^8)))^(1/2)*(((b^4*(-(4*a*c - b^2)^3)^(1/2) - b^7 +
 20*a^3*b*c^3 - 25*a^2*b^3*c^2 + a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) + 9*a*b^5
*c - 3*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4*e^2*f^8 + 16*a^7*c^2*e
^2*f^8 - 8*a^6*b^2*c*e^2*f^8)))^(1/2)*(((b^4*(-(4*a*c - b^2)^3)^(1/2) - b^7
 + 20*a^3*b*c^3 - 25*a^2*b^3*c^2 + a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) + 9*a*b
^5*c - 3*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4*e^2*f^8 + 16*a^7*c^2
*e^2*f^8 - 8*a^6*b^2*c*e^2*f^8)))^(1/2)*(x*(8*a^10*b^3*c^2*e^14*f^20 - 32*a
^11*b*c^3*e^14*f^20) - 32*a^11*b*c^3*d*e^13*f^20 + 8*a^10*b^3*c^2*d*e^13*f^
20) + 16*a^10*c^4*e^12*f^16 + 4*a^8*b^4*c^2*e^12*f^16 - 20*a^9*b^2*c^3*e^12
f^16) + x(4*a^8*c^5*e^12*f^12 + 2*a^6*b^4*c^3*e^12*f^12 - 8*a^7*b^2*c^4*e
^12*f^12) + 4*a^8*c^5*d*e^11*f^12 + 2*a^6*b^4*c^3*d*e^11*f^12 - 8*a^7*b^2*c
^4*d*e^11*f^12)*1i)/(((b^4*(-(4*a*c - b^2)^3)^(1/2) - b^7 + 20*a^3*b*c^3 -
25*a^2*b^3*c^2 + a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) + 9*a*b^5*c - 3*a*b^2*c*(
-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4*e^2*f^8 + 16*a^7*c^2*e^2*f^8 - 8*a^6*b
^2*c*e^2*f^8)))^(1/2)*(((b^4*(-(4*a*c - b^2)^3)^(1/2) - b^7 + 20*a^3*b*c^3
 - 25*a^2*b^3*c^2 + a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) + 9*a*b^5*c - 3*a*b^2*c
*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4*e^2*f^8 + 16*a^7*c^2*e^2*f^8 - 8*a^6
*b^2*c*e^2*f^8)))^(1/2)*(((b^4*(-(4*a*c - b^2)^3)^(1/2) - b^7 + 20*a^3*b*c^
3 - 25*a^2*b^3*c^2 + a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) + 9*a*b^5*c - 3*a*b^2
c(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4*e^2*f^8 + 16*a^7*c^2*e^2*f^8 - 8*a
^6*b^2*c*e^2*f^8)))^(1/2)*(x*(8*a^10*b^3*c^2*e^14*f^20 - 32*a^11*b*c^3*e^14
*f^20) - 32*a^11*b*c^3*d*e^13*f^20 + 8*a^10*b^3*c^2*d*e^13*f^20) - 16*a^10*
c^4*e^12*f^16 - 4*a^8*b^4*c^2*e^12*f^16 + 20*a^9*b^2*c^3*e^12*f^16) + x*(4*
a^8*c^5*e^12*f^12 + 2*a^6*b^4*c^3*e^12*f^12 - 8*a^7*b^2*c^4*e^12*f^12) + 4*
a^8*c^5*d*e^11*f^12 + 2*a^6*b^4*c^3*d*e^11*f^12 - 8*a^7*b^2*c^4*d*e^11*f^12
) - ((b^4*(-(4*a*c - b^2)^3)^(1/2) - b^7 + 20*a^3*b*c^3 - 25*a^2*b^3*c^2 +
a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) + 9*a*b^5*c - 3*a*b^2*c*(-(4*a*c - b^2)^3)
^(1/2))/(8*(a^5*b^4*e^2*f^8 + 16*a^7*c^2*e^2*f^8 - 8*a^6*b^2*c*e^2*f^8)))^(
1/2)*(((b^4*(-(4*a*c - b^2)^3)^(1/2) - b^7 + 20*a^3*b*c^3 - 25*a^2*b^3*c^2
 + a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) + 9*a*b^5*c - 3*a*b^2*c*(-(4*a*c - b^2)^
3)^(1/2))/(8*(a^5*b^4*e^2*f^8 + 16*a^7*c^2*e^2*f^8 - 8*a^6*b^2*c*e^2*f^8)))
^(1/2)*(((b^4*(-(4*a*c - b^2)^3)^(1/2) - b^7 + 20*a^3*b*c^3 - 25*a^2*b^3*c^
2 + a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) + 9*a*b^5*c - 3*a*b^2*c*(-(4*a*c - b^2
)^3)^(1/2))/(8*(a^5*b^4*e^2*f^8 + 16*a^7*c^2*e^2*f^8 - 8*a^6*b^2*c*e^2*f^8)
))^(1/2)*(x*(8*a^10*b^3*c^2*e^14*f^20 - 32*a^11*b*c^3*e^14*f^20) - 32*a^11*
b*c^3*d*e^13*f^20 + 8*a^10*b^3*c^2*d*e^13*f^20) + 16*a^10*c^4*e^12*f^16 + 4
*a^8*b^4*c^2*e^12*f^16 - 20*a^9*b^2*c^3*e^12*f^16) + x*(4*a^8*c^5*e^12*f^12
 + 2*a^6*b^4*c^3*e^12*f^12 - 8*a^7*b^2*c^4*e^12*f^12) + 4*a^8*c^5*d*e^11*f^
12 + 2*a^6*b^4*c^3*d*e^11*f^12 - 8*a^7*b^2*c^4*d*e^11*f^12) + 2*a^6*b*c^5*e
^10*f^8))*((b^4*(-(4*a*c - b^2)^3)^(1/2) - b^7 + 20*a^3*b*c^3 - 25*a^2*b^3*
c^2 + a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) + 9*a*b^5*c - 3*a*b^2*c*(-(4*a*c - b

^2)^3)^(1/2))/(8*(a^5*b^4*e^2*f^8 + 16*a^7*c^2*e^2*f^8 - 8*a^6*b^2*c*e^2*f^
8)))^(1/2)*2i - atan(((-(b^7 + b^4*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3
+ 25*a^2*b^3*c^2 + a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c - 3*a*b^2*c
*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4*e^2*f^8 + 16*a^7*c^2*e^2*f^8 - 8*a^6
*b^2*c*e^2*f^8)))^(1/2)*((-(b^7 + b^4*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c
^3 + 25*a^2*b^3*c^2 + a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c - 3*a*b^
2*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4*e^2*f^8 + 16*a^7*c^2*e^2*f^8 - 8*
a^6*b^2*c*e^2*f^8)))^(1/2)*((x*(8*a^10*b^3*c^2*e^14*f^20 - 32*a^11*b*c^3*e^
14*f^20) - 32*a^11*b*c^3*d*e^13*f^20 + 8*a^10*b^3*c^2*d*e^13*f^20)*(-(b^7 +
b^4*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3 + 25*a^2*b^3*c^2 + a^2*c^2*(-(
4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c - 3*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(8*
(a^5*b^4*e^2*f^8 + 16*a^7*c^2*e^2*f^8 - 8*a^6*b^2*c*e^2*f^8)))^(1/2) - 16*a
^10*c^4*e^12*f^16 - 4*a^8*b^4*c^2*e^12*f^16 + 20*a^9*b^2*c^3*e^12*f^16) + x
*(4*a^8*c^5*e^12*f^12 + 2*a^6*b^4*c^3*e^12*f^12 - 8*a^7*b^2*c^4*e^12*f^12)
+ 4*a^8*c^5*d*e^11*f^12 + 2*a^6*b^4*c^3*d*e^11*f^12 - 8*a^7*b^2*c^4*d*e^11*
f^12)*1i + (-(b^7 + b^4*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3 + 25*a^2*b^
3*c^2 + a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c - 3*a*b^2*c*(-(4*a*c -
b^2)^3)^(1/2))/(8*(a^5*b^4*e^2*f^8 + 16*a^7*c^2*e^2*f^8 - 8*a^6*b^2*c*e^2*
f^8)))^(1/2)*((-(b^7 + b^4*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3 + 25*a^2
*b^3*c^2 + a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c - 3*a*b^2*c*(-(4*a*
c - b^2)^3)^(1/2))/(8*(a^5*b^4*e^2*f^8 + 16*a^7*c^2*e^2*f^8 - 8*a^6*b^2*c*e
^2*f^8)))^(1/2)*((x*(8*a^10*b^3*c^2*e^14*f^20 - 32*a^11*b*c^3*e^14*f^20) -
32*a^11*b*c^3*d*e^13*f^20 + 8*a^10*b^3*c^2*d*e^13*f^20)*(-(b^7 + b^4*(-(4*a
*c - b^2)^3)^(1/2) - 20*a^3*b*c^3 + 25*a^2*b^3*c^2 + a^2*c^2*(-(4*a*c - b^2
)^3)^(1/2) - 9*a*b^5*c - 3*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4*e^
2*f^8 + 16*a^7*c^2*e^2*f^8 - 8*a^6*b^2*c*e^2*f^8)))^(1/2) + 16*a^10*c^4*e^1
2*f^16 + 4*a^8*b^4*c^2*e^12*f^16 - 20*a^9*b^2*c^3*e^12*f^16) + x*(4*a^8*c^5
*e^12*f^12 + 2*a^6*b^4*c^3*e^12*f^12 - 8*a^7*b^2*c^4*e^12*f^12) + 4*a^8*c^5
*d*e^11*f^12 + 2*a^6*b^4*c^3*d*e^11*f^12 - 8*a^7*b^2*c^4*d*e^11*f^12)*1i)/(
(-(b^7 + b^4*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3 + 25*a^2*b^3*c^2 + a^2
c^2(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c - 3*a*b^2*c*(-(4*a*c - b^2)^3)^(1
/2))/(8*(a^5*b^4*e^2*f^8 + 16*a^7*c^2*e^2*f^8 - 8*a^6*b^2*c*e^2*f^8)))^(1/2
)*((-(b^7 + b^4*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3 + 25*a^2*b^3*c^2 +
a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c - 3*a*b^2*c*(-(4*a*c - b^2)^3)
^(1/2))/(8*(a^5*b^4*e^2*f^8 + 16*a^7*c^2*e^2*f^8 - 8*a^6*b^2*c*e^2*f^8)))^(
1/2)*((x*(8*a^10*b^3*c^2*e^14*f^20 - 32*a^11*b*c^3*e^14*f^20) - 32*a^11*b*c
^3*d*e^13*f^20 + 8*a^10*b^3*c^2*d*e^13*f^20)*(-(b^7 + b^4*(-(4*a*c - b^2)^3
)^(1/2) - 20*a^3*b*c^3 + 25*a^2*b^3*c^2 + a^2*c^2*(-(4*a*c - b^2)^3)^(1/2)
- 9*a*b^5*c - 3*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4*e^2*f^8 + 16*
a^7*c^2*e^2*f^8 - 8*a^6*b^2*c*e^2*f^8)))^(1/2) - 16*a^10*c^4*e^12*f^16 - 4*
a^8*b^4*c^2*e^12*f^16 + 20*a^9*b^2*c^3*e^12*f^16) + x*(4*a^8*c^5*e^12*f^12
+ 2*a^6*b^4*c^3*e^12*f^12 - 8*a^7*b^2*c^4*e^12*f^12) + 4*a^8*c^5*d*e^11*f^1
2 + 2*a^6*b^4*c^3*d*e^11*f^12 - 8*a^7*b^2*c^4*d*e^11*f^12) - (-(b^7 + b^4*(
-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3 + 25*a^2*b^3*c^2 + a^2*c^2*(-(4*a*c
- b^2)^3)^(1/2) - 9*a*b^5*c - 3*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b
^4*e^2*f^8 + 16*a^7*c^2*e^2*f^8 - 8*a^6*b^2*c*e^2*f^8)))^(1/2)*((-(b^7 + b^
4*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3 + 25*a^2*b^3*c^2 + a^2*c^2*(-(4*a
*c - b^2)^3)^(1/2) - 9*a*b^5*c - 3*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^
5*b^4*e^2*f^8 + 16*a^7*c^2*e^2*f^8 - 8*a^6*b^2*c*e^2*f^8)))^(1/2)*((x*(8*a^
10*b^3*c^2*e^14*f^20 - 32*a^11*b*c^3*e^14*f^20) - 32*a^11*b*c^3*d*e^13*f^20
+ 8*a^10*b^3*c^2*d*e^13*f^20)*(-(b^7 + b^4*(-(4*a*c - b^2)^3)^(1/2) - 20*a
^3*b*c^3 + 25*a^2*b^3*c^2 + a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c -
3*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4*e^2*f^8 + 16*a^7*c^2*e^2*f^
8 - 8*a^6*b^2*c*e^2*f^8)))^(1/2) + 16*a^10*c^4*e^12*f^16 + 4*a^8*b^4*c^2*e^
12*f^16 - 20*a^9*b^2*c^3*e^12*f^16) + x*(4*a^8*c^5*e^12*f^12 + 2*a^6*b^4*c^
3*e^12*f^12 - 8*a^7*b^2*c^4*e^12*f^12) + 4*a^8*c^5*d*e^11*f^12 + 2*a^6*b^4*
c^3*d*e^11*f^12 - 8*a^7*b^2*c^4*d*e^11*f^12) + 2*a^6*b*c^5*e^10*f^8))*(-(b^
7 + b^4*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3 + 25*a^2*b^3*c^2 + a^2*c^2*
(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c - 3*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/

```
(8*(a^5*b^4*e^2*f^8 + 16*a^7*c^2*e^2*f^8 - 8*a^6*b^2*c*e^2*f^8)))^(1/2)*2i
```

sympy [A] time = 14.01, size = 411, normalized size = 1.74

$$\frac{-a+3bd^2+6bdex+3be^2x^2}{3a^2d^3ef^4+9a^2d^2e^2f^4x+9a^2de^3f^4x^2+3a^2e^4f^4x^3}+\operatorname{RootSum}\left(t^4\left(256a^7c^2e^4f^{16}-128a^6b^2ce^4f^{16}+16a^5b^4e^4f^{16}\right)+t^2\left(-80a^3bc^3e^2f^8+100a^2b^3c^2e^2f^8-36ab^5ce^2f^8+4b^7e^2f^8\right)+c^5,\left(t\mapsto t\log\left(x+\frac{-96t^3a^7bc^2e^3f^{12}+56t^3a^6b^3ce^3f^{12}-8t^3a^5b^5e^3f^{12}-4ta^4c^4ef^4+32ta^3b^2c^3ef^4-40ta^2b^4c^2ef^4+16tab^6cef^4-2tb^8ef^4+a^2c^5d-3ab^2c^4d+b^4c^3d}{a^2c^5e-3ab^2c^4e+b^4c^3e}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*f*x+d*f)**4/(a+b*(e*x+d)**2+c*(e*x+d)**4),x)
```

```
[Out] (-a + 3*b*d**2 + 6*b*d*e*x + 3*b*e**2*x**2)/(3*a**2*d**3*e*f**4 + 9*a**2*d*
*2*e**2*f**4*x + 9*a**2*d*e**3*f**4*x**2 + 3*a**2*e**4*f**4*x**3) + RootSum
(_t**4*(256*a**7*c**2*e**4*f**16 - 128*a**6*b**2*c*e**4*f**16 + 16*a**5*b**
4*e**4*f**16) + _t**2*(-80*a**3*b*c**3*e**2*f**8 + 100*a**2*b**3*c**2*e**2*
f**8 - 36*a*b**5*c*e**2*f**8 + 4*b**7*e**2*f**8) + c**5, Lambda(_t, _t*log(
x + (-96*_t**3*a**7*b*c**2*e**3*f**12 + 56*_t**3*a**6*b**3*c*e**3*f**12 - 8
*_t**3*a**5*b**5*e**3*f**12 - 4*_t*a**4*c**4*e*f**4 + 32*_t*a**3*b**2*c**3*
e*f**4 - 40*_t*a**2*b**4*c**2*e*f**4 + 16*_t*a*b**6*c*e*f**4 - 2*_t*b**8*e*
f**4 + a**2*c**5*d - 3*a*b**2*c**4*d + b**4*c**3*d)/(a**2*c**5*e - 3*a*b**2
*c**4*e + b**4*c**3*e))))
```

3.533 $\int \frac{(df+efx)^4}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx$

Optimal. Leaf size=279

$$\frac{f^4(d+ex)\left(2a+b(d+ex)^2\right)}{2e\left(b^2-4ac\right)\left(a+b(d+ex)^2+c(d+ex)^4\right)}+\frac{f^4\left(b-\frac{4ac+b^2}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\sqrt{c}\,e\left(b^2-4ac\right)\sqrt{b-\sqrt{b^2-4ac}}}+\frac{f^4\left(b\sqrt{b^2-4ac}+4ac+b^{\cdots}\right.}{2\sqrt{2}\sqrt{c}\,e\left(b^2-4ac\right)^{3/2}\cdots}$$

Rubi [A] time = 0.54, antiderivative size = 279, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.121, Rules used = {1142, 1120, 1166, 205}

$$\frac{f^4(d+ex)\left(2a+b(d+ex)^2\right)}{2e\left(b^2-4ac\right)\left(a+b(d+ex)^2+c(d+ex)^4\right)}+\frac{f^4\left(b-\frac{4ac+b^2}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\sqrt{c}\,e\left(b^2-4ac\right)\sqrt{b-\sqrt{b^2-4ac}}}+\frac{f^4\left(b\sqrt{b^2-4ac}+4ac+b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,(d+ex)}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{2\sqrt{2}\sqrt{c}\,e\left(b^2-4ac\right)^{3/2}\sqrt{\sqrt{b^2-4ac}+b}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*f + e*f*x)^4/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2,x]
```

```
[Out] (f^4*(d + e*x)*(2*a + b*(d + e*x)^2))/(2*(b^2 - 4*a*c)*e*(a + b*(d + e*x)^2
 + c*(d + e*x)^4)) + ((b - (b^2 + 4*a*c)/Sqrt[b^2 - 4*a*c])*f^4*ArcTan[(Sqr
t[2]*Sqrt[c]*(d + e*x))/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*Sqrt[c]*(b
^2 - 4*a*c)*Sqrt[b - Sqrt[b^2 - 4*a*c]]*e) + ((b^2 + 4*a*c + b*Sqrt[b^2 - 4
*a*c])*f^4*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b + Sqrt[b^2 - 4*a*c]]])
/(2*Sqrt[2]*Sqrt[c]*(b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]*e)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1120

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> -Simp[(d^3*(d*x)^(m - 3)*(2*a + b*x^2)*(a + b*x^2 + c*x^4)^(p + 1))/(2*
(p + 1)*(b^2 - 4*a*c)), x] + Dist[d^4/(2*(p + 1)*(b^2 - 4*a*c)), Int[(d*x)^
(m - 4)*(2*a*(m - 3) + b*(m + 4*p + 3)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && GtQ[
m, 3] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1142

```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Di
st[u^m/(Coefficient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p,
 x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{(df+efx)^4}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx = \frac{f^4\,\text{Subst}\left(\int \frac{x^4}{\left(a+bx^2+cx^4\right)^2}\,dx,x,d+ex\right)}{e}$$

$$= \frac{f^4(d+ex)\left(2a+b(d+ex)^2\right)}{2\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)} - \frac{f^4\,\text{Subst}\left(\int \frac{2a-bx^2}{a+bx^2+cx^4}\,dx,x,\right.}{2\left(b^2-4ac\right)e}$$

$$= \frac{f^4(d+ex)\left(2a+b(d+ex)^2\right)}{2\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)} - \frac{\left(\left(b^2+4ac-b\sqrt{b^2-4ac}\right)f\right.}{4}$$

$$= \frac{f^4(d+ex)\left(2a+b(d+ex)^2\right)}{2\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)} - \frac{\left(b^2+4ac-b\sqrt{b^2-4ac}\right)f^4}{2\sqrt{2}\sqrt{c}\left(b^2-4ac\right)^{3/2}\sqrt{}}$$

Mathematica [A] time = 0.46, size = 266, normalized size = 0.95

$$\frac{f^4\left(-\frac{2\left(-2a(d+ex)-b(d+ex)^3\right)}{\left(b^2-4ac\right)\left(a+b(d+ex)^2+c(d+ex)^4\right)}+\frac{\sqrt{2}\left(b\sqrt{b^2-4ac}-4ac-b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{c}\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{2}\left(b\sqrt{b^2-4ac}+4ac+b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{c}\left(b^2-4ac\right)^{3/2}\sqrt{\sqrt{b^2-4ac}+b}}\right)}{4e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*f + e*f*x)^4/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2,x]
```

```
[Out] (f^4*((-2*(-2*a*(d + e*x) - b*(d + e*x)^3))/((b^2 - 4*a*c)*(a + b*(d + e*x)
^2 + c*(d + e*x)^4)) + (Sqrt[2]*(-b^2 - 4*a*c + b*Sqrt[b^2 - 4*a*c])*ArcTan
[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[c]*(b^2 -
4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[2]*(b^2 + 4*a*c + b*Sqrt[
b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b + Sqrt[b^2 - 4*a*c]
]])/(Sqrt[c]*(b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]])))/(4*e)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(df+efx)^4}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(d*f + e*f*x)^4/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2,x]
```

```
[Out] IntegrateAlgebraic[(d*f + e*f*x)^4/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2, x
]
```

fricas [B] time = 1.26, size = 2578, normalized size = 9.24

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*f*x+d*f)^4/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="fricas"
)
```

```
[Out] 1/4*(2*b*e^3*f^4*x^3 + 6*b*d*e^2*f^4*x^2 + 2*(3*b*d^2 + 2*a)*e*f^4*x + 2*(b
*d^3 + 2*a*d)*f^4 + sqrt(1/2)*((b^2*c - 4*a*c^2)*e^5*x^4 + 4*(b^2*c - 4*a*c
```

```
^2)*d*e^4*x^3 + (b^3 - 4*a*b*c + 6*(b^2*c - 4*a*c^2)*d^2)*e^3*x^2 + 2*(2*(b
^2*c - 4*a*c^2)*d^3 + (b^3 - 4*a*b*c)*d)*e^2*x + ((b^2*c - 4*a*c^2)*d^4 + a
*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*d^2)*e)*sqrt(-((b^3 + 12*a*b*c)*f^8 + sqrt
(f^16/((b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*e^4))*(b^6*c
- 12*a*b^4*c^2 + 48*a^2*b^2*c^3 - 64*a^3*c^4)*e^2)/((b^6*c - 12*a*b^4*c^2 +
 48*a^2*b^2*c^3 - 64*a^3*c^4)*e^2))*log((3*b^2 + 4*a*c)*e*f^12*x + (3*b^2 +
 4*a*c)*d*f^12 + sqrt(1/2)*((b^4 - 8*a*b^2*c + 16*a^2*c^2)*e*f^8 + 2*sqrt(f
^16/((b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*e^4))*(b^7*c -
12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*e^3)*sqrt(-((b^3 + 12*a*b*c)*
f^8 + sqrt(f^16/((b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*e^4
))*(b^6*c - 12*a*b^4*c^2 + 48*a^2*b^2*c^3 - 64*a^3*c^4)*e^2)/((b^6*c - 12*a
*b^4*c^2 + 48*a^2*b^2*c^3 - 64*a^3*c^4)*e^2))) - sqrt(1/2)*((b^2*c - 4*a*c^
2)*e^5*x^4 + 4*(b^2*c - 4*a*c^2)*d*e^4*x^3 + (b^3 - 4*a*b*c + 6*(b^2*c - 4*
a*c^2)*d^2)*e^3*x^2 + 2*(2*(b^2*c - 4*a*c^2)*d^3 + (b^3 - 4*a*b*c)*d)*e^2*x
 + ((b^2*c - 4*a*c^2)*d^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*d^2)*e)*sqrt(
-((b^3 + 12*a*b*c)*f^8 + sqrt(f^16/((b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^
4 - 64*a^3*c^5)*e^4))*(b^6*c - 12*a*b^4*c^2 + 48*a^2*b^2*c^3 - 64*a^3*c^4)*
e^2)/((b^6*c - 12*a*b^4*c^2 + 48*a^2*b^2*c^3 - 64*a^3*c^4)*e^2))*log((3*b^2
 + 4*a*c)*e*f^12*x + (3*b^2 + 4*a*c)*d*f^12 - sqrt(1/2)*((b^4 - 8*a*b^2*c +
 16*a^2*c^2)*e*f^8 + 2*sqrt(f^16/((b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4
- 64*a^3*c^5)*e^4))*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*
e^3)*sqrt(-((b^3 + 12*a*b*c)*f^8 + sqrt(f^16/((b^6*c^2 - 12*a*b^4*c^3 + 48*
a^2*b^2*c^4 - 64*a^3*c^5)*e^4))*(b^6*c - 12*a*b^4*c^2 + 48*a^2*b^2*c^3 - 64
*a^3*c^4)*e^2)/((b^6*c - 12*a*b^4*c^2 + 48*a^2*b^2*c^3 - 64*a^3*c^4)*e^2)))
 + sqrt(1/2)*((b^2*c - 4*a*c^2)*e^5*x^4 + 4*(b^2*c - 4*a*c^2)*d*e^4*x^3 + (
b^3 - 4*a*b*c + 6*(b^2*c - 4*a*c^2)*d^2)*e^3*x^2 + 2*(2*(b^2*c - 4*a*c^2)*d
^3 + (b^3 - 4*a*b*c)*d)*e^2*x + ((b^2*c - 4*a*c^2)*d^4 + a*b^2 - 4*a^2*c +
(b^3 - 4*a*b*c)*d^2)*e)*sqrt(-((b^3 + 12*a*b*c)*f^8 - sqrt(f^16/((b^6*c^2 -
 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*e^4))*(b^6*c - 12*a*b^4*c^2 +
48*a^2*b^2*c^3 - 64*a^3*c^4)*e^2)/((b^6*c - 12*a*b^4*c^2 + 48*a^2*b^2*c^3 -
 64*a^3*c^4)*e^2))*log((3*b^2 + 4*a*c)*e*f^12*x + (3*b^2 + 4*a*c)*d*f^12 +
sqrt(1/2)*((b^4 - 8*a*b^2*c + 16*a^2*c^2)*e*f^8 - 2*sqrt(f^16/((b^6*c^2 - 1
2*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*e^4))*(b^7*c - 12*a*b^5*c^2 + 48
*a^2*b^3*c^3 - 64*a^3*b*c^4)*e^3)*sqrt(-((b^3 + 12*a*b*c)*f^8 - sqrt(f^16/(
(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*e^4))*(b^6*c - 12*a*
b^4*c^2 + 48*a^2*b^2*c^3 - 64*a^3*c^4)*e^2)/((b^6*c - 12*a*b^4*c^2 + 48*a^2
*b^2*c^3 - 64*a^3*c^4)*e^2))) - sqrt(1/2)*((b^2*c - 4*a*c^2)*e^5*x^4 + 4*(b
^2*c - 4*a*c^2)*d*e^4*x^3 + (b^3 - 4*a*b*c + 6*(b^2*c - 4*a*c^2)*d^2)*e^3*x
^2 + 2*(2*(b^2*c - 4*a*c^2)*d^3 + (b^3 - 4*a*b*c)*d)*e^2*x + ((b^2*c - 4*a*
c^2)*d^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*d^2)*e)*sqrt(-((b^3 + 12*a*b*c
)*f^8 - sqrt(f^16/((b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*e
^4))*(b^6*c - 12*a*b^4*c^2 + 48*a^2*b^2*c^3 - 64*a^3*c^4)*e^2)/((b^6*c - 12
*a*b^4*c^2 + 48*a^2*b^2*c^3 - 64*a^3*c^4)*e^2))*log((3*b^2 + 4*a*c)*e*f^12*
x + (3*b^2 + 4*a*c)*d*f^12 - sqrt(1/2)*((b^4 - 8*a*b^2*c + 16*a^2*c^2)*e*f^
8 - 2*sqrt(f^16/((b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*e^4
))*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*e^3)*sqrt(-((b^3
+ 12*a*b*c)*f^8 - sqrt(f^16/((b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*
a^3*c^5)*e^4))*(b^6*c - 12*a*b^4*c^2 + 48*a^2*b^2*c^3 - 64*a^3*c^4)*e^2)/((
b^6*c - 12*a*b^4*c^2 + 48*a^2*b^2*c^3 - 64*a^3*c^4)*e^2))))/((b^2*c - 4*a*c
^2)*e^5*x^4 + 4*(b^2*c - 4*a*c^2)*d*e^4*x^3 + (b^3 - 4*a*b*c + 6*(b^2*c - 4
*a*c^2)*d^2)*e^3*x^2 + 2*(2*(b^2*c - 4*a*c^2)*d^3 + (b^3 - 4*a*b*c)*d)*e^2*
x + ((b^2*c - 4*a*c^2)*d^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*d^2)*e)
```

giac [B] time = 0.60, size = 1370, normalized size = 4.91

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*f*x+d*f)^4/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="giac")
```

```
[Out] -1/4*(((d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c
))^2*b*f^4*e^2 - 2*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e
^2)*e^(-4)/c))*b*d*f^4*e + b*d^2*f^4 - 2*a*f^4)*log(d*e^(-1) + x + sqrt(1/2
)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))/(2*(d*e^(-1) + sqrt(1/2)
*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^3*c*e^4 - 6*(d*e^(-1) + s
qrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^2*c*d*e^3 - 2*c*d
^3*e - b*d*e + (6*c*d^2*e^2 + b*e^2)*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 + s
qrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))) + ((d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 + s
qrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^2*b*f^4*e^2 - 2*(d*e^(-1) - sqrt(1/2)*sqrt
(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))*b*d*f^4*e + b*d^2*f^4 - 2*a*f^
4)*log(d*e^(-1) + x - sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4
)/c))/(2*(d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)
/c))^3*c*e^4 - 6*(d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2
)*e^(-4)/c))^2*c*d*e^3 - 2*c*d^3*e - b*d*e + (6*c*d^2*e^2 + b*e^2)*(d*e^(-1
) - sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))) + ((d*e^(-1
) + sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^2*b*f^4*e^2
- 2*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))*
b*d*f^4*e + b*d^2*f^4 - 2*a*f^4)*log(d*e^(-1) + x + sqrt(1/2)*sqrt(-(b*e^2
- sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))/(2*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 -
 sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^3*c*e^4 - 6*(d*e^(-1) + sqrt(1/2)*sqrt(-
(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^2*c*d*e^3 - 2*c*d^3*e - b*d*e +
(6*c*d^2*e^2 + b*e^2)*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c
)*e^2)*e^(-4)/c))) + ((d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c
)*e^2)*e^(-4)/c))^2*b*f^4*e^2 - 2*(d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 - sqrt
(b^2 - 4*a*c)*e^2)*e^(-4)/c))*b*d*f^4*e + b*d^2*f^4 - 2*a*f^4)*log(d*e^(-1)
 + x - sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))/(2*(d*e^(
-1) - sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^3*c*e^4 -
6*(d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^2*
c*d*e^3 - 2*c*d^3*e - b*d*e + (6*c*d^2*e^2 + b*e^2)*(d*e^(-1) - sqrt(1/2)*s
qrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))))/(b^2 - 4*a*c) + 1/2*(b*f^
4*x^3*e^3 + 3*b*d*f^4*x^2*e^2 + 3*b*d^2*f^4*x*e + b*d^3*f^4 + 2*a*f^4*x*e +
 2*a*d*f^4)/((c*x^4*e^4 + 4*c*d*x^3*e^3 + 6*c*d^2*x^2*e^2 + 4*c*d^3*x*e + c
*d^4 + b*x^2*e^2 + 2*b*d*x*e + b*d^2 + a)*(b^2*e - 4*a*c*e))
```

maple [C] time = 0.02, size = 695, normalized size = 2.49

$$-\frac{b e^{2} f^{4} x^{3}}{2 \left(c e^{4} x^{4}+4 c d e^{3} x^{3}+6 c d^{2} e^{2} x^{2}+4 c d^{3} e x+b e^{2} x^{2}+c d^{4}+2 b d e x+b d^{2}+a\right) \left(4 a c-b^{2}\right)}-\frac{3 b d e f^{4} x^{2}}{2 \left(c e^{4} x^{4}+4 c d e^{3} x^{3}+6 c d^{2} e^{2} x^{2}+4 c d^{3} e x+b e^{2} x^{2}+c d^{4}+2 b d e x+b d^{2}+a\right) \left(4 a c-b^{2}\right)}-\frac{3 b d^{2} f^{4} x}{2 \left(c e^{4} x^{4}+4 c d e^{3} x^{3}+6 c d^{2} e^{2} x^{2}+4 c d^{3} e x+b e^{2} x^{2}+c d^{4}+2 b d e x+b d^{2}+a\right) \left(4 a c-b^{2}\right)}-\frac{b d^{3} f^{4}}{2 \left(c e^{4} x^{4}+4 c d e^{3} x^{3}+6 c d^{2} e^{2} x^{2}+4 c d^{3} e x+b e^{2} x^{2}+c d^{4}+2 b d e x+b d^{2}+a\right) \left(4 a c-b^{2}\right) e}-\frac{a f^{4} x}{\left(c e^{4} x^{4}+4 c d e^{3} x^{3}+6 c d^{2} e^{2} x^{2}+4 c d^{3} e x+b e^{2} x^{2}+c d^{4}+2 b d e x+b d^{2}+a\right) \left(4 a c-b^{2}\right)}-\frac{a d f^{4}}{\left(c e^{4} x^{4}+4 c d e^{3} x^{3}+6 c d^{2} e^{2} x^{2}+4 c d^{3} e x+b e^{2} x^{2}+c d^{4}+2 b d e x+b d^{2}+a\right) \left(4 a c-b^{2}\right) e}+\frac{f^{4} \left(\sum_{\_R =\mathit{RootOf}\left(\_Z^{4} c e^{4}+4 \_Z^{3} c d e^{3}+c d^{4}+b d^{2}+\left(6 c d^{2} e^{2}+b e^{2}\right) \_Z^{2}+\left(4 c d^{3} e+2 b d e\right) \_Z +a \right)}\frac{\left(-\_R^{2} b e^{2}-2 \_R b d e-b d^{2}+2 a \right) \ln \left(-\_R +x \right)}{2 \_R^{3} c e^{3}+6 \_R^{2} c d e^{2}+6 \_R c d^{2} e+2 c d^{3}+\_R b e+b d}\right)}{4 \left(4 a c-b^{2}\right) e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*f*x+d*f)^4/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x)
```

```
[Out] -1/2*f^4/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d
^4+2*b*d*e*x+b*d^2+a)*b*e^2/(4*a*c-b^2)*x^3-3/2*f^4/(c*e^4*x^4+4*c*d*e^3*x^
3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)*d*b*e/(4*a
*c-b^2)*x^2-3/2*f^4/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*
e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)/(4*a*c-b^2)*x*b*d^2-f^4/(c*e^4*x^4+4*c*d*e
^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)/(4*a*
c-b^2)*x*a-1/2*f^4/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e
^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)/(4*a*c-b^2)*d^3/e*b-f^4/(c*e^4*x^4+4*c*d*e^
3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)/(4*a*c
-b^2)*d/e*a+1/4*f^4/(4*a*c-b^2)/e*sum((-_R^2*b*e^2-2*_R*b*d*e-b*d^2+2*a)/(2
*_R^3*c*e^3+6*_R^2*c*d*e^2+6*_R*c*d^2*e+2*c*d^3+_R*b*e+b*d)*ln(-_R+x),_R=Ro
otOf(_Z^4*c*e^4+4*_Z^3*c*d*e^3+c*d^4+b*d^2+(6*c*d^2*e^2+b*e^2)*_Z^2+(4*c*d^
3*e+2*b*d*e)*_Z+a))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{2} f^{4} \int -\frac{b e^{2} x^{2} + 2 b d e x + b d^{2} - 2 a}{{\left(b^{2} c - 4 a c^{2}\right)} e^{4} x^{4} + 4 {\left(b^{2} c - 4 a c^{2}\right)} d e^{3} x^{3} + {\left(b^{2} c - 4 a c^{2}\right)} d^{4} + {\left(b^{3} - 4 a b c + 6 {\left(b^{2} c - 4 a c^{2}\right)} d^{2}\right)} e^{2} x^{2} + a b^{2} - 4 a^{2} c + {\left(b^{3} - 4 a b c\right)} d^{2} + 2 {\left(2 {\left(b^{2} c - 4 a c^{2}\right)} d^{3} + {\left(b^{3} - 4 a b c\right)} d\right)} e x}\,{d x} + \frac{b e^{3} f^{4} x^{3} + 3 b d e^{2} f^{4} x^{2} + {\left(3 b d^{2} + 2 a\right)} e f^{4} x + {\left(b d^{3} + 2 a d\right)} f^{4}}{2 {\left({\left(b^{2} c - 4 a c^{2}\right)} e^{5} x^{4} + 4 {\left(b^{2} c - 4 a c^{2}\right)} d e^{4} x^{3} + {\left(b^{3} - 4 a b c + 6 {\left(b^{2} c - 4 a c^{2}\right)} d^{2}\right)} e^{3} x^{2} + 2 {\left(2 {\left(b^{2} c - 4 a c^{2}\right)} d^{3} + {\left(b^{3} - 4 a b c\right)} d\right)} e^{2} x + {\left({\left(b^{2} c - 4 a c^{2}\right)} d^{4} + a b^{2} - 4 a^{2} c + {\left(b^{3} - 4 a b c\right)} d^{2}\right)} e\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*f*x+d*f)^4/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="maxima"
)
```

```
[Out] -1/2*f^4*integrate(-(b*e^2*x^2 + 2*b*d*e*x + b*d^2 - 2*a)/((b^2*c - 4*a*c^2
)*e^4*x^4 + 4*(b^2*c - 4*a*c^2)*d*e^3*x^3 + (b^2*c - 4*a*c^2)*d^4 + (b^3 -
4*a*b*c + 6*(b^2*c - 4*a*c^2)*d^2)*e^2*x^2 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b
*c)*d^2 + 2*(2*(b^2*c - 4*a*c^2)*d^3 + (b^3 - 4*a*b*c)*d)*e*x), x) + 1/2*(b
*e^3*f^4*x^3 + 3*b*d*e^2*f^4*x^2 + (3*b*d^2 + 2*a)*e*f^4*x + (b*d^3 + 2*a*d
)*f^4)/((b^2*c - 4*a*c^2)*e^5*x^4 + 4*(b^2*c - 4*a*c^2)*d*e^4*x^3 + (b^3 -
4*a*b*c + 6*(b^2*c - 4*a*c^2)*d^2)*e^3*x^2 + 2*(2*(b^2*c - 4*a*c^2)*d^3 + (
b^3 - 4*a*b*c)*d)*e^2*x + ((b^2*c - 4*a*c^2)*d^4 + a*b^2 - 4*a^2*c + (b^3 -
 4*a*b*c)*d^2)*e)
```

mupad [B] time = 5.09, size = 8025, normalized size = 28.76

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*f + e*f*x)^4/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2,x)
```

```
[Out] atan(((((2048*a^4*c^5*e^12*f^4 + 384*a^2*b^4*c^3*e^12*f^4 - 1536*a^3*b^2*c^
4*e^12*f^4 - 32*a*b^6*c^2*e^12*f^4)/(8*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 -
 12*a*b^4*c)) + ((64*b^9*c^2*d*e^13 - 1024*a*b^7*c^3*d*e^13 + 16384*a^4*b*c
^6*d*e^13 + 6144*a^2*b^5*c^4*d*e^13 - 16384*a^3*b^3*c^5*d*e^13)/(8*(b^6 - 6
4*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) + (x*(16*b^7*c^2*e^14 - 192*a*b^5
*c^3*e^14 - 1024*a^3*b*c^5*e^14 + 768*a^2*b^3*c^4*e^14))/(2*(b^4 + 16*a^2*c
^2 - 8*a*b^2*c)))*(-(b^9*f^8 + f^8*(-(4*a*c - b^2)^9)^(1/2) - 768*a^4*b*c^4
*f^8 - 96*a^2*b^5*c^2*f^8 + 512*a^3*b^3*c^3*f^8)/(32*(b^12*c*e^2 + 4096*a^6
*c^7*e^2 - 24*a*b^10*c^2*e^2 + 240*a^2*b^8*c^3*e^2 - 1280*a^3*b^6*c^4*e^2 +
 3840*a^4*b^4*c^5*e^2 - 6144*a^5*b^2*c^6*e^2)))^(1/2))*(-(b^9*f^8 + f^8*(-(
4*a*c - b^2)^9)^(1/2) - 768*a^4*b*c^4*f^8 - 96*a^2*b^5*c^2*f^8 + 512*a^3*b^
3*c^3*f^8)/(32*(b^12*c*e^2 + 4096*a^6*c^7*e^2 - 24*a*b^10*c^2*e^2 + 240*a^2
*b^8*c^3*e^2 - 1280*a^3*b^6*c^4*e^2 + 3840*a^4*b^4*c^5*e^2 - 6144*a^5*b^2*c
^6*e^2)))^(1/2) - (128*a^3*c^4*d*e^11*f^8 - 4*b^6*c*d*e^11*f^8 + 8*a*b^4*c^
2*d*e^11*f^8)/(8*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) + (x*(b^
4*c*e^12*f^8 + 8*a^2*c^3*e^12*f^8 + 2*a*b^2*c^2*e^12*f^8))/(2*(b^4 + 16*a^2
*c^2 - 8*a*b^2*c)))*(-(b^9*f^8 + f^8*(-(4*a*c - b^2)^9)^(1/2) - 768*a^4*b*c
^4*f^8 - 96*a^2*b^5*c^2*f^8 + 512*a^3*b^3*c^3*f^8)/(32*(b^12*c*e^2 + 4096*a
^6*c^7*e^2 - 24*a*b^10*c^2*e^2 + 240*a^2*b^8*c^3*e^2 - 1280*a^3*b^6*c^4*e^2
 + 3840*a^4*b^4*c^5*e^2 - 6144*a^5*b^2*c^6*e^2)))^(1/2)*1i - ((128*a^3*c^4*
d*e^11*f^8 - 4*b^6*c*d*e^11*f^8 + 8*a*b^4*c^2*d*e^11*f^8)/(8*(b^6 - 64*a^3*
c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) + ((2048*a^4*c^5*e^12*f^4 + 384*a^2*b^4
*c^3*e^12*f^4 - 1536*a^3*b^2*c^4*e^12*f^4 - 32*a*b^6*c^2*e^12*f^4)/(8*(b^6
- 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) - ((64*b^9*c^2*d*e^13 - 1024*a
*b^7*c^3*d*e^13 + 16384*a^4*b*c^6*d*e^13 + 6144*a^2*b^5*c^4*d*e^13 - 16384*
a^3*b^3*c^5*d*e^13)/(8*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) +
(x*(16*b^7*c^2*e^14 - 192*a*b^5*c^3*e^14 - 1024*a^3*b*c^5*e^14 + 768*a^2*b^
3*c^4*e^14))/(2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)))*(-(b^9*f^8 + f^8*(-(4*a*c
- b^2)^9)^(1/2) - 768*a^4*b*c^4*f^8 - 96*a^2*b^5*c^2*f^8 + 512*a^3*b^3*c^3*
f^8)/(32*(b^12*c*e^2 + 4096*a^6*c^7*e^2 - 24*a*b^10*c^2*e^2 + 240*a^2*b^8*c
^3*e^2 - 1280*a^3*b^6*c^4*e^2 + 3840*a^4*b^4*c^5*e^2 - 6144*a^5*b^2*c^6*e^2
)))^(1/2))*(-(b^9*f^8 + f^8*(-(4*a*c - b^2)^9)^(1/2) - 768*a^4*b*c^4*f^8 -
96*a^2*b^5*c^2*f^8 + 512*a^3*b^3*c^3*f^8)/(32*(b^12*c*e^2 + 4096*a^6*c^7*e^
2 - 24*a*b^10*c^2*e^2 + 240*a^2*b^8*c^3*e^2 - 1280*a^3*b^6*c^4*e^2 + 3840*a
^4*b^4*c^5*e^2 - 6144*a^5*b^2*c^6*e^2)))^(1/2) - (x*(b^4*c*e^12*f^8 + 8*a^2
*c^3*e^12*f^8 + 2*a*b^2*c^2*e^12*f^8))/(2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)))*
(-(b^9*f^8 + f^8*(-(4*a*c - b^2)^9)^(1/2) - 768*a^4*b*c^4*f^8 - 96*a^2*b^5*
c^2*f^8 + 512*a^3*b^3*c^3*f^8)/(32*(b^12*c*e^2 + 4096*a^6*c^7*e^2 - 24*a*b^
10*c^2*e^2 + 240*a^2*b^8*c^3*e^2 - 1280*a^3*b^6*c^4*e^2 + 3840*a^4*b^4*c^5*
e^2 - 6144*a^5*b^2*c^6*e^2)))^(1/2)*1i)/((((2048*a^4*c^5*e^12*f^4 + 384*a^2
*b^4*c^3*e^12*f^4 - 1536*a^3*b^2*c^4*e^12*f^4 - 32*a*b^6*c^2*e^12*f^4)/(8*(
```

```
b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) + ((64*b^9*c^2*d*e^13 - 10
24*a*b^7*c^3*d*e^13 + 16384*a^4*b*c^6*d*e^13 + 6144*a^2*b^5*c^4*d*e^13 - 16
384*a^3*b^3*c^5*d*e^13)/(8*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)
) + (x*(16*b^7*c^2*e^14 - 192*a*b^5*c^3*e^14 - 1024*a^3*b*c^5*e^14 + 768*a^
2*b^3*c^4*e^14))/(2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)))*(-(b^9*f^8 + f^8*(-(4*
a*c - b^2)^9)^(1/2) - 768*a^4*b*c^4*f^8 - 96*a^2*b^5*c^2*f^8 + 512*a^3*b^3*
c^3*f^8)/(32*(b^12*c*e^2 + 4096*a^6*c^7*e^2 - 24*a*b^10*c^2*e^2 + 240*a^2*b
^8*c^3*e^2 - 1280*a^3*b^6*c^4*e^2 + 3840*a^4*b^4*c^5*e^2 - 6144*a^5*b^2*c^6
*e^2)))^(1/2))*(-(b^9*f^8 + f^8*(-(4*a*c - b^2)^9)^(1/2) - 768*a^4*b*c^4*f^
8 - 96*a^2*b^5*c^2*f^8 + 512*a^3*b^3*c^3*f^8)/(32*(b^12*c*e^2 + 4096*a^6*c^
7*e^2 - 24*a*b^10*c^2*e^2 + 240*a^2*b^8*c^3*e^2 - 1280*a^3*b^6*c^4*e^2 + 38
40*a^4*b^4*c^5*e^2 - 6144*a^5*b^2*c^6*e^2)))^(1/2) - (128*a^3*c^4*d*e^11*f^
8 - 4*b^6*c*d*e^11*f^8 + 8*a*b^4*c^2*d*e^11*f^8)/(8*(b^6 - 64*a^3*c^3 + 48*
a^2*b^2*c^2 - 12*a*b^4*c)) + (x*(b^4*c*e^12*f^8 + 8*a^2*c^3*e^12*f^8 + 2*a*
b^2*c^2*e^12*f^8))/(2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)))*(-(b^9*f^8 + f^8*(-(
4*a*c - b^2)^9)^(1/2) - 768*a^4*b*c^4*f^8 - 96*a^2*b^5*c^2*f^8 + 512*a^3*b^
3*c^3*f^8)/(32*(b^12*c*e^2 + 4096*a^6*c^7*e^2 - 24*a*b^10*c^2*e^2 + 240*a^2
*b^8*c^3*e^2 - 1280*a^3*b^6*c^4*e^2 + 3840*a^4*b^4*c^5*e^2 - 6144*a^5*b^2*c
^6*e^2)))^(1/2) + ((128*a^3*c^4*d*e^11*f^8 - 4*b^6*c*d*e^11*f^8 + 8*a*b^4*c
^2*d*e^11*f^8)/(8*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) + ((204
8*a^4*c^5*e^12*f^4 + 384*a^2*b^4*c^3*e^12*f^4 - 1536*a^3*b^2*c^4*e^12*f^4 -
 32*a*b^6*c^2*e^12*f^4)/(8*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)
) - ((64*b^9*c^2*d*e^13 - 1024*a*b^7*c^3*d*e^13 + 16384*a^4*b*c^6*d*e^13 +
6144*a^2*b^5*c^4*d*e^13 - 16384*a^3*b^3*c^5*d*e^13)/(8*(b^6 - 64*a^3*c^3 +
48*a^2*b^2*c^2 - 12*a*b^4*c)) + (x*(16*b^7*c^2*e^14 - 192*a*b^5*c^3*e^14 -
1024*a^3*b*c^5*e^14 + 768*a^2*b^3*c^4*e^14))/(2*(b^4 + 16*a^2*c^2 - 8*a*b^2
*c)))*(-(b^9*f^8 + f^8*(-(4*a*c - b^2)^9)^(1/2) - 768*a^4*b*c^4*f^8 - 96*a^
2*b^5*c^2*f^8 + 512*a^3*b^3*c^3*f^8)/(32*(b^12*c*e^2 + 4096*a^6*c^7*e^2 - 2
4*a*b^10*c^2*e^2 + 240*a^2*b^8*c^3*e^2 - 1280*a^3*b^6*c^4*e^2 + 3840*a^4*b^
4*c^5*e^2 - 6144*a^5*b^2*c^6*e^2)))^(1/2))*(-(b^9*f^8 + f^8*(-(4*a*c - b^2)
^9)^(1/2) - 768*a^4*b*c^4*f^8 - 96*a^2*b^5*c^2*f^8 + 512*a^3*b^3*c^3*f^8)/(
32*(b^12*c*e^2 + 4096*a^6*c^7*e^2 - 24*a*b^10*c^2*e^2 + 240*a^2*b^8*c^3*e^2
 - 1280*a^3*b^6*c^4*e^2 + 3840*a^4*b^4*c^5*e^2 - 6144*a^5*b^2*c^6*e^2)))^(1
/2) - (x*(b^4*c*e^12*f^8 + 8*a^2*c^3*e^12*f^8 + 2*a*b^2*c^2*e^12*f^8))/(2*(
b^4 + 16*a^2*c^2 - 8*a*b^2*c)))*(-(b^9*f^8 + f^8*(-(4*a*c - b^2)^9)^(1/2) -
 768*a^4*b*c^4*f^8 - 96*a^2*b^5*c^2*f^8 + 512*a^3*b^3*c^3*f^8)/(32*(b^12*c*
e^2 + 4096*a^6*c^7*e^2 - 24*a*b^10*c^2*e^2 + 240*a^2*b^8*c^3*e^2 - 1280*a^3
*b^6*c^4*e^2 + 3840*a^4*b^4*c^5*e^2 - 6144*a^5*b^2*c^6*e^2)))^(1/2) - (3*a*
b^3*c*e^10*f^12 + 4*a^2*b*c^2*e^10*f^12)/(4*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*
c^2 - 12*a*b^4*c))))*(-(b^9*f^8 + f^8*(-(4*a*c - b^2)^9)^(1/2) - 768*a^4*b*
c^4*f^8 - 96*a^2*b^5*c^2*f^8 + 512*a^3*b^3*c^3*f^8)/(32*(b^12*c*e^2 + 4096*
a^6*c^7*e^2 - 24*a*b^10*c^2*e^2 + 240*a^2*b^8*c^3*e^2 - 1280*a^3*b^6*c^4*e^
2 + 3840*a^4*b^4*c^5*e^2 - 6144*a^5*b^2*c^6*e^2)))^(1/2)*2i - ((x*(2*a*f^4
+ 3*b*d^2*f^4))/(2*(4*a*c - b^2)) + (b*d^3*f^4 + 2*a*d*f^4)/(2*e*(4*a*c - b
^2)) + (b*e^2*f^4*x^3)/(2*(4*a*c - b^2)) + (3*b*d*e*f^4*x^2)/(2*(4*a*c - b^
2)))/(a + x^2*(b*e^2 + 6*c*d^2*e^2) + b*d^2 + c*d^4 + x*(2*b*d*e + 4*c*d^3*
e) + c*e^4*x^4 + 4*c*d*e^3*x^3) + atan(((((2048*a^4*c^5*e^12*f^4 + 384*a^2*
b^4*c^3*e^12*f^4 - 1536*a^3*b^2*c^4*e^12*f^4 - 32*a*b^6*c^2*e^12*f^4)/(8*(b
^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) + ((64*b^9*c^2*d*e^13 - 102
4*a*b^7*c^3*d*e^13 + 16384*a^4*b*c^6*d*e^13 + 6144*a^2*b^5*c^4*d*e^13 - 163
84*a^3*b^3*c^5*d*e^13)/(8*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c))
 + (x*(16*b^7*c^2*e^14 - 192*a*b^5*c^3*e^14 - 1024*a^3*b*c^5*e^14 + 768*a^2
*b^3*c^4*e^14))/(2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)))*((f^8*(-(4*a*c - b^2)^9
)^(1/2) - b^9*f^8 + 768*a^4*b*c^4*f^8 + 96*a^2*b^5*c^2*f^8 - 512*a^3*b^3*c^
3*f^8)/(32*(b^12*c*e^2 + 4096*a^6*c^7*e^2 - 24*a*b^10*c^2*e^2 + 240*a^2*b^8
*c^3*e^2 - 1280*a^3*b^6*c^4*e^2 + 3840*a^4*b^4*c^5*e^2 - 6144*a^5*b^2*c^6*e
^2)))^(1/2))*((f^8*(-(4*a*c - b^2)^9)^(1/2) - b^9*f^8 + 768*a^4*b*c^4*f^8 +
 96*a^2*b^5*c^2*f^8 - 512*a^3*b^3*c^3*f^8)/(32*(b^12*c*e^2 + 4096*a^6*c^7*e
^2 - 24*a*b^10*c^2*e^2 + 240*a^2*b^8*c^3*e^2 - 1280*a^3*b^6*c^4*e^2 + 3840*
```

```
a^4*b^4*c^5*e^2 - 6144*a^5*b^2*c^6*e^2)))^(1/2) - (128*a^3*c^4*d*e^11*f^8 -
 4*b^6*c*d*e^11*f^8 + 8*a*b^4*c^2*d*e^11*f^8)/(8*(b^6 - 64*a^3*c^3 + 48*a^2
*b^2*c^2 - 12*a*b^4*c)) + (x*(b^4*c*e^12*f^8 + 8*a^2*c^3*e^12*f^8 + 2*a*b^2
*c^2*e^12*f^8))/(2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)))*((f^8*(-(4*a*c - b^2)^9
)^(1/2) - b^9*f^8 + 768*a^4*b*c^4*f^8 + 96*a^2*b^5*c^2*f^8 - 512*a^3*b^3*c^
3*f^8)/(32*(b^12*c*e^2 + 4096*a^6*c^7*e^2 - 24*a*b^10*c^2*e^2 + 240*a^2*b^8
*c^3*e^2 - 1280*a^3*b^6*c^4*e^2 + 3840*a^4*b^4*c^5*e^2 - 6144*a^5*b^2*c^6*e
^2)))^(1/2)*1i - ((128*a^3*c^4*d*e^11*f^8 - 4*b^6*c*d*e^11*f^8 + 8*a*b^4*c^
2*d*e^11*f^8)/(8*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) + ((2048
*a^4*c^5*e^12*f^4 + 384*a^2*b^4*c^3*e^12*f^4 - 1536*a^3*b^2*c^4*e^12*f^4 -
32*a*b^6*c^2*e^12*f^4)/(8*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c))
 - ((64*b^9*c^2*d*e^13 - 1024*a*b^7*c^3*d*e^13 + 16384*a^4*b*c^6*d*e^13 + 6
144*a^2*b^5*c^4*d*e^13 - 16384*a^3*b^3*c^5*d*e^13)/(8*(b^6 - 64*a^3*c^3 + 4
8*a^2*b^2*c^2 - 12*a*b^4*c)) + (x*(16*b^7*c^2*e^14 - 192*a*b^5*c^3*e^14 - 1
024*a^3*b*c^5*e^14 + 768*a^2*b^3*c^4*e^14))/(2*(b^4 + 16*a^2*c^2 - 8*a*b^2*
c)))*((f^8*(-(4*a*c - b^2)^9)^(1/2) - b^9*f^8 + 768*a^4*b*c^4*f^8 + 96*a^2*
b^5*c^2*f^8 - 512*a^3*b^3*c^3*f^8)/(32*(b^12*c*e^2 + 4096*a^6*c^7*e^2 - 24*
a*b^10*c^2*e^2 + 240*a^2*b^8*c^3*e^2 - 1280*a^3*b^6*c^4*e^2 + 3840*a^4*b^4*
c^5*e^2 - 6144*a^5*b^2*c^6*e^2)))^(1/2))*((f^8*(-(4*a*c - b^2)^9)^(1/2) - b
^9*f^8 + 768*a^4*b*c^4*f^8 + 96*a^2*b^5*c^2*f^8 - 512*a^3*b^3*c^3*f^8)/(32*
(b^12*c*e^2 + 4096*a^6*c^7*e^2 - 24*a*b^10*c^2*e^2 + 240*a^2*b^8*c^3*e^2 -
1280*a^3*b^6*c^4*e^2 + 3840*a^4*b^4*c^5*e^2 - 6144*a^5*b^2*c^6*e^2)))^(1/2)
 - (x*(b^4*c*e^12*f^8 + 8*a^2*c^3*e^12*f^8 + 2*a*b^2*c^2*e^12*f^8))/(2*(b^4
 + 16*a^2*c^2 - 8*a*b^2*c)))*((f^8*(-(4*a*c - b^2)^9)^(1/2) - b^9*f^8 + 768
*a^4*b*c^4*f^8 + 96*a^2*b^5*c^2*f^8 - 512*a^3*b^3*c^3*f^8)/(32*(b^12*c*e^2
+ 4096*a^6*c^7*e^2 - 24*a*b^10*c^2*e^2 + 240*a^2*b^8*c^3*e^2 - 1280*a^3*b^6
*c^4*e^2 + 3840*a^4*b^4*c^5*e^2 - 6144*a^5*b^2*c^6*e^2)))^(1/2)*1i)/((((204
8*a^4*c^5*e^12*f^4 + 384*a^2*b^4*c^3*e^12*f^4 - 1536*a^3*b^2*c^4*e^12*f^4 -
 32*a*b^6*c^2*e^12*f^4)/(8*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)
) + ((64*b^9*c^2*d*e^13 - 1024*a*b^7*c^3*d*e^13 + 16384*a^4*b*c^6*d*e^13 +
6144*a^2*b^5*c^4*d*e^13 - 16384*a^3*b^3*c^5*d*e^13)/(8*(b^6 - 64*a^3*c^3 +
48*a^2*b^2*c^2 - 12*a*b^4*c)) + (x*(16*b^7*c^2*e^14 - 192*a*b^5*c^3*e^14 -
1024*a^3*b*c^5*e^14 + 768*a^2*b^3*c^4*e^14))/(2*(b^4 + 16*a^2*c^2 - 8*a*b^2
*c)))*((f^8*(-(4*a*c - b^2)^9)^(1/2) - b^9*f^8 + 768*a^4*b*c^4*f^8 + 96*a^2
*b^5*c^2*f^8 - 512*a^3*b^3*c^3*f^8)/(32*(b^12*c*e^2 + 4096*a^6*c^7*e^2 - 24
*a*b^10*c^2*e^2 + 240*a^2*b^8*c^3*e^2 - 1280*a^3*b^6*c^4*e^2 + 3840*a^4*b^4
*c^5*e^2 - 6144*a^5*b^2*c^6*e^2)))^(1/2))*((f^8*(-(4*a*c - b^2)^9)^(1/2) -
b^9*f^8 + 768*a^4*b*c^4*f^8 + 96*a^2*b^5*c^2*f^8 - 512*a^3*b^3*c^3*f^8)/(32
*(b^12*c*e^2 + 4096*a^6*c^7*e^2 - 24*a*b^10*c^2*e^2 + 240*a^2*b^8*c^3*e^2 -
 1280*a^3*b^6*c^4*e^2 + 3840*a^4*b^4*c^5*e^2 - 6144*a^5*b^2*c^6*e^2)))^(1/2
) - (128*a^3*c^4*d*e^11*f^8 - 4*b^6*c*d*e^11*f^8 + 8*a*b^4*c^2*d*e^11*f^8)/
(8*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) + (x*(b^4*c*e^12*f^8 +
 8*a^2*c^3*e^12*f^8 + 2*a*b^2*c^2*e^12*f^8))/(2*(b^4 + 16*a^2*c^2 - 8*a*b^2
*c)))*((f^8*(-(4*a*c - b^2)^9)^(1/2) - b^9*f^8 + 768*a^4*b*c^4*f^8 + 96*a^2
*b^5*c^2*f^8 - 512*a^3*b^3*c^3*f^8)/(32*(b^12*c*e^2 + 4096*a^6*c^7*e^2 - 24
*a*b^10*c^2*e^2 + 240*a^2*b^8*c^3*e^2 - 1280*a^3*b^6*c^4*e^2 + 3840*a^4*b^4
*c^5*e^2 - 6144*a^5*b^2*c^6*e^2)))^(1/2) + ((128*a^3*c^4*d*e^11*f^8 - 4*b^6
*c*d*e^11*f^8 + 8*a*b^4*c^2*d*e^11*f^8)/(8*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c
^2 - 12*a*b^4*c)) + ((2048*a^4*c^5*e^12*f^4 + 384*a^2*b^4*c^3*e^12*f^4 - 15
36*a^3*b^2*c^4*e^12*f^4 - 32*a*b^6*c^2*e^12*f^4)/(8*(b^6 - 64*a^3*c^3 + 48*
a^2*b^2*c^2 - 12*a*b^4*c)) - ((64*b^9*c^2*d*e^13 - 1024*a*b^7*c^3*d*e^13 +
16384*a^4*b*c^6*d*e^13 + 6144*a^2*b^5*c^4*d*e^13 - 16384*a^3*b^3*c^5*d*e^13
)/(8*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) + (x*(16*b^7*c^2*e^1
4 - 192*a*b^5*c^3*e^14 - 1024*a^3*b*c^5*e^14 + 768*a^2*b^3*c^4*e^14))/(2*(b
^4 + 16*a^2*c^2 - 8*a*b^2*c)))*((f^8*(-(4*a*c - b^2)^9)^(1/2) - b^9*f^8 + 7
68*a^4*b*c^4*f^8 + 96*a^2*b^5*c^2*f^8 - 512*a^3*b^3*c^3*f^8)/(32*(b^12*c*e^
2 + 4096*a^6*c^7*e^2 - 24*a*b^10*c^2*e^2 + 240*a^2*b^8*c^3*e^2 - 1280*a^3*b
^6*c^4*e^2 + 3840*a^4*b^4*c^5*e^2 - 6144*a^5*b^2*c^6*e^2)))^(1/2))*((f^8*(-
(4*a*c - b^2)^9)^(1/2) - b^9*f^8 + 768*a^4*b*c^4*f^8 + 96*a^2*b^5*c^2*f^8 -
```

```
512*a^3*b^3*c^3*f^8)/(32*(b^12*c*e^2 + 4096*a^6*c^7*e^2 - 24*a*b^10*c^2*e^
2 + 240*a^2*b^8*c^3*e^2 - 1280*a^3*b^6*c^4*e^2 + 3840*a^4*b^4*c^5*e^2 - 614
4*a^5*b^2*c^6*e^2)))^(1/2) - (x*(b^4*c*e^12*f^8 + 8*a^2*c^3*e^12*f^8 + 2*a*
b^2*c^2*e^12*f^8))/(2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)))*((f^8*(-(4*a*c - b^2
)^9)^(1/2) - b^9*f^8 + 768*a^4*b*c^4*f^8 + 96*a^2*b^5*c^2*f^8 - 512*a^3*b^3
*c^3*f^8)/(32*(b^12*c*e^2 + 4096*a^6*c^7*e^2 - 24*a*b^10*c^2*e^2 + 240*a^2*
b^8*c^3*e^2 - 1280*a^3*b^6*c^4*e^2 + 3840*a^4*b^4*c^5*e^2 - 6144*a^5*b^2*c^
6*e^2)))^(1/2) - (3*a*b^3*c*e^10*f^12 + 4*a^2*b*c^2*e^10*f^12)/(4*(b^6 - 64
*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c))))*((f^8*(-(4*a*c - b^2)^9)^(1/2) -
 b^9*f^8 + 768*a^4*b*c^4*f^8 + 96*a^2*b^5*c^2*f^8 - 512*a^3*b^3*c^3*f^8)/(3
2*(b^12*c*e^2 + 4096*a^6*c^7*e^2 - 24*a*b^10*c^2*e^2 + 240*a^2*b^8*c^3*e^2
- 1280*a^3*b^6*c^4*e^2 + 3840*a^4*b^4*c^5*e^2 - 6144*a^5*b^2*c^6*e^2)))^(1/
2)*2i
```

sympy [B] time = 9.82, size = 641, normalized size = 2.30

$$\frac{-2adf^4 - bd^3f^4 - 3bde^2f^4x^2 - be^3f^4x^3 + x\left(-2aef^4 - 3bd^2ef^4\right)}{8a^2ce - 2ab^2e + 8abcd^2e + 8ac^2d^4e - 2b^3d^2e - 2b^2cd^4e + x^4\left(8ac^2e^5 - 2b^2ce^5\right) + x^3\left(32ac^2de^4 - 8b^2cde^4\right) + x^2\left(8abce^3 + 48ac^2d^2e^3 - 2b^3e^3 - 12b^2cd^2e^3\right) + x\left(16abcde^2 + 32ac^2d^3e^2 - 4b^3de^2 - 8b^2cd^3e^2\right)} + \operatorname{RootSum}\left(t^4\left(1048576a^6c^7e^4 - 1572864a^5b^2c^6e^4 + 983040a^4b^4c^5e^4 - 327680a^3b^6c^4e^4 + 61440a^2b^8c^3e^4 - 6144ab^{10}c^2e^4 + 256b^{12}ce^4\right) + t^2\left(-12288a^4bc^4e^2f^8 + 8192a^3b^3c^3e^2f^8 - 1536a^2b^5c^2e^2f^8 + 16b^9e^2f^8\right) + 16a^3c^2f^{16} + 24a^2b^2cf^{16} + 9ab^4f^{16}, \left(t \mapsto t\log\left(x + \frac{16384t^3a^3bc^4e^3 - 12288t^3a^2b^3c^3e^3 + 3072t^3ab^5c^2e^3 - 256t^3b^7ce^3 + 64ta^2c^2ef^8 - 128tab^2cef^8 - 4tb^4ef^8 + 4acdf^{12} + 3b^2df^{12}}{4acef^{12} + 3b^2ef^{12}}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*f*x+d*f)**4/(a+b*(e*x+d)**2+c*(e*x+d)**4)**2,x)
```

```
[Out] (-2*a*d*f**4 - b*d**3*f**4 - 3*b*d*e**2*f**4*x**2 - b*e**3*f**4*x**3 + x*(-
2*a*e*f**4 - 3*b*d**2*e*f**4))/(8*a**2*c*e - 2*a*b**2*e + 8*a*b*c*d**2*e +
8*a*c**2*d**4*e - 2*b**3*d**2*e - 2*b**2*c*d**4*e + x**4*(8*a*c**2*e**5 - 2
*b**2*c*e**5) + x**3*(32*a*c**2*d*e**4 - 8*b**2*c*d*e**4) + x**2*(8*a*b*c*e
**3 + 48*a*c**2*d**2*e**3 - 2*b**3*e**3 - 12*b**2*c*d**2*e**3) + x*(16*a*b*
c*d*e**2 + 32*a*c**2*d**3*e**2 - 4*b**3*d*e**2 - 8*b**2*c*d**3*e**2)) + Roo
tSum(_t**4*(1048576*a**6*c**7*e**4 - 1572864*a**5*b**2*c**6*e**4 + 983040*a
**4*b**4*c**5*e**4 - 327680*a**3*b**6*c**4*e**4 + 61440*a**2*b**8*c**3*e**4
 - 6144*a*b**10*c**2*e**4 + 256*b**12*c*e**4) + _t**2*(-12288*a**4*b*c**4*e
**2*f**8 + 8192*a**3*b**3*c**3*e**2*f**8 - 1536*a**2*b**5*c**2*e**2*f**8 +
16*b**9*e**2*f**8) + 16*a**3*c**2*f**16 + 24*a**2*b**2*c*f**16 + 9*a*b**4*f
**16, Lambda(_t, _t*log(x + (16384*_t**3*a**3*b*c**4*e**3 - 12288*_t**3*a**
2*b**3*c**3*e**3 + 3072*_t**3*a*b**5*c**2*e**3 - 256*_t**3*b**7*c*e**3 + 64
*_t*a**2*c**2*e*f**8 - 128*_t*a*b**2*c*e*f**8 - 4*_t*b**4*e*f**8 + 4*a*c*d*
f**12 + 3*b**2*d*f**12)/(4*a*c*e*f**12 + 3*b**2*e*f**12))))
```

3.534 $\int \frac{(df+efx)^3}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx$

Optimal. Leaf size=103

$$\frac{f^3\left(2a+b(d+ex)^2\right)}{2e\left(b^2-4ac\right)\left(a+b(d+ex)^2+c(d+ex)^4\right)}-\frac{bf^3\tanh^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{e\left(b^2-4ac\right)^{3/2}}$$

Rubi [A] time = 0.14, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {1142, 1114, 638, 618, 206}

$$\frac{f^3\left(2a+b(d+ex)^2\right)}{2e\left(b^2-4ac\right)\left(a+b(d+ex)^2+c(d+ex)^4\right)}-\frac{bf^3\tanh^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{e\left(b^2-4ac\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*f + e*f*x)^3/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2,x]

[Out] (f^3*(2*a + b*(d + e*x)^2))/(2*(b^2 - 4*a*c)*e*(a + b*(d + e*x)^2 + c*(d +
e*x)^4)) - (b*f^3*ArcTanh[(b + 2*c*(d + e*x)^2)/Sqrt[b^2 - 4*a*c]])/((b^2 -
 4*a*c)^(3/2)*e)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 638

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)*(a + b*x + c*x^2)^(p + 1))/((p +
 1)*(b^2 - 4*a*c)), x] - Dist[((2*p + 3)*(2*c*d - b*e))/((p + 1)*(b^2 - 4*a
*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &&
 NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 1142

```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Di
st[u^m/(Coefficient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p,
 x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

Rubi steps

$$\int \frac{(df+efx)^3}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx = \frac{f^3\operatorname{Subst}\left(\int \frac{x^3}{(a+bx^2+cx^4)^2}\,dx, x, d+ex\right)}{e}$$

$$= \frac{f^3\operatorname{Subst}\left(\int \frac{x}{(a+bx+cx^2)^2}\,dx, x, (d+ex)^2\right)}{2e}$$

$$= \frac{f^3\left(2a+b(d+ex)^2\right)}{2\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)} + \frac{\left(bf^3\right)\operatorname{Subst}\left(\int \frac{1}{a+bx+cx^2}\,dx,\right.}{2\left(b^2-4ac\right)e}$$

$$= \frac{f^3\left(2a+b(d+ex)^2\right)}{2\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)} - \frac{\left(bf^3\right)\operatorname{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx,\right.}{\left(b^2-4ac\right)}$$

$$= \frac{f^3\left(2a+b(d+ex)^2\right)}{2\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)} - \frac{bf^3\tanh^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{3/2}e}$$

Mathematica [A] time = 0.13, size = 103, normalized size = 1.00

$$\frac{f^3\left(\frac{2a+b(d+ex)^2}{\left(b^2-4ac\right)\left(a+(d+ex)^2\left(b+c(d+ex)^2\right)\right)} - \frac{2b\tan^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{4ac-b^2}}\right)}{\left(4ac-b^2\right)^{3/2}}\right)}{2e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*f + e*f*x)^3/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2,x]
```

```
[Out] (f^3*((2*a + b*(d + e*x)^2)/((b^2 - 4*a*c)*(a + (d + e*x)^2*(b + c*(d + e*x
)^2))) - (2*b*ArcTan[(b + 2*c*(d + e*x)^2)/Sqrt[-b^2 + 4*a*c]])/(-b^2 + 4*a
*c)^(3/2)))/(2*e)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(df+efx)^3}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(d*f + e*f*x)^3/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2,x]
```

```
[Out] IntegrateAlgebraic[(d*f + e*f*x)^3/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2, x
]
```

fricas [B] time = 1.21, size = 1077, normalized size = 10.46

$$\left[\frac{\left(b^3-4abc\right)e^2f^3x^2+2\left(b^3-4abc\right)def^3x+\left(2ab^2-8a^2c+\left(b^3-4abc\right)d^2\right)f^3-\left(bce^4f^3x^4+4bcde^3f^3x^3+\left(6bcd^2+b^2\right)e^2f^3x^2+2\left(2bcd^3+b^2d\right)ef^3x+\left(bcd^4+b^2d^2+ab\right)f^3\right)\sqrt{b^2-4ac}\,\log\left(\frac{2c^2e^4x^4+8c^2de^3x^3+2c^2d^4+2\left(6c^2d^2+bc\right)e^2x^2+2bcd^2+4\left(2c^2d^3+bcd\right)ex+b^2-2ac+\left(2ce^2x^2+4cdex+2cd^2+b\right)\sqrt{b^2-4ac}}{ce^4x^4+4cde^3x^3+cd^4+\left(6cd^2+b\right)e^2x^2+bd^2+2\left(2cd^3+bd\right)ex+a}\right)}{2\left(\left(b^4c-8ab^2c^2+16a^2c^3\right)e^5x^4+4\left(b^4c-8ab^2c^2+16a^2c^3\right)de^4x^3+\left(b^5-8ab^3c+16a^2bc^2+6\left(b^4c-8ab^2c^2+16a^2c^3\right)d^2\right)e^3x^2+2\left(2\left(b^4c-8ab^2c^2+16a^2c^3\right)d^3+\left(b^5-8ab^3c+16a^2bc^2\right)d\right)e^2x+\left(ab^4-8a^2b^2c+16a^3c^2+\left(b^4c-8ab^2c^2+16a^2c^3\right)d^4+\left(b^5-8ab^3c+16a^2bc^2\right)d^2\right)e\right)}, \frac{\left(b^3-4abc\right)e^2f^3x^2+2\left(b^3-4abc\right)def^3x+\left(2ab^2-8a^2c+\left(b^3-4abc\right)d^2\right)f^3-2\left(bce^4f^3x^4+4bcde^3f^3x^3+\left(6bcd^2+b^2\right)e^2f^3x^2+2\left(2bcd^3+b^2d\right)ef^3x+\left(bcd^4+b^2d^2+ab\right)f^3\right)\sqrt{-b^2+4ac}\,\arctan\left(-\frac{\left(2ce^2x^2+4cdex+2cd^2+b\right)\sqrt{-b^2+4ac}}{b^2-4ac}\right)}{2\left(\left(b^4c-8ab^2c^2+16a^2c^3\right)e^5x^4+4\left(b^4c-8ab^2c^2+16a^2c^3\right)de^4x^3+\left(b^5-8ab^3c+16a^2bc^2+6\left(b^4c-8ab^2c^2+16a^2c^3\right)d^2\right)e^3x^2+2\left(2\left(b^4c-8ab^2c^2+16a^2c^3\right)d^3+\left(b^5-8ab^3c+16a^2bc^2\right)d\right)e^2x+\left(ab^4-8a^2b^2c+16a^3c^2+\left(b^4c-8ab^2c^2+16a^2c^3\right)d^4+\left(b^5-8ab^3c+16a^2bc^2\right)d^2\right)e\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*f*x+d*f)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="fricas"
)
```

```
[Out] [1/2*((b^3 - 4*a*b*c)*e^2*f^3*x^2 + 2*(b^3 - 4*a*b*c)*d*e*f^3*x + (2*a*b^2
- 8*a^2*c + (b^3 - 4*a*b*c)*d^2)*f^3 - (b*c*e^4*f^3*x^4 + 4*b*c*d*e^3*f^3*x
^3 + (6*b*c*d^2 + b^2)*e^2*f^3*x^2 + 2*(2*b*c*d^3 + b^2*d)*e*f^3*x + (b*c*d
```

```
^4 + b^2*d^2 + a*b)*f^3)*sqrt(b^2 - 4*a*c)*log((2*c^2*e^4*x^4 + 8*c^2*d*e^3
*x^3 + 2*c^2*d^4 + 2*(6*c^2*d^2 + b*c)*e^2*x^2 + 2*b*c*d^2 + 4*(2*c^2*d^3 +
 b*c*d)*e*x + b^2 - 2*a*c + (2*c*e^2*x^2 + 4*c*d*e*x + 2*c*d^2 + b)*sqrt(b^
2 - 4*a*c))/(c*e^4*x^4 + 4*c*d*e^3*x^3 + c*d^4 + (6*c*d^2 + b)*e^2*x^2 + b*
d^2 + 2*(2*c*d^3 + b*d)*e*x + a)))/((b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*e^5*
x^4 + 4*(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*d*e^4*x^3 + (b^5 - 8*a*b^3*c + 1
6*a^2*b*c^2 + 6*(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*d^2)*e^3*x^2 + 2*(2*(b^4
*c - 8*a*b^2*c^2 + 16*a^2*c^3)*d^3 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*d)*e^
2*x + (a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2 + (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3
)*d^4 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*d^2)*e), 1/2*((b^3 - 4*a*b*c)*e^2*
f^3*x^2 + 2*(b^3 - 4*a*b*c)*d*e*f^3*x + (2*a*b^2 - 8*a^2*c + (b^3 - 4*a*b*c
)*d^2)*f^3 - 2*(b*c*e^4*f^3*x^4 + 4*b*c*d*e^3*f^3*x^3 + (6*b*c*d^2 + b^2)*e
^2*f^3*x^2 + 2*(2*b*c*d^3 + b^2*d)*e*f^3*x + (b*c*d^4 + b^2*d^2 + a*b)*f^3)
*sqrt(-b^2 + 4*a*c)*arctan(-(2*c*e^2*x^2 + 4*c*d*e*x + 2*c*d^2 + b)*sqrt(-b
^2 + 4*a*c)/(b^2 - 4*a*c)))/((b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*e^5*x^4 + 4
*(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*d*e^4*x^3 + (b^5 - 8*a*b^3*c + 16*a^2*b
*c^2 + 6*(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*d^2)*e^3*x^2 + 2*(2*(b^4*c - 8*
a*b^2*c^2 + 16*a^2*c^3)*d^3 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*d)*e^2*x + (
a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2 + (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*d^4 +
 (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*d^2)*e)]
```

giac [B] time = 0.52, size = 211, normalized size = 2.05

$$\frac{bf^3\arctan\left(\frac{2\,cd^2f+2\left(fx^2e+2\,dfx\right)ce+bf}{\sqrt{-b^2+4\,ac}\,f}\right)e^{(-1)}}{\left(b^2-4\,ac\right)\sqrt{-b^2+4\,ac}}+\frac{bd^2f^5+\left(fx^2e+2\,dfx\right)bf^4e+2\,af^5}{2\left(cd^4f^2+2\left(fx^2e+2\,dfx\right)cd^2fe+bd^2f^2+\left(fx^2e+2\,dfx\right)^2ce^2+\left(fx^2e+2\,dfx\right)bfe+af^2\right)\left(b^2e-4\,ace\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*f*x+d*f)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="giac")
```

```
[Out] b*f^3*arctan((2*c*d^2*f + 2*(f*x^2*e + 2*d*f*x)*c*e + b*f)/(sqrt(-b^2 + 4*a
*c)*f))*e^(-1)/((b^2 - 4*a*c)*sqrt(-b^2 + 4*a*c)) + 1/2*(b*d^2*f^5 + (f*x^2
*e + 2*d*f*x)*b*f^4*e + 2*a*f^5)/((c*d^4*f^2 + 2*(f*x^2*e + 2*d*f*x)*c*d^2*
f*e + b*d^2*f^2 + (f*x^2*e + 2*d*f*x)^2*c*e^2 + (f*x^2*e + 2*d*f*x)*b*f*e +
 a*f^2)*(b^2*e - 4*a*c*e))
```

maple [C] time = 0.02, size = 500, normalized size = 4.85

$$-\frac{be\,f^3x^2}{2\left(ce^4x^4+4cde^3x^3+6cd^2e^2x^2+4cd^3ex+be^2x^2+cd^4+2bdex+bd^2+a\right)\left(4ac-b^2\right)}-\frac{bd\,f^3x}{\left(ce^4x^4+4cde^3x^3+6cd^2e^2x^2+4cd^3ex+be^2x^2+cd^4+2bdex+bd^2+a\right)\left(4ac-b^2\right)}-\frac{bd^2f^3}{2\left(ce^4x^4+4cde^3x^3+6cd^2e^2x^2+4cd^3ex+be^2x^2+cd^4+2bdex+bd^2+a\right)\left(4ac-b^2\right)e}+\frac{bf^3\left(-\mathrm{RootOf}\left(\_Z^4ce^4+4\_Z^3cde^3+cd^4+bd^2+\left(6cd^2e^2+be^2\right)\_Z^2+\left(4cd^3e+2deb\right)\_Z+a\right)e-d\right)\ln\left(-\mathrm{RootOf}\left(\_Z^4ce^4+4\_Z^3cde^3+cd^4+bd^2+\left(6cd^2e^2+be^2\right)\_Z^2+\left(4cd^3e+2deb\right)\_Z+a\right)+x\right)}{2\left(4ac-b^2\right)e\left(2ce^3\mathrm{RootOf}\left(\_Z^4ce^4+4\_Z^3cde^3+cd^4+bd^2+\left(6cd^2e^2+be^2\right)\_Z^2+\left(4cd^3e+2deb\right)\_Z+a\right)^3+6cde^2\mathrm{RootOf}\left(\_Z^4ce^4+4\_Z^3cde^3+cd^4+bd^2+\left(6cd^2e^2+be^2\right)\_Z^2+\left(4cd^3e+2deb\right)\_Z+a\right)^2+6ecd^2\mathrm{RootOf}\left(\_Z^4ce^4+4\_Z^3cde^3+cd^4+bd^2+\left(6cd^2e^2+be^2\right)\_Z^2+\left(4cd^3e+2deb\right)\_Z+a\right)+2cd^3+be\mathrm{RootOf}\left(\_Z^4ce^4+4\_Z^3cde^3+cd^4+bd^2+\left(6cd^2e^2+be^2\right)\_Z^2+\left(4cd^3e+2deb\right)\_Z+a\right)+bd\right)}-\frac{af^3}{\left(ce^4x^4+4cde^3x^3+6cd^2e^2x^2+4cd^3ex+be^2x^2+cd^4+2bdex+bd^2+a\right)\left(4ac-b^2\right)e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*f*x+d*f)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x)
```

```
[Out] -1/2*f^3/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d
^4+2*b*d*e*x+b*d^2+a)*b*e/(4*a*c-b^2)*x^2-f^3/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*
d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)*b*d/(4*a*c-b^2)*
x-1/2*f^3/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*
d^4+2*b*d*e*x+b*d^2+a)/(4*a*c-b^2)/e*b*d^2-f^3/(c*e^4*x^4+4*c*d*e^3*x^3+6*c
*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)/(4*a*c-b^2)/e*a
+1/2*f^3/(4*a*c-b^2)*b/e*sum((-_R*e-d)/(2*_R^3*c*e^3+6*_R^2*c*d*e^2+6*_R*c*
d^2*e+2*c*d^3+_R*b*e+b*d)*ln(-_R+x),_R=RootOf(_Z^4*c*e^4+4*_Z^3*c*d*e^3+c*d
^4+b*d^2+(6*c*d^2*e^2+b*e^2)*_Z^2+(4*c*d^3*e+2*b*d*e)*_Z+a))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-bf^3\int -\frac{ex+d}{\left(b^2c-4\,ac^2\right)e^4x^4+4\left(b^2c-4\,ac^2\right)de^3x^3+\left(b^2c-4\,ac^2\right)d^4+\left(b^3-4\,abc+6\left(b^2c-4\,ac^2\right)d^2\right)e^2x^2+ab^2-4\,a^2c+\left(b^3-4\,abc\right)d^2+2\left(2\left(b^2c-4\,ac^2\right)d^3+\left(b^3-4\,abc\right)d\right)ex}\,dx+\frac{be^2f^3x^2+2\,bdef^3x+\left(bd^2+2\,a\right)f^3}{2\left(\left(b^2c-4\,ac^2\right)e^5x^4+4\left(b^2c-4\,ac^2\right)de^4x^3+\left(b^3-4\,abc+6\left(b^2c-4\,ac^2\right)d^2\right)e^3x^2+2\left(2\left(b^2c-4\,ac^2\right)d^3+\left(b^3-4\,abc\right)d\right)e^2x+\left(\left(b^2c-4\,ac^2\right)d^4+ab^2-4\,a^2c+\left(b^3-4\,abc\right)d^2\right)e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*f*x+d*f)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="maxima"
)
```

```
[Out] -b*f^3*integrate(-(e*x + d)/((b^2*c - 4*a*c^2)*e^4*x^4 + 4*(b^2*c - 4*a*c^2
)*d*e^3*x^3 + (b^2*c - 4*a*c^2)*d^4 + (b^3 - 4*a*b*c + 6*(b^2*c - 4*a*c^2)*
d^2)*e^2*x^2 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*d^2 + 2*(2*(b^2*c - 4*a*c^
2)*d^3 + (b^3 - 4*a*b*c)*d)*e*x), x) + 1/2*(b*e^2*f^3*x^2 + 2*b*d*e*f^3*x +
 (b*d^2 + 2*a)*f^3)/((b^2*c - 4*a*c^2)*e^5*x^4 + 4*(b^2*c - 4*a*c^2)*d*e^4*
x^3 + (b^3 - 4*a*b*c + 6*(b^2*c - 4*a*c^2)*d^2)*e^3*x^2 + 2*(2*(b^2*c - 4*a
*c^2)*d^3 + (b^3 - 4*a*b*c)*d)*e^2*x + ((b^2*c - 4*a*c^2)*d^4 + a*b^2 - 4*a
^2*c + (b^3 - 4*a*b*c)*d^2)*e)
```

mupad [B] time = 1.90, size = 460, normalized size = 4.47

$$\frac{b\,f^3\,\mathrm{atan}\left(\frac{{\left(4\,a\,c-b^2\right)}^4\,\left(x\,\left(\frac{b^3\,f^6\,\left(2\,b^3\,c^2\,d\,e^9-8\,a\,b\,c^3\,d\,e^9\right)}{a\,e^2\,{\left(4\,a\,c-b^2\right)}^{11/2}}-\frac{2\,b^2\,c^2\,d\,e^7\,f^6}{a\,{\left(4\,a\,c-b^2\right)}^{7/2}}\right)+x^2\,\left(\frac{b^3\,f^6\,\left(2\,b^3\,c^2\,e^{10}-8\,a\,b\,c^3\,e^{10}\right)}{2\,a\,e^2\,{\left(4\,a\,c-b^2\right)}^{11/2}}-\frac{b^2\,c^2\,e^8\,f^6}{a\,{\left(4\,a\,c-b^2\right)}^{7/2}}\right)-\frac{b^3\,f^6\,\left(16\,a^2\,c^3\,e^8-4\,a\,b^2\,c^2\,e^8+8\,a\,b\,c^3\,d^2\,e^8-2\,b^3\,c^2\,d^2\,e^8\right)}{2\,a\,e^2\,{\left(4\,a\,c-b^2\right)}^{11/2}}-\frac{b^2\,c^2\,d^2\,e^6\,f^6}{a\,{\left(4\,a\,c-b^2\right)}^{7/2}}\right)}{2\,b^2\,c^2\,e^6\,f^6}\right)}{e\,{\left(4\,a\,c-b^2\right)}^{3/2}}-\frac{\frac{f^3\,\left(b\,d^2+2\,a\right)}{2\,e\,\left(4\,a\,c-b^2\right)}+\frac{b\,d\,f^3\,x}{4\,a\,c-b^2}+\frac{b\,e\,f^3\,x^2}{2\,\left(4\,a\,c-b^2\right)}}{a+x^2\,\left(6\,c\,d^2\,e^2+b\,e^2\right)+b\,d^2+c\,d^4+x\,\left(4\,c\,e\,d^3+2\,b\,e\,d\right)+c\,e^4\,x^4+4\,c\,d\,e^3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*f + e*f*x)^3/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2,x)
```

```
[Out] (b*f^3*atan(((4*a*c - b^2)^4*(x*((b^3*f^6*(2*b^3*c^2*d*e^9 - 8*a*b*c^3*d*e^
9))/(a*e^2*(4*a*c - b^2)^(11/2)) - (2*b^2*c^2*d*e^7*f^6)/(a*(4*a*c - b^2)^(
7/2))) + x^2*((b^3*f^6*(2*b^3*c^2*e^10 - 8*a*b*c^3*e^10))/(2*a*e^2*(4*a*c -
 b^2)^(11/2)) - (b^2*c^2*e^8*f^6)/(a*(4*a*c - b^2)^(7/2))) - (b^3*f^6*(16*a
^2*c^3*e^8 - 4*a*b^2*c^2*e^8 - 2*b^3*c^2*d^2*e^8 + 8*a*b*c^3*d^2*e^8))/(2*a
*e^2*(4*a*c - b^2)^(11/2)) - (b^2*c^2*d^2*e^6*f^6)/(a*(4*a*c - b^2)^(7/2)))
)/(2*b^2*c^2*e^6*f^6)))/(e*(4*a*c - b^2)^(3/2)) - ((f^3*(2*a + b*d^2))/(2*e
*(4*a*c - b^2)) + (b*d*f^3*x)/(4*a*c - b^2) + (b*e*f^3*x^2)/(2*(4*a*c - b^2
)))/(a + x^2*(b*e^2 + 6*c*d^2*e^2) + b*d^2 + c*d^4 + x*(2*b*d*e + 4*c*d^3*e
) + c*e^4*x^4 + 4*c*d*e^3*x^3)
```

sympy [B] time = 5.29, size = 556, normalized size = 5.40

$$\frac{bf^3\sqrt{-\frac{1}{\left(4ac-b^2\right)^3}}\log\left(\frac{2dx}{e}+x^2+\frac{-16a^2bc^2f^3\sqrt{-\frac{1}{\left(4ac-b^2\right)^3}}+8ab^3cf^3\sqrt{-\frac{1}{\left(4ac-b^2\right)^3}}-b^5f^3\sqrt{-\frac{1}{\left(4ac-b^2\right)^3}}+b^2f^3+2bcd^2f^3}{2bce^2f^3}\right)}{2e}-\frac{bf^3\sqrt{-\frac{1}{\left(4ac-b^2\right)^3}}\log\left(\frac{2dx}{e}+x^2+\frac{16a^2bc^2f^3\sqrt{-\frac{1}{\left(4ac-b^2\right)^3}}-8ab^3cf^3\sqrt{-\frac{1}{\left(4ac-b^2\right)^3}}+b^5f^3\sqrt{-\frac{1}{\left(4ac-b^2\right)^3}}+b^2f^3+2bcd^2f^3}{2bce^2f^3}\right)}{2e}+\frac{-2af^3-bd^2f^3-2bdef^3x-be^2f^3x^2}{8a^2ce-2ab^2e+8abcd^2e+8ac^2d^4e-2b^3d^2e-2b^2cd^4e+x^4\left(8ac^2e^5-2b^2ce^5\right)+x^3\left(32ac^2de^4-8b^2cde^4\right)+x^2\left(8abce^3+48ac^2d^2e^3-2b^3e^3-12b^2cd^2e^3\right)+x\left(16abcde^2+32ac^2d^3e^2-4b^3de^2-8b^2cd^3e^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*f*x+d*f)**3/(a+b*(e*x+d)**2+c*(e*x+d)**4)**2,x)
```

```
[Out] b*f**3*sqrt(-1/(4*a*c - b**2)**3)*log(2*d*x/e + x**2 + (-16*a**2*b*c**2*f**
3*sqrt(-1/(4*a*c - b**2)**3) + 8*a*b**3*c*f**3*sqrt(-1/(4*a*c - b**2)**3) -
 b**5*f**3*sqrt(-1/(4*a*c - b**2)**3) + b**2*f**3 + 2*b*c*d**2*f**3)/(2*b*c
*e**2*f**3))/(2*e) - b*f**3*sqrt(-1/(4*a*c - b**2)**3)*log(2*d*x/e + x**2 +
 (16*a**2*b*c**2*f**3*sqrt(-1/(4*a*c - b**2)**3) - 8*a*b**3*c*f**3*sqrt(-1/
(4*a*c - b**2)**3) + b**5*f**3*sqrt(-1/(4*a*c - b**2)**3) + b**2*f**3 + 2*b
*c*d**2*f**3)/(2*b*c*e**2*f**3))/(2*e) + (-2*a*f**3 - b*d**2*f**3 - 2*b*d*e
*f**3*x - b*e**2*f**3*x**2)/(8*a**2*c*e - 2*a*b**2*e + 8*a*b*c*d**2*e + 8*a
*c**2*d**4*e - 2*b**3*d**2*e - 2*b**2*c*d**4*e + x**4*(8*a*c**2*e**5 - 2*b*
*2*c*e**5) + x**3*(32*a*c**2*d*e**4 - 8*b**2*c*d*e**4) + x**2*(8*a*b*c*e**3
 + 48*a*c**2*d**2*e**3 - 2*b**3*e**3 - 12*b**2*c*d**2*e**3) + x*(16*a*b*c*d
*e**2 + 32*a*c**2*d**3*e**2 - 4*b**3*d*e**2 - 8*b**2*c*d**3*e**2))
```

3.535 $\int \frac{(df+efx)^2}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx$

Optimal. Leaf size=263

$$-\frac{f^2(d+ex)\left(b+2c(d+ex)^2\right)}{2e\left(b^2-4ac\right)\left(a+b(d+ex)^2+c(d+ex)^4\right)}+\frac{\sqrt{c}\,f^2\left(2b-\sqrt{b^2-4ac}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\,e\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{c}\,f^2\left(\sqrt{b^2-4ac}\right.}{\sqrt{2}\,e\left(b^2-4a\right.}$$

Rubi [A] time = 0.37, antiderivative size = 263, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.121, Rules used = {1142, 1119, 1166, 205}

$$-\frac{f^2(d+ex)\left(b+2c(d+ex)^2\right)}{2e\left(b^2-4ac\right)\left(a+b(d+ex)^2+c(d+ex)^4\right)}+\frac{\sqrt{c}\,f^2\left(2b-\sqrt{b^2-4ac}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\,e\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{c}\,f^2\left(\sqrt{b^2-4ac}+2b\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,(d+ex)}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\,e\left(b^2-4ac\right)^{3/2}\sqrt{\sqrt{b^2-4ac}+b}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*f + e*f*x)^2/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2,x]
```

```
[Out] -(f^2*(d + e*x)*(b + 2*c*(d + e*x)^2))/(2*(b^2 - 4*a*c)*e*(a + b*(d + e*x)^
2 + c*(d + e*x)^4)) + (Sqrt[c]*(2*b - Sqrt[b^2 - 4*a*c])*f^2*ArcTan[(Sqrt[2
]*Sqrt[c]*(d + e*x))/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*(b^2 - 4*a*c)^(
3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]*e) - (Sqrt[c]*(2*b + Sqrt[b^2 - 4*a*c])*f^
2*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]
*(b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]*e)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1119

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Simp[(d*(d*x)^(m - 1)*(b + 2*c*x^2)*(a + b*x^2 + c*x^4)^(p + 1))/(2*(p
+ 1)*(b^2 - 4*a*c)), x] - Dist[d^2/(2*(p + 1)*(b^2 - 4*a*c)), Int[(d*x)^(m
- 2)*(b*(m - 1) + 2*c*(m + 4*p + 5)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x
] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && GtQ[m,
1] && LeQ[m, 3] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1142

```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Di
st[u^m/(Coefficient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p,
 x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{(df+efx)^2}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx = \frac{f^2 \text{Subst}\left(\int \frac{x^2}{\left(a+bx^2+cx^4\right)^2}\,dx, x, d+ex\right)}{e}$$

$$= -\frac{f^2(d+ex)\left(b+2c(d+ex)^2\right)}{2\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)} + \frac{f^2 \text{Subst}\left(\int \frac{b-2cx^2}{a+bx^2+cx^4}\,dx, x\right.}{2\left(b^2-4ac\right)e}$$

$$= -\frac{f^2(d+ex)\left(b+2c(d+ex)^2\right)}{2\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)} + \frac{\left(c\left(2b-\sqrt{b^2-4ac}\right)f^2\right)\text{S}}{2(}$$

$$= -\frac{f^2(d+ex)\left(b+2c(d+ex)^2\right)}{2\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)} + \frac{\sqrt{c}\left(2b-\sqrt{b^2-4ac}\right)f^2\,\text{ta}}{\sqrt{2}\left(b^2-4ac\right)^{3/2}\sqrt{b-}}$$

Mathematica [A] time = 0.94, size = 250, normalized size = 0.95

$$-\frac{f^2\left(\frac{b(d+ex)+2c(d+ex)^3}{\left(b^2-4ac\right)\left(a+b(d+ex)^2+c(d+ex)^4\right)}+\frac{\sqrt{2}\sqrt{c}\left(\sqrt{b^2-4ac}-2b\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{2}\sqrt{c}\left(\sqrt{b^2-4ac}+2b\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\left(b^2-4ac\right)^{3/2}\sqrt{\sqrt{b^2-4ac}+b}}\right)}{2e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*f + e*f*x)^2/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2,x]
```

```
[Out] -1/2*(f^2*((b*(d + e*x) + 2*c*(d + e*x)^3)/((b^2 - 4*a*c)*(a + b*(d + e*x)^
2 + c*(d + e*x)^4)) + (Sqrt[2]*Sqrt[c]*(-2*b + Sqrt[b^2 - 4*a*c])*ArcTan[(S
qrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(3/2
)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[2]*Sqrt[c]*(2*b + Sqrt[b^2 - 4*a*c])
*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/((b^2 - 4
*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]])))/e
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(df+efx)^2}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(d*f + e*f*x)^2/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2,x]
```

```
[Out] IntegrateAlgebraic[(d*f + e*f*x)^2/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2, x
]
```

fricas [B] time = 1.32, size = 2600, normalized size = 9.89

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*f*x+d*f)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="fricas"
)
```

```
[Out] -1/4*(4*c*e^3*f^2*x^3 + 12*c*d*e^2*f^2*x^2 + 2*(6*c*d^2 + b)*e*f^2*x + 2*(2
*c*d^3 + b*d)*f^2 + sqrt(1/2)*((b^2*c - 4*a*c^2)*e^5*x^4 + 4*(b^2*c - 4*a*c
```

```
^2)*d*e^4*x^3 + (b^3 - 4*a*b*c + 6*(b^2*c - 4*a*c^2)*d^2)*e^3*x^2 + 2*(2*(b
^2*c - 4*a*c^2)*d^3 + (b^3 - 4*a*b*c)*d)*e^2*x + ((b^2*c - 4*a*c^2)*d^4 + a
*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*d^2)*e)*sqrt(-((b^3 + 12*a*b*c)*f^4 + (a*b
^6 - 12*a^2*b^4*c + 48*a^3*b^2*c^2 - 64*a^4*c^3)*sqrt(f^8/((a^2*b^6 - 12*a^
3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3)*e^4))*e^2)/((a*b^6 - 12*a^2*b^4*c +
48*a^3*b^2*c^2 - 64*a^4*c^3)*e^2))*log((3*b^2*c + 4*a*c^2)*e*f^6*x + (3*b^2
*c + 4*a*c^2)*d*f^6 + 1/2*sqrt(1/2)*((b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*e*f^4
 - (a*b^8 - 8*a^2*b^6*c + 128*a^4*b^2*c^3 - 256*a^5*c^4)*sqrt(f^8/((a^2*b^6
 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3)*e^4))*e^3)*sqrt(-((b^3 + 12*
a*b*c)*f^4 + (a*b^6 - 12*a^2*b^4*c + 48*a^3*b^2*c^2 - 64*a^4*c^3)*sqrt(f^8/
((a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3)*e^4))*e^2)/((a*b^6
- 12*a^2*b^4*c + 48*a^3*b^2*c^2 - 64*a^4*c^3)*e^2))) - sqrt(1/2)*((b^2*c -
4*a*c^2)*e^5*x^4 + 4*(b^2*c - 4*a*c^2)*d*e^4*x^3 + (b^3 - 4*a*b*c + 6*(b^2*
c - 4*a*c^2)*d^2)*e^3*x^2 + 2*(2*(b^2*c - 4*a*c^2)*d^3 + (b^3 - 4*a*b*c)*d)
*e^2*x + ((b^2*c - 4*a*c^2)*d^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*d^2)*e)
*sqrt(-((b^3 + 12*a*b*c)*f^4 + (a*b^6 - 12*a^2*b^4*c + 48*a^3*b^2*c^2 - 64*
a^4*c^3)*sqrt(f^8/((a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3)*e
^4))*e^2)/((a*b^6 - 12*a^2*b^4*c + 48*a^3*b^2*c^2 - 64*a^4*c^3)*e^2))*log((
3*b^2*c + 4*a*c^2)*e*f^6*x + (3*b^2*c + 4*a*c^2)*d*f^6 - 1/2*sqrt(1/2)*((b^
5 - 8*a*b^3*c + 16*a^2*b*c^2)*e*f^4 - (a*b^8 - 8*a^2*b^6*c + 128*a^4*b^2*c^
3 - 256*a^5*c^4)*sqrt(f^8/((a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^
5*c^3)*e^4))*e^3)*sqrt(-((b^3 + 12*a*b*c)*f^4 + (a*b^6 - 12*a^2*b^4*c + 48*
a^3*b^2*c^2 - 64*a^4*c^3)*sqrt(f^8/((a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^
2 - 64*a^5*c^3)*e^4))*e^2)/((a*b^6 - 12*a^2*b^4*c + 48*a^3*b^2*c^2 - 64*a^4
*c^3)*e^2))) + sqrt(1/2)*((b^2*c - 4*a*c^2)*e^5*x^4 + 4*(b^2*c - 4*a*c^2)*d
*e^4*x^3 + (b^3 - 4*a*b*c + 6*(b^2*c - 4*a*c^2)*d^2)*e^3*x^2 + 2*(2*(b^2*c
- 4*a*c^2)*d^3 + (b^3 - 4*a*b*c)*d)*e^2*x + ((b^2*c - 4*a*c^2)*d^4 + a*b^2
- 4*a^2*c + (b^3 - 4*a*b*c)*d^2)*e)*sqrt(-((b^3 + 12*a*b*c)*f^4 - (a*b^6 -
12*a^2*b^4*c + 48*a^3*b^2*c^2 - 64*a^4*c^3)*sqrt(f^8/((a^2*b^6 - 12*a^3*b^4
*c + 48*a^4*b^2*c^2 - 64*a^5*c^3)*e^4))*e^2)/((a*b^6 - 12*a^2*b^4*c + 48*a^
3*b^2*c^2 - 64*a^4*c^3)*e^2))*log((3*b^2*c + 4*a*c^2)*e*f^6*x + (3*b^2*c +
4*a*c^2)*d*f^6 + 1/2*sqrt(1/2)*((b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*e*f^4 + (a
*b^8 - 8*a^2*b^6*c + 128*a^4*b^2*c^3 - 256*a^5*c^4)*sqrt(f^8/((a^2*b^6 - 12
*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3)*e^4))*e^3)*sqrt(-((b^3 + 12*a*b*c
)*f^4 - (a*b^6 - 12*a^2*b^4*c + 48*a^3*b^2*c^2 - 64*a^4*c^3)*sqrt(f^8/((a^2
*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3)*e^4))*e^2)/((a*b^6 - 12*
a^2*b^4*c + 48*a^3*b^2*c^2 - 64*a^4*c^3)*e^2))) - sqrt(1/2)*((b^2*c - 4*a*c
^2)*e^5*x^4 + 4*(b^2*c - 4*a*c^2)*d*e^4*x^3 + (b^3 - 4*a*b*c + 6*(b^2*c - 4
*a*c^2)*d^2)*e^3*x^2 + 2*(2*(b^2*c - 4*a*c^2)*d^3 + (b^3 - 4*a*b*c)*d)*e^2*
x + ((b^2*c - 4*a*c^2)*d^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*d^2)*e)*sqrt
(-((b^3 + 12*a*b*c)*f^4 - (a*b^6 - 12*a^2*b^4*c + 48*a^3*b^2*c^2 - 64*a^4*c
^3)*sqrt(f^8/((a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3)*e^4))*
e^2)/((a*b^6 - 12*a^2*b^4*c + 48*a^3*b^2*c^2 - 64*a^4*c^3)*e^2))*log((3*b^2
*c + 4*a*c^2)*e*f^6*x + (3*b^2*c + 4*a*c^2)*d*f^6 - 1/2*sqrt(1/2)*((b^5 - 8
*a*b^3*c + 16*a^2*b*c^2)*e*f^4 + (a*b^8 - 8*a^2*b^6*c + 128*a^4*b^2*c^3 - 2
56*a^5*c^4)*sqrt(f^8/((a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3
)*e^4))*e^3)*sqrt(-((b^3 + 12*a*b*c)*f^4 - (a*b^6 - 12*a^2*b^4*c + 48*a^3*b
^2*c^2 - 64*a^4*c^3)*sqrt(f^8/((a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 6
4*a^5*c^3)*e^4))*e^2)/((a*b^6 - 12*a^2*b^4*c + 48*a^3*b^2*c^2 - 64*a^4*c^3)
*e^2))))/((b^2*c - 4*a*c^2)*e^5*x^4 + 4*(b^2*c - 4*a*c^2)*d*e^4*x^3 + (b^3
- 4*a*b*c + 6*(b^2*c - 4*a*c^2)*d^2)*e^3*x^2 + 2*(2*(b^2*c - 4*a*c^2)*d^3 +
 (b^3 - 4*a*b*c)*d)*e^2*x + ((b^2*c - 4*a*c^2)*d^4 + a*b^2 - 4*a^2*c + (b^3
 - 4*a*b*c)*d^2)*e)
```

giac [B] time = 0.67, size = 1378, normalized size = 5.24

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*f*x+d*f)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="giac")
```

```
[Out] 1/4*((2*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/
c))^2*c*f^2*e^2 - 4*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*
e^2)*e^(-4)/c))*c*d*f^2*e + 2*c*d^2*f^2 - b*f^2)*log(d*e^(-1) + x + sqrt(1/
2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))/(2*(d*e^(-1) + sqrt(1/2
)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^3*c*e^4 - 6*(d*e^(-1) +
sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^2*c*d*e^3 - 2*c*
d^3*e - b*d*e + (6*c*d^2*e^2 + b*e^2)*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 +
sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))) + (2*(d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2
+ sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^2*c*f^2*e^2 - 4*(d*e^(-1) - sqrt(1/2)*s
qrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))*c*d*f^2*e + 2*c*d^2*f^2 - b
*f^2)*log(d*e^(-1) + x - sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^
(-4)/c))/(2*(d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(
-4)/c))^3*c*e^4 - 6*(d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*
e^2)*e^(-4)/c))^2*c*d*e^3 - 2*c*d^3*e - b*d*e + (6*c*d^2*e^2 + b*e^2)*(d*e^
(-1) - sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))) + (2*(d*
e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^2*c*f^2
*e^2 - 4*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)
/c))*c*d*f^2*e + 2*c*d^2*f^2 - b*f^2)*log(d*e^(-1) + x + sqrt(1/2)*sqrt(-(b
*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))/(2*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*
e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^3*c*e^4 - 6*(d*e^(-1) + sqrt(1/2)*s
qrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^2*c*d*e^3 - 2*c*d^3*e - b*d
*e + (6*c*d^2*e^2 + b*e^2)*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 -
4*a*c)*e^2)*e^(-4)/c))) + (2*(d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2
- 4*a*c)*e^2)*e^(-4)/c))^2*c*f^2*e^2 - 4*(d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2
 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))*c*d*f^2*e + 2*c*d^2*f^2 - b*f^2)*log(d
*e^(-1) + x - sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))/(2
*(d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^3*c
*e^4 - 6*(d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)
/c))^2*c*d*e^3 - 2*c*d^3*e - b*d*e + (6*c*d^2*e^2 + b*e^2)*(d*e^(-1) - sqrt
(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))))/(b^2 - 4*a*c) - 1/
2*(2*c*f^2*x^3*e^3 + 6*c*d*f^2*x^2*e^2 + 6*c*d^2*f^2*x*e + 2*c*d^3*f^2 + b*
f^2*x*e + b*d*f^2)/((c*x^4*e^4 + 4*c*d*x^3*e^3 + 6*c*d^2*x^2*e^2 + 4*c*d^3*
x*e + c*d^4 + b*x^2*e^2 + 2*b*d*x*e + b*d^2 + a)*(b^2*e - 4*a*c*e))
```

maple [C]   time = 0.02, size = 693, normalized size = 2.63

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*f*x+d*f)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x)
```

```
[Out] f^2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*
b*d*e*x+b*d^2+a)*c*e^2/(4*a*c-b^2)*x^3+3*f^2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d
^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)*d*c*e/(4*a*c-b^2)
*x^2+3*f^2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c
*d^4+2*b*d*e*x+b*d^2+a)/(4*a*c-b^2)*x*c*d^2+1/2*f^2/(c*e^4*x^4+4*c*d*e^3*x^
3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)/(4*a*c-b^2
)*x*b+f^2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*
d^4+2*b*d*e*x+b*d^2+a)/(4*a*c-b^2)*d^3/e*c+1/2*f^2/(c*e^4*x^4+4*c*d*e^3*x^3
+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)/(4*a*c-b^2)
*d/e*b+1/4*f^2/(4*a*c-b^2)/e*sum((2*_R^2*c*e^2+4*_R*c*d*e+2*c*d^2-b)/(2*_R^
3*c*e^3+6*_R^2*c*d*e^2+6*_R*c*d^2*e+2*c*d^3+_R*b*e+b*d)*ln(-_R+x),_R=RootOf
(_Z^4*c*e^4+4*_Z^3*c*d*e^3+c*d^4+b*d^2+(6*c*d^2*e^2+b*e^2)*_Z^2+(4*c*d^3*e+
2*b*d*e)*_Z+a))
```

maxima [F]   time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{2} f^2 \int -\frac{2 c e^2 x^2 + 4 c d e x + 2 c d^2 - b}{(b^2 c - 4 a c^2) e^4 x^4 + 4 (b^2 c - 4 a c^2) d e^3 x^3 + (b^2 c - 4 a c^2) d^4 + (b^3 - 4 a b c + 6 (b^2 c - 4 a c^2) d^2) e^2 x^2 + a b^2 - 4 a^2 c + (b^3 - 4 a b c) d^2 + 2 (2 (b^2 c - 4 a c^2) d^3 + (b^3 - 4 a b c) d) e x}\, dx - \frac{2 c e^3 f^2 x^3 + 6 c d e^2 f^2 x^2 + (6 c d^2 + b) e f^2 x + (2 c d^3 + b d) f^2}{2 ((b^2 c - 4 a c^2) e^5 x^4 + 4 (b^2 c - 4 a c^2) d e^4 x^3 + (b^3 - 4 a b c + 6 (b^2 c - 4 a c^2) d^2) e^3 x^2 + 2 (2 (b^2 c - 4 a c^2) d^3 + (b^3 - 4 a b c) d) e^2 x + ((b^2 c - 4 a c^2) d^4 + a b^2 - 4 a^2 c + (b^3 - 4 a b c) d^2) e)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*f*x+d*f)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="maxima"
)
```

```
[Out] 1/2*f^2*integrate(-(2*c*e^2*x^2 + 4*c*d*e*x + 2*c*d^2 - b)/((b^2*c - 4*a*c^
2)*e^4*x^4 + 4*(b^2*c - 4*a*c^2)*d*e^3*x^3 + (b^2*c - 4*a*c^2)*d^4 + (b^3 -
 4*a*b*c + 6*(b^2*c - 4*a*c^2)*d^2)*e^2*x^2 + a*b^2 - 4*a^2*c + (b^3 - 4*a*
b*c)*d^2 + 2*(2*(b^2*c - 4*a*c^2)*d^3 + (b^3 - 4*a*b*c)*d)*e*x), x) - 1/2*(
2*c*e^3*f^2*x^3 + 6*c*d*e^2*f^2*x^2 + (6*c*d^2 + b)*e*f^2*x + (2*c*d^3 + b*
d)*f^2)/((b^2*c - 4*a*c^2)*e^5*x^4 + 4*(b^2*c - 4*a*c^2)*d*e^4*x^3 + (b^3 -
 4*a*b*c + 6*(b^2*c - 4*a*c^2)*d^2)*e^3*x^2 + 2*(2*(b^2*c - 4*a*c^2)*d^3 +
(b^3 - 4*a*b*c)*d)*e^2*x + ((b^2*c - 4*a*c^2)*d^4 + a*b^2 - 4*a^2*c + (b^3
- 4*a*b*c)*d^2)*e)
```

mupad [B] time = 4.40, size = 7835, normalized size = 29.79

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*f + e*f*x)^2/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2,x)
```

```
[Out] ((x*(b*f^2 + 6*c*d^2*f^2))/(2*(4*a*c - b^2)) + (2*c*d^3*f^2 + b*d*f^2)/(2*e
*(4*a*c - b^2)) + (c*e^2*f^2*x^3)/(4*a*c - b^2) + (3*c*d*e*f^2*x^2)/(4*a*c
- b^2))/(a + x^2*(b*e^2 + 6*c*d^2*e^2) + b*d^2 + c*d^4 + x*(2*b*d*e + 4*c*d
^3*e) + c*e^4*x^4 + 4*c*d*e^3*x^3) + atan(((((f^4*(-(4*a*c - b^2)^9)^(1/2))
/32 - (b^9*f^4)/32 + 24*a^4*b*c^4*f^4 + 3*a^2*b^5*c^2*f^4 - 16*a^3*b^3*c^3*
f^4)/(a*b^12*e^2 + 4096*a^7*c^6*e^2 - 24*a^2*b^10*c*e^2 + 240*a^3*b^8*c^2*e
^2 - 1280*a^4*b^6*c^3*e^2 + 3840*a^5*b^4*c^4*e^2 - 6144*a^6*b^2*c^5*e^2))^(
1/2)*((((f^4*(-(4*a*c - b^2)^9)^(1/2))/32 - (b^9*f^4)/32 + 24*a^4*b*c^4*f^4
 + 3*a^2*b^5*c^2*f^4 - 16*a^3*b^3*c^3*f^4)/(a*b^12*e^2 + 4096*a^7*c^6*e^2 -
 24*a^2*b^10*c*e^2 + 240*a^3*b^8*c^2*e^2 - 1280*a^4*b^6*c^3*e^2 + 3840*a^5*
b^4*c^4*e^2 - 6144*a^6*b^2*c^5*e^2))^(1/2)*(((8*b^9*c^2*d*e^13 - 128*a*b^7*
c^3*d*e^13 + 2048*a^4*b*c^6*d*e^13 + 768*a^2*b^5*c^4*d*e^13 - 2048*a^3*b^3*
c^5*d*e^13)/(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c) + (x*(8*b^7*c^
2*e^14 - 96*a*b^5*c^3*e^14 - 512*a^3*b*c^5*e^14 + 384*a^2*b^3*c^4*e^14))/(b
^4 + 16*a^2*c^2 - 8*a*b^2*c))*(((f^4*(-(4*a*c - b^2)^9)^(1/2))/32 - (b^9*f^
4)/32 + 24*a^4*b*c^4*f^4 + 3*a^2*b^5*c^2*f^4 - 16*a^3*b^3*c^3*f^4)/(a*b^12*
e^2 + 4096*a^7*c^6*e^2 - 24*a^2*b^10*c*e^2 + 240*a^3*b^8*c^2*e^2 - 1280*a^4
*b^6*c^3*e^2 + 3840*a^5*b^4*c^4*e^2 - 6144*a^6*b^2*c^5*e^2))^(1/2) + (2*b^7
*c^2*e^12*f^2 + 96*a^2*b^3*c^4*e^12*f^2 - 24*a*b^5*c^3*e^12*f^2 - 128*a^3*b
*c^5*e^12*f^2)/(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) + (16*a^2*
c^5*d*e^11*f^4 + 5*b^4*c^3*d*e^11*f^4 - 24*a*b^2*c^4*d*e^11*f^4)/(b^6 - 64*
a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c) - (x*(4*a*c^4*e^12*f^4 - 5*b^2*c^3*e
^12*f^4))/(b^4 + 16*a^2*c^2 - 8*a*b^2*c))*1i + (((f^4*(-(4*a*c - b^2)^9)^(1
/2))/32 - (b^9*f^4)/32 + 24*a^4*b*c^4*f^4 + 3*a^2*b^5*c^2*f^4 - 16*a^3*b^3*
c^3*f^4)/(a*b^12*e^2 + 4096*a^7*c^6*e^2 - 24*a^2*b^10*c*e^2 + 240*a^3*b^8*c
^2*e^2 - 1280*a^4*b^6*c^3*e^2 + 3840*a^5*b^4*c^4*e^2 - 6144*a^6*b^2*c^5*e^2
))^(1/2)*((((f^4*(-(4*a*c - b^2)^9)^(1/2))/32 - (b^9*f^4)/32 + 24*a^4*b*c^4
*f^4 + 3*a^2*b^5*c^2*f^4 - 16*a^3*b^3*c^3*f^4)/(a*b^12*e^2 + 4096*a^7*c^6*e
^2 - 24*a^2*b^10*c*e^2 + 240*a^3*b^8*c^2*e^2 - 1280*a^4*b^6*c^3*e^2 + 3840*
a^5*b^4*c^4*e^2 - 6144*a^6*b^2*c^5*e^2))^(1/2)*(((8*b^9*c^2*d*e^13 - 128*a*
b^7*c^3*d*e^13 + 2048*a^4*b*c^6*d*e^13 + 768*a^2*b^5*c^4*d*e^13 - 2048*a^3*
b^3*c^5*d*e^13)/(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c) + (x*(8*b^
7*c^2*e^14 - 96*a*b^5*c^3*e^14 - 512*a^3*b*c^5*e^14 + 384*a^2*b^3*c^4*e^14)
)/(b^4 + 16*a^2*c^2 - 8*a*b^2*c))*(((f^4*(-(4*a*c - b^2)^9)^(1/2))/32 - (b^
9*f^4)/32 + 24*a^4*b*c^4*f^4 + 3*a^2*b^5*c^2*f^4 - 16*a^3*b^3*c^3*f^4)/(a*b
^12*e^2 + 4096*a^7*c^6*e^2 - 24*a^2*b^10*c*e^2 + 240*a^3*b^8*c^2*e^2 - 1280
*a^4*b^6*c^3*e^2 + 3840*a^5*b^4*c^4*e^2 - 6144*a^6*b^2*c^5*e^2))^(1/2) - (2
*b^7*c^2*e^12*f^2 + 96*a^2*b^3*c^4*e^12*f^2 - 24*a*b^5*c^3*e^12*f^2 - 128*a
^3*b*c^5*e^12*f^2)/(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) + (16*
a^2*c^5*d*e^11*f^4 + 5*b^4*c^3*d*e^11*f^4 - 24*a*b^2*c^4*d*e^11*f^4)/(b^6 -
 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c) - (x*(4*a*c^4*e^12*f^4 - 5*b^2*c
```

```
^3*e^12*f^4))/(b^4 + 16*a^2*c^2 - 8*a*b^2*c))*1i)/((2*a*c^4*e^10*f^6 + (3*b
^2*c^3*e^10*f^6)/2)/(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c) + (((f
^4*(-(4*a*c - b^2)^9)^(1/2))/32 - (b^9*f^4)/32 + 24*a^4*b*c^4*f^4 + 3*a^2*b
^5*c^2*f^4 - 16*a^3*b^3*c^3*f^4)/(a*b^12*e^2 + 4096*a^7*c^6*e^2 - 24*a^2*b^
10*c*e^2 + 240*a^3*b^8*c^2*e^2 - 1280*a^4*b^6*c^3*e^2 + 3840*a^5*b^4*c^4*e^
2 - 6144*a^6*b^2*c^5*e^2))^(1/2)*((((f^4*(-(4*a*c - b^2)^9)^(1/2))/32 - (b^
9*f^4)/32 + 24*a^4*b*c^4*f^4 + 3*a^2*b^5*c^2*f^4 - 16*a^3*b^3*c^3*f^4)/(a*b
^12*e^2 + 4096*a^7*c^6*e^2 - 24*a^2*b^10*c*e^2 + 240*a^3*b^8*c^2*e^2 - 1280
*a^4*b^6*c^3*e^2 + 3840*a^5*b^4*c^4*e^2 - 6144*a^6*b^2*c^5*e^2))^(1/2)*(((8
*b^9*c^2*d*e^13 - 128*a*b^7*c^3*d*e^13 + 2048*a^4*b*c^6*d*e^13 + 768*a^2*b^
5*c^4*d*e^13 - 2048*a^3*b^3*c^5*d*e^13)/(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2
- 12*a*b^4*c) + (x*(8*b^7*c^2*e^14 - 96*a*b^5*c^3*e^14 - 512*a^3*b*c^5*e^14
 + 384*a^2*b^3*c^4*e^14))/(b^4 + 16*a^2*c^2 - 8*a*b^2*c))*(((f^4*(-(4*a*c -
 b^2)^9)^(1/2))/32 - (b^9*f^4)/32 + 24*a^4*b*c^4*f^4 + 3*a^2*b^5*c^2*f^4 -
16*a^3*b^3*c^3*f^4)/(a*b^12*e^2 + 4096*a^7*c^6*e^2 - 24*a^2*b^10*c*e^2 + 24
0*a^3*b^8*c^2*e^2 - 1280*a^4*b^6*c^3*e^2 + 3840*a^5*b^4*c^4*e^2 - 6144*a^6*
b^2*c^5*e^2))^(1/2) + (2*b^7*c^2*e^12*f^2 + 96*a^2*b^3*c^4*e^12*f^2 - 24*a*
b^5*c^3*e^12*f^2 - 128*a^3*b*c^5*e^12*f^2)/(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c
^2 - 12*a*b^4*c)) + (16*a^2*c^5*d*e^11*f^4 + 5*b^4*c^3*d*e^11*f^4 - 24*a*b^
2*c^4*d*e^11*f^4)/(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c) - (x*(4*
a*c^4*e^12*f^4 - 5*b^2*c^3*e^12*f^4))/(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) - (((
f^4*(-(4*a*c - b^2)^9)^(1/2))/32 - (b^9*f^4)/32 + 24*a^4*b*c^4*f^4 + 3*a^2*
b^5*c^2*f^4 - 16*a^3*b^3*c^3*f^4)/(a*b^12*e^2 + 4096*a^7*c^6*e^2 - 24*a^2*b
^10*c*e^2 + 240*a^3*b^8*c^2*e^2 - 1280*a^4*b^6*c^3*e^2 + 3840*a^5*b^4*c^4*e
^2 - 6144*a^6*b^2*c^5*e^2))^(1/2)*((((f^4*(-(4*a*c - b^2)^9)^(1/2))/32 - (b
^9*f^4)/32 + 24*a^4*b*c^4*f^4 + 3*a^2*b^5*c^2*f^4 - 16*a^3*b^3*c^3*f^4)/(a*
b^12*e^2 + 4096*a^7*c^6*e^2 - 24*a^2*b^10*c*e^2 + 240*a^3*b^8*c^2*e^2 - 128
0*a^4*b^6*c^3*e^2 + 3840*a^5*b^4*c^4*e^2 - 6144*a^6*b^2*c^5*e^2))^(1/2)*(((
8*b^9*c^2*d*e^13 - 128*a*b^7*c^3*d*e^13 + 2048*a^4*b*c^6*d*e^13 + 768*a^2*b
^5*c^4*d*e^13 - 2048*a^3*b^3*c^5*d*e^13)/(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2
 - 12*a*b^4*c) + (x*(8*b^7*c^2*e^14 - 96*a*b^5*c^3*e^14 - 512*a^3*b*c^5*e^1
4 + 384*a^2*b^3*c^4*e^14))/(b^4 + 16*a^2*c^2 - 8*a*b^2*c))*(((f^4*(-(4*a*c
- b^2)^9)^(1/2))/32 - (b^9*f^4)/32 + 24*a^4*b*c^4*f^4 + 3*a^2*b^5*c^2*f^4 -
 16*a^3*b^3*c^3*f^4)/(a*b^12*e^2 + 4096*a^7*c^6*e^2 - 24*a^2*b^10*c*e^2 + 2
40*a^3*b^8*c^2*e^2 - 1280*a^4*b^6*c^3*e^2 + 3840*a^5*b^4*c^4*e^2 - 6144*a^6
*b^2*c^5*e^2))^(1/2) - (2*b^7*c^2*e^12*f^2 + 96*a^2*b^3*c^4*e^12*f^2 - 24*a
*b^5*c^3*e^12*f^2 - 128*a^3*b*c^5*e^12*f^2)/(b^6 - 64*a^3*c^3 + 48*a^2*b^2*
c^2 - 12*a*b^4*c)) + (16*a^2*c^5*d*e^11*f^4 + 5*b^4*c^3*d*e^11*f^4 - 24*a*b
^2*c^4*d*e^11*f^4)/(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c) - (x*(4
*a*c^4*e^12*f^4 - 5*b^2*c^3*e^12*f^4))/(b^4 + 16*a^2*c^2 - 8*a*b^2*c))))*((
f^4*(-(4*a*c - b^2)^9)^(1/2) - b^9*f^4 + 768*a^4*b*c^4*f^4 + 96*a^2*b^5*c^2
*f^4 - 512*a^3*b^3*c^3*f^4)/(32*(a*b^12*e^2 + 4096*a^7*c^6*e^2 - 24*a^2*b^1
0*c*e^2 + 240*a^3*b^8*c^2*e^2 - 1280*a^4*b^6*c^3*e^2 + 3840*a^5*b^4*c^4*e^2
 - 6144*a^6*b^2*c^5*e^2)))^(1/2)*2i + atan(((-((b^9*f^4)/32 + (f^4*(-(4*a*c
 - b^2)^9)^(1/2))/32 - 24*a^4*b*c^4*f^4 - 3*a^2*b^5*c^2*f^4 + 16*a^3*b^3*c^
3*f^4)/(a*b^12*e^2 + 4096*a^7*c^6*e^2 - 24*a^2*b^10*c*e^2 + 240*a^3*b^8*c^2
*e^2 - 1280*a^4*b^6*c^3*e^2 + 3840*a^5*b^4*c^4*e^2 - 6144*a^6*b^2*c^5*e^2))
^(1/2)*((-((b^9*f^4)/32 + (f^4*(-(4*a*c - b^2)^9)^(1/2))/32 - 24*a^4*b*c^4*
f^4 - 3*a^2*b^5*c^2*f^4 + 16*a^3*b^3*c^3*f^4)/(a*b^12*e^2 + 4096*a^7*c^6*e^
2 - 24*a^2*b^10*c*e^2 + 240*a^3*b^8*c^2*e^2 - 1280*a^4*b^6*c^3*e^2 + 3840*a
^5*b^4*c^4*e^2 - 6144*a^6*b^2*c^5*e^2))^(1/2)*(((8*b^9*c^2*d*e^13 - 128*a*b
^7*c^3*d*e^13 + 2048*a^4*b*c^6*d*e^13 + 768*a^2*b^5*c^4*d*e^13 - 2048*a^3*b
^3*c^5*d*e^13)/(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c) + (x*(8*b^7
*c^2*e^14 - 96*a*b^5*c^3*e^14 - 512*a^3*b*c^5*e^14 + 384*a^2*b^3*c^4*e^14))
/(b^4 + 16*a^2*c^2 - 8*a*b^2*c))*(-((b^9*f^4)/32 + (f^4*(-(4*a*c - b^2)^9)^
(1/2))/32 - 24*a^4*b*c^4*f^4 - 3*a^2*b^5*c^2*f^4 + 16*a^3*b^3*c^3*f^4)/(a*b
^12*e^2 + 4096*a^7*c^6*e^2 - 24*a^2*b^10*c*e^2 + 240*a^3*b^8*c^2*e^2 - 1280
*a^4*b^6*c^3*e^2 + 3840*a^5*b^4*c^4*e^2 - 6144*a^6*b^2*c^5*e^2))^(1/2) + (2
*b^7*c^2*e^12*f^2 + 96*a^2*b^3*c^4*e^12*f^2 - 24*a*b^5*c^3*e^12*f^2 - 128*a
```

^3*b*c^5*e^12*f^2)/(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) + (16*
a^2*c^5*d*e^11*f^4 + 5*b^4*c^3*d*e^11*f^4 - 24*a*b^2*c^4*d*e^11*f^4)/(b^6 -
64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c) - (x*(4*a*c^4*e^12*f^4 - 5*b^2*c
^3*e^12*f^4))/(b^4 + 16*a^2*c^2 - 8*a*b^2*c))*1i + (-((b^9*f^4)/32 + (f^4*(
-(4*a*c - b^2)^9)^(1/2))/32 - 24*a^4*b*c^4*f^4 - 3*a^2*b^5*c^2*f^4 + 16*a^3
*b^3*c^3*f^4)/(a*b^12*e^2 + 4096*a^7*c^6*e^2 - 24*a^2*b^10*c*e^2 + 240*a^3*
b^8*c^2*e^2 - 1280*a^4*b^6*c^3*e^2 + 3840*a^5*b^4*c^4*e^2 - 6144*a^6*b^2*c^
5*e^2))^(1/2)*((-((b^9*f^4)/32 + (f^4*(-(4*a*c - b^2)^9)^(1/2))/32 - 24*a^4
*b*c^4*f^4 - 3*a^2*b^5*c^2*f^4 + 16*a^3*b^3*c^3*f^4)/(a*b^12*e^2 + 4096*a^7
*c^6*e^2 - 24*a^2*b^10*c*e^2 + 240*a^3*b^8*c^2*e^2 - 1280*a^4*b^6*c^3*e^2 +
3840*a^5*b^4*c^4*e^2 - 6144*a^6*b^2*c^5*e^2))^(1/2)*(((8*b^9*c^2*d*e^13 -
128*a*b^7*c^3*d*e^13 + 2048*a^4*b*c^6*d*e^13 + 768*a^2*b^5*c^4*d*e^13 - 204
8*a^3*b^3*c^5*d*e^13)/(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c) + (x
*(8*b^7*c^2*e^14 - 96*a*b^5*c^3*e^14 - 512*a^3*b*c^5*e^14 + 384*a^2*b^3*c^4
*e^14))/(b^4 + 16*a^2*c^2 - 8*a*b^2*c))*(-((b^9*f^4)/32 + (f^4*(-(4*a*c - b
^2)^9)^(1/2))/32 - 24*a^4*b*c^4*f^4 - 3*a^2*b^5*c^2*f^4 + 16*a^3*b^3*c^3*f^
4)/(a*b^12*e^2 + 4096*a^7*c^6*e^2 - 24*a^2*b^10*c*e^2 + 240*a^3*b^8*c^2*e^2
- 1280*a^4*b^6*c^3*e^2 + 3840*a^5*b^4*c^4*e^2 - 6144*a^6*b^2*c^5*e^2))^(1/
2) - (2*b^7*c^2*e^12*f^2 + 96*a^2*b^3*c^4*e^12*f^2 - 24*a*b^5*c^3*e^12*f^2
- 128*a^3*b*c^5*e^12*f^2)/(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c))
+ (16*a^2*c^5*d*e^11*f^4 + 5*b^4*c^3*d*e^11*f^4 - 24*a*b^2*c^4*d*e^11*f^4)
/(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c) - (x*(4*a*c^4*e^12*f^4 -
5*b^2*c^3*e^12*f^4))/(b^4 + 16*a^2*c^2 - 8*a*b^2*c))*1i)/((2*a*c^4*e^10*f^6
+ (3*b^2*c^3*e^10*f^6)/2)/(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)
+ (-((b^9*f^4)/32 + (f^4*(-(4*a*c - b^2)^9)^(1/2))/32 - 24*a^4*b*c^4*f^4 -
3*a^2*b^5*c^2*f^4 + 16*a^3*b^3*c^3*f^4)/(a*b^12*e^2 + 4096*a^7*c^6*e^2 - 2
4*a^2*b^10*c*e^2 + 240*a^3*b^8*c^2*e^2 - 1280*a^4*b^6*c^3*e^2 + 3840*a^5*b^
4*c^4*e^2 - 6144*a^6*b^2*c^5*e^2))^(1/2)*((-((b^9*f^4)/32 + (f^4*(-(4*a*c -
b^2)^9)^(1/2))/32 - 24*a^4*b*c^4*f^4 - 3*a^2*b^5*c^2*f^4 + 16*a^3*b^3*c^3*
f^4)/(a*b^12*e^2 + 4096*a^7*c^6*e^2 - 24*a^2*b^10*c*e^2 + 240*a^3*b^8*c^2*e
^2 - 1280*a^4*b^6*c^3*e^2 + 3840*a^5*b^4*c^4*e^2 - 6144*a^6*b^2*c^5*e^2))^(
1/2)*(((8*b^9*c^2*d*e^13 - 128*a*b^7*c^3*d*e^13 + 2048*a^4*b*c^6*d*e^13 + 7
68*a^2*b^5*c^4*d*e^13 - 2048*a^3*b^3*c^5*d*e^13)/(b^6 - 64*a^3*c^3 + 48*a^2
*b^2*c^2 - 12*a*b^4*c) + (x*(8*b^7*c^2*e^14 - 96*a*b^5*c^3*e^14 - 512*a^3*b
*c^5*e^14 + 384*a^2*b^3*c^4*e^14))/(b^4 + 16*a^2*c^2 - 8*a*b^2*c))*(-((b^9*
f^4)/32 + (f^4*(-(4*a*c - b^2)^9)^(1/2))/32 - 24*a^4*b*c^4*f^4 - 3*a^2*b^5*
c^2*f^4 + 16*a^3*b^3*c^3*f^4)/(a*b^12*e^2 + 4096*a^7*c^6*e^2 - 24*a^2*b^10*
c*e^2 + 240*a^3*b^8*c^2*e^2 - 1280*a^4*b^6*c^3*e^2 + 3840*a^5*b^4*c^4*e^2 -
6144*a^6*b^2*c^5*e^2))^(1/2) + (2*b^7*c^2*e^12*f^2 + 96*a^2*b^3*c^4*e^12*f
^2 - 24*a*b^5*c^3*e^12*f^2 - 128*a^3*b*c^5*e^12*f^2)/(b^6 - 64*a^3*c^3 + 48
*a^2*b^2*c^2 - 12*a*b^4*c)) + (16*a^2*c^5*d*e^11*f^4 + 5*b^4*c^3*d*e^11*f^4
- 24*a*b^2*c^4*d*e^11*f^4)/(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c
) - (x*(4*a*c^4*e^12*f^4 - 5*b^2*c^3*e^12*f^4))/(b^4 + 16*a^2*c^2 - 8*a*b^2
*c)) - (-((b^9*f^4)/32 + (f^4*(-(4*a*c - b^2)^9)^(1/2))/32 - 24*a^4*b*c^4*f
^4 - 3*a^2*b^5*c^2*f^4 + 16*a^3*b^3*c^3*f^4)/(a*b^12*e^2 + 4096*a^7*c^6*e^2
- 24*a^2*b^10*c*e^2 + 240*a^3*b^8*c^2*e^2 - 1280*a^4*b^6*c^3*e^2 + 3840*a^
5*b^4*c^4*e^2 - 6144*a^6*b^2*c^5*e^2))^(1/2)*((-((b^9*f^4)/32 + (f^4*(-(4*a
*c - b^2)^9)^(1/2))/32 - 24*a^4*b*c^4*f^4 - 3*a^2*b^5*c^2*f^4 + 16*a^3*b^3*
c^3*f^4)/(a*b^12*e^2 + 4096*a^7*c^6*e^2 - 24*a^2*b^10*c*e^2 + 240*a^3*b^8*c
^2*e^2 - 1280*a^4*b^6*c^3*e^2 + 3840*a^5*b^4*c^4*e^2 - 6144*a^6*b^2*c^5*e^2
))^(1/2)*(((8*b^9*c^2*d*e^13 - 128*a*b^7*c^3*d*e^13 + 2048*a^4*b*c^6*d*e^13
+ 768*a^2*b^5*c^4*d*e^13 - 2048*a^3*b^3*c^5*d*e^13)/(b^6 - 64*a^3*c^3 + 48
*a^2*b^2*c^2 - 12*a*b^4*c) + (x*(8*b^7*c^2*e^14 - 96*a*b^5*c^3*e^14 - 512*a
^3*b*c^5*e^14 + 384*a^2*b^3*c^4*e^14))/(b^4 + 16*a^2*c^2 - 8*a*b^2*c))*(-((
b^9*f^4)/32 + (f^4*(-(4*a*c - b^2)^9)^(1/2))/32 - 24*a^4*b*c^4*f^4 - 3*a^2*
b^5*c^2*f^4 + 16*a^3*b^3*c^3*f^4)/(a*b^12*e^2 + 4096*a^7*c^6*e^2 - 24*a^2*b
^10*c*e^2 + 240*a^3*b^8*c^2*e^2 - 1280*a^4*b^6*c^3*e^2 + 3840*a^5*b^4*c^4*e
^2 - 6144*a^6*b^2*c^5*e^2))^(1/2) - (2*b^7*c^2*e^12*f^2 + 96*a^2*b^3*c^4*e^
12*f^2 - 24*a*b^5*c^3*e^12*f^2 - 128*a^3*b*c^5*e^12*f^2)/(b^6 - 64*a^3*c^3

```
+ 48*a^2*b^2*c^2 - 12*a*b^4*c)) + (16*a^2*c^5*d*e^11*f^4 + 5*b^4*c^3*d*e^11
*f^4 - 24*a*b^2*c^4*d*e^11*f^4)/(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b
^4*c) - (x*(4*a*c^4*e^12*f^4 - 5*b^2*c^3*e^12*f^4))/(b^4 + 16*a^2*c^2 - 8*a
*b^2*c))))*(-(b^9*f^4 + f^4*(-(4*a*c - b^2)^9)^(1/2) - 768*a^4*b*c^4*f^4 -
96*a^2*b^5*c^2*f^4 + 512*a^3*b^3*c^3*f^4)/(32*(a*b^12*e^2 + 4096*a^7*c^6*e^
2 - 24*a^2*b^10*c*e^2 + 240*a^3*b^8*c^2*e^2 - 1280*a^4*b^6*c^3*e^2 + 3840*a
^5*b^4*c^4*e^2 - 6144*a^6*b^2*c^5*e^2)))^(1/2)*2i
```

sympy [B] time = 19.76, size = 646, normalized size = 2.46

$$\frac{bdf^2 + 2cd^3f^2 + 6cde^2f^2x^2 + 2ce^3f^2x^3 + x\left(bef^2 + 6cd^2ef^2\right)}{8a^2ce - 2ab^2e + 8abcd^2e + 8ac^2d^4e - 2b^3d^2e - 2b^2cd^4e + x^4\left(8ac^2e^5 - 2b^2ce^5\right) + x^3\left(32ac^2de^4 - 8b^2cde^4\right) + x^2\left(8abce^3 + 48ac^2d^2e^3 - 2b^3e^3 - 12b^2cd^2e^3\right) + x\left(16abcde^2 + 32ac^2d^3e^2 - 4b^3de^2 - 8b^2cd^3e^2\right)} + \operatorname{RootSum}\left(t^4\left(1048576a^7c^6e^4 - 1572864a^6b^2c^5e^4 + 983040a^5b^4c^4e^4 - 327680a^4b^6c^3e^4 + 61440a^3b^8c^2e^4 - 6144a^2b^{10}ce^4 + 256ab^{12}e^4\right) + t^2\left(-12288a^4bc^4e^2f^4 + 8192a^3b^3c^3e^2f^4 - 1536a^2b^5c^2e^2f^4 + 16b^9e^2f^4\right) + 16a^2c^3f^8 + 24ab^2c^2f^8 + 9b^4cf^8, \left(t \mapsto t\log\left(x + \frac{16384t^3a^5c^4e^3 - 8192t^3a^4b^2c^3e^3 + 512t^3a^2b^6ce^3 - 64t^3ab^8e^3 - 128ta^2bc^2ef^4 - 16tab^3cef^4 - 4tb^5ef^4 + 4ac^2df^6 + 3b^2cdf^6}{4ac^2ef^6 + 3b^2cef^6}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*f*x+d*f)**2/(a+b*(e*x+d)**2+c*(e*x+d)**4)**2,x)
```

```
[Out] (b*d*f**2 + 2*c*d**3*f**2 + 6*c*d*e**2*f**2*x**2 + 2*c*e**3*f**2*x**3 + x*(
b*e*f**2 + 6*c*d**2*e*f**2))/(8*a**2*c*e - 2*a*b**2*e + 8*a*b*c*d**2*e + 8*
a*c**2*d**4*e - 2*b**3*d**2*e - 2*b**2*c*d**4*e + x**4*(8*a*c**2*e**5 - 2*b
**2*c*e**5) + x**3*(32*a*c**2*d*e**4 - 8*b**2*c*d*e**4) + x**2*(8*a*b*c*e**
3 + 48*a*c**2*d**2*e**3 - 2*b**3*e**3 - 12*b**2*c*d**2*e**3) + x*(16*a*b*c*
d*e**2 + 32*a*c**2*d**3*e**2 - 4*b**3*d*e**2 - 8*b**2*c*d**3*e**2)) + RootS
um(_t**4*(1048576*a**7*c**6*e**4 - 1572864*a**6*b**2*c**5*e**4 + 983040*a**
5*b**4*c**4*e**4 - 327680*a**4*b**6*c**3*e**4 + 61440*a**3*b**8*c**2*e**4 -
 6144*a**2*b**10*c*e**4 + 256*a*b**12*e**4) + _t**2*(-12288*a**4*b*c**4*e**
2*f**4 + 8192*a**3*b**3*c**3*e**2*f**4 - 1536*a**2*b**5*c**2*e**2*f**4 + 16
*b**9*e**2*f**4) + 16*a**2*c**3*f**8 + 24*a*b**2*c**2*f**8 + 9*b**4*c*f**8,
 Lambda(_t, _t*log(x + (16384*_t**3*a**5*c**4*e**3 - 8192*_t**3*a**4*b**2*c
**3*e**3 + 512*_t**3*a**2*b**6*c*e**3 - 64*_t**3*a*b**8*e**3 - 128*_t*a**2*
b*c**2*e*f**4 - 16*_t*a*b**3*c*e*f**4 - 4*_t*b**5*e*f**4 + 4*a*c**2*d*f**6
+ 3*b**2*c*d*f**6)/(4*a*c**2*e*f**6 + 3*b**2*c*e*f**6))))
```

3.536 $\int \frac{df+efx}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx$

Optimal. Leaf size=98

$$\frac{2cf\tanh^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{e\left(b^2-4ac\right)^{3/2}}-\frac{f\left(b+2c(d+ex)^2\right)}{2e\left(b^2-4ac\right)\left(a+b(d+ex)^2+c(d+ex)^4\right)}$$

Rubi [A] time = 0.13, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161, Rules used = {1142, 1107, 614, 618, 206}

$$\frac{2cf\tanh^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{e\left(b^2-4ac\right)^{3/2}}-\frac{f\left(b+2c(d+ex)^2\right)}{2e\left(b^2-4ac\right)\left(a+b(d+ex)^2+c(d+ex)^4\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(d*f + e*f*x)/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2,x]
```

```
[Out] -(f*(b + 2*c*(d + e*x)^2))/(2*(b^2 - 4*a*c)*e*(a + b*(d + e*x)^2 + c*(d + e
*x)^4)) + (2*c*f*ArcTanh[(b + 2*c*(d + e*x)^2)/Sqrt[b^2 - 4*a*c]])/((b^2 -
4*a*c)^(3/2)*e)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 614

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[(2*c*(2*p +
 3))/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
egerQ[4*p]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1107

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rule 1142

```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Di
st[u^m/(Coefficient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p,
 x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

Rubi steps

$$\int \frac{df+efx}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx = \frac{f\,\text{Subst}\left(\int \frac{x}{\left(a+bx^2+cx^4\right)^2}\,dx, x, d+ex\right)}{e}$$

$$= \frac{f\,\text{Subst}\left(\int \frac{1}{\left(a+bx+cx^2\right)^2}\,dx, x, (d+ex)^2\right)}{2e}$$

$$= -\frac{f\left(b+2c(d+ex)^2\right)}{2\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)} - \frac{(cf)\,\text{Subst}\left(\int \frac{1}{a+bx+cx^2}\,dx, \right.}{\left(b^2-4ac\right)e}$$

$$= -\frac{f\left(b+2c(d+ex)^2\right)}{2\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)} + \frac{(2cf)\,\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,d\right.}{\left(b^2-4ac\right.}$$

$$= -\frac{f\left(b+2c(d+ex)^2\right)}{2\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)} + \frac{2cf\tanh^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{3/2}e}$$

Mathematica [A] time = 0.12, size = 99, normalized size = 1.01

$$-\frac{f\left(\frac{4c\tan^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}+\frac{b+2c(d+ex)^2}{a+b(d+ex)^2+c(d+ex)^4}\right)}{2e\left(b^2-4ac\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*f + e*f*x)/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2,x]
```

```
[Out] -1/2*(f*((b + 2*c*(d + e*x)^2)/(a + b*(d + e*x)^2 + c*(d + e*x)^4) + (4*c*A
rcTan[(b + 2*c*(d + e*x)^2)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c]))/((b^2
 - 4*a*c)*e)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{df+efx}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(d*f + e*f*x)/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2,x]
```

```
[Out] IntegrateAlgebraic[(d*f + e*f*x)/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2, x]
```

fricas [B] time = 1.11, size = 1066, normalized size = 10.88

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*f*x+d*f)/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="fricas")
```

```
[Out] [-1/2*(2*(b^2*c - 4*a*c^2)*e^2*f*x^2 + 4*(b^2*c - 4*a*c^2)*d*e*f*x + 2*(c^2
*e^4*f*x^4 + 4*c^2*d*e^3*f*x^3 + (6*c^2*d^2 + b*c)*e^2*f*x^2 + 2*(2*c^2*d^3
 + b*c*d)*e*f*x + (c^2*d^4 + b*c*d^2 + a*c)*f)*sqrt(b^2 - 4*a*c)*log((2*c^2
*e^4*x^4 + 8*c^2*d*e^3*x^3 + 2*c^2*d^4 + 2*(6*c^2*d^2 + b*c)*e^2*x^2 + 2*b*
```

```
c*d^2 + 4*(2*c^2*d^3 + b*c*d)*e*x + b^2 - 2*a*c - (2*c*e^2*x^2 + 4*c*d*e*x
+ 2*c*d^2 + b)*sqrt(b^2 - 4*a*c))/(c*e^4*x^4 + 4*c*d*e^3*x^3 + c*d^4 + (6*c
*d^2 + b)*e^2*x^2 + b*d^2 + 2*(2*c*d^3 + b*d)*e*x + a)) + (b^3 - 4*a*b*c +
2*(b^2*c - 4*a*c^2)*d^2)*f)/((b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*e^5*x^4 + 4
*(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*d*e^4*x^3 + (b^5 - 8*a*b^3*c + 16*a^2*b
*c^2 + 6*(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*d^2)*e^3*x^2 + 2*(2*(b^4*c - 8*
a*b^2*c^2 + 16*a^2*c^3)*d^3 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*d)*e^2*x + (
a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2 + (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*d^4 +
 (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*d^2)*e), -1/2*(2*(b^2*c - 4*a*c^2)*e^2*f*
x^2 + 4*(b^2*c - 4*a*c^2)*d*e*f*x - 4*(c^2*e^4*f*x^4 + 4*c^2*d*e^3*f*x^3 +
(6*c^2*d^2 + b*c)*e^2*f*x^2 + 2*(2*c^2*d^3 + b*c*d)*e*f*x + (c^2*d^4 + b*c*
d^2 + a*c)*f)*sqrt(-b^2 + 4*a*c)*arctan(-(2*c*e^2*x^2 + 4*c*d*e*x + 2*c*d^2
 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) + (b^3 - 4*a*b*c + 2*(b^2*c - 4*a*c
^2)*d^2)*f)/((b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*e^5*x^4 + 4*(b^4*c - 8*a*b^
2*c^2 + 16*a^2*c^3)*d*e^4*x^3 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2 + 6*(b^4*c
- 8*a*b^2*c^2 + 16*a^2*c^3)*d^2)*e^3*x^2 + 2*(2*(b^4*c - 8*a*b^2*c^2 + 16*a
^2*c^3)*d^3 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*d)*e^2*x + (a*b^4 - 8*a^2*b^
2*c + 16*a^3*c^2 + (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*d^4 + (b^5 - 8*a*b^3*
c + 16*a^2*b*c^2)*d^2)*e)]
```

giac [B] time = 0.44, size = 211, normalized size = 2.15

$$-\frac{2\,cf\arctan\left(\frac{2\,cd^2f+2\,(fx^2e+2\,dfx)ce+bf}{\sqrt{-b^2+4\,ac}\,f}\right)e^{(-1)}}{(b^2-4\,ac)\sqrt{-b^2+4\,ac}}-\frac{2\,cd^2f^3+2\,(fx^2e+2\,dfx)cf^2e+bf^3}{2\left(cd^4f^2+2\,(fx^2e+2\,dfx)cd^2fe+bd^2f^2+(fx^2e+2\,dfx)^2ce^2+(fx^2e+2\,dfx)bfe+af^2\right)(b^2e-4\,ace)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*f*x+d*f)/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="giac")
```

```
[Out] -2*c*f*arctan((2*c*d^2*f + 2*(f*x^2*e + 2*d*f*x)*c*e + b*f)/(sqrt(-b^2 + 4*
a*c)*f))*e^(-1)/((b^2 - 4*a*c)*sqrt(-b^2 + 4*a*c)) - 1/2*(2*c*d^2*f^3 + 2*(
f*x^2*e + 2*d*f*x)*c*f^2*e + b*f^3)/((c*d^4*f^2 + 2*(f*x^2*e + 2*d*f*x)*c*d
^2*f*e + b*d^2*f^2 + (f*x^2*e + 2*d*f*x)^2*c*e^2 + (f*x^2*e + 2*d*f*x)*b*f*
e + a*f^2)*(b^2*e - 4*a*c*e))
```

maple [C] time = 0.02, size = 484, normalized size = 4.94

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*f*x+d*f)/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x)
```

```
[Out] f/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*
d*e*x+b*d^2+a)*c*e/(4*a*c-b^2)*x^2+2*f/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2
*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)*c*d/(4*a*c-b^2)*x+f/(c*
e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x
+b*d^2+a)/(4*a*c-b^2)/e*c*d^2+1/2*f/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^
2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)/(4*a*c-b^2)/e*b+f*c/(4*a*c
-b^2)/e*sum((_R*e+d)/(2*_R^3*c*e^3+6*_R^2*c*d*e^2+6*_R*c*d^2*e+2*c*d^3+_R*b
*e+b*d)*ln(-_R+x),_R=RootOf(_Z^4*c*e^4+4*_Z^3*c*d*e^3+c*d^4+b*d^2+(6*c*d^2*
e^2+b*e^2)*_Z^2+(4*c*d^3*e+2*b*d*e)*_Z+a))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$2\,cf\int-\frac{ex+d}{(b^2c-4\,ac^2)e^4x^4+4\,(b^2c-4\,ac^2)de^3x^3+(b^2c-4\,ac^2)d^4+(b^3-4\,abc+6\,(b^2c-4\,ac^2)d^2)e^2x^2+ab^2-4\,a^2c+(b^3-4\,abc)d^2+2\,(2\,(b^2c-4\,ac^2)d^3+(b^3-4\,abc)d)ex}\,dx-\frac{2\,ce^2fx^2+4\,cdefx+(2\,cd^2+b)f}{2\left((b^2c-4\,ac^2)e^5x^4+4\,(b^2c-4\,ac^2)de^4x^3+(b^3-4\,abc+6\,(b^2c-4\,ac^2)d^2)e^3x^2+2\,(2\,(b^2c-4\,ac^2)d^3+(b^3-4\,abc)d)e^2x+((b^2c-4\,ac^2)d^4+ab^2-4\,a^2c+(b^3-4\,abc)d^2)e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*f*x+d*f)/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="maxima")
```

```
[Out] 2*c*f*integrate(-(e*x + d)/((b^2*c - 4*a*c^2)*e^4*x^4 + 4*(b^2*c - 4*a*c^2)
*d*e^3*x^3 + (b^2*c - 4*a*c^2)*d^4 + (b^3 - 4*a*b*c + 6*(b^2*c - 4*a*c^2)*d
^2)*e^2*x^2 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*d^2 + 2*(2*(b^2*c - 4*a*c^2
)*d^3 + (b^3 - 4*a*b*c)*d)*e*x), x) - 1/2*(2*c*e^2*f*x^2 + 4*c*d*e*f*x + (2
*c*d^2 + b)*f)/((b^2*c - 4*a*c^2)*e^5*x^4 + 4*(b^2*c - 4*a*c^2)*d*e^4*x^3 +
 (b^3 - 4*a*b*c + 6*(b^2*c - 4*a*c^2)*d^2)*e^3*x^2 + 2*(2*(b^2*c - 4*a*c^2)
*d^3 + (b^3 - 4*a*b*c)*d)*e^2*x + ((b^2*c - 4*a*c^2)*d^4 + a*b^2 - 4*a^2*c
+ (b^3 - 4*a*b*c)*d^2)*e)
```

mupad [B] time = 1.91, size = 442, normalized size = 4.51

$$\frac{\frac{f\,\left(2\,c\,d^2+b\right)}{2\,e\,\left(4\,a\,c-b^2\right)}+\frac{2\,c\,d\,f\,x}{4\,a\,c-b^2}+\frac{c\,e\,f\,x^2}{4\,a\,c-b^2}}{a+x^2\,\left(6\,c\,d^2\,e^2+b\,e^2\right)+b\,d^2+c\,d^4+x\,\left(4\,c\,e\,d^3+2\,b\,e\,d\right)+c\,e^4\,x^4+4\,c\,d\,e^3\,x^3}+\frac{2\,c\,f\,\mathrm{atan}\left(\frac{{\left(4\,a\,c-b^2\right)}^4\,\left(x\,\left(\frac{8\,c^4\,d\,e^7\,f^2}{a\,{\left(4\,a\,c-b^2\right)}^{7/2}}-\frac{8\,b\,c^2\,f^2\,\left(b^3\,c^2\,d\,e^9-4\,a\,b\,c^3\,d\,e^9\right)}{a\,e^2\,{\left(4\,a\,c-b^2\right)}^{11/2}}\right)+x^2\,\left(\frac{4\,c^4\,e^8\,f^2}{a\,{\left(4\,a\,c-b^2\right)}^{7/2}}-\frac{4\,b\,c^2\,f^2\,\left(b^3\,c^2\,e^{10}-4\,a\,b\,c^3\,e^{10}\right)}{a\,e^2\,{\left(4\,a\,c-b^2\right)}^{11/2}}\right)+\frac{4\,c^4\,d^2\,e^6\,f^2}{a\,{\left(4\,a\,c-b^2\right)}^{7/2}}+\frac{4\,b\,c^2\,f^2\,\left(8\,a^2\,c^3\,e^8-2\,a\,b^2\,c^2\,e^8+4\,a\,b\,c^3\,d^2\,e^8-b^3\,c^2\,d^2\,e^8\right)}{a\,e^2\,{\left(4\,a\,c-b^2\right)}^{11/2}}\right)}{8\,c^4\,e^6\,f^2}\right)}{e\,{\left(4\,a\,c-b^2\right)}^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*f + e*f*x)/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2,x)
```

```
[Out] ((f*(b + 2*c*d^2))/(2*e*(4*a*c - b^2)) + (2*c*d*f*x)/(4*a*c - b^2) + (c*e*f
*x^2)/(4*a*c - b^2))/(a + x^2*(b*e^2 + 6*c*d^2*e^2) + b*d^2 + c*d^4 + x*(2*
b*d*e + 4*c*d^3*e) + c*e^4*x^4 + 4*c*d*e^3*x^3) + (2*c*f*atan(((4*a*c - b^2
)^4*(x*((8*c^4*d*e^7*f^2)/(a*(4*a*c - b^2)^(7/2)) - (8*b*c^2*f^2*(b^3*c^2*d
*e^9 - 4*a*b*c^3*d*e^9))/(a*e^2*(4*a*c - b^2)^(11/2))) + x^2*((4*c^4*e^8*f^
2)/(a*(4*a*c - b^2)^(7/2)) - (4*b*c^2*f^2*(b^3*c^2*e^10 - 4*a*b*c^3*e^10))/
(a*e^2*(4*a*c - b^2)^(11/2))) + (4*c^4*d^2*e^6*f^2)/(a*(4*a*c - b^2)^(7/2))
 + (4*b*c^2*f^2*(8*a^2*c^3*e^8 - 2*a*b^2*c^2*e^8 - b^3*c^2*d^2*e^8 + 4*a*b*
c^3*d^2*e^8))/(a*e^2*(4*a*c - b^2)^(11/2))))/(8*c^4*e^6*f^2)))/(e*(4*a*c -
b^2)^(3/2))
```

sympy [B] time = 5.07, size = 525, normalized size = 5.36

$$- \frac{c f \sqrt{- \frac{1}{\left(4 a c - b^{2}\right)^{3}}} \log{\left(\frac{2 d x}{e} + x^{2} + \frac{- 16 a^{2} c^{3} f \sqrt{- \frac{1}{\left(4 a c - b^{2}\right)^{3}}} + 8 a b^{2} c^{2} f \sqrt{- \frac{1}{\left(4 a c - b^{2}\right)^{3}}} - b^{4} c f \sqrt{- \frac{1}{\left(4 a c - b^{2}\right)^{3}}} + b c f + 2 c^{2} d^{2} f}{2 c^{2} e^{2} f} \right)}}{e} + \frac{c f \sqrt{- \frac{1}{\left(4 a c - b^{2}\right)^{3}}} \log{\left(\frac{2 d x}{e} + x^{2} + \frac{16 a^{2} c^{3} f \sqrt{- \frac{1}{\left(4 a c - b^{2}\right)^{3}}} - 8 a b^{2} c^{2} f \sqrt{- \frac{1}{\left(4 a c - b^{2}\right)^{3}}} + b^{4} c f \sqrt{- \frac{1}{\left(4 a c - b^{2}\right)^{3}}} + b c f + 2 c^{2} d^{2} f}{2 c^{2} e^{2} f} \right)}}{e} + \frac{b f + 2 c d^{2} f + 4 c d e f x + 2 c e^{2} f x^{2}}{8 a^{2} c e - 2 a b^{2} e + 8 a b c d^{2} e + 8 a c^{2} d^{4} e - 2 b^{3} d^{2} e - 2 b^{2} c d^{4} e + x^{4} \left(8 a c^{2} e^{5} - 2 b^{2} c e^{5}\right) + x^{3} \left(32 a c^{2} d e^{4} - 8 b^{2} c d e^{4}\right) + x^{2} \left(8 a b c e^{3} + 48 a c^{2} d^{2} e^{3} - 2 b^{3} e^{3} - 12 b^{2} c d^{2} e^{3}\right) + x \left(16 a b c d e^{2} + 32 a c^{2} d^{3} e^{2} - 4 b^{3} d e^{2} - 8 b^{2} c d^{3} e^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*f*x+d*f)/(a+b*(e*x+d)**2+c*(e*x+d)**4)**2,x)
```

```
[Out] -c*f*sqrt(-1/(4*a*c - b**2)**3)*log(2*d*x/e + x**2 + (-16*a**2*c**3*f*sqrt(
-1/(4*a*c - b**2)**3) + 8*a*b**2*c**2*f*sqrt(-1/(4*a*c - b**2)**3) - b**4*c
*f*sqrt(-1/(4*a*c - b**2)**3) + b*c*f + 2*c**2*d**2*f)/(2*c**2*e**2*f))/e +
 c*f*sqrt(-1/(4*a*c - b**2)**3)*log(2*d*x/e + x**2 + (16*a**2*c**3*f*sqrt(-
1/(4*a*c - b**2)**3) - 8*a*b**2*c**2*f*sqrt(-1/(4*a*c - b**2)**3) + b**4*c*
f*sqrt(-1/(4*a*c - b**2)**3) + b*c*f + 2*c**2*d**2*f)/(2*c**2*e**2*f))/e +
(b*f + 2*c*d**2*f + 4*c*d*e*f*x + 2*c*e**2*f*x**2)/(8*a**2*c*e - 2*a*b**2*e
 + 8*a*b*c*d**2*e + 8*a*c**2*d**4*e - 2*b**3*d**2*e - 2*b**2*c*d**4*e + x**
4*(8*a*c**2*e**5 - 2*b**2*c*e**5) + x**3*(32*a*c**2*d*e**4 - 8*b**2*c*d*e**
4) + x**2*(8*a*b*c*e**3 + 48*a*c**2*d**2*e**3 - 2*b**3*e**3 - 12*b**2*c*d**
2*e**3) + x*(16*a*b*c*d*e**2 + 32*a*c**2*d**3*e**2 - 4*b**3*d*e**2 - 8*b**2
*c*d**3*e**2))
```

3.537 $\int \frac{1}{(df+efx)\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx$

Optimal. Leaf size=174

$$\frac{b\left(b^2-6ac\right)\tanh^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{2a^2ef\left(b^2-4ac\right)^{3/2}}-\frac{\log\left(a+b(d+ex)^2+c(d+ex)^4\right)}{4a^2ef}+\frac{\log(d+ex)}{a^2ef}+\frac{-2ac+b^2+bc(d}{2aef\left(b^2-4ac\right)\left(a+b(d+e\right.}$$

Rubi [A] time = 0.30, antiderivative size = 174, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.242, Rules used = {1142, 1114, 740, 800, 634, 618, 206, 628}

$$\frac{b\left(b^2-6ac\right)\tanh^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{2a^2ef\left(b^2-4ac\right)^{3/2}}-\frac{\log\left(a+b(d+ex)^2+c(d+ex)^4\right)}{4a^2ef}+\frac{\log(d+ex)}{a^2ef}+\frac{-2ac+b^2+bc(d+ex)^2}{2aef\left(b^2-4ac\right)\left(a+b(d+ex)^2+c(d+ex)^4\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d*f + e*f*x)*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2),x]
```

```
[Out] (b^2 - 2*a*c + b*c*(d + e*x)^2)/(2*a*(b^2 - 4*a*c)*e*f*(a + b*(d + e*x)^2 +
 c*(d + e*x)^4)) + (b*(b^2 - 6*a*c)*ArcTanh[(b + 2*c*(d + e*x)^2)/Sqrt[b^2
- 4*a*c]])/(2*a^2*(b^2 - 4*a*c)^(3/2)*e*f) + Log[d + e*x]/(a^2*e*f) - Log[a
 + b*(d + e*x)^2 + c*(d + e*x)^4]/(4*a^2*e*f)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 740

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e
)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e
^2)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d +
 e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p +
 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p,
```

```
-1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 1142

```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Di
st[u^m/(Coefficient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p,
 x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

Rubi steps

$$\int \frac{1}{(df+efx)\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x\left(a+bx^2+cx^4\right)^2}\,dx, x, d+ex\right)}{ef}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{x\left(a+bx+cx^2\right)^2}\,dx, x, (d+ex)^2\right)}{2ef}$$

$$= \frac{b^2-2ac+bc(d+ex)^2}{2a\left(b^2-4ac\right)ef\left(a+b(d+ex)^2+c(d+ex)^4\right)} - \frac{\text{Subst}\left(\int \frac{-b^2+\cdots}{x(a+\cdots}\right.}{2a\left(\cdots\right.}$$

$$= \frac{b^2-2ac+bc(d+ex)^2}{2a\left(b^2-4ac\right)ef\left(a+b(d+ex)^2+c(d+ex)^4\right)} - \frac{\text{Subst}\left(\int \left(\frac{-b^2\cdots}{a\cdots}\right.\right.}{\cdots}$$

$$= \frac{b^2-2ac+bc(d+ex)^2}{2a\left(b^2-4ac\right)ef\left(a+b(d+ex)^2+c(d+ex)^4\right)} + \frac{\log(d+ex)}{a^2ef} - \cdots$$

$$= \frac{b^2-2ac+bc(d+ex)^2}{2a\left(b^2-4ac\right)ef\left(a+b(d+ex)^2+c(d+ex)^4\right)} + \frac{\log(d+ex)}{a^2ef} - \cdots$$

$$= \frac{b^2-2ac+bc(d+ex)^2}{2a\left(b^2-4ac\right)ef\left(a+b(d+ex)^2+c(d+ex)^4\right)} + \frac{\log(d+ex)}{a^2ef} - \cdots$$

$$= \frac{b^2-2ac+bc(d+ex)^2}{2a\left(b^2-4ac\right)ef\left(a+b(d+ex)^2+c(d+ex)^4\right)} + \frac{b\left(b^2-6ac\right)\text{ta}\cdots}{2a^2\left(b^2\cdots\right.}$$

Mathematica [A] time = 0.44, size = 238, normalized size = 1.37

$$\frac{\frac{2a\left(-2ac+b^2+bc(d+ex)^2\right)}{\left(b^2-4ac\right)\left(a+(d+ex)^2\left(b+c(d+ex)^2\right)\right)}-\frac{\left(b^2\sqrt{b^2-4ac}-4ac\sqrt{b^2-4ac}-6abc+b^3\right)\log\left(-\sqrt{b^2-4ac}+b+2c(d+ex)^2\right)}{\left(b^2-4ac\right)^{3/2}}+\frac{\left(-b^2\sqrt{b^2-4ac}+4ac\sqrt{b^2-4ac}-6abc+b^3\right)\log\left(\sqrt{b^2-4ac}+b+2c(d+ex)^2\right)}{\left(b^2-4ac\right)^{3/2}}+4\log(d+ex)}{4a^2ef}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d*f + e*f*x)*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2),x]
```

```
[Out] ((2*a*(b^2 - 2*a*c + b*c*(d + e*x)^2))/((b^2 - 4*a*c)*(a + (d + e*x)^2*(b +
 c*(d + e*x)^2))) + 4*Log[d + e*x] - ((b^3 - 6*a*b*c + b^2*Sqrt[b^2 - 4*a*c
] - 4*a*c*Sqrt[b^2 - 4*a*c])*Log[b - Sqrt[b^2 - 4*a*c] + 2*c*(d + e*x)^2])/
(b^2 - 4*a*c)^(3/2) + ((b^3 - 6*a*b*c - b^2*Sqrt[b^2 - 4*a*c] + 4*a*c*Sqrt[
b^2 - 4*a*c])*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*(d + e*x)^2])/(b^2 - 4*a*c)^(
3/2))/(4*a^2*e*f)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(df+efx)\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/((d*f + e*f*x)*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2),
x]
```

```
[Out] IntegrateAlgebraic[1/((d*f + e*f*x)*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2),
 x]
```

fricas [B] time = 2.34, size = 2486, normalized size = 14.29

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*f*x+d*f)/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="fricas"
)
```

```
[Out] [1/4*(2*a*b^4 - 12*a^2*b^2*c + 16*a^3*c^2 + 2*(a*b^3*c - 4*a^2*b*c^2)*e^2*x
^2 + 4*(a*b^3*c - 4*a^2*b*c^2)*d*e*x + 2*(a*b^3*c - 4*a^2*b*c^2)*d^2 + ((b^
3*c - 6*a*b*c^2)*e^4*x^4 + 4*(b^3*c - 6*a*b*c^2)*d*e^3*x^3 + (b^3*c - 6*a*b
*c^2)*d^4 + (b^4 - 6*a*b^2*c + 6*(b^3*c - 6*a*b*c^2)*d^2)*e^2*x^2 + a*b^3 -
 6*a^2*b*c + (b^4 - 6*a*b^2*c)*d^2 + 2*(2*(b^3*c - 6*a*b*c^2)*d^3 + (b^4 -
6*a*b^2*c)*d)*e*x)*sqrt(b^2 - 4*a*c)*log((2*c^2*e^4*x^4 + 8*c^2*d*e^3*x^3 +
 2*c^2*d^4 + 2*(6*c^2*d^2 + b*c)*e^2*x^2 + 2*b*c*d^2 + 4*(2*c^2*d^3 + b*c*d
)*e*x + b^2 - 2*a*c + (2*c*e^2*x^2 + 4*c*d*e*x + 2*c*d^2 + b)*sqrt(b^2 - 4*
a*c))/(c*e^4*x^4 + 4*c*d*e^3*x^3 + c*d^4 + (6*c*d^2 + b)*e^2*x^2 + b*d^2 +
2*(2*c*d^3 + b*d)*e*x + a)) - ((b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*e^4*x^4 +
 4*(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*d*e^3*x^3 + a*b^4 - 8*a^2*b^2*c + 16*
a^3*c^2 + (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*d^4 + (b^5 - 8*a*b^3*c + 16*a^
2*b*c^2 + 6*(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*d^2)*e^2*x^2 + (b^5 - 8*a*b^
3*c + 16*a^2*b*c^2)*d^2 + 2*(2*(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*d^3 + (b^
5 - 8*a*b^3*c + 16*a^2*b*c^2)*d)*e*x)*log(c*e^4*x^4 + 4*c*d*e^3*x^3 + c*d^4
 + (6*c*d^2 + b)*e^2*x^2 + b*d^2 + 2*(2*c*d^3 + b*d)*e*x + a) + 4*((b^4*c -
 8*a*b^2*c^2 + 16*a^2*c^3)*e^4*x^4 + 4*(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*d
*e^3*x^3 + a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2 + (b^4*c - 8*a*b^2*c^2 + 16*a^2
*c^3)*d^4 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2 + 6*(b^4*c - 8*a*b^2*c^2 + 16*a
^2*c^3)*d^2)*e^2*x^2 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*d^2 + 2*(2*(b^4*c -
 8*a*b^2*c^2 + 16*a^2*c^3)*d^3 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*d)*e*x)*l
og(e*x + d))/((a^2*b^4*c - 8*a^3*b^2*c^2 + 16*a^4*c^3)*e^5*f*x^4 + 4*(a^2*b
^4*c - 8*a^3*b^2*c^2 + 16*a^4*c^3)*d*e^4*f*x^3 + (a^2*b^5 - 8*a^3*b^3*c + 1
6*a^4*b*c^2 + 6*(a^2*b^4*c - 8*a^3*b^2*c^2 + 16*a^4*c^3)*d^2)*e^3*f*x^2 + 2
```

```
*(2*(a^2*b^4*c - 8*a^3*b^2*c^2 + 16*a^4*c^3)*d^3 + (a^2*b^5 - 8*a^3*b^3*c +
 16*a^4*b*c^2)*d)*e^2*f*x + (a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2 + (a^2*b^4*
c - 8*a^3*b^2*c^2 + 16*a^4*c^3)*d^4 + (a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2
)*d^2)*e*f), 1/4*(2*a*b^4 - 12*a^2*b^2*c + 16*a^3*c^2 + 2*(a*b^3*c - 4*a^2*
b*c^2)*e^2*x^2 + 4*(a*b^3*c - 4*a^2*b*c^2)*d*e*x + 2*(a*b^3*c - 4*a^2*b*c^2
)*d^2 + 2*((b^3*c - 6*a*b*c^2)*e^4*x^4 + 4*(b^3*c - 6*a*b*c^2)*d*e^3*x^3 +
(b^3*c - 6*a*b*c^2)*d^4 + (b^4 - 6*a*b^2*c + 6*(b^3*c - 6*a*b*c^2)*d^2)*e^2
*x^2 + a*b^3 - 6*a^2*b*c + (b^4 - 6*a*b^2*c)*d^2 + 2*(2*(b^3*c - 6*a*b*c^2)
*d^3 + (b^4 - 6*a*b^2*c)*d)*e*x)*sqrt(-b^2 + 4*a*c)*arctan(-(2*c*e^2*x^2 +
4*c*d*e*x + 2*c*d^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) - ((b^4*c - 8*a*
b^2*c^2 + 16*a^2*c^3)*e^4*x^4 + 4*(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*d*e^3*
x^3 + a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2 + (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)
*d^4 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2 + 6*(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^
3)*d^2)*e^2*x^2 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*d^2 + 2*(2*(b^4*c - 8*a*
b^2*c^2 + 16*a^2*c^3)*d^3 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*d)*e*x)*log(c*
e^4*x^4 + 4*c*d*e^3*x^3 + c*d^4 + (6*c*d^2 + b)*e^2*x^2 + b*d^2 + 2*(2*c*d^
3 + b*d)*e*x + a) + 4*((b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*e^4*x^4 + 4*(b^4*
c - 8*a*b^2*c^2 + 16*a^2*c^3)*d*e^3*x^3 + a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2
+ (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*d^4 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2
+ 6*(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*d^2)*e^2*x^2 + (b^5 - 8*a*b^3*c + 16
*a^2*b*c^2)*d^2 + 2*(2*(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*d^3 + (b^5 - 8*a*
b^3*c + 16*a^2*b*c^2)*d)*e*x)*log(e*x + d))/((a^2*b^4*c - 8*a^3*b^2*c^2 + 1
6*a^4*c^3)*e^5*f*x^4 + 4*(a^2*b^4*c - 8*a^3*b^2*c^2 + 16*a^4*c^3)*d*e^4*f*x
^3 + (a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2 + 6*(a^2*b^4*c - 8*a^3*b^2*c^2 +
 16*a^4*c^3)*d^2)*e^3*f*x^2 + 2*(2*(a^2*b^4*c - 8*a^3*b^2*c^2 + 16*a^4*c^3)
*d^3 + (a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*d)*e^2*f*x + (a^3*b^4 - 8*a^4
*b^2*c + 16*a^5*c^2 + (a^2*b^4*c - 8*a^3*b^2*c^2 + 16*a^4*c^3)*d^4 + (a^2*b
^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*d^2)*e*f)]
```

giac [B] time = 1.43, size = 476, normalized size = 2.74

$$-\frac{\left(a^{2} b^{3} c f e^{3} - 6 a^{3} b c^{2} f e^{3}\right) \sqrt{b^{2} - 4 a c} \log\left(\left|b x^{2} e^{2} + 2 b d x e + \sqrt{b^{2} - 4 a c} x^{2} e^{2} + 2 \sqrt{b^{2} - 4 a c} d x e + b d^{2} + \sqrt{b^{2} - 4 a c} d^{2} + 2 a\right|\right) - \left(a^{2} b^{3} c f e^{3} - 6 a^{3} b c^{2} f e^{3}\right) \sqrt{b^{2} - 4 a c} \log\left(\left|-b x^{2} e^{2} - 2 b d x e + \sqrt{b^{2} - 4 a c} x^{2} e^{2} + 2 \sqrt{b^{2} - 4 a c} d x e - b d^{2} + \sqrt{b^{2} - 4 a c} d^{2} - 2 a\right|\right)}{4 \left(a^{4} b^{4} c f^{2} e^{4} - 8 a^{5} b^{2} c^{2} f^{2} e^{4} + 16 a^{6} c^{3} f^{2} e^{4}\right)} - \frac{e^{(-1)} \log\left(\left|c x^{4} e^{4} + 4 c d x^{3} e^{3} + 6 c d^{2} x^{2} e^{2} + 4 c d^{3} x e + c d^{4} + b x^{2} e^{2} + 2 b d x e + b d^{2} + a\right|\right)}{4 a^{2} f} + \frac{e^{(-1)} \log\left(\left|x e + d\right|\right)}{a^{2} f} + \frac{\left(a b c x^{2} e^{2} + 2 a b c d x e + a b c d^{2} + a b^{2} - 2 a^{2} c\right) e^{(-1)}}{2 \left(c x^{4} e^{4} + 4 c d x^{3} e^{3} + 6 c d^{2} x^{2} e^{2} + 4 c d^{3} x e + c d^{4} + b x^{2} e^{2} + 2 b d x e + b d^{2} + a\right) \left(b^{2} - 4 a c\right) a^{2} f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*f*x+d*f)/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="giac")
```

```
[Out] -1/4*((a^2*b^3*c*f*e^3 - 6*a^3*b*c^2*f*e^3)*sqrt(b^2 - 4*a*c)*log(abs(b*x^2
*e^2 + 2*b*d*x*e + sqrt(b^2 - 4*a*c)*x^2*e^2 + 2*sqrt(b^2 - 4*a*c)*d*x*e +
b*d^2 + sqrt(b^2 - 4*a*c)*d^2 + 2*a)) - (a^2*b^3*c*f*e^3 - 6*a^3*b*c^2*f*e^
3)*sqrt(b^2 - 4*a*c)*log(abs(-b*x^2*e^2 - 2*b*d*x*e + sqrt(b^2 - 4*a*c)*x^2
*e^2 + 2*sqrt(b^2 - 4*a*c)*d*x*e - b*d^2 + sqrt(b^2 - 4*a*c)*d^2 - 2*a)))/(
a^4*b^4*c*f^2*e^4 - 8*a^5*b^2*c^2*f^2*e^4 + 16*a^6*c^3*f^2*e^4) - 1/4*e^(-1
)*log(abs(c*x^4*e^4 + 4*c*d*x^3*e^3 + 6*c*d^2*x^2*e^2 + 4*c*d^3*x*e + c*d^4
 + b*x^2*e^2 + 2*b*d*x*e + b*d^2 + a))/(a^2*f) + e^(-1)*log(abs(x*e + d))/(
a^2*f) + 1/2*(a*b*c*x^2*e^2 + 2*a*b*c*d*x*e + a*b*c*d^2 + a*b^2 - 2*a^2*c)*
e^(-1)/((c*x^4*e^4 + 4*c*d*x^3*e^3 + 6*c*d^2*x^2*e^2 + 4*c*d^3*x*e + c*d^4
+ b*x^2*e^2 + 2*b*d*x*e + b*d^2 + a)*(b^2 - 4*a*c)*a^2*f)
```

maple [C] time = 0.03, size = 714, normalized size = 4.10

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*f*x+d*f)/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x)
```

```
[Out] -1/2/f/a/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d
^4+2*b*d*e*x+b*d^2+a)*b*c*e/(4*a*c-b^2)*x^2-1/f/a/(c*e^4*x^4+4*c*d*e^3*x^3+
6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)*b*c*d/(4*a*c
-b^2)*x-1/2/f/a/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*
x^2+c*d^4+2*b*d*e*x+b*d^2+a)/e/(4*a*c-b^2)*b*c*d^2+1/f/(c*e^4*x^4+4*c*d*e^3
```

```
*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)/e/(4*a*
c-b^2)*c-1/2/f/a/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2
*x^2+c*d^4+2*b*d*e*x+b*d^2+a)/e/(4*a*c-b^2)*b^2-1/2/f/a^2/(4*a*c-b^2)/e*sum
(((4*a*c-b^2)*_R^3*c*e^3+3*(4*a*c-b^2)*_R^2*c*d*e^2+4*a*c^2*d^3-b^2*c*d^3+5
*a*b*c*d-b^3*d+(12*a*c^2*d^2-3*b^2*c*d^2+5*a*b*c-b^3)*_R*e)/(2*_R^3*c*e^3+6
*_R^2*c*d*e^2+6*_R*c*d^2*e+2*c*d^3+_R*b*e+b*d)*ln(-_R+x),_R=RootOf(_Z^4*c*e
^4+4*_Z^3*c*d*e^3+c*d^4+b*d^2+(6*c*d^2*e^2+b*e^2)*_Z^2+(4*c*d^3*e+2*b*d*e)*
_Z+a))+ln(e*x+d)/a^2/e/f
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*f*x+d*f)/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="maxima"
)
```

```
[Out] Timed out
```

mupad [B] time = 11.69, size = 13434, normalized size = 77.21

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((d*f + e*f*x)*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2),x)
```

```
[Out] ((b^2 - 2*a*c + b*c*d^2)/(2*e*(a*b^2 - 4*a^2*c)) + (b*c*e*x^2)/(2*(a*b^2 -
4*a^2*c)) + (b*c*d*x)/(a*b^2 - 4*a^2*c))/(a*f + x^2*(b*e^2*f + 6*c*d^2*e^2*
f) + x*(4*c*d^3*e*f + 2*b*d*e*f) + b*d^2*f + c*d^4*f + c*e^4*f*x^4 + 4*c*d*
e^3*f*x^3) - (log((((a^2*e*f*(-(b^2*(6*a*c - b^2)^2)/(a^4*e^2*f^2*(4*a*c -
b^2)^3))^(1/2) - 1)*(((a^2*e*f*(-(b^2*(6*a*c - b^2)^2)/(a^4*e^2*f^2*(4*a*c
- b^2)^3))^(1/2) - 1)*((2*b*c^2*e^16*(2*b^3 - 10*a*c^2*d^2 + b^2*c*d^2 - 10
*a*b*c))/(a*f*(4*a*c - b^2)) + (b*c^2*e^16*(a^2*e*f*(-(b^2*(6*a*c - b^2)^2)
/(a^4*e^2*f^2*(4*a*c - b^2)^3))^(1/2) - 1)*(a*b + 3*b^2*d^2 + 3*b^2*e^2*x^2
 - 10*a*c*d^2 + 6*b^2*d*e*x - 10*a*c*e^2*x^2 - 20*a*c*d*e*x))/(a^2*f) - (2*
b*c^3*e^18*x^2*(10*a*c - b^2))/(a*f*(4*a*c - b^2)) - (4*b*c^3*d*e^17*x*(10*
a*c - b^2))/(a*f*(4*a*c - b^2))))/(4*a^2*e*f) - (b*c^3*e^15*(4*b^3 - 20*a*c
^2*d^2 + 6*b^2*c*d^2 - 17*a*b*c))/(a^2*f^2*(4*a*c - b^2)^2) + (2*b*c^4*e^17
*x^2*(10*a*c - 3*b^2))/(a^2*f^2*(4*a*c - b^2)^2) + (4*b*c^4*d*e^16*x*(10*a*
c - 3*b^2))/(a^2*f^2*(4*a*c - b^2)^2)))/(4*a^2*e*f) + (b^3*c^5*e^16*x^2)/(a
^3*f^3*(4*a*c - b^2)^3) + (b^2*c^4*e^14*(b^2 - 4*a*c + b*c*d^2))/(a^3*f^3*(
4*a*c - b^2)^3) + (2*b^3*c^5*d*e^15*x)/(a^3*f^3*(4*a*c - b^2)^3))*((b^3*c^5
*e^16*x^2)/(a^3*f^3*(4*a*c - b^2)^3) - ((a^2*e*f*(-(b^2*(6*a*c - b^2)^2)/(a
^4*e^2*f^2*(4*a*c - b^2)^3))^(1/2) + 1)*(((a^2*e*f*(-(b^2*(6*a*c - b^2)^2)/
(a^4*e^2*f^2*(4*a*c - b^2)^3))^(1/2) + 1)*((b*c^2*e^16*(a^2*e*f*(-(b^2*(6*a
*c - b^2)^2)/(a^4*e^2*f^2*(4*a*c - b^2)^3))^(1/2) + 1)*(a*b + 3*b^2*d^2 + 3
*b^2*e^2*x^2 - 10*a*c*d^2 + 6*b^2*d*e*x - 10*a*c*e^2*x^2 - 20*a*c*d*e*x))/(
a^2*f) - (2*b*c^2*e^16*(2*b^3 - 10*a*c^2*d^2 + b^2*c*d^2 - 10*a*b*c))/(a*f*
(4*a*c - b^2)) + (2*b*c^3*e^18*x^2*(10*a*c - b^2))/(a*f*(4*a*c - b^2)) + (4
*b*c^3*d*e^17*x*(10*a*c - b^2))/(a*f*(4*a*c - b^2))))/(4*a^2*e*f) - (b*c^3*
e^15*(4*b^3 - 20*a*c^2*d^2 + 6*b^2*c*d^2 - 17*a*b*c))/(a^2*f^2*(4*a*c - b^2
)^2) + (2*b*c^4*e^17*x^2*(10*a*c - 3*b^2))/(a^2*f^2*(4*a*c - b^2)^2) + (4*b
*c^4*d*e^16*x*(10*a*c - 3*b^2))/(a^2*f^2*(4*a*c - b^2)^2)))/(4*a^2*e*f) + (
b^2*c^4*e^14*(b^2 - 4*a*c + b*c*d^2))/(a^3*f^3*(4*a*c - b^2)^3) + (2*b^3*c^
5*d*e^15*x)/(a^3*f^3*(4*a*c - b^2)^3)))*(2*b^6*e*f - 128*a^3*c^3*e*f + 96*a
^2*b^2*c^2*e*f - 24*a*b^4*c*e*f))/(2*(4*a^2*b^6*e^2*f^2 - 256*a^5*c^3*e^2*f
^2 + 192*a^4*b^2*c^2*e^2*f^2 - 48*a^3*b^4*c*e^2*f^2)) + log(d + e*x)/(a^2*e
*f) + (b*atan((x^2*((((b*(6*a*c - b^2)*((6*a*b^5*c^4*e^17*f + 80*a^3*b*c^6*
e^17*f - 44*a^2*b^3*c^5*e^17*f)/(a^3*b^6*f^3 - 64*a^6*c^3*f^3 - 12*a^4*b^4*
c*f^3 + 48*a^5*b^2*c^2*f^3) - (((2*a^2*b^7*c^3*e^18*f^2 - 36*a^3*b^5*c^4*e^
18*f^2 + 192*a^4*b^3*c^5*e^18*f^2 - 320*a^5*b*c^6*e^18*f^2)/(a^3*b^6*f^3 -
```

```
64*a^6*c^3*f^3 - 12*a^4*b^4*c*f^3 + 48*a^5*b^2*c^2*f^3) + ((2*b^6*e*f - 128
*a^3*c^3*e*f + 96*a^2*b^2*c^2*e*f - 24*a*b^4*c*e*f)*(12*a^3*b^9*c^2*e^19*f^
3 - 184*a^4*b^7*c^3*e^19*f^3 + 1056*a^5*b^5*c^4*e^19*f^3 - 2688*a^6*b^3*c^5
*e^19*f^3 + 2560*a^7*b*c^6*e^19*f^3))/(2*(a^3*b^6*f^3 - 64*a^6*c^3*f^3 - 12
*a^4*b^4*c*f^3 + 48*a^5*b^2*c^2*f^3)*(4*a^2*b^6*e^2*f^2 - 256*a^5*c^3*e^2*f
^2 + 192*a^4*b^2*c^2*e^2*f^2 - 48*a^3*b^4*c*e^2*f^2)))*(2*b^6*e*f - 128*a^3
*c^3*e*f + 96*a^2*b^2*c^2*e*f - 24*a*b^4*c*e*f))/(2*(4*a^2*b^6*e^2*f^2 - 25
6*a^5*c^3*e^2*f^2 + 192*a^4*b^2*c^2*e^2*f^2 - 48*a^3*b^4*c*e^2*f^2))))/(4*a
^2*e*f*(4*a*c - b^2)^(3/2)) - (((b*((2*a^2*b^7*c^3*e^18*f^2 - 36*a^3*b^5*c^
4*e^18*f^2 + 192*a^4*b^3*c^5*e^18*f^2 - 320*a^5*b*c^6*e^18*f^2)/(a^3*b^6*f^
3 - 64*a^6*c^3*f^3 - 12*a^4*b^4*c*f^3 + 48*a^5*b^2*c^2*f^3) + ((2*b^6*e*f -
 128*a^3*c^3*e*f + 96*a^2*b^2*c^2*e*f - 24*a*b^4*c*e*f)*(12*a^3*b^9*c^2*e^1
9*f^3 - 184*a^4*b^7*c^3*e^19*f^3 + 1056*a^5*b^5*c^4*e^19*f^3 - 2688*a^6*b^3
*c^5*e^19*f^3 + 2560*a^7*b*c^6*e^19*f^3))/(2*(a^3*b^6*f^3 - 64*a^6*c^3*f^3
 - 12*a^4*b^4*c*f^3 + 48*a^5*b^2*c^2*f^3)*(4*a^2*b^6*e^2*f^2 - 256*a^5*c^3*e
^2*f^2 + 192*a^4*b^2*c^2*e^2*f^2 - 48*a^3*b^4*c*e^2*f^2)))*(6*a*c - b^2))/(
4*a^2*e*f*(4*a*c - b^2)^(3/2)) + (b*(6*a*c - b^2)*(2*b^6*e*f - 128*a^3*c^3*
e*f + 96*a^2*b^2*c^2*e*f - 24*a*b^4*c*e*f)*(12*a^3*b^9*c^2*e^19*f^3 - 184*a
^4*b^7*c^3*e^19*f^3 + 1056*a^5*b^5*c^4*e^19*f^3 - 2688*a^6*b^3*c^5*e^19*f^3
 + 2560*a^7*b*c^6*e^19*f^3))/(8*a^2*e*f*(4*a*c - b^2)^(3/2)*(a^3*b^6*f^3 -
64*a^6*c^3*f^3 - 12*a^4*b^4*c*f^3 + 48*a^5*b^2*c^2*f^3)*(4*a^2*b^6*e^2*f^2
 - 256*a^5*c^3*e^2*f^2 + 192*a^4*b^2*c^2*e^2*f^2 - 48*a^3*b^4*c*e^2*f^2)))*(
2*b^6*e*f - 128*a^3*c^3*e*f + 96*a^2*b^2*c^2*e*f - 24*a*b^4*c*e*f))/(2*(4*a
^2*b^6*e^2*f^2 - 256*a^5*c^3*e^2*f^2 + 192*a^4*b^2*c^2*e^2*f^2 - 48*a^3*b^4
*c*e^2*f^2)) + (b^3*(6*a*c - b^2)^3*(12*a^3*b^9*c^2*e^19*f^3 - 184*a^4*b^7*
c^3*e^19*f^3 + 1056*a^5*b^5*c^4*e^19*f^3 - 2688*a^6*b^3*c^5*e^19*f^3 + 2560
*a^7*b*c^6*e^19*f^3))/(64*a^6*e^3*f^3*(4*a*c - b^2)^(9/2)*(a^3*b^6*f^3 - 64
*a^6*c^3*f^3 - 12*a^4*b^4*c*f^3 + 48*a^5*b^2*c^2*f^3)))*(3*b^6 - 40*a^3*c^3
 + 69*a^2*b^2*c^2 - 27*a*b^4*c))/(8*a^3*c^2*(4*a*c - b^2)^(7/2)*(6*b^6 - 40
0*a^3*c^3 + 291*a^2*b^2*c^2 - 72*a*b^4*c)) + (3*b*(b^4 + 11*a^2*c^2 - 7*a*b
^2*c)*((((6*a*b^5*c^4*e^17*f + 80*a^3*b*c^6*e^17*f - 44*a^2*b^3*c^5*e^17*f)
/(a^3*b^6*f^3 - 64*a^6*c^3*f^3 - 12*a^4*b^4*c*f^3 + 48*a^5*b^2*c^2*f^3) - (
((2*a^2*b^7*c^3*e^18*f^2 - 36*a^3*b^5*c^4*e^18*f^2 + 192*a^4*b^3*c^5*e^18*f
^2 - 320*a^5*b*c^6*e^18*f^2)/(a^3*b^6*f^3 - 64*a^6*c^3*f^3 - 12*a^4*b^4*c*f
^3 + 48*a^5*b^2*c^2*f^3) + ((2*b^6*e*f - 128*a^3*c^3*e*f + 96*a^2*b^2*c^2*e
*f - 24*a*b^4*c*e*f)*(12*a^3*b^9*c^2*e^19*f^3 - 184*a^4*b^7*c^3*e^19*f^3 +
1056*a^5*b^5*c^4*e^19*f^3 - 2688*a^6*b^3*c^5*e^19*f^3 + 2560*a^7*b*c^6*e^19
*f^3))/(2*(a^3*b^6*f^3 - 64*a^6*c^3*f^3 - 12*a^4*b^4*c*f^3 + 48*a^5*b^2*c^2
*f^3)*(4*a^2*b^6*e^2*f^2 - 256*a^5*c^3*e^2*f^2 + 192*a^4*b^2*c^2*e^2*f^2 -
48*a^3*b^4*c*e^2*f^2)))*(2*b^6*e*f - 128*a^3*c^3*e*f + 96*a^2*b^2*c^2*e*f -
 24*a*b^4*c*e*f))/(2*(4*a^2*b^6*e^2*f^2 - 256*a^5*c^3*e^2*f^2 + 192*a^4*b^2
*c^2*e^2*f^2 - 48*a^3*b^4*c*e^2*f^2)))*(2*b^6*e*f - 128*a^3*c^3*e*f + 96*a^
2*b^2*c^2*e*f - 24*a*b^4*c*e*f))/(2*(4*a^2*b^6*e^2*f^2 - 256*a^5*c^3*e^2*f^
2 + 192*a^4*b^2*c^2*e^2*f^2 - 48*a^3*b^4*c*e^2*f^2)) - (b^3*c^5*e^16)/(a^3*
b^6*f^3 - 64*a^6*c^3*f^3 - 12*a^4*b^4*c*f^3 + 48*a^5*b^2*c^2*f^3) + (b*((b*
((2*a^2*b^7*c^3*e^18*f^2 - 36*a^3*b^5*c^4*e^18*f^2 + 192*a^4*b^3*c^5*e^18*f
^2 - 320*a^5*b*c^6*e^18*f^2)/(a^3*b^6*f^3 - 64*a^6*c^3*f^3 - 12*a^4*b^4*c*f
^3 + 48*a^5*b^2*c^2*f^3) + ((2*b^6*e*f - 128*a^3*c^3*e*f + 96*a^2*b^2*c^2*e
*f - 24*a*b^4*c*e*f)*(12*a^3*b^9*c^2*e^19*f^3 - 184*a^4*b^7*c^3*e^19*f^3 +
1056*a^5*b^5*c^4*e^19*f^3 - 2688*a^6*b^3*c^5*e^19*f^3 + 2560*a^7*b*c^6*e^19
*f^3))/(2*(a^3*b^6*f^3 - 64*a^6*c^3*f^3 - 12*a^4*b^4*c*f^3 + 48*a^5*b^2*c^2
*f^3)*(4*a^2*b^6*e^2*f^2 - 256*a^5*c^3*e^2*f^2 + 192*a^4*b^2*c^2*e^2*f^2 -
48*a^3*b^4*c*e^2*f^2)))*(6*a*c - b^2))/(4*a^2*e*f*(4*a*c - b^2)^(3/2)) + (b
*(6*a*c - b^2)*(2*b^6*e*f - 128*a^3*c^3*e*f + 96*a^2*b^2*c^2*e*f - 24*a*b^4
*c*e*f)*(12*a^3*b^9*c^2*e^19*f^3 - 184*a^4*b^7*c^3*e^19*f^3 + 1056*a^5*b^5*
c^4*e^19*f^3 - 2688*a^6*b^3*c^5*e^19*f^3 + 2560*a^7*b*c^6*e^19*f^3))/(8*a^2
*e*f*(4*a*c - b^2)^(3/2)*(a^3*b^6*f^3 - 64*a^6*c^3*f^3 - 12*a^4*b^4*c*f^3 +
 48*a^5*b^2*c^2*f^3)*(4*a^2*b^6*e^2*f^2 - 256*a^5*c^3*e^2*f^2 + 192*a^4*b^2
*c^2*e^2*f^2 - 48*a^3*b^4*c*e^2*f^2)))*(6*a*c - b^2))/(4*a^2*e*f*(4*a*c - b
```

^2)^(3/2)) + (b^2*(6*a*c - b^2)^2*(2*b^6*e*f - 128*a^3*c^3*e*f + 96*a^2*b^2*c^2*e*f - 24*a*b^4*c*e*f)*(12*a^3*b^9*c^2*e^19*f^3 - 184*a^4*b^7*c^3*e^19*f^3 + 1056*a^5*b^5*c^4*e^19*f^3 - 2688*a^6*b^3*c^5*e^19*f^3 + 2560*a^7*b*c^6*e^19*f^3))/(32*a^4*e^2*f^2*(4*a*c - b^2)^3*(a^3*b^6*f^3 - 64*a^6*c^3*f^3 - 12*a^4*b^4*c*f^3 + 48*a^5*b^2*c^2*f^3)*(4*a^2*b^6*e^2*f^2 - 256*a^5*c^3*e^2*f^2 + 192*a^4*b^2*c^2*e^2*f^2 - 48*a^3*b^4*c*e^2*f^2))))/(8*a^3*c^2*(4*a*c - b^2)^3*(6*b^6 - 400*a^3*c^3 + 291*a^2*b^2*c^2 - 72*a*b^4*c)))*(16*a^6*b^6*f^3*(4*a*c - b^2)^(9/2) - 1024*a^9*c^3*f^3*(4*a*c - b^2)^(9/2) - 192*a^7*b^4*c*f^3*(4*a*c - b^2)^(9/2) + 768*a^8*b^2*c^2*f^3*(4*a*c - b^2)^(9/2)))/(b^6*c^2*e^14 - 12*a*b^4*c^3*e^14 + 36*a^2*b^2*c^4*e^14) + (x*((((((b*(6*a*c - b^2)*((2*(320*a^5*b*c^6*d*e^17*f^2 - 2*a^2*b^7*c^3*d*e^17*f^2 + 36*a^3*b^5*c^4*d*e^17*f^2 - 192*a^4*b^3*c^5*d*e^17*f^2))/(a^3*b^6*f^3 - 64*a^6*c^3*f^3 - 12*a^4*b^4*c*f^3 + 48*a^5*b^2*c^2*f^3) - ((2*b^6*e*f - 128*a^3*c^3*e*f + 96*a^2*b^2*c^2*e*f - 24*a*b^4*c*e*f)*(2560*a^7*b*c^6*d*e^18*f^3 + 12*a^3*b^9*c^2*d*e^18*f^3 - 184*a^4*b^7*c^3*d*e^18*f^3 + 1056*a^5*b^5*c^4*d*e^18*f^3 - 2688*a^6*b^3*c^5*d*e^18*f^3))/((a^3*b^6*f^3 - 64*a^6*c^3*f^3 - 12*a^4*b^4*c*f^3 + 48*a^5*b^2*c^2*f^3)*(4*a^2*b^6*e^2*f^2 - 256*a^5*c^3*e^2*f^2 + 192*a^4*b^2*c^2*e^2*f^2 - 48*a^3*b^4*c*e^2*f^2))))/(4*a^2*e*f*(4*a*c - b^2)^(3/2)) - (b*(6*a*c - b^2)*(2*b^6*e*f - 128*a^3*c^3*e*f + 96*a^2*b^2*c^2*e*f - 24*a*b^4*c*e*f)*(2560*a^7*b*c^6*d*e^18*f^3 + 12*a^3*b^9*c^2*d*e^18*f^3 - 184*a^4*b^7*c^3*d*e^18*f^3 + 1056*a^5*b^5*c^4*d*e^18*f^3 - 2688*a^6*b^3*c^5*d*e^18*f^3))/(4*a^2*e*f*(4*a*c - b^2)^(3/2)*(a^3*b^6*f^3 - 64*a^6*c^3*f^3 - 12*a^4*b^4*c*f^3 + 48*a^5*b^2*c^2*f^3)*(4*a^2*b^6*e^2*f^2 - 256*a^5*c^3*e^2*f^2 + 192*a^4*b^2*c^2*e^2*f^2 - 48*a^3*b^4*c*e^2*f^2)))*(2*b^6*e*f - 128*a^3*c^3*e*f + 96*a^2*b^2*c^2*e*f - 24*a*b^4*c*e*f))/(2*(4*a^2*b^6*e^2*f^2 - 256*a^5*c^3*e^2*f^2 + 192*a^4*b^2*c^2*e^2*f^2 - 48*a^3*b^4*c*e^2*f^2)) + (b*((2*(6*a*b^5*c^4*d*e^16*f - 44*a^2*b^3*c^5*d*e^16*f + 80*a^3*b*c^6*d*e^16*f))/(a^3*b^6*f^3 - 64*a^6*c^3*f^3 - 12*a^4*b^4*c*f^3 + 48*a^5*b^2*c^2*f^3) + (((2*(320*a^5*b*c^6*d*e^17*f^2 - 2*a^2*b^7*c^3*d*e^17*f^2 + 36*a^3*b^5*c^4*d*e^17*f^2 - 192*a^4*b^3*c^5*d*e^17*f^2))/(a^3*b^6*f^3 - 64*a^6*c^3*f^3 - 12*a^4*b^4*c*f^3 + 48*a^5*b^2*c^2*f^3) - ((2*b^6*e*f - 128*a^3*c^3*e*f + 96*a^2*b^2*c^2*e*f - 24*a*b^4*c*e*f)*(2560*a^7*b*c^6*d*e^18*f^3 + 12*a^3*b^9*c^2*d*e^18*f^3 - 184*a^4*b^7*c^3*d*e^18*f^3 + 1056*a^5*b^5*c^4*d*e^18*f^3 - 2688*a^6*b^3*c^5*d*e^18*f^3))/((a^3*b^6*f^3 - 64*a^6*c^3*f^3 - 12*a^4*b^4*c*f^3 + 48*a^5*b^2*c^2*f^3)*(4*a^2*b^6*e^2*f^2 - 256*a^5*c^3*e^2*f^2 + 192*a^4*b^2*c^2*e^2*f^2 - 48*a^3*b^4*c*e^2*f^2)))*(2*b^6*e*f - 128*a^3*c^3*e*f + 96*a^2*b^2*c^2*e*f - 24*a*b^4*c*e*f))/(2*(4*a^2*b^6*e^2*f^2 - 256*a^5*c^3*e^2*f^2 + 192*a^4*b^2*c^2*e^2*f^2 - 48*a^3*b^4*c*e^2*f^2)))*(6*a*c - b^2))/(4*a^2*e*f*(4*a*c - b^2)^(3/2)) + (b^3*(6*a*c - b^2)^3*(2560*a^7*b*c^6*d*e^18*f^3 + 12*a^3*b^9*c^2*d*e^18*f^3 - 184*a^4*b^7*c^3*d*e^18*f^3 + 1056*a^5*b^5*c^4*d*e^18*f^3 - 2688*a^6*b^3*c^5*d*e^18*f^3))/(32*a^6*e^3*f^3*(4*a*c - b^2)^(9/2)*(a^3*b^6*f^3 - 64*a^6*c^3*f^3 - 12*a^4*b^4*c*f^3 + 48*a^5*b^2*c^2*f^3)))*(3*b^6 - 40*a^3*c^3 + 69*a^2*b^2*c^2 - 27*a*b^4*c))/(8*a^3*c^2*(4*a*c - b^2)^(7/2)*(6*b^6 - 400*a^3*c^3 + 291*a^2*b^2*c^2 - 72*a*b^4*c)) + (3*b*(b^4 + 11*a^2*c^2 - 7*a*b^2*c)*((((2*(6*a*b^5*c^4*d*e^16*f - 44*a^2*b^3*c^5*d*e^16*f + 80*a^3*b*c^6*d*e^16*f))/(a^3*b^6*f^3 - 64*a^6*c^3*f^3 - 12*a^4*b^4*c*f^3 + 48*a^5*b^2*c^2*f^3) + (((2*(320*a^5*b*c^6*d*e^17*f^2 - 2*a^2*b^7*c^3*d*e^17*f^2 + 36*a^3*b^5*c^4*d*e^17*f^2 - 192*a^4*b^3*c^5*d*e^17*f^2))/(a^3*b^6*f^3 - 64*a^6*c^3*f^3 - 12*a^4*b^4*c*f^3 + 48*a^5*b^2*c^2*f^3) - ((2*b^6*e*f - 128*a^3*c^3*e*f + 96*a^2*b^2*c^2*e*f - 24*a*b^4*c*e*f)*(2560*a^7*b*c^6*d*e^18*f^3 + 12*a^3*b^9*c^2*d*e^18*f^3 - 184*a^4*b^7*c^3*d*e^18*f^3 + 1056*a^5*b^5*c^4*d*e^18*f^3 - 2688*a^6*b^3*c^5*d*e^18*f^3))/((a^3*b^6*f^3 - 64*a^6*c^3*f^3 - 12*a^4*b^4*c*f^3 + 48*a^5*b^2*c^2*f^3)*(4*a^2*b^6*e^2*f^2 - 256*a^5*c^3*e^2*f^2 + 192*a^4*b^2*c^2*e^2*f^2 - 48*a^3*b^4*c*e^2*f^2)))*(2*b^6*e*f - 128*a^3*c^3*e*f + 96*a^2*b^2*c^2*e*f - 24*a*b^4*c*e*f))/(2*(4*a^2*b^6*e^2*f^2 - 256*a^5*c^3*e^2*f^2 + 192*a^4*b^2*c^2*e^2*f^2 - 48*a^3*b^4*c*e^2*f^2)))*(2*b^6*e*f - 128*a^3*c^3*e*f + 96*a^2*b^2*c^2*e*f - 24*a*b^4*c*e*f))/(2*(4*a^2*b^6*e^2*f^2 - 256*a^5*c^3*e^2*f^2 + 192*a^4*b^2*c^2*e^2*f^2 - 48*a^3*b^4*c*e^2*f^2)) - (2*b^3*c^5*d*e^15)/(a^3*b^

6*f^3 - 64*a^6*c^3*f^3 - 12*a^4*b^4*c*f^3 + 48*a^5*b^2*c^2*f^3) - (b*(6*a*c - b^2)*((b*(6*a*c - b^2)*((2*(320*a^5*b*c^6*d*e^17*f^2 - 2*a^2*b^7*c^3*d*e^17*f^2 + 36*a^3*b^5*c^4*d*e^17*f^2 - 192*a^4*b^3*c^5*d*e^17*f^2))/(a^3*b^6*f^3 - 64*a^6*c^3*f^3 - 12*a^4*b^4*c*f^3 + 48*a^5*b^2*c^2*f^3) - ((2*b^6*e*f - 128*a^3*c^3*e*f + 96*a^2*b^2*c^2*e*f - 24*a*b^4*c*e*f)*(2560*a^7*b*c^6*d*e^18*f^3 + 12*a^3*b^9*c^2*d*e^18*f^3 - 184*a^4*b^7*c^3*d*e^18*f^3 + 1056*a^5*b^5*c^4*d*e^18*f^3 - 2688*a^6*b^3*c^5*d*e^18*f^3))/((a^3*b^6*f^3 - 64*a^6*c^3*f^3 - 12*a^4*b^4*c*f^3 + 48*a^5*b^2*c^2*f^3)*(4*a^2*b^6*e^2*f^2 - 256*a^5*c^3*e^2*f^2 + 192*a^4*b^2*c^2*e^2*f^2 - 48*a^3*b^4*c*e^2*f^2))))/(4*a^2*e*f*(4*a*c - b^2)^(3/2)) - (b*(6*a*c - b^2)*(2*b^6*e*f - 128*a^3*c^3*e*f + 96*a^2*b^2*c^2*e*f - 24*a*b^4*c*e*f)*(2560*a^7*b*c^6*d*e^18*f^3 + 12*a^3*b^9*c^2*d*e^18*f^3 - 184*a^4*b^7*c^3*d*e^18*f^3 + 1056*a^5*b^5*c^4*d*e^18*f^3 - 2688*a^6*b^3*c^5*d*e^18*f^3))/(4*a^2*e*f*(4*a*c - b^2)^(3/2)*(a^3*b^6*f^3 - 64*a^6*c^3*f^3 - 12*a^4*b^4*c*f^3 + 48*a^5*b^2*c^2*f^3)*(4*a^2*b^6*e^2*f^2 - 256*a^5*c^3*e^2*f^2 + 192*a^4*b^2*c^2*e^2*f^2 - 48*a^3*b^4*c*e^2*f^2))))/(4*a^2*e*f*(4*a*c - b^2)^(3/2)) + (b^2*(6*a*c - b^2)^2*(2*b^6*e*f - 128*a^3*c^3*e*f + 96*a^2*b^2*c^2*e*f - 24*a*b^4*c*e*f)*(2560*a^7*b*c^6*d*e^18*f^3 + 12*a^3*b^9*c^2*d*e^18*f^3 - 184*a^4*b^7*c^3*d*e^18*f^3 + 1056*a^5*b^5*c^4*d*e^18*f^3 - 2688*a^6*b^3*c^5*d*e^18*f^3))/(16*a^4*e^2*f^2*(4*a*c - b^2)^3*(a^3*b^6*f^3 - 64*a^6*c^3*f^3 - 12*a^4*b^4*c*f^3 + 48*a^5*b^2*c^2*f^3)*(4*a^2*b^6*e^2*f^2 - 256*a^5*c^3*e^2*f^2 + 192*a^4*b^2*c^2*e^2*f^2 - 48*a^3*b^4*c*e^2*f^2))))/(8*a^3*c^2*(4*a*c - b^2)^3*(6*b^6 - 400*a^3*c^3 + 291*a^2*b^2*c^2 - 72*a*b^4*c)))*(16*a^6*b^6*f^3*(4*a*c - b^2)^(9/2) - 1024*a^9*c^3*f^3*(4*a*c - b^2)^(9/2) - 192*a^7*b^4*c*f^3*(4*a*c - b^2)^(9/2) + 768*a^8*b^2*c^2*f^3*(4*a*c - b^2)^(9/2)))/(b^6*c^2*e^14 - 12*a*b^4*c^3*e^14 + 36*a^2*b^2*c^4*e^14) + (((b*((4*a*b^6*c^3*e^15*f - 33*a^2*b^4*c^4*e^15*f + 68*a^3*b^2*c^5*e^15*f + 6*a*b^5*c^4*d^2*e^15*f + 80*a^3*b*c^6*d^2*e^15*f - 44*a^2*b^3*c^5*d^2*e^15*f)/(a^3*b^6*f^3 - 64*a^6*c^3*f^3 - 12*a^4*b^4*c*f^3 + 48*a^5*b^2*c^2*f^3) - (((4*a^2*b^8*c^2*e^16*f^2 - 52*a^3*b^6*c^3*e^16*f^2 + 224*a^4*b^4*c^4*e^16*f^2 - 320*a^5*b^2*c^5*e^16*f^2 - 320*a^5*b*c^6*d^2*e^16*f^2 + 2*a^2*b^7*c^3*d^2*e^16*f^2 - 36*a^3*b^5*c^4*d^2*e^16*f^2 + 192*a^4*b^3*c^5*d^2*e^16*f^2)/(a^3*b^6*f^3 - 64*a^6*c^3*f^3 - 12*a^4*b^4*c*f^3 + 48*a^5*b^2*c^2*f^3) + ((2*b^6*e*f - 128*a^3*c^3*e*f + 96*a^2*b^2*c^2*e*f - 24*a*b^4*c*e*f)*(4*a^4*b^8*c^2*e^17*f^3 - 48*a^5*b^6*c^3*e^17*f^3 + 192*a^6*b^4*c^4*e^17*f^3 - 256*a^7*b^2*c^5*e^17*f^3 + 2560*a^7*b*c^6*d^2*e^17*f^3 + 12*a^3*b^9*c^2*d^2*e^17*f^3 - 184*a^4*b^7*c^3*d^2*e^17*f^3 + 1056*a^5*b^5*c^4*d^2*e^17*f^3 - 2688*a^6*b^3*c^5*d^2*e^17*f^3))/(2*(a^3*b^6*f^3 - 64*a^6*c^3*f^3 - 12*a^4*b^4*c*f^3 + 48*a^5*b^2*c^2*f^3)*(4*a^2*b^6*e^2*f^2 - 256*a^5*c^3*e^2*f^2 + 192*a^4*b^2*c^2*e^2*f^2 - 48*a^3*b^4*c*e^2*f^2)))*(2*b^6*e*f - 128*a^3*c^3*e*f + 96*a^2*b^2*c^2*e*f - 24*a*b^4*c*e*f))/(2*(4*a^2*b^6*e^2*f^2 - 256*a^5*c^3*e^2*f^2 + 192*a^4*b^2*c^2*e^2*f^2 - 48*a^3*b^4*c*e^2*f^2)))*(6*a*c - b^2))/(4*a^2*e*f*(4*a*c - b^2)^(3/2)) - (((b*(6*a*c - b^2)*((4*a^2*b^8*c^2*e^16*f^2 - 52*a^3*b^6*c^3*e^16*f^2 + 224*a^4*b^4*c^4*e^16*f^2 - 320*a^5*b^2*c^5*e^16*f^2 - 320*a^5*b*c^6*d^2*e^16*f^2 + 2*a^2*b^7*c^3*d^2*e^16*f^2 - 36*a^3*b^5*c^4*d^2*e^16*f^2 + 192*a^4*b^3*c^5*d^2*e^16*f^2)/(a^3*b^6*f^3 - 64*a^6*c^3*f^3 - 12*a^4*b^4*c*f^3 + 48*a^5*b^2*c^2*f^3) + ((2*b^6*e*f - 128*a^3*c^3*e*f + 96*a^2*b^2*c^2*e*f - 24*a*b^4*c*e*f)*(4*a^4*b^8*c^2*e^17*f^3 - 48*a^5*b^6*c^3*e^17*f^3 + 192*a^6*b^4*c^4*e^17*f^3 - 256*a^7*b^2*c^5*e^17*f^3 + 2560*a^7*b*c^6*d^2*e^17*f^3 + 12*a^3*b^9*c^2*d^2*e^17*f^3 - 184*a^4*b^7*c^3*d^2*e^17*f^3 + 1056*a^5*b^5*c^4*d^2*e^17*f^3 - 2688*a^6*b^3*c^5*d^2*e^17*f^3))/(2*(a^3*b^6*f^3 - 64*a^6*c^3*f^3 - 12*a^4*b^4*c*f^3 + 48*a^5*b^2*c^2*f^3)*(4*a^2*b^6*e^2*f^2 - 256*a^5*c^3*e^2*f^2 + 192*a^4*b^2*c^2*e^2*f^2 - 48*a^3*b^4*c*e^2*f^2))))/(4*a^2*e*f*(4*a*c - b^2)^(3/2)) + (b*(6*a*c - b^2)*(2*b^6*e*f - 128*a^3*c^3*e*f + 96*a^2*b^2*c^2*e*f - 24*a*b^4*c*e*f)*(4*a^4*b^8*c^2*e^17*f^3 - 48*a^5*b^6*c^3*e^17*f^3 + 192*a^6*b^4*c^4*e^17*f^3 - 256*a^7*b^2*c^5*e^17*f^3 + 2560*a^7*b*c^6*d^2*e^17*f^3 + 12*a^3*b^9*c^2*d^2*e^17*f^3 - 184*a^4*b^7*c^3*d^2*e^17*f^3 + 1056*a^5*b^5*c^4*d^2*e^17*f^3 - 2688*a^6*b^3*c^5*d^2*e^17*f^3))/(8*a^2*e*f*(4*a*c - b^2)^(3/2)*(a^3*b^6*f^3 - 64*a^6*c^3*f^3 - 12*a^4*b^4*c*f^3 + 48*a^5*b^2*c^

2*f^3)*(4*a^2*b^6*e^2*f^2 - 256*a^5*c^3*e^2*f^2 + 192*a^4*b^2*c^2*e^2*f^2 - 48*a^3*b^4*c*e^2*f^2)))*(2*b^6*e*f - 128*a^3*c^3*e*f + 96*a^2*b^2*c^2*e*f - 24*a*b^4*c*e*f))/(2*(4*a^2*b^6*e^2*f^2 - 256*a^5*c^3*e^2*f^2 + 192*a^4*b^2*c^2*e^2*f^2 - 48*a^3*b^4*c*e^2*f^2)) + (b^3*(6*a*c - b^2)^3*(4*a^4*b^8*c^2*e^17*f^3 - 48*a^5*b^6*c^3*e^17*f^3 + 192*a^6*b^4*c^4*e^17*f^3 - 256*a^7*b^2*c^5*e^17*f^3 + 2560*a^7*b*c^6*d^2*e^17*f^3 + 12*a^3*b^9*c^2*d^2*e^17*f^3 - 184*a^4*b^7*c^3*d^2*e^17*f^3 + 1056*a^5*b^5*c^4*d^2*e^17*f^3 - 2688*a^6*b^3*c^5*d^2*e^17*f^3))/(64*a^6*e^3*f^3*(4*a*c - b^2)^(9/2)*(a^3*b^6*f^3 - 64*a^6*c^3*f^3 - 12*a^4*b^4*c*f^3 + 48*a^5*b^2*c^2*f^3)))*(16*a^6*b^6*f^3*(4*a*c - b^2)^(9/2) - 1024*a^9*c^3*f^3*(4*a*c - b^2)^(9/2) - 192*a^7*b^4*c*f^3*(4*a*c - b^2)^(9/2) + 768*a^8*b^2*c^2*f^3*(4*a*c - b^2)^(9/2))*(3*b^6 - 40*a^3*c^3 + 69*a^2*b^2*c^2 - 27*a*b^4*c))/(8*a^3*c^2*(4*a*c - b^2)^(7/2)*(b^6*c^2*e^14 - 12*a*b^4*c^3*e^14 + 36*a^2*b^2*c^4*e^14)*(6*b^6 - 400*a^3*c^3 + 291*a^2*b^2*c^2 - 72*a*b^4*c)) + (3*b*(b^4 + 11*a^2*c^2 - 7*a*b^2*c)*(16*a^6*b^6*f^3*(4*a*c - b^2)^(9/2) - 1024*a^9*c^3*f^3*(4*a*c - b^2)^(9/2) - 192*a^7*b^4*c*f^3*(4*a*c - b^2)^(9/2) + 768*a^8*b^2*c^2*f^3*(4*a*c - b^2)^(9/2))*((((4*a*b^6*c^3*e^15*f - 33*a^2*b^4*c^4*e^15*f + 68*a^3*b^2*c^5*e^15*f + 6*a*b^5*c^4*d^2*e^15*f + 80*a^3*b*c^6*d^2*e^15*f - 44*a^2*b^3*c^5*d^2*e^15*f)/(a^3*b^6*f^3 - 64*a^6*c^3*f^3 - 12*a^4*b^4*c*f^3 + 48*a^5*b^2*c^2*f^3) - (((4*a^2*b^8*c^2*e^16*f^2 - 52*a^3*b^6*c^3*e^16*f^2 + 224*a^4*b^4*c^4*e^16*f^2 - 320*a^5*b^2*c^5*e^16*f^2 - 320*a^5*b*c^6*d^2*e^16*f^2 + 2*a^2*b^7*c^3*d^2*e^16*f^2 - 36*a^3*b^5*c^4*d^2*e^16*f^2 + 192*a^4*b^3*c^5*d^2*e^16*f^2)/(a^3*b^6*f^3 - 64*a^6*c^3*f^3 - 12*a^4*b^4*c*f^3 + 48*a^5*b^2*c^2*f^3) + ((2*b^6*e*f - 128*a^3*c^3*e*f + 96*a^2*b^2*c^2*e*f - 24*a*b^4*c*e*f)*(4*a^4*b^8*c^2*e^17*f^3 - 48*a^5*b^6*c^3*e^17*f^3 + 192*a^6*b^4*c^4*e^17*f^3 - 256*a^7*b^2*c^5*e^17*f^3 + 2560*a^7*b*c^6*d^2*e^17*f^3 + 12*a^3*b^9*c^2*d^2*e^17*f^3 - 184*a^4*b^7*c^3*d^2*e^17*f^3 + 1056*a^5*b^5*c^4*d^2*e^17*f^3 - 2688*a^6*b^3*c^5*d^2*e^17*f^3))/(2*(a^3*b^6*f^3 - 64*a^6*c^3*f^3 - 12*a^4*b^4*c*f^3 + 48*a^5*b^2*c^2*f^3)*(4*a^2*b^6*e^2*f^2 - 256*a^5*c^3*e^2*f^2 + 192*a^4*b^2*c^2*e^2*f^2 - 48*a^3*b^4*c*e^2*f^2)))*(2*b^6*e*f - 128*a^3*c^3*e*f + 96*a^2*b^2*c^2*e*f - 24*a*b^4*c*e*f))/(2*(4*a^2*b^6*e^2*f^2 - 256*a^5*c^3*e^2*f^2 + 192*a^4*b^2*c^2*e^2*f^2 - 48*a^3*b^4*c*e^2*f^2)))*(2*b^6*e*f - 128*a^3*c^3*e*f + 96*a^2*b^2*c^2*e*f - 24*a*b^4*c*e*f))/(2*(4*a^2*b^6*e^2*f^2 - 256*a^5*c^3*e^2*f^2 + 192*a^4*b^2*c^2*e^2*f^2 - 48*a^3*b^4*c*e^2*f^2)) - (b^4*c^4*e^14 - 4*a*b^2*c^5*e^14 + b^3*c^5*d^2*e^14)/(a^3*b^6*f^3 - 64*a^6*c^3*f^3 - 12*a^4*b^4*c*f^3 + 48*a^5*b^2*c^2*f^3) + (b*((b*(6*a*c - b^2)*((4*a^2*b^8*c^2*e^16*f^2 - 52*a^3*b^6*c^3*e^16*f^2 + 224*a^4*b^4*c^4*e^16*f^2 - 320*a^5*b^2*c^5*e^16*f^2 - 320*a^5*b*c^6*d^2*e^16*f^2 + 2*a^2*b^7*c^3*d^2*e^16*f^2 - 36*a^3*b^5*c^4*d^2*e^16*f^2 + 192*a^4*b^3*c^5*d^2*e^16*f^2)/(a^3*b^6*f^3 - 64*a^6*c^3*f^3 - 12*a^4*b^4*c*f^3 + 48*a^5*b^2*c^2*f^3) + ((2*b^6*e*f - 128*a^3*c^3*e*f + 96*a^2*b^2*c^2*e*f - 24*a*b^4*c*e*f)*(4*a^4*b^8*c^2*e^17*f^3 - 48*a^5*b^6*c^3*e^17*f^3 + 192*a^6*b^4*c^4*e^17*f^3 - 256*a^7*b^2*c^5*e^17*f^3 + 2560*a^7*b*c^6*d^2*e^17*f^3 + 12*a^3*b^9*c^2*d^2*e^17*f^3 - 184*a^4*b^7*c^3*d^2*e^17*f^3 + 1056*a^5*b^5*c^4*d^2*e^17*f^3 - 2688*a^6*b^3*c^5*d^2*e^17*f^3))/(2*(a^3*b^6*f^3 - 64*a^6*c^3*f^3 - 12*a^4*b^4*c*f^3 + 48*a^5*b^2*c^2*f^3)*(4*a^2*b^6*e^2*f^2 - 256*a^5*c^3*e^2*f^2 + 192*a^4*b^2*c^2*e^2*f^2 - 48*a^3*b^4*c*e^2*f^2))))/(4*a^2*e*f*(4*a*c - b^2)^(3/2)) + (b*(6*a*c - b^2)*(2*b^6*e*f - 128*a^3*c^3*e*f + 96*a^2*b^2*c^2*e*f - 24*a*b^4*c*e*f)*(4*a^4*b^8*c^2*e^17*f^3 - 48*a^5*b^6*c^3*e^17*f^3 + 192*a^6*b^4*c^4*e^17*f^3 - 256*a^7*b^2*c^5*e^17*f^3 + 2560*a^7*b*c^6*d^2*e^17*f^3 + 12*a^3*b^9*c^2*d^2*e^17*f^3 - 184*a^4*b^7*c^3*d^2*e^17*f^3 + 1056*a^5*b^5*c^4*d^2*e^17*f^3 - 2688*a^6*b^3*c^5*d^2*e^17*f^3))/(8*a^2*e*f*(4*a*c - b^2)^(3/2)*(a^3*b^6*f^3 - 64*a^6*c^3*f^3 - 12*a^4*b^4*c*f^3 + 48*a^5*b^2*c^2*f^3)*(4*a^2*b^6*e^2*f^2 - 256*a^5*c^3*e^2*f^2 + 192*a^4*b^2*c^2*e^2*f^2 - 48*a^3*b^4*c*e^2*f^2)))*(6*a*c - b^2))/(4*a^2*e*f*(4*a*c - b^2)^(3/2)) + (b^2*(6*a*c - b^2)^2*(2*b^6*e*f - 128*a^3*c^3*e*f + 96*a^2*b^2*c^2*e*f - 24*a*b^4*c*e*f)*(4*a^4*b^8*c^2*e^17*f^3 - 48*a^5*b^6*c^3*e^17*f^3 + 192*a^6*b^4*c^4*e^17*f^3 - 256*a^7*b^2*c^5*e^17*f^3 + 2560*a^7*b*c^6*d^2*e^17*f^3 + 12*a^3*b^9*c^2*d^2*e^17*f^3 - 184*a^4*b^7*c^3*d^2*e^17*f^3 + 1056*a^5*b^5*c^4

```
*d^2*e^17*f^3 - 2688*a^6*b^3*c^5*d^2*e^17*f^3))/(32*a^4*e^2*f^2*(4*a*c - b^
2)^3*(a^3*b^6*f^3 - 64*a^6*c^3*f^3 - 12*a^4*b^4*c*f^3 + 48*a^5*b^2*c^2*f^3)
*(4*a^2*b^6*e^2*f^2 - 256*a^5*c^3*e^2*f^2 + 192*a^4*b^2*c^2*e^2*f^2 - 48*a^
3*b^4*c*e^2*f^2))))/(8*a^3*c^2*(4*a*c - b^2)^3*(b^6*c^2*e^14 - 12*a*b^4*c^3
*e^14 + 36*a^2*b^2*c^4*e^14)*(6*b^6 - 400*a^3*c^3 + 291*a^2*b^2*c^2 - 72*a*
b^4*c)))*(6*a*c - b^2))/(2*a^2*e*f*(4*a*c - b^2)^(3/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*f*x+d*f)/(a+b*(e*x+d)**2+c*(e*x+d)**4)**2,x)
```

```
[Out] Timed out
```

3.538 $\int \frac{1}{(df+efx)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx$

Optimal. Leaf size=360

$$-\frac{3b^2-10ac}{2a^2ef^2\left(b^2-4ac\right)(d+ex)}-\frac{\sqrt{c}\left(\left(3b^2-10ac\right)\sqrt{b^2-4ac}-16abc+3b^3\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\,a^2ef^2\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{c}\left(-\left(3b^2-\right.\right.}{2}$$

Rubi [A] time = 1.60, antiderivative size = 360, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {1142, 1121, 1281, 1166, 205}

$$-\frac{3b^2-10ac}{2a^2ef^2\left(b^2-4ac\right)(d+ex)}-\frac{\sqrt{c}\left(\left(3b^2-10ac\right)\sqrt{b^2-4ac}-16abc+3b^3\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\,a^2ef^2\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{c}\left(-\left(3b^2-10ac\right)\sqrt{b^2-4ac}-16abc+3b^3\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{2\sqrt{2}\,a^2ef^2\left(b^2-4ac\right)^{3/2}\sqrt{\sqrt{b^2-4ac}+b}}+\frac{-2ac+b^2+bc(d+ex)^2}{2aef^2\left(b^2-4ac\right)(d+ex)\left(a+b(d+ex)^2+c(d+ex)^4\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d*f + e*f*x)^2*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2),x]
```

```
[Out] -(3*b^2 - 10*a*c)/(2*a^2*(b^2 - 4*a*c)*e*f^2*(d + e*x)) + (b^2 - 2*a*c + b*
c*(d + e*x)^2)/(2*a*(b^2 - 4*a*c)*e*f^2*(d + e*x)*(a + b*(d + e*x)^2 + c*(d
 + e*x)^4)) - (Sqrt[c]*(3*b^3 - 16*a*b*c + (3*b^2 - 10*a*c)*Sqrt[b^2 - 4*a*
c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(2*Sqr
t[2]*a^2*(b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]*e*f^2) + (Sqrt[c]*
(3*b^3 - 16*a*b*c - (3*b^2 - 10*a*c)*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqr
t[c]*(d + e*x))/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a^2*(b^2 - 4*a*c)^
(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]*e*f^2)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1121

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> -Simp[((d*x)^(m + 1)*(b^2 - 2*a*c + b*c*x^2)*(a + b*x^2 + c*x^4)^(p + 1
))/(2*a*d*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)),
Int[(d*x)^m*(a + b*x^2 + c*x^4)^(p + 1)*Simp[b^2*(m + 2*p + 3) - 2*a*c*(m +
 4*p + 5) + b*c*(m + 4*p + 7)*x^2, x], x], x] /; FreeQ[{a, b, c, d, m}, x]
&& NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p] && (IntegerQ[p] || In
tegerQ[m])
```

Rule 1142

```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Di
st[u^m/(Coefficient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p,
 x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1281

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[(d*(f*x)^(m + 1)*(a + b*x^2 + c*x^4)^(p + 1)
)/(a*f*(m + 1)), x] + Dist[1/(a*f^2*(m + 1)), Int[(f*x)^(m + 2)*(a + b*x^2
+ c*x^4)^p*Simp[a*e*(m + 1) - b*d*(m + 2*p + 3) - c*d*(m + 4*p + 5)*x^2, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[m
, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rubi steps

$$\int \frac{1}{(df+efx)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x^2\left(a+bx^2+cx^4\right)^2}\,dx, x, d+ex\right)}{ef^2}$$

$$= \frac{b^2-2ac+bc(d+ex)^2}{2a\left(b^2-4ac\right)ef^2(d+ex)\left(a+b(d+ex)^2+c(d+ex)^4\right)} - \text{Sub}$$

$$= -\frac{3b^2-10ac}{2a^2\left(b^2-4ac\right)ef^2(d+ex)} + \frac{b^2-2ac+bc(d}{2a\left(b^2-4ac\right)ef^2(d+ex)\left(a+b\right.}$$

$$= -\frac{3b^2-10ac}{2a^2\left(b^2-4ac\right)ef^2(d+ex)} + \frac{b^2-2ac+bc(d}{2a\left(b^2-4ac\right)ef^2(d+ex)\left(a+b\right.}$$

$$= -\frac{3b^2-10ac}{2a^2\left(b^2-4ac\right)ef^2(d+ex)} + \frac{b^2-2ac+bc(d}{2a\left(b^2-4ac\right)ef^2(d+ex)\left(a+b\right.}$$

Mathematica [A] time = 1.47, size = 342, normalized size = 0.95

$$\frac{\frac{2(d+ex)\left(-3abc-2ac^2(d+ex)^2+b^3+b^2c(d+ex)^2\right)}{\left(4ac-b^2\right)\left(a+b(d+ex)^2+c(d+ex)^4\right)}+\frac{\sqrt{2}\sqrt{c}\left(-3b^2\sqrt{b^2-4ac}+10ac\sqrt{b^2-4ac}+16abc-3b^3\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{2}\sqrt{c}\left(-3b^2\sqrt{b^2-4ac}+10ac\sqrt{b^2-4ac}-16abc+3b^3\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\left(b^2-4ac\right)^{3/2}\sqrt{\sqrt{b^2-4ac}+b}}-\frac{4}{d+ex}}{4a^2ef^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d*f + e*f*x)^2*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2),x]
```

```
[Out] (-4/(d + e*x) + (2*(d + e*x)*(b^3 - 3*a*b*c + b^2*c*(d + e*x)^2 - 2*a*c^2*(
d + e*x)^2))/((-b^2 + 4*a*c)*(a + b*(d + e*x)^2 + c*(d + e*x)^4)) + (Sqrt[2
]*Sqrt[c]*(-3*b^3 + 16*a*b*c - 3*b^2*Sqrt[b^2 - 4*a*c] + 10*a*c*Sqrt[b^2 -
4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/((
b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[2]*Sqrt[c]*(3*b^3 -
 16*a*b*c - 3*b^2*Sqrt[b^2 - 4*a*c] + 10*a*c*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqr
t[2]*Sqrt[c]*(d + e*x))/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(3/2)*
Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(4*a^2*e*f^2)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(df+efx)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/((d*f + e*f*x)^2*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2
),x]
```

```
[Out] IntegrateAlgebraic[1/((d*f + e*f*x)^2*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2
), x]
```

fricas [B] time = 1.76, size = 4520, normalized size = 12.56

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*f*x+d*f)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="frica
s")
```

```
[Out] -1/4*(2*(3*b^2*c - 10*a*c^2)*e^4*x^4 + 8*(3*b^2*c - 10*a*c^2)*d*e^3*x^3 + 2
*(3*b^2*c - 10*a*c^2)*d^4 + 2*(3*b^3 - 11*a*b*c + 6*(3*b^2*c - 10*a*c^2)*d^
2)*e^2*x^2 + 4*a*b^2 - 16*a^2*c + 2*(3*b^3 - 11*a*b*c)*d^2 + 4*(2*(3*b^2*c
- 10*a*c^2)*d^3 + (3*b^3 - 11*a*b*c)*d)*e*x + sqrt(1/2)*((a^2*b^2*c - 4*a^3
*c^2)*e^6*f^2*x^5 + 5*(a^2*b^2*c - 4*a^3*c^2)*d*e^5*f^2*x^4 + (a^2*b^3 - 4*
a^3*b*c + 10*(a^2*b^2*c - 4*a^3*c^2)*d^2)*e^4*f^2*x^3 + (10*(a^2*b^2*c - 4*
a^3*c^2)*d^3 + 3*(a^2*b^3 - 4*a^3*b*c)*d)*e^3*f^2*x^2 + (a^3*b^2 - 4*a^4*c
+ 5*(a^2*b^2*c - 4*a^3*c^2)*d^4 + 3*(a^2*b^3 - 4*a^3*b*c)*d^2)*e^2*f^2*x +
((a^2*b^2*c - 4*a^3*c^2)*d^5 + (a^2*b^3 - 4*a^3*b*c)*d^3 + (a^3*b^2 - 4*a^4
*c)*d)*e*f^2)*sqrt(-((a^5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2*c^2 - 64*a^8*c^3)
*e^2*f^4*sqrt((81*b^8 - 918*a*b^6*c + 3051*a^2*b^4*c^2 - 2550*a^3*b^2*c^3 +
 625*a^4*c^4)/((a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)*e
^4*f^8)) + 9*b^7 - 105*a*b^5*c + 385*a^2*b^3*c^2 - 420*a^3*b*c^3)/((a^5*b^6
 - 12*a^6*b^4*c + 48*a^7*b^2*c^2 - 64*a^8*c^3)*e^2*f^4))*log(-(189*b^6*c^3
- 1971*a*b^4*c^4 + 5625*a^2*b^2*c^5 - 2500*a^3*c^6)*e*x - (189*b^6*c^3 - 19
71*a*b^4*c^4 + 5625*a^2*b^2*c^5 - 2500*a^3*c^6)*d + 1/2*sqrt(1/2)*((3*a^5*b
^10 - 55*a^6*b^8*c + 392*a^7*b^6*c^2 - 1344*a^8*b^4*c^3 + 2176*a^9*b^2*c^4
- 1280*a^10*c^5)*e^3*f^6*sqrt((81*b^8 - 918*a*b^6*c + 3051*a^2*b^4*c^2 - 25
50*a^3*b^2*c^3 + 625*a^4*c^4)/((a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2
- 64*a^13*c^3)*e^4*f^8)) - (27*b^11 - 486*a*b^9*c + 3330*a^2*b^7*c^2 - 1054
9*a^3*b^5*c^3 + 14408*a^4*b^3*c^4 - 5200*a^5*b*c^5)*e*f^2)*sqrt(-((a^5*b^6
- 12*a^6*b^4*c + 48*a^7*b^2*c^2 - 64*a^8*c^3)*e^2*f^4*sqrt((81*b^8 - 918*a*
b^6*c + 3051*a^2*b^4*c^2 - 2550*a^3*b^2*c^3 + 625*a^4*c^4)/((a^10*b^6 - 12*
a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)*e^4*f^8)) + 9*b^7 - 105*a*b^5*c
 + 385*a^2*b^3*c^2 - 420*a^3*b*c^3)/((a^5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2*c
^2 - 64*a^8*c^3)*e^2*f^4))) - sqrt(1/2)*((a^2*b^2*c - 4*a^3*c^2)*e^6*f^2*x^
5 + 5*(a^2*b^2*c - 4*a^3*c^2)*d*e^5*f^2*x^4 + (a^2*b^3 - 4*a^3*b*c + 10*(a^
2*b^2*c - 4*a^3*c^2)*d^2)*e^4*f^2*x^3 + (10*(a^2*b^2*c - 4*a^3*c^2)*d^3 + 3
*(a^2*b^3 - 4*a^3*b*c)*d)*e^3*f^2*x^2 + (a^3*b^2 - 4*a^4*c + 5*(a^2*b^2*c -
 4*a^3*c^2)*d^4 + 3*(a^2*b^3 - 4*a^3*b*c)*d^2)*e^2*f^2*x + ((a^2*b^2*c - 4*
a^3*c^2)*d^5 + (a^2*b^3 - 4*a^3*b*c)*d^3 + (a^3*b^2 - 4*a^4*c)*d)*e*f^2)*sq
rt(-((a^5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2*c^2 - 64*a^8*c^3)*e^2*f^4*sqrt((8
1*b^8 - 918*a*b^6*c + 3051*a^2*b^4*c^2 - 2550*a^3*b^2*c^3 + 625*a^4*c^4)/((
a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)*e^4*f^8)) + 9*b^7
 - 105*a*b^5*c + 385*a^2*b^3*c^2 - 420*a^3*b*c^3)/((a^5*b^6 - 12*a^6*b^4*c
+ 48*a^7*b^2*c^2 - 64*a^8*c^3)*e^2*f^4))*log(-(189*b^6*c^3 - 1971*a*b^4*c^4
 + 5625*a^2*b^2*c^5 - 2500*a^3*c^6)*e*x - (189*b^6*c^3 - 1971*a*b^4*c^4 + 5
625*a^2*b^2*c^5 - 2500*a^3*c^6)*d - 1/2*sqrt(1/2)*((3*a^5*b^10 - 55*a^6*b^8
*c + 392*a^7*b^6*c^2 - 1344*a^8*b^4*c^3 + 2176*a^9*b^2*c^4 - 1280*a^10*c^5)
*e^3*f^6*sqrt((81*b^8 - 918*a*b^6*c + 3051*a^2*b^4*c^2 - 2550*a^3*b^2*c^3 +
 625*a^4*c^4)/((a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)*e
^4*f^8)) - (27*b^11 - 486*a*b^9*c + 3330*a^2*b^7*c^2 - 10549*a^3*b^5*c^3 +
14408*a^4*b^3*c^4 - 5200*a^5*b*c^5)*e*f^2)*sqrt(-((a^5*b^6 - 12*a^6*b^4*c +
 48*a^7*b^2*c^2 - 64*a^8*c^3)*e^2*f^4*sqrt((81*b^8 - 918*a*b^6*c + 3051*a^2
*b^4*c^2 - 2550*a^3*b^2*c^3 + 625*a^4*c^4)/((a^10*b^6 - 12*a^11*b^4*c + 48*
a^12*b^2*c^2 - 64*a^13*c^3)*e^4*f^8)) + 9*b^7 - 105*a*b^5*c + 385*a^2*b^3*c
^2 - 420*a^3*b*c^3)/((a^5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2*c^2 - 64*a^8*c^3)
*e^2*f^4))) - sqrt(1/2)*((a^2*b^2*c - 4*a^3*c^2)*e^6*f^2*x^5 + 5*(a^2*b^2*c
 - 4*a^3*c^2)*d*e^5*f^2*x^4 + (a^2*b^3 - 4*a^3*b*c + 10*(a^2*b^2*c - 4*a^3*
```

```
c^2)*d^2)*e^4*f^2*x^3 + (10*(a^2*b^2*c - 4*a^3*c^2)*d^3 + 3*(a^2*b^3 - 4*a^
3*b*c)*d)*e^3*f^2*x^2 + (a^3*b^2 - 4*a^4*c + 5*(a^2*b^2*c - 4*a^3*c^2)*d^4
+ 3*(a^2*b^3 - 4*a^3*b*c)*d^2)*e^2*f^2*x + ((a^2*b^2*c - 4*a^3*c^2)*d^5 + (
a^2*b^3 - 4*a^3*b*c)*d^3 + (a^3*b^2 - 4*a^4*c)*d)*e*f^2)*sqrt(((a^5*b^6 - 1
2*a^6*b^4*c + 48*a^7*b^2*c^2 - 64*a^8*c^3)*e^2*f^4*sqrt((81*b^8 - 918*a*b^6
*c + 3051*a^2*b^4*c^2 - 2550*a^3*b^2*c^3 + 625*a^4*c^4)/((a^10*b^6 - 12*a^1
1*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)*e^4*f^8)) - 9*b^7 + 105*a*b^5*c -
385*a^2*b^3*c^2 + 420*a^3*b*c^3)/((a^5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2*c^2
- 64*a^8*c^3)*e^2*f^4))*log(-(189*b^6*c^3 - 1971*a*b^4*c^4 + 5625*a^2*b^2*c
^5 - 2500*a^3*c^6)*e*x - (189*b^6*c^3 - 1971*a*b^4*c^4 + 5625*a^2*b^2*c^5 -
 2500*a^3*c^6)*d + 1/2*sqrt(1/2)*((3*a^5*b^10 - 55*a^6*b^8*c + 392*a^7*b^6*
c^2 - 1344*a^8*b^4*c^3 + 2176*a^9*b^2*c^4 - 1280*a^10*c^5)*e^3*f^6*sqrt((81
*b^8 - 918*a*b^6*c + 3051*a^2*b^4*c^2 - 2550*a^3*b^2*c^3 + 625*a^4*c^4)/((a
^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)*e^4*f^8)) + (27*b^
11 - 486*a*b^9*c + 3330*a^2*b^7*c^2 - 10549*a^3*b^5*c^3 + 14408*a^4*b^3*c^4
 - 5200*a^5*b*c^5)*e*f^2)*sqrt(((a^5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2*c^2 -
64*a^8*c^3)*e^2*f^4*sqrt((81*b^8 - 918*a*b^6*c + 3051*a^2*b^4*c^2 - 2550*a^
3*b^2*c^3 + 625*a^4*c^4)/((a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*
a^13*c^3)*e^4*f^8)) - 9*b^7 + 105*a*b^5*c - 385*a^2*b^3*c^2 + 420*a^3*b*c^3
)/((a^5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2*c^2 - 64*a^8*c^3)*e^2*f^4))) + sqrt
(1/2)*((a^2*b^2*c - 4*a^3*c^2)*e^6*f^2*x^5 + 5*(a^2*b^2*c - 4*a^3*c^2)*d*e^
5*f^2*x^4 + (a^2*b^3 - 4*a^3*b*c + 10*(a^2*b^2*c - 4*a^3*c^2)*d^2)*e^4*f^2*
x^3 + (10*(a^2*b^2*c - 4*a^3*c^2)*d^3 + 3*(a^2*b^3 - 4*a^3*b*c)*d)*e^3*f^2*
x^2 + (a^3*b^2 - 4*a^4*c + 5*(a^2*b^2*c - 4*a^3*c^2)*d^4 + 3*(a^2*b^3 - 4*a
^3*b*c)*d^2)*e^2*f^2*x + ((a^2*b^2*c - 4*a^3*c^2)*d^5 + (a^2*b^3 - 4*a^3*b*
c)*d^3 + (a^3*b^2 - 4*a^4*c)*d)*e*f^2)*sqrt(((a^5*b^6 - 12*a^6*b^4*c + 48*a
^7*b^2*c^2 - 64*a^8*c^3)*e^2*f^4*sqrt((81*b^8 - 918*a*b^6*c + 3051*a^2*b^4*
c^2 - 2550*a^3*b^2*c^3 + 625*a^4*c^4)/((a^10*b^6 - 12*a^11*b^4*c + 48*a^12*
b^2*c^2 - 64*a^13*c^3)*e^4*f^8)) - 9*b^7 + 105*a*b^5*c - 385*a^2*b^3*c^2 +
420*a^3*b*c^3)/((a^5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2*c^2 - 64*a^8*c^3)*e^2*
f^4))*log(-(189*b^6*c^3 - 1971*a*b^4*c^4 + 5625*a^2*b^2*c^5 - 2500*a^3*c^6)
*e*x - (189*b^6*c^3 - 1971*a*b^4*c^4 + 5625*a^2*b^2*c^5 - 2500*a^3*c^6)*d -
 1/2*sqrt(1/2)*((3*a^5*b^10 - 55*a^6*b^8*c + 392*a^7*b^6*c^2 - 1344*a^8*b^4
*c^3 + 2176*a^9*b^2*c^4 - 1280*a^10*c^5)*e^3*f^6*sqrt((81*b^8 - 918*a*b^6*c
 + 3051*a^2*b^4*c^2 - 2550*a^3*b^2*c^3 + 625*a^4*c^4)/((a^10*b^6 - 12*a^11*
b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)*e^4*f^8)) + (27*b^11 - 486*a*b^9*c +
 3330*a^2*b^7*c^2 - 10549*a^3*b^5*c^3 + 14408*a^4*b^3*c^4 - 5200*a^5*b*c^5)
*e*f^2)*sqrt(((a^5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2*c^2 - 64*a^8*c^3)*e^2*f^
4*sqrt((81*b^8 - 918*a*b^6*c + 3051*a^2*b^4*c^2 - 2550*a^3*b^2*c^3 + 625*a^
4*c^4)/((a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)*e^4*f^8)
) - 9*b^7 + 105*a*b^5*c - 385*a^2*b^3*c^2 + 420*a^3*b*c^3)/((a^5*b^6 - 12*a
^6*b^4*c + 48*a^7*b^2*c^2 - 64*a^8*c^3)*e^2*f^4))))/((a^2*b^2*c - 4*a^3*c^2
)*e^6*f^2*x^5 + 5*(a^2*b^2*c - 4*a^3*c^2)*d*e^5*f^2*x^4 + (a^2*b^3 - 4*a^3*
b*c + 10*(a^2*b^2*c - 4*a^3*c^2)*d^2)*e^4*f^2*x^3 + (10*(a^2*b^2*c - 4*a^3*
c^2)*d^3 + 3*(a^2*b^3 - 4*a^3*b*c)*d)*e^3*f^2*x^2 + (a^3*b^2 - 4*a^4*c + 5*
(a^2*b^2*c - 4*a^3*c^2)*d^4 + 3*(a^2*b^3 - 4*a^3*b*c)*d^2)*e^2*f^2*x + ((a^
2*b^2*c - 4*a^3*c^2)*d^5 + (a^2*b^3 - 4*a^3*b*c)*d^3 + (a^3*b^2 - 4*a^4*c)*
d)*e*f^2)
```

giac [B]   time = 1.02, size = 999, normalized size = 2.78

$$-\frac{\frac{b^2ce^{(-1)}}{(fxe+df)f}-\frac{2ac^2e^{(-1)}}{(fxe+df)f}+\frac{b^3fe^{(-1)}}{(fxe+df)^3}-\frac{3abcfe^{(-1)}}{(fxe+df)^3}}{2(a^2b^2-4a^3c)\left(c+\frac{bf^2}{(fxe+df)^2}+\frac{af^4}{(fxe+df)^4}\right)}-\frac{e^{(-1)}}{(fxe+df)a^2f}+\frac{\left(\left(3a^4b^7-31a^5b^5c+96a^6b^3c^2-80a^7bc^3\right)\sqrt{2ab+2\sqrt{b^2-4ac}\,a}\,f^8e^4+2\left(3a^3b^2c-10a^4c^2\right)\sqrt{2ab+2\sqrt{b^2-4ac}\,a}\,\sqrt{b^2-4ac}\,f^4|a^2b^2f^4e^2-4a^3cf^4e^2|e^2-\left(a^2b^2f^4e^2-4a^3cf^4e^2\right)^2\left(3b^3-13abc\right)\sqrt{2ab+2\sqrt{b^2-4ac}\,a}\right)\arctan\left([illegible]\right)}{16\left(a^5b^2c-4a^6c^2\right)\sqrt{b^2-4ac}\,f^6|a^2b^2f^4e^2-4a^3cf^4e^2||a|}+[illegible]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*f*x+d*f)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="giac"
)
```

```
[Out] -1/2*(b^2*c*e^(-1)/((f*x*e + d*f)*f) - 2*a*c^2*e^(-1)/((f*x*e + d*f)*f) + b
^3*f*e^(-1)/(f*x*e + d*f)^3 - 3*a*b*c*f*e^(-1)/(f*x*e + d*f)^3)/((a^2*b^2 -
 4*a^3*c)*(c + b*f^2/(f*x*e + d*f)^2 + a*f^4/(f*x*e + d*f)^4)) - e^(-1)/((f
```

```
*x*e + d*f)*a^2*f) + 1/16*((3*a^4*b^7 - 31*a^5*b^5*c + 96*a^6*b^3*c^2 - 80*
a^7*b*c^3)*sqrt(2*a*b + 2*sqrt(b^2 - 4*a*c)*a)*f^8*e^4 + 2*(3*a^3*b^2*c - 1
0*a^4*c^2)*sqrt(2*a*b + 2*sqrt(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*f^4*abs(a^
2*b^2*f^4*e^2 - 4*a^3*c*f^4*e^2)*e^2 - (a^2*b^2*f^4*e^2 - 4*a^3*c*f^4*e^2)^
2*(3*b^3 - 13*a*b*c)*sqrt(2*a*b + 2*sqrt(b^2 - 4*a*c)*a))*arctan(2*sqrt(1/2
)*e^(-1)/((f*x*e + d*f)*f*sqrt((a^2*b^3*f^4*e^2 - 4*a^3*b*c*f^4*e^2 + sqrt(
(a^2*b^3*f^4*e^2 - 4*a^3*b*c*f^4*e^2)^2 - 4*(a^3*b^2*f^8*e^4 - 4*a^4*c*f^8*
e^4)*(a^2*b^2*c - 4*a^3*c^2)))/(a^3*b^2*f^8*e^4 - 4*a^4*c*f^8*e^4))))*e^(-3
)/((a^5*b^2*c - 4*a^6*c^2)*sqrt(b^2 - 4*a*c)*f^6*abs(a^2*b^2*f^4*e^2 - 4*a^
3*c*f^4*e^2)*abs(a)) - 1/16*((3*a^4*b^7 - 31*a^5*b^5*c + 96*a^6*b^3*c^2 - 8
0*a^7*b*c^3)*sqrt(2*a*b - 2*sqrt(b^2 - 4*a*c)*a)*f^8*e^4 - 2*(3*a^3*b^2*c -
 10*a^4*c^2)*sqrt(2*a*b - 2*sqrt(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*f^4*abs(
a^2*b^2*f^4*e^2 - 4*a^3*c*f^4*e^2)*e^2 - (a^2*b^2*f^4*e^2 - 4*a^3*c*f^4*e^2
)^2*(3*b^3 - 13*a*b*c)*sqrt(2*a*b - 2*sqrt(b^2 - 4*a*c)*a))*arctan(2*sqrt(1
/2)*e^(-1)/((f*x*e + d*f)*f*sqrt((a^2*b^3*f^4*e^2 - 4*a^3*b*c*f^4*e^2 - sqr
t((a^2*b^3*f^4*e^2 - 4*a^3*b*c*f^4*e^2)^2 - 4*(a^3*b^2*f^8*e^4 - 4*a^4*c*f^
8*e^4)*(a^2*b^2*c - 4*a^3*c^2)))/(a^3*b^2*f^8*e^4 - 4*a^4*c*f^8*e^4))))*e^(
-3)/((a^5*b^2*c - 4*a^6*c^2)*sqrt(b^2 - 4*a*c)*f^6*abs(a^2*b^2*f^4*e^2 - 4*
a^3*c*f^4*e^2)*abs(a))
```

maple [C] time = 0.03, size = 1346, normalized size = 3.74

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*f*x+d*f)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x)
```

```
[Out] -1/f^2/a/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d
^4+2*b*d*e*x+b*d^2+a)*c^2*e^2/(4*a*c-b^2)*x^3+1/2/f^2/a^2/(c*e^4*x^4+4*c*d*
e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)*c*e^
2/(4*a*c-b^2)*x^3*b^2-3/f^2/a/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*
d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)*d*c^2*e/(4*a*c-b^2)*x^2+3/2/f^2/
a^2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*
b*d*e*x+b*d^2+a)*d*c*e/(4*a*c-b^2)*x^2*b^2-3/f^2/a/(c*e^4*x^4+4*c*d*e^3*x^3
+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)/(4*a*c-b^2)
*x*c^2*d^2+3/2/f^2/a^2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x
+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)/(4*a*c-b^2)*x*b^2*c*d^2-3/2/f^2/a/(c*e^
4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b
*d^2+a)/(4*a*c-b^2)*x*b*c+1/2/f^2/a^2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*
x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)/(4*a*c-b^2)*x*b^3-1/f^2/
a/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*
d*e*x+b*d^2+a)*d^3/e/(4*a*c-b^2)*c^2+1/2/f^2/a^2/(c*e^4*x^4+4*c*d*e^3*x^3+6
*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)*d^3/e/(4*a*c-
b^2)*b^2*c-3/2/f^2/a/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b
*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)*d/e/(4*a*c-b^2)*b*c+1/2/f^2/a^2/(c*e^4*x^
4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2
+a)*d/e/(4*a*c-b^2)*b^3-1/4/f^2/a^2/(4*a*c-b^2)/e*sum(((10*a*c-3*b^2)*_R^2*
c*e^2+10*a*c^2*d^2-3*b^2*c*d^2+2*(10*a*c-3*b^2)*_R*c*d*e+13*a*b*c-3*b^3)/(2
*_R^3*c*e^3+6*_R^2*c*d*e^2+6*_R*c*d^2*e+2*c*d^3+_R*b*e+b*d)*ln(-_R+x),_R=Ro
otOf(_Z^4*c*e^4+4*_Z^3*c*d*e^3+c*d^4+b*d^2+(6*c*d^2*e^2+b*e^2)*_Z^2+(4*c*d^
3*e+2*b*d*e)*_Z+a))-1/f^2/a^2/e/(e*x+d)
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*f*x+d*f)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="maxim
a")
```

```
[Out] Timed out
```

mupad [B] time = 7.29, size = 12008, normalized size = 33.36

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((d*f + e*f*x)^2*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2),x)
```

```
[Out] - atan(((-(9*b^13 - 9*b^4*(-(4*a*c - b^2)^9)^(1/2) + 26880*a^6*b*c^6 + 2077
*a^2*b^9*c^2 - 10656*a^3*b^7*c^3 + 30240*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 -
25*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c + 51*a*b^2*c*(-(4*a*c -
b^2)^9)^(1/2))/(32*(a^5*b^12*e^2*f^4 + 4096*a^11*c^6*e^2*f^4 + 240*a^7*b^8*
c^2*e^2*f^4 - 1280*a^8*b^6*c^3*e^2*f^4 + 3840*a^9*b^4*c^4*e^2*f^4 - 6144*a^
10*b^2*c^5*e^2*f^4 - 24*a^6*b^10*c*e^2*f^4)))^(1/2)*((-(9*b^13 - 9*b^4*(-(4
*a*c - b^2)^9)^(1/2) + 26880*a^6*b*c^6 + 2077*a^2*b^9*c^2 - 10656*a^3*b^7*c
^3 + 30240*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 - 25*a^2*c^2*(-(4*a*c - b^2)^9)^
(1/2) - 213*a*b^11*c + 51*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^5*b^12*e
^2*f^4 + 4096*a^11*c^6*e^2*f^4 + 240*a^7*b^8*c^2*e^2*f^4 - 1280*a^8*b^6*c^3
*e^2*f^4 + 3840*a^9*b^4*c^4*e^2*f^4 - 6144*a^10*b^2*c^5*e^2*f^4 - 24*a^6*b^
10*c*e^2*f^4)))^(1/2)*((-(9*b^13 - 9*b^4*(-(4*a*c - b^2)^9)^(1/2) + 26880*a
^6*b*c^6 + 2077*a^2*b^9*c^2 - 10656*a^3*b^7*c^3 + 30240*a^4*b^5*c^4 - 44800
*a^5*b^3*c^5 - 25*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c + 51*a*b^
2*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^5*b^12*e^2*f^4 + 4096*a^11*c^6*e^2*f^4
 + 240*a^7*b^8*c^2*e^2*f^4 - 1280*a^8*b^6*c^3*e^2*f^4 + 3840*a^9*b^4*c^4*e^
2*f^4 - 6144*a^10*b^2*c^5*e^2*f^4 - 24*a^6*b^10*c*e^2*f^4)))^(1/2)*(x*(256*
a^10*b^13*c^2*e^14*f^10 - 6144*a^11*b^11*c^3*e^14*f^10 + 61440*a^12*b^9*c^4
*e^14*f^10 - 327680*a^13*b^7*c^5*e^14*f^10 + 983040*a^14*b^5*c^6*e^14*f^10
- 1572864*a^15*b^3*c^7*e^14*f^10 + 1048576*a^16*b*c^8*e^14*f^10) + 1048576*
a^16*b*c^8*d*e^13*f^10 + 256*a^10*b^13*c^2*d*e^13*f^10 - 6144*a^11*b^11*c^3
*d*e^13*f^10 + 61440*a^12*b^9*c^4*d*e^13*f^10 - 327680*a^13*b^7*c^5*d*e^13*
f^10 + 983040*a^14*b^5*c^6*d*e^13*f^10 - 1572864*a^15*b^3*c^7*d*e^13*f^10)
- 192*a^8*b^13*c^2*e^12*f^8 + 4672*a^9*b^11*c^3*e^12*f^8 - 47360*a^10*b^9*c
^4*e^12*f^8 + 256000*a^11*b^7*c^5*e^12*f^8 - 778240*a^12*b^5*c^6*e^12*f^8 +
 1261568*a^13*b^3*c^7*e^12*f^8 - 851968*a^14*b*c^8*e^12*f^8) + x*(204800*a^
12*c^9*e^12*f^6 + 144*a^6*b^12*c^3*e^12*f^6 - 3264*a^7*b^10*c^4*e^12*f^6 +
30112*a^8*b^8*c^5*e^12*f^6 - 143360*a^9*b^6*c^6*e^12*f^6 + 365568*a^10*b^4*
c^7*e^12*f^6 - 458752*a^11*b^2*c^8*e^12*f^6) + 204800*a^12*c^9*d*e^11*f^6 +
 144*a^6*b^12*c^3*d*e^11*f^6 - 3264*a^7*b^10*c^4*d*e^11*f^6 + 30112*a^8*b^8
*c^5*d*e^11*f^6 - 143360*a^9*b^6*c^6*d*e^11*f^6 + 365568*a^10*b^4*c^7*d*e^1
1*f^6 - 458752*a^11*b^2*c^8*d*e^11*f^6)*1i + (-(9*b^13 - 9*b^4*(-(4*a*c - b
^2)^9)^(1/2) + 26880*a^6*b*c^6 + 2077*a^2*b^9*c^2 - 10656*a^3*b^7*c^3 + 302
40*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 - 25*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) -
213*a*b^11*c + 51*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^5*b^12*e^2*f^4 +
 4096*a^11*c^6*e^2*f^4 + 240*a^7*b^8*c^2*e^2*f^4 - 1280*a^8*b^6*c^3*e^2*f^4
 + 3840*a^9*b^4*c^4*e^2*f^4 - 6144*a^10*b^2*c^5*e^2*f^4 - 24*a^6*b^10*c*e^2
*f^4)))^(1/2)*((-(9*b^13 - 9*b^4*(-(4*a*c - b^2)^9)^(1/2) + 26880*a^6*b*c^6
 + 2077*a^2*b^9*c^2 - 10656*a^3*b^7*c^3 + 30240*a^4*b^5*c^4 - 44800*a^5*b^3
*c^5 - 25*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c + 51*a*b^2*c*(-(4
*a*c - b^2)^9)^(1/2))/(32*(a^5*b^12*e^2*f^4 + 4096*a^11*c^6*e^2*f^4 + 240*a
^7*b^8*c^2*e^2*f^4 - 1280*a^8*b^6*c^3*e^2*f^4 + 3840*a^9*b^4*c^4*e^2*f^4 -
6144*a^10*b^2*c^5*e^2*f^4 - 24*a^6*b^10*c*e^2*f^4)))^(1/2)*((-(9*b^13 - 9*b
^4*(-(4*a*c - b^2)^9)^(1/2) + 26880*a^6*b*c^6 + 2077*a^2*b^9*c^2 - 10656*a^
3*b^7*c^3 + 30240*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 - 25*a^2*c^2*(-(4*a*c - b
^2)^9)^(1/2) - 213*a*b^11*c + 51*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^5
*b^12*e^2*f^4 + 4096*a^11*c^6*e^2*f^4 + 240*a^7*b^8*c^2*e^2*f^4 - 1280*a^8*
b^6*c^3*e^2*f^4 + 3840*a^9*b^4*c^4*e^2*f^4 - 6144*a^10*b^2*c^5*e^2*f^4 - 24
*a^6*b^10*c*e^2*f^4)))^(1/2)*(x*(256*a^10*b^13*c^2*e^14*f^10 - 6144*a^11*b^
11*c^3*e^14*f^10 + 61440*a^12*b^9*c^4*e^14*f^10 - 327680*a^13*b^7*c^5*e^14*
f^10 + 983040*a^14*b^5*c^6*e^14*f^10 - 1572864*a^15*b^3*c^7*e^14*f^10 + 104
8576*a^16*b*c^8*e^14*f^10) + 1048576*a^16*b*c^8*d*e^13*f^10 + 256*a^10*b^13
*c^2*d*e^13*f^10 - 6144*a^11*b^11*c^3*d*e^13*f^10 + 61440*a^12*b^9*c^4*d*e^
```

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13*f^10 - 327680*a^13*b^7*c^5*d*e^13*f^10 + 983040*a^14*b^5*c^6*d*e^13*f^10
 - 1572864*a^15*b^3*c^7*d*e^13*f^10) + 192*a^8*b^13*c^2*e^12*f^8 - 4672*a^9
*b^11*c^3*e^12*f^8 + 47360*a^10*b^9*c^4*e^12*f^8 - 256000*a^11*b^7*c^5*e^12
*f^8 + 778240*a^12*b^5*c^6*e^12*f^8 - 1261568*a^13*b^3*c^7*e^12*f^8 + 85196
8*a^14*b*c^8*e^12*f^8) + x*(204800*a^12*c^9*e^12*f^6 + 144*a^6*b^12*c^3*e^1
2*f^6 - 3264*a^7*b^10*c^4*e^12*f^6 + 30112*a^8*b^8*c^5*e^12*f^6 - 143360*a^
9*b^6*c^6*e^12*f^6 + 365568*a^10*b^4*c^7*e^12*f^6 - 458752*a^11*b^2*c^8*e^1
2*f^6) + 204800*a^12*c^9*d*e^11*f^6 + 144*a^6*b^12*c^3*d*e^11*f^6 - 3264*a^
7*b^10*c^4*d*e^11*f^6 + 30112*a^8*b^8*c^5*d*e^11*f^6 - 143360*a^9*b^6*c^6*d
*e^11*f^6 + 365568*a^10*b^4*c^7*d*e^11*f^6 - 458752*a^11*b^2*c^8*d*e^11*f^6
)*1i)/((-(9*b^13 - 9*b^4*(-(4*a*c - b^2)^9)^(1/2) + 26880*a^6*b*c^6 + 2077*
a^2*b^9*c^2 - 10656*a^3*b^7*c^3 + 30240*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 - 2
5*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c + 51*a*b^2*c*(-(4*a*c - b
^2)^9)^(1/2))/(32*(a^5*b^12*e^2*f^4 + 4096*a^11*c^6*e^2*f^4 + 240*a^7*b^8*c
^2*e^2*f^4 - 1280*a^8*b^6*c^3*e^2*f^4 + 3840*a^9*b^4*c^4*e^2*f^4 - 6144*a^1
0*b^2*c^5*e^2*f^4 - 24*a^6*b^10*c*e^2*f^4)))^(1/2)*((-(9*b^13 - 9*b^4*(-(4*
a*c - b^2)^9)^(1/2) + 26880*a^6*b*c^6 + 2077*a^2*b^9*c^2 - 10656*a^3*b^7*c^
3 + 30240*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 - 25*a^2*c^2*(-(4*a*c - b^2)^9)^(
1/2) - 213*a*b^11*c + 51*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^5*b^12*e^
2*f^4 + 4096*a^11*c^6*e^2*f^4 + 240*a^7*b^8*c^2*e^2*f^4 - 1280*a^8*b^6*c^3*
e^2*f^4 + 3840*a^9*b^4*c^4*e^2*f^4 - 6144*a^10*b^2*c^5*e^2*f^4 - 24*a^6*b^1
0*c*e^2*f^4)))^(1/2)*((-(9*b^13 - 9*b^4*(-(4*a*c - b^2)^9)^(1/2) + 26880*a^
6*b*c^6 + 2077*a^2*b^9*c^2 - 10656*a^3*b^7*c^3 + 30240*a^4*b^5*c^4 - 44800*
a^5*b^3*c^5 - 25*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c + 51*a*b^2
*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^5*b^12*e^2*f^4 + 4096*a^11*c^6*e^2*f^4
 + 240*a^7*b^8*c^2*e^2*f^4 - 1280*a^8*b^6*c^3*e^2*f^4 + 3840*a^9*b^4*c^4*e^2
*f^4 - 6144*a^10*b^2*c^5*e^2*f^4 - 24*a^6*b^10*c*e^2*f^4)))^(1/2)*(x*(256*a
^10*b^13*c^2*e^14*f^10 - 6144*a^11*b^11*c^3*e^14*f^10 + 61440*a^12*b^9*c^4*
e^14*f^10 - 327680*a^13*b^7*c^5*e^14*f^10 + 983040*a^14*b^5*c^6*e^14*f^10 -
 1572864*a^15*b^3*c^7*e^14*f^10 + 1048576*a^16*b*c^8*e^14*f^10) + 1048576*a
^16*b*c^8*d*e^13*f^10 + 256*a^10*b^13*c^2*d*e^13*f^10 - 6144*a^11*b^11*c^3*
d*e^13*f^10 + 61440*a^12*b^9*c^4*d*e^13*f^10 - 327680*a^13*b^7*c^5*d*e^13*f
^10 + 983040*a^14*b^5*c^6*d*e^13*f^10 - 1572864*a^15*b^3*c^7*d*e^13*f^10) +
 192*a^8*b^13*c^2*e^12*f^8 - 4672*a^9*b^11*c^3*e^12*f^8 + 47360*a^10*b^9*c^
4*e^12*f^8 - 256000*a^11*b^7*c^5*e^12*f^8 + 778240*a^12*b^5*c^6*e^12*f^8 -
1261568*a^13*b^3*c^7*e^12*f^8 + 851968*a^14*b*c^8*e^12*f^8) + x*(204800*a^1
2*c^9*e^12*f^6 + 144*a^6*b^12*c^3*e^12*f^6 - 3264*a^7*b^10*c^4*e^12*f^6 + 3
0112*a^8*b^8*c^5*e^12*f^6 - 143360*a^9*b^6*c^6*e^12*f^6 + 365568*a^10*b^4*c
^7*e^12*f^6 - 458752*a^11*b^2*c^8*e^12*f^6) + 204800*a^12*c^9*d*e^11*f^6 +
144*a^6*b^12*c^3*d*e^11*f^6 - 3264*a^7*b^10*c^4*d*e^11*f^6 + 30112*a^8*b^8*
c^5*d*e^11*f^6 - 143360*a^9*b^6*c^6*d*e^11*f^6 + 365568*a^10*b^4*c^7*d*e^11
*f^6 - 458752*a^11*b^2*c^8*d*e^11*f^6) - (-(9*b^13 - 9*b^4*(-(4*a*c - b^2)^
9)^(1/2) + 26880*a^6*b*c^6 + 2077*a^2*b^9*c^2 - 10656*a^3*b^7*c^3 + 30240*a
^4*b^5*c^4 - 44800*a^5*b^3*c^5 - 25*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 213*
a*b^11*c + 51*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^5*b^12*e^2*f^4 + 409
6*a^11*c^6*e^2*f^4 + 240*a^7*b^8*c^2*e^2*f^4 - 1280*a^8*b^6*c^3*e^2*f^4 + 3
840*a^9*b^4*c^4*e^2*f^4 - 6144*a^10*b^2*c^5*e^2*f^4 - 24*a^6*b^10*c*e^2*f^4
)))^(1/2)*((-(9*b^13 - 9*b^4*(-(4*a*c - b^2)^9)^(1/2) + 26880*a^6*b*c^6 + 2
077*a^2*b^9*c^2 - 10656*a^3*b^7*c^3 + 30240*a^4*b^5*c^4 - 44800*a^5*b^3*c^5
 - 25*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c + 51*a*b^2*c*(-(4*a*c
 - b^2)^9)^(1/2))/(32*(a^5*b^12*e^2*f^4 + 4096*a^11*c^6*e^2*f^4 + 240*a^7*b
^8*c^2*e^2*f^4 - 1280*a^8*b^6*c^3*e^2*f^4 + 3840*a^9*b^4*c^4*e^2*f^4 - 6144
*a^10*b^2*c^5*e^2*f^4 - 24*a^6*b^10*c*e^2*f^4)))^(1/2)*((-(9*b^13 - 9*b^4*(
-(4*a*c - b^2)^9)^(1/2) + 26880*a^6*b*c^6 + 2077*a^2*b^9*c^2 - 10656*a^3*b^
7*c^3 + 30240*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 - 25*a^2*c^2*(-(4*a*c - b^2)^
9)^(1/2) - 213*a*b^11*c + 51*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^5*b^1
2*e^2*f^4 + 4096*a^11*c^6*e^2*f^4 + 240*a^7*b^8*c^2*e^2*f^4 - 1280*a^8*b^6*
c^3*e^2*f^4 + 3840*a^9*b^4*c^4*e^2*f^4 - 6144*a^10*b^2*c^5*e^2*f^4 - 24*a^6
*b^10*c*e^2*f^4)))^(1/2)*(x*(256*a^10*b^13*c^2*e^14*f^10 - 6144*a^11*b^11*c
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^3*e^14*f^10 + 61440*a^12*b^9*c^4*e^14*f^10 - 327680*a^13*b^7*c^5*e^14*f^10 + 983040*a^14*b^5*c^6*e^14*f^10 - 1572864*a^15*b^3*c^7*e^14*f^10 + 1048576*a^16*b*c^8*e^14*f^10) + 1048576*a^16*b*c^8*d*e^13*f^10 + 256*a^10*b^13*c^2*d*e^13*f^10 - 6144*a^11*b^11*c^3*d*e^13*f^10 + 61440*a^12*b^9*c^4*d*e^13*f^10 - 327680*a^13*b^7*c^5*d*e^13*f^10 + 983040*a^14*b^5*c^6*d*e^13*f^10 - 1572864*a^15*b^3*c^7*d*e^13*f^10) - 192*a^8*b^13*c^2*e^12*f^8 + 4672*a^9*b^11*c^3*e^12*f^8 - 47360*a^10*b^9*c^4*e^12*f^8 + 256000*a^11*b^7*c^5*e^12*f^8 - 778240*a^12*b^5*c^6*e^12*f^8 + 1261568*a^13*b^3*c^7*e^12*f^8 - 851968*a^14*b*c^8*e^12*f^8) + x*(204800*a^12*c^9*e^12*f^6 + 144*a^6*b^12*c^3*e^12*f^6 - 3264*a^7*b^10*c^4*e^12*f^6 + 30112*a^8*b^8*c^5*e^12*f^6 - 143360*a^9*b^6*c^6*e^12*f^6 + 365568*a^10*b^4*c^7*e^12*f^6 - 458752*a^11*b^2*c^8*e^12*f^6) + 204800*a^12*c^9*d*e^11*f^6 + 144*a^6*b^12*c^3*d*e^11*f^6 - 3264*a^7*b^10*c^4*d*e^11*f^6 + 30112*a^8*b^8*c^5*d*e^11*f^6 - 143360*a^9*b^6*c^6*d*e^11*f^6 + 365568*a^10*b^4*c^7*d*e^11*f^6 - 458752*a^11*b^2*c^8*d*e^11*f^6) + 128000*a^10*c^9*e^10*f^4 + 504*a^6*b^8*c^5*e^10*f^4 - 8112*a^7*b^6*c^6*e^10*f^4 + 48704*a^8*b^4*c^7*e^10*f^4 - 129280*a^9*b^2*c^8*e^10*f^4))*(-(9*b^13 - 9*b^4*(-(4*a*c - b^2)^9)^(1/2) + 26880*a^6*b*c^6 + 2077*a^2*b^9*c^2 - 10656*a^3*b^7*c^3 + 30240*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 - 25*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c + 51*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^5*b^12*e^2*f^4 + 4096*a^11*c^6*e^2*f^4 + 240*a^7*b^8*c^2*e^2*f^4 - 1280*a^8*b^6*c^3*e^2*f^4 + 3840*a^9*b^4*c^4*e^2*f^4 - 6144*a^10*b^2*c^5*e^2*f^4 - 24*a^6*b^10*c*e^2*f^4)))^(1/2)*2i - atan(((-(9*b^13 + 9*b^4*(-(4*a*c - b^2)^9)^(1/2) + 26880*a^6*b*c^6 + 2077*a^2*b^9*c^2 - 10656*a^3*b^7*c^3 + 30240*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 + 25*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c - 51*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^5*b^12*e^2*f^4 + 4096*a^11*c^6*e^2*f^4 + 240*a^7*b^8*c^2*e^2*f^4 - 1280*a^8*b^6*c^3*e^2*f^4 + 3840*a^9*b^4*c^4*e^2*f^4 - 6144*a^10*b^2*c^5*e^2*f^4 - 24*a^6*b^10*c*e^2*f^4)))^(1/2)*((-(9*b^13 + 9*b^4*(-(4*a*c - b^2)^9)^(1/2) + 26880*a^6*b*c^6 + 2077*a^2*b^9*c^2 - 10656*a^3*b^7*c^3 + 30240*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 + 25*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c - 51*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^5*b^12*e^2*f^4 + 4096*a^11*c^6*e^2*f^4 + 240*a^7*b^8*c^2*e^2*f^4 - 1280*a^8*b^6*c^3*e^2*f^4 + 3840*a^9*b^4*c^4*e^2*f^4 - 6144*a^10*b^2*c^5*e^2*f^4 - 24*a^6*b^10*c*e^2*f^4)))^(1/2)*((-(9*b^13 + 9*b^4*(-(4*a*c - b^2)^9)^(1/2) + 26880*a^6*b*c^6 + 2077*a^2*b^9*c^2 - 10656*a^3*b^7*c^3 + 30240*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 + 25*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c - 51*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^5*b^12*e^2*f^4 + 4096*a^11*c^6*e^2*f^4 + 240*a^7*b^8*c^2*e^2*f^4 - 1280*a^8*b^6*c^3*e^2*f^4 + 3840*a^9*b^4*c^4*e^2*f^4 - 6144*a^10*b^2*c^5*e^2*f^4 - 24*a^6*b^10*c*e^2*f^4)))^(1/2)*(x*(256*a^10*b^13*c^2*e^14*f^10 - 6144*a^11*b^11*c^3*e^14*f^10 + 61440*a^12*b^9*c^4*e^14*f^10 - 327680*a^13*b^7*c^5*e^14*f^10 + 983040*a^14*b^5*c^6*e^14*f^10 - 1572864*a^15*b^3*c^7*e^14*f^10 + 1048576*a^16*b*c^8*e^14*f^10) + 1048576*a^16*b*c^8*d*e^13*f^10 + 256*a^10*b^13*c^2*d*e^13*f^10 - 6144*a^11*b^11*c^3*d*e^13*f^10 + 61440*a^12*b^9*c^4*d*e^13*f^10 - 327680*a^13*b^7*c^5*d*e^13*f^10 + 983040*a^14*b^5*c^6*d*e^13*f^10 - 1572864*a^15*b^3*c^7*d*e^13*f^10) - 192*a^8*b^13*c^2*e^12*f^8 + 4672*a^9*b^11*c^3*e^12*f^8 - 47360*a^10*b^9*c^4*e^12*f^8 + 256000*a^11*b^7*c^5*e^12*f^8 - 778240*a^12*b^5*c^6*e^12*f^8 + 1261568*a^13*b^3*c^7*e^12*f^8 - 851968*a^14*b*c^8*e^12*f^8) + x*(204800*a^12*c^9*e^12*f^6 + 144*a^6*b^12*c^3*e^12*f^6 - 3264*a^7*b^10*c^4*e^12*f^6 + 30112*a^8*b^8*c^5*e^12*f^6 - 143360*a^9*b^6*c^6*e^12*f^6 + 365568*a^10*b^4*c^7*e^12*f^6 - 458752*a^11*b^2*c^8*e^12*f^6) + 204800*a^12*c^9*d*e^11*f^6 + 144*a^6*b^12*c^3*d*e^11*f^6 - 3264*a^7*b^10*c^4*d*e^11*f^6 + 30112*a^8*b^8*c^5*d*e^11*f^6 - 143360*a^9*b^6*c^6*d*e^11*f^6 + 365568*a^10*b^4*c^7*d*e^11*f^6 - 458752*a^11*b^2*c^8*d*e^11*f^6)*1i + (-(9*b^13 + 9*b^4*(-(4*a*c - b^2)^9)^(1/2) + 26880*a^6*b*c^6 + 2077*a^2*b^9*c^2 - 10656*a^3*b^7*c^3 + 30240*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 + 25*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c - 51*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^5*b^12*e^2*f^4 + 4096*a^11*c^6*e^2*f^4 + 240*a^7*b^8*c^2*e^2*f^4 - 1280*a^8*b^6*c^3*e^2*f^4 + 3840*a^9*b^4*c^4*e^2*f^4 - 6144*a^10*b^2*c^5*e^2*f^4 - 24*a^6*b^10*c*e^2*f^4)))^(1/2)*((-(9*b^13 + 9*b^4*(-(4

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*a*c - b^2)^9)^(1/2) + 26880*a^6*b*c^6 + 2077*a^2*b^9*c^2 - 10656*a^3*b^7*c
^3 + 30240*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 + 25*a^2*c^2*(-(4*a*c - b^2)^9)^
(1/2) - 213*a*b^11*c - 51*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^5*b^12*e
^2*f^4 + 4096*a^11*c^6*e^2*f^4 + 240*a^7*b^8*c^2*e^2*f^4 - 1280*a^8*b^6*c^3
*e^2*f^4 + 3840*a^9*b^4*c^4*e^2*f^4 - 6144*a^10*b^2*c^5*e^2*f^4 - 24*a^6*b^
10*c*e^2*f^4)))^(1/2)*((-(9*b^13 + 9*b^4*(-(4*a*c - b^2)^9)^(1/2) + 26880*a
^6*b*c^6 + 2077*a^2*b^9*c^2 - 10656*a^3*b^7*c^3 + 30240*a^4*b^5*c^4 - 44800
*a^5*b^3*c^5 + 25*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c - 51*a*b^
2*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^5*b^12*e^2*f^4 + 4096*a^11*c^6*e^2*f^4
 + 240*a^7*b^8*c^2*e^2*f^4 - 1280*a^8*b^6*c^3*e^2*f^4 + 3840*a^9*b^4*c^4*e^
2*f^4 - 6144*a^10*b^2*c^5*e^2*f^4 - 24*a^6*b^10*c*e^2*f^4)))^(1/2)*(x*(256*
a^10*b^13*c^2*e^14*f^10 - 6144*a^11*b^11*c^3*e^14*f^10 + 61440*a^12*b^9*c^4
*e^14*f^10 - 327680*a^13*b^7*c^5*e^14*f^10 + 983040*a^14*b^5*c^6*e^14*f^10
- 1572864*a^15*b^3*c^7*e^14*f^10 + 1048576*a^16*b*c^8*e^14*f^10) + 1048576*
a^16*b*c^8*d*e^13*f^10 + 256*a^10*b^13*c^2*d*e^13*f^10 - 6144*a^11*b^11*c^3
*d*e^13*f^10 + 61440*a^12*b^9*c^4*d*e^13*f^10 - 327680*a^13*b^7*c^5*d*e^13*
f^10 + 983040*a^14*b^5*c^6*d*e^13*f^10 - 1572864*a^15*b^3*c^7*d*e^13*f^10)
+ 192*a^8*b^13*c^2*e^12*f^8 - 4672*a^9*b^11*c^3*e^12*f^8 + 47360*a^10*b^9*c
^4*e^12*f^8 - 256000*a^11*b^7*c^5*e^12*f^8 + 778240*a^12*b^5*c^6*e^12*f^8 -
 1261568*a^13*b^3*c^7*e^12*f^8 + 851968*a^14*b*c^8*e^12*f^8) + x*(204800*a^
12*c^9*e^12*f^6 + 144*a^6*b^12*c^3*e^12*f^6 - 3264*a^7*b^10*c^4*e^12*f^6 +
30112*a^8*b^8*c^5*e^12*f^6 - 143360*a^9*b^6*c^6*e^12*f^6 + 365568*a^10*b^4*
c^7*e^12*f^6 - 458752*a^11*b^2*c^8*e^12*f^6) + 204800*a^12*c^9*d*e^11*f^6 +
 144*a^6*b^12*c^3*d*e^11*f^6 - 3264*a^7*b^10*c^4*d*e^11*f^6 + 30112*a^8*b^8
*c^5*d*e^11*f^6 - 143360*a^9*b^6*c^6*d*e^11*f^6 + 365568*a^10*b^4*c^7*d*e^1
1*f^6 - 458752*a^11*b^2*c^8*d*e^11*f^6)*1i)/((-(9*b^13 + 9*b^4*(-(4*a*c - b
^2)^9)^(1/2) + 26880*a^6*b*c^6 + 2077*a^2*b^9*c^2 - 10656*a^3*b^7*c^3 + 302
40*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 + 25*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) -
213*a*b^11*c - 51*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^5*b^12*e^2*f^4 +
 4096*a^11*c^6*e^2*f^4 + 240*a^7*b^8*c^2*e^2*f^4 - 1280*a^8*b^6*c^3*e^2*f^4
 + 3840*a^9*b^4*c^4*e^2*f^4 - 6144*a^10*b^2*c^5*e^2*f^4 - 24*a^6*b^10*c*e^2
*f^4)))^(1/2)*((-(9*b^13 + 9*b^4*(-(4*a*c - b^2)^9)^(1/2) + 26880*a^6*b*c^6
 + 2077*a^2*b^9*c^2 - 10656*a^3*b^7*c^3 + 30240*a^4*b^5*c^4 - 44800*a^5*b^3
*c^5 + 25*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c - 51*a*b^2*c*(-(4
*a*c - b^2)^9)^(1/2))/(32*(a^5*b^12*e^2*f^4 + 4096*a^11*c^6*e^2*f^4 + 240*a
^7*b^8*c^2*e^2*f^4 - 1280*a^8*b^6*c^3*e^2*f^4 + 3840*a^9*b^4*c^4*e^2*f^4 -
6144*a^10*b^2*c^5*e^2*f^4 - 24*a^6*b^10*c*e^2*f^4)))^(1/2)*((-(9*b^13 + 9*b
^4*(-(4*a*c - b^2)^9)^(1/2) + 26880*a^6*b*c^6 + 2077*a^2*b^9*c^2 - 10656*a^
3*b^7*c^3 + 30240*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 + 25*a^2*c^2*(-(4*a*c - b
^2)^9)^(1/2) - 213*a*b^11*c - 51*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^5
*b^12*e^2*f^4 + 4096*a^11*c^6*e^2*f^4 + 240*a^7*b^8*c^2*e^2*f^4 - 1280*a^8*
b^6*c^3*e^2*f^4 + 3840*a^9*b^4*c^4*e^2*f^4 - 6144*a^10*b^2*c^5*e^2*f^4 - 24
*a^6*b^10*c*e^2*f^4)))^(1/2)*(x*(256*a^10*b^13*c^2*e^14*f^10 - 6144*a^11*b^
11*c^3*e^14*f^10 + 61440*a^12*b^9*c^4*e^14*f^10 - 327680*a^13*b^7*c^5*e^14*
f^10 + 983040*a^14*b^5*c^6*e^14*f^10 - 1572864*a^15*b^3*c^7*e^14*f^10 + 104
8576*a^16*b*c^8*e^14*f^10) + 1048576*a^16*b*c^8*d*e^13*f^10 + 256*a^10*b^13
*c^2*d*e^13*f^10 - 6144*a^11*b^11*c^3*d*e^13*f^10 + 61440*a^12*b^9*c^4*d*e^
13*f^10 - 327680*a^13*b^7*c^5*d*e^13*f^10 + 983040*a^14*b^5*c^6*d*e^13*f^10
 - 1572864*a^15*b^3*c^7*d*e^13*f^10) + 192*a^8*b^13*c^2*e^12*f^8 - 4672*a^9
*b^11*c^3*e^12*f^8 + 47360*a^10*b^9*c^4*e^12*f^8 - 256000*a^11*b^7*c^5*e^12
*f^8 + 778240*a^12*b^5*c^6*e^12*f^8 - 1261568*a^13*b^3*c^7*e^12*f^8 + 85196
8*a^14*b*c^8*e^12*f^8) + x*(204800*a^12*c^9*e^12*f^6 + 144*a^6*b^12*c^3*e^1
2*f^6 - 3264*a^7*b^10*c^4*e^12*f^6 + 30112*a^8*b^8*c^5*e^12*f^6 - 143360*a^
9*b^6*c^6*e^12*f^6 + 365568*a^10*b^4*c^7*e^12*f^6 - 458752*a^11*b^2*c^8*e^1
2*f^6) + 204800*a^12*c^9*d*e^11*f^6 + 144*a^6*b^12*c^3*d*e^11*f^6 - 3264*a^
7*b^10*c^4*d*e^11*f^6 + 30112*a^8*b^8*c^5*d*e^11*f^6 - 143360*a^9*b^6*c^6*d
*e^11*f^6 + 365568*a^10*b^4*c^7*d*e^11*f^6 - 458752*a^11*b^2*c^8*d*e^11*f^6
) - (-(9*b^13 + 9*b^4*(-(4*a*c - b^2)^9)^(1/2) + 26880*a^6*b*c^6 + 2077*a^2
*b^9*c^2 - 10656*a^3*b^7*c^3 + 30240*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 + 25*a
```

```
^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c - 51*a*b^2*c*(-(4*a*c - b^2)
^9)^(1/2))/(32*(a^5*b^12*e^2*f^4 + 4096*a^11*c^6*e^2*f^4 + 240*a^7*b^8*c^2*
e^2*f^4 - 1280*a^8*b^6*c^3*e^2*f^4 + 3840*a^9*b^4*c^4*e^2*f^4 - 6144*a^10*b
^2*c^5*e^2*f^4 - 24*a^6*b^10*c*e^2*f^4)))^(1/2)*((-(9*b^13 + 9*b^4*(-(4*a*c
 - b^2)^9)^(1/2) + 26880*a^6*b*c^6 + 2077*a^2*b^9*c^2 - 10656*a^3*b^7*c^3 +
 30240*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 + 25*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2
) - 213*a*b^11*c - 51*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^5*b^12*e^2*f
^4 + 4096*a^11*c^6*e^2*f^4 + 240*a^7*b^8*c^2*e^2*f^4 - 1280*a^8*b^6*c^3*e^2
*f^4 + 3840*a^9*b^4*c^4*e^2*f^4 - 6144*a^10*b^2*c^5*e^2*f^4 - 24*a^6*b^10*c
*e^2*f^4)))^(1/2)*((-(9*b^13 + 9*b^4*(-(4*a*c - b^2)^9)^(1/2) + 26880*a^6*b
*c^6 + 2077*a^2*b^9*c^2 - 10656*a^3*b^7*c^3 + 30240*a^4*b^5*c^4 - 44800*a^5
*b^3*c^5 + 25*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c - 51*a*b^2*c*
(-(4*a*c - b^2)^9)^(1/2))/(32*(a^5*b^12*e^2*f^4 + 4096*a^11*c^6*e^2*f^4 + 2
40*a^7*b^8*c^2*e^2*f^4 - 1280*a^8*b^6*c^3*e^2*f^4 + 3840*a^9*b^4*c^4*e^2*f^
4 - 6144*a^10*b^2*c^5*e^2*f^4 - 24*a^6*b^10*c*e^2*f^4)))^(1/2)*(x*(256*a^10
*b^13*c^2*e^14*f^10 - 6144*a^11*b^11*c^3*e^14*f^10 + 61440*a^12*b^9*c^4*e^1
4*f^10 - 327680*a^13*b^7*c^5*e^14*f^10 + 983040*a^14*b^5*c^6*e^14*f^10 - 15
72864*a^15*b^3*c^7*e^14*f^10 + 1048576*a^16*b*c^8*e^14*f^10) + 1048576*a^16
*b*c^8*d*e^13*f^10 + 256*a^10*b^13*c^2*d*e^13*f^10 - 6144*a^11*b^11*c^3*d*e
^13*f^10 + 61440*a^12*b^9*c^4*d*e^13*f^10 - 327680*a^13*b^7*c^5*d*e^13*f^10
 + 983040*a^14*b^5*c^6*d*e^13*f^10 - 1572864*a^15*b^3*c^7*d*e^13*f^10) - 19
2*a^8*b^13*c^2*e^12*f^8 + 4672*a^9*b^11*c^3*e^12*f^8 - 47360*a^10*b^9*c^4*e
^12*f^8 + 256000*a^11*b^7*c^5*e^12*f^8 - 778240*a^12*b^5*c^6*e^12*f^8 + 126
1568*a^13*b^3*c^7*e^12*f^8 - 851968*a^14*b*c^8*e^12*f^8) + x*(204800*a^12*c
^9*e^12*f^6 + 144*a^6*b^12*c^3*e^12*f^6 - 3264*a^7*b^10*c^4*e^12*f^6 + 3011
2*a^8*b^8*c^5*e^12*f^6 - 143360*a^9*b^6*c^6*e^12*f^6 + 365568*a^10*b^4*c^7*
e^12*f^6 - 458752*a^11*b^2*c^8*e^12*f^6) + 204800*a^12*c^9*d*e^11*f^6 + 144
*a^6*b^12*c^3*d*e^11*f^6 - 3264*a^7*b^10*c^4*d*e^11*f^6 + 30112*a^8*b^8*c^5
*d*e^11*f^6 - 143360*a^9*b^6*c^6*d*e^11*f^6 + 365568*a^10*b^4*c^7*d*e^11*f^
6 - 458752*a^11*b^2*c^8*d*e^11*f^6) + 128000*a^10*c^9*e^10*f^4 + 504*a^6*b^
8*c^5*e^10*f^4 - 8112*a^7*b^6*c^6*e^10*f^4 + 48704*a^8*b^4*c^7*e^10*f^4 - 1
29280*a^9*b^2*c^8*e^10*f^4))*(-(9*b^13 + 9*b^4*(-(4*a*c - b^2)^9)^(1/2) + 2
6880*a^6*b*c^6 + 2077*a^2*b^9*c^2 - 10656*a^3*b^7*c^3 + 30240*a^4*b^5*c^4 -
 44800*a^5*b^3*c^5 + 25*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c - 5
1*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^5*b^12*e^2*f^4 + 4096*a^11*c^6*e
^2*f^4 + 240*a^7*b^8*c^2*e^2*f^4 - 1280*a^8*b^6*c^3*e^2*f^4 + 3840*a^9*b^4*
c^4*e^2*f^4 - 6144*a^10*b^2*c^5*e^2*f^4 - 24*a^6*b^10*c*e^2*f^4)))^(1/2)*2i
 - ((x*(3*b^3*d - 20*a*c^2*d^3 + 6*b^2*c*d^3 - 11*a*b*c*d))/(a*(a*b^2 - 4*a
^2*c)) - (x^4*(10*a*c^2*e^3 - 3*b^2*c*e^3))/(2*a*(a*b^2 - 4*a^2*c)) - (2*x^
3*(10*a*c^2*d*e^2 - 3*b^2*c*d*e^2))/(a*(a*b^2 - 4*a^2*c)) + (2*a*b^2 - 8*a^
2*c + 3*b^3*d^2 - 10*a*c^2*d^4 + 3*b^2*c*d^4 - 11*a*b*c*d^2)/(2*a*e*(a*b^2
- 4*a^2*c)) + (x^2*(3*b^3*e - 60*a*c^2*d^2*e + 18*b^2*c*d^2*e - 11*a*b*c*e)
)/(2*a*(a*b^2 - 4*a^2*c)))/(x^2*(10*c*d^3*e^2*f^2 + 3*b*d*e^2*f^2) + x*(a*e
*f^2 + 3*b*d^2*e*f^2 + 5*c*d^4*e*f^2) + x^3*(b*e^3*f^2 + 10*c*d^2*e^3*f^2)
+ b*d^3*f^2 + c*d^5*f^2 + a*d*f^2 + c*e^5*f^2*x^5 + 5*c*d*e^4*f^2*x^4)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*f*x+d*f)**2/(a+b*(e*x+d)**2+c*(e*x+d)**4)**2,x)
```

```
[Out] Timed out
```

3.539 $\int \frac{1}{(df+efx)^3\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx$

Optimal. Leaf size=228

$$\frac{b\log\left(a+b(d+ex)^2+c(d+ex)^4\right)}{2a^3ef^3}-\frac{2b\log(d+ex)}{a^3ef^3}-\frac{b^2-3ac}{a^2ef^3\left(b^2-4ac\right)(d+ex)^2}-\frac{\left(6a^2c^2-6ab^2c+b^4\right)\tanh^{-1}\left(\right.}{a^3ef^3\left(b^2-4ac\right)^{3/2}}$$

Rubi [A] time = 0.37, antiderivative size = 228, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.242, Rules used = {1142, 1114, 740, 800, 634, 618, 206, 628}

$$-\frac{\left(6a^2c^2-6ab^2c+b^4\right)\tanh^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{a^3ef^3\left(b^2-4ac\right)^{3/2}}-\frac{b^2-3ac}{a^2ef^3\left(b^2-4ac\right)(d+ex)^2}+\frac{b\log\left(a+b(d+ex)^2+c(d+ex)^4\right)}{2a^3ef^3}-\frac{2b\log(d+ex)}{a^3ef^3}+\frac{-2ac+b^2+bc(d+ex)^2}{2aef^3\left(b^2-4ac\right)(d+ex)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d*f + e*f*x)^3*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2),x]
```

```
[Out] -((b^2 - 3*a*c)/(a^2*(b^2 - 4*a*c)*e*f^3*(d + e*x)^2)) + (b^2 - 2*a*c + b*c
*(d + e*x)^2)/(2*a*(b^2 - 4*a*c)*e*f^3*(d + e*x)^2*(a + b*(d + e*x)^2 + c*(
d + e*x)^4)) - ((b^4 - 6*a*b^2*c + 6*a^2*c^2)*ArcTanh[(b + 2*c*(d + e*x)^2)
/Sqrt[b^2 - 4*a*c]])/(a^3*(b^2 - 4*a*c)^(3/2)*e*f^3) - (2*b*Log[d + e*x])/(
a^3*e*f^3) + (b*Log[a + b*(d + e*x)^2 + c*(d + e*x)^4])/(2*a^3*e*f^3)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 740

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e
)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e
^2)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d +
 e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p +
 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4
```

```
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p,
-1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 1142

```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Di
st[u^m/(Coefficient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p,
 x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

Rubi steps

$$\int \frac{1}{(df+efx)^3\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x^3\left(a+bx^2+cx^4\right)^2}\,dx,x,d+ex\right)}{ef^3}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{x^2\left(a+bx+cx^2\right)^2}\,dx,x,(d+ex)^2\right)}{2ef^3}$$

$$= \frac{b^2-2ac+bc(d+ex)^2}{2a\left(b^2-4ac\right)ef^3(d+ex)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)} - \text{Sul}$$

$$= \frac{b^2-2ac+bc(d+ex)^2}{2a\left(b^2-4ac\right)ef^3(d+ex)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)} - \text{Sul}$$

$$= -\frac{b^2-3ac}{a^2\left(b^2-4ac\right)ef^3(d+ex)^2} + \frac{b^2-2ac+bc(\cdots}{2a\left(b^2-4ac\right)ef^3(d+ex)^2\left(a+\cdots\right.}$$

$$= -\frac{b^2-3ac}{a^2\left(b^2-4ac\right)ef^3(d+ex)^2} + \frac{b^2-2ac+bc(\cdots}{2a\left(b^2-4ac\right)ef^3(d+ex)^2\left(a+\cdots\right.}$$

$$= -\frac{b^2-3ac}{a^2\left(b^2-4ac\right)ef^3(d+ex)^2} + \frac{b^2-2ac+bc(\cdots}{2a\left(b^2-4ac\right)ef^3(d+ex)^2\left(a+\cdots\right.}$$

$$= -\frac{b^2-3ac}{a^2\left(b^2-4ac\right)ef^3(d+ex)^2} + \frac{b^2-2ac+bc(\cdots}{2a\left(b^2-4ac\right)ef^3(d+ex)^2\left(a+\cdots\right.}$$

Mathematica [A] time = 0.52, size = 287, normalized size = 1.26

$$\frac{\frac{\left(6a^2c^2-6ab^2c-4abc\sqrt{b^2-4ac}+b^3\sqrt{b^2-4ac}+b^4\right)\log\left(-\sqrt{b^2-4ac}+b+2c(d+ex)^2\right)}{\left(b^2-4ac\right)^{3/2}}+\frac{\left(-6a^2c^2+6ab^2c-4abc\sqrt{b^2-4ac}+b^3\sqrt{b^2-4ac}-b^4\right)\log\left(\sqrt{b^2-4ac}+b+2c(d+ex)^2\right)}{\left(b^2-4ac\right)^{3/2}}+\frac{a\left(-3abc-2ac^2(d+ex)^2+b^3+b^2c(d+ex)^2\right)}{\left(4ac-b^2\right)\left(a+b(d+ex)^2+c(d+ex)^4\right)}-\frac{a}{(d+ex)^2}-4b\log(d+ex)}{2a^3ef^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d*f + e*f*x)^3*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2),x]
```

```
[Out] (-(a/(d + e*x)^2) + (a*(b^3 - 3*a*b*c + b^2*c*(d + e*x)^2 - 2*a*c^2*(d + e*
x)^2))/((-b^2 + 4*a*c)*(a + b*(d + e*x)^2 + c*(d + e*x)^4)) - 4*b*Log[d + e
*x] + ((b^4 - 6*a*b^2*c + 6*a^2*c^2 + b^3*Sqrt[b^2 - 4*a*c] - 4*a*b*c*Sqrt[
b^2 - 4*a*c])*Log[b - Sqrt[b^2 - 4*a*c] + 2*c*(d + e*x)^2])/(b^2 - 4*a*c)^(
3/2) + ((-b^4 + 6*a*b^2*c - 6*a^2*c^2 + b^3*Sqrt[b^2 - 4*a*c] - 4*a*b*c*Sqr
t[b^2 - 4*a*c])*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*(d + e*x)^2])/(b^2 - 4*a*c)
^(3/2))/(2*a^3*e*f^3)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(df+efx)^3\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/((d*f + e*f*x)^3*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2
),x]
```

```
[Out] IntegrateAlgebraic[1/((d*f + e*f*x)^3*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2
), x]
```

fricas [B] time = 5.90, size = 4604, normalized size = 20.19

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*f*x+d*f)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="frica
s")
```

```
[Out] [-1/2*(2*(a*b^4*c - 7*a^2*b^2*c^2 + 12*a^3*c^3)*e^4*x^4 + 8*(a*b^4*c - 7*a^
2*b^2*c^2 + 12*a^3*c^3)*d*e^3*x^3 + a^2*b^4 - 8*a^3*b^2*c + 16*a^4*c^2 + 2*
(a*b^4*c - 7*a^2*b^2*c^2 + 12*a^3*c^3)*d^4 + (2*a*b^5 - 15*a^2*b^3*c + 28*a
^3*b*c^2 + 12*(a*b^4*c - 7*a^2*b^2*c^2 + 12*a^3*c^3)*d^2)*e^2*x^2 + (2*a*b^
5 - 15*a^2*b^3*c + 28*a^3*b*c^2)*d^2 + 2*(4*(a*b^4*c - 7*a^2*b^2*c^2 + 12*a
^3*c^3)*d^3 + (2*a*b^5 - 15*a^2*b^3*c + 28*a^3*b*c^2)*d)*e*x + ((b^4*c - 6*
a*b^2*c^2 + 6*a^2*c^3)*e^6*x^6 + 6*(b^4*c - 6*a*b^2*c^2 + 6*a^2*c^3)*d*e^5*
x^5 + (b^5 - 6*a*b^3*c + 6*a^2*b*c^2 + 15*(b^4*c - 6*a*b^2*c^2 + 6*a^2*c^3)
*d^2)*e^4*x^4 + (b^4*c - 6*a*b^2*c^2 + 6*a^2*c^3)*d^6 + 4*(5*(b^4*c - 6*a*b
^2*c^2 + 6*a^2*c^3)*d^3 + (b^5 - 6*a*b^3*c + 6*a^2*b*c^2)*d)*e^3*x^3 + (b^5
 - 6*a*b^3*c + 6*a^2*b*c^2)*d^4 + (a*b^4 - 6*a^2*b^2*c + 6*a^3*c^2 + 15*(b^
4*c - 6*a*b^2*c^2 + 6*a^2*c^3)*d^4 + 6*(b^5 - 6*a*b^3*c + 6*a^2*b*c^2)*d^2)
*e^2*x^2 + (a*b^4 - 6*a^2*b^2*c + 6*a^3*c^2)*d^2 + 2*(3*(b^4*c - 6*a*b^2*c^
2 + 6*a^2*c^3)*d^5 + 2*(b^5 - 6*a*b^3*c + 6*a^2*b*c^2)*d^3 + (a*b^4 - 6*a^2
*b^2*c + 6*a^3*c^2)*d)*e*x)*sqrt(b^2 - 4*a*c)*log((2*c^2*e^4*x^4 + 8*c^2*d*
e^3*x^3 + 2*c^2*d^4 + 2*(6*c^2*d^2 + b*c)*e^2*x^2 + 2*b*c*d^2 + 4*(2*c^2*d^
3 + b*c*d)*e*x + b^2 - 2*a*c + (2*c*e^2*x^2 + 4*c*d*e*x + 2*c*d^2 + b)*sqrt
(b^2 - 4*a*c))/(c*e^4*x^4 + 4*c*d*e^3*x^3 + c*d^4 + (6*c*d^2 + b)*e^2*x^2 +
 b*d^2 + 2*(2*c*d^3 + b*d)*e*x + a)) - ((b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3
)*e^6*x^6 + 6*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d*e^5*x^5 + (b^6 - 8*a*b
^4*c + 16*a^2*b^2*c^2 + 15*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^2)*e^4*x^
4 + (b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^6 + 4*(5*(b^5*c - 8*a*b^3*c^2 +
16*a^2*b*c^3)*d^3 + (b^6 - 8*a*b^4*c + 16*a^2*b^2*c^2)*d)*e^3*x^3 + (b^6 -
```

```
8*a*b^4*c + 16*a^2*b^2*c^2)*d^4 + (a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2 + 15*
(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^4 + 6*(b^6 - 8*a*b^4*c + 16*a^2*b^2*
c^2)*d^2)*e^2*x^2 + (a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*d^2 + 2*(3*(b^5*c
- 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^5 + 2*(b^6 - 8*a*b^4*c + 16*a^2*b^2*c^2)*d^
3 + (a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*d)*e*x)*log(c*e^4*x^4 + 4*c*d*e^3*
x^3 + c*d^4 + (6*c*d^2 + b)*e^2*x^2 + b*d^2 + 2*(2*c*d^3 + b*d)*e*x + a) +
4*((b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*e^6*x^6 + 6*(b^5*c - 8*a*b^3*c^2 +
16*a^2*b*c^3)*d*e^5*x^5 + (b^6 - 8*a*b^4*c + 16*a^2*b^2*c^2 + 15*(b^5*c - 8
*a*b^3*c^2 + 16*a^2*b*c^3)*d^2)*e^4*x^4 + (b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c
^3)*d^6 + 4*(5*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^3 + (b^6 - 8*a*b^4*c
+ 16*a^2*b^2*c^2)*d)*e^3*x^3 + (b^6 - 8*a*b^4*c + 16*a^2*b^2*c^2)*d^4 + (a*
b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2 + 15*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*
d^4 + 6*(b^6 - 8*a*b^4*c + 16*a^2*b^2*c^2)*d^2)*e^2*x^2 + (a*b^5 - 8*a^2*b^
3*c + 16*a^3*b*c^2)*d^2 + 2*(3*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^5 + 2
*(b^6 - 8*a*b^4*c + 16*a^2*b^2*c^2)*d^3 + (a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c
^2)*d)*e*x)*log(e*x + d))/((a^3*b^4*c - 8*a^4*b^2*c^2 + 16*a^5*c^3)*e^7*f^3
*x^6 + 6*(a^3*b^4*c - 8*a^4*b^2*c^2 + 16*a^5*c^3)*d*e^6*f^3*x^5 + (a^3*b^5
- 8*a^4*b^3*c + 16*a^5*b*c^2 + 15*(a^3*b^4*c - 8*a^4*b^2*c^2 + 16*a^5*c^3)*
d^2)*e^5*f^3*x^4 + 4*(5*(a^3*b^4*c - 8*a^4*b^2*c^2 + 16*a^5*c^3)*d^3 + (a^3
*b^5 - 8*a^4*b^3*c + 16*a^5*b*c^2)*d)*e^4*f^3*x^3 + (a^4*b^4 - 8*a^5*b^2*c
+ 16*a^6*c^2 + 15*(a^3*b^4*c - 8*a^4*b^2*c^2 + 16*a^5*c^3)*d^4 + 6*(a^3*b^5
 - 8*a^4*b^3*c + 16*a^5*b*c^2)*d^2)*e^3*f^3*x^2 + 2*(3*(a^3*b^4*c - 8*a^4*b
^2*c^2 + 16*a^5*c^3)*d^5 + 2*(a^3*b^5 - 8*a^4*b^3*c + 16*a^5*b*c^2)*d^3 + (
a^4*b^4 - 8*a^5*b^2*c + 16*a^6*c^2)*d)*e^2*f^3*x + ((a^3*b^4*c - 8*a^4*b^2*
c^2 + 16*a^5*c^3)*d^6 + (a^3*b^5 - 8*a^4*b^3*c + 16*a^5*b*c^2)*d^4 + (a^4*b
^4 - 8*a^5*b^2*c + 16*a^6*c^2)*d^2)*e*f^3), -1/2*(2*(a*b^4*c - 7*a^2*b^2*c^
2 + 12*a^3*c^3)*e^4*x^4 + 8*(a*b^4*c - 7*a^2*b^2*c^2 + 12*a^3*c^3)*d*e^3*x^
3 + a^2*b^4 - 8*a^3*b^2*c + 16*a^4*c^2 + 2*(a*b^4*c - 7*a^2*b^2*c^2 + 12*a^
3*c^3)*d^4 + (2*a*b^5 - 15*a^2*b^3*c + 28*a^3*b*c^2 + 12*(a*b^4*c - 7*a^2*b
^2*c^2 + 12*a^3*c^3)*d^2)*e^2*x^2 + (2*a*b^5 - 15*a^2*b^3*c + 28*a^3*b*c^2)
*d^2 + 2*(4*(a*b^4*c - 7*a^2*b^2*c^2 + 12*a^3*c^3)*d^3 + (2*a*b^5 - 15*a^2*
b^3*c + 28*a^3*b*c^2)*d)*e*x + 2*((b^4*c - 6*a*b^2*c^2 + 6*a^2*c^3)*e^6*x^6
 + 6*(b^4*c - 6*a*b^2*c^2 + 6*a^2*c^3)*d*e^5*x^5 + (b^5 - 6*a*b^3*c + 6*a^2
*b*c^2 + 15*(b^4*c - 6*a*b^2*c^2 + 6*a^2*c^3)*d^2)*e^4*x^4 + (b^4*c - 6*a*b
^2*c^2 + 6*a^2*c^3)*d^6 + 4*(5*(b^4*c - 6*a*b^2*c^2 + 6*a^2*c^3)*d^3 + (b^5
 - 6*a*b^3*c + 6*a^2*b*c^2)*d)*e^3*x^3 + (b^5 - 6*a*b^3*c + 6*a^2*b*c^2)*d^
4 + (a*b^4 - 6*a^2*b^2*c + 6*a^3*c^2 + 15*(b^4*c - 6*a*b^2*c^2 + 6*a^2*c^3)
*d^4 + 6*(b^5 - 6*a*b^3*c + 6*a^2*b*c^2)*d^2)*e^2*x^2 + (a*b^4 - 6*a^2*b^2*
c + 6*a^3*c^2)*d^2 + 2*(3*(b^4*c - 6*a*b^2*c^2 + 6*a^2*c^3)*d^5 + 2*(b^5 -
6*a*b^3*c + 6*a^2*b*c^2)*d^3 + (a*b^4 - 6*a^2*b^2*c + 6*a^3*c^2)*d)*e*x)*sq
rt(-b^2 + 4*a*c)*arctan(-(2*c*e^2*x^2 + 4*c*d*e*x + 2*c*d^2 + b)*sqrt(-b^2
+ 4*a*c)/(b^2 - 4*a*c)) - ((b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*e^6*x^6 + 6
*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d*e^5*x^5 + (b^6 - 8*a*b^4*c + 16*a^2
*b^2*c^2 + 15*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^2)*e^4*x^4 + (b^5*c -
8*a*b^3*c^2 + 16*a^2*b*c^3)*d^6 + 4*(5*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)
*d^3 + (b^6 - 8*a*b^4*c + 16*a^2*b^2*c^2)*d)*e^3*x^3 + (b^6 - 8*a*b^4*c + 1
6*a^2*b^2*c^2)*d^4 + (a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2 + 15*(b^5*c - 8*a*
b^3*c^2 + 16*a^2*b*c^3)*d^4 + 6*(b^6 - 8*a*b^4*c + 16*a^2*b^2*c^2)*d^2)*e^2
*x^2 + (a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*d^2 + 2*(3*(b^5*c - 8*a*b^3*c^2
 + 16*a^2*b*c^3)*d^5 + 2*(b^6 - 8*a*b^4*c + 16*a^2*b^2*c^2)*d^3 + (a*b^5 -
8*a^2*b^3*c + 16*a^3*b*c^2)*d)*e*x)*log(c*e^4*x^4 + 4*c*d*e^3*x^3 + c*d^4 +
 (6*c*d^2 + b)*e^2*x^2 + b*d^2 + 2*(2*c*d^3 + b*d)*e*x + a) + 4*((b^5*c - 8
*a*b^3*c^2 + 16*a^2*b*c^3)*e^6*x^6 + 6*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)
*d*e^5*x^5 + (b^6 - 8*a*b^4*c + 16*a^2*b^2*c^2 + 15*(b^5*c - 8*a*b^3*c^2 +
16*a^2*b*c^3)*d^2)*e^4*x^4 + (b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^6 + 4*(
5*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^3 + (b^6 - 8*a*b^4*c + 16*a^2*b^2*
c^2)*d)*e^3*x^3 + (b^6 - 8*a*b^4*c + 16*a^2*b^2*c^2)*d^4 + (a*b^5 - 8*a^2*b
^3*c + 16*a^3*b*c^2 + 15*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^4 + 6*(b^6
- 8*a*b^4*c + 16*a^2*b^2*c^2)*d^2)*e^2*x^2 + (a*b^5 - 8*a^2*b^3*c + 16*a^3*
```

```
b*c^2)*d^2 + 2*(3*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^5 + 2*(b^6 - 8*a*b
^4*c + 16*a^2*b^2*c^2)*d^3 + (a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*d)*e*x)*l
og(e*x + d))/((a^3*b^4*c - 8*a^4*b^2*c^2 + 16*a^5*c^3)*e^7*f^3*x^6 + 6*(a^3
*b^4*c - 8*a^4*b^2*c^2 + 16*a^5*c^3)*d*e^6*f^3*x^5 + (a^3*b^5 - 8*a^4*b^3*c
 + 16*a^5*b*c^2 + 15*(a^3*b^4*c - 8*a^4*b^2*c^2 + 16*a^5*c^3)*d^2)*e^5*f^3*
x^4 + 4*(5*(a^3*b^4*c - 8*a^4*b^2*c^2 + 16*a^5*c^3)*d^3 + (a^3*b^5 - 8*a^4*
b^3*c + 16*a^5*b*c^2)*d)*e^4*f^3*x^3 + (a^4*b^4 - 8*a^5*b^2*c + 16*a^6*c^2
+ 15*(a^3*b^4*c - 8*a^4*b^2*c^2 + 16*a^5*c^3)*d^4 + 6*(a^3*b^5 - 8*a^4*b^3*
c + 16*a^5*b*c^2)*d^2)*e^3*f^3*x^2 + 2*(3*(a^3*b^4*c - 8*a^4*b^2*c^2 + 16*a
^5*c^3)*d^5 + 2*(a^3*b^5 - 8*a^4*b^3*c + 16*a^5*b*c^2)*d^3 + (a^4*b^4 - 8*a
^5*b^2*c + 16*a^6*c^2)*d)*e^2*f^3*x + ((a^3*b^4*c - 8*a^4*b^2*c^2 + 16*a^5*
c^3)*d^6 + (a^3*b^5 - 8*a^4*b^3*c + 16*a^5*b*c^2)*d^4 + (a^4*b^4 - 8*a^5*b^
2*c + 16*a^6*c^2)*d^2)*e*f^3)]
```

giac [B] time = 1.30, size = 687, normalized size = 3.01

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*f*x+d*f)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="giac"
)
```

```
[Out] 1/2*((a^3*b^4*c*f^3*e^3 - 6*a^4*b^2*c^2*f^3*e^3 + 6*a^5*c^3*f^3*e^3)*sqrt(b
^2 - 4*a*c)*log(abs(b*x^2*e^2 + 2*b*d*x*e + sqrt(b^2 - 4*a*c)*x^2*e^2 + 2*s
qrt(b^2 - 4*a*c)*d*x*e + b*d^2 + sqrt(b^2 - 4*a*c)*d^2 + 2*a)) - (a^3*b^4*c
*f^3*e^3 - 6*a^4*b^2*c^2*f^3*e^3 + 6*a^5*c^3*f^3*e^3)*sqrt(b^2 - 4*a*c)*log
(abs(-b*x^2*e^2 - 2*b*d*x*e + sqrt(b^2 - 4*a*c)*x^2*e^2 + 2*sqrt(b^2 - 4*a*
c)*d*x*e - b*d^2 + sqrt(b^2 - 4*a*c)*d^2 - 2*a)))/(a^6*b^4*c*f^6*e^4 - 8*a^
7*b^2*c^2*f^6*e^4 + 16*a^8*c^3*f^6*e^4) + 1/2*b*e^(-1)*log(abs(c*x^4*e^4 +
4*c*d*x^3*e^3 + 6*c*d^2*x^2*e^2 + 4*c*d^3*x*e + c*d^4 + b*x^2*e^2 + 2*b*d*x
*e + b*d^2 + a))/(a^3*f^3) - 2*b*e^(-1)*log(abs(x*e + d))/(a^3*f^3) - 1/2*(
2*a*b^2*c*d^4 - 6*a^2*c^2*d^4 + 2*a*b^3*d^2 - 7*a^2*b*c*d^2 + 2*(a*b^2*c*e^
4 - 3*a^2*c^2*e^4)*x^4 + a^2*b^2 - 4*a^3*c + 8*(a*b^2*c*d*e^3 - 3*a^2*c^2*d
*e^3)*x^3 + (12*a*b^2*c*d^2*e^2 - 36*a^2*c^2*d^2*e^2 + 2*a*b^3*e^2 - 7*a^2*
b*c*e^2)*x^2 + 2*(4*a*b^2*c*d^3*e - 12*a^2*c^2*d^3*e + 2*a*b^3*d*e - 7*a^2*
b*c*d*e)*x)*e^(-1)/((c*x^4*e^4 + 4*c*d*x^3*e^3 + 6*c*d^2*x^2*e^2 + 4*c*d^3*
x*e + c*d^4 + b*x^2*e^2 + 2*b*d*x*e + b*d^2 + a)*(b^2 - 4*a*c)*(x*e + d)^2*
a^3*f^3)
```

maple [C] time = 0.03, size = 1047, normalized size = 4.59

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*f*x+d*f)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x)
```

```
[Out] -1/f^3/a/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d
^4+2*b*d*e*x+b*d^2+a)*c^2*e/(4*a*c-b^2)*x^2+1/2/f^3/a^2/(c*e^4*x^4+4*c*d*e^
3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)*c*e/(4
*a*c-b^2)*x^2*b^2-2/f^3/a/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*
e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)*c^2*d/(4*a*c-b^2)*x+1/f^3/a^2/(c*e^4
*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*
d^2+a)*c*d/(4*a*c-b^2)*x*b^2-1/f^3/a/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x
^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)/e/(4*a*c-b^2)*c^2*d^2+1/2
/f^3/a^2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d
^4+2*b*d*e*x+b*d^2+a)/e/(4*a*c-b^2)*b^2*c*d^2-3/2/f^3/a/(c*e^4*x^4+4*c*d*e^
3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)/e/(4*a
*c-b^2)*b*c+1/2/f^3/a^2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*
x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)/e/(4*a*c-b^2)*b^3+1/f^3/a^3/(4*a*c-b^2
)/e*sum(((4*a*c-b^2)*_R^3*b*c*e^3+3*(4*a*c-b^2)*_R^2*b*c*d*e^2+4*a*b*c^2*d^
```

```
3-b^3*c*d^3-3*a^2*c^2*d+5*a*b^2*c*d-b^4*d+(12*a*b*c^2*d^2-3*b^3*c*d^2-3*a^2
*c^2+5*a*b^2*c-b^4)*_R*e)/(2*_R^3*c*e^3+6*_R^2*c*d*e^2+6*_R*c*d^2*e+2*c*d^3
+_R*b*e+b*d)*ln(-_R+x),_R=RootOf(_Z^4*c*e^4+4*_Z^3*c*d*e^3+c*d^4+b*d^2+(6*c
*d^2*e^2+b*e^2)*_Z^2+(4*c*d^3*e+2*b*d*e)*_Z+a))-1/2/f^3/a^2/e/(e*x+d)^2-2*b
*ln(e*x+d)/a^3/e/f^3
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*f*x+d*f)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="maxim
a")
```

```
[Out] Timed out
```

mupad [B] time = 13.52, size = 14830, normalized size = 65.04

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((d*f + e*f*x)^3*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2),x)
```

```
[Out] ((x*(2*b^3*d - 12*a*c^2*d^3 + 4*b^2*c*d^3 - 7*a*b*c*d))/(4*a^3*c - a^2*b^2)
 - (x^4*(3*a*c^2*e^3 - b^2*c*e^3))/(4*a^3*c - a^2*b^2) - (4*x^3*(3*a*c^2*d*
e^2 - b^2*c*d*e^2))/(4*a^3*c - a^2*b^2) + (a*b^2 - 4*a^2*c + 2*b^3*d^2 - 6*
a*c^2*d^4 + 2*b^2*c*d^4 - 7*a*b*c*d^2)/(2*e*(4*a^3*c - a^2*b^2)) + (x^2*(2*
b^3*e - 36*a*c^2*d^2*e + 12*b^2*c*d^2*e - 7*a*b*c*e))/(2*(4*a^3*c - a^2*b^2
)))/(x^3*(20*c*d^3*e^3*f^3 + 4*b*d*e^3*f^3) + x*(2*a*d*e*f^3 + 4*b*d^3*e*f^
3 + 6*c*d^5*e*f^3) + x^4*(b*e^4*f^3 + 15*c*d^2*e^4*f^3) + x^2*(a*e^2*f^3 +
6*b*d^2*e^2*f^3 + 15*c*d^4*e^2*f^3) + a*d^2*f^3 + b*d^4*f^3 + c*d^6*f^3 + c
*e^6*f^3*x^6 + 6*c*d*e^5*f^3*x^5) + (log((((b + a^3*e*f^3*(-(b^4 + 6*a^2*c^
2 - 6*a*b^2*c)^2/(a^6*e^2*f^6*(4*a*c - b^2)^3))^(1/2))*(((b + a^3*e*f^3*(-(
b^4 + 6*a^2*c^2 - 6*a*b^2*c)^2/(a^6*e^2*f^6*(4*a*c - b^2)^3))^(1/2))*((4*c^
2*e^16*(2*b^5 + 6*a^2*b*c^2 + b^4*c*d^2 - 30*a^2*c^3*d^2 - 10*a*b^3*c + 2*a
*b^2*c^2*d^2))/(a^2*f^3*(4*a*c - b^2)) + (4*c^3*e^18*x^2*(b^4 - 30*a^2*c^2
+ 2*a*b^2*c))/(a^2*f^3*(4*a*c - b^2)) - (2*b*c^2*e^16*(b + a^3*e*f^3*(-(b^4
 + 6*a^2*c^2 - 6*a*b^2*c)^2/(a^6*e^2*f^6*(4*a*c - b^2)^3))^(1/2))*(a*b + 3*
b^2*d^2 + 3*b^2*e^2*x^2 - 10*a*c*d^2 + 6*b^2*d*e*x - 10*a*c*e^2*x^2 - 20*a*
c*d*e*x))/(a^3*f^3) + (8*c^3*d*e^17*x*(b^4 - 30*a^2*c^2 + 2*a*b^2*c))/(a^2*
f^3*(4*a*c - b^2))))/(2*a^3*e*f^3) - (4*c^3*e^15*(3*a*c - b^2)*(4*b^4 + 3*a
^2*c^2 + 6*b^3*c*d^2 - 17*a*b^2*c - 23*a*b*c^2*d^2))/(a^4*f^6*(4*a*c - b^2)
^2) + (4*b*c^4*e^17*x^2*(6*b^4 + 69*a^2*c^2 - 41*a*b^2*c))/(a^4*f^6*(4*a*c
- b^2)^2) + (8*b*c^4*d*e^16*x*(6*b^4 + 69*a^2*c^2 - 41*a*b^2*c))/(a^4*f^6*(
4*a*c - b^2)^2)))/(2*a^3*e*f^3) - (8*c^5*e^16*x^2*(3*a*c - b^2)^3)/(a^6*f^9
*(4*a*c - b^2)^3) + (8*c^4*e^14*(3*a*c - b^2)^2*(b^3 - 3*a*c^2*d^2 + b^2*c*
d^2 - 4*a*b*c))/(a^6*f^9*(4*a*c - b^2)^3) - (16*c^5*d*e^15*x*(3*a*c - b^2)^
3)/(a^6*f^9*(4*a*c - b^2)^3))*(((b - a^3*e*f^3*(-(b^4 + 6*a^2*c^2 - 6*a*b^2
*c)^2/(a^6*e^2*f^6*(4*a*c - b^2)^3))^(1/2))*(((b - a^3*e*f^3*(-(b^4 + 6*a^2
*c^2 - 6*a*b^2*c)^2/(a^6*e^2*f^6*(4*a*c - b^2)^3))^(1/2))*((4*c^2*e^16*(2*b
^5 + 6*a^2*b*c^2 + b^4*c*d^2 - 30*a^2*c^3*d^2 - 10*a*b^3*c + 2*a*b^2*c^2*d^
2))/(a^2*f^3*(4*a*c - b^2)) + (4*c^3*e^18*x^2*(b^4 - 30*a^2*c^2 + 2*a*b^2*c
))/(a^2*f^3*(4*a*c - b^2)) - (2*b*c^2*e^16*(b - a^3*e*f^3*(-(b^4 + 6*a^2*c^
2 - 6*a*b^2*c)^2/(a^6*e^2*f^6*(4*a*c - b^2)^3))^(1/2))*(a*b + 3*b^2*d^2 + 3
*b^2*e^2*x^2 - 10*a*c*d^2 + 6*b^2*d*e*x - 10*a*c*e^2*x^2 - 20*a*c*d*e*x))/(
a^3*f^3) + (8*c^3*d*e^17*x*(b^4 - 30*a^2*c^2 + 2*a*b^2*c))/(a^2*f^3*(4*a*c
- b^2))))/(2*a^3*e*f^3) - (4*c^3*e^15*(3*a*c - b^2)*(4*b^4 + 3*a^2*c^2 + 6*
b^3*c*d^2 - 17*a*b^2*c - 23*a*b*c^2*d^2))/(a^4*f^6*(4*a*c - b^2)^2) + (4*b*
c^4*e^17*x^2*(6*b^4 + 69*a^2*c^2 - 41*a*b^2*c))/(a^4*f^6*(4*a*c - b^2)^2) +
 (8*b*c^4*d*e^16*x*(6*b^4 + 69*a^2*c^2 - 41*a*b^2*c))/(a^4*f^6*(4*a*c - b^2
)^2)))/(2*a^3*e*f^3) - (8*c^5*e^16*x^2*(3*a*c - b^2)^3)/(a^6*f^9*(4*a*c - b
```

```
^2)^3) + (8*c^4*e^14*(3*a*c - b^2)^2*(b^3 - 3*a*c^2*d^2 + b^2*c*d^2 - 4*a*b
*c))/(a^6*f^9*(4*a*c - b^2)^3) - (16*c^5*d*e^15*x*(3*a*c - b^2)^3)/(a^6*f^9
*(4*a*c - b^2)^3)))*(b^7*e*f^3 - 12*a*b^5*c*e*f^3 - 64*a^3*b*c^3*e*f^3 + 48
*a^2*b^3*c^2*e*f^3))/(2*(a^3*b^6*e^2*f^6 - 64*a^6*c^3*e^2*f^6 + 48*a^5*b^2*
c^2*e^2*f^6 - 12*a^4*b^4*c*e^2*f^6)) - (2*b*log(d + e*x))/(a^3*e*f^3) - (at
an(((2*a^9*b^6*f^9*(4*a*c - b^2)^(9/2) - 128*a^12*c^3*f^9*(4*a*c - b^2)^(9/
2) - 24*a^10*b^4*c*f^9*(4*a*c - b^2)^(9/2) + 96*a^11*b^2*c^2*f^9*(4*a*c - b
^2)^(9/2))*(x*((((8*(54*a^3*c^8*d*e^15 - 2*b^6*c^5*d*e^15 + 18*a*b^4*c^6*d*
e^15 - 54*a^2*b^2*c^7*d*e^15))/(a^6*b^6*f^9 - 64*a^9*c^3*f^9 - 12*a^7*b^4*c
*f^9 + 48*a^8*b^2*c^2*f^9) - (((8*(276*a^5*b*c^7*d*e^16*f^3 - 6*a^2*b^7*c^4
*d*e^16*f^3 + 65*a^3*b^5*c^5*d*e^16*f^3 - 233*a^4*b^3*c^6*d*e^16*f^3))/(a^6
*b^6*f^9 - 64*a^9*c^3*f^9 - 12*a^7*b^4*c*f^9 + 48*a^8*b^2*c^2*f^9) - (((8*(
480*a^8*c^7*d*e^17*f^6 - a^4*b^8*c^3*d*e^17*f^6 + 6*a^5*b^6*c^4*d*e^17*f^6
+ 30*a^6*b^4*c^5*d*e^17*f^6 - 272*a^7*b^2*c^6*d*e^17*f^6))/(a^6*b^6*f^9 - 6
4*a^9*c^3*f^9 - 12*a^7*b^4*c*f^9 + 48*a^8*b^2*c^2*f^9) - (4*(b^7*e*f^3 - 12
*a*b^5*c*e*f^3 - 64*a^3*b*c^3*e*f^3 + 48*a^2*b^3*c^2*e*f^3)*(640*a^10*b*c^6
*d*e^18*f^9 + 3*a^6*b^9*c^2*d*e^18*f^9 - 46*a^7*b^7*c^3*d*e^18*f^9 + 264*a^
8*b^5*c^4*d*e^18*f^9 - 672*a^9*b^3*c^5*d*e^18*f^9))/((a^6*b^6*f^9 - 64*a^9*
c^3*f^9 - 12*a^7*b^4*c*f^9 + 48*a^8*b^2*c^2*f^9)*(a^3*b^6*e^2*f^6 - 64*a^6*
c^3*e^2*f^6 + 48*a^5*b^2*c^2*e^2*f^6 - 12*a^4*b^4*c*e^2*f^6)))*(b^7*e*f^3 -
 12*a*b^5*c*e*f^3 - 64*a^3*b*c^3*e*f^3 + 48*a^2*b^3*c^2*e*f^3))/(2*(a^3*b^6
*e^2*f^6 - 64*a^6*c^3*e^2*f^6 + 48*a^5*b^2*c^2*e^2*f^6 - 12*a^4*b^4*c*e^2*f
^6)))*(b^7*e*f^3 - 12*a*b^5*c*e*f^3 - 64*a^3*b*c^3*e*f^3 + 48*a^2*b^3*c^2*e
*f^3))/(2*(a^3*b^6*e^2*f^6 - 64*a^6*c^3*e^2*f^6 + 48*a^5*b^2*c^2*e^2*f^6 -
12*a^4*b^4*c*e^2*f^6)) - (((((8*(480*a^8*c^7*d*e^17*f^6 - a^4*b^8*c^3*d*e^1
7*f^6 + 6*a^5*b^6*c^4*d*e^17*f^6 + 30*a^6*b^4*c^5*d*e^17*f^6 - 272*a^7*b^2*
c^6*d*e^17*f^6))/(a^6*b^6*f^9 - 64*a^9*c^3*f^9 - 12*a^7*b^4*c*f^9 + 48*a^8*
b^2*c^2*f^9) - (4*(b^7*e*f^3 - 12*a*b^5*c*e*f^3 - 64*a^3*b*c^3*e*f^3 + 48*a
^2*b^3*c^2*e*f^3)*(640*a^10*b*c^6*d*e^18*f^9 + 3*a^6*b^9*c^2*d*e^18*f^9 - 4
6*a^7*b^7*c^3*d*e^18*f^9 + 264*a^8*b^5*c^4*d*e^18*f^9 - 672*a^9*b^3*c^5*d*e
^18*f^9))/((a^6*b^6*f^9 - 64*a^9*c^3*f^9 - 12*a^7*b^4*c*f^9 + 48*a^8*b^2*c^
2*f^9)*(a^3*b^6*e^2*f^6 - 64*a^6*c^3*e^2*f^6 + 48*a^5*b^2*c^2*e^2*f^6 - 12*
a^4*b^4*c*e^2*f^6)))*(b^4 + 6*a^2*c^2 - 6*a*b^2*c))/(2*a^3*e*f^3*(4*a*c - b
^2)^(3/2)) - (2*(b^4 + 6*a^2*c^2 - 6*a*b^2*c)*(b^7*e*f^3 - 12*a*b^5*c*e*f^3
 - 64*a^3*b*c^3*e*f^3 + 48*a^2*b^3*c^2*e*f^3)*(640*a^10*b*c^6*d*e^18*f^9 +
3*a^6*b^9*c^2*d*e^18*f^9 - 46*a^7*b^7*c^3*d*e^18*f^9 + 264*a^8*b^5*c^4*d*e^
18*f^9 - 672*a^9*b^3*c^5*d*e^18*f^9))/(a^3*e*f^3*(4*a*c - b^2)^(3/2)*(a^6*b
^6*f^9 - 64*a^9*c^3*f^9 - 12*a^7*b^4*c*f^9 + 48*a^8*b^2*c^2*f^9)*(a^3*b^6*e
^2*f^6 - 64*a^6*c^3*e^2*f^6 + 48*a^5*b^2*c^2*e^2*f^6 - 12*a^4*b^4*c*e^2*f^6
)))*(b^4 + 6*a^2*c^2 - 6*a*b^2*c))/(2*a^3*e*f^3*(4*a*c - b^2)^(3/2)) + ((b^
4 + 6*a^2*c^2 - 6*a*b^2*c)^2*(b^7*e*f^3 - 12*a*b^5*c*e*f^3 - 64*a^3*b*c^3*e
*f^3 + 48*a^2*b^3*c^2*e*f^3)*(640*a^10*b*c^6*d*e^18*f^9 + 3*a^6*b^9*c^2*d*e
^18*f^9 - 46*a^7*b^7*c^3*d*e^18*f^9 + 264*a^8*b^5*c^4*d*e^18*f^9 - 672*a^9*
b^3*c^5*d*e^18*f^9))/(a^6*e^2*f^6*(4*a*c - b^2)^3*(a^6*b^6*f^9 - 64*a^9*c^3
*f^9 - 12*a^7*b^4*c*f^9 + 48*a^8*b^2*c^2*f^9)*(a^3*b^6*e^2*f^6 - 64*a^6*c^3
*e^2*f^6 + 48*a^5*b^2*c^2*e^2*f^6 - 12*a^4*b^4*c*e^2*f^6)))*(3*b^6 - 3*a^3*
c^3 + 36*a^2*b^2*c^2 - 21*a*b^4*c))/(8*a^3*c^2*(4*a*c - b^2)^3*(9*a^4*c^4 -
 6*b^8 - 288*a^2*b^4*c^2 + 382*a^3*b^2*c^3 + 72*a*b^6*c)) - (b*((((((8*(480
*a^8*c^7*d*e^17*f^6 - a^4*b^8*c^3*d*e^17*f^6 + 6*a^5*b^6*c^4*d*e^17*f^6 + 3
0*a^6*b^4*c^5*d*e^17*f^6 - 272*a^7*b^2*c^6*d*e^17*f^6))/(a^6*b^6*f^9 - 64*a
^9*c^3*f^9 - 12*a^7*b^4*c*f^9 + 48*a^8*b^2*c^2*f^9) - (4*(b^7*e*f^3 - 12*a*
b^5*c*e*f^3 - 64*a^3*b*c^3*e*f^3 + 48*a^2*b^3*c^2*e*f^3)*(640*a^10*b*c^6*d*
e^18*f^9 + 3*a^6*b^9*c^2*d*e^18*f^9 - 46*a^7*b^7*c^3*d*e^18*f^9 + 264*a^8*b
^5*c^4*d*e^18*f^9 - 672*a^9*b^3*c^5*d*e^18*f^9))/((a^6*b^6*f^9 - 64*a^9*c^3
*f^9 - 12*a^7*b^4*c*f^9 + 48*a^8*b^2*c^2*f^9)*(a^3*b^6*e^2*f^6 - 64*a^6*c^3
*e^2*f^6 + 48*a^5*b^2*c^2*e^2*f^6 - 12*a^4*b^4*c*e^2*f^6)))*(b^4 + 6*a^2*c^
2 - 6*a*b^2*c))/(2*a^3*e*f^3*(4*a*c - b^2)^(3/2)) - (2*(b^4 + 6*a^2*c^2 - 6
*a*b^2*c)*(b^7*e*f^3 - 12*a*b^5*c*e*f^3 - 64*a^3*b*c^3*e*f^3 + 48*a^2*b^3*c
^2*e*f^3)*(640*a^10*b*c^6*d*e^18*f^9 + 3*a^6*b^9*c^2*d*e^18*f^9 - 46*a^7*b^
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7*c^3*d*e^18*f^9 + 264*a^8*b^5*c^4*d*e^18*f^9 - 672*a^9*b^3*c^5*d*e^18*f^9)
)/(a^3*e*f^3*(4*a*c - b^2)^(3/2)*(a^6*b^6*f^9 - 64*a^9*c^3*f^9 - 12*a^7*b^4
*c*f^9 + 48*a^8*b^2*c^2*f^9)*(a^3*b^6*e^2*f^6 - 64*a^6*c^3*e^2*f^6 + 48*a^5
*b^2*c^2*e^2*f^6 - 12*a^4*b^4*c*e^2*f^6)))*(b^7*e*f^3 - 12*a*b^5*c*e*f^3 -
64*a^3*b*c^3*e*f^3 + 48*a^2*b^3*c^2*e*f^3))/(2*(a^3*b^6*e^2*f^6 - 64*a^6*c^
3*e^2*f^6 + 48*a^5*b^2*c^2*e^2*f^6 - 12*a^4*b^4*c*e^2*f^6)) - (((8*(276*a^5
*b*c^7*d*e^16*f^3 - 6*a^2*b^7*c^4*d*e^16*f^3 + 65*a^3*b^5*c^5*d*e^16*f^3 -
233*a^4*b^3*c^6*d*e^16*f^3))/(a^6*b^6*f^9 - 64*a^9*c^3*f^9 - 12*a^7*b^4*c*f
^9 + 48*a^8*b^2*c^2*f^9) - (((8*(480*a^8*c^7*d*e^17*f^6 - a^4*b^8*c^3*d*e^1
7*f^6 + 6*a^5*b^6*c^4*d*e^17*f^6 + 30*a^6*b^4*c^5*d*e^17*f^6 - 272*a^7*b^2*
c^6*d*e^17*f^6))/(a^6*b^6*f^9 - 64*a^9*c^3*f^9 - 12*a^7*b^4*c*f^9 + 48*a^8*
b^2*c^2*f^9) - (4*(b^7*e*f^3 - 12*a*b^5*c*e*f^3 - 64*a^3*b*c^3*e*f^3 + 48*a
^2*b^3*c^2*e*f^3)*(640*a^10*b*c^6*d*e^18*f^9 + 3*a^6*b^9*c^2*d*e^18*f^9 - 4
6*a^7*b^7*c^3*d*e^18*f^9 + 264*a^8*b^5*c^4*d*e^18*f^9 - 672*a^9*b^3*c^5*d*e
^18*f^9))/((a^6*b^6*f^9 - 64*a^9*c^3*f^9 - 12*a^7*b^4*c*f^9 + 48*a^8*b^2*c^
2*f^9)*(a^3*b^6*e^2*f^6 - 64*a^6*c^3*e^2*f^6 + 48*a^5*b^2*c^2*e^2*f^6 - 12*
a^4*b^4*c*e^2*f^6)))*(b^7*e*f^3 - 12*a*b^5*c*e*f^3 - 64*a^3*b*c^3*e*f^3 + 4
8*a^2*b^3*c^2*e*f^3))/(2*(a^3*b^6*e^2*f^6 - 64*a^6*c^3*e^2*f^6 + 48*a^5*b^2
*c^2*e^2*f^6 - 12*a^4*b^4*c*e^2*f^6)))*(b^4 + 6*a^2*c^2 - 6*a*b^2*c))/(2*a^
3*e*f^3*(4*a*c - b^2)^(3/2)) + ((b^4 + 6*a^2*c^2 - 6*a*b^2*c)^3*(640*a^10*b
*c^6*d*e^18*f^9 + 3*a^6*b^9*c^2*d*e^18*f^9 - 46*a^7*b^7*c^3*d*e^18*f^9 + 26
4*a^8*b^5*c^4*d*e^18*f^9 - 672*a^9*b^3*c^5*d*e^18*f^9))/(a^9*e^3*f^9*(4*a*c
 - b^2)^(9/2)*(a^6*b^6*f^9 - 64*a^9*c^3*f^9 - 12*a^7*b^4*c*f^9 + 48*a^8*b^2
*c^2*f^9)))*(3*b^6 - 49*a^3*c^3 + 72*a^2*b^2*c^2 - 27*a*b^4*c))/(8*a^3*c^2*
(4*a*c - b^2)^(7/2)*(9*a^4*c^4 - 6*b^8 - 288*a^2*b^4*c^2 + 382*a^3*b^2*c^3
 + 72*a*b^6*c))) + x^2*((((4*(54*a^3*c^8*e^16 - 2*b^6*c^5*e^16 + 18*a*b^4*c^
6*e^16 - 54*a^2*b^2*c^7*e^16))/(a^6*b^6*f^9 - 64*a^9*c^3*f^9 - 12*a^7*b^4*c
*f^9 + 48*a^8*b^2*c^2*f^9) + (((4*(6*a^2*b^7*c^4*e^17*f^3 - 65*a^3*b^5*c^5*
e^17*f^3 + 233*a^4*b^3*c^6*e^17*f^3 - 276*a^5*b*c^7*e^17*f^3))/(a^6*b^6*f^9
 - 64*a^9*c^3*f^9 - 12*a^7*b^4*c*f^9 + 48*a^8*b^2*c^2*f^9) + (((4*(480*a^8*
c^7*e^18*f^6 - a^4*b^8*c^3*e^18*f^6 + 6*a^5*b^6*c^4*e^18*f^6 + 30*a^6*b^4*c
^5*e^18*f^6 - 272*a^7*b^2*c^6*e^18*f^6))/(a^6*b^6*f^9 - 64*a^9*c^3*f^9 - 12
*a^7*b^4*c*f^9 + 48*a^8*b^2*c^2*f^9) - (2*(b^7*e*f^3 - 12*a*b^5*c*e*f^3 - 6
4*a^3*b*c^3*e*f^3 + 48*a^2*b^3*c^2*e*f^3)*(3*a^6*b^9*c^2*e^19*f^9 - 46*a^7*
b^7*c^3*e^19*f^9 + 264*a^8*b^5*c^4*e^19*f^9 - 672*a^9*b^3*c^5*e^19*f^9 + 64
0*a^10*b*c^6*e^19*f^9))/((a^6*b^6*f^9 - 64*a^9*c^3*f^9 - 12*a^7*b^4*c*f^9 +
 48*a^8*b^2*c^2*f^9)*(a^3*b^6*e^2*f^6 - 64*a^6*c^3*e^2*f^6 + 48*a^5*b^2*c^2
*e^2*f^6 - 12*a^4*b^4*c*e^2*f^6)))*(b^7*e*f^3 - 12*a*b^5*c*e*f^3 - 64*a^3*b
*c^3*e*f^3 + 48*a^2*b^3*c^2*e*f^3))/(2*(a^3*b^6*e^2*f^6 - 64*a^6*c^3*e^2*f^
6 + 48*a^5*b^2*c^2*e^2*f^6 - 12*a^4*b^4*c*e^2*f^6)))*(b^7*e*f^3 - 12*a*b^5*
c*e*f^3 - 64*a^3*b*c^3*e*f^3 + 48*a^2*b^3*c^2*e*f^3))/(2*(a^3*b^6*e^2*f^6 -
 64*a^6*c^3*e^2*f^6 + 48*a^5*b^2*c^2*e^2*f^6 - 12*a^4*b^4*c*e^2*f^6)) - (((
((4*(480*a^8*c^7*e^18*f^6 - a^4*b^8*c^3*e^18*f^6 + 6*a^5*b^6*c^4*e^18*f^6 +
 30*a^6*b^4*c^5*e^18*f^6 - 272*a^7*b^2*c^6*e^18*f^6))/(a^6*b^6*f^9 - 64*a^9
*c^3*f^9 - 12*a^7*b^4*c*f^9 + 48*a^8*b^2*c^2*f^9) - (2*(b^7*e*f^3 - 12*a*b^
5*c*e*f^3 - 64*a^3*b*c^3*e*f^3 + 48*a^2*b^3*c^2*e*f^3)*(3*a^6*b^9*c^2*e^19*
f^9 - 46*a^7*b^7*c^3*e^19*f^9 + 264*a^8*b^5*c^4*e^19*f^9 - 672*a^9*b^3*c^5*
e^19*f^9 + 640*a^10*b*c^6*e^19*f^9))/((a^6*b^6*f^9 - 64*a^9*c^3*f^9 - 12*a^
7*b^4*c*f^9 + 48*a^8*b^2*c^2*f^9)*(a^3*b^6*e^2*f^6 - 64*a^6*c^3*e^2*f^6 + 4
8*a^5*b^2*c^2*e^2*f^6 - 12*a^4*b^4*c*e^2*f^6)))*(b^4 + 6*a^2*c^2 - 6*a*b^2*
c))/(2*a^3*e*f^3*(4*a*c - b^2)^(3/2)) - ((b^4 + 6*a^2*c^2 - 6*a*b^2*c)*(b^7
*e*f^3 - 12*a*b^5*c*e*f^3 - 64*a^3*b*c^3*e*f^3 + 48*a^2*b^3*c^2*e*f^3)*(3*a
^6*b^9*c^2*e^19*f^9 - 46*a^7*b^7*c^3*e^19*f^9 + 264*a^8*b^5*c^4*e^19*f^9 -
672*a^9*b^3*c^5*e^19*f^9 + 640*a^10*b*c^6*e^19*f^9))/(a^3*e*f^3*(4*a*c - b^
2)^(3/2)*(a^6*b^6*f^9 - 64*a^9*c^3*f^9 - 12*a^7*b^4*c*f^9 + 48*a^8*b^2*c^2*
f^9)*(a^3*b^6*e^2*f^6 - 64*a^6*c^3*e^2*f^6 + 48*a^5*b^2*c^2*e^2*f^6 - 12*a^
4*b^4*c*e^2*f^6)))*(b^4 + 6*a^2*c^2 - 6*a*b^2*c))/(2*a^3*e*f^3*(4*a*c - b^2
)^(3/2)) + ((b^4 + 6*a^2*c^2 - 6*a*b^2*c)^2*(b^7*e*f^3 - 12*a*b^5*c*e*f^3 -
 64*a^3*b*c^3*e*f^3 + 48*a^2*b^3*c^2*e*f^3)*(3*a^6*b^9*c^2*e^19*f^9 - 46*a^
```

7*b^7*c^3*e^19*f^9 + 264*a^8*b^5*c^4*e^19*f^9 - 672*a^9*b^3*c^5*e^19*f^9 + 640*a^10*b*c^6*e^19*f^9))/(2*a^6*e^2*f^6*(4*a*c - b^2)^3*(a^6*b^6*f^9 - 64*a^9*c^3*f^9 - 12*a^7*b^4*c*f^9 + 48*a^8*b^2*c^2*f^9)*(a^3*b^6*e^2*f^6 - 64*a^6*c^3*e^2*f^6 + 48*a^5*b^2*c^2*e^2*f^6 - 12*a^4*b^4*c*e^2*f^6)))*(3*b^6 - 3*a^3*c^3 + 36*a^2*b^2*c^2 - 21*a*b^4*c))/(8*a^3*c^2*(4*a*c - b^2)^3*(9*a^4*c^4 - 6*b^8 - 288*a^2*b^4*c^2 + 382*a^3*b^2*c^3 + 72*a*b^6*c)) - (b*(((((4*(480*a^8*c^7*e^18*f^6 - a^4*b^8*c^3*e^18*f^6 + 6*a^5*b^6*c^4*e^18*f^6 + 30*a^6*b^4*c^5*e^18*f^6 - 272*a^7*b^2*c^6*e^18*f^6))/(a^6*b^6*f^9 - 64*a^9*c^3*f^9 - 12*a^7*b^4*c*f^9 + 48*a^8*b^2*c^2*f^9) - (2*(b^7*e*f^3 - 12*a*b^5*c*e*f^3 - 64*a^3*b*c^3*e*f^3 + 48*a^2*b^3*c^2*e*f^3)*(3*a^6*b^9*c^2*e^19*f^9 - 46*a^7*b^7*c^3*e^19*f^9 + 264*a^8*b^5*c^4*e^19*f^9 - 672*a^9*b^3*c^5*e^19*f^9 + 640*a^10*b*c^6*e^19*f^9))/((a^6*b^6*f^9 - 64*a^9*c^3*f^9 - 12*a^7*b^4*c*f^9 + 48*a^8*b^2*c^2*f^9)*(a^3*b^6*e^2*f^6 - 64*a^6*c^3*e^2*f^6 + 48*a^5*b^2*c^2*e^2*f^6 - 12*a^4*b^4*c*e^2*f^6)))*(b^4 + 6*a^2*c^2 - 6*a*b^2*c))/(2*a^3*e*f^3*(4*a*c - b^2)^(3/2)) - ((b^4 + 6*a^2*c^2 - 6*a*b^2*c)*(b^7*e*f^3 - 12*a*b^5*c*e*f^3 - 64*a^3*b*c^3*e*f^3 + 48*a^2*b^3*c^2*e*f^3)*(3*a^6*b^9*c^2*e^19*f^9 - 46*a^7*b^7*c^3*e^19*f^9 + 264*a^8*b^5*c^4*e^19*f^9 - 672*a^9*b^3*c^5*e^19*f^9 + 640*a^10*b*c^6*e^19*f^9))/(a^3*e*f^3*(4*a*c - b^2)^(3/2)*(a^6*b^6*f^9 - 64*a^9*c^3*f^9 - 12*a^7*b^4*c*f^9 + 48*a^8*b^2*c^2*f^9)*(a^3*b^6*e^2*f^6 - 64*a^6*c^3*e^2*f^6 + 48*a^5*b^2*c^2*e^2*f^6 - 12*a^4*b^4*c*e^2*f^6)))*(b^7*e*f^3 - 12*a*b^5*c*e*f^3 - 64*a^3*b*c^3*e*f^3 + 48*a^2*b^3*c^2*e*f^3))/(2*(a^3*b^6*e^2*f^6 - 64*a^6*c^3*e^2*f^6 + 48*a^5*b^2*c^2*e^2*f^6 - 12*a^4*b^4*c*e^2*f^6)) + (((4*(6*a^2*b^7*c^4*e^17*f^3 - 65*a^3*b^5*c^5*e^17*f^3 + 233*a^4*b^3*c^6*e^17*f^3 - 276*a^5*b*c^7*e^17*f^3))/(a^6*b^6*f^9 - 64*a^9*c^3*f^9 - 12*a^7*b^4*c*f^9 + 48*a^8*b^2*c^2*f^9) + (((4*(480*a^8*c^7*e^18*f^6 - a^4*b^8*c^3*e^18*f^6 + 6*a^5*b^6*c^4*e^18*f^6 + 30*a^6*b^4*c^5*e^18*f^6 - 272*a^7*b^2*c^6*e^18*f^6))/(a^6*b^6*f^9 - 64*a^9*c^3*f^9 - 12*a^7*b^4*c*f^9 + 48*a^8*b^2*c^2*f^9) - (2*(b^7*e*f^3 - 12*a*b^5*c*e*f^3 - 64*a^3*b*c^3*e*f^3 + 48*a^2*b^3*c^2*e*f^3)*(3*a^6*b^9*c^2*e^19*f^9 - 46*a^7*b^7*c^3*e^19*f^9 + 264*a^8*b^5*c^4*e^19*f^9 - 672*a^9*b^3*c^5*e^19*f^9 + 640*a^10*b*c^6*e^19*f^9))/((a^6*b^6*f^9 - 64*a^9*c^3*f^9 - 12*a^7*b^4*c*f^9 + 48*a^8*b^2*c^2*f^9)*(a^3*b^6*e^2*f^6 - 64*a^6*c^3*e^2*f^6 + 48*a^5*b^2*c^2*e^2*f^6 - 12*a^4*b^4*c*e^2*f^6)))*(b^7*e*f^3 - 12*a*b^5*c*e*f^3 - 64*a^3*b*c^3*e*f^3 + 48*a^2*b^3*c^2*e*f^3))/(2*(a^3*b^6*e^2*f^6 - 64*a^6*c^3*e^2*f^6 + 48*a^5*b^2*c^2*e^2*f^6 - 12*a^4*b^4*c*e^2*f^6)))*(b^4 + 6*a^2*c^2 - 6*a*b^2*c))/(2*a^3*e*f^3*(4*a*c - b^2)^(3/2)) + ((b^4 + 6*a^2*c^2 - 6*a*b^2*c)^3*(3*a^6*b^9*c^2*e^19*f^9 - 46*a^7*b^7*c^3*e^19*f^9 + 264*a^8*b^5*c^4*e^19*f^9 - 672*a^9*b^3*c^5*e^19*f^9 + 640*a^10*b*c^6*e^19*f^9))/(2*a^9*e^3*f^9*(4*a*c - b^2)^(9/2)*(a^6*b^6*f^9 - 64*a^9*c^3*f^9 - 12*a^7*b^4*c*f^9 + 48*a^8*b^2*c^2*f^9)))*(3*b^6 - 49*a^3*c^3 + 72*a^2*b^2*c^2 - 27*a*b^4*c))/(8*a^3*c^2*(4*a*c - b^2)^(7/2)*(9*a^4*c^4 - 6*b^8 - 288*a^2*b^4*c^2 + 382*a^3*b^2*c^3 + 72*a*b^6*c))) + (((((4*(36*a^6*c^7*e^15*f^3 + 4*a^2*b^8*c^3*e^15*f^3 - 45*a^3*b^6*c^4*e^15*f^3 + 170*a^4*b^4*c^5*e^15*f^3 - 225*a^5*b^2*c^6*e^15*f^3 - 276*a^5*b*c^7*d^2*e^15*f^3 + 6*a^2*b^7*c^4*d^2*e^15*f^3 - 65*a^3*b^5*c^5*d^2*e^15*f^3 + 233*a^4*b^3*c^6*d^2*e^15*f^3))/(a^6*b^6*f^9 - 64*a^9*c^3*f^9 - 12*a^7*b^4*c*f^9 + 48*a^8*b^2*c^2*f^9) - (((4*(2*a^4*b^9*c^2*e^16*f^6 - 26*a^5*b^7*c^3*e^16*f^6 + 118*a^6*b^5*c^4*e^16*f^6 - 208*a^7*b^3*c^5*e^16*f^6 - 480*a^8*c^7*d^2*e^16*f^6 + 96*a^8*b*c^6*e^16*f^6 + a^4*b^8*c^3*d^2*e^16*f^6 - 6*a^5*b^6*c^4*d^2*e^16*f^6 - 30*a^6*b^4*c^5*d^2*e^16*f^6 + 272*a^7*b^2*c^6*d^2*e^16*f^6))/(a^6*b^6*f^9 - 64*a^9*c^3*f^9 - 12*a^7*b^4*c*f^9 + 48*a^8*b^2*c^2*f^9) + (2*(b^7*e*f^3 - 12*a*b^5*c*e*f^3 - 64*a^3*b*c^3*e*f^3 + 48*a^2*b^3*c^2*e*f^3)*(a^7*b^8*c^2*e^17*f^9 - 12*a^8*b^6*c^3*e^17*f^9 + 48*a^9*b^4*c^4*e^17*f^9 - 64*a^10*b^2*c^5*e^17*f^9 + 640*a^10*b*c^6*d^2*e^17*f^9 + 3*a^6*b^9*c^2*d^2*e^17*f^9 - 46*a^7*b^7*c^3*d^2*e^17*f^9 + 264*a^8*b^5*c^4*d^2*e^17*f^9 - 672*a^9*b^3*c^5*d^2*e^17*f^9))/((a^6*b^6*f^9 - 64*a^9*c^3*f^9 - 12*a^7*b^4*c*f^9 + 48*a^8*b^2*c^2*f^9)*(a^3*b^6*e^2*f^6 - 64*a^6*c^3*e^2*f^6 + 48*a^5*b^2*c^2*e^2*f^6 - 12*a^4*b^4*c*e^2*f^6)))*(b^7*e*f^3 - 12*a*b^5*c*e*f^3 - 64*a^3*b*c^3*e*f^3 + 48*a^2*b^3*c^2*e*f^3))/(2*(a^3*b^6*e^2*f^6 - 64*a^6*c^3*e^2*f^6 + 48*a^5*b^2*c^2*e^2*f^6 - 12 *a^4*b^4*c*e^2*f^6)))*(b^7*e*f^3 - 12*a*b^5*c*e*f^3 - 64*a^3*b*c^3*e*f^3 + 48*a^2*b^3*c^2*e*f^3))/(2*(a^3*b^6*e^2*f^6 - 64*a^6*c^3*e^2*f^6 + 48*a^5*b^2*c^2*e^2*f^6 - 12*a^4*b^4*c*e^2*f^6)) - (4*(2*b^7*c^4*e^14 - 20*a*b^5*c^5*e^14 - 72*a^3*b*c^7*e^14 + 66*a^2*b^3*c^6*e^14 - 54*a^3*c^8*d^2*e^14 + 2*b^6*c^5*d^2*e^14 + 54*a^2*b^2*c^7*d^2*e^14 - 18*a*b^4*c^6*d^2*e^14))/(a^6*b^6*f^9 - 64*a^9*c^3*f^9 - 12*a^7*b^4*c*f^9 + 48*a^8*b^2*c^2*f^9) + (((((4*(2*a^4*b^9*c^2*e^16*f^6 - 26*a^5*b^7*c^3*e^16*f^6 + 118*a^6*b^5*c^4*e^16*f^6 - 208*a^7*b^3*c^5*e^16*f^6 - 480*a^8*c^7*d^2*e^16*f^6 + 96*a^8*b*c^6*e^16*f^6 + a^4*b^8*c^3*d^2*e^16*f^6 - 6*a^5*b^6*c^4*d^2*e^16*f^6 - 30*a^6*b^4*c^5*d^2*e^16*f^6 + 272*a^7*b^2*c^6*d^2*e^16*f^6))/(a^6*b^6*f^9 - 64*a^9*c^3*f^9 - 12*a^7*b^4*c*f^9 + 48*a^8*b^2*c^2*f^9) + (2*(b^7*e*f^3 - 12*a*b^5*c*e*f^3 - 64*a^3*b*c^3*e*f^3 + 48*a^2*b^3*c^2*e*f^3)*(a^7*b^8*c^2*e^17*f^9 - 12*a^8*b^6*c^3*e^17*f^9 + 48*a^9*b^4*c^4*e^17*f^9 - 64*a^10*b^2*c^5*e^17*f^9 + 640*a^10*b*c^6*d^2*e^17*f^9 + 3*a^6*b^9*c^2*d^2*e^17*f^9 - 46*a^7*b^7*c^3*d^2*e^17*f^9 + 264*a^8*b^5*c^4*d^2*e^17*f^9 - 672*a^9*b^3*c^5*d^2*e^17*f^9))/((a^6*b^6*f^9 - 64*a^9*c^3*f^9 - 12*a^7*b^4*c*f^9 + 48*a^8*b^2*c^2*f^9)*(a^3*b^6*e^2*f^6 - 64*a^6*c^3*e^2*f^6 + 48*a^5*b^2*c^2*e^2*f^6 - 12*a^4*b^4*c*e^2*f^6)))*(b^4 + 6*a^2*c^2 - 6*a*b^2*c))/(2*a^3*e*f^3*(4*a*c - b^2)^(3/2)) + ((b^4 + 6*a^2*c^2 - 6*a*b^2*c)*(b^7*e*f^3 - 12*a*b^5*c*e*f^3 - 64*a^3*b*c^3*e*f^3 + 48*a^2*b^3*c^2*e*f^3)*(a^7*b^8*c^2*e^17*f^9 - 12*a^8*b^6*c^3*e^17*f^9 + 48*a^9*b^4*c^4*e^17*f^9 - 64*a^10*b^2*c^5*e^17*f^9 + 640*a^10*b*c^6*d^2*e^17*f^9 + 3*a^6*b^9*c^2*d^2*e^17*f^9 - 46*a^7*b^7*c^3*d^2*e^17*f^9 + 264*a^8*b^5*c^4*d^2*e^17*f^9 - 672*a^9*b^3*c^5*d^2*e^17*f^9))/(a^3*e*f^3*(4*a*c - b^2)^(3/2)*(a^6*b^6*f^9 - 64*a^9*c^3*f^9 - 12*a^7*b^4*c*f^9 + 48*a^8*b^2*c^2*f^9)*(a^3*b^6*e^2*f^6 - 64*a^6*c^3*e^2*f^6 + 48*a^5*b^2*c^2*e^2*f^6 - 12*a^4*b^4*c*e^2*f^6)))*(b^4 + 6*a^2*c^2 - 6*a*b^2*c))/(2*a^3*e*f^3*(4*a*c - b^2)^(3/2)) + ((b^4 + 6*a^2*c^2 - 6*a*b^2*c)^2*(b^7*e*f^3 - 12*a*b^5*c*e*f^3 - 64*a^3*b*c^3*e*f^3 + 48*a^2*b^3*c^2*e*f^3)*(a^7*b^8*c^2*e^17*f^9 - 12*a^8*b^6*c^3*e^17*f^9 + 48*a^9*b^4*c^4*e^17*f^9 - 64*a^10*b^2*c^5*e^17*f^9 + 640*a^10*b*c^6*d^2*e^17*f^9 + 3*a^6*b^9*c^2*d^2*e^17*f^9 - 46*a^7*b^7*c^3*d^2*e^17*f^9 + 264*a^8*b^5*c^4*d^2*e^17*f^9 - 672*a^9*b^3*c^5*d^2*e^17*f^9))/(2*a^6*e^2*f^6*(4*a*c - b^2)^3*(a^6*b^6*f^9 - 64*a^9*c^3*f^9 - 12*a^7*b^4*c*f^9 + 48*a^8*b^2*c^2*f^9)*(a^3*b^6*e^2*f^6 - 64*a^6*c^3*e^2*f^6 + 48*a^5*b^2*c^2*e^2*f^6 - 12*a^4*b^4*c*e^2*f^6)))*(3*b^6 - 3*a^3*c^3 + 36*a^2*b^2*c^2 - 21*a*b^4*c))/(8*a^3*c^2*(4*a*c - b^2)^3*(9*a^4*c^4 - 6*b^8 - 288*a^2*b^4*c^2 + 382*a^3*b^2*c^3 + 72*a*b^6*c)) - (b*((((4*(36*a^6*c^7*e^15*f^3 + 4*a^2*b^8*c^3*e^15*f^3 - 45*a^3*b^6*c^4*e^15*f^3 + 170*a^4*b^4*c^5*e^15*f^3 - 225*a^5*b^2*c^6*e^15*f^3 - 276*a^5*b*c^7*d^2*e^15*f^3 + 6*a^2*b^7*c^4*d^2*e^15*f^3 - 65*a^3*b^5*c^5*d^2*e^15*f^3 + 233*a^4*b^3*c^6*d^2*e^15*f^3))/(a^6*b^6*f^9 - 64*a^9*c^3*f^9 - 12*a^7*b^4*c*f^9 + 48*a^8*b^2*c^2*f^9) - (((4*(2*a^4*b^9*c^2*e^16*f^6 - 26*a^5*b^7*c^3*e^16*f^6 + 118*a^6*b^5*c^4*e^16*f^6 - 208*a^7*b^3*c^5*e^16*f^6 - 480*a^8*c^7*d^2*e^16*f^6 + 96*a^8*b*c^6*e^16*f^6 + a^4*b^8*c^3*d^2*e^16*f^6 - 6*a^5*b^6*c^4*d^2*e^16*f^6 - 30*a^6*b^4*c^5*d^2*e^16*f^6 + 272*a^7*b^2*c^6*d^2*e^16*f^6))/(a^6*b^6*f^9 - 64*a^9*c^3*f^9 - 12*a^7*b^4*c*f^9 + 48*a^8*b^2*c^2*f^9) + (2*(b^7*e*f^3 - 12*a*b^5*c*e*f^3 - 64*a^3*b*c^3*e*f^3 + 48*a^2*b^3*c^2*e*f^3)*(a^7*b^8*c^2*e^17*f^9 - 12*a^8*b^6*c^3*e^17*f^9 + 48*a^9*b^4*c^4*e^17*f^9 - 64*a^10*b^2*c^5*e^17*f^9 + 640*a^10*b*c^6*d^2*e^17*f^9 + 3*a^6*b^9*c^2*d^2*e^17*f^9 - 46*a^7*b^7*c^3*d^2*e^17*f^9 + 264*a^8*b^5*c^4*d^2*e^17*f^9 - 672*a^9*b^3*c^5*d^2*e^17*f^9))/((a^6*b^6*f^9 - 64*a^9*c^3*f^9 - 12*a^7*b^4*c*f^9 + 48*a^8*b^2*c^2*f^9)*(a^3*b^6*e^2*f^6 - 64*a^6*c^3*e^2*f^6 + 48*a^5*b^2*c^2*e^2*f^6 - 12*a^4*b^4*c*e^2*f^6)))*(b^7*e*f^3 - 12*a*b^5*c*e*f^3 - 64*a^3*b*c^3*e*f^3 + 48*a^2*b^3*c^2*e*f^3))/(2*(a^3*b^6*e^2*f^6 - 64*a^6*c^3*e^2*f^6 + 48*a^5*b^2*c^2*e^2*f^6 - 12*a^4*b^4*c*e^2*f^6)))*(b^4 + 6*a^2*c^2 - 6*a*b^2*c))/(2*a^3*e*f^3*(4*a*c - b^2)^(3/2)) - (((((4*(2*a^4*b^9*c^2*e^16*f^6 - 26*a^5*b^7*c^3*e^16*f^6 + 118*a^6*b^5*c^4*e^16*f^6 - 208*a^7*b^3*c^5*e^16*f^6 - 480*a^8*c^7*d^2*e^16*f^6 + 96*a^8*b*c^6*e^16*f^6 + a^4*b^8*c^3*d^2*e^16*f^6 - 6*a^5*b^6*c^4*d^2*e^16*f^6 - 30*a^6*b^4*c^5*d^2*e^16*f^6 + 272*a^7*b^2*c^6*d^2*e^16*f^6))/(a^6*b^6*f^9 - 64*a^9*c^3*f^9 - 12*a^7*b^4*c*f^9 + 48*a^8*b^

```
2*c^2*f^9) + (2*(b^7*e*f^3 - 12*a*b^5*c*e*f^3 - 64*a^3*b*c^3*e*f^3 + 48*a^2
*b^3*c^2*e*f^3)*(a^7*b^8*c^2*e^17*f^9 - 12*a^8*b^6*c^3*e^17*f^9 + 48*a^9*b^
4*c^4*e^17*f^9 - 64*a^10*b^2*c^5*e^17*f^9 + 640*a^10*b*c^6*d^2*e^17*f^9 + 3
*a^6*b^9*c^2*d^2*e^17*f^9 - 46*a^7*b^7*c^3*d^2*e^17*f^9 + 264*a^8*b^5*c^4*d
^2*e^17*f^9 - 672*a^9*b^3*c^5*d^2*e^17*f^9))/((a^6*b^6*f^9 - 64*a^9*c^3*f^9
 - 12*a^7*b^4*c*f^9 + 48*a^8*b^2*c^2*f^9)*(a^3*b^6*e^2*f^6 - 64*a^6*c^3*e^2
*f^6 + 48*a^5*b^2*c^2*e^2*f^6 - 12*a^4*b^4*c*e^2*f^6)))*(b^4 + 6*a^2*c^2 -
6*a*b^2*c))/(2*a^3*e*f^3*(4*a*c - b^2)^(3/2)) + ((b^4 + 6*a^2*c^2 - 6*a*b^2
*c)*(b^7*e*f^3 - 12*a*b^5*c*e*f^3 - 64*a^3*b*c^3*e*f^3 + 48*a^2*b^3*c^2*e*f
^3)*(a^7*b^8*c^2*e^17*f^9 - 12*a^8*b^6*c^3*e^17*f^9 + 48*a^9*b^4*c^4*e^17*f
^9 - 64*a^10*b^2*c^5*e^17*f^9 + 640*a^10*b*c^6*d^2*e^17*f^9 + 3*a^6*b^9*c^2
*d^2*e^17*f^9 - 46*a^7*b^7*c^3*d^2*e^17*f^9 + 264*a^8*b^5*c^4*d^2*e^17*f^9
- 672*a^9*b^3*c^5*d^2*e^17*f^9))/(a^3*e*f^3*(4*a*c - b^2)^(3/2)*(a^6*b^6*f^
9 - 64*a^9*c^3*f^9 - 12*a^7*b^4*c*f^9 + 48*a^8*b^2*c^2*f^9)*(a^3*b^6*e^2*f^
6 - 64*a^6*c^3*e^2*f^6 + 48*a^5*b^2*c^2*e^2*f^6 - 12*a^4*b^4*c*e^2*f^6)))*(
b^7*e*f^3 - 12*a*b^5*c*e*f^3 - 64*a^3*b*c^3*e*f^3 + 48*a^2*b^3*c^2*e*f^3))/
(2*(a^3*b^6*e^2*f^6 - 64*a^6*c^3*e^2*f^6 + 48*a^5*b^2*c^2*e^2*f^6 - 12*a^4*
b^4*c*e^2*f^6)) + ((b^4 + 6*a^2*c^2 - 6*a*b^2*c)^3*(a^7*b^8*c^2*e^17*f^9 -
12*a^8*b^6*c^3*e^17*f^9 + 48*a^9*b^4*c^4*e^17*f^9 - 64*a^10*b^2*c^5*e^17*f^
9 + 640*a^10*b*c^6*d^2*e^17*f^9 + 3*a^6*b^9*c^2*d^2*e^17*f^9 - 46*a^7*b^7*c
^3*d^2*e^17*f^9 + 264*a^8*b^5*c^4*d^2*e^17*f^9 - 672*a^9*b^3*c^5*d^2*e^17*f
^9))/(2*a^9*e^3*f^9*(4*a*c - b^2)^(9/2)*(a^6*b^6*f^9 - 64*a^9*c^3*f^9 - 12*
a^7*b^4*c*f^9 + 48*a^8*b^2*c^2*f^9)))*(3*b^6 - 49*a^3*c^3 + 72*a^2*b^2*c^2
- 27*a*b^4*c))/(8*a^3*c^2*(4*a*c - b^2)^(7/2)*(9*a^4*c^4 - 6*b^8 - 288*a^2*
b^4*c^2 + 382*a^3*b^2*c^3 + 72*a*b^6*c))))/(36*a^4*c^6*e^14 + b^8*c^2*e^14
- 12*a*b^6*c^3*e^14 + 48*a^2*b^4*c^4*e^14 - 72*a^3*b^2*c^5*e^14))*(b^4 + 6*
a^2*c^2 - 6*a*b^2*c))/(a^3*e*f^3*(4*a*c - b^2)^(3/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*f*x+d*f)**3/(a+b*(e*x+d)**2+c*(e*x+d)**4)**2,x)
[Out] Timed out
```

3.540 $\int \frac{1}{(df+efx)^4\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx$

Optimal. Leaf size=423

$$\frac{b\left(5b^2-19ac\right)}{2a^3ef^4\left(b^2-4ac\right)(d+ex)}-\frac{5b^2-14ac}{6a^2ef^4\left(b^2-4ac\right)(d+ex)^3}+\frac{\sqrt{c}\left(28a^2c^2-29ab^2c+b\left(5b^2-19ac\right)\sqrt{b^2-4ac}+5b^4\right)}{2\sqrt{2}\,a^3ef^4\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4a}}}$$

Rubi [A] time = 3.55, antiderivative size = 423, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {1142, 1121, 1281, 1166, 205}

$$\frac{\sqrt{c}\left(28a^2c^2-29ab^2c+b\left(5b^2-19ac\right)\sqrt{b^2-4ac}+5b^4\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\,a^3ef^4\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{c}\left(28a^2c^2-29ab^2c-b\left(5b^2-19ac\right)\sqrt{b^2-4ac}+5b^4\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{2\sqrt{2}\,a^3ef^4\left(b^2-4ac\right)^{3/2}\sqrt{\sqrt{b^2-4ac}+b}}+\frac{b\left(5b^2-19ac\right)}{2a^3ef^4\left(b^2-4ac\right)(d+ex)}-\frac{5b^2-14ac}{6a^2ef^4\left(b^2-4ac\right)(d+ex)^3}+\frac{-2ac+b^2+bc(d+ex)^2}{2aef^4\left(b^2-4ac\right)(d+ex)^3\left(a+b(d+ex)^2+c(d+ex)^4\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d*f + e*f*x)^4*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2),x]
```

```
[Out] -(5*b^2 - 14*a*c)/(6*a^2*(b^2 - 4*a*c)*e*f^4*(d + e*x)^3) + (b*(5*b^2 - 19*
a*c))/(2*a^3*(b^2 - 4*a*c)*e*f^4*(d + e*x)) + (b^2 - 2*a*c + b*c*(d + e*x)^
2)/(2*a*(b^2 - 4*a*c)*e*f^4*(d + e*x)^3*(a + b*(d + e*x)^2 + c*(d + e*x)^4)
) + (Sqrt[c]*(5*b^4 - 29*a*b^2*c + 28*a^2*c^2 + b*(5*b^2 - 19*a*c)*Sqrt[b^2
 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b - Sqrt[b^2 - 4*a*c]]])
/(2*Sqrt[2]*a^3*(b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]*e*f^4) - (S
qrt[c]*(5*b^4 - 29*a*b^2*c + 28*a^2*c^2 - b*(5*b^2 - 19*a*c)*Sqrt[b^2 - 4*a
*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(2*Sq
rt[2]*a^3*(b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]*e*f^4)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1121

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> -Simp[((d*x)^(m + 1)*(b^2 - 2*a*c + b*c*x^2)*(a + b*x^2 + c*x^4)^(p + 1
))/(2*a*d*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)),
Int[(d*x)^m*(a + b*x^2 + c*x^4)^(p + 1)*Simp[b^2*(m + 2*p + 3) - 2*a*c*(m +
 4*p + 5) + b*c*(m + 4*p + 7)*x^2, x], x], x] /; FreeQ[{a, b, c, d, m}, x]
&& NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p] && (IntegerQ[p] || In
tegerQ[m])
```

Rule 1142

```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Di
st[u^m/(Coefficient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p,
 x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1281

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[(d*(f*x)^(m + 1)*(a + b*x^2 + c*x^4)^(p + 1)
)/(a*f*(m + 1)), x] + Dist[1/(a*f^2*(m + 1)), Int[(f*x)^(m + 2)*(a + b*x^2
+ c*x^4)^p*Simp[a*e*(m + 1) - b*d*(m + 2*p + 3) - c*d*(m + 4*p + 5)*x^2, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[m
, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rubi steps

$$\int \frac{1}{(df+efx)^4\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x^4\left(a+bx^2+cx^4\right)^2}\,dx, x, d+ex\right)}{ef^4}$$

$$= \frac{b^2-2ac+bc(d+ex)^2}{2a\left(b^2-4ac\right)ef^4(d+ex)^3\left(a+b(d+ex)^2+c(d+ex)^4\right)} -$$

$$= -\frac{5b^2-14ac}{6a^2\left(b^2-4ac\right)ef^4(d+ex)^3} + \frac{b^2-2ac+}{2a\left(b^2-4ac\right)ef^4(d+ex)^3}$$

$$= -\frac{5b^2-14ac}{6a^2\left(b^2-4ac\right)ef^4(d+ex)^3} + \frac{b\left(5b^2-19ac\right)}{2a^3\left(b^2-4ac\right)ef^4(d+ex)}$$

$$= -\frac{5b^2-14ac}{6a^2\left(b^2-4ac\right)ef^4(d+ex)^3} + \frac{b\left(5b^2-19ac\right)}{2a^3\left(b^2-4ac\right)ef^4(d+ex)}$$

$$= -\frac{5b^2-14ac}{6a^2\left(b^2-4ac\right)ef^4(d+ex)^3} + \frac{b\left(5b^2-19ac\right)}{2a^3\left(b^2-4ac\right)ef^4(d+ex)}$$

Mathematica [A] time = 3.02, size = 387, normalized size = 0.91

$$\frac{\frac{6(d+ex)\left(2a^2c^2-4ab^2c-3abc^2(d+ex)^2+b^4+b^3c(d+ex)^2\right)}{\left(b^2-4ac\right)\left(a+(d+ex)^2\left(b+c(d+ex)^2\right)\right)}+\frac{3\sqrt{2}\sqrt{c}\left(28a^2c^2-29ab^2c-19abc\sqrt{b^2-4ac}+5b^3\sqrt{b^2-4ac}+5b^4\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{3\sqrt{2}\sqrt{c}\left(-28a^2c^2+29ab^2c-19abc\sqrt{b^2-4ac}+5b^3\sqrt{b^2-4ac}-5b^4\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\left(b^2-4ac\right)^{3/2}\sqrt{\sqrt{b^2-4ac}+b}}-\frac{4a}{(d+ex)^3}+\frac{24b}{d+ex}}{12a^3ef^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d*f + e*f*x)^4*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2),x]
```

```
[Out] ((-4*a)/(d + e*x)^3 + (24*b)/(d + e*x) + (6*(d + e*x)*(b^4 - 4*a*b^2*c + 2*
a^2*c^2 + b^3*c*(d + e*x)^2 - 3*a*b*c^2*(d + e*x)^2))/((b^2 - 4*a*c)*(a + (
d + e*x)^2*(b + c*(d + e*x)^2))) + (3*Sqrt[2]*Sqrt[c]*(5*b^4 - 29*a*b^2*c +
 28*a^2*c^2 + 5*b^3*Sqrt[b^2 - 4*a*c] - 19*a*b*c*Sqrt[b^2 - 4*a*c])*ArcTan[
(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(3
/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (3*Sqrt[2]*Sqrt[c]*(-5*b^4 + 29*a*b^2*c
- 28*a^2*c^2 + 5*b^3*Sqrt[b^2 - 4*a*c] - 19*a*b*c*Sqrt[b^2 - 4*a*c])*ArcTan
[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(
3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(12*a^3*e*f^4)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(df+efx)^4\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/((d*f + e*f*x)^4*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2
),x]
```

```
[Out] IntegrateAlgebraic[1/((d*f + e*f*x)^4*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2
), x]
```

fricas [B] time = 2.13, size = 5954, normalized size = 14.08

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*f*x+d*f)^4/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="frica
s")
```

```
[Out] 1/12*(6*(5*b^3*c - 19*a*b*c^2)*e^6*x^6 + 36*(5*b^3*c - 19*a*b*c^2)*d*e^5*x^
5 + 2*(15*b^4 - 62*a*b^2*c + 14*a^2*c^2 + 45*(5*b^3*c - 19*a*b*c^2)*d^2)*e^
4*x^4 + 6*(5*b^3*c - 19*a*b*c^2)*d^6 + 8*(15*(5*b^3*c - 19*a*b*c^2)*d^3 + (
15*b^4 - 62*a*b^2*c + 14*a^2*c^2)*d)*e^3*x^3 + 2*(15*b^4 - 62*a*b^2*c + 14*
a^2*c^2)*d^4 + 2*(45*(5*b^3*c - 19*a*b*c^2)*d^4 + 10*a*b^3 - 40*a^2*b*c + 6
*(15*b^4 - 62*a*b^2*c + 14*a^2*c^2)*d^2)*e^2*x^2 - 4*a^2*b^2 + 16*a^3*c + 2
0*(a*b^3 - 4*a^2*b*c)*d^2 + 4*(9*(5*b^3*c - 19*a*b*c^2)*d^5 + 2*(15*b^4 - 6
2*a*b^2*c + 14*a^2*c^2)*d^3 + 10*(a*b^3 - 4*a^2*b*c)*d)*e*x - 3*sqrt(1/2)*(
(a^3*b^2*c - 4*a^4*c^2)*e^8*f^4*x^7 + 7*(a^3*b^2*c - 4*a^4*c^2)*d*e^7*f^4*x
^6 + (a^3*b^3 - 4*a^4*b*c + 21*(a^3*b^2*c - 4*a^4*c^2)*d^2)*e^6*f^4*x^5 + 5
*(7*(a^3*b^2*c - 4*a^4*c^2)*d^3 + (a^3*b^3 - 4*a^4*b*c)*d)*e^5*f^4*x^4 + (a
^4*b^2 - 4*a^5*c + 35*(a^3*b^2*c - 4*a^4*c^2)*d^4 + 10*(a^3*b^3 - 4*a^4*b*c
)*d^2)*e^4*f^4*x^3 + (21*(a^3*b^2*c - 4*a^4*c^2)*d^5 + 10*(a^3*b^3 - 4*a^4*
b*c)*d^3 + 3*(a^4*b^2 - 4*a^5*c)*d)*e^3*f^4*x^2 + (7*(a^3*b^2*c - 4*a^4*c^2
)*d^6 + 5*(a^3*b^3 - 4*a^4*b*c)*d^4 + 3*(a^4*b^2 - 4*a^5*c)*d^2)*e^2*f^4*x
+ ((a^3*b^2*c - 4*a^4*c^2)*d^7 + (a^3*b^3 - 4*a^4*b*c)*d^5 + (a^4*b^2 - 4*a
^5*c)*d^3)*e*f^4)*sqrt(-((a^7*b^6 - 12*a^8*b^4*c + 48*a^9*b^2*c^2 - 64*a^10
*c^3)*e^2*f^8*sqrt((625*b^12 - 8250*a*b^10*c + 39525*a^2*b^8*c^2 - 83630*a^
3*b^6*c^3 + 76686*a^4*b^4*c^4 - 24108*a^5*b^2*c^5 + 2401*a^6*c^6)/((a^14*b^
6 - 12*a^15*b^4*c + 48*a^16*b^2*c^2 - 64*a^17*c^3)*e^4*f^16)) + 25*b^9 - 31
5*a*b^7*c + 1386*a^2*b^5*c^2 - 2415*a^3*b^3*c^3 + 1260*a^4*b*c^4)/((a^7*b^6
 - 12*a^8*b^4*c + 48*a^9*b^2*c^2 - 64*a^10*c^3)*e^2*f^8))*log((1125*b^8*c^4
 - 12325*a*b^6*c^5 + 43410*a^2*b^4*c^6 - 50421*a^3*b^2*c^7 + 9604*a^4*c^8)*
e*x + (1125*b^8*c^4 - 12325*a*b^6*c^5 + 43410*a^2*b^4*c^6 - 50421*a^3*b^2*c
^7 + 9604*a^4*c^8)*d + 1/2*sqrt(1/2)*((5*a^7*b^11 - 94*a^8*b^9*c + 700*a^9*
b^7*c^2 - 2576*a^10*b^5*c^3 + 4672*a^11*b^3*c^4 - 3328*a^12*b*c^5)*e^3*f^12
*sqrt((625*b^12 - 8250*a*b^10*c + 39525*a^2*b^8*c^2 - 83630*a^3*b^6*c^3 + 7
6686*a^4*b^4*c^4 - 24108*a^5*b^2*c^5 + 2401*a^6*c^6)/((a^14*b^6 - 12*a^15*b
^4*c + 48*a^16*b^2*c^2 - 64*a^17*c^3)*e^4*f^16)) - (125*b^14 - 2425*a*b^12*
c + 18940*a^2*b^10*c^2 - 75579*a^3*b^8*c^3 + 160932*a^4*b^6*c^4 - 172990*a^
5*b^4*c^5 + 79408*a^6*b^2*c^6 - 10976*a^7*c^7)*e*f^4)*sqrt(-((a^7*b^6 - 12*
a^8*b^4*c + 48*a^9*b^2*c^2 - 64*a^10*c^3)*e^2*f^8*sqrt((625*b^12 - 8250*a*b
^10*c + 39525*a^2*b^8*c^2 - 83630*a^3*b^6*c^3 + 76686*a^4*b^4*c^4 - 24108*a
^5*b^2*c^5 + 2401*a^6*c^6)/((a^14*b^6 - 12*a^15*b^4*c + 48*a^16*b^2*c^2 - 6
4*a^17*c^3)*e^4*f^16)) + 25*b^9 - 315*a*b^7*c + 1386*a^2*b^5*c^2 - 2415*a^3
*b^3*c^3 + 1260*a^4*b*c^4)/((a^7*b^6 - 12*a^8*b^4*c + 48*a^9*b^2*c^2 - 64*a
^10*c^3)*e^2*f^8))) + 3*sqrt(1/2)*((a^3*b^2*c - 4*a^4*c^2)*e^8*f^4*x^7 + 7*
(a^3*b^2*c - 4*a^4*c^2)*d*e^7*f^4*x^6 + (a^3*b^3 - 4*a^4*b*c + 21*(a^3*b^2*
c - 4*a^4*c^2)*d^2)*e^6*f^4*x^5 + 5*(7*(a^3*b^2*c - 4*a^4*c^2)*d^3 + (a^3*b
```

```
^3 - 4*a^4*b*c)*d)*e^5*f^4*x^4 + (a^4*b^2 - 4*a^5*c + 35*(a^3*b^2*c - 4*a^4
*c^2)*d^4 + 10*(a^3*b^3 - 4*a^4*b*c)*d^2)*e^4*f^4*x^3 + (21*(a^3*b^2*c - 4*
a^4*c^2)*d^5 + 10*(a^3*b^3 - 4*a^4*b*c)*d^3 + 3*(a^4*b^2 - 4*a^5*c)*d)*e^3*
f^4*x^2 + (7*(a^3*b^2*c - 4*a^4*c^2)*d^6 + 5*(a^3*b^3 - 4*a^4*b*c)*d^4 + 3*
(a^4*b^2 - 4*a^5*c)*d^2)*e^2*f^4*x + ((a^3*b^2*c - 4*a^4*c^2)*d^7 + (a^3*b^
3 - 4*a^4*b*c)*d^5 + (a^4*b^2 - 4*a^5*c)*d^3)*e*f^4)*sqrt(-((a^7*b^6 - 12*a
^8*b^4*c + 48*a^9*b^2*c^2 - 64*a^10*c^3)*e^2*f^8*sqrt((625*b^12 - 8250*a*b^
10*c + 39525*a^2*b^8*c^2 - 83630*a^3*b^6*c^3 + 76686*a^4*b^4*c^4 - 24108*a^
5*b^2*c^5 + 2401*a^6*c^6)/((a^14*b^6 - 12*a^15*b^4*c + 48*a^16*b^2*c^2 - 64
*a^17*c^3)*e^4*f^16)) + 25*b^9 - 315*a*b^7*c + 1386*a^2*b^5*c^2 - 2415*a^3*
b^3*c^3 + 1260*a^4*b*c^4)/((a^7*b^6 - 12*a^8*b^4*c + 48*a^9*b^2*c^2 - 64*a^
10*c^3)*e^2*f^8))*log((1125*b^8*c^4 - 12325*a*b^6*c^5 + 43410*a^2*b^4*c^6 -
 50421*a^3*b^2*c^7 + 9604*a^4*c^8)*e*x + (1125*b^8*c^4 - 12325*a*b^6*c^5 +
43410*a^2*b^4*c^6 - 50421*a^3*b^2*c^7 + 9604*a^4*c^8)*d - 1/2*sqrt(1/2)*((5
*a^7*b^11 - 94*a^8*b^9*c + 700*a^9*b^7*c^2 - 2576*a^10*b^5*c^3 + 4672*a^11*
b^3*c^4 - 3328*a^12*b*c^5)*e^3*f^12*sqrt((625*b^12 - 8250*a*b^10*c + 39525*
a^2*b^8*c^2 - 83630*a^3*b^6*c^3 + 76686*a^4*b^4*c^4 - 24108*a^5*b^2*c^5 + 2
401*a^6*c^6)/((a^14*b^6 - 12*a^15*b^4*c + 48*a^16*b^2*c^2 - 64*a^17*c^3)*e^
4*f^16)) - (125*b^14 - 2425*a*b^12*c + 18940*a^2*b^10*c^2 - 75579*a^3*b^8*c
^3 + 160932*a^4*b^6*c^4 - 172990*a^5*b^4*c^5 + 79408*a^6*b^2*c^6 - 10976*a^
7*c^7)*e*f^4)*sqrt(-((a^7*b^6 - 12*a^8*b^4*c + 48*a^9*b^2*c^2 - 64*a^10*c^3
)*e^2*f^8*sqrt((625*b^12 - 8250*a*b^10*c + 39525*a^2*b^8*c^2 - 83630*a^3*b^
6*c^3 + 76686*a^4*b^4*c^4 - 24108*a^5*b^2*c^5 + 2401*a^6*c^6)/((a^14*b^6 -
12*a^15*b^4*c + 48*a^16*b^2*c^2 - 64*a^17*c^3)*e^4*f^16)) + 25*b^9 - 315*a*
b^7*c + 1386*a^2*b^5*c^2 - 2415*a^3*b^3*c^3 + 1260*a^4*b*c^4)/((a^7*b^6 - 1
2*a^8*b^4*c + 48*a^9*b^2*c^2 - 64*a^10*c^3)*e^2*f^8))) + 3*sqrt(1/2)*((a^3*
b^2*c - 4*a^4*c^2)*e^8*f^4*x^7 + 7*(a^3*b^2*c - 4*a^4*c^2)*d*e^7*f^4*x^6 +
(a^3*b^3 - 4*a^4*b*c + 21*(a^3*b^2*c - 4*a^4*c^2)*d^2)*e^6*f^4*x^5 + 5*(7*(
a^3*b^2*c - 4*a^4*c^2)*d^3 + (a^3*b^3 - 4*a^4*b*c)*d)*e^5*f^4*x^4 + (a^4*b^
2 - 4*a^5*c + 35*(a^3*b^2*c - 4*a^4*c^2)*d^4 + 10*(a^3*b^3 - 4*a^4*b*c)*d^2
)*e^4*f^4*x^3 + (21*(a^3*b^2*c - 4*a^4*c^2)*d^5 + 10*(a^3*b^3 - 4*a^4*b*c)*
d^3 + 3*(a^4*b^2 - 4*a^5*c)*d)*e^3*f^4*x^2 + (7*(a^3*b^2*c - 4*a^4*c^2)*d^6
 + 5*(a^3*b^3 - 4*a^4*b*c)*d^4 + 3*(a^4*b^2 - 4*a^5*c)*d^2)*e^2*f^4*x + ((a
^3*b^2*c - 4*a^4*c^2)*d^7 + (a^3*b^3 - 4*a^4*b*c)*d^5 + (a^4*b^2 - 4*a^5*c)
*d^3)*e*f^4)*sqrt(((a^7*b^6 - 12*a^8*b^4*c + 48*a^9*b^2*c^2 - 64*a^10*c^3)*
e^2*f^8*sqrt((625*b^12 - 8250*a*b^10*c + 39525*a^2*b^8*c^2 - 83630*a^3*b^6*
c^3 + 76686*a^4*b^4*c^4 - 24108*a^5*b^2*c^5 + 2401*a^6*c^6)/((a^14*b^6 - 12
*a^15*b^4*c + 48*a^16*b^2*c^2 - 64*a^17*c^3)*e^4*f^16)) - 25*b^9 + 315*a*b^
7*c - 1386*a^2*b^5*c^2 + 2415*a^3*b^3*c^3 - 1260*a^4*b*c^4)/((a^7*b^6 - 12*
a^8*b^4*c + 48*a^9*b^2*c^2 - 64*a^10*c^3)*e^2*f^8))*log((1125*b^8*c^4 - 123
25*a*b^6*c^5 + 43410*a^2*b^4*c^6 - 50421*a^3*b^2*c^7 + 9604*a^4*c^8)*e*x +
(1125*b^8*c^4 - 12325*a*b^6*c^5 + 43410*a^2*b^4*c^6 - 50421*a^3*b^2*c^7 + 9
604*a^4*c^8)*d + 1/2*sqrt(1/2)*((5*a^7*b^11 - 94*a^8*b^9*c + 700*a^9*b^7*c^
2 - 2576*a^10*b^5*c^3 + 4672*a^11*b^3*c^4 - 3328*a^12*b*c^5)*e^3*f^12*sqrt(
(625*b^12 - 8250*a*b^10*c + 39525*a^2*b^8*c^2 - 83630*a^3*b^6*c^3 + 76686*a
^4*b^4*c^4 - 24108*a^5*b^2*c^5 + 2401*a^6*c^6)/((a^14*b^6 - 12*a^15*b^4*c +
 48*a^16*b^2*c^2 - 64*a^17*c^3)*e^4*f^16)) + (125*b^14 - 2425*a*b^12*c + 18
940*a^2*b^10*c^2 - 75579*a^3*b^8*c^3 + 160932*a^4*b^6*c^4 - 172990*a^5*b^4*
c^5 + 79408*a^6*b^2*c^6 - 10976*a^7*c^7)*e*f^4)*sqrt(((a^7*b^6 - 12*a^8*b^4
*c + 48*a^9*b^2*c^2 - 64*a^10*c^3)*e^2*f^8*sqrt((625*b^12 - 8250*a*b^10*c +
 39525*a^2*b^8*c^2 - 83630*a^3*b^6*c^3 + 76686*a^4*b^4*c^4 - 24108*a^5*b^2*
c^5 + 2401*a^6*c^6)/((a^14*b^6 - 12*a^15*b^4*c + 48*a^16*b^2*c^2 - 64*a^17*
c^3)*e^4*f^16)) - 25*b^9 + 315*a*b^7*c - 1386*a^2*b^5*c^2 + 2415*a^3*b^3*c^
3 - 1260*a^4*b*c^4)/((a^7*b^6 - 12*a^8*b^4*c + 48*a^9*b^2*c^2 - 64*a^10*c^3
)*e^2*f^8))) - 3*sqrt(1/2)*((a^3*b^2*c - 4*a^4*c^2)*e^8*f^4*x^7 + 7*(a^3*b^
2*c - 4*a^4*c^2)*d*e^7*f^4*x^6 + (a^3*b^3 - 4*a^4*b*c + 21*(a^3*b^2*c - 4*a
^4*c^2)*d^2)*e^6*f^4*x^5 + 5*(7*(a^3*b^2*c - 4*a^4*c^2)*d^3 + (a^3*b^3 - 4*
a^4*b*c)*d)*e^5*f^4*x^4 + (a^4*b^2 - 4*a^5*c + 35*(a^3*b^2*c - 4*a^4*c^2)*d
^4 + 10*(a^3*b^3 - 4*a^4*b*c)*d^2)*e^4*f^4*x^3 + (21*(a^3*b^2*c - 4*a^4*c^2
```

```
)*d^5 + 10*(a^3*b^3 - 4*a^4*b*c)*d^3 + 3*(a^4*b^2 - 4*a^5*c)*d)*e^3*f^4*x^2
 + (7*(a^3*b^2*c - 4*a^4*c^2)*d^6 + 5*(a^3*b^3 - 4*a^4*b*c)*d^4 + 3*(a^4*b^
2 - 4*a^5*c)*d^2)*e^2*f^4*x + ((a^3*b^2*c - 4*a^4*c^2)*d^7 + (a^3*b^3 - 4*a
^4*b*c)*d^5 + (a^4*b^2 - 4*a^5*c)*d^3)*e*f^4)*sqrt(((a^7*b^6 - 12*a^8*b^4*c
 + 48*a^9*b^2*c^2 - 64*a^10*c^3)*e^2*f^8*sqrt((625*b^12 - 8250*a*b^10*c + 3
9525*a^2*b^8*c^2 - 83630*a^3*b^6*c^3 + 76686*a^4*b^4*c^4 - 24108*a^5*b^2*c^
5 + 2401*a^6*c^6)/((a^14*b^6 - 12*a^15*b^4*c + 48*a^16*b^2*c^2 - 64*a^17*c^
3)*e^4*f^16)) - 25*b^9 + 315*a*b^7*c - 1386*a^2*b^5*c^2 + 2415*a^3*b^3*c^3
- 1260*a^4*b*c^4)/((a^7*b^6 - 12*a^8*b^4*c + 48*a^9*b^2*c^2 - 64*a^10*c^3)*
e^2*f^8))*log((1125*b^8*c^4 - 12325*a*b^6*c^5 + 43410*a^2*b^4*c^6 - 50421*a
^3*b^2*c^7 + 9604*a^4*c^8)*e*x + (1125*b^8*c^4 - 12325*a*b^6*c^5 + 43410*a^
2*b^4*c^6 - 50421*a^3*b^2*c^7 + 9604*a^4*c^8)*d - 1/2*sqrt(1/2)*((5*a^7*b^1
1 - 94*a^8*b^9*c + 700*a^9*b^7*c^2 - 2576*a^10*b^5*c^3 + 4672*a^11*b^3*c^4
- 3328*a^12*b*c^5)*e^3*f^12*sqrt((625*b^12 - 8250*a*b^10*c + 39525*a^2*b^8*
c^2 - 83630*a^3*b^6*c^3 + 76686*a^4*b^4*c^4 - 24108*a^5*b^2*c^5 + 2401*a^6*
c^6)/((a^14*b^6 - 12*a^15*b^4*c + 48*a^16*b^2*c^2 - 64*a^17*c^3)*e^4*f^16))
 + (125*b^14 - 2425*a*b^12*c + 18940*a^2*b^10*c^2 - 75579*a^3*b^8*c^3 + 160
932*a^4*b^6*c^4 - 172990*a^5*b^4*c^5 + 79408*a^6*b^2*c^6 - 10976*a^7*c^7)*e
*f^4)*sqrt(((a^7*b^6 - 12*a^8*b^4*c + 48*a^9*b^2*c^2 - 64*a^10*c^3)*e^2*f^8
*sqrt((625*b^12 - 8250*a*b^10*c + 39525*a^2*b^8*c^2 - 83630*a^3*b^6*c^3 + 7
6686*a^4*b^4*c^4 - 24108*a^5*b^2*c^5 + 2401*a^6*c^6)/((a^14*b^6 - 12*a^15*b
^4*c + 48*a^16*b^2*c^2 - 64*a^17*c^3)*e^4*f^16)) - 25*b^9 + 315*a*b^7*c - 1
386*a^2*b^5*c^2 + 2415*a^3*b^3*c^3 - 1260*a^4*b*c^4)/((a^7*b^6 - 12*a^8*b^4
*c + 48*a^9*b^2*c^2 - 64*a^10*c^3)*e^2*f^8))))/((a^3*b^2*c - 4*a^4*c^2)*e^8
*f^4*x^7 + 7*(a^3*b^2*c - 4*a^4*c^2)*d*e^7*f^4*x^6 + (a^3*b^3 - 4*a^4*b*c +
 21*(a^3*b^2*c - 4*a^4*c^2)*d^2)*e^6*f^4*x^5 + 5*(7*(a^3*b^2*c - 4*a^4*c^2)
*d^3 + (a^3*b^3 - 4*a^4*b*c)*d)*e^5*f^4*x^4 + (a^4*b^2 - 4*a^5*c + 35*(a^3*
b^2*c - 4*a^4*c^2)*d^4 + 10*(a^3*b^3 - 4*a^4*b*c)*d^2)*e^4*f^4*x^3 + (21*(a
^3*b^2*c - 4*a^4*c^2)*d^5 + 10*(a^3*b^3 - 4*a^4*b*c)*d^3 + 3*(a^4*b^2 - 4*a
^5*c)*d)*e^3*f^4*x^2 + (7*(a^3*b^2*c - 4*a^4*c^2)*d^6 + 5*(a^3*b^3 - 4*a^4*
b*c)*d^4 + 3*(a^4*b^2 - 4*a^5*c)*d^2)*e^2*f^4*x + ((a^3*b^2*c - 4*a^4*c^2)*
d^7 + (a^3*b^3 - 4*a^4*b*c)*d^5 + (a^4*b^2 - 4*a^5*c)*d^3)*e*f^4)
```

giac [B] time = 0.71, size = 2002, normalized size = 4.73

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*f*x+d*f)^4/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="giac"
)
```

```
[Out] -1/4*((5*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)
/c))^2*b^3*c*e^2 - 19*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c
)*e^2)*e^(-4)/c))^2*a*b*c^2*e^2 - 10*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 + s
qrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))*b^3*c*d*e + 38*(d*e^(-1) + sqrt(1/2)*sqrt(
-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))*a*b*c^2*d*e + 5*b^3*c*d^2 - 19*
a*b*c^2*d^2 + 5*b^4 - 24*a*b^2*c + 14*a^2*c^2)*log(d*e^(-1) + x + sqrt(1/2)
*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))/(2*(d*e^(-1) + sqrt(1/2)*
sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^3*c*e^4 - 6*(d*e^(-1) + sq
rt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^2*c*d*e^3 - 2*c*d^
3*e - b*d*e + (6*c*d^2*e^2 + b*e^2)*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 + sq
rt(b^2 - 4*a*c)*e^2)*e^(-4)/c))) + (5*(d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 +
sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^2*b^3*c*e^2 - 19*(d*e^(-1) - sqrt(1/2)*sq
rt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^2*a*b*c^2*e^2 - 10*(d*e^(-1)
 - sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))*b^3*c*d*e + 3
8*(d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))*a*
b*c^2*d*e + 5*b^3*c*d^2 - 19*a*b*c^2*d^2 + 5*b^4 - 24*a*b^2*c + 14*a^2*c^2)
*log(d*e^(-1) + x - sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/
c))/(2*(d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c
))^3*c*e^4 - 6*(d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*
```

```
e^(-4)/c))^2*c*d*e^3 - 2*c*d^3*e - b*d*e + (6*c*d^2*e^2 + b*e^2)*(d*e^(-1)
- sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))) + (5*(d*e^(-1
) + sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^2*b^3*c*e^2
- 19*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))
^2*a*b*c^2*e^2 - 10*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*
e^2)*e^(-4)/c))*b^3*c*d*e + 38*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^
2 - 4*a*c)*e^2)*e^(-4)/c))*a*b*c^2*d*e + 5*b^3*c*d^2 - 19*a*b*c^2*d^2 + 5*b
^4 - 24*a*b^2*c + 14*a^2*c^2)*log(d*e^(-1) + x + sqrt(1/2)*sqrt(-(b*e^2 - s
qrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))/(2*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 - sq
rt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^3*c*e^4 - 6*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*
e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^2*c*d*e^3 - 2*c*d^3*e - b*d*e + (6*
c*d^2*e^2 + b*e^2)*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e
^2)*e^(-4)/c))) + (5*(d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)
*e^2)*e^(-4)/c))^2*b^3*c*e^2 - 19*(d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 - sqrt
(b^2 - 4*a*c)*e^2)*e^(-4)/c))^2*a*b*c^2*e^2 - 10*(d*e^(-1) - sqrt(1/2)*sqrt
(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))*b^3*c*d*e + 38*(d*e^(-1) - sqr
t(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))*a*b*c^2*d*e + 5*b^3
*c*d^2 - 19*a*b*c^2*d^2 + 5*b^4 - 24*a*b^2*c + 14*a^2*c^2)*log(d*e^(-1) + x
 - sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))/(2*(d*e^(-1)
- sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^3*c*e^4 - 6*(d
*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^2*c*d*
e^3 - 2*c*d^3*e - b*d*e + (6*c*d^2*e^2 + b*e^2)*(d*e^(-1) - sqrt(1/2)*sqrt(
-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))))/(a^3*b^2*f^4 - 4*a^4*c*f^4) +
 1/2*(b^3*c*x^3*e^3 - 3*a*b*c^2*x^3*e^3 + 3*b^3*c*d*x^2*e^2 - 9*a*b*c^2*d*x
^2*e^2 + 3*b^3*c*d^2*x*e - 9*a*b*c^2*d^2*x*e + b^3*c*d^3 - 3*a*b*c^2*d^3 +
b^4*x*e - 4*a*b^2*c*x*e + 2*a^2*c^2*x*e + b^4*d - 4*a*b^2*c*d + 2*a^2*c^2*d
)/((a^3*b^2*f^4*e - 4*a^4*c*f^4*e)*(c*x^4*e^4 + 4*c*d*x^3*e^3 + 6*c*d^2*x^2
*e^2 + 4*c*d^3*x*e + c*d^4 + b*x^2*e^2 + 2*b*d*x*e + b*d^2 + a)) + 1/3*(6*b
*x^2*e^2 + 12*b*d*x*e + 6*b*d^2 - a)*e^(-1)/((x*e + d)^3*a^3*f^4)
```

maple [C] time = 0.03, size = 1569, normalized size = 3.71

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*f*x+d*f)^4/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x)

[Out] 3/2/f^4/a^2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+
c*d^4+2*b*d*e*x+b*d^2+a)*b*c^2*e^2/(4*a*c-b^2)*x^3-1/2/f^4/a^3/(c*e^4*x^4+4
*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)
*b^3*c*e^2/(4*a*c-b^2)*x^3+9/2/f^4/a^2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2
*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)*d*b*c^2*e/(4*a*c-b^2)*x
^2-3/2/f^4/a^3/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x
^2+c*d^4+2*b*d*e*x+b*d^2+a)*d*b^3*c*e/(4*a*c-b^2)*x^2+9/2/f^4/a^2/(c*e^4*x^
4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2
+a)/(4*a*c-b^2)*x*b*c^2*d^2-3/2/f^4/a^3/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^
2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)/(4*a*c-b^2)*x*b^3*c*d^
2-1/f^4/a/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*
d^4+2*b*d*e*x+b*d^2+a)/(4*a*c-b^2)*x*c^2+2/f^4/a^2/(c*e^4*x^4+4*c*d*e^3*x^3
+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)/(4*a*c-b^2)
*x*b^2*c-1/2/f^4/a^3/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b
*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)/(4*a*c-b^2)*x*b^4+3/2/f^4/a^2/(c*e^4*x^4+
4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a
)*d^3/e/(4*a*c-b^2)*b*c^2-1/2/f^4/a^3/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*
x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)*d^3/e/(4*a*c-b^2)*b^3*c-
1/f^4/a/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^
4+2*b*d*e*x+b*d^2+a)*d/e/(4*a*c-b^2)*c^2+2/f^4/a^2/(c*e^4*x^4+4*c*d*e^3*x^3
+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)*d/e/(4*a*c-
b^2)*b^2*c-1/2/f^4/a^3/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x
+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)*d/e/(4*a*c-b^2)*b^4+1/4/f^4/a^3/(4*a*c-
```

```
b^2)/e*sum(((19*a*c-5*b^2)*_R^2*b*c*e^2+19*a*b*c^2*d^2-5*b^3*c*d^2+2*(19*a*
c-5*b^2)*_R*b*c*d*e-14*a^2*c^2+24*a*b^2*c-5*b^4)/(2*_R^3*c*e^3+6*_R^2*c*d*e
^2+6*_R*c*d^2*e+2*c*d^3+_R*b*e+b*d)*ln(-_R+x),_R=RootOf(_Z^4*c*e^4+4*_Z^3*c
*d*e^3+c*d^4+b*d^2+(6*c*d^2*e^2+b*e^2)*_Z^2+(4*c*d^3*e+2*b*d*e)*_Z+a))-1/3/
f^4/a^2/e/(e*x+d)^3+2/f^4/a^3*b/e/(e*x+d)
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*f*x+d*f)^4/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="maxim
a")
```

```
[Out] Timed out
```

mupad [B] time = 10.45, size = 13781, normalized size = 32.58

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((d*f + e*f*x)^4*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2),x)
```

```
[Out] atan(((-(25*b^15 - 25*b^6*(-(4*a*c - b^2)^9)^(1/2) - 80640*a^7*b*c^7 + 6366
*a^2*b^11*c^2 - 35767*a^3*b^9*c^3 + 116928*a^4*b^7*c^4 - 219744*a^5*b^5*c^5
 + 215040*a^6*b^3*c^6 + 49*a^3*c^3*(-(4*a*c - b^2)^9)^(1/2) - 615*a*b^13*c
- 246*a^2*b^2*c^2*(-(4*a*c - b^2)^9)^(1/2) + 165*a*b^4*c*(-(4*a*c - b^2)^9)
^(1/2))/(32*(a^7*b^12*e^2*f^8 + 4096*a^13*c^6*e^2*f^8 + 240*a^9*b^8*c^2*e^2
*f^8 - 1280*a^10*b^6*c^3*e^2*f^8 + 3840*a^11*b^4*c^4*e^2*f^8 - 6144*a^12*b^
2*c^5*e^2*f^8 - 24*a^8*b^10*c*e^2*f^8)))^(1/2)*((-(25*b^15 - 25*b^6*(-(4*a*
c - b^2)^9)^(1/2) - 80640*a^7*b*c^7 + 6366*a^2*b^11*c^2 - 35767*a^3*b^9*c^3
 + 116928*a^4*b^7*c^4 - 219744*a^5*b^5*c^5 + 215040*a^6*b^3*c^6 + 49*a^3*c^
3*(-(4*a*c - b^2)^9)^(1/2) - 615*a*b^13*c - 246*a^2*b^2*c^2*(-(4*a*c - b^2)
^9)^(1/2) + 165*a*b^4*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^7*b^12*e^2*f^8 + 4
096*a^13*c^6*e^2*f^8 + 240*a^9*b^8*c^2*e^2*f^8 - 1280*a^10*b^6*c^3*e^2*f^8
+ 3840*a^11*b^4*c^4*e^2*f^8 - 6144*a^12*b^2*c^5*e^2*f^8 - 24*a^8*b^10*c*e^2
*f^8)))^(1/2)*((-(25*b^15 - 25*b^6*(-(4*a*c - b^2)^9)^(1/2) - 80640*a^7*b*c
^7 + 6366*a^2*b^11*c^2 - 35767*a^3*b^9*c^3 + 116928*a^4*b^7*c^4 - 219744*a^
5*b^5*c^5 + 215040*a^6*b^3*c^6 + 49*a^3*c^3*(-(4*a*c - b^2)^9)^(1/2) - 615*
a*b^13*c - 246*a^2*b^2*c^2*(-(4*a*c - b^2)^9)^(1/2) + 165*a*b^4*c*(-(4*a*c
- b^2)^9)^(1/2))/(32*(a^7*b^12*e^2*f^8 + 4096*a^13*c^6*e^2*f^8 + 240*a^9*b^
8*c^2*e^2*f^8 - 1280*a^10*b^6*c^3*e^2*f^8 + 3840*a^11*b^4*c^4*e^2*f^8 - 614
4*a^12*b^2*c^5*e^2*f^8 - 24*a^8*b^10*c*e^2*f^8)))^(1/2)*(x*(256*a^15*b^13*c
^2*e^14*f^20 - 6144*a^16*b^11*c^3*e^14*f^20 + 61440*a^17*b^9*c^4*e^14*f^20
- 327680*a^18*b^7*c^5*e^14*f^20 + 983040*a^19*b^5*c^6*e^14*f^20 - 1572864*a
^20*b^3*c^7*e^14*f^20 + 1048576*a^21*b*c^8*e^14*f^20) + 1048576*a^21*b*c^8*
d*e^13*f^20 + 256*a^15*b^13*c^2*d*e^13*f^20 - 6144*a^16*b^11*c^3*d*e^13*f^2
0 + 61440*a^17*b^9*c^4*d*e^13*f^20 - 327680*a^18*b^7*c^5*d*e^13*f^20 + 9830
40*a^19*b^5*c^6*d*e^13*f^20 - 1572864*a^20*b^3*c^7*d*e^13*f^20) - 917504*a^
19*c^9*e^12*f^16 + 320*a^12*b^14*c^2*e^12*f^16 - 7936*a^13*b^12*c^3*e^12*f^
16 + 82816*a^14*b^10*c^4*e^12*f^16 - 468480*a^15*b^8*c^5*e^12*f^16 + 153600
0*a^16*b^6*c^6*e^12*f^16 - 2867200*a^17*b^4*c^7*e^12*f^16 + 2719744*a^18*b^
2*c^8*e^12*f^16) - x*(401408*a^16*c^10*e^12*f^12 - 400*a^9*b^14*c^3*e^12*f^
12 + 9440*a^10*b^12*c^4*e^12*f^12 - 92816*a^11*b^10*c^5*e^12*f^12 + 488096*
a^12*b^8*c^6*e^12*f^12 - 1458688*a^13*b^6*c^7*e^12*f^12 + 2401280*a^14*b^4*
c^8*e^12*f^12 - 1871872*a^15*b^2*c^9*e^12*f^12) - 401408*a^16*c^10*d*e^11*f
^12 + 400*a^9*b^14*c^3*d*e^11*f^12 - 9440*a^10*b^12*c^4*d*e^11*f^12 + 92816
*a^11*b^10*c^5*d*e^11*f^12 - 488096*a^12*b^8*c^6*d*e^11*f^12 + 1458688*a^13
*b^6*c^7*d*e^11*f^12 - 2401280*a^14*b^4*c^8*d*e^11*f^12 + 1871872*a^15*b^2*
c^9*d*e^11*f^12)*1i + (-(25*b^15 - 25*b^6*(-(4*a*c - b^2)^9)^(1/2) - 80640*
a^7*b*c^7 + 6366*a^2*b^11*c^2 - 35767*a^3*b^9*c^3 + 116928*a^4*b^7*c^4 - 21
```

9744*a^5*b^5*c^5 + 215040*a^6*b^3*c^6 + 49*a^3*c^3*(-(4*a*c - b^2)^9)^(1/2)
 - 615*a*b^13*c - 246*a^2*b^2*c^2*(-(4*a*c - b^2)^9)^(1/2) + 165*a*b^4*c*(-
(4*a*c - b^2)^9)^(1/2))/(32*(a^7*b^12*e^2*f^8 + 4096*a^13*c^6*e^2*f^8 + 240
*a^9*b^8*c^2*e^2*f^8 - 1280*a^10*b^6*c^3*e^2*f^8 + 3840*a^11*b^4*c^4*e^2*f^
8 - 6144*a^12*b^2*c^5*e^2*f^8 - 24*a^8*b^10*c*e^2*f^8)))^(1/2)*((-(25*b^15
- 25*b^6*(-(4*a*c - b^2)^9)^(1/2) - 80640*a^7*b*c^7 + 6366*a^2*b^11*c^2 - 3
5767*a^3*b^9*c^3 + 116928*a^4*b^7*c^4 - 219744*a^5*b^5*c^5 + 215040*a^6*b^3
*c^6 + 49*a^3*c^3*(-(4*a*c - b^2)^9)^(1/2) - 615*a*b^13*c - 246*a^2*b^2*c^2
*(-(4*a*c - b^2)^9)^(1/2) + 165*a*b^4*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^7*
b^12*e^2*f^8 + 4096*a^13*c^6*e^2*f^8 + 240*a^9*b^8*c^2*e^2*f^8 - 1280*a^10*
b^6*c^3*e^2*f^8 + 3840*a^11*b^4*c^4*e^2*f^8 - 6144*a^12*b^2*c^5*e^2*f^8 - 2
4*a^8*b^10*c*e^2*f^8)))^(1/2)*((-(25*b^15 - 25*b^6*(-(4*a*c - b^2)^9)^(1/2)
 - 80640*a^7*b*c^7 + 6366*a^2*b^11*c^2 - 35767*a^3*b^9*c^3 + 116928*a^4*b^7
*c^4 - 219744*a^5*b^5*c^5 + 215040*a^6*b^3*c^6 + 49*a^3*c^3*(-(4*a*c - b^2)
^9)^(1/2) - 615*a*b^13*c - 246*a^2*b^2*c^2*(-(4*a*c - b^2)^9)^(1/2) + 165*a
*b^4*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^7*b^12*e^2*f^8 + 4096*a^13*c^6*e^2*
f^8 + 240*a^9*b^8*c^2*e^2*f^8 - 1280*a^10*b^6*c^3*e^2*f^8 + 3840*a^11*b^4*c
^4*e^2*f^8 - 6144*a^12*b^2*c^5*e^2*f^8 - 24*a^8*b^10*c*e^2*f^8)))^(1/2)*(x*
(256*a^15*b^13*c^2*e^14*f^20 - 6144*a^16*b^11*c^3*e^14*f^20 + 61440*a^17*b^
9*c^4*e^14*f^20 - 327680*a^18*b^7*c^5*e^14*f^20 + 983040*a^19*b^5*c^6*e^14*
f^20 - 1572864*a^20*b^3*c^7*e^14*f^20 + 1048576*a^21*b*c^8*e^14*f^20) + 104
8576*a^21*b*c^8*d*e^13*f^20 + 256*a^15*b^13*c^2*d*e^13*f^20 - 6144*a^16*b^1
1*c^3*d*e^13*f^20 + 61440*a^17*b^9*c^4*d*e^13*f^20 - 327680*a^18*b^7*c^5*d*
e^13*f^20 + 983040*a^19*b^5*c^6*d*e^13*f^20 - 1572864*a^20*b^3*c^7*d*e^13*f
^20) + 917504*a^19*c^9*e^12*f^16 - 320*a^12*b^14*c^2*e^12*f^16 + 7936*a^13*
b^12*c^3*e^12*f^16 - 82816*a^14*b^10*c^4*e^12*f^16 + 468480*a^15*b^8*c^5*e^
12*f^16 - 1536000*a^16*b^6*c^6*e^12*f^16 + 2867200*a^17*b^4*c^7*e^12*f^16 -
 2719744*a^18*b^2*c^8*e^12*f^16) - x*(401408*a^16*c^10*e^12*f^12 - 400*a^9*
b^14*c^3*e^12*f^12 + 9440*a^10*b^12*c^4*e^12*f^12 - 92816*a^11*b^10*c^5*e^1
2*f^12 + 488096*a^12*b^8*c^6*e^12*f^12 - 1458688*a^13*b^6*c^7*e^12*f^12 + 2
401280*a^14*b^4*c^8*e^12*f^12 - 1871872*a^15*b^2*c^9*e^12*f^12) - 401408*a^
16*c^10*d*e^11*f^12 + 400*a^9*b^14*c^3*d*e^11*f^12 - 9440*a^10*b^12*c^4*d*e
^11*f^12 + 92816*a^11*b^10*c^5*d*e^11*f^12 - 488096*a^12*b^8*c^6*d*e^11*f^1
2 + 1458688*a^13*b^6*c^7*d*e^11*f^12 - 2401280*a^14*b^4*c^8*d*e^11*f^12 + 1
871872*a^15*b^2*c^9*d*e^11*f^12)*1i)/((-(25*b^15 - 25*b^6*(-(4*a*c - b^2)^9
)^(1/2) - 80640*a^7*b*c^7 + 6366*a^2*b^11*c^2 - 35767*a^3*b^9*c^3 + 116928*
a^4*b^7*c^4 - 219744*a^5*b^5*c^5 + 215040*a^6*b^3*c^6 + 49*a^3*c^3*(-(4*a*c
 - b^2)^9)^(1/2) - 615*a*b^13*c - 246*a^2*b^2*c^2*(-(4*a*c - b^2)^9)^(1/2)
+ 165*a*b^4*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^7*b^12*e^2*f^8 + 4096*a^13*c
^6*e^2*f^8 + 240*a^9*b^8*c^2*e^2*f^8 - 1280*a^10*b^6*c^3*e^2*f^8 + 3840*a^1
1*b^4*c^4*e^2*f^8 - 6144*a^12*b^2*c^5*e^2*f^8 - 24*a^8*b^10*c*e^2*f^8)))^(1
/2)*((-(25*b^15 - 25*b^6*(-(4*a*c - b^2)^9)^(1/2) - 80640*a^7*b*c^7 + 6366*
a^2*b^11*c^2 - 35767*a^3*b^9*c^3 + 116928*a^4*b^7*c^4 - 219744*a^5*b^5*c^5
+ 215040*a^6*b^3*c^6 + 49*a^3*c^3*(-(4*a*c - b^2)^9)^(1/2) - 615*a*b^13*c -
 246*a^2*b^2*c^2*(-(4*a*c - b^2)^9)^(1/2) + 165*a*b^4*c*(-(4*a*c - b^2)^9)^
(1/2))/(32*(a^7*b^12*e^2*f^8 + 4096*a^13*c^6*e^2*f^8 + 240*a^9*b^8*c^2*e^2*
f^8 - 1280*a^10*b^6*c^3*e^2*f^8 + 3840*a^11*b^4*c^4*e^2*f^8 - 6144*a^12*b^2
*c^5*e^2*f^8 - 24*a^8*b^10*c*e^2*f^8)))^(1/2)*((-(25*b^15 - 25*b^6*(-(4*a*c
 - b^2)^9)^(1/2) - 80640*a^7*b*c^7 + 6366*a^2*b^11*c^2 - 35767*a^3*b^9*c^3
+ 116928*a^4*b^7*c^4 - 219744*a^5*b^5*c^5 + 215040*a^6*b^3*c^6 + 49*a^3*c^3
*(-(4*a*c - b^2)^9)^(1/2) - 615*a*b^13*c - 246*a^2*b^2*c^2*(-(4*a*c - b^2)^
9)^(1/2) + 165*a*b^4*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^7*b^12*e^2*f^8 + 40
96*a^13*c^6*e^2*f^8 + 240*a^9*b^8*c^2*e^2*f^8 - 1280*a^10*b^6*c^3*e^2*f^8 +
 3840*a^11*b^4*c^4*e^2*f^8 - 6144*a^12*b^2*c^5*e^2*f^8 - 24*a^8*b^10*c*e^2*
f^8)))^(1/2)*(x*(256*a^15*b^13*c^2*e^14*f^20 - 6144*a^16*b^11*c^3*e^14*f^20
 + 61440*a^17*b^9*c^4*e^14*f^20 - 327680*a^18*b^7*c^5*e^14*f^20 + 983040*a^
19*b^5*c^6*e^14*f^20 - 1572864*a^20*b^3*c^7*e^14*f^20 + 1048576*a^21*b*c^8*
e^14*f^20) + 1048576*a^21*b*c^8*d*e^13*f^20 + 256*a^15*b^13*c^2*d*e^13*f^20
 - 6144*a^16*b^11*c^3*d*e^13*f^20 + 61440*a^17*b^9*c^4*d*e^13*f^20 - 327680

*a^18*b^7*c^5*d*e^13*f^20 + 983040*a^19*b^5*c^6*d*e^13*f^20 - 1572864*a^20*b^3*c^7*d*e^13*f^20) - 917504*a^19*c^9*e^12*f^16 + 320*a^12*b^14*c^2*e^12*f^16 - 7936*a^13*b^12*c^3*e^12*f^16 + 82816*a^14*b^10*c^4*e^12*f^16 - 468480*a^15*b^8*c^5*e^12*f^16 + 1536000*a^16*b^6*c^6*e^12*f^16 - 2867200*a^17*b^4*c^7*e^12*f^16 + 2719744*a^18*b^2*c^8*e^12*f^16) - x*(401408*a^16*c^10*e^12*f^12 - 400*a^9*b^14*c^3*e^12*f^12 + 9440*a^10*b^12*c^4*e^12*f^12 - 92816*a^11*b^10*c^5*e^12*f^12 + 488096*a^12*b^8*c^6*e^12*f^12 - 1458688*a^13*b^6*c^7*e^12*f^12 + 2401280*a^14*b^4*c^8*e^12*f^12 - 1871872*a^15*b^2*c^9*e^12*f^12) - 401408*a^16*c^10*d*e^11*f^12 + 400*a^9*b^14*c^3*d*e^11*f^12 - 9440*a^10*b^12*c^4*d*e^11*f^12 + 92816*a^11*b^10*c^5*d*e^11*f^12 - 488096*a^12*b^8*c^6*d*e^11*f^12 + 1458688*a^13*b^6*c^7*d*e^11*f^12 - 2401280*a^14*b^4*c^8*d*e^11*f^12 + 1871872*a^15*b^2*c^9*d*e^11*f^12) - (-(25*b^15 - 25*b^6*(-(4*a*c - b^2)^9)^(1/2) - 80640*a^7*b*c^7 + 6366*a^2*b^11*c^2 - 35767*a^3*b^9*c^3 + 116928*a^4*b^7*c^4 - 219744*a^5*b^5*c^5 + 215040*a^6*b^3*c^6 + 49*a^3*c^3*(-(4*a*c - b^2)^9)^(1/2) - 615*a*b^13*c - 246*a^2*b^2*c^2*(-(4*a*c - b^2)^9)^(1/2) + 165*a*b^4*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^7*b^12*e^2*f^8 + 4096*a^13*c^6*e^2*f^8 + 240*a^9*b^8*c^2*e^2*f^8 - 1280*a^10*b^6*c^3*e^2*f^8 + 3840*a^11*b^4*c^4*e^2*f^8 - 6144*a^12*b^2*c^5*e^2*f^8 - 24*a^8*b^10*c*e^2*f^8)))^(1/2)*((-(25*b^15 - 25*b^6*(-(4*a*c - b^2)^9)^(1/2) - 80640*a^7*b*c^7 + 6366*a^2*b^11*c^2 - 35767*a^3*b^9*c^3 + 116928*a^4*b^7*c^4 - 219744*a^5*b^5*c^5 + 215040*a^6*b^3*c^6 + 49*a^3*c^3*(-(4*a*c - b^2)^9)^(1/2) - 615*a*b^13*c - 246*a^2*b^2*c^2*(-(4*a*c - b^2)^9)^(1/2) + 165*a*b^4*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^7*b^12*e^2*f^8 + 4096*a^13*c^6*e^2*f^8 + 240*a^9*b^8*c^2*e^2*f^8 - 1280*a^10*b^6*c^3*e^2*f^8 + 3840*a^11*b^4*c^4*e^2*f^8 - 6144*a^12*b^2*c^5*e^2*f^8 - 24*a^8*b^10*c*e^2*f^8)))^(1/2)*((-(25*b^15 - 25*b^6*(-(4*a*c - b^2)^9)^(1/2) - 80640*a^7*b*c^7 + 6366*a^2*b^11*c^2 - 35767*a^3*b^9*c^3 + 116928*a^4*b^7*c^4 - 219744*a^5*b^5*c^5 + 215040*a^6*b^3*c^6 + 49*a^3*c^3*(-(4*a*c - b^2)^9)^(1/2) - 615*a*b^13*c - 246*a^2*b^2*c^2*(-(4*a*c - b^2)^9)^(1/2) + 165*a*b^4*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^7*b^12*e^2*f^8 + 4096*a^13*c^6*e^2*f^8 + 240*a^9*b^8*c^2*e^2*f^8 - 1280*a^10*b^6*c^3*e^2*f^8 + 3840*a^11*b^4*c^4*e^2*f^8 - 6144*a^12*b^2*c^5*e^2*f^8 - 24*a^8*b^10*c*e^2*f^8)))^(1/2)*(x*(256*a^15*b^13*c^2*e^14*f^20 - 6144*a^16*b^11*c^3*e^14*f^20 + 61440*a^17*b^9*c^4*e^14*f^20 - 327680*a^18*b^7*c^5*e^14*f^20 + 983040*a^19*b^5*c^6*e^14*f^20 - 1572864*a^20*b^3*c^7*e^14*f^20 + 1048576*a^21*b*c^8*e^14*f^20) + 1048576*a^21*b*c^8*d*e^13*f^20 + 256*a^15*b^13*c^2*d*e^13*f^20 - 6144*a^16*b^11*c^3*d*e^13*f^20 + 61440*a^17*b^9*c^4*d*e^13*f^20 - 327680*a^18*b^7*c^5*d*e^13*f^20 + 983040*a^19*b^5*c^6*d*e^13*f^20 - 1572864*a^20*b^3*c^7*d*e^13*f^20) + 917504*a^19*c^9*e^12*f^16 - 320*a^12*b^14*c^2*e^12*f^16 + 7936*a^13*b^12*c^3*e^12*f^16 - 82816*a^14*b^10*c^4*e^12*f^16 + 468480*a^15*b^8*c^5*e^12*f^16 - 1536000*a^16*b^6*c^6*e^12*f^16 + 2867200*a^17*b^4*c^7*e^12*f^16 - 2719744*a^18*b^2*c^8*e^12*f^16) - x*(401408*a^16*c^10*e^12*f^12 - 400*a^9*b^14*c^3*e^12*f^12 + 9440*a^10*b^12*c^4*e^12*f^12 - 92816*a^11*b^10*c^5*e^12*f^12 + 488096*a^12*b^8*c^6*e^12*f^12 - 1458688*a^13*b^6*c^7*e^12*f^12 + 2401280*a^14*b^4*c^8*e^12*f^12 - 1871872*a^15*b^2*c^9*e^12*f^12) - 401408*a^16*c^10*d*e^11*f^12 + 400*a^9*b^14*c^3*d*e^11*f^12 - 9440*a^10*b^12*c^4*d*e^11*f^12 + 92816*a^11*b^10*c^5*d*e^11*f^12 - 488096*a^12*b^8*c^6*d*e^11*f^12 + 1458688*a^13*b^6*c^7*d*e^11*f^12 - 2401280*a^14*b^4*c^8*d*e^11*f^12 + 1871872*a^15*b^2*c^9*d*e^11*f^12) + 1800*a^9*b^9*c^6*e^10*f^8 - 29080*a^10*b^7*c^7*e^10*f^8 + 176032*a^11*b^5*c^8*e^10*f^8 - 473216*a^12*b^3*c^9*e^10*f^8 + 476672*a^13*b*c^10*e^10*f^8))*(-(25*b^15 - 25*b^6*(-(4*a*c - b^2)^9)^(1/2) - 80640*a^7*b*c^7 + 6366*a^2*b^11*c^2 - 35767*a^3*b^9*c^3 + 116928*a^4*b^7*c^4 - 219744*a^5*b^5*c^5 + 215040*a^6*b^3*c^6 + 49*a^3*c^3*(-(4*a*c - b^2)^9)^(1/2) - 615*a*b^13*c - 246*a^2*b^2*c^2*(-(4*a*c - b^2)^9)^(1/2) + 165*a*b^4*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^7*b^12*e^2*f^8 + 4096*a^13*c^6*e^2*f^8 + 240*a^9*b^8*c^2*e^2*f^8 - 1280*a^10*b^6*c^3*e^2*f^8 + 3840*a^11*b^4*c^4*e^2*f^8 - 6144*a^12*b^2*c^5*e^2*f^8 - 24*a^8*b^10*c*e^2*f^8)))^(1/2)*2i - ((x^4*(15*b^4*e^3 + 14*a^2*c^2*e^3 + 225*b^3*c*d^2*e^3 - 62*a*b^2*c*e^3 - 855*a*b*c^2*d^2*e^3))/(6*a*(4*a^3*c - a^2*b^2)) + (3*x^5*(5*b^3*c*d*e^4 - 19*a*b*c^2*d*e^4))/(a*(4*a^3*c - a^2*b^2))

+ (2*x^3*(15*b^4*d*e^2 + 14*a^2*c^2*d*e^2 + 75*b^3*c*d^3*e^2 - 62*a*b^2*c*
d*e^2 - 285*a*b*c^2*d^3*e^2))/(3*a*(4*a^3*c - a^2*b^2)) + (x*(30*b^4*d^3 +
45*b^3*c*d^5 + 28*a^2*c^2*d^3 + 10*a*b^3*d - 40*a^2*b*c*d - 124*a*b^2*c*d^3
- 171*a*b*c^2*d^5))/(3*a*(4*a^3*c - a^2*b^2)) + (x^6*(5*b^3*c*e^5 - 19*a*b
*c^2*e^5))/(2*a*(4*a^3*c - a^2*b^2)) + (x^2*(90*b^4*d^2*e + 10*a*b^3*e + 84
*a^2*c^2*d^2*e - 40*a^2*b*c*e + 225*b^3*c*d^4*e - 372*a*b^2*c*d^2*e - 855*a
*b*c^2*d^4*e))/(6*a*(4*a^3*c - a^2*b^2)) + (8*a^3*c - 2*a^2*b^2 + 15*b^4*d^
4 + 10*a*b^3*d^2 + 15*b^3*c*d^6 + 14*a^2*c^2*d^4 - 40*a^2*b*c*d^2 - 62*a*b^
2*c*d^4 - 57*a*b*c^2*d^6)/(6*a*e*(4*a^3*c - a^2*b^2)))/(x*(3*a*d^2*e*f^4 +
5*b*d^4*e*f^4 + 7*c*d^6*e*f^4) + x^4*(35*c*d^3*e^4*f^4 + 5*b*d*e^4*f^4) + x
^2*(10*b*d^3*e^2*f^4 + 21*c*d^5*e^2*f^4 + 3*a*d*e^2*f^4) + x^5*(b*e^5*f^4 +
21*c*d^2*e^5*f^4) + x^3*(a*e^3*f^4 + 10*b*d^2*e^3*f^4 + 35*c*d^4*e^3*f^4)
+ a*d^3*f^4 + b*d^5*f^4 + c*d^7*f^4 + c*e^7*f^4*x^7 + 7*c*d*e^6*f^4*x^6) +
atan(((-(25*b^15 + 25*b^6*(-(4*a*c - b^2)^9)^(1/2) - 80640*a^7*b*c^7 + 6366
*a^2*b^11*c^2 - 35767*a^3*b^9*c^3 + 116928*a^4*b^7*c^4 - 219744*a^5*b^5*c^5
+ 215040*a^6*b^3*c^6 - 49*a^3*c^3*(-(4*a*c - b^2)^9)^(1/2) - 615*a*b^13*c
+ 246*a^2*b^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 165*a*b^4*c*(-(4*a*c - b^2)^9)
^(1/2))/(32*(a^7*b^12*e^2*f^8 + 4096*a^13*c^6*e^2*f^8 + 240*a^9*b^8*c^2*e^2
*f^8 - 1280*a^10*b^6*c^3*e^2*f^8 + 3840*a^11*b^4*c^4*e^2*f^8 - 6144*a^12*b^
2*c^5*e^2*f^8 - 24*a^8*b^10*c*e^2*f^8)))^(1/2)*((-(25*b^15 + 25*b^6*(-(4*a*
c - b^2)^9)^(1/2) - 80640*a^7*b*c^7 + 6366*a^2*b^11*c^2 - 35767*a^3*b^9*c^3
+ 116928*a^4*b^7*c^4 - 219744*a^5*b^5*c^5 + 215040*a^6*b^3*c^6 - 49*a^3*c^
3*(-(4*a*c - b^2)^9)^(1/2) - 615*a*b^13*c + 246*a^2*b^2*c^2*(-(4*a*c - b^2)
^9)^(1/2) - 165*a*b^4*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^7*b^12*e^2*f^8 + 4
096*a^13*c^6*e^2*f^8 + 240*a^9*b^8*c^2*e^2*f^8 - 1280*a^10*b^6*c^3*e^2*f^8
+ 3840*a^11*b^4*c^4*e^2*f^8 - 6144*a^12*b^2*c^5*e^2*f^8 - 24*a^8*b^10*c*e^2
f^8)))^(1/2)((-(25*b^15 + 25*b^6*(-(4*a*c - b^2)^9)^(1/2) - 80640*a^7*b*c
^7 + 6366*a^2*b^11*c^2 - 35767*a^3*b^9*c^3 + 116928*a^4*b^7*c^4 - 219744*a^
5*b^5*c^5 + 215040*a^6*b^3*c^6 - 49*a^3*c^3*(-(4*a*c - b^2)^9)^(1/2) - 615*
a*b^13*c + 246*a^2*b^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 165*a*b^4*c*(-(4*a*c
- b^2)^9)^(1/2))/(32*(a^7*b^12*e^2*f^8 + 4096*a^13*c^6*e^2*f^8 + 240*a^9*b^
8*c^2*e^2*f^8 - 1280*a^10*b^6*c^3*e^2*f^8 + 3840*a^11*b^4*c^4*e^2*f^8 - 614
4*a^12*b^2*c^5*e^2*f^8 - 24*a^8*b^10*c*e^2*f^8)))^(1/2)*(x*(256*a^15*b^13*c
^2*e^14*f^20 - 6144*a^16*b^11*c^3*e^14*f^20 + 61440*a^17*b^9*c^4*e^14*f^20
- 327680*a^18*b^7*c^5*e^14*f^20 + 983040*a^19*b^5*c^6*e^14*f^20 - 1572864*a
^20*b^3*c^7*e^14*f^20 + 1048576*a^21*b*c^8*e^14*f^20) + 1048576*a^21*b*c^8*
d*e^13*f^20 + 256*a^15*b^13*c^2*d*e^13*f^20 - 6144*a^16*b^11*c^3*d*e^13*f^2
0 + 61440*a^17*b^9*c^4*d*e^13*f^20 - 327680*a^18*b^7*c^5*d*e^13*f^20 + 9830
40*a^19*b^5*c^6*d*e^13*f^20 - 1572864*a^20*b^3*c^7*d*e^13*f^20) - 917504*a^
19*c^9*e^12*f^16 + 320*a^12*b^14*c^2*e^12*f^16 - 7936*a^13*b^12*c^3*e^12*f^
16 + 82816*a^14*b^10*c^4*e^12*f^16 - 468480*a^15*b^8*c^5*e^12*f^16 + 153600
0*a^16*b^6*c^6*e^12*f^16 - 2867200*a^17*b^4*c^7*e^12*f^16 + 2719744*a^18*b^
2*c^8*e^12*f^16) - x*(401408*a^16*c^10*e^12*f^12 - 400*a^9*b^14*c^3*e^12*f^
12 + 9440*a^10*b^12*c^4*e^12*f^12 - 92816*a^11*b^10*c^5*e^12*f^12 + 488096*
a^12*b^8*c^6*e^12*f^12 - 1458688*a^13*b^6*c^7*e^12*f^12 + 2401280*a^14*b^4*
c^8*e^12*f^12 - 1871872*a^15*b^2*c^9*e^12*f^12) - 401408*a^16*c^10*d*e^11*f
^12 + 400*a^9*b^14*c^3*d*e^11*f^12 - 9440*a^10*b^12*c^4*d*e^11*f^12 + 92816
*a^11*b^10*c^5*d*e^11*f^12 - 488096*a^12*b^8*c^6*d*e^11*f^12 + 1458688*a^13
*b^6*c^7*d*e^11*f^12 - 2401280*a^14*b^4*c^8*d*e^11*f^12 + 1871872*a^15*b^2*
c^9*d*e^11*f^12)*1i + (-(25*b^15 + 25*b^6*(-(4*a*c - b^2)^9)^(1/2) - 80640*
a^7*b*c^7 + 6366*a^2*b^11*c^2 - 35767*a^3*b^9*c^3 + 116928*a^4*b^7*c^4 - 21
9744*a^5*b^5*c^5 + 215040*a^6*b^3*c^6 - 49*a^3*c^3*(-(4*a*c - b^2)^9)^(1/2)
- 615*a*b^13*c + 246*a^2*b^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 165*a*b^4*c*(-
(4*a*c - b^2)^9)^(1/2))/(32*(a^7*b^12*e^2*f^8 + 4096*a^13*c^6*e^2*f^8 + 240
*a^9*b^8*c^2*e^2*f^8 - 1280*a^10*b^6*c^3*e^2*f^8 + 3840*a^11*b^4*c^4*e^2*f^
8 - 6144*a^12*b^2*c^5*e^2*f^8 - 24*a^8*b^10*c*e^2*f^8)))^(1/2)*((-(25*b^15
+ 25*b^6*(-(4*a*c - b^2)^9)^(1/2) - 80640*a^7*b*c^7 + 6366*a^2*b^11*c^2 - 3
5767*a^3*b^9*c^3 + 116928*a^4*b^7*c^4 - 219744*a^5*b^5*c^5 + 215040*a^6*b^3
*c^6 - 49*a^3*c^3*(-(4*a*c - b^2)^9)^(1/2) - 615*a*b^13*c + 246*a^2*b^2*c^2

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*(-(4*a*c - b^2)^9)^(1/2) - 165*a*b^4*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^7*
b^12*e^2*f^8 + 4096*a^13*c^6*e^2*f^8 + 240*a^9*b^8*c^2*e^2*f^8 - 1280*a^10*
b^6*c^3*e^2*f^8 + 3840*a^11*b^4*c^4*e^2*f^8 - 6144*a^12*b^2*c^5*e^2*f^8 - 2
4*a^8*b^10*c*e^2*f^8)))^(1/2)*((-(25*b^15 + 25*b^6*(-(4*a*c - b^2)^9)^(1/2)
 - 80640*a^7*b*c^7 + 6366*a^2*b^11*c^2 - 35767*a^3*b^9*c^3 + 116928*a^4*b^7
*c^4 - 219744*a^5*b^5*c^5 + 215040*a^6*b^3*c^6 - 49*a^3*c^3*(-(4*a*c - b^2)
^9)^(1/2) - 615*a*b^13*c + 246*a^2*b^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 165*a
*b^4*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^7*b^12*e^2*f^8 + 4096*a^13*c^6*e^2*
f^8 + 240*a^9*b^8*c^2*e^2*f^8 - 1280*a^10*b^6*c^3*e^2*f^8 + 3840*a^11*b^4*c
^4*e^2*f^8 - 6144*a^12*b^2*c^5*e^2*f^8 - 24*a^8*b^10*c*e^2*f^8)))^(1/2)*(x*
(256*a^15*b^13*c^2*e^14*f^20 - 6144*a^16*b^11*c^3*e^14*f^20 + 61440*a^17*b^
9*c^4*e^14*f^20 - 327680*a^18*b^7*c^5*e^14*f^20 + 983040*a^19*b^5*c^6*e^14*
f^20 - 1572864*a^20*b^3*c^7*e^14*f^20 + 1048576*a^21*b*c^8*e^14*f^20) + 104
8576*a^21*b*c^8*d*e^13*f^20 + 256*a^15*b^13*c^2*d*e^13*f^20 - 6144*a^16*b^1
1*c^3*d*e^13*f^20 + 61440*a^17*b^9*c^4*d*e^13*f^20 - 327680*a^18*b^7*c^5*d*
e^13*f^20 + 983040*a^19*b^5*c^6*d*e^13*f^20 - 1572864*a^20*b^3*c^7*d*e^13*f
^20) + 917504*a^19*c^9*e^12*f^16 - 320*a^12*b^14*c^2*e^12*f^16 + 7936*a^13*
b^12*c^3*e^12*f^16 - 82816*a^14*b^10*c^4*e^12*f^16 + 468480*a^15*b^8*c^5*e^
12*f^16 - 1536000*a^16*b^6*c^6*e^12*f^16 + 2867200*a^17*b^4*c^7*e^12*f^16 -
 2719744*a^18*b^2*c^8*e^12*f^16) - x*(401408*a^16*c^10*e^12*f^12 - 400*a^9*
b^14*c^3*e^12*f^12 + 9440*a^10*b^12*c^4*e^12*f^12 - 92816*a^11*b^10*c^5*e^1
2*f^12 + 488096*a^12*b^8*c^6*e^12*f^12 - 1458688*a^13*b^6*c^7*e^12*f^12 + 2
401280*a^14*b^4*c^8*e^12*f^12 - 1871872*a^15*b^2*c^9*e^12*f^12) - 401408*a^
16*c^10*d*e^11*f^12 + 400*a^9*b^14*c^3*d*e^11*f^12 - 9440*a^10*b^12*c^4*d*e
^11*f^12 + 92816*a^11*b^10*c^5*d*e^11*f^12 - 488096*a^12*b^8*c^6*d*e^11*f^1
2 + 1458688*a^13*b^6*c^7*d*e^11*f^12 - 2401280*a^14*b^4*c^8*d*e^11*f^12 + 1
871872*a^15*b^2*c^9*d*e^11*f^12)*1i)/((-(25*b^15 + 25*b^6*(-(4*a*c - b^2)^9
)^(1/2) - 80640*a^7*b*c^7 + 6366*a^2*b^11*c^2 - 35767*a^3*b^9*c^3 + 116928*
a^4*b^7*c^4 - 219744*a^5*b^5*c^5 + 215040*a^6*b^3*c^6 - 49*a^3*c^3*(-(4*a*c
 - b^2)^9)^(1/2) - 615*a*b^13*c + 246*a^2*b^2*c^2*(-(4*a*c - b^2)^9)^(1/2)
- 165*a*b^4*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^7*b^12*e^2*f^8 + 4096*a^13*c
^6*e^2*f^8 + 240*a^9*b^8*c^2*e^2*f^8 - 1280*a^10*b^6*c^3*e^2*f^8 + 3840*a^1
1*b^4*c^4*e^2*f^8 - 6144*a^12*b^2*c^5*e^2*f^8 - 24*a^8*b^10*c*e^2*f^8)))^(1
/2)*((-(25*b^15 + 25*b^6*(-(4*a*c - b^2)^9)^(1/2) - 80640*a^7*b*c^7 + 6366*
a^2*b^11*c^2 - 35767*a^3*b^9*c^3 + 116928*a^4*b^7*c^4 - 219744*a^5*b^5*c^5
+ 215040*a^6*b^3*c^6 - 49*a^3*c^3*(-(4*a*c - b^2)^9)^(1/2) - 615*a*b^13*c +
 246*a^2*b^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 165*a*b^4*c*(-(4*a*c - b^2)^9)^
(1/2))/(32*(a^7*b^12*e^2*f^8 + 4096*a^13*c^6*e^2*f^8 + 240*a^9*b^8*c^2*e^2*
f^8 - 1280*a^10*b^6*c^3*e^2*f^8 + 3840*a^11*b^4*c^4*e^2*f^8 - 6144*a^12*b^2
*c^5*e^2*f^8 - 24*a^8*b^10*c*e^2*f^8)))^(1/2)*((-(25*b^15 + 25*b^6*(-(4*a*c
 - b^2)^9)^(1/2) - 80640*a^7*b*c^7 + 6366*a^2*b^11*c^2 - 35767*a^3*b^9*c^3
+ 116928*a^4*b^7*c^4 - 219744*a^5*b^5*c^5 + 215040*a^6*b^3*c^6 - 49*a^3*c^3
*(-(4*a*c - b^2)^9)^(1/2) - 615*a*b^13*c + 246*a^2*b^2*c^2*(-(4*a*c - b^2)^
9)^(1/2) - 165*a*b^4*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^7*b^12*e^2*f^8 + 40
96*a^13*c^6*e^2*f^8 + 240*a^9*b^8*c^2*e^2*f^8 - 1280*a^10*b^6*c^3*e^2*f^8 +
 3840*a^11*b^4*c^4*e^2*f^8 - 6144*a^12*b^2*c^5*e^2*f^8 - 24*a^8*b^10*c*e^2*
f^8)))^(1/2)*(x*(256*a^15*b^13*c^2*e^14*f^20 - 6144*a^16*b^11*c^3*e^14*f^20
 + 61440*a^17*b^9*c^4*e^14*f^20 - 327680*a^18*b^7*c^5*e^14*f^20 + 983040*a^
19*b^5*c^6*e^14*f^20 - 1572864*a^20*b^3*c^7*e^14*f^20 + 1048576*a^21*b*c^8*
e^14*f^20) + 1048576*a^21*b*c^8*d*e^13*f^20 + 256*a^15*b^13*c^2*d*e^13*f^20
 - 6144*a^16*b^11*c^3*d*e^13*f^20 + 61440*a^17*b^9*c^4*d*e^13*f^20 - 327680
*a^18*b^7*c^5*d*e^13*f^20 + 983040*a^19*b^5*c^6*d*e^13*f^20 - 1572864*a^20*
b^3*c^7*d*e^13*f^20) - 917504*a^19*c^9*e^12*f^16 + 320*a^12*b^14*c^2*e^12*f
^16 - 7936*a^13*b^12*c^3*e^12*f^16 + 82816*a^14*b^10*c^4*e^12*f^16 - 468480
*a^15*b^8*c^5*e^12*f^16 + 1536000*a^16*b^6*c^6*e^12*f^16 - 2867200*a^17*b^4
*c^7*e^12*f^16 + 2719744*a^18*b^2*c^8*e^12*f^16) - x*(401408*a^16*c^10*e^12
*f^12 - 400*a^9*b^14*c^3*e^12*f^12 + 9440*a^10*b^12*c^4*e^12*f^12 - 92816*a
^11*b^10*c^5*e^12*f^12 + 488096*a^12*b^8*c^6*e^12*f^12 - 1458688*a^13*b^6*c
^7*e^12*f^12 + 2401280*a^14*b^4*c^8*e^12*f^12 - 1871872*a^15*b^2*c^9*e^12*f
```

```
^12) - 401408*a^16*c^10*d*e^11*f^12 + 400*a^9*b^14*c^3*d*e^11*f^12 - 9440*a
^10*b^12*c^4*d*e^11*f^12 + 92816*a^11*b^10*c^5*d*e^11*f^12 - 488096*a^12*b^
8*c^6*d*e^11*f^12 + 1458688*a^13*b^6*c^7*d*e^11*f^12 - 2401280*a^14*b^4*c^8
*d*e^11*f^12 + 1871872*a^15*b^2*c^9*d*e^11*f^12) - (-(25*b^15 + 25*b^6*(-(4
*a*c - b^2)^9)^(1/2) - 80640*a^7*b*c^7 + 6366*a^2*b^11*c^2 - 35767*a^3*b^9*
c^3 + 116928*a^4*b^7*c^4 - 219744*a^5*b^5*c^5 + 215040*a^6*b^3*c^6 - 49*a^3
*c^3*(-(4*a*c - b^2)^9)^(1/2) - 615*a*b^13*c + 246*a^2*b^2*c^2*(-(4*a*c - b
^2)^9)^(1/2) - 165*a*b^4*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^7*b^12*e^2*f^8
+ 4096*a^13*c^6*e^2*f^8 + 240*a^9*b^8*c^2*e^2*f^8 - 1280*a^10*b^6*c^3*e^2*f
^8 + 3840*a^11*b^4*c^4*e^2*f^8 - 6144*a^12*b^2*c^5*e^2*f^8 - 24*a^8*b^10*c*
e^2*f^8)))^(1/2)*((-(25*b^15 + 25*b^6*(-(4*a*c - b^2)^9)^(1/2) - 80640*a^7*
b*c^7 + 6366*a^2*b^11*c^2 - 35767*a^3*b^9*c^3 + 116928*a^4*b^7*c^4 - 219744
*a^5*b^5*c^5 + 215040*a^6*b^3*c^6 - 49*a^3*c^3*(-(4*a*c - b^2)^9)^(1/2) - 6
15*a*b^13*c + 246*a^2*b^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 165*a*b^4*c*(-(4*a
*c - b^2)^9)^(1/2))/(32*(a^7*b^12*e^2*f^8 + 4096*a^13*c^6*e^2*f^8 + 240*a^9
*b^8*c^2*e^2*f^8 - 1280*a^10*b^6*c^3*e^2*f^8 + 3840*a^11*b^4*c^4*e^2*f^8 -
6144*a^12*b^2*c^5*e^2*f^8 - 24*a^8*b^10*c*e^2*f^8)))^(1/2)*((-(25*b^15 + 25
*b^6*(-(4*a*c - b^2)^9)^(1/2) - 80640*a^7*b*c^7 + 6366*a^2*b^11*c^2 - 35767
*a^3*b^9*c^3 + 116928*a^4*b^7*c^4 - 219744*a^5*b^5*c^5 + 215040*a^6*b^3*c^6
 - 49*a^3*c^3*(-(4*a*c - b^2)^9)^(1/2) - 615*a*b^13*c + 246*a^2*b^2*c^2*(-(
4*a*c - b^2)^9)^(1/2) - 165*a*b^4*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^7*b^12
*e^2*f^8 + 4096*a^13*c^6*e^2*f^8 + 240*a^9*b^8*c^2*e^2*f^8 - 1280*a^10*b^6*
c^3*e^2*f^8 + 3840*a^11*b^4*c^4*e^2*f^8 - 6144*a^12*b^2*c^5*e^2*f^8 - 24*a^
8*b^10*c*e^2*f^8)))^(1/2)*(x*(256*a^15*b^13*c^2*e^14*f^20 - 6144*a^16*b^11*
c^3*e^14*f^20 + 61440*a^17*b^9*c^4*e^14*f^20 - 327680*a^18*b^7*c^5*e^14*f^2
0 + 983040*a^19*b^5*c^6*e^14*f^20 - 1572864*a^20*b^3*c^7*e^14*f^20 + 104857
6*a^21*b*c^8*e^14*f^20) + 1048576*a^21*b*c^8*d*e^13*f^20 + 256*a^15*b^13*c^
2*d*e^13*f^20 - 6144*a^16*b^11*c^3*d*e^13*f^20 + 61440*a^17*b^9*c^4*d*e^13*
f^20 - 327680*a^18*b^7*c^5*d*e^13*f^20 + 983040*a^19*b^5*c^6*d*e^13*f^20 -
1572864*a^20*b^3*c^7*d*e^13*f^20) + 917504*a^19*c^9*e^12*f^16 - 320*a^12*b^
14*c^2*e^12*f^16 + 7936*a^13*b^12*c^3*e^12*f^16 - 82816*a^14*b^10*c^4*e^12*
f^16 + 468480*a^15*b^8*c^5*e^12*f^16 - 1536000*a^16*b^6*c^6*e^12*f^16 + 286
7200*a^17*b^4*c^7*e^12*f^16 - 2719744*a^18*b^2*c^8*e^12*f^16) - x*(401408*a
^16*c^10*e^12*f^12 - 400*a^9*b^14*c^3*e^12*f^12 + 9440*a^10*b^12*c^4*e^12*f
^12 - 92816*a^11*b^10*c^5*e^12*f^12 + 488096*a^12*b^8*c^6*e^12*f^12 - 14586
88*a^13*b^6*c^7*e^12*f^12 + 2401280*a^14*b^4*c^8*e^12*f^12 - 1871872*a^15*b
^2*c^9*e^12*f^12) - 401408*a^16*c^10*d*e^11*f^12 + 400*a^9*b^14*c^3*d*e^11*
f^12 - 9440*a^10*b^12*c^4*d*e^11*f^12 + 92816*a^11*b^10*c^5*d*e^11*f^12 - 4
88096*a^12*b^8*c^6*d*e^11*f^12 + 1458688*a^13*b^6*c^7*d*e^11*f^12 - 2401280
*a^14*b^4*c^8*d*e^11*f^12 + 1871872*a^15*b^2*c^9*d*e^11*f^12) + 1800*a^9*b^
9*c^6*e^10*f^8 - 29080*a^10*b^7*c^7*e^10*f^8 + 176032*a^11*b^5*c^8*e^10*f^8
 - 473216*a^12*b^3*c^9*e^10*f^8 + 476672*a^13*b*c^10*e^10*f^8))*(-(25*b^15
+ 25*b^6*(-(4*a*c - b^2)^9)^(1/2) - 80640*a^7*b*c^7 + 6366*a^2*b^11*c^2 - 3
5767*a^3*b^9*c^3 + 116928*a^4*b^7*c^4 - 219744*a^5*b^5*c^5 + 215040*a^6*b^3
*c^6 - 49*a^3*c^3*(-(4*a*c - b^2)^9)^(1/2) - 615*a*b^13*c + 246*a^2*b^2*c^2
*(-(4*a*c - b^2)^9)^(1/2) - 165*a*b^4*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^7*
b^12*e^2*f^8 + 4096*a^13*c^6*e^2*f^8 + 240*a^9*b^8*c^2*e^2*f^8 - 1280*a^10*
b^6*c^3*e^2*f^8 + 3840*a^11*b^4*c^4*e^2*f^8 - 6144*a^12*b^2*c^5*e^2*f^8 - 2
4*a^8*b^10*c*e^2*f^8)))^(1/2)*2i
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*f*x+d*f)**4/(a+b*(e*x+d)**2+c*(e*x+d)**4)**2,x)
[Out] Timed out
```

3.541 $\int \frac{(df+efx)^4}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^3}\,dx$

Optimal. Leaf size=353

$$-\frac{f^4(d+ex)\left(-4ac+7b^2+12bc(d+ex)^2\right)}{8e\left(b^2-4ac\right)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)}+\frac{f^4(d+ex)\left(2a+b(d+ex)^2\right)}{4e\left(b^2-4ac\right)\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}+\frac{3\sqrt{c}f^4\left(-2b\sqrt{b^2-4a\right.}{4\sqrt{2}e\left(b^2-\right.}$$

Rubi [A] time = 0.87, antiderivative size = 353, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {1142, 1120, 1178, 1166, 205}

$$-\frac{f^4(d+ex)\left(-4ac+7b^2+12bc(d+ex)^2\right)}{8e\left(b^2-4ac\right)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)}+\frac{f^4(d+ex)\left(2a+b(d+ex)^2\right)}{4e\left(b^2-4ac\right)\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}+\frac{3\sqrt{c}f^4\left(-2b\sqrt{b^2-4ac}+4ac+3b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{4\sqrt{2}e\left(b^2-4ac\right)^{5/2}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{3\sqrt{c}f^4\left(2b\sqrt{b^2-4ac}+4ac+3b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{4\sqrt{2}e\left(b^2-4ac\right)^{5/2}\sqrt{\sqrt{b^2-4ac}+b}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*f + e*f*x)^4/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3,x]
```

```
[Out] (f^4*(d + e*x)*(2*a + b*(d + e*x)^2))/(4*(b^2 - 4*a*c)*e*(a + b*(d + e*x)^2
 + c*(d + e*x)^4)^2) - (f^4*(d + e*x)*(7*b^2 - 4*a*c + 12*b*c*(d + e*x)^2))
/(8*(b^2 - 4*a*c)^2*e*(a + b*(d + e*x)^2 + c*(d + e*x)^4)) + (3*Sqrt[c]*(3*
b^2 + 4*a*c - 2*b*Sqrt[b^2 - 4*a*c])*f^4*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))
/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(4*Sqrt[2]*(b^2 - 4*a*c)^(5/2)*Sqrt[b - Sqrt
[b^2 - 4*a*c]]*e) - (3*Sqrt[c]*(3*b^2 + 4*a*c + 2*b*Sqrt[b^2 - 4*a*c])*f^4*
ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(4*Sqrt[2]
*(b^2 - 4*a*c)^(5/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]*e)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1120

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> -Simp[(d^3*(d*x)^(m - 3)*(2*a + b*x^2)*(a + b*x^2 + c*x^4)^(p + 1))/(2*
(p + 1)*(b^2 - 4*a*c)), x] + Dist[d^4/(2*(p + 1)*(b^2 - 4*a*c)), Int[(d*x)^
(m - 4)*(2*a*(m - 3) + b*(m + 4*p + 3)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && GtQ[
m, 3] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1142

```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Di
st[u^m/(Coefficient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p,
 x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1178

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[(x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*(a + b*x^2 +
 c*x^4)^(p + 1))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2
 - 4*a*c)), Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p + 7
)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
LtQ[p, -1] && IntegerQ[2*p]
```

Rubi steps

$$\int \frac{(df+efx)^4}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^3}\,dx = \frac{f^4\,\text{Subst}\left(\int \frac{x^4}{\left(a+bx^2+cx^4\right)^3}\,dx, x, d+ex\right)}{e}$$

$$= \frac{f^4(d+ex)\left(2a+b(d+ex)^2\right)}{4\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)^2} - \frac{f^4\,\text{Subst}\left(\int \frac{2a-5bx^2}{\left(a+bx^2+cx^4\right)^2}\right.}{4\left(b^2-4ac\right)}$$

$$= \frac{f^4(d+ex)\left(2a+b(d+ex)^2\right)}{4\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)^2} - \frac{f^4(d+ex)\left(7b^2-4ac\right.}{8\left(b^2-4ac\right)^2e\left(a+b(d\right.}$$

$$= \frac{f^4(d+ex)\left(2a+b(d+ex)^2\right)}{4\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)^2} - \frac{f^4(d+ex)\left(7b^2-4ac\right.}{8\left(b^2-4ac\right)^2e\left(a+b(d\right.}$$

$$= \frac{f^4(d+ex)\left(2a+b(d+ex)^2\right)}{4\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)^2} - \frac{f^4(d+ex)\left(7b^2-4ac\right.}{8\left(b^2-4ac\right)^2e\left(a+b(d\right.}$$

Mathematica [A] time = 4.45, size = 331, normalized size = 0.94

$$\frac{f^4\left(\frac{(d+ex)\left(4ac-7b^2-12bc(d+ex)^2\right)}{\left(b^2-4ac\right)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)}-\frac{2\left(-2a(d+ex)-b(d+ex)^3\right)}{\left(b^2-4ac\right)\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}+\frac{3\sqrt{2}\sqrt{c}\left(-2b\sqrt{b^2-4ac}+4ac+3b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\left(b^2-4ac\right)^{5/2}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{3\sqrt{2}\sqrt{c}\left(2b\sqrt{b^2-4ac}+4ac+3b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\left(b^2-4ac\right)^{5/2}\sqrt{\sqrt{b^2-4ac}+b}}\right)}{8e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*f + e*f*x)^4/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3,x]
```

```
[Out] (f^4*((-2*(-2*a*(d + e*x) - b*(d + e*x)^3))/((b^2 - 4*a*c)*(a + b*(d + e*x)
^2 + c*(d + e*x)^4)^2) + ((d + e*x)*(-7*b^2 + 4*a*c - 12*b*c*(d + e*x)^2))/
((b^2 - 4*a*c)^2*(a + b*(d + e*x)^2 + c*(d + e*x)^4)) + (3*Sqrt[2]*Sqrt[c]*
(3*b^2 + 4*a*c - 2*b*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/
Sqrt[b - Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(5/2)*Sqrt[b - Sqrt[b^2 - 4*a*
c]]) - (3*Sqrt[2]*Sqrt[c]*(3*b^2 + 4*a*c + 2*b*Sqrt[b^2 - 4*a*c])*ArcTan[(S
qrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(5/2
)*Sqrt[b + Sqrt[b^2 - 4*a*c]])))/(8*e)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(df+efx)^4}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(d*f + e*f*x)^4/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3,x]
```

```
[Out] IntegrateAlgebraic[(d*f + e*f*x)^4/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3, x
]
```

fricas [B] time = 1.41, size = 6770, normalized size = 19.18

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*f*x+d*f)^4/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="fricas"
)
```

```
[Out] -1/16*(24*b*c^2*e^7*f^4*x^7 + 168*b*c^2*d*e^6*f^4*x^6 + 2*(252*b*c^2*d^2 +
19*b^2*c - 4*a*c^2)*e^5*f^4*x^5 + 10*(84*b*c^2*d^3 + (19*b^2*c - 4*a*c^2)*d
)*e^4*f^4*x^4 + 2*(420*b*c^2*d^4 + 5*b^3 + 16*a*b*c + 10*(19*b^2*c - 4*a*c^
2)*d^2)*e^3*f^4*x^3 + 2*(252*b*c^2*d^5 + 10*(19*b^2*c - 4*a*c^2)*d^3 + 3*(5
*b^3 + 16*a*b*c)*d)*e^2*f^4*x^2 + 2*(84*b*c^2*d^6 + 5*(19*b^2*c - 4*a*c^2)*
d^4 + 3*a*b^2 + 12*a^2*c + 3*(5*b^3 + 16*a*b*c)*d^2)*e*f^4*x + 2*(12*b*c^2*
d^7 + (19*b^2*c - 4*a*c^2)*d^5 + (5*b^3 + 16*a*b*c)*d^3 + 3*(a*b^2 + 4*a^2*
c)*d)*f^4 + 3*sqrt(1/2)*((b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*e^9*x^8 + 8*(
b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d*e^8*x^7 + 2*(b^5*c - 8*a*b^3*c^2 + 16
*a^2*b*c^3 + 14*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^2)*e^7*x^6 + 4*(14*(
b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^3 + 3*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b
*c^3)*d)*e^6*x^5 + (b^6 - 6*a*b^4*c + 32*a^3*c^3 + 70*(b^4*c^2 - 8*a*b^2*c^
3 + 16*a^2*c^4)*d^4 + 30*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^2)*e^5*x^4
+ 4*(14*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^5 + 10*(b^5*c - 8*a*b^3*c^2
+ 16*a^2*b*c^3)*d^3 + (b^6 - 6*a*b^4*c + 32*a^3*c^3)*d)*e^4*x^3 + 2*(14*(b^
4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^6 + a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2
+ 15*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^4 + 3*(b^6 - 6*a*b^4*c + 32*a^3
*c^3)*d^2)*e^3*x^2 + 4*(2*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^7 + 3*(b^5
*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^5 + (b^6 - 6*a*b^4*c + 32*a^3*c^3)*d^3 +
 (a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*d)*e^2*x + ((b^4*c^2 - 8*a*b^2*c^3 +
16*a^2*c^4)*d^8 + 2*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^6 + a^2*b^4 - 8*
a^3*b^2*c + 16*a^4*c^2 + (b^6 - 6*a*b^4*c + 32*a^3*c^3)*d^4 + 2*(a*b^5 - 8*
a^2*b^3*c + 16*a^3*b*c^2)*d^2)*e)*sqrt(-((b^5 + 40*a*b^3*c + 80*a^2*b*c^2)*
f^8 + (a*b^10 - 20*a^2*b^8*c + 160*a^3*b^6*c^2 - 640*a^4*b^4*c^3 + 1280*a^5
*b^2*c^4 - 1024*a^6*c^5)*sqrt(f^16/((a^2*b^10 - 20*a^3*b^8*c + 160*a^4*b^6*
c^2 - 640*a^5*b^4*c^3 + 1280*a^6*b^2*c^4 - 1024*a^7*c^5)*e^4))*e^2)/((a*b^1
0 - 20*a^2*b^8*c + 160*a^3*b^6*c^2 - 640*a^4*b^4*c^3 + 1280*a^5*b^2*c^4 - 1
024*a^6*c^5)*e^2))*log(27*(5*b^4*c + 40*a*b^2*c^2 + 16*a^2*c^3)*e*f^12*x +
27*(5*b^4*c + 40*a*b^2*c^2 + 16*a^2*c^3)*d*f^12 + 27/2*sqrt(1/2)*((b^8 - 8*
a*b^6*c + 128*a^3*b^2*c^3 - 256*a^4*c^4)*e*f^8 - (a*b^13 - 8*a^2*b^11*c - 8
0*a^3*b^9*c^2 + 1280*a^4*b^7*c^3 - 6400*a^5*b^5*c^4 + 14336*a^6*b^3*c^5 - 1
2288*a^7*b*c^6)*sqrt(f^16/((a^2*b^10 - 20*a^3*b^8*c + 160*a^4*b^6*c^2 - 640
*a^5*b^4*c^3 + 1280*a^6*b^2*c^4 - 1024*a^7*c^5)*e^4))*e^3)*sqrt(-((b^5 + 40
*a*b^3*c + 80*a^2*b*c^2)*f^8 + (a*b^10 - 20*a^2*b^8*c + 160*a^3*b^6*c^2 - 6
40*a^4*b^4*c^3 + 1280*a^5*b^2*c^4 - 1024*a^6*c^5)*sqrt(f^16/((a^2*b^10 - 20
*a^3*b^8*c + 160*a^4*b^6*c^2 - 640*a^5*b^4*c^3 + 1280*a^6*b^2*c^4 - 1024*a^
7*c^5)*e^4))*e^2)/((a*b^10 - 20*a^2*b^8*c + 160*a^3*b^6*c^2 - 640*a^4*b^4*c
^3 + 1280*a^5*b^2*c^4 - 1024*a^6*c^5)*e^2))) - 3*sqrt(1/2)*((b^4*c^2 - 8*a*
b^2*c^3 + 16*a^2*c^4)*e^9*x^8 + 8*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d*e^
8*x^7 + 2*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3 + 14*(b^4*c^2 - 8*a*b^2*c^3 +
 16*a^2*c^4)*d^2)*e^7*x^6 + 4*(14*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^3
+ 3*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d)*e^6*x^5 + (b^6 - 6*a*b^4*c + 32
*a^3*c^3 + 70*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^4 + 30*(b^5*c - 8*a*b^
3*c^2 + 16*a^2*b*c^3)*d^2)*e^5*x^4 + 4*(14*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*
c^4)*d^5 + 10*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^3 + (b^6 - 6*a*b^4*c +
 32*a^3*c^3)*d)*e^4*x^3 + 2*(14*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^6 +
a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2 + 15*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3
)*d^4 + 3*(b^6 - 6*a*b^4*c + 32*a^3*c^3)*d^2)*e^3*x^2 + 4*(2*(b^4*c^2 - 8*a
```

*b^2*c^3 + 16*a^2*c^4)*d^7 + 3*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^5 + (
b^6 - 6*a*b^4*c + 32*a^3*c^3)*d^3 + (a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*d)
*e^2*x + ((b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^8 + 2*(b^5*c - 8*a*b^3*c^2
 + 16*a^2*b*c^3)*d^6 + a^2*b^4 - 8*a^3*b^2*c + 16*a^4*c^2 + (b^6 - 6*a*b^4*
c + 32*a^3*c^3)*d^4 + 2*(a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*d^2)*e)*sqrt(-
((b^5 + 40*a*b^3*c + 80*a^2*b*c^2)*f^8 + (a*b^10 - 20*a^2*b^8*c + 160*a^3*b
^6*c^2 - 640*a^4*b^4*c^3 + 1280*a^5*b^2*c^4 - 1024*a^6*c^5)*sqrt(f^16/((a^2
*b^10 - 20*a^3*b^8*c + 160*a^4*b^6*c^2 - 640*a^5*b^4*c^3 + 1280*a^6*b^2*c^4
 - 1024*a^7*c^5)*e^4))*e^2)/((a*b^10 - 20*a^2*b^8*c + 160*a^3*b^6*c^2 - 640
*a^4*b^4*c^3 + 1280*a^5*b^2*c^4 - 1024*a^6*c^5)*e^2))*log(27*(5*b^4*c + 40*
a*b^2*c^2 + 16*a^2*c^3)*e*f^12*x + 27*(5*b^4*c + 40*a*b^2*c^2 + 16*a^2*c^3)
*d*f^12 - 27/2*sqrt(1/2)*((b^8 - 8*a*b^6*c + 128*a^3*b^2*c^3 - 256*a^4*c^4)
*e*f^8 - (a*b^13 - 8*a^2*b^11*c - 80*a^3*b^9*c^2 + 1280*a^4*b^7*c^3 - 6400*
a^5*b^5*c^4 + 14336*a^6*b^3*c^5 - 12288*a^7*b*c^6)*sqrt(f^16/((a^2*b^10 - 2
0*a^3*b^8*c + 160*a^4*b^6*c^2 - 640*a^5*b^4*c^3 + 1280*a^6*b^2*c^4 - 1024*a
^7*c^5)*e^4))*e^3)*sqrt(-((b^5 + 40*a*b^3*c + 80*a^2*b*c^2)*f^8 + (a*b^10 -
 20*a^2*b^8*c + 160*a^3*b^6*c^2 - 640*a^4*b^4*c^3 + 1280*a^5*b^2*c^4 - 1024
*a^6*c^5)*sqrt(f^16/((a^2*b^10 - 20*a^3*b^8*c + 160*a^4*b^6*c^2 - 640*a^5*b
^4*c^3 + 1280*a^6*b^2*c^4 - 1024*a^7*c^5)*e^4))*e^2)/((a*b^10 - 20*a^2*b^8*
c + 160*a^3*b^6*c^2 - 640*a^4*b^4*c^3 + 1280*a^5*b^2*c^4 - 1024*a^6*c^5)*e^
2))) + 3*sqrt(1/2)*((b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*e^9*x^8 + 8*(b^4*c
^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d*e^8*x^7 + 2*(b^5*c - 8*a*b^3*c^2 + 16*a^2*
b*c^3 + 14*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^2)*e^7*x^6 + 4*(14*(b^4*c
^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^3 + 3*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)
*d)*e^6*x^5 + (b^6 - 6*a*b^4*c + 32*a^3*c^3 + 70*(b^4*c^2 - 8*a*b^2*c^3 + 1
6*a^2*c^4)*d^4 + 30*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^2)*e^5*x^4 + 4*(
14*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^5 + 10*(b^5*c - 8*a*b^3*c^2 + 16*
a^2*b*c^3)*d^3 + (b^6 - 6*a*b^4*c + 32*a^3*c^3)*d)*e^4*x^3 + 2*(14*(b^4*c^2
 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^6 + a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2 + 15*
(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^4 + 3*(b^6 - 6*a*b^4*c + 32*a^3*c^3)
*d^2)*e^3*x^2 + 4*(2*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^7 + 3*(b^5*c -
8*a*b^3*c^2 + 16*a^2*b*c^3)*d^5 + (b^6 - 6*a*b^4*c + 32*a^3*c^3)*d^3 + (a*b
^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*d)*e^2*x + ((b^4*c^2 - 8*a*b^2*c^3 + 16*a^
2*c^4)*d^8 + 2*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^6 + a^2*b^4 - 8*a^3*b
^2*c + 16*a^4*c^2 + (b^6 - 6*a*b^4*c + 32*a^3*c^3)*d^4 + 2*(a*b^5 - 8*a^2*b
^3*c + 16*a^3*b*c^2)*d^2)*e)*sqrt(-((b^5 + 40*a*b^3*c + 80*a^2*b*c^2)*f^8 -
 (a*b^10 - 20*a^2*b^8*c + 160*a^3*b^6*c^2 - 640*a^4*b^4*c^3 + 1280*a^5*b^2*
c^4 - 1024*a^6*c^5)*sqrt(f^16/((a^2*b^10 - 20*a^3*b^8*c + 160*a^4*b^6*c^2 -
 640*a^5*b^4*c^3 + 1280*a^6*b^2*c^4 - 1024*a^7*c^5)*e^4))*e^2)/((a*b^10 - 2
0*a^2*b^8*c + 160*a^3*b^6*c^2 - 640*a^4*b^4*c^3 + 1280*a^5*b^2*c^4 - 1024*a
^6*c^5)*e^2))*log(27*(5*b^4*c + 40*a*b^2*c^2 + 16*a^2*c^3)*e*f^12*x + 27*(5
*b^4*c + 40*a*b^2*c^2 + 16*a^2*c^3)*d*f^12 + 27/2*sqrt(1/2)*((b^8 - 8*a*b^6
*c + 128*a^3*b^2*c^3 - 256*a^4*c^4)*e*f^8 + (a*b^13 - 8*a^2*b^11*c - 80*a^3
*b^9*c^2 + 1280*a^4*b^7*c^3 - 6400*a^5*b^5*c^4 + 14336*a^6*b^3*c^5 - 12288*
a^7*b*c^6)*sqrt(f^16/((a^2*b^10 - 20*a^3*b^8*c + 160*a^4*b^6*c^2 - 640*a^5*
b^4*c^3 + 1280*a^6*b^2*c^4 - 1024*a^7*c^5)*e^4))*e^3)*sqrt(-((b^5 + 40*a*b^
3*c + 80*a^2*b*c^2)*f^8 - (a*b^10 - 20*a^2*b^8*c + 160*a^3*b^6*c^2 - 640*a^
4*b^4*c^3 + 1280*a^5*b^2*c^4 - 1024*a^6*c^5)*sqrt(f^16/((a^2*b^10 - 20*a^3*
b^8*c + 160*a^4*b^6*c^2 - 640*a^5*b^4*c^3 + 1280*a^6*b^2*c^4 - 1024*a^7*c^5
)*e^4))*e^2)/((a*b^10 - 20*a^2*b^8*c + 160*a^3*b^6*c^2 - 640*a^4*b^4*c^3 +
1280*a^5*b^2*c^4 - 1024*a^6*c^5)*e^2))) - 3*sqrt(1/2)*((b^4*c^2 - 8*a*b^2*c
^3 + 16*a^2*c^4)*e^9*x^8 + 8*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d*e^8*x^7
 + 2*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3 + 14*(b^4*c^2 - 8*a*b^2*c^3 + 16*a
^2*c^4)*d^2)*e^7*x^6 + 4*(14*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^3 + 3*(
b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d)*e^6*x^5 + (b^6 - 6*a*b^4*c + 32*a^3*
c^3 + 70*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^4 + 30*(b^5*c - 8*a*b^3*c^2
 + 16*a^2*b*c^3)*d^2)*e^5*x^4 + 4*(14*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*
d^5 + 10*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^3 + (b^6 - 6*a*b^4*c + 32*a
^3*c^3)*d)*e^4*x^3 + 2*(14*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^6 + a*b^5

```
- 8*a^2*b^3*c + 16*a^3*b*c^2 + 15*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^4
 + 3*(b^6 - 6*a*b^4*c + 32*a^3*c^3)*d^2)*e^3*x^2 + 4*(2*(b^4*c^2 - 8*a*b^2*
c^3 + 16*a^2*c^4)*d^7 + 3*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^5 + (b^6 -
 6*a*b^4*c + 32*a^3*c^3)*d^3 + (a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*d)*e^2*
x + ((b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^8 + 2*(b^5*c - 8*a*b^3*c^2 + 16
*a^2*b*c^3)*d^6 + a^2*b^4 - 8*a^3*b^2*c + 16*a^4*c^2 + (b^6 - 6*a*b^4*c + 3
2*a^3*c^3)*d^4 + 2*(a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*d^2)*e)*sqrt(-((b^5
 + 40*a*b^3*c + 80*a^2*b*c^2)*f^8 - (a*b^10 - 20*a^2*b^8*c + 160*a^3*b^6*c^
2 - 640*a^4*b^4*c^3 + 1280*a^5*b^2*c^4 - 1024*a^6*c^5)*sqrt(f^16/((a^2*b^10
 - 20*a^3*b^8*c + 160*a^4*b^6*c^2 - 640*a^5*b^4*c^3 + 1280*a^6*b^2*c^4 - 10
24*a^7*c^5)*e^4))*e^2)/((a*b^10 - 20*a^2*b^8*c + 160*a^3*b^6*c^2 - 640*a^4*
b^4*c^3 + 1280*a^5*b^2*c^4 - 1024*a^6*c^5)*e^2))*log(27*(5*b^4*c + 40*a*b^2
*c^2 + 16*a^2*c^3)*e*f^12*x + 27*(5*b^4*c + 40*a*b^2*c^2 + 16*a^2*c^3)*d*f^
12 - 27/2*sqrt(1/2)*((b^8 - 8*a*b^6*c + 128*a^3*b^2*c^3 - 256*a^4*c^4)*e*f^
8 + (a*b^13 - 8*a^2*b^11*c - 80*a^3*b^9*c^2 + 1280*a^4*b^7*c^3 - 6400*a^5*b
^5*c^4 + 14336*a^6*b^3*c^5 - 12288*a^7*b*c^6)*sqrt(f^16/((a^2*b^10 - 20*a^3
*b^8*c + 160*a^4*b^6*c^2 - 640*a^5*b^4*c^3 + 1280*a^6*b^2*c^4 - 1024*a^7*c^
5)*e^4))*e^3)*sqrt(-((b^5 + 40*a*b^3*c + 80*a^2*b*c^2)*f^8 - (a*b^10 - 20*a
^2*b^8*c + 160*a^3*b^6*c^2 - 640*a^4*b^4*c^3 + 1280*a^5*b^2*c^4 - 1024*a^6*
c^5)*sqrt(f^16/((a^2*b^10 - 20*a^3*b^8*c + 160*a^4*b^6*c^2 - 640*a^5*b^4*c^
3 + 1280*a^6*b^2*c^4 - 1024*a^7*c^5)*e^4))*e^2)/((a*b^10 - 20*a^2*b^8*c + 1
60*a^3*b^6*c^2 - 640*a^4*b^4*c^3 + 1280*a^5*b^2*c^4 - 1024*a^6*c^5)*e^2))))
/((b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*e^9*x^8 + 8*(b^4*c^2 - 8*a*b^2*c^3 +
 16*a^2*c^4)*d*e^8*x^7 + 2*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3 + 14*(b^4*c^
2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^2)*e^7*x^6 + 4*(14*(b^4*c^2 - 8*a*b^2*c^3 +
 16*a^2*c^4)*d^3 + 3*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d)*e^6*x^5 + (b^6
 - 6*a*b^4*c + 32*a^3*c^3 + 70*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^4 + 3
0*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^2)*e^5*x^4 + 4*(14*(b^4*c^2 - 8*a*
b^2*c^3 + 16*a^2*c^4)*d^5 + 10*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^3 + (
b^6 - 6*a*b^4*c + 32*a^3*c^3)*d)*e^4*x^3 + 2*(14*(b^4*c^2 - 8*a*b^2*c^3 + 1
6*a^2*c^4)*d^6 + a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2 + 15*(b^5*c - 8*a*b^3*c
^2 + 16*a^2*b*c^3)*d^4 + 3*(b^6 - 6*a*b^4*c + 32*a^3*c^3)*d^2)*e^3*x^2 + 4*
(2*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^7 + 3*(b^5*c - 8*a*b^3*c^2 + 16*a
^2*b*c^3)*d^5 + (b^6 - 6*a*b^4*c + 32*a^3*c^3)*d^3 + (a*b^5 - 8*a^2*b^3*c +
 16*a^3*b*c^2)*d)*e^2*x + ((b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^8 + 2*(b^
5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^6 + a^2*b^4 - 8*a^3*b^2*c + 16*a^4*c^2
+ (b^6 - 6*a*b^4*c + 32*a^3*c^3)*d^4 + 2*(a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^
2)*d^2)*e)
```

giac [B] time = 0.80, size = 1844, normalized size = 5.22

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*f*x+d*f)^4/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="giac")
```

```
[Out] 3/16*((4*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)
/c))^2*b*c*f^4*e^2 - 8*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*
c)*e^2)*e^(-4)/c))*b*c*d*f^4*e + 4*b*c*d^2*f^4 - b^2*f^4 - 4*a*c*f^4)*log(d
*e^(-1) + x + sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))/(2
*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^3*c
*e^4 - 6*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)
/c))^2*c*d*e^3 - 2*c*d^3*e - b*d*e + (6*c*d^2*e^2 + b*e^2)*(d*e^(-1) + sqrt
(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))) + (4*(d*e^(-1) - sq
rt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^2*b*c*f^4*e^2 - 8*
(d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))*b*c*
d*f^4*e + 4*b*c*d^2*f^4 - b^2*f^4 - 4*a*c*f^4)*log(d*e^(-1) + x - sqrt(1/2)
*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))/(2*(d*e^(-1) - sqrt(1/2)*
sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^3*c*e^4 - 6*(d*e^(-1) - sq
rt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^2*c*d*e^3 - 2*c*d^
```

```
3*e - b*d*e + (6*c*d^2*e^2 + b*e^2)*(d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 + sq
rt(b^2 - 4*a*c)*e^2)*e^(-4)/c))) + (4*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 -
sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^2*b*c*f^4*e^2 - 8*(d*e^(-1) + sqrt(1/2)*s
qrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))*b*c*d*f^4*e + 4*b*c*d^2*f^4
 - b^2*f^4 - 4*a*c*f^4)*log(d*e^(-1) + x + sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^
2 - 4*a*c)*e^2)*e^(-4)/c))/(2*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2
 - 4*a*c)*e^2)*e^(-4)/c))^3*c*e^4 - 6*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 -
sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^2*c*d*e^3 - 2*c*d^3*e - b*d*e + (6*c*d^2*
e^2 + b*e^2)*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^
(-4)/c))) + (4*(d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*
e^(-4)/c))^2*b*c*f^4*e^2 - 8*(d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2
- 4*a*c)*e^2)*e^(-4)/c))*b*c*d*f^4*e + 4*b*c*d^2*f^4 - b^2*f^4 - 4*a*c*f^4)
*log(d*e^(-1) + x - sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/
c))/(2*(d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c
))^3*c*e^4 - 6*(d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*
e^(-4)/c))^2*c*d*e^3 - 2*c*d^3*e - b*d*e + (6*c*d^2*e^2 + b*e^2)*(d*e^(-1)
- sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))))/(b^4 - 8*a*b
^2*c + 16*a^2*c^2) - 1/8*(12*b*c^2*f^4*x^7*e^7 + 84*b*c^2*d*f^4*x^6*e^6 + 2
52*b*c^2*d^2*f^4*x^5*e^5 + 420*b*c^2*d^3*f^4*x^4*e^4 + 420*b*c^2*d^4*f^4*x^
3*e^3 + 252*b*c^2*d^5*f^4*x^2*e^2 + 84*b*c^2*d^6*f^4*x*e + 12*b*c^2*d^7*f^4
 + 19*b^2*c*f^4*x^5*e^5 - 4*a*c^2*f^4*x^5*e^5 + 95*b^2*c*d*f^4*x^4*e^4 - 20
*a*c^2*d*f^4*x^4*e^4 + 190*b^2*c*d^2*f^4*x^3*e^3 - 40*a*c^2*d^2*f^4*x^3*e^3
 + 190*b^2*c*d^3*f^4*x^2*e^2 - 40*a*c^2*d^3*f^4*x^2*e^2 + 95*b^2*c*d^4*f^4*
x*e - 20*a*c^2*d^4*f^4*x*e + 19*b^2*c*d^5*f^4 - 4*a*c^2*d^5*f^4 + 5*b^3*f^4
*x^3*e^3 + 16*a*b*c*f^4*x^3*e^3 + 15*b^3*d*f^4*x^2*e^2 + 48*a*b*c*d*f^4*x^2
*e^2 + 15*b^3*d^2*f^4*x*e + 48*a*b*c*d^2*f^4*x*e + 5*b^3*d^3*f^4 + 16*a*b*c
*d^3*f^4 + 3*a*b^2*f^4*x*e + 12*a^2*c*f^4*x*e + 3*a*b^2*d*f^4 + 12*a^2*c*d*
f^4)/((c*x^4*e^4 + 4*c*d*x^3*e^3 + 6*c*d^2*x^2*e^2 + 4*c*d^3*x*e + c*d^4 +
b*x^2*e^2 + 2*b*d*x*e + b*d^2 + a)^2*(b^4*e - 8*a*b^2*c*e + 16*a^2*c^2*e))
```

maple [C] time = 0.05, size = 3432, normalized size = 9.72

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*f*x+d*f)^4/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x)
```

```
[Out] -95/4*f^4/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*
d^4+2*b*d*e*x+b*d^2+a)^2*e^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x^3*b^2*c*d^2-2*f^4
/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d
*e*x+b*d^2+a)^2*e^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x^3*a*b*c-63/2*f^4/(c*e^4*x^
4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2
+a)^2*d^5*e/(16*a^2*c^2-8*a*b^2*c+b^4)*x^2*b*c^2+5*f^4/(c*e^4*x^4+4*c*d*e^3
*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*d^3*e
/(16*a^2*c^2-8*a*b^2*c+b^4)*x^2*a*c^2-95/4*f^4/(c*e^4*x^4+4*c*d*e^3*x^3+6*c
*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*d^3*e/(16*a^2
*c^2-8*a*b^2*c+b^4)*x^2*b^2*c-6*f^4/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^
2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2/(16*a^2*c^2-8*a*b^2*c+b^
4)*x*a*b*c*d^2-2*f^4/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b
*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2/(16*a^2*c^2-8*a*b^2*c+b^4)*d^3/e*a*b*c-
105/2*f^4/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*
d^4+2*b*d*e*x+b*d^2+a)^2*c^2*d^3*e^3/(16*a^2*c^2-8*a*b^2*c+b^4)*x^4*b+5/2*f
^4/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b
*d*e*x+b*d^2+a)^2*c^2*d*e^3/(16*a^2*c^2-8*a*b^2*c+b^4)*x^4*a-95/8*f^4/(c*e^
4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b
*d^2+a)^2*c*d*e^3/(16*a^2*c^2-8*a*b^2*c+b^4)*x^4*b^2-6*f^4/(c*e^4*x^4+4*c*d
*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*d
*e/(16*a^2*c^2-8*a*b^2*c+b^4)*x^2*a*b*c+3/16*f^4/(16*a^2*c^2-8*a*b^2*c+b^4)
/e*sum((-4*_R^2*b*c*e^2-8*_R*b*c*d*e-4*b*c*d^2+4*a*c+b^2)/(2*_R^3*c*e^3+6*_
R^2*c*d*e^2+6*_R*c*d^2*e+2*c*d^3+_R*b*e+b*d)*ln(-_R+x),_R=RootOf(_Z^4*c*e^4
```

```
+4*_Z^3*c*d*e^3+c*d^4+b*d^2+(6*c*d^2*e^2+b*e^2)*_Z^2+(4*c*d^3*e+2*b*d*e)*_Z
+a))-15/8*f^4/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^
2+c*d^4+2*b*d*e*x+b*d^2+a)^2*d*e/(16*a^2*c^2-8*a*b^2*c+b^4)*x^2*b^3-21/2*f^
4/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*
d*e*x+b*d^2+a)^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x*b*c^2*d^6+5/2*f^4/(c*e^4*x^4+
4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a
)^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x*a*c^2*d^4-95/8*f^4/(c*e^4*x^4+4*c*d*e^3*x^
3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2/(16*a^2*
c^2-8*a*b^2*c+b^4)*x*b^2*c*d^4-3/2*f^4/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2
*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2/(16*a^2*c^2-8*a*b^2*c
+b^4)*d^7/e*b*c^2+1/2*f^4/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*
e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2/(16*a^2*c^2-8*a*b^2*c+b^4)*d^5/e*a
*c^2-19/8*f^4/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^
2+c*d^4+2*b*d*e*x+b*d^2+a)^2/(16*a^2*c^2-8*a*b^2*c+b^4)*d^5/e*b^2*c-3/2*f^4
/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d
*e*x+b*d^2+a)^2/(16*a^2*c^2-8*a*b^2*c+b^4)*d/e*a^2*c-3/8*f^4/(c*e^4*x^4+4*c
*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2
/(16*a^2*c^2-8*a*b^2*c+b^4)*d/e*a*b^2+1/2*f^4/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*
d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*c^2*e^4/(16*a^
2*c^2-8*a*b^2*c+b^4)*x^5*a-19/8*f^4/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^
2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*c*e^4/(16*a^2*c^2-8*a*b^
2*c+b^4)*x^5*b^2-3/2*f^4/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e
*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*c^2*e^6*b/(16*a^2*c^2-8*a*b^2*c+b^4
)*x^7-5/8*f^4/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^
2+c*d^4+2*b*d*e*x+b*d^2+a)^2*e^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x^3*b^3-15/8*f^
4/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*
d*e*x+b*d^2+a)^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x*b^3*d^2-3/2*f^4/(c*e^4*x^4+4*
c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^
2/(16*a^2*c^2-8*a*b^2*c+b^4)*x*a^2*c-3/8*f^4/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d
^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2/(16*a^2*c^2-8*a
*b^2*c+b^4)*x*a*b^2-5/8*f^4/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^
3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2/(16*a^2*c^2-8*a*b^2*c+b^4)*d^3/e
*b^3-105/2*f^4/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x
^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*e^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x^3*b*c^2*d^4+
5*f^4/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+
2*b*d*e*x+b*d^2+a)^2*e^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x^3*a*c^2*d^2-21/2*f^4/
(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*
e*x+b*d^2+a)^2*c^2*d*e^5*b/(16*a^2*c^2-8*a*b^2*c+b^4)*x^6-63/2*f^4/(c*e^4*x
^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^
2+a)^2*c^2*e^4/(16*a^2*c^2-8*a*b^2*c+b^4)*x^5*b*d^2
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*f*x+d*f)^4/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="maxima"
)
```

```
[Out] Timed out
```

mupad [B] time = 7.52, size = 13840, normalized size = 39.21

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*f + e*f*x)^4/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3,x)
```

```
[Out] atan(((-(9*(b^15*f^8 + f^8*(-(4*a*c - b^2)^15)^(1/2) - 81920*a^7*b*c^7*f^8
- 560*a^2*b^11*c^2*f^8 + 4160*a^3*b^9*c^3*f^8 - 11520*a^4*b^7*c^4*f^8 - 102
4*a^5*b^5*c^5*f^8 + 61440*a^6*b^3*c^6*f^8 + 20*a*b^13*c*f^8))/(512*(a*b^20*
```

e^2 + 1048576*a^11*c^10*e^2 - 40*a^2*b^18*c*e^2 + 720*a^3*b^16*c^2*e^2 - 76
80*a^4*b^14*c^3*e^2 + 53760*a^5*b^12*c^4*e^2 - 258048*a^6*b^10*c^5*e^2 + 86
0160*a^7*b^8*c^6*e^2 - 1966080*a^8*b^6*c^7*e^2 + 2949120*a^9*b^4*c^8*e^2 -
2621440*a^10*b^2*c^9*e^2)))^(1/2)*((((1024*b^15*c^2*d*e^13 - 28672*a*b^13*c
^3*d*e^13 - 16777216*a^7*b*c^9*d*e^13 + 344064*a^2*b^11*c^4*d*e^13 - 229376
0*a^3*b^9*c^5*d*e^13 + 9175040*a^4*b^7*c^6*d*e^13 - 22020096*a^5*b^5*c^7*d*
e^13 + 29360128*a^6*b^3*c^8*d*e^13)/(128*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8
*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c
)) + (x*(128*b^11*c^2*e^14 - 2560*a*b^9*c^3*e^14 - 131072*a^5*b*c^7*e^14 +
20480*a^2*b^7*c^4*e^14 - 81920*a^3*b^5*c^5*e^14 + 163840*a^4*b^3*c^6*e^14))
/(16*(b^8 + 256*a^4*c^4 + 96*a^2*b^4*c^2 - 256*a^3*b^2*c^3 - 16*a*b^6*c)))*
(-(9*(b^15*f^8 + f^8*(-(4*a*c - b^2)^15)^(1/2) - 81920*a^7*b*c^7*f^8 - 560*
a^2*b^11*c^2*f^8 + 4160*a^3*b^9*c^3*f^8 - 11520*a^4*b^7*c^4*f^8 - 1024*a^5*
b^5*c^5*f^8 + 61440*a^6*b^3*c^6*f^8 + 20*a*b^13*c*f^8))/(512*(a*b^20*e^2 +
1048576*a^11*c^10*e^2 - 40*a^2*b^18*c*e^2 + 720*a^3*b^16*c^2*e^2 - 7680*a^4
*b^14*c^3*e^2 + 53760*a^5*b^12*c^4*e^2 - 258048*a^6*b^10*c^5*e^2 + 860160*a
^7*b^8*c^6*e^2 - 1966080*a^8*b^6*c^7*e^2 + 2949120*a^9*b^4*c^8*e^2 - 262144
0*a^10*b^2*c^9*e^2)))^(1/2) - (786432*a^6*c^8*e^12*f^4 - 192*b^12*c^2*e^12*
f^4 - 15360*a^2*b^8*c^4*e^12*f^4 + 245760*a^4*b^4*c^6*e^12*f^4 - 786432*a^5
*b^2*c^7*e^12*f^4 + 3072*a*b^10*c^3*e^12*f^4)/(128*(b^12 + 4096*a^6*c^6 + 2
40*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 2
4*a*b^10*c)))*(-(9*(b^15*f^8 + f^8*(-(4*a*c - b^2)^15)^(1/2) - 81920*a^7*b*
c^7*f^8 - 560*a^2*b^11*c^2*f^8 + 4160*a^3*b^9*c^3*f^8 - 11520*a^4*b^7*c^4*f
^8 - 1024*a^5*b^5*c^5*f^8 + 61440*a^6*b^3*c^6*f^8 + 20*a*b^13*c*f^8))/(512*
(a*b^20*e^2 + 1048576*a^11*c^10*e^2 - 40*a^2*b^18*c*e^2 + 720*a^3*b^16*c^2*
e^2 - 7680*a^4*b^14*c^3*e^2 + 53760*a^5*b^12*c^4*e^2 - 258048*a^6*b^10*c^5*
e^2 + 860160*a^7*b^8*c^6*e^2 - 1966080*a^8*b^6*c^7*e^2 + 2949120*a^9*b^4*c^
8*e^2 - 2621440*a^10*b^2*c^9*e^2)))^(1/2) + (18432*a^4*c^7*d*e^11*f^8 + 936
*b^8*c^3*d*e^11*f^8 - 6912*a*b^6*c^4*d*e^11*f^8 + 11520*a^2*b^4*c^5*d*e^11*
f^8)/(128*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*
a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c)) + (x*(144*a^2*c^5*e^12*f^8 +
117*b^4*c^3*e^12*f^8 + 72*a*b^2*c^4*e^12*f^8))/(16*(b^8 + 256*a^4*c^4 + 96
*a^2*b^4*c^2 - 256*a^3*b^2*c^3 - 16*a*b^6*c)))*1i + (-(9*(b^15*f^8 + f^8*(-
(4*a*c - b^2)^15)^(1/2) - 81920*a^7*b*c^7*f^8 - 560*a^2*b^11*c^2*f^8 + 4160
*a^3*b^9*c^3*f^8 - 11520*a^4*b^7*c^4*f^8 - 1024*a^5*b^5*c^5*f^8 + 61440*a^6
*b^3*c^6*f^8 + 20*a*b^13*c*f^8))/(512*(a*b^20*e^2 + 1048576*a^11*c^10*e^2 -
40*a^2*b^18*c*e^2 + 720*a^3*b^16*c^2*e^2 - 7680*a^4*b^14*c^3*e^2 + 53760*a
^5*b^12*c^4*e^2 - 258048*a^6*b^10*c^5*e^2 + 860160*a^7*b^8*c^6*e^2 - 196608
0*a^8*b^6*c^7*e^2 + 2949120*a^9*b^4*c^8*e^2 - 2621440*a^10*b^2*c^9*e^2)))^(
1/2)*((((1024*b^15*c^2*d*e^13 - 28672*a*b^13*c^3*d*e^13 - 16777216*a^7*b*c^
9*d*e^13 + 344064*a^2*b^11*c^4*d*e^13 - 2293760*a^3*b^9*c^5*d*e^13 + 917504
0*a^4*b^7*c^6*d*e^13 - 22020096*a^5*b^5*c^7*d*e^13 + 29360128*a^6*b^3*c^8*d
e^13)/(128(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 384
0*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c)) + (x*(128*b^11*c^2*e^14 -
2560*a*b^9*c^3*e^14 - 131072*a^5*b*c^7*e^14 + 20480*a^2*b^7*c^4*e^14 - 8192
0*a^3*b^5*c^5*e^14 + 163840*a^4*b^3*c^6*e^14))/(16*(b^8 + 256*a^4*c^4 + 96*
a^2*b^4*c^2 - 256*a^3*b^2*c^3 - 16*a*b^6*c)))*(-(9*(b^15*f^8 + f^8*(-(4*a*c
- b^2)^15)^(1/2) - 81920*a^7*b*c^7*f^8 - 560*a^2*b^11*c^2*f^8 + 4160*a^3*b
^9*c^3*f^8 - 11520*a^4*b^7*c^4*f^8 - 1024*a^5*b^5*c^5*f^8 + 61440*a^6*b^3*c
^6*f^8 + 20*a*b^13*c*f^8))/(512*(a*b^20*e^2 + 1048576*a^11*c^10*e^2 - 40*a^
2*b^18*c*e^2 + 720*a^3*b^16*c^2*e^2 - 7680*a^4*b^14*c^3*e^2 + 53760*a^5*b^1
2*c^4*e^2 - 258048*a^6*b^10*c^5*e^2 + 860160*a^7*b^8*c^6*e^2 - 1966080*a^8*
b^6*c^7*e^2 + 2949120*a^9*b^4*c^8*e^2 - 2621440*a^10*b^2*c^9*e^2)))^(1/2) +
(786432*a^6*c^8*e^12*f^4 - 192*b^12*c^2*e^12*f^4 - 15360*a^2*b^8*c^4*e^12*
f^4 + 245760*a^4*b^4*c^6*e^12*f^4 - 786432*a^5*b^2*c^7*e^12*f^4 + 3072*a*b^
10*c^3*e^12*f^4)/(128*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6
*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c)))*(-(9*(b^15*f^8
+ f^8*(-(4*a*c - b^2)^15)^(1/2) - 81920*a^7*b*c^7*f^8 - 560*a^2*b^11*c^2*f^
8 + 4160*a^3*b^9*c^3*f^8 - 11520*a^4*b^7*c^4*f^8 - 1024*a^5*b^5*c^5*f^8 + 6

```
1440*a^6*b^3*c^6*f^8 + 20*a*b^13*c*f^8))/(512*(a*b^20*e^2 + 1048576*a^11*c^
10*e^2 - 40*a^2*b^18*c*e^2 + 720*a^3*b^16*c^2*e^2 - 7680*a^4*b^14*c^3*e^2 +
 53760*a^5*b^12*c^4*e^2 - 258048*a^6*b^10*c^5*e^2 + 860160*a^7*b^8*c^6*e^2
 - 1966080*a^8*b^6*c^7*e^2 + 2949120*a^9*b^4*c^8*e^2 - 2621440*a^10*b^2*c^9*
e^2)))^(1/2) + (18432*a^4*c^7*d*e^11*f^8 + 936*b^8*c^3*d*e^11*f^8 - 6912*a*
b^6*c^4*d*e^11*f^8 + 11520*a^2*b^4*c^5*d*e^11*f^8)/(128*(b^12 + 4096*a^6*c^
6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^
5 - 24*a*b^10*c)) + (x*(144*a^2*c^5*e^12*f^8 + 117*b^4*c^3*e^12*f^8 + 72*a*
b^2*c^4*e^12*f^8))/(16*(b^8 + 256*a^4*c^4 + 96*a^2*b^4*c^2 - 256*a^3*b^2*c^
3 - 16*a*b^6*c)))*1i)/((-(9*(b^15*f^8 + f^8*(-(4*a*c - b^2)^15)^(1/2) - 819
20*a^7*b*c^7*f^8 - 560*a^2*b^11*c^2*f^8 + 4160*a^3*b^9*c^3*f^8 - 11520*a^4*
b^7*c^4*f^8 - 1024*a^5*b^5*c^5*f^8 + 61440*a^6*b^3*c^6*f^8 + 20*a*b^13*c*f^
8))/(512*(a*b^20*e^2 + 1048576*a^11*c^10*e^2 - 40*a^2*b^18*c*e^2 + 720*a^3*
b^16*c^2*e^2 - 7680*a^4*b^14*c^3*e^2 + 53760*a^5*b^12*c^4*e^2 - 258048*a^6*
b^10*c^5*e^2 + 860160*a^7*b^8*c^6*e^2 - 1966080*a^8*b^6*c^7*e^2 + 2949120*a
^9*b^4*c^8*e^2 - 2621440*a^10*b^2*c^9*e^2)))^(1/2)*((((1024*b^15*c^2*d*e^13
 - 28672*a*b^13*c^3*d*e^13 - 16777216*a^7*b*c^9*d*e^13 + 344064*a^2*b^11*c^
4*d*e^13 - 2293760*a^3*b^9*c^5*d*e^13 + 9175040*a^4*b^7*c^6*d*e^13 - 220200
96*a^5*b^5*c^7*d*e^13 + 29360128*a^6*b^3*c^8*d*e^13)/(128*(b^12 + 4096*a^6*
c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*
c^5 - 24*a*b^10*c)) + (x*(128*b^11*c^2*e^14 - 2560*a*b^9*c^3*e^14 - 131072*
a^5*b*c^7*e^14 + 20480*a^2*b^7*c^4*e^14 - 81920*a^3*b^5*c^5*e^14 + 163840*a
^4*b^3*c^6*e^14))/(16*(b^8 + 256*a^4*c^4 + 96*a^2*b^4*c^2 - 256*a^3*b^2*c^3
 - 16*a*b^6*c)))*(-(9*(b^15*f^8 + f^8*(-(4*a*c - b^2)^15)^(1/2) - 81920*a^7
*b*c^7*f^8 - 560*a^2*b^11*c^2*f^8 + 4160*a^3*b^9*c^3*f^8 - 11520*a^4*b^7*c^
4*f^8 - 1024*a^5*b^5*c^5*f^8 + 61440*a^6*b^3*c^6*f^8 + 20*a*b^13*c*f^8))/(5
12*(a*b^20*e^2 + 1048576*a^11*c^10*e^2 - 40*a^2*b^18*c*e^2 + 720*a^3*b^16*c
^2*e^2 - 7680*a^4*b^14*c^3*e^2 + 53760*a^5*b^12*c^4*e^2 - 258048*a^6*b^10*c
^5*e^2 + 860160*a^7*b^8*c^6*e^2 - 1966080*a^8*b^6*c^7*e^2 + 2949120*a^9*b^4
*c^8*e^2 - 2621440*a^10*b^2*c^9*e^2)))^(1/2) - (786432*a^6*c^8*e^12*f^4 - 1
92*b^12*c^2*e^12*f^4 - 15360*a^2*b^8*c^4*e^12*f^4 + 245760*a^4*b^4*c^6*e^12
*f^4 - 786432*a^5*b^2*c^7*e^12*f^4 + 3072*a*b^10*c^3*e^12*f^4)/(128*(b^12 +
 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 614
4*a^5*b^2*c^5 - 24*a*b^10*c)))*(-(9*(b^15*f^8 + f^8*(-(4*a*c - b^2)^15)^(1/
2) - 81920*a^7*b*c^7*f^8 - 560*a^2*b^11*c^2*f^8 + 4160*a^3*b^9*c^3*f^8 - 11
520*a^4*b^7*c^4*f^8 - 1024*a^5*b^5*c^5*f^8 + 61440*a^6*b^3*c^6*f^8 + 20*a*b
^13*c*f^8))/(512*(a*b^20*e^2 + 1048576*a^11*c^10*e^2 - 40*a^2*b^18*c*e^2 +
720*a^3*b^16*c^2*e^2 - 7680*a^4*b^14*c^3*e^2 + 53760*a^5*b^12*c^4*e^2 - 258
048*a^6*b^10*c^5*e^2 + 860160*a^7*b^8*c^6*e^2 - 1966080*a^8*b^6*c^7*e^2 + 2
949120*a^9*b^4*c^8*e^2 - 2621440*a^10*b^2*c^9*e^2)))^(1/2) + (18432*a^4*c^7
*d*e^11*f^8 + 936*b^8*c^3*d*e^11*f^8 - 6912*a*b^6*c^4*d*e^11*f^8 + 11520*a^
2*b^4*c^5*d*e^11*f^8)/(128*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^
3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c)) + (x*(144*a
^2*c^5*e^12*f^8 + 117*b^4*c^3*e^12*f^8 + 72*a*b^2*c^4*e^12*f^8))/(16*(b^8 +
 256*a^4*c^4 + 96*a^2*b^4*c^2 - 256*a^3*b^2*c^3 - 16*a*b^6*c))) - (-(9*(b^1
5*f^8 + f^8*(-(4*a*c - b^2)^15)^(1/2) - 81920*a^7*b*c^7*f^8 - 560*a^2*b^11*
c^2*f^8 + 4160*a^3*b^9*c^3*f^8 - 11520*a^4*b^7*c^4*f^8 - 1024*a^5*b^5*c^5*f
^8 + 61440*a^6*b^3*c^6*f^8 + 20*a*b^13*c*f^8))/(512*(a*b^20*e^2 + 1048576*a
^11*c^10*e^2 - 40*a^2*b^18*c*e^2 + 720*a^3*b^16*c^2*e^2 - 7680*a^4*b^14*c^3
*e^2 + 53760*a^5*b^12*c^4*e^2 - 258048*a^6*b^10*c^5*e^2 + 860160*a^7*b^8*c^
6*e^2 - 1966080*a^8*b^6*c^7*e^2 + 2949120*a^9*b^4*c^8*e^2 - 2621440*a^10*b^
2*c^9*e^2)))^(1/2)*((((1024*b^15*c^2*d*e^13 - 28672*a*b^13*c^3*d*e^13 - 167
77216*a^7*b*c^9*d*e^13 + 344064*a^2*b^11*c^4*d*e^13 - 2293760*a^3*b^9*c^5*d
*e^13 + 9175040*a^4*b^7*c^6*d*e^13 - 22020096*a^5*b^5*c^7*d*e^13 + 29360128
*a^6*b^3*c^8*d*e^13)/(128*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3
*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c)) + (x*(128*b^
11*c^2*e^14 - 2560*a*b^9*c^3*e^14 - 131072*a^5*b*c^7*e^14 + 20480*a^2*b^7*c
^4*e^14 - 81920*a^3*b^5*c^5*e^14 + 163840*a^4*b^3*c^6*e^14))/(16*(b^8 + 256
*a^4*c^4 + 96*a^2*b^4*c^2 - 256*a^3*b^2*c^3 - 16*a*b^6*c)))*(-(9*(b^15*f^8
```

+ f^8*(-(4*a*c - b^2)^15)^(1/2) - 81920*a^7*b*c^7*f^8 - 560*a^2*b^11*c^2*f^
8 + 4160*a^3*b^9*c^3*f^8 - 11520*a^4*b^7*c^4*f^8 - 1024*a^5*b^5*c^5*f^8 + 6
1440*a^6*b^3*c^6*f^8 + 20*a*b^13*c*f^8))/(512*(a*b^20*e^2 + 1048576*a^11*c^
10*e^2 - 40*a^2*b^18*c*e^2 + 720*a^3*b^16*c^2*e^2 - 7680*a^4*b^14*c^3*e^2 +
53760*a^5*b^12*c^4*e^2 - 258048*a^6*b^10*c^5*e^2 + 860160*a^7*b^8*c^6*e^2
- 1966080*a^8*b^6*c^7*e^2 + 2949120*a^9*b^4*c^8*e^2 - 2621440*a^10*b^2*c^9*
e^2)))^(1/2) + (786432*a^6*c^8*e^12*f^4 - 192*b^12*c^2*e^12*f^4 - 15360*a^2
*b^8*c^4*e^12*f^4 + 245760*a^4*b^4*c^6*e^12*f^4 - 786432*a^5*b^2*c^7*e^12*f
^4 + 3072*a*b^10*c^3*e^12*f^4)/(128*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2
- 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c)))*(
-(9*(b^15*f^8 + f^8*(-(4*a*c - b^2)^15)^(1/2) - 81920*a^7*b*c^7*f^8 - 560*a
^2*b^11*c^2*f^8 + 4160*a^3*b^9*c^3*f^8 - 11520*a^4*b^7*c^4*f^8 - 1024*a^5*b
^5*c^5*f^8 + 61440*a^6*b^3*c^6*f^8 + 20*a*b^13*c*f^8))/(512*(a*b^20*e^2 + 1
048576*a^11*c^10*e^2 - 40*a^2*b^18*c*e^2 + 720*a^3*b^16*c^2*e^2 - 7680*a^4*
b^14*c^3*e^2 + 53760*a^5*b^12*c^4*e^2 - 258048*a^6*b^10*c^5*e^2 + 860160*a^
7*b^8*c^6*e^2 - 1966080*a^8*b^6*c^7*e^2 + 2949120*a^9*b^4*c^8*e^2 - 2621440
*a^10*b^2*c^9*e^2)))^(1/2) + (18432*a^4*c^7*d*e^11*f^8 + 936*b^8*c^3*d*e^11
*f^8 - 6912*a*b^6*c^4*d*e^11*f^8 + 11520*a^2*b^4*c^5*d*e^11*f^8)/(128*(b^12
+ 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6
144*a^5*b^2*c^5 - 24*a*b^10*c)) + (x*(144*a^2*c^5*e^12*f^8 + 117*b^4*c^3*e^
12*f^8 + 72*a*b^2*c^4*e^12*f^8))/(16*(b^8 + 256*a^4*c^4 + 96*a^2*b^4*c^2 -
256*a^3*b^2*c^3 - 16*a*b^6*c))) + (135*b^5*c^3*e^10*f^12 + 1080*a*b^3*c^4*e
^10*f^12 + 432*a^2*b*c^5*e^10*f^12)/(64*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*
c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c)
)))*(-(9*(b^15*f^8 + f^8*(-(4*a*c - b^2)^15)^(1/2) - 81920*a^7*b*c^7*f^8 -
560*a^2*b^11*c^2*f^8 + 4160*a^3*b^9*c^3*f^8 - 11520*a^4*b^7*c^4*f^8 - 1024*
a^5*b^5*c^5*f^8 + 61440*a^6*b^3*c^6*f^8 + 20*a*b^13*c*f^8))/(512*(a*b^20*e^
2 + 1048576*a^11*c^10*e^2 - 40*a^2*b^18*c*e^2 + 720*a^3*b^16*c^2*e^2 - 7680
*a^4*b^14*c^3*e^2 + 53760*a^5*b^12*c^4*e^2 - 258048*a^6*b^10*c^5*e^2 + 8601
60*a^7*b^8*c^6*e^2 - 1966080*a^8*b^6*c^7*e^2 + 2949120*a^9*b^4*c^8*e^2 - 26
21440*a^10*b^2*c^9*e^2)))^(1/2)*2i - ((x^3*(5*b^3*e^2*f^4 + 16*a*b*c*e^2*f^
4 - 40*a*c^2*d^2*e^2*f^4 + 190*b^2*c*d^2*e^2*f^4 + 420*b*c^2*d^4*e^2*f^4))/
(8*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (5*x^4*(19*b^2*c*d*e^3*f^4 - 4*a*c^2*d
*e^3*f^4 + 84*b*c^2*d^3*e^3*f^4))/(8*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (x^5
*(19*b^2*c*e^4*f^4 - 4*a*c^2*e^4*f^4 + 252*b*c^2*d^2*e^4*f^4))/(8*(b^4 + 16
*a^2*c^2 - 8*a*b^2*c)) + (x*(3*a*b^2*f^4 + 12*a^2*c*f^4 + 15*b^3*d^2*f^4 -
20*a*c^2*d^4*f^4 + 95*b^2*c*d^4*f^4 + 84*b*c^2*d^6*f^4 + 48*a*b*c*d^2*f^4))
/(8*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (x^2*(15*b^3*d*e*f^4 - 40*a*c^2*d^3*e
*f^4 + 190*b^2*c*d^3*e*f^4 + 252*b*c^2*d^5*e*f^4 + 48*a*b*c*d*e*f^4))/(8*(b
^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (5*b^3*d^3*f^4 - 4*a*c^2*d^5*f^4 + 19*b^2*c
*d^5*f^4 + 12*b*c^2*d^7*f^4 + 3*a*b^2*d*f^4 + 12*a^2*c*d*f^4 + 16*a*b*c*d^3
*f^4)/(8*e*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (3*b*c^2*e^6*f^4*x^7)/(2*(b^4
+ 16*a^2*c^2 - 8*a*b^2*c)) + (21*b*c^2*d*e^5*f^4*x^6)/(2*(b^4 + 16*a^2*c^2
- 8*a*b^2*c)))/(x^2*(6*b^2*d^2*e^2 + 28*c^2*d^6*e^2 + 2*a*b*e^2 + 12*a*c*d^
2*e^2 + 30*b*c*d^4*e^2) + x^6*(28*c^2*d^2*e^6 + 2*b*c*e^6) + x*(4*b^2*d^3*e
+ 8*c^2*d^7*e + 8*a*c*d^3*e + 12*b*c*d^5*e + 4*a*b*d*e) + x^3*(4*b^2*d*e^3
+ 56*c^2*d^5*e^3 + 8*a*c*d*e^3 + 40*b*c*d^3*e^3) + x^5*(56*c^2*d^3*e^5 + 1
2*b*c*d*e^5) + x^4*(b^2*e^4 + 70*c^2*d^4*e^4 + 2*a*c*e^4 + 30*b*c*d^2*e^4)
+ a^2 + b^2*d^4 + c^2*d^8 + c^2*e^8*x^8 + 2*a*b*d^2 + 2*a*c*d^4 + 2*b*c*d^6
+ 8*c^2*d*e^7*x^7) + atan((((9*(f^8*(-(4*a*c - b^2)^15)^(1/2) - b^15*f^8 +
81920*a^7*b*c^7*f^8 + 560*a^2*b^11*c^2*f^8 - 4160*a^3*b^9*c^3*f^8 + 11520*
a^4*b^7*c^4*f^8 + 1024*a^5*b^5*c^5*f^8 - 61440*a^6*b^3*c^6*f^8 - 20*a*b^13*
c*f^8))/(512*(a*b^20*e^2 + 1048576*a^11*c^10*e^2 - 40*a^2*b^18*c*e^2 + 720*
a^3*b^16*c^2*e^2 - 7680*a^4*b^14*c^3*e^2 + 53760*a^5*b^12*c^4*e^2 - 258048*
a^6*b^10*c^5*e^2 + 860160*a^7*b^8*c^6*e^2 - 1966080*a^8*b^6*c^7*e^2 + 29491
20*a^9*b^4*c^8*e^2 - 2621440*a^10*b^2*c^9*e^2)))^(1/2)*((((1024*b^15*c^2*d*
e^13 - 28672*a*b^13*c^3*d*e^13 - 16777216*a^7*b*c^9*d*e^13 + 344064*a^2*b^1
1*c^4*d*e^13 - 2293760*a^3*b^9*c^5*d*e^13 + 9175040*a^4*b^7*c^6*d*e^13 - 22
020096*a^5*b^5*c^7*d*e^13 + 29360128*a^6*b^3*c^8*d*e^13)/(128*(b^12 + 4096*

a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*
b^2*c^5 - 24*a*b^10*c)) + (x*(128*b^11*c^2*e^14 - 2560*a*b^9*c^3*e^14 - 131
072*a^5*b*c^7*e^14 + 20480*a^2*b^7*c^4*e^14 - 81920*a^3*b^5*c^5*e^14 + 1638
40*a^4*b^3*c^6*e^14))/(16*(b^8 + 256*a^4*c^4 + 96*a^2*b^4*c^2 - 256*a^3*b^2
*c^3 - 16*a*b^6*c)))*((9*(f^8*(-(4*a*c - b^2)^15)^(1/2) - b^15*f^8 + 81920*
a^7*b*c^7*f^8 + 560*a^2*b^11*c^2*f^8 - 4160*a^3*b^9*c^3*f^8 + 11520*a^4*b^7
*c^4*f^8 + 1024*a^5*b^5*c^5*f^8 - 61440*a^6*b^3*c^6*f^8 - 20*a*b^13*c*f^8))
/(512*(a*b^20*e^2 + 1048576*a^11*c^10*e^2 - 40*a^2*b^18*c*e^2 + 720*a^3*b^1
6*c^2*e^2 - 7680*a^4*b^14*c^3*e^2 + 53760*a^5*b^12*c^4*e^2 - 258048*a^6*b^1
0*c^5*e^2 + 860160*a^7*b^8*c^6*e^2 - 1966080*a^8*b^6*c^7*e^2 + 2949120*a^9*
b^4*c^8*e^2 - 2621440*a^10*b^2*c^9*e^2)))^(1/2) - (786432*a^6*c^8*e^12*f^4
- 192*b^12*c^2*e^12*f^4 - 15360*a^2*b^8*c^4*e^12*f^4 + 245760*a^4*b^4*c^6*e
^12*f^4 - 786432*a^5*b^2*c^7*e^12*f^4 + 3072*a*b^10*c^3*e^12*f^4)/(128*(b^1
2 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 -
6144*a^5*b^2*c^5 - 24*a*b^10*c)))*((9*(f^8*(-(4*a*c - b^2)^15)^(1/2) - b^15
*f^8 + 81920*a^7*b*c^7*f^8 + 560*a^2*b^11*c^2*f^8 - 4160*a^3*b^9*c^3*f^8 +
11520*a^4*b^7*c^4*f^8 + 1024*a^5*b^5*c^5*f^8 - 61440*a^6*b^3*c^6*f^8 - 20*a
*b^13*c*f^8))/(512*(a*b^20*e^2 + 1048576*a^11*c^10*e^2 - 40*a^2*b^18*c*e^2
+ 720*a^3*b^16*c^2*e^2 - 7680*a^4*b^14*c^3*e^2 + 53760*a^5*b^12*c^4*e^2 - 2
58048*a^6*b^10*c^5*e^2 + 860160*a^7*b^8*c^6*e^2 - 1966080*a^8*b^6*c^7*e^2 +
2949120*a^9*b^4*c^8*e^2 - 2621440*a^10*b^2*c^9*e^2)))^(1/2) + (18432*a^4*c
^7*d*e^11*f^8 + 936*b^8*c^3*d*e^11*f^8 - 6912*a*b^6*c^4*d*e^11*f^8 + 11520*
a^2*b^4*c^5*d*e^11*f^8)/(128*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*
a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c)) + (x*(144
*a^2*c^5*e^12*f^8 + 117*b^4*c^3*e^12*f^8 + 72*a*b^2*c^4*e^12*f^8))/(16*(b^8
+ 256*a^4*c^4 + 96*a^2*b^4*c^2 - 256*a^3*b^2*c^3 - 16*a*b^6*c)))*1i + ((9*
(f^8*(-(4*a*c - b^2)^15)^(1/2) - b^15*f^8 + 81920*a^7*b*c^7*f^8 + 560*a^2*b
^11*c^2*f^8 - 4160*a^3*b^9*c^3*f^8 + 11520*a^4*b^7*c^4*f^8 + 1024*a^5*b^5*c
^5*f^8 - 61440*a^6*b^3*c^6*f^8 - 20*a*b^13*c*f^8))/(512*(a*b^20*e^2 + 10485
76*a^11*c^10*e^2 - 40*a^2*b^18*c*e^2 + 720*a^3*b^16*c^2*e^2 - 7680*a^4*b^14
*c^3*e^2 + 53760*a^5*b^12*c^4*e^2 - 258048*a^6*b^10*c^5*e^2 + 860160*a^7*b^
8*c^6*e^2 - 1966080*a^8*b^6*c^7*e^2 + 2949120*a^9*b^4*c^8*e^2 - 2621440*a^1
0*b^2*c^9*e^2)))^(1/2)*((((1024*b^15*c^2*d*e^13 - 28672*a*b^13*c^3*d*e^13 -
16777216*a^7*b*c^9*d*e^13 + 344064*a^2*b^11*c^4*d*e^13 - 2293760*a^3*b^9*c
^5*d*e^13 + 9175040*a^4*b^7*c^6*d*e^13 - 22020096*a^5*b^5*c^7*d*e^13 + 2936
0128*a^6*b^3*c^8*d*e^13)/(128*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280
*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c)) + (x*(12
8*b^11*c^2*e^14 - 2560*a*b^9*c^3*e^14 - 131072*a^5*b*c^7*e^14 + 20480*a^2*b
^7*c^4*e^14 - 81920*a^3*b^5*c^5*e^14 + 163840*a^4*b^3*c^6*e^14))/(16*(b^8 +
256*a^4*c^4 + 96*a^2*b^4*c^2 - 256*a^3*b^2*c^3 - 16*a*b^6*c)))*((9*(f^8*(-
(4*a*c - b^2)^15)^(1/2) - b^15*f^8 + 81920*a^7*b*c^7*f^8 + 560*a^2*b^11*c^2
*f^8 - 4160*a^3*b^9*c^3*f^8 + 11520*a^4*b^7*c^4*f^8 + 1024*a^5*b^5*c^5*f^8
- 61440*a^6*b^3*c^6*f^8 - 20*a*b^13*c*f^8))/(512*(a*b^20*e^2 + 1048576*a^11
*c^10*e^2 - 40*a^2*b^18*c*e^2 + 720*a^3*b^16*c^2*e^2 - 7680*a^4*b^14*c^3*e^
2 + 53760*a^5*b^12*c^4*e^2 - 258048*a^6*b^10*c^5*e^2 + 860160*a^7*b^8*c^6*e
^2 - 1966080*a^8*b^6*c^7*e^2 + 2949120*a^9*b^4*c^8*e^2 - 2621440*a^10*b^2*c
^9*e^2)))^(1/2) + (786432*a^6*c^8*e^12*f^4 - 192*b^12*c^2*e^12*f^4 - 15360*
a^2*b^8*c^4*e^12*f^4 + 245760*a^4*b^4*c^6*e^12*f^4 - 786432*a^5*b^2*c^7*e^1
2*f^4 + 3072*a*b^10*c^3*e^12*f^4)/(128*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c
^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c))
)*((9*(f^8*(-(4*a*c - b^2)^15)^(1/2) - b^15*f^8 + 81920*a^7*b*c^7*f^8 + 560
*a^2*b^11*c^2*f^8 - 4160*a^3*b^9*c^3*f^8 + 11520*a^4*b^7*c^4*f^8 + 1024*a^5
*b^5*c^5*f^8 - 61440*a^6*b^3*c^6*f^8 - 20*a*b^13*c*f^8))/(512*(a*b^20*e^2 +
1048576*a^11*c^10*e^2 - 40*a^2*b^18*c*e^2 + 720*a^3*b^16*c^2*e^2 - 7680*a^
4*b^14*c^3*e^2 + 53760*a^5*b^12*c^4*e^2 - 258048*a^6*b^10*c^5*e^2 + 860160*
a^7*b^8*c^6*e^2 - 1966080*a^8*b^6*c^7*e^2 + 2949120*a^9*b^4*c^8*e^2 - 26214
40*a^10*b^2*c^9*e^2)))^(1/2) + (18432*a^4*c^7*d*e^11*f^8 + 936*b^8*c^3*d*e^
11*f^8 - 6912*a*b^6*c^4*d*e^11*f^8 + 11520*a^2*b^4*c^5*d*e^11*f^8)/(128*(b^
12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 -

6144*a^5*b^2*c^5 - 24*a*b^10*c)) + (x*(144*a^2*c^5*e^12*f^8 + 117*b^4*c^3*e^12*f^8 + 72*a*b^2*c^4*e^12*f^8))/(16*(b^8 + 256*a^4*c^4 + 96*a^2*b^4*c^2 - 256*a^3*b^2*c^3 - 16*a*b^6*c)))*1i)/(((9*(f^8*(-(4*a*c - b^2)^15)^(1/2) - b^15*f^8 + 81920*a^7*b*c^7*f^8 + 560*a^2*b^11*c^2*f^8 - 4160*a^3*b^9*c^3*f^8 + 11520*a^4*b^7*c^4*f^8 + 1024*a^5*b^5*c^5*f^8 - 61440*a^6*b^3*c^6*f^8 - 20*a*b^13*c*f^8))/(512*(a*b^20*e^2 + 1048576*a^11*c^10*e^2 - 40*a^2*b^18*c*e^2 + 720*a^3*b^16*c^2*e^2 - 7680*a^4*b^14*c^3*e^2 + 53760*a^5*b^12*c^4*e^2 - 258048*a^6*b^10*c^5*e^2 + 860160*a^7*b^8*c^6*e^2 - 1966080*a^8*b^6*c^7*e^2 + 2949120*a^9*b^4*c^8*e^2 - 2621440*a^10*b^2*c^9*e^2)))^(1/2)*((((1024*b^15*c^2*d*e^13 - 28672*a*b^13*c^3*d*e^13 - 16777216*a^7*b*c^9*d*e^13 + 344064*a^2*b^11*c^4*d*e^13 - 2293760*a^3*b^9*c^5*d*e^13 + 9175040*a^4*b^7*c^6*d*e^13 - 22020096*a^5*b^5*c^7*d*e^13 + 29360128*a^6*b^3*c^8*d*e^13)/(128*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c)) + (x*(128*b^11*c^2*e^14 - 2560*a*b^9*c^3*e^14 - 131072*a^5*b*c^7*e^14 + 20480*a^2*b^7*c^4*e^14 - 81920*a^3*b^5*c^5*e^14 + 163840*a^4*b^3*c^6*e^14))/(16*(b^8 + 256*a^4*c^4 + 96*a^2*b^4*c^2 - 256*a^3*b^2*c^3 - 16*a*b^6*c)))*((9*(f^8*(-(4*a*c - b^2)^15)^(1/2) - b^15*f^8 + 81920*a^7*b*c^7*f^8 + 560*a^2*b^11*c^2*f^8 - 4160*a^3*b^9*c^3*f^8 + 11520*a^4*b^7*c^4*f^8 + 1024*a^5*b^5*c^5*f^8 - 61440*a^6*b^3*c^6*f^8 - 20*a*b^13*c*f^8))/(512*(a*b^20*e^2 + 1048576*a^11*c^10*e^2 - 40*a^2*b^18*c*e^2 + 720*a^3*b^16*c^2*e^2 - 7680*a^4*b^14*c^3*e^2 + 53760*a^5*b^12*c^4*e^2 - 258048*a^6*b^10*c^5*e^2 + 860160*a^7*b^8*c^6*e^2 - 1966080*a^8*b^6*c^7*e^2 + 2949120*a^9*b^4*c^8*e^2 - 2621440*a^10*b^2*c^9*e^2)))^(1/2) - (786432*a^6*c^8*e^12*f^4 - 192*b^12*c^2*e^12*f^4 - 15360*a^2*b^8*c^4*e^12*f^4 + 245760*a^4*b^4*c^6*e^12*f^4 - 786432*a^5*b^2*c^7*e^12*f^4 + 3072*a*b^10*c^3*e^12*f^4)/(128*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c)))*((9*(f^8*(-(4*a*c - b^2)^15)^(1/2) - b^15*f^8 + 81920*a^7*b*c^7*f^8 + 560*a^2*b^11*c^2*f^8 - 4160*a^3*b^9*c^3*f^8 + 11520*a^4*b^7*c^4*f^8 + 1024*a^5*b^5*c^5*f^8 - 61440*a^6*b^3*c^6*f^8 - 20*a*b^13*c*f^8))/(512*(a*b^20*e^2 + 1048576*a^11*c^10*e^2 - 40*a^2*b^18*c*e^2 + 720*a^3*b^16*c^2*e^2 - 7680*a^4*b^14*c^3*e^2 + 53760*a^5*b^12*c^4*e^2 - 258048*a^6*b^10*c^5*e^2 + 860160*a^7*b^8*c^6*e^2 - 1966080*a^8*b^6*c^7*e^2 + 2949120*a^9*b^4*c^8*e^2 - 2621440*a^10*b^2*c^9*e^2)))^(1/2) + (18432*a^4*c^7*d*e^11*f^8 + 936*b^8*c^3*d*e^11*f^8 - 6912*a*b^6*c^4*d*e^11*f^8 + 11520*a^2*b^4*c^5*d*e^11*f^8)/(128*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c)) + (x*(144*a^2*c^5*e^12*f^8 + 117*b^4*c^3*e^12*f^8 + 72*a*b^2*c^4*e^12*f^8))/(16*(b^8 + 256*a^4*c^4 + 96*a^2*b^4*c^2 - 256*a^3*b^2*c^3 - 16*a*b^6*c))) - ((9*(f^8*(-(4*a*c - b^2)^15)^(1/2) - b^15*f^8 + 81920*a^7*b*c^7*f^8 + 560*a^2*b^11*c^2*f^8 - 4160*a^3*b^9*c^3*f^8 + 11520*a^4*b^7*c^4*f^8 + 1024*a^5*b^5*c^5*f^8 - 61440*a^6*b^3*c^6*f^8 - 20*a*b^13*c*f^8))/(512*(a*b^20*e^2 + 1048576*a^11*c^10*e^2 - 40*a^2*b^18*c*e^2 + 720*a^3*b^16*c^2*e^2 - 7680*a^4*b^14*c^3*e^2 + 53760*a^5*b^12*c^4*e^2 - 258048*a^6*b^10*c^5*e^2 + 860160*a^7*b^8*c^6*e^2 - 1966080*a^8*b^6*c^7*e^2 + 2949120*a^9*b^4*c^8*e^2 - 2621440*a^10*b^2*c^9*e^2)))^(1/2)*((((1024*b^15*c^2*d*e^13 - 28672*a*b^13*c^3*d*e^13 - 16777216*a^7*b*c^9*d*e^13 + 344064*a^2*b^11*c^4*d*e^13 - 2293760*a^3*b^9*c^5*d*e^13 + 9175040*a^4*b^7*c^6*d*e^13 - 22020096*a^5*b^5*c^7*d*e^13 + 29360128*a^6*b^3*c^8*d*e^13)/(128*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c)) + (x*(128*b^11*c^2*e^14 - 2560*a*b^9*c^3*e^14 - 131072*a^5*b*c^7*e^14 + 20480*a^2*b^7*c^4*e^14 - 81920*a^3*b^5*c^5*e^14 + 163840*a^4*b^3*c^6*e^14))/(16*(b^8 + 256*a^4*c^4 + 96*a^2*b^4*c^2 - 256*a^3*b^2*c^3 - 16*a*b^6*c)))*((9*(f^8*(-(4*a*c - b^2)^15)^(1/2) - b^15*f^8 + 81920*a^7*b*c^7*f^8 + 560*a^2*b^11*c^2*f^8 - 4160*a^3*b^9*c^3*f^8 + 11520*a^4*b^7*c^4*f^8 + 1024*a^5*b^5*c^5*f^8 - 61440*a^6*b^3*c^6*f^8 - 20*a*b^13*c*f^8))/(512*(a*b^20*e^2 + 1048576*a^11*c^10*e^2 - 40*a^2*b^18*c*e^2 + 720*a^3*b^16*c^2*e^2 - 7680*a^4*b^14*c^3*e^2 + 53760*a^5*b^12*c^4*e^2 - 258048*a^6*b^10*c^5*e^2 + 860160*a^7*b^8*c^6*e^2 - 1966080*a^8*b^6*c^7*e^2 + 2949120*a^9*b^4*c^8*e^2 - 2621440*a^10*b^2*c^9*e^2)))^(1/2) + (786432*a^6*c^8*e^12*f^4 - 192*b^12*c^2*e^12*f^4

```
- 15360*a^2*b^8*c^4*e^12*f^4 + 245760*a^4*b^4*c^6*e^12*f^4 - 786432*a^5*b^2
*c^7*e^12*f^4 + 3072*a*b^10*c^3*e^12*f^4)/(128*(b^12 + 4096*a^6*c^6 + 240*a
^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*
b^10*c)))*((9*(f^8*(-(4*a*c - b^2)^15)^(1/2) - b^15*f^8 + 81920*a^7*b*c^7*f
^8 + 560*a^2*b^11*c^2*f^8 - 4160*a^3*b^9*c^3*f^8 + 11520*a^4*b^7*c^4*f^8 +
1024*a^5*b^5*c^5*f^8 - 61440*a^6*b^3*c^6*f^8 - 20*a*b^13*c*f^8))/(512*(a*b^
20*e^2 + 1048576*a^11*c^10*e^2 - 40*a^2*b^18*c*e^2 + 720*a^3*b^16*c^2*e^2 -
 7680*a^4*b^14*c^3*e^2 + 53760*a^5*b^12*c^4*e^2 - 258048*a^6*b^10*c^5*e^2 +
 860160*a^7*b^8*c^6*e^2 - 1966080*a^8*b^6*c^7*e^2 + 2949120*a^9*b^4*c^8*e^2
 - 2621440*a^10*b^2*c^9*e^2)))^(1/2) + (18432*a^4*c^7*d*e^11*f^8 + 936*b^8*
c^3*d*e^11*f^8 - 6912*a*b^6*c^4*d*e^11*f^8 + 11520*a^2*b^4*c^5*d*e^11*f^8)/
(128*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b
^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c)) + (x*(144*a^2*c^5*e^12*f^8 + 117*
b^4*c^3*e^12*f^8 + 72*a*b^2*c^4*e^12*f^8))/(16*(b^8 + 256*a^4*c^4 + 96*a^2*
b^4*c^2 - 256*a^3*b^2*c^3 - 16*a*b^6*c))) + (135*b^5*c^3*e^10*f^12 + 1080*a
*b^3*c^4*e^10*f^12 + 432*a^2*b*c^5*e^10*f^12)/(64*(b^12 + 4096*a^6*c^6 + 24
0*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24
*a*b^10*c))))*((9*(f^8*(-(4*a*c - b^2)^15)^(1/2) - b^15*f^8 + 81920*a^7*b*c
^7*f^8 + 560*a^2*b^11*c^2*f^8 - 4160*a^3*b^9*c^3*f^8 + 11520*a^4*b^7*c^4*f^
8 + 1024*a^5*b^5*c^5*f^8 - 61440*a^6*b^3*c^6*f^8 - 20*a*b^13*c*f^8))/(512*(
a*b^20*e^2 + 1048576*a^11*c^10*e^2 - 40*a^2*b^18*c*e^2 + 720*a^3*b^16*c^2*e
^2 - 7680*a^4*b^14*c^3*e^2 + 53760*a^5*b^12*c^4*e^2 - 258048*a^6*b^10*c^5*e
^2 + 860160*a^7*b^8*c^6*e^2 - 1966080*a^8*b^6*c^7*e^2 + 2949120*a^9*b^4*c^8
*e^2 - 2621440*a^10*b^2*c^9*e^2)))^(1/2)*2i
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*f*x+d*f)**4/(a+b*(e*x+d)**2+c*(e*x+d)**4)**3,x)
[Out] Timed out
```

3.542 $\int \frac{(df+efx)^3}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^3}\,dx$

Optimal. Leaf size=159

$$-\frac{3bf^3\left(b+2c(d+ex)^2\right)}{4e\left(b^2-4ac\right)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)}+\frac{f^3\left(2a+b(d+ex)^2\right)}{4e\left(b^2-4ac\right)\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}+\frac{3bcf^3\tanh^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{e\left(b^2-4ac\right)^{5/2}}$$

Rubi [A] time = 0.20, antiderivative size = 159, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {1142, 1114, 638, 614, 618, 206}

$$-\frac{3bf^3\left(b+2c(d+ex)^2\right)}{4e\left(b^2-4ac\right)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)}+\frac{f^3\left(2a+b(d+ex)^2\right)}{4e\left(b^2-4ac\right)\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}+\frac{3bcf^3\tanh^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{e\left(b^2-4ac\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*f + e*f*x)^3/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3,x]
```

```
[Out] (f^3*(2*a + b*(d + e*x)^2))/(4*(b^2 - 4*a*c)*e*(a + b*(d + e*x)^2 + c*(d +
e*x)^4)^2) - (3*b*f^3*(b + 2*c*(d + e*x)^2))/(4*(b^2 - 4*a*c)^2*e*(a + b*(d
 + e*x)^2 + c*(d + e*x)^4)) + (3*b*c*f^3*ArcTanh[(b + 2*c*(d + e*x)^2)/Sqrt
[b^2 - 4*a*c]])/((b^2 - 4*a*c)^(5/2)*e)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 614

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[(2*c*(2*p +
 3))/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
egerQ[4*p]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 638

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)*(a + b*x + c*x^2)^(p + 1))/((p +
 1)*(b^2 - 4*a*c)), x] - Dist[((2*p + 3)*(2*c*d - b*e))/((p + 1)*(b^2 - 4*a
*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &&
 NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 1142

```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Di
st[u^m/(Coefficient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p,
 x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

Rubi steps

$$\int \frac{(df+efx)^3}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^3}\,dx = \frac{f^3\,\text{Subst}\left(\int \frac{x^3}{\left(a+bx^2+cx^4\right)^3}\,dx, x, d+ex\right)}{e}$$

$$= \frac{f^3\,\text{Subst}\left(\int \frac{x}{\left(a+bx+cx^2\right)^3}\,dx, x, (d+ex)^2\right)}{2e}$$

$$= \frac{f^3\left(2a+b(d+ex)^2\right)}{4\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)^2} + \frac{\left(3bf^3\right)\text{Subst}\left(\int \frac{1}{\left(a+bx+cx^2\right)^2}\right.}{4\left(b^2-4ac\right)}$$

$$= \frac{f^3\left(2a+b(d+ex)^2\right)}{4\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)^2} - \frac{3bf^3\left(b+2c(d+\right.}{4\left(b^2-4ac\right)^2e\left(a+b(d+e\right.}$$

$$= \frac{f^3\left(2a+b(d+ex)^2\right)}{4\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)^2} - \frac{3bf^3\left(b+2c(d+\right.}{4\left(b^2-4ac\right)^2e\left(a+b(d+e\right.}$$

$$= \frac{f^3\left(2a+b(d+ex)^2\right)}{4\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)^2} - \frac{3bf^3\left(b+2c(d+\right.}{4\left(b^2-4ac\right)^2e\left(a+b(d+e\right.}$$

Mathematica [A] time = 0.21, size = 149, normalized size = 0.94

$$\frac{f^3\left(\frac{\left(b^2-4ac\right)\left(2a+b(d+ex)^2\right)}{\left(a+(d+ex)^2\left(b+c(d+ex)^2\right)\right)^2} - \frac{12bc\tan^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}} - \frac{3b\left(b+2c(d+ex)^2\right)}{a+b(d+ex)^2+c(d+ex)^4}\right)}{4e\left(b^2-4ac\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*f + e*f*x)^3/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3,x]
```

```
[Out] (f^3*((-3*b*(b + 2*c*(d + e*x)^2))/(a + b*(d + e*x)^2 + c*(d + e*x)^4) + ((
b^2 - 4*a*c)*(2*a + b*(d + e*x)^2))/(a + (d + e*x)^2*(b + c*(d + e*x)^2))^2
 - (12*b*c*ArcTan[(b + 2*c*(d + e*x)^2)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*
a*c]))/(4*(b^2 - 4*a*c)^2*e)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(df+efx)^3}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(d*f + e*f*x)^3/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3,x]
```

```
[Out] IntegrateAlgebraic[(d*f + e*f*x)^3/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3, x
]
```

fricas [B] time = 1.44, size = 3843, normalized size = 24.17

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*f*x+d*f)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="fricas"
)
```

```
[Out] [-1/4*(6*(b^3*c^2 - 4*a*b*c^3)*e^6*f^3*x^6 + 36*(b^3*c^2 - 4*a*b*c^3)*d*e^5
*f^3*x^5 + 9*(b^4*c - 4*a*b^2*c^2 + 10*(b^3*c^2 - 4*a*b*c^3)*d^2)*e^4*f^3*x
^4 + 12*(10*(b^3*c^2 - 4*a*b*c^3)*d^3 + 3*(b^4*c - 4*a*b^2*c^2)*d)*e^3*f^3*
x^3 + 2*(b^5 + a*b^3*c - 20*a^2*b*c^2 + 45*(b^3*c^2 - 4*a*b*c^3)*d^4 + 27*(
b^4*c - 4*a*b^2*c^2)*d^2)*e^2*f^3*x^2 + 4*(9*(b^3*c^2 - 4*a*b*c^3)*d^5 + 9*
(b^4*c - 4*a*b^2*c^2)*d^3 + (b^5 + a*b^3*c - 20*a^2*b*c^2)*d)*e*f^3*x + (6*
(b^3*c^2 - 4*a*b*c^3)*d^6 + a*b^4 + 4*a^2*b^2*c - 32*a^3*c^2 + 9*(b^4*c - 4
*a*b^2*c^2)*d^4 + 2*(b^5 + a*b^3*c - 20*a^2*b*c^2)*d^2)*f^3 - 6*(b*c^3*e^8*
f^3*x^8 + 8*b*c^3*d*e^7*f^3*x^7 + 2*(14*b*c^3*d^2 + b^2*c^2)*e^6*f^3*x^6 +
4*(14*b*c^3*d^3 + 3*b^2*c^2*d)*e^5*f^3*x^5 + (70*b*c^3*d^4 + 30*b^2*c^2*d^2
 + b^3*c + 2*a*b*c^2)*e^4*f^3*x^4 + 4*(14*b*c^3*d^5 + 10*b^2*c^2*d^3 + (b^3
*c + 2*a*b*c^2)*d)*e^3*f^3*x^3 + 2*(14*b*c^3*d^6 + 15*b^2*c^2*d^4 + a*b^2*c
 + 3*(b^3*c + 2*a*b*c^2)*d^2)*e^2*f^3*x^2 + 4*(2*b*c^3*d^7 + 3*b^2*c^2*d^5
+ a*b^2*c*d + (b^3*c + 2*a*b*c^2)*d^3)*e*f^3*x + (b*c^3*d^8 + 2*b^2*c^2*d^6
 + 2*a*b^2*c*d^2 + (b^3*c + 2*a*b*c^2)*d^4 + a^2*b*c)*f^3)*sqrt(b^2 - 4*a*c
)*log((2*c^2*e^4*x^4 + 8*c^2*d*e^3*x^3 + 2*c^2*d^4 + 2*(6*c^2*d^2 + b*c)*e^
2*x^2 + 2*b*c*d^2 + 4*(2*c^2*d^3 + b*c*d)*e*x + b^2 - 2*a*c + (2*c*e^2*x^2
+ 4*c*d*e*x + 2*c*d^2 + b)*sqrt(b^2 - 4*a*c))/(c*e^4*x^4 + 4*c*d*e^3*x^3 +
c*d^4 + (6*c*d^2 + b)*e^2*x^2 + b*d^2 + 2*(2*c*d^3 + b*d)*e*x + a)))/((b^6*
c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*e^9*x^8 + 8*(b^6*c^2 - 12
*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d*e^8*x^7 + 2*(b^7*c - 12*a*b^5*c
^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4 + 14*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^
2*c^4 - 64*a^3*c^5)*d^2)*e^7*x^6 + 4*(14*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b
^2*c^4 - 64*a^3*c^5)*d^3 + 3*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^
3*b*c^4)*d)*e^6*x^5 + (b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 -
 128*a^4*c^4 + 70*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^
4 + 30*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*d^2)*e^5*x^4
+ 4*(14*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^5 + 10*(b^
7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*d^3 + (b^8 - 10*a*b^6*c
 + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*d)*e^4*x^3 + 2*(a*b^7 - 1
2*a^2*b^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3 + 14*(b^6*c^2 - 12*a*b^4*c^3 +
48*a^2*b^2*c^4 - 64*a^3*c^5)*d^6 + 15*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^
3 - 64*a^3*b*c^4)*d^4 + 3*(b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c
^3 - 128*a^4*c^4)*d^2)*e^3*x^2 + 4*(2*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*
c^4 - 64*a^3*c^5)*d^7 + 3*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b
*c^4)*d^5 + (b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c
^4)*d^3 + (a*b^7 - 12*a^2*b^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3)*d)*e^2*x +
 ((b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^8 + a^2*b^6 - 12
*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3 + 2*(b^7*c - 12*a*b^5*c^2 + 48*a^2
*b^3*c^3 - 64*a^3*b*c^4)*d^6 + (b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*
b^2*c^3 - 128*a^4*c^4)*d^4 + 2*(a*b^7 - 12*a^2*b^5*c + 48*a^3*b^3*c^2 - 64*
a^4*b*c^3)*d^2)*e), -1/4*(6*(b^3*c^2 - 4*a*b*c^3)*e^6*f^3*x^6 + 36*(b^3*c^2
 - 4*a*b*c^3)*d*e^5*f^3*x^5 + 9*(b^4*c - 4*a*b^2*c^2 + 10*(b^3*c^2 - 4*a*b*
c^3)*d^2)*e^4*f^3*x^4 + 12*(10*(b^3*c^2 - 4*a*b*c^3)*d^3 + 3*(b^4*c - 4*a*b
^2*c^2)*d)*e^3*f^3*x^3 + 2*(b^5 + a*b^3*c - 20*a^2*b*c^2 + 45*(b^3*c^2 - 4*
a*b*c^3)*d^4 + 27*(b^4*c - 4*a*b^2*c^2)*d^2)*e^2*f^3*x^2 + 4*(9*(b^3*c^2 -
4*a*b*c^3)*d^5 + 9*(b^4*c - 4*a*b^2*c^2)*d^3 + (b^5 + a*b^3*c - 20*a^2*b*c^
2)*d)*e*f^3*x + (6*(b^3*c^2 - 4*a*b*c^3)*d^6 + a*b^4 + 4*a^2*b^2*c - 32*a^3
*c^2 + 9*(b^4*c - 4*a*b^2*c^2)*d^4 + 2*(b^5 + a*b^3*c - 20*a^2*b*c^2)*d^2)*
```

```
f^3 - 12*(b*c^3*e^8*f^3*x^8 + 8*b*c^3*d*e^7*f^3*x^7 + 2*(14*b*c^3*d^2 + b^2
*c^2)*e^6*f^3*x^6 + 4*(14*b*c^3*d^3 + 3*b^2*c^2*d)*e^5*f^3*x^5 + (70*b*c^3*
d^4 + 30*b^2*c^2*d^2 + b^3*c + 2*a*b*c^2)*e^4*f^3*x^4 + 4*(14*b*c^3*d^5 + 1
0*b^2*c^2*d^3 + (b^3*c + 2*a*b*c^2)*d)*e^3*f^3*x^3 + 2*(14*b*c^3*d^6 + 15*b
^2*c^2*d^4 + a*b^2*c + 3*(b^3*c + 2*a*b*c^2)*d^2)*e^2*f^3*x^2 + 4*(2*b*c^3*
d^7 + 3*b^2*c^2*d^5 + a*b^2*c*d + (b^3*c + 2*a*b*c^2)*d^3)*e*f^3*x + (b*c^3
*d^8 + 2*b^2*c^2*d^6 + 2*a*b^2*c*d^2 + (b^3*c + 2*a*b*c^2)*d^4 + a^2*b*c)*f
^3)*sqrt(-b^2 + 4*a*c)*arctan(-(2*c*e^2*x^2 + 4*c*d*e*x + 2*c*d^2 + b)*sqrt
(-b^2 + 4*a*c)/(b^2 - 4*a*c)))/((b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 -
64*a^3*c^5)*e^9*x^8 + 8*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c
^5)*d*e^8*x^7 + 2*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4 + 1
4*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^2)*e^7*x^6 + 4*(
14*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^3 + 3*(b^7*c -
12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*d)*e^6*x^5 + (b^8 - 10*a*b^6*
c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4 + 70*(b^6*c^2 - 12*a*b^4*
c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^4 + 30*(b^7*c - 12*a*b^5*c^2 + 48*a^2*
b^3*c^3 - 64*a^3*b*c^4)*d^2)*e^5*x^4 + 4*(14*(b^6*c^2 - 12*a*b^4*c^3 + 48*a
^2*b^2*c^4 - 64*a^3*c^5)*d^5 + 10*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 -
64*a^3*b*c^4)*d^3 + (b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 1
28*a^4*c^4)*d)*e^4*x^3 + 2*(a*b^7 - 12*a^2*b^5*c + 48*a^3*b^3*c^2 - 64*a^4*
b*c^3 + 14*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^6 + 15*
(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*d^4 + 3*(b^8 - 10*a*
b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*d^2)*e^3*x^2 + 4*(2*
(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^7 + 3*(b^7*c - 12*
a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*d^5 + (b^8 - 10*a*b^6*c + 24*a^2
*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*d^3 + (a*b^7 - 12*a^2*b^5*c + 48*a
^3*b^3*c^2 - 64*a^4*b*c^3)*d)*e^2*x + ((b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2
*c^4 - 64*a^3*c^5)*d^8 + a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c
^3 + 2*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*d^6 + (b^8 -
10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*d^4 + 2*(a*b^7
- 12*a^2*b^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3)*d^2)*e)]
```

giac [**B**] time = 0.78, size = 447, normalized size = 2.81

$$-\frac{3\,bcf^3\arctan\left(\frac{2\,cd^2f+2\,(fx^2e+2\,dfx)ce+bf}{\sqrt{-b^2+4\,ac}\,f}\right)e^{(-1)}}{(b^4-8\,ab^2c+16\,a^2c^2)\sqrt{-b^2+4\,ac}}-\frac{6\,bc^2d^6f^7+18\,(fx^2e+2\,dfx)bc^2d^4f^6e+9\,b^2cd^4f^7+18\,(fx^2e+2\,dfx)^2bc^2d^2f^5e^2+18\,(fx^2e+2\,dfx)b^2cd^2f^6e+2\,b^3d^2f^7+10\,abcd^2f^7+6\,(fx^2e+2\,dfx)^3bc^2f^4e^3+9\,(fx^2e+2\,dfx)^2b^2cf^5e^2+2\,(fx^2e+2\,dfx)b^3f^6e+10\,(fx^2e+2\,dfx)abcf^6e+ab^2f^7+8\,a^2cf^7}{4\,\left(cd^4f^2+2\,(fx^2e+2\,dfx)cd^2fe+bd^2f^2+(fx^2e+2\,dfx)^2ce^2+(fx^2e+2\,dfx)bfe+af^2\right)^2(b^4e-8\,ab^2ce+16\,a^2c^2e)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*f*x+d*f)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="giac")
```

```
[Out] -3*b*c*f^3*arctan((2*c*d^2*f + 2*(f*x^2*e + 2*d*f*x)*c*e + b*f)/(sqrt(-b^2
+ 4*a*c)*f))*e^(-1)/((b^4 - 8*a*b^2*c + 16*a^2*c^2)*sqrt(-b^2 + 4*a*c)) - 1
/4*(6*b*c^2*d^6*f^7 + 18*(f*x^2*e + 2*d*f*x)*b*c^2*d^4*f^6*e + 9*b^2*c*d^4*
f^7 + 18*(f*x^2*e + 2*d*f*x)^2*b*c^2*d^2*f^5*e^2 + 18*(f*x^2*e + 2*d*f*x)*b
^2*c*d^2*f^6*e + 2*b^3*d^2*f^7 + 10*a*b*c*d^2*f^7 + 6*(f*x^2*e + 2*d*f*x)^3
*b*c^2*f^4*e^3 + 9*(f*x^2*e + 2*d*f*x)^2*b^2*c*f^5*e^2 + 2*(f*x^2*e + 2*d*f
*x)*b^3*f^6*e + 10*(f*x^2*e + 2*d*f*x)*a*b*c*f^6*e + a*b^2*f^7 + 8*a^2*c*f^
7)/((c*d^4*f^2 + 2*(f*x^2*e + 2*d*f*x)*c*d^2*f*e + b*d^2*f^2 + (f*x^2*e + 2
*d*f*x)^2*c*e^2 + (f*x^2*e + 2*d*f*x)*b*f*e + a*f^2)^2*(b^4*e - 8*a*b^2*c*e
 + 16*a^2*c^2*e))
```

maple [**C**] time = 0.05, size = 2181, normalized size = 13.72

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*f*x+d*f)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x)
```

```
[Out] -3/2*f^3/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d
^4+2*b*d*e*x+b*d^2+a)^2*c^2*e^5*b/(16*a^2*c^2-8*a*b^2*c+b^4)*x^6-9*f^3/(c*e
^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+
```

```
b*d^2+a)^2*e^4*b*c^2*d/(16*a^2*c^2-8*a*b^2*c+b^4)*x^5-45/2*f^3/(c*e^4*x^4+4
*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)
^2*b*c^2*e^3/(16*a^2*c^2-8*a*b^2*c+b^4)*x^4*d^2-9/4*f^3/(c*e^4*x^4+4*c*d*e^
3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*b^2*
c*e^3/(16*a^2*c^2-8*a*b^2*c+b^4)*x^4-30*f^3/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^
2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*c^2*d^3*e^2*b/(1
6*a^2*c^2-8*a*b^2*c+b^4)*x^3-9*f^3/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2
+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*c*d*e^2*b^2/(16*a^2*c^2-8
*a*b^2*c+b^4)*x^3-45/2*f^3/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3
*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*b*e/(16*a^2*c^2-8*a*b^2*c+b^4)*x^
2*c^2*d^4-27/2*f^3/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e
^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*b^2*e/(16*a^2*c^2-8*a*b^2*c+b^4)*x^2*c*d^
2-5/2*f^3/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*
d^4+2*b*d*e*x+b*d^2+a)^2*b*e/(16*a^2*c^2-8*a*b^2*c+b^4)*x^2*a*c-1/2*f^3/(c*
e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x
+b*d^2+a)^2*b^3*e/(16*a^2*c^2-8*a*b^2*c+b^4)*x^2-9*f^3/(c*e^4*x^4+4*c*d*e^3
*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*d^5*b
/(16*a^2*c^2-8*a*b^2*c+b^4)*x*c^2-9*f^3/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^
2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*d^3*b^2/(16*a^2*c^2-
8*a*b^2*c+b^4)*x*c-5*f^3/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e
*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*d*b/(16*a^2*c^2-8*a*b^2*c+b^4)*x*a*
c-f^3/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+
2*b*d*e*x+b*d^2+a)^2*d*b^3/(16*a^2*c^2-8*a*b^2*c+b^4)*x-3/2*f^3/(c*e^4*x^4+
4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a
)^2/(16*a^2*c^2-8*a*b^2*c+b^4)/e*b*c^2*d^6-9/4*f^3/(c*e^4*x^4+4*c*d*e^3*x^3
+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2/(16*a^2*c
^2-8*a*b^2*c+b^4)/e*b^2*c*d^4-5/2*f^3/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*
x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2/(16*a^2*c^2-8*a*b^2*c+
b^4)/e*a*b*c*d^2-1/2*f^3/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e
*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2/(16*a^2*c^2-8*a*b^2*c+b^4)/e*b^3*d^
2-2*f^3/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^
4+2*b*d*e*x+b*d^2+a)^2/(16*a^2*c^2-8*a*b^2*c+b^4)/e*a^2*c-1/4*f^3/(c*e^4*x^
4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2
+a)^2/(16*a^2*c^2-8*a*b^2*c+b^4)/e*a*b^2+3/2*f^3*b*c/(16*a^2*c^2-8*a*b^2*c+
b^4)/e*sum((-_R*e-d)/(2*_R^3*c*e^3+6*_R^2*c*d*e^2+6*_R*c*d^2*e+2*c*d^3+_R*b
*e+b*d)*ln(-_R+x),_R=RootOf(_Z^4*c*e^4+4*_Z^3*c*d*e^3+c*d^4+b*d^2+(6*c*d^2*
e^2+b*e^2)*_Z^2+(4*c*d^3*e+2*b*d*e)*_Z+a))
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*f*x+d*f)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="maxima"
)
```

```
[Out] Timed out
```

mupad [B] time = 4.02, size = 1267, normalized size = 7.97

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*f + e*f*x)^3/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3,x)
```

```
[Out] - ((a*b^2*f^3 + 8*a^2*c*f^3 + 2*b^3*d^2*f^3 + 9*b^2*c*d^4*f^3 + 6*b*c^2*d^6
*f^3 + 10*a*b*c*d^2*f^3)/(4*e*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (x^2*(b^3*e
*f^3 + 27*b^2*c*d^2*e*f^3 + 45*b*c^2*d^4*e*f^3 + 5*a*b*c*e*f^3))/(2*(b^4 +
16*a^2*c^2 - 8*a*b^2*c)) + (9*x^4*(b^2*c*e^3*f^3 + 10*b*c^2*d^2*e^3*f^3))/(
```

```
4*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (3*d*x^3*(3*b^2*c*e^2*f^3 + 10*b*c^2*d^
2*e^2*f^3))/(b^4 + 16*a^2*c^2 - 8*a*b^2*c) + (d*x*(b^3*f^3 + 9*b^2*c*d^2*f^
3 + 9*b*c^2*d^4*f^3 + 5*a*b*c*f^3))/(b^4 + 16*a^2*c^2 - 8*a*b^2*c) + (3*b*c
^2*e^5*f^3*x^6)/(2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (9*b*c^2*d*e^4*f^3*x^5
)/(b^4 + 16*a^2*c^2 - 8*a*b^2*c))/(x^2*(6*b^2*d^2*e^2 + 28*c^2*d^6*e^2 + 2*
a*b*e^2 + 12*a*c*d^2*e^2 + 30*b*c*d^4*e^2) + x^6*(28*c^2*d^2*e^6 + 2*b*c*e^
6) + x*(4*b^2*d^3*e + 8*c^2*d^7*e + 8*a*c*d^3*e + 12*b*c*d^5*e + 4*a*b*d*e)
 + x^3*(4*b^2*d*e^3 + 56*c^2*d^5*e^3 + 8*a*c*d*e^3 + 40*b*c*d^3*e^3) + x^5*
(56*c^2*d^3*e^5 + 12*b*c*d*e^5) + x^4*(b^2*e^4 + 70*c^2*d^4*e^4 + 2*a*c*e^4
 + 30*b*c*d^2*e^4) + a^2 + b^2*d^4 + c^2*d^8 + c^2*e^8*x^8 + 2*a*b*d^2 + 2*
a*c*d^4 + 2*b*c*d^6 + 8*c^2*d*e^7*x^7) - (3*b*c*f^3*atan(((b^4*(4*a*c - b^2
)^5 + 16*a^2*c^2*(4*a*c - b^2)^5 - 8*a*b^2*c*(4*a*c - b^2)^5)*(x^2*((9*b^2*
c^4*e^8*f^6)/(a*(4*a*c - b^2)^(9/2)*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (9*b^
3*c^2*f^6*(2*b^5*c^2*e^10 - 16*a*b^3*c^3*e^10 + 32*a^2*b*c^4*e^10))/(2*a*e^
2*(4*a*c - b^2)^(15/2)*(b^4 + 16*a^2*c^2 - 8*a*b^2*c))) + x*((9*b^3*c^2*f^6
*(2*b^5*c^2*d*e^9 - 16*a*b^3*c^3*d*e^9 + 32*a^2*b*c^4*d*e^9))/(a*e^2*(4*a*c
 - b^2)^(15/2)*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (18*b^2*c^4*d*e^7*f^6)/(a*
(4*a*c - b^2)^(9/2)*(b^4 + 16*a^2*c^2 - 8*a*b^2*c))) + (9*b^2*c^4*d^2*e^6*f
^6)/(a*(4*a*c - b^2)^(9/2)*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (9*b^3*c^2*f^6
*(64*a^3*c^4*e^8 + 4*a*b^4*c^2*e^8 - 32*a^2*b^2*c^3*e^8 + 2*b^5*c^2*d^2*e^8
 - 16*a*b^3*c^3*d^2*e^8 + 32*a^2*b*c^4*d^2*e^8))/(2*a*e^2*(4*a*c - b^2)^(15
/2)*(b^4 + 16*a^2*c^2 - 8*a*b^2*c))))/(18*b^2*c^4*e^6*f^6)))/(e*(4*a*c - b^
2)^(5/2))
```

sympy [B] time = 14.58, size = 1794, normalized size = 11.28

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*f*x+d*f)**3/(a+b*(e*x+d)**2+c*(e*x+d)**4)**3,x)
```

```
[Out] 3*b*c*f**3*sqrt(-1/(4*a*c - b**2)**5)*log(2*d*x/e + x**2 + (-192*a**3*b*c**
4*f**3*sqrt(-1/(4*a*c - b**2)**5) + 144*a**2*b**3*c**3*f**3*sqrt(-1/(4*a*c
- b**2)**5) - 36*a*b**5*c**2*f**3*sqrt(-1/(4*a*c - b**2)**5) + 3*b**7*c*f**
3*sqrt(-1/(4*a*c - b**2)**5) + 3*b**2*c*f**3 + 6*b*c**2*d**2*f**3)/(6*b*c**
2*e**2*f**3))/(2*e) - 3*b*c*f**3*sqrt(-1/(4*a*c - b**2)**5)*log(2*d*x/e + x
**2 + (192*a**3*b*c**4*f**3*sqrt(-1/(4*a*c - b**2)**5) - 144*a**2*b**3*c**3
*f**3*sqrt(-1/(4*a*c - b**2)**5) + 36*a*b**5*c**2*f**3*sqrt(-1/(4*a*c - b**
2)**5) - 3*b**7*c*f**3*sqrt(-1/(4*a*c - b**2)**5) + 3*b**2*c*f**3 + 6*b*c**
2*d**2*f**3)/(6*b*c**2*e**2*f**3))/(2*e) + (-8*a**2*c*f**3 - a*b**2*f**3 -
10*a*b*c*d**2*f**3 - 2*b**3*d**2*f**3 - 9*b**2*c*d**4*f**3 - 6*b*c**2*d**6*
f**3 - 36*b*c**2*d*e**5*f**3*x**5 - 6*b*c**2*e**6*f**3*x**6 + x**4*(-9*b**2
*c*e**4*f**3 - 90*b*c**2*d**2*e**4*f**3) + x**3*(-36*b**2*c*d*e**3*f**3 - 1
20*b*c**2*d**3*e**3*f**3) + x**2*(-10*a*b*c*e**2*f**3 - 2*b**3*e**2*f**3 -
54*b**2*c*d**2*e**2*f**3 - 90*b*c**2*d**4*e**2*f**3) + x*(-20*a*b*c*d*e*f**
3 - 4*b**3*d*e*f**3 - 36*b**2*c*d**3*e*f**3 - 36*b*c**2*d**5*e*f**3))/(64*a
**4*c**2*e - 32*a**3*b**2*c*e + 128*a**3*b*c**2*d**2*e + 128*a**3*c**3*d**4
*e + 4*a**2*b**4*e - 64*a**2*b**3*c*d**2*e + 128*a**2*b*c**3*d**6*e + 64*a*
*2*c**4*d**8*e + 8*a*b**5*d**2*e - 24*a*b**4*c*d**4*e - 64*a*b**3*c**2*d**6
*e - 32*a*b**2*c**3*d**8*e + 4*b**6*d**4*e + 8*b**5*c*d**6*e + 4*b**4*c**2*
d**8*e + x**8*(64*a**2*c**4*e**9 - 32*a*b**2*c**3*e**9 + 4*b**4*c**2*e**9)
+ x**7*(512*a**2*c**4*d*e**8 - 256*a*b**2*c**3*d*e**8 + 32*b**4*c**2*d*e**8
) + x**6*(128*a**2*b*c**3*e**7 + 1792*a**2*c**4*d**2*e**7 - 64*a*b**3*c**2*
e**7 - 896*a*b**2*c**3*d**2*e**7 + 8*b**5*c*e**7 + 112*b**4*c**2*d**2*e**7)
 + x**5*(768*a**2*b*c**3*d*e**6 + 3584*a**2*c**4*d**3*e**6 - 384*a*b**3*c**
2*d*e**6 - 1792*a*b**2*c**3*d**3*e**6 + 48*b**5*c*d*e**6 + 224*b**4*c**2*d*
*3*e**6) + x**4*(128*a**3*c**3*e**5 + 1920*a**2*b*c**3*d**2*e**5 + 4480*a**
2*c**4*d**4*e**5 - 24*a*b**4*c*e**5 - 960*a*b**3*c**2*d**2*e**5 - 2240*a*b*
*2*c**3*d**4*e**5 + 4*b**6*e**5 + 120*b**5*c*d**2*e**5 + 280*b**4*c**2*d**4
*e**5) + x**3*(512*a**3*c**3*d*e**4 + 2560*a**2*b*c**3*d**3*e**4 + 3584*a**
```

```
2*c**4*d**5*e**4 - 96*a*b**4*c*d*e**4 - 1280*a*b**3*c**2*d**3*e**4 - 1792*a
*b**2*c**3*d**5*e**4 + 16*b**6*d*e**4 + 160*b**5*c*d**3*e**4 + 224*b**4*c**
2*d**5*e**4) + x**2*(128*a**3*b*c**2*e**3 + 768*a**3*c**3*d**2*e**3 - 64*a*
*2*b**3*c*e**3 + 1920*a**2*b*c**3*d**4*e**3 + 1792*a**2*c**4*d**6*e**3 + 8*
a*b**5*e**3 - 144*a*b**4*c*d**2*e**3 - 960*a*b**3*c**2*d**4*e**3 - 896*a*b*
*2*c**3*d**6*e**3 + 24*b**6*d**2*e**3 + 120*b**5*c*d**4*e**3 + 112*b**4*c**
2*d**6*e**3) + x*(256*a**3*b*c**2*d*e**2 + 512*a**3*c**3*d**3*e**2 - 128*a*
*2*b**3*c*d*e**2 + 768*a**2*b*c**3*d**5*e**2 + 512*a**2*c**4*d**7*e**2 + 16
*a*b**5*d*e**2 - 96*a*b**4*c*d**3*e**2 - 384*a*b**3*c**2*d**5*e**2 - 256*a*
b**2*c**3*d**7*e**2 + 16*b**6*d**3*e**2 + 48*b**5*c*d**5*e**2 + 32*b**4*c**
2*d**7*e**2))
```

3.543 $\int \frac{(df+efx)^2}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^3}\,dx$

Optimal. Leaf size=375

$$\frac{f^2(d+ex)\left(c\left(20ac+b^2\right)(d+ex)^2+b\left(8ac+b^2\right)\right)}{8ae\left(b^2-4ac\right)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)}-\frac{f^2(d+ex)\left(b+2c(d+ex)^2\right)}{4e\left(b^2-4ac\right)\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}+\frac{\sqrt{c}\,f^2\left(\frac{b\left(b^2-52ac\right)}{\sqrt{b^2-4ac}}\right.}{8\sqrt{2}\,ae\,(}$$

Rubi [A] time = 0.97, antiderivative size = 375, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {1142, 1119, 1178, 1166, 205}

$$\frac{f^2(d+ex)\left(c\left(20ac+b^2\right)(d+ex)^2+b\left(8ac+b^2\right)\right)}{8ae\left(b^2-4ac\right)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)}-\frac{f^2(d+ex)\left(b+2c(d+ex)^2\right)}{4e\left(b^2-4ac\right)\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}+\frac{\sqrt{c}\,f^2\left(\frac{b\left(b^2-52ac\right)}{\sqrt{b^2-4ac}}+20ac+b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{8\sqrt{2}\,ae\left(b^2-4ac\right)^2\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{c}\,f^2\left(-\frac{b\left(b^2-52ac\right)}{\sqrt{b^2-4ac}}+20ac+b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{8\sqrt{2}\,ae\left(b^2-4ac\right)^2\sqrt{\sqrt{b^2-4ac}+b}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*f + e*f*x)^2/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3,x]
```

```
[Out] -(f^2*(d + e*x)*(b + 2*c*(d + e*x)^2))/(4*(b^2 - 4*a*c)*e*(a + b*(d + e*x)^
2 + c*(d + e*x)^4)^2) + (f^2*(d + e*x)*(b*(b^2 + 8*a*c) + c*(b^2 + 20*a*c)*
(d + e*x)^2))/(8*a*(b^2 - 4*a*c)^2*e*(a + b*(d + e*x)^2 + c*(d + e*x)^4)) +
 (Sqrt[c]*(b^2 + 20*a*c + (b*(b^2 - 52*a*c))/Sqrt[b^2 - 4*a*c])*f^2*ArcTan[
(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(8*Sqrt[2]*a*(b^2
 - 4*a*c)^2*Sqrt[b - Sqrt[b^2 - 4*a*c]]*e) + (Sqrt[c]*(b^2 + 20*a*c - (b*(b
^2 - 52*a*c))/Sqrt[b^2 - 4*a*c])*f^2*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqr
t[b + Sqrt[b^2 - 4*a*c]]])/(8*Sqrt[2]*a*(b^2 - 4*a*c)^2*Sqrt[b + Sqrt[b^2 -
 4*a*c]]*e)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1119

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Simp[(d*(d*x)^(m - 1)*(b + 2*c*x^2)*(a + b*x^2 + c*x^4)^(p + 1))/(2*(p
+ 1)*(b^2 - 4*a*c)), x] - Dist[d^2/(2*(p + 1)*(b^2 - 4*a*c)), Int[(d*x)^(m
- 2)*(b*(m - 1) + 2*c*(m + 4*p + 5)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x
] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && GtQ[m,
1] && LeQ[m, 3] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1142

```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Di
st[u^m/(Coefficient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p,
 x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1178

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[(x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*(a + b*x^2 +
 c*x^4)^(p + 1))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2
 - 4*a*c)), Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p + 7
)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
LtQ[p, -1] && IntegerQ[2*p]
```

Rubi steps

$$\int \frac{(df+efx)^2}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^3}\,dx = \frac{f^2\,\text{Subst}\left(\int \frac{x^2}{\left(a+bx^2+cx^4\right)^3}\,dx, x, d+ex\right)}{e}$$

$$= -\frac{f^2(d+ex)\left(b+2c(d+ex)^2\right)}{4\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)^2} + \frac{f^2\,\text{Subst}\left(\int \frac{b-10cx^2}{\left(a+bx^2+cx^4\right)\ldots}\right.}{4\left(b^2-4ac\ldots\right.}$$

$$= -\frac{f^2(d+ex)\left(b+2c(d+ex)^2\right)}{4\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)^2} + \frac{f^2(d+ex)\left(b\left(b^2+8ac\ldots\right.\right.}{8a\left(b^2-4ac\right)^2e\left(a+\ldots\right.}$$

$$= -\frac{f^2(d+ex)\left(b+2c(d+ex)^2\right)}{4\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)^2} + \frac{f^2(d+ex)\left(b\left(b^2+8ac\ldots\right.\right.}{8a\left(b^2-4ac\right)^2e\left(a+\ldots\right.}$$

$$= -\frac{f^2(d+ex)\left(b+2c(d+ex)^2\right)}{4\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)^2} + \frac{f^2(d+ex)\left(b\left(b^2+8ac\ldots\right.\right.}{8a\left(b^2-4ac\right)^2e\left(a+\ldots\right.}$$

Mathematica [A] time = 4.48, size = 385, normalized size = 1.03

$$\frac{f^2\left(-\frac{4\left(b(d+ex)+2c(d+ex)^3\right)}{\left(b^2-4ac\right)\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}+\frac{2(d+ex)\left(8abc+20ac^2(d+ex)^2+b^3+b^2c(d+ex)^2\right)}{a\left(b^2-4ac\right)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)}+\frac{\sqrt{2}\sqrt{c}\left(b^2\sqrt{b^2-4ac}+20ac\sqrt{b^2-4ac}-52abc+b^3\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{a\left(b^2-4ac\right)^{5/2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{2}\sqrt{c}\left(b^2\sqrt{b^2-4ac}+20ac\sqrt{b^2-4ac}+52abc-b^3\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{a\left(b^2-4ac\right)^{5/2}\sqrt{\sqrt{b^2-4ac}+b}}\right)}{16e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*f + e*f*x)^2/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3,x]
```

```
[Out] (f^2*((-4*(b*(d + e*x) + 2*c*(d + e*x)^3))/((b^2 - 4*a*c)*(a + b*(d + e*x)^
2 + c*(d + e*x)^4)^2) + (2*(d + e*x)*(b^3 + 8*a*b*c + b^2*c*(d + e*x)^2 + 2
0*a*c^2*(d + e*x)^2))/(a*(b^2 - 4*a*c)^2*(a + b*(d + e*x)^2 + c*(d + e*x)^4
)) + (Sqrt[2]*Sqrt[c]*(b^3 - 52*a*b*c + b^2*Sqrt[b^2 - 4*a*c] + 20*a*c*Sqrt
[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b - Sqrt[b^2 - 4*a*c
]]])/(a*(b^2 - 4*a*c)^(5/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[2]*Sqrt[c]
*(-b^3 + 52*a*b*c + b^2*Sqrt[b^2 - 4*a*c] + 20*a*c*Sqrt[b^2 - 4*a*c])*ArcTa
n[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(a*(b^2 - 4*a*c
)^(5/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]])))/(16*e)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(df+efx)^2}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(d*f + e*f*x)^2/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3,x]
```

```
[Out] IntegrateAlgebraic[(d*f + e*f*x)^2/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3, x
]
```

fricas [**B**] time = 2.20, size = 7838, normalized size = 20.90

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*f*x+d*f)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="fricas"
)
```

```
[Out] 1/16*(2*(b^2*c^2 + 20*a*c^3)*e^7*f^2*x^7 + 14*(b^2*c^2 + 20*a*c^3)*d*e^6*f^
2*x^6 + 2*(2*b^3*c + 28*a*b*c^2 + 21*(b^2*c^2 + 20*a*c^3)*d^2)*e^5*f^2*x^5
+ 10*(7*(b^2*c^2 + 20*a*c^3)*d^3 + 2*(b^3*c + 14*a*b*c^2)*d)*e^4*f^2*x^4 +
2*(35*(b^2*c^2 + 20*a*c^3)*d^4 + b^4 + 5*a*b^2*c + 36*a^2*c^2 + 20*(b^3*c +
 14*a*b*c^2)*d^2)*e^3*f^2*x^3 + 2*(21*(b^2*c^2 + 20*a*c^3)*d^5 + 20*(b^3*c
+ 14*a*b*c^2)*d^3 + 3*(b^4 + 5*a*b^2*c + 36*a^2*c^2)*d)*e^2*f^2*x^2 + 2*(7*
(b^2*c^2 + 20*a*c^3)*d^6 + 10*(b^3*c + 14*a*b*c^2)*d^4 - a*b^3 + 16*a^2*b*c
 + 3*(b^4 + 5*a*b^2*c + 36*a^2*c^2)*d^2)*e*f^2*x + 2*((b^2*c^2 + 20*a*c^3)*
d^7 + 2*(b^3*c + 14*a*b*c^2)*d^5 + (b^4 + 5*a*b^2*c + 36*a^2*c^2)*d^3 - (a*
b^3 - 16*a^2*b*c)*d)*f^2 + sqrt(1/2)*((a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c
^4)*e^9*x^8 + 8*(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*d*e^8*x^7 + 2*(a*b
^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3 + 14*(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^
3*c^4)*d^2)*e^7*x^6 + 4*(14*(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*d^3 +
3*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*d)*e^6*x^5 + (a*b^6 - 6*a^2*b^4*
c + 32*a^4*c^3 + 70*(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*d^4 + 30*(a*b^
5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*d^2)*e^5*x^4 + 4*(14*(a*b^4*c^2 - 8*a^2
*b^2*c^3 + 16*a^3*c^4)*d^5 + 10*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*d^
3 + (a*b^6 - 6*a^2*b^4*c + 32*a^4*c^3)*d)*e^4*x^3 + 2*(a^2*b^5 - 8*a^3*b^3*
c + 16*a^4*b*c^2 + 14*(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*d^6 + 15*(a*
b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*d^4 + 3*(a*b^6 - 6*a^2*b^4*c + 32*a^4
*c^3)*d^2)*e^3*x^2 + 4*(2*(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*d^7 + 3*
(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*d^5 + (a*b^6 - 6*a^2*b^4*c + 32*a^
4*c^3)*d^3 + (a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*d)*e^2*x + ((a*b^4*c^2
- 8*a^2*b^2*c^3 + 16*a^3*c^4)*d^8 + a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2 + 2*
(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*d^6 + (a*b^6 - 6*a^2*b^4*c + 32*a^
4*c^3)*d^4 + 2*(a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*d^2)*e)*sqrt(-((b^7 -
 35*a*b^5*c + 280*a^2*b^3*c^2 + 1680*a^3*b*c^3)*f^4 + (a^3*b^10 - 20*a^4*b^
8*c + 160*a^5*b^6*c^2 - 640*a^6*b^4*c^3 + 1280*a^7*b^2*c^4 - 1024*a^8*c^5)*
sqrt((b^4 - 50*a*b^2*c + 625*a^2*c^2)*f^8/((a^6*b^10 - 20*a^7*b^8*c + 160*a
^8*b^6*c^2 - 640*a^9*b^4*c^3 + 1280*a^10*b^2*c^4 - 1024*a^11*c^5)*e^4))*e^2
)/((a^3*b^10 - 20*a^4*b^8*c + 160*a^5*b^6*c^2 - 640*a^6*b^4*c^3 + 1280*a^7*
b^2*c^4 - 1024*a^8*c^5)*e^2))*log((35*b^6*c^2 - 1491*a*b^4*c^3 + 15000*a^2*
b^2*c^4 + 10000*a^3*c^5)*e*f^6*x + (35*b^6*c^2 - 1491*a*b^4*c^3 + 15000*a^2
*b^2*c^4 + 10000*a^3*c^5)*d*f^6 + 1/2*sqrt(1/2)*((b^11 - 53*a*b^9*c + 940*a
^2*b^7*c^2 - 6832*a^3*b^5*c^3 + 21824*a^4*b^3*c^4 - 25600*a^5*b*c^5)*e*f^4
- (a^3*b^14 - 38*a^4*b^12*c + 480*a^5*b^10*c^2 - 2720*a^6*b^8*c^3 + 6400*a^
7*b^6*c^4 + 1536*a^8*b^4*c^5 - 32768*a^9*b^2*c^6 + 40960*a^10*c^7)*sqrt((b^
4 - 50*a*b^2*c + 625*a^2*c^2)*f^8/((a^6*b^10 - 20*a^7*b^8*c + 160*a^8*b^6*c
^2 - 640*a^9*b^4*c^3 + 1280*a^10*b^2*c^4 - 1024*a^11*c^5)*e^4))*e^3)*sqrt(-
((b^7 - 35*a*b^5*c + 280*a^2*b^3*c^2 + 1680*a^3*b*c^3)*f^4 + (a^3*b^10 - 20
*a^4*b^8*c + 160*a^5*b^6*c^2 - 640*a^6*b^4*c^3 + 1280*a^7*b^2*c^4 - 1024*a^
8*c^5)*sqrt((b^4 - 50*a*b^2*c + 625*a^2*c^2)*f^8/((a^6*b^10 - 20*a^7*b^8*c
+ 160*a^8*b^6*c^2 - 640*a^9*b^4*c^3 + 1280*a^10*b^2*c^4 - 1024*a^11*c^5)*e^
4))*e^2)/((a^3*b^10 - 20*a^4*b^8*c + 160*a^5*b^6*c^2 - 640*a^6*b^4*c^3 + 12
80*a^7*b^2*c^4 - 1024*a^8*c^5)*e^2))) - sqrt(1/2)*((a*b^4*c^2 - 8*a^2*b^2*c
^3 + 16*a^3*c^4)*e^9*x^8 + 8*(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*d*e^8
```

x^7 + 2(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3 + 14*(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*d^2)*e^7*x^6 + 4*(14*(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*d^3 + 3*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*d)*e^6*x^5 + (a*b^6 - 6*a^2*b^4*c + 32*a^4*c^3 + 70*(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*d^4 + 30*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*d^2)*e^5*x^4 + 4*(14*(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*d^5 + 10*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*d^3 + (a*b^6 - 6*a^2*b^4*c + 32*a^4*c^3)*d)*e^4*x^3 + 2*(a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2 + 14*(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*d^6 + 15*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*d^4 + 3*(a*b^6 - 6*a^2*b^4*c + 32*a^4*c^3)*d^2)*e^3*x^2 + 4*(2*(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*d^7 + 3*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*d^5 + (a*b^6 - 6*a^2*b^4*c + 32*a^4*c^3)*d^3 + (a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*d)*e^2*x + ((a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*d^8 + a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2 + 2*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*d^6 + (a*b^6 - 6*a^2*b^4*c + 32*a^4*c^3)*d^4 + 2*(a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*d^2)*e)*sqrt(-((b^7 - 35*a*b^5*c + 280*a^2*b^3*c^2 + 1680*a^3*b*c^3)*f^4 + (a^3*b^10 - 20*a^4*b^8*c + 160*a^5*b^6*c^2 - 640*a^6*b^4*c^3 + 1280*a^7*b^2*c^4 - 1024*a^8*c^5)*sqrt((b^4 - 50*a*b^2*c + 625*a^2*c^2)*f^8/((a^6*b^10 - 20*a^7*b^8*c + 160*a^8*b^6*c^2 - 640*a^9*b^4*c^3 + 1280*a^10*b^2*c^4 - 1024*a^11*c^5)*e^4))*e^2)/((a^3*b^10 - 20*a^4*b^8*c + 160*a^5*b^6*c^2 - 640*a^6*b^4*c^3 + 1280*a^7*b^2*c^4 - 1024*a^8*c^5)*e^2))*log((35*b^6*c^2 - 1491*a*b^4*c^3 + 15000*a^2*b^2*c^4 + 10000*a^3*c^5)*e*f^6*x + (35*b^6*c^2 - 1491*a*b^4*c^3 + 15000*a^2*b^2*c^4 + 10000*a^3*c^5)*d*f^6 - 1/2*sqrt(1/2)*((b^11 - 53*a*b^9*c + 940*a^2*b^7*c^2 - 6832*a^3*b^5*c^3 + 21824*a^4*b^3*c^4 - 25600*a^5*b*c^5)*e*f^4 - (a^3*b^14 - 38*a^4*b^12*c + 480*a^5*b^10*c^2 - 2720*a^6*b^8*c^3 + 6400*a^7*b^6*c^4 + 1536*a^8*b^4*c^5 - 32768*a^9*b^2*c^6 + 40960*a^10*c^7)*sqrt((b^4 - 50*a*b^2*c + 625*a^2*c^2)*f^8/((a^6*b^10 - 20*a^7*b^8*c + 160*a^8*b^6*c^2 - 640*a^9*b^4*c^3 + 1280*a^10*b^2*c^4 - 1024*a^11*c^5)*e^4))*e^3)*sqrt(-((b^7 - 35*a*b^5*c + 280*a^2*b^3*c^2 + 1680*a^3*b*c^3)*f^4 + (a^3*b^10 - 20*a^4*b^8*c + 160*a^5*b^6*c^2 - 640*a^6*b^4*c^3 + 1280*a^7*b^2*c^4 - 1024*a^8*c^5)*sqrt((b^4 - 50*a*b^2*c + 625*a^2*c^2)*f^8/((a^6*b^10 - 20*a^7*b^8*c + 160*a^8*b^6*c^2 - 640*a^9*b^4*c^3 + 1280*a^10*b^2*c^4 - 1024*a^11*c^5)*e^4))*e^2)/((a^3*b^10 - 20*a^4*b^8*c + 160*a^5*b^6*c^2 - 640*a^6*b^4*c^3 + 1280*a^7*b^2*c^4 - 1024*a^8*c^5)*e^2))) + sqrt(1/2)*((a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*e^9*x^8 + 8*(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*d*e^8*x^7 + 2*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3 + 14*(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*d^2)*e^7*x^6 + 4*(14*(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*d^3 + 3*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*d)*e^6*x^5 + (a*b^6 - 6*a^2*b^4*c + 32*a^4*c^3 + 70*(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*d^4 + 30*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*d^2)*e^5*x^4 + 4*(14*(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*d^5 + 10*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*d^3 + (a*b^6 - 6*a^2*b^4*c + 32*a^4*c^3)*d)*e^4*x^3 + 2*(a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2 + 14*(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*d^6 + 15*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*d^4 + 3*(a*b^6 - 6*a^2*b^4*c + 32*a^4*c^3)*d^2)*e^3*x^2 + 4*(2*(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*d^7 + 3*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*d^5 + (a*b^6 - 6*a^2*b^4*c + 32*a^4*c^3)*d^3 + (a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*d)*e^2*x + ((a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*d^8 + a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2 + 2*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*d^6 + (a*b^6 - 6*a^2*b^4*c + 32*a^4*c^3)*d^4 + 2*(a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*d^2)*e)*sqrt(-((b^7 - 35*a*b^5*c + 280*a^2*b^3*c^2 + 1680*a^3*b*c^3)*f^4 - (a^3*b^10 - 20*a^4*b^8*c + 160*a^5*b^6*c^2 - 640*a^6*b^4*c^3 + 1280*a^7*b^2*c^4 - 1024*a^8*c^5)*sqrt((b^4 - 50*a*b^2*c + 625*a^2*c^2)*f^8/((a^6*b^10 - 20*a^7*b^8*c + 160*a^8*b^6*c^2 - 640*a^9*b^4*c^3 + 1280*a^10*b^2*c^4 - 1024*a^11*c^5)*e^4))*e^2)/((a^3*b^10 - 20*a^4*b^8*c + 160*a^5*b^6*c^2 - 640*a^6*b^4*c^3 + 1280*a^7*b^2*c^4 - 1024*a^8*c^5)*e^2))*log((35*b^6*c^2 - 1491*a*b^4*c^3 + 15000*a^2*b^2*c^4 + 10000*a^3*c^5)*e*f^6*x + (35*b^6*c^2 - 1491*a*b^4*c^3 + 15000*a^2*b^2*c^4 + 10000*a^3*c^5)*d*f^6 + 1/2*sqrt(1/2)*((b^11 - 53*a*b^9*c + 940*a^2*b^7*c^2 - 6832*a^3*b^5*c^3 + 21824*a^4*b^3*c^4

- 25600*a^5*b*c^5)*e*f^4 + (a^3*b^14 - 38*a^4*b^12*c + 480*a^5*b^10*c^2 - 2720*a^6*b^8*c^3 + 6400*a^7*b^6*c^4 + 1536*a^8*b^4*c^5 - 32768*a^9*b^2*c^6 + 40960*a^10*c^7)*sqrt((b^4 - 50*a*b^2*c + 625*a^2*c^2)*f^8/((a^6*b^10 - 20*a^7*b^8*c + 160*a^8*b^6*c^2 - 640*a^9*b^4*c^3 + 1280*a^10*b^2*c^4 - 1024*a^11*c^5)*e^4))*e^3)*sqrt(-((b^7 - 35*a*b^5*c + 280*a^2*b^3*c^2 + 1680*a^3*b*c^3)*f^4 - (a^3*b^10 - 20*a^4*b^8*c + 160*a^5*b^6*c^2 - 640*a^6*b^4*c^3 + 1280*a^7*b^2*c^4 - 1024*a^8*c^5)*sqrt((b^4 - 50*a*b^2*c + 625*a^2*c^2)*f^8/((a^6*b^10 - 20*a^7*b^8*c + 160*a^8*b^6*c^2 - 640*a^9*b^4*c^3 + 1280*a^10*b^2*c^4 - 1024*a^11*c^5)*e^4))*e^2)/((a^3*b^10 - 20*a^4*b^8*c + 160*a^5*b^6*c^2 - 640*a^6*b^4*c^3 + 1280*a^7*b^2*c^4 - 1024*a^8*c^5)*e^2))) - sqrt(1/2)*((a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*e^9*x^8 + 8*(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*d*e^8*x^7 + 2*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3 + 14*(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*d^2)*e^7*x^6 + 4*(14*(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*d^3 + 3*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*d)*e^6*x^5 + (a*b^6 - 6*a^2*b^4*c + 32*a^4*c^3 + 70*(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*d^4 + 30*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*d^2)*e^5*x^4 + 4*(14*(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*d^5 + 10*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*d^3 + (a*b^6 - 6*a^2*b^4*c + 32*a^4*c^3)*d)*e^4*x^3 + 2*(a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2 + 14*(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*d^6 + 15*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*d^4 + 3*(a*b^6 - 6*a^2*b^4*c + 32*a^4*c^3)*d^2)*e^3*x^2 + 4*(2*(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*d^7 + 3*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*d^5 + (a*b^6 - 6*a^2*b^4*c + 32*a^4*c^3)*d^3 + (a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*d)*e^2*x + ((a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*d^8 + a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2 + 2*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*d^6 + (a*b^6 - 6*a^2*b^4*c + 32*a^4*c^3)*d^4 + 2*(a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*d^2)*e)*sqrt(-((b^7 - 35*a*b^5*c + 280*a^2*b^3*c^2 + 1680*a^3*b*c^3)*f^4 - (a^3*b^10 - 20*a^4*b^8*c + 160*a^5*b^6*c^2 - 640*a^6*b^4*c^3 + 1280*a^7*b^2*c^4 - 1024*a^8*c^5)*sqrt((b^4 - 50*a*b^2*c + 625*a^2*c^2)*f^8/((a^6*b^10 - 20*a^7*b^8*c + 160*a^8*b^6*c^2 - 640*a^9*b^4*c^3 + 1280*a^10*b^2*c^4 - 1024*a^11*c^5)*e^4))*e^2)/((a^3*b^10 - 20*a^4*b^8*c + 160*a^5*b^6*c^2 - 640*a^6*b^4*c^3 + 1280*a^7*b^2*c^4 - 1024*a^8*c^5)*e^2))*log((35*b^6*c^2 - 1491*a*b^4*c^3 + 15000*a^2*b^2*c^4 + 10000*a^3*c^5)*e*f^6*x + (35*b^6*c^2 - 1491*a*b^4*c^3 + 15000*a^2*b^2*c^4 + 10000*a^3*c^5)*d*f^6 - 1/2*sqrt(1/2)*((b^11 - 53*a*b^9*c + 940*a^2*b^7*c^2 - 6832*a^3*b^5*c^3 + 21824*a^4*b^3*c^4 - 25600*a^5*b*c^5)*e*f^4 + (a^3*b^14 - 38*a^4*b^12*c + 480*a^5*b^10*c^2 - 2720*a^6*b^8*c^3 + 6400*a^7*b^6*c^4 + 1536*a^8*b^4*c^5 - 32768*a^9*b^2*c^6 + 40960*a^10*c^7)*sqrt((b^4 - 50*a*b^2*c + 625*a^2*c^2)*f^8/((a^6*b^10 - 20*a^7*b^8*c + 160*a^8*b^6*c^2 - 640*a^9*b^4*c^3 + 1280*a^10*b^2*c^4 - 1024*a^11*c^5)*e^4))*e^3)*sqrt(-((b^7 - 35*a*b^5*c + 280*a^2*b^3*c^2 + 1680*a^3*b*c^3)*f^4 - (a^3*b^10 - 20*a^4*b^8*c + 160*a^5*b^6*c^2 - 640*a^6*b^4*c^3 + 1280*a^7*b^2*c^4 - 1024*a^8*c^5)*sqrt((b^4 - 50*a*b^2*c + 625*a^2*c^2)*f^8/((a^6*b^10 - 20*a^7*b^8*c + 160*a^8*b^6*c^2 - 640*a^9*b^4*c^3 + 1280*a^10*b^2*c^4 - 1024*a^11*c^5)*e^4))*e^2)/((a^3*b^10 - 20*a^4*b^8*c + 160*a^5*b^6*c^2 - 640*a^6*b^4*c^3 + 1280*a^7*b^2*c^4 - 1024*a^8*c^5)*e^2))))/((a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*e^9*x^8 + 8*(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*d*e^8*x^7 + 2*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3 + 14*(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*d^2)*e^7*x^6 + 4*(14*(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*d^3 + 3*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*d)*e^6*x^5 + (a*b^6 - 6*a^2*b^4*c + 32*a^4*c^3 + 70*(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*d^4 + 30*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*d^2)*e^5*x^4 + 4*(14*(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*d^5 + 10*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*d^3 + (a*b^6 - 6*a^2*b^4*c + 32*a^4*c^3)*d)*e^4*x^3 + 2*(a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2 + 14*(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*d^6 + 15*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*d^4 + 3*(a*b^6 - 6*a^2*b^4*c + 32*a^4*c^3)*d^2)*e^3*x^2 + 4*(2*(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*d^7 + 3*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*d^5 + (a*b^6 - 6*a^2*b^4*c + 32*a^4*c^3)*d^3 + (a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*d)*e^2*x + ((a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*d^8

```
+ a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2 + 2*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*
b*c^3)*d^6 + (a*b^6 - 6*a^2*b^4*c + 32*a^4*c^3)*d^4 + 2*(a^2*b^5 - 8*a^3*b^
3*c + 16*a^4*b*c^2)*d^2)*e)
```

giac [B] time = 0.79, size = 2527, normalized size = 6.74

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*f*x+d*f)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="giac")
```

```
[Out] -1/16*(((d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/
c))^2*b^2*c*f^2*e^2 + 20*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*
a*c)*e^2)*e^(-4)/c))^2*a*c^2*f^2*e^2 - 2*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2
 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))*b^2*c*d*f^2*e - 40*(d*e^(-1) + sqrt(1/
2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))*a*c^2*d*f^2*e + b^2*c*d
^2*f^2 + 20*a*c^2*d^2*f^2 + b^3*f^2 - 16*a*b*c*f^2)*log(d*e^(-1) + x + sqrt
(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))/(2*(d*e^(-1) + sqrt(
1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^3*c*e^4 - 6*(d*e^(-1)
 + sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^2*c*d*e^3 - 2
*c*d^3*e - b*d*e + (6*c*d^2*e^2 + b*e^2)*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2
 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))) + ((d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2
 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^2*b^2*c*f^2*e^2 + 20*(d*e^(-1) - sqrt(
1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^2*a*c^2*f^2*e^2 - 2*(
d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))*b^2*c
*d*f^2*e - 40*(d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e
^(-4)/c))*a*c^2*d*f^2*e + b^2*c*d^2*f^2 + 20*a*c^2*d^2*f^2 + b^3*f^2 - 16*a
*b*c*f^2)*log(d*e^(-1) + x - sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2
)*e^(-4)/c))/(2*(d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)
*e^(-4)/c))^3*c*e^4 - 6*(d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a
*c)*e^2)*e^(-4)/c))^2*c*d*e^3 - 2*c*d^3*e - b*d*e + (6*c*d^2*e^2 + b*e^2)*(
d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))) + ((
d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^2*b^2
*c*f^2*e^2 + 20*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)
*e^(-4)/c))^2*a*c^2*f^2*e^2 - 2*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b
^2 - 4*a*c)*e^2)*e^(-4)/c))*b^2*c*d*f^2*e - 40*(d*e^(-1) + sqrt(1/2)*sqrt(-
(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))*a*c^2*d*f^2*e + b^2*c*d^2*f^2 +
20*a*c^2*d^2*f^2 + b^3*f^2 - 16*a*b*c*f^2)*log(d*e^(-1) + x + sqrt(1/2)*sqr
t(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))/(2*(d*e^(-1) + sqrt(1/2)*sqrt
(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^3*c*e^4 - 6*(d*e^(-1) + sqrt(1
/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^2*c*d*e^3 - 2*c*d^3*e
- b*d*e + (6*c*d^2*e^2 + b*e^2)*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b
^2 - 4*a*c)*e^2)*e^(-4)/c))) + ((d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b
^2 - 4*a*c)*e^2)*e^(-4)/c))^2*b^2*c*f^2*e^2 + 20*(d*e^(-1) - sqrt(1/2)*sqrt
(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^2*a*c^2*f^2*e^2 - 2*(d*e^(-1)
- sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))*b^2*c*d*f^2*e
- 40*(d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))
*a*c^2*d*f^2*e + b^2*c*d^2*f^2 + 20*a*c^2*d^2*f^2 + b^3*f^2 - 16*a*b*c*f^2)
*log(d*e^(-1) + x - sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/
c))/(2*(d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c
))^3*c*e^4 - 6*(d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*
e^(-4)/c))^2*c*d*e^3 - 2*c*d^3*e - b*d*e + (6*c*d^2*e^2 + b*e^2)*(d*e^(-1)
- sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))))/(a*b^4 - 8*a
^2*b^2*c + 16*a^3*c^2) + 1/8*(b^2*c^2*f^2*x^7*e^7 + 20*a*c^3*f^2*x^7*e^7 +
7*b^2*c^2*d*f^2*x^6*e^6 + 140*a*c^3*d*f^2*x^6*e^6 + 21*b^2*c^2*d^2*f^2*x^5*
e^5 + 420*a*c^3*d^2*f^2*x^5*e^5 + 35*b^2*c^2*d^3*f^2*x^4*e^4 + 700*a*c^3*d^
3*f^2*x^4*e^4 + 35*b^2*c^2*d^4*f^2*x^3*e^3 + 700*a*c^3*d^4*f^2*x^3*e^3 + 21
*b^2*c^2*d^5*f^2*x^2*e^2 + 420*a*c^3*d^5*f^2*x^2*e^2 + 7*b^2*c^2*d^6*f^2*x*
e + 140*a*c^3*d^6*f^2*x*e + b^2*c^2*d^7*f^2 + 20*a*c^3*d^7*f^2 + 2*b^3*c*f^
2*x^5*e^5 + 28*a*b*c^2*f^2*x^5*e^5 + 10*b^3*c*d*f^2*x^4*e^4 + 140*a*b*c^2*d
```

```
*f^2*x^4*e^4 + 20*b^3*c*d^2*f^2*x^3*e^3 + 280*a*b*c^2*d^2*f^2*x^3*e^3 + 20*
b^3*c*d^3*f^2*x^2*e^2 + 280*a*b*c^2*d^3*f^2*x^2*e^2 + 10*b^3*c*d^4*f^2*x*e
+ 140*a*b*c^2*d^4*f^2*x*e + 2*b^3*c*d^5*f^2 + 28*a*b*c^2*d^5*f^2 + b^4*f^2*
x^3*e^3 + 5*a*b^2*c*f^2*x^3*e^3 + 36*a^2*c^2*f^2*x^3*e^3 + 3*b^4*d*f^2*x^2*
e^2 + 15*a*b^2*c*d*f^2*x^2*e^2 + 108*a^2*c^2*d*f^2*x^2*e^2 + 3*b^4*d^2*f^2*
x*e + 15*a*b^2*c*d^2*f^2*x*e + 108*a^2*c^2*d^2*f^2*x*e + b^4*d^3*f^2 + 5*a*
b^2*c*d^3*f^2 + 36*a^2*c^2*d^3*f^2 - a*b^3*f^2*x*e + 16*a^2*b*c*f^2*x*e - a
*b^3*d*f^2 + 16*a^2*b*c*d*f^2)/((c*x^4*e^4 + 4*c*d*x^3*e^3 + 6*c*d^2*x^2*e^
2 + 4*c*d^3*x*e + c*d^4 + b*x^2*e^2 + 2*b*d*x*e + b*d^2 + a)^2*(a*b^4*e - 8
*a^2*b^2*c*e + 16*a^3*c^2*e))
```

maple [C] time = 0.05, size = 4751, normalized size = 12.67

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*f*x+d*f)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x)
```

```
[Out] 7/8*f^2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^
4+2*b*d*e*x+b*d^2+a)^2*c^2*d*e^5/(16*a^2*c^2-8*a*b^2*c+b^4)/a*x^6*b^2+21/8*
f^2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*
b*d*e*x+b*d^2+a)^2*c^2*e^4/(16*a^2*c^2-8*a*b^2*c+b^4)/a*x^5*b^2*d^2+35/8*f^
2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*
d*e*x+b*d^2+a)^2*c^2*d^3*e^3/(16*a^2*c^2-8*a*b^2*c+b^4)/a*x^4*b^2+5/4*f^2/(
c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e
*x+b*d^2+a)^2*c*d*e^3/(16*a^2*c^2-8*a*b^2*c+b^4)/a*x^4*b^3+35/8*f^2/(c*e^4*
x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d
^2+a)^2*e^2/(16*a^2*c^2-8*a*b^2*c+b^4)/a*x^3*b^2*c^2*d^4+3/8*f^2/(c*e^4*x^4
+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+
a)^2/(16*a^2*c^2-8*a*b^2*c+b^4)/a*x*b^4*d^2+105/2*f^2/(c*e^4*x^4+4*c*d*e^3*
x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*d^5*e/
(16*a^2*c^2-8*a*b^2*c+b^4)*x^2*c^3+35/2*f^2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^
2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2/(16*a^2*c^2-8*a*
b^2*c+b^4)*x*b*c^2*d^4+15/8*f^2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*
c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x
*b^2*c*d^2+7/2*f^2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e
^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2/(16*a^2*c^2-8*a*b^2*c+b^4)*d^5/e*b*c^2+5/
8*f^2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+
2*b*d*e*x+b*d^2+a)^2/(16*a^2*c^2-8*a*b^2*c+b^4)*d^3/e*b^2*c+35/2*f^2/(c*e^4
*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*
d^2+a)^2*c^3*d*e^5/(16*a^2*c^2-8*a*b^2*c+b^4)*x^6+2*f^2/(c*e^4*x^4+4*c*d*e^
3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2/(16*
a^2*c^2-8*a*b^2*c+b^4)*a*x*b*c+9/2*f^2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2
*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2/(16*a^2*c^2-8*a*b^2*c
+b^4)*a*d^3/e*c^2+1/8*f^2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*
e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2/(16*a^2*c^2-8*a*b^2*c+b^4)/a*d^3/e
*b^4+175/2*f^2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x
^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*c^3*d^3*e^3/(16*a^2*c^2-8*a*b^2*c+b^4)*x^4+17
5/2*f^2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^
4+2*b*d*e*x+b*d^2+a)^2*e^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x^3*c^3*d^4+5/8*f^2/(
c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e
*x+b*d^2+a)^2*e^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x^3*b^2*c+105/2*f^2/(c*e^4*x^4
+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+
a)^2*c^3*e^4/(16*a^2*c^2-8*a*b^2*c+b^4)*x^5*d^2+7/2*f^2/(c*e^4*x^4+4*c*d*e^
3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*c^2*
e^4/(16*a^2*c^2-8*a*b^2*c+b^4)*x^5*b+9/2*f^2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d
^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*e^2/(16*a^2*c^2
-8*a*b^2*c+b^4)*a*x^3*c^2+1/8*f^2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+
4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*e^2/(16*a^2*c^2-8*a*b^2*c+
b^4)/a*x^3*b^4+27/2*f^2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*
```

```
x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2/(16*a^2*c^2-8*a*b^2*c+b^4)*a*x*c^2*d
^2+5/2*f^2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c
*d^4+2*b*d*e*x+b*d^2+a)^2*c^3*e^6/(16*a^2*c^2-8*a*b^2*c+b^4)*x^7+35/2*f^2/(
c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e
*x+b*d^2+a)^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x*c^3*d^6+5/2*f^2/(c*e^4*x^4+4*c*d
*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2/(
16*a^2*c^2-8*a*b^2*c+b^4)*d^7/e*c^3-1/8*f^2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^
2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2/(16*a^2*c^2-8*a*
b^2*c+b^4)*d/e*b^3+1/8*f^2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3
*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2/(16*a^2*c^2-8*a*b^2*c+b^4)/a*d^7/
e*b^2*c^2+1/4*f^2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^
2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2/(16*a^2*c^2-8*a*b^2*c+b^4)/a*d^5/e*b^3*c+2
*f^2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2
*b*d*e*x+b*d^2+a)^2/(16*a^2*c^2-8*a*b^2*c+b^4)*a*d/e*b*c+35/2*f^2/(c*e^4*x^
4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2
+a)^2*c^2*d*e^3/(16*a^2*c^2-8*a*b^2*c+b^4)*x^4*b+35*f^2/(c*e^4*x^4+4*c*d*e^
3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*e^2/
(16*a^2*c^2-8*a*b^2*c+b^4)*x^3*b*c^2*d^2+35*f^2/(c*e^4*x^4+4*c*d*e^3*x^3+6*
c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*d^3*e/(16*a^
2*c^2-8*a*b^2*c+b^4)*x^2*b*c^2+15/8*f^2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^
2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*d*e/(16*a^2*c^2-8*a*
b^2*c+b^4)*x^2*b^2*c+1/4*f^2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d
^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*c*e^4/(16*a^2*c^2-8*a*b^2*c+b^4
)/a*x^5*b^3+1/8*f^2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*
e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*c^2*e^6/(16*a^2*c^2-8*a*b^2*c+b^4)/a*x^7
*b^2+27/2*f^2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^
2+c*d^4+2*b*d*e*x+b*d^2+a)^2*d*e/(16*a^2*c^2-8*a*b^2*c+b^4)*a*x^2*c^2+3/8*f
^2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b
*d*e*x+b*d^2+a)^2*d*e/(16*a^2*c^2-8*a*b^2*c+b^4)/a*x^2*b^4+7/8*f^2/(c*e^4*x
^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^
2+a)^2/(16*a^2*c^2-8*a*b^2*c+b^4)/a*x*b^2*c^2*d^6+5/4*f^2/(c*e^4*x^4+4*c*d*
e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2/(1
6*a^2*c^2-8*a*b^2*c+b^4)/a*x*b^3*c*d^4+5/2*f^2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c
*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*e^2/(16*a^2*c
^2-8*a*b^2*c+b^4)/a*x^3*b^3*c*d^2+21/8*f^2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2
*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*d^5*e/(16*a^2*c^2
-8*a*b^2*c+b^4)/a*x^2*b^2*c^2+5/2*f^2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*
x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*d^3*e/(16*a^2*c^2-8*a*
b^2*c+b^4)/a*x^2*b^3*c-1/8*f^2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c
*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x*
b^3+1/16*f^2/(16*a^2*c^2-8*a*b^2*c+b^4)/a/e*sum(((20*a*c+b^2)*_R^2*c*e^2+20
*a*c^2*d^2+b^2*c*d^2+2*(20*a*c+b^2)*_R*c*d*e-16*a*b*c+b^3)/(2*_R^3*c*e^3+6*
_R^2*c*d*e^2+6*_R*c*d^2*e+2*c*d^3+_R*b*e+b*d)*ln(-_R+x),_R=RootOf(_Z^4*c*e^
4+4*_Z^3*c*d*e^3+c*d^4+b*d^2+(6*c*d^2*e^2+b*e^2)*_Z^2+(4*c*d^3*e+2*b*d*e)*_
Z+a))
```

maxima [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*f*x+d*f)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="maxima"
)
```

```
[Out] Timed out
```

mupad [**B**] time = 7.94, size = 16025, normalized size = 42.73

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*f + e*f*x)^2/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3,x)

[Out] atan((((((67108864*a^9*b*c^9*d*e^13 - 4096*a^2*b^15*c^2*d*e^13 + 114688*a^3
*b^13*c^3*d*e^13 - 1376256*a^4*b^11*c^4*d*e^13 + 9175040*a^5*b^9*c^5*d*e^13
 - 36700160*a^6*b^7*c^6*d*e^13 + 88080384*a^7*b^5*c^7*d*e^13 - 117440512*a^
8*b^3*c^8*d*e^13)/(512*(a^2*b^12 + 4096*a^8*c^6 - 24*a^3*b^10*c + 240*a^4*b
^8*c^2 - 1280*a^5*b^6*c^3 + 3840*a^6*b^4*c^4 - 6144*a^7*b^2*c^5)) + (x*(262
144*a^7*b*c^7*e^14 - 256*a^2*b^11*c^2*e^14 + 5120*a^3*b^9*c^3*e^14 - 40960*
a^4*b^7*c^4*e^14 + 163840*a^5*b^5*c^5*e^14 - 327680*a^6*b^3*c^6*e^14))/(32*
(a^2*b^8 + 256*a^6*c^4 - 16*a^3*b^6*c + 96*a^4*b^4*c^2 - 256*a^5*b^2*c^3)))
*(-(b^17*f^4 + b^2*f^4*(-(4*a*c - b^2)^15)^(1/2) - 1720320*a^8*b*c^8*f^4 +
1140*a^2*b^13*c^2*f^4 - 10160*a^3*b^11*c^3*f^4 + 34880*a^4*b^9*c^4*f^4 + 43
776*a^5*b^7*c^5*f^4 - 680960*a^6*b^5*c^6*f^4 + 1863680*a^7*b^3*c^7*f^4 - 55
*a*b^15*c*f^4 - 25*a*c*f^4*(-(4*a*c - b^2)^15)^(1/2))/(512*(a^3*b^20*e^2 +
1048576*a^13*c^10*e^2 - 40*a^4*b^18*c*e^2 + 720*a^5*b^16*c^2*e^2 - 7680*a^6
*b^14*c^3*e^2 + 53760*a^7*b^12*c^4*e^2 - 258048*a^8*b^10*c^5*e^2 + 860160*a
^9*b^8*c^6*e^2 - 1966080*a^10*b^6*c^7*e^2 + 2949120*a^11*b^4*c^8*e^2 - 2621
440*a^12*b^2*c^9*e^2)))^(1/2) - (122880*a^3*b^9*c^4*e^12*f^2 - 9216*a^2*b^1
1*c^3*e^12*f^2 - 819200*a^4*b^7*c^5*e^12*f^2 + 2949120*a^5*b^5*c^6*e^12*f^2
 - 5505024*a^6*b^3*c^7*e^12*f^2 + 256*a*b^13*c^2*e^12*f^2 + 4194304*a^7*b*c
^8*e^12*f^2)/(512*(a^2*b^12 + 4096*a^8*c^6 - 24*a^3*b^10*c + 240*a^4*b^8*c^
2 - 1280*a^5*b^6*c^3 + 3840*a^6*b^4*c^4 - 6144*a^7*b^2*c^5)))*(-(b^17*f^4 +
 b^2*f^4*(-(4*a*c - b^2)^15)^(1/2) - 1720320*a^8*b*c^8*f^4 + 1140*a^2*b^13*
c^2*f^4 - 10160*a^3*b^11*c^3*f^4 + 34880*a^4*b^9*c^4*f^4 + 43776*a^5*b^7*c^
5*f^4 - 680960*a^6*b^5*c^6*f^4 + 1863680*a^7*b^3*c^7*f^4 - 55*a*b^15*c*f^4
 - 25*a*c*f^4*(-(4*a*c - b^2)^15)^(1/2))/(512*(a^3*b^20*e^2 + 1048576*a^13*c
^10*e^2 - 40*a^4*b^18*c*e^2 + 720*a^5*b^16*c^2*e^2 - 7680*a^6*b^14*c^3*e^2
 + 53760*a^7*b^12*c^4*e^2 - 258048*a^8*b^10*c^5*e^2 + 860160*a^9*b^8*c^6*e^2
 - 1966080*a^10*b^6*c^7*e^2 + 2949120*a^11*b^4*c^8*e^2 - 2621440*a^12*b^2*c
^9*e^2)))^(1/2) + (204800*a^5*c^8*d*e^11*f^4 - 16*b^10*c^3*d*e^11*f^4 + 672
*a*b^8*c^4*d*e^11*f^4 - 28160*a^2*b^6*c^5*d*e^11*f^4 + 209920*a^3*b^4*c^6*d
*e^11*f^4 - 479232*a^4*b^2*c^7*d*e^11*f^4)/(512*(a^2*b^12 + 4096*a^8*c^6 -
24*a^3*b^10*c + 240*a^4*b^8*c^2 - 1280*a^5*b^6*c^3 + 3840*a^6*b^4*c^4 - 614
4*a^7*b^2*c^5)) + (x*(800*a^3*c^6*e^12*f^4 - b^6*c^3*e^12*f^4 - 1472*a^2*b^
2*c^5*e^12*f^4 + 34*a*b^4*c^4*e^12*f^4))/(32*(a^2*b^8 + 256*a^6*c^4 - 16*a^
3*b^6*c + 96*a^4*b^4*c^2 - 256*a^5*b^2*c^3)))*(-(b^17*f^4 + b^2*f^4*(-(4*a*
c - b^2)^15)^(1/2) - 1720320*a^8*b*c^8*f^4 + 1140*a^2*b^13*c^2*f^4 - 10160*
a^3*b^11*c^3*f^4 + 34880*a^4*b^9*c^4*f^4 + 43776*a^5*b^7*c^5*f^4 - 680960*a
^6*b^5*c^6*f^4 + 1863680*a^7*b^3*c^7*f^4 - 55*a*b^15*c*f^4 - 25*a*c*f^4*(-(
4*a*c - b^2)^15)^(1/2))/(512*(a^3*b^20*e^2 + 1048576*a^13*c^10*e^2 - 40*a^4
*b^18*c*e^2 + 720*a^5*b^16*c^2*e^2 - 7680*a^6*b^14*c^3*e^2 + 53760*a^7*b^12
*c^4*e^2 - 258048*a^8*b^10*c^5*e^2 + 860160*a^9*b^8*c^6*e^2 - 1966080*a^10*
b^6*c^7*e^2 + 2949120*a^11*b^4*c^8*e^2 - 2621440*a^12*b^2*c^9*e^2)))^(1/2)*
1i + ((((67108864*a^9*b*c^9*d*e^13 - 4096*a^2*b^15*c^2*d*e^13 + 114688*a^3*
b^13*c^3*d*e^13 - 1376256*a^4*b^11*c^4*d*e^13 + 9175040*a^5*b^9*c^5*d*e^13
 - 36700160*a^6*b^7*c^6*d*e^13 + 88080384*a^7*b^5*c^7*d*e^13 - 117440512*a^8
*b^3*c^8*d*e^13)/(512*(a^2*b^12 + 4096*a^8*c^6 - 24*a^3*b^10*c + 240*a^4*b^
8*c^2 - 1280*a^5*b^6*c^3 + 3840*a^6*b^4*c^4 - 6144*a^7*b^2*c^5)) + (x*(2621
44*a^7*b*c^7*e^14 - 256*a^2*b^11*c^2*e^14 + 5120*a^3*b^9*c^3*e^14 - 40960*a
^4*b^7*c^4*e^14 + 163840*a^5*b^5*c^5*e^14 - 327680*a^6*b^3*c^6*e^14))/(32*(
a^2*b^8 + 256*a^6*c^4 - 16*a^3*b^6*c + 96*a^4*b^4*c^2 - 256*a^5*b^2*c^3)))*
(-(b^17*f^4 + b^2*f^4*(-(4*a*c - b^2)^15)^(1/2) - 1720320*a^8*b*c^8*f^4 + 1
140*a^2*b^13*c^2*f^4 - 10160*a^3*b^11*c^3*f^4 + 34880*a^4*b^9*c^4*f^4 + 437
76*a^5*b^7*c^5*f^4 - 680960*a^6*b^5*c^6*f^4 + 1863680*a^7*b^3*c^7*f^4 - 55*
a*b^15*c*f^4 - 25*a*c*f^4*(-(4*a*c - b^2)^15)^(1/2))/(512*(a^3*b^20*e^2 + 1
048576*a^13*c^10*e^2 - 40*a^4*b^18*c*e^2 + 720*a^5*b^16*c^2*e^2 - 7680*a^6*
b^14*c^3*e^2 + 53760*a^7*b^12*c^4*e^2 - 258048*a^8*b^10*c^5*e^2 + 860160*a^
9*b^8*c^6*e^2 - 1966080*a^10*b^6*c^7*e^2 + 2949120*a^11*b^4*c^8*e^2 - 26214
40*a^12*b^2*c^9*e^2)))^(1/2) + (122880*a^3*b^9*c^4*e^12*f^2 - 9216*a^2*b^11
*c^3*e^12*f^2 - 819200*a^4*b^7*c^5*e^12*f^2 + 2949120*a^5*b^5*c^6*e^12*f^2

- 5505024*a^6*b^3*c^7*e^12*f^2 + 256*a*b^13*c^2*e^12*f^2 + 4194304*a^7*b*c^
8*e^12*f^2)/(512*(a^2*b^12 + 4096*a^8*c^6 - 24*a^3*b^10*c + 240*a^4*b^8*c^2
 - 1280*a^5*b^6*c^3 + 3840*a^6*b^4*c^4 - 6144*a^7*b^2*c^5)))*(-(b^17*f^4 +
b^2*f^4*(-(4*a*c - b^2)^15)^(1/2) - 1720320*a^8*b*c^8*f^4 + 1140*a^2*b^13*c
^2*f^4 - 10160*a^3*b^11*c^3*f^4 + 34880*a^4*b^9*c^4*f^4 + 43776*a^5*b^7*c^5
*f^4 - 680960*a^6*b^5*c^6*f^4 + 1863680*a^7*b^3*c^7*f^4 - 55*a*b^15*c*f^4 -
 25*a*c*f^4*(-(4*a*c - b^2)^15)^(1/2))/(512*(a^3*b^20*e^2 + 1048576*a^13*c^
10*e^2 - 40*a^4*b^18*c*e^2 + 720*a^5*b^16*c^2*e^2 - 7680*a^6*b^14*c^3*e^2 +
 53760*a^7*b^12*c^4*e^2 - 258048*a^8*b^10*c^5*e^2 + 860160*a^9*b^8*c^6*e^2
- 1966080*a^10*b^6*c^7*e^2 + 2949120*a^11*b^4*c^8*e^2 - 2621440*a^12*b^2*c^
9*e^2)))^(1/2) + (204800*a^5*c^8*d*e^11*f^4 - 16*b^10*c^3*d*e^11*f^4 + 672*
a*b^8*c^4*d*e^11*f^4 - 28160*a^2*b^6*c^5*d*e^11*f^4 + 209920*a^3*b^4*c^6*d*
e^11*f^4 - 479232*a^4*b^2*c^7*d*e^11*f^4)/(512*(a^2*b^12 + 4096*a^8*c^6 - 2
4*a^3*b^10*c + 240*a^4*b^8*c^2 - 1280*a^5*b^6*c^3 + 3840*a^6*b^4*c^4 - 6144
*a^7*b^2*c^5)) + (x*(800*a^3*c^6*e^12*f^4 - b^6*c^3*e^12*f^4 - 1472*a^2*b^2
*c^5*e^12*f^4 + 34*a*b^4*c^4*e^12*f^4))/(32*(a^2*b^8 + 256*a^6*c^4 - 16*a^3
*b^6*c + 96*a^4*b^4*c^2 - 256*a^5*b^2*c^3)))*(-(b^17*f^4 + b^2*f^4*(-(4*a*c
 - b^2)^15)^(1/2) - 1720320*a^8*b*c^8*f^4 + 1140*a^2*b^13*c^2*f^4 - 10160*a
^3*b^11*c^3*f^4 + 34880*a^4*b^9*c^4*f^4 + 43776*a^5*b^7*c^5*f^4 - 680960*a^
6*b^5*c^6*f^4 + 1863680*a^7*b^3*c^7*f^4 - 55*a*b^15*c*f^4 - 25*a*c*f^4*(-(4
*a*c - b^2)^15)^(1/2))/(512*(a^3*b^20*e^2 + 1048576*a^13*c^10*e^2 - 40*a^4*
b^18*c*e^2 + 720*a^5*b^16*c^2*e^2 - 7680*a^6*b^14*c^3*e^2 + 53760*a^7*b^12*
c^4*e^2 - 258048*a^8*b^10*c^5*e^2 + 860160*a^9*b^8*c^6*e^2 - 1966080*a^10*b
^6*c^7*e^2 + 2949120*a^11*b^4*c^8*e^2 - 2621440*a^12*b^2*c^9*e^2)))^(1/2)*1
i)/(((((67108864*a^9*b*c^9*d*e^13 - 4096*a^2*b^15*c^2*d*e^13 + 114688*a^3*b
^13*c^3*d*e^13 - 1376256*a^4*b^11*c^4*d*e^13 + 9175040*a^5*b^9*c^5*d*e^13 -
 36700160*a^6*b^7*c^6*d*e^13 + 88080384*a^7*b^5*c^7*d*e^13 - 117440512*a^8*
b^3*c^8*d*e^13)/(512*(a^2*b^12 + 4096*a^8*c^6 - 24*a^3*b^10*c + 240*a^4*b^8
*c^2 - 1280*a^5*b^6*c^3 + 3840*a^6*b^4*c^4 - 6144*a^7*b^2*c^5)) + (x*(26214
4*a^7*b*c^7*e^14 - 256*a^2*b^11*c^2*e^14 + 5120*a^3*b^9*c^3*e^14 - 40960*a^
4*b^7*c^4*e^14 + 163840*a^5*b^5*c^5*e^14 - 327680*a^6*b^3*c^6*e^14))/(32*(a
^2*b^8 + 256*a^6*c^4 - 16*a^3*b^6*c + 96*a^4*b^4*c^2 - 256*a^5*b^2*c^3)))*(
-(b^17*f^4 + b^2*f^4*(-(4*a*c - b^2)^15)^(1/2) - 1720320*a^8*b*c^8*f^4 + 11
40*a^2*b^13*c^2*f^4 - 10160*a^3*b^11*c^3*f^4 + 34880*a^4*b^9*c^4*f^4 + 4377
6*a^5*b^7*c^5*f^4 - 680960*a^6*b^5*c^6*f^4 + 1863680*a^7*b^3*c^7*f^4 - 55*a
*b^15*c*f^4 - 25*a*c*f^4*(-(4*a*c - b^2)^15)^(1/2))/(512*(a^3*b^20*e^2 + 10
48576*a^13*c^10*e^2 - 40*a^4*b^18*c*e^2 + 720*a^5*b^16*c^2*e^2 - 7680*a^6*b
^14*c^3*e^2 + 53760*a^7*b^12*c^4*e^2 - 258048*a^8*b^10*c^5*e^2 + 860160*a^9
*b^8*c^6*e^2 - 1966080*a^10*b^6*c^7*e^2 + 2949120*a^11*b^4*c^8*e^2 - 262144
0*a^12*b^2*c^9*e^2)))^(1/2) - (122880*a^3*b^9*c^4*e^12*f^2 - 9216*a^2*b^11*
c^3*e^12*f^2 - 819200*a^4*b^7*c^5*e^12*f^2 + 2949120*a^5*b^5*c^6*e^12*f^2 -
 5505024*a^6*b^3*c^7*e^12*f^2 + 256*a*b^13*c^2*e^12*f^2 + 4194304*a^7*b*c^8
*e^12*f^2)/(512*(a^2*b^12 + 4096*a^8*c^6 - 24*a^3*b^10*c + 240*a^4*b^8*c^2
- 1280*a^5*b^6*c^3 + 3840*a^6*b^4*c^4 - 6144*a^7*b^2*c^5)))*(-(b^17*f^4 + b
^2*f^4*(-(4*a*c - b^2)^15)^(1/2) - 1720320*a^8*b*c^8*f^4 + 1140*a^2*b^13*c^
2*f^4 - 10160*a^3*b^11*c^3*f^4 + 34880*a^4*b^9*c^4*f^4 + 43776*a^5*b^7*c^5*
f^4 - 680960*a^6*b^5*c^6*f^4 + 1863680*a^7*b^3*c^7*f^4 - 55*a*b^15*c*f^4 -
25*a*c*f^4*(-(4*a*c - b^2)^15)^(1/2))/(512*(a^3*b^20*e^2 + 1048576*a^13*c^1
0*e^2 - 40*a^4*b^18*c*e^2 + 720*a^5*b^16*c^2*e^2 - 7680*a^6*b^14*c^3*e^2 +
53760*a^7*b^12*c^4*e^2 - 258048*a^8*b^10*c^5*e^2 + 860160*a^9*b^8*c^6*e^2 -
 1966080*a^10*b^6*c^7*e^2 + 2949120*a^11*b^4*c^8*e^2 - 2621440*a^12*b^2*c^9
*e^2)))^(1/2) + (204800*a^5*c^8*d*e^11*f^4 - 16*b^10*c^3*d*e^11*f^4 + 672*a
*b^8*c^4*d*e^11*f^4 - 28160*a^2*b^6*c^5*d*e^11*f^4 + 209920*a^3*b^4*c^6*d*e
^11*f^4 - 479232*a^4*b^2*c^7*d*e^11*f^4)/(512*(a^2*b^12 + 4096*a^8*c^6 - 24
*a^3*b^10*c + 240*a^4*b^8*c^2 - 1280*a^5*b^6*c^3 + 3840*a^6*b^4*c^4 - 6144*
a^7*b^2*c^5)) + (x*(800*a^3*c^6*e^12*f^4 - b^6*c^3*e^12*f^4 - 1472*a^2*b^2*
c^5*e^12*f^4 + 34*a*b^4*c^4*e^12*f^4))/(32*(a^2*b^8 + 256*a^6*c^4 - 16*a^3*
b^6*c + 96*a^4*b^4*c^2 - 256*a^5*b^2*c^3)))*(-(b^17*f^4 + b^2*f^4*(-(4*a*c
- b^2)^15)^(1/2) - 1720320*a^8*b*c^8*f^4 + 1140*a^2*b^13*c^2*f^4 - 10160*a^

3*b^11*c^3*f^4 + 34880*a^4*b^9*c^4*f^4 + 43776*a^5*b^7*c^5*f^4 - 680960*a^6
*b^5*c^6*f^4 + 1863680*a^7*b^3*c^7*f^4 - 55*a*b^15*c*f^4 - 25*a*c*f^4*(-(4*
a*c - b^2)^15)^(1/2))/(512*(a^3*b^20*e^2 + 1048576*a^13*c^10*e^2 - 40*a^4*b
^18*c*e^2 + 720*a^5*b^16*c^2*e^2 - 7680*a^6*b^14*c^3*e^2 + 53760*a^7*b^12*c
^4*e^2 - 258048*a^8*b^10*c^5*e^2 + 860160*a^9*b^8*c^6*e^2 - 1966080*a^10*b^
6*c^7*e^2 + 2949120*a^11*b^4*c^8*e^2 - 2621440*a^12*b^2*c^9*e^2)))^(1/2) -
((((67108864*a^9*b*c^9*d*e^13 - 4096*a^2*b^15*c^2*d*e^13 + 114688*a^3*b^13*
c^3*d*e^13 - 1376256*a^4*b^11*c^4*d*e^13 + 9175040*a^5*b^9*c^5*d*e^13 - 367
00160*a^6*b^7*c^6*d*e^13 + 88080384*a^7*b^5*c^7*d*e^13 - 117440512*a^8*b^3*
c^8*d*e^13)/(512*(a^2*b^12 + 4096*a^8*c^6 - 24*a^3*b^10*c + 240*a^4*b^8*c^2
 - 1280*a^5*b^6*c^3 + 3840*a^6*b^4*c^4 - 6144*a^7*b^2*c^5)) + (x*(262144*a^
7*b*c^7*e^14 - 256*a^2*b^11*c^2*e^14 + 5120*a^3*b^9*c^3*e^14 - 40960*a^4*b^
7*c^4*e^14 + 163840*a^5*b^5*c^5*e^14 - 327680*a^6*b^3*c^6*e^14))/(32*(a^2*b
^8 + 256*a^6*c^4 - 16*a^3*b^6*c + 96*a^4*b^4*c^2 - 256*a^5*b^2*c^3)))*(-(b^
17*f^4 + b^2*f^4*(-(4*a*c - b^2)^15)^(1/2) - 1720320*a^8*b*c^8*f^4 + 1140*a
^2*b^13*c^2*f^4 - 10160*a^3*b^11*c^3*f^4 + 34880*a^4*b^9*c^4*f^4 + 43776*a^
5*b^7*c^5*f^4 - 680960*a^6*b^5*c^6*f^4 + 1863680*a^7*b^3*c^7*f^4 - 55*a*b^1
5*c*f^4 - 25*a*c*f^4*(-(4*a*c - b^2)^15)^(1/2))/(512*(a^3*b^20*e^2 + 104857
6*a^13*c^10*e^2 - 40*a^4*b^18*c*e^2 + 720*a^5*b^16*c^2*e^2 - 7680*a^6*b^14*
c^3*e^2 + 53760*a^7*b^12*c^4*e^2 - 258048*a^8*b^10*c^5*e^2 + 860160*a^9*b^8
*c^6*e^2 - 1966080*a^10*b^6*c^7*e^2 + 2949120*a^11*b^4*c^8*e^2 - 2621440*a^
12*b^2*c^9*e^2)))^(1/2) + (122880*a^3*b^9*c^4*e^12*f^2 - 9216*a^2*b^11*c^3*
e^12*f^2 - 819200*a^4*b^7*c^5*e^12*f^2 + 2949120*a^5*b^5*c^6*e^12*f^2 - 550
5024*a^6*b^3*c^7*e^12*f^2 + 256*a*b^13*c^2*e^12*f^2 + 4194304*a^7*b*c^8*e^1
2*f^2)/(512*(a^2*b^12 + 4096*a^8*c^6 - 24*a^3*b^10*c + 240*a^4*b^8*c^2 - 12
80*a^5*b^6*c^3 + 3840*a^6*b^4*c^4 - 6144*a^7*b^2*c^5)))*(-(b^17*f^4 + b^2*f
^4*(-(4*a*c - b^2)^15)^(1/2) - 1720320*a^8*b*c^8*f^4 + 1140*a^2*b^13*c^2*f^
4 - 10160*a^3*b^11*c^3*f^4 + 34880*a^4*b^9*c^4*f^4 + 43776*a^5*b^7*c^5*f^4
 - 680960*a^6*b^5*c^6*f^4 + 1863680*a^7*b^3*c^7*f^4 - 55*a*b^15*c*f^4 - 25*a
*c*f^4*(-(4*a*c - b^2)^15)^(1/2))/(512*(a^3*b^20*e^2 + 1048576*a^13*c^10*e^
2 - 40*a^4*b^18*c*e^2 + 720*a^5*b^16*c^2*e^2 - 7680*a^6*b^14*c^3*e^2 + 5376
0*a^7*b^12*c^4*e^2 - 258048*a^8*b^10*c^5*e^2 + 860160*a^9*b^8*c^6*e^2 - 196
6080*a^10*b^6*c^7*e^2 + 2949120*a^11*b^4*c^8*e^2 - 2621440*a^12*b^2*c^9*e^2
)))^(1/2) + (204800*a^5*c^8*d*e^11*f^4 - 16*b^10*c^3*d*e^11*f^4 + 672*a*b^8
*c^4*d*e^11*f^4 - 28160*a^2*b^6*c^5*d*e^11*f^4 + 209920*a^3*b^4*c^6*d*e^11*
f^4 - 479232*a^4*b^2*c^7*d*e^11*f^4)/(512*(a^2*b^12 + 4096*a^8*c^6 - 24*a^3
*b^10*c + 240*a^4*b^8*c^2 - 1280*a^5*b^6*c^3 + 3840*a^6*b^4*c^4 - 6144*a^7*
b^2*c^5)) + (x*(800*a^3*c^6*e^12*f^4 - b^6*c^3*e^12*f^4 - 1472*a^2*b^2*c^5*
e^12*f^4 + 34*a*b^4*c^4*e^12*f^4))/(32*(a^2*b^8 + 256*a^6*c^4 - 16*a^3*b^6*
c + 96*a^4*b^4*c^2 - 256*a^5*b^2*c^3)))*(-(b^17*f^4 + b^2*f^4*(-(4*a*c - b^
2)^15)^(1/2) - 1720320*a^8*b*c^8*f^4 + 1140*a^2*b^13*c^2*f^4 - 10160*a^3*b^
11*c^3*f^4 + 34880*a^4*b^9*c^4*f^4 + 43776*a^5*b^7*c^5*f^4 - 680960*a^6*b^5
*c^6*f^4 + 1863680*a^7*b^3*c^7*f^4 - 55*a*b^15*c*f^4 - 25*a*c*f^4*(-(4*a*c
 - b^2)^15)^(1/2))/(512*(a^3*b^20*e^2 + 1048576*a^13*c^10*e^2 - 40*a^4*b^18*
c*e^2 + 720*a^5*b^16*c^2*e^2 - 7680*a^6*b^14*c^3*e^2 + 53760*a^7*b^12*c^4*e
^2 - 258048*a^8*b^10*c^5*e^2 + 860160*a^9*b^8*c^6*e^2 - 1966080*a^10*b^6*c^
7*e^2 + 2949120*a^11*b^4*c^8*e^2 - 2621440*a^12*b^2*c^9*e^2)))^(1/2) + (800
0*a^3*c^7*e^10*f^6 - 35*b^6*c^4*e^10*f^6 + 12720*a^2*b^2*c^6*e^10*f^6 - 84*
a*b^4*c^5*e^10*f^6)/(256*(a^2*b^12 + 4096*a^8*c^6 - 24*a^3*b^10*c + 240*a^4
*b^8*c^2 - 1280*a^5*b^6*c^3 + 3840*a^6*b^4*c^4 - 6144*a^7*b^2*c^5))))*(-(b^
17*f^4 + b^2*f^4*(-(4*a*c - b^2)^15)^(1/2) - 1720320*a^8*b*c^8*f^4 + 1140*a
^2*b^13*c^2*f^4 - 10160*a^3*b^11*c^3*f^4 + 34880*a^4*b^9*c^4*f^4 + 43776*a^
5*b^7*c^5*f^4 - 680960*a^6*b^5*c^6*f^4 + 1863680*a^7*b^3*c^7*f^4 - 55*a*b^1
5*c*f^4 - 25*a*c*f^4*(-(4*a*c - b^2)^15)^(1/2))/(512*(a^3*b^20*e^2 + 104857
6*a^13*c^10*e^2 - 40*a^4*b^18*c*e^2 + 720*a^5*b^16*c^2*e^2 - 7680*a^6*b^14*
c^3*e^2 + 53760*a^7*b^12*c^4*e^2 - 258048*a^8*b^10*c^5*e^2 + 860160*a^9*b^8
*c^6*e^2 - 1966080*a^10*b^6*c^7*e^2 + 2949120*a^11*b^4*c^8*e^2 - 2621440*a^
12*b^2*c^9*e^2)))^(1/2)*2i + atan((((((67108864*a^9*b*c^9*d*e^13 - 4096*a^2
*b^15*c^2*d*e^13 + 114688*a^3*b^13*c^3*d*e^13 - 1376256*a^4*b^11*c^4*d*e^13

+ 9175040*a^5*b^9*c^5*d*e^13 - 36700160*a^6*b^7*c^6*d*e^13 + 88080384*a^7*
b^5*c^7*d*e^13 - 117440512*a^8*b^3*c^8*d*e^13)/(512*(a^2*b^12 + 4096*a^8*c^
6 - 24*a^3*b^10*c + 240*a^4*b^8*c^2 - 1280*a^5*b^6*c^3 + 3840*a^6*b^4*c^4 -
6144*a^7*b^2*c^5)) + (x*(262144*a^7*b*c^7*e^14 - 256*a^2*b^11*c^2*e^14 + 5
120*a^3*b^9*c^3*e^14 - 40960*a^4*b^7*c^4*e^14 + 163840*a^5*b^5*c^5*e^14 - 3
27680*a^6*b^3*c^6*e^14))/(32*(a^2*b^8 + 256*a^6*c^4 - 16*a^3*b^6*c + 96*a^4
*b^4*c^2 - 256*a^5*b^2*c^3)))*(-(b^17*f^4 - b^2*f^4*(-(4*a*c - b^2)^15)^(1/
2) - 1720320*a^8*b*c^8*f^4 + 1140*a^2*b^13*c^2*f^4 - 10160*a^3*b^11*c^3*f^4
+ 34880*a^4*b^9*c^4*f^4 + 43776*a^5*b^7*c^5*f^4 - 680960*a^6*b^5*c^6*f^4 +
1863680*a^7*b^3*c^7*f^4 - 55*a*b^15*c*f^4 + 25*a*c*f^4*(-(4*a*c - b^2)^15)
^(1/2))/(512*(a^3*b^20*e^2 + 1048576*a^13*c^10*e^2 - 40*a^4*b^18*c*e^2 + 72
0*a^5*b^16*c^2*e^2 - 7680*a^6*b^14*c^3*e^2 + 53760*a^7*b^12*c^4*e^2 - 25804
8*a^8*b^10*c^5*e^2 + 860160*a^9*b^8*c^6*e^2 - 1966080*a^10*b^6*c^7*e^2 + 29
49120*a^11*b^4*c^8*e^2 - 2621440*a^12*b^2*c^9*e^2)))^(1/2) - (122880*a^3*b^
9*c^4*e^12*f^2 - 9216*a^2*b^11*c^3*e^12*f^2 - 819200*a^4*b^7*c^5*e^12*f^2 +
2949120*a^5*b^5*c^6*e^12*f^2 - 5505024*a^6*b^3*c^7*e^12*f^2 + 256*a*b^13*c
^2*e^12*f^2 + 4194304*a^7*b*c^8*e^12*f^2)/(512*(a^2*b^12 + 4096*a^8*c^6 - 2
4*a^3*b^10*c + 240*a^4*b^8*c^2 - 1280*a^5*b^6*c^3 + 3840*a^6*b^4*c^4 - 6144
*a^7*b^2*c^5)))*(-(b^17*f^4 - b^2*f^4*(-(4*a*c - b^2)^15)^(1/2) - 1720320*a
^8*b*c^8*f^4 + 1140*a^2*b^13*c^2*f^4 - 10160*a^3*b^11*c^3*f^4 + 34880*a^4*b
^9*c^4*f^4 + 43776*a^5*b^7*c^5*f^4 - 680960*a^6*b^5*c^6*f^4 + 1863680*a^7*b
^3*c^7*f^4 - 55*a*b^15*c*f^4 + 25*a*c*f^4*(-(4*a*c - b^2)^15)^(1/2))/(512*(
a^3*b^20*e^2 + 1048576*a^13*c^10*e^2 - 40*a^4*b^18*c*e^2 + 720*a^5*b^16*c^2
*e^2 - 7680*a^6*b^14*c^3*e^2 + 53760*a^7*b^12*c^4*e^2 - 258048*a^8*b^10*c^5
*e^2 + 860160*a^9*b^8*c^6*e^2 - 1966080*a^10*b^6*c^7*e^2 + 2949120*a^11*b^4
*c^8*e^2 - 2621440*a^12*b^2*c^9*e^2)))^(1/2) + (204800*a^5*c^8*d*e^11*f^4 -
16*b^10*c^3*d*e^11*f^4 + 672*a*b^8*c^4*d*e^11*f^4 - 28160*a^2*b^6*c^5*d*e^
11*f^4 + 209920*a^3*b^4*c^6*d*e^11*f^4 - 479232*a^4*b^2*c^7*d*e^11*f^4)/(51
2*(a^2*b^12 + 4096*a^8*c^6 - 24*a^3*b^10*c + 240*a^4*b^8*c^2 - 1280*a^5*b^6
*c^3 + 3840*a^6*b^4*c^4 - 6144*a^7*b^2*c^5)) + (x*(800*a^3*c^6*e^12*f^4 - b
^6*c^3*e^12*f^4 - 1472*a^2*b^2*c^5*e^12*f^4 + 34*a*b^4*c^4*e^12*f^4))/(32*(
a^2*b^8 + 256*a^6*c^4 - 16*a^3*b^6*c + 96*a^4*b^4*c^2 - 256*a^5*b^2*c^3)))*
(-(b^17*f^4 - b^2*f^4*(-(4*a*c - b^2)^15)^(1/2) - 1720320*a^8*b*c^8*f^4 + 1
140*a^2*b^13*c^2*f^4 - 10160*a^3*b^11*c^3*f^4 + 34880*a^4*b^9*c^4*f^4 + 437
76*a^5*b^7*c^5*f^4 - 680960*a^6*b^5*c^6*f^4 + 1863680*a^7*b^3*c^7*f^4 - 55*
a*b^15*c*f^4 + 25*a*c*f^4*(-(4*a*c - b^2)^15)^(1/2))/(512*(a^3*b^20*e^2 + 1
048576*a^13*c^10*e^2 - 40*a^4*b^18*c*e^2 + 720*a^5*b^16*c^2*e^2 - 7680*a^6*
b^14*c^3*e^2 + 53760*a^7*b^12*c^4*e^2 - 258048*a^8*b^10*c^5*e^2 + 860160*a^
9*b^8*c^6*e^2 - 1966080*a^10*b^6*c^7*e^2 + 2949120*a^11*b^4*c^8*e^2 - 26214
40*a^12*b^2*c^9*e^2)))^(1/2)*1i + ((((67108864*a^9*b*c^9*d*e^13 - 4096*a^2*
b^15*c^2*d*e^13 + 114688*a^3*b^13*c^3*d*e^13 - 1376256*a^4*b^11*c^4*d*e^13
+ 9175040*a^5*b^9*c^5*d*e^13 - 36700160*a^6*b^7*c^6*d*e^13 + 88080384*a^7*b
^5*c^7*d*e^13 - 117440512*a^8*b^3*c^8*d*e^13)/(512*(a^2*b^12 + 4096*a^8*c^6
- 24*a^3*b^10*c + 240*a^4*b^8*c^2 - 1280*a^5*b^6*c^3 + 3840*a^6*b^4*c^4 -
6144*a^7*b^2*c^5)) + (x*(262144*a^7*b*c^7*e^14 - 256*a^2*b^11*c^2*e^14 + 51
20*a^3*b^9*c^3*e^14 - 40960*a^4*b^7*c^4*e^14 + 163840*a^5*b^5*c^5*e^14 - 32
7680*a^6*b^3*c^6*e^14))/(32*(a^2*b^8 + 256*a^6*c^4 - 16*a^3*b^6*c + 96*a^4*
b^4*c^2 - 256*a^5*b^2*c^3)))*(-(b^17*f^4 - b^2*f^4*(-(4*a*c - b^2)^15)^(1/2
) - 1720320*a^8*b*c^8*f^4 + 1140*a^2*b^13*c^2*f^4 - 10160*a^3*b^11*c^3*f^4
+ 34880*a^4*b^9*c^4*f^4 + 43776*a^5*b^7*c^5*f^4 - 680960*a^6*b^5*c^6*f^4 +
1863680*a^7*b^3*c^7*f^4 - 55*a*b^15*c*f^4 + 25*a*c*f^4*(-(4*a*c - b^2)^15)^
(1/2))/(512*(a^3*b^20*e^2 + 1048576*a^13*c^10*e^2 - 40*a^4*b^18*c*e^2 + 720
*a^5*b^16*c^2*e^2 - 7680*a^6*b^14*c^3*e^2 + 53760*a^7*b^12*c^4*e^2 - 258048
*a^8*b^10*c^5*e^2 + 860160*a^9*b^8*c^6*e^2 - 1966080*a^10*b^6*c^7*e^2 + 294
9120*a^11*b^4*c^8*e^2 - 2621440*a^12*b^2*c^9*e^2)))^(1/2) + (122880*a^3*b^9
*c^4*e^12*f^2 - 9216*a^2*b^11*c^3*e^12*f^2 - 819200*a^4*b^7*c^5*e^12*f^2 +
2949120*a^5*b^5*c^6*e^12*f^2 - 5505024*a^6*b^3*c^7*e^12*f^2 + 256*a*b^13*c^
2*e^12*f^2 + 4194304*a^7*b*c^8*e^12*f^2)/(512*(a^2*b^12 + 4096*a^8*c^6 - 24
*a^3*b^10*c + 240*a^4*b^8*c^2 - 1280*a^5*b^6*c^3 + 3840*a^6*b^4*c^4 - 6144*

a^7*b^2*c^5)))*(-(b^17*f^4 - b^2*f^4*(-(4*a*c - b^2)^15)^(1/2) - 1720320*a^8*b*c^8*f^4 + 1140*a^2*b^13*c^2*f^4 - 10160*a^3*b^11*c^3*f^4 + 34880*a^4*b^9*c^4*f^4 + 43776*a^5*b^7*c^5*f^4 - 680960*a^6*b^5*c^6*f^4 + 1863680*a^7*b^3*c^7*f^4 - 55*a*b^15*c*f^4 + 25*a*c*f^4*(-(4*a*c - b^2)^15)^(1/2))/(512*(a^3*b^20*e^2 + 1048576*a^13*c^10*e^2 - 40*a^4*b^18*c*e^2 + 720*a^5*b^16*c^2*e^2 - 7680*a^6*b^14*c^3*e^2 + 53760*a^7*b^12*c^4*e^2 - 258048*a^8*b^10*c^5*e^2 + 860160*a^9*b^8*c^6*e^2 - 1966080*a^10*b^6*c^7*e^2 + 2949120*a^11*b^4*c^8*e^2 - 2621440*a^12*b^2*c^9*e^2)))^(1/2) + (204800*a^5*c^8*d*e^11*f^4 - 16*b^10*c^3*d*e^11*f^4 + 672*a*b^8*c^4*d*e^11*f^4 - 28160*a^2*b^6*c^5*d*e^11*f^4 + 209920*a^3*b^4*c^6*d*e^11*f^4 - 479232*a^4*b^2*c^7*d*e^11*f^4)/(512*(a^2*b^12 + 4096*a^8*c^6 - 24*a^3*b^10*c + 240*a^4*b^8*c^2 - 1280*a^5*b^6*c^3 + 3840*a^6*b^4*c^4 - 6144*a^7*b^2*c^5)) + (x*(800*a^3*c^6*e^12*f^4 - b^6*c^3*e^12*f^4 - 1472*a^2*b^2*c^5*e^12*f^4 + 34*a*b^4*c^4*e^12*f^4))/(32*(a^2*b^8 + 256*a^6*c^4 - 16*a^3*b^6*c + 96*a^4*b^4*c^2 - 256*a^5*b^2*c^3)))*(-(b^17*f^4 - b^2*f^4*(-(4*a*c - b^2)^15)^(1/2) - 1720320*a^8*b*c^8*f^4 + 1140*a^2*b^13*c^2*f^4 - 10160*a^3*b^11*c^3*f^4 + 34880*a^4*b^9*c^4*f^4 + 43776*a^5*b^7*c^5*f^4 - 680960*a^6*b^5*c^6*f^4 + 1863680*a^7*b^3*c^7*f^4 - 55*a*b^15*c*f^4 + 25*a*c*f^4*(-(4*a*c - b^2)^15)^(1/2))/(512*(a^3*b^20*e^2 + 1048576*a^13*c^10*e^2 - 40*a^4*b^18*c*e^2 + 720*a^5*b^16*c^2*e^2 - 7680*a^6*b^14*c^3*e^2 + 53760*a^7*b^12*c^4*e^2 - 258048*a^8*b^10*c^5*e^2 + 860160*a^9*b^8*c^6*e^2 - 1966080*a^10*b^6*c^7*e^2 + 2949120*a^11*b^4*c^8*e^2 - 2621440*a^12*b^2*c^9*e^2)))^(1/2)*1i)/(((((67108864*a^9*b*c^9*d*e^13 - 4096*a^2*b^15*c^2*d*e^13 + 114688*a^3*b^13*c^3*d*e^13 - 1376256*a^4*b^11*c^4*d*e^13 + 9175040*a^5*b^9*c^5*d*e^13 - 36700160*a^6*b^7*c^6*d*e^13 + 88080384*a^7*b^5*c^7*d*e^13 - 117440512*a^8*b^3*c^8*d*e^13)/(512*(a^2*b^12 + 4096*a^8*c^6 - 24*a^3*b^10*c + 240*a^4*b^8*c^2 - 1280*a^5*b^6*c^3 + 3840*a^6*b^4*c^4 - 6144*a^7*b^2*c^5)) + (x*(262144*a^7*b*c^7*e^14 - 256*a^2*b^11*c^2*e^14 + 5120*a^3*b^9*c^3*e^14 - 40960*a^4*b^7*c^4*e^14 + 163840*a^5*b^5*c^5*e^14 - 327680*a^6*b^3*c^6*e^14))/(32*(a^2*b^8 + 256*a^6*c^4 - 16*a^3*b^6*c + 96*a^4*b^4*c^2 - 256*a^5*b^2*c^3)))*(-(b^17*f^4 - b^2*f^4*(-(4*a*c - b^2)^15)^(1/2) - 1720320*a^8*b*c^8*f^4 + 1140*a^2*b^13*c^2*f^4 - 10160*a^3*b^11*c^3*f^4 + 34880*a^4*b^9*c^4*f^4 + 43776*a^5*b^7*c^5*f^4 - 680960*a^6*b^5*c^6*f^4 + 1863680*a^7*b^3*c^7*f^4 - 55*a*b^15*c*f^4 + 25*a*c*f^4*(-(4*a*c - b^2)^15)^(1/2))/(512*(a^3*b^20*e^2 + 1048576*a^13*c^10*e^2 - 40*a^4*b^18*c*e^2 + 720*a^5*b^16*c^2*e^2 - 7680*a^6*b^14*c^3*e^2 + 53760*a^7*b^12*c^4*e^2 - 258048*a^8*b^10*c^5*e^2 + 860160*a^9*b^8*c^6*e^2 - 1966080*a^10*b^6*c^7*e^2 + 2949120*a^11*b^4*c^8*e^2 - 2621440*a^12*b^2*c^9*e^2)))^(1/2) - (122880*a^3*b^9*c^4*e^12*f^2 - 9216*a^2*b^11*c^3*e^12*f^2 - 819200*a^4*b^7*c^5*e^12*f^2 + 2949120*a^5*b^5*c^6*e^12*f^2 - 5505024*a^6*b^3*c^7*e^12*f^2 + 256*a*b^13*c^2*e^12*f^2 + 4194304*a^7*b*c^8*e^12*f^2)/(512*(a^2*b^12 + 4096*a^8*c^6 - 24*a^3*b^10*c + 240*a^4*b^8*c^2 - 1280*a^5*b^6*c^3 + 3840*a^6*b^4*c^4 - 6144*a^7*b^2*c^5)))*(-(b^17*f^4 - b^2*f^4*(-(4*a*c - b^2)^15)^(1/2) - 1720320*a^8*b*c^8*f^4 + 1140*a^2*b^13*c^2*f^4 - 10160*a^3*b^11*c^3*f^4 + 34880*a^4*b^9*c^4*f^4 + 43776*a^5*b^7*c^5*f^4 - 680960*a^6*b^5*c^6*f^4 + 1863680*a^7*b^3*c^7*f^4 - 55*a*b^15*c*f^4 + 25*a*c*f^4*(-(4*a*c - b^2)^15)^(1/2))/(512*(a^3*b^20*e^2 + 1048576*a^13*c^10*e^2 - 40*a^4*b^18*c*e^2 + 720*a^5*b^16*c^2*e^2 - 7680*a^6*b^14*c^3*e^2 + 53760*a^7*b^12*c^4*e^2 - 258048*a^8*b^10*c^5*e^2 + 860160*a^9*b^8*c^6*e^2 - 1966080*a^10*b^6*c^7*e^2 + 2949120*a^11*b^4*c^8*e^2 - 2621440*a^12*b^2*c^9*e^2)))^(1/2) + (204800*a^5*c^8*d*e^11*f^4 - 16*b^10*c^3*d*e^11*f^4 + 672*a*b^8*c^4*d*e^11*f^4 - 28160*a^2*b^6*c^5*d*e^11*f^4 + 209920*a^3*b^4*c^6*d*e^11*f^4 - 479232*a^4*b^2*c^7*d*e^11*f^4)/(512*(a^2*b^12 + 4096*a^8*c^6 - 24*a^3*b^10*c + 240*a^4*b^8*c^2 - 1280*a^5*b^6*c^3 + 3840*a^6*b^4*c^4 - 6144*a^7*b^2*c^5)) + (x*(800*a^3*c^6*e^12*f^4 - b^6*c^3*e^12*f^4 - 1472*a^2*b^2*c^5*e^12*f^4 + 34*a*b^4*c^4*e^12*f^4))/(32*(a^2*b^8 + 256*a^6*c^4 - 16*a^3*b^6*c + 96*a^4*b^4*c^2 - 256*a^5*b^2*c^3)))*(-(b^17*f^4 - b^2*f^4*(-(4*a*c - b^2)^15)^(1/2) - 1720320*a^8*b*c^8*f^4 + 1140*a^2*b^13*c^2*f^4 - 10160*a^3*b^11*c^3*f^4 + 34880*a^4*b^9*c^4*f^4 + 43776*a^5*b^7*c^5*f^4 - 680960*a^6*b^5*c^6*f^4 + 1863680*a^7*b^3*c^7*f^4 - 55*a*b^15*c*f^4 + 25*a*c*f^4*(-(4*a*c - b^2)^15)^(1/2))/(512*(a^3*b^20*e^2 + 104

8576*a^13*c^10*e^2 - 40*a^4*b^18*c*e^2 + 720*a^5*b^16*c^2*e^2 - 7680*a^6*b^14*c^3*e^2 + 53760*a^7*b^12*c^4*e^2 - 258048*a^8*b^10*c^5*e^2 + 860160*a^9*b^8*c^6*e^2 - 1966080*a^10*b^6*c^7*e^2 + 2949120*a^11*b^4*c^8*e^2 - 2621440*a^12*b^2*c^9*e^2)))^(1/2) - ((((67108864*a^9*b*c^9*d*e^13 - 4096*a^2*b^15*c^2*d*e^13 + 114688*a^3*b^13*c^3*d*e^13 - 1376256*a^4*b^11*c^4*d*e^13 + 9175040*a^5*b^9*c^5*d*e^13 - 36700160*a^6*b^7*c^6*d*e^13 + 88080384*a^7*b^5*c^7*d*e^13 - 117440512*a^8*b^3*c^8*d*e^13)/(512*(a^2*b^12 + 4096*a^8*c^6 - 24*a^3*b^10*c + 240*a^4*b^8*c^2 - 1280*a^5*b^6*c^3 + 3840*a^6*b^4*c^4 - 6144*a^7*b^2*c^5)) + (x*(262144*a^7*b*c^7*e^14 - 256*a^2*b^11*c^2*e^14 + 5120*a^3*b^9*c^3*e^14 - 40960*a^4*b^7*c^4*e^14 + 163840*a^5*b^5*c^5*e^14 - 327680*a^6*b^3*c^6*e^14))/(32*(a^2*b^8 + 256*a^6*c^4 - 16*a^3*b^6*c + 96*a^4*b^4*c^2 - 256*a^5*b^2*c^3)))*(-(b^17*f^4 - b^2*f^4*(-(4*a*c - b^2)^15)^(1/2) - 1720320*a^8*b*c^8*f^4 + 1140*a^2*b^13*c^2*f^4 - 10160*a^3*b^11*c^3*f^4 + 34880*a^4*b^9*c^4*f^4 + 43776*a^5*b^7*c^5*f^4 - 680960*a^6*b^5*c^6*f^4 + 1863680*a^7*b^3*c^7*f^4 - 55*a*b^15*c*f^4 + 25*a*c*f^4*(-(4*a*c - b^2)^15)^(1/2))/(512*(a^3*b^20*e^2 + 1048576*a^13*c^10*e^2 - 40*a^4*b^18*c*e^2 + 720*a^5*b^16*c^2*e^2 - 7680*a^6*b^14*c^3*e^2 + 53760*a^7*b^12*c^4*e^2 - 258048*a^8*b^10*c^5*e^2 + 860160*a^9*b^8*c^6*e^2 - 1966080*a^10*b^6*c^7*e^2 + 2949120*a^11*b^4*c^8*e^2 - 2621440*a^12*b^2*c^9*e^2)))^(1/2) + (122880*a^3*b^9*c^4*e^12*f^2 - 9216*a^2*b^11*c^3*e^12*f^2 - 819200*a^4*b^7*c^5*e^12*f^2 + 2949120*a^5*b^5*c^6*e^12*f^2 - 5505024*a^6*b^3*c^7*e^12*f^2 + 256*a*b^13*c^2*e^12*f^2 + 4194304*a^7*b*c^8*e^12*f^2)/(512*(a^2*b^12 + 4096*a^8*c^6 - 24*a^3*b^10*c + 240*a^4*b^8*c^2 - 1280*a^5*b^6*c^3 + 3840*a^6*b^4*c^4 - 6144*a^7*b^2*c^5)))*(-(b^17*f^4 - b^2*f^4*(-(4*a*c - b^2)^15)^(1/2) - 1720320*a^8*b*c^8*f^4 + 1140*a^2*b^13*c^2*f^4 - 10160*a^3*b^11*c^3*f^4 + 34880*a^4*b^9*c^4*f^4 + 43776*a^5*b^7*c^5*f^4 - 680960*a^6*b^5*c^6*f^4 + 1863680*a^7*b^3*c^7*f^4 - 55*a*b^15*c*f^4 + 25*a*c*f^4*(-(4*a*c - b^2)^15)^(1/2))/(512*(a^3*b^20*e^2 + 1048576*a^13*c^10*e^2 - 40*a^4*b^18*c*e^2 + 720*a^5*b^16*c^2*e^2 - 7680*a^6*b^14*c^3*e^2 + 53760*a^7*b^12*c^4*e^2 - 258048*a^8*b^10*c^5*e^2 + 860160*a^9*b^8*c^6*e^2 - 1966080*a^10*b^6*c^7*e^2 + 2949120*a^11*b^4*c^8*e^2 - 2621440*a^12*b^2*c^9*e^2)))^(1/2) + (204800*a^5*c^8*d*e^11*f^4 - 16*b^10*c^3*d*e^11*f^4 + 672*a*b^8*c^4*d*e^11*f^4 - 28160*a^2*b^6*c^5*d*e^11*f^4 + 209920*a^3*b^4*c^6*d*e^11*f^4 - 479232*a^4*b^2*c^7*d*e^11*f^4)/(512*(a^2*b^12 + 4096*a^8*c^6 - 24*a^3*b^10*c + 240*a^4*b^8*c^2 - 1280*a^5*b^6*c^3 + 3840*a^6*b^4*c^4 - 6144*a^7*b^2*c^5)) + (x*(800*a^3*c^6*e^12*f^4 - b^6*c^3*e^12*f^4 - 1472*a^2*b^2*c^5*e^12*f^4 + 34*a*b^4*c^4*e^12*f^4))/(32*(a^2*b^8 + 256*a^6*c^4 - 16*a^3*b^6*c + 96*a^4*b^4*c^2 - 256*a^5*b^2*c^3)))*(-(b^17*f^4 - b^2*f^4*(-(4*a*c - b^2)^15)^(1/2) - 1720320*a^8*b*c^8*f^4 + 1140*a^2*b^13*c^2*f^4 - 10160*a^3*b^11*c^3*f^4 + 34880*a^4*b^9*c^4*f^4 + 43776*a^5*b^7*c^5*f^4 - 680960*a^6*b^5*c^6*f^4 + 1863680*a^7*b^3*c^7*f^4 - 55*a*b^15*c*f^4 + 25*a*c*f^4*(-(4*a*c - b^2)^15)^(1/2))/(512*(a^3*b^20*e^2 + 1048576*a^13*c^10*e^2 - 40*a^4*b^18*c*e^2 + 720*a^5*b^16*c^2*e^2 - 7680*a^6*b^14*c^3*e^2 + 53760*a^7*b^12*c^4*e^2 - 258048*a^8*b^10*c^5*e^2 + 860160*a^9*b^8*c^6*e^2 - 1966080*a^10*b^6*c^7*e^2 + 2949120*a^11*b^4*c^8*e^2 - 2621440*a^12*b^2*c^9*e^2)))^(1/2) + (8000*a^3*c^7*e^10*f^6 - 35*b^6*c^4*e^10*f^6 + 12720*a^2*b^2*c^6*e^10*f^6 - 84*a*b^4*c^5*e^10*f^6)/(256*(a^2*b^12 + 4096*a^8*c^6 - 24*a^3*b^10*c + 240*a^4*b^8*c^2 - 1280*a^5*b^6*c^3 + 3840*a^6*b^4*c^4 - 6144*a^7*b^2*c^5))))*(-(b^17*f^4 - b^2*f^4*(-(4*a*c - b^2)^15)^(1/2) - 1720320*a^8*b*c^8*f^4 + 1140*a^2*b^13*c^2*f^4 - 10160*a^3*b^11*c^3*f^4 + 34880*a^4*b^9*c^4*f^4 + 43776*a^5*b^7*c^5*f^4 - 680960*a^6*b^5*c^6*f^4 + 1863680*a^7*b^3*c^7*f^4 - 55*a*b^15*c*f^4 + 25*a*c*f^4*(-(4*a*c - b^2)^15)^(1/2))/(512*(a^3*b^20*e^2 + 1048576*a^13*c^10*e^2 - 40*a^4*b^18*c*e^2 + 720*a^5*b^16*c^2*e^2 - 7680*a^6*b^14*c^3*e^2 + 53760*a^7*b^12*c^4*e^2 - 258048*a^8*b^10*c^5*e^2 + 860160*a^9*b^8*c^6*e^2 - 1966080*a^10*b^6*c^7*e^2 + 2949120*a^11*b^4*c^8*e^2 - 2621440*a^12*b^2*c^9*e^2)))^(1/2)*2i + ((x^5*(2*b^3*c*e^4*f^2 + 21*b^2*c^2*d^2*e^4*f^2 + 28*a*b*c^2*e^4*f^2 + 420*a*c^3*d^2*e^4*f^2))/(8*a*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (x^3*(b^4*e^2*f^2 + 36*a^2*c^2*e^2*f^2 + 35*b^2*c^2*d^4*e^2*f^2 + 5*a*b^2*c*e^2*f^2 + 700*a*c^3*d^4*e^2*f^2 + 20*b^3*c*d^2*e^2*f^2 + 280*a*b*c^2*d^2*e^2*f^2))/(8*a*(b^4 + 16*a^2*c^2 -

```
 8*a*b^2*c)) + (x*(3*b^4*d^2*f^2 - a*b^3*f^2 + 140*a*c^3*d^6*f^2 + 10*b^3*c
*d^4*f^2 + 108*a^2*c^2*d^2*f^2 + 7*b^2*c^2*d^6*f^2 + 16*a^2*b*c*f^2 + 15*a*
b^2*c*d^2*f^2 + 140*a*b*c^2*d^4*f^2))/(8*a*(b^4 + 16*a^2*c^2 - 8*a*b^2*c))
+ (x^2*(3*b^4*d*e*f^2 + 108*a^2*c^2*d*e*f^2 + 420*a*c^3*d^5*e*f^2 + 20*b^3*
c*d^3*e*f^2 + 21*b^2*c^2*d^5*e*f^2 + 15*a*b^2*c*d*e*f^2 + 280*a*b*c^2*d^3*e
*f^2))/(8*a*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (b^4*d^3*f^2 + 20*a*c^3*d^7*f
^2 + 2*b^3*c*d^5*f^2 + 36*a^2*c^2*d^3*f^2 + b^2*c^2*d^7*f^2 - a*b^3*d*f^2 +
 16*a^2*b*c*d*f^2 + 5*a*b^2*c*d^3*f^2 + 28*a*b*c^2*d^5*f^2)/(8*a*e*(b^4 + 1
6*a^2*c^2 - 8*a*b^2*c)) + (7*x^6*(20*a*c^3*d*e^5*f^2 + b^2*c^2*d*e^5*f^2))/
(8*a*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (5*x^4*(7*b^2*c^2*d^3*e^3*f^2 + 2*b^
3*c*d*e^3*f^2 + 140*a*c^3*d^3*e^3*f^2 + 28*a*b*c^2*d*e^3*f^2))/(8*a*(b^4 +
16*a^2*c^2 - 8*a*b^2*c)) + (f^2*x^7*(20*a*c^3*e^6 + b^2*c^2*e^6))/(8*a*(b^4
 + 16*a^2*c^2 - 8*a*b^2*c)))/(x^2*(6*b^2*d^2*e^2 + 28*c^2*d^6*e^2 + 2*a*b*e
^2 + 12*a*c*d^2*e^2 + 30*b*c*d^4*e^2) + x^6*(28*c^2*d^2*e^6 + 2*b*c*e^6) +
x*(4*b^2*d^3*e + 8*c^2*d^7*e + 8*a*c*d^3*e + 12*b*c*d^5*e + 4*a*b*d*e) + x^
3*(4*b^2*d*e^3 + 56*c^2*d^5*e^3 + 8*a*c*d*e^3 + 40*b*c*d^3*e^3) + x^5*(56*c
^2*d^3*e^5 + 12*b*c*d*e^5) + x^4*(b^2*e^4 + 70*c^2*d^4*e^4 + 2*a*c*e^4 + 30
*b*c*d^2*e^4) + a^2 + b^2*d^4 + c^2*d^8 + c^2*e^8*x^8 + 2*a*b*d^2 + 2*a*c*d
^4 + 2*b*c*d^6 + 8*c^2*d*e^7*x^7)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*f*x+d*f)**2/(a+b*(e*x+d)**2+c*(e*x+d)**4)**3,x)
```

```
[Out] Timed out
```

3.544 $\int \frac{df+efx}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^3}\,dx$

Optimal. Leaf size=153

$$-\frac{6c^2 f \tanh^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{e\left(b^2-4ac\right)^{5/2}}+\frac{3cf\left(b+2c(d+ex)^2\right)}{2e\left(b^2-4ac\right)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)}-\frac{f\left(b+2c(d+ex)^2\right)}{4e\left(b^2-4ac\right)\left(a+b(d+ex)^2+c(d+\right.}$$

Rubi [A] time = 0.19, antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161, Rules used = {1142, 1107, 614, 618, 206}

$$-\frac{6c^2 f \tanh^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{e\left(b^2-4ac\right)^{5/2}}+\frac{3cf\left(b+2c(d+ex)^2\right)}{2e\left(b^2-4ac\right)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)}-\frac{f\left(b+2c(d+ex)^2\right)}{4e\left(b^2-4ac\right)\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(d*f + e*f*x)/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3,x]
```

```
[Out] -(f*(b + 2*c*(d + e*x)^2))/(4*(b^2 - 4*a*c)*e*(a + b*(d + e*x)^2 + c*(d + e
*x)^4)^2) + (3*c*f*(b + 2*c*(d + e*x)^2))/(2*(b^2 - 4*a*c)^2*e*(a + b*(d +
e*x)^2 + c*(d + e*x)^4)) - (6*c^2*f*ArcTanh[(b + 2*c*(d + e*x)^2)/Sqrt[b^2
- 4*a*c]])/((b^2 - 4*a*c)^(5/2)*e)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 614

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[(2*c*(2*p +
 3))/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
egerQ[4*p]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1107

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rule 1142

```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Di
st[u^m/(Coefficient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p,
 x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

Rubi steps

$$\int \frac{df+efx}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^3}\,dx = \frac{f\,\text{Subst}\left(\int \frac{x}{\left(a+bx^2+cx^4\right)^3}\,dx,x,d+ex\right)}{e}$$

$$= \frac{f\,\text{Subst}\left(\int \frac{1}{\left(a+bx+cx^2\right)^3}\,dx,x,(d+ex)^2\right)}{2e}$$

$$= -\frac{f\left(b+2c(d+ex)^2\right)}{4\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)^2} - \frac{(3cf)\,\text{Subst}\left(\int \frac{1}{\left(a+bx+cx^2\right)^2}\right.}{2\left(b^2-4ac\right.}$$

$$= -\frac{f\left(b+2c(d+ex)^2\right)}{4\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)^2} + \frac{3cf\left(b+2c(d\right.}{2\left(b^2-4ac\right)^2e\left(a+b(d+\right.}$$

$$= -\frac{f\left(b+2c(d+ex)^2\right)}{4\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)^2} + \frac{3cf\left(b+2c(d\right.}{2\left(b^2-4ac\right)^2e\left(a+b(d+\right.}$$

$$= -\frac{f\left(b+2c(d+ex)^2\right)}{4\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)^2} + \frac{3cf\left(b+2c(d\right.}{2\left(b^2-4ac\right)^2e\left(a+b(d+\right.}$$

Mathematica [A] time = 0.18, size = 148, normalized size = 0.97

$$\frac{f\left(\frac{24c^2\tan^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}+\frac{\left(b^2-4ac\right)\left(-b-2c(d+ex)^2\right)}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}+\frac{6c\left(b+2c(d+ex)^2\right)}{a+b(d+ex)^2+c(d+ex)^4}\right)}{4e\left(b^2-4ac\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*f + e*f*x)/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3,x]
```

```
[Out] (f*(((b^2 - 4*a*c)*(-b - 2*c*(d + e*x)^2))/(a + b*(d + e*x)^2 + c*(d + e*x)
^4)^2 + (6*c*(b + 2*c*(d + e*x)^2))/(a + b*(d + e*x)^2 + c*(d + e*x)^4) + (
24*c^2*ArcTan[(b + 2*c*(d + e*x)^2)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c]
))/(4*(b^2 - 4*a*c)^2*e)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{df+efx}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(d*f + e*f*x)/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3,x]
```

```
[Out] IntegrateAlgebraic[(d*f + e*f*x)/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3, x]
```

fricas [B] time = 1.59, size = 3748, normalized size = 24.50

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*f*x+d*f)/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="fricas")
```

```
[Out] [1/4*(12*(b^2*c^3 - 4*a*c^4)*e^6*f*x^6 + 72*(b^2*c^3 - 4*a*c^4)*d*e^5*f*x^5
 + 18*(b^3*c^2 - 4*a*b*c^3 + 10*(b^2*c^3 - 4*a*c^4)*d^2)*e^4*f*x^4 + 24*(10
*(b^2*c^3 - 4*a*c^4)*d^3 + 3*(b^3*c^2 - 4*a*b*c^3)*d)*e^3*f*x^3 + 4*(b^4*c
+ a*b^2*c^2 - 20*a^2*c^3 + 45*(b^2*c^3 - 4*a*c^4)*d^4 + 27*(b^3*c^2 - 4*a*b
*c^3)*d^2)*e^2*f*x^2 + 8*(9*(b^2*c^3 - 4*a*c^4)*d^5 + 9*(b^3*c^2 - 4*a*b*c^
3)*d^3 + (b^4*c + a*b^2*c^2 - 20*a^2*c^3)*d)*e*f*x + 12*(c^4*e^8*f*x^8 + 8*
c^4*d*e^7*f*x^7 + 2*(14*c^4*d^2 + b*c^3)*e^6*f*x^6 + 4*(14*c^4*d^3 + 3*b*c^
3*d)*e^5*f*x^5 + (70*c^4*d^4 + 30*b*c^3*d^2 + b^2*c^2 + 2*a*c^3)*e^4*f*x^4
+ 4*(14*c^4*d^5 + 10*b*c^3*d^3 + (b^2*c^2 + 2*a*c^3)*d)*e^3*f*x^3 + 2*(14*c
^4*d^6 + 15*b*c^3*d^4 + a*b*c^2 + 3*(b^2*c^2 + 2*a*c^3)*d^2)*e^2*f*x^2 + 4*
(2*c^4*d^7 + 3*b*c^3*d^5 + a*b*c^2*d + (b^2*c^2 + 2*a*c^3)*d^3)*e*f*x + (c^
4*d^8 + 2*b*c^3*d^6 + 2*a*b*c^2*d^2 + (b^2*c^2 + 2*a*c^3)*d^4 + a^2*c^2)*f)
*sqrt(b^2 - 4*a*c)*log((2*c^2*e^4*x^4 + 8*c^2*d*e^3*x^3 + 2*c^2*d^4 + 2*(6*
c^2*d^2 + b*c)*e^2*x^2 + 2*b*c*d^2 + 4*(2*c^2*d^3 + b*c*d)*e*x + b^2 - 2*a*
c - (2*c*e^2*x^2 + 4*c*d*e*x + 2*c*d^2 + b)*sqrt(b^2 - 4*a*c))/(c*e^4*x^4 +
 4*c*d*e^3*x^3 + c*d^4 + (6*c*d^2 + b)*e^2*x^2 + b*d^2 + 2*(2*c*d^3 + b*d)*
e*x + a)) + (12*(b^2*c^3 - 4*a*c^4)*d^6 - b^5 + 14*a*b^3*c - 40*a^2*b*c^2 +
 18*(b^3*c^2 - 4*a*b*c^3)*d^4 + 4*(b^4*c + a*b^2*c^2 - 20*a^2*c^3)*d^2)*f)/
((b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*e^9*x^8 + 8*(b^6*c^
2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d*e^8*x^7 + 2*(b^7*c - 12*a
*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4 + 14*(b^6*c^2 - 12*a*b^4*c^3 + 48*
a^2*b^2*c^4 - 64*a^3*c^5)*d^2)*e^7*x^6 + 4*(14*(b^6*c^2 - 12*a*b^4*c^3 + 48
*a^2*b^2*c^4 - 64*a^3*c^5)*d^3 + 3*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 -
 64*a^3*b*c^4)*d)*e^6*x^5 + (b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2
*c^3 - 128*a^4*c^4 + 70*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c
^5)*d^4 + 30*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*d^2)*e^
5*x^4 + 4*(14*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^5 +
10*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*d^3 + (b^8 - 10*a
*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*d)*e^4*x^3 + 2*(a*b
^7 - 12*a^2*b^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3 + 14*(b^6*c^2 - 12*a*b^4*
c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^6 + 15*(b^7*c - 12*a*b^5*c^2 + 48*a^2*
b^3*c^3 - 64*a^3*b*c^4)*d^4 + 3*(b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3
*b^2*c^3 - 128*a^4*c^4)*d^2)*e^3*x^2 + 4*(2*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^
2*b^2*c^4 - 64*a^3*c^5)*d^7 + 3*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64
*a^3*b*c^4)*d^5 + (b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128
*a^4*c^4)*d^3 + (a*b^7 - 12*a^2*b^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3)*d)*e
^2*x + ((b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^8 + a^2*b^
6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3 + 2*(b^7*c - 12*a*b^5*c^2 +
48*a^2*b^3*c^3 - 64*a^3*b*c^4)*d^6 + (b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 3
2*a^3*b^2*c^3 - 128*a^4*c^4)*d^4 + 2*(a*b^7 - 12*a^2*b^5*c + 48*a^3*b^3*c^2
 - 64*a^4*b*c^3)*d^2)*e), 1/4*(12*(b^2*c^3 - 4*a*c^4)*e^6*f*x^6 + 72*(b^2*c
^3 - 4*a*c^4)*d*e^5*f*x^5 + 18*(b^3*c^2 - 4*a*b*c^3 + 10*(b^2*c^3 - 4*a*c^4
)*d^2)*e^4*f*x^4 + 24*(10*(b^2*c^3 - 4*a*c^4)*d^3 + 3*(b^3*c^2 - 4*a*b*c^3)
*d)*e^3*f*x^3 + 4*(b^4*c + a*b^2*c^2 - 20*a^2*c^3 + 45*(b^2*c^3 - 4*a*c^4)*
d^4 + 27*(b^3*c^2 - 4*a*b*c^3)*d^2)*e^2*f*x^2 + 8*(9*(b^2*c^3 - 4*a*c^4)*d^
5 + 9*(b^3*c^2 - 4*a*b*c^3)*d^3 + (b^4*c + a*b^2*c^2 - 20*a^2*c^3)*d)*e*f*x
 - 24*(c^4*e^8*f*x^8 + 8*c^4*d*e^7*f*x^7 + 2*(14*c^4*d^2 + b*c^3)*e^6*f*x^6
 + 4*(14*c^4*d^3 + 3*b*c^3*d)*e^5*f*x^5 + (70*c^4*d^4 + 30*b*c^3*d^2 + b^2*
c^2 + 2*a*c^3)*e^4*f*x^4 + 4*(14*c^4*d^5 + 10*b*c^3*d^3 + (b^2*c^2 + 2*a*c^
3)*d)*e^3*f*x^3 + 2*(14*c^4*d^6 + 15*b*c^3*d^4 + a*b*c^2 + 3*(b^2*c^2 + 2*a
*c^3)*d^2)*e^2*f*x^2 + 4*(2*c^4*d^7 + 3*b*c^3*d^5 + a*b*c^2*d + (b^2*c^2 +
2*a*c^3)*d^3)*e*f*x + (c^4*d^8 + 2*b*c^3*d^6 + 2*a*b*c^2*d^2 + (b^2*c^2 + 2
*a*c^3)*d^4 + a^2*c^2)*f)*sqrt(-b^2 + 4*a*c)*arctan(-(2*c*e^2*x^2 + 4*c*d*e
*x + 2*c*d^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) + (12*(b^2*c^3 - 4*a*c^
4)*d^6 - b^5 + 14*a*b^3*c - 40*a^2*b*c^2 + 18*(b^3*c^2 - 4*a*b*c^3)*d^4 + 4
*(b^4*c + a*b^2*c^2 - 20*a^2*c^3)*d^2)*f)/((b^6*c^2 - 12*a*b^4*c^3 + 48*a^2
*b^2*c^4 - 64*a^3*c^5)*e^9*x^8 + 8*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4
 - 64*a^3*c^5)*d*e^8*x^7 + 2*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^
3*b*c^4 + 14*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^2)*e^
```

```
7*x^6 + 4*(14*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^3 +
3*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*d)*e^6*x^5 + (b^8
- 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4 + 70*(b^6*c^2
- 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^4 + 30*(b^7*c - 12*a*b^5*c^
2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*d^2)*e^5*x^4 + 4*(14*(b^6*c^2 - 12*a*b^4
*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^5 + 10*(b^7*c - 12*a*b^5*c^2 + 48*a^2
*b^3*c^3 - 64*a^3*b*c^4)*d^3 + (b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*
b^2*c^3 - 128*a^4*c^4)*d)*e^4*x^3 + 2*(a*b^7 - 12*a^2*b^5*c + 48*a^3*b^3*c^
2 - 64*a^4*b*c^3 + 14*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5
)*d^6 + 15*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*d^4 + 3*(
b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*d^2)*e^3*
x^2 + 4*(2*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^7 + 3*(
b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*d^5 + (b^8 - 10*a*b^6
*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*d^3 + (a*b^7 - 12*a^2*b
^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3)*d)*e^2*x + ((b^6*c^2 - 12*a*b^4*c^3 +
 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^8 + a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2
 - 64*a^5*c^3 + 2*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*d^
6 + (b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*d^4
+ 2*(a*b^7 - 12*a^2*b^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3)*d^2)*e)]
```

giac [B] time = 0.69, size = 445, normalized size = 2.91

$$\frac{6\,c^{2}f\arctan\left(\frac{2\,cd^{2}f+2\,(fx^{2}e+2\,dfx)ce+bf}{\sqrt{-b^{2}+4\,ac}\,f}\right)e^{(-1)}}{(b^{4}-8\,ab^{2}c+16\,a^{2}c^{2})\sqrt{-b^{2}+4\,ac}}+\frac{12\,c^{3}d^{6}f^{5}+36\,(fx^{2}e+2\,dfx)c^{3}d^{4}f^{4}e+18\,bc^{2}d^{4}f^{5}+36\,(fx^{2}e+2\,dfx)^{2}c^{3}d^{2}f^{3}e^{2}+36\,(fx^{2}e+2\,dfx)bc^{2}d^{2}f^{4}e+4\,b^{2}cd^{2}f^{5}+20\,ac^{2}d^{2}f^{5}+12\,(fx^{2}e+2\,dfx)^{3}c^{3}f^{2}e^{3}+18\,(fx^{2}e+2\,dfx)^{2}bc^{2}f^{3}e^{2}+4\,(fx^{2}e+2\,dfx)b^{2}cf^{4}e+20\,(fx^{2}e+2\,dfx)ac^{2}f^{4}e-b^{3}f^{5}+10\,abcf^{5}}{4\,\left(cd^{4}f^{2}+2\,(fx^{2}e+2\,dfx)cd^{2}fe+bd^{2}f^{2}+(fx^{2}e+2\,dfx)^{2}ce^{2}+(fx^{2}e+2\,dfx)bfe+af^{2}\right)^{2}(b^{4}e-8\,ab^{2}ce+16\,a^{2}c^{2}e)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*f*x+d*f)/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="giac")
```

```
[Out] 6*c^2*f*arctan((2*c*d^2*f + 2*(f*x^2*e + 2*d*f*x)*c*e + b*f)/(sqrt(-b^2 + 4
*a*c)*f))*e^(-1)/((b^4 - 8*a*b^2*c + 16*a^2*c^2)*sqrt(-b^2 + 4*a*c)) + 1/4*
(12*c^3*d^6*f^5 + 36*(f*x^2*e + 2*d*f*x)*c^3*d^4*f^4*e + 18*b*c^2*d^4*f^5 +
 36*(f*x^2*e + 2*d*f*x)^2*c^3*d^2*f^3*e^2 + 36*(f*x^2*e + 2*d*f*x)*b*c^2*d^
2*f^4*e + 4*b^2*c*d^2*f^5 + 20*a*c^2*d^2*f^5 + 12*(f*x^2*e + 2*d*f*x)^3*c^3
*f^2*e^3 + 18*(f*x^2*e + 2*d*f*x)^2*b*c^2*f^3*e^2 + 4*(f*x^2*e + 2*d*f*x)*b
^2*c*f^4*e + 20*(f*x^2*e + 2*d*f*x)*a*c^2*f^4*e - b^3*f^5 + 10*a*b*c*f^5)/(
(c*d^4*f^2 + 2*(f*x^2*e + 2*d*f*x)*c*d^2*f*e + b*d^2*f^2 + (f*x^2*e + 2*d*f
*x)^2*c*e^2 + (f*x^2*e + 2*d*f*x)*b*f*e + a*f^2)^2*(b^4*e - 8*a*b^2*c*e + 1
6*a^2*c^2*e))
```

maple [C] time = 0.05, size = 2132, normalized size = 13.93

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*f*x+d*f)/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x)
```

```
[Out] 3*f/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*
b*d*e*x+b*d^2+a)^2*c^3*e^5/(16*a^2*c^2-8*a*b^2*c+b^4)*x^6+18*f/(c*e^4*x^4+4
*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)
^2*e^4*c^3*d/(16*a^2*c^2-8*a*b^2*c+b^4)*x^5+45*f/(c*e^4*x^4+4*c*d*e^3*x^3+6
*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*c^3*e^3/(16
*a^2*c^2-8*a*b^2*c+b^4)*x^4*d^2+9/2*f/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*
x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*c^2*e^3/(16*a^2*c^2-8*
a*b^2*c+b^4)*x^4*b+60*f/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*
x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*c^3*d^3*e^2/(16*a^2*c^2-8*a*b^2*c+b^
4)*x^3+18*f/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+
c*d^4+2*b*d*e*x+b*d^2+a)^2*c^2*d*e^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x^3*b+45*f/
(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*
e*x+b*d^2+a)^2*c^3*e/(16*a^2*c^2-8*a*b^2*c+b^4)*x^2*d^4+27*f/(c*e^4*x^4+4*c
*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2
*c^2*e/(16*a^2*c^2-8*a*b^2*c+b^4)*x^2*b*d^2+5*f/(c*e^4*x^4+4*c*d*e^3*x^3+6*
```

```
c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*c^2*e/(16*a^
2*c^2-8*a*b^2*c+b^4)*x^2*a+f/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d
^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*c*e/(16*a^2*c^2-8*a*b^2*c+b^4)*
x^2*b^2+18*f/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2
+c*d^4+2*b*d*e*x+b*d^2+a)^2*c^3*d^5/(16*a^2*c^2-8*a*b^2*c+b^4)*x+18*f/(c*e^
4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b
*d^2+a)^2*c^2*d^3/(16*a^2*c^2-8*a*b^2*c+b^4)*x*b+10*f/(c*e^4*x^4+4*c*d*e^3*
x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*c^2*d/
(16*a^2*c^2-8*a*b^2*c+b^4)*x*a+2*f/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2
+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*c*d/(16*a^2*c^2-8*a*b^2*c
+b^4)*x*b^2+3*f/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*
x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2/(16*a^2*c^2-8*a*b^2*c+b^4)/e*c^3*d^6+9/2*f/(
c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e
*x+b*d^2+a)^2/(16*a^2*c^2-8*a*b^2*c+b^4)/e*b*c^2*d^4+5*f/(c*e^4*x^4+4*c*d*e
^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2/(16
*a^2*c^2-8*a*b^2*c+b^4)/e*a*c^2*d^2+f/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*
x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2/(16*a^2*c^2-8*a*b^2*c+
b^4)/e*b^2*c*d^2+5/2*f/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x
+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2/(16*a^2*c^2-8*a*b^2*c+b^4)/e*a*b*c-1/
4*f/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*
b*d*e*x+b*d^2+a)^2/(16*a^2*c^2-8*a*b^2*c+b^4)/e*b^3+3*f*c^2/(16*a^2*c^2-8*a
*b^2*c+b^4)/e*sum((_R*e+d)/(2*_R^3*c*e^3+6*_R^2*c*d*e^2+6*_R*c*d^2*e+2*c*d^
3+_R*b*e+b*d)*ln(-_R+x),_R=RootOf(_Z^4*c*e^4+4*_Z^3*c*d*e^3+c*d^4+b*d^2+(6*
c*d^2*e^2+b*e^2)*_Z^2+(4*c*d^3*e+2*b*d*e)*_Z+a))
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*f*x+d*f)/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="maxima")
[Out] Timed out
```

mupad [B] time = 3.99, size = 1199, normalized size = 7.84

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*f + e*f*x)/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3,x)
[Out] ((x^2*(5*a*c^2*e*f + b^2*c*e*f + 45*c^3*d^4*e*f + 27*b*c^2*d^2*e*f))/(b^4 +
 16*a^2*c^2 - 8*a*b^2*c) + (12*c^3*d^6*f - b^3*f + 20*a*c^2*d^2*f + 4*b^2*c
*d^2*f + 18*b*c^2*d^4*f + 10*a*b*c*f)/(4*e*(b^4 + 16*a^2*c^2 - 8*a*b^2*c))
+ (9*x^4*(10*c^3*d^2*e^3*f + b*c^2*e^3*f))/(2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c
)) + (2*d*x*(9*c^3*d^4*f + 5*a*c^2*f + b^2*c*f + 9*b*c^2*d^2*f))/(b^4 + 16*
a^2*c^2 - 8*a*b^2*c) + (6*d*x^3*(10*c^3*d^2*e^2*f + 3*b*c^2*e^2*f))/(b^4 +
16*a^2*c^2 - 8*a*b^2*c) + (3*c^3*e^5*f*x^6)/(b^4 + 16*a^2*c^2 - 8*a*b^2*c)
+ (18*c^3*d*e^4*f*x^5)/(b^4 + 16*a^2*c^2 - 8*a*b^2*c))/(x^2*(6*b^2*d^2*e^2
+ 28*c^2*d^6*e^2 + 2*a*b*e^2 + 12*a*c*d^2*e^2 + 30*b*c*d^4*e^2) + x^6*(28*c
^2*d^2*e^6 + 2*b*c*e^6) + x*(4*b^2*d^3*e + 8*c^2*d^7*e + 8*a*c*d^3*e + 12*b
*c*d^5*e + 4*a*b*d*e) + x^3*(4*b^2*d*e^3 + 56*c^2*d^5*e^3 + 8*a*c*d*e^3 + 4
0*b*c*d^3*e^3) + x^5*(56*c^2*d^3*e^5 + 12*b*c*d*e^5) + x^4*(b^2*e^4 + 70*c^
2*d^4*e^4 + 2*a*c*e^4 + 30*b*c*d^2*e^4) + a^2 + b^2*d^4 + c^2*d^8 + c^2*e^8
*x^8 + 2*a*b*d^2 + 2*a*c*d^4 + 2*b*c*d^6 + 8*c^2*d*e^7*x^7) + (6*c^2*f*atan
(((b^4*(4*a*c - b^2)^5 + 16*a^2*c^2*(4*a*c - b^2)^5 - 8*a*b^2*c*(4*a*c - b^
2)^5)*(x^2*((36*c^6*e^8*f^2)/(a*(4*a*c - b^2)^(9/2)*(b^4 + 16*a^2*c^2 - 8*a
*b^2*c)) + (36*b*c^4*f^2*(b^5*c^2*e^10 - 8*a*b^3*c^3*e^10 + 16*a^2*b*c^4*e^
10))/(a*e^2*(4*a*c - b^2)^(15/2)*(b^4 + 16*a^2*c^2 - 8*a*b^2*c))) + x*((72*
```

```
c^6*d*e^7*f^2)/(a*(4*a*c - b^2)^(9/2)*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (72
*b*c^4*f^2*(b^5*c^2*d*e^9 - 8*a*b^3*c^3*d*e^9 + 16*a^2*b*c^4*d*e^9))/(a*e^2
*(4*a*c - b^2)^(15/2)*(b^4 + 16*a^2*c^2 - 8*a*b^2*c))) + (36*c^6*d^2*e^6*f^
2)/(a*(4*a*c - b^2)^(9/2)*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (36*b*c^4*f^2*(
32*a^3*c^4*e^8 + 2*a*b^4*c^2*e^8 - 16*a^2*b^2*c^3*e^8 + b^5*c^2*d^2*e^8 - 8
*a*b^3*c^3*d^2*e^8 + 16*a^2*b*c^4*d^2*e^8))/(a*e^2*(4*a*c - b^2)^(15/2)*(b^
4 + 16*a^2*c^2 - 8*a*b^2*c))))/(72*c^6*e^6*f^2)))/(e*(4*a*c - b^2)^(5/2))
```

sympy [B] time = 13.97, size = 1707, normalized size = 11.16

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*f*x+d*f)/(a+b*(e*x+d)**2+c*(e*x+d)**4)**3,x)
```

```
[Out] -3*c**2*f*sqrt(-1/(4*a*c - b**2)**5)*log(2*d*x/e + x**2 + (-192*a**3*c**5*f
*sqrt(-1/(4*a*c - b**2)**5) + 144*a**2*b**2*c**4*f*sqrt(-1/(4*a*c - b**2)**
5) - 36*a*b**4*c**3*f*sqrt(-1/(4*a*c - b**2)**5) + 3*b**6*c**2*f*sqrt(-1/(4
*a*c - b**2)**5) + 3*b*c**2*f + 6*c**3*d**2*f)/(6*c**3*e**2*f))/e + 3*c**2*
f*sqrt(-1/(4*a*c - b**2)**5)*log(2*d*x/e + x**2 + (192*a**3*c**5*f*sqrt(-1/
(4*a*c - b**2)**5) - 144*a**2*b**2*c**4*f*sqrt(-1/(4*a*c - b**2)**5) + 36*a
*b**4*c**3*f*sqrt(-1/(4*a*c - b**2)**5) - 3*b**6*c**2*f*sqrt(-1/(4*a*c - b*
*2)**5) + 3*b*c**2*f + 6*c**3*d**2*f)/(6*c**3*e**2*f))/e + (10*a*b*c*f + 20
*a*c**2*d**2*f - b**3*f + 4*b**2*c*d**2*f + 18*b*c**2*d**4*f + 12*c**3*d**6
*f + 72*c**3*d*e**5*f*x**5 + 12*c**3*e**6*f*x**6 + x**4*(18*b*c**2*e**4*f +
 180*c**3*d**2*e**4*f) + x**3*(72*b*c**2*d*e**3*f + 240*c**3*d**3*e**3*f) +
 x**2*(20*a*c**2*e**2*f + 4*b**2*c*e**2*f + 108*b*c**2*d**2*e**2*f + 180*c*
*3*d**4*e**2*f) + x*(40*a*c**2*d*e*f + 8*b**2*c*d*e*f + 72*b*c**2*d**3*e*f
+ 72*c**3*d**5*e*f))/(64*a**4*c**2*e - 32*a**3*b**2*c*e + 128*a**3*b*c**2*d
**2*e + 128*a**3*c**3*d**4*e + 4*a**2*b**4*e - 64*a**2*b**3*c*d**2*e + 128*
a**2*b*c**3*d**6*e + 64*a**2*c**4*d**8*e + 8*a*b**5*d**2*e - 24*a*b**4*c*d*
*4*e - 64*a*b**3*c**2*d**6*e - 32*a*b**2*c**3*d**8*e + 4*b**6*d**4*e + 8*b*
*5*c*d**6*e + 4*b**4*c**2*d**8*e + x**8*(64*a**2*c**4*e**9 - 32*a*b**2*c**3
*e**9 + 4*b**4*c**2*e**9) + x**7*(512*a**2*c**4*d*e**8 - 256*a*b**2*c**3*d*
e**8 + 32*b**4*c**2*d*e**8) + x**6*(128*a**2*b*c**3*e**7 + 1792*a**2*c**4*d
**2*e**7 - 64*a*b**3*c**2*e**7 - 896*a*b**2*c**3*d**2*e**7 + 8*b**5*c*e**7
+ 112*b**4*c**2*d**2*e**7) + x**5*(768*a**2*b*c**3*d*e**6 + 3584*a**2*c**4*
d**3*e**6 - 384*a*b**3*c**2*d*e**6 - 1792*a*b**2*c**3*d**3*e**6 + 48*b**5*c
*d*e**6 + 224*b**4*c**2*d**3*e**6) + x**4*(128*a**3*c**3*e**5 + 1920*a**2*b
*c**3*d**2*e**5 + 4480*a**2*c**4*d**4*e**5 - 24*a*b**4*c*e**5 - 960*a*b**3*
c**2*d**2*e**5 - 2240*a*b**2*c**3*d**4*e**5 + 4*b**6*e**5 + 120*b**5*c*d**2
*e**5 + 280*b**4*c**2*d**4*e**5) + x**3*(512*a**3*c**3*d*e**4 + 2560*a**2*b
*c**3*d**3*e**4 + 3584*a**2*c**4*d**5*e**4 - 96*a*b**4*c*d*e**4 - 1280*a*b*
*3*c**2*d**3*e**4 - 1792*a*b**2*c**3*d**5*e**4 + 16*b**6*d*e**4 + 160*b**5*
c*d**3*e**4 + 224*b**4*c**2*d**5*e**4) + x**2*(128*a**3*b*c**2*e**3 + 768*a
**3*c**3*d**2*e**3 - 64*a**2*b**3*c*e**3 + 1920*a**2*b*c**3*d**4*e**3 + 179
2*a**2*c**4*d**6*e**3 + 8*a*b**5*e**3 - 144*a*b**4*c*d**2*e**3 - 960*a*b**3
*c**2*d**4*e**3 - 896*a*b**2*c**3*d**6*e**3 + 24*b**6*d**2*e**3 + 120*b**5*
c*d**4*e**3 + 112*b**4*c**2*d**6*e**3) + x*(256*a**3*b*c**2*d*e**2 + 512*a*
*3*c**3*d**3*e**2 - 128*a**2*b**3*c*d*e**2 + 768*a**2*b*c**3*d**5*e**2 + 51
2*a**2*c**4*d**7*e**2 + 16*a*b**5*d*e**2 - 96*a*b**4*c*d**3*e**2 - 384*a*b*
*3*c**2*d**5*e**2 - 256*a*b**2*c**3*d**7*e**2 + 16*b**6*d**3*e**2 + 48*b**5
*c*d**5*e**2 + 32*b**4*c**2*d**7*e**2))
```

3.545 $\int \frac{1}{(df+efx)\left(a+b(d+ex)^2+c(d+ex)^4\right)^3}\,dx$

Optimal. Leaf size=270

$$-\frac{\log\left(a+b(d+ex)^2+c(d+ex)^4\right)}{4a^3ef}+\frac{\log(d+ex)}{a^3ef}+\frac{16a^2c^2+2bc\left(b^2-7ac\right)(d+ex)^2-15ab^2c+2b^4}{4a^2ef\left(b^2-4ac\right)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)}+\frac{b\left(30a^2c^2\right.}{}$$

Rubi [A] time = 0.50, antiderivative size = 270, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {1142, 1114, 740, 822, 800, 634, 618, 206, 628}

$$\frac{16a^2c^2+2bc\left(b^2-7ac\right)(d+ex)^2-15ab^2c+2b^4}{4a^2ef\left(b^2-4ac\right)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)}+\frac{b\left(30a^2c^2-10ab^2c+b^4\right)\tanh^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{2a^3ef\left(b^2-4ac\right)^{5/2}}-\frac{\log\left(a+b(d+ex)^2+c(d+ex)^4\right)}{4a^3ef}+\frac{\log(d+ex)}{a^3ef}+\frac{-2ac+b^2+bc(d+ex)^2}{4aef\left(b^2-4ac\right)\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d*f + e*f*x)*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3),x]
```

```
[Out] (b^2 - 2*a*c + b*c*(d + e*x)^2)/(4*a*(b^2 - 4*a*c)*e*f*(a + b*(d + e*x)^2 +
 c*(d + e*x)^4)^2) + (2*b^4 - 15*a*b^2*c + 16*a^2*c^2 + 2*b*c*(b^2 - 7*a*c)
*(d + e*x)^2)/(4*a^2*(b^2 - 4*a*c)^2*e*f*(a + b*(d + e*x)^2 + c*(d + e*x)^4
)) + (b*(b^4 - 10*a*b^2*c + 30*a^2*c^2)*ArcTanh[(b + 2*c*(d + e*x)^2)/Sqrt[
b^2 - 4*a*c]])/(2*a^3*(b^2 - 4*a*c)^(5/2)*e*f) + Log[d + e*x]/(a^3*e*f) - L
og[a + b*(d + e*x)^2 + c*(d + e*x)^4]/(4*a^3*e*f)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 740

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e
)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e
^2)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d +
 e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p +
 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
```

```
b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p,
-1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 822

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(f*(b*c*d - b^2*e +
2*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x)*(a
+ b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), x]
 + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m +
2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*m
 + b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) - f
*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1
] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 1142

```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Di
st[u^m/(Coefficient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p,
 x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

Rubi steps

$$\int \frac{1}{(df+efx)\left(a+b(d+ex)^2+c(d+ex)^4\right)^3}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x\left(a+bx^2+cx^4\right)^3}\,dx, x, d+ex\right)}{ef}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{x\left(a+bx+cx^2\right)^3}\,dx, x, (d+ex)^2\right)}{2ef}$$

$$= \frac{b^2-2ac+bc(d+ex)^2}{4a\left(b^2-4ac\right)ef\left(a+b(d+ex)^2+c(d+ex)^4\right)^2} - \frac{\text{Subst}\left(\int \cdots\right.}{\cdots}$$

$$= \frac{b^2-2ac+bc(d+ex)^2}{4a\left(b^2-4ac\right)ef\left(a+b(d+ex)^2+c(d+ex)^4\right)^2} + \frac{2b^4-15ab\cdots}{4a^2\left(b^2-\cdots\right.}$$

$$= \frac{b^2-2ac+bc(d+ex)^2}{4a\left(b^2-4ac\right)ef\left(a+b(d+ex)^2+c(d+ex)^4\right)^2} + \frac{2b^4-15ab\cdots}{4a^2\left(b^2-\cdots\right.}$$

$$= \frac{b^2-2ac+bc(d+ex)^2}{4a\left(b^2-4ac\right)ef\left(a+b(d+ex)^2+c(d+ex)^4\right)^2} + \frac{2b^4-15ab\cdots}{4a^2\left(b^2-\cdots\right.}$$

$$= \frac{b^2-2ac+bc(d+ex)^2}{4a\left(b^2-4ac\right)ef\left(a+b(d+ex)^2+c(d+ex)^4\right)^2} + \frac{2b^4-15ab\cdots}{4a^2\left(b^2-\cdots\right.}$$

$$= \frac{b^2-2ac+bc(d+ex)^2}{4a\left(b^2-4ac\right)ef\left(a+b(d+ex)^2+c(d+ex)^4\right)^2} + \frac{2b^4-15ab\cdots}{4a^2\left(b^2-\cdots\right.}$$

$$= \frac{b^2-2ac+bc(d+ex)^2}{4a\left(b^2-4ac\right)ef\left(a+b(d+ex)^2+c(d+ex)^4\right)^2} + \frac{2b^4-15ab\cdots}{4a^2\left(b^2-\cdots\right.}$$

Mathematica [A] time = 3.93, size = 394, normalized size = 1.46

$$\frac{\frac{a^2\left(2ac-b^2-bc(d+ex)^2\right)}{\left(4ac-b^2\right)\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}+\frac{a\left(16a^2c^2-15ab^2c-14abc^2(d+ex)^2+2b^4+2b^3c(d+ex)^2\right)}{\left(b^2-4ac\right)^2\left(a+(d+ex)^2\left(b+c(d+ex)^2\right)\right)}-\frac{\left(16a^2c^2\sqrt{b^2-4ac}+30a^2bc^2-10ab^3c-8ab^2c\sqrt{b^2-4ac}+b^4\sqrt{b^2-4ac}+b^5\right)\log\left(-\sqrt{b^2-4ac}+b+2c(d+ex)^2\right)}{\left(b^2-4ac\right)^{5/2}}+\frac{\left(-16a^2c^2\sqrt{b^2-4ac}+30a^2bc^2-10ab^3c+8ab^2c\sqrt{b^2-4ac}-b^4\sqrt{b^2-4ac}+b^5\right)\log\left(\sqrt{b^2-4ac}+b+2c(d+ex)^2\right)}{\left(b^2-4ac\right)^{5/2}}+4\log(d+ex)}{4a^3ef}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d*f + e*f*x)*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3),x]

[Out] ((a^2*(-b^2 + 2*a*c - b*c*(d + e*x)^2))/((-b^2 + 4*a*c)*(a + b*(d + e*x)^2
+ c*(d + e*x)^4)^2) + (a*(2*b^4 - 15*a*b^2*c + 16*a^2*c^2 + 2*b^3*c*(d + e*
x)^2 - 14*a*b*c^2*(d + e*x)^2))/((b^2 - 4*a*c)^2*(a + (d + e*x)^2*(b + c*(d
 + e*x)^2))) + 4*Log[d + e*x] - ((b^5 - 10*a*b^3*c + 30*a^2*b*c^2 + b^4*Sqr
t[b^2 - 4*a*c] - 8*a*b^2*c*Sqrt[b^2 - 4*a*c] + 16*a^2*c^2*Sqrt[b^2 - 4*a*c]
)*Log[b - Sqrt[b^2 - 4*a*c] + 2*c*(d + e*x)^2])/(b^2 - 4*a*c)^(5/2) + ((b^5
 - 10*a*b^3*c + 30*a^2*b*c^2 - b^4*Sqrt[b^2 - 4*a*c] + 8*a*b^2*c*Sqrt[b^2 -
 4*a*c] - 16*a^2*c^2*Sqrt[b^2 - 4*a*c])*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*(d
+ e*x)^2])/(b^2 - 4*a*c)^(5/2))/(4*a^3*e*f)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(df+efx)\left(a+b(d+ex)^2+c(d+ex)^4\right)^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/((d*f + e*f*x)*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3),
x]
```

```
[Out] IntegrateAlgebraic[1/((d*f + e*f*x)*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3),
 x]
```

fricas [B] time = 7.39, size = 9926, normalized size = 36.76

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*f*x+d*f)/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="fricas"
)
```

```
[Out] [1/4*(2*(a*b^5*c^2 - 11*a^2*b^3*c^3 + 28*a^3*b*c^4)*e^6*x^6 + 12*(a*b^5*c^2
 - 11*a^2*b^3*c^3 + 28*a^3*b*c^4)*d*e^5*x^5 + (4*a*b^6*c - 45*a^2*b^4*c^2 +
 132*a^3*b^2*c^3 - 64*a^4*c^4 + 30*(a*b^5*c^2 - 11*a^2*b^3*c^3 + 28*a^3*b*c
^4)*d^2)*e^4*x^4 + 3*a^2*b^6 - 33*a^3*b^4*c + 108*a^4*b^2*c^2 - 96*a^5*c^3
+ 2*(a*b^5*c^2 - 11*a^2*b^3*c^3 + 28*a^3*b*c^4)*d^6 + 4*(10*(a*b^5*c^2 - 11
*a^2*b^3*c^3 + 28*a^3*b*c^4)*d^3 + (4*a*b^6*c - 45*a^2*b^4*c^2 + 132*a^3*b^
2*c^3 - 64*a^4*c^4)*d)*e^3*x^3 + (4*a*b^6*c - 45*a^2*b^4*c^2 + 132*a^3*b^2*
c^3 - 64*a^4*c^4)*d^4 + 2*(a*b^7 - 10*a^2*b^5*c + 23*a^3*b^3*c^2 + 4*a^4*b*
c^3 + 15*(a*b^5*c^2 - 11*a^2*b^3*c^3 + 28*a^3*b*c^4)*d^4 + 3*(4*a*b^6*c - 4
5*a^2*b^4*c^2 + 132*a^3*b^2*c^3 - 64*a^4*c^4)*d^2)*e^2*x^2 + 2*(a*b^7 - 10*
a^2*b^5*c + 23*a^3*b^3*c^2 + 4*a^4*b*c^3)*d^2 + 4*(3*(a*b^5*c^2 - 11*a^2*b^
3*c^3 + 28*a^3*b*c^4)*d^5 + (4*a*b^6*c - 45*a^2*b^4*c^2 + 132*a^3*b^2*c^3 -
 64*a^4*c^4)*d^3 + (a*b^7 - 10*a^2*b^5*c + 23*a^3*b^3*c^2 + 4*a^4*b*c^3)*d)
*e*x + ((b^5*c^2 - 10*a*b^3*c^3 + 30*a^2*b*c^4)*e^8*x^8 + 8*(b^5*c^2 - 10*a
*b^3*c^3 + 30*a^2*b*c^4)*d*e^7*x^7 + 2*(b^6*c - 10*a*b^4*c^2 + 30*a^2*b^2*c
^3 + 14*(b^5*c^2 - 10*a*b^3*c^3 + 30*a^2*b*c^4)*d^2)*e^6*x^6 + 4*(14*(b^5*c
^2 - 10*a*b^3*c^3 + 30*a^2*b*c^4)*d^3 + 3*(b^6*c - 10*a*b^4*c^2 + 30*a^2*b^
2*c^3)*d)*e^5*x^5 + (b^5*c^2 - 10*a*b^3*c^3 + 30*a^2*b*c^4)*d^8 + (b^7 - 8*
a*b^5*c + 10*a^2*b^3*c^2 + 60*a^3*b*c^3 + 70*(b^5*c^2 - 10*a*b^3*c^3 + 30*a
^2*b*c^4)*d^4 + 30*(b^6*c - 10*a*b^4*c^2 + 30*a^2*b^2*c^3)*d^2)*e^4*x^4 + a
^2*b^5 - 10*a^3*b^3*c + 30*a^4*b*c^2 + 2*(b^6*c - 10*a*b^4*c^2 + 30*a^2*b^2
*c^3)*d^6 + 4*(14*(b^5*c^2 - 10*a*b^3*c^3 + 30*a^2*b*c^4)*d^5 + 10*(b^6*c -
 10*a*b^4*c^2 + 30*a^2*b^2*c^3)*d^3 + (b^7 - 8*a*b^5*c + 10*a^2*b^3*c^2 + 6
0*a^3*b*c^3)*d)*e^3*x^3 + (b^7 - 8*a*b^5*c + 10*a^2*b^3*c^2 + 60*a^3*b*c^3)
*d^4 + 2*(a*b^6 - 10*a^2*b^4*c + 30*a^3*b^2*c^2 + 14*(b^5*c^2 - 10*a*b^3*c^
3 + 30*a^2*b*c^4)*d^6 + 15*(b^6*c - 10*a*b^4*c^2 + 30*a^2*b^2*c^3)*d^4 + 3*
(b^7 - 8*a*b^5*c + 10*a^2*b^3*c^2 + 60*a^3*b*c^3)*d^2)*e^2*x^2 + 2*(a*b^6 -
 10*a^2*b^4*c + 30*a^3*b^2*c^2)*d^2 + 4*(2*(b^5*c^2 - 10*a*b^3*c^3 + 30*a^2
*b*c^4)*d^7 + 3*(b^6*c - 10*a*b^4*c^2 + 30*a^2*b^2*c^3)*d^5 + (b^7 - 8*a*b^
5*c + 10*a^2*b^3*c^2 + 60*a^3*b*c^3)*d^3 + (a*b^6 - 10*a^2*b^4*c + 30*a^3*b
^2*c^2)*d)*e*x)*sqrt(b^2 - 4*a*c)*log((2*c^2*e^4*x^4 + 8*c^2*d*e^3*x^3 + 2*
c^2*d^4 + 2*(6*c^2*d^2 + b*c)*e^2*x^2 + 2*b*c*d^2 + 4*(2*c^2*d^3 + b*c*d)*e
*x + b^2 - 2*a*c + (2*c*e^2*x^2 + 4*c*d*e*x + 2*c*d^2 + b)*sqrt(b^2 - 4*a*c
))/(c*e^4*x^4 + 4*c*d*e^3*x^3 + c*d^4 + (6*c*d^2 + b)*e^2*x^2 + b*d^2 + 2*(
2*c*d^3 + b*d)*e*x + a)) - ((b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a
^3*c^5)*e^8*x^8 + 8*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*
d*e^7*x^7 + 2*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4 + 14*(b
^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^2)*e^6*x^6 + 4*(14*(
b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^3 + 3*(b^7*c - 12*a
```

```
*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*d)*e^5*x^5 + (b^6*c^2 - 12*a*b^4*
c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^8 + (b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2
 + 32*a^3*b^2*c^3 - 128*a^4*c^4 + 70*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c
^4 - 64*a^3*c^5)*d^4 + 30*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b
*c^4)*d^2)*e^4*x^4 + a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3 +
 2*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*d^6 + 4*(14*(b^6*
c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^5 + 10*(b^7*c - 12*a*b^
5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*d^3 + (b^8 - 10*a*b^6*c + 24*a^2*b^4
*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*d)*e^3*x^3 + (b^8 - 10*a*b^6*c + 24*a^
2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*d^4 + 2*(a*b^7 - 12*a^2*b^5*c + 4
8*a^3*b^3*c^2 - 64*a^4*b*c^3 + 14*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4
- 64*a^3*c^5)*d^6 + 15*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^
4)*d^4 + 3*(b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^
4)*d^2)*e^2*x^2 + 2*(a*b^7 - 12*a^2*b^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3)*
d^2 + 4*(2*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^7 + 3*(
b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*d^5 + (b^8 - 10*a*b^6
*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*d^3 + (a*b^7 - 12*a^2*b
^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3)*d)*e*x)*log(c*e^4*x^4 + 4*c*d*e^3*x^3
 + c*d^4 + (6*c*d^2 + b)*e^2*x^2 + b*d^2 + 2*(2*c*d^3 + b*d)*e*x + a) + 4*(
(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*e^8*x^8 + 8*(b^6*c^2
 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d*e^7*x^7 + 2*(b^7*c - 12*a*
b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4 + 14*(b^6*c^2 - 12*a*b^4*c^3 + 48*a
^2*b^2*c^4 - 64*a^3*c^5)*d^2)*e^6*x^6 + 4*(14*(b^6*c^2 - 12*a*b^4*c^3 + 48*
a^2*b^2*c^4 - 64*a^3*c^5)*d^3 + 3*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 -
64*a^3*b*c^4)*d)*e^5*x^5 + (b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^
3*c^5)*d^8 + (b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*
c^4 + 70*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^4 + 30*(b
^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*d^2)*e^4*x^4 + a^2*b^6
 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3 + 2*(b^7*c - 12*a*b^5*c^2 + 4
8*a^2*b^3*c^3 - 64*a^3*b*c^4)*d^6 + 4*(14*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*
b^2*c^4 - 64*a^3*c^5)*d^5 + 10*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*
a^3*b*c^4)*d^3 + (b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*
a^4*c^4)*d)*e^3*x^3 + (b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 -
 128*a^4*c^4)*d^4 + 2*(a*b^7 - 12*a^2*b^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3
 + 14*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^6 + 15*(b^7*
c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*d^4 + 3*(b^8 - 10*a*b^6*c
 + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*d^2)*e^2*x^2 + 2*(a*b^7 -
 12*a^2*b^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3)*d^2 + 4*(2*(b^6*c^2 - 12*a*b
^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^7 + 3*(b^7*c - 12*a*b^5*c^2 + 48*a^
2*b^3*c^3 - 64*a^3*b*c^4)*d^5 + (b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3
*b^2*c^3 - 128*a^4*c^4)*d^3 + (a*b^7 - 12*a^2*b^5*c + 48*a^3*b^3*c^2 - 64*a
^4*b*c^3)*d)*e*x)*log(e*x + d))/((a^3*b^6*c^2 - 12*a^4*b^4*c^3 + 48*a^5*b^2
*c^4 - 64*a^6*c^5)*e^9*f*x^8 + 8*(a^3*b^6*c^2 - 12*a^4*b^4*c^3 + 48*a^5*b^2
*c^4 - 64*a^6*c^5)*d*e^8*f*x^7 + 2*(a^3*b^7*c - 12*a^4*b^5*c^2 + 48*a^5*b^3
*c^3 - 64*a^6*b*c^4 + 14*(a^3*b^6*c^2 - 12*a^4*b^4*c^3 + 48*a^5*b^2*c^4 - 6
4*a^6*c^5)*d^2)*e^7*f*x^6 + 4*(14*(a^3*b^6*c^2 - 12*a^4*b^4*c^3 + 48*a^5*b^
2*c^4 - 64*a^6*c^5)*d^3 + 3*(a^3*b^7*c - 12*a^4*b^5*c^2 + 48*a^5*b^3*c^3 -
64*a^6*b*c^4)*d)*e^6*f*x^5 + (a^3*b^8 - 10*a^4*b^6*c + 24*a^5*b^4*c^2 + 32*
a^6*b^2*c^3 - 128*a^7*c^4 + 70*(a^3*b^6*c^2 - 12*a^4*b^4*c^3 + 48*a^5*b^2*c
^4 - 64*a^6*c^5)*d^4 + 30*(a^3*b^7*c - 12*a^4*b^5*c^2 + 48*a^5*b^3*c^3 - 64
*a^6*b*c^4)*d^2)*e^5*f*x^4 + 4*(14*(a^3*b^6*c^2 - 12*a^4*b^4*c^3 + 48*a^5*b
^2*c^4 - 64*a^6*c^5)*d^5 + 10*(a^3*b^7*c - 12*a^4*b^5*c^2 + 48*a^5*b^3*c^3
- 64*a^6*b*c^4)*d^3 + (a^3*b^8 - 10*a^4*b^6*c + 24*a^5*b^4*c^2 + 32*a^6*b^2
*c^3 - 128*a^7*c^4)*d)*e^4*f*x^3 + 2*(a^4*b^7 - 12*a^5*b^5*c + 48*a^6*b^3*c
^2 - 64*a^7*b*c^3 + 14*(a^3*b^6*c^2 - 12*a^4*b^4*c^3 + 48*a^5*b^2*c^4 - 64*
a^6*c^5)*d^6 + 15*(a^3*b^7*c - 12*a^4*b^5*c^2 + 48*a^5*b^3*c^3 - 64*a^6*b*c
^4)*d^4 + 3*(a^3*b^8 - 10*a^4*b^6*c + 24*a^5*b^4*c^2 + 32*a^6*b^2*c^3 - 128
*a^7*c^4)*d^2)*e^3*f*x^2 + 4*(2*(a^3*b^6*c^2 - 12*a^4*b^4*c^3 + 48*a^5*b^2*
c^4 - 64*a^6*c^5)*d^7 + 3*(a^3*b^7*c - 12*a^4*b^5*c^2 + 48*a^5*b^3*c^3 - 64
```

*a^6*b*c^4)*d^5 + (a^3*b^8 - 10*a^4*b^6*c + 24*a^5*b^4*c^2 + 32*a^6*b^2*c^3 - 128*a^7*c^4)*d^3 + (a^4*b^7 - 12*a^5*b^5*c + 48*a^6*b^3*c^2 - 64*a^7*b*c ^3)*d)*e^2*f*x + (a^5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2*c^2 - 64*a^8*c^3 + (a ^3*b^6*c^2 - 12*a^4*b^4*c^3 + 48*a^5*b^2*c^4 - 64*a^6*c^5)*d^8 + 2*(a^3*b^7 *c - 12*a^4*b^5*c^2 + 48*a^5*b^3*c^3 - 64*a^6*b*c^4)*d^6 + (a^3*b^8 - 10*a^ 4*b^6*c + 24*a^5*b^4*c^2 + 32*a^6*b^2*c^3 - 128*a^7*c^4)*d^4 + 2*(a^4*b^7 - 12*a^5*b^5*c + 48*a^6*b^3*c^2 - 64*a^7*b*c^3)*d^2)*e*f), 1/4*(2*(a*b^5*c^2 - 11*a^2*b^3*c^3 + 28*a^3*b*c^4)*e^6*x^6 + 12*(a*b^5*c^2 - 11*a^2*b^3*c^3 + 28*a^3*b*c^4)*d*e^5*x^5 + (4*a*b^6*c - 45*a^2*b^4*c^2 + 132*a^3*b^2*c^3 - 64*a^4*c^4 + 30*(a*b^5*c^2 - 11*a^2*b^3*c^3 + 28*a^3*b*c^4)*d^2)*e^4*x^4 + 3*a^2*b^6 - 33*a^3*b^4*c + 108*a^4*b^2*c^2 - 96*a^5*c^3 + 2*(a*b^5*c^2 - 1 1*a^2*b^3*c^3 + 28*a^3*b*c^4)*d^6 + 4*(10*(a*b^5*c^2 - 11*a^2*b^3*c^3 + 28* a^3*b*c^4)*d^3 + (4*a*b^6*c - 45*a^2*b^4*c^2 + 132*a^3*b^2*c^3 - 64*a^4*c^4)*d)*e^3*x^3 + (4*a*b^6*c - 45*a^2*b^4*c^2 + 132*a^3*b^2*c^3 - 64*a^4*c^4)* d^4 + 2*(a*b^7 - 10*a^2*b^5*c + 23*a^3*b^3*c^2 + 4*a^4*b*c^3 + 15*(a*b^5*c^ 2 - 11*a^2*b^3*c^3 + 28*a^3*b*c^4)*d^4 + 3*(4*a*b^6*c - 45*a^2*b^4*c^2 + 13 2*a^3*b^2*c^3 - 64*a^4*c^4)*d^2)*e^2*x^2 + 2*(a*b^7 - 10*a^2*b^5*c + 23*a^3 *b^3*c^2 + 4*a^4*b*c^3)*d^2 + 4*(3*(a*b^5*c^2 - 11*a^2*b^3*c^3 + 28*a^3*b*c ^4)*d^5 + (4*a*b^6*c - 45*a^2*b^4*c^2 + 132*a^3*b^2*c^3 - 64*a^4*c^4)*d^3 + (a*b^7 - 10*a^2*b^5*c + 23*a^3*b^3*c^2 + 4*a^4*b*c^3)*d)*e*x + 2*((b^5*c^2 - 10*a*b^3*c^3 + 30*a^2*b*c^4)*e^8*x^8 + 8*(b^5*c^2 - 10*a*b^3*c^3 + 30*a^ 2*b*c^4)*d*e^7*x^7 + 2*(b^6*c - 10*a*b^4*c^2 + 30*a^2*b^2*c^3 + 14*(b^5*c^2 - 10*a*b^3*c^3 + 30*a^2*b*c^4)*d^2)*e^6*x^6 + 4*(14*(b^5*c^2 - 10*a*b^3*c^ 3 + 30*a^2*b*c^4)*d^3 + 3*(b^6*c - 10*a*b^4*c^2 + 30*a^2*b^2*c^3)*d)*e^5*x^ 5 + (b^5*c^2 - 10*a*b^3*c^3 + 30*a^2*b*c^4)*d^8 + (b^7 - 8*a*b^5*c + 10*a^2 *b^3*c^2 + 60*a^3*b*c^3 + 70*(b^5*c^2 - 10*a*b^3*c^3 + 30*a^2*b*c^4)*d^4 + 30*(b^6*c - 10*a*b^4*c^2 + 30*a^2*b^2*c^3)*d^2)*e^4*x^4 + a^2*b^5 - 10*a^3* b^3*c + 30*a^4*b*c^2 + 2*(b^6*c - 10*a*b^4*c^2 + 30*a^2*b^2*c^3)*d^6 + 4*(1 4*(b^5*c^2 - 10*a*b^3*c^3 + 30*a^2*b*c^4)*d^5 + 10*(b^6*c - 10*a*b^4*c^2 + 30*a^2*b^2*c^3)*d^3 + (b^7 - 8*a*b^5*c + 10*a^2*b^3*c^2 + 60*a^3*b*c^3)*d)* e^3*x^3 + (b^7 - 8*a*b^5*c + 10*a^2*b^3*c^2 + 60*a^3*b*c^3)*d^4 + 2*(a*b^6 - 10*a^2*b^4*c + 30*a^3*b^2*c^2 + 14*(b^5*c^2 - 10*a*b^3*c^3 + 30*a^2*b*c^4)*d^6 + 15*(b^6*c - 10*a*b^4*c^2 + 30*a^2*b^2*c^3)*d^4 + 3*(b^7 - 8*a*b^5*c + 10*a^2*b^3*c^2 + 60*a^3*b*c^3)*d^2)*e^2*x^2 + 2*(a*b^6 - 10*a^2*b^4*c + 30*a^3*b^2*c^2)*d^2 + 4*(2*(b^5*c^2 - 10*a*b^3*c^3 + 30*a^2*b*c^4)*d^7 + 3* (b^6*c - 10*a*b^4*c^2 + 30*a^2*b^2*c^3)*d^5 + (b^7 - 8*a*b^5*c + 10*a^2*b^3 *c^2 + 60*a^3*b*c^3)*d^3 + (a*b^6 - 10*a^2*b^4*c + 30*a^3*b^2*c^2)*d)*e*x)* sqrt(-b^2 + 4*a*c)*arctan(-(2*c*e^2*x^2 + 4*c*d*e*x + 2*c*d^2 + b)*sqrt(-b^ 2 + 4*a*c)/(b^2 - 4*a*c)) - ((b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64* a^3*c^5)*e^8*x^8 + 8*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5) *d*e^7*x^7 + 2*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4 + 14*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^2)*e^6*x^6 + 4*(14* (b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^3 + 3*(b^7*c - 12* a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*d)*e^5*x^5 + (b^6*c^2 - 12*a*b^4 *c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^8 + (b^8 - 10*a*b^6*c + 24*a^2*b^4*c^ 2 + 32*a^3*b^2*c^3 - 128*a^4*c^4 + 70*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2* c^4 - 64*a^3*c^5)*d^4 + 30*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3* b*c^4)*d^2)*e^4*x^4 + a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3 + 2*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*d^6 + 4*(14*(b^6 *c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^5 + 10*(b^7*c - 12*a*b ^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*d^3 + (b^8 - 10*a*b^6*c + 24*a^2*b^ 4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*d)*e^3*x^3 + (b^8 - 10*a*b^6*c + 24*a ^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*d^4 + 2*(a*b^7 - 12*a^2*b^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3 + 14*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^6 + 15*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c ^4)*d^4 + 3*(b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c ^4)*d^2)*e^2*x^2 + 2*(a*b^7 - 12*a^2*b^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3) *d^2 + 4*(2*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^7 + 3* (b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*d^5 + (b^8 - 10*a*b^

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6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*d^3 + (a*b^7 - 12*a^2*
b^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3)*d)*e*x)*log(c*e^4*x^4 + 4*c*d*e^3*x^
3 + c*d^4 + (6*c*d^2 + b)*e^2*x^2 + b*d^2 + 2*(2*c*d^3 + b*d)*e*x + a) + 4*
((b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*e^8*x^8 + 8*(b^6*c^
2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d*e^7*x^7 + 2*(b^7*c - 12*a
*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4 + 14*(b^6*c^2 - 12*a*b^4*c^3 + 48*
a^2*b^2*c^4 - 64*a^3*c^5)*d^2)*e^6*x^6 + 4*(14*(b^6*c^2 - 12*a*b^4*c^3 + 48
*a^2*b^2*c^4 - 64*a^3*c^5)*d^3 + 3*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 -
 64*a^3*b*c^4)*d)*e^5*x^5 + (b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a
^3*c^5)*d^8 + (b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4
*c^4 + 70*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^4 + 30*(
b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*d^2)*e^4*x^4 + a^2*b^
6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3 + 2*(b^7*c - 12*a*b^5*c^2 +
48*a^2*b^3*c^3 - 64*a^3*b*c^4)*d^6 + 4*(14*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2
*b^2*c^4 - 64*a^3*c^5)*d^5 + 10*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64
*a^3*b*c^4)*d^3 + (b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128
*a^4*c^4)*d)*e^3*x^3 + (b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3
- 128*a^4*c^4)*d^4 + 2*(a*b^7 - 12*a^2*b^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^
3 + 14*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^6 + 15*(b^7
*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*d^4 + 3*(b^8 - 10*a*b^6*
c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*d^2)*e^2*x^2 + 2*(a*b^7
- 12*a^2*b^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3)*d^2 + 4*(2*(b^6*c^2 - 12*a*
b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^7 + 3*(b^7*c - 12*a*b^5*c^2 + 48*a
^2*b^3*c^3 - 64*a^3*b*c^4)*d^5 + (b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^
3*b^2*c^3 - 128*a^4*c^4)*d^3 + (a*b^7 - 12*a^2*b^5*c + 48*a^3*b^3*c^2 - 64*
a^4*b*c^3)*d)*e*x)*log(e*x + d))/((a^3*b^6*c^2 - 12*a^4*b^4*c^3 + 48*a^5*b^
2*c^4 - 64*a^6*c^5)*e^9*f*x^8 + 8*(a^3*b^6*c^2 - 12*a^4*b^4*c^3 + 48*a^5*b^
2*c^4 - 64*a^6*c^5)*d*e^8*f*x^7 + 2*(a^3*b^7*c - 12*a^4*b^5*c^2 + 48*a^5*b^
3*c^3 - 64*a^6*b*c^4 + 14*(a^3*b^6*c^2 - 12*a^4*b^4*c^3 + 48*a^5*b^2*c^4 -
64*a^6*c^5)*d^2)*e^7*f*x^6 + 4*(14*(a^3*b^6*c^2 - 12*a^4*b^4*c^3 + 48*a^5*b
^2*c^4 - 64*a^6*c^5)*d^3 + 3*(a^3*b^7*c - 12*a^4*b^5*c^2 + 48*a^5*b^3*c^3 -
 64*a^6*b*c^4)*d)*e^6*f*x^5 + (a^3*b^8 - 10*a^4*b^6*c + 24*a^5*b^4*c^2 + 32
*a^6*b^2*c^3 - 128*a^7*c^4 + 70*(a^3*b^6*c^2 - 12*a^4*b^4*c^3 + 48*a^5*b^2*
c^4 - 64*a^6*c^5)*d^4 + 30*(a^3*b^7*c - 12*a^4*b^5*c^2 + 48*a^5*b^3*c^3 - 6
4*a^6*b*c^4)*d^2)*e^5*f*x^4 + 4*(14*(a^3*b^6*c^2 - 12*a^4*b^4*c^3 + 48*a^5*
b^2*c^4 - 64*a^6*c^5)*d^5 + 10*(a^3*b^7*c - 12*a^4*b^5*c^2 + 48*a^5*b^3*c^3
 - 64*a^6*b*c^4)*d^3 + (a^3*b^8 - 10*a^4*b^6*c + 24*a^5*b^4*c^2 + 32*a^6*b^
2*c^3 - 128*a^7*c^4)*d)*e^4*f*x^3 + 2*(a^4*b^7 - 12*a^5*b^5*c + 48*a^6*b^3*
c^2 - 64*a^7*b*c^3 + 14*(a^3*b^6*c^2 - 12*a^4*b^4*c^3 + 48*a^5*b^2*c^4 - 64
*a^6*c^5)*d^6 + 15*(a^3*b^7*c - 12*a^4*b^5*c^2 + 48*a^5*b^3*c^3 - 64*a^6*b*
c^4)*d^4 + 3*(a^3*b^8 - 10*a^4*b^6*c + 24*a^5*b^4*c^2 + 32*a^6*b^2*c^3 - 12
8*a^7*c^4)*d^2)*e^3*f*x^2 + 4*(2*(a^3*b^6*c^2 - 12*a^4*b^4*c^3 + 48*a^5*b^2
*c^4 - 64*a^6*c^5)*d^7 + 3*(a^3*b^7*c - 12*a^4*b^5*c^2 + 48*a^5*b^3*c^3 - 6
4*a^6*b*c^4)*d^5 + (a^3*b^8 - 10*a^4*b^6*c + 24*a^5*b^4*c^2 + 32*a^6*b^2*c^
3 - 128*a^7*c^4)*d^3 + (a^4*b^7 - 12*a^5*b^5*c + 48*a^6*b^3*c^2 - 64*a^7*b*
c^3)*d)*e^2*f*x + (a^5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2*c^2 - 64*a^8*c^3 + (
a^3*b^6*c^2 - 12*a^4*b^4*c^3 + 48*a^5*b^2*c^4 - 64*a^6*c^5)*d^8 + 2*(a^3*b^
7*c - 12*a^4*b^5*c^2 + 48*a^5*b^3*c^3 - 64*a^6*b*c^4)*d^6 + (a^3*b^8 - 10*a
^4*b^6*c + 24*a^5*b^4*c^2 + 32*a^6*b^2*c^3 - 128*a^7*c^4)*d^4 + 2*(a^4*b^7
- 12*a^5*b^5*c + 48*a^6*b^3*c^2 - 64*a^7*b*c^3)*d^2)*e*f)]
```

giac [B] time = 1.70, size = 1044, normalized size = 3.87

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*f*x+d*f)/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="giac")

[Out] -1/4*((a^3*b^7*c*f*e^3 - 14*a^4*b^5*c^2*f*e^3 + 70*a^5*b^3*c^3*f*e^3 - 120*
a^6*b*c^4*f*e^3)*sqrt(b^2 - 4*a*c)*log(abs(b*x^2*e^2 + 2*b*d*x*e + sqrt(b^2
```

```
 - 4*a*c)*x^2*e^2 + 2*sqrt(b^2 - 4*a*c)*d*x*e + b*d^2 + sqrt(b^2 - 4*a*c)*d
^2 + 2*a)) - (a^3*b^7*c*f*e^3 - 14*a^4*b^5*c^2*f*e^3 + 70*a^5*b^3*c^3*f*e^3
 - 120*a^6*b*c^4*f*e^3)*sqrt(b^2 - 4*a*c)*log(abs(-b*x^2*e^2 - 2*b*d*x*e +
sqrt(b^2 - 4*a*c)*x^2*e^2 + 2*sqrt(b^2 - 4*a*c)*d*x*e - b*d^2 + sqrt(b^2 -
4*a*c)*d^2 - 2*a)))/(a^6*b^8*c*f^2*e^4 - 16*a^7*b^6*c^2*f^2*e^4 + 96*a^8*b^
4*c^3*f^2*e^4 - 256*a^9*b^2*c^4*f^2*e^4 + 256*a^10*c^5*f^2*e^4) - 1/4*e^(-1
)*log(abs(c*x^4*e^4 + 4*c*d*x^3*e^3 + 6*c*d^2*x^2*e^2 + 4*c*d^3*x*e + c*d^4
 + b*x^2*e^2 + 2*b*d*x*e + b*d^2 + a))/(a^3*f) + e^(-1)*log(abs(x*e + d))/(
a^3*f) + 1/4*(2*a*b^3*c^2*d^6 - 14*a^2*b*c^3*d^6 + 4*a*b^4*c*d^4 - 29*a^2*b
^2*c^2*d^4 + 16*a^3*c^3*d^4 + 2*a*b^5*d^2 - 12*a^2*b^3*c*d^2 - 2*a^3*b*c^2*
d^2 + 2*(a*b^3*c^2*e^6 - 7*a^2*b*c^3*e^6)*x^6 + 3*a^2*b^4 - 21*a^3*b^2*c +
24*a^4*c^2 + 12*(a*b^3*c^2*d*e^5 - 7*a^2*b*c^3*d*e^5)*x^5 + (30*a*b^3*c^2*d
^2*e^4 - 210*a^2*b*c^3*d^2*e^4 + 4*a*b^4*c*e^4 - 29*a^2*b^2*c^2*e^4 + 16*a^
3*c^3*e^4)*x^4 + 4*(10*a*b^3*c^2*d^3*e^3 - 70*a^2*b*c^3*d^3*e^3 + 4*a*b^4*c
*d*e^3 - 29*a^2*b^2*c^2*d*e^3 + 16*a^3*c^3*d*e^3)*x^3 + 2*(15*a*b^3*c^2*d^4
*e^2 - 105*a^2*b*c^3*d^4*e^2 + 12*a*b^4*c*d^2*e^2 - 87*a^2*b^2*c^2*d^2*e^2
+ 48*a^3*c^3*d^2*e^2 + a*b^5*e^2 - 6*a^2*b^3*c*e^2 - a^3*b*c^2*e^2)*x^2 + 4
*(3*a*b^3*c^2*d^5*e - 21*a^2*b*c^3*d^5*e + 4*a*b^4*c*d^3*e - 29*a^2*b^2*c^2
*d^3*e + 16*a^3*c^3*d^3*e + a*b^5*d*e - 6*a^2*b^3*c*d*e - a^3*b*c^2*d*e)*x)
*e^(-1)/((c*x^4*e^4 + 4*c*d*x^3*e^3 + 6*c*d^2*x^2*e^2 + 4*c*d^3*x*e + c*d^4
 + b*x^2*e^2 + 2*b*d*x*e + b*d^2 + a)^2*(b^2 - 4*a*c)^2*a^3*f)
```

maple [C] time = 0.08, size = 4606, normalized size = 17.06

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*f*x+d*f)/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x)
```

```
[Out] -1/2/f/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4
+2*b*d*e*x+b*d^2+a)^2*e/(16*a^2*c^2-8*a*b^2*c+b^4)*x^2*b*c^2-1/f/(c*e^4*x^4
+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+
a)^2*d/(16*a^2*c^2-8*a*b^2*c+b^4)*x*b*c^2-1/2/f/(c*e^4*x^4+4*c*d*e^3*x^3+6*
c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2/e/(16*a^2*c^
2-8*a*b^2*c+b^4)*b*c^2*d^2+1/2/f/a^2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x
^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*e/(16*a^2*c^2-8*a*b^2*c
+b^4)*x^2*b^5+1/f/a^2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+
b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*d/(16*a^2*c^2-8*a*b^2*c+b^4)*x*b^5+1/2
/f/a^2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4
+2*b*d*e*x+b*d^2+a)^2/e/(16*a^2*c^2-8*a*b^2*c+b^4)*b^5*d^2+ln(e*x+d)/a^3/e/
f-7/2/f/a/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*
d^4+2*b*d*e*x+b*d^2+a)^2*c^3*e^5*b/(16*a^2*c^2-8*a*b^2*c+b^4)*x^6+1/2/f/a^2
/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d
*e*x+b*d^2+a)^2*c^2*e^5*b^3/(16*a^2*c^2-8*a*b^2*c+b^4)*x^6+6/f*a/(c*e^4*x^4
+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+
a)^2/e/(16*a^2*c^2-8*a*b^2*c+b^4)*c^2+3/4/f/a/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*
d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2/e/(16*a^2*c^2-
8*a*b^2*c+b^4)*b^4+4/f/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x
+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*e^3*c^3/(16*a^2*c^2-8*a*b^2*c+b^4)*x^
4+16/f/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4
+2*b*d*e*x+b*d^2+a)^2*d^3/(16*a^2*c^2-8*a*b^2*c+b^4)*x*c^3+4/f/(c*e^4*x^4+4
*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)
^2/e/(16*a^2*c^2-8*a*b^2*c+b^4)*c^3*d^4-21/4/f/(c*e^4*x^4+4*c*d*e^3*x^3+6*c
*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2/e/(16*a^2*c^2
-8*a*b^2*c+b^4)*b^2*c-1/2/f/a^3/(16*a^2*c^2-8*a*b^2*c+b^4)/e*sum((c*e^3*(16
*a^2*c^2-8*a*b^2*c+b^4)*_R^3+3*c*d*e^2*(16*a^2*c^2-8*a*b^2*c+b^4)*_R^2+e*(4
8*a^2*c^3*d^2-24*a*b^2*c^2*d^2+3*b^4*c*d^2+23*a^2*b*c^2-9*a*b^3*c+b^5)*_R+1
6*a^2*c^3*d^3-8*a*b^2*c^2*d^3+b^4*c*d^3+23*a^2*b*c^2*d-9*a*b^3*c*d+b^5*d)/(
2*_R^3*c*e^3+6*_R^2*c*d*e^2+6*_R*c*d^2*e+2*c*d^3+_R*b*e+b*d)*ln(-_R+x),_R=R
ootOf(_Z^4*c*e^4+4*_Z^3*c*d*e^3+c*d^4+b*d^2+(6*c*d^2*e^2+b*e^2)*_Z^2+(4*c*d
```

```
^3*e+2*b*d*e)*_Z+a))+10/f/a^2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*
d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*c^2*d^3*e^2/(16*a^2*c^2-8*a*b^
2*c+b^4)*x^3*b^3-29/f/a/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*
x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*c^2*d*e^2/(16*a^2*c^2-8*a*b^2*c+b^4)
*x^3*b^2+4/f/a^2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2
*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*c*d*e^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x^3*b^4-
105/2/f/a/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*
d^4+2*b*d*e*x+b*d^2+a)^2*e/(16*a^2*c^2-8*a*b^2*c+b^4)*x^2*b*c^3*d^4+15/2/f/
a^2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*
b*d*e*x+b*d^2+a)^2*e/(16*a^2*c^2-8*a*b^2*c+b^4)*x^2*b^3*c^2*d^4-87/2/f/a/(c
*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*
x+b*d^2+a)^2*e/(16*a^2*c^2-8*a*b^2*c+b^4)*x^2*b^2*c^2*d^2+6/f/a^2/(c*e^4*x^
4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2
+a)^2*e/(16*a^2*c^2-8*a*b^2*c+b^4)*x^2*b^4*c*d^2-21/f/a/(c*e^4*x^4+4*c*d*e^
3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*b*c^
3*d*e^4/(16*a^2*c^2-8*a*b^2*c+b^4)*x^5+3/f/a^2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c
*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*b^3*c^2*d*e^4
/(16*a^2*c^2-8*a*b^2*c+b^4)*x^5-105/2/f/a/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*
e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*e^3*c^3/(16*a^2*c^
2-8*a*b^2*c+b^4)*x^4*b*d^2+15/2/f/a^2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*
x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*e^3*c^2/(16*a^2*c^2-8*
a*b^2*c+b^4)*x^4*b^3*d^2-70/f/a/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*
c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*c^3*d^3*e^2/(16*a^2*c^2-8*a*
b^2*c+b^4)*x^3*b+16/f/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+
b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*c^3*d*e^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x
^3+24/f/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^
4+2*b*d*e*x+b*d^2+a)^2*e/(16*a^2*c^2-8*a*b^2*c+b^4)*x^2*c^3*d^2+1/f/a^2/(c*
e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x
+b*d^2+a)^2*e^3*c/(16*a^2*c^2-8*a*b^2*c+b^4)*x^4*b^4-3/f/a/(c*e^4*x^4+4*c*d
*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*e
/(16*a^2*c^2-8*a*b^2*c+b^4)*x^2*b^3*c-21/f/a/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d
^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*d^5/(16*a^2*c^2
-8*a*b^2*c+b^4)*x*b*c^3+3/f/a^2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*
c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*d^5/(16*a^2*c^2-8*a*b^2*c+b^
4)*x*b^3*c^2-29/f/a/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*
e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*d^3/(16*a^2*c^2-8*a*b^2*c+b^4)*x*b^2*c^2
+4/f/a^2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d
^4+2*b*d*e*x+b*d^2+a)^2*d^3/(16*a^2*c^2-8*a*b^2*c+b^4)*x*b^4*c-6/f/a/(c*e^4
*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*
d^2+a)^2*d/(16*a^2*c^2-8*a*b^2*c+b^4)*x*b^3*c-7/2/f/a/(c*e^4*x^4+4*c*d*e^3*
x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2/e/(16*
a^2*c^2-8*a*b^2*c+b^4)*b*c^3*d^6+1/2/f/a^2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2
*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2/e/(16*a^2*c^2-8*a
*b^2*c+b^4)*b^3*c^2*d^6-29/4/f/a/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4
*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2/e/(16*a^2*c^2-8*a*b^2*c+b^4
)*b^2*c^2*d^4+1/f/a^2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+
b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2/e/(16*a^2*c^2-8*a*b^2*c+b^4)*b^4*c*d^4
-3/f/a/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4
+2*b*d*e*x+b*d^2+a)^2/e/(16*a^2*c^2-8*a*b^2*c+b^4)*b^3*c*d^2-29/4/f/a/(c*e^
4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b
*d^2+a)^2*e^3*c^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x^4*b^2
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*f*x+d*f)/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="maxima"
)
```

[Out] Timed out

mupad [B] time = 18.49, size = 22621, normalized size = 83.78

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((d*f + e*f*x)*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3),x)
```

```
[Out] ((x^2*(b^5*e + 48*a^2*c^3*d^2*e + 15*b^3*c^2*d^4*e - 6*a*b^3*c*e - a^2*b*c^
2*e + 12*b^4*c*d^2*e - 105*a*b*c^3*d^4*e - 87*a*b^2*c^2*d^2*e))/(2*(a^2*b^4
 + 16*a^4*c^2 - 8*a^3*b^2*c)) + (x^4*(4*b^4*c*e^3 + 16*a^2*c^3*e^3 - 29*a*b
^2*c^2*e^3 + 30*b^3*c^2*d^2*e^3 - 210*a*b*c^3*d^2*e^3))/(4*(a^2*b^4 + 16*a^
4*c^2 - 8*a^3*b^2*c)) + (x^3*(16*a^2*c^3*d*e^2 + 10*b^3*c^2*d^3*e^2 + 4*b^4
*c*d*e^2 - 29*a*b^2*c^2*d*e^2 - 70*a*b*c^3*d^3*e^2))/(a^2*b^4 + 16*a^4*c^2
- 8*a^3*b^2*c) + (3*x^5*(b^3*c^2*d*e^4 - 7*a*b*c^3*d*e^4))/(a^2*b^4 + 16*a^
4*c^2 - 8*a^3*b^2*c) + (x^6*(b^3*c^2*e^5 - 7*a*b*c^3*e^5))/(2*(a^2*b^4 + 16
*a^4*c^2 - 8*a^3*b^2*c)) + (x*(b^5*d + 4*b^4*c*d^3 + 16*a^2*c^3*d^3 + 3*b^3
*c^2*d^5 - 29*a*b^2*c^2*d^3 - 6*a*b^3*c*d - a^2*b*c^2*d - 21*a*b*c^3*d^5))/
(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c) + (3*a*b^4 + 24*a^3*c^2 + 2*b^5*d^2 -
21*a^2*b^2*c + 4*b^4*c*d^4 + 16*a^2*c^3*d^4 + 2*b^3*c^2*d^6 - 2*a^2*b*c^2*d
^2 - 29*a*b^2*c^2*d^4 - 12*a*b^3*c*d^2 - 14*a*b*c^3*d^6)/(4*e*(a^2*b^4 + 16
*a^4*c^2 - 8*a^3*b^2*c)))/(x^3*(56*c^2*d^5*e^3*f + 4*b^2*d*e^3*f + 40*b*c*d
^3*e^3*f + 8*a*c*d*e^3*f) + x^2*(6*b^2*d^2*e^2*f + 28*c^2*d^6*e^2*f + 2*a*b
*e^2*f + 12*a*c*d^2*e^2*f + 30*b*c*d^4*e^2*f) + x*(4*b^2*d^3*e*f + 8*c^2*d^
7*e*f + 4*a*b*d*e*f + 8*a*c*d^3*e*f + 12*b*c*d^5*e*f) + x^4*(b^2*e^4*f + 70
*c^2*d^4*e^4*f + 2*a*c*e^4*f + 30*b*c*d^2*e^4*f) + x^5*(56*c^2*d^3*e^5*f +
12*b*c*d*e^5*f) + a^2*f + x^6*(28*c^2*d^2*e^6*f + 2*b*c*e^6*f) + b^2*d^4*f
+ c^2*d^8*f + c^2*e^8*f*x^8 + 2*a*b*d^2*f + 2*a*c*d^4*f + 2*b*c*d^6*f + 8*c
^2*d*e^7*f*x^7) - (log((((a^3*e*f*(-(b^2*(b^4 + 30*a^2*c^2 - 10*a*b^2*c)^2)
/(a^6*e^2*f^2*(4*a*c - b^2)^5))^(1/2) + 1)*(((a^3*e*f*(-(b^2*(b^4 + 30*a^2*
c^2 - 10*a*b^2*c)^2)/(a^6*e^2*f^2*(4*a*c - b^2)^5))^(1/2) + 1)*((2*b*c^2*e^
16*(2*b^5 + 46*a^2*b*c^2 + b^4*c*d^2 + 10*a^2*c^3*d^2 - 18*a*b^3*c - 2*a*b^
2*c^2*d^2))/(a^2*f*(4*a*c - b^2)^2) + (b*c^2*e^16*(a^3*e*f*(-(b^2*(b^4 + 30
*a^2*c^2 - 10*a*b^2*c)^2)/(a^6*e^2*f^2*(4*a*c - b^2)^5))^(1/2) + 1)*(a*b +
3*b^2*d^2 + 3*b^2*e^2*x^2 - 10*a*c*d^2 + 6*b^2*d*e*x - 10*a*c*e^2*x^2 - 20*
a*c*d*e*x))/(a^3*f) + (2*b*c^3*e^18*x^2*(b^4 + 10*a^2*c^2 - 2*a*b^2*c))/(a^
2*f*(4*a*c - b^2)^2) + (4*b*c^3*d*e^17*x*(b^4 + 10*a^2*c^2 - 2*a*b^2*c))/(a
^2*f*(4*a*c - b^2)^2)))/(4*a^3*e*f) + (b*c^3*e^15*(7*a*c - b^2)*(4*b^5 + 71
*a^2*b*c^2 + 6*b^4*c*d^2 + 80*a^2*c^3*d^2 - 33*a*b^3*c - 47*a*b^2*c^2*d^2))
/(a^4*f^2*(4*a*c - b^2)^4) - (b*c^4*e^17*x^2*(6*b^6 - 560*a^3*c^3 + 409*a^2
*b^2*c^2 - 89*a*b^4*c))/(a^4*f^2*(4*a*c - b^2)^4) - (2*b*c^4*d*e^16*x*(6*b^
6 - 560*a^3*c^3 + 409*a^2*b^2*c^2 - 89*a*b^4*c))/(a^4*f^2*(4*a*c - b^2)^4))
)/(4*a^3*e*f) - (b^3*c^5*e^16*x^2*(7*a*c - b^2)^3)/(a^6*f^3*(4*a*c - b^2)^6
) + (b^2*c^4*e^14*(7*a*c - b^2)^2*(b^4 + 16*a^2*c^2 + b^3*c*d^2 - 8*a*b^2*c
 - 7*a*b*c^2*d^2))/(a^6*f^3*(4*a*c - b^2)^6) - (2*b^3*c^5*d*e^15*x*(7*a*c -
 b^2)^3)/(a^6*f^3*(4*a*c - b^2)^6))*(((a^3*e*f*(-(b^2*(b^4 + 30*a^2*c^2 - 1
0*a*b^2*c)^2)/(a^6*e^2*f^2*(4*a*c - b^2)^5))^(1/2) - 1)*(((a^3*e*f*(-(b^2*(
b^4 + 30*a^2*c^2 - 10*a*b^2*c)^2)/(a^6*e^2*f^2*(4*a*c - b^2)^5))^(1/2) - 1)
*((2*b*c^2*e^16*(2*b^5 + 46*a^2*b*c^2 + b^4*c*d^2 + 10*a^2*c^3*d^2 - 18*a*b
^3*c - 2*a*b^2*c^2*d^2))/(a^2*f*(4*a*c - b^2)^2) - (b*c^2*e^16*(a^3*e*f*(-(
b^2*(b^4 + 30*a^2*c^2 - 10*a*b^2*c)^2)/(a^6*e^2*f^2*(4*a*c - b^2)^5))^(1/2)
 - 1)*(a*b + 3*b^2*d^2 + 3*b^2*e^2*x^2 - 10*a*c*d^2 + 6*b^2*d*e*x - 10*a*c*
e^2*x^2 - 20*a*c*d*e*x))/(a^3*f) + (2*b*c^3*e^18*x^2*(b^4 + 10*a^2*c^2 - 2*
a*b^2*c))/(a^2*f*(4*a*c - b^2)^2) + (4*b*c^3*d*e^17*x*(b^4 + 10*a^2*c^2 - 2
*a*b^2*c))/(a^2*f*(4*a*c - b^2)^2)))/(4*a^3*e*f) - (b*c^3*e^15*(7*a*c - b^2
)*(4*b^5 + 71*a^2*b*c^2 + 6*b^4*c*d^2 + 80*a^2*c^3*d^2 - 33*a*b^3*c - 47*a*
b^2*c^2*d^2))/(a^4*f^2*(4*a*c - b^2)^4) + (b*c^4*e^17*x^2*(6*b^6 - 560*a^3*
c^3 + 409*a^2*b^2*c^2 - 89*a*b^4*c))/(a^4*f^2*(4*a*c - b^2)^4) + (2*b*c^4*d
*e^16*x*(6*b^6 - 560*a^3*c^3 + 409*a^2*b^2*c^2 - 89*a*b^4*c))/(a^4*f^2*(4*a
```

*c - b^2)^4)))/(4*a^3*e*f) - (b^3*c^5*e^16*x^2*(7*a*c - b^2)^3)/(a^6*f^3*(4*a*c - b^2)^6) + (b^2*c^4*e^14*(7*a*c - b^2)^2*(b^4 + 16*a^2*c^2 + b^3*c*d^2 - 8*a*b^2*c - 7*a*b*c^2*d^2))/(a^6*f^3*(4*a*c - b^2)^6) - (2*b^3*c^5*d*e^15*x*(7*a*c - b^2)^3)/(a^6*f^3*(4*a*c - b^2)^6)))*(2*b^10*e*f - 2048*a^5*c^5*e*f + 320*a^2*b^6*c^2*e*f - 1280*a^3*b^4*c^3*e*f + 2560*a^4*b^2*c^4*e*f - 40*a*b^8*c*e*f))/(2*(4*a^3*b^10*e^2*f^2 - 4096*a^8*c^5*e^2*f^2 + 640*a^5*b^6*c^2*e^2*f^2 - 2560*a^6*b^4*c^3*e^2*f^2 + 5120*a^7*b^2*c^4*e^2*f^2 - 80*a^4*b^8*c*e^2*f^2)) + log(d + e*x)/(a^3*e*f) - (b*atan((x*((((((b*((2*(5120*a^10*b*c^9*d*e^17*f^2 + 2*a^4*b^13*c^3*d*e^17*f^2 - 36*a^5*b^11*c^4*d*e^17*f^2 + 276*a^6*b^9*c^5*d*e^17*f^2 - 1216*a^7*b^7*c^6*d*e^17*f^2 + 3456*a^8*b^5*c^7*d*e^17*f^2 - 6144*a^9*b^3*c^8*d*e^17*f^2))/(a^6*b^12*f^3 + 4096*a^12*c^6*f^3 - 24*a^7*b^10*c*f^3 + 240*a^8*b^8*c^2*f^3 - 1280*a^9*b^6*c^3*f^3 + 3840*a^10*b^4*c^4*f^3 - 6144*a^11*b^2*c^5*f^3) - ((2*b^10*e*f - 2048*a^5*c^5*e*f + 320*a^2*b^6*c^2*e*f - 1280*a^3*b^4*c^3*e*f + 2560*a^4*b^2*c^4*e*f - 40*a*b^8*c*e*f)*(163840*a^13*b*c^9*d*e^18*f^3 - 12*a^6*b^15*c^2*d*e^18*f^3 + 328*a^7*b^13*c^3*d*e^18*f^3 - 3840*a^8*b^11*c^4*d*e^18*f^3 + 24960*a^9*b^9*c^5*d*e^18*f^3 - 97280*a^10*b^7*c^6*d*e^18*f^3 + 227328*a^11*b^5*c^7*d*e^18*f^3 - 294912*a^12*b^3*c^8*d*e^18*f^3))/((4*a^3*b^10*e^2*f^2 - 4096*a^8*c^5*e^2*f^2 + 640*a^5*b^6*c^2*e^2*f^2 - 2560*a^6*b^4*c^3*e^2*f^2 + 5120*a^7*b^2*c^4*e^2*f^2 - 80*a^4*b^8*c*e^2*f^2)*(a^6*b^12*f^3 + 4096*a^12*c^6*f^3 - 24*a^7*b^10*c*f^3 + 240*a^8*b^8*c^2*f^3 - 1280*a^9*b^6*c^3*f^3 + 3840*a^10*b^4*c^4*f^3 - 6144*a^11*b^2*c^5*f^3)))*(b^4 + 30*a^2*c^2 - 10*a*b^2*c))/(4*a^3*e*f*(4*a*c - b^2)^(5/2)) - (b*(b^4 + 30*a^2*c^2 - 10*a*b^2*c)*(2*b^10*e*f - 2048*a^5*c^5*e*f + 320*a^2*b^6*c^2*e*f - 1280*a^3*b^4*c^3*e*f + 2560*a^4*b^2*c^4*e*f - 40*a*b^8*c*e*f)*(163840*a^13*b*c^9*d*e^18*f^3 - 12*a^6*b^15*c^2*d*e^18*f^3 + 328*a^7*b^13*c^3*d*e^18*f^3 - 3840*a^8*b^11*c^4*d*e^18*f^3 + 24960*a^9*b^9*c^5*d*e^18*f^3 - 97280*a^10*b^7*c^6*d*e^18*f^3 + 227328*a^11*b^5*c^7*d*e^18*f^3 - 294912*a^12*b^3*c^8*d*e^18*f^3))/(4*a^3*e*f*(4*a*c - b^2)^(5/2)*(4*a^3*b^10*e^2*f^2 - 4096*a^8*c^5*e^2*f^2 + 640*a^5*b^6*c^2*e^2*f^2 - 2560*a^6*b^4*c^3*e^2*f^2 + 5120*a^7*b^2*c^4*e^2*f^2 - 80*a^4*b^8*c*e^2*f^2)*(a^6*b^12*f^3 + 4096*a^12*c^6*f^3 - 24*a^7*b^10*c*f^3 + 240*a^8*b^8*c^2*f^3 - 1280*a^9*b^6*c^3*f^3 + 3840*a^10*b^4*c^4*f^3 - 6144*a^11*b^2*c^5*f^3)))*(2*b^10*e*f - 2048*a^5*c^5*e*f + 320*a^2*b^6*c^2*e*f - 1280*a^3*b^4*c^3*e*f + 2560*a^4*b^2*c^4*e*f - 40*a*b^8*c*e*f))/(2*(4*a^3*b^10*e^2*f^2 - 4096*a^8*c^5*e^2*f^2 + 640*a^5*b^6*c^2*e^2*f^2 - 2560*a^6*b^4*c^3*e^2*f^2 + 5120*a^7*b^2*c^4*e^2*f^2 - 80*a^4*b^8*c*e^2*f^2)) - (b*((2*(6*a^2*b^11*c^4*d*e^16*f - 137*a^3*b^9*c^5*d*e^16*f + 1217*a^4*b^7*c^6*d*e^16*f - 5256*a^5*b^5*c^7*d*e^16*f + 11024*a^6*b^3*c^8*d*e^16*f - 8960*a^7*b*c^9*d*e^16*f))/(a^6*b^12*f^3 + 4096*a^12*c^6*f^3 - 24*a^7*b^10*c*f^3 + 240*a^8*b^8*c^2*f^3 - 1280*a^9*b^6*c^3*f^3 + 3840*a^10*b^4*c^4*f^3 - 6144*a^11*b^2*c^5*f^3) - (((2*(5120*a^10*b*c^9*d*e^17*f^2 + 2*a^4*b^13*c^3*d*e^17*f^2 - 36*a^5*b^11*c^4*d*e^17*f^2 + 276*a^6*b^9*c^5*d*e^17*f^2 - 1216*a^7*b^7*c^6*d*e^17*f^2 + 3456*a^8*b^5*c^7*d*e^17*f^2 - 6144*a^9*b^3*c^8*d*e^17*f^2))/(a^6*b^12*f^3 + 4096*a^12*c^6*f^3 - 24*a^7*b^10*c*f^3 + 240*a^8*b^8*c^2*f^3 - 1280*a^9*b^6*c^3*f^3 + 3840*a^10*b^4*c^4*f^3 - 6144*a^11*b^2*c^5*f^3) - ((2*b^10*e*f - 2048*a^5*c^5*e*f + 320*a^2*b^6*c^2*e*f - 1280*a^3*b^4*c^3*e*f + 2560*a^4*b^2*c^4*e*f - 40*a*b^8*c*e*f)*(163840*a^13*b*c^9*d*e^18*f^3 - 12*a^6*b^15*c^2*d*e^18*f^3 + 328*a^7*b^13*c^3*d*e^18*f^3 - 3840*a^8*b^11*c^4*d*e^18*f^3 + 24960*a^9*b^9*c^5*d*e^18*f^3 - 97280*a^10*b^7*c^6*d*e^18*f^3 + 227328*a^11*b^5*c^7*d*e^18*f^3 - 294912*a^12*b^3*c^8*d*e^18*f^3))/((4*a^3*b^10*e^2*f^2 - 4096*a^8*c^5*e^2*f^2 + 640*a^5*b^6*c^2*e^2*f^2 - 2560*a^6*b^4*c^3*e^2*f^2 + 5120*a^7*b^2*c^4*e^2*f^2 - 80*a^4*b^8*c*e^2*f^2)*(a^6*b^12*f^3 + 4096*a^12*c^6*f^3 - 24*a^7*b^10*c*f^3 + 240*a^8*b^8*c^2*f^3 - 1280*a^9*b^6*c^3*f^3 + 3840*a^10*b^4*c^4*f^3 - 6144*a^11*b^2*c^5*f^3)))*(2*b^10*e*f - 2048*a^5*c^5*e*f + 320*a^2*b^6*c^2*e*f - 1280*a^3*b^4*c^3*e*f + 2560*a^4*b^2*c^4*e*f - 40*a*b^8*c*e*f))/(2*(4*a^3*b^10*e^2*f^2 - 4096*a^8*c^5*e^2*f^2 + 640*a^5*b^6*c^2*e^2*f^2 - 2560*a^6*b^4*c^3*e^2*f^2 + 5120*a^7*b^2*c^4*e^2*f^2 - 80*a^4*b^8*c*e^2*f^2)))*(b^4 + 30*a^2*c^2 - 10*a*b^2*c))/(4*a^3*e*f*(4*a*c - b^2)^(5/2)) + (b^3*(b^4 + 30*a^2*c^2 - 10*a*b^2*c)^3*(163840*a^13* b*c^9*d*e^18*f^3 - 12*a^6*b^15*c^2*d*e^18*f^3 + 328*a^7*b^13*c^3*d*e^18*f^3 - 3840*a^8*b^11*c^4*d*e^18*f^3 + 24960*a^9*b^9*c^5*d*e^18*f^3 - 97280*a^10*b^7*c^6*d*e^18*f^3 + 227328*a^11*b^5*c^7*d*e^18*f^3 - 294912*a^12*b^3*c^8*d*e^18*f^3))/(32*a^9*e^3*f^3*(4*a*c - b^2)^(15/2)*(a^6*b^12*f^3 + 4096*a^12*c^6*f^3 - 24*a^7*b^10*c*f^3 + 240*a^8*b^8*c^2*f^3 - 1280*a^9*b^6*c^3*f^3 + 3840*a^10*b^4*c^4*f^3 - 6144*a^11*b^2*c^5*f^3)))*(3*b^8 + 160*a^4*c^4 + 180*a^2*b^4*c^2 - 325*a^3*b^2*c^3 - 39*a*b^6*c))/(8*a^3*c^2*(4*a*c - b^2)^(13/2)*(6*b^10 - 6400*a^5*c^5 + 960*a^2*b^6*c^2 - 3850*a^3*b^4*c^3 + 7775*a^4*b^2*c^4 - 120*a*b^8*c)) + (3*b*((2*(b^9*c^5*d*e^15 - 21*a*b^7*c^6*d*e^15 + 147*a^2*b^5*c^7*d*e^15 - 343*a^3*b^3*c^8*d*e^15))/(a^6*b^12*f^3 + 4096*a^12*c^6*f^3 - 24*a^7*b^10*c*f^3 + 240*a^8*b^8*c^2*f^3 - 1280*a^9*b^6*c^3*f^3 + 3840*a^10*b^4*c^4*f^3 - 6144*a^11*b^2*c^5*f^3) - (((2*(6*a^2*b^11*c^4*d*e^16*f - 137*a^3*b^9*c^5*d*e^16*f + 1217*a^4*b^7*c^6*d*e^16*f - 5256*a^5*b^5*c^7*d*e^16*f + 11024*a^6*b^3*c^8*d*e^16*f - 8960*a^7*b*c^9*d*e^16*f))/(a^6*b^12*f^3 + 4096*a^12*c^6*f^3 - 24*a^7*b^10*c*f^3 + 240*a^8*b^8*c^2*f^3 - 1280*a^9*b^6*c^3*f^3 + 3840*a^10*b^4*c^4*f^3 - 6144*a^11*b^2*c^5*f^3) - (((2*(5120*a^10*b*c^9*d*e^17*f^2 + 2*a^4*b^13*c^3*d*e^17*f^2 - 36*a^5*b^11*c^4*d*e^17*f^2 + 276*a^6*b^9*c^5*d*e^17*f^2 - 1216*a^7*b^7*c^6*d*e^17*f^2 + 3456*a^8*b^5*c^7*d*e^17*f^2 - 6144*a^9*b^3*c^8*d*e^17*f^2))/(a^6*b^12*f^3 + 4096*a^12*c^6*f^3 - 24*a^7*b^10*c*f^3 + 240*a^8*b^8*c^2*f^3 - 1280*a^9*b^6*c^3*f^3 + 3840*a^10*b^4*c^4*f^3 - 6144*a^11*b^2*c^5*f^3) - ((2*b^10*e*f - 2048*a^5*c^5*e*f + 320*a^2*b^6*c^2*e*f - 1280*a^3*b^4*c^3*e*f + 2560*a^4*b^2*c^4*e*f - 40*a*b^8*c*e*f)*(163840*a^13*b*c^9*d*e^18*f^3 - 12*a^6*b^15*c^2*d*e^18*f^3 + 328*a^7*b^13*c^3*d*e^18*f^3 - 3840*a^8*b^11*c^4*d*e^18*f^3 + 24960*a^9*b^9*c^5*d*e^18*f^3 - 97280*a^10*b^7*c^6*d*e^18*f^3 + 227328*a^11*b^5*c^7*d*e^18*f^3 - 294912*a^12*b^3*c^8*d*e^18*f^3))/((4*a^3*b^10*e^2*f^2 - 4096*a^8*c^5*e^2*f^2 + 640*a^5*b^6*c^2*e^2*f^2 - 2560*a^6*b^4*c^3*e^2*f^2 + 5120*a^7*b^2*c^4*e^2*f^2 - 80*a^4*b^8*c*e^2*f^2)*(a^6*b^12*f^3 + 4096*a^12*c^6*f^3 - 24*a^7*b^10*c*f^3 + 240*a^8*b^8*c^2*f^3 - 1280*a^9*b^6*c^3*f^3 + 3840*a^10*b^4*c^4*f^3 - 6144*a^11*b^2*c^5*f^3)))*(2*b^10*e*f - 2048*a^5*c^5*e*f + 320*a^2*b^6*c^2*e*f - 1280*a^3*b^4*c^3*e*f + 2560*a^4*b^2*c^4*e*f - 40*a*b^8*c*e*f))/(2*(4*a^3*b^10*e^2*f^2 - 4096*a^8*c^5*e^2*f^2 + 640*a^5*b^6*c^2*e^2*f^2 - 2560*a^6*b^4*c^3*e^2*f^2 + 5120*a^7*b^2*c^4*e^2*f^2 - 80*a^4*b^8*c*e^2*f^2)))*(2*b^10*e*f - 2048*a^5*c^5*e*f + 320*a^2*b^6*c^2*e*f - 1280*a^3*b^4*c^3*e*f + 2560*a^4*b^2*c^4*e*f - 40*a*b^8*c*e*f))/(2*(4*a^3*b^10*e^2*f^2 - 4096*a^8*c^5*e^2*f^2 + 640*a^5*b^6*c^2*e^2*f^2 - 2560*a^6*b^4*c^3*e^2*f^2 + 5120*a^7*b^2*c^4*e^2*f^2 - 80*a^4*b^8*c*e^2*f^2)) - (b*((b*((2*(5120*a^10*b*c^9*d*e^17*f^2 + 2*a^4*b^13*c^3*d*e^17*f^2 - 36*a^5*b^11*c^4*d*e^17*f^2 + 276*a^6*b^9*c^5*d*e^17*f^2 - 1216*a^7*b^7*c^6*d*e^17*f^2 + 3456*a^8*b^5*c^7*d*e^17*f^2 - 6144*a^9*b^3*c^8*d*e^17*f^2))/(a^6*b^12*f^3 + 4096*a^12*c^6*f^3 - 24*a^7*b^10*c*f^3 + 240*a^8*b^8*c^2*f^3 - 1280*a^9*b^6*c^3*f^3 + 3840*a^10*b^4*c^4*f^3 - 6144*a^11*b^2*c^5*f^3) - ((2*b^10*e*f - 2048*a^5*c^5*e*f + 320*a^2*b^6*c^2*e*f - 1280*a^3*b^4*c^3*e*f + 2560*a^4*b^2*c^4*e*f - 40*a*b^8*c*e*f)*(163840*a^13*b*c^9*d*e^18*f^3 - 12*a^6*b^15*c^2*d*e^18*f^3 + 328*a^7*b^13*c^3*d*e^18*f^3 - 3840*a^8*b^11*c^4*d*e^18*f^3 + 24960*a^9*b^9*c^5*d*e^18*f^3 - 97280*a^10*b^7*c^6*d*e^18*f^3 + 227328*a^11*b^5*c^7*d*e^18*f^3 - 294912*a^12*b^3*c^8*d*e^18*f^3))/((4*a^3*b^10*e^2*f^2 - 4096*a^8*c^5*e^2*f^2 + 640*a^5*b^6*c^2*e^2*f^2 - 2560*a^6*b^4*c^3*e^2*f^2 + 5120*a^7*b^2*c^4*e^2*f^2 - 80*a^4*b^8*c*e^2*f^2)*(a^6*b^12*f^3 + 4096*a^12*c^6*f^3 - 24*a^7*b^10*c*f^3 + 240*a^8*b^8*c^2*f^3 - 1280*a^9*b^6*c^3*f^3 + 3840*a^10*b^4*c^4*f^3 - 6144*a^11*b^2*c^5*f^3)))*(b^4 + 30*a^2*c^2 - 10*a*b^2*c))/(4*a^3*e*f*(4*a*c - b^2)^(5/2)) - (b*(b^4 + 30*a^2*c^2 - 10*a*b^2*c)*(2*b^10*e*f - 2048*a^5*c^5*e*f + 320*a^2*b^6*c^2*e*f - 1280*a^3*b^4*c^3*e*f + 2560*a^4*b^2*c^4*e*f - 40*a*b^8*c*e*f)*(163840*a^13*b*c^9*d*e^18*f^3 - 12*a^6*b^15*c^2*d*e^18*f^3 + 328*a^7*b^13*c^3*d*e^18*f^3 - 3840*a^8*b^11*c^4*d*e^18*f^3 + 24960*a^9*b^9*c^5*d*e^18*f^3 - 97280*a^10*b^7*c^6*d*e^18*f^3 + 227328*a^11*b^5*c^7*d*e^18*f^3 - 294912*a^12*b^3*c^8*d*e^18*f^3))/(4*a^3*e*f*(4*a*c - b^2)^(5/2)*(4*a^3*b^10*e^2*f^2 - 4096*a^8*c^5*e^2*f^2 + 640*a^5*b^6*c^2*e^2*f^2 - 2560*a^6*b^4*c^3*e^2*f^2 + 5120*a^7*b^2*c^4*e^2*f^2 - 8

0*a^4*b^8*c*e^2*f^2)*(a^6*b^12*f^3 + 4096*a^12*c^6*f^3 - 24*a^7*b^10*c*f^3 + 240*a^8*b^8*c^2*f^3 - 1280*a^9*b^6*c^3*f^3 + 3840*a^10*b^4*c^4*f^3 - 6144 *a^11*b^2*c^5*f^3)))*(b^4 + 30*a^2*c^2 - 10*a*b^2*c))/(4*a^3*e*f*(4*a*c - b ^2)^(5/2)) + (b^2*(b^4 + 30*a^2*c^2 - 10*a*b^2*c)^2*(2*b^10*e*f - 2048*a^5* c^5*e*f + 320*a^2*b^6*c^2*e*f - 1280*a^3*b^4*c^3*e*f + 2560*a^4*b^2*c^4*e*f - 40*a*b^8*c*e*f)*(163840*a^13*b*c^9*d*e^18*f^3 - 12*a^6*b^15*c^2*d*e^18*f ^3 + 328*a^7*b^13*c^3*d*e^18*f^3 - 3840*a^8*b^11*c^4*d*e^18*f^3 + 24960*a^9 *b^9*c^5*d*e^18*f^3 - 97280*a^10*b^7*c^6*d*e^18*f^3 + 227328*a^11*b^5*c^7*d *e^18*f^3 - 294912*a^12*b^3*c^8*d*e^18*f^3))/(16*a^6*e^2*f^2*(4*a*c - b^2)^ 5*(4*a^3*b^10*e^2*f^2 - 4096*a^8*c^5*e^2*f^2 + 640*a^5*b^6*c^2*e^2*f^2 - 25 60*a^6*b^4*c^3*e^2*f^2 + 5120*a^7*b^2*c^4*e^2*f^2 - 80*a^4*b^8*c*e^2*f^2)*(a^6*b^12*f^3 + 4096*a^12*c^6*f^3 - 24*a^7*b^10*c*f^3 + 240*a^8*b^8*c^2*f^3 - 1280*a^9*b^6*c^3*f^3 + 3840*a^10*b^4*c^4*f^3 - 6144*a^11*b^2*c^5*f^3)))*(b^6 - 45*a^3*c^3 + 40*a^2*b^2*c^2 - 11*a*b^4*c))/(8*a^3*c^2*(4*a*c - b^2)^6 *(6*b^10 - 6400*a^5*c^5 + 960*a^2*b^6*c^2 - 3850*a^3*b^4*c^3 + 7775*a^4*b^2 *c^4 - 120*a*b^8*c)))*(16*a^9*b^12*f^3*(4*a*c - b^2)^(15/2) + 65536*a^15*c^ 6*f^3*(4*a*c - b^2)^(15/2) - 384*a^10*b^10*c*f^3*(4*a*c - b^2)^(15/2) + 384 0*a^11*b^8*c^2*f^3*(4*a*c - b^2)^(15/2) - 20480*a^12*b^6*c^3*f^3*(4*a*c - b ^2)^(15/2) + 61440*a^13*b^4*c^4*f^3*(4*a*c - b^2)^(15/2) - 98304*a^14*b^2*c ^5*f^3*(4*a*c - b^2)^(15/2)))/(b^10*c^2*e^14 - 20*a*b^8*c^3*e^14 + 160*a^2* b^6*c^4*e^14 - 600*a^3*b^4*c^5*e^14 + 900*a^4*b^2*c^6*e^14) + (x^2*((((((b* ((2*a^4*b^13*c^3*e^18*f^2 - 36*a^5*b^11*c^4*e^18*f^2 + 276*a^6*b^9*c^5*e^18 *f^2 - 1216*a^7*b^7*c^6*e^18*f^2 + 3456*a^8*b^5*c^7*e^18*f^2 - 6144*a^9*b^3 *c^8*e^18*f^2 + 5120*a^10*b*c^9*e^18*f^2)/(a^6*b^12*f^3 + 4096*a^12*c^6*f^3 - 24*a^7*b^10*c*f^3 + 240*a^8*b^8*c^2*f^3 - 1280*a^9*b^6*c^3*f^3 + 3840*a^ 10*b^4*c^4*f^3 - 6144*a^11*b^2*c^5*f^3) + ((2*b^10*e*f - 2048*a^5*c^5*e*f + 320*a^2*b^6*c^2*e*f - 1280*a^3*b^4*c^3*e*f + 2560*a^4*b^2*c^4*e*f - 40*a*b ^8*c*e*f)*(12*a^6*b^15*c^2*e^19*f^3 - 328*a^7*b^13*c^3*e^19*f^3 + 3840*a^8* b^11*c^4*e^19*f^3 - 24960*a^9*b^9*c^5*e^19*f^3 + 97280*a^10*b^7*c^6*e^19*f^ 3 - 227328*a^11*b^5*c^7*e^19*f^3 + 294912*a^12*b^3*c^8*e^19*f^3 - 163840*a^ 13*b*c^9*e^19*f^3))/(2*(4*a^3*b^10*e^2*f^2 - 4096*a^8*c^5*e^2*f^2 + 640*a^5 *b^6*c^2*e^2*f^2 - 2560*a^6*b^4*c^3*e^2*f^2 + 5120*a^7*b^2*c^4*e^2*f^2 - 80 *a^4*b^8*c*e^2*f^2)*(a^6*b^12*f^3 + 4096*a^12*c^6*f^3 - 24*a^7*b^10*c*f^3 + 240*a^8*b^8*c^2*f^3 - 1280*a^9*b^6*c^3*f^3 + 3840*a^10*b^4*c^4*f^3 - 6144* a^11*b^2*c^5*f^3)))*(b^4 + 30*a^2*c^2 - 10*a*b^2*c))/(4*a^3*e*f*(4*a*c - b^ 2)^(5/2)) + (b*(b^4 + 30*a^2*c^2 - 10*a*b^2*c)*(2*b^10*e*f - 2048*a^5*c^5*e *f + 320*a^2*b^6*c^2*e*f - 1280*a^3*b^4*c^3*e*f + 2560*a^4*b^2*c^4*e*f - 40 *a*b^8*c*e*f)*(12*a^6*b^15*c^2*e^19*f^3 - 328*a^7*b^13*c^3*e^19*f^3 + 3840* a^8*b^11*c^4*e^19*f^3 - 24960*a^9*b^9*c^5*e^19*f^3 + 97280*a^10*b^7*c^6*e^1 9*f^3 - 227328*a^11*b^5*c^7*e^19*f^3 + 294912*a^12*b^3*c^8*e^19*f^3 - 16384 0*a^13*b*c^9*e^19*f^3))/(8*a^3*e*f*(4*a*c - b^2)^(5/2)*(4*a^3*b^10*e^2*f^2 - 4096*a^8*c^5*e^2*f^2 + 640*a^5*b^6*c^2*e^2*f^2 - 2560*a^6*b^4*c^3*e^2*f^2 + 5120*a^7*b^2*c^4*e^2*f^2 - 80*a^4*b^8*c*e^2*f^2)*(a^6*b^12*f^3 + 4096*a^ 12*c^6*f^3 - 24*a^7*b^10*c*f^3 + 240*a^8*b^8*c^2*f^3 - 1280*a^9*b^6*c^3*f^3 + 3840*a^10*b^4*c^4*f^3 - 6144*a^11*b^2*c^5*f^3)))*(2*b^10*e*f - 2048*a^5* c^5*e*f + 320*a^2*b^6*c^2*e*f - 1280*a^3*b^4*c^3*e*f + 2560*a^4*b^2*c^4*e*f - 40*a*b^8*c*e*f))/(2*(4*a^3*b^10*e^2*f^2 - 4096*a^8*c^5*e^2*f^2 + 640*a^5 *b^6*c^2*e^2*f^2 - 2560*a^6*b^4*c^3*e^2*f^2 + 5120*a^7*b^2*c^4*e^2*f^2 - 80 *a^4*b^8*c*e^2*f^2)) + (b*((8960*a^7*b*c^9*e^17*f - 6*a^2*b^11*c^4*e^17*f + 137*a^3*b^9*c^5*e^17*f - 1217*a^4*b^7*c^6*e^17*f + 5256*a^5*b^5*c^7*e^17*f - 11024*a^6*b^3*c^8*e^17*f)/(a^6*b^12*f^3 + 4096*a^12*c^6*f^3 - 24*a^7*b^1 0*c*f^3 + 240*a^8*b^8*c^2*f^3 - 1280*a^9*b^6*c^3*f^3 + 3840*a^10*b^4*c^4*f^ 3 - 6144*a^11*b^2*c^5*f^3) + (((2*a^4*b^13*c^3*e^18*f^2 - 36*a^5*b^11*c^4*e ^18*f^2 + 276*a^6*b^9*c^5*e^18*f^2 - 1216*a^7*b^7*c^6*e^18*f^2 + 3456*a^8*b ^5*c^7*e^18*f^2 - 6144*a^9*b^3*c^8*e^18*f^2 + 5120*a^10*b*c^9*e^18*f^2)/(a^ 6*b^12*f^3 + 4096*a^12*c^6*f^3 - 24*a^7*b^10*c*f^3 + 240*a^8*b^8*c^2*f^3 - 1280*a^9*b^6*c^3*f^3 + 3840*a^10*b^4*c^4*f^3 - 6144*a^11*b^2*c^5*f^3) + ((2 *b^10*e*f - 2048*a^5*c^5*e*f + 320*a^2*b^6*c^2*e*f - 1280*a^3*b^4*c^3*e*f + 2560*a^4*b^2*c^4*e*f - 40*a*b^8*c*e*f)*(12*a^6*b^15*c^2*e^19*f^3 - 328*a^7

*b^13*c^3*e^19*f^3 + 3840*a^8*b^11*c^4*e^19*f^3 - 24960*a^9*b^9*c^5*e^19*f^
3 + 97280*a^10*b^7*c^6*e^19*f^3 - 227328*a^11*b^5*c^7*e^19*f^3 + 294912*a^1
2*b^3*c^8*e^19*f^3 - 163840*a^13*b*c^9*e^19*f^3))/(2*(4*a^3*b^10*e^2*f^2 -
4096*a^8*c^5*e^2*f^2 + 640*a^5*b^6*c^2*e^2*f^2 - 2560*a^6*b^4*c^3*e^2*f^2 +
5120*a^7*b^2*c^4*e^2*f^2 - 80*a^4*b^8*c*e^2*f^2)*(a^6*b^12*f^3 + 4096*a^12
*c^6*f^3 - 24*a^7*b^10*c*f^3 + 240*a^8*b^8*c^2*f^3 - 1280*a^9*b^6*c^3*f^3 +
3840*a^10*b^4*c^4*f^3 - 6144*a^11*b^2*c^5*f^3)))*(2*b^10*e*f - 2048*a^5*c^
5*e*f + 320*a^2*b^6*c^2*e*f - 1280*a^3*b^4*c^3*e*f + 2560*a^4*b^2*c^4*e*f -
40*a*b^8*c*e*f))/(2*(4*a^3*b^10*e^2*f^2 - 4096*a^8*c^5*e^2*f^2 + 640*a^5*b
^6*c^2*e^2*f^2 - 2560*a^6*b^4*c^3*e^2*f^2 + 5120*a^7*b^2*c^4*e^2*f^2 - 80*a
^4*b^8*c*e^2*f^2)))*(b^4 + 30*a^2*c^2 - 10*a*b^2*c))/(4*a^3*e*f*(4*a*c - b^
2)^(5/2)) - (b^3*(b^4 + 30*a^2*c^2 - 10*a*b^2*c)^3*(12*a^6*b^15*c^2*e^19*f^
3 - 328*a^7*b^13*c^3*e^19*f^3 + 3840*a^8*b^11*c^4*e^19*f^3 - 24960*a^9*b^9*
c^5*e^19*f^3 + 97280*a^10*b^7*c^6*e^19*f^3 - 227328*a^11*b^5*c^7*e^19*f^3 +
294912*a^12*b^3*c^8*e^19*f^3 - 163840*a^13*b*c^9*e^19*f^3))/(64*a^9*e^3*f^
3*(4*a*c - b^2)^(15/2)*(a^6*b^12*f^3 + 4096*a^12*c^6*f^3 - 24*a^7*b^10*c*f^
3 + 240*a^8*b^8*c^2*f^3 - 1280*a^9*b^6*c^3*f^3 + 3840*a^10*b^4*c^4*f^3 - 61
44*a^11*b^2*c^5*f^3)))*(3*b^8 + 160*a^4*c^4 + 180*a^2*b^4*c^2 - 325*a^3*b^2
*c^3 - 39*a*b^6*c))/(8*a^3*c^2*(4*a*c - b^2)^(13/2)*(6*b^10 - 6400*a^5*c^5
+ 960*a^2*b^6*c^2 - 3850*a^3*b^4*c^3 + 7775*a^4*b^2*c^4 - 120*a*b^8*c)) + (
3*b*((b^9*c^5*e^16 - 21*a*b^7*c^6*e^16 + 147*a^2*b^5*c^7*e^16 - 343*a^3*b^3
*c^8*e^16)/(a^6*b^12*f^3 + 4096*a^12*c^6*f^3 - 24*a^7*b^10*c*f^3 + 240*a^8*
b^8*c^2*f^3 - 1280*a^9*b^6*c^3*f^3 + 3840*a^10*b^4*c^4*f^3 - 6144*a^11*b^2*
c^5*f^3) + (((8960*a^7*b*c^9*e^17*f - 6*a^2*b^11*c^4*e^17*f + 137*a^3*b^9*c
^5*e^17*f - 1217*a^4*b^7*c^6*e^17*f + 5256*a^5*b^5*c^7*e^17*f - 11024*a^6*b
^3*c^8*e^17*f)/(a^6*b^12*f^3 + 4096*a^12*c^6*f^3 - 24*a^7*b^10*c*f^3 + 240*
a^8*b^8*c^2*f^3 - 1280*a^9*b^6*c^3*f^3 + 3840*a^10*b^4*c^4*f^3 - 6144*a^11*
b^2*c^5*f^3) + (((2*a^4*b^13*c^3*e^18*f^2 - 36*a^5*b^11*c^4*e^18*f^2 + 276*
a^6*b^9*c^5*e^18*f^2 - 1216*a^7*b^7*c^6*e^18*f^2 + 3456*a^8*b^5*c^7*e^18*f^
2 - 6144*a^9*b^3*c^8*e^18*f^2 + 5120*a^10*b*c^9*e^18*f^2)/(a^6*b^12*f^3 + 4
096*a^12*c^6*f^3 - 24*a^7*b^10*c*f^3 + 240*a^8*b^8*c^2*f^3 - 1280*a^9*b^6*c
^3*f^3 + 3840*a^10*b^4*c^4*f^3 - 6144*a^11*b^2*c^5*f^3) + ((2*b^10*e*f - 20
48*a^5*c^5*e*f + 320*a^2*b^6*c^2*e*f - 1280*a^3*b^4*c^3*e*f + 2560*a^4*b^2*
c^4*e*f - 40*a*b^8*c*e*f)*(12*a^6*b^15*c^2*e^19*f^3 - 328*a^7*b^13*c^3*e^19
*f^3 + 3840*a^8*b^11*c^4*e^19*f^3 - 24960*a^9*b^9*c^5*e^19*f^3 + 97280*a^10
*b^7*c^6*e^19*f^3 - 227328*a^11*b^5*c^7*e^19*f^3 + 294912*a^12*b^3*c^8*e^19
*f^3 - 163840*a^13*b*c^9*e^19*f^3))/(2*(4*a^3*b^10*e^2*f^2 - 4096*a^8*c^5*e
^2*f^2 + 640*a^5*b^6*c^2*e^2*f^2 - 2560*a^6*b^4*c^3*e^2*f^2 + 5120*a^7*b^2*
c^4*e^2*f^2 - 80*a^4*b^8*c*e^2*f^2)*(a^6*b^12*f^3 + 4096*a^12*c^6*f^3 - 24*
a^7*b^10*c*f^3 + 240*a^8*b^8*c^2*f^3 - 1280*a^9*b^6*c^3*f^3 + 3840*a^10*b^4
*c^4*f^3 - 6144*a^11*b^2*c^5*f^3)))*(2*b^10*e*f - 2048*a^5*c^5*e*f + 320*a^
2*b^6*c^2*e*f - 1280*a^3*b^4*c^3*e*f + 2560*a^4*b^2*c^4*e*f - 40*a*b^8*c*e*
f))/(2*(4*a^3*b^10*e^2*f^2 - 4096*a^8*c^5*e^2*f^2 + 640*a^5*b^6*c^2*e^2*f^2
- 2560*a^6*b^4*c^3*e^2*f^2 + 5120*a^7*b^2*c^4*e^2*f^2 - 80*a^4*b^8*c*e^2*f
^2)))*(2*b^10*e*f - 2048*a^5*c^5*e*f + 320*a^2*b^6*c^2*e*f - 1280*a^3*b^4*c
^3*e*f + 2560*a^4*b^2*c^4*e*f - 40*a*b^8*c*e*f))/(2*(4*a^3*b^10*e^2*f^2 - 4
096*a^8*c^5*e^2*f^2 + 640*a^5*b^6*c^2*e^2*f^2 - 2560*a^6*b^4*c^3*e^2*f^2 +
5120*a^7*b^2*c^4*e^2*f^2 - 80*a^4*b^8*c*e^2*f^2)) - (b*((b*((2*a^4*b^13*c^3
*e^18*f^2 - 36*a^5*b^11*c^4*e^18*f^2 + 276*a^6*b^9*c^5*e^18*f^2 - 1216*a^7*
b^7*c^6*e^18*f^2 + 3456*a^8*b^5*c^7*e^18*f^2 - 6144*a^9*b^3*c^8*e^18*f^2 +
5120*a^10*b*c^9*e^18*f^2)/(a^6*b^12*f^3 + 4096*a^12*c^6*f^3 - 24*a^7*b^10*c
*f^3 + 240*a^8*b^8*c^2*f^3 - 1280*a^9*b^6*c^3*f^3 + 3840*a^10*b^4*c^4*f^3 -
6144*a^11*b^2*c^5*f^3) + ((2*b^10*e*f - 2048*a^5*c^5*e*f + 320*a^2*b^6*c^2
*e*f - 1280*a^3*b^4*c^3*e*f + 2560*a^4*b^2*c^4*e*f - 40*a*b^8*c*e*f)*(12*a^
6*b^15*c^2*e^19*f^3 - 328*a^7*b^13*c^3*e^19*f^3 + 3840*a^8*b^11*c^4*e^19*f^
3 - 24960*a^9*b^9*c^5*e^19*f^3 + 97280*a^10*b^7*c^6*e^19*f^3 - 227328*a^11*
b^5*c^7*e^19*f^3 + 294912*a^12*b^3*c^8*e^19*f^3 - 163840*a^13*b*c^9*e^19*f^
3))/(2*(4*a^3*b^10*e^2*f^2 - 4096*a^8*c^5*e^2*f^2 + 640*a^5*b^6*c^2*e^2*f^2
- 2560*a^6*b^4*c^3*e^2*f^2 + 5120*a^7*b^2*c^4*e^2*f^2 - 80*a^4*b^8*c*e^2*f

^2)*(a^6*b^12*f^3 + 4096*a^12*c^6*f^3 - 24*a^7*b^10*c*f^3 + 240*a^8*b^8*c^2
*f^3 - 1280*a^9*b^6*c^3*f^3 + 3840*a^10*b^4*c^4*f^3 - 6144*a^11*b^2*c^5*f^3
)))*(b^4 + 30*a^2*c^2 - 10*a*b^2*c))/(4*a^3*e*f*(4*a*c - b^2)^(5/2)) + (b*(
b^4 + 30*a^2*c^2 - 10*a*b^2*c)*(2*b^10*e*f - 2048*a^5*c^5*e*f + 320*a^2*b^6
*c^2*e*f - 1280*a^3*b^4*c^3*e*f + 2560*a^4*b^2*c^4*e*f - 40*a*b^8*c*e*f)*(1
2*a^6*b^15*c^2*e^19*f^3 - 328*a^7*b^13*c^3*e^19*f^3 + 3840*a^8*b^11*c^4*e^1
9*f^3 - 24960*a^9*b^9*c^5*e^19*f^3 + 97280*a^10*b^7*c^6*e^19*f^3 - 227328*a
^11*b^5*c^7*e^19*f^3 + 294912*a^12*b^3*c^8*e^19*f^3 - 163840*a^13*b*c^9*e^1
9*f^3))/(8*a^3*e*f*(4*a*c - b^2)^(5/2)*(4*a^3*b^10*e^2*f^2 - 4096*a^8*c^5*e
^2*f^2 + 640*a^5*b^6*c^2*e^2*f^2 - 2560*a^6*b^4*c^3*e^2*f^2 + 5120*a^7*b^2*
c^4*e^2*f^2 - 80*a^4*b^8*c*e^2*f^2)*(a^6*b^12*f^3 + 4096*a^12*c^6*f^3 - 24*
a^7*b^10*c*f^3 + 240*a^8*b^8*c^2*f^3 - 1280*a^9*b^6*c^3*f^3 + 3840*a^10*b^4
*c^4*f^3 - 6144*a^11*b^2*c^5*f^3)))*(b^4 + 30*a^2*c^2 - 10*a*b^2*c))/(4*a^3
*e*f*(4*a*c - b^2)^(5/2)) - (b^2*(b^4 + 30*a^2*c^2 - 10*a*b^2*c)^2*(2*b^10*
e*f - 2048*a^5*c^5*e*f + 320*a^2*b^6*c^2*e*f - 1280*a^3*b^4*c^3*e*f + 2560*
a^4*b^2*c^4*e*f - 40*a*b^8*c*e*f)*(12*a^6*b^15*c^2*e^19*f^3 - 328*a^7*b^13*
c^3*e^19*f^3 + 3840*a^8*b^11*c^4*e^19*f^3 - 24960*a^9*b^9*c^5*e^19*f^3 + 97
280*a^10*b^7*c^6*e^19*f^3 - 227328*a^11*b^5*c^7*e^19*f^3 + 294912*a^12*b^3*
c^8*e^19*f^3 - 163840*a^13*b*c^9*e^19*f^3))/(32*a^6*e^2*f^2*(4*a*c - b^2)^5
*(4*a^3*b^10*e^2*f^2 - 4096*a^8*c^5*e^2*f^2 + 640*a^5*b^6*c^2*e^2*f^2 - 256
0*a^6*b^4*c^3*e^2*f^2 + 5120*a^7*b^2*c^4*e^2*f^2 - 80*a^4*b^8*c*e^2*f^2)*(a
^6*b^12*f^3 + 4096*a^12*c^6*f^3 - 24*a^7*b^10*c*f^3 + 240*a^8*b^8*c^2*f^3 -
 1280*a^9*b^6*c^3*f^3 + 3840*a^10*b^4*c^4*f^3 - 6144*a^11*b^2*c^5*f^3)))*(b
^6 - 45*a^3*c^3 + 40*a^2*b^2*c^2 - 11*a*b^4*c))/(8*a^3*c^2*(4*a*c - b^2)^6*
(6*b^10 - 6400*a^5*c^5 + 960*a^2*b^6*c^2 - 3850*a^3*b^4*c^3 + 7775*a^4*b^2*
c^4 - 120*a*b^8*c)))*(16*a^9*b^12*f^3*(4*a*c - b^2)^(15/2) + 65536*a^15*c^6
f^3(4*a*c - b^2)^(15/2) - 384*a^10*b^10*c*f^3*(4*a*c - b^2)^(15/2) + 3840
*a^11*b^8*c^2*f^3*(4*a*c - b^2)^(15/2) - 20480*a^12*b^6*c^3*f^3*(4*a*c - b^
2)^(15/2) + 61440*a^13*b^4*c^4*f^3*(4*a*c - b^2)^(15/2) - 98304*a^14*b^2*c^
5*f^3*(4*a*c - b^2)^(15/2)))/(b^10*c^2*e^14 - 20*a*b^8*c^3*e^14 + 160*a^2*b
^6*c^4*e^14 - 600*a^3*b^4*c^5*e^14 + 900*a^4*b^2*c^6*e^14) - (((b*((4*a^2*b
^12*c^3*e^15*f - 93*a^3*b^10*c^4*e^15*f + 854*a^4*b^8*c^5*e^15*f - 3889*a^5
*b^6*c^6*e^15*f + 8808*a^6*b^4*c^7*e^15*f - 7952*a^7*b^2*c^8*e^15*f - 8960*
a^7*b*c^9*d^2*e^15*f + 6*a^2*b^11*c^4*d^2*e^15*f - 137*a^3*b^9*c^5*d^2*e^15
*f + 1217*a^4*b^7*c^6*d^2*e^15*f - 5256*a^5*b^5*c^7*d^2*e^15*f + 11024*a^6*
b^3*c^8*d^2*e^15*f)/(a^6*b^12*f^3 + 4096*a^12*c^6*f^3 - 24*a^7*b^10*c*f^3 +
 240*a^8*b^8*c^2*f^3 - 1280*a^9*b^6*c^3*f^3 + 3840*a^10*b^4*c^4*f^3 - 6144*
a^11*b^2*c^5*f^3) - (((4*a^4*b^14*c^2*e^16*f^2 - 100*a^5*b^12*c^3*e^16*f^2
 + 1052*a^6*b^10*c^4*e^16*f^2 - 5952*a^7*b^8*c^5*e^16*f^2 + 19072*a^8*b^6*c^
6*e^16*f^2 - 32768*a^9*b^4*c^7*e^16*f^2 + 23552*a^10*b^2*c^8*e^16*f^2 + 512
0*a^10*b*c^9*d^2*e^16*f^2 + 2*a^4*b^13*c^3*d^2*e^16*f^2 - 36*a^5*b^11*c^4*d
^2*e^16*f^2 + 276*a^6*b^9*c^5*d^2*e^16*f^2 - 1216*a^7*b^7*c^6*d^2*e^16*f^2
 + 3456*a^8*b^5*c^7*d^2*e^16*f^2 - 6144*a^9*b^3*c^8*d^2*e^16*f^2)/(a^6*b^12*
f^3 + 4096*a^12*c^6*f^3 - 24*a^7*b^10*c*f^3 + 240*a^8*b^8*c^2*f^3 - 1280*a^
9*b^6*c^3*f^3 + 3840*a^10*b^4*c^4*f^3 - 6144*a^11*b^2*c^5*f^3) + ((2*b^10*e
*f - 2048*a^5*c^5*e*f + 320*a^2*b^6*c^2*e*f - 1280*a^3*b^4*c^3*e*f + 2560*a
^4*b^2*c^4*e*f - 40*a*b^8*c*e*f)*(4*a^7*b^14*c^2*e^17*f^3 - 96*a^8*b^12*c^3
*e^17*f^3 + 960*a^9*b^10*c^4*e^17*f^3 - 5120*a^10*b^8*c^5*e^17*f^3 + 15360*
a^11*b^6*c^6*e^17*f^3 - 24576*a^12*b^4*c^7*e^17*f^3 + 16384*a^13*b^2*c^8*e^
17*f^3 - 163840*a^13*b*c^9*d^2*e^17*f^3 + 12*a^6*b^15*c^2*d^2*e^17*f^3 - 32
8*a^7*b^13*c^3*d^2*e^17*f^3 + 3840*a^8*b^11*c^4*d^2*e^17*f^3 - 24960*a^9*b^
9*c^5*d^2*e^17*f^3 + 97280*a^10*b^7*c^6*d^2*e^17*f^3 - 227328*a^11*b^5*c^7*
d^2*e^17*f^3 + 294912*a^12*b^3*c^8*d^2*e^17*f^3))/(2*(4*a^3*b^10*e^2*f^2 -
4096*a^8*c^5*e^2*f^2 + 640*a^5*b^6*c^2*e^2*f^2 - 2560*a^6*b^4*c^3*e^2*f^2 +
 5120*a^7*b^2*c^4*e^2*f^2 - 80*a^4*b^8*c*e^2*f^2)*(a^6*b^12*f^3 + 4096*a^12
*c^6*f^3 - 24*a^7*b^10*c*f^3 + 240*a^8*b^8*c^2*f^3 - 1280*a^9*b^6*c^3*f^3 +
 3840*a^10*b^4*c^4*f^3 - 6144*a^11*b^2*c^5*f^3)))*(2*b^10*e*f - 2048*a^5*c^
5*e*f + 320*a^2*b^6*c^2*e*f - 1280*a^3*b^4*c^3*e*f + 2560*a^4*b^2*c^4*e*f -
 40*a*b^8*c*e*f))/(2*(4*a^3*b^10*e^2*f^2 - 4096*a^8*c^5*e^2*f^2 + 640*a^5*b

^6*c^2*e^2*f^2 - 2560*a^6*b^4*c^3*e^2*f^2 + 5120*a^7*b^2*c^4*e^2*f^2 - 80*a
^4*b^8*c*e^2*f^2)))*(b^4 + 30*a^2*c^2 - 10*a*b^2*c))/(4*a^3*e*f*(4*a*c - b^
2)^(5/2)) - (((b*((4*a^4*b^14*c^2*e^16*f^2 - 100*a^5*b^12*c^3*e^16*f^2 + 10
52*a^6*b^10*c^4*e^16*f^2 - 5952*a^7*b^8*c^5*e^16*f^2 + 19072*a^8*b^6*c^6*e^
16*f^2 - 32768*a^9*b^4*c^7*e^16*f^2 + 23552*a^10*b^2*c^8*e^16*f^2 + 5120*a^
10*b*c^9*d^2*e^16*f^2 + 2*a^4*b^13*c^3*d^2*e^16*f^2 - 36*a^5*b^11*c^4*d^2*e
^16*f^2 + 276*a^6*b^9*c^5*d^2*e^16*f^2 - 1216*a^7*b^7*c^6*d^2*e^16*f^2 + 34
56*a^8*b^5*c^7*d^2*e^16*f^2 - 6144*a^9*b^3*c^8*d^2*e^16*f^2)/(a^6*b^12*f^3
+ 4096*a^12*c^6*f^3 - 24*a^7*b^10*c*f^3 + 240*a^8*b^8*c^2*f^3 - 1280*a^9*b^
6*c^3*f^3 + 3840*a^10*b^4*c^4*f^3 - 6144*a^11*b^2*c^5*f^3) + ((2*b^10*e*f -
2048*a^5*c^5*e*f + 320*a^2*b^6*c^2*e*f - 1280*a^3*b^4*c^3*e*f + 2560*a^4*b
^2*c^4*e*f - 40*a*b^8*c*e*f)*(4*a^7*b^14*c^2*e^17*f^3 - 96*a^8*b^12*c^3*e^1
7*f^3 + 960*a^9*b^10*c^4*e^17*f^3 - 5120*a^10*b^8*c^5*e^17*f^3 + 15360*a^11
*b^6*c^6*e^17*f^3 - 24576*a^12*b^4*c^7*e^17*f^3 + 16384*a^13*b^2*c^8*e^17*f
^3 - 163840*a^13*b*c^9*d^2*e^17*f^3 + 12*a^6*b^15*c^2*d^2*e^17*f^3 - 328*a^
7*b^13*c^3*d^2*e^17*f^3 + 3840*a^8*b^11*c^4*d^2*e^17*f^3 - 24960*a^9*b^9*c^
5*d^2*e^17*f^3 + 97280*a^10*b^7*c^6*d^2*e^17*f^3 - 227328*a^11*b^5*c^7*d^2*
e^17*f^3 + 294912*a^12*b^3*c^8*d^2*e^17*f^3))/(2*(4*a^3*b^10*e^2*f^2 - 4096
*a^8*c^5*e^2*f^2 + 640*a^5*b^6*c^2*e^2*f^2 - 2560*a^6*b^4*c^3*e^2*f^2 + 512
0*a^7*b^2*c^4*e^2*f^2 - 80*a^4*b^8*c*e^2*f^2)*(a^6*b^12*f^3 + 4096*a^12*c^6
*f^3 - 24*a^7*b^10*c*f^3 + 240*a^8*b^8*c^2*f^3 - 1280*a^9*b^6*c^3*f^3 + 384
0*a^10*b^4*c^4*f^3 - 6144*a^11*b^2*c^5*f^3)))*(b^4 + 30*a^2*c^2 - 10*a*b^2*
c))/(4*a^3*e*f*(4*a*c - b^2)^(5/2)) + (b*(b^4 + 30*a^2*c^2 - 10*a*b^2*c)*(2
*b^10*e*f - 2048*a^5*c^5*e*f + 320*a^2*b^6*c^2*e*f - 1280*a^3*b^4*c^3*e*f +
2560*a^4*b^2*c^4*e*f - 40*a*b^8*c*e*f)*(4*a^7*b^14*c^2*e^17*f^3 - 96*a^8*b
^12*c^3*e^17*f^3 + 960*a^9*b^10*c^4*e^17*f^3 - 5120*a^10*b^8*c^5*e^17*f^3 +
15360*a^11*b^6*c^6*e^17*f^3 - 24576*a^12*b^4*c^7*e^17*f^3 + 16384*a^13*b^2
*c^8*e^17*f^3 - 163840*a^13*b*c^9*d^2*e^17*f^3 + 12*a^6*b^15*c^2*d^2*e^17*f
^3 - 328*a^7*b^13*c^3*d^2*e^17*f^3 + 3840*a^8*b^11*c^4*d^2*e^17*f^3 - 24960
*a^9*b^9*c^5*d^2*e^17*f^3 + 97280*a^10*b^7*c^6*d^2*e^17*f^3 - 227328*a^11*b
^5*c^7*d^2*e^17*f^3 + 294912*a^12*b^3*c^8*d^2*e^17*f^3))/(8*a^3*e*f*(4*a*c
- b^2)^(5/2)*(4*a^3*b^10*e^2*f^2 - 4096*a^8*c^5*e^2*f^2 + 640*a^5*b^6*c^2*e
^2*f^2 - 2560*a^6*b^4*c^3*e^2*f^2 + 5120*a^7*b^2*c^4*e^2*f^2 - 80*a^4*b^8*c
*e^2*f^2)*(a^6*b^12*f^3 + 4096*a^12*c^6*f^3 - 24*a^7*b^10*c*f^3 + 240*a^8*b
^8*c^2*f^3 - 1280*a^9*b^6*c^3*f^3 + 3840*a^10*b^4*c^4*f^3 - 6144*a^11*b^2*c
^5*f^3)))*(2*b^10*e*f - 2048*a^5*c^5*e*f + 320*a^2*b^6*c^2*e*f - 1280*a^3*b
^4*c^3*e*f + 2560*a^4*b^2*c^4*e*f - 40*a*b^8*c*e*f))/(2*(4*a^3*b^10*e^2*f^2
- 4096*a^8*c^5*e^2*f^2 + 640*a^5*b^6*c^2*e^2*f^2 - 2560*a^6*b^4*c^3*e^2*f^
2 + 5120*a^7*b^2*c^4*e^2*f^2 - 80*a^4*b^8*c*e^2*f^2)) + (b^3*(b^4 + 30*a^2*
c^2 - 10*a*b^2*c)^3*(4*a^7*b^14*c^2*e^17*f^3 - 96*a^8*b^12*c^3*e^17*f^3 + 9
60*a^9*b^10*c^4*e^17*f^3 - 5120*a^10*b^8*c^5*e^17*f^3 + 15360*a^11*b^6*c^6*
e^17*f^3 - 24576*a^12*b^4*c^7*e^17*f^3 + 16384*a^13*b^2*c^8*e^17*f^3 - 1638
40*a^13*b*c^9*d^2*e^17*f^3 + 12*a^6*b^15*c^2*d^2*e^17*f^3 - 328*a^7*b^13*c^
3*d^2*e^17*f^3 + 3840*a^8*b^11*c^4*d^2*e^17*f^3 - 24960*a^9*b^9*c^5*d^2*e^1
7*f^3 + 97280*a^10*b^7*c^6*d^2*e^17*f^3 - 227328*a^11*b^5*c^7*d^2*e^17*f^3
+ 294912*a^12*b^3*c^8*d^2*e^17*f^3))/(64*a^9*e^3*f^3*(4*a*c - b^2)^(15/2)*(
a^6*b^12*f^3 + 4096*a^12*c^6*f^3 - 24*a^7*b^10*c*f^3 + 240*a^8*b^8*c^2*f^3
- 1280*a^9*b^6*c^3*f^3 + 3840*a^10*b^4*c^4*f^3 - 6144*a^11*b^2*c^5*f^3)))*(
3*b^8 + 160*a^4*c^4 + 180*a^2*b^4*c^2 - 325*a^3*b^2*c^3 - 39*a*b^6*c)*(16*a
^9*b^12*f^3*(4*a*c - b^2)^(15/2) + 65536*a^15*c^6*f^3*(4*a*c - b^2)^(15/2)
- 384*a^10*b^10*c*f^3*(4*a*c - b^2)^(15/2) + 3840*a^11*b^8*c^2*f^3*(4*a*c -
b^2)^(15/2) - 20480*a^12*b^6*c^3*f^3*(4*a*c - b^2)^(15/2) + 61440*a^13*b^4
*c^4*f^3*(4*a*c - b^2)^(15/2) - 98304*a^14*b^2*c^5*f^3*(4*a*c - b^2)^(15/2)
))/(8*a^3*c^2*(4*a*c - b^2)^(13/2)*(b^10*c^2*e^14 - 20*a*b^8*c^3*e^14 + 160
*a^2*b^6*c^4*e^14 - 600*a^3*b^4*c^5*e^14 + 900*a^4*b^2*c^6*e^14)*(6*b^10 -
6400*a^5*c^5 + 960*a^2*b^6*c^2 - 3850*a^3*b^4*c^3 + 7775*a^4*b^2*c^4 - 120*
a*b^8*c)) - (3*b*((((4*a^2*b^12*c^3*e^15*f - 93*a^3*b^10*c^4*e^15*f + 854*a
^4*b^8*c^5*e^15*f - 3889*a^5*b^6*c^6*e^15*f + 8808*a^6*b^4*c^7*e^15*f - 795
2*a^7*b^2*c^8*e^15*f - 8960*a^7*b*c^9*d^2*e^15*f + 6*a^2*b^11*c^4*d^2*e^15*

f - 137*a^3*b^9*c^5*d^2*e^15*f + 1217*a^4*b^7*c^6*d^2*e^15*f - 5256*a^5*b^5
*c^7*d^2*e^15*f + 11024*a^6*b^3*c^8*d^2*e^15*f)/(a^6*b^12*f^3 + 4096*a^12*c
^6*f^3 - 24*a^7*b^10*c*f^3 + 240*a^8*b^8*c^2*f^3 - 1280*a^9*b^6*c^3*f^3 + 3
840*a^10*b^4*c^4*f^3 - 6144*a^11*b^2*c^5*f^3) - (((4*a^4*b^14*c^2*e^16*f^2
- 100*a^5*b^12*c^3*e^16*f^2 + 1052*a^6*b^10*c^4*e^16*f^2 - 5952*a^7*b^8*c^5
*e^16*f^2 + 19072*a^8*b^6*c^6*e^16*f^2 - 32768*a^9*b^4*c^7*e^16*f^2 + 23552
*a^10*b^2*c^8*e^16*f^2 + 5120*a^10*b*c^9*d^2*e^16*f^2 + 2*a^4*b^13*c^3*d^2*
e^16*f^2 - 36*a^5*b^11*c^4*d^2*e^16*f^2 + 276*a^6*b^9*c^5*d^2*e^16*f^2 - 12
16*a^7*b^7*c^6*d^2*e^16*f^2 + 3456*a^8*b^5*c^7*d^2*e^16*f^2 - 6144*a^9*b^3*
c^8*d^2*e^16*f^2)/(a^6*b^12*f^3 + 4096*a^12*c^6*f^3 - 24*a^7*b^10*c*f^3 + 2
40*a^8*b^8*c^2*f^3 - 1280*a^9*b^6*c^3*f^3 + 3840*a^10*b^4*c^4*f^3 - 6144*a^
11*b^2*c^5*f^3) + ((2*b^10*e*f - 2048*a^5*c^5*e*f + 320*a^2*b^6*c^2*e*f - 1
280*a^3*b^4*c^3*e*f + 2560*a^4*b^2*c^4*e*f - 40*a*b^8*c*e*f)*(4*a^7*b^14*c^
2*e^17*f^3 - 96*a^8*b^12*c^3*e^17*f^3 + 960*a^9*b^10*c^4*e^17*f^3 - 5120*a^
10*b^8*c^5*e^17*f^3 + 15360*a^11*b^6*c^6*e^17*f^3 - 24576*a^12*b^4*c^7*e^17
*f^3 + 16384*a^13*b^2*c^8*e^17*f^3 - 163840*a^13*b*c^9*d^2*e^17*f^3 + 12*a^
6*b^15*c^2*d^2*e^17*f^3 - 328*a^7*b^13*c^3*d^2*e^17*f^3 + 3840*a^8*b^11*c^4
*d^2*e^17*f^3 - 24960*a^9*b^9*c^5*d^2*e^17*f^3 + 97280*a^10*b^7*c^6*d^2*e^1
7*f^3 - 227328*a^11*b^5*c^7*d^2*e^17*f^3 + 294912*a^12*b^3*c^8*d^2*e^17*f^3
))/(2*(4*a^3*b^10*e^2*f^2 - 4096*a^8*c^5*e^2*f^2 + 640*a^5*b^6*c^2*e^2*f^2
- 2560*a^6*b^4*c^3*e^2*f^2 + 5120*a^7*b^2*c^4*e^2*f^2 - 80*a^4*b^8*c*e^2*f^
2)*(a^6*b^12*f^3 + 4096*a^12*c^6*f^3 - 24*a^7*b^10*c*f^3 + 240*a^8*b^8*c^2*
f^3 - 1280*a^9*b^6*c^3*f^3 + 3840*a^10*b^4*c^4*f^3 - 6144*a^11*b^2*c^5*f^3)
))*(2*b^10*e*f - 2048*a^5*c^5*e*f + 320*a^2*b^6*c^2*e*f - 1280*a^3*b^4*c^3*
e*f + 2560*a^4*b^2*c^4*e*f - 40*a*b^8*c*e*f))/(2*(4*a^3*b^10*e^2*f^2 - 4096
*a^8*c^5*e^2*f^2 + 640*a^5*b^6*c^2*e^2*f^2 - 2560*a^6*b^4*c^3*e^2*f^2 + 512
0*a^7*b^2*c^4*e^2*f^2 - 80*a^4*b^8*c*e^2*f^2)))*(2*b^10*e*f - 2048*a^5*c^5*
e*f + 320*a^2*b^6*c^2*e*f - 1280*a^3*b^4*c^3*e*f + 2560*a^4*b^2*c^4*e*f - 4
0*a*b^8*c*e*f))/(2*(4*a^3*b^10*e^2*f^2 - 4096*a^8*c^5*e^2*f^2 + 640*a^5*b^6
*c^2*e^2*f^2 - 2560*a^6*b^4*c^3*e^2*f^2 + 5120*a^7*b^2*c^4*e^2*f^2 - 80*a^4
*b^8*c*e^2*f^2)) - (b^10*c^4*e^14 - 22*a*b^8*c^5*e^14 + 177*a^2*b^6*c^6*e^1
4 - 616*a^3*b^4*c^7*e^14 + 784*a^4*b^2*c^8*e^14 + b^9*c^5*d^2*e^14 + 147*a^
2*b^5*c^7*d^2*e^14 - 343*a^3*b^3*c^8*d^2*e^14 - 21*a*b^7*c^6*d^2*e^14)/(a^6
*b^12*f^3 + 4096*a^12*c^6*f^3 - 24*a^7*b^10*c*f^3 + 240*a^8*b^8*c^2*f^3 - 1
280*a^9*b^6*c^3*f^3 + 3840*a^10*b^4*c^4*f^3 - 6144*a^11*b^2*c^5*f^3) + (b*(
(b*((4*a^4*b^14*c^2*e^16*f^2 - 100*a^5*b^12*c^3*e^16*f^2 + 1052*a^6*b^10*c^
4*e^16*f^2 - 5952*a^7*b^8*c^5*e^16*f^2 + 19072*a^8*b^6*c^6*e^16*f^2 - 32768
*a^9*b^4*c^7*e^16*f^2 + 23552*a^10*b^2*c^8*e^16*f^2 + 5120*a^10*b*c^9*d^2*e
^16*f^2 + 2*a^4*b^13*c^3*d^2*e^16*f^2 - 36*a^5*b^11*c^4*d^2*e^16*f^2 + 276*
a^6*b^9*c^5*d^2*e^16*f^2 - 1216*a^7*b^7*c^6*d^2*e^16*f^2 + 3456*a^8*b^5*c^7
*d^2*e^16*f^2 - 6144*a^9*b^3*c^8*d^2*e^16*f^2)/(a^6*b^12*f^3 + 4096*a^12*c^
6*f^3 - 24*a^7*b^10*c*f^3 + 240*a^8*b^8*c^2*f^3 - 1280*a^9*b^6*c^3*f^3 + 38
40*a^10*b^4*c^4*f^3 - 6144*a^11*b^2*c^5*f^3) + ((2*b^10*e*f - 2048*a^5*c^5*
e*f + 320*a^2*b^6*c^2*e*f - 1280*a^3*b^4*c^3*e*f + 2560*a^4*b^2*c^4*e*f - 4
0*a*b^8*c*e*f)*(4*a^7*b^14*c^2*e^17*f^3 - 96*a^8*b^12*c^3*e^17*f^3 + 960*a^
9*b^10*c^4*e^17*f^3 - 5120*a^10*b^8*c^5*e^17*f^3 + 15360*a^11*b^6*c^6*e^17*
f^3 - 24576*a^12*b^4*c^7*e^17*f^3 + 16384*a^13*b^2*c^8*e^17*f^3 - 163840*a^
13*b*c^9*d^2*e^17*f^3 + 12*a^6*b^15*c^2*d^2*e^17*f^3 - 328*a^7*b^13*c^3*d^2
*e^17*f^3 + 3840*a^8*b^11*c^4*d^2*e^17*f^3 - 24960*a^9*b^9*c^5*d^2*e^17*f^3
 + 97280*a^10*b^7*c^6*d^2*e^17*f^3 - 227328*a^11*b^5*c^7*d^2*e^17*f^3 + 294
912*a^12*b^3*c^8*d^2*e^17*f^3))/(2*(4*a^3*b^10*e^2*f^2 - 4096*a^8*c^5*e^2*f
^2 + 640*a^5*b^6*c^2*e^2*f^2 - 2560*a^6*b^4*c^3*e^2*f^2 + 5120*a^7*b^2*c^4*
e^2*f^2 - 80*a^4*b^8*c*e^2*f^2)*(a^6*b^12*f^3 + 4096*a^12*c^6*f^3 - 24*a^7*
b^10*c*f^3 + 240*a^8*b^8*c^2*f^3 - 1280*a^9*b^6*c^3*f^3 + 3840*a^10*b^4*c^4
*f^3 - 6144*a^11*b^2*c^5*f^3)))*(b^4 + 30*a^2*c^2 - 10*a*b^2*c))/(4*a^3*e*f
*(4*a*c - b^2)^(5/2)) + (b*(b^4 + 30*a^2*c^2 - 10*a*b^2*c)*(2*b^10*e*f - 20
48*a^5*c^5*e*f + 320*a^2*b^6*c^2*e*f - 1280*a^3*b^4*c^3*e*f + 2560*a^4*b^2*
c^4*e*f - 40*a*b^8*c*e*f)*(4*a^7*b^14*c^2*e^17*f^3 - 96*a^8*b^12*c^3*e^17*f
^3 + 960*a^9*b^10*c^4*e^17*f^3 - 5120*a^10*b^8*c^5*e^17*f^3 + 15360*a^11*b^

```
6*c^6*e^17*f^3 - 24576*a^12*b^4*c^7*e^17*f^3 + 16384*a^13*b^2*c^8*e^17*f^3
- 163840*a^13*b*c^9*d^2*e^17*f^3 + 12*a^6*b^15*c^2*d^2*e^17*f^3 - 328*a^7*b
^13*c^3*d^2*e^17*f^3 + 3840*a^8*b^11*c^4*d^2*e^17*f^3 - 24960*a^9*b^9*c^5*d
^2*e^17*f^3 + 97280*a^10*b^7*c^6*d^2*e^17*f^3 - 227328*a^11*b^5*c^7*d^2*e^1
7*f^3 + 294912*a^12*b^3*c^8*d^2*e^17*f^3))/(8*a^3*e*f*(4*a*c - b^2)^(5/2)*(
4*a^3*b^10*e^2*f^2 - 4096*a^8*c^5*e^2*f^2 + 640*a^5*b^6*c^2*e^2*f^2 - 2560*
a^6*b^4*c^3*e^2*f^2 + 5120*a^7*b^2*c^4*e^2*f^2 - 80*a^4*b^8*c*e^2*f^2)*(a^6
*b^12*f^3 + 4096*a^12*c^6*f^3 - 24*a^7*b^10*c*f^3 + 240*a^8*b^8*c^2*f^3 - 1
280*a^9*b^6*c^3*f^3 + 3840*a^10*b^4*c^4*f^3 - 6144*a^11*b^2*c^5*f^3)))*(b^4
 + 30*a^2*c^2 - 10*a*b^2*c))/(4*a^3*e*f*(4*a*c - b^2)^(5/2)) + (b^2*(b^4 +
30*a^2*c^2 - 10*a*b^2*c)^2*(2*b^10*e*f - 2048*a^5*c^5*e*f + 320*a^2*b^6*c^2
*e*f - 1280*a^3*b^4*c^3*e*f + 2560*a^4*b^2*c^4*e*f - 40*a*b^8*c*e*f)*(4*a^7
*b^14*c^2*e^17*f^3 - 96*a^8*b^12*c^3*e^17*f^3 + 960*a^9*b^10*c^4*e^17*f^3 -
 5120*a^10*b^8*c^5*e^17*f^3 + 15360*a^11*b^6*c^6*e^17*f^3 - 24576*a^12*b^4*
c^7*e^17*f^3 + 16384*a^13*b^2*c^8*e^17*f^3 - 163840*a^13*b*c^9*d^2*e^17*f^3
 + 12*a^6*b^15*c^2*d^2*e^17*f^3 - 328*a^7*b^13*c^3*d^2*e^17*f^3 + 3840*a^8*
b^11*c^4*d^2*e^17*f^3 - 24960*a^9*b^9*c^5*d^2*e^17*f^3 + 97280*a^10*b^7*c^6
*d^2*e^17*f^3 - 227328*a^11*b^5*c^7*d^2*e^17*f^3 + 294912*a^12*b^3*c^8*d^2*
e^17*f^3))/(32*a^6*e^2*f^2*(4*a*c - b^2)^5*(4*a^3*b^10*e^2*f^2 - 4096*a^8*c
^5*e^2*f^2 + 640*a^5*b^6*c^2*e^2*f^2 - 2560*a^6*b^4*c^3*e^2*f^2 + 5120*a^7*
b^2*c^4*e^2*f^2 - 80*a^4*b^8*c*e^2*f^2)*(a^6*b^12*f^3 + 4096*a^12*c^6*f^3 -
 24*a^7*b^10*c*f^3 + 240*a^8*b^8*c^2*f^3 - 1280*a^9*b^6*c^3*f^3 + 3840*a^10
*b^4*c^4*f^3 - 6144*a^11*b^2*c^5*f^3)))*(b^6 - 45*a^3*c^3 + 40*a^2*b^2*c^2
- 11*a*b^4*c)*(16*a^9*b^12*f^3*(4*a*c - b^2)^(15/2) + 65536*a^15*c^6*f^3*(4
*a*c - b^2)^(15/2) - 384*a^10*b^10*c*f^3*(4*a*c - b^2)^(15/2) + 3840*a^11*b
^8*c^2*f^3*(4*a*c - b^2)^(15/2) - 20480*a^12*b^6*c^3*f^3*(4*a*c - b^2)^(15/
2) + 61440*a^13*b^4*c^4*f^3*(4*a*c - b^2)^(15/2) - 98304*a^14*b^2*c^5*f^3*(
4*a*c - b^2)^(15/2)))/(8*a^3*c^2*(4*a*c - b^2)^6*(b^10*c^2*e^14 - 20*a*b^8*
c^3*e^14 + 160*a^2*b^6*c^4*e^14 - 600*a^3*b^4*c^5*e^14 + 900*a^4*b^2*c^6*e^
14)*(6*b^10 - 6400*a^5*c^5 + 960*a^2*b^6*c^2 - 3850*a^3*b^4*c^3 + 7775*a^4*
b^2*c^4 - 120*a*b^8*c)))*(b^4 + 30*a^2*c^2 - 10*a*b^2*c))/(2*a^3*e*f*(4*a*c
 - b^2)^(5/2))
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*f*x+d*f)/(a+b*(e*x+d)**2+c*(e*x+d)**4)**3,x)
[Out] Timed out
```

3.546 $\int \frac{1}{(df+efx)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)^3}\,dx$

Optimal. Leaf size=499

$$-\frac{3\left(5b^2-12ac\right)\left(b^2-5ac\right)}{8a^3ef^2\left(b^2-4ac\right)^2(d+ex)}+\frac{36a^2c^2+bc\left(5b^2-32ac\right)(d+ex)^2-35ab^2c+5b^4}{8a^2ef^2\left(b^2-4ac\right)^2(d+ex)\left(a+b(d+ex)^2+c(d+ex)^4\right)}-\frac{3\sqrt{c}\left(\frac{b\left(124a^2c^2-47ab^2c+\right.}{\sqrt{b^2-4ac}}\right.}{8\sqrt{2}}$$

Rubi [A] time = 1.09, antiderivative size = 499, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {1142, 1121, 1277, 1281, 1166, 205}

$$\frac{36a^2c^2+bc\left(5b^2-32ac\right)(d+ex)^2-35ab^2c+5b^4}{8a^2ef^2\left(b^2-4ac\right)^2(d+ex)\left(a+b(d+ex)^2+c(d+ex)^4\right)}-\frac{3\sqrt{c}\left(\frac{b\left(124a^2c^2-47ab^2c+5b^4\right)}{\sqrt{b^2-4ac}}+\left(5b^2-12ac\right)\left(b^2-5ac\right)\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{8\sqrt{2}a^3ef^2\left(b^2-4ac\right)^2\sqrt{b-\sqrt{b^2-4ac}}}-\frac{3\sqrt{c}\left(\left(5b^2-12ac\right)\left(b^2-5ac\right)-\frac{124a^2bc^2-47ab^3c+5b^5}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{8\sqrt{2}a^3ef^2\left(b^2-4ac\right)^2\sqrt{\sqrt{b^2-4ac}+b}}-\frac{3\left(5b^2-12ac\right)\left(b^2-5ac\right)}{8a^3ef^2\left(b^2-4ac\right)^2(d+ex)}+\frac{-2ac+b^2+bc(d+ex)^2}{4aef^2\left(b^2-4ac\right)(d+ex)\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d*f + e*f*x)^2*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3),x]
```

```
[Out] (-3*(5*b^2 - 12*a*c)*(b^2 - 5*a*c))/(8*a^3*(b^2 - 4*a*c)^2*e*f^2*(d + e*x))
 + (b^2 - 2*a*c + b*c*(d + e*x)^2)/(4*a*(b^2 - 4*a*c)*e*f^2*(d + e*x)*(a +
b*(d + e*x)^2 + c*(d + e*x)^4)^2) + (5*b^4 - 35*a*b^2*c + 36*a^2*c^2 + b*c*
(5*b^2 - 32*a*c)*(d + e*x)^2)/(8*a^2*(b^2 - 4*a*c)^2*e*f^2*(d + e*x)*(a + b
*(d + e*x)^2 + c*(d + e*x)^4)) - (3*Sqrt[c]*((5*b^2 - 12*a*c)*(b^2 - 5*a*c)
 + (b*(5*b^4 - 47*a*b^2*c + 124*a^2*c^2))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2
]*Sqrt[c]*(d + e*x))/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(8*Sqrt[2]*a^3*(b^2 - 4*
a*c)^2*Sqrt[b - Sqrt[b^2 - 4*a*c]]*e*f^2) - (3*Sqrt[c]*((5*b^2 - 12*a*c)*(b
^2 - 5*a*c) - (5*b^5 - 47*a*b^3*c + 124*a^2*b*c^2)/Sqrt[b^2 - 4*a*c])*ArcTa
n[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(8*Sqrt[2]*a^3*
(b^2 - 4*a*c)^2*Sqrt[b + Sqrt[b^2 - 4*a*c]]*e*f^2)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1121

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> -Simp[((d*x)^(m + 1)*(b^2 - 2*a*c + b*c*x^2)*(a + b*x^2 + c*x^4)^(p + 1
))/(2*a*d*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)),
Int[(d*x)^m*(a + b*x^2 + c*x^4)^(p + 1)*Simp[b^2*(m + 2*p + 3) - 2*a*c*(m +
 4*p + 5) + b*c*(m + 4*p + 7)*x^2, x], x], x] /; FreeQ[{a, b, c, d, m}, x]
&& NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p] && (IntegerQ[p] || In
tegerQ[m])
```

Rule 1142

```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Di
st[u^m/(Coefficient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p,
 x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
```

```
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1277

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> -Simp[((f*x)^(m + 1)*(a + b*x^2 + c*x^4)^(p + 1)*
(d*(b^2 - 2*a*c) - a*b*e + (b*d - 2*a*e)*c*x^2))/(2*a*f*(p + 1)*(b^2 - 4*a*
c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(f*x)^m*(a + b*x^2 + c*x^
4)^(p + 1)*Simp[d*(b^2*(m + 2*(p + 1) + 1) - 2*a*c*(m + 4*(p + 1) + 1)) - a
*b*e*(m + 1) + c*(m + 2*(2*p + 3) + 1)*(b*d - 2*a*e)*x^2, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, m}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && Intege
rQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1281

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[(d*(f*x)^(m + 1)*(a + b*x^2 + c*x^4)^(p + 1)
)/(a*f*(m + 1)), x] + Dist[1/(a*f^2*(m + 1)), Int[(f*x)^(m + 2)*(a + b*x^2
+ c*x^4)^p*Simp[a*e*(m + 1) - b*d*(m + 2*p + 3) - c*d*(m + 4*p + 5)*x^2, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[m
, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rubi steps

$$\int \frac{1}{(df+efx)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)^3}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x^2\left(a+bx^2+cx^4\right)^3}\,dx,x,d+ex\right)}{ef^2}$$

$$= \frac{b^2-2ac+bc(d+ex)^2}{4a\left(b^2-4ac\right)ef^2(d+ex)\left(a+b(d+ex)^2+c(d+ex)^4\right)^2} - \text{Sul}$$

$$= \frac{b^2-2ac+bc(d+ex)^2}{4a\left(b^2-4ac\right)ef^2(d+ex)\left(a+b(d+ex)^2+c(d+ex)^4\right)^2} + \frac{}{8a^2}$$

$$= -\frac{3\left(5b^2-12ac\right)\left(b^2-5ac\right)}{8a^3\left(b^2-4ac\right)^2ef^2(d+ex)} + \frac{b^2-2ac+bc}{4a\left(b^2-4ac\right)ef^2(d+ex)\left(a+\right.}$$

$$= -\frac{3\left(5b^2-12ac\right)\left(b^2-5ac\right)}{8a^3\left(b^2-4ac\right)^2ef^2(d+ex)} + \frac{b^2-2ac+bc}{4a\left(b^2-4ac\right)ef^2(d+ex)\left(a+\right.}$$

$$= -\frac{3\left(5b^2-12ac\right)\left(b^2-5ac\right)}{8a^3\left(b^2-4ac\right)^2ef^2(d+ex)} + \frac{b^2-2ac+bc}{4a\left(b^2-4ac\right)ef^2(d+ex)\left(a+\right.}$$

Mathematica [A] time = 6.21, size = 575, normalized size = 1.15

$$-\frac{1}{a^3ef^2(d+ex)}+\frac{-3abc(d+ex)-2ac^2(d+ex)^3+b^3(d+ex)+b^2c(d+ex)^3}{4a^2ef^2\left(4ac-b^2\right)\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}-\frac{3\sqrt{c}\left(60a^2c^2\sqrt{b^2-4ac}+124a^2bc^2-47ab^3c-37ab^2c\sqrt{b^2-4ac}+5b^4\sqrt{b^2-4ac}+5b^5\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{8\sqrt{2}a^3ef^2\left(b^2-4ac\right)^{5/2}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{3\sqrt{c}\left(60a^2c^2\sqrt{b^2-4ac}-124a^2bc^2+47ab^3c-37ab^2c\sqrt{b^2-4ac}+5b^4\sqrt{b^2-4ac}-5b^5\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{8\sqrt{2}a^3ef^2\left(b^2-4ac\right)^{5/2}\sqrt{\sqrt{b^2-4ac}+b}}+\frac{-84a^2bc^2(d+ex)-52a^2c^3(d+ex)^3+52ab^3c(d+ex)+47ab^2c^2(d+ex)^3-7b^5(d+ex)-7b^4c(d+ex)^3}{8a^3ef^2\left(4ac-b^2\right)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d*f + e*f*x)^2*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3),x]
```

```
[Out] -(1/(a^3*e*f^2*(d + e*x))) + (b^3*(d + e*x) - 3*a*b*c*(d + e*x) + b^2*c*(d
+ e*x)^3 - 2*a*c^2*(d + e*x)^3)/(4*a^2*(-b^2 + 4*a*c)*e*f^2*(a + b*(d + e*x
)^2 + c*(d + e*x)^4)^2) + (-7*b^5*(d + e*x) + 52*a*b^3*c*(d + e*x) - 84*a^2
*b*c^2*(d + e*x) - 7*b^4*c*(d + e*x)^3 + 47*a*b^2*c^2*(d + e*x)^3 - 52*a^2*
c^3*(d + e*x)^3)/(8*a^3*(-b^2 + 4*a*c)^2*e*f^2*(a + b*(d + e*x)^2 + c*(d +
e*x)^4)) - (3*Sqrt[c]*(5*b^5 - 47*a*b^3*c + 124*a^2*b*c^2 + 5*b^4*Sqrt[b^2
- 4*a*c] - 37*a*b^2*c*Sqrt[b^2 - 4*a*c] + 60*a^2*c^2*Sqrt[b^2 - 4*a*c])*Arc
Tan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(8*Sqrt[2]*a^
3*(b^2 - 4*a*c)^(5/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]*e*f^2) - (3*Sqrt[c]*(-5*b
^5 + 47*a*b^3*c - 124*a^2*b*c^2 + 5*b^4*Sqrt[b^2 - 4*a*c] - 37*a*b^2*c*Sqrt
[b^2 - 4*a*c] + 60*a^2*c^2*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d +
e*x))/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(8*Sqrt[2]*a^3*(b^2 - 4*a*c)^(5/2)*Sqrt
[b + Sqrt[b^2 - 4*a*c]]*e*f^2)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(df+efx)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/((d*f + e*f*x)^2*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3
),x]
```

```
[Out] IntegrateAlgebraic[1/((d*f + e*f*x)^2*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3
), x]
```

fricas [B] time = 3.61, size = 10518, normalized size = 21.08

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*f*x+d*f)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="frica
s")
```

```
[Out] -1/16*(6*(5*b^4*c^2 - 37*a*b^2*c^3 + 60*a^2*c^4)*e^8*x^8 + 48*(5*b^4*c^2 -
37*a*b^2*c^3 + 60*a^2*c^4)*d*e^7*x^7 + 2*(30*b^5*c - 227*a*b^3*c^2 + 392*a^
2*b*c^3 + 84*(5*b^4*c^2 - 37*a*b^2*c^3 + 60*a^2*c^4)*d^2)*e^6*x^6 + 12*(28*
(5*b^4*c^2 - 37*a*b^2*c^3 + 60*a^2*c^4)*d^3 + (30*b^5*c - 227*a*b^3*c^2 + 3
92*a^2*b*c^3)*d)*e^5*x^5 + 6*(5*b^4*c^2 - 37*a*b^2*c^3 + 60*a^2*c^4)*d^8 +
2*(15*b^6 - 91*a*b^4*c + 25*a^2*b^2*c^2 + 324*a^3*c^3 + 210*(5*b^4*c^2 - 37
*a*b^2*c^3 + 60*a^2*c^4)*d^4 + 15*(30*b^5*c - 227*a*b^3*c^2 + 392*a^2*b*c^3
)*d^2)*e^4*x^4 + 2*(30*b^5*c - 227*a*b^3*c^2 + 392*a^2*b*c^3)*d^6 + 8*(42*(
5*b^4*c^2 - 37*a*b^2*c^3 + 60*a^2*c^4)*d^5 + 5*(30*b^5*c - 227*a*b^3*c^2 +
392*a^2*b*c^3)*d^3 + (15*b^6 - 91*a*b^4*c + 25*a^2*b^2*c^2 + 324*a^3*c^3)*d
)*e^3*x^3 + 16*a^2*b^4 - 128*a^3*b^2*c + 256*a^4*c^2 + 2*(15*b^6 - 91*a*b^4
*c + 25*a^2*b^2*c^2 + 324*a^3*c^3)*d^4 + 2*(84*(5*b^4*c^2 - 37*a*b^2*c^3 +
60*a^2*c^4)*d^6 + 25*a*b^5 - 194*a^2*b^3*c + 364*a^3*b*c^2 + 15*(30*b^5*c -
 227*a*b^3*c^2 + 392*a^2*b*c^3)*d^4 + 6*(15*b^6 - 91*a*b^4*c + 25*a^2*b^2*c
^2 + 324*a^3*c^3)*d^2)*e^2*x^2 + 2*(25*a*b^5 - 194*a^2*b^3*c + 364*a^3*b*c^
2)*d^2 + 4*(12*(5*b^4*c^2 - 37*a*b^2*c^3 + 60*a^2*c^4)*d^7 + 3*(30*b^5*c -
227*a*b^3*c^2 + 392*a^2*b*c^3)*d^5 + 2*(15*b^6 - 91*a*b^4*c + 25*a^2*b^2*c^
2 + 324*a^3*c^3)*d^3 + (25*a*b^5 - 194*a^2*b^3*c + 364*a^3*b*c^2)*d)*e*x -
3*sqrt(1/2)*((a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*e^10*f^2*x^9 + 9*(a
^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d*e^9*f^2*x^8 + 2*(a^3*b^5*c - 8*a
^4*b^3*c^2 + 16*a^5*b*c^3 + 18*(a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d
^2)*e^8*f^2*x^7 + 14*(6*(a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d^3 + (a
^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*d)*e^7*f^2*x^6 + (a^3*b^6 - 6*a^4*
b^4*c + 32*a^6*c^3 + 126*(a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d^4 + 4
2*(a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*d^2)*e^6*f^2*x^5 + (126*(a^3*b
^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d^5 + 70*(a^3*b^5*c - 8*a^4*b^3*c^2 +
```

16*a^5*b*c^3)*d^3 + 5*(a^3*b^6 - 6*a^4*b^4*c + 32*a^6*c^3)*d)*e^5*f^2*x^4 + 2*(a^4*b^5 - 8*a^5*b^3*c + 16*a^6*b*c^2 + 42*(a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d^6 + 35*(a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*d^4 + 5*(a^3*b^6 - 6*a^4*b^4*c + 32*a^6*c^3)*d^2)*e^4*f^2*x^3 + 2*(18*(a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d^7 + 21*(a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b *c^3)*d^5 + 5*(a^3*b^6 - 6*a^4*b^4*c + 32*a^6*c^3)*d^3 + 3*(a^4*b^5 - 8*a^5 *b^3*c + 16*a^6*b*c^2)*d)*e^3*f^2*x^2 + (a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2 + 9*(a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d^8 + 14*(a^3*b^5*c - 8*a^4 *b^3*c^2 + 16*a^5*b*c^3)*d^6 + 5*(a^3*b^6 - 6*a^4*b^4*c + 32*a^6*c^3)*d^4 + 6*(a^4*b^5 - 8*a^5*b^3*c + 16*a^6*b*c^2)*d^2)*e^2*f^2*x + ((a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d^9 + 2*(a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c ^3)*d^7 + (a^3*b^6 - 6*a^4*b^4*c + 32*a^6*c^3)*d^5 + 2*(a^4*b^5 - 8*a^5*b^3 *c + 16*a^6*b*c^2)*d^3 + (a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*d)*e*f^2)*sqr t(-(25*b^11 - 495*a*b^9*c + 3894*a^2*b^7*c^2 - 15015*a^3*b^5*c^3 + 27720*a^ 4*b^3*c^4 - 18480*a^5*b*c^5 + (a^7*b^10 - 20*a^8*b^8*c + 160*a^9*b^6*c^2 - 640*a^10*b^4*c^3 + 1280*a^11*b^2*c^4 - 1024*a^12*c^5)*e^2*f^4*sqrt((625*b^1 2 - 12250*a*b^10*c + 94725*a^2*b^8*c^2 - 351310*a^3*b^6*c^3 + 591886*a^4*b^ 4*c^4 - 312300*a^5*b^2*c^5 + 50625*a^6*c^6)/((a^14*b^10 - 20*a^15*b^8*c + 1 60*a^16*b^6*c^2 - 640*a^17*b^4*c^3 + 1280*a^18*b^2*c^4 - 1024*a^19*c^5)*e^4 *f^8)))/((a^7*b^10 - 20*a^8*b^8*c + 160*a^9*b^6*c^2 - 640*a^10*b^4*c^3 + 12 80*a^11*b^2*c^4 - 1024*a^12*c^5)*e^2*f^4))*log(-27*(4125*b^10*c^4 - 77825*a *b^8*c^5 + 571030*a^2*b^6*c^6 - 1957349*a^3*b^4*c^7 + 2835000*a^4*b^2*c^8 - 810000*a^5*c^9)*e*x - 27*(4125*b^10*c^4 - 77825*a*b^8*c^5 + 571030*a^2*b^6 *c^6 - 1957349*a^3*b^4*c^7 + 2835000*a^4*b^2*c^8 - 810000*a^5*c^9)*d + 27/2 *sqrt(1/2)*((5*a^7*b^16 - 152*a^8*b^14*c + 2006*a^9*b^12*c^2 - 14960*a^10*b ^10*c^3 + 68640*a^11*b^8*c^4 - 197120*a^12*b^6*c^5 + 342528*a^13*b^4*c^6 - 323584*a^14*b^2*c^7 + 122880*a^15*c^8)*e^3*f^6*sqrt((625*b^12 - 12250*a*b^1 0*c + 94725*a^2*b^8*c^2 - 351310*a^3*b^6*c^3 + 591886*a^4*b^4*c^4 - 312300* a^5*b^2*c^5 + 50625*a^6*c^6)/((a^14*b^10 - 20*a^15*b^8*c + 160*a^16*b^6*c^2 - 640*a^17*b^4*c^3 + 1280*a^18*b^2*c^4 - 1024*a^19*c^5)*e^4*f^8)) - (125*b ^17 - 3775*a*b^15*c + 49360*a^2*b^13*c^2 - 362733*a^3*b^11*c^3 + 1623534*a^ 4*b^9*c^4 - 4463140*a^5*b^7*c^5 + 7146736*a^6*b^5*c^6 - 5684672*a^7*b^3*c^7 + 1324800*a^8*b*c^8)*e*f^2)*sqrt(-(25*b^11 - 495*a*b^9*c + 3894*a^2*b^7*c^ 2 - 15015*a^3*b^5*c^3 + 27720*a^4*b^3*c^4 - 18480*a^5*b*c^5 + (a^7*b^10 - 2 0*a^8*b^8*c + 160*a^9*b^6*c^2 - 640*a^10*b^4*c^3 + 1280*a^11*b^2*c^4 - 1024 *a^12*c^5)*e^2*f^4*sqrt((625*b^12 - 12250*a*b^10*c + 94725*a^2*b^8*c^2 - 35 1310*a^3*b^6*c^3 + 591886*a^4*b^4*c^4 - 312300*a^5*b^2*c^5 + 50625*a^6*c^6) /((a^14*b^10 - 20*a^15*b^8*c + 160*a^16*b^6*c^2 - 640*a^17*b^4*c^3 + 1280*a ^18*b^2*c^4 - 1024*a^19*c^5)*e^4*f^8)))/((a^7*b^10 - 20*a^8*b^8*c + 160*a^9 *b^6*c^2 - 640*a^10*b^4*c^3 + 1280*a^11*b^2*c^4 - 1024*a^12*c^5)*e^2*f^4))) + 3*sqrt(1/2)*((a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*e^10*f^2*x^9 + 9 *(a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d*e^9*f^2*x^8 + 2*(a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3 + 18*(a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d^2)*e^8*f^2*x^7 + 14*(6*(a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d^3 + (a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*d)*e^7*f^2*x^6 + (a^3*b^6 - 6*a ^4*b^4*c + 32*a^6*c^3 + 126*(a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d^4 + 42*(a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*d^2)*e^6*f^2*x^5 + (126*(a^ 3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d^5 + 70*(a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*d^3 + 5*(a^3*b^6 - 6*a^4*b^4*c + 32*a^6*c^3)*d)*e^5*f^2*x^ 4 + 2*(a^4*b^5 - 8*a^5*b^3*c + 16*a^6*b*c^2 + 42*(a^3*b^4*c^2 - 8*a^4*b^2*c ^3 + 16*a^5*c^4)*d^6 + 35*(a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*d^4 + 5*(a^3*b^6 - 6*a^4*b^4*c + 32*a^6*c^3)*d^2)*e^4*f^2*x^3 + 2*(18*(a^3*b^4*c^ 2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d^7 + 21*(a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^ 5*b*c^3)*d^5 + 5*(a^3*b^6 - 6*a^4*b^4*c + 32*a^6*c^3)*d^3 + 3*(a^4*b^5 - 8* a^5*b^3*c + 16*a^6*b*c^2)*d)*e^3*f^2*x^2 + (a^5*b^4 - 8*a^6*b^2*c + 16*a^7* c^2 + 9*(a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d^8 + 14*(a^3*b^5*c - 8* a^4*b^3*c^2 + 16*a^5*b*c^3)*d^6 + 5*(a^3*b^6 - 6*a^4*b^4*c + 32*a^6*c^3)*d^ 4 + 6*(a^4*b^5 - 8*a^5*b^3*c + 16*a^6*b*c^2)*d^2)*e^2*f^2*x + ((a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d^9 + 2*(a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*

b*c^3)*d^7 + (a^3*b^6 - 6*a^4*b^4*c + 32*a^6*c^3)*d^5 + 2*(a^4*b^5 - 8*a^5*
b^3*c + 16*a^6*b*c^2)*d^3 + (a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*d)*e*f^2)*
sqrt(-(25*b^11 - 495*a*b^9*c + 3894*a^2*b^7*c^2 - 15015*a^3*b^5*c^3 + 27720
*a^4*b^3*c^4 - 18480*a^5*b*c^5 + (a^7*b^10 - 20*a^8*b^8*c + 160*a^9*b^6*c^2
 - 640*a^10*b^4*c^3 + 1280*a^11*b^2*c^4 - 1024*a^12*c^5)*e^2*f^4*sqrt((625*
b^12 - 12250*a*b^10*c + 94725*a^2*b^8*c^2 - 351310*a^3*b^6*c^3 + 591886*a^4
*b^4*c^4 - 312300*a^5*b^2*c^5 + 50625*a^6*c^6)/((a^14*b^10 - 20*a^15*b^8*c
+ 160*a^16*b^6*c^2 - 640*a^17*b^4*c^3 + 1280*a^18*b^2*c^4 - 1024*a^19*c^5)*
e^4*f^8)))/((a^7*b^10 - 20*a^8*b^8*c + 160*a^9*b^6*c^2 - 640*a^10*b^4*c^3 +
 1280*a^11*b^2*c^4 - 1024*a^12*c^5)*e^2*f^4))*log(-27*(4125*b^10*c^4 - 7782
5*a*b^8*c^5 + 571030*a^2*b^6*c^6 - 1957349*a^3*b^4*c^7 + 2835000*a^4*b^2*c^
8 - 810000*a^5*c^9)*e*x - 27*(4125*b^10*c^4 - 77825*a*b^8*c^5 + 571030*a^2*
b^6*c^6 - 1957349*a^3*b^4*c^7 + 2835000*a^4*b^2*c^8 - 810000*a^5*c^9)*d - 2
7/2*sqrt(1/2)*((5*a^7*b^16 - 152*a^8*b^14*c + 2006*a^9*b^12*c^2 - 14960*a^1
0*b^10*c^3 + 68640*a^11*b^8*c^4 - 197120*a^12*b^6*c^5 + 342528*a^13*b^4*c^6
 - 323584*a^14*b^2*c^7 + 122880*a^15*c^8)*e^3*f^6*sqrt((625*b^12 - 12250*a*
b^10*c + 94725*a^2*b^8*c^2 - 351310*a^3*b^6*c^3 + 591886*a^4*b^4*c^4 - 3123
00*a^5*b^2*c^5 + 50625*a^6*c^6)/((a^14*b^10 - 20*a^15*b^8*c + 160*a^16*b^6*
c^2 - 640*a^17*b^4*c^3 + 1280*a^18*b^2*c^4 - 1024*a^19*c^5)*e^4*f^8)) - (12
5*b^17 - 3775*a*b^15*c + 49360*a^2*b^13*c^2 - 362733*a^3*b^11*c^3 + 1623534
*a^4*b^9*c^4 - 4463140*a^5*b^7*c^5 + 7146736*a^6*b^5*c^6 - 5684672*a^7*b^3*
c^7 + 1324800*a^8*b*c^8)*e*f^2)*sqrt(-(25*b^11 - 495*a*b^9*c + 3894*a^2*b^7
*c^2 - 15015*a^3*b^5*c^3 + 27720*a^4*b^3*c^4 - 18480*a^5*b*c^5 + (a^7*b^10
- 20*a^8*b^8*c + 160*a^9*b^6*c^2 - 640*a^10*b^4*c^3 + 1280*a^11*b^2*c^4 - 1
024*a^12*c^5)*e^2*f^4*sqrt((625*b^12 - 12250*a*b^10*c + 94725*a^2*b^8*c^2 -
 351310*a^3*b^6*c^3 + 591886*a^4*b^4*c^4 - 312300*a^5*b^2*c^5 + 50625*a^6*c
^6)/((a^14*b^10 - 20*a^15*b^8*c + 160*a^16*b^6*c^2 - 640*a^17*b^4*c^3 + 128
0*a^18*b^2*c^4 - 1024*a^19*c^5)*e^4*f^8)))/((a^7*b^10 - 20*a^8*b^8*c + 160*
a^9*b^6*c^2 - 640*a^10*b^4*c^3 + 1280*a^11*b^2*c^4 - 1024*a^12*c^5)*e^2*f^4
))) + 3*sqrt(1/2)*((a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*e^10*f^2*x^9
+ 9*(a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d*e^9*f^2*x^8 + 2*(a^3*b^5*c
 - 8*a^4*b^3*c^2 + 16*a^5*b*c^3 + 18*(a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*
c^4)*d^2)*e^8*f^2*x^7 + 14*(6*(a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d^
3 + (a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*d)*e^7*f^2*x^6 + (a^3*b^6 -
6*a^4*b^4*c + 32*a^6*c^3 + 126*(a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d
^4 + 42*(a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*d^2)*e^6*f^2*x^5 + (126*
(a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d^5 + 70*(a^3*b^5*c - 8*a^4*b^3*
c^2 + 16*a^5*b*c^3)*d^3 + 5*(a^3*b^6 - 6*a^4*b^4*c + 32*a^6*c^3)*d)*e^5*f^2
x^4 + 2(a^4*b^5 - 8*a^5*b^3*c + 16*a^6*b*c^2 + 42*(a^3*b^4*c^2 - 8*a^4*b^
2*c^3 + 16*a^5*c^4)*d^6 + 35*(a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*d^4
 + 5*(a^3*b^6 - 6*a^4*b^4*c + 32*a^6*c^3)*d^2)*e^4*f^2*x^3 + 2*(18*(a^3*b^4
*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d^7 + 21*(a^3*b^5*c - 8*a^4*b^3*c^2 + 16
*a^5*b*c^3)*d^5 + 5*(a^3*b^6 - 6*a^4*b^4*c + 32*a^6*c^3)*d^3 + 3*(a^4*b^5 -
 8*a^5*b^3*c + 16*a^6*b*c^2)*d)*e^3*f^2*x^2 + (a^5*b^4 - 8*a^6*b^2*c + 16*a
^7*c^2 + 9*(a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d^8 + 14*(a^3*b^5*c -
 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*d^6 + 5*(a^3*b^6 - 6*a^4*b^4*c + 32*a^6*c^3)
d^4 + 6(a^4*b^5 - 8*a^5*b^3*c + 16*a^6*b*c^2)*d^2)*e^2*f^2*x + ((a^3*b^4*
c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d^9 + 2*(a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a
^5*b*c^3)*d^7 + (a^3*b^6 - 6*a^4*b^4*c + 32*a^6*c^3)*d^5 + 2*(a^4*b^5 - 8*a
^5*b^3*c + 16*a^6*b*c^2)*d^3 + (a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*d)*e*f^
2)*sqrt(-(25*b^11 - 495*a*b^9*c + 3894*a^2*b^7*c^2 - 15015*a^3*b^5*c^3 + 27
720*a^4*b^3*c^4 - 18480*a^5*b*c^5 - (a^7*b^10 - 20*a^8*b^8*c + 160*a^9*b^6*
c^2 - 640*a^10*b^4*c^3 + 1280*a^11*b^2*c^4 - 1024*a^12*c^5)*e^2*f^4*sqrt((6
25*b^12 - 12250*a*b^10*c + 94725*a^2*b^8*c^2 - 351310*a^3*b^6*c^3 + 591886*
a^4*b^4*c^4 - 312300*a^5*b^2*c^5 + 50625*a^6*c^6)/((a^14*b^10 - 20*a^15*b^8
*c + 160*a^16*b^6*c^2 - 640*a^17*b^4*c^3 + 1280*a^18*b^2*c^4 - 1024*a^19*c^
5)*e^4*f^8)))/((a^7*b^10 - 20*a^8*b^8*c + 160*a^9*b^6*c^2 - 640*a^10*b^4*c^
3 + 1280*a^11*b^2*c^4 - 1024*a^12*c^5)*e^2*f^4))*log(-27*(4125*b^10*c^4 - 7
7825*a*b^8*c^5 + 571030*a^2*b^6*c^6 - 1957349*a^3*b^4*c^7 + 2835000*a^4*b^2

*c^8 - 810000*a^5*c^9)*e*x - 27*(4125*b^10*c^4 - 77825*a*b^8*c^5 + 571030*a
^2*b^6*c^6 - 1957349*a^3*b^4*c^7 + 2835000*a^4*b^2*c^8 - 810000*a^5*c^9)*d
+ 27/2*sqrt(1/2)*((5*a^7*b^16 - 152*a^8*b^14*c + 2006*a^9*b^12*c^2 - 14960*
a^10*b^10*c^3 + 68640*a^11*b^8*c^4 - 197120*a^12*b^6*c^5 + 342528*a^13*b^4*
c^6 - 323584*a^14*b^2*c^7 + 122880*a^15*c^8)*e^3*f^6*sqrt((625*b^12 - 12250
*a*b^10*c + 94725*a^2*b^8*c^2 - 351310*a^3*b^6*c^3 + 591886*a^4*b^4*c^4 - 3
12300*a^5*b^2*c^5 + 50625*a^6*c^6)/((a^14*b^10 - 20*a^15*b^8*c + 160*a^16*b
^6*c^2 - 640*a^17*b^4*c^3 + 1280*a^18*b^2*c^4 - 1024*a^19*c^5)*e^4*f^8)) +
(125*b^17 - 3775*a*b^15*c + 49360*a^2*b^13*c^2 - 362733*a^3*b^11*c^3 + 1623
534*a^4*b^9*c^4 - 4463140*a^5*b^7*c^5 + 7146736*a^6*b^5*c^6 - 5684672*a^7*b
^3*c^7 + 1324800*a^8*b*c^8)*e*f^2)*sqrt(-(25*b^11 - 495*a*b^9*c + 3894*a^2*
b^7*c^2 - 15015*a^3*b^5*c^3 + 27720*a^4*b^3*c^4 - 18480*a^5*b*c^5 - (a^7*b^
10 - 20*a^8*b^8*c + 160*a^9*b^6*c^2 - 640*a^10*b^4*c^3 + 1280*a^11*b^2*c^4
- 1024*a^12*c^5)*e^2*f^4*sqrt((625*b^12 - 12250*a*b^10*c + 94725*a^2*b^8*c^
2 - 351310*a^3*b^6*c^3 + 591886*a^4*b^4*c^4 - 312300*a^5*b^2*c^5 + 50625*a^
6*c^6)/((a^14*b^10 - 20*a^15*b^8*c + 160*a^16*b^6*c^2 - 640*a^17*b^4*c^3 +
1280*a^18*b^2*c^4 - 1024*a^19*c^5)*e^4*f^8)))/((a^7*b^10 - 20*a^8*b^8*c + 1
60*a^9*b^6*c^2 - 640*a^10*b^4*c^3 + 1280*a^11*b^2*c^4 - 1024*a^12*c^5)*e^2*
f^4))) - 3*sqrt(1/2)*((a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*e^10*f^2*x
^9 + 9*(a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d*e^9*f^2*x^8 + 2*(a^3*b^
5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3 + 18*(a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a
^5*c^4)*d^2)*e^8*f^2*x^7 + 14*(6*(a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)
*d^3 + (a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*d)*e^7*f^2*x^6 + (a^3*b^6
- 6*a^4*b^4*c + 32*a^6*c^3 + 126*(a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4
)*d^4 + 42*(a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*d^2)*e^6*f^2*x^5 + (1
26*(a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d^5 + 70*(a^3*b^5*c - 8*a^4*b
^3*c^2 + 16*a^5*b*c^3)*d^3 + 5*(a^3*b^6 - 6*a^4*b^4*c + 32*a^6*c^3)*d)*e^5*
f^2*x^4 + 2*(a^4*b^5 - 8*a^5*b^3*c + 16*a^6*b*c^2 + 42*(a^3*b^4*c^2 - 8*a^4
*b^2*c^3 + 16*a^5*c^4)*d^6 + 35*(a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*
d^4 + 5*(a^3*b^6 - 6*a^4*b^4*c + 32*a^6*c^3)*d^2)*e^4*f^2*x^3 + 2*(18*(a^3*
b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d^7 + 21*(a^3*b^5*c - 8*a^4*b^3*c^2 +
16*a^5*b*c^3)*d^5 + 5*(a^3*b^6 - 6*a^4*b^4*c + 32*a^6*c^3)*d^3 + 3*(a^4*b^
5 - 8*a^5*b^3*c + 16*a^6*b*c^2)*d)*e^3*f^2*x^2 + (a^5*b^4 - 8*a^6*b^2*c + 1
6*a^7*c^2 + 9*(a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d^8 + 14*(a^3*b^5*
c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*d^6 + 5*(a^3*b^6 - 6*a^4*b^4*c + 32*a^6*c
^3)*d^4 + 6*(a^4*b^5 - 8*a^5*b^3*c + 16*a^6*b*c^2)*d^2)*e^2*f^2*x + ((a^3*b
^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d^9 + 2*(a^3*b^5*c - 8*a^4*b^3*c^2 + 1
6*a^5*b*c^3)*d^7 + (a^3*b^6 - 6*a^4*b^4*c + 32*a^6*c^3)*d^5 + 2*(a^4*b^5 -
8*a^5*b^3*c + 16*a^6*b*c^2)*d^3 + (a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*d)*e
*f^2)*sqrt(-(25*b^11 - 495*a*b^9*c + 3894*a^2*b^7*c^2 - 15015*a^3*b^5*c^3 +
27720*a^4*b^3*c^4 - 18480*a^5*b*c^5 - (a^7*b^10 - 20*a^8*b^8*c + 160*a^9*b
^6*c^2 - 640*a^10*b^4*c^3 + 1280*a^11*b^2*c^4 - 1024*a^12*c^5)*e^2*f^4*sqrt
((625*b^12 - 12250*a*b^10*c + 94725*a^2*b^8*c^2 - 351310*a^3*b^6*c^3 + 5918
86*a^4*b^4*c^4 - 312300*a^5*b^2*c^5 + 50625*a^6*c^6)/((a^14*b^10 - 20*a^15*
b^8*c + 160*a^16*b^6*c^2 - 640*a^17*b^4*c^3 + 1280*a^18*b^2*c^4 - 1024*a^19
*c^5)*e^4*f^8)))/((a^7*b^10 - 20*a^8*b^8*c + 160*a^9*b^6*c^2 - 640*a^10*b^4
*c^3 + 1280*a^11*b^2*c^4 - 1024*a^12*c^5)*e^2*f^4))*log(-27*(4125*b^10*c^4
- 77825*a*b^8*c^5 + 571030*a^2*b^6*c^6 - 1957349*a^3*b^4*c^7 + 2835000*a^4*
b^2*c^8 - 810000*a^5*c^9)*e*x - 27*(4125*b^10*c^4 - 77825*a*b^8*c^5 + 57103
0*a^2*b^6*c^6 - 1957349*a^3*b^4*c^7 + 2835000*a^4*b^2*c^8 - 810000*a^5*c^9)
*d - 27/2*sqrt(1/2)*((5*a^7*b^16 - 152*a^8*b^14*c + 2006*a^9*b^12*c^2 - 149
60*a^10*b^10*c^3 + 68640*a^11*b^8*c^4 - 197120*a^12*b^6*c^5 + 342528*a^13*b
^4*c^6 - 323584*a^14*b^2*c^7 + 122880*a^15*c^8)*e^3*f^6*sqrt((625*b^12 - 12
250*a*b^10*c + 94725*a^2*b^8*c^2 - 351310*a^3*b^6*c^3 + 591886*a^4*b^4*c^4
- 312300*a^5*b^2*c^5 + 50625*a^6*c^6)/((a^14*b^10 - 20*a^15*b^8*c + 160*a^1
6*b^6*c^2 - 640*a^17*b^4*c^3 + 1280*a^18*b^2*c^4 - 1024*a^19*c^5)*e^4*f^8))
+ (125*b^17 - 3775*a*b^15*c + 49360*a^2*b^13*c^2 - 362733*a^3*b^11*c^3 + 1
623534*a^4*b^9*c^4 - 4463140*a^5*b^7*c^5 + 7146736*a^6*b^5*c^6 - 5684672*a^
7*b^3*c^7 + 1324800*a^8*b*c^8)*e*f^2)*sqrt(-(25*b^11 - 495*a*b^9*c + 3894*a

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^2*b^7*c^2 - 15015*a^3*b^5*c^3 + 27720*a^4*b^3*c^4 - 18480*a^5*b*c^5 - (a^7
*b^10 - 20*a^8*b^8*c + 160*a^9*b^6*c^2 - 640*a^10*b^4*c^3 + 1280*a^11*b^2*c
^4 - 1024*a^12*c^5)*e^2*f^4*sqrt((625*b^12 - 12250*a*b^10*c + 94725*a^2*b^8
*c^2 - 351310*a^3*b^6*c^3 + 591886*a^4*b^4*c^4 - 312300*a^5*b^2*c^5 + 50625
*a^6*c^6)/((a^14*b^10 - 20*a^15*b^8*c + 160*a^16*b^6*c^2 - 640*a^17*b^4*c^3
 + 1280*a^18*b^2*c^4 - 1024*a^19*c^5)*e^4*f^8)))/((a^7*b^10 - 20*a^8*b^8*c
+ 160*a^9*b^6*c^2 - 640*a^10*b^4*c^3 + 1280*a^11*b^2*c^4 - 1024*a^12*c^5)*e
^2*f^4))))/((a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*e^10*f^2*x^9 + 9*(a^
3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d*e^9*f^2*x^8 + 2*(a^3*b^5*c - 8*a^
4*b^3*c^2 + 16*a^5*b*c^3 + 18*(a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d^
2)*e^8*f^2*x^7 + 14*(6*(a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d^3 + (a^
3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*d)*e^7*f^2*x^6 + (a^3*b^6 - 6*a^4*b
^4*c + 32*a^6*c^3 + 126*(a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d^4 + 42
*(a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*d^2)*e^6*f^2*x^5 + (126*(a^3*b^
4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d^5 + 70*(a^3*b^5*c - 8*a^4*b^3*c^2 + 1
6*a^5*b*c^3)*d^3 + 5*(a^3*b^6 - 6*a^4*b^4*c + 32*a^6*c^3)*d)*e^5*f^2*x^4 +
2*(a^4*b^5 - 8*a^5*b^3*c + 16*a^6*b*c^2 + 42*(a^3*b^4*c^2 - 8*a^4*b^2*c^3 +
 16*a^5*c^4)*d^6 + 35*(a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*d^4 + 5*(a
^3*b^6 - 6*a^4*b^4*c + 32*a^6*c^3)*d^2)*e^4*f^2*x^3 + 2*(18*(a^3*b^4*c^2 -
8*a^4*b^2*c^3 + 16*a^5*c^4)*d^7 + 21*(a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*
c^3)*d^5 + 5*(a^3*b^6 - 6*a^4*b^4*c + 32*a^6*c^3)*d^3 + 3*(a^4*b^5 - 8*a^5*
b^3*c + 16*a^6*b*c^2)*d)*e^3*f^2*x^2 + (a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2
+ 9*(a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d^8 + 14*(a^3*b^5*c - 8*a^4*
b^3*c^2 + 16*a^5*b*c^3)*d^6 + 5*(a^3*b^6 - 6*a^4*b^4*c + 32*a^6*c^3)*d^4 +
6*(a^4*b^5 - 8*a^5*b^3*c + 16*a^6*b*c^2)*d^2)*e^2*f^2*x + ((a^3*b^4*c^2 - 8
*a^4*b^2*c^3 + 16*a^5*c^4)*d^9 + 2*(a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^
3)*d^7 + (a^3*b^6 - 6*a^4*b^4*c + 32*a^6*c^3)*d^5 + 2*(a^4*b^5 - 8*a^5*b^3*
c + 16*a^6*b*c^2)*d^3 + (a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*d)*e*f^2)
```

giac [B] time = 1.42, size = 1658, normalized size = 3.32

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*f*x+d*f)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="giac"
)
```

```
[Out] -1/8*(7*b^4*c^2*e^(-1)/((f*x*e + d*f)*f) - 47*a*b^2*c^3*e^(-1)/((f*x*e + d*
f)*f) + 52*a^2*c^4*e^(-1)/((f*x*e + d*f)*f) + 14*b^5*c*f*e^(-1)/(f*x*e + d*
f)^3 - 99*a*b^3*c^2*f*e^(-1)/(f*x*e + d*f)^3 + 136*a^2*b*c^3*f*e^(-1)/(f*x*
e + d*f)^3 + 7*b^6*f^3*e^(-1)/(f*x*e + d*f)^5 - 43*a*b^4*c*f^3*e^(-1)/(f*x*
e + d*f)^5 + 25*a^2*b^2*c^2*f^3*e^(-1)/(f*x*e + d*f)^5 + 68*a^3*c^3*f^3*e^(
-1)/(f*x*e + d*f)^5 + 9*a*b^5*f^5*e^(-1)/(f*x*e + d*f)^7 - 66*a^2*b^3*c*f^5
*e^(-1)/(f*x*e + d*f)^7 + 108*a^3*b*c^2*f^5*e^(-1)/(f*x*e + d*f)^7)/((a^3*b
^4 - 8*a^4*b^2*c + 16*a^5*c^2)*(c + b*f^2/(f*x*e + d*f)^2 + a*f^4/(f*x*e +
d*f)^4)^2) - e^(-1)/((f*x*e + d*f)*a^3*f) + 3/64*((5*a^6*b^13 - 112*a^7*b^1
1*c + 1030*a^8*b^9*c^2 - 4928*a^9*b^7*c^3 + 12736*a^10*b^5*c^4 - 16384*a^11
*b^3*c^5 + 7680*a^12*b*c^6)*sqrt(2*a*b + 2*sqrt(b^2 - 4*a*c)*a)*f^8*e^4 + 2
*(5*a^4*b^6*c - 57*a^5*b^4*c^2 + 208*a^6*b^2*c^3 - 240*a^7*c^4)*sqrt(2*a*b
+ 2*sqrt(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*f^4*abs(a^3*b^4*f^4*e^2 - 8*a^4*
b^2*c*f^4*e^2 + 16*a^5*c^2*f^4*e^2)*e^2 - (a^3*b^4*f^4*e^2 - 8*a^4*b^2*c*f^
4*e^2 + 16*a^5*c^2*f^4*e^2)^2*(5*b^5 - 42*a*b^3*c + 92*a^2*b*c^2)*sqrt(2*a*
b + 2*sqrt(b^2 - 4*a*c)*a))*arctan(2*sqrt(1/2)*e^(-1)/((f*x*e + d*f)*f*sqrt
((a^3*b^5*f^4*e^2 - 8*a^4*b^3*c*f^4*e^2 + 16*a^5*b*c^2*f^4*e^2 + sqrt((a^3*
b^5*f^4*e^2 - 8*a^4*b^3*c*f^4*e^2 + 16*a^5*b*c^2*f^4*e^2)^2 - 4*(a^4*b^4*f^
8*e^4 - 8*a^5*b^2*c*f^8*e^4 + 16*a^6*c^2*f^8*e^4)*(a^3*b^4*c - 8*a^4*b^2*c^
2 + 16*a^5*c^3)))/(a^4*b^4*f^8*e^4 - 8*a^5*b^2*c*f^8*e^4 + 16*a^6*c^2*f^8*e
^4))))*e^(-3)/((a^7*b^6*c - 12*a^8*b^4*c^2 + 48*a^9*b^2*c^3 - 64*a^10*c^4)*
sqrt(b^2 - 4*a*c)*f^6*abs(a^3*b^4*f^4*e^2 - 8*a^4*b^2*c*f^4*e^2 + 16*a^5*c^
2*f^4*e^2)*abs(a)) - 3/64*((5*a^6*b^13 - 112*a^7*b^11*c + 1030*a^8*b^9*c^2
```

```
- 4928*a^9*b^7*c^3 + 12736*a^10*b^5*c^4 - 16384*a^11*b^3*c^5 + 7680*a^12*b*
c^6)*sqrt(2*a*b - 2*sqrt(b^2 - 4*a*c)*a)*f^8*e^4 - 2*(5*a^4*b^6*c - 57*a^5*
b^4*c^2 + 208*a^6*b^2*c^3 - 240*a^7*c^4)*sqrt(2*a*b - 2*sqrt(b^2 - 4*a*c)*a
)*sqrt(b^2 - 4*a*c)*f^4*abs(a^3*b^4*f^4*e^2 - 8*a^4*b^2*c*f^4*e^2 + 16*a^5*
c^2*f^4*e^2)*e^2 - (a^3*b^4*f^4*e^2 - 8*a^4*b^2*c*f^4*e^2 + 16*a^5*c^2*f^4*
e^2)^2*(5*b^5 - 42*a*b^3*c + 92*a^2*b*c^2)*sqrt(2*a*b - 2*sqrt(b^2 - 4*a*c)
*a))*arctan(2*sqrt(1/2)*e^(-1)/((f*x*e + d*f)*f*sqrt((a^3*b^5*f^4*e^2 - 8*a
^4*b^3*c*f^4*e^2 + 16*a^5*b*c^2*f^4*e^2 - sqrt((a^3*b^5*f^4*e^2 - 8*a^4*b^3
*c*f^4*e^2 + 16*a^5*b*c^2*f^4*e^2)^2 - 4*(a^4*b^4*f^8*e^4 - 8*a^5*b^2*c*f^8
*e^4 + 16*a^6*c^2*f^8*e^4)*(a^3*b^4*c - 8*a^4*b^2*c^2 + 16*a^5*c^3)))/(a^4*
b^4*f^8*e^4 - 8*a^5*b^2*c*f^8*e^4 + 16*a^6*c^2*f^8*e^4))))*e^(-3)/((a^7*b^6
*c - 12*a^8*b^4*c^2 + 48*a^9*b^2*c^3 - 64*a^10*c^4)*sqrt(b^2 - 4*a*c)*f^6*a
bs(a^3*b^4*f^4*e^2 - 8*a^4*b^2*c*f^4*e^2 + 16*a^5*c^2*f^4*e^2)*abs(a))
```

maple [C] time = 0.07, size = 7019, normalized size = 14.07

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*f*x+d*f)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x)
[Out] result too large to display
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*f*x+d*f)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="maxim
a")
[Out] Timed out
```

mupad [B] time = 15.40, size = 20580, normalized size = 41.24

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((d*f + e*f*x)^2*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3),x)
[Out] - atan(((-(9*(25*b^21 - 25*b^6*(-(4*a*c - b^2)^15)^(1/2) + 18923520*a^10*b*
c^10 + 17794*a^2*b^17*c^2 - 188095*a^3*b^15*c^3 + 1299860*a^4*b^13*c^4 - 61
26640*a^5*b^11*c^5 + 19905600*a^6*b^9*c^6 - 43904256*a^7*b^7*c^7 + 62684160
*a^8*b^5*c^8 - 52039680*a^9*b^3*c^9 + 225*a^3*c^3*(-(4*a*c - b^2)^15)^(1/2)
 - 995*a*b^19*c - 694*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 245*a*b^4*c*(
-(4*a*c - b^2)^15)^(1/2)))/(512*(a^7*b^20*e^2*f^4 + 1048576*a^17*c^10*e^2*f
^4 + 720*a^9*b^16*c^2*e^2*f^4 - 7680*a^10*b^14*c^3*e^2*f^4 + 53760*a^11*b^1
2*c^4*e^2*f^4 - 258048*a^12*b^10*c^5*e^2*f^4 + 860160*a^13*b^8*c^6*e^2*f^4
- 1966080*a^14*b^6*c^7*e^2*f^4 + 2949120*a^15*b^4*c^8*e^2*f^4 - 2621440*a^1
6*b^2*c^9*e^2*f^4 - 40*a^8*b^18*c*e^2*f^4)))^(1/2)*(x*(271790899200*a^20*c^
14*e^12*f^6 - 230400*a^9*b^22*c^3*e^12*f^6 + 9861120*a^10*b^20*c^4*e^12*f^6
 - 191038464*a^11*b^18*c^5*e^12*f^6 + 2207803392*a^12*b^16*c^6*e^12*f^6 - 1
6878108672*a^13*b^14*c^7*e^12*f^6 + 89374851072*a^14*b^12*c^8*e^12*f^6 - 33
3226967040*a^15*b^10*c^9*e^12*f^6 + 869815812096*a^16*b^8*c^10*e^12*f^6 - 1
543847804928*a^17*b^6*c^11*e^12*f^6 + 1747313491968*a^18*b^4*c^12*e^12*f^6
- 1101055131648*a^19*b^2*c^13*e^12*f^6) - (-(9*(25*b^21 - 25*b^6*(-(4*a*c -
 b^2)^15)^(1/2) + 18923520*a^10*b*c^10 + 17794*a^2*b^17*c^2 - 188095*a^3*b^
15*c^3 + 1299860*a^4*b^13*c^4 - 6126640*a^5*b^11*c^5 + 19905600*a^6*b^9*c^6
 - 43904256*a^7*b^7*c^7 + 62684160*a^8*b^5*c^8 - 52039680*a^9*b^3*c^9 + 225
*a^3*c^3*(-(4*a*c - b^2)^15)^(1/2) - 995*a*b^19*c - 694*a^2*b^2*c^2*(-(4*a*
```

c - b^2)^15)^(1/2) + 245*a*b^4*c*(-(4*a*c - b^2)^15)^(1/2)))/(512*(a^7*b^20
*e^2*f^4 + 1048576*a^17*c^10*e^2*f^4 + 720*a^9*b^16*c^2*e^2*f^4 - 7680*a^10
*b^14*c^3*e^2*f^4 + 53760*a^11*b^12*c^4*e^2*f^4 - 258048*a^12*b^10*c^5*e^2*
f^4 + 860160*a^13*b^8*c^6*e^2*f^4 - 1966080*a^14*b^6*c^7*e^2*f^4 + 2949120*
a^15*b^4*c^8*e^2*f^4 - 2621440*a^16*b^2*c^9*e^2*f^4 - 40*a^8*b^18*c*e^2*f^4
)))^(1/2)*((-(9*(25*b^21 - 25*b^6*(-(4*a*c - b^2)^15)^(1/2) + 18923520*a^10
*b*c^10 + 17794*a^2*b^17*c^2 - 188095*a^3*b^15*c^3 + 1299860*a^4*b^13*c^4 -
 6126640*a^5*b^11*c^5 + 19905600*a^6*b^9*c^6 - 43904256*a^7*b^7*c^7 + 62684
160*a^8*b^5*c^8 - 52039680*a^9*b^3*c^9 + 225*a^3*c^3*(-(4*a*c - b^2)^15)^(1
/2) - 995*a*b^19*c - 694*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 245*a*b^4*
c*(-(4*a*c - b^2)^15)^(1/2)))/(512*(a^7*b^20*e^2*f^4 + 1048576*a^17*c^10*e^
2*f^4 + 720*a^9*b^16*c^2*e^2*f^4 - 7680*a^10*b^14*c^3*e^2*f^4 + 53760*a^11*
b^12*c^4*e^2*f^4 - 258048*a^12*b^10*c^5*e^2*f^4 + 860160*a^13*b^8*c^6*e^2*f
^4 - 1966080*a^14*b^6*c^7*e^2*f^4 + 2949120*a^15*b^4*c^8*e^2*f^4 - 2621440*
a^16*b^2*c^9*e^2*f^4 - 40*a^8*b^18*c*e^2*f^4)))^(1/2)*(x*(262144*a^15*b^23*
c^2*e^14*f^10 - 11534336*a^16*b^21*c^3*e^14*f^10 + 230686720*a^17*b^19*c^4*
e^14*f^10 - 2768240640*a^18*b^17*c^5*e^14*f^10 + 22145925120*a^19*b^15*c^6*
e^14*f^10 - 124017180672*a^20*b^13*c^7*e^14*f^10 + 496068722688*a^21*b^11*c
^8*e^14*f^10 - 1417339207680*a^22*b^9*c^9*e^14*f^10 + 2834678415360*a^23*b^
7*c^10*e^14*f^10 - 3779571220480*a^24*b^5*c^11*e^14*f^10 + 3023656976384*a^
25*b^3*c^12*e^14*f^10 - 1099511627776*a^26*b*c^13*e^14*f^10) - 109951162777
6*a^26*b*c^13*d*e^13*f^10 + 262144*a^15*b^23*c^2*d*e^13*f^10 - 11534336*a^1
6*b^21*c^3*d*e^13*f^10 + 230686720*a^17*b^19*c^4*d*e^13*f^10 - 2768240640*a
^18*b^17*c^5*d*e^13*f^10 + 22145925120*a^19*b^15*c^6*d*e^13*f^10 - 12401718
0672*a^20*b^13*c^7*d*e^13*f^10 + 496068722688*a^21*b^11*c^8*d*e^13*f^10 - 1
417339207680*a^22*b^9*c^9*d*e^13*f^10 + 2834678415360*a^23*b^7*c^10*d*e^13*
f^10 - 3779571220480*a^24*b^5*c^11*d*e^13*f^10 + 3023656976384*a^25*b^3*c^1
2*d*e^13*f^10) - 245760*a^12*b^23*c^2*e^12*f^8 + 10911744*a^13*b^21*c^3*e^1
2*f^8 - 220397568*a^14*b^19*c^4*e^12*f^8 + 2673082368*a^15*b^17*c^5*e^12*f^
8 - 21630025728*a^16*b^15*c^6*e^12*f^8 + 122607894528*a^17*b^13*c^7*e^12*f^
8 - 496773365760*a^18*b^11*c^8*e^12*f^8 + 1438679826432*a^19*b^9*c^9*e^12*f
^8 - 2918430277632*a^20*b^7*c^10*e^12*f^8 + 3949222428672*a^21*b^5*c^11*e^1
2*f^8 - 3208340570112*a^22*b^3*c^12*e^12*f^8 + 1185410973696*a^23*b*c^13*e^
12*f^8) + 271790899200*a^20*c^14*d*e^11*f^6 - 230400*a^9*b^22*c^3*d*e^11*f^
6 + 9861120*a^10*b^20*c^4*d*e^11*f^6 - 191038464*a^11*b^18*c^5*d*e^11*f^6 +
 2207803392*a^12*b^16*c^6*d*e^11*f^6 - 16878108672*a^13*b^14*c^7*d*e^11*f^6
 + 89374851072*a^14*b^12*c^8*d*e^11*f^6 - 333226967040*a^15*b^10*c^9*d*e^11
*f^6 + 869815812096*a^16*b^8*c^10*d*e^11*f^6 - 1543847804928*a^17*b^6*c^11*
d*e^11*f^6 + 1747313491968*a^18*b^4*c^12*d*e^11*f^6 - 1101055131648*a^19*b^
2*c^13*d*e^11*f^6)*1i + (-(9*(25*b^21 - 25*b^6*(-(4*a*c - b^2)^15)^(1/2) +
18923520*a^10*b*c^10 + 17794*a^2*b^17*c^2 - 188095*a^3*b^15*c^3 + 1299860*a
^4*b^13*c^4 - 6126640*a^5*b^11*c^5 + 19905600*a^6*b^9*c^6 - 43904256*a^7*b^
7*c^7 + 62684160*a^8*b^5*c^8 - 52039680*a^9*b^3*c^9 + 225*a^3*c^3*(-(4*a*c
- b^2)^15)^(1/2) - 995*a*b^19*c - 694*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2)
 + 245*a*b^4*c*(-(4*a*c - b^2)^15)^(1/2)))/(512*(a^7*b^20*e^2*f^4 + 1048576
*a^17*c^10*e^2*f^4 + 720*a^9*b^16*c^2*e^2*f^4 - 7680*a^10*b^14*c^3*e^2*f^4
+ 53760*a^11*b^12*c^4*e^2*f^4 - 258048*a^12*b^10*c^5*e^2*f^4 + 860160*a^13*
b^8*c^6*e^2*f^4 - 1966080*a^14*b^6*c^7*e^2*f^4 + 2949120*a^15*b^4*c^8*e^2*f
^4 - 2621440*a^16*b^2*c^9*e^2*f^4 - 40*a^8*b^18*c*e^2*f^4)))^(1/2)*(x*(2717
90899200*a^20*c^14*e^12*f^6 - 230400*a^9*b^22*c^3*e^12*f^6 + 9861120*a^10*b
^20*c^4*e^12*f^6 - 191038464*a^11*b^18*c^5*e^12*f^6 + 2207803392*a^12*b^16*
c^6*e^12*f^6 - 16878108672*a^13*b^14*c^7*e^12*f^6 + 89374851072*a^14*b^12*c
^8*e^12*f^6 - 333226967040*a^15*b^10*c^9*e^12*f^6 + 869815812096*a^16*b^8*c
^10*e^12*f^6 - 1543847804928*a^17*b^6*c^11*e^12*f^6 + 1747313491968*a^18*b^
4*c^12*e^12*f^6 - 1101055131648*a^19*b^2*c^13*e^12*f^6) - (-(9*(25*b^21 - 2
5*b^6*(-(4*a*c - b^2)^15)^(1/2) + 18923520*a^10*b*c^10 + 17794*a^2*b^17*c^2
 - 188095*a^3*b^15*c^3 + 1299860*a^4*b^13*c^4 - 6126640*a^5*b^11*c^5 + 1990
5600*a^6*b^9*c^6 - 43904256*a^7*b^7*c^7 + 62684160*a^8*b^5*c^8 - 52039680*a
^9*b^3*c^9 + 225*a^3*c^3*(-(4*a*c - b^2)^15)^(1/2) - 995*a*b^19*c - 694*a^2

*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 245*a*b^4*c*(-(4*a*c - b^2)^15)^(1/2))
)/(512*(a^7*b^20*e^2*f^4 + 1048576*a^17*c^10*e^2*f^4 + 720*a^9*b^16*c^2*e^2
*f^4 - 7680*a^10*b^14*c^3*e^2*f^4 + 53760*a^11*b^12*c^4*e^2*f^4 - 258048*a^
12*b^10*c^5*e^2*f^4 + 860160*a^13*b^8*c^6*e^2*f^4 - 1966080*a^14*b^6*c^7*e^
2*f^4 + 2949120*a^15*b^4*c^8*e^2*f^4 - 2621440*a^16*b^2*c^9*e^2*f^4 - 40*a^
8*b^18*c*e^2*f^4)))^(1/2)*((-(9*(25*b^21 - 25*b^6*(-(4*a*c - b^2)^15)^(1/2)
 + 18923520*a^10*b*c^10 + 17794*a^2*b^17*c^2 - 188095*a^3*b^15*c^3 + 129986
0*a^4*b^13*c^4 - 6126640*a^5*b^11*c^5 + 19905600*a^6*b^9*c^6 - 43904256*a^7
*b^7*c^7 + 62684160*a^8*b^5*c^8 - 52039680*a^9*b^3*c^9 + 225*a^3*c^3*(-(4*a
*c - b^2)^15)^(1/2) - 995*a*b^19*c - 694*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1
/2) + 245*a*b^4*c*(-(4*a*c - b^2)^15)^(1/2)))/(512*(a^7*b^20*e^2*f^4 + 1048
576*a^17*c^10*e^2*f^4 + 720*a^9*b^16*c^2*e^2*f^4 - 7680*a^10*b^14*c^3*e^2*f
^4 + 53760*a^11*b^12*c^4*e^2*f^4 - 258048*a^12*b^10*c^5*e^2*f^4 + 860160*a^
13*b^8*c^6*e^2*f^4 - 1966080*a^14*b^6*c^7*e^2*f^4 + 2949120*a^15*b^4*c^8*e^
2*f^4 - 2621440*a^16*b^2*c^9*e^2*f^4 - 40*a^8*b^18*c*e^2*f^4)))^(1/2)*(x*(2
62144*a^15*b^23*c^2*e^14*f^10 - 11534336*a^16*b^21*c^3*e^14*f^10 + 23068672
0*a^17*b^19*c^4*e^14*f^10 - 2768240640*a^18*b^17*c^5*e^14*f^10 + 2214592512
0*a^19*b^15*c^6*e^14*f^10 - 124017180672*a^20*b^13*c^7*e^14*f^10 + 49606872
2688*a^21*b^11*c^8*e^14*f^10 - 1417339207680*a^22*b^9*c^9*e^14*f^10 + 28346
78415360*a^23*b^7*c^10*e^14*f^10 - 3779571220480*a^24*b^5*c^11*e^14*f^10 +
3023656976384*a^25*b^3*c^12*e^14*f^10 - 1099511627776*a^26*b*c^13*e^14*f^10
) - 1099511627776*a^26*b*c^13*d*e^13*f^10 + 262144*a^15*b^23*c^2*d*e^13*f^1
0 - 11534336*a^16*b^21*c^3*d*e^13*f^10 + 230686720*a^17*b^19*c^4*d*e^13*f^1
0 - 2768240640*a^18*b^17*c^5*d*e^13*f^10 + 22145925120*a^19*b^15*c^6*d*e^13
*f^10 - 124017180672*a^20*b^13*c^7*d*e^13*f^10 + 496068722688*a^21*b^11*c^8
*d*e^13*f^10 - 1417339207680*a^22*b^9*c^9*d*e^13*f^10 + 2834678415360*a^23*
b^7*c^10*d*e^13*f^10 - 3779571220480*a^24*b^5*c^11*d*e^13*f^10 + 3023656976
384*a^25*b^3*c^12*d*e^13*f^10) + 245760*a^12*b^23*c^2*e^12*f^8 - 10911744*a
^13*b^21*c^3*e^12*f^8 + 220397568*a^14*b^19*c^4*e^12*f^8 - 2673082368*a^15*
b^17*c^5*e^12*f^8 + 21630025728*a^16*b^15*c^6*e^12*f^8 - 122607894528*a^17*
b^13*c^7*e^12*f^8 + 496773365760*a^18*b^11*c^8*e^12*f^8 - 1438679826432*a^1
9*b^9*c^9*e^12*f^8 + 2918430277632*a^20*b^7*c^10*e^12*f^8 - 3949222428672*a
^21*b^5*c^11*e^12*f^8 + 3208340570112*a^22*b^3*c^12*e^12*f^8 - 118541097369
6*a^23*b*c^13*e^12*f^8) + 271790899200*a^20*c^14*d*e^11*f^6 - 230400*a^9*b^
22*c^3*d*e^11*f^6 + 9861120*a^10*b^20*c^4*d*e^11*f^6 - 191038464*a^11*b^18*
c^5*d*e^11*f^6 + 2207803392*a^12*b^16*c^6*d*e^11*f^6 - 16878108672*a^13*b^1
4*c^7*d*e^11*f^6 + 89374851072*a^14*b^12*c^8*d*e^11*f^6 - 333226967040*a^15
*b^10*c^9*d*e^11*f^6 + 869815812096*a^16*b^8*c^10*d*e^11*f^6 - 154384780492
8*a^17*b^6*c^11*d*e^11*f^6 + 1747313491968*a^18*b^4*c^12*d*e^11*f^6 - 11010
55131648*a^19*b^2*c^13*d*e^11*f^6)*1i)/((-(9*(25*b^21 - 25*b^6*(-(4*a*c - b
^2)^15)^(1/2) + 18923520*a^10*b*c^10 + 17794*a^2*b^17*c^2 - 188095*a^3*b^15
*c^3 + 1299860*a^4*b^13*c^4 - 6126640*a^5*b^11*c^5 + 19905600*a^6*b^9*c^6 -
 43904256*a^7*b^7*c^7 + 62684160*a^8*b^5*c^8 - 52039680*a^9*b^3*c^9 + 225*a
^3*c^3*(-(4*a*c - b^2)^15)^(1/2) - 995*a*b^19*c - 694*a^2*b^2*c^2*(-(4*a*c
- b^2)^15)^(1/2) + 245*a*b^4*c*(-(4*a*c - b^2)^15)^(1/2)))/(512*(a^7*b^20*e
^2*f^4 + 1048576*a^17*c^10*e^2*f^4 + 720*a^9*b^16*c^2*e^2*f^4 - 7680*a^10*b
^14*c^3*e^2*f^4 + 53760*a^11*b^12*c^4*e^2*f^4 - 258048*a^12*b^10*c^5*e^2*f^
4 + 860160*a^13*b^8*c^6*e^2*f^4 - 1966080*a^14*b^6*c^7*e^2*f^4 + 2949120*a^
15*b^4*c^8*e^2*f^4 - 2621440*a^16*b^2*c^9*e^2*f^4 - 40*a^8*b^18*c*e^2*f^4))
)^(1/2)*(x*(271790899200*a^20*c^14*e^12*f^6 - 230400*a^9*b^22*c^3*e^12*f^6
+ 9861120*a^10*b^20*c^4*e^12*f^6 - 191038464*a^11*b^18*c^5*e^12*f^6 + 22078
03392*a^12*b^16*c^6*e^12*f^6 - 16878108672*a^13*b^14*c^7*e^12*f^6 + 8937485
1072*a^14*b^12*c^8*e^12*f^6 - 333226967040*a^15*b^10*c^9*e^12*f^6 + 8698158
12096*a^16*b^8*c^10*e^12*f^6 - 1543847804928*a^17*b^6*c^11*e^12*f^6 + 17473
13491968*a^18*b^4*c^12*e^12*f^6 - 1101055131648*a^19*b^2*c^13*e^12*f^6) - (
-(9*(25*b^21 - 25*b^6*(-(4*a*c - b^2)^15)^(1/2) + 18923520*a^10*b*c^10 + 17
794*a^2*b^17*c^2 - 188095*a^3*b^15*c^3 + 1299860*a^4*b^13*c^4 - 6126640*a^5
*b^11*c^5 + 19905600*a^6*b^9*c^6 - 43904256*a^7*b^7*c^7 + 62684160*a^8*b^5*
c^8 - 52039680*a^9*b^3*c^9 + 225*a^3*c^3*(-(4*a*c - b^2)^15)^(1/2) - 995*a*

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b^19*c - 694*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 245*a*b^4*c*(-(4*a*c -
 b^2)^15)^(1/2)))/(512*(a^7*b^20*e^2*f^4 + 1048576*a^17*c^10*e^2*f^4 + 720*
a^9*b^16*c^2*e^2*f^4 - 7680*a^10*b^14*c^3*e^2*f^4 + 53760*a^11*b^12*c^4*e^2
*f^4 - 258048*a^12*b^10*c^5*e^2*f^4 + 860160*a^13*b^8*c^6*e^2*f^4 - 1966080
*a^14*b^6*c^7*e^2*f^4 + 2949120*a^15*b^4*c^8*e^2*f^4 - 2621440*a^16*b^2*c^9
*e^2*f^4 - 40*a^8*b^18*c*e^2*f^4)))^(1/2)*((-(9*(25*b^21 - 25*b^6*(-(4*a*c
- b^2)^15)^(1/2) + 18923520*a^10*b*c^10 + 17794*a^2*b^17*c^2 - 188095*a^3*b
^15*c^3 + 1299860*a^4*b^13*c^4 - 6126640*a^5*b^11*c^5 + 19905600*a^6*b^9*c^
6 - 43904256*a^7*b^7*c^7 + 62684160*a^8*b^5*c^8 - 52039680*a^9*b^3*c^9 + 22
5*a^3*c^3*(-(4*a*c - b^2)^15)^(1/2) - 995*a*b^19*c - 694*a^2*b^2*c^2*(-(4*a
*c - b^2)^15)^(1/2) + 245*a*b^4*c*(-(4*a*c - b^2)^15)^(1/2)))/(512*(a^7*b^2
0*e^2*f^4 + 1048576*a^17*c^10*e^2*f^4 + 720*a^9*b^16*c^2*e^2*f^4 - 7680*a^1
0*b^14*c^3*e^2*f^4 + 53760*a^11*b^12*c^4*e^2*f^4 - 258048*a^12*b^10*c^5*e^2
*f^4 + 860160*a^13*b^8*c^6*e^2*f^4 - 1966080*a^14*b^6*c^7*e^2*f^4 + 2949120
*a^15*b^4*c^8*e^2*f^4 - 2621440*a^16*b^2*c^9*e^2*f^4 - 40*a^8*b^18*c*e^2*f^
4)))^(1/2)*(x*(262144*a^15*b^23*c^2*e^14*f^10 - 11534336*a^16*b^21*c^3*e^14
*f^10 + 230686720*a^17*b^19*c^4*e^14*f^10 - 2768240640*a^18*b^17*c^5*e^14*f
^10 + 22145925120*a^19*b^15*c^6*e^14*f^10 - 124017180672*a^20*b^13*c^7*e^14
*f^10 + 496068722688*a^21*b^11*c^8*e^14*f^10 - 1417339207680*a^22*b^9*c^9*e
^14*f^10 + 2834678415360*a^23*b^7*c^10*e^14*f^10 - 3779571220480*a^24*b^5*c
^11*e^14*f^10 + 3023656976384*a^25*b^3*c^12*e^14*f^10 - 1099511627776*a^26*
b*c^13*e^14*f^10) - 1099511627776*a^26*b*c^13*d*e^13*f^10 + 262144*a^15*b^2
3*c^2*d*e^13*f^10 - 11534336*a^16*b^21*c^3*d*e^13*f^10 + 230686720*a^17*b^1
9*c^4*d*e^13*f^10 - 2768240640*a^18*b^17*c^5*d*e^13*f^10 + 22145925120*a^19
*b^15*c^6*d*e^13*f^10 - 124017180672*a^20*b^13*c^7*d*e^13*f^10 + 4960687226
88*a^21*b^11*c^8*d*e^13*f^10 - 1417339207680*a^22*b^9*c^9*d*e^13*f^10 + 283
4678415360*a^23*b^7*c^10*d*e^13*f^10 - 3779571220480*a^24*b^5*c^11*d*e^13*f
^10 + 3023656976384*a^25*b^3*c^12*d*e^13*f^10) + 245760*a^12*b^23*c^2*e^12*
f^8 - 10911744*a^13*b^21*c^3*e^12*f^8 + 220397568*a^14*b^19*c^4*e^12*f^8 -
2673082368*a^15*b^17*c^5*e^12*f^8 + 21630025728*a^16*b^15*c^6*e^12*f^8 - 12
2607894528*a^17*b^13*c^7*e^12*f^8 + 496773365760*a^18*b^11*c^8*e^12*f^8 - 1
438679826432*a^19*b^9*c^9*e^12*f^8 + 2918430277632*a^20*b^7*c^10*e^12*f^8 -
 3949222428672*a^21*b^5*c^11*e^12*f^8 + 3208340570112*a^22*b^3*c^12*e^12*f^
8 - 1185410973696*a^23*b*c^13*e^12*f^8) + 271790899200*a^20*c^14*d*e^11*f^6
 - 230400*a^9*b^22*c^3*d*e^11*f^6 + 9861120*a^10*b^20*c^4*d*e^11*f^6 - 1910
38464*a^11*b^18*c^5*d*e^11*f^6 + 2207803392*a^12*b^16*c^6*d*e^11*f^6 - 1687
8108672*a^13*b^14*c^7*d*e^11*f^6 + 89374851072*a^14*b^12*c^8*d*e^11*f^6 - 3
33226967040*a^15*b^10*c^9*d*e^11*f^6 + 869815812096*a^16*b^8*c^10*d*e^11*f^
6 - 1543847804928*a^17*b^6*c^11*d*e^11*f^6 + 1747313491968*a^18*b^4*c^12*d*
e^11*f^6 - 1101055131648*a^19*b^2*c^13*d*e^11*f^6) - (-(9*(25*b^21 - 25*b^6
*(-(4*a*c - b^2)^15)^(1/2) + 18923520*a^10*b*c^10 + 17794*a^2*b^17*c^2 - 18
8095*a^3*b^15*c^3 + 1299860*a^4*b^13*c^4 - 6126640*a^5*b^11*c^5 + 19905600*
a^6*b^9*c^6 - 43904256*a^7*b^7*c^7 + 62684160*a^8*b^5*c^8 - 52039680*a^9*b^
3*c^9 + 225*a^3*c^3*(-(4*a*c - b^2)^15)^(1/2) - 995*a*b^19*c - 694*a^2*b^2*
c^2*(-(4*a*c - b^2)^15)^(1/2) + 245*a*b^4*c*(-(4*a*c - b^2)^15)^(1/2)))/(51
2*(a^7*b^20*e^2*f^4 + 1048576*a^17*c^10*e^2*f^4 + 720*a^9*b^16*c^2*e^2*f^4
- 7680*a^10*b^14*c^3*e^2*f^4 + 53760*a^11*b^12*c^4*e^2*f^4 - 258048*a^12*b^
10*c^5*e^2*f^4 + 860160*a^13*b^8*c^6*e^2*f^4 - 1966080*a^14*b^6*c^7*e^2*f^4
 + 2949120*a^15*b^4*c^8*e^2*f^4 - 2621440*a^16*b^2*c^9*e^2*f^4 - 40*a^8*b^1
8*c*e^2*f^4)))^(1/2)*(x*(271790899200*a^20*c^14*e^12*f^6 - 230400*a^9*b^22*
c^3*e^12*f^6 + 9861120*a^10*b^20*c^4*e^12*f^6 - 191038464*a^11*b^18*c^5*e^1
2*f^6 + 2207803392*a^12*b^16*c^6*e^12*f^6 - 16878108672*a^13*b^14*c^7*e^12*
f^6 + 89374851072*a^14*b^12*c^8*e^12*f^6 - 333226967040*a^15*b^10*c^9*e^12*
f^6 + 869815812096*a^16*b^8*c^10*e^12*f^6 - 1543847804928*a^17*b^6*c^11*e^1
2*f^6 + 1747313491968*a^18*b^4*c^12*e^12*f^6 - 1101055131648*a^19*b^2*c^13*
e^12*f^6) - (-(9*(25*b^21 - 25*b^6*(-(4*a*c - b^2)^15)^(1/2) + 18923520*a^1
0*b*c^10 + 17794*a^2*b^17*c^2 - 188095*a^3*b^15*c^3 + 1299860*a^4*b^13*c^4
- 6126640*a^5*b^11*c^5 + 19905600*a^6*b^9*c^6 - 43904256*a^7*b^7*c^7 + 6268
4160*a^8*b^5*c^8 - 52039680*a^9*b^3*c^9 + 225*a^3*c^3*(-(4*a*c - b^2)^15)^(
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1/2) - 995*a*b^19*c - 694*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 245*a*b^4
c(-(4*a*c - b^2)^15)^(1/2)))/(512*(a^7*b^20*e^2*f^4 + 1048576*a^17*c^10*e
^2*f^4 + 720*a^9*b^16*c^2*e^2*f^4 - 7680*a^10*b^14*c^3*e^2*f^4 + 53760*a^11
*b^12*c^4*e^2*f^4 - 258048*a^12*b^10*c^5*e^2*f^4 + 860160*a^13*b^8*c^6*e^2*
f^4 - 1966080*a^14*b^6*c^7*e^2*f^4 + 2949120*a^15*b^4*c^8*e^2*f^4 - 2621440
*a^16*b^2*c^9*e^2*f^4 - 40*a^8*b^18*c*e^2*f^4)))^(1/2)*((-(9*(25*b^21 - 25*
b^6*(-(4*a*c - b^2)^15)^(1/2) + 18923520*a^10*b*c^10 + 17794*a^2*b^17*c^2 -
188095*a^3*b^15*c^3 + 1299860*a^4*b^13*c^4 - 6126640*a^5*b^11*c^5 + 199056
00*a^6*b^9*c^6 - 43904256*a^7*b^7*c^7 + 62684160*a^8*b^5*c^8 - 52039680*a^9
*b^3*c^9 + 225*a^3*c^3*(-(4*a*c - b^2)^15)^(1/2) - 995*a*b^19*c - 694*a^2*b
^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 245*a*b^4*c*(-(4*a*c - b^2)^15)^(1/2)))/
(512*(a^7*b^20*e^2*f^4 + 1048576*a^17*c^10*e^2*f^4 + 720*a^9*b^16*c^2*e^2*f
^4 - 7680*a^10*b^14*c^3*e^2*f^4 + 53760*a^11*b^12*c^4*e^2*f^4 - 258048*a^12
*b^10*c^5*e^2*f^4 + 860160*a^13*b^8*c^6*e^2*f^4 - 1966080*a^14*b^6*c^7*e^2*
f^4 + 2949120*a^15*b^4*c^8*e^2*f^4 - 2621440*a^16*b^2*c^9*e^2*f^4 - 40*a^8*
b^18*c*e^2*f^4)))^(1/2)*(x*(262144*a^15*b^23*c^2*e^14*f^10 - 11534336*a^16*
b^21*c^3*e^14*f^10 + 230686720*a^17*b^19*c^4*e^14*f^10 - 2768240640*a^18*b^
17*c^5*e^14*f^10 + 22145925120*a^19*b^15*c^6*e^14*f^10 - 124017180672*a^20*
b^13*c^7*e^14*f^10 + 496068722688*a^21*b^11*c^8*e^14*f^10 - 1417339207680*a
^22*b^9*c^9*e^14*f^10 + 2834678415360*a^23*b^7*c^10*e^14*f^10 - 37795712204
80*a^24*b^5*c^11*e^14*f^10 + 3023656976384*a^25*b^3*c^12*e^14*f^10 - 109951
1627776*a^26*b*c^13*e^14*f^10) - 1099511627776*a^26*b*c^13*d*e^13*f^10 + 26
2144*a^15*b^23*c^2*d*e^13*f^10 - 11534336*a^16*b^21*c^3*d*e^13*f^10 + 23068
6720*a^17*b^19*c^4*d*e^13*f^10 - 2768240640*a^18*b^17*c^5*d*e^13*f^10 + 221
45925120*a^19*b^15*c^6*d*e^13*f^10 - 124017180672*a^20*b^13*c^7*d*e^13*f^10
+ 496068722688*a^21*b^11*c^8*d*e^13*f^10 - 1417339207680*a^22*b^9*c^9*d*e^
13*f^10 + 2834678415360*a^23*b^7*c^10*d*e^13*f^10 - 3779571220480*a^24*b^5*
c^11*d*e^13*f^10 + 3023656976384*a^25*b^3*c^12*d*e^13*f^10) - 245760*a^12*b
^23*c^2*e^12*f^8 + 10911744*a^13*b^21*c^3*e^12*f^8 - 220397568*a^14*b^19*c^
4*e^12*f^8 + 2673082368*a^15*b^17*c^5*e^12*f^8 - 21630025728*a^16*b^15*c^6*
e^12*f^8 + 122607894528*a^17*b^13*c^7*e^12*f^8 - 496773365760*a^18*b^11*c^8
*e^12*f^8 + 1438679826432*a^19*b^9*c^9*e^12*f^8 - 2918430277632*a^20*b^7*c^
10*e^12*f^8 + 3949222428672*a^21*b^5*c^11*e^12*f^8 - 3208340570112*a^22*b^3
*c^12*e^12*f^8 + 1185410973696*a^23*b*c^13*e^12*f^8) + 271790899200*a^20*c^
14*d*e^11*f^6 - 230400*a^9*b^22*c^3*d*e^11*f^6 + 9861120*a^10*b^20*c^4*d*e^
11*f^6 - 191038464*a^11*b^18*c^5*d*e^11*f^6 + 2207803392*a^12*b^16*c^6*d*e^
11*f^6 - 16878108672*a^13*b^14*c^7*d*e^11*f^6 + 89374851072*a^14*b^12*c^8*d
*e^11*f^6 - 333226967040*a^15*b^10*c^9*d*e^11*f^6 + 869815812096*a^16*b^8*c
^10*d*e^11*f^6 - 1543847804928*a^17*b^6*c^11*d*e^11*f^6 + 1747313491968*a^1
8*b^4*c^12*d*e^11*f^6 - 1101055131648*a^19*b^2*c^13*d*e^11*f^6) + 191102976
000*a^17*c^14*e^10*f^4 + 2851200*a^9*b^16*c^6*e^10*f^4 - 92568960*a^10*b^14
*c^7*e^10*f^4 + 1312630272*a^11*b^12*c^8*e^10*f^4 - 10611136512*a^12*b^10*c
^9*e^10*f^4 + 53445353472*a^13*b^8*c^10*e^10*f^4 - 171591892992*a^14*b^6*c^
11*e^10*f^4 + 342580396032*a^15*b^4*c^12*e^10*f^4 - 388363714560*a^16*b^2*c
^13*e^10*f^4))*(-(9*(25*b^21 - 25*b^6*(-(4*a*c - b^2)^15)^(1/2) + 18923520*
a^10*b*c^10 + 17794*a^2*b^17*c^2 - 188095*a^3*b^15*c^3 + 1299860*a^4*b^13*c
^4 - 6126640*a^5*b^11*c^5 + 19905600*a^6*b^9*c^6 - 43904256*a^7*b^7*c^7 + 6
2684160*a^8*b^5*c^8 - 52039680*a^9*b^3*c^9 + 225*a^3*c^3*(-(4*a*c - b^2)^15
)^(1/2) - 995*a*b^19*c - 694*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 245*a*
b^4*c*(-(4*a*c - b^2)^15)^(1/2)))/(512*(a^7*b^20*e^2*f^4 + 1048576*a^17*c^1
0*e^2*f^4 + 720*a^9*b^16*c^2*e^2*f^4 - 7680*a^10*b^14*c^3*e^2*f^4 + 53760*a
^11*b^12*c^4*e^2*f^4 - 258048*a^12*b^10*c^5*e^2*f^4 + 860160*a^13*b^8*c^6*e
^2*f^4 - 1966080*a^14*b^6*c^7*e^2*f^4 + 2949120*a^15*b^4*c^8*e^2*f^4 - 2621
440*a^16*b^2*c^9*e^2*f^4 - 40*a^8*b^18*c*e^2*f^4)))^(1/2)*2i - atan(((-(9*(
25*b^21 + 25*b^6*(-(4*a*c - b^2)^15)^(1/2) + 18923520*a^10*b*c^10 + 17794*a
^2*b^17*c^2 - 188095*a^3*b^15*c^3 + 1299860*a^4*b^13*c^4 - 6126640*a^5*b^11
*c^5 + 19905600*a^6*b^9*c^6 - 43904256*a^7*b^7*c^7 + 62684160*a^8*b^5*c^8 -
52039680*a^9*b^3*c^9 - 225*a^3*c^3*(-(4*a*c - b^2)^15)^(1/2) - 995*a*b^19*
c + 694*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 245*a*b^4*c*(-(4*a*c - b^2)

^15)^(1/2)))/(512*(a^7*b^20*e^2*f^4 + 1048576*a^17*c^10*e^2*f^4 + 720*a^9*b
^16*c^2*e^2*f^4 - 7680*a^10*b^14*c^3*e^2*f^4 + 53760*a^11*b^12*c^4*e^2*f^4
- 258048*a^12*b^10*c^5*e^2*f^4 + 860160*a^13*b^8*c^6*e^2*f^4 - 1966080*a^14
*b^6*c^7*e^2*f^4 + 2949120*a^15*b^4*c^8*e^2*f^4 - 2621440*a^16*b^2*c^9*e^2*
f^4 - 40*a^8*b^18*c*e^2*f^4)))^(1/2)*(x*(271790899200*a^20*c^14*e^12*f^6 -
230400*a^9*b^22*c^3*e^12*f^6 + 9861120*a^10*b^20*c^4*e^12*f^6 - 191038464*a
^11*b^18*c^5*e^12*f^6 + 2207803392*a^12*b^16*c^6*e^12*f^6 - 16878108672*a^1
3*b^14*c^7*e^12*f^6 + 89374851072*a^14*b^12*c^8*e^12*f^6 - 333226967040*a^1
5*b^10*c^9*e^12*f^6 + 869815812096*a^16*b^8*c^10*e^12*f^6 - 1543847804928*a
^17*b^6*c^11*e^12*f^6 + 1747313491968*a^18*b^4*c^12*e^12*f^6 - 110105513164
8*a^19*b^2*c^13*e^12*f^6) - (-(9*(25*b^21 + 25*b^6*(-(4*a*c - b^2)^15)^(1/2
) + 18923520*a^10*b*c^10 + 17794*a^2*b^17*c^2 - 188095*a^3*b^15*c^3 + 12998
60*a^4*b^13*c^4 - 6126640*a^5*b^11*c^5 + 19905600*a^6*b^9*c^6 - 43904256*a^
7*b^7*c^7 + 62684160*a^8*b^5*c^8 - 52039680*a^9*b^3*c^9 - 225*a^3*c^3*(-(4*
a*c - b^2)^15)^(1/2) - 995*a*b^19*c + 694*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(
1/2) - 245*a*b^4*c*(-(4*a*c - b^2)^15)^(1/2)))/(512*(a^7*b^20*e^2*f^4 + 104
8576*a^17*c^10*e^2*f^4 + 720*a^9*b^16*c^2*e^2*f^4 - 7680*a^10*b^14*c^3*e^2*
f^4 + 53760*a^11*b^12*c^4*e^2*f^4 - 258048*a^12*b^10*c^5*e^2*f^4 + 860160*a
^13*b^8*c^6*e^2*f^4 - 1966080*a^14*b^6*c^7*e^2*f^4 + 2949120*a^15*b^4*c^8*e
^2*f^4 - 2621440*a^16*b^2*c^9*e^2*f^4 - 40*a^8*b^18*c*e^2*f^4)))^(1/2)*((-(
9*(25*b^21 + 25*b^6*(-(4*a*c - b^2)^15)^(1/2) + 18923520*a^10*b*c^10 + 1779
4*a^2*b^17*c^2 - 188095*a^3*b^15*c^3 + 1299860*a^4*b^13*c^4 - 6126640*a^5*b
^11*c^5 + 19905600*a^6*b^9*c^6 - 43904256*a^7*b^7*c^7 + 62684160*a^8*b^5*c^
8 - 52039680*a^9*b^3*c^9 - 225*a^3*c^3*(-(4*a*c - b^2)^15)^(1/2) - 995*a*b^
19*c + 694*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 245*a*b^4*c*(-(4*a*c - b
^2)^15)^(1/2)))/(512*(a^7*b^20*e^2*f^4 + 1048576*a^17*c^10*e^2*f^4 + 720*a^
9*b^16*c^2*e^2*f^4 - 7680*a^10*b^14*c^3*e^2*f^4 + 53760*a^11*b^12*c^4*e^2*f
^4 - 258048*a^12*b^10*c^5*e^2*f^4 + 860160*a^13*b^8*c^6*e^2*f^4 - 1966080*a
^14*b^6*c^7*e^2*f^4 + 2949120*a^15*b^4*c^8*e^2*f^4 - 2621440*a^16*b^2*c^9*e
^2*f^4 - 40*a^8*b^18*c*e^2*f^4)))^(1/2)*(x*(262144*a^15*b^23*c^2*e^14*f^10
- 11534336*a^16*b^21*c^3*e^14*f^10 + 230686720*a^17*b^19*c^4*e^14*f^10 - 27
68240640*a^18*b^17*c^5*e^14*f^10 + 22145925120*a^19*b^15*c^6*e^14*f^10 - 12
4017180672*a^20*b^13*c^7*e^14*f^10 + 496068722688*a^21*b^11*c^8*e^14*f^10 -
1417339207680*a^22*b^9*c^9*e^14*f^10 + 2834678415360*a^23*b^7*c^10*e^14*f^
10 - 3779571220480*a^24*b^5*c^11*e^14*f^10 + 3023656976384*a^25*b^3*c^12*e^
14*f^10 - 1099511627776*a^26*b*c^13*e^14*f^10) - 1099511627776*a^26*b*c^13*
d*e^13*f^10 + 262144*a^15*b^23*c^2*d*e^13*f^10 - 11534336*a^16*b^21*c^3*d*e
^13*f^10 + 230686720*a^17*b^19*c^4*d*e^13*f^10 - 2768240640*a^18*b^17*c^5*d
*e^13*f^10 + 22145925120*a^19*b^15*c^6*d*e^13*f^10 - 124017180672*a^20*b^13
*c^7*d*e^13*f^10 + 496068722688*a^21*b^11*c^8*d*e^13*f^10 - 1417339207680*a
^22*b^9*c^9*d*e^13*f^10 + 2834678415360*a^23*b^7*c^10*d*e^13*f^10 - 3779571
220480*a^24*b^5*c^11*d*e^13*f^10 + 3023656976384*a^25*b^3*c^12*d*e^13*f^10)
- 245760*a^12*b^23*c^2*e^12*f^8 + 10911744*a^13*b^21*c^3*e^12*f^8 - 220397
568*a^14*b^19*c^4*e^12*f^8 + 2673082368*a^15*b^17*c^5*e^12*f^8 - 2163002572
8*a^16*b^15*c^6*e^12*f^8 + 122607894528*a^17*b^13*c^7*e^12*f^8 - 4967733657
60*a^18*b^11*c^8*e^12*f^8 + 1438679826432*a^19*b^9*c^9*e^12*f^8 - 291843027
7632*a^20*b^7*c^10*e^12*f^8 + 3949222428672*a^21*b^5*c^11*e^12*f^8 - 320834
0570112*a^22*b^3*c^12*e^12*f^8 + 1185410973696*a^23*b*c^13*e^12*f^8) + 2717
90899200*a^20*c^14*d*e^11*f^6 - 230400*a^9*b^22*c^3*d*e^11*f^6 + 9861120*a^
10*b^20*c^4*d*e^11*f^6 - 191038464*a^11*b^18*c^5*d*e^11*f^6 + 2207803392*a^
12*b^16*c^6*d*e^11*f^6 - 16878108672*a^13*b^14*c^7*d*e^11*f^6 + 89374851072
*a^14*b^12*c^8*d*e^11*f^6 - 333226967040*a^15*b^10*c^9*d*e^11*f^6 + 8698158
12096*a^16*b^8*c^10*d*e^11*f^6 - 1543847804928*a^17*b^6*c^11*d*e^11*f^6 + 1
747313491968*a^18*b^4*c^12*d*e^11*f^6 - 1101055131648*a^19*b^2*c^13*d*e^11*
f^6)*1i + (-(9*(25*b^21 + 25*b^6*(-(4*a*c - b^2)^15)^(1/2) + 18923520*a^10*
b*c^10 + 17794*a^2*b^17*c^2 - 188095*a^3*b^15*c^3 + 1299860*a^4*b^13*c^4 -
6126640*a^5*b^11*c^5 + 19905600*a^6*b^9*c^6 - 43904256*a^7*b^7*c^7 + 626841
60*a^8*b^5*c^8 - 52039680*a^9*b^3*c^9 - 225*a^3*c^3*(-(4*a*c - b^2)^15)^(1/
2) - 995*a*b^19*c + 694*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 245*a*b^4*c

*(-(4*a*c - b^2)^15)^(1/2)))/(512*(a^7*b^20*e^2*f^4 + 1048576*a^17*c^10*e^2
*f^4 + 720*a^9*b^16*c^2*e^2*f^4 - 7680*a^10*b^14*c^3*e^2*f^4 + 53760*a^11*b
^12*c^4*e^2*f^4 - 258048*a^12*b^10*c^5*e^2*f^4 + 860160*a^13*b^8*c^6*e^2*f^
4 - 1966080*a^14*b^6*c^7*e^2*f^4 + 2949120*a^15*b^4*c^8*e^2*f^4 - 2621440*a
^16*b^2*c^9*e^2*f^4 - 40*a^8*b^18*c*e^2*f^4)))^(1/2)*(x*(271790899200*a^20*
c^14*e^12*f^6 - 230400*a^9*b^22*c^3*e^12*f^6 + 9861120*a^10*b^20*c^4*e^12*f
^6 - 191038464*a^11*b^18*c^5*e^12*f^6 + 2207803392*a^12*b^16*c^6*e^12*f^6 -
 16878108672*a^13*b^14*c^7*e^12*f^6 + 89374851072*a^14*b^12*c^8*e^12*f^6 -
333226967040*a^15*b^10*c^9*e^12*f^6 + 869815812096*a^16*b^8*c^10*e^12*f^6 -
 1543847804928*a^17*b^6*c^11*e^12*f^6 + 1747313491968*a^18*b^4*c^12*e^12*f^
6 - 1101055131648*a^19*b^2*c^13*e^12*f^6) - (-(9*(25*b^21 + 25*b^6*(-(4*a*c
 - b^2)^15)^(1/2) + 18923520*a^10*b*c^10 + 17794*a^2*b^17*c^2 - 188095*a^3*
b^15*c^3 + 1299860*a^4*b^13*c^4 - 6126640*a^5*b^11*c^5 + 19905600*a^6*b^9*c
^6 - 43904256*a^7*b^7*c^7 + 62684160*a^8*b^5*c^8 - 52039680*a^9*b^3*c^9 - 2
25*a^3*c^3*(-(4*a*c - b^2)^15)^(1/2) - 995*a*b^19*c + 694*a^2*b^2*c^2*(-(4*
a*c - b^2)^15)^(1/2) - 245*a*b^4*c*(-(4*a*c - b^2)^15)^(1/2)))/(512*(a^7*b^
20*e^2*f^4 + 1048576*a^17*c^10*e^2*f^4 + 720*a^9*b^16*c^2*e^2*f^4 - 7680*a^
10*b^14*c^3*e^2*f^4 + 53760*a^11*b^12*c^4*e^2*f^4 - 258048*a^12*b^10*c^5*e^
2*f^4 + 860160*a^13*b^8*c^6*e^2*f^4 - 1966080*a^14*b^6*c^7*e^2*f^4 + 294912
0*a^15*b^4*c^8*e^2*f^4 - 2621440*a^16*b^2*c^9*e^2*f^4 - 40*a^8*b^18*c*e^2*f
^4)))^(1/2)*((-(9*(25*b^21 + 25*b^6*(-(4*a*c - b^2)^15)^(1/2) + 18923520*a^
10*b*c^10 + 17794*a^2*b^17*c^2 - 188095*a^3*b^15*c^3 + 1299860*a^4*b^13*c^4
 - 6126640*a^5*b^11*c^5 + 19905600*a^6*b^9*c^6 - 43904256*a^7*b^7*c^7 + 626
84160*a^8*b^5*c^8 - 52039680*a^9*b^3*c^9 - 225*a^3*c^3*(-(4*a*c - b^2)^15)^
(1/2) - 995*a*b^19*c + 694*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 245*a*b^
4*c*(-(4*a*c - b^2)^15)^(1/2)))/(512*(a^7*b^20*e^2*f^4 + 1048576*a^17*c^10*
e^2*f^4 + 720*a^9*b^16*c^2*e^2*f^4 - 7680*a^10*b^14*c^3*e^2*f^4 + 53760*a^1
1*b^12*c^4*e^2*f^4 - 258048*a^12*b^10*c^5*e^2*f^4 + 860160*a^13*b^8*c^6*e^2
*f^4 - 1966080*a^14*b^6*c^7*e^2*f^4 + 2949120*a^15*b^4*c^8*e^2*f^4 - 262144
0*a^16*b^2*c^9*e^2*f^4 - 40*a^8*b^18*c*e^2*f^4)))^(1/2)*(x*(262144*a^15*b^2
3*c^2*e^14*f^10 - 11534336*a^16*b^21*c^3*e^14*f^10 + 230686720*a^17*b^19*c^
4*e^14*f^10 - 2768240640*a^18*b^17*c^5*e^14*f^10 + 22145925120*a^19*b^15*c^
6*e^14*f^10 - 124017180672*a^20*b^13*c^7*e^14*f^10 + 496068722688*a^21*b^11
*c^8*e^14*f^10 - 1417339207680*a^22*b^9*c^9*e^14*f^10 + 2834678415360*a^23*
b^7*c^10*e^14*f^10 - 3779571220480*a^24*b^5*c^11*e^14*f^10 + 3023656976384*
a^25*b^3*c^12*e^14*f^10 - 1099511627776*a^26*b*c^13*e^14*f^10) - 1099511627
776*a^26*b*c^13*d*e^13*f^10 + 262144*a^15*b^23*c^2*d*e^13*f^10 - 11534336*a
^16*b^21*c^3*d*e^13*f^10 + 230686720*a^17*b^19*c^4*d*e^13*f^10 - 2768240640
*a^18*b^17*c^5*d*e^13*f^10 + 22145925120*a^19*b^15*c^6*d*e^13*f^10 - 124017
180672*a^20*b^13*c^7*d*e^13*f^10 + 496068722688*a^21*b^11*c^8*d*e^13*f^10 -
 1417339207680*a^22*b^9*c^9*d*e^13*f^10 + 2834678415360*a^23*b^7*c^10*d*e^1
3*f^10 - 3779571220480*a^24*b^5*c^11*d*e^13*f^10 + 3023656976384*a^25*b^3*c
^12*d*e^13*f^10) + 245760*a^12*b^23*c^2*e^12*f^8 - 10911744*a^13*b^21*c^3*e
^12*f^8 + 220397568*a^14*b^19*c^4*e^12*f^8 - 2673082368*a^15*b^17*c^5*e^12*
f^8 + 21630025728*a^16*b^15*c^6*e^12*f^8 - 122607894528*a^17*b^13*c^7*e^12*
f^8 + 496773365760*a^18*b^11*c^8*e^12*f^8 - 1438679826432*a^19*b^9*c^9*e^12
*f^8 + 2918430277632*a^20*b^7*c^10*e^12*f^8 - 3949222428672*a^21*b^5*c^11*e
^12*f^8 + 3208340570112*a^22*b^3*c^12*e^12*f^8 - 1185410973696*a^23*b*c^13*
e^12*f^8) + 271790899200*a^20*c^14*d*e^11*f^6 - 230400*a^9*b^22*c^3*d*e^11*
f^6 + 9861120*a^10*b^20*c^4*d*e^11*f^6 - 191038464*a^11*b^18*c^5*d*e^11*f^6
 + 2207803392*a^12*b^16*c^6*d*e^11*f^6 - 16878108672*a^13*b^14*c^7*d*e^11*f
^6 + 89374851072*a^14*b^12*c^8*d*e^11*f^6 - 333226967040*a^15*b^10*c^9*d*e^
11*f^6 + 869815812096*a^16*b^8*c^10*d*e^11*f^6 - 1543847804928*a^17*b^6*c^1
1*d*e^11*f^6 + 1747313491968*a^18*b^4*c^12*d*e^11*f^6 - 1101055131648*a^19*
b^2*c^13*d*e^11*f^6)*1i)/((-(9*(25*b^21 + 25*b^6*(-(4*a*c - b^2)^15)^(1/2)
+ 18923520*a^10*b*c^10 + 17794*a^2*b^17*c^2 - 188095*a^3*b^15*c^3 + 1299860
*a^4*b^13*c^4 - 6126640*a^5*b^11*c^5 + 19905600*a^6*b^9*c^6 - 43904256*a^7*
b^7*c^7 + 62684160*a^8*b^5*c^8 - 52039680*a^9*b^3*c^9 - 225*a^3*c^3*(-(4*a*
c - b^2)^15)^(1/2) - 995*a*b^19*c + 694*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/

2) - 245*a*b^4*c*(-(4*a*c - b^2)^15)^(1/2)))/(512*(a^7*b^20*e^2*f^4 + 10485
76*a^17*c^10*e^2*f^4 + 720*a^9*b^16*c^2*e^2*f^4 - 7680*a^10*b^14*c^3*e^2*f^
4 + 53760*a^11*b^12*c^4*e^2*f^4 - 258048*a^12*b^10*c^5*e^2*f^4 + 860160*a^1
3*b^8*c^6*e^2*f^4 - 1966080*a^14*b^6*c^7*e^2*f^4 + 2949120*a^15*b^4*c^8*e^2
*f^4 - 2621440*a^16*b^2*c^9*e^2*f^4 - 40*a^8*b^18*c*e^2*f^4)))^(1/2)*(x*(27
1790899200*a^20*c^14*e^12*f^6 - 230400*a^9*b^22*c^3*e^12*f^6 + 9861120*a^10
*b^20*c^4*e^12*f^6 - 191038464*a^11*b^18*c^5*e^12*f^6 + 2207803392*a^12*b^1
6*c^6*e^12*f^6 - 16878108672*a^13*b^14*c^7*e^12*f^6 + 89374851072*a^14*b^12
*c^8*e^12*f^6 - 333226967040*a^15*b^10*c^9*e^12*f^6 + 869815812096*a^16*b^8
*c^10*e^12*f^6 - 1543847804928*a^17*b^6*c^11*e^12*f^6 + 1747313491968*a^18*
b^4*c^12*e^12*f^6 - 1101055131648*a^19*b^2*c^13*e^12*f^6) - (-(9*(25*b^21 +
 25*b^6*(-(4*a*c - b^2)^15)^(1/2) + 18923520*a^10*b*c^10 + 17794*a^2*b^17*c
^2 - 188095*a^3*b^15*c^3 + 1299860*a^4*b^13*c^4 - 6126640*a^5*b^11*c^5 + 19
905600*a^6*b^9*c^6 - 43904256*a^7*b^7*c^7 + 62684160*a^8*b^5*c^8 - 52039680
*a^9*b^3*c^9 - 225*a^3*c^3*(-(4*a*c - b^2)^15)^(1/2) - 995*a*b^19*c + 694*a
^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 245*a*b^4*c*(-(4*a*c - b^2)^15)^(1/2
)))/(512*(a^7*b^20*e^2*f^4 + 1048576*a^17*c^10*e^2*f^4 + 720*a^9*b^16*c^2*e
^2*f^4 - 7680*a^10*b^14*c^3*e^2*f^4 + 53760*a^11*b^12*c^4*e^2*f^4 - 258048*
a^12*b^10*c^5*e^2*f^4 + 860160*a^13*b^8*c^6*e^2*f^4 - 1966080*a^14*b^6*c^7*
e^2*f^4 + 2949120*a^15*b^4*c^8*e^2*f^4 - 2621440*a^16*b^2*c^9*e^2*f^4 - 40*
a^8*b^18*c*e^2*f^4)))^(1/2)*((-(9*(25*b^21 + 25*b^6*(-(4*a*c - b^2)^15)^(1/
2) + 18923520*a^10*b*c^10 + 17794*a^2*b^17*c^2 - 188095*a^3*b^15*c^3 + 1299
860*a^4*b^13*c^4 - 6126640*a^5*b^11*c^5 + 19905600*a^6*b^9*c^6 - 43904256*a
^7*b^7*c^7 + 62684160*a^8*b^5*c^8 - 52039680*a^9*b^3*c^9 - 225*a^3*c^3*(-(4
*a*c - b^2)^15)^(1/2) - 995*a*b^19*c + 694*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^
(1/2) - 245*a*b^4*c*(-(4*a*c - b^2)^15)^(1/2)))/(512*(a^7*b^20*e^2*f^4 + 10
48576*a^17*c^10*e^2*f^4 + 720*a^9*b^16*c^2*e^2*f^4 - 7680*a^10*b^14*c^3*e^2
*f^4 + 53760*a^11*b^12*c^4*e^2*f^4 - 258048*a^12*b^10*c^5*e^2*f^4 + 860160*
a^13*b^8*c^6*e^2*f^4 - 1966080*a^14*b^6*c^7*e^2*f^4 + 2949120*a^15*b^4*c^8*
e^2*f^4 - 2621440*a^16*b^2*c^9*e^2*f^4 - 40*a^8*b^18*c*e^2*f^4)))^(1/2)*(x*
(262144*a^15*b^23*c^2*e^14*f^10 - 11534336*a^16*b^21*c^3*e^14*f^10 + 230686
720*a^17*b^19*c^4*e^14*f^10 - 2768240640*a^18*b^17*c^5*e^14*f^10 + 22145925
120*a^19*b^15*c^6*e^14*f^10 - 124017180672*a^20*b^13*c^7*e^14*f^10 + 496068
722688*a^21*b^11*c^8*e^14*f^10 - 1417339207680*a^22*b^9*c^9*e^14*f^10 + 283
4678415360*a^23*b^7*c^10*e^14*f^10 - 3779571220480*a^24*b^5*c^11*e^14*f^10
+ 3023656976384*a^25*b^3*c^12*e^14*f^10 - 1099511627776*a^26*b*c^13*e^14*f^
10) - 1099511627776*a^26*b*c^13*d*e^13*f^10 + 262144*a^15*b^23*c^2*d*e^13*f
^10 - 11534336*a^16*b^21*c^3*d*e^13*f^10 + 230686720*a^17*b^19*c^4*d*e^13*f
^10 - 2768240640*a^18*b^17*c^5*d*e^13*f^10 + 22145925120*a^19*b^15*c^6*d*e^
13*f^10 - 124017180672*a^20*b^13*c^7*d*e^13*f^10 + 496068722688*a^21*b^11*c
^8*d*e^13*f^10 - 1417339207680*a^22*b^9*c^9*d*e^13*f^10 + 2834678415360*a^2
3*b^7*c^10*d*e^13*f^10 - 3779571220480*a^24*b^5*c^11*d*e^13*f^10 + 30236569
76384*a^25*b^3*c^12*d*e^13*f^10) + 245760*a^12*b^23*c^2*e^12*f^8 - 10911744
*a^13*b^21*c^3*e^12*f^8 + 220397568*a^14*b^19*c^4*e^12*f^8 - 2673082368*a^1
5*b^17*c^5*e^12*f^8 + 21630025728*a^16*b^15*c^6*e^12*f^8 - 122607894528*a^1
7*b^13*c^7*e^12*f^8 + 496773365760*a^18*b^11*c^8*e^12*f^8 - 1438679826432*a
^19*b^9*c^9*e^12*f^8 + 2918430277632*a^20*b^7*c^10*e^12*f^8 - 3949222428672
*a^21*b^5*c^11*e^12*f^8 + 3208340570112*a^22*b^3*c^12*e^12*f^8 - 1185410973
696*a^23*b*c^13*e^12*f^8) + 271790899200*a^20*c^14*d*e^11*f^6 - 230400*a^9*
b^22*c^3*d*e^11*f^6 + 9861120*a^10*b^20*c^4*d*e^11*f^6 - 191038464*a^11*b^1
8*c^5*d*e^11*f^6 + 2207803392*a^12*b^16*c^6*d*e^11*f^6 - 16878108672*a^13*b
^14*c^7*d*e^11*f^6 + 89374851072*a^14*b^12*c^8*d*e^11*f^6 - 333226967040*a^
15*b^10*c^9*d*e^11*f^6 + 869815812096*a^16*b^8*c^10*d*e^11*f^6 - 1543847804
928*a^17*b^6*c^11*d*e^11*f^6 + 1747313491968*a^18*b^4*c^12*d*e^11*f^6 - 110
1055131648*a^19*b^2*c^13*d*e^11*f^6) - (-(9*(25*b^21 + 25*b^6*(-(4*a*c - b^
2)^15)^(1/2) + 18923520*a^10*b*c^10 + 17794*a^2*b^17*c^2 - 188095*a^3*b^15*
c^3 + 1299860*a^4*b^13*c^4 - 6126640*a^5*b^11*c^5 + 19905600*a^6*b^9*c^6 -
43904256*a^7*b^7*c^7 + 62684160*a^8*b^5*c^8 - 52039680*a^9*b^3*c^9 - 225*a^
3*c^3*(-(4*a*c - b^2)^15)^(1/2) - 995*a*b^19*c + 694*a^2*b^2*c^2*(-(4*a*c -

b^2)^15)^(1/2) - 245*a*b^4*c*(-(4*a*c - b^2)^15)^(1/2)))/(512*(a^7*b^20*e^2*f^4 + 1048576*a^17*c^10*e^2*f^4 + 720*a^9*b^16*c^2*e^2*f^4 - 7680*a^10*b^14*c^3*e^2*f^4 + 53760*a^11*b^12*c^4*e^2*f^4 - 258048*a^12*b^10*c^5*e^2*f^4 + 860160*a^13*b^8*c^6*e^2*f^4 - 1966080*a^14*b^6*c^7*e^2*f^4 + 2949120*a^15*b^4*c^8*e^2*f^4 - 2621440*a^16*b^2*c^9*e^2*f^4 - 40*a^8*b^18*c*e^2*f^4)))^(1/2)*(x*(271790899200*a^20*c^14*e^12*f^6 - 230400*a^9*b^22*c^3*e^12*f^6 + 9861120*a^10*b^20*c^4*e^12*f^6 - 191038464*a^11*b^18*c^5*e^12*f^6 + 2207803392*a^12*b^16*c^6*e^12*f^6 - 16878108672*a^13*b^14*c^7*e^12*f^6 + 89374851072*a^14*b^12*c^8*e^12*f^6 - 333226967040*a^15*b^10*c^9*e^12*f^6 + 869815812096*a^16*b^8*c^10*e^12*f^6 - 1543847804928*a^17*b^6*c^11*e^12*f^6 + 1747313491968*a^18*b^4*c^12*e^12*f^6 - 1101055131648*a^19*b^2*c^13*e^12*f^6) - (-(9*(25*b^21 + 25*b^6*(-(4*a*c - b^2)^15)^(1/2) + 18923520*a^10*b*c^10 + 17794*a^2*b^17*c^2 - 188095*a^3*b^15*c^3 + 1299860*a^4*b^13*c^4 - 6126640*a^5*b^11*c^5 + 19905600*a^6*b^9*c^6 - 43904256*a^7*b^7*c^7 + 62684160*a^8*b^5*c^8 - 52039680*a^9*b^3*c^9 - 225*a^3*c^3*(-(4*a*c - b^2)^15)^(1/2) - 995*a*b^19*c + 694*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 245*a*b^4*c*(-(4*a*c - b^2)^15)^(1/2)))/(512*(a^7*b^20*e^2*f^4 + 1048576*a^17*c^10*e^2*f^4 + 720*a^9*b^16*c^2*e^2*f^4 - 7680*a^10*b^14*c^3*e^2*f^4 + 53760*a^11*b^12*c^4*e^2*f^4 - 258048*a^12*b^10*c^5*e^2*f^4 + 860160*a^13*b^8*c^6*e^2*f^4 - 1966080*a^14*b^6*c^7*e^2*f^4 + 2949120*a^15*b^4*c^8*e^2*f^4 - 2621440*a^16*b^2*c^9*e^2*f^4 - 40*a^8*b^18*c*e^2*f^4)))^(1/2)*((-(9*(25*b^21 + 25*b^6*(-(4*a*c - b^2)^15)^(1/2) + 18923520*a^10*b*c^10 + 17794*a^2*b^17*c^2 - 188095*a^3*b^15*c^3 + 1299860*a^4*b^13*c^4 - 6126640*a^5*b^11*c^5 + 19905600*a^6*b^9*c^6 - 43904256*a^7*b^7*c^7 + 62684160*a^8*b^5*c^8 - 52039680*a^9*b^3*c^9 - 225*a^3*c^3*(-(4*a*c - b^2)^15)^(1/2) - 995*a*b^19*c + 694*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 245*a*b^4*c*(-(4*a*c - b^2)^15)^(1/2)))/(512*(a^7*b^20*e^2*f^4 + 1048576*a^17*c^10*e^2*f^4 + 720*a^9*b^16*c^2*e^2*f^4 - 7680*a^10*b^14*c^3*e^2*f^4 + 53760*a^11*b^12*c^4*e^2*f^4 - 258048*a^12*b^10*c^5*e^2*f^4 + 860160*a^13*b^8*c^6*e^2*f^4 - 1966080*a^14*b^6*c^7*e^2*f^4 + 2949120*a^15*b^4*c^8*e^2*f^4 - 2621440*a^16*b^2*c^9*e^2*f^4 - 40*a^8*b^18*c*e^2*f^4)))^(1/2)*(x*(262144*a^15*b^23*c^2*e^14*f^10 - 11534336*a^16*b^21*c^3*e^14*f^10 + 230686720*a^17*b^19*c^4*e^14*f^10 - 2768240640*a^18*b^17*c^5*e^14*f^10 + 22145925120*a^19*b^15*c^6*e^14*f^10 - 124017180672*a^20*b^13*c^7*e^14*f^10 + 496068722688*a^21*b^11*c^8*e^14*f^10 - 1417339207680*a^22*b^9*c^9*e^14*f^10 + 2834678415360*a^23*b^7*c^10*e^14*f^10 - 3779571220480*a^24*b^5*c^11*e^14*f^10 + 3023656976384*a^25*b^3*c^12*e^14*f^10 - 1099511627776*a^26*b*c^13*e^14*f^10) - 1099511627776*a^26*b*c^13*d*e^13*f^10 + 262144*a^15*b^23*c^2*d*e^13*f^10 - 11534336*a^16*b^21*c^3*d*e^13*f^10 + 230686720*a^17*b^19*c^4*d*e^13*f^10 - 2768240640*a^18*b^17*c^5*d*e^13*f^10 + 22145925120*a^19*b^15*c^6*d*e^13*f^10 - 124017180672*a^20*b^13*c^7*d*e^13*f^10 + 496068722688*a^21*b^11*c^8*d*e^13*f^10 - 1417339207680*a^22*b^9*c^9*d*e^13*f^10 + 2834678415360*a^23*b^7*c^10*d*e^13*f^10 - 3779571220480*a^24*b^5*c^11*d*e^13*f^10 + 3023656976384*a^25*b^3*c^12*d*e^13*f^10) - 245760*a^12*b^23*c^2*e^12*f^8 + 10911744*a^13*b^21*c^3*e^12*f^8 - 220397568*a^14*b^19*c^4*e^12*f^8 + 2673082368*a^15*b^17*c^5*e^12*f^8 - 21630025728*a^16*b^15*c^6*e^12*f^8 + 122607894528*a^17*b^13*c^7*e^12*f^8 - 496773365760*a^18*b^11*c^8*e^12*f^8 + 1438679826432*a^19*b^9*c^9*e^12*f^8 - 2918430277632*a^20*b^7*c^10*e^12*f^8 + 3949222428672*a^21*b^5*c^11*e^12*f^8 - 3208340570112*a^22*b^3*c^12*e^12*f^8 + 1185410973696*a^23*b*c^13*e^12*f^8) + 271790899200*a^20*c^14*d*e^11*f^6 - 230400*a^9*b^22*c^3*d*e^11*f^6 + 9861120*a^10*b^20*c^4*d*e^11*f^6 - 191038464*a^11*b^18*c^5*d*e^11*f^6 + 2207803392*a^12*b^16*c^6*d*e^11*f^6 - 16878108672*a^13*b^14*c^7*d*e^11*f^6 + 89374851072*a^14*b^12*c^8*d*e^11*f^6 - 333226967040*a^15*b^10*c^9*d*e^11*f^6 + 869815812096*a^16*b^8*c^10*d*e^11*f^6 - 1543847804928*a^17*b^6*c^11*d*e^11*f^6 + 1747313491968*a^18*b^4*c^12*d*e^11*f^6 - 1101055131648*a^19*b^2*c^13*d*e^11*f^6) + 191102976000*a^17*c^14*e^10*f^4 + 2851200*a^9*b^16*c^6*e^10*f^4 - 92568960*a^10*b^14*c^7*e^10*f^4 + 1312630272*a^11*b^12*c^8*e^10*f^4 - 10611136512*a^12*b^10*c^9*e^10*f^4 + 53445353472*a^13*b^8*c^10*e^10*f^4 - 171591892992*a^14*b^6*c^11*e^10*f^4 + 342580396032*a^15*b^4*c^12*e^10*f^4 - 388363714560*a^16*b^2*c^13*e^10*f^4))

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*(-(9*(25*b^21 + 25*b^6*(-(4*a*c - b^2)^15)^(1/2) + 18923520*a^10*b*c^10 +
17794*a^2*b^17*c^2 - 188095*a^3*b^15*c^3 + 1299860*a^4*b^13*c^4 - 6126640*a
^5*b^11*c^5 + 19905600*a^6*b^9*c^6 - 43904256*a^7*b^7*c^7 + 62684160*a^8*b^
5*c^8 - 52039680*a^9*b^3*c^9 - 225*a^3*c^3*(-(4*a*c - b^2)^15)^(1/2) - 995*
a*b^19*c + 694*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 245*a*b^4*c*(-(4*a*c
 - b^2)^15)^(1/2)))/(512*(a^7*b^20*e^2*f^4 + 1048576*a^17*c^10*e^2*f^4 + 72
0*a^9*b^16*c^2*e^2*f^4 - 7680*a^10*b^14*c^3*e^2*f^4 + 53760*a^11*b^12*c^4*e
^2*f^4 - 258048*a^12*b^10*c^5*e^2*f^4 + 860160*a^13*b^8*c^6*e^2*f^4 - 19660
80*a^14*b^6*c^7*e^2*f^4 + 2949120*a^15*b^4*c^8*e^2*f^4 - 2621440*a^16*b^2*c
^9*e^2*f^4 - 40*a^8*b^18*c*e^2*f^4)))^(1/2)*2i - ((x^4*(15*b^6*e^3 + 324*a^
3*c^3*e^3 + 450*b^5*c*d^2*e^3 + 25*a^2*b^2*c^2*e^3 + 12600*a^2*c^4*d^4*e^3
+ 1050*b^4*c^2*d^4*e^3 - 91*a*b^4*c*e^3 - 3405*a*b^3*c^2*d^2*e^3 + 5880*a^2
*b*c^3*d^2*e^3 - 7770*a*b^2*c^3*d^4*e^3))/(8*a*(a^2*b^4 + 16*a^4*c^2 - 8*a^
3*b^2*c)) + (x^6*(30*b^5*c*e^5 - 227*a*b^3*c^2*e^5 + 392*a^2*b*c^3*e^5 + 50
40*a^2*c^4*d^2*e^5 + 420*b^4*c^2*d^2*e^5 - 3108*a*b^2*c^3*d^2*e^5))/(8*a*(a
^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)) + (x*(30*b^6*d^3 + 90*b^5*c*d^5 + 648*a
^3*c^3*d^3 + 720*a^2*c^4*d^7 + 60*b^4*c^2*d^7 + 25*a*b^5*d - 681*a*b^3*c^2*
d^5 + 1176*a^2*b*c^3*d^5 - 444*a*b^2*c^3*d^7 + 50*a^2*b^2*c^2*d^3 - 194*a^2
*b^3*c*d + 364*a^3*b*c^2*d - 182*a*b^4*c*d^3))/(4*a*(a^2*b^4 + 16*a^4*c^2 -
 8*a^3*b^2*c)) + (3*x^5*(1680*a^2*c^4*d^3*e^4 + 140*b^4*c^2*d^3*e^4 + 30*b^
5*c*d*e^4 - 227*a*b^3*c^2*d*e^4 + 392*a^2*b*c^3*d*e^4 - 1036*a*b^2*c^3*d^3*
e^4))/(4*a*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)) + (3*x^8*(60*a^2*c^4*e^7 +
 5*b^4*c^2*e^7 - 37*a*b^2*c^3*e^7))/(8*a*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*
c)) + (x^2*(90*b^6*d^2*e + 25*a*b^5*e + 1944*a^3*c^3*d^2*e + 5040*a^2*c^4*d
^6*e + 420*b^4*c^2*d^6*e - 194*a^2*b^3*c*e + 364*a^3*b*c^2*e + 450*b^5*c*d^
4*e - 546*a*b^4*c*d^2*e - 3405*a*b^3*c^2*d^4*e + 5880*a^2*b*c^3*d^4*e - 310
8*a*b^2*c^3*d^6*e + 150*a^2*b^2*c^2*d^2*e))/(8*a*(a^2*b^4 + 16*a^4*c^2 - 8*
a^3*b^2*c)) + (x^3*(15*b^6*d*e^2 + 324*a^3*c^3*d*e^2 + 150*b^5*c*d^3*e^2 +
2520*a^2*c^4*d^5*e^2 + 210*b^4*c^2*d^5*e^2 - 91*a*b^4*c*d*e^2 + 25*a^2*b^2*
c^2*d*e^2 - 1135*a*b^3*c^2*d^3*e^2 + 1960*a^2*b*c^3*d^3*e^2 - 1554*a*b^2*c^
3*d^5*e^2))/(2*a*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)) + (3*x^7*(60*a^2*c^4
*d*e^6 + 5*b^4*c^2*d*e^6 - 37*a*b^2*c^3*d*e^6))/(a*(a^2*b^4 + 16*a^4*c^2 -
8*a^3*b^2*c)) + (8*a^2*b^4 + 128*a^4*c^2 + 15*b^6*d^4 - 64*a^3*b^2*c + 25*a
*b^5*d^2 + 30*b^5*c*d^6 + 324*a^3*c^3*d^4 + 180*a^2*c^4*d^8 + 15*b^4*c^2*d^
8 - 194*a^2*b^3*c*d^2 + 364*a^3*b*c^2*d^2 - 227*a*b^3*c^2*d^6 + 392*a^2*b*c
^3*d^6 - 111*a*b^2*c^3*d^8 + 25*a^2*b^2*c^2*d^4 - 91*a*b^4*c*d^4)/(8*a*e*(a
^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))/(x^3*(10*b^2*d^2*e^3*f^2 + 84*c^2*d^6*
e^3*f^2 + 2*a*b*e^3*f^2 + 20*a*c*d^2*e^3*f^2 + 70*b*c*d^4*e^3*f^2) + x^6*(8
4*c^2*d^3*e^6*f^2 + 14*b*c*d*e^6*f^2) + x^2*(10*b^2*d^3*e^2*f^2 + 36*c^2*d^
7*e^2*f^2 + 6*a*b*d*e^2*f^2 + 20*a*c*d^3*e^2*f^2 + 42*b*c*d^5*e^2*f^2) + x^
4*(5*b^2*d*e^4*f^2 + 126*c^2*d^5*e^4*f^2 + 10*a*c*d*e^4*f^2 + 70*b*c*d^3*e^
4*f^2) + x^7*(36*c^2*d^2*e^7*f^2 + 2*b*c*e^7*f^2) + x^5*(b^2*e^5*f^2 + 126*
c^2*d^4*e^5*f^2 + 2*a*c*e^5*f^2 + 42*b*c*d^2*e^5*f^2) + x*(a^2*e*f^2 + 5*b^
2*d^4*e*f^2 + 9*c^2*d^8*e*f^2 + 6*a*b*d^2*e*f^2 + 10*a*c*d^4*e*f^2 + 14*b*c
*d^6*e*f^2) + a^2*d*f^2 + b^2*d^5*f^2 + c^2*d^9*f^2 + c^2*e^9*f^2*x^9 + 2*a
*b*d^3*f^2 + 2*a*c*d^5*f^2 + 2*b*c*d^7*f^2 + 9*c^2*d*e^8*f^2*x^8)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*f*x+d*f)**2/(a+b*(e*x+d)**2+c*(e*x+d)**4)**3,x)
```

```
[Out] Timed out
```

3.547 $\int \frac{1}{(df+efx)^3\left(a+b(d+ex)^2+c(d+ex)^4\right)^3}\,dx$

Optimal. Leaf size=343

$$\frac{3b\log\left(a+b(d+ex)^2+c(d+ex)^4\right)}{4a^4ef^3}-\frac{3b\log(d+ex)}{a^4ef^3}-\frac{3\left(b^2-5ac\right)\left(b^2-2ac\right)}{2a^3ef^3\left(b^2-4ac\right)^2(d+ex)^2}+\frac{20a^2c^2+3bc\left(b^2-6ac\right)(}{4a^2ef^3\left(b^2-4ac\right)^2(d+ex)^2(}$$

Rubi [A] time = 0.59, antiderivative size = 343, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {1142, 1114, 740, 822, 800, 634, 618, 206, 628}

$$\frac{20a^2c^2+3bc\left(b^2-6ac\right)(d+ex)^2-20ab^2c+3b^4}{4a^2ef^3\left(b^2-4ac\right)^2(d+ex)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)}-\frac{3\left(30a^2b^2c^2-20a^3c^3-10ab^4c+b^6\right)\tanh^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{2a^4ef^3\left(b^2-4ac\right)^{5/2}}-\frac{3\left(b^2-5ac\right)\left(b^2-2ac\right)}{2a^3ef^3\left(b^2-4ac\right)^2(d+ex)^2}+\frac{3b\log\left(a+b(d+ex)^2+c(d+ex)^4\right)}{4a^4ef^3}-\frac{3b\log(d+ex)}{a^4ef^3}+\frac{-2ac+b^2+bc(d+ex)^2}{4aef^3\left(b^2-4ac\right)(d+ex)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d*f + e*f*x)^3*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3),x]
```

```
[Out] (-3*(b^2 - 5*a*c)*(b^2 - 2*a*c))/(2*a^3*(b^2 - 4*a*c)^2*e*f^3*(d + e*x)^2)
+ (b^2 - 2*a*c + b*c*(d + e*x)^2)/(4*a*(b^2 - 4*a*c)*e*f^3*(d + e*x)^2*(a +
 b*(d + e*x)^2 + c*(d + e*x)^4)^2) + (3*b^4 - 20*a*b^2*c + 20*a^2*c^2 + 3*b
*c*(b^2 - 6*a*c)*(d + e*x)^2)/(4*a^2*(b^2 - 4*a*c)^2*e*f^3*(d + e*x)^2*(a +
 b*(d + e*x)^2 + c*(d + e*x)^4)) - (3*(b^6 - 10*a*b^4*c + 30*a^2*b^2*c^2 -
20*a^3*c^3)*ArcTanh[(b + 2*c*(d + e*x)^2)/Sqrt[b^2 - 4*a*c]])/(2*a^4*(b^2 -
 4*a*c)^(5/2)*e*f^3) - (3*b*Log[d + e*x])/(a^4*e*f^3) + (3*b*Log[a + b*(d +
 e*x)^2 + c*(d + e*x)^4])/(4*a^4*e*f^3)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 740

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e
)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e
^2)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d +
 e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p +
```

```
 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p,
-1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 822

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(f*(b*c*d - b^2*e +
2*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x)*(a
+ b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), x]
 + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m +
2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*m
 + b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) - f
*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1
] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 1142

```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Di
st[u^m/(Coefficient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p,
 x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

Rubi steps

$$\int \frac{1}{(df+efx)^3\left(a+b(d+ex)^2+c(d+ex)^4\right)^3}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x^3\left(a+bx^2+cx^4\right)^3}\,dx, x, d+ex\right)}{ef^3}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{x^2\left(a+bx+cx^2\right)^3}\,dx, x, (d+ex)^2\right)}{2ef^3}$$

$$= \frac{b^2-2ac+bc(d+ex)^2}{4a\left(b^2-4ac\right)ef^3(d+ex)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)^2} - \text{Su}\ldots$$

$$= \frac{b^2-2ac+bc(d+ex)^2}{4a\left(b^2-4ac\right)ef^3(d+ex)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)^2} + \frac{\ldots}{4a\ldots}$$

$$= \frac{b^2-2ac+bc(d+ex)^2}{4a\left(b^2-4ac\right)ef^3(d+ex)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)^2} + \frac{\ldots}{4a\ldots}$$

$$= -\frac{3\left(b^2-5ac\right)\left(b^2-2ac\right)}{2a^3\left(b^2-4ac\right)^2 ef^3(d+ex)^2} + \frac{b^2-2ac+b\ldots}{4a\left(b^2-4ac\right)ef^3(d+ex)^2\left(a\ldots\right.}$$

$$= -\frac{3\left(b^2-5ac\right)\left(b^2-2ac\right)}{2a^3\left(b^2-4ac\right)^2 ef^3(d+ex)^2} + \frac{b^2-2ac+b\ldots}{4a\left(b^2-4ac\right)ef^3(d+ex)^2\left(a\ldots\right.}$$

$$= -\frac{3\left(b^2-5ac\right)\left(b^2-2ac\right)}{2a^3\left(b^2-4ac\right)^2 ef^3(d+ex)^2} + \frac{b^2-2ac+b\ldots}{4a\left(b^2-4ac\right)ef^3(d+ex)^2\left(a\ldots\right.}$$

$$= -\frac{3\left(b^2-5ac\right)\left(b^2-2ac\right)}{2a^3\left(b^2-4ac\right)^2 ef^3(d+ex)^2} + \frac{b^2-2ac+b\ldots}{4a\left(b^2-4ac\right)ef^3(d+ex)^2\left(a\ldots\right.}$$

Mathematica [A] time = 6.16, size = 509, normalized size = 1.48

$$-\frac{3b\log(d+ex)}{a^4ef^3} - \frac{1}{2a^3ef^3(d+ex)^2} + \frac{-3abc-2ac^2(d+ex)^2+b^3+b^2c(d+ex)^2}{4a^2ef^3\left(4ac-b^2\right)\left(a+b(d+ex)^2+c(d+ex)^4\right)^2} + \frac{-46a^2bc^2-28a^2c^3(d+ex)^2+29ab^3c+26ab^2c^2(d+ex)^2-4b^5-4b^4c(d+ex)^2}{4a^3ef^3\left(4ac-b^2\right)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)} + \frac{3\left(-20a^3c^3+30a^2b^2c^2+16a^2bc^2\sqrt{b^2-4ac}-10ab^4c+b^5\sqrt{b^2-4ac}-8ab^3c\sqrt{b^2-4ac}+b^6\right)\log\left(-\sqrt{b^2-4ac}+b+2c(d+ex)^2\right)}{4a^4ef^3\left(b^2-4ac\right)^{5/2}} + \frac{3\left(20a^3c^3-30a^2b^2c^2+16a^2bc^2\sqrt{b^2-4ac}+10ab^4c+b^5\sqrt{b^2-4ac}-8ab^3c\sqrt{b^2-4ac}-b^6\right)\log\left(\sqrt{b^2-4ac}+b+2c(d+ex)^2\right)}{4a^4ef^3\left(b^2-4ac\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d*f + e*f*x)^3*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3),x]
```

```
[Out] -1/2*1/(a^3*e*f^3*(d + e*x)^2) + (b^3 - 3*a*b*c + b^2*c*(d + e*x)^2 - 2*a*c
^2*(d + e*x)^2)/(4*a^2*(-b^2 + 4*a*c)*e*f^3*(a + b*(d + e*x)^2 + c*(d + e*x
)^4)^2) + (-4*b^5 + 29*a*b^3*c - 46*a^2*b*c^2 - 4*b^4*c*(d + e*x)^2 + 26*a*
b^2*c^2*(d + e*x)^2 - 28*a^2*c^3*(d + e*x)^2)/(4*a^3*(-b^2 + 4*a*c)^2*e*f^3
*(a + b*(d + e*x)^2 + c*(d + e*x)^4)) - (3*b*Log[d + e*x])/(a^4*e*f^3) + (3
*(b^6 - 10*a*b^4*c + 30*a^2*b^2*c^2 - 20*a^3*c^3 + b^5*Sqrt[b^2 - 4*a*c] -
8*a*b^3*c*Sqrt[b^2 - 4*a*c] + 16*a^2*b*c^2*Sqrt[b^2 - 4*a*c])*Log[b - Sqrt[
b^2 - 4*a*c] + 2*c*(d + e*x)^2])/(4*a^4*(b^2 - 4*a*c)^(5/2)*e*f^3) + (3*(-b
^6 + 10*a*b^4*c - 30*a^2*b^2*c^2 + 20*a^3*c^3 + b^5*Sqrt[b^2 - 4*a*c] - 8*a
*b^3*c*Sqrt[b^2 - 4*a*c] + 16*a^2*b*c^2*Sqrt[b^2 - 4*a*c])*Log[b + Sqrt[b^2
 - 4*a*c] + 2*c*(d + e*x)^2])/(4*a^4*(b^2 - 4*a*c)^(5/2)*e*f^3)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(df+efx)^3 \left(a+b(d+ex)^2+c(d+ex)^4\right)^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/((d*f + e*f*x)^3*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3
),x]
```

```
[Out] IntegrateAlgebraic[1/((d*f + e*f*x)^3*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3
), x]
```

fricas [B] time = 22.61, size = 15231, normalized size = 44.41

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*f*x+d*f)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="frica
s")
```

```
[Out] [-1/4*(6*(a*b^6*c^2 - 11*a^2*b^4*c^3 + 38*a^3*b^2*c^4 - 40*a^4*c^5)*e^8*x^8
 + 48*(a*b^6*c^2 - 11*a^2*b^4*c^3 + 38*a^3*b^2*c^4 - 40*a^4*c^5)*d*e^7*x^7
+ 3*(4*a*b^7*c - 45*a^2*b^5*c^2 + 162*a^3*b^3*c^3 - 184*a^4*b*c^4 + 56*(a*b
^6*c^2 - 11*a^2*b^4*c^3 + 38*a^3*b^2*c^4 - 40*a^4*c^5)*d^2)*e^6*x^6 + 6*(56
*(a*b^6*c^2 - 11*a^2*b^4*c^3 + 38*a^3*b^2*c^4 - 40*a^4*c^5)*d^3 + 3*(4*a*b^
7*c - 45*a^2*b^5*c^2 + 162*a^3*b^3*c^3 - 184*a^4*b*c^4)*d)*e^5*x^5 + 2*a^3*
b^6 - 24*a^4*b^4*c + 96*a^5*b^2*c^2 - 128*a^6*c^3 + 6*(a*b^6*c^2 - 11*a^2*b
^4*c^3 + 38*a^3*b^2*c^4 - 40*a^4*c^5)*d^8 + (6*a*b^8 - 60*a^2*b^6*c + 158*a
^3*b^4*c^2 + 44*a^4*b^2*c^3 - 400*a^5*c^4 + 420*(a*b^6*c^2 - 11*a^2*b^4*c^3
 + 38*a^3*b^2*c^4 - 40*a^4*c^5)*d^4 + 45*(4*a*b^7*c - 45*a^2*b^5*c^2 + 162*
a^3*b^3*c^3 - 184*a^4*b*c^4)*d^2)*e^4*x^4 + 3*(4*a*b^7*c - 45*a^2*b^5*c^2 +
 162*a^3*b^3*c^3 - 184*a^4*b*c^4)*d^6 + 4*(84*(a*b^6*c^2 - 11*a^2*b^4*c^3 +
 38*a^3*b^2*c^4 - 40*a^4*c^5)*d^5 + 15*(4*a*b^7*c - 45*a^2*b^5*c^2 + 162*a^
3*b^3*c^3 - 184*a^4*b*c^4)*d^3 + 2*(3*a*b^8 - 30*a^2*b^6*c + 79*a^3*b^4*c^2
 + 22*a^4*b^2*c^3 - 200*a^5*c^4)*d)*e^3*x^3 + 2*(3*a*b^8 - 30*a^2*b^6*c + 7
9*a^3*b^4*c^2 + 22*a^4*b^2*c^3 - 200*a^5*c^4)*d^4 + (9*a^2*b^7 - 104*a^3*b^
5*c + 394*a^4*b^3*c^2 - 488*a^5*b*c^3 + 168*(a*b^6*c^2 - 11*a^2*b^4*c^3 + 3
8*a^3*b^2*c^4 - 40*a^4*c^5)*d^6 + 45*(4*a*b^7*c - 45*a^2*b^5*c^2 + 162*a^3*
b^3*c^3 - 184*a^4*b*c^4)*d^4 + 12*(3*a*b^8 - 30*a^2*b^6*c + 79*a^3*b^4*c^2
+ 22*a^4*b^2*c^3 - 200*a^5*c^4)*d^2)*e^2*x^2 + (9*a^2*b^7 - 104*a^3*b^5*c +
 394*a^4*b^3*c^2 - 488*a^5*b*c^3)*d^2 + 2*(24*(a*b^6*c^2 - 11*a^2*b^4*c^3 +
 38*a^3*b^2*c^4 - 40*a^4*c^5)*d^7 + 9*(4*a*b^7*c - 45*a^2*b^5*c^2 + 162*a^3
*b^3*c^3 - 184*a^4*b*c^4)*d^5 + 4*(3*a*b^8 - 30*a^2*b^6*c + 79*a^3*b^4*c^2
+ 22*a^4*b^2*c^3 - 200*a^5*c^4)*d^3 + (9*a^2*b^7 - 104*a^3*b^5*c + 394*a^4*
b^3*c^2 - 488*a^5*b*c^3)*d)*e*x + 3*((b^6*c^2 - 10*a*b^4*c^3 + 30*a^2*b^2*c
^4 - 20*a^3*c^5)*e^10*x^10 + 10*(b^6*c^2 - 10*a*b^4*c^3 + 30*a^2*b^2*c^4 -
20*a^3*c^5)*d*e^9*x^9 + (2*b^7*c - 20*a*b^5*c^2 + 60*a^2*b^3*c^3 - 40*a^3*b
*c^4 + 45*(b^6*c^2 - 10*a*b^4*c^3 + 30*a^2*b^2*c^4 - 20*a^3*c^5)*d^2)*e^8*x
^8 + 8*(15*(b^6*c^2 - 10*a*b^4*c^3 + 30*a^2*b^2*c^4 - 20*a^3*c^5)*d^3 + 2*(
b^7*c - 10*a*b^5*c^2 + 30*a^2*b^3*c^3 - 20*a^3*b*c^4)*d)*e^7*x^7 + (b^8 - 8
*a*b^6*c + 10*a^2*b^4*c^2 + 40*a^3*b^2*c^3 - 40*a^4*c^4 + 210*(b^6*c^2 - 10
*a*b^4*c^3 + 30*a^2*b^2*c^4 - 20*a^3*c^5)*d^4 + 56*(b^7*c - 10*a*b^5*c^2 +
30*a^2*b^3*c^3 - 20*a^3*b*c^4)*d^2)*e^6*x^6 + (b^6*c^2 - 10*a*b^4*c^3 + 30*
a^2*b^2*c^4 - 20*a^3*c^5)*d^10 + 2*(126*(b^6*c^2 - 10*a*b^4*c^3 + 30*a^2*b^
2*c^4 - 20*a^3*c^5)*d^5 + 56*(b^7*c - 10*a*b^5*c^2 + 30*a^2*b^3*c^3 - 20*a^
3*b*c^4)*d^3 + 3*(b^8 - 8*a*b^6*c + 10*a^2*b^4*c^2 + 40*a^3*b^2*c^3 - 40*a^
4*c^4)*d)*e^5*x^5 + 2*(b^7*c - 10*a*b^5*c^2 + 30*a^2*b^3*c^3 - 20*a^3*b*c^4
)*d^8 + (2*a*b^7 - 20*a^2*b^5*c + 60*a^3*b^3*c^2 - 40*a^4*b*c^3 + 210*(b^6*
c^2 - 10*a*b^4*c^3 + 30*a^2*b^2*c^4 - 20*a^3*c^5)*d^6 + 140*(b^7*c - 10*a*b
```

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^5*c^2 + 30*a^2*b^3*c^3 - 20*a^3*b*c^4)*d^4 + 15*(b^8 - 8*a*b^6*c + 10*a^2*
b^4*c^2 + 40*a^3*b^2*c^3 - 40*a^4*c^4)*d^2)*e^4*x^4 + (b^8 - 8*a*b^6*c + 10
*a^2*b^4*c^2 + 40*a^3*b^2*c^3 - 40*a^4*c^4)*d^6 + 4*(30*(b^6*c^2 - 10*a*b^4
*c^3 + 30*a^2*b^2*c^4 - 20*a^3*c^5)*d^7 + 28*(b^7*c - 10*a*b^5*c^2 + 30*a^2
*b^3*c^3 - 20*a^3*b*c^4)*d^5 + 5*(b^8 - 8*a*b^6*c + 10*a^2*b^4*c^2 + 40*a^3
*b^2*c^3 - 40*a^4*c^4)*d^3 + 2*(a*b^7 - 10*a^2*b^5*c + 30*a^3*b^3*c^2 - 20*
a^4*b*c^3)*d)*e^3*x^3 + 2*(a*b^7 - 10*a^2*b^5*c + 30*a^3*b^3*c^2 - 20*a^4*b
*c^3)*d^4 + (45*(b^6*c^2 - 10*a*b^4*c^3 + 30*a^2*b^2*c^4 - 20*a^3*c^5)*d^8
 + a^2*b^6 - 10*a^3*b^4*c + 30*a^4*b^2*c^2 - 20*a^5*c^3 + 56*(b^7*c - 10*a*b
^5*c^2 + 30*a^2*b^3*c^3 - 20*a^3*b*c^4)*d^6 + 15*(b^8 - 8*a*b^6*c + 10*a^2*
b^4*c^2 + 40*a^3*b^2*c^3 - 40*a^4*c^4)*d^4 + 12*(a*b^7 - 10*a^2*b^5*c + 30*
a^3*b^3*c^2 - 20*a^4*b*c^3)*d^2)*e^2*x^2 + (a^2*b^6 - 10*a^3*b^4*c + 30*a^4
*b^2*c^2 - 20*a^5*c^3)*d^2 + 2*(5*(b^6*c^2 - 10*a*b^4*c^3 + 30*a^2*b^2*c^4
 - 20*a^3*c^5)*d^9 + 8*(b^7*c - 10*a*b^5*c^2 + 30*a^2*b^3*c^3 - 20*a^3*b*c^4
)*d^7 + 3*(b^8 - 8*a*b^6*c + 10*a^2*b^4*c^2 + 40*a^3*b^2*c^3 - 40*a^4*c^4)*
d^5 + 4*(a*b^7 - 10*a^2*b^5*c + 30*a^3*b^3*c^2 - 20*a^4*b*c^3)*d^3 + (a^2*b
^6 - 10*a^3*b^4*c + 30*a^4*b^2*c^2 - 20*a^5*c^3)*d)*e*x)*sqrt(b^2 - 4*a*c)*
log((2*c^2*e^4*x^4 + 8*c^2*d*e^3*x^3 + 2*c^2*d^4 + 2*(6*c^2*d^2 + b*c)*e^2*
x^2 + 2*b*c*d^2 + 4*(2*c^2*d^3 + b*c*d)*e*x + b^2 - 2*a*c + (2*c*e^2*x^2 +
4*c*d*e*x + 2*c*d^2 + b)*sqrt(b^2 - 4*a*c))/(c*e^4*x^4 + 4*c*d*e^3*x^3 + c*
d^4 + (6*c*d^2 + b)*e^2*x^2 + b*d^2 + 2*(2*c*d^3 + b*d)*e*x + a)) - 3*((b^7
*c^2 - 12*a*b^5*c^3 + 48*a^2*b^3*c^4 - 64*a^3*b*c^5)*e^10*x^10 + 10*(b^7*c^
2 - 12*a*b^5*c^3 + 48*a^2*b^3*c^4 - 64*a^3*b*c^5)*d*e^9*x^9 + (2*b^8*c - 24
*a*b^6*c^2 + 96*a^2*b^4*c^3 - 128*a^3*b^2*c^4 + 45*(b^7*c^2 - 12*a*b^5*c^3
 + 48*a^2*b^3*c^4 - 64*a^3*b*c^5)*d^2)*e^8*x^8 + 8*(15*(b^7*c^2 - 12*a*b^5*c
^3 + 48*a^2*b^3*c^4 - 64*a^3*b*c^5)*d^3 + 2*(b^8*c - 12*a*b^6*c^2 + 48*a^2*
b^4*c^3 - 64*a^3*b^2*c^4)*d)*e^7*x^7 + (b^9 - 10*a*b^7*c + 24*a^2*b^5*c^2 +
 32*a^3*b^3*c^3 - 128*a^4*b*c^4 + 210*(b^7*c^2 - 12*a*b^5*c^3 + 48*a^2*b^3*
c^4 - 64*a^3*b*c^5)*d^4 + 56*(b^8*c - 12*a*b^6*c^2 + 48*a^2*b^4*c^3 - 64*a^
3*b^2*c^4)*d^2)*e^6*x^6 + (b^7*c^2 - 12*a*b^5*c^3 + 48*a^2*b^3*c^4 - 64*a^3
*b*c^5)*d^10 + 2*(126*(b^7*c^2 - 12*a*b^5*c^3 + 48*a^2*b^3*c^4 - 64*a^3*b*c
^5)*d^5 + 56*(b^8*c - 12*a*b^6*c^2 + 48*a^2*b^4*c^3 - 64*a^3*b^2*c^4)*d^3 +
 3*(b^9 - 10*a*b^7*c + 24*a^2*b^5*c^2 + 32*a^3*b^3*c^3 - 128*a^4*b*c^4)*d)*
e^5*x^5 + 2*(b^8*c - 12*a*b^6*c^2 + 48*a^2*b^4*c^3 - 64*a^3*b^2*c^4)*d^8 +
(2*a*b^8 - 24*a^2*b^6*c + 96*a^3*b^4*c^2 - 128*a^4*b^2*c^3 + 210*(b^7*c^2 -
 12*a*b^5*c^3 + 48*a^2*b^3*c^4 - 64*a^3*b*c^5)*d^6 + 140*(b^8*c - 12*a*b^6*
c^2 + 48*a^2*b^4*c^3 - 64*a^3*b^2*c^4)*d^4 + 15*(b^9 - 10*a*b^7*c + 24*a^2*
b^5*c^2 + 32*a^3*b^3*c^3 - 128*a^4*b*c^4)*d^2)*e^4*x^4 + (b^9 - 10*a*b^7*c
 + 24*a^2*b^5*c^2 + 32*a^3*b^3*c^3 - 128*a^4*b*c^4)*d^6 + 4*(30*(b^7*c^2 - 1
2*a*b^5*c^3 + 48*a^2*b^3*c^4 - 64*a^3*b*c^5)*d^7 + 28*(b^8*c - 12*a*b^6*c^2
 + 48*a^2*b^4*c^3 - 64*a^3*b^2*c^4)*d^5 + 5*(b^9 - 10*a*b^7*c + 24*a^2*b^5*
c^2 + 32*a^3*b^3*c^3 - 128*a^4*b*c^4)*d^3 + 2*(a*b^8 - 12*a^2*b^6*c + 48*a^
3*b^4*c^2 - 64*a^4*b^2*c^3)*d)*e^3*x^3 + 2*(a*b^8 - 12*a^2*b^6*c + 48*a^3*b
^4*c^2 - 64*a^4*b^2*c^3)*d^4 + (a^2*b^7 - 12*a^3*b^5*c + 48*a^4*b^3*c^2 - 6
4*a^5*b*c^3 + 45*(b^7*c^2 - 12*a*b^5*c^3 + 48*a^2*b^3*c^4 - 64*a^3*b*c^5)*d
^8 + 56*(b^8*c - 12*a*b^6*c^2 + 48*a^2*b^4*c^3 - 64*a^3*b^2*c^4)*d^6 + 15*(
b^9 - 10*a*b^7*c + 24*a^2*b^5*c^2 + 32*a^3*b^3*c^3 - 128*a^4*b*c^4)*d^4 + 1
2*(a*b^8 - 12*a^2*b^6*c + 48*a^3*b^4*c^2 - 64*a^4*b^2*c^3)*d^2)*e^2*x^2 + (
a^2*b^7 - 12*a^3*b^5*c + 48*a^4*b^3*c^2 - 64*a^5*b*c^3)*d^2 + 2*(5*(b^7*c^2
 - 12*a*b^5*c^3 + 48*a^2*b^3*c^4 - 64*a^3*b*c^5)*d^9 + 8*(b^8*c - 12*a*b^6*
c^2 + 48*a^2*b^4*c^3 - 64*a^3*b^2*c^4)*d^7 + 3*(b^9 - 10*a*b^7*c + 24*a^2*b
^5*c^2 + 32*a^3*b^3*c^3 - 128*a^4*b*c^4)*d^5 + 4*(a*b^8 - 12*a^2*b^6*c + 48
*a^3*b^4*c^2 - 64*a^4*b^2*c^3)*d^3 + (a^2*b^7 - 12*a^3*b^5*c + 48*a^4*b^3*c
^2 - 64*a^5*b*c^3)*d)*e*x)*log(c*e^4*x^4 + 4*c*d*e^3*x^3 + c*d^4 + (6*c*d^2
 + b)*e^2*x^2 + b*d^2 + 2*(2*c*d^3 + b*d)*e*x + a) + 12*((b^7*c^2 - 12*a*b^
5*c^3 + 48*a^2*b^3*c^4 - 64*a^3*b*c^5)*e^10*x^10 + 10*(b^7*c^2 - 12*a*b^5*c
^3 + 48*a^2*b^3*c^4 - 64*a^3*b*c^5)*d*e^9*x^9 + (2*b^8*c - 24*a*b^6*c^2 + 9
6*a^2*b^4*c^3 - 128*a^3*b^2*c^4 + 45*(b^7*c^2 - 12*a*b^5*c^3 + 48*a^2*b^3*c
^4 - 64*a^3*b*c^5)*d^2)*e^8*x^8 + 8*(15*(b^7*c^2 - 12*a*b^5*c^3 + 48*a^2*b^
```

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3*c^4 - 64*a^3*b*c^5)*d^3 + 2*(b^8*c - 12*a*b^6*c^2 + 48*a^2*b^4*c^3 - 64*a
^3*b^2*c^4)*d)*e^7*x^7 + (b^9 - 10*a*b^7*c + 24*a^2*b^5*c^2 + 32*a^3*b^3*c^
3 - 128*a^4*b*c^4 + 210*(b^7*c^2 - 12*a*b^5*c^3 + 48*a^2*b^3*c^4 - 64*a^3*b
*c^5)*d^4 + 56*(b^8*c - 12*a*b^6*c^2 + 48*a^2*b^4*c^3 - 64*a^3*b^2*c^4)*d^2
)*e^6*x^6 + (b^7*c^2 - 12*a*b^5*c^3 + 48*a^2*b^3*c^4 - 64*a^3*b*c^5)*d^10 +
 2*(126*(b^7*c^2 - 12*a*b^5*c^3 + 48*a^2*b^3*c^4 - 64*a^3*b*c^5)*d^5 + 56*(
b^8*c - 12*a*b^6*c^2 + 48*a^2*b^4*c^3 - 64*a^3*b^2*c^4)*d^3 + 3*(b^9 - 10*a
*b^7*c + 24*a^2*b^5*c^2 + 32*a^3*b^3*c^3 - 128*a^4*b*c^4)*d)*e^5*x^5 + 2*(b
^8*c - 12*a*b^6*c^2 + 48*a^2*b^4*c^3 - 64*a^3*b^2*c^4)*d^8 + (2*a*b^8 - 24*
a^2*b^6*c + 96*a^3*b^4*c^2 - 128*a^4*b^2*c^3 + 210*(b^7*c^2 - 12*a*b^5*c^3
+ 48*a^2*b^3*c^4 - 64*a^3*b*c^5)*d^6 + 140*(b^8*c - 12*a*b^6*c^2 + 48*a^2*b
^4*c^3 - 64*a^3*b^2*c^4)*d^4 + 15*(b^9 - 10*a*b^7*c + 24*a^2*b^5*c^2 + 32*a
^3*b^3*c^3 - 128*a^4*b*c^4)*d^2)*e^4*x^4 + (b^9 - 10*a*b^7*c + 24*a^2*b^5*c
^2 + 32*a^3*b^3*c^3 - 128*a^4*b*c^4)*d^6 + 4*(30*(b^7*c^2 - 12*a*b^5*c^3 +
48*a^2*b^3*c^4 - 64*a^3*b*c^5)*d^7 + 28*(b^8*c - 12*a*b^6*c^2 + 48*a^2*b^4*
c^3 - 64*a^3*b^2*c^4)*d^5 + 5*(b^9 - 10*a*b^7*c + 24*a^2*b^5*c^2 + 32*a^3*b
^3*c^3 - 128*a^4*b*c^4)*d^3 + 2*(a*b^8 - 12*a^2*b^6*c + 48*a^3*b^4*c^2 - 64
*a^4*b^2*c^3)*d)*e^3*x^3 + 2*(a*b^8 - 12*a^2*b^6*c + 48*a^3*b^4*c^2 - 64*a^
4*b^2*c^3)*d^4 + (a^2*b^7 - 12*a^3*b^5*c + 48*a^4*b^3*c^2 - 64*a^5*b*c^3 +
45*(b^7*c^2 - 12*a*b^5*c^3 + 48*a^2*b^3*c^4 - 64*a^3*b*c^5)*d^8 + 56*(b^8*c
 - 12*a*b^6*c^2 + 48*a^2*b^4*c^3 - 64*a^3*b^2*c^4)*d^6 + 15*(b^9 - 10*a*b^7
*c + 24*a^2*b^5*c^2 + 32*a^3*b^3*c^3 - 128*a^4*b*c^4)*d^4 + 12*(a*b^8 - 12*
a^2*b^6*c + 48*a^3*b^4*c^2 - 64*a^4*b^2*c^3)*d^2)*e^2*x^2 + (a^2*b^7 - 12*a
^3*b^5*c + 48*a^4*b^3*c^2 - 64*a^5*b*c^3)*d^2 + 2*(5*(b^7*c^2 - 12*a*b^5*c^
3 + 48*a^2*b^3*c^4 - 64*a^3*b*c^5)*d^9 + 8*(b^8*c - 12*a*b^6*c^2 + 48*a^2*b
^4*c^3 - 64*a^3*b^2*c^4)*d^7 + 3*(b^9 - 10*a*b^7*c + 24*a^2*b^5*c^2 + 32*a^
3*b^3*c^3 - 128*a^4*b*c^4)*d^5 + 4*(a*b^8 - 12*a^2*b^6*c + 48*a^3*b^4*c^2 -
 64*a^4*b^2*c^3)*d^3 + (a^2*b^7 - 12*a^3*b^5*c + 48*a^4*b^3*c^2 - 64*a^5*b*
c^3)*d)*e*x)*log(e*x + d))/((a^4*b^6*c^2 - 12*a^5*b^4*c^3 + 48*a^6*b^2*c^4
- 64*a^7*c^5)*e^11*f^3*x^10 + 10*(a^4*b^6*c^2 - 12*a^5*b^4*c^3 + 48*a^6*b^2
*c^4 - 64*a^7*c^5)*d*e^10*f^3*x^9 + (2*a^4*b^7*c - 24*a^5*b^5*c^2 + 96*a^6*
b^3*c^3 - 128*a^7*b*c^4 + 45*(a^4*b^6*c^2 - 12*a^5*b^4*c^3 + 48*a^6*b^2*c^4
 - 64*a^7*c^5)*d^2)*e^9*f^3*x^8 + 8*(15*(a^4*b^6*c^2 - 12*a^5*b^4*c^3 + 48*
a^6*b^2*c^4 - 64*a^7*c^5)*d^3 + 2*(a^4*b^7*c - 12*a^5*b^5*c^2 + 48*a^6*b^3*
c^3 - 64*a^7*b*c^4)*d)*e^8*f^3*x^7 + (a^4*b^8 - 10*a^5*b^6*c + 24*a^6*b^4*c
^2 + 32*a^7*b^2*c^3 - 128*a^8*c^4 + 210*(a^4*b^6*c^2 - 12*a^5*b^4*c^3 + 48*
a^6*b^2*c^4 - 64*a^7*c^5)*d^4 + 56*(a^4*b^7*c - 12*a^5*b^5*c^2 + 48*a^6*b^3
*c^3 - 64*a^7*b*c^4)*d^2)*e^7*f^3*x^6 + 2*(126*(a^4*b^6*c^2 - 12*a^5*b^4*c^
3 + 48*a^6*b^2*c^4 - 64*a^7*c^5)*d^5 + 56*(a^4*b^7*c - 12*a^5*b^5*c^2 + 48*
a^6*b^3*c^3 - 64*a^7*b*c^4)*d^3 + 3*(a^4*b^8 - 10*a^5*b^6*c + 24*a^6*b^4*c^
2 + 32*a^7*b^2*c^3 - 128*a^8*c^4)*d)*e^6*f^3*x^5 + (2*a^5*b^7 - 24*a^6*b^5*
c + 96*a^7*b^3*c^2 - 128*a^8*b*c^3 + 210*(a^4*b^6*c^2 - 12*a^5*b^4*c^3 + 48
*a^6*b^2*c^4 - 64*a^7*c^5)*d^6 + 140*(a^4*b^7*c - 12*a^5*b^5*c^2 + 48*a^6*b
^3*c^3 - 64*a^7*b*c^4)*d^4 + 15*(a^4*b^8 - 10*a^5*b^6*c + 24*a^6*b^4*c^2 +
32*a^7*b^2*c^3 - 128*a^8*c^4)*d^2)*e^5*f^3*x^4 + 4*(30*(a^4*b^6*c^2 - 12*a^
5*b^4*c^3 + 48*a^6*b^2*c^4 - 64*a^7*c^5)*d^7 + 28*(a^4*b^7*c - 12*a^5*b^5*c
^2 + 48*a^6*b^3*c^3 - 64*a^7*b*c^4)*d^5 + 5*(a^4*b^8 - 10*a^5*b^6*c + 24*a^
6*b^4*c^2 + 32*a^7*b^2*c^3 - 128*a^8*c^4)*d^3 + 2*(a^5*b^7 - 12*a^6*b^5*c +
 48*a^7*b^3*c^2 - 64*a^8*b*c^3)*d)*e^4*f^3*x^3 + (a^6*b^6 - 12*a^7*b^4*c +
48*a^8*b^2*c^2 - 64*a^9*c^3 + 45*(a^4*b^6*c^2 - 12*a^5*b^4*c^3 + 48*a^6*b^2
*c^4 - 64*a^7*c^5)*d^8 + 56*(a^4*b^7*c - 12*a^5*b^5*c^2 + 48*a^6*b^3*c^3 -
64*a^7*b*c^4)*d^6 + 15*(a^4*b^8 - 10*a^5*b^6*c + 24*a^6*b^4*c^2 + 32*a^7*b^
2*c^3 - 128*a^8*c^4)*d^4 + 12*(a^5*b^7 - 12*a^6*b^5*c + 48*a^7*b^3*c^2 - 64
*a^8*b*c^3)*d^2)*e^3*f^3*x^2 + 2*(5*(a^4*b^6*c^2 - 12*a^5*b^4*c^3 + 48*a^6*
b^2*c^4 - 64*a^7*c^5)*d^9 + 8*(a^4*b^7*c - 12*a^5*b^5*c^2 + 48*a^6*b^3*c^3
- 64*a^7*b*c^4)*d^7 + 3*(a^4*b^8 - 10*a^5*b^6*c + 24*a^6*b^4*c^2 + 32*a^7*b
^2*c^3 - 128*a^8*c^4)*d^5 + 4*(a^5*b^7 - 12*a^6*b^5*c + 48*a^7*b^3*c^2 - 64
*a^8*b*c^3)*d^3 + (a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)*d)
*e^2*f^3*x + ((a^4*b^6*c^2 - 12*a^5*b^4*c^3 + 48*a^6*b^2*c^4 - 64*a^7*c^5)*
```

d^10 + 2*(a^4*b^7*c - 12*a^5*b^5*c^2 + 48*a^6*b^3*c^3 - 64*a^7*b*c^4)*d^8 + (a^4*b^8 - 10*a^5*b^6*c + 24*a^6*b^4*c^2 + 32*a^7*b^2*c^3 - 128*a^8*c^4)*d^6 + 2*(a^5*b^7 - 12*a^6*b^5*c + 48*a^7*b^3*c^2 - 64*a^8*b*c^3)*d^4 + (a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)*d^2)*e*f^3), -1/4*(6*(a*b^6*c^2 - 11*a^2*b^4*c^3 + 38*a^3*b^2*c^4 - 40*a^4*c^5)*e^8*x^8 + 48*(a*b^6*c^2 - 11*a^2*b^4*c^3 + 38*a^3*b^2*c^4 - 40*a^4*c^5)*d*e^7*x^7 + 3*(4*a*b^7*c - 45*a^2*b^5*c^2 + 162*a^3*b^3*c^3 - 184*a^4*b*c^4 + 56*(a*b^6*c^2 - 11*a^2*b^4*c^3 + 38*a^3*b^2*c^4 - 40*a^4*c^5)*d^2)*e^6*x^6 + 6*(56*(a*b^6*c^2 - 11*a^2*b^4*c^3 + 38*a^3*b^2*c^4 - 40*a^4*c^5)*d^3 + 3*(4*a*b^7*c - 45*a^2*b^5*c^2 + 162*a^3*b^3*c^3 - 184*a^4*b*c^4)*d)*e^5*x^5 + 2*a^3*b^6 - 24*a^4*b^4*c + 96*a^5*b^2*c^2 - 128*a^6*c^3 + 6*(a*b^6*c^2 - 11*a^2*b^4*c^3 + 38*a^3*b^2*c^4 - 40*a^4*c^5)*d^8 + (6*a*b^8 - 60*a^2*b^6*c + 158*a^3*b^4*c^2 + 44*a^4*b^2*c^3 - 400*a^5*c^4 + 420*(a*b^6*c^2 - 11*a^2*b^4*c^3 + 38*a^3*b^2*c^4 - 40*a^4*c^5)*d^4 + 45*(4*a*b^7*c - 45*a^2*b^5*c^2 + 162*a^3*b^3*c^3 - 184*a^4*b*c^4)*d^2)*e^4*x^4 + 3*(4*a*b^7*c - 45*a^2*b^5*c^2 + 162*a^3*b^3*c^3 - 184*a^4*b*c^4)*d^6 + 4*(84*(a*b^6*c^2 - 11*a^2*b^4*c^3 + 38*a^3*b^2*c^4 - 40*a^4*c^5)*d^5 + 15*(4*a*b^7*c - 45*a^2*b^5*c^2 + 162*a^3*b^3*c^3 - 184*a^4*b*c^4)*d^3 + 2*(3*a*b^8 - 30*a^2*b^6*c + 79*a^3*b^4*c^2 + 22*a^4*b^2*c^3 - 200*a^5*c^4)*d)*e^3*x^3 + 2*(3*a*b^8 - 30*a^2*b^6*c + 79*a^3*b^4*c^2 + 22*a^4*b^2*c^3 - 200*a^5*c^4)*d^4 + (9*a^2*b^7 - 104*a^3*b^5*c + 394*a^4*b^3*c^2 - 488*a^5*b*c^3 + 168*(a*b^6*c^2 - 11*a^2*b^4*c^3 + 38*a^3*b^2*c^4 - 40*a^4*c^5)*d^6 + 45*(4*a*b^7*c - 45*a^2*b^5*c^2 + 162*a^3*b^3*c^3 - 184*a^4*b*c^4)*d^4 + 12*(3*a*b^8 - 30*a^2*b^6*c + 79*a^3*b^4*c^2 + 22*a^4*b^2*c^3 - 200*a^5*c^4)*d^2)*e^2*x^2 + (9*a^2*b^7 - 104*a^3*b^5*c + 394*a^4*b^3*c^2 - 488*a^5*b*c^3)*d^2 + 2*(24*(a*b^6*c^2 - 11*a^2*b^4*c^3 + 38*a^3*b^2*c^4 - 40*a^4*c^5)*d^7 + 9*(4*a*b^7*c - 45*a^2*b^5*c^2 + 162*a^3*b^3*c^3 - 184*a^4*b*c^4)*d^5 + 4*(3*a*b^8 - 30*a^2*b^6*c + 79*a^3*b^4*c^2 + 22*a^4*b^2*c^3 - 200*a^5*c^4)*d^3 + (9*a^2*b^7 - 104*a^3*b^5*c + 394*a^4*b^3*c^2 - 488*a^5*b*c^3)*d)*e*x + 6*((b^6*c^2 - 10*a*b^4*c^3 + 30*a^2*b^2*c^4 - 20*a^3*c^5)*e^10*x^10 + 10*(b^6*c^2 - 10*a*b^4*c^3 + 30*a^2*b^2*c^4 - 20*a^3*c^5)*d*e^9*x^9 + (2*b^7*c - 20*a*b^5*c^2 + 60*a^2*b^3*c^3 - 40*a^3*b*c^4 + 45*(b^6*c^2 - 10*a*b^4*c^3 + 30*a^2*b^2*c^4 - 20*a^3*c^5)*d^2)*e^8*x^8 + 8*(15*(b^6*c^2 - 10*a*b^4*c^3 + 30*a^2*b^2*c^4 - 20*a^3*c^5)*d^3 + 2*(b^7*c - 10*a*b^5*c^2 + 30*a^2*b^3*c^3 - 20*a^3*b*c^4)*d)*e^7*x^7 + (b^8 - 8*a*b^6*c + 10*a^2*b^4*c^2 + 40*a^3*b^2*c^3 - 40*a^4*c^4 + 210*(b^6*c^2 - 10*a*b^4*c^3 + 30*a^2*b^2*c^4 - 20*a^3*c^5)*d^4 + 56*(b^7*c - 10*a*b^5*c^2 + 30*a^2*b^3*c^3 - 20*a^3*b*c^4)*d^2)*e^6*x^6 + (b^6*c^2 - 10*a*b^4*c^3 + 30*a^2*b^2*c^4 - 20*a^3*c^5)*d^10 + 2*(126*(b^6*c^2 - 10*a*b^4*c^3 + 30*a^2*b^2*c^4 - 20*a^3*c^5)*d^5 + 56*(b^7*c - 10*a*b^5*c^2 + 30*a^2*b^3*c^3 - 20*a^3*b*c^4)*d^3 + 3*(b^8 - 8*a*b^6*c + 10*a^2*b^4*c^2 + 40*a^3*b^2*c^3 - 40*a^4*c^4)*d)*e^5*x^5 + 2*(b^7*c - 10*a*b^5*c^2 + 30*a^2*b^3*c^3 - 20*a^3*b*c^4)*d^8 + (2*a*b^7 - 20*a^2*b^5*c + 60*a^3*b^3*c^2 - 40*a^4*b*c^3 + 210*(b^6*c^2 - 10*a*b^4*c^3 + 30*a^2*b^2*c^4 - 20*a^3*c^5)*d^6 + 140*(b^7*c - 10*a*b^5*c^2 + 30*a^2*b^3*c^3 - 20*a^3*b*c^4)*d^4 + 15*(b^8 - 8*a*b^6*c + 10*a^2*b^4*c^2 + 40*a^3*b^2*c^3 - 40*a^4*c^4)*d^2)*e^4*x^4 + (b^8 - 8*a*b^6*c + 10*a^2*b^4*c^2 + 40*a^3*b^2*c^3 - 40*a^4*c^4)*d^6 + 4*(30*(b^6*c^2 - 10*a*b^4*c^3 + 30*a^2*b^2*c^4 - 20*a^3*c^5)*d^7 + 28*(b^7*c - 10*a*b^5*c^2 + 30*a^2*b^3*c^3 - 20*a^3*b*c^4)*d^5 + 5*(b^8 - 8*a*b^6*c + 10*a^2*b^4*c^2 + 40*a^3*b^2*c^3 - 40*a^4*c^4)*d^3 + 2*(a*b^7 - 10*a^2*b^5*c + 30*a^3*b^3*c^2 - 20*a^4*b*c^3)*d)*e^3*x^3 + 2*(a*b^7 - 10*a^2*b^5*c + 30*a^3*b^3*c^2 - 20*a^4*b*c^3)*d^4 + (45*(b^6*c^2 - 10*a*b^4*c^3 + 30*a^2*b^2*c^4 - 20*a^3*c^5)*d^8 + a^2*b^6 - 10*a^3*b^4*c + 30*a^4*b^2*c^2 - 20*a^5*c^3 + 56*(b^7*c - 10*a*b^5*c^2 + 30*a^2*b^3*c^3 - 20*a^3*b*c^4)*d^6 + 15*(b^8 - 8*a*b^6*c + 10*a^2*b^4*c^2 + 40*a^3*b^2*c^3 - 40*a^4*c^4)*d^4 + 12*(a*b^7 - 10*a^2*b^5*c + 30*a^3*b^3*c^2 - 20*a^4*b*c^3)*d^2)*e^2*x^2 + (a^2*b^6 - 10*a^3*b^4*c + 30*a^4*b^2*c^2 - 20*a^5*c^3)*d^2 + 2*(5*(b^6*c^2 - 10*a*b^4*c^3 + 30*a^2*b^2*c^4 - 20*a^3*c^5)*d^9 + 8*(b^7*c - 10*a*b^5*c^2 + 30*a^2*b^3*c^3 - 20*a^3*b*c^4)*d^7 + 3*(b^8 - 8*a*b^6*c + 10*a^2*b^4*c^2 + 40*a^3*b^2*c^3 - 40*a^4*c^4)*d^5 + 4*(a*b^7 - 10*a^2*b^5*c + 30*a^3*b^3*c^2 - 20*a^4*b*c^3)*d^3 + (a^2*b^6 - 10*a^3*b

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^4*c + 30*a^4*b^2*c^2 - 20*a^5*c^3)*d)*e*x)*sqrt(-b^2 + 4*a*c)*arctan(-(2*c
*e^2*x^2 + 4*c*d*e*x + 2*c*d^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) - 3*(
(b^7*c^2 - 12*a*b^5*c^3 + 48*a^2*b^3*c^4 - 64*a^3*b*c^5)*e^10*x^10 + 10*(b^
7*c^2 - 12*a*b^5*c^3 + 48*a^2*b^3*c^4 - 64*a^3*b*c^5)*d*e^9*x^9 + (2*b^8*c
- 24*a*b^6*c^2 + 96*a^2*b^4*c^3 - 128*a^3*b^2*c^4 + 45*(b^7*c^2 - 12*a*b^5*
c^3 + 48*a^2*b^3*c^4 - 64*a^3*b*c^5)*d^2)*e^8*x^8 + 8*(15*(b^7*c^2 - 12*a*b
^5*c^3 + 48*a^2*b^3*c^4 - 64*a^3*b*c^5)*d^3 + 2*(b^8*c - 12*a*b^6*c^2 + 48*
a^2*b^4*c^3 - 64*a^3*b^2*c^4)*d)*e^7*x^7 + (b^9 - 10*a*b^7*c + 24*a^2*b^5*c
^2 + 32*a^3*b^3*c^3 - 128*a^4*b*c^4 + 210*(b^7*c^2 - 12*a*b^5*c^3 + 48*a^2*
b^3*c^4 - 64*a^3*b*c^5)*d^4 + 56*(b^8*c - 12*a*b^6*c^2 + 48*a^2*b^4*c^3 - 6
4*a^3*b^2*c^4)*d^2)*e^6*x^6 + (b^7*c^2 - 12*a*b^5*c^3 + 48*a^2*b^3*c^4 - 64
*a^3*b*c^5)*d^10 + 2*(126*(b^7*c^2 - 12*a*b^5*c^3 + 48*a^2*b^3*c^4 - 64*a^3
*b*c^5)*d^5 + 56*(b^8*c - 12*a*b^6*c^2 + 48*a^2*b^4*c^3 - 64*a^3*b^2*c^4)*d
^3 + 3*(b^9 - 10*a*b^7*c + 24*a^2*b^5*c^2 + 32*a^3*b^3*c^3 - 128*a^4*b*c^4)
*d)*e^5*x^5 + 2*(b^8*c - 12*a*b^6*c^2 + 48*a^2*b^4*c^3 - 64*a^3*b^2*c^4)*d^
8 + (2*a*b^8 - 24*a^2*b^6*c + 96*a^3*b^4*c^2 - 128*a^4*b^2*c^3 + 210*(b^7*c
^2 - 12*a*b^5*c^3 + 48*a^2*b^3*c^4 - 64*a^3*b*c^5)*d^6 + 140*(b^8*c - 12*a*
b^6*c^2 + 48*a^2*b^4*c^3 - 64*a^3*b^2*c^4)*d^4 + 15*(b^9 - 10*a*b^7*c + 24*
a^2*b^5*c^2 + 32*a^3*b^3*c^3 - 128*a^4*b*c^4)*d^2)*e^4*x^4 + (b^9 - 10*a*b^
7*c + 24*a^2*b^5*c^2 + 32*a^3*b^3*c^3 - 128*a^4*b*c^4)*d^6 + 4*(30*(b^7*c^2
 - 12*a*b^5*c^3 + 48*a^2*b^3*c^4 - 64*a^3*b*c^5)*d^7 + 28*(b^8*c - 12*a*b^6
*c^2 + 48*a^2*b^4*c^3 - 64*a^3*b^2*c^4)*d^5 + 5*(b^9 - 10*a*b^7*c + 24*a^2*
b^5*c^2 + 32*a^3*b^3*c^3 - 128*a^4*b*c^4)*d^3 + 2*(a*b^8 - 12*a^2*b^6*c + 4
8*a^3*b^4*c^2 - 64*a^4*b^2*c^3)*d)*e^3*x^3 + 2*(a*b^8 - 12*a^2*b^6*c + 48*a
^3*b^4*c^2 - 64*a^4*b^2*c^3)*d^4 + (a^2*b^7 - 12*a^3*b^5*c + 48*a^4*b^3*c^2
 - 64*a^5*b*c^3 + 45*(b^7*c^2 - 12*a*b^5*c^3 + 48*a^2*b^3*c^4 - 64*a^3*b*c^
5)*d^8 + 56*(b^8*c - 12*a*b^6*c^2 + 48*a^2*b^4*c^3 - 64*a^3*b^2*c^4)*d^6 +
15*(b^9 - 10*a*b^7*c + 24*a^2*b^5*c^2 + 32*a^3*b^3*c^3 - 128*a^4*b*c^4)*d^4
 + 12*(a*b^8 - 12*a^2*b^6*c + 48*a^3*b^4*c^2 - 64*a^4*b^2*c^3)*d^2)*e^2*x^2
 + (a^2*b^7 - 12*a^3*b^5*c + 48*a^4*b^3*c^2 - 64*a^5*b*c^3)*d^2 + 2*(5*(b^7
*c^2 - 12*a*b^5*c^3 + 48*a^2*b^3*c^4 - 64*a^3*b*c^5)*d^9 + 8*(b^8*c - 12*a*
b^6*c^2 + 48*a^2*b^4*c^3 - 64*a^3*b^2*c^4)*d^7 + 3*(b^9 - 10*a*b^7*c + 24*a
^2*b^5*c^2 + 32*a^3*b^3*c^3 - 128*a^4*b*c^4)*d^5 + 4*(a*b^8 - 12*a^2*b^6*c
+ 48*a^3*b^4*c^2 - 64*a^4*b^2*c^3)*d^3 + (a^2*b^7 - 12*a^3*b^5*c + 48*a^4*b
^3*c^2 - 64*a^5*b*c^3)*d)*e*x)*log(c*e^4*x^4 + 4*c*d*e^3*x^3 + c*d^4 + (6*c
*d^2 + b)*e^2*x^2 + b*d^2 + 2*(2*c*d^3 + b*d)*e*x + a) + 12*((b^7*c^2 - 12*
a*b^5*c^3 + 48*a^2*b^3*c^4 - 64*a^3*b*c^5)*e^10*x^10 + 10*(b^7*c^2 - 12*a*b
^5*c^3 + 48*a^2*b^3*c^4 - 64*a^3*b*c^5)*d*e^9*x^9 + (2*b^8*c - 24*a*b^6*c^2
 + 96*a^2*b^4*c^3 - 128*a^3*b^2*c^4 + 45*(b^7*c^2 - 12*a*b^5*c^3 + 48*a^2*b
^3*c^4 - 64*a^3*b*c^5)*d^2)*e^8*x^8 + 8*(15*(b^7*c^2 - 12*a*b^5*c^3 + 48*a^
2*b^3*c^4 - 64*a^3*b*c^5)*d^3 + 2*(b^8*c - 12*a*b^6*c^2 + 48*a^2*b^4*c^3 -
64*a^3*b^2*c^4)*d)*e^7*x^7 + (b^9 - 10*a*b^7*c + 24*a^2*b^5*c^2 + 32*a^3*b^
3*c^3 - 128*a^4*b*c^4 + 210*(b^7*c^2 - 12*a*b^5*c^3 + 48*a^2*b^3*c^4 - 64*a
^3*b*c^5)*d^4 + 56*(b^8*c - 12*a*b^6*c^2 + 48*a^2*b^4*c^3 - 64*a^3*b^2*c^4)
*d^2)*e^6*x^6 + (b^7*c^2 - 12*a*b^5*c^3 + 48*a^2*b^3*c^4 - 64*a^3*b*c^5)*d^
10 + 2*(126*(b^7*c^2 - 12*a*b^5*c^3 + 48*a^2*b^3*c^4 - 64*a^3*b*c^5)*d^5 +
56*(b^8*c - 12*a*b^6*c^2 + 48*a^2*b^4*c^3 - 64*a^3*b^2*c^4)*d^3 + 3*(b^9 -
10*a*b^7*c + 24*a^2*b^5*c^2 + 32*a^3*b^3*c^3 - 128*a^4*b*c^4)*d)*e^5*x^5 +
2*(b^8*c - 12*a*b^6*c^2 + 48*a^2*b^4*c^3 - 64*a^3*b^2*c^4)*d^8 + (2*a*b^8 -
 24*a^2*b^6*c + 96*a^3*b^4*c^2 - 128*a^4*b^2*c^3 + 210*(b^7*c^2 - 12*a*b^5*
c^3 + 48*a^2*b^3*c^4 - 64*a^3*b*c^5)*d^6 + 140*(b^8*c - 12*a*b^6*c^2 + 48*a
^2*b^4*c^3 - 64*a^3*b^2*c^4)*d^4 + 15*(b^9 - 10*a*b^7*c + 24*a^2*b^5*c^2 +
32*a^3*b^3*c^3 - 128*a^4*b*c^4)*d^2)*e^4*x^4 + (b^9 - 10*a*b^7*c + 24*a^2*b
^5*c^2 + 32*a^3*b^3*c^3 - 128*a^4*b*c^4)*d^6 + 4*(30*(b^7*c^2 - 12*a*b^5*c^
3 + 48*a^2*b^3*c^4 - 64*a^3*b*c^5)*d^7 + 28*(b^8*c - 12*a*b^6*c^2 + 48*a^2*
b^4*c^3 - 64*a^3*b^2*c^4)*d^5 + 5*(b^9 - 10*a*b^7*c + 24*a^2*b^5*c^2 + 32*a
^3*b^3*c^3 - 128*a^4*b*c^4)*d^3 + 2*(a*b^8 - 12*a^2*b^6*c + 48*a^3*b^4*c^2
- 64*a^4*b^2*c^3)*d)*e^3*x^3 + 2*(a*b^8 - 12*a^2*b^6*c + 48*a^3*b^4*c^2 - 6
4*a^4*b^2*c^3)*d^4 + (a^2*b^7 - 12*a^3*b^5*c + 48*a^4*b^3*c^2 - 64*a^5*b*c^
```

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3 + 45*(b^7*c^2 - 12*a*b^5*c^3 + 48*a^2*b^3*c^4 - 64*a^3*b*c^5)*d^8 + 56*(b
^8*c - 12*a*b^6*c^2 + 48*a^2*b^4*c^3 - 64*a^3*b^2*c^4)*d^6 + 15*(b^9 - 10*a
*b^7*c + 24*a^2*b^5*c^2 + 32*a^3*b^3*c^3 - 128*a^4*b*c^4)*d^4 + 12*(a*b^8 -
 12*a^2*b^6*c + 48*a^3*b^4*c^2 - 64*a^4*b^2*c^3)*d^2)*e^2*x^2 + (a^2*b^7 -
12*a^3*b^5*c + 48*a^4*b^3*c^2 - 64*a^5*b*c^3)*d^2 + 2*(5*(b^7*c^2 - 12*a*b^
5*c^3 + 48*a^2*b^3*c^4 - 64*a^3*b*c^5)*d^9 + 8*(b^8*c - 12*a*b^6*c^2 + 48*a
^2*b^4*c^3 - 64*a^3*b^2*c^4)*d^7 + 3*(b^9 - 10*a*b^7*c + 24*a^2*b^5*c^2 + 3
2*a^3*b^3*c^3 - 128*a^4*b*c^4)*d^5 + 4*(a*b^8 - 12*a^2*b^6*c + 48*a^3*b^4*c
^2 - 64*a^4*b^2*c^3)*d^3 + (a^2*b^7 - 12*a^3*b^5*c + 48*a^4*b^3*c^2 - 64*a^
5*b*c^3)*d)*e*x)*log(e*x + d))/((a^4*b^6*c^2 - 12*a^5*b^4*c^3 + 48*a^6*b^2*
c^4 - 64*a^7*c^5)*e^11*f^3*x^10 + 10*(a^4*b^6*c^2 - 12*a^5*b^4*c^3 + 48*a^6
*b^2*c^4 - 64*a^7*c^5)*d*e^10*f^3*x^9 + (2*a^4*b^7*c - 24*a^5*b^5*c^2 + 96*
a^6*b^3*c^3 - 128*a^7*b*c^4 + 45*(a^4*b^6*c^2 - 12*a^5*b^4*c^3 + 48*a^6*b^2
*c^4 - 64*a^7*c^5)*d^2)*e^9*f^3*x^8 + 8*(15*(a^4*b^6*c^2 - 12*a^5*b^4*c^3 +
 48*a^6*b^2*c^4 - 64*a^7*c^5)*d^3 + 2*(a^4*b^7*c - 12*a^5*b^5*c^2 + 48*a^6*
b^3*c^3 - 64*a^7*b*c^4)*d)*e^8*f^3*x^7 + (a^4*b^8 - 10*a^5*b^6*c + 24*a^6*b
^4*c^2 + 32*a^7*b^2*c^3 - 128*a^8*c^4 + 210*(a^4*b^6*c^2 - 12*a^5*b^4*c^3 +
 48*a^6*b^2*c^4 - 64*a^7*c^5)*d^4 + 56*(a^4*b^7*c - 12*a^5*b^5*c^2 + 48*a^6
*b^3*c^3 - 64*a^7*b*c^4)*d^2)*e^7*f^3*x^6 + 2*(126*(a^4*b^6*c^2 - 12*a^5*b^
4*c^3 + 48*a^6*b^2*c^4 - 64*a^7*c^5)*d^5 + 56*(a^4*b^7*c - 12*a^5*b^5*c^2 +
 48*a^6*b^3*c^3 - 64*a^7*b*c^4)*d^3 + 3*(a^4*b^8 - 10*a^5*b^6*c + 24*a^6*b^
4*c^2 + 32*a^7*b^2*c^3 - 128*a^8*c^4)*d)*e^6*f^3*x^5 + (2*a^5*b^7 - 24*a^6*
b^5*c + 96*a^7*b^3*c^2 - 128*a^8*b*c^3 + 210*(a^4*b^6*c^2 - 12*a^5*b^4*c^3
+ 48*a^6*b^2*c^4 - 64*a^7*c^5)*d^6 + 140*(a^4*b^7*c - 12*a^5*b^5*c^2 + 48*a
^6*b^3*c^3 - 64*a^7*b*c^4)*d^4 + 15*(a^4*b^8 - 10*a^5*b^6*c + 24*a^6*b^4*c^
2 + 32*a^7*b^2*c^3 - 128*a^8*c^4)*d^2)*e^5*f^3*x^4 + 4*(30*(a^4*b^6*c^2 - 1
2*a^5*b^4*c^3 + 48*a^6*b^2*c^4 - 64*a^7*c^5)*d^7 + 28*(a^4*b^7*c - 12*a^5*b
^5*c^2 + 48*a^6*b^3*c^3 - 64*a^7*b*c^4)*d^5 + 5*(a^4*b^8 - 10*a^5*b^6*c + 2
4*a^6*b^4*c^2 + 32*a^7*b^2*c^3 - 128*a^8*c^4)*d^3 + 2*(a^5*b^7 - 12*a^6*b^5
*c + 48*a^7*b^3*c^2 - 64*a^8*b*c^3)*d)*e^4*f^3*x^3 + (a^6*b^6 - 12*a^7*b^4*
c + 48*a^8*b^2*c^2 - 64*a^9*c^3 + 45*(a^4*b^6*c^2 - 12*a^5*b^4*c^3 + 48*a^6
*b^2*c^4 - 64*a^7*c^5)*d^8 + 56*(a^4*b^7*c - 12*a^5*b^5*c^2 + 48*a^6*b^3*c^
3 - 64*a^7*b*c^4)*d^6 + 15*(a^4*b^8 - 10*a^5*b^6*c + 24*a^6*b^4*c^2 + 32*a^
7*b^2*c^3 - 128*a^8*c^4)*d^4 + 12*(a^5*b^7 - 12*a^6*b^5*c + 48*a^7*b^3*c^2
- 64*a^8*b*c^3)*d^2)*e^3*f^3*x^2 + 2*(5*(a^4*b^6*c^2 - 12*a^5*b^4*c^3 + 48*
a^6*b^2*c^4 - 64*a^7*c^5)*d^9 + 8*(a^4*b^7*c - 12*a^5*b^5*c^2 + 48*a^6*b^3*
c^3 - 64*a^7*b*c^4)*d^7 + 3*(a^4*b^8 - 10*a^5*b^6*c + 24*a^6*b^4*c^2 + 32*a
^7*b^2*c^3 - 128*a^8*c^4)*d^5 + 4*(a^5*b^7 - 12*a^6*b^5*c + 48*a^7*b^3*c^2
- 64*a^8*b*c^3)*d^3 + (a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3
)*d)*e^2*f^3*x + ((a^4*b^6*c^2 - 12*a^5*b^4*c^3 + 48*a^6*b^2*c^4 - 64*a^7*c
^5)*d^10 + 2*(a^4*b^7*c - 12*a^5*b^5*c^2 + 48*a^6*b^3*c^3 - 64*a^7*b*c^4)*d
^8 + (a^4*b^8 - 10*a^5*b^6*c + 24*a^6*b^4*c^2 + 32*a^7*b^2*c^3 - 128*a^8*c^
4)*d^6 + 2*(a^5*b^7 - 12*a^6*b^5*c + 48*a^7*b^3*c^2 - 64*a^8*b*c^3)*d^4 + (
a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)*d^2)*e*f^3)]
```

giac [B] time = 1.63, size = 1735, normalized size = 5.06

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*f*x+d*f)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="giac"
)
```

```
[Out] 3/4*((a^4*b^8*c*f^3*e^3 - 14*a^5*b^6*c^2*f^3*e^3 + 70*a^6*b^4*c^3*f^3*e^3 -
 140*a^7*b^2*c^4*f^3*e^3 + 80*a^8*c^5*f^3*e^3)*sqrt(b^2 - 4*a*c)*log(abs(b*
x^2*e^2 + 2*b*d*x*e + sqrt(b^2 - 4*a*c)*x^2*e^2 + 2*sqrt(b^2 - 4*a*c)*d*x*e
 + b*d^2 + sqrt(b^2 - 4*a*c)*d^2 + 2*a)) - (a^4*b^8*c*f^3*e^3 - 14*a^5*b^6*
c^2*f^3*e^3 + 70*a^6*b^4*c^3*f^3*e^3 - 140*a^7*b^2*c^4*f^3*e^3 + 80*a^8*c^5
*f^3*e^3)*sqrt(b^2 - 4*a*c)*log(abs(-b*x^2*e^2 - 2*b*d*x*e + sqrt(b^2 - 4*a
*c)*x^2*e^2 + 2*sqrt(b^2 - 4*a*c)*d*x*e - b*d^2 + sqrt(b^2 - 4*a*c)*d^2 - 2
```

```
*a)))/(a^8*b^8*c*f^6*e^4 - 16*a^9*b^6*c^2*f^6*e^4 + 96*a^10*b^4*c^3*f^6*e^4
 - 256*a^11*b^2*c^4*f^6*e^4 + 256*a^12*c^5*f^6*e^4) - 1/4*(6*b^4*c^2*x^8*e^
8 - 42*a*b^2*c^3*x^8*e^8 + 60*a^2*c^4*x^8*e^8 + 48*b^4*c^2*d*x^7*e^7 - 336*
a*b^2*c^3*d*x^7*e^7 + 480*a^2*c^4*d*x^7*e^7 + 168*b^4*c^2*d^2*x^6*e^6 - 117
6*a*b^2*c^3*d^2*x^6*e^6 + 1680*a^2*c^4*d^2*x^6*e^6 + 336*b^4*c^2*d^3*x^5*e^
5 - 2352*a*b^2*c^3*d^3*x^5*e^5 + 3360*a^2*c^4*d^3*x^5*e^5 + 420*b^4*c^2*d^4
*x^4*e^4 - 2940*a*b^2*c^3*d^4*x^4*e^4 + 4200*a^2*c^4*d^4*x^4*e^4 + 336*b^4*
c^2*d^5*x^3*e^3 - 2352*a*b^2*c^3*d^5*x^3*e^3 + 3360*a^2*c^4*d^5*x^3*e^3 + 1
68*b^4*c^2*d^6*x^2*e^2 - 1176*a*b^2*c^3*d^6*x^2*e^2 + 1680*a^2*c^4*d^6*x^2*
e^2 + 48*b^4*c^2*d^7*x*e - 336*a*b^2*c^3*d^7*x*e + 480*a^2*c^4*d^7*x*e + 6*
b^4*c^2*d^8 - 42*a*b^2*c^3*d^8 + 60*a^2*c^4*d^8 + 12*b^5*c*x^6*e^6 - 87*a*b
^3*c^2*x^6*e^6 + 138*a^2*b*c^3*x^6*e^6 + 72*b^5*c*d*x^5*e^5 - 522*a*b^3*c^2
*d*x^5*e^5 + 828*a^2*b*c^3*d*x^5*e^5 + 180*b^5*c*d^2*x^4*e^4 - 1305*a*b^3*c
^2*d^2*x^4*e^4 + 2070*a^2*b*c^3*d^2*x^4*e^4 + 240*b^5*c*d^3*x^3*e^3 - 1740*
a*b^3*c^2*d^3*x^3*e^3 + 2760*a^2*b*c^3*d^3*x^3*e^3 + 180*b^5*c*d^4*x^2*e^2
- 1305*a*b^3*c^2*d^4*x^2*e^2 + 2070*a^2*b*c^3*d^4*x^2*e^2 + 72*b^5*c*d^5*x*
e - 522*a*b^3*c^2*d^5*x*e + 828*a^2*b*c^3*d^5*x*e + 12*b^5*c*d^6 - 87*a*b^3
*c^2*d^6 + 138*a^2*b*c^3*d^6 + 6*b^6*x^4*e^4 - 36*a*b^4*c*x^4*e^4 + 14*a^2*
b^2*c^2*x^4*e^4 + 100*a^3*c^3*x^4*e^4 + 24*b^6*d*x^3*e^3 - 144*a*b^4*c*d*x^
3*e^3 + 56*a^2*b^2*c^2*d*x^3*e^3 + 400*a^3*c^3*d*x^3*e^3 + 36*b^6*d^2*x^2*e
^2 - 216*a*b^4*c*d^2*x^2*e^2 + 84*a^2*b^2*c^2*d^2*x^2*e^2 + 600*a^3*c^3*d^2
*x^2*e^2 + 24*b^6*d^3*x*e - 144*a*b^4*c*d^3*x*e + 56*a^2*b^2*c^2*d^3*x*e +
400*a^3*c^3*d^3*x*e + 6*b^6*d^4 - 36*a*b^4*c*d^4 + 14*a^2*b^2*c^2*d^4 + 100
*a^3*c^3*d^4 + 9*a*b^5*x^2*e^2 - 68*a^2*b^3*c*x^2*e^2 + 122*a^3*b*c^2*x^2*e
^2 + 18*a*b^5*d*x*e - 136*a^2*b^3*c*d*x*e + 244*a^3*b*c^2*d*x*e + 9*a*b^5*d
^2 - 68*a^2*b^3*c*d^2 + 122*a^3*b*c^2*d^2 + 2*a^2*b^4 - 16*a^3*b^2*c + 32*a
^4*c^2)/((a^3*b^4*f^3*e - 8*a^4*b^2*c*f^3*e + 16*a^5*c^2*f^3*e)*(c*x^5*e^5
+ 5*c*d*x^4*e^4 + 10*c*d^2*x^3*e^3 + 10*c*d^3*x^2*e^2 + 5*c*d^4*x*e + c*d^5
 + b*x^3*e^3 + 3*b*d*x^2*e^2 + 3*b*d^2*x*e + b*d^3 + a*x*e + a*d)^2) + 3/4*
b*e^(-1)*log(abs(c*x^4*e^4 + 4*c*d*x^3*e^3 + 6*c*d^2*x^2*e^2 + 4*c*d^3*x*e
+ c*d^4 + b*x^2*e^2 + 2*b*d*x*e + b*d^2 + a))/(a^4*f^3) - 3*b*e^(-1)*log(ab
s(x*e + d))/(a^4*f^3)
```

maple [C] time = 0.08, size = 5737, normalized size = 16.73

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*f*x+d*f)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x)

[Out] result too large to display
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*f*x+d*f)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="maxim
a")

[Out] Timed out
```

mupad [B] time = 24.91, size = 25334, normalized size = 73.86

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((d*f + e*f*x)^3*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3),x)

[Out] (log(((27*c^5*e^16*x^2*(b^4 + 10*a^2*c^2 - 7*a*b^2*c)^3)/(a^9*f^9*(4*a*c -
b^2)^6) - ((3*b - 3*a^4*e*f^3*(-(b^6 - 20*a^3*c^3 + 30*a^2*b^2*c^2 - 10*a*b
```

^4*c)^2/(a^8*e^2*f^6*(4*a*c - b^2)^5))^(1/2))*((9*c^3*e^15*(b^4 + 10*a^2*c^2 - 7*a*b^2*c)*(4*b^6 - 10*a^3*c^3 + 6*b^5*c*d^2 + 71*a^2*b^2*c^2 - 33*a*b^4*c - 47*a*b^3*c^2*d^2 + 90*a^2*b*c^3*d^2))/(a^6*f^6*(4*a*c - b^2)^4) - ((3*b - 3*a^4*e*f^3*(-(b^6 - 20*a^3*c^3 + 30*a^2*b^2*c^2 - 10*a*b^4*c)^2/(a^8*e^2*f^6*(4*a*c - b^2)^5))^(1/2))*((6*c^2*e^16*(2*b^7 - 20*a^3*b*c^3 + b^6*c*d^2 + 46*a^2*b^3*c^2 + 100*a^3*c^4*d^2 - 18*a*b^5*c - 2*a*b^4*c^2*d^2 - 30*a^2*b^2*c^3*d^2))/(a^3*f^3*(4*a*c - b^2)^2) + (b*c^2*e^16*(3*b - 3*a^4*e*f^3*(-(b^6 - 20*a^3*c^3 + 30*a^2*b^2*c^2 - 10*a*b^4*c)^2/(a^8*e^2*f^6*(4*a*c - b^2)^5))^(1/2))*(a*b + 3*b^2*d^2 + 3*b^2*e^2*x^2 - 10*a*c*d^2 + 6*b^2*d*e*x - 10*a*c*e^2*x^2 - 20*a*c*d*e*x))/(a^4*f^3) + (6*c^3*e^18*x^2*(b^6 + 100*a^3*c^3 - 30*a^2*b^2*c^2 - 2*a*b^4*c))/(a^3*f^3*(4*a*c - b^2)^2) + (12*c^3*d*e^17*x*(b^6 + 100*a^3*c^3 - 30*a^2*b^2*c^2 - 2*a*b^4*c))/(a^3*f^3*(4*a*c - b^2)^2)))/(4*a^4*e*f^3) + (9*b*c^4*e^17*x^2*(6*b^8 + 900*a^4*c^4 + 479*a^2*b^4*c^2 - 1100*a^3*b^2*c^3 - 89*a*b^6*c))/(a^6*f^6*(4*a*c - b^2)^4) + (18*b*c^4*d*e^16*x*(6*b^8 + 900*a^4*c^4 + 479*a^2*b^4*c^2 - 1100*a^3*b^2*c^3 - 89*a*b^6*c))/(a^6*f^6*(4*a*c - b^2)^4)))/(4*a^4*e*f^3) + (27*c^4*e^14*(b^4 + 10*a^2*c^2 - 7*a*b^2*c)^2*(b^5 + 16*a^2*b*c^2 + b^4*c*d^2 + 10*a^2*c^3*d^2 - 8*a*b^3*c - 7*a*b^2*c^2*d^2))/(a^9*f^9*(4*a*c - b^2)^6) + (54*c^5*d*e^15*x*(b^4 + 10*a^2*c^2 - 7*a*b^2*c)^3)/(a^9*f^9*(4*a*c - b^2)^6))*((27*c^5*e^16*x^2*(b^4 + 10*a^2*c^2 - 7*a*b^2*c)^3)/(a^9*f^9*(4*a*c - b^2)^6) - ((3*b + 3*a^4*e*f^3*(-(b^6 - 20*a^3*c^3 + 30*a^2*b^2*c^2 - 10*a*b^4*c)^2/(a^8*e^2*f^6*(4*a*c - b^2)^5))^(1/2))*((9*c^3*e^15*(b^4 + 10*a^2*c^2 - 7*a*b^2*c)*(4*b^6 - 10*a^3*c^3 + 6*b^5*c*d^2 + 71*a^2*b^2*c^2 - 33*a*b^4*c - 47*a*b^3*c^2*d^2 + 90*a^2*b*c^3*d^2))/(a^6*f^6*(4*a*c - b^2)^4) - ((3*b + 3*a^4*e*f^3*(-(b^6 - 20*a^3*c^3 + 30*a^2*b^2*c^2 - 10*a*b^4*c)^2/(a^8*e^2*f^6*(4*a*c - b^2)^5))^(1/2))*((6*c^2*e^16*(2*b^7 - 20*a^3*b*c^3 + b^6*c*d^2 + 46*a^2*b^3*c^2 + 100*a^3*c^4*d^2 - 18*a*b^5*c - 2*a*b^4*c^2*d^2 - 30*a^2*b^2*c^3*d^2))/(a^3*f^3*(4*a*c - b^2)^2) + (b*c^2*e^16*(3*b + 3*a^4*e*f^3*(-(b^6 - 20*a^3*c^3 + 30*a^2*b^2*c^2 - 10*a*b^4*c)^2/(a^8*e^2*f^6*(4*a*c - b^2)^5))^(1/2))*(a*b + 3*b^2*d^2 + 3*b^2*e^2*x^2 - 10*a*c*d^2 + 6*b^2*d*e*x - 10*a*c*e^2*x^2 - 20*a*c*d*e*x))/(a^4*f^3) + (6*c^3*e^18*x^2*(b^6 + 100*a^3*c^3 - 30*a^2*b^2*c^2 - 2*a*b^4*c))/(a^3*f^3*(4*a*c - b^2)^2) + (12*c^3*d*e^17*x*(b^6 + 100*a^3*c^3 - 30*a^2*b^2*c^2 - 2*a*b^4*c))/(a^3*f^3*(4*a*c - b^2)^2)))/(4*a^4*e*f^3) + (9*b*c^4*e^17*x^2*(6*b^8 + 900*a^4*c^4 + 479*a^2*b^4*c^2 - 1100*a^3*b^2*c^3 - 89*a*b^6*c))/(a^6*f^6*(4*a*c - b^2)^4) + (18*b*c^4*d*e^16*x*(6*b^8 + 900*a^4*c^4 + 479*a^2*b^4*c^2 - 1100*a^3*b^2*c^3 - 89*a*b^6*c))/(a^6*f^6*(4*a*c - b^2)^4)))/(4*a^4*e*f^3) + (27*c^4*e^14*(b^4 + 10*a^2*c^2 - 7*a*b^2*c)^2*(b^5 + 16*a^2*b*c^2 + b^4*c*d^2 + 10*a^2*c^3*d^2 - 8*a*b^3*c - 7*a*b^2*c^2*d^2))/(a^9*f^9*(4*a*c - b^2)^6) + (54*c^5*d*e^15*x*(b^4 + 10*a^2*c^2 - 7*a*b^2*c)^3)/(a^9*f^9*(4*a*c - b^2)^6)))*(6*b^11*e*f^3 - 120*a*b^9*c*e*f^3 - 6144*a^5*b*c^5*e*f^3 + 960*a^2*b^7*c^2*e*f^3 - 3840*a^3*b^5*c^3*e*f^3 + 7680*a^4*b^3*c^4*e*f^3))/(2*(4*a^4*b^10*e^2*f^6 - 4096*a^9*c^5*e^2*f^6 + 640*a^6*b^6*c^2*e^2*f^6 - 2560*a^7*b^4*c^3*e^2*f^6 + 5120*a^8*b^2*c^4*e^2*f^6 - 80*a^5*b^8*c*e^2*f^6)) - ((x^4*(6*b^6*e^3 + 100*a^3*c^3*e^3 + 180*b^5*c*d^2*e^3 + 14*a^2*b^2*c^2*e^3 + 4200*a^2*c^4*d^4*e^3 + 420*b^4*c^2*d^4*e^3 - 36*a*b^4*c*e^3 - 1305*a*b^3*c^2*d^2*e^3 + 2070*a^2*b*c^3*d^2*e^3 - 2940*a*b^2*c^3*d^4*e^3))/(4*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)) + (3*x^6*(4*b^5*c*e^5 - 29*a*b^3*c^2*e^5 + 46*a^2*b*c^3*e^5 + 560*a^2*c^4*d^2*e^5 + 56*b^4*c^2*d^2*e^5 - 392*a*b^2*c^3*d^2*e^5))/(4*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)) + (x*(12*b^6*d^3 + 36*b^5*c*d^5 + 200*a^3*c^3*d^3 + 240*a^2*c^4*d^7 + 24*b^4*c^2*d^7 + 9*a*b^5*d - 261*a*b^3*c^2*d^5 + 414*a^2*b*c^3*d^5 - 168*a*b^2*c^3*d^7 + 28*a^2*b^2*c^2*d^3 - 68*a^2*b^3*c*d + 122*a^3*b*c^2*d - 72*a*b^4*c*d^3))/(2*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)) + (3*x^5*(560*a^2*c^4*d^3*e^4 + 56*b^4*c^2*d^3*e^4 + 12*b^5*c*d*e^4 - 87*a*b^3*c^2*d*e^4 + 138*a^2*b*c^3*d*e^4 - 392*a*b^2*c^3*d^3*e^4))/(2*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)) + (3*x^8*(10*a^2*c^4*e^7 + b^4*c^2*e^7 - 7*a*b^2*c^3*e^7))/(2*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)) + (x^2*(36*b^6*d^2*e + 9*a*b^5*e + 600*a^3*c^3*d^2*e + 1680*a^2*c^4*d^6*e + 168*b^4*c^2*d^6*e - 68*a^2*b^3*c*e + 122*a^3*b*c^2*e + 180*b^5*c*d^4*e - 216*a*b^4*c*d^2*e - 1305*a*b^3*

c^2*d^4*e + 2070*a^2*b*c^3*d^4*e - 1176*a*b^2*c^3*d^6*e + 84*a^2*b^2*c^2*d^
2*e))/(4*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)) + (x^3*(6*b^6*d*e^2 + 100*a^
3*c^3*d*e^2 + 60*b^5*c*d^3*e^2 + 840*a^2*c^4*d^5*e^2 + 84*b^4*c^2*d^5*e^2 -
36*a*b^4*c*d*e^2 + 14*a^2*b^2*c^2*d*e^2 - 435*a*b^3*c^2*d^3*e^2 + 690*a^2*
b*c^3*d^3*e^2 - 588*a*b^2*c^3*d^5*e^2))/(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c
) + (12*x^7*(10*a^2*c^4*d*e^6 + b^4*c^2*d*e^6 - 7*a*b^2*c^3*d*e^6))/(a^3*b^
4 + 16*a^5*c^2 - 8*a^4*b^2*c) + (2*a^2*b^4 + 32*a^4*c^2 + 6*b^6*d^4 - 16*a^
3*b^2*c + 9*a*b^5*d^2 + 12*b^5*c*d^6 + 100*a^3*c^3*d^4 + 60*a^2*c^4*d^8 + 6
*b^4*c^2*d^8 - 68*a^2*b^3*c*d^2 + 122*a^3*b*c^2*d^2 - 87*a*b^3*c^2*d^6 + 13
8*a^2*b*c^3*d^6 - 42*a*b^2*c^3*d^8 + 14*a^2*b^2*c^2*d^4 - 36*a*b^4*c*d^4)/(
4*e*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))/(x^4*(15*b^2*d^2*e^4*f^3 + 210*c
^2*d^6*e^4*f^3 + 2*a*b*e^4*f^3 + 30*a*c*d^2*e^4*f^3 + 140*b*c*d^4*e^4*f^3)
+ x^7*(120*c^2*d^3*e^7*f^3 + 16*b*c*d*e^7*f^3) + x*(6*b^2*d^5*e*f^3 + 10*c^
2*d^9*e*f^3 + 2*a^2*d*e*f^3 + 8*a*b*d^3*e*f^3 + 12*a*c*d^5*e*f^3 + 16*b*c*d
^7*e*f^3) + x^3*(20*b^2*d^3*e^3*f^3 + 120*c^2*d^7*e^3*f^3 + 8*a*b*d*e^3*f^3
+ 40*a*c*d^3*e^3*f^3 + 112*b*c*d^5*e^3*f^3) + x^2*(a^2*e^2*f^3 + 15*b^2*d^
4*e^2*f^3 + 45*c^2*d^8*e^2*f^3 + 12*a*b*d^2*e^2*f^3 + 30*a*c*d^4*e^2*f^3 +
56*b*c*d^6*e^2*f^3) + x^5*(6*b^2*d*e^5*f^3 + 252*c^2*d^5*e^5*f^3 + 12*a*c*d
*e^5*f^3 + 112*b*c*d^3*e^5*f^3) + x^8*(45*c^2*d^2*e^8*f^3 + 2*b*c*e^8*f^3)
+ x^6*(b^2*e^6*f^3 + 210*c^2*d^4*e^6*f^3 + 2*a*c*e^6*f^3 + 56*b*c*d^2*e^6*f
^3) + a^2*d^2*f^3 + b^2*d^6*f^3 + c^2*d^10*f^3 + c^2*e^10*f^3*x^10 + 2*a*b*
d^4*f^3 + 2*a*c*d^6*f^3 + 2*b*c*d^8*f^3 + 10*c^2*d*e^9*f^3*x^9) - (3*b*log(
d + e*x))/(a^4*e*f^3) + (3*atan((x^2*((((((54*a^3*b^13*c^4*e^17*f^3 - 1233*
a^4*b^11*c^5*e^17*f^3 + 11583*a^5*b^9*c^6*e^17*f^3 - 57204*a^6*b^7*c^7*e^17
*f^3 + 156276*a^7*b^5*c^8*e^17*f^3 - 223200*a^8*b^3*c^9*e^17*f^3 + 129600*a
^9*b*c^10*e^17*f^3)/(a^9*b^12*f^9 + 4096*a^15*c^6*f^9 - 24*a^10*b^10*c*f^9
+ 240*a^11*b^8*c^2*f^9 - 1280*a^12*b^6*c^3*f^9 + 3840*a^13*b^4*c^4*f^9 - 61
44*a^14*b^2*c^5*f^9) - (((153600*a^13*c^10*e^18*f^6 + 6*a^6*b^14*c^3*e^18*f
^6 - 108*a^7*b^12*c^4*e^18*f^6 + 588*a^8*b^10*c^5*e^18*f^6 + 792*a^9*b^8*c^
6*e^18*f^6 - 22272*a^10*b^6*c^7*e^18*f^6 + 100608*a^11*b^4*c^8*e^18*f^6 - 1
99680*a^12*b^2*c^9*e^18*f^6)/(a^9*b^12*f^9 + 4096*a^15*c^6*f^9 - 24*a^10*b^
10*c*f^9 + 240*a^11*b^8*c^2*f^9 - 1280*a^12*b^6*c^3*f^9 + 3840*a^13*b^4*c^4
*f^9 - 6144*a^14*b^2*c^5*f^9) + ((6*b^11*e*f^3 - 120*a*b^9*c*e*f^3 - 6144*a
^5*b*c^5*e*f^3 + 960*a^2*b^7*c^2*e*f^3 - 3840*a^3*b^5*c^3*e*f^3 + 7680*a^4*
b^3*c^4*e*f^3)*(12*a^9*b^15*c^2*e^19*f^9 - 328*a^10*b^13*c^3*e^19*f^9 + 384
0*a^11*b^11*c^4*e^19*f^9 - 24960*a^12*b^9*c^5*e^19*f^9 + 97280*a^13*b^7*c^6
*e^19*f^9 - 227328*a^14*b^5*c^7*e^19*f^9 + 294912*a^15*b^3*c^8*e^19*f^9 - 1
63840*a^16*b*c^9*e^19*f^9))/(2*(4*a^4*b^10*e^2*f^6 - 4096*a^9*c^5*e^2*f^6 +
640*a^6*b^6*c^2*e^2*f^6 - 2560*a^7*b^4*c^3*e^2*f^6 + 5120*a^8*b^2*c^4*e^2*
f^6 - 80*a^5*b^8*c*e^2*f^6)*(a^9*b^12*f^9 + 4096*a^15*c^6*f^9 - 24*a^10*b^1
0*c*f^9 + 240*a^11*b^8*c^2*f^9 - 1280*a^12*b^6*c^3*f^9 + 3840*a^13*b^4*c^4*
f^9 - 6144*a^14*b^2*c^5*f^9)))*(6*b^11*e*f^3 - 120*a*b^9*c*e*f^3 - 6144*a^5
*b*c^5*e*f^3 + 960*a^2*b^7*c^2*e*f^3 - 3840*a^3*b^5*c^3*e*f^3 + 7680*a^4*b^
3*c^4*e*f^3))/(2*(4*a^4*b^10*e^2*f^6 - 4096*a^9*c^5*e^2*f^6 + 640*a^6*b^6*c
^2*e^2*f^6 - 2560*a^7*b^4*c^3*e^2*f^6 + 5120*a^8*b^2*c^4*e^2*f^6 - 80*a^5*b
^8*c*e^2*f^6)))*(6*b^11*e*f^3 - 120*a*b^9*c*e*f^3 - 6144*a^5*b*c^5*e*f^3 +
960*a^2*b^7*c^2*e*f^3 - 3840*a^3*b^5*c^3*e*f^3 + 7680*a^4*b^3*c^4*e*f^3))/(
2*(4*a^4*b^10*e^2*f^6 - 4096*a^9*c^5*e^2*f^6 + 640*a^6*b^6*c^2*e^2*f^6 - 25
60*a^7*b^4*c^3*e^2*f^6 + 5120*a^8*b^2*c^4*e^2*f^6 - 80*a^5*b^8*c*e^2*f^6))
- (27000*a^6*c^11*e^16 + 27*b^12*c^5*e^16 - 567*a*b^10*c^6*e^16 + 4779*a^2*
b^8*c^7*e^16 - 20601*a^3*b^6*c^8*e^16 + 47790*a^4*b^4*c^9*e^16 - 56700*a^5*
b^2*c^10*e^16)/(a^9*b^12*f^9 + 4096*a^15*c^6*f^9 - 24*a^10*b^10*c*f^9 + 240
*a^11*b^8*c^2*f^9 - 1280*a^12*b^6*c^3*f^9 + 3840*a^13*b^4*c^4*f^9 - 6144*a^
14*b^2*c^5*f^9) + (3*((3*((153600*a^13*c^10*e^18*f^6 + 6*a^6*b^14*c^3*e^18*
f^6 - 108*a^7*b^12*c^4*e^18*f^6 + 588*a^8*b^10*c^5*e^18*f^6 + 792*a^9*b^8*c
^6*e^18*f^6 - 22272*a^10*b^6*c^7*e^18*f^6 + 100608*a^11*b^4*c^8*e^18*f^6 -
199680*a^12*b^2*c^9*e^18*f^6)/(a^9*b^12*f^9 + 4096*a^15*c^6*f^9 - 24*a^10*b
^10*c*f^9 + 240*a^11*b^8*c^2*f^9 - 1280*a^12*b^6*c^3*f^9 + 3840*a^13*b^4*c^
4*f^9 - 6144*a^14*b^2*c^5*f^9) + ((6*b^11*e*f^3 - 120*a*b^9*c*e*f^3 - 6144*

a^5*b*c^5*e*f^3 + 960*a^2*b^7*c^2*e*f^3 - 3840*a^3*b^5*c^3*e*f^3 + 7680*a^4
*b^3*c^4*e*f^3)*(12*a^9*b^15*c^2*e^19*f^9 - 328*a^10*b^13*c^3*e^19*f^9 + 38
40*a^11*b^11*c^4*e^19*f^9 - 24960*a^12*b^9*c^5*e^19*f^9 + 97280*a^13*b^7*c^
6*e^19*f^9 - 227328*a^14*b^5*c^7*e^19*f^9 + 294912*a^15*b^3*c^8*e^19*f^9 -
163840*a^16*b*c^9*e^19*f^9))/(2*(4*a^4*b^10*e^2*f^6 - 4096*a^9*c^5*e^2*f^6
+ 640*a^6*b^6*c^2*e^2*f^6 - 2560*a^7*b^4*c^3*e^2*f^6 + 5120*a^8*b^2*c^4*e^2
*f^6 - 80*a^5*b^8*c*e^2*f^6)*(a^9*b^12*f^9 + 4096*a^15*c^6*f^9 - 24*a^10*b^
10*c*f^9 + 240*a^11*b^8*c^2*f^9 - 1280*a^12*b^6*c^3*f^9 + 3840*a^13*b^4*c^4
*f^9 - 6144*a^14*b^2*c^5*f^9)))*(b^6 - 20*a^3*c^3 + 30*a^2*b^2*c^2 - 10*a*b
^4*c))/(4*a^4*e*f^3*(4*a*c - b^2)^(5/2)) + (3*(b^6 - 20*a^3*c^3 + 30*a^2*b^
2*c^2 - 10*a*b^4*c)*(6*b^11*e*f^3 - 120*a*b^9*c*e*f^3 - 6144*a^5*b*c^5*e*f^
3 + 960*a^2*b^7*c^2*e*f^3 - 3840*a^3*b^5*c^3*e*f^3 + 7680*a^4*b^3*c^4*e*f^3
)*(12*a^9*b^15*c^2*e^19*f^9 - 328*a^10*b^13*c^3*e^19*f^9 + 3840*a^11*b^11*c
^4*e^19*f^9 - 24960*a^12*b^9*c^5*e^19*f^9 + 97280*a^13*b^7*c^6*e^19*f^9 - 2
27328*a^14*b^5*c^7*e^19*f^9 + 294912*a^15*b^3*c^8*e^19*f^9 - 163840*a^16*b*
c^9*e^19*f^9))/(8*a^4*e*f^3*(4*a*c - b^2)^(5/2)*(4*a^4*b^10*e^2*f^6 - 4096*
a^9*c^5*e^2*f^6 + 640*a^6*b^6*c^2*e^2*f^6 - 2560*a^7*b^4*c^3*e^2*f^6 + 5120
*a^8*b^2*c^4*e^2*f^6 - 80*a^5*b^8*c*e^2*f^6)*(a^9*b^12*f^9 + 4096*a^15*c^6*
f^9 - 24*a^10*b^10*c*f^9 + 240*a^11*b^8*c^2*f^9 - 1280*a^12*b^6*c^3*f^9 + 3
840*a^13*b^4*c^4*f^9 - 6144*a^14*b^2*c^5*f^9)))*(b^6 - 20*a^3*c^3 + 30*a^2*
b^2*c^2 - 10*a*b^4*c))/(4*a^4*e*f^3*(4*a*c - b^2)^(5/2)) + (9*(b^6 - 20*a^3
*c^3 + 30*a^2*b^2*c^2 - 10*a*b^4*c)^2*(6*b^11*e*f^3 - 120*a*b^9*c*e*f^3 - 6
144*a^5*b*c^5*e*f^3 + 960*a^2*b^7*c^2*e*f^3 - 3840*a^3*b^5*c^3*e*f^3 + 7680
*a^4*b^3*c^4*e*f^3)*(12*a^9*b^15*c^2*e^19*f^9 - 328*a^10*b^13*c^3*e^19*f^9
+ 3840*a^11*b^11*c^4*e^19*f^9 - 24960*a^12*b^9*c^5*e^19*f^9 + 97280*a^13*b^
7*c^6*e^19*f^9 - 227328*a^14*b^5*c^7*e^19*f^9 + 294912*a^15*b^3*c^8*e^19*f^
9 - 163840*a^16*b*c^9*e^19*f^9))/(32*a^8*e^2*f^6*(4*a*c - b^2)^5*(4*a^4*b^1
0*e^2*f^6 - 4096*a^9*c^5*e^2*f^6 + 640*a^6*b^6*c^2*e^2*f^6 - 2560*a^7*b^4*c
^3*e^2*f^6 + 5120*a^8*b^2*c^4*e^2*f^6 - 80*a^5*b^8*c*e^2*f^6)*(a^9*b^12*f^9
 + 4096*a^15*c^6*f^9 - 24*a^10*b^10*c*f^9 + 240*a^11*b^8*c^2*f^9 - 1280*a^1
2*b^6*c^3*f^9 + 3840*a^13*b^4*c^4*f^9 - 6144*a^14*b^2*c^5*f^9)))*(3*b^8 + 1
0*a^4*c^4 + 120*a^2*b^4*c^2 - 145*a^3*b^2*c^3 - 33*a*b^6*c))/(8*a^3*c^2*(4*
a*c - b^2)^6*(100*a^6*c^6 - 6*b^12 - 960*a^2*b^8*c^2 + 3840*a^3*b^6*c^3 - 7
675*a^4*b^4*c^4 + 6100*a^5*b^2*c^5 + 120*a*b^10*c)) + (b*((3*((54*a^3*b^13*
c^4*e^17*f^3 - 1233*a^4*b^11*c^5*e^17*f^3 + 11583*a^5*b^9*c^6*e^17*f^3 - 57
204*a^6*b^7*c^7*e^17*f^3 + 156276*a^7*b^5*c^8*e^17*f^3 - 223200*a^8*b^3*c^9
*e^17*f^3 + 129600*a^9*b*c^10*e^17*f^3)/(a^9*b^12*f^9 + 4096*a^15*c^6*f^9 -
 24*a^10*b^10*c*f^9 + 240*a^11*b^8*c^2*f^9 - 1280*a^12*b^6*c^3*f^9 + 3840*a
^13*b^4*c^4*f^9 - 6144*a^14*b^2*c^5*f^9) - (((153600*a^13*c^10*e^18*f^6 + 6
*a^6*b^14*c^3*e^18*f^6 - 108*a^7*b^12*c^4*e^18*f^6 + 588*a^8*b^10*c^5*e^18*
f^6 + 792*a^9*b^8*c^6*e^18*f^6 - 22272*a^10*b^6*c^7*e^18*f^6 + 100608*a^11*
b^4*c^8*e^18*f^6 - 199680*a^12*b^2*c^9*e^18*f^6)/(a^9*b^12*f^9 + 4096*a^15*
c^6*f^9 - 24*a^10*b^10*c*f^9 + 240*a^11*b^8*c^2*f^9 - 1280*a^12*b^6*c^3*f^9
 + 3840*a^13*b^4*c^4*f^9 - 6144*a^14*b^2*c^5*f^9) + ((6*b^11*e*f^3 - 120*a*
b^9*c*e*f^3 - 6144*a^5*b*c^5*e*f^3 + 960*a^2*b^7*c^2*e*f^3 - 3840*a^3*b^5*c
^3*e*f^3 + 7680*a^4*b^3*c^4*e*f^3)*(12*a^9*b^15*c^2*e^19*f^9 - 328*a^10*b^1
3*c^3*e^19*f^9 + 3840*a^11*b^11*c^4*e^19*f^9 - 24960*a^12*b^9*c^5*e^19*f^9
+ 97280*a^13*b^7*c^6*e^19*f^9 - 227328*a^14*b^5*c^7*e^19*f^9 + 294912*a^15*
b^3*c^8*e^19*f^9 - 163840*a^16*b*c^9*e^19*f^9))/(2*(4*a^4*b^10*e^2*f^6 - 40
96*a^9*c^5*e^2*f^6 + 640*a^6*b^6*c^2*e^2*f^6 - 2560*a^7*b^4*c^3*e^2*f^6 + 5
120*a^8*b^2*c^4*e^2*f^6 - 80*a^5*b^8*c*e^2*f^6)*(a^9*b^12*f^9 + 4096*a^15*c
^6*f^9 - 24*a^10*b^10*c*f^9 + 240*a^11*b^8*c^2*f^9 - 1280*a^12*b^6*c^3*f^9
+ 3840*a^13*b^4*c^4*f^9 - 6144*a^14*b^2*c^5*f^9)))*(6*b^11*e*f^3 - 120*a*b^
9*c*e*f^3 - 6144*a^5*b*c^5*e*f^3 + 960*a^2*b^7*c^2*e*f^3 - 3840*a^3*b^5*c^3
*e*f^3 + 7680*a^4*b^3*c^4*e*f^3))/(2*(4*a^4*b^10*e^2*f^6 - 4096*a^9*c^5*e^2
*f^6 + 640*a^6*b^6*c^2*e^2*f^6 - 2560*a^7*b^4*c^3*e^2*f^6 + 5120*a^8*b^2*c^
4*e^2*f^6 - 80*a^5*b^8*c*e^2*f^6)))*(b^6 - 20*a^3*c^3 + 30*a^2*b^2*c^2 - 10
*a*b^4*c))/(4*a^4*e*f^3*(4*a*c - b^2)^(5/2)) - (((3*((153600*a^13*c^10*e^18
*f^6 + 6*a^6*b^14*c^3*e^18*f^6 - 108*a^7*b^12*c^4*e^18*f^6 + 588*a^8*b^10*c

^5*e^18*f^6 + 792*a^9*b^8*c^6*e^18*f^6 - 22272*a^10*b^6*c^7*e^18*f^6 + 1006
08*a^11*b^4*c^8*e^18*f^6 - 199680*a^12*b^2*c^9*e^18*f^6)/(a^9*b^12*f^9 + 40
96*a^15*c^6*f^9 - 24*a^10*b^10*c*f^9 + 240*a^11*b^8*c^2*f^9 - 1280*a^12*b^6
*c^3*f^9 + 3840*a^13*b^4*c^4*f^9 - 6144*a^14*b^2*c^5*f^9) + ((6*b^11*e*f^3
- 120*a*b^9*c*e*f^3 - 6144*a^5*b*c^5*e*f^3 + 960*a^2*b^7*c^2*e*f^3 - 3840*a
^3*b^5*c^3*e*f^3 + 7680*a^4*b^3*c^4*e*f^3)*(12*a^9*b^15*c^2*e^19*f^9 - 328*
a^10*b^13*c^3*e^19*f^9 + 3840*a^11*b^11*c^4*e^19*f^9 - 24960*a^12*b^9*c^5*e
^19*f^9 + 97280*a^13*b^7*c^6*e^19*f^9 - 227328*a^14*b^5*c^7*e^19*f^9 + 2949
12*a^15*b^3*c^8*e^19*f^9 - 163840*a^16*b*c^9*e^19*f^9))/(2*(4*a^4*b^10*e^2*
f^6 - 4096*a^9*c^5*e^2*f^6 + 640*a^6*b^6*c^2*e^2*f^6 - 2560*a^7*b^4*c^3*e^2
*f^6 + 5120*a^8*b^2*c^4*e^2*f^6 - 80*a^5*b^8*c*e^2*f^6)*(a^9*b^12*f^9 + 409
6*a^15*c^6*f^9 - 24*a^10*b^10*c*f^9 + 240*a^11*b^8*c^2*f^9 - 1280*a^12*b^6*
c^3*f^9 + 3840*a^13*b^4*c^4*f^9 - 6144*a^14*b^2*c^5*f^9)))*(b^6 - 20*a^3*c^
3 + 30*a^2*b^2*c^2 - 10*a*b^4*c))/(4*a^4*e*f^3*(4*a*c - b^2)^(5/2)) + (3*(b
^6 - 20*a^3*c^3 + 30*a^2*b^2*c^2 - 10*a*b^4*c)*(6*b^11*e*f^3 - 120*a*b^9*c*
e*f^3 - 6144*a^5*b*c^5*e*f^3 + 960*a^2*b^7*c^2*e*f^3 - 3840*a^3*b^5*c^3*e*f
^3 + 7680*a^4*b^3*c^4*e*f^3)*(12*a^9*b^15*c^2*e^19*f^9 - 328*a^10*b^13*c^3*
e^19*f^9 + 3840*a^11*b^11*c^4*e^19*f^9 - 24960*a^12*b^9*c^5*e^19*f^9 + 9728
0*a^13*b^7*c^6*e^19*f^9 - 227328*a^14*b^5*c^7*e^19*f^9 + 294912*a^15*b^3*c^
8*e^19*f^9 - 163840*a^16*b*c^9*e^19*f^9))/(8*a^4*e*f^3*(4*a*c - b^2)^(5/2)*
(4*a^4*b^10*e^2*f^6 - 4096*a^9*c^5*e^2*f^6 + 640*a^6*b^6*c^2*e^2*f^6 - 2560
*a^7*b^4*c^3*e^2*f^6 + 5120*a^8*b^2*c^4*e^2*f^6 - 80*a^5*b^8*c*e^2*f^6)*(a^
9*b^12*f^9 + 4096*a^15*c^6*f^9 - 24*a^10*b^10*c*f^9 + 240*a^11*b^8*c^2*f^9
- 1280*a^12*b^6*c^3*f^9 + 3840*a^13*b^4*c^4*f^9 - 6144*a^14*b^2*c^5*f^9)))*
(6*b^11*e*f^3 - 120*a*b^9*c*e*f^3 - 6144*a^5*b*c^5*e*f^3 + 960*a^2*b^7*c^2*
e*f^3 - 3840*a^3*b^5*c^3*e*f^3 + 7680*a^4*b^3*c^4*e*f^3))/(2*(4*a^4*b^10*e^
2*f^6 - 4096*a^9*c^5*e^2*f^6 + 640*a^6*b^6*c^2*e^2*f^6 - 2560*a^7*b^4*c^3*e
^2*f^6 + 5120*a^8*b^2*c^4*e^2*f^6 - 80*a^5*b^8*c*e^2*f^6)) + (27*(b^6 - 20*
a^3*c^3 + 30*a^2*b^2*c^2 - 10*a*b^4*c)^3*(12*a^9*b^15*c^2*e^19*f^9 - 328*a^
10*b^13*c^3*e^19*f^9 + 3840*a^11*b^11*c^4*e^19*f^9 - 24960*a^12*b^9*c^5*e^1
9*f^9 + 97280*a^13*b^7*c^6*e^19*f^9 - 227328*a^14*b^5*c^7*e^19*f^9 + 294912
*a^15*b^3*c^8*e^19*f^9 - 163840*a^16*b*c^9*e^19*f^9))/(64*a^12*e^3*f^9*(4*a
c - b^2)^(15/2)(a^9*b^12*f^9 + 4096*a^15*c^6*f^9 - 24*a^10*b^10*c*f^9 + 2
40*a^11*b^8*c^2*f^9 - 1280*a^12*b^6*c^3*f^9 + 3840*a^13*b^4*c^4*f^9 - 6144*
a^14*b^2*c^5*f^9)))*(3*b^8 + 190*a^4*c^4 + 180*a^2*b^4*c^2 - 335*a^3*b^2*c^
3 - 39*a*b^6*c))/(8*a^3*c^2*(4*a*c - b^2)^(13/2)*(100*a^6*c^6 - 6*b^12 - 96
0*a^2*b^8*c^2 + 3840*a^3*b^6*c^3 - 7675*a^4*b^4*c^4 + 6100*a^5*b^2*c^5 + 12
0*a*b^10*c)))*(16*a^12*b^12*f^9*(4*a*c - b^2)^(15/2) + 65536*a^18*c^6*f^9*(
4*a*c - b^2)^(15/2) - 384*a^13*b^10*c*f^9*(4*a*c - b^2)^(15/2) + 3840*a^14*
b^8*c^2*f^9*(4*a*c - b^2)^(15/2) - 20480*a^15*b^6*c^3*f^9*(4*a*c - b^2)^(15
/2) + 61440*a^16*b^4*c^4*f^9*(4*a*c - b^2)^(15/2) - 98304*a^17*b^2*c^5*f^9*
(4*a*c - b^2)^(15/2)))/(10800*a^6*c^8*e^14 + 27*b^12*c^2*e^14 - 540*a*b^10*
c^3*e^14 + 4320*a^2*b^8*c^4*e^14 - 17280*a^3*b^6*c^5*e^14 + 35100*a^4*b^4*c
^6*e^14 - 32400*a^5*b^2*c^7*e^14) - (x*((((2*(27000*a^6*c^11*d*e^15 + 27*b^
12*c^5*d*e^15 - 567*a*b^10*c^6*d*e^15 + 4779*a^2*b^8*c^7*d*e^15 - 20601*a^3
*b^6*c^8*d*e^15 + 47790*a^4*b^4*c^9*d*e^15 - 56700*a^5*b^2*c^10*d*e^15))/(a
^9*b^12*f^9 + 4096*a^15*c^6*f^9 - 24*a^10*b^10*c*f^9 + 240*a^11*b^8*c^2*f^9
- 1280*a^12*b^6*c^3*f^9 + 3840*a^13*b^4*c^4*f^9 - 6144*a^14*b^2*c^5*f^9) -
(((2*(129600*a^9*b*c^10*d*e^16*f^3 + 54*a^3*b^13*c^4*d*e^16*f^3 - 1233*a^4
*b^11*c^5*d*e^16*f^3 + 11583*a^5*b^9*c^6*d*e^16*f^3 - 57204*a^6*b^7*c^7*d*e
^16*f^3 + 156276*a^7*b^5*c^8*d*e^16*f^3 - 223200*a^8*b^3*c^9*d*e^16*f^3))/(
a^9*b^12*f^9 + 4096*a^15*c^6*f^9 - 24*a^10*b^10*c*f^9 + 240*a^11*b^8*c^2*f^
9 - 1280*a^12*b^6*c^3*f^9 + 3840*a^13*b^4*c^4*f^9 - 6144*a^14*b^2*c^5*f^9)
- (((2*(153600*a^13*c^10*d*e^17*f^6 + 6*a^6*b^14*c^3*d*e^17*f^6 - 108*a^7*b
^12*c^4*d*e^17*f^6 + 588*a^8*b^10*c^5*d*e^17*f^6 + 792*a^9*b^8*c^6*d*e^17*f
^6 - 22272*a^10*b^6*c^7*d*e^17*f^6 + 100608*a^11*b^4*c^8*d*e^17*f^6 - 19968
0*a^12*b^2*c^9*d*e^17*f^6))/(a^9*b^12*f^9 + 4096*a^15*c^6*f^9 - 24*a^10*b^1
0*c*f^9 + 240*a^11*b^8*c^2*f^9 - 1280*a^12*b^6*c^3*f^9 + 3840*a^13*b^4*c^4*
f^9 - 6144*a^14*b^2*c^5*f^9) - ((6*b^11*e*f^3 - 120*a*b^9*c*e*f^3 - 6144*a^

```
5*b*c^5*e*f^3 + 960*a^2*b^7*c^2*e*f^3 - 3840*a^3*b^5*c^3*e*f^3 + 7680*a^4*b
^3*c^4*e*f^3)*(163840*a^16*b*c^9*d*e^18*f^9 - 12*a^9*b^15*c^2*d*e^18*f^9 +
328*a^10*b^13*c^3*d*e^18*f^9 - 3840*a^11*b^11*c^4*d*e^18*f^9 + 24960*a^12*b
^9*c^5*d*e^18*f^9 - 97280*a^13*b^7*c^6*d*e^18*f^9 + 227328*a^14*b^5*c^7*d*e
^18*f^9 - 294912*a^15*b^3*c^8*d*e^18*f^9))/((4*a^4*b^10*e^2*f^6 - 4096*a^9*
c^5*e^2*f^6 + 640*a^6*b^6*c^2*e^2*f^6 - 2560*a^7*b^4*c^3*e^2*f^6 + 5120*a^8
*b^2*c^4*e^2*f^6 - 80*a^5*b^8*c*e^2*f^6)*(a^9*b^12*f^9 + 4096*a^15*c^6*f^9
- 24*a^10*b^10*c*f^9 + 240*a^11*b^8*c^2*f^9 - 1280*a^12*b^6*c^3*f^9 + 3840*
a^13*b^4*c^4*f^9 - 6144*a^14*b^2*c^5*f^9)))*(6*b^11*e*f^3 - 120*a*b^9*c*e*f
^3 - 6144*a^5*b*c^5*e*f^3 + 960*a^2*b^7*c^2*e*f^3 - 3840*a^3*b^5*c^3*e*f^3
+ 7680*a^4*b^3*c^4*e*f^3))/(2*(4*a^4*b^10*e^2*f^6 - 4096*a^9*c^5*e^2*f^6 +
640*a^6*b^6*c^2*e^2*f^6 - 2560*a^7*b^4*c^3*e^2*f^6 + 5120*a^8*b^2*c^4*e^2*f
^6 - 80*a^5*b^8*c*e^2*f^6)))*(6*b^11*e*f^3 - 120*a*b^9*c*e*f^3 - 6144*a^5*b
*c^5*e*f^3 + 960*a^2*b^7*c^2*e*f^3 - 3840*a^3*b^5*c^3*e*f^3 + 7680*a^4*b^3*
c^4*e*f^3))/(2*(4*a^4*b^10*e^2*f^6 - 4096*a^9*c^5*e^2*f^6 + 640*a^6*b^6*c^2
*e^2*f^6 - 2560*a^7*b^4*c^3*e^2*f^6 + 5120*a^8*b^2*c^4*e^2*f^6 - 80*a^5*b^8
*c*e^2*f^6)) - (3*((3*((2*(153600*a^13*c^10*d*e^17*f^6 + 6*a^6*b^14*c^3*d*e
^17*f^6 - 108*a^7*b^12*c^4*d*e^17*f^6 + 588*a^8*b^10*c^5*d*e^17*f^6 + 792*a
^9*b^8*c^6*d*e^17*f^6 - 22272*a^10*b^6*c^7*d*e^17*f^6 + 100608*a^11*b^4*c^8
*d*e^17*f^6 - 199680*a^12*b^2*c^9*d*e^17*f^6))/(a^9*b^12*f^9 + 4096*a^15*c^
6*f^9 - 24*a^10*b^10*c*f^9 + 240*a^11*b^8*c^2*f^9 - 1280*a^12*b^6*c^3*f^9 +
 3840*a^13*b^4*c^4*f^9 - 6144*a^14*b^2*c^5*f^9) - ((6*b^11*e*f^3 - 120*a*b^
9*c*e*f^3 - 6144*a^5*b*c^5*e*f^3 + 960*a^2*b^7*c^2*e*f^3 - 3840*a^3*b^5*c^3
*e*f^3 + 7680*a^4*b^3*c^4*e*f^3)*(163840*a^16*b*c^9*d*e^18*f^9 - 12*a^9*b^1
5*c^2*d*e^18*f^9 + 328*a^10*b^13*c^3*d*e^18*f^9 - 3840*a^11*b^11*c^4*d*e^18
*f^9 + 24960*a^12*b^9*c^5*d*e^18*f^9 - 97280*a^13*b^7*c^6*d*e^18*f^9 + 2273
28*a^14*b^5*c^7*d*e^18*f^9 - 294912*a^15*b^3*c^8*d*e^18*f^9))/((4*a^4*b^10*
e^2*f^6 - 4096*a^9*c^5*e^2*f^6 + 640*a^6*b^6*c^2*e^2*f^6 - 2560*a^7*b^4*c^3
*e^2*f^6 + 5120*a^8*b^2*c^4*e^2*f^6 - 80*a^5*b^8*c*e^2*f^6)*(a^9*b^12*f^9 +
 4096*a^15*c^6*f^9 - 24*a^10*b^10*c*f^9 + 240*a^11*b^8*c^2*f^9 - 1280*a^12*
b^6*c^3*f^9 + 3840*a^13*b^4*c^4*f^9 - 6144*a^14*b^2*c^5*f^9)))*(b^6 - 20*a^
3*c^3 + 30*a^2*b^2*c^2 - 10*a*b^4*c))/(4*a^4*e*f^3*(4*a*c - b^2)^(5/2)) - (
3*(b^6 - 20*a^3*c^3 + 30*a^2*b^2*c^2 - 10*a*b^4*c)*(6*b^11*e*f^3 - 120*a*b^
9*c*e*f^3 - 6144*a^5*b*c^5*e*f^3 + 960*a^2*b^7*c^2*e*f^3 - 3840*a^3*b^5*c^3
*e*f^3 + 7680*a^4*b^3*c^4*e*f^3)*(163840*a^16*b*c^9*d*e^18*f^9 - 12*a^9*b^1
5*c^2*d*e^18*f^9 + 328*a^10*b^13*c^3*d*e^18*f^9 - 3840*a^11*b^11*c^4*d*e^18
*f^9 + 24960*a^12*b^9*c^5*d*e^18*f^9 - 97280*a^13*b^7*c^6*d*e^18*f^9 + 2273
28*a^14*b^5*c^7*d*e^18*f^9 - 294912*a^15*b^3*c^8*d*e^18*f^9))/(4*a^4*e*f^3*
(4*a*c - b^2)^(5/2)*(4*a^4*b^10*e^2*f^6 - 4096*a^9*c^5*e^2*f^6 + 640*a^6*b^
6*c^2*e^2*f^6 - 2560*a^7*b^4*c^3*e^2*f^6 + 5120*a^8*b^2*c^4*e^2*f^6 - 80*a^
5*b^8*c*e^2*f^6)*(a^9*b^12*f^9 + 4096*a^15*c^6*f^9 - 24*a^10*b^10*c*f^9 + 2
40*a^11*b^8*c^2*f^9 - 1280*a^12*b^6*c^3*f^9 + 3840*a^13*b^4*c^4*f^9 - 6144*
a^14*b^2*c^5*f^9)))*(b^6 - 20*a^3*c^3 + 30*a^2*b^2*c^2 - 10*a*b^4*c))/(4*a^
4*e*f^3*(4*a*c - b^2)^(5/2)) + (9*(b^6 - 20*a^3*c^3 + 30*a^2*b^2*c^2 - 10*a
*b^4*c)^2*(6*b^11*e*f^3 - 120*a*b^9*c*e*f^3 - 6144*a^5*b*c^5*e*f^3 + 960*a^
2*b^7*c^2*e*f^3 - 3840*a^3*b^5*c^3*e*f^3 + 7680*a^4*b^3*c^4*e*f^3)*(163840*
a^16*b*c^9*d*e^18*f^9 - 12*a^9*b^15*c^2*d*e^18*f^9 + 328*a^10*b^13*c^3*d*e^
18*f^9 - 3840*a^11*b^11*c^4*d*e^18*f^9 + 24960*a^12*b^9*c^5*d*e^18*f^9 - 97
280*a^13*b^7*c^6*d*e^18*f^9 + 227328*a^14*b^5*c^7*d*e^18*f^9 - 294912*a^15*
b^3*c^8*d*e^18*f^9))/(16*a^8*e^2*f^6*(4*a*c - b^2)^5*(4*a^4*b^10*e^2*f^6 -
4096*a^9*c^5*e^2*f^6 + 640*a^6*b^6*c^2*e^2*f^6 - 2560*a^7*b^4*c^3*e^2*f^6 +
 5120*a^8*b^2*c^4*e^2*f^6 - 80*a^5*b^8*c*e^2*f^6)*(a^9*b^12*f^9 + 4096*a^15
*c^6*f^9 - 24*a^10*b^10*c*f^9 + 240*a^11*b^8*c^2*f^9 - 1280*a^12*b^6*c^3*f^
9 + 3840*a^13*b^4*c^4*f^9 - 6144*a^14*b^2*c^5*f^9)))*(3*b^8 + 10*a^4*c^4 +
120*a^2*b^4*c^2 - 145*a^3*b^2*c^3 - 33*a*b^6*c))/(8*a^3*c^2*(4*a*c - b^2)^6
*(100*a^6*c^6 - 6*b^12 - 960*a^2*b^8*c^2 + 3840*a^3*b^6*c^3 - 7675*a^4*b^4*
c^4 + 6100*a^5*b^2*c^5 + 120*a*b^10*c)) + (b*((((3*((2*(153600*a^13*c^10*d*
e^17*f^6 + 6*a^6*b^14*c^3*d*e^17*f^6 - 108*a^7*b^12*c^4*d*e^17*f^6 + 588*a^
8*b^10*c^5*d*e^17*f^6 + 792*a^9*b^8*c^6*d*e^17*f^6 - 22272*a^10*b^6*c^7*d*e
```

^17*f^6 + 100608*a^11*b^4*c^8*d*e^17*f^6 - 199680*a^12*b^2*c^9*d*e^17*f^6))
/(a^9*b^12*f^9 + 4096*a^15*c^6*f^9 - 24*a^10*b^10*c*f^9 + 240*a^11*b^8*c^2*
f^9 - 1280*a^12*b^6*c^3*f^9 + 3840*a^13*b^4*c^4*f^9 - 6144*a^14*b^2*c^5*f^9
) - ((6*b^11*e*f^3 - 120*a*b^9*c*e*f^3 - 6144*a^5*b*c^5*e*f^3 + 960*a^2*b^7
*c^2*e*f^3 - 3840*a^3*b^5*c^3*e*f^3 + 7680*a^4*b^3*c^4*e*f^3)*(163840*a^16*
b*c^9*d*e^18*f^9 - 12*a^9*b^15*c^2*d*e^18*f^9 + 328*a^10*b^13*c^3*d*e^18*f^
9 - 3840*a^11*b^11*c^4*d*e^18*f^9 + 24960*a^12*b^9*c^5*d*e^18*f^9 - 97280*a
^13*b^7*c^6*d*e^18*f^9 + 227328*a^14*b^5*c^7*d*e^18*f^9 - 294912*a^15*b^3*c
^8*d*e^18*f^9))/((4*a^4*b^10*e^2*f^6 - 4096*a^9*c^5*e^2*f^6 + 640*a^6*b^6*c
^2*e^2*f^6 - 2560*a^7*b^4*c^3*e^2*f^6 + 5120*a^8*b^2*c^4*e^2*f^6 - 80*a^5*b
^8*c*e^2*f^6)*(a^9*b^12*f^9 + 4096*a^15*c^6*f^9 - 24*a^10*b^10*c*f^9 + 240*
a^11*b^8*c^2*f^9 - 1280*a^12*b^6*c^3*f^9 + 3840*a^13*b^4*c^4*f^9 - 6144*a^1
4*b^2*c^5*f^9)))*(b^6 - 20*a^3*c^3 + 30*a^2*b^2*c^2 - 10*a*b^4*c))/(4*a^4*e
f^3(4*a*c - b^2)^(5/2)) - (3*(b^6 - 20*a^3*c^3 + 30*a^2*b^2*c^2 - 10*a*b^
4*c)*(6*b^11*e*f^3 - 120*a*b^9*c*e*f^3 - 6144*a^5*b*c^5*e*f^3 + 960*a^2*b^7
*c^2*e*f^3 - 3840*a^3*b^5*c^3*e*f^3 + 7680*a^4*b^3*c^4*e*f^3)*(163840*a^16*
b*c^9*d*e^18*f^9 - 12*a^9*b^15*c^2*d*e^18*f^9 + 328*a^10*b^13*c^3*d*e^18*f^
9 - 3840*a^11*b^11*c^4*d*e^18*f^9 + 24960*a^12*b^9*c^5*d*e^18*f^9 - 97280*a
^13*b^7*c^6*d*e^18*f^9 + 227328*a^14*b^5*c^7*d*e^18*f^9 - 294912*a^15*b^3*c
^8*d*e^18*f^9))/(4*a^4*e*f^3*(4*a*c - b^2)^(5/2)*(4*a^4*b^10*e^2*f^6 - 4096
*a^9*c^5*e^2*f^6 + 640*a^6*b^6*c^2*e^2*f^6 - 2560*a^7*b^4*c^3*e^2*f^6 + 512
0*a^8*b^2*c^4*e^2*f^6 - 80*a^5*b^8*c*e^2*f^6)*(a^9*b^12*f^9 + 4096*a^15*c^6
*f^9 - 24*a^10*b^10*c*f^9 + 240*a^11*b^8*c^2*f^9 - 1280*a^12*b^6*c^3*f^9 +
3840*a^13*b^4*c^4*f^9 - 6144*a^14*b^2*c^5*f^9)))*(6*b^11*e*f^3 - 120*a*b^9*
c*e*f^3 - 6144*a^5*b*c^5*e*f^3 + 960*a^2*b^7*c^2*e*f^3 - 3840*a^3*b^5*c^3*e
*f^3 + 7680*a^4*b^3*c^4*e*f^3))/(2*(4*a^4*b^10*e^2*f^6 - 4096*a^9*c^5*e^2*f
^6 + 640*a^6*b^6*c^2*e^2*f^6 - 2560*a^7*b^4*c^3*e^2*f^6 + 5120*a^8*b^2*c^4*
e^2*f^6 - 80*a^5*b^8*c*e^2*f^6)) - (3*((2*(129600*a^9*b*c^10*d*e^16*f^3 + 5
4*a^3*b^13*c^4*d*e^16*f^3 - 1233*a^4*b^11*c^5*d*e^16*f^3 + 11583*a^5*b^9*c^
6*d*e^16*f^3 - 57204*a^6*b^7*c^7*d*e^16*f^3 + 156276*a^7*b^5*c^8*d*e^16*f^3
 - 223200*a^8*b^3*c^9*d*e^16*f^3))/(a^9*b^12*f^9 + 4096*a^15*c^6*f^9 - 24*a
^10*b^10*c*f^9 + 240*a^11*b^8*c^2*f^9 - 1280*a^12*b^6*c^3*f^9 + 3840*a^13*b
^4*c^4*f^9 - 6144*a^14*b^2*c^5*f^9) - (((2*(153600*a^13*c^10*d*e^17*f^6 + 6
*a^6*b^14*c^3*d*e^17*f^6 - 108*a^7*b^12*c^4*d*e^17*f^6 + 588*a^8*b^10*c^5*d
*e^17*f^6 + 792*a^9*b^8*c^6*d*e^17*f^6 - 22272*a^10*b^6*c^7*d*e^17*f^6 + 10
0608*a^11*b^4*c^8*d*e^17*f^6 - 199680*a^12*b^2*c^9*d*e^17*f^6))/(a^9*b^12*f
^9 + 4096*a^15*c^6*f^9 - 24*a^10*b^10*c*f^9 + 240*a^11*b^8*c^2*f^9 - 1280*a
^12*b^6*c^3*f^9 + 3840*a^13*b^4*c^4*f^9 - 6144*a^14*b^2*c^5*f^9) - ((6*b^11
*e*f^3 - 120*a*b^9*c*e*f^3 - 6144*a^5*b*c^5*e*f^3 + 960*a^2*b^7*c^2*e*f^3 -
 3840*a^3*b^5*c^3*e*f^3 + 7680*a^4*b^3*c^4*e*f^3)*(163840*a^16*b*c^9*d*e^18
*f^9 - 12*a^9*b^15*c^2*d*e^18*f^9 + 328*a^10*b^13*c^3*d*e^18*f^9 - 3840*a^1
1*b^11*c^4*d*e^18*f^9 + 24960*a^12*b^9*c^5*d*e^18*f^9 - 97280*a^13*b^7*c^6*
d*e^18*f^9 + 227328*a^14*b^5*c^7*d*e^18*f^9 - 294912*a^15*b^3*c^8*d*e^18*f^
9))/((4*a^4*b^10*e^2*f^6 - 4096*a^9*c^5*e^2*f^6 + 640*a^6*b^6*c^2*e^2*f^6 -
 2560*a^7*b^4*c^3*e^2*f^6 + 5120*a^8*b^2*c^4*e^2*f^6 - 80*a^5*b^8*c*e^2*f^6
)*(a^9*b^12*f^9 + 4096*a^15*c^6*f^9 - 24*a^10*b^10*c*f^9 + 240*a^11*b^8*c^2
*f^9 - 1280*a^12*b^6*c^3*f^9 + 3840*a^13*b^4*c^4*f^9 - 6144*a^14*b^2*c^5*f^
9)))*(6*b^11*e*f^3 - 120*a*b^9*c*e*f^3 - 6144*a^5*b*c^5*e*f^3 + 960*a^2*b^7
*c^2*e*f^3 - 3840*a^3*b^5*c^3*e*f^3 + 7680*a^4*b^3*c^4*e*f^3))/(2*(4*a^4*b^
10*e^2*f^6 - 4096*a^9*c^5*e^2*f^6 + 640*a^6*b^6*c^2*e^2*f^6 - 2560*a^7*b^4*
c^3*e^2*f^6 + 5120*a^8*b^2*c^4*e^2*f^6 - 80*a^5*b^8*c*e^2*f^6)))*(b^6 - 20*
a^3*c^3 + 30*a^2*b^2*c^2 - 10*a*b^4*c))/(4*a^4*e*f^3*(4*a*c - b^2)^(5/2)) +
 (27*(b^6 - 20*a^3*c^3 + 30*a^2*b^2*c^2 - 10*a*b^4*c)^3*(163840*a^16*b*c^9*
d*e^18*f^9 - 12*a^9*b^15*c^2*d*e^18*f^9 + 328*a^10*b^13*c^3*d*e^18*f^9 - 38
40*a^11*b^11*c^4*d*e^18*f^9 + 24960*a^12*b^9*c^5*d*e^18*f^9 - 97280*a^13*b^
7*c^6*d*e^18*f^9 + 227328*a^14*b^5*c^7*d*e^18*f^9 - 294912*a^15*b^3*c^8*d*e
^18*f^9))/(32*a^12*e^3*f^9*(4*a*c - b^2)^(15/2)*(a^9*b^12*f^9 + 4096*a^15*c
^6*f^9 - 24*a^10*b^10*c*f^9 + 240*a^11*b^8*c^2*f^9 - 1280*a^12*b^6*c^3*f^9
+ 3840*a^13*b^4*c^4*f^9 - 6144*a^14*b^2*c^5*f^9)))*(3*b^8 + 190*a^4*c^4 + 1

80*a^2*b^4*c^2 - 335*a^3*b^2*c^3 - 39*a*b^6*c))/(8*a^3*c^2*(4*a*c - b^2)^(1 3/2)*(100*a^6*c^6 - 6*b^12 - 960*a^2*b^8*c^2 + 3840*a^3*b^6*c^3 - 7675*a^4* b^4*c^4 + 6100*a^5*b^2*c^5 + 120*a*b^10*c)))*(16*a^12*b^12*f^9*(4*a*c - b^2)^(15/2) + 65536*a^18*c^6*f^9*(4*a*c - b^2)^(15/2) - 384*a^13*b^10*c*f^9*(4 *a*c - b^2)^(15/2) + 3840*a^14*b^8*c^2*f^9*(4*a*c - b^2)^(15/2) - 20480*a^1 5*b^6*c^3*f^9*(4*a*c - b^2)^(15/2) + 61440*a^16*b^4*c^4*f^9*(4*a*c - b^2)^(15/2) - 98304*a^17*b^2*c^5*f^9*(4*a*c - b^2)^(15/2)))/(10800*a^6*c^8*e^14 + 27*b^12*c^2*e^14 - 540*a*b^10*c^3*e^14 + 4320*a^2*b^8*c^4*e^14 - 17280*a^3 *b^6*c^5*e^14 + 35100*a^4*b^4*c^6*e^14 - 32400*a^5*b^2*c^7*e^14) + (((((36* a^3*b^14*c^3*e^15*f^3 - 14400*a^10*c^10*e^15*f^3 - 837*a^4*b^12*c^4*e^15*f^ 3 + 8046*a^5*b^10*c^5*e^15*f^3 - 40941*a^6*b^8*c^6*e^15*f^3 + 116532*a^7*b^ 6*c^7*e^15*f^3 - 177588*a^8*b^4*c^8*e^15*f^3 + 119520*a^9*b^2*c^9*e^15*f^3 + 129600*a^9*b*c^10*d^2*e^15*f^3 + 54*a^3*b^13*c^4*d^2*e^15*f^3 - 1233*a^4* b^11*c^5*d^2*e^15*f^3 + 11583*a^5*b^9*c^6*d^2*e^15*f^3 - 57204*a^6*b^7*c^7* d^2*e^15*f^3 + 156276*a^7*b^5*c^8*d^2*e^15*f^3 - 223200*a^8*b^3*c^9*d^2*e^1 5*f^3)/(a^9*b^12*f^9 + 4096*a^15*c^6*f^9 - 24*a^10*b^10*c*f^9 + 240*a^11*b^ 8*c^2*f^9 - 1280*a^12*b^6*c^3*f^9 + 3840*a^13*b^4*c^4*f^9 - 6144*a^14*b^2*c ^5*f^9) - (((12*a^6*b^15*c^2*e^16*f^6 - 300*a^7*b^13*c^3*e^16*f^6 + 3156*a^ 8*b^11*c^4*e^16*f^6 - 17976*a^9*b^9*c^5*e^16*f^6 + 59136*a^10*b^7*c^6*e^16* f^6 - 109824*a^11*b^5*c^7*e^16*f^6 + 101376*a^12*b^3*c^8*e^16*f^6 + 153600* a^13*c^10*d^2*e^16*f^6 - 30720*a^13*b*c^9*e^16*f^6 + 6*a^6*b^14*c^3*d^2*e^1 6*f^6 - 108*a^7*b^12*c^4*d^2*e^16*f^6 + 588*a^8*b^10*c^5*d^2*e^16*f^6 + 792 *a^9*b^8*c^6*d^2*e^16*f^6 - 22272*a^10*b^6*c^7*d^2*e^16*f^6 + 100608*a^11*b ^4*c^8*d^2*e^16*f^6 - 199680*a^12*b^2*c^9*d^2*e^16*f^6)/(a^9*b^12*f^9 + 409 6*a^15*c^6*f^9 - 24*a^10*b^10*c*f^9 + 240*a^11*b^8*c^2*f^9 - 1280*a^12*b^6* c^3*f^9 + 3840*a^13*b^4*c^4*f^9 - 6144*a^14*b^2*c^5*f^9) + ((6*b^11*e*f^3 - 120*a*b^9*c*e*f^3 - 6144*a^5*b*c^5*e*f^3 + 960*a^2*b^7*c^2*e*f^3 - 3840*a^ 3*b^5*c^3*e*f^3 + 7680*a^4*b^3*c^4*e*f^3)*(4*a^10*b^14*c^2*e^17*f^9 - 96*a^ 11*b^12*c^3*e^17*f^9 + 960*a^12*b^10*c^4*e^17*f^9 - 5120*a^13*b^8*c^5*e^17* f^9 + 15360*a^14*b^6*c^6*e^17*f^9 - 24576*a^15*b^4*c^7*e^17*f^9 + 16384*a^1 6*b^2*c^8*e^17*f^9 - 163840*a^16*b*c^9*d^2*e^17*f^9 + 12*a^9*b^15*c^2*d^2*e ^17*f^9 - 328*a^10*b^13*c^3*d^2*e^17*f^9 + 3840*a^11*b^11*c^4*d^2*e^17*f^9 - 24960*a^12*b^9*c^5*d^2*e^17*f^9 + 97280*a^13*b^7*c^6*d^2*e^17*f^9 - 22732 8*a^14*b^5*c^7*d^2*e^17*f^9 + 294912*a^15*b^3*c^8*d^2*e^17*f^9))/(2*(4*a^4* b^10*e^2*f^6 - 4096*a^9*c^5*e^2*f^6 + 640*a^6*b^6*c^2*e^2*f^6 - 2560*a^7*b^ 4*c^3*e^2*f^6 + 5120*a^8*b^2*c^4*e^2*f^6 - 80*a^5*b^8*c*e^2*f^6)*(a^9*b^12* f^9 + 4096*a^15*c^6*f^9 - 24*a^10*b^10*c*f^9 + 240*a^11*b^8*c^2*f^9 - 1280* a^12*b^6*c^3*f^9 + 3840*a^13*b^4*c^4*f^9 - 6144*a^14*b^2*c^5*f^9)))*(6*b^11 *e*f^3 - 120*a*b^9*c*e*f^3 - 6144*a^5*b*c^5*e*f^3 + 960*a^2*b^7*c^2*e*f^3 - 3840*a^3*b^5*c^3*e*f^3 + 7680*a^4*b^3*c^4*e*f^3))/(2*(4*a^4*b^10*e^2*f^6 - 4096*a^9*c^5*e^2*f^6 + 640*a^6*b^6*c^2*e^2*f^6 - 2560*a^7*b^4*c^3*e^2*f^6 + 5120*a^8*b^2*c^4*e^2*f^6 - 80*a^5*b^8*c*e^2*f^6)))*(6*b^11*e*f^3 - 120*a* b^9*c*e*f^3 - 6144*a^5*b*c^5*e*f^3 + 960*a^2*b^7*c^2*e*f^3 - 3840*a^3*b^5*c ^3*e*f^3 + 7680*a^4*b^3*c^4*e*f^3))/(2*(4*a^4*b^10*e^2*f^6 - 4096*a^9*c^5*e ^2*f^6 + 640*a^6*b^6*c^2*e^2*f^6 - 2560*a^7*b^4*c^3*e^2*f^6 + 5120*a^8*b^2* c^4*e^2*f^6 - 80*a^5*b^8*c*e^2*f^6)) - (27*b^13*c^4*e^14 - 594*a*b^11*c^5*e ^14 + 43200*a^6*b*c^10*e^14 + 5319*a^2*b^9*c^6*e^14 - 24732*a^3*b^7*c^7*e^1 4 + 62748*a^4*b^5*c^8*e^14 - 82080*a^5*b^3*c^9*e^14 + 27000*a^6*c^11*d^2*e^ 14 + 27*b^12*c^5*d^2*e^14 + 4779*a^2*b^8*c^7*d^2*e^14 - 20601*a^3*b^6*c^8*d ^2*e^14 + 47790*a^4*b^4*c^9*d^2*e^14 - 56700*a^5*b^2*c^10*d^2*e^14 - 567*a* b^10*c^6*d^2*e^14)/(a^9*b^12*f^9 + 4096*a^15*c^6*f^9 - 24*a^10*b^10*c*f^9 + 240*a^11*b^8*c^2*f^9 - 1280*a^12*b^6*c^3*f^9 + 3840*a^13*b^4*c^4*f^9 - 614 4*a^14*b^2*c^5*f^9) + (3*((3*((12*a^6*b^15*c^2*e^16*f^6 - 300*a^7*b^13*c^3* e^16*f^6 + 3156*a^8*b^11*c^4*e^16*f^6 - 17976*a^9*b^9*c^5*e^16*f^6 + 59136* a^10*b^7*c^6*e^16*f^6 - 109824*a^11*b^5*c^7*e^16*f^6 + 101376*a^12*b^3*c^8* e^16*f^6 + 153600*a^13*c^10*d^2*e^16*f^6 - 30720*a^13*b*c^9*e^16*f^6 + 6*a^ 6*b^14*c^3*d^2*e^16*f^6 - 108*a^7*b^12*c^4*d^2*e^16*f^6 + 588*a^8*b^10*c^5* d^2*e^16*f^6 + 792*a^9*b^8*c^6*d^2*e^16*f^6 - 22272*a^10*b^6*c^7*d^2*e^16*f ^6 + 100608*a^11*b^4*c^8*d^2*e^16*f^6 - 199680*a^12*b^2*c^9*d^2*e^16*f^6)/(

a^9*b^12*f^9 + 4096*a^15*c^6*f^9 - 24*a^10*b^10*c*f^9 + 240*a^11*b^8*c^2*f^9 - 1280*a^12*b^6*c^3*f^9 + 3840*a^13*b^4*c^4*f^9 - 6144*a^14*b^2*c^5*f^9) + ((6*b^11*e*f^3 - 120*a*b^9*c*e*f^3 - 6144*a^5*b*c^5*e*f^3 + 960*a^2*b^7*c^2*e*f^3 - 3840*a^3*b^5*c^3*e*f^3 + 7680*a^4*b^3*c^4*e*f^3)*(4*a^10*b^14*c^2*e^17*f^9 - 96*a^11*b^12*c^3*e^17*f^9 + 960*a^12*b^10*c^4*e^17*f^9 - 5120*a^13*b^8*c^5*e^17*f^9 + 15360*a^14*b^6*c^6*e^17*f^9 - 24576*a^15*b^4*c^7*e^17*f^9 + 16384*a^16*b^2*c^8*e^17*f^9 - 163840*a^16*b*c^9*d^2*e^17*f^9 + 12*a^9*b^15*c^2*d^2*e^17*f^9 - 328*a^10*b^13*c^3*d^2*e^17*f^9 + 3840*a^11*b^11*c^4*d^2*e^17*f^9 - 24960*a^12*b^9*c^5*d^2*e^17*f^9 + 97280*a^13*b^7*c^6*d^2*e^17*f^9 - 227328*a^14*b^5*c^7*d^2*e^17*f^9 + 294912*a^15*b^3*c^8*d^2*e^17*f^9))/(2*(4*a^4*b^10*e^2*f^6 - 4096*a^9*c^5*e^2*f^6 + 640*a^6*b^6*c^2*e^2*f^6 - 2560*a^7*b^4*c^3*e^2*f^6 + 5120*a^8*b^2*c^4*e^2*f^6 - 80*a^5*b^8*c*e^2*f^6)*(a^9*b^12*f^9 + 4096*a^15*c^6*f^9 - 24*a^10*b^10*c*f^9 + 240*a^11*b^8*c^2*f^9 - 1280*a^12*b^6*c^3*f^9 + 3840*a^13*b^4*c^4*f^9 - 6144*a^14*b^2*c^5*f^9)))*(b^6 - 20*a^3*c^3 + 30*a^2*b^2*c^2 - 10*a*b^4*c))/(4*a^4*e*f^3*(4*a*c - b^2)^(5/2)) + (3*(b^6 - 20*a^3*c^3 + 30*a^2*b^2*c^2 - 10*a*b^4*c)*(6*b^11*e*f^3 - 120*a*b^9*c*e*f^3 - 6144*a^5*b*c^5*e*f^3 + 960*a^2*b^7*c^2*e*f^3 - 3840*a^3*b^5*c^3*e*f^3 + 7680*a^4*b^3*c^4*e*f^3)*(4*a^10*b^14*c^2*e^17*f^9 - 96*a^11*b^12*c^3*e^17*f^9 + 960*a^12*b^10*c^4*e^17*f^9 - 5120*a^13*b^8*c^5*e^17*f^9 + 15360*a^14*b^6*c^6*e^17*f^9 - 24576*a^15*b^4*c^7*e^17*f^9 + 16384*a^16*b^2*c^8*e^17*f^9 - 163840*a^16*b*c^9*d^2*e^17*f^9 + 12*a^9*b^15*c^2*d^2*e^17*f^9 - 328*a^10*b^13*c^3*d^2*e^17*f^9 + 3840*a^11*b^11*c^4*d^2*e^17*f^9 - 24960*a^12*b^9*c^5*d^2*e^17*f^9 + 97280*a^13*b^7*c^6*d^2*e^17*f^9 - 227328*a^14*b^5*c^7*d^2*e^17*f^9 + 294912*a^15*b^3*c^8*d^2*e^17*f^9))/(8*a^4*e*f^3*(4*a*c - b^2)^(5/2)*(4*a^4*b^10*e^2*f^6 - 4096*a^9*c^5*e^2*f^6 + 640*a^6*b^6*c^2*e^2*f^6 - 2560*a^7*b^4*c^3*e^2*f^6 + 5120*a^8*b^2*c^4*e^2*f^6 - 80*a^5*b^8*c*e^2*f^6)*(a^9*b^12*f^9 + 4096*a^15*c^6*f^9 - 24*a^10*b^10*c*f^9 + 240*a^11*b^8*c^2*f^9 - 1280*a^12*b^6*c^3*f^9 + 3840*a^13*b^4*c^4*f^9 - 6144*a^14*b^2*c^5*f^9)))*(b^6 - 20*a^3*c^3 + 30*a^2*b^2*c^2 - 10*a*b^4*c))/(4*a^4*e*f^3*(4*a*c - b^2)^(5/2)) + (9*(b^6 - 20*a^3*c^3 + 30*a^2*b^2*c^2 - 10*a*b^4*c)^2*(6*b^11*e*f^3 - 120*a*b^9*c*e*f^3 - 6144*a^5*b*c^5*e*f^3 + 960*a^2*b^7*c^2*e*f^3 - 3840*a^3*b^5*c^3*e*f^3 + 7680*a^4*b^3*c^4*e*f^3)*(4*a^10*b^14*c^2*e^17*f^9 - 96*a^11*b^12*c^3*e^17*f^9 + 960*a^12*b^10*c^4*e^17*f^9 - 5120*a^13*b^8*c^5*e^17*f^9 + 15360*a^14*b^6*c^6*e^17*f^9 - 24576*a^15*b^4*c^7*e^17*f^9 + 16384*a^16*b^2*c^8*e^17*f^9 - 163840*a^16*b*c^9*d^2*e^17*f^9 + 12*a^9*b^15*c^2*d^2*e^17*f^9 - 328*a^10*b^13*c^3*d^2*e^17*f^9 + 3840*a^11*b^11*c^4*d^2*e^17*f^9 - 24960*a^12*b^9*c^5*d^2*e^17*f^9 + 97280*a^13*b^7*c^6*d^2*e^17*f^9 - 227328*a^14*b^5*c^7*d^2*e^17*f^9 + 294912*a^15*b^3*c^8*d^2*e^17*f^9))/(32*a^8*e^2*f^6*(4*a*c - b^2)^5*(4*a^4*b^10*e^2*f^6 - 4096*a^9*c^5*e^2*f^6 + 640*a^6*b^6*c^2*e^2*f^6 - 2560*a^7*b^4*c^3*e^2*f^6 + 5120*a^8*b^2*c^4*e^2*f^6 - 80*a^5*b^8*c*e^2*f^6)*(a^9*b^12*f^9 + 4096*a^15*c^6*f^9 - 24*a^10*b^10*c*f^9 + 240*a^11*b^8*c^2*f^9 - 1280*a^12*b^6*c^3*f^9 + 3840*a^13*b^4*c^4*f^9 - 6144*a^14*b^2*c^5*f^9)))*(3*b^8 + 10*a^4*c^4 + 120*a^2*b^4*c^2 - 145*a^3*b^2*c^3 - 33*a*b^6*c)*(16*a^12*b^12*f^9*(4*a*c - b^2)^(15/2) + 65536*a^18*c^6*f^9*(4*a*c - b^2)^(15/2) - 384*a^13*b^10*c*f^9*(4*a*c - b^2)^(15/2) + 3840*a^14*b^8*c^2*f^9*(4*a*c - b^2)^(15/2) - 20480*a^15*b^6*c^3*f^9*(4*a*c - b^2)^(15/2) + 61440*a^16*b^4*c^4*f^9*(4*a*c - b^2)^(15/2) - 98304*a^17*b^2*c^5*f^9*(4*a*c - b^2)^(15/2)))/(8*a^3*c^2*(4*a*c - b^2)^6*(10800*a^6*c^8*e^14 + 27*b^12*c^2*e^14 - 540*a*b^10*c^3*e^14 + 4320*a^2*b^8*c^4*e^14 - 17280*a^3*b^6*c^5*e^14 + 35100*a^4*b^4*c^6*e^14 - 32400*a^5*b^2*c^7*e^14)*(100*a^6*c^6 - 6*b^12 - 960*a^2*b^8*c^2 + 3840*a^3*b^6*c^3 - 7675*a^4*b^4*c^4 + 6100*a^5*b^2*c^5 + 120*a*b^10*c)) + (b*((3*((36*a^3*b^14*c^3*e^15*f^3 - 14400*a^10*c^10*e^15*f^3 - 837*a^4*b^12*c^4*e^15*f^3 + 8046*a^5*b^10*c^5*e^15*f^3 - 40941*a^6*b^8*c^6*e^15*f^3 + 116532*a^7*b^6*c^7*e^15*f^3 - 177588*a^8*b^4*c^8*e^15*f^3 + 119520*a^9*b^2*c^9*e^15*f^3 + 129600*a^9*b*c^10*d^2*e^15*f^3 + 54*a^3*b^13*c^4*d^2*e^15*f^3 - 1233*a^4*b^11*c^5*d^2*e^15*f^3 + 11583*a^5*b^9*c^6*d^2*e^15*f^3 - 57204*a^6*b^7*c^7*d^2*e^15*f^3 + 156276*a^7*b^5*c^8*d^2*e^15*f^3 - 223200*a^8*b^3*c^9*d^2*e^15*f^3)/(a^9*b^12*f^9 + 4096*a^15*c^6*f^9 - 24*a^10*b^10*c*f^9 + 24

0*a^11*b^8*c^2*f^9 - 1280*a^12*b^6*c^3*f^9 + 3840*a^13*b^4*c^4*f^9 - 6144*a
^14*b^2*c^5*f^9) - (((12*a^6*b^15*c^2*e^16*f^6 - 300*a^7*b^13*c^3*e^16*f^6
+ 3156*a^8*b^11*c^4*e^16*f^6 - 17976*a^9*b^9*c^5*e^16*f^6 + 59136*a^10*b^7*
c^6*e^16*f^6 - 109824*a^11*b^5*c^7*e^16*f^6 + 101376*a^12*b^3*c^8*e^16*f^6
+ 153600*a^13*c^10*d^2*e^16*f^6 - 30720*a^13*b*c^9*e^16*f^6 + 6*a^6*b^14*c^
3*d^2*e^16*f^6 - 108*a^7*b^12*c^4*d^2*e^16*f^6 + 588*a^8*b^10*c^5*d^2*e^16*
f^6 + 792*a^9*b^8*c^6*d^2*e^16*f^6 - 22272*a^10*b^6*c^7*d^2*e^16*f^6 + 1006
08*a^11*b^4*c^8*d^2*e^16*f^6 - 199680*a^12*b^2*c^9*d^2*e^16*f^6)/(a^9*b^12*
f^9 + 4096*a^15*c^6*f^9 - 24*a^10*b^10*c*f^9 + 240*a^11*b^8*c^2*f^9 - 1280*
a^12*b^6*c^3*f^9 + 3840*a^13*b^4*c^4*f^9 - 6144*a^14*b^2*c^5*f^9) + ((6*b^1
1*e*f^3 - 120*a*b^9*c*e*f^3 - 6144*a^5*b*c^5*e*f^3 + 960*a^2*b^7*c^2*e*f^3
- 3840*a^3*b^5*c^3*e*f^3 + 7680*a^4*b^3*c^4*e*f^3)*(4*a^10*b^14*c^2*e^17*f^
9 - 96*a^11*b^12*c^3*e^17*f^9 + 960*a^12*b^10*c^4*e^17*f^9 - 5120*a^13*b^8*
c^5*e^17*f^9 + 15360*a^14*b^6*c^6*e^17*f^9 - 24576*a^15*b^4*c^7*e^17*f^9 +
16384*a^16*b^2*c^8*e^17*f^9 - 163840*a^16*b*c^9*d^2*e^17*f^9 + 12*a^9*b^15*
c^2*d^2*e^17*f^9 - 328*a^10*b^13*c^3*d^2*e^17*f^9 + 3840*a^11*b^11*c^4*d^2*
e^17*f^9 - 24960*a^12*b^9*c^5*d^2*e^17*f^9 + 97280*a^13*b^7*c^6*d^2*e^17*f^
9 - 227328*a^14*b^5*c^7*d^2*e^17*f^9 + 294912*a^15*b^3*c^8*d^2*e^17*f^9))/(
2*(4*a^4*b^10*e^2*f^6 - 4096*a^9*c^5*e^2*f^6 + 640*a^6*b^6*c^2*e^2*f^6 - 25
60*a^7*b^4*c^3*e^2*f^6 + 5120*a^8*b^2*c^4*e^2*f^6 - 80*a^5*b^8*c*e^2*f^6)*(
a^9*b^12*f^9 + 4096*a^15*c^6*f^9 - 24*a^10*b^10*c*f^9 + 240*a^11*b^8*c^2*f^
9 - 1280*a^12*b^6*c^3*f^9 + 3840*a^13*b^4*c^4*f^9 - 6144*a^14*b^2*c^5*f^9))
)*(6*b^11*e*f^3 - 120*a*b^9*c*e*f^3 - 6144*a^5*b*c^5*e*f^3 + 960*a^2*b^7*c^
2*e*f^3 - 3840*a^3*b^5*c^3*e*f^3 + 7680*a^4*b^3*c^4*e*f^3))/(2*(4*a^4*b^10*
e^2*f^6 - 4096*a^9*c^5*e^2*f^6 + 640*a^6*b^6*c^2*e^2*f^6 - 2560*a^7*b^4*c^3
*e^2*f^6 + 5120*a^8*b^2*c^4*e^2*f^6 - 80*a^5*b^8*c*e^2*f^6)))*(b^6 - 20*a^3
*c^3 + 30*a^2*b^2*c^2 - 10*a*b^4*c))/(4*a^4*e*f^3*(4*a*c - b^2)^(5/2)) - ((
(3*((12*a^6*b^15*c^2*e^16*f^6 - 300*a^7*b^13*c^3*e^16*f^6 + 3156*a^8*b^11*c
^4*e^16*f^6 - 17976*a^9*b^9*c^5*e^16*f^6 + 59136*a^10*b^7*c^6*e^16*f^6 - 10
9824*a^11*b^5*c^7*e^16*f^6 + 101376*a^12*b^3*c^8*e^16*f^6 + 153600*a^13*c^1
0*d^2*e^16*f^6 - 30720*a^13*b*c^9*e^16*f^6 + 6*a^6*b^14*c^3*d^2*e^16*f^6 -
108*a^7*b^12*c^4*d^2*e^16*f^6 + 588*a^8*b^10*c^5*d^2*e^16*f^6 + 792*a^9*b^8
*c^6*d^2*e^16*f^6 - 22272*a^10*b^6*c^7*d^2*e^16*f^6 + 100608*a^11*b^4*c^8*d
^2*e^16*f^6 - 199680*a^12*b^2*c^9*d^2*e^16*f^6)/(a^9*b^12*f^9 + 4096*a^15*c
^6*f^9 - 24*a^10*b^10*c*f^9 + 240*a^11*b^8*c^2*f^9 - 1280*a^12*b^6*c^3*f^9
+ 3840*a^13*b^4*c^4*f^9 - 6144*a^14*b^2*c^5*f^9) + ((6*b^11*e*f^3 - 120*a*b
^9*c*e*f^3 - 6144*a^5*b*c^5*e*f^3 + 960*a^2*b^7*c^2*e*f^3 - 3840*a^3*b^5*c^
3*e*f^3 + 7680*a^4*b^3*c^4*e*f^3)*(4*a^10*b^14*c^2*e^17*f^9 - 96*a^11*b^12*
c^3*e^17*f^9 + 960*a^12*b^10*c^4*e^17*f^9 - 5120*a^13*b^8*c^5*e^17*f^9 + 15
360*a^14*b^6*c^6*e^17*f^9 - 24576*a^15*b^4*c^7*e^17*f^9 + 16384*a^16*b^2*c^
8*e^17*f^9 - 163840*a^16*b*c^9*d^2*e^17*f^9 + 12*a^9*b^15*c^2*d^2*e^17*f^9
- 328*a^10*b^13*c^3*d^2*e^17*f^9 + 3840*a^11*b^11*c^4*d^2*e^17*f^9 - 24960*
a^12*b^9*c^5*d^2*e^17*f^9 + 97280*a^13*b^7*c^6*d^2*e^17*f^9 - 227328*a^14*b
^5*c^7*d^2*e^17*f^9 + 294912*a^15*b^3*c^8*d^2*e^17*f^9))/(2*(4*a^4*b^10*e^2
*f^6 - 4096*a^9*c^5*e^2*f^6 + 640*a^6*b^6*c^2*e^2*f^6 - 2560*a^7*b^4*c^3*e^
2*f^6 + 5120*a^8*b^2*c^4*e^2*f^6 - 80*a^5*b^8*c*e^2*f^6)*(a^9*b^12*f^9 + 40
96*a^15*c^6*f^9 - 24*a^10*b^10*c*f^9 + 240*a^11*b^8*c^2*f^9 - 1280*a^12*b^6
*c^3*f^9 + 3840*a^13*b^4*c^4*f^9 - 6144*a^14*b^2*c^5*f^9)))*(b^6 - 20*a^3*c
^3 + 30*a^2*b^2*c^2 - 10*a*b^4*c))/(4*a^4*e*f^3*(4*a*c - b^2)^(5/2)) + (3*(
b^6 - 20*a^3*c^3 + 30*a^2*b^2*c^2 - 10*a*b^4*c)*(6*b^11*e*f^3 - 120*a*b^9*c
*e*f^3 - 6144*a^5*b*c^5*e*f^3 + 960*a^2*b^7*c^2*e*f^3 - 3840*a^3*b^5*c^3*e*
f^3 + 7680*a^4*b^3*c^4*e*f^3)*(4*a^10*b^14*c^2*e^17*f^9 - 96*a^11*b^12*c^3*
e^17*f^9 + 960*a^12*b^10*c^4*e^17*f^9 - 5120*a^13*b^8*c^5*e^17*f^9 + 15360*
a^14*b^6*c^6*e^17*f^9 - 24576*a^15*b^4*c^7*e^17*f^9 + 16384*a^16*b^2*c^8*e^
17*f^9 - 163840*a^16*b*c^9*d^2*e^17*f^9 + 12*a^9*b^15*c^2*d^2*e^17*f^9 - 32
8*a^10*b^13*c^3*d^2*e^17*f^9 + 3840*a^11*b^11*c^4*d^2*e^17*f^9 - 24960*a^12
*b^9*c^5*d^2*e^17*f^9 + 97280*a^13*b^7*c^6*d^2*e^17*f^9 - 227328*a^14*b^5*c
^7*d^2*e^17*f^9 + 294912*a^15*b^3*c^8*d^2*e^17*f^9))/(8*a^4*e*f^3*(4*a*c -
b^2)^(5/2)*(4*a^4*b^10*e^2*f^6 - 4096*a^9*c^5*e^2*f^6 + 640*a^6*b^6*c^2*e^2

```
*f^6 - 2560*a^7*b^4*c^3*e^2*f^6 + 5120*a^8*b^2*c^4*e^2*f^6 - 80*a^5*b^8*c*e
^2*f^6)*(a^9*b^12*f^9 + 4096*a^15*c^6*f^9 - 24*a^10*b^10*c*f^9 + 240*a^11*b
^8*c^2*f^9 - 1280*a^12*b^6*c^3*f^9 + 3840*a^13*b^4*c^4*f^9 - 6144*a^14*b^2*
c^5*f^9)))*(6*b^11*e*f^3 - 120*a*b^9*c*e*f^3 - 6144*a^5*b*c^5*e*f^3 + 960*a
^2*b^7*c^2*e*f^3 - 3840*a^3*b^5*c^3*e*f^3 + 7680*a^4*b^3*c^4*e*f^3))/(2*(4*
a^4*b^10*e^2*f^6 - 4096*a^9*c^5*e^2*f^6 + 640*a^6*b^6*c^2*e^2*f^6 - 2560*a^
7*b^4*c^3*e^2*f^6 + 5120*a^8*b^2*c^4*e^2*f^6 - 80*a^5*b^8*c*e^2*f^6)) + (27
*(b^6 - 20*a^3*c^3 + 30*a^2*b^2*c^2 - 10*a*b^4*c)^3*(4*a^10*b^14*c^2*e^17*f
^9 - 96*a^11*b^12*c^3*e^17*f^9 + 960*a^12*b^10*c^4*e^17*f^9 - 5120*a^13*b^8
*c^5*e^17*f^9 + 15360*a^14*b^6*c^6*e^17*f^9 - 24576*a^15*b^4*c^7*e^17*f^9 +
 16384*a^16*b^2*c^8*e^17*f^9 - 163840*a^16*b*c^9*d^2*e^17*f^9 + 12*a^9*b^15
*c^2*d^2*e^17*f^9 - 328*a^10*b^13*c^3*d^2*e^17*f^9 + 3840*a^11*b^11*c^4*d^2
*e^17*f^9 - 24960*a^12*b^9*c^5*d^2*e^17*f^9 + 97280*a^13*b^7*c^6*d^2*e^17*f
^9 - 227328*a^14*b^5*c^7*d^2*e^17*f^9 + 294912*a^15*b^3*c^8*d^2*e^17*f^9))/
(64*a^12*e^3*f^9*(4*a*c - b^2)^(15/2)*(a^9*b^12*f^9 + 4096*a^15*c^6*f^9 - 2
4*a^10*b^10*c*f^9 + 240*a^11*b^8*c^2*f^9 - 1280*a^12*b^6*c^3*f^9 + 3840*a^1
3*b^4*c^4*f^9 - 6144*a^14*b^2*c^5*f^9)))*(3*b^8 + 190*a^4*c^4 + 180*a^2*b^4
*c^2 - 335*a^3*b^2*c^3 - 39*a*b^6*c)*(16*a^12*b^12*f^9*(4*a*c - b^2)^(15/2)
 + 65536*a^18*c^6*f^9*(4*a*c - b^2)^(15/2) - 384*a^13*b^10*c*f^9*(4*a*c - b
^2)^(15/2) + 3840*a^14*b^8*c^2*f^9*(4*a*c - b^2)^(15/2) - 20480*a^15*b^6*c^
3*f^9*(4*a*c - b^2)^(15/2) + 61440*a^16*b^4*c^4*f^9*(4*a*c - b^2)^(15/2) -
98304*a^17*b^2*c^5*f^9*(4*a*c - b^2)^(15/2)))/(8*a^3*c^2*(4*a*c - b^2)^(13/
2)*(10800*a^6*c^8*e^14 + 27*b^12*c^2*e^14 - 540*a*b^10*c^3*e^14 + 4320*a^2*
b^8*c^4*e^14 - 17280*a^3*b^6*c^5*e^14 + 35100*a^4*b^4*c^6*e^14 - 32400*a^5*
b^2*c^7*e^14)*(100*a^6*c^6 - 6*b^12 - 960*a^2*b^8*c^2 + 3840*a^3*b^6*c^3 -
7675*a^4*b^4*c^4 + 6100*a^5*b^2*c^5 + 120*a*b^10*c)))*(b^6 - 20*a^3*c^3 + 3
0*a^2*b^2*c^2 - 10*a*b^4*c))/(2*a^4*e*f^3*(4*a*c - b^2)^(5/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*f*x+d*f)**3/(a+b*(e*x+d)**2+c*(e*x+d)**4)**3,x)
```

```
[Out] Timed out
```

3.548 $\int (2+3x)^6 \left(1+(2+3x)^7+(2+3x)^{14}\right) \, dx$

Optimal. Leaf size=34

$$\frac{1}{63}(3x+2)^{21}+\frac{1}{42}(3x+2)^{14}+\frac{1}{21}(3x+2)^7$$

Rubi [A] time = 0.04, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1390, 14}

$$\frac{1}{63}(3x+2)^{21}+\frac{1}{42}(3x+2)^{14}+\frac{1}{21}(3x+2)^7$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 3*x)^6*(1 + (2 + 3*x)^7 + (2 + 3*x)^14),x]
```

```
[Out] (2 + 3*x)^7/21 + (2 + 3*x)^14/42 + (2 + 3*x)^21/63
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 1390

```
Int[(u_)^(m_.)*((a_.) + (c_.)*(v_)^(n2_.) + (b_.)*(v_)^(n_))^(p_.), x_Symbo
l] :> Dist[u^m/(Coefficient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^n + c*x^(
2*n))^p, x], x, v], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && L
inearPairQ[u, v, x]
```

Rubi steps

$$\begin{aligned}\int (2+3x)^6\left(1+(2+3x)^7+(2+3x)^{14}\right)dx &= \frac{1}{3}\text{Subst}\left(\int x^6\left(1+x^7+x^{14}\right)dx,x,2+3x\right)\\ &= \frac{1}{3}\text{Subst}\left(\int \left(x^6+x^{13}+x^{20}\right)dx,x,2+3x\right)\\ &= \frac{1}{21}(2+3x)^7+\frac{1}{42}(2+3x)^{14}+\frac{1}{63}(2+3x)^{21}\end{aligned}$$

Mathematica [A] time = 0.01, size = 34, normalized size = 1.00

$$\frac{1}{63}(3x+2)^{21}+\frac{1}{42}(3x+2)^{14}+\frac{1}{21}(3x+2)^7$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 3*x)^6*(1 + (2 + 3*x)^7 + (2 + 3*x)^14),x]
```

```
[Out] (2 + 3*x)^7/21 + (2 + 3*x)^14/42 + (2 + 3*x)^21/63
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (2+3x)^6\left(1+(2+3x)^7+(2+3x)^{14}\right)dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(2 + 3*x)^6*(1 + (2 + 3*x)^7 + (2 + 3*x)^14),x]

[Out] IntegrateAlgebraic[(2 + 3*x)^6*(1 + (2 + 3*x)^7 + (2 + 3*x)^14), x]

fricas [**B**] time = 1.17, size = 104, normalized size = 3.06

$$\frac{1162261467}{7}\,x^{21} + 2324522934\,x^{20} + 15496819560\,x^{19} + 65431015920\,x^{18} + 196293047760\,x^{17} + 444930908256\,x^{16} + 790988281344\,x^{15} + \frac{15819767221203}{14}\,x^{14} + 1318314865122\,x^{13} + 1269491970942\,x^{12} + 1015602174288\,x^{11} + 677082445416\,x^{10} + 376174427616\,x^{9} + 173635132896\,x^{8} + 66158154783\,x^{7} + 20588764518\,x^{6} + 5149786572\,x^{5} + 1010576952\,x^{4} + 149902032\,x^{3} + 15808800\,x^{2} + 1056832\,x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2+3*x)^6*(1+(2+3*x)^7+(2+3*x)^14),x, algorithm="fricas")

[Out] 1162261467/7*x^21 + 2324522934*x^20 + 15496819560*x^19 + 65431015920*x^18 + 196293047760*x^17 + 444930908256*x^16 + 790988281344*x^15 + 15819767221203/14*x^14 + 1318314865122*x^13 + 1269491970942*x^12 + 1015602174288*x^11 + 677082445416*x^10 + 376174427616*x^9 + 173635132896*x^8 + 66158154783*x^7 + 20588764518*x^6 + 5149786572*x^5 + 1010576952*x^4 + 149902032*x^3 + 15808800*x^2 + 1056832*x

giac [**A**] time = 0.35, size = 28, normalized size = 0.82

$$\frac{1}{63}\,(3\,x+2)^{21} + \frac{1}{42}\,(3\,x+2)^{14} + \frac{1}{21}\,(3\,x+2)^{7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2+3*x)^6*(1+(2+3*x)^7+(2+3*x)^14),x, algorithm="giac")

[Out] 1/63*(3*x + 2)^21 + 1/42*(3*x + 2)^14 + 1/21*(3*x + 2)^7

maple [**B**] time = 0.00, size = 105, normalized size = 3.09

$$\frac{1162261467}{7}\,x^{21} + 2324522934\,x^{20} + 15496819560\,x^{19} + 65431015920\,x^{18} + 196293047760\,x^{17} + 444930908256\,x^{16} + 790988281344\,x^{15} + \frac{15819767221203}{14}\,x^{14} + 1318314865122\,x^{13} + 1269491970942\,x^{12} + 1015602174288\,x^{11} + 677082445416\,x^{10} + 376174427616\,x^{9} + 173635132896\,x^{8} + 66158154783\,x^{7} + 20588764518\,x^{6} + 5149786572\,x^{5} + 1010576952\,x^{4} + 149902032\,x^{3} + 15808800\,x^{2} + 1056832\,x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((3*x+2)^6*(1+(3*x+2)^7+(3*x+2)^14),x)

[Out] 1162261467/7*x^21+2324522934*x^20+15496819560*x^19+65431015920*x^18+196293047760*x^17+444930908256*x^16+790988281344*x^15+15819767221203/14*x^14+1318314865122*x^13+1269491970942*x^12+1015602174288*x^11+677082445416*x^10+376174427616*x^9+173635132896*x^8+66158154783*x^7+20588764518*x^6+5149786572*x^5+1010576952*x^4+149902032*x^3+15808800*x^2+1056832*x

maxima [**B**] time = 0.54, size = 104, normalized size = 3.06

$$\frac{1162261467}{7}\,x^{21} + 2324522934\,x^{20} + 15496819560\,x^{19} + 65431015920\,x^{18} + 196293047760\,x^{17} + 444930908256\,x^{16} + 790988281344\,x^{15} + \frac{15819767221203}{14}\,x^{14} + 1318314865122\,x^{13} + 1269491970942\,x^{12} + 1015602174288\,x^{11} + 677082445416\,x^{10} + 376174427616\,x^{9} + 173635132896\,x^{8} + 66158154783\,x^{7} + 20588764518\,x^{6} + 5149786572\,x^{5} + 1010576952\,x^{4} + 149902032\,x^{3} + 15808800\,x^{2} + 1056832\,x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2+3*x)^6*(1+(2+3*x)^7+(2+3*x)^14),x, algorithm="maxima")

[Out] 1162261467/7*x^21 + 2324522934*x^20 + 15496819560*x^19 + 65431015920*x^18 + 196293047760*x^17 + 444930908256*x^16 + 790988281344*x^15 + 15819767221203/14*x^14 + 1318314865122*x^13 + 1269491970942*x^12 + 1015602174288*x^11 + 677082445416*x^10 + 376174427616*x^9 + 173635132896*x^8 + 66158154783*x^7 + 20588764518*x^6 + 5149786572*x^5 + 1010576952*x^4 + 149902032*x^3 + 15808800*x^2 + 1056832*x

mupad [**B**] time = 1.58, size = 29, normalized size = 0.85

$$\frac{(3\,x+2)^7\,\left(3\,(3\,x+2)^7+2\,(3\,x+2)^{14}+6\right)}{126}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x + 2)^6*((3*x + 2)^7 + (3*x + 2)^14 + 1),x)
```

```
[Out] ((3*x + 2)^7*(3*(3*x + 2)^7 + 2*(3*x + 2)^14 + 6))/126
```

sympy [B] time = 0.10, size = 107, normalized size = 3.15

$$\frac{1162261467x^{21}}{7} + 2324522934x^{20} + 15496819560x^{19} + 65431015920x^{18} + 196293047760x^{17} + 444930908256x^{16} + 790988281344x^{15} + \frac{15819767221203x^{14}}{14} + 1318314865122x^{13} + 1269491970942x^{12} + 1015602174288x^{11} + 677082445416x^{10} + 376174427616x^{9} + 173635132896x^{8} + 66158154783x^{7} + 20588764518x^{6} + 5149786572x^{5} + 1010576952x^{4} + 149902032x^{3} + 15808800x^{2} + 1056832x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*x)**6*(1+(2+3*x)**7+(2+3*x)**14),x)
```

```
[Out] 1162261467*x**21/7 + 2324522934*x**20 + 15496819560*x**19 + 65431015920*x**
18 + 196293047760*x**17 + 444930908256*x**16 + 790988281344*x**15 + 1581976
7221203*x**14/14 + 1318314865122*x**13 + 1269491970942*x**12 + 101560217428
8*x**11 + 677082445416*x**10 + 376174427616*x**9 + 173635132896*x**8 + 6615
8154783*x**7 + 20588764518*x**6 + 5149786572*x**5 + 1010576952*x**4 + 14990
2032*x**3 + 15808800*x**2 + 1056832*x
```

3.549 $\int (2+3x)^6 \left(1+(2+3x)^7+(2+3x)^{14}\right)^2 \, dx$

Optimal. Leaf size=56

$$\frac{1}{105}(3x+2)^{35}+\frac{1}{42}(3x+2)^{28}+\frac{1}{21}(3x+2)^{21}+\frac{1}{21}(3x+2)^{14}+\frac{1}{21}(3x+2)^7$$

Rubi [A] time = 0.10, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1390, 1352, 611}

$$\frac{1}{105}(3x+2)^{35}+\frac{1}{42}(3x+2)^{28}+\frac{1}{21}(3x+2)^{21}+\frac{1}{21}(3x+2)^{14}+\frac{1}{21}(3x+2)^7$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 3*x)^6*(1 + (2 + 3*x)^7 + (2 + 3*x)^14)^2,x]
```

```
[Out] (2 + 3*x)^7/21 + (2 + 3*x)^14/21 + (2 + 3*x)^21/21 + (2 + 3*x)^28/42 + (2 +
 3*x)^35/105
```

Rule 611

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegr
and[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
0] && IGtQ[p, 0] && (EqQ[a, 0] ||  !PerfectSquareQ[b^2 - 4*a*c])
```

Rule 1352

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a,
b, c, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

Rule 1390

```
Int[(u_)^(m_.)*((a_.) + (c_.)*(v_)^(n2_.) + (b_.)*(v_)^(n_))^(p_.), x_Symbo
l] :> Dist[u^m/(Coefficient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^n + c*x^(
2*n))^p, x], x, v], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && L
inearPairQ[u, v, x]
```

Rubi steps

$$\begin{aligned}
\int (2+3x)^6 \left(1+(2+3x)^7+(2+3x)^{14}\right)^2 \, dx &= \frac{1}{3}\text{Subst}\left(\int x^6 \left(1+x^7+x^{14}\right)^2 \, dx, x, 2+3x\right)\\
&= \frac{1}{21}\text{Subst}\left(\int \left(1+x+x^2\right)^2 \, dx, x, (2+3x)^7\right)\\
&= \frac{1}{21}\text{Subst}\left(\int \left(1+2x+3x^2+2x^3+x^4\right) dx, x, (2+3x)^7\right)\\
&= \frac{1}{21}(2+3x)^7+\frac{1}{21}(2+3x)^{14}+\frac{1}{21}(2+3x)^{21}+\frac{1}{42}(2+3x)^{28}+
\end{aligned}$$

Mathematica [B] time = 0.01, size = 188, normalized size = 3.36

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 3*x)^6*(1 + (2 + 3*x)^7 + (2 + 3*x)^14)^2,x]
```

```
[Out] 17451466816*x + 443569828128*x^2 + 7299544818384*x^3 + 87406679578680*x^4 +
 (4057390785756924*x^5)/5 + 6077684727888102*x^6 + 37727143432895007*x^7 +
197897276851452864*x^8 + 889942562270387136*x^9 + (17344958593049772048*x^1
0)/5 + 11821487501620716192*x^11 + 35454069480572048124*x^12 + 940692639189
29616324*x^13 + 221699757548270194389*x^14 + 465517091041681015296*x^15 + 8
72775774067455498528*x^16 + 1463104032160519033200*x^17 + 21945771660147522
40080*x^18 + 2945285062308448290360*x^19 + 3534290697929473864098*x^20 + (2
6506949038858918036881*x^21)/7 + 3614565944605222108800*x^22 + 306451507651
2846852480*x^23 + 2298383223254096766840*x^24 + (7584660010542711771792*x^2
5)/5 + 875152864622814086340*x^26 + 437576396725285446564*x^27 + (262545832
6972530284475*x^28)/14 + 67899784121041365504*x^29 + (101849676181562048256
*x^30)/5 + 4928210137817518464*x^31 + 924039400840784712*x^32 + 12600537284
1925188*x^33 + 11118121133111046*x^34 + (16677181699666569*x^35)/35
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (2+3x)^6 \left(1+(2+3x)^7+(2+3x)^{14}\right)^2 \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(2 + 3*x)^6*(1 + (2 + 3*x)^7 + (2 + 3*x)^14)^2,x]
```

```
[Out] IntegrateAlgebraic[(2 + 3*x)^6*(1 + (2 + 3*x)^7 + (2 + 3*x)^14)^2, x]
```

fricas [B] time = 1.00, size = 174, normalized size = 3.11

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*x)^6*(1+(2+3*x)^7+(2+3*x)^14)^2,x, algorithm="fricas")
```

```
[Out] 16677181699666569/35*x^35 + 11118121133111046*x^34 + 126005372841925188*x^3
3 + 924039400840784712*x^32 + 4928210137817518464*x^31 + 101849676181562048
256/5*x^30 + 67899784121041365504*x^29 + 2625458326972530284475/14*x^28 + 4
37576396725285446564*x^27 + 875152864622814086340*x^26 + 758466001054271177
1792/5*x^25 + 2298383223254096766840*x^24 + 3064515076512846852480*x^23 + 3
614565944605222108800*x^22 + 26506949038858918036881/7*x^21 + 3534290697929
473864098*x^20 + 2945285062308448290360*x^19 + 2194577166014752240080*x^18
+ 1463104032160519033200*x^17 + 872775774067455498528*x^16 + 46551709104168
1015296*x^15 + 221699757548270194389*x^14 + 94069263918929616324*x^13 + 354
54069480572048124*x^12 + 11821487501620716192*x^11 + 17344958593049772048/5
*x^10 + 889942562270387136*x^9 + 197897276851452864*x^8 + 37727143432895007
*x^7 + 6077684727888102*x^6 + 4057390785756924/5*x^5 + 87406679578680*x^4 +
 7299544818384*x^3 + 443569828128*x^2 + 17451466816*x
```

giac [A] time = 0.42, size = 46, normalized size = 0.82

$$\frac{1}{105}\,(3\,x+2)^{35} + \frac{1}{42}\,(3\,x+2)^{28} + \frac{1}{21}\,(3\,x+2)^{21} + \frac{1}{21}\,(3\,x+2)^{14} + \frac{1}{21}\,(3\,x+2)^{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*x)^6*(1+(2+3*x)^7+(2+3*x)^14)^2,x, algorithm="giac")
```

```
[Out] 1/105*(3*x + 2)^35 + 1/42*(3*x + 2)^28 + 1/21*(3*x + 2)^21 + 1/21*(3*x + 2)
^14 + 1/21*(3*x + 2)^7
```

maple [B] time = 0.00, size = 175, normalized size = 3.12

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x+2)^6*(1+(3*x+2)^7+(3*x+2)^14)^2,x)
```

```
[Out] 17451466816*x+16677181699666569/35*x^35+678997841210413655504*x^29+262545832
6972530284475/14*x^28+875152864622814086340*x^26+229838322325409676684 0*x^2
4+361456594460522210880 0*x^22+353429069792947386409 8*x^20+2194577166014752 2
40080*x^18+872775774067455498528*x^16+1111812113311104 6*x^34+12600537284192
5188*x^33+924039400840784712*x^32+4928210137817518464*x^31+1018496761815620
48256/5*x^30+437576396725285446564*x^27+889942562270387136*x^9+118214875016
20716192*x^11+940692639189296163 24*x^13+465517091041681015296*x^15+14631040
32160519033200*x^17+2945285062308448290360*x^19+265069490388589180368 81/7*x
^21+306451507651284685248 0*x^23+758466001054271177 1792/5*x^25+8740667957868
0*x^4+443569828128*x^2+6077684727888102*x^6+197897276851452864*x^8+17344958
593049772048/5*x^10+35454069480572048124*x^12+221699757548270194389*x^14+72
99544818384*x^3+4057390785756924/5*x^5+37727143432895007*x^7
```

maxima [B] time = 0.77, size = 174, normalized size = 3.11

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*x)^6*(1+(2+3*x)^7+(2+3*x)^14)^2,x, algorithm="maxima")
```

```
[Out] 16677181699666569/35*x^35 + 11118121133111046*x^34 + 126005372841925188*x^3
3 + 924039400840784712*x^32 + 4928210137817518464*x^31 + 101849676181562048
256/5*x^30 + 67899784121041365504*x^29 + 26254583269725302844 75/14*x^28 + 4
37576396725285446564*x^27 + 875152864622814086340*x^26 + 758466001054271177
1792/5*x^25 + 2298383223254096766840*x^24 + 3064515076512846852480*x^23 + 3
614565944605222108800*x^22 + 26506949038858918036881/7*x^21 + 3534290697929
473864098*x^20 + 2945285062308448290360*x^19 + 2194577166014752240080*x^18
+ 1463104032160519033200*x^17 + 872775774067455498528*x^16 + 46551709104168
1015296*x^15 + 221699757548270194389*x^14 + 94069263918929616324*x^13 + 354
54069480572048124*x^12 + 11821487501620716192*x^11 + 17344958593049772048/5
*x^10 + 889942562270387136*x^9 + 197897276851452864*x^8 + 37727143432895007
*x^7 + 6077684727888102*x^6 + 4057390785756924/5*x^5 + 87406679578680*x^4 +
 7299544818384*x^3 + 443569828128*x^2 + 17451466816*x
```

mupad [B] time = 1.60, size = 46, normalized size = 0.82

$$\frac{(3\,x+2)^7}{21}+\frac{(3\,x+2)^{14}}{21}+\frac{(3\,x+2)^{21}}{21}+\frac{(3\,x+2)^{28}}{42}+\frac{(3\,x+2)^{35}}{105}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x + 2)^6*((3*x + 2)^7 + (3*x + 2)^14 + 1)^2,x)
```

```
[Out] (3*x + 2)^7/21 + (3*x + 2)^14/21 + (3*x + 2)^21/21 + (3*x + 2)^28/42 + (3*x
 + 2)^35/105
```

sympy [B] time = 0.15, size = 187, normalized size = 3.34

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*x)**6*(1+(2+3*x)**7+(2+3*x)**14)**2,x)
```

```
[Out] 16677181699666569*x**35/35 + 11118121133111046*x**34 + 126005372841925188*x
**33 + 924039400840784712*x**32 + 4928210137817518464*x**31 + 1018496761815
62048256*x**30/5 + 67899784121041365504*x**29 + 2625458326972530284475*x**2
8/14 + 437576396725285446564*x**27 + 875152864622814086340*x**26 + 75846600
```

10542711771792*x**25/5 + 2298383223254096766840*x**24 + 3064515076512846852
480*x**23 + 3614565944605222108800*x**22 + 26506949038858918036881*x**21/7
+ 3534290697929473864098*x**20 + 2945285062308448290360*x**19 + 21945771660
14752240080*x**18 + 1463104032160519033200*x**17 + 872775774067455498528*x*
*16 + 465517091041681015296*x**15 + 221699757548270194389*x**14 + 940692639
18929616324*x**13 + 35454069480572048124*x**12 + 11821487501620716192*x**11
 + 17344958593049772048*x**10/5 + 889942562270387136*x**9 + 197897276851452
864*x**8 + 37727143432895007*x**7 + 6077684727888102*x**6 + 405739078575692
4*x**5/5 + 87406679578680*x**4 + 7299544818384*x**3 + 443569828128*x**2 + 1
7451466816*x

Chapter 4

Appendix

Local contents

4.1 Download section

The following zip files contain the raw integrals used in this test.

This is a subset of Rubi test suite thanks to Albert Rich, which includes only the algebraic integrals with elementray optimal antiderivatives. It also includes a subset of a test file provided thanks to Sam Blake.

Mathematica format Mathematica_syntax_CAS_integration_elementary_version.zip

Maple and Mupad format Maple_syntax_CAS_integration_elementary_version.zip

Sympy format SYMPY_syntax_CAS_integration_elementary_version.zip

Sage math format SAGE_syntax_CAS_integration_elementary_version.zip

4.2 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.2.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*          GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] :=
  If[ExpnType[result]<=ExpnType[optimal],
    If[FreeQ[result,Complex] || Not[FreeQ[optimal,Complex]],
      If[LeafCount[result]<=2*LeafCount[optimal],
        "A",
      "B"],
    "C"],
  If[FreeQ[result,Integrate] && FreeQ[result,Int],
    "C",
  "F"]]


(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
```

```
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
        Exp,Log,
        Sin,Cos,Tan,Cot,Sec,Csc,
        ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
        Sinh,Cosh,Tanh,Coth,Sech,Csch,
        ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
        Erf, Erfc, Erfi,
        FresnelS, FresnelC,
        ExpIntegralE, ExpIntegralEi, LogIntegral,
        SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
        Gamma, LogGamma, PolyGamma,
        Zeta, PolyLog, ProductLog,
        EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
```

```
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.2.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems

GradeAntiderivative := proc(result,optimal)
local leaf_count_result, leaf_count_optimal,ExpnType_result,ExpnType_optimal,
   debug:=false;

      leaf_count_result:=leafcount(result);
      #do NOT call ExpnType() if leaf size is too large. Recursion problem
      if leaf_count_result > 500000 then
          return "B";
      fi;

      leaf_count_optimal:=leafcount(optimal);

      ExpnType_result:=ExpnType(result);
      ExpnType_optimal:=ExpnType(optimal);

      if debug then
         print("ExpnType_result",ExpnType_result," ExpnType_optimal=",
   ExpnType_optimal);
      fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

  #This check below actually is not needed, since I only
  #call this grading only for passed integrals. i.e. I check
  #for "F" before calling this. But no harm of keeping it here.
  #just in case.


  if not type(result,freeof('int')) then
     return "F";
  end if;


  if ExpnType_result<=ExpnType_optimal then
      if debug then
           print("ExpnType_result<=ExpnType_optimal");
      fi;
```

```
      if is_contains_complex(result) then
         if is_contains_complex(optimal) then
            if debug then
               print("both result and optimal complex");
            fi;
            #both result and optimal complex
            if leaf_count_result<=2*leaf_count_optimal then
               return "A";
            else
               return "B";
            end if
         else #result contains complex but optimal is not
            if debug then
               print("result contains complex but optimal is not");
            fi;
            return "C";
         end if
      else # result do not contain complex
           # this assumes optimal do not as well
           if debug then
               print("result do not contain complex, this assumes optimal do
   not as well");
           fi;
           if leaf_count_result<=2*leaf_count_optimal then
              if debug then
                 print("leaf_count_result<=2*leaf_count_optimal");
              fi;
              return "A";
           else
              if debug then
                 print("leaf_count_result>2*leaf_count_optimal");
              fi;
              return "B";
           end if
      end if
  else #ExpnType(result) > ExpnType(optimal)
      if debug then
         print("ExpnType(result) > ExpnType(optimal)");
      fi;
      return "C";
  end if

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
```

```
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:
```

```
AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.2.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
```

```
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
           return True
        else:
           return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn
   )))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,
   ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or
   type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,
   expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType
   ,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
```

```
def grade_antiderivative(result,optimal):

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        return "F"

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as
   well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```

4.2.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
```

```
    else:
        return False

def is_elementary_function(func):
    debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    debug=False
    if debug: print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
           'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','
    sinh_integral'
           'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
           'polylog','lambert_w','elliptic_f','elliptic_e',
           'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','
    hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in
    sagemath

def is_atom(expn):

    debug=False
    if debug: print ("Enter is_atom")


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-
    sagemath-equivalent-to-atomic-type-in-maple/
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
```

```
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens")
    :
            return expn in expn.parent().base_ring() or expn in expn.parent().
    gens()
        return False

    except AttributeError as error:
        return False


def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],
    Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.
    args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer
    )
            return expnType(expn.operands()[0])   #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],
    Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],
    Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(
    expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()
    [1])) #max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1]))
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #
    isinstance(expn,Add) or isinstance(expn,Mul)
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn
    .func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(
    expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #
    is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
    expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
    expn.args)))
```

```
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
    expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    if debug: print ("Enter grade_antiderivative for sagemath")

    leaf_count_result  = tree_size(result) #leaf_count(result)
    leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

    if debug: print ("leaf_count_result=", leaf_count_result, "
    leaf_count_optimal=",leaf_count_optimal)


    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",
    expnType_optimal)

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as
    well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```